Energy Efficient Cooperative Spectrum Sensing Techniques in Cognitive Radio Networks

Masters Thesis
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A thesis presented for the degree of
MSc Eng
in
Electronic Engineering

School of Electrical, Electronic & Computer Engineering
Durban
South Africa

Thesis submitted July, 2017
Energy Efficient Cooperative Spectrum Sensing Techniques in Cognitive Radio Networks

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A thesis submitted in fulfilment of the requirement for the degree of

MASTERS IN ENGINEERING
(ELECTRONIC)

School of Electrical, Electronic & Computer Engineering
University of KwaZulu-Natal
South Africa

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Thesis submitted JULY, 2017
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2. Edwin Kataka, Tom Walingo, "Energy efficient statistical cooperative spectrum sensing in cognitive radio networks", South African Institute of Electrical Engineers (SAIEE), 2017 (Accepted for Publication in SAIEE Journal).

Dedication

....To my mother and children; Gonzalez, Ratzinger, Nympha, Charles, Loice and Damaris.....
For always sacrificing and pushing me to do my best. I pray that you grow to share my exploratious life with your mother.....
Acknowledgments

This thesis is dedicated to all who helped in making my MSc program a successful journey. First and foremost, I thank God for giving me strength, ability, patience and the finances to complete this study. Secondly, I would like to give my sincere gratitude to my supervisor, Dr. Tom Walingo, for believing in me, offering me the opportunity, and providing me with useful directions and feedback towards improving my research work. I thank Dr. Remmy Musumpuka, for his assistance in compiling my thesis. My most heartfelt and sincere thanks go to my loving family for their support and encouragement in every aspect of my life. Without their love and care, I would not have been able to complete this degree. Special thanks to all my friends and colleagues in UKZN (Howard College) who made the whole two years enjoyable. I finally dedicate this thesis to my son Charles Kataka JNR and her sister Tracy Loice.
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<td>ADC</td>
<td>Analog to Digital Converter</td>
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<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<td>CCSS</td>
<td>Central Cooperative Spectrum Sensing</td>
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<td>CR</td>
<td>Cognitive Radio</td>
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<td>CRN</td>
<td>Cognitive Radio Networks</td>
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<td>CSS</td>
<td>Cooperative Spectrum Sensing</td>
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<td>CSI</td>
<td>Channel State Information</td>
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<td>DARPA</td>
<td>Defense Advanced Research Projects Agency</td>
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<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
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<tr>
<td>DSP</td>
<td>Digital Signal Processing</td>
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<td>ED</td>
<td>Energy Detection</td>
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<tr>
<td>EGC</td>
<td>Equal Gain Combining</td>
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<tr>
<td>erfc</td>
<td>Complimentary error function</td>
</tr>
<tr>
<td>$E_t$</td>
<td>Power consumed by SU during transmission</td>
</tr>
<tr>
<td>FC</td>
<td>Fusion Centre</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>GHOST</td>
<td>Goodness-of-Fit Testing</td>
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<tr>
<td>$H_0$</td>
<td>Null Hypothesis</td>
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<tr>
<td>Acronym</td>
<td>Definition</td>
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<td>$H_1$</td>
<td>Alternative Hypothesis</td>
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<td>HOS</td>
<td>Higher Order Statistics</td>
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<td>IF</td>
<td>Intermediate Frequency</td>
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<td>kurt</td>
<td>Kurtosis</td>
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<tr>
<td>kurtT</td>
<td>Transformed kurtosis</td>
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<td>JB</td>
<td>Jarque Bera</td>
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<td>maj</td>
<td>Majority rule</td>
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<td>MDF</td>
<td>Matched Filter Detection</td>
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<td>MF</td>
<td>Matched Filter</td>
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<td>MRC</td>
<td>Maximum Ratio Combining</td>
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<td>NR</td>
<td>Newton Raphason</td>
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<td>NP</td>
<td>Neyman Pearson</td>
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<td>omnbt</td>
<td>Omnibus test</td>
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<td>$P_d$</td>
<td>Local probability of detection</td>
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<td>$P_{fa}$</td>
<td>Local probability of false alarm</td>
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<tr>
<td>$P_{md}$</td>
<td>Local probability of misdetection</td>
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<tr>
<td>PU</td>
<td>Primary User</td>
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<tr>
<td>PU$_{TX}$</td>
<td>Primary User Transmitter</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
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<td>PSK</td>
<td>Phase shift keying modulation</td>
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<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<td>$Q_d$</td>
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<td>$Q_{fa}$</td>
<td>Global probability of false alarm</td>
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<td>Description</td>
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<tr>
<td>QoS</td>
<td>Quality of Service</td>
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<td>RF</td>
<td>Radio frequency</td>
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<td>SC</td>
<td>Selection Combining</td>
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<tr>
<td>RMTO</td>
<td>Restrained multichannel threshold optimization</td>
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<td>ROC</td>
<td>Receiver operating characteristics</td>
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<td>SFC</td>
<td>Spectral Correlation Function</td>
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<td>skew</td>
<td>Skewness</td>
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<tr>
<td>skewT</td>
<td>Transformed skewness</td>
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<tr>
<td>SLC</td>
<td>Square Law Combining</td>
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<td>SPTF</td>
<td>Spectrum Policy Task Force</td>
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<td>SU</td>
<td>Secondary users</td>
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<td>Wireless Sensing Network</td>
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Abstract

The demand for spectrum is increasing particularly due to the accelerating growth in wireless data traffic generated by smart phones, tablets and other internet access devices. Most of prime spectrum is already licensed. The licensed spectrum is underutilized or used inefficiently, i.e. spectrum sits idle at any given time and location. Opportunistic Spectrum Access (OSA) is proposed as a solution to provide access to the temporarily unused spectrum commonly known as white spaces to improve spectrum utilization, increase spectrum efficiency and reduce spectrum scarcity. The aim of this research is to investigate potential impact of cooperative spectrum sensing techniques technologies on spectrum management. To fulfill this we focused on two spectrum sensing techniques namely; Firstly energy efficient statistical cooperative spectrum sensing in cognitive radio networks, this work exploits the higher order statistical (HOS) tests to detect the status of PU signal by a group of SUs. Secondly, an optimal energy based cooperative spectrum sensing in cognitive radio networks was investigated. In this work the performance of optimal hard fusion rules are employed in SU’s selection criteria and fusion of the decisions under Gaussian channel and Rayleigh channels. To optimize on the energy a two stage fusion and selection strategy is adopted to minimize the number of collaborating SUs.
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Preface

“We cannot seek achievement for ourselves and forget about progress and prosperity of our community... Our ambitions must be broad enough to include the aspirations and needs of others, for their sakes and for our own”

— Cesar Chavez

Matsaniza Edwin Kataka
University of KwaZulu-Natal, November 7, 2017
Part I

Introduction
1 Introduction

The license of electromagnetic spectrum is a preserve of governments world over for purposes of wireless communication band allocations. The licensed radio spectrum is within the range of frequencies between 3000 Hz and 300 GHz [38]. Spectrum scarcity is the main problem as the demand for additional bandwidth unreservedly increases. Research has shown that the licensed spectrum is relatively unused. The report published by Federal Communications Commission (FCC) revealed by Spectrum Policy Task Force (SPTF) established that to large extend, some allocated spectral bands are underutilized while others are extensively used most of the time [35].

Cognitive Radio (CR) is an intelligent electronic gadget employed in wireless communication technologies with the ability to sense and adopt to its surrounding environment. The CR technology is envisaged to enable identification, use and management of idle channels. CR has the ability to sense the environment and reconfigure its internal status to the statistical changes in the incoming radio frequency stimuli. It responds by making corresponding changes in certain operation parameters (e.g., transmit-power, carrier-frequency, and modulation strategy) in real-time [13]. What makes CR better than normal radio is its ability to opportunistically, quickly and autonomously access the vacant bands without interfering with the primary users which are licensed to transmit on these channels. It can transmit and receive signals simultaneously and therefore automatically detect (sense the existence of) idle channels in a wireless spectrum. The CR must understand the primary user’s channel status, and consequently create a knowledge data base that can be used later to determine network decisions through the cognition cycle by changing their transceiver parameters [19].

The spectrum utilization can be improved greatly by allowing secondary users to dy-
2. Cognitive Radio Cycle

Dynamically access spectrum holes temporally unoccupied by the primary user in the geographical environment of interest. A spectrum hole is a theoretical hyperspace occupied by radio signals which has dimensions of location, angle of arrival, frequency, time, energy and possibly many others parameters. A radio built on cognitive radio concept has the ability to sense and understand its local radio spectrum environment. It does this by identifying the spectrum holes in radio spectrum space and developing autonomous decisions models on how to access spectrum. The CR using dynamic spectrum access has the potential to significantly improve spectrum efficiency utilization, resulting in easier and flexible spectrum access for current or future wireless services. An illustration of hole concept is shown in Fig. 1 [3]. Based on this model, the CR dynamically evaluates the available channel selection alternatives, access and opportunistically use the channels for the period that the licensed users are idle.

![Fig. 1: Illustration of Spectrum Hole Concept [3].](image)

2 Cognitive Radio Cycle

The cognition capability of a CR allows it to continually observe the dynamically changing surrounding environment in order to interactively come up with the prudent transmission strategies to be used. The block diagram in Fig. 2 describes the four main components of the cognitive radio cycle [22].

![Diagram of Cognitive Radio Cycle](image)
2. Cognitive Radio Cycle

![Functional Architecture of Cognitive Radio](image)

**2.1 Spectrum Sensing**

Spectrum sensing is one of the most critical functions of a CR. Spectrum sensing refers to the ability of a cognitive radio to sense the spectral band in order to capture the parameters related to cumulative power levels and user activities of a licensed primary users. A CR must make up to date real-time decisions about which primary user channel is idle, when and for how long. The sensed spectrum information must be adequate enough for the CR to reach accurate conclusions regarding the radio environment [21]. Furthermore, spectrum sensing must be quick to track the temporal variations of the radio environment. Such requirements of spectrum sensing puts stringent requirements on the hardware implementation of cognitive radios in terms of the sensing bandwidth, the processing power and the radio frequency (RF) circuitry.

**2.2 Spectrum Analysis**

Spectrum analysis takes care of estimation of the channel state information (CSI), it refers to the existence of spectral opportunities in the surrounding radio network based on the sensed wireless communication parameters. A spectral band opportunity is conventionally defined as a band of frequencies that are not being used by the licensed primary user of that band at a particular time in a defined geographic environment. However, such a definition covers three dimension of the spectrum space: frequency, time, and space [22].
2. Cognitive Radio Cycle

2.3 Spectrum Access Decisions

In the cognition cycle CR should be able to make decisions from a set of transmission actions based on the outcome of the spectrum sensing and analysis procedures. The CR utilizes the information collected regarding the PU’s channel opportunities identified as available for the secondary users to opportunistically access [21]. The set of transceiver parameters to be decided on depends on the inherent transceiver architecture. Examples include; which spectrum is more favorable for the preceding transmission, the maximum transmission power, modulation rate, the angle of arrival for directional transmissions, the time instant a transmission over a certain band should start, the spread spectrum hopping scheme, and the number and identity of the antennas [22]. Based on the sensed spectrum information and the transceiver architecture, CR defines the values of the parameters to be configured for an upcoming transmission.

2.4 Adaptability (Reconfiguration) of Cognitive Radio

A key feature that distinguishes CR from an integrated radio is it’s ability to adapt and reconfigure its transceiver parameters based on the assessment of surrounding radio environment. While today’s radios have considerable flexibility in terms of their ability to reconfigure some transmission parameters such as the transmission rate and power, they are typically designed to operate over certain frequency band(s) according to a certain communication protocols [21]. However CR transceiver are robust and agile towards the utilizing the emerging spectral opportunities over a wider spectrum range. For instance, a cognitive radio must be able to configure the transmission bandwidth to adapt to spectral opportunities of different sizes. Furthermore, CR is not restricted to a certain communication protocol only but should be able to adjust and adapt. It must determine the appropriate communication protocol to be used over different spectral opportunities based on its recognition of the radio environment [21, 22].

This work focuses on spectrum sensing, which is the pivot component of the cognitive radio cycle. The CR is faced with a myriad of sensing challenges which this work endeavors to address in order to improve on it core mandate of effective and reliable detection of the licensed primary user with minimal interference.
3 Spectrum Sensing Challenges

The major challenge in CR is that the secondary users need to detect the presence of licensed primary users on spectrum with precision and quit the spectral band as quickly as possible when the primary user emerges to transmit on its channel in order to avoid interference [1]. Spectrum sensing in cognitive radio networks is challenged by several sources of uncertainties ranging from channel randomness at device level to network-level uncertainties. Such uncertainties usually have implications in terms of the required channel uncertainty, noise uncertainty, and detection sensitivity.

3.1 Channel Uncertainty

The hidden node problem is a classic issue with radio systems that opportunistically share the same spectral resources and can result in significant performance degradation. The reason for the degradation is due to the fact that an interfering node (or node pair) may be unaware that they are causing interference to another transmission, which is normally an essential prerequisite for radio coexistence etiquette. This is caused by many factors including severe multipath fading or shadowing observed by secondary users while scanning the primary users’ transmission channels [31]. Multipath fading and shadowing attenuates the signal power as it travels through space. The attenuation is exponentially proportional to the distance the signal travels. The energy loss along the path from the transmitter to the receiver is defined as path loss in wireless communication. The block diagram in Fig. 3 illustrates the concept of interference as a function
of the distance in a cognitive radio network. In this network only one secondary user (SU) detects a number of primary users (PUs) as a receiver of transmitted information data from the primary user transmitter ($PU_{TX}$). The interference range of the secondary user, is determined as follows [14]

$$\lambda = \frac{P_p L(D)}{P_s L(d) + P_b}$$

(1)

where $\lambda$ is the threshold determined by the regulating bodies, $P_p$ and $P_s$ denotes the transmitted power of the primary user and secondary user respectively, $D$ is the coverage radius between the primary user and the primary transmitter ($PU_{TX}$), $L(D)$ is the path loss (including shadowing and multipath fading effects) at distance range $D$, $L(d)$ is the path loss at the distance range $d$ and $P_b$ is the power of background interference. Since path loss varies with frequency, terrain characteristics and antenna heights, these parameters should be taken into account. Here, the CR device causes unwanted interference to the primary user (receiver) as the primary transmitter’s signal can not be detected because of the locations of SU’s devices. Under channel fading or shadowing, a low received SNR of the PU signal does not necessarily mean that the PU is located out of the secondary user’s interference range, as the PU may be undergoing a deep fade due to shadow obstacles [2]. Therefore, spectrum sensing mitigates channel uncertainty in this respect the CRs have the capacity and sensitivity to differentiate between a faded or shadowed primary signal from a white space.

### 3.2 Noise Uncertainty

Spectrum sensing is further challenged by noise uncertainty when energy detection is used as the underlying sensing technique. More specifically, a very weak primary signal will be indistinguishable from noise if its SNR falls below a certain threshold determined by the level of noise uncertainty [14]. Feature detectors, on the other hand, are not susceptible to this limitation due to their ability to differentiate between signal and noise [2]

### 3.3 Detection Sensitivity

Detection sensitivity drops quickly with the increase of the averaged noise power fluctuations and becomes worse at low SNR. Interference due to a cognitive radio network is deemed harmful if it causes the SNR at any primary receiver to fall below a certain threshold ($\lambda$) set up by the wireless communication regulatory bodies world over. This
threshold depends on the receiver’s robustness toward interference and varies from one primary band or service to another [14]. It should, however, come as no surprise that this threshold in general may depend on the characteristics of the interfering signal (e.g., signal waveform, and intermittent interference). This may in turn influence cognitive radio’s choice of transmission waveform in certain licensed bands. Building on the above definition, the interference range of a secondary transmitter may be defined as the maximum distance from a primary receiver at which the incurred interference is still considered harmful [2]. The interference range depends not only on the secondary user’s transmitted power, but also on the primary user’s interference tolerance.

This works addresses channel uncertainty and noise uncertainty issues in part. To mitigate the impact of these challenges, cooperative spectrum sensing has been employed to improve the detection performance by exploiting spatial diversity. In this work a group of SUs collaborate to determine the final decision on the presence or absence of the PU. In cooperative spectrum sensing, detection performance and relaxed sensitivity requirement can be realized. The noise uncertainty and detection sensitivity are specifically addressed by utilization of higher order statistics (HOS) test to detect the PU.

4 Spectrum Sensing Techniques

Spectrum sensing refers to a process of detecting spectrum holes in an opportunistic manner without causing interference to the primary user. The block diagram of Fig. 4 describes the spectrum sensing techniques. They are classified into three: non-cooperative, cooperative detection and interference detection. These approaches fall under the category of spectrum overlay wherein SUs only transmit over the spectrum when the licensed PUs are not using the band [29, 44].

4.1 Non-Cooperative Detection

Non cooperative detection, also called transmitter detection, is based on the sensing the signals from a primary user through the local observation by secondary users. Transmitter detection is classified into three main detection schemes; energy detection, matched filter detection and cyclostationary feature detection [3]
4. Spectrum Sensing Techniques

![Diagram of Spectrum Sensing Techniques]

**Fig. 4: Classification of Spectrum Sensing Techniques.**

4.1.1 Energy Detection

It is a non-coherent sensing technique that detects the primary signal based on the sensed energy. Due to its simplicity and non requirement on a priori knowledge of primary user signal energy detection (ED) is the most popular sensing technique [44]. The block diagram of Fig. 5 describes the energy detection technique. In this model the signal $X(t)$ is passed through radio frequency (RF) and intermediate frequency (IF) pre-processing stages where it is demodulated, amplified, converted from analog to digital and selected by a band pass filter [29]. The PU signal is computed as the energy spectral density or power spectral density (PSD) measured over a specific time interval. Summation or integration of the spectral components yields the total power which is measured as a statistical phenomena. The received primary user signal is expressed as [15]

$$E\{x(t)\} = \frac{1}{T} \int_{t-T}^{t} x^2(\tau)d\tau$$

where $E$ is the energy of the input signal $x(t)$ at any time over a period interval $T$. The input signal $x(t)$ consists either of noise alone or a signal plus noise. The output from the integrator block is then compared to a predefined threshold whose value is based on
the channel conditions. The major drawback of the energy detector is its inability to distinguish between different sources of received energy, i.e., it cannot distinguish between noise and licensed user's signal. This makes it unreliable technique to be employed in detecting the presence of the primary user especially at low SNR conditions. Energy detection is simple to implement, and hence widely adopted in spectral sensing. Part of this work is focused on energy detection in a cooperative spectrum sensing network.

