A Case Study of Mathematics Teaching and Learning at a Rural School in Kwa Zulu-Natal

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I, Linda van Laren (Student number 9903766), declare that this dissertation is my own work, and has not been submitted previously for any degree in any university.

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Abstract

The purpose of this study was not only to examine the teaching strategies that a teacher at a rural school in Kwa Zulu-Natal used in a Grade 8 mathematics classroom but also to examine the learning outcomes and to explore possible links between teacher strategies and learning outcomes. This study made use of both quantitative as well as qualitative data collected over a two-week period.

In order to explore these foci it was necessary to look to theoretical frameworks that are considered as powerful in the context in which the learning was taking place. The theoretical framework provided by social linguist, Vygotsky, as well behaviourist and constructivist frameworks provided pertinent information linking possible teacher perspectives and teaching strategies employed by the teacher. Vygotsky's theory was investigated to shed light on the issue of using a second language as the medium of instruction in mathematics. The South African Outcomes Based Education system appears to be based on a constructivist/behaviourist model so these approaches were compared.

The literature survey comprises readings in the teaching of elementary algebra, assessment and teaching mathematics to second language learners. These topics provide useful, current alternatives as well as insights into how and why elementary algebra programmes are developed in a particular manner. A variety of different approaches to the teaching and learning of elementary algebra also provide useful lenses that were used to probe the manner in which the Grade 8 learners at Angaziwa High School were taught.

Assessment plays a vital role in teaching and learning thus the test devised by the teacher was thoroughly examined and used to interpret what learners had understood by the topic "solving linear equations". The techniques that the learners applied in solving linear equations possibly emanated from their trying to implement rules provided by the teacher and/or their attempting to use their own intuitive arithmetic methods to obtain solutions. Despite all the detailed step by

step chalkboard illustrations and verbal instructions presented by the teacher, the learners made use of a variety of incorrect methods of solving and setting out solutions. It is contended that sole use of teacher-centred teaching techniques in the teaching of mathematics provides few opportunities for learners to reflect or develop their metacognitive abilities.

This study has recommended that teachers of mathematics should be afforded the opportunity to participate in in-service programmes so that enthusiastic teachers, such as the teacher at Angaziwa High School, may develop a variety of teaching strategies which will provide avenues for the teacher to become a reflective practitioner.

Dedication

To

MY HUSBAND,

(Henk)

MY DAUGHTERS,

(Louise and Liska)

MY MOTHER

(Audrey Botha)

and

MY MOTHER-IN-LAW

(Riet van Laren)

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Chapter 1: This Study

1.1. Rationale

Since starting as a Mathematics and Mathematics Education lecturer twenty-six years ago, I have observed many student teachers teaching mathematics (and other subjects) at primary and secondary schools for approximately five weeks each year. I have had the opportunity of working at two different colleges of education in Kwa Zulu-Natal and have usually observed student teachers in urban/township schools that are in close proximity to these two urban colleges of education. During the practice teaching periods the strategies employed by students teaching mathematics were the focus of my attention. In addition to this focus, however, it is also necessary to examine how learners respond to the teacher strategies. In this study I decided to investigate teacher strategies, learner outcomes as well as possible relationships existing between teacher strategies and learner responses.

Until 1992, the two colleges of education at which I have been employed served only "white" student teachers. Since then I have been afforded the opportunity to visit a greater variety of classrooms to observe and tutor student teachers other than those destined for white provincial and private schools. As a rule, students have been allowed to choose where they would prefer to do their practice teaching so that most students are observed in the type of school they themselves attended and these schools cater mainly for learners of the same racial/language group as the student. The chosen schools are usually located in and around the immediate vicinity of the urban colleges of education.

To improve my practice as a student teacher evaluator, it was valuable to take part in a two-week, in-depth study of the teaching and learning practice that occurred in a classroom in a different, rural setting. Furthermore, the teacher strategies could be examined in conjunction with learner outcomes instead of focusing predominantly on the teacher strategies. By being with a particular teacher and a specific class, instead of observing isolated lessons being taught by a number of student teachers in a variety of classes, it was possible to become well acquainted with the mathematics activities concerned with teaching and learning in a rural school. This opportunity to scrutinize mathematics being taught in a "black" rural school may possibly be described as a unique experience for a "white" South African educator.

Furthermore, this study will form part of a national study that is concerned with examining classroom practice. Twenty-four schools were chosen in South Africa in which to observe the teaching of mathematics to Grade 8 learners. In August/September 1999, at each school, one set of learners together with their mathematics teacher was observed for a two-week period. The study involved primarily, select teachers based on results obtained from The Third Mathematics and Science Study-Repeat (TIMSS-R) that took place in 1998/99. The provinces that were targeted were the Northern Cape, Western Cape, Free State and Kwa Zulu-Natal. In each province six classes, each at a different school, were chosen for this project. The findings from all these provinces will result in a fairly generalizable study on classroom practice. The project leader of this Human Sciences Research Council (HSRC) study is Botshabelo Maja and the deputy leader is Colleen Hughes. To date this report has not been published.

This research should be seen as an attempt to arrive at some understanding of classroom practices and learners' performance in a unique setting. The teacher, and hence the school, were chosen by the HSRC project leaders. The fact that South African learners performed poorly in the TIMSS conducted in 1995 cannot be overlooked. The South African learners' performance was rated the lowest of the forty-one countries that participated in the TIMSS. In the TIMSS the learner outcomes were assessed but a more comprehensive study of the situation at this particular rural school would need to examine the teacher's classroom performance as well as the learners' attainment in the specific learning context. Furthermore, the whole spectrum of the learners' experience needs to be recognised in studying learners' mathematics achievement. Perhaps this study could form a starting point for a possible transformation process involving classroom practices currently in use in South Africa. Hence project benefits from this case study in Kwa Zulu-Natal can be gauged at various levels:

National Government: This project is located within the Presidential Education Initiative Projects and the National System of Innovation and it will directly influence the government's prioritisation of areas of need in it's attempt to improve education and, in particular, mathematics attainment in South Africa.

National Department of Education: The findings of this study could assist the National department in decision-making when addressing the issue of attending to the quality of mathematics achievement in South Africa.

Provincial Education Department: The dire need for interaction between departments at provincial level could be facilitated by this initiative as it allows for comparative assessments of quality of education to be made. The fact that the Kwa Zulu-Natal department of education receives the least amount of money per annum per school-going learner cannot be ignored.

Schools: Individual schools may learn from these research findings and position themselves appropriately in their attempts to reflect on present practices in mathematics classrooms.

Knowledge: This study will contribute to knowledge of the efficiency of mathematics teaching and learning in South Africa.

1.2 <u>The school context</u>

The rural school that was selected by the HSRC study was remote. For the purposes of this study the school has been named Angaziwa High School. ("Angaziwa" means "unknown" in isiZulu.) Despite various inquiries amongst colleagues and friends, no one was able to give a clear indication of the exact location of the school. Eventually an ex-colleague and friend, Ms Pumla Mfeka (Mathematics Supervisor for North Durban region), was contacted and fortunately she knew precisely where the school was situated. Ms Mfeka kindly volunteered to take me to the school, as map details were unavailable. Location details could not be defined, as written or drawn information was scant because of the lack of road signs in the area.

On Friday, 13 August 1999, Ms Mfeka and the researcher travelled the 68 kilometres from Durban to Angaziwa High School. This journey takes one and a quarter hours to complete as 26,3 km of the road consists of rough, dusty gravel. When we arrived at Angaziwa School at 10h45, only a few learners were still milling around the school. Some learners were sweeping the empty classroom that served as the headmaster's office while the headmaster was away collecting "redeployment letters". The teacher who was to be observed explained that, owing to redeployment, three mathematics teachers had been removed from Angaziwa High School and that she was the only teacher who was to teach 135 learners both mathematics and physical science. For the purposes of this study the teacher will be named Ms Fundisi. ("Fundisi" means "teacher" in isiZulu.) The HSRC project was discussed with Ms Fundisi and she acknowledged that she knew about the study as she had received a letter from the HSRC. The two-week long daily visits were arranged to start on 17 August 1999 and thirty-five of the Grade 8 learners would be selected to be taught by Ms Fundisi.

The researcher was warned that no schooling would take place if it rained during the two-week observation period as the gravel roads to the school become impassable to vehicles in wet weather. There was no running water at the school and the rain water tanks had run dry. The solar panel that was used to operate the telephone had been stolen. A high well-kept fence enclosed the school but the gates were not locked. There was a groundsman who tidied up the veld inside the fence that surrounded the school, but there was an assortment of cattle, goats and chickens grazing within the school fence. The outside toilets were not flushable and were situated approximately 50 metres from the main school building. There was no electricity at the school. The poem written by the researcher's daughter, Liska van Laren, describes the setting at Angaziwa High School. (See Appendix I.)

The headmaster said he had been at the school since 1990. The school buildings consisted of two distinct constructions. According to the headmaster, the older, single-storied section may have been built in the seventies whereas the newer, double-storied section may have been constructed in1983. The older structure is used to store old, dilapidated school furniture and part of this building serves as an assembly hall. The new face brick, double-storied structure serves as the main school building that houses the classrooms.

The 455 to 460 learners at the school come from five feeder primary schools in the surrounding area. There were four Grade 8 classes, three Grade 9 classes and one each of Grades 10, 11 and 12. There were eleven teachers and a headmaster who also assisted with physical science teaching. The only mathematics teacher left after the redeployment exercise was Ms Fundisi. Twenty-two learners took mathematics in Grade 10, forty learners in Grade 11 and twenty-one in Grade 12.

In the Grade 8 class that was observed for two weeks, thirteen of the learners had the same surname. Later the researcher discovered that the name of this area is synonymous with a certain Eastern-Nguni clan. Whenever one mentions the name of the rural area in which the Angaziwa High School is situated, people who come from that clan keenly relate the story of a well-known cattle "dealer" of the area. In A.T. Bryant's book, *Olden times in Zululand and Natal* (1965:497), a similar anecdote is recorded. There was a man of that region who was known to be a very successful, crafty cattle-lifter who, on the instruction of Shaka (king of the Zulus), was bound horizontally to the cattle kraal gate. Shaka's entire herd of cattle was then chased out of the kraal and the man was trampled, poked and kicked to death. People of this area often tell of this cruel episode.

The timetable at Angaziwa High School was devised by the headmaster and implemented by the teachers and was displayed in an empty classroom that was adjacent to the headmaster's office. This classroom served as the teachers' staffroom. The learners stayed in their classrooms throughout the day whilst the teachers moved from classroom to classroom according to the timetable. Occasionally a school bell was used to indicate the commencement of a particular period.

Ms Fundisi does not live in this rural area but comes from a town situated approximately forty kilometres from the school. Every day all the teachers travel together from urban areas to Angaziwa High School in shared transport.

Ms Fundisi had had six years of teaching experience when the research was undertaken. She obtained a three-year Secondary School Teaching Diploma at one of Kwa Zulu-Natal's colleges of education and this was followed by a fourth year diploma at another college of education in the same province. In 1999 and 2000 she was studying, on a part-time basis, for a Further Education Diploma at the University of Natal. Her subject specialisation for her Further Education diploma is Physical Science.

Chapter 2: Literature Survey

2.1. Introduction

In order to explore the critical questions, a number of relevant topics were researched in pertinent literature. Particular aspects considered for this investigation related to:

1. relevant learning theories
2. the teaching of mathematics to second language learners
3. assessment
4. the teaching of elementary algebra.

The learning theories selected are those currently considered significant in the learning of mathematics. Vygotsky, behaviourism, Piaget and constructivism offer learning theories that are seen as the most powerful in explaining the social interaction and cognitive development in the mathematics classroom. These theories give a perspective on the events observed in this classroom-based research project. Furthermore, the theoretical framework makes visible the assumptions and beliefs that the researcher brings to the study.

A literature survey of research on teaching mathematics to second language learners was done to provide a lens through which the observed practices could be analysed. The test devised by the teacher and completed by the learners played a pivotal role in classification and analysis in this research project. Aspects relating to the observed assessment of mathematics thus need to be considered so as to link this research to current thinking in assessment and the learning of mathematics.

The part of the survey that deals with the teaching of algebra contains a phenomenological analysis and a mathematical pedagogical analysis of elementary algebra. The topic taught by the teacher was "Solving linear equations" but the manner in which the elementary algebraic concepts were conveyed would impinge on the observed teaching strategies employed by the teacher. Furthermore, analysis from other empirical research was sought to shed light on relevant aspects in the teaching and learning of algebra.

2.2. Relevant learning theories

2.2.1. Lev Vygotsky – Language and Thought

A significant aspect of any classroom situation is undoubtedly the social interactions that take place. Not only is there the interaction between learners but also interactions between the learners and the teacher. Although communications may be physical, the usual interchange of knowledge, skills, attitudes and values amongst the individuals in the group are through symbolic, socially defined means. No one can ignore the importance of the type or quality of the language usage in the classroom. The exclusive use of English in the mathematics classroom of Zulu-speaking learners can also not be overlooked.

In the past decade much emphasis has been placed on the social interaction taking place in the mathematics classroom. There have been numerous articles in recent journals (Vace (1993); Garrison (1997); Sfard *et al* (1998); Atkins (1999); Robinson & Adin (1999); Reinhart (2000)) discussing the influence of language interaction on the development of mathematical thinking, learning and understanding. The foundations of much of this research stem from work done by Lev Semenovich Vygotsky (1896 – 1934). During the 1920s and 1930s this Russian psychologist developed a theory explaining how children conceptualize the meaning of words. Vygotsky placed social interaction and communication at the centre of the process of conceptualization. Talking was seen as a vehicle for the internalization of spoken words. The significance of the words would become meaningful when communication of concepts took place.

Furthermore, according to Vygotsky, children learn new vocabulary by reflecting on and visualizing the meanings of the words as they talk and discuss concepts.

It is thus through vocalization of thought that children are best able to reason for themselves. For Vygotsky the proper unit for the analysis of verbal thought is the meaning of words and the meaning of a word is a generalisation of a concept. In his opinion, generalisations and concepts are acts of thought and there are thus very close relationships between language and thought. Furthermore, Vygotsky did not recognise concepts as conditioned associations as was suggested by Pavlov's study on conditioned reflexes. Langer (1969:80) quotes from Vygotsky's *Thought and Language* (1962:82 – 83)

> A concept is more than the sum of certain associative bonds formed by memory, more than a mere mental habit; it is a complex and genuine act of thought that cannot be taught by drilling but can be accomplished only when the child's mental development itself has reached the requisite level... [of] deliberate attention, logical memory, abstraction, the ability to compare and to differentiate.

It would thus be difficult to justify requiring learners merely to repeat a word a number of times as a strategy to bring about understanding of a concept; the chorusing of words cannot guarantee learners' mental development.

The importance of the presence of the "knowledgeable other" in the discussion of new words is essential for the positioning of the learner in what Vygotsky called, "the zone of proximal development" (ZDP). It is here that the child's current understanding and potential understanding is located (Vygotsky, 1978). The knowledgeable person would thus need to enter the child's ZDP to enhance the meanings of what is familiar and understood by the child. In the ZDP there is a rich fount of disorganized informal concepts that may be distilled by interaction with the systematic, formal reasoning of a knowledgeable person. In order to develop and supplement the child's existing knowledge base and move the learner from the known to the unknown, a teacher needs to interact with the learner within the learner's ZDP.

Children therefore develop and extend language through their experiences gained by talking. In order to develop, clarify and generalize meanings of words they need to learn the words as symbols of experienced concepts.

Communication with other people is, however, a requirement so that there is reaction to the children's word usage. Mathematical language building would thus also require communication of concepts perceived by the learners. Second language learners would experience an added complication. Possibly these learners would need to interact within their ZDP using their preferred mode of communication before it is likely that they would be able to interact in another language. Here learners would not only have to deal with the unfamiliar sound of the words but also their own understandings might not be easily communicated within the unaccustomed knowledge base of the knowledgeable other. It is thus not only the sound of the word that is significant, but also the understanding of the concepts behind the words that has to be developed and vocalized. The meanings of the words of the second language learners has to filter through thought, familiar language, and familiar experience in addition to unfamiliar language, unfamiliar experience and unfamiliar means of communication.

Many first language learners are confounded by new mathematical language when presented with vocabulary that is not within their experience. According to Steele (1999:39), "A teacher needs to encourage students to think about, talk about, and learn mathematical vocabulary in a way that is consistent with Vygotsky's ideas". This task may be particularly problematic for second language learners who are not afforded the opportunity to discuss concepts.

When students use unfamiliar language to describe their thinking it may be difficult for the teacher to gain information about what they have understood. It is possible that the teacher never enters the ZDP of the learners. The opportunities to explore, investigate and explain their ideas are hampered by the translation process from what is in the ZDP of the learner.

If the teacher introduces new words by requiring the learners to repeat out loud or chorus the new words, the learners do not have the opportunity to make the terminology their own. According to Steinbring (1999:54), "Strictly fixed readings of mathematical signs may cause a paralysis of mathematical communication " that "may also lead to a transformed, a ritualised communication". He considers direct, intentional teaching not to be the most successful approach. The vocabulary would probably remain "detached" and outside the ZDP of each learner. Here there is no negotiation of the shared meaning of the mathematical vocabulary and the only person gaining from the vocalization of the terminology is perhaps the teacher. The vocabulary acquired by learners does not enhance the language development if it is only seen as a skill to be used to communicate "correctly" pronounced words. The mathematical concepts need to be developed in conjunction with meaningful social interaction in the classroom.

Vygotsky saw cognition as internalization of social interaction. According to Wilson *et al* (1993) Vygotsky was concerned with culturally situated learning. Vygotsky believed that educational interactions reflect the surrounding culture. This culturally situated learning would be difficult to define when considering learners being taught in an unfamiliar, second language. Wilson *et al* (1993) consider that instructional methods based on Vygotsky would emphasize the need for social interaction in problem-solving environments. Unlike behaviourist approaches, however, Vygotsky-influenced instructional methods cannot be reduced to a procedural set of rules. Wilson *et al* (1993) highlight some teaching strategies consistent with Vygotsky's Theory of Mind, namely:

1. Primacy of the social. Vygotsky's claim that cognition is the internalization of social interaction is a powerful idea. ...

2. Motivation and attitude development. A social/cultural approach to cognition provides a fresh and much needed slant on questions of attitude development and motivation. ...

3. The role of dialogue. Dialogue – the two-way interactive exchange between two speakers...

4. The zone of proximal development. The zone of proximal development was conceived in terms of the added capacity a child has when supported in performance by a teacher or more skilled peers.....

Here teachers are provided with some guidelines, but what appears to be central to the teaching situation is social interaction. Guerra (1999) quotes Vygotsky

(1962:55) to indicate how Vygotsky proposed concept formation would be developed in learners. "A problem must arise that cannot be solved otherwise than through the formation of new concepts." Perhaps the vehicle for concept formation was seen as problem solving.

The use of technological devices, such as calculators and computers, may be seen as another form of interaction and would therefore not be in opposition to the theory advocated by Vygotsky. Sutherland & Balacheff (1999:1), suggest that there should be a move from a focus on the meanings constructed by students to a focus on the teacher as the facilitator of the learning situation. Interactions between teacher, students and the computer may also be considered as significant in a learning situation. Another useful aspect this theory takes into account is the inclusion of aspects of motivation and attitude. This facet of human interaction mentioned in Vygotsky's theory is often overlooked in other theories.

When, however, the mathematical language is explicitly taught and prescribed and not developed through personal sensemaking, the object of attention in the mathematical learning process becomes the spoken word. This would probably not be considered as social interaction. Adler (1999:47) points out

1. ...that explicit language teaching where teachers attend to pupils' verbal expressions as a public resource for class teaching, appears to be a primary condition of access to mathematics.
2. ...the possibility in explicit language teaching may focus too much on what is said and how it is said.

This is of particular significance when the learners are themselves not proficient in the language being spoken. It would be possible to feel excluded from the mathematics because of inadequacies in the "medium". Adler (1999:62) considers that language itself became"visible" and the focus of attention. "It is no longer the medium of expression, it is the message". Emphasis on the mathematical words would thus detract from the building of mathematical knowledge.

2.2.2. Behaviourism

Although behaviourism emerged from experiments done on animals by Pavlov (1906) & Skinner (1953) (in Langer 1969:52 – 53), subtle, moderate forms remain common in education. Here learners are seen as possessing little knowledge and it is the task of the teacher to transmit whatever knowledge the learners need into the empty, "black box" minds of the learners. Knowledge is seen in terms of behavioural responses to external stimuli. Only what can be observed as behaviour is counted as learning. Learning comes about by reinforcement of predetermined observable actions. Each lesson is prepared in terms of objectives that are couched in terms of behaviour that should be observed by the end of the lesson. There is strict control of the learning environment and the teacher has the sole responsibility of breaking down the desired knowledge into appropriate stimuli that are transmitted in a sequential, hierarchical manner. Learners are taught to mimic responses provided by models and these responses are controlled by reinforcement. Carefully graded activities are prepared for learners to complete in a structured environment. Behavioural performance is used to measure learning. Much criticism has been leveled at behaviourism as the teacher became the focal point of the learning process. In the learning process, overt, observable behaviour of the learners is seen to be of paramount importance at the expense of their covert, mental behaviour. In the learning environment the learner is not afforded enough credit as the teacher is deemed to be in charge of the whole learning and teaching arena.

Behaviourist approaches have been heavily criticised as learners are often considered to be unable to bring their own meaning when a mathematical problem is to be solved or learners' meanings and experience are simply disregarded. Furthermore, incorrect or partially incorrect understandings are readily passed on to learners when teachers impart knowledge. Treating learners as passive recipients whose behaviour can be manipulated could be seen as viewing the aims of education as the passing on of a known body of knowledge. The clear-cut instructional methods with exact strategies for teaching and testing learned material ignores the existence of individual learners in the classroom. The attempt at ensuring conformity amongst learners leads to a reduction of approaches to procedural recipe type teaching. The rapidly changing and growing body of knowledge learners will be encountering in the future cannot be anticipated. Learners need to develop their innate abilities as problem solvers and not rely solely on external control of their behaviour. If teachers insist that learners use a prescribed step by step method to solve a problem the learners may attempt to rely more on their memory of the procedure at the expense of using rational thinking.

2.2.3. Jean Piaget's kinds of knowledge

In contrast to behaviourism and the social theory of Vygotsky, Piaget (Kamii, 1985, in Murray *et al*, 1998:7) distinguished three kinds of knowledge. In addition to social knowledge (knowledge formed by people entering into conventions), physical knowledge (knowledge gained by observing and interacting with physical phenomena) and logico-mathematical knowledge (knowledge produced by human reflection) are considered. According to Piaget, more than just knowledge of social origin is necessary to acquire new personal mathematical knowledge. However, the modes in which individuals acquire knowledge suggested by Vygotsky and Piaget are somewhat similar. Piaget suggests assimilation of new experiences into already-existing mental schemes and accommodation that involves the resculpturing and extension of new knowledge. To assimilate a new concept the child restructures the appropriate environmental cues so that it becomes coherent within existing schemes. A scheme functions as a form of action that structures experience into percepts and concepts. These schemes of action have self-constructive capacities from the onset and are in contrast with habits, which are the result of conditioned associations between unrelated elements. The function of accommodation is to modify and extend the child's schemes so that they will be consistent with the character of the physical environment.

According to Murray *et al* (1998:8) the acquisition of physical and social knowledge is facilitated by co-operating with knowledgeable others and logico-mathematical knowledge should be constructed on an individual basis by making use of appropriate tasks.

2.2.4. Constructivism

The manner in which the learner makes sense of acquired knowledge is given particular significance by constructivists (Human, 1996). The knowledge experienced as social knowledge would be adequate for the development of socially determined mathematical language, but unsatisfactory for inducing personal logical justification of mathematical knowledge. Constructivists would object to practices dominated by social interaction to bring about construction of authentic mathematical knowledge. In the classroom there would thus be pressure on the teacher to devise problems that require suitable mathematical knowledge elements in order to bring about satisfactory modes of sensemaking that require logical justification. In a classroom where there is predominantly "chalk and talk" by the teacher, the mathematics learnt may not be perceived as a sensemaking process that requires individual justification. According to Human (1996:1), to constructivists making sense of mathematics would include "the *assignment of meanings* (to symbols, procedures, concepts, propositions, etc.), the *experiencing of purposes* for mathematical knowledge elements, and the personal *production of logical justifications.*" The quality of learning is thus heavily dependent on the manner in which the teacher presents the mathematics.

Cobb (in Sfard *et al*, 1998:46) distinguishes two aspects of classroom conversations that he and his colleagues have found to be potentially productive for students' learning: "calculational discourse" and "conceptual discourse". The former refers to discussions in which "the primary topic of conversation is any type of calculational process". This, however, does not merely focus on the procedural manipulation of conventional symbols that do not necessarily mean anything. "Rules without reason" would not qualify as "calculational discourse". Steinbring (1999:53) believes that "When mathematical knowledge is reduced to its formal terminology and its logical consistency with reference to fixed referents then the mathematical discourse is in danger of turning into a communication about the definite 'correct' interpretation of mathematical signs what in the end is decided by the teacher's authority". Conceptual discourse, according to Cobb, refers to discourse in which "the reasons for calculating in particular ways" becomes the topic of conversation. Cobb (in Sfard *et al,* 1998:48) pointed out that reflection is enabled by participation in discourse but that students should be allowed to work individually on the understanding that they may talk to peers of their choosing as the need arises.

According to Human (1999:9), "Constructivism suggests an alternative strategy, namely to recognize that learners do not acquire knowledge by assimilating given information, but through sensegiving and generative personal constructions, and to endeavour to guide these constructive processes rather than to try to prescribe the knowledge to be constructed". He advocates the problem-centred approach as a strategy "to guide learners' constructive processes with respect to mathematics towards rational sensemaking, and away from submergement." There is, however, a difference between merely providing problems for learners to solve and the problem-centred approach. A problem-centred approach uses a problem as the starting or focal point requiring individual sensemaking and forms an integral part of a theme to be taught. It is, however, not an easy task to devise such an appropriate problem. The problems may be of a "real-life" interest to the learners or the problems may have been of interest to someone in the past or the problems may be only of pure mathematical interest. In general, when learners problem-solve the activity may not always be a novel, non-routine situation for each learner. An added difficulty in devising appropriate problem-centred situations is that some learners may see the problem as a routine, recall situation or an exercise, whereas for others it could be the required "novel" type problem appropriate for the problem-centred approach. Murray (1994:7) describes a "Problem-Centred Approach" as one whereby:

1. Pupils are confronted with problems that they perceive as meaningful, but which they cannot solve with ease using known procedures.

2. The teacher does not demonstrate a method, nor does s/he supply hints or ask leading questions.

3. Pupils are required to explain, justify and argue in a mutually-supportive, non-critical atmosphere. Mistakes are accepted as part of the learning process, and competition is regarded as contraproductive.

4. Teachers have to select the problems they pose in such a way that all concepts and procedures mentioned in the syllabus are covered, and that pupils are given opportunities to *develop* these concepts over a period of time.

Mousley *et al* (1992) studied 11 teachers in Australia and Malaysia who strove to create classroom conditions in which the learners "owned" the mathematics by being involved in reflective problem-solving activities in which they constructed mathematical concepts and relationships. These researchers found that even teachers who try to develop learning environments that provide rich interactive dialogue still end up leading instead of facilitating.

Sometimes the problems used in a problem-centred approach are "word" problems (story sums) and there may be sensemaking required in "real-life" terms. Thus second language learners would have more than just the mathematics to contend with. In general these word problems are "dense" and specific meanings are associated with very carefully chosen words.

Perhaps the manner in which Andrew Wiles, a researcher at Princeton, developed the proof of Fermat's Last Theorem (FLT) would illustrate how a problem-centred approach attempts to model the way in which mathematicians discover and invent new results and proofs. For seven years he pursued a proof in almost complete isolation. Finally, he announced that he had a "proof" on 23 June 1993. Unfortunately Wiles' "proof" was found to be wanting and in 1994, Wiles acknowledged that a gap existed. However, together with Richard Taylor, Wiles came up with a different strategy to circumvent the problem with his first attempt at proving FLT. Together their papers were published in the May 1995 issue of *Annals of Mathematics*. It would be impracticable to consider that learners should grapple with a single problem for so long in isolation, but this example illustrates how a mathematician could solve a problem, first by individual investigation and then, when the need arises, by looking for further insight by talking to a peer. Perhaps this method is rather idealistic as there are very few school learners who are destined to become mathematicians of such note, but it would seem reasonable to afford each individual in the mathematics classroom the opportunity to make sense of a problem for her/himself before engaging in discussion with others.

Despite the fact that constructivism recognises the learner as being at the centre of the learning in the mathematics classroom, there have been criticisms from social, cultural and political fronts. Zevenbergen (1996:95) argues that constructivism favours the individual construction of meaning and in doing so "the social and political contexts in which mathematical knowledge is located is ignored." She considers that scientific knowledge confers more status and power to those who are able to operate with such forms of knowledge and this effectively excludes and marginalises groups of people who are not involved with empirical sciences. Students taking science degrees at university may seem to be "superior" to students taking other degrees. Often mathematics as a matriculation subject is one of the pre-requisites for some of the more prestigious degrees. Thus learners who are, for some reason or other, not able at mathematics may see themselves as lacking some natural talent and worthless in the wider society. No longer can the "useless" teacher be blamed for being incapable or unwilling to explain the mathematics content as it is the individual's capacity to construct knowledge that is lacking.

According to Zevenbergen (1996:105), learners whose social/cultural background is similar to that of the formal school context would be considered to construct knowledge which is deemed valid and valuable. In addition to the social and cultural backdrop, groups whose language is dissimilar to that of formal schooling will be at a distinct disadvantage when it comes to reproducing the specialised language of mathematics. Non-mother tongue learners could thus be seen as incapable of constructing any worthwhile knowledge as these learners may find it difficult to articulate mathematical concepts constructed. According to Zevenbergen (1996:107) "Constructivism does not call into question the actual linguistic code of mathematics." This may, however, also be seen as a criticism of Vygotsky's language and thought theory and not of constructivism, as the language issue is at stake here and not the learner's construction of meaning.

Zevenbergen (1996:110) criticises constructivist theories because schools do not recognise nor reward the individual construction of meaning but particular constructions of knowledge. Once again, this criticism should be aimed at the manner in which schools operate their reward system and not constructivist theories. It would be the knowledge displayed that is in keeping with the culture of the school system that would be valued and reinforced. Zevenbergen (1996: 105) argues that success in constructing meanings which are in keeping with those of the formal mathematics curriculum is not really of importance as an individual attribute but more of a social talent. So, although the learning process appears to be learner-centred, it is actually the teacher and society that legitimate the knowledge constructed by the learners. Hence the teacher and society would, once again, play a significant role in making decisions about which particular knowledge should be rewarded.

