THE BASIS OF LEGITIMISATION OF MATHEMATICAL LITERACY IN SOUTH AFRICA

by

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Submitted in fulfilment of the academic
requirements for the degree of Doctor of Philosophy,
in the Graduate Programme in the
School of Education, University of KwaZulu-Natal,
Pietermaritzburg, South Africa.

March 2015

DECLARATION

Submitted in fulfilment of the requirements for the degree of Doctor of Philosophy, in the Graduate Programme in the School of Education, University of KwaZulu-Natal, Pietermaritzburg, South Africa.

I, **Marc Philip North,** declare that

1. The research reported in this thesis, except where otherwise indicated, is my original research.
2. This thesis has not been submitted for any degree or examination at any other university.
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    a. Their words have been re-written but the general information attributed to them has been referenced
    b. Where their exact words have been used, then their writing has been placed in italics and inside quotation marks, or in block quotes, and referenced.
5. This thesis does not contain text, graphics or tables copied and pasted from the Internet, unless specifically acknowledged, and the source being detailed in the thesis and in the References sections.

Student Name: **Marc Philip North**

Date: **10 March 2015**

Name of Supervisor: **Professor Iben Maj Christiansen**

Signature: ____________

ABSTRACT

This study is an exercise in knowledge production: the purpose is to present a theoretical language of the structure of participation in a conception of the knowledge domain of mathematical literacy – and in the associated practices of the South African school subject Mathematical Literacy – in which an orientation for life-preparedness is prioritised. This orientation is presented as an alternative to the current structure of mathematically-legitimised forms of participation in the subject which, I argue, promote educational disadvantage.

This intention is guided by the following two (paraphrased) questions:
In what ways does Dowling's (1998) language of description provide a means for problematising current practices in the subject Mathematical Literacy?; and, *What characterises mathematical literacy as a knowledge domain?*

To facilitate use of the language for empirical analysis, two further (paraphrased) questions are posed: *What would constitute an external language that would enable a (re)description of an empirical practice in the subject in terms of mathematical literacy as a knowledge domain?*; and, *(a) How can the external and internal languages be used to determine the dominant basis of legitimisation in a segment of the Mathematical Literacy curriculum, a textbook, teacher education course notes, and national assessments?; (b) How can identification of the dominant basis of legitimisation be used to determine coherence or disjunction within and between practices/discourse in the subject?*

Through a methodology of textual analysis I argue that the developed language facilitates identification of the prioritisation of different domains of practice in the texts, with only the texts from the teacher education course reflecting an orientation for life-preparedness. Implications of the disjuncture between these texts for practice and policy are highlighted, together with the potential consequence of emphasis on primarily mathematised forms of participation. I also offer suggested policy and practice implications for the adoption of a life-preparedness orientation. I conclude by arguing that the empirical analysis demonstrates the coherence of the developed language for identifying the structure of participation in practices that draw on the knowledge domain of mathematical literacy: this is the key finding. However, the language is not without deficiencies and limitations; these are identified and discussed.

PREFACE

The work described in this thesis was carried out in the School of Education at the Pietermaritzburg Campus of the University of KwaZulu-Natal from January 2011 to October 2014 under the supervision of Associate Professor Iben Maj Christiansen.

Ethical clearance was granted for this thesis by the Research Office of the University of KwaZulu-Natal. The supplied Protocol Reference number for the Ethical Clearance Approval is HSS/0066/013D (see Appendix D on page 496 below).

This PhD thesis is the outcome of a Masters project which was upgraded into the PhD programme. Formal acceptance of this upgrade was communicated via letter correspondence from the School of Education on 15 August 2012. This letter is available on request.

The upgrade of this thesis from the Masters to the PhD programme has contributed to the length of the thesis. Given this length, I applied formally in writing to the School of Education for permission to submit an extended length thesis. My application was supported in writing by my supervisor Professor Iben Christiansen. Copies of the submitted letters of application and accompanying e-mail correspondence are available on request. All examiners approached for examination of this thesis were made aware of the length of the thesis, and the appointed examiners acknowledged willingness to examine a thesis of this length.

A Turnitin report regarding plagiarism was supplied (as a separate document) at the time of submission of this thesis for examination.

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Mom, this is for you … because I know you would have understood just how important this is to me.

ACKNOWLEDGEMENTS

To the *Sasol Inzalo Foundation*: thank you for your generous funding and support. To Cynthia and Marietjie, in particular –this would have been impossible without you. Thank you so very much.

To my 'Super-Professor' *Iben Christiansen*: thank you for your friendship, care, concern, encouragement, and support. I could not have asked for a more committed supervisor, despite everything that you have had to deal with in the past few years. You are an inspiration to me.

To *Viren*: thank you for being my friend and for always being prepared to put up with my moaning and groaning. I cannot wait to walk across the stage with you.

To *Julie:* thank you for listening to me for so many hours, and for your constant words of wisdom, support and encouragement. You have been an invaluable source of inspiration, friendship and counsel, and I have cherished our sessions together dearly.

To *Dad*: thank you for never questioning my decisions, for always supporting us in our choices, and especially for your contribution to my gene pool which, ultimately, made this possible! Dad, I know that you don't always understand the work I do, but this has never stopped you from supporting me. Thank you.

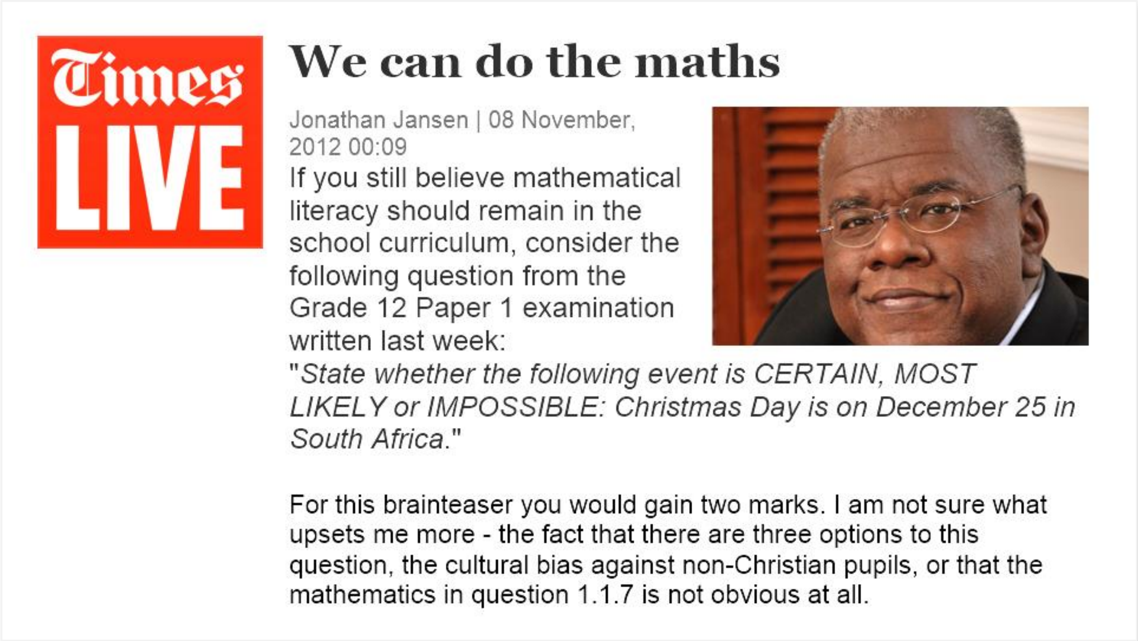
To *Manny-Moo*, *Squiggy-Wig* and *Noo-Noo-Frog*: thank you for always being so excited about my studying even though you could not possibly have had any clue what it was actually all about. Thank you, too, for giving up so many hours of our play time together while I was working. I love you all so much.

To *Kate* – my dear and beautiful wife: thank you so very much for your unconditional love, constant support, and for the many many many cups of tea that you have made for me over the past four years. Thank you, too, for always being prepared to embark on adventures with me. You are, without a shadow of a doubt, my soul-mate.

And, to *God the Father*: I know that we may not always have been on the best of terms over the past few years, but I also know from the depths of my being that you are always by my side and that without you none of this would have been possible.

INTRODUCTION AND STUDY OVERVIEW

BACKGROUND INFORMATION, MOTIVATION AND GUIDING RESEARCH QUESTIONS


Times LIVE

We can do the maths

Jonathan Jansen | 08 November, 2012 00:09

If you still believe mathematical literacy should remain in the school curriculum, consider the following question from the Grade 12 Paper 1 examination written last week:

"*State whether the following event is CERTAIN, MOST LIKELY or IMPOSSIBLE: Christmas Day is on December 25 in South Africa*."

For this brainteaser you would gain two marks. I am not sure what upsets me more - the fact that there are three options to this question, the cultural bias against non-Christian pupils, or that the mathematics in question 1.1.7 is not obvious at all.

Figure 1: Newspaper extract showing prevailing criticism of the subject Mathematical Literacy in South Africa (Jansen, 2012)

At the beginning of Grade 10 in the secondary school system in South Africa, learners are required to choose between participation in either Core Mathematics or Mathematical Literacy. Core Mathematics is characterised by the study of scientific and abstract mathematical contents; Mathematical Literacy, on the other hand, is characterised by engagement with mathematical contents in everyday settings. In recent years, the subject Mathematical Literacy has come under increasing scrutiny and criticism – captured, in part, in the newspaper extract shown in Figure 1 above – from various sectors in society. My involvement in the educational terrain of the subject Mathematical Literacy in South Africa has led me to believe that this current criticism is grounded in three main concerns relating to the structure, status and practices of the subject. Firstly, although Mathematical Literacy is positioned as a compulsory alternative choice to the subject Core Mathematics, Mathematical Literacy is perceived to be a significantly easier qualification – and the extraordinarily high pass rate of 87,1% in the subject compared to 59,1% in Core Mathematics (DBE, 2014a, p. 125 & 159) in the 2013 academic year provides some level of validation of this concern. Critics argue that this distinction in the accessibility of the two subjects is contributing to the exodus of increasing numbers of learners from Core Mathematics to Mathematical Literacy such that enrolment in Core Mathematics has decreased significantly since the introduction of Mathematical Literacy (from 60,1% in 2006 to 42,7% in 2013)[1]. Secondly, there is criticism of the quality (or 'currency') of the Mathematical Literacy qualification, accompanied by concern that participation in the

[1] These values were determined using statistics given in (DoE, 2008b, p. 27) and (DBE, 2014a, p. 125 & 159).

subject does not afford access to various avenues of tertiary study and/or career choice (and, certainly not to the same and equally varied and comprehensive avenues of study and/or career choice as Core Mathematics). Mathematical Literacy, then, is perceived as a limiting qualification – hence, why increasing enrolment figures in the subject at the expense of enrolment in Core Mathematics is of such concern. It is this concern that contributed to the decision by the current Minister of Education to institute a ministerial panel to investigate, amongst other things, "the currency of Mathematics and Mathematical Literacy and whether this is the best option for the South African schooling system in terms of preparing learners for the workplace and for higher education studies." (DBE, 2013a). The third concern draws directly from the two previous concerns and is of particular relevance to this study. This concern relates specifically to a mismatch between the stated intention and current legitimised forms of participation for/in the subject. The stated intention is for preparing and empowering participants for everyday life, the demands of the workplace, and for effective participation in a democratic society (DBE, 2011a, p. 8) – what Venkat (2010, p. 55) refers to as a life-preparedness orientation. Current legitimised forms of participation, by contrast, prioritise mathematised forms of engagement with contextual problem-solving scenarios. As a result of this mismatch, participation in the subject Mathematical Literacy is reserved primarily for 'weaker' learners who are unable to cope with the demands of scientific mathematics contents. In the South African context, such learners are, predominantly, Black learners who are located in poorly resourced schools situated in lower socio-economic environments. The positioning of Mathematical Literacy as qualification that involves engagement with only limited forms of mathematical participation has serious consequences for the (increasing numbers of) 'weaker' learners who engage in the subject. These learners are not only denied access to an educational experience that would better prepare them for life and the world of work, but also to a vast array of study and career opportunities which would facilitate upward social and economic mobility. As such, I contend that the existing structure of mathematically legitimised participation in the subject Mathematical Literacy in the South African secondary schooling system contributes to the (re)production and sustainment of educational, social and economic disadvantage.

In light of the contents of the discussion above, this Interpretivist-oriented study is motivated in large part by a two-fold intention. Firstly, to come to understanding in a clearer and more comprehensive way – and from a theoretically informed perspective – the current nature of practice in the specific empirical terrain of the subject-matter domain of Mathematical Literacy, and, particularly, the problematic nature of this practice. This dimension of the study is directed by the following research question:

Research Question #1

In what ways does Dowling's (1998) language of description provide a means for problematising current teaching and learning practices associated with the school subject Mathematical Literacy?

As suggested in the above research question, I employ Paul Dowling's (1998) work on 'domains of mathematical practice' to investigate and problematise the structure of current practice and participation in the subject-matter domain of Mathematical Literacy (c.f. Part 3 on page 132 below). In employing this work, I argue that current practices in the subject (classified as 'Public' and 'Expressive domain' of mathematics practices) imbue a form of participation that denies access to apprenticeship in the domain of mathematics, and, instead, position participants, primarily, in the roles of dependents or objects in the learning process. In these positions, participants are denied access to the esoteric domain mathematical principles that define and structure the basis of legitimate participation in the practices of the subject. Participants are also presented with mathematised and mythologised representations of reality that do not provide adequate preparation for engagement in the practices of the world outside and beyond the classroom. And, it is in this 'no-mans-land' that the practices of the subject-matter domain of Mathematical Literacy embody a limiting and disadvantage-sustaining qualification.

In a response to this theoretically described state of affairs, the second intention for the study is focused on describing an alternative structure of practice, participation and knowledge in the subject that alleviates and negates the challenges and concerns related to the current structure of practice and participation. This intention for the study is directed by the following research question:

Research Question #2
What characterises mathematical literacy as a knowledge domain? What might the regulating principles of mathematical literacy be when taking into account previous work on the nature of mathematical literacy?

Crucially, this second intention represents the central component of this study: namely, the development of a theoretical language for describing the practices and associated forms of knowledge and legitimate participation that characterise a particular conception of the knowledge domain of mathematical literacy (c.f. Part 4 on page 181 below). This conception of the knowledge domain of mathematical literacy is characterised by an agenda for contextual sense-making practices and an intention for critical evaluation of existing mathematical and contextual structures, which, in combination, give rise to a life-preparedness orientation for the subject. This life preparedness orientation posits mathematically literate behaviour, as a particular form of enacted literacy, as comprising both the ability to interpret, understand, and critically evaluate and engage contextual sense-making practices (and the use of mathematics in this process), together with the ability to communicate results, decisions, workings and conclusions in a contextually and mathematically appropriate and accessible manner. In developing and presenting this theoretical language of description, I argue that since Mathematical Literacy is separated from the domain of scientific mathematics, a description of practice for the knowledge domain of mathematical literacy is required in which participation is legitimised according to something other than mathematically dominant structures. Instead, the esoteric domain of mathematical literacy is characterised by four domains of practice, including (i) Everyday, (ii) Mathematical Competency, and (iii) Modelling knowledge and practices, and a domain of (iv) Reasoning and Reflection that facilitates critical engagement with both contextual and mathematical contents and structures encountered in problem-solving processes. These domains of practice are, further, grounded in a Contextual Domain – constituted as the public domain of mathematical literacy – which facilitates and necessitates engagement with authentic real-world environments and resources. Crucially, it is through engagement with all of these domains of practice that the life-preparedness orientation conceptualised for the alternative description of the knowledge domain of mathematical literacy is facilitated. And, since current practices in the subject-matter domain of Mathematical Literacy are dominated by mathematised forms of participation, I argue that the characterisation of this life-preparedness orientation through engagement in the four domains of practice serves as a potential means to negate and alleviate existing concerns with these current practices.

Importantly, notice that the combination of the first and second research questions signifies two different levels of engagement in this study. Specifically, the particular *empirical terrain* under investigation is that of the subject-matter domain of Mathematical Literacy. However, to facilitate a description of the practices of this terrain engagement is necessary with the *knowledge domain* on which the practices of this empirical domain are based. The theoretical language of description developed in the study reflects engagement in the knowledge domain of mathematical literacy; and, the necessity for this engagement at the level of knowledge production is facilitated through theoretical analysis of current practices in the empirical terrain of the subject. To facilitate the use of this theoretical language of knowledge for analysis of practices relating to the empirical terrain of the subject-matter domain of Mathematical Literacy, a further element is necessary – an 'external' dimension or component of the language of description. The development of this external component is directed by a third research question:

Research Question #3

What would constitute an external language of description that would enable a (re)description of an empirical space within the terrain of the subject Mathematical Literacy in terms of reference of mathematical literacy as a knowledge domain?

The developed and presented external component of the language of description draws predominantly on the work of Anna Sfard (2007, 2008) and involves identification and analysis of the unique and distinctive *discursive resources* that characterise practices and forms of discourse associated with the reconceptualised knowledge domain of mathematical literacy. I argue that each of the domains of practice of the knowledge domain of mathematical literacy are characterised by uniquely identifiable discursive resources. Identification of the structure of the discursively mediated practices that characterise the subject Mathematical Literacy in relation to the discursively mediated practices that characterise the domains of practice of the knowledge domain of mathematical literacy serves a crucial function. Namely, this process provides a means for comparing and distinguishing the structure of legitimate participation and knowledge in different empirical practices and activities. In other words, the components of the external dimension of the language of description provide a lens for analysis of empirical practices in the subject through the components of the internal dimension of the language. This application of the external dimension of the language for the analysis of empirical practices associated with the subject-matter domain of Mathematical Literacy is directed by the fourth research question:

Research Question #4

a. *How can the external language of description, in conjunction with the internal language of description for mathematical literacy, be used to determine the dominant basis of legitimisation in a segment of the Mathematical Literacy curriculum, a textbook, course notes for a teacher education course, and national assessment items?*

b. *How can identification of the dominant basis of legitimisation in these empirical spaces be used to determine coherence or disjunction within and between practices and discourse associated with the subject Mathematical Literacy?*

As highlighted in the components of the research question given above, the utility and validity of the internal dimension of the language of description as an instrument for analysis and comparison of the structure of participation in practices associated with the knowledge domain of mathematical literacy is demonstrated in relation to various textual resources encountered in the terrain of the subject-matter domain of Mathematical Literacy (c.f. Part 7 on page 381). As such, the primary methodology employed in the empirical analysis phase of the study is that of textual analysis, accompanied by a method of discourse analysis – which, in turn, employs elements of the field of semiotics through focus on the signifiers in a text (c.f. Part 6 on page 360).

Although mentioned briefly above, it is important to reiterate that the primary component of this study is the development of a theoretical language of description. In other words, this study is an attempt at the production and description of the components and characteristics of a domain of knowledge (and of the practices and associated with that domain and the forms of legitimate and endorsed participation that characterise those practices). In this sense, the developed language of description is an analytical concept and not an empirical substance. This theoretical language has been developed in response to a particular problematic scenario existing in the specific empirical terrain of the subject-matter domain of Mathematical Literacy. As such, this empirical terrain again provides a necessary and appropriate site for testing of the utility and validity of the theoretical language as a means for describing and comparing the structure of legitimate participation prioritised in different empirical practices. I make this point deliberately to emphasise that it is the development, theorisation and testing of the theoretical language of description that is of central concern. The empirical terrain and practices of the subject-matter domain of Mathematical Literacy, by contrast, provide a site of motivation and application for this theory-development process and, additionally, for testing and illustrating the descriptive power of the language. It is in light of this motivation, intention and structure that the title of the study is posed as: *The basis of legitimisation of Mathematical Literacy in South Africa*.

Since no attempt has been made either in the South African literature or in international work to describe an alternative structure of knowledge for the knowledge domain of mathematical literacy in specific relation to the empirical terrain of the subject-matter domain of Mathematical Literacy in South Africa, this study constitutes an original work. There do exist a small number of South African studies which identify, highlight and describe existing structures of participation in certain practices in the subject (see, for example, (Geldenhuys, 2009; Venkatakrishnan & Graven, 2007)) and possible problematic intentions and forms of participation in the subject (see, for example, (Christiansen, 2006; Julie, 2006)). However, there are no studies that reconceptualise the domain of knowledge on which the practices in the subject-matter domain of Mathematical Literacy are based to promote an alternative and more empowering form of participation in the subject. All of the above signifies the originality, utility and importance of the contribution of this study to the existing body of literature on the subject-matter domain of Mathematical Literacy. Furthermore, this study also contributes to a wider and more general (and largely international) body of literature that offer descriptions of practices and associate forms of participation and knowledge that characterise differing conceptions of mathematically literate (and numerate and/or quantitatively literate) behaviour.

SCOPE, KEY FINDINGS AND LIMITATIONS OF THE STUDY

The scope of this study also gives rise to the key limitations of the study. Namely, the developed language of description has specific application to a domain of knowledge (and associated practices) characterised by a particular structure of relationship between mathematical and contextual terrains, practices and knowledge. Specifically, the presented theoretical language of the knowledge domain of mathematical literacy describes a structure of participation for practices encountered outside of the domain of scientific mathematics contents. In this reconceptualised knowledge domain, a life-preparedness orientation and associated critical contextual sense-making practices (instead of mathematical structures and a goal for the development of mathematical knowledge) are cited as the primary goal of pedagogic practice. It could be argued, then, that the language has a limited and restricted scope of applicability and transferability. In response, my position is that the language reflects an attempt to supplement existing theories that explore legitimised forms of participation involving engagement with both mathematical and contextual elements and makes no claims to be able to offer descriptions of practices that are legitimised according to esoteric domain mathematical structures.

A second issue worth consideration stems from my commitment to an Interpretivist paradigmatic orientation. This paradigmatic orientation is accompanied by a commitment to an ontological position on the socially constructed nature of knowledge. This orientation is further characterised by a sociological impetus for analysing structures to unveil elements of educational disadvantage which are supported by the structure but which may not be immediately obvious and visible. A chosen methodology of textual analysis that is highly interpretive and subjective is the final component of this orientation. The consequence of the above is that any deductions made in the study are linked to a high degree to my own ontological and epistemological perspectives – my particular 'world-view' – of the research process and also of my place in the world. This, again, impacts on the extent to which any deductions made in the study are able to be generalised to a different empirical terrain and/or practice.

Despite these limitations, I argue (in Part 7 of the study – see page 381 below) that the demonstrated textual analysis process illustrates the utility and validity of the language of description. In particular, this process illustrates the utility and validity of the language for identifying, describing and comparing the criteria according to which participation in different empirical activities relating to the knowledge domain of mathematical literacy is legitimised and endorsed. In specific relation to the analysed empirical texts, I argue that it is only the texts associated with the teacher education course which reflect the life-preparedness orientation espoused for and prioritised in the presented language of description for the knowledge domain of mathematical literacy. The curriculum document for the subject also reflects an expectation for engagement with all of the domains of practice of the described knowledge domain of mathematical literacy, but falls short of this expectation when describing the contents of particular curriculum topics and sections. The exemplar examination papers, by contrast, prioritise an exclusive emphasis on Mathematical Competency domain practices at the expense of engagement in any of the other domains (which reflects, in large part, consistency with the structure of practice and participation currently identified – in Part 2 of the study – as contributing to the concerns raised about the subject). Finally, the textbook is characterised by a form of participation that reflects consistency with the Everyday and Mathematical Competency domains of practice, but with no expectation whatsoever for Modelling related practices. I conclude the study by arguing that although the language of description is able to highlight differences in the structure of legitimate and endorsed practice, participation and knowledge prioritised in the texts, there are also several components of the language that requires reworking and/or modification (in future research projects) to further enhance the utility and validity of the language.

STRUCTURE OF THE STUDY

The study is structured in a way that reflects the sequence of the stated research questions and which facilitates the various intentions described above. In this regard, the study is comprised of seven 'Parts', with each part further comprising several chapters. In Part 1 I highlight theoretical considerations of importance in the study, including my commitment to an Interpretivist paradigmatic orientation. The implication of this orientation on the ontological and epistemological perspectives that inform the research process and on the chosen methodology of textual analysis are also discussed. Various considerations involved in the development of a theoretical language are explored. In Part 2 I highlight different perspectives on the structure of knowledge and behaviour associated with differing conceptions of mathematical literacy, numeracy and quantitative literacy. I introduce the empirical terrain of the subject-matter domain of Mathematical Literacy, distinguish mathematical literacy from scientific mathematics, and highlight areas of similarity and divergence to more general perspectives on mathematical literacy, numeracy and quantitative literacy. I rationalise my privileging of the term mathematical literacy and emphasise that this construct is comprised of a collection of various literacies – including numeracy, quantitative literacy, statistical literacy, textual literacy, spatial literacy, and context literacy. I argue that this collection of literacies facilitates a form of mathematically literate behaviour oriented towards life-preparedness, which, in turn, is characterised by the ability to interpret, understand, engage and critically analyse contextual problem-solving scenarios, and to communicate working, results and decisions in a contextually appropriate and accessible manner. In some ways, this part of the study provides a review of relevant literature on the domain of mathematical literacy. In Part 3 I employ Dowling's (1998) theoretical 'domains of mathematical practice' schematic to problematise current practices in the subject. Certain limitations of Dowling's theoretical language are discussed. In Part 4 I identify the need and rationale for an alternative description of the structure of knowledge in the knowledge and associated subject domain of Mathematical Literacy, and present a theoretical description of the internal components of this knowledge domain. This part of the study represents, in part, the theoretical framework dimension of the study. In Part 5 I present an external component of the language of description. This external component facilitates the use of the theoretical components of the internal dimension of the language for analysis of empirical practices and resources relating to the subject-matter domain of Mathematical Literacy. I also reflect on certain limitations of the external component of the language encountered during the utilisation of this component in the analysis of empirical text (the process of which is described in Part 7). Prior to utilisation of the internal and external components of the language of description in analysis of empirical textual resources for the subject-matter domain of Mathematical Literacy in Part 7, in Part 6 I discuss methodological considerations for the methodology of textual analysis and highlight the central roles of the methods of discourse analysis and semiotics in the textual analysis process employed in Part 7 of this study. In both of these parts I highlight limitations and issues concerning validity in relation to both the employed methodology of textual analysis and the developed theoretical language. Finally, I conclude the study by reflecting and sharing on my experiences in the development of the theoretical language of description and highlight various limitations of the study.

PART 1
THEORETICAL (AND OTHER) ANTECEDENTS

INTRODUCTION AND OVERVIEW

The intention of this part of the study is to outline certain theoretical considerations that have informed the development of this study. In particular, the key area of focus of this study: namely, the development of a 'language of description' of the structure of knowledge associated with a form of the knowledge domain of mathematical literacy (and associated practices and forms of participation in the South African high school subject-matter domain of Mathematical Literacy) that promotes the development of a life-preparedness orientation. The discussion in this regard moves from a macro to a micro level of analysis. Namely, from consideration of the positioning of the study as a whole within a particular paradigm and educational framework, to a more localised discussion on the contents of the study. This latter level of analysis is achieved through exploration of the components and theoretical grounding of the language of description developed in the study.

This part of the study is divided into four chapters (Chapter 1 to Chapter 4). In Chapter 1 I position the study within an interpretive paradigm in the field of mathematics education, and highlight the ontological, epistemological and methodological positions that stem from this paradigmatic orientation. In Chapter 2 I argue that the key intention of this study is the development of a 'language of description' (both internal and external) of the structure of knowledge associated with a particular form of the knowledge domain of mathematical literacy and explore more precisely what is meant by a language of description. In this chapter I also explore various facets requiring consideration with respect to the development of such a language, including identifying the claims and concepts of the language and the mode of theorising adopted in the language. In Chapter 3 I expand on the characterisation of the study as the development of a language of description of a knowledge structure by locating the study in the Field of Knowledge Production within Bernstein's (2000) pedagogic device, and discuss the limiting perspective offered by the study as a result of this categorisation. In the final chapter (Chapter 4) I conceptualise and clarify the dominant orientation that characterises the language of description for the knowledge domain of mathematical literacy – namely, a 'life-preparedness orientation'.

This chapter structure and the positioning of this study within the field of educational research are illustrated in Figure 2 below, with the hierarchy of the diagram intended to illustrate the movement from a macro to a micro level of discussion.

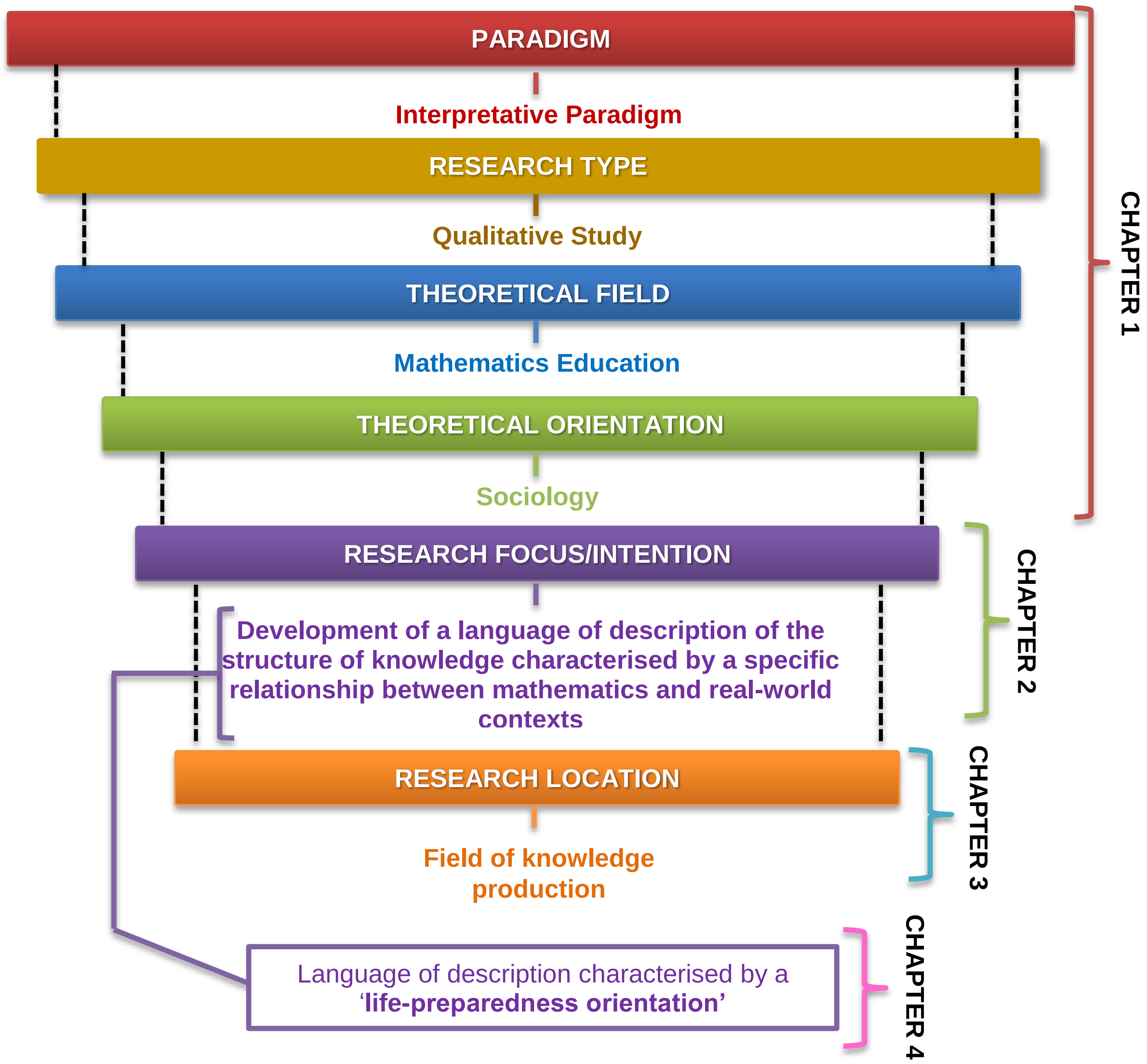


Figure 2: Overview and chapter structure of <u>Part 1</u> of the study

CHAPTER 1
RESEARCH METHODS, PARADIGMS AND MATHEMATICS EDUCATION THEORIES

In this chapter I position this study within an interpretive paradigmatic orientation and outline the ontological, epistemological and methodological positions that characterise this paradigmatic orientation. I argue further that this study is driven by a sociological impetus and is characterised by a qualitative research process that is located in the field of mathematics education.

