A STUDY OF ACHIEVEMENT IN MATHEMATICS WITH SPECIAL REFERENCE TO THE RELATIONSHIP BETWEEN ATTITUDES AND ATTAINMENT

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*To my
parents and late grand-parents
for their faith in education,
despite great odds.*

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It is hereby declared that this is my own work, both in conception and execution and that the opinions expressed or conclusions reached are not to be regarded as reflecting the views of the above-mentioned persons or institutions.

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SUMMARY

This study was designed to explore the patterns and trends in attitudes towards mathematics and attainment in mathematics among Indian pre-matriculants. It also aimed at investigating the relationships between attitudes and attainment and the effects of differences in sex, grade, levels and certain background variables on mathematics attainment and attitudes.

A set of 24 multiple-choice mathematics test items, compiled to test cognitive outcomes in mathematics at three taxonomic levels, served as a measure of attainment in mathematics. An attitude scale of 48 Likert-type items comprising six dimensions was developed to measure pupil's attitudes towards mathematics (affective outcomes). In addition, questionnaires were used to collect data on selected background variables and teachers' ratings of the pupils.

The test, attitude scale and questionnaires were administered to 680 pupils selected randomly from 17 secondary schools in Durban. The 53 teachers responsible for the 151 mathematics class units (Std 9 and Std 10) at these schools also participated in the research project. The data was subjected to statistical analyses (item analyses, correlational analyses, z-scores and ANOVAS) by computerization.

The reliability and validity of both the mathematics test and the attitude scale were demonstrated. The potential value of these instruments as measures of cognitive and affective outcomes in mathematics has been presented and argued. The significances of the relationships in respect of the background variables (including sex, grade and levels) and mathematics attitudes and attainment have been carefully documented (see summary - section 6.4).

The relationship between attitudes towards mathematics and attainment in mathematics was found to be positive and significant, with no difference between males and females. It was also demonstrated that attainment in mathematics might be predicted from attitude and ability (IQ) scores, and a regression equation was derived for this purpose. Finally, consideration was given to implications of the major findings and problems for future research.

CHAPTER ONE

1. ATTITUDES AND ATTAINMENT AS RELATED TO THE PROBLEM TO BE STUDIED

1.1 INTRODUCTION

In this section the theoretical framework for the research will be presented in order to provide a meaningful reference for the exploration and delimitation of the problem to be studied. The relevant aspects of this broad overview in the form of a conceptual framework will be elaborated upon in the subsequent sections and chapters.

1.1.1 The Three Domains of Educational Outcomes

In this study education is seen as a deliberate process for changing behaviour patterns of human beings (Furst, 1958). Within this context the three-fold division of educational objectives in terms of cognitive, affective and psychomotor domains, as proposed and popularized by Bloom et al (1956), is regarded as an acceptable model for curriculum designs and instructional decisions.

These authors claim that this division is not new and that philosophers (since Greek times) and psychologists have repeatedly used similar tripartite organizations. Even the ancient teachings of Hinduism, steeped in Vedantic Philosophy, have recognized that

> "education is the development of the whole man;
> the head, heart and hand must be trained through
> scientific and practical education".
> (Sivananda on Hinduism, undated: 26)

This is a clear reference to the three main divisions of interacting behaviours : thinking, feeling and acting technically termed *cognitive*, *affective* and *psychomotor* domains respectively.

These three domains are briefly explained as follows (Krathwohl et al, 1964):

Cognitive : outcomes range from simple recall of material learned to highly original and creative ways of combining and synthesizing new ideas (commonly refers to remembering, understanding, analysing and applying).

Affective : outcomes emphasize a feeling tone, an emotion or a degree of acceptance or rejection (commonly refers to attitudes, interests, appreciations, values and emotions).

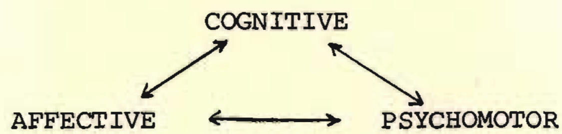
Psychomotor: outcomes involving manipulation of objects or acts requiring neuromuscular co-ordination (commonly refers to manipulative skills).

The authors of the Taxonomy of Educational Objectives see a natural link between these divisions and the practice of education as

> "most of the objectives stated by teachers in our institutions, as well as those found in literature, could be placed rather easily in one of three major domains .... the teachers and curriculum workers who state objectives do make distinctions between problem solving and attitudes, between acting and thinking or feeling" (Krathwohl et al, 1964:6-7).

Recent writers (e.g. Mathews, 1974:172; Fraser and Gillam, 1972:26-37) have recognized this as a useful framework for describing and understanding educational outcomes. In a previous study this researcher also made use of this model for demarcating objectives in mathematics learning (Moodley, 1975).

The three interacting domains of behaviour may thus be represented as follows:



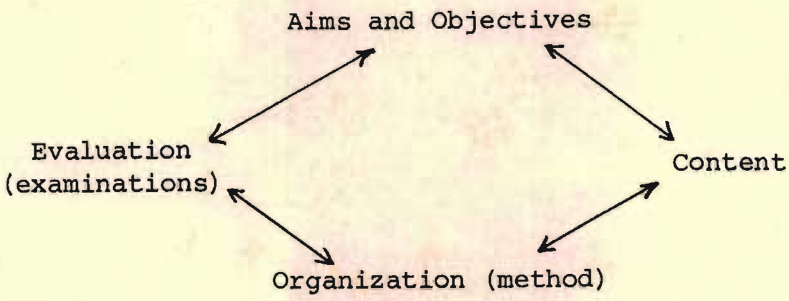
MODEL A

The interactions are depicted because the principle that educational outcomes contain elements of all three behaviours is accepted. It is clear that a single behaviour can rarely be said to be solely thinking, feeling or acting and each person responds as a total 'organism' (Mathews, 1974; Krathwohl, 1964). It is also accepted that one of the three types of behaviour may be predominant depending on the task at hand. For example, mathematics tasks such as calculations and proofs in geometry are predominantly cognitive. Within the context of the present research the cognitive outcomes and affective outcomes are restricted to performance in mathematics and attitudes toward mathematics respectively. Little work has been forthcoming in the psychomotor domain (Fraser and Gillam, 1972). Since it also has little relevance for the present study of mathematics achievement in the normal school, it has been excluded.

1.1.2 Educational Outcomes (Objectives) and the Curriculum

'Curriculum' in the present study is not restricted to the content of the teaching/learning programme (which is usually referred to as the syllabus). A review of the works of curriculum theorists such as Taba (1962), Tyler (1949), Furst (1958) and Wheeler (1967) reveals that aims and objectives, content (learning experiences), organization (method) and evaluation (examinations) form the basic components of any subject curriculum. The following interpretation shows the interaction and continuous reciprocal relationships among these components (Moodley, 1975:20):

MODEL B

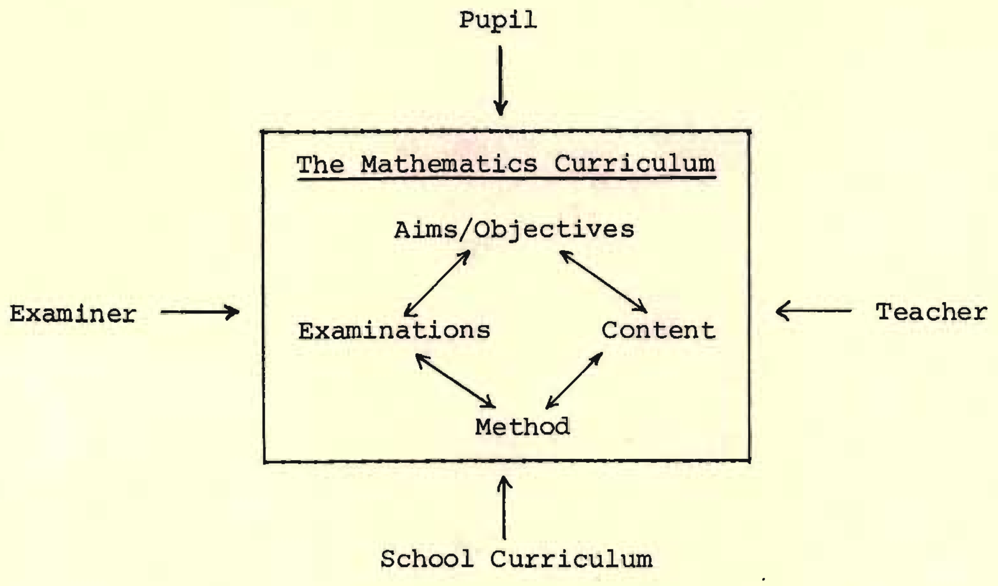


The aims and objectives serve as the bases for developing learning experiences and evaluation procedures which in turn provide feedback on the effectiveness of the curriculum (Furst, 1958). The link between this model and that presented in the previous section is that the objectives component of the curriculum must be seen in terms of the interaction among the three domains of educational outcomes. In the present study, however, such an interaction is considered in terms of the cognitive and affective domains only. Further, attention is focussed on the mathematics curriculum to the exclusion of other subjects offered within the school curriculum.

1.1.3 Attitudes and Attainment

Any evaluation of the subject curriculum must take into account the interaction between it and the following: pupil, teacher, school curriculum and examiner. Thus the context in which the mathematics curriculum is seen may be shown in the following way:

MODEL C



It must be noted that teachers usually act as examiners for all internal school examinations and the above representation also provides for external examinations as it obtains at the matriculation level in South Africa.

For classroom practice the most important interactions with the curriculum involve the pupils and the teachers. Therefore, the present study looks in particular at

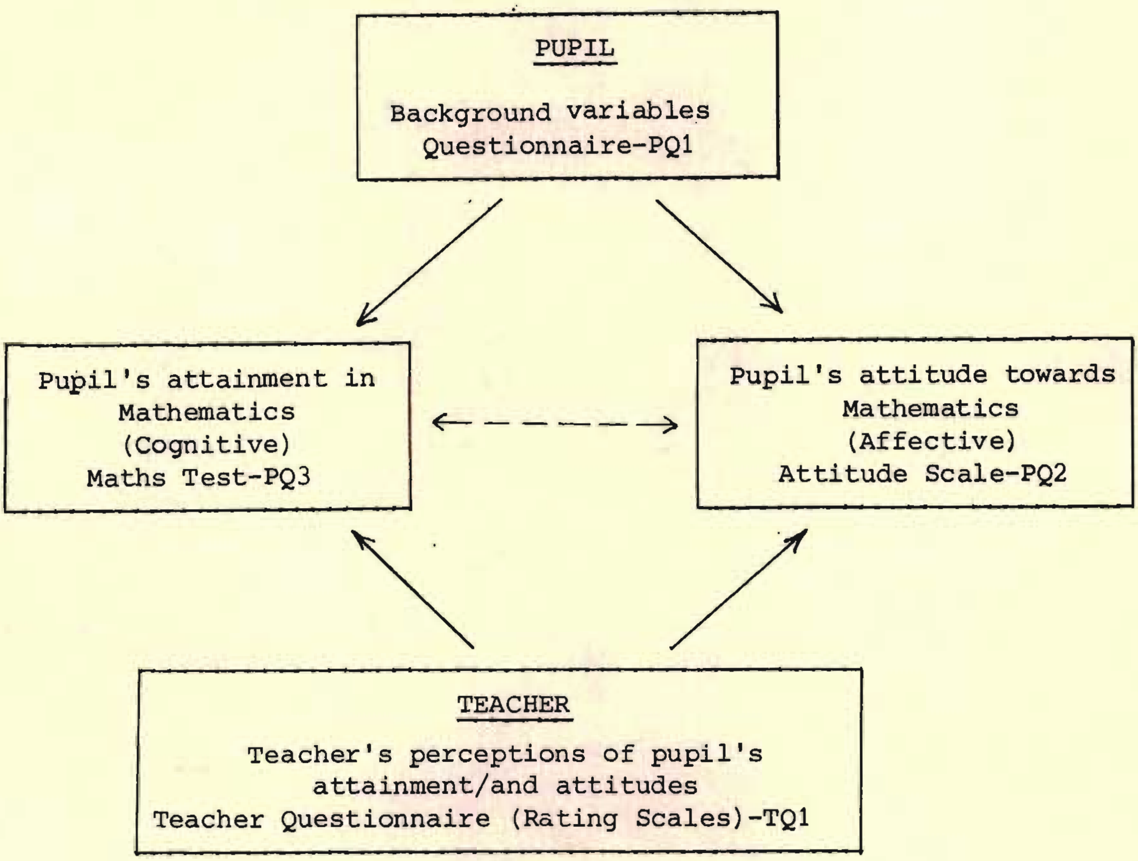
(i) the pupils' attainment in mathematics and their attitudes towards mathematics,

(ii) the teachers' perceptions of the pupils' attainment and attitudes.

Attainment in mathematics is given by measures of cognitive learning outcomes in mathematics (tests) while attitude towards mathematics is gauged from measures of affective learning outcomes (attitude scales).

Finally, within the present development, a study of attainment and attitude towards mathematics may thus be seen as follows:

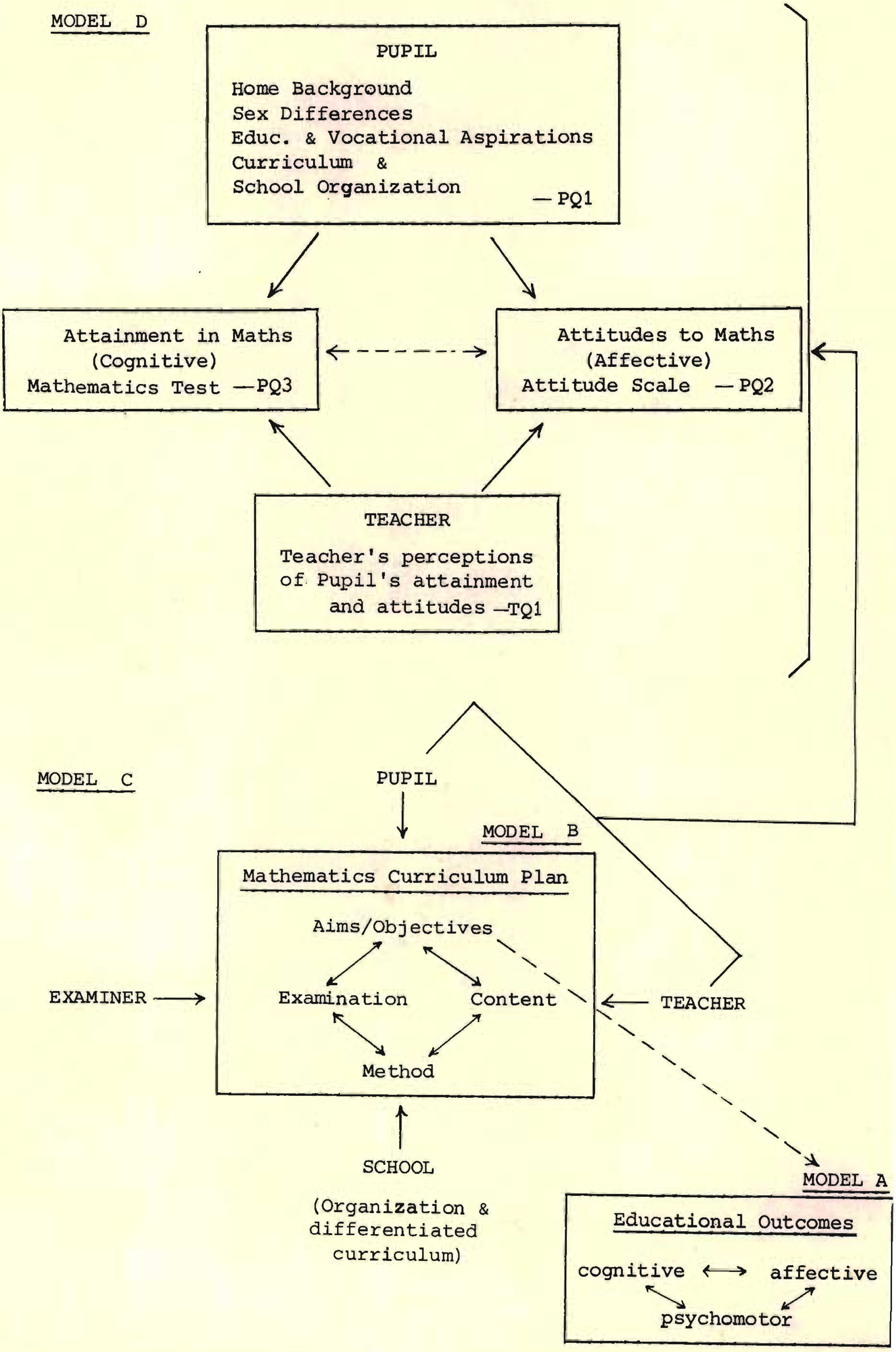
MODEL D



1.1.4 Conceptual Framework for the Present Research

Synthesizing all four representations (Models A, B, C and D) in the previous three sections and looking at the whole framework, a comprehensive picture emerges (as shown in Model E). While the present study touches on aspects of Models A, B, and C, it focusses attention on Model D of the total representation (Model E).

MODEL E : THE CONCEPTUAL FRAMEWORK FOR THE RESEARCH



1.1.5 Outline of Chapters in this Report

The rest of this chapter includes a discussion of the nature and background of the problem and sets down the variables studied, the purposes, limitations and design of study.

Chapter Two considers cognitive learning outcomes with special reference to mathematics and the testing of mathematical achievement. It also deals with the compilation of the mathematics test which was used as a measure of attainment in mathematics.

In Chapter Three affective learning outcomes are discussed with special reference to attitudes towards mathematics. Methods of measuring attitudes towards mathematics are considered. It also includes the compilation of the attitude scale which was used to measure attitudes towards mathematics.

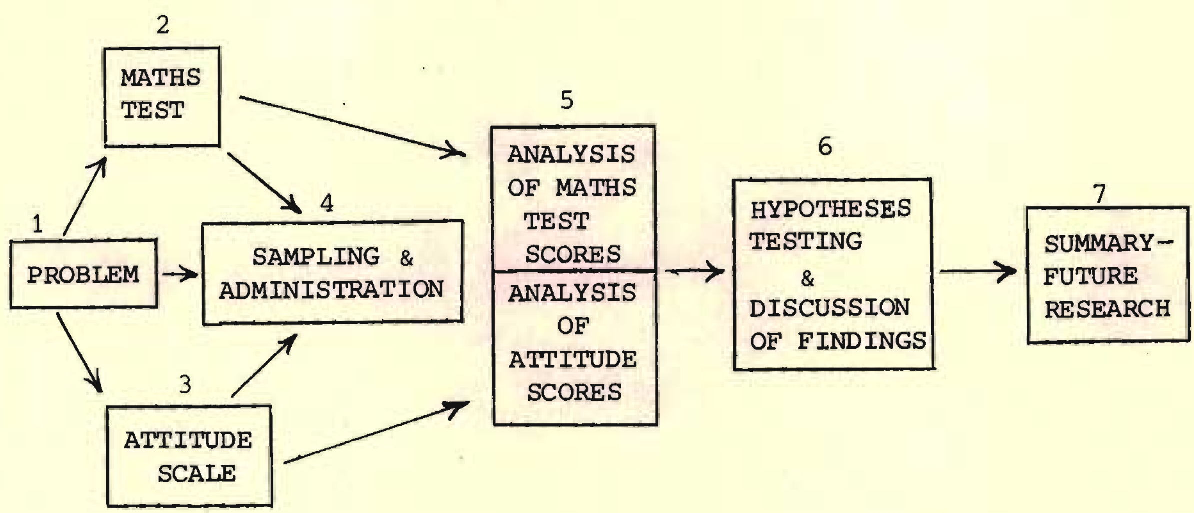
Chapter Four sets out the sampling procedures used and the administration of test, scales and questionnaires. It also includes a discussion on the handling and processing of data.

In Chapter Five statistical analyses of the data from both the mathematics test and the attitude scale are presented. Evidence of the quality of these instruments is argued in terms of *(inter alia)* distribution, reliability, validity and discrimination.

Chapter Six presents a detailed analysis and discussion (based on the findings) regarding social factors, curriculum and school organization, and teachers' perceptions as related to attainment and attitudes. It finally deals with the question of relationships between attainment in mathematics and attitudes towards mathematics.

In Chapter Seven, the concluding chapter, a summary of the major findings, recommendations and problems for future research are presented.

The following flow chart shows the development of the chapters:



1.2 NATURE AND BACKGROUND OF PROBLEM TO BE STUDIED

Historically, the past two decades have seen the mathematics curricula the world over in a state of flux and 'modernization'. These changes in curricula have brought with them certain changes in emphases in instructional strategies. Since the late 1950's, two special issues of Educational Studies in Mathematics have been devoted to assessment of changes in mathematics education in 16 countries (Freudenthal, 1978). Several recent studies of the transition from traditional to modern mathematics have expressed concern at the lack of emphasis on computational ability and decline in mathematics achievement (e.g. Hammons, 1972; Starr, 1977; Maffei, 1977; Kapur, 1978).

Moreover, the concern expressed in recent literature relating to achievement in school mathematics may be summarised as follows:

> "Public dissatisfaction with declining achievement scores in mathematics shows a need for changing roles

for federal agencies, professional organisations, administrators, supervisors, teachers and parents in curriculum development and teacher pre-service and inservice education"

(Gibney and Karns, 1979:359)

In submitting evidence to the Committee of Enquiry into the Teaching of Mathematics in the United Kingdom, the Association of Teachers of Mathematics (1979:63) commented as follows:

"Increased demands that mathematics teaching in all its aspects be improved imply considerable changes in attitudes towards mathematics and its teaching."

Recently, in the Senior Certificate Examinations (Department of Indian Education, 1979) the highest percentage failure (below F symbol) was for mathematics: Higher Grade - 31,44% and Standard Grade - 33,34% (Centre for Tertiary Education, 1980). In 1980 although some improvement was noted, mathematics still remained as the subject with the highest percentage failure (below F symbol) : Higher Grade - 27,21% and Standard Grade - 27,50% (Centre for Tertiary Education, 1981). A scrutiny of the entry and pass ratios for Standard Grade and Higher Grade mathematics at matriculation level over the past 5 years reveals a declining trend in HG entry and pass (See Table 1.1). As expected, there was a corresponding increasing trend in the SG entry and pass. It was evident that fewer and fewer candidates were entering and passing mathematics on the HG.

In view of the key role mathematics plays in tertiary education and in the many vocations, this trend in achievement in mathematics has given rise to great concern (DIE Report, 1980). It is this concern that provides, in part, the motivation for a study of achievement in mathematics.

TABLE 1.1

DEPARTMENT OF INDIAN EDUCATION - MATHEMATICS ENTRY AND PASS RATIOS FOR HIGHER GRADE (HG) AND STANDARD GRADE (SG) AT STD 10 LEVEL

	1976	1977	1978	1979	1980
	HG : SG	HG : SG	HG : SG	HG : SG	HG : SG
ENTRY RATIO*	45,57:54,43	42,52:57,48	38,74:61,26	32,60:67,40	31,17:68,83
PASS RATIO**	35,88:64,12	30,40:69,60	26,91:73,09	25,20:74,80	20,83:79,17

* ratio of HG to SG of all those who entered for mathematics

** ratio of HG to SG of all those who passed mathematics.

Apart from changes in curriculum and teaching strategies, there are several intellective (Aiken, 1971) and non-intellective (Aiken, 1970c) factors which affect achievement in mathematics. The multidimensional nature of the problems in mathematics achievement has been recognized and recent researches have been moving away from looking into purely cognitive factors affecting mathematics achievement. Several studies linking attitudes towards mathematics and achievement have been reported (Stephens, 1960; Aiken, 1961, 1970, 1976, 1979a; Kempa and McGough, 1977; Neale 1969; Sandman, 1974; Hunkler, 1977; Degnan, 1967; Burbank, 1970; Starr, 1977). The IEA (Husén, 1967) and NLSMA (Wilson, 1971) studies, both well known researches into achievement in mathematics, incorporated the affective component.

The areas covered by these researches include, among others,

methods of measuring attitudes towards mathematics,

the effect of attitudes on achievement in mathematics,

the relationships of mathematics attitudes to ability,

to parental attitudes and expectations and to

teacher characteristics, attitudes and behaviour

and the effects of modern mathematics curricula and other

curriculum practices on attitudes.

Leake (1970) concludes from his own research and a review of other studies that

> "changes in attitude toward mathematics involve a complex interaction among student and teacher characteristics, course content, method of instruction, instructional materials, parental and peer support and methods of measuring these changes".

When attitude scores are used as predictors of achievement in mathematics, a low but significant positive correlation is found (Neale, 1969). While attitude toward mathematics is directly related to both actual and aspired marks in mathematics courses (Spickerman, 1970), it is somewhat inversely related to grade levels (Callahan, 1971; Evans, 1972).

On the international front, the Second IEA International Mathematics Study is currently in progress. In a recent publication (IEA, 1979:7) which outlines and motivates the project, the following observation is made:

"In view of the importance of mathematics in society and in the schools, it seems obvious that the efficacy of mathematics teaching and learning deserves continued and sustained scrutiny. The purpose of the Second International Study is to compare and contrast the varieties of curricula, instructional practices and student outcomes (both attitudinal and cognitive) in an international context."

A previous research by this researcher focussed on the construction and use of an evaluation instrument to measure attainment of cognitive objectives in mathematics learning (Moodley, 1975). This research developed a scheme of objectives which is being currently used for the teaching, learning and examining of mathematics at the senior secondary level in the Department of Indian Education. Since its introduction other Departments of Education in South Africa have shown a great deal of interest in the use of this scheme (Meeting of Mathematics Examiners, 1978). Although this study was restricted to the cognitive domain of learning outcomes in mathematics, it suggested that the affective component may well have a strong input into achievement in mathematics (Moodley, 1975:73).

A recent survey by the Institute for Planning Research (Kriel, 1978) pointed out that

"It is an unfortunate fact that there are, as a result of various factors, numerous people who have a negative attitude towards mathematics, ranging from dislike to little confidence in their own mathematical abilities."

The report went on to explain that students who take subjects like mathematics, physics or statistics in spite of their negative attitude to anything mathematical, are most likely to drop out or fail.

Moreover, in South Africa, there is an absence of research literature on achievement in mathematics and attitudes towards mathematics. Initial research in this area is, therefore, bound to create new avenues for research and development. Further, any knowledge of the ways in which attitudes influence performance in mathematics can be invaluable to curriculum and instructional decisions.

It has long been felt that affective objectives in general and the development of attitudes in particular have been either ignored or least emphasized by teachers and curriculum developers the world over. In this regard Bloom (1974:416) cautions as follows:

> "If individuals are to continue learning in the major fields and areas of interest introduced by schools, much will depend on the affective qualities that schools have developed in them, whether they were developed intentionally or not. This is an area that warrants cooperative research throughout the world."

In another study on the use of the semantic differential in measuring attitudes, Scharf (1971:641) argues that

> "given the widespread belief that children do, indeed, have a stable measurable attitude toward the learning of mathematics, and that this attitude has some effect upon their immediate achievement in the subject or, more importantly, in terms of our increasingly technological society, on their long-range academic and vocational goals, continued investigation in this area is justifiable".

This researcher concurs with both these viewpoints. It is within this context that research is needed to find answers to some of the many questions relating to achievement in mathematics and attitudes toward mathematics:

What are the patterns in mathematics attainment?

What are the trends in attitudes toward mathematics?

Are there relationships between these attitudes and attainment in mathematics?

Do curricular differences (curriculum biases) result in attitudinal/achievement differences?

Are there sex differences in attitudes and attainment?

Do differences in choice of grades result in attitudinal/achievement differences?

While attempting to find possible answers to some of the above questions, this research study concerns itself with the main problem of exploring possible relationships between attitudes towards mathematics and attainment in mathematics. In general, this study is motivated by this researcher's recognition of the importance of attitudes toward mathematics since this subject, like any other aspect of the school curriculum, is intended as a foundation for further learning. In this context the IEA study (Husén, 1967a:73) observed that

> "if, while learning mathematics, the student acquires a dislike for the subject, further learning is unlikely and part of the purpose of instruction is lost".

It is therefore hoped that this research will contribute to a better understanding of the role of attitudes towards mathematics in mathematics education.

1.3 PURPOSE OF THIS STUDY

In the light of what has been presented in the previous section and in the light of the researcher's own experiences (as examiner of matriculation mathematics), it is felt that a reasonably comprehensive view of achievement is necessary - especially at this stage of research and development in school methematics in South Africa. To this end the present research aims to explore as comprehensively as possible the patterns and trends in attitudes towards mathematics and mathematics attainment among Indian pre-matriculants. In particular this study proposes:

(i) to develop and administer a mathematics test to discover any patterns in mathematics attainment;

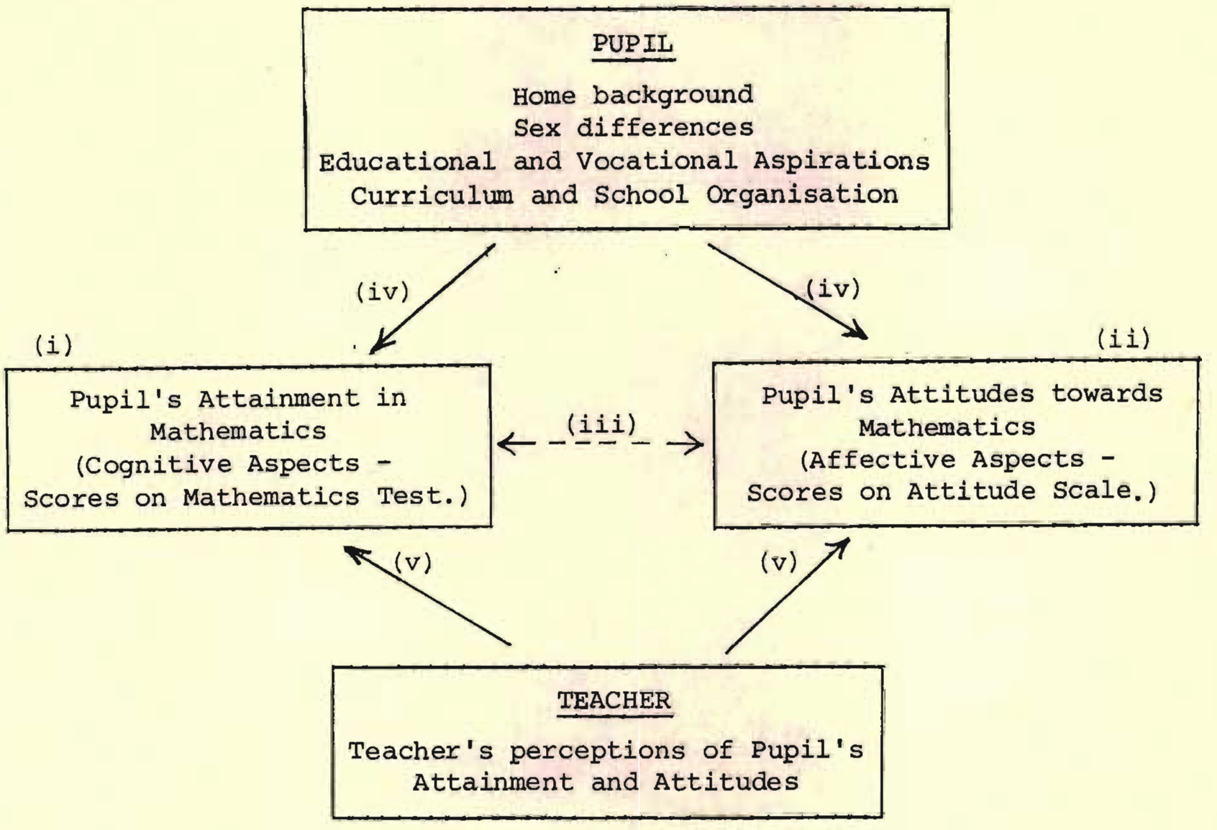
(ii) to develop and administer an attitude scale to measure attitudes toward mathematics;

(iii) to investigate possible relationships between these attitudes and attainment in mathematics;

(iv) to investigate differences in sex, grade, curricula, levels, cognitive preferences etc. which point to differences in attitudes to mathematics and attainment in mathematics;

(v) to investigate possible relationships between teachers' perceptions of pupils' attainment and attitudes and the pupils' actual attainment and attitudes.

These objectives ((i) ... (v)) of the present research can be mapped on to MODEL D of the conceptual framework proposed earlier as follows:



From this plan it is clear that the research design must establish (i) and (ii) and investigate a possible connection between these two. In this process the influences of the pupil's background and the teacher's perceptions ((iv) and (v)) have to be monitored.

Apart from exposing and increasing the understanding of the problems underlying attitudes and achievement in mathematics, it is considered that this study will yield some suggestions in respect of

Methods of teaching and learning mathematics in our schools,

Curriculum development in mathematics,

Counselling of students taking mathematics, within the differentiated curriculum,

Improvement of attitudes towards mathematics by techniques for developing positive attitudes and modifying negative attitudes,

Examinations in mathematics and

Future problems for research.

1.4 VARIABLES TO BE STUDIED

The variables selected for study must be seen in relation to the conceptual model for the present research as presented in the previous section. These variables (together with the respective instruments) fall into four categories as follows:

a. Pupil background (Pupil Questionnaire - PQ1).

b. Pupil's Attitude toward Mathematics (Attitude Scale - PQ2).

c. Pupil's Attainment in Mathematics (Mathematics Test - PQ3).

d. Teacher's Perceptions of Pupils' Attitudes and Attainment (Teacher Questionnaire - TQ1).

1.4.1 Pupil Variables (Home Background, Sex, Educational and Vocational Aspirations, Curriculum and School Organization):—

1. Sex.
2. Grade in Mathematics and Reasons for Choice (HG/SG).
3. Level (Std 9/Std 10).
4. Course Selection - mathematical and science bias within the differentiated curriculum.
5. Size of Mathematics Class.
6. Homework in Mathematics.
7. Preference for Mathematics (relative to other subjects).
8. Performance in Mathematics (relative to other subjects).
9. Content Preference (Algebra, Geometry, Trigonometry).
10. Cognitive Preference (symbolic, verbal, diagrammatic).
11. Parents' Educational and Occupational Levels.
12. Educational and Vocational Aspirations.
13. IQ (General Ability).

1.4.2 Attitudes toward Mathematics

A total attitude variable and six component attitude variables are considered:

1. Total Attitude (including all six components).
2. Views on Mathematics Teaching.
3. Views on School Learning.
4. Attitudes to Difficulties in Learning Mathematics.
5. Attitudes to Place and Importance of Mathematics in Society.
6. Enjoyment of Mathematics and Motivation.
7. General Attitude to School, Man and his Environment.

1.4.3 Attainment in Mathematics

A total mathematics achievement score and three subscores are obtained as follows:

1. Total Score on Mathematics Test.

   Subscores on Mathematics Test:—
2. Knowledge and Skills;
3. Comprehension Abilities;
4. Application and Higher Abilities.

1.4.4 Teachers' Perceptions

1. Pupil's Ability in terms of Objectives.
2. Pupil's Ability in terms of Content Areas in Mathematics.
3. Overall Estimate of Performance Level.
4. Pupil's Attitude towards Mathematics.

1.5 FORMULATION OF HYPOTHESES

In the light of the conceptual scheme put forward earlier and the stated purpose of this study, an attempt to establish any interrelationships between attitude to mathematics and attainment in mathematics will generate several hypotheses. Observations including pupil-background variables and teachers' perceptions will further increase the number of hypotheses which may be tested. The hypotheses fall into four main categories as per variables presented in the preceding section and, in general, they may be stated in terms of the following two types:

(i) those that deal with differences between two groups in respect of a single variable;

(ii) those that deal with relationships between two variables in respect of a single group.

Examples of hypotheses of the first type:

(a) there is no difference in attitude scores between HG and SG.

(b) there is no difference in attainment in mathematics between High scorers and Low scorers on Attitude Scale.

(c) there is no difference in time spent on mathematics Homework between High Scorers and Low Scorers on Mathematics Test.

Examples of hypotheses of the second type:

(a) there is no significant relationship between attitude towards mathematics and general ability (IQ).

(b) there is no significant relationship between the teacher's perception of the pupil's attitude and the pupil's attainment in mathematics.

(c) there is no significant relationship between the mathematical bias in the curriculum and attainment in mathematics.

(d) there is no significant relationship between attitudes toward mathematics and mathematics attainment.

In order to avoid unnecessary repetition, the hypotheses will not be stated here as they will be formulated and discussed fully during the course of the analysis of the data in chapter six. No attempt is made in this study to state every possible hypothesis within the conceptual framework. Only those that relate closely to the main theme of this study and its stated objectives will be considered.

1.6 ASSUMPTIONS AND LIMITATIONS

For a meaningful appraisal of findings, a research study needs to take into account the assumptions on which it is based and the limitations within which it is conducted. The present investigation of the problem outlined rests on certain assumptions and limitations in respect of the theory and practice of education:

(i) The theoretical framework within which this research study is set is presented in a previous section of this chapter. It accepts the view that education is a deliberate "process for changing behaviour patterns of human beings" (Furst, 1958:2).

(ii) The study is restricted to the cognitive and affective domains of educational outcomes in mathematics.

(iii) In the affective domain attention is focussed on attitudes towards nathematics from among the several non-intellective variables such as personality, motivation, anxiety etc.

(iv) The sample for the study is delimited as follows:

Std 9 and Std 10 pupils taking the Ordinary Course in mathematics within the Differentiated System of Education in Indian High schools in the Durban and District area.

(v) Items for the attitude scale were selected from overseas researches and hence certain assumptions about its composition, reliability and validity were made prior to administration.

(vi) Although the mathematics test items used in this study were carefully selected (and modified where necessary) from the IEA study (Husén, 1967b), certain assumptions regarding reliability, validity and discrimination were made prior to administration of the test in its final form.

(vii) This study employs survey-type techniques for exploring the problem (outlined earlier) in a static design. Thus no conclusion can be made on developments or changes in attitudes and attainment as is done in a longitudinal study.

1.7 OUTLINE OF DESIGN AND PROCEDURES

(i) A set of 24 mathematics test items was compiled to serve as a measure of attainment in mathematics. The items were selected from among IEA items (Husén, 1967b) to test cognitive outcomes in mathematics at three levels:

Knowledge and Skills,

Comprehension,

Selection-Application and Analysis-Synthesis.

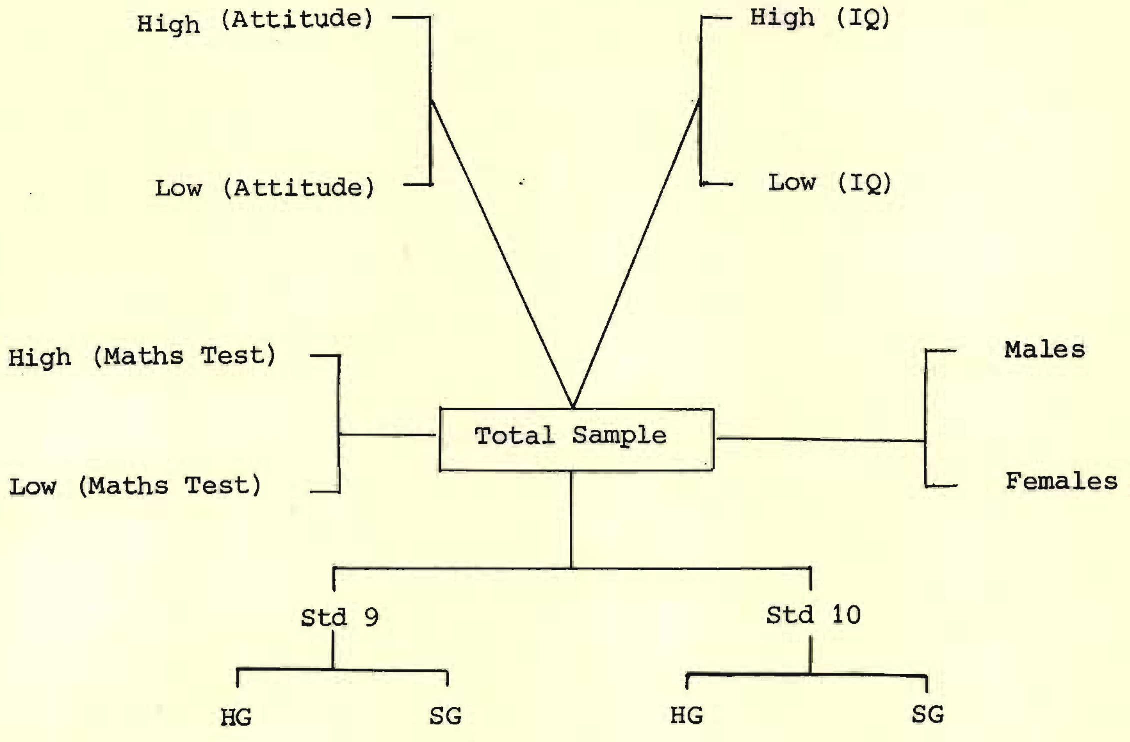
(ii) A set of 48 items on 'attitudes towards mathematics' was compiled to serve as attitude scales in six dimensions in order to establish a measure of pupils' attitudes towards mathematics.

(iii) A pupil questionnaire (PQ1) was produced to gather quantitative data on pupil background as related to mathematics achievement e.g. choice of grade in mathematics, homework, preference for mathematics in school curriculum, content preference, vocational and educational aspirations, parents' level of education etc.

(iv) A teacher questionnaire (TQ1) was compiled to gather quantitative information on the teachers' perceptions of the pupils' attainment and attitude. Suitable rating scales for attitude toward mathematics and attainment of cognitive objectives in mathematics were included in TQ1. Teachers were also required to rate the performance of their pupils in terms of content areas and to give a global performance mark.

(v) The Pupil Questionnaire (PQ1), Attitude Scale (PQ2) and Mathematics Test (PQ3) were administered to a randomly selected sample of 680 Indian pre-matriculants (Stds 9 and 10) at 17 secondary schools in the Durban Municipal area. In addition, 53 mathematics teachers completed rating scales (TQ1) for each of their pupils selected to take part in this project. All the data were coded, punched and processed by computer (UNIVAC 90/30; FORTRAN IV).

(vi) The data was gathered for variables in four major categories: attainment, attitude, pupil background and teachers' perceptions. The data was analysed to yield means, standard deviations and standard errors of means (for significance of differences) for the various subsamples:



One-way ANOVAS and z - Scores were used to test several hypotheses regarding attitude to mathematics and attainment in mathematics in relation to the above partitions of the sample and subsamples.

(vii) Intercorrelations of the variables in the four major categories were computed to test the significance of the interrelationships. Where necessary the effects of certain variables were partialled out.

(viii) Item analysis data, reliability and validity coefficients were determined in order to provide evidence regarding the quality of the mathematics test items.

(ix) In respect of the attitude scale, item correlations, reliability coefficients and intercorrelations between the various dimensions were computed in order to establish the quality of the scale.

CHAPTER TWO

2. COGNITIVE LEARNING OUTCOMES AND THE TESTING OF MATHEMATICAL ACHIEVEMENT

As mentioned in the previous chapter, one of the purposes of this study (within the conceptual framework) has been to investigate the patterns of attainment in mathematics. In order to accomplish this aspect an instrument for the measurement of mathematical achievement had to be developed. This gave rise to the question of what mathematical abilities such an instrument should test. In the present chapter the framework for the mathematical abilities to be tested is presented and the compilation of the mathematics test is discussed.

2.1 OBJECTIVES IN MATHEMATICS LEARNING

In a previous study (Moodley, 1975) a scheme of objectives for mathematics learning at the senior secondary level was suggested. This scheme was used as the basis for the construction and validation of mathematics test items. Since this classification is also used in this study to compile a set of test items, the background to its development is presented here.

2.1.1 Aims and Objectives

A distinction between aims and objectives may be made as follows (Wood, 1968):

> aims can be thought of as general declarations of intent which give shape and direction to education while objectives are regarded as statements of what the pupils should be able to do at the end of a course of study i.e. they are statements of expected outcomes or "intended behaviours" (Bloom et al, 1956).

The well-known curriculum models (Tyler, 1949; Furst, 1958; Wheeler, 1967) make the basic assumption that the general aim of education is to change behaviour, and they emphasize the essential nature of aims and objectives in the curriculum process. Aims reflecting value judgments are necessary but they are insufficient for classroom practice (Taba, 1962:196). In order to make aims more practically feasible it is necessary to describe in detail and to specify the expected outcomes. In this context Brubacher (1962:118) has pointed out that

> "in spite of their prime importance, the ultimate aims of education mark out the teacher's tasks in only the most general outlines" and that in order to be effective in the classroom these aims must be broken down into "more immediate, specific or proximate objectives for the pupil and teacher to pursue".

In sharp contrast to the above viewpoint, curriculum researchers have observed that syllabuses were vague and were mainly concerned with covering topics. Wood (1968:93) claimed that the GCE or CSE syllabus

> "encourages a shallow ephemeral style of learning largely because it offers little or no guidance to the teacher on

the learning possibilities inherent in each topic."

Similar observations have been made in South Africa where

> "the senior secondary mathematics syllabus sets down, in addition to a listing of subject matter, certain general aims ... and there seems to be a lack of clarity among teachers, examiners and pupils as to the depth of treatment of the subject matter in mathematics" (Moodley, 1975:22).

In the same study the researcher explored the need for objectives and how they should be derived and stated; and proceeded to develop a suitable classification of cognitive objectives for use in the teaching of senior secondary school mathematics.

2.1.2 A Model for Classification of Objectives

The need for classifying the numerous objectives of a course of instruction has been recognized and the need for a comprehensive model of levels of performance from the lowest to the highest has been suggested (Avital and Shettleworth, 1968:4). It is within such a framework that the teacher or researcher will be able to construct the objectives to include the full range of performance.

The Taxonomy of Educational Objectives in the Cognitive Domain (Bloom et al, 1956) has emerged as one of the most useful models of classification of objectives based on one of the three interacting areas of human behaviour (cognitive, affective and psychomotor). Since most behavioural outcomes in mathematics seem to have cognitive origins (Fraser and Gillam, 1972; Wood, 1972:31; Wilson, 1971:648; Weaver, 1971), this classification has special significance for mathematics.

