New models for quark stars with charge and anisotropy

by

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As the candidate's supervisor I have approved this thesis for submission.

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Abstract

We find new classes of exact solutions for the Einstein-Maxwell field equations. The solutions are obtained by considering charged anisotropic matter with a linear equation of state consistent with quark stars. The field equations are integrated by specifying forms for the measure of anisotropy and one of the gravitational potentials which are physically reasonable. A general feature of our models is that isotropic pressures are regained when certain parameters vanish; this behaviour is missing in most previous treatments. Particular models found in our results generalize the models of Mak and Harko, Komathiraj and Maharaj, Misner and Zapolsky, and the earlier results of Einstein. The graphical and physical analyses indicate that the gravitational potentials, the matter variables, the electric field and the mass are well behaved. In performing physical analysis we regain masses and radii of stellar objects consistent with observations. It is also shown that other masses and radii may be generated which are in acceptable ranges consistent with observed values of stellar objects. In particular we have established that our model is consistent with the stellar object SAXJ1808.4-3658. A study of the mass-radius relation indicates the effect of the electromagnetic field and anisotropy on the mass of the relativistic star.

Dedication

To my wife Rehema, and our two little boys Daniel and Elijah this work is dedicated for all support that contribute to the peace, joy and love in my life.

Preface

This work described in this thesis was carried out in the School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal from January 2012 to September 2014, under the supervision of Prof. Sunil D. Maharaj and Prof. Subharthi Ray.

This work represents my own original work and has not otherwise been submitted in any form for any degree at any tertiary institution. Where use has been made of the work of others it is duly acknowledged in the text.

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I, Jefta Mvukaye Sunzu declare that

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Declaration of Publications

Details of contribution to publications that form part and/or include research presented in this thesis.

Publication 1:

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Publication 2:

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Publication 3:

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To all I say thank you and may the Lord God bless you.

Chapter 1

Introduction

The Einstein-Maxwell equations describe charged gravitating matter which are important in relativistic astrophysics, and they model compact objects such as neutron stars, gravastars, dark energy stars and quark stars. In the study of such astrophysical compact objects, the Einstein-Maxwell field equations in static spherical spacetimes provide the basis of investigation, and they have therefore attracted the attention of many researchers. With the help of these field equations, researchers have discovered different structures and properties of relativistic stellar bodies relevant in astrophysical studies. For example, the exact models for these field equations obtained by Sunzu *et al* (2014) generated new masses and radii for quark stars which are in acceptable ranges consistent with observed values of stellar objects. The stellar masses generated range from $1.28994M_{\odot}$ to $1.73268M_{\odot}$ with radius varying from 5.77km to 7.61km. The solutions to the field equations in static spacetimes obtained by Thirukkanesh and Maharaj (2008) describe realistic compact anisotropic spheres whose properties are relevant to stellar bodies such as SAXJ1808.4-3658. These solutions contain masses and central densities that correspond to realistic stellar bodies. The models for compact spheres obtained by Chaisi and Maharaj (2005) generate surface redshifts and masses which correspond to realistic stellar objects such as Her X-1 and Vela X-1. Stable models for neutron stars highlighted by Astashenok *et al* (2013) provide evidence for the existence of stable star configurations at high central densities for a stellar object with maximum mass $1.9M_{\odot}$ and minimum radius 9km. Recently the models for charged matter generated by Rahaman *et al* (2012) describe ultra compact astrophysical objects. The relativistic solutions obtained by Kalam *et al* (2012) describe charged compact objects and are comparable with well known stars. Other models with astrophysical significance include: solutions and relativistic models highlighted by Murad and Pant (2014), models for stability of strange stars illustrated by Sinha *et al* (2002), general relativistic model for SAX J1808.4-3658 generated by Sharma *et al* (2002) and the exact solutions obtained by Sharma *et al* (2006). These studies indicate that the Einstein-Maxwell field equations have many applications in the modelling of relativistic astrophysical objects.

Pressure anisotropy is an important ingredient in many stellar systems in the absence of charge. Since the pioneering paper by Bowers and Liang (1974), who were the first to consider pressure anisotropy in the study of anisotropic spheres in general relativity, there has been extensive research in this direction. It was established by Dev and Gleiser (2002) that pressure anisotropy has a significant effect on the structure and properties of stellar spheres. The maximum value for $\frac{2M}{R}$ was found to be either greater or less than $\frac{8}{9}$ for anisotropic spheres and less than $\frac{8}{9}$ for isotropic objects. In particular it was shown that both the maximum mass and the redshift vary with the magnitude of the pressure anisotropy. For a positive measure of anisotropy, the stability of the sphere is enhanced when compared to isotropic configurations, and anisotropic distributions are stable for smaller adiabatic index values as shown by Dev and Gleiser (2003). The results generated in Gleiser and Dev (2004) indicate that pressure anisotropy may significantly affect the physical structure of the stellar object which may cause several observational effects. In their paper it was indicated that the surface redshift of the star may be arbitrary large ($z_s \geq 2$) and that stellar objects which are observed at large redshifts may be closer than they appear due to anisotropic distortions. It was highlighted that stars may be more stable if the pressure anisotropy exists near its core. Recently, models obtained by Kalam *et al* (2013b) for uncharged anisotropic stars were shown to be compatible with strange star candidates Her X-1, SAXJ1808.4-3658 and 4U 1820-30. Other uncharged anisotropic models in spherically symmetry spacetimes include the relativistic compact nonsingular models for anisotropic stars obtained by Mak and Harko (2003, 2005) and Harko and Mak (2002), new anisotropic models generated by Maharaj and Chaisi (2006a, 2006b), solutions generated by Kalam *et al* (2013a), compact models developed by Karmakar *et al* (2007), relativistic strange star models found by Paul *et al* (2011), and solutions contained in Chaisi and Maharaj (2005, 2006a, 2006b). It is interesting to note the paper of Ivanov (2010) who showed that anisotropic models with heat flow can absorb the addition of charge, viscosity and convert null fluids to a perfect fluid.

It is important for many applications to include the electric field in stellar models. In particular, models with electric field present permit causal signals over a wide range of parameters as illustrated by Sharma *et al* (2001). It has been shown by Ivanov (2002) that the presence of the electric field significantly affects the redshift, luminosity and mass of the compact object. Most of the models that include an electromagnetic field distribution are isotropic; these include the new classes of solutions obtained by Maharaj and Komathiraj (2007), Komathiraj and Maharaj (2007a, 2007b), Thirukkanesh and Maharaj (2006, 2009) and Maharaj and Thirukkanesh (2009a). The isotropic charged solutions obtained by Chattopadhyay *et al* (2012) contain masses, radii and compactification consistent with compact X-ray pulsars HER-1 and SAXJ1808.4-3658. Other stellar models that describe charged bodies with isotropic pressures are given by Gupta and Maurya (2011a, 2011b), Murad and Fatema (2013), Pant and Negi (2012), Mehta *et al* (2013), and Bijalwan (2011). There are fewer research papers that include both anisotropic pressures and electromagnetic field distributions. The presence of pressure anisotropy with an electric field enhances the stability of a configuration under radial adiabatic perturbations compared to the matter with isotropic pressures. Stellar models containing both pressure anisotropy and electric field include compact objects admitting a one-parameter group of conformal motions of Esculpi and Aloma (2010), the generalized isothermal models of Maharaj and Thirukkanesh (2009b), the stellar models of Thirukkanesh and Maharaj (2008), the regular compact models of Mafa Takisa and Maharaj (2013a), some simple models for quark stars of Maharaj *et al* (2014) and models for quark stars generated by Sunzu *et al* (2014). Other charged anisotropic models are those of Rahaman *et al* (2012) and Maurya and Gupta (2012). However most of these models have the anisotropy parameter always present, and they do not contain isotropic solutions as a special case. It is important to build physical stellar models in which the anisotropy vanishes for an equilibrium configuration.

Different forms of the barotropic equation of state have been applied with the field equations to find exact models that govern compact relativistic gravitating objects such as dark energy stars and quark strange stars (hybrid stars). Thirukkanesh and Ragel (2012) have found exact solutions for the uncharged anisotropic sphere with the polytropic equation of state for particular choices of the polytropic index. Mafa Takisa and Maharaj (2013b) used the general polytropic equation of state, and obtained exact solutions for the field equations in the presence of the electromagnetic field and anisotropic pressures. Shibata (2004) studied the stability of rotating bodies, and Lai and Xu (2009) indicated that large amounts of gravitational energy are released in the gravitational collapse of polytropes. Other treatments on polytropes include the results of Tooper (1964), Nilsson and Ugla (2001), Kinasiewicz and Mach (2007) and Heinzle *et al* (2003). Maharaj and Mafa Takisa (2012) and Feroze and Siddiqui (2011) found exact solutions of the Einstein-Maxwell field equations for charged anisotropic stars using a quadratic equation of state. There have been many anisotropic and charged exact models with a linear equation of state: Mafa Takisa and Maharaj (2013a) generated compact exact models with regular distributions, Thirukkanesh and Maharaj (2008) found models consistent with dark energy stars and quark stars, Maharaj and Thirukkanesh (2009b) generated anisotropic isothermal models, Sharma and Maharaj (2007) found models consistent with quark matter, and Esculpi and Aloma (2010) generated conformally invariant spheres. However, in general, most of these models do not regain charged isotropic models. Some analytical solutions to the field equations with a linear quark equation of state for charged isotropic stars were found by Komathiraj and Maharaj (2007c). Using the same equation of state, Sotani and Harada (2003), Sotani *et al* (2004), and Bombaci (2000) analysed quark stars with isotropic pressures. There has been an extension of the linear quark equation to include anisotropic pressures in modeling the behavior of strange stars by Rahaman *et al* (2012), Kalam *et al*

(2013b), and Mak and Harko (2002).

It should be noted that the microscopic effects of the strange quark matter coming from strong interactions of quantum chromodynamics (QCD) (e.g., see Dong *et al* (2013) and Dey *et al* (1998)) are all encrypted in the final form in the equation of state of matter. We study the general relativistic behaviour of these equations of states, by employing a linear approximation for strange matter. Such approximations of the equation of states can be found in the literature in the study of various properties of compact stars. The linear approximation of the strange quark matter equation of state has been used by Zdunik (2000) to study the quasi periodic oscillation (QPO) frequencies in the Lower Mass X-ray Binaries (LMXBs). Gondek-Rosinaka *et al* (2000) used the linear approximation of strange matter to compute the mass shedding limit of strange stars.

We also comment on the origin of equations of state and modified theories of gravity. Recently a class of exact isotropic solutions of Einstein's equations for non-rotating relativistic stars has also been studied by Murad and Pant (2014). They also comment that as strange stars are not purely gravitationally bound; they are bound by strong interactions. Study of the same in the light of modified gravity theories should not produce any difference in the mass-radius relation. In this context, although Astashenok *et al* (2013) showed that there is an increase in the mass of neutron stars in the $f(R) = R + R\left(e^{-R/R_0} - 1\right)$ gravity model, Ganguly *et al* (2014) showed that for the $f(R) = R + \alpha R^2$ model (and subsequently many other $f(R)$ models where the uniqueness theorem is valid) the existence of compact astrophysical objects is highly unnatural. This is because the equation of state of a compact star should be completely determined by the physics of nuclear matter at high density, and not only by the theory of gravity.

The objective of this thesis is to find new classes of exact solutions of the Einstein-Maxwell system of field equations with a linear quark equation of state for charged anisotropic stars. We seek to generate solutions with astrophysical significance in which we regain previously researched models and use our results to obtain masses of stars consistent with observations. In order to achieve this objective our thesis is arranged in the following manner: In Chapter 2 we give the basic equations important in the our thesis. We derive the Einstein-Maxwell field equations by aid of the different types of tensors in differential geometry and general relativity. In Chapter 3 we seek to find new classes of some simple models for charged anisotropic quark stars. We generate two classes of exact solutions in term of the elementary functions. We also seek to regain previous charged isotropic models as a special case. In this chapter we discuss the physical analysis of the gravitational potentials, matter variables, electric field and the mass and indicate that these variables are well behaved. In Chapter 4 we perform a detailed physical analysis of a nonsingular model obtained in the previous chapter. We regain masses and radii consistent with different stellar objects obtained by other researchers. Other masses and radii generated are in acceptable ranges and consistent with observations. The mass-radius relationship is given by comparing and considering the charged matter with anisotropic and isotropic pressures. In Chapter 5 we analyse two relativistic models. The first model is regular, throughout the interior, in the matter variables and gravitational potentials; it contains the Einstein model as a limiting case and we can generate finite masses for the star. The second model is a generalized metric that admits a singularity in some of the matter variables at the centre of the stellar object. However a graphical analysis indicates matter variables and the mass are well behaved. We give the conclusion in Chapter 6.

Chapter 2

Basic equations

2.1 Introduction

The Einstein-Maxwell equations form a system of field equations which are essential for studying relativistic astrophysical models. In this chapter we briefly review basic equations and derive the Einstein-Maxwell equations necessary for this thesis. In order to derive these field equations we employ our knowledge of tensor analysis, differential geometry and the theory of general relativity. In the theory of general relativity spacetime is considered to be a four-dimensional differentiable manifold endowed with a symmetric, nonsingular metric tensor field with signature $(-+++)$. More detailed information on differential geometry and manifolds is given by Hawking and Ellis (1973), Wald (1981), Misner *et al* (1973) and Stephani *et al* (2003).

The Riemann tensor, which is obtained from the metric tensor, describes the curvature of the spacetime manifold. The Einstein tensor, derived from the Riemann tensor and the Ricci scalar, describes the geometry of the gravitational field. The matter content and electromagnetic distribution comprises a relativistic fluid governed by the energy momentum tensor. The Einstein field equations which describe the influence and the behaviour of the gravitational field on the matter content is obtained by equating the Einstein tensor and the energy momentum tensor. The electromagnetic field is governed by the Maxwell's equations. The Einstein-Maxwell field equations depend on the nature of the fluid under consideration. Neutral/charged fluids may be isotropic or anisotropic. In this thesis we consider charged anisotropic matter. On physical grounds we also assume the spacetime to be spherically symmetric. For many different physical applications of the field equations in various spacetimes the reader is referred to Krasinski (1997).

2.2 The metric tensor and connection

The line element which governs the invariant distance between neighbouring points on the manifold is defined as

$$ds^2 = g_{ab}dx^a dx^b, \tag{2.1}$$

where g_{ab} represents the metric tensor field components with standard spherical coordinate $x^a = (t, r, \theta, \phi)$. For a static spherically symmetric spacetimes we have

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{2.2}$$

We can write the metric tensor as

$$g_{ab} = \begin{pmatrix} -e^{2\nu} & 0 & 0 & 0 \\ 0 & e^{2\lambda} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}. \tag{2.3}$$

The metric connection Γ, or the Christoffel symbol of the second kind, is of great importance in computing other tensors which lead to the field equations. It is given by

$$\Gamma^a{}_{bc} = \frac{1}{2}g^{ad}\left(g_{cd,b} + g_{db,c} - g_{bc,d}\right), \tag{2.4}$$

which is symmetric. Using (2.4) we obtain the following nonvanishing components of the metric connection

$$
\begin{aligned}
\Gamma^0{}_{01} &= \nu', \\
\Gamma^1{}_{00} &= \nu' e^{2(\nu-\lambda)}, \\
\Gamma^1{}_{11} &= \lambda', \\
\Gamma^1{}_{22} &= -re^{-2\lambda}, \\
\Gamma^1{}_{33} &= -re^{-2\lambda}\sin^2\theta, \\
\Gamma^2{}_{12} &= \frac{1}{r}, \\
\Gamma^2{}_{33} &= -\sin\theta\cos\theta, \\
\Gamma^3{}_{13} &= \frac{1}{r}, \\
\Gamma^3{}_{23} &= \cot\theta,
\end{aligned}
\tag{2.5}
$$

for the metric (2.2).

2.3 Curvature tensors

The Riemann tensor is expressed in terms of the metric connection and it is used to express the Ricci tensor upon contraction. It is given by

$$R^a{}_{bcd} = \Gamma^a{}_{bd,c} - \Gamma^a{}_{bc,d} + \Gamma^a{}_{ec}\Gamma^e{}_{bd} - \Gamma^a{}_{ed}\Gamma^e{}_{bc}. \tag{2.6}$$

The Ricci tensor is defined by $R^c{}_{acb} = R_{ab}$. From (2.6) we see that the Ricci tensor is given by

$$R_{ab} = \Gamma^c{}_{ab,c} - \Gamma^c{}_{ac,b} + \Gamma^c{}_{dc}\Gamma^d{}_{ab} - \Gamma^c{}_{db}\Gamma^d{}_{ac}. \tag{2.7}$$

Using the relevant metric connections in (2.5) we obtain the following components of the Ricci tensor

$$
\begin{aligned}
R_{00} &= e^{2(\nu-\lambda)}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{2\nu'}{r}\right), && (2.8a)\\
R_{11} &= -\left(\nu'' + \nu'^2 - \nu'\lambda' - \frac{2\lambda'}{r}\right), && (2.8b)\\
R_{22} &= 1 - e^{-2\lambda}\left(1 + r\nu' + r\lambda' - 2r\lambda'\right), && (2.8c)\\
R_{33} &= \sin^2\theta R_{22}, && (2.8d)\\
R_{ab} &= 0, \text{ for } a \neq b. && (2.8e)
\end{aligned}
$$

We now define the the Ricci or curvature scalar which is needed for defining the Einstein tensor. It is given by

$$R = g^{ab}R_{ab}. \tag{2.9}$$

Using (2.3), (2.8) and (2.9) we obtain

$$R = 2\left[\frac{1}{r^2} - e^{2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{2\nu'}{r} - \frac{2\lambda'}{r} + \frac{1}{r^2}\right)\right]. \tag{2.10}$$

The Einstein tensor is essential for formulating the field equations. It is expressed in terms of Ricci and metric tensors as

$$G_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}. \tag{2.11}$$

With the help of (2.3), (2.8) and (2.10) we generate the following components of the Einstein tensor

$$
\begin{aligned}
G_{00} &= \frac{1}{r^2}e^{2\nu}\left(1 - e^{-2\lambda} + \frac{2\lambda'}{r}e^{2(\nu-\lambda)}\right), && (2.12a)\\
G_{11} &= -\frac{1}{r^2}e^{2\lambda}\left(1 - e^{-2\lambda}\right) + \frac{2\nu}{r}, && (2.12b)\\
G_{22} &= r^2e^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu'}{r} - \frac{\lambda'}{r}\right), && (2.12c)\\
G_{33} &= \sin^2\theta G_{22}, && (2.12d)\\
G_{ab} &= 0, \text{ for } a \neq b. && (2.12e)
\end{aligned}
$$

We note that ${G^{ab}}_{;b} = 0$ is a conservation law for the Einstein tensor.

2.4 Energy momentum tensor

The matter tensor identifies the matter distribution. For uncharged matter it is given by

$$M_{ab} = (\rho + p)u_a u_b + p g_{ab} + q_a u_b + q_b u_a + \pi_{ab}. \tag{2.13}$$

In the above ρ is the energy density, p is the isotropic pressure, q is the heat flow vector, π_{ab} is the stress tensor ($\pi^a{}_{ab}u^b = 0 = \pi^a{}_a$) and $\mathbf{u}$ is a unit, timelike vector ($u^a u_a = -1$). We are considering static stellar models with no heat flow so that $\mathbf{q} = 0$. Then the energy momentum tensor becomes

$$M_{ab} = (\rho + p)u_a u_b + p g_{ab} + \pi_{ab}. \tag{2.14}$$

We take the matter fluid to be comoving so that the four-velocity becomes $u^a = e^{-\nu}\delta^a_0$. The isotropic pressure p is defined in terms of the radial pressure p_r and the tangential pressure p_t by

$$p = \frac{1}{3}\left(p_r + 2p_t\right). \tag{2.15}$$

Then the anisotropic stress tensor π_{ab} is given by

$$\pi_{ab} = (p_r - p_t)\left(n_a n_b - \frac{1}{3}h_{ab}\right), \tag{2.16}$$

where $h_{ab} = u_a u_b + g_{ab}$ is the projection tensor, $\mathbf{n}$ is a unit radial vector such that $n^a = e^{-\lambda}\delta^a_1$, $n^a u_a = 0$ and $n^a n_a = 1$. Using (2.14), (2.15) and (2.16) we obtain the matter energy tensor

$$M_{ab} = \begin{pmatrix} \rho e^{2\nu} & 0 & 0 & 0 \\ 0 & p_r e^{2\lambda} & 0 & 0 \\ 0 & 0 & p_t r^2 & 0 \\ 0 & 0 & 0 & p_t r^2 \sin^2\theta \end{pmatrix} \tag{2.17}$$

for a neutral anisotropic fluid.

For a charged fluid the total energy momentum tensor is given by

$$T_{ab} = M_{ab} + E_{ab}, \tag{2.18}$$

where **E** is the electromagnetic field tensor defined in terms of the skew-symmetric electromagnetic field component **F**. The tensor **E** is given by

$$E_{ab} = F_{ac}F^c_b - \frac{1}{4}g_{ab}F_{cd}F^{cd}. \tag{2.19}$$

The tensor field **F** is defined in terms of the four-potential **A** by

$$F_{ab} = A_{b;a} - A_{a;b}. \tag{2.20}$$

For simplicity we choose the potential

$$A^a = (\phi(r), 0, 0, 0) = \phi(r)\delta^a_0. \tag{2.21}$$

From (2.20) and (2.21) we obtain the nonvanishing components

$$F_{01} = -F_{10} = -\phi'(r). \tag{2.22}$$

Hence it is easy to show that

$$F^{01} = -F^{10} = \phi'(r)e^{-2(\nu+\lambda)}. \tag{2.23}$$

We define

$$\phi'(r) = E(r)e^{(\nu+\lambda)} \longrightarrow E(r) = \phi'(r)e^{-(\nu+\lambda)}, \tag{2.24}$$

where $E(r)$ is the electric field intensity.

From (2.19), (2.23) and (2.24) we obtain the following form for the electromagnetic field tensor

$$E_{ab} = \frac{1}{2}E^2(r)\begin{pmatrix} e^{2\nu} & 0 & 0 & 0 \\ 0 & -e^{2\lambda} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}. \tag{2.25}$$

Using (2.17), (2.18) and (2.25) we obtain the total energy momentum tensor in the form

$$T_{ab} = \begin{pmatrix} e^{2\nu}\left(\rho + \frac{1}{2}E^2\right) & 0 & 0 & 0 \\ 0 & e^{2\lambda}\left(p_r - \frac{1}{2}E^2\right) & 0 & 0 \\ 0 & 0 & r^2\left(p_t + \frac{1}{2}E^2\right) & 0 \\ 0 & 0 & 0 & r^2\left(p_t + \frac{1}{2}E^2\right)\sin^2\theta \end{pmatrix}, \tag{2.26}$$

for a charged anisotropic fluid.

2.5 Field equations for anisotropic matter

To generate the Einstein field equations, for a neutral anisotropic fluid, we equate the Einstein tensor **G** found in (2.12) and the energy momentum tensor **M** in (2.17). This gives the system

$$
\begin{aligned}
\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\lambda'}{r}e^{-2\lambda} &= \rho, && \text{(2.27a)}\\
-\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\nu'}{r}e^{-2\lambda} &= p_r, && \text{(2.27b)}\\
e^{-2\lambda}\left(\nu''+\nu'^2-\nu'\lambda'+\frac{\nu'}{r}-\frac{\lambda'}{r}\right) &= p_t. && \text{(2.27c)}
\end{aligned}
$$

Equating (2.12) and (2.26) we generate the Einstein field equations for a charged anisotropic fluid as the system

$$
\begin{aligned}
\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\lambda'}{r}e^{-2\lambda} &= \rho+\frac{1}{2}E^2, && \text{(2.28a)}\\
-\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\nu'}{r}e^{-2\lambda} &= p_r-\frac{1}{2}E^2, && \text{(2.28b)}\\
e^{-2\lambda}\left(\nu''+\nu'^2-\nu'\lambda'+\frac{\nu'}{r}-\frac{\lambda'}{r}\right) &= p_t+\frac{1}{2}E^2. && \text{(2.28c)}
\end{aligned}
$$

In order to complete the system of field equations for a charged fluid, we require the Maxwell's equation

$$
\begin{aligned}
F_{ab;c}+F_{bc;a}+F_{ca;b} &= 0, && \text{(2.29a)}\\
F^{ab}{}_{;b} &= J^a. && \text{(2.29b)}
\end{aligned}
$$

The four-current **J** is defined by

$$J^a = \sigma\mu^a, \tag{2.30}$$

and σ is the proper charge density. It is easy to check that (2.29a) is identically satisfied for the spherically symmetric line element (2.2) and the components (2.22). From (2.29b) we generate the result

$$\sigma = e^{-\lambda}\left(E'+\frac{2}{r}E\right) = \frac{1}{r^2}e^{-\lambda}\left(r^2E\right)'. \tag{2.31}$$

Equation (2.31) is the only condition that arises from Maxwell's equations. The system of equations (2.28) and the condition (2.31) together constitute the Einstein-Maxwell system for a charged anisotropic fluid.