1.1 Positioning this study in an interpretive paradigm: considerations of ontological, epistemological and methodological orientations

A paradigm may be viewed as a set of *basic beliefs* (or metaphysics) that deals with ultimates or first principles. It represents a *worldview* that defines, for its holder, the nature of the "world," the individual's place in it, and the range of possible relationships to that world and its parts … (Guba & Lincoln, 1994, p. 107) (emphases in original)

Drawing from the above, a paradigm is a particular way of looking at the world, informed by the values and assumptions of the researcher, which directs the focus and form of the intellectual structure on which the research process is based (Kuhn, 1962). The particular paradigm in which a researcher is positioned, then, affects not only what is perceived to fall "within and outside the limits of legitimate inquiry" (Guba & Lincoln, 1994, p. 108). Instead, what is also affected is how the researcher views the world and their own role or subjectivity in the research process, the kinds of questions that are asked with respect to a particular inquiry and how these questions are to be scrutinized, and the type of methodology that is chosen by the researcher to gather information relating to these questions.

Guba and Lincoln (1994, p. 108) argue that the basic beliefs that define the dominant view or perspective in a paradigm are encapsulated through the responses that would be given by proponents of a particular paradigm to three fundamental questions. Firstly, the *ontological question* – "What is the form and nature of reality and, therefore, what is there that can be known about it?" Secondly, the *epistemological question* – "What is the nature of the relationship between knower or would-be-knower and what can be known?": namely, how does the researcher perceive knowledge is produced, acquired and communicated (Scotland, 2012, p. 9) and their influence over the knowledge being collected or uncovered, or do they perceive that they are external to it and/or do not influence it (University of South Hampton, 2011). And thirdly, the *methodological question* – "How can the enquirer (would-be-knower) go about finding out whatever he or she believes can be known?" (All quotations above: Guba & Lincoln, 1994, p. 108). In short, the questions address the issue of what – in the eyes of the researcher - constitutes reality, how does the researcher position themselves with respect to this reality, and how will the researcher go about finding out something about the reality? These three questions provide a "holistic view of how we view knowledge, how we see ourselves in relation to this knowledge and the methodological strategies we use to un/discover it." (University of South Hampton, 2011) – in other words, these three questions provide a view or perspective of the researcher on "what is", "what it means to know”, and the “why, what, from where, when and how” of the data collection process (Scotland, 2012, p. 9). Collectively, then, these three questions provide not only a point of analysis for individual paradigms but also points of comparison between different paradigms.

The specific paradigm that is reflected in this study, and which represents my own world view on the topic under scrutiny in this study, is that of the *Interpretive (or Social Constructivism) paradigm*. From this paradigmatic orientation, the dominant *ontological* position is that of relativism (Scotland, 2012, p. 11): namely, ‘reality’ is viewed as a social construction such that different cultures are seen to make sense of the world differently as influenced by the specific social, economic, political and cultural environment and issues which define and affect participation in the culture at particular points in time. As such, from this ontological perspective there is no single or accurate reality. Rather, an experience of reality is dependent on the culture and context in which a person finds themselves at a particular point in time – reality is “socially and experientially based” (Guba & Lincoln, 1994, p. 110); and this reality shifts over time, between cultures, and even within a culture between different groups (McKee, 2003, pp. 9-10). In other words, reality is subjective, is individually constructed, and there are multiple possible realities for differing individuals, groups, and cultures (Scotland, 2012, p. 11). This viewpoint is supported by constructivist and sociological perspectives that promote the notion of the social construction of knowledge and of an individual’s relation to that knowledge (McKee, 2003, pp. 9-10).

This ontological perspective of multiple realities gives rise to an *epistemological* position that views the potential for the existence of ‘multiple knowledges’ (Guba & Lincoln, 1994, p. 113). Accordingly, the world is seen to not exist independently of our descriptions of it: “Meaning is not discovered; it is constructed through the interaction between consciousness and the world.” (Scotland, 2012, p. 11). Subjectivity is, thus, a key component of the interpretative epistemology, characterised by a research process involving constant interaction between the researcher and the object of investigation (Guba & Lincoln, 1994, p. 111). Furthermore, this position is accompanied with explicit recognition of the influence of the researcher’s own values and social, economic, cultural and political orientations on choices and decisions made during the research process and, hence, on the intention, structure and outcome of this process (Scotland, 2012, p. 12). Given this adherence to relativism and a perspective on the potential for multiple knowledges, the primary aim of the interpretivist research process is understanding of the ways in which different individuals or groups of people perceive their reality and their place in that reality (Guba & Lincoln, 1994, p. 113). Importantly, this intention for understanding can also be accompanied by intentions for “Advocacy and activism” (Guba & Lincoln, 1994, p. 113). In other words, interpretative research need not be exclusively descriptive and can, instead, be accompanied by critical components for change. This issue is particularly pertinent to the sociological impetus of this study (see page 15 below) in which a key intention in the study is the modification of the existing structure of knowledge in the subject Mathematical Literacy to overcome perceived structures of educational disadvantage afforded through participation in the subject.

Methodologically, Interpretivism promotes the use of research methods that facilitate investigation, analysis, interpretation and understanding of how different people and groups perceive reality and their place in that reality, together with social and cultural conditions that influence perceptions of reality (Scotland, 2012, p. 12). Interpretive methodology also promotes the use of methods that facilitate interpretation of varied and multiple representations of reality from an individual's perspective through direct interaction between researcher and the object of investigation (Guba & Lincoln, 1994, p. 111). It is as a result of this positioning of the researcher as an active transactional participant in the construction and understanding of meaning in socially constructed representations of reality that lead Guba and Lincoln (1994, p. 111) to categorise Interpretivist methodology as "Hermeneutical and dialectical". In other words, understanding of how others associate meaning to their own representations of reality is able to be deemed through derivation of expressed and/or hidden meaning in texts (i.e. hermeneutics) or through dialogue between researcher and the objects of the research process.

Having outlined the general characteristics of the ontological, epistemological and methodological positions associated with the interpretative paradigm, it now becomes possible to reflect on the significance of these positions with respect to the commitment in this study to an interpretative paradigmatic orientation. *Ontologically*, the research process in this study is driven by an intention to interpret, analyse and understand the dominant perception of the 'reality' of the structure of knowledge and participation in the subject-matter domain of Mathematical Literacy. This commitment is further characterised by an intention to understand how this reality is differently experienced by various role-players who participate in the subject (e.g. teachers vs. curriculum developers vs. examiners of national exam papers vs. textbook authors vs. teacher trainers). Understanding of how this reality is different to the perceived reality in other subjects (such as Core [scientific] Mathematics) is also in focus. This ontological position recognises that these multiple representations of reality of knowledge and participation for the subject are directly influenced by various political, social, economic and cultural factors, and, particularly, by pressure within the South African education system to prioritise mathematical knowledge as a preferenced form or structure of knowledge. However, in interpreting existing perceptions of reality for the subject Mathematical Literacy, there is also an impetus for 'activism' (Guba & Lincoln, 1994, p. 113). Namely, my own position is that the existing reality – which I claim is dominated by a prioritising of mathematical knowledge structures as the basis for legitimate participation in the subject – supports the promotion of Myths of Reference, Participation and Emancipation (Dowling, 1998)[2]. Consequently, a degree of educational disadvantage with respect to future career, work and social advancement opportunity is sustained and reproduced through participation in the subject. As such, a key intention of this study is for the development of an alternative 'reality' of the structure of knowledge and participation for the subject-matter domain of Mathematical Literacy in which a life-preparedness orientation is prioritised over the development of mathematical knowledge. This alternative reality is theorised and discussed in detail in Part 4 of the study where I present an alternative 'language of description' for the knowledge domain of mathematical literacy.

Drawing from an *epistemological* position that views the potential for the existence of 'multiple knowledges', the research process in this study acknowledges varying possible interpretations and descriptions of the ways in which role-players in the subject Mathematical Literacy (as the object of study) perceive the structure of legitimate participation in the subject. The particular interpretation adopted in this study, as informed by my historical, social, economic, political and theoretical orientation, construes existing structures of participation as problematic in relation to issues of educational, social and economic positioning and access. As such, my interpretation leads

[2] Refer to Part 3, Chapter 10 and sub-section 10.2.2 starting on page 156 below for a detailed discussion of these concepts.

me to question current pedagogic and assessment priorities in the subject and to present an alternative understanding of a structure of participation which aims to negate existing forms of positioning. It is this interpretation which has directly informed the structure of the developed alternative internal language of description for the knowledge domain of mathematical literacy (see Part 4 starting on page 181 below) and the promotion of a 'life-preparedness' (Venkat, 2010) orientation3 for the subject over the prioritising of mathematising processes in this alternative language of description. This is evidenced through the composition of the language of description of knowledge domains of practice that prioritise modelling processes (the 'Modelling' domain of practice) and understanding of patterns and structures of everyday practices (the 'Everyday' domain of practice) alongside mathematical structures (the 'Mathematical Competency' domain of practice). This interpretation of the structure of participation for the subject has also directly informed the structure of the external language of description developed to facilitate the means through which the internal language (and characteristics of the associated domains of practice) can be used as a lens for analysis of empirical resources relating to the subject (c.f. Part 5 starting on page 253). This external language of description focuses on identification and interpretation of the discursive resources (signifiers [words/vocabulary and visual mediators], routines and narratives) embedded and indexed in discursive practices in the subject, and is accompanied by recognition of the highly interpretive and subjective nature of this process of engaging with discursive resources and of any assumptions or conclusions made as a result of this process. My commitment to an interpretivist paradigmatic orientation has, thus, influenced the deliberately construed and highly interpretive description of the structure of knowledge and participation in the subject-matter domain of Mathematical Literacy adopted in this study, and also the "Transactional and subjectivist" (Guba & Lincoln, 1994, p. 111) role of me as the researcher in this process.

Methodologically, the ontological and epistemological orientations adopted in this study necessitate a specific methodology. In particular, this methodology must facilitate interpretation and analysis of how the practices of participants in the subject either reflect coherence or divergence with the alternative conception of knowledge and participation developed for the subject-matter domain of Mathematical Literacy (presented through the internal language of description and operationalised through the external language of description). This methodological intention is accompanied by a descriptive rather than normative approach in this sense that there is recognition that the structure of knowledge and participation as presented in the internal language of description for the domain of mathematical literacy presented a particular, a limited, and only one of many possible 'world-views' of the structure of knowledge and participation for the subject. The specific site of empirical analysis involves textual resources (i.e. a textbook section, a section of the curriculum, Grade 12 national examinations, and course notes for a teacher education course) that reflect different aspects of knowledge and participation in relation to the subject-matter domain of Mathematical Literacy. As such, the methodology selected and employed to facilitate analysis of these empirical resources in a way that is consistent with interpretivist ontological and epistemological orientations is that of textual analysis, as a sub-set of a wider methodology of discourse analysis (c.f. Part 6 starting on page 360). This method is drawn from the realm of hermeneutics (which is identified by both Guba and Lincoln (1994, p. 111) and Scotland (2012, p. 12) as reflecting interpretivist intentions). Hermeneutics involves analysis of the discursive resources (signifiers [words/vocabulary and visual mediators], routines and endorsed narratives) embedded

[3] Refer to Chapter 4 on page 26 below for an elaborated discussion of the notion and components of a life-preparedness orientation.

in (and associated discourse communicated through) textual resources for the subject. This process also involves comparison of these discursive resources to the characteristics of the discursive resources of the domains of practice that constitute the internal language of description for the knowledge domain of mathematical literacy. This methodology provides the means for identification and interpretation of possible 'world-views' of the authors of these texts and, specifically, of how these authors perceive the structure of legitimate knowledge and the criteria for legitimate participation in practices involving the use of mathematics in contextualised problem situations. And since this methodology recognises the interpretative, subjective and transactional nature of the investigative research process, the methodology is entirely consistent with interpretivist ontological and epistemological orientations.

Ontologically, epistemologically and methodologically, then, this study is driven by an interpretivist paradigmatic orientation, albeit with an impetus for activism directed towards understanding how knowledge and practices associated with the subject-matter domain of Mathematical Literacy can be changed to facilitate that participation in the subject does not reinforce social and/or educational disadvantage and positioning.

1.2 Positioning this study as a form of qualitative research

> Despite many proposed differences between quantitative and qualitative epistemologies, ultimately, the heart of the quantitative-qualitative "debate" is philosophical, not methodological. (Krauss, 2005, p. 759)

As suggested by the above quotation, although quantitative and qualitative studies are often differentiated according to the type of methodology employed in gathering and analysing information about an aspect of reality, these differences in methodology are the result of a deeper and, ultimately, epistemological difference between the nature of quantitative and qualitative research.

For Krauss (2005, p. 759), "Epistemology poses the following questions: What is the relationship between the knower and what is known? How do we know what we know? What counts as knowledge?" The difference between quantitative and qualitative research is then defined in large part by the differential answers to each of these questions for each type of research. For quantitative research, the object of study is viewed as independent of the researcher and, so, it is possible to use a quantitative analysis of the object, deemed through observation rather than direct interaction, and to make connections across multiple different realities. Qualitative research, on the other hand, deems that knowledge is directly attached to specific instances or phenomenon and that it is not possible to separate meaning from this instance. As such, it is only through direct interaction with the phenomenon and the object(s) or subject(s) under investigation that a more complete view of the phenomenon is gained. From this perspective, an exclusively numerically based analysis of a situation that does not taken into account the specifics of the individual object(s) or subject(s) in the phenomenon is incapable of providing adequate interpretation or description of the situation. Furthermore, given the distinctive nature of each phenomenon and the need for direct intervention in the phenomenon, it is not possible for a single tool to facilitate connections across multiple realities since every reality is unique and different (Krauss, 2005, pp. 759-760).

In the previous section it has already been established that this study falls within an interpretivist paradigm, hereby acknowledging the role of the researcher in shaping the ultimate form and assumptions of the study. Following on this line of thinking, there should be no doubt in the mind of the reader that that this study is directly influenced by my own interests, perspectives, and values. For this reason, the object of scrutiny in this study – namely, the structure of knowledge that characterises the specific relationship between mathematical content and real-world contexts envisioned for pedagogic practice in the subject Mathematical Literacy – simply does not avail itself to a quantitative analysis. Rather, a *qualitative analysis* is necessitated by the distinctive and unique nature of the subject Mathematical Literacy in the South African context. This form of analysis is driven by my own personal interest in understanding the way in which different groups of participants perceive 'reality' for the subject − and particularly for preferencing a version of 'reality' in which preparation for life is prioritised over the learning of mathematical knowledge.

In summary, then, this study is informed by a *qualitative interpretivist paradigmatic* research framework.

1.3 Positioning this study within the field of the sociology of mathematics education

Alongside an interpretivist paradigmatic orientation, the study also falls within a particular theoretical field: namely, *mathematics education*. And within this field, the study is directed in large part by a *sociological* impetus.

According to Ensor and Galant (2005),

> Sociology, broadly speaking, is the study of social actors, of the groups to which they belong (social groups, such as families, social classes and age groups, or cultural groups such as sports and leisure clubs), of the relationships between these groups, and the distribution across them of symbolic and material resources. The objects of sociological study are thus individuals, groups and/or institutions and their distinctive practices. (p. 282)

Dowling (1998, p. 1) offers a similar explanation by emphasising that the focus of a sociological study is the nature of the relationships between individuals and groups and how those relationships are produced and reproduced through different activities, practices and actions.

Reflecting on these explanations in relation to the impetus for and intention of this study: a key impetus for this study is concern over the existing structure of knowledge and participation in the subject-matter domain of Mathematical Literacy and relations of social and/or educational disadvantage that arise from this structure. This impetus has given rise to an intention for the development of an alternative language of description of a structure of knowledge for the knowledge domain of mathematical literacy that aims to negate existing concerns through the promotion of a life-preparedness orientation over the prioritising of mathematisation processes. This impetus and intention are, thus, driven by a sociological concern over the way in which the relationship between the mathematical and the contextual terrains are conceptualised in the structure of legitimate knowledge and participation in the subject. The impact of different conceptualisations of this relationship (and associated criteria for legitimate knowledge and participation) on issues of social positioning, educational ad/disadvantage, and career mobility or restriction are also under analysis.

CHAPTER 2
THIS STUDY AS THE DEVELOPMENT OF A 'LANGUAGE OF DESCRIPTION'

The primary intention of this study is the development and attempted validation of a 'language of description' of the structure of knowledge associated with a form of the knowledge domain of mathematical literacy that promotes the development of a life-preparedness orientation. This language of description comprises a *conceptual framework* that encapsulates aspects associated with mathematically literate behaviour in general and, specifically, with the structure of knowledge and the criteria for legitimate participation in the school-based subject-matter domain of Mathematical Literacy. In light of this, it is feasible to make a diversion into certain considerations relating to the concept of a 'theory'. This includes consideration of Bernstein's (2000) notions of the internal and external languages of description of a theory and Jablonka and Bergsten's (2010) work on different modes of categorisation of a theory. A discussion of the components of a theory (including the claims and concepts that characterise and constitute the theory) are also provided. Finally, the relevance of these theoretical and research based considerations to the language of description developed in this study are considered.

2.1 Internal and external languages of description

2.1.1 General theory on languages of description

Bernstein (1996, 2000), in arguing that an accurate analysis of an empirical space is impossible without an underlying theoretical basis (Morais, 2002, p. 564), presents the concept of *internal* and *external languages of description* to describe the relationship between a theory and the description that the theory provides of a specific empirical activity or resource under investigation (Jablonka & Bergsten, 2010, p. 39).

> Briefly, a language of description is a translation device whereby one language is transformed into another. We can distinguish between internal and external languages of description. The internal language of description refers to the syntax whereby a conceptual language is created. The external language of description refers to the syntax whereby the internal language of description can describe something other than itself. (Bernstein, 2000, p. 132)

In alternative terms, the internal language of description represents the language, components and/or various facets that make up the *theory* – that which is <u>internal</u> to the theory, and the external language of description represents the *methodology* used to establish a description of an empirical activity through the 'gaze' of the theory – that which is <u>external</u> to the theory:

> a theory includes an organised system of theoretical entities, basic principles [internal language of description], and a relation to an empirical field in the form of a more or less explicitly developed methodology [external language of description]. (Jablonka & Bergsten, 2010, p. 27, text in brackets inserted by me)

The internal language of description − the theory − comprises a conceptual language, operating at a high level of abstraction, with explicit theoretical indicators, constructs, grammars and syntax (i.e. rules for describing the components of the theory) and with descriptions of the relationship between different entities in the theory (Jablonka & Bergsten, 2010, p. 39; Morais, 2002, p. 564). All of these are *internal* to the theory. Given that the theory operates at a high level of abstraction, in order to extend the theory to describe something other than itself, an external language of description must be developed for "transforming observed empirical instances of a phenomenon of interest into theoretically relevant data." (Jablonka & Bergsten, 2010, p. 39). The external language of description is comprised of concepts, models and propositions derived from the internal language but with greater applicability (i.e. lower abstraction) (Morais, 2002, p. 564), which are external to the theory, and which work towards bringing the theory closer to the data under analysis (Hoadley, 2006a, p. 24). In Bernstein's words,

> Internal languages are the condition for constructing invisibles, external languages are the means of making those invisibles visible … the external language of description (L2) is the means by which the internal language of description (L1) is activated as a reading device and vice versa. (Bernstein, 2000, p. 133)

And further:

> One of the difficulties of much social theory is that these theories have a powerful and persuasive internal conceptual language but reduced powers to provide externally unambiguous descriptions of the phenomena of their concern. Thus, researchers have difficulty in using the theory to generate the language which will transform the language of enactment, that is the text they are studying (interviews, visual, graphic representations, etc.) into a language which can be read by the theory. These theories rarely generate a language of description: a language which can transform the language of enactment, into a language which the theory can directly read. (Bernstein & Solomon, 1999, p. 274)

Morais (2002, p. 564) illustrates this relationship between internal and external language of description in the following diagram (Figure 3 below), albeit directed specifically towards the social relations of pedagogic activity (which is the primary focus of her own work):

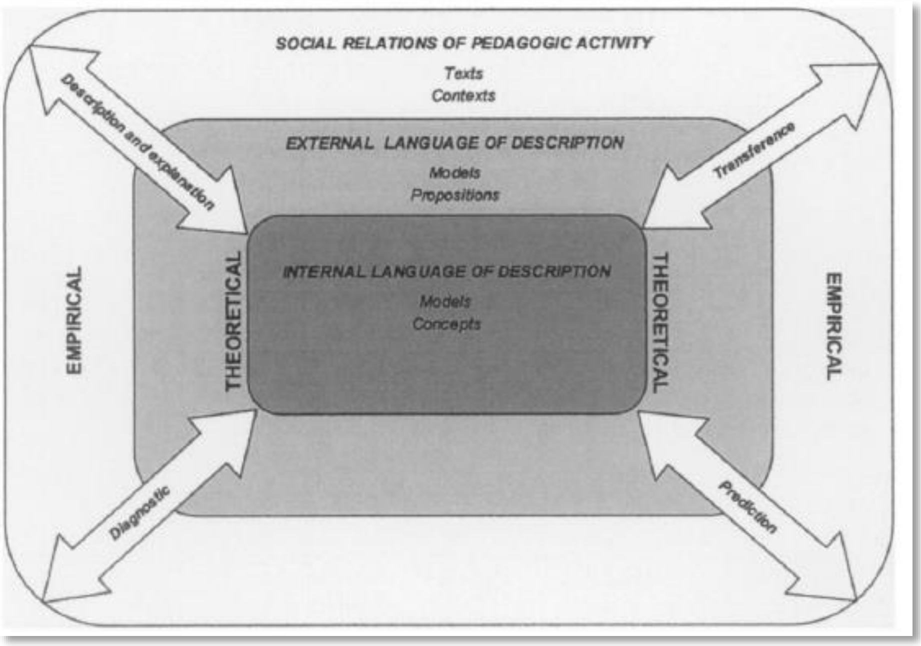


Figure 3: Relationship between internal and external languages of description

Notice that the arrows on the diagrams are bidirectional − flowing from the theoretical to the empirical and from the empirical to the theoretical. This illustrates the “dialogic relationship” (Dowling, 1998, p. 124) between the internal and external languages of description: the internal language of description directs the external language of description in describing the empirical; inversely, the results from the analysis of the empirical (re)inform the theory so that the theory is transformed into a more accurate and precise description of the empirical (Jablonka & Bergsten, 2010, p. 39; Morais, 2002, p. 564).

The extensive quotation below concludes this brief sojourn into the notion of languages of description and illustrates in a comprehensive way the intricate and complex nature of the interaction between theory and method in analysing an empirical space.

> If verticality has to do with how theory develops internally, with what Bernstein later called the internal language of description, grammaticality (in the external sense) has to do with how theory deals with the world, or how theoretical statements deal with their empirical predicates, the external language of description (Bernstein, 2000). The stronger the (external) grammaticality of a language, the more stably it is able to generate empirical correlates and the more unambiguous because more restricted the field of referents; the weaker it is, the weaker is its capacity to stably identify empirical correlates and the more ambiguous because much broader is the field of referents, thus depriving such weak grammar knowledge structures of a principal means of generating progress, namely empirical disconfirmation: ‘Weak powers of empirical descriptions removes a crucial resource for either development or rejection of a particular language and so contribute to its stability as a frozen form’ (Bernstein, 2000, pp. 167-168). In other words, grammaticality determines the capacity of a theory or a language to progress through worldly corroboration; verticality determines the capacity of a theory or language to progress integratively through explanatory sophistication. Together, we may say that these two criteria determine the capacity of a particular knowledge structure to progress. (Muller, 2007, p. 12) (emphasis in original text)

2.1.2 The internal and external languages of description of this study

The *internal language of description* developed in this study comprises a theory that encapsulates and describes various components considered crucial for the promotion of a particular conception of knowledge and behaviour. This conception of knowledge and behaviour is associated, at a general level, with the knowledge domain of mathematical literacy, and, at a more localised level, with the practices and structure of participation that are envisioned to stem from engagement with the components of this knowledge domain in the setting of the subject-matter domain of Mathematical Literacy. At the foundation of the internal language of description is an expression of a conceived of relationship between mathematical content and knowledge and real-world contexts (and problems encountered in such contexts). Specifically, this conceived of relationship is characterised by an explicit expectation for the subordination of the mathematical terrain to a prioritising of sense-making of contextual situations and an associated life-preparedness orientation. The structure and orientation of the internal language of description is driven in large part by a response to a critical analysis of existing knowledge structures and criteria for participation in the subject through the lens of Paul

Dowling's (1998) Social Activity Theory.[4] A key argument in this theory is that mathematical and everyday practices are incommensurable and that the inclusion of everyday contexts in the mathematical classroom under the guise of making mathematics 'relevant' neither enhances understanding of mathematics nor understanding of the everyday. Instead, the inclusion of contexts are seen to inhibit the ability to 'see' the mathematics clearly; and the everyday contexts included in the classroom are not realistic − they are mathematised and, therefore, mythologised contexts (1995a, p. 9; 1995b, p. 209; 1998, p. 33). The utilisation of this theory to critically analyse current practices in the subject-matter domain of Mathematical Literacy reveals stark similarities to the problematic scenarios identified by Dowling in his own work. It is, thus, in an attempt to address these issues that the incentive for an alternative language of description of the structure of knowledge for the domain of mathematical literacy arises. That promotes a form of participation in the subject that prioritises a life-preparedness orientation. This life-preparedness orientation is deliberately characterised by a perspective in which the utilisation of mathematical structures is seen as being in service to a larger goal for the sense-making of contextual situations and life-preparation (as opposed to a priority for the development of mathematical knowledge and processes of mathematisation).

The internal language, however, remains entirely in the realm of the theoretical. As such, to facilitate the use of the internal language in interpretation and analysis of specific empirical practices relating to the subject-matter domain of Mathematical Literacy, an external language of description is necessary. And, since the empirical practices under investigation in the study involve analysis of textual resources relating to the subject, the external language of description comprises a methodology that involves identification and analysis of the discursive resources – specifically, words/vocabulary, visual mediators, routines and endorsed narratives (c.f. Sfard, 2008) – that characterise these various textual resources.[5] Comparison of the discursive resources in the empirical textual resources to the discursive characteristics of practices associated with the various domains of practice of the developed internal language of description then provides a means for identifying and/or constructing the specific type of knowledge and practices prioritised in a segment of practice relating to the subject. And in so doing, the external language of description provides the means through which the internal language can be employed as a lens for analysing empirical practices associated with the subject-matter domain of Mathematical Literacy.

2.2 The claims and concepts of the language of description

The discussion above highlighted macro-level descriptions and differences between internal and external languages of description. The discussion below now shifts to a more micro-level analysis by focussing on the specific components of an *internal* language of description.

As was mentioned above, Jablonka and Bergsten (2010, p. 27) argue that a theory comprises a system of theoretical entities, basic principles and a methodology that links

[4] See Part 3 starting on page 132 for a detailed discussion of Dowling's theory and page 167 for a discussion of the relevance of his work to the structure and contents of the internal language of description developed in this study.
[5] See Part 5 starting on page 253 for a discussion of the external language of description for this study. For a discussion of the primary methodology (of textual analysis) and associated methods (draw from the field of semiotics) employed in this study in the analysis of empirical resources, see Part 6 starting on page 360.

the theoretical entities or principles to a specific empirical activity or field. Palm (2009, p. 6), in drawing on the work of Niss (2007, p. 1308), argues in a similar way that a theory is a system of interrelated *concepts* and *claims*. The concepts comprise an organised network, linked through a hierarchy, and commonly positioned in a research framework (which can be theoretical, practical or conceptual). The claims – or *theoretical entities* (Jablonka & Bergsten, 2010, p. 27) – of a theory refer to a domain or class of domains consisting of objects, processes, situations and phenomena: the claims comprise a 'theoretical manifesto' (Dowling, 1994, p. 125) of hypotheses, assumptions or axioms about the domain or class of domains which are taken as fundamental, or statements about the domain which draw from or are based on the fundamental claims. These statements often evolve through application of the theory to a specific empirical space as the theory is modified to ensure an effective and comprehensive reading of the space. Thus, a theory can comprise a set of claims about an object and a framework that facilitates analysis of that object.

2.2.1 Claim

In contrast to Dowling's argument that mathematical and everyday practices are incommensurate, my language of description presents the claim (i.e. theoretical proposition) in the form of a hypothesis that: *mathematics is useful and empowering for making sense of real-world contexts and/or problems encountered in real-world settings*. This claim, however, requires several qualifications.

Firstly, the claim is only valid if the motivation and focus of the problem-solving process involving the use of mathematics in contextual situations is for the sense-making of the contextual situations or problems encountered in those situations and not for the learning of mathematical content (which is the focal point of Dowling's theorising). As such, where the domain of Dowling's theory is mathematical knowledge as employed within the site of high school Mathematics, the domain of my language of description is the structure of knowledge associated with a conception of mathematically literate behaviour that is separated from or positioned outside of the domain of scientific mathematics and where engagement with mathematical and contextual structures are (supposedly) equally valued and prioritised. And the above claim for the language of description developed in this study is only valid in this 'external-to-scientific-mathematics' domain.[6] Importantly, the stated claim is also grounded on the assumption that since a primary intention in the language of description is on the *use* of mathematics as a tool for making sense of real-world situations and not on the learning of mathematical content, a necessary level of mathematical competency is already in place. As a consequence, a format for the subject-matter domain of Mathematical Literacy that is aligned to the structure of knowledge outlined in the language of description presented in this study[7] should not have as a *primary* or *ultimate* goal the learning of mathematical content or the development of mathematical competency, although this may occur during the process of preparation for life.

[6] In Part 5 in this study I refer to the knowledge domain of mathematical literacy as a 'blended' domain – namely, as the blending of knowledge and practices associated with both mathematical and contextual structures. See Chapter 17 and sub-section 17.2.1 on page 272 for a discussion of this point.

[7] Or, for that matter, any course that aims to develop mathematically literate behaviour, but where this process takes place outside of the domain of scientific mathematics or separated from the teaching of formal mathematical content.

Secondly, building and extending on from the previous qualification is the recognition and acceptance that although mathematics is useful for making sense of and/or for solving everyday real-world problems, *mathematics is not enough*. Rather, mathematical solutions and models are limited, and there are commonly a variety of extra-mathematical factors which influence the decisions that people make in solving problems in real-world situations. In other words, mathematics is viewed as just one of many tools that can be employed in developing an understanding of a context and associated problems. As such, effective pedagogic practices associated with the a format of the subject Mathematical Literacy that is aligned to the structure of knowledge outlined in the language of description developed in this study must give recognition, exposure, and credence to informal and non-mathematical techniques, structures and considerations which affect and reflect the reality of participation in real-world problem scenarios.

Thirdly, for mathematics to be useful as a tool for the sense-making of real-world situations and problems encountered in those situations, there is an inherent expectation that the person solving the problems is able to apply the mathematical content in a variety of real-world contexts and for a variety of problems. This includes the use of integrated content and skills, developing appropriate models, and, crucially, the ability to identify which techniques and content are appropriate for use in a particular setting and the limitations of the mathematical solution and/or model. As such, the usefulness of mathematics for making sense of the real-world is reliant on the ability of the practitioner to mathematise and model (albeit, in reference to the second qualification above, with an understanding of the limitations of the mathematical solution and/or model).

The final qualification, which relates to the claim and to the previous three qualifications, involves the issue of a 'critical gaze'. Namely, the usefulness of mathematical content and techniques for modelling and sense-making of everyday situations cannot end at the level of 'sense-making'. Rather, sense-making must be accompanied by a critical gaze – a level of 'reflective knowing' (Skovsmose, 1992, 1994b) − that offers a critique of existing structures and recognition of alternative approaches to problem-solving scenarios. This critical gaze, taken together with the combination of calculation and sense-making techniques, provides the means for developing a more comprehensive and critical understanding of the structure of participation in contextual environments and of possible alternative forms of participation in those environments.