The Taxonomy identifies six major categories of cognitive behaviours as follows:

1.0 Knowledge

2.0 Comprehension

3.0 Application

4.0 Analysis

5.0 Synthesis

6.0 Evaluation.

This classification is not arbitrary. It represents "the hierarchical order of the different classes of objectives which range from the simple to the complex (Bloom et al, 1956:18). This structure also suggests a relationship between complexity of behaviour and facility of problem solving. Gagne (1977:152) supports this view and suggests that

> "learning hierarchies imply that learning has a cumulative character, in which the acquisition of specific rules establishes the possibility of transfer of learning to a number of more complex 'higher order' rules".

The Taxonomy has proved to be a valuable tool for educational research workers, teachers and testers. In a detailed review of several studies which have shown how the Taxonomy has been utilized and studied this researcher concluded that

> "the greatest value of the Taxonomy lies in the field of evaluation" (Moodley, 1975:44).

Several studies in mathematics (Husén, 1967a; Wood, 1968; Romberg and Wilson, 1968; Moodley, 1975) illustrate the use of the Taxonomy, or an adaptation of it, in test construction.

In assessing the Taxonomy this researcher has observed that

> "while there is little doubt about its communicability and usefulness in education and educational research, few research studies concerning the existence of empirical foundations for the Taxonomy are available" (Moodley, 1975:44).

However, a review of the available validation studies (Kropp et al, 1966; Smith, 1968; Stoker and Kropp, 1964; Schools Council, 1970) revealed that they support the basic assumptions on which the Taxonomy is founded. Further, the previous research also demonstrated that

> "it is possible to devise test items which test at least two taxonomic levels of mathematical ability" (Moodley, 1975:152).

2.1.3 Classification of Objectives Used in this Study

Since mathematics is a logical, deductive system where complex proofs and solutions are built on simple definitions axioms and theorems, the hierarchical structure of the cognitive abilities ranging from the simplest to the complex in the Taxonomy readily renders it adapatble to mathematical performance. Several of these adaptations have been used in mathematics education:

The Indian National Council of Educational Research (Wood, 1968),

The International Study of Achievement in Mathematics (Husén, 1967a),

The Ontario Institute for Studies in Education: Objectives for Mathematics Learning (Avital and Shettleworth, 1968),

National Longitudinal Study of Mathematical Abilities (Romberg and Wilson, 1968), Item Bank Project (Wood, 1972).

The following scheme of objectives for mathematics learning was suggested for the senior secondary course (Moodley, 1975:58-59):

A. KNOWLEDGE

1. *Specific Facts*: the ability to recall definitions of terms, notation (symbols), formulae.

2. *Universal facts or generalizations*: the ability to recall axioms/postulates, theorems, conventions, methods, techniques, patterns, structure, conditions (criteria), classifications.

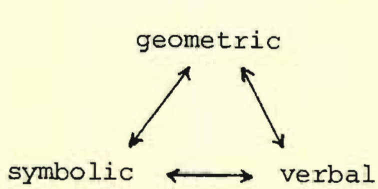
B. SKILLS

1. *Manipulative skills*: the ability to handle instruments, draw graphs/figures, read tables.

2. *Computational skills*: the ability to perform operations, factorise, solve, substitute, change subject of formula.

C. COMPREHENSION

1. *Translation*: the ability to translate from the verbal to the symbolic and vice-versa, from the geometric to the verbal and vice-versa, from the symbolic to the geometrical and vice-versa:



2. *Interpretation*: the ability to illustrate terms/concepts, to explain mathematical terms, notation, concepts and principles in own words.

3. *Extrapolation*: the ability to perceive and extend a trend pattern/idea.

D. SELECTION - APPLICATION

1. *Selection*: the ability to select appropriately the principle, method, formula, axiom or theorem required for the solution of a problem; the ability to reduce an unfamiliar situation to a familiar situation.

2. *Application*: the ability to apply correctly a principle, method, formula, axiom or theorem in a problem situation.

E. ANALYSIS - SYNTHESIS

1. *Analysis*: the ability to analyse data (parts) with the view of forming relationships; to compare related mathematical concepts/terms; to discriminate between concepts/terms.

2. *Synthesis*: the ability to generalize; to establish relationships; to construct problems/solutions/proofs.

3. *Evaluation*: the ability to check the validity of a solution, proof or generalization.

This scheme includes all the important behaviours relevant to mathematics, which have been presented in the Taxonomy and other classifications used in secondary school mathematics. (For details of illustrative test items in each category of the scheme see Moodley, 1975:61-75).

This classification has already been used with a great measure of success in two major ways:-

(i) the construction and validation of mathematics test items for research (Moodley, 1975);

(ii) the redrafting of the senior secondary mathematics syllabus in terms of objectives by the Mathematics Subject Committee of the Department of Indian Education (DIE, 1976).

Further, it is the considered opinion of the writer that teachers and examiners in the Department of Indian Education have been influenced by the syllabus in terms of objectives. A comparative study of the Mathematics Examination Papers (Senior Certificate) prior to 1977 and since that year shows the shift in emphasis to a covering of a wider range of abilities.

There are two good reasons why this classification of objectives lends itself to application in this study. Firstly, the scheme has been used by all the schools at the senior secondary level. Therefore, any test items selected in accordance with this scheme are bound to acknowledge the teaching/learning situations to which it has been applied. Thus test items selected on this basis may be expected to contribute positively to the content validity of the test.

Secondly, a comparison of this classification with the more detailed scheme used in the IEA study (Husén, 1967a:93) from which the test items were selected, shows a rough correspondence as follows:-

IEA Scheme of Objectives (Husén, 1967a:93)	Classification used in this Study
1. Ability to remember or recall definitions, notations, operations and concepts.	A. Knowledge
2. Ability to manipulate and compute rapidly and accurately.	B. Skills
3. Ability to interpret symbolic data. 4. Ability to put data into symbols. 5. Ability to follow proofs.	C. Comprehension
6. Ability to construct proofs. 7. Ability to apply concepts to mathematical problems. 8. Ability to apply concepts to non-mathematical problems.	D. Selection - Application
9. Ability to analyse and determine the operations which may be applied. 10. Ability to invent mathematical generalizations.	E. Analysis - Synthesis

In the IEA study the above objectives were derived from the following major categories of cognitive behaviours which were

> "accepted as desirable by most teachers of mathematics regardless of their nationality" (Husén, 1967a:81):—

A. Knowledge and information : recall of definitions, notation, concepts.

B. Techniques and skills : solutions.

C. Translation of data into symbols or schema and vice versa.

D. Comprehension : capacity to analyse problems, to follow reasoning.

E. Inventiveness : reasoning creatively in mathematics.

Behaviours A and B were classified as "lower mental processes" while D and E were regarded as "higher mental processes". Behaviour C was regarded as more difficult to classify into one of the two levels because of the problem of determining whether the situation confronted by the student was well known or new to him.

In the present research the two extreme levels of cognitive outcomes in mathematics were recognized and catered for as follows:

lower level mathematical abilities: { Knowledge; Skills }

higher level mathematical abilities: { Selection - Application; Analysis - Synthesis }

Furthermore, 'comprehension' was retained as the 'middle level'. 'Comprehension' is characterized by the following three types of behaviour : Translation, Interpretation and Extrapolation. Since all mathematical problems in the higher abilities level require one or more of these behaviours before solution is reached, 'comprehension' was recognized as a separate, important level in mathematical abilities. It was also felt that a consideration of three levels rather than two extreme levels will yield more definitive information on the taxonomic levels of mathematical abilities.

The following three levels then formed the basis for the selection of the test items:

Level 1 : lower level - Knowledge and Skills.

Level 2 : middle level - Comprehension.

Level 3 : higher level - Selection - Application and Analysis - Synthesis.

2.2 COMPILATION OF THE MATHEMATICS TEST

One of the two major variables in this study was attainment in mathematics (cognitive abilities). A test instrument was necessary to gather data on this variable but a suitable mathematics test for the present research design was not available. The idea of constructing a test was considered but discarded in favour of compiling a suitable set of items from the several items used in the IEA study (Husén, 1967b:312-359). In this section the development of this test will be discussed.

2.2.1 Identification of Purposes of the Test

Within the framework of the present research design the mathematics test had to perform the following essential functions:

(i) to measure cognitive abilities in mathematics in at least three levels : knowledge - skills, comprehension, and application and higher abilities;

(ii) to serve as a criterion measure for both Std 9 and Std 10;

(iii) to serve as a screening device to distinguish between the high and low achievers in the sample and subsamples of the target population.

2.2.2 Specifications for the Test

Any test item is completely specified by a blueprint in terms of the content and the objectives which state the behaviours to be exhibited by the pupil (Wood, 1972:34; Merwin, 1961:44). In this study, while fulfilling the purposes as identified above, the test had to comply with specifications in respect of the content of the mathematics course and its objectives.

2.2.2.1 Mathematics Courses in the System of Differentiated Education

In order to understand the context in which the mathematics course is studied in the schools it is useful to see it in relation to the courses offered in the System of Differentiated Education. Act 39 of 1967 made provision for a differentiated system of education for Whites in South Africa as follows:

> "education shall be provided in accordance with the ability and aptitude of and interest shown by the pupil ... " (cited by Behr, 1978).

Following this provision the Department of Indian Education (DIE) introduced this system in Indian Schools in 1973 (DIE, 1972). It is a four-phase system with most of the differentiation taking place in the Fourth Phase (Senior Secondary Phase - Stds 8, 9 and 10). The differentiation in this phase takes place in two ways:

(i) choice of areas of study e.g. Humanities, Technical, Science etc.;

(ii) choice of grades on which a subject may be taken in the ordinary course: Higher Grade (HG) or Standard Grade (SG).

Further, a practical course is offered to weaker pupils, who cannot cope with the ordinary course (academic programme), from Std 6 to Std 10.

Thus, we find that there are three distinct mathematics courses offered in the senior secondary phase:

Ordinary Course : Higher Grade - preparation for further study in mathematics.

Standard Grade - suitable for those who do not wish to study higher mathematics.

Practical Course : Practical Mathematics - vocationally orientated.

These courses, within the system of differentiated education, are best illustrated diagrammatically (See Fig. 2.1 below).

Phase →	1 - Junior Primary	2 - Senior Primary	3 - Junior Secondary	4 - Senior Secondary
Year of Schooling	1 2 3	4 5 6	7 8 9	10 11 12
Class/ Standard	I II 1	2 3 4	5 6 7	8 9 10
Mathematics Courses	——→	——→	——→ - - - →	HG ——→ SG ——→ - - - →
——→ Ordinary Course			-----→ Practical Course	

FIG. 2.1: MATHEMATICS COURSES IN THE SYSTEM OF DIFFERENTIATED EDUCATION

2.2.2.2 Course Content

The course content was obviously restricted (as per target population) to experiences of pupils taking the ordinary mathematics course within the system of differentiated education. Further, every piece of content used had to be within the experience range of both levels (Std 9 and Std 10) and both grades (HG and SG).

In general, there was a vast difference between Std 9 and Std 10 in terms of content material covered. The pupils in Std 10 were close to completing the Matriculation syllabus while those in Std 9 had spent about four months on the first few sections. It was therefore clear that any plan to compile a test had to take into account the type of content covered by the Std 9 pupils in both grades while keeping in mind the objectives to be tested. It was ensured that no unfamiliar content which might reduce test validity crept into the items.

The possibility of compiling two tests in order to cater adequately for the two levels was considered but regarded as unsatisfactory. It was necessary in this study to draw comparisons between the various subsamples in terms of attainment and attitudes. Therefore one test suitable to each of the groups was the answer. In the IEA study (Husén, 1967a:104) where the target populations were defined at four levels a similar problem was overcome by designing two common tests : one common to the first three population levels and another common to the last three levels; whereas, all the other tests taken were different for the four population levels.

The test developed here was not meant to be an examination for purposes of promotion or certification. Therefore, no effort was made to 'cover' a particular range of content material within the experiences of the subjects in the sample. However, every effort had to be made to test a fair range of the cognitive abilities in mathematics. For this purpose any piece of content within the experience range of all the Std 9 pupils was acceptable.

2.2.2.3 Levels of Objectives

The objectives for mathematics learning have been discussed in section 2.1 where a classification scheme used to redraft the Higher Grade Mathematics Syllabus (DIE, 1976) has been presented. The objectives in this classification were stipulated according to the following major categories:

A. Knowledge

B. Skills

C. Comprehension

D. Selection - Application

E. Analysis - Synthesis .

Since the syllabus in terms of the above categories of objectives had been in use for the past four years, it was clear that this scheme should form the basis for selection of suitable test items.

For purposes of this research 3 levels of mathematical abilities were distinguished:

1. lower level - knowledge and skills (A and B)
2. middle level - comprehension (C)
3. higher level - application and higher abilities (D and E).

It was decided that the items selected should adequately cover these levels in order to provide a satisfactory measure of attainment in mathematics over a wide range of abilities. Several studies (e.g. Husén, 1967a; Klein, 1972; Lessinger, 1963; Lewis, 1965; Romberg and

Wilson, 1968; Moodley, 1975) have demonstrated that items may be constructed or classified to test the different levels of objectives in the cognitive domain.

2.2.3 Mathematics Test Items

Having decided on the specifications for the test in terms of the content material and cognitive objectives, the next step was to compile a suitable set of mathematics test items.

2.2.3.1 Preliminary Search for Test Items

Apart from the 20-item mathematics test developed by this researcher (Moodley, 1975) no suitable locally developed tests were available for this research. Although this 20-item test was developed on the basis of the same scheme of objectives as used here it was felt that the items were not entirely suitable. The items which had been designed to measure

> "the extent to which senior secondary pupils are achieving certain objectives including those which go beyond the mere recall of formal knowledge",

had been restricted to the Std 8 Standard Grade Syllabus (Moodley, 1975:25-6). Since Std 10 pupils on both grades were included in the present sample this test was not acceptable.

There were two options:

(i) to develop and validate a test suitable for this research,

(ii) to select and possibly modify items used in recognized researches or projects.

The first option, though attractive, was not practicable in terms of the time and effort which went beyond the physical scope of this research. The present study had in itself grown into a fairly comprehensive project to be handled by one researcher. Therefore, the second option was explored.

A cursory study of the items used in the following overseas projects/ materials was made:

(i) National Longitudinal Study of Mathematical Abilities (Wilson, 1971).

(ii) CSE: Experimental Examinations in Mathematics (Secondary Schools Examinations Council, 1964).

(iii) International Study of Achievement in Mathematics (IEA) (Husén, 1967b).

(iv) The Ontario Institute for Studies in Education - Objectives for Mathematics Learning (Avital and Shettleworth, 1968).

This researcher was attracted by the extensive list of 174 items used in the IEA study. The comprehensive coverage of the objectives and the availability of item analysis data for each of these items were viewed as being most useful in the selection of items for the present study.

It was thus decided that a careful study should be made of the IEA items and that a suitable set of items should be selected from among them.

2.2.3.2 IEA Study and Test Development

The international Project for the Evaluation of Educational Achievement (IEA) which embarked upon a study of achievement in mathematics in 12 countries has been reported in two volumes : International Study of Achievement in Mathematics (Husén, 1967a,b). It was a comparative study which aimed at investigating

> "the efforts of the differences among school systems of the world and, in particular, differences in the achievement, interests and attitudes of the students" (Husén, 1967a:28).

Four target populations were identified and the information gathered comprised more than two hundred variables and a great deal of qualitative data. Among these variables were those relating to cognitive abilities in mathematics which were measured by tests specifically developed for the study. It was observed that

> "the overall objective in the test construction was to arrive at internationally valid measuring instruments covering a wide range of content and objectives within which each country could find its own emphases" (Husén, 1967a:47).

In order to assess the quality of test items it was necessary to consider the bases on which the IEA tests were developed.

The development of mathematics test items for the IEA study involved the following sequence of procedures (Husén, 1967a:90):-

1. Identification of a purpose or purposes that the test is to serve.

2. Development of a general plan and detailed specifications for the form and content of the test.

3. Preparation of test exercises conforming to the specifications.

4. Review and editing of the test exercises by persons competent in the substantive field and individuals sensitive to common editorial shortcomings of tests.

5. Preliminary trial of the exercises to determine whether each is of an appropriate difficulty level and whether each differentiates between the more able and the less able students.

6. Final selection of those exercises that are to make up the test, in view of their conformity to the plan and specifications and their statistical characteristics as shown in the preliminary try-out.

7. Organization of the items into appropriate units and reproduction of them in appropriate format for use.

In all, 10 separately prepared test booklets (A, B, C, 3, 4, 5, 6, 7, 8 and 9) graded according to difficulty were produced with a total of 174 items. Test 3 was made up of items from A, B and C.

The target populations were tested at 4 levels and the number of tests given to any one group was three or four. In order that "there be some thread of continuity to tie together the results from the different groups" the first three groups were all to take Test 3 while the last three groups were all to take Test 5.

Total number of items administered to each of the 4 levels was as follows: 70, 50, 58, and 69. The following corresponding reliability coefficients (average for the 12 countries) were reported 0,92, 0,91, 0,88 and 0,84. All the test items except those from

Test 4 have been reported in Vol. II of the IEA study. In addition, each item carries a facility index and a discrimination index based on the average for the 12 countries (Husén, 1967b:312-359).

2.2.3.3 Selection of Test Items for the Present Study

Although 174 items were used in the IEA study, Test 4 comprising 18 items was not reproduced in the report. Therefore, the selection had to be made from the available 156 items.

The preliminary selection of items took into account the following considerations:

(i) objectives tested by items,

(ii) coverage of content tested by the items,

(iii) item analysis data which was available,

(iv) the possibility of metrication of certain items which were otherwise very good,

(v) the construction of distractors for some of the good items for which they were not available.

In making these considerations and certain decisions that went with them, the researcher relied heavily on his own experiences

(i) with research into objectives and the construction and validation of mathematics test items (Moodley, 1975),

(ii) as sub-examiner for matriculation mathematics,

(iii) as convener of a panel of examiners of matriculation mathematics,

(iv) as a teacher of mathematics and methodology of mathematics.

A careful study was made of the objectives in mathematics learning and of the content material as presented in the syllabus and the text books popularly used (Nero and Malan, 1974; Dreyer, 1979; Gonin et al, 1976). Then, every item in the IEA study was worked out in order to get as precise an idea as possible of the nature of each item in terms of the content and objectives it was testing. On this basis the first round of preliminary selection of 'eligible' items was made. This resulted in 97 items as follows:-

IEA Test	A	B	C	5	6	7	8	9
No. of items 'eligible'	21	18	17	13	15	4	6	3

In order to produce a test which was a suitable screening device it was essential that the items should be discriminative. In this regard the available item analysis data provided a reasonably good guide. Although it was recognized that item analysis data are related to the particular sample for which they have been calculated (Conrad, 1951:253), it was decided that the data be used as a guide in selecting the items. This decision was based on the fact that

(i) the discrimination indices in the IEA study reflected the averages over 12 countries and thousands of pupils,

(ii) it was better to have some knowledge of the discriminating power of the items than none at all.

The next round of preliminary selection was thus based mainly on item discrimination. It was decided that items with indices of 0,25 and over should be accepted (Morrison, 1974:50). On this basis 50 items were accepted as eligible. This had to be reduced in order to arrive at a single test of suitable length.

This reduction was borne in mind while several other considerations were made in the next round of selection:

(i) It was necessary to have a balance in terms of the difficulty of questions.

(ii) An attempt was made to get a reasonable coverage of geometric and algebraic questions.

(iii) There had to be a balance in terms of the 3 levels of objectives defined earlier.

(iv) There were some items which had no distractors but required a single answer. These had to be studied carefully and, where possible, distractors were constructed (e.g. see items 18, 19 Appendix 3).

(v) There were other items which needed minor changes in respect of metrication (e.g. see items 6,23 Appendix 3).

(vi) In striving for a further degree of perfection and greater reliability the cut-off point for the discrimination index was raised to 0,30.

All these aspects were carefully monitored during this round of selection. In all, 32 items passed these restrictions and requirements. They were distributed over the 3 levels of objectives as follows:

1. Knowledge and Skills (10 items)
2. Comprehension (12 items)

3. Application and Higher Abilities (10 items).

These 32 items made up the 'trial test'.

2.2.3.4 Trial of Test

No attempt was made to produce item analysis data for the 'trial test'. It was argued that the strength of the test must be judged by the painstaking process of selection (from an internationally recognized set of items) which was employed. However, a small group of 2 weak, 2 average and 2 bright pupils were given the test in order to gauge

(i) time and length,
(ii) difficulties in technical presentation,
(iii) content coverage.

It was not the purpose of this trial to validate this test but rather to improve its validity and reliability.

Each pupil took more than two hours to complete the test. The final test had to be shorter as the questionnaire and scale also had to be administered. Further, the schools would be reluctant to allow long hours of testing etc. It was, therefore, necessary for the test to be further reduced.

The pupils were required to indicate items which concerned material not covered in class. Six items were thought to be beyond the content experience of the Std 9 pupils, while the Std 10 pupils had no such difficulty.

In terms of technical production the notations in items 1 and 15 and the label in the diagram of item 24 were queried (see details in next section).

2.2.3.5 Judging Items in 'Trial Test'

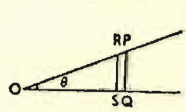
Seven teachers of mathematics from 4 different schools were required to judge the items in terms of the objectives they were testing. Six of these teachers were currently engaged in teaching Std 9 and Std 10 mathematics. The one was a senior lecturer at a College of Education, who had been involved in the redrafting of the syllabus in terms of objectives.

They were required to "study each item and indicate which of the objectives it best tests" (Appendix 8). A short explanation of each of the 5 categories of objectives was supplied. Since this was in accordance with the syllabus used by the teachers since 1977 (DIE, 1976), they experienced few problems. They were also asked to indicate any item involving content which might not have been covered by their pupils.

Although the teachers were required to distinguish among 5 categories of objectives, such a distinction was not necessary for this test as the 3 levels of objectives sufficed. However, the distinction facilitated the placing of items within 3 levels. Since Knowledge and Skills constituted one level and Application and Higher Abilities another, any disagreement among the teachers in respect of the categories within each of these levels was ignored. However, differences between two levels were not acceptable. Items were then accepted on the basis of majority agreement in respect of the levels of objectives: e.g. Item 18

In the figure below, $PQ \perp OQ$, and $RS \perp OQ$. If the measure of OQ and of OR equal 1 and θ is the measure of $\angle POQ$, then the measure of segment PQ is equal to

A. $\sin\theta$
B. $\cos\theta$
C. $\tan\theta$
D. $2\sin\theta$
E. $1-\cos\theta$



was rejected because there was uncertainty as to whether it was testing Knowledge or Application.

The teachers also pointed out that certain items on inequalities, rationalization of roots, similarity, geometric progression and double angles in trigonometry might not have been covered by most Std 9 classes. This corroborated with information from pupils who attempted the trial test.

In terms of technical presentation the teachers queried the following four items in the trial test: 1, 11, 15 and 24 (the pupils had queried 1, 15 and 24). These were attended to as follows:—

Item 1:

In the division on the right, the correct answer is   0,004/24,56

A. 0,614
B. 6,14
C. 61,4
D. 614
E. 6140

The division sign was inserted to avoid confusion with the square root sign (initially the points had been replaced by commas). (Item 1 - Appendix 3)

Item 11:

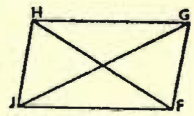
The distance between two towns, *A* and *B*, is 150 kilometers. This distance is represented on a certain map by a length of 30 centimeters. The scale of this map is

A. 1/500,000   D. 1/5,000
B. 30/150   E. 1/200,000
C. 1/20,000

The comma in each of the distractors was removed (Item 10 - Appendix 3).

Item 15:

In the figure on the right, *FGHJ* is a parallelogram. Which of the following statements is a condition which implies that *FGHJ* is a rectangle?



A. $JF = GH$
B. $\angle HJG = \angle JGF$
C. $\angle HJF = \angle JHG$
D. $\angle HJF$ and $\angle JGH$ are supplementary
E. *HF* and *JG* are perpendicular bisectors of each other

The use of letter 'J' led to some confusion and was changed to a clearer form. $\angle$ JGH in Distractor D was changed to $\angle$ JHG to improve its function (Item 15 - Appendix 3).

In Item 24 "PO$\perp$LM" was corrected in accordance with the label in the diagram to read "PQ$\perp$LM" (see Item 20 - Appendix 3).

Taking into account all the above considerations in terms of the trial and judging of items, 24 items were found to be acceptable. However, the balance desired in terms of objectives was not achieved. In order to accomplish this it was necessary to exclude one item testing 'lower level' and to add one item testing 'higher level'. A previously discarded item (because of lack of distractors) with D=0,36 and F=37 was found for this purpose. Distractors were constructed and it was agreed that it best tested applicational ability (see Item 18, Appendix 3).

Thus, in the final selection there were 8 items in each of 3 hierarchical levels of objectives. It was interesting to note that a general pattern of distribution of these items as per IEA tests emerged (See Table 2.1 below). All the items in the Knowledge-Skills level came from tests A and B, while most of them in the Application and Higher Abilities level came from tests 5, 6 and 7. The Comprehension items were spread across five tests. No items were selected from tests 8 and 9 which were based mainly on determinants, matrices, calculus etc. as these aspects were not part of the course content.

TABLE 2.1

DISTRIBUTION OF SELECTED IEA TEST ITEMS OVER THREE LEVELS OF OBJECTIVES

IEA TESTS → / LEVELS ↓	A	B	C	5	6	7	8	9	TOT.
1. Knowledge-Skills	4	4							8
2. Comprehension	2	1	2	2	1				8
3. Application-Higher Abilities	1	1		1	4	1			8

The 24 items were then organized in a meaningful way according to the following considerations:-

(i) grading of items in terms of the complexity of the levels of objectives e.g. first 8 items testing Knowledge-Skills etc.,

(ii) matching of items in terms of difficulty and content within each level,

(iii) balancing of odd and even-numbered items as far as possible to attain reasonably equivalent forms for split-half reliability (Fraser and Gillam, 1972:131-2).

The final form of the test together with instructions on the front page of the Test Booklet is presented in Appendix 3. A new answer sheet for the 24-item test was constructed in accordance with the multiple-choice format of the test.

2.2.3.6 General Comments on the Test

The quality of the test may be gauged from a study of the items themselves and from the several considerations that have been made in selecting the items for this study. Each successive round of selection involved greater and greater refinement mainly in terms of course content, spread of objectives and item analysis data.

Since the items were not developed per se by this researcher, statistical evidence in respect of its validity and reliability etc. could only be produced after final administration of the test. However, it must be pointed out that the average reliability coefficients of tests from which these items have been selected ranged from 0,73 to 0,96 for the various groups (Husén, 1967a:107).

Davis (1951:266) considered that

> "statistical data are at best merely a guide in putting a test together and cannot take the place of scholarship, ingenuity and painstaking effort on the part of the item writer".

In this connection we note that the items used in the present study were prepared by internationally recognized experts such as Hartung, Fehr, Pidgeon, Servais et al. (Husén, 1967a:92). It was also expected that the following considerations would contribute positively to the reliability of the test:

(i) use of multiple choice format to ensure objectivity in scoring (Fraser and Gillam, 1972:127);

(ii) use of 5 alternatives in the multiple-choice format to minimise the probability of guessing (Lindvall, 1967:42);

(iii) timing of test for 80 mins. to enable pupils to consider all items and to deter them from blind guessing which reduces reliability (Ebel, 1965:342; Davis, 1959:271). (Pupils were also advised on the amount of time to be spent on the items in each level.);

(iv) the high cut-off point of 0,30 on the Discrimination Index (Ebel, 1965:364; Furst, 1958:324). See Table 2.2 below.

TABLE 2.2

ITEM ANALYSIS DATA ON SELECTED IEA TEST ITEMS

(HUSéN, 1967b : 312-359)

Item No.[1]	Test/Item (IEA)	Facility Index (F) %	Discrimination Index (D)
1	B/3	58	0,43
2	B/13	59	0,38
3	A/6	46	0,46
4	A/6	27	0,54
5	B/9	38	0,51
6	A/8	42	0,37
7	B/22	60	0,36
8	A/20	64	0,40
9	A/10	20	0,34
10	A/22	26	0,33
11	C/22	84	0,33
12	C/16	51	0,32
13	B/20	46	0,43
14	5/6	69	0,43
15	6/14	42	0,37
16	5/8	74	0,41
17	6/7	42	0,32
18	6/5	37	0,36
19	A/19	69	0,48
20	B/18	51	0,32
21	6/3	48	0,37
22	7/6	47	0,36
23	6/16	43	0,37
24	5/12	40	0,37

(1) Refers to items in the test compiled for this study - see Appendix 3.

The validity of the test may be gauged from:

(i) the measures adopted to judge items in terms of the objectives they were testing and the content material they covered,

(ii) the high discriminating power of each item (greater then 0,30 - See Table 2.2),

(iii) the procedures involved in the initial devëlopment and trial of the test items by the IEA team (Husén, 1967a),

(iv) the selection of items involving several important considerations including relevance of objectives tested and the content covered by the items which would ensure a reasonable measure of objective and curricular validity (Fraser and Gillam, 1972: 134-5).

As mentioned earlier in this chapter, the item analysis data were used as guidelines (among several other considerations) in the selection of the items. It was also recognized that such data were relative to the population for which they were derived (Conrad, 1959: 253). However, it must be emphasized that this population was spread across 12 countries (Husén, 1967). On this basis it would be worth noting that for the 24 items which were carefully selected here the mean Discrimination Index was 0,39 and the mean Facility Index was 49,29 (See Table 2.2). Since tests with items of 50% difficulty tend to discriminate maximally (Davis, 1959:309-310), it was expected that this would contribute positively to the reliability of the present test (Ebel, 1965:364) and hence probably to its validity (Ebel, 1965: 357).

In respect of the format and layout of the test (See Appendix 3) the items were grouped according to the levels of objectives they were testing. This was essential and meaningful (Furst, 1958:279) within the context of the present research which was designed to detect any patterns in attainment of cognitive outcomes. The positions of correct responses were randomly varied to avoid 'response sets' which might have an adverse effect on reliability. Finally, every precaution was taken to assemble the items into a legible, attractive and economical format in order to ensure that the validity and reliability of the test results were not negatively affected by this aspect (Spaulding, 1951:454; Furst, 1958:279).

In general, it was considered that a reasonably satisfactory screening device and measure of cognitive outcomes in mathematics had been developed. The claims made here in respect of the quality of the instrument will be further substantiated through statistical analyses of the test results which will be presented in chapter five.

CHAPTER THREE

3. AFFECTIVE LEARNING OUTCOMES AND THE MEASUREMENT OF ATTITUDES TOWARD MATHEMATICS

One major aspect of this study was to measure effectively the cognitive learning outcomes in mathematics. Another major part of this research concerned itself with the measurement of attitudes toward mathematics. In this chapter the affective learning outcomes with special reference to attitudes in general and attitudes towards mathematics in particular are discussed. The methods of measuring attitudes and the development of the scale for measuring attitudes toward mathematics are presented.

3.1 AFFECTIVE LEARNING OUTCOMES WITH SPECIAL REFERENCE TO ATTITUDES

3.1.1 The Affective Domain

As mentioned earlier the Affective Domain is one of a tripartite organization of educational outcomes: cognitive, affective and psychomotor domains (Bloom et al, 1956; Krathwohl et al, 1964). These three interacting areas of human behaviour roughly correspond to thinking, feeling and acting respectively. These authors clearly recognize that no objective in one class is entirely devoid of some components of the other two classes. They maintain that the value of the present classification lies in:

(i) greater precision with which objectives are likely to be stated,

(ii) increased communicability of the objectives,

(iii) the extent to which evaluation evidence will become available to appraise students' progress towards the objectives (Krathwohl et al, 1964:8).

The Taxonomy of Educational Objectives in the Affective Domain (Krathwohl et al, 1964) was an attempt at ordering the numerous affective objectives in some logical way. Such objectives which were seen to emphasize a feeling tone, an emotion, or a degree of acceptance or rejection are commonly expressed in literature as "interests, attitudes, appreciations, values and emotional sets or biases" (Krathwohl et al, 1964: 7). Other writers have referred to them collectively as non-intellective variables (Finger and Schlessor, 1965; Aiken, 1970c) or affective factors (Khan, 1969; Aiken, 1970a).

The need for such a classification arose mainly out of the need for a

> "systematic effort to collect evidence of growth in affective objectives which is in any way parallel to the very great and systematic efforts to evaluate cognitive achievement" (Krathwohl et al, 1964:16).

The most difficult task that faced the authors was the search for a continuum which would provide a means of ordering and relating the different kinds of affective behaviour. Detailed analyses of materials relating to educational outcomes dealing with interests, attitudes, values, appreciation etc. led to the formulation of a hierarchical continuum of internalization from the lowest to the highest (Krathwohl et al, 1964):-

1.0 Receiving (attending)
   1.1 Awareness
   1.2 Willingness to receive
   1.3 Controlled or selected attention
2.0 Responding
   2.1 Acquiescence in responding
   2.2 Willingness to respond
   2.3 Satisfaction in response
3.0 Valuing
   3.1 Acceptance of a value
   3.2 Preference for a value
   3.3 Commitment (conviction)
4.0 Organization
   4.1 Conceptualization of a value
   4.2 Organization of a value system
5.0 Characterization by a value or a value complex
   5.1 Generalized set
   5.2 Characterization.

These five levels describe the degree to which an individual receives, accepts and adopts an educational experience. They range from the simple receiving of an educational experience to the acceptance of a way of life (Mathews, 1974:177).

Each category of the Taxonomy is described and supplemented by illustrative educational objectives and test items. A serious limitation in compiling the 'test items' has been the lack of evaluation material in the affective domain. This has been largely due to the fact that the typical school examines for mainly cognitive outcomes (Mathews, 1974:174; Krathwohl et al, 1964:96; Bloom et al, 1974:226). In fact, it was the

> "absence of instruments to measure affective outcomes of instruction in the various physical and biological sciences,

in mathematics, and in the social studies"
which forced the authors of the taxonomy to devise their own illustrations for these areas (Krathwohl et al, 1964:96).

The main value of the taxonomy lies in the fact that it provides a useful framework for defining (more clearly) and communicating objectives in the affective domain. The taxonomy is a result of logical and psychological analysis based on experiences of examiners, educators, curriculum workers and psychologists rather than on empirical evidence. However, its usefulness in educational research has been demonstrated by studies which have used it as a frame of reference for stating affective objectives and developing attitude scales: Lewy (1966), cited by Bloom et al (1971:232-235), used the taxonomy for selecting affective objectives for reading, music and mathematics; Adams and Von Brock (1967) devised a mathematics attitude scale based on the taxonomy; Connelly (1973) used the taxonomy to develop an instrument to evaluate attitudes of prospective elementary teachers of mathematics.

Above all, the authors maintain that just as the taxonomy in the cognitive domain provided the basis for exposing the overwhelming emphasis on 'knowledge' objectives at the expense of the other higher abilities, the taxonomy in the affective domain should help to "highlight the current emphasis on cognitive objectives at the *expense* of the affective" (Krathwohl et al, 1964:57).

This resumé of the taxonomy has been presented because of its increasing importance as a method of ordering affective outcomes in education and because it provides a useful framework against which a measurement of attitudes may be seen.

3.1.2 Relationship between Affective and Cognitive Domains

The authors of the taxonomic classifications (Bloom et al, 1956; Krathwohl et al, 1964) have readily conceded that the division of educational outcomes into the three domains is arbitrary, but they regard these divisions as 'natural'. Natural, in the sense that teachers and educators have tended to divide their objectives into these categories, either explicitly or implicitly (Krathwohl et al, 1964:47).

The present study concerns itself with aspects of the cognitive and affective domains as evidenced by measures of attainment in mathematics and attitude towards mathematics. It is also accepted that the one domain cannot be seen exclusively separate from the other (Mathews, 1974:172). In this regard the authors of the Taxonomy have cited Sheerer (1954) in asserting that

> "behaviour may be conceptualized as being embedded in a cognitive-emotional-motivational-matrix in which no true separation is possible" (Krathwohl et al, 1964:45).

They further demonstrate the roughly parallel steps in the two domains by explaining the 'level-to-level' correspondence between the two taxonomic structures. A similar illustration of the relationship between the cognitive and affective domains has been given by (Pierce and Lorber, 1977:72). It is argued that the attainment of cognitive objectives leads to attainment of affective objectives and vice versa, while there is also the possibliity of seeking both types of behaviours simultaneously (Krathwohl et al, 1964:49-60).

Other writers (Aiken, 1970b:558; Neale, 1969) have also argued that there is a reciprocal relationship between attitudes and performance whereby attitudes affect achievement and achievement in turn affects attitudes. Gagne (1977:235-6) has suggested that attitudes have both cognitive and affective components.

In the same context Messick (1979) maintains that the so-called cognitive measures of intellective abilities, information processing skills and subject matter achievement frequently entail motivational, attentional and affective aspects. Thus the distinction is not categorical but one of degree in the relative balance between intellective and non-intellective factors.

3.1.3 Attitudes and the Affective Domain

The authors of the taxonomy in the affective domain carefully analysed commonly used terms such as interest, attitude, appreciation and value in terms of the taxonomic framework and found a wide range of meanings. A comparison of the range of meanings for any one term with the structure of the taxonomy revealed that each term took on meanings across a section of the classification (Krathwohl et al, 1964:36-37). Of special interest to the present study is the interpretation given to the term "attitude" which ranges from a "willingness to respond" to "conceptualization of a value" (See continuum presented in 3.1.1).

3.1.4 Definition and Nature of Attitude

In order to clarify the way in which the concept attitude is conceived and applied in this study a review of the use of the term is presented here.

In a historical survey of the concept of attitude, Allport (1935) traced it to one of its earliest uses by Herbert Spencer (1862) and suggested that the following three convergent trends emerged:

(i) After the breakdown of intellectualistic psychology the phenomena of determination came slowly but certainly to be admitted to unquestioned standing in experimental psychology. Attitudes came into fashion.

(ii) Under the influence of psychoanalytic theory the dynamic and unconscious character of attitudes became more fully realized.

(iii) In sociological writing there was a gradual turning of interest to attitudes considered as the concrete representations of culture.

Further, Allport (1935) (in Fishbein, 1967:7-8) reviewed a representative selection of definitions and characterizations of attitude advanced by the many early workers (e.g. Baldwin, 1901; Thomas and Znaniecki, 1918; Kohler, 1929; Bogardus, 1931; Droba, 1933) and concluded that each of the definitions regarded "the essential feature of attitude as a preparation or readiness for response". In an effort to include the recognized types of attitudes he put forth the following definition:

> "An attitude is a mental and neural state of readiness, organized through experience, exerting a directive or dynamic influence upon the individual's response to all objects and situations with which it is related." (Allport (1935) in Fishbein (1967:8)).

In general, the definitions and uses of the term 'attitude' tend to vary from operational to metatheoretical. Shaw and Wright (1967:2) consider (after Cardano, 1955) that despite variation in the multitude of definitions one characteristic is agreed upon:

"Attitude entails an existing predisposition to respond to social objects which in interaction with situational and other dispositional variables, guides and directs the overt behaviour of the individual."

It is claimed that the variance in the definition and use of the concept attitude may be attributed to:

(i) specificity versus generality in the determination of behaviour,

(ii) predispositions to respond to social versus non-social aspects,

(iii) different theoretical conceptions of the composition of an attitude (Shaw and Wright, 1967:2-3).

One of the crucial aspects of the present research was to arrive at a measure of attitudes (specifically) toward mathematics. This required an operational definition of attitude which would relate to the measurement of the construct. In this regard Fishbein and Ajzen (1975:6) have suggested that an explicit conceptual definition of attitude would lead to an adequate basis for the development of measurement procedures. Thus, following the approach of Shaw and Wright (1967:3) the following restrictions were accepted by this researcher:

(i) an attitude is considered to have a *specific* referent (or class of referents),

(ii) an attitude involves predispositions to only *social* objects,

(iii) an attitude is seen in terms of an affective component based upon evaluative concepts which are closely related to other cognitions and overt behaviour.

In terms of the above restrictions these writers advance the following definition:

"A relatively enduring system of evaluative, affective reactions based upon and reflecting the evaluative concepts or beliefs which have been learned about the social object or class of social objects" (Shaw and Wright, 1967:3).

This view of attitude is similar to those expressed by Osgood et al (1957), Rhine (1958) and Anderson and Fishbein (1965). The definition has the advantage of relating the theoretical construct of attitude as closely as possible to the operation in the form of an attitude scale. Such a scale would reflect the varying degrees of positivity and negativity in attitudes. Messick (1979:284-5) concurs with this view and adds that "attitudes are thus always directed favourably or unfavourably". In fact, the polarity in the direction of attitudes has been recognized as their most distinctive feature in the definitions of Bogardus (1931) and Thurstone (1932) as cited by Allport (1935). It is this positivity and negativity that renders attitudes towards some social object measurable. Pertinent to the present study is the fact that investigators who constructed and/or used scales to measure attitudes towards mathematics have assumed the favourable - unfavourable polarity (Aiken and Dreger, 1961; Fedon, 1958; Dutton, 1951; Dutton and Blum, 1968; Husén, 1967a). In the present study the positive-negative polarity in attitudes towards mathematics is also assumed.

In order to apply the definition of attitude as developed here, it was necessary to recognize mathematics as the specific attitude referent. Further, since school is clearly a social object and mathematics is an activity which is an integral part of the school curriculum, it must also be regarded as a social object. Moreover mathematics has been considered as "one of the most original creations of the human spirit" (Whitehead, 1925 cited by Bell, 1937:11).

Similarly, in discussing the significance of mathematics in society, Human (1975:11) considers that it

> "constitutes the foundation of man's handling of quantitative aspects of situations in which he finds himself".

Other recent writers (Williams, 1978; Griffiths and Howson, 1974) have also paid attention to the sociological considerations in mathematics education. The view that mathematics is a social object is consistent with the distinction made between social and non-social objects by Shaw and Wright (1967:2) who consider social objects to "include interactions with persons and person produced objects, events and situations".

Thus, in this study the attitudinal referent was mathematics which is both a specific referent and a social object. The above conception of attitude was consistent with and relevant to this research design which included a measurement of attitudes towards mathematics.

3.1.5 Relevance of Attitudes for Education

If education is intended to produce behaviour changes in children (Furst, 1958:2) and behaviour is seen in terms of the three interacting domains including the'affective domain' (Bloom et al, 1956; Krathwohl et al, 1964), then attitudes which are embedded in the affective domain must clearly be relevant to education.

Lewis (1974:155) maintains that

> "just as distinctive kinds of abilities exist, and need to be taken into account in the practice of education, so also do several kinds of attitudes .... Thus we may need to take account of pupils' attitudes towards different school subjects."

In a study of affective correlates (including attitudes and motivation) of academic achievement, Khan (1969:270) points out the need for comprehensive and systematic research on affective variables which may have

> "potential significance in educational decision making, guidance and placement of students, and identification of high and low achievers".

Messick (1979), in discussing the potential educational import of non-cognitive personal characteristics such as attitudes, interests, motives, curiosity and values, points out the close relationship between these concepts and observes that "educationally relevant attitudes include orientations toward learning, school, subject matter, and self".

Gagne (1977:253) considers that

> "the learning and modification of attitudes, referred to by some as affective domain of objectives, is surely of great importance to educational programs of almost every kind".

It is thus desirable that students acquire positive attitudes to a particular subject studied and to learning activities in general.

3.2 ATTITUDES TOWARDS MATHEMATICS

3.2.1 Introduction

In the previous sections of this chapter the concept of attitude has been developed within the theoretical framework of this study. The definition adopted here (from among a multitude of definitions and uses of the term) has been carefully restricted: attitude is seen to have a specific referent which is a social object and it is based on evaluative concepts learned about the object, which are related to the cognitions and overt behaviour. In the present research the specific attitudinal referent is mathematics which is a social object born out of purposive human endeavour. In operational terms, attitude towards mathematics refers to the positivity or negativity in reactions to mathematics.

This approach to the use of the concept of attitude is closely related to that of Aiken (1970b) who has been conducting research into attitudes towards mathematics over the past two decades. He refers to attitude, in general, as

> "a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept or another person" (Aiken, 1970b:551).

In this section attention will be given to research studies relating to aspects of attitudes towards mathematics. In particular, the relationship between attitudes towards mathematics and achievement in mathematics, attitude as a moderator variable, and sex differences in attitudes towards mathematics will be discussed.

3.2.2 Studies in Attitudes towards Mathematics

Among the multitude of research studies and articles relating to non-intellective factors in mathematics achievement, 'attitudes towards mathematics' features prominently.

A survey of recent literature reveals that several aspects of attitudes towards mathematics have been studied fairly extensively in the USA (e.g. Aiken and Dreger 1961, Cristantiello, 1962; Anttonen, 1969; Aiken, 1970b, 1972a, 1976; Neale 1969; Evans, 1972; Spickerman, 1970; Moore, 1972; Sandman, 1974) and to a lesser degree in other countries (Aiken, 1979a (Iran); Kulkarni and Naidu, 1970 (India); Nevin, 1973 (Ireland); Muckerjee, 1978 (Nigeria); Marjoribanks, 1976 (Australia)).

In general, many of these studies have been prompted by the debate on whether modern curricula have fostered more positive attitudes. In order to consider questions relating to the possible causes of negative attitudes and how to remedy them, information relating to the following three factors was needed:

* biological inheritance and home background of the pupil
* attitudes and training of teachers
* content, organization, grades and adaptability of the curriculum (Aiken, 1970b:551).

In accordance with these factors, the studies in attitudes towards mathematics seem to fall mainly into the following broad categories:

1. Methods of measuring attitudes towards mathematics.
2. Relationship of attitudes to achievement in mathematics.
3. Relationship between attitudes towards mathematics and personality and social factors including sex differences.
4. Stability of attitudes towards mathematics.
5. Teacher characteristics and attitudes.
6. Instructional method and curriculum.