Chapter 3

Simple models for quark stars

3.1 Introduction

The first study of quark stars was performed by Itoh (1970) for static matter in equilibrium. The physical processes governing the behaviour of quark matter with ultrahigh densities is still under investigation, with special interest in the equation of state for quark matter. The phenomenology of the MIT bag model indicates that a linear form for the equation of state is possible with a nonzero bag constant. This is shown in the works by Chodos *et al* (1974), Farhi and Jaffe (1984) and Witten (1984). The review of Weber (2005) highlights models of compact astrophysical objects composed of strange quark stars. Some recent investigations for compact objects with a quark equation of state include the treatments of Kalam *et al* (2013b) and Mafa Takisa and Maharaj (2013a). The effect of the electromagnetic field on quark star was studied by Mak and Harko (2004) in the presence of a conformal symmetry. Sharma and Maharaj (2007) considered the role of anisotropy for a specified mass distribution. Charged anisotropic matter with a linear equation of state, extendible to the more general nonlinear case, was analysed by Varela *et al* (2010). Other papers containing interesting features relating to charge and anisotropy are given by Thirukkanesh and Maharaj (2008), Esculpi and Aloma (2010) and Maurya and Gupta (2012).

Mak and Harko (2004) found strange quark stars with isotropic pressures in the presence of charge. Komathiraj and Maharaj (2007c) presented a method of solving the Einstein-Maxwell system to produce new models of charged quark stars. In the present work we show that the Komathiraj and Maharaj method allows us to integrate the Einstein-Maxwell equations with anisotropic pressures and charge. Therefore we are able to generate new quark stars which are charged and anisotropic. Two new classes of solutions to the field equations are obtained by specifying the measure of anisotropy. Earlier solutions are shown to be contained in our results. A notable feature of our models is that we get the anisotropy to vanish, for particular parameter values, and isotropic pressures are regained. In many previous investigations the anisotropy is always present which is not desirable. A physical analysis indicates that the gravitational potentials and the matter variables are well behaved, and we can generate masses consistent with observations.

3.2 The model

We intend to model the stellar interior with quark matter in general relativity. The spacetime geometry is static and spherically symmetric. The interior spacetime is represented by the line element

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{3.1}$$

where $\nu(r)$ and $\lambda(r)$ are arbitrary functions representing gravity. The exterior spacetime is given by the Reissner-Nordstrom line element

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{3.2}$$

where M and Q are the total mass and charge of the star respectively. The energy momentum tensor for anisotropic charged fluid matter is of the form

$$T_{ab} = \text{diag}\left(-\rho - \frac{1}{2}E^2, p_r - \frac{1}{2}E^2, p_t + \frac{1}{2}E^2, p_t + \frac{1}{2}E^2\right). \tag{3.3}$$

In the above ρ is energy density, p_r is the radial pressure, p_t is the tangential pressure, and E is the electric field intensity. These quantities are measured relative to a comoving unit timelike fluid four-velocity u^a.

Then the Einstein-Maxwell equations can be written as

$$\begin{aligned}
\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\lambda'}{r}e^{-2\lambda} &= \rho+\frac{1}{2}E^2, && (3.4a)\\
-\frac{1}{r^2}\left(1-e^{-2\lambda}\right)+\frac{2\nu'}{r}e^{-2\lambda} &= p_r-\frac{1}{2}E^2, && (3.4b)\\
e^{-2\lambda}\left(\nu''+\nu'^2-\nu'\lambda'+\frac{\nu'}{r}-\frac{\lambda'}{r}\right) &= p_t+\frac{1}{2}E^2, && (3.4c)\\
\sigma &= \frac{1}{r^2}e^{-\lambda}\left(r^2E\right)', && (3.4d)
\end{aligned}$$

where primes denote differentiation with respect to radial coordinate r. The function σ represents the proper charge density. We are using the units where the coupling constant $\frac{8\pi G}{c^4}$ and the speed of light c are unity. The mass contained within the radius r of the charged sphere is given by

$$M(r)=\frac{1}{2}\int_0^r \omega^2\left(\rho_*+E^2\right)d\omega, \tag{3.5}$$

where ρ_* is the energy density when the electric field $E=0$. For a quark star we assume a linear relationship between the radial pressure and the energy density

$$p_r=\frac{1}{3}\left(\rho-4B\right), \tag{3.6}$$

where B is the bag constant. To transform the field equations to a more convenient form we introduce new variables defined by

$$x=Cr^2,\quad Z(x)=e^{-2\lambda(r)},\quad A^2y^2(x)=e^{2\nu(r)}, \tag{3.7}$$

where A and C are arbitrary constants. This transformation was first suggested by Durgapal and Bannerji (1983). Applying this transformation, the line element in (3.1) becomes

$$ds^2=-A^2y^2dt^2+\frac{1}{4xCZ}dx^2+\frac{x}{C}(d\theta^2+\sin^2\theta d\phi^2). \tag{3.8}$$

Then the field equations (3.4) are transformed to

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{C} + \frac{E^2}{2C}, \tag{3.9a}$$

$$4Z\frac{\dot{y}}{y} + \frac{Z-1}{x} = \frac{p_r}{C} - \frac{E^2}{2C}, \tag{3.9b}$$

$$4xZ\frac{\ddot{y}}{y} + \left(4Z + 2x\dot{Z}\right)\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{C} + \frac{E^2}{2C}, \tag{3.9c}$$

$$\frac{\sigma^2}{C} = \frac{4Z}{x}\left(x\dot{E} + E\right)^2, \tag{3.9d}$$

where dots represent derivatives with respect to the variable x. The mass function (3.5) becomes

$$M(x) = \frac{1}{4C^{\frac{3}{2}}}\int_0^x \sqrt{\omega}\left(\rho_* + E^2\right) d\omega, \tag{3.10}$$

where

$$\rho_* = \left(\frac{1-Z}{x} - 2\dot{Z}\right) C. \tag{3.11}$$

The Einstein-Maxwell field equations (3.9) for quark matter have the following form

$$\rho = 3p_r + 4B, \tag{3.12a}$$

$$\frac{p_r}{C} = Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C}, \tag{3.12b}$$

$$p_t = p_r + \Delta, \tag{3.12c}$$

$$\begin{aligned}\Delta &= \frac{4xCZ\ddot{y}}{y} + C\left(2x\dot{Z} + 6Z\right)\frac{\dot{y}}{y} \\ &\quad + C\left(2\left(\dot{Z} + \frac{B}{C}\right) + \frac{Z-1}{x}\right),\end{aligned} \tag{3.12d}$$

$$\frac{E^2}{2C} = \frac{1-Z}{x} - 3Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C}, \tag{3.12e}$$

$$\sigma = 2\sqrt{\frac{ZC}{x}}\left(x\dot{E} + E\right). \tag{3.12f}$$

The quantity $\Delta = p_t - p_r$ is the measure of anisotropy. This system consists of eight variables (ρ, p_r, p_t, E, Z, y, σ, Δ) in six equations. It is apparent that if we specify two of these variables then the system may be integrated. The gravitational behavior of the anisotropic charged quark star is governed by the system (3.12). For $\Delta = 0$ we have the isotropic model that was described by Komathiraj and Maharaj (2007c).

For neutral fluids with isotropic pressures ($\Delta = 0,\ E = 0$) there is no freedom in the system (3.12) as the equation of state has been specified. For a charged fluid with anisotropic pressures ($\Delta \neq 0,\ E \neq 0$), with the linear equation of state, there are two degrees of freedom because of the appearance of new matter quantities, the electric field and anisotropy. From a mathematical viewpoint any two of the eight variables may be chosen to integrate the system (3.12); the choice should be carefully made on physical grounds so that a well behaved model results.

In order to find exact solutions to this model we have to specify two quantities: we choose the potential y and the quantity Δ. We specify the metric function

$$y = (a + x^m)^n \,, \tag{3.13}$$

where a, m and n are constants. A similar choice was made by Komathiraj and Maharaj (2007c). The choice guarantees that the metric function y is regular and well behaved within the interior. It remains nonsingular at the centre of the star. Note that special cases of the potential y corresponds to known quark models, e.g. when $m = \frac{1}{2}$, $n = 1$ we regain the Mak and Harko (2004) quark star model and when $m = 1$, $n = 2$ we regain the Komathiraj and Maharaj (2007c) model for a quark star with isotropic pressures. We expect that the potential (3.13) is therefore likely to produce new solutions to the Einstein-Maxwell system when charge and anisotropy are present. Also we specify the measure of anisotropy in the form

$$\Delta = A_0 + A_1 x + A_2 x^2 + A_3 x^3, \tag{3.14}$$

where $A_0,\ A_1,\ A_2,$ and A_3 are arbitrary constants. This choice is physically reasonable and ensures that we regain isotropic pressures when $A_0 = A_1 = A_2 = A_3 = 0$. Note that we have effectively taken three orders of a Taylor expansion for Δ in terms of the radial coordinate. This form of Δ enables us to integrate the Einstein-Maxwell system; higher order terms lead to expressions which are not integrable. An important point to note is that the form (3.14) allows us to regain isotropic pressures by setting parameters to vanish. In most other treatments involving anisotropic stellar configurations this is not the case as indicated in the works of Dev and Gleiser (2002), Esculpi and Aloma

(2010), Harko and Mak (2002), and Mak and Harko (2003). The recent strange quark models of Kalam *et al* (2013b) and Paul *et al* (2011) also have a nonzero anisotropy throughout the star. In our model the choice (3.14) enables us to regain isotropic pressures.

Substituting (3.13) in (3.12d) we obtain the first order differential equation

$$\dot{Z} + \frac{\left[a^2 + 2a\left(mn(1+2m)+1\right)x^m + \left(2mn(2mn+1)+1\right)x^{2m}\right]Z}{2x\left(a+(1+mn)x^m\right)(a+x^m)} = \frac{\left(1 - \frac{2xB}{C} + \frac{x\Delta}{C}\right)(a+x^m)}{2x\left(a+(1+mn)x^m\right)}. \tag{3.15}$$

To make the equation easily integrable we decompose by partial fractions the coefficient of Z which gives

$$\dot{Z} + \left(\frac{1}{2x} + \frac{2m(n-1)x^{m-1}}{a+x^m} + \frac{m\left(4(1+mn)-3n\right)x^{m-1}}{2\left(a+(1+mn)x^m\right)}\right)Z = \frac{\left(1 - \frac{2xB}{C} + \frac{x\Delta}{C}\right)(a+x^m)}{2x\left(a+(1+mn)x^m\right)}. \tag{3.16}$$

Substituting (3.14) in (3.16) we obtain the differential equation

$$\dot{Z} + \left(\frac{1}{2x} + \frac{2m(n-1)x^{m-1}}{a+x^m} + \frac{m\left(4(1+mn)-3n\right)x^{m-1}}{2\left(a+(1+mn)x^m\right)}\right)Z = \frac{\left(1 - \frac{2xB}{C} + \frac{(A_0+A_1x+A_2x^2+A_3x^3)x}{C}\right)(a+x^m)}{2x\left(a+(1+mn)x^m\right)}. \tag{3.17}$$

Once (3.17) is integrated we can directly find the remaining quantities ρ, p_r, p_t, E^2 and σ from the system (3.12). In order to find an exact solution to (3.17) we need to specify values for the constants m and n.

3.3 Generalized Komathiraj-Maharaj model

We can find an exact solution of (3.17) when $m = \frac{1}{2}$ and $n = 1$. In this case the metric function in (3.13) becomes

$$y = a + \sqrt{x}.$$

For this choice of m and n, (3.17) becomes

$$\dot{Z} + \left(\frac{1}{2x} + \frac{3}{2\sqrt{x}(2a+3\sqrt{x})}\right)Z = \frac{\left(1 - \frac{2xB}{C} + \frac{(A_0+A_1x+A_2x^2+A_3x^3)x}{C}\right)(a+\sqrt{x})}{x\left(2a+3\sqrt{x}\right)}. \tag{3.18}$$

Solving (3.18) we obtain the solution

$$Z = \left[\left(3\left(2a+\sqrt{x}\right) - \frac{Bx}{C}\left(4a+3\sqrt{x}\right)\right) + \frac{3F(x)}{C} + \frac{k}{\sqrt{x}}\right]\frac{1}{3\left(2a+3\sqrt{x}\right)}, \tag{3.19}$$

where

$$\begin{aligned} F(x) &= A_0\left(\frac{2}{3}ax + \frac{1}{2}x^{\frac{3}{2}}\right) + A_1\left(\frac{2}{5}ax^2 + \frac{1}{3}x^{\frac{5}{2}}\right) + A_2\left(\frac{2}{7}ax^3 + \frac{1}{4}x^{\frac{7}{2}}\right) \\ &\quad + A_3\left(\frac{2}{9}ax^4 + \frac{1}{5}x^{\frac{9}{2}}\right), \end{aligned}$$

and k is a constant of integration. In order to avoid the singularity in the potential Z we should set $k = 0$. Note that $F(x) = 0$ at the centre of the star and this condition is satisfied for isotropic pressures.

The potentials and matter variables are given by

$$e^{2\nu} = A^2\left(a+\sqrt{x}\right)^2, \tag{3.20a}$$

$$e^{2\lambda} = \frac{3\left(2a+3\sqrt{x}\right)}{3\left(2a+\sqrt{x}\right) - \frac{Bx}{C}\left(4a+3\sqrt{x}\right) + \frac{3F(x)}{C}}, \tag{3.20b}$$

$$\begin{aligned} \rho &= \frac{3C\left(6a^2+10a\sqrt{x}+3x\right)}{2\sqrt{x}(a+\sqrt{x})(2a+3\sqrt{x})^2} + \frac{B\left(16a^3+47a^2\sqrt{x}+48ax+18x^{\frac{3}{2}}\right)}{2(a+\sqrt{x})(2a+3\sqrt{x})^2} \\ &\quad -3G(x)\left(\frac{1}{2\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right), \end{aligned} \tag{3.20c}$$

$$\begin{aligned} p_r &= \frac{C\left(6a^2+10a\sqrt{x}+3x\right)}{2\sqrt{x}(a+\sqrt{x})(2a+3\sqrt{x})^2} - \frac{B\left(\frac{16}{3}a^3+27a^2\sqrt{x}+40ax+18x^{\frac{3}{2}}\right)}{2(a+\sqrt{x})(2a+3\sqrt{x})^2} \\ &\quad -G(x)\left(\frac{1}{2\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right), \end{aligned} \tag{3.20d}$$

$$\begin{aligned} p_t &= \frac{C\left(6a^2+10a\sqrt{x}+3x\right)}{2\sqrt{x}(a+\sqrt{x})(2a+3\sqrt{x})^2} - \frac{B\left(\frac{16}{3}a^3+27a^2\sqrt{x}+40ax+18x^{\frac{3}{2}}\right)}{2(a+\sqrt{x})(2a+3\sqrt{x})^2} \\ &\quad +H(x)\left(\frac{1}{2\left(a+\sqrt{x}\right)\left(2a+3\sqrt{x}\right)^2}\right), \end{aligned} \tag{3.20e}$$

$$\Delta = A_0 + A_1x + A_2x^2 + A_3x^3, \tag{3.20f}$$

$$\begin{aligned} E^2 &= \left[C\left(-2a^2-2a\sqrt{x}+3x\right) + Bx\left(a^2+2a\sqrt{x}\right) - J(x)\right] \\ &\quad \times\left(\frac{1}{\sqrt{x}(a+\sqrt{x})(2a+3\sqrt{x})^2}\right). \end{aligned} \tag{3.20g}$$

In this case the line element becomes

$$\begin{aligned} ds^2 &= -A^2\left(a+\sqrt{x}\right)^2 dt^2 + \left(\frac{3\left(2a+3\sqrt{x}\right)}{3\left(2a+\sqrt{x}\right)-\frac{Bx}{C}\left(4a+3\sqrt{x}\right)+\frac{3F(x)}{C}}\right) dr^2 \\ &\quad +r^2(d\theta^2+\sin^2\theta d\phi^2). \end{aligned} \tag{3.21}$$

To simplify the relevant expressions we have set

$$\begin{aligned} G(x) &= A_0\left(\frac{4}{3}a^3+\frac{5}{2}a^2\sqrt{x}+ax\right) \\ &\quad +A_1\left(\frac{8}{5}a^3x+\frac{64}{15}a^2x^{\frac{3}{2}}+\frac{18}{5}ax^2+x^{\frac{5}{2}}\right) \\ &\quad +A_2\left(\frac{12}{7}a^3x^2+\frac{141}{28}a^2x^{\frac{5}{2}}+\frac{67}{14}ax^3+\frac{3}{2}x^{\frac{7}{2}}\right) \\ &\quad +A_3\left(\frac{16}{9}a^3x^3+\frac{82}{15}a^2x^{\frac{7}{2}}+\frac{82}{15}ax^4+\frac{9}{5}x^{\frac{9}{2}}\right), \\ H(x) &= A_0\left(\frac{20}{3}a^3+\frac{59}{2}a^2\sqrt{x}+41ax+18x^{\frac{3}{2}}\right) \\ &\quad +A_1\left(\frac{32}{5}a^3x+\frac{416}{15}a^2x^{\frac{3}{2}}+\frac{192}{5}ax^2+17x^{\frac{5}{2}}\right) \\ &\quad +A_2\left(\frac{44}{7}a^3x^2+\frac{755}{28}a^2x^{\frac{5}{2}}+\frac{521}{14}ax^3+\frac{33}{2}x^{\frac{7}{2}}\right) \\ &\quad +A_3\left(\frac{56}{9}a^3x^3+\frac{398}{15}a^2x^{\frac{7}{2}}+\frac{548}{15}ax^4+\frac{81}{5}x^{\frac{9}{2}}\right), \\ J(x) &= A_0\left(4a^3\sqrt{x}+\frac{33}{2}a^2x+22ax^{\frac{3}{2}}+9x^2\right) \\ &\quad +A_1\left(\frac{16}{5}a^3x^{\frac{3}{2}}+\frac{64}{5}a^2x^2+\frac{84}{5}ax^{\frac{5}{2}}+7x^3\right) \\ &\quad +A_2\left(\frac{20}{7}a^3x^{\frac{5}{2}}+\frac{313}{28}a^2x^3+\frac{101}{7}ax^{\frac{7}{2}}+6x^4\right) \\ &\quad +A_3\left(\frac{8}{3}a^3x^{\frac{7}{2}}+\frac{154}{15}a^2x^4+\frac{196}{15}ax^{\frac{9}{2}}+\frac{27}{5}x^5\right). \end{aligned}$$

The exact solution (3.20) and (3.21) is a new model for a charged anisotropic quark star.

If we set $A_0 = A_1 = A_2 = A_3 = 0$, then we regain the first Komathiraj and

Maharaj (2007c) isotropic exact solution given by

$$\begin{aligned}
e^{2\nu} &= A^2\left(a+\sqrt{x}\right)^2, \\
e^{2\lambda} &= \frac{3\left(2a+3\sqrt{x}\right)}{3\left(2a+\sqrt{x}\right)-\frac{Bx}{C}\left(4a+3\sqrt{x}\right)}, \\
\rho &= \frac{3C\left(6a^2+10a\sqrt{x}+3x\right)}{2\sqrt{x}(a+\sqrt{x})(2a+3\sqrt{x})^2}+\frac{B\left(16a^3+47a^2\sqrt{x}+48ax+18x^{\frac{3}{2}}\right)}{2(a+\sqrt{x})(2a+3\sqrt{x})^2}, \\
p &= \frac{C\left(6a^2+10a\sqrt{x}+3x\right)}{2\sqrt{x}(a+\sqrt{x})(2a+3\sqrt{x})^2}-\frac{B\left(\frac{16}{3}a^3+27a^2\sqrt{x}+40ax+18x^{\frac{3}{2}}\right)}{2(a+\sqrt{x})(2a+3\sqrt{x})^2}, \\
E^2 &= \frac{\left[C\left(-2a^2-2a\sqrt{x}+3x\right)+Bx\left(a^2+2a\sqrt{x}\right)\right]}{\sqrt{x}(a+\sqrt{x})(2a+3\sqrt{x})^2}.
\end{aligned}$$

The line element corresponding to this solution is

$$\begin{aligned}
ds^2 &= -A^2\left(a+\sqrt{x}\right)^2 dt^2+\left(\frac{3\left(2a+3\sqrt{x}\right)}{3\left(2a+\sqrt{x}\right)-\frac{Bx}{C}\left(4a+3\sqrt{x}\right)}\right)dr^2 \\
&\quad +r^2(d\theta^2+\sin^2\theta d\phi^2),
\end{aligned} \tag{3.23}$$

with isotropic pressures and with equation of state $p=\frac{1}{3}(\rho-4B)$. We observe that when we set $G(x)=0$, $H(x)=0$ and $J(x)=0$ in (3.20) we obtain expressions for the energy density ρ, the pressure p and electric field E^2 which are identical to those in the Komathiraj and Maharaj (2007c) model. Furthermore if we let $a=0$ in (3.23) we obtain the Mak and Harko (2004) line element

$$ds^2=-A^2Cr^2dt^2+\left(\frac{3}{1-Br^2}\right)dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{3.24}$$

with the matter variables

$$\rho=\frac{1}{2r^2}+B,\quad p=\frac{1}{6r^2}-B,\quad E^2=\frac{1}{3r^2}.$$

On setting $B=0$ we regain the Misner and Zapolsky (1964) particular solution with the equation of state $p=\frac{1}{3}\rho$. Note that the class of solutions found in this section contains a singularity in the electric field at the centre. This feature is also present in the Mak and Harko (2004) model for a quark star. However the total charge and mass remains finite which is a good feature of this class of models.

3.4 Nonsingular quark model

We can find another exact solution by choosing $m = 1$ and $n = 2$. For this choice (3.13) gives the metric function

$$y(x) = (a + x)^2 . \tag{3.25}$$

The differential equation (3.17) becomes

$$\dot{Z} + \left(\frac{1}{2x} + \frac{2}{a+x} + \frac{3}{a+3x}\right) Z = \frac{\left(1 - \frac{2xB}{C} + \frac{(A_0 + A_1 x + A_2 x^2 + A_3 x^3)x}{C}\right)(a+x)}{2x\,(a+3x)}. \tag{3.26}$$

Equation (3.26) is integrated to yield the solution

$$\begin{aligned} Z &= \frac{(35a^3 + 35a^2 x + 21ax^2 + 5x^3)}{35(a+x)^2(a+3x)} \\ &\quad + \frac{\frac{315L(x)}{C} - 2BxC\,(105a^3 + 189a^2 x + 135ax^2 + 35x^3) + \frac{k}{\sqrt{x}}}{315(a+x)^2(a+3x)}. \end{aligned} \tag{3.27}$$

In the above we have set

$$\begin{aligned} L(x) &= A_0\left(\frac{1}{3}a^3 x + \frac{3}{5}a^2 x^2 + \frac{3}{7}ax^3 + \frac{1}{9}x^4\right) \\ &\quad + A_1\left(\frac{1}{5}a^3 x^2 + \frac{3}{7}a^2 x^3 + \frac{1}{3}ax^4 + \frac{1}{11}x^5\right) \\ &\quad + A_2\left(\frac{1}{7}a^3 x^3 + \frac{1}{3}a^2 x^4 + \frac{3}{11}ax^5 + \frac{1}{13}x^6\right) \\ &\quad + A_3\left(\frac{1}{9}a^3 x^4 + \frac{3}{11}a^2 x^5 + \frac{3}{13}ax^6 + \frac{1}{15}x^7\right), \end{aligned}$$

and k is a constant of integration. In order to avoid a singularity in the metric function Z, we set $k = 0$. Note that $L(x) = 0$ at the centre of the star and this condition is satisfied for isotropic pressures.