2.2.2 Concepts

The claim (theoretical proposition) of the internal theoretical language is accompanied by a *conceptual framework* (c.f. F. Lester, 2005, pp. 458-460)[8] that identifies different

[8] F. Lester (2005, pp. 458-460) defines a research framework as "a basic structure of the ideas (i.e. abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated.", and distinguishes three different frameworks: theoretical, practical and conceptual. The framework that comprises the components of the internal language of description developed in this study falls within the description of a *conceptual framework*: "A conceptual framework is an argument including different points of view and culminating in a series of reasons for adopting some points – i.e. some ideas or concepts – and not others." (Eisenhart, 1991, p. 209). Importantly, there are two key elements of conceptual frameworks which are particularly characteristic of the developed internal language presented in this study. Firstly, a conceptual framework is built on an array of previous and current research from various sources, rather than on a single theory (F. Lester, 2005, p. 460). Secondly, the framework need not be limited to drawing on theories; rather, the local knowledge of practitioners who participate in activities in the terrain under investigation can also be used to inform the questions raised by the researcher and the structure and contents of the framework (Eisenhart, 1991, p. 209).

concepts, the collective and integrative of which are seen to characterise a particular structure of knowledge and participation for the subject Mathematical Literacy that prioritise a life-preparedness orientation. These concepts are borne out of analysis of various perspectives of the components/traits/behaviours/knowledge associated with mathematically literate behaviour. As such, the conceptual framework that characterises the language of description (and the various concepts that make up that conceptual framework) reflects a conglomerate of views and opinions. This conglomerate has have been translated into a language for describing the structure of knowledge needed in the knowledge domain of mathematical literacy (and associated forms of participation in the subject-matter domain of Mathematical Literacy) to facilitate a life-preparedness orientation for the subject.

The conceptual framework envisioned for the theoretical language of description for the structure of knowledge in the knowledge domain of mathematical literacy comprises five interrelated components:

- Contextual domain of reconstituted real-world contexts;
- Everyday domain of practice;
- Mathematical Competency domain of practice;
- Modelling domain of practice;
- A domain of practice involving Reasoning and Reflection on both contextual and mathematical elements.

Each of these domains of practice is discussed and theorised in detail in Part 4 of the study (c.f. Chapter 14 and section 14.4 on page 194 below).

CHAPTER 3
POSITIONING THIS STUDY IN THE FIELD OF KNOWLEDGE PRODUCTION

A key component of this study is concerned with knowledge production (specifically related to a particular form of relationship between the mathematical terrain and the terrain of the real-world) and also with how that knowledge is internalised, recontextualised and enacted in practice (through participation in the subject-matter domain of Mathematical Literacy). For this reason it is appropriate to locate the study in relation to the fields that make up Bernstein's pedagogic device. My intention for doing this is to highlight the limited field of application of this study and, so, to acknowledge the restrictions of the study for making claims beyond this limited field.

To describe the "systemic and institutionalised" ways in which knowledge is converted and recontextualised from the field where it is produced, into the schooling system, and then again within that schooling system (Bertram, 2009, p. 47), and the consequent relations of power and control that develop through the recontextualisation process (Singh, 2002, p. 571), Bernstein (1996) presents the notion of the *pedagogic device*. According to Maton and Muller (2007),

> … the pedagogic device forms the basis of his [Bernstein's] account of: the ordered regulation and distribution of society's worthwhile store of knowledge, ordered by a specifiable set of *distributive rules*; the transformation of this store into a pedagogic discourse, a form amenable to pedagogic transmission, ordered by a specifiable set of *recontextualizing rules*; and the further transformation of this pedagogic discourse into a set of evaluative criteria to be attained, ordered by a specifiable set of *evaluative rules*. (p. 19, emphasis in original text)

Each of these rules is associated with a specific field of activity involving the production and recontextualisation of knowledge, thus giving rise to three primary site of knowledge production / recontextualisation: *field of production*; *field of recontextualisation* (comprising the sub-fields of the official recontextualisation field and the pedagogic recontextualisation field); and the *field of reproduction.*[9]

[9] It is important to point out that the fields (and also the rules that govern how behaviour is transformed in a particular field) are hierarchically related (Singh, 2002, p. 573 & 574) in the sense that a body of knowledge must first be produced before it can be recontextualised or reproduced (and/or evaluated). Reflecting this discussion on the empirical terrain of the subject Mathematical Literacy, a key challenge encountered in the subject is that the knowledge domain on which the subject is based is not clearly and explicitly defined, nor are the rules for recontextualisation and evaluation. For some, the recontextualisation and evaluation rules are based in mathematical structures, and for others they are to be found in the domain of the contextual world. The result is that the knowledge domain of mathematical literacy is interpreted and understood differently by and for different people, with participation in the practices of the subject differently legitimised and evaluated by different role-players (which an analysis of curriculum intention and examination focus will quickly reveal – see (North, 2010) for an illustration of this issue). The learners in the subject are, then, caught in the middle of this process, having to deduce the rules and structures of recontextualisation and evaluation according to which their participation is to be endorsed. The primary intention of this study, then, is to more clearly and explicitly define the components of the knowledge domain of mathematical literacy and to provide a means through which this internal dimension is able to be employed in analysis of empirical practices in the subject (through a developed external dimension of the language). However, identification of the rules through which the knowledge store for mathematical literacy is recontextualised and evaluated in pedagogic and empirical practices is beyond the scope of this study and, instead, is reserved for future research endeavours.

The *field of production* is the field in which discipline-specific or domain-specific expert knowledge (Singh, 2002, p. 572) is constructed and positioned by role-players such as academics and professionals in the field.[10] Since the primary intention of this study is the development of a theoretical language to describe a particular conception of the knowledge required to engage both mathematical and contextual domains of practice, a major component of this study is, thus, located in the field of knowledge production. This knowledge production is in specific relation to domain of knowledge associated with a specific conception of mathematically literate behaviour and also to the localised empirical site of practice of the secondary school subject Mathematical Literacy in South Africa.

To facilitate the development of this language of description and to provide a rationale of the need for this language, it is also necessary to temporarily shift the object of study. This shift in the object of study facilitates analysis of literature relating to conceptions of mathematically literate, numerate and/or quantitatively literate behaviour in general and, specifically, to the South African conception of mathematical literacy embodied in the secondary school subject Mathematical Literacy. This latter site of analysis requires a reading of various curriculum and related supporting documents for the subject. Thus, Part 2 of the study (c.f. page 29 below) – in which the bulk of the analysis described above takes place – facilitates a shift of focus in the object of study to a site characterised by Bernstein as the *official recontextualising field* (as a sub-field of the field of recontextualisation). Once the internal language of description is developed, a gaze is cast from this internal language – through an external language and associated methodology of textual analysis – over empirical textual resources (textbook section, curriculum section, national examinations, and course notes for a teacher education course) relating to the subject-matter domain of Mathematical Literacy (c.f. Part 7 starting on page 381). This process is also located in field of recontextualisation, albeit this time in the *pedagogic recontextualising sub-field*. A word of clarification and caution is, however, necessary. It is essential to reiterate that the primary intention of this study is the development of a theoretical language to describe a particular knowledge structure – which takes place in the field of production. Any attempt to shift the object of study outside of this field – for example, through the analysis of curriculum documents or pedagogic texts – is always in service to this knowledge development intention. As such, although focus shifts to different sites in the official recontextualising field in the analysis of subject-related documents, this shift is made to further facilitate and strengthen the developed internal language of description.

As regards the *field of reproduction*: this is the field in which teachers make use of the "privileged and privileging texts created in the field of recontextualisation" (Singh, 2002, p. 577) to interpret and implement the curriculum through classroom and assessment practices. Although in this study I draw on my own experiences of teaching in the subject-matter domain of Mathematical Literacy and on my interactions with other teachers of the subject in providing an analysis of the literature and curriculum documents and in outlining approaches to pedagogic practice for the subject, the key area of focus of this study does not fall within this field of reproduction. Rather, I step periodically and somewhat superficially into this field to inform the knowledge construction process of a

[10] Bernstein distinguishes between two primary forms of knowledge – hierarchical and horizontal knowledge structures. These knowledge forms are differentially distributed to different groups of people to produce differential power relationships (Singh, 2002, p. 573). Hierarchical and horizontal knowledge structures, together with horizontal and vertical discourses, are discussed briefly in Part 3 of this study (c.f. Chapter 10 and sub-section 10.1.1.2 starting on page 150 below).

language of description within the field of production and the testing of this language in the field of recontextualisation.

The positioning of this study in relation to Bernstein's pedagogic device is illustrated in the diagram shown in Figure 4 below:

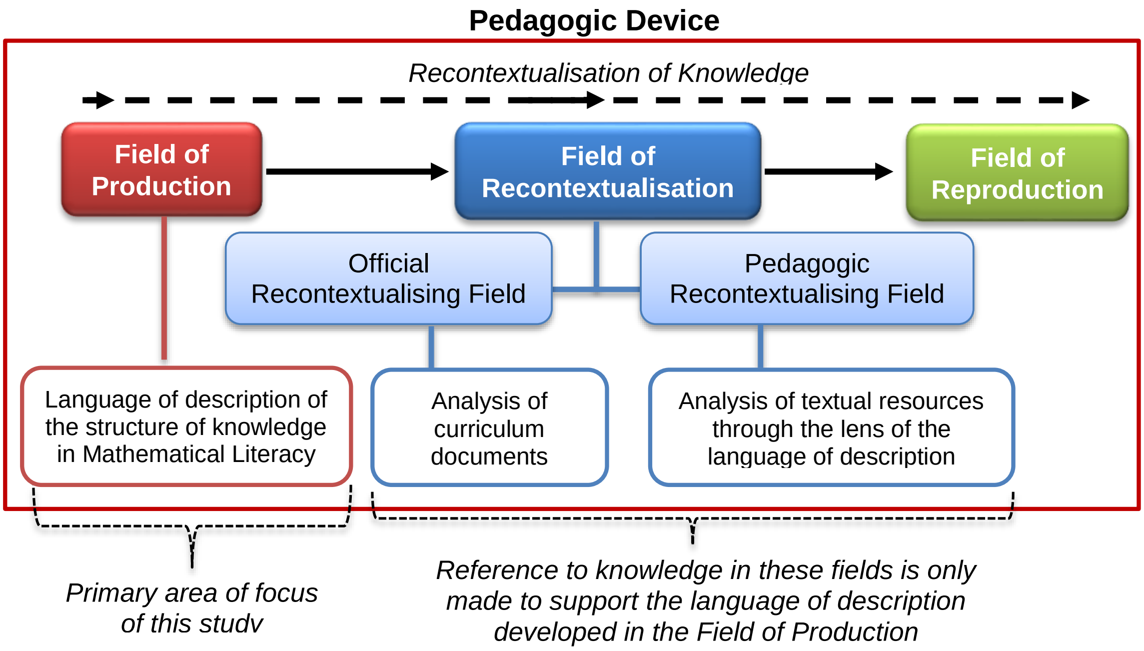


Figure 4: Positioning of this study in relation to Bernstein's Pedagogic Device

Importantly, the location of this study primarily within the field of knowledge production signifies that any claims made only provide a limited perspective on the practices associated, specifically, with the subject-matter domain Mathematical Literacy and, more generally, on the traits associated with mathematically literate behaviour that are seen to stem from participation in this subject. In particular, since the study does not delve into the field of reproduction, little or no validated commentary is possible regarding the classroom practice of teachers or learners and/or how such participants may further re-contextualise the legitimised knowledge developed in the field of knowledge production.

CHAPTER 4
CONCEPTUALISING AND CLARIFYING THE NOTIONS OF A ‘LIFE-PREPAREDNESS ORIENTATION’ AND ‘PRACTICE’

4.1 ‘Life-preparedness orientation’

The key focus of this study is the theorisation of a theoretical language for describing the structure of knowledge in a conception of the knowledge domain of mathematical literacy that prioritises a *life-preparedness orientation*. The notion of a life-preparedness orientation is, thus, a central component of the language of description, and the structure of legitimate participation envisioned for the subject Mathematical Literacy is directly linked to the components of this orientation. In this chapter I elaborate and clarify the precise meaning of the notion of a life-preparedness orientation.

The phrase ‘life-preparedness orientation’ is originally coined by Venkat (2010), and my usage of the term in this study is based primarily on this original conceptualisation. Briefly, a life-preparedness orientation prioritises understanding of the world through any and all available means and resources – some of which may include formal mathematics, but others which may utilise informal or everyday reasoning and forms of participation and communication. A life-preparedness orientation is directed towards preparation for effective functioning in everyday life through exposure to existing everyday forms of knowledge and participation and to possible alternative forms derived through mathematically informed considerations. A life-preparedness orientation recognises that a mathematically based view of the world provides only one view – and often a limited view − of the world, and that although mathematical knowledge can facilitate an alternative understanding of the world, the structures that define action, decision-making, communication and endorsed participation in the world are often not mathematically based. As such, preparation for the world must acknowledge the role of non-mathematical factors and considerations which might affect decision-making and the limitations of a mathematical gaze in reflecting reality. In this orientation, although the role of mathematics in facilitating an alternative view of the world is recognised, mathematics is always seen as in service to a goal for a better understanding of an aspect of reality and of the structure of legitimate forms of participation in that reality. Preparation for life and not the learning of mathematical knowledge is the ultimate goal, and the terrain of the contextual and not of the mathematical constitutes the organising principle of legitimate participation. Translating this to the terrain of the subject Mathematical Literacy, my key argument is that a conception of this subject that aims to prepare participants for a potentially more empowered functioning in the real-world – for life-preparation – must facilitate several facets. These include: engagement with authentic contexts, recognition of the limited role of formal mathematical structures and techniques in adequately describing real-world situations, and acknowledgement that any mathematics appropriated and/or used must be in service to a goal for broader contextual sense-making practices.

In Part 2, Chapter 5 and sub-section 5.2.2.3 of the study (c.f. page 47 below) I extend the conceptualisation of the notion of a life-preparedness orientation. I achieve this by positing that, at a general level, this type of orientation is characterised by a structure of participation that explicitly prioritises an <u>agenda</u> (namely, specific areas of prioritising that are internal to a practice) for contextual sense-making practices (as opposed to an agenda for literacy in mathematics, contextualised calculations or modelling). I argue further that this orientation is also characterised by a range of possible <u>intentions</u>

(namely, the specific external purpose of the practice) – including intentions for the development of Human Capital, Cultural Identity, Social Change, Environmental Awareness, and Critically Evaluating Structures. The format of the life-preparedness orientation conceptualised and promoted in the internal language of description is, then, posited as characterised by the specific combination of an agenda of Contextual Sense-Making Practices and an intention for Critically Evaluating (contextual and mathematical) Structures encountered in real-world problem-solving scenarios. In Part 4 (c.f. page 181) I then argue that this particular conception of a life-preparedness orientation is facilitated through engagement with different domains of practice − comprising the Everyday, Mathematical Competency, Modelling knowledge, and Reasoning and Reflection domains – in relation to a Contextual domain of reconstituted real-world contexts. The collective of these domains comprises the components of the developed internal language of description of a structure of knowledge for the knowledge domain of mathematical literacy (and associated forms of participation in the subject-matter domain of Mathematical Literacy), and engagement with this collective is presented as facilitating life-preparedness.

In short and in summary, a life-preparedness orientation for the subject Mathematical Literacy is facilitated through understanding of how people function in daily-life and of how mathematics can be used to describe a particular form of participation in the world, and of the ability to engage mathematical elements encountered in contextual environments and to do so in a critical way.

4.2 The notion of 'practice' as employed in this study

In the discussion so far, and for the remainder of the discussion in the study, I have made reference to the notion of 'practice(s)', both generally in relation to the knowledge domain of mathematical literacy and also more specifically in relation to the subject Mathematical Literacy. By 'practice' I am referring to all activities associated with these sites, including physical actions (such as problem-solving experiences and pedagogic interactions) and also resources (such as textual resources, technology resources, and any other resources which inform legitimate participation in a contextual situation). However, the empirical analysis component of this study is focussed specifically on analysis of textual resources and of the forms of practice that are legitimised in those resources for successful participation in the subject Mathematical Literacy. Thus, while the developed language of description of the knowledge domain of mathematical literacy presents a particular structuring of knowledge associated with participation in all forms and sites of practice (in relation to a specific format of the interaction between the contextual and mathematical terrains), exemplification of the utility of language is restricted to practices embedded in and espoused through textual resources. This contributes to a key limitation of this study – namely, that the conclusions presented with regards to practices in the subject Mathematical Literacy are limited to those involving engagement with specific textual resources. No validated commentary is able to be supplied regarding other sites of pedagogic practice, including about interactions between various role-players in the subject (e.g. teachers, learners) or about other sites of engagement (e.g. professional development initiatives for pre- and in-service teachers).

WHERE TO FROM HERE

Having now specified certain theoretical antecedents that have informed the orientation, structure and approach adopted in this study, focus in Part 2 of the study shifts to discussion of the empirical terrain that is the focus of this study – namely, the subject-matter domain of Mathematical Literacy. The structure of legitimate participation in the subject (and the structure of knowledge required to facilitate this legitimate participation) is of particular concern in this impending discussion. This discussion is precluded by focus on general conceptions of mathematical literacy, numeracy and/or quantitative literacy and the structure of participation and behaviour prioritised in these different conceptions, together with clarification of the criteria according to which participation in the subject is legitimised. A framework to facilitate comparison of different conceptions of mathematically literate, numerate and/or quantitatively literate behaviour is also presented.

PART 1
THEORETICAL (AND OTHER) ANTECEDENTS

INTRODUCTION AND OVERVIEW

The intention of this part of the study is to outline certain theoretical considerations that have informed the development of this study. In particular, the key area of focus of this study: namely, the development of a 'language of description' of the structure of knowledge associated with a form of the knowledge domain of mathematical literacy (and associated practices and forms of participation in the South African high school subject-matter domain of Mathematical Literacy) that promotes the development of a life-preparedness orientation. The discussion in this regard moves from a macro to a micro level of analysis. Namely, from consideration of the positioning of the study as a whole within a particular paradigm and educational framework, to a more localised discussion on the contents of the study. This latter level of analysis is achieved through exploration of the components and theoretical grounding of the language of description developed in the study.

This part of the study is divided into four chapters (Chapter 1 to Chapter 4). In Chapter 1 I position the study within an interpretive paradigm in the field of mathematics education, and highlight the ontological, epistemological and methodological positions that stem from this paradigmatic orientation. In Chapter 2 I argue that the key intention of this study is the development of a 'language of description' (both internal and external) of the structure of knowledge associated with a particular form of the knowledge domain of mathematical literacy and explore more precisely what is meant by a language of description. In this chapter I also explore various facets requiring consideration with respect to the development of such a language, including identifying the claims and concepts of the language and the mode of theorising adopted in the language. In Chapter 3 I expand on the characterisation of the study as the development of a language of description of a knowledge structure by locating the study in the Field of Knowledge Production within Bernstein's (2000) pedagogic device, and discuss the limiting perspective offered by the study as a result of this categorisation. In the final chapter (Chapter 4) I conceptualise and clarify the dominant orientation that characterises the language of description for the knowledge domain of mathematical literacy – namely, a 'life-preparedness orientation'.

This chapter structure and the positioning of this study within the field of educational research are illustrated in Figure 2 below, with the hierarchy of the diagram intended to illustrate the movement from a macro to a micro level of discussion.

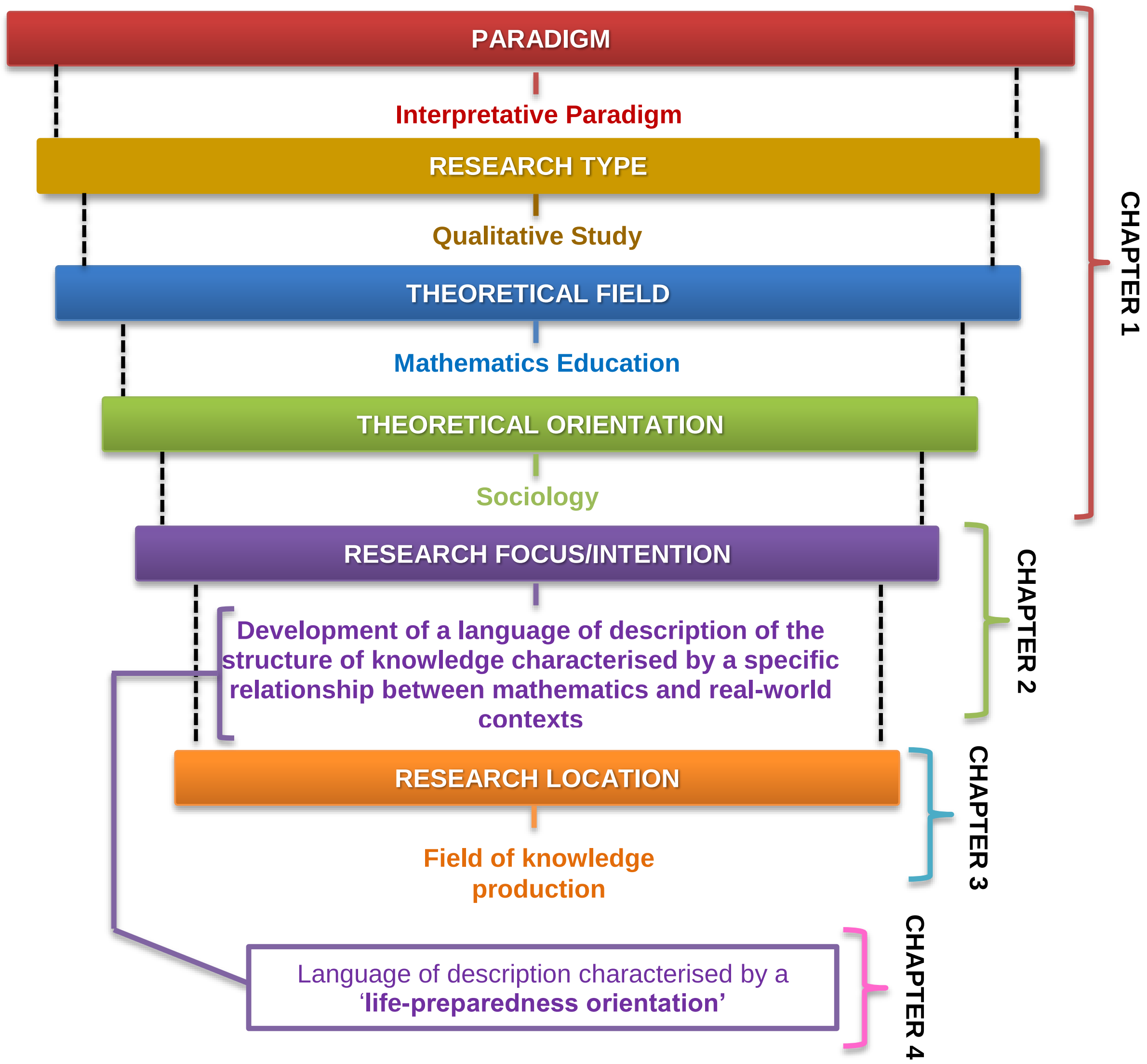


Figure 2: Overview and chapter structure of <u>Part 1</u> of the study

CHAPTER 1
RESEARCH METHODS, PARADIGMS AND MATHEMATICS EDUCATION THEORIES

In this chapter I position this study within an interpretive paradigmatic orientation and outline the ontological, epistemological and methodological positions that characterise this paradigmatic orientation. I argue further that this study is driven by a sociological impetus and is characterised by a qualitative research process that is located in the field of mathematics education.

1.1 Positioning this study in an interpretive paradigm: considerations of ontological, epistemological and methodological orientations

A paradigm may be viewed as a set of *basic beliefs* (or metaphysics) that deals with ultimates or first principles. It represents a *worldview* that defines, for its holder, the nature of the "world," the individual's place in it, and the range of possible relationships to that world and its parts … (Guba & Lincoln, 1994, p. 107) (emphases in original)

Drawing from the above, a paradigm is a particular way of looking at the world, informed by the values and assumptions of the researcher, which directs the focus and form of the intellectual structure on which the research process is based (Kuhn, 1962). The particular paradigm in which a researcher is positioned, then, affects not only what is perceived to fall "within and outside the limits of legitimate inquiry" (Guba & Lincoln, 1994, p. 108). Instead, what is also affected is how the researcher views the world and their own role or subjectivity in the research process, the kinds of questions that are asked with respect to a particular inquiry and how these questions are to be scrutinized, and the type of methodology that is chosen by the researcher to gather information relating to these questions.

Guba and Lincoln (1994, p. 108) argue that the basic beliefs that define the dominant view or perspective in a paradigm are encapsulated through the responses that would be given by proponents of a particular paradigm to three fundamental questions. Firstly, the *ontological question* – "What is the form and nature of reality and, therefore, what is there that can be known about it?" Secondly, the *epistemological question* – "What is the nature of the relationship between knower or would-be-knower and what can be known?": namely, how does the researcher perceive knowledge is produced, acquired and communicated (Scotland, 2012, p. 9) and their influence over the knowledge being collected or uncovered, or do they perceive that they are external to it and/or do not influence it (University of South Hampton, 2011). And thirdly, the *methodological question* – "How can the enquirer (would-be-knower) go about finding out whatever he or she believes can be known?" (All quotations above: Guba & Lincoln, 1994, p. 108). In short, the questions address the issue of what – in the eyes of the researcher - constitutes reality, how does the researcher position themselves with respect to this reality, and how will the researcher go about finding out something about the reality? These three questions provide a "holistic view of how we view knowledge, how we see ourselves in relation to this knowledge and the methodological strategies we use to un/discover it." (University of South Hampton, 2011) – in other words, these three questions provide a view or perspective of the researcher on "what is", "what it means to know”, and the “why, what, from where, when and how” of the data collection process (Scotland, 2012, p. 9). Collectively, then, these three questions provide not only a point of analysis for individual paradigms but also points of comparison between different paradigms.

The specific paradigm that is reflected in this study, and which represents my own world view on the topic under scrutiny in this study, is that of the *Interpretive (or Social Constructivism) paradigm*. From this paradigmatic orientation, the dominant *ontological* position is that of relativism (Scotland, 2012, p. 11): namely, ‘reality’ is viewed as a social construction such that different cultures are seen to make sense of the world differently as influenced by the specific social, economic, political and cultural environment and issues which define and affect participation in the culture at particular points in time. As such, from this ontological perspective there is no single or accurate reality. Rather, an experience of reality is dependent on the culture and context in which a person finds themselves at a particular point in time – reality is “socially and experientially based” (Guba & Lincoln, 1994, p. 110); and this reality shifts over time, between cultures, and even within a culture between different groups (McKee, 2003, pp. 9-10). In other words, reality is subjective, is individually constructed, and there are multiple possible realities for differing individuals, groups, and cultures (Scotland, 2012, p. 11). This viewpoint is supported by constructivist and sociological perspectives that promote the notion of the social construction of knowledge and of an individual’s relation to that knowledge (McKee, 2003, pp. 9-10).

This ontological perspective of multiple realities gives rise to an *epistemological* position that views the potential for the existence of ‘multiple knowledges’ (Guba & Lincoln, 1994, p. 113). Accordingly, the world is seen to not exist independently of our descriptions of it: “Meaning is not discovered; it is constructed through the interaction between consciousness and the world.” (Scotland, 2012, p. 11). Subjectivity is, thus, a key component of the interpretative epistemology, characterised by a research process involving constant interaction between the researcher and the object of investigation (Guba & Lincoln, 1994, p. 111). Furthermore, this position is accompanied with explicit recognition of the influence of the researcher’s own values and social, economic, cultural and political orientations on choices and decisions made during the research process and, hence, on the intention, structure and outcome of this process (Scotland, 2012, p. 12). Given this adherence to relativism and a perspective on the potential for multiple knowledges, the primary aim of the interpretivist research process is understanding of the ways in which different individuals or groups of people perceive their reality and their place in that reality (Guba & Lincoln, 1994, p. 113). Importantly, this intention for understanding can also be accompanied by intentions for “Advocacy and activism” (Guba & Lincoln, 1994, p. 113). In other words, interpretative research need not be exclusively descriptive and can, instead, be accompanied by critical components for change. This issue is particularly pertinent to the sociological impetus of this study (see page 15 below) in which a key intention in the study is the modification of the existing structure of knowledge in the subject Mathematical Literacy to overcome perceived structures of educational disadvantage afforded through participation in the subject.

Methodologically, Interpretivism promotes the use of research methods that facilitate investigation, analysis, interpretation and understanding of how different people and groups perceive reality and their place in that reality, together with social and cultural conditions that influence perceptions of reality (Scotland, 2012, p. 12). Interpretive methodology also promotes the use of methods that facilitate interpretation of varied and multiple representations of reality from an individual's perspective through direct interaction between researcher and the object of investigation (Guba & Lincoln, 1994, p. 111). It is as a result of this positioning of the researcher as an active transactional participant in the construction and understanding of meaning in socially constructed representations of reality that lead Guba and Lincoln (1994, p. 111) to categorise Interpretivist methodology as "Hermeneutical and dialectical". In other words, understanding of how others associate meaning to their own representations of reality is able to be deemed through derivation of expressed and/or hidden meaning in texts (i.e. hermeneutics) or through dialogue between researcher and the objects of the research process.

Having outlined the general characteristics of the ontological, epistemological and methodological positions associated with the interpretative paradigm, it now becomes possible to reflect on the significance of these positions with respect to the commitment in this study to an interpretative paradigmatic orientation. *Ontologically*, the research process in this study is driven by an intention to interpret, analyse and understand the dominant perception of the 'reality' of the structure of knowledge and participation in the subject-matter domain of Mathematical Literacy. This commitment is further characterised by an intention to understand how this reality is differently experienced by various role-players who participate in the subject (e.g. teachers vs. curriculum developers vs. examiners of national exam papers vs. textbook authors vs. teacher trainers). Understanding of how this reality is different to the perceived reality in other subjects (such as Core [scientific] Mathematics) is also in focus. This ontological position recognises that these multiple representations of reality of knowledge and participation for the subject are directly influenced by various political, social, economic and cultural factors, and, particularly, by pressure within the South African education system to prioritise mathematical knowledge as a preferenced form or structure of knowledge. However, in interpreting existing perceptions of reality for the subject Mathematical Literacy, there is also an impetus for 'activism' (Guba & Lincoln, 1994, p. 113). Namely, my own position is that the existing reality – which I claim is dominated by a prioritising of mathematical knowledge structures as the basis for legitimate participation in the subject – supports the promotion of Myths of Reference, Participation and Emancipation (Dowling, 1998)[2]. Consequently, a degree of educational disadvantage with respect to future career, work and social advancement opportunity is sustained and reproduced through participation in the subject. As such, a key intention of this study is for the development of an alternative 'reality' of the structure of knowledge and participation for the subject-matter domain of Mathematical Literacy in which a life-preparedness orientation is prioritised over the development of mathematical knowledge. This alternative reality is theorised and discussed in detail in Part 4 of the study where I present an alternative 'language of description' for the knowledge domain of mathematical literacy.

Drawing from an *epistemological* position that views the potential for the existence of 'multiple knowledges', the research process in this study acknowledges varying possible interpretations and descriptions of the ways in which role-players in the subject Mathematical Literacy (as the object of study) perceive the structure of legitimate participation in the subject. The particular interpretation adopted in this study, as informed by my historical, social, economic, political and theoretical orientation, construes existing structures of participation as problematic in relation to issues of educational, social and economic positioning and access. As such, my interpretation leads

[2] Refer to Part 3, Chapter 10 and sub-section 10.2.2 starting on page 156 below for a detailed discussion of these concepts.

me to question current pedagogic and assessment priorities in the subject and to present an alternative understanding of a structure of participation which aims to negate existing forms of positioning. It is this interpretation which has directly informed the structure of the developed alternative internal language of description for the knowledge domain of mathematical literacy (see Part 4 starting on page 181 below) and the promotion of a 'life-preparedness' (Venkat, 2010) orientation3 for the subject over the prioritising of mathematising processes in this alternative language of description. This is evidenced through the composition of the language of description of knowledge domains of practice that prioritise modelling processes (the 'Modelling' domain of practice) and understanding of patterns and structures of everyday practices (the 'Everyday' domain of practice) alongside mathematical structures (the 'Mathematical Competency' domain of practice). This interpretation of the structure of participation for the subject has also directly informed the structure of the external language of description developed to facilitate the means through which the internal language (and characteristics of the associated domains of practice) can be used as a lens for analysis of empirical resources relating to the subject (c.f. Part 5 starting on page 253). This external language of description focuses on identification and interpretation of the discursive resources (signifiers [words/vocabulary and visual mediators], routines and narratives) embedded and indexed in discursive practices in the subject, and is accompanied by recognition of the highly interpretive and subjective nature of this process of engaging with discursive resources and of any assumptions or conclusions made as a result of this process. My commitment to an interpretivist paradigmatic orientation has, thus, influenced the deliberately construed and highly interpretive description of the structure of knowledge and participation in the subject-matter domain of Mathematical Literacy adopted in this study, and also the "Transactional and subjectivist" (Guba & Lincoln, 1994, p. 111) role of me as the researcher in this process.