It is apparent that any study on attitudes toward mathematics is bound to be related to one or more of the above aspects. In the present study the researcher focussed attention on the relationship between attitudes towards mathematics and achievement in mathematics. In doing so it became necessary to consider the methods used in measuring attitudes towards mathematics in particular and the other aspects in general - the latter, merely to provide a more complete background to the research in the field of attitudes towards mathematics. In the sections that follow research relating to the above will be reviewed. The methods of measuring attitudes will be dealt with in the next major section together with scales used in this study.

3.2.3 Relationship between Attitudes toward Mathematics and Achievement in Mathematics

On the question of relationships in general between school-related attitudes and academic achievement, several studies have documented positive relationships (e.g. Brodie, 1964; Holtzman and Brown, 1968; Finger and Schlessor, 1968 ; Khan and Roberts, 1969 ; Williams, 1970; Marjoribanks, 1976; Husén, Fagerlind and Liljefors, 1974). In this section studies concerning relationships in particular between attitudes towards mathematics and achievement in mathematics will be considered.

The measurement of attitudes towards mathematics would not receive the attention of researchers if it were not thought that attitudes affect performance in some way (Aiken, 1970b). As mentioned earlier, the relationship between attitude and performance seems to be a reciprocal one where attitudes affect achievement and vice versa (Neale, 1969). However, there is little research basis for believing that attitudes towards mathematics and achievement in mathematics are causally related (Knaupp, 1973). This relationship appears to be of special importance to mathematics as shown by a study (Brown and Abel, 1965) which found that the correlation between pupil attitude towards a subject and achievement in that subject was higher for arithmetic than for spelling, reading or language.

In reviewing studies on attitudes towards mathematics at the elementary school level, Aiken (1970b:559) observed that the correlations between attitude and achievement, though statistically significant in certain instances, were typically not very large. Several other studies also reported similar relationships at the primary school level (Lindgren et al, 1964; Anttonen, 1969; Evans, 1972; Moore, 1972). A major problem in researching this area at this level relates to the use of attitude scales which present reading and interpretation problems (Knaupp, 1973; Aiken, 1970a). Research workers have recognized this problem and have attempted the design of attitudinal devices specially suited for young children (Scharf, 1971; Dunlap, 1976). These problems tend to be less serious at the high school level (Aiken, 1970b).

At the high school and college level attitude scores tend to contribute something over and above ability test scores to the prediction of achievement in mathematics (Aiken and Dreger, 1961). Several studies on the relationship between attitudes and attainment at this level have been reported. Anttonen (1969), in his longitudinal study, reported moderate correlations of attitude scores with mathematics grade-point averages and standardized test scores for eleventh and twelfth grades. He also observed that achievement was greater for students whose attitudes had remained favourable over a period of time. In comparing low and high achievers in mathematics, Degnan (1967) found that the high achievers had a more positive attitude towards mathematics and also gave mathematics a significantly higher preference ranking than the low achievers. Substantial relationships between attitudes and achievement in mathematics have been reported by Aiken and Dreger (1961). Other studies have also produced evidence in respect of this relationship at the high school and college level (Stephens, 1960; Spickerman, 1970; Burbank, 1970; Aiken, 1971; Callahan, 1971; Whipkey, 1970; Wilson, 1973; Husén, 1967a; Johnson, 1977).

A few general studies have reported no significant relationships between attitude scores and achievement (Jackson and Lahaderne, 1967; Goldfried and D'Zurilla, 1973).

In general, these studies highlight the importance of attitudes toward mathematics and report low to moderate correlations between attitude and achievement. However, these relationships make no pronouncements on the question of causality.

3.2.4 Attitude as a Moderator Variable

Although attitude scores have positive weights in regression equations for predicting mathematical achievement, attitude measures may be used more efficiently as moderator variables in predicting attainment in mathematics from ability scores (Aiken, 1970c). In an earlier study by Aiken and Dreger (1961), a multiple-correlation approach was used to predict achievement. However, another approach to prediction is to view attitude as a moderator variable by correlating ability and achievement separately at each level of attitude (Aiken, 1970c). In this way it is possible to ascertain to what extent the correlation varies with the level of attitude.

Chirstantiello's (1962) study is a good example of the use of the moderator variable. He found that among the high, middle and low groups on attitude towards mathematics the correlation between ability scores and mathematics achievement was higher for the *middle* group than for the high and low groups. Aiken (1970b) explains that mathematical ability may be a less important determiner of achievement of students who have more extreme attitudes towards mathematics than those who have more moderate attitudes.

In another study, Marjoribanks (1976:659) used complex multiple-regression models to generate regression surfaces and by including measures of cognitive ability observed that:

(i) at each attitude level, increases in cognitive ability are related to increases in academic achievement,

(ii) at each level of ability, increases in attitude scores, in general, are related to increases in achievement.

Other studies have also considered the relative contributions of ability, achievement and attitude in predicting achievement in mathematics (Neale, 1969; Hunkler, 1977; Cappadona and Kerzner-Lipsky, 1979). In the present study, attempts were made to discover how the relationship between ability scores and achievement in mathematics varies with different levels of attitudes towards mathematics.

3.2.5 Sex Differences in Attitudes towards Mathematics and Achievement in Mathematics

As mentioned earlier, this study concerned itself (in the main) with the question of the relationship between attitudes towards mathematics and mathematical achievement. However, it was recognized that sex can be an important moderator variable in the prediction of achievement from measures of attitudes (Aiken, 1970b). Thus, in investigating this relationship, provision was made in this study for an analysis by sex in addition to class level and grade.

Educational research has shown continued interest in sex and personality variables. Several recent studies and articles have considered sex differences in mathematical achievement and attitudes towards mathematics either separately or in combination (e.g. Nevin, 1973; Keeves, 1973; Fennema, 1974; Hilton and Berglund, 1974; Ernest, 1976; Fennema and Sherman 1978; Burton, 1978; Luchins, 1979; Clements, 1979; Roach, 1979; Stamp, 1979; Atkinson, 1979).

Sex differences in both attitudes and achievement in mathematics tend to appear more frequently at the high school and college level in favour of the males (Keeves, 1973; Hilton and Berglund, 1974; Nevin, 1973). While the differences at the elementary school level are not apparent, they tend to become more marked as children progress through school and are more predominant at the secondary school and college level (Ernest, 1976; Fennema and Sherman, 1977).

Behr (1973) found that the relationship between attitude and achievement was somewhat higher for girls than for boys, indicating that mathematics scores are more predictable from attitude scores for girls than for boys. Although boys have been traditionally viewed as better performers in mathematics, some studies (Meyer and Bendig (1961) cited by Aiken, 1970b; Clements, 1979; Roach, 1979 (Jamaicans)) have reported a superiority among girls.

In general, when sex differences are perceived the studies tend to advance one or more of the following reasons to explain them: biological, socio-cultural pressures, and sex-role identification (Burton, 1978). Most studies, however, emphasize socio-cultural influences rather than intrinsic biological differences (Aiken, 1976; Fennema and Sherman, 1977, 1978; Poffenberger and Norton, 1959; Keeves, 1973; Burton, 1978).

3.3 MEASUREMENT OF ATTITUDES TOWARD MATHEMATICS

Mathews (1974:174) observes that

> "the almost total absence of assessment of attitudes in the outcome of education is all the more remarkable when one contrasts the emphasis given to it in statements about the aims and objectives of education".

This has been largely due to the fact that the "extreme complexity of these noncognitive outcomes" (Wilson, 1971:685) has rendered them somewhat more difficult to measure than the cognitive objectives.

However, if attitude objectives are accepted as appropriate for school mathematics then some provision for their testing should be made (Fraser and Gillam, 1972:78; Corcoran and Gibb, 1961:121). This would entail the development of suitable instruments to measure such attitudes.

A realization of the fact that attitudes towards mathematics affect achievement in some way must be followed by determining the prevailing attitudes through measurement. The information gathered in this way can serve a useful purpose in making meaningful decisions regarding the educational process.

In this section the methods of appraising attitudes, attitude scaling procedures and scales used in this study will be discussed.

3.3.1 Methods of Appraising Attitudes

It is maintained that inferences may be made about an underlying attitude of a person by observing what a person says, does or writes in relevant situations (Corcoran and Gibb, 1961; Aiken, 1979b:208). Though not perfect, several methods of measuring attitudes based on such inferences have emerged.

The many devices that have been developed to gather information on attitudes of individuals and groups range from the more direct self-report questionnaires and scales to the highly indirect projective techniques (Aiken, 1979b:208). Corcoran and Gibb (1961:107) consider that, in general, the various techniques for measuring attitudes fall into three main categories:-

(i) Self-report methods such as questionnaires, attitude scales, sentence completion, projective techniques, and content analysis of essays. Here the subject reports his own attitude by responding to a rating scale or describes his feelings by writing an essay.

(ii) Observational methods where another person observes and records behaviour which is indicative of evidence of interest or a attitude.

(iii) Interviews where another person interviews and notes responses using a structured or unstructured questionnaire.

Self-report procedures have been better developed and more utilized than other appraisal methods and more examples of such instruments can be found. Nunnally (1978:591), in considering the various approaches to the measurement of attitudes, claims that most measures of attitudes are based on self-report and that from evidence concerning validity of the different approaches "the self-report offers the most valid approach currently available".

Self-report methods involving scales are usually non-disguised (or direct) techniques where no attempt is made to conceal the purpose of the measurement from the subjects. The other self-report procedures tend to disguise the purpose in order to obtain a less-distorted measure of attitudes (Fishbein and Ajzen, 1975:89).

The present study made use of the attitude scale, which has emerged as the most popular psychometric device for determining attitudes (Aiken, 1979b), to measure directly the pupils' attitudes toward mathematics. The possibility of some distortion in attitude responses was recognized and every attempt was made during the administration of the scale "to provide an atmosphere in which the student can feel confident in expressing himself" (Corcoran and Gibb, 1961:107). The observational method was also used (but to a much lesser degree) where teachers reported on their observations of their pupils' attitudes (see TQ1 - Appendix 4).

3.3.2 Attitude Scales

A psychometric device such as the attitude scale usually comprises a series of positive and negative statements. In general, all scales require the subject to agree or disagree with the items which indicate some degree of favourableness or unfavourableness toward the attitude object (Fishbein and Ajzen, 1975:81). This approach is consistent with the notion of an attitude in terms of an evaluative dimension ranging from favourableness to unfavourableness (discussed earlier in this chapter)

Corcoran and Gibb (1961) recognize two important dimensions of an attitude which are catered for in the above approach:

(i) direction (as outlined above in terms of polarities: positive-negative, favourable-unfavourable and like-dislike)

(ii) intensity (how strongly does one feel about an attitude).

For example, a student who states "Mathematics stimulates me and it is my best subject" shows a stronger feeling than one who says "Mathematics is a very worthwhile and necessary subject".

3.3.3 Standard Attitude-Scaling Procedures.

A major purpose of any procedure used to construct an attitude scale is to select items which permit accurate inferences about the respondent's attitude. Most attitude-scaling procedures arrive at an attitude score on the basis of a person's responses to the items which are statements of belief or intentions (Fishbein and Ajzen, 1975). For example, the following items may be used to measure attitude towards mathematics by requiring the student to indicate his agreement or disagreement with each item:

"Mathematics helps to develop the mind and teaches a person to think"

"I have seldom liked studying mathematics".

Since the present study makes use of an attitude scale to measure attitudes towards mathematics, the major standard scaling procedures will be reviewed so that the procedures used in this study may be viewed in relation to them. Shaw and Wright (1967) describe and comment on the quality and use of some 176 of these scales covering attitudes to a great variety of issues, concepts, groups and institutions.

Historically, designing of attitude scales may be traced to Bogardus (1925) and it was mainly due to the work of Thurstone (1928), Likert (1932), Guttman (1944) and Osgood (1963) that the assessment of attitudes assumed increasing importance in the field of psychological measurement.

3.3.3.1 Bogardus's Measure of Social Distance

One of the earliest scales to be designed was the Bogardus Social Distance Scale. Bogardus (1925) attempted to measure social distance or prejudice toward members of various national, religious and racial groups by developing an instrument of seven intentional items. The respondents were required to indicate whether or not they would admit members of a particular group e.g. Germans, Jews etc. (Bogardus, 1925:72).

It was assumed that the seven items implied increasing degrees of social distance and a score was obtained by simply counting the number of settings from which members of a given group would be excluded. Although this scale has proved valuable in research relating to regional differences and prejudice, it has been regarded as 'crude' by present-day standards of attitude-instrument development (Aiken, 1979b: 211).

3.3.3.2 Thurstone's Equal-Appearing Interval Scale

This procedure involves the collection of a large number of 'opinion' statements, the responses to which have been assumed by Thurstone (1928:78) to be expressions of the person's attitudes: "An opinion symbolizes an attitude". These items which express positive and negative attitudes toward some group, institution or concept (i.e. favourableness or unfavourableness toward the attitude object) are sorted into 11 categories by several judges. The categories range from least favourable (1) to most favourable (11). A scale value (medium-category score) is computed for each item. The final scale is made up of about 20 items with scale values at approximately equal distances on the attitude continuum. Respondents are required to endorse only those items with which they agree and the score is the sum or mean of the scale values of the endorsed statements.

Although this method of attitude scaling has been widely used since its inception, some reservations have been expressed about the assumptions underlying the model (Nunnally, 1978:603; Lewis, 1974:159).

In respect of application of this method to measurement of attitudes towards mathematics it is noted that a 15-item scale has been developed by Dutton (1951) to measure attitudes of prospective elementary school teachers towards arithmetic. This scale has been used both in the junior high school (Dutton, 1968) and in the elementary school (Fedon, 1958).

3.3.3.3 Likert's Method of Summated Ratings

In this method of attitude-scale construction (Likert, 1932) a large number of statements which are indicative of favourableness or unfavourableness toward an attitude object are administered to a group. The respondents indicate whether they strongly agree, agree, are undecided, disagree or strongly disagree with each of the items. Very often a three-point scale - agree, uncertain, disagree - is used (Corcoran and Gibb, 1972:109).

For scoring purposes the responses to positive items are weighted 5, 4, 3, 2 and 1 from strongly agree to strongly disagree. Scoring is reversed for unfavourable items so that agreement with an unfavourable item results in a low score. A respondent's total score is obtained by summing across all his item scores.

Items for the final scale are selected on the basis of internal consistency and discriminating ability. An item is said to meet the criterion of internal consistency if the item score correlates significantly with the attitude score (Fishbein and Ajzen, 1975:72). Further, an item contributes to internal consistency if it discriminates between two extreme groups i.e. if the mean score on the item is higher for the 'favourable' group than for the 'unfavourable' group.

The Likert-type scales have become increasingly popular among workers in the field of attitude measurement. Its advantages and uses will be considered in a later section.

3.3.3.4 Guttman's Scalogram Analysis

Although Guttman's Scalogram Analysis (Guttman, 1944) is less frequently employed (Aiken, 1970b, 1979b), the technique is reviewed here because the IEA Study (Husén, 1967a:118), from which items were drawn for this study, used the technique. However, for purposes of scoring, Likert's method of summated ratings was used (Husén, 1967a:121).

An assumption underlying this method is that all items comprising the scale are on a single dimension so that a respondent who endorses an item will endorse all items with lower scale values than this one. Some writers feel that "such a restriction is more likely to be satisfied for cognitive test items than for affective items like attitude statements" (Aiken, 1970b:554).

Perfect Guttman scales are seldom obtained and deviations occur which indicate that the items are not unidimensional (Fishbein and Ajzen, 1975:67). Guttman (1944:107) proposed a coefficient of reproducibility as a measure of the degree of such deviations and hence of the approximation to perfection.

An example of a Guttman scale used in the measurement of attitudes toward mathematics is that of Anttonen (1969) who arranged 94 attitude-scale items into 15 Guttman-type scales.

3.3.3.5 Osgood's Semantic Differential

Osgood (1963) developed the technique for purposes of measuring meaning rather than for the assessment of attitudes. However, the semantic differential has been used in a limited way to assess attitudes towards mathematics (Scharf, 1971; McCallon and Brown, 1971).

This technique requires the respondent to differentiate between given concepts against a set of bipolar adjectives (e.g. good-bad, fair-unfair, hard-soft ... Osgood (1963)). The respondent rates each concept against each bipolar scale of seven intervals, e.g.

Concept : Taking a mathematics test is ...

Scales : BAD ___ ___ ___ ___ ___ ___ ___ GOOD

HAPPY ___ ___ ___ ___ ___ ___ ___ SAD

(Scharf, 1971:643)

In this way a profile of ratings is obtained for each concept and it is assumed that two concepts are similar if their profiles are similar (Fishbein and Ajzen, 1975:75). The scales are scored from $^+3$ to $^-3$ ranging from positive to negative side of scale. The total attitude score (as in the case of the Likert scale) is the sum of these values.

3.3.3.6 General Comments

All the standard scaling procedures reviewed above have a fundamental similarity in that

"the resultant attitude score represents an individual's location on a bipolar evaluative dimension vis-a-vis a given object" (Fishbein and Ajzen, 1975:79).

Having reviewed major types of scales, the decision to use a particular type in this study rested on its suitability for the present design and the frequency and success with which it has been used in measuring attitudes towards mathematics and in other similar studies.

It has been found that Likert-type scales are comparable in reliability to Thurstone-type scales and since prior judgement by unbiased judges is not necessary, it is regarded as an easier scale to construct (Aiken, 1970b:554). For the same reason many researchers in the field of measurement of attitudes towards mathematics have shown preference for Likert scales. For example, both the scales on attitudes towards mathematics, included in their book on attitude scales by Shaw and Wright (1967:237-243), are Likert scales (one by Gladstone et al (1960) and the other by Aiken (1963)). Another example is that of Dutton and Blum (1968) who reformulated some of the items from Dutton's earlier Thurstone-type scale (Dutton, 1951) to form a Likert scale.

In a study of the reliabilities of four arithmetic attitude scales and an investigation of component mathematics attitudes, Evans (1972) compared Dutton-Thurstone, Dutton-Likert, Antonnen revised Hoyt and Semantic Differential scales. The Dutton-Likert and the Antonnen revised Hoyt performed the best in terms of test-retest reliability.

Nunnally (1978:604) suggests that Likert scales have several attractive advantages over all other methods as they

(i) follow from an appealing model,

(ii) are rather easy to construct,

(iii) are usually highly reliable,

(iv) can be adapted to measure many different kinds of attitudes,

(v) have produced meaningful results in many studies to date.

In general, Likert-type scales have been widely used with meaningful results in several studies concerning attitudes towards mathematics (e.g. Aiken and Dreger, 1961; Aiken, 1963, 1974, 1979; Dutton and Blum, 1969; Husén, 1967a; Sandman, 1974; Hunkler and Quast, 1972; Michaels and Forsyth, 1977; Kempa and McGough, 1977; Sovchik, 1980).

On the basis of the foregoing arguments and the success with which studies in mathematics have utilized the Likert scale, it was decided that the Likert-type scale should be used in the present study. In this way, attitude scores were arrived at through summated ratings.

3.4 ATTITUDE SCALES USED IN THIS STUDY

As mentioned in the previous section this researcher settled for the widely used Likert approach which had been successfully applied to numerous studies in attitudes towards mathematics. In addition, a decision had to be made between

(i) a global scale where a general attitude towards mathematics is treated as a unidimensional construct; and

(ii) multifaceted scales where attitude towards mathematics is viewed as a multidimensional construct.

In the following sections this decision and a discussion of the scales employed in this study will be presented.

3.4.1 A Multi-dimensional Attitude Scale

Although a majority of the earlier investigations have dealt with attitudes towards mathematics in general, recently greater interest has been shown in studying specific aspects of the attitudes. Corcoran and Gibb (1967:106) suggest that, while there is good reason to begin with a general measure of attitude toward mathematics, more specific attitudes also merit study. Such aspects may refer to attitudes to specific topics in mathematics (e.g. Metric System attitude scale - Sovchik, 1980; Attitudes toward use of calculators - Bitter, 1980) and to attitudes to mathematics teachers, difficulty in mathematics, enjoyment of mathematics, value of mathematics etc.

Thus, scales used to measure attitudes towards mathematics fall into two broad categories:

(i) those that view attitude to mathematics as a unidimensional construct; and

(ii) those that regard attitude towards mathematics as a multidimensional construct.

Studies which use the first type yield a single score for a general attitude toward mathematics (e.g. Dutton, 1951; Aiken, 1963; Gladstone et al, 1960 (cited by Shaw and Wright, 1967); Aiken, 1972b). Studies using multidimensional scales yield a separate score for each dimension or facet of attitude towards mathematics that is assessed (e.g. Husén, 1967a; Aiken, 1974, 1979a; Burek, 1975; Michaels and Forsyth, 1977; Kempa and McGough, 1977; Sandman, 1974; Bowling, 1977; Corbitt, 1979).

Sandman's (1974) inventory attempted to measure six constructs of mathematics attitudes: perception of the mathematics teacher; anxiety towards mathematics; value of mathematics in society; self concept in mathematics; enjoyment of mathematics and motivation in mathematics. Six eight-item scales were constructed and administered to eighth and and eleventh graders.

Michaels and Forsyth (1977) constructed items to measure four constructs: enjoyment of word problems; enjoyment of pictorial problems; appreciation of utility of mathematics; security with mathematics.

Aiken's (1979) study in Iranian middle schools utilized a four-component Likert-type scale relating to enjoyment of mathematics, motivation in mathematics, importance of mathematics and freedom from fear of mathematics.

The value of assessment of attitudes toward mathematics lies not only in the possibility for prediction of achievement but also in evaluation of the curriculum process in mathematics. Corcoran and Gibb (1972:121) maintain that

> "if students' attitudes towards mathematics learning are important both as indicators of what they have learned and as elements in motivation for further learning, attitude appraisal should not be left out of the evaluation program".

Though useful, assessment of attitudes on a unidimensional scale yields a single score and has limited value for evaluation of the effectiveness of a programme. In this context, Michaels and Forsyth (1977:1044) suggest that the

> "identification and treatment of individual students' problems with mathematics would probably be easier if more specific rather than general information on those students' attitudes were available".

Aiken (1970b:589) also emphasized the "diagnostic usefulness" of such scales.

It must also be pointed out that a multi-dimensional scale can provide a single score as well as several subscores. Thus it does not exclude the functions of a uni-dimensional scale. This researcher was attracted by the prospects of securing qualitative attitude data through multi-dimensional scales and the subsequent value of that data for teachers of mathematics and school counsellors. It was therefore decided that a multidimensional scale modelled on the lines of the IEA study (Husén, 1967a) and Aiken's (1979) study should be compiled and employed. A brief review of the scales in these two studies follows.

3.4.2 Scales used in the IEA Study (Husén, 1967a:109-122)

Since the majority of the items used in the attitude scales in the present study were drawn from the IEA Study, a review of this aspect of that study will provide a useful background.

The hypotheses formulated for the IEA Study involved the measurement of both cognitive (through mathematics tests) and affective (through attitude scales) outcomes. The following seven final attitude scales were used:

- Views about Mathematics Teaching (11 items);
- Views about School Learning (11 items);
- Attitudes towards Mathematics as a Process (8 items);
- Attitudes about the Difficulties of Learning Mathematics (7 items);
- Attitudes towards the Place of Mathematics in Society (8 items);
- Attitudes towards School and School Learning (11 items);
- Attitudes towards Man and his Environment (9 items).

In general, the scales were designed to measure student attitude towards some aspect of mathematics, school or life.

The first step in the construction of the scales was the definition of the underlying continuum for each scale. For example, statements in the scale measuring attitudes about the difficulties of learning mathematics range from a view that mathematics is a subject which could be learnt only by an elitist few to a view that anyone can master it. The items were then constructed to cover each continuum. They were initially screened, then ranked by a panel of three judges and selected for trial when complete agreement was reached by these judges. After field testing in seven countries among about 4000 students (Husén, 1967a:115), Guttman Scale Analysis was applied. Items which showed disagreement with the judges' original ranking and those whose positions in the scale fluctuated from country to country were eliminated. In addition, certain items owing to special problems in content, translation etc. were also eliminated. In this way one third of the items used in the pilot instruments were discarded.

Each statement in a scale was a declarative sentence e.g. "Mathematics is of great importance to a country's development", and the student was required to indicate whether he agreed, disagreed or was uncertain about the statement. Certain ststements e.g. "Very few people can learn mathematics" were arbitrarily designated as 'unfavourable' while others (such as the previous example above) were regarded as 'favourable'. For purposes of scoring, agreement with a 'favourable' item was awarded two points, agreement with an 'unfavourable' item zero, and 'uncertainty' was given one. The total score on each scale was then obtained by tallying the points.

It is clear that stringent requirements were placed on the items before they were selected for use in the final scales. The scales served the useful purpose of producing information on student attitudes towards the various aspects of mathematics, school and life. It must be noted that almost 50% of the items were geared to measure student attitudes towards school and life in general. This emphasis is understandable as the study aimed at comparing the various educational systems. The study excluded such personality variables as 'anxiety', 'self concept', 'security' etc. as related to mathematics. It also made no specific provision for such facets as 'enjoyment of mathematics' and 'motivation in mathematics'. These aspects have assumed importance in recent studies (Sandman, 1974; Michaels and Forsyth, 1977; Aiken, 1972, 1974, 1979a).

In general, the items in the IEA scales provide a useful basis for the study of attitudes towards mathematics. A recent study by Kempa and McGough (1977) has used an inventory comprising 33 Likert-type items based on the IEA inventory and embracing four sub-scales.

3.4.3 <u>Scales used in Aiken's Studies (1974, 1979a)</u>

In recognition of the fact that attitude towards mathematics is composed of several dimensions, Aiken (1979a) designed a four-component Likert-type questionnaire of 24 items to measure the following constructs:

Enjoyment of Mathematics;

Motivation in Mathematics;

Importance of Mathematics;

Freedom from Fear of Mathematics.

The items of any one scale were spread out evenly among items of the other scales e.g. items 1, 5, 9, 13, 17 and 21 constituted the 'Enjoyment of Mathematics" scale. In all, 5 scores were obtained for each student: 4 sub-scores and a total score.

These scales were administered to 300 boys and girls ranging from grades 6 to 8 in Iranian Middle schools. Intercorrelations between the 24 items and the 5 scores suggested that scores on the 'Motivation' scale were too closely related especially to scores on the 'Enjoyment' scale (Aiken, 1979a:232).

In an earlier study, Aiken (1974:67) concluded that

> "there is a general psychological dimension of 'enjoyment of mathematics', which encompasses not only a liking for mathematics problems but also a liking for mathematical terms, symbols, and routine computations".

Aiken (1974) constructed an 11-item Likert-type 'Enjoyment of Mathematics' scale and obtained a Coefficient Alpha of 0,95 which was indicative of a very high internal-consistency reliability. In correlating the individual item score with the total score, it was found that the item,

> "Mathematics is enjoyable and stimulating to me"

gained the highest correlation of 0,91.

In general, although Aiken's (1979a) study did not include all the facets used in the IEA study, it provided a useful store of items which (when seen complementary to IEA items) produced interesting possibilities for the compilation of scales for research into attitudes towards mathematics. The present study makes use of items from both sets of scales.

3.4.4 Dimensions of Attitudes toward Mathematics in the Present Study

From a review of the various studies involving multi-dimensional constructs of attitudes towards mathematics (Husén, 1967a; Aiken, 1972, 1974, 1979a; Sandman, 1974; Michaels and Forsyth, 1977; Kempa and McGough, 1977), it was clear that there was some overlap in items as well as facets measured. Since such personality factors as 'anxiety', 'self concept' etc. were not considered in the present study, it was found that the items used in the IEA study (Husén, 1967a) and Aiken's study (1979a) reasonably covered the various dimensions which appeared with some consistency (though with varying titles) in different multi-dimensional scales. Modelled on these two studies, the present study sought to investigate attitudes toward mathematics in terms of the following six aspects:

Mathematics Teaching;

School Learning;

Difficulties in Learning Mathematics;

Place or Importance of Mathematics in Society;

School and Life in General;

Enjoyment of Mathematics.

This conceptual model of six factors makes provision for certain specific as well as general aspects of attitudes. Four of the scales relate specifically to attitudes toward mathematics while the two relate to school and life in general. In this way a range of student attitudes and their relationship to achievement in mathematics could be studied.

There seems to be no hard and fast rule about the number of items to be included in each sub-scale. Most studies have a varying number of items in each scale: 5 to 12 items (Kempa and McCough, 1977), 7 to 11 items (Husén, 1967a), 6 to 10 items (Michaels and Forsyth, 1977). However, Aiken (1979a) used 6 items in each of 4 scales. There is merit in using an equal number of items in each scale because:

(i) a balance between the dimensions is maintained;

(ii) scores, means and standard deviations may be readily and easily compared.

In addition, an even number of items in each scale lends itself to split-half reliabilities for both the subscores and the total score. With the above considerations in mind, it was decided that 8 items should be used in each of the 6 scales in the present study.

3.4.5 Selection of Items

As mentioned earlier, items for this study were selected from the expertly constructed scales used in the IEA Study (Husén, 1967a) and Aiken's (1979a) study which have been reviewed in a previous section. It was recognized that these sets of scales had been successfully applied: the IEA scales had been used in 12 countries while items in Aiken's scale had been used in studies in USA (Aiken, 1972, 1974) before being used in Iranian middle schools in the present form (Aiken, 1979a:231).

However, it was felt that the items should be subjected to further scrutiny in order to gauge their suitability for use in the local situation. In general, the method of selection involved a preliminary selection of items for each of the scales and scrutiny by 4 judges (mathematics teachers who had judged the mathematics test items). An item was finally selected if all four judges agreed on its relevance in a particular scale and its suitability for use in the schools. The judges were not required to rank the position of any item as there was no need to calculate scale values. However, wherever a whole scale or a large portion of it was accepted the relative positions of the items were retained. Further, the pupils who took the mathematics 'trial test' were required to comment on the clarity of communication of each item in the set of items from which selections were made. Finally each scale was compiled as follows:-

3.4.5.1 Subscale 1 : Mathematics Teaching

Two of the 11 items in the IEA scale (Husén, 1967a:116) were found to be somewhat overlapping viz.

"My mathematics teacher wants us to discover mathematical principles and ideas for ourselves" and

"My mathematics teacher explains the basic ideas; we are expected to develop the methods of solution ourselves" .

A similar overlap was found between two other items. Thus two items were excluded and a further one was dropped on the advice of the judges. The remaining 8 items were incorporated in the final scale as items 1, 7, 13, 19, 25, 31, 37 and 43 in PQ2 (See Appendix 2).

3.4.5.2 Subscale 2 : School Learning

As in the previous scale the IEA scale (Husén, 1967a:117) consisted of 11 items. The one item seemed obviously overlapping with another: "Most of our classroom work is listening to the teacher" was not requiring anything new when compared with "The pupils spend most of their class time listening to the teachers and taking notes". Thus the former was dropped after the preliminary selection. The remaining 10 were found to be suitable and in order to get 8 items two further items were excluded after consultation with the judges. The items which made up the final scale are 2, 8, 14, 20, 26, 32, 38 and 44 in PQ2 (See Appendix 2).

3.4.5.3 Subscale 3 : Difficulties of Learning Mathematics

The IEA Scale on this dimension consisted of 7 items (Husén, 1967a:119).

Although all the items were thought to be suitable after preliminary selection, there was disagreement about one item: "Anyone can learn mathematics". It was thought to be somewhat vague. This item was omitted in order to avoid any confusion. A further two items were selected from Aiken's (1979a:231) scale after a careful study of 4 items which were closest to the 'difficulty' scale. To maintain a balance the one item chosen was positive ('I am very calm when studying mathematics') while the other was negative ('Mathematics is one of my most dreaded subjects'). The 8 items selected appear as 3, 9, 15, 21, 27, 33, 39, and 45 in PQ2 (See Appendix 2).

3.4.5.4 Subscale 4 : Place or Importance of Mathematics in Society

The IEA scale comprised 8 items (Husén, 1967a:120). In the preliminary selection it was felt that three of the items which included the terminology 'advanced mathematics' might cause some problems in interpretation. This suspicion was confirmed by the queries made by the pupils who were required to comment on the items in the draft scales. These items were excluded. In order to replace them, three items from Aiken's (1979a:231) scale on "Importance of Mathematics" were selected. The decisions were guided largely by whether the items overlapped with those already selected. The 8 items which made up the final scale are numbered 4, 10, 16, 22, 28, 34, 40 and 46 in PQ2 (See Appendix 2).

3.4.5.5 Subscale 5 : School and Life in General

Items for this general scale were selected from the two IEA scales (Husén, 1967a:121-122):

Attitudes Towards School and School Learning, and

Attitudes Towards Man and his Environment. The decision not to have two separate scales was made on the basis that this dimension was subordinate to the main aspects dealing with attitudes specific to mathematics in this study. Since almost all items were reasonably suitable, it was decided that four from each scale should be selected to maintain a balance in terms of the two dimensions defined in the IEA study. Further balance was attained by selecting an equal number of negative and positive items. The 8 items which were agreed upon appear as items 5, 11, 17, 23, 29, 35, 41, and 47 in PQ2 (Appendix 2).

3.4.5.6 Subscale 6 : Enjoyment of Mathematics

The selection of items for this scale was based entirely on Aiken's (1979a) scale which included 6 items on 'Enjoyment of Mathematics' and 6 items on 'Motivation in Mathematics'. As pointed out earlier, Aiken (1979a:232) found that the motivation variable was too closely related to enjoyment to be viewed as a separate factor. All six items (3 positive and 3 negative) on the 'enjoyment' scale were found to be suitable. Two further items (one positive and the other negative) from the 'motivation' scale which were indicative of the liking for mathematics were selected. The final set of 8 items are numbered 6, 12, 18, 24, 30, 36, 42 and 48 in PQ2 (Appendix 2).

3.4.6 Format of Final Scale

The items of each scale were not presented separately but were distributed among items of the other scales "to minimize the likelihood of the formation of response sets" (Husén, 1967a:120). The distribution of the items was, however, not random but according to a pattern which was not apparent to anyone looking at the scale for the first time. Items of each scale were placed at intervals of 6 e.g. items of the first scale were numbered 1, 7, 13, 19, 25 etc. and those of the second scale 2, 8, 14, 20 etc. This procedure was similar to that followed by Aiken (1979a) for the four subscales.

There were 21 items which were arbitrarily designated as negative items e.g.

"Very few people can learn mathematics"

"Mathematics is not a very interesting subject".

Such items were randomly placed in the attitude scale (indicated by * in PQ2 - see Appendix 2).

For each statement in the scale the pupil was required to indicate whether he agreed, disagreed or was uncertain by using the following key:

agree = 2, uncertain = 1, disagree = 0

In the instructions to the pupil it was made clear that

"there is no right or wrong answer to any statement - it is just what *you* sincerely feel" (Appendix 2).

For scoring purposes the above weights had to be reversed for 'unfavourable' (or negative) items. Thus the possible score for any sub-scale could range from 0 to 16 while that for the total scale from 0 to 96.

3.4.7 General Comments on Attitude Scale

The items for the attitude scale in the present study were not developed and validated by this researcher but were selected from scales which had been successfully used in the IEA study (Husén, 1967a) and Aiken's (1979a) study. The quality of the attitude instrument must therefore be viewed against the several considerations that had been made in the initial development of the items and scales (reviewed in sections 3.4.2 and 3.4.3) and in the selection of the items from those scales.

It is useful to note that the scales from which the items were selected for this study have been reported to have high internal-consistency coefficients. The coefficients of reproducibility for the IEA scales (Husén, 1967a:118) averaged close to 0,90. Aiken (1979a:230-232) reported alpha coefficients ranging from 0,50 to 0,86 for the sub-scores and from 0,81 to 0,91 for total scores.

In addition to the fact that the items had been expertly developed and successfully applied in the original studies (Husén, 1967; Aiken, 1979a, 1974), it was expected that the following considerations would contribute positively to the quality of the scale as a reliable and valid measure of attitudes:

(i) use of Likert-type scales ensures reasonably high reliability (Evans, 1972; Nunnally, 1978),

(ii) provision of an atmosphere for free expression by students in order to prevent any distortive effects (Corcoran and Gibb, 1972:107),

(iii) careful selection of items based on judgment of relevance, suitability and clarity of communication,

(iv) no changes made to any of the items selected,

(v) careful distribution of items in each scale to prevent response sets.

In general, it was considered that a reasonably satisfactory device for measuring attitudes had been compiled. However, it was recognized that reliability and validity data should be produced for the instrument in terms of the population for which it is used. The relevant statistical analysis will be presented in chapter five.

CHAPTER FOUR

4. SELECTION OF SAMPLE, ADMINISTRATION OF QUESTIONNAIRE, ATTITUDE SCALE AND TEST, AND HANDLING OF DATA

The selection of a random and representative sample was central to this research design. The careful administration of the questionnaires, scale and test was of paramount importance in securing information as accurately as possible. In this chapter the procedures used in sampling and in the administration of the materials will be discussed. The procedures and problems of handling the data are also presented.

4.1 SAMPLING

4.1.1 Definition of Target Population

The population under study needs to be clearly defined so that the nature of the units that comprise it may be easily identified.

The target population was identified by the following considerations:

(i) Type of Schools: Indian Secondary Schools offering Mathematics at senior secondary level.

(ii) Levels of Schooling: Pupils following the ordinary course (within the System of Differentiated Education) in Standards 9 and 10 (DIE, 1972).

(iii) Geographical: Indian Group Areas in the Durban Municipality.

The target population was thus defined as follows:

All pupils in Standards Nine and Ten taking the ordinary course in Mathematics in Indian Secondary schools within the Durban Municipality (See Fig 4.1).

The pre-matriculation levels (Standards 9 and 10) were specifically chosen for study in view of the declining trends in the matriculation results (see Table 1.1). Both the levels were considered in order to study any differences in attitudes and attainment that may result from an extra year of schooling.

In view of the fact that the greatest proportion of the Indian population in South Africa is concentrated in the Durban Metropolis, it was expected that any generalizations drawn would have obvious wider implications and applicability.

4.1.2 Selection of Sample

It was decided that the sampling design employed should ensure a random selection as far as it was possible. Cognisance was taken of the

"extreme practical significance that all mathematical sampling theory is based finally on the assumption of *random selection*" (Lindquist, 1940:24).

The present study aimed at securing a large but manageable sample of between 600 and 700 to represent the characteristics of the population as defined earlier (Van Dalen, 1962:250). Thus a small sample, which would result in inadequate data in respect of this research design and in the application of small sample theory with its numerous limitations, was avoided. The large sample, however, was not to be restricted to a few schools by selecting all the pupils in each of those schools.

In order to select a large and representative sample there had to be

(i) a wide coverage of the areas in which the schools were situated;

(ii) a complete coverage of the mathematics class units (both grades - Higher and Standard; both levels - Std 9 and Std 10).

4.1.2.1 Selection of Schools

Twenty six schools were identified in accordance with the defined population. These schools were spread over 16 regions or zones. (See Fig. 4.1 for a diagrammatic representation).

At least one school was chosen randomly from each of the areas except Durban Central where there were four schools and the only mixed school was selected. Both the schools in the Westcliff/Silverglen zone were chosen because they served Kharwastan/Woodhurst where there were no senior secondary classes. The newly developed residential suburb of Phoenix was excluded as there were no schools with Std 9 and Std 10 classes taking the ordinary course in mathematics.

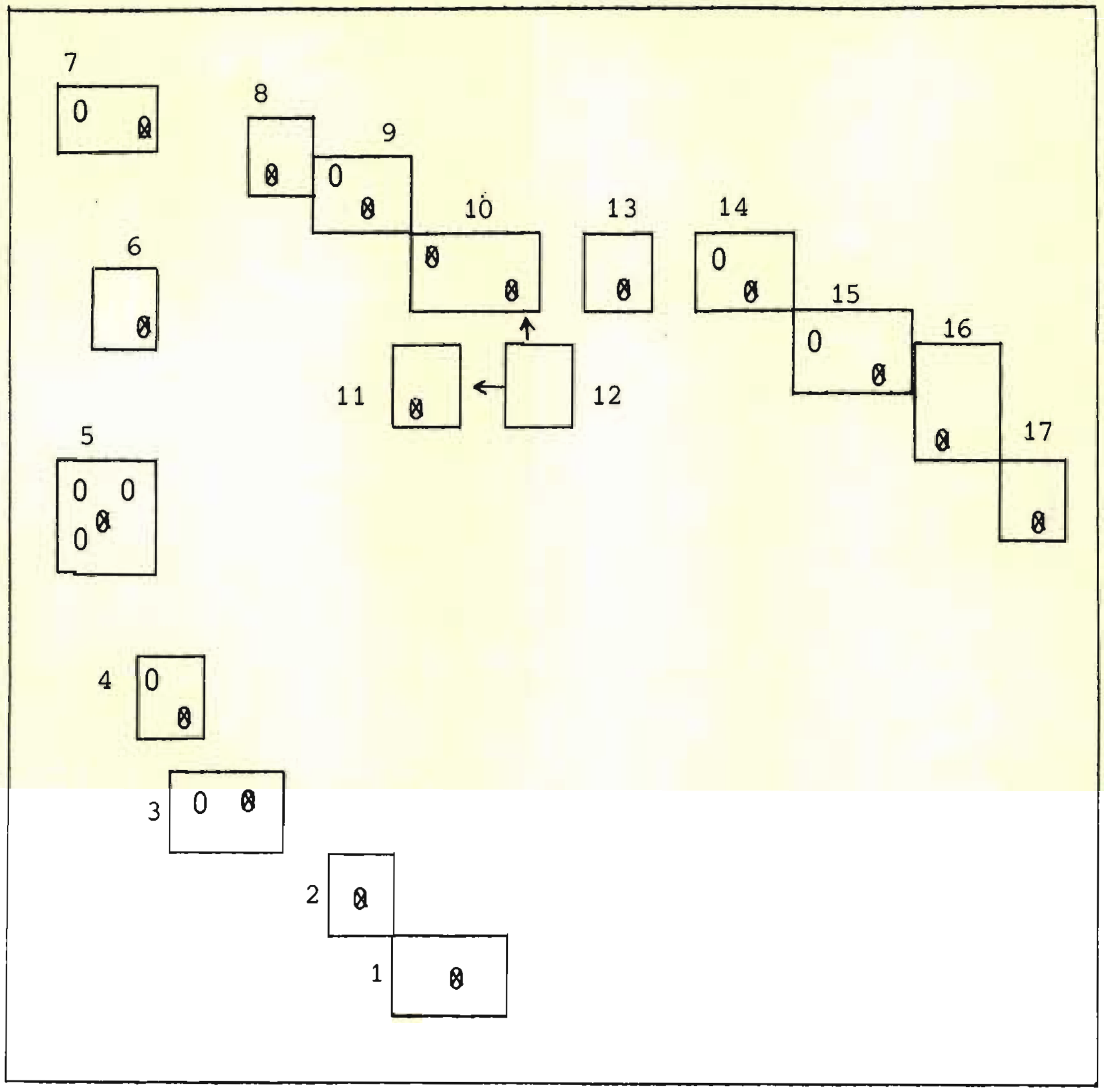


FIG. 4.1

DIAGRAMMATIC REPRESENTATION OF THE DISTRIBUTION OF SCHOOLS ACCORDING TO INDIAN AREAS IN THE DURBAN MINICIPALITY

(SCHOOLS SELECTED SHOWN BY 'Ø')

Key to Areas Served by the School:

1. Avoca - Effingham Heights - Greenwood Park - Red Hill
2. Sea Cow Lake - Newlands
3. Springfield - Reservoir Hills
4. Sydenham - Asherville
5. Durban Central
6. Clairwood - Jacobs
7. Merebank - Merewent
8. Mobeni Heights - Havenside
9. Bayview - Silverglen
10. Silverglen - Westcliff
11. Umhlatuzana Township
12. Kharwastan - Woodhurst
13. Witteklip
14. Arena Park
15. Montford
16. Crossmoor
17. Shallcross (also serves Marianhill)

It was felt that this approach to selecting schools for the study would ensure a wide coverage of the Indian areas in the Durban Metropolis. Fig. 4.1 shows the distribution of the schools according to the areas in which they are situated.

4.1.2.2 Selection of Pupils

Since a representative cross-section of all the schools was far more useful to this research design, a completely random selection from the total population in the 17 schools was rejected to avoid an undue proportion from a single school or area. Thus a stratified random sampling design was employed to get a more representative sample (Van Dalen, 1962:252). The two major stratifications used were levels (Std 9 and Std 10) and grades (Higher and Standard). This fitted in with the organisation of class units within schools. In all schools there were separate class units for Std 9 and Std 10. Within these 2 levels there was a further stratification (in most schools) into separate grades. The mathematics class unit became the smallest "intact group" from which the sample was to be selected. The decision to select from every class unit also ensured the participation of every mathematics teacher at the Stds. 9 and 10 levels.

The problem arose as to what proportion of the class unit should be selected in order to stay within the limits of a large, manageable sample. Therefore, data had to be gathered on each of the 17 schools with respect to the number of pupils in each of the mathematics class units in these schools. On the basis of this information (see Table 4.1), it was decided that a one-fifth proportion would yield the required sample.

TABLE 4.1

DISTRIBUTION OF MATHEMATICS CLASS UNITS, TEACHERS AND PUPILS

SCHOOL	MATHS CLASS UNITS		MATHS TEACHERS	POPULATION		PUPILS IN SAMPLE	
	Std 10	Std 9		Std 10	Std 9	Std 10	Std 9
01	5	6	4	127	149	22	27
02	3	6	3	80	104	16	24
03	4	5	2	89	149	17	20
04	6	5	3	112	159	23	19
05	3	4	3	52	109	12	22
06	2	4	3	49	54	12	15
07	4	4	3	78	120	16	22
08	5	4	4	99	99	20	21
09	6	6	4	154	167	23	22
10	4	6	3	78	133	17	24
11	3	3	3	70	94	13	20
12	4	4	3	98	106	16	24
13	4	4	3	104	146	20	22
14	3	6	3	66	153	14	23
15	5	8	5	96	173	18	29
16	2	4	2	50	112	11	28
17	6	3	2	149	114	27	21
	69	82		1551	2141	297	383
TOTAL	151		53	3692		680	

The method of selection then became evident and further simplified by the fact that the names of pupils were arranged in a strict alphabetical list for each mathematics class unit in the mathematics teacher's mark book. Every fifth name on this alphabetical list was selected. In the event of an absentee or school leaver, the next name was chosen. This principle was generally applied except in a few schools (with large numbers) where, owing to lack of accommodation in a separate room, a slightly lower proportion of one-sixth was applied.