The potentials and matter variables become

$$
\begin{aligned}
e^{2\nu} &= A^2\,(a+x)^4\,, && (3.28a)\\
e^{2\lambda} &= 315(a+x)^2(a+3x)\left[9\left(35a^3+35a^2x+21ax^2+5x^3\right)\right.\\
&\quad \left.-\frac{2Bx}{C}\left(105a^3+189a^2x+135ax^2+35x^3\right)+\frac{315L(x)}{C}\right]^{-1}, && (3.28b)\\
\rho &= \frac{3C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35(a+x)^3(a+3x)^2}\\
&\quad +\frac{B\left(210a^5+798a^4x+1476a^3x^2+2540a^2x^3+2090ax^4+630x^5\right)}{105(a+x)^3(a+3x)^2}\\
&\quad +\frac{3\Psi(x)}{105(a+x)^3(a+3x)^2}, && (3.28c)\\
p_r &= \frac{C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35(a+x)^3(a+3x)^2}\\
&\quad -\frac{B\left(70a^5+994a^4x+3708a^3x^2+\frac{16780}{3}a^2x^3+\frac{11770}{3}ax^4+1050x^5\right)}{105(a+x)^3(a+3x)^2}\\
&\quad +\frac{\Psi(x)}{105(a+x)^3(a+3x)^2}, && (3.28d)\\
p_t &= \frac{C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35(a+x)^3(a+3x)^2}\\
&\quad -\frac{B\left(70a^5+994a^4x+3708a^3x^2+\frac{16780}{3}a^2x^3+\frac{11770}{3}ax^4+1050x^5\right)}{105(a+x)^3(a+3x)^2}\\
&\quad +\frac{\Omega(x)}{105(a+x)^3(a+3x)^2}, && (3.28e)\\
\Delta &= A_0+A_1x+A_2x^2+A_3x^3, && (3.28f)\\
E^2 &= \frac{C\left(1764a^3x+13068a^2x^2+12204ax^3+3780x^4\right)-\Lambda(x)}{315(a+x)^3(a+3x)^2}\\
&\quad -\frac{B\left(168a^4x+1296a^3x^2+6528a^2x^3+7280ax^4+2520x^5\right)}{315(a+x)^3(a+3x)^2}. && (3.28g)
\end{aligned}
$$

For this model the line element becomes

$$
\begin{aligned}
ds^2 &= -A^2\,(a+x)^4\,dt^2\\
&\quad +315(a+x)^2(a+3x)\left[9\left(35a^3+35a^2x+21ax^2+5x^3\right)\right.\\
&\quad \left.-\frac{2Bx}{C}\left(105a^3+189a^2x+135ax^2+35x^3\right)+\frac{315L(x)}{C}\right]^{-1}dr^2\\
&\quad +r^2(d\theta^2+\sin^2\theta d\phi^2). && (3.29)
\end{aligned}
$$

For simplicity we have set

$$
\begin{aligned}
\Psi(x) \;=\;& A_0\left(-\frac{35}{2}a^5+\frac{49}{2}a^4x+279a^3x^2+\frac{1145}{3}a^2x^3+\frac{1375}{6}ax^4+\frac{105}{2}x^5\right)\\
&+A_1x\left(-21a^5-57a^4x+20a^3x^2+\frac{1360}{11}a^2x^3+105ax^4+\frac{315}{11}x^5\right)\\
&+A_2x^2\left(-\frac{45}{2}a^5-\frac{185}{2}a^4x-\frac{1145}{11}a^3x^2-\frac{315}{13}a^2x^3+\frac{7245}{286}ax^4+\frac{315}{26}x^5\right)\\
&-A_3x^3\left(\frac{70}{3}a^5+\frac{3710}{33}a^4x+\frac{2310}{13}a^3x^2+\frac{17206}{143}a^2x^3+\frac{392}{13}ax^4\right),\\
\Omega(x) \;=\;& A_0\left(\frac{175}{2}a^5+\frac{1939}{2}a^4x+3429a^3x^2+\frac{15635}{3}a^2x^3+\frac{22165}{6}ax^4+\frac{1995}{2}x^5\right)\\
&+A_1x\left(84a^5+888a^4x+3170a^3x^2+\frac{54490}{11}a^2x^3+3570ax^4+\frac{10710}{11}x^5\right)\\
&+A_2x^2\left(\frac{165}{2}a^5+\frac{1705}{2}a^4x+\frac{33505}{11}a^3x^2+\frac{62474}{13}a^2x^3\right.\\
&\left.+\frac{998235}{286}ax^4+\frac{24885}{26}x^5\right)+A_3x^3\left(\frac{245}{3}a^5+\frac{27475}{33}a^4x\right.\\
&\left.+\frac{38640}{13}a^3x^2+\frac{673484}{143}a^2x^3+\frac{44653}{13}ax^4+945x^5\right),\\
\Lambda(x) \;=\;& A_0\left(315a^5+2751a^4x+8802a^3x^2+11226a^2x^3+6755ax^4+1575x^5\right)\\
&+A_1x\left(252a^5+2124a^4x+6732a^3x^2+\frac{100380}{11}a^2x^3\right.\\
&\left.+\frac{63000}{11}ax^4+\frac{15120}{11}x^5\right)+A_2x^2\left(225a^5+1845a^4x+\frac{63210}{11}a^3x^2\right.\\
&\left.+\frac{1133370}{143}a^2x^3+\frac{55755}{11}ax^4+\frac{16065}{13}x^5\right)+A_3x^3\left(210a^5+\frac{18550}{11}a^4x\right.\\
&\left.+\frac{738360}{143}a^3x^2+\frac{78624}{11}a^2x^3+\frac{59934}{13}ax^4+1134x^5\right),
\end{aligned}
$$

in the above. The mass function giving the total mass within a sphere of radius x is given by

$$\begin{aligned}
M(x) &= \left(\left(\frac{1268}{96525}a - \frac{1}{30}x^2 - \frac{14}{585}ax \right) A_3 \right. \\
&\quad \left. - \left(\frac{4}{91}x + \frac{74}{2145}a \right) A_2 - \frac{7}{110}A_1 \right) \frac{x^{\frac{5}{2}}}{C^{\frac{3}{2}}} \\
&\quad - \left(\frac{1}{9}B + \frac{1}{9}A_0 + \frac{2}{33}aA_1 - \frac{10}{429}a^2A_2 + \frac{14}{1287}a^3A_3 \right) \left(\frac{x}{C} \right)^{\frac{3}{2}} \\
&\quad - \sqrt{\frac{a}{C^3}} \left(\frac{62}{105}aB + \frac{93}{35}C - \frac{31}{105}aA_0 + \frac{31}{385}a^2A_1 \right. \\
&\quad \left. - \frac{31}{1001}a^3A_2 + \frac{31}{2145}a^4A_3 \right) \arctan \sqrt{\frac{x}{a}} \\
&\quad + \frac{\sqrt{3a}}{3C^{\frac{3}{2}}} \left(\frac{188}{315}aB + \frac{129}{35}C - \frac{94}{315}aA_0 + \frac{59}{1155}a^2A_1 \right. \\
&\quad \left. - \frac{100}{9009}a^3A_2 + \frac{157}{57915}a^4A_3 \right) \arctan \sqrt{\frac{3x}{a}} \\
&\quad + \left(\frac{76}{189}aB + \frac{8}{9}C - \frac{38}{189}aA_0 + \frac{52}{693}a^2A_1 \right. \\
&\quad \left. - \frac{934}{27027}a^3A_2 + \frac{3088}{173745}a^4A_3 \right) \sqrt{\frac{x}{C^3}} \\
&\quad - \left(\frac{6}{35}a^2B + \frac{27}{35}aC - \frac{3}{35}a^2A_0 + \frac{9}{385}a^3A_1 \right. \\
&\quad \left. - \frac{9}{1001}a^4A_2 + \frac{3}{715}a^5A_3 \right) \frac{\sqrt{x}}{(a+x)C^{\frac{3}{2}}} \\
&\quad - \left(\frac{4}{105}a^3B + \frac{6}{35}a^2C - \frac{2}{105}a^3A_0 + \frac{2}{385}a^4A_1 \right. \\
&\quad \left. - \frac{2}{1001}a^5A_2 + \frac{2}{2145}a^6A_3 \right) \frac{\sqrt{x}}{(a+x)^2C^{\frac{3}{2}}} \\
&\quad - \left(\frac{188}{945}a^2B + \frac{43}{35}aC - \frac{94}{945}a^2A_0 + \frac{59}{3465}a^3A_1 \right. \\
&\quad \left. - \frac{100}{27027}a^4A_2 + \frac{157}{173745}a^5A_3 \right) \frac{\sqrt{x}}{(a+3x)C^{\frac{3}{2}}}. \qquad (3.30)
\end{aligned}$$

The exact solution (3.28) and (3.29) is a new model for the Einstein-Maxwell system with charge and anisotropy.

If we set $A_0 = A_1 = A_2 = A_3 = 0$, then we regain the second Komathiraj and

Maharaj (2007c) nonsingular exact model given by

$$
\begin{aligned}
e^{2\nu} &= A^2\left(a+x\right)^4, \\
e^{2\lambda} &= \frac{315(a+x)^2(a+3x)}{9\left(35a^3+35a^2x+21ax^2+5x^3\right)-\dfrac{2Bx}{C}\left(105a^3+189a^2x+135ax^2+35x^3\right)}, \\
\rho &= \frac{3C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35(a+x)^3(a+3x)^2} \\
&\quad + \frac{B\left(210a^5+798a^4x+1476a^3x^2+2540a^2x^3+2090ax^4+630x^5\right)}{105(a+x)^3(a+3x)^2}, \\
p &= \frac{C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35(a+x)^3(a+3x)^2} \\
&\quad - \frac{B\left(70a^5+994a^4x+3708a^3x^2+\frac{16780}{3}a^2x^3+\frac{11770}{3}ax^4+1050x^5\right)}{105(a+x)^3(a+3x)^2}, \\
E^2 &= \frac{C\left(1764a^3x+13068a^2x^2+12204ax^3+3780x^4\right)}{315(a+x)^3(a+3x)^2} \\
&\quad - \frac{B\left(168a^4x+1296a^3x^2+6528a^2x^3+7280ax^4+2520x^5\right)}{315(a+x)^3(a+3x)^2}.
\end{aligned}
$$

The line element for this isotropic nonsingular model becomes

$$
\begin{aligned}
ds^2 = &- A^2\left(a+x\right)^4 dt^2 \\
&+ \frac{315(a+x)^2(a+3x)dr^2}{9\left(35a^3+35a^2x+21ax^2+5x^3\right)-\dfrac{2Bx}{C}\left(105a^3+189a^2x+135ax^2+35x^3\right)} \\
&+ r^2(d\theta^2+\sin^2\theta d\phi^2).
\end{aligned}
$$

The potentials, matter variables, including the electric field, remain finite at the centre so that our model in the system (3.28) is nonsingular. At the centre ($x = 0$) we have

$$
\begin{aligned}
e^{2\nu(0)} &= A^2a^4, \\
e^{2\lambda(0)} &= 1, \\
\rho(0) &= 2\left(\frac{6C}{a} + B - \frac{1}{4}A_0\right), \\
p_r(0) &= \frac{1}{3}\left(\frac{12C}{a} - 2B - \frac{A_0}{2}\right) = \frac{1}{3}\left(\rho_0 - 4B\right), \\
p_t(0) &= \frac{1}{3}\left(\frac{12C}{a} - 2B + \frac{5A_0}{2}\right) = \frac{1}{3}\left(\rho_0 - 4B\right) + A_0, \\
\Delta(0) &= A_0, \\
E^2(0) &= -A_0, \\
M(0) &= 0.
\end{aligned}
$$

For stability we should have $\Delta(0) = 0$ so that $A_0 = 0$. This ensures that the anisotropy and the electric field vanish at the centre. Consequently the class of solutions found in this section are good candidates to produce charged anisotropic stars with physically reasonable interior distributions.

3.5 Discussion

In this section we indicate that the exact solutions of the field equations in this chapter are well behaved. To do this we generate graphical plots for the gravitational potentials, matter variables and the electric field. The Python programming language was used to generate plots for the particular choices $a = 0.2$, $A = 0.69$, $B = 0.198$, $C = 1$, $A_0 = 0.0$, $A_1 = 0.6$, $A_2 = 0.15$, and $A_3 = -0.7$. The graphical plots generated are for the potential $e^{2\nu}$ (Fig. 3.1), potential $e^{2\lambda}$ (Fig. 3.2), energy density ρ (Fig. 3.3), radial pressure p_r (Fig. 3.4), tangential pressure p_t (Fig. 3.5), measure of anisotropy Δ (Fig. 3.6), the electric field E^2 (Fig. 3.7) and the mass M (Fig. 3.8). All figures are plotted against the radial coordinate r. These quantities are regular and well behaved in the stellar interior. The energy density, the radial pressure and the tangential pressure are decreasing functions as we approach the boundary from the centre. In general the measure of anisotropy Δ is finite and continuous. We observe that Δ increases from the centre until it attains a maximum value and decreases sharply towards the surface of the star. This profile is similar to that obtained by Sharma and Maharaj (2007) and Mafa Takisa and Maharaj (2013a, 2013b). The electric field E^2 is finite and regular at the centre. It increases from the centre and then decreases after reaching a maximum value. We observe in Fig. 3.8 that the mass increases with radial distance monotonically.

Finally we note for the values $a = 0.0278$, $B = 0.0064$, $C = 0.0005$, $A_0 = 0.0000$, $A_1 = 0.0107$, $A_2 = 0.0134$, and $A_3 = 0.0107$ we can generate a quark star with radius $R = 9.46$km and mass $M = 2.86M_\odot$. These figures correspond to a distribution with a linear quark equation of state. They are consistent with the values found by Mak and Harko (2004). Other values of the parameters produce radii and masses consistent with previous investigations. A detailed analysis of the physical features of the models found here is undertaken in subsequent chapters.

The interior solutions obtained in this chapter match the exterior Reissner-Nordstrom spacetime (3.2) across the boundary $r = R$. This generates the condition

$$\left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right) = A^2 y^2,$$

and

$$\left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right)^{-1} = e^{2\lambda},$$

which relate the constants a, A, B, C, A_0, A_1, A_2 and A_3. There are sufficient free parameters in the model to ensure the continuity of the metric coefficients at the boundary of the star.