Methodologically, the ontological and epistemological orientations adopted in this study necessitate a specific methodology. In particular, this methodology must facilitate interpretation and analysis of how the practices of participants in the subject either reflect coherence or divergence with the alternative conception of knowledge and participation developed for the subject-matter domain of Mathematical Literacy (presented through the internal language of description and operationalised through the external language of description). This methodological intention is accompanied by a descriptive rather than normative approach in this sense that there is recognition that the structure of knowledge and participation as presented in the internal language of description for the domain of mathematical literacy presented a particular, a limited, and only one of many possible 'world-views' of the structure of knowledge and participation for the subject. The specific site of empirical analysis involves textual resources (i.e. a textbook section, a section of the curriculum, Grade 12 national examinations, and course notes for a teacher education course) that reflect different aspects of knowledge and participation in relation to the subject-matter domain of Mathematical Literacy. As such, the methodology selected and employed to facilitate analysis of these empirical resources in a way that is consistent with interpretivist ontological and epistemological orientations is that of textual analysis, as a sub-set of a wider methodology of discourse analysis (c.f. Part 6 starting on page 360). This method is drawn from the realm of hermeneutics (which is identified by both Guba and Lincoln (1994, p. 111) and Scotland (2012, p. 12) as reflecting interpretivist intentions). Hermeneutics involves analysis of the discursive resources (signifiers [words/vocabulary and visual mediators], routines and endorsed narratives) embedded

[3] Refer to Chapter 4 on page 26 below for an elaborated discussion of the notion and components of a life-preparedness orientation.

in (and associated discourse communicated through) textual resources for the subject. This process also involves comparison of these discursive resources to the characteristics of the discursive resources of the domains of practice that constitute the internal language of description for the knowledge domain of mathematical literacy. This methodology provides the means for identification and interpretation of possible 'world-views' of the authors of these texts and, specifically, of how these authors perceive the structure of legitimate knowledge and the criteria for legitimate participation in practices involving the use of mathematics in contextualised problem situations. And since this methodology recognises the interpretative, subjective and transactional nature of the investigative research process, the methodology is entirely consistent with interpretivist ontological and epistemological orientations.

Ontologically, epistemologically and methodologically, then, this study is driven by an interpretivist paradigmatic orientation, albeit with an impetus for activism directed towards understanding how knowledge and practices associated with the subject-matter domain of Mathematical Literacy can be changed to facilitate that participation in the subject does not reinforce social and/or educational disadvantage and positioning.

1.2 Positioning this study as a form of qualitative research

> Despite many proposed differences between quantitative and qualitative epistemologies, ultimately, the heart of the quantitative-qualitative "debate" is philosophical, not methodological. (Krauss, 2005, p. 759)

As suggested by the above quotation, although quantitative and qualitative studies are often differentiated according to the type of methodology employed in gathering and analysing information about an aspect of reality, these differences in methodology are the result of a deeper and, ultimately, epistemological difference between the nature of quantitative and qualitative research.

For Krauss (2005, p. 759), "Epistemology poses the following questions: What is the relationship between the knower and what is known? How do we know what we know? What counts as knowledge?" The difference between quantitative and qualitative research is then defined in large part by the differential answers to each of these questions for each type of research. For quantitative research, the object of study is viewed as independent of the researcher and, so, it is possible to use a quantitative analysis of the object, deemed through observation rather than direct interaction, and to make connections across multiple different realities. Qualitative research, on the other hand, deems that knowledge is directly attached to specific instances or phenomenon and that it is not possible to separate meaning from this instance. As such, it is only through direct interaction with the phenomenon and the object(s) or subject(s) under investigation that a more complete view of the phenomenon is gained. From this perspective, an exclusively numerically based analysis of a situation that does not taken into account the specifics of the individual object(s) or subject(s) in the phenomenon is incapable of providing adequate interpretation or description of the situation. Furthermore, given the distinctive nature of each phenomenon and the need for direct intervention in the phenomenon, it is not possible for a single tool to facilitate connections across multiple realities since every reality is unique and different (Krauss, 2005, pp. 759-760).

In the previous section it has already been established that this study falls within an interpretivist paradigm, hereby acknowledging the role of the researcher in shaping the ultimate form and assumptions of the study. Following on this line of thinking, there should be no doubt in the mind of the reader that that this study is directly influenced by my own interests, perspectives, and values. For this reason, the object of scrutiny in this study – namely, the structure of knowledge that characterises the specific relationship between mathematical content and real-world contexts envisioned for pedagogic practice in the subject Mathematical Literacy – simply does not avail itself to a quantitative analysis. Rather, a *qualitative analysis* is necessitated by the distinctive and unique nature of the subject Mathematical Literacy in the South African context. This form of analysis is driven by my own personal interest in understanding the way in which different groups of participants perceive 'reality' for the subject − and particularly for preferencing a version of 'reality' in which preparation for life is prioritised over the learning of mathematical knowledge.

In summary, then, this study is informed by a *qualitative interpretivist paradigmatic* research framework.

1.3 Positioning this study within the field of the sociology of mathematics education

Alongside an interpretivist paradigmatic orientation, the study also falls within a particular theoretical field: namely, *mathematics education*. And within this field, the study is directed in large part by a *sociological* impetus.

According to Ensor and Galant (2005),

> Sociology, broadly speaking, is the study of social actors, of the groups to which they belong (social groups, such as families, social classes and age groups, or cultural groups such as sports and leisure clubs), of the relationships between these groups, and the distribution across them of symbolic and material resources. The objects of sociological study are thus individuals, groups and/or institutions and their distinctive practices. (p. 282)

Dowling (1998, p. 1) offers a similar explanation by emphasising that the focus of a sociological study is the nature of the relationships between individuals and groups and how those relationships are produced and reproduced through different activities, practices and actions.

Reflecting on these explanations in relation to the impetus for and intention of this study: a key impetus for this study is concern over the existing structure of knowledge and participation in the subject-matter domain of Mathematical Literacy and relations of social and/or educational disadvantage that arise from this structure. This impetus has given rise to an intention for the development of an alternative language of description of a structure of knowledge for the knowledge domain of mathematical literacy that aims to negate existing concerns through the promotion of a life-preparedness orientation over the prioritising of mathematisation processes. This impetus and intention are, thus, driven by a sociological concern over the way in which the relationship between the mathematical and the contextual terrains are conceptualised in the structure of legitimate knowledge and participation in the subject. The impact of different conceptualisations of this relationship (and associated criteria for legitimate knowledge and participation) on issues of social positioning, educational ad/disadvantage, and career mobility or restriction are also under analysis.

CHAPTER 2
THIS STUDY AS THE DEVELOPMENT OF A 'LANGUAGE OF DESCRIPTION'

The primary intention of this study is the development and attempted validation of a 'language of description' of the structure of knowledge associated with a form of the knowledge domain of mathematical literacy that promotes the development of a life-preparedness orientation. This language of description comprises a *conceptual framework* that encapsulates aspects associated with mathematically literate behaviour in general and, specifically, with the structure of knowledge and the criteria for legitimate participation in the school-based subject-matter domain of Mathematical Literacy. In light of this, it is feasible to make a diversion into certain considerations relating to the concept of a 'theory'. This includes consideration of Bernstein's (2000) notions of the internal and external languages of description of a theory and Jablonka and Bergsten's (2010) work on different modes of categorisation of a theory. A discussion of the components of a theory (including the claims and concepts that characterise and constitute the theory) are also provided. Finally, the relevance of these theoretical and research based considerations to the language of description developed in this study are considered.

2.1 Internal and external languages of description

2.1.1 General theory on languages of description

Bernstein (1996, 2000), in arguing that an accurate analysis of an empirical space is impossible without an underlying theoretical basis (Morais, 2002, p. 564), presents the concept of *internal* and *external languages of description* to describe the relationship between a theory and the description that the theory provides of a specific empirical activity or resource under investigation (Jablonka & Bergsten, 2010, p. 39).

> Briefly, a language of description is a translation device whereby one language is transformed into another. We can distinguish between internal and external languages of description. The internal language of description refers to the syntax whereby a conceptual language is created. The external language of description refers to the syntax whereby the internal language of description can describe something other than itself. (Bernstein, 2000, p. 132)

In alternative terms, the internal language of description represents the language, components and/or various facets that make up the *theory* – that which is internal to the theory, and the external language of description represents the *methodology* used to establish a description of an empirical activity through the 'gaze' of the theory – that which is external to the theory:

> a theory includes an organised system of theoretical entities, basic principles [internal language of description], and a relation to an empirical field in the form of a more or less explicitly developed methodology [external language of description]. (Jablonka & Bergsten, 2010, p. 27, text in brackets inserted by me)

The internal language of description − the theory − comprises a conceptual language, operating at a high level of abstraction, with explicit theoretical indicators, constructs, grammars and syntax (i.e. rules for describing the components of the theory) and with descriptions of the relationship between different entities in the theory (Jablonka & Bergsten, 2010, p. 39; Morais, 2002, p. 564). All of these are *internal* to the theory. Given that the theory operates at a high level of abstraction, in order to extend the theory to describe something other than itself, an external language of description must be developed for "transforming observed empirical instances of a phenomenon of interest into theoretically relevant data." (Jablonka & Bergsten, 2010, p. 39). The external language of description is comprised of concepts, models and propositions derived from the internal language but with greater applicability (i.e. lower abstraction) (Morais, 2002, p. 564), which are external to the theory, and which work towards bringing the theory closer to the data under analysis (Hoadley, 2006a, p. 24). In Bernstein's words,

> Internal languages are the condition for constructing invisibles, external languages are the means of making those invisibles visible … the external language of description (L2) is the means by which the internal language of description (L1) is activated as a reading device and vice versa. (Bernstein, 2000, p. 133)

And further:

> One of the difficulties of much social theory is that these theories have a powerful and persuasive internal conceptual language but reduced powers to provide externally unambiguous descriptions of the phenomena of their concern. Thus, researchers have difficulty in using the theory to generate the language which will transform the language of enactment, that is the text they are studying (interviews, visual, graphic representations, etc.) into a language which can be read by the theory. These theories rarely generate a language of description: a language which can transform the language of enactment, into a language which the theory can directly read. (Bernstein & Solomon, 1999, p. 274)

Morais (2002, p. 564) illustrates this relationship between internal and external language of description in the following diagram (Figure 3 below), albeit directed specifically towards the social relations of pedagogic activity (which is the primary focus of her own work):

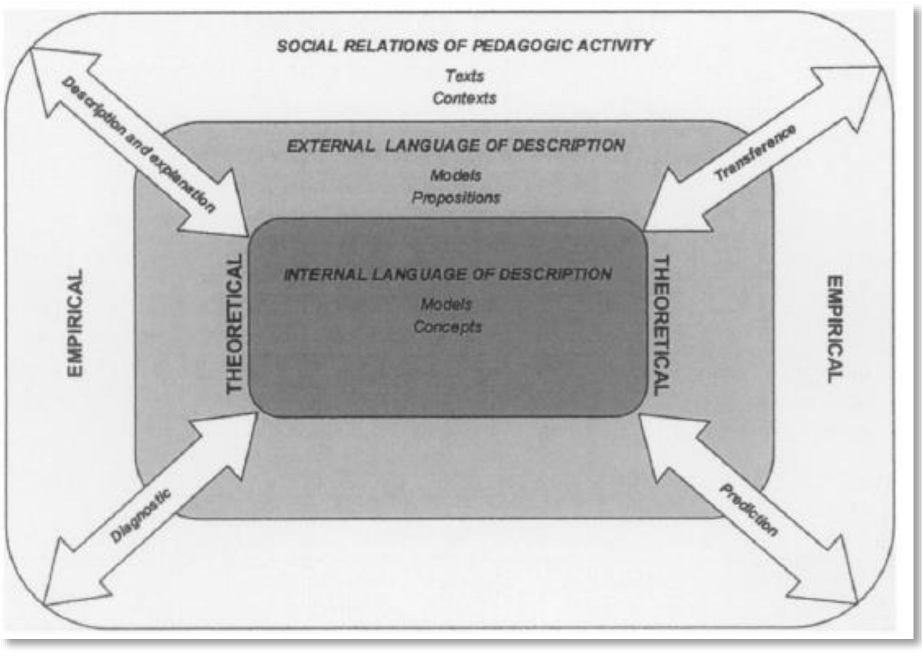


Figure 3: Relationship between internal and external languages of description

Notice that the arrows on the diagrams are bidirectional − flowing from the theoretical to the empirical and from the empirical to the theoretical. This illustrates the "dialogic relationship" (Dowling, 1998, p. 124) between the internal and external languages of description: the internal language of description directs the external language of description in describing the empirical; inversely, the results from the analysis of the empirical (re)inform the theory so that the theory is transformed into a more accurate and precise description of the empirical (Jablonka & Bergsten, 2010, p. 39; Morais, 2002, p. 564).

The extensive quotation below concludes this brief sojourn into the notion of languages of description and illustrates in a comprehensive way the intricate and complex nature of the interaction between theory and method in analysing an empirical space.

> If verticality has to do with how theory develops internally, with what Bernstein later called the <u>internal</u> language of description, <u>grammaticality</u> (in the external sense) has to do with how theory deals with the world, or how theoretical statements deal with their empirical predicates, the <u>external</u> language of description (Bernstein, 2000). The stronger the (external) grammaticality of a language, the more stably it is able to generate empirical correlates and the more unambiguous because more restricted the field of referents; the weaker it is, the weaker is its capacity to stably identify empirical correlates and the more ambiguous because much broader is the field of referents, thus depriving such weak grammar knowledge structures of a principal means of generating progress, namely empirical disconfirmation: 'Weak powers of empirical descriptions removes a crucial resource for either development or rejection of a particular language and so contribute to its stability as a frozen form' (Bernstein, 2000, pp. 167-168). In other words, grammaticality determines the capacity of a theory or a language to progress through worldly corroboration; verticality determines the capacity of a theory or language to progress integratively through explanatory sophistication. Together, we may say that these two criteria determine the capacity of a particular knowledge structure to progress. (Muller, 2007, p. 12) (emphasis in original text)

2.1.2 <u>The internal and external languages of description of this study</u>

The *internal language of description* developed in this study comprises a theory that encapsulates and describes various components considered crucial for the promotion of a particular conception of knowledge and behaviour. This conception of knowledge and behaviour is associated, at a general level, with the knowledge domain of mathematical literacy, and, at a more localised level, with the practices and structure of participation that are envisioned to stem from engagement with the components of this knowledge domain in the setting of the subject-matter domain of Mathematical Literacy. At the foundation of the internal language of description is an expression of a conceived of relationship between mathematical content and knowledge and real-world contexts (and problems encountered in such contexts). Specifically, this conceived of relationship is characterised by an explicit expectation for the subordination of the mathematical terrain to a prioritising of sense-making of contextual situations and an associated life-preparedness orientation. The structure and orientation of the internal language of description is driven in large part by a response to a critical analysis of existing knowledge structures and criteria for participation in the subject through the lens of Paul

Dowling's (1998) Social Activity Theory.[4] A key argument in this theory is that mathematical and everyday practices are incommensurable and that the inclusion of everyday contexts in the mathematical classroom under the guise of making mathematics 'relevant' neither enhances understanding of mathematics nor understanding of the everyday. Instead, the inclusion of contexts are seen to inhibit the ability to 'see' the mathematics clearly; and the everyday contexts included in the classroom are not realistic − they are mathematised and, therefore, mythologised contexts (1995a, p. 9; 1995b, p. 209; 1998, p. 33). The utilisation of this theory to critically analyse current practices in the subject-matter domain of Mathematical Literacy reveals stark similarities to the problematic scenarios identified by Dowling in his own work. It is, thus, in an attempt to address these issues that the incentive for an alternative language of description of the structure of knowledge for the domain of mathematical literacy arises. That promotes a form of participation in the subject that prioritises a life-preparedness orientation. This life-preparedness orientation is deliberately characterised by a perspective in which the utilisation of mathematical structures is seen as being in service to a larger goal for the sense-making of contextual situations and life-preparation (as opposed to a priority for the development of mathematical knowledge and processes of mathematisation).

The internal language, however, remains entirely in the realm of the theoretical. As such, to facilitate the use of the internal language in interpretation and analysis of specific empirical practices relating to the subject-matter domain of Mathematical Literacy, an external language of description is necessary. And, since the empirical practices under investigation in the study involve analysis of textual resources relating to the subject, the external language of description comprises a methodology that involves identification and analysis of the discursive resources – specifically, words/vocabulary, visual mediators, routines and endorsed narratives (c.f. Sfard, 2008) – that characterise these various textual resources.[5] Comparison of the discursive resources in the empirical textual resources to the discursive characteristics of practices associated with the various domains of practice of the developed internal language of description then provides a means for identifying and/or constructing the specific type of knowledge and practices prioritised in a segment of practice relating to the subject. And in so doing, the external language of description provides the means through which the internal language can be employed as a lens for analysing empirical practices associated with the subject-matter domain of Mathematical Literacy.

2.2 The claims and concepts of the language of description

The discussion above highlighted macro-level descriptions and differences between internal and external languages of description. The discussion below now shifts to a more micro-level analysis by focussing on the specific components of an *internal* language of description.

As was mentioned above, Jablonka and Bergsten (2010, p. 27) argue that a theory comprises a system of theoretical entities, basic principles and a methodology that links

[4] See Part 3 starting on page 132 for a detailed discussion of Dowling's theory and page 167 for a discussion of the relevance of his work to the structure and contents of the internal language of description developed in this study.
[5] See Part 5 starting on page 253 for a discussion of the external language of description for this study. For a discussion of the primary methodology (of textual analysis) and associated methods (draw from the field of semiotics) employed in this study in the analysis of empirical resources, see Part 6 starting on page 360.

the theoretical entities or principles to a specific empirical activity or field. Palm (2009, p. 6), in drawing on the work of Niss (2007, p. 1308), argues in a similar way that a theory is a system of interrelated *concepts* and *claims*. The concepts comprise an organised network, linked through a hierarchy, and commonly positioned in a research framework (which can be theoretical, practical or conceptual). The claims – or *theoretical entities* (Jablonka & Bergsten, 2010, p. 27) – of a theory refer to a domain or class of domains consisting of objects, processes, situations and phenomena: the claims comprise a 'theoretical manifesto' (Dowling, 1994, p. 125) of hypotheses, assumptions or axioms about the domain or class of domains which are taken as fundamental, or statements about the domain which draw from or are based on the fundamental claims. These statements often evolve through application of the theory to a specific empirical space as the theory is modified to ensure an effective and comprehensive reading of the space. Thus, a theory can comprise a set of claims about an object and a framework that facilitates analysis of that object.

2.2.1 Claim

In contrast to Dowling's argument that mathematical and everyday practices are incommensurate, my language of description presents the claim (i.e. theoretical proposition) in the form of a hypothesis that: *mathematics is useful and empowering for making sense of real-world contexts and/or problems encountered in real-world settings*. This claim, however, requires several qualifications.

Firstly, the claim is only valid if the motivation and focus of the problem-solving process involving the use of mathematics in contextual situations is for the sense-making of the contextual situations or problems encountered in those situations and not for the learning of mathematical content (which is the focal point of Dowling's theorising). As such, where the domain of Dowling's theory is mathematical knowledge as employed within the site of high school Mathematics, the domain of my language of description is the structure of knowledge associated with a conception of mathematically literate behaviour that is separated from or positioned outside of the domain of scientific mathematics and where engagement with mathematical and contextual structures are (supposedly) equally valued and prioritised. And the above claim for the language of description developed in this study is only valid in this 'external-to-scientific-mathematics' domain.[6] Importantly, the stated claim is also grounded on the assumption that since a primary intention in the language of description is on the *use* of mathematics as a tool for making sense of real-world situations and not on the learning of mathematical content, a necessary level of mathematical competency is already in place. As a consequence, a format for the subject-matter domain of Mathematical Literacy that is aligned to the structure of knowledge outlined in the language of description presented in this study[7] should not have as a *primary* or *ultimate* goal the learning of mathematical content or the development of mathematical competency, although this may occur during the process of preparation for life.

[6] In Part 5 in this study I refer to the knowledge domain of mathematical literacy as a 'blended' domain – namely, as the blending of knowledge and practices associated with both mathematical and contextual structures. See Chapter 17 and sub-section 17.2.1 on page 272 for a discussion of this point.
[7] Or, for that matter, any course that aims to develop mathematically literate behaviour, but where this process takes place outside of the domain of scientific mathematics or separated from the teaching of formal mathematical content.

Secondly, building and extending on from the previous qualification is the recognition and acceptance that although mathematics is useful for making sense of and/or for solving everyday real-world problems, *mathematics is not enough*. Rather, mathematical solutions and models are limited, and there are commonly a variety of extra-mathematical factors which influence the decisions that people make in solving problems in real-world situations. In other words, mathematics is viewed as just one of many tools that can be employed in developing an understanding of a context and associated problems. As such, effective pedagogic practices associated with the a format of the subject Mathematical Literacy that is aligned to the structure of knowledge outlined in the language of description developed in this study must give recognition, exposure, and credence to informal and non-mathematical techniques, structures and considerations which affect and reflect the reality of participation in real-world problem scenarios.

Thirdly, for mathematics to be useful as a tool for the sense-making of real-world situations and problems encountered in those situations, there is an inherent expectation that the person solving the problems is able to apply the mathematical content in a variety of real-world contexts and for a variety of problems. This includes the use of integrated content and skills, developing appropriate models, and, crucially, the ability to identify which techniques and content are appropriate for use in a particular setting and the limitations of the mathematical solution and/or model. As such, the usefulness of mathematics for making sense of the real-world is reliant on the ability of the practitioner to mathematise and model (albeit, in reference to the second qualification above, with an understanding of the limitations of the mathematical solution and/or model).

The final qualification, which relates to the claim and to the previous three qualifications, involves the issue of a 'critical gaze'. Namely, the usefulness of mathematical content and techniques for modelling and sense-making of everyday situations cannot end at the level of 'sense-making'. Rather, sense-making must be accompanied by a critical gaze – a level of 'reflective knowing' (Skovsmose, 1992, 1994b) − that offers a critique of existing structures and recognition of alternative approaches to problem-solving scenarios. This critical gaze, taken together with the combination of calculation and sense-making techniques, provides the means for developing a more comprehensive and critical understanding of the structure of participation in contextual environments and of possible alternative forms of participation in those environments.

2.2.2 <u>Concepts</u>

The claim (theoretical proposition) of the internal theoretical language is accompanied by a *conceptual framework* (c.f. F. Lester, 2005, pp. 458-460)[8] that identifies different

[8] F. Lester (2005, pp. 458-460) defines a research framework as "a basic structure of the ideas (i.e. abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated.", and distinguishes three different frameworks: theoretical, practical and conceptual. The framework that comprises the components of the internal language of description developed in this study falls within the description of a *conceptual framework*: "A conceptual framework is an argument including different points of view and culminating in a series of reasons for adopting some points – i.e. some ideas or concepts – and not others." (Eisenhart, 1991, p. 209). Importantly, there are two key elements of conceptual frameworks which are particularly characteristic of the developed internal language presented in this study. Firstly, a conceptual framework is built on an array of previous and current research from various sources, rather than on a single theory (F. Lester, 2005, p. 460). Secondly, the framework need not be limited to drawing on theories; rather, the local knowledge of practitioners who participate in activities in the terrain under investigation can also be used to inform the questions raised by the researcher and the structure and contents of the framework (Eisenhart, 1991, p. 209).

concepts, the collective and integrative of which are seen to characterise a particular structure of knowledge and participation for the subject Mathematical Literacy that prioritise a life-preparedness orientation. These concepts are borne out of analysis of various perspectives of the components/traits/behaviours/knowledge associated with mathematically literate behaviour. As such, the conceptual framework that characterises the language of description (and the various concepts that make up that conceptual framework) reflects a conglomerate of views and opinions. This conglomerate has have been translated into a language for describing the structure of knowledge needed in the knowledge domain of mathematical literacy (and associated forms of participation in the subject-matter domain of Mathematical Literacy) to facilitate a life-preparedness orientation for the subject.

The conceptual framework envisioned for the theoretical language of description for the structure of knowledge in the knowledge domain of mathematical literacy comprises five interrelated components:

- Contextual domain of reconstituted real-world contexts;
- Everyday domain of practice;
- Mathematical Competency domain of practice;
- Modelling domain of practice;
- A domain of practice involving Reasoning and Reflection on both contextual and mathematical elements.

Each of these domains of practice is discussed and theorised in detail in Part 4 of the study (c.f. Chapter 14 and section 14.4 on page 194 below).

CHAPTER 3
POSITIONING THIS STUDY IN THE FIELD OF KNOWLEDGE PRODUCTION

A key component of this study is concerned with knowledge production (specifically related to a particular form of relationship between the mathematical terrain and the terrain of the real-world) and also with how that knowledge is internalised, recontextualised and enacted in practice (through participation in the subject-matter domain of Mathematical Literacy). For this reason it is appropriate to locate the study in relation to the fields that make up Bernstein's pedagogic device. My intention for doing this is to highlight the limited field of application of this study and, so, to acknowledge the restrictions of the study for making claims beyond this limited field.

To describe the "systemic and institutionalised" ways in which knowledge is converted and recontextualised from the field where it is produced, into the schooling system, and then again within that schooling system (Bertram, 2009, p. 47), and the consequent relations of power and control that develop through the recontextualisation process (Singh, 2002, p. 571), Bernstein (1996) presents the notion of the *pedagogic device*. According to Maton and Muller (2007),

> … the pedagogic device forms the basis of his [Bernstein's] account of: the ordered regulation and distribution of society's worthwhile store of knowledge, ordered by a specifiable set of *distributive rules*; the transformation of this store into a pedagogic discourse, a form amenable to pedagogic transmission, ordered by a specifiable set of *recontextualizing rules*; and the further transformation of this pedagogic discourse into a set of evaluative criteria to be attained, ordered by a specifiable set of *evaluative rules*. (p. 19, emphasis in original text)

Each of these rules is associated with a specific field of activity involving the production and recontextualisation of knowledge, thus giving rise to three primary site of knowledge production / recontextualisation: *field of production*; *field of recontextualisation* (comprising the sub-fields of the official recontextualisation field and the pedagogic recontextualisation field); and the *field of reproduction.*[9]

[9] It is important to point out that the fields (and also the rules that govern how behaviour is transformed in a particular field) are hierarchically related (Singh, 2002, p. 573 & 574) in the sense that a body of knowledge must first be produced before it can be recontextualised or reproduced (and/or evaluated). Reflecting this discussion on the empirical terrain of the subject Mathematical Literacy, a key challenge encountered in the subject is that the knowledge domain on which the subject is based is not clearly and explicitly defined, nor are the rules for recontextualisation and evaluation. For some, the recontextualisation and evaluation rules are based in mathematical structures, and for others they are to be found in the domain of the contextual world. The result is that the knowledge domain of mathematical literacy is interpreted and understood differently by and for different people, with participation in the practices of the subject differently legitimised and evaluated by different role-players (which an analysis of curriculum intention and examination focus will quickly reveal – see (North, 2010) for an illustration of this issue). The learners in the subject are, then, caught in the middle of this process, having to deduce the rules and structures of recontextualisation and evaluation according to which their participation is to be endorsed. The primary intention of this study, then, is to more clearly and explicitly define the components of the knowledge domain of mathematical literacy and to provide a means through which this internal dimension is able to be employed in analysis of empirical practices in the subject (through a developed external dimension of the language). However, identification of the rules through which the knowledge store for mathematical literacy is recontextualised and evaluated in pedagogic and empirical practices is beyond the scope of this study and, instead, is reserved for future research endeavours.

The *field of production* is the field in which discipline-specific or domain-specific expert knowledge (Singh, 2002, p. 572) is constructed and positioned by role-players such as academics and professionals in the field.[10] Since the primary intention of this study is the development of a theoretical language to describe a particular conception of the knowledge required to engage both mathematical and contextual domains of practice, a major component of this study is, thus, located in the field of knowledge production. This knowledge production is in specific relation to domain of knowledge associated with a specific conception of mathematically literate behaviour and also to the localised empirical site of practice of the secondary school subject Mathematical Literacy in South Africa.

To facilitate the development of this language of description and to provide a rationale of the need for this language, it is also necessary to temporarily shift the object of study. This shift in the object of study facilitates analysis of literature relating to conceptions of mathematically literate, numerate and/or quantitatively literate behaviour in general and, specifically, to the South African conception of mathematical literacy embodied in the secondary school subject Mathematical Literacy. This latter site of analysis requires a reading of various curriculum and related supporting documents for the subject. Thus, Part 2 of the study (c.f. page 29 below) – in which the bulk of the analysis described above takes place – facilitates a shift of focus in the object of study to a site characterised by Bernstein as the *official recontextualising field* (as a sub-field of the field of recontextualisation). Once the internal language of description is developed, a gaze is cast from this internal language – through an external language and associated methodology of textual analysis – over empirical textual resources (textbook section, curriculum section, national examinations, and course notes for a teacher education course) relating to the subject-matter domain of Mathematical Literacy (c.f. Part 7 starting on page 381). This process is also located in field of recontextualisation, albeit this time in the *pedagogic recontextualising sub-field*. A word of clarification and caution is, however, necessary. It is essential to reiterate that the primary intention of this study is the development of a theoretical language to describe a particular knowledge structure – which takes place in the field of production. Any attempt to shift the object of study outside of this field – for example, through the analysis of curriculum documents or pedagogic texts – is always in service to this knowledge development intention. As such, although focus shifts to different sites in the official recontextualising field in the analysis of subject-related documents, this shift is made to further facilitate and strengthen the developed internal language of description.

As regards the *field of reproduction*: this is the field in which teachers make use of the "privileged and privileging texts created in the field of recontextualisation" (Singh, 2002, p. 577) to interpret and implement the curriculum through classroom and assessment practices. Although in this study I draw on my own experiences of teaching in the subject-matter domain of Mathematical Literacy and on my interactions with other teachers of the subject in providing an analysis of the literature and curriculum documents and in outlining approaches to pedagogic practice for the subject, the key area of focus of this study does not fall within this field of reproduction. Rather, I step periodically and somewhat superficially into this field to inform the knowledge construction process of a

[10] Bernstein distinguishes between two primary forms of knowledge – hierarchical and horizontal knowledge structures. These knowledge forms are differentially distributed to different groups of people to produce differential power relationships (Singh, 2002, p. 573). Hierarchical and horizontal knowledge structures, together with horizontal and vertical discourses, are discussed briefly in Part 3 of this study (c.f. Chapter 10 and sub-section 10.1.1.2 starting on page 150 below).

language of description within the field of production and the testing of this language in the field of recontextualisation.

