The History of Mathematics as a Pedagogical Tool: Exploring learner perspectives with regard to the Theorem of Pythagoras

By

Winfilda Kapofu

Student Number: 214585621

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Supervisor: Dr Jayaluxmi Naidoo

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Dedication

To my husband Lifeas K. Kapofu, with love, for without his understanding, support and prayer, this would not have been possible.

To my children, Chantelle and Christian, and my mother for their encouragement, time patience and support.

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Abbreviations

CAPS: Curriculum and Assessment Policy Statement

CDE: Centre for Development and Enterprise

FET: Further Education and Training

GET: General Education and Training

NCATE: National Council for Accreditation of Teacher Education

NCTM: National Council of Teachers of Mathematics

NRC: National Research Council

RNCS: Revised National Curriculum Statement

TIMSS: Third International Mathematics and Science Study

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Abstract

This study sought to ascertain whether the inclusion of the history of the Theorem of Pythagoras has any positive influence on learners' perceptions of the theorem and its applications, on geometry and on mathematics in general. To accomplish this aim, this study had to establish learner perceptions before and after the incorporation of the history of mathematics in the lessons on the Pythagorean Theorem. Changes in perceptions after the inclusion of the history of mathematics were regarded as indicative of the influence of the history of mathematics.

The study was informed by the constructivist's perspective and the genetic approach to teaching and learning. These played a pivotal role in the design of learning materials used during the lessons in which the history of mathematics was incorporated. A case study approach where grade 11 mathematics learners' perceptions on mathematics, geometry and Pythagoras' Theorem were sought, was used. Data on learners' perceptions was collected using qualitative data collection methods, that is, focus group discussions, group interviews and journal entries made by participants after being exposed to lessons in which the history of mathematics was integrated. The collected data was analysed using NVivo software.

A qualitative interpretation of analysed results indicated a change in learner perceptions of mathematics, geometry and the theorem of Pythagoras. The change was perceived to have occurred when learners were exposed to learning situations which incorporated the history of mathematics. Notable changes in learner perceptions included affirmations of the increased levels of motivation and claims of preparedness to work hard even in the face of adversity. Despite challenging tasks, learners now regarded mathematics and geometry as challenges that needed to be faced head-on and conquered. It also seemed, from the inclusion of the history of mathematics that learners had learnt that failure is part of the learning process. Learners also confirmed that they had gained confidence in dealing with proofs, enjoyed making their own discoveries and solving mathematical problems in general.

Findings from this study were crucial in that they revealed that the history of mathematics has great, yet untapped potential to resolve the challenges of learner apathy and facilitate greater rate of uptake in mathematics at FET phase in South Africa.

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CHAPTER 1

1.1 INTRODUCTION

The recurring economic challenge facing South Africa is a shortage of skilled manpower in the field of science and technology. Research findings indicate that this shortage stems from there being fewer graduates from mathematics related areas of study such as engineering fields and information technology from tertiary institutions (Alexander; Lotriet & Pieterse, 2014). As observed by Torerai (2013), this challenge has its roots in secondary school where most learners are opting for mathematical literacy rather than mathematics. Of these few, only a small percentage meets the university requirements where good passes in mathematics and science are pre-requisites (Alexander, Lotriet & Pieterse, 2014). This trend is unsustainable considering the current skills shortages with South Africa's information, communication and technology industry. These sentiments were echoed by the Minister of Higher and Tertiary Education in his statement: "South Africa is not only suffering skills shortage it has also destroyed the training capacity over the past decade and a half further complicating the situation" (Mantashe, 2008, p. 2)

It is therefore of paramount importance that mathematics educators and researchers take steps to redress this situation and in light of this, research such as this is, one small step in such an overture.

1.2 BACKGROUND TO STUDY

A dwindling interest in the mathematics related subjects is a concerning development in developing countries like South Africa. Statistics on South Africa show a steady decline in the number of learners who take up mathematics at grade ten, opting for the less challenging subject mathematical literacy (Torerai, 2013). The effect of this decline was observed by Radebe (2013) who noted that there were about 300 000 learners who wrote mathematics in 2011 compared to 225 000 in 2013. One of the reasons given for this decline is that the subject has been negatively perceived and regarded as difficult such that most learners fear that they will not pass it, despite the pass mark being as low as 30%. The results of 2013 where only 26.1% of those who sat matric mathematics examinations passed with 50% or more, indicates to a large extent the source of this fear and negative perceptions (Barry, 2014). Elaborating on this statistic (Barry, ibid) purports that the diagnostic report compiled by chief markers, internal moderators and subject specialists revealed that learners' algebraic skills were poor for example, they struggled with grade 11 and 12

mathematics because they could not perform basic grade 8 to 10 mathematical operations implying that they had also not mastered the skills at this level.

In the Third International Mathematics and Science Study (TIMSS) conducted in 1995 just after the advent of democracy, South African mathematics learners came last out of forty two countries, with a mean score of 351 (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996). This mean score was significantly lower than the international benchmark of 513. Less than 2% of these learners reached or exceeded the international mean score (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996). TIMSS conducted in 1999 revealed that grade 8 learners from South Africa once again performed poorly. This time, their mean score of 275 was below the international mean of 487. South Africa's mean was lower than that of some developing countries such as Morocco, Tunisia, Chile, Indonesia, Malaysia, and the Philippines (Howie, 2001; Naidoo, 2004; Mji & Makgato, 2006). In 2003 a decade into the democratic dispensation another TIMSS was conducted and no improvement was recorded for South African learners (Reddy, 2004). Eight years later, in 2011, the same international study further revealed a disconcerting picture in which South African learners were dismally outperformed by their peers from other countries with the same economic disposition as South Africa (McCarthy & Oliphant, 2013).

Table 1.1 South African Grade 9, 2011 TIMSS Mathematics results (McCarthy & Oliphant, 2013, p. 7)

Range	Percentage of scores
Less than 30%	91.9
30 – 39%	3.8
40 – 49%	2.1
50 – 59%	1.1
60 – 69%	0.6
70 – 79%	0.3
80 and over	0.2
Total	100

To illustrate these results, Table 1 has been extracted from McCarthy and Oliphant's 2013 report for CDE (Centre for Development and Enterprise). From Table 1 it can be noted that only about 2.2 percent of the South African learners scored 50% and above, almost 92 percent of the learners scored marks that were 30% or less.

Liu (2003) states that many people view mathematics as a rigid and dry subject mainly because of its rigorous and abstract features. The status quo as well as the potency of the history of mathematics as a pedagogical tool is presented by Speranza (1994, p. 238) who argues that:

> Mathematics lessons in secondary schools are very often not about anything. You collect like terms, or learn the laws of indices with no perception why anyone needs to do such things. Such procedural and instrumental approaches to mathematics … make it empty and dull.

Speranza (1994, ibid) highlighted that, many textbooks focus on developing learners' skills and concepts with very little or no reference to the history of mathematics. He further argues that the formation of learner attitudes towards and perception of mathematics is an essential aspect that is accommodated by the history of mathematics. Lakatos (1976, p. 142) posited that contemporary text materials usually present mathematical concepts in a superficial manner which "… hides the struggle, hides the adventure. The whole story vanishes".

This research sought to find ways by which the flame of interest in the subject may be rekindled in learners' minds and dispel some of the fears and insecurities associated with the uptake of mathematics at FET phase by using the history of mathematics as a pedagogical tool and as a vehicle for change of learner perceptions and attitude towards the subject and in the process work towards improving the quality of mathematics teaching.

The Department of Education and Training (DET, 2002, p. 6) maintains that "… the teaching and learning of mathematics can enable the learner to develop an awareness of the diverse historical, social and cultural practices of mathematics…", and (DET, 2005, p. 99) also gives the following as some of the grade 8 Assessment Standards, "Describes and illustrates the historical and cultural developments of numbers; Describes and illustrates ways of measuring in different cultures throughout history (e.g. determining right angles using knotted string leading to the Theorem of Pythagoras)", these standards acknowledge the importance of the history of mathematics in the

learning of mathematics. Such standards are encapsulated in the Curriculum and Assessment Policy Statement (CAPS, 2011) in one of its aims which states that mathematics education should be able to show mathematics as a human creation by including the history of mathematics.

An obvious effort has been made by some textbooks where authors include materials with historical flavour. For example, in Carter, Dunne, Morgan and Smuts (2006, p. 8) an illustration of the ancient Indian numerals used in the eastern part of the Arabic empire in the years between 969 and 1082 is given. Laridon (2006) on the other hand provides a summary of the origins of Pythagoras' theorem. Although these textbooks and the curriculum statement provide historical snippets, the history of mathematics seem not to have been fore grounded much in practice.

This research attempted to bring life back to mathematics in classrooms through the integration of the history of mathematics. This study postulates that through the use of the history of mathematics learners' perceptions of the subject may be improved or turned around, to some extent.

1.3 THE RATIONALE

For several years South Africa has been importing skilled human resources in the fields of science and technology from other countries. Ramsuran (2005) confirms this fact by highlighting that currently, South Africa has no capacity to expand economically without importing foreign scientific and technological expertise. According to findings by Dinaledi Zenex Foundation (2007), such a situation poses a threat to South Africa's future economic prosperity. This lack of capacity derives largely from the fact that many South African school leavers are not endowed with basic mathematical skills hence are unable to pursue science related careers. Thus, this research hopes to contribute in stemming this problem to some extent through using some content from history of mathematics to positively influence learners' perspectives on Pythagoras' Theorem, geometry and mathematics in general.

The study is undertaken from the view that if learners have the historical background information to concepts to be learnt, their perceptions of these concepts will be positively enhanced. According to the mathematics results analysis report (Department of Education, 2014) mathematics is a subject that depends on the understanding of basic concepts, if learners have not mastered the

basics, they get lost and confused easily which affects their attitude towards mathematics. In this study it is envisaged and contended that when learners have the right perspective on mathematics, the zeal to learn will grow thus igniting an inquisitive mind in the learner which is a prerequisite for success in mathematics.

1.4 PURPOSE OF STUDY

This study sought to explore learners' perceptions of the Theorem of Pythagoras. Despite the theorem being learnt and taught at GET phase, in this study it was investigated using grade 11 learners since it provides a foundation for sections such as analytical geometry and trigonometry dealt with at this level. The study is focused on ascertaining whether the inclusion of some historical content on Pythagoras' Theorem had any positive influence on learners' perceptions and perspectives of the theorem and its applications, geometry and mathematics in general. It also seeks to establish which aspects of using some historical content on Pythagoras' Theorem has most influence. This will be done through the use of the genetic approach to learning and teaching of the Theorem of Pythagoras while incorporating some historical content on the theorem and geometry.

1.5 RESEARCH OBJECTIVES

This study sought:

1. To explore learners' perceptions of the Pythagorean Theorem.
2. To determine the ways in the incorporation of some history of Pythagoras' Theorem that influence learners' perceptions of geometry in general as well as their perceptions of the theorem itself.
3. To establish what aspects of the use of some history of Pythagoras' Theorem have the most influence on learners' perceptions.

1.6 CRITICAL RESEARCH QUESTION

Can the history of mathematics serve as a pedagogical tool in teaching Pythagoras' Theorem at grade 11?

Sub-Research Questions

To address the critical question the following sub-research questions needed to be answered:

1. What are learners' perceptions of the Pythagorean Theorem?
2. How does the incorporation of the history of Pythagoras' Theorem influence learners' perceptions of geometry in general as well as their perceptions of the theorem itself?
3. What aspects of the use of the history of Pythagoras' Theorem have the most influence on learners' perceptions?

1.7 AN OVERVIEW OF UPCOMING CHAPTERS

In Chapter 2: A review of literature on history of mathematics as a pedagogical tool, history of geometry as a branch of mathematics and the teaching methods that are currently being employed in teaching of mathematics and some pedagogical interventions and their impart, in the South African context.

Chapter 3: A discussion of the theoretical framework of the study.

Chapter 4: A discussion focussing on research design and methodology outlining the nature of research, data collection and analysis techniques that will be used.

Chapter 5: Provides a chronicle and documentation of the extent to which the inclusion of some historical content on the Theorem of Pythagoras impacted learners' perspectives and perceptions of the theorem itself, geometry and mathematics in general, after careful analysis of the transcribed interviews and learners' journals.

Chapter 6: Comment on the implications of this study on how mathematics should be taught in South African classrooms and recommendations for future research in this field.

1.8 CONCLUSION

In this chapter a brief introduction, background to and purpose of this study were outlined as well as an overview of upcoming chapters.

CHAPTER 2
REVIEW OF LITERATURE

2.1 INTRODUCTION

Since this study considered the influence of the history of mathematics on learners' perceptions of geometry while teaching Pythagoras' Theorem, it is important to provide a background of the history of geometry and the pedagogy associated with it. The first section of this review considered the intersection of the history of mathematics and mathematics education. The second section placed geometry within mathematics education. The third section explored the history of geometry and the fourth section, considered the aspects in the use of the history of mathematics, the fifth section dealt with learning of geometry. Lastly in the sixth section pedagogy and geometry were explored.

2.2 THE HISTORY OF MATHEMATICS AND MATHEMATICS EDUCATION

In this study the history of mathematics is regarded as the study of past events and the evolution of mathematics, that is, the study of mathematical discoveries such as the theorems, postulates and axioms which are still in use today. History of mathematics has been characterised as a living science that "creates a bridge from the past to the future" (Reimer & Reimer, 1995, p. 107).

The importance and role of the history of mathematics in mathematics education has not escaped acknowledgement by international professional councils such as the National Council of Teachers of Mathematics (NCTM), the National Council for Accreditation of Teacher Education (NCATE) and the National Research Council (NRC). The centrality of history of mathematics in mathematics education has also been supported by research studies (Furinghetti, 2000; Siu, 2004; Weng Kin, 2008; Horton & Panasuk, 2013). Furinghetti (2000) purports that the history of mathematics is instrumental in the development of learners' conceptions of mathematics. Furinghetti (ibid) further argues that the history of mathematics is excellent for encouraging flexibility and open mindedness when dealing with mathematical problems. Horton and Panasuk (2013) support these sentiments by arguing that the history of mathematics provides a rich foundation for a more grounded understanding and conceptual development in mathematics. They further posit that the history of mathematics connects learners to the great mathematicians and

what they endured while making their inventions. Hence the history of mathematics may provide a new perspective about mathematics as a human endeavour.

Elaborating on the importance of the history of mathematics as a pedagogical tool, Horton and Panasuk (ibid) highlight that teaching mathematics as a conceptual isolate hedged from its history negatively impacts on the learning of mathematics through the deprivation of learners from experiencing the subject as a human construct evolved over centuries. Such a context that negates the origins of mathematics and its history has been associated with dislike and fear of the subject (Smith, 1996). On the other hand teaching that is characterised by the integration of the history of mathematics has been observed to allow learners to recognise and acknowledge the humanity of the subject through an exploration of diverse contexts and cultures that have contributed in the evolution of mathematical concepts (Grugnetti, 2000). Such teaching has been observed to inspire learners and stimulate classroom discussions, whilst reconstructing mathematics as a people-centred subject which deserves to be taught in like manner.

Bidwell (1993) recognises the ability of the history of mathematics to humanise mathematics which is perceived by learners as an island that is "…closed, dead, emotionless and all discovered…" (p. 461). Arguing on that same point Bidwell (ibid) posits that the integration of the history of mathematics in mathematics lessons may help rescue learners from this island to a mainland of open, lively, emotional and interesting mathematics. The importance of including the humanistic aspect is highlighted by Tymoczko (1993) when he argues that without the history of mathematics, educators would not be able to teach learners to appreciate, love or understand mathematical concepts. Swetz (1984) contends that historical content can breathe life into mathematics lessons, and if integrated appropriately into the teaching and learning of mathematics, history of mathematics may substantially humanise mathematics.

Exposing high school learners to the history of mathematics has been observed to have a profound effect on mathematics education. Scholars argue that the inclusion of mathematics at high school level has a profound effect because: firstly, it is at this level that learners begin to appreciate the conceptual intensity of mathematics and realise the extensive nature of its applications (Swetz, Fauvel, Bekken, Johansson and Katz, 1995). Secondly, through the integration of history of mathematics in mathematics lessons, learners may begin to feel that they are not alone in their

struggle with mathematical problems during the learning processes (Ernest, 1998; Fauvel, 1991) thus reducing mathematical anxiety and avoidance of the subject (Marshall, 2000). Thirdly, past obstacles in mathematics development may help explain what today learners find difficult to comprehend and articulate thus saving them from unnecessary anguish. Fourthly, through the study of the lives and work of some of the great mathematicians in history, learners may learn to appreciate the work ethic and requisite determination as well as the importance of tenacity in pursuance of one's dreams (Shortsberger, 2000; Tzanakis & Arcavi, 2000). Fifthly, history of mathematics has so many emotional instances whose inclusion in mathematics lessons have the capacity to captivate learners' attention and curiosity about mathematics (Barbin, 2000; Rubinstein & Schwartz, 2000). Lastly, learners may begin to appreciate how mathematical precepts, means of communication and conveying mathematical knowledge have endured many centuries of evolution and refinement (Tzanakis & Arcavi, 2000; Oliveira & Ponte, 1999).

It has been proposed by several scholars that integration of the history of mathematics in mathematics education has the following benefits: (i) raising learners' motivation and instilling positive dispositions towards mathematics, (ii) assisting in the resolution of difficult and confusing concepts that learners face, as they explore the historical development of mathematics, (iii) promoting the evolution of learners' mathematical thought and the capacity to reason through the use of historically- nuanced problems and (iv) provision of a pedagogical template by which practitioners may structure teaching contexts (Liu & Po-Hung, 2003; McBride & Rollins 1977; Wong, 2003, 2004). Even though according to Wilson and Chauvot (2000) use of the history of mathematics is instrumental for effective teaching and learning Ho (2008, p. 3) points out that so far "…little has been done to help the learners to develop a positive attitude towards the subject…" To have a positive attitude towards mathematics entails, possessing a specific mental schema and worldview and philosophy towards mathematics, having genuine interest and enjoyment in learning mathematics. It also includes the development of mathematical qualities, nurturing confidence in the application of mathematics and cultivating a tenacity and purposefulness in tackling mathematical problems.

The dominant perception amongst mathematics education researchers is that mathematics can be more interesting if the personalities of mathematicians are revealed with historical problems

awakening and maintaining interest in the subject. Through their work, researchers like, McBride and Rollins (1977), Ho (2008), Lim and Pang (2002), Ng (2006) have demonstrated the positive influence of the inclusion of the history of mathematics. In their study involving college algebra classes, McBride and Rollins (1977) observed a marked positive change in students' attitudes towards mathematics when history of mathematics was included. According to Ho (2008) learners who had experienced the lesson in which the history of mathematics was incorporated performed significantly better in their test scores than those who had not. In a separate study, Taiwanese learners after being exposed to a problem based course that utilised a historical approach were observed to have embraced active views about mathematical thinking which enabled them to adopt multiple approaches in solving given problems. These learners were also observed to be more willing to think and try out problems, (Liu 2003).

A review of literature however indicates no evidence that this kind of research has been done in South Africa. Considering all these positive and encouraging findings it is however interesting to ascertain whether within the South African context, the history of mathematics would yield similar results, which is, what prompted this study.

2.3 ASPECTS IN THE USE OF THE HISTORY OF MATHEMATICS

The history of mathematics may be incorporated into the mathematics curriculum taking cognisance of the importance of these aspects, that is, historical narratives, discovery learning, cooperative learning and the use of technology. These aspects in the use of the history of mathematics in this study were presented with historical narratives in subsection (2.3.1), discovery learning in subsection (2.3.2), cooperative learning in subsection (2.3.3) and technology in subsection (2.3.4).

2.3.1 The use of historical narratives within mathematics classrooms

According to Liu (2003), mathematical historical narratives reveal mathematical personalities in the history of mathematical development. De Villiers (2008) conceptualises historical narratives as an aspect of the history of mathematics that can be utilised in teaching mathematics. Many researchers of mathematics education and mathematics teachers are of the opinion that mathematics could be made more exciting by the inclusion of historical narratives (De Villiers,

2007; Ho, 2008; Horton & Panasuk, 2013; Liu, 2003). The value of historical narratives in the teaching of mathematics lies in their ability to reveal the human aspect of mathematics. According to Philippou and Christou (1998), through historical narratives learners discover the fallibility of great mathematicians, a discovery which sustains their perseverance in problem solving. According to these scholars historical narratives allow learners to realise that even great mathematicians made mistakes and did not always get the solutions right the first time. Arguing on the same point Liu (2003), posits that current conceptual difficulties experienced by learners can be resolved easily as learners are exposed to how past obstacles were overcame as mathematics developed. The overarching notion from the scholarly work above is that through the use of historical narratives learners are prompted to reflect on mathematical concepts and processes. Through this engagement learner motivation is aroused and sustained, whilst their confidence is built.

In Ho's (2008) research on: *Using history of mathematics in the teaching and learning of mathematics in Singapore*, findings illustrated benefits of historical narratives similar to those cited earlier. Ho (ibid) while teaching matrices, used the historical element of the *'Life story of forgetful Sylvester'* and on vectors he used '*Descartes' dream in the furnace and Descartes' secret notebook'*. In his findings he reported that after the use of historical content learners' perspective on mathematics was that it was interesting and learners were motivated to learn it. It was this study's endeavour to find out if the same impact would be experienced by learners when exposed to mathematical historical narratives in the South African context.

2.3.2 The use of discovery learning within mathematics classrooms
According to Freudenthal (1991), the history of mathematics may be learnt when learners make their own re-inventions of the axioms and theorems that have already been discovered in the development of mathematical concepts. This aspect of the history of mathematics De Villiers (2008), equates to discovery learning. According to Bruner (1961), discovery learning is an inquiry-based learning technique through which learners are encouraged actively question, formulate their solution and deduce principles from their lived experiences or from presented practical examples. In discovery learning the learner uses their experiences and prior knowledge to explore facts and relationships.

Bicknell-Holmes and Hoffman (2000), conceptualise discovery learning as a learner-driven technique grounded on tenets of exploration, problem-solving and generalisation of created knowledge. There are many merits to discovery learning as proposed by Bicknell-Holmes and Hoffman (ibid). These scholars argue that discovery learning ensures that learners are actively engaged in the learning activity. The scholars also posit that, discovery learning fosters curiosity and enables the development of life long skills. Bicknell-Holmes and Hoffman (2000), further maintain that in the discovery learning technique learners take ownership of and personalise their experiences. According to the two scholars discovery learning provides high motivation for the learners as they are involved in the discoveries.

While researching on the integration of the history of mathematics in mathematics, Goktepe and Ozdemir (2013) made the findings that learners enjoyed making their own discoveries as they challenged the learners' thinking capabilities. Another researcher Albayrak (2008), used the discovery learning aspect of the history of mathematics in teaching the volume of pyramids, cones and spheres. Findings were that the learners who were exposed to the history of mathematics had positive thoughts about the concepts taught as compared to the other learners who were not exposed. This study sought to discover if the inclusion of this aspect of the history of mathematics in the South African context, would yield similar results as those found elsewhere, the world over.

2.3.3 The use of cooperative learning within mathematics classrooms

Felder and Brent (2007) regard cooperative learning as an organised and structured way of using small groups to enhance learning and interdependence among learners. It is on this premise that in this study cooperative learning was understood as an instructional strategy in which learning takes place when learners work together on a common task in small groups. It is a technique that allows learners to learn from each other. In cooperative learning each individual has responsibilities and is held accountable for assisting in the completion of the task at hand. Success therefore depends on the cooperation of every member of the group.

Cooperative learning has been known to have several benefits: Firstly, it is interactive such that learners are engaged, active participants in the learning process. Secondly, cooperative learning

allows discussion and critical thinking, so the learners learn more and remember what they have learnt for a very long time. Thirdly, working cooperatively keeps the weak learners engaged whereas if they were working as individuals they were most likely to give up when they encountered difficulties. Fourthly, when strong learners are immersed in group tasks with their less academically endowed peers, it has been observed that their weak areas become exposed and hence are rendered opportunity to remedy gaps in their mathematics schema. Through learners' relationships and experiences with their peers among other things learners acquire knowledge and develop perceptions on mathematics (Anderson, 2007).

Learning in children awakens internal psycho-cognitive processes that are activated only when the children interact with significant others in their environment and as they actively engage with peers at the same level of psychological development (Vygotsky, 1978). He emphasised that children develop in a social net formed by their interactions and relationships with other children. Even though Vygotsky's (ibid) theories were built around children, the same theories hold for learners' development since they are children. Social environment gives learners an open area of communication. This allows them to express and discuss their ideas as well as contribute to each other's understanding. Observation in research has been that as learners scaffold each other's learning and capitalise on learning opportunities presented in group settings their creativity is triggered and have a greater likelihood of completing given tasks (Tharpe & Gallimore, 1988). The premise behind this reasoning is that, learners who understand a task and are more capable will assist the less capable ones and in the end all the learners would have understood the concept. The more capable learner will build on the competences of the less capable learners.

Thus, this study set out to establish whether or not the use of collaborative learning as an aspect of the history of mathematics would have the same impact on learners in South Africa.

2.3.4 The use of technology within mathematics classrooms

Even though technology on its own is not a direct aspect of the history of mathematics, De Villiers (2008), contends that the history of mathematics can be broadened to include non-historical material as long as it is based on an analysis of the historical development of that particular concept

or theorem. As such it was for this reason that I incorporated technology as an aspect of the history of mathematics.

