

**THE APPLICATION OF ARTIFACTS IN THE  
TEACHING AND LEARNING OF GRADE 9  
GEOMETRY.**

**By**

**Z. M. M. JOJO**

**A Dissertation submitted to the Faculty of Education**

**of**

**University of KwaZulu-Natal**

**In partial fulfilment of the requirements for the**

**Degree of Master of Education  
(Mathematics Education)**

**Supervisor: Dr D. Brijlall**

**Co-supervisor: Mr A. Maharaj**

**March 2005**

## **PREFACE**

The research described in this dissertation was carried out at Mashsha Junior Secondary School from January 2003 to June 2004 under the supervision of Dr D. Brijlall and Mr. A. Maharaj of University of KwaZulu Natal, Durban.

These studies represent the original work by the author and have not been submitted in any form for any degree or diploma to any tertiary institution. Where use has been made of the work of others, it is duly acknowledged in the text.



.....

Z. M. M. JOJO

**Student Number: 201506651**

**Durban**

**MARCH 2005**

## **ACKNOWLEDGEMENTS**

I wish to put on record my profound appreciation and sincere thanks to all those who helped and advised me in pursuing this work. The diversity of experience and academic support of the 2001 group of students for this initial Master of Education (Mathematics Education) programme broadened my perspective on Mathematics Education and challenged many of my long held ideas. I am grateful to Mr A. Maharaj and Dr D. Brjlall, for their role as facilitators throughout this dissertation. Special thanks go to Mr D. Luthuli who initially guided me through this research. His untiring support, patience and advice are duly acknowledge.

I would also like to put on record my appreciation to the learners at Mashsha Junior Secondary School for their willingness to allow their work to form part of this study.

Finally, my sincere gratitude is also extended to my children, Onesimo, Sinesipho, Siphesihle, Gcina and Lutho for their support during the course of this study, especially my husband, Gugulethu, for helping and encouraging me in an attempt to see me through.

## **ABSTRACT**

The main focus of the study was to explore how the experiences that the learners went through in the Technology class during the construction and design of artifacts, could be used to inform the teaching of Geometry in the mainstream Mathematics classes. It was important to find out how the teaching of Geometry would allow the learners to both reflect and utilize the Geometry they know, as a starting point or springboard for further study of Geometry.

Data was collected through observations, structured and semi-structured interviews of a sample of twenty grade 9 learners of Mashsha Junior Secondary School of Margate in Kwazulu Natal. It was collected through observation of drawings and completely constructed double-storey artifacts at different intervals of designing. Observations and notes on every activity done by the learners for example, measurements, comparisons, estimations, scaling, drawings use of symmetry and perspective drawing were kept and analyzed. Data for the interviews was collected in the form of drawings, photographs, transcriptions of video and audiotapes. The observations in particular were looking for the Geometry in finished artifacts. Interviews with the learners were directed at how each learner started drawing a house to the finish. When and how scale drawing, projections, angles made and length preservation were used by the learner, was of utmost importance.

It is believed that grade 9 learners of Mashsha have Geometric experiences which can be used to inform the teaching of Geometry in mainstream mathematics. It was found that

this experience brought by the learners from the Technology construction of artifacts could cause the learners to find mainstream mathematics interesting and challenging. It is also believed that the use of projective Geometry already employed by the learners can be incorporated in mainstream mathematics so as to improve how learners understand Euclidean Geometry. In this way, it is believed, that the teaching of Geometry will allow the learners to utilize and reflect the Geometry already known to them. This Geometry would therefore be used as a starting point for further study of Geometry. Suggestions for further research and recommendations for the improvement of Geometry teaching and learning have also been made.

# TABLE OF CONTENTS

TITLE	PAGE NUMBER	
TITLE PAGE		
PREFACE	(i)	
ACKNOWLEDGEMENTS	(ii)	
ABSTRACT	(iii)	
TABLE OF CONTENTS	(v)	
LIST OF TABLES	(x)	
LIST OF FIGURES	(xi)	
<b>CONTENTS:</b>		
<b>CHAPTER 1</b>	<b>INTRODUCTION AND OVERVIEW</b>	
1.1	Introduction	1
1.2	Motivation	3
1.2.1	Researcher's personal experience and interest	3
1.2.2	Transformation in Education Curriculum	4
1.2.3	Learners' Geometrical understanding by use of cultural practices	6
1.3	Design in Technology	9
1.4	Research questions and focus of enquiry	10
1.5	Terminology and concepts	10
1.5.1	Mathematics	10
1.5.2	Technology	11
1.5.3	Design	11
1.5.4	Perspective Drawing	11

1.5.5	Scale Drawing	11
1.5.6	Orthographic Projections	11
1.5.7	Isometric Projections	11
<b>CHAPTER 2</b>	<b>REVIEW OF THE LITERATURE</b>	
2.1	Introduction	12
2.2	Constructivism	12
2.2.1	Knowledge acquiring perception by the learner	13
2.2.2	Teaching and learning	16
2.2.2.1	Understanding of concepts	17
2.2.2.2	Interaction with learners in class	18
2.2.2.3	Learners' self regulated activities	19
2.3	Learners' reflection of Geometric knowledge	20
2.3.1	Conceptions about Mathematics and Technology learning	20
2.3.2	Conceptions about Technology teaching	22
2.3.2.1	Design	22
2.3.2.2	Drawing	23
2.3.3	The effect of communication in the teaching of both Mathematics and Technology	27
2.4	Conclusion	29

## **CHAPTER 3 RESEARCH METHODOLOGY AND PROCEDURE**

3.1	Introduction	30
3.2	Reasons for choosing research methods	30
3.2.1	Teacher as a researcher	30
3.2.2	Focus of enquiry	32
3.2.3	Action research	32
3.3	The suitability of methodology with reference to the study	34
3.3.1	Learners' action process	34
3.3.2	Major issues associated with the research project	36
3.4	Participants in the study	37
3.5	Research instruments	37
3.6	Reflection	38

## **CHAPTER 4 RESULTS AND ANALYSIS**

4.1	Introduction	39
4.2	Contextual information	39
4.3	The teaching of Technology in Grade 9 at Mashsha J.S.S.	42
4.4	The description of the classroom and the activities done by the learners	45
4.5	Geometry used and revealed by learners in the drawing of artefacts	48
4.5.1	Shape	48
4.5.2	Angles	49
4.5.3	Congruency	50
4.5.4	Parallelism	50
4.5.5	Similarity and Comparisons	51

4.5.6	Perspective drawing	52
4.5.7	Isometries and Orthographic projections	54
4.5.8	Use of particular Geometric shapes	54
4.6	More Geometry in finished models	58
4.7	Geometry used ties up with Geometry in mainstream	
	Mathematics	62
4.8	Summary	64
<b>CHAPTER 5 DISCUSSION</b>		
5.1	Introduction	65
5.2	Geometric representations drawn by learners	65
5.3	Diagrams to represent objects	69
5.4	Geometry manifest in finished artefacts	70
5.4.1	Shape	71
5.4.2	Angles	72
5.4.3	Congruency	72
5.4.4	Symmetry	72
5.4.5	Projections	73
5.4.6	Similarity	73
5.4.7	Estimations	73
5.4.8	Measurement	73
5.5	Implementation of Geometric manifestations in mainstream	
	Classes	74
5.6	Summary	76
<b>CHAPTER 6</b>		
6.1	Introduction	77

6.2	Validity of empirical data	77
6.2.1	Gender performance in drawing and construction	77
6.2.2	Order of data collection	78
6.3	Limitations of the study	78
6.4	Reflective ability as a source of knowledge	79
6.5	Implications of the study for further research	81
6.6	Way forward	82
6.5	Summary	83
<b>REFERENCES</b>		84
<b>APPENDIX A</b>	Interviews on how learners drew and constructed double-storey houses	93
<b>APPENDIX B</b>	Observations of how learners constructed and designed double-storey houses	99
<b>APPENDIX C</b>	The drawing of some of the learners' houses	106
<b>APPENDIX D</b>	Artifacts constructed by the learners	108

## **LIST OF TABLES**

4.1 ORDER IN WHICH SYLLABUS TOPICS WERE TAUGHT	41
4.3 ACTIVITIES DONE BY LEARNERS	44
4.5 WORKPLAN FOLLOWED BY LEARNERS	46

## **LIST OF FIGURES**

4.2 DIAGRAM OF THE TECHNOLOGY CLASSROOM	42
4.4 PERCENTAGE OF TIME SPENT ON EACH OF THE ACTIVITIES	45
4.6 THE SITTING ARRANGEMENT OF LEARNERS	47
4.7 THE FRONT FACE OF SONWABILE'S HOUSE	55
4.8 SONWABILE'S WINDOW	55
4.9 DIAGRAM OF HOW LEARNERS STARTED DRAWING IN %	56
4.10 ONE OF THE HOUSES DONE BY LEARNERS	62

# CHAPTER 1

## INTRODUCTION AND OVERVIEW

### 1.1 INTRODUCTION

This chapter provides an overview of the study. The motivation for doing this research is discussed and the research questions and key terms are introduced. In conclusion, the significance of the results of this study in the current era in South Africa is indicated.

This study, deriving as it is from the perspective of ethnomathematics, seeks to increase and encourage the integration of Mathematics with other learning areas, (DOE, 2002) and National Curriculum Statements of 2003, (DOE, 2003). This study attempts to identify how Mathematics teachers can use the learners' experiences when constructing and designing artifacts in the Technology classroom to effect change in the way we understand and teach Geometry in formal Mathematics classes. The study therefore aims to find out, which of the learners' experiences in the Technology classroom can inform new ways of teaching Geometry in a manner that will enrich the mainstream Mathematics curriculum.

A group of 20 grade 9 learners from Mashsha Junior Secondary School in Margate of KwaZulu-Natal, with the age ranging from thirteen to nineteen, were sampled, observed and interviewed as they constructed and designed some artifacts. Two different tasks involving the construction and design of models of double-storey houses or flats with two or three floors, *izitezi*, as commonly known to the learners, were given

to them as a project to be done in Technology. Indeed, these are kinds of buildings that are either found within the learner's neighboring environment or in Margate, a town that they previously visited. Looking at what had happened in their Mathematics classroom, these activities were given to the aforementioned learners in Technology. The aim was to seek answers to the following questions:

- (1) Can learners' experiences in their Technology classroom inform the teaching of Geometry in mainstream Mathematics?
- (2) Does the Geometry teaching in formal Mathematics not restrict the learners to focus only on proof, whilst neglecting the other aspects of Geometry, like drawing?
- (3) How would the teaching of Geometry allow the learners to both reflect and utilize what they already know, as a starting point or springboard for the further study of Geometry?
- (4) Would the experience that the learners bring from the Technology class cause the learners to find mainstream Geometry interesting and challenging?
- (5) Would the approach of Geometry via Technology show how mainstream Geometry is used in practice or real life?
- (6) How can a different Geometry like Projective Geometry, ideas of which the learners already employ in the Technology classroom, be incorporated into mainstream Mathematics?

The analysis of the learners' activities as well as the observed artifacts revealed that certain Geometrical concepts, ideas and principles, were embedded in the construction

of artifacts. The educational implications of these findings for making Geometry more meaningful and accessible to learners, is discussed.

## **1.2 Motivation**

There are three motivational factors that came to mind and persuaded this research. They include: (a) researcher's personal experience and interest, (b) transformation in Education curriculum and (c) learners' geometrical understanding by use of cultural practices in Technology.

### **1.2.1 Researcher's personal experience and interest**

The researcher has been a Mathematics teacher of Grades 10 – 12 learners, from 1986 to 2001. In 2002, she had the privilege of teaching Technology for the first time to Grade 9 learners at Mashsha Junior Secondary School. She also taught Mathematics to Grade 9 and 10 learners at this school.

When she started teaching Technology, she discovered that there was a lot of classical Euclidean Geometry and Projective Geometry that featured. In Technology, Geometry arose in a different context from the manner in which the learners experienced the subject in a formal Mathematics classroom. In Mathematics, no reference was made in describing these concepts, to the objects in the students' own classrooms and homes. In Technology, however, learners experience Geometry through producing drawings of actual objects found in the learner's environment. Therefore, in the Technology classroom, the learners use points, lines, triangles, angles and quadrilaterals to produce drawings of artifacts and complete structures like models of houses.

During Technology classes, it was noticed that the learners get to know Perspective Drawing, Isometric Drawing, Orthographic Projection and Scale Drawing through activities that were different from those in a normal Mathematics classroom. For example, in producing a drawing of an object for Perspective drawing, the learners were required to draw an object as seen from afar or as would be seen at a nearby distance. The application of Isometric Drawing by learners, becomes manifest when the learners are asked to draw diagrams of finished structures like houses. Learners do these tasks on their own and without prior formal tutoring.

These observations from the Technology class led to an interest of how the learners' experiences of drawing artifacts could be used to help them make more meaning of Geometry. It was noticed that the same learners enjoyed and had fun whilst they designed and constructed artifacts in Technology. This study therefore aimed to find out which of the learners' experiences in the Technology classroom could inform and bring new ways of teaching Geometry to enrich the mainstream Mathematics curriculum.

### **1.2.2. Transformation in Education curriculum**

This study was conducted within an outcomes based education (OBE) environment. The OBE approach has been spelled out in the Revised National Curriculum Statement of 2002 (DOE, 2002) and National Curriculum Statements of 2003, (DOE, 2003). Outcomes-based education forms the foundation of the current curriculum in South Africa. It encourages a learner-centered and activity based approach to education. Amongst the developmental outcomes, for the learners to learn effectively in Mathematics, they should be able to reflect on and explore a variety of strategies. They should also be culturally and aesthetically sensitive across a range of social contexts.

DOE (2002) defines Mathematics as a human activity that involves observing, representing and investigating patterns and quantitative relationships in (a) physical and social phenomena and (b) between the Mathematical objects themselves. Amongst the skills that the learners should acquire in the learning of Mathematics, are the ability to interpret, calculate, measure, estimate, compare and contrast, as well as visualize and represent with confidence and accuracy. Geometry in particular, is about visual imagery that can be used by the learners, (Gattegno, 1965). Learners should be encouraged to represent what they see in their environment. On the other hand Duval (1995) suggested that Geometry involves three kinds of cognitive processes working together, namely:

(a) visualization processes regarding space representations, (b) construction processes by tools (e.g. rulers, compasses) and (c) the reasoning processes.

According to him, any activity in Geometry should involve a communication between these three processes, even though the different processes can be performed apart from each other. Nonetheless, for a learner to be able to see a structure or object, does not necessarily depend on whether the learner is able to construct the structure. Rather, learners should always have an access to figures, no matter which way they were constructed.

Amongst other key principles of OBE, mentioned in DOE (2002), integration of learning areas is encouraged. Also, this document urges that at some point in the teaching of Mathematics, special attention should be paid to the use of Mathematical models in both Science and Technology. The learners also have to be encouraged by the teachers to examine the relationship between Mathematics, Science and Technology. It

is this belief that has also triggered the researcher to want to use the Geometric experiences of the learners to inform the teaching of Mathematics in the mainstream. It is the researcher's view that OBE is a way of teaching, which is not different from the way teachers were teaching Mathematics or any learning area some time ago. OBE requires a mixture of content, and activities in class where the teacher continuously assesses whether the way he/she teaches Mathematics is appropriate to achieve the intended outcome of each lesson. OBE focuses more on what the learners do and learn, than on what the teachers do. The learners must leave the class with useful skills and knowledge applicable to everyday life. This study is an effort to explore how the skills and experiences that the learners go through in Technology can help to improve the understanding of Geometry in Mathematics.

### **1.2.3 Learners' Geometrical understanding by use of cultural practices**

Clithroe, et al (2000) asserted that one of the aims of teaching Technology is to teach learners to communicate using drawings, designs and pattern formation. Drawings and shape form the basis of Geometry. The grade 9 learners are required to analyze, identify, classify, and represent drawings in their study of Geometry in Mathematics, DOE (2002). Geometry in Mathematics is introduced via abstract definitions of concepts, instead of using the environment in which the Geometric concepts are manifest. For example, concepts like points, lines, triangles, angles, quadrilaterals and the relationships between angles is defined without reference to the concrete reality in which the concepts exist, like referring to the learners' own classroom and homes. In Technology however, learners experience Geometry through drawings of actual objects that they see in their neighboring environment. The designs are done in the form of a

diagram drawn to illustrate the intended product. The design specifications in Technology include stimulation of the learners mind to think of:

- (a) choosing the suitable material for the product,
- (b) the best size of the object,
- (c) the best shape of the product and
- (d) the effect that the product will have on the environment.

Through designing, the learner goes through Geometric processes on his own like estimation, comparisons, measurement, drawings and projections.

Geometry teaching can therefore focus on the mastery of Geometry content using culturally meaningful and relevant activities familiar to the learners such as constructing and designing artifacts. Mogari (2000) suggested that Geometry content would be meaningful, if it can be taught in a practical and familiar context that motivates the learners. He also proposed that Mathematics should not be taught in complete isolation from other subjects but should demonstrate its value across the curriculum. Our classroom practice in the teaching of Mathematics should allow integration with other learning areas.