The positioning of this study in relation to Bernstein's pedagogic device is illustrated in the diagram shown in Figure 4 below:

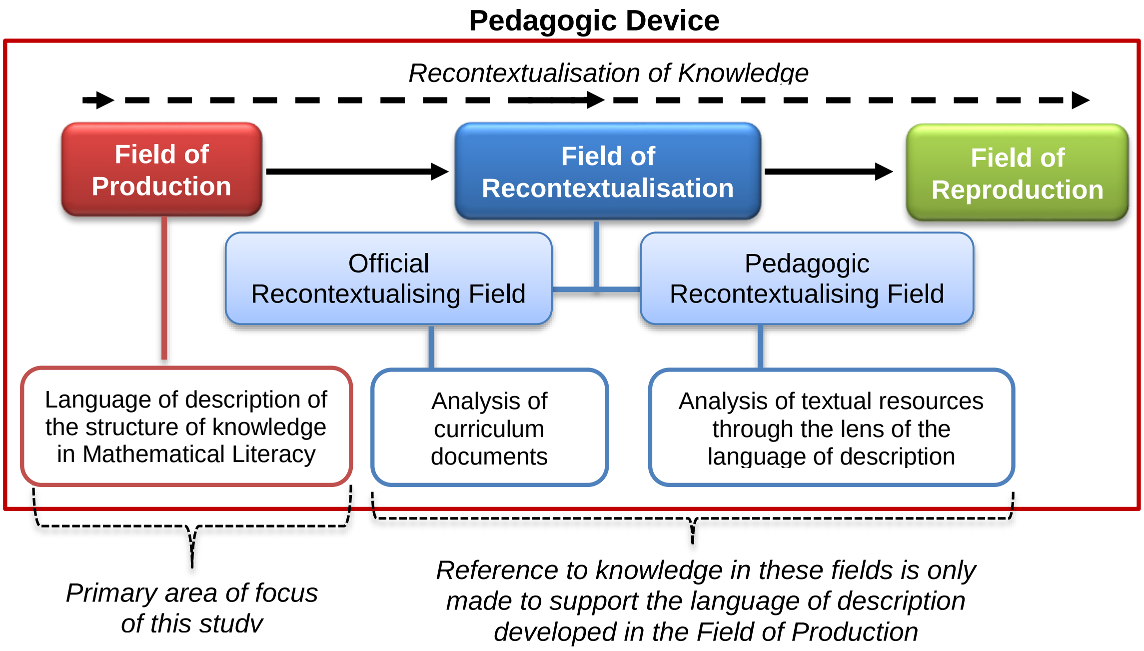


Figure 4: Positioning of this study in relation to Bernstein's Pedagogic Device

Importantly, the location of this study primarily within the field of knowledge production signifies that any claims made only provide a limited perspective on the practices associated, specifically, with the subject-matter domain Mathematical Literacy and, more generally, on the traits associated with mathematically literate behaviour that are seen to stem from participation in this subject. In particular, since the study does not delve into the field of reproduction, little or no validated commentary is possible regarding the classroom practice of teachers or learners and/or how such participants may further re-contextualise the legitimised knowledge developed in the field of knowledge production.

CHAPTER 4
CONCEPTUALISING AND CLARIFYING THE NOTIONS OF A ‘LIFE-PREPAREDNESS ORIENTATION’ AND ‘PRACTICE’

4.1 ‘Life-preparedness orientation’

The key focus of this study is the theorisation of a theoretical language for describing the structure of knowledge in a conception of the knowledge domain of mathematical literacy that prioritises a *life-preparedness orientation*. The notion of a life-preparedness orientation is, thus, a central component of the language of description, and the structure of legitimate participation envisioned for the subject Mathematical Literacy is directly linked to the components of this orientation. In this chapter I elaborate and clarify the precise meaning of the notion of a life-preparedness orientation.

The phrase ‘life-preparedness orientation’ is originally coined by Venkat (2010), and my usage of the term in this study is based primarily on this original conceptualisation. Briefly, a life-preparedness orientation prioritises understanding of the world through any and all available means and resources – some of which may include formal mathematics, but others which may utilise informal or everyday reasoning and forms of participation and communication. A life-preparedness orientation is directed towards preparation for effective functioning in everyday life through exposure to existing everyday forms of knowledge and participation and to possible alternative forms derived through mathematically informed considerations. A life-preparedness orientation recognises that a mathematically based view of the world provides only one view – and often a limited view − of the world, and that although mathematical knowledge can facilitate an alternative understanding of the world, the structures that define action, decision-making, communication and endorsed participation in the world are often not mathematically based. As such, preparation for the world must acknowledge the role of non-mathematical factors and considerations which might affect decision-making and the limitations of a mathematical gaze in reflecting reality. In this orientation, although the role of mathematics in facilitating an alternative view of the world is recognised, mathematics is always seen as in service to a goal for a better understanding of an aspect of reality and of the structure of legitimate forms of participation in that reality. Preparation for life and not the learning of mathematical knowledge is the ultimate goal, and the terrain of the contextual and not of the mathematical constitutes the organising principle of legitimate participation. Translating this to the terrain of the subject Mathematical Literacy, my key argument is that a conception of this subject that aims to prepare participants for a potentially more empowered functioning in the real-world – for life-preparation – must facilitate several facets. These include: engagement with authentic contexts, recognition of the limited role of formal mathematical structures and techniques in adequately describing real-world situations, and acknowledgement that any mathematics appropriated and/or used must be in service to a goal for broader contextual sense-making practices.

In Part 2, Chapter 5 and sub-section 5.2.2.3 of the study (c.f. page 47 below) I extend the conceptualisation of the notion of a life-preparedness orientation. I achieve this by positing that, at a general level, this type of orientation is characterised by a structure of participation that explicitly prioritises an <u>agenda</u> (namely, specific areas of prioritising that are internal to a practice) for contextual sense-making practices (as opposed to an agenda for literacy in mathematics, contextualised calculations or modelling). I argue further that this orientation is also characterised by a range of possible <u>intentions</u>

(namely, the specific external purpose of the practice) – including intentions for the development of Human Capital, Cultural Identity, Social Change, Environmental Awareness, and Critically Evaluating Structures. The format of the life-preparedness orientation conceptualised and promoted in the internal language of description is, then, posited as characterised by the specific combination of an agenda of Contextual Sense-Making Practices and an intention for Critically Evaluating (contextual and mathematical) Structures encountered in real-world problem-solving scenarios. In Part 4 (c.f. page 181) I then argue that this particular conception of a life-preparedness orientation is facilitated through engagement with different domains of practice − comprising the Everyday, Mathematical Competency, Modelling knowledge, and Reasoning and Reflection domains – in relation to a Contextual domain of reconstituted real-world contexts. The collective of these domains comprises the components of the developed internal language of description of a structure of knowledge for the knowledge domain of mathematical literacy (and associated forms of participation in the subject-matter domain of Mathematical Literacy), and engagement with this collective is presented as facilitating life-preparedness.

In short and in summary, a life-preparedness orientation for the subject Mathematical Literacy is facilitated through understanding of how people function in daily-life and of how mathematics can be used to describe a particular form of participation in the world, and of the ability to engage mathematical elements encountered in contextual environments and to do so in a critical way.

4.2 The notion of 'practice' as employed in this study

In the discussion so far, and for the remainder of the discussion in the study, I have made reference to the notion of 'practice(s)', both generally in relation to the knowledge domain of mathematical literacy and also more specifically in relation to the subject Mathematical Literacy. By 'practice' I am referring to all activities associated with these sites, including physical actions (such as problem-solving experiences and pedagogic interactions) and also resources (such as textual resources, technology resources, and any other resources which inform legitimate participation in a contextual situation). However, the empirical analysis component of this study is focussed specifically on analysis of textual resources and of the forms of practice that are legitimised in those resources for successful participation in the subject Mathematical Literacy. Thus, while the developed language of description of the knowledge domain of mathematical literacy presents a particular structuring of knowledge associated with participation in all forms and sites of practice (in relation to a specific format of the interaction between the contextual and mathematical terrains), exemplification of the utility of language is restricted to practices embedded in and espoused through textual resources. This contributes to a key limitation of this study – namely, that the conclusions presented with regards to practices in the subject Mathematical Literacy are limited to those involving engagement with specific textual resources. No validated commentary is able to be supplied regarding other sites of pedagogic practice, including about interactions between various role-players in the subject (e.g. teachers, learners) or about other sites of engagement (e.g. professional development initiatives for pre- and in-service teachers).

WHERE TO FROM HERE

Having now specified certain theoretical antecedents that have informed the orientation, structure and approach adopted in this study, focus in Part 2 of the study shifts to discussion of the empirical terrain that is the focus of this study – namely, the subject-matter domain of Mathematical Literacy. The structure of legitimate participation in the subject (and the structure of knowledge required to facilitate this legitimate participation) is of particular concern in this impending discussion. This discussion is precluded by focus on general conceptions of mathematical literacy, numeracy and/or quantitative literacy and the structure of participation and behaviour prioritised in these different conceptions, together with clarification of the criteria according to which participation in the subject is legitimised. A framework to facilitate comparison of different conceptions of mathematically literate, numerate and/or quantitatively literate behaviour is also presented.

PART 2
REVIEW OF THE LITERATURE

INTRODUCTION AND OVERVIEW

Given that the empirical terrain of focus in this study is the secondary school subject-matter domain of Mathematical Literacy in South Africa, my intention in this chapter is to provide background information about the subject, to identify the rationale or intention for the introduction of the subject in the South African curriculum framework, and to provide clarification on the intended and implemented structure of knowledge and legitimate participation in the subject. I also investigate areas of divergence and commonality in the structure of knowledge and legitimate participation in the subject to existing local (South African) and international perspectives on forms and characteristics of mathematically literate, numerate and/or quantitatively literate behaviour. All of this is achieved through investigation of the dominant agendas and intentions prioritised in different conceptions of mathematical literacy, and the resultant structure of legitimate participation that is endorsed.

A key point that I highlight throughout this discussion is that there is a dominant orientation permeating much of the international literature on mathematical literacy, numeracy and/or quantitative literacy, and also within the literature on the South African school subject Mathematical Literacy, for the prioritisation of mathematical knowledge, techniques and structures (i.e. mathematisation processes and/or the imposition of a 'mathematical gaze'[11]). This prioritisation occurs over an agenda for contextually relevant/appropriate/legitimate forms of participation and contextual sense-making practices in the development of mathematically literate behaviour: namely, the positioning of the mathematical terrain rather than the contextual terrain as the organising principle of the learning process[12]. In the South African context – where the development of mathematically literate behaviour has been extricated and separated from the domain of scientific mathematics – this emphasis on mathematised forms of participation is disadvantaging learners who engage in the subject Mathematical Literacy and is contributing to the re(production) of a degree of educational and social disadvantage. The problematic nature of current practices in the subject is be analysed and theorised in detail through the work of Paul Dowling (1998) in Part 3 and Chapter 11 of the study (starting on page 167).

Although my primary intention in this part of the study is for analysis and discussion of the dominant agendas and intentions that characterise differing conceptions of mathematical literacy, a secondary intention also exists. Namely, in Chapter 5 below I present and utilise an analytic framework that I have developed to facilitate comparison and categorisation of differing perspectives and conceptions of the characteristics of mathematically literate behaviour. One dimension of this framework is focused on the dominant agenda and intention prioritised in a particular conception of mathematical literacy and the structure of legitimate knowledge and participation associated with these. I contend that the components of this dimension provide a particularly important means

[11] Refer to Part 3, Chapter 10 and sub-section 10.2.2 (starting on page 156) for an elaborated discussion of the notion of a 'mathematical gaze' as described from the perspective of Dowling (1998).
[12] The idea for distinguishing between the mathematical terrain and the contextual terrain in terms of the dominant *organising principle* of an activity or learning process is drawn from the work of Ginsburg, Manly, and Schmitt (2006).

for differentiating different conceptions of mathematical literacy, and do so in a way that is able to account for both internal and external impetus that impact on the structure of participation or behaviour that is endorsed. Thus, the discussion in this part of the study provides an opportunity to demonstrate this framework and to test if the framework is able to provide a form of categorisation that illustrates differences in the structure of participation or behaviour legitimised in different conceptions of mathematical literacy.

Having presented the components of the analytic framework in Chapter 5, in Chapter 6 I then utilise the framework to provide an analysis and comparison of international literature and perspectives[13] on general conceptions of mathematical literacy, numeracy and/or quantitative literacy, and the types of behaviour envisioned for these different conceptions.

In Chapter 7 I outline different meanings attached to the terms mathematical literacy, numeracy and/or quantitative literacy within the literature and provide a justification and motivation for my own privileging of the term 'mathematical literacy'.

The discussion in Chapter 8 shifts to a specific focus on the South African conception of mathematically literate behaviour as encompassed in the school subject Mathematical Literacy. After providing a brief history of the subject, I use the same analytic framework outlined in Chapter 5 to analyse and discuss various facets of the subject. I also point out areas of commonality and divergence to international perspectives on mathematical literacy, numeracy and/or quantitative literacy.

The structure of this part of the study is illustrated in Figure 5 below:

[13] By 'international' I am referring to literature that reference conceptions of mathematical literacy, numeracy and/or quantitative literacy that do not relate to the South African conception of mathematical literacy as encompassed in the South African school subject Mathematical Literacy. It is necessary to make this distinction because South Africa is the only country where the development of mathematically literate behaviour is given its own domain as a fully-fledged subject. In all other countries where mathematical literacy, numeracy and/or quantitative literacy is acknowledged and encouraged, it is done so in the context of the normal teaching of the discipline of mathematics. This important different in the way in which mathematical literacy in South Africa is conceptualised and positioned in the school and/or subject framework in comparison to international conceptions of mathematical literacy, numeracy and/or quantitative literacy makes an independent analysis of these two spaces necessary.

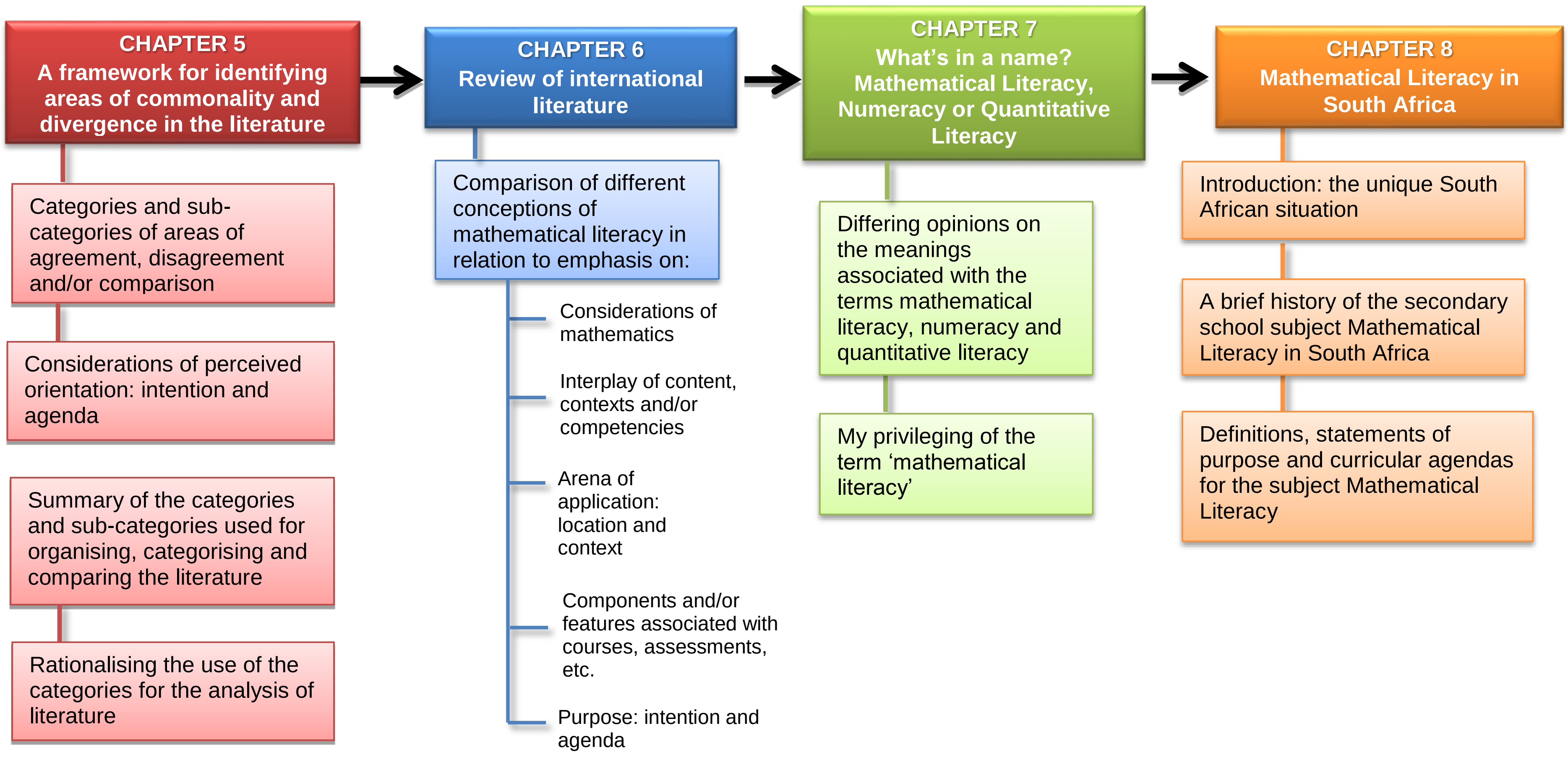


Figure 5: Overview and chapter structure of Part 2 of the study

CHAPTER 5
A FRAMEWORK FOR ORGANISING THE LITERATURE ON MATHEMATICALLY LITERATE, NUMERATE AND/OR QUANTITATIVELY LITERATE BEHAVIOUR[14]

5.1 Justifying the need for a framework

Consider the following selection of definitions or expressions of mathematical literacy, numeracy and quantitative literacy:

> Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems. (DoE, 2003a, p. 9)

> *Mathematical literacy* is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 1999, p. 41; 2003, p. 24; 2006, p. 72; 2009, p. 84) (emphasis in original texts)

> Numeracy is the ability of a person to make effective use of appropriate mathematical competencies for successful participation in everyday life, including personal life, at school, at work and in the wider community. It involves understanding real-life contexts, applying appropriate mathematical competencies, communicating the results of these to others, and critically evaluating mathematically based statements and results. (Neill, 2001, p. 7)

> For me, the key area of distinction [between mathematical and quantitative literacy] is signaled by the term literacy itself, which implies an integrated ability to function seamlessly within a given community of practice. Literacy as generally understood in the verbal world thus means something qualitatively different from the kinds of skills acquired in formal English courses. For one thing, it is profoundly social, and is therefore a moving target because its contents depend on a particular social context. (Ewell, 2001, p. 37)

> The central argument is that it is not possible to promote a conception of mathematical literacy without at the same time – implicitly or explicitly – promoting a particular social practice. (Jablonka, 2003, p. 75)

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[14] A qualification is necessary here. By using the term 'behaviour' I am deliberately making a distinction between literature relating to the broader notions of mathematical literacy, numeracy and quantitative literacy, much of which is from an international perspective, and literature relating specifically to the South African school subject Mathematical Literacy. The distinction, then, is between a form of behaviour and the contents of a school subject.

> The main goal of a criticalmathematical literacy is not to understand mathematical concepts better, although that is needed to achieve the goal. Rather, it is to understand how to use mathematical ideas in struggles to make the world better. In other words, the question to be investigated about my criticalmathematical literacy curriculum is not "Do the *real* real-life mathematical word problems make the mathematics more clear?" The key research questions are "Do the *real* real-life mathematical word problems make the social justice issues more clear?" and, "Does that clarity lead to actions for social justice?*"* (Frankenstein, 2009a, pp. 1-2, emphasis in original text)

> It is argued that mathematical literacy focussing on citizenship should refer to the aim of critically evaluating aspects of a surrounding culture – a culture that is more or less colonised by practices that involve mathematics. Thus the ability to understand and evaluate these practices should form a component of mathematical literacy. (Jablonka, 2003, p. 76)

Even amongst this small selection of definitions or expressions significant differences emerge with respect to how different authors conceptualise the notions of mathematical literacy, numeracy and/or quantitative literacy and the facets or characteristics associated with these different conceptualisations. For example, the first three quotations clearly posit mathematical literacy, numeracy and/or quantitative literacy as a form of 'behaviour' − as a "way of thinking; a way of behaving; a way of relating to the numerically-based world in which we live." (Brombacher, 2007, p. 2) − which enables a more effective functioning in the world. The fourth quotation embodies a different agenda – a literacy agenda − positioning mathematical literacy, numeracy and/or quantitative literacy as the mathematical cousin to written and verbal literacy. This quotation, together with the fifth quotation, also promotes the notion of mathematical literacy, numeracy and/or quantitative literacy as a socially embedded practice that is defined and influenced by and inseparable from the social practice and/or context in which it is encountered. The sixth and seventh quotations posit a different conception – a critical conception − where the primary intention for the development of mathematically literate, numerate and/or quantitatively literate behaviour is as an enabler for challenging existing societal structures. This conception calls for a different type of critical behaviour than is suggested in some of the other quotations, in that the intention is not so much to become a critical consumer or user of mathematics as to become a critical citizen – namely, a citizen who has the capacity to analyse and challenge existing models and structures that underpin societal, economic and political structures and decision-making processes.

The discussion above highlights the complexity involved in trying to make sense of the enormous body of literature relating to conceptions of mathematically literate, numerate and quantitatively literate behaviour. One facet of this complexity stems from the differential usage of three different terms – mathematical literacy, numeracy and quantitatively literate − to describe what, in many ways, amounts to the same beast: namely, the relationship between school mathematics and out-of-school contexts. A second factor is the widely varying opinions on the purpose, intention, dominant areas of focus, sites of practice, components, and structure of legitimate participation associated with differing conceptions of mathematically literate, numerate and/or quantitatively literate behaviour. Jablonka (2003) provides a possible reason for this lack of consensus:

> Any attempt at defining 'mathematical literacy' faces the problem that it cannot be conceptualised exclusively in terms of mathematical knowledge, because it is about an individual's capacity to *use* and *apply* this knowledge. Thus is has to be conceived of in functional terms as applicable to the situations in which this knowledge is to be used. (p. 78, emphasis in original text)

Different conceptions of mathematical literacy, numeracy and/or quantitative literacy, and associated forms of knowledge and characteristics of behaviour, then, are "culturally attuned" (de Lange, 2003, p. 87) and may vary with respect to "the culture and the context of the stakeholders who promote it." (Jablonka, 2003, p. 76). Steen (2003b) argues similarly:

> To the degree that numeracy and mathematics are important features of our culture, differences in national traditions will necessarily create significant differences in both the objectives and outcomes of mathematics education. (p. 212)

In sum, being mathematically literate, quantitatively literate and/or numerate means different things to different people in different contexts, cultures and time periods.

For the reasons given above, to make sense of the plethora of definitions, expressions and descriptions of mathematical literacy, numeracy and/or quantitative literacy it has been necessary to devise a framework according to which the literature can be organised, categorised and compared to identify areas of commonality and divergence. In the discussion below I outline the structure of this framework and explain how I intend to use the framework to analyse the relevant literature.

It is important to point out that this framework is not a theoretically derived framework based on a particular theoretical language or structure. Rather, the framework is an analytical framework that is used to separate the readings and their constituent elements in order to analyse and compare them. The components of the framework have been derived from the literature itself: a reading of the literature has allowed for identification of areas of commonality and divergence, which, in turn, have provided the components and/or categories for comparison.

5.2 A framework for identifying areas of commonality and divergence in the literature

5.2.1 Categories and sub-categories of areas of agreement, disagreement and/or comparison

The work of Neill (2001) provides an appropriate initial framework for identifying areas of commonality and/or divergence in the literature relating to mathematical literacy, numeracy and quantitative literacy. Through analysis of various definitions of these concepts in academic literature, Neill identifies three primary themes common to most definitions: (i) reference to context or location[15]; (ii) reference to mathematics; and (iii) mention of particular 'strands of mathematics' through which the relationship between specific mathematical content and real-world problems is established (Neill, 2001, pp. 4-6). However, Neill's analysis is dated (having been conducted in 2001) and the body of literature on mathematical literacy, numeracy and quantitative literacy has grown tremendously since that period; furthermore, views on these conceptions have also altered and transformed. For example, while it may still be useful to know that 'reference to mathematics' is a prevalent theme in the literature on mathematical literacy, numeracy and quantitative literacy, what is perhaps currently of more relevance is the differential privileging of mathematical content and knowledge over everyday forms of knowledge and participation in different conceptions and expressions. For this reason, while Neill's analysis provides a suitable starting point, it is necessary to expand on this analysis to accommodate subsequent changes in research, opinion and thought on the structure of knowledge and characteristics of behaviour associated with differing conceptions of mathematical literacy, numeracy and quantitative literacy.

In light of the above and through a comprehensive reading of the literature on conceptions of mathematical literacy, numeracy and/or quantitative literacy, the following categories and sub-categories emerge as areas of agreement, disagreement and/or comparison:

- <u>Category 1</u> − Considerations of *mathematics*[16]:
    - Reference to the inclusion of mathematical content, knowledge and/or techniques.
    - Reference to the 'use' or 'application' of mathematical content, knowledge and/or techniques to solve problems.
    - Description of specific strands of mathematical content.
    - Description of the scope of applicable mathematical content, ranging from basic number concepts to understanding of abstract principles.
    - Recognition of contextual forms of participation and less formal mathematical techniques (e.g. estimation) and/or non-mathematical (situational) techniques.
    - Reference to distinguishing characteristics from scientific mathematics.

- <u>Category 2</u> − Interplay of *content, contexts* and/or *competencies*:
    - Specification of the way in which the role of content, contexts and/or competencies is construed in the development of mathematically literate, numerate and/or quantitatively literate behaviour.

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[15] Neill (2001, p. 4) distinguishes between location and context as follows: "Location refers to the places in which a person may apply numeracy such as at home, at work, in society, etc. … Context refers to the specific problems, situations, or tasks that the person applies numeracy to." For instance, 'home' is a location which can give rise to different contexts such as cooking or construction. Location and context, thus, refer to different levels of categorisation, with location at a macro-level and contexts at a more micro-level and embedded within the macro-level. Neill goes on to argue that only very few of the definitions analysed emphasise both the locational and contextual facets of numeracy, and, thus, presents an alternate definition of numeracy that emphasises both.

[16] Importantly, the fact that I have deliberately started with a description of the 'mathematical considerations' must not be interpreted as a prioritising of mathematical considerations over the other categories of considerations listed. My decision to do so is driven by the fact that most expressions of mathematical literacy, numeracy and/or quantitative literacy describe these conceptualisations in terms of the *use-value* of mathematical content − or as mathematical content in use in real-world contexts. As such, in developing a framework it made sense to me to begin by first developing an understanding of the nature of this mathematics before investigating perceptions of how the mathematics is to be integrated within context.

    - Specific mention of skills, competencies, and/or traits associated with mathematically literate, numerate and/or quantitatively literate behaviour (where appropriate).

- Category 3 − Consideration of the *arena of application* of mathematical content:
    - Reference to location (i.e. the place in which a person may apply mathematically literate, numerate and/or quantitatively literate behaviour: e.g. home, work, community).
    - Reference to context (i.e. the specific problems or situations encountered within a location requiring the use of mathematically literate, numerate and/or quantitatively literate behaviour: e.g. working with the context of electricity billing in the location of the home).

- Category 4 − Specified *components* of mathematical literacy, numeracy and/or quantitative literacy with respect to courses, assessments and/or pedagogic practice that promote the development of specific forms of knowledge and behaviour.

- Category 5 − Consideration of the perceived *orientation* – as embodied in an indication of the dominant agenda and intention − of a particular conception of mathematical literacy, numeracy and/or quantitative literacy.

This final category of perceived orientation – and the embedded sub-categories of agenda and intention – requires further elaboration.

5.2.2 Considerations of perceived 'orientation': dominant agendas and intentions

Jablonka (2003) identifies and distinguishes between different 'perspectives' and 'conceptions' of mathematical literacy. The various 'perspectives' are differentiated by the extent to which there is a prioritising or privileging of the mathematical terrain or the real-world terrain (and of associated forms of knowledge, participation and communication for each of these), or of something in-between; and, by consequence, the extent to which a particular conception of mathematical literacy, numeracy and/or quantitative literacy is directed towards the learning of mathematics or towards a life-preparedness goal. Alternatively, this can be described as a distinction in the extent to which mathematical or contextual knowledge structures (and associated legitimised forms of participation and communication) are positioned as the organising principle of the leaning process. I use the term *agenda* to refer to the dominant domain of prioritising – mathematical, contextual, or a combination of both − within a particular form of mathematical literacy, numeracy and/or quantitative literacy.

The 'conceptions', on the other hand, describe a broader *intention* behind the promotion of a particular conception of mathematical literacy, an intention that is driven by possible career, economic, social, environmental and/or political motivations. This intention is what it is envisioned participants will be able to 'do' once they have developed a level of competence – the final outcome or goal for which participants are being imbued with a particular form of mathematically literate, numerate and/or quantitatively literate behaviour. The intentions define what mathematically literate, numerate and/or quantitatively literate behaviour is prioritised for: for example, to facilitate functioning in a business environment, or the solving of environmentally based problems.

Where the 'conceptions' emulate a macro-level view of the intention of a particular form of mathematically literate behaviour, the 'perspectives' or agendas embody a micro-level consideration of the foregrounding of a particular domain of knowledge and associated criteria for successful and legitimate participation in a practice. In overly simplistic terms, the intentions embody the question of 'why' and the agendas the question of 'what': why is a particular form of mathematical literacy, numeracy and/or quantitative literacy being developed – for preparation for the workplace, or for challenging existing societal structures, or for something else?; and what constitutes mathematically literate behaviour – engagement with mathematics, engagement with context, a combination of both, or something else? The intentions, then, characterise an *external* impetus for the development of a form of behaviour, knowledge and participation; while the agendas characterise that which is *internal* to the specific form of knowledge development.

'*Intention*' (conceptions)
- refers to 'why' a form of behaviour is promoted and the *external* (macro-level) impetus for the development of that behaviour.

'*Agenda'* (perspectives)
- refers to 'what' constitutes the dominant organising principle *internally* (micro-level view) in the development of a form of knowledge or behaviour.

Taken together, identification of the dominant agenda and intention inherent in a particular conception of mathematical literacy, numeracy and/or quantitative literacy provides a lens for establishing whether mathematical learning or contextual sense-making practices (and an associated goal for life-preparedness) is prioritised. Identification of the dominant agenda and intention also brings to light what it is envisioned participants 'will be able to do' with their mathematical or life-preparedness knowledge and skills. The collective of these categories thus constitutes evidence of a particular *orientation* espoused for mathematical literacy, numeracy and/or quantitative literacy.

In the pages below, the various agendas and intentions for mathematical literacy, numeracy and/or quantitative literacy are discussed in detail and presented in the form of a 2-dimensional cross-product or matrix. I argue that consideration of these two dimensions provides insight into the perceived orientation promoted for a particular form of knowledge and behaviour involving the relationship between mathematics and real-world contexts or problems.

It is my intention to use this cross-product in conjunction with the other categories identified above to identify common and divergent statements of purpose in the literature on mathematical literacy, numeracy and quantitative literacy.

5.2.2.1 Dominant agendas in mathematical literacy and numeracy[17]

Julie (2006, p. 62) argues that the various definitions on mathematical literacy can be seen to be on a continuum, with mathematical literacy for entry into mathematics on one end of the spectrum and mathematical literacy for critical interaction with mathematical structures and installations in society on the other end. Jablonka (2003, p. 76) illuminates

[17] In identifying the different perspectives, Jablonka (2003, p. 76) initially refers specifically to mathematical literacy and numeracy and not to quantitative literacy. Only at a later point in the discussion does she refer to quantitative literacy. At no point does she explicitly distinguish between the three or even acknowledge a deliberate interchangeable usage of the three terms. Despite this inconsistency, my intention is to use these different perspectives to organise and identity common threads in the literature that refers to mathematical literacy, numeracy *and* quantitative literacy.

the in-between categories by suggesting that the following groupings can be identified from the multitude of different perspectives of numeracy and mathematical literacy in the literature: mathematical literacy or numeracy is seen as:

(i) basic mathematical competence: i.e. “knowledge and understanding of fundamental mathematical notions” (Jablonka, 2003, p. 76);
(ii) the ability to perform mathematical calculations[18] in everyday contexts;
(iii) the ability to develop mathematical models of both simple and complex real-world contexts;
(iv) the ability to understand and evaluate existing mathematical knowledge, and models and structures developed by others which promote a particular value system and/or perspective.

I contend that the movement from perspectives (i) to (iv) is characterised by a shift in prioritisation of the mathematical terrain to the terrain of real-world. While the first perspective has as an explicit goal the development of mathematical knowledge and techniques, the fourth perspective is oriented towards sense-making of real-world environments and the forms of knowledge that facilitate legitimate and endorsed participation and communication in those environments. This movement is also characterised by a heightened degree of critical engagement with contextual components and awareness of the role of mathematics in informing and shaping preferenced interpretations of real-world practices. Perspectives (ii) and (iii) reflect moderated versions of perspectives (i) and (iv), and exhibit differential degrees of prioritisation of the mathematical terrain and the real-world terrain respectively.

