A CASE STUDY EXPLORING THE PEDAGOGICAL CONTENT KNOWLEDGE OF INTERMEDIATE PHASE MATHEMATICS TEACHERS

By

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ABSTRACT

The poor performance of South African learners in mathematics has been a cause of great concern. Various factors have been identified which contribute to learners' underperformance and amongst these factors, is that of teacher knowledge. Shulman (1986) who was the initial researcher on teacher knowledge, categorized seven knowledge domains. These knowledge domains include: content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational purposes and values and pedagogical content knowledge (PCK). PCK is deemed as the knowledge domain that makes "teachers, teachers" rather than subject experts (Cochran, King and DeRuiter, 1993). This study has therefore focused on this specific knowledge domain.

The purpose of this study was to explore the pedagogical content knowledge (PCK) of two Grade Six mathematics teachers from the Umgungundlovu district in KwaZulu- Natal. The study focused on the categories of PCK that the teachers drew on when teaching 2D and 3D shapes. The teachers were purposively selected on the basis of having five years or more of teaching experience and their willingness to participate in the research. The study was guided by a qualitative case study method and located within the interpretive paradigm. Structured lesson observations and video stimulated interviews were used as data collection methods. The video stimulated interviews were transcribed and Ball et al's. (2008) framework on Mathematics Knowledge for Teaching (MKT) with a specific focus on PCK was used to analyze the data. The data was coded according to the three categories of PCK in the MKT framework. These categories are comprised of Knowledge of Content and Curriculum (KCC), Knowledge of Content and Teaching (KCT) and Knowledge of Content and Students.

The findings of this study suggest that the teachers implemented KCC, KCS and KCT in their teaching of 2D and 3D shapes. However, it was also observed that both teachers failed to address the incorrect responses provided by their learners. The issue of time constraints was identified as a possible factor that could have contributed to the teachers' inability to probe deeper into the learners' misconceptions. In addition to this, the study also aimed to describe the nature of the relationship between the teachers' PCK and pedagogic decision making. The results of this study

also suggest that the teachers' KCS and KCC greatly influenced their pedagogic decision making and a call for further research in this area is suggested.

DECLARATION

Submitted in partial fulfilment of the requirements for the degree of

Masters in Education, in the Graduate Programme in the College of Humanities,

University of KwaZulu-Natal, Pietermaritzburg, South Africa.

I, Leena Nadas declare that:

1. The research reported in this thesis, except where otherwise indicated, and is my original research.
2. This thesis has not been submitted for any degree or examination at any other university.
3. This thesis does not contain other persons’ data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons.
4. This thesis does not contain other persons' writing, unless specifically acknowledged as being sourced from other researchers. Where other written sources have been quoted, then:

a. Their words have been re-written but the general information attributed to them has been referenced

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5. This thesis does not contain text, graphics or tables copied and pasted from the Internet, unless specifically acknowledged, and the source being detailed in the thesis and in the References sections.

Leena Nadas | 16 March 2020

Student Name | Date

DEDICATION

This dissertation is dedicated to my mother, Elizabeth Pillay and my loving husband, Shaun Nadas, who have been both a pillar of strength and encouragement to me. Thank you for your unwavering love and support during this journey.

ACKNOWLEDGEMENTS

"I can do all things through Christ who strengthens me"

Thank you God Almighty for being my source of strength and thank you for giving me the courage to continue even when I felt like giving up. You make all things possible.

My heartfelt thanks and appreciation is owed to the following people who have contributed greatly to this study. Without your support and guidance, the completion of this study would not have been possible.

- Professor Carol Bertram: Thank you for your patience, support and thorough feedback throughout this study. Your positive attitude and professional guidance have been instrumental in helping me achieve academic success, and for this, I am truly grateful.

- The KwaZulu-Natal Department of Education for granting me permission to conduct this research.

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- To my amazing son, Nathan Nadas: Thank you for being my greatest inspiration.

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PREFACE

The work described in this dissertation was carried out from January 2018 to December 2019, under the supervision of Professor C. Bertram of the Pietermaritzburg campus of the University of the University of KwaZulu-Natal.

This study represents the original work completed by the author and has not been submitted in any form for any diploma or degree to any other tertiary institution. Where the author has made use of the work of other authors, this has been duly acknowledged in the text.

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Leena Nadas

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Date

As the candidate's supervisor I agree to the submission of this dissertation.

16 March 2020

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Prof. Carol Bertram
Supervisor
Pietermaritzburg Campus

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Date

LIST OF ACRONYMS

ACRONYM	DESCRIPTION
PCK	Pedagogical Content Knowledge
DOE	Department of Education
CDE	Centre for Development and Enterprise
TIMSS	Trends in International Mathematics and Science Studies
SACMEQ	Southern and Eastern African Consortium for Monitoring Educational Quality
MKT	Mathematics Knowledge for Teaching
SMK	Subject Matter Knowledge
KCS	Knowledge of Content and Students
KCC	Knowledge of Content and Curriculum
KCT	Knowledge of Content and Teaching
GPK	General Pedagogic Knowledge
CCK	Common Content Knowledge
MfT	Mathematics for Teaching
PLC	Professional Learning Communities

TABLE OF CONTENTS

Contents

CHAPTER 1: INTRODUCTION

1.1. Introduction

The underperformance of South African learners in mathematics has been a cause of great concern in our education system. Various factors have been identified in contributing to learners' underperformance and amongst these factors is that of teacher's knowledge. This study focuses on the aspect of teacher knowledge with a specific focus on pedagogical content knowledge (PCK). It explores the PCK of two purposively selected Grade Six mathematics teachers in order to gain a better understanding of their PCK. This chapter introduces the focus and purpose of this study. It further sets the background to this study and also provides an overview of the succeeding chapters.

1.2. Focus and purpose of the study

This study focuses on the phenomenon of teacher knowledge, specifically the PCK of selected intermediate phase teachers. While Shulman (1986) has categorized seven knowledge domains, this study focuses on PCK as it extends beyond a teacher's knowledge of subject matter. Instead it involves making use of the most useful forms of illustrations in order to help learners understand and comprehend concepts that are taught. The purpose of this study was twofold. Firstly, it aimed to explore the pedagogical content knowledge (PCK) of two Grade Six mathematics teachers. Secondly, it aimed to understand the relationship between the teachers' PCK and their pedagogic decisions.

1.3. Rationale and significance of the study

For many years I have taught learners who do not have English as their home language. I assumed that the language barrier contributed greatly to learner difficulties in understanding and grasping mathematical concepts. Having recently transferred to a school where ninety percent of the learners' home language is English, I noticed that some learners still experience difficulties in grasping certain concepts. Thus, it appears that the issue of language is not the only reason for learners' experiencing difficulties in mathematics. However, other factors such as teachers' weak subject matter knowledge and knowledge of pedagogy also contribute to the learners' lack of understanding (Venkat & Spaull, 2014).

Grossman (1990) argues that a teacher's knowledge plays a pivotal role in effective teaching. Similarly, Attard (2011) argues that one of the most powerful influences of student engagement in mathematics is the teacher and teacher practices. Etkina (2010) contends that teachers should possess the skills and abilities to integrate a subject's content knowledge with their knowledge of the learners they teach. It is further argued that while teachers may have higher qualifications in mathematics, it does not necessarily lead to strong learning outcomes for their students. This is a result of teachers having a weak understanding of how students learn (Attard, 2011). Educator knowledge has consistently been linked to the learners' poor performance in mathematics and science. Taylor (2018) points out that teachers' disciplinary knowledge is the foundation on which effective pedagogy rests. He further adds that the majority of South African teachers possess weak disciplinary knowledge which adds to the poor performance of South African learners. I therefore consider it important to explore and understand the pedagogical content knowledge (PCK) of intermediate phase mathematics teachers as it can help to address the challenges that teachers within this phase face.

In addition to exploring the selected teachers' PCK, I was also interested to understand the nature of the relationship between a teachers' PCK and their pedagogic actions. Barendsen and Henze (2017) conducted a study in Netherlands in order to investigate the relationship between science teachers' PCK and their classroom actions. They argue that while PCK has been researched extensively, "very little empirical evidence has been found to determine how this knowledge actually informs teachers' actions in the classroom" (Barendsen & Henze, 2017, p.1). They therefore call for more qualitative studies to examine the relationship between teacher knowledge and classroom practice. Based on this notion, I thought it would be interesting and informative to see if there is indeed a relationship that exists between a teacher's PCK and pedagogical decision making, hence the reason for exploring the second research question.

1.4 Background information

Over the past two decades the South African government has made relatively large investments in the education system compared to its neighbouring countries (Chisholm & Wildeman, 2013). A wave of curriculum reform was implemented post-apartheid in order to bridge the gap in the inequality of education which was brought about during the apartheid era. However, despite its investments, South African learners continue to underperform in mathematics. According to the report by The Centre for Development and Enterprise (CDE), South Africa significantly underperforms in education especially in the teaching and learning of mathematics. The authors further maintain that "of the full complement of pupils who start school, only 50 per cent will make it to Grade 12 and only 12 per cent will qualify for university entrance" (McCarthy & Oliphant, 2013, p.1). These statistics are alarming and of great concern for South African teachers and learners. Trends in International Mathematics and Science Studies (TIMSS, 2007) has revealed that South African learners have the lowest performance among all 21 middle-income countries that have participated (McCarthy & Oliphant, 2013). From these statistics, it is clear that mathematics learning and teaching is a genuine problem that exists in South African schools.

Makgato and Mji (2006) identified a number of factors that affect learners' performance in mathematics and science. These factors include learners' lack of motivation in learning mathematics because they are scared of the high failure rate, lack of parental involvement and lack of resources such as textbooks. However, amongst these factors is the lack of teacher knowledge (Venkat & Spaull, 2014). Stols et al. (2015) argue that teachers' lack of mathematical knowledge and the skills needed to apply what they know in the classroom has been seen as a contributing factor to learners' poor performance in mathematics. Similarly, numerous studies have been conducted in South Africa in order to understand why our learners are underperforming in mathematics (Venkat & Spaull, 2014; Hugo, Jack, Wedekind & Wilson, 2010; Carnoy et al. 2008; Shepherd, 2013). One of the common findings amongst these studies is the lack of teachers' content knowledge.

1.4.1. Studies on South African teachers' poor content knowledge

Taylor (2018) asserts that the poor disciplinary knowledge held by the majority of South African primary school teachers has been documented in a number of research studies. In addition to this, he argues that the conclusions of these studies were also confirmed by the Southern African Consortium for Monitoring Educational Quality (SACMEQ 111, 2007) educator test results. The literature below, highlights some of the findings on South African teachers' poor content knowledge and its relationship to learner performance.

Venkat and Spaull (2014) conducted an empirical study by analyzing the SACMEQ 2007 mathematics teachers' test data. The results of the test data which included a sample of 401 Grade Six mathematics teachers, revealed that 79% of the Grade Six mathematics teachers showed content knowledge levels below the Grade Six or Seven band, alluding to teachers having a poor level of content knowledge.

Hugo, Jack, Wedekind and Wilson (2010) carried out a study in order to develop a more detailed picture of the quality of teaching and learning delivered in primary school classrooms in KwaZulu-Natal. The team participated in an international comparative study examining Grade Six level mathematics. Forty schools were sampled and were tested against Grade Five level. The findings revealed that 1 870 learners performed poorly in the test and the teacher test revealed that despite the teachers being qualified, on average only 47% of the teachers managed to get each test item correct. The video recording of lessons which focused on teacher knowledge revealed absent or incorrect content knowledge and the teachers showed almost no ability to recognise why students might be doing something incorrectly.

A pilot study of Grade Six mathematics teachers in Gauteng was carried out by Carnoy et al. (2008). The purpose of this study was to understand student academic performance in South Africa. Forty- nine teachers participated in this study and the findings of the study revealed that on average the teachers scored 60 % on both parts of the test, which is regarded as low, considering the teachers were tested on a Grade Five level test. The study therefore revealed that the teacher participants had a relatively low level of mathematical knowledge.

Shepherd (2013) also carried out a study using a nationally representative dataset – the 2007 wave of the Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ). She hoped to build on the findings of two case studies compiled by Carnoy et al. (2008) and Carnoy and Arends (2012). These studies were carried out in the Gauteng and North West provinces, respectively. The findings of these case studies showed evidence of a positive relationship between teacher knowledge and learner performance (Shepherd, 2013). Shepherd (2013) argues that the dataset used in her study is unique as the teachers were asked to complete a subject specific test. The findings of her study revealed that not all teachers with poor content knowledge are ineffective and that not all teachers with good content knowledge are effective.

The findings of Shepherd (2013) study is in line with the point that Shulman (1986) made when he argued that content knowledge is not enough without pedagogical skill. Shepherd (2013) further argues that a deep knowledge and understanding of subject matter is important, but more importantly is the teachers' ability to transfer information in a way that is meaningful to the learners. Therefore, a teacher's PCK plays a crucial role in student learning.

1.5 Context of the study

This study was conducted in two schools under the Umgungundlovu district which is home to 502 Ordinary Public Schools and 45 Independent Ordinary Schools. Like many other educational districts in KwaZulu- Natal, public schools in the Umgungundlovu district faces many challenges such as poor learner performance in mathematics and literacy, over-crowded classrooms, lack of skilled mathematics teachers, lack of school resources, violence in schools, the use of drugs and so forth. As mentioned in the introduction, these factors also contribute to the low performance of South African learners.

The schools in this study are referred to as School A and School B. School A is a well-resourced ex-model C school that has a learner population of 615 learners. The majority of the learners come from middle class and high income homes. There are approximately 27 learners in each class and each learner has access to textbooks as well as exposure to technological teaching resources. Grade Six mathematics is taught by a male teacher and one of the major challenges faced by this teacher is the language barrier as many of these learners are not English home language learners. School B in contrast to School A is comprised of learners who come from poor socio-economic environments. Many of the parents are unable to pay school fees and learners have access to a feeding scheme. School B has a learner population of 1000 learners and the classrooms are often over crowded. The classroom sizes range between 39-44 learners in a class. Grade Six mathematics is also taught by a male teacher who has 34 years of teaching experience.

1.5. Research questions

In this study, I sought to explore the PCK of two Grade Six mathematics teachers and the relationship between their pedagogic decision making and their PCK. This study aimed to develop an understanding of these teachers' PCK through lesson observations and video stimulated recall interviews. It is hoped that this study will further help the participants to reflect on their practices and make them more cognisant of why they do what they do. In addition to this, the findings of this study will be made available to the Department of Education which may help inform future departmental teacher training programs in mathematics. My research questions are:

1. What pedagogical knowledge do intermediate phase teachers show in their teaching of mathematics?

2. What is the relationship between teachers' pedagogic decision making and PCK?

1.6. A brief review of the related literature and conceptual framework

The literature reviewed for this study emphasizes the important role that teacher knowledge plays in students' achievements. In addition to this, teacher knowledge is defined and described according to various scholars. It further provides a detailed description of PCK by reviewing both international and national studies. The importance of PCK is discussed and a justification for choosing Ball, Thames and Phelps' (2008) Mathematics Knowledge for Teaching (MKT) framework is then provided.

Shulman (1987) who was the initial researcher on teacher knowledge defines teacher knowledge as knowledge exclusively applied to teaching. He argued that the teaching profession requires a specific knowledge base which includes an understanding of the subject content, skills and methods to deliver this content as well as an understanding of learners' preconceptions or misconceptions of the content that needs to be taught. Shulman (1986) categorised teacher knowledge into 7 domains i.e. content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational purposes and values and pedagogical content knowledge (PCK). This study focuses specifically on PCK.

"Pedagogical content knowledge is the intersection of content and pedagogy" (Ijeh & Nkopodi, 2013. p 475). Shulman (1987) asserts that PCK involves using the most useful forms of representations and explanations in order to make a concept understandable to others. It also involves having an understanding of what makes learning certain concepts easy or difficult and having knowledge of the preconceptions and misconceptions that students bring with them. Many researchers both internationally and locally have built on Shulman's (1987) concept of PCK. A set of authors who have added to Shulman's (1987) definition of PCK is Ball, Thames and Phelps (2008). Their framework has been used for this study.

1.6.1 Conceptual framework

This study made use of Ball, Thames and Phelps' (2008) Mathematics Knowledge for Teaching (MKT) framework. The authors define MKT as "knowledge that is needed to perform the recurrent tasks of teaching students mathematics" (Ball et al., 2008. p 398). This framework is similar to

PCK, however, it focuses specifically on mathematics. MKT comprises of both subject matter knowledge (SMK) and PCK. The PCK component of this framework is made up of three categories which are: knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum (KCC). This framework was used in my data analysis by matching the teachers' activities to each of the characteristics in each category of PCK.

1.7 Methodological approach

This study is underpinned by the interpretive paradigm as it relies "on the participants' views of a situation being studied" (Creswell, 2003, p. 8). The findings of the study are explained through the participants' view. This is done by analyzing the participants' responses from the semi structured interviews. Bertram and Christiansen (2014) argue that researchers working within the interpretive paradigm aim to gain a better understanding of how people make sense of the context in which they work. The participants in School A and B were observed within their working contexts and a structured observation schedule was used in order to generate data.

Qualitative research is often described as a naturalistic, interpretative approach which concerns itself with exploring a phenomenon from the inside and it takes into account the views of the participant and uses this as a starting point (Ritchie et al., 2013). It also involves using a variety of data collection methods and it describes routines problems and meanings in individuals' lives (Denzin & Lincoln, 2005).A qualitative approach was used in this study to explore the characteristics of PCK that the selected Grade Six teachers used in their teaching of space and shape in mathematics.

A case study is an approach that is used when one wants to gain an in-depth understanding of a complex issue in a real- life context (Zainal, 2007; Bertram & Christiansen, 2014). This study used a case study approach to explore the PCK of two Grade Six mathematics teachers while teaching the concept of 2D and 3D shapes. I observed and video recorded one lesson for each of the two educators. This provided me with rich, in-depth data. The data collection methods used were video stimulated interviews and lesson observations.

This study employed purposive sampling. The two teachers were chosen on the basis of their experience which is five years and more and their willingness to participate in this study. Only one Grade Six mathematics teacher is used from each school. The schools were also selected on their willingness to participate in this study.

1.8. Overview of dissertation

Chapter One highlights the purpose and focus of this study. It describes and discusses the background to this study, the research questions and methodological approach. It further presents a brief overview of the literature and conceptual framework used.

Chapter Two provides an in-depth explanation of pedagogical content knowledge (PCK) by drawing on contemporary literature. It further describes the influence of PCK on the teaching and learning of mathematics and thereafter discusses Ball and her colleagues (2008) framework of PCK.

Chapter Three presents a description of the methodology used. This includes the research paradigm, the research style, methods of data collection, analysis of data, ethical issues and trustworthiness of the research.

Chapter Four discusses and presents the findings of this study as obtained from the analysis of the data. Each case is examined according to the data collected from the lesson observations and video-stimulated interviews.

Chapter Five provides a summary of the study. It further describes and discusses the PCK of the selected intermediate phase mathematics teachers and the nature of the relationship between the teachers' PCK and pedagogic decisions. Recommendations on how to overcome challenges faced within this phase is also offered.

1.9 Conclusion

This chapter has provided an overview of the background to this study and has further described the context of the study. The focus, purpose and rationale of the study was discussed and the research questions were stated. A brief description of the literature reviewed was presented and the chapter further provided a description of the research methodology which guided this study. In addition to this, a brief overview of the succeeding chapters was provided.