It is a good setting for the learners to do Mathematics in an environment that allows flexibility and practical activity. Observations of the grade 9 learners in the researcher's Technology class when they were drawing, designing, and constructing artifacts, indicated that there was some Geometry that they used and applied. They measured angles and lines using suitable scaling, compared and estimated as they proceeded with constructions without being told to do so by the teacher. Anderson (1980) argued that teaching Mathematics in a way detached from cultural aspects, and in a purely abstract,

symbolic and meaningless way, is not only useless, but also very harmful to the learner, to society, to Mathematics itself and to future generations. It therefore concerned the researcher whether the Geometry taught in formal Mathematics, was not restricting the learners to focus only on proof, whilst neglecting the other aspects of Geometry, like drawing. How would the teaching of Geometry allow the learners to both reflect and utilize what they already know as a starting point or springboard for the further study of Geometry?

The learning of Mathematics can be fun and enjoyable if taught in a familiar context related to the culture of the learners. Geometry, in Technology, arises in a different context from the manner in which the learners experience it in a formal Mathematics classroom. They experience Geometry through producing drawings of actual objects found in class and around the learners' environment. Given that different cultures engage in different mathematically related activities, there has been a need to start looking at the possibility of using mathematically related activities in the teaching and learning of Technology, with the aim of informing the teaching of Geometry in the mainstream Mathematics. At present, Geometry education in South Africa is in a state of turmoil (Mogari, 1998). A great number of learners perform poorly in Euclidean Geometry (Mogari, 1998). This observation alone, suggested that there is a need to seek an alternative approach to the study of shape and space-related Mathematics. Technology, allows the learners to represent what they see through drawings and constructions. This is where they show their Geometric expertise without guidance from the teacher.

### **1.3 Design in Technology**

Learners are encouraged in Technology, to be innovative by drawing information and to using materials from their immediate environment. They do these constructions of artifacts, for this study, by using design. Design is a human process that shapes the world we live in, (DOE, 2002) and (DOE, 2003). Through design, learners acquire skills, knowledge, attitudes and values that can be transferred and applied across the curriculum. Design in technology opens up an exciting world of creative and personal exploration for the learners. The learners in Technology develop new ways of interacting with their world, they communicate ideas effectively, and develop their perceptual skills. They are also encouraged at all times to investigate and experiment with the creative possibilities of the various materials and tools at their disposal.

Learners experience Geometry in these investigations and experiments using various materials. Learners measure, compare, estimate, visualize, interpret, represent and describe the properties of artifacts. They use a variety of contexts, i.e. Orthographic and Isometric, to represent spatial aspects through diagrams and models. They also analyze shapes geometrically, in order to solve problems and investigate situations. In this way, Technology provides a chance for the development of intellectual and practical skills in learners through participation and creativity in a range of design activities.

Mathematics is not merely a body of facts, but a way of thinking about the world (Smithies,1970). Learning Mathematics should therefore, not only focus at acquiring knowledge about facts or even understanding them, but should also involve the ability to do Mathematics. When the learners designed and constructed artifacts in Technology, it was observed how they used and did Geometry.

#### **1.4 Research questions and focus of enquiry**

The focus of the study was to explore how the experiences that the learners go through in the Technology classroom during the construction and design of artifacts, can be used to inform the teaching of Geometry in the mainstream Mathematics classes. The research questions that this study addresses include:

1. *What kind of Geometric representations of finished artifacts are the learners able to draw on their own without prior instruction?*
2. *How do the learners represent existing objects through Geometric diagrams?*
3. *What Geometry is manifest in the finished artifacts that the learners construct?*
4. *Can the Geometric manifestations tie up and be implemented in the mainstream Mathematics classes?*

#### **1.5 Terminology and concepts**

##### **1.5.1 Mathematics**

Mathematics in this study refers to Mathematical Literacy, Mathematics and Mathematical Sciences. DOE (1997) defined this as, ‘the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem solving, logical thinking, etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction.’

### **1.5.2 Technology**

This is a learning area that employs the use of knowledge, skills and resources to meet human needs and wants and to recognize and solve problems by investigating, designing, developing and evaluating products and systems, (Bosch, 2001).

### **1.5.3 Design**

Design is a creative, intellectual, problem-solving process involving problem identification, planning, research, innovation, conceptualization, prototyping and critical reflection. This process typically results in environments, systems, services, and products that may be unique. Design is concerned with issues of purpose, functionality and aesthetics in shaping the social, cultural and physical environment to the benefit of the nation.

### **1.5.4 Perspective Drawing**

This is the art of making objects appear to be receding when we are drawing on a piece of paper, (Nkabinde et al, 2001).

### **1.5.5 Scale Drawing**

This is the count by which a picture is accurately enlarged or reduced to fit a certain given working space (Bosch, 2001).

### **1.5.6 Orthographic Projections**

This is the view and representation of an object at right angles, (Nkabinde et al, 2001).

### **1.5.7 Isometric Projections**

This refers to the drawings of 3-dimensional objects, showing proper thickness or depth on a piece of flat paper.

## **CHAPTER 2**

### **REVIEW OF THE LITERATURE**

#### **2.1 INTRODUCTION**

In this chapter, the theories and literature that guided the thinking and analysis of data in this study are discussed. Attention was also focused on some of the studies carried out in teaching of Geometry. At the time of the study there existed no South African research in the area of investigating Geometry that has been used by learners in the design and construction of artifacts.

#### **2.2 Constructivism**

‘Piaget is undoubtedly the father of constructivism, but those who have followed him have given us many useful insights which have led to a great change to the way Mathematics is viewed today when comparing with the way it was in his heyday,’ writes Clayton (1994). According to constructivist, von Glasersfeld (1992), reflective ability is a major source of knowledge on all levels of Mathematics. It is therefore important that the learners are led to talk about their thoughts to each other and to the teacher. To talk about what one is doing, ensures that one is examining it. In this study the result of such examinations led the learners to discuss their view of the problem and their own tentative approaches. Their self- confidence was raised while they reflected and devised more viable conceptual strategies of solving a problem. The first part of this chapter therefore focuses on how constructivism informed the teaching of Mathematics in this study.

### **2.2.1 Knowledge acquiring perception by the learner**

Mc Dermott (1992) indicated that according to the constructivist perspective, knowledge is perceived as a learner's action upon an object. The abstractions made are constructed by the learner and the learner discovers the properties of the object from the action. This discovery is not a passive process but a construction made by the learner from what he already knows. In Technology, this becomes much evident, because learners are just introduced to a problem, like the one of designing and constructing artifacts. How they do their designs, is left unto them using the knowledge that they already have. Interestingly enough, each learner comes up with a drawing, of say the same artifact drawn differently, depending on the way the individual sees it.

Learning starts at a physical level where the learner does some action and develops into an internalization process. The learner becomes the one who is active and is assisted by the teacher. Nonetheless, learning should take place according to the developmental stages of the learner out of which cognitive structures will develop. In constructivism, the basic concept is that knowledge is created actively and all Mathematical concepts are constructed and are based on the experiences that the learners have (Clayton, 1994). They are not transmitted by the learner's teacher. It is therefore under this idea that it is an accepted view that learners actively construct their Mathematical ways of knowing as they strive to be effective by restoring coherence to the worlds of their personal experience, (Cobb, 1994). Thus, in constructivist approaches, learners are assumed to construct their own Mathematical conceptual understandings as they take part in cultural practices, and whilst they interact with each other.

It is Piaget's view that all learners are capable of learning to reason abstractly (Mc Dermott, 1992). This should not lead teachers to think that they can force learners to memorize work without understanding. Rather, learners should be encouraged to use their own understanding or their instincts to invent their own procedures of problem solving. Piaget (as cited in Clayton, 1994) also claimed that the learner is active when acquiring new knowledge. Teachers should therefore no longer view teaching as a transfer of knowledge to passive, silent learners. Rather learners must take part in the learning process by asking questions, solving problems and making sense of what is to be learnt.

Although the Piagetian theory accepts that a learner enters a learning situation with pre-conceived ideas which may not match the correct concept, this teaching style does not threaten Mathematical knowledge, because the teacher still has a chance to alter a wrong conception until it matches the correct one. In this theory, ideas are constructed and made meaningful when the learners integrate them into the existing structures of knowledge. Mc Dermott (1992) argued that no one true reality exist, only individual interpretations of the world exist. These interpretations are shaped by experience and social interactions because learning is a social process. Numerous studies, (e.g. Cobb 1994, von Glasersfeld 1992), documented significant qualitative differences in the understandings learners develop in instructional situations, and these understandings are frequently different from those intended by the teacher, (Confrey, 1990).

The classroom is seen as a culture where learners are involved in sharing, inventions, evaluations, negotiations, explorations and social interactions. Indeed, this kind of classroom prevailed during Technology lessons. Learners shared ideas, explored various options during the design process and eventually came up with a new product to solve a given problem.

Ernest (1996) classified constructivism as being weak, social and radical. Von Glasersfeld (1992) claimed that knowledge is seen as constructed by the individual on the strength of the individual's world. This therefore means that there is no particular knowledge that can claim to be unique. Thus, each individual will make meaning of knowledge acquired based on what he already knows. No matter how satisfactory a response may be to a problem, it cannot be regarded as the only solution. This also applies to Mathematics, where in some cases there is more than one solution to a problem. Technology on the other hand, allows the learners to explore all possible solutions to a problem given, in their own ways, without prior guidance from the teacher.

Social constructivism on the other hand, concerns the learners' sense of functioning in the Mathematics class, (Simon, 1995). These social norms also include expectations that the community members have about the teacher and learners, the conception of what it means to do Mathematics in the community and how Mathematical validity is established. The community members would congratulate a teacher who teaches Mathematics such that the learners are able to apply it to everyday situations.

Technology mostly operates and exposes the learners to the solution of problems related to the learners' everyday living.

Rogoff (1990) pointed out that children are already active participants in the social practice and that this implies that they engage in and contribute to the development of classroom mathematical practices from the outset. It therefore seems reasonable to suggest that Mathematical learning is a process of active construction that occurs when learners engage in classroom mathematical practices, frequently while interacting with other learners. Geometry teachers should therefore allow more of this interaction in class so that the learners can share their experiences and compare their representations. It can therefore be said that conceptual knowledge cannot be transferred from one person to the other, but must be actively built up by every learner based on his or her own experience. The teacher becomes less a dispenser of knowledge, and more a critical facilitator of learning. Learners' ideas are respected and valued, and the child is seen as an active participant in the learning situation, rather than a receiver of knowledge. This means that computational procedures should not be imposed on learners, but the learners should be encouraged to construct their own conceptually based reasoning.

### **2.2.2 Teaching and learning**

Here the teaching and learning of mathematics and Technology in relation to constructivists' approaches is discussed. Attention is focused on: (a) understanding of concepts, (b) interaction with learners in class and (c) learners' self-regulated activities.

### **2.2.2.1. Understanding of concepts**

In constructivist teaching, the learners construct their Mathematics in such a way that their ways of knowing is a result of the fact that there are qualitative differences in the understanding of concepts that the learners develop when they are taught Mathematics (Cobb, 1994). It seems as if such differences are in most cases different from those intended by the teacher. Within constructivism, there is a generally accepted view that learners actively construct their mathematical ways of knowing as they strive to be effective by restoring coherence to the worlds of their personal experience, (Cobb, 1994). Thus, learners are assumed to construct their own mathematical conceptual understandings as they take part in cultural practices and when they interact with each other. It therefore isn't obvious for a teacher in Mathematics to observe or measure the understanding of a concept by the learners. In a Technology classroom though, it is easy to see the qualitative differences in the understanding of concepts because when the learners are given a task like that of drawing or designing an artifact, they represent it in different ways. The final product is a direct result of how they see and interpret what they see. Thus, the learners' drawings differ depending on the way in which each learner sees the object.

Constructivists (e.g. Cobb, 1994; Von Glasersfeld, 1984; Confrey, 1990 and Steffe, 1992) focused on the quality of individual learner interpretive activity. This refers to development of ways of knowing by the learners and the interactive performance of learners in the Mathematics classroom. In the Technology classroom, the way the

learner represents the artifact in the form of a drawing, is a reflection of how the learner interprets the artifact itself.

#### **2.2.2.2 Interaction with learners in class**

Social constructivists, (e.g. Ernest, 1996; Simon, 1995; Cobb; Yackel & Wood 1991) see Mathematics classroom learning as focusing on the individual's knowledge of and about Mathematics. They also claim that the members of the classroom community have no direct access to each other's understanding. However, they agree that the learners in the classroom share some aspects of knowledge. In their representation of artifacts by means of a drawing, the learners in a Technology classroom came up with their own ways of how to arrange and join lines together, in order to end up with an appropriate representation of an object. As the learners were drawing or building the artifacts in the Technology classroom, they interacted freely with each other in an informal setting. They exchanged ideas amongst themselves and, as a result, their collective personal knowledge was enriched.

In the Technology classroom, the learners often revealed things that the teachers were not aware the learners had knowledge of. The learners argued amongst themselves until they agreed on a common issue, and in this way, helped eliminate various misunderstandings that arose regarding a given task. Also, activities took place in an informal setting. Cobb (1994), Voigt (1992), and Hatano (1996) in their theory of social constructivism have argued that these are ideal situations, under which ideas are exchanged among learners. These informal activities that the learners engaged in, were not under strict supervision, guidance or detailed instruction from the teacher. There

were also no restrictions on how the artifacts should be constructed or represented by means of a drawing. The learners were free to operate on their own and make their own decisions.

### **2.2.2.3. Learners' self-regulated activities**

Cobb, Yackel & Wood (1992) suggested that learning should be viewed as an active, constructive process in which learners attempt to resolve problems that arise as they participate in the mathematical practices of the classroom. Such a view emphasizes that the learning-teaching process is interactive in nature and involves the implicit and explicit negotiation of mathematical meanings. These meanings are negotiated between both the teacher and the learner. Hence, according to the social constructivists, as far as learning is concerned, knowledge cannot be instructed or transmitted by the teacher, it can only be constructed by the learner. Individuals are much better at remembering information they create for themselves actively than information that they obtain passively.

In Technology learning was open for the learners. The teacher only introduced the learners to a problem situation, how they go about solving the problem was left for the learner to sort out without any further guidance. This was because the learning of Technology involved a process of self-organization, whereby the learner re-organized his/her activities and decided where to start and which steps to follow in constructing an artifact. The Mathematics teachers, also, ought to find out how and under what instructional conditions learners become self-regulated.

As a consequence of the events that took place in a Technology class, this study also seeks to find out how learners learn and monitor their own process of knowledge building and skill acquisition. During the construction and design of artifacts, the learners were able to self-regulate. They knew where to start to draw, if expected to do so. The learners themselves chose the design of the artifacts. In addition, the study seeks to find out how the transition from teacher-regulated learners to self-regulated learners occurs. From experience in the Technology classroom, the researcher now wants to find out more about conditions under which learners will monitor their own progress in completing the given tasks in mathematics.

### **2.3 Learners' reflection of Geometric knowledge**

Here focus is on how learners reflected on their knowledge of Geometry regarding:

- (a) conceptions about Mathematics and Technology learning,
- (b) conceptions about Mathematics and Technology teaching and
- (c) the effect of communication in the teaching of both Mathematics and Technology.

#### **2.3.1 Conceptions about Mathematics and Technology learning**

DOE (2003) emphasizes one of the purposes of teaching Mathematics, as the establishment of proper connections between Mathematics as a discipline and the application of Mathematics in the real world context. It further indicates that Mathematical modeling provides the learners with the means to analyze and describe their world mathematically, so as to allow the learner to deepen their understanding of Mathematics while adding to their Mathematical tools for solving real-world problems. Somerville (1958) indicated that the crude ideas of shape, bulk superficial extent and length, become analyzed, refined and made abstract, and they lead to the conception of

geometrical figures. This implies that the learners start with immature conceptions of shape that display shallow impressions about geometric figures over a wide range, and these develop into abstractions at a later stage. Geometry for an individual begins by intuition and develops by a co-ordination of the senses of touch and sight. This is what learners experience in Technology, they draw what they see. They then analyze and interpret the figure seen in terms of design and drawing. Onwu (1992) suggested that Mathematics in school should be set in a practical and familiar context that will motivate the learners and make Geometry content meaningful. Mogari (1998), also argues that the learning of Mathematics can and will remain fun and enjoyable if Mathematics is taught in a context familiar to the learners.

Mogari (1998) further argues that the learners do not enter the classroom with blank minds, instead they bring with them ideas and experiences about Mathematical principles, practices and concepts. A familiar and relaxed atmosphere in class can stimulate the learners to use what they already know for the construction of new knowledge in Geometry. De Villiers (1994) suggests that we ought to stop teaching Mathematics in complete isolation from other subjects. He asserts that the teachers of Mathematics should demonstrate the value of Mathematics across the curriculum.