Taylor (1996:157) refers to research on teachers' thinking as having shown that beliefs below the surface of consciousness are very influential in maintaining teachers' established classroom roles. If the teacher maintains the traditional behaviourist role of informer and controller then few opportunities become available for learners to develop mathematical concepts on their own. What the teacher believes may not be in keeping with what she says or perhaps what the teacher believes to be constructivism may not be constructivism at all. It could thus follow that learners would not readily be given the opportunity to exercise self-determination in respect of their learning activities.

2.3. Teaching mathematics to second language learners

MacGregor and Price (1999:449) investigated language proficiency and algebra learning and conclude that if learners are proficient in language it enables them to "use language as an organizer of knowledge and a tool for reasoning." These authors point out (1999:450) that it has been shown "that students who performed poorly in mathematics tended to have low levels of competence in their mother tongues. A level of language proficiency in at least one language is a necessary foundation for academic learning." These authors investigated three components of metalinguistic awareness – awareness of symbol, syntax, and ambiguity – to ascertain students' success in learning the notation of algebra. They found that very few students with low metalinguistic scores achieved high algebra scores. Hence, it would be necessary to ascertain mother-tongue language proficiency of learners when embarking on ascertaining possible reasons for inadequacies in mathematics when learners are taught mathematics. Furthermore, learners being taught in a second language would have to cope with the mastery of an additional language before this new language may be used as a composer of knowledge and as an instrument for thinking.

Bishop (1992:176) refers to research with second-language learners where problems of learning mathematics through a second language are described as "formidable, and do not just relate to the linguistic aspect." Language is characterised as being a product of, and a carrier of, cultural and societal assumptions and history, and "what" it describes can be just as incomprehensible to a non-speaker as "how" it describes. He suggests that bilingual learners should not have to do everything in the "official" language and small-group work allows use of more familiar language.

Laridon (1993:42) points to studies that indicate that thorough bilingualism is fundamental to enhance cognitive ability to cope with the learning of mathematics through a non-mother tongue medium. Furthermore, constructivism relies heavily on efficient communication among learners and between learners and their teacher. He thus considers the language issue fundamental to the development of reformed learner-centred curricula.

Setati (1999:179) points out that there are benefits that result from alternating between two or more languages (code-switching) in the mathematics classroom. This author highlighted other studies that have shown that the use of the learners' first language in mathematics teaching and learning provides the support needed while the learners continue to develop proficiency in the second language. Setati (1998:40) considers that the extensive use of the first language is not really permissible in South African classrooms but it is the "best means available to teachers to foster mathematical understanding..." She is of the opinion that it is an educational resource and the use of the learners' first language is "also a key to the world and culture of the learners involved. It enables the participants to make relevant connections with their lives beyond the school."

Moschkovich (1999) also suggests strategies for supporting a mathematical discussion among English second language learners. A teacher could introduce students to concepts and terms in the familiar language and later conduct lessons in English. The learners would, however, also need to be surrounded by materials in both languages. Moschkovich indicates that communication amongst learners also needs to be fostered so that learners should be grouped in mathematics lessons. Strategies she suggests to support student participation in mathematical discussions included "establishing and modeling consistent norms for discussions, revoicing student contributions, building on what students say and probing what students mean" (Moschkovich 1999:18). She adds that a teacher should not focus primarily on vocabulary development but instead on mathematical content and arguments whilst interpreting, clarifying and rephrasing what students say. She also advocates a discourse approach to learning mathematics. By this she means considering the different ways of talking about mathematical objects and points of view of mathematical situations that students bring to classroom discussions. According to Moschkovich a discourse approach to learning mathematics can also help to shift the focus of mathematics instruction for English language learners from language development to mathematical content. So instead of requiring learners to chorus technical, mathematical words, they should rather be given the opportunity to participate in discussions. Students would need to clarify, accept and build on their responses and there should also be revoicing of student statements. After all, in a mathematics classroom, the mathematics content is more important than the "correct" pronunciation of the English words of mathematical terminology.

Brodie (1991:17) points to research which showed that students learning mathematics in a language, which is not their mother tongue, may be faced with difficulties such as:

- differences between ordinary English and mathematical English
- the Greek or Latin roots of mathematical terms
- the lack of accessibility of "logical connectives in the mathematical reasoning process"
- the absence of context in many algebraic problems.

However, these obstacles mentioned by Brodie may not be limited to second-language learners. All these problems may be equally pertinent to first-language learners of mathematics. She suggests the following techniques and activities to try to integrate language and mathematics:

- holding mathematics discussions
- explicitly teaching mathematical language
- developing concepts before naming them
- encouraging students' questions
- asking open-ended questions
- teaching the history of mathematics
- encouraging students to verbalise their sense of pattern and generality before using symbols.

Perhaps not only second-language learners, but all mathematics learners, would benefit from these suggestions.

Adler (1998:25) interviewed six teachers in three different urban multilingual contexts in South Africa and she found that some mathematics teachers who teach mathematics in a language that is neither the teacher's nor the pupils' first language, consider that this places additional and complex demands on the teaching and learning of mathematics. Other teachers believed that English as the language of instruction is not the problem but that mathematics is difficult for everyone, irrespective of their main language. She found that some teachers considered that both the medium of instruction and the fact that mathematics is a difficult subject is of concern in the mathematics classroom. Adler refers to the "three-dimensional dynamic at play in the teaching and learning of mathematics in multilingual classrooms". This points to the access to the language of learning, (English), the access to the language of mathematics and to "classroom cultural processes". It would appear that teachers who teach mathematics through the medium of English to Zulu speaking learners, with limited knowledge of English, have a challenging situation at hand.

2.4. Assessment

Barnes (1969:17) suggested that the types of questions posed by teachers may be categorised into factual ("What?"), reasoning ("How?" and "Why?"), open or social questions. This classification of questions provided insight into the types of thinking required in lessons. The categorisation of questions was not specifically designed to classify questions posed in mathematics lessons.

Moodley (1992a:101), on the other hand, used Bloom's 1956 Taxonomy of Educational Objectives: Cognitive Domain to suggest a model of levels of performance from lowest to highest. The six major categories of cognitive behaviours suggested by Bloom are "Knowledge, Comprehension, Application, Analysis, Synthesis and Evaluation". Moodley developed these into an evaluation instrument for use in mathematics classrooms at the senior secondary school level (now known as the Further Education and Training Band). The levels Moodley (1992a:102) proposes are:

KNOWLEDGE (specific facts, universal facts/generalizations)

SKILLS (manipulative, computational)

COMPREHENSION (translation, interpretation, extrapolation)

SELECTION-APPLICATION

ANALYSIS-SYNTHESIS (analysis, synthesis, evaluation)

Moodley (1992b:137) points out that teachers need to develop in their learners a wide range of mathematical abilities. He suggests that teachers should ask both:

> lower order questions – requiring recall of facts and generalisations, manipulative and computational skills and higher order questions – requiring comprehension, selection, application analysis, synthesis, evaluation.

According to Moodley, lower order questions (LOQ) are characterised by responses that show duplication of information previously presented. Whereas responses to higher order questions (HOQ) will need to show a transformation of the information so as to explain, apply, analyse and evaluate for the preparation of new content. Furthermore, he considers LOQ to be indicative of a teaching style, which emphasises a "showing and telling", and "seeing and following". In opposition to this behaviouristic style, a learner-centred approach that is characterised by problem solving and mathematical thinking is more likely to develop when HOQ are used.

Du Toit (1992:112) considered evaluation to be an essential aspect of instruction and posited that evaluation should not be considered as a disconnected part of the teaching and learning programme. He regarded an effective evaluation programme as a means of determining to what extent learners have achieved the necessary outcomes of instruction. He suggested five graded cognitive levels of performance that may be used to classify mathematics questions, namely: Knowledge, Computational Skill, Comprehension, Application and Inventiveness. He described these as follows:

> "KNOWLEDGE
>
> The learner is required only to recall a fact and no understanding of the knowledge is necessary."
>
> "COMPUTATIONAL SKILL"
>
> Here "straightforward manipulation according to rules and theorems that the learner has already learned" is tested.
>
> "COMPREHENSION
>
> The question requires an understanding of the underlying concepts and he is required to interpret the significance of the data."
>
> "APPLICATION
>
> The question requires that the learner uses relevant ideas, principles or methods known to him and applies them to new situations." Here the solution requires the combination of more than one line of thought.
>
> "INVENTIVENESS
>
> The question is a non-routine application." Here the learner is required to develop her/his own strategy for solving the problem that s/he has not attempted before. (Du Toit 1992:113).

Du Toit (1992:114) distinguished different mathematical processes that he considered significant in the teaching of mathematics. These he listed as: *Abstracting, Generalising, Classifying, Translating* and *Validating.* These he described as:

> "*Abstracting"* whereby learners become aware of similarities among their experiences and a permanent mental change results.
>
> "*Generalising"* whereby an observed common abstraction is seen as valid for a greater variety of situations.
>
> "*Classifying"* whereby identification of categories of concepts emerge.
>
> "*Translating"* whereby change from one symbolic form to another is required.
>
> "*Validating"* whereby validity of a proposition is determined. (Du Toit 1992:115).

Du Toit therefore did not consider mathematics as a set of concepts, rules, theorems or structures but as a variety of processes. When teaching mathematics concepts, teachers should not just identify the range of subordinate skills that build towards higher level problems and provide reinforcement that would later be reflected in tests. Activities should rather be designed to provide learners with the opportunities to make use of a variety of processes.

Leder and Forgasz (1992:17) suggest that assessment drives the curriculum and testing drives instruction and point out that "children learn well what teachers teach and assess well". However, Grouws and Meier (1992:94) point to research that showed that the nature of the connections between teaching, testing and learning is still unclear. Ledger and Forgasz (1992) state that reforms in assessment must accompany any curriculum reforms if learners are to benefit from changes in teaching styles. It would appear that careful investigation of assessment techniques used by teachers could give critical insight into the beliefs that teachers hold about the nature of mathematics and how learning should occur.

Yackel, Cobb and Wood (1992) compared traditional forms of assessment with those of socio-constructivist forms and concluded that cheating and/or copying, which are problems in traditional behaviourist classrooms, are not issues when the learners' thinking rather than their answers are the focus. Mousley *et al* (1992:137) also emphasise that diagnosis of individual understandings as well as strengths and weaknesses necessitates both informal and frequent dialogue between teachers and learners. These dialogues are considered to provide teachers with opportunities to afford feedback, encouragement and assistance as well as to draw out an individual learner's thoughts and ideas.

Doig and Masters (1992:285) believe that learners' errors are the means through which teachers are able to see how the learners are thinking. These authors consider that evaluation should be used as a means of improving instruction, learning programmes as well as learning and should thus not focus only on the learner's ability. Therefore, assessment should shape and guide instruction and not remain segregated from it.

In classrooms with large numbers of learners, however, individual learners interacting with the teacher may seldom occur without making use of group teaching. By teaching small groups of learners at a time, whilst other groups are solving problems without assistance, the teacher may be able to provide time for each learner in a large class. When learners are able to interact with the teacher and other learners in small groups, the teacher may be able to observe and listen to individuals. Adopting socio-constructivism would not involve evaluating the learner by judging the correctness of the learner's answers but rather evaluating activities to see which would encourage the learner's further conceptual development.

Grouws and Meier (1992:98) suggest alternate methods of assessment to improve learning. These assessment tasks would need to consist of authentic student products. Their list of alternatives include:

- learner portfolios
- learner writings about mathematics
- learner investigations in mathematics
- open ended questions
- performance tasks
- observations
- interviews
- learner self-assessment.

All these ideas are considered to provide suitable evidence of learner concept development. These authors provide an example of a six-point rubric scoring technique to assess problem solving. Each problem would need a specific rubric that includes the exact nature of the responses expected for a given score. Although the assessment becomes more qualitative than quantitative, it would still be up to the teacher to evaluate each of the pieces of work constituting a learner profile and the responsibility would ultimately still rest on the teacher to decide on the correctness of the learner's responses. Instead of making the learners more reliant on their own reflections of mathematical thinking, a teacher-centred situation would prevail.

Clarke (1992:156) considers that assessment tasks should maximise the opportunities for learners to express the outcomes of their learning. He is against assessment that merely constrains learners solely to "mimicry of taught procedures". According to Clarke (1992:163) good assessment is synonymous with good instruction and it should anticipate action. He also suggests alternative types of assessment such as:

- annotated classlists (where the teacher identifies significant moments in specific learner's thinking)
- student work folios
- practical tests
- student-constructed tests
- student self-assessment.

Yet most of these forms of assessment would rely on the teacher's authority and, to a lesser extent, on learner reflection. A teacher-centred modus operandi would thus still predominate.

Bishop (1992:190) argues that for learners the familiarity of format of tests is an aspect that has not been researched. If an unfamiliar format is used in a test situation, learners may experience increased levels of anxiety and perform poorly. External examinations may also cause some learners an immense amount of stress.

If the processes learners use to solve problems in a test cannot be observed then the test does not reveal anything about learners' strategies. De Lange (1992:314) points out that this lack of information about a learner's thinking strategies may result in drawing wrong conclusions about the learner's performance. De Lange, however, considers written tests an essential part of a learner's evaluation package and insists that tests should, thus, not be omitted. Tests would be part of the "balanced package" that consists of written tests, individual observation and interviews. In the 1980s researchers in the Netherlands devised new test formats in developmental research. The following principles were followed:

1. Tests should be an integrated part of the learning process, so tests should improve learning.
2. Tests should enable students to show what they know, rather than what they do not know – testing should be positive.
3. Tests should operationalise all goals.
4. The quality of the test is not in the first place dictated by its potential for objective scoring.
5. Tests should fit into the constraints of school practice. (De Lange, 1992:314).

Although these principles appear to be sound, it is still the test that is considered to be at the centre of the assessment procedure and not the learner.

As long as learners are expected to write external mathematics examinations, such as the matriculation examination in South Africa, teachers will persist in placing a great deal of emphasis on written tests. Learners will continue to be subjected to stressful, time-restricted examinations where they will have to demonstrate mastery of pre-specified bodies of facts and skills. Learners will work through past examination questions in order to practice facts and algorithms to be committed to memory. The learners will then recall and apply these rules when required. In mathematics matriculation examinations not much emphasis has been placed on multiple-choice items, yet, there is inevitably only one correct answer to each problem presented to the learners. Although there has been growing awareness in recent decades that the learning of mathematics rarely occurs as a passive, receptive process, little is being done to make learning more meaningful where learners develop their own interpretations, approaches and ways of viewing phenomena. In practice, most mathematics testing continues to reflect a view of mathematics learning as a process of recalling isolated facts and algorithms that have been demonstrated by the teacher and ensuring that the correct method is recalled and applied as quickly as possible without allowing sufficient time for reflection.

2.5. The teaching of elementary algebra

2.5.1. The historical development of algebra

Sfard (1995:15) delineates three stages in the historical development of algebra.

- Stage 1: From antiquity to renaissance – toward the science of generalized numerical computations
- Stage 2: From Viète to Peacock – algebra as a science of universal computations
- Stage 3: From Galois to Bourbaki – algebra as a science of abstract structures.

Sfard links these stages to difficulties experienced in the knowledge formation of individuals learning algebra. These stages correspond more or less to what is taught in primary, secondary and tertiary institutions. By detecting recurrent phenomena in the development of abstract concepts in algebra, Sfard finds that mathematicians met the many ways in which new ideas were disclosed and evolved with distrust and reluctance. She suggests that this could have stemmed from the inability to reify a process. (Reification is an act of turning computational operations into permanent object-like entities.) This natural resistance to upheavals "in tacit epistemological and ontological assumptions" that blocked the historical growth of mathematics is thought to be unavoidable even in the mathematics classroom. Sfard admits, however, to using only a very general view of algebra to present this dual perspective.

There has been much controversy about what "algebra" is and therefore there is not much unanimity amongst historians about the origins of algebra. Sfard considers that one of the notable characteristics that make algebra different from arithmetic is generality. Hogben (1945:302) classified the mathematics taught in our foundation and intermediate schools, that is made up partly of rules for calculation based on Hindu and Arab algorithms and partly of the solution of numerical problems, as arithmetic. When mathematics involves using the abstract number symbols it may be called algebra. Mathematicians use the term "algebra" to mean rules for solving problems about numbers, whether the rules are written out in full (*rhetorical* algebra), or more or less simplified by abbreviations (*syncopated* algebra), or expressed with the aid of letters and operative signs exclusively (*symbolic* algebra). Hogben (1945:303) gives the following examples to show the transition from pure rhetorical algebra to modern algebraic shorthand:

Regiomontanus, A.D. 1464:

3 Census et 6 demptis 5 rebus acquatur zero.

Pacioli, A.D. 1494:

3 Census p 6 de 5 rebus ae 0.

Vieta, A.D. 1591:

3 in A quad – 5 in A plano + 6 aequatur 0.

Stevinus, A.D. 1585:

$3 \, (2) - 5 \, (1) + 6 \, (\cdot) = 0.$

Descartes, A.D. 1637:

$3x^2 - 5x + 6 = 0.$

Algebraic notation is a relatively recent invention. Hogben (1945) noted that the simple and consistent rules for using abstract numbers and the shorthand symbols for mathematical verbs and operators evolved very slowly. He suggested that this is because of the individualistic manner in which each mathematician used a shorthand which only he himself understood. When a mathematician attempted to explain his methods to other people, he had to resort to everyday language. Until the 16th century reckoning processes were presented either verbally or in a mixture of words and symbols. The ancient mathematicians, however, usually explained their computational methods through concrete numerical examples rather than by universal rules.

Using the rhetorical and syncopated expressions places a considerable burden on the working memory and is more cumbersome and less effective than the modern symbolic approach employed. Sfard considers this to be one of the reasons why learners often revert to algebraic symbolism once they have been introduced to it. She also points to several research studies that showed that learners may do better with verbal than with symbolic methods even if they have had several years of symbolic algebra behind them. (Sfard 1995:21). Furthermore, current studies on visualization showed that making use of graphical representations to support explanations might be purposeful in abstract algebra.

Only during the 16th century were letters employed in a manner that made a real difference to algebraic manipulations. A French mathematician, François Viète (1540 – 1602), was the first to replace numbers with symbols. He was the inventor of parametric equations i.e. equations with literal coefficients. Previous mathematicians used letters in algebra to symbolise the unknown quantities that were being sought. Viète now denoted these unknown quantities by vowels and those numbers that were assumed to be known and provided he represented by consonants. Viète considered arithmetic to be the science of concrete numbers and algebra the science of types of things rather than the things themselves. In algebra symbolically represented equations now became objects of investigation in their own right and the purely mechanical method of solving problems by reverse calculations was replaced by formal manipulations of given formulae. Manipulating equations with literal coefficients is seen as conceptually more advanced than using equations with numerical coefficients. Sfard (1995:26) recounts an anecdote revealing how her learners found working with parametric equations considerably more challenging.

When solving equations in a rhetorical way, reversing computation processes or undoing what was done to the unknown was used. Sfard points to evidence that has been collected that shows that learners experience difficulty when the transition from such a working backward technique to the method involving making use of inverse operations on both sides of an equation are encouraged. Here learners are asked to reason in terms of the forward operations that represent the structure of the problem rather than in terms of the reverse processes of computation.

The exact meaning of a variable cannot easily be explained through a rigorous definition and may well be one of the most problematic concepts in the whole of mathematics. Some mathematicians just consider a variable to be something that changes. However through persistent usage of variables an exact definition became unnecessary.

Until the 1800s algebra had been regarded as "universal arithmetic", but, the broadening of the scope of the concept of algebra loosened it from restraints on its meaning. With George Peacock's notion (1791 – 1858) of the "principle of permanence" the concept of algebraically equivalent algebraic expressions was developed. According to Sfard (1995:28) this principle may be formulated as follows: "If a number does not obey a law, the number rather than the law would be the one to go." Hence a variable should not be seen as a generalized number but must be treated as an object in it own right. Variables are thus just symbols that can be manipulated, but that denote nothing physical. This "dearithmetization" of algebra brought about its full reification.

Sfard's research, which attempted to find out learners' implicit beliefs about the meaning of symbolic formulae and manipulations, led her to advocate courses which take the historical facts into consideration and compromise the modern definitions for the sake of an easier more accessible operational approach.

Sfard points out that Hamilton's invention of quaternions in the 1850s brought about the development of modern algebra. Another milestone in the history of abstract algebra was the emergence of the concept of a group suggested by Joseph Louis Lagrange (1736 – 1813) and Paolo Ruffini (1765 – 1822). Austin Louis Cauchy (1789 – 1857) later made steps toward reification of the process of rearranging a sequence of entities. Evariste Galois (1811 – 1832) was the mathematician who eventually defined the notion of a group but Arthur Cayley (1821 – 1895) ultimately shifted the emphasis from the manipulated entities to the operations themselves. After the development of the concept of group, algebra became a science of abstract structures. Sfard (1995:33) describes the development of mathematics as not "being the servant of natural science and from then on [it] was developed for its own sake." In the future, Sfard sees the computer as a powerful means of providing mathematicians with further means of reification. This machine will undoubtedly allow for the evolvement of even more theoretical mathematical structures.

The idea of a function is one of the important as well as basic ideas of mathematics. The historical development of the idea of a function therefore deserves mention. The concept of a function has also gradually developed over the years. Shuard and Neill (1977:18) give the following list of definitions to show how mathematicians groped with developing a precise definition of a function:

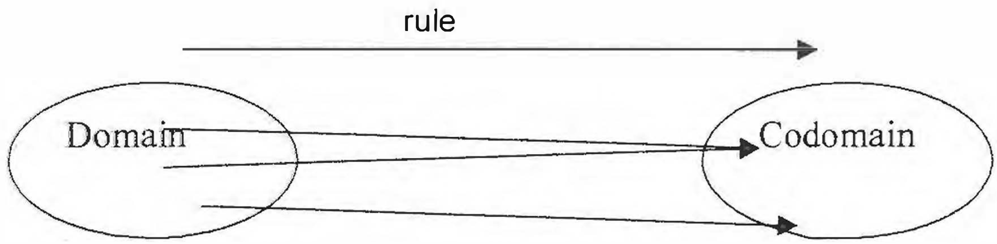
> A quantity composed in any manner of a variable and any constants. (Jean Bernoulli, 1718)
> Any analytic expression whatsoever made up from that variable quantity and from numbers or constant quantities. (Euler, 1748)
> Quantities dependent on others, such that as the second change, so do the first, are said to be functions. (Euler)
> If a variable y is related to a variable x, so that whenever a numerical value is assigned to x there is a rule according to which a *unique* value of y is determined, then y is said to be a function of the independent variable x (Dirichlet, 1837)

According to Shuard and Neill these early definitions strive to express in words the idea of the dependence of one quantity on another, together with the idea that the second quantity is *uniquely* determined from the first by some *rule*. Elementary modern treatments of the concept of a function describe three constituent parts. Shuard and Neill (1977:18) define and represent these parts of a function as follows:

(i) A starting set, called the *domain* of the function, whose members are the admissible values of x;

(ii) A target set, called the *codomain* of the function, a single member of which is attached to each x;

(iii) A set of arrows or a *rule*, to show which member of the codomain depends on each member of the domain.



This view of a function as having three parts, a domain (input), a codomain (output, range) and a rule is considered to be helpful to learners in understanding the power of the function concept, and how functions are used in modelling real situations. When the rule linking sets of numbers representing the domain and codomain is sought, then the modelling aspect of a function is used. When, however, elements of the domain together with the rule are used to obtain the codomain elements then "substitution" is said to occur. The "working backwards" from the codomain using the rule to obtain the domain is the process required when equations are "solved".

This development of the history of algebra reflects how some of the abstract processes have emerged. Learners studying algebra may have to trace this path for themselves and what mathematicians found challenging will almost certainly prove to be difficult for learners. Sfard (1995:34) emphasises that those who teach must be familiar with the history of mathematics in order to understand the problems that learners experience with concepts such as a variable. What may appear to be trivial to the teacher may be difficult for the learner who has a different ontological perspective. She suggests that learners should even work with algebraic techniques and manipulate abstract objects even if the learner has doubts as to their meaning. Sfard cautions teachers to be patient with gaps in learners' understanding as the concepts "will eventually become easier to reify – and to accept."

2.5.2. A phenomenological and pedagogical analysis of elementary algebra

> Maths was a total bore. I dropped it in standard 7 cause like alphabets and numbers going together doesn't really excite me.

Comment about school algebra made by a first year Intermediate phase student teacher in one of researcher's tutorials in January 1999

An aspect of vital importance in the mathematics classroom is the kind of mathematical understanding promoted. Skemp (1976) distinguished two types of understanding, namely "relational" and "instrumental". The kind of learning which leads to instrumental understanding of mathematics consists of the learning of a large number of fixed "recipes" which learners use on data to answer the questions. The step by step procedure does not necessarily allow for awareness of the overall relationships between successive stages and the final outcome. Here the order of the procedure is of prime importance in a kind of "flow chart" progression. The learner becomes dependent on his/her memory and not on making sense of the situation. In contrast, relational understanding in mathematics consists of developing a conceptual structure from which a learner is able to select appropriate strategies. According to De Villiers (1999), there needs to be discussions involving aspects of algebraic thinking before learners are merely told or shown how to manipulate symbols. In order to solve algebraic equations there should be some clarity about:

- how the balancing of equations maintains equivalence
- the meaning of the variable
- the meaning of algebraic expressions
- the manner in which inverse operations "undo" each other
- the meaning of the solution(s)
- the meaning of equation in terms of a function.

After learners are allowed to become familiar with such investigations they may be able to manipulate mathematical content fluently and may, when the need arises, be able to consciously reflect on legitimate meanings of algebraic symbols. Learners would then not be deprived of understanding the meaning and logical relationships between the mathematical content.

In a "chalk and talk", teacher-centred learning environment, where the solution to a linear equation is presented in a step by step, procedural manner the learners have little choice of strategy to employ. Only instrumental understanding is guaranteed when the teacher merely demonstrates algorithms. With this lack of insight into how the various steps are logically related, together with a limited amount of opportunity to try a variety of practice examples, relational mathematical understanding would certainly be hampered.

Furthermore, according to De Villiers (1999), learners with instrumental understanding may not understand the function or the purpose of the mathematical content. To promote relational understanding and allow learners to see the interconnections that exist in the mathematical content, learners should be encouraged to make use of mechanical methods such as:

- guess and check
- tables
- graphs
- "function machines" and inverse operations
- iteration or numerical methods
- manipulation practice
- solving word problems.

Being able to do an algorithm does not necessarily ensure that concepts, propositions, symbolic representation and mathematical processes are understood. De Villiers suggests that mathematics teaching should begin with promoting functional understanding, followed by relational understanding and lastly instrumental understanding. Bazzini (1999:263) considers that a "passage from natural to symbolic language is a key point in the development of algebraic thinking..."

Fey (1992:37) questions the assumption behind design of curricular sequences in algebra. He argues that the capabilities of current and projected computing technology challenge the assumption that technical skills must be mastered before applications and problem solving can be tackled. The algebra learning programme should focus on the processes of expressing and interpreting quantitative relations in symbolic form rather than concentrate on symbol manipulation.

In a report prepared by The Royal Society and Joint Mathematical Council (JMC) of the United Kingdom, under the chair of Sutherland (1997:13), the following diagram is presented as an overview of the essential elements of a pre-16 algebra curriculum and the interrelationship between these elements.

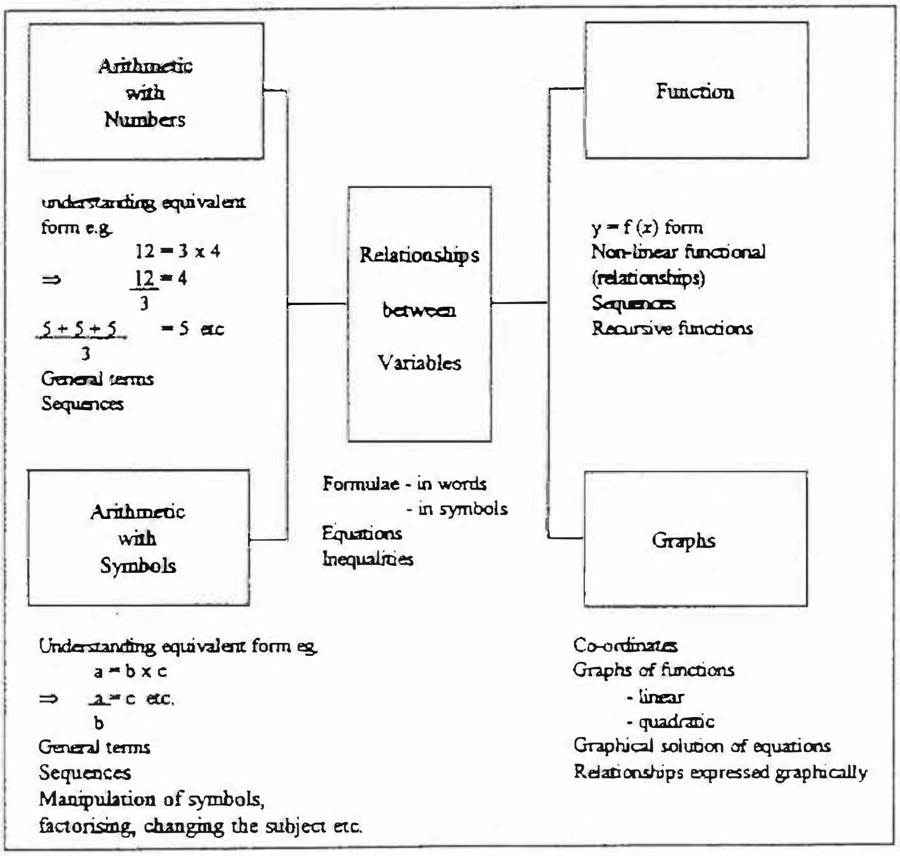


Figure 1 *Diagrammatic view of algebra for pre-16-year-old learners as suggested by The Royal Society/JMC report, "Teaching and Learning Algebra pre-19" (1997:13)*

Brown (1999:154) notes that algebra should arise from complex situations. She sees the need for algebra as a means of providing learners with empowerment and it would evolve as a kind of language that emerges from situations and contexts that are already laden with meaning. Brown (1999:155) uses the following definition of algebraic activity that is also used by The Royal Society/JMC report (1997:12):

(i) Generational activities which involve: generalizing from arithmetic, from patterns and sequences, generating symbolic expressions and equations which represent quantitative situations, generating expressions of the rules governing numerical relationships.