4.1.2 Matched Filter Detector

In cognitive radio networks, matched filter detection is obtained by correlating a known signal or template with an unknown signal to determine the presence of the template in the unknown signal conditions. This is equivalent to convolving the unknown signal with a conjugated time-reversed version of the template [44]. The block diagram of Fig. 6 shows matched filter (MF) [26]. The primary signal $x(t)$ is converted from analog to digital and subsequently passed through the band pass filter to select the desired primary signal. The hypothesis statistical test $H_0$ shows that the primary user is absent and $H_1$ when it is present. In a scenario where secondary user has a priori knowledge of primary user signal, matched filter detection (MFD) is considered to be the most appropriate detection scheme. The matched filter is the optimal linear filter on maximizing the signal to noise ratio (SNR) in the presence of additive stochastic noise. In this form of detection paradigm the PU transmitter sends a pilot stream simultaneously with the data, the SU receiver therefore has a perfect knowledge of the pilot stream to verify its transmission on the frequency band [34]. However, the most significant disadvantage of MFD is that a CR would need a dedicated receiver for every type of primary user. The sensing decision is based on the knowledge of the statistical distribution of the autocorrelation function. For random noise, the first lag of the autocorrelation is very small or negative, but when there is a signal the autocorrelation at the first lag will represent a significant value. In signal processing, for a given signal $Y[n]$, as a general convolution...
sum equation is expressed as [26]

\[ Y[n] = \sum h[n - k]x[k] \]  

(3)

where \( x \) is the unknown signal (vector) and is convolved with the \( h \), the impulse response of matched filter that is matched to the reference signal for maximizing the SNR. Due to the fact that MFD requires a prior knowledge of every primary signal, if the information is not accurate MFD performs very poorly. Also the most significant disadvantage is that CR needs a dedicated receiver for every type of primary user. This technique is not used much in cooperative sensing and therefore not considered in this work.

4.1.3 Cyclostationary Feature Detection

Cyclostationary is a statistical process with properties to exploits the inherent cyclostationary characteristics of the received PU’s signal. The scheme deals with the periodicity inherent in sinusoidal carriers, spreading code, hopping sequences or cyclic prefixes and pulse trains of the primary signals. Such features have a periodic statistics and spectral correlation that cannot be found in any interference signal or stationary noise. That is why the cyclostationary feature detection method possesses higher noise immunity than any other spectrum sensing method [44]. The block diagram of Fig. 7 shows the stages of cyclostationary detection scheme [5]. The signal \( x(t) \) is passed through the filter circuit which selects the center frequency, and bandwidth of interest (primary user channel). The ADC electronic circuit reconverts the signal from analog to digital. Fast Fourier transform (FFT) algorithm computes the discrete Fourier transform (DFT) of a sequence i.e., converts the signal of interest from its original domain (often time or space) to a representation in the frequency and vice versa. The signal in cyclostationary processes is periodic in time duration \( T \), which also possess a periodic autocorrelation function. The primary user signal is then averaged over a given period of time. Feature detector, is used to extract the signal features achieved by decimation of the cyclic spectrum. In
4. Spectrum Sensing Techniques

this method, the cyclic spectral correlation function (or SCF) is the parameter that is used for detecting the primary user signals. The cyclic SCF of received signal can be formulated as [5, 26]

\[
S_{yy}(f) = \sum_{\tau=-\infty}^{\infty} R_{yy}^\alpha(\tau)e^{-j2\pi f \tau}
\]

where \(R_{yy}^\alpha(\tau)\) is the cyclic autocorrelation function obtained from the conjugate time varying autocorrelation function of PU’s signal \(s(t)\) periodic in time \((t)\). When the parameter \(\alpha\) is the cyclic frequency and equal to zero, the SCF becomes power spectral density. A peak cyclic SCF value implies that the primary user is present on that band. Although this scheme requires a priori knowledge of the signal characteristics, it’s capable of distinguishing the CR transmissions from other types of PU signals [44]. It has an advantage over energy detection since it eliminates the synchronization requirement when applied in cooperative spectrum sensing networks. Moreover, cognitive radio users may not be required to keep silent during cooperative sensing and thus improving the overall cognitive radio throughput. This method has its own shortcomings owing to its high computational complexity and long sensing time [26]. Due to these issues, this detection method is less common than energy detection in cooperative sensing and is not featured in this work.

4.2 Interference Based Detection

Interference based detection model attempts to regulate interference at the primary receiver. The CR users are allowed to transmit on the spectrum band as long as they do not exceed the interference temperature limit. That is, during the interference based detection, the CRs have to measure the interference temperature and adjust their transmission in a way to avoid raising the interference temperature over the interference temperature limit [44]. Typically, CR user-transmitters control their interference by regulating their transmission power (their out-of-band emissions) based on their locations with respect to primary users. The CR users are allowed to coexist and transmit simultaneously with primary users using low transmit power that is restricted by the interference temperature level so as not to cause harmful interference [26].

However, the main drawback of interference based detection is that the CR users cannot transmit their data with higher power even if the licensed system is completely idle since they are not allowed to transmit with higher than the preset power to limit the interference at primary users. It is noted that the CR users in this method are required
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to know the location and corresponding upper level of allowed transmit power levels, otherwise they will interfere with the primary user transmissions [5]. This spectrum scheme is seldom used and is not part of this work.

4.3 Cooperative Spectrum Sensing

In cooperative spectrum sensing (CSS) paradigm, each secondary user shares the information acquired from sensing the primary user. In this technique, a group of secondary users collectively gather the information concerning channel status and spectrum map used as a database of the information [20]. A group secondary users detect the PU at regular sensing intervals and forwards their decisions to the cluster center or fusion center (cognitive radio nerve center), where the final global decision is made about the primary user channel status and feedback given back to the respective secondary users hence allocating them the spectrum.

In wireless communication the detection performance on large extent depends on a number of factors, among them effects such as shadowing, multipath fading, and hidden nodes problem [31]. Under cooperative spectrum sensing some SUs may suffer from the receiver uncertainty challenges because they are not aware of the PU’s presence and as a consequence, those SUs experiencing uncertainty problem end up interfering with the signal reception at PU receiver. However, due to spatial diversity, it is rare for all spatially distributed SUs in a CR network to simultaneously experience fading or receiver uncertainty problems.

To alleviate fading problems SUs which observe a strong PU signal can be allowed to collaborate and share the detection results with those SUs observing weak signal. The combined cooperative decision derived from the spatially collected individual SUs data is used to overcome the shortcomings of individual decisions made by only one SU [20]. Thus, the overall detection performance can be greatly improved. This is the reason why cooperative spectrum sensing is an attractive and efficient approach to mitigate multipath fading and shadowing and receiver uncertainty problems. Cooperative spectrum sensing can be classified into three categories depending on how cooperating SUs share the sensing data in the network; distributed, relay-assisted and centralized schemes [40].
4. Spectrum Sensing Techniques

4.3.1 Distributed (Decentralized) Spectrum Sensing

Distributed, also referred to as decentralized, spectrum detection scheme refers to the set of algorithms where cognitive sensors group themselves based on a distribution statistics. They combine their individual results and communicate among themselves regarding presence or absence of white spaces in the cognitive radio network [44]. The CR network shown in Fig. 8 describes a decentralized spectrum sensing [40], this sensing architecture does not rely on a fusion center in making global collaborative decisions. In this scenario, SUs exchange the sensing observations in a given cluster and converge to a unified decision. Based on a distributed decision algorithm, each SU sends its own sensing data to other users, combines its data with the received sensing data, and decides whether or not the primary user’s transmission is present by using a local decision criterion. If the decision threshold is not met, SUs send their combined results to other users again, and iterate this process until the decision algorithm converges and a final decision is made. Authors in [43] argue that this approach greatly increases detection reliability and certainty. An efficient decentralized scheme requires a user selection protocol responsible for determining how many and which SU are going to collaborate so that not all SUs participate simultaneously. The criteria is pegged on the SUs proximity to the PU and its SNR at the time of transmitting. However, they agree the scheme may add a lot of system overhead and compromise throughput of the SU network. In decentralized cooperative sensing, SUs sense the presence of PU periodically, the information

![Decentralized Spectrum Sensing Network](image-url)
sensed becomes obsolete very fast due to factors like mobility, and channel impairments. Decentralized cooperative spectrum sensing raises new challenges including detection delay, coordination algorithms complexity, and asynchronous sensing design. These schemes are susceptible to higher error probability than the other two schemes and are not the focus of this work.

4.3.2 Relay-Assisted Cooperative Sensing

The relay-assisted cooperative sensing can exist in a distributed scheme. In fact, when the sensing results need to be forwarded by multiple hops to reach the intended receiver node, all the intermediate hops acts as relays [45]. In practice both sensing channel and reporting channels are imperfect and SUs that observe strong PU signals may suffer from a weak reporting channel. Those SUs with a strong reporting channel can serve as relays to assist in forwarding the sensing results of those SUs with strong sensing channel but with weak reporting channel to the fusion center [47]. The relay-assisted cooperative sensing scheme requires more than one antenna at the transmitter. However, many wireless devices are limited by antenna size and hardware design complexity [37]. This cooperative spectrum sensing scheme is not the focus of this work due to its complexity in implementation.

4.3.3 Centralized Spectrum Sensing

In this paradigm there is a master node (cognitive radio) within the spatially spaced secondary users network that collects the sensing information from all the sense nodes or secondary users within the network. It then analyzes the PU’s signal to determine the spectral bands that are idle [35]. Fig. 9 shows centralized cooperative spectrum sensing model, where a group of spatially distributed secondary users sense the primary user channel and report their observed individual local decisions to central processor or the fusion centre. The absence or presence of the PU on channel can be modeled by null hypothesis \( H_0 \) and alternative hypothesis \( H_1 \) test statistics respectively [3]. In this scheme FC controls a three-step cycle of cooperative spectrum sensing. Firstly, the FC selects a frequency band of interest to be sensed and commands all collaborating SUs to individually perform local sensing. Secondly, all cooperating SUs transmit their sensed data results via the control channel to the FC. Thirdly the FC integrates and fuses the received local sensing information from the SUs to perform a global decision on the presence or absence of PUs, and retransmits the decision back to the SUs, instructing
them on status of the channel [45]. One of the chief problems with non-cooperative spectrum sensing is that even though the secondary user may not be able to detect the PU it may still interfere with it. By using a centralized cooperative sensing system, it is possible to reduce the possibility of this happening because a greater number of SUs will build up a more accurate detection picture of the primary user transmissions conditions.

The CR perform spectral sensing at periodic intervals, the challenge with this is that the sensed information from the PU become obsolete fast due to factors like mobility and fading issues. The CR then needs, a big capacity to hold the data as it updates its current information. It is without doubt that this greatly increases storage and sensory space resulting in the data overheads, this has a ripple effect on the system in terms of data throughput, and energy consumption. Even with this challenges cooperative sensing based on hard fusion schemes can be implemented without incurring much overheads because only approximate sensing digital data is required. It therefore eliminates the need for complex signal processing schemes at the receiver and reduce the data over load. In centralized spectrum sensing, SUs collaborate in the sensing and decision making. This helps to acquire accurate information which further reduces the false alarms and subsequently maximizes the system reliability, hence this is the focus of this work.

5 Higher Order Statistics Detection Techniques

In literature a number of new spectrum sensing paradigms based on higher order statistic (HOS) have been proposed. These algorithms perform non-Gaussianity check on the signal distribution, and they are based on the fact that noise follows Gaussian dis-
Higher Order Statistics Detection Techniques

Distribution whereas the signal does not [42]. Estimated value of third and fourth order cumulants are employed in Gaussian tests on the real and imaginary parts of the FFT spectrum. Some of the known HOS tests include skewness, kurtosis, Jarque Bera and omnibus tests. These methods are evaluated by simulation and the results have shown increased robustness against noise uncertainty as compared to energy detection schemes. This has shown a paradigm shift towards achieving robust spectrum sensing based on goodness-of-fit. The major challenge in cognitive radio networks is the noise uncertainty, however HOS performs extremely well in this conditions and is the focus this work.

5.1 Kurtosis and Skewness

The skewness and kurtosis of a received signal has predetermined statistical pattern which is compared to a set threshold to determine the deviation from a normal distribution [18]. These techniques are used to measure the statistical properties of a set of randomly distributed data samples. Kurtosis as a fourth and skewness third moments both measures the degree of departure from Gaussian probability distribution function (PDF). Large positive values of kurtosis indicate a highly peaked PDF that is much narrower than a Gaussian distribution while negative kurtosis indicates a broad PDF that is much wider and flatter than a Gaussian distribution. The main difference between kurtosis and skewness is that while kurtosis measures the peakness of a distribution relative to the Gaussian distribution, skewness on the other hand measures the statistical asymmetry of the distribution [18].

Fig. 10 shows a block diagram of kurtosis and skewness tests as utilized in spectrum sensing [33]. The received primary user signal to be tested by this model is converted from analog to digital by an analog to digital converter (ADC) circuit, the power spectral density (PSD) of the received signal is then calculated. Periodogram of the estimated PSD is employed to accurately determine the frequency-domain statistical properties of primary user’s signal to be detected in the network.

The statistical tests are derived by calculating a Fast Fourier Transform (FFT) over the digital signal samples which is compared to a predetermined threshold ($\lambda$) value of the Gaussian noise. Thresholding is calculated based on fixed probability of false alarm and usually 10 percent is the commonly accepted standard value [8, 10]. This threshold set-
5. Higher Order Statistics Detection Techniques

Fig. 10: Block diagram for kurtosis & skewness detection tests

ing is done on every detected sample frame size \( (N) \) such as the number of FFT points. As noise varies for each hardware, this threshold setting method will guarantee noise adaptability. It’s assumed that the noise is stationary and follows the additive white Gaussian noise (AWGN) distribution property. The AWGN is a channel model where the only impairment to communication is noise; with a constant spectral density. In this model noise possesses zero mean, and is assumed to be white over the bandwidth of consideration; i.e. samples of the noise process are uncorrelated [18].

5.2 Jarque Bera Test

The \( JB \) test is HOS spectrum sensing technique employed to compute normality test on a given data sample in order to determine how close the data is to a normal distribution. It is non-parametric tests most preferred because it does not require previous information of the PU’s status on the channel to make a decision (blind detection) [8]. The \( JB \) test is a asymptotically chi-squared distributed with two degrees of freedom. It uses the unbiased samples of skewness and kurtosis to verify the adherence to Gaussian distribution [18]. Fig.11 shows a block diagram of \( JB \) spectrum sensing scheme [11], where the PU signal \( X(t) \) is demodulated through RF and IF preprocessing stages and converted from analog to digital by ADC electronic circuit. The signal is shifted to a base band to reduce the sampling frequency needed to obtain Nyquist digital samples of the spectral band. The FFT points are then calculated, subdivided into \( N \) frames of \( \text{NFFT} \) points and concatenated. The \( JB \) values are compared with a predefined threshold to distinguish between occupied spectrum and white space. The threshold is calculated from empirical estimation of system’s noise, that acknowledges the distribution of the
5. Higher Order Statistics Detection Techniques

Signal $Y(n)$, as being Gaussian (channel is idle) or non-Gaussian (channel occupied) [18]. Considering a AWGN channel where the PU’s transmitted signal is well defined, the probability distribution for an idle channel is given as a Gaussian random variable with zero mean and unit variance. Under normal distribution the statistical values are known, the SUs can therefore use this statistic to determine the presence or absence of the primary user on the channel. Authors in [8] have shown that JB test as applied in spectrum sensing algorithms can obtain better detection performance than existing higher order statistics (HOS) methods, since it is more robust to noise uncertainty even when investigated on a small sample size.

5.3 Omnibus ($K^2$) test

Omnibus ($K^2$) is applied to assess the normality of random variables by calculating kurtosis and skewness [39]. Omnibus is a moment test derived from the recognition of departure to normality from a Gaussian distribution. A statistical test implemented on the overall hypothesis that tends to find general significance between parameters’ variance, while examining parameters of the same type. In statistical research test, a random sample from a population distributed with unspecified mean and variance, omnibus $K^2$ will test whether the explained variance in a set of data is significantly greater than the unexplained variance. A successful omnibus $K^2$ test would lead one to reject the null hypothesis if and only if the data comes from another distribution [30]. It is formulated based on the standardized third and fourth moments to assess the normality of random variables. Generally omnibus $K^2$ statistic integrates the standardized sample moments of transformed skewness and kurtosis into normal variants respectively. It combines them into single test statistic designed to detect a broad range of departures from a specific null hypothesis [25]. A correctly sized omnibus test as a chi-squared
distribution test can be specified to determine the information originating from two moments. The main advantage of this the test is the simplicity provided by the \( \chi^2 \) distribution statistical framework [30].

6 Spectrum Sensing over Fading Channels

Many wireless communication networks are subjected to fading caused by multipath propagation due to reflections, refractions and scattering by buildings and other large structures. In PU detection, flat fading delivers the poorest performance since frequency selectivity provides multiple “observations” simultaneously. Furthermore, in a composite fading/shadowing environment, apart from the multipath fading, wireless signals may undergo shadowing process typically modeled as a Log-normal distribution and multipath fading which can be modeled as a Rayleigh, Rice or Nakagami distributions [23]. In this environment the receiver does not average out the envelope fading due to multipath but rather reacts to the instantaneous composite multipath/shadowed signal. This is often the scenario in congested areas with slow moving objects. Therefore, some practical communication channels can be modeled as a multipath fading superimposed on Log-normal shadowing.

6.1 Rayleigh Fading Channels

This is a statistical distribution commonly employed to model the signal amplitude variation when the signal is not received on a line-of-sight path between the transmitting antenna and the receiver. The channel fading amplitude follows the distribution function of a statistical time varying nature of the received envelope of a flat fading or the envelope of an individual multipath component. This fading model considers urban multipath features, including effects of the ionosphere and troposphere. When this model is employed, attenuation of the signal is Rayleigh distributed and therefore the SNR at every node follows an exponential distribution [7]. Due to the hidden terminal problem, a cognitive radio may fail to identify the presence of the PU and then will allow erroneous access the licensed channel, causing interference to the licensed system. In order to deal with the hidden terminal problem in cognitive radio networks, multiple cognitive users can cooperate to conduct spectrum sensing [31].
6.2 **Nakagami-\( M \) Fading**

This static technique is preferred to model multipath propagation in indoor mobile communications and radio links for ionosphere communications. The Nakagami fading distribution is a convenient model for analyzing the performance of digital communication systems over generalized fading channels. This fading distribution is assumed in the analysis of many terrestrial wireless communication systems. It is flexible and embraces scattered, reflected and direct components of the original transmitted signal [7, 48].

6.3 **Lognormal Fading**

This statistical technique models the envelope of received signal when affected by shadowing effect due to blockage caused by buildings and hills among others obstructing objects. The probability density function, empirically models an outdoor and indoor wireless propagation environments. In the presence of lognormal channel interference, computing the outage probability (or its bounds) often involves calculating the mean and variance of the sum of lognormal random variables [48]. Several approximate methods have been suggested in the literature to compute both the outage probability and the underlying lognormal sum distribution. Shadowing is usually statistically independent however in some cases it may be statistically correlated. Estimation of outage probability requires pdf of sum of lognormal random variables representing the shadowing, heuristics or local optimization algorithms can be used to find (local) solutions to the problem of minimizing the total transmitter power subject to outage probability constraints [17].