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction

This chapter focuses on the concept of teacher knowledge and its influence on student learning. It further provides a brief description of the domains of knowledge and then provides an in-depth explanation of the concept of pedagogical content knowledge (PCK). It reiterates the importance of PCK in the teaching of mathematics and further discusses the rationale for selecting Ball, Thames and Phelps' (2008) Mathematics Knowledge for Teaching (MKT) framework for this study. The second aim of this study was to explore the reasons for the teachers' pedagogic decisions. A brief literature review is provided to discuss teachers' pedagogic decision making and its connection to teacher knowledge.

2.2. Teacher knowledge

"To be a teacher requires extensive and highly organized bodies of knowledge" (Shulman, 1985. p. 47)

As mentioned in Chapter One, the poor performance of South African learners in mathematics has been a cause for great concern in the educational system. Studies conducted concerning learners poor performance revealed that one of the common findings that contributed to this problem was teacher's lack of understanding of fundamental mathematics (Venkat & Spaull, 2015). Pournara, Hodgen, Adler and Pillay (2015) carried out a study in which Grade 10 teachers from five secondary schools in Johannesburg participated in mathematical development programs. The findings of the study revealed that the students of the teachers who participated in the developmental programs outperformed the control group of learners. This implies that by developing the mathematical knowledge of teachers, student learning can be enhanced. They therefore argue that attending to mathematical teachers' knowledge can in fact have a positive impact on student learning. Similarly, Hill, Rowan and Ball (2005, p.14) also argue "that the quality of mathematics teaching depends on teachers' knowledge of the content". It is no wonder that the study of teachers' knowledge has taken a central position in educational research for the past two decades (Liu, 2010). The importance of teacher's knowledge cannot be over emphasized as "teachers employ their knowledge base when they teach students in the classroom" (Sothayapetch, Lavonen & Juuti, 2013, p. 84). It is for this very reason that the primary focus of my study is teacher knowledge with a specific focus on pedagogical content knowledge (PCK).

2.3. Defining teacher knowledge

Grossman and Richert (1988, p. 54) define teacher knowledge as "a body of professional knowledge that encompasses both knowledge of general pedagogical principles and skills and knowledge of the subject matter to be taught". This implies that teachers need to have knowledge of the strategies used to teach as well as knowledge of the content that is taught.

Shulman was the initial researcher who described the knowledge base for teachers and the different domains of knowledge (Bertram, 2011). Various studies on teacher knowledge draw on Shulman's domains of knowledge in order to discuss the concept of teacher knowledge (Bertram, 2011; Grossman, 1990; Banks, Leach & Moon, 2005; Liu, 2010).

Shulman (1987) defines teacher knowledge as knowledge exclusively applied to teaching. Shulman (1987) also argues that the teaching profession requires a specific knowledge base. This knowledge base includes an understanding of the subject content, skills and methods to deliver this content as well as an understanding of learners' preconceptions or misconceptions of the content that needs to be taught. While this provides a skeletal overview of Shulman's (1987) argument for teacher knowledge, Shulman also argues that the knowledge base for teachers' is not cast in stone, "instead as more is learned about teaching, researchers will come to know and identify new categories of performance and understandings of good teachers, and will have to reconsider and redefine other domains" (Shulman, 1987, p.12).

2.4. Domains of knowledge

Various studies define teacher knowledge by describing the different domains of knowledge (Shulman, 1987; Grossman, 1990; Cogill, 2008). Shulman (1987) who carried out the first key study on teacher knowledge, categorized teacher knowledge into seven categories. These seven categories include content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational purposes and values and pedagogical content knowledge.

Grossman's (1990) summarized Shulman's domains and categorized teacher knowledge into four domains, namely general pedagogic knowledge, subject matter knowledge, knowledge of context and pedagogical content knowledge. The first type of knowledge as argued by Grossman (1990) is ***General Pedagogic Knowledge*** (GPK). According to Grossman (1990), GPK comprises of a body of general knowledge and skills pertaining to teaching, as well as knowledge and beliefs concerning student learning. The general body of knowledge refers to the principles of instruction for such as academic instruction and wait time whereas knowledge and skills refer to classroom management and knowledge and beliefs refers to the aims and purposes of education.

Bertram (2011) maintains that GPK also comprises of knowledge of classroom management and organization, knowledge of different teaching methods, as well as knowledge of assessment strategies and classroom dialogues. Similarly, Shulman (1987, p.8) describes GPK as involving "broad principles and strategies of classroom management and organization that appear to transcend subject matter." In addition to this, Morine-Deshimer and Kent (1999) further divide GPK into three main categories namely; classroom management, classroom communication and instructional model. Classroom management involves the general principles that teachers incorporate in order to promote student achievement. Classroom communication involves the interaction between student and teacher as teaching and learning cannot occur without proper communication. An instructional model includes the instructional tools and methods teachers use to deliver the curriculum or make a concept understandable to learners.

The second type of knowledge that Grossman (1990) describes is ***Subject Matter Knowledge.*** Grossman (1990) maintains that subject matter knowledge includes knowledge of content as well as knowledge of the substantive and syntactic structures of that discipline. Shulman (1986) maintains that substantive structures are the different ways in which the principles and concepts of a particular discipline is organized to include its facts. The syntactic structure of a discipline refers to the ways in which truth or falsehood are established. This implies that when there are contrasting claims about a phenomenon, the syntax of the discipline serves as a guide in order to assess which claim is more valid (Shulman, 1986). Shulman (1986) further asserts that teachers should not only possess the knowledge of defining concepts within a discipline, but in addition to this they must be able to explain why a particular phenomenon is considered valid and further explain why it is worth studying. Grossman (1990, p. 7) also argues that "without knowledge of the structure of a discipline, teachers may misrepresent the content and nature of the discipline".

The third type of knowledge as argued by Grossman (1990) is ***Knowledge of Context***. Grossman (1990) asserts that in order for a teacher's knowledge to be of use in the classroom, it needs to be context specific. This implies that the teacher's knowledge needs to be specific to the learners' needs as well as the district in which they work and operate. Grossman (1990) further describes knowledge of contexts by maintaining that teachers should have knowledge of their district policies and requirements as well as knowledge of the learners within their schools, this includes their backgrounds and communities as well as knowledge of the school culture. Grossman (1990) also maintains that teachers should also have knowledge of their students' weaknesses and strengths.

The fourth domain of teacher knowledge as categorized by Grossman (1990) is ***pedagogical content knowledge*** (PCK). Grossman (1990) who also expanded on Shulman's concept of PCK, identified four components to this concept. These four components include a) conceptions of teaching purposes/knowledge and beliefs about the purposes for teaching a subject at different grade levels, 2) knowledge of students, including students' understanding, conceptions, and misconceptions of particular topics in a subject matter, 3) curricular knowledge, which includes knowledge of curriculum materials available for teaching particular subject matter knowledge and about the horizontal and vertical curricula for a subject; as well as 4) knowledge of instructional strategies and representations for teaching particular topics.

2.5. Pedagogical content knowledge (PCK)

In 1986, Lee Shulman introduced the concept of PCK. He defines PCK as comprising:

> [t]he most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations - in a word, the ways of representing and formulating the subject that make it comprehensible to others. It also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners (Shulman, 1986, p. 9 and 10).

According to Hu Jing-Jing (2014, p. 412), Shulman's (1986) definition of PCK can be clarified into three components i.e. "(1) knowledge of topics regularly taught in one's subject area, (2) knowledge of forms of representation of those ideas, and (3) knowledge of students' understanding of the topics". Many researchers have adapted or expanded on Shulman's definition of PCK, which is evident from the literature to follow.

Krauss et al. (2008) who has added to Shulman's (1986) definition of PCK argues that his definition of PCK is generic. This implies that Shulman's (1986) definition encompasses all teaching subjects. However, research has identified various characteristics that are specifically important in the teaching of mathematics. These aspects can be used to conceptualize PCK for mathematics teaching. Krauss et al. (2008) argues that because tasks play a vital role for laying the foundation for mathematics learning, knowledge about the potential of mathematical tasks is the most important aspect of mathematics PCK. In addition to having knowledge of appropriate mathematical tasks, understanding learners existing conceptions and prior knowledge is also important. Krauss et al. (2008) maintains that learners' mistakes can provide valuable and insightful information about the learners' implicit knowledge. The authors also maintain that learners' construction of knowledge often relies on instructional guidance and support materials. This may require additional explanation and mathematical-specific instructional tools. The knowledge of mathematical-specific instructional tools is therefore also important. Krauss et al. (2008) further maintains that while the latter two are part of Shulman's (1986) generic definition of PCK, they have added knowledge of the potential of mathematical tasks.

Cochran, King and DeRuiter (1993, p. 5) maintain that PCK "is a form of knowledge that makes teachers, teachers, rather than subject area experts". Cochran et al. (1993) developed their own model of PCK which comprises of four areas of knowledge. These areas of knowledge include 1) "content (subject area) knowledge, 2) pedagogical knowledge, 3) knowledge of students (e.g., their prior subject area knowledge, motivation, and backgrounds), and 4) knowledge of the environmental context (e.g. knowledge of the school climate, parental concerns, legal issues, and the social context of the community)" (Cochran et al., 1993, p. 12). Their definition of PCK differs from Shulman's in that they have placed greater emphasis on the environmental contexts of learning and the teachers' knowledge of students.

2.5.1 Mathematics knowledge for teaching

Another set of authors who researched mathematics teacher knowledge is Ball, Thames and Phelps (2008). Ball et al. (2008) who expanded on Shulman's (1986) concept of teacher knowledge introduced the concept of Mathematics Knowledge for Teaching (MKT). They define MKT as "mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" (2008, p. 399). MKT comprises of 4 domains which are: 1) Common content knowledge (CCK); 2) Specialized content knowledge (SCK); 3) Knowledge of content and students (KCS) and 4) Knowledge of content and teaching (KCT). Ball et al. (2008) argue that KCS, KCT and

Knowledge of curriculum are the knowledge domains that make up PCK. These important components of MKT distinguish it from all other conceptions of content knowledge and pedagogical content knowledge

According to Ball et al. (2008), KCS is a knowledge form that includes knowing about mathematics and knowing about students. They argue that teachers should be able to anticipate what learners are likely to think and what they will find confusing. This implies that teachers should have knowledge of learners' preconceptions and misconceptions of mathematical concepts. Ball et al. (2008, p. 401) further argue that "knowledge of students and content is an amalgam, involving a particular mathematical idea or procedure and familiarity with what students often think or do".

Whilst KCS deals with understanding learners' preconceptions and misconceptions, KCT is a combination of knowing about students and knowing about teaching. Ball et al. (2008) maintains that KCT involves having mathematical knowledge of designing instructions. This includes using the most appropriate examples to introduce a concept and later incorporating suitable examples to take learners' deeper into a concept. KCT also includes understanding which materials and resources would be most suitable and effective to teach a concept.

The third component of Ball et al's. (2008) PCK framework, incorporates Shulman's (1986) curricular knowledge. Curricular knowledge includes having knowledge of the programs designed for teaching a particular subject at the different levels. It also includes having knowledge of the instructional materials that can be used in order to facilitate the stipulated curriculum. In addition to this, teachers should also have knowledge of which materials support or does not support the stipulated curriculum. The framework is illustrated in Figure 1.

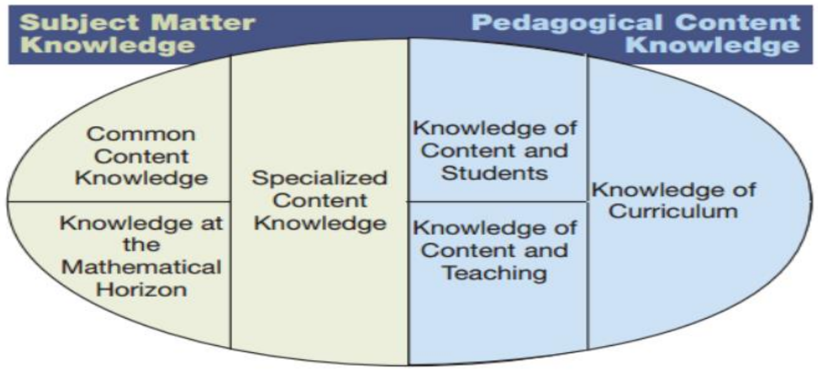


Figure 1: Ball et al. (2008) Mathematical Knowledge for Teaching Framework

Kazima, Pillay and Adler (2008) who were interested in investigating the mathematical knowledge that teachers needed to know and know how to use, carried out two case studies in South African schools. They use the term Mathematics for Teaching (MfT) to refer to "specialized mathematical knowledge that teachers (need to) know and know how to use" (Kazima et al., p.284). They argue that teachers need more than knowledge of topics taught in mathematics, instead the teacher needs mathematical knowledge that is useful and usable in teaching. Kazima et al. (2008) further argues that while Shulman (1987) distinguishes PCK from subject matter knowledge (SMK), the boundaries between these two knowledge domains is not clear in the practice of teaching mathematics and therefore "refer to the more inclusive notion of mathematics for teaching" (p. 284). Kazima et al. (2008) drew on Ball et al's. (2004) framework in order to analyze the mathematical work of teaching in their study. However, they condensed Ball et al's. (2004) framework from eight aspects of mathematical work of teaching to six aspects. They maintained that Ball et al's. (2004) framework was a practice- based notion of MfT and they found it useful in their study.

While the context of Kazima et al's. (2008) study is located within South Africa, which would make it a suitable framework for my study, I have chosen to use Ball and her colleagues' (2008) MKT framework. This framework was selected on the basis that it specifically relates to mathematics teaching. In addition to this, Ball and her colleagues' work was also found to be useful in the analysis for Kazima et al's. (2008) study.

In addition to selecting Ball et al's. (2008) MKT framework for this study, I have chosen to specifically focus on the PCK aspect of this framework. This decision was influenced by the fact that literature pointed towards the need for teachers to have more than just subject matter knowledge. Teachers also need to have a sound knowledge of how students learn, the misconceptions that they bring with them about a particular concept as well as knowledge of the most effective teaching strategies. The SMK part of the framework was also omitted as I felt that given the vast years of experience that both participants held in teaching mathematics, they would have a deep understanding of the knowledge and skills required to teach this subject. The literature that follows, further elaborates on the importance of PCK and its influence on student learning, hence the reason for focusing specifically on this knowledge domain.

2.6. The importance of PCK

Initially, it was believed that if a teacher had a good knowledge or understanding of a subject then he/she was qualified to teach that subject (Shulman, 1986). However, Shulman (1987) built on Dewey's (1902) conception of teacher knowledge which took into consideration the ways in which teachers took a specific subject matter and made it a part of the learners' experiences. This implies that teachers needed more than just a good knowledge of the subject matter; instead the ways in which teachers taught a concept and made it accessible to the learners, taking into account their prior conceptions or misconceptions are all part of teacher knowledge and the knowledge base for teachers.

Similarly, Attard (2011, p.70) argues that "teachers with higher mathematical qualifications do not necessarily produce strong learning outcomes in their students as a result of weak understandings of how students learn and the pedagogies that are appropriate for particular mathematics content." This implies that teachers need more than subject matter knowledge in order to produce strong learning outcomes. They also need to have knowledge of how students learn and the strategies used to teach a particular concept.

Kazima, Pillay and Adler (2008) conducted a study on mathematics for teaching. Their study included a sample of two secondary schools in Gauteng and the aim of their study was to explore the mathematical knowledge for teaching. Based on their study, the authors argue that:

> The teaching of mathematics does not only require the teacher to be knowledgeable about the topic that is to be taught in the sense that the teacher is proficient in solving any problem within the topic. The teacher needs to know and be able to do more than doing the mathematics for him or herself. The teacher needs mathematical knowledge that is useful and useable for teaching”. For example, a teacher must be able to select and clarify appropriate mathematical goals for any lesson taught, and link these with the approach used to teach an idea or concept (p. 283).

Pournara et al. (2015) similarly, argue that teachers need more than a sound content knowledge in order to teach mathematics.

From the review of literature, it is evident that teachers need more than just subject matter knowledge to teach mathematics. In addition, they need to have knowledge of the most useful forms of illustrations in order to make concepts, within a specific subject, comprehensible to others. In addition to this, they also need to understand what makes learning a particular concept easier. Teachers should also possess knowledge of the conceptions and misconceptions learners bring with them regarding subject matter or concepts and they should then be able to use teaching strategies in order to address these misconceptions thereby creating a correct understanding of the concept that is taught (Shulman, 1986). This type of knowledge is what Shulman (1986) terms as pedagogical content knowledge (PCK).

2.6.1 PCK and its link to learner achievement

Lange, Kleickmann and Moller (2012) carried out a study in Germany which aimed to explore whether the PCK of elementary science teachers influenced student learning on the related topics. The sample in their study included 60 fourth-grade classes and their science teachers. Tests were used as part of the data collection instruments. The findings of the study revealed that the science teachers’ PCK was significantly related to the students’ achievement.

Sadler, Sonnert, Coyle, Cook-Smith and Miller (2013) also carried out a study in the United States, in which they examined the relationship between teacher knowledge and 9 556 middle school students. Test items were administered several times during the year to both the teachers and learners. The results of the study revealed that the teachers who could identify students 'misconceptions had larger classroom gains than the teachers who only had knowledge of the correct answer. As mentioned previously, PCK also involves knowing about students' misconceptions. We can therefore argue that the teachers PCK influenced their classroom gains.

2.7. Pedagogic decision making

Pedagogic Decision Making (PDM) has been defined as the process of thinking and reasoning for choosing a particular action from the given alternatives, in the hope that it will bring about effective and meaningful learning for the students (Saad, Ratnavadivel, Hin, Nagappan, Yasin and Radzi, 2009). Simply put, PDM refers to the decisions that teachers make in their teaching in order to ensure that effective learning takes place. These decisions include choosing amongst suitable teaching strategies, instructional tools, teaching resources, asking the right questions, selecting suitable examples etc. with the intention of conveying the curriculum in a way that is most meaningful to the learner. Various researchers describe PDM in different ways. Prachagool, Nuangchalerm, Subramanian and Dostal (2016, p.4) maintains that PDM concerns itself with the "beliefs, efficacy and actions that teachers expose to the classroom". Parmigiani's (2012, p. 172) asserts that PDM refers to "teachers' instructional decisions that enhance the activities in the classroom". This study however, focuses specifically on the instructional decisions that teachers make when facilitating their lessons. These decisions include the sequencing of concepts, selection teaching strategies, instructional tools (text books, worksheets), choice of examples and classroom activities.

2.7.1 Importance of decision making

Over four decades ago, Bishop (1976, p. 42) argued that decision making is at "the heart of the teaching process". Similarly, Watson (2018, p. 1) argues that while "teacher knowledge and beliefs are important", teacher decision making and the actions they take during their lessons, influences the learning environment. Saad et al. (2009) argues that teachers do more than merely deliver the curriculum. Instead they help to redefine it and reinterpret it. They further argue that a teachers' ability "to structure materials, ask higher-order questions, use student ideas, and probe student comments have also been found to be important variables in what students learn" (Saad et al., 2009, p.1). In addition to this, they maintain that it is what teachers think, believe and do at classroom level that will shape the type of learning that learners receive. However, despite the importance that teacher decision making plays in student learning, teacher decision making has been regarded as a neglected area of research since the mid 1980's (Watson, 2018).