(ii) Transformational activities which involve: manipulating and simplifying algebraic expressions to include collecting like terms factorizing, working with inverse operations, solving equations and inequalities with an emphasis on the notion of equations as independent 'objects' which could themselves be manipulated, working with the unknown, shifting between different representations of, function, including tabular, graphical and symbolic.

(iii) Global, meta-level activities which involve: awareness of mathematical structure, awareness of constraints of the problem situation, anticipation and working backward, problem-solving, explaining and justifying.

The Mathematics Learning and Teaching Initiative (MALATI) project (2000:7) use experimentation with numbers through the vehicles of generalisation, structure and statements about numbers and operations to develop an understanding of the letter as a number from the perspective of known and unknown and variable. This project suggests using experimentation within the context of number to develop simultaneously the notion of function by making use of:

1. Generalisations of number patterns expressed in different ways, for example, in tables or pictures
2. Algebraic language
3. Structure of algebraic expressions
4. Transformation (manipulation) of algebraic expressions.

MALATI's pedagogical approach on "structures" within the context of numerical expressions is based on the recommendations of Sfard and Linchevski (1994b) who recommend that the development of algebraic concepts should be from an "operational (process-oriented) conception to a structural conception".

In MALATI 's rationale for school algebra (2000:13) it points out that:

> Algebra is a language and a tool to study the nature of the relationship between specific variables in a situation. The power of Algebra is that it provides us with *models* to describe and analyse such situations and that it provides us with the analytical tools to obtain additional, unknown information about the situation.

The following diagram is given as a summary of how MALATI (2000:15) consider the relationship between the "Problem Situation", "Mathematical Model", "Equivalent Mathematical Model" and "More Information of Model" interact.

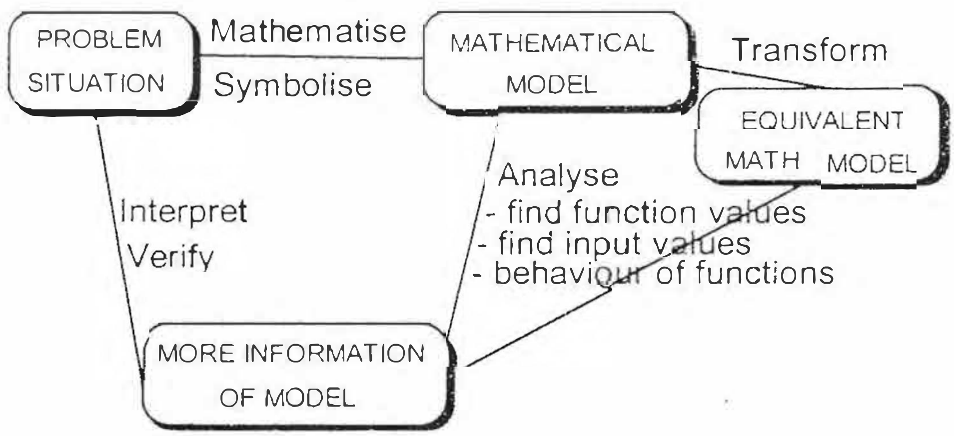


Figure 2 *Diagrammatic view of how learning of Algebra should be developed as suggested by MALATI (2000:15)*

Da Silva & Baldino (1999:329) argue that the discontinuity between arithmetic and algebra is radical and the use of words such as "cut", "gap', "dichotomy" or "duality" are not enough to explain the difference. Furthermore, according to Linchevski & Herscovics (1996) (in Da Silva & Baldino, 1999:329) "attempts to teach algebra starting from arithmetic lead to difficulties, if not to impossibility". Da Silva & Baldino (1999:333) believe that if the aim is to enable students to think algebraically then it is best to "start by assigning them typical tasks of the algebraic domain." They go on to suggest a "manipulative-computerized puzzle" to solve linear systems of two equations in two unknowns to teach introductory algebra courses.

Sfard (1994a:286) uses the term "reification" to denote the switch in a pupil's conception that is necessary to turn a process into an object. From examination of much research gleaned from the literature, she has become convinced that "reification is inherently difficult and that many students never develop a fully-blown structural conception of the most important mathematical concepts taught at schools, the concept of function being probably the most problematic of all." If this were the case, then the activities suggested by Sutherland would appear to be difficult for learners to master. Sfard (1994a:306) points out that "For the majority of pupils, it seems, an equation and inequation are meaningless strings of symbols to which certain well-defined procedures are routinely applied." Sfard (1995:34) studied the history of algebra and describes it as a "long sequence of acts of creation" where increasing abstractness has been brought into existence. According to Sfard, "Students who learn algebra have to recreate these objects for themselves."

In an article entitled "Let's not teach algebra to eighth graders!" published in 1985, Prevost (1985:587) concludes, after he conducted an "exhaustive" study in New Hamsphire, that "only about half of the students who take algebra as eighth graders continue their study of mathematics through a fifth year." He used anecdotal evidence he obtained during this study that indicated that learners who do better at other academic subjects often stop their study of mathematics because of their comparatively lower levels of achievement at mathematics. He suggests that instead of teaching algebra, teachers should rather design challenging, engaging alternative pre-algebra courses. He stated emphatically; "*You don't have to teach algebra to eighth graders!*" (1985:587).

Other authors reported that the use of spreadsheets in the early stages of learning algebra is beneficial. Friedlander (1999:344) notes that "The observation of the cognitive processes involved in solving an algebraic problem with Excel, showed some significant advantages for using spreadsheets in algebra." Friedlander points out that other authors, such as Sutherland and Rojano (in Friedlander, 1999:337), found that processes such as naming a variable, representing and testing mathematical relationships, generalising from arithmetic and extending informal arithmetic strategies, are facilitated by work with spreadsheets. A number in a cell can have several meanings. Sutherland & Balacheff (1999:22) see that it could be "a specified number or a cell representing a general number, or a cell representing an unknown number or a cell representing a relationship between numbers." Ainley (1999) also reckons that the spreadsheet offered an environment with interesting algebraic opportunities. She emphasises the fact that the spreadsheet provided a strong visual image of the cell as a "container" for a number.

Sutherland (1999:182) is of the opinion that "Computer-based environments can motivate young people to engage with challenging mathematical problems which they might otherwise avoid." Sutherland (1995:285) concludes that there are a number of possible reasons why computer environments support learners to develop an algebraic approach to problem solving in mathematics, for example:

> The most important is that pupils use the computer-based symbolic language to construct their own mathematical generalization, which derives from their previous experience of arithmetic. In addition the computer frees pupils from the process activity of evaluating an expression, thus enabling them to focus more on the structural aspects of the situation.

Trigueros and Ursini (1999:273) do not consider that students' difficulties with understanding a variable are of a cognitive or epistemological nature but that they are a consequence of current didactical approaches. As a variable is a multifaceted concept, these authors emphasise that a variable must be understood as an unknown, a general number and in functional relationships. If the learners do not understand its usefulness in "functional understandings", they are forced to memorise techniques.

2.5.3. "Misconceptions", errors and deficiencies in elementary algebra

Olivier (1992:196) considers misconceptions to be crucially important in the light of constructivist theory. Misconceptions are deemed to form part of a learner's conceptual structure that will interact with new concepts and influence new learning negatively as "misconceptions *generate* errors". He also distinguished between slips, errors and misconceptions. Olivier (1992:197) describes the differences:

> Slips are wrong answers due to *processing*; they are not systematic, but are sporadically carelessly made by both experts and novices; they are easily detected and are spontaneously corrected. ... Errors are wrong answers due to *planning*; they are systematic in that they are applied regularly in the same circumstances. Errors are *symptoms* of the underlying conceptual structures that are the *cause* of errors. It is these underlying beliefs and principles in the cognitive structure that are the cause of systematic conceptual errors that I shall call *misconceptions*.

Olivier (1992:207) states that "for the most part, children do not make mistakes because they are stupid – their mistakes are *rational* and *meaningful* efforts to cope with mathematics. These mistakes are derivations from what they have been taught and he calls for educators to show *"empathy"* with children who make errors and develop misconceptions.

Kuchemann (1981) identifies many limited conceptions of the concept of a variable or unknown or generalised number, namely:

- Letter evaluation. Here learners assign numerical values to letters at the outset of a problem. For example, when asked to describe the expression 3 + 2x children often assign a value to x, such as 1, and compute the answer. Thus 3 + 2x = 3 + 2 x 1 = 5.

- Letter not used. Learners ignore the letters, or acknowledge their existence but do not give them meaning. Learners tend to "conjoin" expressions. For example, the algebraic expression 3x + 4y is equated to 7xy.
- Letter used as objects. Learners regard the letter as shorthand for an object or as an object in its own right. For example, 3a + 2b represent adding 3 apples and 2 bananas.
- Letters used as a specific unknown or constant. Learners perceive the letter as a specific but unknown, fixed number. For example, the expression A + B + C would never equal A + D + C as B cannot equal D. B and D are acknowledged as unknowns but they must always be different values from each other as different letters are used to represent them.
- Letters used as a generalised number. Learners perceive the letter as representing several values rather than just one. For example, if learners are asked to say something about x in the equation x + y = 8 where x is less than y, they will list more than one of the whole numbers which will satisfy the condition instead of writing x < 4.

Furthermore, Kuchemann (1981:118) is of the opinion that:

> In algebra and in the other topics investigated, the research has found children frequently tackle mathematics problems with methods that have little or nothing to do with what has been taught. This may be because mathematics teaching is often seen as an initiation into rules and procedures which, though very powerful (and therefore attractive to teachers), are often seen by children as meaningless. It follows that children's methods and their levels of understanding need to be taken far more into account, however difficult this may be in practice.

In Warren's literature survey (1999:313), she found yet another interpretation of an unknown/variable/generalised number, namely:

- Assigning the letter as a subdivisional label. For example, 3a refers to the first part of the problem.

Warren (1999:314) also summarises misconceptions when examining expressions, namely:

- Closure. Some learners exhibit a need to have a single "answer". For example, x + y is conjoined to become xy.
- Equal sign. In arithmetic "=" tends to mean for learners "to compute", "makes", or "here comes the answer", or a place for the answer. Learners fail to recognise the equality relation between left and right hand side of the equation.

Olivier (1988) examined pupils' interpretation of literal symbols in elementary algebra; namely that different literal symbols necessarily represent different values. He states that the underlying causes for the misconceptions stem from the fact that the appropriate cognitive structures necessary for assimilation are not yet available to the students. Learners therefore have to be provided with problems in which experience interacts with their existing concepts and through "conflict", construction of this algebraic concept develops. Learners who viewed letters as objects had to make more errors before letters could be seen as representing a number of objects. These learners did not undergo conceptual changes by directly telling them about misconceptions.

Human (1989:32) gives a list of four prominent deficiencies in learners' understanding of manipulative algebra. These are summarised as:

1. Learners fail to simplify complex expressions when these are to be evaluated for different values of the variable(s).
2. There is widespread occurrence of conjoining (e.g. $5x^2 + 3x = 8x^3$). Human considers the use of conjoining to be related to the fact that learners do not recognise the essential nature and purpose of manipulation, i.e. to construct equivalent expressions.
3. There is often failure to distinguish between manipulated expressions and equations.

4. There is the tendency to interpret the = sign as a "do something" signal instead of a symbol indicating that two different expressions are equivalent.

Human (1989:33) ascribes these deficiencies to early algebraic manipulations (calculations) where there was little or no emphasis on meaning and utility of equivalent expressions. This induces learners to assimilate algebraic manipulation into a purely arithmetical framework that suppresses and even precludes the understanding needed to prevent or overcome the four deficiencies listed above. The characterisation of manipulative processes as "calculations" forces learners to interpret algebraic manipulations as arithmetic calculations where a single number is obtained at the end of a "calculation".

Human (1989:34) provides an alternative approach for initiation to algebraic manipulation by developing the following understandings/skills in the specific order listed below:

1. The ability to evaluate algebraic expressions for different rational values of the variable.
2. Awareness of the phenomenon that two different algebraic expressions may be equivalent.
3. Awareness of the utility of equivalent expressions in the sense that one expression may be easier to evaluate than its equivalent counterpart(s).
4. Some understanding of the relationships between equivalence of expressions and general properties of rational numbers.
5. The ability to construct expressions equivalent to given expressions by applying the following number properties:
   - the rearrangeability of additions and subtractions
   - the rearrangeability of multiplications
   - the distributive property.
6. Self-reliance in the recognition of possibilities to simplify algebraic expressions which are to be evaluated.

Sutherland (1999:182), states that:

> Symbolic algebra is not likely to evolve from classroom work that is pre-dominantly driven by pupil's own approaches to solving problems, because symbolic algebra is more like a language which can only be learned through exposure to people using this language. So teachers will have to find ways of using symbolic algebra themselves so that pupils can leam what symbol use in mathematics means. The 'old' way of doing this was to enter the classroom, demonstrate a set of problems on the blackboard, and then ask students to carry out more, similar problems (a 'drill and practice' approach). This method is so alienating for so many pupils, that it is an ineffective way of teaching.

Sutherland (1999:182) suggests that the most important job of the teacher is to devolve the responsibility for solving mathematics problems to learners. She indicated that perhaps if the teacher worked with the whole class, drawing on the learners' own awareness of mathematical structure, but at the same time transforming these perceptions through the use of algebraic language, there would be use of the algebraic language as a means of justification and proof within the context of mathematical activities.

Chapter 3: Methodology

3.1. This case study

The method of educational research employed in this project may be considered as a case study as the researcher observed the characteristics of an individual unit – one class of grade 8 learners and a specific teacher. The case study approach provided an opportunity for the researcher to study in some depth teacher strategies, learning outcomes and the relationship between the teacher's strategies and learners' performance. According to Cohen and Manion (1994; 106), using a case study allows for deep investigation and intensive analysis of the "multifarious phenomena that constitute the life cycle of the unit with a view to establishing generalizations about the wider population to which that unit belongs". Another strength of a case study approach is that it permits the researcher to focus on a specific example and this allows for attempts to identify the various interactive processes at work.

The researcher could, however, not undertake such an investigation without being influenced by her previous knowledge, skills, attitudes and values. What the researcher brought to the study was a myriad of experiences that influenced the manner in which observations were interpreted. Undoubtedly the years of experience at being a teacher educator influenced the positioning of the researcher in this investigation. To understand the nature of the teaching and learning that took place at Angaziwa High School the researcher's experience, reasoning and research were, however, at the researcher's disposal. These complementary and overlapping categories provided evidence where interpretations in this novel rural context were sought.

One of the most significant issues in a case study is the method of observation used by the researcher to gain knowledge of the manner in which learners learn and the means by which the teacher achieve her goals. Although there are two principal types of observation that may be employed, participant observation and non-participant observation, it would be difficult to label the method of observation procedure in this research as exclusively one or the other. The researcher did not engage in the classroom activities that were to be observed but at the same time was not a non-participant observer who stood aloof from the activities being investigated at Angaziwa High School. Not only was the researcher conspicuous but she was asked by the teacher to administer the test that was set by the teacher.

During observations the researcher sat close to the learners at the back of the classroom and concentrated mainly on what, why and how the teacher was teaching the mathematics. [The poem written by the researcher's daughter, Louise van Laren, aptly describes how the researcher felt being a "visitor" at Angaziwa High School. (See Appendix II.)]. The teacher was the central focus of the observation as the schedule and the audio recording used attempted to tap on the knowledge, skills and values displayed by the teacher.

3.2. <u>Data collected</u>

As much information as possible was collected during the two-week period from 17 August – 27 August 1999. On 24 August some teacher unions declared a stay-away; so no lesson was observed on that day. Everything that was said or written by the teacher as well as the learners was collected. Sources included:

- observation schedules that were completed during and after each lesson
- audio tape recordings of the teacher's oral exposition
- audio tape recordings of the learners' responses during lesson presentations
- hand written copies of the teacher's chalkboard work
- the learners' written responses to an algebra test set by the teacher and completed by on 20 August 1999
- the learners' class work and homework completed in an exercise book
- personal journal entries made after each trip to the school.

3.3. Limitations

During the observation period a representative from the Human Sciences Research Council (HSRC) made video recordings of the lessons taught on 18 August and 23 August 1999. Three attempts to obtain copies of these videos have failed. There has been no response to my faxes sent to the HSRC. These videos would have enhanced the qualitative observations made, but because during recording, the camera was aimed only at the teacher no further information about the learners would have been obtained.

In order to gain additional data about the teacher, a detailed questionnaire was compiled. (See Appendix III.) The researcher telephoned the teacher to ask whether she would be willing to complete it. Despite sending three copies of the questionnaire to the teacher, together with self-addressed envelopes, no response has to date been received. The teacher obviously did not want to complete the questionnaire. On 13 July 2000 the teacher was telephoned to ask for a telephonic interview. An appointment was set up for 09h00 on 14 July 2000, but when the researcher telephoned, the teacher had "gone to visit her sister in hospital". The researcher did not pursue this matter, as the teacher probably did not want to discuss further details with the researcher.

Unfortunately it was not possible to interview the learners. Having interviews with each learner, or at least a sample of the learners, would have provided meaningful insight into how the learners understood the concepts taught by the teacher.

The only information that was available to the researcher to gain knowledge of the manner in which learners responded to the teacher's strategies was the test scripts of the learners and the written work completed in their exercise books. Although the language usage by the teacher was carefully documented by means of a tape-recorder, the learners' reflections and meanings that they would have expressed though talk and discussion were not recorded; only the guided, chorused and mimicked responses of the whole class of learners could be accurately recorded for analysis.

There was no opportunity to make eye contact with the learners or interpret the emotions displayed by the learners. The research material used for investigation was based on some of the concepts and skills gained by the learners but the research instruments used could not tap values and attitudes gained. Much more interaction with the learners would have given insight into what mathematics was learnt. Without careful examination of learner reflection, the researcher examined the product of the learning instead of the processes of learning. In this research project too much emphasis was placed on the transmission and the medium of transmission at the expense of the learning process.

In comparison with other studies in the field of language usage and algebraic thinking, this research has limitations. The "voices" of the teacher and those of the learners are not clearly articulated. What the learners have written is examined in this analysis instead of consideration also being given to what they think. There should have been an attempt made to listen carefully to what the teacher and students had to say when asked explicitly about their perceptions about language usage and about the meaning of the learners' attempts at solutions to problems posed by the teacher. Valuable information may have been gained from the learners, especially if they could have explained the methods/strategies they used to "solve" the test questions. Even if these learners are Zulu speaking, an interview situation would have highlighted their interpretations better than merely examining what they wrote. Perhaps an interpreter could have been employed to translate interviews with learners to try to ascertain how the learners were reasoning. Interviews could have tapped into significant knowledge, information, values, preferences, attitudes and beliefs. In this study too much reliance has been made on what the researcher presumes the learners were thinking and this is a serious drawback.

The methods employed in introducing learners to algebra prior to the observation period were not discussed with the teacher. It is not clear from the two weeks at Angaziwa what the background knowledge of the learners was. The only means of ascertaining what had been done in previous lessons/years on the topic was hearing the teacher's incidental comments during lessons that were usually in the vein of "as I told you before".

Although no research method can claim to extricate "the truth, the whole truth, and nothing but the truth", making use of an observation schedule prepared by another person possibly implanted preconceived notions that inevitably biased some of the researcher's conclusions. For example, after each of the nine criteria that deals with the teacher's instructional practices, five graded responses are offered. These five options are not completely distinct but Cheryl Reeves had obviously decided what the appropriate approach was and rated this possibility as "best" response.

The design of the observation schedule was an attempt to classify human behaviour and thought. The person doing the research is made into an observer set on discovering general laws governing human behaviour. This schedule was an attempt at "straight jacketing" a human into being an observer who regards the person/researcher as devoid of subjective human qualities.

The response mode in Cheryl Reeves' observation schedule comprised fill-in as well as ranking modes. Tuckman (1972, in Cohen and Manion, 1994:285) pointed to the fact that these modes are difficult to score and difficult to complete. The researcher found this to be the case as it was impossible to memorise the entire nine-page schedule. Prior to each observed lesson the densely packed schedule was re-read and, in order to be as objective as possible, the researcher did not refer to what was recorded in the observation schedule for previous lessons. This proved to be time consuming and often the schedule had to be completed after the observed lesson. The completion of the observation schedules was, however, done immediately after observation in order to obtain an accurate reflection of the information obtained

For triangulation purposes in social studies where human behaviour is of concern, at least two different methods of data collection should be used. This case study would have been well served by including the two video recordings that were made by the HSRC at Angaziwa High School. These recordings were not made available to the researcher. The information gained from the data collected would have been enhanced if the camera had focused on the behaviour of the teacher as well as the learners. In the data analysis, however, the researcher did make use of both hand-written comments as well as audio tape recordings of lessons. These two methods of collecting data did provide a measure of confidence in the data analysed.

The test that was set by the teacher played a pivotal role in this study. This test was marked by the researcher and used in various sections of the data analysis. If this test had been marked by the teacher, additional insights into important aspects of teaching and learning would have been gained. This would have allowed for further integration and/or contrasting of the mathematics learning outcomes and teacher strategies. Different "actors" in assessment, a vital part of teaching, may have brought different meanings and also resulted in a richer research experience.

According to Adelman *et al* (1980, in Cohen and Manion, 1994;123) case study data is difficult to organise so the collection and subsequently the analysis of the data was set out in a rather fragmented manner that stays close to the categories provided in the observation schedule. Cheryl Reeves' observation schedule purports to make use of both quantitative and qualitative data but to quantify and describe any study of human behaviour such as teaching strategies remains a mammoth challenge. Making use of both types of data allows for a deeper study in the analysis of the complex problems of teaching, learning and human interaction in a classroom. Cohen and Manion (1994;27) suggest that interpretation of qualitative data relies on making use of "ourselves as a key to understanding of others and, conversely, our understanding of others as a way of finding out about ourselves." This appears to be a relatively easy task, but humans are often unsure of themselves let alone of interpreting the actions of others.

Chapter 4: Teaching Strategies used by Teacher

What teaching strategies were used by the teacher to engage the class of Grade 8 learners with mathematics concepts and processes at Angaziwa High School in Kwa Zulu-Natal?

4.1. Introduction

An attempt was made to gain as much qualitative information as possible about the teaching environment at Angaziwa High School. Here Ms Fundisi's instructional approaches as well as how she went about engaging her learners in learning mathematics concepts and processes were observed. Areas that were studied included surveying the constant physical school/classroom constraints, classroom interactions, activities learners did in their lessons and Ms Fundisi's approach to assessment. The teacher's perception of the learning situation was also an important aspect and her awareness of the individuals in the classroom was also considered. The teacher's method of lesson planning and how she saw herself in terms of being a specialist mathematics teacher were also explored.

An observation schedule devised for the HSRC by Cheryl Reeves was used to address these areas. (See Appendix IV.) This schedule not only required the ticking off of given appropriate options but also required written selection and completion of relevant choices. On this schedule the following sub-questions were posed and answered for each lesson observed:

- What were the length of lessons, number of learners, classroom conditions and lesson topic?
- Did the teacher make the mathematics concepts and processes to be learnt explicit?
- Did the teacher introduce learners to the new/additional language they needed in order to discuss and think about the mathematics concepts or processes to be learnt?

- How did the teacher introduce mathematics concepts and demonstrate processes?
- Did the teacher demonstrate how the mathematics concepts or processes to be learnt work?
- Did the teacher assist learners to engage with and interpret written mathematical texts/representations related to the concepts or processes to be learnt?
- Did the teacher provide learners with opportunities to express their current understanding of the mathematics concepts or processes to be learnt?
- Did the teacher provide the learners with opportunities to revise mathematics concepts or practise processes to be learnt?
- Did the teacher encourage learners to discuss the mathematics concepts or processes to be learnt with each other?
- Did the teacher structure mathematics activities through which learners experiment with using the mathematics concepts and processes to solve problems?
- Did the teacher assess whether learners had learnt the mathematics concepts or processes?
- How many learners were absent from the lesson prior to the structured interview?
- What criteria were used for grouping the learners?
- What was the purpose or goal of the lesson prior to the structured interview?
- What information was used to plan for and during the lesson?
- Were there any adverse factors affecting the lesson?
- Did the teacher enjoy teaching mathematics and did she consider herself to be a mathematics subject specialist?
- What were the teacher's qualifications, teaching experience and ambitions for the future?

The sources of information required for completion of the schedule were obtained from:

- two structured teacher interviews that took place on 18 August and on 23 August 1999
- informal discussions with the teacher
- observations recorded on the schedule during each lesson
- reflections recorded after each lesson
- additional qualitative, personal journal entries made after each trip to the school.

Each day, care was taken not to refer back to previous schedules so that each day was observed without bias. A trial run of the schedule was also made on 12 August 1999 in order to become familiar with the types of questions posed. A student teacher consented to my use of the observation schedule at a lesson taught by him whilst he was doing his stint of practice teaching at a secondary school at Kwandengezi, Kwa Zulu-Natal.

In order to focus on the lessons pertaining to the teaching of algebra, concentration was on the first four lesson observations. Each day a mathematics lesson in a Grade 8 class that was taught by Ms Fundisi was observed.

4.2. Analysis of data

4.2.1. Establishing the lesson context

What were the length of lessons, number of learners, classroom conditions and lesson topic?

4.2.1.1. The learning environment

The classroom in which Ms Fundisi was to teach mathematics to a class of Grade 8 learners was bright but without electricity. The handles of the classroom door had been removed. At the back of the classroom there was an unused notice board. An empty steel cabinet was adjacent to the classroom door but the doors of the cabinet had been dismantled at its hinges. The chalkboard was in good condition. The teacher did not have a special teacher's desk but used an empty learner's desk on which to place her teaching notes. There was sufficient seating for the 38 – 51 learners who attended the mathematics lessons. The learners were permitted to sit wherever they pleased. Throughout the two-week observation period some of the twenty-five boys and twenty-four girls attended lessons. The temperature in the classroom was comfortable and there was adequate ventilation. The only noise heard was that of the odd vehicle that passed on the gravel road outside the school fence that was approximately twenty metres from the school building. The school bell was used only occasionally. The learners did not move from their classroom as the teachers rotated according to the timetable posted in the staffroom.

4.2.1.2. Classroom organisation

The learners were seated in pairs at two-seater desks. All the learners were seated facing the teacher. There was ample space between the columns of desks to allow for free movement of the teacher and learners between desks.

4.2.1.3. Lesson topics

During this two-week period two different topics were addressed. During the first week solutions of linear equations using word problems as well as manipulation of algebraic equations were tackled. In the second week angles related to triangles and parallel lines were covered. In order to concentrate on the algebra taught, the lessons dealing with geometry are not included in this report.

4.2.1.4. Lesson structure

The introductory lesson on solution of algebraic equations dealt with the terminology, the balance algorithm and use of inverse operations to isolate the unknown. The second and third lessons involved formulation of algebraic equations from word problems and their solution. Three 35-minute periods were devoted to linear equations. The learners were tested during the fourth lesson.

During lesson 3 the learners were told of the test that was to occur the following day. It appeared that the topic was a further development/extension of work done previously as the teacher often reminded the learners of what had been said before. For example, in lesson 3 the teacher said: "We did this rule of signs at the beginning of the year."

The number of minutes spent on whole class teaching of linear equations was 20 minutes for each of the three lessons. Thereafter learners were to work alone with "no noise" for 15 minutes. The teacher used all of the lesson time for teaching/instruction and delivered presentations at a brisk, lively pace. The teacher determined the pace at which the whole class learning/teaching took place.

4.2.1.5. Organisation and use of textbooks/technology and other material resources

Only one textbook was used during all the lessons. The teacher had only one copy of the textbook. The title of her textbook was *Mathematics in Action 7, New Syllabus* by Fletcher, Fletcher and Roos (1986). Chapter 6 dealt with solution of linear equations. No photocopies of the chapters could be made at Angaziwa High School, as there are no photocopying facilities available at the school. No use was made of worksheets.

Extensive use was made of the chalkboard. On the whole, the chalkboard work was accurate. The board was systematically sectioned and cleared, neat chalkboard work was well-displayed. Throughout the lesson presentation, step-by-step details of the topic development were written up. Meanings of terms were written up, for example in lesson 1: "unknown - don't know" and "coefficient (next to variable)" In lesson 3, for example, this was recorded to show learners what section was being taught:

1 unknown *Problems leading to equations*

$x + 5 = 21$

$2x + 4 = 60$

The teacher went on, using the chalkboard to demonstrate using graded problems, the exact expanded procedure to be followed:

$2x = 7 + x$ *Collect like terms*

$2x - x = 7 + x - x$ *Change additive inverses*

$x = 7$

The teacher reminded the learners of the necessity to balance the equation by saying:

> *We are going to put minus x again on this side because what we do on this side we must also do on the other side.*

No other materials/resources/apparatus, other than the chalkboard, were used to demonstrate to the whole class. Calculators were not mentioned in any lesson, but during the test written on 20 August 1999, some learners did use their own calculators. Calculators were owned by nine of the 38 learners who sat for the test. No computers could be operated in the entire school, as there was no electricity or telephone. The researcher provided exercise books for the two weeks. All the learners did, however, appear to possess their own pencils/pens.

4.2.1.6. Organisation of the task/activities

Usually the learners worked together as a class with the teacher assisting the whole class. The whole class was often asked to chorus steps required to solve problems whilst the teacher was at the chalkboard. When the learners were allowed to attempt solutions individually, they were told to “Write first step and raise up your hand” when finished. The teacher then checked the step and clearly stated whether the attempt was correct or incorrect. The teacher would use “good”, “very good”, “wrong”, “very wrong” or “very, very wrong”, as feedback without entering into a discussion with the individual learners.

4.2.1.7. Language(s) of learning and teaching

The teacher did not code switch during any of the observed lesson presentations. Only English was used as an instruction medium. When asked, the teacher confirmed that only English was spoken during mathematics lessons. The teacher did, however, speak to the learners in isiZulu about matters not directly related to mathematics, for example, at the commencement of a lesson. The learners did not take down the notes that were made on the chalkboard during lesson presentations. During presentations the learners concentrated on and interacted only with the educator. Usually the learners communicated with the teacher using one or a few words, i.e. not in complete sentences. Only work allocated as "classwork" or "homework" was written down in mathematics notation. There were, however, word problems written and solved using a combination of mathematics and English. For example, in lesson 2 this was the problem given for "homework": "Share 27 buns between two boys so that one gets 3 more (*than*) the other".

No full sentences were written in English. Interactions were limited to English in teacher-learner interactions. Very little learner-learner interaction was allowed, but when this took place it was in the vernacular.

4.2.1.8. Learner participation and involvement

Although the learners were mostly observing the teacher and responding to the teacher, they were all actively following the lesson and able to contribute by chorusing and answering of questions posed. The learners did not seem to be perturbed about the researcher's presence in the classroom and remained focused on the educator throughout lesson presentations. Hence they were frequently listening to the teacher, observing the chalkboard work and responding to teacher instructions. The learners never copied down teacher's notes, read any mathematical text on their own, discussed ideas with their peers or wrote their own notes. One test on solving linear equations was written during lesson 4. This test was not marked or returned by the time the researcher left. The homework was discussed and corrected only during Lesson 3.