The use of technological devices has become the order of the day in homes and at work places. Research has shown that technological devices have pedagogical currency and enhance learning opportunities when considered in teaching-context development (Means & Haertel, 2004). Technological devices when appropriately incorporated can reduce inauthentic labour that is characteristic of tedious computations and as such maximise opportunities for learners to focus on target concepts and engagements that lead to conceptual development in mathematics. Thus, technology has the potential to present mathematics in ways that help learners understand learnt concepts (Kaput, 2007).

In his research, Yevdokimov (2006) investigated the use of e-book in the incorporation of the history of mathematics. His findings were that the use of electronic learning textbook problems in the history of mathematics presented learners with opportunities for modelling mathematical problems.

Bearing in mind the advantages of integrating technology in lessons, even though there were no computers at the school where the study was conducted some Geogebra application software was used in presenting some materials on the proofs of Pythagoras' Theorem. This study sought to discover the influence of this aspect of the history of mathematics on learner perceptions of mathematics, geometry and Pythagoras' Theorem.

2.4 GEOMETRY IN MATHEMATICS EDUCATION

According to the Tabak (2004) geometry is the study of those geometric properties of a figure that do not change when the shape or object is rotated or translated. This definition is in many ways similar to Bassarear's (2012) conceptualisation of geometry as the study of shapes, how these shapes are related and their properties. Geometric properties include points, lines, planes, angles, curves, surfaces and bodies in three or more dimensions. Felix Klein (1872) regarded geometry as the study of those properties of the given geometric space that remain invariant under certain group transformations. In other words, every geometry is the invariant theory of the given transformation

group (Klein, ibid). This means that there is a geometry corresponding to every group of transformations acting on a space. Simple and complex, concrete and abstract, all these contrasting descriptions characterise geometry. It is a discipline whose history is as old as civilisation.

Geometry is an important discipline of mathematics and has been identified as a field of mathematics with immense potential of making the subject alive (Chambers, 2008). It involves mathematical exploration and connects mathematics with the domains of culture, history, art and design. According to Chambers (2008), the inherent capacity of geometry in facilitating learner motivation lies in its synergy with these vital human constructs. A general consensus exists that learning of and constructions in geometry enhance the development of the thinking abilities in learners (Hanna, 1998). The view that constructions in geometry enhance development of thinking abilities in learners is upheld in the mathematics curriculum (2005). In this document it is argued that, reasoning is fundamental to mathematical activity and active learners of mathematics must be able to question, examine, conjecture and experiment and these skills can be acquired through learning geometry. This promotes logical thinking in learners which trains the mind in clear and rigorous thinking. Once learners have mastered the skill to logical and clear thinking they can then also apply it to other domains, (Gonzalez and Herbst, 2006). When learners carry out geometry proofs, they are provided with the opportunity to experience mathematical activity similar to that of mathematicians, which is innovation and discovery (Gonzalez and Herbst, ibid). Concurring with this view, the NCTM (2000) argues that learning geometry enhances learners' ability to analyse three dimensional geometric shapes and develops the ability to construct and interpret mathematical arguments. Hence it provides them with a rare opportunity to apply their understandings of geometric objects to novel situations outside the immediate teaching-learning context.

Van De Walle (1994) outlines some of the benefits of learning geometry. Firstly, geometry helps learners better appreciate the world they live in, that is, the solar system and the synthetic universe. Secondly, geometric exploration may develop problem solving skills such as spatial reasoning. Thirdly, geometry facilitates the study of other areas of mathematics such as ratio and proportion. Finally, geometry can also be used by people in their professional lives, for example artists, land developers, home designers and decorators and lastly geometry is fun and enjoyable and can be

used to entice learners to appreciate learning mathematics. Gonzales and Herbst (2006) add on positing that geometry allows learners to connect with the real world if their experiences are matched with the demands of their careers.

According to Sarama and Clements (2009) there is no mathematical component more permeating than geometry, which they purport lies at the heart of natural science disciplines as well as art and architecture. It also lies at the heart of mathematics. Hence the centrality of geometry in mathematics education cannot be over emphasised. To show how important geometry is to mathematics education in South Africa, CAPS (2011) has re-introduced Euclidean Geometry in the FET phase main stream mathematics curriculum.

Considering all this, it is therefore paramount that geometry be effectively taught and learners develop an understanding and liking of geometrical content. It can also then be argued that teaching approaches need to encourage logical reasoning and development of skills to apply geometry through modelling and problem solving in novel situations among other things. It is in this regard that a historical approach in teaching geometry needs to be explored. This calls for a deeper exploration of what geometry is.

2.5 THE STORY OF GEOMETRY: THALES TO EUCLID

Since in this study, an experiment is to be carried out on the influence of the history of mathematics on learners' perceptions of geometry while teaching the Pythagoras' Theorem, it is important to provide a brief background of geometry. According to Eves (1976, p. 167) the word "*geometry*" means "*measurement of the earth*", it is derived from the Greek roots *"geo"* meaning *"earth"* and *"metrein"* meaning *"measure"* . The general assumption is that scientific geometry arose from practical necessity to assist engineering and agricultural pursuits (Eves, ibid). For example, for the Egyptians the need to develop fast and accurate surveying techniques was the motivation for the development of their geometry. This need resulted in the evolution of geometric techniques that were a precursor of modern mensuration and consisted of concepts and techniques involved in measurement (Eves, ibid). He also suggests that geometry arose from the practical necessity to assist engineering and agricultural pursuits. In Cajori (1907, p. 11) the famous Greek historian Herodotus, of the fifth century B.C., further confirms such origins when he writes:

> They said also that this king [Sesostris] divided the land among all Egyptians so as to give each one a quadrangle of equal size… But everyone from whose part the river tore away… He then sent the overseers, who had to measure out by how much the land had become smaller, in order that the owner might pay on what was left, in proportion to the entire tax imposed. In this way, it appears to me, geometry originated…

Early Egyptian surveying practices thus marked the beginning of geometry as a science. According to (Danziger, 2004) geometric civilisation occurred along the Nile river in Egypt, the Hwang Ho and Yangtze of eastern Asia, the Ganges and Indus of south-central Asia and the Euphrates and Tigris of Mesopotamia to mention a few. Eves (1976) noted that Egyptian interests in geometry did not extend much beyond what was needed for practical purposes. They developed formulas, some of which were more accurate than others, to measure certain simple areas and simple volumes. The skill with which they did this made a great impression on their neighbours the Greeks. From Egypt the Greek philosopher and mathematician Thales of Miletus (650 – 546 B.C.E.) introduced geometric ideas to Greece which led to the evolution of Greek deductive proofs. Thales is generally accredited as being the first mathematician to use deductive proof in a modern sense and the first of the "*Seven Wise Men*" (Danziger, 2004). According to Greek accounts, he was the first in a long chain of Greek mathematicians and philosophers. Lamphier (2004, p. 2) credits Thales with the following five geometric propositions and their proofs. The propositions are:

i.   A circle is bisected by any diameter.
ii. The base angles of an isosceles triangle are equal.
iii. The vertical angles between two intersecting straight lines are equal.
iv. Two triangles are congruent if they have two angles and one side equal.
v.   An angle in a semicircle is a right angle.

The Pythagorean Theorem is one of the most important propositions in the entire world of geometry. The theorem was named after Pythagoras (572 B.C.) born on the island of Samos. It is however claimed that the theorem was discovered well before his time, in Egypt and Mesopotamia, not to mention China. Pythagoras founded a scholarly society called the Pythagorean brotherhood. It is the Pythagorean Society that is credited with producing the first known proof of the Pythagorean Theorem (Burton, 2011). According to Davis and Hersh (1980, p. 147) the theorem

states that, *"In right-angled triangles the square on the side opposite the right angle equal the sum of the squares on the sides containing the right angle"* Proposition 47 of Euclid's 'Elements' Book 1. The Pythagorean Theorem has two fundamental aspects; one is about areas and the other is about lengths. Hence this landmark theorem connects Geometry and Algebra. The converse of Pythagoras's theorem is also true. According to Burton (2011) it states that *"If in a triangle the square on one of the sides equals the sum of the squares on the remaining two sides of the triangle, the angle contained by the remaining two sides of the triangle is right"* (p. 158). It was first mentioned and proved by Euclid (Danziger, 2004).

Hippocrates of Chios born around (440 B.C.) is credited with two contributions to geometry. One of which is the composition of the Elements of Geometry which was the first attempt at developing theorems of geometry precisely and logically from given axioms and postulates. Plato lived between (427 – 347 B.C.) and studied mathematics under Theodorus, of Cyrene in North Africa. His influence in mathematics is largely due to his conviction that the study of mathematics provided rigours for training the mind. The famous sign above his academy door read *"let no one ignorant of geometry enter here"*.

As Tabak (2004) posits, geometry of the ancient Greeks, is called Euclidean geometry, named after the prominent Greek mathematician, Euclid of Alexandria (300 – B.C.E.). According to Tabak (ibid) Euclid's genius was not so much in inventing new mathematics but it was in systematising the known Greek geometry of that time into an axiomatic-deductive system with the publication of his *"Elements"*. The publication was divided into thirteen books that contain four hundred and sixty five propositions from plane and solid geometry to number theory. He is one of the best known mathematicians in history, to be more precise, Euclid is one of the best-known names in the history of mathematics (Tabak, ibid). Ancient Greek geometry is a record of mathematical excellence and longevity that no other culture has ever matched. The creation of the idea of deductive proof, and working hard to place geometry on a firm logical foundation, are all Greek attributes (Danziger, 2004). Starting almost from scratch, the Greeks asked profound questions about the nature of mathematics and what it entailed to understand mathematics. They invented the notion of proof, and worked tirelessly to position geometry on a firm logical foundation. Many Greek mathematical discoveries, the proofs as well as the statements of the

results themselves still sound modern to today's reader. Today mathematicians recognise Euclidean geometry as one of the many types of geometry that are in existence.

2.6 LEARNING GEOMETRY

There are many definitions of learning. According to Merriam Webster's learners' dictionary (2016) learning is a process by which behavioural modification is acquired through experience. Van Rossam and Hamer (2010, p. 2), described five way in which learning may be conceptualised, these have been outlined as:

    i.   Learning as the increase of knowledge

    ii.   Learning as memorising

    iii.   Learning as the acquisition of facts, procedures etcetera, which can be retained and / or utilised in practice

    iv.   Learning as the abstraction of meaning

    v.   Learning as an interpretive process aimed at the understanding of reality.

In this study however learning is understood as the acquisition of conceptual understanding and skills that arises from experience, deliberate study or being subjected to training. This definition subscribes to the definitions cited already and to the Oxford dictionary's (2006) definition of the verb to 'learn' as meaning, to gain knowledge of or skill in something through either studying, experiences or being taught.

As alluded to above geometry is an important branch of mathematics as it provides opportunities for people to comprehend the world by comparing shapes, their relatedness and their properties (Gunhan, 2014). It is for this reason that the study of geometry becomes fundamental. This perspective is shared by Luneta (2015), who further posits that the applicability of geometry in real life, makes it a central aspect of mathematics that should be learnt and mastered by learners. It is of paramount importance to learn geometry for geometry itself, provides a culturally and historically rich context within which mathematics can be done (Jones, 2002). To add on to these characteristics, geometry has links with art and design.

The existence of such qualities in this section of mathematics paves a way for it to be possible for learners to engage in exploration activities making geometry motivating with a capacity to arouse

learner interest (Chambers, 2008). With this capacity geometry is instrumental in bringing the subject to life (Chambers, ibid). Hence when geometry is presented to learners, it is important that it be done in a way that arouses curiosity and encourages exploration. This method of disseminating geometric knowledge, Jones (2002), believes may enhance learners' learning capabilities and improve on their attitudes towards geometry. For the teacher to achieve this goal he or she should be able to recognise interesting geometrical problems and theorems, furthermore he or she should also have an appreciation of the history and cultural context of geometry.

In order to have a clear understanding of how geometric concepts may be learnt it is important that the Van Hiele's model on the levels of geometric thinking be examined. In the field of geometry, Van Hiele's is one of the most well defined models for learners' levels of thinking (Senk, 1989). According to Abdul Halim and Effandi (2012) the levels are visualisation and recognition, analysis, informal deduction, formal deduction and rigor. Abdul Halim and Effandi (ibid) contend that in level zero, the first level is visualisation and recognition. According to van Hiele's levels of geometric thinking, learners at the visualisation level can recognise shapes by appearance but do not as yet perceive the properties of these figures. The second level, level one of the Van Hiele's model is analysis. The analysis stage is predominantly descriptive. At level one, Abdul Halim and Effandi (2012) purport that learners are able to identify and name properties of certain geometric shapes. At this level however learners are not conscious of the relationships between these properties. At the second level, a learner may be able to list most or all of the properties of a geometric figure but may not be able to discern which properties best describe the figure.

The third level is known as informal deduction or abstraction, at which level learners are able to comprehend relationships between properties and corresponding geometric shapes. At this level learners can meaningfully define and present non-formal arguments for their reasoning. Learners at this level, however, do not understand the meaning and importance of formal deduction. The fourth level of the Van Hiele's model for learners' levels of thinking is formal deduction. Learners at this level can operate and make formal deductions and can elucidate axioms, proofs, postulates and theorems. Learners at level four, due to their ability for formal reasoning can construct proofs and know the meaning of necessary and sufficient conditions.

The fifth and final level four of the Van Hiele's model is rigour. Learners at level five can understand aspects of deduction that is, they can establish and compare mathematical systems. They are able to employ indirect proof and proof by contrapositive. They can also master non-Euclidean mathematical conventions.

Van Hiele (1984) also identified some generalities that characterise the model. The first property is that the levels are sequential. This means that a learner has to go through all the levels in the order discussed above, they cannot skip a level. According to Van Hiele (ibid) these levels are progressive, hence a learner has to acquire competency in one level before moving on to the one above it. The second property is advancement, progress or lack of progress from level to level depends largely on the content and methods of instruction and not on the ages of the learners. The third property is intrinsic or extrinsic, that is the inherent object at a preceding level becomes the object of study at the level above it. The fourth level is linguistics, meaning that each level has its own language symbols and relational systems connecting all the symbols at that level. This implies that a relation that may be correct at one level may be altered at another. The fifth property is mismatch, the learner and the instruction must be at the same level for the desired learning and progress to occur. These properties may be useful to the educator when making instructional decisions.

According to the Van Hiele's (1984) model, moving from one level to the next depends mainly on the teaching approach used than on the age of the learner. When the traditional approach is employed, the learners cannot perceive, as the teacher, in a geometric situation, hence they do not benefit much from such modes of learning. Research has it that when such a method is used most lower secondary learners perform at the first and second levels with almost 40 percent of learners completing secondary school before they reach level two (Jones, 2002). Van Hiele contended that by using the traditional approach the level of geometric thinking of secondary school learners will not reach the desired level. According to this model it is impossible for a learner to skip a level. Usiskin's (1982) research findings revealed that 70 percent who graduate from their school were either in the first, second or third level of the Van Hiele's levels of thinking and were not at the desired levels that is, fourth and fifth. Usiskin (ibid) discovered that almost half of the learners

managed to finish their secondary school's curriculum with their geometric thinking at the level of primary school learners.

Geometry can however be learnt effectively if in lesson preparation the teacher bears in mind and highlights where appropriate the key ideas in geometry that are invariance, symmetry and transformation (Jones, ibid). He argues that symmetry can be used to simplify and make arguments more powerful. Jones also suggests the use of dynamic software as a tool that may help captivate learners and stimulate interest in them. Jones (2002) argues that effective teaching methods encourage learners to recognise connections between representations of geometric ideas in geometry and other areas of mathematics. According to (Jones, ibid) such approaches and innovations in teaching foster learner retention of mathematical knowledge and skills which equips them with the capacity to deal with new problems confidently. It is therefore important that geometry be effectively learnt if learners are to realise the fruits of learning geometry. With such a model of learning the question that is aroused then becomes, can the history of mathematics address the various levels of the Van Hiele's model?

2.7 PEDAGOGY AND GEOMETRY

Within the context of this study pedagogy is understood as the method and practice of teaching. This definition subscribes to Alexander's (2003, p.3), definition which states that "pedagogy is the act of teaching together with its attendant discourse". Leach and Moon (1999) go a step further to define a pedagogical setting as "the practice that a teacher, together with a particular group of learners creates, enacts and experiences" (p. 267). This definition suggests that learners are active participants in the teaching and learning processes and should not be excluded in the formation of concepts. Bhowmik, Banerjee and Banerjee (2013) emphasise that pedagogy should involve a wide range of teaching strategies that enhance intellectual involvement. Bhowmik, Banerjee and Banerjee (ibid) contend further that pedagogical practice improves learners' and educators' confidence, it adds on to their sense of purpose for their presence at school. Thus enhancing community confidence in the quality of learning and teaching in school.

Research institutions such as the Royal Society Mathematical Council of the United Kingdom (2001) and Atiyah (2001) recognise geometry as one of the most as a crucial aspect of mathematics

education and mathematics itself. For this reason it can be argued that teaching strategies need to encourage the development of deductive reasoning, development of skills for utilising geometry in solving problems in a range of contexts (Jones, 2000). According to Jones (ibid) such teaching strategies raise and recognise the contributions made by geometry in the historical and cultural development of the society.

Over the past three decades mathematics education has promoted developing an understanding of geometric concepts, procedures and applications through problem solving (NCTM, 2000). Research observation has been that teacher-centred approaches which are stoic and fact-laden are the mainstay of geometry teaching in schools (Indradevi, 1998; Lim & Hwa, 2007). This implies that learners are just passive recipients. Regarding the South African context, Wessels (2001) concedes that Euclidean geometry is a complete disaster mainly because it is badly taught, to which Fish (1996) agrees, lamenting that teachers are poorly equipped to teach geometry hence their failure to transmit geometric content to learners. Researchers in South Africa have noted the dominance of traditional teaching approaches, dominated by the teacher and where learners are passive recipients of knowledge (De Villiers, 1997). Such scholars have attributed the poor understanding of geometry among South African learners to such approaches.

Freudenthal (1973) strongly criticised the traditional practice of providing geometry definitions to learners arguing that learners should be involved in the formulation of geometric definitions and building of concepts. Ohtani (1996) perceived and argued that the traditional approach of simply giving definitions to learners by teachers is just so that the teachers may have control over learners, achieve a degree of uniformity in learners' work and avoid problematic interactions with the learners. Earlier on making the same argument Vinner (1991) posited that conceptual understanding could not be based on a superficial recollection of geometric definitions.

To remedy such shortcomings inductive approaches and / or investigative approaches have been recommended by other mathematics educators such as Serra (1997) to replace the traditional teaching approach that emphasises the mastery of content without the development of skills and learners' critical thinking capabilities. Like several researchers who have attempted to develop different pedagogical strategies that may improve learners' understanding of geometry in South

Africa (Siyepu & Mtonjeni, 2014), this research seeks to develop one such strategy through the use of some content from the history of mathematics in teaching the Pythagorean Theorem.

2.8 CONCLUSION

This chapter sought to provides an overview of the literature on (i) history of mathematics and mathematics education, (ii) geometry in mathematics education, (iii) the history of geometry: from Thales to Euclid, (iv) learning and geometry and (v) pedagogy and geometry. The next chapter places this work within a theoretical framework which will be discussed.

CHAPTER 3

THEORETICAL FRAMEWORK

3.1 INTRODUCTION

The theoretical framework within which this research was embedded was the genetic approach with emerging themes from a constructivists' perspective. The genetic approach and the constructivists' perspective were conflated in this study as they seemed to subscribe to the same general ideology relating to how learners acquire knowledge. On the basis of this theory overlap the theoretical framework for this research was constructivist in nature with reconstructive perspectives.

3.2 OPTING FOR A DIFFERENT LENS – THE GENETIC APPROACH

The genetic approach is one of the four ways through which the history of mathematics has been integrated in mathematics education. De Villiers (2008) describes four ways in which the history of mathematics may be integrated by classroom teachers. Such ways according to De Villiers (ibid), include: (i) teachers making complete presentations of the historical development of mathematical concepts or precepts; (ii) teacher teaching mathematics using sequenced annotated presentations of the mathematical milestones; (iii) the teaching of mathematics using historically-nuanced simulations other than the actual developmental trends. According to De Villiers such historically-nuanced simulations in the classroom provide the advantage of hindsight in the exploration of the development of mathematical thought and processes; (iv) making presentations devoid of historical material but premised on an analytical exploration of the historical progression of mathematical thought, an approach De Villiers (2008) refers to as the *genetic approach*.

It is, the genetic approach which was chosen for this study because it encompasses the other three approaches. This approach posits that individuals acquire mathematical knowledge in the same way in which mankind developed it (Polya, 1963; Freudenthal, 1991), that is, through discovery or invention. Thus, mathematical knowledge within the current context may be developed or enhanced when instruction is structured in such a way that learners undergo what their forerunners experienced when they invented mathematical theorems, axioms concepts and algorithms. This aspect framed the general scope of this study as well as its methods. With regard to scope it entailed that learners had to be presented with tasks that prompted them to make their own discoveries

about Pythagoras' Theorem, and opportunities to apply the theorem to a variety of interesting and challenging situations. I hasten to point out that in the study's overall scope it was not necessary for participants to experience all the motions their predecessors (original discoverers) went through, but only the aspects that were relevant and applicable to their context and curriculum were carefully selected.

The overall spirit of the genetic approach was reconstructive such that Human (1978, p. 182) referred to it as the "reconstructive approach". In this approach it is contended that mathematical content should not be directly presented to learners as a finished, polished terminal product of mathematical engagement, but rather the content should be newly re-constructively packaged during teaching and learning. This aspect influenced and informed lesson structure as well as the design of learning activities designed to incorporate the history of mathematics. From this perspective learners were presented with learning situations in which they participated actively in the construction and development of the Pythagoras' Theorem and concepts like Pythagorean triples, distance formula and other related concepts in mathematical activities in this study.

Freudenthal (1991) identified the principal tenets of the genetic approach as rediscovery or reinvention, which means teaching of mathematics in which discoveries stand for what they really are, that is, for discoveries. The implication of these tenets for this research study was that learners had to be actively involved in the re-invention or re-discovery process that the mathematicians and scholars who discovered and proved the Pythagorean Theorem experienced. Activities were designed and presented in such a way that learners' experiences during their discoveries made them part of the creation of mathematical content, thus the learning process in this context was just like a re-invention process all over again. As an example in Task 1 and 2 of the learning activities learners invented the Pythagorean Theorem through a series of designed group and individual cut and paste and written activities. Learners were presented with tasks which led them to invent the theorem themselves and the teacher's function in the classroom context became that of facilitator of the learning experiences. In assuming this role and guided by the genetic approach the teacher and learners ceased to slavishly adhere to the historical order of mathematical development of the Pythagorean Theorem but to the broad general lines along which creative mathematical thought had evolved, and along which learners were most naturally likely to learn.

The overall impetus of the research was pedagogical improvement and thus the research pursuit was an exploration of how teaching could follow an improved and better guided path hence the genetic approach. According to Safuanov (2005) the genetic teaching of mathematics has the following characteristics which were crucial and framed the instruments and learning activities design. Firstly, the genetic approach based teaching has to be based on learners' existing knowledge, their previous experiences and their levels of thinking. Taking this into consideration, when creating the learning activities (Tasks 1-5) learners' previous experiences and pre-existing knowledge were taken into account and their levels of thinking put into consideration.

Secondly, the genetic approach based teaching has to involve variability in terms of context structure and contextual mathematical and non-mathematical problems which have to resonate with learners' experiences that are to be used for the study of new concepts. When tasks on application of Pythagoras' Theorem were designed in this study a wide range of contextual problems were chosen for the learners to solve. Under a genetic paradigm the teacher should give minimum but effective assistance during lessons. Cognisant of this positionality of the teacher in most of the tasks given to learners in this study, the teacher took the role of the facilitator allowing learners to investigate and discover the theorem on their own.

Thirdly, under the genetic paradigm learners should be constantly motivated to ensure mental stimulation and cognitive activity. Activities such as the one on Bhaskara (Task 1.1), Rene Descartes (Task 3.1), and the cut and paste tasks were framed by this idea. The activities were carefully chosen because of their ability to have participants identifying with the mathematicians thus keeping them interested, motivated, and actively involved. Also during this teaching experiment, problem-oriented teaching, continuity and visual representations, enrichment and transformation principles of genetic teaching as suggested in genetic-based pedagogical approaches were adopted.

The section above sought to provide an outline of how the history of mathematics in the form of the genetic approach provided the philosophical foundations that influenced the scope and the design process of this study. These philosophies dove-tailed with the second framework that was

conflated with the genetic approach, and rendered the nature of this research to be constructive and reconstructive, which is the constructivist philosophy.

3.3 THEMES FROM CONSTRUCTIVISM

This research study sought to explore learner perceptions and perspectives of the Pythagoras' Theorem. The belief and possibility of this exploration was premised on constructivist ideas, thus the learning philosophy that framed the study was constructivism. Constructivism has been regarded in two main ways, that is, as a philosophy and a perspective of learning. Glasersfeld (1989) managed to conjugate the two views of constructivism and this conjugated view was the one adopted and framed this study. According to Glasersfeld (1989) constructivism encompasses psychological theories with the same epistemological foundations on which assumptions about knowing and learning as well as epistemological acquisition that emphasises how knowledge is constructed rather than how it is transmitted are premised. According to Glasersfeld (ibid) all learning involves mental constructions as a learner creates and adjusts his internal mental schema to accommodate his ever growing and ever-evolving epistemic banks. Hence, according to constructivists, learners construct their own knowledge and do not just, mirror or reflect what they have been taught or what they have read.