2.7.2 PCK and pedagogical decision making

A number of factors such as "classroom context, teachers' experiences and values, content knowledge and pedagogy and the individual students" have shown to influence teacher decision making (Parmigiani, 2012, p. 172). This study however, focuses on exploring the ways in which teachers' PCK influences their pedagogical decision making. I was interested in finding out how teachers utilized their knowledge of the curriculum, the students and teaching to help inform their instructional practices. In addition to this, as mentioned in Chapter One, Barendsen and Henze (2017) argue that there is minimal empirical evidence that determines how teacher knowledge actually informs a teacher's actions in the classroom. The authors therefore suggest that more qualitative studies should be carried out to investigate the relationship between teacher knowledge and classroom practice.

In light of the fact that teacher decision making plays a central role in the teaching and learning process and that there is a lack of studies determining its relationship to teacher knowledge, I thought it would be worthwhile to explore this gap in the educational field.

2.8. Conclusion

In this chapter, I sought to provide a description of teacher knowledge by reviewing both international and local literature. This is followed by an in depth look at PCK as defined by various scholars. I then argue for the use of Ball and her colleagues MKT framework for this study. In addition to this, I have discussed and described pedagogical decision making and its importance in the teaching and learning process. The chapter ended by discussing the need to explore the relationship between teachers' PCK and pedagogic decisions. In the succeeding chapter, I discuss the methodological approach that was undertaken by this study.

CHAPTER 3: RESEARCH METHODOLOGY

3.1. Introduction

This chapter focuses on the research procedure that was used to undertake this study. The chapter begins by providing an explanation of the research paradigm; the methodological approach; the research design and case studies. It also includes a discussion on the main questions guiding this study, the research sample as well as the data collection methods. The suitability of the research paradigm and procedures used in this study is also discussed and justified. To conclude a discussion on the trustworthiness of the study as well as ethical considerations is provided.

3.2. Methodological approach

Cohen, Manion and Morrrison (2007) assert that methodology refers to a variety of approaches that are utilized in educational research in order to gather data, which is later interpreted to provide explanations. Brynard et al. (2014) puts forth a similar argument in that he defines research methodology as focusing on the process of a research and the decisions that a researcher has to make in order to carry out his/her research.

Mackenzie and Knipe (2006, p. 197) argue that "methodology is the overall approach to research linked to the paradigm or theoretical framework while the method refers to the systematic procedures or tools which are used for collection and analysis of data". In the same vein, Pavan and Kulkarni (2014) argue that research methods refer to the techniques used by a researcher to conduct research whereas methodology refers to a way of solving problems in a systematic manner. They further argue that research methodology not only refers to the research method but also the logic behind using a particular technique.

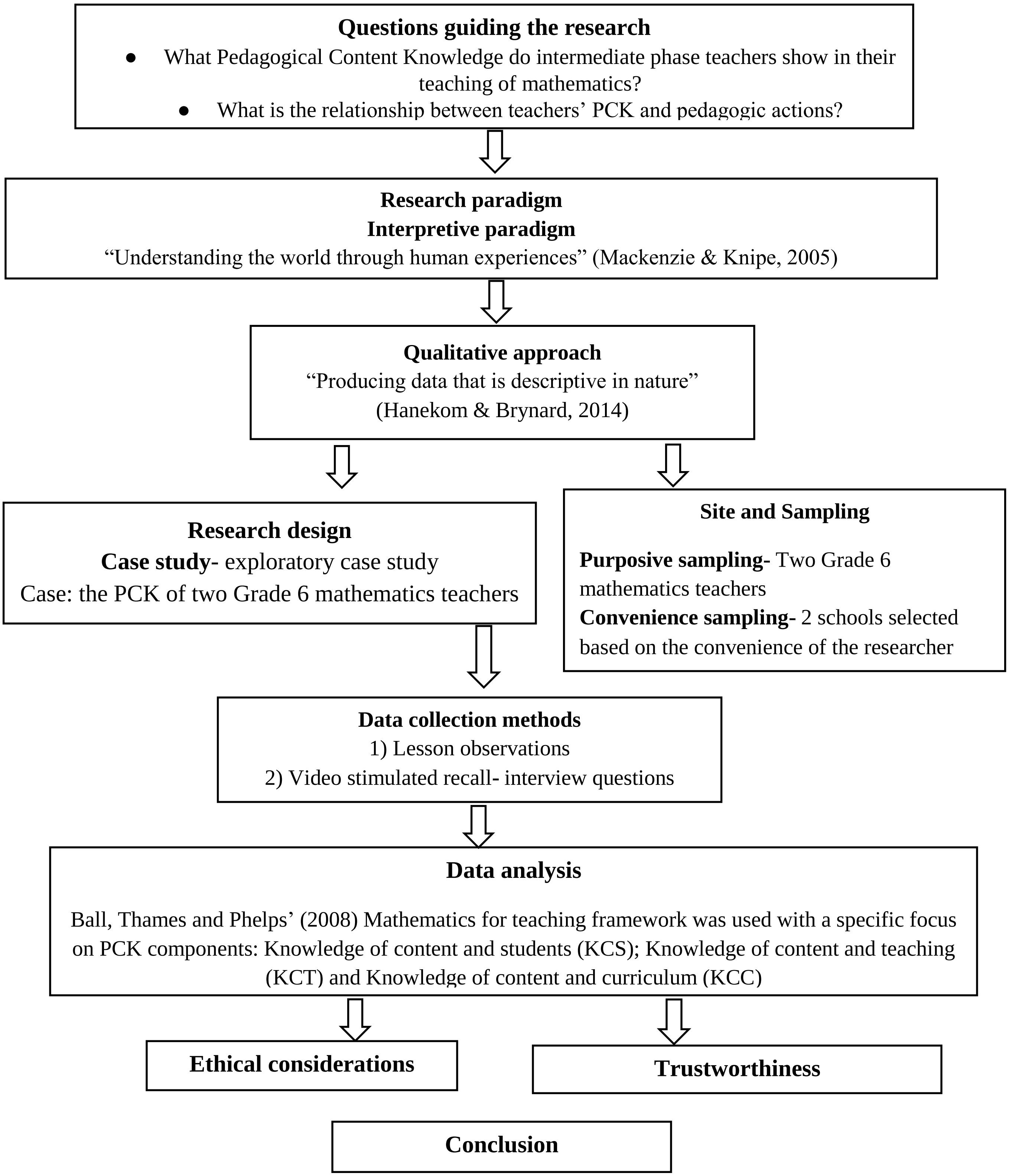


Figure 2 : Outline of Chapter Three

3.3. Research paradigm

Bogdan and Biklen (1998) define a paradigm as a collection of rational beliefs or thoughts that positions or aligns a researcher's thinking and research. Rahi (2017, p. 1) defines the term paradigm as "essential collection and beliefs shared by scientists, it is a set of agreements of how a problem is to be understood, how we view the world and thus go about conducting research."

3.3.1 An interpretive paradigm

This study is underpinned by the interpretive paradigm. Mackenzie and Knipe (2005) assert that researchers working within this paradigm try to understand the world through human experiences which implies that they believe reality is socially constructed. Similarly, Bertram and Christiansen (2014) argue that the interpretive paradigm is unlike the positivist paradigm in that its aim is not to predict and control but rather to understand how people make meaning of their world. They further argue that the purpose of research conducted within this paradigm, "is to develop a greater understanding of how people make sense of the contexts in which they live and work" (2014, p. 26). Hence, I have chosen to work within this paradigm as the purpose of my study was not to make predictions, instead it aimed to explore and understand the PCK of selected intermediate phase mathematics teachers, and how these teachers made sense of their pedagogic actions. Mack (2010) alleges that research within this paradigm is subjective and is observed from the inside, through the direct experience of people. This paradigm allowed me to gain a deeper understanding of the selected teachers PCK by observing their lessons and carrying out interviews which further supported the aim and purpose of this study.

3.4. A qualitative approach

A qualitative approach was used for this study which enabled me to gather rich, in-depth data, in its natural setting which is in line with the characteristics of an interpretive paradigm. Brynard, Hanekom and Brynard (2014) define a qualitative methodology as research that produces data that is descriptive in nature. The data generated is often from the participant's written expressions, perceptions and experiences. Brynard et al. (2014) further argues that research within this type of approach focuses on peoples' real-life experiences and it allows the researcher to get to know people personally and see them for the way they are.

Krauss (2005) puts forth a similar argument as Brynard et al. (2014) in that he contends that researchers within the qualitative approach work under different epistemological assumptions compared to quantitative researchers. For example, they believe that the most beneficial way to understand a phenomenon is to observe it in its own context. They believe that by quantifying the data, it limits it in nature and only a minor part of a reality is viewed. Krauss (2005) further argues that qualitative researchers immerse themselves into the phenomenon being studied and are part of the research.

A qualitative approach was therefore suitable for this study as the population sample consisted of only two teachers who were observed within their own working contexts. Lesson observations and video stimulated interviews were used to get an in-depth view of the participants' reality and the data that was generated from the interviews were the participants' own responses.

3.5. Research design

A research design is a plan of how to go about answering a research question (Mujtaba & Scharf, 2007). It consists of clear research objectives, which arise from the research questions, it clearly stipulates the data collection methods, and further outlines the limitations of the study and ethical issues that are relevant to the research. Similarly, Pavan and Kulkarni (2014, p.171) maintain that a research design "is the blueprint of research". They contend that the research plan helps to guide the researcher on how to gather and analyze data.

3.6. Case study approach

This study uses a case study research design. A case study is an approach that is used when one wants to gain an in- depth understanding of a complex issue in a real-life context (Crowe et al., 2011; Zainal, 2007; Bertram & Christiansen, 2014). This research style is used by researchers working within the interpretive paradigm and is a study of one particular case within a context (Bertram & Christiansen, 2014). This study focused on one particular case which is the PCK of two Grade Six mathematics teachers and this was carried out in their real-life teaching context.

Bertram and Christiansen (2014, p. 42) contend that "case studies aim to describe what it is like to be in a particular situation and are therefore descriptive in nature". Case studies aim to capture a close-up view of a participant's lived experiences and their feelings and thoughts about a particular situation (Cohen et al., 2007). Because case studies involve observing a phenomenon in its natural setting, it is therefore regarded as a naturalistic design (Crowe et al., 2011). The teachers in this study were observed within their working environments in order to gain a better understanding of the PCK that they displayed in their teaching of mathematics. This gave me the opportunity to get an in-depth of the participant's lived reality and to keep a detailed record of their lessons.

3.6.1 Types of case studies

Yin (2003) has proposed three purposes of case studies, which are: exploratory; descriptive and explanatory. An exploratory case study aims at exploring a particular phenomenon that is of interest to the researcher. Zainal (2007) describes an exploratory case study as:

> A case study that sets out to explore any phenomenon in the data which serves as a point of interest to the researcher. For instance, a researcher conducting an exploratory case study on individual's reading process may ask general questions, such as, "Does a student use any strategies when he reads a text?" and "if so, how often?". These general questions are meant to open up the door for further examination of the phenomenon observed. In this case study, prior fieldwork and small-scale data collection may be conducted before the research questions and hypotheses are proposed (p.3).

Descriptive case studies, however set out to describe a particular phenomenon whereas explanatory case studies "examine the data closely both at surface and deeper levels in order to explain the phenomena in the data (Zainal, 2007, p.3). Yin (2003, p. 7) avers that there are three conditions that determine the type of strategy to use. These conditions include (a) the type of research question asked, (b) the degree to which the researcher has control over actual behavioural events, and (c) "the degree of focus on contemporary as opposed to historical events". This study made use of an exploratory case study as the aim of the study was to explore the PCK of the selected Grade Six mathematics teachers. In addition to this, using an exploratory case study was more suitable in answering the research question "what pedagogical content knowledge do intermediate phase teachers show in their teaching of mathematics?"

3.6.2 Strengths of a case study

Zainal (2007) argues that case studies allow for the researcher to obtain data from a real-life context and observation takes place within the context of the phenomenon being studied. This contrasts with experiments, which isolates a phenomenon from its context, thereby focusing on a limited number of variables. In addition to this, Zainal (2007) also argues that a case study allows for both quantitative and qualitative approaches to data collection. Using multiple approaches of data collection methods allow for triangulation of data.

While Zainal (2007) has identified the above strengths of a case study, Cohen et al. (2007, p. 256) have also highlighted the following strengths of a case study:

- Researchers make use of everyday language which allows the results to be understood by a wide range of audience.
- The case studies speak for themselves.
- They pick- up unique features that may otherwise be lost in large scale surveys.
- They are strong on reality.
- They provide insights into other similar situations and cases, thereby assisting interpretation of other similar cases.
- They can be undertaken by a single researcher without needing a full research team.

3.6.3 Weaknesses of a case study

Yin (1984) argues that there are three types of criticism against case studies. Firstly, researchers have argued that case studies lack the rigor that is required in research. Their views were biased and influenced the findings of the research. Secondly, the use of small sample sizes does not allow for generalization. Thirdly, a case study has been viewed as being long, time consuming and producing a large amount of documentation.

Cohen et al. (2007, p. 256) also argue that a case study has the following weaknesses:

- The results may not be generalizable.
- They are not easily open to cross-checking, hence they may be selective, biased and subjective.
- They are prone to problems of observer bias, despite attempts made to address reflexivity.

3.6.3 Refuting the criticism of case studies

Merriam (2009) maintains that in spite of the shortcomings, many researchers have used a case study design in PCK studies due to it being descriptive and detailed in nature. While case studies do have their limitations, its strengths for a qualitative research study far outweigh their weaknesses and therefore, I selected it as my research method.

3.7. Research setting

This study was conducted in two primary schools in the Umgungundlovu District. School A is a well-resourced ex-model C school that comprises of 615 learners who come from middle to higher income homes. This school would be classified as a quintile five school, as it is a well- resourced whose current school fees are between R22 000 to R23 000 per annum. There are approximately 27 learners in each class and each learner has access to textbooks as well as exposure to the internet and technological teaching resources. School B however comprises of learners who come from poorer socio-economic environments. School B would be regarded as a Quintile four school. This implies that it is a less resourced school that School A and it receives greater funding from the DOE. While the school fees are between R1 600 to R1 700 per annum, many of the parents are unable to pay the school fees. The learners are also provided with a feeding scheme and there are almost twice the number of learners in a class in School B than there are in School A.

3.8. Research sampling

This study employed purposive sampling and convenience sampling. Convenience sampling was used in the selection of the schools for this study. This type of sampling is referred to “researching subjects of the population that are easily accessible to the researcher” (Etikan et al., 2016, p. 2). The criteria for this sampling method also include: 1) geographical proximity; 2) availability at any given time and 3) willingness to participate in the research (Etikan et al., 2016). Being based in the Umgungundlovu district made it easier for me to access these schools which are situated within the same location. In addition to this, the principals of the selected schools granted me permission to carry out the research in their schools. This sampling technique was therefore appropriate for this study because the schools were chosen on the basis of accessibility and willingness to participate in the research.

Purposive sampling was used to select the teachers as participants. According to Etikan, Musa and Alkassin (2016) in purposive sampling the researcher deliberately makes a choice in selecting a participant due to the qualities that the participant possesses. They further maintain that the “researcher decides what is to be known and sets out to find people who can or are willing to provide information by virtue of knowledge or experience” (2016, p. 2). Etikan et al. (2016) also argue that sampling is used in qualitative research to identify and select information-rich cases. The sample used in this study comprised of two Grade Six mathematics teachers from two different schools.

The participants in this study were selected based on having taught mathematics for 5 years or more. Purposive sampling was suitable for this study as the participants are knowledgeable about the Mathematics curriculum and they are currently teaching Grade Six mathematics. In addition to this, they were willing to participate in the research.

Purposive and convenience sampling do have their limitations in that it is not representative of the wider Grade Six mathematics teacher population and the findings from this study cannot be generalized. However, this limitation is minimized by using schools from two different socio-economic contexts.

3.9. Research participants

The participants' biographical details include their ages, teaching qualifications, subject majors and teaching experience in mathematics. This information is presented in the table below.

Table 1: Participants' biographical details

Description	**Teacher A**	**Teacher B**
Age	34	59
Gender	Male	Male
Educational qualifications	Bachelor of Education: Further Education Phase- Grades 10-12	Diploma in Higher Education- Secondary School
Qualification majors	Geography, English and Technology	Accounting, Economics, Business Economics, English. Adult Basic Education
Mathematics teaching experience	7 years	34 years
Name of School	School A	School B
Location of school	Urban area- Quintile 5	Urban area- less - resourced school, Quintile 4

Neither Teacher A nor Teacher B are qualified to teach primary school mathematics as both have qualifications to teach at secondary school level. In addition to this, neither teacher A nor B specialized in teaching mathematics. Teacher B has 27 years more experience than Teacher A in teaching mathematics, which suggests that his knowledge of the subject and students should be greater. Both participants teach in schools where the majority of learners do not speak English as their home language. The participants also work in two different contexts. Teacher A teaches in a well- resourced school, where the majority of the parent community come from well earning homes, whereas Teacher B works in a community of middle class to low income homes. Many of the learners that Teacher B teaches come from communities with poor socio-economic factors.

3.10. Data collection methods

This study employed structured lesson observations as well as video stimulated recall interviews as methods of data collection.

3.10.1 Observations

"Classroom observation is a method of directly observing teacher practice as it unfolds in real time" (Hora & Ferrare, 2013, p.1). This allows the researcher to observe firsthand what transpires during the lesson. Cohen and Manion (2007) maintain that in structured observations, the researcher knows in advance what they are looking for. A structured lesson observation (refer to Appendix 4), which was adapted from Ball et al.'s. (2008) MKT framework was used during the lesson observation. A structured observation schedule was suitable for this study as I used the characteristics for each of the PCK domains in order to identify the type of knowledge the teachers utilized in their teaching. I observed one lesson for each educator and this helped me generate data on the teaching strategies that they used, how they addressed learners' prior misconceptions and the examples they used to make concepts understandable to the learners. I looked for characteristics of KCT, KCC and KCS when observing the lessons. The lessons were captured in detail in order to help make sense of the findings.

3.10.2 Video stimulated recall interviews

This study also used video stimulated recall interviews. Visual stimulated recall "involves video recording an activity, then replaying the recordings to the participants so that they can comment on matters of interest" (Rowe, 2009, p. 427). I video recorded one lesson for each teacher. Thereafter, I watched the recorded lessons and this helped to inform my interview questions. The teachers also had the opportunity to view the recorded lessons and to reflect on the reasons for their pedagogic actions and together we engaged in a meaningful conversation on their teaching practice. Using video stimulated recall, helped me to probe deeper into the reasons why the teachers organized their lessons the way they did, it also provided insight into the reasons behind the examples and analogies they used. In addition to this, I was also able to gain a better understanding on how the teachers' addressed the prior misunderstandings learners had about the topics that were taught, and how these misunderstandings were rectified. Rowe (2009) contends that one of the benefits of using video-stimulated recall is that it provides the researcher with an insider's perspective. This implies that the researcher gets to see things from the participants' point of view. This is in keeping with the interpretive paradigm characteristics and was therefore a suitable method of data collection for this study. In addition to this, the interview process was also recorded and then transcribed. This helped me capture verbatim what was said between the researcher and the participant. This ensured that the researcher captured the participants' true response.

3.11. Data analysis

Cohen et al. (2007, p. 461) assert that:

> [q]ualitative data analysis involves organizing, accounting for and explaining the data; in short, making sense of data in terms of the participants' definitions of the situation, noting patterns, themes, categories and regularities.

Mouton (2001, p. 108) describes it as a "process of bringing order, structure and meaning to the data collected by breaking it up into manageable themes, patterns, trends and relationships".