7 **Cooperative Spectrum Fusion Techniques**

In cooperative spectrum sensing, the major challenge is how SUs share information amongst themselves to make the final decisions on whether PU is active or not on the channel. The shared information transmitted by individual SUs are combined to make a global decision at the fusion center (FC). In literature two main fusion techniques namely, soft or hard decisions have been proposed to determine the global decisions done at the fusion center in a cooperative spectrum sensing networks [12].
7. Cooperative Spectrum Fusion Techniques

7.1 Soft Fusion Decision Schemes

This refers to a spectrum sensing technique where SUs send their instantaneous received signal-to-noise ratios (SNRs) or any other detection metric to a central unit known as the fusion center. The sensing results in form of likelihood ratios are combined using soft combination strategies to fuse the observed instantaneous energy [28]. Algorithms such as equal gain combining (EGC), maximal ratio combining (MRC), square law combining (SLC) and selection combining (SC) approaches have been adopted. In all cases, the observed energies from \( N \) number of cooperative users are scaled by weight factors and added up. The decision is a result of the weighted sum expressed as [12]

\[
X = \sum_{j=1}^{N} w_j X_j
\]

where \( X_j \) is the observed energy of the \( j \)-th secondary user and \( w_j \) denotes the weight factor for the \( j \)-th secondary user. The resulting decision statistic is compared to a decision threshold (\( \lambda \)) to decide between \( H_1 \) (the channel is occupied) and \( H_0 \) (the channel is idle)

\[
\begin{align*}
X > \lambda & \quad \text{Accept alternative hypothesis } H_1 \\
X < \lambda & \quad \text{Accept null hypothesis } H_0
\end{align*}
\]

The threshold (\( \lambda \)) is set to achieve the desired probability of false alarm or miss detection. The main difference between MRC and EGC fusion techniques is on how the weights are evaluated. The MRC is expressed as [16]

\[
X_{MRCj} = \frac{X_j}{\sqrt{\sum_{k=1}^{N} X_k^2}} \quad 1 \leq j \leq N
\]

where \( X_j \) represents the measured instantaneous signal to noise ratio of the \( j \)-th SU. The SUs with strong signals are then amplified, while weak signals are attenuated. MRC shows in practice the optimal performance but it is hardly employed as it requires prior knowledge on the estimated channel gain. Similarly the weights of EGC soft combination technique formulated as in eqn. (8) [1]

\[
W_{EGCj} = \frac{1}{\sqrt{N}} \quad 1 \leq j \leq N
\]

where \( N \) is the number of samples of the PU signal over a given period of time. Secondary users are assigned same weights based on the number of \( N \) collaborating SUs. If the channel state information (CSI) between the primary users and the secondary users is perfectly known, MRC could achieve higher probability of detection hence achieves
optimal performance at low SNR as compared to EGC. However, EGC is less complex in design since it does not require channel estimation [1, 16]. In [41], authors studied collaborative detection in wireless transmissions using soft decision and the likelihood ratio test. It was shown that soft decision combination in spectrum sensing achieved more precise and reliable PU detection than hard decision combination. However this comes at the cost of large overheads in terms of bandwidth which is already a scare resource.

7.2 Hard Fusion Decisions Schemes

In this scheme a group of SU performs local spectrum sensing to determine the presence or absence of the PU on channel and retransmits their individual decisions in binary logic form to the FC. At the FC, decisions are collated, analyzed and integrated to make the final global decision on the status of PU [6]. Three hard combining decision rules used to arrive at the global decision include; Majority, OR and AND rules [41]. Due to cost implied on the bandwidth, the hard decision combination is preferred since utilizes less spectrum as compared to soft fusion decision hence remains an attractive option in CSS networks, this has informed the choice of hard fusion schemes over soft combination schemes in this work.

7.2.1 Majority Counting Rule

This is also called \( k \) out of \( n \) counting rule where the FC decides on the presence of the PU on condition that \( k \) or more number of SUs out of the total \( n \) collaborate to determine the final decision on the status of PU on the channel. Therefore if \( k \) number of SU or more decide in favor of PU’s presence then the global decision reached at the FC is a binary 1 formulated by a null hypothesis test \((H_0 / H_1)\), implying that the PU is transmitting on the channel [4].

7.2.2 Logical OR Rule

This is a hard logic combination rule made by the FC in a central cooperative spectrum sensing network confirming the presence of PU on the channel on condition that at least one or more SUs declares presence of PU. Since SUs transmits on a licensed frequency band may cause interference to the PUs, the risk of SUs causing interference to the PU under the logical OR fusion rule is greatly increased [7, 12].
7.2.3 Logical AND Rule

In this hard fusion scheme all SUs in a central cooperative spectrum sensing network (CCSS) must declare and report the presence of PU on the channel to the FC before it confirms that indeed the PU is transmitting on the channel (binary 1), otherwise the global decision at the FC will show absence of PU (binary 0). Therefore, global decision is given by the hypothesis \((H_1)\) only if all of the SUs decide on presence of PU \((H_1/H_1)\) [4].

8 Optimization by Lagrange Criterion

In this work Lagrange criterion is used to optimize the number of participating SUs in cooperative spectrum sensing. Limiting the number of SUs communicating is important in minimizing the energy consumption in the cognitive radio network (CRN). An adaptive distributed iterative algorithm is proposed to solve this problem by using Lagrange dual theory and logarithmic transformation. In [46], the authors investigated the performance in CRN based on the Lagrange criterion algorithm in optimal resource allocation and indeed guaranteed high QoS as compared to the other optimization algorithms. A Lagrange criterion problem can be formulated as

\[
\begin{align*}
\text{Maximize} & \quad \frac{f(x)}{g(x)} \\
\text{Subject to} & \quad h_i(x) \leq 0 \quad \forall \ i = 1, 2, \ldots, N
\end{align*}
\]

where \(f(\cdot), g(\cdot)\) and \(h_i(\cdot), i = 1, 2, \ldots, N\), denote real valued functions which are defined on the set, \(X\) of \(\mathbb{R}^n\). Lagrange as a function program is a concave fractional program if it satisfies the following two conditions

1. \(f(\cdot)\) is the concave and \(g(\cdot)\) is the convex on \(X\)
2. \(f(\cdot)\) is positive on \(S\) if \(g(\cdot)\) is not affine,

where \(S = \{x \in X : h_i(x) \leq 0 \ \forall \ i = 1, 2, \ldots, N\}\). It is noted that in a concave fractional program, any local maximum is a global maximum i.e. a differentiable concave fractional program solution of the Lagrange condition provides maximum solution. It can be seen that the function in the numerator is concave function and the denominator is affine and all the constraints are affine. The optimal problem of eqn. (9) above is differentiable and satisfies the Lagrange criterion. This optimization technique has been used in this research work.
9 Neyman-Pearson Optimization

The optimal fusion strategy based on hard fusion schemes in CCSS network is important in minimizing the probability of false alarm or maximizing probability of detection. Neyman-Pearson (NP) criterion is used to solve this optimization problem. In this detection paradigm unknown deterministic signals developed as a binary hypothesis test problem represented by $H_0$ as a default model also called the null hypothesis is compared to $H_1$ also called the alternative hypothesis as a likelihood ratio test. The binary hypothesis statistics test is solved by the Neyman-Pearson criterion wherein the performance of the system is expressed in terms of false alarm and detection probability [48].

The NP test compares the likelihood ratio of a set threshold to the optimal threshold as a function of the prior probabilities and the costs assignment on different errors. The choice of costs is subjective and depends on the nature of the problem, but the prior probabilities must be known [12]. NP just like other statistics, needs to preselect a threshold ($\lambda$) to balance the trade-off between false positive error and false negative error. In the NP classification setting, the threshold can be optimized by introducing a risk to the objective function as described in eqn. (10). Let $\alpha$ be the risk of false positive and $\beta$ be the risk of false negative, the formulation of NP classification can be formulated as follows [36]

$$\min_{\lambda} (f(\lambda) - \alpha) + \beta,$$

Subject to $\beta \leq \alpha$, (10)

The optimal $\lambda$ can be selected as a trade-off problem. A classic result due to NP has showed that the solution to this type of likelihood ratio test is optimal [32]. This work focuses on both Lagrange and NP optimization techniques used to achieve a two stage spectrum sensing optimization paradigm.

10 Energy Efficiency in Cooperative Spectral Sensing Networks

Cognitive radio networks are considered as a novel and reliable paradigm shift in energy efficient wireless communication systems. The SU devices are powered by batteries and often embedded into the system permanently, it is often impractical to charge or replace the exhausted battery. Energy-efficiency is therefore an important component for cognitive radio operations and communications over the wireless channels. While
energy efficiency is the most important parameter in designing secondary user detection networks, other quality of service QoS parameters such as throughput and system reliability are investigated in this work [7]. An energy efficiency metric can be defined as the effective throughput per one unit of transmitted power. That implies, we can call a scheme energy-efficient or green if we can reduce the total network power without introducing significant impact on the network throughput. Energy efficiency is measured in bits per Joule. This means energy is required in Joules to transfer one bit from one point to the other [27]. The number of SUs determine the total energy consumed in the CRN, an efficient CSS network is one where a minimum number of SU collaborate to make the final decision with high reliability and probability of detection. The term green is synonymous to energy-efficiency for wireless sensing network (WSN) design, since maximizing energy efficiency reduces the power usage in a WSN life cycle, and subsequently reduces air pollutants.

11 Performance Metrics in CSS Network

The reliability of the spectral band information availability is defined by the performance metrics. In cooperative spectrum sensing they are specified by the following general metrics; local probability of detection ($P_d$), local probability of false alarm ($P_{fa}$), local probability of misdetection ($P_m$), global probability of detection ($Q_d$), global probability of false alarm ($Q_{fa}$) and global probability of misdetection ($Q_m$).

11.1 The Local Probability of Detection ($P_d$)

In opportunistic spectrum sensing, local probability of detection ($P_d$) specifies that SU in a cognitive radio network makes the correct decisions on the presence or absence of primary user (PU) on the channel. This is informed by the ($H_1|H_1$) hypothesis test, where the SUs correctly determines the status of PU. The ($P_d$) is an indicator on the level of interference protection provided to the PU. Hence, a large ($P_d$) denotes precise sensing; which translate to small chance(s) of interference [24].

11.2 The Local Probability of False Alarm ($P_{fa}$)

The local probability of false alarm ($P_{fa}$) event occurs when the SU assumes that the PU is transmitting on the channel when in fact it is not. This is represented by the hypothesis ($H_1|H_0$) where the SU makes a decision of presence of the PU ($H_1$) when actually it is idle $H_0$. When a false alarm event occurs, the SU would not exploit the free
spectrum, thus missing a chance to utilize the free channel. The $P_{fa}$ should be kept as small as possible in order to prevent underutilization of transmission opportunities. The performance of the spectrum sensing technique is usually influenced by the $P_{fa}$, since this is the most essential metric [9, 24].

11.3 The Local Probability of Misdetection ($P_m$)

The probability of declaring the PU is idle ($H_0$), when it is indeed transmitting on the channel ($H_1$), is referred to as the probability of missed detection ($P_m$) represented by hypothesis ($H_0|H_1$). A high ($P_m$) implies an increase in the chance of interference to PU by the corresponding SU. If the detection fails, or a miss detection occurs, the SU initiates a transmission resulting in interference with the PU signal; contravening the opportunistic access concept. In essence, the spectrum sensing method should record a high probability of detection (minimal misdetection probability) and low probability of false alarm [9, 24].

11.4 The Global Probabilities in CCSS Networks

The global probability of detection ($Q_d$) is the joint probability for all the SUs in a CSS network carried out at the fusion center or the cluster head in a cognitive radio network. The global detection ($Q_d$) is a joint probability of correctly determining the presence of the PU on the channel after a summation of individual local probabilities of detection ($P_d$) done by individual SUs in the cognitive radio network. The joint probability is determined by two forms of fusion schemes; soft and hard fusion rules as described in previous sections of this work. The global probability of misdetection ($Q_m$) is a joint probability of SUs wrongly determining the absence of the PU on the channel when in fact the PU is transmitting. This will definitely cause interference of the SUs to a transmitting PU in cooperative spectrum sensing networks. It should be minimized as much as possible to improve on the detection reliability of the SUs. The global probability of false alarm ($Q_{fa}$) on the other hand is the joint probability summed from individual SUs local false alarm probabilities. This decision makes the spectrum to be underutilized and hence should be minimized as much as possible to make efficient use of the spectrum [45]. This work addresses the methods utilized to minimize the global probability false ($Q_{fa}$) and maximize the global probability of detection ($Q_d$) as applied on the hard fusion rules using Neyman Pearson and Lagrange criterions.

The SUs performance is analyzed by depicting the receiver operating characteristics
12. Problem Statement

Spectrum resource scarcity is the greatest challenge in wireless communication due to growth of demand for the spectrum. However most of the frequency band is left underutilized and therefore the need for opportunistic spectrum access and hence the inception of cognitive radio network. This problem can be solved by allowing cognitive users (unlicensed users) to occupy the spectral band at the time when the primary users (licensed users) are not transmitting on the channel. However, it is difficult for a single SU to make a right decision due to multi-path fading, noise uncertainty, hidden nodes and shadow effects in wireless environment. Cooperative spectrum sensing (CSS) is employed in this work to alleviate this problem. CSS utilize multiple secondary users (SUs) to sense the vacant spectrum and send their decision to the fusion center (FC) for a final global decision to be made regarding the presence of the primary user (PU). Too many secondary users also increase the total energy consumption in CSS network. This work optimizes on the number of SUs employed in detection of a PU in order to minimize the energy consumption in the CRN. Spectrum sensing in cognitive radio networks has raised a number of concerns such as noise uncertainty and sensing interference. HOS tests are preferred in local sensing because they perform better under noise uncertainties as compared to energy detection techniques.

13. Scope of the Study

This work focuses on centralized cooperative spectrum sensing based on energy detection and higher order statistical (HOS) detection tests. A group of spatially placed SUs sense a PU in a cooperative spectrum network. The models are analyzed based on the local probability of detection ($P_d$) and probability of false alarm ($P_{fa}$) performance metrics. The decisions of SUs are transmitted to the FC center through wireless fading channels where global detection probability $Q_d$ is made. Optimization is done on the
hard fusion techniques to improve on the spectrum sensing and energy consumption in CRN.

14 Specific Objectives

The following is an enumerated summary of the main objectives of this thesis:

1. To derive a hard fusion strategy to be utilized in the fusion of the secondary users decisions at the fusion center in cooperative spectrum sensing network.

2. To derive an selection criteria of collaborating SUs in cooperative spectrum sensing networks to achieve optimal energy efficiency.

3. To investigate on performance of energy efficient higher order statistics (HOS) techniques over wireless cooperative spectrum sensing schemes in cognitive radio networks.

15 Thesis Organization

This thesis is composed of two parts: Part I presents a general introduction to the cognitive radio networks, cooperative spectrum sensing techniques and optimization strategies. Part II is focused on paper A, titled, "Optimal Energy Based Cooperative Spectrum Sensing under Gaussian and Rayleigh channels". Paper B, titled, "Energy Efficient Higher Order Statistical (HOS) tests in a centralized cooperative spectrum sensing network".

16 Research contribution

The following papers are the main contributions related to this thesis:

16.1 Paper A: Optimal Energy Based Cooperative Spectrum Sensing Schemes in Cognitive Radio Networks

Abstract

Cooperative spectrum sensing (CSS) alleviates the problems of multipath, shadowing and hidden nodes experienced in wireless communication. Both the selection criterion of collaborating secondary users and the fusion schemes used in CSS affect the
reliability of detecting the status of primary user (PU) on the channel. This paper investigates the performance of optimal energy based hard fusion schemes as employed in secondary users’ selection criteria and fusion under Additive White Gaussian Noise and Rayleigh faded channels. To minimize energy not all SUs participate in detecting the PU on the channel. This is achieved by a two tier optimization paradigm. Firstly, by optimal selection of secondary users (SUs) in the network using Lagrange criterion and secondly by optimizing on the energy based hard fusion techniques achieved by Newton-Raphson optimization criterion. The results indicate that an optimal energy based majority counting fusion rule shows greater detection capability than the AND & OR energy based detection schemes, and reduces overall system energy consumption in CSS networks.


Abstract

Cooperative spectrum sensing (CSS) alleviates the problem of imperfect detection of primary users (PU)’s in cognitive radio (CR) networks by exploiting spatial diversity of the different secondary users (SUs). The efficiency of CSS depends on the accuracy of the SUs in detecting the PU and accurate decision making at the fusion center (FC). This work exploits the higher order statistical (HOS) tests of the PU signal for blind detection by the SUs and combination of their decision statistics to make a global decision at the FC. To minimize energy, a two stage optimization paradigm is carried out, firstly by optimal iterative selection of SUs in the network using Lagrange criterion and secondly optimized fusion techniques achieved by Neyman Pearson. The probability of detecting the PU based on HOS and hard fusion schemes is investigated. The results indicate that the Omnibus HOS test based detection and optimized majority fusion rule greatly increases the probability of detecting the PU and reduces the overall system energy consumption.

17 Conclusion

This work presented two journal papers; The first paper is titled "Optimal Energy Based Cooperative Sensing Schemes in Cognitive Radio Networks", in this paper a two-stage optimization detection scheme was modeled. Performance analysis on energy based
hard fusion techniques were investigated and from the simulated results $k$ out of $n$ counting rule showed better detection performance both in AWGN and Rayleigh channels as compared to AND & OR logic rules. The second paper titled, "Energy Efficient Statistical Cooperative Spectrum Sensing in Cognitive Radio Networks", in this model the performance of HOS tests in PU detection was done. The simulated results showed that optimal $k$ out of $n$ based omnibus ($K^2$) statistics test was superior to the other HOS tests operating under noisy conditions. The overall system energy was tremendously reduced in the network due to the two-stage optimization since fewer cooperative SU made the final decision on the status of the PU on the channel but still maintained reliable decision outcomes. From the two papers it was observed that energy in cooperative spectrum sensing network was reduced by employing an optimal number of SUs from the total number of SUs in the network.

18 Future Work

This work has not compared the energy detection as presented in the paper A with the higher order statistics detection schemes as presented in paper B, this can be done in future work. The complexity of the models was not done in the two papers and this is proposed for future work.

References


References


References


References


References


Part II

Papers
Optimal Energy Based Cooperative Spectrum Sensing Schemes in Cognitive Radio Networks

Kataka Edwin Matsanza and Tom M. Walingo, Member IEEE

This paper is under review

*International Journal of Future Generation, Communication & Networking (IJFCN), 2017*
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The layout has been revised.
1. Introduction

Abstract

Cooperative spectrum sensing (CSS) alleviates the problems of multipath, shadowing and hidden nodes experienced in wireless communication. Both the selection criterion of collaborating secondary users and the fusion schemes used in CSS affect the reliability of detecting the status of primary user (PU) on the channel. This paper investigates the performance of optimal energy based hard fusion schemes as employed in secondary users’ selection criteria and fusion under Additive White Gaussian Noise and Rayleigh faded channels. To minimize energy not all SUs participate in detecting the PU on the channel. This is achieved by a two tier optimization paradigm. Firstly, by optimal selection of secondary users (SUs) in the network using Lagrange criterion and secondly by optimizing on the energy based hard fusion techniques achieved by Newton-Raphson optimization criterion. The results indicate that an optimal energy based majority counting fusion rule shows greater detection capability than the AND & OR energy based detection schemes, and reduces overall system energy consumption in CSS networks.