During the observation period classwork/homework was never assessed to generate marks. The learners were occasionally encouraged to ask questions, for example, "Any questions?" "Confused or tired?" To this the learners responded, "Yes, confused". They were often asked whether or not they understood, for example, "Hands up if you don't see this statement". The learners were not afraid to say if they were unsure of the work. The learners did not, however, indicate specific concepts that they were unsure of. They just chorused "Yes" or "No".

4.2.1.9. Assessment

During the lessons the learners were assessed by their oral responses to questions. The teacher told the learners directly if they were wrong, but remediation was usually handled as a whole class discussion accompanied by a chalkboard solution. There was one written test on solutions of linear equations. The researcher was asked to write the test up on the board for the learners to complete. The researcher also invigilated this test. The teacher inquired whether the researcher wanted to mark the test, but the researcher declined the offer. The researcher did, however, photocopy the learners' scripts so that the learners' responses could be analysed at a later stage. The test written in lesson 4 consisted of:

Question 1

Solve these equations by writing all the steps:

(a) $x + 20 = 36$

(b) $x - 9 = 1$

(c) $2x = 10$

(d) $3x + 7 = 25$

(e) *A man owns 48 sheep. How many more does he need to have 96 sheep?*

<u>Question 2</u>

(a) Give the additive inverses of the following:

1. +4 *2. -3* *3. -2*

(b) Find three consecutive natural numbers whose sum is 42.

The learners completed this test in 35 minutes. The learners' performance and the level of understanding indicated by their responses to this test will be analysed in chapter 5.

4.2.2. Teacher's instructional practices

4.2.2.1. Explicitness of mathematics concepts and processes

Did the teacher make the mathematics concepts and processes to be learnt explicit?

Throughout the observation period, the learners were explicitly told exactly how to go about solving linear equations. The learners were not given the opportunity to devise their own methods of solution. The purpose or reasons for learning the processes were not spelled out, but there were problems linking solution of linear equations to "real-life" word problems. For example, in lesson 2 the teacher posed the following problem:

> *Nomsa owns 56 chickens. How many more chickens must she buy to Have 100 chickens altogether?*

The learners were showed how to link related familiar mathematics concepts and processes to the new concepts and processes. For example, in lesson 1 the teacher made a chalkboard summary where a numerical example using a "Place holder" was used to introduce the concept of an "Unknown" in an equation. The teacher used the numerical example $\square + 3 = 9$ to try to explain the understanding of a letter symbol. The teacher emphasized that any one of the four operations could be involved in the equation. This was written on the chalkboard:

Place holder	*□ + 3 = 9*	*<u>Calculate</u>*
equal sign		*unknown - don't know*
addition		*x + 4 = 6 equation*
subtraction		
multiplication		
division		

New terms required were clearly displayed, explained by the teacher and chorused by the learners. The teacher focused on the terminology before developing the concepts informally. For example, in lesson 2 these words were written on the chalkboard and explained orally:

<u>New terms</u>

1. *additive inverses*
2. *multiplicative inverses*
3. *variables*
4. *co-efficients*

At the commencement of lesson 1 learners were introduced to the term "variable". The teacher said:

> *So in standard five, you've learnt about eh..place holders where you use a symbol for a place holder, eh.. equal signs, addition, subtraction - ...*

and went on to repeat

> *..So in standard five you have learnt about when we will use a place holder to get the number instead of the value, a place holder plus or a place base plus three equal to 9, where you were required to find the number.*

The example written on the chalkboard to illustrate this was "□ + 3 = 9". Later on the teacher added:

> *..But now we are in standard six we have to find or we have to calculate that value instead of the place holder. Right. What we are going to do is calculate. Let's say we are given an unknown, an unknown is a number*

which we don't know. Right. So let's say that we are given an unknown, x, right, plus four equals to six. So, we are trying to find the value of the unknown, the value of x.

The example provided on the chalkboard is "x + 4 = 6".

The teacher introduced the learners to the term "variable" using examples where a specific unknown is required.

Later on in Lesson 1 the teacher reminded the learners that an unknown is synonymous with a variable. The teacher did not explain that the meaning of x changes according to the context in which the x is being used. The learners were prompted as follows:

Note: "T" indicates what the teacher said; "L" indicates learners' chorused response; "..." indicates pause.

T: *...Two x plus four is equal to six. So here we see this is the unknown. You see. Look at the board please. This is the unknown, right? The unknown or variable, this is the...*

L: *Unknown.*

T: *This is the...*

L: *Variable.*

T: *This is the...*

T: *Variable.*

The teacher did not explain that 2x is a shorthand notation for "two multiplied by x". The teacher gave a definition of a "coefficient" in the following manner:

T: *And last time I told that a number next to a variable is called what? A coefficient . Do you remember this word? All of you say co...*

L: *efficient*

T: *Again.*

L: *Coefficient.*

T: *A coefficient is always next to a variable. So we are having a variable x and a coefficient two. Two is the coefficient and x is the variable.*

The learners were not reminded of the meaning of 2x but were told to make use of the multiplicative inverse to solve for x. The teacher emphasised the terminology before dealing with the concept. The learners were not given the opportunity to reflect on the problem before being told how to proceed. No opportunity was provided for the learners to make sense of the problem for themselves or to develop their own solutions. The purpose of the "multiplicative inverse" was developed as follows:

T: *So they say find the value of x not the value of 2x. Right, what are we going to do now? So now we are having 2x is equal to two. This is not the answer. You see this is not the answer, they say find the value of x not the value of two x. What are you going to do now? Right? I told you there is something called what ...*

L: *Additive inverse.*

T: *Read this word. Multiplicative inverse. Class...*

L: *Multiplicative inverse.*

T: *Again.*

L: *Multiplicative inverse.*

T: *Again.*

L: *Multiplicative inverse.*

T: *So, here in this case of a variable and a coefficient, you are going to use the multiplicative inverse so that you can get the value of the unknown. So here you are having the coefficient two and the variable x, so the multiplicative inverse of this two will be what? Two. So that number will divide that number and will be left with the variable. So that will mean the coefficient of x is 2. If you were given four x, what will be the multiplicative inverse of this one? Sorry... What will be the multiplicative inverse of four?*

.L: (incomprehensible)...............

T: *Of four? Here we are having two then we use two. So, of four will be?*

L: *Four.*

T: *Four. What will be the multiplicative, multiplicative inverse of 3x?*

L: *Three.*

T: *So if you are given eight x, we are going to divide by...*

L: *Eight.*

T: *If you are given ten x , you are going to divide by...*

L: *Ten.*

T: *Right. So let's divide by two. Two into two how many times...*

The approach used by Ms Fundisi did not include an explanation/demonstration of the importance of "balancing" the equation. The teacher did not afford the learners the opportunity to try and solve any problems by themselves, nor was this exercise linked to a "real world" situation to give it meaning.

4.2.2.2. Introduction of new/additional language

Did the teacher introduce learners to the new/additional language they needed in order to discuss and think about the mathematics concepts or processes to be learnt?

The teacher was meticulous about explaining new/additional language but the focus was on the terminology and not on the concept being conveyed. These second language learners were, however, just told the mathematical meaning of the terms. There was no link between the usual "English" meaning and the "mathematical" meaning. For example, the word "variable" may in everyday language was used as in "variable weather". The everyday usage of variable, meaning fluctuating/changeable/inconstant, was not linked to the meaning of the term "variable" in mathematics. There are examples of deliberate introduction to correct mathematical language in each lesson. For example in lesson 1, "additive inverses" were explained by way of examples on the chalkboard. The teacher used the following explanation:

Note: "T" indicates what the teacher said; "L" indicates learners' chorused response; "L1" indicates a learner's response; "..." indicates pause.

T: *...for example, what is positive one plus negative one? Hands up. What is the answer here? Yes? Yes?*

L1: *Zero.*

T: *Yes, Zero. What is negative four plus positive four? Class?*

L: *Zero.*

T: *Zero.. What is negative 100 plus positive 100? All of you it is..?*

L: *Zero.*

T: *Zero. So, let's say here, let's say here that given this four and that given this six. You want to remove this positive four before in order to get what, zero, you see what we call an additive inverse. We call what?*

L: *Additive inverse.*

T: *Additive inverse of for example of positive one is negative one. What goes with x and positive one and negative one is ... you get what?*

L: *Zero.*

T: *Which means that x is in the opposite one is negative one and again opposite one is ...*

L: *negative one.*

T: *because they give you what?*

L: *Zero.*

T: *So, if you use the additive inverse, you get what?... Zero.*

Chorusing of new jargon was frequently used to reinforce "correct" pronunciation. Eventually, however, learners will be expected to know how to solve linear equations and saying the terminology properly will not assist in the solution of problems. To develop relational understanding the learners would need to understand the mathematical concepts fully and this will not be achieved by repeatedly saying the terms. For example, in lesson 1 the word "variable" was repeated by the learners three times and "coefficient" twice.

4.2.2.3. Introduction of concepts and demonstration of processes

How did the teacher introduce mathematics concepts and demonstrate processes?

The teacher consistently used a systematic, logical development of each topic as was set out in the textbook she used. Emphasis was placed on instrumental understanding rather than on relational or conceptual understanding.

Below is the step by step explanation provided by the teacher (T) together with the chorusing by learners (L) extracted from Lesson 3 ("..." indicates that the teacher paused for the learners to continue the "sentence"):

T: *Let's try and solve eh, two or more unknowns, for example. Let's try this one, with more than one unknown. Two x equals to seven plus x. Speak all of you.*

L: *Two x equals to seven plus x.*

T: *Again.*

L: *Two x equals to seven plus x.*

T: *OK. First of all, I told you that the first step is to...*

L: *Collect like terms.*

T: *The first step is to...*

L: *Collect like terms.*

T: *All of you.*

L: *Collect like terms.*

T: *Again.*

L: *Collect like terms.*

T: *Are there any like terms here? Are there any like terms here, Six C?*

L: *Yes.*

T: *Speak aloud. Yes or no?*

L: *Yes.*

T: *So let's collect like terms. So we are having two x and we are having one x on the right hand side. Right? So, let us collect like terms. So, it will be two x.. and if we collect like terms this side. What does the signs do? The signs... What does the signs do? The signs...*

L: *Change.*

T: *The signs...*

L: *Change.*

T: *Right. By changing it means we use what?*

L: *Additive inverses.*

T: *We use what?*

L: *Additive inverses.*

T: *We use what?*

L: *Additive inverses.*

The teacher gave no explanation as to the purpose or value of solving equations, i.e. the teacher's explanations did not promote relational or functional understanding.

T: *So let's use those additive inverses. So, we are going.... want to bring this positive x....one x to this side. Right? We are going to use the additive inverses so let's bring it. Here we are having two x, here we are having seven plus x. We want to remove this plus x. So we are going to use* *minus x. We are going to put minus x again on this side because what we do on this side we must also do on the other side. Right? So, now we are having two x minus one x equals to seven plus x minus x. Which is? All of you.*

L: *Two x minus x equals to seven.*

T: *Two x minus one x is?*

L: *Equals to seven.*

T: *What is the answer? Two x minus one x, class?*

L: *Equals to seven.*

The teacher went on to show how one of the x's are eliminated by crossing off an x. The x's were treated as objects.

T: *Two x minus one x! Here I am having two x's minus one x.*
(On board: x, x)

L: *One x.*

T: *One x equals to...*

L: *Seven*

T: *So the value of x is*

L: *Seven.*

At this point, in this particular lesson, the teacher solved the equation 6x + 11 = 11x – 14 in a similar step by step manner and whilst explaining the procedure wrote on the chalkboard:

$$6x + 11 = 11x - 14$$

$$6x - 11x + 11 - 11 = 11x - 11x - 11 - 14$$

$$\frac{-5x}{5} = \frac{-25}{5}$$

Then the chalkboard was used for explanation/revision purposes by recording:

1 x 1 = 1 *- x + = -*

$1 \times 2 = 2$ $+ \times + = +$

$2 \times 2 = 4$ $- \times - = +$

The teacher used only the positive or negative signs to indicate what the sign of the products are. The teacher detached the signs and used only the signs to perform the operation. This is how the teacher reminded the class of the rules for multiplication of positive and negative "signs":

T: *..We know that a negative sign multiplied by a positive sign is...*

L: *Negative*

T: *And positive multiplied by a positive is...*

L: *Positive*

T: *Negative multiplied by a negative is...*

L: *Positive*

Thereafter the solution was completed by writing:

$$- x = -5$$
$$-1 \times -x = -1 \times -5$$
$$x = 5$$

After the teacher has explained the solution she asked the learners "Are you happy? Are you happy? Are you happy?" i.e. If they could replicate the procedure used to solve the equation $6x + 11 = 11x - 14$. The learners chorused "No" and the teacher patiently demonstrated the solution to $13x + 22 = 6x – 6$ using the same procedure. In the same lesson the teacher also showed the learners how to go about solving the equation $5x + 8 = 2x + 41$.

The learners were not given the opportunity to try any problems on their own; they were expected to be able to follow, repeat and memorise what was required after the teacher had illustrated the manipulation techniques.

At the close of the lesson, the chalkboard was used to provide a "Linear Equations class exercise $6x + 5 = 2x + 11$" to be solved by the learners on their own.

In lesson 2 the "Class exercise" was

Find 3 consecutive even numbers whose sum is 27

Learners were allowed to try this problem but were told to "Write up the first step and raise up your hand" and "...let's see after the first step, don't continue." The learners were expected to follow or copy the procedure the teacher had demonstrated. After the learners had experienced difficulties solving this problem the teacher developed the "solution" with the learners responding in unison and being prompted by the teacher.

The teacher often focused the learners' attention on the relationships between the new mathematics concepts and the mathematics representations by dealing with numerical examples to illustrate. For example, in lesson 2 the learners were reminded that "even numbers" were "2; 4; 6", but the link between using a variable x and representing consecutive even numbers as x, x + 2 and x + 4 was not made.

The chalkboard working displayed was

Let the 1st number be x

Let the 2nd number be x + 2

Let the 3rd number be x + 4

$$x + x + 2 + x + 4 = 27$$

$$3x + 6 = 27$$

$$3x + 6 - 6 = 27 - 6$$

$$3x = 21$$

$$\frac{3x}{3} = \frac{21}{3}$$

$$x = 7$$

The teacher did not realize that this answer was odd instead of even and that the problem therefore had no solution. Her focus was clearly only on instrumental procedure. If she had viewed the problem relationally she may have realised that no solution was possible since the sum of three even numbers has to be even.

4.2.2.4. Assistance provided for engagement with written text representations

Did the teacher assist learners to engage with and interpret written mathematical texts/representations related to the concepts or processes to be learnt?

The only text available for interaction and interpretation was the written text on the chalkboard. This interpretation was never on an individual sense-making basis by the learner, but through the medium of the teacher's spoken word. The teacher, however, made every effort to allow for whole class engagement using these chalkboard summaries/texts that were developed systematically throughout each lesson. No textbooks or worksheets were available for learners to develop their own interpretations. No provision was made for learners to observe and recognise emerging patterns for themselves.

The comprehension of the chalkboard text was regularly tested as the systematic usage of the chalkboard allowed learners to refer back to previously solved problems. The notes were not written down in their exercise books and were therefore not available to learners for further reference at a later stage. The new terminology was seen and repeated, but the learners did not have the opportunity to engage with the text at a personal level.

4.2.2.5. Provision of opportunities to express learners' current understandings

Did the teacher provide learners with opportunities to express their current understanding of the mathematics concepts or processes to be learnt?

The most common technique used by the educator to provide opportunities to express current understanding was questioning followed by chorus answering by the learners. The answers needed to be the correct/exact response i.e. the range of responses was limited to one and the teacher made use of only convergent questioning. During the algebra lessons, the teacher did not use the learners' own expressions of their understandings as tools for teaching, but rather the responses to the questions as guidelines for further teaching or repetition. The learners did not use full sentences to communicate their difficulties to the teacher.

Learners were always given opportunities to practice pronunciation of new words, learnt by chorusing the terms and seeing them written on the chalkboard. Ample provision for learners to express their uncertainties about the understanding of mathematical concepts was made. The learners willingly expressed their difficulties. The responses, however, were limited to "yes" or "no", without elaborating on explicit concerns.

Whenever topics were taught, the teacher made summaries of prior knowledge which required building on and moving beyond their new understandings of the mathematics concepts. For example, in lesson 3, the learners were reminded that "substituting" meant "put a number instead of x".

4.2.2.6. Provision made for learners to revise concepts or practice processes

Did the teacher provide the learners with opportunities to revise mathematics concepts or practise processes to be learnt?

During each lesson opportunities for practice were provided. The teacher first demonstrated the graded problems that were similar to those the learners were required to practise. This was in the form of whole class practice where the teacher, together with the learners, completed a chalkboard solution. Here the learners would be told "Class do together". At other times the teacher asked "Write the first step and raise up your hand". At most three problems were written up for homework.

4.2.2.7. Encouragement of discussion of mathematics concepts or processes

Did the teacher encourage learners to discuss the mathematics concepts or processes to be learnt with each other?

Without exception, this large class of learners (38 – 51) was not encouraged to discuss new mathematics concepts or processes with each other. The learners were always focused on the teacher and obediently followed instructions - even when doing work on their own they were urged not to chatter or "make a noise".

4.2.2.8. Activities provided to solve problems

Did the teacher structure mathematics activities through which learners experiment with using the mathematics concepts and processes to solve problems?

Most of the problems the learners solved could be described as routine/textbook type of examples. The exercises involved applying the step by step algorithm demonstrated by the teacher. Although the solutions to the problems were not immediately obvious to the learners, they were of a similar type. The only slightly "different" type of problem posed for homework at the close of lesson 1 was an example where the variable was on both sides of the equality i.e. $2x = 7 + x$. This single homework problem was unlike the types discussed in class. During class, examples were limited to equations where the variable was on only one side of the equality *i.e.* $2x + 4 = 6$, $x + 14 = 28$, $6x = 18$. This example would not, however, constitute a completely different "process" type problem.

4.2.2.9. Assessment of mathematics concepts, principles or strategies learnt

Did the teacher assess whether learners had learnt the mathematics concepts or processes?

The teacher's feedback to the learners was always accurate, as the answers required were limited to a single possibility. The teacher feedback did not involve remediation or in-depth discussions with individuals. The learners were not given much opportunity to state their personal insights/understandings so that there was no scope for extension of particular awareness to "push" their learning further. The correction of the test written in lesson 4 might have provided valuable insights into individual perceptions and interpretations but feedback from this test was not available to the learners whilst the researcher was at the school.

4.2.3. Teacher's responses to post-lesson interviews

4.2.3.1. Number of learners absent from the class

How many learners were absent from the lesson prior to the structured interview?

On both occasions when the teacher was asked how many learners were absent from her class, she reported that there were three absent. This was said without hesitation but these figures could not have been accurate as this would mean that there are 54 or 51 learners in Grade 8 C. When the researcher asked if she could have a copy of the class list there didn't appear to be one available. During the observed lessons there was never mention made of who was present/absent from the lesson. The learners were not asked who was away from school that day. Perhaps this was because the headmaster selected only some of the 135 grade 8 learners for the observation lessons. After the redeployment of the three other mathematics teachers, all these learners were being taught together by Ms Fundisi in the school hall. Sometimes learners were addressed by their names and at other times the teacher just pointed to the learner who was to respond to a question.

4.2.3.2. Criteria used for grouping the learners

What criteria were used for grouping the learners?

The learners were not grouped in any manner. The girls and boys were permitted to sit next to whomever they pleased. There were usually unoccupied desks at the front of the classroom. The learners were even permitted to sit three to a double desk during the test. The type of grouping arrangement used by the teacher is immaterial in this learning environment as the learners are not encouraged to communicate with each other during mathematics lessons.

4.2.3.3. Purpose or goals of lessons

What was the purpose or goal of the lesson prior to the structured interview?

Whilst conducting the interview, the teacher was asked what the purpose of the observed lesson was. The teacher decided that she wanted to complete this section of the form. This is what she wrote in the observation schedule:

18/8/1999 *I wanted the children to have quick minds and to let them use the correct calculations when they are calculating on their own.*

4.2.3.4. Resources mostly used for planning the lessons

What information was used to plan for and during the lessons?

The teacher indicated that the resource she used most for planning was "teacher guides or teacher edition of textbooks". The only textbook mentioned by the teacher was the *Mathematics in Action, New Syllabus* by Fletcher, Fletcher and Roos. This book was first published in 1986 and the edition used by the teacher was reprinted in 1995. The teacher used an exercise book to jot down rough "lesson plans". The teacher indicated that the document she mainly used to plan the grade 8 mathematics programme was the Departmental Mathematics syllabus for standard 6 (1995).

4.2.3.5. Adverse factors affecting the lessons

Were there any adverse factors affecting the lesson?

On the interview forms the teacher indicated that there were no adverse factors affecting the school, her, or the learners on that particular day. It was, nonetheless, obvious from my initial interview on 13 August 1999, that the redeployment of the other mathematics and science teachers had an overall impact on the number of lessons that were to be taught by the teacher. The teacher was now required to teach all the mathematics (grades 8 – 12) in the school as well as grade 11 and 12 science.

On 20 August 1999, the teacher was visibly unhappy as she was crying when I arrived at the school. She explained that this was because of a rude grade 12 learner. She did not elaborate on how the learner had upset her but on this day the researcher wrote up the test on the chalkboard for the learners to complete. The researcher also invigilated this test.

During the first week of my observation the teacher told me she was feeling ill but she came to school to teach because she knew I would be disappointed if she did not arrive. This showed real commitment to the research process and to her teaching.

When I made tape recordings of her lessons, the teacher organised the switching on and off of the machine and she did not appear to be unduly nervous about the video recording sessions. It looked as if the teacher looked forward to the HSRC video recording of lessons which took place on 18 August and 23 August 1999.

4.2.3.6. The teacher's attitude towards teaching mathematics and her confidence in her subject content

Did the teacher enjoy teaching mathematics and did she consider herself to be a mathematics subject specialist?

When the teacher was asked whether she enjoyed teaching mathematics her response was an emphatic "Yes". This showed in the lesson presentations. The subject was taught in an enthusiastic, interested, lively manner.

The teacher considered herself to be a mathematics teaching specialist probably because her major subjects in her teaching qualification were mathematics and science. When the teacher was asked why she considered herself as a mathematics subject specialist the teacher wrote:

18/8/2000 *I want to help the young ones especially to have sharp minds and use Maths effectively as it is the most important thing in their daily lives.*

23/8/2000 *I like students to enjoy and like Maths, as they are going to face the outside world.*

Here the teacher may have linked her enthusiasm for the subject to her reasons for being a subject specialist. She appeared to want her learners to enjoy the same pleasure she experiences when she teaches mathematics. The teacher connected mathematics with "daily living". Perhaps she sees mathematics as a subject that is important because of its use in everyday activities.

The teacher wanted the following fact to be known about the lesson taught on 18 August. She wrote:

> *I think the researchers should know that 1st the pupils have* [need to] *understood the lesson. And that their* [they] *can apply* [it] *in their daily activities.*

From this response it seems that the teacher is keen for learners to have full understanding of mathematical concepts. This was also apparent in the manner in which she explained the mathematical procedures. Once again, she made mention of the significance of mathematics as a subject that is applicable to "daily activities".

4.2.3.7. Teacher's qualifications, teaching experience and ambitions

What were the teacher's qualifications, teaching experience and ambitions for the future?

She had spent three years training at one of the Kwa Zulu-Natal Colleges and one year at another College in the same province. Ms Fundisi holds a Secondary Teaching Diploma. She had gained six years of teaching experience. The only school she had taught at was Angaziwa High School. At the time of the interview the teacher was doing part-time studying for a Further Education Diploma in science through the University of Natal. Unfortunately Ms Fundisi was not asked whether she belonged to any mathematics teacher organisation or had attended any in-service mathematics training courses. During a telephone conversation in March 2000 the teacher told me she had received an award from the department because of the achievement of her science matriculants in 1999. Ms Fundisi appears to be keen to improve her qualifications as a science teacher.

4.3. Discussion

4.3.1. Establishing the lesson context

The physical conditions that prevailed at this rural school cannot be ignored. The fact that no water was available for drinking purposes must influence the learning environment. It is ironic that there was an urgent need for rain to fill the water tanks but that rain, in turn, prevented teaching. In wet weather the dirt road to the school becomes too treacherous for the teachers to travel.

There is an ongoing controversy about the role that factors such as availability of water, power and telephones play in education. From the 1996 Census and 1996 School Register of Needs it has been estimated that in Kwa Zulu-Natal there are 1 233 schools without water, 3 197 without power and 3 421 without telephones in Kwa Zulu-Natal. According to Wilson (1999), no firm conclusions have been drawn, but he considers that "other less tangible factors such as the culture of learning, teacher motivation and community support are deemed to be as important, if not more so, in determining school performance." He draws this conclusion from the fact that several of the top-performing schools in South Africa have achieved good results despite lacking many of the amenities traditionally available in schools in well-resourced areas. Nevertheless, one cannot deny that there needs to be an equitable distribution of resources amongst schools, and electricity and water for all schools cannot be considered as an unfair demand.

The school building was in fairly good condition but the visible state of the classroom was not welcoming. Broken handles, empty dismantled cupboards and bleak notice boards did nothing to enhance the atmosphere. The room was probably not seen as "belonging" to anyone; not even the teacher had her own desk. The teacher could, however, decide on the topics she wanted to teach and may be seen to "own" the mathematical content imparted/taught by her.

There was a conspicuous absence of any forms of written language in the school building. There were empty notice boards devoid of any cuttings, circulars, warnings, and records of achievements or posters. Some graffiti did, however, occur on the walls. The absence of mathematics textbooks had a serious impact on the learning, as the learners could not interact with the text on a personal basis. Only the teacher was in possession of a textbook and the teacher may thus be seen to be in control of what was revealed; only what the teacher saw as significant was divulged. The chalkboard writing of the teacher became the only written mathematics "text" that was observed by the learners. The chalkboard may thus be seen as an "extension" of the teacher's "voice" to communicate exactly what was significant. This, too, signaled to the learners that the teacher was totally in charge of not only what mathematics they hear in lessons, but also what they are permitted to see.

4.3.2. Analysis of text used by teacher

The Chapter entitled "Linear Equations" in the textbook by Fletcher, Fletcher and Roos (1986) used by Ms Fundisi is subdivided into:

6.1 Revision: Simple equations
6.2 Problems leading to equations
6.3 Simple equations involving brackets
6.4 Simple equations involving fractions.

The teacher used only sections 6.1 and 6.2 during the observation period. In this textbook the traditional approach is observed. These authors started the section with developing the skills and only presented the contextualised problems only after the learners had mastered the rules using decontextualised problems. The application problems were introduced only after the manipulation techniques had been systematically supplied.

The concept of an "unknown" is not dealt with in this chapter but the textbook explanation, given in Chapter 1, is as follows: "In an open sentence such as

$x + 4 = 9$; x is used to represent the number which would make the open sentence true. x is called the variable." Perhaps this is what guided Ms Fundisi when she told the learners that an "unknown" is synonymous with a variable.

The revision examples provided are not appropriate in that far more economical means, such as knowledge of arithmetic, could have been used to solve them. The problem is firstly given in terms of an "English" sentence and then the algebraic solution follows. For example:

Example 1:

What must be added to 15 to give 21?

$x + 15 = 21$

$x + 15 - 15 = 21 - 15$ $\quad (15 - 15 = 0)$ $\quad$ Check: When $x = 6$;

$x = 6$ $\quad\quad$ $x + 15 = 6 + 15 = 21$

Remember:

If one operation is performed on one side of an equation, the same operation must be performed on the other side.

The inverse of ^{+}x is ^{-}x, and $^{+}x + ^{-}x = 0$

The inverse of $\times\ x$ is $\div\ x$, and $x \div x = 1$

The advice given for **"More difficult examples"** such as $2x = 7 + x$ is

Note 1:

With practice, some of the steps in the above examples can be omitted

Note 2:

Always check by substituting your solution for x in the original equation.

The **"More difficult examples"** are not accompanied by English sentences. Five of the problems dealt with in Lesson 3, namely: $2x = 7 + x$, $6x + 11 = 11x - 14$, $13x + 22 = 6x - 6$, $5x + 8 = 2x + 41$ and $6x + 5 = 2x + 11$ are listed in the book. Only $2x = 7 + x$ appears as a worked example. The problem with no solution, "Find three consecutive even numbers whose sum is 27.", does not appear in the textbook and may have been made up by the teacher. In the chapter entitled "Linear Equations" there is no mention made of alternative ways of solving equations using, for example, guessing and checking, tables, graphs, "function machines", iteration or other numerical methods; the only strategy used to solve the linear equation was the "isolation of the unknown" using the balance algorithm. Substitution methods were omitted and these could have illustrated simply and effectively the meaning of algebraic letter symbols.

The teacher did not encourage the learners to omit any of the steps or check the answers by substitution. Ms Fundisi did not start each problem with the equivalent "English" sentence as in the "simple" problems provided in the textbook, but the procedure used for solving the equations was followed rigidly. The teacher graded the problems in a manner similar to that used in the textbook i.e. demonstrating the easier problems first and then moving on to the more complicated problems. The worked examples in the textbook were given in the following graded sequence:

- What must be added to 15 to give 21? $x + 15 = 21$
- What must be subtracted from 21 to give 15? $21 - x = 15$
- From what number can 17 be subtracted to leave 6? $x - 17 = 6$
- By what number must 7 be multiplied to give a product of 42? $7x = 42$
- By what number must 54 be divided to give a quotient of 6? $54 \div x = 6$ or $54/x = 6$
- $\frac{8x}{3} = \frac{2}{9}$
- $2x = 7 + x$
- $3x = 8 - x$
- $18x - 7 = 4x + 49$
- $7x + 15 = 15x - 5$ (solution is a mixed number)
- The sum of two numbers is 86 and their difference is 10. Find the numbers.
- Find three consecutive odd numbers whose sum is 21.
- A father is now five times as old as his son. In five years' time, he will be three times as old. What are their present ages?
- $2(4x - 3) - 5(x + 3) = 10x$
- $5(2x + 5) + 3(2x - 5) = 6(4x - 5)$

- $\frac{2x-3}{3} = \frac{x+1}{4}$

These examples appear to indicate a behaviouristic approach. Initially the textbook provides problems to practice the balance algorithm and later on in the chapter there are "word problems" to be solved. The manner in which this section is organised is, thus, not in keeping with a "problem-centred approach".