Bertram and Christiansen (2014) maintain that there are two broad approaches to qualitative data analysis i.e. inductive reasoning and deductive reasoning. In inductive reasoning the researcher starts looking for patterns and irregularities in the raw data. The researcher then tries to formulate a hypothesis that they can explore whereas with deductive reasoning the researcher may have a theoretical framework or set of concepts beforehand, "and uses this framework to analyze the data" Bertram & Christiansen, 2014, p. 117). This study adopted a deductive reasoning approach for the lesson observations. I used predetermined categories from Ball et al.'s (2008) MKT framework with a specific focus on the characteristics of PCK to explore the PCK categories that the participants employed when teaching mathematics. Inductive reasoning will be used for analyzing the video stimulated recall interviews.

3.12. Trustworthiness

Trustworthiness is a way in which researchers can convince themselves and their readers that the findings of their research are worthy (Nowell, Norris, White & Moules, 2017). Golafshani (2003) argues that credibility, transferability, neutrality or confirmability are essential criteria to ensure the quality of a research study. Similarly, Bertram and Christiansen (2014) argue that the trustworthiness of the research can be enhanced through credibility (to what extent do the findings reflect participants' lived reality), transferability is the extent to which it can be transferred to another context and confirmability/dependability refers to the way in which the researcher addresses their own biases in the study.

The following strategies were implemented in order to ensure the trustworthiness of the research. The observed lessons were recorded and thereafter transcribed. This ensured that a proper account of the lessons was captured. The semi-structured interviews were also recorded and then transcribed. This captured verbatim what the participants said and did, thereby ensuring that the data collected was a true reflection of the participants' reality and views. Ball et al.'s. (2008) framework was used to code the data, which also strengthened the trustworthiness of this research as it is a well-established framework which is based on work that has already been undertaken in the field of mathematical knowledge for teaching and it has been cited in various scholarly literature. Member checking was also carried out in order to ensure the data reflects the participants view accurately. In addition to this, triangulation of data sources was used to strengthen the findings of this study. Triangulation refers to making use of two or more data collection methods in a study in order to develop a better understanding of the phenomenon being studied (Cohen, Manion & Morrison, 2007). The data generated from the lesson observations and transcribed interviews were combined and rigorously examined before drawing any conclusion.

3.13 Ethical issues

Bertram and Christiansen (2014) assert that ethics are important aspects of research and that all research studies should follow certain ethical principles namely; autonomy, non-maleficence and beneficence. Autonomy refers to respecting the participants in the study by obtaining their voluntary consent and where participants should be able to withdraw from the research at any time. Non-maleficence means to “do no harm”. Researchers need to consider the effects of their study and whether it would cause harm to other parties, be it emotionally, physically or socially. Beneficence refers to the research being of benefit to the participants. This can either benefit the participants directly or indirectly.

In order to adhere to these three principles, I firstly obtained ethical clearance from the KZN Department of Education (refer to Appendix 1) and from UKZN Human Sciences Ethics Committee (Appendix 2). Written consent was obtained from the principals of the schools and the participants who were involved in the study (Appendix 3, Appendix 4, Appendix 5 & Appendix 6). The informed consent letters clearly outlined the purpose of the research and what it entailed. The principal and participants were also informed that their participation in this study was on a voluntary basis and that they could withdraw from the study at any time. The principals and participants were further informed of how the results of this research could possibly benefit them as it could help address the challenges that we face in the teaching and learning of intermediate phase mathematics. Pseudonyms were used to protect the identity of the school and the teachers.

3.14 Conclusion

Shoaib and Mujtaba (2016, p. 83) argue that “choosing an appropriate methodology for a research can enhance its quality”. They further argue that the research question is the determining factor for the type of methodology used. This study was based on exploring the pedagogical content knowledge (PCK) of intermediate phase mathematics teachers and it did not seek to make generalizations, instead it sought to explore and understand a phenomenon. The research therefore lends itself to an interpretive paradigm and made use of a qualitative approach. A qualitative approach deals with understanding human experiences and produces data that is descriptive in nature. This type of data can be generated by using a case study. It is therefore argued that the research paradigm, the methodological approach and methods suit the purpose of the research questions. This is guided by the idea of "fitness for purpose", "which means that the methods of data collection must match the kind of data that the researcher wants to collect and the kind of data that needs to be collected will be informed by the research question and the style of research" (Bertram & Christiansen, p. 41).

This chapter has provided a detailed account of the research approach and research design used in this study. The data collection instruments and a description of the data analysis was provided. Ethical considerations and trustworthiness of the research were also explained. To conclude, this chapter also justified the use of the research paradigm and approach for this study.

The succeeding chapter will present and discuss the data collected.

CHAPTER 4: DATA ANALYSIS AND FINDINGS

4.1. Introduction

This chapter presents the data analysis and findings on the pedagogical content knowledge of two Grade Six mathematics teacher's teaching of 2D and 3D shapes. As discussed in Chapter Three, data was collected using lesson observations and video stimulated recall interviews. The data collected for this study was rigorously reviewed through repeated reading. Ibrahim (2013) maintains that data analysis involves organizing the data in a proper way and further maintains that qualitative data collection is often dependent on interpretation. The data collected was coded by looking for characteristics of PCK based on Ball, Thames and Phelps' (2008) Mathematics Knowledge for Teaching (MKT) framework.

This chapter starts off by providing a brief description of the topics that were taught during the lesson observations, thereafter the biographical details of the two Grade Six mathematics teachers is provided. Each teachers' lesson observation is then presented and analysed according to the three categories of PCK which are knowledge of content and curriculum (**KCC**), knowledge of content and students (**KCS**) and knowledge of content and teaching **(KCT**). The transcribed interviews were also coded according to the three categories of PCK. Sutton and Austin (2015, p. 227) maintain that the most important part of data analysis in a qualitative study "is to be true to the participant", therefore the analysis of the data collected uses direct quotes from the participants in order to show their perspectives and the meanings that they ascribe to their lived realities. The data collection and analysis were undertaken to address the following main questions:

1. What pedagogical knowledge do intermediate phase teachers show in their teaching of mathematics?

2. What is the relationship between teachers' pedagogic decision making and PCK?

4.1.1 Space and Shape

The selection of the topics that were taught during the lesson observation was based on the time that the data was collected. The Curriculum Assessment and Policy Statement (CAPS) is the prescribed curriculum document for South African public schools. Each term, teachers are required to teach a specific concept during a specific week. At the time of my data collection, both Teacher A and Teacher B were teaching concepts under the content area of Space and Shape (geometry). Space and Shape is one of the five content areas in mathematics. These content areas include the following categories:

- Numbers, Operations and Relationships;
- Patterns, Functions and Algebra;
- Space and Shape (Geometry);
- Measurement; and
- Data Handling (CAPS, 2011).

Space and Shape is an important knowledge component in the teaching and learning of mathematics. The learners are required to "describe and represent characteristics and relationship between two-dimensional shapes and three-dimensional objects in a variety of positions, as well as able to analyse and explain the properties of two-dimensional shapes and three-dimensional shapes" (Kotze, 2007, p. 21). The table below demonstrates the structuring of Space and Shape in the grade six syllabus as per the CAPS (2011) document.

Table 2: Grade six space and shape content area

Categories of Space and Shape	
Range of shapes	• Recognize, visualize and name 2-D shapes in the environment and geometric settings, focusing on: -regular and irregular polygons - triangles, squares, rectangles, parallelograms, other quadrilaterals, pentagons, hexagons, heptagons, octagons -circles - similarities and differences between rectangles and parallelograms

Characteristics of shapes	Describe, sort and compare 2-D shapes in terms of - number of sides -lengths of sides - sizes of angles ◊ acute ◊ right ◊ obtuse ◊ straight ◊ reflex ◊ revolution
Angles	• Recognize and name the following angles in 2-D shapes: - acute - right - obtuse - straight - reflex - revolution
Range of objects	Recognize, visualize and name 3-D objects in the environment and geometric settings, focusing on -rectangular prisms -cubes - tetrahedrons -pyramids -similarities and differences between tetrahedrons and other pyramids
Characteristics of objects	• Describe, sort and compare 3-D objects in terms of - number and shape of faces - number of vertices - number of edges **Further activities** • Make 3-D models using: -drinking straws, toothpicks etc -- nets

Adapted from the CAPS (2011) document.

The teachers in this study taught lessons based on the properties of two-dimensional and three-dimensional shapes, calculating the surface area of a square-based pyramid and the values of the sum of interior angles in two-dimensional shapes. This is further described in section 4.4 under the discussion of their lesson observations.

4.2. Participant’s biographical details

The sample in this study consisted of two Grade Six mathematics teachers. To protect their identities, they were given the pseudonyms Teacher A and Teacher B respectively. Teacher A teaches in an ex-model C school whilst Teacher B teaches in an under-resourced school.

Teacher A is a male teacher who is 34 years old. He has 10 years of teaching experience; however he has taught mathematics for only 7 years. He has a Bachelors’ Degree in Education. He specialized in the Further Education and Training Phase (FET), which is Grades 10 to 12. He majored in English, geography and technology. While he did not specialize in mathematics, his technology subject did involve some mathematics. When I enquired about why he chose to teach mathematics his response was:

> *One reason was because I had to. I was teaching in MC and I was doing Grade 10 geography and 11 geography and then the position opened up in the Grade Seven class and in Grade Seven, you had to teach majority of the subjects in that school. And obviously, math was one of it. So, I had to teach it. But when I did start teaching it, I really enjoyed it.*

Although Teacher A only started teaching mathematics because he had to teach it, Teacher A articulated that once he started teaching it, he actually enjoyed it.

Teacher B is also a male teacher who is 59 years old. He is the Head of Department (HOD) at School B and has a Higher Diploma in Education. He specialized in Secondary Education with majors in accounting, economics, business economics, English and adult basic education. He has 34 years of teaching experience in mathematics and his response for choosing to teach this subject was as follows:

> *The reason was simple, I worked for four years and I used to do all the payments at the workplace. So, I had the proper figures and I was able to multiply and add quickly, you know, we think, I can problem solve on the spur of the moment. And the principal felt I could teach maths and I didn't make a mistake by doing that, I’ll be honest.*

Teacher B, like Teacher A is not a mathematics qualified teacher, however both teachers have expressed their passion for teaching this subject as they both find it enjoyable.

4.3. The data analysis process

Creswell (1997) argues that data analysis is carried out in order to develop a detailed knowledge of the phenomenon being studied. This study aimed to explore the PCK of two Grade Six mathematics teachers and it followed the approach used by Sutton and Austin (2015, p. 228) who argue that data coding involves identifying "topics, issues, similarities, and differences that are revealed through the participants' narratives and interpreted by the researcher". They further maintain that this can be achieved through note making along the margins of the transcribed interviews. Relevant texts, patterns or themes can also be highlighted. This study employed a similar coding approach. The data collected from the lesson observations and video stimulated interviews were manually transcribed. Coding was then carried out by looking for characteristics of Ball, Thames and Phelps' (2008) Mathematics Knowledge for Teaching (MKT) framework, with a specific focus on PCK.

The interview transcripts and lesson observations were repeatedly read so that a clear and deep understanding of the teachers' PCK could be obtained. Relevant texts in the interview transcripts were then highlighted using three different colours to represent the three different categories of PCK (KCT, KCS and KCT). This was also accompanied by note making along the margins. For the lesson observation, notes were also made alongside each activity that the teacher had carried out. This was done in order to identify and discuss the component of PCK that was used (refer to Table 2).Whilst this is a qualitative study, tables and diagrams have been used to represent certain aspects of information that I felt would be easily understood when represented in this manner.

Table 2 below displays the three components of PCK with their characteristics as identified by Ball, Thames and Phelps (2008). This was used to identify the components of PCK that the participants' utilized in their teaching of Grade Six mathematics.

Table 3: Categories of PCK and their characteristics (adapted from Ball et al. 2008, p. 401-402)

KCT: Knowledge of content and Teaching **Combines knowing about teaching and knowing about mathematics.**	**KCS- Knowledge of Content and Students** **Knowledge that combines knowing about mathematics and knowing about students**	**KCC- Knowledge of Content and Curriculum** **Knowledge of the content requirements of the curriculum and the materials that can be used to teach that particular content**
• sequence mathematical content • present mathematical ideas • select examples to take students deeper into mathematical content • select appropriate representations to illustrate the content • ask productive mathematical questions • recognise what is involved in using a particular representation • modify tasks to be either easier or harder • use appropriate teaching strategies • respond to students' why questions • choose and develop useable definitions • provide suitable examples	• anticipate what students are likely to think and do • predict what students will find interesting and motivating when choosing an example • anticipate what a student will find difficult and easy when completing a task • anticipate students' emerging and incomplete ideas • recognise and articulate misconceptions students carry about particular mathematics content	• articulate the topics in the curriculum • articulate the competencies related to each topic in the mathematics curriculum • articulate and demonstrate a familiarity with the structure of the mathematics curriculum • link representations to underlying ideas and to other representations • knowledgeability of available materials (e.g. textbooks) and their purposes

4.4 Teacher A: Pedagogical Content Knowledge

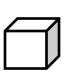
This section presents the findings on Teacher A's PCK. A description of the lesson observation is provided and links are made to the three components of PCK namely; knowledge of content and teaching (KCT); knowledge of content and students (KCS) and knowledge of content and curriculum (KCC). In addition to this, a detailed description of the findings for the semi-structured interview is then provided according to the three categories of PCK.

4.4.1 Description of the classroom observation

The purpose of the classroom observation was to explore the different components of PCK that the teacher uses when teaching a specific topic. In this case the surface area of a square-based pyramid. The observation focused on how the teacher conveyed the content in terms of the instructional strategies and representations that he used to make the concept understandable to the learners, his understanding of the learners' preconceptions and misconceptions, how he addressed this as well as his ability to take the learners deeper into the concept.

The lesson observation data has been divided into two columns. On the left-hand column a description of the teaching and learning activities is provided. Classroom activities are described as episodes. On the right-hand column, categories of PCK is identified and explained. The PCK categories identified are coded as A1; A2; A3 and so on, whereas the teaching and learning activities have been coded as Episode 1, Episode 2 and so forth. The symbol "A" is used to represent Teacher A and numbers are used to represent the activity. The same coding is used for Teacher B and the symbol "A" is replaced with a "B". Under the findings/data analysis section, these codes will be used when referring to the category of PCK and Episode1, Episode 2 etc. will be used to refer to the teaching and learning activity.

Table 4: Lesson observation of Teacher A

Description of lesson observation: Teacher A **Grade Six (School A)**	**Analysis of lesson using categories of PCK**
Classroom environment: There are 27 learners in the class. Two learners are seated per desk. There is ample space between the desks. This allows the teacher to move freely from row to row. The class is not overcrowded. The teacher has a white board and markers as well as a marker remover. The teacher is visible to all the learners. The class is well lit and there are charts on the wall. The class is clean and tidy.	The classroom environment is highly conducive to learning.
Lesson observation: Topic: Area of a square based pyramid **07:54 to 8:15 (20minutes)** **Episode 1: Activity 1 Quiz (5 minutes)** Teacher starts the lesson with a quiz which recaps work already learned. The teacher uses the whiteboard to write down the questions for the quiz. Learners copied each question into their books, and did the calculation. 1. △ B= 10cm H= 5cm find the A=_____ 2. □ L= 8cm W= 4cm find the A= ___ 3. □ L=_____W= 3cm A= 33cm^{2}. Find the length. 4. □ L= 12cm B= 6cm find the A= ___. 5. □ B= 18 cm H= 10cm find the A= ___. 6. △ W= 4cm L= ___ A= 37cm^{2}. (Mixed number or decimal) 7.  Work out the surface area if one edge is 2cm (Think about how many faces the cube has). 8. □ Square: A= 16cm^{2} work out L and W.	**A1.** **Knowledge of content and teaching**. In this activity Teacher A has displayed characteristics of **KCT**: He used examples **to take the learners deeper into the mathematical content**. He firstly started the quiz by using 2 dimensional shapes (these shapes have only 2 dimensions which are length and width) and then moved on to 3 dimensional shapes (these shapes have 3 dimensions which are length, width and height). A higher level of cognitive engagement is needed to work out the area for 3D shapes. The same approach is used in the structuring of his questions. He first provides the learners with both measurements (the length and width) thereafter he omits a measurement and provides the area. This requires the learners to think more critically in order to find the missing length. He moves from an easier level to a more critical engagement. --- **A2.** Teacher A also displays knowledge of content and teaching (**KCT)** when writing down question 7 on the board. He explains to the learners that when working out the surface area

9.☐ L= 9cm W= 4cm find the A= ____. 10.☐ The measurement for one side is given = 7cm find the A=____. The teacher makes use of diagrams to show the rectangles, triangles etc. He plots the measurements next to the given sides. The teacher explains to the learners that they are required to find the missing information. The teacher starts the quiz off by first using 2D shapes**. Initially the questions provide at least 2 measurements (length and width), however as he moves along in the quiz, he omits a measurement from the 2D shape and provides them with one measurement and the Area.** The learner is then required to find the missing measurement. The teacher then moves on to using 3D shapes as evident in question 7. **The teacher reminds the learners' that for question 3 there are two ways to get the answer and suggested that they do not use substitution.** The teacher then walks around the class and assists the learners while they are doing calculations in their books.	for the cuboid, they must think about how many faces it has. In other words, because all the faces of the cuboid are the same size, all you have to do is find the surface area of one face at multiply it by the number of faces that it has .In this way the teacher is assisting the learners or **modifying the tasks to make it easier which is KCT.** **A3.** **Knowledge of content and students** In question 3, Teacher A reminds the learners not to use substitution. **He anticipated that the learners are likely to do this** and therefore reminded them not to do so. This is evidence of **KCS**.
Episode 2 (Marking the quiz) After completing the quiz, learners move places to sit next to whomever they choose. They exchanged exercise books and mark each other's work. The teacher asks Learner A how to get to answer 1. Learner A responds ½ base x height, which is correct. The teacher then works out the answers to the questions and in doing so, he explains his answers. For question 6 the teacher explains to the learners that the answer has a fraction- he asks the learners to convert it into a decimal and he explains how to get to the decimal. For question 8 the teacher selects a learner who he thinks is not really paying attention, to answer the question. The learner responds with the correct answer. The teacher explains that the area is a squared number (16cm^2) and that a squared number is a number multiplied by itself. Learner B asks the teacher "what if you put 4x4? The teacher says that he will accept this answer. The teacher then walks around to check the learners' marks.	**A4.** Teacher A accepts the answer 4 x 4 from the learner, however there is no probing from the teacher or a further explanation demanded by the teacher.

8:15 to 8:50 (35 minutes)

Episode 3 Teacher explanation

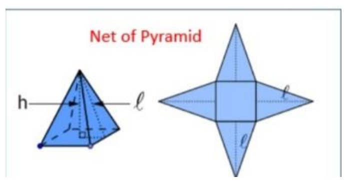
The teacher reminds the learners that the day before they discussed the square based pyramid. He explains that the learners will now cut the net and construct the 3D shape. Each learner receives a worksheet with the outline of the net.

While the learners are cutting out the net, the teacher explains what a net is. **He explains that it is different from a fishing net instead it is a pattern in 2 Dimensional that can be folded to make 3D shape. The teacher then asks the learners "How do we know that this is a pyramid?"**

Learner C responds "It has triangular faces for its sides".

The teacher also asks the learners the following questions:

- How many faces does the shape have?
- Are all the faces identical?
- What are the shapes of the faces?
- How many edges does the shape have?
- What can we say about the triangles?



One of the learners' responded incorrectly to the question on the number of edges for the square based pyramid. The teacher then asks another learner and that learner responds correctly.