1 Introduction

Spectrum sensing is the first step towards efficient utilization of the available spectrum resource in cognitive radio network (CRN). The non-cooperative spectrum sensing (NCSS) comprises of energy detection, matched filter and cyclostationary feature detection techniques [1]. In NCSS schemes, only one SU detects and determines the presence or absence of the PU on the channel. Energy detection is the most commonly used spectrum sensing technique as it is easy to implement and does not require a priori knowledge. However, it is well known that the performance of NCSS energy detection is very vulnerable to multipath fading, shadowing, hidden nodes and noise uncertainty due to the fact that the detection decisions are made only by a single SU [2]. This has necessitated a spectrum sensing paradigm shift to cooperative spectrum sensing where multiple SUs share their decisions to make a unified final decision. The concept behind CSS is to improve the sensing performance by making use of the spatial diversity in the observations of spatially distributed SUs in a geographical environment [3]. Centralized cooperative spectrum sensing employs a central identity called fusion center (FC) to collate and control all the decision processes of secondary users (SUs) [4]. Depending on the form in which the SUs transmit the PU’s signal information to the FC, soft and hard combination are utilized. In the soft data combination scheme, each SU transmits the real value of its sensing data to the fusion center. Practically a large number bits
1. Introduction

are required since it measures the instantaneous signal energy over a period of time, resulting into large communication bandwidth. This has necessitated the adoption of hard combination schemes in which only one-bit local decision is forwarded to the FC by individual SUs for decision making. Hard fusion decisions consists of AND, OR and majority rules depending on how the SUs are selected to make the final decisions [5]. In this paper, we model a two stage CSS energy detection scheme based on optimal majority fusion rule in both Gaussian and Rayleigh channels. This has not been adequately addressed in literature. This is realized by a two level optimization, firstly an optimal selection of the SUs that qualify to participate in detection process in a CRN is done. To achieve this, an iterative optimization threshold algorithm is employed on the SUs’ signal to noise ratio (SNR) based on majority rule also called \( r \) out of \( n \) counting rule. This is actualized by Lagrange optimization criterion, where the probability of detection is maximized subject to minimized error probability cost function. It should be noted that those SUs that do not meet this threshold are rejected at this point in time. Secondly those SUs selected during the first level optimization are subjected to the second level optimization process. This is realized by a prudent and optimal choice of majority counting rule. A strategic \( k \) out of \( n \) counting rule is employed to determine the combinatorial order of the different ways the selected SUs’ decisions combine to make the final global decision. Neyman Pearson optimization criterion is employed to actualize this objective. Newton Pearson optimization is numerically determined by an iterative Newton Raphson algorithm search on \( k \) out of \( n \) counting rule. The objective function is to maximize on the probability of detection subject to minimized probability of false alarm. The selection of fewer cooperating SUs at the two tiers in the sensing, fusion and optimization leads to a reduction of about 30 percent energy consumption in CRN. The power demand that maximizes the energy-efficiency of this model is formulated by the optimization on the ratio of network throughput and the energy objective function. The number of cooperating SUs is minimized by \( k \) out of \( n \) fusion counting rule with a constraint on the probability of detection and false alarm while maximizing the throughput of the cognitive radio network. In summary, we propose a two level optimization energy efficient CSS model on a Rayleigh and Gaussian distributed wireless channels.

The rest of this paper is structured as follows. Section II, presents the related work, section III, describes the system models, section IV, describes local spectrum sensing techniques, section V, presents the fusion schemes, section VI, shows the energy efficiency on CSS network. Simulation results illustrating the effectiveness of the proposed
2. Related Work

Spectrum sensing schemes have fairly been studied in literature. In [6], authors proposed an improved model of energy detection scheme used in the spectrum sensing. The improved detection technique employed the classical energy detection algorithm. In [7], authors investigated the performance of a CSS scheme where a group of SUs cooperated to detect the presence or absence of PU in Rayleigh fading channel environment. They made comparative study on the three main hard fusion techniques i.e. OR-logic, AND-logic and Majority-logic to make global decisions at the FC. In [8] the authors formulated Barlett’s estimate used as an energy decision statistic. The authors analyzed the performance for PU’s signal under Rician and Rayleigh fading channels. The reliability of their method was compared with periodogram techniques. The models stated in [6–8] are not optimal, the number of SUs employed to make final decisions on the presence of PU are unlimited and as a consequence a large amount of energy is wasted in the spectrum sensing network. This compromises the system energy consumption hence the efficiency of the models. They also assumed that the SUs have the same signal to noise ratio (SNR). In a practical situation SUs experience different signal strengths (SNRs) depending on their actual positions with respect to the FC. If those SUs with low SNR are allowed to participate in the decision making, then they compromise on the reliability of the final decisions. This challenges have been addressed in our model. In [9] the authors proposed and analyzed the different hard decision fusion rules based on energy detection with an aim to minimize the total error rate in centralized CSS network in both AWGN and Rayleigh fading channels. The authors in [10], proposed on the optimality of k out of n fusion strategy and cooperative-user number. The optimizing on fusion strategy was done under both the Neyman-Pearson (N-P) and Bayesian criteria. The models in [9, 10] considered SUs with same SNR which practically is not correct due to the effect of fading and shadowing experienced in a CSS network. The optimization was derived from fixed decision thresholds which definitely may compromise on the reliability of final decisions made on the presence of the SUs on the channel. In [11], the authors proposed and optimized the detection threshold in order to minimize both the error detection probabilities of single-channel and multichannel cooperative spectrum sensing. In single-channel cooperative spectrum sensing, the iterative optimal thresholds with AND logic, OR logic, and k out of n
2. Related Work

counting rule are respectively proposed. In multichannel cooperative spectrum sensing, the non-restrained multichannel threshold optimization (NRMTO) and the restrained multichannel threshold optimization (RMTO) was proposed. In [12], the authors proposed a dynamic threshold energy detection algorithm, in which, two threshold levels are fixed based upon the average energy received from the primary user (PU) during a specified period of observation. In [13], authors proposed selection technique based on iteratively setting different thresholds for different SNR of SUs in CSS with OR logic fusion technique done at the FC. In the models in [11–13] optimization was done only on the SUs’ selection criteria. However, in our model we proposed a two tier optimization strategy, firstly on the optimal SUs’ selection criteria based on iterative decision thresholding and secondly an optimized fusion technique applied in CSS network. Authors in [14], proposed a strategy to minimize the number of SUs making decisions in the centralized CSS system. They proposed scheme based on maximizing the throughput and minimizing the number of transmitting SUs. In [15], authors improved the energy efficiency in cognitive radio by optimization of the fusion rule (FR) by which the individual results were processed. They optimized the \( k \text{ out of } n \) by maximizing energy efficiency and detection accuracy. In [16], the authors investigated on throughput optimization of the hard fusion based sensing using the \( k \text{ out of } n \) rule. They maximized the throughput of the cognitive radio network subject to a constraint on the probability of detection and energy consumption per cognitive radio in order to derive the optimal number of users. In [17], authors proposed an energy efficient setup. The number of cooperating cognitive radios was minimized in a \( k \text{ out of } n \) fusion rule with constraint on the probability of detection, false alarm and throughput. However, unlike the models in [14–17] that proposed one stage optimization this work proposes a two stage optimization paradigm. This paper proposes a two level optimization, firstly optimal selection of SUs based on SUs’ SNR to participate in the decision making and secondly optimization on hard fusion rules. This positively improved on the energy consumption in the CCSS network and detection reliability.
3. System Model

3.1 Practical Cooperative Sensing Scheme

The system model in Fig. A.1 shows a group of spatially distributed SUs which observe a physical phenomenon on the presence or absence of the PU. The sample observations $(y_1, y_2, ..., y_n)$ of the PU are received by individuals SUs through the licensed sensing channels. The SUs then make local decisions $(u_1, u_2, ..., u_n)$ and retransmits their decisions through the reporting channels to the fusion centre (FC). In this model all SUs are assumed to be synchronized with the FC to detect the channel or the frequency band of interest. The FC finally combines the reported local sensing decisions to make cooperative global decision $(u)$ that is relayed back to the SUs for necessary assignment of the channel to the various SUs depending on available resource allocation schemes which are not part of this work.

![Fig. A.1: The practical cognitive radio network](image)

3.2 Proposed Cooperative Spectrum Scheme

In the proposed lower level model of fig. A.2 secondary users $(SU_1, SU_2, ..., SU_n)$ sense the presence or absence of a single PU on the channel independently. The SUs execute the detection individually based on the measured energy $(ED_1, ED_2, ..., ED_n)$. The sensed instantaneous energy of the PU’s received signal is integrated to determine the detection decision hypothesis test statistics. The local decision data is then transmitted over either a Gaussian distributed or Rayleigh fading channels $(CH_1, CH_2, ..., CH_n)$. The FC is the nerve center of the cognitive radio network where hard fusion decisions $(u_1, u_2, ..., u_n)$ are fused to form a global decision $u$. In this model decisions are taken only from a selected number of SUs for the purpose of achieving energy optimality and
4. Local Spectrum Sensing

4.1 Energy Detection Hypothesis Test

The measured energy decision test statistics of the PU signal \( (Y(y_i)) \) during the sensing observation time as detected by the \( i \)-th SU signal is given in algorithm 1, as in [11, 18].

\[
Y(y_i) = \sum_{m=1}^{M} |y_i(m)|^2 \quad m = 1, ..., M \tag{A.1}
\]

where \( M \) the number of digital samples, \( y_i(m) \) is the received PU signal, \( m \) is binary digit of either 0 or 1 numbering \( M \). The spectrum sensing phenomena can be formulated as a binary hypothesis testing problem with two hypothesis \( H_0 \) and \( H_1 \) given as [4]

\[
H_0 : y(m) = w(m) \quad m = 0, ..., M - 1 \tag{A.2}
\]

\[
H_1 : y(m) = s(m) + w(m) \quad m = 0, ..., M - 1 \tag{A.3}
\]

where \( y(m) \) is the received signal, \( w(m) \) is the noise and \( s(m) \) is the PU signal. In order to derive the detection and false alarm probabilities, a probability density function (PDF) of the test statistic is developed for both \( H_0 \) and \( H_1 \) as

\[
Y(y_i) < \lambda_{d,i} \quad H_0 \tag{A.4}
\]

\[
Y(y_i) > \lambda_{d,i} \quad H_1 \tag{A.5}
\]

where \( Y(y_i) \) is the energy test statistic for the binary hypothesis test and \( \lambda_{d,i} \) is the decision threshold for the \( i \)-th SU.
4. Local Spectrum Sensing

4.2 Additive White Gaussian Noise Statistics (AWGN)

The wireless channel (\(CH_1, CH_2, ..., CH_n\)) in Fig. A.2 can be modeled as AWGN, where white noise is the only impairment to the equality of the transmitted signal with zero mean and unity variance. The test statistics (\(Y(y_i)\)) can be accurately approximated as

Algorithm 1 Energy detection algorithm

Input: \(\gamma = -15 : 2 : 25, M = 10^3, P_{fa} = 0.01 : 0.01 : 1\)

Output: \(P_d = \text{decision}\{H_1, H_0\}\)

\(s(m) \leftarrow \text{generate M random data eqn. (A.3)}\)

\(y(m) \leftarrow \text{modulate the signal (PSK mod) and add noise}\)

Initialize: \(P_{fa} = 0.01, \gamma = -5\)

Simulated probability of detection(\(P_d\))

for \(i = \text{length}(P_{fa}), j = \text{length}(\gamma)\)

while \(i \leftarrow 0, j \leftarrow 0\) do

\(y_i(m) \leftarrow \text{calculate energy statistics eqn.(A.1)}\)

\(\lambda_{d,i} \leftarrow \text{calculate the threshold eqn. (A.9)}\)

FFT on energy statistics

\(\text{FFT}(y_i(m)) \leftarrow \text{FFT}\{y_{i+1},..y_{MFFT-1}\}\)

\(y_{FFT} \leftarrow \text{break FFT}(y_i(m), M_{FFT}, M)\)

\(Y(y_i(m)) \leftarrow \text{concatenation of } y_{FFT}\)

\(Y(y_i(m)) = \text{real parts}(y_{FFT}) + \text{img. parts}(y_{FFT})\)

if average\((Y(y_i(m))) \geq \lambda_{d,i}\) then

\(\text{decision} = H_1\)

increment counter \(\leftarrow H_1 = H_1 + 1\)

else if average\((Y(y_i(m))) \leq \lambda_{d,i}\) \n
\(\text{decision} = H_0\)

increment counter \(\leftarrow i = i + 1, j = j + 1\)

probability of detection\((P_d) = \text{sum}(H_1_{MFFT})\)

Plot \(P_d vs \gamma_i\)

end if

end while

normal distribution by [18]

\[
Y(y_i) \approx \begin{cases} M\sigma_z^2, 2M\sigma_z^4 & \text{ } H_0 \\ M\sigma_z^2 + M\gamma_i\sigma_z^2, 2M\sigma_z^2 + 4M\gamma_i\sigma_z^4 & \text{ } H_1 \end{cases} \quad (A.6)
\]
4. Local Spectrum Sensing

where $\sigma^2$ is the noise variance $\sigma^4$ is the square of noise variance, $\gamma_i$ is SNR of the $i$-th SU, $M$ is the number of digital samples. In testing the $(H_0)$ and $(H_1)$, two types of errors are formulated; the probability of false alarm ($P_{fa}$) and probability of detection ($P_d$). In AWGN distribution, the $P_{fa}$ is statistically formulated as

$$P_{fa} = \text{Prob}(Y(y_i) < \lambda_d) \quad H_0 \quad (A.7)$$

Similarly probability of detection ($P_d$) is given as

$$P_d = \text{Prob}(Y(y_i) > \lambda_d) \quad H_1 \quad (A.8)$$

It should be noted that if $M$ is large, then by using central limit theory, the energy based metric in equation (A.6) can be approximated as Gaussian random process. Based on the test statics $Y(y_i)$ the probability of false alarm ($P_{fa,i}$) for the $i$-th SU can be formulated as [18]

$$P_{fa,i} = Q\left(\frac{\lambda_{d,i} - 2M}{\sqrt{4M}}\right) \quad (A.9)$$

where $M$ is the number of data samples, $\lambda_{d,i}$ is the decision threshold for the $i$-th SU and $Q(\cdot)$ is the the Gaussian Q-function. Similarly the probability of misdetection ($P_{md,i} = 1 - P_{d,i}$) for the $i$-th SU is expressed as [16]

$$P_{md,i} = 1 - Q\left(\frac{\lambda_{d,i} - 2M(1 - \gamma)}{\sqrt{4M(1 + 2\gamma)}}\right) \quad (A.10)$$

where $P_{fa,i}$ and $P_{md,i}$ represent the individual SU probabilities of false alarm and misdetection on the local decisions ($u_1, u_2, ..., u_n$) as shown in Fig. A.2.

4.3 Rayleigh Fading Channel Statistics.

The wireless channel $(CH_1, CH_2, ..., CH_n)$ in fig. A.2 can be modeled as a Rayleigh fading channel. If the signal amplitude follows a Rayleigh distribution, then the SNR will also follow an exponential probability density function (PDF), given by [7]

$$f(\gamma_i) = \frac{1}{\gamma_i} \exp\left(-\frac{\gamma_i}{\bar{\gamma}}\right) \quad \gamma_i \geq 0 \quad (A.11)$$

where $\bar{\gamma}$ is the average SNR and $\gamma_i$ is the instantaneous SNR for the $i$-th SU. In the Rayleigh fading channel the probability of misdetection of the $i$-th SU is formulated in [10].

$$P_{md,i} = 1 - e^{-\frac{\lambda_{d,i}}{2\bar{\gamma}}} \sum_{s=0}^{U-2} \left(\frac{1}{s!}\right)\left(\frac{\lambda_{d,i}}{2}\right)^s - \left(\frac{1 - \bar{\gamma}}{\bar{\gamma}}\right)^{U-1} \ast \left[e^{-\frac{\lambda_{d,i}}{2\bar{\gamma}}} + e^{-\frac{\lambda_{d,i}}{2\bar{\gamma}}} \sum_{s=0}^{U-2} \left(\frac{1}{s!}\right)\left(\frac{\lambda_{d,i}}{2(1 + \bar{\gamma})}\right)^s\right] \quad (A.12)$$
where \( \lambda_{d,i} \) is the decision threshold for the \( i \)-th SU, \( U = WT \) is the product of spectrum sensing time (\( T \)) and channel bandwidth (\( W \)) over Raleigh fading channel. Under the Rayleigh fading channel, the probability of false alarm for \( i \)-th SU \( P_{fa,i} \) is given by [18]

\[
P_{fa,i} = \frac{\Gamma(U, \frac{\lambda_{d,i}}{2})}{\Gamma(U)}
\]  

(A.13)

where \( \lambda_{d,i} \) is the decision threshold, \( U \) is the time bandwidth product, \( \Gamma(\cdot, \cdot) \) is the incomplete gamma function and \( \Gamma(\cdot) \) is the gamma function. The probabilities given in equations (A.12) and (A.13) are local probabilities misdetection and false alarm of \((u_1, u_2, ..., u_n)\) decisions made by SUs shown in fig. A.2.

5 Fusion Schemes

5.1 First Stage Optimization on SU’s Selection Criteria

The aim is to iteratively select the number of SUs subject to minimizing the local error detection \( P_e \). The global probability misdetection \( Q_{md} \) and false alarm \( Q_{fa} \) are formulated as a result of the individual local probabilities \( P_{md,i} \) and \( P_{fa,i} \) for the \( i \)-th SU respectively. The decisions from \( n \) number of SUs are selected from a larger sample of \( N \) SUs in a centralized CSS network. The criteria on selection is based on SUs’ decrementing SNR as formulated in algorithm 2. The error detection \( P_{e,i} \) for the \( i \)-th SU is expressed as

\[
P_{e,i} = P(H_0)P_{fa,i} + P(H_1)P_{md,i}
\]  

(A.14)

where \( P(H_0) \) is the null hypothesis, and \( P(H_1) \) the alternative hypothesis. The sum of global probability of false alarm \( (Q_{fa}) \) and misdetection \( (Q_{md}) \) are formulated as cost functions subject to the global decremental error probability \( (Q_e) \). The minimization problem is formulated based on the work done in [11, 15, 17]

Min \( \lambda \left( Q_{md}(\lambda_{d,i}^*) \text{ and } Q_{fa}(\lambda_{d,i}^*) \right) \)  

Subject to \( Q_e > 0 \)  

(A.15)

where \( \lambda_{d,i}^* \) is the optimal decision threshold on the \( i \)-th SU in the network. Considering equations (A.9), (A.10),(A.12) and (A.13) in AWGN and Rayleigh channels respectively, the optimal global decision threshold \( (\lambda_{d,i}^*) \) is formulated as

\[
\lambda_{d,i}^* = \arg \min_{\lambda_d} \left( P_{e,i} = (\beta P_{fa,i} + P_{md,i})P(H_1) \right)
\]  

(A.16)
Algorithm 2 First Stage Optimal Selection of SUs

Input: $N = 18$, $SNR = -15 : 2 : 5$

Output: $\lambda^*_d, n, Q_e(r,n)$

initialize: $n = 1, r = \frac{N}{2}$

$N \leftarrow$ sort all SUs in descending SNR

calculate the following parameters

step 1: $P_{fa,1}$ and $P_{d,1} \leftarrow 1^{st}$ iter. eqn. (A.9), (A.10), (A.12)&(A.13)

step 2: $\lambda^*_d \leftarrow$ the threshold of $i$th SU, eqn. (A.19), (A.27)

step 3: $Q_{fa}^{c,n} \leftarrow$ cal. false alarm, eqn. (A.22), (A.26), (A.29)

step 4: $Q_{gd}^{c,n} \leftarrow$ cal. detection prob, eqn. (A.20), (A.25), (A.28)

step 5: $Q_e^{c,n} \leftarrow$ the decremental error, eqn. (A.24)

for $i = \text{length} (n)$ and $r = \text{length} (\frac{N}{2})$

while $n \leq N, n \leftarrow 0$ do

if $Q_e^{c,n} \geq 0$ then

$i = n + 1$

increment counter $\leftarrow n = n + 1$

$\lambda^*_n \leftarrow$ cal. the optimal threshold

go to step 3

else ($Q_e^{c,n} \leq 0$)

$n = n - 1 \leftarrow$ delete the SU from the list

go to step 1

else $\leftarrow$ soln. found

$n = n + 1$

end if

end while
where \( \beta = \frac{P(H_0)}{P(H_1)} \) is the detection factor. Consequently from equation (A.16), the threshold is maximized as follows

\[
\lambda_{d,i}^* = \arg \max_{\lambda_{d,i}} (P_{d,i} - \beta P_{fa,i} - 1)P(H_1)
\]

\[
= \arg \max_{\lambda_{d,i}} (P_{d,i} - \beta P_{fa,i})
\]  

(A.17)

where \( P_{d,i} = 1 - P_{md,i} \) is the local probability of detection for the \( i \)-th SU. By the Lagrange theorem the threshold is maximized by differentiating in parts as follows [11]

\[
\frac{\partial P_{d,i}}{\partial \lambda_{d,i}} \bigg|_{\lambda_{d,i}^*} = \beta \frac{\partial P_{fa,i}}{\partial \lambda_{d,i}} \bigg|_{\lambda_{d,i}^*} \quad i = 1, ..., n,
\]

(A.18)

where \( n \) is the number of SUs selected to participate in fusion and \( \lambda_{d,i}^* \) is derived as

\[
\lambda_{d,i}^* = \frac{\sigma_s^2}{\beta} + \sigma_s^2 \sqrt{\frac{1}{4} + \frac{\gamma_i}{2}} + \frac{4\gamma_i + 2 \log (\beta \sqrt{2\gamma_i} + 1 * \psi)}{U \gamma_i}
\]

(A.19)

where \( \sigma_s^2 \) is the noise variance, \( r \in [1, n] \), \( \psi = \frac{Q_{fa}^{(r-1,n-1)} - Q_{fa}^{(r,n-1)} - Q_{d}^{(r-1,n-1)} - Q_{d}^{(r,n-1)}}{Q_{d}^{(r-1,n-1)} - Q_{d}^{(r,n-1)}} \) is the decremental detection factor, \( \gamma_i \) is the SNR of the \( i \)-th SU and \( U = 2TW \) is time bandwidth product.