With this in mind – and reflecting back on my usage of the term ‘agenda’ to refer to the dominant area of prioritising in the development of a particular form of mathematically literate, numerate and/or quantitatively literate knowledge or behaviour − it proves useful to view the different perspectives offered by Jablonka as constituting a *spectrum of*

[18] Jablonka (2003, p. 76) uses the word ‘basic’ in describing the type of mathematical calculations envisioned in this perspective: “basic computational and geometrical skills in everyday contexts”. I have deliberately excluded this word from the description of this perspective, as a reading of the literature quickly highlights the dissention amongst authors of the meaning of ‘basic’ when referring to mathematical content in the context of mathematical literacy, numeracy and quantitative literacy. Furthermore, in many cases the mathematical content specified is certainly not basic (in terms of how a non-mathematician might conceptualise basic content) and, rather, involves abstract algebraic, geometric and trigonometric concepts. Exclusion of the word ‘basic’, then, facilitates for the inclusion of a broader range of literature in this category.

agendas[19]. This spectrum describes the differential relationship between a prioritising of exclusively mathematical goals on the one extreme of the spectrum and the evaluation of real-world contexts and problems on the other.

My own reading of the literature has, however, prompted an expansion of the four perspectives identified by Jablonka. This is achieved through the inclusion of sub-categories for some of the perspectives. This expanded spectrum of agendas is illustrated in Figure 6 below.

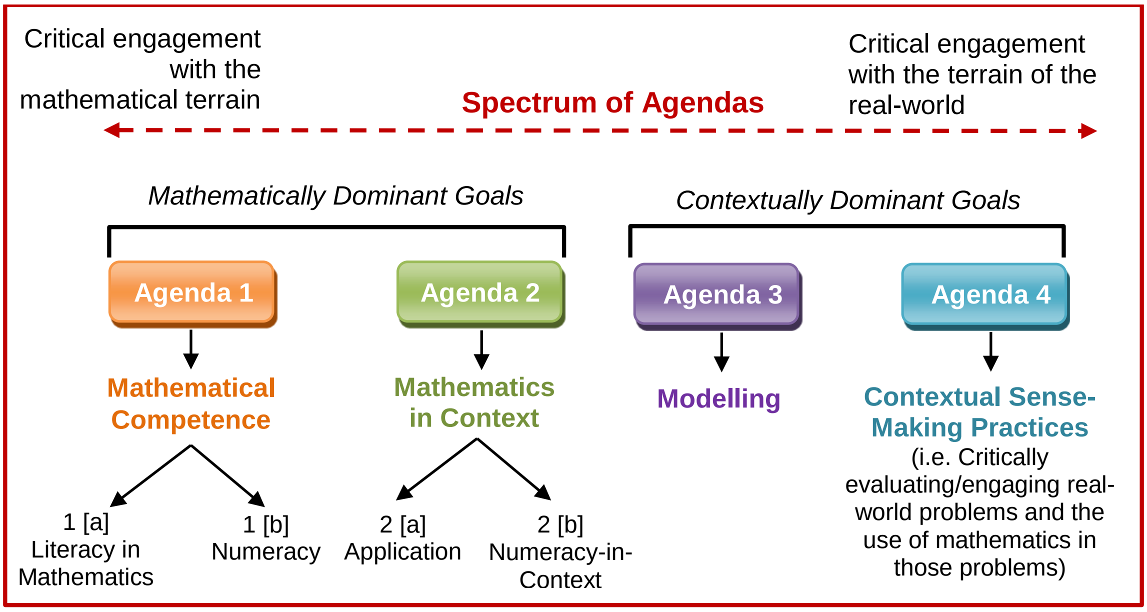

Figure 6: 'Spectrum of Agendas' in the literature on mathematical literacy, numeracy and quantitative literacy

Some clarification is necessary. Firstly, with respect to Agenda 1 – with a primary agenda for the development of mathematical competence, a distinction is made between *Literacy in mathematics* and *Numeracy*. Some conceptions of mathematical literacy, numeracy and/or quantitative literacy equate mathematically literate, numerate and/or quantitatively literate behaviour with literacy, competency and efficiency in the understanding of complex and abstract mathematical concepts. In other words, to be mathematically literate is seen to equate to being able to demonstrate understanding of abstract

[19] The phrase is borrowed from the work of Venkatakrishnan and Graven (2007) who identify four different agendas in the teaching of the subject Mathematical Literacy in South Africa. These agendas encapsulate differential pedagogic practices in the teaching of the subject, practices which "traverse across the purpose of contexts and degree of integration of contexts within pedagogic situations." (Venkatakrishnan & Graven, 2007, p. 77). The agendas are: (i) content driven; (ii) mainly content driven; (iii) content and context driven; and (iv) context driven. Hechter (2011) has expanded this line of research through utilisation of the spectrum of agendas framework to identify and classify five different question types used in classroom-based Mathematical Literacy assessment tasks. These include: (i) purely mathematical questions; (ii) mathematical questions where the context is in service to the mathematics; (iii) dialectical questions where both content and context are prioritised; (iv) contextual questions where the mathematics is treated in service to the context; and (v) purely contextual questions. The first four question types correlate roughly to pedagogic practices in each of the four pedagogic agendas, but Hechter claims that the contextual questions are positioned outside of this framework. It is important to recognise that although there are similarities between the perspectives identified by Jablonka, the spectrum of agendas identified by Venkat and Graven, and the classification of questions offered by Hechter, each of these authors is operating at different levels of analysis. Jablonka is referring to broad notions or understandings of conceptions of mathematical literacy, while Venkat and Graven and Hechter are operating at the level of the classroom – the former in terms of general pedagogic practices and the latter in terms of specific textual assessment practices.

mathematical knowledge. Numeracy[20], by contrast, is seen to refer to the ability to manage calculations involving foundational and elementary mathematical concepts and principles, such as the ability to work with ratio and proportion, equations, and percentages.

Agenda 2 encapsulates those perspectives who promote a primary agenda for the utility of mathematics in extra-mathematical contextual settings[21]. Here a distinction is made between *Application* and *Numeracy-in-Context*. Starting with the Numeracy-in-Context dimension, this perspective promotes the development of skills associated with the use of elementary mathematics in solving contextual tasks encountered in (supposedly)[22] everyday contexts: for example, the use of ratio and proportion to determine best buy options.

Elaboration of the characteristics of the *Application* perspective as a sub-category of Agenda 2 requires a brief caveat. In the context of the spectrum of agendas above (as well as throughout the contents of this larger study), 'application' refers to the imposition[23] of (all forms of) mathematics – the application of mathematics – to contextual structures. The direction of movement in an application is from mathematics to a context (W. Blum, et al., 2002, pp. 153-154). As suggested by Stillman (2012, p. 2), "With *applications* the direction (mathematics → reality) is the focus. 'Where can I use this particular piece of mathematical knowledge?' The model is already learnt and built." This conception of application is to be contrasted with the *Modelling* perspective that comprises Agenda 3. The current usage of the term 'modelling' in this spectrum (as well as in the larger study), denotes the process involved in moving from a particular problem situation based in an *authentic* real-life context (as opposed to a mathematised situation) to a reconstruction of that context, and where the reconstruction is commonly grounded in mathematical

[20] Some authors would disagree ardently (and perhaps violently) with the way in which I have equated Numeracy to refer to competency with basic mathematical principles, and would instead argue that Numeracy involves significantly more than just the ability to understand basic mathematical concepts and to perform simple calculations (see, for example, (Neill, 2001), (Gal & Tout, 2012) and (Hogan & Thornton, 2012)). Although I acknowledge these differing perspectives, my own position is that the term 'Numeracy' provides a suitable descriptor for engagement with elementary mathematical contents, and that other terms such as 'Application' and 'Modelling' provide alternative descriptors for different forms and levels of engagement with mathematical contents.

[21] Extra-mathematical settings refer to contextual settings that exist in a domain *outside* of mathematics. So, a context drawn from a real-life context such as shopping would be included here. Extra-mathematical settings are contrasted with intra-mathematical contextual settings, which refer to settings that exist and develop exclusively *within* the domain of mathematics. A particular type of exclusively mathematical problem involving only mathematical entities and signifiers (e.g. factorisation) would classify as an intra-mathematical problem. I make this distinction to clarify that the Agenda 2 perspective is characterised by attempts to move mathematics beyond or outside of its own domain.

[22] I have used the word 'supposedly' here to emphasise, as Dowling (1998) consistently points out, that although many contexts drawn from the real-world into the mathematics classroom have a base in reality, they are quickly mathematised and recontextualised according to mathematical principles, knowledge and structures. The consequence is that these contexts no longer adequately reflect how a person might act, think, or communicate in that context in their daily lives. Such contexts are then 'advertised' as everyday contexts, but are in fact nothing more that mythologised representations of reality.

[23] Note that the word 'imposition' as employed here does not denote a negative connotation and is not to be equated with impressions of colonisation or subordination. Rather, the word has been deliberately employed to emphasise the particular directional flow of mathematics being placed in or on something else, as opposed to something else drawing in mathematics.

structures[24]. In modelling processes, the direction of movement is from reality to mathematics (Stillman, 2012, p. 2). This reconstruction provides an alternative view[25] of the situation and of possible alternative forms of legitimate participation and communication in the situation, and can sometimes facilitate a different or broader understanding of the situation. Importantly, this conception of modelling does not include word problems or other problem situations which involve "nothing more than a 'dressing up' of a purely mathematical problem in the words of a segment of the real world." (W. Blum, et al., 2002, p. 153). In other words, modelling as conceived of in this immediate discussion (and extended study) involves the accessing of mathematical resources in making sense of authentic real-world situations and not for the promotion of mathematical learning through exposure to mathematised situations (W. Blum, et al., 2002, pp. 153-154).

The distinction made between applications and modelling in terms of the direction of movement between the real and mathematical worlds is important in that it suggests a significant difference in the ultimate goal of the processes involved. In applications, the goal is to impose a specific mathematical concept on a situation − to mathematise[26] the situation. In modelling, by contrast, mathematics is simply a tool for providing an alternative view or understanding of a situation: the mathematised view is not the ultimate goal but rather a means to an end, where the end is a broader, alternative or more in-depth understanding of the structure of participation in a particular real-world setting. In the

[24] A Modelling agenda also comprises a key dimension of the internal language of description of the structure of knowledge for the knowledge domain of mathematical literacy to be presented at a later point in the study. The specific components or stages envisioned for this Modelling agenda are discussed in detail as part of the discussion of this modelling dimension of the internal language. See Part 4, Chapter 14 and sub-section 14.4.4 (starting on page 220) for this elaborated discussion.

[25] To clarify the meaning of 'alternative view' in the way it is referenced here, consider the following example. A mathematical model can be constructed to show a comparison of the monthly costs on two different cell phone contract systems. The model, thus, provides a particular perspective on costing or pricing issues that could be used to facilitate a decision-making process on the most appropriate cell phone choice for an individual with particular needs. However, this is only one perspective or view of the situation and does not account for other issues that might affect choice − such as the colour or features of the phone, the particular financial situation or constraints of the person exploring the different contracts, and so on. In other words, although the model provides a view of the situation from a particular (mathematically based) perspective and, in so doing, provides access to an alternative understanding of the situation, the model is only one of several possible perspectives which may cast a gaze over the situation and which may be drawn on to facilitate appropriate and legitimate forms of participation in the situation.

[26] The tern 'mathematise' originally stems from the work of Freudenthal (1968) and was employed to describe the activity of (re)organising reality or even mathematical contents (van den Heuvel-Panhuizen, 2003, p. 11). This latter aspect of mathematisation (of mathematical contents) is referred to as 'vertical mathematisation' (c.f. (Treffers, 1987) – cited in (van den Heuvel-Panhuizen, 2003); also, see sub-section 14.4.4.2 on page 223 below for a more detailed discussion of this concept). The current usage of the term mathematisation in much of the literature on mathematical literacy, numeracy and/or quantitative literacy, is more commonly associated with activities involving the (re)organisation of reality according to mathematical structures and principles than with the (re)organisation of mathematical contents.

modelling process, the mathematical approach can be ignored for a different perspective; in the application process, an accurate mathematical approach is the ultimate goal.[27 & 28]

On the extreme right of the spectrum of agendas is positioned Agenda 4. This perspective promotes a dominant agenda for engagement in *contextual sense-making practices*. Namely, utilisation of a variety of skills, techniques and knowledge forms to make sense of contextual situations and of forms of appropriate and legitimate participation in those contexts, and also to analyse existing structures and to question the underlying assumptions (both mathematical and other) that influence the nature of participation in these structures. In this agenda, the primary goal in a problem-solving process is the development of a broader and/or more complete understanding of a contextual situation or the successful completion of a real-world task. Mathematics is seen as simply one of many tools and considerations that may be imported into and utilised in the problem-solving process to facilitate understanding of the context or completion of the task. In this agenda, authentic real-world contextual situations – and appropriate and legitimate forms of participation in those situations – function as the organising principle of the learning process.

Further clarification of the distinction between Agenda's 3 and 4 is necessary. As envisioned here, the process of modelling (Agenda 3) involves the development of mathematically structured or informed models to represent real-world situations and the interpretation of those models to deepen understanding of possible forms of legitimate participation and engagement in the situations. Although there is clearly a motivation for enhanced engagement with real-world situations in the Modelling agenda, there remains an emphasis on the mathematical structure of the model and the specifics of the mathematical knowledge and techniques employed in the construction of the model. Agenda 4, by contrast, is concerned more with developing a comprehensive understanding of a particular real-world situation through consideration of existing forms of legitimate knowledge and communication that facilitate endorsed participation in the situation, together with possible alternative forms of communication and legitimised participation facilitated through engagement with mathematical structures in the situation. Agenda 4 is also concerned with critical evaluation of existing models (mathematical and others) that claim to provide an enhanced view the structure of legitimate knowledge and participation in a situation and with evaluation of the values and perspectives embodied

[27] Note that the distinction that I am making here between modelling and application is specific to the discussion of the spectrum of agendas and also to the language of description of the structure of knowledge for the knowledge domain of mathematical literacy to be presented in Part 4. There is every possibility that a real-world practitioner (e.g. an engineer) who applies scientific mathematical principles to solve a problem (e.g. involving the construction of a bridge) would disagree with this distinction.

[28] It is inevitable that some people will disagree with this distinction between applications and modelling with respect to the direction of movement from mathematics to reality. In fact, in many of the texts read, the terms application and modelling are used interchangeably – together with 'problem-solving' – to refer to the same process of relating mathematics to a real-world situation. I have simply chosen to make an explicit distinction between these practices to emphasise that there are two possible directions of movement and that I am most interested in the one from reality to mathematics – and I am preferencing the word 'modelling' to represent this direction of movement.

In reference to 'problem-solving' as distinct from application or modelling, I take problem-solving to refer to the solving of problems *within* a particular domain of knowledge or practice – while both application and modelling involve a movement *outside* of a domain to another domain. In the realm of esoteric mathematics instruction, problem-solving would refer to the solving of mathematically based problems through the utilisation of appropriate mathematical knowledge and techniques. In the realm of a contextual domain, problem-solving would refer to the solving of a contextual problem through the utilisation of appropriate contextual knowledge and resources (of which mathematical knowledge may be one such resource). In this sense, problem-solving is an integral part of each of the agendas for mathematical literacy, numeracy and/or quantitative literacy identified on the spectrum.

in the models which preference particular forms of participation. In Agenda 4 questions are raised as to why a particular form of knowledge (mathematical or other) has been used to describe or make sense of a situation, why specific variables in the situation have been included and others excluded, and the implications of these selection or exclusion strategies on the view afforded by the model. In Agenda 4, mathematical decision-making and solution strategies are questioned, alongside acknowledgement of the restricted view that such strategies afford of real-world situations. This does not suggest that critical evaluation of the suitability and viability of constructed models does not form part of the modelling process. Rather, that Agenda 4 prioritises the opportunity for a critical 'outsiders' view of a situation, removed from ambitions for representing the world mathematically, and directed towards deepening an understanding of a real-world situation through a deliberate questioning of the models and structures that claim to represent and describe legitimate forms of participation in the situation.

Crucially, however, in as much as the contextual terrain dominates in Agenda 4, contextual sense-making practices are facilitated in part through engagement with mathematical techniques, knowledge and forms of working, and also with modelling processes. In other words, investigation of possible alternative (and mathematised) forms of participation in a contextual environment is only possible if a degree of mathematical understanding is already in place, and mathematised descriptions of segments of real-world practice can only be considered if modelling processes are available. As such, the agenda of Contextual Sense-Making Practices is supported by elements of the Numeracy-in-Context (Agenda 2 [b]) and Modelling (Agenda 3) agendas. Crucially, however, these two latter agendas are subordinated and in service to the dominant agenda for contextual sense-making practices: in short, any mathematics employed is in service to a broader goal for understanding the context and possible forms of legitimate and endorsed participation in the context.

Notice that Agendas 1 and 2 have been grouped under the banner of 'Mathematically Dominant Goals' and Agendas 3 and 4 under 'Contextually Dominant Goals'. These distinctive groupings have been included to emphasise the overarching goals prioritised in the learning processes, the organising principles that dominate and dictate the structure of knowledge and participation, and the type of behaviour that is expected will develop – all in relation to practices aligned to each agenda. As such, a learning process dominated by Agendas 1 or 2 will prioritise the development of mathematical knowledge or the utilisation of mathematical skills as a primary outcome. By contrast, a learning process dominated by Agendas 3 or 4 will prioritise enhanced functioning in real-world settings as a primary goal.

As a final comment, the agendas are not mutually exclusive in the sense that it is highly likely that an individual may operate in more than one agenda in a single instance of practice depending on their needs or objectives. To illustrate, if we shift the agendas to the level of classroom practice, a teacher may position themselves in each of the agendas during different phases of a lesson depending on what it is they hope to achieve during the lesson. So, they may begin by teaching an un-contextualised mathematical concept. Thereafter they may move towards a contextualised application of the concept through exposure to real-world components. The concept may then be combined with other concepts in the construction of a model to highlight particular elements of the situation and/or to investigate a particular form of participation in the situation. Finally, a critical discussion may ensue regarding the validity of the model for describing the situation. By operating in this way the teacher will have traversed the entire spectrum. However, there remains a sense in which a particular agenda dominates with respect to an individual's understanding of and emphasis in a particular form of mathematically literate, numerate and/or quantitatively literate behaviour. For some the contextual components are priority; for others the learning of mathematics dominates − and any alternative agendas employed are done so in service to this dominant goal.[29]

5.2.2.2 'Conceptions' of mathematical literacy

Having described different perspectives of numeracy and mathematical literacy, Jablonka (2003) commits to a privileging of the term 'mathematical literacy' in order to "focus attention on its connection to mathematics and to being literate", as opposed to numerate practices of working with and calculations on numbers (Jablonka, 2003, p. 77). She then identifies five different 'conceptions' of mathematical literacy[30], each of which represents a particular view on "how the relationship between mathematics, the surrounding culture, and the curriculum" is conceived (Jablonka, 2003, p. 80). Hence, my assertion above (c.f. page 37) that the conceptions (or 'intentions' as I refer to them) exemplify an external impetus – namely, that which is beyond the realm of the mathematics classroom.

The five conceptions of mathematical literacy as described by Jablonka are: *mathematical literacy for* (i) Developing Human Capital, (ii) Cultural Identity, (iii) Social Change, (iv) Environmental Awareness, and (v) for Evaluating Mathematics (Jablonka, 2003, pp. 80-97). Importantly, these categorisations are not mutually exclusive and it is highly likely that a strand of research of mathematical literacy, quantitative literacy and/or numeracy comprises elements of more than one conception. However, a particular conception dominates in a strand of research or in a particular educational positioning, and it is according to this dominance that a particular strand is classified. Since it is my intention to use these different conceptions as part of a framework for categorising and organising the literature relating to mathematical literacy, numeracy and quantitative literacy, it is necessary to discuss in detail each of the conceptions identified Jablonka.

[29] It is, perhaps, worth mentioning that the language of description of the structure of knowledge for the knowledge domain of mathematical literacy presented in Part 4 in this study prioritises, primarily, Agenda 4 (Contextualised Sense-Making practices). However, there is also recognition that such practices are informed by a certain degree of mathematical competency (Agenda 2 − specifically the Numeracy-in-Context dimension) and that Modelling (Agenda 3) processes provide an integral means for generating descriptions of elements of contextual practice. In relation to the components of the internal language of description, Agenda 4 is captured in the Everyday domain of practice of the language, Agenda 2 [b] is captured in the Mathematical Competency domain of practice of the language, and Agenda 3 in the Modelling domain of practice of the language. Agenda 1, which is dominated entirely by a mathematical orientation and with no consideration given to contextual elements, is not accounted for in the language of description. The language of description also contains a further domain – that of Contextual Reasoning and Mathematical Reflection. This domain shares consistency with elements of an 'Intention' for critically Evaluating Structures (both mathematical and contextual). See sub-section 5.2.2.2 immediately below on 'Conceptions of mathematical literacy' for an elaborated discussion of the characteristics of this intention.

[30] By providing a definition of mathematical literacy that prioritises a 'literacy' agenda, Jablonka essentially restricts notions of mathematical literacy to a particular type, hereby excluding conceptions of numeracy, mathematical literacy and/or quantitative literacy that do not emphasise a literacy agenda. However, when presenting the different conceptions of mathematical literacy she then does not limit discussion to conceptions that fall exclusively within the definition provided: a seeming anomaly in her writing. The relevance of this to my intended use of Jablonka's conceptions is that I, too, have not worked exclusively with literature that promotes a 'literacy' agenda. Instead, I have used the different conceptions as a means of characterising and organising the multitude of literature relating to 'mathematical literacy', 'numeracy' *and* 'quantitative literacy', irrespective of the agenda promoted or areas of focus in that literature.

The first conception – *mathematical literacy for developing Human Capital* – comprises the "mathematisation and modelling perspective" (Jablonka, 2003, p. 80): namely, that the primary intention of mathematical literacy is to provide participants with a core set of mathematical knowledge and skills that can be used to solve any problem encountered in daily life, including in employment situations. In this conception, the development of mathematical knowledge and skills is prioritised and these knowledge or skills are seen to transcend any particular contexts such that a participant is able to use the *same* knowledge and skills to solve problems in *any* context − thus assuming fairly unhindered transfer. "This conception of mathematical literacy aims to look at the world through mathematical eyes" (Jablonka, 2003, p. 80), or, in the words of Dowling (1998, p. 10), through a 'mathematical gaze'. Furthermore, this conception promotes the ideology that inculcation of this set of transcendent knowledge and skills empowers the individual by better equipping them for participation in the world, hereby increasing their marketability and enhancing their worth as a primary form of human capital (Jablonka, 2003, pp. 80-82).

The second conception – *mathematical literacy for Cultural Identity* (also referred to as Ethnomathematics) – encompasses the view that far from mathematics being a universal language which transcends cultures and contexts, rather the techniques and knowledge used to solve problems are "embedded in different social activities" (Jablonka, 2003, p. 82). This means that these techniques are specific to the situations in which the problems are encountered and are informed by the beliefs and values of the participants in those practices. As such, "The official mathematics curriculum does not usually reflect the ethnomathematical techniques used in the workplace. This means that these competencies [for solving problems in out-of-school contexts] do not develop from learning mathematics at school." (Jablonka, 2003, p. 82). In this conception, a proper understanding of out-of-school practices is only seen to be possible through analysis of the techniques involved in such practices and not through the imposition of a mathematical gaze on the practices.

"Another strand of ethnomathematical research consists of uncovering the latent mathematical content that is hidden in traditional artefacts of indigenous people." (Jablonka, 2003, p. 83). So, the patterns embroidered on blankets produced by Xhosa weavers provide evidence of an understanding of symmetry and transformation geometry amongst the participants in that practice. The argument presented for this strand is that by exposing the mathematical ideas already in use in contexts and tasks specific to a particular culture provides a celebration of that culture and an alternative to 'imposed' Western mathematical ideas and knowledge. This, in turn, helps learners to see that mathematics exists in and originates from their own cultures: hence, mathematical literacy for developing cultural identity (Jablonka, 2003, pp. 83-84). Furthermore, making participants in such practices aware of the mathematical ideas in use in familiar cultural practices and by expanding knowledge of those ideas empowers the participants through the generation of a broader world-view. The reverse is also true: "Not unpacking or further developing the mathematics can have the effect of disempowering individuals by excluding them from academic mathematics, which means excluding them from career options." (Jablonka, 2003, p. 83).[31]

The third conception – *mathematical literacy for Social Change* − has as a primary concern the use of mathematics education to promote critical citizenship. Namely, "Mathematical literacy then is a competency for re-interpreting parts of reality and participating in a process of pursuing a different reality." (Jablonka, 2003, p. 85). In this conception, mathematics – and particularly (but not exclusively) statistics − is seen as an essential instrument for critically analysing existing social and political structures and for challenging social inequalities (Jablonka, 2003, p. 85). In so doing, mathematics is seen as devoid of political, social or cultural flavour, as an objective entity that exists outside of the influences of these aspects of society and, thus, is able to cast a critical gaze on existing structures.

The fourth conception – *mathematical literacy for Environmental Awareness* – extends the notion of problem-solving in out-of-school contexts from personal, workplace and economic contexts to global environmental problems (Jablonka, 2003, p. 86). Mathematics is seen as a key tool for modelling and investigating various environmental issues, from pollution and global warming to water shortages and population growth. Clearly, this conception of mathematical literacy envisions necessary integration of contents from across a range of disciplines, including mathematical, geographical, historical, and scientific (Jablonka, 2003, p. 86).

The fifth and final conception – *mathematical literacy for Evaluating Mathematics*[32] – is the view privileged by Jablonka over all other conceptions:

> It is argued that mathematical literacy focussing on citizenship should refer to the aim of critically evaluating aspects of the surrounding culture – a culture that is more or less colonised by practices that involve mathematics. Thus the ability to understand and evaluate these practices should form a component of mathematical literacy. (Jablonka, 2003, p. 76).

In this conception, mathematics is not a neutral concept or an objective entity. Rather, "all applications of mathematics are value-driven" (Jablonka, 2003, p. 90) and mathematics can and is used to promote particular social, economic and political viewpoints, and, in the context of schooling, can even produce and reinforce social class (c.f. Dowling, 1998). As such, to be a critical citizen entails having an understanding of the colonising power of mathematics (Jablonka, 2003, p. 89) and the ability to interpret,

[31] Reflecting forward to Part 3 (c.f. page 132 below) of the study where Dowling's (1998) language of description is used to describe, theorise and problematise practices involving both mathematical and contextual elements, Dowling explicitly rejects Ethnomathematics and, by implication, the conception of the development of Cultural Identity through mathematics. For Dowling, Ethnomathematics embodies the 'Myth of Emancipation' (c.f. page 161 below) – namely, the myth that revealing cultural practices through a mathematical lens empowers the participants who are engaged in those cultural practices. Instead, argues Dowling, the reverse is true − not emancipation, but a form of colonisation: the cultural practices are mathematised, recontextualised and ultimately colonised through the lens of a Westernised mathematical gaze, and it is only through the gaze that the structures of the practice are able to be revealed and more empowered functioning is facilitated. For Dowling, this conception denies the celebration of cultural practices in their own right and prioritises instead the valuing of Western-based mathematical descriptions of the practices.

[32] This conception of mathematical literacy also bears resemblance to what is referred to as 'critical mathematics education'. However, critical mathematics education also includes aspects of the previous two conceptions of mathematical literacy and is not reflected in entirety in characteristics of this conception.

analyse and evaluate existing societal structures that make use of mathematical arguments, knowledge and practices. Jablonka likens the critical citizen to a ‘consumer’ of mathematics rather than a developer of mathematics (Jablonka, 2003, p. 89 & 90). In other words, a critical citizen is not so much concerned with learning mathematical principles and/or knowledge but rather with how mathematics is used to promote a particular reality. The critical citizen is not so much concerned with *what* mathematics is used in a situation as with *how* the mathematics is used. It is for this reason that Jablonka (2003) argues that

> The ability to evaluate critically can neither be considered mathematics, nor automatically follows from a high degree of mathematical knowledge. Consciousness of the values and perceptions of mathematical knowledge associated with distinct mathematical practices and their history can compensate to a large extent for a lack of detailed expert knowledge. (p. 98)

In terms of my own intended use of this conception, I argue that a form of mathematically literate behaviour that is driven by this intention is also characterised by a capacity for critically evaluating contextual structures, and the factors and considerations which affect how people think, act and behave in certain contextual environments. Often these factors have nothing to do with mathematical considerations or with the formatting power of mathematics[33], and, as such, a different form of critical evaluation is necessary – namely, a form of critical evaluation that understands the underlying contextual factors that inform and direct the criteria according to which participation in a contextual environment is legitimated and endorsed. As such, it is my intention for the remainder of this study to refer to this conception of mathematical literacy as characterised by an intention for Critically Evaluating (Mathematical and Contextual) Structures.

And although a direct one-to-one mapping is not guaranteed, there is a high degree of correlation between this conception and *Agenda 4* − in which a life-preparedness agenda is promoted through the prioritisation of critical engagement in contextual sense-making practices. In other words, if a conception of *mathematical literacy for Evaluating Structures* is dominant, then high priority is placed in this conception on a form of participation that involves the critical evaluation of the use of mathematics in describing real-world situations.

5.2.2.3 A proposed structure for identifying dominant agendas and statements of intention in the literature relating to mathematical literacy, numeracy and quantitative literacy

Taken together, the ‘perspectives’ and ‘conceptions’ of mathematical literacy described above provide the space illustrated in Figure 7 on the pahe below.

This space provides a two-dimensional framework for categorising the literature relating to mathematical literacy, numeracy and quantitative literacy with respect to:
(i) the dominant ‘perspective’ or *agenda* in a body of literature in terms of the extent to which the literature prioritises mathematical knowledge and techniques over emphasis on contextual sense-making practices that facilitate empowered and/or alternative forms of

[33] Skovsmose (1994b, p. 36) argues that mathematics does not only comprise descriptive potential, but also the potential for directing action. This action is commonly evidenced in relation to the structuring and constitution of social phenomenon and activities (Skovsmose, 1998, p. 197) – namely, in the ‘formatting’ of such phenomenon.

functioning in real-life settings: i.e. the organising principle according to which the structure of knowledge is legitimised and participation with that knowledge is endorsed; (ii) the explicit or implicit *intention* promoted within a particular 'conception' of mathematical literacy with respect to future goal or outcome – i.e. what participants will be able to do as a result of exposure to the legitimised structure of knowledge and associated endorsed form of participation.