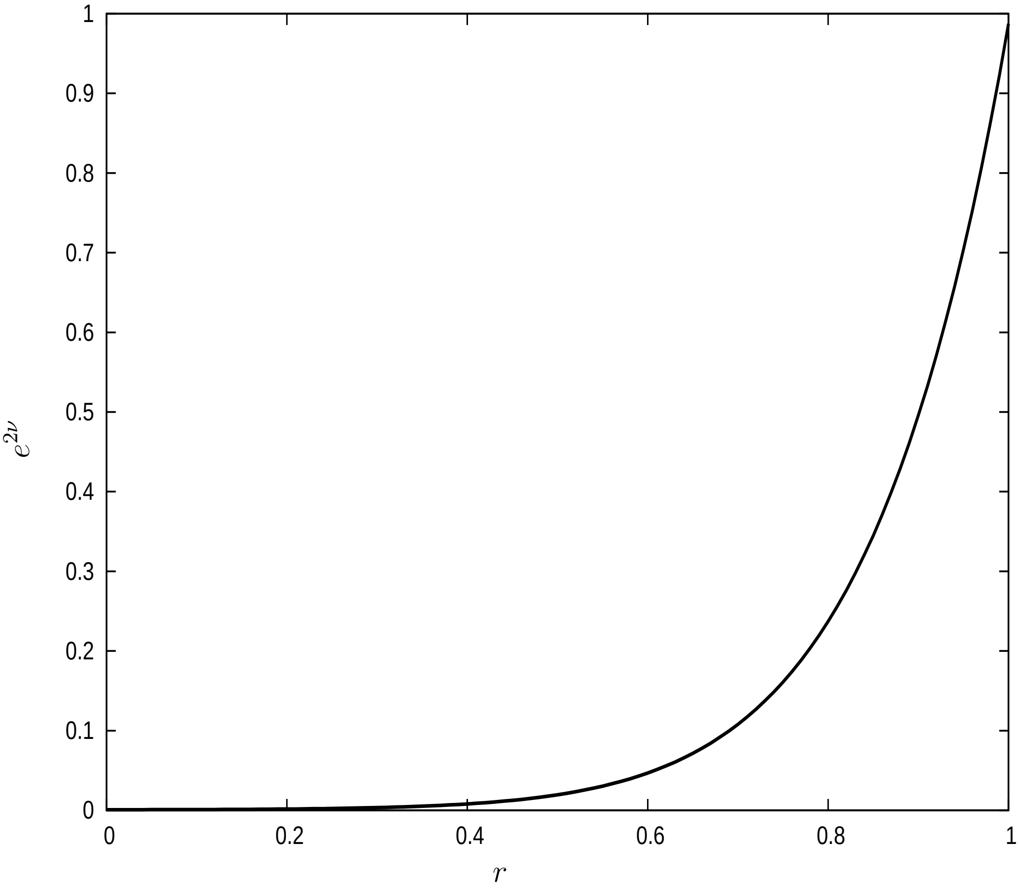

Figure 3.1: The potential $e^{2\nu}$ against the radial distance r

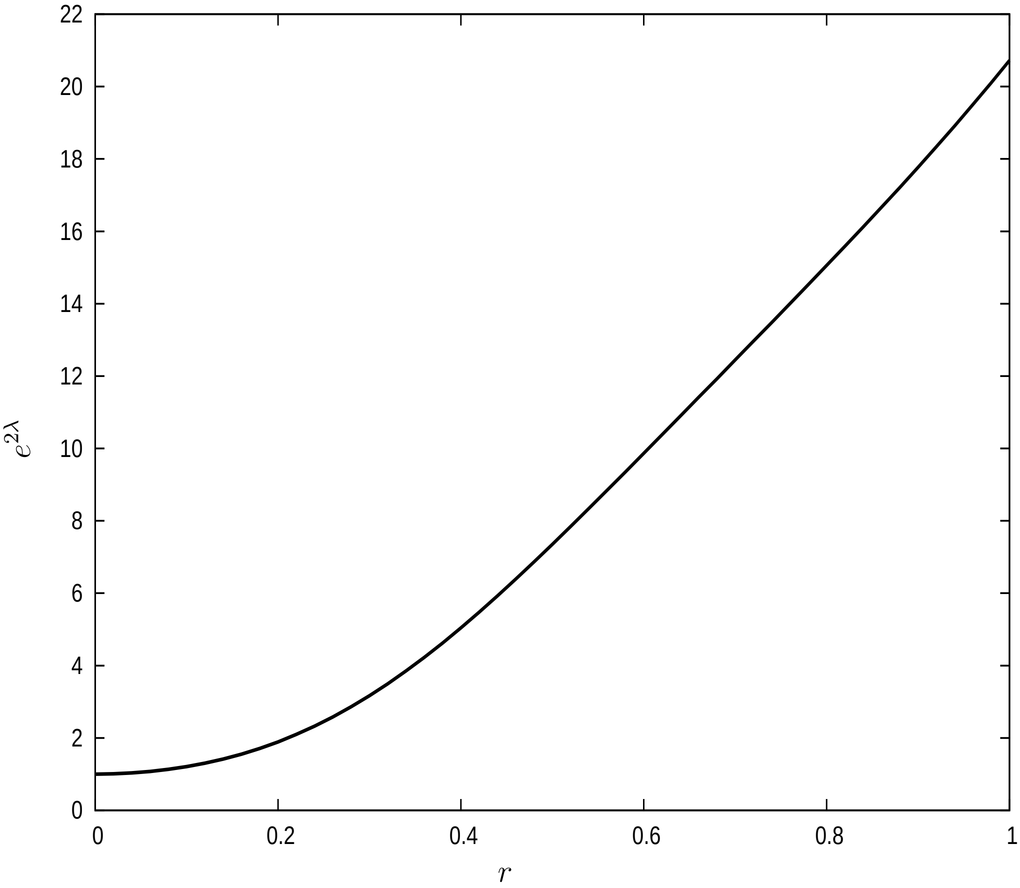

Figure 3.2: The potential $e^{2\lambda}$ against the radial distance r

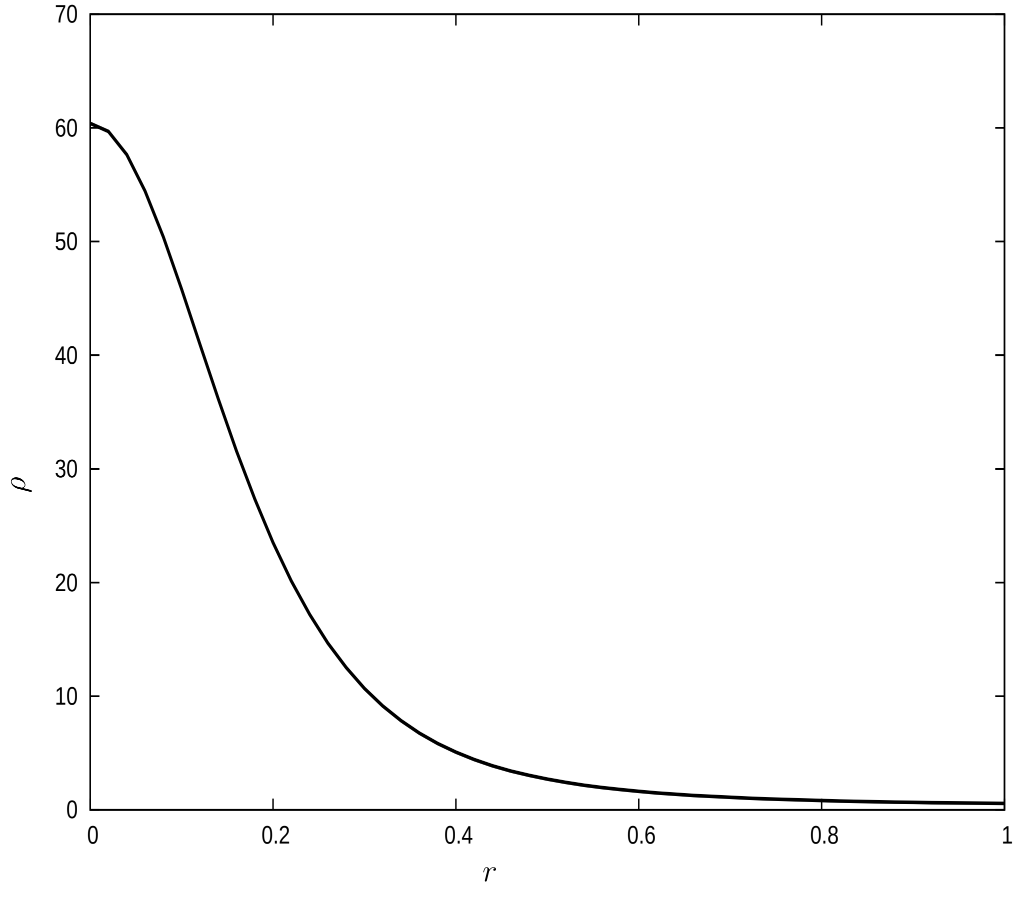

Figure 3.3: The energy density ρ against the radial distance r

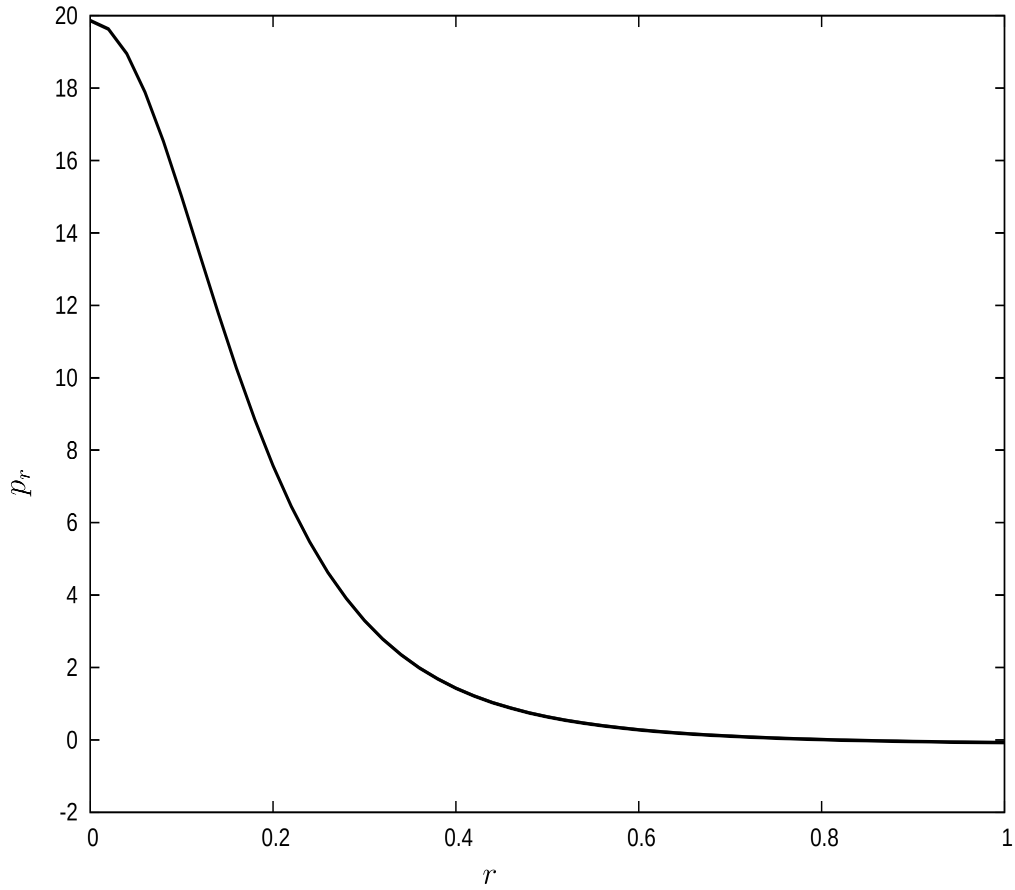

Figure 3.4: The radial pressure p_r against the radial distance r

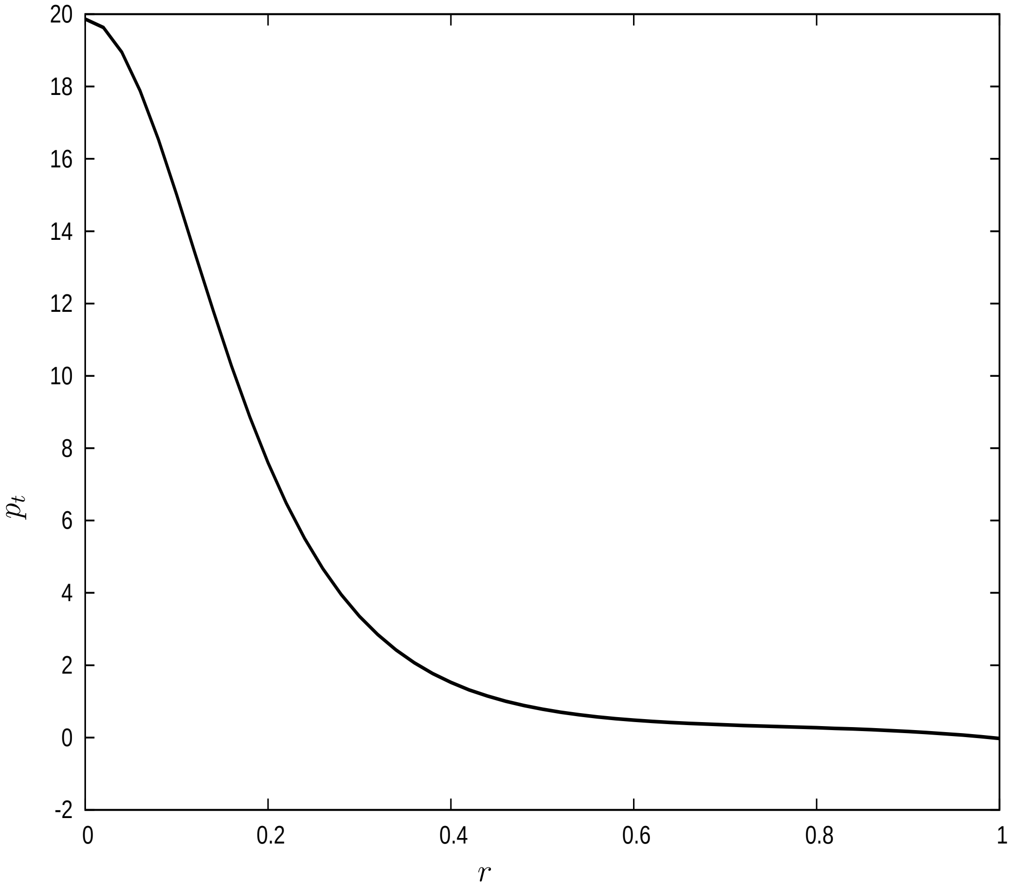

Figure 3.5: The tangential pressure p_t against radial distance r

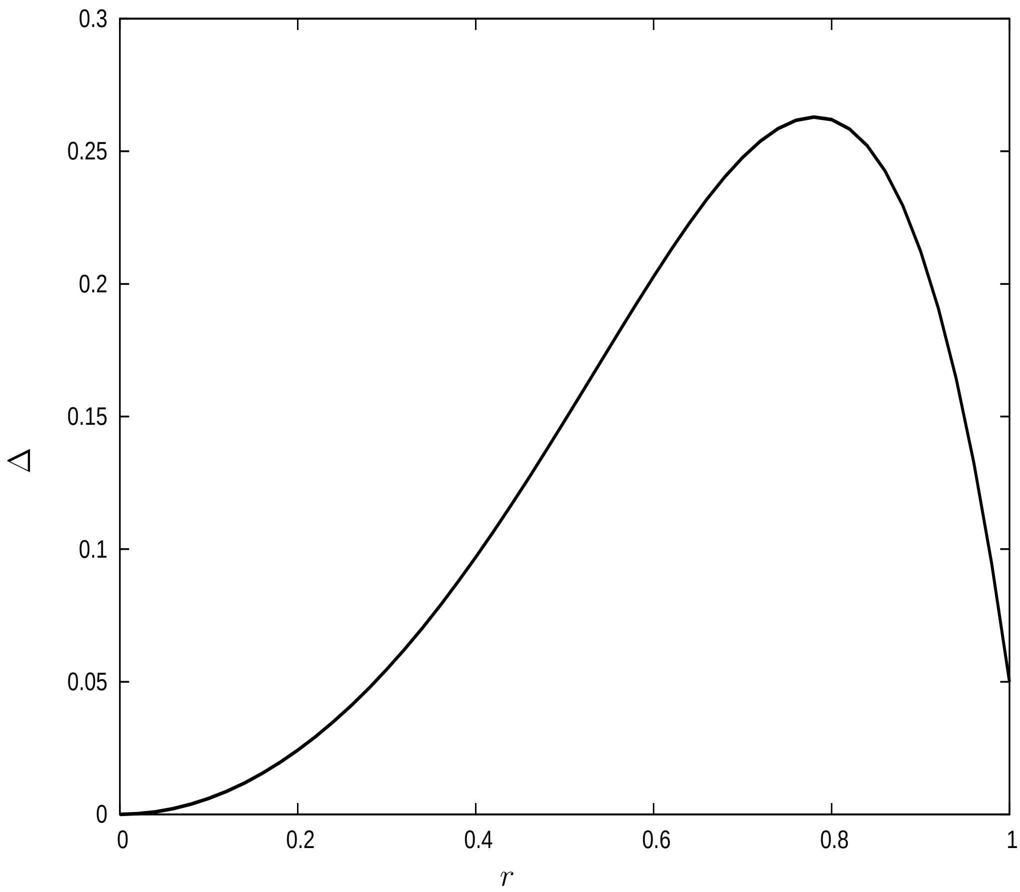

Figure 3.6: The measure of anisotropy Δ against the radial distance r

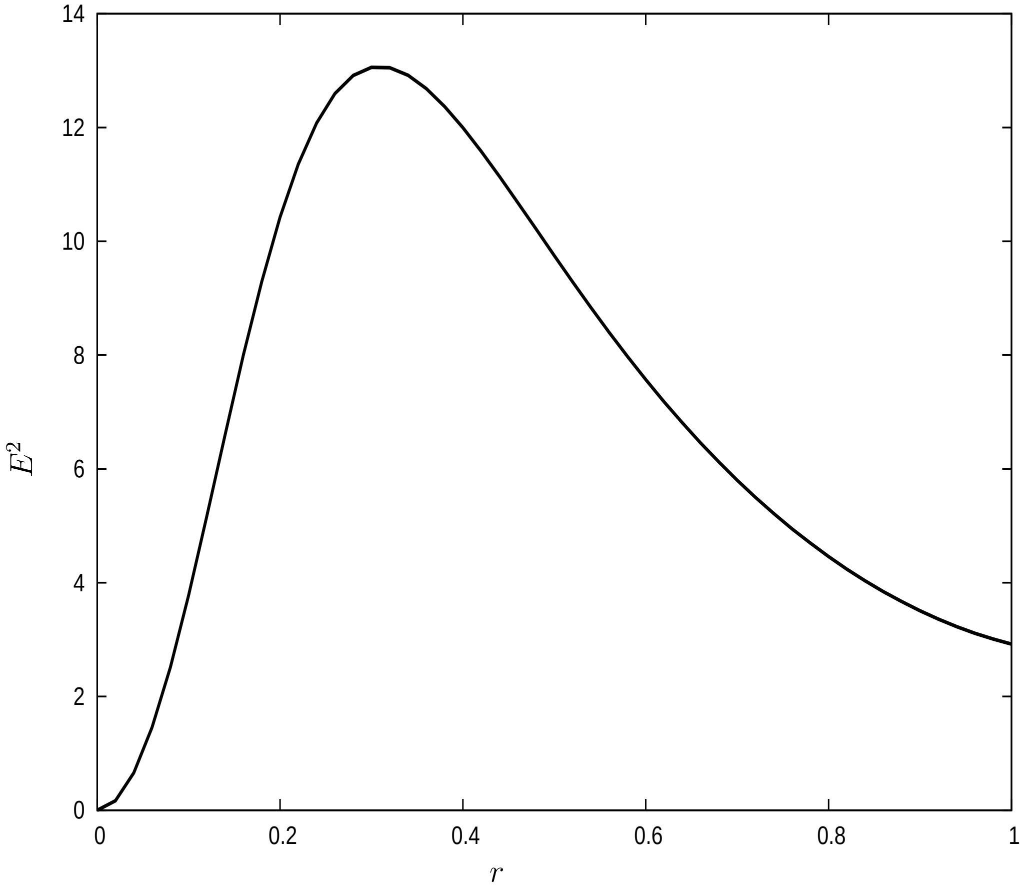

Figure 3.7: The electric field E^2 against radial distance r

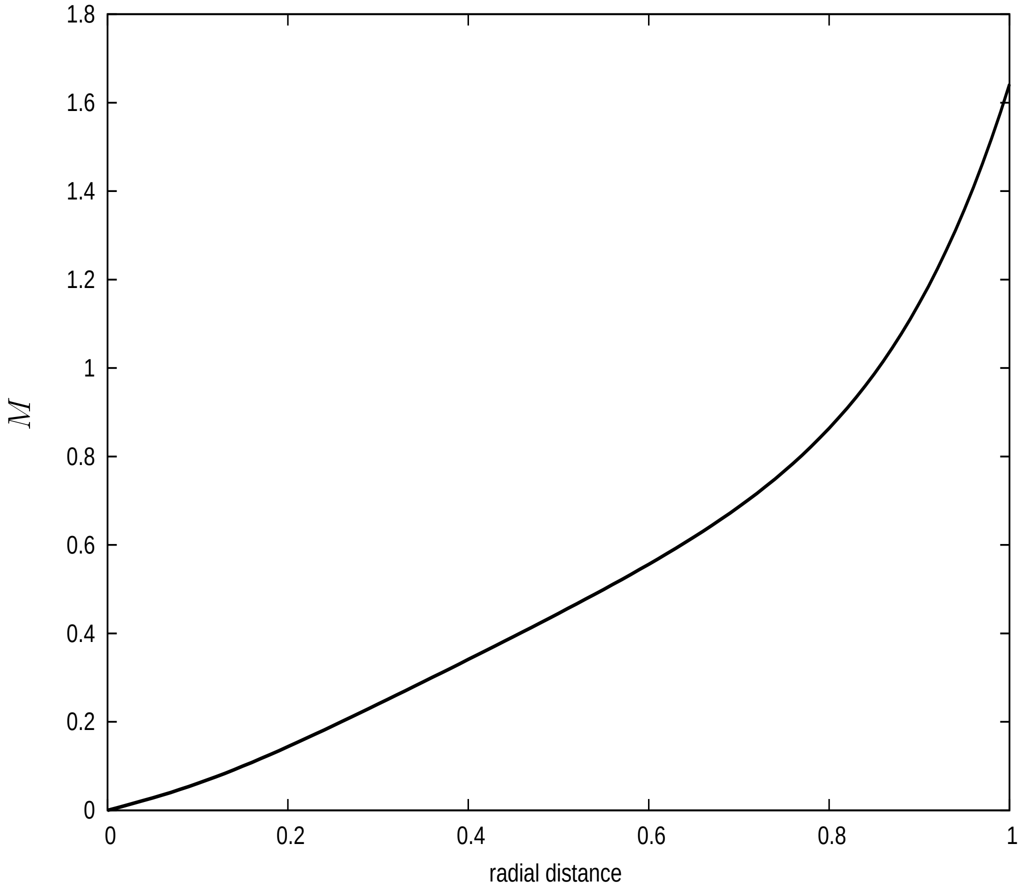

Figure 3.8: The mass M against radial distance r

Chapter 4

Charged anisotropic models I

4.1 Introduction

Since the pioneering paper by Bowers and Liang (1974), who were the first to consider pressure anisotropy in the study of anisotropic spheres in general relativity, there has been much extensive research in this direction. It has been indicated by Dev and Gleiser (2002, 2003) and Gleiser and Dev (2004) that pressure anisotropy has a significant effect on the configurations, structures and properties of stellar bodies. In particular it was established that both the maximum mass and redshift vary with the magnitude of the pressure anisotropy. In the study of these stellar objects it has also been indicated that the electric field is an important ingredient to be included in the models for many applications. It has been shown by Ivanov (2002) that the presence of electromagnetic distribution affects the redshift, luminosity and the maximum mass of compact relativistic stellar objects. It was illustrated by Sharma *et al* (2001) that stellar objects with an electric field allow causal signals over a wide range of parameters compared to uncharged stellar objects. In addition, it has been shown by Esculpi and Aloma (2010) that the presence of pressure anisotropy with an electric field enhances the stability of a configuration under radial adiabatic perturbations compared to matter with isotropic pressures. There are fewer research papers that include both anisotropic pressures and electromagnetic field distributions. These include solutions obtained by Thirukkanesh and Maharaj (2008), the models generated by Rahaman *et al* (2012), the compact models of Mafa Takisa and Maharaj (2013a, 2013b), regular models of Maharaj and Mafa Takisa (2012), models for quark stars obtained by Maharaj *et al* (2014), and Maurya and Gupta (2012). However most of these models have the anisotropy parameter always present, and they do not contain isotropic solutions as a special case. It is important to build physical stellar models in which the anisotropy vanishes for an equilibrium configuration.

There have been some anisotropic and charged exact models with a linear equation of state: Maharaj and Thirukkanesh (2009b) found solutions for the Einstein-Maxwell field equations that generalize isothermal models, Esculpi and Aloma (2010) generated solutions for conformally invariant relativistic compact spheres, Thirukkanesh and Maharaj (2008) found classes of new solutions of field equations for a compact relativistic objects consistent with dark energy stars and quark stars, Mafa Takisa and Maharaj (2013a) generated compact exact models with regular distributions and Sharma and Maharaj (2007) found anisotropic models consistent with relativistic quark matter. However, in general, most of these models do not regain charged isotropic models. Some analytical solutions to the field equations with a linear quark equation of state for charged isotropic stars were found by Komathiraj and Maharaj (2007c). Using the same equation of state with isotropic pressures, Sotani and Harada (2003) generated models with nonradial oscillations of quark stars, Sotani *et al* (2004) found models that restrict quark matter by gravitational wave observation, and Bombaci (2000) generated models which indicate that X-ray pulsars SAX J1808.4-3658 and 4U 1728-34 are likely to be strange star candidates that do exist in the universe. There has been an extension of the linear quark equation to include anisotropic pressures in modeling the behaviour of strange stars by Rahaman *et al* (2012), Kalam *et al* (2013b), and Mak and Harko (2002).

The objective of this chapter is to perform a detailed physical analysis of the particular exact solutions to the Einstein-Maxwell system of equations with a linear quark equation of state for charged anisotropic stars obtained by Maharaj et al (2014).

In performing such a physical analysis we seek to regain models with masses and radii obtained by other researchers, and show that other new masses generated in our model are in acceptable ranges. We also seek to compare masses and radii by considering anisotropic and isotropic pressures. This analysis shows that the relevant class of exact solutions with a quark equation of state has astrophysical significance.

4.2 The model

We model the stellar interior with quark matter in general relativity. The spacetime geometry is spherically symmetric and given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{4.1}$$

where $\nu(r)$ and $\lambda(r)$ are the gravitational potentials. The Reissner-Nordstrom line element

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{4.2}$$

describes the exterior spacetime. The quantities M and Q define the total mass and charge of the star, respectively. The energy momentum tensor is defined by

$$T_{ab} = \mathrm{diag}\left(-\rho - \frac{1}{2}E^2, p_r - \frac{1}{2}E^2, p_t + \frac{1}{2}E^2, p_t + \frac{1}{2}E^2\right), \tag{4.3}$$

in the presence of charge and anisotropy. The energy density (ρ), the radial pressure (p_r), the tangential pressure (p_t), and the electric field intensity (E) are measured relative to a comoving fluid four-velocity $u^a(u^a u_a = -1)$.

The Einstein-Maxwell field equations are given by

$$\begin{aligned}
\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} &= \rho + \frac{1}{2}E^2, && (4.4a)\\
-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} &= p_r - \frac{1}{2}E^2, && (4.4b)\\
e^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu'}{r} - \frac{\lambda'}{r}\right) &= p_t + \frac{1}{2}E^2, && (4.4c)\\
\sigma &= \frac{1}{r^2}e^{-\lambda}\left(r^2 E\right)', && (4.4d)
\end{aligned}$$

where primes denote differentiation with respect to the radial coordinate r. The function σ represents the proper charge density. The equation of state is linear and of the form

$$p_r = \frac{1}{3}(\rho - 4B), \tag{4.5}$$

where B is a constant related to the surface density of the stellar body representing a sharp surface. If we consider the MIT bag model for quark stars, then B can also be identified with the bag constant.

We introduce a new independent variable x and define the metric functions $Z(x)$ and $y(x)$ as

$$x = Cr^2, \quad Z(x) = e^{-2\lambda(r)}, \quad A^2y^2(x) = e^{2\nu(r)}, \tag{4.6}$$

where A and C are arbitrary constants (see Durgapal and Bannerji (1983)). With this transformation the line element in (4.1) becomes

$$ds^2 = -A^2y^2dt^2 + \frac{1}{4xCZ}dx^2 + \frac{x}{C}(d\theta^2 + \sin^2\theta d\phi^2). \tag{4.7}$$

The Einstein-Maxwell field equations (4.4) become

$$\begin{aligned}
\rho &= 3p_r + 4B, && (4.8a)\\
\frac{p_r}{C} &= Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C}, && (4.8b)\\
p_t &= p_r + \Delta, && (4.8c)\\
\Delta &= \frac{4xCZ\ddot{y}}{y} + C\left(2x\dot{Z} + 6Z\right)\frac{\dot{y}}{y} \\
&\quad + C\left(2\left(\dot{Z} + \frac{B}{C}\right) + \frac{Z-1}{x}\right), && (4.8d)\\
\frac{E^2}{2C} &= \frac{1-Z}{x} - 3Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C}, && (4.8e)\\
\sigma &= 2\sqrt{\frac{ZC}{x}}\left(x\dot{E} + E\right), && (4.8f)
\end{aligned}$$

where dots represent derivatives with respect to the variable x. The quantity $\Delta = p_t - p_r$ is called the measure of anisotropy. We introduce the mass function given by

$$M(x) = \frac{1}{4C^{\frac{3}{2}}}\int_0^x \sqrt{\omega}\left(\rho_* + E^2\right)d\omega, \tag{4.9}$$

where

$$\rho_* = \left(\frac{1-Z}{x} - 2\dot{Z}\right) C, \tag{4.10}$$

is the energy density when the electric field $E = 0$.

Some solutions to the system (4.8), applicable to quark matter, were presented in Maharaj *et al* (2014). In that model it was assumed that

$$\begin{aligned} y &= (a + x^m)^n \,, \\ \Delta &= A_0 + A_1 x + A_2 x^2 + A_3 x^3. \end{aligned}$$

For particular choices of the parameters m and n it is possible to integrate the Einstein-Maxwell system exactly. The choice of anisotropy ensures isotropic pressures can be regained. To ensure that the anisotropy vanishes at the stellar centre we should set $A_0 = 0$. Here we consider a particular solution of Maharaj *et al* (2014) that enables us to perform a detailed physical analysis. The particular solution that we utilize can be written in terms of analytical functions and is given by:

$$
\begin{aligned}
e^{2\nu} &= A^2\left(a+x\right)^4, \\
e^{2\lambda} &= 315(a+x)^2(a+3x)\Big[9\left(35a^3+35a^2x+21ax^2+5x^3\right) \\
&\quad -\frac{2Bx}{C}\left(105a^3+189a^2x+135ax^2+35x^3\right)+\frac{315L(x)}{C}\Big]^{-1}, \\
\rho &= \frac{3C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35(a+x)^3(a+3x)^2} \\
&\quad +\frac{3\Psi(x)+B\left(210a^5+798a^4x+1476a^3x^2+2540a^2x^3+2090ax^4+630x^5\right)}{105(a+x)^3(a+3x)^2}, \\
p_r &= \frac{C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35(a+x)^3(a+3x)^2} \\
&\quad +\frac{\Psi(x)-B\left(70a^5+994a^4x+3708a^3x^2+\frac{16780}{3}a^2x^3+\frac{11770}{3}ax^4+1050x^5\right)}{105(a+x)^3(a+3x)^2}, \\
p_t &= \frac{C\left(140a^4+434a^3x+318a^2x^2+150ax^3+30x^4\right)}{35(a+x)^3(a+3x)^2} \\
&\quad +\frac{\Omega(x)-B\left(70a^5+994a^4x+3708a^3x^2+\frac{16780}{3}a^2x^3+\frac{11770}{3}ax^4+1050x^5\right)}{105(a+x)^3(a+3x)^2}, \\
\Delta &= A_1x+A_2x^2+A_3x^3, \\
E^2 &= \frac{C\left(1764a^3x+13068a^2x^2+12204ax^3+3780x^4\right)-\Lambda(x)}{315(a+x)^3(a+3x)^2} \\
&\quad -\frac{B\left(168a^4x+1296a^3x^2+6528a^2x^3+7280ax^4+2520x^5\right)}{315(a+x)^3(a+3x)^2},
\end{aligned}
$$

where

$$\begin{aligned}
L(x) &= A_1\left(\frac{1}{5}a^3x^2+\frac{3}{7}a^2x^3+\frac{1}{3}ax^4+\frac{1}{11}x^5\right)\\
&\quad +A_2\left(\frac{1}{7}a^3x^3+\frac{1}{3}a^2x^4+\frac{3}{11}ax^5+\frac{1}{13}x^6\right)\\
&\quad +A_3\left(\frac{1}{9}a^3x^4+\frac{3}{11}a^2x^5+\frac{3}{13}ax^6+\frac{1}{15}x^7\right),\\
\Psi(x) &= A_1x\left(-21a^5-57a^4x+20a^3x^2+\frac{1360}{11}a^2x^3+105ax^4+\frac{315}{11}x^5\right)\\
&\quad +A_2x^2\left(-\frac{45}{2}a^5-\frac{185}{2}a^4x-\frac{1145}{11}a^3x^2-\frac{315}{13}a^2x^3+\frac{7245}{286}ax^4+\frac{315}{26}x^5\right)\\
&\quad -A_3x^3\left(\frac{70}{3}a^5+\frac{3710}{33}a^4x+\frac{2310}{13}a^3x^2+\frac{17206}{143}a^2x^3+\frac{392}{13}ax^4\right),\\
\Omega(x) &= A_1x\left(84a^5+888a^4x+3170a^3x^2+\frac{54490}{11}a^2x^3+3570ax^4+\frac{10710}{11}x^5\right)\\
&\quad +A_2x^2\left(\frac{165}{2}a^5+\frac{1705}{2}a^4x+\frac{33505}{11}a^3x^2+\frac{62474}{13}a^2x^3\right.\\
&\quad \left.+\frac{998235}{286}ax^4+\frac{24885}{26}x^5\right)+A_3x^3\left(\frac{245}{3}a^5+\frac{27475}{33}a^4x\right.\\
&\quad \left.+\frac{38640}{13}a^3x^2+\frac{673484}{143}a^2x^3+\frac{44653}{13}ax^4+945x^5\right),\\
\Lambda(x) &= A_1x\left(252a^5+2124a^4x+6732a^3x^2+\frac{100380}{11}a^2x^3\right.\\
&\quad \left.+\frac{63000}{11}ax^4+\frac{15120}{11}x^5\right)+A_2x^2\left(225a^5+1845a^4x+\frac{63210}{11}a^3x^2\right.\\
&\quad \left.+\frac{1133370}{143}a^2x^3+\frac{55755}{11}ax^4+\frac{16065}{13}x^5\right)+A_3x^3\left(210a^5+\frac{18550}{11}a^4x\right.\\
&\quad \left.+\frac{738360}{143}a^3x^2+\frac{78624}{11}a^2x^3+\frac{59934}{13}ax^4+1134x^5\right).
\end{aligned}$$

With this exact solution the line element (4.1) becomes

$$\begin{aligned}
ds^2 &= -A^2\left(a+x\right)^4dt^2\\
&\quad +315(a+x)^2(a+3x)\left[9\left(35a^3+35a^2x+21ax^2+5x^3\right)\right.\\
&\quad \left.-\frac{2Bx}{C}\left(105a^3+189a^2x+135ax^2+35x^3\right)+\frac{315L(x)}{C}\right]^{-1}dr^2\\
&\quad +r^2(d\theta^2+\sin^2\theta d\phi^2).
\end{aligned}$$

The mass function (4.9) becomes

$$\begin{aligned}
M(x) \;=\; & \left(\left(\frac{1268}{96525}a - \frac{1}{30}x^2 - \frac{14}{585}ax\right)A_3\right.\\
& \left. - \left(\frac{4}{91}x + \frac{74}{2145}a\right)A_2 - \frac{7}{110}A_1\right)\frac{x^{\frac{5}{2}}}{C^{\frac{3}{2}}}\\
& - \left(\frac{1}{9}B + \frac{1}{9}A_0 + \frac{2}{33}aA_1 - \frac{10}{429}a^2A_2 + \frac{14}{1287}a^3A_3\right)\left(\frac{x}{C}\right)^{\frac{3}{2}}\\
& - \sqrt{\frac{a}{C^3}}\left(\frac{62}{105}aB + \frac{93}{35}C - \frac{31}{105}aA_0 + \frac{31}{385}a^2A_1\right.\\
& \left. - \frac{31}{1001}a^3A_2 + \frac{31}{2145}a^4A_3\right)\arctan\sqrt{\frac{x}{a}}\\
& + \frac{\sqrt{3a}}{3C^{\frac{3}{2}}}\left(\frac{188}{315}aB + \frac{129}{35}C - \frac{94}{315}aA_0 + \frac{59}{1155}a^2A_1\right.\\
& \left. - \frac{100}{9009}a^3A_2 + \frac{157}{57915}a^4A_3\right)\arctan\sqrt{\frac{3x}{a}}\\
& + \left(\frac{76}{189}aB + \frac{8}{9}C - \frac{38}{189}aA_0 + \frac{52}{693}a^2A_1\right.\\
& \left. - \frac{934}{27027}a^3A_2 + \frac{3088}{173745}a^4A_3\right)\sqrt{\frac{x}{C^3}}\\
& - \left(\frac{6}{35}a^2B + \frac{27}{35}aC - \frac{3}{35}a^2A_0 + \frac{9}{385}a^3A_1\right.\\
& \left. - \frac{9}{1001}a^4A_2 + \frac{3}{715}a^5A_3\right)\frac{\sqrt{x}}{(a+x)C^{\frac{3}{2}}}\\
& - \left(\frac{4}{105}a^3B + \frac{6}{35}a^2C - \frac{2}{105}a^3A_0 + \frac{2}{385}a^4A_1\right.\\
& \left. - \frac{2}{1001}a^5A_2 + \frac{2}{2145}a^6A_3\right)\frac{\sqrt{x}}{(a+x)^2C^{\frac{3}{2}}}\\
& - \left(\frac{188}{945}a^2B + \frac{43}{35}aC - \frac{94}{945}a^2A_0 + \frac{59}{3465}a^3A_1\right.\\
& \left. - \frac{100}{27027}a^4A_2 + \frac{157}{173745}a^5A_3\right)\frac{\sqrt{x}}{(a+3x)C^{\frac{3}{2}}}. \qquad (4.12)
\end{aligned}$$

Note that this generalized class of models with a quark equation of state contains the nonsingular solutions of Komathiraj and Maharaj (2007c) with isotropic pressures.