The teacher then explains that one side of the square (the base of the pyramid is a square) is 7cm. The teacher then quickly changes that measurement to 8cm as it would be easier to halve an even number than an odd number when working out the formula for the square based pyramid. The teacher then reminds the learners on how to work out the area of the square.

The teacher then explains to the learners that when working out the height of the triangle they need to measure from the base of the triangle. He also explains to the learners that the reason why he is showing this to them is because a lot of them make a mistake when measuring the height of the triangle and he wants to correct this misconception.

The teacher then demonstrates using illustrations on the board that the height of a triangular face is measured from the base of the pyramid and not from the centre of the square.

The teacher now moves on to explain how to work out the surface area of the square based pyramid.

The teacher writes the formula on the board. ½ base x height. The teacher explains step by step how to work out the formula for the surface area of the square based pyramid.

A5.

Teacher A develops useable definitions by explaining the meaning of a net, this is evidence of knowledge of content and teaching (**KCT**).

A6.

Teacher A has displayed knowledge of content and curriculum (KCC). The learners needed to first know about the properties of a square based pyramid (what makes it a square based pyramid) before working out its' surface area. The teacher therefore asks questions related to its properties.

A7.

Teacher A also makes use of nets (which were constructed into the 3D shape) to help explain the properties of the 3D shape being discussed. The learners were able to use the 3D shape to count how many faces the square based pyramid has and the shapes of its faces. This helped the learners to answer the questions asked by the teacher. This teaching material was suitable for the lesson. This is evidence of knowledge of materials and their purposes when teaching, which is a characteristic of **knowledge of curriculum and content** (**KCC**).

A8.

Teacher A changes the measurement for the square base from 7cm to 8cm as it would be easier for the learners to halve an even number than an odd number. This shows evidence of knowledge of content and teaching (**KCT) (modify tasks to be easier)** as well as evidence of knowledge of content and students **KCS (anticipate what a student will find difficult and easy).**

All the information discussed with the learners at the beginning of lesson was required in order to work out the surface area of the square based pyramid. Step 1: A of a △= ½ base x height ½ (8) x 12 = 4 x 12 = 48cm x 4 = 192 cm^2 The teacher walks around the class to check that everyone is working out the area of the triangle correctly. The teacher then explains that they now have to work out the area of the square. He uses the formula below to show the learners how to find the area of the square. Step 2: A of a □ = L x B = 8 x 8 = 64 cm^2 Step 3: Add the area of the triangle and square 192 + 64 = 256 cm^2 The teacher explains this step by step to the learners.	**A9.** Teacher A shows knowledge of learners' misconceptions when he reinforces and explains the correct way of measuring the height of the triangle. This is evidence of **Knowledge of content and students (KCS).** **A10.** Teacher A uses his **content knowledge and procedural knowledge** in order to explain the formula for the surface area of the square based pyramid.
Episode 4 (8:50 to 9:10) **Learner activity** The teacher then puts another example on the board. The height of the triangle= 14cm The base of the triangle is 6cm The learners work independently to solve this example. After a few minutes the teacher writes the answer on the board and at the same time he explains how to calculate the surface area of this square based pyramid. A of a △ = ½ base x height 3 x 14cm 42cm ^{2} Multiply the Area of 1 △ x 4 = 168cm ^{2} A of a □ = L x B 6cm x 6cm $36cm^2$ Add the area of the triangle and square= 168cm ^{2} + $36cm^2$ = 204 cm^2 The teacher informs the learners that if they any queries they could come and see him later.	**A11**. Teacher A uses his **content knowledge and knowledge of procedures i**n order to explain how to calculate the area of the square based pyramid.

4.4.2 Summary of Teacher A's lesson observation

The table below is a summary of Teacher A's lesson observation. The lesson is divided into four episodes. The first episode is a quiz, the second episode is the peer assessment of the quiz and teacher feedback, the third episode is the teacher's explanation of calculating the surface area of the square-based pyramid and the fourth episode involves the learners working out an example on their own.

Table 5: Summary of Teacher A's lesson observation

Episode	**Description**
Episode 1 **Quiz activity**	In this episode the teacher starts the lesson with a quiz. The quiz is a recap of previous lessons and is based on calculating the area of various 2D shapes (triangle, rectangle, square and a cuboid). Initially the teacher provides the learners with the measurements of two sides, thereafter the learners needed to calculate the area for these shapes. The teacher then moves on to provide the learners with only one measurement and the area of the shape. The learners then needed to find the missing length using inverse operation. The quiz is sequenced from lower order to higher order questions. The learners work independently to calculate their answers.
Episode 2 **Peer assessment and teacher feedback**	During this episode the learners move places and sit next to whomever they choose. The learners exchange exercise books and mark each other's work. The teacher provides the learners with the correct answers and at the same time explains to the learners how he arrived at the correct answers. The teacher also responds to questions that the learners posed.
Episode 3 **Teacher explanation of calculating surface area of a square-based pyramid**	The learners were given the net of a square based pyramid. The learners cut out the net and constructed the 3D shape. The teacher asked the learners a few questions based on this shape, for example; How many faces does the shape have? Are all the faces identical? The learners used their 3D shape to assist them in answering these questions. The teacher then explains to the learners that when calculating the height of the pyramid they need to measure the height from the base of the pyramid (Teacher A articulated that this was a common misconception that learners have and he wanted to rectify

	this misconception). The teacher then explains to the learners how to find the Area of a square-based pyramid by using its formula.
Episode 4 **Learner activity**	This is the concluding episode. The teacher puts an example on the whiteboard. The learners work independently to calculate the area of the square-based pyramid. The teacher then provides the learners with the correct answer.

4.5 An analysis of Teacher A's PCK based on the lesson observation and semi-structured interview data

The following section provides a synthesized analysis of the PCK of Teacher A drawing on the data generated from the lesson observation and the interview.

4.5.1 Knowledge of Content and Curriculum

Gulpric Chua (2018, p.5) describes KCC as knowledge that "reflects a teachers' broader knowledge of how topics are relevant to others within the same subject matter but beyond the grade level". This implies that a teacher who is able to demonstrate how one topic within the same subject is relevant to another topic, does in fact possess characteristics of KCC. Teacher A articulated this characteristic as he demonstrated in his teaching that the properties of a 2D shape is related to working out the surface area of a 3D shape. Teacher A structured and sequenced his lesson by firstly re-capping the properties of 2D shapes. This was done in the form of a quiz as evident in his lesson observation (refer to Table 3 Episode 1). In order for the learners to work out the surface area of a square based pyramid, they would firstly need to have knowledge of the properties of a triangle and square, which Teacher A mentioned was done in the previous lessons. The following statement is evidence of that:

> *[T]he formula for the triangles we had done area of 2D shapes before. We had looked at squares, rectangles, triangles, and also circles.*

His knowledge was also evident during the lesson observation when he asked the learners questions related to their prior knowledge and to the topic he was about to teach (refer to Table 3, Episode 3, A6). In addition to this Teacher A mentioned that *"We had looked at squares, rectangles, triangles, and also circles,* ***because they don't necessarily need to know it, but it's nice just to push them and see what they are capable of".*** The highlighted statement is in line with what Gulpric Chua (2018) argues when he stated that apart from relating the topics within a subject, the teacher should extend it beyond the "grade level", which is what Teacher A employed in his teaching.

Teacher A drew on his knowledge of content to skillfully demonstrate to the learners how to calculate the area of a square based pyramid as evident in his lesson observation. He used physical resources such as nets, diagrams on the board and 3D shapes to firstly highlight the properties of a square based pyramid and he then used an explanation to calculate the area of a square based pyramid.

4.5.2 Knowledge of Content and Teaching

Ball and her colleagues argue that this category of PCK combines knowing about teaching and knowing about mathematics. This implies that the teacher should have knowledge of the teaching strategies, resources and materials used to effectively teach a concept as well as a deep knowledge of the content matter. Teacher A displayed characteristics of KCT as he made use of relevant and effective teaching materials (the net of the square based pyramid, Table 3, Episode 3). Teacher A said using the net of the square based pyramid was a suitable resource, and the following statement is evidence of this:

> *So, the resources I used, and I think there was just the nets, and then I gave them the net again which they had to cut and fold into the 3D shape itself. I don't necessarily like to use a textbook too much. I find that, I know last year, I used a textbook quite a lot. I relied on it and the kids got quite bored of it and it did not allow kids to move forward or further than other ones.*

The following statement is further evidence of knowing about teaching, as Teacher A knew the type of resource that would be required to teach his lesson effectively:

> *I didn't use power point or anything like that because I didn't think it was necessary. I think that the physical objects that they were holding, the 3D shape was enough for them.*

This is also in line with how he executed his teaching during his lesson observation (refer to Table 3, Episode 3, A7). The learners used the net (which they constructed into a 3D shape) to answer the questions which were posed by Teacher A and by using the net, they were easily able to identify the number of faces, vertices etc. of the 3D shape. Teacher A also demonstrated a good understanding of the mathematical content as he drew on this knowledge of content to explain to the learners how to calculate the surface area of the square based pyramid (refer to Table3, Episode 3 and 4, A10 and A11).

4.5.3 Knowledge of Content and Students

Knowledge of Content and Students has been described as knowing about mathematics and knowing about students (Ball et al., 2008). Gulpric Chua (2018) maintains that this knowledge category includes the teacher's ability to design lessons that will stimulate the learners as well as address any misconceptions that learners bring with them about a particular concept. He further states that having knowledge of what may work for the learners or what may impede on their development in understanding a concept, also demonstrates a high level of proficiency in this knowledge category by the teacher.

Teacher A demonstrated an adequate level of KCS in that he was able to identify some of the misconceptions that the learners had with regards to finding the area of the square based pyramid. Teacher A identified the following misconceptions about the topic in his interview:

> *I think that's where a lot of them struggle with is working out the area of the triangular faces.*

One of the major reasons I also did it was because a lot of the time I've seen kids make mistakes when it comes to the heights of the triangle, so its half base times height and often I've seen kids measure height from the centre of the pyramid, that's why I was saying to them that the centre of the pyramid to its highest point, that's not the height of that triangular face

Just using simple words such as vertical, horizontal and diagonal. They don't understand what that word means.

Teacher A stated in his interview that the learners often made the mistake of measuring the height of the pyramid from the centre of its base which is incorrect and that he tries to correct this misconception in his teaching, which is in line with his lesson observation (refer to Table 3, Episode 3, A9).

While Teacher A has displayed adequate knowledge of the learners' misconceptions, his ability to deal with the learners' mistakes appear to be limited. During the lesson observation a learner responded incorrectly to a question posed by Teacher A (refer to Table 3, Episode 3, A7). Teacher A simply moved on by allowing another learner to answer the question. Teacher A did not try and understand or address the learner's mistake or misconception but instead moved on to another learner who provided the correct answer to the question. During the interview, I asked Teacher A if he thought that the purpose of his lesson was achieved and his response was:

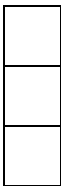
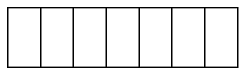
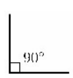
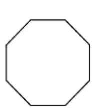
I think to an extent. I think the unfortunate thing, because of time constraints you can't necessarily sit on it and there are students that are going to not pick up on it.

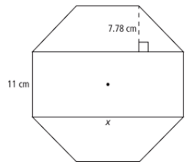
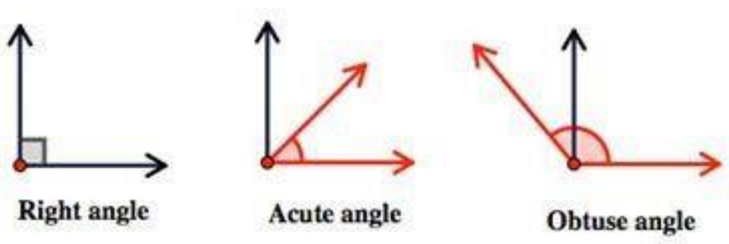
I presume that the issue of time constraints could be a contributing factor as to why Teacher A did not take the time to address the learner's mistake and he simply moved on. This issue will be further discussed in Chapter Five.

4.6. Teacher B: Pedagogical Content Knowledge

Table 6: Lesson observation of Teacher B

Description of lesson observation: Teacher B **Grade: 6** **School B:**	**Categories of PCK**
Classroom environment: There are 34 learners in the class even though this is a school that receives very little school fees as the learners come from very poor communities. Two learners are seated per desk. There is ample space between the desks. This allows the teacher to move freely from row to row. The class is not overcrowded and the furniture is in a fairly good condition. The teacher has a chalk board, duster and chalk. Learners have departmental workbooks and worksheets are used as resources. The teacher is visible to all the learners. The class is well lit and there are charts on the wall.	The classroom is conducive to learning.
Lesson observation: Topic: finding the value of angles **8:00 to 8: 20** **<u>Episode 1 Re-cap 2D shapes</u>** Teacher B begins the lesson with a re-cap of 2D shapes. The teacher asks the learners to tell him something about a 2D shape. A learner responds by saying that it has 2 dimensions. Teacher B then describes a 2D shape by explaining to the learners that it is a flat shape and that it is a polygon (this is a re-cap of the work that has already been done). Teacher B then asks the learners for examples of triangles, he gives them a clue that one of the names of a triangle starts with an "E" (for equilateral). Teacher B then lists the different triangles (isosceles, scalene and equilateral). He explains to the learners that the "tri" in triangles stands for the number three and triangles represent a three-sided figure. Teacher B then discusses quadrilaterals. He makes use of real-life objects to discuss this 2D shape by informing the learners that it is a shape that has 4 straight sides and ask them to think of a quad bike that has four wheels. Teacher B asks the learners to describe other polygons example: 5 sides, 6 sides, 7 sides, 8 sides, 9 sides. The learners respond (chorus like): pentagons, hexagons, heptagons, octagons and nonagons. Teacher B then moves on to ask the learners "what is a line?" There is no response from the learners. So, Teacher B explains that a line is a combination of dots. Teacher B then asked the learners to "name 3 types of lines." A learner responds vertical line. Teacher B then calls out another learner to draw a vertical line on the chalk board. The learner draws it incorrectly. The teacher corrects this misconception by explaining what a vertical line is. Teacher B explains that a vertical line goes from North to South	**B1.** Teacher B sequences the lesson by first starting off with the learners' prior knowledge. Properties of 2D shapes are taught in grades 4, 5 and 6. The teacher re-caps this information and reinforces. Learners need to know this information before they can work out the values of the angles of a 2D shape. Teacher B has shown evidence of **knowledge of content and teaching (KCT)** by s**equencing the mathematical content** in this manner. He also displayed knowledge of content and curriculum **(KCC).** He showed competency in knowing what prior knowledge the learners' needed before introducing the new concept. **B2.** Teacher B gives the learners a clue that one of the triangles start with an "E" (equilateral triangle) in this way, Teacher B is assisting the learners to find the answer (**making it easier**) which is evidence of **Knowledge of content and teaching(KCT).** **B3.** **Teacher B uses content knowledge to explain and describe the properties of the 2D shapes.** **Teacher B has shown evidence of knowledge of content and teaching (KCT)** when explaining the 3 types of lines used to construct or draw a shape. Teacher B choose and used useable definitions (he explained to the learners that a vertical line goes from north to south and a horizontal line goes from east to

while a horizontal line goes from east to west (horizontal for horizon) and draws it on the board while explaining to the learners. Teacher B then asks the learners to name the third type of line. A learner responds that it is a diagonal line. Teacher B calls the learner out and asks the learner to draw the line on the board. The learner draws the line correctly. Teacher further explains horizontal and vertical lines by using the diagram of a table:   Horizontal lines are rows Vertical lines are columns Teacher B then asks a learner to draw a shape that uses vertical, diagonal and horizontal lines. The learner draws an octagon. The teacher then explains to the class that when drawing a shape, you use a combination of all three lines.	west and related this term to the word horizon. Teacher B also showed evidence of **KCT** in using appropriate representations to illustrate the content (he drew the lines on the board while explaining it to the learners. In this way the learner could attach the illustration to the word. **B4**. Teacher B showed evidence of **knowledge of content and students (KCS)** by explaining the different lines that make up a 2D shape as he knew that many learners had misconceptions or did not really understand and know what a vertical, horizontal or diagonal line is. **B5**. Teacher B uses a discussion/explaining technique as well as questioning technique to re-cap the concepts in the first part of his lesson. He also gets the learners involved by getting them to come to the board and draw the different lines that were discussed. Teacher B has shown evidence of **knowledge of content and teaching (KCT)** as he asks productive mathematical questions, and uses appropriate teaching strategies.
<u>Episode 2: Re-cap of value of angles</u> Teacher B asked the learners "What instrument do we use to measure angles?" A learner responds "We use degrees". Another learner shouts out "centimetres!" Teacher B shows the learners a semi-circle instrument and explains to the learners that it is protractor and we use it to measure angles. Teacher B then asks for a learner to come out and draw a right angle, which the learner does correctly. The teacher then draws an angle on the board **L** and he asks the learners if it is correct. No learners respond. He then explains that it is incorrect as it does not have an angle indicator.  Teacher B then draws a rectangle on the board and asks for its properties. A learner lists the properties of a rectangle (2 opposite sides are equal, 2 long and to short equal sides, all straight sides and four 90˚). Teacher B explains that the sum total of the angles of a rectangle is 360˚ (90˚+ 90˚+90˚+90˚= 360˚). Teacher B then draws an octagon on the board and asks for the sum total of the angles of an octagon. The learners are unable to give a correct answer. Teacher B then draws a trapezium on the board and asks for its sum total of the angles and the learners respond by saying 360 ˚. 	**B6.** Teacher B does not probe deeper by asking the learners about why they had this misconception when it comes to measuring angles. Instead Teacher B moves on by showing the learners a protractor and discussing it as a measuring instrument. **B7.** Teacher B structured this part of the lesson in such a way that by re-capping and teaching the properties of a rectangle first, he used this as a basis to show learners how they could divide other shapes such as hexagons and octagons into quadrilaterals and calculate the sum values of the interior angles. This is evidence of **knowledge of content and teaching (KCT)** as teacher B was able to sequence mathematical content, present mathematical ideas, and select examples to take students deeper into mathematical content. He also made use of appropriate representations.

Teacher B explains that the octagon is made up of 2 trapeziums and 1 rectangle. He explains this by drawing it on the board. He then explains that each trapezium has a sum total of 360 ˚ angles. So, the two trapeziums = 720 ˚ and the rectangle also has a sum total of 360˚ angles. So, 720 ˚ + 180 ˚= 1080 ˚. Teacher B explains to the learners how to work out the sum total of a shape by breaking the shape up as illustrated in the diagram below.  	
Acute angle, obtuse angle and right angle. Teacher B explains that an acute is smaller than a right angle and an obtuse angle is larger than a right angle.  Line of symmetry Teacher B asks the learners "what is a line of symmetry?" The learner responds that it is a line that divides a shape in half. Teacher B corrects this misconception by asking another learner and that learner responds that "it is a line that divides a shape into two equal halves". Teacher B further adds to this by using a piece of page and folding it into two equal halves.	**B8.** Teacher B recaps the three different angles and then discusses lines of symmetry. Teacher B corrects a learner's misconception of a line of symmetry as the learner states that it divides the shape in half. Instead Teacher B explains that it divides the shape into two equal halves. Teacher B demonstrates this by folding a page into two equal parts. This is evidence of **knowledge of content and students (KCS) as well as knowledge of content and teaching (KCT).**
<u>Episode 3: Learner Activity in the whole class</u> **8:21 to 8: 40** Teacher B gives learners strips that are equal in length. Learners are asked to create a triangle from the strips. The learners are then randomly chosen to come forward and pick a name of a 2D shape from the box. The learners then had to use the strips to create the 2D shape. Teacher B explains that he is going to use the shapes that the learners create to discuss the properties of the 2D shapes.	**B9.** In this activity Teacher B gives the learners equal length of strips to create their shapes. In the interview Teacher B states that he used the strips as he knew from past experience that when the learners draw shapes on the board, it is so tiny that the learners at the back are unable to see the shapes. By using the strips of equal lengths, the learners were able to create a

Teacher B Then starts by discussing the angles of the rectangle. He explains that it has four 90˚-degree angles.