### 5.1.1 Majority Counting Rule

The optimal SU selection in CSS network can be iteratively found by utilizing the \( r \) out of \( n \) counting rule in algorithm 2. The global probability of detection \((Q_{gd})\) can be formulated as [19]

\[
Q_{gd}^{(r,n)} = \sum_{B=0}^{2^n-1} \prod_{r=1}^{n} (P_{d,r})^{B_{(n,2)}} (1 - P_{d,r})^{1 - B_{(n,2)}}
\]

(A.20)

where \( P_{d,r} \) is the probability of detection for the \( r \)-th SU in the \( r \) out of \( n \) counting rule, where \( r \in [i,n] \), \( B_{n,2} \) is the \( n \)-th bit binary vector representing the binary transform of an integer number \( B \in \{0, 1, ..., 2^n - 1\} \) SUs to be selected and \( B_{n,2}' \) is the \( r \)-th bit of the \( B_{n,2} \). However, \( n \in [1, N] \), where \( N \) is the total number of SUs in the CSS network. The value of \( Q_{gd}^{(r,n)} \) in equation (A.20) can be iteratively derived as follows

\[
Q_{gd}^{(r,n)} = Q_{gd}^{(r-1,n-1)} (P_{d,r}) + Q_{gd}^{(r,n-1)} (1 - P_{d,r})
\]

(A.21)

Similarly the global probability of false alarm \((Q_{fa})\) is given as

\[
Q_{fa}^{(r,n)} = \sum_{B=0}^{2^n-1} \prod_{r=1}^{n} (P_{fa,r})^{B_{(n,2)}} (1 - P_{fa,r})^{1 - B_{(n,2)}}
\]

(A.22)

where \( P_{fa,r} \) is the local probability of false alarm for the \( r \)-th SU. Similarly \( Q_{fa}^{(r,n)} \) can be iteratively derived from the equation (A.22) as

\[
Q_{fa}^{(r,n)} = Q_{fa}^{(r-1,n-1)} (P_{fa,r}) + Q_{fa}^{(r,n-1)} (1 - P_{fa,r})
\]

(A.23)
The global decremented error probability is expressed as

\[
Q_c^{(r,n)} = P(H_1)P_{d,r} \left( Q_{gd}^{(r-1,n-1)} - Q_{gd}^{(r,n-1)} \right) - P(H_0)P_{fa,r} \left( Q_{fa}^{(r-1,n-1)} - Q_{fa}^{(r,n-1)} \right),
\]  

(A.24)

where the \( P(H_0) \) and \( P(H_1) \) are probabilities of false alarm and probability of detection respectively, \( n \) is the number of SUs selected to participate from a total of \( N \) SUs in the CSS network.

### 5.1.2 Logic AND Rule

The AND logic is hard fusion scheme which is employed at the FC to make global decisions on the status of the PU on the channel in algorithm 2. Here the decision is given as a binary 1 only if all the SUs detect the presence of PU. Otherwise the decision is binary 0 representing the absence of the PU. The global probability of detection \( (Q_{gd}) \) determined at the FC is expressed as [11]

\[
Q_{gd}^{(n)} = \prod_{i=1}^{n} P_{d,i}, \quad n \in \{ i = 1, 2, .., N \}
\]

(A.25)

where \( Q_{gd}^{(n)} \) is iteratively derived as follows \( Q_{gd}^{(n)} = Q_{gd}^{(n-1)} P_{d,n} \) but \( P_{d,n} \) is the probability of detection for the \( n \)-th SU and \( N \) is the total number of SUs in the CSS network. Similarly the global probability of false alarm \( Q_{fa}^{(n)} \) is expressed as

\[
Q_{fa}^{(n)} = \prod_{i=1}^{n} P_{fa,i}, \quad n \in \{ i = 1, 2, .., N \}
\]

(A.26)

where \( Q_{fa}^{(n)} \) is iteratively expressed as \( Q_{fa}^{(n)} = Q_{fa}^{(n-1)} P_{fa,n} \) but \( P_{fa,n} \) is the probability of false alarm for the \( n \)-th SU. The optimal decision threshold \( \lambda_{d,i}^{*} \) is given as [11]

\[
\lambda_{d,i}^{*} = \frac{\sigma^2}{2} + \sigma^2 \sqrt{\frac{1}{4} + \frac{\gamma_i}{2} + \frac{4\gamma_i + 2}{U \gamma_i} \log \left( \beta \sqrt{2\gamma_i + 1} \ast \Psi \right)}
\]

(A.27)

where \( \Psi = \frac{Q_{fa}^{(n-1)}}{Q_{gd}^{(n)}} \) is the preceding detection factor and \( U \) is the time bandwidth product.

### 5.1.3 Logic OR Rule

The OR rule is a hard fusion technique utilized to determine the global decision at the FC. In this scheme the global decision is given as binary 1 when at least one of the SUs detect the presence of the PU on the channel. The global probability detection \( (Q_{gd}) \) in a CSS network with \( N \) SUs is formulated by [11]

\[
Q_{gd}^{(n)} = 1 - \prod_{i=1}^{n} P_{d,i}, \quad n \in \{ i = 1, 2, .., N \}
\]

(A.28)
where \( n \) is the SUs selected from a total of \( N \) SUs in CSS network, \( Q_{gd}^{(n)} \) is iteratively derived as 
\[
Q_{gd}^{(n)} = Q_{gd}^{(n-1)} + \left( 1 - Q_{gd}^{(n-1)} \right) P_{d,n},
\]
where \( P_{d,n} \) is the local probability of detection for the \( n \)-th SU. Subsequently the global probability of false alarm \( (Q_{fa}) \) is given as
\[
Q_{fa}^{(n)} = 1 - \prod_{i=1}^{n} P_{fa,i}, \quad n \in \{1, 2, ..., N\}
\]
where \( Q_{fa}^{(n)} = Q_{fa}^{(n-1)} + \left( 1 - Q_{fa}^{(n-1)} \right) P_{fa,n} \). The optimal threshold \( (\lambda_d^*) \) for OR rule is same as that in AND rule formulated in eqn. (A.27) but \( \Psi = \frac{1 - Q_{fa}^{(n-1)}}{1 - Q_{gd}^{(n)}} \) is the preceding detection factor.

5.2 Second Stage Optimal Strategy

At the FC a numerical iterative as in algorithm 3 is employed to find the optimal number of SUs in a \( k \) strategy. It should be noted that \( k \) SUs are selected from a subset \( k \in [1, n] \) of a larger set of \( n \in [1, N] \) SUs, where the larger set compromises of \( n \) selected SUs from first optimization stage and \( N \) is the total number of SUs in CSS network before selection. The objective is to determine an optimal combinatorial strategy of \( k \) out of \( n \) counting rule subject to minimal probability of false alarm. Optimization is achieved by Neyman-Person (N-P) criterion. To achieve this an upper-threshold of global probability false alarm \( (Q_f) \) of less than utilization level \( (\epsilon) \) is formulated based on the work done in [7, 14, 15]

\[
\begin{align*}
\text{Maximize} & \quad (Q_d) \\
\text{Subject to} & \quad Q_f < \epsilon
\end{align*}
\]

The global probability of false alarm is formulated as
\[
Q_f = \sum_{k=j}^{n} \binom{n}{k} \left( P_{fa,j}^k \right) (1 - P_{fa,i})^{n-k}
\]
where \( k = 1, ..., n \), and \( P_{fa,i} \) is the probability false alarm of the \( i \)-th SU. Similarly the global probability of detection \( (Q_d) \) is given as
\[
Q_d = \sum_{j=k}^{n} \binom{n}{k} \left( P_{d,j}^k \right) (1 - P_{d,i})^{n-k}
\]
The roots of \( P_{fa,i} \) is found by optimizing \( Q_f \), this is achieved by differentiating eqn. (A.31) as follows
\[
\frac{Q_f(P_{fa,i})}{d(P_{fa,i})} = n \binom{n-1}{k} \left( P_{fa,i}^k \right) (1 - P_{fa,i})^{n-k-1} > 0
\]
From equations (A.9) and (A.10) the following eqn. (A.34) must hold true

\[ \frac{P_{d,i}}{P_{fa,i}} > \frac{d(P_{d,i})}{d(P_{fa,i})} > \frac{1-P_{d,i}}{1-P_{fa,i}} \]  

(A.34)

The goal is to find the optimal \( k \) out of \( n \) defined by differentiating \( Q_d \) with respect to \( Q_f \), formulated as

\[ \frac{Q_d}{Q_f} = \frac{\frac{d(Q_d)}{d(P_{fa,i})}}{\frac{d(Q_f)}{d(P_{fa,i})}} = \frac{P_{d,i}(1-P_{d,i})^{n-k-1}}{P_{fa,i}(1-P_{fa,i})^{n-k-1}} \left( \frac{d(P_{d,i})}{d(P_{fa,i})} \right) > 0 \]  

(A.35)

Similarly the equations (A.31),(A.32),(A.33), (A.34) and (A.35) can be formulated as an integrated optimal solution to \( k \) out of \( n \) counting rule as follows [15].

\[ \frac{Q_d(k)}{Q_f(k)} = \frac{\partial Q_d(k)}{\partial P_{fa,i}(k)} \frac{\partial P_{fa,i}}{\partial Q_f} = \frac{\partial Q_d(k)}{\partial P_{fa,i}(k)} \frac{\partial P_{fa,i}}{\partial P_{fa,i}} \]  

(A.36)

From the above equation it is true to say \( Q_d(k) \) is linearly increasing function of \( Q_f(k) \). The procedure is to determine the values for all \( k \in [1, n] \) are the roots of \( Q_f(k, P_{fa,i}) \). However, the closed form solution of the eqn. (A.36) can be complicated hence the need for a numerical search to achieve the solution. An explicit optimal solution can be iteratively obtained by utilizing the Newton-Raphson (NR) criterion as expressed in algorithm 3 in reference [14]. The algorithm is broken down as follows; For each \( P_{fa,i} \) determine the corresponding \( P_{d,i} \) and \( Q_d(k, P_{fa,i}) \). Compare the listed values of global \( Q_d(k, P_{fa,i}) \) for all the numbers of \( k \) SUs and select the highest among the list, this gives the optimal number of \( k \) for the optimal \( k \) out of \( n \) rule.

6 Energy Efficiency

In order to achieve a good tradeoff between these contrasting objectives of throughput and energy consumption, it is more convenient to optimize the parameters of the \( k \) out of \( n \) for the maximum energy efficiency (\( \eta \)).

6.1 Energy Optimization Setup

The global probability of false alarm (\( Q_f \)) determines the throughput which shows the chances of fully utilizing the spectrum in the cognitive radio network. The optimization problem can be formulated by minimizing the global probability of false alarm (\( Q_f \)), subject to set global of detection (\( Q_d \)) threshold as follows [9, 15, 20]

\[ \min(k, Q_f, \lambda^{*}_{d,i}) \]  

Subject to \( Q_d \geq \alpha \quad 1 \leq k \leq n \)  

(A.37)
Algorithm 3 Second stage optimization by NR criterion

Input: \( k, n = 14, P_{fa,i} = 0.01 : 0.01 : 1 \)

Output: \( Q_{d(k,n), k^{opt}} \)

Initialize: \( \epsilon = 0.01, k = 1, P_{fa,i} = 0.01, i = 0, j = 0 \)

Function: \( f(P_{fa,i}) = Q_{f(i,j)}(P_{fa,i}) \leq \epsilon \rightarrow \text{eqn. (A.31)} \)

for \( k_j \leftarrow \{ j = \text{length}(n) \} \), \( P_{fa,i} \leftarrow \{ i = \text{length}(P_{fa}) \} \)

cal. initial probability \( (Q_{f(P_{fa,1})}) \)

while \( |(f(P_{fa,i}))| > \epsilon \) do

\( P_{fa,i} \leftarrow P_{fa,i+1} - \frac{1}{f'(P_{fa,i})}f'(P_{fa,i+1}) \)

increment counter \( \leftarrow i = i + 1, j = j + 1 \)

return \( \leftarrow \text{roots of}(P_{fa,i}) \) for \( \in [k, n] \)

For all \( (P_{fa,i}) \) calc. \( Q_{d(k,n), (P_{fa,i})} \leftarrow \text{eqn. (A.10), (A.32)} \)

choose \( Q_{d,k} \leftarrow \text{Max. Then} \)

\( k = k^{opt} \leftarrow \text{optimal number of k SUs} \)

end while

where \( \alpha \) is the target performance, \( k \) is the participating SUs from \( n \) total number of SU in \( k \) out of \( n \) counting rule. The average throughput of the cognitive radio network (CRN) in [21] is given as

\[
\phi(\lambda_i, k, n, \tau) = Pr(H_0)(1-Q_f)(\Gamma - \tau)C
\]  \hspace{1cm} (A.38)

where \( \lambda_i \) is the decision threshold for \( i\text{-th} \) SU, \( Pr(H_0) \) denotes the probabilities of the PU not transmitting, \( C \) is the rate when a SU occupies spectrum to transmit data with no interference from PU given as \( C = \log_2(1 - \text{SNRs}) \), \( \Gamma = T - nr \) indicate the maximum value of the sensing time \( (\tau) \), \( r \) is the time taken by each SU to send its sensing results to the FC and \( T \) is the length of sensing frame. The average energy consumed in CRN can be expressed as

\[
Y(\lambda_i, k, n, \tau) = x + y(1-Q_d) + z(1-Q_f)
\]  \hspace{1cm} (A.39)

where \( x = n(E_s \tau + E_i r) \), \( y = Pr(H_1)E_i(\Gamma - \tau) \), \( z = Pr(H_0)E_i(\Gamma - \tau) \), \( E_s \) is the power consumed by each SU in the process of spectrum sensing, \( E_i \) is the power consumed by each SU to send its sensing results to the FC. The energy efficiency can be given as [20]

\[
\eta(\lambda_i, k, n, \tau) = \frac{\phi(\lambda_i, k, n, \tau)}{Y(\lambda_i, k, n, \tau)}
\]  \hspace{1cm} (A.40)
7. Simulation Results

The optimal efficiency can be numerically formulated under the constrains on probability of detection and false alarm expressed as [11, 17, 20]

$$\max_{\tau} \eta(\tau) = \left( \frac{\omega(\tau)(1 - Q_f(\tau))}{x(\tau) + (1 - Q_d(\tau)) y(\tau) + (1 - Q_d(\tau)) z(\tau)} \right)$$ (A.41)

Subject to $0 \leq \tau \leq \Gamma$

where $\omega(\tau) = P_r(H_0)(\Gamma - \tau)C$, $x(\tau) = N(E_s \tau + E_t \tau)$, $y(\tau) = P_r(H_1)E_t(\Gamma - \tau)$ and $z(\tau) = P_r(H_0)E_t(\Gamma - \tau)$.

7 Simulation Results

In order to evaluate the performance of energy detection based on hard fusion techniques in CSS network, this paper considered a cognitive radio network with 18 SUs transmitting on PSK modulated signal built in matlab software for analysis. It should be noted that any other modulation scheme can be used to model the SU signal. In all subsequent figures, the numerical results are plotted on receiver operating characteristics curves (ROC). Simulation results are denoted with discrete marks on the curves, simulation parameters are give in table A.1.

<table>
<thead>
<tr>
<th>Simulation Parameters</th>
<th>Actual Values Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(H_0)$ and $P(H_1)$</td>
<td>0.5</td>
</tr>
<tr>
<td>Frequency range</td>
<td>0-2000</td>
</tr>
<tr>
<td>Monte Carlo trials</td>
<td>$10^3$ to $10^4$</td>
</tr>
<tr>
<td>Noise variance ($\sigma^2$)</td>
<td>1</td>
</tr>
<tr>
<td>FFT</td>
<td>2048</td>
</tr>
<tr>
<td>Average SNR ($\bar{\gamma}$)</td>
<td>-5</td>
</tr>
<tr>
<td>Mean</td>
<td>0</td>
</tr>
<tr>
<td>Time bandwidth product</td>
<td>10</td>
</tr>
<tr>
<td>$E_s, E_t$</td>
<td>1 Joule, 500 mJoule</td>
</tr>
<tr>
<td>$T, r, \tau$</td>
<td>200ms, 100µs, 10ms</td>
</tr>
<tr>
<td>Data rate (R)</td>
<td>500 kbps</td>
</tr>
</tbody>
</table>

In Fig. A.3, the ROC curves show probability of detection $P_d$ against the SNR, energy detection statistics in both AWGN and Rayleigh channels as shown in algorithm 1. From
the plot, as expected the probability of detecting the PU increases with increase in SNR. The energy detection statistics test in AWGN channel showed higher probability of detection as compared to Rayleigh channel progressively from a low SNR of about -15dB to -8dB then rapidly thereafter with maximum detection probability attained at -5dB. In summary the deduced performance of energy based detection test under AWGN was better than in Rayleigh channel for all ranges of SNR. The results of the test conform to those in [7, 11, 20, 21].