Thus, our classroom practice in the teaching of Mathematics should allow integration with other learning areas. Technology is one learning area that makes this integration possible. Technology allows learners to use Mathematics whilst they draw freely on their own. The learners utilized and encountered Mathematics in an informal environment and in a practical manner. Mogari (2002) in his study on research

involving Ethnomathematics in a Mathematics classroom has argued that it has become necessary for Mathematics educators to seek ways that will redress the quality of instruction in Geometry. In Technology, the learners used Geometrical ideas that revealed their knowledge of Parallelism, Similarity, Congruency, Scale Drawing and comparisons of sides of an existing object or artifact that they had to design or construct. The Geometry that the learners were exposed to, was that of the artifacts or objects that the learners saw in their immediate environment.

### **2.3.2 Conceptions about Technology teaching**

Here the focus is on how Technology teaching is related to the teaching of Mathematics, covering mainly design and drawing.

#### **2.3.2.1 Design**

In Technology, we design and make artifacts. Designing involves developing ideas about products and artifacts DOE (2002). The designs start with drawings and making of models. A designer must set down his ideas by making sketches and models. All artifacts that are done in Technology are planned or designed. Its shape, lines, colors, tones and textures must be well arranged. A finished picture has its format, which is the shape of the paper or board on which the artifact is made. The shape and mass of an artifact are of more relevance to the interests of the study. Here, learners instinctively represent the things they see by Geometrical symbols. Different Geometric shapes which include squares, circles, triangular shapes, oval rectangular shapes are a combination of types of figures used by the learners as they draw the artifacts. In graphic communication, how the shapes are portrayed has a special significance. It is

interesting to note that an inverted triangular shape signifies tension while rectangular forms suggest stability and protection of artifacts.

### **2.3.2.2 Drawing**

Nxawe (1995) argued that the Euclidean understanding of Geometry is a logico-deductive system, with a handful of units of knowledge to be mastered or memorized. Gattegno (1980) argues that in this approach Geometry can often be seen as an endless set of algorithms and theorems where the learners will be given units of information, one after the other, and the learners memorize a definition a theorem and a proof, without really having a grasp of what the problem was all about. During the construction and design of artifacts by learners in Technology, the learners get to reveal most of the Geometry they know and this helps to inform the teachers.

Wilson (1977) specified that the ability to draw, sketch and use geometrical instruments is one of the most important objectives in the teaching of Geometry. He indicated that the process of drawing could contribute usefully to most aspects of Geometry and indeed of Mathematics. Drawing serves as a powerful tool in the knowledge and understanding of geometrical figures and the awareness of the links between Geometry with the real world.

Wilson also claims that scale drawing provides an appropriate way of producing Mathematical models that are accessible to all learners. This includes a wide range of practical situations like plans of buildings, drawings of maps, etc. In Technology, learners used scale drawing as they made plans of buildings and also during the

construction of artifacts. This skill was not taught to the learners. The drawings of artifacts in our investigation informed this belief.

The artifacts they drew, were found in their neighboring environment and they enabled the learners to acquire a conscious awareness of the Geometry that is to be found in their real world. Nonetheless, in the process of drawing artifacts, the learners in Technology had a chance of revealing the knowledge and understanding of some Geometric concepts without being formally taught. In Mathematics, learners have to memorize a definition, a theorem and a proof, without really having known what the problem is. The problem with this view is that it treats Geometry as an obvious deductive process that is static and hence encourages rigid thinking. Here the interesting approach where the learners expose their drawing skills, and the meaning implied thereof by the drawings, is left out. Geometry involves the world around us, and we can be Geometric by actually accessing the world through our minds.

Alexandrov (1995) suggested that Geometry is a union of logical imagination and practice. Each geometric concept should be demonstrated and understood in material form as a reflection of reality not as drawings on paper or blackboard. It seems that problems with drawings on paper arise because the learner is not aware of their origin. The learner cannot associate the drawing with any of his surrounding world. Geometric concepts should also be seen as embedded in the learner's surrounding world. In Technology, the learners were given a chance to draw on paper artifacts from their immediate world. They further developed their designs into constructions of the same

artifacts. The learners also built real world and material objects from their diagrammatic representations. All the artifacts that the learners produced reflected the reality in their immediate environment.

It is therefore proposed that perspective drawing could be one way of introducing Geometry. This may be partly because central to good perspective is basic drawing ability, which is not a manual dexterity but rather, it is a process in the mind that hinges on the way we see things. It is one way of helping learners to constantly develop their visual skills and allow them to draw what they see and have their sensitivity to three-dimensional form. This command of three-dimensional form allows the learners to visualize and draw products as if they are transparent.

We may notice that the further away an object is from us, the smaller it appears to be. We have also noticed how parallel railway lines appear to meet at a far distance. Perspective drawing is the art of making objects appear to be receding, when we are drawing on a flat piece of paper, (Nkabinde et al, 2001). Here figures are drawn as the eye would see different objects at different distances from the viewer. Geometry in the mainstream Mathematics never allows the learners to explore this skill through description or drawing. Perhaps, perspective drawing may serve to extract the Geometry that each learner already knows, even before he learns about axioms, theorems and proof. Kaput (1992) noted that, "Technology changes the way learners think about Mathematics and brings about opportunities for new content, new curricular and new teaching strategies." This implies that the availability and use of modern Technology allows easier ways of teaching Mathematics. In a Technology class, the

learners remained motivated and faced continuous challenges as they pursued their avenues of interest. The learners also attained ownership of concepts that they discovered as they worked on their projects. The issue that arises from this about Mathematics teaching and learning, is: *Can the drawing and design of artifacts by learners in Technology help to inform the teachers of interesting, novel and challenging ways, in which the learners can be taught Geometry?*

An informal approach to Geometry that is practical, intuitive and experimental has to give the learners imagery and dynamics, through the drawing and design of artifacts. The learners have to talk in a Mathematics classroom, not to wait to be told facts by the teacher. They have to be aware of the Geometric experience and articulate it, for a different experience. Such activities as those of designing artifacts encourage learners to develop and explore their own artistic creativity and to exercise their spatial sense within Mathematics. Instead of the teacher demonstrating computational methods of solving problems in a Mathematic class, the teacher should present all Mathematical activity as problems to be solved. The teacher as in Technology must challenge the learners and expect them to make some progress in solving the problems in their own way.

Learners in Mathematics, have to accept this challenge and take responsibility for their own learning. Specific strategies are never illustrated or prescribed by the teacher, and the learning process should not be structured to lead to a set of procedures or methods or

strategies that the teacher approves of. Learners must invent their own ways of doing Geometry, and must be free to choose those strategies.

Technology encourages negotiation, interaction and communication between the learners and the teacher, and among learners themselves in their construction of artifacts. Learners are also expected to demonstrate and explain their methods of construction to their peers, both verbally and in writing or in drawing. During the lesson, the learners are encouraged to discuss, compare, monitor, reflect on different strategies, justify their own to peers, and try to make sense of other learners' explanations, and if necessary, modify their own strategies. The teacher in Technology spends a lot of time listening to the learners, acknowledging their explanations and justifications, and facilitating discussions, reflection, monitoring, and self-regulation, when necessary. These discussions, explanations, and attempts to follow other people's logic force the learners to think at a higher, more critical level, and help them to be aware of errors in their own designs.

### **2.3.3 The effect of communication in the teaching of both Mathematics and Technology**

The world trend in Education towards the recognition of the learners, their experiences including language of communication in the classroom, is a major concern that affects the teaching of Mathematics (Adler, 1991). Specific questions that may be asked in this regard include whether the learners can construct effective meanings in Mathematics in communicative settings, when the language of instruction and communication is not

their mother tongue. Various studies, for example Berry (1985) have been conducted on this issue and have recommended that a policy to accommodate the second language speakers, be drawn and applied to promote their understanding of Mathematics. This would imply a completely different classroom culture, in which both the teachers and learners are active participants and meaning-makers in the teaching and learning process.

A constructivist approach emphasizes communication and negotiation and hence the use of language in the construction of meaning. According to Steffe et al (1988), communication through discussions with others is important for construction and reconstruction of concepts. The implication of this is that the learners must have some form of verbal interaction in order to share their meaning of most Mathematical concepts. In formal Mathematics classrooms, concepts are introduced in Mathematical language, for example a point that sometimes is interpreted as a dot, whilst the meaning of position might give a more clear meaning to the learners. Also the diagrams and properties of various quadrilaterals with some characteristics may be seen as something different from what the learners already know. In Technology though, there is no control over the language that is used by the learners as they do their constructions. There is also no particular sequence in which the constructions and designs are done. The learner decides from what he sees and uses his/her interpretation, with no communication problem, to represent the desired end product. The activities done are always centered on the necessity for exchange of information, which in turn facilitates participation in their own social worlds of experience.

## **2.4 Conclusion**

Alexandrov (1995) claimed that Geometry is a union of logic, imagination and practice. Every concept of Geometry should be demonstrated and understood in material form as a reality not merely drawn on paper or a chalkboard, but rather seen somewhere in the surrounding world. The design and construction of artifacts, where Geometry is used in Technology, is another example of practical operations in Geometry. Geometry provides a means whereby the learners perceive their environment and how they can express themselves.

In conclusion, from the literature reviewed, it can be deduced that both teacher and the learner share a responsibility for the understanding of geometry in Mathematics. The learning and understanding of Geometry can be effective if the teachers may allow the learners to reveal the Geometry that they already know through diagrammatic representation of what they see around them.

## **CHAPTER 3**

### **RESEARCH METHODOLOGY AND PROCEDURE**

#### **3.1 INTRODUCTION**

This chapter focuses on the methods and procedures of choosing the population, sample, sample instruments, the history of qualitative research and the suitability of methodology with reference to the study.

#### **3.2 Reasons for choosing research methods**

The history of qualitative research that focuses on the teacher as a researcher, the focus of enquiry and action research are discussed.

##### **3.2.1. Teacher as a researcher**

Knowledge generated by the teachers' research in their classrooms, promotes and increases the knowledge base about teaching as a professional and academic activity (Adler, 1997). Moreover, such research activity contributes positively to the motion of teacher development, and leads to issues of ownership of research and its findings on the part of the teacher (Lerman, 1993).

Qualitative research involves the process of determining the value or level of a particular attribute for a non-numerical data, using observational data analysis of non-statistical nature (Bailey, 1987). This research might be conducted either formally or as a practical inquiry by the teacher in his/her own classroom. Practical enquiry is more likely to lead to immediate and meaningful change in the classroom than formal

research (Lerman, 1993). Practical enquiry includes the notions of teaching as research, the teacher as a reflective practitioner, and action research. The notion of teachers as researchers, inquirers and producers of knowledge assume that an inquiring stance towards their work, especially in difficult contexts should help teachers to be more critical and reflective in their teaching.

Along with the practical inquiry, should be the development of teacher research skills. Thus as teachers work on curriculum development, even though they are actively involved in the project, there is no attempt to develop their research skills, other than discussions and reflections on their work in class. The value of the teacher as researcher is widely recognized. Lerman (1993) suggested the following as spin-offs of such a research:

- (a) The classroom is a very rich environment, and extremely complex. In a sense, a teacher is well placed to appreciate this complexity, and to be able to interpret nuances of interactions in the classroom.
- (b) The teacher is emancipating in the classroom. He/she only puts into practice the decisions of others in relation to teaching styles, but are also innovators and reflectors of what takes place in their classrooms.
- (c) Through sharpening attention to incidents in the classroom, teachers can continue to learn about their teaching and improve on their practice in an ongoing manner.
- (d) Teacher research leads to ownership as teachers start to pose their own questions in their classrooms.

The usual slogan that is often heard from “experienced” teachers: “I know what I am doing in my class, I am an experienced teacher, “ or “We’ve always done it this way, it works and there is no reason for changes...” is, a recipe for stagnation. It prevents the teachers from trying out new methods of teaching and from reflecting on, and improving

on these methods. It is quite reasonable and justifiable to raise skeptical questions regarding claims about the success of certain teaching methods. What is needed is proof of validation of the nature of these methods and their effectiveness. To provide this proof, we turn to an increasingly popular method of research in the social sciences, which has been the qualitative measure under which this study has been conducted. The researcher herself is the teacher, and the study was conducted in her classroom.

### **3.2.2 Focus of enquiry**

A qualitative study involving the use of structured and semi-structured classroom interviews was carried out, to establish what transpired in and out of the Technology classes during the construction and design of artifacts by Grade 9 learners. The main focus of the study was to explore how the experience that the learners go through in the Technology class during the construction and design of artifacts can be used to inform the teaching of Geometry in the mainstream Mathematics classes. Data was collected through observing what the learners did as they constructed and represented artifacts by means of drawings. This was also done through interviews with learners about how and why they did what they were observed to be doing. Data was also collected through video and audiotapes, which were later transcribed. Responses to questions (discussed on pages 1 and 2 of chapter 1) assisted the research to focus on the experiences of the learners.

### **3.2.3 Action Research**

Kemmis and Mc Taggard (1988) pointed out that the linking of the terms 'action' and 'research' highlights the essential feature of the approach, trying out ideas in practice as a means of improvement and as a means of increasing knowledge about the curriculum,

teaching and learning in the classroom. The result is improvement in what happens in the classroom and school, better articulation and justification of the educational reason for what goes on.

The purpose of action research, according to Lampert (1990) is not to derive new theories that can then be applied to reform practice, but subject theory to the conditions of practice and examine practical action in a concrete situation so that theory and practice develop interactively. In action research teachers become reflective practitioners: a teacher subjects her conscious and deliberate actions in the classroom, and the resulting pupils' actions and learning, to observation and critical analysis and then stands back and reflects on the results of these actions. Reflection of this results in further action being planned with the purpose of improving the teaching method and facilitating better learning for the pupils. In this sense therefore, action research has to be an on-going process in a spiral: planning, action, observation, reflection, re-planning and further action.

Action research encourages teachers to investigate their own practice on the job and to become adventurous and critical in their thinking. An important aspect of action research is that it resolves to give reasoned justification to claims to professional knowledge about teaching and learning. According to Mc Niff (1988) it is the ability to explain the process and present evidence to back up claims of improvement that is inherent in the notion of action research and the teacher's educational improvement.

### **3.3 The suitability of methodology with reference to the study**

In this section, focus is on the suitability of methods used regarding the learners' action process and the major issues associated with the research process.

#### **3.3.1 Learners' action process**

The first step in this action process was the recognition and acceptance that the methods teachers use in the teaching of Geometry in Mathematics, require evaluation and constant reappraisal. This process began with a general idea that some kind of improvement or change in the teaching of Geometry was necessary. When deciding where to make the improvements, it was diagnosed that the Grade 9 learners of Mashsha J.S. in Margate of Kwa-Zulu Natal, had a problem in understanding, applying and relating Geometric concepts in the mathematics classroom. This was the case, despite the fact that the same learners used the same geometry in their own way to design and construct artifacts in the Technology classroom. Learners could not relate the two processes and the learning areas, Mathematics and Technology.

The researcher therefore thought of some tasks in Technology, to reveal and use the geometry that the learners know, so as to inform the teaching of Geometry in the mainstream mathematics. Two different tasks involving the constructions and design of models of double-story houses or flats were given to the Grade 9 learners, as a project in Technology. The learners were familiar with these types of houses, as they see them very often in a nearby town, Margate. Drawing, the researcher knew, was the learners' first step in designing. Some of the geometric features, as the learners drew, were surely

going to be revealed at this stage. The learners had to draw the concepts formed in their minds of a double-storey house that they have seen. Some drew their dream houses. Those learners who were to construct or build models of the houses had to first have a plan of the house. Material could not just be cut and pasted without the learner knowing the dimensions of the house.

The planned action of capturing the data included the observation of the learners by the researcher to look for:

- (a) Geometry revealed and used by the learners in their drawings,
- (b) Perspective drawing application,
- (c) Isometrism,
- (d) Use of Orthographic projections and
- (e) Different Geometric shapes used.

The circumstances under which the afore-mentioned would be used were also of importance to the researcher. The strategies used by the learners in their drawings and constructions, were also of significance to the study. Thus the study used an interpretive methodology, which attempted to develop an in-depth analysis and description of what occurs in Grade 9 technology classroom at Mashsha.