In all, 680 pupils from each of 151 mathematics class units in the 17 schools were selected and they made up the subjects in the sample studied in this research (See Table 4.1). In addition, since rating scales (TQ1) had to be completed for each of the pupils by their respective teachers, all the mathematics teachers (n=53) took part in this project.

4.1.3 Evaluation of Sampling

In general it was felt that, with the co-operation of pupils, teachers and principals, the sampling design had been successfully applied. The final sample (n = 680) with major subsamples in respect of levels, grades and sex reflected a fairly large proportion of the population under study, viz., 18.42% (See Table 4.2).

From Table 4.2 it is clear that the ratio HG:SG in the sample was similar to that in the population. Further, the ratio Std 9 : Std 10 was almost equal for both the sample and the population. This was indicative of the consistency in sampling and of the representativeness of the sample in terms of grades and levels (Van Dalen, 1962). The sample percentage of the population for the various grades and levels ranged from 17,77 to 19,15 . It was indicative of the fact that almost equal proportions of the population were represented in the selected sample. Thus it was clear that any comparisons between grades or levels would not be affected by undue proportion of any one type of unit in the sample.

TABLE 4.2

SAMPLE AND POPULATION PROPORTIONS IN RESPECT OF LEVELS AND GRADES

	STD 9			STD 10			TOTAL
	HG	SG	TOTAL	HG	SG	TOTAL	
POPULATION (17 SCHOOLS)	1066 49,78%:	1075 50,22%	2141 57,99%	530 34,17%:	1021 65,83%	1551 42,01%	3692
SAMPLE (17 SCHOOLS)	192 50,13%:	191 49,87%	383 56,33%	104 35,02%:	193 64,98%	297 43,67%	680
% Population in Sample	18,01	17,77	17,89	19,62	18,90	19,15	18,42

TABLE 4.3

MATHEMATICS AT STD 9 AND STD 10 LEVELS: COMPARATIVE PROPORTIONS OF NATIONAL POPULATION AND TARGET POPULATION (17 SELECTED SCHOOLS)

	STD 9	STD 10
National Population (Indian)	6673* (59,22%)	4595* (40,78%)
Target Population: 17 Schools	2141 (57,99%)	1551 (42,01%)
% Target Population	32,08%	33,75%

* These figures apply as at March 1980 (DIE - Examinations and Statistics)

A study of Table 4.3 reveals the closeness in the proportions of Std 9 and Std 10 for the National Population and Target Population. Further, the target population constituted 32,08% of the National Population for Std 9 and 33,75% for Std 10. Since only 17 out of 26 eligible schools in the Durban Municipality made up the target population, these high proportions are attributed to the large concentration of Indian South Africans in Durban. Representative samples drawn from such large proportions of the National Population only serve to increase the significance of the generalizations that may ensue from the research.

TABLE 4.4

DISTRIBUTION OF SUBSAMPLES OVER LEVELS, GRADES AND SEX

	HG	SG	MALE	FEMALE	TOTAL
Std 9	192	191	220	163	383
Std 10	104	193	178	119	297
TOTAL	296	384	398	282	680

Table 4.4 shows the distribution of the subsamples over grades, levels and sex in 14 subcells. The male and female proportions were 57,45% and 42,55% for Std 9, and 59,94% and 40,06% for Std 10. (The relative proportions for the grades and levels are shown in Table 4.2). The male and female proportions were almost similar for the two levels.

It must also be pointed out that the teachers who took part in this research constituted 100% of the mathematics teaching staff at the

Std 9 - Std 10 levels in these schools.

The 17 schools selected were indicative of the excellent geographic coverage of the Indian areas in the Durban Metropolis. Further, the pupils were selected randomly from every mathematics class unit in these schools. Thus the final sample was considered to be randomly selected and representative of the population under study.

4.2 ADMINISTRATION OF QUESTIONNAIRES, ATTITUDE SCALE AND MATHEMATICS TEST

Having decided upon the sampling design and the sample, the next step was to administer the research instruments. In this study, three questionnaires (PQ1, TQ1 and TQ2), one attitude scale (PQ2) and a mathematics test (PQ3) were used to gather data. A brief description of the administration of these questionnaires, scale and test will be presented in this section.

4.2.1 Preliminary Arrangements

Through a letter setting out a motivated request together with the nature and scope of the research project, prior approval for use of the schools was obtained from the Director of Indian Education.

The principals of the selected schools were informed of the research project and the participation of pupils and teachers in their schools (see Appendix 6). In particular, it was emphasized that "there will be minimal disruption of classes" and that "teachers will *not* be involved in any testing or marking". The immediate value of this exercise for pupils and teachers was also pointed out.

Final arrangements were then made through the principal of each school in respect of the following:

(i) date and time of administration of the materials;

(ii) room and seating arrangements;

(iii) selection of pupils.

There was no need for any prior preparation on the part of the pupils. No special requests in respect of writing materials, mathematics instruments etc. were necessary.

Since all administration of materials and supervision were to be carried out personally by this researcher, the arrangements were greatly facilitated.

4.2.2 Administration of Materials

The date and time were chosen to fit in with the school programme. Although PQ1, PQ2 and PQ3 were timed to take ± 140 mins. (about 4 periods), in practice this was not possible. Much time was spent on getting the pupils, selected from the several class units, to assemble in the testing room and on checking absentees and replacements. The names of pupils had to be called out while they were in their own form groups in the very first period (registration). This prevented further delay in locating them in the various 'split' groups after the registration period.

In all the schools the administration of the instruments had to be spread over 5 periods. The first two periods were spent on PQ1 and PQ2, then there was a special break and the next three periods were spent on administering PQ3. In every school the administration of all the materials was completed well within the period, 09h00 to 12h00.

4.2.2.1 PQ1 - Pupil Questionnaire (See Appendix 1)

Since PQ1 required information of a general nature regarding age, sex, reasons for choosing mathematics, homework, educational and vocational aspirations etc. (see Appendix 1), it was chosen to be the first to be administered. Prior to administering PQ1, the nature and value of the research project was briefly outlined. It was explained that this research did not concern itself with how weak or how good each pupil was in mathematics, but rather with what the trends were. It was therefore necessary to get as true a picture as possible and this could only be achieved by honest, frank and sincere responses from them. It was also pointed out that their participation was entirely optional and anyone with the slightest inclination towards not taking the questionnaire or test was welcome to leave. It was interesting to note that only one pupil exercised this option. Pupils were also given the assurance that their responses would be treated confidentially.

Pupils were assisted, question by question, in completing PQ1. In this way any repetition of clarifications on a particular question was avoided and the number of incomplete questionnaires was kept to a minimum.

In view of all the precautions taken by this researcher, it must be assumed that the information gathered through PQ1 reflects accurately as possible the position of the pupils in regard to mathematics, home background and aspirations etc.

4.2.2.2 PQ2 : Attitude Scale (See Appendix 2)

Since PQ2 was an attitude scale requiring responses to the way pupils felt about mathematics, it was decided that it should be administered before the test itself. This ensured that there was no distortive effect on the responses as a result of having liked or disliked the mathematics test.

Prior to administration of the scale, its nature was clearly and briefly outlined. Special mention was made of the fact that some statements related specifically to mathematics while others were general, e.g.

> "The pupils spend most of their time listening to their teachers and taking notes." is a general statement while "Mathematics is not a very interesting subject." refers *specifically* to mathematics.

As the trial of PQ2 had shown, any such confusion had to be clarified.

The pupils were also urged to consider each statement carefully and to respond as honestly as possible. It was emphasized that there was no right or wrong answer to any statement. Once the scales were handed out, no discussion was allowed.

Although 30 mins. were allotted, all of them completed the scale well within this period. Before the scales were collected, the pupils had to check that every 'block' had been filled.

They were given sufficient time to consider each item. In order to minimise any distortion that might occur every attempt was made "to provide an atmosphere in which the student can feel confident to express himself freely" (Corcoran and Gibb, 1961:107). There were no queries and no incomplete parts. The scales had been successfully administered.

4.2.2.3 PQ3 : Mathematics Test (See Appendix 3)

The mathematics test was the last to be administered. This was done when the pupils returned from the special break arranged for them.

Each pupil was provided with a specially prepared Answer Sheet (See Appendix 3) and paper for rough work. They were not allowed to write on the Test Booklet.

It was impressed upon them that this was not an examination, but that they would write under test conditions. They were urged to do their best. The Test Booklets were handed out and the pupils were required to read carefully the instructions on the front page. They were given 80 mins. to complete the test and advised to spend

2 mins. each on the first 8,

3 mins. each on the next 8, and

5 mins. each on the last 8.

This helped them to adjust their rate of work to the time available (Ebel, 1965:236).

Almost all of them completed the test well within 80 mins. Some completed it within an hour. A few of them were allowed an extra 5 mins. To reduce to a minimum any temptation to guess blindly, it was essential that the pupils were given enough time to consider all the items. Further, they were advised against blind guessing but were allowed to make "rational guesses" (Ebel, 1965:237). At the end of the test the pupils were required to indicate how they found the test by ticking one of the following:

too easy	☐	1
easy	☐	2
about right	☐	3
difficult	☐	4
too difficult	☐	5

They were also free to make any general comment if they so wished.

The Test Booklets and Answer Sheets were collected separately and checked to ensure that the numbers tallied.

From the interest shown (after the test) in some of the test items and in the research project itself and from such general comments as "challenging", "good mixture of questions", "more tests should be like this" and "difficult but made me think", it was apparent that the test had been well received.

4.2.2.4 TQ1 : Rating by Teacher (See Appendix 4)

TQ1 was designed to gather information on the teacher's assessment of the pupil's

(a) ability in terms of cognitive objectives,

(b) ability in terms of content areas,

(c) attitude towards mathematics,

(d) overall performance level in mathematics.

In addition, the pupil's IQ score was also required.

The mathematics teacher was required to complete one TQ1 for each pupil selected from his class units.

Several teachers personally acknowledged the need for such a research study and the researcher was assured of their co-operation at all times. Apart from the fact that 39 pupils had no IQ scores, there were no problems with the completion and return of all the TQ1 scales.

4.2.2.5 TQ2 : Teacher Questionnaire (See Appendix 9)

TQ2 was a short questionnaire designed to obtain information on teaching experience, teaching load, and professional and academic background of the mathematics teachers of the pupils selected for this study. Each mathematics teacher completed a TQ2.

4.2.3 General Comments

A great deal of time had been spent on the planning and preparation of the instruments, the preliminary arrangements for the field work and on the administration of the materials. However, the enormous amount of useful data gathered more than justified the time spent.

In general, the administration of the materials proceeded as planned and was considered to be an unqualified success. This was attributed to the following:

(i) advance planning of preliminary arrangements;

(ii) careful preparation of all materials;

(iii) the excellent co-operation received from pupils, teachers and principals;

(iv) personal administration of all the materials by this researcher.

The collection of data as accurately as possible is an essential prerequisite for any research. It was felt that all precautions taken in the preparation and administration of the materials ought to have contributed significantly to satisfying this condition.

4.3 HANDLING OF DATA

It was planned that the data should be collected in such a way that it could be processed by computerization. As mentioned earlier, the questionnaires, scale and test were geared to this end. In this section the procedures and problems of handling the data (from the point of collection to the time when feedback from computer print-outs were obtained) are discussed.

4.3.1 Collection and Collation

Since PQ1, PQ2 and PQ3 were administered separately they had to be collected separately. Each of them bore the full name of the pupil for identification. PQ2 and PQ3 (Answer Sheet) were checked and collected on the day of administration. PQ1 was checked but not collected as TQ1 which was attached to it had to be completed by the mathematics teacher. PQ1 (together with TQ1 and TQ2) was collected on another day by special arrangement with the Head of Department.

In order to facilitate handling, the three sets were filed separately for each school. The files were labelled 01 to 17. For each school each of the three sets was sorted alphabetically according to the surnames of the pupils and the three sets were then collated. Finally, all the data for a particular pupil could be seen as a totality (PQ1, TQ1, PQ2 and Answer Sheet). This amounted to 9 pages of data for each pupil, which we shall refer to as the data schedule. This procedure was repeated for each of the 17 files. All the data schedules were then ready for coding and punching.

4.3.2 Coding and Punching

Data for processing must be carefully stored so that it may be easily retrieved when needed. To ensure greater flexibility in the analysis of the data and to get as much as possible from the data collected, it was necessary to store each piece of information as a separate unit.

Each data schedule was serialised by means of a six-digit identification code. The first two digits identified the school, the next digit the teacher and the next two digits the pupil in that school. The last digit indicated the data card number (1 or 2). e.g. 023071 refers to pupil number 7 in school number 2.

Most of PQ1 was precoded numerically (See Appendix 1). Wherever quantitative data was required, coding was not necessary. However, in some parts (e.g. 3, 19, 22, 23) the verbal responses had to be coded. In question 3, for example, it was necessary to know the mathematical bias in the curriculum and also to consider separately those taking both Biology and Physical Science and those who were taking either one of these or none. These two aspects were coded separately i.e. two columns on the data card were set aside. Certain other parts (5(a), 5(b), 16) were also coded clearly (in red) to minimise any error during transfer to data cards.

It was not necessary to code data in TQ1 and PQ2. (See Appendices 4 and 2 respectively). The data on the answer sheet had to be coded. The responses were coded according to the choices : A = 1, B = 2 ... and O for omission etc. In this way provision was made for the percentage response to each distractor and item analysis data. The prescoring and supplying of a single score, which would have considerably limited the information gained from the test, was avoided. Great care had to be taken in this coding as it was in fact a scoring procedure. This part was also double checked to ensure marker reliability.

Since the data schedules were carefully coded and checked, the data could be punched directly on to cards. The transfer of data to data processing sheets prior to punching was thus avoided. This not only saved a great deal of time but also eliminated the further possibility of error in such a transfer.

PQ1 and TQ1 carried 59 bits of information. In all, 71 columns on the data card had to be set aside for storage. PQ2 and the Answer Sheet carried a further 73 bits of information and required as many columns. It was thus clear that two cards for each data schedule had to be used. In all, 680 pairs of cards were punched, using a total of 150 columns in each pair.

All cards were read into the computer and a complete listing of data was obtained (separately for each school). This was used for checking whether the information from the data schedules had been correctly transferred. There was no 'short cut' to this task. The serial numbers helped greatly in locating cards with errors. Finally, all the cards with errors were located and replaced by cards with correctly transferred information.

The great deal of time and effort spent on the task of coding and punching was rewarded and justified by the knowledge that all the information collected had been transferred as accurately as possible to the data cards.

4.3.3 Scoring and Programming

Certain parts of the data schedule had to be scored by computer before any further calculations could be carried out. (See 3 of TQ1 in Appendix 4, PQ2 in Appendix 2 and Answer sheet (PQ3) in Appendix 3).

In 3 of TQ1 certain negative statements had to be scored in reverse e.g. disagreement with a negative statement would have had a response of '0' which had to be reversed to '2'. A similar procedure was adopted for certain negative statements in PQ2. All such statements had to be identified clearly so that they could be provided for in the scoring procedures.

In the case of the responses to the Test (PQ3) is recorded (and coded) on the answer sheet, the correct answers were identified so that the total scores and subscores could be worked out. No provision was made for correction for guessing (reasons to be discussed in Chapter 5).

The programmes were written in Fortran IV for use on the Univac 90/30, then tried for small samples of data cards and finally checked by calculations on electronic mini calculators. Since a great deal of data had to be handled for each calculation, the programmes had to be split up into several sub-programmes. These could then be run in shorter periods of time within each working day.

During interpretation of the results obtained from the analyses, new questions arose which were answered by slightly modified programmes.

4.3.4 General Comments

It was clear that the data collected in this research could not have been handled without computer assistance. The handling of data for computerization has been presented here, in reasonable detail, in order

(i) to illustrate how data processing was accomplished in this study;

(ii) to stimulate large scale educational research (through computer assistance) by individual researchers.

CHAPTER FIVE

5. STATISTICAL ANALYSES OF DATA FROM MATHEMATICS TEST AND ATTITUDE SCALE

5.1 INTRODUCTION

The two measures of attainment in mathematics and attitudes towards mathematics were central to the gathering of data in the present research design. The theoretical formulations leading to the development of the instruments for each of these measures have been presented in chapters two and three respectively.

The quality of an instrument may be assessed by

(i) a study of the instrument itself in terms of its specifications and all the considerations made in developing it;

(ii) statistical evidence arising out of an analysis of the data it provides.

In the present study a great deal of reliance has been placed on the former. However, some statistical evidence has also been presented but it must be recognized that such evidence was dependent upon the samples to which the original instruments had been administered (Conrad, 1951:253; Thorndike, 1951:604). For this reason and in view of the importance of the roles of these instruments, it was decided that further empirical evidence should be gathered in respect of the population being investigated. It was felt that this would not only enable a more accurate judgement of the findings arising from the research but also offer possibilities for future use of such instruments.

In this chapter statistical evidence in respect of the quality of the mathematics test and the mathematics attitude scale will be presented and discussed.

5.2 THE MATHEMATICS TEST (PQ3)

5.2.1 Scoring Procedures and Correction for Guessing

The response to each question on the multiple-choice format with five alternatives was punched directly on to the data card. Scoring was then done by computer where the correct responses were incorporated into the programme. It was therefore possible not only to retrieve subscores and total scores on the test for each individual but also the percentage response for each alternative. Thus, marker unreliability was eliminated and it was expected that the objectivity in scoring attained in this study would contribute positively to test reliability (Wesman, 1968).

On the question of correction for guessing, there seems to be no finality. In a recent extensive review of research studies on this matter, Diamond and Evans (1973:181) decided to "report the findings and let the reader make his own decision". In this context Traxler (1951:347) observed that

> "after more than three decades of experience with multiple-response tests, there is still not complete

agreement among test specialists concerning this question".

However, in this study, the decision not to use a correction formula for guessing was made (after careful consideration) for the following reasons:

(i) Pupils were advised against blind guessing.

(ii) The pupils were allowed sufficient time to consider all items, thus minimizing blind guessing. It was essentially a *power* test and not a speed test.

(iii) Correction for guessing is based on the assumption that all wrong answers and right answers are due to guessing (Diamond and Evans, 1973:181; Traxler, 1951:348; Ebel, 1965:225). It was felt that the distractors in the present test were reasonably functional so that they might be selected through incorrect reasoning or misinformation. In which case, the application of the correction formula would seriously overcorrect (Traxler, 1951:341).

In addition to these considerations in the preparation and administration of the test, it was argued that

(i) corrected scores have the same relative rank positions as the uncorrected scores,

(ii) the possibility of getting a respectable score on blind guessing alone was extremely small (Ebel, 1965:229) - especially with questions involving a large number of choices as was the case in this study,

(iii) correction complicates the scoring task and tends to lower the accuracy of the scores (Ebel, 1965:232).

5.2.2 Distribution of Test Scores

In interpreting reliability data, it is essential to consider both descriptive and statistical characteristics of the groups studied. In respect of the latter the number of cases, means, standard deviations (or variances) are of particular importance (Thorndike, 1951:607; Downie and Heath, 1970:101). Further, for any correlational analysis to be meaningful a knowledge of the manner of distribution of the observations is also essential. Table 5.1 and Figure 5.1 show the frequency distribution of the scores for the respective samples.

The test scores (as shown in Table 5.1 and Figure 5.1) range from a low of 2 to a high of 24 on the twenty four-item instrument. The relatively high standard deviations which ranged from 3,80 to 4,57 were indicative of the high degree of variability of the scores. Such variability was bound to contribute positively to the discriminating power and reliability of the test (Secondary Schools Examinations Council, 1964:41; Ebel, 1965:302).

It has been estimated that a standard deviation of one-sixth of the range between the highest possible score and the expected chance score (in this case 3,17) is generally satisfactory (Ebel, 1965:302). Although it was found that the standard deviation was somewhat smaller for Std 9 than for Std 10 or the whole sample, all values were appreciably higher than 3,17.

For meaningful and valid inferences to be drawn from statistical analyses of observations made on a sample,

(i) the sample has to be randomly selected (Lindquist, 1940:24; Ilersic, 1964:226);

TABLE 5.1

FREQUENCY DISTRIBUTION OF TEST SCORES

TEST SCORE	STD 9	STD 10	TOTAL
0	0	0	0
1	0	0	0
2	7	2	9
3	16	5	21
4	24	9	33
5	37	11	48
6	36	22	58
7	45	27	72
8	33	21	54
9	37	24	61
10	44	27	71
11	26	25	51
12	20	26	46
13	10	16	26
14	13	27	40
15	14	4	18
16	5	13	18
17	5	7	12
18	5	2	7
19	5	11	16
20	1	9	10
21	0	5	5
22	0	1	1
23	0	2	2
24	0	1	1
N	383	297	680
$\overline{x}$	8,69 (36,20%)	11,00 (45,83%)	9,70 (40,42%)
SD	3,80	4,57	4,31

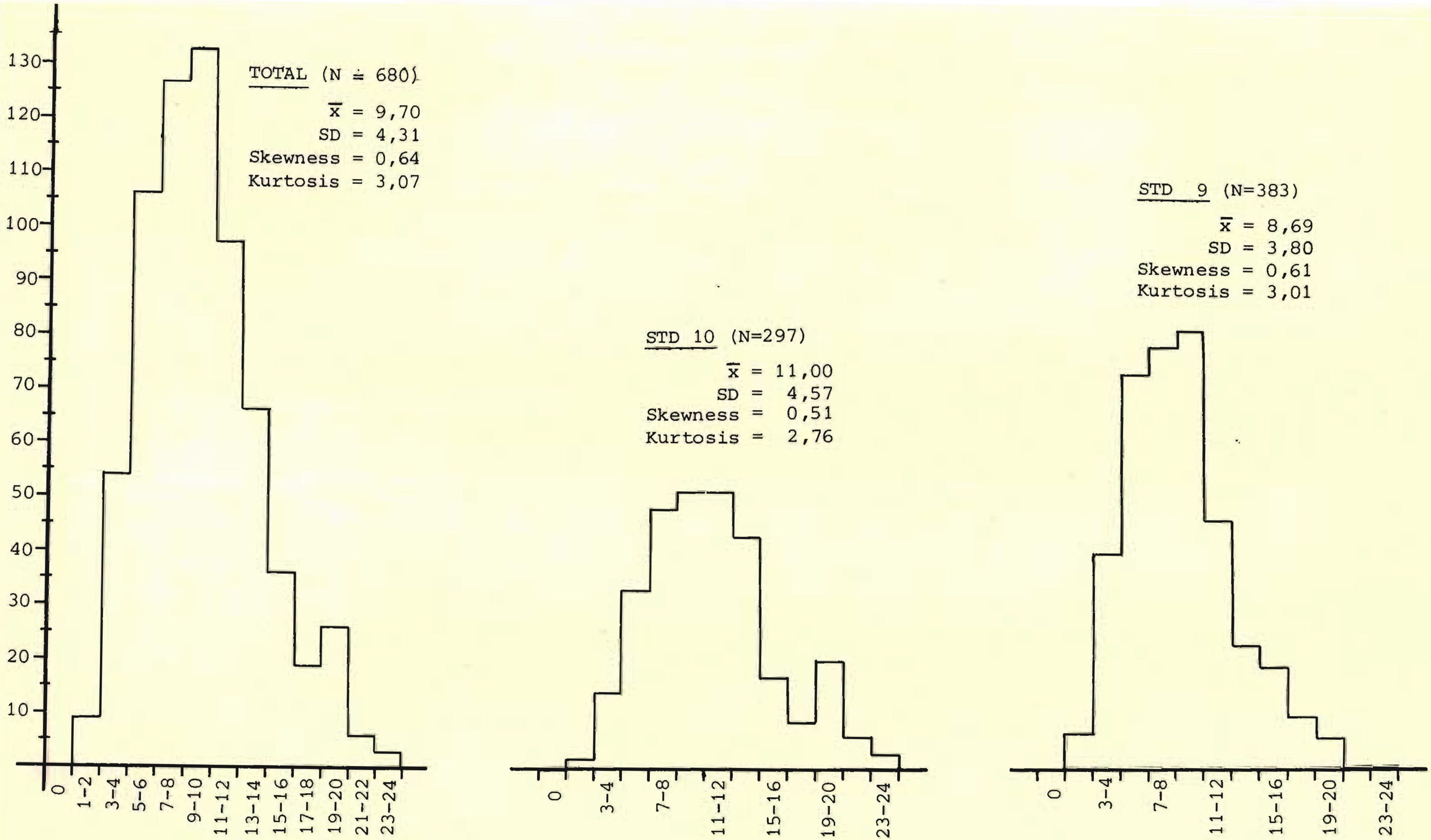


FIG. 5.1 FREQUENCY DISTRIBUTIONS OF MATHEMATICS TEST SCORES

(ii) the observations have to be reasonably normally distributed (DuBois, 1965:285).

In the present study provisions for the former had been made in the sampling (discussed in chapter 4). In respect of the latter a study of the distribution of the test scores revealed that 70,44% of the scores fell within one standard deviation of the mean, while in a perfectly normal distribution 68,26% fall within these limits. Table 5.2 shows the details for the other intervals. Further, values for Skewness and Kurtosis (shown in Fig. 5 - see Appendix 5 for details) were indicative of the closeness to a normal distribution. In general, it was clear that the distribution of scores for this test fairly closely approximated to that of a normal distribution. Mathematics achievement being one of the most crucial variables in this study, it was essential for the test scores to meet with this requirement for purposes of a correlational analysis (Nunnally, 1978:138).

TABLE 5.2

A COMPARISON OF THE DISTRIBUTION OF TEST SCORES AND NORMAL DISTRIBUTION

SD Intervals	Distribution of Test Scores	Perfectly Normal Distribution [x]
$^{-}1\sigma$ — $^{+}1\sigma$	70,44%	68,26%
$^{-}2\sigma$ — $^{+}2\sigma$	94,85%	95,46%
$^{-}3\sigma$ — $^{+}3\sigma$	99,56%	99,72%

[x] Ebel (1965:249)

The mean scores of the test ranged from 36,20% (8,69) to 45,83% (11,00). Although the test appeared to be somewhat difficult these means compared favourably with the highest total mean of 37,83% on the 69-item test used in the IEA study (Husén, 1967b:22-25). The standard error of the mean for the whole sample was 0,165 which yielded an unbiased estimate of the population mean of $9{,}70 \pm (2{,}58)(0{,}165) = 9{,}70 \pm 0{,}43$ ($p < 0{,}01$).

In general, it was felt that the distribution and variability of the scores were very satisfactory and were bound to contribute positively to the discriminating ability, reliability and (possibly) validity of the test.

5.2.3 Reliability

The reliability of a test is the consistency with which the test measures whatever it is intended to measure (Ebel, 1965:310; Fraser and Gillam, 1972:126). In this study, reliability estimates for the mathematics test were obtained in two ways : split-half method and Kuder-Richardson Formula 20.

5.2.3.1 Split-half Reliability

By this method reliability is measured through a correlation of two sets of scores. Since the items were organized in such a way that the even-numbered items were matched as far as possible with the odd-numbered items in terms of ability levels and difficulty (discussed in Chapter 2), the split-half method recommended itself. It was possible to treat the two halves of the test for purposes of correlation as two equivalent tests administered simultaneously. The Pearson Product-Moment correlation (see 3.1 of Appendix 5) was calculated for the three samples and corrected by the use of the Spearman-Brown correction formula (see 3.2.1 of Appendix 5). The details are presented in Table 5.3.

TABLE 5.3

RELAIBILITY COEFFICIENTS - MATHEMATICS TEST SCORES

SAMPLE	r	N	$\overline{x}$	SD	r (Spearman-Brown correction)
Std 9	0,63	383	8,69	3,80	0,77 (p < 0,001)
Std 10	0,67	297	11,00	4,57	0,80 (p < 0,001)
Total	0,67	680	9,70	4,31	0,80 (p < 0,001)

5.2.3.2 Kuder-Richardson Formula 20

The application of the Kuder-Richardson Formula 20 requires that the test be a power test and not a speed test (Thorndike, 1951:587). In this study the pupils were given enough time to consider every item. It was thus possible to use the item analysis data that was available for the total sample to get a second estimate of test reliability by the use of the Kuder-Richardson Formula 20 (see 3.2.3 of Appendix 5). While the split-half method depended on the choice of the particular split, the K-R formula 20 depended on the proportion of candidates responding to the items, the variance of the total test scores and the number of items in the test. The K-R formula 20 is a method of measuring not only the internal consistency but also the extent to which the test items are measuring the same general factor (mathematical ability) i.e. the extent to which the test posesses homogeneity of content (Thorndike, 1951:588; Schools Council, 1965:11; Fraser and Gillam, 1972:132). In this way a value of $r = 0,804$ ($p < 0,001$) was obtained for the test.

5.2.3.3 General

Though not a sufficient condition, reliability is a necessary condition for validity. Thus while statistical evidence is central to establishing reliability of a test, it does contribute to assessing validity. Ebel (1965:310) considers that

> "since a test must be reliable if it is to be valid, the statistical analysis of the quality of educational achievement test scores ordinarily places primary emphases on reliability of those scores".

Both the methods yielded a high reliability coefficient (r=0,80) which was significant ($p < 0,001$). This reliability value compared favourably with those obtained in the IEA study (Husén, 1967a:107) where it was found that the reliability coefficients on the 70-item test ranged from 0,732 to 0,958 for the different countries. It must be noted that the present study used a test about one third the size of the above test. Thus if the Spearman-Brown formula for predicting increase in reliability (See 3.2.2 of Appendix 5) resulting from lengthening a test by adding items like those in the original test is applied, the reliability coefficient rises to 0,923 (Thorndike, 1951:602).

In his research on the quality of standardized high school mathematics tests in USA, Petrosko (1978:143) evaluated 522 tests and observed that in more than 73% of the cases the internal consistency coefficient was below 0,70 (or was unreported).

In standardizing the Senior Aptitude Tests for Indian South Africans (SATISA), De Villiers (1977:36) reported reliability coefficients ranging from 0,57 to 0,83 (Std 9) and from 0,69 to 0,85 (Std 10) for the tests on Verbal Reasoning, Numerical Reasoning, Spatial Perception and Classification.

The size of the reliability coefficient which is acceptable depends on the context in which it is used. Kelly (cited by Thorndike, 1951:609) suggested a minimum correlation of 0,50 to evaluate level of group accomplishment. Since, in this study, group accomplishment in mathematics was of major concern, the significantly high reliability coefficient of 0,80 was accepted as a good indication of the internal consistency of the test.

The standard error of measurement (Se) (See 3.2.4 of Appendix 5) gives an indication of the absolute consistency of the test i.e. the degree of accuracy that can be expected in the test scores. The relatively high standard error of 1,927 (based on r=0,80 and SD=4,31) obtained for this test must be attributed to the high variability of the scores. However, it was found that this value was less than the estimated standard error (2,116) for a twenty four-item test. It must be pointed out that good tests may have larger probable errors than poor ones because of the greater variability in scores (Ebel, 1965:303).

In addition to the statistical evidence presented here, claims for test reliability may also be made on the basis of the several considerations made in the compilation and administration of the test (discussed earlier in section 2.2.3.6).

5.2.4 Validity

Validity refers to the extent to which a test measures what it sets out to measure. It is associated with a particular purpose and is indicative of how well the test serves the purpose for which it is used (Cureton, 1951:621). The purpose is associated with both the function to be appraised and the groups in which the appraisal is made. The present study concerned itself with the appraisal of mathematical ability (resulting from previous instruction in mathematics) among a randomly selected group of Std 9 and Std 10 pupils of mathematics.

Measurements of validity are essentially measurements of correlations between the test scores and criterion scores. This view is consistent with Cureton's (1951:625) operational conception of validity : "an estimate of the correlation between raw test scores and the 'true' (that is, perfectly reliable) criterion scores". One of the major problems with the measurement of validity is that

> "criterion scores that measure exactly the same thing as the test is intended to measure are seldom available, even in unreliable form, for most classroom tests of educational achievement" (Ebel, 1965:378).

In Petrosko's (1978:142) evaluation of some 522 standardized mathematics tests, "a test received the highest rating when criteria were closely related to the test's educational objectives" and it was found that in more than 83% of the cases either no study was cited at all or an irrelevant study was cited as criterion.

Hence, in the present study, crude estimates of validity were obtained by correlating the test scores with several 'criterion' scores which appeared to be most closely related to the test:

(i) Teacher's overall assessment of pupil's *performance* in mathematics ($r=0,50$; $p<0,001$).

(ii) Teacher's assessment of pupil's mathematical ability in terms of *objectives* ($r=0,49$; $p<0,001$).

(iii) Teacher's rating of pupil's level of attainment in terms of the various *content* areas (Algebra, Geometry and Trigonometry) ($r=0,47$; $p<0,001$).

(iv) Pupil's own *estimate of his performance* at the time of testing ($r=0,48$; $p<0,001$).

(v) Pupil's *general ability* score (combined verbal and non-verbal score on GTISA) ($r=0,59$; $p<0,001$).

It was clear from the above correlations that there was a substantial, significant relationship between the mathematics test scores and the above 'criterion' scores, but this was far from a complete relationship. It was recognized that the criterion scores were themselves not perfectly reliable measures of the attributes assessed by the test. Therefore, to compensate for any possible unreliability an attempt was made to estimate validity by combining two measures in order to yield a correlation with the 'true' criterion score (Cureton, 1951:680; Fraser and Gillam, 1972:133). For this purpose intercorrelations of the sets of scores were computed (See Table 5.4).

TABLE 5.4

INTERCORRELATIONS OF MATHEMATICS TEST SCORES AND CRITERION SCORES

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
(i) Teacher's rating of overall performance	-	0,83	0,82	0,71	0,42	0,50
(ii) Teacher's rating in terms of objectives		-	0,92	0,64	0,39	0,49
(iii) Teacher's rating in terms of content-areas			-	0,64	0,36	0,47
(iv) Pupil's estimate of his performance				-	0,37	0,48
(v) Pupil's general ability (IQ)					-	0,59
(vi) Mathematics Test						-
All correlations significant : $p<0,001$.						

It is clear from Table 5.4 that the teachers' ratings of the pupils in terms of

overall performance,

performance in terms of objectives,

and performance in terms of content areas

showed a high degree of consistency. The intercorrelations which ranged from 0,82 to 0,92 were also indicative of a substantial overlap in the assessments.

Under these circumstances, it was sufficient to use only one of these ratings (for combining) to yield estimates of validity. 'Overall performance' was chosen because its correlation with the test was slightly higher. The substantial relationship between the IQ scores and the mathematics test (MT) scores ($r=0,59$) showed that the general ability (IQ) which was a combined score of verbal and non-verbal ability was accounting for some 35% of the variance in the test scores. IQ was therefore used to estimate validity. In addition, pupils' estimates of their levels of performance which correlated substantially with the MT scores ($r=0,48$ $p < 0,001$) was also used.

Using Cureton's (1951:680) method for estimation of validity, combinations of

(a) rating of overall performance by teachers,

(b) estimate of performance level by pupil,

and (c) IQ (combined verbal and non-verbal - GTISA)

were used to estimate validity coefficients for

(d) the Mathematics Test.

The following estimates of validity coefficients (see 3.3 of Appendix 5) emerged:

$$r_{d(a,b)} = 0,58 \qquad (p < 0,001)$$
$$r_{d(a,c)} = 0,70 \qquad (p < 0,001)$$
$$r_{d(b,c)} = 0,77 \qquad (p < 0,001).$$

These values compare very favourably with those found in other studies. The validity coefficients obtained by correlating total scores on GTISA with examinations ranged from 0,36 to 0,60 (NBESR, 1968). The correlations between total scores on SATISA (Verbal Reasoning, Numerical Reasoning, Spatial Perception and Classification) with normalised examination marks in mathematics ranged (in general) from 0,20 to 0,41 (DeVilliers, 1977:63-66). In a Schools Council (1970) study correlations between experimental mathematics tests and GCE and CSE examinations ranged from 0,523 to 0,647. In an earlier Schools Council (1965:12-13) study crude estimates of validity using teachers' forecasts and ranking ranged from 0,49 to 0,72. Some statistical evidence for the IEA tests (Husén, 1967a:108) was obtained by a follow-up study of the performance in GCE for different examination boards in England. A value of 0,65 was reported as being representative of the data taken as a whole. Nunnally (1978:90) considers it reasonable

> "to expect only moderate correlations between a criterion and either an individual predictor test or combination of predictor tests ....
> people are far too complex to permit a highly accurate estimate of their proficiency in most performance-related situations from any practicable collection of test materials".

In view of the above it was considered that the validity estimates ranging from 0,58 to 0,77 in the present study were highly satisfactory.

Further, evidence of construct validity came from the specifications used in compiling the test. We recall that the items were selected to test three hierarchical levels of mathematical ability:

(i) lower level - knowledge and skills;

(ii) comprehension level;

(iii) higher abilities - selection-application, and analysis-synthesis.

The results obtained in terms of difficulty of items shown by mean percentage of sample (N=680) responding correctly to each of the sets of items for the three levels were as follows:

(i) 58,46%

(ii) 35,83%

(iii) 26,97% .

This pattern was remarkably consistent with the theoretical formulation of hierarchical order (Bloom et al, 1956:18) and the relationship between complexity of behaviour and facility of problem-solving, which formed the basis for the specifications of the test (discussed fully in chapter 2). This finding also lent support to the assumption that the items were, in general, testing a range of objectives from low to high.

The mean scores on the test for the three levels were also compared. The details are shown in Table 5.5. The significant differences between the mean scores show that the items testing lower level objectives were easier to attain than those testing higher level objectives. This served to support further the equivalent nature of item complexity and item difficulty as postulated by Bloom et al (1956) and hence the assumption underlying the hierarchical structure of a taxonomic classification. This result is consistent with findings of other research studies (Smith, 1968; Stoker and Kropp, 1964; Schools Council, 1970; Moodley, 1975). A further examination of means at each level of objectives for HG amd SG revealed that the pattern remained invariant

TABLE 5.5

COMPARISON OF MEAN SCORES IN RESPECT OF LEVELS OF OBJECTIVES

LEVELS OF OBEJCTIVES	N	$\bar{x}$	DIFFERENCE BETWEEN MEANS	r	z-SCORE (Correlated data)
(a) Lower Level Objectives (Items 1-8)	680	4,68			
			1,81	r_{ba}=0,60	28,91 (p < 0,001)
(b) Comprehension Level (Items 9-16)	680	2,87			
			0,71	r_{bc}=0,47	10,52 (p < 0,001)
(c) Higher Level Objectives (Items 17-24)	680	2,16			

and at each level the SG score was significantly ($p < 0{,}001$) lower than the HG score.

As observed earlier, statistical evidence in respect of validity must be interpreted with caution in the absence of perfectly reliable criterion measures. However, apart from the statistical evidence, the claims for validity in this study must also rest on the soundness and appropriateness of the methods employed in the initial development of the test items for the IEA study (Husén, 1967a) and on the methods of selection of items and test specifications employed by this researcher. All the considerations made in the compilation of the test items (discussed in Chapter 2) must be interpreted as contributing positively to the degree of validity.

5.2.5 Item Analysis

Item analysis provides quantitative data on the quality of the test items in respect of their difficulty levels and discriminating power.

Such data obtained from the IEA study (Husén, 1967b:312-358) have already been presented in chapter two for those items selected for use in this study.

However, it was essential to recognize that item analysis data are dependent on the characteristics of the sample tested (Conrad, 1951:253). Therefore, it was decided that new data should be gathered after the final administration of the test. This served the useful purpose of providing evidence concerning the quality of the test in respect of the sample studied.

The first step in item analysis is to identify the upper and lower criterion groups. Kelly (cited by Ebel, 1965:349) suggested 27% as the optimal size of each group which would ensure that the groups are as large as possible and (at the same time) as different as possible. In this study the upper and lower 27% groups were identified and used in computing the Difficulty and Discrimination Indices (See 4 of Appendix 5).

5.2.5.1 Difficulty

The index of item difficulty (F) is given by the percentage of correct item responses for both upper and lower groups. Thus the higher the value of this index the easier the item.

The difficulty indices (also known as facility indices) for the mathematics test are shown in Table 5.6. The difficulty (F) values range from 13 to 83 (for total sample) with a mean of 42,42. It was considered that the wide range of items from the easy to the difficult ought to have contributed significantly to the high reliability of the test scores (Ebel, 1965:363). Since this was a power test designed to

TABLE 5.6

DIFFICULTY INDICES FOR THE MATHEMATICS TEST ITEMS

ITEM	STD 9	STD 10	TOTAL SAMPLE
1	34	45	38
2	65	76	70
3	80	86	83
4	71	82	76
5	69	81	75
6	47	47	47
7	33	42	39
8	27	53	40
9	61	66	62
10	27	32	29
11	40	45	42
12	13	26	19
13	25	42	34
14	33	47	44
15	22	28	24
16	53	64	58
17	30	43	36
18	34	45	37
19	30	40	33
20	39	47	45
21	27	34	31
22	11	15	13
23	17	28	21
24	17	24	22

test at least three levels of mathematical abilities, it was expected that the F values would range from low to high.

An examination of the pattern of the range of values revealed that there was a consistent increase in difficulty with the increase in level of complexity. The mean difficulty values for the three levels are shown in Table 5.7.

Moreover, the difference between any two consecutive means for a particular group was significant at $p < 0,001$. In addition, the consistent difference between means of Std 9 and Std 10 at each level showed that the items were somewhat more difficult for the former than for the latter.

TABLE 5.7

MEAN DIFFICULTY VALUES FOR THE THREE LEVELS OF ABILITIES

ABILITY LEVELS	STD 9	STD 10	TOTAL
Lower Level (Knowledge and Skills)	53,25	64,00	58,50
Comprehension Level	34,25	43,75	39,00
Higher Level (Selection-Application, Analysis and Synthesis)	25,63	34,50	29,75
All Levels	37,71	47,42	42,42

Since difficulty values (as supplied by IEA study) were not regulated during the course of selection of items for this study, it was clear that this finding further confirmed the relationship between complexity of behaviour and facility of problem solving as postulated by Bloom et al (1956).

It was interesting to note that the mean difficulty value (42,42) closely approximated to the mean score (40,42%) on the test. This observation was consistent with educational measurement theory that

> "the mean score on a test is determined completely by the mean difficulty of the items composing it" (Ebel, 1965:300).

In general, the pupils' reactions to the test were considered to be favourable as a significantly ($\chi^2 = 1020{,}37$; $p < 0{,}001$) high proportion (64,56%) regarded the test as being "about right". The reactions were as follows:

Too easy	:	0%
Easy	:	4,71%
About right	:	64,56%
Difficult	:	28,38%
Too difficult	:	2,35% .

5.2.5.2 Discrimination

The discrimination index (D) of an item shows the extent to which it differentiates between the high scorers and the low scorers - in the present study the high and low groups comprised the top 27% and the bottom 27% on the total test scores. Theoretically, the D values may range from -1 to 1 i.e. from perfectly non-discriminating to perfectly discriminating. (See 4 of Appendix 5).

The D values for the present test (shown in Table 5.8) ranged from 0,07 to 0,66 with a mean discrimination index of 0,44 for the total sample. More than 83% of the items had a D value of 0,30 and higher. In accordance with limits suggested by Ebel (1965:283) for evaluation of discrimination indices the items in the test compared very favourably:

High (0,40 and up)	:	15 items
Moderate (0,20 to 0,39)	:	6 items
Low (0,01 to 0,19)	:	3 items
Zero or Negative	:	0 items .

TABLE 5.8

DISCRIMINATION INDICES FOR THE MATHEMATICS TEST ITEMS

ITEM	STD 9	STD 10	TOTAL
1	0,58	0,65	0,64
2	0,42	0,21	0,36
3	0,21	0,15	0,19
4	0,50	0,36	0,44
5	0,50	0,32	0,43
6	0,54	0,71	0,66
7	0,55	0,63	0,61
8	0,41	0,69	0,61
9	0,71	0,69	0,68
10	0,25	0,46	0,38
11	0,49	0,57	0,54
12	0,13	0,40	0,28
13	0,38	0,49	0,51
14	0,32	0,60	0,45
15	0,37	0,44	0,40
16	0,53	0,52	0,57
17	0,40	0,44	0,43
18	0,36	0,35	0,34
19	0,46	0,60	0,51
20	0,29	0,51	0,39
21	0,41	0,57	0,47
22	0,01	0,13	0,07
23	0,11	0,31	0,19
24	0,30	0,39	0,32

Several other researchers have suggested that D values of 0,20 and higher are acceptable (Davis cited by Klein, 1972:44; Macintosh and Morrison cited by Behr, 1971:131; Furst, 1958:314; Garrett and Woodworth, 1964:368).

In addition to the item analysis data, it was found that the test differentiated between HG and SG pupils at both levels (See Table 5.9).

TABLE 5.9

DIFFERENCE BETWEEN MEAN TEST SCORES FOR HG AND SG OVER THE TWO LEVELS

LEVEL	$\bar{x}$(SG)	$\bar{x}$(HG)	DIFFERENCE	z-SCORE
Std 9 (N=383)	6,72 (N=191)	10,65 (N=192)	3,93	36,06 ($p < 0,001$)
Std 10 (N=297)	8,78 (N=193)	15,13 (N=104)	6,35	31,67 ($p < 0,001$)

At both levels the HG performed significantly better than the SG. Since, in general, the more capable students are placed in the HG classes, the above findings were indicative of the discriminating power of the test.

Further, the test also differentiated between Std 9 and Std 10 pupils in each of the two grades (See Table 5.10). The Std 10 pupils performed significantly better than the Std 9 pupils in each of the grades. Since the former had had one more year of schooling and experience with mathematics, this difference was expected. Thus, this was interpreted as an indication of the discriminating ability of the test.

TABLE 5.10

DIFFERENCE BETWEEN MEAN TEST SCORES FOR STD 9 AND STD 10 OVER THE TWO GRADES

GRADES	$\bar{x}$ (STD 9)	$\bar{x}$ (STD 10)	DIFFERENCE	z-SCORE
SG (N=384)	6,72 (N=191)	8,78 (N=193)	2,06	24,38 ($p < 0,001$)
HG (N=296)	10,65 (N=192)	15,13 (N=104)	4,48	19,91 ($p < 0,001$)

From all the above evidence it was concluded that a fairly highly discriminating mathematics test had been compiled for this study.