These ideas provided thinking tools which were critical both in research design and analysis as well as the teaching approaches that were opted for in this research. By construing learners as active and not passive recipients of knowledge constructivism provided a link between the genetic approach which was the essence of history of mathematics in this study with the teaching and learning context of this study. A principal tenet of constructivism is that learners are not blank slates and as such are endowed with ideas about all phenomena they encounter both familiar and unfamiliar. Thus, in the design of this study with this idea, constructivism provided a frame through which learner ideas could be solicited both before and after intervention using the genetic approach.

Another of the constructivists' ideas is that all knowledge is unique to the individual despite the method through which it is acquired. This may be taken to imply that learners in the same geometry lesson do not construct the same knowledge of geometry even though they were exposed to the

same teaching strategies. This led to the solicitation of individual learner perceptions and perspectives on the Theorem of Pythagoras through the use of journals which the whole class kept for the entire duration of the data collection period. Also in the focus group discussions held, each participant was given an opportunity to share their perceptions and perspectives because using a constructivist frame, every learner's input was invaluable.

Constructivists view learning as an active process governed by discovering principles involving harnessing and channelling of concepts and facts by the learners to the generation of new schemas (Ackerman, 1996). This view resonated with the genetic approach and as such was invoked and framed the design of learning tasks in which learners worked in groups, as individuals and as a class in a variety of tasks which led them to the re-discovery of the Theorem of Pythagoras and its applications. The tasks were structured in such a way that learners worked on their own with minimum assistance from the teacher. Such an approach was in line with the constructivists' view of a teacher, that is, her role needed to be that of a facilitator, meaning that I needed to help learners get their own understanding of the content.

Constructivists argue that even though knowledge is in some way personal and individualistic learners construct their knowledge through their interaction with the physical world and their involvement in social and cultural settings. Learning can be viewed in this respect as a social process. Learners learn from each other and the teacher through discussions, communication and sharing ideas. During data collection I took this into cognisance and organised focus group discussions as opposed to interviewing individuals since I believed that participants learn better when they actively engage with their own worldviews through reflecting on their own thinking and trying reconcile their thinking with that of their peers and by striving to get a common and shared meaning.

Thus, together with the genetic approach, constructivism provided the thinking tools and a lens through which this research study was viewed and conducted. These two formed the philosophical reasons that governed my thought patterns and provided theoretical justifications for the manner in which this study was done. Conducting this study however, was not just governed by philosophies but also by other thinking tools in the form of concepts. Below I present some of

these concepts which were critical in making sense of the research approach, paradigm, methodology and methods.

3.4 THE CONCEPTUAL FRAMEWORK - PERCEPTIONS AND UNDERSTANDING

This conceptual framework presents concepts I considered critical in this study. The close relationship between learners' mathematical perceptions and perspectives and their learning of the subject has been widely acknowledged (for example, Pehkonen and Torner, 1998; Schoenfeld, 1992). As such within this conceptual framework I explore and describe perceptions as a concept which was important for this study. This study sought to explore what learner perceptions were and how these perceptions were influenced by the integration of history of mathematics vis-à-vis the Pythagoras' Theorem. Hence, perceptions were the fundamental variable for this study and their conceptualisation study was important. In this section I explore perceptions, firstly, their nature, then their importance and factors affecting them and lastly how perceptions change.

3.4.1 Nature of perceptions

The basic idea rooted for in constructivism is that learners bring their own ideas to a lesson. The ideas learners bring are built from socially shared mathematical experiences, contextually shared mathematical experiences and their previous mathematical experiences. These ideas are captured in their perceptions (Sharma, 2015). The symbiosis between learners' perceptions on mathematics and their learning of mathematics has been widely acknowledged (Pehkonen and Torner, 1998; Schoenfeld, 1992). My understanding of perception is based on the ideas of Sharma (2015) who views perception as the process by which individuals organise and interpret the signals they receive through their sensory organs to give meaning to their environment. Individuals are unique thus they hold different perceptions of the same object and what they perceive may not be an exact duplicate of reality. Taking this into cognisance the way learners viewed Pythagoras' Theorem as solicited in data collection was regarded as their perceptions of the theorem. It was with this understanding that this study sought to establish learners' perceptions of Pythagoras' Theorem before and after exposure to some lessons on Pythagoras' Theorem in which the history of the theorem had been included. This was done initially to establish what the learners' perceptions of Pythagoras' Theorem were. After integration of history of Pythagoras' Theorem the aim became to see how the incorporation of the history of Pythagoras' Theorem had influenced learners'

perceptions and which aspects of the history of Pythagoras' Theorem had the most influence on these learners' perceptions.

The fact that perceptions form the basis upon which individuals give meaning to their environment renders perceptions as a schema which connects it with knowledge and understanding. When connected to understanding, perceptions have quantitative, hence measurable attributes and qualitative attributes that can be deciphered and qualified based on the intricacy of the connections that an individual is able to make (Van de Walle, 2004). According to Van de Walle (ibid) these connections are incumbent and dependant on the proliferation of novel and appropriate ideas as well as the creation of new connections. Hence by implication from a genetic-cum-constructivist perspective learners have some pre-existing knowledge embodied in their perceptions and are therefore not blank slates. This means that by nature perceptions are knowledge connections which constitute individuals' schema or body of knowledge.

According to Hiebert and Carpenter (1992) conceptual understanding in mathematics is synonymous with the building of mathematical structures or mathematical schema. In trying to comprehend this definition it can be argued that mathematics as the whole can be contemplated and understood through a mastery of its parts. Thus, the degree of one's understanding of a target concept, idea or procedure largely hinges on the number and strength of its connections to existing networks - perception Hiebert and Carpenter (ibid). According to Grossman (1986) this means that he or she uses the existing schema to give meaning to new experiences. Further, commenting on the nature and location of perceptions from a constructivist position, Skemp (1979) remarks that conceptual structures are a major factor of human progress. He is of the notion that, since our existing conceptual structures serve either to promote or restrict the assimilation of new concepts, then the quality of what learners already know is an important factor in their ability to understand. In support of this argument Nickerson (1985) posits that a person is more likely to understand better if he embeds a new concept in a rich conceptual context. The implication of this understanding on research rationale and design was that mathematics lessons needed to be built on the mathematical knowledge that the learners already possessed in their schema. This conceptual understanding was also foregrounded in the first research question and was the basis upon which the first phase of this study was premised.

3.4.2 How perceptions change

Perceptions are not static but rather dynamic progressing in cycles of constructive reconstruction (Grossman, 1986). Such dynamic cycles cause perceptions to change. Within teaching and learning contexts learners add new knowledge to the pre-existing body of knowledge through the integration of new understandings in existent cognitive schemas, a process which constitutes a perceptual change. As a result epistemologically knowledge construction occurs as learners try to organise, structure and restructure their perceptions in light of available perceptual structures. At the end of this restructuring a new schema emerges which will be transformed as new stimuli are encountered. The manner in which perceptions change is suggested by Grossman (1986) who argues that when a person understands something he or she assimilates it into an appropriate schema. When this happens the individual forms more numerous strong connections between the new concept and the already existing networks in his schema thus rearranging it. He argues further that the re-organisation of schema and bridge-formation is intertwined with formation and changes in perceptions. Nickerson (1985) affirms this when he posits that perception formation in our daily lives is enhanced by the learner's ability to structure conceptual bridges transcending conceptual domains. Emphasising the importance of enriching learning environments Wong (2001) suggests that learner experiences in mathematics lessons have an influence on the formation of their view on mathematics, which means that different experiences in mathematics lessons bring about different views of mathematics. The question that surfaced as a rationale from Wong's emphasis was, whether the integration of history of Pythagoras' Theorem would influence learners' perceptions of the theorem cognisant of the fact that perceptions were malleable.

3.4.3 Factors affecting perceptions

There are a number of factors that operate to shape perceptions. These factors may be resident in the situation or context in which the object is being perceived, in the object that is being perceived or in the perceiver (Wong, 2001). Situational factors which may also be referred to as contextual factors are time, work setting and social setting (Robbins, 1991). For example what learners perceive is influenced by the environment in which they are learning, which generates the signal received by their sensory organs. The implication for this study was that when learners were learning about Pythagoras' Theorem there was a need to ensure that the lesson environment was conducive to learning to enhance learners' perceptions of the theorem. I ensured that there was

enough time allocated for each learning task, such that learners would not feel that they were hard pressed for time to complete tasks.

This was achieved by carrying out lessons after school on Fridays or during learners' free periods on Wednesdays and focus group discussions were held during break periods. These time slots were especially chosen to eliminate the formality of the general lesson environment. A classroom which was slightly removed from other classrooms was selected to minimise disruptions. Learners were provided with a variety of tasks, some of which were group cut and paste, others were simply group discussions and individual activities, which promoted interaction amongst the learners. When group discussions were held it was ensured that the furniture was arranged in a way that learners were as close to each other as possible and facing each other thus promoting meaningful discussions. Charts related to the content being covered were positioned on the walls to provide a stimulating environment for the learners. It was the researcher's endeavour to investigate the influence of such a context on learner perceptions.

Factors residing in the perceiver are experiences, expectations, attitudes, motives and interests (Wong, 2001). These are the principal factors whose influence this study sought to solicit. It was my view that the formation of learners' perceptions of the Pythagoras' Theorem was influenced by their experiences, attitudes and interests in the learning context. The aim of this study in light of factors affecting perception was therefore to explore whether the inclusion of history of mathematics would influence learner perceptions in terms of their interests, experiences and attitudes. In this study effort was made in lesson presentations to make the lessons different and exciting through using cut and paste activities, group discussions, individual activities and use of Geogebra software designed activities. The intended effect was that if learners' learning experiences were improved, interest aroused maybe a positive impact on their attitude towards the Theorem of Pythagoras and would be observed hence a positive influence on their perceptions of the theorem.

Factors which may be resident in the object are novelty, motion, size, background, proximity and similarity. Characteristics of the object being observed influence what is perceived because objects are not looked at in isolation. It was therefore important that I organised the components of the

lessons and the content of the history of Pythagoras' Theorem such that they blended being conceptually linked. This was done in order to improve learner perception of the theorem as well as facilitate perceptual change through progressive bridge-formations. The relationship of the theorem to its background was also important since people tend to group together things that are similar and those that are in close proximity, which also has an impact on their perception hence the integration of history of Pythagoras' Theorem was chronologically arranged.

3.4.4 Importance of perceptions

Perceptions are important because they shape a person's behaviour (Springer, 1992). As acknowledged by Torerai (2013) fewer learners choose mathematics at grade 10 opting for mathematical literacy which they perceive as less difficult. If a learner has not mastered the basic concepts in mathematics, she/he may find it difficult to focus, pay attention and participate in a lesson because she/he would get lost and confused easily (Department of Education, 2014). Perceptions have an influence also on one's performance in mathematics (Pehkonen & Torner, 1998). If for example fewer and weak connections are formed in a learner's schema then is may have a negative impact on her/his performance in the subject. It is the fear and negative perception of mathematics which led to a decline in mathematics performance and quality of passes at matric level (Barry, 2014). In my perspective mathematical beliefs which are built on mathematical perceptions, are important for they form the frame for an individual's knowledge structure. This view is also shared by Pehkonen and Torner (1998) who argue that this frame has a broad influence on the individual's mathematical understanding.

3.5 CONCLUSION

The theoretical framework which was extensively discussed through the interrogation of the genetic approach, constructivism and perceptions framed the research design and methodology of this research which will be dealt with in the next chapter.

CHAPTER 4

RESEARCH DESIGN AND METHODOLOGY

4.1 INTRODUCTION

The main objective of this study was to investigate the possibility of using some content from the history of mathematics to influence learners' views and perceptions of geometry. In particular the study sought to explore whether learners' opinions and perceptions of geometry could be positively changed when Pythagoras' Theorem was taught with the inclusion of some historical content. In this chapter I provide a synthesis of the nature of research undertaken, the research tools used to gather information and how the collected data was analysed. Furthermore it covers issues of ethics and outlines the research limitations.

4.2 DESIGN OF STUDY

According to Guba (1981) research design needs to be premised on a paradigm whose assumptions converge and resonate with the phenomenon being investigated. Subscribing to the view of many other researchers (Van Rensburg & Smit, 2004; Denzin & Lincoln, 2003; Richardson, 1995) who contend that human phenomena is best studied using the qualitative approach this research study was embedded in a qualitative perspective and grounded within an interpretive paradigm.

4.2.1 Research approach

The qualitative perspective upholds the following basic assumptions upon which this study was premised. The first assumption is that, when trying to decipher and interpret human phenomena, things are best studied in their natural settings (Denzin & Lincoln, 2005). This research study sought to understand the meaning through perspectives that learners in the study attached to the events, situations and actions that they were involved with in the mathematics teaching and learning context. The study was also focussed on soliciting learners' accounts of their experiences in the mathematics lessons on Pythagoras' Theorem. It sought to establish the influence of this context on their perceptions and perspectives of Pythagoras' Theorem.

The second assumption specifies that knowledge is constructed socially as active self-determining individuals become involved in the research process (Schwandt, 2000). Hence in this research study it was my belief that the best people to inform us about the phenomenon under study were

the learners themselves. It was my view as I embarked on this study that it was the learners' views about the history of mathematics and its influence on the learning of Pythagoras Theorem. In line with this notion learner views were the primary source of data.

 The third assumption is that researchers should strive to get the insiders' perspective and interpretation of experiences as they try to conceptualise the complex world of their participants (Schwandt, ibid). Heeding and embracing this assumption this study became an endeavour to explore the complex world of the lived experiences of grade eleven learners as they learnt Pythagoras' Theorem. I sought to investigate the influence of the inclusion of the history of mathematics on learner perceptions, without manipulating the study situation or learners' natural setting.

4.2.2 Research paradigm

According to Kuhn (1962) a paradigm is an amalgam of worldviews and pragmatic techniques that are held by members of a given research community. This constellation constitutes particular understandings of reality and scaffolds the way a community organises itself (Capra, 1996). This study adopted the views above and regarded a paradigm as a system of beliefs that guides the way things are done in a community. Guba (1990) posits that paradigms may be distinguished through their ontology, epistemology and methodology. These three, in his opinion form a holistic view of how knowledge is viewed.

There are many definitions of ontology for example, Grix (2004) defines ontology as the study of "claims and assumptions made about what exists, what it looks like, what units make it up and how these units interact with each other", (p. 59). In short it is the study of existence. This study however upheld the perspective that ontology is the view of reality that is held by the people. Epistemology is the theory of knowledge (Crotty, 1998) and forms the theoretical foundations for methodology. This may be perceived as the theory of what is and how knowledge is acquired. Combined together ontological assumptions, epistemological assumptions and methodology make up a paradigm.

The interpretivist paradigm encompasses all approaches which foreground the meaningful nature of human character and acknowledge the significance of human participation in socio-cultural life (Elster, 2007). Interpretive methods deal with the 'whats' and 'hows' of social reality and adopt the view that knowledge of reality is socially constructed by people. It revolves around how humans logically structure their experiences and their worlds, and in the process shape the meanings they ascribe to their reality (Gubrium & Holstein, 2002). The table (4.1) outlines a summary of the characteristics of the interpretive paradigm as applied in this study.

Table 4.1 Characteristics of the interpretive paradigm (Mlenzana, 2013, p. 20)

Feature	Description
Purpose of research	• Understand and interpret learners' perceptions and perspectives of the Theorem of Pythagoras before and after the inclusion of the history of mathematics in their lessons.
Ontology	• There are multiple realities. • Reality can be explored, and constructed through human interactions and meaningful actions. • Many social realities exist due to varying human experiences, including people's knowledge, views and interpretations. • People make sense of their social worlds in the natural setting by means of daily routines, conversations and interacting with others around them.
Epistemology	• Events are understood through the mental processes of interpretation that is influenced by interaction with social contexts. • Those active in the research process socially construct knowledge by experiencing the real life or natural settings
Methodology	• More personal and interactive mode of data collection • Inquirer and the inquired-into are interlocked in an interactive process of talking and listening, reading and writing cutting and pasting. Processing of data collected by group interviews, focus group discussions and journal entries and reflective sessions.

In this study such questions as, 'what are learner perceptions and perspectives of the Theorem of Pythagoras', 'how does the incorporation of the history of the Theorem of Pythagoras influence learner perspectives of geometry and the theorem itself were interrogated. Learners' experiences were investigated through seeking their perspectives and perceptions of the Theorem of Pythagoras before and after the inclusion of the history of mathematics in the context of teaching and learning the theorem. This was done to discover what reality learners constructed from their experiences while learning the Theorem of Pythagoras, and what meaning could be read from these experiences and realities. In general interpretivists foreground qualitative data in pursuit of knowledge (Kaplan & Maxwell, 1994).

4.3 RESEARCH METHODOLOGY

With regards to methodology this study was conducted as a case study. Case studies have been conceptualised in myriad ways. For Thomas (2011) the distinguishing feature of case studies is the foregrounding of a 'case' which is the object of study. Thomas (ibid) posits that a case is the essence of inquiry into any human phenomenon as it provides a lens for analysis and narrows the focus of the research study. In this conceptualisation, a case serves as the manifest object within which research is conducted and hence serves to illuminate the path to understanding the phenomenon being explored (Thomas, ibid). On the other hand Yin (2014) conceptualises a case study as an empirical exploration of contemporary human phenomenon within real-life settings. According to Yin (ibid) case studies have utility in addressing the 'how', 'what', and 'why' questions of observed human phenomena. Yin is of the notion that the power of the case study methodology lies in its accommodation of multiple sources of evidence. From the conceptualisations above this study regards a case study as a holistic analysis human nature, actions, resultant events, their decisions-making process, and human institutional arrangements using multiple methods. The latter provides multiple lens for interrogating human phenomenon, a major feature of a case study methodology which yields a more clear and balanced view of puzzling human phenomenon being studied (Johansson, 2003). A case study by virtue of methodological triangulation allows the researcher to explore and unravel myriad complex variables that interact to produce unique and distinct cases that warrant a case study (Yin, 2014).

The understanding of a case study for this study was guided by Yin's (2014) definition. That is the 'how', 'why', and 'what', questions prompted the birth of this research. To answer the 'what' question this investigation sought to interrogate learners' perceptions and perspectives of the Theorem of Pythagoras and geometry in general. The 'how' question was attended to when the history of mathematics was included in the lessons on Pythagoras' Theorem to investigate how the history of mathematics influenced learners' perceptions and perspectives. The 'why' question was answered when the study sought to discover why the inclusion of the history of mathematics influenced learners' perceptions and perspectives in the way it did. Furthermore the investigation was carried out on grade 11 mathematics learners in an all- girls' high school setting which qualified as a real life context since no special alterations were made to their context. A case study approach was therefore the ideal and most responsive methodology in exploring learner perceptions and how they were influenced by the inclusion of history of mathematics. As an interpretive form of research, the case study allowed this exploration of intricate details and meanings of experiences of the research participants (Duff & Chapelle, 2003).

Within this context learners' perceptions and perspectives of the Theorem of Pythagoras were solicited using group interviews conducted before and after the lessons on Pythagoras' Theorem where the history of the theorem was integrated. An account of learners' reflexive experiences was obtained from their journal entries which they made on a regular basis throughout the data collection period. Learners' responses in the learning activities conducted during the teaching and learning section of data collection were also analysed.

4.4 DATA COLLECTION

Since this study is a case study that is deeply embedded in constructivist perspectives and the genetic approach, it was appropriate that qualitative data collection techniques be employed. This section is divided into several subsections. The first subsection (4.4.1), presented the setting of the study. The second subsection (4.4.2), gave a description of the sampling techniques used to select the sample of participants for the interviews and focus groups. The third section (4.4.3) outlined the data collection methods that were used in this study.

4.4.1 Setting

Warrenview girls' high school is situated in the central business district of one of the prominent cities of the province of KwaZulu-Natal, in South Africa. Despite it being at the heart of the city the school seems to attract learners from townships and surrounding low income flats. This may be attributed to the fact that the school charges very low fees as compared to most of the neighbouring schools and most of the residential areas around the school are low income. The other contributing factor may be the education departmental policy that learners should be enrolled at schools close to their place of residence.

The school was founded as an Indian school in 1906 with only white and Indian educators. Over time the demographics of the school have changed. At the time when this research study was conducted, the school had an enrolment of about eight hundred learners. These eight hundred learners in the school were made up of mainly African and very few Indian learners (about eight per entire grade) and one white learner. This shift may have been caused by the change in the target market of the school. Instead of accommodating Indian girl learners, it now catered for all races, as long as they lived within a certain radius from the school.

The educator population has not been spared in the evolution. Instead of having only Indian and white educators as when it was in the beginning, Warrenview girls' high school had, at the time this study was undertaken, a staff compliment of twenty eight educators. There were twenty two Indian and six African educators. Of the twenty eight educators six teach mathematics, three of whom take both General Education and Training phase and Further Education and Training phases. The other three were trained to teach the senior and intermediate phases. At the time of the study one educator was taking the two grade 12 mathematics classes. The remaining educators taught grade 10 and 11 mathematics and a few classes of either grade 8 or 9.

Warrenview girls' school follows the Department of Education's Curriculum and Assessment Policy Statements (CAPS) programme. It runs General Education and Training (GET) and Further Education and Training (FET) phases. In grades 8 and 9 all learners do mathematics and English is the language of instruction. There are six class for each stream with an average of about thirty five learners in a class. At grade 10 level, learners have the option to choose between mathematics

and mathematical literacy. There are two classes of mathematics per grade for the FET phase and four of mathematical literacy. The numbers of learners in the mathematics classes at FET phase range from twenty one to thirty five whereas the mathematical literacy classes are always over flowing. At grade 10 the classes are somewhat fuller (i.e. around thirty five) but the numbers dwindle as the learners climb up the academic ladder. The mathematics class sizes become smaller as some learners change courses to mathematical literacy combinations after failing to progress into the next grade. Another factor may be the high school dropout rate.

The language of instruction is English except for the IsiZulu and Afrikaans subjects. At FET phase learners take English as a first additional language, with their home language subject being either IsiZulu or Afrikaans. Since learners take English as a first additional language, some of the learners face an uphill challenge in comprehending the demand of mathematical questions or activity.

The infrastructure at Warrenview girls' high school was still in good condition. The classrooms however were originally meant to accommodate between twenty and twenty five learners. Now that the enrolment has increased the learners are somewhat crowded in the small spaces especially in lower grades. The school had a non-functional library which was used only to house the duplicators for running out worksheets and examination papers. Worksheets were used to supplement textbooks since there were not enough. Sometimes the books in the set allocated to a teacher for a particular grade were not enough for a class, such that the learners had to share a text book one between two learners. Learners bought their own stationery, mathematical instruments and calculators. As a result of their challenging economic circumstances most learners depended on borrowing and sharing calculators and mathematical instruments.

About fifteen years ago the school had a computer laboratory. At the time the study was conducted there was no evidence that the school used a computer laboratory. For the purposes of this study the teacher and researcher had to bring her personal technological devices.

4.4.2 Sampling

The definition of sampling is standard across all disciplines that collect data by the sampling method (Latham, 2000). Latham (ibid) goes on further to purport that, it is the creative definition

of a sample that vary for purposes of creating understanding. For the purposes of this study Lohr's (1999) definition of sampling as, taking a representative selection of the population was adopted. It is also from this understanding that for this study a sample was regarded as a subgroup of a population, a unit that is selected to represent the characteristics of the entire population.

According to Latham (2000) there are two types of sampling: - random sampling (probability sampling) and non-random (non-probability) sampling. Of these techniques non-probability sampling was the preferred method for this study. There are several methods of non-probability sampling but in this study purposive sampling was the preferred method. Purposive sampling is sometimes referred to as judgemental sampling (Babbie, 1990). Sampling becomes purposive when a sample is selected on the basis of one's knowledge of the population and the nature and purpose of the research study (Frey, Carl & Gary, 2000). This was the preferred method because a small subset (eight focus group members and eight participants in the group interviews), of a larger population (mathematics learners at Warrenview girls' high school) was being studied. Many members of the subset easily identified with the larger population but the enumeration of all of them was impossible. That is a case of one grade 11 mathematics class of thirty learners at Warrenview girls' high school was studied out of all the grade 11 mathematics learners at the school and in general.

A sample of individuals who were capable of contributing to the achievement of research objectives was purposively selected from the case. A girls' school was especially chosen because (from the researcher's experience as a mathematics learner and teacher) very few girls opt to take up mathematics at grade 10 so it was more sensible to solicit their perspectives and perceptions as representatives of the entire grade 11 learner population within the school. Learners selected for the group interviews were of mixed abilities and were selected by their peers to represent their groups and to present the group's perceptions and perspectives.

4.4.3 Data collection methods

To answer the research questions discussed earlier on in chapter 1 of this study several data collection techniques were employed. These included group interviews, focus group and journal entries. The methods mentioned will be described as used below.

4.4.3.1 *Group interviews*

Learners' perspectives and perceptions on Pythagoras' Theorem were solicited for through pre-instruction and post-instruction interviews which were conducted using group interviews. Interview guides were used to facilitate this process *(see Appendix 5)*. The group consisted of eight learners who were identified as group representatives in the various group activities performed during the teaching and learning sessions. Group interviewing was the preferred method since these group meetings tended to produce a wide range of responses and rich information as learners' insights triggered the sharing of other diverse personal experiences, perceptions and perspectives of the Theorem of Pythagoras (Mathers; Fox & Hunn, 2002). Having more than one interviewee present helped the learners to complement each other with additional information which led to a more complete and reliable record. This method was convenient since it was quicker and conserved time (Cohen & Manion, 2011).