Ambrose et al (2002) suggested that the Spatial Intelligence is one of seven intelligences that involved picturing things in the head. This includes the abilities to imagine folding and turning an object, to visualize the 3-D objects associated with 2-D drawings and to recognize pictures of the same object drawn from the different perspectives. Ambrose et al (ibid) further speculated that boys exhibit advanced performance in this regard. This advantage that the boys have over the girls may result from the boys' play preferences,

such as building blocks, negotiating computer environments and playing with miniature cars. Indeed, in the Technology classroom, real 3-D objects are represented by 2-D drawings. Although this study does not focus on gender issues regarding the expertise that boys have in producing drawing of artifacts, it was noticed that boys do produce good diagrammatic representations of artifacts than what girls come up with. Based on this research experience, an important question that needs to be answered, is: *As happens in Technology, should learners in the Mathematics classroom be allowed to draw freely in order to help develop their spatial abilities.*

### **3.3.2 Major issues associated with the research project**

Mashesha J.S. is situated in rural area, 25 km inland away from Margate, a town located in the South Coast of Kwa-Zulu Natal. The area has no electricity supply. It was therefore hard to collect data through a video tape recorder or any other equipment that uses electricity. To conduct the research, a generator was hired each time an electrical equipment was used. This entailed enormous expense in running this project, for example, transporting the generator from Margate to the school.

Schools are normally expected to buy their own tools for use in the Technology classroom. However, the school is an impoverished community, the parents cannot afford monies to buy tools and to pay for the replenishment of consumables. For use in their Technology classrooms, the learners needed equipment like pairs of scissors, saws, pairs of pliers and drawing pins. The parents were unable to provide these. The school itself could not afford to buy all these essentials for Technology from its own financial resources. There was also no special classroom dedicated to Technology lesson, as the

curriculum requires. The school timetable did not cater for more time for the activities that were done in Technology. There are only 4 hours allowed for Technology in an eight-day cycle and this time was not enough. Thus, the study had to be carried out over a longer period than was anticipated.

### **3.4 Participants in the study**

The main participants in the study were 20 Grade 9 learners of Mashsha J.S. This is a sample out of a total of 81 learners in the Grade 9 classroom. The 81 learners consist of 37 boys and 44 girls whose ages range from 13 – 19 years. The researcher has taught Technology and Mathematics to these learners for almost a year. She had an idea of what the learners had already done in the Technology class when they were constructing and designing artifacts.

### **3.5 Research Instruments**

Structured and semi-structured classroom interviews with the learners were the main instruments used to establish what transpired in and out of the Technology classes during the construction and design of artifacts by the Grade 9 learners. Data had been collected through observations at different stages of the designing of artifacts. Data from these interviews had been collected in the form of drawings, photographs, transcription of video and audio-taping.

Observations on every activity that the learners did e.g. measurement, comparisons, estimations, use of Symmetry was done and notes on every activity were kept. The

observations in particular were looking for the Geometry that appeared in the finished artifact. They also took note of the estimations, scale of drawing, measurements and comparisons used by the learners as they built the models. It was of importance also whether the completed artifacts represented a true model of the double-storey house or flat. The interviews with the learners were directed at how each learner started drawing a house to the finish. It was also of utmost importance to note where scale drawing, projections, angles made and length preservation were used by the learner.

### **3.6 Reflection**

This involved the researcher looking back at what had happened in the Technology classroom and deciding on what could inform future action. Discussions were held with the learners by the observer and the responses of learners, expressions of the views on what was observed, and how these reflections could inform the next action were noted.

## CHAPTER 4

### RESULTS AND ANALYSIS

#### 4.1 INTRODUCTION

This chapter provides a description of the classroom context in which the problem tasks were used. It gives a complete picture of what was happening in the classroom. Consequently, a detailed description of the physical classroom, the participants, the activities done by the learners in the classroom and the response of the participants to these activities are given. Where necessary, the participants' own word responses and explanations to questions were documented. The descriptive account has been supported with summary tables and other data developed from the analysis.

The resulting description forms part of the answer to the first research question which asked: *What kind of Geometric representations of finished artifacts are the learners able to draw on their own and without prior instructions?* This description served as the foundation on which the analysis and interpretation on the role of activities were based.

#### 4.2 Contextual Information

Many of the learners were orphans and others had one or both parents who worked as domestic workers in Margate. None of the learners had parents who were professionals in any field. All the learners in this school were black. The school had practically no resources and no teaching facilities except the chalkboard. The area was generally impoverished leading to certain problems discussed in the previous chapter.

There was no special classroom dedicated to Technology lessons, as the curriculum requires. The school schedule also, did not cater for more time for the activities that were done in Technology. There were only four hours allowed for Technology in an eight- day cycle. Also, schools were expected to buy their own tools for use in the Technology classroom. Mashsha J.S. could not afford to buy all these essentials for Technology from its own financial resources.

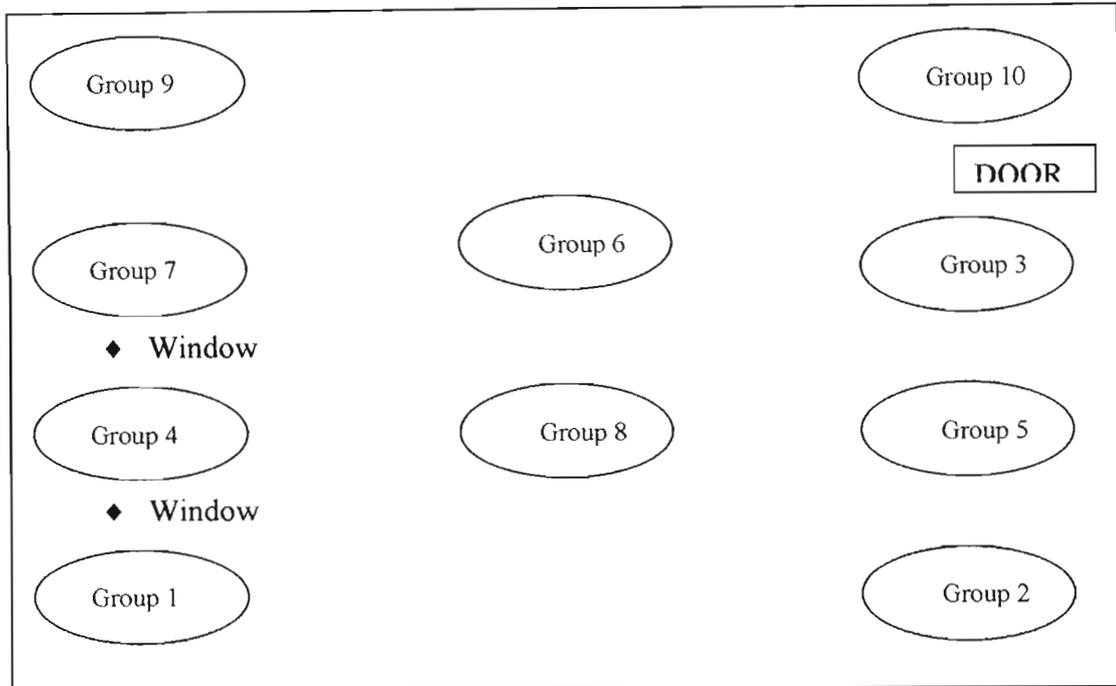
The school academic year for the learners was broken into four terms, each of about 10 weeks. For a variety of reasons, Mathematics and Technology teaching did not take place during some days as educators attended provincial departmental curriculum workshops. Activities were assigned to the learners and were monitored by other teachers. There was not a concise syllabus for Grade 9 Technology although the textbooks specify topics that could be taught to learners at this grade. However, Table 4.1 shows that the order of the syllabus was changed to suit the availability of resources. This was an internal arrangement in the Mathematics, Science and Technology department. It was educationally sound to teach 'Graphic Communications' in Technology, during the first term, before any Geometry was taught to Grade 9 learners.

Table 4.1: Order in which the syllabus topics were taught.

Term	Grade 9 Technology	Grade 9 Mathematics
1 <sup>st</sup> Term	Introduction to Technology activities based on our environment, as the phase organizer	Exponents, addition and subtraction of algebraic expressions, simplification of algebraic expressions
2 <sup>nd</sup> Term	Technology and communication, Graphic communication.	Statistics, probability and the drawing of all statistical graphs, working with data
3 <sup>rd</sup> Term	Textile Technology and food Technology	Quadrilaterals, the area and perimeter of triangles, quadrilaterals, theorems proofs and their applications
4 <sup>th</sup> Term	Culture and Society in processing materials, Hydraulics and pneumatics	Graphs, tables and rules, financial mathematics, congruency and volumes

The teaching of Mathematics and Technology took place in the two Grade 9 classrooms with 40 learners in the ‘A’ class and 41 learners in the ‘B’ class respectively. Learners sat in groups of five to six in well distributed single desks and chairs arranged in irregular pentagons or rectangles. These classrooms were large with only the chalkboard, movable single desks and chairs inside, no tables for the teacher and no cupboards to keep the equipment. The same educator handled Mathematics and Technology in these Grade 9 classes. Figure 4.2 shows the seating arrangement of learners.

Fig. 4.2: Diagram of the Technology classroom.



### 4.3 The teaching of Technology in grade 9 at Mashsha J.S.S.

One of the aims of teaching Technology is for learners to communicate using drawings, designs and follow certain patterns, (Clithroe, 2000). The grade 9 Mathematics syllabus also required the learners to analyze, identify, classify and represent drawings in their study of Geometry (DOE, 2002 and DOE, 2003). Clithroe (2000) identified seven steps that should be followed by learners in any Technological problem. These were:

- (a) identifying the problem,
- (b) considering solutions,
- (c) making an informed choice,
- (d) developing a design,
- (e) making a solution according to the design,
- (f) testing and evaluating the design and

(g) lastly recording and communicating the process.

The Technological process was similar to problem-solving process in Mathematics. Amongst other questions considered by the learner in making design specifications were: (a) what materials could be used, (b) the best size of the product, (c) the best shape of the product and (d) what the product would look like. The learner had to choose the best size, best shape and material to use when solving a given task or problem. The learners mind was stimulated, weighing possibilities and deciding suitable solutions to the problem.

After the learner had decided on the design specifications, he/she was then encouraged to take his/her thinking further. The learner had to develop a work plan. This involved the learner deciding which activity in the task was to be done first, tasks that could be done together or tasks that depended on others being completed. The learner also had to decide on the materials needed for the task, and how long it would take each task to be completed. In this manner the learner took ownership of the task. The teacher's duty was to provide the task and specify the time available for the learners to complete the task. The learner activities are summarized in Table 4.3.

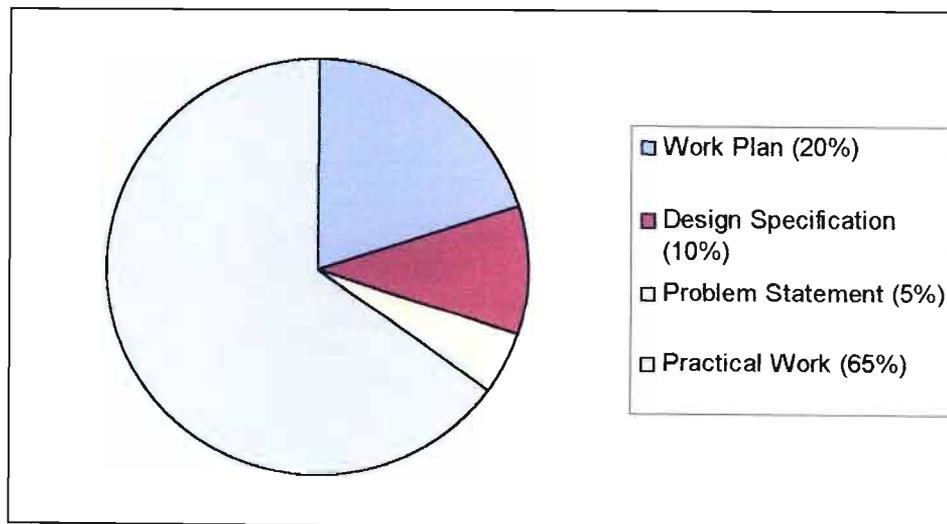
Table 4.3. Activities done by learners

Task number	Description	Time to do the task (in days)	Number of days left
1	Make a drawing of a double-storey house or flat	2	34
2	Decide on the best design and draw a detailed orthographic plan	4	30
3	Choose and collect the material to be used in constructing the artifact	2	28
4	Construct the artifact	20	8
5	Submission of the artifact for observation purposes	5	3
6	Complete finish and touches	3	0

The end products would then be submitted to the teacher for evaluation. The start of the Technology lessons could not follow a predictable pattern of 'straight to work', or continue where the teacher had left off the previous lesson. Rather, the teacher would spend about half an hour, looking at the various designs made by the learners. Of most interest, were the explanations given by the learners when they tried to convince the teacher and other learners in the groups, why they did what they did in a particular way. Although there had always been chaos and noise in the class, learners argued amongst themselves and moved up and down the classroom, each learner was proud to bring an end product that represented his own thinking. Knowledge was acquired through learner's action upon an object (Mc Dermott, 1992). The learner abstracted and discovered the properties of the action from his action. The teacher did not do much of

the teaching except introduction of new terms and their meaning to the learners. The teacher reported and assessed what the learners presented. The Technology classroom represented a culture where learners were involved in sharing, inventions, evaluations, negotiations, explorations and social interactions. When the bell rang after an hour, there would be papers scattered all over the floor, and it usually took another 10 minutes to prepare for the next lesson. By the end of the lesson, the learners would still be doing the Technological activities. The learners would then take their unfinished products home. This is because the school did not have cupboards or a special Technology classroom where they kept their work. Thus in a particular week period where a new topic was introduced, four major activities predominated, namely: (a) the statement of the problem by the teacher, (b) development of design specifications, (c) developing a work plan and (d) practical work or making the product.

Figure 4.4. Percentage of time spent on each of the activities.



#### 4.4 The description of the classroom and the activities done by the learners

The data collection was achieved through the activities described in Chapter 3. The

work plan that was followed by the learners can be summarized in the Table 4.5.

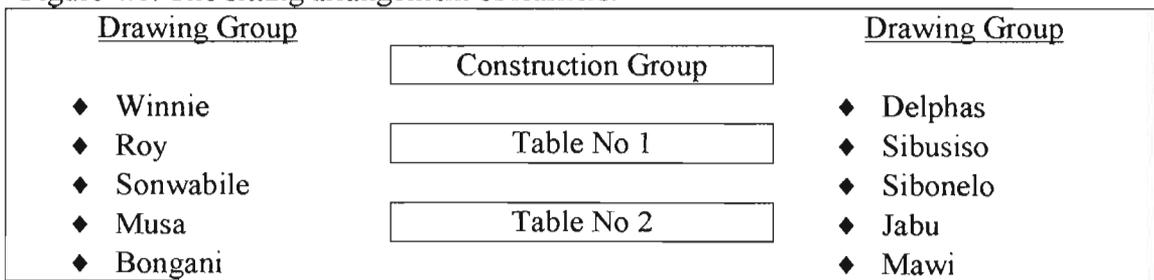
Table 4.5: Work plan followed by the learners

Design brief	Make a model of a double-storey house which has been seen either in Margate or in the neighboring environment
Design specifications	Models must be neat, accurate, proportional
Materials to be used	Pieces of cardboard, glue, cellotape, rulers, pair of scissors, pritt, glass paper, pencils, strings, measuring tape, pins, needles etc.
The size of the model	To be decided by the learner, not very small and not very big.
The shape of the model	To represent the actual double-storey house
The cost of the material	Minimal, with the learners and the teacher bringing all the material from home.

For the purposes of this study, a day was arranged with the principal of this school, where two hours were to be set aside for the observation of Grade 9 learners as they constructed and designed the artifacts. On this particular day, a video camera was used to capture activities that were done by 20 sampled Grade 9 learners for the purposes of the research study. This was not a normal teaching day at the school as the learners had just finished writing their June examinations. Observations took place on one of the free days when the examination timetable had provided nothing for Grade 9 learners. Ten of the 20 learners, namely, Mawi, Musa, Isaac, Sonwabile, Sbusiso, Oscar, Delphas, Bongani, Winnie and Jabu, were asked to redo the design and construction of double storey houses and flats.

The classroom was arranged the previous day to suit the activities to be done during the observation period. Five single desks and chairs were arranged neatly in a line next to 2 oppositely faced walls of the classroom. Thus, each learner who was to draw, had his own desk, 5 on each side of the classroom. Each learner was supplied with two A4 sized photocopy papers, a lead pencil, eraser, sharpener, 30 cm ruler and a packet of koki pens for decorations. A schematic diagram that shows where each of the learners sat when they were drawing has been provided in Fig.4.6.

Figure 4.6: The sitting arrangement of learners.



The observation schedule was used specifically to look for:

- (a) Geometry used and revealed by the learners
- (b) Reflection and application of perspective drawing
- (c) Use of particular Geometric shapes
- (d) Use of Isometrism and Orthographic projections
- (e) Strategies used by the learners to draw.

**Assertion 1:** Mashsha grade 9 learners could use Geometric representations and projections when they drew artifacts on their own without following any guidelines.

An interpretive method which attempts to develop an in depth analysis and description of what occurred in Grade 9 Technology classroom at Mashsha was used in this study. From time to time, progress of their drawings was monitored. Each one of them was

interviewed on how they did the drawings, the first lines they drew, whether they represented the roof or the floor. The following questions were posed to learners:

- (a) Was there any significance about where you started drawing?
- (b) Where did you start your drawing?
- (c) Did you involve any measurements whilst drawing?