5.2.5.3 Response Options

In the present test the multiple-choice format with five response options was used : one correct response and four distractors. Percentage response to each distractor was calculated in order to gain some insight into the extent to which these were functional. Table 5.11 shows the percentage responses. It was also found that of the 96 distractors, 13 attracted below 3% response each (i.e. below 20 responses). The following is a distribution of percentage responses for distractors:-

%-Response	No. of Distractors
40^+	2
30 - 39	6
20 - 29	14
10 - 19	34
3 - 10	27
below 3	13

It was interesting to note that 10 of the 13 distractors were found to be among the items testing lower level obejctives and none were among those testing higher abilities. This may be explained by the fact that knowledge and skill items (low level) tend to be easier and the correct responses are more apparent. There is also the difficulty of finding attractive response options at this level. In general, a study of the distribution of the percentage response to the distractors showed that they were fairly functional.

TABLE 5.11

PERCENTAGE RESPONSE TO EACH RESPONSE OPTION

ITEM	O (OMIT)	RESPONSE OPTIONS 1	2	3	4	5
1	0,44	21,03	32,21	4,71	8,09	33,53
2	0,59	18,09	4,26	2,06	73,68	1,32
3	0	0,15	2,06	1,76	86,62	9,41
4	0,29	2,35	81,03	1,47	2,21	12,65
5	0,29	3,24	4,85	1,18	11,62	78,82
6	2,35	15,59	42,65	22,94	10,29	6,18
7	0,88	34,85	29,56	15,74	0,74	18,24
8	7,79	10,59	16,91	7,79	36,47	20,44
9	0,44	12,50	3,38	1,47	65,15	17,06
10	1,18	26,62	45,74	1,62	23,38	1,47
11	6,03	15,00	9,85	16,76	40,59	11,76
12	1,91	17,06	3,97	7,79	14,12	55,15
13	8,97	5,88	15,15	32,21	28,38	9,41
14	2,21	18,09	12,06	15,44	16,91	35,29
15	0,59	28,53	7,50	18,24	17,79	27,35
16	0,44	58,24	10,59	3,53	17,06	10,15
17	5,29	20,29	33,97	33,82	3,82	2,79
18	4,12	11,62	15,44	12,06	15,15	41,62
19	6,47	30,44	12,94	29,41	18,53	2,21
20	3,82	38,82	28,09	4,26	6,47	18,53
21	3,24	24,12	17,50	8,53	8,68	37,94
22	5,74	32,06	11,47	20,15	9,41	21,18
23	6,18	23,38	28,82	13,09	20,44	8,09
24	3,09	36,03	5,74	16,03	5,15	33,97

5.2.6 General Comments

In the foregoing sections statistical evidence has been presented and discussed in respect of reliability, validity, difficulty, discrimination and response options in order to present a complete picture of the quality of the test.

Taking into account both the considerations made in the development and compilation of the test items and the statistical evidence presented, it was considered that a highly reliable and valid test of mathematical abilities had been compiled. It must also be pointed out that the items were capable of testing a range of abilities from low to high. It was clearly demonstrated that the test differentiated between three taxonomic levels of mathematical abilities. Further, item analyses revealed that the test was of a reasonable level of difficulty, that the items were highly discriminating and that the distractors were fairly functional.

In view of the above qualities of the test, it was felt that the provision of norms would increase the usefulness of the test to researchers and teachers. The norms are presented in chapter seven.

5.3 THE ATTITUDE SCALE

5.3.1 Distribution of Scores

As mentioned earlier, a Likert-type scale with 48 items was used to obtain attitude scores through summated ratings. The whole scale was conceived as 6 eight-item subscales. Each item was scored on a three-point scale (2, 1, 0) with unfavourable or negative items being scored in the reverse. Thus the scores on the total scale could range theoretically from 0 to 96, while those for each subscale could range from 0 to 16.

As in the case of the mathematics test, it was necessary to present the statistical characteristics of the sample studied, in order to study the reliability of the attitude scores. Table 5.12 shows the frequency distribution of the attitude scores together with the number of cases, means and standard deviations. Figure 5.2 shows the distributions for the respective samples.

TABLE 5.12

FREQUENCY DISTRIBUTION OF ATTITUDE SCORES

SCORES	STD 9	STD 10	TOTAL SAMPLE
0 - 5	0	0	0
6 - 10	0	0	0
11 - 15	0	0	0
16 - 20	0	0	0
21 - 25	0	0	0
26 - 30	0	0	0
31 - 35	1	2	3
36 - 40	3	2	5
41 - 45	2	7	9
46 - 50	10	10	20
51 - 55	17	8	25
56 - 60	31	22	53
61 - 65	44	20	64
66 - 70	67	55	122
71 - 75	80	59	139
76 - 80	82	63	145
81 - 85	38	39	77
86 - 90	8	9	17
91 - 95	0	1	1
N	383	297	680
$\bar{x}$	70,11	70,48	70,27
SD	9,78	10,93	10,30
Skewness*	-0,79	-0,96	-0,87
Kurtosis*	3,49	3,84	3,71

* (see 9.1 and 9.2 of Appendix 5)

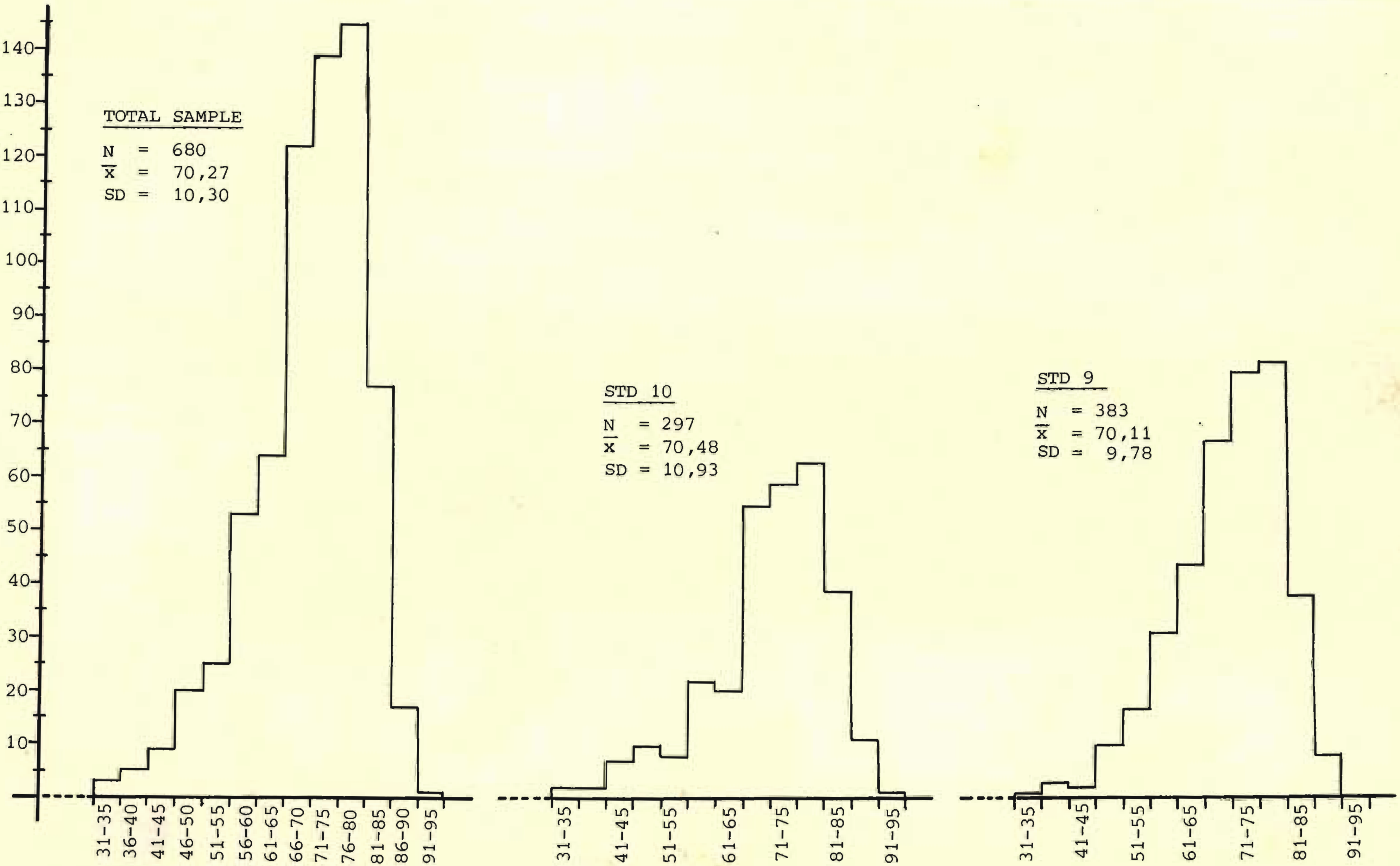


FIG. 5.2 FREQUENCY DISTRIBUTIONS OF ATTITUDE SCORES

It was observed that the distribution was somewhat negatively skewed; which was not unexpected for attitude scores for a group of pupils who were all taking mathematics. However, the standard deviations ranging from 9,78 to 10,93 for the range 31-92 were indicative of the reasonable amount of variability in the attitude scores. The standard error of the mean (total sample) was 0,395 and yielded an unbiased estimate of the population mean of 70,27 ± 1,02 ($p < 0,01$).

The means and the standard deviations for the subscores were also examined (See Table 5.13). It was observed that

(i) though slight, the scores for the Std 10 pupils were almost always showing greater variability than those for the Std 9;

(ii) in all three groups the dimension 'enjoyment of mathematics' showed greater variability than any of the others.

TABLE 5.13

ATTITUDE SCORES - MEANS AND STANDARD DEVIATIONS FOR THE SUBSCORES

	STD 9 (N=383)		STD 10 (N=297)		TOTAL (N=680)	
	$\bar{x}$	SD	$\bar{x}$	SD	$\bar{x}$	SD
Mathematics Teaching	11,61	2,22	11,50	2,44	11,56	2,32
School Learning	10,23	2,28	10,64	2,43	10,41	2,36
Difficulty of Mathematics	12,31	2,93	12,38	2,91	12,34	2,92
Importance of Mathematics	11,80	2,62	11,58	2,76	11,70	2,68
School and Life	12,49	2,32	12,63	2,40	12,55	2,36
Enjoyment of Mathematics	11,67	3,86	11,76	4,48	11,71	4,14
Total Score	70,11	9,77	70,48	10,93	70,27	10,30

5.3.2 Reliability

As explained in section 3.3.4.6, the items were so organized that there was balance between the first half and second half of the scale. Each half contained one half of the number of items in each subscale. Therefore, the split-half method of calculating reliability was employed by correlating the scores on the first 24 items with the scores on the remaining 24 items.

The Pearson Product-Moment correlation was calculated separately for Std 9, Std 10 and the total sample. The Spearman-Brown correction was applied to get a reliability estimate of the whole scale. The details are set out in Table 5.14.

TABLE 5.14

RELIABILITY COEFFICIENTS - ATTITUDE SCALE

SAMPLE	r	N	$\bar{x}$	SD	r (Spearman-Brown correction)	
STD 9	0,64	383	70,11	9,78	0,78	($p < 0,001$)
STD 10	0,72	297	70,48	10,93	0,84	($p < 0,001$)
TOTAL	0,67	680	70,27	10,30	0,80	($p < 0,001$)

A further estimate of reliability was obtained by the application of Cronbach's Coefficient Alpha (Guilford, 1954:385) using the variances of the total scores and that of the two parallel half scores (See 7 of Appendix 5). In this way an alpha coefficient of 0,804 was obtained for N=680.

These values for reliability compare very favourably with those obtained in other studies. In constructing and validating an instrument measuring attitudes towards mathematics, Michaels and Forsyth (1977:1047) found reliability estimates ranging from 0,51 to 0,78. In another validation study of a reading attitude scale, Roettger et al (1979:140) reported a split-half reliability estimate of 0,52. Aiken (1979a) found alpha coefficients for four part scores on a mathematics attitude scale to range from 0,50 to 0,86 and from 0,81 to 0,91 for total scores.

Corcoran and Gibb (1961:120-121) have noted that measurements of attitudes seem to be less reliable than those of achievement areas, and that measures of internal consistency

> "often yield reliabilities in the .70's and even in the .80's with well-constructed attitude scales in which a well-defined attitude object is appraised".

The reliability estimates obtained in this study, ranging from 0,78 to 0,84, comfortably surpassed Kelly's criteria for minimum acceptable reliability coefficients "to evaluate level of group accomplishment" (Thorndike, 1951:609).

In addition, it was found that the correlation between scores on the 21 negative items and the 27 positive items in the attitude scale was 1,00 (rounded off to 2 decimal places from the actual reading of 0,9999801...). This almost perfect relationship showed that the pupils responded honestly and consistently to the items in the scale. Adams and Von Brock (1967:248) used the same method and reported a correlation of 0,70 for their 35-item attitude scale. It was thus felt that the attitude responses in the present study were highly distortion-free.

In general, it was considered that these reliability estimates were acceptable and that they were indicative of the high degree of internal consistency with which the scale measured.

5.3.3 Intercorrelations of the Subscores

Intercorrelations of the subscores were computed to see the degree to which the various dimensions overlapped. The intercorrelations are shown in Table 5.15.

TABLE 5.15

INTERCORRELATIONS OF SUBSCORES AND TOTAL SCORES

	1	2	3	4	5	6	7
1. Mathematics Teaching	-						
2. School Learning	0,26	-					
3. Difficulty of Maths.	0,19	0,09	-				
4. Importance of Maths.	0,11	0,09	0,34	-			
5. School and Life	0,17	0,18	0,19	0,14	-		
6. Enjoyment of Maths.	0,28	0,14	0,63	0,32	0,20	-	
7. Total Score	0,51	0,43	0,73	0,56	0,48	0,80	-
(For $r > 0,13$; $p < 0,01$)							

The 'difficulty' and 'enjoyment' variables were substantially related (r=0,63) - the one was accounting for 40% of the variance in the scores of the other. The generally low correlations were interpreted as the existence of a fair degree of independence between the dimensions measured (Michaels and Forsyth, 1978; Aiken and Dreger, 1961). Moreover, it could be safely assumed that no two dimensions were measuring identical variables (Aiken, 1974:70).

5.3.4 Item-Subscore Correlations

Each item score was correlated with the total score of each of the subscales. Since the scores on the items were not dichotomous and were on a multi-point scale (2,1,0), the product-moment correlation was used (Nunnally, 1978:132-139). Table 5.16 shows the correlations together with the items belonging to each subscale.

A study of the correlations in Table 5.16 revealed a generally consistent pattern: the items constituting a particular subscale correlated generally more highly with the total score for that dimension than with the other subscores. Such correlations were also generally high and significant ($p < 0{,}001$). For example, the average correlation of the 8 items in the dimension, mathematics teaching, with the total score on this dimension was 0,44 ($p < 0{,}001$); while that for the remaining 40 items with the same dimension was 0,11 ($p < 0{,}01$). Table 5.17 presents the average correlations for the 6 subscales (dimensions).

From the patterns in the correlations observed in Tables 5.16 and 5.17 it was inferred that the items in a particular dimension were measuring, in general, an attribute similar to that measured by items in *that* subscale but somewhat dissimilar to that measured by items not in it. This was also interpreted as evidence of construct validity (Shaw and Wright, 1967:120). Several items in dimension 6 were correlating highly with dimension 3. This was due to the overlap between these dimensions which was observed in the intercorrelations between the subscales (See Table 5.15).

TABLE 5.16

ITEM CORRELATIONS WITH SUBSCORES

SUBSCALE →	1	2	3	4	5	6
	Maths. Teaching:	School Learning:	Difficulty of Maths.:	Importance of Maths.:	School and Life :	Enjoyment of Maths.:
ITEM ↓	1, 7, 13, 19, 25, 31, 37, 43.	2, 8, 14, 20, 26, 32, 38, 44.	3, 9, 15, 21,27, 33, 39, 45.	4, 10, 16, 22, 28, 34, 40, 46.	5, 11, 17, 23, 29, 35, 41, 47.	6, 12, 18, 24, 30, 36, 42, 48.
1	0,50	0,14	0,12	0,15	0,11	0,14
2	-0,10	0,18	-0,16	-0,07	-0,11	-0,15
3	0,07	0,05	0,56	0,15	0,14	0,25
4	-0,03	0,01	-0,01	0,40	-0,05	-0,08
5	0,06	0,14	0,13	0,14	0,57	0,14
6	0,20	0,09	0,50	0,18	0,11	0,70
7	0,36	0,19	0,15	0,17	0,18	0,17
8	0,04	0,44	0,04	0,07	0,10	0,05
9	0,14	0,10	0,56	0,19	0,15	0,25
10	0,04	0,14	0,18	0,51	0,15	0,18
11	0,18	0,17	0,09	0,04	0,62	0,12
12	0,19	0,10	0,49	0,24	0,12	0,78
13	0,51	0,18	0,12	0,02	0,15	0,22
14	0,08	0,46	0,04	0,05	0,09	0,04
15	0,12	0,09	0,52	0,19	0,08	0,27
16	0,09	0,01	0,16	0,50	0,01	0,12
17	0,16	0,14	0,04	0,02	0,52	0,14
18	0,13	0,12	0,39	0,22	0,10	0,68
19	0,53	0,11	0,07	0,03	0,04	0,13
20	0,21	0,34	0,05	0,04	0,12	0,11
21	0,06	-0,03	0,51	0,22	0,05	0,22
22	0,08	0,06	0,24	0,55	0,13	0,28
23	0,19	0,12	0,12	0,09	0,63	0,17
24	0,24	0,13	0,56	0,31	0,16	0,85
25	0,33	0,13	0,07	0,04	0,04	0,03
26	0,23	0,39	0,10	0,05	0,11	0,09
27	-0,05	-0,10	0,40	0,06	0,00	0,11
28	0,04	0,05	0,12	0,56	0,03	0,10
29	0,00	0,06	0,16	0,05	0,42	0,11
30	0,24	0,14	0,53	0,30	0,17	0,78
31	0,36	0,03	0,00	-0,02	0,03	0,04
32	0,11	0,42	0,08	0,00	0,11	0,08
33	0,10	0,05	0,61	0,19	0,17	0,40
34	0,03	0,03	0,24	0,52	0,10	0,22
35	-0,00	-0,02	0,01	0,09	0,26	-0,06
36	0,25	0,11	0,47	0,27	0,19	0,74
37	0,61	0,15	0,16	0,08	0,07	0,19
38	0,12	0,42	0,07	0,05	0,10	0,08
39	0,17	0,06	0,54	0,19	0,10	0,43
40	0,10	-0,02	0,30	0,44	0,09	0,34
41	0,01	0,04	0,01	0,01	0,37	0,05
42	0,17	0,08	0,21	0,07	0,17	0,48
43	0,30	0,04	-0,01	-0,02	0,02	0,05
44	0,16	0,43	0,08	0,08	0,06	0,14
45	0,16	0,13	0,61	0,27	0,13	0,69
46	0,16	0,04	0,21	0,52	0,14	0,23
47	0,06	0,02	0,19	0,09	0,28	0,11
48	0,18	0,02	0,42	0,23	0,12	0,67

Correlations for items within the respective subscale are underlined.

TABLE 5.17

AVERAGE CORRELATIONS OF ITEMS IN EACH SUBSCALE AND OF REMAINING ITEMS WITH THAT SUBSCALE (DIMENSION)

DIMENSION (SUBSCALE)	8 ITEMS IN SUBSCALE	REMAINING 40 ITEMS
1. Mathematics Teaching	0,44	0,11
2. School Learning	0,39	0,08
3. Difficulty	0,54	0,17
4. Importance	0,50	0,11
5. School and Life	0,46	0,09
6. Enjoyment	0,71	0,16

5.3.5 Relative Attitude Index

While the correlational analyses revealed relationships in respect of subscores, total scores and item responses, the degree of response to each item was not known. In order to ascertain this a relative attitude index (RAI) was computed for each item. For this purpose the researcher used a modified version of Onibokun's (1974:195) Relative Habitability Index as follows:

$$\text{RAI}\ (\%) = \frac{\sum_{i=1}^{3} i.n_i}{3\sum_{i=1}^{3} n_i} \times 100$$

where n = number of responses to each of the 3 categories (agree, uncertain, disagree)

i = 1, 2, 3.

Accurate information regarding each test item is obtained by caluclating the Difficulty and Duscrimination indices but little information is forthcoming for the individual attitude item. The IEA study (Husén, 1967a:116-122) provided a crude measure of the 'favourableness' of each item by reporting "the percentage 'favourable' response". It is not clear how this percentage was arrived at and whether it took into account the responses to all three categories (agree, uncertain, disagree). The use of the RAI in this study was an attempt to provide evidence regarding the responses to each item. In order to employ this formula the following weights had to be used: disagree = 1, uncertain = 2, agree = 3. This obviated the lowering of accuracy resulting from weighting disagree = 0 etc. The RAI for each item of the attitude scale is shown in Table 5.18.

It follows from the theoretical formulation that the values of RAI can range from 33,33% (where each response is weighted 1) to 100% (where each response is weighted 3). However, the observed values for RAI ranged from a low of 43,23% to a high of 96,42%. The items placed into three equal categories (high, medium and low) according to the RAI were distributed as follows:

78 - 100	(high)	-	36 items
55 - 77	(medium)	-	11 items
33 - 54	(low)	-	1 item.

It was clear that the attitudes were, in general, more favourable than unfavourable. This was consistent with the somewhat negatively skewed distribution of attitude scores observed earlier on.

TABLE 5.18

RELATIVE ATTITUDE INDICES - ATTITUDE SCALE

ITEM NO.	RESPONSE WEIGHTS 1	2	3	RAI %
1	75	53	552	90,05
2	188	237	255	69,95
3	104	206	370	79,71
4	184	127	369	75,74
5	77	134	469	85,88
6	82	119	479	86,13
7	19	35	626	96,42
8	317	178	185	60,20
9	59	87	534	89,95
10	113	161	406	81,03
11	84	111	485	86,32
12	95	111	474	85,25
13	81	80	519	88,14
14	104	109	467	84,46
15	43	138	499	89,20
16	29	173	478	88,68
17	29	100	551	92,25
18	185	130	365	75,49
19	134	44	502	84,71
20	79	141	460	85,34
21	63	210	407	83,53
22	91	187	402	81,91
23	50	104	526	90,00
24	129	169	382	79,07
25	19	27	634	96,81
26	37	95	548	91,71
27	48	163	469	87,30
28	95	131	454	84,26
29	81	142	457	85,10
30	47	136	498	88,82
31	70	156	454	85,48
32	252	213	215	64,85
33	43	90	547	91,37
34	227	319	134	62,11
35	75	182	423	83,73
36	70	100	510	88,24
37	239	190	251	67,25
38	111	47	522	86,81
39	181	172	327	74,31
40	28	87	565	92,99
41	234	228	218	65,88
42	199	168	313	72,25
43	538	82	60	43,23
44	230	148	302	70,20
45	113	113	454	83,38
46	23	157	500	90,05
47	19	47	614	95,83
48	86	200	394	81,76

5.3.6 Validity

It has been recognized that, for the various procedures of attitude measurement, the problem of validity (i.e. "the degree to which the scale measures what it is supposed to measure" (Shaw and Wright, 1967:17)) has not been completely solved. There are two major approaches to validating attitude scales (Corcoran and Gibb, 1972:120):-

(i) a logical judgement that the information obtained is relevant to the attitude presumed to be measured;

(ii) validation of attitude measures in terms of the extent to which the responses are related to outside criterion measures.

The former approach refers to content validity and involves a subjective, judgemental procedure. In this regard the claims for validity of the attitude measure used in this study must rest on

(i) the procedures involved in the original development and use of the items in the IEA (Husén, 1967a) and Aiken (1979a) studies (discussed fully in chapter three).

(ii) the manner in which the items were selected and compiled for this study (also discussed in chapter three).

In addition to a logical judgement of the scale, statistical evidence for validity was obtained by correlations with criterion-related measures.

5.3.6.1 Criterion-related Measures

As in the case of the test, criterion-related measures which were themselves reliable were not available. In order to obtain some measure of the estimate of validity for the present attitude scale, the total attitude scores were correlated with the following (See Table 5.19):

(a) pupil's self-rating of liking for mathematics in relation to other subjects;

(b) teacher's rating of pupil's attitude towards mathematics.

TABLE 5.19

INTERCORRELATIONS OF ATTITUDE SCORE (c) AND CRITERION MEASURES (a and b)

	a	b	c
a	-		
b	0,35	-	
c	0,46	0,37	-
N = 680 All correlations significant : $p < 0{,}001$			

Combining the two criterion measures an estimate of validity was obtained as follows (Cureton, 1951:680):

$$r_{c(a,b)} = \frac{r_{ca}\, r_{cb}}{r_{ab}} = 0{,}70$$

These values compare favourably with those reported in other validation studies. In validating a reading attitude scale, Roettger et al (1979:140) found correlations ranging from 0,27 to 0,44 for students' and teachers' ratings. Michaels and Forsyth (1977:1047), in an attempt to provide evidence of 'convergent' validity (1), reported significant correlations ($p < 0,05$) ranging from 0,11 to 0,25.

5.3.6.2 Further Evidence of Validity

Another approach to seeking evidence for validity is to hypothesize, on the basis of theoretical considerations, certain relationships that would be expected between the attitude and other related behaviour. In the present study it was hypothesized that there would be a significant difference in attitudes between HG and SG. A valid scale should therefore yield different scores for these two groups (See Table 5.20).

From Table 5.20 it was clear that there were significant differences in attitudes between HG and SG at both levels. It was thus inferred that the scale exhibited a measure of construct validity (Shaw and Wright, 1967:19).

TABLE 5.20

DIFFERENCES IN ATTITUDES BETWEEN HG AND SG

	$\bar{x}$ (HG)	$\bar{x}$ (SG)	DIFFERENCE	z-SCORE
Std 9	74,24	65,96	8,28	9,097 ($p < 0,001$)
Std 10	74,69	68,22	6,47	5,274 ($p < 0,001$)

(1) This involves the multitrait-multimethod matrix analysis suggested by Campbell and Fiske (1959) where at least two attitudes and at least two methods of measuring each of them are used (cited by Fishbein and Ajzen, 1975:111).

Finally, evidence for construct validity may also be inferred from internal consistency measures. In this study the item correlations with subscores (as shown in 5.3.4) provided supportive evidence.

5.3.7 General Comments

In the preceding sections statistical evidence has been presented with respect to reliability and validity of the scale, the variability of the scores, item correlations and relative attitude indices, in order to judge the quality of the attitude scale compiled in this study.

Considering the procedures involved in the initial development of the items and in the selection by this researcher and the statistical evidence presented above, it was felt that a highly reliable and reasonably valid measure of attitudes towards mathematics had been developed. It was also demonstrated that, while the whole scale was measuring a general attitude towards mathematics, the subscales were measuring certain specific aspects of attitudes towards mathematics. However, a substantial degree of overlap was found between two of the subscales.

It was also observed that the attitude scale was capable of discriminating between two different groups (e.g. HG and SG). Hence, it was considered that a highly satisfactory screening device had been compiled.

Finally, it must be pointed out that this researcher steered clear of using achievement measure as a criterion for validity because this would have been a grave logical error within the present research design which has attempted to investigate the relationship between attitude and achievement. Attempts by some researchers (Aiken and Dreger, 1961; Lindgren et al, 1964) to demonstrate construct validity by showing that scores on their measure of attitude towards mathematics correlate with those on a measure of mathematics attainment may be severely criticised. For example, Michaels and Forsyth (1978:25) have considered this type of validity evidence to be weak

> "because attitude and achievement are not the same thing and the relationship between them is poorly understood".

Moreover, the present study set out (in the first place) to investigate *this* particular relationship.

CHAPTER SIX

6. ANALYSES OF DATA AND DISCUSSION OF HYPOTHESES IN RESPECT OF ATTITUDES TOWARD MATHEMATICS AND ATTAINMENT IN MATHEMATICS

In addition to the development and administration of the mathematics test and the attitude scale, data on selected background variables was gathered in this study in order to explore their relationships with mathematics attainment and attitudes toward mathematics. These variables included curriculum and school organisation, social factors, sex differences, educational and vocational aspirations, home background, and teachers' perceptions of pupils' attitudes and attainment. In this chapter, hypotheses in respect of these variables and mathematics attitudes and attainment are presented and investigated; and finally, the relationship between attitudes toward mathematics and attainment in mathematics is closely examined.

6.1 CURRICULUM AND SCHOOL ORGANIZATION AS RELATED TO ATTITUDES AND ATTAINMENT

In this section the results of statistical analyses of data in respect of curriculum and school organization are presented and discussed. In particular, the influence of such variables as curriculum choice, grades, levels, class size and homework on attitudes towards mathematics and attainment in mathematics is explored.

6.1.1 Curriculum Choice

Pupils in the senior secondary phase may choose from the following five areas of study within the framework of the system of differentiated education : Humanities, Science, Technical, Commerce and Home Economics. For each of these directions the two official languages (English and Afrikaans) are compulsory and four others may be selected from the subject groupings as stipulated by the Joint Matriculation Board. Further, the subjects selected may be taken on either the Higher Grade (HG) or the Standard Grade (SG). The pass requirements for matriculation exemption have been summarised as follows (Centre for Tertiary Education, 1981:23):-

(a) Subject Groupings:

GROUPS →	A Official Languages	B Maths	C Sciences	D Third Language	E Humanities	F Remainder
Main Offerings	Eng. First Language Afrikaans Second Language HG	Maths HG/SG	Phys. Sc. Biology HG/SG	Arabic SG.	History Geography Economics HG/SG	Other Subjects; also Geography if not taken under E.

(b) Pass Requirements:

(i) Candidate must pass in 5 subjects selected from at least 4 groups.

(ii) Of the 5 subjects passed 4 must be on the HG, and these must include the 2 official languages, and 2 other subjects from 2 different groups.

In this research, the following aspects relating to the selection and study of mathematics within the school curriculum were examined in relation to attitudes toward mathematics and attainment in mathematics:

(i) Reasons for choosing mathematics;

(ii) Reasons for choice of grade;

(iii) Mathematics bias within the school curriculum;

(iv) Science bias within the school curriculum;

(v) Preference for mathematics in relation to other subjects in the curriculum;

(vi) Performance in mathematics in relation to other subjects in the curriculum;

(vii) Content preferences;

(viii) Cognitive preferences.

6.1.1.1 Reasons for Choosing Mathematics

The pupils were asked to indicate the main reason for choosing mathematics as part of the curriculum (see 5(a) of PQ1 - Appendix 1). Initially these reasons had been compiled from the responses of some 200 senior secondary pupils who had been asked to write down the main reason for taking mathematics. In this way 8 categories of reasons were developed and used as the classification variable. The differences between the means in respect of attitudes toward mathematics and achievement in mathematics were examined (see Table 6.1).

TABLE 6.1

REASONS FOR CHOOSING MATHEMATICS, AND MEAN ATTITUDE AND ATTAINMENT SCORES

REASONS	No.	%	ATTITUDE $\bar{x}$	ATTITUDE SD	ATTAINMENT $\bar{x}$	ATTAINMENT SD
1. Enjoyable and interesting	176	25,88	76,22	6,08	11,32	4,18
2. Influenced by parents/ friends	16	2,35	67,50	8,93	7,44	2,81
3. Advised by teacher/ counsellor	13	1,91	65,92	10,91	7,69	2,30
4. Part of course I chose	151	22,21	65,93	10,86	8,19	3,70
5. Necessary for matric exemption	80	11,76	69,63	8,69	9,04	3,61
6. Necessary for further study/good job	223	32,79	70,56	9,61	9,78	4,32
7. No option	18	2,65	53,00	13,66	7,89	3,63
8. Other	3	0,44	74,00	6,16	9,00	1,63

It was evident that all the reasons for choosing mathematics had been accommodated as only 0,44% indicated 'other' reasons. Since a great majority (92,64%) gave reasons 1, 4, 5 and 6 for choosing mathematics, the 4 groups (n=630) constituted in this way were subjected to one-way ANOVAS (see 8 of Appendix 5) in respect of attitudes toward mathematics and attainment in mathematics, in order to determine the significance of the differences (see Tables 6.2 and 6.3).

TABLE 6.2

ONE-WAY ANOVA - MEAN ATTITUDE SCORES OVER (FOUR) MAIN REASONS FOR CHOOSING MATHEMATICS

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	8865,76	3	2955,25	35,45
Within Groups	52182,29	626	83,36	($p < 0,001$)
Total	61048,05	629		

TABLE 6.3

ONE-WAY ANOVA - MEAN ATTAINMENT SCORES OVER (FOUR) MAIN REASONS FOR CHOOSING MATHEMATICS

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	841,70	3	280,57	27,58
Within Groups	6367,42	626	10,17	($p < 0,001$)
Total	7209,12	629		

For the 4 groups considered, the differences between the means were significant ($p < 0,001$) in respect of both attitudes and attainment. It was found that those who gave 'I find it enjoyable and interesting' as the main reason for choosing mathematics performed significantly better on the mathematics test ($p < 0,001$), and that they also had significantly more positive attitudes ($p < 0,001$) than the others. A further analysis according to sex, levels (Std 9 and Std 10) and grades (SG and HG) revealed a consistent pattern as above i.e. irrespective of sex, level or grade, those pupils who found mathematics enjoyable and interesting performed significantly better and were more well disposed towards mathematics than the others.

It was also found that those who took mathematics because it was part of a 'package' course they had selected from those available at the school performed significantly worse and were far less positive in their attitudes than the others. A disturbing fact is that a relatively high proportion (22,21%) fell into this category. Adding to this those (2,65%) who had no other option, it would appear that some 25% of the sample was 'forced' to take mathematics. This may be due to the fact that it is uneconomical and impracticable for a school to offer the whole range of packages to enable the pupils to select 'freely' within the system of differentiated education.

Further, a relatively high proportion (44,55%) chose mathematics because it was necessary for matric exemption or further study and/or a good job. These proportions may well account for the relatively large percentage of students entering for mathematics in the Department of Indian Education (DIE). In 1980, 84,97% of the matriculants had entered for mathematics (Centre for Tertiary Education, 1981:25). In 1979, 70,10% entered as compared with an average of 50,50% for the other Departments of Education in RSA (DIE Report, 1980). In the four White Provincial Departments an average of 56,23%, over a period of 14 years (1964-1977), took mathematics (Jansen, 1980:77).

From the foregoing, the following reasons may be advanced for the relatively high entry into mathematics in the Department of Indian Education:

(i) limited 'package' courses offered by the school;

(ii) demands for matric exemption and further study/good jobs; which are probably due to socio-economic demands placing a premium on high-status jobs, and to limited job opportunities for Indian pupils.

In the latter context, TASA (1980) pointed out "that job opportunities for Indian work seekers were at a premium".

Thus it is possible to explain partly the low percentage entry in HG mathematics which has given rise to concern among education planners (DIE Report, 1980). It would appear from the findings here that it is not the low HG entries but rather the unrealistically larger numbers taking mathematics (many for wrong reasons) which inflate the SG numbers and result in a lowering of the HG : SG entry and (hence) pass ratios.

In a related study, Entwistle and Duckworth (1977) presented research evidence on factors associated with subject choice and identified attitudes as an important correlate of science choice. Further support for the above findings comes from Brodie (1964) who found that attitudinal contrast consistently favoured the 'satisfied' students in academic achievement. In general, the findings in the present study point to an important link between choice of mathematics and both attitudes and attainment.

6.1.1.2 Reasons for Choice of Grade

Categories of reasons for choosing a particular grade were compiled separately for each grade from the responses of 200 senior pupils of mathematics. These categories were then used to classify the pupils into groups. The means in respect of attitudes towards mathematics and attainment in mathematics were examined for the various categories (see Table 6.4).

TABLE 6.4

REASONS FOR CHOOSING HG, AND MEAN ATTITUDE AND ATTAINMENT SCORES

REASONS	No.	%	ATTITUDE $\overline{x}$	ATTITUDE SD	ATTAINMENT $\overline{x}$	ATTAINMENT SD
1. HG is more challenging	95	32,09	76,79	6,44	12,69	4,07
2. Necessary for matric exemption	70	23,65	71,06	9,29	10.80	3,98
3. I can easily manage the HG	27	9,12	75,89	5,94	12,85	4,11
4. Influenced by parents/ friends	2	0,68	65,50	2,50	10,00	0
5. Advised by teacher/ counsellor	9	3,04	76,89	7,85	9,33	2,58
6. Necessary for further study	91	30,74	74,24	8,14	12,42	4,36
7. Other	2	0,68	63,00	4,00	10,00	3,00
Total	296	100,00				

Since 95,60% of the HG sample chose this particular grade for reasons 1, 2, 3 and 6, these categories were included in further analyses (n=283) while the others (4, 5 and 7, where the proportions were small) were excluded. Using reasons for choice of grade as a classification variable, one-way ANOVAS were computed in respect of attitudes toward mathematics and attainment in mathematics (see Tables 6.5 and 6.6).

TABLE 6.5

ONE-WAY ANOVA - MEAN ATTITUDE SCORES OVER (FOUR) MAIN REASONS FOR CHOICE OF HG

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	1384,55	3	461,52	7,59
Within Groups	16963,56	279	60,80	($p < 0{,}001$)
Total	18348,11	282		

TABLE 6.6

ONE-WAY ANOVA - MEAN ATTAINMENT SCORES OVER (FOUR) MAIN REASONS FOR CHOICE OF HG

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	175,14	3	58,38	3,35
Within Groups	4868,45	279	17,45	($p < 0{,}025$)
Total	5043,59	282		

The ANOVAS revealed significant differences between means for attitude and attainment. However, a further observation of the differences between means revealed that, for both attitudes and attainment, there were no significant differences between any two of the groups 1, 3 and 6. It was found that subjects in these groups performed significantly better ($p < 0{,}01$) than those in group 2 ('necessary for matric exemption'). They also possessed significantly more positive attitudes than those in group 2. It was evident that 23,65% of the HG who opted for HG mathematics because it was 'necessary for matric exemption' were less positively inclined towards the subject and attained lower test scores than those who found it challenging, easily manageable and necessary for further study.

TABLE 6.7

REASONS FOR CHOOSING SG, AND MEAN ATTITUDE AND ATTAINMENT SCORES

REASONS	No.	%	ATTITUDE $\bar{x}$	ATTITUDE SD	ATTAINMENT $\bar{x}$	ATTAINMENT SD
1. I cannot manage the HG	167	43,49	64,60	10,86	7,32	2,92
2. Influenced by parents/ friends	0	0	0	0	0	0
3. Chances of passing SG better	127	33,07	70,20	8,22	7,98	2,90
4. Advised by teacher/counsellor	11	2,86	65,91	11,41	7,09	2,84
5. Easier to get a good mark	46	11,98	70,93	9,53	8,28	3,42
6. No other available option	27	7,03	61,19	14,49	7,96	3,07
7. Other	6	1,56	70,33	6,94	7,83	2,54
Total	384	99,99				

Since a great majority of the SG (95,58%) gave reasons 1, 3, 5 or 6 and very small proportions of the sample gave reasons 2, 4 and 7, the latter were excluded from further analyses. One-way ANOVAS (see Tables 6.8 and 6.9) applied to the means in these 4 categories (n=367) revealed significant differences ($p < 0,01$) for attitudes but no significant differences ($p > 0,05$) for attainment in mathematics.

TABLE 6.8

ONE-WAY ANOVA - MEAN ATTITUDE SCORES OVER (FOUR) MAIN REASONS FOR CHOICE OF SG.

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	3881,91	3	1293,97	12,20
Within Groups	38123,78	363	106,04	($p < 0,001$)
Total	42005,69	366		

TABLE 6.9

ONE-WAY ANOVA - MEAN ATTAINMENT SCORES OVER (FOUR) MAIN REASONS FOR CHOICE OF SG.

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	51,29	3	17,10	1,89
Within Groups	3284,49	363	9,05	($p > 0,10$)
Total	3336,78	366		

A further examination of the means in respect of attitudes showed that those who 'cannot manage HG'and those who had 'no other available option' were poorly disposed towards mathematics as compared with those who thought the 'chances of passing SG are better' and that it was 'easier to get a good mark'. Thus it would appear that while reasons for choice affect attitudes towards mathematics in SG, there seems to be no such influence on performance, since the observed differences were not significant.

6.1.1.3 Mathematics Bias within the School Curriculum

It was hypothesized that attitudes towards mathematics and attainment in mathematics would be positively related to the mathematics bias in the curriculum as inferred from choice of subjects for the Senior Certificate. In order to study this aspect, data on the three subjects other than English, Afrikaans and Mathematics (selected for the Senior Certificate) was collected. From consultation with teachers of mathematics and other subjects, it was felt that those pupils who had selected subjects such as Physical Science, Biology, Technical Drawing, Accounting and Geography showed a bias towards mathematics (see 3 of PQ1 - Appendix 1). The mathematics bias was thus designated in accordance with the number of the above subjects included in the curriculum.

Product-moment correlations revealed a strong positive relationship between mathematics bias and performance in mathematics ($r=0,31$; $p < 0,001$). There was a mild but positive relationship between attitudes toward mathematics and mathematics bias ($r=0,11$; $p < 0,05$). In general, it was clear that those with a strong leaning towards mathematics (as indicated by the selection of subjects) tend to possess more positive attitudes and score more highly on the mathematics test. In a similar study of mathematical bias and attitudes towards mathematics Kempa and McGough (1977) found that attitudes to mathematics were strongly correlated with students' mathematical bias as inferred from their choice of sixth-form subjects.

6.1.1.4 Science Bias within the School Curriculum

Science bias was indicated by the number of science subjects included in the curriculum: no science, one of Biology or Physical Science, and both Biology and Physical Science. It was hypothesized that students taking both Physical Science and Biology would exhibit higher attitude scores and mathematics attainment scores than those taking one or no science. Science bias was used as a classification variable and the means in respect of attitudes and attainment were examined in the various categories (see Table 6.10).

TABLE 6.10

SCIENCE BIAS, AND MEAN ATTITUDE AND ATTAINMENT SCORES

SCIENCE BIAS	No.	%	ATTITUDE $\bar{x}$	ATTITUDE SD	ATTAINMENT $\bar{x}$	ATTAINMENT SD
No Science	101	14,85	69,23	9,85	8,06	3,30
One of Biology or Physical Science	347	51,03	68,25	10,65	8,06	3,17
Both Biology and Physical Science	232	34,12	73,77	8,97	12,53	4,16
Total	680	100,00				

It was found that students taking Physical Science and Biology showed significantly more positive attitudes toward mathematics ($p < 0,001$) and performed significantly better on the mathematics test ($p < 0,001$) than those taking no science or one science. This finding is consistent with the generally known fact that the better students take both Physical Science and Biology as they aspire to enter the medical or science faculty at university. This is borne out by the fact that 9 out of 10 of the top Indian Matriculants in the 1980 Senior Certificate examinations had included both Biology and Physical Science in their curricula (Centre for Tertiary Education, 1981). It could also be interpreted as a reflection of the science student's recognition of mathematics as an important service subject (Kempa and McGough, 1977:301). In a study of a new approach to assessment of mathematical competence, Ekenstam and Nilsson (1979:64) observed that

> "students going to the science line of senior high school had scored higher (in mathematics) than the others".

In general, the findings in this study pointed to a positive relationship between science bias and attitudes toward mathematics, and between science bias and attainment in mathematics. In distinguishing between those with a strong leaning to science and those with little or no science bias, the discriminating ability of the attitude scale and the mathematics test was demonstrated.

6.1.1.5 Preference for Mathematics within the Curriculum

Kane (1968) measured attitudes toward mathematics of college students by the extent to which mathematics was preferred over three other subjects. In the present study pupils were required to indicate their preference for mathematics within the chosen curriculum by stating the number of subjects they liked better than they liked mathematics (see 13a of PQ1 - Appendix 1). It was hypothesized that there will be a positive relationship between preference for mathematics and

(i) attitudes toward mathematics,
(ii) performance in mathematics.

The correlation between preference for mathematics and attitudes toward mathematics was found to be 0,46 ($p < 0,001$). Preference for mathematics was also positively and significantly related to performance in mathematics ($r=0,46$; $p < 0,001$). These findings are consistent with those of Degnan (1967) who reported that high achievers in mathematics had a more positive attitude towards the subject and also gave mathematics a significantly higher preference ranking than the low achievers.

In addition, it was found that preference for mathematics was most highly related to 'enjoyment of mathematics' among the six dimensions of the attitude scale ($r=0,59$; $p<0,001$). This finding was consistent with that of Kempa and McGough (1977:304) who reported that

> "of the four attitude measures employed in this study, 'liking/ enjoyment of mathematics' was found to be the one most strongly differentiating between student groups with different mathematical biases and those in different achievement categories".

In general, it was concluded that those who showed a preference for mathematics within the curriculum were well disposed towards it, enjoyed it and attained high scores. Furthermore, students' preference for mathematics emerged as a satisfactory predictor of attainment in mathematics and as a measure of attitudes towards mathematics.

6.1.1.6 <u>Performance in Mathematics within the Curriculum</u>

Data was gathered on the relative performance in mathematics (as perceived by the students) within the curriculum by requiring them to indicate the number of subjects in which they did better than they did in mathematics (see 13(b) of PQ1 - Appendix 1). It was hypothesized that there will be a significant positive relationship between the relative performance in mathematics and

(i) attitudes toward mathematics,

(ii) attainment in mathematics.

It was found that performance in mathematics within the curriculum as perceived by the students correlated significantly with their attitudes toward mathematics ($r=0,36$; $p < 0,001$). It was also significantly related to attainment in mathematics ($r=0,42$; $p < 0,001$). A substantial relationship between preference for mathematics and relative performance was observed ($r=0,70$; $p < 0,001$). This was indicative of the fact that students do better in subjects for which they show greater preference. The relative performance in mathematics within the curriculum yielded yet another guide to predicting attainment in mathematics.