In this chapter there is a glaring absence of explanations/discussions. The mathematics is "displayed" as a "follow my lead" type of "game". The rules of the game are not negotiated but set out in neat "do as I say" rules. The manner in which this chapter is set out gives the impression that there is a division between understanding mathematics and "doing" mathematics. If the teacher used only this textbook she would have to "make up" for herself how to explain the "rules of the game". This textbook is available in both of the public libraries visited by the researcher in July 2000 (Hillcrest and Cato Ridge). The manner in which these textbooks present mathematics is outdated and they should be replaced with textbooks that apply more suitable, less behaviouristic approaches to mathematics.

4.3.3. Teacher's instructional practices

The teacher's step-by-step method was developed as the only method of solving linear equations. The learners had no opportunity to make sense of the mathematical concepts for themselves. The mathematical information was imparted as if it were purely a set of rules to which to adhere. The physical knowledge required in order to understand the notions of equivalence and balancing of equations was not developed at all. In the observed lessons the learners were not afforded the opportunity to experiment with "function machines", "flow diagrams" or even "balancing" numerical systems when dealing with algebraic problems. During the observation period at Angaziwa High School the teacher did not consider the variable as an unknown, a general number and as in functional relationships. The teacher did, however, remind the learners that "x" may represent an unknown or a variable but the distinction was not developed.

The teacher started teaching the topic by using decontextualised problems, but this, according to observations made by De Villiers (1992a:3) leads to irrelevant and meaningless teaching. He observed how learners struggled to make sense of various arithmetical calculations. Although the learners were able to solve straightforward numerical calculations, they were unable to write "a little story describing a problem situation" to which the calculations would produce the answers of the numerical calculations. De Villiers found that very few learners were able to devise appropriate problems, in fact, of the class of 30 learners only 3 wrote stories involving the correct operation. De Villiers concluded that decontextualised calculations lead to inapplicable learning as there is no point in being able to add, subtract, multiply and divide numbers without knowing what it means nor for what it is useful in a problem solving situation.

The learners were taught in a manner that suggested that each individual learner should think and do exactly as the teacher does. It appeared that if the teacher believed that knowledge could be transferred from one person to another. In such a learning environment each learner could not develop individual mathematical knowledge nor be in control of the material presented.

The fact that the writing was always neatly displayed and organised in an orderly manner on the chalkboard may also have led the learners to believe that there is no effort required in finding solutions to problems in mathematics. The systematic procedure exhibited by the teacher did not give the impression that active wrestling and grappling with problems may be necessary to accomplish the task. The manner in which the teacher presented the work could lead the learners to think that everything done in mathematics may be as simple, neat, straightforward and effortless as illustrated by the teacher. The organisation of the learning was developed in a similar manner to that demonstrated in the textbook. The problems were graded from what may appear to the teacher to be simple linear equations to more complex equations. After each set of worked examples the teacher provided a limited number of exercise examples for practice. There was, though, not much drill and practice of the procedures demonstrated.

The teacher often wrote up the meaning of terms on the chalkboard. The learners, second language learners, had very little opportunity to interact with the English language, let alone technical mathematical terms. The fact that the teacher made the meaning visible on the chalkboard was no guarantee that the concept would be understood. The words could be seen as symbols to explain symbols that had not been internalised by the learners. What was seen on the chalkboard could be easily vocalized by the learners but this did not indicate that what was seen was available for individual use in a meaningful manner. The recording of terms such as “collect like terms” and “change additive inverses” may have provided little or no assistance in developing the learners' understanding of what the purpose of solving linear equations was. According to Olivier (1989:26) the use of the terminology “like and unlike terms” emphasizes that variables should be treated as objects which can never be the same. This may lead to a common misconception of the concept of a variable. The teacher involved in this project may have considered knowing the terminology an important aspect in mathematics, but more than just seeing and defining a mathematics term would be required to promote comprehension.

Although lessons were was observed for two weeks, it was difficult to judge how much of the necessary mathematical background knowledge the learners possessed. How much the learners knew about operations with integers or acquaintance with their previous knowledge of algebra would have been beneficial. Perhaps the learners needed more background knowledge of working with integers. It appeared that the learners were expected to build onto their previous knowledge without a firm foundation.

Barnes (1969:17) suggested that the types of questions posed by teachers may be categorised into factual ("What?"), reasoning ("How?" and "Why?"), "open" or "social" questions. It was found that all the questions posed by Ms Fundisi were of the factual type. Furthermore, the responses required were usually not more than chorusing of three words at a time. No full sentences were used. In the lessons observed there was a total absence of reasoning questions. The teacher covertly signaled to the learners what their roles as learners were. The "hidden" role of the learners was merely to echo or chorus what the teacher had given and this role did not involve reasoning. The learners were to accept the factual model presented to them passively and then reproduce it to match it against the teacher's model only to be judged right or wrong.

The type of "questions" posed by the teacher at Angaziwa High School may be classified as lower order questions (LOQ) and, according to Moodley (1992b:137), this would be indicative of a teacher-centred teaching style. The type of question posed determines the kind of thinking required by the learners. "Good thinking" can be developed only in learners when appropriate questions are posed. Perhaps this thinking leads to learners becoming aware of processes such as planning, monitoring and evaluating themselves. It is important to be aware of learners' "metacognition". Fortunato *et al* (1991:39) consider "metacognition" to be the general awareness of cognitive activities engaged in during a task or it may refer to the thoughts of learners or knowledge learners have about their thoughts. It is therefore essential that a range of both LOQ and higher order questions (HOQ) be posed to develop good thinking and, in turn, metacognition.

The style of teaching employed did not permit the learner to generate a sequence of ideas for him or herself. No discussion amongst the learners or between the teacher and the learners took place. The learners' responses to "questions" were guided by step-by-step prompts. The method of questioning facilitated lesson planning but obscured any issues that needed to be raised to ensure understanding.

There was demand for "correct" pronunciation by requiring the learners to repeat technical jargon. These second language learners may not have been able to explore the personal meanings attached to the terms repeated. Any parent is only too aware of the "Why? Why?" stage toddlers go through when first learning to speak. Perhaps these learners would also have benefited from the opportunity to ask some "why?" reasoning questions.

Du Toit (1992:114) distinguished various mathematical processes that he considered significant in the teaching of mathematics. These he listed as: *Abstracting, Generalising, Classifying, Translating* and *Validating.* None of these mathematical processes were observed in the teaching of mathematics at Angaziwa High School as the teacher possibly saw the nature of mathematics as a set of concepts, rules and structures that needed to be passed on as a fixed body of knowledge.

The teacher did pose questions that required translation of verbal symbolic symbols to algebraic symbols. For example, "Nomsa owns 56 chickens. How many more chickens must she buy to have 100 chickens altogether?", "Find three consecutive natural numbers whose sum is 63", "Find three consecutive even numbers whose sum is 27." and "Share 27 buns between two boys so that one gets 3 more (than) the other". The learners would have to change these word problems into algebraic symbols to solve the problems as the teacher required, but these problems could have been solved using easier methods.

The language used by mathematics teachers includes specific terminology almost exclusive to mathematics. This specific group of learners would not be familiar with these forms of language. This difficulty is further compounded by the fact that these learners probably have little or no chance of hearing the English language, let alone this specialist mathematical language. This teacher was not unaware of the crucial role that language plays in mathematics, but more than just chorusing the nomenclature would be imperative for understanding. Unfortunately the desire to teach terminology could prevent the teacher from perceiving her true task. The terminology should not just be used in order to substitute a phrase with a term but the learner should be able to "use the words to think with" (Barnes, 1975:50). Perhaps the teacher saw the value of the term for its own sake and this substitution then served as an explanation.

During the lessons the learners were focused on listening to the teacher for at least 20 minutes in a 35-minute lesson. Perhaps there was too much "teachers talk" because of the teacher's enthusiasm when trying to impart knowledge. The talking may have stopped the teacher from perceiving the needs of the learners. The extensive use of language in this classroom may be seen as an instrument of teaching and not as a vehicle for active learning. In order for learners to become problem solvers, teaching should result in understanding as well as in gaining know-how. The learners need to have the wisdom to tackle new problems and not only provide known answers to established problems.

The emphasis on "teacher talk" may also become an agonizing exercise in which the learners become even more aware of their language inadequacies. These learners do not use English in their homes and may even feel excluded from the classroom/teacher talk. If these learners were expected to use known language to grapple with new mathematical experiences then they would be at a distinct disadvantage.

The learners may, on the other hand, expect the teacher to "stand and deliver" as this method of teaching may be seen by learners as the only way in which competent teachers behave. Perhaps the learners may feel "short changed" if the teacher is not constantly telling the learners how to go about "doing" the mathematics. The teacher kept the learners focused on her and the chalkboard throughout presentations and this may be what these learners would demand of a good mathematics teacher.

Discussion amongst learners was not encouraged consequently, these learners did not have the opportunity to develop a sense of ownership of their own learning of the mathematical concepts. Through learners talking about mathematical problems, they could not only have been afforded the opportunity to clarify the processes for themselves, but may also have used their peers as evaluators of their thinking. It was not just mathematical language that the learners did not explore but language that was required to make sense of the meaning of the new processes learnt. The learners did not have the opportunity to use language to assimilate or synthesize new knowledge or to develop their mathematical reasoning. In such a large class it would be impossible for the teacher to communicate frequently with each learner but, if the learners had had the chance to communicate with peers, at least some "mathematics talk" could have taken place.

Usually schools are perceived as places where mountains of faint blue-lined exercise books are filled, particularly in the mathematics classroom. This was, however, not the case at the observed school. The absence of pencil and paper manipulations of mathematical symbols during lessons and for homework may also stifle learning. It is necessary for the learners to try writing their ideas or at least to make copies of teacher solutions presented on the chalkboard. The learners needed to make sense of the methods used by the teacher. The number of examples the learners were given to try on their own was not sufficient for them to observe and formulate patterns and generalizations for themselves. The low level of learner involvement in writing activities was a matter of concern. Eventually these learners would be expected to communicate their ideas on paper for evaluation purposes. Here the learners would need to make explicit their mathematical insight as individuals. This would be a difficult task if the learners had so little practice. Mathematical writing has particular rules to which both teachers and learners must adhere. The solution of mathematical problems done by someone else always appears to be relatively easy until the solution is tackled in isolation. The task certainly becomes more challenging when done alone. How can the learners be expected to become adept at completing written solutions when their main role in the classroom is that of listening and chorusing?

The teacher was usually accurate in solving problems selected by her, but, the problem with no solution (Find three consecutive even numbers whose sum is 27) was solved by the teacher on the chalkboard without herself reflecting on the accuracy of this solution. (The teacher's solution of this problem is shown in section 4.2.2.3.) The teacher did not ever check solutions by substitution to see whether or not the solution could to be correct. Once the step-by-step method had been completed there was no doubt as to the truth of the answer.
The teacher was enthusiastic about teaching mathematics and she considered herself to be a mathematics subject specialist but perhaps she would benefit from further studies of the subject matter to develop increased relational understanding.

Many of the problems chosen as examples to illustrate the method advocated by the teacher were not appropriate. The learners might have profited by looking at alternate strategies of solving relatively simple problems such as "$x - 9 = 1$". Substitution might be the easiest method of finding the value of x instead of insisting on the use of the step-by-step method. The learners may have been more successful in solving this problem by trying various values for x. The insistence of the "sledge hammer" to "crack" the solution appeared to disempower rather than empower the learners.

De Villiers (1991:4) observed how learners solved problems and concluded that learners often prefer to use their own informal methods to approach problems rather than to use algebraic methods. He found that narrow, prescriptive approaches, where the teacher insists on a specific algebraic approach, stifle thinking and appear to be a hindrance rather than an aid to solving problems. Learners often forget procedures but may be able to reason out the solution using their own strategies. De Villiers (1992b:3) also found that younger children are often able to make sense of word problems more readily than older children. He suggests that this is because of the manner in which children are taught. He found that "prescriptive teaching" stifles children's natural creativity and ability to solve problems. Learners who are taught in a prescriptive manner may become pre-occupied with trying to remember and comply with rules instead of thinking about the problem that they are attempting to solve. Unfortunately, Ms Fundisi's learners will not develop what she considers to be the purpose of her lessons, viz. "quick minds", if she emphasises the cumbersome, uneconomical, prescriptive methods she demonstrates.

Chapter 5: Learning Outcomes in relation to Algebra

What are the mathematics learning outcomes in relation to algebra of the learners in Grade 8 C at Angaziwa High School?

5.1. Introduction

Here the focus was to determine how the learners interpreted, performed and learnt in the specific classroom environment being observed.

The sources used to address this question were:

- learner scripts of the test completed on 20 August 1999
- learner scripts of work completed during lessons and at home.

The teacher set the test but asked the researcher to write it up on the chalkboard and invigilate whilst the learners completed the test. The teacher had not marked the test written on 20 August 1999 by the time the data collection was completed on 27 August 1999. The teacher asked if the researcher was willing to mark the tests but the offer was declined as this may have interfered with the HSRC research process. Later, the researcher prepared a possible mark memorandum and the learners' tests were marked by the researcher.
The test written in lesson 4, on 20 August 1999 consisted of:

Question 1
Solve these equations by writing all the steps:
(a) $x + 20 = 36$
(b) $x - 9 = 1$
(c) $2x = 10$
(d) $3x + 7 = 25$
(e) A man owns 48 sheep. How many more does he need to have 96 sheep?

<u>Question 2</u>

(a) Give the additive inverses of the following:

1. +4 2. -3 3. -2

(b) Find three consecutive natural numbers whose sum is 42.

Below is a possible "model" answer prepared by researcher using the methods demonstrated by the teacher:

Question 1

(a) $x + 20 = 36$

$x + 20 - 20 = 36 - 20$ ✓ m

$x = 16$ ✓ a

(2)

(b) $x - 9 = 1$

$x - 9 + 9 = 1 + 9$ ✓ m

$x = 10$ ✓ a

(2)

(c) $2x = 10$

$\frac{2x}{2} = \frac{10}{2}$ ✓ m

$x = 5$ ✓ a

(2)

(d) $3x + 7 = 25$

$3x + 7 - 7 = 25 - 7$ ✓ m

$\frac{3x}{3} = \frac{18}{3}$ ✓ m

$x = 6$ ✓ a

(3)

(e) $x + 48 = 96$ ✓ a

$x + 48 - 48 = 96 - 48$ ✓ m

$x = 48$ ✓ a

(3)

Question 2

(a) The additive inverse of:

1. +4 is –4 ✓ a 2. -3 is +3 ✓ a 3. -2 is +2 ✓ a

(3)

(b) Let the first number be x

Let the second number be x + 1 ✓ m

Let the third number be x + 2

$x + x + 1 + x + 2 = 42$ ✓[m]

$3x + 3 = 42$ ✓[a]

$3x + 3 - 3 = 42 - 3$ ✓[m]

$\frac{3x}{3} = \frac{39}{3}$ ✓[a]

$x = 13$ ✓[a]

First number is 13
Second number is $x + 1 = 14$ ✓[a]
Third number is $x + 2 = 15$

(7)

Total [22]

Note: ✓[a] indicates a mark allocated for accuracy
✓[m] indicates a mark allocated for method

5.2. Analysis of data

5.2.1. Using the test prepared by the teacher

5.2.1.1. Analysis of test

The questions in the test were categorised according to the different cognitive levels suggested by Du Toit (1992).

Knowledge: All the questions required the learners to recall the knowledge gained by observing the step-by-step techniques demonstrated by the teacher. Question 2 (a) in particular called upon the learners to remember what an additive inverse is and how to obtain an additive inverse.

Computational Skill: Questions 1 (a) through to 1 (d), as well as question 2 (a) were designed to require straightforward manipulation on decontextualised problems according to rules that the learners should have remembered.

Comprehension: Questions 1 (e) and 2 (b) required understanding of the underlying concepts and required interpretation of the significance of the data. Learners were not given the equation to solve in a decontextualized format. Here the learners had to decide how to formulate an equation as well as solve the equation.

Application: There were no questions that required the learners to apply relevant ideas, principles or known methods to new situations. There are thus no questions that required the combination of more than one line of thought.
Inventiveness: No non-routine application questions were posed. The learners did not have to develop their own techniques for solving the problems. All the questions prepared for the test were similar to questions demonstrated by the teacher on the chalkboard.

Question 1 (e) is not an equation so the instruction "Solve these equations by writing all the steps" is not applicable to this word problem.
A classification of addition and subtraction word problems was devised by the Unit for Research on Mathematics Teaching at the University of Stellenbosch (RUMEUS) and used in a document developed by Du Toit *et al* (1993) of the former Cape Education Department. These word problems were subdivided into *Change, Combine (part-part-whole), Compare* or *Equalize* categories that may be identified as follows:

Change:	Start with a single collection and either add to it or remove from it to result in a larger or smaller collection. Here action is implied.
Combine: (part-part-whole)	Start with more than one collection that are united or separated to find the whole or parts of the whole. Here a static situation is implied.
Compare:	Start with two or more collections. It is implied that the difference will persist i.e. the operation addition or subtraction is merely to determine the extent of the difference. Here a static situation is implied.
Equalize:	Start with two or more collections. It is implied that the difference between the sets will be removed. Here action is implied.

Using this classification of addition and subtraction word problems proposed by RUMEUS (du Toit *et al*, 1993) questions 1 (e) and 2 (b) may be described as *Combine* problems. In each case information is given about the whole and the various parts required to make up the whole is to be determined. In both questions, there is a static situation as no action is implied. All the questions, except 1 (e) are decontextualised problems.

5.2.1.2. Analysis of test scores

The marks gained by the learners were grouped using the class intervals 0% to 19%, 20% to 39% etc up to 80% to 99%. Table 1 shows the number of learners with test scores in each class interval.

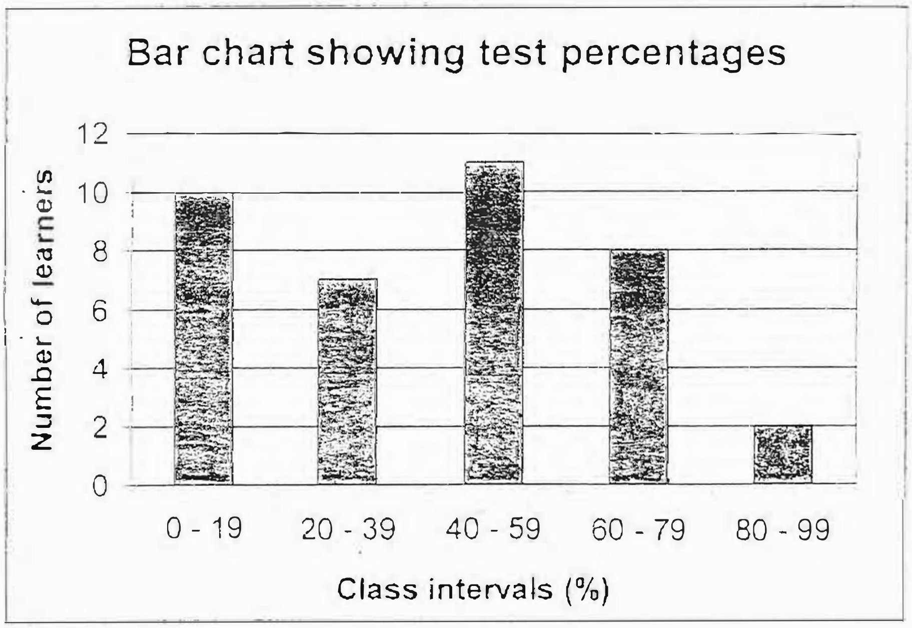


Table 1 *Bar chart showing learners' performance in a test that was set by the teacher but marked by the researcher*

The modal class was 40% - 59%. 42% of the learners' scores was less than the mean. Both the mean and the median are 41%. The range of the percentages is 0% - 86%. The standard deviation is 35,24% which indicates that there is a wide spread of the percentages about the mean. The inter-quartile range is 23% - 59%, i.e. the middle half of the learners scored between 23% and 59%.

The marks gained for each of the seven questions were then added and expressed as a percentage of the total possible marks for that particular question. Using this data it was possible to grade the questions according to those questions at which learners performed better or worse. Table 2 shows this percentage for each particular question.

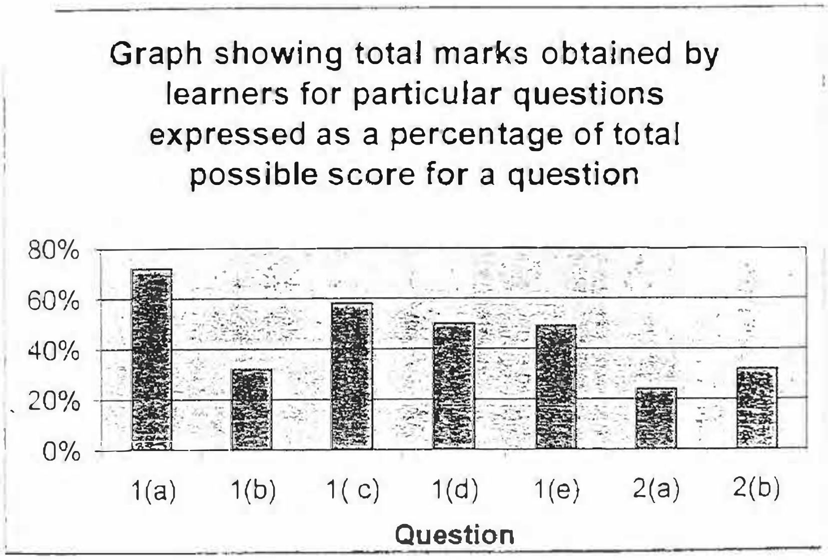


Table 2 *Bar chart showing marks gained for each test question expressed as a percentage of the total possible score for each question*

The learners' test scripts were then analysed in an attempt to explain possible methods the learners used to answer the questions and the conceptions that may have guided them. Possible trends were sought in the manner in which the learners tackled the problems posed by the teacher. Making use of interviews with the learners would have enhanced the analysis. If each learner were given the opportunity to explain how she or he understood the problem and solution, then a more accurate, meaningful analysis could have been prepared. It would have been useful to listen to how learners obtained the correct as well as incorrect solutions. As the school is situated in such a remote area, it was not possible for the researcher to revisit the school.

In order to ensure the anonymity of each learner, names were changed to letters of the alphabet. When the scripts were collected those of learners seated next to each other were kept together. Hence, for example, learners designated M1, M2 and M3 were sitting at the same double desk.

5.2.1.3. Analysis of responses to test questions

The analysis of the questions is ordered according to marks gained in the test. (See Table 2.) The questions were graded on a scale from 1 to 7 according to the marks obtained. Section 5.2.1.3.1. deals with the question in which learners fared the worst and section 5.2.1.3.7. deals with the question in which the learners fared the best.

5.2.1.3.1. Test question 2 (a)

Of all the questions in the test, learners scored the least number of marks for Question 2 (a). This is how the question was displayed on the chalkboard.

<u>*Question 2*</u>

(a) Give the additive inverses of the following

1. +4   *2. -3*   *3. -2*

This setting out of the question seemed to confuse the learners. Eleven out of the 38 learners who completed the test used the subsection numbers as part of the question. The labels for the subsections, namely "2." and "3.", were considered to be integers that should be given additive inverses. Learners who used the subsection numbers in this way mostly found the additive inverses of five numbers instead of just the required three. The first subsection, label "1.", was not used to give an additive inverse. Perhaps this was an unfamiliar format or perhaps the "full stop (.)" after the subsection numeral was not clearly visible on the chalkboard. For example, learner M3 wrote:

<u>Questions 2</u>

a, -4 , +2 , +3 , -3 , +2

Learner Q3 just wrote "*multiplicative inverse*" but produced no numerical answer. Perhaps Learner P1 thought that one of the given three possibilities should be chosen, as in a multiple-choice question, and wrote only "2 –3" as the selected solution.

Eight learners (24% of the learners who attempted the problem) obtained the correct solutions but five of these learners used the "is equal to (=)" sign as meaning, "do something" and wrote "+4 = -4". These learners may not have used the "=" sign to indicate an equality relation. Five learners (13%) made no attempt at the problem.

The learners had a variety of their own interpretations as to what was required in this question. Seven learners (21% of the learners who attempted this problem) appeared to interpret "additive inverse" as having something to do with "addition". For example, Learner P2 doubled each integer in the following manner:

Question 2

1. ~~-4 2, +3 3, +2~~ (1) $+4 + {}^{+}4 = +8$

(2) $-3 + {}^{-}3 = {}^{-}6$

(3) $-2 + {}^{-}2 = {}^{-}4$

Nine learners (27% of the learners who attempted the solution) looked at the "signs" of the integers given. For example Learner A2 appeared to classify the integers according to whether they were "Negetive" (sic) or positive and recorded:

Questions 2

(a) $^{-}3$ and $^{-}2$ = Negetive

(b) $^{+}4$, 2 and 3 = Positive

Detailed analysis of the meanings of "additive inverse" yielded the following:

Learners	Learner's response	Interpretation of response
O2	*+4 = +4; -3 = -3; -2 = -2*	Placed an equals sign between the numerals
P2	*+4 + +4 = + 8; -3 + -3 = -6;* *-2 + -2 = -4*	Doubled the given numbers
G2	*a) negative* *b) positive* *c)*	Seemed to realise that additive inverse implies having an opposite sign
B2 K2	*a) +4* *1. +4 2) -3 = 0 3) -2 = 0*	Only positive integers were additive inverses - perhaps the negative integers were "nothing" i.e. 0.
G1 A2	*1) +4 = Positive* *2) -3 = Negative* *3) -2 = Negative* *(a) -3 and -2 = Negetive* (sic) *(b) +4, 2 and 3 = Positive*	Sorted the integers according to whether they were positive or negative
A1	*1. Production* *2. Available* *3. Coe'fficient*	Considered terminology to be required - perhaps business economics terminology?
E1 E3	*1. +4 2. 3- 3. 2-* *1. +4 2. 3+ 3. 2-*	Only the position of the "-" needed to be changed
E2 I1 D1	*+4 + 2 = +6 -3 + 3 = -6 - -2 = -4* *+4 + 2 + -3 + 3 + -2 = +14* *+4 + -3 + -2 = +7 + -2 = +9*	"Addition" of all integers was attempted
O1	*a) +4 x -4 -2 + +3 +3 + -2*	Changed the signs of everything and placed a "x" or a "+" sign between the integers
N1	*(a) +4 2. -3 3. -2* *1.2 4. -6 6. -4*	Halved the first integer but doubled the others

Table 3 Table showing learners' possible interpretations of "additive inverse"

A3	(1) +4 = +2 (2) -3 = -1 (3) -2 = +4	**Perhaps** "added" or "subtracted" 2 from the integers given?
D2	(a) let 1st number x let 2nd number x+2 let 3rd number x + 4 $x + x + 2 + x + 4 = 4$ $3x + 6 - 6 = 4$ $\frac{3x}{3} = \frac{4}{3}$ $x = 1\frac{1}{3}$	**Perhaps** linked question 2 (a) and (b) where "three consecutive natural numbers" were required?
K1	1. 1 2 3 = +4 2. 1 2 = -3 3. 1 = -2	**Perhaps** "counted" until the number was reached?
M1	1 +4 2 –3 3 –2 1 2 3 4	**Perhaps** wrote down the question and selected, in order, the positive integers?
M2	(a) +4 2. -3 3. -2 2 1 -1	**Perhaps** found the "difference" between adjacent integers given?
N2	a) +4 2 -3 3 -2 123456789	**Perhaps** "counted on" according to the positive integers in the question?
P1	(a) 2 -3	**Perhaps** "selected" the correct response as in a multiple-choice question?
Q1	a) 1, 2, 3, 4 b) 2, 4, 6, 8	**Perhaps** wrote the positive integers and doubled them?
Q3	a) Multiplicative inverse	**Perhaps** the "." was seen as a multiplication sign?

Table 4 *Table showing learners' responses to finding of "additive inverses" that are uninterpretable*

5.2.1.3.2. Test question 1 (b)

The question was:

<u>*Question 1*</u>

Solve these equations by writing all the steps:

(b) $x - 9 = 1$

All 38 learners attempted this problem. Four learners (11%) obtained a correct solution using the setting out that was demonstrated to them during lesson presentations. Two learners, H1 and Q2 also obtained a correct answer but gave the final solution as "= 10". Perhaps the is equal to symbol ("=") was a way of showing that the answer was to follow. Learner H1 wrote:

(b) $x - 9 = 1$

$x - 9 + 9 = 1 + 9$

$= 10.$

Seven learners appeared to have developed their own strategy or used their knowledge of arithmetic. These learners may have used substitution to obtain the solution but unsuccessfully attempted to write what the teacher required. For example, Learner K1 perhaps attempted to use the teacher's method but possibly solved the problem mentally by inspection but then incorrectly tried to write the solution in the way the teacher required. Learner K1 recorded:

(b) $x - 9 = 1$

$x - 9 - 9 = 9$

$x = 10$

The other six learners did the following:

Learners	Learner responses
C1, C2, D1	$x - 9 = 1$ $x - 9 + 1$ $x = 10$
A3, F2	$x - 9 = 1 - 1$ $x = 10$
N1	$x - 9 = 1$ $x - 9 = 1 - 9$ $x = 10$

Table 5 *Table showing learners' responses to solving $x - 9 = 1$ where they possibly used their knowledge of arithmetic*

Three learners subtracted the same amount from each side of the equation instead of adding nine to each side. They balanced the equation but this did not assist in isolating the unknown. For example, learner E1 recorded:

b) $x - 9 = 1$

$x - 9 - 9 = 1 - 9$

$x = {}^{-}8$

Seventeen learners (45%) developed alternative incorrect strategies to "balance" the equation. These learners did not add or subtract the same amount from each side of the equation. The most common "type" of balancing used was adding 9 to the left-hand side (LHS) of the equation and subtracting 9 from the right hand side (RHS) of the equation. Perhaps the learners did not realise that equivalence has to be maintained when the balance algorithm was used or perhaps adding 9 and subtracting 9 was seen as "balancing" i.e. adding nine and subtracting nine results in no change. There were, however, many variations of "balancing" and these methods are detailed below:

Learners	**Sample** of each learner's response
A2, F1, M2, O1, Q1	*x - 9 + 9 = 1 - 9 or* *x + 9 - 9 = 1 - 9*
M1, L1	*x + 9 - 9 = 1 - 9*
G1	*x - 9 - 9 = 9 - 1*
A3, F2	*x - 9 = 1 - 1*
Q3	*x - 9 + 9 = 1 - 10*
A1	*x - 9 + 9 = 1 + 1*
K1	*x - 9 - 9 = 9*
N1, O2	*x - 9 = 1 - 9*
J2	*x + 9 - 9 = 9 - 1*
E2	*x - 9 + x = x - x + 1*

Table 6 *Table showing a sample of learners' responses where methods of "balancing" was unsuccessfully attempted*

Five learners incorrectly added integers. Perhaps these learners did not see the significance/purpose of adding the additive inverse to each side of the equation. Learners D2, I2 and P2 may have replaced an is equal to symbol ("=") with a plus sign ("+") and replaced the minus sign ("-") with an is equal to symbol ("="). Learner P2's solution was:

b) $x - 9 = 1$

$x - 9 + 9 = 1 + 9$

$x = 18 + 10$

$x = \underline{28}$

Learners “added” the integers as follows:

Learners	**Sample** of learner responses
D2, I2, P2	*x* – 9 + 9 = 1 + 9 *x* = 18 + 10 *x* = 28
B2	*x* – 9 + 9 = 1 + 9 *x* – 18 = 10 *x* – 18 + 18 = 10 + 10
K2	*x* – 9 + 9 = 1 + 9 *x* = 18 + 1 *x* = 19

Table 7 *Table showing a sample of learners' responses where "addition" of Integers was unsuccessfully attempted*

Learner P1 replaced the minus sign (”-“) with a plus sign (“+”) and proceeded as follows:

(b) $x - 9 = 1$

$x + 9 = 1$

$8x + 9 = 1$

$8x + 9 - 9 = 1 + 9$

$\frac{8x}{2} = \frac{10}{2}$

$x = 5$

Learner G2 renamed the equation as an expression. This learner found a “solution” by manipulating the symbols in the following manner:

$x - 9 - 9 - 1$

$9 - 1 = 8$

~~b) $x - 9 = 1$~~

~~$x10x - 9 = 1$~~

5.2.1.3.3. Test question 2 (b)

This is how the question was written on the chalkboard:

Question 2

(b) Find three consecutive natural numbers whose sum is 42.