4.3 Stellar masses

Our exact solutions are more general than earlier treatments and have the flexibility of allowing for fine-tuning of the parameters. The right choice of parameters in the multi-dimensional parameter space enables us to regain the stellar masses of compact bodies previously identified by many other research groups. To start with, we make the following transformations:
$\tilde{A}_1 = A_1R^2$, $\tilde{A}_2 = A_2R^2$, $\tilde{A}_3 = A_3R^2$, $\tilde{A}_3 = A_3R^2$, $\tilde{B} = BR^2$, $\tilde{C} = CR^2$ $\tilde{a} = aR^2$, where R takes the same unit as x, and in order to match with the realistic units, it is renormalised by a factor of 43.245, i.e., $R = 43.245x$. In the literature, we find many observed and analysed compact star masses, varying from $0.9M_\odot$ to $2.01M_\odot$. The studies of charged stars however allow for more mass in the stable configuration. In our present study, we aim to regain masses of some of the observed compact stellar bodies for the uncharged cases identified to be strange stars, thereby narrowing our parameter ranges. For the charged cases, we follow the same exercise to regain the values of the theoretically obtained masses for charged stars.

In particular, for the electrically charged strange quark stars, we have regained the mass $M = 2.86M_\odot$ with radius $r = 9.46$km consistent with mass and radius obtained by Mak and Harko (2004), the mass $M = 2.02M_\odot$ with radius $r = 10.99$km consistent with the object found by Negreiros *et al* (2009), and the mass $M = 0.94M_\odot$ with radius $r = 7.07$km consistent with the particular results obtained by Thirukkanesh and Maharaj (2008) and Mafa Takisa and Maharaj (2013a). Charged compact stars have been identified as quark stars: the mass $M = 1.67M_\odot$ with radius of 9.4km consistent with the star PSR J1903+327 is discussed by Freire *et al* (2011) and Gangopathyay *et al* (2013), and the mass $M = 1.433M_\odot$ with radius of 7.07km was found by Dey *et al* (1998) in their strange star models. Parameter values which give these masses and radii in our model are given in Table 4.1.

Table 4.1: Various parameter values for particular stellar objects

$\tilde{a}$	$\tilde{B}$	$\tilde{C}$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$	r(km)	$\frac{M}{M_\odot}$	Model
52	12	1	20	25	20	9.46	2.86	Mak and Harko (2004)
350	12	1	250	280	290	10.99	2.02	Negreiros *et al* (2009)
350	12	1	230	235	240	9.40	1.67	Gangopathyay *et al* (2013)
202	12	1	25	20	20	7.07	1.433	Dey *et al* (1998)
350	12	1	289	200	260	7.07	0.94	Thirukkanesh and Maharaj (2008)

Table 4.2: Masses and radii for isotropic and anisotropic stars for different choice of parameters

Name	$\tilde{a}$	$\tilde{B}$	$\tilde{C}$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$	$r_{(\Delta\neq 0)}$	$r_{(\Delta=0)}$	$\left(\frac{M}{M_\odot}\right)_{\Delta\neq 0}$	$\left(\frac{M}{M_\odot}\right)_{\Delta=0}$
R1	285	12	1	25	20	25	6.84	6.85	1.28994	1.31530
R2	100	12	1	20	5	10	6.67	6.68	1.56259	1.56730
R3	260	10	1	35	25	30	7.59	7.61	1.58585	1.61878
R4	260	10	1	20	30	20	7.60	7.61	1.60033	1.61878
R5	200	10	1	40	30	40	7.57	7.59	1.66749	1.69064
R6	35	12	1	25	10	15	5.78	5.77	1.73268	1.72885

Table 4.3: Variation of parameter $\tilde{A}_1$ for $\tilde{a} = 260$, $\tilde{B} = 10$, $\tilde{C} = 1$, $\tilde{A}_2 = 15$, $\tilde{A}_3 = 20$

$\tilde{A}_1$	$r_{(\Delta\neq 0)}$	$r_{(\Delta=0)}$	$\left(\frac{M}{M_\odot}\right)_{\Delta\neq 0}$	$\left(\frac{M}{M_\odot}\right)_{\Delta=0}$
5	7.6100		1.61456	
10	7.6100		1.61087	
15	7.6100		1.60718	
20	7.6100	7.6100	1.60349	1.61878
25	7.6100		1.59981	
30	7.6100		1.59612	
35	7.6100		1.59243	
40	7.6100		1.58874	

Table 4.4: Variation of parameter $\tilde{A}_2$ for $\tilde{a} = 260$, $\tilde{B} = 10$, $\tilde{C} = 1$, $\tilde{A}_1 = 20$, $\tilde{A}_3 = 20$

$\tilde{A}_2$	$r_{(\Delta\neq 0)}$	$r_{(\Delta=0)}$	$\left(\frac{M}{M_\odot}\right)_{\Delta\neq 0}$	$\left(\frac{M}{M_\odot}\right)_{\Delta=0}$
5	7.6000		1.60033	
10	7.6000		1.60033	
15	7.6000		1.60033	
20	7.6000		1.60033	
25	7.6000	7.6100	1.60033	1.61878
30	7.6000		1.60033	
35	7.6000		1.60033	
40	7.6000		1.60033	
100	7.6000		1.60033	

Table 4.5: Variation of parameter $\tilde{A}_3$ for $\tilde{a} = 260$, $\tilde{B} = 10$, $\tilde{C} = 1$, $\tilde{A}_1 = 20$, $\tilde{A}_2 = 15$

$\tilde{A}_3$	$r_{(\Delta\neq0)}$	$r_{(\Delta=0)}$	$\left(\frac{M}{M_\odot}\right)_{\Delta\neq0}$	$\left(\frac{M}{M_\odot}\right)_{\Delta=0}$
5	7.6000		1.60073	
10	7.6000		1.60060	
15	7.6000		1.60046	
20	7.6000		1.60033	
25	7.6000	7.6100	1.60020	1.61878
30	7.6000		1.60006	
35	7.6000		1.59993	
40	7.6000		1.59980	
100	7.6000		1.59820	

4.4 Physical analysis

Although we could regain the values of masses and radii of many previously obtained stellar models, a systematic study of the variation of the anisotropic parameters in our model is also necessary. To this end, we study the effect of the anisotropic parameters $\tilde{A}_1$, $\tilde{A}_2$ and $\tilde{A}_3$ on masses and radii of stellar bodies, by varying one parameter at a time and keeping the others fixed. Also, to make the effects more pronounced, we have chosen a few sets of parameters so as to give a value of the mass-radius relation in the *acceptable* range. The surface of the anisotropic star is considered to be the point of vanishing radial pressure. Also, in our most general solution we have set $\tilde{A}_1 = \tilde{A}_2 = \tilde{A}_3 = 0$ so as to obtain the isotropic model.

Table 4.2 shows different masses and radii for isotropic and anisotropic stars for different choices of parameters. This study shows that the masses and radii of the anisotropic stars are less than the corresponding quantities for isotropic stars for most of the values of the parameters chosen. However, the model indicates that there are also some values of the parameters which give the mass and radius of the anisotropic star greater than the corresponding value for the isotropic star. This is shown in the last row (R6) which indicates small values of radii but greater masses for both anisotropic and isotropic stars. The masses and radii indicated in Table 4.2 are in the acceptable range for the quark stars as studied by Gangopadhyay *et al* (2013).

The effect of the parameter $\tilde{A}_1$ on the mass and radius of the anisotropic star is shown in Table 4.3. As $\tilde{A}_1$ increases the mass of the anisotropic star decreases while the radius remains constant. The corresponding mass and radius for the isotropic case are $1.61878 M_\odot$ and 7.61km respectively.

In Table 4.4 it is observed that variation of the parameter $\tilde{A}_2$ in the range indicated in this table does not visibly alter the mass and radius for both anisotropic and isotropic stars. The mass of the anisotropic star is constant around $1.60033 M_\odot$ with radius 7.60km while the isotropic star has the mass $1.61878 M_\odot$ with radius 7.61km. The difference in mass between the isotropic and anisotropic cases here is due to the presence of the other anisotropic parameters $\tilde{A}_1$ and $\tilde{A}_3$.

In Table 4.5, we see that the variation of the parameter $\tilde{A}_3$ does not affect the radius of the star. Rather there is a decrease in the mass of the star with an increase of $\tilde{A}_3$. Here also, the masses and radii remain in the acceptable range of values for quark stars.

In Fig. 4.1 and Fig. 4.2 we learn that the mass of the star decreases linearly with an increase of the anisotropic parameters $\tilde{A}_1$ and $\tilde{A}_3$. In order to compare the variation of the mass throughout the interior of anisotropic and isotropic stars, we have plotted in Fig. 4.3-4.8 the mass against radial distance using the parameter values in Table 4.2. In general, we see that the masses for the isotropic cases are larger than their anisotropic counterparts, except for the parameter sets R2 and R6, where the two graphs appear to overlap. In Fig. 4.9 and Fig. 4.10 we have plotted graphs for the mass-radius relationship for anisotropic and isotropic stars separately as indicated in Table 4.2. In general, it is shown that different values of parameters give different values of masses and radii for both anisotropic and isotropic stars. However some graphs appear to overlap which implies that different set of parameters values could give almost the same values of mass and the radius.

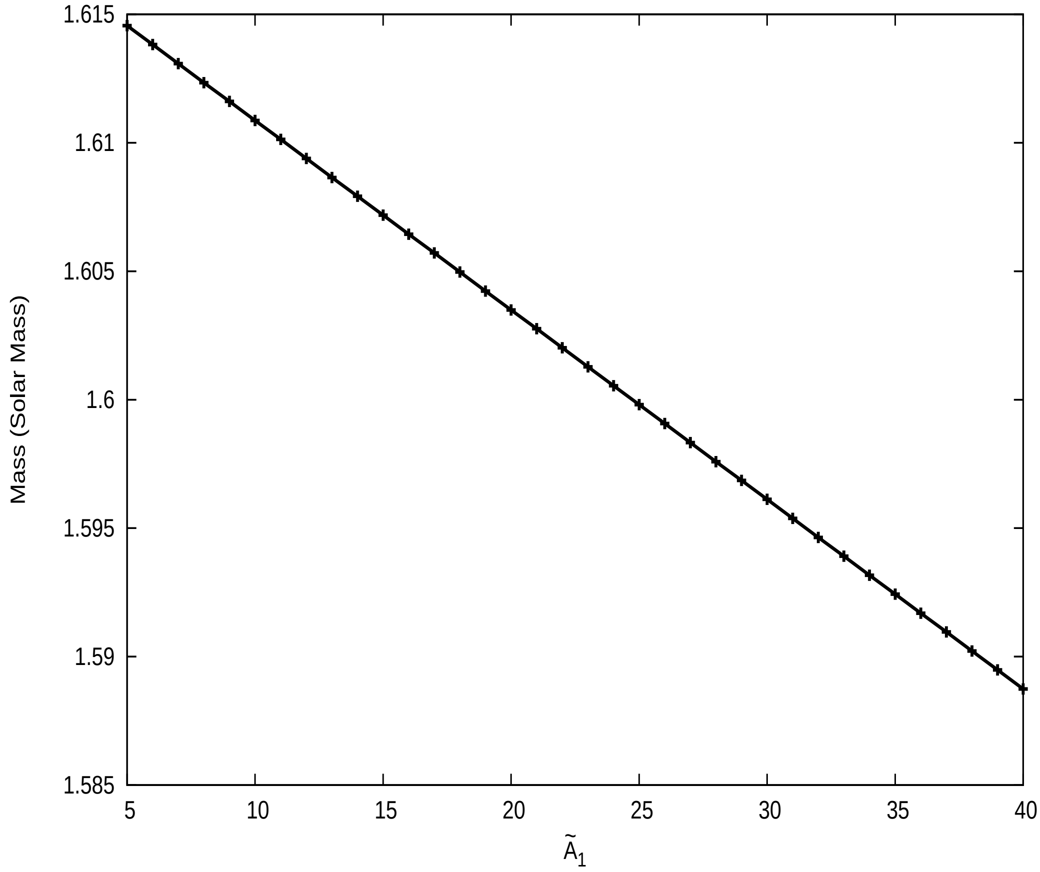

Figure 4.1: Variation of the mass with the parameter $\tilde{A}_1$, keeping other parameters constant

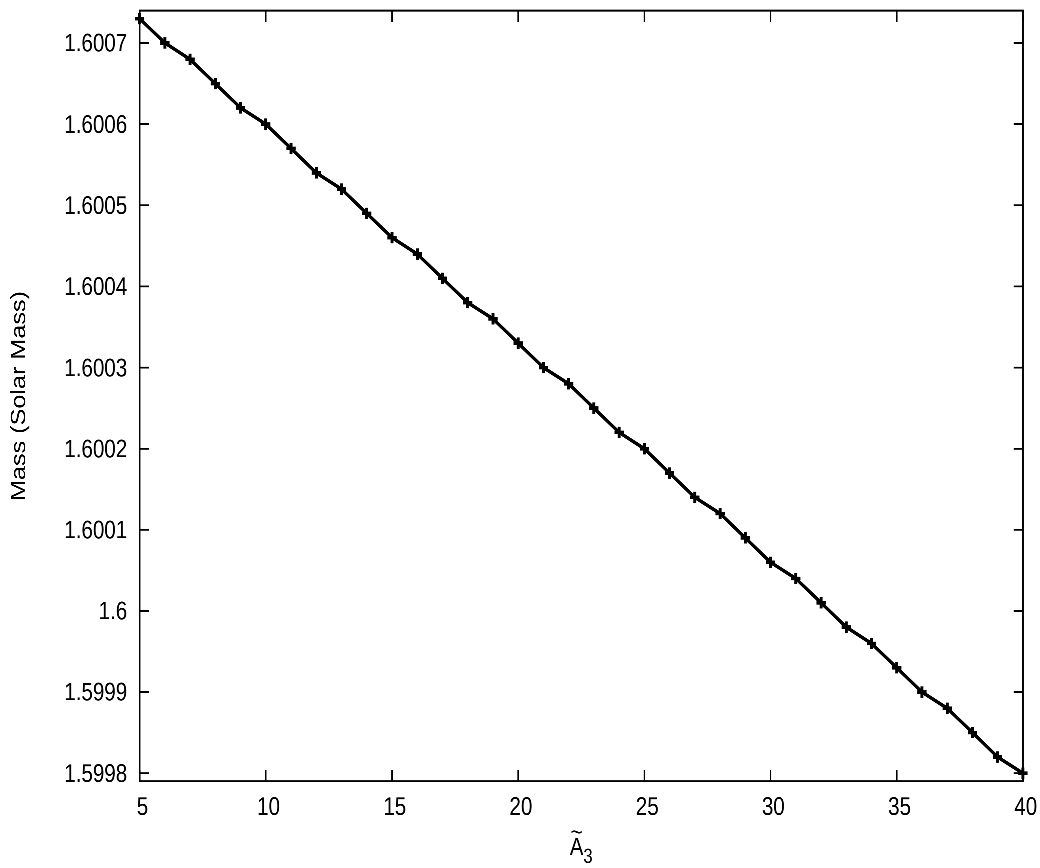

Figure 4.2: Variation of the mass with the parameter $\tilde{A}_3$, keeping other parameters constant

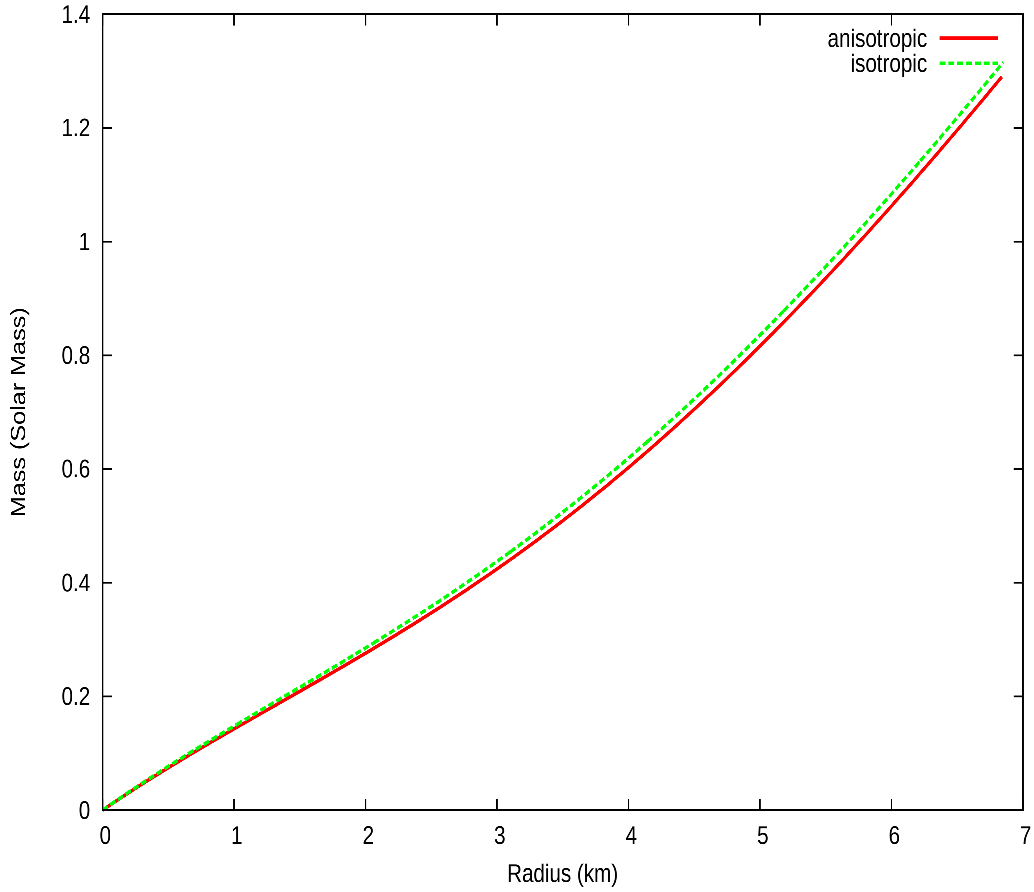

Figure 4.3: The mass-radius relation using parametric values indicated by $R1$. Here we see that there is an increase in the values for the isotropic case as compared to the anisotropic ones

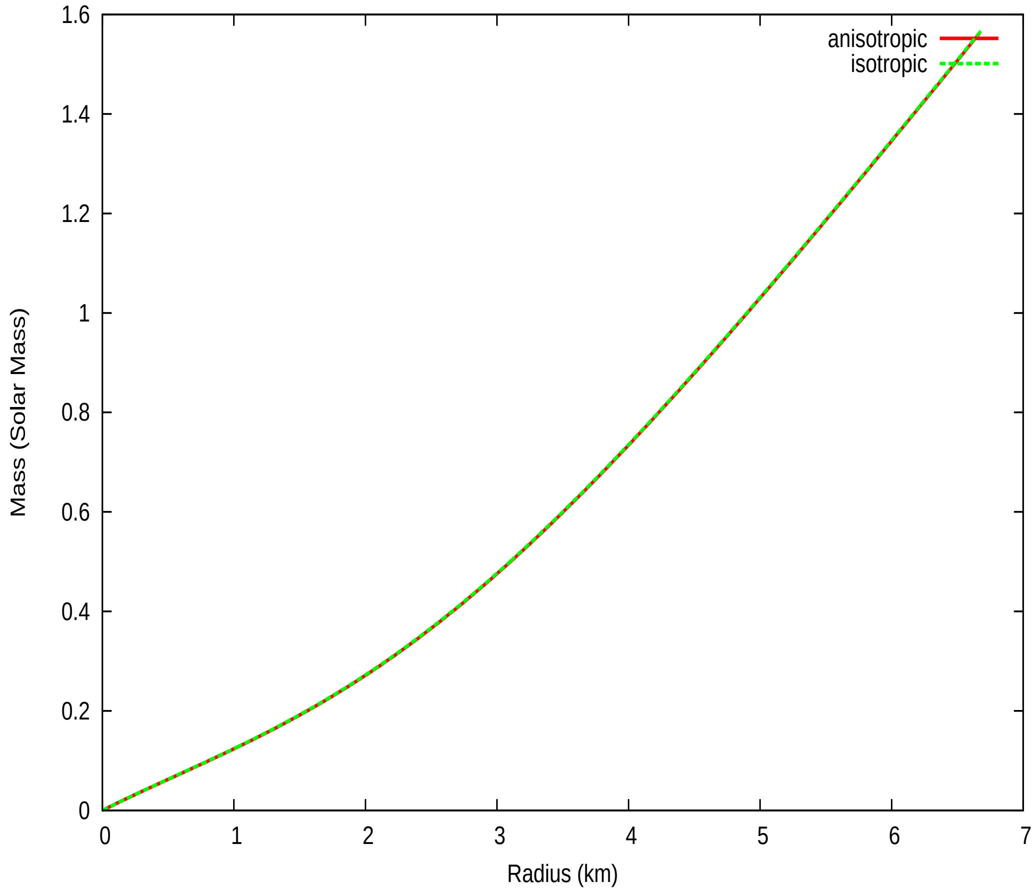

Figure 4.4: The mass-radius relation using parametric values indicated by $R2$. Clearly, for these choices of parameter sets, there is not much difference between the anisotropic and the isotropic cases