The first group interview was opened by a round of introductions, even though the learners were familiar with each other and the interviewer. This was carried out to create a conducive and relaxed atmosphere for all parties, interviewees and interviewer. This was followed by a series of questions asked before tasks on reinvention of the Pythagorean Theorem were administered in Learning activity 1 *(see Appendix 6 which contains a few of the designed learning materials)* was carried out. This interview was conducted on 23rd of July 2015. The questions in the interview were formulated to glean learners' perceptions and perspectives on geometry prior to the inclusion of specific aspects of the history of mathematics in learning.

The second interview was held on the 17th of August 2015 after the activities which were designed to allow learners to rediscover the Theorem of Pythagoras on their own with little help from the teacher. In this post-instruction group interview questions were asked to establish if reinventing the Pythagorean Theorem for themselves had any influence on any of the learners' perceptions and perspectives of the Theorem of Pythagoras. The third interview was carried out after the tasks of Learning activity 2 *(see Appendix 7 which is a description of the learning interactions)* were experienced by the learners. The interview was carried out on the 20th of August 2015. The activities involved the converse of the Theorem of Pythagoras, application of the theorem and the

derivation of the formula used in the generation of the Pythagorean Triples. This interview sought to find out if the inclusion of the history of Pythagoras' Theorem in the form of various proofs of the theorem had any influence on the way learners applied the theorem to solve given problems. The fourth interview was conducted on the 2nd of September 2015 but was curtailed because all grade 11learners were called to attend an emergency principal's meeting at the time the lesson was scheduled to take place. It was then continued on the 3rd of September 2015, which was carried out after tasks in Learning activities 3 and 4 *(see Appendix 7)* were administered to the learners. These activities included rediscovering the distance formula, research work on some great mathematicians in history and application of Pythagoras' Theorem in solving trigonometric problems. The interview focused on finding out if the mathematicians' life stories, experiences and discoveries had some, if any, impact on learners' perceptions and perspectives of mathematics. All this is outlined in the group interview guide, which covered all the interview sessions held *(see Appendix 5)*. Pre and post interviews were held in order to compare learners' perceptions and perspectives before and after the inclusion of the history of Pythagoras' Theorem to ascertain if its inclusion had some if any, influence on learners' perceptions and perspectives.

The fifth interview was held on the 17th of September 2015, after the activities in which the mathematics application Geogebra was used. During the lesson Geogebra software was utilised to draw diagrams which were used by learners to prove the Theorem of Pythagoras. This method was used because in this day where we are at the cutting edge of technology dynamic mathematics software such as Geogebra is regarded as a powerful teaching and learning medium that enhances conceptual development, enriches visualization of geometry and creates opportunities for creative thinking (Sanders, 1998). In support of this argument (Clements; Sarama; Yelland & Glass, 2008) purport that dynamic environment improves learners' ability to concentrate on interrelationships of geometric shapes. The interview endeavoured to find out if the use of technology in the inclusion of the history of Pythagoras' Theorem had any impact on the way learners perceive the theorem, geometry and mathematics in general. As a result a total of six interview sessions were held with this group.

4.4.3.2 *Focus groups interviews*

A focus group is a contrived setting that brings together a deliberately selected group of individuals who are part of a bigger population to discuss a particular topic (Cohen and Manion, 2011). Focus groups have been observed to have utility in cases where the research endeavour is to gather qualitative data and generate data quickly at a low cost (Robson, 2002). This research study, subscribes to this definition of a focus group and its uses and three focus group discussions were held with a group of eight learners representing learners who were struggling with mathematics. These learners were identified using the grade 10 term one, term two, term three and term four results. The focus group was also chosen for its ability to allow participants' views to be captured as they interact with each other. The first dialogue held on the 19th of August was opened by asking the learners what came to their minds when the word *mathematics* was mentioned to them. This question was focussed on soliciting for learners' perspectives and perceptions of mathematics in general as they interacted with each other. The discussion digressed to learners' perspectives and perceptions of geometry and the Theorem of Pythagoras before the inclusion of the history of mathematics in the content on the theorem.

The second meeting was held on the 3rd of September 2015 after the administration of tasks in learning activities 1, 2, 3 and 4 during the lessons that were conducted on reinventing the Pythagorean Theorem and its applications. Learners were discussing their experiences during the lessons when they were taught Pythagoras' Theorem and its history. The purpose of this dialogue was to determine (i) if the inclusion of the history of mathematics in their lessons had some influence on their perceptions and perspectives of the Theorem of Pythagoras and geometry and (ii) what aspects of the use of the history of Pythagoras' Theorem had the most influence on learners' perceptions and perspectives.

A third dialogue was held on the 16th of September where learners shared their views and opinions on the methods of teaching and learning employed during their lessons on the Theorem of Pythagoras. It also sought their perspectives on the methods they most enjoyed and why, how they wished to be taught and the resources they longed to use in their mathematics lessons. The focus of this dialogue was learners' perceptions and perspectives on the methods of teaching and learning

(that is the genetic approach) employed in the teaching of the Theorem of Pythagoras in which the history of mathematics was included.

4.4.3.3 *Journal entries*

Data was also collected using learners' journal entries, which they kept for the entire duration of data collection of this research. This approach was used to obtain perceptions and perspectives on the theorem of Pythagoras from learners who were not included in group interviews as well as triangulate learner responses after the focus groups. This method of data collection was administered in order to gather learners' perspectives on the teaching methods employed by the educator. It was my view that the influence of the history of mathematics on learner perceptions and perspectives could be gleaned from learners' documented experiences and opinions since some learners may have been shy or uncomfortable to critique the educator's teaching method in her presence during the interviews. The journals entries were utilised periodically (after set of tasks), as a reflection tool further prompting group discussions.

4.4.3.4 *Lessons on Pythagoras' Theorem*

Mathematics lessons on Pythagoras' Theorem in which historical content was included in the designing of learning activities in such a way that history of mathematics was not visualised as the main element of the lessons were conducted (*see Appendix 7)*. The purpose of these lessons was comparative of perceptions and perspectives of learners on Pythagoras' Theorem, geometry and mathematics in general before and after the inclusion of the history of mathematics in their lessons. All data collected during group interviews and focus group discussions was video / audio-taped and transcribed. Learner journals were collected and entries were also transcribed and analysed. Table (4.2) presents a synopsis of the data collection process in relation to the research questions.

Table 4.2 Research methods used in the study

Research questions	Data collection methods	Instruments/Tools
What are learners' perspectives and perceptions of the Pythagorean Theorem?	-Pre-instruction focus groups -journal entries analysis	-focus group meetings guide - learners' journals -video / audio recordings

How does the incorporation of the history of Pythagoras' Theorem influence learners' perspectives of geometry in general as well as their perceptions of the theorem itself?	-post-instruction focus groups -journal entries analysis	-focus group meetings guide -learners' journals -video / audio recordings
What aspects of the use of the history of Pythagoras' Theorem have the most influence on learners' perspectives and perceptions?	-post-instruction focus groups -journal entry analysis	-focus group meetings guide -learners journals -video / audio recordings

4.5 DATA ANALYSIS

Qualitative data analysis, according to Bogdan and Biklen (2003) involves data fragmentation into manageable units in an attempt at extracting meaning. Bogdan and Biklen (ibid) posit that this would be accompanied by coding, synthesising the data and the search for patterns. For this research study, data analysis was guided by the interpretive approach discussed above. This meant that the search for meaning of collected data was, carried out through direct interpretation of learners' experiences perceptions and perspectives. Under the interpretive paradigm data analysis is done to establish patterns, themes and meanings inherent in the data. The patterns and themes which emerge help to explain and identify causal links in the data as postulated by Yin (2014). Data analysis in this study was systematic and organised so that information could be easily accessed and traced back to the source.

4.5.1 Data transcription

Data analysis began informally during interviews and focus group dialogue and reviews of the journal entries and continued after the completion of the data collection exercise. Data

transcription was necessary so as to have a permanent written record of the interviews, discussions and journal entries which can be shared with anyone who might be interested in the research study. Data from video and audio recordings of interviews and focus group discussion was transcribed using the NVivo 10 software programme. Data analysis continued with the transcription of journal entries in which participants transcribed their lived experiences during the lessons on the Theorem of Pythagoras. Data from journals was presented in table form. The interviewees were given pseudonyms in the transcripts for anonymity purposes. The transcripts were given titles and dated using the dates of the interviews. The transcripts were read and re-read checked for spelling mistakes, inconsistencies in statements and sentences to get an overall picture of learners' perceptions and perspectives. During this initial reading insights that emerged were written down as memos. This to a large extent influenced the steps that followed in the data analysis because the relevance of the study began to unfold even in these early stages. After the initial reading the data was edited to eliminate obvious repetitions and redundancies, the researcher however took care not to lose important data. After transcription each learner was given the opportunity to check the transcript(s) of the interviews and journal entries depending on where their input was. This was in a bid to check the accuracy of the transcription and to ensure that learner perceptions, perspectives and experiences had been captured correctly.

4.5.2 Organising and coding of the data generated

The data from the interviews, focus group discussions and journal entries' transcripts were analysed to identify sections that were relevant to the study. Based on this analysis a classification system of issues was developed using NVivo 10 software programme in which the data was put under related nodes. The nodes used are listed below:

(i)   Learner perceptions and perspectives of mathematics
(ii)   Learner perceptions and perspectives of geometry
(iii)   Learner perceptions and perspectives of Pythagoras' Theorem
(iv)   Learner perceptions and perspectives of the history of mathematics
(v)   Learner perceptions and perspectives of the history of geometry
(vi)   Learner perceptions and perspectives of the history of the Theorem of Pythagoras
(vii)   Learner perceptions and perspectives of pedagogy
(viii) Group dynamics

(ix)   Aspects of the history of Pythagoras' Theorem that had the most influence

Formulation of nodes was greatly influenced and informed by the theoretical framework of this study which was discussed in chapter three. The data was put into categories and organised in search of patterns, critical themes and crucial meanings that emerged from them so that comparisons and contrasts could be made. This involved interacting with the data comparing learners' views searching for similarities and differences, categorising according to their similarities thus organising them. Categorisation and organisation were thus done to excavate the complex threads of the data and in the process try to make sense of them.

4.6 ETHICAL CONSIDERATIONS

Permission to conduct this research was granted by the university, the Department of Education, the school governing body, the parents or guardians of the participants and the participants themselves in the form of consent letters (see Appendices 1, 2, 3 & 4).

This research involved minor children which necessitated the use of pseudonyms and the location of the research site was not disclosed in this thesis. The autonomy of the participants was safeguarded and guaranteed through the use of informed consent. Participants had to complete a consent form which specified the following issues: (i) nature and purpose of this research, (ii) the identity and institutional association of the researcher, supervisors and their contact details. (iii) Participants were also informed that their involvement was voluntary and they were free to withdraw from the research at any time without negative consequences to them, (iv) they were given the assurance that their responses were to be treated in the strictest of confidence, (v) and participant anonymity was ensured by the use of pseudonyms.

Collected data as part of ethical considerations was to be kept under lock and key for a period of five years. After the duration of five years it will then be disposed of by shredding and incineration. Lastly in the subsequent dissemination of my research findings, in the form of the finished thesis, oral presentations and publications anonymity will always be maintained through the use of pseudonyms.

4.7 RESEARCH TRUSTWORTHINESS

In qualitative research, trustworthiness is inherent in what is done at every stage of the research process. It can be regarded as a fit between collected and recorded data with what actually transpires in the research setting. According to Bogdan and Biklen (1992) trustworthiness is a measure of the degree of accuracy and comprehensiveness of coverage. In concurrence with this view, Guba and Lincoln (1985) defined the trustworthiness and credibility of research as the truthful description and explanation of naturally occurring phenomena. In qualitative research and this understanding of trustworthiness research focus ceases to be striving for uniformity. Such a context allows different researchers exploring similar case to come up with different findings and the research remains trustworthy. For example this allows many different interpretations of data from interviews as there are researchers (Kvale, 1996). With the same understanding of trustworthiness as discussed above, this research, in pursuit of a trustworthy study employed four criteria as proposed by Guba (1981) that is credibility, transferability, dependability and conformability.

To ensure credibility of this research, member checks of data collected, interpretations and verification of themes formed were performed. Member checks are critical in ensuring a study's trustworthiness (Guba &Lincoln, 1985). Checks concerning accuracy of data were conducted during the course and at the end of data collection. Once transcription was complete learners were tasked to read transcripts from group interviews, focus group dialogue and journal entries in which they participated to ensure that their views are correctly captured and represented. Here the emphasis rested on whether the learners considered that their words matched what they actually intended, since, video / audio recording was used.

Trustworthiness and confirmability were also achieved by the triangulation of methods, that is, adoption of different angles from which to view the same phenomenon. This was achieved through using different data collection strategies, such as group interviews, focus group, journal entries and learners' responses from the various learning activities. Triangulation was used in order to promote confirmability since it reduces researcher bias (Yin, 2014). To this end, beliefs underpinning decisions made and methods adopted were acknowledged within the research report.

Guba and Lincoln (1985) emphasise the interconnection of credibility and dependability as they stress that in practice a demonstration of one of the two constructs confirms the achievement of the other. In order to enhance trustworthiness, confirmability, credibility and dependability aspects, in this study I presented an audit trail of the processes and steps undertaken in its conduct. Reporting with this much detail was done to enable future researchers repeat the work and not necessarily to gain the same results.

4.8 CONCLUSION

This chapter sought to outline the research design and the data collection methods that were used. The next chapter presents the study findings.

CHAPTER FIVE

FINDINGS, ANALYSIS AND INTERPRETATION

5.1 INTRODUCTION

In the previous chapter, Chapter four, I presented the research methods that were employed in this study for the attainment of the set research objectives and addressing the study's research questions. The focus of this chapter, Chapter five, is to present the findings as analysed. This chapter has been organised and divided into sections. The first section (5.2), provides an outline of participants' profiles. The second section (5.3), focuses on answering research question one: - what are learners' perceptions of the Pythagorean Theorem? The third section (5.4), addresses research question two: - how does the incorporation of the history of Pythagoras' Theorem influence learners' perceptions of geometry in general as well as their perceptions of the theorem itself? The fourth section (5.5), endeavours to address research question three: - what aspects of the use of the history of Pythagoras' Theorem had the most influence on learner perceptions?

5.2 ABOUT THE PARTICIPANTS

In order to analyse and interpret data collected, learners' background information was gleaned and is presented below as I introduce the participants in this study. In Table 5.1 participants' gender, number of years in the grade and mathematical capabilities are outlined. Information in Table 5.1 also includes learners who took part in the group interviews, focus group discussions or were a part of the class who participated in the teaching and learning of the Pythagorean Theorem in which the history of the theorem was incorporated. At this moment it is of utmost importance that I mention that anonymity of participants and the school at which the research was conducted was maintained by the adoption of pseudonyms for both participants and the school.

Table 5.1 Learners' profiles

Pseudonym	Gender: F / M	Number of years in grade	Mathematical capabilities
Phume	F	1	Below average
Winnie	F	2	Below average
Wendy	F	1	Above average
Pamela	F	1	Average
Phumla	F	1	Average
Nonhlanhla	F	1	Average
Londy	F	1	Average
Nozipho	F	1	Below average

Asanda	F	1	Below average
Minenhle	F	1	Above average
Jackie	F	1	Below average
Nolwazi	F	1	Below average
Sbu	F	1	Below average
Feryn	F	1	Below average
Lorraine	F	1	Above average
Pinky	F	1	Average
Maxine	F	1	Below average
Sammy	F	1	Below average
Sandra	F	1	Average
Thandi	F	2	Below average
Pearl	F	1	Below average
Asanda	F	1	Average
Pretty	F	1	Average
Stabile	F	1	Average
Judith	F	2	Below average
Nandie	F	1	Below average
Belinda	F	2	Average
Ayanda	F	1	Above average
Samke	F	1	Below average

The information about the number of years in the same grade and ability level, is based on the learners' grade 10 or grade 11 end of year (2014) marks. Four of the participants were repeating grade 11 as they had not qualified to be promoted into grade 12. The promotion policy at Further Education and training (FET) phase is such that, a learner would not qualify to be promoted into the next grade if they had failed their home language, life orientation or more than two of the subjects they were learning. In this study, in terms of ability, below average means that the end of year marks for the learner were below thirty percent. Average marks ranged from thirty one percent to forty nine percent and above average represented marks above fifty percent.

5.3 Learners' profiles

Profiles of sixteen learners who participated in either group interviews or focus group discussions have been presented below to provide further background information on the participants in this study. It is of utmost importance that I mention that, all in all, there were thirty participants in this study and as such it would not be feasible and viable to present all these profiles hence a sample of sixteen learners was considered. These were the learners who were involved in either the focus group discussions or the group interviews. The profiles of some learners who participated in the teaching and learning of Pythagoras' Theorem and made contributions through journal entries

which were captured in this chapter, have not been included in this section as they were not participants in the group interviews and focus discussions.

Ayanda

My name is Ayanda Mchunu and I live in Beverley Hills. I'm currently doing grade 11 at Warrenview Girls High School where I do the following subjects: Pure mathematics, Life orientation, Life sciences, Accounting, Business studies English and IsiZulu. My hobbies are tennis, reading and swimming. One of my fascinations is the Egyptian Ancient Work and I would like to be an Archaeologist as a hobby during my leisure time. I find the ancient Egyptian hieroglyphics very extraordinary and the gods of Egypt and their powers fascinating.

On my family topic, well I value my family very much both maternal and paternal. I have three siblings. If I could describe myself in a few words I would say I'm kind, short tempered, a bit immature but very honest. I attend extra tuition on Saturdays in a bid to improve on my mathematics, physics and accounting performance. When I grow up I would like to be a virologist which is a person that works with viruses in laboratories. Some of the universities I plan going to are University of Cape Town, University of KwaZulu-Natal, Stellenbosch, Rhodes and Witwatersrand. When I am older I would like to own my own house and car and probably build my own family.

Maxine

My name is Maxine Nxumalo a learner at Warrenview Girls High School in grade 11. I enjoy reading and listening to good music sometimes even lazing around.

I do subjects like Mathematics, Accounting, Life sciences and Business studies. Out of all the subjects I do I find Life sciences interesting, because for some reason it's fascinating for me. Once I'm grown I would like to have a career in the science field.

As a person I can say that I am funny and very fun to be around, I believe I was a born entertainer. I work hard in things that sometimes are not important. I am one person who loses focus easily and lacks concentration skills. In a debate I like for my point to get across to my opponents and if I feel no one heard it or argues against my point, I will debate them for hours.

Based on previous and current challenges, academically I feel mathematics and accounting, basically things involving numbers and calculations are quite a challenge for me. Hard work and dedication is what is required for me to overcome all the challenges that come with these subjects.

Sbu

My name is Sbu Xulu a learner at Warrenview Girls High School and at present I'm in grade 11. I am fifteen years old. I enjoy practising mathematics and listening to music. At school I do Mathematics, Accounting, Life Sciences and Business besides the four compulsory subjects. I am an easy learner to get along with, I sometimes work hard to achieve the best. I am dedicated to my school work. I like working with numbers, even though sometimes I find it hard to work with them. I grew up in a family where not everything was easy and that makes me to strive for a better future.

Out of the subjects I am doing the subject what I find challenging is mathematics, even though it is challenging, I'm not willing to give up because I believe hard work and dedication pays. In primary school I used to get good grades in mathematics but that all changed when I came to high school where my strengths are now in Business studies and Life sciences.

Jackie

I am Jackie Mahlabela, I attend school at Warrenview Girls High School where I'm doing grade 11. I stay in Marandelas with my parents and siblings. I like writing poems, short stories and reading novels. I have no dislikes. My hobbies are reciting poems and singing. In school I have some strong points as well as weak points. My strength lies in the languages both isiZulu and English. I am weak in mathematics even though I do not dislike it. On average I think I spend about five hours on my school work.

Phume

My name is Phume Mkhize aged sixteen, I am in grade 11 at Warrenview Girls High School. I live in Westley with my father and grandparents, my mother passed on when I was in grade five. I like playing netball and dancing to and listening to music. My favourite subject is Life sciences, even though mathematics is not a favourite subject to me I have some topics I enjoy, topics such as

algebra and analytic geometry but I do not like trigonometry. I would like to be a nurse when I am done with matric because I like helping old and sick people.

Nolwazi

I am Nolwazi Makhoba, a grade 11 learner at Warrenview Girls School. I come from a family of four children with me being the eldest. I was born with a weak heart such that I constantly visit the hospital for check-ups and medication. Both my parents are still alive and stay with me.

In school I chose to do Mathematics, Accounting Life sciences and Business studies besides the subjects that are done by everyone. I made this choice because when I grow up I would like to be a doctor or have a career in the health sciences so that I can help people who are sick like me.

Phumla

My name is Phumla Jali, I'm sixteen years old. I am a girl who is open to other people' suggestions and enjoy talking with other people. I am humble and respect other people. My favourite sport is girl soccer and tennis. I like people who do not think that they are better than others and dislike those who think they are better than other people. I love music and cooking. My hobbies are singing and dancing.

When I grow up I want to be a doctor. If I do not qualify I would love to be a pilot. Most of the time I am usually sick, I have asthma and sinus. My favourite subject is science, I like this subject because I usually discover new things about it. I learn things that I didn't know at all.

Minenhle

My name is Minenhle Zulu I am fifteen years old. I am in grade 11 at WGHS. I stay in Waterkloof with my parents and siblings. My favourite subject is physical sciences because it challenges my mind and keeps me busy all the time. The section I really like in this subject is the physics section. The subject I do not like is life sciences because it has long notes and it takes a lot of my time. My likes are listening to music, watching television and reading novels. I dislike people who tease others making them feel uncomfortable. I also do not like people who fight. I am a shy and quite person who does not like the company of many people. My role model is my mother, she is a

strong woman who overcame many obstacles in her life while raising us and she does everything in her power to make sure that we are okay. My favourite teacher is Mrs Mkhize, my mathematics and form teacher. I like her because she is polite, teaches us well explaining to us clearly when we do not understand and always shows us the way when we go astray.

Londy

My name is Londy Mnguni, I am sixteen years old. I come from a family of five, two siblings and grandparents and I. We had to live with our grandparents because both our father and mother are deceased. When I am not in school I have to help with house chores because my grandmother is now old and weak. I attend school at Warren Girls High School and I am in a science class. My favourite subjects are Geography and Mathematics. I like Geography because it tells us about the world and Mathematics because I want to be a radiologist after completing matric. My hobbies are watching movies and listening to music.

Wendy

My name is Wendy Nzama I am fifteen years of age. My favourite subjects are Physical sciences and Mathematics. I love mathematics because it always challenges my mind and physics is just in my heart and it keeps me wondering. My worst subject is life sciences because it is hard.

My hobby is to watch television with my friends and family. I like laughing a lot and chatting with people and respect adults. I dislike people when they shout at me and people who are talkative. My role model is Connie Ferguson who is acting the role of Karabo in Generations. The animal that I am scared of is a frog but I like fish. My strength is cooking, cleaning and art. When I grow up I would like to be an actress.

Pinky

My name is Pinky Mbili and I am 16 years old. I am doing grade 11 at Warren Girls High School and live right in town. My favourite subject is Mathematics even though it is difficult, it exercises my mind and keeps busy. The subject I do not like is Life sciences because it has a lot of notes. My hobbies are listening to music and singing with my family.

My likes are to make beats and chat with my friends. I dislike lying to my parents and seeing other people unhappy. My role model is Sindi Dlathu who acts in uMuvhango the role of Thandaza Mokoena. When I grow up I would like to be an actor, acting is my talent and I love it. I also have other options like being a social worker or a teacher because I also like helping children.

Samke

My name is Samke Nzuza, I live in Fowleni with some relatives from my mother's side, who helped me find a place to stay for the year. I like staying with other people and share with them good things that will make my future brighter and work hard for things that will help me in the future.

My subjects at school are Physical sciences, Mathematics, Life Orientation, Geography, Life sciences and the languages. I chose these subjects because they open doors to my future. In school I am struggling to get good grade in mathematics and physical science. When I grow up I would like to be a doctor specialising in bones. I want to make my family proud especially my mother who is always encouraging me to do well.

Nonhlanhla

My name is Nonhlanhla Khumalo I am sixteen years of age. I am in a science class at Warrenview Girls High School, where I'm doing Mathematics, Life sciences, Physical sciences, Geography, Life orientation and the two languages IsiZulu and English. I am interested in architecture, music and dancing. My favourite subjects are English and Physical sciences. Mathematics is not my worst subject, but it can be very difficult at times.

My hobbies are reading novels, listening to music and playing with friend. I love and care for those who care about me. I dislike discrimination of people and those people who think they can control other people.

My role model is Oprah Winfrey because she is making a difference in other people's lives and she lives just the way I love to live. My strengths are cooking, designing, creative writing. I am a

very shy person when it comes to standing in front of people and I love to think that I am the most respectful person. Chatting with people makes my day just so special.

Asanda

My name is Asanda Wanda, I am sixteen years of old. I live with my mother two little sisters whom I like spending time with. In my spare time I enjoy chatting with people and cooking. My favourite subject is science because we discover new things we did not know about. When I finish matric I would like to become a research scientist.

Winnie

I am Winnie Ndlovu and I am seventeen. I like listening to music and watching movies. At school I do Mathematics, Physical sciences, Life sciences and Geography besides the four compulsory subjects. Mathematics is my favourite subject even though I find it difficult. My dream is to become an engineer one day, drive an expensive car and live a comfortable life. I also would like to help people who are in need.