Learners Q1 and Q 3 did not attempt this problem. Only one learner, E3, (3% of the learners who attempted this problem) solved the problem as demonstrated by the teacher with correct substitution to find "three consecutive natural numbers". Learner L1 found the first of the three consecutive numbers but used the equals sign as "the answer to follow" to obtain the other two numbers by substitution. Nine learners (25% of the learners who attempted this problem) found the first number, using the teacher's method but did not continue to find the second and third numbers.

Learner L2 was able to set up the first equation correctly as "x + x + 1 + x + 2 = 42" but simplified this as "3x + 2 = 42" instead of 3x + 3 = 42. Perhaps this was just a slip or error. This learner then obtained "x = 13 1/3". In trying to find a solution, the learner seems to have changed 13 1/3 back to 40/3, simply ignoring the denominator to write "x = 40" as the first number. The other two numbers were given as 41 and 42. Learner L2 wrote:

b) let the 1st number be x
let the 2nd number be $x+1$
let the 3rd number be $x+2$

$x + x + \cancel{1} + x + 2 = 42$

$3x + 2 = 42$

$3x + 2 - 2 = 42 - 2$

$\frac{3x}{3} = \frac{40}{3}$

$x = 13\frac{1}{3}$ NO

$x = 40$ 1st NO

$x + 1 = 40$ ✓ NO

$40 + 1 = 41$

$x + 2 = 40$

$40 + 2 = 42$ 3rd NO

Learner D2 found the correct initial equation but subtracted 3 from 42 to obtain 14. The solution was then "correctly" solved using "3x = 14" to obtain "x = 4 2/3". This learner did not realise that this solution was not a natural number. Learner D2 recorded:

b) Let the 1st number be x
Let the 2nd number be $x+1$
Let the 3rd number be $x+2$

~~$3x + 42 = 42 + 3x + 3x = 42 - 3$~~
$42x + 3x + 42 = 3x - 3x = 42 + 3$
$39 - 3$
$\frac{39}{3}$

Learner E2 correctly represented the three consecutive numbers as "x, x + 1" and "x + 2". Perhaps this learner understood the concept of consecutive numbers but was not able to use this information to set up the required equation. Learner E2 wrote:

b) let 1st number x
let 2nd number $x+1$
let 3rd numbe $x+2$
$\cancel{3}\ x+x+1+x+2 = \cancel{44}\ 42$
$3x+3-3 = 42-3$
$\frac{3x}{3} = \frac{14}{3}$
$x = 4\frac{2}{3}$

Learners translated the problem in mathematical terms using a variety of different assumptions about what "consecutive" means. For example,
Learners (14% of the learners who attempted the problem) interpreted "consecutive" as "x, x + 2 and x + 4". Other responses were as follows:

Learners	**Sample** of learner responses
G1	*Let 1st number be x* *Let 2nd number be x+ 2* *Let 3rd number be x + 3*
M3	*Next First number = x* *Next Second number = 2* *Next third number = 4*
O2	*Let 1st number be x* *Let 2nd number be x + 3* *Let 3rd number be x + 6*

Table 8 *Table showing a sample of learners' responses where term "consecutive" translated into mathematical terms in test question 2(b)*

Only Learners G2 and I2 linked their first steps of their solution to the sum being 42 and then set up the equation as "$x + x + 2 + x + 4 = 42$".
Five learners did not link their naming of the consecutive numbers directly to the sum being 42. These learners went on to write:

Learners	**Sample** of learner responses
D1	$x + x + 2 + x + 4$
G1	$x + x + x + 2 + x + 3 = 42$
M3	$x + x + x + 2 + 4 = 42$
O2	$3x = 42$

Table 9 *Table showing sample of learners' responses where learners set out their first expression/equation in question 2(b)*

Learners N1, N2 and J2 represented three consecutive numbers as "x + 3" in mathematical terms and wrote the equation as "x + 3 = 42". For example, learner N1 wrote:

(b) $x + 3 = 42$

$x + 3 - 3 = 42 - 3$

$x \neq 42$

$\underline{42}$

Five learners (14% of the learners who attempted this problem) interchanged x + 3 and 3x and also manipulated and interpret terms in the expressions in a variety of ways. For example, learner J2 renamed x + 3 as " x + x + x + 3" and this, in turn was conjoined to become "x3". Possibly this learner lacked understanding of the meaning of algebraic symbols. Learner J2 wrote:

b) $x + 3 = 42$

$x + x + x + 3 - 3 = 42 - 3$

$x3 + 42 - 3$

$\underline{x3 = 39}$

3

Learner P1 "simplified" *3x – 42* to obtain "x = -39" i.e. the expression became an equation and the minus sign ("-") was replaced with an is equal to sign ("="). Learner F2 considered "*x* + *x* + 4 + x + 4" to be synonymous with "4x + 4 - 4 = 42 –". Perhaps this learner conjoined x + 4 as 4x. Learner F2 wrote:

(b) Let the 1st number

Let the 2nd number

Let the 3rd number

$x + x + 4 + x + 4$

$4x + 4 - 4 = 42 -$

$\frac{4x}{4} \cdot \frac{34}{4}$

$\underline{x = 4}$

When learner E2 came to connecting numbers x, x + 1 and x + 2 with the fact that their sum is 42, he interpreted the information given as "42x – 3x + 42 = 3x – 3x = 42 + 3" i.e. separating the coefficient from the unknown. (See page 106.) Learner O2 replaced 3x with "3x + x + x + 3". Learner D1 replaced the expression "3x + 2 + 4" with the equation "3x = 6" and then gave the answer as "12".

Learners M1, N2, K1 possibly tried various incorrect numerical manipulations to arrive at possible solutions. For example, Learner M1 wrote:

B Let Stad with first number 1
Let stad with second number 2
Let Stad with the thirth number3
~~1+2+3—48~~ 1+1
2+3
5−5
0 − ~~4~~ 1+1+2+5
= 9

Five learners (14% of the learners who attempted the problem) answered by listing a set of numbers. Perhaps the words “natural numbers” urged them to list numbers. For example learners C1 and C2 listed the first three multiples of 14 instead of “consecutive natural numbers” i.e. “{14, 28, 42,,,}”. Learner M2 listed the three consecutive even numbers before 42 i.e. “{36, 38, 40}”. Learner Q3 wrote:

b) 1 - 2 - 3 - 4
(ii) 2 + 4 + 6 - 8
iii) 1 - 3 - 5 - 7

Perhaps seeing the words “natural numbers” reminded Learner A3 of these “laws” previously taught by the teacher. Learner A3 attempted the solution by writing:

(b) (i) Communitative law
(ii) Distributive law
(iii) Associative law

5.2.1.3.4. Test question 1 (e)

The problem was:

<u>*Question 1*</u>

Solve these equations by writing all the steps:

(e) A man owns 48 sheep. How many more does he need to have 96 sheep?

Learner F1 did not attempt a solution. This learner merely wrote down the problem. Learner B1 may also only have written down the problem and recorded:

(e) he need a 48 sheeps to have a 96 sheeps?

Eleven learners (30% of the learners who attempted a solution) correctly set up the equation and solved it as the teacher required using "all the steps". The only mistake learner J2 made was 96 – 48 = 58 instead of 48. Possibly learners A3, C1, C2 and E2 (11% of the learners who attempted a solution) obtained the correct solution by using their knowledge of numbers and not by "writing all the steps". Learner C1 wrote:

(e) 48 sHeep
+48 sHeep
96 sHeep

Three of the learners obtained a solution of 144. This was possibly because the learners just looked for the numbers given in the problem, did not read the problem and just proceeded to add the numbers 48 and 96 to obtain 144. Perhaps the learners correctly recognised that the problem involves an increase situation and therefore thought they should add the numbers. Learner P1 recorded:

(e) let 48 st
let 2 end nember
let the 4 end nember
let the 6 end nember
~~96~~ 48 + 96 = 144

Learners A1 and M2 tried various numerical calculations only to obtain incorrect answers. For example, learner A1 found "x = -42" and did not realise that this could not be a sensible answer to the problem. Learner A1 wrote:

(e) $48 = 96$

$\cancel{48 \quad 48}$ $\quad 48 - 48 = 96 - 48$

$x = {}^{-}42$

Twelve learners (32% of the learners who attempted a solution) started off in a similar manner by possibly trying to explain what the unknown represented. As this was a word problem, learners may have considered it necessary to set out the problem with these initial steps. Perhaps these learners also found the solution by inspection and then tried to write it using the teacher's format. For example, learner N1 wrote:

(e) A man owns 48 Sheep how many more does he need to have 9 Sheep?

Let the 1st number is $x + 0$

Let the 2nd Number is $x + 1$

Let the 3rd Number is $x = 2$

$x + 48 = 96$

$x + 48 - 48 = 96 - 48$

$x + 6 - 48$

$\underline{x = 48}$

However, only learners D2, G1 and G2 went on to use the first three steps recorded to formulate their first equations. For example, learner G1 recorded:

Let 1st number be x

Let 2nd number $x+2$

Let 3rd number $x+4$

$x+x+x+2+x+4 = 48-96$

$x+6 = 96 - \cancel{86} = 96-48$

$\cancel{x+6x} = \frac{38}{\cancel{6}}$ $\quad \frac{}{6}$

$\underline{x = 3}$

Learners F2 and L1, after writing down incorrect algebraic expressions in terms of x went on to replace the x symbols with the number one. Perhaps these learners had difficulty working with symbols as variables so they substituted numerical values for these symbols. Kuchemann (1981) identifies this tendency of learners as "letter evaluation". For example, learner L1 recorded:

e. 48

x + 48 + x + 96

+ 1 + 48 + 1 + 96

2 + 48 + 48 = 96 − 48

x 48

Learners Q1 and Q2 seemed to have used the x to representing "sheep" and set up the equation as "48x + x = 96", i.e. the "x" stands for sheep. Kuchemann (1981) classified this misuse of a variable as "letter used as objects". These learners may have interpreted the unknown "x" as standing for an object in its own right in much the same way as letters are used to represent abbreviations. For example, instead of writing "metres", one uses the letter "m" as its abbreviation.

5.2.1.3.5. Test question 1 (d)

The question was:

<u>*Question 1*</u>

Solve these equations by writing all the steps:

(d) *3x + 7 = 25*

One learner, O1, omitted this problem. Eleven learners (30% of the learners who attempted this problem) solved the equation as the teacher required. Learner M2 obtained the correct solution but set out the final answer as "= x 6". It appeared that this learner was using the equals sign as "here comes the solution". Learner F1 also gave the solution as "x 6" but did not use the equals sign.

Six learners (16% of the learners who attempted this question) used a variety of "combining" methods to join the terms. This tendency of learners to combine terms is known as "conjoining" (Kuchemann, 1981). Perhaps these learners wanted to have a single "answer" and used this type of "closure". For example learner G2 joined 3x + 7 to become 10 and wrote "3x + 7 + 15 = 25", i.e. 10 + 15 = 25. Learner I1 changed the "3x = 18" to "3x + 18" and conjoined this as "x21", probably by adding the 18 and the coefficient of x, namely 3. Learner M1 may have "combined" 7 - 3x to obtain "=x4" and recorded:

$3x+7=25$

$3x+7-3x$

$= x\ 4$

Eight learners did not use "balancing of the equation" i.e. adding or subtracting the same amount from both sides of the equation. For example, learner N1 wrote "3x + 7 + 7 = 25 – 7", i.e. the seven was added to one side of the equation and subtracted from the other side. Perhaps this learner considered the adding 7 to one side of the equation and subtracting 7 from the other side is same as adding zero. (Learner N1 did not use this balancing strategy in any other problems.) These learners did not use the notion of an "equation" as an equivalence relationship.

Two learners, A3 and Q2 used arithmetic manipulations to solve the problem. Perhaps these learners had difficulty working with symbols as variables so they substituted numerical values for these symbols. Learner Q2 wrote "3x 6 + 7 = 25 – 7", but gave the answer as 18. This learner may have seen that if 6 were substituted for x then the correct solution would be found. This learner wrote:

d $3x+7=25$

$3 \times x + 7 = 25$

$3 \times 6 + 7 = 25 - 7$

$= 18$

Two learners, D1 and G1 made mistakes in subtracting or dividing numbers. Learner D1 gave the answer to 18 ÷ 3 as 16. Learner G1 incorrectly solved 25 – 7 as 8 instead of 18.

5.2.1.3.6. Test question 1 (c)

The question was:

<u>Question 1</u>

Solve these equations by writing all the steps:

(c) *2x = 10*

All thirty-eight learners attempted this problem. Fifteen learners (39%) solved this problem accurately. Learners H1, I1, N2 and M2 obtained the correct answer but did not write this as x = 5. For example, learner M2 gave the final solution as "= x 5" and learner I 1 wrote "x5".

Nine learners each used their own interpretation of 2x. For example, learner L1 renamed 2x as "2 – x" and learner M1 changed 2x to "2 + 1" and learner K1 considered 2x to be "x + 2" and wrote:

$$
\begin{aligned}
&(c)\ 2x = 10\\
&x + 2 - 2 = 10\\
&x = 8.
\end{aligned}
$$

Eight learners "balanced the equation" using a variety of incorrect, yet interesting methods. For example, learner P1 may have known that the solution was 5 but wrote:

$$
\begin{aligned}
&(c)\ 2x = 10\\
&2x \times 5 = 10 \div 5\\
&2x \times 5 = 10\\
&\underline{x = 5}
\end{aligned}
$$

Learner A3 used an incorrect numerical example to solve the equation and wrote:

$$(c)\quad 2x = 10$$
$$2 \times 2 + 6 = 10$$

5.2.1.3.7. Test question 1 (a)

Learners scored the most marks for Question 1 (a). The question was:

<u>*Question 1*</u>

Solve these equations by writing all the steps:

(a) x + 20 = 36

This equation presented the fewest difficulties for the learners. All the learners attempted this problem and eighteen learners (47%) solved this correctly. Learners A1, I1, N2, and Q2 found the correct solution but incorrectly used "is equal to" symbol as a "now follows the answer" symbol. For example, Learner A1 gave the solution as "= x = 16".

$$(a)\ x + 20 = 36$$
$$x + 20 - 20 = 36 - 20$$
$$= x = 16$$

Six learners (16%) of the learners may have used their knowledge of arithmetic to solve the problem correctly, but also unsuccessfully attempted the method demonstrated by the teacher. Learner J1 used an incorrect "balancing" by adding 20 to the left-hand side and subtracting 20 from the right-hand side but still obtained 16 as the solution. (Learner J1 used this strategy of adding to the LHS and subtracting from the RHS in questions 1 (d), 1 (e) and 2 (a). Hence this was a consistent error/misconception that was not just a random, careless mistake.) Learners F1 and F2 subtracted 20 from the left-hand side and 2 from the right-hand side. Learners C1 and C2 used the left-hand side of the equation and wrote "x + 20 – 20" and did not write the right-hand side at all but managed to find the correct solution. Learner K1 subtracted 20 from the left-hand side of the equation only and left the right-hand side as 36 but also gave the solution as x = 16.

Learner L1 copied down the equation to be solved as "x + 30 = 36" but managed to solve this equation correctly.

Learner D1 subtracted 20 from the left-hand side of the equation and 36 from the right-hand side but did not proceed from that step.

Learners E1, I2, P2 and Q3 balanced the equation correctly by subtracting 20 from each side of the equation. These learners did not, however, write the correct answer for 36 – 20. Perhaps these learners considered the "steps" to be adequate and may not have realised that the value of the unknown was required.

Learner G2 "solved" by substitution but wrote "6 + 20 = 36". Perhaps this learner had difficulty working with symbols as variables so she substituted numerical values for these symbols. The value of x was not indicated.

Learners E2, P1 used various forms of "conjoining" when they interpreted the symbols. For example, learner E2 considered "x + 20" to be "x20". She then replaced an is equal to symbol ("=") with a plus sign ("+") and wrote:

$$x + 20 = 36$$
$$x20 + 36$$
$$\underline{56}$$

Learner P1 may have realised that 20 had to be subtracted in order to obtain the value of the unknown but used the 16 to "set up" another new equation i.e. wrote "16x + 20 = 36". Subtracting 20 from the left-hand side and adding 20 to the right-hand side was used to solve this equation. A calculator might then have been used to find the final solution of 3,5. Learner P1 thus appeared to know that the solution was 16 but used various steps only to arrive at an incorrect solution. Learner P1 wrote:

$$
\begin{aligned}
&(a)\ x + 20 = 36 \\
&x - 20 = 36 \\
&16x + 20 = 36. \\
&16x + 20 - 20 = 36 + 20 \\
&16x = 56 \\
&\underline{x = 3{,}5}
\end{aligned}
$$

5.2.2. Using the learners' classwork and homework

A number of the learners completed the test on 20 August 1999 but did not submit their exercise books on 27 August 1999. Only thirty of the thirty-eight learners who sat for the test handed in their exercise books. Learners recorded only examples that were given as class exercises or homework exercises. In their exercise books the learners did not record the solutions demonstrated by the teacher on the chalkboard.

In lesson 1 learners solved x + 14 = 28 and were instructed to "Write the first step and raise up your hand." They then attempted to solve 6x = 18 and x + x + x + x + 4 = 10. One problem, 2x = 7 + x, was given as homework. In lesson 2 the class exercise was "Find 3 consecutive even numbers whose sum is 27". The learners did not manage to do this on their own so the teacher demonstrated the "solution" to this fallacious problem on the chalkboard. (See section 4.2.2.3.) The homework was "Share 27 buns between two boys so that one gets 3 more [than] the other." In lesson 3 the class exercise was 6x + 5 = 2x + 11 and no homework was given, but the learners were told of the test that was to be written the following day. In the three days of observation the learners were given seven examples to solve on their own. The data available for analysis is thus restricted.

Using the classification of addition and subtraction word problems proposed by RUMEUS (du Toit *et al*, 1993) the problem attempted by the learners in lesson 2 involving the "consecutive even numbers" may be described as a decontextualised *Combine* problem. In this case information is given about the whole and the various parts required to make up the whole are to be determined. The sharing problem also attempted in lesson 2 may be categorised as a *Change* problem as it involves starting with a single collection and two smaller collections are formed. The action implied here is "sharing". The four problems mentioned in lesson 1, as well as the one problem given for homework in lesson 3, are decontextualised problems.

In order to ascertain whether or not learners consistently recorded solutions as displayed in their tests, work completed as classwork and homework was analysed. These were categorised according to common trends that were observed in the test. The use of the equals sign at the end of a solution, the use of arithmetic to solve a problem, the variety of methods used to "balance" the equations, the setting out of "word" problems, the interpretation and manipulation of the algebraic symbols and the ability to add and subtract integers were considered. The learners who went about solving the test questions and the classwork / homework in a similar manner are isolated below.

5.2.2.1. The use of the equals sign at the end of a solution

Seven learners often omitted the equals symbol, and merely wrote the numerical answer or wrote the equals symbol before the x indicating that the answer was to follow. For example, learner L1 solved x + x + x + 4 = 10 and wrote the final solution as "= x 2".

1) $x + x + x + 4 = 10$

$3x + 4 = 10$

$3x + 4 - 4 = 10 - 4$

$\frac{3x}{3} = \frac{6}{3}$

$\underline{= x\ 2}$

Learner H1's final solution to the same problem did not indicate that the unknown was x. This is how the solution was recorded:

(iii) $x + x + x + 4 = 10$

$x + x + x + 4 - 4 = 10 - 4$

$x + x + x + 4 = 10$

$3x + 4 = 10$

$3 \quad 3 \quad x$

1

$x + x + x + 4 = 10$

$3x + 4 = 10$

$3x + 4 - 4 = 10 - 4$

$\frac{3x}{3} = \frac{6}{3}$

2

Learner I1 solved x + 14 = 28 and recorded the solution as "x 14", i.e. without an equals symbol.

5.2.2.2. The use of arithmetic to solve the problem

Eight learners used arithmetic substitutions to find solutions to algebraic equations. For example, learner D1 clearly knew that in the equation 6x = 18 the value of the unknown x must be 3 but since the teacher marked this solution as incorrect, the learner subsequently unsuccessfully attempted the solution by "balancing". This learner recorded:

(ii) $6x = 18$

~~$6 \times 3 = 18$~~ $\quad 6 \times 3 = 18 \quad = \quad \frac{6x}{6} = 18$

5.2.2.3. The variety of methods used to "balance" the equations

Nine learners did not balance the equations. For example, in the equation x + 14 = 28, 14 was added to the RHS and subtracted from the LHS. Learner G1 added and subtracted 7 from the RHS and subtracted 2 from the LHS. She recorded:

9) $2x = 7 + x$

$2x + 7 - 7 = 7 - 2$

$\frac{2}{2} = \frac{7}{2}$

$x = \frac{1}{2}$

Learner H1 divided each term on the RHS of the equation by 3 and left the LHS unchanged. This learner wrote:

(iii) $x + x + x + 4 = 10$

$x + x + x + 4 - 4 = 10 - 4$

$x + x + x + 4 = 10$

$\underline{3x} + \underline{4} = 10$

$3 \quad 3$

1

$x + x + x + 4 = 10$

$3x + 4 = 10$

$3x + 4 - 4 = 10 - 4$

$\frac{3x}{3} = \frac{6}{3}$

2

5.2.2.4. The setting out of "word" problems

Twelve learners all started setting out the problem, "Share 27 buns between two boys so that one gets 3 more [than] the other", in a similar manner by writing "Let the 1st number be x", etc. The variations are detailed below:

Learners	**Sample** of each learners' response
Q1	*Let the 1st number = x* *Let the 2nd number = x + 1* *Let the 3rd number = x + 3*
D2, E1	*Let 1st number be x* *Let 2nd number be x + 3* *Let 3rd number be x + 6*
F2, G1, G2, L1, M1, M2, N1, N2, O1, Q2	*Let the 1st number be x* *Let the 2nd number be x + 2* *Let the 3rd number be x + 4*

Table 10 *Table showing sample of learners' responses where term "consecutive" translated into mathematical terms in exercise example*

Learners D2, G1, G2, F2, L1, Q1 and Q2 went on to use the first three steps recorded to formulate their first equations. For example, learner Q1 wrote:

Share 27 buns between two boys so that one ... more the other

Let the 1st number = x

Let the 2nd number = $x + 1$

Let the 3rd number = $x + 3$

$$3x + 4 = 27$$

$$3.\ x + 4 - 4 = 27 - 4$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{23}{3}$$

$$x = 7\frac{1}{2}$$

5.2.2.5. The interpretation and manipulation of the algebraic symbols

On attempting the solution to 6x = 18, learner M1 replaced "6x" with "6 + x" and recorded:

11.\ $6x = 18$ $\qquad$ $x = 18$

$6 + x = 18$

$x = 18$ $\qquad$ $x = 3$

In her solution learner M1 replaced the symbol x with the number minus one on the right hand side of the equation:

$2x = 7 - x$ $\qquad$ $2x = 7 + x$

$2x = 7 + x$ $\qquad$ $2x = 7 - 1$

$2x$ ... $\qquad$ $\frac{2x}{2}$ $\frac{6}{2}$

$x = 3$

Learner I1 renamed "7 + x" as "7x" in the following example:

$$2x = 7 + x$$

$$\frac{2x}{2} \quad \frac{7x}{2}$$

$4x$.

Perhaps learner Q3 considered "6x + 12" to be equivalent to 18 by conjoining (Kuchemann, 1981) and manipulated terms in the following manner:

(1) $6x = 18$

$6x + 12 = 18 - 12$

$6x = 6$

5.2.2.6. The ability to add and subtract integers

Learners I1, B2 and D1 incorrectly added, subtracted or divided integers.

Learner I1 considered 11 - 5 to be 16 and recorded:

1. $6x + 5 = 2x + 11$

$6x - 2x + 5 = 2x - 2x + 11$

$4x + 5 = 11$

$4x + 5 - 5 = 11 - 5$

$= \underline{16}$

4

$x = 4$

Learner B2 considered 7 - 2 to be 6 and wrote:

(i) $2x = 7 + x$

$2x + 2 - 2 = 7 - 2$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2}$

$x = 3$

Learner D1 consistently considered 18 ÷ 3 to be 16 and wrote:

$\cancel{3}x \rightarrow \frac{\cancel{3}x}{\cancel{3}} = \frac{\cancel{18}}{\cancel{3}}$

16

5.3. Discussion

The common trends that were observed in the test and in the learners' classwork and homework have been documented in the literature. These may be classified as the use of the equals sign at the end of a solution, the use of arithmetic to solve a problem, the "balancing" of equations, the setting out and translation of word problems, the interpretation and manipulation of the algebraic symbols, the performing of operations using integers and the interpretation of subdivisional labels.

- The use of the equals sign at the end of a solution was discussed by Human (1989). He states that learners sometimes consider the is equal to sign ("=") to mean "do something" instead of using the symbol to indicate that two different expressions are equivalent.
- The use of arithmetic in order to solve a problem was also observed by De Villiers (1991). It seems that learners were able to solve some problems without relying on unnecessary, cumbersome procedures.
- The variety of methods used to "balance" the equations indicates that learners needed to develop understandings as well as skills in relation to making equivalent expressions/equations (Human, 1989).
- Difficulties experienced in the setting out of "word" problems point to the fact that learners needed to have a clear concept of the meaning of the variable (De Villiers, 1999). The learners possibly experienced difficulties in translating a word problem from the language of everyday life into the language of algebra because of the complex nature of the transition from rhetorical to symbolical algebra (Hogben, 1945). The problem-centred approach, according to Smit (1995:1), counteracts postponement and suspension of sensemaking that exists in traditional drill approaches. Learners may therefore have benefited by being allowed to make sense of word problems before practising skills and procedures.
- The obstacles encountered in interpretation and manipulation of the algebraic symbols may possibly have been overcome by developing relational rather than an instrumental understanding (De Villiers, 1999). The learners may

have been able to see the interconnections that exist in the mathematical content if the learners were permitted to use shorter, quicker and more economical methods.

- The difficulties experienced in addition and subtraction of integers may have occurred as unforced errors or perhaps as a result of limited experience at performing operations with integers.
- The setting out of a question in an unfamiliar format may confuse learners. Warren (1999) points to literature that indicates that the assigning a letter as a subdivisional label, such as 3a, may bewilder learners. From this research it appears that using numerical subdivisional labels was also a stumbling block.

Chapter 6: Teacher Strategies and Learners' Performance

What is the relationship between the teacher's strategies and her learners' performance?

6.1 Introduction

In order to attempt to identify whether teacher strategies contributed to the development of misconceptions and to explain why the learners performed better at some test problems than at others, the lesson transcripts were analysed according to how similar problems were demonstrated and explained by the teacher during lesson presentations. The lesson transcripts were sectioned and analysed according to how the learners were taught to solve problems that were similar to specific test questions. The analysis of the transcriptions of lessons is partitioned into sections 1 to 7 according to explanations of problems similar to those in the test. 6.2.1. indicates the section dealing with explanations of problems similar to those in which learners fared the worst in the test with 6.2.7. being the section dealing with problems similar to those in which the learners fared the best in the test.

6.2. Analysis of data

6.2.1. Test question 2 (a)

The learners scored the least number of marks in the test for the question that was displayed on the chalkboard as:

Question 2

(a) Give the additive inverses of the following:

1. +4 *2. -3* *3. –2*

The teacher often mentioned "additive inverses" during lessons. The table below indicates the number of times the term "additive inverses" was used during the three lessons.

Lesson	Number of times mentioned by teacher	Number of times echoed by learners
1	8	4
2	5	1
3	13	3

Table 11 *Table showing number of times term "additive inverse" was used by teacher and learners in various lessons*

The learners thus heard the term "additive inverse" 26 times and chorused the term 8 times during the three lesson presentations but 13% of the learners did not even attempt this problem. The learners were introduced to the term "additive inverses" in lesson 1 in the following manner:

T: *...So we have to calculate using what... for example, what is positive one plus negative one? Hands up. What is the answer here? Yes? Yes?*

L1: *Zero.*

T: *Yes, zero. What is negative four plus positive four? Class?*

L: *Zero.*

T: *Zero. What is negative 100 plus positive 100? All of you it is..?*

L: *Zero.*

T: *Zero. So, let's say here, let's say here that given this four and that given this six. You want to remove this positive four before in order to get what? Zero. You see what we call an additive inverse. We call what?*

(On the board the teacher demonstrated using the example x + 4 = 6.)

L: *Additive inverse.*

T: *Additive inverse of, for example, of positive one is negative one. What goes with x and positive one and negative one is '... you get what?*

L: *Zero.*

T: *Which means that x is in the opposite one is negative one and again opposite one is...*

L: *Negative one.*

T: *Because they give you what?*

L: *Zero.*

T: *So, if you use the additive inverse, you get what?... Zero.*

Here the teacher emphasised that the sum of a number and its additive inverse is zero. The teacher did not explain why the additive inverses are useful i.e. the functional understanding was not considered. The teacher placed much emphasis on the instrumental understanding required for the technique to be demonstrated.