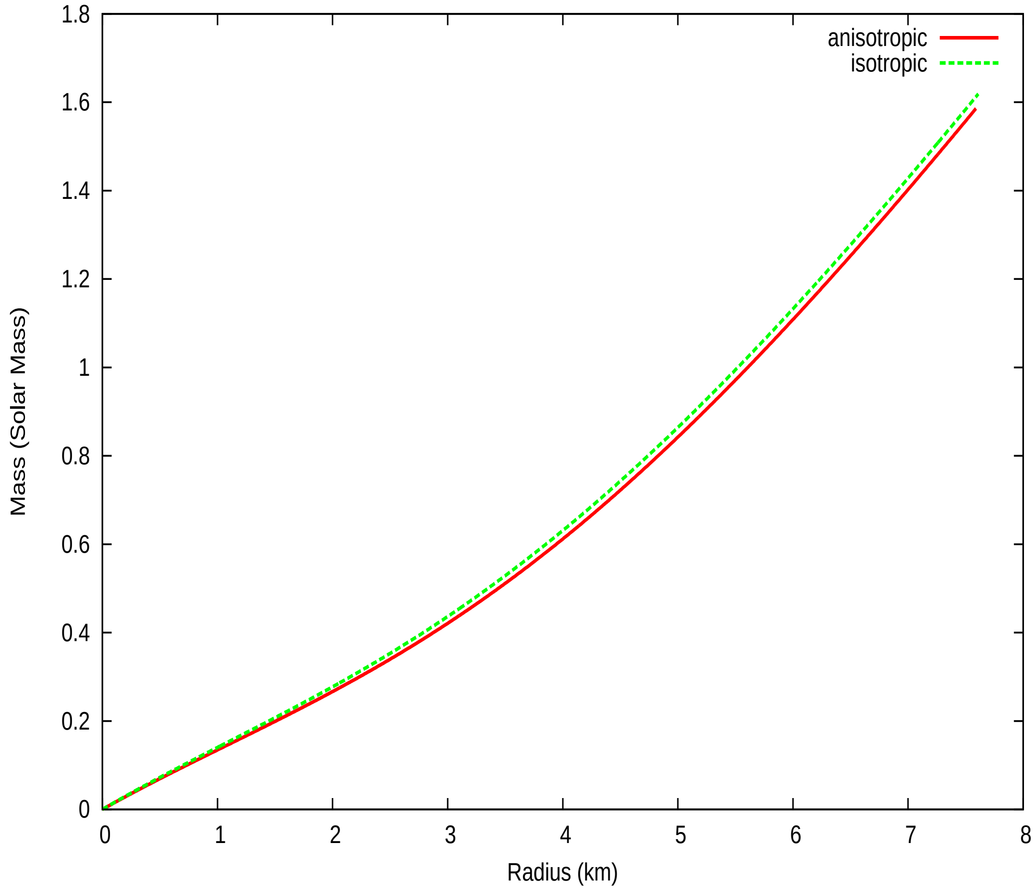

Figure 4.5: The mass-radius relation using parametric values labelled by $R3$

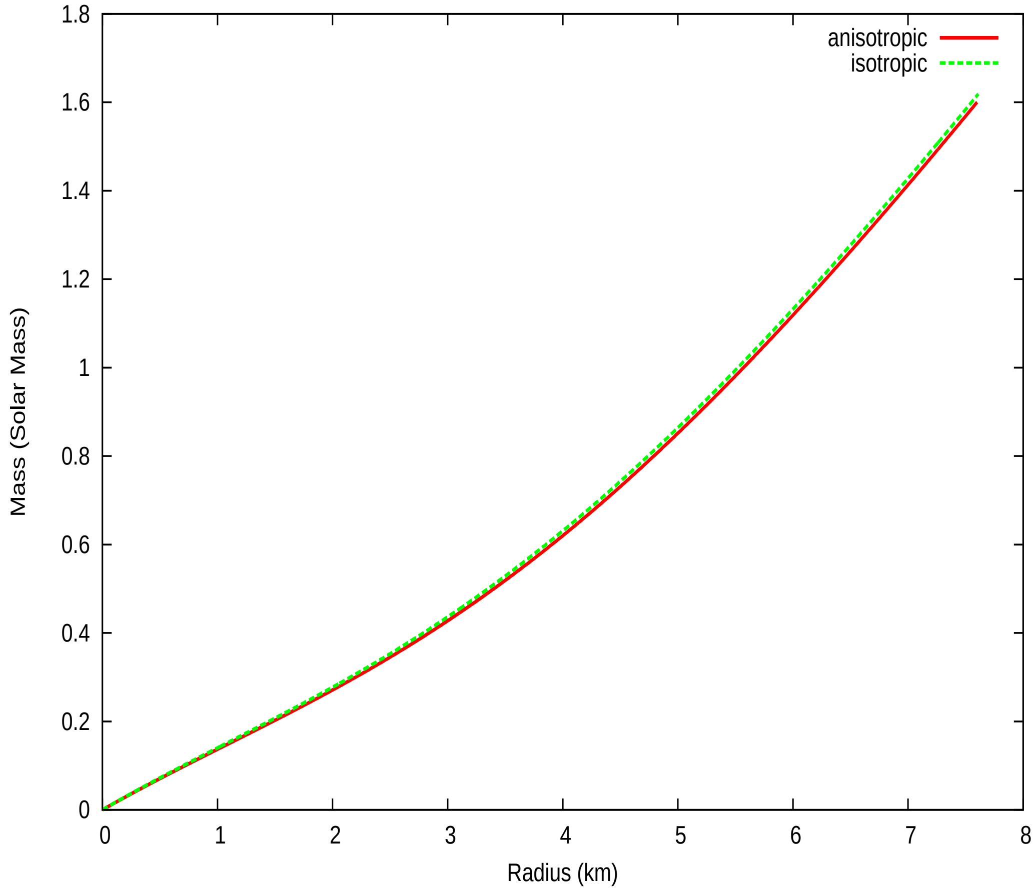

Figure 4.6: The mass-radius relation using parametric values indicated by $R4$

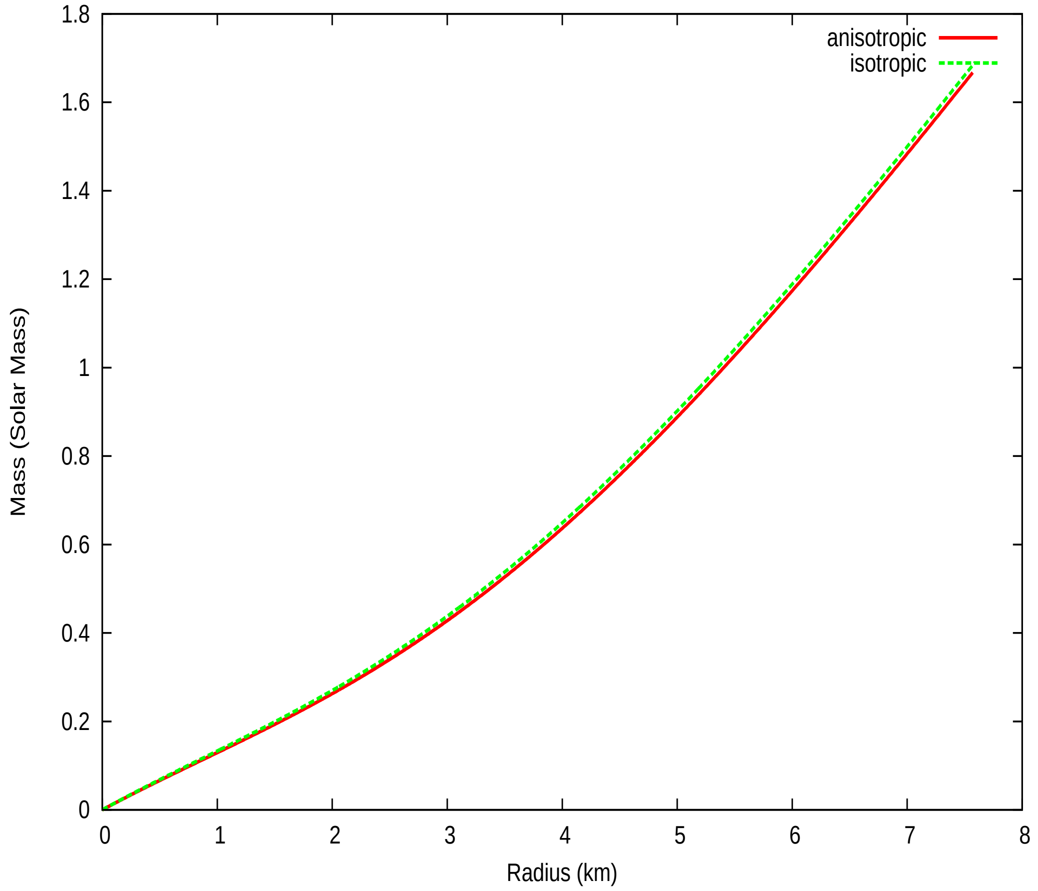

Figure 4.7: The mass-radius relation using parametric values given by $R5$

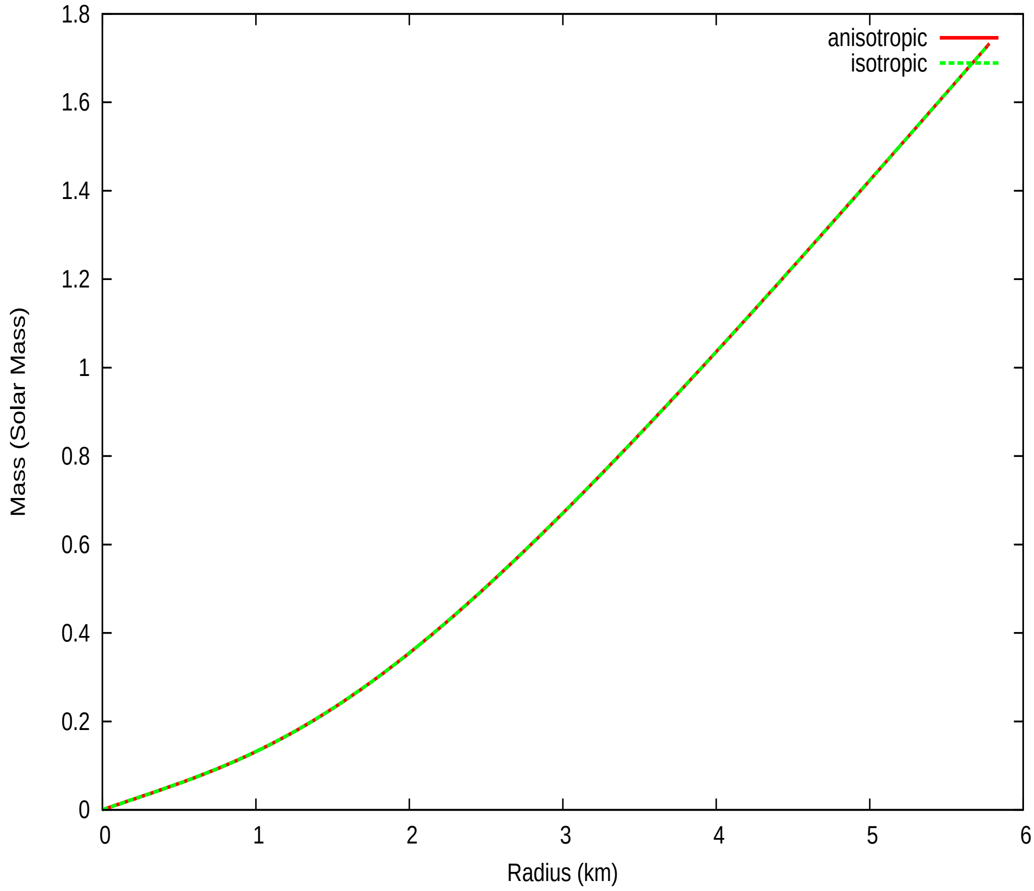

Figure 4.8: The mass-radius relation using parametric values labelled by $R6$

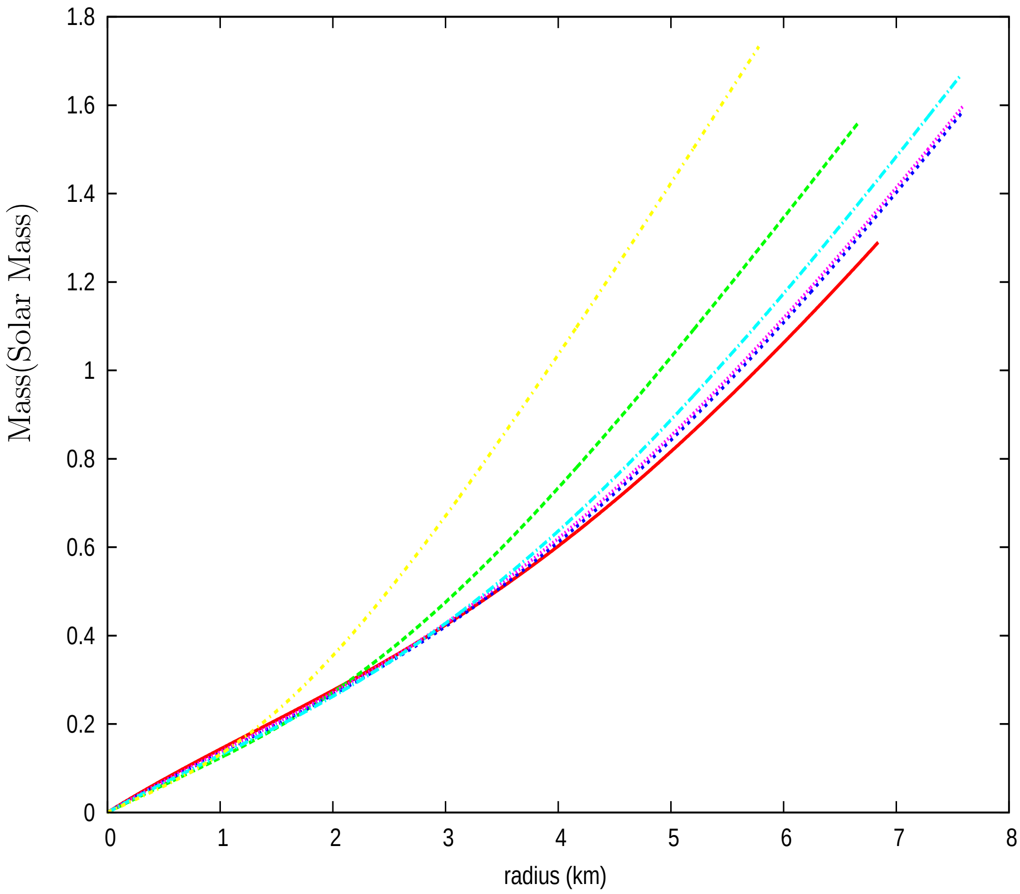

Figure 4.9: Masses and radii of anisotropic stars at different set of parameters as indicated in the Table 4.2.

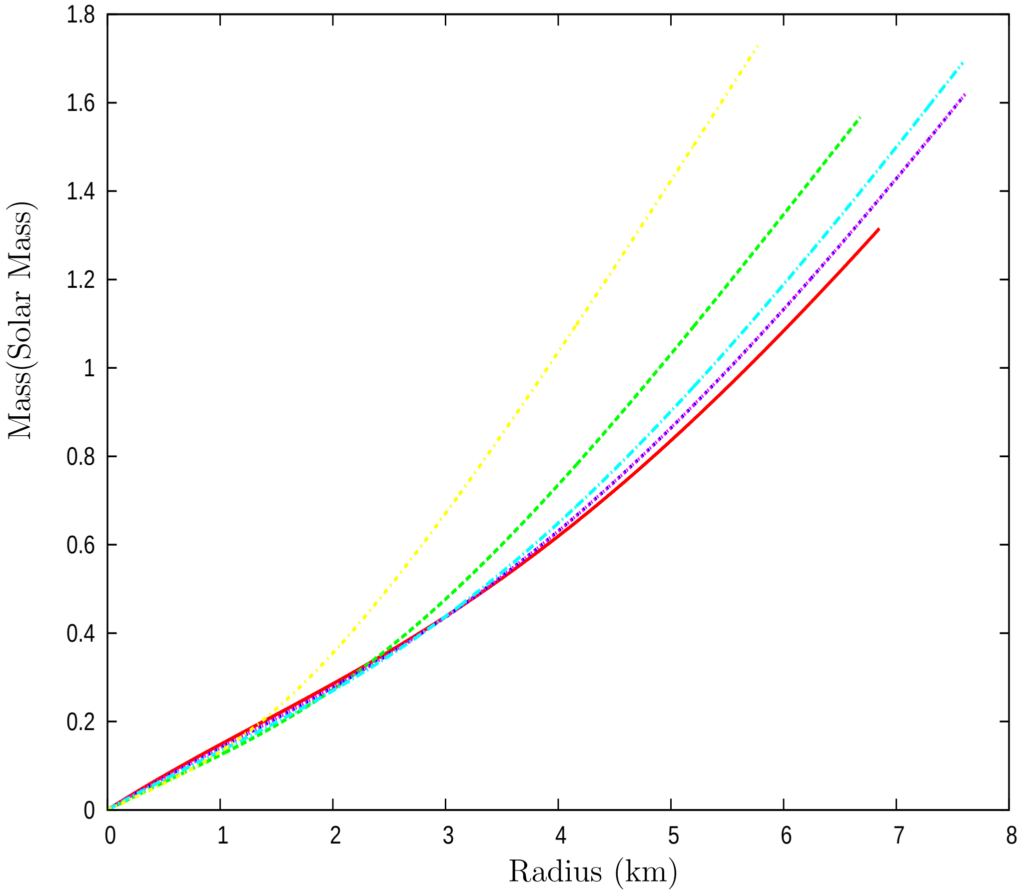

Figure 4.10: Masses and radii of isotropic stars at different set of parameters as indicated in the Table 4.2.

Chapter 5

Charged anisotropic models II

5.1 Introduction

The nonlinear Einstein-Maxwell field equations are necessary for the description of the behaviour of relativistic gravitating matter with or without electromagnetic field distributions, and they are tools for modeling relativistic compact objects such as dark energy stars, gravastars, quark stars, black holes and neutron stars. With the help of diverse solutions of the field equations and different matter configurations, the structure and properties of relativistic stellar bodies have been investigated. This is reflected in several investigations over the recent past. Models of neutral compact spheres with isotropic pressures have been studied by Murad and Pant (2014), Mak and Harko (2005), and Sharma *et al* (2006). The case of neutral anisotropic matter was investigated by Paul *et al* (2011), Harko and Mak (2002) and Kalam *et al* (2012, 2013a, 2013b). Charged isotropic compact models are highlighted by Gupta and Maurya (2011a, 2011b), Negreiros *et al* (2009), Murad and Fatema (2013), and Bijalwan (2011). The general model with charge and anisotropy was analysed by Esculpi and Aloma (2010), Mafa Takisa and Maharaj (2013a) and Rahaman *et al* (2012). Several interesting features of exact solutions to the Einstein-Maxwell system for charged anisotropic quark stars were highlighted in the treatments of Maharaj *et al* (2014) and Sunzu *et al* (2014).

The effect of the electromagnetic distribution and pressure anisotropy are important ingredients to be considered when undertaking studies of relativistic stellar objects. Ivanov (2002) highlighted the fact that the presence of charge in a compact stellar matter contributes to changes in the mass, redshift and luminosity. It was shown by Sharma *et al* (2001) that charged models could allow causal signals in the stellar interior over a wide range of parameters. On the other hand, Dev and Gleiser (2002) demonstrated that pressure anisotropy affects the physical properties, stability and structure of stellar matter. The stability of stellar bodies is improved for positive measure of anisotropy when compared to configurations of isotropic stellar objects. Furthermore the maximum mass and the redshift depend on the magnitude of the pressure anisotropy as illustrated by Dev and Gleiser (2003) and Gleiser and Dev (2004). They also showed that the presence of anisotropic pressures in charged matter enhances the stability of the configuration under radial adiabatic perturbations when compared to isotropic matter. There have been many recent investigations which include the presence of charge and anisotropy in the stellar interior. For example, Maharaj and Mafa Takisa (2012) presented regular models for charged anisotropic stellar bodies, generalized isothermal models were found by Maharaj and Thirukkanesh (2009b), and superdense models were investigated by Maurya and Gupta (2012). Other new exact solutions for charged anisotropic stars are contained in the treatment of Mafa Takisa and Maharaj (2013b). Some other models describing anisotropic static spheres with variable energy density include the works of Cosenza *et al* (1981), Gokhroo and Mehra (1994), and Herrera and Santos (1994).

On physical grounds for a stellar model we should include a barotropic equation of state so that the radial pressure is a function of the energy density. Exact models of charged anisotropic matter with a quadratic equation of state were found by Feroze and Siddiqui (2011). Using the same equation of state, Maharaj and Mafa Takisa (2012) generated regular models for charged anisotropic stars. A strange star model with a quadratic equation of state was recently generated by Malaver (2014). Polytropic models were analysed by Mafa Takisa and Maharaj (2013b) for charged matter with anisotropic stresses. Malaver (2013a, 2013b) found charged stellar models with a van der Waals and generalized van der Waals equation of state respectively. Anisotropic models with a modified van der Waals equation of state are contained in the paper by Thirukkanesh and Ragel (2014). Other relativistic stellar models with a van der Waals equation of state are studied in the treatment of Lobo (2007). However for a quark star we require a linear equation of state. The first treatment of quark stars was undertaken by Itoh (1970) for hydrostatic matter in equilibrium. Since then there have been many investigations on the study of structure and properties of quark matter by adopting a linear equation of state. It has been shown by Witten (1984), Chodos (1974), Farhi and Jaffe (1984) that quark matter could be studied with the aid of the phenomenology of the MIT bag model; these studies indicate that a linear quark matter equation of state with a nonzero bag constant can be used. The review by Weber (2005) described the astrophysical phenomenology of compact quark stars. The study of nonradial oscillations of quark stars was performed by Sotani *et al* (2004) and Sotani and Harada (2003). Charged isotropic models for quark stars are described by Mak and Harko (2004) and Komathiraj and Maharaj (2007c). Particular models have been analysed to study the effect of both the electric field and the anisotropy in quark stars including those generated by Rahaman *et al* (2012), Varela *et al* (2010), Thirukkanesh and Maharaj (2008), Maharaj and Thirukkanesh (2009b) and Esculpi and Aloma (2010). However most charged anisotropic models of quark stars have anisotropy always present and do not regain isotropic pressures as a special case. Charged anisotropic models for quark stars that allow anisotropy to vanish have been found in the papers by Maharaj *et al* (2014) and Sunzu *et al* (2014).

The objective of this chapter is to find new exact solutions to the Einstein-Maxwell system of equations with a linear quark matter equation of state for charged anisotropic stars. We build new models by specifying a particular form for one of the gravitational potentials and the measure of anisotropy. A particular model in our results allows us to regain Einstein results with isotropic pressures as a special case. We also seek to generate masses and radii consistent with observed stellar objects and indicate that the gravitational potentials, matter variables and the electric field are well behaved.

5.2 Fundamental equations

We intend to describe stellar structure with quark matter in a general relativistic setting. The spacetime manifold must be static and spherically symmetric. The interior spacetime is given by the metric

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{5.1}$$

where $\nu(r)$ and $\lambda(r)$ are arbitrary functions. The Reissner-Nordstrom line element describes the exterior spacetime

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{5.2}$$

where M and Q represent total mass and charge as measured by an observer at infinity. The energy momentum tensor

$$T_{ab} = \text{diag}\left(-\rho - \frac{1}{2}E^2, p_r - \frac{1}{2}E^2, p_t + \frac{1}{2}E^2, p_t + \frac{1}{2}E^2\right) \tag{5.3}$$

describes anisotropic charged matter. The energy density ρ, the radial pressure p_r, the tangential pressure p_t, and the electric field intensity E are measured relative to a vector $\mathbf{u}$. The vector u^a is comoving, unit and timelike.

The Einstein-Maxwell equations with matter and charge can be written as

$$\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho + \frac{1}{2}E^2, \tag{5.4a}$$

$$-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2, \tag{5.4b}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu'}{r} - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2, \tag{5.4c}$$

$$\sigma = \frac{1}{r^2}e^{-\lambda}\left(r^2E\right)', \tag{5.4d}$$

where primes indicate differentiation with respect to the radial coordinate r. The quantity σ denotes the proper charge density. Note that we are using units where the coupling constant $\frac{8\pi G}{c^4} = 1$ and the speed of light $c = 1$. The mass contained within the charged sphere is defined by

$$M(r) = \frac{1}{2}\int_0^r \omega^2\left(\rho_* + E^2\right)d\omega, \tag{5.5}$$

where ρ_* is the energy density when the electric field $E = 0$. For a quark star we have a linear relationship between the radial pressure and the energy density

$$p_r = \frac{1}{3}(\rho - 4B), \tag{5.6}$$

where B is the bag constant.