6.1.1.7 Content Preferences

A measure of the attitudes towards Algebra, Geometry and Trigonometry was obtained by requiring the pupils to indicate the extent of their 'liking' for each of these content areas on a 5-point scale (see 14 of PQ1 - Appendix 1). The differences between the mean preferences for these 3 sections in mathematics were investigated (see Table 6.11).

TABLE 6.11

DIFFERENCES BETWEEN MEAN PREFERENCES FOR CONTENT AREAS

(correlated data)

CONTENT PREFERENCE	n	$\bar{x}$	SD	r	z-SCORE
Algebra	680	3,06	0,93		
				0,15	4,22 ($p < 0,001$)
Trigonometry	680	2,84	0,96		
				-0,03	6,96 ($p < 0,001$)
Geometry	680	2,46	1,17		

The differences between the means were significant ($p < 0,001$). The pupils showed the greatest preference for Algebra and the least for

Geometry. These results were compared with the teachers' ratings of pupils' performance in each of these content areas on a 5-point scale. The pattern was identical with that observed above i.e. the performance was rated highest for Algebra, next for Trigonometry and lowest for Geometry (see Table 6.11). It was clear that the pupils were showing greater preference (as expected) for areas in which they were experiencing greater success. The implication is that they were experiencing greater success in Algebra than in Geometry. A possible explanation for this lies in the fact that there seems to be greater scope for techniques and skills in School Algebra (which are easily attainable lower level objectives) than there is in Geometry. This researcher's experience with matriculation examinations in mathematics is consistent with this view.

Further, the content preferences were correlated with the teachers' ratings for the three sections and with the attitude scores (see Table 6.12).

TABLE 6.12

CORRELATIONS OF PREFERENCES WITH TEACHERS' RATINGS AND ATTITUDE SCORES

Content Preference	Teachers' Ratings of Performance	Attitude Scores
Algebra	0,28	0,37
Geometry	0,32	0,28
Trigonometry	0,17	0,23

n=680. All correlations significant: $p<0,01$.

It was evident that there were significant positive relationships between the preferences and the teachers' ratings of the performance. As expected, since content preference itself was a measure of attitude, there was a significant positive relationship between preference ratings and total attitude scores. In general, it was clear that attitudes towards specific content areas in mathematics were positively related to performance in those areas.

6.1.1.8 Cognitive Preferences

This researcher's experiences as examiner for matriculation mathematics have shown that verbally-presented problems are generally avoided. There were, of course, no scientific grounds for any claims in this respect. As a preliminary investigation in this study, information on students' preference (liking) for different modes of presenting mathematical information was obtained and examined. The students were required to indicate their liking for the following types of presentations of problems on a 5-point scale (see 15 of PQ1 - Appendix 1):

(i) diagrammatic
(ii) symbolic
(iii) verbal.

The responses were taken as indication of their attitudes toward the modes of presentation.

It was observed that the students' preferences for geometry and algebra correlated significantly highly with preferences for the diagrammatic and symbolic modes of presentation respectively ($r=0{,}59$ and $r=0{,}56$; $p<0{,}001$). This result was indicative of the relationships perceived by students between algebra and symbolic mode, and between geometry and diagrammatic mode.

It was hypothesized that there would be no significant differences between the preferences for the various modes of presentation (see Table 6.13 for the means).

TABLE 6.13

DIFFERENCES BETWEEN MEAN PREFERENCES FOR MODES OF PRESENTATION

(correlated data)

COGNITIVE PREFERENCES	n	$\bar{x}$	SD	r	z-SCORE
Symbolic	680	2,90	1,02		
				-0,10	4,07 ($p < 0,001$)
Diagrammatic	680	2,67	1,16		
				0,03	6,46 ($p < 0,001$)
Verbal	680	2,31	1,17		

The null hypothesis was rejected ($p < 0,001$). It was further observed that the verbal mode of presentation was least preferred even when the data was examined separately over levels, grades and sex. In this context Dutton and Blum (1968) found that one of the most frequent reasons given by students for disliking arithmetic was that 'word problems that were frustrating'.

It is also possible that the pupils experience difficulty in translating mathematical information from verbal to geometric or symbolic forms. This would appear to be a serious problem in view of the fact that 'translating' ability has been recognized as an important and integral part of classifications of objectives in mathematics teaching and learning (Husén, 1967a; Romberg and Wilson, 1968; Wood, 1968; Moodley, 1975). It has also been suggested that

"A great deal of mathematical performance depends on translation behaviour which is essential to develop a fluent use of mathematical language" (Moodley, 1975:64).

In a more detailed study, Sherril (1974) used a final test of 20 problems selected from Y- and Z- population test batteries of the National Longitudinal Study of Mathematical Abilities and constructed three equivalent forms:

(a) only verbal statement of problem;

(b) verbal statement plus accurate pictorial representation;

(c) verbal statement plus distorted pictorial representation.

It was found that

> "the subjects' achievement in solving printed word problems is affected by the presence of a pictorial representation of the problem situation, regardless of the accuracy of the pictorial representation" (Sherril, 1974:280).

Sherill (1974:281) explained that

> "the subjects tended to place emphasis on the pictorial representation rather than the prose description of the problem situation".

The results of this exploratory aspect of the present study seem to be consistent with the above findings and confirm the researcher's own observations of examinations. Within this context, there has been a move by matriculation mathematics examiners in recent years to supply diagrams for verbally stated problems but this practice is questionable (e.g. *vide* DIE, HG Mathematics Paper II 1979, 1980).

The data in Table 6.13 also shows that the symbolic mode of presentation was most preferred by the pupils. This finding is consistent with that of Kempa and McGough (1977:304) who explained that

> "the leaning to symbolic thinking modes which, it may safely be argued, is a concomitant mathematical ability, expresses itself also in the preference for a symbol-based communication mode".

Furthermore, correlations between mode of presentation and attitudes toward mathematics and attainment in mathematics (see Table 6.14) revealed that preference for the diagrammatic mode of representation of problems seemed to be the best predictor of performance in mathematics. It was also observed that the relationships between the preferences for the various modes and attitudes were substantial, while those for symbolic and verbal modes and attainment were considerably lower.

TABLE 6.14

CORRELATIONS BETWEEN COGNITIVE PREFERENCE AND ATTITUDES AND BETWEEN COGNITIVE PREFERENCE AND ATTAINMENT

COGNITIVE PREFERENCE	ATTITUDE	ATTAINMENT
Symbolic	0,29*	0,11
Diagrammatic	0,29*	0,29*
Verbal	0,20*	0,08
*significant at $p < 0,001$.	n = 680	

In general, a consideration of cognitive preferences within the framework of the present research design has been exploratory in nature and a more in-depth study is necessary. The researches of Sherril (1974), Kempa and McGough (1977), Days et al (1979), Malpas and Brown (1974) and other related studies (Kempa and Dube, 1973; Tamir, 1976; Heath, 1964) show that this area is open for further research.

6.1.2 Grades (HG and SG)

Grades in this study refer to the differentiated syllabus: Higher Grade (HG) and Standard Grade (SG). In the HG, the subject matter

> "lends itself particularly to preparing and selecting prospective university students ... for further study in mathematics, physical sciences and other courses of study for which mathematical methods and techniques are essential" (DIE-HG Syllabus, 1976:4).

Students who are less able and who do not wish to proceed further with mathematics usually take it on the Standard Grade. In the present study the differences in attitudes and attainment between the two grades were investigated. It was hypothesized that:

(i) there will be no significant difference in attitudes toward mathematics between HG and SG students.

(ii) there will be no significant difference in mathematics achievement between HG and SG students.

The means and differences are shown in Tables 6.15 and 6.16.

TABLE 6.15

DIFFERENCE BETWEEN MEANS OF ATTITUDE SCORES FOR HG AND SG

LEVEL	GRADE	n	$\bar{x}$	SD	DIF.	z-SCORE
STD 9	SG	191	65,96	10,18	8,28	9,10 ($p < 0,001$)
	HG	192	74,24	7,30		
STD 10	SG	193	68,22	11,04	6,47	5,27 ($p < 0,001$)
	HG	104	74,69	9,39		

TABLE 6.16

DIFFERENCE BETWEEN MEANS OF ATTAINMENT SCORES FOR HG AND SG

LEVEL	GRADE	n	$\bar{x}$	SD	DIF.	z-SCORE
STD 9	SG	191	6,72	2,64	3,93	11,90 ($p < 0,001$)
	HG	192	10,65	3,76		
STD 10	SG	193	8,78	3,12	6,35	14,18 ($p < 0,001$)
	HG	104	15,13	3,92		

The differences between means for HG and SG in respect of attitudes and attainment were consistently significant ($p < 0,001$) at each level. Thus both the null hypotheses were rejected. In all cases the HG exhibited significantly more positive attitudes and performed significantly better on the mathematics test than the SG. These findings are consistent with the general practice in schools whereby the better candidates are usually placed in the HG. Considering these results in conjunction with the declining trend in entry and pass ratios for HG (shown in Table 1.1), it would appear that there may be a declining trend in attitudes towards mathematics.

Further, the effect of instruction in mixed grade classes (where pupils from two different grades are taught in a single class unit) on attitudes towards mathematics and attainment in mathematics was also examined. The differences in means between S(HS) (standard grades from mixed grade classes) and SO (standard grades only from single grade classes) and between H(HS) (higher grades from mixed grade classes) and HO (higher grades only from single grade classes) are shown in Tables 6.17 and 6.18.

TABLE 6.17

DIFFERENCE BETWEEN MEANS FOR MIXED GRADES AND SINGLE GRADES IN RESPECT OF ATTITUDES

GRADES		No.	%	$\bar{x}$	SD	DIF.	z-SCORE
SG	S(HS)	106	27,60	67,41	9,60	0,43	0,34 ($p > 0,05$)
	SO	278	72,40	66,98	11,07		
HG	H(HS)	102	34,46	73,64	6,87	1,16	1,26 ($p > 0,05$)
	HO	194	65,54	74,80	8,65		

TABLE 6.18

DIFFERENCE BETWEEN MEANS FOR MIXED GRADES AND SINGLE GRADES IN RESPECT OF ATTAINMENT

GRADES		No.	%	$\bar{x}$	SD	DIF.	z-SCORE
SG	S(HS)	106	27,60	7,21	2,68	0,68	2,11 ($p > 0,05$)
	SO	278	72,40	7,89	3,10		
HG	H(HS)	102	34,46	10,13	3,61	2,91	6,21 ($p < 0,001$)
	HO	194	65,54	13,04	4,13		

It was hypothesized that there would be no difference between means in respect of attitudes for those who were taught in mixed-grade classes and those in single-grade classes. It was found (see Table 6,17) that the differences in means between S(HS) and SO and between H(HS) and HO were not significant ($p > 0,05$). Thus the above null hypothesis was accepted. It was evident that instruction in mixed or single grade classes had no effect on attitudes towards mathematics.

It was also hypothesized that there would be no difference between means in respect of achievement in mathematics for those who were taught in mixed and single-grade classes. It was found (see Table 6.18) that the difference in means between S(HS) and SO was not significant ($p > 0,05$) but the difference between H(HS) and HO was significant ($p < 0,001$) in favour of HO with 12 percentage points higher. Thus, while instruction in mixed-grade classes had a negative effect on HG pupils, it made no difference to the SG pupils. These results were consistent with an earlier finding by this researcher (Moodley, 1975:154). It would appear that, in view of the difficulties of coping with instruction at two different grades for the two vastly differing ability groups, the instruction is aimed mainly at the standard grades and the performance of higher grades in the mixed-grade classes is affected negatively. Although it was found that a smaller proportion (30,58%) as compared with 53,5% in 1975 had been taught in mixed-grade classes, more than 34% of the HG in the sample was affected in this way.

6.1.3 Levels (Std 9 and Std 10)

In this study the two levels of schooling in the senior secondary phase (viz. Std 9 and Std 10) were considered. The following null hypotheses were tested:

(i) that there is no difference between the two levels in respect of attitudes towards mathematics.

(ii) that there is no difference between the two levels in respect of attainment in mathematics.

Tables 6.19 and 6.20 show the means and differences for the two levels for attitudes and attainment respectively.

TABLE 6.19

DIFFERENCE BETWEEN MEANS FOR STD 9 AND STD 10 IN RESPECT OF ATTITUDES TOWARD MATHEMATICS

LEVELS	n	$\bar{x}$	SD	DIF.	z-SCORE
STD 9	383	70,11	9,78	0,38	0,468 (p > 0,05)
STD 10	297	70,49	10,94		

TABLE 6.20

DIFFERENCE BETWEEN MEANS FOR STD 9 AND STD 10 IN RESPECT OF ATTAINMENT IN MATHEMATICS

LEVELS	n	$\bar{x}$	SD	DIF.	z-SCORE
STD 9	383	8,69	3,80	2,31	6,997 (p < 0,001)
STD 10	297	11,00	4,57		

The first null hypothesis was accepted (p > 0,05). An examination of the means over the two levels revealed no significant differences in attitudes between the two levels. It was evident that the difference of a year of schooling between the two levels had no effect on attitudes towards mathematics. Although a longitudinal study is necessary to make any claims about the stabilising of attitudes, it would appear from these results that attitudes may be formed before Std 9. Several studies have reported that attitudes may be formed in the earlier primary grades (Dutton, 1956; Fedon, 1958; Stright, 1960; Taylor, 1970). Reys and Delon (1968) reported that attitudes toward arithmetic reached the highest development during the junior high school years. Callahan, (1971) also observed that grades six and seven were most important for developing attitudes. Other attempts by researchers to pinpoint the critical period in the formation of attitudes toward mathematics have not reached consensus (Tulock, 1957; Poffenberger and Norton, 1959).

The second null hypothesis was rejected at $p < 0{,}001$ (see Table 6.20). A further examination of the means, over the two grades separately, revealed significant ($p < 0{,}001$) differences in mathematics achievement between Std 9 and Std 10. In both instances the pupils in Std 10 performed better than those in Std 9. It was noted that the difference of one year of schooling influenced the performance (mean ages of pupils in Std 9 and Std 10 were 16,59 and 17,52 respectively). Similar differences in performance between Std 9 and Std 10 were noted in a standardization of senior aptitude tests for Indian South Africans (De Villiers, 1977:31).

6.1.4 Class Size

The relationships between class size and mathematics achievement and between between class size and attitudes toward mathematics were investigated. It was hypothesized that

(i) there is no relationship between size of class and performance in mathematics.

(ii) there is no relationship between size of class and attitudes toward mathematics.

The mean sizes of class for the whole sample and subsamples at the two levels were obtained (see Table 6.21).

TABLE 6.21

MEAN SIZE OF MATHEMATICS CLASS AND CORRELATIONS WITH ATTITUDES AND ATTAINMENT

Sample	n	$\overline{x}$ (Class Size)	SD	Correlation with Attitudes	Correlation with Attainment
Total	680	26,34	7,20	-0,03	-0,11*
Std 9	383	28,37	6,62	0,02	0,10*
Std 10	297	23,37	7,08	-0,08	-0,14*

* $p < 0,05$; all others $p > 0,05$.

It was found that the class units in Std 10 were significantly smaller than those in Std 9 (z-Score = 7,73, $p < 0,001$). This may be due to the further selection at the end of the Std 9 examinations. In the IEA study (Husén, 1967b:79) the average class size for the 12 countries ranged from 15 in Germany to 41 in Japan. A further examination of class size over grades revealed that there was no difference ($p > 0,05$) in class size at the Std 9 level while there was a significant difference ($p < 0,001$) at the Std 10 level, with smaller classes in HG than in SG. This is consistent with the smaller proportion of candidates entering the HG at the Std 10 level.

The product-moment correlations (see Table 6.21) revealed that there was no relationship between class size and attitudes toward mathematics. However, there was a weak positive relationship between class size and attainment for Std 9, while that for Std 10 was negative. In general, these relationships were fairly weak. This was consistent with the IEA findings (Husén, 1967b:85) that "in the majority of the cases ... size of class is not related to mathematics achievement". It was also concluded in that study that, where the correlations were negative for the older and smaller classes, a reduction in class size might operate to advantage only when the size is reduced to twenty or fewer. In another study, Gibney and Karns (1979) observed that research during the two decades (1955-1975) provided little evidence that mathematics achievement was influenced by class size.

The present findings clearly support the findings of the other researchers. It was thus evident that the cause of variance in attitudes and attainment must be sought outside the variable - class size.

6.1.5 Homework

The Mathematical Association of America and the National Council of Teachers of Mathematics (1978:230), in their recommendations for preparation of high school students for college mathematics, pointed out that

> "homework and drill are very important pedagogical tools used to help the student gain understanding as well as proficiency in the skills of arithmetic and algebra".

In the present study, the relationships between homework and attitudes and between homework and attainment were examined in relation to

(i) approximate number of hours a week devoted to all homework;

(ii) approximate number of hours a week devoted to mathematics homework;

(iii) number of days per week (on an average) homework is done in mathematics.

The means for amount of homework were examined for the whole sample and over levels and grades (see Table 6.22).

TABLE 6.22

MEANS FOR HOMEWORK OVER LEVELS AND GRADES

HOMEWORK	Total Sample n=680 $\bar{x}$	HIGHER GRADE Std 10 $\bar{x}$	Std 9 $\bar{x}$	STANDARD GRADE Std 10 $\bar{x}$	Std 9 $\bar{x}$
All Homework (hrs. p.w.)	19,98	24,08	18,65	27,73	16,31
Maths. Homework (hrs. p.w.)	5,07	5,95	4,84	5,47	4,43
Maths. Homework (days p.w.)	4,26	4,84	4,15	4,26	4,07

In general, it was evident that the Std 10 and HG were spending more time on homework than the Std 9 and SG respectively. On an average, the pre-matriculants were spending one quarter of all their homework time on mathematics. It was expected that pupils in Std 10 who were preparing for the final Senior Certificate examination would spend more time on homework than those in Std 9. Since the HG syllabus was more extensive and demanding than the SG, it was also expected that the former would spend more time than the latter.

The relationships between amount of homework and attitude and between amount of homework and attainment are shown in Table 6.23.

TABLE 6.23

CORRELATIONS BETWEEN HOMEWORK AND ATTITUDES AND BETWEEN HOMEWORK AND ATTAINMENT

HOMEWORK	ATTITUDE	ATTAINMENT	n = 680
All homework (hrs. p.w.)	0,20 *	0,15 +	*$p < 0{,}01$
Maths. Homework (hrs. p.w.)	0,25 *	0,12 +	
Maths. Homework (days p.w.)	0,27 *	0,11 +	+$p < 0{,}05$

It was found that there was, in general, a positive relationship between homework and attitude and between homework and attainment. The latter relationship was, however, consistently lower. The low but positive relationship between homework and attainment is supported by research. After an extensive review of research on homework from 1900 - 1959, Goldstein (1960) concluded that the available evidence favoured homework as an important factor in increasing student achievement. In a similar review of homework research in mathematics from 1960 - 1977, Austin (1979) concludes that homework is preferable to non-homework for achievement in mathematics.

However, the present study does not support Austin's (1979) declaration, after the works of Maertens (1968) and Maertens and Johnston (1972) : that there is no relation between homework and attitudes toward mathematics. The present research contends that those who are more favourably disposed towards mathematics tend to do more mathematics homework.

6.2 SOCIAL AND OTHER FACTORS AS RELATED TO MATHEMATICS ATTITUDES AND ATTAINMENT

In this section data concerning sex differences, educational and vocational aspirations and home background are examined in relation to attitudes toward mathematics and attainment in mathematics.

6.2.1 Sex Differences in Attitudes toward Mathematics and Achievement in Mathematics

As mentioned earlier, this study recognized that sex can be an important moderator variable in the prediction of achievement from measures of attitudes (Aiken, 1970b). Research studies relating to sex differences in attitudes toward mathematics and achievement in mathematics have been briefly reviewed in section 3.2.5. Certain findings relating to sex differences in respect of some of the variables considered have already been reported in the preceding sections of this chapter. In this section the findings relating specifically to sex differences in attitudes toward mathematics and achievement in mathematics will be considered.

It was hypothesized that

(i) there will be no difference between the sexes in attitude towards mathematics;

(ii) there will be no difference between the sexes in mathematics achievement.

A simple two-way partition of the sample in terms of males and females over each of the levels was used and the differences between the means were examined (see Table 6.24 and Table 6.25).

6.2.1.1 Sex Differences in Attitudes toward Mathematics

The first null hypothesis, that there is no difference between the sexes in respect of attitudes toward mathematics, was accepted (see Table 6.24). This result is supported by several studies which also found no significant differences between the sexes (Keller, 1974; Roberts, 1970; McClure, 1971; Jacobs, 1974; Aiken, 1972a; Dutton and Blum, 1968; Holmes, 1979). In a study of attitudes toward mathematics, Ernest (1976) observed that there were no differences in sex in respect of preference patterns in mathematics and concluded that

> "there is nothing intrinsic in arithmetic or mathematics that makes it more appealing or enjoyable to one sex than the the other" (Ernest, 1976:597).

Sherman and Fennema (1977) found few sex-related differences in attitudes towards mathematics when tenth and eleventh grade girls and boys were equated for cognitive variables and intent to study mathematics.

TABLE 6.24

SEX DIFFERENCES IN ATTITUDES TOWARD MATHEMATICS

		n	$\overline{x}$	SD	DIF.	z-SCORE
STD 10	Males	178	70,33	11,53	0,40	0,32 ($p > 0{,}05$)
	Females	119	70,73	9,98		
STD 9	Males	220	69,83	9,35	0,67	0,65 ($p > 0{,}05$)
	Females	163	70,50	10,31		

A closer examination of the means for the subscales in the present study revealed that the boys scored significantly ($z = 3{,}30$; $p < 0{,}001$) higher than the girls on the dimension "Difficulties in Learning

Mathematics". This was a clear indication that the girls were experiencing more difficulties in learning mathematics than the boys. Thus, although there were no sex differences for the overall attitude scale, this last result may be the basis for predicting that the girls may do less well than the boys on the mathematics test.

Other studies have reported differences between attitudes to favour boys over girls (Keeves, 1973; Niven, 1973; Simpson, 1974; Hilton and Berglund, 1974). Fennema and Sherman (1977) have found that males have higher scores than females on most of the affective factors when intent and cognitive variables are uncontrolled. Nelson (1979:4794) reported that

> "few sex differences existed in attitudes towards mathematics among Afro-American students, and when sex differences are found, the differences favour males".

Yet others have reported more favourable school-related attitudes in girls than in boys (Barker-Lunn, 1972; Fitt, 1956; Wisenthal, 1965).

In general, the results of research in this area are somewhat conflicting. They seem to be dependent on the instruments used and the samples studied. While more work needs to be done to generalize the findings of the present study, it is clear that, at least for the population under study, the differences in attitudes towards mathematics as measured by the attitude scale are not sex-linked.

6.2.1.2 Sex Differences in Mathematics Achievement

The second null hypothesis, that there is no difference between sexes in respect of mathematics achievement, was rejected at $p < 0,05$ for Std 10 and at $p < 0,001$ for Std 9 (see Table 6.25). In both cases the difference was in favour of the males. It was clear that the difference was more marked at the Std 9 level. As explained earlier this may be attributed to greater homogeneity in the final year of schooling (Std 10) than in the penultimate year (Std 9).

TABLE 6.25

SEX DIFFERENCES IN MATHEMATICS ACHIEVEMENT

		n	$\overline{x}$	SD	DIF.	z-SCORE
Std 10	Males	178	11,51	4,60	1,27	2,36 ($p < 0,05$)
	Females	119	10,24	4,41		
Std 9	Males	220	9,25	3,76	1,32	3,45 ($p < 0,001$)
	Females	163	7,93	3,72		

Several studies support such differences in mathematical performance in favour of boys (Husén, 1967b; Ernest, 1976; Aiken, 1971; Astin, 1974; Hilton and Berglund, 1974; Burton, 1978; Keeves, 1973; Stones, 1978; Fennema, 1974). Moreover, Husén (1967:241) found that boys show a greater variability than the girls. In considering quantitative ability, Maccoby and Jacklin (1974) concluded that there are no consistent sex-related differences prior to adolescence, but when differences are found they favour boys and after the age of 13 males are superior to females.

However, some studies have failed to show such differences. In a research study of sex-related differences in mathematics achievement, spatial visualization and affective factors, Fennema and Sherman (1977: 69) have produced data which do not support

"either the expectations that males are invariably superior

> in mathematics achievement and spatial visualization or the idea that differences between the sexes increase with age and/or mathematics difficulty".

These researchers have observed that the pattern of differences in mathematics achievement, spatial visualization and affective variables strongly suggests the influence of socio-cultural factors. In a further study of mathematics achievement of students in grades 6-12, Fennema and Sherman (1978) found that when relevant factors are controlled, sex-related differences in favour of males do not appear often and when they do they are not large. Similar findings were reported by Holmes (1979).

A few recent studies have reported a superiority among girls. Clements (1979:321) observed that girls in grades 5-8 performed significantly better than boys on mathematical tests involving calculations and applications but points out that a much smaller proportion of girls than boys were studying mathematics. Roach (1979) found that Jamaican girls did better than boys in mathematics and attributes the difference to a social-cultural basis whereby academic activities are considered feminine. Wozencraft (1963) found that girls in third and sixth grades were better at arithmetic than boys.

Murphy (1978:263) observed sex differences in examination performance in GCE 'O' level and concludes that sex-role stereotyping has a considerable influence within education while

> "the extent to which these stereotypes are built on actual differences in ability and the extent to which they are created by society remains unresolved, although it seems likely that the social-cultural influence plays the bigger role".

Fox et al (1979) present accumulated evidence supporting the hypothesis that sex-role socialization inhibits female performance in mathematics and suggest that the subtle ways in which socialization forces interact should be further researched.

Since, in this study, there was no significant difference between the mean IQ scores for the males and females, it was felt that socio-cultural factors (and sex-role identification) rather than differences in ability may be largely responsible for the differences observed in mathematics achievement.

Although it has been pointed out that

> "any discussion of the development of sex-related differences is drawn, inevitably, into the nature-nurture controversey"

and that it remains unresolved (Wittig and Peterson, 1979:11), the weight of evidence including the findings of this research strongly points to socio-cultural influences.

6.2.1.3 General Comments

While the results of research into sex differences "are very mixed and often confusing" (Ernest, 1976:599), the evidence (in general) points to the existence of differences which seem to have their origins in biological or socio-cultural factors and sex role identification (Burton, 1978; Keeves, 1973; Murphy, 1978). The data in the present study support such differences for achievement in mathematics but no significant sex differences have been observed for overall attitudes. It must be pointed out, however, that girls seem to be experiencing more difficulties than boys in learning mathematics.

Keeves (1973:60) suggested that the

> "interaction between achievement and attitudes could well contribute to the sex differences reported, and any discussion

of possible causes should take account of both aspects".

Thus, it would seem that a complex set of forces involving biological, socio-cultural factors and sex-role identification imposed on children by the home, peers and school are at work in determining the sex differences in attitudes towards mathematics and achievement in mathematics.

6.2.2 Educational Aspirations of Pre-matriculants

In this study, attitudes towards mathematics and achievement in mathematics were examined in terms of the educational aspirations of pre-matriculants. The following aspects which were used in the IEA study (Husén, 1967) were considered as indicators of aspirations:

(i) further education institution the pupils wished to enter after school;

(ii) number of years of full-time further education they desired;

(iii) desire to study more mathematics and/or subjects involving mathematics.

In this section a discussion of the differences between means for attitudes toward mathematics and attainment in mathematics in respect of the above aspects will be presented.

6.2.2.1 Choice of Further Education Institution

The subjects in this study were required to indicate what they intended to do (in terms of further education) after they left school (see 16 of PQ1 - Appendix 1). The institution they desired to enter was used as a classification variable in order to observe the differences between the means in respect of attitudes and attainment (see Table 6.26).

TABLE 6.26

INSTITUTIONS OF ENTRY AFTER SCHOOL AND MEAN ATTITUDE AND ATTAINMENT SCORES

INSTITUTION	No.	%	ATTITUDE		ATTAINMENT	
			$\bar{x}$	SD	$\bar{x}$	SD
1. University (full time)	246	36,18	72,11	9,88	10,74	4,11
2. University (part time)	50	7,35	71,30	8,08	8,88	3,02
3. Medical School	60	8,82	75,23	7,47	13,63	4,32
4, College of Teacher Education	130	19,12	68,55	11,34	8,18	3,31
5. Technikon	116	17,06	68,41	9,61	8,27	3,60
6. Full-time Employment	60	8,82	65,65	10,63	7,00	3,20
7. No further study	0	0	0	0	0	0
8. Other	18	2,65	65,89	11,90	9,61	4,41
Total	680	100,00				

The five categories (n=602) concerning *further education* were subjected to one-way ANOVAS (see Tables 6.27 and 6.28) in order to determine the significance of the differences in respect of attitudes and attainment.

TABLE 6.27

ONE-WAY ANOVA - MEAN SCORES ON ATTITUDES TOWARD MATHEMATICS OVER THE FIVE CATEGORIES OF CHOICE OF FURTHER EDUCATION INSTITUTION

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	2929,77	4	732,44	9,24
Within Groups	47342,94	597	79,30	($p < 0{,}001$)
Total	50272,71	601		

TABLE 6.28

ONE-WAY ANOVA - MEAN SCORES ON ATTAINMENT IN MATHEMATICS OVER THE FIVE CATEGORIES OF CHOICE OF FURTHER EDUCATION INSTITUTION

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	2856,26	4	714,07	49,25
Within Groups	8658,87	597	14,50	($p < 0{,}001$)
Total	11515,13	601		

From the ANOVAS it was clear that the differences between the means for the five categories in respect of both attitudes and attainment were significant ($p < 0{,}001$). It was further observed that those who opted for Medical School performed significantly better on the mathematics test ($z=4{,}68$; $p < 0{,}001$) and were significantly more well-disposed towards mathematics ($z=2{,}69$; $p < 0{,}01$) than those who aspired to go to

University, College of Education or Technikon. A similar difference was also found between the University (full time) group and those who wished to enter the College of Education or Technikon.

It was observed that almost 90% of the sample wished to pursue some form of tertiary education with almost equal proportions intending to attend a College of Education (19,12%) or Technikon (17,06%). Although a small proportion expressed the desire to enter Medical School, this group constituted by far the better candidates in the sample studied. A comparable trend has been observed in White education in RSA: in expressing his concern for the dearth of mathematics and physical science teachers, the Scientific Adviser to the Prime Minister pointed out that

> "the best talent in the natural sciences is already being drawn away to medicine and engineering and the natural science faculties are left with the weaker candidates" (Meiring Naude, 1975:126).

6.2.2.2 Years of Further Education Desired

A measure of the students' educational aspirations was obtained by requiring them to indicate the number of years of further education they would like to complete. It was hypothesized that

(i) there is no relationship between educational aspiration and attitudes towards mathematics;

(ii) there is no relationship between educational aspiration and performance in mathematics.

The relevant correlations are reported in Table 6.29.

TABLE 6.29

CORRELATIONS BETWEEN EDUCATIONAL ASPIRATIONS AND ATTITUDES AND BETWEEN EDUCATIONAL ASPIRATIONS AND ATTAINMENT

SAMPLE	n	ATTITUDE	ATTAINMENT	
Total	680	0,21	0,27	* significant at $p < 0,05$;
Std 9	383	0,30	0,30	
Std 10	297	0,11*	0,31	all others significant at $p < 0,001$.

It was evident that those with higher scores in mathematics had greater aspirations for further education. This finding was consistent with the observation made in the IEA study (Husén, 1967b:251).

Although attitudes toward mathematics correlated positively with educational aspiration, the correlation was lower for Std 10 than for Std 9. This may possibly be due to the greater homogeneity of the older students in respect of aspirations.

6.2.2.3 Desire to Take More Mathematics and/or Subjects Involving Mathematics

The differences in attitudes and attainment between those who wished to study more mathematics and/or subjects involving mathematics and those who wished to study other subjects but not mathematics were investigated. It was hypothesized that there is no difference between these two groups in respect of

(i) attitudes towards mathematics;

(ii) attainment in mathematics.

The respective means are shown in Table 6.30.

TABLE 6.30

MEAN DIFFERENCES IN ATTITUDES AND ATTAINMENT BETWEEN THOSE WHO WOULD LIKE TO TAKE MORE MATHEMATICS AND OTHERS

	n	ATTITUDE $\bar{x}$	ATTITUDE SD	ATTAINMENT $\bar{x}$	ATTAINMENT SD
No mathematics	284	64,63	10,98	8,08	3,54
More mathematics	396	74,33	7,52	10,67	4,25
		z=12,88 ($p < 0,001$)		z=8,72 ($p < 0,001$)	

Both the above null hypotheses were rejected ($p < 0,001$). Those who wished to study more mathematics and/or subjects involving mathematics exhibited significantly more positive attitudes ($p < 0,001$) and performed significantly better on the mathematics test ($p < 0,001$) than those who did not intend to study mathematics any further. The above findings lend support to Aiken's (1970a, 1972b) observation that attitude scales may be good predictors of choice i.e. that attitudes toward mathematics would be important in determining whether a student elects to take further courses in mathematics. In another study Kempa and McGough (1977:301) suggested that

> "it is unlikely that a student with little liking for mathematics would study mathematics or mathematics-related disciplines at the sixth form level".

6.2.3 Vocational Aspirations

A measure of the students' vocational aspirations was obtained by requiring them to state the occupations they would like to enter. These responses were coded into five categories using the IEA classification of occupational categories with slight modifications (Husén, 1967a:144). The occupational levels were as follows:-

1. Unskilled manual workers, labourers, factory hands etc.
2. Skilled manual workers e.g. carpenter, driver, mechanic.
3. Clerical and sales workers (lower levels of white collar workers) e.g. salesmen, clerks, shop assistants, typists, cashiers etc.
4. Sub-professional and technical e.g. technicians, nurses, designers, social workers, supervisors (foremen) and small working proprieters.
5. Higher professional and technical e.g. engineers, doctors, lawyers, principals, lecturers etc.
   and Administrative and Executive e.g. managers, directors etc.

The means in respect of attitudes toward mathematics and attainment in mathematics were examined for the five categories (see Table 6.31).

TABLE 6.31

VOCATIONAL ASPIRATIONS, AND MEAN ATTITUDE AND ATTAINMENT SCORES

OCCUPATIONAL LEVELS	NO.	%	ATTITUDE		ATTAINMENT	
			$\overline{x}$	SD	$\overline{x}$	SD
1. Unskilled	0	0	0	0	0	0
2. Skilled manual	50	7,35	66,34	10,04	8,04	3,49
3. Clerical and sales	26	3,82	64,27	10,96	6,42	2,42
4. Sub-professional and Technical	136	20,00	69,63	10,34	8,91	3,92
5. Higher professional and Technical	468	68,82	71,22	10,05	10,13	4,23
Total	680	99,99				

As expected, none of the pre-matriculants wished to enter occupations in category 1, while the greatest proportion aspired for the higher level vocations. In order to determine the significance of the differences between the means one-way ANOVAS, using occupational level as a classification variable, were computed (see Tables 6,32 and 6,33). Occupational level 1 was omitted in the analysis.

TABLE 6.32

ONE-WAY ANOVA - MEAN ATTITUDE SCORES OVER OCCUPATIONAL LEVELS

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	2186,29	3	728,76	7,04
Within Groups	69970,95	676	103,51	($p < 0,001$)
Total	72167,24	679		

TABLE 6.33

ONE-WAY ANOVA - MEAN ATTAINMENT SCORES OVER OCCUPATIONAL LEVELS

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	580,75	3	193,58	11,65
Within Groups	11224,98	676	16,61	($p < 0,001$)
Total	11805,73	679		

The ANOVAS revealed significant differences between means for attitudes toward mathematics and attainment in mathematics ($p < 0,001$). A further examination of the means revealed that those who opted for category 3 (smallest proportion) obtained the lowest means for both attitudes and attainment. Moreover, those who aspired to category 5 (higher professional, technical and administrative) obtained higher scores. In general, it was evident that those who aspired to higher vocations appeared to be more positive towards mathematics and also attained higher scores on the mathematics test.

In a slightly different approach to this problem, the IEA study (Husén, 1967b:252) used vocational aspirations as a dichotomous variable in which the student's response was categorized as to whether it was or was not a professional or technical type of occupation for which higher mathematics would be a requirement. There was a significant positive relationship with achievement in mathematics for 10 of the 12 countries in respect of the 13-year olds and for Belgium, England and United States in the case of the final years.

6.2.4 Home Background

In this study, fathers' occupation and parents' level of education were used as measures of socio-economic status which was indicative of the home background (Kulkarni and Naidu, 1970; Husén, 1967b; Keeves, 1975). Mother's occupation was not considered since a very large percentage of them (78,68%) fell into the category: housewife or never worked or not known.

It was hypothesized that fathers' occupation and parents' level of education will be positively related to

(i) pupils' attitudes toward mathematics;
(ii) pupils' attainment in mathematics.

In this section these hypotheses in respect of the two variables are examined.

6.2.4.1 Father's Occupational Status

The five categories which were used to determine the vocational aspirations of the pupils were used to classify the occupations of the fathers. The means in respect of attitudes toward mathematics and attainment in mathematics were examined in the various categories (see Table 6.34).

TABLE 6.34

MEANS IN RESPECT OF ATTITUDES AND ATTAINMENT ACCORDING TO FATHERS' OCCUPATIONAL STATUS

OCCUPATIONAL STATUS	n	%	ATTITUDE		ATTAINMENT	
			$\bar{x}$	SD	$\bar{x}$	SD
0. Not known	28	4,12	70,89	9,50	8,46	3,01
1. Manual unskilled	21	3,09	71,76	10,03	7,71	2,95
2. Manual skilled	219	32,21	70,78	9,79	9,12	3,96
3. Sales, clerical	130	19,12	70,92	9,96	9,52	3,81
4. Sub-professional, technical	204	30,00	69,27	10,95	9,65	4,28
5. Higher professional	78	11,47	68,83	10,61	11,78	4,78
Total	680	100,00				

Using occupational status as the classification variable, one-way ANOVAS were applied to both attitude means and attainment means in order to determine the significance of the differences (see Tables 6.35

and 6.36). Category 0 (not known) was omitted in the analysis as its contribution would be meaningless in the present context.

TABLE 6.35

ONE-WAY ANOVA - MEAN SCORES ON ATTITUDES TOWARD MATHEMATICS OVER FATHERS' OCCUPATIONAL STATUS

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	377,44	4	94,36	0,88
Within Groups	69239,42	647	107,02	($p > 0,10$)
Total	69616,86	651		

TABLE 6.36

ONE-WAY ANOVA - MEAN SCORES ON ATTAINMENT IN MATHEMATICS OVER FATHER'S OCCUPATIONAL STATUS

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	496,54	4	124,14	7,29
Within Groups	11023,24	647	17,04	($p < 0,001$)
Total		651		

It is clear from Table 6.35 that the occupational level of the father had no significant influence on the students' attitudes toward mathematics. However, from Table 6.36, it is evident that the differences in means of attainment in mathematics were significant for the various levels.

The general pattern was that pupils of fathers in the higher occupational levels performed significantly better than those with fathers in the lower occupational levels.

6.2.4.2 Parents' Level of Education

Six levels of education of the fathers and mothers were used to classify pupils in terms of attitude and attainment scores. It was hypothesized that the parents' level of education is positively related to

(i) pupils' attitudes towards mathematics;

(ii) pupils' attainment in mathematics.

The means in respect of attitude and attainment scores were examined for the various levels of parents' education (see Table 6.37).

TABLE 6.37

MEAN SCORES IN RESPECT OF ATTITUDES AND ATTAINMENT ACCORDING TO FATHERS' LEVEL OF EDUCATION

LEVEL OF EDUCATION	n	%	ATTITUDE		ATTAINMENT	
			$\overline{x}$	SD	$\overline{x}$	SD
1. No school education	14	2,06	67,43	10,04	9,29	2,96
2. Primary school	204	30	70,78	9,87	9,13	3,86
3. High school	374	55	69,86	10,31	9,35	4,16
4. Diploma (post matric)	35	5,15	70,29	11,92	11,11	3,36
5. University degree	28	4,12	73,93	7,44	11,96	4,75
6. More than one univ. degree	25	3,68	69,88	12,65	12,24	5,25
Total	680	100,01				

Using fathers' level of education as a classification variable, one-way ANOVAS were applied to each set of means in order to determine the significance of the differences (see Tables 6.38 and 6.39).

TABLE 6.38

ONE-WAY ANOVA - MEAN SCORES ON ATTITUDES TOWARD MATHEMATICS OVER FATHERS' EDUCATIONAL LEVEL

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	607,72	5	121,54	1,14
Within Groups	71562,50	674	106,18	($p > 0,10$)
Total	72170,22	679		

TABLE 6.39

ONE-WAY ANOVA - MEAN SCORES ON ATTAINMENT IN MATHEMATICS OVER FATHERS' EDUCATIONAL LEVEL

SOURCE OF VARIANCE	SUM OF SQUARES	d.f.	MEAN SQUARE	F-ratio
Between Groups	479,67	5	95,93	5,70
Within Groups	11350,42	674	16,84	($p < 0,001$)
Total	11830,09	679		

It was observed (Table 6.38) that father's level of education had no significant effect ($p > 0,10$) on the pupils' attitudes toward mathematics. However, it was found that the mean differences in mathematics attainment were significant ($p < 0,001$) for the various levels (see Table 6.39). Further, it was observed that the correlation between father's level of education and attitudes toward mathematics was 0,02 ($p > 0,05$), while the relationship with achievement in mathematics was given by $r=0,18$ ($p < 0,01$). Since a similar pattern of relationships emerged between mother's educational level and attitudes toward mathematics ($r=0,02$; $p > 0,05$), and achievement in mathematics ($r=0,13$; $p < 0,01$), a separate data analysis has not been presented here.

It was thus concluded that parents' level of education had no effect on the pupils' attitudes toward mathematics. However, there was clear indication that the higher the level of education of the parents the higher the level of mathematics attainment of the pupils.

6.2.4.3 General Comments

The findings in this study are, in general, consistent with those of other researches. The National Council of Educational Research and Training (NCERT - India), as part of a major survey in mathematics achievement, investigated the relationship between socio-economic variables and mathematics achievement. The results revealed that the "level of father's occupation has got definite influence on the performance of students" (Kulkarni and Naidu, 1970:56). The IEA study (Husén, 1967b:254) reported that for all 12 countries under study

> "there is a significant relationship between the parents' characteristics (education and occupational status) and the students' mathematics achievement".

However, in the same study it was observed:

> "we did not find strong relationships between students' interest in mathematics and father's level of education" (Husén, 1967b:254).

In another study on the profile of high and low achievers in mathematics, Gordon (1978:4640) found that

> "the vocational status of the students' parents is related to the students' achievement in mathematics".

Austin and Postlethwaite (1974) observed that there was an interaction between home background and attainment of instructional objectives in mathematics. Wimbush (1977) found that socio-economic status was one of the best single predictors of achievement in mathematics.

In studies of a general nature, it has been shown that a child's school achievement is determined to some extent by his parents' attitudes which are partly determined by their material circumstances and by their socio-economic status (Plowden (1967) cited by Barker-Lunn, 1972:73). Moreover, Douglas (1964:170-1) found that middle class children generally did better than the manual working class children on the eleven-year tests. Lavin (1967:125) observed from review of research in this area that "the higher one's social status, the higher his level of performance".

It would appear that parents of higher occupational status recognize the value of success in mathematics and hence probably exert greater pressure on their children to succeed. Finalyson (1971:61) explains that certain aspects of the socio-economic background of parents determine the degree to which they value education and suggests that

> "children of parents with working-class values therefore do less well than the children of parents with middle class values who foster behaviour and attitudes considered conducive to successful school performance".

Douglas (1964:83) found that middle class parents show a greater interest in their children's education than the working class parents.

From the findings in the present study it would appear that, while the parents see the importance of good performance in mathematics, they may not (in general) be aware of the role of attitudes towards mathematics. It is also probable that the parents in the higher socio-economic levels can afford to provide extra tuition for their children hence enhancing their performance.

6.3 TEACHERS' PERCEPTIONS OF PUPILS' ATTITUDES TOWARD MATHEMATICS AND ACHIEVEMENT IN MATHEMATICS

This study recognized the importance of the role the teacher can play in rating the pupil's attainment in mathematics and attitude towards mathematics. The teachers involved in this study were required to rate

(i) pupils' attainment of objectives in mathematics;
(ii) pupils' attainment in mathematics in terms of content areas;
(iii) pupils' attitudes toward mathematics;
(iv) pupils' ovaerall performance in mathematics.

In this way data was gathered (see TQ1 - Appendix 4) to investigate how the mathematics teacher perceived the pupils' attitude towards mathematics and achievement in mathematics. In this section some background information on the teachers is provided and the teachers' ratings are discussed.

6.3.1 Background Information on Mathematics Teachers

As pointed out earlier, every mathematics teacher taking Std 9 and/or Std 10 mathematics in the Ordinary Course in the 17 high schools was involved in this study. There were 151 class units which were taught by 53 teachers. It would be useful to consider the academic and professional background of these teachers before discussing their ratings of the pupils (see Table 6.40, and TQ2 of Appendix 9).

TABLE 6.40

ACADEMIC AND PROFESSIONAL BACKGROUND OF THE MATHEMATICS TEACHERS (n=53)

	No.	%
Males	47	88,68
Females	6	11,32
Received special training to teach mathematics	45	84,91
No special training to teach mathematics	8	15,09
Professional Qualifications:		
Nil	1	1,89
Two-year Teacher's Diploma	2	3,77
Three-year Teacher's Diploma	42	79,25
One-year Post-graduate Teacher's Diploma	8	15,09
Academic Qualifications:		
Highest Degree: Nil	30	56,60
Bachelors Degree	13	24,53
Honours Degree	1	1,89
BEd	9	16,98
MEd	0	0 0
Highest Qualification in Mathematics		
Nil	4	7,55
Mathematics I or equivalent	32	60,38
Mathematics II	7	13,21
Mathematics III	10	18,87
Mathematics (Hons)	0	0
Mean Years of Teaching Experience		9,72
Mean Years of Experience in the Teaching of Mathematics		8,87

It was evident that the teaching force involved in this study consisted predominantly of males. It was also observed that a great majority of the teachers had entered the profession during the course of the last 10 years (mean years of teaching experience = 9,72) and most of them had been engaged in the teaching of mathematics most of the time during this period (mean years of teaching mathematics = 8,87).