![Energy detection in AWGN and Rayleigh channel](image)

Fig. A.3: Energy detection test in AWGN and Rayleigh channels

Fig. A.4, shows a graph of global probability of detection $Q_d$ against probability of false alarm $Q_f$ in a two tier optimization hard fusion schemes over AWGN channel as shown in algorithm 3. It should be noted that optimal combination of $k$ out of $n$ is $k = 10$ and $n = 14$, as determined by the algorithm 2. It can be deduced from the plot that optimal combination of (10 out of 14) counting rule showed a probability of detection of about 0.95 at a defined probability of false alarm of 0.10 which is within the IEEE 802.22 regulation standards [5]. From the plot, the optimal (10 out of 14) counting rule combination strategy displayed higher probability of detection as compared to AND fusion rule. The performance was followed by the AND rule which showed about 0.7 probability of detection at 0.1 and lastly OR fusion rule which displayed 0.5 detection probability. Theoretically OR rule should have higher detection but this is under a fixed probability of false alarm. In summary $k$ out of $n$ counting rule displayed the highest detection probability for all ranges of false alarm.
7. Simulation Results

Fig. A.4: The performance of hard fusion rules in AWGN channel

Fig. A.5, shows a plot of global probability of misdetection $Q_m$ against global false alarm $Q_f$ in a two tier optimization energy detection tests over AWGN channel. The plot shows effect of interference of the SUs on the licensed PU also referred to as misdetection analyzed as a function of probability of false alarm. From this plot, it can be deduced that the optimal 10 out of 14 based on a two stage optimization global detection scheme displayed lowest probability of misdetection to that of the optimal AND rule. However, AND rule deduced a lower misdetection to that of the OR rule respectively. From the plots in fig. A.4 and A.5 it can be shown that the optimal $k$ out of $n$ counting is a superior fusion technique in terms of providing higher detection of the PU with the lowest misdetection in AWGN channel hence would therefore be preferred to AND & OR fusion rules.

In fig. A.6, the ROC shows global probability detection $Q_d$ against global false alarm $Q_f$ as determined in second stage optimization strategy and shown in algorithm 3 over Rayleigh channel. It should be noted that optimal combination of $k$ out of $n$ is given as $k = 10$ and $n = 14$. From the plot it can be inferred that the optimal 10 out of 14 counting rule showed the highest probability of detection. At 0.1 probability of false alarm the optimal 10 out of 14 counting rule strategy presented about 0.6 probability of detection. This was followed by optimal AND fusion rule with probability of detection of 0.5 and lastly the OR fusion rule with 0.3. From this plot, it can be concluded that $k$ out of $n$ is the most reliable hard fusion technique with the highest probability of de-
7. Simulation Results

Fig. A.5: The performance of hard fusion schemes in AWGN channel.

Fig. A.6: The performance of optimal hard fusion techniques in Rayleigh channel.
7. Simulation Results

detection all ranges of probability of false alarm.

![Performance of hard fusion schemes in Rayleigh channel](image)

**Fig. A.7:** The comparative performance of hard fusion schemes in Rayleigh channel

In fig. A.7, the ROC curves shows global probability of misdetection $Q_m$ against global false alarm $Q_f$ in two tier optimization hard fusion schemes over Rayleigh fading channel. The plot display the levels of interference of the SUs to the PU in the utilization of the channel. From this plot it can be inferred that 10 out of 14 counting rule has the lowest degree of misdetection. This is followed by AND and lastly the OR fusion rule. From the fig. A.6 and fig. A.7 it can be concluded that $k$ out of $n$ has the highest probability of detecting the presence of the PU on the channel and with the lowest misdetection as compared to AND & OR fusion rules. It must be noted here that $k$ out of $n$ performed better in AWGN channel as compared to the Rayleigh channel.

![Graph of global probability detection against global false alarm](image)

**Fig. A.8:** shows a graph of global probability detection $Q_d$ against global false alarm $Q_f$, two tier optimization $k$ out of $n$ counting rule as compared to single stage energy detection test over Rayleigh fading channel. From this plot, it can be deduced that 10 out of 14 counting rule was better than single stage optimization. At probability of 0.1 the two stage showed probability of detection of 0.85 against 0.5 for single stage. From this plot it can be deduced that a two stage $k$ out of $n$ counting rule showed the highest probability of detection.

**Fig. A.9:** shows a plot of energy efficiency against the number of SUs for different optimal hard fusion rules under three scenarios; optimal majority counting rule and total number of 14 SUs utilized in OR & AND fusion rules. From the plot it can be observed
7. Simulation Results

Fig. A.8: The optimal counting rule based on two stage against single stage in Rayleigh channel.

Fig. A.9: Comparison on energy efficiency in hard fusion schemes
that the optimal efficiency is given as $2 \times 10^4$ Bits per Joule, that is when 8 SUs employed. However from this plot it can be shown that for the optimal $10 \text{ out of } 14$ counting rule, the energy efficiency reduced to about $1.8 \times 10^4$ Bits per Joule. This is when 10 SUs are employed in the final decision as arrived at in the first optimization stage. It is still better than AND & OR fusion techniques in terms of the throughput. This was followed by AND rule which delivered about $1.4 \times 10^4$ Bits per Joule when 10 SUs are used and lastly OR fusion rule with about $1.5 \times 10^4$ Bits per Joule for 10 SUs employed. It should also be noted that OR fusion technique shows improved efficiency as the number of SUs increase as observed with more than 12 SUs. It outperforms the majority counting rule & AND fusion rule when a larger number of SUs are employed. In conclusion an optimal $k \text{ out of } n$ counting hard fusion rule displayed the most efficient energy technique which delivered the highest throughputs with minimum number of cooperating SUs in the CSS network.

8 Conclusion

In the proposed energy detection model, an optimal $k \text{ out of } n$ counting rule showed to be better than other hard fusion rules in detection reliability both in AWGN and Rayleigh channels. Another advantage of this model was on the overall reduction in energy consumption in the network due to the two tier optimization strategy. Fewer SUs were employed to determine the final global decision on the presence or absence of the PU on the channel but still maintained high throughput and energy efficiency.
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Paper B

ENERGY EFFICIENT STATISTICAL COOPERATIVE SPECTRUM SENSING IN COGNITIVE RADIO NETWORKS

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The paper has been Accepted for Publication in SAIEE Journal
South African Institute of Electrical Engineers (SAIEE), 2017.
1. Introduction

Abstract

Cooperative spectrum sensing (CSS) alleviates the problem of imperfect detection of primary users (PUs) in cognitive radio (CR) networks by exploiting spatial diversity of the different secondary users (SUs). The efficiency of CSS depends on the accuracy of the SUs in detecting the PU and accurate decision making at the fusion center (FC). This work exploits the higher order statistical (HOS) tests of the PU signal for blind detection by the SUs and combination of their decision statistics to make a global decision at the FC. To minimize energy, a two stage optimization paradigm is carried out, firstly by optimal iterative selection of SUs in the network using Lagrange criterion and secondly optimized fusion techniques achieved by Neyman Pearson. The probability of detecting the PU based on HOS and hard fusion schemes is investigated. The results indicate that the Omnibus HOS test based detection and optimized majority fusion rule greatly increases the probability of detecting the PU and reduces the overall system energy consumption.

1 Introduction

Cooperative spectrum sensing (CSS) utilizes multiple secondary users (SUs) to sense the vacant spectrum and send their decision to the fusion centre (FC) for a final global decision to be made regarding the presence of the primary user (PU) on the channel. CSS overcomes the challenges of wireless channel characteristics such as multipath fading, shadowing or hidden terminal problem experienced when only one SU is employed to detect the PU. This is due to the spatial diversity of the different SUs cooperating to make the final decision on the status of the PU on the channel [1, 2]. A number of spectrum detection schemes have been proposed to detect the presence or absence of PU, among them include energy, matched filter and cyclostationary methods [3]. In most practical systems the transmission channels are usually noisy hence causing tremendous reduction in signal to noise ratio (SNR) of the PU received signals. This has prompted the need for the higher order statistical (HOS) detection techniques which have very high sensitivity at low SNR signal condition while maintaining reasonable circuit complexity [4]. CSS can generally be divided into two detection stages; local update stage and global fusion stage. At the local update stage, the individual SU’s detect the received PU’s signals based on HOS. The SU then computes a local decision and sends it to the FC for fusion. The commonly used metrics that utilize the HOS properties to detect the PU’s received signals include Jarque-Bera, kurtosis, skewness and omnibus
1. Introduction

tests. These statistical tests are utilized to determine the probability distribution function (PDF) of a group of data samples. This is crucial for benchmarking the distribution in order to make an informed inference on a physical phenomena (existence of PU on the channel) [5]. In this paper, the performance analysis of the HOS tests on the PU signal is investigated with aim of selecting the best statistical technique in determining the status of the PU on the channel. This has not been adequately addressed in literature.
The global fusion stage is performed at the fusion centre where either soft or hard combination schemes are employed to fuse the received signals from individual SUs [6]. Furthermore to reduce energy consumption in the cooperative network not all the SU need to report their individual decisions. To optimize on the number of SUs selected to participate in the fusion process, this paper proposes a two stage optimization strategy. The first stage is to select the SUs which qualify to transmit their individual decision data to the fusion centre. To achieve this an iterative optimization threshold algorithm is employed and determined based on the SUs’ SNR. However, this is at the cost of minimizing on the error probability formulated by the Lagrange optimization criterion. The rest of SUs that do not meet this threshold are rejected at this sensing point in time (they are not allowed to transmit). Those SUs selected during the first optimization stage are subjected to the second stage optimization process, realized by a prudent and optimal choice of hard fusion criteria taken to fuse the SUs’ binary decisions. A strategic $k$ out of $n$ counting rule is adopted to determine the optimal combinatorial order of the SUs to be considered for final global fusion. To realize this, Neyman- Pearson optimization criterion is employed through an iterative Bisection numerical search algorithm formulated on $k$ out of $n$ rule. The cost function is to maximize the probability of detection subject to minimizing of the probability of false alarm. In summary, a hybrid detection strategy of HOS local detection test and optimal global fusion technique was implemented. The simulated results show an optimal $k$ out of $n$ fusion rule based on omnibus test perform better than other HOS tests in terms of detection probability. In this model, not all SUs participate in detection at any one sensing time frame hence great energy cost saving in the cooperative spectrum sensing network.
The rest of the paper is organized as follows. Section II presents the related work, section III describes the system model, section IV is devoted on local spectrum sensing, section V focuses on the fusion techniques, section VI presents the energy efficiency. Simulation results illustrating the effectiveness of the scheme are given in section VII and finally, section VIII, draws the conclusions.
2. Related Work

Cooperative spectrum sensing schemes have not been exhaustively studied in the current literatures. In [7], authors investigated the performance of energy based CSS scheme where a group of SUs cooperated to detect the presence or absence of primary user (PU) in fading channel environment. They also made comparative study on the three main hard fusion techniques i.e. OR-logic, AND-logic and Majority-logic to make global decisions at the fusion centre. In [8], authors proposed selection technique based on iteratively setting different thresholds for different signal to noise ratio (SNR) of SUs in cooperative spectrum sensing with OR logic fusion technique done at the fusion centre. This scheme highly outperformed the traditional energy spectrum sensing with the same threshold in terms of reduced probability of false alarm. Higher order test (HOS) have been utilized in literature to analyze data distribution and its degree of departure from the normal distribution. The concept of separation is based on the maximization of the non-Gaussian property of separated signals to improve the robustness against noise uncertainty. The authors in [9], proposed kurtosis and skewness (goodness-of-fit) test to check the non-Gaussianity of an averaged periodogram of received SUs signal. This is computed from the Fast Fourier transform (FFT) of the PU signal to justify its existence and hence the availability or not of the spectrum for a cognitive radio transmission. Their findings showed improved detection of the PU signals especially under very low SNR conditions i.e the SUs are able to detect the primary channel with certainty even under very noisy environment. In [10], authors proposed Jarque-Bera tests based spectrum sensing algorithm and compared it to a kurtosis & skewness combination test statistics. From their simulated results they concluded that Jarque-Bera showed better detection performance than the kurtosis & skewness in terms of the reliability i.e. improved probability of detection for different values of SUs' SNR. In the emerging research on spectrum sensing schemes, researchers considered a number of modulation schemes on multipath fading channel based on Jarque-Bera test in detection of the primary user. These schemes were considered to transcend the absence of a priori information of the spectrum occupancy under additive white Gaussian noise channel [4]. In [11], authors showed Jarque-Bera as having rather poor small data sample properties, slow convergence of the test statistic to its limiting distribution. In their findings the power of the statistical tests showed the same eccentric form, the reason being skewness and kurtosis are not independently distributed, and the sample kurtosis especially at-
2. Related Work

tains normality very gradually. However, the JB test is simple to calculate and its power has proved to match other powerful statistical tests. A genuine omnibus test is consistent to any departure from the null hypothesis. In [12], authors formulated omnibus test which is based on the standardized third and fourth moments. This was done to assess the normality of random variables by calculating the transformed samples of kurtosis & skewness. In the computational economics these authors showed omnibus’s simplicity provided by the chi-squared framework. In this work the omnibus test is applied in CSS and compared to other well known Jarque-Bera, kurtosis and skewness tests.

Fusion of the decisions received at the fusion centre with a view to make the final global decision on the status of the primary user is also another important challenge that has not been exhaustively studied. Fusion techniques are classified into soft and hard combination schemes. In hard decision strategy the FC combines binary decisions using standard hard decision rules to achieve the global decision. Three hard combining decision rules used to arrive at the final decision are classified as AND, OR and majority also called \( k \) out of \( n \) counting rule [13]. In [14], authors made a comparative study of the performance of the three hard fusion techniques. In their findings they concluded that AND rule was the most reliable fusion scheme followed by majority and the lastly the OR rule. Another comparative study on the performance of hard fusion schemes and soft decision schemes was done by authors in [15]. In their study they confirmed earlier research done to justify that soft fusion decision reported better PU signal detection, albeit having significant data communication overheads. Hard combination schemes however have attracted most attention from researchers since these fusion schemes are easy to implement by simple logics gates. The authors in [16], proposed strategies on how the AND, majority and OR fusion rules are optimized based on the Neyman-Pearson criterion. Under this strategy the sensing objective was to maximize the probability of detection with the constraint on the probability of false alarm of less than 10 percent. Their findings showed AND rule had higher detection performance than the other two.

In our proposed energy detection model as shown in the previous journal paper, an optimal \( k \) out of \( n \) counting rule showed to be better than other hard fusion rules in detection performance both in AWGN and Rayleigh channels. Another advantage of this model was on the overall reduction in energy consumption in the network due to the two tier optimization strategy. Fewer SUs were employed determine the final global decision on the presence or absence of the PU on the channel but still maintained high throughput and energy efficiency. Spectrum sensing in the IEEE 802.22 standard, for ex-
ample requires stringent sensing of a false alarm probability of less than 0.1 for a signal as low as -20 dB (SNR) [17]. In [18], authors proposed an iterative threshold cooperative spectrum technique. Their objective was to optimize the thresholds of the cooperative spectrum sensing with different fusion rules including AND logic & OR logic. This was done in order to obtain the optimal SUs in cooperative spectrum sensing and their optimal thresholds. Their algorithm achieved better detection performance for SUs’ with different SNR. The optimal scheme also employed fewer SUs in collaborative sensing at the fusion center. In [19], the authors proposed an optimized detection threshold in order to minimize both the error detection probabilities of single-channel and multi-channel cooperative spectrum sensing. In single-channel cooperative spectrum sensing, they performed an iterative optimal thresholds with AND logic, OR logic and $k$ out of $n$ rule respectively. Their findings showed a great decrease in the error on detecting PU status on the channel. Energy efficiency in the cognitive radio network is defined as the ratio of throughput (average amount of successfully delivered bits transmitted from SUs to the fusion center) to the total average energy consumption in the system [20]. In order to reduce the energy consumed in spectrum sensing network, not all SUs in each cluster send their sensed results to the fusion center of local cluster. In [21], authors optimized $k$ out of $n$ by allowing those SUs with reliable sensing results to transmit to the FC. This showed some reduction in energy consumption of the cognitive radio network. In this paper an optimal $k$ out of $n$ is applied to improve on the probability of detection and reduce on the energy system consumption by employing fewer SUs in the final detection on the presence or absence of the PU. To minimize energy a two tier optimization paradigm is employed; firstly, by optimal selection of secondary users (SUs) in the network using Lagrange criterion and secondly by optimizing on the energy based hard fusion techniques achieved by Newton-Raphson optimization criterion. The results indicate that an optimal energy based majority counting fusion rule shows greater detection capability than AND & OR based energy detection schemes and also overall system energy consumption in CSS networks is reduced since not all SUs participate in the sensing of the PU.

**Notations** : $E[\cdot]$ is the expectant operator, $\text{var}$ is the variance, $\text{Im}[\cdot]$ and $\text{Re}[\cdot]$ are the imaginary and real parts of the signal $X(\cdot)$, $\text{erfc}(\cdot)$ is complementary error function and $h$ is the circular Gaussian channel.
3 System Model

3.1 Practical Cooperative Sensing Model

The system model in Fig. B.1 shows a practical CSS network. In this scheme, a group of SUs sense the spectral band to determine the presence or absence of PU. They receive this information through the control channel and independently analyze it by utilizing the statistical properties of the received PU’s signal and subsequently communicate their individual decisions through the reporting channel to the FC. At the fusion centre, the decisions from individual SUs are integrated together to finally make the global decision on whether the PU is transmitting on the channel or not. The fusion center then allocates the idle channel to the SUs depending their demands against the available bandwidth.

3.2 Proposed Cooperative Spectrum Model

In the proposed lower level system model of fig. B.2, the secondary users \((SU_1, SU_2, ..., SU_n)\) collectively sense the PU channel based on HOS tests namely, kurtosis \& skewness \((kurt & skew)\), omnibus \((omnb)\) and Jarque-Bera \((JB)\) statistics tests. The hard binary local decisions made by SUs are transmitted over wireless Gaussian channel represented as \((CH_1, CH_2, ..., CH_n)\) to the data FC. The binary data \((b_1, b_2, ..., b_n)\) is fused to achieve the final global decision on the presence or absence of the primary user.

Fig. B.1: A practical cognitive radio network
4. Local Spectrum Sensing

4.1 Spectrum Sensing Hypothesis

Generally the spectrum sensing problem can be formulated by the following two hypothesis [4, 9]

\[ H_0 : x(t) = w(t) \quad t = 0, \ldots, T - 1 \]  \hspace{1cm} (B.1)
\[ H_1 : x(t) = s(t) + w(t) \quad t = 0, \ldots, T - 1 \]  \hspace{1cm} (B.2)

where \( H_0 \) and \( H_1 \) are null and alternative hypothesis respectively, \( t \) is the digital samples numbering \( T \), \( w(t) \) is the additive white Gaussian noise, \( s(t) \) is the PU’s signal and \( x(t) \) is the signal received at the fusion centre. The received signal plus additive white Gaussian noise \( x(t) \) as function of SNR (\( \gamma \)) is given as

\[ x(t) = f[(s(t) + w(t)), \gamma] \]  \hspace{1cm} (B.3)

where \( \gamma \) is the PU signal to noise ratio (SNR). The probability of detection is formulated as hypothesis test \( P_d = \text{Prob}(\text{Signal Detected} \mid H_1) \), whereas the probability of false detection is determined as \( P_f = \text{Prob}(\text{Signal not Detected} \mid H_1) \). Another form of formulation is thresholding on the statistical test parameter. To detect the PU’s spectrum effectively there is need to first estimate and analyze the power spectral density (PSD) of the SU’s received signal. A strategic periodogram PSD estimation technique can be used to accurately present the frequency-domain statistical properties of a signal [9]. Based on the periodogram method and as formulated in algorithm 1, the received signal \( x(t) \) of \( T \) samples is firstly subdivided into \( L \) smaller segments. Then the \( i-th \) segment signal
4. Local Spectrum Sensing

can be formulated as [9]

\[ x_i(t) = x[t + iM] \quad \text{(B.4)} \]

where \( i = 0, \ldots, T - 1 \) is the number of data samples, \( M = T/L \) is the length of each segment and \( t = 0, \ldots, M - 1 \) are the Fast Fourier transforms (FFT) points in one segment. Performing FFT on signal sample \( x_i(t) \), periodogram of the \( i \)-th SU, \( y_i(t) \) is given by

\[ y_i(t) = \frac{1}{M} \left| \sum_{t=0}^{M-1} x_i[t] e^{-j\omega t M} \right|^2 \quad \text{(B.5)} \]

where \( i \in [t, T] \) is the number of samples, \( M \) is the length of each segment representing the elements of discrete Fourier transform (DFT) and \( \omega = 2\pi f \). The function \( y_i(t) \) is modeled as the PU signal and is utilized in the next section to determine the skewness and kurtosis.