We transform the field equations to an equivalent form by introducing a new independent variable x and defining metric functions $Z(x)$ and $y(x)$ as

$$x = Cr^2, \quad Z(x) = e^{-2\lambda(r)}, \quad A^2y^2(x) = e^{2\nu(r)}, \tag{5.7}$$

where A and C are arbitrary constants. With this transformation the line element in (5.1) becomes

$$ds^2 = -A^2y^2dt^2 + \frac{1}{4xCZ}dx^2 + \frac{x}{C}(d\theta^2 + \sin^2\theta d\phi^2). \tag{5.8}$$

Then the field equations (5.4) are transformed to

$$\begin{aligned}
\frac{1-Z}{x} - 2\dot{Z} &= \frac{\rho}{C} + \frac{E^2}{2C}, , &(5.9a)\\
4Z\frac{\dot{y}}{y} + \frac{Z-1}{x} &= \frac{p_r}{C} - \frac{E^2}{2C}, &(5.9b)\\
4xZ\frac{\ddot{y}}{y} + \left(4Z + 2x\dot{Z}\right)\frac{\dot{y}}{y} + \dot{Z} &= \frac{p_t}{C} + \frac{E^2}{2C}, &(5.9c)\\
\frac{\sigma^2}{C} &= \frac{4Z}{x}\left(x\dot{E} + E\right)^2. &(5.9d)
\end{aligned}$$

The mass function (5.5) becomes

$$M(x) = \frac{1}{4C^{\frac{3}{2}}}\int_0^x \sqrt{\omega}\left(\rho_* + E^2\right)d\omega, \tag{5.10}$$

where

$$\rho_* = \left(\frac{1-Z}{x} - 2\dot{Z}\right)C, \tag{5.11}$$

and a dot represents differentiation with respect to the variable x.

Then we can write the Einstein-Maxwell field equations (5.9), with the quark equation of state (5.6), in the following form

$$\begin{aligned}
\rho &= 3p_r + 4B, && (5.12a)\\
\frac{p_r}{C} &= Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C}, && (5.12b)\\
p_t &= p_r + \Delta, && (5.12c)\\
\Delta &= \frac{4xCZ\ddot{y}}{y} + C\left(2x\dot{Z} + 6Z\right)\frac{\dot{y}}{y} \\
&\quad + C\left(2\left(\dot{Z} + \frac{B}{C}\right) + \frac{Z-1}{x}\right), && (5.12d)\\
\frac{E^2}{2C} &= \frac{1-Z}{x} - 3Z\frac{\dot{y}}{y} - \frac{\dot{Z}}{2} - \frac{B}{C}, && (5.12e)\\
\sigma &= 2\sqrt{\frac{ZC}{x}}\left(x\dot{E} + E\right). && (5.12f)
\end{aligned}$$

The gravitational behaviour of the anisotropic charged quark star is governed by the system (5.12). The quantity $\Delta = p_t - p_r$ is called the measure of anisotropy. The system of equations (5.12) consists of eight variables ($\rho,\ p_r,\ p_t,\ E,\ Z,\ y,\ \sigma,\ \Delta$) in six equations. The advantage of the Einstein-Maxwell system (5.12) is that it has a simple representation: it is given in terms of the matter variables ($\rho,\ p_r,\ p_t,\ \Delta$), the charged quantities (E, σ) and the gravitational potentials Z and y. We rewrite (5.12d) in a more simplified form as

$$\dot{Z} + \frac{\left(4x^2\ddot{y} + 6x\dot{y} + y\right)}{2x\left(x\dot{y} + y\right)}Z = \frac{\left(\frac{x\Delta}{C} + 1 - \frac{2xB}{C}\right)y}{2x\left(x\dot{y} + y\right)}. \qquad (5.13)$$

This is a highly nonlinear equation in general. However if y and Δ are given functions then the form (5.13) of the field equation is linear in the variable Z. In order to find exact solutions to this model we will specify the two quantities y and Δ.

We choose the metric function as

$$y = \frac{1 - ax^m}{1 + bx^n}, \qquad (5.14)$$

where a, b, m and n are constants. This choice guarantees that the metric function y is continuous and well behaved within the interior of the star for a range of values of m and n. The metric function y is also finite at the centre of the star. We specify the measure of anisotropy in the form

$$\Delta = A_1 x + A_2 x^2 + A_3 x^3, \qquad (5.15)$$

where A_1, A_2, and A_3 are arbitrary constants. A similar choice of anisotropy was made by Maharaj *et al* (2014). This choice is physically reasonable as it is continuous and well behaved throughout the interior of the star. It is finite at the centre of the star. It is possible to regain isotropic pressures when $A_1 = A_2 = A_3 = 0$. We then have $\Delta = 0$ and the anisotropy vanishes. Substituting (5.14) and (5.15) in (5.13) we obtain the first order differential equation

$$\dot{Z} + \frac{[g(x) + ax^m\left[-g(x) + 4(m + bmx^n)^2 - 2m(1 + bx^n)(b(4n-1)x^n - 1)\right]]}{2x(1 + bx^n)[b(n-1)x^n - 1 + ax^m(1 + m + bmx^n - b(n-1)x^n)]} Z$$
$$= \frac{-\left(\frac{(A_1x + A_2x^2 + A_3x^3)x}{C} + 1 - \frac{2xB}{C}\right)(1 - ax^m)(1 + bx^n)}{2x[b(n-1)x^n - 1 + ax^m(1 + m + bmx^n - b(n-1)x^n)]}, \tag{5.16}$$

where we have set

$$g(x) = 2b(-1 + n + 2n^2)x^n - b^2(1 - 2n + 4n^2)x^{2n} - 1,$$

for convenience.

5.3 A regular model

A solution to (5.16) is desirable. We can find a nonsingular exact model for the choice of values of the parameter $m = 1$, $n = \frac{1}{2}$ and $a = b = 0$. With these values the potential $y = 1$ and (5.16) becomes

$$\dot{Z} + \frac{1}{2x}Z = \frac{A_1x + A_2x^2 + A_3x^3}{2C} + \frac{1}{2x} - \frac{B}{C}. \tag{5.17}$$

Solving the above differential equation we obtain

$$Z = 1 + \frac{x}{C}\left(-\frac{2B}{3} + \frac{A_1x}{5} + \frac{A_2x^2}{7} + \frac{A_3x^3}{9}\right). \tag{5.18}$$

Using the system (5.12) we obtain the exact solution describing the potentials and matter variables as

$$
\begin{aligned}
e^{2\nu} &= A^2, && (5.19a)\\
e^{2\lambda} &= \frac{315}{315+\frac{x}{C}\left(-210B+63A_1x+45A_2x^2+35A_3x^3\right)}, && (5.19b)\\
\rho &= 2B-\left(\frac{3A_1x}{5}+\frac{9A_2x^2}{14}+\frac{2A_3x^3}{3}\right), && (5.19c)\\
p_r &= -\left(\frac{2B}{3}+\frac{A_1x}{5}+\frac{3A_2x^2}{14}+\frac{2A_3x^3}{9}\right), && (5.19d)\\
p_t &= -\frac{2B}{3}+\frac{4A_1x}{5}+\frac{11A_2x^2}{14}+\frac{7A_3x^3}{9}, && (5.19e)\\
\Delta &= A_1x+A_2x^2+A_3x^3, && (5.19f)\\
E^2 &= -\left(\frac{4A_1x}{5}+\frac{5A_2x^2}{7}+\frac{2A_3x^3}{3}\right). && (5.19g)
\end{aligned}
$$

This model admits no singularity in the interior in the potentials and in the matter variables. In addition $\Delta = 0$ and $E^2 = 0$ at the stellar centre.

With this model the line element (5.8) becomes

$$
\begin{aligned}
ds^2 &= -A^2dt^2\\
&\quad+\frac{1}{4xC}\left(\frac{315}{315+\frac{x}{C}\left(-210B+63A_1x+45A_2x^2+35A_3x^3\right)}\right)dx^2\\
&\quad+\frac{x}{C}(d\theta^2+\sin^2\theta d\phi^2). \qquad (5.20)
\end{aligned}
$$

The mass function (5.10) becomes

$$
M(x)=\left(\frac{x}{C}\right)^{\frac{3}{2}}\left(\frac{1}{3}B-\frac{9}{50}A_1x-\frac{6}{49}A_2x^2-\frac{5}{54}A_3x^3\right). \qquad (5.21)
$$

In this exact solution we regain the special case of vanishing anisotropy and charge: $\Delta = 0$ and $E^2 = 0$. Then the potentials and matter variables become

$$
\begin{aligned}
e^{2\nu} &= A^2,\ e^{2\lambda}=\frac{315C}{315C-210Bx},\\
\rho &= 2B,\ p_r=p_t=-\frac{2B}{3}, \qquad (5.22)
\end{aligned}
$$

with the line element

$$
ds^2=-A^2dt^2+\left(\frac{315}{4x(315C-210Bx)}\right)dx^2+\frac{x}{C}(d\theta^2+\sin^2\theta d\phi^2), \qquad (5.23)
$$

in terms of the variable x. Note that we can write (5.23) in the equivalent form

$$ds^2 = -A^2dt^2 + \left(1 - \frac{r^2}{\Gamma^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{5.24}$$

where $\Gamma^2 = \frac{315}{210B}$. We observe that (5.24) is the familiar uncharged Einstein model with isotropic pressure and the equation of state $p_r = p_t = -\frac{1}{3}\rho$. We can therefore interpret the exact solution (5.19) as a generalized Einstein model with charge and anisotropy. This possibility arises only because the energy density at the boundary is a nonzero constant in a quark star.

The solutions found in this section do represent finite masses that can be related to observed objects. To show this we introduce the transformations
$\tilde{A}_1 = A_1R^2$, $\tilde{A}_2 = A_2R^2$, $\tilde{A}_3 = A_3R^2$, $\tilde{B} = BR^2$, $\tilde{C} = CR^2$.
Based on these transformations we choose values of parameters to generate stellar masses and radii in Table 5.1. For computation purposes we have set $R = 43.245$.

Therefore we generate masses in the range $0.94M\odot - 2.86M\odot$ contained in the investigations of Mak and Harko (2004), Negreiros *et al* (2009), Freire *et al* (2011), Sunzu *et al* (2014), Dey *et al* (1998) and Thirukkanesh and Maharaj (2008). Therefore the exact solutions of this section do produce finite masses consistent with physically reasonable astronomical objects.

Table 5.1: Particular stellar objects obtained for various parameters for a regular model

$\tilde{B}$	$\tilde{C}$	$\tilde{A}_1$	$\tilde{A}_2$	$\tilde{A}_3$	r(km)	$\frac{M}{M_\odot}$	Model
28.0	1.0	1.1	2.2	1.8	9.46	2.86	Mak and Harko (2004)
13.0	1.0	11.0	9.0	5.0	10.99	2.02	Negreiros *et al* (2009)
17.0	1.0	13.5	10.0	8.0	9.40	1.67	Freire *et al* (2011)
30.54	1.0	20.51	25.0	30.0	7.60	1.60033	Sunzu *et al* (2014)
34.0	1.0	28.6	35.0	20.0	7.07	1.433	Dey *et al* (1998)
33.93	1.0	40.4	24.0	20.0	6.84	1.28994	Sunzu *et al* (2014)
22.18	1.0	10.5	4.0	5.0	7.07	0.94	Thirukkanesh and Maharaj (2008)

5.4 Generalized models

It is possible that other exact solutions exist, in addition to those found above, and which may be obtained using the approach in this chapter. Clearly these new solutions will correspond to different matter distributions, and consequently have different energy density profiles to the Einstein-Maxwell model considered in Section 5.3. The choice of parameters we made in Section 5.3 led to constant y. Here we again choose $m = 1$, $n = \frac{1}{2}$ but we take $a = b^2$. Then the gravitational potential y is no longer constant. Consequently (5.16) can be written in the form

$$\begin{aligned} \dot{Z} + \frac{(1-3b\sqrt{x})\,Z}{x(2-3b\sqrt{x})} &= \frac{(b\sqrt{x}-1)\,[C + x\,(\Delta - 2B)]}{Cx(3b\sqrt{x}-2)} \\ &= \frac{(b\sqrt{x}-1)\,[C + x\,(A_1 x + A_2 x^2 + A_3 x^3 - 2B)]}{Cx(3b\sqrt{x}-2)}. \end{aligned} \tag{5.25}$$

Equation (5.25) is more complicated than (5.17) but it can be integrated. Solving (5.25) we obtain the function

$$Z = \left[2 - b\sqrt{x} + \frac{x}{C}\left(B\left(b\sqrt{x} - \frac{4}{3}\right) + f(x)\right)\right]\left(\frac{1}{2-3b\sqrt{x}}\right), \tag{5.26}$$

where

$$f(x) = A_1 x\left(\frac{2}{5} - \frac{b\sqrt{x}}{3}\right) + A_2 x^2\left(\frac{2}{7} - \frac{b\sqrt{x}}{4}\right) + A_3 x^3\left(\frac{2}{9} - \frac{b\sqrt{x}}{5}\right).$$

Note that when $f(x) = 0$ then we have isotropic pressures. The function (5.26) demonstrates that there are other exact solutions to the differential equation (5.13) in terms of elementary functions.

Using the field equations indicated in the system (5.12) we obtain the following exact solution

$$
\begin{aligned}
e^{2\nu} &= A^2\left(\frac{1-b^2x}{1+b\sqrt{x}}\right)^2, && (5.27a)\\
e^{2\lambda} &= \frac{2-3b\sqrt{x}}{2-b\sqrt{x}+\frac{x}{C}\left[B\left(b\sqrt{x}-\frac{4}{3}\right)+f(x)\right]}, && (5.27b)\\
\rho &= \frac{3C\left(\frac{6b}{\sqrt{x}}-10b^2+3b^3\sqrt{x}\right)}{2(2-3b\sqrt{x})^2(b\sqrt{x}-1)}\\
&\quad+\frac{B\left(-16+47b\sqrt{x}-48b^2x+18b^3x^{\frac{3}{2}}\right)+3f_r(x)}{2(2-3b\sqrt{x})^2(b\sqrt{x}-1)}, && (5.27c)\\
p_r &= \frac{C\left(\frac{6b}{\sqrt{x}}-10b^2+3b^3\sqrt{x}\right)}{2(2-3b\sqrt{x})^2(b\sqrt{x}-1)}\\
&\quad+\frac{B\left(\frac{16}{3}-27b\sqrt{x}+40b^2x-18b^3x^{\frac{3}{2}}\right)+f_r(x)}{2(2-3b\sqrt{x})^2(b\sqrt{x}-1)}, && (5.27d)\\
p_t &= \frac{C\left(\frac{6b}{\sqrt{x}}-10b^2+3b^3\sqrt{x}\right)}{2(2-3b\sqrt{x})^2(b\sqrt{x}-1)}\\
&\quad+\frac{B\left(\frac{16}{3}-27b\sqrt{x}+40b^2x-18b^3x^{\frac{3}{2}}\right)+f_t(x)}{2(2-3b\sqrt{x})^2(b\sqrt{x}-1)}, && (5.27e)\\
\Delta &= A_1x+A_2x^2+A_3x^3, && (5.27f)\\
E^2 &= \frac{C\left(2b^2+3b^3\sqrt{x}-\frac{2b}{\sqrt{x}}\right)+B\left(b\sqrt{x}-2b^2x\right)+f_e(x)}{(2-3b\sqrt{x})^2(b\sqrt{x}-1)}, && (5.27g)
\end{aligned}
$$

where we have set

$$
\begin{aligned}
f_r(x) &= A_1 x\left(\frac{8}{5}-\frac{64}{15}b\sqrt{x}+\frac{18}{5}b^2x-b^3x^{\frac{3}{2}}\right)\\
&\quad +A_2x^2\left(\frac{12}{7}-\frac{141}{28}b\sqrt{x}+\frac{67}{14}b^2x-\frac{3}{2}b^3x^{\frac{3}{2}}\right)\\
&\quad +A_3x^3\left(\frac{16}{9}-\frac{82}{15}b\sqrt{x}+\frac{82}{15}b^2x-\frac{9}{5}b^3x^{\frac{3}{2}}\right),\\
f_t(x) &= A_1 x\left(-\frac{32}{5}+\frac{416}{15}b\sqrt{x}-\frac{192}{5}b^2x+17b^3x^{\frac{3}{2}}\right)\\
&\quad +A_2x^2\left(-\frac{44}{7}+\frac{755}{28}b\sqrt{x}-\frac{521}{14}b^2x+\frac{33}{2}b^3x^{\frac{3}{2}}\right)\\
&\quad +A_3x^3\left(-\frac{56}{9}+\frac{398}{15}b\sqrt{x}-\frac{548}{15}b^2x+\frac{81}{5}b^3x^{\frac{3}{2}}\right),\\
f_e(x) &= A_1 x\left(\frac{16}{5}-\frac{64}{5}b\sqrt{x}+\frac{84}{5}b^2x-7b^3x^{\frac{3}{2}}\right)\\
&\quad +A_2x^2\left(\frac{20}{7}-\frac{313}{28}b\sqrt{x}+\frac{101}{7}b^2x-6b^3x^{\frac{3}{2}}\right)\\
&\quad +A_3x^3\left(\frac{8}{3}-\frac{154}{15}b\sqrt{x}+\frac{196}{15}b^2x-\frac{27}{5}b^3x^{\frac{3}{2}}\right),
\end{aligned}
$$

for convenience.

Based on our exact solution in the system (5.27), the line element in (5.8) becomes

$$
\begin{aligned}
ds^2 &= -A^2\left(\frac{1-b^2x}{1+b\sqrt{x}}\right)^2 dt^2\\
&\quad +\frac{1}{4xC}\left(\frac{2-3b\sqrt{x}}{2-b\sqrt{x}+\frac{x}{C}\left(B\left(b\sqrt{x}-\frac{4}{3}\right)+f(x)\right)}\right)dx^2\\
&\quad +\frac{x}{C}(d\theta^2+\sin^2\theta d\phi^2).
\end{aligned}
\tag{5.28}
$$

The mass function has the form

$$
\begin{aligned}
M(x) = \quad & \frac{x^{\frac{5}{2}}}{b^4 C^{\frac{3}{2}}}\left(-\frac{2b^4 A_1}{15} + \frac{47b^2 A_2}{2520} + \frac{113 A_3}{12150}\right) \\
& -\frac{x^{\frac{7}{2}}}{b^2 C^{\frac{3}{2}}}\left(\frac{5b^2 A_2}{56} - \frac{5A_3}{378}\right) \\
& +\frac{x^3}{b^6 C^{\frac{3}{2}}}\left(\frac{b^6 B}{6} + \frac{b^4 A_1}{30} + \frac{b^2 A_2}{56} + \frac{A_3}{90}\right) \\
& +\frac{\ln(1-b\sqrt{x})}{b^9 C^{\frac{3}{2}}}\left(\frac{3b^8 C}{2} - \frac{b^6 B}{2} + \frac{b^4 A_1}{10} + \frac{3b^2 A_2}{56} + \frac{A_3}{30}\right) \\
& +\left(\frac{2b^8 C}{3} - \frac{4b^6 B}{27} + \frac{64b^4 A_1}{3645} + \frac{80b^2 A_2}{15309} + \frac{512 A_3}{295245}\right) \\
& \times\frac{1}{2b^9 C^{\frac{3}{2}}}\left(\frac{2}{3b\sqrt{x}-2} + 1\right) \\
& +\frac{x^3}{3b^3 C^{\frac{3}{2}}}\left(\frac{13b^2 A_3}{240}x - \frac{b^3}{5}x^{\frac{3}{2}} + \frac{b^2 A_2}{14} + \frac{17A_3}{540}\right) \\
& +\left(b^8 C - \frac{2b^6 B}{9} + \frac{32b^4 A_1}{1215} + \frac{40b^2 A_2}{5103} + \frac{256 A_3}{98415}\right) \\
& \times\frac{1}{b^9 C^{\frac{3}{2}}}\ln\left(\frac{2}{2-3b\sqrt{x}}\right) \\
& +\frac{x^2}{b^5 C^{\frac{3}{2}}}\left(\frac{13b^4 A_1}{360} + \frac{101b^2 A_2}{6048} + \frac{55A_3}{5832}\right) \\
& +\frac{x}{b^7 C^{\frac{3}{2}}}\left(-\frac{b^6 B}{6} + \frac{13b^4 A_1}{324} + \frac{649b^2 A_2}{27216} + \frac{2059 A_3}{131220}\right) \\
& +\frac{\sqrt{x}}{b^8 C^{\frac{3}{2}}}\left(\frac{b^8 C}{2} - \frac{5b^6 B}{18} + \frac{179b^4 A_1}{2430} + \frac{1867b^2 A_2}{40824} + \frac{6049 A_3}{196830}\right). \qquad (5.29)
\end{aligned}
$$

Therefore we have obtained another exact solution to the Einstein-Maxwell system of equations (5.12) with a quark equation of state. Other solutions to (5.16) are possible for different choices of parameters m, n, a and b. It is not clear that other choices are likely to easily produce tractable forms for the gravitational potential Z. The advantage of the exact solutions (5.19) and (5.27) is that they have a simple form. They are expressed in terms of elementary functions. The model (5.27) is singular at the centre. This is a feature that is shared with the quark star model of Mak and Harko (2004) but the stellar mass and electric field remain finite.

5.5 Discussion

In this section we indicate that the exact solution of the field equations (5.27) is well behaved away from the centre. To do this we consider the behaviour of the gravitational potentials, matter variables and the electric field. We note that $\rho' < 0$, $p_r' < 0$, and $p_t' < 0$, so that the energy density, radial pressure and the tangential pressure are decreasing functions. The gradients are greatest in the central core regions. This happens because the profiles for ρ, p_r and p_t are dominated by the term containing the factor $x^{-\frac{1}{2}}$. Other choices for the parameters m, n, a, and b in (5.16) could lead to models with gradients where the rate of change is more gradual. The Python programming language was used to generate graphical plots for the particular choices $b = \pm 0.5$, $A = 0.664$, $B = 0.198$, $C = 1$, $A_1 = -0.6$, $A_2 = -0.15$, and $A_3 = 0.2$. The graphical plots generated are for the potential $e^{2\nu}$ (Fig. 5.1), potential $e^{2\lambda}$ (Fig. 5.2), energy density ρ (Fig. 5.3), radial pressure p_r (Fig. 5.4), tangential pressure p_t (Fig. 5.5), measure of anisotropy Δ (Fig. 5.6), the electric field E^2 (Fig. 5.7) and the mass M (Fig. 5.8). All figures are plotted against the radial coordinate r. These quantities are regular and well behaved in the stellar interior except for the energy density, radial pressure, tangential pressure and electric field which are divergent at the centre. In this case our exact solutions may describe the outer regions, away from the centre, in a core envelope model. However, note that the gravitational potentials, the measure of anisotropy and the mass remain finite, regular and well behaved throughout the interior of the stellar structure. In general the measure of anisotropy Δ is finite and a continuous decreasing function. A similar profile of the anisotropy was obtained by Kalam *et al* (2013b) and Karmakar *et al* (2007). The mass is an increasing function of the radial distance as indicated in Fig. 5.8.