Pamela

My name is Pamela Jaca I am 16 years old and stay with my parents in South gate. My favourite subject is Geography because we study about the things we are experiencing in our lives. I like being with people and spend time with them and motivate those who are facing bad situations. I dislike seeing other people unhappy. My goal is to go to university and study law because I want to be a lawyer someday.

These were the profiles of the selected learners who participated in the focus and group interviews. In the following section (5.3) I address the first research question: what are learners' perceptions of the Pythagorean Theorem? As I address this research question, I present the findings, my interpretations and discuss them.

5.4 LEARNER PERCEPTIONS OF THE PYTHAGOREAN THEOREM

The identification of learner perceptions of the Pythagorean Theorem was not done in isolation. Literature reveals that learner perceptions are complex and are intertwined with other dominant

perceptions (Anguilar, Rosas & Zavaleta, 2012; Hannula, 2007). This notion influenced my approach in this study in that, searching for learner perceptions had to consider learner perceptions of mathematics. It was my view that there was a possibility that perhaps learner perceptions of the theorem also derived from their perceptions of mathematics. Taking this possibility into account, this study therefore solicited and here begins by presenting learner perceptions of mathematics before presenting their perceptions of the Pythagorean Theorem.

5.4.1 Learner perceptions of mathematics

Overall learner perceptions of mathematics were negative. Findings from group interviews, focus group discussions and journal entries portrayed mathematics as a difficult, disillusioning, demanding and confusing subject among other things.

Grade 11 learners at Warrenview Girls High School (WGHS) perceived mathematics as a difficult subject. Learner sentiments as captured in Asanda's focus group contribution affirm this perception.

According to Asanda mathematics:

"*…is very hard and confusing …*" (Focus Group Discussion, 19th of August 2015).

Asanda's perception of mathematics supported the view posited by Curtis (2016) who acknowledges that mathematics is perceived as hard and discusses some of the reasons why it is perceived as such. Curtis (ibid) cited the absence of the bigger picture and an inferiority complex in some of the learners when faced with mathematical problems, as some of the reasons that caused mathematics to be viewed as difficult by learners. Learners are not exposed to most of the information pertaining to a topic or section in mathematics, they are not given a bird's eye view of the section. Learners are only given information and work that is in line with their mathematics syllabus and is examinable, such that background information on the taught concepts, in most cases is not revealed to them. Curtis (2016), highlighted another reason for the negative perception as the inferiority complex that grips the learners when they are faced with unfamiliar mathematical situations and problems.

The view that mathematics is hard was confirmed by Nolwazi in the same focus group discussion where she posited that:

"It is a little bit confusing and distressful because one minute you are in class and you understand everything...the next minute you go home and you attempt the homework and eish you can't remember anything. You don't know whether to start from A or whether to start from Z" (Focus Group Discussion, 19th of August 2015).

From the above statement Nolwazi admitted that mathematics was a difficult subject by adding another dimension onto their perceptions of mathematics. Nolwazi brought forward the view that mathematics is confusing and distressful. Similar views were echoed in learners' profiles where some learners indicated their willingness to learn mathematics even though it was difficult for them to master some concepts. Such learner perceptions resonated with findings by Nardi and Steward (2003) who note that mathematics is considered a difficult learning area in schools. This perception is attributed to fear of the subject and the feeling of inferiority that grips some learners when confronted with mathematical concepts (Curtis, 2016). Some of the causes of this fear of mathematics were observed by Ireland (2016), to be (i) previous bad experiences in mathematics, (ii) lack of self confidence in performing mathematical operations and doubting one's self. (iii) learners experiencing difficulties in comprehension of mathematical notation and symbols, (iv) over reliance by learners on gadgets such as calculators such that the mind gets out of tune and begins to lose its edge to store mathematical information. Beilock and Willingham (2014) add to these observations by citing mathematical anxiety as another contributing factor to the fear of mathematics. However, an intriguing finding from this study was that the difficulty of mathematics was connected with it being regarded as confusing and distressful.

Mathematics was perceived as frustrating by learners of WGHS. When learners failed to solve problems posed to them by their educator or during their own individual endeavors, a sense of frustration set in, as they found it difficult to progress to the next level in their work.

Ayanda confirmed this perception in a group interview.

In this group interview Ayanda described her frustrations with mathematics as follows:

"When you are having a sum and you can't figure it out. You are trying to do it but you are not getting it, it frustrates you. You are so angry that you can't get the sum..." (Group Interview, 23rd of July 2015).

This sounds like the learners are left powerless and defeated when they fail to perform certain calculations in mathematics effectively. Radford (2015) had similar findings where some of the participants expressed that they hated giving their answers, were frustrated with mathematics to the extent of breaking down in tears as a manifestation of their disappointment.

Presenting her contribution on the same perspective another learner Maxine, (Group Interview, 23rd of July 2015) shared a similar perception of mathematics. She presented this argument when she purported that, there were some problems in mathematics which were very difficult to work out despite which method one tried to use. Maxine further confirmed this perspective as she went on to cite geometry as one of the sections in mathematics that is frustrating for the learners. She argued that:

"There are some sums that are very difficult to work out, it's like Euclidean geometry, and it's so confusing you feel frustrated." (Group Interview, 23rd of July 2015).

With the same perception, in that same group interview, Nolwazi concurred with the view posited by Ayanda and Maxine and findings by Radford (2015). She cemented this perception when she lamented that: *"A lot of times you are depressed a lot after trying to find a solution to a problem and failing"*.

It can then be gleaned from the learners' perceptions that mathematics is associated with disheartenment and loss of morale as problems in mathematics have a tendency of not working out as expected by the learners. These emerging views are not unique to this research study. This perspective is captured by researcher Sam (2002), in his study when he claims that many learners are frustrated by mathematics and feel powerless when presented with mathematical ideas. It is however of interest to note the psycho-social impact emerging apart from mathematics being confusing and frustrating which, is a novel dimension. From literature scanned, this dimension has not been associated with previous studies of learner perceptions of mathematics in South Africa. Learners at WGHS found mathematics to be time consuming and demanding. This perception seemed to have been built from the fact that learners struggled over the same problem for long periods of time sometimes without success. In the learners' opinion multi-tasking was impossible

while engaging in mathematical activities because in their view mathematical problems required total concentration and focus.

Expressing her misgivings about mathematics taking up most of her time Jackie had this to say:

"I find mathematics demanding, maths needs your time, like your 99.9% time…" (Focus Group Discussion, 19th of August 2015).

The view that mathematics is demanding posited by Jackie, may be construed as meaning that, mathematics is demanding and left the learners with very little or no time for anything else since it takes almost all of their time. Confirming that mathematics is demanding Jackie went on to further lament how the subject can consume one's life and not allow one to engage in other activities simultaneously.

In that same focus group, Jackie's contribution relayed this point of view:

"You have to think about it, in your mind you have to think only about maths. You cannot think about two things at the same time. It needs your full time." (Group Interview, 19th of August 2015).

These contributions were interpreted as implying that mathematics leaves one very little time for anything else. Thus, the perception of learners at WGHS was that mathematics requires one's total commitment, focus and takes up much of one's time, which is in line with the observations made by Ashbacher (2015). Lamenting on how demanding mathematics is, Weidman (2016) posits that, to do mathematics a person must immerse himself or herself   completely in a situation studying it from all angles, working on it day and night while devoting every scrap of available energy to understanding it. Ireland (2016) likened mathematics to sport, where constant and consistent practice is required for one to be and remain on top of one's game, which implies much of time has to be invested in the subject for learners to be able to master mathematical concepts.

One crucial perception of learners at WGHS was that mathematics is numbers. Mathematics was associated with numbers by the grade 11 learners. For the learners anything that was presented to them as mathematics had to be in numerical form. This view was presented by learners when they were asked to share their view on mathematics in a group interview (23rd of July 2015). In her contribution, in this group interview Pinky revealed that when the word "mathematics" was mentioned to her numbers came to mind. Pinky's remark helped reveal why the learners viewed

mathematics as complex. They had an aversion for numbers as testified by Maxine in her focus group contribution (17th September 2015) where she confessed that she had no interest in anything with numbers. Learners also could not relate anything that was not purely numbers to mathematics. As a result mathematics sections such as algebra and word problems compelled learners to switch off their minds and not actively engage in the learning process.

5.4.2 Learner perceptions of geometry

Another notable finding was that whilst the participants held negative perceptions of mathematics the learners at WGHS had some topics in mathematics which they considered as favourites and about which they enjoyed learning. What could be summed up from this observation was that not all sections of mathematics were viewed the same by the learners.

The perceptions of geometry of the grade 11 learners at WGHS were not much different from their perceptions of mathematics, that is, they were on generally negative. Learners' perception of geometry was, that it was difficult and confusing. When asked to name their favourite topics in mathematics they made mention of number patterns, probability and financial mathematics (Group Interview, 23rd July 2015). This could be attributed to the fact that these sections involve as little algebra as possible. The algebra in these sections is mainly confined to the formulae they will use repeatedly. By this the learners were confessing that learning that involved abstract thinking made them uncomfortable. Since none of the grade 11 learners mentioned geometry, as one of their favourite sections of study in mathematics I delved into finding out the reasons for this omission. Upon further inquiry it was noted that, in their opinion, it was sections like geometry that made mathematics unbearable. On asking whether the learners considered geometry as one of their favoured sections, their emphatic response was a unanimous **"NO!"** from all the members of the group (Group Interview, 23rd July 2015). This revealed that geometry was far from being even just a likable section. When exposed to further probing, as a reason why they disliked geometry, Maxine revealed the reason.

In a group interview Maxine remarked that: *"…mathematics is very difficult, especially geometry…"* (Group Interview, 19th August 2015).

Maxine's contribution affirms the perception that the learners at WGHS held similar perceptions of geometry and mathematics. They perceived both mathematics and geometry as difficult. This was however not surprising since geometry is a branch of mathematics. Wendy elaborated further on the reason for their negative perception of geometry.

In her contribution on the reasons as to why the WGHS learners did not like geometry Wendy posited that:

"Euclidean geometry is so difficult" (Group Interview, 23rd of July 2015).

All the learners were in agreement with this perception that geometry was Euclidean geometry and it was very difficult. In the learners' minds, their understanding of geometry was limited to Euclidean geometry only. It was clear that what came to their minds when the topic of geometry cropped up in a conversation or in a lesson on geometry was basically Euclidean geometry and nothing else. To further advance Wendy's perception of Euclidean geometry, Ayanda in the same group interview volunteered an explanation for the learners' negative perceptions of Euclidean geometry.

She remarked that:

"…with the circles and angles in between, it has theorems so it's like mixed." (Group Interview, 23rd July 2015)

This perception was an indicator that the learners had problems with calculating given angles while justifying their answers using appropriate theorems. Since the theorems are either in words and / or symbols learners would activate their defence mechanism, that is, subconsciously switch off. Work by other researchers such as Hanna and De Villiers (2012) affirmed this view point as they purport that, it has always been known that learners have difficulties with Euclidean geometry.

From Ayanda and other learners' remarks, it was my interpretation that learners perceived geometry as the mathematics that dealt with shapes, finding lengths of sides, calculating missing angles with reasons and proofs. It was their view that geometry dealt with finding the length of sides and missing angles, which they perceived was not a problem as long as the question was (i) numeric in nature such that their solution would be a numerical value and (ii) did not ask them to give reasons to justify how they arrived at their solutions. It was when the question or activity required them to provide reasons and justifications for their solutions that geometry became a

"…nightmare…" to the learners, (Sbu, Focus Group, 19th August 2015). Research shows invariably that learners do not comprehend the need for proofs in mathematics and cannot distinguish between various forms of mathematical reasoning (Jones, 2002).

The view that geometry became difficult when the aspect of critical thinking was introduced was captured in Sbu's focus group contribution.

In that discussion Sbu argued that geometry is:

"…all about proving…" (19th of August 2015).

It was in that instance where they had to think critically and carry out proofs of how certain theorems and axioms were arrived at, that, in Sbu's opinion, geometry became unbearably difficult. Jones (2002) attributes difficulties learners experience in geometry to the complexity of learning to prove the given concepts, theorems and axioms. The absence of a numerical solution, and the presence of reasons to justify the steps taken to arrive at some conclusion in the proofs seem to be too much for the learners.

Similar views were echoed by Samke (Focus Group Discussion, 19th of August 2015).

Samke shared her understanding of geometry as:

"…proving this is this and this is equal to this and this is what this is."

Concurring with this perspective Sbu posited that:

"…yes, just solve for x, proving with reasons, no it's difficult."

These perceptions confirmed observations made by Jones (2002) that geometry becomes difficult when learners are unable to distinguish by intuition what it is they have been given from what is to be proved. The perception that geometry is difficult posited by learners affirmed the research findings by Mji and Makgato (2006) whilst studying factors associated with high school learners' poor mathematics and physical sciences performance. Even though they were investigating a different phenomenon South Africa they found out that learners considered geometry to be a difficult section in learning of mathematics. In a separate study Van der Sandt (2007), made similar

observations and findings were consistent with those of this study as far as conceding that learners in South Africa regarding geometry as a difficult area of learning at high school level.

What clearly emerged from this study was that learners of WGHS' perceptions of geometry were in many ways similar to their perceptions of mathematics, that is, negative. Since the learners had a negative perception of geometry it is my view that this perception was now being generalized by the learners in their view of mathematics. My position is based on learners' contributions that they enjoyed the section on analytic geometry which they did not consider as geometry.

The main reason for the perception that analytic geometry was not a section of geometry was that there was no reasoning or proofs required from them in this section, they claim that it was simply application of the formulae learnt.

According to Wendy (Group Interview, 19th of August 2015) what is good about analytic geometry is that:

"In analytic geometry the formulae do not change."

In concurrence with Wendy, Ayanda (Group Interview, 19th of August 2015) posited that she liked analytic geometry and her perception of analytical geometry was that:

"…its formulae are so easy to remember such that you cannot forget them."

In the same group interview Sbu added that:

"… it is easy to remember as opposed to other sections of mathematics like Euclidean Geometry."

Participants at WGHS agreed with Wendy, and seemed to appreciate mathematics if it did not involve critical thinking. To the learners it was important that they continued using the same formulae which they will have mastered either by cramming or repeated use, as they were reluctant to venture into new situations while applying acquired knowledge.

5.4.3 Learner perceptions of the Pythagorean Theorem

It was surprising to note that despite all the negativity on learners' perceptions of mathematics and geometry, grade 11 learners at WGHS had a positive view of the Theorem of Pythagoras, a topic in geometry.

They all concurred with Pinky (Group Interview, 23rd of July 2015) who was of the perception that Pythagoras' Theorem:

"…is very easy and easy to apply…" when using numerical values.

When asked about the origins of the Pythagoras' Theorem and its proofs, it was observed that learners had no knowledge of the origins of the theorem and could not prove that it was mathematically true. Despite not knowing the historical foundations of the Pythagorean theorem learners expressed that they were comfortable and could deploy the theorem in solving for missing sides of triangles. It was the learners' view that the Pythagorean Theorem was easier because they knew the formula and it did not change. For them it was simply regurgitation of the formula already in their minds. Ayanda demonstrated this aspect in the same interview as she quickly recited the theorem.

 Without any effort she said:

"…r squared is equal to x squared plus y squared."

It showed that learners were familiar with the Theorem. This could be easily understood since the Theorem of Pythagoras is taught in the General Education and Training Phase (GET) phase. The ease with which the learners could recite the theorem also affirmed my perception mentioned earlier that the learners had a positive perception of mathematics if it was simple application of learnt concepts.

Even though learners thought that the theorem was true and could apply it, they could not describe how it came about and how they may prove that the Theorem of Pythagoras held true. They had no knowledge of how it could be shown that the theorem held. It was the researcher view that if learners knew how to prove that the Pythagorean Theorem was true, then they may find it less challenging to apply it. Learners also expressed that they had difficulties applying the Pythagorean Theorem to determine the missing sides in given right shapes leaving their solutions in algebraic

form. The researcher further held the view that knowing how to prove the Pythagorean Theorem, may assist learners to identify instances and situations when the theorem could be applicable. Which in my opinion may result in it being easier for them to calculate the missing sides of right angled triangles. As I discussed these aspects with them, learners registered a strong resentment in using algebra in the application of the theorem. This view I associated with their view of mathematics, in which they had an aversion to algebra.

5.5 INFLUENCE OF THE HISTORY OF PYTHAGORAS' THEOREM ON LEARNERS' PERCEPTIONS

The history of Pythagoras' Theorem was incorporated as was discussed in the study's theoretical framework and research design. The genetic approach combined with some themes from the constructivists' perspective were used in designing of learning activities and cascading the knowledge on the Pythagorean Theorem, its proofs and various applications.

5.5.1 The use of historical narratives within mathematics classrooms

The incorporation of the history of mathematics in the lessons on the Theorem of Pythagoras involved the use of historical narratives on Bhaskara, Pythagoras and several other mathematicians who proved that the theorem holds. This method seemed well received by the grade 11 learners at WGHS as evidenced in their reflections captured in their journal entries. It seemed the inclusion of the history of mathematics positively impacted learner' perceptions of mathematics, geometry and the Theorem of Pythagoras itself.

According to the learners, the use of narratives from the history of the Theorem of Pythagoras made the lessons very exciting and informative. This perspective was captured in group interviews and journal entries in which learners wrote their reflections on their experiences during the lessons in which the history of the theorem was integrated. Lorraine was so engrossed in the story of Bhaskara, his life, achievements and his family.

Lorraine confessed that it was:

"Very interesting to find out about Bhaskara and his daughter..." (Group Interview, 20th of August 2015).

This in a way indicated that she now associated the theorem with real human beings and not just seeing it in isolation, an aspect which humanised the subject (Bidwell, 1993).

This view was also affirmed by Phumla who posited in her journal that:

"The lesson was good and exciting and it changed the way we look at things. We learnt about the mathematician Bhaskara which we did not know so it was good to know it for the first time..." (Journal Entry, 5th of August 2015).

Phumla acknowledged that, the history of the Theorem of Pythagoras gave her background information on the theorem which changed the way she looked at the theorem and its applications. Learning about the mathematicians who invented theorems and axioms made learning exciting for the learners as well as changed their perception of the Theorem of Pythagoras. Researchers such as Liu and Po-Hung (2003) made similar observations and cited motivation and change of learner perception of mathematics as positive benefits of incorporating the history of mathematics in the mathematics education in the form of historical narratives.

It was my finding that the history of the Theorem of Pythagoras awakened in the learners a desire to know more about the history of the mathematicians and their accomplishments. Sarah thought that the same lesson was interesting because she had learnt about the mathematician Bhaskara and expressed that the lesson had left her in suspense such that wished to read more about the story of Bhaskara and his work (Journal Entry, 5th of August 2015). Sarah's contribution made the researcher realize that a quest to learn more had been awakened in some of the learners. This view is confirmed by Barbin (2000) who is of the notion that the history of mathematics has the power to captivate learners' attention and curiosity about the subject.

The method of using stories fascinated the learners because they had not thought of mathematics having a theoretical or non-numerical aspect or background to it. For them it had always been presented as a finished product without any human element (Liu, 2003). Learners had not been exposed to this aspect of mathematics. This was evident in Sbu's journal entry (28th July 2015) where she highlighted that her perception of mathematics had been influenced in some way by the use of historical narratives during the lesson.

According to Sbu the lesson was:

"…quite interesting because I learnt that mathematics is not only about numbers… it was quite interesting to learn about the history of Pythagoras' Theorem."

In this entry Sbu admitted to the fact that her perception of mathematics had changed to include other aspects of mathematics other than numbers. This meant that her perspective of mathematics had been broadened by the use of the history of Pythagoras' Theorem. While reflecting on the lesson of the same day Feryn wrote, as she reflected information which showed that she shared the same perspective with Sbu.

Her journal entry read:

"It was a very informative lesson, I found it very much interesting to learn about Bhaskara's diagram and how the title of his book came about…" (Journal Entry, 28th of July 2015).

What I could infer from Feryn's was that the history of the Theorem of Pythagoras made learning different, exciting and worthwhile. This perception is confirmed by Horton and Panasuk (2013) who contend that including the history of mathematics in mathematics education renders excitement and emotion to the subject. This aspect of the history of mathematics had the same influence on the WGHS grade 11 learners as described by Bidwell (1993) that is, humanising the mathematics, a dimension which is missing when mathematics is perceived as cold, dull faceless numbers. Below in Figure 5.1 learners engage in work involving Bhaskara's proof and a story of his life.
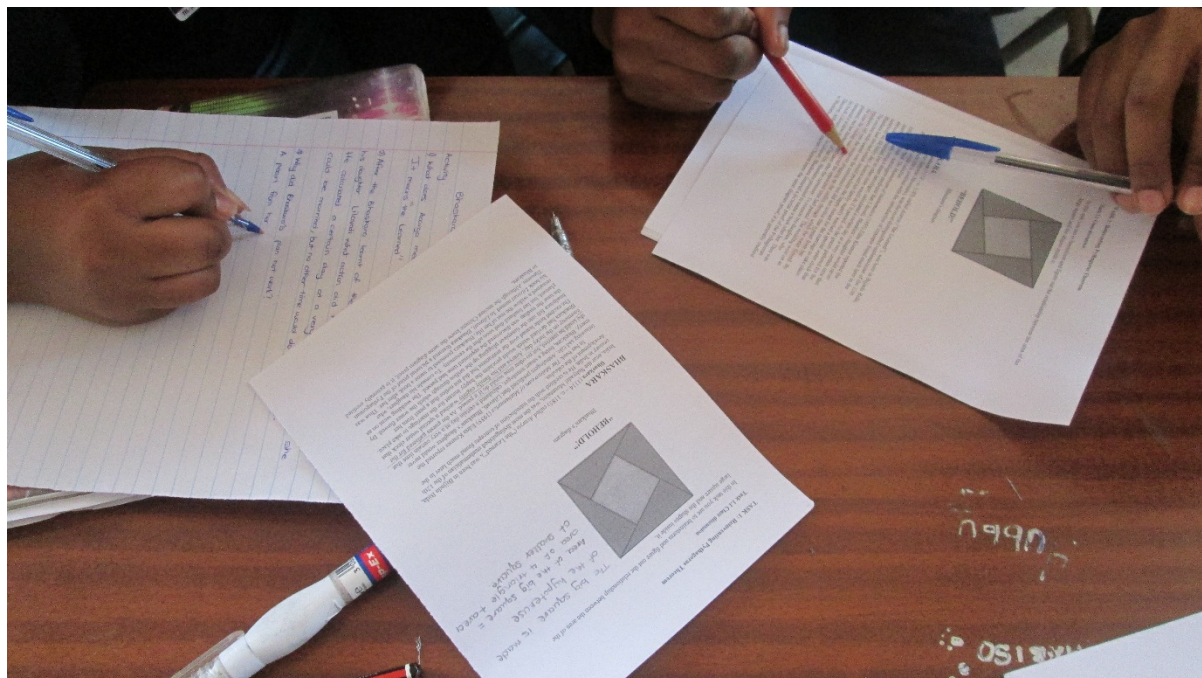
Figure 5.1 A learning activity on Bhaskara's diagram and some literature about his life

The stories about the mathematicians, their lives, pursuits and achievements served as motivating tools to the participants. From the sentiments posited by the learners, it was found that the hardships, hard work and accomplishments of the mathematicians the learners researched on and learnt about, helped learners to change their perception of Pythagoras' Theorem and geometry. This was made evident in their group interview contributions they made on 3rd of September 2015. In this group interview Nolwazi reminisced that the research she had conducted on Descartes had encouraged her to persevere even though she was faced with challenges and difficulties in mathematics and in real life. Nolwazi as a sickly child could identify with Descartes easily and felt challenged by his determination.

Through their experiences with the history of mathematics as historical narratives, learners felt that they were not alone in their struggles with mathematics as they could identify and relate with some characters in the history of mathematics. This perspective affirms the argument posited by Ernest (1998) and Fauvel (1991) that, learners' feel that their struggles are not unique to them and this realisation reduces anxiety in the learners.

Reflecting on her work on the research on the great mathematician Euclid, Lorraine claimed that Euclid's work gave her inspiration to persevere and do her best (Group Interview, 3rd of September 2015). Thus, learners at WGHS seemed more determined to do their best even in the face of challenges and setbacks. It seemed, etched within their minds was the view that, through hard work one could achieve great things. In a group interview Maxine shared her experiences while researching on the work of Mary Mirzakhani, a modern mathematician who won several awards of recognition of her great work.

Maxine reflected:

"Maryam Mirzakhani really showed me that through hard work and perseverance you can achieve a whole lot more."

Inclusion of the history of mathematics led learners to develop an appreciation of hard work and its benefits as evidenced in Maxine's contribution. These findings confirmed the notion that the inclusion of the history of mathematics is associated with learners' appreciation of the power of hard word work and determination as well as persistence in pursuit of one's dreams, (Shortsberger, 2000; Tzanakis and Arcavi, 2000).

Through historical narratives learners at WGHS perceived that mathematical greatness is not achieved in one day, one has to work for prolonged periods of time to improve and perfect one's mathematical skills. Through interpretation of historical narratives, learners were of the view that they learnt to have patience with and not to be too hard on themselves when solving mathematical problems as suggested by Phume (Group Interview, 3rd of September 2015). She claims that mathematical concepts require time, they cannot be mastered over a single activity but through consistent hard work since the mathematicians themselves took years to accomplish the mathematical theorems, axioms and postulates that are in use today (Group Interview, 3rd of September 2015). These findings confirmed findings by other researchers such as Ho (2008) who argues that the inclusion of the history of mathematics fosters a positive attitude towards the subject which may in learners developing an appreciation of the qualities of the subject such as hard work as well as fostering genuine enjoyment of mathematics.