I got various answers as to where each one of the learners started their drawings. Here are some of their responses:

Musa: *I started with these two parallelograms here.* (pointing at the roof of his drawing).

Sbusiso: *At the roof.*

Bongani: *I started with this bottom line.*

## **4.5 Geometry used and revealed by the learners in the drawing of artifacts**

### **4.5.1 Shape**

It was observed that each learner had a particular shape in his mind of the double-storey house that he was drawing. It was either his dream house, or a house that he had seen around Margate. Each learner could explain in detail why certain lines in his drawing were projected the way they were in their diagrams. They used a combination of geometrical structures like parallelograms, trapezia, triangles, rectangles, squares, pentagons and some interesting curved garage representations. Sonwabile's response, when he was asked where he had seen the house he was drawing, was:

*I would like a house like this, masengime kahle (when I have my own moneys).*

Some learners used measurements in the course of their drawings so as to elucidate some important features of their roofs. Here are some of their explanations:

Sonwabile: *When I draw the gable part of the roof, I measured so as to make sure that both lines on either sides were equal.*

It also shows that he knew a property of a gable. Even in the construction of models representing their double-storey house drawings, various geometric shapes were used. It appeared though that these were not intended shapes except that the said shapes were appropriate and necessary to represent certain parts in the diagram. When Isaac was asked whether there were any geometric shapes that he could identify in his complete structure, his response was:

*Yes, the doors, garages and windows are rectangles, and I would say that this side of the house is a pentagon, since it has five sides,... my roof, if you look from this side, (pointing) is in the form of a trapezium.*

Researcher: *Were these intended shapes or are they shapes that we just get from finished structures?*

Isaac: *No teacher, I was just cutting out the cardboard, such that the structure represents what I have in my diagram.*

Some of the learners explored and enjoyed using different shapes from the normally seen and known shapes of parts of their drawings. They were creative. Musa said:

*I did not want the usual rectangular type of garage that you normally see in any house, I wanted mine to be different ... ..and I thought of two five sided garages.*

#### **4.5.2 Angles**

In each of the houses drawn, the lines drawn formed 90 degrees with each other. In the parallelograms drawn to represent the roofs, opposite angles were equal. Also, when

adding the angles around each point of intersection of the lines, a revolution was formed. Vertically opposite angles were spotted from their drawings. When Mawi was asked whether he remembered particular geometric relationships between the angles formed in his drawing, he said:

*Yes, alternating angles. And later,*

*No, they are vertically opposite angles.*

It also appeared that he did not think about the type of angles made whilst he was drawing, except that, 'he was just drawing.' Other angles that were spotted in their drawings, included straight angles, corresponding angles, co-interior angles, vertically opposite angles, alternating angles, and right angles.

#### **4.5.3 Congruency**

It was observed that when some of the learners were asked about the relationships between different shapes in their diagrams, they indicated that the structures were equal in all respects. Most of the time they drew the particular shapes to be equal in size intentionally. Thus, it was not by chance that the structures were equal, but were intended and made by the learner to be equal. Musa gave this explanation regarding the sizes of his drawn garages:

*Yes I wanted them to appear equal in size, same height and same width.*

#### **4.5.4 Parallelism**

In various stages of the drawing of the houses, most learners used parallel lines. Most of their roofs depicted a gable shape and were drawn in the form of parallelograms. When Sonwabile was asked why he drew a shorter line than the original one in his diagram, his response was:

*I drew two other equal lines on both ends of that line, but to be shorter and parallel to the former line.*

He even used his hands to show that the lines were meant to be parallel. He further justified his action by explaining that:

*Ngiyaqondanisa ngoba ngifuna the walls engizozakha, zibukeke sengathi zibhekene (I draw them parallel because I want the walls to appear as if they are facing each other.)*

Bongani used his hands, lifted them parallel to each other to show that if one is standing next to the garage, he would see the other side of the wall.

#### **4.5.5 Similarity and Comparisons**

In most parts, for example the roofs, the learners drew some shapes to be similar. The rooms and windows were drawn to be similar. For example, Sonwabile drew his house such that it had similar faces on either sides of his front view. The explanation that he gave was:

*I wanted to build this wall to face the one from just like the one on the other side.*

Oscar, who started his house by drawing two parallelograms, gave this explanation for their size:

*Yes they are equal when I look at them, I must draw them sengathi ziyashiyana, khona kuzobonakala ukuthi nguroof wenye indlu lo, nomunye welinye ikamera (as if parallel to each other, so that this can be one roof for one room and the other for a separate room).*

The use of the similar shapes and congruent lines also enabled the learners to compare the shapes that they drew. Sbusiso showed the significance of where he had started with his drawing by saying:

*No, kodwa uma uwazi usize weroof yakho, kubalula ukuthi ukale amawalls namafloors wakho ( But, if you know the size of your roof, it becomes easier to measure your walls and floors).*

Bongani , on the other hand, said:

*I drew this vertical line to be the longest, so that I can compare the other vertical lines and the height of my double-storey to be equal to the first one.*

#### **4.5.6 Perspective Drawing**

When Musa was asked why he had drawn one of his garages to be half, his response was:

*No, this is meant to be half because, if you are looking at this house from this side, you can not see the other part of the garage.*

Researcher: *In other words, where is the person standing when looking at your house?*

Musa: *Uma ume ( when you are standing) in front.*

Researcher: *But now you said, there was a side of the garage that will not be seen.*

Musa: *Kushukuthi yona indlu iyaqhubeka ngokwamehlo (showing with his hands) but now the problem is the paper, you cannot represent the whole house on paper. Some of the things kumele uqhubeke uzibona ngamehlo (will be projected by your eyes).*

This shows clearly that he used his drawing to project his house to face to the left hand side. What was of interest was that the house was drawn such that all the features in the house could be seen if a person stands in front of the house. The person is not drawn but by imagination in Musa's mind projecting lines were drawn such they instructed the viewer to stand at a particular point or position when looking at the house. Also, when the ruler was used to measure if the sides of the roof were equal, which came out as not equal, Musa gave this explanation:

*No they might not be equal, but a person standing here, from the front, will see them as equal. Only, the other one is above and further away from the person looking from the front.*

Thus, perspective drawing was also one of the properties used easily by Musa. He used various explanations that showed that he understood and used perspective drawing freely. Some reflections of this skill are contained in the following responses:

Musa : *This is just a corner, and this is my front window, and this is the door, but you can't see it when looking from your side, so you will come here through the verandah.*

Bongani: *No the other one shows a corner and should be drawn such that lo line ongemuva uvele obonisa (the line behind shows) the corner at the back of the flat. Futhi idistance phakathi kwalo line nalo (i.e.between this line and that one) funeka ibe the same, ilingane (the distance between these lines must be equal). Then to propel the front, I drew another vertical line (i.e. 5 on the diagram)*

*so as to complete the front side, because the other side elihamba naleli eliseceleni (i.e. Sida A) Bheka mhem alizikuvela. Bheka mhem kanje (look teacher, this way, showing with his hands).*

#### **4.5.7 Isometries and Orthographic projections**

Isometries were also used by the learners in their drawings to indicate proper thickness and depth of certain parts and levels of floors. Here are some of the responses they gave when I asked how they arranged their roofs:

Musa : *For this roof to be drawn, there must be this room on top of the garage, so that this other roof will be joined to the bottom of the room above the garage, of this other room on the side of the steps... ..The wall of the room on top of the garage ends, then start the roof of the first room, to correspond to the one on the first floor.*

Orthographic projections could be seen clearly in Bongani and Oscar's drawings. Bongani drew flats with three floors. All the floors were well represented with each floor at 90 degrees to each other. (See Appendix 3 of Bongani's drawing).

#### **4.5.8 Use of particular Geometric shapes**

Generally, all the learners used a combination of shapes in their completed drawings. Sonwabile drew two rectangles, one on top of each other, and a pentagon on top of the rectangles to represent the front faces of his house. His windows were rectangles with an oval shape on top, the verandah, in the form of a trapezium. Small, elongated parallelograms were used as gutters around his roof. The following Figure 4-7 shows one of the front faces, while Figure 4-8 shows the shape of one of his windows.

Fig. 4.7 The front face of Sonwabile's house & Fig.4.8 Sonwabile's window

Fig 4.7

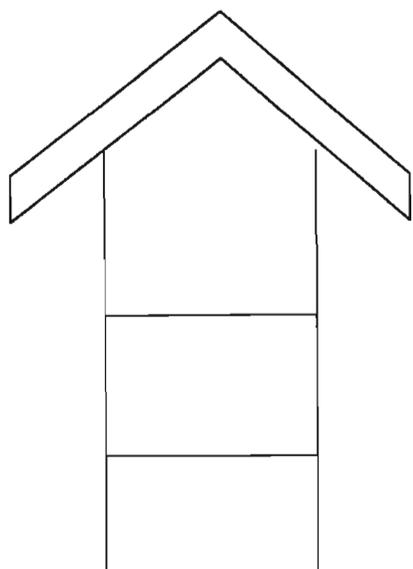
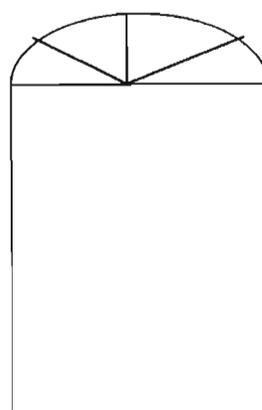


Fig.4.8



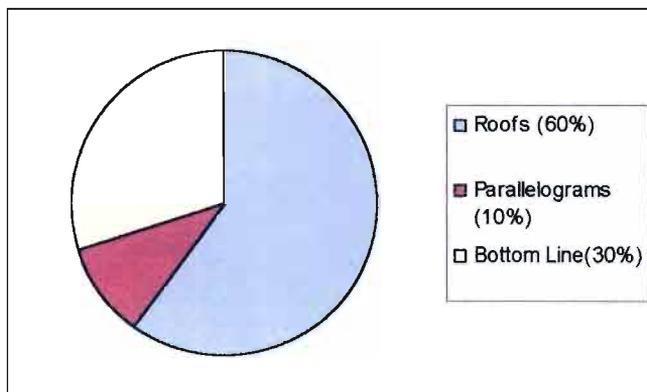
Musa, on the other hand, used a number of parallelograms, to project his roofs. His garages were full pentagons and a half pentagon. Isaac's doors, garages and windows were rectangles, one of the faces of his house was a pentagon, and his roof was a trapezium.

**Assertion 2:** Learners did not use scale drawing or preserve lengths when representing existing objects through diagrams, scale drawing was used when designing and constructing models.

As I interviewed the learners from time to time whilst they were drawing, I was introduced to interesting ways of doing a drawing. Oscar, Sibusiso, Sonwabile, Winnie, Jabu and Isaac, started their drawings at the roof. They started their roofs with either parallelograms or rectangles. Musa, Mawi, Bongani and Delphas said that they started

with the foundation of the house, some with lines and Musa started by drawing two garages of same size and shape. Figure 4.9 shows how the learners started their drawings in % form.

Figure 4.9. Diagram of how learners started drawing in %



Each one of the learners who drew was interviewed and a record of observations was kept namely:

- (a) Notes on every activity that the learners did when they drew. Records of measurements, where used, comparisons and estimations done whilst the learners designed and constructed the models.
- (b) Interviews with learners made whilst they drew diagrams.
- (c) How they started drawing, and the steps that they went through until the activity of drawing artifacts was completed.
- (d) Whether or not scale drawing was used in preserving lengths and other dimensions of the actual objects.

This part of semi-structured interviews took place whilst the learners were drawing their artifacts. The essence of this exercise was to get answers to the following questions:

- (a) How do you draw that?
- (b) Why are you drawing that?
- (c) How do you know that?
- (d) Where did you get the idea from?
- (e) What made you think of drawing this type of a house?

The learners were at ease and answered every question freely and talked freely amongst themselves as they were drawing. Sometimes they explained things to the researcher voluntarily. From the answers given by the learners when asked where they got the idea of drawing a type of a house, it was evident that one, Sonwabile, was drawing his dream house. He claimed that was the type of house he would buy or build as soon as he got money. This implies that he was using his imagination of a certain house that would suit him when he grows up. This was his idea. Musa drew a house that he had seen and appreciated next to the beach in Margate.

Sixty percent of the learners started their drawings at the roof while 30% of them had started drawing with garages at the bottom and worked their way up the double-storey house. Some reasons given for starting with the roof of the house were:

*It is easier to draw the house, if you start with the roof.*

One learner claimed that, if he knew the size of his roof, then it would be easy to measure the walls and floors of his house. It also appeared from the answers given by the learners, on whether they used specific measurements when drawing various aspects

of the house, that 40% of them used no particular measurements except that that they relied on estimations. Nonetheless, 60% of the learners indicated that the measurements of the other parts of the house depended on the roof or garage already drawn. It also appeared that even if no measurements were done at the beginning of the drawing, the next part to be drawn would be measured to compare with the first part already drawn. Some explanations in this regard were:

Sonwabile: *No, I did not measure it but when I draw the gable part of the roof, I measured to make sure that both lines on either sides of the roof were equal.*

Musa: *I proceeded to the one on top, almost the same measurement as the one next to the garage.*

It appears from the answers given by the learners that no scale drawing was used when drawing. Rather the drawing of the house was done to fit the A4 size paper on which the learners drew. Nonetheless, the learners were able to represent the actual artifact with its external features on paper.

#### **4.6 More Geometry in finished models**

This section is concerned with the geometry used by the learners whilst constructing the models as elicited and inferred from their responses when they were interviewed. This information was sought in response to the following research question:

*'What Geometry is manifest in the finished artifacts?'*

**Assertion 3:** Learners used measurements based on a chosen scale to produce finished double-storey model houses which revealed some Geometry.

The main instrument that was used at this stage was classroom observations, using a video camera. Unfortunately, it was not possible for the learners to finish the artifacts within the two hours allocated for observation. They left their unfinished models and continued with them for three other days until they finished them. The learners did this either after school or during their spare time. The activities done by the learners to produce the finished artifacts and their responses to questions as they were interviewed were recorded and the notes on their responses were analyzed.

The following instructions were specified by the researcher to the learners when they started building their double-storey models. They were told that the artifacts should:

- (a) Be made of cardboard, paper and or glass-paper.
- (b) Be strong enough to handle.
- (c) Be able to stand on a flat surface.
- (d) Be neatly finished and attractive.

Each time the researcher came to class, she observed the double-storey models. The researcher thought that this would help in identifying different Geometry related shapes from a number of double-storey models. This was done to get a first hand account from the learners and broader picture of the artifacts.

In the middle of the classroom, tables were haphazardly put to allow a space of about a meter apart, between them (see figure 4.6). Here ten other learners were constructing and designing new artifacts in the form of double-storey houses and flats. Various cardboard strips, most of which were collected and brought to school by the researcher,

were scattered all over the floor. Each one of the ten learners would from time to time go and cut a piece of cardboard, measure and fit it into the house being constructed.

Learners were supplied with rulers, lead pencils, cutting knives, glues ( shoe glue) and glass paper for windows. The researcher moved around and observed each step that was followed by each learner in the constructions. The learners' names were Sandile, Wonder, Ndumiso, Mzikayifani, Isaac, Armstrong, Morris, Innocent, Mawi and Oscar. As the researcher moved around the desks, it was noticed that some learners had started to construct their houses. When Wonder was asked how far he had gone with the building process, his response was:

*I am busy making a list of measurements that I must cut out from the big cardboard*

When he was further asked why the measurements were important, he said:

*Wonder: In actual fact teacher, in the drawing, my floor is only 20cm in length. Now, 20cm is too short when I use cardboard, I have to increase that and cut out something that can be seen.*

It was evident from Wonder's response that he could not cut out his material without doing measurements according to a particular scale that he chose. His response to the question: *Do you have to measure each aspect of the house?* was:

*Oh yes, so that I can convert each measurement of each part of the house, and know how much the breadth and length of my windows are and how far apart they should be. Futhi ke mhem (also my teacher): Before the house building, one has to measure each item so as to plan, the amount of material needed, the*

*appropriate scale and the complications that one might have in building the house. At some stage where I see that, umangiqhathanisa la masayidi awaqondani, kushukuthi ngeke axhumane, ngizobona ke mina njengo builder ukuthi ngenza kanjani (I will see what I do as a builder if the sides do not compare or are not equal).*

The responses received from the learners showed that most of them used certain measurements and a chosen scale before they cut out the material to be used . Some selected extracts from typical responses are:

Isaac: *I measured two rooms to be equal, after which I marked where to make the garage.*

Delphas: *Yes, all the sides have been measured*

Some of the learners' responses indicated that some comparisons done by referring to the diagrams already drawn. The model had to show the features in the diagram. Here is one of the learners' explanations:

*Every measurement in the diagram has been multiplied by three for the model.*

Some learners' responses indicated that they sometimes drew on the cardboard before cutting out either a window, a door or any part of the house. It was also of importance to observe congruency of some features of the house whilst they were cutting the pieces. They also made sure that the distance from one edge of a side of the house was the same as the distance to the other edge. Learners made estimates to cut out parts like the roof to suit the walls that were already erected. When the cardboard walls were placed on the flat surface, a learner would ask another one to hold the walls straight, or at a certain angle whilst putting glue to paste it on. In this exercise, some angles were made, e.g. 90

degrees between the floor and walls, and various acute angles to link with the roof. Features like the walls at the back of the house, which were not shown in the diagrams were cut out by the learners to be shown in the finished artifacts and various explanations were given in this regard.