While 98,11% of them were qualified teachers (with 94,34% having attained a minimum of 3 years professional training), some 15,09% had not received any special training to teach mathematics. It was also noted that 92,45% possessed at least Mathematics I (or equivalent) and 43,40% held at least a bachelor's degree. Taking Mathemathics I (or equivalent) as a minimum academic qualification and a three-year diploma as a minimum professional qualification, it was found that 90,57% of these teachers satisfied this requirement. However, if Mathematics II is taken as a minimum qualification, then only 32,08% satisfy the requirement. This figure, compared with those for the White education departments (1972), revealed a serious shortage of well-qualified mathematics teachers in the RSA (van den Berg, 1976:8):-

Orange Free State	57,6%
Natal	30,1%
Cape Province	65,3%
Transvaal	40,3%

Of the 53 teachers involved in this study 20,75% held promotion posts (Head of Department or Deputy Principal) and the remaining 79,25% were ordinary teachers. It must be noted that there were no Heads of Department (specifically) for mathematics at 6 of the 17 schools.

It was also found that the mean teaching load of the ordinary teachers was 38,31 periods per week while their mean mathematics teaching load was 36,74 p.p.w. The negligible difference is indicative of the fact that the mathematics teacher was teaching mainly mathematics. It was therefore clear that the services of the mathematics teachers were being maximally utilized. This was expected in view of the shortage experienced in this field.

6.3.2 Teachers' Ratings in terms of Objectives in Mathematics Learning

The teachers indicated on a 5-point scale (see 2 of TQ1 - Appendix 4) the extent to which pupils attained the following objectives in mathematics: Knowledge, Skills, Comprehension, Selection-Application and Analysis-Synthesis. The mean ratings for each of these levels were observed (see Table 6.41).

TABLE 6.41

DIFFERENCES BETWEEN MEAN RATINGS IN TERMS OF OBJECTIVES (n=680)

OBJECTIVES	$\overline{x}$	SD	DIF.	r	z-SCORE
KNOWLEDGE	2,51	0,89			
			0,17	0,78	8,54 ($p < 0{,}001$)
SKILLS	2,34	0,89			
			0,15	0,79	7,72 ($p < 0{,}001$)
COMPREHENSION	2,19	0,88			
			0,13	0,84	7,66 ($p < 0{,}001$)
SELECTION-APPLICATION	2,06	0,91			
			0,16	0,88	10,89 ($p < 0{,}001$)
ANALYSIS-SYNTHESIS	1,90	0,87			

The differences between the means were not only significant ($p < 0{,}001$) but they also revealed a strict hierarchical pattern with mean ratings higher for lower level objectives and lower for higher level objectives. This finding was consistent with the theoretical formulation of hierarchical levels of objectives (Bloom et al, 1956) and with the pattern of performance in the mathematics test found in this study (see discussion in section 5.2.4). This result was indicative of the clarity with which the teachers were distinguishing between the various levels of objectives in mathematics. This positive trend must be attributed to the use of objectives (as set down in the syllabus (DIE, 1976)) in recent years.

There was also a high degree of consistency in rating by the teachers as seen from the substantial intercorrelations in Table 6.42.

TABLE 6.42

INTERCORRELATIONS OF RATINGS BY TEACHERS IN TERMS OF OBJECTIVES (n=680)

	1	2	3	4	5	6
1. Knowledge	-					
2. Skills	0,78	-				
3. Comprehension	0,73	0,79	-			
4. Selection-Application	0,68	0,74	0,84	-		
5. Analysis-Synthesis	0,63	0,72	0,82	0,88	-	
6. Total	0,85	0,90	0,93	0,92	0,90	-
All correlations significant at $p < 0{,}001$.						

6.3.3 Teachers' Ratings in terms of Content Areas (Algebra, Geometry and Trigonometry)

The teachers were also required to rate pupils' performance on the 5-point scale in each of the above content areas. The differences between the means (see Table 6.43) were significant ($p < 0{,}001$).

It was also observed that the differences were strikingly consistent with those found for pupils' content preferences (see section 6.1.1.7). The pupils showed a greater preference for Algebra than for Geometry and the teachers had (independently) rated their performance higher in

Algebra than in Geometry. The pattern of these differences was invariant when the data was analysed separately over grades, levels and sex.

TABLE 6.43

DIFFERENCES BETWEEN MEAN RATINGS BY TEACHERS IN TERMS OF CONTENT AREAS (correlated data)

	$\bar{x}$	SD	DIF.	r	z-SCORE	
Algebra	2,55	0,81				
			0,26	0,72	11,58	($p < 0,001$)
Trigonometry	2,29	0,86				
			0,25	0,71	10,94	($p < 0,001$)
Geometry	2,04	0,90				

Further, the intercorrelations of the ratings showed a high degree of consistency in the teachers' ratings (see Table 6.44).

TABLE 6.44

INTERCORRELATIONS OF RATINGS BY TEACHERS IN TERMS OF CONTENT AREAS (n=680)

	1	2	3
1. Algebra	-		
2. Trigonometry	0,72	-	
3. Geometry	0,72	0,71	-
All correlations significant at $p < 0,001$.			

6.3.4 Teachers' Ratings of Pupils' Attitudes toward Mathematics

The teachers were supplied with 10 statements (5 negative and 5 positive) about the pupils' attitudes toward mathematics and required to rate the attitudes on a 3-point Likert-type scale (see 3 of TQ1 - Appendix 4). The summated rating was taken as a measure of the teacher's perception of the pupils' attitude towards mathematics. The teachers' ratings were found to correlate significantly positively with the total scores on the attitude scale ($r=0,35$; $p < 0,001$) and with pupils' liking for mathematics in relation to other subjects in the school curriculum ($r=0,44$; $p < 0,001$). These ratings also correlated more highly with the 'enjoyment' dimension of the attitude scale ($r=0,38$; $p < 0,001$) than with any other dimension.

The mean attitude ratings by the teachers were investigated over grades, levels and sex (see Table 6.45). There were significant differences between the means in respect of grades even when examined separately over the two levels. The pattern of differentiation made by the teachers in respect of grades and levels was strikingly consistent with the findings obtained by the use of the attitude scale. The teachers clearly recognized the HG pupils as being more positively disposed towards mathematics than their SG counterparts, while they perceived no differences in attitudes between the levels.

However, there was a tendency for teachers to rate the attitudes of girls towards mathematics more highly than those of the boys ($p < 0,05$). There is support for this observation as far as general attitudes are concerned: girls are more conforming and amenable to discipline and order than boys (Fitt, 1956; Wisenthal, 1965) and "teachers tend to find girls more of a pleasure than boys" (Barker Lunn, 1972:72). Therefore, a possible explanation is that the teachers' ratings in this study were somewhat 'coloured' by the general attitudes displayed by the pupils.

TABLE 6.45

DIFFERENCES BETWEEN MEANS FOR TEACHERS RATINGS OF ATTITUDES OVER GRADES, LEVELS AND SEX

		n	$\bar{x}$	SD	DIF.	z-SCORE
STD 10	HG	104	15,16	5,71	4,10	5,51 ($p < 0,001$)
	SG	193	11,06	6,75		
STD 9	HG	192	14,06	6,08	4,49	6,90 ($p < 0,001$)
	SG	191	9,57	6,62		
STD 10		297	12,50	6,70	0,68	1,31 ($p > 0,05$)
STD 9		383	11,82	6,74		
MALES		398	11,61	6,90	1,22	2,36 ($p < 0,05$)
FEMALES		282	12,83	6,42		

6.3.5 Teachers' Ratings of Pupils' Overall Performance in Mathematics

A further measure of a pupil's performance in mathematics was obtained by requiring the teacher to estimate the pupil's performance level in mathematics based on tests, assignments and examinations (see 4 of TQ1 - Appendix 4). These ratings were found to correlate significantly highly with total scores on the mathematics test ($r=0,50$; $p < 0,001$) and with pupils' own ratings of performance ($r=0,70$; $p < 0,001$).

The mean performance ratings by the teachers were examined over grades, levels and sex (see Table 6.46). The pattern of differentiation made by the teachers in respect of grades (over separate levels) was consistent with the findings obtained by the use of the mathematics test. The teachers clearly recognized the HG pupils as better performers in mathematics than the SG.

TABLE 6.46

DIFFERENCE BETWEEN MEANS FOR TEACHERS' RATINGS OF MATHEMATICS PERFORMANCE OVER GRADES, LEVELS AND SEX

		n	$\bar{x}$	SD	DIF	z-SCORE
STD 10	HG	104	54,84	15,18	12,10	6,58 ($p < 0,001$)
	SG	193	42,74	14,70		
STD 9	HG	192	50,90	13,82	11,09	8,49 ($p < 0,001$)
	SG	191	39,81	11,53		
MALES		398	45,60	14,30	1,13	0,96 ($p > 0,05$)
FEMALES		282	46,73	15,56		

In respect of sex, however, the teachers' perceptions of the pupils' performance differed from that obtained by the mathematics test which differentiated in favour of males. A possible explanation is that the teachers may have tended to rate girls slightly higher because they tended to recognize the more positive attitudes of girls.

6.3.6 General Comments

The intercorrelations of the teachers' ratings in terms of objectives, content areas, overall performance and attitudes towards mathematics showed a high degree of interrelationship (see Table 6.47).

TABLE 6.47

INTERCORRELATIONS OF TEACHERS' RATINGS OF PUPILS' MATHEMATICAL ACHIEVEMENT AND ATTITUDES TOWARD MATHEMATICS (n=680)

	1	2	3	4
1. Objectives	-			
2. Content Areas	0,92	-		
3. Overall Performance	0,83	0,82	-	
4. Attitudes	0,72	0,70	0,74	-
All correlations significant at $p < 0{,}001$.				

In addition to the hypothesized relationship between attitudes and attainment, the significant and substantial relationship between the teachers' ratings of attitudes and their ratings of performance may be attributed to the fact that the teachers' assessment of performance is influenced by his perception of the pupil's attitude and vice versa. Teachers tend to rate more highly those who show more positive attitudes or conversely

> "teachers varied in their attitude to pupils of different ability, and the general trend was for a less favourable response to duller pupils" (Barker Lunn, 1972:72).

The manner in which the interplay of attitudes and achievement (as observed by the teachers' ratings) manifested itself in this study is consistent with Neale's (1969) suggestion that there is a mutual relationship between attitudes and achievement in mathematics.

The generally striking consistency in patterns observed between the teachers' ratings and the results obtained through this researcher's instruments indicates a need to explore more fully the use of teachers' ratings in evaluation programmes. In this context Capadona and Kerzner-Lipsky (1979:144) suggested that

> "teachers and administrators consider teachers' ratings as the most significant and economic method for prediction of mathematical achievement in the 7th grade".

In this researcher's view there would appear to be room for a far greater involvement of the class teacher in assessing pupil achievement.

6.4 RELATIONSHIP BETWEEN ATTITUDES TOWARDS MATHEMATICS AND ATTAINMENT IN MATHEMATICS

Several studies which have examined the existence of a relationship between attitudes toward mathematics and attainment in mathematics have already been reviewed and/or mentioned in chapter three. The results of the present study in this context will be presented and discussed in the sections which follow.

The preceding sections of this chapter examined the manner in which attitudes toward mathematics and attainment in mathematics were related to several variables involving social factors, aspirations, curriculum and school organization and teachers' ratings (a summary is presented in Table 6.48). It was found that several of these variables were related to *both* attitudes toward mathematics and attainment in mathematics. This provided further support for investigating the relationship between attitudes and attainment.

TABLE 6.48

SUMMARY - SIGNIFICANCE OF DIFFERENCES AND RELATIONSHIPS IN RESPECT OF SELECTED BACKGROUND VARIABLES, ATTITUDES AND ATTAINMENT

VARIABLES	ATTITUDES		ATTAINMENT	
	Difference	Relationship	Difference	Relationship
Reasons for choosing Maths	S($p < 0,001$)		S($p < 0,025$)	
Reasons for choice of HG	S($p < 0,001$)		S($p < 0,025$)	
Reasons for choice of SG	S($p < 0,001$)		NS	
Mathematics Bias		S($p < 0,001$)		S($p < 0,05$)
Science Bias	S($p < 0,001$)		S($p < 0,001$)	
Preference for Maths		S($p < 0,001$)		S($p < 0,001$)
Self-assessment of Performance		S($p < 0,001$)		S($p < 0,001$)
Content Preference		S($p < 0,01$)		S($p < 0,01$)
Cognitive Preference (Diagram)		S($p < 0,001$)		S($p < 0,001$)
Grades (HG/SG)	S($p < 0,001$)		S($p < 0,001$)	
Levels (Std 9/ Std 10)	NS		S($p < 0,001$)	
Class Size		NS		NS
Homework		S($p < 0,001$)		S($p < 0,05$)
Sex	NS		S($p < 0,05$)	
Father's Occupational Status	NS		S($p < 0,01$)	
Parents' Level of Education	NS		S($p < 0,01$)	
Educational Aspirations:				
Further Education Institution	S($p < 0,01$)		S($p < 0,01$)	
Yrs. of Further Education		S($p < 0,05$)		S($p < 0,001$)
More Mathematics	S($p < 0,001$)		S($p < 0,001$)	
Vocational Aspirations	S($p < 0,01$)		S($p < 0,01$)	
Teachers' Ratings		S($p < 0,001$)		S($p < 0,001$)

S = Significant, NS = Not Significant, Alpha level : $p < 0,05$.

6.4.1 Attitude, Attainment and Ability

In order to arrive at as accurate a picture as possible of the relationship between attitudes and attainment, ability (IQ) was included as a further important variable in the examination of this relationship. This decision was influenced by the weight of research studies which have used IQ (ability scores) in investigating academic achievement. Marjoribanks (1976:654) suggested that inconsistencies in findings on the relationship between attitudes and attainment often occur because of

> "the failure of most of the studies to include in their analyses an examination of the cognitive abilities of the children".

Barakat (1951:239) concluded that

> "a basic or general ability, roughly identifiable with innate intelligence, appears to play by far the largest part in mathematical attainments of every kind".

Aiken (1976) observed that ability is usually a more important predictor of achievement than attitudes. Cattell and Butcher (1968:13) strongly suggest

> "that general ability is the most important predictor of school achievement in our studies".

Several other research studies on achievement and/or attitudes have included ability as a variable (e.g. Stephens, 1960; Cristantiello, 1962; Williams, 1970; Botes, 1976; Gilbert, 1977; Hunkler, 1977; Cappadona and Kerzner-Lipsky, 1979).

IQ scores (verbal and non-verbal ability) were obtained from school records on GTISA (NBESR, 1968) for 641 of the total sample of 680 pupils. Mean IQ scores according grade, level and sex are shown in Table 6.49. As expected, there were no differences in mean IQ between the levels and between the sexes. However, there were significant differences

($p < 0{,}001$) between HG and SG in respect of IQ, even when examined separately over the levels, with more able pupils in the HG.

TABLE 6.49

MEAN IQ SCORES ACCORDING TO GRADE, LEVEL AND SEX

SAMPLE	n	$\overline{x}$	SD	z-SCORE FOR DIFFERENCES
TOTAL	641	117,03	11,57	
Std 9	361	116,66	11,05	0,91 (p > 0,05)
Std 10	280	117,51	12,20	
Males	369	116,65	11,42	0,96 (p > 0,05)
Females	272	117,54	11,75	
Std 9 (HG)	183	121,00	9,92	10,27 (p < 0,001)
Std 9 (SG)	178	112,19	10,35	
Std 10 (HG)	98	124,68	11,57	7,82 (p < 0,001)
Std 10 (SG)	182	113,64	10,68	

Pearson product-moment correlations were used to determine the interactions among attitudes, attainment and ability. The intercorrelations were computed separately for males and females (see Table 6.50).

It was found that the correlations were generally positive and significant and the magnitudes of the relationships were in descending order as follows:

(i) Attainment X Ability,

(ii) Attitude X Attainment,

(iii) Attitude X Ability.

TABLE 6.50

INTERCORRELATIONS : ATTITUDES, ATTAINMENT AND ABILITY

	ALL			FEMALES			MALES		
	1	2	3	1	2	3	1	2	3
1. Attitudes toward Maths.	-	27	18*	-	28	22	-	27	14*
2. Attainment in Maths.	27	-	59	28	-	64	27	-	57
3. Ability Scores (IQ)	18*	59	-	22	64		14*	57	-
n	680	680	641	282	282	272	398	398	369

Correlations with decimals omitted. *significant at $p < 0,01$; all others significant at $p < 0,001$.

While attitude towards mathematics was positively (and significantly) related to attainment it was clear that ability emerged as the more important predictor of mathematical attainment. There were no differences between males and females in the extent of the relationship between attitudes and achievement. In both cases the relationships were positive and significant. Thus the findings in this study failed to support the distinction found by Aiken and Dreger (1961) in favour of females: Although the relationship between attitude and ability was considerably lower, it was found that it was much higher for the 'enjoyment of mathematics' dimension of the attitude scale (r=0,25). Kempa and McGough (1977:301) reported similar findings.

It must be pointed out that the relationships found here were obtained without any control of variables other than that exercised through random sampling. It was thus decided that the relationship between any two of these variables should be examined by controlling the third. This was achieved through the technique of partial correlation (Guilford and Fruchter, 1973) and the calculation of first-order correlations. It was thus found that, when ability was partialled out, the relationship between attitude and attainment remained positive and significant ($r_{12,3}$=0,21; $p < 0,01$). Similarly, when attitude was held constant the relationship between ability and attainment remained positive and significant ($r_{23,1}$=0,57; $p < 0,001$).

In general, the relationship between attitudes toward mathematics and attainment in mathematics was found to be positive and significant. This result is consistent with the findings of several research studies (Stephens, 1960; Aiken and Dreger, 1961; Bassham et al, 1964; Spickerman, 1970; Burbank, 1970; Whipkey, 1970; Husén, 1967b; Fenneman, 1974; Johnson, 1977; Gordon, 1978).

The finding that ability emerged as an important predictor of achievement in mathematics is also supported by several research studies (e.g. Barakat, 1951; Aiken, 1976; Marjoribanks, 1976; Gilbert, 1977; Hunkler, 1977; Gordon, 1978).

6.4.2 Attitude as a Moderator Variable

Aiken (1970c) suggested that attitude measures may be used more efficiently as moderator variables in predicting mathematics attainment from ability scores. In this approach ability and attainment are correlated separately at each level of attitude. For this purpose the frequency distribution of attitude scores was used to split the sample into high, middle and low attitude groups. A perfectly equitable distribution of numbers over the three groups was not accomplished as pupils with the same score had to be kept within the same group.

The non-availability of IQ scores for some also affected the numbers.

The groups were defined as follows:

High : 77 - 96, n = 203

Middle : 67 - 76, n = 275

Low : 0 - 66, n = 202.

The correlations between IQ and attainment for each of the groups are shown in Table 6.51.

TABLE 6.51

CORRELATIONS BETWEEN ABILITY (IQ) AND ATTAINMENT AT EACH ATTITUDE LEVEL

ATTITUDE LEVEL	n	r
High	195	0,496 ($p < 0,001$)
Middle	258	0,619 ($p < 0,001$)
Low	188	0,571 ($p < 0,001$)

Although the correlations were significant at each level, the ability scores correlated somewhat more highly with mathematics attainment for the middle range of attitudes. This trend was similar to that observed by Cristantiello (1962) who found that correlation between ability scores and mathematics grades were significantly more positive for students with middle attitude scores. In a related study, Jackson (1968), cited by Aiken (1970b), suggested that the middle range of attitude scores have little relation to achievement and that only at the extremes attitude affects achievement. A similar pattern emerged in this study (see Table 6.52). Although not all correlations were significant, in the middle range of attitudes the relationship between attitude and achievement was markedly smaller (and close to zero) than at the extremes.

TABLE 6.52

CORRELATIONS BETWEEN ATTITUDE AND ATTAINMENT AT EACH ATTITUDE LEVEL

ATTITUDE LEVEL	n	r
High	203	0,175 ($p < 0,05$)
Middle	275	0,027 ($p > 0,10$)
Low	202	0,118 ($p > 0,05$)

The results of the researches by Cristantiello (1962) and Jackson (1968) led Aiken (1970b) to conclude that, in the middle range of attitude scores, ability scores rather than attitude scores will be more accurate predictors or determiners of achievement. Thus, it may be useful to take cognisance of the prevailing attitudes of a given population toward mathematics when using ability scores to predict achievement.

6.4.3 Differences between Means for Extreme Groups according to Attitude, Attainment and Ability

Using high, middle and low groups in respect of attitude, ability and attainment scores, it was possible to observe for the extreme groups of any one variable the corresponding differences between the groups in respect of the other two variables. For attitudes, the groups were as defined in the previous section. For attainment and ability, the groups were similarly defined (see Table 6.53). The high and low groups were used in the analyses as the extreme groups.

TABLE 6.53

DEFINITION OF HIGH, MIDDLE AND LOW GROUPS IN RESPECT OF ATTAINMENT AND ABILITY (IQ)

LEVELS	ATTAINMENT		ABILITY (IQ)	
	Range	n	Range	n
High	12-24	202	122-145	217
Middle	8-11	237	112-121	213
Low	1- 7	241	72-111	211

6.4.3.1 Extreme Attitude Groups

The differences between extreme attitude groups were examined in respect of attainment in mathematics and ability (IQ). Table 6.54 shows the means.

TABLE 6.54

DIFFERENCES BETWEEN MEANS FOR EXTREME ATTITUDE GROUPS IN RESPECT OF ATTAINMENT AND ABILITY

ATTITUDE LEVEL	ATTITUDES		ATTAINMENT			ABILITY (IQ)		
	n	$\overline{x}$	n	$\overline{x}$	z-SCORE	n	$\overline{x}$	z-SCORE
High	203	80,86	203	10,89	7,30	195	119,52	4,48
Low	202	57,54	202	8,10	($p < 0{,}001$)	188	114,58	($p < 0{,}001$)

It was found that the high attitude group performed significantly better than the low attitude group on the mathematics test ($p < 0{,}001$). Also, the high attitude group had significantly higher ability scores than the low attitude group ($p < 0{,}001$). It was thus evident that those who were positively inclined towards mathematics were not only brighter but also the higher achievers in mathematics than those with low attitude scores. These findings are consistent with those of a related study (Marjoribanks, 1976:659) which reported that

> "at each attitude level increases in cognitive ability are related to increases in academic achievement".

In another related study, Williams (1970) reported that students who were dissatisfied with school obtained significantly lower scores on all ability, achievement and personality variables than students of positive orientation. The findings here may be summarized mathematically as follows:

$$\text{d.extr.}\left[\text{ATTITUDE}\right] \Longrightarrow \text{d}\left[\text{ATTAINMENT}\right] \wedge \text{d}\left[\text{ABILITY}\right]$$

In other words, if there is a difference between extreme attitude groups then there are corresponding significant differences in attainment and ability.

6.4.3.2 Extreme Attainment Groups

Next, the differences between extreme attainment groups were examined in respect of attitudes towards mathematics and ability (see Table 6.55).

An examination of the extreme attainment groups revealed that the high attainment group possessed significantly more positive attitudes than the low attainment group ($p < 0{,}001$). Also, the high attainment group had significantly higher ability scores than the low attainment group

($p < 0,001$). It was therefore concluded that the high achievers in mathematics were more positive in their attitudes toward mathematics and possessed higher ability (IQ) than the low achievers. This result is supported by Stephens (1960) who found that there was a highly significant difference in attitudes towards arithemtic between high and low achievers. In a related study, Degnan (1967:59) produced data which showed that a

> "a group of 'achievers' obtained a mean score of 63,9 indicating a much more positive attitude toward arithmetic than that held by the group of 'underachievers' who obtained a mean score of 28,6".

TABLE 6.55

DIFFERENCES BETWEEN MEANS FOR EXTREME ATTAINMENT GROUPS IN RESPECT OF ATTITUDES AND ABILITY

ATTAINMENT LEVEL	ATTAINMENT		ATTITUDES			ABILITY		
	n	$\bar{x}$	n	$\bar{x}$	z-SCORE	n	$\bar{x}$	z-SCORE
High	202	15,04	202	74,06	7,53	192	125,64	14,26
Low	241	5,41	241	67,02	($p < 0,001$)	227	111,27	($p < 0,001$)

In general, the present findings may be depicted as follows:

$$\text{d.extr.}[\text{ATTAINMENT}] \Longrightarrow \text{d}[\text{ATTITUDE}] \wedge \text{d}[\text{ABILITY}]$$

In other words, if there is a difference between extreme attainment groups then there are corresponding significant differences in attitudes and ability.

6.4.3.3 Extreme Ability Groups

Finally, the differences between the extreme ability groups were examined in respect of attitudes toward mathematics and attainment in mathematics (Table 6.56 shows the means).

TABLE 6.56

DIFFERENCES BETWEEN MEANS FOR EXTREME ABILITY (IQ) GROUPS IN RESPECT OF ATTITUDES AND ATTAINMENT

ABILITY LEVEL	ABILITY (IQ)		ATTITUDES			ATTAINMENT		
	n	$\bar{x}$	n	$\bar{x}$	z-SCORE	n	$\bar{x}$	z-SCORE
High	217	129,77	217	72,40	4,50	217	12,77	15,18
Low	211	104,62	211	68,03	($p < 0,001$)	211	7,21	($p < 0,001$)

It was evident that the high ability group performed significantly better than the low ability group on the mathematics test ($p < 0,001$). It was also found that the high ability group possessed significantly more positive attitudes than the low ability group ($p < 0,001$). It was therefore concluded that the brighter pupils were not only more positive in their attitudes toward mathematics but were also higher achievers in mathematics than the low ability group.

These findings are, in general, consistent with those of Marjoribanks (1976:659) who concluded that

> "at each level of ability, increases in attitude scores, in general, are related to increases in achievement".

Sociometric studies (Buswell, 1953; Heber, 1956) have suggested that the brighter child's greater popularity, due partly to his greater self confidence, contributes to the formation of more positive attitudes. Barakat (1951) investigated the factors underlying mathematical abilities of Grammar School pupils and concluded that general ability (IQ) appears to play the most important role in mathematical attainments. The findings in the present study may be represented as follows:

$$\text{d.extr.}[\text{ABILITY (IQ)}] \Longrightarrow \text{d}[\text{ATTITUDES}] \quad \wedge \quad \text{d}[\text{ATTAINMENT}]$$

In other words, the difference between extreme ability groups implies that there are corresponding significant differences in attitudes and attainment.

6.4.4 A Multiple Regression Equation for Predicting Attainment from Attitude and Ability Scores

It was observed earlier that ability (as measured by GTISA) accounted for 34,81% of the variance in attainment scores. When attainment and attitudes were combined a multiple correlation coefficient of 0,613 (between attainment (X_1) and a combination of attitudes (X_2) and ability (X_3)) was obtained (see 10 of Appendix 5). Although a combination of the two variables accounted for a greater percentage (37,58%) of the variance in attainment scores, it was evident that there were other variables than attitude and ability which influenced attainment in mathematics.

Since this was a multiple-prediction problem involving three variables, a multiple regression equation for prediction was derived from the data (shown in Table 6.57) by using the method outlined by Guilford and Fruchter (1973:361-363) (see 11 of Appendix 5).

TABLE 6.57

ATTAINMENT, ATTITUDES AND ABILITY : INTERCORRELATIONS, MEANS AND STANDARD DEVIATIONS

VARIABLE	CORRELATIONS X_1	X_2	X_3	$\overline{x}$	SD
X_1 : Attainment	-			9,70	4,31
X_2 : Attitude	0,27	-		70,28	10,30
X_3 : Ability (IQ)	0,59	0,18	-	117,03	11,57
n=680 All correlations significant at $p < 0{,}01$.					

The multiple regression equation was then found to be:

$$X_1 = -19{,}75 + 0{,}071X_2 + 0{,}209X_3$$

where X_1 is the predicted mathematics attainment score;

X_2 is the attitude score;

X_3 is the IQ score.

For example, the predicted score on the mathematics test for a pupil with IQ score of 120 and attitude score of 80 will be given by:

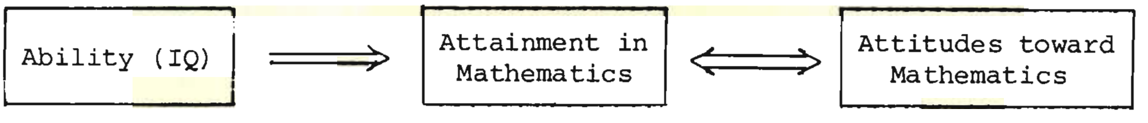
$$X_1 = -19{,}75 + 0{,}071(80) + 0{,}209(120)$$
$$= 11{,}01$$

Using this score against the norms for the test (see Table 7.1), it is possible to predict the percentile rank of the individual in respect of performance in mathematics.

6.4.5 General Comments

The three-way analysis of the means for the extreme groups and the multiple regression equation presented in the preceding sections revealed the interactions among the three variables: attitude, attainment and ability. From the relations that have been found, it is clear that these variables are somewhat mutually related. However, the extent of the interactions vary as observed from the correlations in section 6.4.1. The findings in this study thus strongly point to a reciprocal relationship between attitudes and attainment which is consistent with the suggestions of Neale (1969) and Aiken (1970b) and with the theoretical formulation of the relationships between cognitive and affective objectives (Krathwohl et al, 1964:49-60).

Now, given that basic or general ability is "roughly identifiable with innate intelligence" (Barakat, 1951:239) and taking into account that the largest relationship was found between ability (IQ) and attainment, the relationship may be depicted as follows:



Thus it may be postulated that, in general, those with higher abilities will experience greater success in mathematics and this will result in more positive attitudes towards mathematics which in turn will lead to better performance in mathematics.

CHAPTER SEVEN

MAJOR CONTRIBUTIONS OF THIS STUDY, IMPLICATIONS AND POINTERS FOR FURTHER RESEARCH

The previous chapter dealt in detail with the hypotheses concerning several background variables, attitudes toward mathematics and attainment in mathematics. In this final chapter of this research report, the major contributions of the present study, their implications and certain problems for future research will be presented under the following headings:

(i) The Mathematics Test.

(ii) The Attitude Scale.

(iii) Curriculum and School Organization as related to Attitudes and Attainment in Mathematics.

(iv) Social and Other Factors as related to Attitudes and Attainment in Mathematics.

(v) Teachers' Background and Perceptions of Pupils' Attitudes and Attainment.

(vi) Relationship between Attitudes toward Mathematics and Attainment in Mathematics.

(vii) Other Research Problems.

(viii) General.

7.1 THE MATHEMATICS TEST

From the considerations made in developing the test (see chapter two) and from the psychometric data gathered (see chapter five), it was evident that a highly reliable and valid test of mathematical abilities had been compiled. The test was of a reasonable level of difficulty with highly discriminating items and generally functional distractors. The ability of the test to differentiate between three taxonomic levels of objectives in mathematics learning was also demonstrated.

Although the test is criterion-referenced as compared with norm-referenced in terms of the explanation given by Davies (1976:91-92), the total score can be used, without loss of generality, to compare pupils' performances. This is possible since all schools follow a common syllabus which sets down the criteria for evaluation in terms of a classification of objectives (DIE HG Mathematics Syllabus, 1976). In order to increase the usefulness of the test to teachers, counsellors and researchers, norms were drawn up in terms of z-scores and percentiles (see Table 7.1). In view of the difference in performance between Std 9 and Std 10, it was considered necessary to produce two separate sets of norms.

The test may be used to evaluate both individual and group accomplishment in mathematics. It must be pointed out that the use of the test should be limited to Std 9 and Std 10 Indian pupils in an urban setting. Its use in Std 8 or in a rural population or among other race groups needs to be studied. While it is expected that the norms may differ for the different race groups due to differences in provision of facilities, staffing, guidance etc, it is hypothesized that the broad trends in mathematics achievement will remain unchanged e.g. differences in attainment of objectives in terms of taxonomic levels;

TABLE 7.1

MATHEMATICS TEST NORMS FOR INDIAN PRE-MATRICULANTS.

RAW SCORES	STD 9		STD 10	
	z-SCORE	PERCENTILE	z-SCORE	PERCENTILE
1	-2,02	0	-2,19	0
2	-1,76	1,82	-1,97	0,67
3	-1,50	6,00	-1,75	2,36
4	-1,23	12,27	-1,53	5,39
5	-0,97	21,93	-1,31	9,09
6	-0,71	31,33	-1,09	16,50
7	-0,44	43,08	-0,88	25,59
8	-0,18	51,70	-0,66	32,66
9	0,08	61,36	-0,44	40,74
10	0,34	72,85	-0,22	49,83
11	0,61	79,63	0	58,25
12	0,87	84,86	0,22	67,00
13	1,13	87,47	0,44	72,39
14	1,40	90,86	0,66	81,48
15	1,66	94,52	0,88	82,83
16	1,92	95,82	1,09	87,21
17	2,19	97,13	1,31	89,56
18	2,45	98,43	1,53	90,24
19	2,71	99,74	1,75	93,94
20	2,98		1,97	96,97
21	3,24		2,19	98,65
22	3,50	100,00	2,41	98,99
23	3,77		2,63	99,66
24	4,03		2,84	100,00

differences in mathematical performance between the sexes.

The mathematics test instrument will not only be of value to research workers but will also enable teachers and counsellors to determine the level of performance of their pupils for purposes of guidance and selection. In particular, the test may be used

(i) to judge a pupil's performance against the population norms;

(ii) to gain insights into weaknesses and strengths of pupils in respect of attainment of objectives in mathematics learning in relation to at least three taxonomic levels;

(iii) to differentiate between HG and SG by using the means obtained in this study as guidelines and hence to assist pupils in choosing the grade in which they are likely to experience maximum success.

The test norms may be used to predict achievement in mathematics from attitude scores (on this researcher's scale) and IQ scores by using the regression equation derived in this study.

7.2 THE ATTITUDE SCALE

From the considerations made in the development of the Likert-type scale and from the subsequent statistical evidence which was produced, it was concluded that an attitude scale with a high reliability and a very satisfactory validity had been compiled. While the whole scale was found to be a highly satisfactory screening device, the 'enjoyment' dimension of the scale emerged as the best measure with a high variability. Except for a degree of overlap between two dimensions of the scale ('enjoyment' and 'difficulty'), it was demonstrated that the subscales were somewhat independent and were measuring certain specific attitudes.

In view of the essential link between attainment and attitudes which has been established in this study, the attitude scale can serve to "create a balance between the cognitive and affective domains" (Fellows, 1973). The use of such a scale by researchers, teachers and curriculum developers will not only bring about an awareness of the existence of attitudes but also show children that their feelings about mathematics are valued by their teachers.

Stephens (1960:352) has indicated that

> "a measure of attitudes might give added information which would make selection of students more accurate".

In this context, it must be pointed out that the scale developed and used in this study differentiated significantly between attitudes of HG and SG pupils of mathematics. The following three levels are suggested for purposes of rough screening based on the attitude scores obtained from the use of the present scale:

High	: 77 - 96
Middle	: 67 - 76
Low	: 0 - 66

These levels have been successfully used in this study to differentiate between extreme attitude groups in respect of ability and attainment (see section 6.4.3.1). It is suggested that pupils who score low on the scale should be singled out for special attention. Their attitudes may be improved through individualized instruction (Whitley, 1979), simple graded exercises which provide success experiences (Aiken, 1970b), recreational activities in mathematics (Tulock, 1957), and topics from history of mathematics (McBride and Rollins, 1977; Koop and Fraser, 1978). It is expected that these approaches may result in improved attainment which in turn may lead to better attitudes.

As in the case of the study by Michaels and Forsyth (1977), this study provided

> "some support for viewing attitudes toward mathematics as a multidimensional construct".

Several other overseas studies have recognized the value of multidimensional scales (Husén, 1967b; Aiken, 1974, 1979a; Burek, 1975; Sandman, 1974; Kempa and McGough, 1977; Corbitt, 1979). The scores on the six dimensions of the attitude scale may be used to provide a mathematics attitude profile of a pupil. However, the subscales developed here need to be studied further before they can be used as *separate* measures.

It is expected that the total attitude scale and the subscales will provide teachers and counsellors with a satisfactory measuring device for identifying some of the areas of negative attitudes toward mathematics. In general, the attitude scale offers a method of measuring and describing essential attitudinal differences and can be used in research to investigate the effect of differences in methods on attitudes toward mathematics. It may also be possible to isolate other factors which affect attitudes toward mathematics.

7.3 CURRICULUM AND SCHOOL ORGANIZATION AS RELATED TO ATTITUDES TOWARD MATHEMATICS AND ATTAINMENT IN MATHEMATICS

7.3.1 Reasons for Choosing Mathematics

Pupils who chose mathematics because they found it enjoyable and interesting performed significantly better and were more well-disposed towards mathematics than the others. The exact opposite was found in respect of those (22,21%) who took mathematics because it was part of a 'package' course. A sizeable proportion (23,65%) of the HG who opted for HG mathematics because it was "necessary for matric exemption" were less positively inclined towards the subject and attained lower test scores than those who found it challenging, easily manageable and necessary for further study. It was also concluded that the unrealistically larger numbers taking mathematics (many for wrong reasons) eventually inflate the SG entries and result in a lowering of the HG : SG entry and pass ratios.

Teachers, counsellors, subject advisers and planners cannot afford to ignore the reasons for which pupils choose mathematics. It would appear that the problem of restricted 'package' courses offered by the school and the clamour for matric exemption result in many pupils being 'forced' to take mathematics. These pupils are bound to become dissatisfied, develop negative attitudes (if not already acquired) and perform poorly. While the problem of job opportunities seems to lie outside the school system, ways and means of offering a wider range of courses within the system of differentiated education need to be explored and employed in order to save pupils from being 'forced' into mathematics, especially HG mathematics.

7.3.2 Mathematics and Science Bias within the Curriculum

Those with a strong leaning towards mathematics (as indicated by the selection of subjects) tend to possess more positive attitudes and score more highly in mathematics. It must be pointed out that this relationship might have been even stronger if it were not for the unrealistically high proportion of pupils entering for mathematics (discussed in the previous section).

It was also found that pupils taking both physical science and biology displayed significantly more positive attitudes towards mathematics and performed significantly better in mathematics than those taking no science or one science subject.

In general, it would appear that those with a strong leaning towards science or mathematics-orientated subjects experience greater success in mathematics. It is therefore suggested that choice of mathematics as a subject must not be seen in isolation but rather in relation to the other subjects in the school curriculum. Students selecting courses with strong science or mathematics bias are more likely to succeed in mathematics.

7.3.3 Preference for Mathematics within the School Curriculum

It was found that those who showed a greater preference for mathematics within the curriculum were more well-disposed towards mathematics, enjoyed it and attained higher scores on the test. The relative preference for mathematics within the curriculum emerged as a crude measure of attitudes toward mathematics. It is thus possible for teachers and counsellors to use a measure of relative preference for mathematics (see section 6.1.1.5) as a quick but rough estimate of the prevailing attitudes of pupils.

7.3.4 Preference for Content Areas

In general, it was found that the students showed a greater preference for algebra than for geometry and were rated as experiencing more success in algebra than in geometry by their teachers. This was probably due to the fact that the algebra syllabus provides greater scope for techniques and skills (which are easily attainable lower level objectives) than geometry.

This trend needs to be further investigated as there would appear to be less emphasis on higher level abilities to which a great deal of geometry readily lends itself. As Geometry (including Trigonometry and Vectors) constitutes 50% of the examination, it would pay teachers and pupils to increase their efforts in this area. In particular, teachers should emphasize the discovery approach to learning geometry. For example, all theorems in geometry lend themselves to discovery via induction : practical work ⟶ discovery ⟶ generalization (statement of theorem) ⟶ simple applications/proofs ⟶ proof of theorem ⟶ more complex applications/proofs.

7.3.5 Cognitive Preferences

There were significant differences between the preferences for the various modes of presenting mathematical information, with the verbal mode being least preferred. While the symbolic mode was most preferred, students showed distinctly greater preference for the diagrammatic over the verbal mode. These findings were, in general, consistent with those of Dutton and Blum (1968), Sherril (1974) and Kempa and McGough (1977).

It would appear that students are experiencing difficulty in translating mathematical information from one form to another. Since this is a crucial aspect of learning mathematics and higher abilities in mathematics depend almost entirely upon it, teachers need to place greater emphasis on these abilities. Although the recent trend by several mathematics examiners to supply diagrams in the geometry paper may be well-intentioned in terms of saving time in the examination, this researcher regards the practice as questionable. It is strongly felt that this would discourage pupils from learning and exercising their ability to translate, particularly from the verbal to the diagrammatic form. This may lead to an eventual loss of such a crucial ability.

Teachers and examiners should make greater provision for training in translating abilities. It is suggested that pupils be given translation exercises of the various types (see C1 of section 2.1.3) without requiring any solutions or proofs:

e.g. (i) The chord AB of a circle with centre O is perpendicular to the diameter COD and meets the diameter in the mid point M of OC. The semicircle on AB cuts OD in E while AE produced meets the circle O in P.

Draw a diagram to accurately depict the above.

(ii) The product of two consecutive natural numbers is 4 more than 4 times their sum.

Write an equation in x which would enable you to find the numbers.

(Exercise adapted from Malan et al, 1974).

7.3.6 Grades

HG students exhibited significantly more positive attitudes and performed significantly better on the mathematics test than the SG. A further examination of the attitudes and attainment of the students from mixed and single-grade classes revealed that instruction in mixed-grade classes had a negative effect on the attainment of HG students, but it made no difference to the SG. There were no significant differences in respect of attitudes.

The mean attainment of HG from the mixed-grades classes was 12 percentage points lower than that of HG from single grade classes. It was evident that an average HG student (40% level of performance) who found himself in a mixed-grade class was bound to achieve some 12 percentage points lower and hence fail. In the opinion of this researcher, no further research is necessary to prove this point. Everything else being equal, it is hypothesized that the instruction of HG and SG in separate classes will result in an improvement in the performance of HG students.

7.3.7 Levels

The present study revealed that there was no significant difference in attitudes toward mathematics between Std 9 and Std 10. Attitudes that prevailed in Std 9 presumably continued through to Std 10.

It is possible that attitudes become somewhat stabilised before the penultimate year of schooling. It is therefore recommended that any specific attempts to foster or develop positive attitudes should begin well before Std 9. Moreover, researches (e.g. Stright, 1960; Reys and Delon, 1968; Callahan, 1971) suggest that attitudes are developed earlier at the primary and junior high school level.

7.3.8 Class Size

Apart from the finding that the Std 9 mathematics classes were significantly larger than those in Std 10, this study revealed that the size of the classes had no significant effect on either attitudes towards mathematics or attainment in mathematics. Although this finding was consistent with previous research findings which provided little evidence that mathematics attainment was influenced by class size

(Gibney and Karns, 1979), it has been hypothesized by the IEA study (Husén, 1967b:85) that class size may operate to advantage only when the size is reduced to twenty or fewer.

While the immediate problem (as seen by this researcher) is to eliminate mixed-grade instruction in mathematics, keeping classes to a manageable size for specialized and more individualized instruction should not be ignored.

7.3.9 Homework

A weak but positive relationship was found between homework and mathematics attainment and this relationship was consistently lower than that between homework and attitudes. Contrary to previous findings (Austin, 1979) that there was no relation between homework and attitudes, this study revealed that those who were favourably disposed towards mathematics tended to do more mathematics homework than the others. The apparent discrepancy in the findings could possibly be due to motivational factors within the socio-cultural context.

7.4 SOCIAL AND OTHER FACTORS

7.4.1 Sex Differences

This study revealed that there were no sex differences in attitudes towards mathematics. In general, the results of researches in this area seem to be somewhat conflicting. While more work needs to be done before generalizing the present findings, it must be pointed out that, for the population under study, there were clearly no sex-linked differences in attitudes toward mathematics as measured by the attitude scale. However, it was observed that on the "difficulties in learning mathematics" dimension of the scale the girls scored significantly lower than the boys. It was thus evident that the girls were experiencing more difficulties than the boys.

The results of researches into sex differences in mathematics attainment are also "very mixed and often confusing" (Ernest, 1976:599). In general, the weight of evidence points to socio-cultural rather than biological factors as underlying causes of the sex differences. The data in the present study revealed significant sex differences in mathematics achievement in favour of males.

Since this research showed no sex differences in general ability and in attitudes toward mathematics, it is hypothesized that females may have as much mathematical potential as many males. It is considered that the generalized belief that females cannot do well in mathematics may be contributing largely to the differences in performance. Such beliefs may be countered by teachers and counsellors who should make the girls aware of the full range of career opportunities available to them and of the role of mathematics.

It would also be useful to follow the suggestion of Fox et al (1979) and conduct research into the ways in which socialization forces interact to produce such differences.

7.4.2 Educational and Vocational Aspirations of Pre-matriculants

Almost 90% of the sample expressed the desire to pursue some form of tertiary education. Although a relatively small proportion aspired to enter medical school, this group not only performed significantly better on the mathematics test but also possessed significantly more positive attitudes than those who desired to go to university, college of education or technikon.

In general, it was concluded that educational aspirations of the pre-matriculants was positively related to attitudes and attainment. Since unrealistic aspirations would have been characterised by negative or extremely low correlations, it was felt that the pupils' mathematics attainment was realistically matched with their aspirations. It is thus suggested that school counsellors should pay special attention to pupils' educational aspirations in decision-making.

It was also evident that, in general, the better mathematics students were attracted to medicine. Although those who aspired to higher vocations appeared to be more positive towards mathematics and also attained higher scores in mathematics, the relatively low means (see Table 6.31) must be attributed to the high proportion who aspired to higher professional/technical jobs. It was evident that the vocational aspirations were not realistically matched with either the attitudes towards mathematics or attainment in mathematics. It would appear that vocational guidance in schools is not having the desired effect. This problem needs to be investigated.