5.5.2 The use of discovery learning within mathematics classrooms

The use of discovery learning, which is a method that is highly favoured by both the genetic approach and constructivists' ideology was used as an aspect of the history of mathematics as lessons were conducted with grade 11 mathematics learners at WGHS. When this method was used learners had to reinvent the Pythagorean Theorem on their own using minimum assistance from the teacher. This method required the learners to apply their problem solving skills and to logically create sound proofs of the theorem.

Before the lessons in which the history of the Theorem of Pythagoras was incorporated were taught learners had a negative attitude towards proofs as captured in Sbu's contribution on what learners' views of geometry were. In the interview held on (19th August 2015) Sbu claimed that geometry was all about proofs and reasons which was what made geometry difficult.

Now after the lessons commenting on the fact that they had to prove that the Theorem of Pythagoras holds for themselves, Ayanda reminisced that the history of the theorem:

"…helped me understand the roots of what I am doing because we sometimes ask ourselves this question - why do we have to apply Pythagoras' Theorem?" (Group Interview, 3rd of September 2015).

This was an indicator that the participants appreciated the inclusion of the history of the theorem because it made them realise why it was necessary to learn Pythagoras' Theorem and carry out its proofs and application. It was also be observed that there was a change in perception on proofs. This affirms the research findings made by Horton and Panasuk (2013), that the history of mathematics provided a rich foundation for conceptualising the development of mathematical concepts. The significance of these parallel findings was that it broadened the utility of history of mathematics in pedagogical settings in other areas of the mathematics curriculum and not necessarily the Pythagorean Theorem as was in this study.

Discovery learning involving the use of the history of Pythagoras' Theorem is fun as claimed by the participants from WGHS. This is confirmed for example by Winnie when she commented on a lesson in which the learners had to discover the Pythagorean Theorem.

Winnie's perception of the lesson was positioned as she claimed that:

"The lesson was very good…" (Journal Entry, 28th of July 2015).

This was after they had learnt how to make their own equations using the shapes which was a new experience in mathematics for them. Winnie went further to describe her experience during the lesson:

"I learnt a new work and also had fun."

This perception was shared by Nonhlanhla as captured in journal entry contribution (28th of July 2015) where she indicated that the lesson was:

"…interesting, as I usually learn to use the theorems not actually proving or looking at how they were invented."

It is clear that what made the lesson interesting in Nonhlanhla's opinion is that she learnt about the origins of the theorem and proved it. Through discovery learning the inclusion of the history of the Theorem of Pythagoras, proofs were no longer scary to the learners but interesting and fun. Participants at WGHS expressed their appreciation of the experience of discovering things themselves instead of having the results being imposed on them. These sentiments are reflected in learners' perceptions on the method that was employed by the teacher shared by some of the learners.

Phume (Journal Entry, 28th of July 2015) preferred this method of learning as indicated in her argument that:

"…I liked the method because she lets us do the work without her input and we liked it because there were no rules she gave us on how to answer the question so we did the work according to our understanding…"

Expressing her excitement on the discovery learning method Jackie documented that:

"The teacher lets us do the work ourselves rather than just feeding us on information. The method she used made it fun for us to understand what we were learning." (Journal Entry, 28th July 2015).

Whilst investigating the influence of discovery learning on student success and the acquisition of inquiry learning skills Balim (2009) expressed that activities on discovery learning made lessons much more exciting for the learners. Although the focus of my investigation was different consistent with findings by Balim (2009) the effects of discovery learning in both contexts were found to be similar.

Nozipho, who had not invented or proved the Theorem of Pythagoras before expressed her enthusiasm at having re-discovered the theorem for herself.

She maintained that:

"I really learnt something very good because I didn't know how the Pythagoras' Theorem came about." (Journal Entry, 5th of August 2015).

From Nozipho's contribution I interpreted that the history of mathematics through the employment of discovery learning assists learners in discovering and learning about the origins of the concepts at hand.

WGHS participants found out that discovery learning encouraged creative thinking. To substantiate this perception some extracts of learners' contributions are presented.

From Sammy's point of view the lesson on reinvention of the Pythagorean Theorem was:

"Mind awakening…" (Journal Entry, 5th of August 2015).

Reflecting on the same lesson Lorraine (Journal Entry, 5th of August), thought that:

"The lesson was very interesting and quite challenging, the shapes and squares were just about complicated."

Ayanda in her response in the group interview (5th August 2015) purported that discovering the Theorem of Pythagoras:

"Was very interesting and challenging like we said before that maths, you find maths quite frustrating so this time, being able to do it by ourselves, on our own, it was like a victory, like you have won a battle…you could do it yourself, you could prove it right without actually getting frustrated, being stressed trying to figure out a way forward."

Ayanda and her peers' contributions show a change of perception that mathematics was no longer frustrating and distressing, it was now viewed as interesting and mind awakening. This change in

my opinion was attributed to the learning activities learners at WGHS were exposed to in which they experienced some victories over the subject. The change in their view may be taken to insinuate that if other topics were taught in the same manner and learners went through similar experiences even these topics and sections of mathematics could also be interesting. Even though they may be challenging learners would still enjoy the learning experience and small victories in their discoveries or inventions.

Pretty commenting on the same lesson thought the lesson was interesting because it opened their minds to think outside the box which she thought was important in understanding mathematics and other subjects. Learners were of the perception that discovery learning in which they used several ways to reinvent the Theorem of Pythagoras challenged their thinking capabilities and made them exercise their minds which in their opinion was good. This change in perception confirmed the notion posited by Furinghetti (2000) that the history of mathematics encourages flexibility and open mindedness when dealing with mathematical problems.

5.5.3 The use of cooperative learning within mathematics classrooms

As part of the genetic approach to learning, which is an aspect of the history of mathematics, cooperative learning was used in designing and teaching the activities on Pythagoras' Theorem. Cooperative learning involved learners engaging in group tasks, working in pairs or participating in class discussions.

Learners carried out paper cutting activities in groups of five. They were excited to engage in the paper cutting activities as captured in their reflections on the activities and the lesson which were documented in their journal entries. These activities were exciting to the learners because as they claimed they were doing it themselves using their psychomotor skills. The method appealed to learners as recorded in Phume's journal entry (28th of July 2015).

In the entry she purported that:

"The lesson was fun and realistic because we were using objects we can see and we were doing our work all by ourselves, interesting because we were proving what we did, on our own."

Concurring with Phume, Maxine claimed that the same lesson was fun as she remarked that:

"I liked using my hands, so I enjoyed it."

Sammy regarded the same lesson as *"awesome"* because according to her:

"We got to cut and compare shapes."

Samke (Journal Entry, 5th of August 2015) went on further to explain why she thought this lesson was interesting. She elaborated that:

"The lesson was very fun because we got to cut shapes out and figure out how they connect to each other and we got to see things about shapes that I did not know."

From all these contributions I gleaned that what made the lesson *"fun"* and *"interesting,"* as the learners put it, was the fact that they were involved in the lesson and they did the work themselves using concrete objects they could see, touch, cut and paste which made the lessons quite appealing to the learners. This was in confirmation of the ideas put forward by Seefeldt and Wasik (2006) who posit that children learn better when they manipulate concrete objects in their environment in group settings. Figure 5.2 that follows shows learners engaged in cooperative learning.
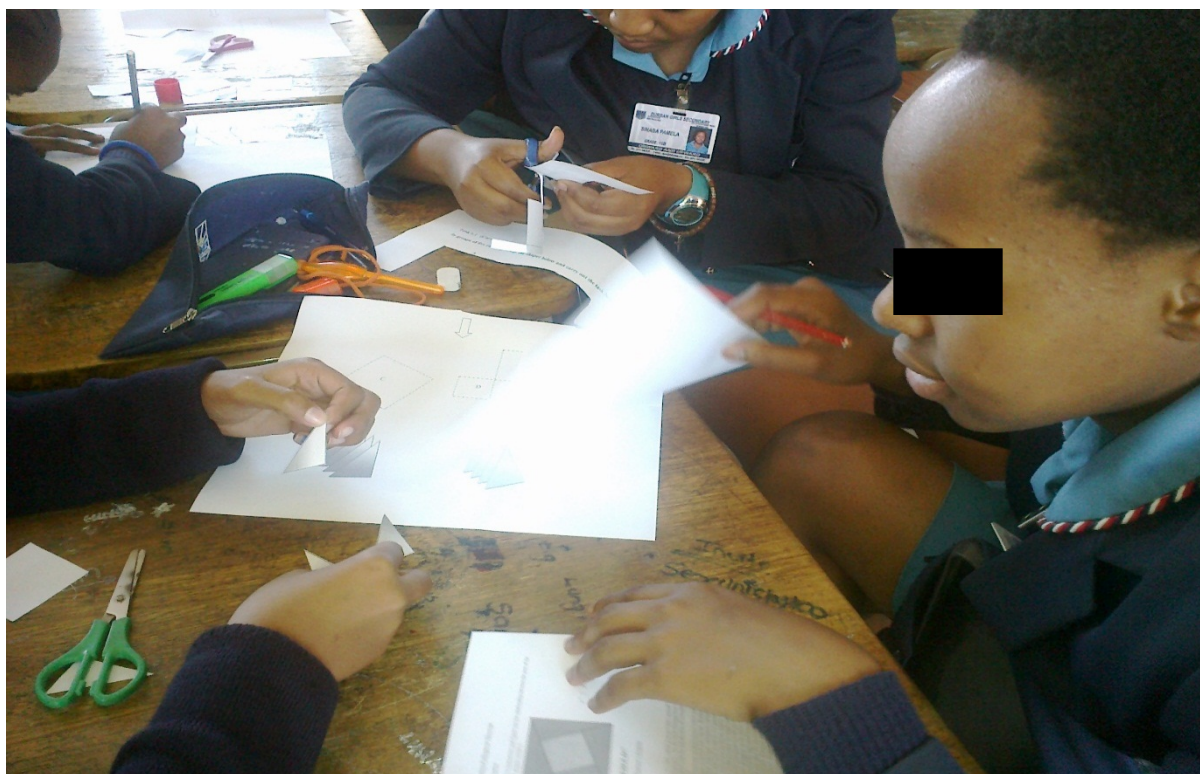
Figure 5.2 Learners engaged in a cut and paste activity

The grade 11 learners at WGHS enjoyed working in groups and they claimed that this method of learning presented many benefits. As Vygotsky (1978) puts it, collaborative learning harnesses learners' focus and allows them to share their world views in comfortable environments. In affirmation participants at WGHS claimed that group work encouraged sharing of ideas. As they worked in groups they discussed both their understanding of the tasks at hand and their individual solutions to the questions in the tasks.

Learners posited that it was through these group discussions that better understanding of questions and quality solutions to questions emerged. Working in group Nonhlanhla purported, made her enjoy the lesson because she was afforded the opportunity to work as a team with other learners to synthesise their ideas and conceive more meaningful and refined solutions to the tasks in which they were engaged (Journal Entry, 28th of July 2015). Feryn viewed group work as a method that simplified learning.

She remarked that:

"…group work makes it easier for us to understand, through sharing each other's ideas and having different perspectives on how to comprehend the questions."

These findings are similar to those obtained by Aschermann (2001) in her study on collaborative learning amongst children. Even though the essence of the study was different her findings on the benefits of corporative learning were confirmed by the findings in this study.

The learners maintained that through working in groups those learners who were shy and who would want to keep to themselves got the chance to gain confidence and interact with other learners. Londy expressed her joy in engaging in group activities and posited that group work provided her with the opportunity to express herself to her group members with confidence, which translated in her gaining confidence in dealing with mathematics problems.

As she reflected on the lesson she went on to describe her experiences as:

"…fun, I could express myself very well and maybe it will help me stop being shy to answer questions. A very good way of gaining confidence in mathematics…" (Journal Entry, 28th of July 2015).

Gillies (2004) maintains that learners are more perceptive of the needs and challenges of their fellow learners and are often prepared to offer unsolicited assistance. Such an environment breeds healthy social relations and provides opportunities that nurture learner potential (Smith & MacGregor, 1992).

Group work provided learners with the opportunity to participate in the activities of the lesson, since there were only a few learners in a group each member got a chance to take part. This benefit of group work was shared by Jackie in her journal entry dated 28th of July 2015.

In her journal entry, Jackie maintained that:
"The method used made it fun for us to understand what we were learning… we worked in groups and we were all part of what was being taught for the day."

Subscribing to the same view Wendy posited that:
"The lesson was fun and all of us as learners participated in the lesson and helped one another by sharing ideas…" (Journal Entry, 5th of August 2015).

Hence group work encouraged interaction and the development of interpersonal relationships amongst the learners which resulted in the lessons on the Theorem of Pythagoras being exciting and fun (Gillies, 2004).

Figure 5.3 Learners engaged in group activities

5.5.4 The use of technology within the classroom

The genetic approach as a theoretical framework calls for novel approaches to teaching, which are context-based. As I developed and integrated the history of mathematics technology integration was an imperative. Technology had the potential to capture learners' attention and maintain learners' interest hence it had a motivating effect on the learners (Shelly, Cashman, Gunter & Gunter, 2004). Learners had a lesson in which the laptop, projector and a whiteboard were used to present diagrams on the proof of the Pythagorean Theorem which were drawn using the Geogebra application. The learners used these diagrams to re-discover the proofs firstly using a trapezium that had been divided to form various sizes of triangles and secondly using rectangles assembled together to prove the Pythagorean Theorem.

In spite of having professed that they did not enjoy working with numbers, algebraic terms and expressions before the lessons in which the history of Pythagoras' Theorem was incorporated participants at WGHS echoed a different view after the lessons. It was interesting to note that learners now confessed that they were completely engrossed in the lesson and its activities when

the history of Pythagoras' Theorem was used in collaboration with the use of the Geogebra application. When asked if the use of the computer, projector and white board had any effect on the lesson and their view of the Pythagoras' Theorem learners indicated that technology made the lesson more interesting and captivating.

Ayanda's response to the question was:

"It was an exciting method because you were not only teaching us about mathematics but giving a view about how you can use devices like the laptop and the projector to actually make mathematics exciting apart from the daily sequences of just using the chalkboard and writing down notes and trying to figure things out. I think you gave us a perspective of things we usually don't get from mathematics..." (Group Interview, 17th of September 2015).

The diagrams presented in Figures 5.4 and 5.5 were presented to learners in the lesson in which technological devices were used while teaching and learning of the Pythagorean Theorem proofs. The lesson aimed at demonstrating to learners that the Pythagorean Theorem can be proved using shapes other than squares.
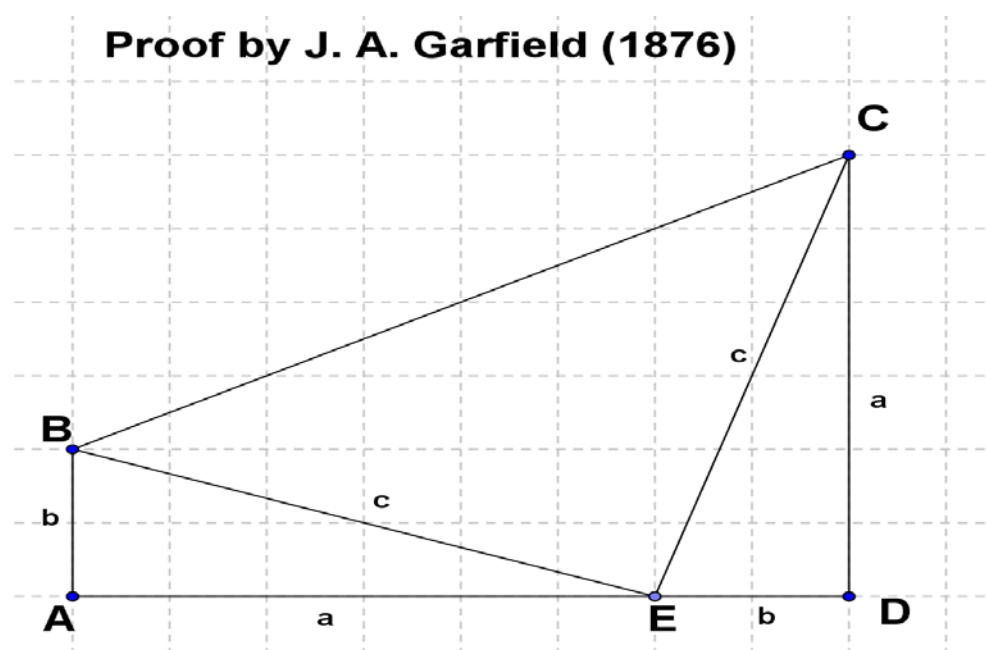
Figure 5.4 Proving the Pythagorean Theorem using a trapezium (drawn using Geogebra software)
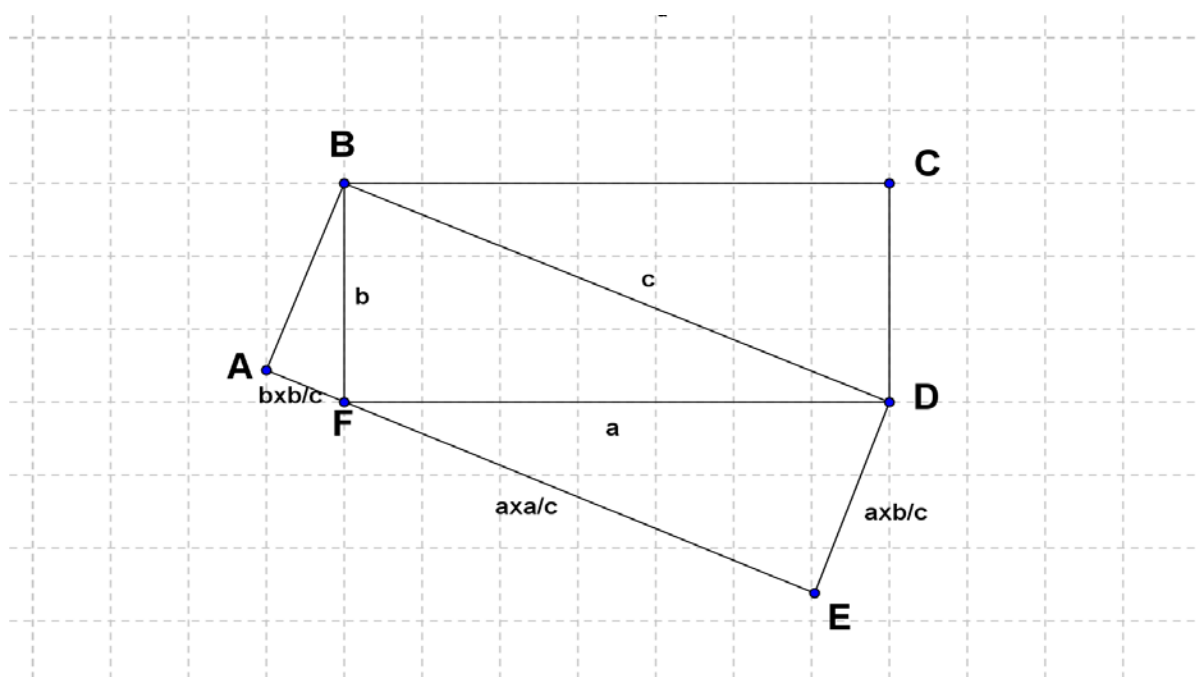
Figure 5.5 Proving the Pythagorean Theorem using rectangles (drawn using Geogebra software)

Learners viewed the use of the Geogebra application on a laptop and projector as a way of concretising things even though they could not touch and manipulate the shapes themselves using their hands. In the subsequent interview after the lesson in which the gadgets mentioned before were used to teach some proofs of the theorem of Pythagoras' Theorem Pinky's remarks on the use of technology were that:

"It got more people to concentrate on what they were doing..." (Group Interview, 17th of September 2015).

This perception affirmed the observations made by Naidoo (2006) that visual tools have utility in communicating mathematical ideas as they make mathematics easier, more concrete, accessible and comprehensible.

Phume supported these observations by claiming that:

"Some of us like to look at things or watch, so a visual board makes an impact in our understanding of the Pythagoras' Theorem because those shapes stayed in our minds…" (Group Interview, 17th of September 2015).

What emerged from these remarks was that learners have different styles but the majority of the WGHS grade 11 learners were visual learners. Learners' were captivated with work that was presented in an array of colours. Affirming this perception Phume (Group Interview, 17th September 2015) cited black and white hues, the colours dominant in their textbooks as boring to the mind because they were used to these colours.

Phume claimed that:
"Some other colours blue, some green … that made us interested and we were part of the lesson…"

In that same group interview Pinky added to this perspective highlighting that:
"…using technology to study is the best and fun way you can learn things."

These views by learners support research findings made by Naidoo (2012) who claimed that, the use of vivid colours had the power to create exciting, interesting and fun mathematics teaching and learning contexts. In this era where the youth is synonymous with technological astuteness using it in the teaching and learning process takes advantage of what most learners were brought up with and find difficult to resist (Bester & Brand, 2013).

Confirming this perspective Ayanda argued that:
"…we are a very lazy generation, we sit around a TV, so using the projector is a way of turning something that is a habit to us and something we usually do and make it into something that can benefit us rather than destroy us…" (Group Interview, 17th of September 2015).

All these contributions by learners were affirming the observations that were made by Naidoo (2012). It was my interpretation that the incorporation of the history of Pythagoras' Theorem enhanced with the use of appropriate technology had led to change in learners as they showed an interest in the Pythagorean Theorem proofs.

5.6 ASPECTS OF THE HISTORY OF MATHEMATICS THAT HAD THE MOST INFLUENCE ON LEARNER PERCEPTIONS

Findings were that not all aspects of the history of mathematics had equal influenced on learner perceptions. In this section I present observed aspects of the history of mathematics that were adjudged to have had a greater influence on learner perceptions in this study. These aspects have been ranked in order influence from the most to the least influential. It is important to note that even though discovery learning had significant influence on learners' perceptions of mathematics, geometry and the Pythagorean Theorem, it had the least influence so it was therefore not included in this section.

5.6.1 The use of historical narratives in the teaching and learning of mathematics

The history of mathematics is full of emotional instances whose inclusion in the mathematics classroom has the potential to capture learners' attention and curiosity about mathematics (Barbin, 2000). Using stories from the history of mathematics seemed to have remarkable impact on learner perceptions of the Pythagorean Theorem. From my observation and background information on the learners this may be attributed to the fact that most of the learners enjoyed reading and poetry as hobbies. Learners' inclination towards reading and poetry provided rich ground for their understanding and appreciation of the literature on the history of mathematics as indicated by Lorraine and Sandra in the findings.

During and after the lessons in which the history of Pythagoras' Theorem was integrated in the lessons participants at WGHS began to associate mathematics with real people after learning about the inventers of the theorems, axioms propositions and postulates used in mathematics today. They no longer treated mathematics in isolation but linked it with real life experiences since mathematical concepts originated in a bid to solve problems that communities faced at particular times. Londy (Focus Group Discussion, 19th August 2015) remarked that her perception of mathematics had been altered such that she did not regard it as invincible anymore. Londy argued that she now realised that if mathematicians did all they did and they were mere humans nothing also should stop her from doing the same thus her realisation that it can be done.

Pinky (Group Interview, 20th August 2015) outside the focus group, also pointed out that the story of mathematician Mary Fairfax Somerville taught her that there is always a way of overcoming obstacles and hurdles in life and mathematics, making all things possible. Findings by researcher Ho (2008) that the history of mathematics motivates learners to persevere affirms what was contained in Londy and Pinky's perceptions after the incorporation of the history of mathematics.

Researching on some modern mathematicians served as a motivational tool that prompted learners to realise that there are still things to discover and even today some people are still working hard and persevering.

These sentiments were captured in Maxine's contribution (Group Interview, 3rd September 2015) that:

"Through learning mam you can achieve many things, she (Maryam Mirzakhani) won the highest prize in the mathematics field, a medal, yes she won that…. So she really showed me that through hard work and perseverance you can achieve a whole lot more."

Before exposure to mathematical content in which the history of mathematics had been infused learners associated mathematics with numbers. After exposure to lessons in which historical narratives had been used learners began to perceive the subject as not constituted by numbers and mathematical problems to be solved only as a result learners took interest in it instantly.

Thandi (Journal Entry, 28th July 2015) reflecting on the lesson in which historical narratives were used remarked that:

"It was a great lesson and I really did enjoy a different learning style without the sums and all is very interesting."

The fact that learners were not dealing with numbers made it less intimidating for them. Nolwazi's input points to this effect when she says:

"I think I like comprehensions, yes it is based on mathematics, but its only just comprehensions on the people that created the maths."

In my interpretation the use of the historical narratives had the most influence because learners were motivated to learn and the interest was maintained since learners now waited eagerly for the

next lesson to find out which mathematician they would be learning about. Learners no longer dragged their feet to a mathematics class, for they did not want to miss out on the activities of the day.

5.5.2 The use of cooperative learning in mathematics classrooms
The integration of the history of mathematics through cooperative learning on proofs of Pythagoras' Theorem had great influence on learner perceptions of the theorem because learners, working as a team in small groups shared ideas and discussed the demands of each activity. Learners perceived the demands of the activities differently but through discussion they interrogated every perception and created a more refined understanding of the requirements of the activities at hand. Through collaboration as an aspect of the history of mathematics leaners realised that they did not have to work in isolation but had their peers to critically analyse their ideas and work.