On the chalkboard the teacher wrote the following examples, together with the term "additive inverse":

$$^{+}1 + {}^{-}1 = 0$$
$$^{-}4 + {}^{+}4 = 0$$
$$^{-}100 + {}^{+}100 = 0$$
additive inverse

The teacher used these examples to demonstrate the theory that was later to be applied in solving the equation. In the example that followed, the teacher demonstrated how to solve for x using the equation x + 4 = 6. She said:

> *...Right, so let's write the unknown x, so the additive inverse of positive four is negative four. ...*

The teacher thus renamed the "plus four" as "positive four". The teacher then used the example, "x minus two equals to four", but asked for the additive inverse of "negative two". This is what the teacher said:

> *Let's say you are given this. Let's say you are given x minus two equals to four. Find the value of x. Again you use whatever? You use the additive inverse of? Negative two. What is the additive inverse of negative two? Hands up. Hands up so I can see. Yes.*

Once again the teacher renamed the "minus two" as "negative two".

The teacher often renamed the "minus" as "negative and "plus" as "positive" during presentations. The teacher did not, however, use the notation using the superscript, i.e. $^{+}4$ or $^{-}2$, again after the three examples were written on the chalkboard when the teacher introduced the learners to this term in lesson 1. The learners always made use of additive inverses whilst dealing with equations. The "+" and "-" signs seen in equations did not represent "positive" or "negative" but were perhaps interpreted as symbols indicating an operation.

In the test question 2 (a) the learners were asked to find the additive inverses of integers that were not part of an equation. The learners, other than simply observing the three examples written by the teacher on the chalkboard, did not use the notation used in the test question. The additive inverses of the integers given in the test were not written with a superscript but as *+4, -3 and –2.* Perhaps this is one of the reasons why only 87% of the learners attempted this test question and only 24% of these learners gave the correct solution. The notation used in the test may well have been unfamiliar to the learners.

During the observed lessons the learners were reminded to "remove" terms as well as "bring" terms to the other side using additive inverses. The teacher treated the terms as objects, or the unknown as "the thing that we don't know", and used additive inverses in order to isolate the unknown. The teacher did not stress the importance of ensuring that equivalence is maintained when the balance algorithm is employed. The teacher emphasised the fact that the sum of the number and its additive inverse is zero and thus the unknown becomes isolated on one side of the equation. During lesson presentations the additive inverses of terms were not dealt with as representing integers but as objects that had to be removed. The learners always found the additive inverses of terms in an equation by dealing with the unknown as if it were an object.

The setting out of the question may also have confused the 33% of the learners who attempted this test problem. The subdivision of the question (a), using numbers, may have been an unfamiliar format. This particular setting out of subsections was only seen in the test and was not used by the teacher to set out problems during lesson presentations.

6.2.2. Test question 1 (b)

The test question was:

Solve these equations by writing all the steps:

(b) $x - 9 = 1$.

An example similar to this was demonstrated during lesson 1. The solution of the linear equation $x - 2 = 4$ was the second example presented by the teacher. The teacher described the step by step procedure as follows:

T: *Let's say you are given this. Let's say you are given x minus two equals to four. Find the value of x. Again you use whatever?.. You use the additive inverse of? Negative two. What is the additive inverse of negative two? Hands up. Hands up so I can see. Yes.*

L 2: *Two.*

T: *The additive inverse of negative two. Speak aloud.*

L: *Positive two.*

T: *So it will be x minus ...All of you...*

L: *Two.*

T: *Plus, let's work together, equals to...*

L: Four.

T: *All of you.*

L: *Plus.*

T: *What did you put here? Four. All of you.*

L: *Plus.*

T: *Plus.*

L: Two.

T: *So x is equal to four plus two is?*

L: *Six.*

T: *Is it easy?*

L: *Yes.*

The learners said that this solution was easy, but solutions to mathematics problems may have appeared to be easy when demonstrated and where no reflection is required on the part of the learners who are merely observing the demonstrated procedure. The learners were not given the opportunity to suggest a possible method of solution as they were told to give the additive inverse of "negative two" and reminded to "add" two.

In lesson 1 the learners were also reminded of what an equation is. The learners were not told why they must balance the equation. This was how the learners were prompted:

T: *...mind you, I told that in mathematics what you do on the right hand side you must do on the left hand side, so this is an equation, you see this is an equation . An...*

L: *Equation.*

T: *An equation normally has two sides, right? So the equal sign that divides these two sides, right? So, this is the right hand side and this is the...*

L: *Left hand side.*

T: *Left hand side of the equation. So, here we have put negative four on the left hand side of the equation, right? So, what you do on the left hand side you also do on the...*

L: *Right hand side.*

T: *Right hand side. We have placed here negative four. It means here we are going to put again negative four. Siya bona?* (Do you see?)

L: *Yebo.* (Yes.)

Here the learners were told how to balance an equation but no mention was made of equivalence, i.e. that the resulting equation has the same solution as the original equation. The learners were instructed to manipulate according to the rules provided by the teacher. Perhaps the learners did not know why the same amount had to be added or subtracted from each side of the equation as 45% of the learners developed alternative incorrect strategies to "balance" the equation.

The meaning given to the equals sign is that of a "barrier" as the teacher considers the sign to "divide" the two sides of an equation. An equation is not portrayed as one continuous entity but as two separate parts, each part of the equation requiring manipulation. In the test responses the learners used a variety of incorrect methods to "balance" the equations. Learners' attempts at balancing the equation $x - 9 = 1$ are illustrated in 5.2.1.3.2. The need to ensure that equivalent equations are found when solving equations was not emphasised and was not made clear to the learners.

Unfortunately the word "divide" also has other connotations in mathematics and use of this specific word in explaining what an equation is could have misled the learners. The emphasis on using the exact terminology required by the teacher during lesson presentations could perhaps make the learners see an equation as involving the division process. However, in the test analysis the learners did not appear to interpret an equation as involving division.

6.2.3. Test question 2 (b)

The test question was:

Find three consecutive natural numbers whose sum is 42.

An example similar to this was demonstrated during lesson 2. The problem demonstrated was "Find three consecutive natural numbers whose sum is 63". The teacher introduced the problem in lesson 2 as follows:

T: *Find three consecutive natural numbers whose sum is sixty-three. Read all of you.*

L: *Find three consecutive natural numbers whose sum is sixty-three.*

T: *Come again.*

L: *Find three consecutive natural numbers whose sum is sixty-three.*

T: *OK. They say find how many numbers?...*

L: *Three.*

The learners are then reminded of what Natural numbers are by asking:

T: *What, which numbers?*

L: *Natural numbers.*

T: *Do you know natural numbers?*

L: *Yes.*

T: *Can you give me natural numbers between zero and ten? Hands up. Natural numbers between zero and ten. Hands up. I said hands up. Yes,* L3.

L3: *1; 2; 3; 4; 5; 6; 7; 8; 9.*

The teachers then went on to explain the meanings of "consecutive" and "sum".

T: *Good. They say give natural numbers. So, three natural numbers, right? So, first of all here, if I should get this correct, I have to understand English. Each and every word here. Find three, OK, we know that there are three. Consecutive, let's look at this separate word "consecutive". Find how many numbers?... Three. Those numbers which are... consecutive. It means the numbers which follow one another. So, it means if the first natural number is one, so, it will be followed by two,* *right? And a third. So, find three consecutive natural numbers whose sum is...*

L: *63.*

T: *What is the sum? Do you know the sum? What is the sum? Hands up. Yes.*

L4: *It is the number you get when you add.*

T: *Is it the number? Stand up. The sum...*

L4: *The sum is the number.*

T: *Is it the number or the answer?*

L4: *It is the answer.*

T: *Can you help her? Is it the number or the answer?*

L: *Answer.*

T: *OK. Come again. The sum is the...*

L: *The sum is the answer when we added...we add.*

T: *When we... the answer you get, when we...*

L: *Add.*

T: *All of you. When we...*

L: *Add.*

The teacher did not explain the term "consecutive" by giving examples from "everyday" usage for example, "It rained for three consecutive days". No numerical examples of three consecutive numbers were given as illustrations of the term. The teacher defined the term as numbers that "follow one another".

The teacher was not satisfied with L4's answer to the question "What is the sum?" L4 said "It is the number you get when you add." but the teacher wanted the exact response "The sum is the answer when we add". The learners were then quizzed to ascertain whether they remembered what a "quotient", "difference" and "product" meant and then returned to solve the problem "Find three consecutive natural numbers whose sum is 42.".

T: *So here we will get the sum. It means after you get those three numbers you are going to get the sum. So, because we are told in mathematics for the thing that we don't know, for the unknown we use x. Cause we don't know those three numbers, we are going to find first the first number. So, we are going to let the first number, be what... All of you. Be...*

L: *x.*

T: *Because they are natural numbers, we are going to let what be the second number be... what?...Be..*

L: *x plus...* (mixed response)

T: *Be... x plus first natural number, which is one. Again the third number. Let the third number be what?...*

L: *x plus two.*

The teacher wrote on the chalkboard:

Let the 1st number be x
Let the 2nd number be x + 1
Let the 3rd number be x + 2

The teacher then continued with the solution by linking the three consecutive numbers as follows:

T: *So now we are having how many numbers? One, two, three. They are going to total up to sixty-three. So let us add them. It will be...All of you...*

L: x

T: *Plus.*

L: *x plus one.*

T: *Plus.*

L: *Two.*

T: *Is it two or x you are going to add to x, plus x plus one plus...*

L: *x plus two.*

T: *So it will be...All of you.*

L: *x plus x plus one plus x plus two.*

T: *Equal to...*

L: *Sixty-three.*

The teacher completed the solution and substituted the value for x into the expressions x + 1 and x + 2 but did not check whether the sum of the numerical answers, i.e. 20, 21 and 22, was 63. The learners were not given the opportunity to copy down the solution to this problem. Thus if the learners wanted to revise for their test, this problem and its solution were not recorded in their exercise books. This example demonstrated on the chalkboard was almost identical to the test problem and the learners would therefore have had to rely on what they remembered/understood to solve the test problem.

At the end of the demonstration the learners were asked, "Is this clear?" and they chorused "Yes". Yet when the teacher asked this question again, the teacher appeared to sense that this problem was not fully understood by the learners. The teacher indicated this when she said:

> *...Is it clear? Are you happy? Can you write on your own? No. Is it clear? Yes. Can you write on your own? No. What is clear if you can't write? Is it clear? Yes. Hands up. So let's try this class exercise. Open up your exercise books. Write today's date. Let us hear whether you are able to say "hard". You want you to show me what was clear. Try that one. Find three consecutive even numbers, not natural numbers now, even numbers, whose sum is 27.*

The learners did, however, have the solution to the "class exercise" "Find 3 consecutive even numbers whose sum is 27" written in their exercise books. The learners were not encouraged to write down the word problem but to start with the solution that was recorded in their exercise books for them to refer to at a later stage. 67% of the learners who sat for the test and handed in their exercise book did not write down the original problem. The solution to this problem was demonstrated on the chalkboard because many of the learners were not getting the first three statements correct. The teacher decided to do the problem on the chalkboard after these negative, rather disparaging, comments:

> *...I said the even number, that why I said the first step. Not the natural numbers, not the odd numbers but even numbers. You are comfortable with even numbers. This is good, continue. This is wrong. Read the statement very carefully, find three even numbers. Maybe I can send you back to standard five. I'm going to send you to attend standard five or even standard four. Wrong. Maybe, if I had a stick you will find the even numbers. Maybe your tears will help you. I think so. I said are you clear with this. You said yes. OK, right, you say. So I was teaching only this one. Good. Second one. The second witness. Where did you get what? Where are other witnesses? I'm giving you only three minutes. Wrong. Maybe, if I had a stick, you would find those even numbers. On this side everybody is dead. Incorrect. What's wrong with you? Correct. Correct. Find what you need. OK. Are you going to get the answer right? Correct. Good boy, continue. After you have waked up from the grave. Just call me I'm coming. Wrong. Wrong. This is wrong. Others have finished number one. So let's do together. Right.*

Note: Here teacher moved around amongst learners to correct attempts. Sometimes her comments to individual learners were indistinct.

The teacher recorded the first three statements on the chalkboard as:

> *Let the 1st number be x*
> *Let the 2nd number be x + 2*
> *Let the 3rd number be x + 4*

According to the teacher these three expressions represented the three consecutive even numbers called for in the class exercise. In the test, four learners used the teacher's interpretation of "consecutive even numbers" to mean "consecutive". This was perhaps because the learners had these statements recorded in their exercise books and they considered this to be what "consecutive" meant.

25% of the learners managed to obtain the correct value for the first of the three consecutive numbers but did not substitute to find the other two consecutive numbers. Perhaps this may be ascribed to the fact that the teacher demonstrated substitution for only three of the problems dealt with during the three observed lessons. The teacher substituted to find the three consecutive numbers in "Find three consecutive natural numbers whose sum is 42" and used substitution to find the "three consecutive even numbers whose sum is 27". Only one problem was checked using substitution. After solving the problem "Nomsa owns 56 chickens. How many more chickens must she buy to have 100 chickens altogether?", the teacher checked the solution by substitution. No other problems were checked by substitution into the original problem. The learners were thus not encouraged to use substitution to check their answers.

6.2.4. Test question 1 (e)

The test question was:

Solve these equations by writing all the steps:

(d) *A man owns 48 sheep. How many more does he need to have 96 sheep?*

An example similar to this was demonstrated during lesson 2. The solution of the word problem "Nomsa owns 56 chickens. How many more chickens must she buy to have 100 chickens altogether" was the first contextualized problem presented by the teacher. The teacher described the step-by-step procedure as follows:

T: *I'm having a problem here to do. Here maybe it could be a problem. Let's read this. Nomsa owns fifty-six chickens. How many more chickens must she buy to have a hundred chickens altogether? Read all of you.*

L: *Nomsa owns fifty-six chickens. How many more chickens must she buy to have a hundred chickens altogether?*

T: *Nomsa owns how many chickens?*

L: *Fifty-six chickens.*

T: *OK. So we know the number of chickens Nomsa owns. So we know how many chickens?*

L: *Fifty-six chickens.*

T: *Fifty-six chickens. So our problem is that we don't know the number of chickens she must buy to have, how many chickens?*

L: *A hundred chickens.*

T: *A hundred altogether. We don't know the number of chickens she must buy. So, I told you that for the unknown we use what?...x. So we are going to say x plus the number of chickens she had, fifty-six...*

L: *Chickens.*

Here the teacher recorded x + 56 = 100 on the chalkboard instead of the way she vocalised the problem which was 56 + x = 100. The teacher did not emphasise that x represented the **unknown number** of chickens, x just represented the unknown. The solution was continued as follows:

T: *Chickens equal to...*

L: *A hundred*

T: *Then it will be easy. Solve for the unknown, right?*

L: *Yebo.* (Yes.)

Once again the learners considered the solution of the equation to be an easy procedure as they were now familiar with manoeuvring terms in decontextualised problems. The solution was continued as follows:

T: *Let's work together. It will be...*

L: *x plus fifty-six minus fifty-six is equal to a hundred minus fifty-six.*

T: *Right. Next step will be...*

L: *x is equal to...*

T: *A hundred minus fifty-six is...*

L: *Forty-four.*

T: *Forty...*

L: *Four.*

The teacher wrote on the chalkboard:

$$x + 56 = 100$$
$$x + 56 - 56 = 100 - 56$$
$$x = 44$$

The teacher went on to check the solution and confirm that the learners were able to see (understand?) the procedure as follows:

T: *So now we know the number of chickens Nomsa must have. So, as she was having fifty-six, when we added fifty-six and this forty-four, you get...*

L: *A hundred.*

T: *So it means this is correct. Any questions?*

L: *No.*

T: *OK. No questions. Any questions?*

L: *No questions.*

The learners again confirmed that they had no questions, but the step-by-step procedure demonstrated probably appeared easy to follow whilst the teacher was ordering the maneuvers in a logical sequence. The marks obtained for the similar test question showed that 60% of the learners who attempted this test question were not able to find the equation needed and/or to use the teacher's step-by-step method when it came to doing the problem on their own.

During lesson 1 the teacher used only decontextualised problems to facilitate the learning of the algorithm. Only during lesson 2 were the contextualised problems considered. The learners using their knowledge of arithmetic could easily solve the word problems chosen by the teacher. Perhaps this is why 11% of the learners who attempted a solution to the test question just gave the correct solution without using the step-by-step procedure taught.

Although the teacher referred to x only as the unknown, 32% of the learners who attempted the solution to the test question tried to explain what the unknown represented. These learners started their working by writing "Let 1st number be x" etc. Two learners interpreted the unknown "x" as standing for an object in its own right. (See 5.2.1.3.4.) On completion of the solution the teacher did, however, state that the number of chickens was found and the solution was confirmed according to the information given.

6.2.5. Test question 1 (d)

The test question was:

Solve these equations by writing all the steps:

(c) $3x + 7 = 25$

60% of the learners who attempted this test question obtained incorrect solutions, but it is surprising that this problem was tackled with more success than the "easier" problem $x - 9 = 1$. Perhaps this is because the teacher demonstrated three problems similar to this test question, $3x + 7 = 25$. During lesson 1 the teacher used $2x + 4 = 6$, $3x - 4 = 8$, and $3x + 4 = 10$ as examples to illustrate the procedure. The teacher did these three examples after illustrating how to solve $x + 4 = 6$, $x - 2 = 4$, $4 + x = 6$ and $x + 11 = 22$. Perhaps the learners had become more familiar with the procedure required by the time this type of problem was presented. The teacher had thus started with easy problems and then progressed to solving more "complicated" problems. The learners also had the opportunity to record the solution to $3x + 4 = 10$ in their exercise books. This problem is very similar to the test question.

6.2.6. Test question 1 (c)

The test question was:

Solve these equations by writing all the steps:

(c) $2x = 10$

The procedure required to solve for x was dealt with in lesson 1. The teacher first gave the learners the name of the term "multiplicative inverse" and then told them how to find the multiplicative inverses of a variety of numbers. This is how the teacher introduced the "removing" of the co-efficient:

T: *So they say find the value of x not the value of two x. Right, what are we going to do now? So now we are having two x is equal to two. This is not the answer. You see this is not the answer, they say find the value of x not the value of two x. What are you going to do now? Right? I told you there is something called what ...*

L: *Additive inverse*

T: *Read this word. Multiplicative inverse. Class...*

(On the board the teacher wrote "multiplicative inverse")

L: *Multiplicative inverse.*

T: *Again.*

L: *Multiplicative inverse.*

T: *Again.*

L: *Multiplicative inverse.*

The teacher went on to remind the learners of the position of a coefficient in relation to the variable. During the explanation the terms variable and unknown were used interchangeably. The learners were not reminded of the fact that 2x means "two multiplied by x" or shorthand for "x + x". Perhaps the learners were unsure of what 2x meant and that is why 24% of the learners used alternative incorrect interpretations for 2x.

The division process was used as if it were a mechanism by which one cancels the coefficient, since the teacher considered that the unknown is what is "left". A few examples were given to reinforce which multiplicative inverse was to be used in specific cases. In lesson 1 the teacher explained the procedure used to isolate of the variable in the equation 2x = 2 as follows:

T: *So, here in this case of a variable and a coefficient, you are going to use the multiplicative inverse so that you can get the value of the unknown. So here you are having the coefficient two and the variable x, so the multiplicative inverse of this two will be what? Two. So that number will divide that number and will be left with the variable. So that will mean the coefficient of x is two. If you were given four x, what will be the multiplicative inverse of this one? Sorry... What will be the multiplicative Inverse of four?*

L: (Indecipherable responses)

T: *Of four? Here we are having two then we use two. So, of four will be?*

L: *Four.*

T: *Four. What will be the multiplicative, multiplicative inverse of three x?*

L: *Three.*

T: *So if you are given eight x, we are going to divide by...*

L: *Eight.*

T: *If you are given ten x , you are going to divide by...*

L: *Ten.*

T: *Right. So let's divide by two. Two into two how many times...*

L: *One.*

T: *So we are left with... we left with...*

L: *x.*

T: *Equal to...*

L: *Two.*

When the teacher divided the LHS of the equation by two, she did not indicate that two divided by two is one but recorded on the chalkboard:

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{2}{2}$$
$$x = 1$$

The teacher cancelled the twos on the LHS of the equation as if the numbers were being eliminated and did not indicate that two divided into both twos to obtain a quotient of 1.

The teacher then reminded the learners of the difference between the terms "multiplicative inverse" and "additive inverse" as follows:

T: *So now we have the value of x which is two. Not the value of two x. So it differs in a way...So for you today you need to know the difference between the additive inverse and the...*

L: *Multiplicative inverse.*

T: *And the...*

L: *Multiplicative inverse.*

Throughout the presentations the teacher placed much emphasis on the pronunciation and labelling of mathematical terminology. Knowing the name of the term "multiplicative inverse" did not, however, assist the learners in solving the equations.

The second example attempted by the learners in their exercise books was $6x = 18$. The teacher gave the following instructions to the learners whilst she went round checking solutions. The learners were not given the instruction "Solve for x" but the problem and solution were recorded on the chalkboard. The teacher did not interact with individual learners but said:

T: *So here we are given six x equals to...*

L: *Eighteen.*

T: *Let's look at the board. You are given...*

L: *Six x equals to eighteen.*

T: *So don't be using multiplicative inverse only. You can divide by six both sides. So then you cancel. Six into eighteen, how many times?*

L: *Three.*

Thus the learners had the solution to an example similar to the test question in their exercise books that could be used to refer to if they wished.

6.2.7. Test question 1 (a)

The test question was:

Solve these equations by writing all the steps:

(a) x + 20 = 36

This was the question in which the learners experienced the most success and 47% of the learners obtained the correct solution. The teacher demonstrated two similar problems on the chalkboard during lesson 1. The two problems were x + 4 = 6 and x + 11 = 22.

The learners did have the solution to a problem similar to this test question recorded in their exercise book. The first problem they attempted on their own was x + 14 = 28. Whilst the learners were trying to solve this problem, the teacher went round the class and gave individual comments indicating whether or not the step completed was correct. The teacher did not guide the learners in solving this problem. The teacher did not interact with the learners, she merely told them whether the solution was right or wrong. There was no written instruction on the chalkboard, i.e. the teacher did not write "Solve for x" but told the learners to solve for x and wrote only the problem x + 14 = 28 on the chalkboard. The teacher said:

T: *OK, so let's quick open your first page of exercise book. Write your name in front. At the front of the exercise book write your name and you surname please. Write your name. So I'm giving you only two minutes to* *do this one. Write today's date on top please. Write today's date. Try and solve this one. So you write today's date, write the topic "linear equations" and then you solve for x. Then x plus fourteen equals to twenty-eight. Write the first step and raise up your hand. Quickly write the first step and raise up your hand. Has anyone finished the first step? Don't be afraid. Raise up your hand after the first step. OK. Try this one. Try this one. I know some of you will say "We didn't do this one", but we did it. We did this one. Use the short method and get the answer. Has anyone seen the first step of this? I'm giving your only two minutes for the first step of this.*

Whilst the teacher walked around amongst the learners marking their work with a pencil she commented on their attempts using the following comments according to whether the work was correct or incorrect.

Wrong.
Very good.
Very wrong.
Very, very wrong.
Very good.
Very wrong.
Very wrong.
Very wrong.
Very, very good.
And I see some of you are finding problems with this one. So it is ....

Here the teacher urged the learners to solve the problem quickly using the "short method". The teacher probably referred to the method she had showed the learners as the "short method". If the learners had, however, used their knowledge of arithmetic, the method advocated by the teacher would prove to be somewhat "longer". Furthermore, using the balance algorithm causes the suspension of the meaning of variables whereas using inspection reinforces the meaning of the variable and of an algebraic expression.

6.3. Discussion

It appears that listening and chorusing was not enough to ensure that learners would be competent at mathematics. Perhaps the learners may have been gaining some social knowledge by these passive modes of tackling mathematics, but, to enable learners to gain logico-mathematical knowledge it would appear that more active reflection and participation were required.

The unfamiliar notation used as subdivisional labels, such as 2 (a) 1., may be seen to supply the learners with social knowledge that was foreign. The learners were not provided with any written mathematical text so the methods or types of setting out of subdivisional labels on the chalkboard were the only text to which learners were accustomed. Without worksheets or textbooks the learners had little experience of various types of setting out of problems.

The learners often chorused whilst the teacher prompted them during lesson presentations but this may be inadequate when learners were expected to **write** tests on their own and not as an oral group effort. When revision for tests was needed, the learners were also disadvantaged as they had very little written reference material. The teacher did not permit writing down of exercises whilst lesson presentations took place. The learners experienced more difficulties in solving the problem $x - 9 = 1$ than with the problem $3x + 7 = 25$. Perhaps the learners made use of their written solution of the example $3x + 4 = 10$ (similar to $3x + 7 = 25$) in their exercise books to revise for the test. This written recipe may have provided the learners with the guidance they required for revision purposes.

Throughout the observation period very little variety in methods of solution or methods of presentation of problems was seen. The learners were not encouraged to make use of substitution to solve or check solutions, although using a substitution method would have been appropriate for many of the rather trivial problems that the learners were expected to solve using an elaborate "step-by-step" method. It appears that many learners used their own intuitive techniques to find the solutions but were hampered by the fact that they were expected to use the method upon which the teacher insisted.

Chapter 7: Concluding Remarks

7.1. Synthesis

In South Africa there appears to be two dominant views on the nature of mathematics and the one adopted influences the manner in which the subject is taught. These outlooks, behaviourist and constructivist, may be compared in the following manner:

	Behaviourism	Constructivism
Nature of Mathematics	Set of concepts, rules, theorems and structures	A variety of processes e.g. generalising, classifying, formalising, organising, abstracting, translating, validating, conjecturing, reflecting, modelling and exploring pattern
How learners are seen	As "empty vessels" that need to be "filled" with knowledge	As active mathematical thinkers who try to construct meaning and reflect
What "learning" means	Behaviour modification by positive and negative reinforcement	Structuring and restructuring of conceptual schemas via the processes of assimilation and accommodation
What "learning" is about	Emphasising procedures and manipulation techniques	A process where reflecting on physical and mental actions occurs
What "teaching" is about	Showing, explaining and telling	Challenging, questioning and guiding
What the result of teaching should be	Stockpiling of knowledge	Conceptual restructuring by accommodation and assimilation
What learners should do	Follow, repeat, memorise and practice	Do, investigate, think and apply
What activities should be assessed	Overt, observable behaviours	Overt (observable) and covert thinking

How learners should be assessed	By completing graded test problems similar to those practised in class in order to determine how much the learners know/remember	By discussing/using processes necessary to solve novel problems in order to examine learners thinking

Table 12 A comparison between behaviourism and constructivism

The recently introduced Outcomes Based Education (OBE) in South African schools appears to be based on constructivist theory but also has a behaviourist flavour to it. In OBE there appears to be emphasis on observable learner behaviour, but for meaningful learning to take place it is necessary for the learners to be actively involved in mathematization and reflection. It would appear that mathematical concept formation would be hampered in a classroom that is completely teacher-centred and where no allowances are made for any meaningful social interaction or problem solving.

Learners sitting passively at their desks, collectively chorusing words (not sentences), would be totally unsatisfactory method to engage learners in making sense of the mathematics; it would rather serve as an effective way of submerging any constructive processes that may be taking place. Such a teacher-centred method does not allow for development of sensemaking or enhance learners' innate ability to solve problems. According to a research survey by Schoenfeld (1991, in Wilson *et al*, 1993), there were negative consequences of traditional mathematics teaching in schools. It was found that where students were steeped in procedure-orientated practices the learners became willing to engage in mathematical activities that are nonsense. For example, using the balance algorithm to solve $4 + x = 6$ would be an "over kill". Schoenfeld found that teachers unwittingly assisted their learners in the suspension of sensemaking by providing them with rules to memorise and use. This type of "problem-solving" ultimately becomes "finding a solution" and not necessarily "understanding the problem".

The theories proposed by Vygotsky and Piaget, in particular, have yielded insight into and understanding of what was observed at Angaziwa High School. There appear to be serious drawbacks in teaching mathematics to Zulu speaking learners through the medium of English only. This study cannot provide recommendations as no comparisons between using this medium of instruction and other strategies for teaching mathematics to second language learners was conducted. Perhaps code switching is a possible solution, but some learners may be inclined to ignore the teacher's explanations in English and concentrate only when isiZulu is used. The speaking of English would then serve only as a means of providing the learners with the opportunity to hear and perhaps acquire a second language, English. Making use of isiZulu in the mathematics classroom may allow learners to believe that the mathematics content "belongs" to them and perhaps they may not feel excluded from the subject. All communications within the mathematics classroom are meant to provide the learners with a means of understanding the mathematics and, if this is the focus of language used, questions need to be asked about the current exclusive use of English in mathematics classrooms. Unfortunately only English and Afrikaans are used in the setting of matriculation examination papers and this probably drives language policies in high schools.

The learners in the classroom observed never had the opportunity to explain or say exactly what their understandings of "solving linear equations" were. The teacher asked the learners if they understood and if they did not, they simply replied, in unison, "no". The learners were seldom treated as individuals but rather as a collective group. By making exclusive use of teaching to the whole class, the teacher may have overlooked the fact that the personal mathematical development of each learner should be the focus in the learning situation. How each learner makes sense of the mathematics cannot be ascertained from this practice. Perhaps the teacher usually teaches very large classes and this is the only strategy that enables her to control and teach simultaneously. On the 13 August 1999, Ms Fundisi told the researcher that she had to teach 135 learners as one group since three mathematics teachers had been re-deployed from the school, yet there were only between 38 and 51learners in a class (numbers varied daily) whilst the researcher was at Angaziwa.

The manner in which the teacher tackled the solving of linear equations probably fits into the "instrumental" mode described by Skemp. There was no development of "functional" or "relational" understanding as described by De Villiers (1999). The teacher did make use of inverse operations to explain how the unknown should be isolated but no mention was made of the concept of "equivalence". The teacher used the "drill and practice" method whereby the solutions to a few linear equations were demonstrated and then the learners were given some to try on their own. As the learners were not provided with textbook or photocopied exercises, the practice involved trying at most three examples individually. Often teachers advocate a "Practice makes perfect" approach when encouraging learners to study mathematics. In solving mathematical problems, however, it may not always be possible to apply techniques in a routine manner, particularly when non-routine problems are posed. None of the problems used by the teacher at Angaziwa High School could be considered as non-standard, so perhaps more practising of routine problems would have benefited these learners.

The April 2000 issue of *Mathematics Education Dialogues* (Vol 3 Issue 2), a publication of the National Council of Teachers of Mathematics (NCTM), is devoted to discussions about teaching algebra. This issue is entitled "Algebra? A gate! A barrier! A mystery!" and highlights the controversy surrounding what algebra is, when it should be taught and to which learners. In the article there is no unanimity about numerous questions raised about the teaching of algebra. It does, however, appear possible to provide suitable learning experiences for Senior Phase learners to develop concepts required for understanding algebra. The solution of equations may be introduced in appropriate ways so that the learners become aware of the meaning and use of equivalent expressions.