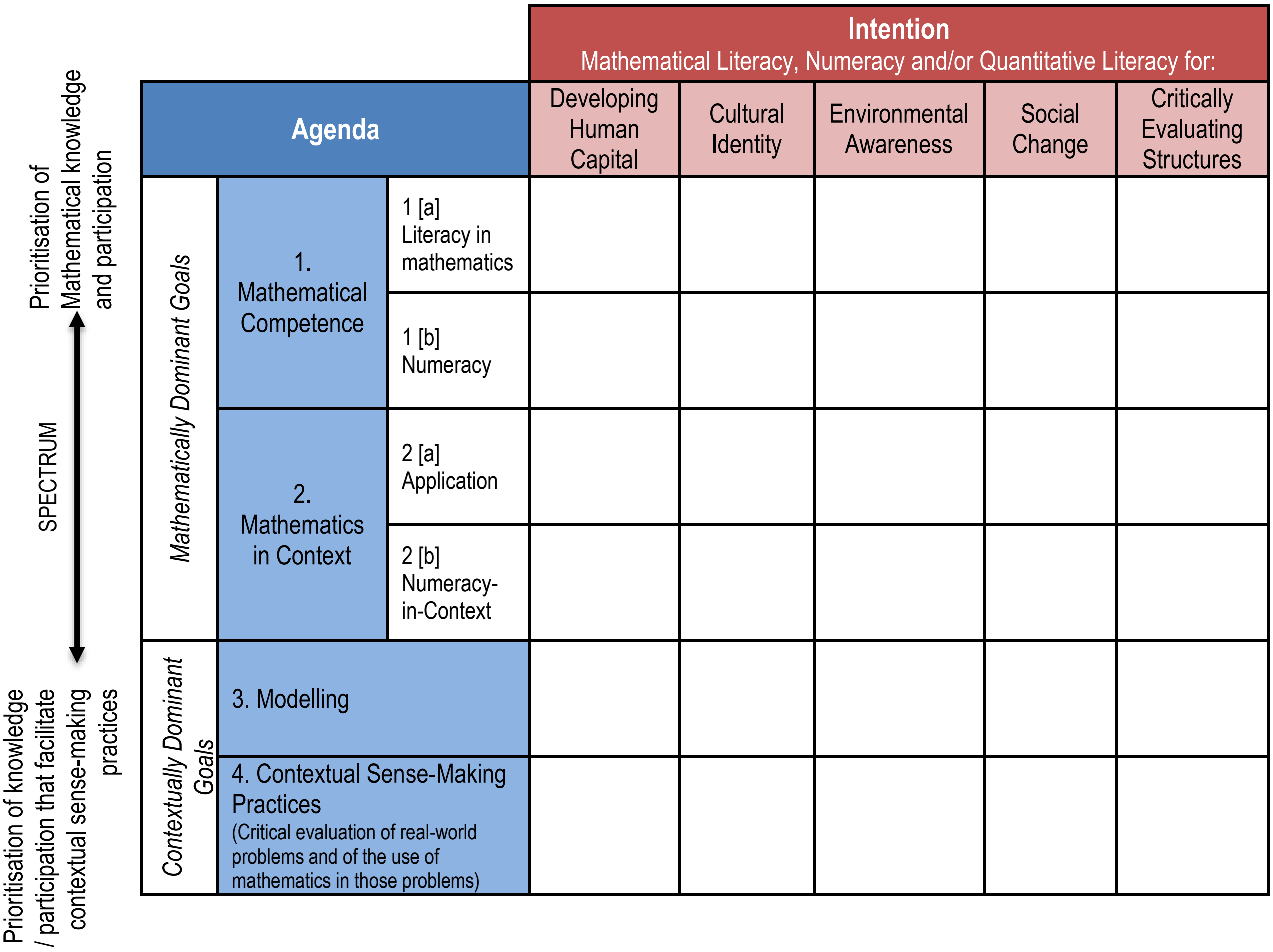

Figure 7: Framework of agendas and statements of intention for mathematical literacy, numeracy and quantitative literacy

And, as discussed above, the combination of these two components provides insight into the particular *orientation* for different descriptions of mathematical literacy, numeracy and/or quantitative literacy contained in the literature. Namely, these components provide an indication of the function or purpose towards which a particular description of mathematical literacy, numeracy and/or quantitative literacy is orientated and the structure of knowledge and participation that must be engaged with for this function or purpose to come to fruition.[34]

5.2.3 Summary of the categories and sub-categories to be used for organising, categorising and comparing the literature on mathematical literacy, numeracy and quantitative literacy

The categories and sub-categories identified in the pages above as areas of agreement, disagreement and/or comparison in the literature on mathematical literacy, numeracy and/or quantitative literacy are summarised in Table 1 below.

[34] The language of description of the structure of knowledge (and associated forms of legitimate participation and communication) for the knowledge domain of mathematical literacy developed and presented in Part 4 in this study (see page 181 below) is characterised by a life-preparedness orientation. In terms of the immediate discussion above, a life-preparedness orientation as envisioned in this study is characterised by a structure of participation that explicitly prioritises an agenda for contextual sense-making practices (Agenda 4) together with *any* of the identified intentions. This means that it is possible for a conception of mathematical literacy, numeracy and/or quantitative literacy to promote a life-preparedness orientation in service to an intention for, for example, the development of Human Capital or for Cultural Awareness. I also contend that a life-preparedness orientation – and the ability to successfully engage in contextual sense-making practices − is characterised and supported by a certain degree of mathematical competency (Agenda 2 [b]) and by the capacity to generate (and be critical of) descriptions of real-world practices using modelling processes (Agenda 3). However, in a life-preparedness orientation, these two agendas are subordinate and in service to the dominant agenda for contextual sense-making practices.

In Part 4 I argue that the specific life-preparedness orientation that is envisioned in the developed language of description is characterised by a form of participation that prioritises a dominant agenda of *Contextual Sense-Making Practices* (Agenda 4) and by a dominant intention for critically *Evaluating Structures* (both contextual and mathematical) (Intention 5). For me, this particular format of a life-preparedness orientation facilitates a form of real-world functioning that is capable of employing both mathematical and contextual techniques to critically engage with, understand, and, if necessary, challenge existing forms of participation in real-world practices. However, and as argued above, this dominant agenda is to be supported by agendas for Numeracy-in-Context and Modelling. Translating this to the language of description, the Numeracy-in-Context agenda is embodied in a Mathematical Competency domain of practice, the Modelling agenda in the Modelling domain of practice, while the dominant intention for Critical Evaluation of Structures is embodied in a Contextual Reasoning and Mathematical Reflection domain of practice. I argue that when this collective is taken together with a further domain of practice that prioritises engagement with Everyday forms of knowledge and participation, then a life-preparedness orientation that facilitates contextual sense-making practices emerges.

Table 1: Categories and sub-categories for identifying areas of commonality and divergence in the literature on mathematical literacy, numeracy and quantitative literacy

<table>
<tr><th>Category</th><th colspan="3">Sub-category</th></tr>
<tr><td rowspan="6">Considerations of mathematics</td><td colspan="3">(i) Reference to mathematical content</td></tr>
<tr><td colspan="3">(ii) Reference to the ‘use’ or ‘application’ of mathematical content/knowledge/techniques</td></tr>
<tr><td colspan="3">(iii) Strands of mathematical content</td></tr>
<tr><td colspan="3">(iv) Scope of mathematical content</td></tr>
<tr><td colspan="3">(v) Recognition of contextual forms of participation and less formal mathematical techniques and/or non-mathematical (situational) techniques</td></tr>
<tr><td colspan="3">(vi) Distinguished from scientific mathematics</td></tr>
<tr><td rowspan="2">Interplay of content, contexts and/or competencies</td><td colspan="3">(i) Perceived role of content, contexts and/or competencies</td></tr>
<tr><td colspan="3">(ii) Specified skills, competencies and/or traits</td></tr>
<tr><td rowspan="2">Arena of application</td><td colspan="3">(i) Location</td></tr>
<tr><td colspan="3">(ii) Context</td></tr>
<tr><td>Components</td><td colspan="3">Specified components and/or features associated with courses, assessments and/or pedagogic practice</td></tr>
<tr><td rowspan="9">Orientation</td><td rowspan="3">Intention
(i.e. ML for)</td><td>(i) Human Capital</td><td>(ii) Cultural Identity</td></tr>
<tr><td>(iii) Environmental Awareness</td><td>(iv) Social Change</td></tr>
<tr><td colspan="2">(iv) Evaluating Structures</td></tr>
<tr><td rowspan="6">Agenda</td><td rowspan="2">Agenda 1: Mathematical Competence</td><td>Literacy in Mathematics</td></tr>
<tr><td>Numeracy</td></tr>
<tr><td rowspan="2">Agenda 2: Mathematics in context</td><td>Application</td></tr>
<tr><td>Numeracy-in-Context</td></tr>
<tr><td colspan="2">Agenda 3: Modelling</td></tr>
<tr><td colspan="2">Agenda 4: Contextual Sense-Making Practices</td></tr>
</table>

5.2.4 Rationalising the intended use of these categories and sub-categories in the analysis of the literature

My immediate application of the framework in the pages below involves structuring a discussion on features of commonality and divergence on areas of focus, facets and/or components of mathematical literacy, numeracy and quantitative literacy expressed in *international* bodies of literature. Thereafter, I use this same framework again to organise and define opinion on the *South African subject* Mathematical Literacy, hereby providing a platform for analysis and comparison of international perspectives to perspectives of the local subject. In Part 4 of the study, components of this framework are also employed in reference to the domains of practice that characterise the envisioned structure of knowledge and participation for the knowledge domain of mathematical literacy (as described in the developed or presented internal language of description for this knowledge domain). The framework, thus, provides a ‘common denominator’ against which to compare and contrast differing views, structures and conceptions of mathematical literacy and associated knowledge and behaviour forms.

As regards the specific 2-dimensional framework developed to identify, locate, and compare dominant statements of *Orientation* − comprising intention(s) and agenda(s) – in the literature, my intention is to position each individual piece of literature read within a cell in the framework. This makes it possible to identify any trends in the whole collection of literature in terms of whether a particular agenda and intention dominate. The categorisation of the literature in this framework is not accompanied by any form of quantitative analysis, simply because the quantity of literature read is insufficient to warrant viable and reliable quantitative analysis.

A word of caution is, however, necessary. It is my belief that use of the framework (of orientations) in the way described above provides useful and relevant information about the literature read for this study. However, I acknowledge that my reading of the literature for this study is limited and that a wider reading of the literature may result in the generation of a different positioning of the distribution of the literature across the framework. Furthermore, I recognise that the conducted analysis is only valid in the context of the arguments and discussions presented in this study and in relation to the limited body of literature read for this study. It is also necessary to acknowledge that there is every possibility that it may not be possible to categorise certain literature in table, specifically literature that only provides reference to one dimension – for example, either intention or agenda – but not to both.

CHAPTER 6
AREAS OF COMMONALITY AND DIVERGENCE IN THE LITERATURE

In this chapter I provide an analysis of the international literature and international perspectives on mathematical literacy, numeracy and/or quantitative literacy. This is done to identify areas of commonality and divergence in the literature with respect to how different perspectives and conceptions of mathematical literacy, numeracy and/or quantitative literacy conceptualise the relationship between the mathematical and the contextual terrains in problem-solving processes involving real-world contexts. The dominant structure of knowledge and forms of behaviour that facilitate legitimised and endorsed participation in these processes is also investigated. This analysis reveals a dominant emphasis and prioritisation of agendas that promote a form of behaviour that is legitimised primarily by mathematised forms of participation and by mathematical structures and knowledge. In short, in most conceptions of mathematical literacy, numeracy and/or quantitative literacy it is the mathematical terrain (and not the contextual terrain) that is posited as the organising principle of a practice.

The framework of categories and sub-categories identified and discussed in Chapter 5 informs the structure and sequence of the discussion of the results of this analysis process in the pages below.

6.1 Category 1 − Considerations of mathematics

In terms of (i) <u>reference to mathematical content, knowledge and/or techniques</u>, there is overwhelming and almost unanimous agreement in the literature that, irrespective of the name used, mathematical literacy, numeracy and/or quantitative literacy involves mathematical content, knowledge and skills. The selection of quotations below illustrate this point:

> Numeracy is the ability of a person to make effective use of appropriate mathematical competencies for successful participation in everyday life, including personal life, at school, at work and in the wider community. It involves understanding real-life contexts, applying appropriate mathematical competencies, communicating the results of these to others, and critically evaluating mathematically based statements and results. (Neill, 2001, p. 7)

> [Numeracy]: the ability to access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life. (OECD, 2012a, p. 34)

> Mathematical Literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well founded judgements and to use and engage with mathematics in ways that meets the needs of the individual's life as a constructive, concerned and reflective citizen. (OECD, 2009, p. 84)[35]

> The concept of quantitative literacy is rooted in the connection between mathematics and reason. (Richards, 2001, p. 35)

> … my general sense of what quantitative literacy should be: the predilection and ability to make use of various modes of mathematical thought and knowledge to make sense of situations we encounter as we make our way through the world. (Schoenfeld, 2001, p. 51)

> Quantitative Literacy is the ability to identify, understand and use quantitative arguments in everyday contexts. (Hughes-Hallett, 2003, p. 91)

All of the quotations also hint at a further element of common agreement with respect to mathematical literacy, numeracy and/or quantitative literacy. Namely, (ii) reference to the 'use' or 'application' of mathematical content/knowledge/techniques to solve problems. There is widespread acknowledgement and agreement that mathematical literacy, numeracy and/or quantitative literacy, involves more than simply mathematical content. Rather, as suggested by Neill (2001, p. 2), "Numerate behaviour can then be analogously defined as: *The standard mathematical tools, especially when used for other than mathematical purposes*." In other words, a key intention in the development of mathematically literate, numerate and/or quantitatively literate behaviour is the ability to *use* mathematical content, knowledge, and techniques to solve problems:

> The ability to use mathematics to solve problems is a primary goal of becoming mathematically literate. (Pugalee, 1999, p. 3)

> The test of numeracy, as of any literacy, is whether a person naturally uses appropriate skills in many different contexts. (L. A. Steen, et al., 2001, p. 6)

> Quantitative literacy is not about how much mathematics a person knows but about how well it can be used. (Hughes-Hallett, 2003, p. 91)

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[35] There have been five PISA studies − in 1999, 2003, 2006, 2009 and 2012 (OECD, 1999, 2003, 2006, 2009, 2012b). All of the pre-2012 studies are comprised of three components: Reading, Mathematics (or Mathematical Literacy) and Science. The 2012 study, by contrast, includes two additional components – one on Problem-Solving and the second on Financial Literacy. In each year of implementation, one domain is given the opportunity for revision, and this works on a rotational system. For the Mathematics domain, this reformulation occurred in 2006 and again in 2012 (Stacey, 2012) (note that the 2012 study was under construction at the time of writing of the original draft of this part of the study, and at the time of finalising this study the 2012 results has not yet been released to the public). The importance of this point is that for the Mathematics component of the studies, in most parts the contents − and especially the theoretical framework that underpins the domain – have remained largely unchanged since the inception of the original study in 1999, bar minor tweaking. For this reason, rather than always referencing all four studies, I commonly only reference the most recent study – namely, the one conducted in 2009 (which is identical to the 2006 study), and this reference must be interpreted to be representative of all previous studies. However, in the event that the 2009 document contains information that is different to the 1999 document or to any of the other documents, then in such cases I reference the immediately relevant document and, where necessary, state the difference.

> Any attempt at defining 'mathematical literacy' faces the problem that it cannot be conceptualised exclusively in terms of mathematical knowledge, because it is about an individual's capacity to *use* and *apply* this knowledge. (Jablonka, 2003, p. 78, emphasis in original text)

> An important part of mathematical literacy is using, doing and recognizing mathematics in a variety of situations.
> (de Lange, 2003, p. 80)

> Thus, literacy in mathematics is about the functionality of the mathematics you have learned at school. (de Lange, 2006, p. 16)

Moreover, the 'problems' that are the focus of this application of mathematics are to be based in extra-mathematical and/or real-world contexts and experiences (as opposed to esoteric mathematical situations). Mathematical literacy, numeracy and/or quantitative literacy, thus, embody a relationship between mathematical content and knowledge and extra-mathematical contexts, situations and problems:

> A quantitatively literate person is a person who, with understanding, can both read and represent quantitative information arising in his or her everyday life.
> (Richardson & McCallum, 2003, p. 99)

> … being numerate involves more than just knowing mathematics. It implies that to organise their lives as individuals, as workers, and as citizens, adults need to feel confident of their own mathematical capacities and be able to make effective decisions in mathematical situations in real life. (Van Groenestijn, 2003, p. 230)

> **Numerate behavior** is observed when people manage a situation or solve a problem in a real context ... (Gal, van Groenestijn, Manly, Schmitt, & Tout, 2005, p. 152)

> An important commonality in the above descriptions of numeracy is the presence of mathematical elements in real situations, and the notion that these can be used or addressed by a person in a goal-oriented way, dependent on the needs and interests of the individual within the given context (home, community, workplace, etc.), as well as on his or her dispositions. (Gal et al., 2005, p. 151)

> … mathematical literacy refers to the competency to handle situations in work, leisure, home and the public domain which involves what mathematicians would consider 'mathematical' competencies. (Christiansen, 2007, p. 92)

> Numerate behavior involves managing a situation or solving a problem in a real context, by responding to mathematical content/information/ideas represented in multiple ways. (OECD, 2012a, p. 34)

While there is agreement in the various conceptions of mathematical literacy, numeracy and/or quantitatively literacy on the relationship of the mathematical terrain to the real-world, such agreement is lacking with respect to opinions on the scope, location and specific contexts in which such real-world problems are to be explored. This particular area of divergence is discussed in more detail in the pages below in the section that deals with *Arena of application* (c.f. page 68 below).

Consideration of the (iii) strands of mathematical content emphasised in the literature is the next sub-category for discussion. Where reference to categories or strands of content is made[36], different content groupings or strands are employed and emphasised by different authors. Steen (1990), as an early proponent of quantitative literacy, identifies and emphasises six content strands: *Quantity*, *Dimension*, *Pattern*, *Shape*, *Uncertainty*, and *Change*. In later work, L. A. Steen, et al. (2001, pp. 15-17) modifies his original thinking to focus instead on 'Skills of Quantitative Literacy' rather than specific content strands, and includes as part of these skills the categories of *Arithmetic*, *Data*, use of *Computers*, *Modeling*, *Statistics*, *Chance,* and *Reasoning*. Importantly, while these categories define some content to be included in the development of quantitative literacy, they also extend beyond simple content categories to include mathematical and logical skills required for solving problems based in context (L. A. Steen, et al., 2001, p. 17). Following on from the early work of Steen, the OECD-PISA assessment frameworks highlight four content categories, comprising the labels *Quantity*, *Change and Relationship*, *Space and Shape*, and *Uncertainty* (OECD, 2009, pp. 93-104)[37]. The ALL and PIAAC frameworks[38] (Gal et al., 2005; PIAAC Numeracy Expert Group, 2009) place emphasis on the strands of *Quantity and Number*; *Dimension and Shape*; *Patterns, Functions and Relationships; Change;* [these previous two strands are combined into a single strand of Patterns, Relationships and Change in PIAAC] and *Data and Chance*. Closer to home, the curriculum statement document for the South African subject-matter domain of Mathematical Literacy (DoE, 2003a) categorises the curriculum for the subject according to the four strands[39] *Numbers and Operations in Context*, *Patterns and*

[36] Explicit reference to specific strands of mathematical content is more prevalent in studies, reports and frameworks that have been written to explicitly define a structure for pedagogic and/or assessment practice associated with mathematically literate, numerate and/or quantitatively literate behaviour. For example:

- the OECD-PISA assessment frameworks (1999, 2003, 2006, 2009, 2012b);
- the International Adult Literacy Survey (ALL) (Gal et al., 2005);
- the NCSALL Report titled *The Components of Numeracy* (Ginsburg et al., 2006);
- Neill's (2001) description of components of numeracy for the New Zealand Curriculum;
- and the NCED publication *Mathematics and Democracy* (L. A. Steen, et al., 2001) that outlines the components of quantitative literacy from an American perspective.

There is, however, a multitude of literature that provide alternative focus on unpacking expressions and descriptions of behaviour associated with conceptions of mathematical literacy, numeracy and/or quantitative literacy rather than identifying specific categories of content and arenas of application through and within which such behaviour is to be developed. Pugalee (1999), Jablonka (2003) and Frankenstein (2009a) are examples of authors who contribute to this body of literature.

[37] The 1999 PISA framework (OECD, 1999, pp. 47-50) uses different categories to subsequent PISA frameworks, with emphasis on Chance, Change and Growth, Space and Shape, Quantitative Reasoning, Uncertainty, and Dependency and Relationships.

[38] ALL stands for Adult Literacy and Lifeskills Survey and was developed as an international study to measure the numeracy and literacy levels of adults. The ALL study is the successor of the IALS study (International Adult Literacy Survey) which represented the world's first large-scale, comparative assessment of adult literacy (see (Organization for Economic Co-operation and Development (OECD) & Statistics Canada, 2000) and (Kirsch, Jungblut, & Mosenthal, 1998)). The latest successor to the ALL study is the Programme for the International Assessment of Adult Competencies (PIAAC), developed in 2009 and currently in implementation phase. See Gal and Tout (2012) for a comprehensive discussion of the history of the ALL study and of the key theoretical components of the study.

[39] The curriculum document uses the word 'Learning Outcome' rather than strand and explains the meaning of a Learning Outcome as follows:

> *A learning Outcome is a statement of an intended result of learning and teaching. It describes the knowledge, skills and values that learners should acquire by the end of the Further Education and Training band.* (DoE, 2003a, p. 7)

Importantly, each Learning Outcome is an overarching category that comprises not only content but also contexts and competencies that are to be developed in each grade.

Relationships, *Space, Shape and Measurement*, and *Data Handling*. The more recent Curriculum and Assessment Policy Statement[40] (DBE, 2011a) for the subject makes use of the five topics of *Numbers and calculations with numbers*, *Patterns, relationships and representations*, *Measurement*, *Maps, plans and other representations of the physical world*, *Data Handling*, and *Probability*.

Irrespective of the titles used for the different content strands, the following commonalities emerge:

- emphasis on number concepts, including number formats (e.g. percentages ratios, decimals, fractions), and rules and techniques for calculations involving numbers;
- emphasis on relationships between quantities and representations of those relationships (e.g. in tables, graphs and equations);
- emphasis on how different quantities change in relation to each other and ways of measuring and representing that change;
- emphasis on concepts related to measurement, including physical measurement and calculations involving measured values (e.g. area calculations);
- emphasis on 2-D and 3-D space, including visualisation of 2-and 3-D shapes, calculations for such shapes (e.g. volume calculations), and visualisation for other 2- and 3-D object like maps and plans;
- emphasis on working with statistical data and the use of statistical tools (e.g. tables, graphs, measures) to interpret and make sense of such data;
- emphasis on the notion of chance (likelihood or probability).

Another observation is necessary, in that irrespective of the title used to describe the particular organisation of the content through which the development of mathematically literate, numerate and/or quantitatively literate behaviour is to be explored, this organisation is distinctly different from the traditional content strands of Algebra, Geometry, Trigonometry, and Calculus found in historical mathematics classroom. Rather, conceptions of mathematical literacy, numeracy and/or quantitative literacy deliberately organise content through reference to ‘big ideas’ (OECD, 1999) (Gal et al., 2005), ‘overarching ideas’ (OECD, 2003, 2006, 2009), or ‘phenomenological categories’ (de Lange, 2003, 2006) that contain not only reference to mathematical content but also to contexts of application, problem situations, and skills required for solving problems in such contexts and situations. ‘Content’ in mathematical literacy, numeracy and/or quantitative literacy is, thus, not only mathematical content; rather, it implies a whole spectrum of components – including mathematical, contextual, and competencies (OECD, 2009, p. 90) − required for solving problems based in real-life situations.

Both de Lange (2003, p. 78) and the OECD-PISA documents (OECD, 2003, p. 34) offer a motivation for this move:

> Mathematical concepts, structures, and ideas have been invented as tools to organize the phenomena in the natural, social and mental worlds. In the real world, the phenomena that lend themselves to mathematical treatment do not come organized as they are in the school curriculum structures. Rarely do real-life problems arise in ways and contexts that allow their understanding and solutions to be achieved through an application of knowledge from a single content strand.

[40] A detailed discussion of differences and shifts between the NCS and the CAPS structures is provided at a later point in this part of the study (c.f. sub-section 8.3.2 starting on page 122 below).

The use of phenomenological categories "encompassing set[s] of phenomena and concepts that make sense together and may be encountered across a multitude of quite different situations" (de Lange, 2006, p. 21; OECD, 2006, p. 83) is a deliberate attempt to make provision for an approach to solving real-world problems that requires integrated use of a range of mathematical components as well as consideration of other phenomena. In other words, the use of 'big ideas' or overarching categories allows for the content component of an activity to be organised in terms of the phenomena to be described by that content:

> PISA therefore identifies mathematical content by listing a small set of overarching ideas that represent broad categories of real-world phenomena through which opportunities to explore and use mathematics arise in our interactions with the world. (OECD, 2009, p. 91)

In summary, the use of phenomenological categories allows for a prioritising of not only mathematical but also contextual and competency related considerations in interactions with real-world phenomenon.

Descriptions of the (iv) <u>scope of mathematical content</u>[41] appropriate for mathematical literacy, numeracy and/or quantitative literacy is an area of significant divergence in the literature. Some authors suggest that only basic or elementary mathematics is required for sense-making practices of problems situated in real-world scenarios. For example, Christiansen (2007) suggests that,

> Thus, all of these examples illustrate how mathematics beyond simple arithmetic is not really central to performance in everyday situations, because whatever little mathematics is used, it is subordinated to the principles of the activity. (p. 97)

Steen (1999, 2003b; 2001) follows suit, emphasising the need for elementary and/or basic mathematical content and skills – such as arithmetic, percentages, ratios, simple algebra, measurement, estimation, logic, data analysis, and geometric reasoning − over abstract concepts. Others who offer similar opinion include the ALL framework (Gal et al., 2005),

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[41] In identifying differential emphasis on different forms or scope of mathematical content in the literature, I run the risk of significantly undermining how different authors identify the complex interplay between mathematical content and real-world problem situations. For example, to say that Steen prioritises a notion of numeracy or quantitative literacy that comprises only *basic* mathematical content in no way gives credence to the complex interaction between such content and the intricacies of real-world problem situations that Steen conceptualises and verbalises as being part of numerate and/or quantitatively literate behaviour. As eloquently summarised by L. A. Steen, et al. (2001),

> Typical numeracy challenges involve real data and uncertain procedures but require primarily elementary mathematics. In contrast, typical school mathematics problems involve simplified numbers and straightforward procedures but require sophisticated abstract concepts. (p. 6)

While not intending to underscore the intricate relationship between content and application, the differential emphasis placed by different authors on the scope of mathematical content required in the development of mathematically literate, numerate and/or quantitatively literate behaviour suggests variation in the way in which these authors view the role and scope of such behaviour. For example, an author who views trigonometry as an essential component of interactions with the real-world clearly has a different 'mathematical gaze' than the author who suggests that only basic arithmetic is required. This distinction has implications for the extent to which authors prioritise the development of mathematical knowledge as a component of mathematically literate, numerate and/or quantitatively literate behaviour, as well as the scope of the real-world that they deem appropriate for consideration in the development of such behaviour.

Hughes-Hallett (2003), Richardson and McCallum (2003), and Howe (2003, p. 185) – who suggests as a requirement “comfort with numbers”.

Other authors suggest that application in real-world situations exists on a hierarchy, with some types of application requiring relatively elementary mathematical content and others requiring more complex and abstract content. Neill (2001, p. 11), for example, suggests that “Numeracy has a hierarchy of levels ranging from the understanding and use of a few basic ideas of number through to complex mathematical applications.” De Lange (2003, p. 81) argues similarly for a distinction to be made between basic and advanced levels of mathematical literacy. The basic level is perceived as a requirement for all learners up to a particular age at school and irrespective of their future career ambitions, and the advanced level defined by the requirements for participation in post-school social, economic and workplace community of practices. The OECD-PISA frameworks (OECD, 1999, 2003, 2006, 2009) also adopt a wider view of the scope of mathematical content relevant for the development of mathematically literate behaviour. This is particularly evident in some of the examples of problem situations or types shown in the frameworks, which require the use of esoteric mathematical concepts such as trigonometry and geometric reasoning. This allowance for a wider scope of mathematical content beyond elementary concepts stems from a key area of focus of the study to measure “ability to pose, formulate, solve, and interpret problems using mathematics within a variety of situations or contexts”, but where ‘contexts’ refers to both purely mathematical situations as well as real-world situations (OECD, 2009, p. 85).

Worth noting is the general lack of clarification provided of terms like ‘basic’, ‘elementary’, ‘advanced’ and ‘abstract’ in descriptions of the scope of mathematical content. This lack of clarity gives rise to a host of questions and uncertainties, such as: what constitutes ‘basic’ or ‘elementary’ mathematics?; and when does basic or elementary mathematics stop being such and turn instead into ‘advanced’ mathematics?; what constitutes non-abstract mathematics, and how are abstract and non-abstract mathematical contents to be differentiated? Added to this is the level of generality (or lack of specificity) used to identify the scope of real-world situations to which this mathematics can be applied. General phrases such as ‘the world’, ‘everyday contexts’, ‘the real-world’, ‘the individual’s life’, ‘personal life’, ‘the workplace’, and ‘society’, are commonly branded as possible arenas of application of the mathematical content. Yet, within each of these arenas there is tremendous variation in the possible problem scenarios and in the level of abstraction and complexity of the mathematical content required to model and/or make sense of such situations. The point is simply this: there is no certainty or agreement over what constitutes appropriate and sufficient mathematical content for the development of mathematically literate behaviour, of the limit of such content, or of the scope of suitable real-world contexts.

Instead, different authors promote different priorities and scope of mathematical content depending on the particular conception of legitimate knowledge and associated behaviour encapsulated and/or promoted in that literature and the particular use-value envisioned for that behaviour in a particular society. Being mathematically literate in South Africa, and the mathematical content and contexts of application appropriate to the development of such behaviour, is very different to being mathematically literate in any other country. It is, perhaps, in the light of considerations such as this that Ewell (2001) makes the following statement:

> For one thing, it [a literacy] is profoundly social, and is therefore a moving target because its contents depend on a particular social context. For instance, it is easy to imagine literacies being quite different from one another in different historical periods or cultural contexts. So a literacy is not just an applied version of a discipline. Instead, it would seem to flow out of a specific set of symbolic and communication needs embedded deeply in a particular social environment or community of practice. (p. 37)

In my mind, (v) recognition of contextual forms of participation and less formal mathematical techniques and/or non-mathematical (situational) techniques is an essential issue for analysis in discussions pertaining to the literature on mathematical literacy, numeracy and quantitative literacy. This is because evidence of such recognition highlights acknowledgement that "people learn other or additional knowledge and competencies [that are often of a non-mathematical nature] in the contexts where they need them." (Christiansen, 2007, p. 94) and that "When confronted with a mathematical-like dilemma they [participants in everyday life] resort to qualitative ways of dealing with these dilemmas and utilise all sorts of contextually driven procedures to resolve these dilemmas." (Julie, 2006, p. 68). This same perspective is encapsulated in the PIAAC framework (2009, p. 30) where the argument is presented that "Proper interpretation of mathematical information or quantitative messages by adults depends on their ability to place messages in a context and access their world knowledge, as well as rely on their personal experiences and practices …". This recognition implies acceptance of contextual forms of knowledge and participation and of non-mathematical considerations in problem-solving practices involving real-word situations – which constitutes a direct denial of the dominance of a 'mathematical gaze' on the world and, so, an inclination towards greater prioritising of contextual sense-making practices contextual environments (and an associated life-preparedness orientation) over exclusively mathematical goals.

With this in mind, much of the literature does not promote a conception of mathematical literacy, numeracy and/or quantitative literacy that makes explicit allowance for extensive usage of less formal mathematical or non-mathematical (situational) techniques and contextual forms of knowledge, participation and communication. As is illustrated in the following quotation by Van Groenestijn (2003), primary emphasis within much of the literature is on the *mathematical* component of the application and associated behaviour rather than on real-world considerations:

> Hence, numeracy courses embedded in school programs must focus on problem-solving activities in which students can apply their acquired *mathematical insights and skills* and learn how to manage such situations. (p. 233)

And where allowance is made for the inclusion of informal or localised techniques, a cautionary tone is inevitably employed. Neill (2001, p. 2), for example, recognises that "numeracy is grounded in a common sense approach to problem solving", but only if the 'common sense' is grounded in a deeper understanding of the mathematical: "However, street-wise strategies without robust, general applicable, and efficient strategies to back them up may leave students somewhat deficient." Neill (2001, p. 12).

Richardson and McCallum (2003) come closer than most others to recognising the importance of contextual considerations, knowledge and forms of participation alongside mathematical issues in practices involving real-world problem-solving scenarios,

> A quantitatively literate person must be able to think mathematically in context. This requires a dual duty, marrying the mathematical meaning of symbols and operations to their contextual meaning, and thinking simultaneously about both. (p. 101)

but then shift into a prioritising of the mathematical terrain by arguing that it is the mathematics, engaged in context, that provides the 'power' needed to move the 'engine' that facilitates understanding of the context (Richardson & McCallum, 2003, p. 101).