4.2 Spectrum Sensing HOS Techniques

4.2.1 Skewness and Kurtosis

The estimated skewness (skew) is defined as third standard moment of a random variable \( x_i(t) \) of a Gaussian distribution. Estimated kurtosis (kurt) on the other hand is given by fourth standard moment of a random distribution. The value tends to 3 as the sample size considered for the test increases [20]. For given sample set of \( y_i(t) \) the estimated sample of skew is given as

\[ \text{skew}(y(t)) = \frac{1}{M} \sum_{i=0}^{M-1} (y_i(t) - \bar{y})^3 \left( \frac{1}{M} \sum_{i=0}^{M-1} (y_i(t) - \bar{y})^2 \right)^{-3/2} \quad \text{(B.6)} \]

where \( \bar{y} \) is the mean of a given signal data. Similarly, the estimated kurt of a random sample is formulated as

\[ \text{kurt}(y(t)) = \frac{1}{M} \sum_{i=0}^{M-1} (y_i(t) - \bar{y})^4 \left( \frac{1}{M} \sum_{i=0}^{M-1} (y_i(t) - \bar{y})^2 \right)^{-2} \quad \text{(B.7)} \]

The test statistics \( ST(s_t) \) of the periodogram (power spectral density) is represented as the square root of the sum of squares of \( \text{skew}(y(t)) \) and \( \text{kurt}(y(t)) \) as used in algorithm 1. When the value of test statistics is larger than a set threshold \( T_\lambda \), the distribution of the received signal’s averaged periodogram deviates from the AWGN’s power spectral density which is an indicator of the presence of PU’s signal. The test statistics of the periodogram estimate can be formulated as

\[ ST(s_t) = \sqrt{\text{skew}(y(t))^2 + \text{kurt}(y(t))^2} \quad \text{(B.8)} \]
Algorithm 1 Algorithm for HOS test detection

Input: $M = M_{FFT}$, $T = 3000$, $\gamma_j = -30 : 5$, $P_f = 0.1 : 1$

Output: $P_{d,kurt& skew}, P_{d,JB}, P_{d,omnb}$

1. $x(t) \leftarrow$ generate $T$ random data, eqn. (B.2)
2. $x_i(t) \leftarrow$ modulate $x(t)$ (16 QAM) plus noise, eqn. (B.4)
3. fast Fourier transform on modulated signal
4. $y_i(t) \leftarrow$ FFT on $x_i(t) \mod$, eqn. (B.5)
5. $y_i(t) \leftarrow$ concatenation of $y_{FFT}$
6. $y_i(t) = $ real parts ($y_{FFT}$) + imaginary parts ($y_{FFT}$)
7. for $j = \text{length} (\gamma)$, $i = \text{length} (M_{FFT})$ do
8. Calculate kurtosis & skewness
   $\text{skew}(y(t)) \leftarrow$ skewness test, eqn. (B.6)
   $\text{kurt}(y(t)) \leftarrow$ kurtosis test, eqn. (B.7)
9. while $\gamma_j \leq 0$, $n \leftarrow 0$ do
   10. $S_t \leftarrow$ test statistics, eqn. (B.8) & $T_\lambda \leftarrow$ thr’d, eqn. (B.9)
   11. if $ST(S_t) \geq T_\lambda$ then
      12. decision = $H_1$ increment counter $\leftarrow H_1 = H_1 + 1$
   13. else $[ST(S_t) \leq T_\lambda]$
      14. decision = $H_0$ (discard) incr. count $\leftarrow i = i + 1, j = j + 1$
   15. $P_{d,kurt& skew} = \text{sum} \left( \frac{H_1}{M_{FFT}} \right)$
   16. end if
17. end while
18. Jarque Bera & Omnibus $K^2$ test
19. while $\gamma_j \leq 0$, $n \leftarrow 0$ do
20. $\text{JB} \& K^2 \leftarrow$ test statistics, eqn. (B.11) (B.14)
21. $\text{JB}_\lambda \& K^2_\lambda \leftarrow$ the threshold, eqn.(B.13)(B.16)
22. if $\text{JB} \geq \text{JB}_\lambda \& K^2 \geq K^2_\lambda$ then
23. decision = $H_1$ incr. counter $\leftarrow H_1 = H_1 + 1$
24. else $[\text{JB} \leq \text{JB}_\lambda \& K^2 \leq K^2_\lambda]$
25. decision = $H_0$ incr. count $\leftarrow i = i + 1, j = j + 1$
26. $P_{d,JB} \& P_{d,K^2} = \text{sum} \left( \frac{H_1}{M_{FFT}} \right)$
27. end if
28. end while
4. Local Spectrum Sensing

where skew(y(t)) and kurt(y(t)) are the test statistics for skew and kurt respectively of the signal x(t). For a given probability of false alarm \( \text{P}_f \), the threshold \( T_\lambda \) for skew and kurt tests the null hypothesis \( (H_0) \). This is a chi-squared distribution defined as \( P_f = 1 - f(T_\lambda : H_0) \) and hence is formulated as [9]

\[
T_\lambda = \sqrt{-\log(P_f)} \tag{B.9}
\]

In order to derive the probability of detection \( (P_d) \) and \( (P_f) \), the PDF for the test statistic is developed for both \( H_0 \) and \( H_1 \) as

\[
\begin{cases}
ST(S_t) \geq T_\lambda & H_1 \\
ST(S_t) < T_\lambda & H_0
\end{cases} \tag{B.10}
\]

4.2.2 Jarque-Bera (JB)

The Jarque Bera statistic has asymptotic chi-squared distribution with two degrees of freedom [10], formulated by considering the estimated skew and kurt on the transmitted PU signal, defined as [11]

\[
JB = \frac{M}{6} \left[ \text{skew}^2 + \left( \frac{\text{kurt}^2 - 3}{4} \right)^2 \right] \tag{B.11}
\]

where \( M=M_{\text{FFT}} \) is the number FFT points. In order to derive the \( P_d \) and \( P_f \) the hypothesis tests \( H_1 \) and \( H_0 \) are formulated as

\[
\begin{cases}
JB \geq JB_\lambda & H_1 \\
JB < JB_\lambda & H_0
\end{cases} \tag{B.12}
\]

For a given probability of false alarm \( (P_f) \), the threshold for JB test based on null hypothesis \( (H_0) \), for an \( M_{\text{FFT}} \) points is expressed as [11]

\[
JB_\lambda = 0.0688 M_{\text{FFT}} \tag{B.13}
\]

For the null hypothesis to be accepted the test statistics must be smaller than a critical value that is positive and near zero. Higher values of JB indicate the sample do not follow the Gaussian distribution. The probability of detection is iteratively determined as shown in pseudo code for algorithm 1.

4.2.3 Omnibus \( (K^2) \) Test

Omnibus is defined as the square root of a transformed skewness \( (\text{skewT}) \) and kurtosis \( (\text{kurtT}) \) test statistics. The asymptotic normal values for \( (\text{skew}) \) and \( (\text{kurt}) \) are used to
4. Local Spectrum Sensing

construct a chi-squared test involving the first two moments of the asymptotic distributions [12], mathematically expressed as

\[ K^2 = \sqrt{\text{skewT}^2 + \text{kurtT}^2} \]  \hspace{1cm} (B.14)

The hypothetical omnibus test is derived by comparing to defined threshold \( (K^2_\lambda) \) formulated as

\[
\begin{cases}
K^2 \geq K^2_\lambda & H_1 \\
K^2 < K^2_\lambda & H_0
\end{cases}
\]  \hspace{1cm} (B.15)

For a predetermined \( P_f \) the threshold for omnibus test is a fixed value determined by

\[ K^2_\lambda = 0.0688 \ M_{FFT} \]  \hspace{1cm} (B.16)

where \( M_{FFT} \) is the number of FFT points. The \( (\text{skewT}) \) on the estimated data sample is given as [11, 12]

\[ \text{skewT} = \delta \log \left[ \frac{Y}{\Phi} + \sqrt{\left( \frac{Y}{\Phi} \right)^2 + 1} \right] \]  \hspace{1cm} (B.17)

where \( \Phi = \sqrt{\frac{2}{W^2-1}} \) is a small deviation from the critical value on the skewness of the estimated distributed random data, \( W^2 = (\sqrt{4B_2-4} - 1) \) is a constant of normalization on skewness, \( \delta = \frac{1}{\sqrt{\log W}} \) is the skewness parameter and \( (Y) \) is the estimated skewness value of the random distributed data given as

\[ Y = \text{skew} \left[ \frac{(M+1)(M+3)}{6(M-2)} \right] \]  \hspace{1cm} (B.18)

where \( \text{skew} = \text{skew}(y(t)) \) is estimated skewness of the sampled signal data as given in eqn. (B.7), \( M \) is the number FFT data sample points. The skewness as a function of the variance \( \mu_2(\text{skew}) \) is formulated as

\[ \mu_2(\text{skew}) = B_2 = \frac{3(M^2 + 27M - 70)(M+1)(M+3)}{(M-2)(M+5)(M+7)(M+9)} \]  \hspace{1cm} (B.19)

The transformed kurtosis \( (\text{kurtT}) \) on the random distributed received PU’s signal is also formulated as [11, 12]

\[ \text{kurtT} = \frac{(1 - \frac{2}{\sqrt{D}})^{\frac{1 - \hat{\rho}}{1 + x \sqrt{\frac{2}{\sqrt{D} - 2}}}}}{\frac{2}{\sqrt{D}}} \]  \hspace{1cm} (B.20)

where \( D \) is a constant that denotes the degrees of freedom for the chi-squared distribution. Solving for \( D \) to equate the third moment of theoretical and sampling distributions, it is possible then to compute \( D \) as follows

\[ D = 6 + \frac{8}{B_1} \left[ \frac{2}{B_1} + \sqrt{1 + \frac{4}{B_1}} \right] \]  \hspace{1cm} (B.21)
where $B_1 = \mu_1(\text{kurt})$ is the kurtosis as a function of the mean ($\mu_1$), given as

$$
\mu_1(\text{kurt}) = B_1 = \frac{6(M^2 - 5M + 2)}{(M + 7)(M + 9)} \frac{6(M + 3)(M + 5)}{M(M - 2)(M - 3)}
$$  \hspace{1cm} (B.22)

where $\text{kurt} = \text{kurt}(y(t))$ is the estimated kurtosis given in eqn. (B.7) and $M$ is the number of samples. It is possible to standardize kurtosis by formulating the expression as

$$
x = \frac{\text{kurt} - \mathbb{E}[\text{kurt}]}{\sqrt{\text{var}[\text{kurt}]}}
$$  \hspace{1cm} (B.23)

where the mean as a function of kurtosis is given as $\mathbb{E}[\text{kurt}] = \frac{24M(M-2)(M-3)}{(M+1)^2(M+3)(M+5)}$ and variance as a function of kurtosis is expressed as $\text{var}[\text{kurt}] = \frac{3(M-1)}{M+1}$, are all computed to determine transformed estimated kurtosis.

5 Fusion Schemes

5.1 Fusion Strategy Hypothesis Tests

The null hypothesis ($H_0$) for decision statistics of the omnibus test can be derived as

$$
\begin{align*}
K^2 &\geq H_1 \quad \lambda \\
K^2 &< H_0 \quad \lambda
\end{align*}
$$  \hspace{1cm} (B.24)

where $\lambda$ is the decision threshold which has to be optimized. The cost functions are formulated in terms of probability of misdetection and false alarm as conditioned on the channel, the probability of misdetection is formulated as [22]

$$
P_{m,i|\gamma,\theta} = 1 - \frac{1}{2}\text{erfc} \left( \frac{\lambda_i - K^2}{\sqrt{2}\sigma_1(\gamma, \theta)} \right) + \frac{1}{2}\text{erfc} \left( \frac{\lambda_i + K^2}{\sqrt{2}\sigma_1(\gamma, \theta)} \right)
$$  \hspace{1cm} (B.25)

where $\gamma = |h|^2 \left( \frac{\mathbb{E}[|x(t)|^2]}{\mathbb{E}[|w(t)|^2]} \right)$ is given as the instantaneous SNR. The instantaneous channel phase angle $\theta$ is defined as $\theta = \tan^{-1} \left( \frac{\mathbb{E}[\text{Im}[y(t)^\dagger]]}{\mathbb{E}[\text{Re}[w(t)^\dagger]]} \right)$, $w(t)$ is the AWGN. The probability of misdetection ($P_{m,i|\gamma,\theta}$) is the sum of the lower bound probability $P_{m,1|\gamma,\theta} = \frac{1}{2}\text{erfc} \left( \frac{\lambda_i - K^2}{\sqrt{2}\sigma_1(\gamma, \theta)} \right)$ and upper bound probability $P_{m,2|\gamma,\theta} = \frac{1}{2}\text{erfc} \left( \frac{\lambda_i + K^2}{\sqrt{2}\sigma_1(\gamma, \theta)} \right)$. Unlike in [22], this paper uses omnibus test ($K^2$) instead of kurtosis. $\lambda_i$ is the decision threshold, $\sigma_1(\lambda, \theta)$ is expressed in terms of instantaneous SNR and phase angle of a circular Gaussian channel and is given as,

$$
\sigma_1(\lambda, \theta) =
= a_{00} + a_{10} \gamma + [a_{20} + a_{21} \sin^2(2\theta)] \gamma^2 + [a_{30} + a_{31} \sin^2(2\theta)] \gamma^3
+ [a_{40} + a_{42} \sin^2(2\theta)] \gamma^4
$$  \hspace{1cm} (B.26)
5. Fusion Schemes

The following constants; \(a_{00}, a_{10}, a_{20}, a_{21}, a_{30}, a_{31}, a_{40}, a_{41} \) \& \(a_{42}\) are given in table B.1. The conditional (on the channel) probability of false alarm is given as

\[
P_{f,i|\gamma,\theta} = \frac{1}{2} \text{erfc} \left( \frac{\lambda_i - \mu_0}{\sqrt{2\sigma_0}} \right) + \frac{1}{2} \text{erfc} \left( \frac{\lambda_i + \mu_0}{\sqrt{2\sigma_0}} \right) \tag{B.27}
\]

where \(\theta\) is the phase angle, \(\gamma\) is the SNR of the signal, \(\sigma_0\) is the modulation constant and \(\mu_0\) is the mean of the data distribution as given in table B.1.

**Table B.1: Modulation Constants**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Actual values used</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_{00}, a_{10}, a_{20}, a_{21})</td>
<td>(\frac{24\rho_0^8}{M}, \frac{96\rho_0^8}{M}, \frac{46\rho_0^8}{M}, -\frac{48.96\rho_0^8}{M})</td>
</tr>
<tr>
<td>(a_{30}, a_{31}, a_{40}, a_{41})</td>
<td>(\frac{33.28\rho_0^8}{M}, \frac{128.64\rho_0^8}{M}, \frac{10.33\rho_0^8}{M}, -\frac{1.93\rho_0^8}{M})</td>
</tr>
<tr>
<td>(a_{42}, \sigma_0, \mu_0)</td>
<td>(\frac{1.74\rho_0^8}{M}, \frac{24\rho_0^8}{M}, 1)</td>
</tr>
</tbody>
</table>

5.2 First Stage Optimization on SU Selection Criteria

The aim of the first stage optimization is to iteratively select \(n\) SUs in \(\forall n \in [1, N]\) SUs, in an \(r\) out of \(n\) counting rule where \(r\) is the number of SUs that form the combinatorial \(n\) fusion order and \(N\) is the total number of SUs in CSS network. The criteria on selection is based on SUs’ decrementing SNR as formulated in algorithm 2. The error probability is further expressed as

\[
P_{e,i} = P(H_0)Q_f + P(H_1)Q_m \tag{B.28}
\]

where \(P(H_0)\) is the null hypothesis, \(P(H_1)\) is the alternative hypothesis, \(Q_f\) is the global probability of false alarm and \(Q_m\) is probability of misdetection. The sum of probability of false alarm and misdetection is derived as a cost function to determine the global decremental error probability \((Q_e)\) in the detection of the primary user in CSS network. The minimization problem is formulated as [15, 16, 18, 19]

\[
\min_{\lambda} \left( Q_m(\lambda^{opt}) \right) \quad \text{and} \quad Q_f(\lambda^{opt}) \tag{B.29}
\]

Subject to \(Q_e > 0\)

where \(\lambda^{opt}\) is the optimal decision threshold. Considering eqn. (B.25) and eqn. (B.27), the optimal threshold is formulated as

\[
\lambda_i^* = \arg \min_{\lambda} \left( P_{e,i} = (\beta P_{f,i|\gamma,\theta} + P_{m,i|\gamma,\theta})P(H_1) \right) \tag{B.30}
\]

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Algorithm 2 First Stage optimal selection of SUs

**Input:** \( N = 15, \text{SNR} = -30 : 2 : -5.0 \)

**Output:** \( \lambda_n^{opt}, n, \ r = \frac{N}{2} \)

**initialize:** \( n = 1 \leftarrow \) sort all SUs in descending order SNR

**calculate the following**

**step1:** \( \lambda_i^{*} \leftarrow \) the threshold of \( i \)th SU, eqn. (B.34)

**step2:** \( P_{r,n}^{*} \leftarrow \) the error detection, eqn. (B.28)

**step3:** \( P_{f,1|\gamma,\theta} \) and \( P_{d,1|\gamma,\theta} \leftarrow 1^{st} \) iterate, eqn. (B.27) & (B.31)

**step4:** \( Q_{d}^{r,n} \leftarrow \) the detection prob, eqn. (B.35)

**step5:** \( Q_{f}^{r,n} \leftarrow \) the false alarm, eqn. (B.36)

**step6:** \( Q_{c}^{r,n} \leftarrow \) the decremental error, eqn. (B.41)

**for** \( i = \text{length (n)} \) and \( r = \text{length} \left( \frac{N}{2} \right) \)

**while** \( n \leq m, \ n \leftarrow 0 \) **do**

**if** \( Q_{c}^{r,n} \geq 0 \) **then**

\( i = n + 1 \)

**increment counter** \( \leftarrow n = n + 1 \)

\( \lambda_n^{opt} \leftarrow \) the optimal threshold, eqn. (B.40)

**go to step 4**

**else** \( \{Q_{c}^{r,n} \leq 0\} \)

\( n = n - 1 \leftarrow \) delete the SU

**go to step 4 otherwise have attained the solution**

**end if**

**end while**
where $\beta = \frac{P(H_0)}{P(H_1)}$ is the detection factor, $P_{f,i|\gamma,\theta}$ is the false alarm and $P_{m,i|\gamma,\theta}$ is the misdetection of the $i$th SU. From eqn. (B.25), the probability of detection is similarly given as

$$P_{d,i|\gamma,\theta} = 1 - P_{m,i|\gamma,\theta} \quad (B.31)$$

Consequently from eqn. (B.30), the threshold is maximized as follows

$$\lambda_i^* = \arg \max_{\lambda} \left( (P_{d,i|\gamma,\theta} - \beta P_{f,i|\gamma,\theta} - 1) P(H_1) \right) = \arg \max_{\lambda} \left( P_{d,i|\gamma,\theta} - \beta P_{f,i|\gamma,\theta} \right) \quad (B.32)$$

By the Lagrange theorem, the maximum threshold is obtained by differentiating by parts as follows

$$\frac{\partial P_{d,i|\gamma,\theta}}{\partial \lambda_i} \bigg|_{\lambda_i^*} = \beta \frac{\partial P_{f,i|\gamma,\theta}}{\partial \lambda_i} \quad (B.33)$$

where $i = 1, \ldots, n$ is the number of SUs selected to participate in fusion and $\lambda_i^*$ is the initial optimal threshold derived as

$$\lambda_i^* = \frac{\sigma_s^2}{2} + \sigma_s^2 \sqrt{\frac{1}{4} + \frac{\gamma_i}{2} + \frac{4\gamma_i + 2}{M\gamma_i} \log (\beta \sqrt{2\gamma_i + 1})} \quad (B.34)$$

where $\sigma_s^2$ is the noise variance, $\gamma_i$ is the SNR of the $i$th SU and $M$ is the number of signal data samples. The global probability of detection in $r$ out of $n$ rule is derived as

$$Q_{d}^{(r,n)} = \sum_{j=r}^{n} \binom{n}{j} \prod_{i=1}^{j} P_{d,i|\gamma,\theta} \prod_{i=j+1}^{n} (1 - P_{d,i|\gamma,\theta}) \quad (B.35)$$

where $n \in \{j = 1, \ldots, N\}$, $N$ is the total number of SUs, $P_{d,i|\gamma,\theta} = 1 - P_{m,i|\gamma,\theta}$ is probability of detection as given in eqn. (B.25), $r$ is the actual number of SUs that form $r$ out of $n$ counting rule and $n$ is the total number of SUs selected to participate in decision making.