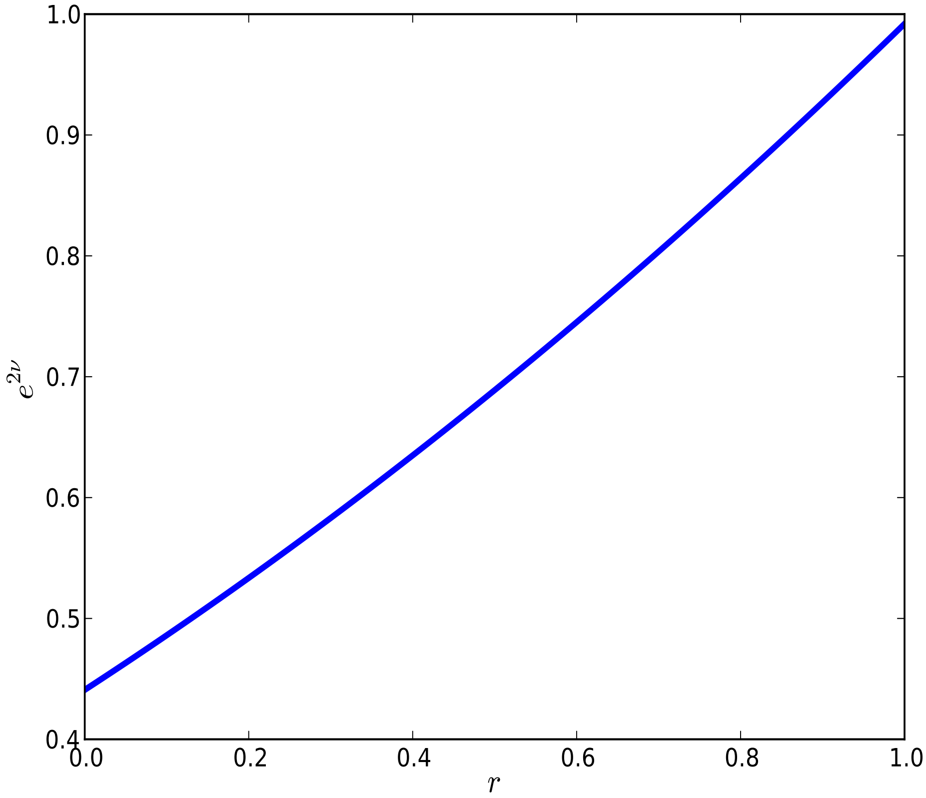

Figure 5.1: The potential $e^{2\nu}$ against radial distance r

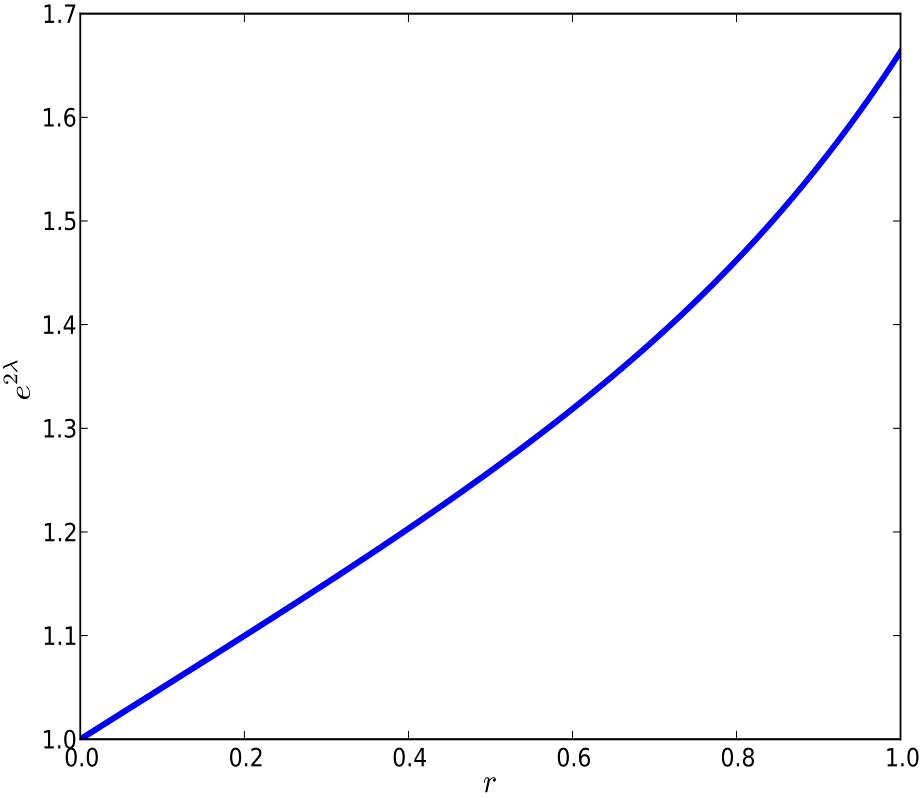

Figure 5.2: The potential $e^{2\lambda}$ against radial distance r

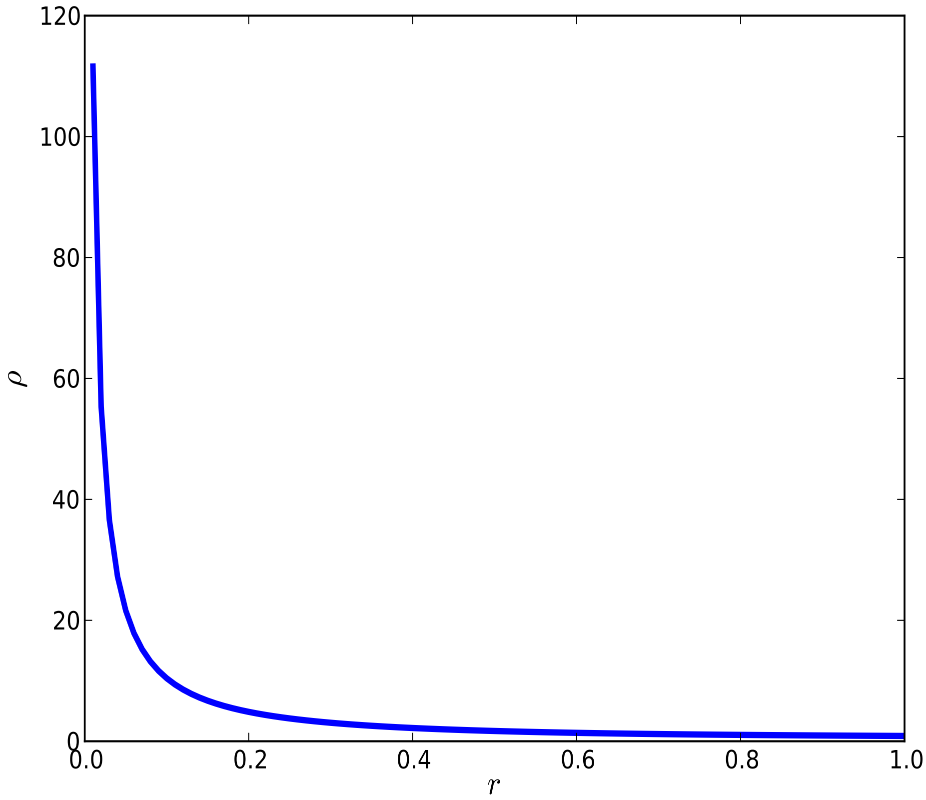

Figure 5.3: The energy density ρ against radial distance r

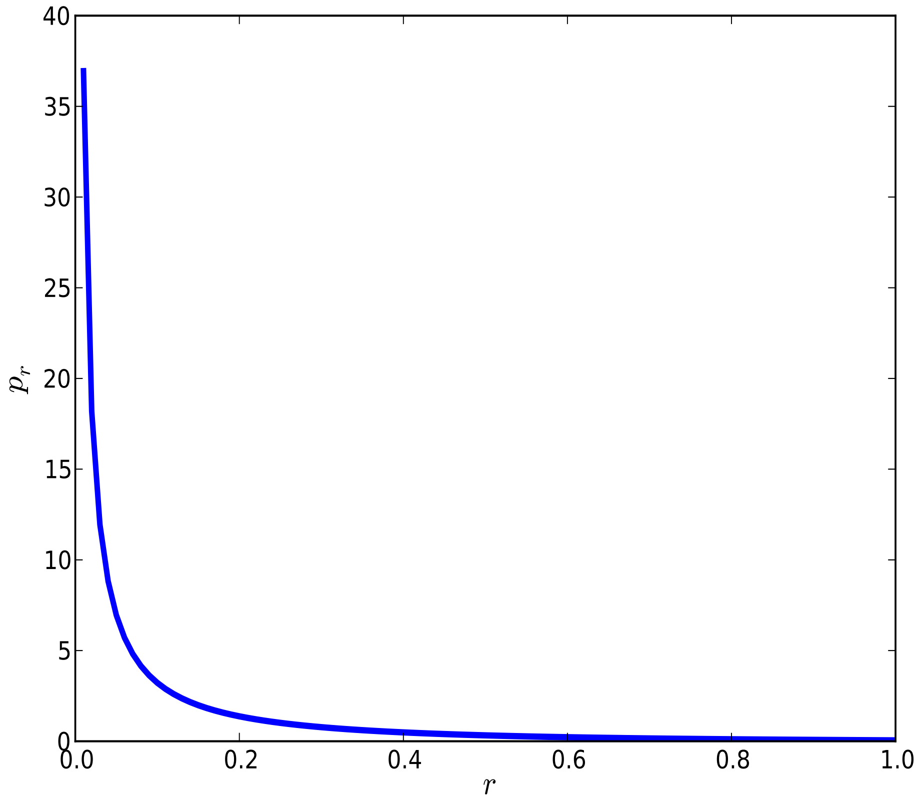

Figure 5.4: The radial pressure p_r against radial distance r

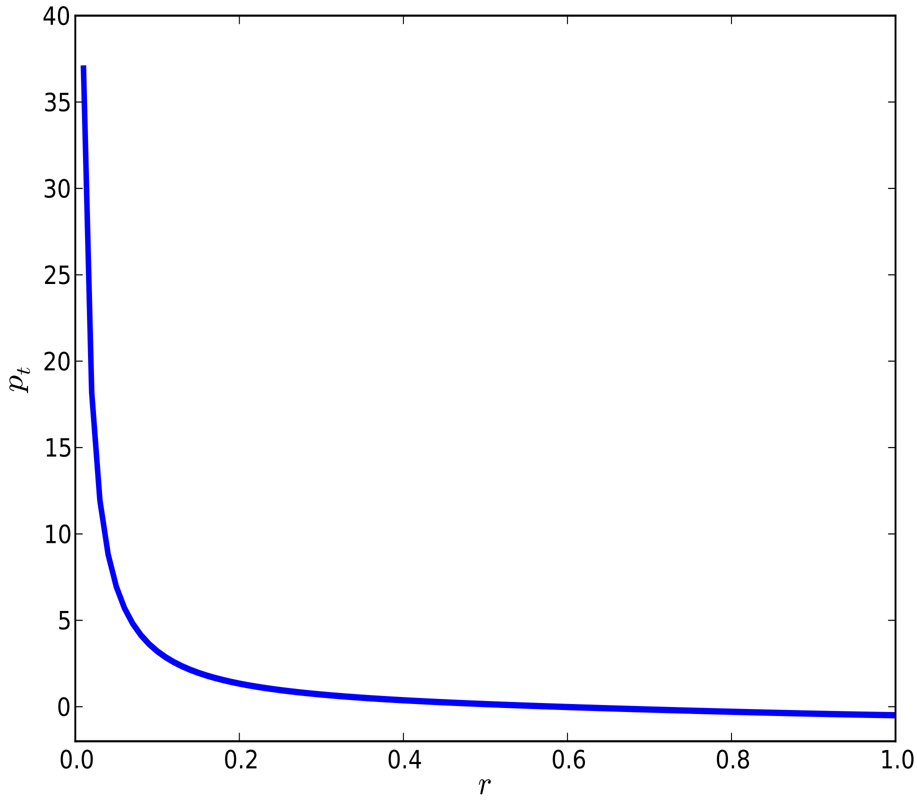

Figure 5.5: The tangential pressure p_t against radial distance r

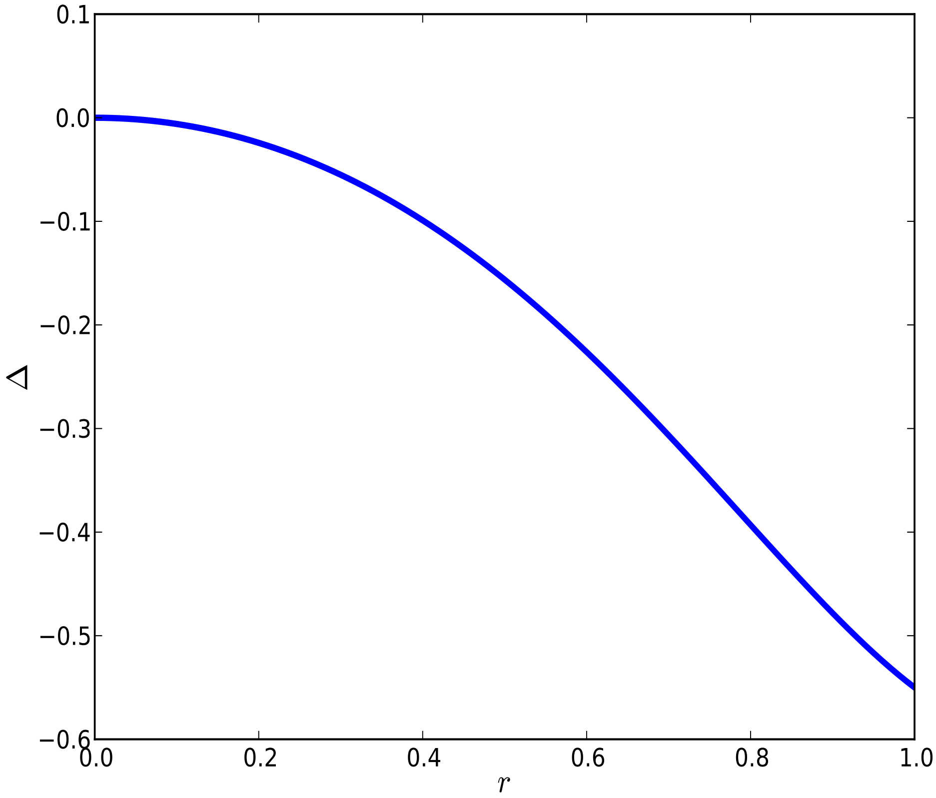

Figure 5.6: The anisotropy Δ against radial distance r

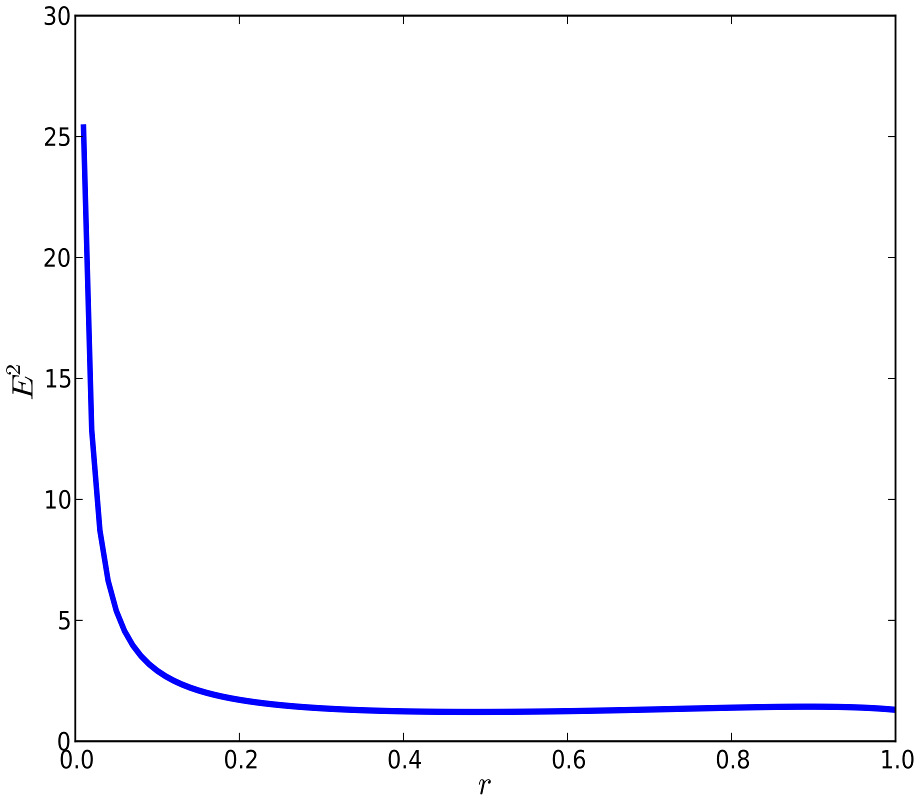

Figure 5.7: The electric field E^2 against radial distance r

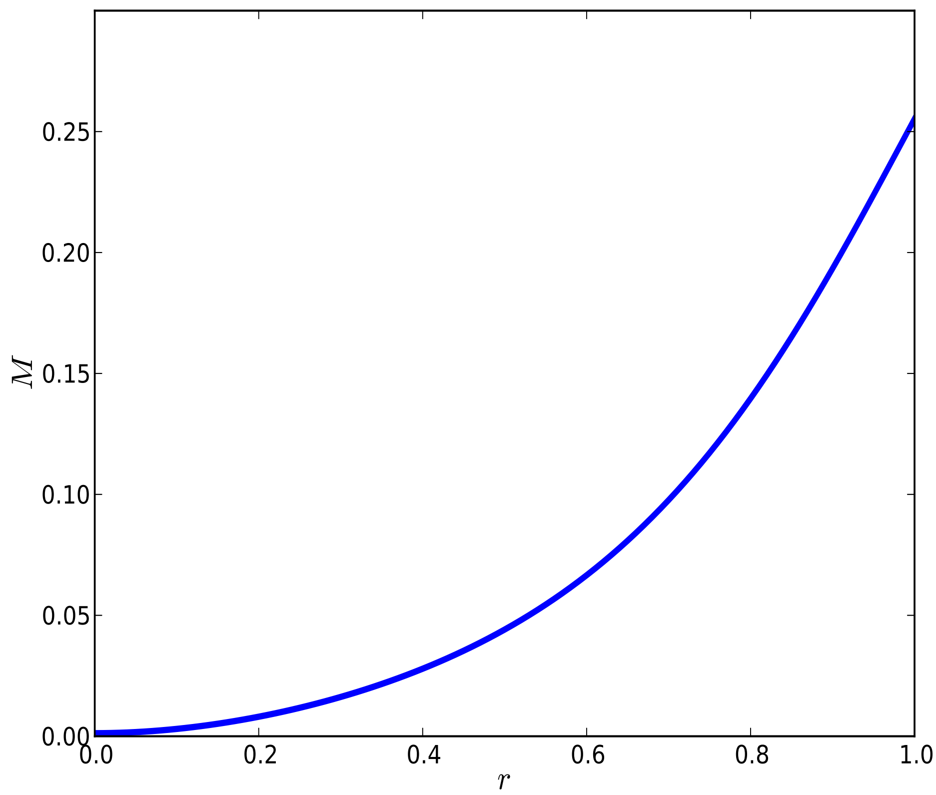

Figure 5.8: The mass M against radial distance r

Chapter 6

Conclusion

The main purpose of this thesis was to generate exact models that provide new solutions of the Einstein-Maxwell field equations that may be adopted to describe relativistic quark strange stars in spherically symmetric spacetimes. The models were generated by considering a charged and anisotropic matter distribution. For consistency with quark matter we have incorporated the linear form of the equation of state in the MIT bag model which relates the radial pressure and the energy density in

$$p_r = \frac{1}{3}\left(\rho - 4B\right).$$

The solutions were generated by integrating the master differential equations after specifying a new form for the measure of anisotropy and one of the gravitational potentials which are physically viable. The solutions obtained in this thesis generalize the models obtained by other researchers in the past and do contain isotropic models as a special case. In performing the physical and graphical analyses we have indicated that the potentials, matter variables, and electric field are well behaved. We have shown that the mass functions in our models generate masses consistent with observations of known stellar quark stars. Other stellar masses generated for quark stars are in acceptable ranges.

We now provide an overview of the main results generated during the course of this research

- In Chapter 2 we discussed briefly aspects of differential geometry as applied in general relativity. This was necessary for later chapters. We indicated how the Einstein-Maxwell equation for charged anisotropic matter are derived in spherically symmetric spacetimes with the aid of curvature tensors.

- Our objective in Chapter 3 was to generate exact solutions for the Einstein-Maxwell field equations for the charged and anisotropic quark stars. We chose the gravitational potential

  $$y = (a + x^m)^n .$$

  We indicated that when the measure of anisotropy is

  $$\Delta = A_0 + A_1 x + A_2 x^2 + A_3 x^3,$$

  the master equation becomes a linear differential equation in the form

  $$\begin{aligned} \dot{Z} &+ \left(\frac{1}{2x} + \frac{2m(n-1)x^{m-1}}{a + x^m} + \frac{m\,(4(1+mn) - 3n)\,x^{m-1}}{2\,(a + (1+mn)x^m)} \right) Z \\ &= \frac{\left(1 - \frac{2xB}{C} + \frac{(A_0 + A_1 x + A_2 x^2 + A_3 x^3)x}{C}\right)(a + x^m)}{2x\,(a + (1+mn)x^m)}. \end{aligned}$$

  We generated two new classes of exact solutions depending on the choice of the values for the constants m and n. For $m = \frac{1}{2}$ and $n = 1$ we obtained anisotropic charged exact solutions that generalize the exact isotropic models of Komathiraj and Maharaj (2007c), Mak and Harko (2004) and Misner and Zapolsky (1964). When $m = 1$ and $n = 2$ we generated a second class of charged anisotropic nonsingular exact solutions which generalize the isotropic model of Komathiraj and Maharaj (2007c). The physical and graphical analyses of the nonsingular model indicated that the gravitational potentials, matter variables, the electric field and the mass function are well behaved.

- In Chapter 4 we performed a detailed physical analysis of a nonsingular class of exact models for charged, anisotropic matter obtained by Maharaj *et al* (2014). Using this system of exact solutions and choosing suitable values of parameters appearing in the mass function, we generated masses and radii consistent with previously observed stellar objects by other researchers. We regained masses and radii of earlier investigations. In particular we regained the results:

(i) mass $M = 2.86M_\odot$ with radius $r = 9.46$km consistent with the mass and radius obtained by Mak and Harko (2004),

(ii) mass $M = 2.02M_\odot$ with radius $r = 10.99$km consistent with the object found by Negreiros *et al* (2009),

(iii) the mass $M = 1.433M_\odot$ with radius $r = 7.07$km consistent with the particular results obtained by Thirukkanesh and Maharaj (2008) and Mafa Takisa and Maharaj (2013a),

(iv) the mass $M = 1.67M_\odot$ with radius of 9.4km consistent with the star PSR J1903+327 discussed by Freire *et al* (2011) and Gangopathyay *et al* (2013), and

(v) the mass $M = 1.433M_\odot$ with radius of 7.07 km found by Dey *et al* (1998) in their strange star models.

For the anisotropic case we have also generated new masses ranging from $1.28994M_\odot$ to $1.73268M_\odot$ with radius of the range 5.78km to 7.61km and for the isotropic case the masses generated are from $1.31530M_\odot$ to $1.72885M_\odot$ with radius varying from 5.77km to 7.61km. In order to compare isotropic and anisotropic models, we created graphical plots for the mass-radius relation. In general results indicated that masses of a stellar object with isotropic pressure are greater than the anisotropic objects. It was shown that different sets of parameters values give different masses and radii for the stellar objects. It is interesting that masses and radii are in acceptable ranges for the quark stars. By varying the parameters in the solution we were in a position to generate a variety of masses and radii with values acceptable for a quark star. Our solutions generated are suitable for describing observed astronomical objects. In particular our model is a good candidate for the object SAXJ1808.4-3658.

- The objective in Chapter 5 was to find new exact solutions of the Einstein-

Maxwell field equations for charged anisotropic quark matter with a new choice for one of the gravitational potentials. In this chapter we again considered a linear equation of state consistent with quark matter. The new form of the gravitational potential selected for the metric function was the form

$$y = \frac{1 - ax^m}{1 + bx^n}.$$

The choice for the measure of anisotropy was

$$\Delta = A_1 x + A_2 x^2 + A_3 x^3.$$

This choice guaranteed that the metric function y is continuous and well behaved, and the choice of the anisotropy ensured that the model produces isotropic pressures as a special case when parameters are set to vanish. After substituting the function y in the transformed field equation we obtained the first order differential equation given by

$$\dot{Z} + \frac{[(gx) + ax^m [-g(x) + 4(m + bmx^n)^2 - 2m(1 + bx^n)(b(4n-1)x^n - 1)]]}{2x(1 + bx^n)[b(n-1)x^n - 1 + ax^m(1 + m + bmx^n - b(n-1)x^n)]} Z$$
$$= \frac{-\left(\frac{x\Delta}{C} + 1 - \frac{2xB}{C}\right)(1 - ax^m)(1 + bx^n)}{2x[b(n-1)x^n - 1 + ax^m(1 + m + bmx^n - b(n-1)x^n)]},$$

where

$$g(x) = 2b(-1 + n + 2n^2)x^n - b^2(1 - 2n + 4n^2)x^{2n} - 1.$$

This is a very complicated equation but solution can be found. Exact solutions of the above differential equation were found by selecting values for the parameters a, b, m and n. Two classes of exact solution to the Einstein-Maxwell system were found:

(i) when we set $m = 1$, $n = \frac{1}{2}$ and $a = b = 0$ we obtained a regular model which generalizes the Einstein model with charge and anisotropy. In this model we generated masses and radii consistent with stellar objects investigated by other researchers in the range $0.94M\odot - 2.86M\odot$. We regained the results of Mak and Harko (2004), Negreiros *et al* (2009), Freire *et al* (2011), Sunzu *et al* (2014), Dey *et al* (1998) and Thirukkanesh and Maharaj (2008),

(ii) when $m = 1$, $n = \frac{1}{2}$ and $a = b^2$ we generated a second generalized model with a singularity in some matter variables at the centre of the stellar object. However the gravitational potentials, the anisotropy and the mass remained finite throughout the interior of the stellar body. A graphical analysis indicated that the potentials, matter variables and the mass are well behaved.

Finally we point out that particular results found in this research are good candidates for stellar objects, in particular SAXJ808.4-3658. Different masses and radii consistent with astronomical objects studied by other researchers have been found. We believe that our stellar models may facilitate studies of anisotropic quark stars with an electromagnetic field distribution and provide room for further studies with relativistic matter distributions. This may be achieved with a specific equation of state, spacetime geometry, anisotropy and metric functions different from what we have considered in this thesis.

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