Pentagon: Teacher B asks the learners “what type of interior angles does this shape have?” a learner responds that it has acute angles as well as obtuse angles.

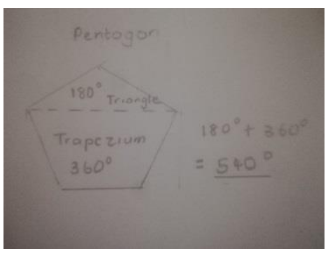
Teacher B then asks the learners what is the sum of the interior angles. The learners are unable to give a correct answer.

Teacher B then uses the diagram on the board to explain to the learners that the pentagon is made up from a triangle and trapezium. (as illustrated in the diagram below).

The sum of the angles of a triangle = 180˚

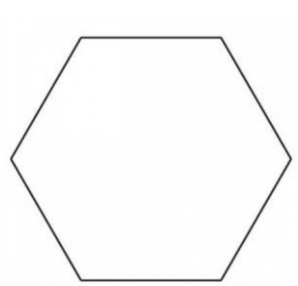
Teacher B explains why the sum the triangle = 180 ˚. He does so by drawing a rectangle on the board. He divides the rectangle with a diagonal line to form two triangles. The rectangle is divided equally so each half is = 180 ˚. Each half also forms a triangle. So, the angles of a triangle add up to 180 ˚.

Teacher B then explains that the sum of the angles of the trapezium = 360˚. So, 360 ˚+ 180 ˚= 540 ˚



Teacher B then draws the following shapes on the board:

Hexagon and heptagon. Using the method that Teacher B had taught the learners, the learners had to calculate the sum of the interior angles.



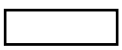
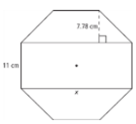
A learner divides the shape in the centre (horizontally) to make two trapeziums. The learner then says that 360˚ + 360˚ (because 360˚ is the total of the interior angles of a trapezium) = 720˚. So, the sum of the angles of the hexagon is 720˚.Teacher B uses the same approach to demonstrate to the learners how to calculate the sum of the interior angles of the heptagon.

Teacher B then emphasizes that this approach can only be used when calculating the sum of the interior angles of a regular polygon (a 2D shape with all equal sides)

perfect regular shape. The learners also selected strips based on the number of sides the shape has. They were able to identify the property of a shape (for example, they were able to identify that a pentagon has five sides and therefore chose five strips). This is evidence of **knowledge of content and students (KCS) as well as Knowledge of Content and Teaching (KCT)**.

B10.

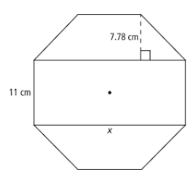
Teacher B also uses the shapes constructed by the learners to discuss the properties and angles of these shapes. This shows evidence of **knowledge of content and teaching (KCT)** as the teacher was able to sequence the mathematical concepts as well as take the learners deeper into the concept.

<u>Episode 4: Re capping the lesson</u> **8:41 to 8:51** Teacher B uses flash cards that have the properties of different 2D shapes. Teacher B sticks these on the board. The learners then have to guess which shape these properties belong to. Teacher B then does a re-cap of the lesson. He names the three angles discussed i.e. acute angle, obtuse angle and right angle. Teacher B then reminds the learners that: △ = all angles of a triangle add up to 180˚  = all angles add up to 360˚  = two trapeziums and one rectangle so = 360˚ + 360˚+ 360˚= 1080˚ **<u>Episode 5: Learner activity</u>** Worksheets are handed out to the learners. The activities of the worksheet include properties of 2D shapes and a recap of the angles. Learners completed this for homework.	**B11.** Teacher B makes use of flash cards and worksheets to reconcile the concepts that were taught. These resources and materials were suitable for teaching this concept. Teacher B has therefore shown evidence of **knowledge of content and teaching (KCT) and knowledge of curriculum and content (KCC).**

4.6.1 Summary of Teacher B’s lesson observation

As with Teacher A, the table below is a summary of Teacher B’s lesson observation. In the first episode, Teacher B starts the lesson with a re-cap of 2D shapes. The second episode is a re-cap of angles and their values. The third episode is a whole class activity in which the selected learners use equal lengths of strips to construct 2D shapes. The fourth episode is the teacher’s explanation of calculating the sum of the interior angles of 2D shapes. The fifth episode is a homework activity.

Table 7: Summary of Teacher B's lesson observation

Episode	**Description**
Episode 1 **Re-cap 2D shapes**	The lesson started off with a re-cap of 2D shapes. The teacher discusses the different types of triangles and explains that the word “tri” stands for three and that the triangle is a three-sided figure. Teacher B then discusses the different types of polygons in order of the number of sides they have (quadrilaterals, pentagons, hexagon, Heptagons and so forth.) The teacher then discusses and explains the concept of lines (horizontal, vertical and diagonal). He explains to the learners that these lines are used to construct a shape.
Episode 2 **Re-cap of angles and their values**	The teacher introduces the protractor as an instrument for measuring angles. He then re-caps the three types of angles (right angle, acute angle and obtuse angle). The teacher then explains to the learners that the sum of the interior angles of the rectangle/square is 360° as it has 4 right angles ($4 \times 90^\circ$). He also explains that the sum of angles for the trapezium also adds up to 360°. He then uses this information to explain to the learners that the sum of the interior angles of an octagon is 1080° because the octagon can be divided into two trapeziums and a rectangle (refer to diagram below). Therefore $360^\circ + 360^\circ + 360^\circ = 1080^\circ$ 
Episode 3 **Whole class activity**	The teacher randomly selected learners and gave them equal lengths of strips. The selected learners were then asked to pick a name of a shape from the box on the teacher’s table. The learners were then required to use the strips to construct their selected shape on the board. The teacher also explains to the learners that the sum of angles of a triangle is 180°.
Episode 4 **Sum of interior angles of a 2D shape**	Using the shapes that the learners constructed on the board, the teacher explained to the learners that a regular pentagon can be divided into a triangle and trapezium so therefore the sum of its interior angles are $180^\circ + 360^\circ = 540^\circ$.The teacher then explained

	how to calculate the sum of the interior angles of an hexagon and heptagon.
Episode 5 **Lesson re-cap**	The teacher handed out flash cards to a few learners. These flash cards contained properties of 2D shapes. The learners then had to match the properties of the 2D shape to its corresponding shape. The teacher then did a quick re-cap of the lesson's concepts.
Episode 6 **Learner activities**	The learners are handed worksheets. The worksheets contained activities relating to the properties of 2D shapes and the sum of angles of 2D shapes. This was completed for homework as the lesson had ended.

4.6 Analysis of Teacher B's PCK based on the lesson observations and semi-structured interview

A summary of Teacher B's PCK based on his lesson observation and semi structured interview will be provided. This will be carried out in the same manner as done with Teacher A.

4.6.1 Knowledge of Content and Curriculum

Teacher B demonstrated that he has a deep knowledge of both content and curriculum in his teaching of 2D shapes and the total values of their angles. Teacher B structured his lesson by starting off with what the learners should know thereafter moving to the unknown. In order to teach the sum of the interior angles of a 2D shape the learners needed to have a prior knowledge of the properties of 2D shapes. They needed to know the different types of polygons e.g. triangles; quadrilaterals, pentagons etc. Teacher B started his lesson by re-capping the properties of 2D shapes as evident in his interview and this was in line with his lesson observation (refer to Table 5, Episode 1 & 2). Teacher B was also able to accurately identify the prior knowledge that learners needed before teaching this concept as evident in the statements below:

> *They must be able to identify the shapes and their properties and then from there they must try and ascertain the length, width and so forth.*

They must understand the word tri, it means three. So, any three- sided figure is a triangle. And we did different types of triangles, which we discussed.

And any four-sided figure, as long as there is four lines in any direction is called a quadrilateral. But I just went to the basic one, the rectangle, square, rhombus parallelogram and trapezium.

Similar to Teacher A, Teacher B also demonstrated through his teaching how the properties of a 2D shape is linked to working out the sum of its interior angles which Gulpric Chua (2018) describes as a characteristic of KCC. In addition to this, Teacher B went beyond the grade level in his teaching of this topic, this is evident from his response to the question related to the purpose of his lesson:

*(*The lesson's purpose was to teach)

The value of the angle, it is not needed in Grade Six, but I felt that children should know it. They only know right angle.

A search through the Curriculum and Assessment Policy confirmed that the content to be taught to the Grade Six's with regards to shape and space does not include the values of the angles of 2D shapes. Teacher B has therefore demonstrated that he does possess the characteristics of KCC.

4.6.2 Knowledge of Content and Teaching

The diagram below outlines how Teacher B skillfully sequenced his teaching in order to take the learners from the known to the unknown thereby developing the learners understanding of the new concept. This diagram is based on Teacher's B lesson observation and it is an indication that Teacher B possesses a deep understanding about knowing mathematics and knowing about teaching mathematics.

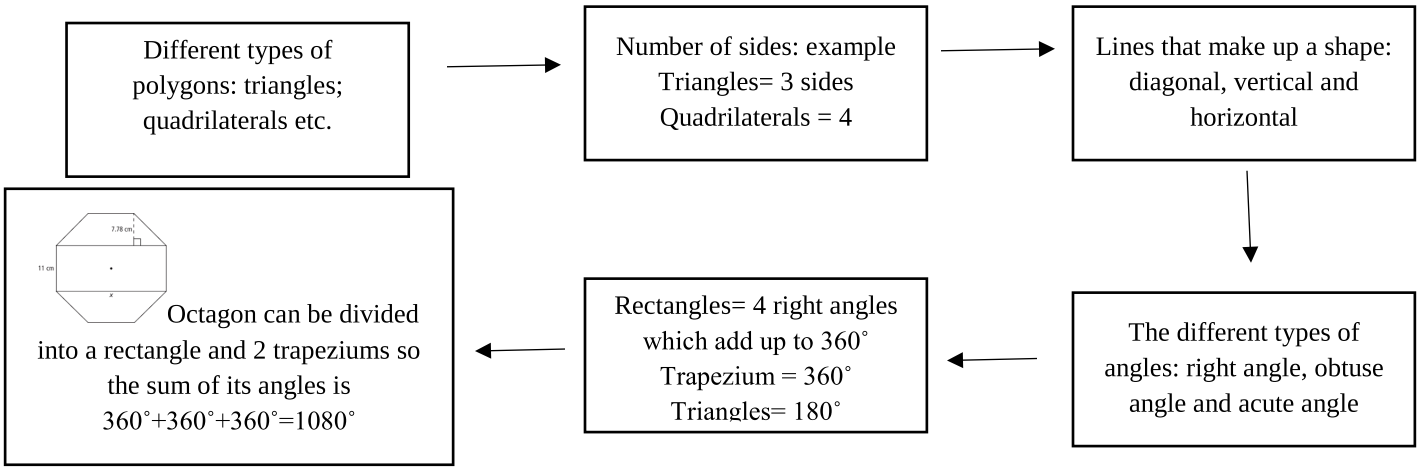


Figure 3: Diagram outlining Teacher B's lesson

Teacher B also used a range of teaching strategies when he facilitated his lesson. These strategies included an explanation, demonstration and learner interaction (refer to Table 5).

Teacher B was also able to anticipate what the learners would find difficult (drawing of the 2D shapes) and what they would find interesting, therefore he was able to select relevant resources. The following response to the question based on his selection of resources is evidence of this which is also in line with his lesson observation (refer to Table5, Episode 3, B9).

> *We normally take things for granted. The child knows what a pentagon is, the child knows what a heptagon etc. is, but when you call them to the board to draw it, they can't draw it. The lines are all distorted and so forth. So instead of giving you a perfect image, they give you a distorted image. So, I thought, okay, let me get the strips. And then let them use the strips and then they can get it in that way. It was actually more exciting than drawing it.*

Based on the observed lesson and semi-structured interview, Teacher B displayed a deep knowledge of content and teaching mathematics. It is presumed that his vast years of experience in teaching this subject across the spectrum has contributed to this knowledge category and his skillful application in teaching this topic.

4.6.3 Knowledge of Content and Students

Teacher B was able to confidently articulate the misconceptions that the learners may have with regards to the topic that was taught. During the interview Teacher B was able to identify the following misconceptions:

"What I know the misconception they have is a shape is the shape to them"- (this implies that the learners are unable to differentiate between a regular polygon which is made up of equal sides and an irregular polygon, which does not have all equal sides).

> *We were teaching them regular shapes today, or perfect shapes, where we can teach line of symmetry, where we can teach right angles, etc. They do an irregular shape; they do an irregular pentagon and as long as it has 5 sides for them, it's a pentagon*
>
> *For them, everything is the same but the properties vary from a regular and irregular figure, especially with the line of symmetry and the interior angles*
>
> *Although we use the terminology in the class and today you saw some of them didn't even know what a vertical line or diagonal line is*

The fourth misconception that Teacher B identified was also articulated by Teacher A for the teaching of the content on space and shape. Teacher A also stated that:

> *Just using simple words such as vertical, horizontal and diagonal, they don't understand what that word means.*

It seems that an understanding of mathematical terminology is definitely a challenge for the learners. As noted in both the description of the classes, Teacher A and Teacher B both teach learners for whose home language is not English. This could be a contributing factor to their misconceptions.

While Teacher B is able to identify the misconceptions that learners may have about this topic, like Teacher A, he scarcely elaborated on his learners' misconceptions (refer to Table 5, Episode 2, B6). There were very few, if not any incidents where both Teacher A and Teacher B addressed the mistakes or incorrect answers that the learners produced in their lessons. In both cases, the teachers moved on to ask another learner who gave a correct response.

4.7. The relationship between a teacher's pedagogical decision making and PCK

Barendsen and Henze (2017) conducted a study in the Netherlands which investigated the relationship between Science teachers' PCK and their classroom actions. They argue that while PCK has been researched extensively, "very little empirical evidence has been found to determine how this knowledge actually informs teachers' actions in the classroom" (2017, p.1). They therefore call for more qualitative studies to examine the relationship between teacher knowledge and classroom practice. Based on this notion, I thought it would be interesting to see if there is indeed a relationship that exists between a teacher's PCK and pedagogical decision making, hence the reason for exploring the second research question.

The primary focus of this study was to explore the PCK of selected Grade Six mathematics teachers. The data collection instruments (semi-structured interviews and lesson observations) focused mainly on the first research question. I had hoped that the analysis of the data would reveal some valuable information relating to the teachers' PCK and its relationship with their pedagogic decisions.

In trying to explore the relationship between the teachers PCK and pedagogic decisions, I approached this study from the standpoint that I would let the data speak for itself. I went in with no preconceived notions as I was unable to find relevant literature that I thought would assist me with this research question.

From my data analysis it appears that a possible relationship does exist between the teachers' PCK and pedagogic decisions. A justification for this statement will now be provided:

4.7.1 Teacher A and Teacher B's KCS and its relationship with pedagogic decision making.

As mentioned in Table 2, KCS has been defined by Ball and her colleagues as knowledge that combines knowing about mathematics and knowing about students. This category of PCK involves amongst other characteristics:

- Recognising and articulating learners' misconceptions
- Predicting what students will find interesting

Teacher A and Teacher B used their knowledge about the learners' misconceptions to guide and influence the way in which they taught their lessons. Teacher A stated that one of the misconceptions that learners have with regards to calculating the surface area of a square based pyramid, is that they measure the height of the pyramid from its centre. The statement below from his interview is evidence of this:

> *One of the major reasons I also did it was because a lot of the time I've seen kids make mistakes when it comes to the heights of the triangle, so its half base times height and often I've seen kids measure height from the centre of the pyramid, that's why I was saying to them that the centre of the pyramid to its highest point, that's not the height of that triangular face.*

Teacher A used this knowledge about his learners to guide his teaching and he ensured that he addressed this in his teaching (refer to Table 3 Episode 3, A 9). Teacher A uses a diagram to demonstrate to the learners that the height of the pyramid is measured from its base and not from the centre of the square. It could be argued that Teacher A's knowledge of the learners' misconception influenced the decisions that he made when teaching this topic.

Similarly, Teacher B stated that one of the misconceptions that learners have about 2D shapes is that they do not understand what the words diagonal, vertical and horizontal means. Teacher B further stated that the learners were unable to draw the shapes properly on the board. The following statements are evidence of this:

> *Although we use the terminology in the class and today you saw some of them didn't even know what a vertical line or diagonal line is.*

> *We normally take things for granted. The child knows what a pentagon is, the child knows what a heptagon etc. is, but when you call them to the board to draw it, they can't draw it. The lines are all distorted and so forth. So instead of giving you a perfect image, they give you a distorted image. So, I thought, okay, let me get the strips. And then let them use the strips and then they can get it in that way. It was actually more exciting than drawing it.*

Teacher B's knowledge of the common errors made by his learners and their misconceptions seemed to influence the way he taught his lessons. Teacher B made use of equal lengths of strips to get the learners to construct the shapes on the board (refer to Table 5, Episode 4, B9). This suggests that Teacher B's KCS influenced his pedagogic decisions in the classroom.

In addition to this, Teacher B who has 34 years of teaching experience was able to skillfully sequence his teaching as illustrated in figure 2. Teacher B's vast experience in teaching could have contributed to his deep knowledge of the content and curriculum (KCC) and therefore he was able to structure his lesson in this manner. This could also imply that a teachers' KCC can also influence their pedagogical decisions regarding the structuring and sequencing of a lesson.

It could also be argued that Teacher B's knowledge of curriculum and content (KCC) i.e. how topics are linked in a subject, also assisted him in skillfully sequencing his teaching. This implies that a teachers' KCC can also influence their pedagogical decisions regarding the structuring of a lesson.

4.8 Conclusion

This chapter presented the findings of two case studies which set out to explore the PCK of two Grade Six mathematics teachers' teaching of the value of angles and the surface area of a square based pyramid. The findings revealed that the teachers displayed an adequate knowledge of the three categories of PCK, however they failed to address the incorrect answers given by the learners during their teaching. Teacher A mentioned in his interview that due to time constraints he could not really "sit" on a topic to ensure that all the learners understood the concept, and this could be a possible reason why the teachers made the decision to not address the learners mistakes on an individual basis. I further discuss this in Chapter Five.

The study further aimed at determining the nature of the relationship between a teachers' PCK and their pedagogic actions. The findings suggests that the teachers' knowledge about content and students (KCS) and knowledge of content and curriculum (KCC) seems to influence their pedagogic decisions. I believe that the findings of this study addressed both the research questions.

The next chapter presents a discussion on the findings, recommendations for further research and a conclusion.

CHAPTER 5: DISCUSSION OF FINDINGS AND RECOMMENDATIONS

5.1 Introduction

The main purpose of this chapter is to report on the findings of this study and to provide recommendations for future research. This study aimed to explore the PCK of selected Grade Six Mathematics teachers and the nature of the relationship between their PCK and pedagogic decisions. The study was guided by the following research questions:

1. What pedagogical knowledge do intermediate phase teachers show in their teaching of mathematics?
2. What is the relationship between teachers' pedagogic decision making and PCK?