When learners were confronted with the question as to why they did not consider geometry as a favourite subject before attending lessons in which the history of mathematics was included, their major reason was they considered proofs to be difficult and confusing. After engaging in several lessons in which learners collaboratively worked on their own discoveries of the Pythagorean Theorem, learners still considered proofs to be challenging but now they had an appreciation for them, enjoyed proving the Pythagorean Theorem in various ways and stood up to the challenges and seemed unfazed. According to Ayanda (Group Interview, 17th of August 2015), proofs were no longer frustrating and difficult but simply a challenge that needed to be subdued and conquered. It may be deduced that the incorporation of the history of mathematics using collaborative learning had some significant positive influence on learner perceptions of proofs by helping them overcome their fear of proofs and reduce mathematical anxiety (Marshall, 2000).

Working in groups made learners aware that they did not have to struggle alone. Working with others assisted them in that often peers have a better understanding of the challenges being experienced by their fellow learners and are in a better position to offer assistance in that regard. Feryn (Journal Entry, 28th July 2015) was of the opinion that group work activities were more

helpful because they helped them understand the work they were doing and gave the learners the opportunity to interact.

Discovery learning involving group work activities, a tenet of the genetic approach to learning presented a somewhat informal working environment for the learners. The informal environment created made learners comfortable and seemed to reduce their anxiety in learning proofs. Within this collaborative environment learners seemed better disposed to learning and as a result their perception on Pythagoras' Theorem was positively influenced in some way. Collaboratively experiencing mathematics in the making made learners view mathematics as a dynamic human enterprise (Grugnetti & Rogers, 2000). Reinventing the Pythagorean Theorem collaboratively, learners at WGHS realised that what they have achieved was what they once considered to be the work of great mathematicians and began to regard themselves as such. This boosted their confidence in solving problems on Pythagoras' Theorem and geometry in general.

5.6.3 Technology integration in teaching and learning

At this stage let me hasten to say, even though technology is not a direct aspect of the history of mathematics, its integration with the history of Pythagoras' Theorem had tremendous influence on learner perceptions of geometry. Learners appreciated the use of technology since we are in the era when technology is the order of the day. It was a good experience for them since they enjoyed watching movies, using cell phones and other electronic gadgets. Use of technological devices in the learning of the Pythagorean Theorem in which its history was incorporated, was simply bringing their favourite gadgets into the lesson. The diagrams presented to them on the white board were large, vivid and colourful. This had an impact on their perception of the lesson work since learners reiterated that bright colours appealed to them.

5.7 CONCLUSION

In this chapter the findings of the study were presented, analysed and interpreted. The next and last chapter of this research study endeavours to present concluding remarks, recommendations to educators and curriculum developers and discusses the limitations of the study.

CHAPTER SIX

CONCLUDING REMARKS, IMPLICATIONS AND LIMITATIONS OF STUDY

6.1 INTRODUCTION

Chapter five dealt with the presentation and discussion of this study's findings. This chapter provides concluding remarks and discusses the implications of the research study findings and limitations. Recommendations are also suggested for possible studies within mathematics education.

6.2 CONCLUDING REMARKS

This study sought to ascertain whether the inclusion of the history of the Theorem of Pythagoras had any positive influence on learners' perceptions of the theorem and its applications, on geometry and on mathematics in general. To accomplish this aim this study had to establish before and after the incorporation of the history of mathematics in the lessons on the Pythagorean Theorem. Changes in learner perceptions after the inclusion of the history of mathematics were regarded as indicative of the influence of the history of mathematics. After interrogating learner perceptions the study also endeavoured to establish which aspects of using the history of Pythagorean Theorem had the most influence on learners' perceptions of the theorem, geometry and mathematics.

An interpretation of analysed results indicated that learners' perceptions of mathematics before the lessons in which the history of mathematics was incorporated were that, mathematics was a difficult subject and geometry was a very difficult section of mathematics. After the inclusion of the history of mathematics learners' perceptions were that mathematics and geometry were not as difficult as they had initially indicated. Despite challenging tasks, learners now regarded mathematics and geometry as challenges that needed to be faced head-on and conquered.

The influence of the history of mathematics was observed in that before the use of the history of mathematics participants considered mathematics to be a frustrating subject, as learners failed to

get expected solutions to problems. It was noted however that after the lessons in which the history of mathematics was included learners no longer considered mathematics to be frustrating even after failing to get the expected solutions to given problems. It seemed, from the inclusion of the history of mathematics learners had learnt that failure is part of the learning process. Despite sometimes failing to get expected solutions to the tasks they were assigned to, learners reiterated that the lessons were exciting and interesting even though sometimes they had difficulties trying to invent their own proofs for Pythagoras' Theorem. Before the inclusion of the history of mathematics lessons on Pythagoras' Theorem, learners perceived mathematics as time consuming and demanding such that it required total commitment and putting in long hours of hard work. After the incorporation of the history of mathematics in their lessons even though learners still considered mathematics to be a subject that required time and concentration they seemed to appreciate hard work. The participants now regarded hard work as an investment and enjoyed it without negativity. Through historical narratives learners saw the mathematicians as their role models, considering the long periods of hard work that were put in by the mathematicians in the history of mathematics. This aspect encouraged learners to work hard in the face of adversity.

The perception of learners regarding proofs before the lessons on Pythagoras' Theorem were conducted was that it was the proofs that made geometry difficult. Learners did not appreciate the benefits of proofs. After the integration of the history of mathematics, solicited learner perceptions of proofs were found to have changed as learners now had an appreciation of the reasoning behind conducting proofs. Learners no longer regarded proofs negatively but now enjoyed making their own discoveries through proving the existing Theorem of Pythagoras.

6.3 IMPLICTIONS OF STUDY'S FINDINGS

After careful analysis and interpretation of the findings from this study I came up with some implications that may be useful for practice, mathematics teachers, policy makers and further study in South Africa. These implications are laid out as follows: section (6.3.1):- implications for practice, section (6.3.2):- implications for educators, section (6.3.3):- implications for policy makers and section (6.3.4):- implications for future research.

6.3.1 Implication for practice

This study has some implications for mathematical didactics. Considering the findings of this study it is imperative that the history of mathematics be considered as a tool in mathematics education. The incorporation of the history of mathematics in this study, brought about positive changes in learners' perceptions of mathematics, geometry and Pythagoras' Theorem. The history of mathematics could be considered therefore as a vehicle for improvement of motivation amongst learners of mathematics, if used effectively (Liu, 2003). The history of mathematics if used appropriately, may assist learners to develop positive perceptions towards mathematics and an interest in learning mathematics. If implemented appropriately the integration of the history of mathematics may assist learners in appreciating the benefits of consistent hard work and perseverance in mathematics and in life in general.

Observations of practice have been that teachers meet learners and start engaging them in learning activities without finding out their perceptions of what they are going to learn about. The findings of this study made me realise that it is important to solicit learners' perceptions on topics or sections to be taught before teaching them. The solicitation of learners' perceptions before the teaching of a section assists teachers to identify the deeply held views that may exist amongst the learners concerning that section of mathematics or any other subject. Such an exercise will serve to authenticate if "real" learning has taken place. So establishing learner perceptions helps the teachers in designing appropriate learning tasks that may assist learners in turning around their perceptions of some topic or section, if there existed any negative perceptions. Instead of giving tests after a topic has been taught it may also be of paramount importance to assess learners by checking if there has been a positive perception change after a teaching and learning exercise.

6.3.2 Implications for educators

It was observed that the use of narratives from the history of mathematics yielded positive influence on learners' perceptions of mathematics, geometry and Pythagoras' Theorem. The observation was that the use of narratives from the history of mathematics captured and maintained learners' interest in mathematics. My recommendation in this regard is that practising educators use appropriate historical narratives to introduce concepts to be learnt. In this regard educators

need to show clearly the correlation between the mathematicians' lives, challenges they faced and their work.

An additional observation was that, employing the history of mathematics using the genetic approach may help the learners to develop an appreciation of mathematics, its origins and its importance in impacting community development. The history of mathematics assisted learners to develop positive perceptions about proofs in geometry and appreciate the reasons for conducting proofs in mathematics. Hence, if used properly the history of mathematics could yield similar results in other fields of mathematics, that is awaken a desire in learners to become heroes of their communities by finding mathematical solutions to solve everyday problems. This reason would prompt learners to work hard and persevere in the face of challenges.

Discovery learning, an aspect of the history of mathematics assisted in developing of reasoning skills, inquisitive minds and confidence in mathematics by the learners. Considering this finding the educators of mathematics may be recommended to use the history of mathematics in which learners make their own discoveries in mathematics as this assists learners in developing inquiring minds and confidence in handling mathematical situations. To facilitate discovery instruction must be structured without a single method in mind. Learners need to be presented with opportunities to explore multiple possibilities. As they discover connections and conceptual conceptions, learners do not only get the intellectual affirmation, but also their sense of competency is conduced.

Triangulating mathematics, technology integration and the history of mathematics had a profound influence on learner perceptions of mathematics, geometry and the Theorem of Pythagoras. For practising educators combining technology and the history of mathematics in mathematics education assists in keeping learners captivated in mathematics lessons.

6.3.3 Implications for policy makers

My findings had some implications for the policy on South African mathematics education. The incorporation of the history of mathematics in the mathematics curriculum could have similar benefits as those established in this study. Instead of just having snippets of the history of mathematics in some of the textbooks it may be worthwhile investing in the integration of the

history of mathematics in mathematics education at policy level. Learners find it interesting to learn the history of mathematics but if it is not examinable educators may consider it time wasting as they are in a race to complete the syllabus (Horton & Panasuk, 2013). However making the history of mathematics part of the syllabus and examinable, will be a worthwhile avenue to explore at policy level or in curriculum development.

My recommendation is that the structure of learners' examinations in mathematics be tailored to include comprehension questions and essays to cater for learners who are not good with numbers. The inclusion of the mathematics would be even more beneficial this way more than just learning it for perception change and motivation.

Making resources of the history of mathematics readily available in schools and libraries would expose learners to the information on the subject such that reading about it becomes a part of their daily routine.

6.3.4 Implications for future research

For future research it may be necessary to explore the theoretical aspects of the integration of the history of mathematics. Addressing the "why" question would serve as a better premise to argue for further theoretical exploration for the possibilities and dilemmas for the integration of the history of mathematics.

6.4 LIMITATIONS OF STUDY

While conducting this study some limiting circumstances were encountered which will be discussed in this section. Firstly, a large sample or multiple case studies would have been ideal for this study, but due to accessibility constraints a single case of only thirty grade 11 learners was studied instead. Secondly, due to time constraints only a unit on Pythagoras' Theorem was taught instead of the whole section of grade 11 geometry.

The researcher recognised that there were some limitations and weakness using interviews as a data collecting method (Marshall & Rossman, 1995) with regard to: (i) its dependency upon the full cooperation of the participants, (ii) some of the participants who may have been uncomfortable

sharing all that was hoped to be explored and (iii) there was a possibility that some elements of the interview responses may not have been properly comprehended by the researcher.

6.5 CONCLUSION

This study sought to investigate the possibility of incorporating the history of mathematics in the mathematics curriculum, to find out if it had some influence on learners' perceptions of Pythagoras' Theorem, geometry and mathematics in general. When I set out on this investigation, the inclusion of the history of mathematics was just a notion I was entertaining. However after analysing and interpretation of the results I am truly convinced that infusion of the history of mathematics in mathematics education has astounding benefits even for South Africa. Similar studies have been carried out by researchers such as McBride and Rollins (1977), Philippou and Christou (1998), but no record of such a study had yet been carried out in South Africa. I conclude this study by proposing that, as observed elsewhere the history of mathematics has great yet untapped potential to resolve the challenges of learner apathy and facilitate greater rate of uptake in mathematics at FET phase in South Africa.

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Appendix 1

UNIVERSITY OF ™ KWAZULU-NATAL

INYUVESI YAKWAZULU-NATALI

14 October 2015

Mrs Winfileda Kapofu 2145856621
School of Education
Edgewood Campus

Dear Mrs Kapofu

Protocol reference number: HSS/1207/015M
Project title: Exploring the teaching of the Teorem of Pythagoras from a historical perspectives

Full Approval – Expedited Application
In response to your application received on 20 August 2015, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol have been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....
Dr Shenuka Singh(Chair)
Humanities & Social Sciences Research Ethics Committee

/pm

cc Supervisor: Prof M de Villiers & Dr J Naidoo
cc. Academic Leader: Professor P Morojele
cc. School Administrstor: Ms T Khumalo

Humanities & Social Sciences Research Ethics Committee
Dr Shenuka Singh (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Telephone: +27 (0) 31 260 3587/8350/4557 Facsimile: +27 (0) 31 260 4609 Email: ximbap@ukzn.ac.za / snymanm@ukzn.ac.za / mohunp@ukzn.ac.za
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Appendix 2

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Enquiries: Nomangisi Ngubane   Tel: 033 392 1004   Ref.:2/4/8/547

Mrs W Kapofu
19 Ethelbert Mews
36 Ethelbert Road
QUEENSBURGH

Dear Mrs Kapofu

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"EXPLORING THE TEACHING OF THE THEOREM OF PYTHAGORAS FROM A HISTORICAL PERSPECTIVE"**, in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions   where the intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 12 October 2015 to 31 October 2016.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Connie Kehologile at the contact numbers below.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report / dissertation / thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

UMlazi District

Nkosinathi S.P. Sishi, PhD
Head of Department: Education
Date: 09 October 2015
KWAZULU-NATAL DEPARTMENT OF EDUCATION

POSTAL: Private Bag X 9137, Pietermaritzburg, 3200, KwaZulu-Natal, Republic of South Africa   ...dedicated to service and performance
PHYSICAL: 247 Burger Street, Anton Lembede House, Pietermaritzburg, 3201. Tel. 033 392 1004**beyond the call of duty**
EMAIL ADDRESS: kehologile.connie@kzndoe.gov.za / Nomangisi.Ngubane@kzndoe.gov.za
CALL CENTRE: 0860 596 363; Fax: 033 392 1203 WEBSITE: WWW.kzneducation.gov.za

Appendix 3

Angela Bryan & Associates

6 La Vigna
Plantations
47 Shongweni Road
Hillcrest

`

Date: 28 November 2016

To whom it may concern

This is to certify that the Masters Dissertation, Title: The History of Mathematics as a Pedagogical Tool: Exploring Learner Perspectives with Regard to the Theorem of Pythagoras written by Winfida Kapofu has been edited by me for language.

Please contact me should you require any further information.

Kind Regards

Angela Bryan

angelakirbybryan@gmail.com

0832983312

Appendix 4

Turnitin report

The History of Mathematics as a Pedagogical Tool: Exploring learner perspectives with regards to the Theorem of Pythagoras

ORIGINALITY REPORT

%14	%12	%3	%7
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	www.coursehero.com Internet Source	<%1
2	www.acci.org.za Internet Source	<%1
3	146.230.128.141 Internet Source	<%1
4	129.63.176.150 Internet Source	<%1
5	files.eric.ed.gov Internet Source	<%1
6	www.scribd.com Internet Source	<%1
7	www.mathunion.org Internet Source	<%1
8	frink.machighway.com Internet Source	<%1

Appendix 5

Parental and participants consent letters

Dear Parent / Guardian

I am a Master's student in the School of Humanities at the University of KwaZulu Natal. The purpose of writing you this letter is to seek your consent to your child / ward's participation in my Master's research project.

The title of my project is: The History of Mathematics as a Pedagogical tool: Exploring learner perspectives in regard to the Theorem of Pythagoras. The purpose of this study is to find out if the inclusion of some historical content on Pythagoras' Theorem positively influence learners' perspectives of the theorem. If so then the history of mathematics may be used to influence learners' perspectives of mathematics in general in the South African context.

The research will involve interviewing your child /ward in focus groups, before and after the inclusion of some historical content on the Theorem of Pythagoras in lessons in order to find out her perspective of the Theorem of Pythagoras. The interviews will take place at school during normal school hours and will be between twenty five to thirty minutes long. Your child /ward's participation in this research will be treated confidentially and all information will be kept anonymously.

This project will be supervised by Professor Michael de Villiers of University of KwaZulu-Natal. If you have any comments or questions about this research please could you contact my supervisor, using the contact details provided below.

Contact details: Prof Michael de Villiers

Mathematics Education (Edgewood Campus)

University of KwaZulu-Natal

Private Bag X03

3605 ASHWOOD, South Africa

Tel: 027-(0)31-2607252 (w)

Fax: 027-(0)31-2603697 (w)

This research has been approved by the University of KwaZulu-Natal Ethics Committee. Many thanks in advance for your consideration of this project. Please let me know if you need more information. I would appreciate it if you could complete the attached permission slip and return it.

Regards,

W. Kapofu

Researcher

L. Bugatti

Principal [Warrenview Girls High School]

Parental consent form

I _____ hereby consent to my child /ward's participation in the research.
I have been fully informed about the study and know that I may contact the researcher for information or explanations at any time.

I understand that:

a. My child/ward's participation in this research will involve taking part in recorded interviews, in which she will be asked question which will be transcribed later. The interviews will be fully anonymous when transcribed. The video / audio files will also then be destroyed.

b. Her participation in this study is entirely voluntary and that she can withdraw from this study at any time without giving a reason.

c. Her participation will be treated confidentially and all information will be stored anonymously and securely. All information appearing in the final report will be anonymous.

d. I am free to ask any questions at any time. I am free to discuss any questions or comments I would like to make with the researcher's supervisor.

e. I am free to contact the KwaZulu Natal University Ethics Committee to discuss any complaints I might have.

Consenting Parent / Guardian full name_____

Signature_____   Date_____

Appendix 6

Learner's Letter of Consent

I _____ hereby consent to participate in the research.
I have been fully informed about the study and know that I may contact the researcher for information or explanations at any time.
I understand that:

a. My participation in this research will involve taking part in recorded interviews, in which I will be asked question which will be transcribed later. The interviews will be fully anonymous when transcribed. The video / audio files will be also then be destroyed.

b. My participation in this study is entirely voluntary and that I can withdraw from this study at any time without giving a reason.

c. My participation will be treated confidentially and all information will be stored anonymously and securely. All information appearing in the final report will be anonymous.

d. I am free to ask any questions at any time. I am free to discuss any questions or comments I would like to make with the researcher's supervisor.

Consenting Learner (full name) _____

Signature_____   Date_____

Appendix 7

<u>Instrument 1: Group interviews' Guide (video recorded)</u>

<u>Purpose:</u> The purpose of these video recorded focus groups discussions is to get an understanding of learners' perspectives on geometry through their own voices also to capture their perceptions and experiences of geometry before and after the inclusion of some history content in the lessons on Pythagoras' Theorem. This instrument will assist in addressing all the research questions.

1.   <u>Questions before Task 1</u>
1.1.   Let's do a quick round of introductions. Can each one of you tell the group your name, favourite section in mathematics and how it became your favourite?
1.2.   **If it is geometry**: what is your view of geometry?
1.2.1. Which topics in geometry do you like best?
1.2.2. **If it is not geometry**: What made you decide that geometry is not your favourite Section?
1.3.   When did you become aware of the Theorem of Pythagoras?
1.4.   How did you learn about the Theorem?
1.5.   Think back what is the Theorem of Pythagoras about?
1.6.   Do you think Pythagoras' Theorem true?
1.7.   Can you describe how you can prove that the theorem is mathematically true?
1.8.   What have been your experiences with the application of Pythagoras' Theorem solving mathematical problems? Why was that?
1.9.   Given the chance what would you change about the way you were taught the Theorem of Pythagoras?

2. <u>Questions after Task 1</u>

Questions 1.5 to 1.9 will be repeated excluding question 1.8.

2.1.   Was your understanding of Pythagoras' Theorem changed in some way by discovering the theorem yourself and learning about some of the mathematicians who proved it?
2.1.1. **If yes**, in what way?
2.1.2. **If not**, why?

3. Questions after Task 2

Question 1.8 will be repeated.

3.1. Has your new understanding of the theorem influenced the way in which you solved **task 2** problems? How so?

3.1.1. How has the historical information on Pythagoras the mathematician influenced your understanding of the theorem?

Questions before Task 3 and 4

3.2. Is it possible to use Pythagoras' Theorem in other areas of mathematics?

3.3. **If yes**, in which areas?

4. Questions after Task 3 and 4

4.1. Has the knowledge of Rene Descartes' history helped you in understanding how the Cartesian plane came about?

4.1.1. **Yes / No**, in what way?

4.2. How did discovering the distance formula yourself influence your understanding of **Task (3.4)**?

4.3. What have you learnt from the life of Rene Descartes?

4.4. What do the solutions on **"as the crow flies"** and **"taxi geometry"** tell you about different approaches to the same problem in mathematics?

4.5. Which activities did you enjoy the most during all the lessons we have had on the Theorem of Pythagoras and its application?

4.5.1. Why was that so?

4.6. Has there been any change in your perception of the Theorem of Pythagoras after you attended these lessons? **Yes / No**

4.6.1. **Why** your response in **(4.6)** above?

4.7. Does this view have any influence on the way you perceive geometry?

4.8. After your research on the lives and work of some of the great mathematicians what is your view of geometry?

4.9. Why was your understanding of the theorem changed and your general outlook of geometry?

Instrument 3: Journal (*Kept by learners for the entire duration of data collection*)

Today's lesson

What it was about?

What I think of the lesson:

What I liked about my teacher's method of teaching the lesson:

What I found challenging about the lesson:

What I would change about my teacher's method of teaching the lesson:

Appendix 8

TASK 1: Reinventing Pythagoras' Theorem

In this task learners are to brainstorm and figure out the relationship between the area of the large square and the shapes inside it.

Task 1.1 Class discussion
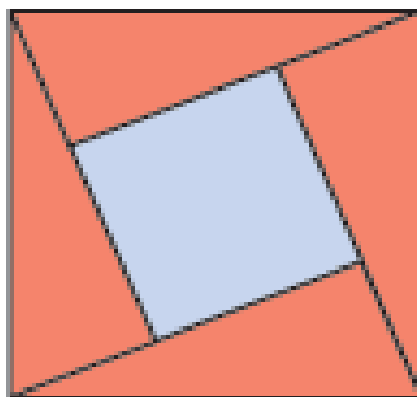
"BEHOLD!"

Bhaskara's diagram

BHASKARA

Bhaskara (1114 – c. 1185) called *Acarya* ("the Learned"), was born in Bijjada Bida, India, near the Sahyadri Mountains, was the most distinguished mathematician of the 12th century in India. He is credited with the introduction of concepts found much later in the development of the calculus.

In her book *The Mainstream of Mathematics* (1955) Edna Kramer reported the following sad tale. Astrologers predicted that Lilavati, Bhaskara's daughter would never marry. Bhaskara, being a smart astrologer, calculated a certain day at a very certain time that she could be married, but no other time would do if it passed. As the guests gathered for the ceremony on the lucky day, Acarya and his family eagerly watched a special water clock that Bhaskara had devised which would proclaim the happy instant for the marriage to take place. The excited bride leaned over the instrument but did not notice that a pearl from her headpiece fell into the timepiece plugging up the orifice through which the water flowed. By the time the mishap was discovered the appointed time had passed. The wedding went on as planned, but her husband died soon after the ceremony. To console his daughter, who remained a widow the rest of her life, Bhaskara promised to name a book after her. Thus was his book *Lilavati* named. In *Lilavati*, Bhaskara featured a pictorial proof of the Pythagorean Theorem. Although the ancient Chinese knew the same diagram proof, it is generally credited to Bhaskara**.**

Task 1.2 Learners work in groups of five. (Cut and paste activity)

A   B

C

E

D

+   +
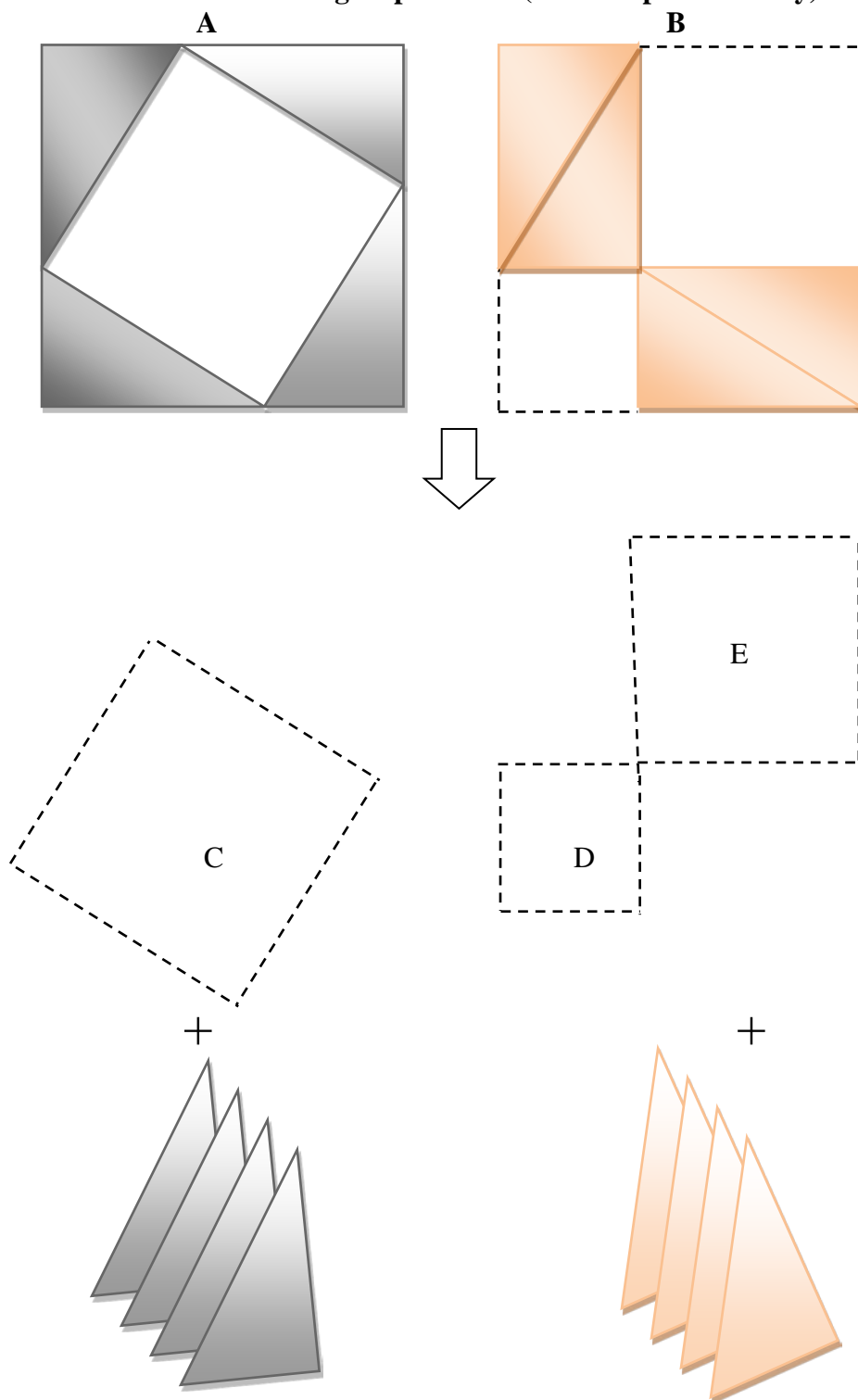
Looking at the two identical squares A and B above:

- 1.2.1. What is the relationship between the dark and the light triangles?
- 1.2.2. What can you say is the relationship between square C and squares D and E?
- 1.2.3. Write an equation for the components of square A and B and simplify it if possible.