Isaac: *I just cut a cardboard that I estimated to fit that roof, taking care that the front projection of the roof had to be longer than the back because it has to accommodate the verandah.*

Delphas: *mh...mh...one for the front side and the other for the back. You will not see the back side in the drawing, it's not shown.*

However, it is encouraging to note that the finished artifacts, to about 90% of accuracy, represented what was on the drawings. (see the diagram portrayed below in fig.4.10. It is also important to note that the houses were built according to some plans, measured and done according to certain chosen scale throughout. Some projections were also reflected on some parts of the houses.

Figure 4.10. One of the houses drawn by the learners.



#### 4.7 Geometry used ties up with Geometry in mainstream Mathematics

This section is concerned with whether the Geometric manifestations revealed by the learners in drawings and finished artifacts were of any use to the mainstream Mathematics. The information presented was in response to the following research question: *Can these Geometric manifestations tie up and be implemented in the mainstream classes?*

**Assertion 4:** The Geometric manifestations in the drawings and finished artifacts are no different from the Geometry used in the main stream Mathematics.

When Bongani was asked to explain how he came up with his complete double-storey house drawing, his response was:

*I drew this vertical line to be the longest, so that I can compare the other vertical lines and the height of my house to be equal to the first one.*

Is that not what Geometry is about, comparisons of lines drawn, heights, congruency and lengths? This response shows that Bongani understands his drawing, and could argue for the intentions that he had, when he propelled the lines the way he did. When the learners were given chances to explain how they constructed the artifacts, Isaac revealed that the congruency done through the actual measurement of windows and the division of the floor into equal parts, serving as his foundation, was used. This is how he put his response:

*I measured two rooms to be equal after which I marked where to make the garage. I then divided the floor into equal parts, before I cut out the equal windows. Then I cut two equal sides for the front and the back.*

Here the learners were actively involved in using Geometry to build their artifacts. This approach allowed learners to be part of the learning process. It is unlike figures that we, Mathematics teachers come with from textbooks, draw them on the chalkboard, and teach Geometry in isolation from reality.

#### **4.8 Summary**

Measurement and construction of right angles, parallel lines and congruent figures, for example are activities that are normally carried out with the use of modern scientific equipment like set squares, compass, protractors and rulers. Most of the time this equipment is available in economically advantaged schools, and unfortunately, Mashsha J.S. is not one of them. An activity like the one of drawing and construction of artifacts, is not only an activity that can be used in Technology, but rather embodies Mathematically related activities like measurements and projections. This study has shown that the learners know and use these Geometric concepts with understanding. Their explanations and understandings can be used in the mainstream Mathematics. These results and their implications for the teaching of Geometry in Mathematics will further be discussed in chapter five.

# CHAPTER 5

## DISCUSSION

### 5.1 INTRODUCTION

This study was done over a period of six months, which included the encounter on the learners' drawings. All the participants were grade 9 learners. No other research done in this field at the school. All of the results were important to inform the future development of Geometry teaching in Mathematics. The following discussion of the results centres on each of the research questions and their assertions related to:

- (1) geometric representations drawn by learners,
- (2) diagrams to represent objects,
- (3) geometry manifest in finished artifacts and
- (3) implementation of geometric manifestations in mainstream classes.

### 5.2 Geometric representations drawn by learners

**Assertion 1:** Mashsha J.S. grade 9 learners used Geometric representations and projections when they drew artifacts on their own without following any guidelines.

The learners' explanations when they tried to convince the researcher and other learners in the groups on how they represented their artifacts in the form of drawings were quite interesting. Various responses were gathered and analyzed. This discussion shows that the learners could represent the finished artifacts in the form of a drawing without problems.

Participants were asked to draw artifacts representing double-storey houses or flats. The examination of the drawings focused on: (a) where the drawing started, (b) the reasons for this and (c) whether, any measurements were used by the learner. These were then classified according to the positions where the drawings were started. This was because the development of a drawing that started with a roof, say of parallelogram shape, was different from developing a line at the foot of the house. The researcher therefore expected that the learners would produce different diagrams, some being a true reflection of the finished artifact, while most of them showed interesting combinations of Geometric representations.

From the responses to the initial activity of the study, it was clear that the learners could identify most Geometric shapes in their complete drawings. They knew the properties of most shapes. They used the different shapes in various parts of their drawings to get a desired complete shape of the double-storey houses. The complete shape of the drawing showed a combination of Geometric structures like rectangles, squares, triangles, trapeziums, pentagons, parallelograms and some well-designed curves, all proportionally arranged in the diagrams. Mogari (1998) in his study of the Geometry embedded in the construction of miniature wire cars suggests the use of more interactive teaching approach with the aim of inducing effective learning. Perhaps, an approach to Geometry where the learners are presented with certain objects like boxes of matches, and cereal boxes to draw or give a description of, would challenge the learners in many ways.

Despite the chaos and noise made by the learners whilst drawing the double-storey houses and flats in the classroom, many of the learners did appear to enjoy drawing the houses of their dreams. Initially when I had asked the learners to draw any artifact, some drew cars, people, their school, and houses. This shows that drawing these artifacts for this study was not a big deal for them. The learners used their own understanding and instincts to draw the diagrams. Nobody instructed them to draw the artifacts from certain directions or such that a certain feature of the artifact was left out of the complete structure. The learner himself produced a drawing. This is in line with suggestions made by Rogoff (1990) as discussed in the theoretical framework in chapter 2. Each learner could explain how he ended up with a certain drawing. This shows that the learner understood what he/she did. The construction of artifacts served as a good exercise to show that the learners enter a classroom with pre-conceived ideas of Geometry, (Piaget, 1980). Individual interpretations of how the learners see the world around them were made clear from their drawings. This supports the views of Mc Dermott (1992) discussed in the theoretical framework. Some had dream houses in which they would like to live as soon as they had money. Some learners could extend and build on what the human eye could see. He could create and project a drawing to spell out certain features when the complete house was viewed from a particular angle. This is creativity and this is Geometry. This is also in line with the constructivists view: (Cobb, 1994) discussed previously.

Polya (1957) emphasized the drawing of a picture as an important discovery device in problem solving. He indicated that even if your problem is not a problem of Geometry,

one could try to represent the problem in the form of a figure. Technology also supports the use of drawing as the first step into solving a problem. Currently Geometry teaching devalues the visual processing of information and appears to focus on training the learners to write rigorous mathematics in the form of theorems, proofs and solution of problems, (Luthuli,1996). He has claimed that these proofs are done from diagrams already drawn in textbooks of origin not known to the learner. Thus the demand for the solution of ready-made problems by school pupils reduces the learners to non-participants in the evolution of Mathematics. This contradicts the views of social constructivists, Von Glasersfeld (1984), Confrey (1990) and Steffe (1992), discussed in the theoretical framework. It is evident from the previous chapter that the drawing and making of models of double-storey houses were two activities in which the learners in this study delighted. The interpretation of drawings has been overemphasized in South Africa in Euclidean Geometry. The use of models is not adequately exploited.

The activity of drawing artifacts was worthwhile since at the very least, after participating in the drawing of artifacts, the learners developed the idea that Geometry in the mainstream Mathematics was not new to them and that it has real life applications. Further, the relationship between Geometric shapes used by the learners when drawing in Technology, enriched and changed the attitude towards Geometry because it encompassed features and lines that they were familiar with.

### 5.3 Diagrams to represent objects

**Assertion 2:** Learners did not use scale drawing or preserve the lengths when they represented existing objects through diagrams, scale drawing was used when designing and constructing models.

The learners participating in this study did their drawings based on estimations. The main aim was to represent the whole of the double-storey house on the given A4 size paper. About 60% of the participants though, used comparisons and measurements to produce features of the drawings that corresponded to the first part of the drawing already drawn. The remaining participants indicated that they used no particular measurements, but relied on imaginary estimations using their eyes and images formed in their minds, which concurs with Piaget (1980) and Alexandrov (1995) in the theoretical framework. Geometrical thinking involves images whether these are thought of as internalized experiences of the world of mental constructions that are imposed upon it, (Mason, 1989). These learners showed a skill of thinking geometrically and to work in some way with such images. A diagram has many advantages over verbal communication, for example it can show positional relationships more easily and clearly than a verbal description. Also, it can clarify ideas in person's minds and solve problems that lend themselves to visual representations. When drawing a diagram, the information was organized in a spatial form by the learner. The visual part of learner's mind was allowed to become more involved in representing the houses on paper. Through the diagrams

made by the learners, it was obvious that they knew more Geometry than anticipated, (Cobb, 1994).

#### 5.4 Geometry manifest in finished artifacts?

**Assertion 3:** Learners used measurements based on a chosen scale to produce finished double-storey models, which revealed some Geometry.

The researcher looked at the different orientations of diagrams, some parts of their drawings shaded to bring out the perspective as clearly as possible. This often included seeing a learner trying to visualize and orient a model in the same way as the diagram on paper. As opposed to algebra, which is a more methodical, structured and ‘law bound’ part of Mathematics, Geometry is more spatial, visual, insightful and philosophical aspect of Mathematics, (Wilson, 1977). The drawings of artifacts that the learners produced in Technology have never been formally taught to the learners. The task of drawing double-storey houses which was given to the learners, embodies aspects of Geometry like scale drawing, symmetry, parallelism, congruency and similarity. From their drawings, it was observed how the mentioned Geometrical concepts were utilized. Learners displayed Perspective drawing, Scale drawing, Orthographic projections and Isometry without knowing the formal definitions of the concepts.

The learners were in control of how they made the designs and how these designs were developed into double-storey finished model houses. Anyone who makes objects to his own designs comes in time to have a feeling for shapes and sizes, and this is an

indispensable part of Geometrical education (Wilson, 1977). Responses from the learners indicated that, as the learners used measurements and adjusted the scale to produce artifacts that could be seen, they were acquainted with sizes and shape. Each learner had a list of chosen measurements that were proportional to each other. The measurement on the roof corresponded with the measurement for the floor, the different faces, windows and doors. They came up with interesting Geometric representations of double-storey houses. It was noticed that drawing and the making of models were two activities in which the learners displayed their drawing and construction skills. Wilson (1977) indicated that Geometry is very close to the natural impulses of the learner. The Geometry revealed in the complete artifacts included: (a) shape, (b) angles, (c) congruency, (d) symmetry, (e) projections, (f) similarity, (g) estimations and (h) measurement.

#### 5.4.1 Shape

Learners could recognize, describe, identify, and interpret the different shapes displayed in the finished artifacts. They gave satisfactory explanations of how they produced certain shapes and how those shapes compared with others in the completely finished artifact. They also explained why certain figures or shapes at some instances were not used since the desired designs would not be acquired. Much Geometry lies so close to our level of awareness that we do not think of it as Geometry at all (Wilson, 1977). He further indicated that, it is an advantage of Geometry that many of its problems can be treated either in a primitive way that everyone can understand, or in a deeper theoretical manner that is perhaps more instructive but accessible to fewer people. The finished artifacts indeed revealed the use of Geometry to produce proportional structures.

#### 5.4.2 Angles

As the learners pasted each floor to a side, he/she would ensure that the corners made 90 degrees with the floor. This was done by ensuring that after the connecting glue was dry, the side of the house erected would be left and viewed from afar, whether it would wobble or stand straight. If there was a movement towards the inside of the house, one learner explained, that this meant that the angle made between the side and floor was less than 90 degrees. At 90 degrees the erected side stood straight. If the side movement was toward the outside of the house, this meant that the angle formed between the side and the floor was more than 90 degrees.

#### 5.4.3 Congruency

The learners made the triangular faces that supported the roofs of the finished artifacts to be equal. This was done by cutting out a triangle of one side and putting it on top of the other side, before pasting it to the roof. In the same way the corresponding faces of the house were also made equal. The distance from the ground floor to the end of the first floor and the distance between the first floor and second floor were also measured to be equal in the complete artifacts. The windows were also equal to each other.

#### 5.4.4 Symmetry

Various features of the houses were placed symmetrically in the finished artifacts. This was done through the measurement of the distance from the wall and the main door on either sides of the front face, before cutting out the windows. The windows in the finished artifacts showed reflection even from one floor to the other. It was noticed that one window was placed on the left hand side of the second floor of the house to be in line

with the one downstairs and correspond also in position with the one in the opposite room. This is a mathematical skill. This is symmetry.

#### **5.4.5 Projections**

Orthographic projections where the learners showed expertise of representing all floors and walls at 90 degrees to each other could be seen in the completed models. Isometry was also projected in these artifacts where the depth and thickness of the walls could be spelled out from the complete structures.

#### **5.4.6 Similarity**

The window type and size were cut out by the learners to be similar to each other and were placed on either side of the doors. For a particular floor suitable types of windows were cut. Also, in the case of a double-storey model house with two garages, the garages were of similar shape. Various faces in the models were also cut such that they were similar to each other.

#### **5.4.7 Estimations**

The learners could recognize the proportionality of space to be used as they cut out windows and doors into the skeleton of walls in their buildings. In most instances, like the measurements of the roof of the model house, learners used estimated measurements. The explanation was that the roof had to project over the front walls so that it could be steep enough to cater for rainy days. They used appropriate scale though and in their adjustments, length was preserved.

#### **5.4.8 Measurement**

The measuring device used was a ruler. A learner would make a mark on the big cardboard strip, draw a line and then cut out a particular measured piece of cardboard.

The cut piece was then compared to a piece representing a side wall, for example, to assure that the two strips match. The first part that the learners measured was the floor of the house that served as a base.

These were some of the mathematical concepts, ideas and principles identified during the construction of double-storey houses.

### **5.5 Implementation of Geometric manifestations in the mainstream classes?**

**Assertion 4:** The Geometric manifestations in the drawings and finished artifacts are no different from the Geometry used in the mainstream Mathematics.

Learners used arguments in their explanations that revealed knowledge of Geometry. Each learner was able to report on what he/she had done and considered whilst drawing. They used mathematical conventions, rules and techniques to build arguments that could not be refuted, in explaining why they constructed and drew diagrams the way they did. This therefore shows that they understood all the stages that they went through whilst constructing the double-storey houses. For example, learners measured strips of cardboard using rulers in the same way they would measure a side of a rectangle for example. One learner claimed that he had to divide the floor into four equal parts, before he could cut out two windows on the same strip. This is congruency. Congruency focuses on geometric figures which have the same shape and size. The learner even put

an already cut side on top of a big flat cardboard to cut out a shape of the same size as the other side.

The expansions by means of adjusting the scale of the drawing to three times that of the artifact, was a form of size change used by learners without any effort. Learners also used proportional contractions to represent real double-storey houses on A4 size papers. In the size change, with shape preserved, the pre-image of the house and image were similar. Similarity covers comparisons of triangles, quadrilaterals and circles in the grade 9 syllabus. At this stage the fact that similar triangles are equiangular is still underemphasized. The drawings of the learners though, revealed that the learners are aware and can use this idea. The cut pieces of cardboard were of similar shapes and different sizes because angles were equal.

The learners compared lines drawn, heights and lengths of sides of the models. The corners had to be straight and perpendicular to the floor. The walls were constructed straight up to support the angular roofs. There was equal space between the ground and the first floor and second floor. Also, the distance between the floors was the same in the finished artifacts. The distance between the windows was accurately measured to be the same in the different floors. Parallelism was also used to make sure that the distance between the floors was the same. Walls were erected parallel to each other in most artifacts.

All the above include the Geometry done in the mainstream Mathematics. The skills used by the learners in dealing with the procedures of constructing artifacts were mostly Geometrically inclined. The learners had some pre-conceived notions about the use of shapes in their finished artifacts. These properties were not previously taught in class. Mostly learners were able to represent the real houses in the form of drawings and models. This shows that the learners knew Geometry of mainstream Mathematics. The Geometry needs to be restructured and presented to the learners in a familiar context.

### **5.6 Summary**

In conclusion, it was evident from the activities done by the learners that most Geometry was known and used. This involved the combination of cultural aspects involving ethnomathematics and the allowance given to learners to operate in a free classroom environment. In their finished artifacts the use of parallelism, comparisons, similarity, congruency, effective scale reduction and expansion, isometry, projections and an overall shape of the artifact, were prominent. Geometric properties were combined nicely and accurately to show the delight of each learner in making his own model. Recommendations and the implications of this study in Geometry teaching are further discussed in chapter 6.