Learners may make mistakes no matter what teaching strategies and terminologies are employed, but perhaps if more "functional understanding" is promoted to develop the concept, and fewer "drill and practice" methods are employed to introduce the topic, learners may understand more about these abstract algebraic ideas. Perhaps some learners are not ready for the kinds of abstract formulation required of them in present methods of teaching algebra.

In what Lakatos (1989:36) calls a "deductivist style", the learners of mathematics at Angaziwa High School were obliged to attend mathematics lessons where a conjuring act was performed. Here the audience was never allowed to ask questions about the background of the act or about how the teacher's sleight –of-hand had been prepared. In this style, mathematics is presented as an ever-increasing set of endless, permanent facts and an authoritarian air is secured for the subject because the learners are not exposed to the reasons behind developing the set of procedures displayed.

7.2. Recommendations and conclusion

Unfortunately, learners at Angaziwa High School would not be able to make use of computers to improve their algebraic skills, as there is no electricity available at the school, but a more learner-centred approach may have benefited them. Perhaps allowing learners to develop their own strategies, together with peers, then as a whole class by means of discussions, would have allowed for a manageable plan to develop for the solving of linear equations. This sentiment was echoed by two first year student teachers that are currently at a College of Education in Kwa Zulu-Natal. When newly matriculated learners were asked what they liked about high school mathematics, two students responded:

- I liked the fact that we would be given work to do and then allowed to do it, so we didn't have to sit and listen to the teacher all the time, as well as the fact that the teacher was available to help us.
- I liked being left alone to solve a Mathematical Problem, because when a teacher used to confront me I used to go blank so on my own I was able to try.

This research has exposed the need for in-service opportunities. Teachers of mathematics need to have the chance to develop their teaching skills. Perhaps teachers' personal theory about the nature of mathematics needs to be explored, as these assumptions will certainly effect the manner in which the teacher will present mathematics to learners. According to Fey (1999:15) our teaching traditions encourage learners to acquire routine procedural skills "through a passive classroom routine of listening and practising". If we were to focus our teaching on enhancing learner thinking then teachers making use of the NCTM Standards proposals should bear in mind that:

> Mathematical ideas should be developed through student work on interesting and challenging problem-solving tasks, often in problems that have authentic contexts.
> Students should very often collaborate in mathematical problem-solving and explorations with the teacher acting as stimulant and guide rather than as an expositor.
> Procedural knowledge should be developed on a foundation of conceptual understanding.
> Technology can be a powerful tool in helping students to learn mathematical ideas.
> Discourse about mathematical understandings is a powerful strategy for developing and assessing student understanding.
> Heterogeneous grouping of students will yield greatest overall mathematical achievement by all students. (Fey, 1999:18)

These recommendations can only be achieved though helping mathematics teachers to develop their pedagogical content knowledge though appropriate in-service mathematics education courses or by providing incentives for teachers to belong to and be active members of appropriate mathematics teacher organisations.

Adelman *et al* (1980, in Cohen and Manion, 1994:123) list many advantages of case studies. This case study, captured at Angaziwa High School, may be considered as "a step to action" as it certainly begins in a world of action and contributes to it. The learners and the teacher observed at Angaziwa may benefit from this research project if Ms Fundisi were to discuss the contents of this thesis with the researcher. In addition, validity would be best achieved if the teacher and the learners were to check the contents of this research study. As a teacher-educator the researcher has gained valuable insight into recent ideas on the teaching and learning of algebra which will undoubtedly enrich the researcher's student teachers of mathematics. This in-depth examination of strategies used by a teacher to engage learners with mathematics concepts and processes, the resultant mathematical learning outcomes, and the relationship between teacher strategies and learners' performance has shown that for effective teaching of "solution of linear equations" student teachers would need to develop a repertoire of teaching and assessment strategies.

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Appendix I

Over the Hills and Far Away

I am sitting in a room a brown brick room.
No kettles whistling whilst waiting.
No colourful cooking books.
No printing press or paper.
No tills at the tuck shop.
No technical teachers.

Teachers muttering while also coughing and
spluttering.
I learned a lot and saw the goats, cows and chickens.

It was like a farm.
Pleasant and calm.

Liska van Laren (9 years) 14 December 2000

Appendix II

Isivakashi
(The Visitor)

i, a teaspoon of white sugar,
sit
Squeezed
between
Coffee and cream
The dull classroom cup is filled.
Turbulent particles, *vibrating*
At their own frequency
Motionless-
I take it all in…

I yearn for Hot water
I long to be carried in the swirl
Of challenged particles-
To be stirred
But
There is no electricity here
No sophistication

Particles mull around

Weak beverage!
The cup tips
Coffee and cream rush past me-
Oblivious?
All that remains…

a teaspoon
Of
Soggy
white sugar, i

Louise van Laren (17 years) 14 December 2000

Appendix III

c/o Edgewood College of Education
P. Bag X03
ASHWOOD
3605

30 October 1999

Dear ,

Please would you be so kind as to complete this questionnaire for me? The first day we met (13 August 1999), your answered some of these questions but now that I know you a bit better, I have compiled a more appropriate set of questions. This information I need for a further in depth profile of you. I would appreciate it if you could post the questionnaire, as soon as you have completed it, in the stamped addressed envelope.

On the 15 October I submitted my report on my visit to High School to the Human Sciences Research Council. I have not heard anything from them so I presume my report was in order. They will use the results for their report that will only be available in March 2000. I am under the impression that they will contact you once they have collected all their data.

Once again, thank you for being so co-operative and for making my visit to such a unique experience.

Yours sincerely,

Linda van Laren

A. BIOGRAPHICAL INFORMATION

Surname:..Initials:....................................

Contact Postal Address..

....

....Code:.....................................

Home Telephone Number:...

1. Qualifications:

Name of diploma	Where and when completed

2. Major methods qualified in ....

....

3. Minor methods qualified in ....

....

4. Diplomas obtained:

Name of diploma	Name of institution

B. BACKGROUND INFORMATION

1. Tick the description that best describes where you attended school as a pupil.

A medium or large city, e.g. Durban, Pietermaritzburg	
A suburban area, e.g. Umlazi	
A rural area or small town within 100 km of a large city, e.g. Ixopo	
A remote rural area, e.g. Nongoma	

Where did you attend high school?..

When did you pass Std 10? ....

2. Tick the description that best describes where you stayed before attending college.

A medium or large city, e.g. Durban, Pietermaritzburg	
A suburban area, e.g. Umlazi	
A rural area or small town within 100 km of a large city, e.g. Ixopo	
A remote rural area, e.g. Nongoma	

3. Tick the description that best describes where you stayed during your your study for your diplomas.

Name of diploma	Where stayed
	Residence (on campus) Private residence outside campus
	Residence (on campus) Private residence outside campus

4. 1. Tick work experience gained before study for first education diploma:

None	
As a teacher	
In industry or private sector	
Other (please specify)	

4\. 2. Tick work experience gained before study for second education diploma:

None	
As a teacher	
In industry or private sector	
Other (please specify)	

5\. What was your main reason for wanting to become a teacher?

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....

....

6\. What was your main reason for wanting to become a high school Mathematics teacher?

....

....

7\. Reasons for doing a teaching diploma.

To what extent do you agree with the following statements. Use the scale
1 (strongly agree),
2 (agree),
3 (undecided),
4 (disagree) and
5 (strongly disagree).

I pursued a teaching diploma because: -

	1	2	3	4	5
1. I was encouraged by my parents					
2. Education was my first choice of careers					
3. I was influenced by a former teacher					
4. I felt it was an easy diploma to get					
5. I wanted to be a teacher					
6. I enjoyed working with teenagers					
7. I was attracted to the time schedule of school, i.e. school time, vacations etc.					
8. I enjoyed school					

9. I enjoyed previous experience in teaching					
10. Teaching is a secure career					
11. I wanted to have an impact on students' lives					
12. I wanted to teach my subject specialization					
13. I liked the respect that accompanies a teaching post					
14. I was unable to finish another diploma/degree					
15. I did not meet entry qualifications for another tertiary institution I preferred					
16. Teaching is a positive way to contribute to society					
17. Teachers' salaries are attractive					
18. Teaching gives me an opportunity to work in my community					
19. Teaching is a good career to combine with raising a family (or other pursuits like running a tuck shop)					

C. CURRENT EMPLOYMENT STATUS

1. Please tick the categories that apply to your employment situation:

Employment as a full-time permanent teacher	
Employment as a full-time temporary teacher	

2. List the subjects that you teach.

....

3. Complete details of what <u>currently</u> enrolled for further study.

Name of diploma/degree (include specialisation subject(s))	Name of institution

4. Give reasons for why you would want to acquire a further qualification.

....

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5. Give reasons for your choice of subject for your further qualification.

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6. If you could start your career again, would you still want to become a teacher? Please explain your answer.

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Appendix IV

Fieldworker's name: ....

Province: .... School: ....

Teacher's name: ....

Today's date: .... Venue /Location: ....

Lesson No: .... Number of learners actually present at the lesson: ....

Approximate length of lesson observed: .... minutes

Time lesson begins: .... Time lesson ends: ....

OBSERVATION SCHEDULE

PART ONE: ESTABLISHING THE LESSON CONTEXT

This schedule to be completed by the fieldworker before, while and after observing the lesson. Please tick (✓) or cross (x) relevant blocks and comment where necessary.

The learning environment

1. In the classroom/room, is/are there:

Tick one box in each row

a)	cupboards/storage space?	Yes 1	No 2
b)	usable chalkboards?	Yes 1	No 2
c)	a table for the teacher?	Yes 1	No 2
d)	sufficient seating or desks or writing surface(s) per learner?	Yes 1	No 2
e)	sufficient space for the teacher to organise different activities or seating arrangements?	Yes 1	No 2
f)	adequate lighting?	Yes 1	No 2
g)	adequate ventilation?	Yes 1	No 2
h)	a comfortable temperature?	Yes 1	No 2
i)	noise or outside distraction?	Yes 1	No 2

Comments on physical condition of classroom *(e.g. evidence of care/neglect, e.g. vandalism, cleanliness, etc.)*

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Classroom organisation

2. Are learners seated:

Tick one box only

alone at individual desks/tables?	1
in pairs at 2 seater desks/tables?	2
in groups at desks/tables grouped together?	3
other, specify	4

3. Are all/most of the learners seated facing the teacher/front of the classroom? . . . . . *Tick one box in each row* Yes [1] No [2]

4. In the course of the lesson, does the teacher: *Tick one box only*
   - remain in one place? . . . . . [1]
   - move around the class? . . . . . [2]
   - both of the above? . . . . . [3]
   - other, specify? . . . . . [4]

Any other comments you wish to make?: ....

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....

Lesson topic

5. What is the Maths topic addressed in the lesson (i.e. what is being taught)? *[if the topic is not clear, state this.]* ....

   ....

6. Was this lesson: *Tick one box only*
   - an introductory lesson? . . . . . [1]
   - a continuation of a previous lesson? . . . . . [2]
   - the end of a series of lessons? . . . . . [3]
   - other, specify? . . . . . [4]

Lesson structure

7. Describe the sequence of the lesson activities and estimate the number of minutes spent on each activity. Ignore activities that are not applicable:

	Sequence of activities	*Estimated no of minutes*
a) whole class teaching		
b) whole class discussion?		
c) learners working in groups/pairs		
d) learners working alone		
e) organisation of learners/distribution of textbooks, notebooks, apparatus, collection of homework, etc.?		
f) disruptions/interruptions (e.g. intercom announcements, teacher having to leave the room, etc)		
g) other, specify		

8. How does the teacher pace the lesson in terms of available time? *Tick one box only*
   - very efficiently? . . . . . [1]
   - efficiently? . . . . . [2]
   - inefficiently? . . . . . [3]

Any other comments?: ....

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Organisation and use of textbooks/technology and other material resources

Tick one box in each row

9.	Is/are textbooks(s) used during the lessons?	Yes	1	No	2
10	Are mathematics worksheet(s) used?	Yes	1	No	2
11	If yes to 9/10, were you able to get a copy/photocopy of the relevant pages from the teacher to attach to this schedule?	Yes	1	No	2

12. If yes to 9/10, is there a textbook/worksheet? *Tick one box only*

for the teacher only?	1
per group of learners?	2
per desk/table?	3
per learner?	4
other, specify	5

13.	Does the teacher write activities/exercises/work on the chalkboard?	Yes	1	No	2

14. If yes, write down the activities/work on the board here: ....

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15.	Is use made of other support material/resources/apparatus, e.g. overhead projector?	Yes	1	No	2

If yes, specify: ....

....

16. If teacher uses material/resources/apparatus to demonstrate, does the teacher demonstrate to: *Tick one box only*

the whole class?	1
a group of learners at a time?	2
other, specify?	3

17.	If teacher uses material/resources/apparatus to demonstrate, are all learners able to see the teacher's demonstrations?	Yes	1	No	2
18.	Is use made of calculators during the lesson?	Yes	1	No	2
19.	If yes, do the learners themselves use calculators?	Yes	1	No	2

20. If yes, is there a calculator: *Tick one box only*

per learner?	1
per pair of learners?	2
per group of learners?	3
other, specify?	4

21. Is use made of computers during the lesson? Yes 1 No 2

If yes, provide details: ....

....

22. Do learners have the necessary writing equipment (pens, paper, etc.) for the lesson? *Tick one box only*

all	1
Most (at least three quarters of the class)	2
Some (at least half the class)	3
Few (less than half the class)	4
None	5

Other comments?: ....

....

....

Organisation of the task/activities

23 Does the teacher organise tasks/activities so that learners work: *Tick relevant boxes*

Individually without assistance from the teacher?	1
individually with assistance from the teacher?	2
together as a class with the teacher assisting the whole class?	3
together as a class with learners responding to one another?	4
in pairs or small groups without assistance from the teacher?	5
in pairs or small groups with assistance from the teacher?	6
other, specify?	7
teacher does not organise tasks/activities	8

Other comments?: ....

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Language(s) of learning and teaching

24. Activities are written in: *Tick one box only*

English	1
the vernacular	2
Maths terminology / numbers / Maths notation only	3
English / the vernacular but mainly English	4
English / the vernacular but mainly the vernacular	5
activities not used	6

25. Learners complete or write activities in: *Tick one box only*
- English 1
- the vernacular 2
- Maths terminology / numbers / Maths notation only 3
- English / the vernacular but mainly English 4
- English / the vernacular but mainly the vernacular 5
- activities not used 6

26. Teacher instructs in: *Tick one box only*
- English 1
- the vernacular 2
- the vernacular with Maths terminology / numbers / Maths notation in English 3
- English / the vernacular but mainly English 4
- English / the vernacular but mainly the vernacular 5

27. In teacher-learner interactions, learners mainly use: *Tick one box only*
- English 1
- the vernacular 2
- the vernacular with Maths terminology / numbers / Maths notation in English 3
- English / the vernacular but mainly English 4
- English / the vernacular but mainly the vernacular 5
- no teacher-learner interaction 6

28. In learner-learner interactions, learners mainly use: *Tick one box only*
- English 1
- the vernacular 2
- the vernacular with Maths terminology / numbers / Maths notation in English 3
- English / the vernacular but mainly English 4
- English / the vernacular but mainly the vernacular 5
- learners do not interact 6

Any other comments?:..

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Learner participation and involvement

29. Do all learners participate actively in the lesson? *Tick one box only*
- all 1
- most (about three quarters) 2
- some (about half) 3
- few (less than half) 4
- none 5
- other, specify 6

30. Indicate the type and extent of learner involvement during the lesson *(ignore activities that are not applicable)*

	not at all		occasionally / some of the time		frequently / a large proportion of the time		all the time	
a) Listening to the teacher		1		2		3		4
b) Observing demonstrations . . .		1		2		3		4
c) Copying down teacher's notes		1		2		3		4
d) Memorising and or repeating words or Maths terms		1		2		3		4
e) Responding to teacher's questions		1		2		3		4
f) Asking questions		1		2		3		4
g) Completing tasks/activities in their exercise books		1		2		3		4
h) Reading textbooks/books, etc.		1		2		3		4
i) Discussing with their peers . . .		1		2		3		4
j) Writing their own notes		1		2		3		4
k) Using calculators		1		2		3		4
l) Writing a test		1		2		3		4
m) Marking/reviewing of own/ other homework/classwork . . .		1		2		3		4
n) Other, specify		1		2		3		4

Any comments?: ....

....

....

....

Assessment

31. How are learners assessed? *Tick one box only*

learners' oral responses	1
learners' written work	2
both	3

32. How are learners provided with feedback?: *Tick one box only*

individually	1
as a class	2
both	3

PART TWO: TEACHER'S INSTRUCTIONAL PRACTICES

CRITERION 1
DOES THE TEACHER MAKE THE MATHEMATICS CONCEPTS OR PROCESSES TO BE LEARNT EXPLICIT?

1	2	3	4	5
♦ No concepts or processes made explicit	♦ Wrong concepts or processes made explicit or uses incorrect representations or definitions.	♦ Concepts or processes made clear/explicit. ♦ Purpose or reason for learning them not made clear.	♦ Concepts or processes made clear/explicit. ♦ Purpose or reason for learning them made clear. ♦ Does not assist learners to link related/familiar Maths concepts and processes to the new concepts and processes.	♦ Concepts or processes made clear/explicit. ♦ Purpose or reason for learning them made clear. ♦ Assists learners to link related/ familiar Maths concepts and processes to the new concepts and processes.

Explain (using specific examples): ....

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CRITERION 2

DOES THE TEACHER PROVIDE LEARNERS WITH OPPORTUNITIES TO EXPRESS THEIR CURRENT UNDERSTANDINGS OF THE MATHS CONCEPTS OR PROCESSES TO BE LEARNT?

1	2	3	4	5
♦ No opportunities for learners to express their current understandings of the maths concepts or processes to be learnt.	♦ Provides opportunities for learners to express their current understandings of the maths concepts or processes to be learnt. ♦ Does not use learners' expressions of their understandings as tools for teaching.	♦ Provides opportunities for learners to express their current understandings of the maths concepts or processes to be learnt. ♦ Uses learners' expressions of their understandings as tools for consolidating their existing mathematical understandings. ♦ Does not use learners expression as tools for 'sorting out' differences between their existing understandings and the new maths concepts or processes.	♦ Provides opportunities for learners to express their current understandings of the maths concepts or processes to be learnt. ♦ Uses learners' expressions of their understandings as tools for consolidating their existing mathematical understandings and for 'sorting out' differences between their existing understandings and the new maths concepts or processes. ♦ Does not build on and move beyond their new understandings of the maths concepts or processes.	♦ Provides opportunities for learners to express their current understandings of the maths concepts or processes to be learnt. ♦ Uses learners' expressions of their understandings as tools for consolidating their existing mathematical understandings and for 'sorting out' differences between their existing understandings and the new maths concepts or processes. ♦ Builds on and moves beyond their new understandings of the maths concepts or processes.

Explain (using specific examples): ....

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CRITERION 3
DOES THE TEACHER INTRODUCE LEARNERS TO THE NEW/ADDITIONAL LANGUAGE THEY NEED IN ORDER TO DISCUSS AND THINK ABOUT THE MATHEMATICS CONCEPTS OR PROCESSES TO BE LEARNT?

1	2	3	4	5
♦ Does not deliberately introduce learners to new/additional maths language. (For example, by not providing them with new maths terminology and definitions.)	♦ Introduces learners to incorrect/inappropriate maths language. (For example, incorrect definitions or modelling technically/ mathematically incorrect or inappropriate language.)	♦ Deliberately introduces learners to appropriate and correct new/additional maths language algorithm ♦ Focuses on form rather than meaning. (For example, engages learners in surface articulation of maths language related to the concepts or processes through involving learners in verbally repeating new maths terminology or in labelling.)	♦ Deliberately introduces learners to appropriate and correct new/additional maths language. ♦ Focuses on meaning rather than form. (For example, by making connections/ differences between related terms in learners' primary language and the language of learning (English) explicit, or by making connections/ differences between learners' existing knowledge of maths language and the new maths language explicit.)	♦ Deliberately introduces learners to appropriate and correct new/additional maths language. ♦ Focuses on meaning rather than form. ♦ Provides learners with the opportunity to practise using new maths language to formalise their thinking and understanding of the concepts or processes. (For example, by asking individual learners to explain why they think what they do and using those aspects of their explanations that are useful to provide them with the maths language they need to formalise their thinking and understanding of the concepts or processes; or by using whole class discussion to elicit learners' understandings and provide the class with more appropriate maths language, etc.)

Explain (using specific examples): ....

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CRITERION 4
DOES THE TEACHER DEMONSTRATE HOW THE MATHEMATICS CONCEPTS OR PROCESSES TO BE LEARNT WORK?

1	2	3	4	5
♦ Does not demonstrate how new mathematics concepts or processes work.	♦ Uses unfamiliar mathematical imagery, abstractions or representations to demonstrate how new mathematics concepts or processes work. ♦ Emphasises procedural understanding (how to do). ♦ Does not try to emphasise conceptual understanding.	♦ Uses different (familiar/ unfamiliar) forms of mathematical imagery, abstractions or representations to demonstrate how new mathematics concepts or processes work. (For example, graphs, number lines, pictures, diagrams and symbols.) ♦ Tries to emphasise conceptual understanding over procedural understanding. ♦ Does not focus learners' attention on the relationships between the representations and the new maths concepts or processes.	♦ Uses different forms of mathematical imagery, abstractions or representations to demonstrate how new mathematics concepts or processes work. ♦ Emphasises conceptual understanding. ♦ Focuses learners' attention on the relationships between the new mathematics concepts or processes and the mathematical representations / abstractions / imagery. ♦ Does not illustrate how the new maths concepts or processes become explanatory rules or can be generalised and applied to solve problems that are similar in mathematical content and structure.	♦ Uses multiple forms of mathematical imagery, abstractions, representations to demonstrate how new mathematics concepts or processes work. ♦ Emphasises conceptual understanding ♦ Focuses learners' attention on the relationships between the new mathematics concepts or processes and the representations / abstractions / imagery. ♦ Illustrates how the new mathematics concepts or processes become explanatory rules or can be generalised and applied to solve problems that are similar in mathematical content and structure.

Explain (using specific examples): ....

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CRITERION 5
DOES THE TEACHER PROVIDE LEARNERS WITH OPPORTUNITIES TO PRACTISE USING THE MATHEMATICS CONCEPTS OR PROCESSES TO BE LEARNT?

1	2	3	4	5
♦ Does not provide learners with opportunities to practise using new mathematics concepts or processes themselves. (For example, by doing most of the mental work and solving problems for the class.)	♦ Provides learners with opportunities to practise the new mathematics concepts and processes ♦ Does not provide learners with an appropriate level. (For example, by providing activities / exercises that are pitched at too high / too low a starting point for the learners.)	♦ Provides learners with opportunities to practise using new mathematics concepts or processes. ♦ Provides learners with an appropriate level. ♦ Does not provide learners with opportunities to develop greater levels of independent competence by giving them opportunities to use new maths concepts or processes in terms of incremental complexity.	♦ Provides learners with opportunities to practise using new maths concepts or processes ♦ Provides appropriate level. ♦ Assists learners to develop greater levels of independent competence by giving them opportunities to use new maths concepts or processes in terms of incremental complexity. (For example, engaging learners in using increasingly complex examples that assist them to develop their understanding and use of new concepts or processes in progressively difficult ways.) ♦ Does not provide learners who demonstrate competence / mastery with opportunities to complete additional activities using new maths concepts or processes in a variety of other applications.	♦ Provides learners with opportunities to participate in practising using new maths concepts of processes in a variety of ways that emphasise conceptual understanding. ♦ Provides appropriate level. ♦ Assists learners to develop greater levels of independent competence by giving the learners opportunities to practice using new maths concepts or processes in terms of incremental complexity. ♦ Provides learners who demonstrate competence / mastery with opportunities to complete additional activities using new maths concepts or processes in a variety of other applications. (For example, using and applying concepts and process to solve everyday/ real life problems.)

Explain (using specific examples): ....

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CRITERION 6

DOES THE TEACHER ASSIST LEARNERS TO ENGATE WITH AND INTERPRET (MAKE SENSE OF/DECODE) WRITTEN MATHEMATICS TEXT(S)/REPRESENTATIONS RELATED TO THE CONCEPTS OR PROCESSES TO BE LEARNT?

1	2	3	4	5
♦ No written mathematical text and/or representations are provided.	♦ Learners provided with written mathematical text/ representations but tells them what the text/ representations mean. ♦ Does not provide learners with opportunities to engage with or interpret the text/representation themselves.	♦ Learners provided with written mathematical text/ representations. ♦ Provides learners with opportunities to engage with (interact with) and interpret (make their own sense of) the text/ representations. ♦ Does not test their comprehension of the text/ representations.	♦ Provides learners with written mathematical text/ representations. ♦ Provides them with opportunities to engage with and make sense of the text/ representations themselves. ♦ Tests their comprehension of the text/ representations. ♦ Does not assist learners to develop the strategies they need to engage with and interpret text/ representations independently.	♦ Provides learners with opportunities to engage, interact with and make sense of the text/ representations themselves. ♦ Tests their comprehension of the text(s)/ representations. ♦ Assists learners to develop the strategies they need in order to do this independently. (For example, by assisting them to use their prior knowledge of maths and language; use semantic (contextual) and syntactic (structural) clues and cues; talk about/respond to text/ representations as they read them; collaborate with each other in sorting out their understanding of the text/ representations; communicate their understandings of the text/ representations in their primary language and the language of learning; and use their own words to summarise what they have read, or restate what they see as key ideas, etc.)

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Explain (using specific examples): ....

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CRITERION 7
DOES THE TEACHER ENCOURAGE LEARNERS TO DISCUSS THE MATHEMATICS CONCEPTS OR PROCESSES TO BE LEARNT WITH EACH OTHER?

1	2	3	4	5
♦ Does not encourage learners to discuss new mathematics concepts or processes with each other.	♦ Encourages learners to check/correct one another's answers. ♦ Does not encourage them to help one another.	♦ Encourages learners to discuss new mathematics concepts or processes with each other by encouraging them to help one another. ♦ Does not structure discussion / tasks so that learners can benefit from each other's thinking/discourse (maths language).	♦ Encourages learners to discuss the mathematics concepts or processes together by encouraging them to help one another. ♦ Structures the discussion / tasks so that learners can benefit from each other's thinking/discourse. (For example, asking learners to present their answers and thinking to the whole class and involving the whole class in deciding on the best solution(s). ♦ Does not make explicit the strategies learners need to work or solve problems collaboratively.	♦ Encourages learners to discuss the mathematics concepts or processes together by encouraging them to help one another. ♦ Structures the discussion / tasks so that learners can benefit from each other's thinking/discourse. ♦ Makes explicit strategies learners need to work or solve problems collaboratively. (For example, how to share ideas, how to negotiate, how to explain their thinking, how to evaluate each other's method, etc.)

xxxiii

Explain (using specific examples): ....
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CRITERION 8
DOES THE TEACHER STRUCTURE MATHEMATICS ACTIVITIES THROUGH WHICH LEARNERS EXPERIMENT WITH USING THE MATHEMATICS CONCEPTS AND PROCESSES TO SOLVE PROBLEMS?

1	2	3	4	5
• Does not structure mathematics activities or tasks in ways which provide learners with opportunities to experiment with using the mathematics concepts and processes to solve problems. (For example, by not providing the learners with the opportunity to grapple with problems themselves.)	• Structures mathematics activities or tasks in ways which provide learners with opportunities to experiment with using their current mathematical and everyday knowledge of the mathematics concepts and processes to solve **routine** problems even if they are not using the most efficient or effective ways of solving the problems. (For example, by providing them with opportunities to experiment with using algorithms but allowing them to use concrete or physical representations such as counting fingers to calculate.)	• Structures mathematics activities or tasks in ways which provide learners with opportunities to experiment with using the mathematics concepts and processes to solve **routine** problems more efficiently and effectively. (For example, by providing them with opportunities to experiment with using algorithms and encouraging them to estimate and calculate mentally.) • Does not provide learners with opportunities to experiment with using the concepts and processes to solve **novel** problems (problems for which learners cannot immediately solve using a routine method).	• Structures mathematics activities or tasks in ways which provide learners with opportunities to experiment with using the new concepts, principles or strategies to solve **routine** problems more efficiently or effectively. • Provides learners with opportunities to experiment with using the mathematics concepts and processes to solve **novel** problems (problems which have no obvious solution) • Does not assist learners to develop the strategies they need to solve novel problems independently.	• Structures mathematics activities or tasks in ways which provide learners with opportunities to experiment with using the new concepts, principles or strategies to solve **routine** problems more efficiently or effectively. • Provides learners with opportunities to experiment with using the mathematics concepts and processes to solve **novel** problems. • Assists learners to develop the strategies they need to solve novel problems independently. (For example, hypothesising, predicting, estimating, investigating, exploring, and discovering patterns and connections through matching, ordering, sorting, etc.)

Explain (using specific examples): ....

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CRITERION 9
DOES THE TEACHER ASSESS WHETHER LEARNERS HAVE LEARNT THE MATHEMATICS CONCEPTS, PRINCIPLES OR STRATEGIES?

1	2	3	4	5
♦ Does not assess whether learners have learnt the mathematics concepts, principles or strategies during the course of the lesson.	♦ Incorrectly assesses whether learners have learnt the mathematics concepts or processes during the course of the lesson. (For example by mismanaging the assessment, mismatching the activities/tasks with the concepts or processes to be assessed; by assessing the wrong concepts, principles or strategies; by failing to recognise emerging understandings and abilities, etc.)	♦ Correctly assesses whether learners have learnt the mathematics concepts or processes during the course of the lesson. ♦ Informs learners about whether their responses are correct or incorrect. ♦ Does not use this information to identify learners' misconceptions and provide them with feedback about what they must do to improve their learning.	♦ Correctly assesses whether learners have learnt the mathematics concepts or processes during the course of the lesson. ♦ Informs learners about whether their responses are correct or incorrect. ♦ Uses this information to identify learners' misconceptions and provide them with feedback about what they must do to improve their learning. ♦ Does not use learners' insights to develop or 'push' their learning further.	♦ Correctly assesses whether learners have learnt the mathematics concepts or processes during the course of the lesson. ♦ Informs learners about whether their responses are correct or incorrect. ♦ Uses this information to identify learners' misconceptions and provide them with feedback about what they must do to improve their learning. ♦ Uses learners insights to develop or 'push' their learning further.

xxxv

Explain (using specific examples): ....

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