Similarly, the global probability of false alarm is formulated as

$$Q_{f}^{(r,n)} = \sum_{j=r}^{n} \binom{n}{j} \prod_{i=1}^{j} P_{f,i|\gamma,\theta} \prod_{i=j+1}^{n} (1 - P_{f,i|\gamma,\theta}) \quad (B.36)$$

where $n \in \{j = 1, \ldots, N\}$, $P_{f,i|\gamma,\theta}$ is probability of false alarm as given in eqn. (B.27). The selection criteria is done by the iterative calculation of global probability detection and false alarm simultaneously, as performed in algorithm 3. The minimization problem stated in eqn. (B.29) is formulated mathematically as

$$Q_{d}^{r,n} = Q_{d}^{(r-1,n-1)}(P_{d,n|\gamma,\theta}) + Q_{d}^{(r,n-1)}(1 - P_{d,n|\gamma,\theta}) \quad (B.37)$$

where $Q_{d} = 1 - Q_{m}$ is the global probability of detection, the probability of false alarm is similarly derived as

$$Q_{f}^{r,n} = Q_{f}^{(r-1,n-1)}(P_{f,n|\gamma,\theta}) + Q_{f}^{(r,n-1)}(1 - P_{f,n|\gamma,\theta}) \quad (B.38)$$
The final iteration gives the optimal threshold $\lambda_{opt}^{n}$ given for $n$ number of SUs, formulated as

$$Q^{(r,n-1)}d \frac{\partial P_{d,n|\gamma,\theta}}{\partial \lambda_n} |_{\lambda_{opt}^{n}} = \beta Q^{(r,n-1)}f \frac{\partial P_{f,n|\gamma,\theta}}{\partial \lambda_n} |_{\lambda_{opt}^{n}} \quad (B.39)$$

where the optimal threshold is given in this scenario as

$$\lambda_{opt}^{n} = \frac{\sigma^2_s}{2} + \sigma^2_s \left( \frac{1}{4} + \frac{\gamma_n}{2} + \frac{4\gamma_i + 2}{M\gamma_n} \log \left( \beta \sqrt{2\gamma_i + 1} \ast B \right) \right) \quad (B.40)$$

where $B = \frac{Q^{(r-1,n-1)}d - Q^{(r,n-1)}d}{Q^{(r-1,n-1)}f - Q^{(r,n-1)}f}$ is the detection factor, $\gamma_n$ is the SNR for the $n$-th SU, $\sigma^2_s$ is the noise variance and $M$ is the signal data samples. The decremented detection error is expressed as

$$Q^{(r,n)}_e = P(H_1)P_{d,n|\gamma,\theta} \left( Q^{(r-1,n-1)}d - Q^{(r,n-1)}d \right) - P(H_0)P_{f,n|\gamma,\theta} \left( Q^{(r-1,n-1)}f - Q^{(r,n-1)}f \right) \quad (B.41)$$

where the $P(H_0)$ and $P(H_1)$ are the weights for probability of false ($P_{f,n|\gamma,\theta}$) and probability of detection ($P_{d,n|\gamma,\theta}$) respectively, $n$ is the number of SUs participating in detection of the presence or absence of the PU on the channel, $\gamma$ is the SNR and $\theta$ is the uniformly distributed phase angle.

### 5.3 Second Stage Optimal Strategy

At the FC, a specific $k$ out of $n$ strategy is employed to process the SUs’ received decisions at the FC. Where $k$ is number of SUs in the range of $(1 \leq k \leq n)$ and $n$ is the total number of SUs selected from a total of $N$ as realized in the first optimization stage. The idea behind this rule is to find the number of SUs whose local binary decisions is 1. If this number is larger than or equal $k$, then the spectrum is said to be used otherwise the spectrum is unused. An iterative algorithm search to find an optimal number of $k$ SUs in $k$ out of $n$ combinatorial order is done at the FC. To achieve this an upper-threshold of global probability false alarm ($Q_f$) of less than utilization level ($\epsilon$) is set. The maximization problem can be formulated as [7, 15, 16]

$$\text{Maximize } (Q_d(k))_{1 \leq k \leq n} \quad (B.42)$$

$$\text{Subject to } Q_f(k) < \epsilon \quad (B.43)$$

The global probability of false alarm $Q_f$ based on $k$ out of $n$ counting rule is formulated in algorithm 3 and mathematically derived as

$$Q_f(k) = \sum_{j=k}^{n} \binom{n}{j} \left( \frac{p_{j|\gamma,\theta}^k}{1 - P_{j|\gamma,\theta}} \right)^{n-k} = \epsilon \quad (B.43)$$
where \( e \) is the utilization level, \( k \) is number of SUs selected to participate in the \( k \text{ out of } n \) fusion process, \( n \) is number of SUs iteratively found in the first optimization stage section 5.2. The derivative of global probability of false alarm \( (Q_f) \) as function of \((P_f)\) is derived as

\[
\frac{\partial Q_f(P_f)}{\partial(P_f)} = n\left(\frac{n-1}{k-1}\right)p^k_{f,i|\gamma,\theta} \left(1 - P_{f,i|\gamma,\theta}\right)^{n-k-1}
\]

\[
= n\varphi(k-1, n-1, P_{f,i|\lambda,\theta}) > 0
\]

From eqn. (B.44) it follows that \( \varphi \) is the binomial cumulative function given as

\[
\varphi = \binom{n-1}{k-1} p^k_{f,i|\gamma,\theta} \left(1 - P_{f,i|\gamma,\theta}\right)^{n-k}
\]

Subsequently the global probability of detection in \( k \text{ out of } n \) case is given as

\[
Q_d(k) = \sum_{j=k}^{n} \binom{n}{j} p^k_{d,i|\gamma,\theta} \left(1 - P_{d,i|\gamma,\theta}\right)^{n-k} > 0
\]

To optimize the eqn. (B.46), we differentiate by parts the function as follows

\[
\frac{\partial Q_d(P_d)}{\partial(P_d)} = n\left(\frac{n-1}{k-1}\right)p^k_{d,i|\gamma,\theta} \left(1 - P_{d,i|\gamma,\theta}\right)^{n-k-1} > 0
\]

From eqn. (B.25) and eqn. (B.27) the following probabilities must hold true.

\[
\frac{P_{d,i|\gamma,\theta}}{P_{f,i|\gamma,\theta}} > \frac{\partial(P_{d,i|\gamma,\theta})}{\partial(P_{f,i|\gamma,\theta})} > \frac{1 - P_{d,i|\gamma,\theta}}{1 - P_{f,i|\gamma,\theta}}
\]

Similarly the above equation can be further formulated as follows

\[
\frac{Q_d(k)}{Q_f(k)} = \frac{\partial Q_d(k)}{\partial P_f(k)} * \frac{\partial P_f}{\partial Q_f} = \frac{\partial Q_d(k)}{\partial P_f(k)} * \frac{\partial P_f}{\partial Q_f}
\]

From the above equation it is true to say \( Q_d(k) \) is linearly increasing function of \( Q_f(k) \).

For all \( k \in [1,n] \) then the roots of \( Q_f(k, P_f) \) are formulated in Bisection algorithm 3.

The algorithm is broken down as follows; for each \( P_{f,i|\gamma,\theta} \) determine the corresponding \( P_{d,i|\gamma,\theta} \) and \( Q_d(k, P_f) \), select the highest global probability, the value of \( k \) is the optimal number of SUs.

6 Energy Efficiency

Energy efficiency is the ratio of throughput to average energy consumed during the cooperative spectrum sensing time. The throughput \( \overline{\text{THR}} \) is formulated as [21]

\[
\overline{\text{THR}} = P(H_0)(1 - Q_f) R_I
\]
Algorithm 3 Second Stage Bisection Algorithm

Input: $P_f = P_{f,i|\gamma,\theta}$, $\epsilon = 0.001$

Output: $k$, $Q_d(k)$

$n \leftarrow$ from algorithm 2

initialize: endpoints $\leftarrow P_{f,L} = 0.01, P_{f,U} = 0.1$

for $i =$ length ($P_f$) and $k =$ length ($n$)
  while $Q_f(k) \leq \epsilon$, $k \leftarrow 1$ from eqn. B.43 do
    if $P_{f,L} \leq P_{f,U}$, $Q_f(P_{f,L}) \leq 0$ and $Q_f(P_{f,U}) > 0$ then
      mid ($P_f$) = \frac{P_{f,U} - P_{f,L}}{2}
      condition: if $Q_f(mid(P_f)) = 0$ then
        solution is found
      else
        Determine the following:
        $P_{d,1|\gamma,\theta}$ $\leftarrow$ cal. detection probability, eqn. (B.31)
        $Q_d(1)$ $\leftarrow$ cal. the false alarm, eqn. (B.43)
        $Q_d(1)$ $\leftarrow$ cal. detection probability, eqn. (B.46)
      else \{$Q_f(P_{f,L}) > 0$ and $Q_f(P_{f,U}) < 0$\}
      mid ($P_f$) = \frac{P_{f,U} - mid(P_f)}{2}
      if sign $Q_f(mid(P_f))$ = sign $Q_f(P_{f,U})$ then
        $P_{f,L}$ $\leftarrow$ mid ($P_f$)
      else
        $P_{f,U}$ $\leftarrow$ mid ($P_f$)
      end if
    end if
  end while
where $R$ is the data rate, $t$ is the transmission time length, $P(H_0)$ is the probability that the spectrum is not being used, $Q_f$ is the global probability of false alarm. The average energy consumed in the network by all SUs $E_c$ is derived as

$$E_c = n e_{su} + P_u e_{st} \quad (B.51)$$

where $n$ is the total number of SUs selected from first optimization stage, $e_{su}$ is the energy consumed during CSS by all the SUs, $e_{st}$ is the energy consumed during data transmission, $P_u$ is the probability of identifying if the spectrum is idle, given as

$$P_u = P(H_0)(1 - Q_f) + P(H_1)(1 - Q_d) \quad (B.52)$$

where $P(H_1) = 1 - P(H_0)$ is the probability of the spectrum being used, $Q_f$ is the global probability of false alarm and $Q_d$ is the probability of detection. Note that the energy consumption during transmission occurs only if the spectrum is identified as unused. The efficiency ($\eta$) can be formulated as [20, 21]

$$\eta = \frac{THR}{E_c} = \frac{P(H_0)(1 - Q_f)Rt}{n e_{su} + (1 - P_0 Q_f - P_1 Q_d) e_{st}} \quad (B.53)$$

where $n$ is number of SUs in equation (B.53), computed as

$$n = \ln \left( \frac{P(H_1)(1 - Q_f)e_{st}}{N e_{su} + P(H_1)(1 - Q_d)e_{st}} \right) - k \ln \left( \frac{P_y(1 - P_x)}{P_x(1 - P_y)} \right) \quad (B.54)$$

where $N$ is the total SUs in CSS network, $k$ is the number of SUs in the $k$ out of $n$ counting rule. A noisy channel is modeled as binary symmetric channel with error probability ($P_e$) and it is the same among all SUs. $p_x = P_{d|i|\gamma,\theta}(1 - P_e) + (1 - P_{d|i|\gamma,\theta})P_e$ is the probability of receiving a local binary decision of 1 when the spectrum is busy and $p_y = P_{f|i|\gamma,\theta}(1 - P_e) + (1 - P_{f|i|\gamma,\theta})P_e$ is the probability of receiving a local binary decision of “1” when the spectrum is idle.

### 7 Simulation Results

In order to evaluate the HOS test for cooperative spectrum sensing capability, we considered a cognitive radio network with 15 SUs transmitting on 16 QAM constellation modulated signal built in matlab software for analysis. It should be noted that any other modulation scheme can be used to model the PU signal. In all subsequent figures, the numerical results are plotted on receiver operating characteristics curves (ROC). Simulation results are denoted with discrete marks on the curves. The simulation parameters are given in table B.2.
7. Simulation Results

Table B.2: Simulation parameters

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Actual values used</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(H_0)$ and $P(H_1)$</td>
<td>0.5</td>
</tr>
<tr>
<td>Frequency range</td>
<td>0-800</td>
</tr>
<tr>
<td>Monte Carlo trials</td>
<td>$10^3$ to $10^4$</td>
</tr>
<tr>
<td>Noise variance $\sigma_n$</td>
<td>1</td>
</tr>
<tr>
<td>phase angle</td>
<td>$0 \leq \theta \leq 2\pi$</td>
</tr>
<tr>
<td>Range of $\delta$</td>
<td>$0 \leq \delta \leq 1$</td>
</tr>
<tr>
<td>mean ($\mu_0$) for ($H_0$)</td>
<td>0</td>
</tr>
<tr>
<td>$e_{st}$, $e_{su}$</td>
<td>1 Joule, 100 mJoule</td>
</tr>
<tr>
<td>Transmission time (t)</td>
<td>0.5 sec</td>
</tr>
<tr>
<td>Data rate (R)</td>
<td>100 kbps</td>
</tr>
</tbody>
</table>

In Fig. B.3, the ROC curves shows the probability of detection ($P_d$) against SNR as formulated in the algorithm 1 for omnibus ($omnb$), Jarque Bera ($JB$), kurtosis & skewness ($kurt & skew$) and kurtosis ($kurt$) test statistics. In this scheme 2048 FFT sample points were considered. From the plot, as expected, the probability of detection increased with increase in SNR starting from a low SNR. The $omnb$ test displayed the highest probability of detection progressively from a low SNR up to about -16 dB. The plot shows that $omnb$ performs better at low SNR. This was followed by JB, then $kurt & skew$. The results of the other HOS tests are close to those in [9, 10, 20].

In Fig. B.4, the graph illustrates the probability of detection ($P_d$) against SNR for the HOS tests considered under a smaller data sample of 512 FFT points. The plot shows $omnb$ still has higher detection probability for all ranges of SNR and even better under extremely low SNR (-30dB). The $omnb$ test technique therefore tends to suppress the Gaussian noise showing an improved performance. From the two results displayed in fig. (B.3) and (B.4), it can be concluded that omnibus is a superior statistical test for both small and big data sample at low SNRs.

In Fig. B.5, the shows the global probability of detection ($Q_d$) against false alarm ($Q_f$) as discribed in the second stage optimization, for optimal $k$ out of $n$ counting rule based on HOS tests. The rules are for omnibus and majority rule ($omnb$ and $maj$), Jarque-Bera.
7. Simulation Results

Fig. B.3: Detection probability for HOS tests against a range of SNR in 2048 FFT data points

Fig. B.4: Detection probability for HOS tests against a range of SNR in 512 FFT data points
7. Simulation Results

Fig. B.5: Global probability of detection against false alarm for HOS tests

and majority ($JB$ and $maj$), kurtosis & skewness and majority ($kurt$ & skew and $maj$). The optimal number of 8 out of 10 SUs was determined by a two stage optimization as given in algorithms (2) and (3). From ROC curves, it can observed that a combination of $omnb$ and $maj$ displays a higher probability of detection for a false alarm of less than 0.1. This is as per the requirement of IEE 802.22 standards [17]. The performance was then followed by $JB$ and $maj$ and lastly $kurt$ skew and $maj$.

Fig. B.6, shows global probability of misdetection ($Q_m$) against false alarm ($Q_f$), comparative performance for HOS based optimal majority rules; $omnb$ and $maj$, $JB$ and $maj$, $kurt$ skew and $maj$ and lastly $kurt$ and $maj$ is done. The optimal number of 8 out of 10 SUs was realized in the algorithm 3. From the plot, it can be deduced that $omnb$ and $maj$ combination strategy displayed the lowest probability of misdetection for all values of probability of false alarm as compared to the three other combinations. In conclusion, based on the fig. (B.5) and (B.6), $omnb$ and $maj$ rule showed the highest probability of detection and the lowest misdetection as compared to all the other HOS based majority rule for all ranges of false alarm.

Fig. B.7, shows performance of a hybrid spectrum sensing scheme of $k$ out of $n$ counting rule, based omnibus test for different numbers of SUs. The plot shows the comparative performance of different numbers of SUs as selected in single stage compared to two stage optimization. Where $n = 10$, $k = 5$ and $k = 8$ respectively. From this plot, it can be deduced that omnibus a local detection test based on a two stage optimization
7. Simulation Results

Fig. B.6: Global probability of misdetection against false alarm for HOS tests

Fig. B.7: Comparative analysis on single and two stage optimization
global detection scheme displayed higher probability of detection to that of single stage optimization for all ranges of false alarm.

![Energy efficiency for k out of n counting rules](image)

**Fig. B.8:** Energy efficiency in *k* out of *n* counting rule.

Fig. B.8, shows the energy efficiency for different *k* out of *n* counting rules representing three scenarios. The first case is when all the SUs in the cooperative spectrum sensing *N* = 15 participate in the detection of the PU. The second case is when an optimal number of SUs as found in the first optimization stage *n* = 10 and the third case is when *n* = 8 just for the purpose of benchmarking. From this plot the optimal case showed the greatest energy efficiency of about 2 * 10^4 Bits per Joule. This was achieved when *k* = 8 SUs in the combinatorial order of 8 out of 10 counting rule. Note that due to the *k* out of *n* rule the number of *k* can only go up to *n* number of SUs.
8 Conclusion

In the proposed hybrid model, an optimal \( k \) out of \( n \) based omnibus (\( K^2 \)) statistics test was shown to be more superior to the other HOS tests. This model would be preferred to detect the PU in cognitive radio networks operating under noisy conditions. Another advantage of this model is the overall reduction in energy consumption in the network due to the two stage optimization. Fewer SUs make the final decision on the status of the PU on the channel but still maintain reliable decision outcomes.
References


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References