Task 1.3 (Cut and paste activity)

In their already formed groups learners perform the task below:

    1.3.1. Given four identical squares with similar colours as shown in **diagram 1**, you are required to cut triangles from the squares and rearrange them.

    1.3.2. Show that the triangles form three squares (with each square having the same shade) and describe the relationship between the areas of the dark triangles is equal to that of the light coloured triangles in the squares.

    1.3.3. Paste the squares in such a way that they form a triangle at the centre, measure the angle formed by the two shorter sides.

Note: learners will only be provided with shapes in diagram 1 and they have to figure out how they can rearrange the triangles.
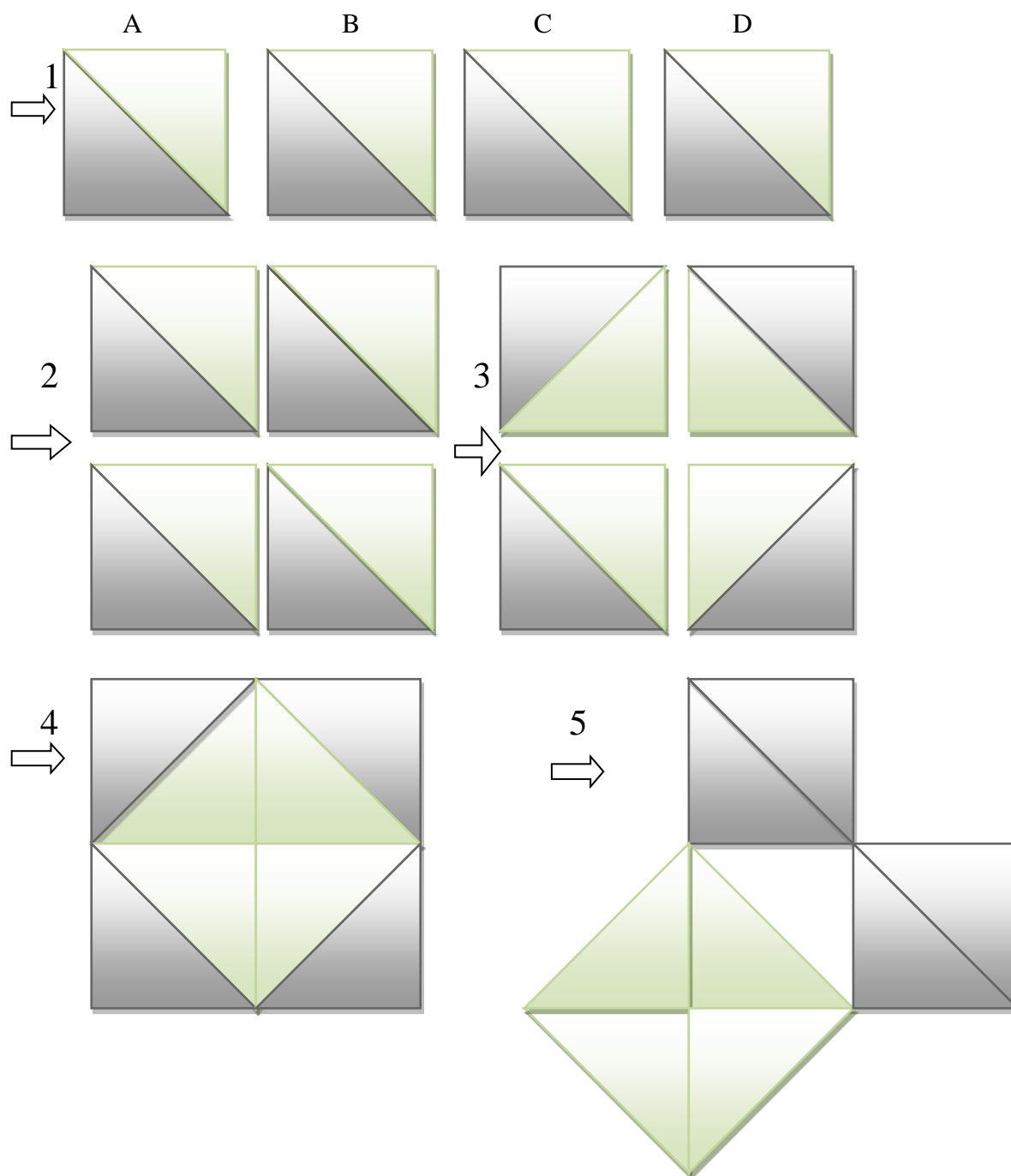
Task 1. 4

To modernise task 1.2 and 1.3(Cut and paste activity)

Given that: *a; b* and *c* are the measurements of the given triangles below, go through the motions step by step (from the left to the right) until you get to the last diagram in the 4ᵗʰ row 3ʳᵈ column cutting and pasting, making careful observations of relationships that may exist in the figures as you go along.
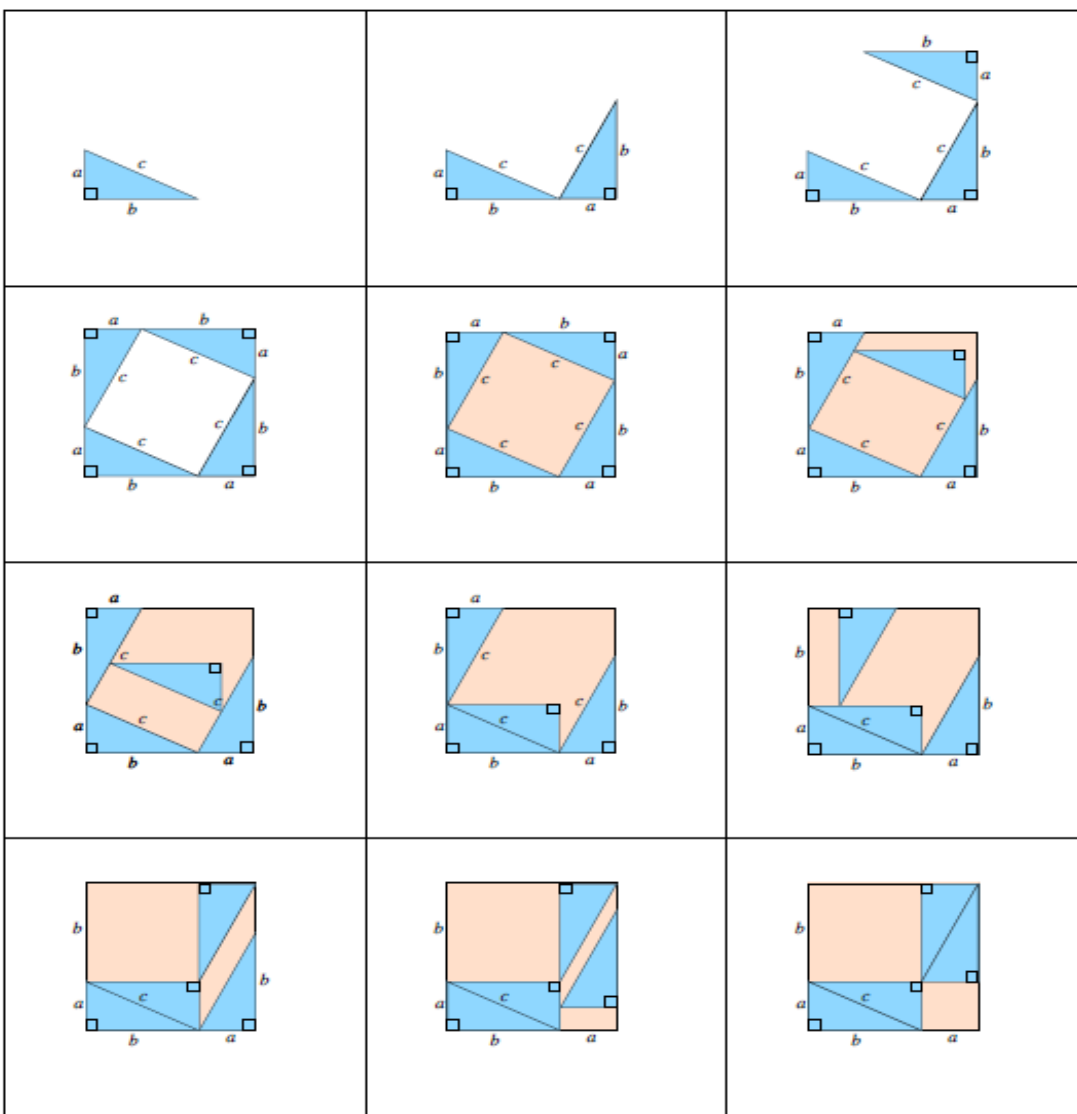
In terms of *a, b* and *c*:

    1.4.1. Calculate the area of the horizontal rectangle.

    1.4.2. What is the relationship between the areas of the horizontal and the vertical rectangles in the last diagram?

    1.4.3. Find the area of the formed smaller square.

    1.4.4. Calculate the area of the formed larger square.

    1.4.5. Is there a relationship between the sum of the areas of the interior components and that of the large outer square? If so, express the relationship between the areas of

the large outer square and the squares and rectangles inside it in the form of an equation.

1.4.6. Considering the 4th diagram in the 2nd row 1st column find the area of the inside square in terms of **c.**

1.4.7. Is there another way of finding the area of that same square without using **c?** Express the area of that same square in terms of **a** and **b.**

1.4.8. Using your findings from (g) and (h) show that $a^2 + b^2 = c^2$.

Task 1.5

Modern form of Bhaskara's diagram: *Learners work in groups of five.*

A learner Nonhle in grade 10 from Emalahleni high school drew the following diagram and prescribed value a, value b and a value c to the side of her triangles.
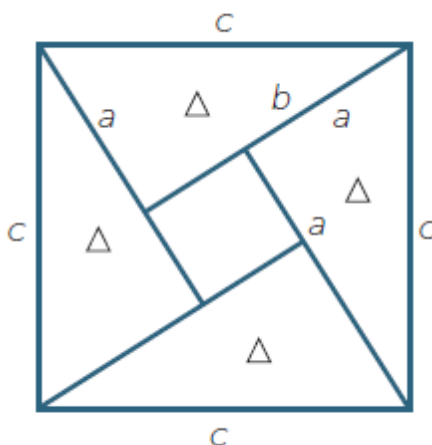
1.5.1. If all the triangles are right angled calculate the area of one of the triangles.
1.5.2. Determine the area covered by the four triangles.
1.5.3. What is the size of each of the sides of the square (in terms of a and b) at the centre of the diagram?
1.5.4. What is the area of the small square in the centre?
1.5.5. Hence express the area of the large square in terms of a and b.
1.5.6. Using the area of the large square form an equation in terms of a, b and c.
1.5.7. Calculate the value of c.
1.5.8. Using the diagram above prove that in any right angled triangle with sides a, b and hypotenuse c, $a^2 + b^2 = c^2$.

Considering results from the five tasks learners will be asked to come up with a generalisation on the relationship between the hypotenuse and the two shorter sides of a right angled triangle.

Task 1.6: Euclid's proof of Pythagorean Theorem
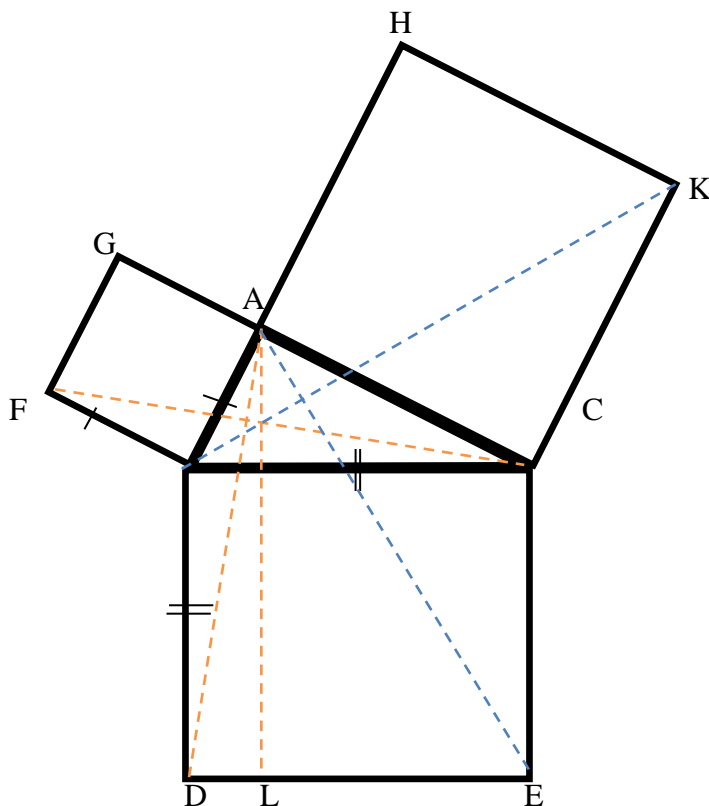
1.6.1. Read and transcribe the information on Euclid's proof given below onto the   diagram above.

By assumption, Euclid knew that angle BAC was a right angle. He constructed the squares on the three sides. He then drew AL through A and parallel to BD. He also drew lines AD and FC.
It was critical for Euclid to establish that CA and AG lie on the same straight line. This he did by noting that angle GAB was a right angle by the construction of the square, while angle BAC was a right by hypothesis. Since these two angles sum to two right angles it meant that GAC was itself a straight line. Interestingly, it was in proving this apparently minor technical point that he made his one and only use of the fact that angle BAC is right.
Now Euclid looked at the two slender triangles ABD and FBC. Their shorter sides AB and FB, respectively were equal since they were the sides of the same square; their longer sides BD and BC were equal for the same reason. And what about the corresponding included angles?
Notice that triangle ABD is the sum of angle ABC and the square's right angle CBD, while angle FBC is the sum of angle ABC and the square's right angle FBA and thus angle ABD = angle FBC.

    1.6.2.   What do you think Euclid had established at this point concerning the narrow triangles ABD and FBC? Give a reason for your suggestion.

    1.6.3   Considering your answer to (1.4.2) what conclusion can you draw about the area of the two triangles ABD and FBC?

Next Euclid observed that triangle ABD and rectangle BDLM shared the same base (BD) and fell within the same parallels (BD and AL), and thus the area of BDLM was twice the area of triangle ABD. Similarly, triangle FBC and square ABFG shared base BF. In addition, Euclid had taken pains to prove that GAC was a straight line, so the triangle and the square both lay between parallels BF and GC again the area of square ABFG was twice that of triangle FBC.

1.6.4. If you combined these results and the previously established results from (1.6.1 and 1.6.2), write an equation showing the relationship between the areas of rectangle BDLM and triangle ABD.

1.6.5. Write another equation to illustrate the relationship between the areas of rectangle BDLM and triangle FBC.

1.6.6. Considering the two equations in (1.6.4. and 1.6.5.) write an equation to relate the areas of rectangle BDLM and square ABFG.

This was half of Euclid's mission that is to find the relationship between areas of rectangle BDLM and square ABFG. Following the similar steps, he first drew AE and BK, and proved that BAH was a straight line, of triangle ACE and triangle BCK.

1.6.7. What do you think Euclid had established at this point concerning the narrow triangles ACE and BCK? Give a reason for your suggestion.

1.6.8. Considering your answer to (1.6.7) what conclusion can you draw about the area of the two triangles ABD and FBC?

1.6.9. If you combined these results and the previously established results from (1.6.7 and 1.6.8), write an equation showing the relationship between the areas of rectangle CELM and triangle ACE

1.6.10. Write another equation to illustrate the relationship between the areas of rectangle CELM and triangle BCK.

1.6.11. Considering the two equations in (1.6.9. and 1.6.10.) write an equation to relate the areas of rectangle CELM and square ACKH.

1.6.12. Write an equation showing the relationship the area of the square BCED and the areas of rectangles BCED and CELM and

1.6.13. Find the last equation relating the areas of the square BCED and squares ABFG and ACKH.

NAMING OUR INVENTION

PYTHAGORAS

Pythagoras was a Greek mathematician and philosopher, born on the island of Samos (ca. 582 BC). He founded a number of schools, one in particular in a town in southern Italy called Crotone, whose members eventually became known as the Pythagoreans. The inner-circle at the school, the *Mathematikoi*, lived at the school, rid themselves of all personal possessions, were vegetarians, and observed a strict vow of silence. They studied mathematics, philosophy, and music, and held the belief that numbers constitute the true nature of things, giving numbers a mystical or even spiritual quality. Today, nothing is known of Pythagoras's writings, perhaps due to the secrecy and silence of the Pythagorean society. However, one of the most famous theorems in all of mathematics does bear his name, the *Pythagoras Theorem.* Pythagoras was not the first in history to know about the remarkable theorem that bears his name, but it is thought that he was the first to formally prove it using deductive geometry and the first to actively 'market' it (using today's terms) throughout the ancient world. . There are many ways of proving the theorem but only three have been used in this unit.

Therefore in modern terms: Let **c** represent the length of the Hypotenuse, the side of a right triangle directly opposite the right angle of the triangle. The remaining sides of the right triangle are called the legs of the right triangle, whose lengths are designated by latters **a** and **b**. The relationship involving the legs and the hypotenuse of the right triangle, given by
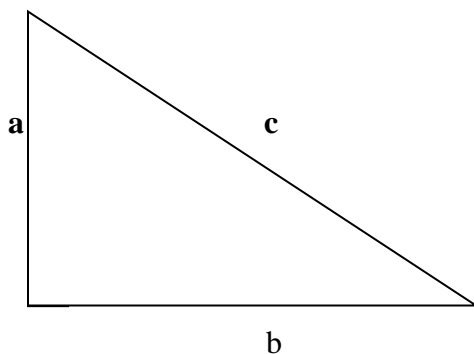
Task 1.7

To demonstrate that shapes other than squares can also be used to prove Pythagoras' Theorem. (*Using Geogebra*)

1.7.1.

President J. A. Garfield (1876) used the **Figure 1** shown on the screen to Prove Pythagoras' Theorem. Using the same diagram and your knowledge of finding the areas of a trapezium and a triangle:

1.7.1.1. Find the area of the Trapezium in terms of **a, b** and **c.**
1.7.1.2. Find the total area of the three triangles in terms of **a, b** and **c.**
1.7.1.3. Equate the area of the trapezium to the total area of the three triangles and simplify.
1.7.1.4. Does this give you the Pythagoras' Theorem?

1.7.2. Study the **Figure 2** shown on the screen carefully and apply your knowledge of area of rectangles to assist you to:
1.7.2.1. Find the total area in two different ways taking the various shapes into consideration in terms of **a, b** and **c**.
1.7.2.2. Equate the two total areas from 1.7.2.1.
1.7.2.3. Show that the above equation can be simplified to $c^2 = a^2 + b^2$

Appendix 9

Lesson Interaction (*Details of the activities were provided in each learning task*)

Learning Tasks: Educator Input	Student Activities	Teacher's Support and Actions	Checking Understanding
Task 1: Discovering Pythagoras' Theorem – If a square is draw from each side of any right angled triangle, the sum of area of the two smaller squares is equal to the area of the larger square.(Lesson of about 120minutes)			
- gives the Bhaskara diagram with the word "BEHOLD!" under it to the learners and asks them to study it carefully. Learners are asked to relay their observations. -organises learners into groups of five and issues out cardboard and diagrams of triangles -gives out four identical squares to each group and cardboard. -ask the learners to draw diagonals on all four squares and shade one	Task 1.1 Class discussion on the observations made from the diagram concerning the relationships that exist among the components of the diagram. Task 1.2 -Cutting and pasting activity from the task sheet. Task1.3 -draw diagonals on the squares and shade the triangles. -carry out the instructions given on task 1.3. Task 1.4	Educator directs the discussion and asks probing questions for elaboration purposes. -moves about assisting learners who may need guidance. -summarises the findings from the activity -monitors progress -class discussion of learners' responses to task 1.3 written activity	Written work -answer questions in task1.2 as individuals Written work -answer questions in task1.3 as individuals

| triangle from each with the same shading.

-gives each learner a task page with instructions and shapes.

-learners are issued with a modern version of Bhaskara's diagram and are asked to answer questions from task 1.5.

-shares the stories of the two mathematicians, Bhaskara and Pythagoras and asks learners to read more about them from the internet. | -using their triangles they go through all the steps on the instruction sheet until they get to step 9
-make careful observations as they go along.

Task 1.5

-as individuals learners write solutions to task 1.5 questions.

-research for more information on the two mathematicians and their work. | -monitors progress

-picks learners at random to give feedback to task 1.4 written activity which is followed by a class discussion of what will have been presented.

-summary of the finding is written on the chalkboard and learners peer mark each other's work.

-educator summarises all the work that was done in this lesson and formally introduces learners to the Pythagoras' Theorem by bringing to their attention that what they discovered, just like the mathematicians in ancient times, is what is called the **Pythagoras' Theorem** | Written work
-answer questions in task1.4 as individuals

Written work
-answer questions in task1.5 as individuals

Research work

-a one – two page essay on the

 life and work of either

Bhaskara or Pythagoras. |
| **Task 2: General application of Pythagoras' Theorem (60minutes)** | | | |

-question and answer recap session on the discoveries made by the learners in their previous lessons. -educator reminds the learners of the research work that will be due in a week's time. -works out 3 problems on the chalkboard on application of Pythagoras' Theorem to find the missing side on right angled triangles. -gives out task 2.1 for learners to do as individuals. -A quick revision of the work on measurement and then learners are given Task 2.2 as homework.	Respond to questions asked by the educator. -work together with the educator on the 3 examples. Task 2.1 -individual written activity Task 2.2	Moves around marking learners work and assisting where necessary.	Written work -answer questions in task2.1 Homework Task 2.2

Task 3: Discovering the distance formula (120 minutes).			
-gives feedback on the activities written in the previous lesson

-introduces the origins of the Cartesian plane.
-distribute papers with the history of Rene Descartes and comprehension questions.

-educator gives an overview of the Cartesian plane and how points may be located on a Cartesian plane.
-asks learners in their usual groups of fives to draw a straight line PQ and asks them to drop a perpendicular from P and a horizon broken line which meet at R | -do corrections if there is need.

Task 3.1
-read the story and then answer comprehension questions

Task 3.2
-in pairs learners work on the *as the crow flies task.*

-class discussion of solutions to the

Task 3.3

-in terms of x and y learners find the shortest distance between PQ

- group feedback and class discussion of group findings | -moves about monitoring progress

Assists the learners only when necessary.

-Moves about monitoring progress | Written work

Task 3.1 on the mathematician and philosopher Rene Descartes.

Written work

Task 3.2

Written work

Task3.3 |

forming a right angled triangle. -sums up the discussion by writing the general distance formula on the chalkboard. -hands out individual written task to the learners -gives out literature on taxicab geometry.	Task 3.4 Individual activity. Group work -after reading given literature on taxicab geometry learners discuss in groups the similarities and differences between how the distance is calculated in *as the crow flies and taxicab metric.* Task 3.5 an individual activity	-comments on feedback and asks probing questions where necessary. -goes around marking learners' work and checking progress -asks group representatives to give feedback and then sums up the main points. -moves about checking learners' work	Written work Task 3.4 -individual task -Written work Task 3.5

Task 4: Application of Pythagoras' Theorem (60minutes)

-shares the story of Hipparchus and his contribution to geometry. -hand out task 4.1 to learners on trigonometry.	-research on the internet for Hipparchus' history and contribution to trigonometry and submit an essay in a week's time. Task 4.1 -individual task	-moves about monitoring progress and checking learners' work -asks learners to make presentations of their work on the chalkboard	Research work on Hipparchus Written work Task 4.1
-issues out task 4.2	Task 4.2 -in pairs learners discuss and write the solutions to the	-gives a summary of all the work on Pythagoras theorem	Written work Task 4.2