# CHAPTER 6

## CONCLUSIONS

### **6.1 INTRODUCTION**

In this chapter the validity of the empirical data, limitations of the research for present practice and policy, learners' reflective ability as a source of knowledge and implications of the study for further research are discussed. Suggestions on answers to the questions in the first chapter are made. It is hoped to improve the way Geometry is taught at school.

### **6.2 Validity of the empirical data**

The participants had limited experience with drawing and construction of artifacts. It would therefore not have been realistic to expect that they would have represented and described the artifacts in a consistent, coherent and articulate manner. Also as second language speakers, some of their descriptions and explanations had to be translated as they were given in their home language, Zulu. The following aspects of the study were problematic in generating authentic empirical information:

#### **6.2.1 Gender performance in drawing and construction**

Boys produced more beautiful drawings full of art and colour than girls. This is in agreement with Ambrose (2002) who had speculated that boys exhibited advanced drawings of pictures of the same object when drawn from different perspectives. Nonetheless, it is hoped that the discussion on how the artifacts were drawn and constructed was sufficient to indicate the extent to which learners use Geometry to represent their thinking.

### **6.2.2 Order of data collection**

The order in which the data was collected was important. In order to access the Geometry used by the learners in finished artifacts, it was essential to have got them to draw their designs first. It was at that stage where their investigating skills were evident.

When the learners made the artifacts, it was noticed that to a greater extent the design specifications mentioned before were satisfied. The learners could measure windows and walls, cut out roofs, join sides together, shape the house, form a roof, combine floors, walls and roof, and lastly, assemble all parts together to produce a complete double-storey house. At a later stage, the learners analyzed their drawings and constructions through communication with other learners and with the researcher during interviews. They could justify and identify some Geometric shapes used in their drawing. Some knew that, if two parallelograms of equal sizes were placed slightly to overlap each other by a certain distance, then the result of the shape formed would be a gable roof. Explanations were also given regarding lines drawn shorter than others to elucidate similar faces of the double-storeys. When some lines in the drawing were thickened and colored, isometry was revealed. Drawings done in perspective by each drawer instructed the viewer as to the angle from which he should stand in order to see certain features of the figure. Windows were also measured to be equal in size for certain floors.

### **6.3 Limitations of the study**

The main limitation of this research is the small sample size which impacts on generalizing the data. It is not certain that all Grade 9 learners beyond the school context in which the research was conducted, can draw and construct double-storey artifacts.

#### **6.4 Reflective ability as a source of knowledge**

Learners should be involved and be taught how to reflect, refine generalize and to ask questions, claims Nicholson (1992). It has been noticed that the skill of reflecting on what the learners were doing was greatly employed during the drawing and construction of artifacts. Learners should be allowed more time to think about a given problem in Geometry. They should also ask questions about the problem or given figure because their questions are relevant and have a purpose of creating an understanding of the drawing. The implication of this for our teaching is that we must provide opportunities for learners to struggle with the problem of communicating their diverse and personal intuitive mathematical thought. The outcome of this activity would be the learners' own construction of notations, proofs, definitions and the appreciation of Geometry as a whole. Lessons should be conducted in a manner that enables the learners to proceed from their own intuitions to develop the ideas required by the particular syllabus. The information learnt is at the disposal of the learner, he/she owns it and can use it in ways that he/she chooses.

Herbst (2002), points out that the stability brought about by using traditional approaches in teaching Geometric deductions can actually reduce the learners' participation in the development of new ideas. Learners in this study could explain and describe their drawings because the ideas of what kind of a drawing or structure a learner produced belonged to him. Teachers are therefore encouraged to use new approaches to the teaching of Geometry that will enhance the development of new Geometric ideas that can be used by the learners. The activities described in chapter 3 of this study are some

effective and flexible ways that can be used by the teachers to extract and find out the Geometry known to the learners and build on it.

Visual perception depends on transformation since the variety of retinal images derived from any object seen at different angles at different times have to be linked in the mind if recognition is to take place. Also imagination is a personal world in which transformation rules unchallenged. Thus, a bridge exists between what the teachers know, what the learners can do and some of the Mathematics the teachers would like them to learn. The idea of transformation becomes a tool that a teacher can use, a tool that does not dehumanize learning because it corresponds to one aspect of the voluntary activities of the child's mind.

Learners gathered information on how they could build a double-storey house and what material was available for use in the process. They accessed information regarding the size of the artifact, recorded measurements and predicted the different walls to be constructed before the others were done. They identified the congruent faces of the building, compared similar sides and used scale drawing to represent their creativity in the form of drawings. Management of resources, knowledge and skills related to graphics e.g. use of color, rendering techniques, two-dimensional and three-dimensional drawings, planning, sketching, drawing, calculating and construction of models were used by the learners. Furthermore, the learners justified the choices made (see interviews in chapter 4). This is Geometry, enriching reasoning skills.

## **6.5 Implications of the study for further research**

The study dealt with:

- (a) how learners learn and monitor their own process of knowledge building and skill acquisition in Geometry,
- (b) the conditions under which learners monitor their progress in completing given tasks in mainstream mathematics,
- (c) how the experience that the learners go through in the Technology classroom during the construction of artifacts can inform the teaching of mainstream mathematics,
- (d) how would the teaching of Geometry allow the learners to both reflect and utilize what they already know as a starting point or springboard for the further study of Geometry and
- (e) how a different Geometry, like projective geometry could be incorporated into mainstream mathematics.

These are themes for further research:

- (a) how could learners be taught how to reflect, refine, generalize and ask questions during mathematics lessons,
- (b) transformation as a tool used by teachers to enhance the voluntary activities of the child's mind
- (c) how teachers could make learners to communicate their diverse and personal intuitive mathematical thought and
- (d) which part of the projective geometry could inform the teaching of Geometry in the mainstream mathematics.

## 6.6 Way forward

The situation of very low performance in Geometry has to be overcome by both Mathematics teachers and learners. This can be accomplished in various ways:

(a) Development and use of Mathematical skills displayed by learners in other learning areas to enhance appreciation and their attitude towards Geometry.

(b) Representation of various artifacts and objects in the form of design and drawing from the learners' own perspective.

(c) Learners should be regarded and accepted as intelligent and creative individuals whose questions are valued. Teachers should therefore afford the learners more choice in discussions. This would give the learners effective decision making powers in Geometry.

(d) Learners should also describe and represent their experiences with shape, space and time irrespective of the prescription of the syllabus per grade.

(e) Learners should be allowed to use any language to communicate Mathematical ideas, concepts, generalizations and thought processes. This will include the use of language to express Mathematical observations, interpretation and analysis of models.

(f) Learners should be afforded an opportunity to actually do the mathematics based on ethnomathematics, construct objects and comment on their presentations.

(g) Teachers must introduce and present Geometry in an interesting and joyful manner. OBE has introduced a new thinking in our education. What a wonderful opportunity to introduce too, a refreshingly useful, enjoyable and meaningful Geometry, based on learners' experiences.

## **6.7 Summary**

This research study has attempted to deal fairly with ways in which Geometry can be approached by teachers using experiences that the learners went through during the constructions and designs of models of double-storey artifacts in Technology. It used as its basis one sample of grade 9 learners in one school in Margate. It may be used by people interested in further research in Mathematics Education. Mathematics teachers themselves may want to scrutinize the suggestions made in this study for improvement in their Geometry teaching strategies at school. Other research could be undertaken to determine if there are some other ways in which Geometry can be taught successfully to learners.

## REFERENCES:

- Adler, J. (1997). Professionalism in Process: *Mathematics teacher as researcher from a South African Perspective*; Proceedings of the First Annual Meeting of SAARMSTE.
- Alexandrov, A. D. (1995). Geometry as an element of culture; *in selected lectures, ICME*, Russia: St' Petersburg Department of the Steklov Mathematical Institute.
- Ambrose, et al, (2002). *Teaching children Mathematics in Developing Spatial understanding through building Polyhedrons*; Vol8, no8. Reston: NCTM Academy.
- Anderson, D. (1980). For the Learning of Mathematics; *Short Communication presented at the fourth International Congress on Mathematics Education* August, California: FLM Public Association.
- Bailey, G. (1987). *Methods of Social Research*; New York: The free press Macmillan Inc.
- Bereiter, C. (1994). Constructivism, Socio-culturalism and Popper's World 3; *Educational Research Volume 23*, No 9 pp 21 – 23, Center for Applied Cognitive Science, Toronto Ontario.

Berry, J.W. (1985). 'Learning Mathematics in a second language: some cross-cultural issues', *For the learning of Mathematics*, 5(2) pp18-23.

Bosch, D.J. (2001). *Technology for all*; Manzini: Macmillan Boleswa Publishers.

Clayton, C. (1994). *How do learning theories affect what happens in my classroom*; Cemp VI, Jan/Feb Vol 1, Radmaste: University of Witwatersrand.

Clements, D.H. and Battista, M.T. (1992): Geometry and spatial reasoning; (*In Gouws, D.A. ed. Handbook of research on Mathematics teaching and learning*, New York; Macmillan/NCTM pp 420 – 436).

Clithroe, F. et al (2000). *Designing and Planning in Dynamic Technology*; Cape Town: Kagiso Education.

Cobb, P. (1994): Where is the mind? Constructivist and Socio-cultural Perspective in Mathematical Development; *Educational Researcher*, Vol 23 No 7; pp 13 – 20. Vanderbilt University: Nashville.

Cobb, P.; Yackel, E.; & Wood, T.; (1991). A constructivist approach to second grade Mathematics, *in von Glasersveld, ed, Constructivism in Mathematics Education*; Dodrecht, The Netherlands: Kluwer.

Cobb, P.; Yackel, E.; & Wood, T., (1992). A constructivist alternative to the representational view of mind in Mathematics Education *in Journal for Research in Mathematics Education* 23.

Confrey, J. (1990): A review of research on student conceptions in Mathematics, Science and programming; *In C.B. Cazden (Ed) Review of Research in Education (Vol. 16 pp 3 – 55)* Washington D.C.: American Education Research Association.

Department of Education, (1997). *Curriculum 2005: Lifelong learning for the 21<sup>st</sup> century*; February. Cape Town: department of Education.

Department of Education, (2002). *Revised National Curriculum Statement Grade 9*; Cape Town: Department of Education.

Department of Education, (2003). *Revised National Curriculum Statement Grade 9*; Cape Town: Department of Education.

De Villiers, M. (1994). Exploring loci in a sketchboard; *Pythagoras*. Centrahill: AMESA.

Dowling, P. (1995). Against utility in School Mathematics and Educational Research, Cape Town: *Proceedings of the 3<sup>rd</sup> Annual Meeting of SAARMSTE*, Vol 1 (pp 174 – 191).

Duval, R. (1995). Why teach Geometry: *In Perspectives on the teaching of Geometry for the 21<sup>st</sup> century*; Carmelo Mammana: Department of Mathematics, University of Catania.

Ernest, P. (1996). 'Varieties of constructivism: A Framework for comparison', in L.P. Steffe, P. Nesher, P. Cobb, G.A. Goldin & Greer, eds., *Theories of Mathematical learning*, Mahwah, Lawrence Erlbaum Associates.

Fischbein, E. (1990). Mathematics and Cognition, a research synthesis by the international group for the Psychology of Mathematics Education; *ICMI Study series*. Cambridge: Cambridge University press.

Freudenthal, H.(1991). Revisiting Mathematics Education; *China lectures*: Kluwer.

Gattegno, C. (1965). 'Mathematics and Imagery' in *Mathematics Teaching* 33, Association of teachers of Mathematics; UK: Derby.

Gattegno, C. (1980). 'The foundations of Geometry.' in *For the learning of Mathematics* 1 (1) pp10-16.

- Gouws, D.A. (1992). *Handbook of research on mathematics teaching and learning*; New York: Macmillan.
- Hanna, G. (2000). Proof, explanation and exploration; *An overview in Educational studies in Mathematics* 44(1/2); pp 5-23.
- Hatano, G. (1996). A concept of knowledge acquisition and its implications for Mathematics Education; in L.P. Steffe; P. Nesher; P. Cobb; J. Kilpatrick and B. Greer, eds. *Theories of mathematical learning* pp 197 – 217. Mahwah, N.J.: Lawrence Erlbaum Associates.
- Herbst, P.G. (2002). Establishing a custom of proving in American School Geometry; Evolution of the two column proof in early twentieth century in *Educational studies in Mathematics* 49(3); pp283-312.
- Kaput, J.J. (1989). Supporting concrete visual thinking in multiplicative reasoning; *Focus of learning problems in Mathematics*, 11 (1); pp 35 – 47.
- Kemmis, S. McTaggart, R. (ed) (1988). *The action research planner*; Australia; Deakin University.
- Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematics knowing and teaching. *American Educational research journal* ; 27: 1 29-63.

Lerman, S. (1993). Constructive Classroom Research in Mathematics Learning; *Proceedings of the first annual meeting of SAARMSTE*.

Luthuli, D. V. (1996). Geometric riders and proof; *Pythagoras*; Centrahill: Amesa.

Mason, J. (1989). Geometry: what, where and how? *In Mathematics teaching 129*.

Mc Dermott, (1992). *Constructivist Perspective in Goldfields college*, Radmaste; Johannesburg : Education Mathematics Project.

Mcniff, J. (1988). Action – research: *Principles and practice*; Macmillan Educational Ltd.

Mogari, D. (1998). Geometrical constructs & pupil's construction of miniature wire cars; *Pythagorus 46/47*, Centrahill: Amesa.

Mogari, D. (2000). Problems associated with the use of Ethnomathematics approach, in Mahlomaholo, S.; Nkoane, M.; Smit, k.(eds). *Proceedings of the 8<sup>th</sup> Annual Conference of the Southern African Association for Research in Mathematics and Science Education*, South Africa: University of Port Elizabeth.

- Mogari, D. (2002). Research involving Ethnomathematics in a Mathematics classroom; Durban: *Saarmste Proceedings of the 10<sup>th</sup> annual conference*.
- Mudaly, V.; de Villiers, N. (2000). Learners' needs for conviction and explanation within the context of dynamic geometry in *Pythagorus*; August Centrahill: Amesa.
- Nicholson, M.J. (1992). Problem solving in *Moodley, M.; Njisane, R.A.; and Presmeg, N.C. (1992) (eds). Mathematics Education*; Pietermaritzburg: Shuter and Shooter.
- Nkabinde, T.C., & Mfusi, M.T. (2001). Graphic communication in *Unlocking Technology*; Cape Town: Nasou.
- Nxawe, M. (1995). *Tesselations*; Cape Town; The Mathematics Education Project.
- Onwu, G.O.M. (1994). Conducive Classroom Environment for Science, Technology and Mathematics Education: *Implications for the learner*. A lead paper in Proceedings Science.
- Piaget, J. (1980). Adaptation and intelligence: *Organic selection and photocopy*; Chicago: University of Chicago press.

Polya, G. (1957). *How to solve it*; New York: Doubleday & Co. Inc.

Rogoff, B. (1990). Apprenticeship in thinking cognitive development in social context in Cobb P. (1994) Where is the mind? Constructivists and socio-cultural perspectives on Mathematical Development; *Educational Researcher Vol. 23 No. 7 pp 16*, Nashville: vanderbilt University.

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a constructivist perspective. *Journal for research in Mathematics Education*;26, 114-115.

Smithies, F. (1970). *Mathematical Gazette* 389.

Sommerville, D.M.Y. (1958). *Visualization and institution in elementary Mathematics*; Cape Town: Mathematics Education Project.

Steffe, L.; Leino, J.; Lowenthal, F.D.; Ohuche, R.O.; & Perry, B.; (1988). Action Group1: Early Childhood Years; In Hirst, A.; & Hirst, K. (eds). *Proceedings of the Sixth International Congress on Mathematics Education*: Janos Bolyai Mathematical Society. 101-115,70(4), 413-425.

Steffe, L.P. (1992). Prospects for alternative epistemologies in Education. *In L.P. Steffe (eds), Constructivism in Education*. Hillsdale, N.J.: Lawrence Erlbaum Associates.

Voigt, J. (1992). Negotiation of Mathematical meaning in classroom processes. *Paper presented at the International Congress on Mathematical Education; Quebec City*.

Von Glasersfeld, E. (1984). An introduction to radical constructivism. *In P. Watzlawick (Ed), The invented reality*. New York: Norton.

Von Glasersfeld (1992). Constructivism reconstructed, *a reply to Suchting: Science and Education*; pp 370 – 384.

Wilson, W.W. (1977). *The Mathematics curriculum*, University of Nottingham, London: Blackie and son LTD.

## APPENDIX A

### **Interviews on how learners drew and constructed double-storey houses.**

In a classroom, at Mashsha J.S. in the outskirts of Margate, a video camera was used to capture activities that were done by 20 sampled Grade 9 learners for the purposes of the research study. 10 of the 20 learners, namely: Mawi; Musa; Isaac; Sonwabile; Sbusiso; Oscar; Delphas; Winnie and Jabu were asked to redo the design and construction of double storey houses and flats.