7.4.3 Home Background

It was found that both fathers' occupational status and parents' level of education had no significant effect on the attitudes toward mathematics. However, pupils of fathers in the higher occupational levels performed significantly better on the mathematics test than those with fathers in the lower occupational levels. It was also found that the higher the educational level of the parents the higher the attainment in mathematics.

It was thus reasonable to conclude that pupils who come from poor socio-economic and/or educational background would, in general, perform worse than their counterparts from better home backgrounds. Teachers, counsellors and subject advisers need to consider these factors when teaching and testing. In view of these findings, any comparison of performances of individuals and/or schools without taking into account the socio-economic and educational backgrounds would be meaningless. Moreover, the appraisal of teachers on the basis of their pupils' performances would be a questionable practice.

7.5 TEACHERS' BACKGROUND AND PERCEPTIONS OF PUPILS' ATTITUDES AND ATTAINMENT

7.5.1 Teachers' Academic and Professional Background

An examination of the teachers' qualifications and experience revealed that 92,45% of them possessed at least Mathematics I (or equivalent), while only 32,08% possessed at least Mathematics II. It was observed that the services of the mathematics teachers were maximally utilized as they were engaged in mathematics teaching (on an average) for 95,90% of their teaching time. It was evident that their services in the second subject for which they had trained were hardly used. In the light of this, the question of requiring all senior secondary teachers to be qualified to teach in *two* different disciplines should be scrutinized. The possibility of training mathematics teachers for this phase by giving undivided attention to the one special method should be considered. In this way their academic level may be raised to Mathematics II (or equivalent) and their professional competence improved. This suggestion implies a longer period of training in the case of the diploma students, which is consistent with the requirement of four-year training for White teachers (Act 39 of 1967).

7.5.2 Teachers' Ratings

The teachers' ratings of pupils' attainment of objectives in mathematics revealed:

(i) a high degree of consistency;

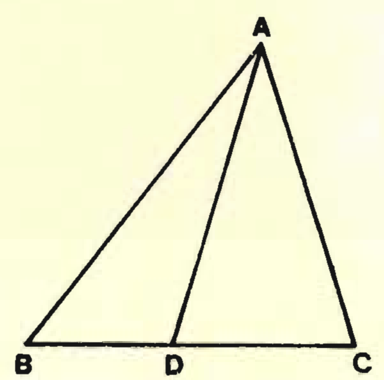
(ii) a strict hierarchical pattern, with mean ratings higher for lower level objectives and lower for higher level objectives, which was consistent with the theoretical formulation of the taxonomic structure of objectives (Bloom et al, 1956).

It was concluded that the teachers were clearly distinguishing between the various levels of objectives in mathematics. This is a positive trend which should be maintained.

However, teachers should place greater emphasis on higher level abilities in their instructional programmes by using carefully constructed exercises with built-in graded questions which lead the pupils from the simple to the more complex abilities. These exercises also have the advantage of motivating pupils to tackle more complex problems. The following is an example of such an exercise which has been taken from the Senior Certificate Examination (DIE, HG Paper 2, Dec. 1980):

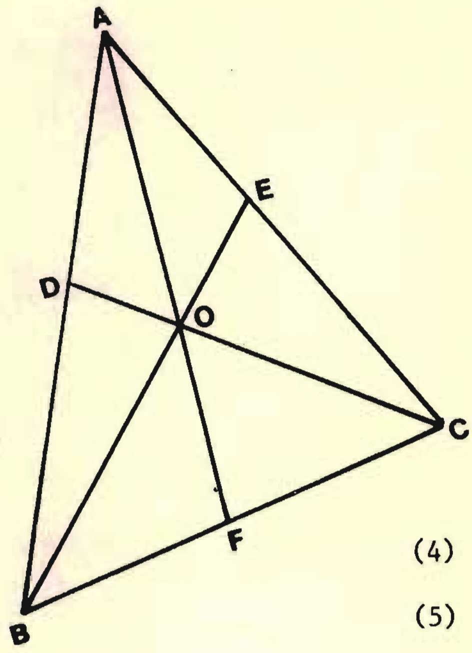
(a) With respect to the figure, complete the statement:

$$\frac{\text{area } \Delta ABD}{\text{area } \Delta ADC} = \frac{BD}{..} \quad .$$



(1)

(b) ABC is a triangle.
D and E are points on AB and AC respectively such that $\frac{AD}{DB} = \frac{AE}{EC}$.
DC and BE intersect at O . AO is produced to cut BC in F .
Prove that



(i) area ΔDOB = area ΔEOC , (4)

(ii) area ΔAOD = area ΔAOE , (5)

(iii) BF = FC . (5)

Total Marks : 15

In the teaching situation, such problems must be followed by several other exercises which do not provide 'lead-in' questions. These will obviously demand higher level abilities such as analysis-synthesis. Consider, for example, the solution to (b)(iii) without the 'lead-ins' (a), (b)(i) and (b)(ii) in the above problem.

In general, the striking consistency in patterns observed between teachers' ratings and results obtained through the researcher's instruments points to the need to explore more fully the use of teachers' ratings in evaluation programmes. In this context, Naidoo (1981) observed that

> "a great deal of assessing, testing and examining rightly takes place on the initiative of principals or individual teachers or under arrangements made by subject advisers and circuit inspectors. Apart from performance in the Senior Certificate Examination, this activity is not co-ordinated nationally".

These thoughts were based on observations in Britain which were summarized as follows by the Schools Council:

> "it is accepted that the teacher is likely to know more about his pupils than an external examiner, ... he can also put his own pupils in an order of merit more accurately than any examination" (cited by Naidoo, 1981).

It is thus recommended that provision for greater involvement of the class teacher in evaluation programmes should be considered. A starting point would seem to be to devise and employ methods of combining teachers' marks (perhaps converted to standard scores to eliminate differences between classes and between schools) with external examination marks.

7.6 RELATIONSHIP BETWEEN ATTITUDES TOWARD MATHEMATICS AND MATHEMATICS ATTAINMENT

In general, the relationship between attitudes toward mathematics and attainment in mathematics was found to be positive and significant with no difference between males and females in the extent of the relationship. It was also found that general ability emerged as an important predictor of achievement in mathematics.

Using attitude as a moderator variable, it was observed that:

(i) in the middle range of attitude scores, ability scores were somewhat more highly correlated with attainment in mathematics;

(ii) in the middle range of attitude scores the relationship between attitude and achievement was markedly smaller (and close to zero) than at the extremes.

These findings suggest that the use of ability scores to predict achievement in mathematics may be enhanced by taking into account the attitudes towards mathematics (as measured by this researcher's scale) of a given population.

This study examined the means of extreme groups in respect of attitudes, attainment and ability to observe for any one variable the corresponding differences between the groups for each of the other two variables. It was consistently found that the differences between the extreme groups for any one variable revealed significant differences in respect of each of the other two variables.

In view of these mutual relationships, a multiple regression equation was derived to predict attainment in mathematics from attitude and ability scores. The use of this equation together with the norms for the mathematics test (see Table 7.1) should enable teachers and counsellors to predict the percentile rank of a pupil in respect of performance in mathematics.

In general, the findings pointed to a reciprocal relationship between attitudes and attainment, while ability remained the best predictor of performance in mathematics. A demonstration of this relationship in the present study serves to underline the potential educational significance of attitudes to ultimate achievement in mathematics. Moreover, teachers and counsellors should be encouraged to use mathematics attitude measures (such as the one developed by this researcher) in educational decision-making. Mathematics educators are strongly advised to take cognisance of the attitudes of pupils when designing and implementing mathematics instructional programmes.

7.7 OTHER RESEARCH PROBLEMS

In the preceding sections, implications of the present study were considered specifically in relation to the various aspects of the research. Arising out of the study as a whole or a combination of several parts of it, there are certain implications for research in general. These research problems will be briefly presented in the following sections.

7.7.1 Causality

The nature and design of the present research precluded any pronouncements on the question of causality in respect of attitudes and attainment. Although this research has postulated a reciprocal relationship between attitudes and attainment, it did not indicate which is the stronger cause. It is suggested that the problem of causality may be approached through studies making use of cross-lagged correlation analyses. Such designs which have been used in recent studies (Quinn, 1978; Burek, 1975) to examine the causal relationship between mathematics achievement and attitudes seem to be promising. Another approach would be to use control and experimental groups in a pre-test - post-test design by manipulating attitudes and observing changes in attainment.

7.7.2 Other Non-intellective Factors

While this research has attempted to study several background factors affecting attitudes and attainment, it has been restricted to *attitudes towards mathematics* among several other non-intellective factors. Further research needs to consider such variables as motivation, anxiety, personality, interests, aptitudes and study habits. Studies which have examined one or more of these aspects have revealed considerable scope for research (e.g. Cohen and Carry, 1978; Finger and Schlessor, 1965; Taylor et al, 1976; Messick, 1979; Kifer, 1975; Cappadona and Kerzner-Lipsky, 1979; Khan, 1969; James, 1976).

Researches designed to discover the effects of these variables on attitudes and attainment will provide useful clues for improving attitudes and (hence) attainment in mathematics.

7.7.3 Teachers' Attitudes

The present study focussed attention on pupils' attitudes toward mathematics and attainment in mathematics, using teachers' ratings for purposes of validation and comparison. It did not, however, include a study of the effect of teachers' attitudes, experience and qualification on the attitudes and attainment of pupils. It has been stressed that

> "teacher attitude and effectiveness in a particular subject are important determinants of student attitudes and performance". (Aiken, 1970b:572)

Some of the studies worth noting in this context are Dutton (1951), Stright (1960), Peskin (1965), Dale (1966), Hunkler and Quast (1972), Wardrop (1972), Phillips (1973), Gilbert (1976), Campbell and Schoen (1977), and McMahon (1979). Since no research has been conducted in this area in the RSA, future studies should be designed to investigate the effects of the attitudes of mathematics teachers on pupils' attitudes and attainment.

7.7.4 Longitudinal Study

Although the present research made possible a comparison between Std 9 and Std 10 pupils' attitudes and attainment, its static design precluded an investigation of the developmental aspects of attitudes. Longitudinal studies such as that of Anttonen (1968) or a series of short term studies should be employed to assess the distribution and stability of attitudes toward mathematics. In this way it may be possible to pinpoint the period of schooling during which attitudes are optimally developed and become stabilised.

7.7.5 Attitude Change and Development

Since this study has carefully established a positive relationship between attitudes and attainment, ways and means of developing positive attitudes and modifying negative attitudes need to be discovered and employed. Studies should be designed to observe changes in attitudes and their effects on attainment. Aiken (1970b) observed that, while there was considerable interest in this area, there has been little substantive research to investigate techniques of improving pupil attitudes toward mathematics.

However, a few recent studies have highlighted the scope for research into these aspects (McMillan, 1976; Muckerjee, 1978; McBride and Rollins, 1977). For example, Whitley (1979:188) showed that

> "exposure to individualized instruction had a favourable impact on pupil attitudes at each of the three grade levels".

It must be emphasized that research must not only devise techniques but also demonstrate their effectiveness.

7.8 GENERAL

In general, the findings of this study and their implications justify continued research into the interrelationships between intellective and non-intellective variables and their effects on attitudes towards mathematics and attainment in mathematics. Finally, it is hoped that the contributions of this study will not only stimulate research in mathematics education but also encourage teachers and advisers in their efforts to improve the teaching and learning of mathematics in our schools.

APPENDIX 1 : PUPIL QUESTIONNAIRE (PQ1)

PQ1 To be completed by pupils taking Maths Test PQ3 (confidential)

1. Full Name .... Male ☐ 1 Female ☐ 0

2. Age .... yrs (to nearest birthday) Standard ☐

3. Which subjects (other than English, Afrikaans and Mathematics) are you taking for the Matriculation Examinations? ....

4. On what grade are you taking mathematics HG ☐ 1 SG ☐ 0

5. (a) What is your main reason for choosing mathematics?

I find it enjoyable and interesting	☐	1
influenced by my parents/friends	☐	2
advised by my teacher/counsellor	☐	3
part of course I chose	☐	4
necessary for matric exemption	☐	5
necessary for further study/good job	☐	6
no other available option	☐	7
other (state)	☐	8

(b) What is your main reason for choosing this particular grade?

Higher Grade			Standard Grade		
HG is more challenging	1	☐	I cannot manage the HG	1	☐
necessary for matric exemption	2	☐	influenced by parents/friends	2	☐
I can easily manage the HG	3	☐	chances of passing on SG are better	3	☐
influenced by parents/friends	4	☐	advised by teacher/counsellor	4	☐
advised by teacher/counsellor	5	☐	easier to get a good mark	5	☐
necessary for further study	6	☐	no other available option	6	☐
other (state)	7	☐	other (state)	7	☐

. Estimate your level of performance in mathematics right now? ....%

What do you hope to achieve in the final examinations? ....%

APPENDIX 1 (Continued)

7. What is the total number of pupils in your present mathematics class? ....

8. Are all the pupils in your mathematics class studying it on the same grade? Yes [ ] No [ ]

9. How many hours a week (approximately) do you devote to all your homework? ....

10. How many hours a week (approximately) do you devote to your mathematics homework? ....

11. On an average how many days a week do you do work in mathematics at home? ....

12. Have you been a member of any mathematics club or have attended special lectures on mathematics? Yes [ ] No [ ]

13. Of the six examination subjects you are taking

(a) how many subjects do you like better than mathematics? ...

(b) in how many subjects do you do better than you do in mathematics? ....

FOR THE FOLLOWING 2 QUESTIONS USE

4 for "strongly like" 3 for "like"
2 for "undecided" 1 for "dislike"
0 for "strongly dislike"

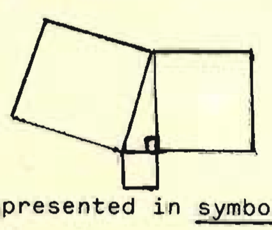
14. Indicate your liking for the following sections of mathematics:-

(1) Algebra [ ] (2) Geometry & Vectors [ ] (3) Trigonometry [ ]

15. Indicate your liking for the following types of problems in mathematics:-

(i) problems which are presented in the form of a diagram

eg.



(ii) problems which are presented in symbols

eg. $c^2 = a^2 + b^2$

(iii) problems which are presented in words

eg. In a right angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

APPENDIX 1 (Continued)

16. When you leave school what do you intend to do? (Mark your first choice ...1 and second choice ...2)

University full-time	☐ 1
University part-time	☐ 2
Medical School	☐ 3
College of Teacher Education	☐ 4
Technikon (Advanced Technical Education)	☐ 5
Full-time employment	☐ 6
No further study	☐ 7
Other (state)	☐ 8

17. After this year how many years of full-time education can you complete? .... ☐

18. How many more years of full-time education would you like to complete? .. ☐

19. What occupation would you like to enter? ....

20. After completing school would you like to study:

more mathematics and subjects involving mathematics ☐

other subjects but not mathematics? ☐

21. What is the level of education of your parents?

	FATHER		MOTHER
no school education		1	
primary school		2	
high school		3	
diploma after passing high school		4	
university degree		5	
more than one university degree		6	

22. Father's occupation ....

23. Mother's occupation ....

24. How many persons make up your family? ....

APPENDIX 2 : ATTITUDE SCALE (PQ2)

PQ2: Attitude Scale - to be completed by pupils.

Here are some statements about mathematics, school and life in general. For each of them merely indicate whether you agree, disagree or are uncertain by using the following key: agree = 2; uncertain = 1; disagree = 0.

There is no right or wrong answer to any statement - it is just what you sincerely feel.

1. My mathematics teacher shows us different ways of solving the same problem. .... ☐

*2. In our school we get a great deal of practice and drill until we are almost perfect in our learning. .... ☐

*3. Very few people can learn mathematics. .... ☐

4. More of the most able people should be encouraged to become mathematicians and mathematics teachers. .... ☐

*5. Most school learning has little value for a person. .... ☐

*6. Mathematics is not a very interesting subject. .... ☐

*7. My mathematics teacher does not like pupils to ask questions after he has given an explanation. .... ☐

*8. The pupils spend most of their class time listening to the teachers and taking notes. .... ☐

9. Almost anyone can learn mathematics if he is willing to study. ... ☐

*10. Outside of science and engineering there is little need for mathematics in most jobs. .... ☐

*11. I am bored most of the time in school. .... ☐

APPENDIX 2 (Continued)

12. I have usually enjoyed studying mathematics in school. .... ☐

*13. My mathematics teacher wants pupils to solve problems only by the procedures he teaches. .... ☐

*14. Our teachers wants us to do most of our learning from the textbook which is used in the course. .... ☐

15. Any person of average intelligence can learn to understand a good deal of mathematics. .... ☐

16. Mathematics is of great importance to a country's development. .... ☐

17. I enjoy most of my school work and want to get as much additional education as possible. .... ☐

*18. I have seldom liked studying mathematics. .... ☐

19. My mathematics teacher expects us to learn how to solve problems by ourselves but helps when we have difficulties. .... ☐

20. We are expected to learn and discover many ideas for ourselves at school. .... ☐

21. Even complex mathematics can be made understandable and useful to every high school pupil. .... ☐

*22. Mathematics is not useful for the problems of everyday life. ...... ☐

23. I find school interesting and challenging. .... ☐

24. Mathematics is enjoyable and stimulating to me. .... ☐

APPENDIX 2 (Continued)

25. My mathematics teacher requires us not only to master the steps in solving problems, but also to understand the reasoning involved. ... ☐

26. In our school we are expected to develop a thorough understanding of ideas and not just to memorize information. .... ☐

27. Almost all pupils can learn complex mathematics if it is properly taught. .... ☐

28. It is important to know mathematics in order to get a good job. .... ☐

*29. Success depends to a large part on luck and fate. .... ☐

*30. Mathematics is dull and boring. .... ☐

31. My mathematics course requires more thinking about the methods of solving problems than memorization of rules and formulae. .... ☐

*32. Our teachers believe in strict discipline and each pupil does exactly what he is told to do. .... ☐

*33. Only people with a very special talent can learn mathematics. ...... ☐

*34. Other subjects are more important to people than mathematics. ...... ☐

35. By improving industrial and agricultural methods, poverty can be eliminated in the world. .... ☐

36. I like trying to solve new problems in mathematics. .... ☐

APPENDIX 2 (Continued)

37. My mathematics teacher wants us to discover mathematical principles and ideas for ourselves. .... ☐

38. We do not use just one textbook for most of our subjects. Various sources and books from which we can learn are suggested to us. ..... ☐

39. I am very calm when studying mathematics. .... ☐

40. Mathematics helps to develop the mind and teaches a person to think.. ☐

*41. Education can only help people develop their natural abilities; it cannot change people in any fundamental way. .... ☐

*42. I am not motivated to work very hard on mathematics lessons. .... ☐

*43. Most of the problems my mathematics teacher assigns are to give us practice in using a particular rule or formulae. .... ☐

44. Much of our classroom work is discussing ideas and problems with the teacher and other pupils. .... ☐

*45. Mathematics is one of my most dreaded subjects. .... ☐

46. Mathematics has contributed greatly to the advancement of civilization. .... ☐

47. With hard work anyone can succeed. .... ☐

48. I plan to take as much mathematics as I can during my education. .... ☐

Note: Negative items are shown by *.

APPENDIX 3 : MATHEMATICS TEST (PQ3)

MATHEMATICS TEST BOOKLET

READ CAREFULLY:

1. This is *not* an examination. It is a mathematics test designed to test your knowledge, skill, understanding and ability to apply in mathematics.

2. HOW TO ANSWER:

   (a) The actual writing down of the answer is simple. Each question is supplied with 5 probable answers marked (A), (B), (C), (D), (E). Only *ONE* of them is correct. When you have carefully thought about the question select the correct answer and put " ✓ " on your answer sheet in the relevant block. Use a pencil.

   If you selected (D) then your answer sheet for the particular question will look like this:

A	B	C	✓D	E

   (b) If you change your mind erase and place " ✓ " correctly and clearly.

   (c) If you are uncertain do not guess but leave a blank. Guessing will be of no help.

   (d) You will be supplied with paper for rough work. Please do not tear any part of this booklet or write on it.

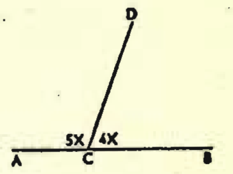
— p. 2

APPENDIX 3 (Continued)

1. In the division on the right, the correct answer is $0{,}004\overline{)24{,}56}$

A. 0,614
B. 6,14
C. 61,4
D. 614
E. 6140

2. If AB is a straight line, what is the measure in degrees of angle BCD in the figure on the right?



A. 20 B. 40 C. 50 D. 80 E. 100

3. The value of $2^3 \times 3^2$ is

A. 30 B. 36 C. 64 D. 72 E. none of these

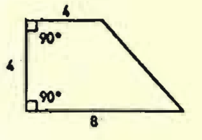
4. What is the square root of 12×75?

A. 6.25 B. 30 C. 87 D. 625 E. 900

5. Which of the following is *false* when a and b are different real numbers:

A. $(a+b)+c=a+(b+c)$
B. $ab=ba$
C. $a+b=b+a$
D. $(ab)c=a(bc)$
E. $a-b=b-a$

6. There is a brass plate of the shape and dimensions shown in the adjoining figure. What is its area in square units?



A. 16 B. 24 C. 32 D. 64 E. 96

7. The expression $\frac{a}{b-c}+\frac{a}{c-b}$ where $a \neq 0$, is equal to

A. 0
B. $\frac{2a}{b-c}$
C. $\frac{a}{b^2-c^2}$
D. $\frac{a}{2b}$
E. $2a$

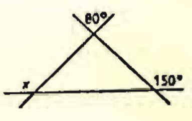
8. In the solution of the following system of equations,

$$\left.\begin{array}{l} 2x+y=7 \\ x-4y=4 \end{array}\right\}$$

the value of y is equal to

A. $-\frac{5}{3}$ B. -9 C. $\frac{1}{9}$ D. $-\frac{1}{9}$ E. $\frac{5}{3}$

9. Three straight lines intersect as shown in the figure on the right. What is x equal to in degrees?

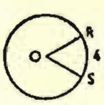


A. 30 B. 50 C. 60 D. 110 E. 150

10. The distance between two towns, A and B, is 150 kilometers. This distance is represented on a certain map by a length of 30 centimeters. The scale of this map is

A. 1/500 000
B. 30/150
C. 1/20 000
D. 1/5 000
E. 1/200 000

11. The length of the circumference of the circle on the right with center at O is 24 and the length of arc RS is 4. What is the measure in degrees of the central angle ROS?

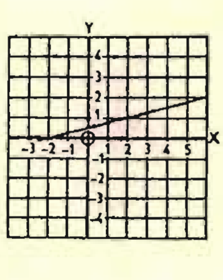


A. 24 B. 30 C. 45 D. 60 E. 90

12. A factory produces m units per week. How many units per week will it produce after production is increased p per-cent?

A. $100p+m$
B. $100m+mp$
C. $\frac{m+mp}{100}$
D. $m+\frac{mp}{100}$
E. $\frac{p}{100}+$

13. The equation of the line shown in the graph is

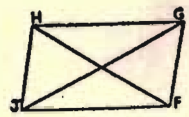


A. $x+4y=4$
B. $2x-y=4$
C. $2x=y-2$
D. $x-4y+2=0$
E. $4x-y=2$

14. If $xy=1$ and x is greater than 0, which of the following statements is true?

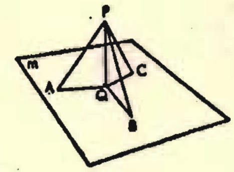
A. When x is greater than 1, y is negative.
B. When x is greater than 1, y is greater than 1.
C. When x is less than one, y is less than 1.
D. As x increases, y increases.
E. As x increases, y decreases.

15. In the figure on the right, $FGHJ$ is a parallelogram. Which of the following statements is a condition which implies that $FGHJ$ is a rectangle?



A. $JF = GH$
B. $\angle HJG = \angle JGF$
C. $\angle HJF = \angle JHG$
D. $\angle HJF$ and $\angle JHG$ are supplementary
E. HF and JG are perpendicular bisectors of each other

16. In the figure on the right, m represents a plane, and PQ is a line segment which is perpendicular to the plane at the point Q. Points A, B and C lie on the plane. If $QA = QB = QC$, then the triangles PQA, PQB, and PQC are



A. congruent by SAS
B. congruent by SSA
C. congruent by ASA
D. similar but not congruent
E. neither similar nor congruent

17. A number is the multiplicative inverse of another number if the product of the two numbers is 1. Which of the following sets of numbers is identical to the set of its multiplicative inverses?

A. $\{1, 2, 3\}$
B. $\{1, \frac{1}{2}\}$
C. $\{1, 2, \frac{1}{2}\}$
D. $\{2, 3, 5, \frac{1}{2}, \frac{1}{3}\}$
E. $\{2, 3, \frac{2}{3}\}$

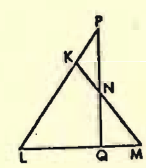
18. A certain number of students are to be accommodated in a hostel. If 2 students share each room, then 2 students will be left without any room. If 3 students share each room, then 2 rooms will be left unoccupied. How many rooms are there in the hostel?

A. 24
B. 4
C. 18
D. 12
E. 8

19. The lengths of the sides of a triangle XYZ are 4, 7, and 10. If a similar triangle has a perimeter of 147, what is the length of its shortest side?

A. 7
B. 14
C. 28
D. 21
E. 70

20. In $\triangle KLM$ on the right, $KL = KM$, $PQ \perp LM$, and LP is a straight line. Then $\triangle NKP$ is isosceles because



A. $\angle P = \angle KNP$, since both are complements of the equal angles L and M.
B. $NK = PK$, since $\angle P = \angle M$.
C. its sides are parallel to the sides of $\triangle KLM$.
D. its sides are perpendicular to the sides of $\triangle KLM$.
E. $\angle P = \angle KNP$, since both are half the supplement of angle M.

21. Of three wires, each 36 cm long, one is bent into a square, another into a rectangle with length and width in the ratio of 2: 1 and the third into an equilateral triangle. Which one of the following statements describes the correct relationship between the enclosed areas?

A. The area of the square is the greatest and that of the triangle is the least.
B. The area of the rectangle is the greatest and that of the triangle is the least.
C. The area of the triangle is the greatest and that of the square is the least.
D. The area of the triangle is the greatest and that of the rectangle is the least.
E. The areas of the square and the rectangle are the same, but the area of the triangle is less than that of the square or the rectangle.

22. A wholesale merchant bought a television set at a certain price and then sold it to a retail merchant at an increase of P per cent of this price. The retail merchant sold the set to a consumer for P per cent more than he paid for it. If the customer paid 65 per cent more than the price originally paid by the wholesale merchant, then P satisfies the equation:

A. $1 + \frac{2P}{100} = 1,65$
B. $\left(1 + \frac{P}{100}\right)^2 = 1,65$
C. $1 + \left(\frac{P}{100}\right)^2 = 1,65$
D. $1 + P^2 = 1,65$
E. $1 + 2P = 1,65$

23. A freight train traveling at 50 km per hour leaves a station 3 hours before an express train which travels in the same direction at 90 km an hour. How many hours will it take the express train to overtake the freight train?

A. $\frac{5}{9}$ B. $\frac{9}{5}$ C. $\frac{12}{5}$ D. $\frac{15}{4}$ E. $\frac{18}{4}$

24. A square plate of the largest possible size is cut from a circular plate of 16 cm diameter. The area of the square plate (in sq cm) will be

A. 64 B. 96 C. 128 D. 192 E. 256

APPENDIX 3 (Continued)

MATHEMATICS TEST - ANSWER SHEET

Full Name: ------------------------------------

1.	A	B	C	D	*E	2.	A	B	C	*D	E
3.	A	B	C	*D	E	4.	A	*B	C	D	E
5.	A	B	C	D	*E	6.	A	*B	C	D	E
7.	*A	B	C	D	E	8.	A	B	C	*D	E
9.	A	B	C	*D	E	10.	*A	B	C	D	E
11.	A	B	C	*D	E	12.	A	B	C	*D	E
13.	A	B	C	*D	E	14.	A	B	C	D	*E
15.	A	B	*C	D	E	16.	*A	B	C	D	E
17.	A	B	*C	D	E	18.	A	B	C	D	*E
19.	A	B	*C	D	E	20.	*A	B	C	D	E
21.	*A	B	*C	D	E	22.	A	*B	C	D	E
23.	A	B	C	*D	E	24.	A	B	*C	D	E

Note: Correct answers are shown by *.

APPENDIX 4 : TEACHER QUESTIONNAIRE (TQ1)

TQ 1 : Information about pupil's performance and attitude to be completed by mathematics teacher (Confidential)

1. Name of Pupil: ....

2. Rating of pupil's mathematical ability : Use the following key to rate:-

   1 = Weak　　2 = Satisfactory
   3 = Fair to Good　　4 = Very good
   5 = Excellent

   (a) Ability in terms of objectives:

   Knowledge: ability to remember or recall definitions, notations, operations, concepts, axioms and theorems .... ☐

   Skill: ability to manipulate and compute rapidly and accurately. ☐

   Comprehension: ability to interpret verbal, symbolic, geometric, forms and to translate from one form to the other; to follow proofs of theorems .... ☐

   Application: ability to select and apply concepts/ operations/ theorems to mathematical and or non-mathematical problems ...... ☐

   Analysis-synthesis: ability to analyse problems, to construct and evaluate proofs and solutions .... ☐

   (b) Ability in terms of content areas:

   Algebra .... ☐

   Geometry(+ Vectors) ..... ☐

   Trignometry .... ☐

3. Use a tick (✓) to indicate whether you agree, disagree or are undecided in respect of each of the following statements about this pupil:-

	2 AGREE	1 UN-DECIDED	0 DIS-AGREE
Likes to solve new problems in mathematics			
Does not do his homework			
Is motivated to work very hard			
Displays a lack of interest in mathematics			
Finds mathematics enjoyable and stimulating			
Does not take his work seriously			
Becomes uneasy and confused during maths lessons			
Is a pleasure to teach			
Rarely takes part in discussions			
Adopts a generally very positive attitude to maths			

4. Overall estimate of pupil's performance level in maths (based on tests, assignments and examinations) .... ________ %

5. IQ Score (GTISA - Verbal and Non-Verbal) ________

APPENDIX 5 : STATISTICAL METHODS USED IN THIS STUDY

1. Standard Error of Mean ($S_{\bar{x}}$) $= \dfrac{SD}{\sqrt{N-1}}$, where $SD = \sqrt{\dfrac{\Sigma(x-\bar{x})^2}{N}}$

   (Downie and Heath, 1970:161)

   The standard error of mean yields a confidence interval for the mean as follows:

   $\bar{x} \pm (2,58)S_{\bar{x}}$ $(p < 0,01)$

   (Downie and Heath, 1970:164-165)

   This interval enables an unbiased estimation of the population mean (m) from the sample mean ($\bar{x}$),

2. Differences between Means

   Since all samples in this study were considered to be large, the z-score was computed to test significance of the difference between means (Downie and Heath, 1970:178).

   The z-score is interpreted by the use of the normal probability tables (Downie and Heath, 1970:303-309). The z-score is calculated differently for uncorrelated data and correlated data.

   2.1 Differences between Means - Uncorrelated Data for Large Sample (z-Test)

   Firstly, the standard error of the difference between two means, $\bar{x}_1$ and $\bar{x}_2$ is defined by the following formula:

$$S_{D_{\bar{x}}} = \sqrt{S_{\bar{x}_1}^2 + S_{\bar{x}_2}^2}$$

   where $S_{\bar{x}_1} = \dfrac{(SD)_1}{\sqrt{N_1 - 1}}$, $S_{\bar{x}_2} = \dfrac{(SD)_2}{\sqrt{N_2 - 1}}$

   are the standard errors of the two sample means. (Downie and Heath, 1970:172).

   Secondly, the z-score is given by

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{S_{D_{\bar{x}}}}$$

   (Downie and Heath, 1970:172).

   From normal probability tables for $z > 2,58$ and $z > 3,30$ it is seen that $p < 0,01$ and $p < 0,001$ respectively.

APPENDIX 5 (Continued)

2.2 Differences between Means - Correlated Data for Large Sample (z-Test)

When two sets of scores for the same sample are considered the data are said to be correlated. In this case the standard error of the difference between the two means, $\bar{x}_1$ and $\bar{x}_2$ is defined by:

$$S_{D_{\bar{x}}} = \sqrt{S_{\bar{x}_1}^2 + S_{\bar{x}_2}^2 - 2(r)(S_{\bar{x}_1})(S_{\bar{x}_2})}$$

Where $S_{\bar{x}_1}$ and $S_{\bar{x}_2}$ are the standard errors of the means, and

r = coefficient of correlation for the two sets of scores.

The z-score is then given by

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{S_{D_{\bar{x}}}}$$

$$= \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{S_{\bar{x}_1}^2 + S_{\bar{x}_2}^2 - 2(r)(S_{\bar{x}_1})(S_{\bar{x}_2})}}$$

(Downie and Heath, 1970:174-175).

3. Correlation Coefficients

3.1 Pearson Product-Moment Correlation Coefficient.

The machine formula for the Pearson product-moment correlation coefficient is given by

$$r = \frac{\Sigma xy - \frac{\Sigma x \, \Sigma y}{N}}{\sqrt{\left[\Sigma x^2 - \frac{(\Sigma x)^2}{N}\right]\left[\Sigma y^2 - \frac{(\Sigma y)^2}{N}\right]}}$$

where x, y are pairs of scores and N = number of pairs.

(Downie and Heath, 1970:93).

APPENDIX 5 (Continued)

3.2 Reliability Coefficient

The reliability coefficient of a test indicates the consistency with which it yields its results. The following two methods have been used to obtain the reliability coefficients in this study:

(i) the split-half method;

(ii) Kuder-Richardson Formula No. 20.

3.2.1 The Split-Half Method

This method is used to simulate the effect of two equivalent tests by dividing the given test into halves, e.g. even-numbered items and odd-numbered items (as was the case in this study). A Pearson product-moment correlation is computed between the scores on the even-numbered items and the scores on the odd-numbered items to yield the coefficient of internal consistency. Since this coefficient measures the reliability of a test which is half as long as the actual test the correlation is corrected by using the Spearman-Brown formula,

$$r = \frac{2r_{12}}{1 + r_{12}}$$

where r_{12}=reliability coefficient obtained by correlating the scores on the odd-numbered items with the scores on the even-numbered items or first half with second half.
(Downie and Heath, 1970:244).

3.2.2 Generalized Spearman-Brown Formula

$$r_n = \frac{nr_{12}}{1+(n-1)r_{12}}$$

where r_n is the reliability of a test n times the length of the test from which the observed correlation r_{12} was obtained. (Thorndike, 1951:581).

3.2.3 Kuder-Richardson Formula 20

The Kuder-Richardson formula No. 20, which also yields a coefficient of internal consistency, is easily applied to item analysis data.

APPENDIX 5 (Continued)

This formula is defined as follows:

$$r_{tt} = \frac{k}{k-1}\left[1 - \frac{\sum_{i=1}^{k} p_i q_i}{(SD)^2}\right],$$

where r_{tt} = reliability of the total test,

k = number of items in the test,

$(SD)^2$ = the variance of the test ,

p_i = proportion of candidates who correctly responded to the i-th item, and

q_i = $1 - p_i$.

(Thorndike in Lindquist, 1951:587).

3.2.4 The Standard Error of Measurement

The Standard error of measurement provides an indication of the accuracy of the test scores and is estimated by the following formula:

$$Se = SD\sqrt{1 - r}$$

where Se = standard error of measurement

SD = standard deviation of the test

r = reliability of the test.

$Se = 0.432\sqrt{k}$, where k = number of items on the test, is also regarded as a good estimate of the standard error of measurement.

(Downie and Heath, 1970:247-248).

3.3 Validity Coefficient

The validity coefficient of a test indicates the extent to which it measures what it sets out to measure. Measurements of validity are essentially measurements of correlation between the scores on the test and criterion scores in the attribute which the test attempts to measure. Since criterion scores are themselves often not reliable measures, it is

APPENDIX 5 (Continued)

necessary to have two parallel, but not necessarily equivalent, sets of criterion scores. Given a set of scores x (whose validity is to be estimated) and two sets of corresponding criterion scores a and b, the validity coefficient of the correlation between the test and the "true" criterion score is defined by

$$r_{x,ab} = \frac{r_{xa} \cdot r_{xb}}{r_{ab}},$$

where r_{xa} = correlation between the test and criterion score a,

r_{xb} = correlation between the test and criterion score b,

r_{ab} = correlation between the criterion score a and criterion score b.

(Cureton in Lindquist, 1951:680).

3.4 Levels of Significance for Correlation coefficients are obtained from a table of values, e.g. (Edwards, 1967, Table VI, p. 426).

4. Item Analysis

Item analysis provides quantitative information in respect of the difficulty and discriminating power (the extent to which an item differentiates between the weaker and brighter candidates) of each item. The Difficulty (or Facility) Index (F) and Discriminating Index (D) are calculated for each item.

In order to compute the F and D values the scores are arranged from the highest to the lowest. The upper 27 percent and the lower 27 percent of the scores are used to make the two extreme groups as large and different as possible (Ebel, 1965:349). For each item the number of correct responses from each of the upper and lower groups is counted.

The Difficulty Index and Discrimination Index for each item are given by the following formulae:

$$F = \left[\frac{(R_u + R_e)}{2n}\right] \times 100,$$

$$D = \frac{R_u - R_e}{n},$$

APPENDIX 5 (Continued)

where F = Difficulty Index,

D = Discrimination Index,

R_u = number of correct responses for upper 27% group,

R_e = number of correct responses for lower 27% group, and

n = number of candidates in each group.

(The above formulae have been adapted from the steps outlined by Ebel, 1965:347 and Husén, 1967a:101).

5. Chi Square (χ^2)

The Chi-square statistic is a test of significance which compares observed frequencies (O) with expected frequencies (E).

The general formula for chi square is given by

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E)^2}{E}$$

where O = observed frequency

E = expected frequency, and

n = number of frequencies.

The χ^2 is used to test the null hypothesis that the observed frequencies do not differ from the expected frequencies by chance. The level of significance is read from probability tables for (n - 1) degrees of freedom. (Downie and Heath, 1970:197-199 & 311).

6. Relative Attitude Index (RAI)

$$RAI = \frac{\sum_{i=1}^{3} i.n_i}{3\sum_{i=1}^{3} n_i} \times 100$$

where n = number of responses to each of the three categories: agree, uncertain, disagree .

i = 1, 2, 3.

This method, which has been adapted from Onibokun's (1974:195) Relative Habitability Index, enables one to compare attitude statements in terms of the degree of favourableness in responses.

7. Coefficient Alpha

Cronbach's coefficient alpha was obtained by using the variances of the total scores and that of the two parallel halves:

APPENDIX 5 (Continued)

$$\alpha = \left[\frac{n}{n-1}\right]\left[1 - \frac{\Sigma \sigma_i^2}{\sigma_t^2}\right]$$

where, for the special case, n=2 and i=1, 2

σ_t^2 = variance of total scores

(Guilford, 1954:385)

8. One-way ANOVA (samples of unequal size).

Sum of squares for the *between-groups* is given by

$$(SS)_b = \Sigma n_s (M_s - M_t)^2$$

where n_s = number of cases in a specified group

M_s = mean of this group

M_t = mean of all observations.

Sum of squares for the *within-groups* is given by

$$(SS)_w = \Sigma x_s^2 = \Sigma(X_s - M_s)^2$$

where x_s is the deviation of a score in a particular group from the mean of that group.

$$\text{F-ratio} = \frac{(SS)b}{(SS)w}$$

d.f. for between-groups is (no. of groups - 1)
d.f. for within-groups is $\Sigma(n_s - 1)$

(See Guilford and Fruchter, 1973:230-239).

Significance levels for the F-Distribution were read from tables supplied by Slakter (1972:437-442).

9. Skewness and Kurtosis

9.1 Skewness

Skewness shows the extent to which the distribution of a set of scores deviates from the normal distribution in terms of the symmetry about the mean and is given by

$$S = \frac{\frac{\Sigma x^3}{n}}{\sigma_x^3}$$

APPENDIX 5 (Continued)

where x = deviation scores

n = number of cases

σ_x = standard deviation for the scores.

For a normal distribution of scores $S = 0{,}0$.

(Ghiselli, 1964:58).

9.2 Kurtosis

Kurtosis is the criterion for peakedness or flatness of a distribution curve and is given by

$$K = \frac{\frac{\Sigma x^4}{n}}{\sigma_x^4}$$

where x, n, σ_x are as in 9.1

For normal distribution of scores $K = 3$

(Ghiselli, 1964:58).

10. Partial Correlation

A partial correlation between two variables is one that nullifies the effects of a third variable (or a number of other variables) upon both the variables being correlated. The correlation between variables 1 and 2 with 3 held constant is given by:

$$r_{12.3} = \frac{r_{12} - r_{13}\, r_{23}}{\sqrt{(1-r_{13}^2)\,(1-r_{23}^2)}}$$

(Guilford and Fruchter, 1973:312)

11. Multiple Correlation

The coefficient of multiple correlation indicates the strength of relationship between one variable and two or more others combined with optional weights. The multiple correlation between variable 1 and variables 2 and 3 combined is given by

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}\, r_{13}\, r_{23}}{1 - r_{23}^2}}$$

(Guilford and Fruchter, 1973:360-361)

APPENDIX 5 (Continued)

12. The Multiple Regression Equation

For the three-variable problem, the regression equation has the general form:

$$X_1' = a + b_{12.3} X_2 + b_{13.2} X_3$$

where X_1' is the predicted value of X_1 from X_2 and X_3.

$$b_{12.3} = \frac{\sigma_1}{\sigma_2} \left[\frac{r_{12} - r_{13} r_{23}}{1 - r_{23}^2} \right]$$

$$b_{13.2} = \frac{\sigma_1}{\sigma_3} \left[\frac{r_{13} - r_{12} r_{23}}{1 - r_{23}^2} \right]$$

$$a = \overline{X}_1 - b_{12.3} \overline{X}_2 - b_{13.2} \overline{X}_3$$

(Guilford and Fruchter, 1973:361-363).

APPENDIX 6 : LETTER TO PRINCIPAL

UNIVERSITY OF DURBAN-WESTVILLE

DIVISION OF MATHEMATICS EDUCATION

25 April 1980

The Principal

....

Dear

The Division of Mathematics Education, in consultation with the Department of Indian Education, is undertaking a comprehensive research study of the state of achievement in mathematics in Std 9 and 10 in our schools.

Further to permission granted by the Department, I wish to inform you that your school has been selected for this purpose. I want to assure you that there will be minimal disruption of classes during tests as only about 1/5 of the Std 9 and 10 mathematics students will be selected. Teachers will not be involved in any testing or marking. However, the Mathematics teacher will be required to rate his pupils (± 5 mins each) and fill in one questionnaire (± 3 mins).

Apart from contributing to research, teachers are bound to benefit from this exercise in that they will evaluate their pupils' abilities/ weaknesses and their own attitudes/approaches to their pupils. Pupils, on the other hand, will be engaged in thinking about their performance, future careers/studies and their attitudes to mathematics and the school. The test, based on the syllabus, will form an essential part of the revision/learning programme in mathematics.

It is hoped that this research project will advance some possible solutions to the many problems currently experienced by our teachers and pupils of mathematics.

I shall communicate personally with you regarding the dates and times of the testing etc.

I am looking forward to working at your school.

Thank you.

Yours faithfully

M MOODLEY

APPENDIX 7 : LETTER TO TEACHER

UNIVERSITY OF DURBAN-WESTVILLE

DIVISION OF MATHEMATICS EDUCATION

25 April 1980

....

Dear Colleague

The Division of Mathematics Education, in consultation with the Department of Indian Education, is undertaking a comprehensive research study of the state of achievement in mathematics in our schools.

Your valued experience and assistance is sought for a small but very important part of this program. You will be required to complete

1. a questionnaire (TQ2) .... ± 3 mins
2. one rating scale (TQ1) for each of a small sample from your Std 9/10 classes .... ± 5 mins.

Please be free to respond as frankly as possible. All responses will be treated strictly confidential.

It is hoped that the research findings will advance some possible solutions to the many problems experienced by our teachers and pupils of mathematics. At any rate, I want to assure you that your participation in this project will certainly contribute to your thoughts about your teaching, your pupils and mathematics education in general.

My personal thanks to you for making time available on your busy schedule.

Yours sincerely

____

M MOODLEY

APPENDIX 8 : RATING OF TEST ITEMS ACCORDING TO OBJECTIVES

Please refer to the items in MATHS TEST PQ3

A. Study each item and indicate which of the following abilities it best tests in Stds. 9/10:

1. KNOWLEDGE - ability to remember or recall defns., notations, operations, concepts, axioms and theorems.

2. SKILLS - ability to manipulate and compute rapidly and accurately.

3. COMPREHENSION - ability to interpret verbal, symbolic, geometric forms and to translate from one to the other; to follow proofs.

4. APPLICATION - ability to select and apply concepts/operations/ theorems to mathematical and non-mathematical problems.

5. ANALYSIS-SYNTHESIS - ability to analyse problems, to construct and evaluate proofs and solutions.

B. Please indicate any item(s) which your pupils in Stds 9/10 may not be able to attempt because the content might not have been covered.

APPENDIX 9 : TEACHER QUESTIONNAIRE (TQ2)

TQ 2 : To be completed by mathematics teacher (Use " ✓ " where possible) (all information will be treated strictly confidential).

1. Male: ____ Female: ____
2. Years of teaching experience .... ____
3. Years of experience in the teaching of mathematics ____
4. Have you received special training to teach mathematics?

   Yes ____ No ____
5. Professional qualifications:

   Two Year Teacher's Diploma 1 ____

   Three Year Teacher's Diploma 2 ____

   One Year Post-graduate Teacher's Diploma 3 ____
6. Academic qualifications:

   (a) Highest Degree held:

   Bachelors Degree .... 1 ____

   Honours Degree .... 2 ____

   B Ed .... 3 ____

   M Ed .... 4 ____

   (b) Highest qualification in Mathematics:

   Mathematics I or equivalent .... 1 ____

   Mathematics II .... 2 ____

   Mathematics III .... 3 ____

   Mathematics (Hons) .... 4 ____
7. Total teaching Load: No. of periods (per week) ____
8. No. of these periods used for mathematics teaching: ____

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