5.2 Discussion and findings of this study

The summary of the findings is based on the PCK of two Grade Six mathematics teachers' teaching of 2D and 3D shapes. The aim of the research was achieved and that the research questions were answered. In addition to this, an attempt is also made to link the findings of this research with other scholarly literature. The findings for this research will be discussed in order of the questions posed. I will firstly discuss the findings of the teachers' PCK and thereafter, the relationship between the teachers' PCK and pedagogic actions will be discussed.

5.2.1 Findings for Research Question One

In order to answer the research questions, data collected from lesson observations and video stimulated interviews were analyzed and coded using Ball, Thames and Phelps' (2008) MKT framework with a specific focus on PCK and its categories (KCC; KCT and KCS).

5.2.1.1. Knowledge of Curriculum and Content

KCC is a knowledge domain which Ball and her colleagues included in the MKT framework and provisionally placed under the PCK domain. While Ball and her colleagues do not elaborate on this knowledge category, they strongly relate it to Shulman's (1986) conception of curricular knowledge. Shulman (1986) describes curricular knowledge as knowledge that involves knowing how "the topics of subjects are arranged within a school year and over longer periods of time and ways of using curriculum resources, such as textbooks, to organize a program of study for students" (Hill, Schilling & Ball, 2004, p. 13). Teachers who demonstrate these characteristics would be considered to possess KCC.

In this study, both Teacher A and Teacher B have demonstrated adequate Knowledge of Curriculum and Content. Both participants sequenced the concepts taught in their lesson in such way that the current topic was built on all other concepts that the learners needed to know before-hand. Prior knowledge was central to both Teacher A and Teacher B's teaching. This demonstrates that the teachers are aware of how topics are arranged in a school year. In addition to this, during the interview, Teacher A and B were clearly able to articulate how the topics in Grade Six are linked to the topics in Grade Five and Grade Seven , which also indicates that they have a sound knowledge of the curriculum across the strand.

5.2.1.2. Knowledge of Content and Teaching

Ball and her colleagues argue that KCT requires that teachers have:

> A mathematical knowledge of the design of instruction. Teachers sequence particular content for instruction. They choose which examples to start with and which examples to use to take students deeper into the content. Teachers evaluate the instructional advantages and disadvantages of representations used to teach a specific idea and identify what different methods and procedures afford instructionally. Each of these tasks requires an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning (Ball et al., 2008, p. 401).

Knowledge of Content and Teaching therefore involves knowing about mathematics and knowing about teaching (Ball et al., 2008). Ball and her colleagues further argue that this knowledge domain involves knowing the procedures and pedagogical principles needed to teach a specific mathematical idea and is therefore an amalgamation of content knowledge and teaching. Similarly, Nolan, Dempsey, Lovatt and O'Shea (2015) describes this knowledge category as a combination of mathematical knowledge and instructional choices in order to facilitate learning. This knowledge category therefore focuses on the teachers' teaching techniques and choice of instructional tools.

Teacher A used a questioning technique as one of his teaching strategies. He started off his lesson with a quiz and possibly used this technique to help his learners recall previous knowledge which was needed to build on his impending lesson. This teaching strategy has been regarded as an important diagnostic tool to analyze the students understanding of concepts (McCarthy, Sithole, McCarthy, Cho, & Gyan, 2016).Teacher A thereafter used an explanation technique to teach the new concept. To help facilitate the learning process, Teacher A gave his learners the net of a square based pyramid as he felt that it was sufficient to teach the lesson. Teacher A's instructional choices and teaching strategies were well suited for his lesson which is an indication that Teacher A did in fact display KCT.

Teacher B also used a questioning technique to recap on previous lessons. A discussion and explanation were also used as a teaching strategy, however unlike Teacher A, Teacher B made use of activities that involved more hands-on student engagement. Teacher B actively engaged the learners in the lesson by getting them to construct various regular 2D shapes on the chalkboard using equal lengths of strips of paper. The learners were also involved in activities in which they had to match various properties of 2D shapes to its respective shape.

In summary, both Teacher A and Teacher B displayed a deep knowledge of the content that was taught. It is presumed that the teachers' many years of experience could have contributed to their deep knowledge of the content. In addition to this, they also made use of relevant teaching strategies and teaching aids. However, during the lesson observation it was noted that both teachers only pointed to the learners who raised their hands when questions were asked. As a result, this could have possibly limited both the teachers' knowledge of some of the misconceptions that passive learners could have had, thereby hindering them from moving forward in the learning process. McCarthy et al. (2016, p. 80) similarly argues that while using a questioning technique is an effective strategy to enhance student learning, if not used properly, it could "have a negative impact on the learning process".

5.2.1.3. Knowledge of Content and Students

According to Gulpric Chua (2018), KCS includes the teacher's ability to design lessons that the learners would find stimulating as well as being able to predetermine the conceptions learners may bring with them about a particular concept and how to help them move past these conceptions. This category of PCK is therefore based on knowing about Mathematics and knowing about students.

When interviewed, both Teacher A and Teacher B were easily able to articulate the misconceptions that the learners' have about the topics that were taught. During the lesson observation it was also observed that both teachers used the learners' misconceptions to guide their teaching. This is in line with the argument that Sapire, Shalem, Wilson-Thompson, and Paulsen (2016, p. 5) puts forth when they state that, "sizing up the source of students' errors or recognizing common misinterpretations of topics is related to the teachers' knowledge of errors". They further argue that from an error analysis point of view, knowledge of student errors also involves the teacher's ability to teach a topic from the perspective of how learners learn or "the mistakes or misconceptions that commonly arise during the process of learning the topic" (Sapire et al., 2016, p. 5).

Teacher A and Teacher B used a similar approach in their teaching. Both teachers used their knowledge of the common misconceptions of their students to teach the concept. This suggests that Teacher A and B have a good understanding of student errors and used this as a foundation on which to teach their lesson.

However, I also noted that while both Teacher A and Teacher B were easily able to articulate the misconceptions that their learners had regarding the topics that were taught, the teachers did not engage in the incorrect responses that they received from their students during their lessons.

Teacher A and Teacher B simply moved on to the next learner who raised their hand to answer the question when they received an incorrect response from another learner. Both teachers did not probe deeper in order to address the source of the misconception or misunderstanding. This is partially similar to the findings of Barendsen and Henze's (2017) study that was conducted in the Netherlands. Barendsen and Henze (2017) found that the science teacher (their participant) scarcely elaborated on the students' ideas and missed opportunities to address the students' misconceptions. Barendsen and Henze's (2017) participant also articulated that the reason why his lessons were teacher dominated was because he felt under pressure due to time constraints. Similarly, Teacher A mentioned in his interview that due to time constraints, he cannot often "sit" on a topic. I therefore think that the issue of time constraints could be a contributing factor as to why the teachers did not pursue or delve into the learner's incorrect responses. Instead they choose to provide a general explanation of the concept with the hope that it would correct any misunderstandings that the other learners may have.

Hoadley (2012), who reported on the findings of a number of small scale and large-scale studies carried out in the South African classrooms also found that some of the common findings of these studies were the lack of learner feedback and the issue of time. Hoadley (2012) highlighted that learner pacing had a crucial effect on time. Hoadley (2003) argues that "pacing in a working classroom was extremely slow" and that the class moved at the pace of the weakest learner (as cited in Hoadley, 2012, p. 193). The slow pace at which learners move affects curriculum coverage and erodes into the teachers' instructional time. As a result, teachers often then find themselves under pressure.

Hoadley (2012) also highlighted that lack of learner feedback was one of the issues identified in South African literacy classrooms. Hoadley (2008) argues that many teachers adopt a chorus technique during reading aloud activities. She further maintains that teachers failed to engage learners in extended reading for meaning activities (as cited in Hoadley, 2012). Sapire et al. (2016, p. 1) who also carried out a study in South Africa dealing with student errors, argues that many of the teachers in their study "ignored their learners' errors or only dealt with it partially".

The above literature confirms that lack of learner feedback/ lack of engaging in student errors and the issue of time constraints are factors that are prevalent in both international and national studies. This is also consistent with the findings of this study.

5.2.2 Findings for Research Question Two

Research Question Two explores the nature of the relationship between the teachers' PCK and their pedagogic decision making. The data collected from the lesson observations and video stimulated interviews were also used to answer this research question.

Pedagogical decision making is an important part of the teaching role and it helps teachers' succeed in their teaching (Prachagool, Nuangchalerm, Subramaniam & Dostál, 2016). It is the essence of teachers' professional practice and it refers to a process whereby teachers use their reasoning and thinking skills to choose amongst available alternatives in the teaching process, with the hope that it will have a meaningful effect on student learning (Saad, Ratnavadivel, Hin, Nagappan, Yasin & Radzi, 2009). While teacher knowledge has been viewed as an important factor which influences learner performance, the decisions that teachers make in their teaching also influences learning outcomes. I therefore felt it necessary to explore the relationship between the teachers' PCK and pedagogic actions.

Based on the observed lessons and video stimulated interviews, Teacher A and Teacher B used their knowledge of the learners' misconceptions to structure and organise their lessons. According to Ball and her colleagues (2008), KCS involves, amongst other characteristics, recognizing and articulating learners' misconceptions as well as predicting what students will find interesting. As mentioned previously, Teacher A and Teacher B were easily able to articulate the misconceptions that their learners have about the topics that were taught and they used this as a basis on which to plan and execute their lessons. The following actions and phrases were evidence of this.

Teacher A articulated that the learners often measure the height of the square based pyramid from the centre of its square base, which is incorrect and therefore leads to an incorrect answer. He therefore structured his teaching in order to address this misconception. Similarly, Teacher B was also easily able to articulate the misconceptions that the learners had about the topic that he was teaching and used this as a foundation on which to teach his lesson. Teacher B stated that many of his learners did not even understand what the words diagonal, vertical and horizontal mean. He therefore explained these terminologies to the learners and thereafter used equal strips of paper to get the learners to construct 2D shapes on the chalkboard. Teacher B also stated that the learners were unable to draw the 2D shapes properly and therefore gave them equal strips of paper so that they could construct a regular 2D shape.

In addition to this, Teacher B was also able to use his Knowledge of Content and Curriculum to skillfully sequence his lesson. Teacher B's knowledge of how topics are linked in a subject, assisted him with sequencing the lesson in such a manner that it took the learners from the known to the unknown.

In summary, both Teacher A and Teacher B used their Knowledge of Content and Students (KCS) to help inform their teaching and decisions in the classroom. In addition to this, Teacher B's Knowledge of Content and Curriculum also assisted him to skillfully organize and sequence his lesson. It can therefore be argued that both teachers KCS and Teacher B's KCC influenced their pedagogic decisions. This implies that what the teacher knows about mathematics and their students, can in fact influence the decisions they make in the classroom and teaching.

5.3 Summary of findings

In this study it was established that both Teacher A and Teacher B employed all three knowledge categories of Ball et al's. (2008) PCK domain to a certain degree. While the teachers displayed a deep knowledge of KCC and KCT, they used their knowledge of the learners' misconceptions to guide their teaching. However, the findings of this study also revealed that both teachers didn't address the incorrect responses provided by their learners. Time constraint was identified as a possible contributing factor to this issue. In addition to this, the nature of the relationship between the teachers' PCK and pedagogic decision making, could be described in the following manner:

What teachers know about their students and mathematics; will influence how they teach.

This study found that a possible relationship existed between the teachers KCS and KCC. This was observed through their lesson observations and video stimulated interviews.

5.4 Limitations of this study

This study involved a small sample of teachers who were selected because of their willingness to participate in the research. However, the sample does include teachers from different teaching contexts as described in Chapter Three. The results from this study can therefore not be generalized but could be transferred to a similar context.

The participants in this study were also not trained in teaching mathematics. However, I felt that it was not a necessary requirement for this study. Kind (2009) maintains that PCK is tacit knowledge, which is gained through experience. The participants in this study have extensive years of teaching experience in mathematics, which has contributed greatly to their knowledge of the subject and their ability to skillfully teach their lessons as demonstrated in Chapter Four. In addition to this, Attard (2011) argues that having higher mathematical qualifications does not necessarily lead to strong learning outcomes in students (as mentioned in Chapter Two). Instead, teachers also need to have a good knowledge of how students learn and what they would find difficult and how to address students' misconceptions about a particular concept. Given that the focus of this study was on the teachers' PCK, a criteria for specialized training in mathematics was therefore not required.

Another limitation to this study was the issue of time. There was also not enough time to conduct this study. As a full-time departmental paid teacher I was only able to take three days leave to carry out my data collection, which is in fact a short period of time to do so.

Lastly, I found that working with Ball et al.'s. (2008) PCK categories proved to be quite a challenge. It was rather difficult to distinguish between the categories of PCK that was being implemented by the teacher during their teaching and learning activities. I found that the different categories of PCK seemed to overlap in their characteristics. There are no clear boundaries as to where one category of PCK ends and the other begins. Ball et al. (2008, p. 402) also acknowledged that "it is not always easy to discern where one of our categories divides from the next, and this affects the precision of our definitions".

5.5 Recommendations

The findings of this study revealed that the participants implemented the different categories of PCK to a certain degree. However, both participants failed to engage their students' incorrect answers and provide clear feedback as to why these answers were incorrect. Brodie (2013) argues that errors are a normal part of learning. She further argues that even experienced mathematicians make mistakes, however in doing so, creates new knowledge in mathematics. She therefore argues that if teachers look for ways to try and understand why learners have made these errors, "they may come to value learners' thinking and find ways to engage their current knowledge in order to create new knowledge" (Brodie, 2013, p. 9). Brodie (2013) suggests that teachers use professional learning communities (PLC) to help them find ways to engage with student errors. Stoll and Louis (2008) maintains that PLC's "usually refers to teachers 'critically interrogating their practice in ongoing, reflective and collaborative ways' in order to promote and enhance student learning" (as cited in Brodie, 2013, p. 6). It is therefore suggested that teachers in collaboration with the DOE, work together to form PLC's in which they can engage with student errors with the common goal of helping them to inform their practice, develop their knowledge and enhance student learning.

In addition to this, the issue of time constraints was also identified as a possible reason for the lack of engagement with student errors. It is also suggested that departmental workshops should be held to help the teachers deal with challenges (such as time constraints) that they face in the mathematics classroom.

Further research should be carried out in order to explore the extent to which other factors such as time constraints impacts on a teacher's PCK. In addition to this, more qualitative studies should be carried out to investigate the extent to which a teacher's knowledge actually influences their pedagogic decision making.

5.6 Conclusion

The following study effectively explored the PCK of two grade six mathematics teachers. Ball et al.'s (2008) MKT framework was used to analyse the PCK that the participants used in their teaching of 2D and 3D shapes. The findings of study revealed that both participants displayed a deep knowledge of content and students (KCS) and knowledge of content and teaching (KCT), as identified by Ball et al. (2008). However, it was also noted that while the participants utilized both these knowledge domains in their teaching, they failed to address their learners' incorrect responses. Time-constraints was identified a possible contributing factor towards this issue. It was suggested that the participants engage in professional learning communities (PLC) in order to find ways to engage with student errors (Brodie, 2013) In addition to this, the study also aimed to explore the nature of the relationship that existed between the participants' PCK and their pedagogic actions. The study revealed that a possible relationship existed between the participants' knowledge of content and students (KCS) and their knowledge of content and curriculum (KCC), as it was observed that the participants drew on these two knowledge domains in order to inform their actions in the classroom. Recommendations for further research on the relationship between a teacher's PCK and pedagogic decision making was further suggested and engagement in PLC's was suggested as a way of addressing the challenges that these teachers experienced in engaging with student errors.

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APPENDIX 1: Permission to conduct research

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Enquiries: Phindile Duma	Tel: 033 392 1063	Ref.:2/4/8/1748

Mrs L Nadas
Private Bag X9065
Pietermaritzburg
3200

Dear Mrs Nadas

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"EXPLORING THE PEDAGOGICAL CONTENT KNOWLEDGE OF INTERMEDIATE PHASE IN MATHEMATICS TEACHERS",** in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 26 March 2019 to 01 September 2021.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma at the contact numbers below.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

UMgungundlovu District

Dr. EV Nzama
Head of Department: Education
Date: 27 March 2019

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KWAZULU-NATAL DEPARTMENT OF EDUCATION
Postal Address: Private Bag X9137 • Pietermaritzburg • 3200 • Republic of South Africa
Physical Address: 247 Burger Street • Anton Lembede Building • Pietermaritzburg • 3201
Tel.: +27 33 392 1063 • **Fax.:** +27 033 392 1203• **Email:** Phindile.Duma@kzndoe.gov.za •**Web:** www.kzneducation.gov.za
Facebook: KZNDOE....Twitter: @DBE_KZN....Instagram: kzn_education....Youtube:kzndoe

APPENDIX 2: Ethical Clearance

UNIVERSITY OF KWAZULU-NATAL™
INYUVESI YAKWAZULU-NATALI

17 June 2019

Mrs Leena Nadas (208516756)
School of Education
Pietermaritzburg Campus

Dear Mrs Nadas,

Protocol reference number: HSS/0121/019M
Project title: A case study exploring the Pedagogical Content Knowledge (PCK) of Intermediate Phase Mathematics teachers

Approval Notification – Expedited Application

In response to your application received on 14 February 2019, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol has been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 1 year from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Dr Rosemary Sibanda (Chair)

/ms

Cc Supervisor: Professor Carol Bertram
cc Academic Leader Research: Dr Ansurie Pillay
cc School Administrator: Ms Sheryl Jeenarain

Humanities & Social Sciences Research Ethics Committee
Dr Rosemary Sibanda (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Telephone: +27 (0) 31 260 3587/8350/4557 **Facsimile:** +27 (0) 31 260 4609 **Email:** ximbap@ukzn.ac.za / snymanm@ukzn.ac.za / mohunp@ukzn.ac.za
Website: www.ukzn.ac.za

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APPENDIX 3: Letter to the Principal

Consent form for school Principal

XXXXXXXXXXX

XXXXXXXXXXX

Pietermaritzburg

3 201

11 February 2019

XXXXX Primary School

____

____

3202

Dear Principal

RE: request to conduct research at your school.

Research project title: Exploring the pedagogical content knowledge of Intermediate Phase Mathematics teachers.

My name is Leena Nadas and I am a Master of Education student from the University of Kwa-Zulu Natal. I will be conducting a research project titled: exploring the pedagogical content knowledge of intermediate phase mathematics teachers. I am an educator from Ridge View Primary School and I humbly request your assistance in this research process by being granted permission to conduct research in your school. The study involves an exploration into the pedagogical content knowledge of intermediate phase mathematics teachers. The aim and purpose of this research is to explore the pedagogical content knowledge (PCK) of selected intermediate phase mathematics teachers, to find out the reasons for their pedagogic actions and to develop an understanding of their teacher knowledge. It is hoped that the findings of this study can help in addressing the challenges that teachers' face within this phase and can inform us on how we can improve our practice.

The participants in my study will be intermediate phase mathematics teachers from your school. I will observe one lesson of each teacher and they will also be required to participate in individual interviews that may last between 45-60 minutes. These interviews will be carried out at the participant's convenience and will in no way intrude on their teaching and learning time. I will audio record their interviews as well as video record one lesson of each teacher.

The school will not receive any material gains from their participation in this research, however the results from this research may assist teachers in improving their practices. Participation in this study is on a voluntary basis and participants are free to withdraw from this study at any time. The participants' identity will remain anonymous. This will be achieved through the use of pseudonyms. All responses from the participants will be treated with strict confidence.

This study is carried under the supervision of Prof. Carol Bertram at UKZN, Pietermaritzburg. (College of Education, Humanities). If you need any clarification about the project, feel free to contact my supervisor.