On this particular day, the said learners started their designs by making drawings of the double-storey houses for flats that they were going to construct at a later stage. At the end of the section, the learners were expected to have completed drawing of each house. This was not a normal school technology period but 2 hours had been set aside and arranged with the principal, so as to enable me to observe all that the learners did.

The classroom was arranged the previous day to suit the activities to be done during the observation period. Five single desks and chairs were arranged neatly on a line next to 2 oppositely faced walls of the classroom. Thus, each learner who was to draw, had his own desk, 5 on each side of the classroom. Each learner was supplied with 2 A4 sized photocopy papers, a lead pencil; eraser; sharpener; 30 cm ruler and a packet of koki pens for decorations from time to time, I went from desk to desk monitoring the progress of their drawings. I interviewed each one of them on how they did the drawings, the first lines they drew, whether they represented the roof or the floor. The following questions are some of the questions that I asked each one of them as they were drawing:

- Was there any significance about where you started drawing?

- Where did you start your drawing?
- Did you involve any measurements whilst drawing?

I got various different answers as to where each one of the learners started their drawings:

**Oscar:** “I started with these 2 parallelograms here.” (pointing at the roof)

**Researcher:** “Are the parallelograms related in size?”

**Oscar:** “Yes – they are equal but umangizibheka (when I look at them), I must draw them sengathi ziyashiyana, khona kuzobonakala ukuthi ngu roof wenye indlu lo, nomunye welinye ikamera.(as if parallel to each other, so that this can be one roof for one room and the other for a separate room.)”

**Researcher:** “Which part will you draw after the roof?”

**Oscar:** “The different walls, ngigcine ngedoors namawindi (the doors and windows will be drawn later).”

**Researcher:** “How will you do that?”

**Oscar:** “Ngizobona ispace enginaso (I will look at the space that I have), besengikala ukuthi ngibeke (then I will measure such that I have) two equal windows and a door on each side.”

**Researcher:** “Where did you start with the drawing?”

**Sbusiso:** “At the roof.”

**Researcher:** “Why?”

**Sbu’:** “Because its easier to draw the house if your start with the roof.”

**Researcher:** “Is there anything significant about where you started your drawing?”

**Sbu’:** “No, kodwa uma uwazi usize weroof yakho, kubalula ukuthi ukale amawalls namafloors wakho. (but if you know the size of your roof it becomes easier to measure your walls and floors)”

**Researcher:** “Sonwabile, you’ve done a lot with your drawing! What is left before you complete it?”

**Sonwa’:** “I have to fit in the doors and windows and then to finish ups like the shading and decorations.”

**Researcher:** “Is this your dream house?”

**Sonwa’:** “Oh, yes, I would like a house like this masengime kahle (when I have my own moneys).”

**Researcher:** “Take me through the steps that you have followed in drawing this house.”

**Sonwa’:** “Ngiqale ngalo line ophezulu weroof.”(I started with this upper line)

**Researcher:** “Okay, let me write 1 to indicate your starting line. Did you measure it?”

**Sonwa’:** “No, I did not measure it but when I draw the gable part of the roof, I measured so as to make sure that both lines on either sides were equal.”

**Researcher:** “Let us put 2 to indicate these lines. Can I also measure and verify that they are equal?”

**Sonwa’:** “Hau! Khululeka mhem, uzozibonela.(No just relax teacher you will see for yourself)”

**Researcher:** “Go on.”

**Sonwa’:** “I then drew another line esiphakathini (in the middle) from both ends of line 1, ukugqibezela (to finish up) iborder line yeroof yam.”

**Researcher:** “Alright – let’s call it line 3, go on....”

**Sonwa’:** “Nga draw(a) (I drew) two other equal lines on both ends on line 3, kodwa (but) to be shorter and kuqondane no (parallel to) line 2.”

**Researcher:** “Why is that the case?”

**Sonwa’:** “Ngiyaqondanisa ngoba ngifuna the walls engizozakha, zibukeke sengathizibhekene (I draw them parallel because I want the walls to appear as if they are facing each other).”

**Researcher:** “And then?”

**Sonwa’:** “I then finished off my roof by drawing these two equal lines to form two incomplete triangles at the end.”

**Researcher:** “Why triangles?”

**Sonwa’:** “I wanted ukuthi ngakhe loludonga (to build this wall) to face the one from just like the one on the other side.”

**Researcher:** “Okay, Sonwabile, carry on drawing, I’ll come back to you, let me see what the others are doing.”

**Researcher:** “Bongani, can we now analyze your diagram? Take me through the steps that you followed, where exactly did you start with your drawing.”

**Bongani:** “Okay, when I started with this bottom line, (pointing) I drew a straight line. “

**Researcher:** “Let’s put one there; and then; did you measure it?”

**Bongani:** “No, I didn’t.”

**Researcher:** “And then...”

**Bongani:** “I drew this other line such that I show the other side, lingasitheli, kuvele isayidi elilodwa.( only one side should appear)”

**Researcher:** “Why is that?”

**Bongani:** “So that indlu yam ivele sengathi ibheka le (my house appears as if its facing that side, pointing to he left).”

**Researcher:** “How?”

**Bongani:** (He lifted his hands parallel to each other to show that if one is standing next to the garage, then he will see the other side of the wall i.e. No 2)  
“Kanje mhem (this way teacher).”

**Researcher:** “Okay, and then...”

**Bongani:** “I drew this vertical line to be the longest, so that I can compare the other vertical lines and the height of my double-storey to be equal to the first one.”

**Researcher:** “But they don’t look equal.”

**Bongani:** “No the other one shows a corner and should be drawn such that lo line ongemuva uvele obonisa (the line behind shows) the corner at the back of the flat. Futhi idistance phakathi kwalo line nalo (i.e. 3 and 4) funeka ibe the same, ilingane(the distance between these lines must be equal). Then to propel the front, I drew another vertical line (i.e. 5 on the diagram) so as to complete the front side, because the other side elihamba naleli

eliseceleni (i.e. Sida A) alizikuvela. Bheka mhem kanje.(which is a partner to this one on this side cannot be shown.)”

**Researcher:** “Alright Bongani, I’ll come back to you, just finish up your drawing.”

## APPENDIX B

### **Observations of how learners constructed and designed double-storey houses**

In the middle of the classroom, 10 small tables were haphazardly put, to allow a space of about a meter apart, in between them. Here 10 other learners were busy constructing and designing new artifacts in the form of double-storey houses and flats. Various card box strips were scattered all around the floor. Each one of them would from time to time go and cut a piece of card box, measured and fitting to the house that the learner was constructing.

The learners were supplied with rulers, lead pencils, cutting knives, glue (shoe glue) and glass paper for windows. The learners had been instructed to construct and design new artifacts in the form of double-storey houses or flats. I moved around and observed each step that was followed by each learner in the construction of a double-storey house. The learners' names were Sandile; Wonder; Ndumiso, Mzikayifam; Susan; Armstrong; Morris; Innocent and Oscar.

As I was moving around I noticed that Wonder had started constructing his house. He had a sketch diagram of a complete double-storey house that he had drawn before.

**Researcher:** "How far have you gone now?"

**Wonder:** "I am busy making a list of measurements that I must cut out from the card box."

**Researcher:** "Why so?"

**Wonder:** “Phela mhem (In actual fact teacher)! In the drawing, my floor is only 20 cm in length. Now 20 cm is quite short uma ngisebenzisa (when I used) card box – I have to increase that and cut out into ezobonakala (that can be seen).”

**Researcher:** “How are you going to do that?”

**Wonder:** “Ngizosebenzisa iscale sika (I will use a scale of) 1cm: 3 cm so that since the floor on the drawing is 20 cm it now will be 60 cm. This is the scale that I will use throughout the house so that everything is proportional.”

**Researcher:** “Why do you use such a big scale? Can’t you reduce your scale to a smaller measurement?”

**Wonder:** “No! You can’t be able to build another house on top of a smaller house; the bottom floor must be bigger so that it can be able to support the houses that are going to be built on top of it.”

**Researcher:** “Do you have to measure each aspect of the house?”

**Wonder:** “Oh yes, so that I can convert each measurement of each part of the house, and know how much the breadth and length of my windows are and how far apart they should be. Futhi ke mhem (also my teacher): Before the house building, one has to measure each item so as to plan, the amount of material needed, the appropriate scale and the complications that one might have in building the house. At some stage where I see that, umangiqhathanisa la masayidi awaqondani, kushukuthi ngeke axhumane, ngizobona ke mina njengo builder ukuthi ngenza kanjani (I will see what I do as a builder if the sides do not compare or are not equal).”

**Researcher:** Alright Wonder, I will come back to you.

.....  
**Researcher:** You are busy , you are trying to represent your drawing in the form of a model. Where are you going to start with your building?

**Isaac:** I will start by cutting out the floor.

**Researcher:** Where is that part your drawing?

**Isaac:** Here.... (pointing at A in the diagram)

**Researcher:** How is your measurement of this floor comparing with your floor in the model.

**Isaac:** Every measurement in the diagram has been multiplied by 3 for the model

**Researcher:** Alright, continue.....

**Isaac:**I measured 2 rooms to be equal after which I marked where to make the garage.

**Researcher:** Okay, go on. Are the garages equal?

**Isaac:** Yes teacher.

**Researcher:** mh....., okay,

**Isaac:** I divided the floor into equal parts, before I cut out the two equal windows. Then I cut two equal sides for the front and back

**Researcher:** Now wait, you mean the sides of the roof. What is the width of your windows?

**Isaac:** 7cm

**Researcher:** Okay, what shape are the windows?

**Isaac:** Rectangles

**Researcher:** How many sides did you cut?

**Isaac:** Two for sides, one for the front and for the back is 4 sides.

**Researcher:** Alright , and what is the relationship between the sides?

**Isaac:** The two sides are equal.

**Researcher:** So when cutting them out, you cut them out such that they are equal.

**Isaac:** Yes.

**Researcher:** Okay, go on.

**Isaac:** After that you stick them with glue, that takes something like 45 minutes, for glue to stick.

**Researcher:** Right, let's go to the roofs.

**Isaac:** I just cut out a cardboard that I estimated to fit that roof, taking care that the front projection of the roof had to be longer than the back because it has to accommodate the verandah.

**Researcher:** Did you do all of this in one day?

**Isaac:** No.

**Researcher:** How long did it take?

**Isaac:** Five days, because I have to stick it together.

**Researcher:** Tell me about the outside building now.

**Isaac:** I now erected another floor on the side.

**Researcher:** Yes.....

**Isaac:** I then cut out the front view and back view

**Researcher:** Okay.....

**Isaac:** I then opened out its windows and inserted the curtains, I then opened the door and then came back to the roof, I cut it nicely.

**Researcher:** Was this room done separately?

**Isaac:** Yes, I did it alone and then came back with it as I finished it , to stick it to the other rooms.

**Researcher:** Are the measurements of this room related to the first house?

**Isaac:** Yes.

**Researcher;** Okay, how many floors is this room?

**Isaac:** Two floors.

**Researcher:** Now, are these any particular geometric shapes that you can identify in your complete structure?

**Isaac:** Yes, the doors garages, windows, are rectangles, and I would say that this side is a pentagon, since it has 5 sides, my roof, if you look from this side, (pointing) is in the form of a trapezium.

**Researcher:** Were these intended shapes or are they shapes that we just get from the finished structure?

**Isaac:** No teacher, I was just cutting out the cardboard, such that the structure represents what I have in my diagram.

**Researcher:** Do you want to assure me that what you have drawn is exactly this at your level best?

**Isaac:** Well, thank you very much Isaac, you have tried.

.....

**Researcher:** Tell me Delphas, how did you start your house?

**Delphas:** I started at the floor and then erected the sides to be equal.

**Researcher:** Where do you cut them ?

**Delphas:** From a bigger cardboard.

**Researcher:** And then .....

**Delphas:** I constructed two equal poles for the verandah..... then I took the roof of the first floor as my base.

**Researcher:** mhm.....

**Delphas:** I cut out two garages.

**Researcher:** Are you doing or cutting all this, one by one as seen in your diagram?

**Delphas:** Yes, one by one, I will stick them together after I have finished measuring.

**Researcher:** How do you open the window?

**Delphas:** I first draw a window on the cardboard and then cut it out, opening a space, and then stick the transparent paper from the inside.

**Researcher:** What's happening in this space?

**Delphas:** There is nothing here, because I intend opening another room. So I cut out two equal sides, one for the front and the other for the back. (you will not see the back side from the drawing, it's not shown.

**Researcher:** Alright.....

**Delphas:** I also cut out a roof which will fit the two sides that I have built, I then connect and stick together all the parts of the first house with glue.

**Researcher:** Are you doing this all by yourself?

**Delphas:** No, I just need somebody to hold it on for me. When the first house is complete, I then start to cut out the front view of the other room.

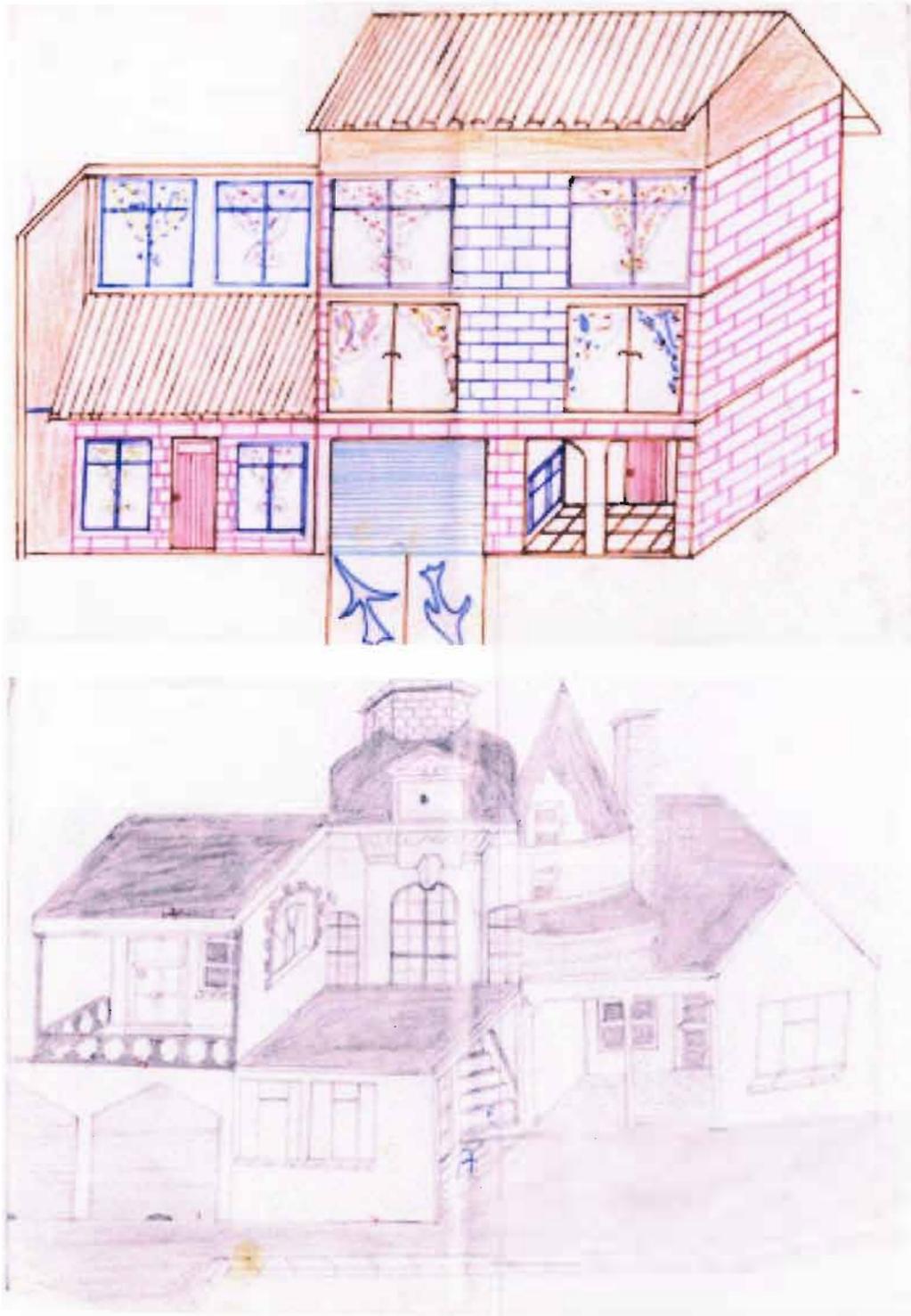
**Researcher:** Do you do any measurements before you cut?

**Delphas:** Yes, teacher, all the sides have been measured.

**Researcher:** Thank you Delphas, just continue with your construction I will come back.

## APPENDIX C

The drawings of some of the learner's houses





## APPENDIX D

Pictures of some artifacts constructed by learners