Researcher

Leena Nadas

076 292 6014

alvaleen@gmail.com

Supervisor

Professor Carol Bertram

Faculty of Education

UKZN

Thank you for considering my application. I am hoping to receive a positive response from you.

Yours faithfully

…………………..

Leena Nadas

APPENDIX 4: Participant's consent letter

UKZN HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE (HSSREC)

APPLICATION FOR ETHICS APPROVAL

For research with human participants

Information Sheet and Consent to Participate in Research

Project title: A case study exploring the pedagogical content knowledge of Intermediate Phase Mathematics teachers.

Date: 12/03/2019

Dear Sir/Madam

My name is Leena Nadas, Master of Education student from University of Kwa-Zulu Natal, School of Education, Pietermaritzburg Campus.

You are being invited to consider participating in a study that involves an exploration into the pedagogical content knowledge of Intermediate Phase Mathematics teachers at Grade Six level. The aim and purpose of this research is to explore the Pedagogical Content Knowledge (PCK) of selected intermediate phase mathematics teachers and to find out the reasons for their pedagogic actions in their teaching of mathematics.

Please be advised that you are not forced to participate in this study, the decision is yours and is highly respected. However, it will be appreciated to share your views and experiences. If you decide to participate, please understand that you can withdraw at any stage of the research for any reason. There won't be any penalties or prejudice and your identity will be protected. Confidentiality will be highly observed and your identity as participants will be protected. Therefore, pseudo names will be used for you and your school to ensure the integrity and confidentiality.

The study is expected to enroll three teachers from two different schools. It will involve the following procedures, observations, stimulated video recall and interviews. I humbly ask for your permission to do video recording and audio recording. I will be asking you questions and I request that you be as open and honest as possible in your answers. I will request that after the video-recorded lesson we view the video together so that we can discuss and reflect on your lesson and pedagogic choices. The observations will take 30-40 minutes and the interviews 30-45 minutes. The information you provide will remain confidential and you are assured that there won't be any comebacks from the answers you give. Please also note that there won't be any remuneration for participating in this study.

Please know that there are no anticipated risks or harm to you. The purpose of the observation is not meant to assess or criticize you in any way and the outcomes of the research will not be used anywhere. The study might be beneficial to you as it will give the opportunity to reflect on classroom practice in terms of content knowledge, knowledge of student thinking and their difficulties so that you can think on how to improve in future.

If possible, I would like to come back when the study is completed to inform you of the outcome of the findings.

In the event of any problems or concerns/questions you may contact the researcher at alvaleen@gmail.com or my supervisor Prof. Carol Bertram, BertramC@ukzn.ac.za, and or the UKZN Humanities & Social Sciences Research Ethics Committee, contact details as follows:

HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS ADMINISTRATION

Research Office, Westville Campus

Govan Mbeki Building

Private Bag X 54001
Durban
4000

KwaZulu-Natal, SOUTH AFRICA

Tel: 27 31 2604557- Fax: 27 31 2604609

Email: HSSREC@ukzn.ac.za

CONSENT

I (Name)……………………………………………… have been informed about the study, "*Exploring the pedagogical content knowledge of Intermediate Phase Mathematics teachers".,* *by Leena Nadas.*

I understand the purpose and procedures of the study.

I have been given an opportunity to answer questions about the study and have had answers to my satisfaction.

I declare that my participation in this study is entirely voluntary and that I may withdraw at any time without affecting any of the benefits that I usually am entitled to.

I have been informed about any available compensation or medical treatment if injury occurs to me as a result of study-related procedures.

If I have any further questions/concerns or queries related to the study I understand that I may contact the researcher at alvaleen@gmail.com or Prof Carol Bertram at (033) 260 5349.

If I have any questions or concerns about my rights as a study participant, or if I am concerned about an aspect of the study or the researchers then I may contact:

HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS ADMINISTRATION

Research Office, Westville Campus

Govan Mbeki Building

Private Bag X 54001
Durban
4000

KwaZulu-Natal, SOUTH AFRICA

Tel: 27 31 2604557 - Fax: 27 31 2604609

Email: HSSREC@ukzn.ac.za

I hereby provide consent to:

Audio-record my interview	YES / NO
Video-record my lesson	YES / NO
Lesson Observation:	YES /NO

APPENDIX 5: Learner assent form

UKZN HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE (HSSREC)

APPLICATION FOR ETHICS APPROVAL

For research with human participants

Information Sheet and Consent for learner to Participate in Research

Project title: A case study exploring the pedagogical content knowledge of Intermediate Phase Mathematics teachers.

Date: 17/07/2019

Dear Sir/Madam

My name is Leena Nadas, Master of Education student from University of Kwa-Zulu Natal, School of Education, Pietermaritzburg Campus.

My study focuses on the practices of your mathematics teacher, who has agreed to participate in the study.

The study involves the observation of one maths lesson and the video recording of one maths lesson. The focus of the observation and video recording will be on the teacher's practice and not on you. However, ethical research practices require that you are aware of the study and give your consent to participate. Please know that there are no anticipated risks or harm to you.

The data will not be made public in any way and will only be used for research purposes. The video recording will be deleted when the study is completed.

The school and teacher's identity will not be made public.

In the event of any problems or concerns/questions you may contact the researcher at alvaleen@gmail.com or my supervisor Prof. Carol Bertram, BertramC@ukzn.ac.za, and or the

UKZN Humanities & Social Sciences Research Ethics Committee, contact details below.

LEARNER ASSENT FORM

I (Name) ………………………………………………, a learner in Grade Six …………….

have been informed about the study, "*Exploring the pedagogical content knowledge of Intermediate Phase Mathematics teachers", by Leena Nadas.*

I understand the purpose and procedures of the study.

I have been given an opportunity to answer questions about the study and have had answers to my satisfaction.

I declare that my participation in this study is entirely voluntary and that Imay withdraw at any time without affecting any of the benefits that I usually am entitled to.

If I have any further questions/concerns or queries related to the study I understand that I may contact the researcher at alvaleen@gmail.com or Prof Carol Bertram at (033) 260 5349.

If I have any questions or concerns about my rights as a study participant, or if I am concerned about an aspect of the study or the researchers then I may contact:

HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS ADMINISTRATION

Research Office, Westville Campus

Govan Mbeki Building

Private Bag X 54001
Durban
4000 KwaZulu-Natal, SOUTH AFRICA

Tel: 27 31 2604557 - Fax: 27 31 2604609 Email: HSSREC@ukzn.ac.za

I hereby assent to be present during:
Video-recording of a maths lesson YES / NO
Lesson Observation of a maths lesson: YES /NO

____________________ ________________________

Signature of Learner Date

____________________ ________________________

Signature of Witness Date

APPENDIX 6: Parent consent form

UKZN HUMANITIES AND SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE (HSSREC)

APPLICATION FOR ETHICS APPROVAL

For research with human participants

Information Sheet and Consent for your Child/ ward to Participate in Research

Project title: Exploring the pedagogical content knowledge of Intermediate Phase Mathematics teachers.

Date: 17/07/2019

Dear Sir/Madam

My name is Leena Nadas, Master of Education student from University of Kwa-Zulu Natal, School of Education, Pietermaritzburg Campus.

My study involves an exploration into the pedagogical content knowledge of Intermediate Phase Mathematics teachers at Grade Six level. Your child's/ward's maths teacher has agreed to participate in the study.

The study involves the observation of one maths lesson and the video recording of one maths lesson. Your child/ward is a student in this class, and thus I am requesting your consent.

The focus of the observation and video recording will be on the teacher's practice and not on your child /ward. However, ethical research practices require that you are aware of the study and give your consent on behalf of your child/ward. Please know that there are no anticipated risks or harm to your child/ward.

The data will not be made public in any way and will only be used for research purposes. The video recording will be deleted when the study is completed.

Confidentiality will be highly observed and the school and teacher's identity will be protected. Therefore, pseudonyms will be used for the teacher and the school to ensure the integrity and confidentiality.

In the event of any problems or concerns/questions you may contact the researcher at alvaleen@gmail.com or my supervisor Prof. Carol Bertram, BertramC@ukzn.ac.za, and or the UKZN Humanities & Social Sciences Research Ethics Committee, contact details below.

---PARENTAL/ GUARDIAN CONSENT FORM FOR CHILD's PARTICIPATION

I (Name)………………………………………………… the parent/guardian of ………………………………… (child/ward) in Grade Six …………….have been informed about the study, "*Exploring the pedagogical content knowledge of Intermediate Phase Mathematics teachers", by Leena Nadas.*

I understand the purpose and procedures of the study.I have been given an opportunity to answer questions about the study and have had answers to my satisfaction.

I declare that my child's/ward's participation in this study is entirely voluntary and that s/he may withdraw at any time without affecting any of the benefits that I usually am entitled to.

If I have any further questions/concerns or queries related to the study I understand that I may contact the researcher at alvaleen@gmail.com or Prof Carol Bertram at (033) 260 5349.

If I have any questions or concerns about my child's rights as a study participant, or if I am concerned about an aspect of the study or the researchers then I may contact:

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Research Office, Westville Campus

Govan Mbeki Building

Private Bag X 54001
Durban
4000 KwaZulu-Natal, SOUTH AFRICA

Tel: 27 31 2604557 - Fax: 27 31 2604609 Email: HSSREC@ukzn.ac.za

I hereby provide consent for my child to be present during:

Video-recording of a maths lesson	YES / NO
Lesson Observation of a maths lesson:	YES /NO

______________________ ______________________

Signature of Parent/Guardian Date

______________________ ______________________

Signature of Witness Date

APPENDIX 7: Structured observation schedule

DATE:	
SCHOOL:	
NAME OF EDUCATOR:	
GRADE:	
OBSERVER:	
TOPIC:	
OBJECTIVES:	
DESCRIPTION OF CLASS	

Lesson observation	**Categories of PCK and its description**
Concept: ____ **Lesson Introduction: Presentation of the content**	**KCT: Knowledge of content and Teaching** **Combines knowing about teaching and knowing about mathematics.** • sequence mathematical content

	• present mathematical ideas • select examples to take students deeper into mathematical content • select appropriate representations to illustrate the content • ask productive mathematical questions • recognise what is involved in using a particular representation • modify tasks to be either easier or harder • use appropriate teaching strategies • respond to students' why questions • choose and develop useable definitions • provide suitable examples
Lesson observation	**Categories of PCK and its description**
Learner activities:	**KCS- Knowledge of Content and Students** **knowledge that combines knowing about mathematics and knowing about students** • anticipate what students are likely to think and do • predict what students will find interesting and motivating when choosing an example • anticipate what a student will find difficult and easy when completing a task • anticipate students' emerging and incomplete ideas • recognise and articulate misconceptions students carry about particular mathematics content

Conclusion of the lesson:	**KCC- Knowledge of Content and Curriculum** **Knowledge of the content requirements of the curriculum and the materials that can be used to teach that particular content** • articulate the topics in the curriculum • articulate the competencies related to each topic in the mathematics curriculum • articulate and demonstrate a familiarity with the structure of the mathematics curriculum • link representations to underlying ideas and to other representations • knowledgeability of available materials (e.g. textbooks) and their purposes

APPENDIX 8: Semi structured interview schedule

Interview schedule for teachers:

Biographical details

Name of teacher: ____

School: ____

Grade taught: _______________________

Time: ____________________________

1. For how long have you been teaching?
2. For how long have you been teaching mathematics?
3. What are your academic qualifications?
4. What subjects did you major in?
5. Why did you chose to teach Mathematics?
6. What was the purpose of the lesson?
7. Do you think this was achieved?
8. Relates to Q7 (Why or Why not?)
9. Why did you select the resources you used?
10. Why did you use the examples that you did for this lesson?
11. Why did you ask these questions?
12. What are some of the misconceptions that learners often have for this topic?
13. What concepts or knowledge must learners have before they can understand this topic?
14. What is your understanding of how this topic links to Grade Five and Grade Seven?

APPENDIX 9: Key statements from Teacher A's interview

Teacher A

Categories of PCK **Ball, Thames and Phelps (2008)**	**Key statements from the interview transcripts**
Knowledge of Content and Students (KCS) 1. What are some of the misconceptions that learners have for this topic?	*I think that's where a lot of them struggle with is working out the area of the triangular faces. I thought I would just - use it for this lesson* *One of the major reasons I also did it was because a lot of the time I've seen kids make mistakes when it comes to the heights of the triangle, so its half base times height and often I've seen kids measure height from the centre of the pyramid, that's why I was saying to them that the centre of the pyramid to its highest point, that's not the height of that triangular face* *-Just using simple words such as vertical, horizontal and diagonal.* *They don't understand what that word means*
Knowledge of Content and Curriculum 2. What was the purpose of your lesson?	*We had looked at squares, rectangles, triangles, and also circles, because they don't necessarily need to know it, but it's nice just to push them and see what they are capable of. So we had learnt the formula for triangles* *So we've been doing a number of 3 dimensional shapes and we've been trying to work out the surface area .It is part of the syllabus but it can be used in many different ways* *it was more about, getting them to understand the concept of a square based pyramid and that it is not the same as a cuboid or a*

	triangular prism, where it's all just rectangles and squares, it's has triangular faces
3. Do you think you achieved your lesson's objective?	*I think to an extent. I think the unfortunate thing, because of time constraints you can't necessarily sit on it. And there are students that are going to not pick up on it*
4. Why did you select the resources that you used?	*I don't necessarily like to use a textbook too much. I find that I know last year, I use a textbook quite a lot. And I relied on it and the kids got quite bored of it and allow kids to move forward or further than other ones*
5. What concepts and knowledge must learners have, before they could understand the topic of working out the surface area?	*To understand the properties of like a parallelogram versus a rectangle, or a square versus rhombus, they need to know the differences. And they should know the properties of a rhombus. And they should know that opposite angles are equal, all sides* *are equal, and so on, so on. So that's reinforcing something that they should have already known at the beginning of the year. And most of them do, hopefully know that. Also, the parallel lines that will comes with 2d shapes and also area of 2d shapes, being able to use the formula to be able to solve that and properties of 3d shapes, as well faces, vertices and edge, faces definitely".*
6. What is your understanding of how this topic links with Grade Five and Grade Seven?	*With grade seven, I know they've just been doing surface area and they do take it a little bit further. But the way that they do it will show they're working, it's a lot more structured. With me it's more abstract, is it more trying just to get them to understand how to solve that area*
Knowledge of Content and Teaching (KCT) 6. Why did you select the resources that you used?	*Every child learns in a different way and some of them are more visual and having that actual shape that they have built up there, they can actually see the faces that they have to measure* *I didn't use power point or anything like that because I didn't think it was necessary. I think that the physical objects that they were holding, the 3D shape was enough for them*

7. You started the quiz with the learners by firstly giving them the measurements they needed to work out the answers then as you went along you gave them the area but left out certain measurements. Why did you do that?	*Going forward, that's easy, but going backwards is much harder. It's also, if you want to really understand the subject, mathematics, you need to be able to work in both directions*

APPENDIX 10: Key statements from Teacher B's interview

Categories of PCK **Ball, Thames and Phelps (2008)**	**Key statements from the interview transcripts**
Knowledge of Content and Students (KCS) 1. What are some of the misconceptions that learners have for this topic?	*What I know the misconception they have is a shape is the shape to them* *We were teaching them regular shapes today, or perfect shapes, where we can teach line of symmetry, where we can teach right angles, etc. They do an irregular shape, they do an irregular pentagon and as long as it has 5 sides for them it's a pentagon* *For them, everything is the same but the properties vary from a regular and irregular figure. Especially with the line of symmetry and the interior angles* *Although we use the terminology in the class and today you saw some of them didn't even know what a vertical line or diagonal line is*
Knowledge of Content and Curriculum 2. What was the purpose of your lesson? I probed further: Which aspect of 2D shapes?	*right we are governed by the departments' syllabus and the syllabus for this week was teaching 2d Shapes* *The value of the angle, It is not needed in Grade Six. But I felt that children should know. They only know right angle.*
3. Do you think you achieved your lesson's objective?	*I did achieve more than what I expected, because my aim was to teach them the properties of the shapes and the angle that's all. Either a right angle, acute angle, obtuse angle, and the, 3d shape, the triangle and the quadrilateral, 180 at 360, that's all. But by breaking it up the child see the rectangle, they could see a triangle in that pentagon, or whatever. That's how we were able to achieve it and I really enjoyed*

4. Why did you use the examples that you used?	*The reason is simply because on Monday, I asked them what is 2D and they couldn't talk about it and they did not know what's length and width. So using that concept, I began to draw diagrams, showing them the length or the width of a 2 D shape, and I also indicated that it's flat not the reason why I use those three lines, vertical, horizontal, diagonal, the combination of all three, or combination of two, one, which we use to draw up the regular shapes. That's The reason why I given them the terminology, I explained what's a vertical line, a diagonal line and a horizontal line*
4. What concepts and knowledge must learners have, before they could understand the topic of working out the surface area?	*they must be able to identify the shapes and their properties and then from there they must try and ascertain the length, width and so forth* *they must understand the word tri, it means three. So any three sided figure is a triangle. And we did different types of triangles, which we discussed* *and any four sided figure, as long as there is four lines in any direction is called a quadrilateral. But I just went to the basic one, the rectangle, square, rhombus parallelogram and trapezium*
5. What is your understanding of how this topic links with Grade Five and Grade Seven?	*In Grade Five it's just basic, you just introduce this topic. The child must just know how many lines, that's all right. With grade six they must know the angles and such in grade seven they must know what the interior angles are equal to but because of my curiosity and then suddenly had this light, I said let me go with all the interior angles. So the child has learned more than what the child has expected* *The child in Grade Six must know there's so many sides and lines of symmetry. So we went beyond because of curiosity and I really enjoyed it, that part, especially So not much difference between five and six and seven with 2D shape. The same properties, you can't change the properties, Just same shapes. But what do we do in grade five, we go with the different triangles, grade six, we go the different quadrilaterals, in grade five we only do more or less only squares and rectangles. And in grade six and seven, we go more into some*

	the pentagon, heptagon and so forth, We even go to a 10 sided figure which the child must be able to draw it and know the properties and lines of symmetry
Question asked to probe further. Why do you think it's important to know grade five and grade seven, If you teaching grade six, to you know, the year before, the curriculum and the year after?	*From my experience you need to know, grade five, grade seven and if you teaching grade six, you should know, where the child should have been in grade five, and if they not there you bring them to grade six, and the high flyers you bring them up to base grade seven because when they go to grade seven, they know what's happening",*
Knowledge of Content and Teaching (KCT)	
6. You asked the learners certain questions. For example, what is a polygon? Name three types of lines and name the properties of the shapes? Why did you ask those questions?	*"The reason is, as I mentioned, these are all associated with shapes, The shape is made up of lines. The vertical, horizontal and diagonal lines. The shapes are made up of that. The line of symmetry line of symmetry divides an object into 2 equal parts, mirror image. That's why I folded that one paper and showed them, so that they could understand each of the shape can have more than one*
7. Why did you select the resources that you used?	*We normally take things for granted. The child knows what a pentagon is, the child knows what a heptagon etc. is, but when you call them to the board to draw it, they can't draw it., The lines are all distorted and so forth. So instead of giving you a perfect image, they give you a distorted image. So I thought, Okay, let me get the strips. And then let them use the strips and then they can get it In that way. it was actually more exciting than drawing it*

APPENDIX 11: Digital receipt for thesis editing

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RECEIPT NO:05 **20/01/2020**

	Amount
Payment received 11/01/2020	800
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