The discussion above has illustrated that my reading of the literature reveals that although at times consideration is given to contextual forms of knowledge and participation (accompanied by consideration of less formal mathematical techniques and/or non-mathematical considerations), it is predominantly the mathematical component of the problem-solving process that is prioritised. Mathematics is not seen as only one of many possible tools that can be employed to enhance understanding of a context; it is the primary and dominant tool. As conceded by L. A. Steen, et al. (2001),

> Quantitatively literate citizens need … a predisposition to look at the world through mathematical eyes … (p. 2)

As a final comment on this issue, it is perhaps worth mentioning that South Africa is the only country where the task for the development of mathematically literate behaviour is embodied in an individualised fully-fledged subject (called Mathematical Literacy) that stands separate from (and largely in opposition to) the teaching and learning of traditional mathematics (called Core Mathematics). In other countries where the development of mathematically literate, numerate or quantitatively literate behaviour is prioritised, it is viewed as a component of the ordinary teaching of mathematics and largely as the responsibility of the mathematics teacher.[42] In such spaces, the intention behind the promotion of a form of mathematically literate behaviour for mathematics learners is to develop better and more functional mathematical knowledge, competencies and applications. It is this point that best highlights a possible reason for the commonly restricted or absent recognition of contextual forms of knowledge or participation and non-mathematical or informal mathematical techniques for solving real-world problems in many international conceptions of mathematical literacy, numeracy and/or quantitative literacy – and, instead, the predominance of the mathematical terrain and mathematically driven goals. Life-preparation is not prioritised simply because it is the development of improved Mathematical practices − and not enhanced functioning in real-world situations − that is the primary goal.

[42] Although, some, like Hogan and Thornton (2012), argue that numerate behaviour develops across the school curriculum and, so, is the responsibility of every teacher in every subject.

With respect to whether and/or how mathematical literacy, numeracy and/or quantitative literacy are (vi) distinguished from scientific mathematics[43], the quotations below illustrate key areas of differentiation.[44]

> Mathematics does not need to look at just the real world, but can look at purely abstract constructs and ideas regardless of their potential applications. Numeracy, on the other hand, is the application of mathematics to real-life situations. (Neill, 2001, p. 10)

> … unlike mathematics, which is primarily about a Platonic realm of abstract structures, numeracy often is anchored in data derived from and attached to the empirical world. (L. A. Steen, 2003a, pp. 62-63)

> Numeracy connotes mathematical topics woven in the context of work, community, and personal life. Moreover, numeracy requires the ability and inclination to explore this situational mathematical content, thus is owned differently by each person. Unlike pure mathematics, numeracy has a distinctly personal element. (Ginsburg et al., 2006, p. 1)

> Typical numeracy challenges involve real data and uncertain procedures but require primarily elementary mathematics. In contrast, typical school mathematics problems involve simplified numbers and straightforward procedures but require sophisticated abstract concepts. (L. A. Steen, et al., 2001, p. 6)

> Another contrast with mathematics, statistics, and most sciences is that numeracy grows more horizontally than vertically. Mathematics climbs the latter of abstraction to see, from sufficient height, common patterns in seemingly different things. Abstraction is what gives mathematics its power; it is what enables methods derived in one context to be applied in others. But abstraction is not the focus of numeracy. Instead, numeracy clings to specifics, marshalling all relevant aspects of setting and context to reach conclusions. (L. A. Steen, et al., 2001, pp. 17-18)

> Mathematics asks students to rise above context, while quantitative literacy asks students to stay in context. Mathematics is about general principles that can be applied in a range of contexts; quantitative literacy is about seeing every context through a quantitative lens. (Hughes-Hallett, 2001, p. 93)

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[43] In employing the term 'scientific mathematics', I am referring to mathematical knowledge and content characterised by a distinctly intra-mathematical base. In Footnote 21 on page 40 above, I distinguished between intra-mathematical and extra-mathematical settings or contexts, with extra-mathematical settings existing in a domain *outside* of the mathematical terrain and intra-mathematical settings existing exclusively *within* the domain of mathematics. The same principle can be applied to the way in which I distinguish here between 'scientific mathematics' and the form of knowledge and behaviour associated with conceptions of mathematical literacy, numeracy and/or quantitative literacy. 'Scientific mathematics', in the way in which the term is being employed in this study, refers to mathematical content and knowledge that is exclusively intra-mathematical in nature and which is comprised of signifiers and objects that are drawn exclusively from the domain of mathematics. This is contrasted with mathematical literacy, numeracy and/or quantitative literacy which make an explicit claim to extra-mathematical components drawn from outside of the domain of mathematics.

[44] However, as is discussed in Chapter 7 below (c.f. page 83), not all authors place equal emphasis on the distinction between mathematical literacy, numeracy and quantitative literacy and scientific mathematics. For example, some such as L. A. Steen, et al. (2001), Steenken (2003), Hughes-Hallett (2003), and Kirst (2003) associate a closer connection of mathematical literacy – as a form a literacy with mathematical concepts – to scientific mathematics than either numeracy or quantitative literacy. For more juicy details, just keep reading …

> … in comparison with traditional school mathematics, ML is less formal, more intuitive, less abstract and more contextual, less symbolic and more concrete. ML also focuses more attention and emphasis on reasoning, thinking, and interpreting as well as on other very mathematical competencies. (de Lange, 2003, p. 77)

From these definitions, the following key areas of distinction can be identified (summarised in Table 2 below):

Table 2: Comparing mathematical literacy, numeracy and/or quantitative literacy and scientific mathematics

Mathematical Literacy, numeracy and/or quantitative literacy	(Scientific) Mathematics
• emphasise the relationship between mathematical content and real-world problem situations	• does not necessarily include a link to a real-world problem situation
• sometimes include focus on context-specific or localised considerations and knowledge	• involves the learning of generalizable principles and procedures which have application across a range of problems
• commonly, though not exclusively, involves the use of elementary mathematical content and sometimes excludes the learning of abstract concepts	• involves the learning of increasingly abstract mathematical content
• mathematics has a 'use value'	• mathematics for increased knowledge

There is little or no dissention that mathematical literacy, numeracy and/or quantitative literacy are distinct in purpose, focus and content from traditional scientific mathematics. However, this does not mean that mathematical literacy, numeracy and/or quantitative literacy and scientific mathematics are seen as incommensurable. Rather, the reverse is true in most conceptions: being mathematically literate is perceived as an enhanced form of mathematical behaviour characterised by the capacity to solve both intra-and extra-mathematical problems. This is particularly true in many conceptions of mathematical literacy, numeracy and/or quantitative literacy in which the responsibility for the development of this type of behaviour falls within the domain of the mathematics classroom.

6.2 Category 2 − Interplay of content, contexts and/or competencies

As has already been discussed, a defining feature of mathematically literate, numerate and/or quantitatively literate behaviour involves an interplay between mathematical content and the usage of that content in solving problems related to real-life considerations. Brombacher[45] (2007, p. 14) makes use of the diagram shown in

[45] Although Brombacher's work stems from a South African perspective and is written with a particular view towards the subject-matter domain of Mathematical Literacy, much of the content of his writing, by his own admission, is informed by international perspectives relating to conceptions of mathematically literate, numerate and/or quantitatively literate behaviour. It is for this reason that I deem it appropriate to cite his work as part of this more general discussion on international perspectives of mathematical literacy, numeracy and/or quantitative literacy.

Figure 8 on the page below to provide clarity on the specific roles of content and context − and the relationship between these two facets − in the development of mathematically literate, numerate and/or quantitatively literate behaviour, and to illustrate the perspective that

> … the attributes of Mathematical Literacy are developed through interplay between content and context. Content enables us to work on finding solutions to problems that are interesting and relevant, while context gives meaning to the mathematical knowledge and skills (content) that we are teaching. (Brombacher, 2007, p. 129)

This dual emphasis on the interplay between content and context is a common feature of much of the literature.

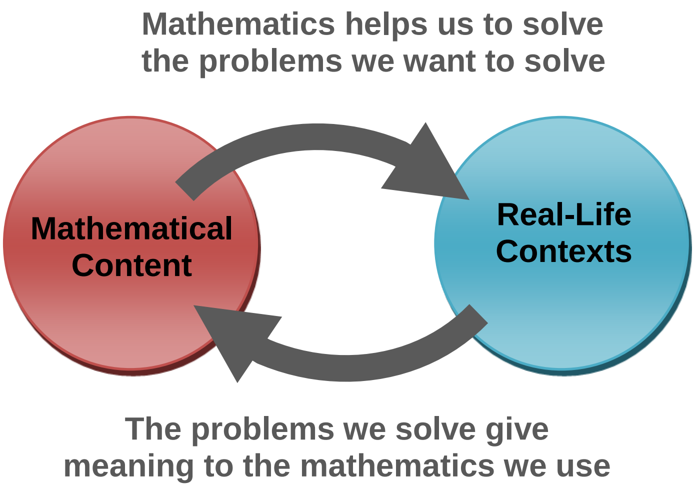

Figure 8: Interplay of content and context in Mathematical Literacy

Despite this emphasis on the duality of the content-context relationship, it is my contention that in much of the literature read there is a prioritising of the content component (i.e. the mathematical terrain) over considerations for genuine sense-making of the contextual terrain and of the structure of legitimate and appropriate knowledge and participation in that terrain. In other words, the contexts that are deemed appropriate are those which allow for a particular type of mathematical exploration of for exploration of a particular mathematical concept. Primary focus is on how particular mathematical knowledge and techniques can be applied in a situation or context. In other words, on how mathematics can cast a gaze outside of its own domain. By contrast, lesser and often no emphasis is placed on the use of a variety of appropriate tools and techniques, including mathematical and/or non-mathematical (i.e. situational) techniques, to facilitate a deeper understanding of legitimate forms of knowledge and participation in a situation or context. Contextual and/or qualitative considerations which may affect how a person might actually solve a problem in an everyday situation are not given credence, and contextually derived narratives are not considered as reflecting appropriate or valid solution strategies. Instead, mathematical structures comprise the organising principle of the activity and, as such, it is mathematically generated solutions derived through mathematically structured routines that are endorsed. Packer (2003, p. 36) suggests as much when, in presenting an argument for a canon of empirical mathematical problems that can be used to develop and measure quantitatively literate behaviour, states that,

> The challenge is to identify important, frequently encountered problems that cannot be efficiently solved without using mathematics.

As does Van Groenestijn (2003):

> Achieving numeracy is a matter of learning how to *use* mathematics in real life and how to *manage* mathematical situations. (p. 233, emphasis in original text)

From one definition or expression to the next, time and time again, the organising principle is the use of mathematical knowledge and techniques to solve problems:

> The ability to use mathematics to solve problems is a primary goal of becoming mathematically literate. (Pugalee, 1999, p. 3)

> A quantitatively literate person is a person who, with understanding, can both read and represent quantitative information arising in his or her everyday life. (Richardson & McCallum, 2003, p. 99)

> Hence, numeracy courses embedded in school programs must focus on problem-solving activities in which students can apply their acquired mathematical insights and skills and learn how to manage such situations. (Van Groenestijn, 2003, p. 233)

> Quantitative Literacy is the ability to identify, understand and use quantitative arguments in everyday contexts. (Hughes-Hallett, 2003, p. 91)

This is not to suggest that such emphasis on mathematics as the organising principle is not deliberate or that such authors are unknowingly prioritising mathematical structures over the contextual terrain. The focus on mathematics and the prioritising of mathematical knowledge, techniques and skills is explicitly acknowledged. And this is not surprising if one recognises, as has already been suggested above, that for many the development of mathematically literate behaviour is encompassed in the teaching and learning of scientific mathematics and is signified through empowered and enhanced mathematical ability.

Alongside an emphasis on the interplay of mathematical content and real-world contexts is a further call, from some authors and frameworks[46], for the central role of the development of skills or competencies – or what the PIAAC framework refers to as 'enabling processes' (PIAAC Numeracy Expert Group, 2009, pp. 29-31) – as characteristic of mathematically literate, numerate and/or quantitatively literate behaviour. De Lange (2003, p. 88) is particularly ardent in this respect, arguing that "the desired competencies, not the mathematical content, are the main criteria …". This perspective is reflected in the OECD-PISA frameworks (OECD, 2003, 2006, 2009, 2012b) where competencies are viewed as the central component that makes it possible for mathematics to be applied and successfully utilised to solve problems encountered in real-world scenarios:

[46] For example: Pugalee (1999); Neill (2001); L. A. Steen, et al. (2001); Niss (2003); Ginsburg et al. (2006); OECD-PISA frameworks (1999, 2003, 2006, 2009); ALL frameworks (Gal et al., 2005) & (Van Groenestijn, 2003); and the SCANS framework (SCANS, 1991) & (Packer, 2003).

> While situations or contexts define the real-world problem areas, and overarching ideas reflect the way in which we look at the world with "mathematical glasses", the competencies are the core of mathematical literacy. Only when certain competencies are available to students will they be in a position to successfully solve given problems. (OECD, 2003, p. 32)

Importantly, in many of the conceptions that emphasise the central role of competencies, these competencies are presented as the component that binds the content and the contexts together. As suggested by Ginsburg et al. (2006, p. 3), the competencies (i.e. the cognitive and affective component of numeracy) are the "processes that enable an individual to solve problems, and thereby, link the content and context." Brombacher (2007) offers a similar perspective,

> the purpose of the content and the contexts in the mathematical literacy classroom is to develop life skills and competencies. These are the competencies that the individual needs to participate in her/his world as a self-managing individual; as a contributing worker; as a life-long learner; and as a critical citizen. (p. 15)

and illustrates the components of this perspective by means of the diagram shown in Figure 9 on the page below.

I interpret this particular conception of the relationship between content, contexts and competencies to suggest that the content and contexts are perceived as tools used to facilitate the development of a set of skills that have application across a wide range of problems and situations. The competencies, and not the contexts or the content, are, thus, the central component in the development of mathematically literate behaviour. De Lange (2003) again echoes a similar sentiment:

> … to effectively transfer their knowledge from one area of application to another, students need experience solving problems in many different situations and contexts … . Making competencies a central emphasis facilitates this process: competencies are independent of the area of application. (p. 80)

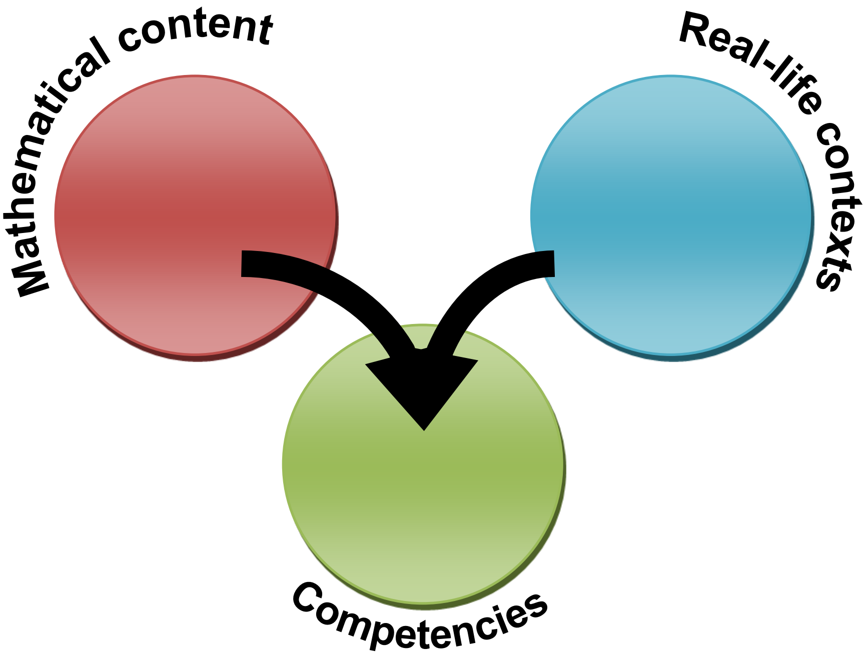

Figure 9: Diagrammatic representation of the relationship between content, context and skills

As an illustration, consider a task in which a learner is instructed on the process for drawing a linear graph to represent costs for a pre-paid electricity scenario. From the position of the conception or perspective described above, the 'linear' and 'electricity' components are secondary to the development of a graph drawing ability. In other words, heightened emphasis is placed on the ability to draw an appropriate graph (irrespective of the shape or name of the graph) to represent a real-life situation (irrespective of the specific nature of the situation). Of lesser importance is the ability to draw linear graphs to represent a pre-paid electricity situation.

A further facet of this emphasis on competencies is the belief that such competencies are universally applicable across a wide range of contexts and problem situations and are not bounded by any particular strand of content or context. As suggested by Howe (2003, p. 185), "This leads me to suspect that there are certain skills that are to some extent context free and that support the ability to deal with quantitative information in a variety of contexts."

It is beyond the scope of this study to provide a comprehensive list of all competencies emphasised in the literature. However, it may prove useful to identify particular skills that are emphasised consistently, since this provides an indication of the types of skills most commonly associated with mathematically literate, numerate and/or quantitatively literate behaviour and, as such, adds further insight into understanding how such behaviour is conceptualised. Common skills include:[47 & 48]

- fluency with mathematical concepts and tools;
- conceptual understanding;
- problem-solving, application and/or modelling;
- reasoning, insight, and/or reflection;
- communication – including communication of mathematical ideas through appropriate usage of operators and symbols, as well as critical communication through the offering of opinions, decision-making, and so on; and
- attitudes, dispositions, beliefs and/or values.

It is, perhaps, in an attempt to encapsulate this emphasis on such competencies as those listed above that Bass (2003) provides the following wide-ranging definition or expression of quantitative literacy:

> QL appears to be some sort of constellation of knowledge, skills, habits of mind, and dispositions that provide the resources and capacity to deal with the quantitative aspects of understanding, making sense of, participating in, and solving problems in the worlds that we inhabit, for example, the workplace, the demands of responsible citizenship in a democracy, personal concerns, and cultural enrichment. (p. 247)

Does this emphasis on competencies and the positioning of mathematical and contextual components as tools negate the prioritising of mathematics as the organising principle and the imposition of a mathematical gaze over incorporated real-world scenarios? Not necessarily, since many of the skills are still grounded in a mathematical base and with a mathematical bias. For example, consider the skill of 'communication'. The type of communication that is envisioned involves, primarily, communication appropriate to the domain of mathematics. Namely, the use of appropriate symbols and mathematical notation, the correct layout of calculations and answers, critical interpretation and comparison of mathematical solutions and options, and the generation of narratives to problem scenarios – through the use of mathematical techniques and routines − that are endorsed primarily according to mathematical knowledge and structures. More general communication skills that reflect common everyday communication strategies – such as the ability to write a paragraph, formulate an argument or opinion, or develop a presentation – are downplayed; instead, the skills that are prioritised are those concerned primarily with the way in which mathematical ideas are communicated in a mathematically logical, appropriate and legitimate way.

[47] The literature consulted in identifying this list of skills includes: (de Lange, 2003, 2006); (Hughes-Hallett, 2001, 2003); (Gal et al., 2005); (Niss, 2003); (Neill, 2001); (OECD, 2009); (Pugalee, 1999); (Pugalee, Douville, Lock, & Wallace, 2002); (Packer, 2003); (Richards, 2001); (Richardson & McCallum, 2003); (Schoenfeld, 2001, p. 53); (L. A. Steen, et al., 2001); (Van Groenestijn, 2003).

[48] The description by Kilpatrick et al., (2001) of the 'Strands of Mathematical Proficiency' provides detailed discussions of many of the skills listed here. Kilpatrick et al., refer to the strands of (i) 'procedural fluency' − which reflects the first skill category listed above; (ii) 'conceptual understanding'; (iii) 'strategic competence' − which reflects in the problem-solving and/or modelling category; 'adaptive reasoning' − which reflects in the reasoning, insight, reflection category; and productive disposition – which reflects in the attitudes, dispositions, beliefs and/or values category.

The Strands of Mathematical Proficiency are referenced specifically in relation to the development of *mathematical knowledge* and there is overlap between these strands and most of the skills identified as essential in the development of mathematical literacy, numeracy and/or quantitative literacy behaviour. This provides evidence of the dominance of the mathematical terrain and mathematical goals in many conception of mathematical literacy, numeracy and/or quantitative literacy.

It is also important to point out, though, that not all agree with the conceptualisation of a generalisable and widely applicable set of competencies as a core component of mathematically literate, numerate and/or quantitatively literate behaviour. This is particularly true for perspectives that position mathematical literacy, numeracy and/or quantitative literacy as a socially or culturally situated practice. Jablonka (2003), for example, argues that:

> The assumption that it makes sense to search for a universalistic applicable cannon of mathematical skills that can be separated from the context of their use is doubtful from the perspective of the socio-cultural view of mathematics. It is doubtful whether mathematical skills can be separated from the social dimensions of action and from the purposes and goals of the activity in which they are embedded. … Such a description ignores the interests and values involved in posing and solving particular problems by means of mathematics. (p. 79)

Ewell (2001, p. 37) offers a similar suggestion, first equating quantitative literacy to a type of literacy and then emphasising the socially or culturally situated nature of literacy. And Frith and Prince (2006, 2009) follow suite, warning against viewing mathematical literacy as a constellation of skills which have application outside of a particular social setting.

These alternative perspectives indicate, once again, the differential emphasis and sometimes lack of consensus over the components and key areas of focus in descriptions of mathematically literate, numerate and/or quantitatively literate behaviour.

6.3 Category 3 - Arena of application

As has been discussed above, there is general consensus in the literature that mathematical literacy, numeracy and/or quantitative literacy involves the use and application of mathematical content in engaging with problems scenarios. A key question then becomes: in which domain are those problems situated and what precisely is scope of the arena of application of such content? Only in real-world contexts? Or are purely mathematical contexts also deemed appropriate? And do the contexts have to be authentic? Or can they be artificially constructed and/or contrived? These are the questions that I address in the discussion in this section.

For the most part, there is general consensus within the literature that mathematically literate, numerate and/or quantitatively literate behaviour involves engagement with problem situations related, primarily, to real-world contexts. As suggested by Van Groenestijn (2003),

> Although there are differences in wording, these definitions [of numeracy, quantitative literacy and mathematical literacy] have a common intention. All three focus on the competencies of individuals to make sensible use in real-life situations of the mathematics they learned in school. (p. 230)

The OECD-PISA frameworks (OECD, 1999, 2003, 2006, 2009) are an obvious exception to this, where both 'intra-mathematical' and 'extra-mathematical'[49] tasks are deemed appropriate.

However, the 'real-world' is a big place and, so, what scope of this world is envisioned to be appropriate for the development of mathematically literate, numerate and/or quantitatively literate behaviour? Analysis of the sub-categories of *Location* and *Context* identified by Neill (2001) provide some insight in this regard and indicate areas of commonality within the literature. In particular, the (i) locations are largely organised into four overarching categories: (1) 'personal life' and/or the 'home'; (2) the 'workplace'; (3) the 'community'; and (4) 'society'.[50] The rationale behind these categories is, seemingly, for an outward expansion of world-view from possibly familiar locations (issues relating to personal life) to likely less familiar locations (national and global issue).

In contrast to the consistent reference to common locations, references to specific (ii) contexts are considerably more varied and it is more difficult to identify categories or trends within references to contexts. This is to be expected, especially if one takes seriously de Lange (2003, pp. 87-88) contention that mathematical literacy means different things to different people in different cultures and, so, needs to be 'culturally attuned' to the needs of a specific population. This implies that the specific problems relevant to the location of, for example, the 'home' differ and vary from one country, population and/or community to another depending on the needs of that group at a particular point in time. For example, personal finance in Zimbabwe – and the contexts that are deemed suitable and relevant to exploring this issue − may mean something completely different to what would be constituted as relevant personal finance contexts in New Zealand.

A related issue with respect to the types of contexts deemed appropriate is the level of 'authenticity' of such contexts. Namely, the extent to which the contexts genuinely and realistically represent the structure of participation that is considered to be legitimate in a particular real-life scenario (as legitimated and endorsed by those who participate in the context on a regular basis). The level of authenticity is also reflected in the accompanying structures of knowledge and forms of communication that are similarly considered legitimate and appropriate for use in that context. And, an equally important issue is the

[49] "If a task refers only to mathematical objects, symbols or structures, and makes no reference to matters outside the mathematical world, the context of the task is considered as intra-mathematical, and the task will be classified as belonging to a 'scientific' situation type. … More typically, problems encountered in the day-to-day experience of the student are not stated in explicit mathematical terms. They refer to real-world objects. These tasks are called 'extra-mathematical', and the student must translate these problem contexts into a mathematical form." (OECD, 2003, p. 33)

[50] Common alternative terms to those listed here include: 'individual's life', 'private life', 'family life', 'everyday life', and 'day-to-day life'; 'educational life' and 'occupational life'; 'public life', 'citizenship', 'national issues' and 'global issues'.

Clearly there is the potential for each of these categories to comprise sub-categories. For example, the category of 'personal life' could include the sub-categories of 'school', 'personal finance', 'personal health', and so on.

Again, the OECD-PISA frameworks are an exception here, referring to the consistent situation types of personal, educational or occupational, public but also including the additional and lesser emphasised domain of 'scientific'.

The ALL and PIAAC frameworks (Gal et al., 2005; PIAAC Numeracy Expert Group, 2009) and the work by Ginsburg et al. (2006) also offer a different perspective, with both including the additional location of 'further learning'. A possible reason for this is that these two studies deal primarily with adult numeracy rather than with school-based numeracy potential implication for problems that do not exhibit a particular level of authenticity. The work of du Feu (2001) provides important insight into this issue.

Du Feu (2001, p. 2) argues that mathematical questions involving the use of or reference to contexts can be classified into one of five categories: (i) *context-free*: this would contain purely esoteric mathematical contents with no reference to a real-world situation; (ii) *real*: these are genuine or real contexts with real names, messy numbers and data (e.g. an unmodified cell phone bill); (iii) *cleaned*: "These are essentially real contexts, but where the mathematical model has been simplified in order to make the question accessible to the user or possible in the time constraints of examinations." (du Feu, 2001, p. 2) (e.g. a cell phone bill that a teacher has (re)developed and simplified from an original 'real' resource); (iv) *parable*: these are fictitious contexts, using fictitious names and people, and where it is obvious that the situation is not real. Such problems are often written in the form "A person has to …" and the intention of such problems is to teach a particular mathematical concept or to make a specific point (hence the name 'parable'); (v) *contrived*: these are contexts that are constructed to fit a particular mathematical concept – irrespective of how appropriate or reflective this is of how the situation actually works in real-life. These types of contexts offer the pretence of reality (through the inclusion of real names, photographs, and other localising resources) by presenting the contexts (and problem-solving activities related to those contexts) as reflective of the structure of participation that is legitimated and endorsed in the real-world.

Having identified these different types of contexts, du Feu (2001, p. 4) goes on to problematise the usage of both parable and, particularly, contrived contexts: "I do not think that contrived examples have any place in mathematics testing or textbooks". Furthermore, learners who employ real-world techniques and considerations in contrived contexts (by believing that the contexts are realistic) rather than mathematical considerations and calculations are likely to be penalised for their non-mathematical techniques. This sends a strong message that what happens in the mathematics classroom and in the real-world are completely divorced from each other (du Feu, 2001, p. 3), which, in turn, has implications for the extent to which learners are able to successfully employ appropriate techniques in solving real real-life problems beyond school:

> If the mathematics teaching does not differentiate between real and imaginary, students are likely to suspend belief and not try to use mathematics when they encounter real problems later on. (du Feu, 2001, p. 3)

As regards the usage of the other three types of contexts, du Feu (2001, p. 4) argues that all three are appropriate but that a balanced approach must be employed.

Alongside du Feu's concerns regarding the use of contrived contexts lies another issue for consideration. Namely, that allowance for a predominance of context-free, cleaned and parable contexts signifies a prioritising of mathematical considerations and knowledge over real-world forms of participation and authentic sense-making of real-world problems. Context-free problems clearly have a mathematical prioritising agenda, as do parable contexts with an explicitly mathematical bias.

As regards cleaned contexts, an important caveat is necessary. As mentioned previously, the level of authenticity of a context refers to the extent to which a context employed within the setting of a learning process or task accurately reflects the reality of how the context is experienced by participants who engage in that context in their daily lives.

However, a call for a high level of authenticity must also be accompanied by acknowledgement and recognition that any context must, inevitably, undergo some degree of recontextualisation when it is incorporated into a learning process or system, together with some form of selection of contextual elements worthy of exploration and other elements that are appropriate to be ignored. It would be naïve of me to suggest that it is possible to study a real-life setting in all aspects and to take into consideration every possible influence and permutation which may determine functioning in that context. Rather, my call for a high degree of authenticity can be rephrased as an appeal to focus attention on how a person might think, act and respond in a particular context and how that context may be experienced by participants in the real-world. Of lesser importance is the extent to which the context illustrates the utility of mathematics or how it is experienced from a reconstructed and mathematically biased perspective. That said, if contexts are cleaned to reduce the complexity of the contextual elements in order to make the mathematical elements more prominent or to ensure that mathematical engagement with the context yields mathematically manageable and sensible results, then for me this signifies a prioritising of the mathematical terrain over the contextual terrain. The result is a consequent reduction in the degree of life-preparedness facilitated. Importantly, this is in no way intended to imply that context-free, parable or cleaned contexts must be avoided. Rather, I simply want to emphasise the point that if there is an intention to promote a life-preparedness agenda where an interest in contextual sense-making practices and contextually appropriate forms of knowledge and participation are prioritised over mathematical structures, then the level of authenticity of the contexts referenced is of critical importance. If, however, the dominant priority involves mathematical considerations and the development of mathematical knowledge, then the usage of different types of contexts − other than contrived (for reasons discussed above) − becomes appropriate. It is also not my intention to deny the utility and suitability of cleaned contexts in the development of behaviour driven by a contextually dominant agenda, but, rather, to caution that such contexts must be used alongside (and not as a replacement of) authentic contexts and problem situations. This ensures that an experience of the 'messiness' and complexity of real-world participation is included in the learning process, hereby facilitating access to a heightened degree of life-preparedness.

In light of both du Feu's concerns and the issue raised above, it becomes important to determine the extent to which 'authenticity' is prioritised in the literature, since this provides further insight into whether mathematical or contextual or life-preparedness agendas predominate. As has already been mentioned, the OECD-PISA frameworks (OECD, 1999, 2003, 2006, 2009, 2012b) include the possibility of both intra-and-extra mathematical problem situations, which suggests the possibility of the inclusion of context-free, parable, cleaned and real contexts within the PISA assessment items. These frameworks exhibit an explicitly mathematical agenda, with a primary focus on assessing the extent to which learners are able to use specific mathematical content, knowledge and skills to solve problems that have some connection to reality. As the following quotation suggests, it is the mathematical component and not the context that is of primary concerns in these assessments:[51]

> The problem derives its quality not primarily from its closeness to the real world but from the fact that it is *mathematically interesting* and calls on competencies that are related to mathematical literacy. (OECD, 2009, p. 93, my emphasis)

The ALL (and PIAAC) framework by contrasts, places explicit emphasis on the use of tasks with a high degree of realism and highlights this issue as a key area of distinction to the OECD-PISA framework: "PISA puts only partial emphasis on the realism of tasks." (Gal et al., 2005, p. 149). A justification for this distinctive emphasis on realism is also provided:

> The philosophy behind the design of mathematical assessments for PISA, GED, and similar assessments is based on assumptions about what it means to "know math" or "be able to do math" in a schooling context; hence, the assessment design assumes that it is legitimate to use a certain degree of formalization of math symbols and to present contrived math tasks. This assumption does not fit the assessment of skills of adults who may have been out of school for many years.[52] (Gal et al., 2005, p. 149)

Unfortunately, no explanation of the term 'realistic' is provided in the ALL framework and, so, it is not possible to determine whether realistic problems include only 'real' contexts or also 'cleaned' contexts. However, the examples of 'realistic' resources provided in the framework certainly suggest a deliberate emphasis on the prioritising of authentic contexts and the sense-making of legitimate and appropriate forms of participation in those contexts – and, certainly, to a greater extent than is the case in the PISA frameworks. It is worth pointing out that this emphasis on engagement with authentic contexts is accompanied in the PIAAC framework with a cautionary note on two fronts. Firstly, recognition is given of the complexity of engagement with authentic

[51] It is, perhaps, worth mentioning that one of the primary areas for consideration in the revision of the 2012 OECD-PISA framework and associated test items is a move towards a reduction in both contextual elements and associated text, and greater explicitness of the mathematical components of each problem-task situation. This call has come in the wake of protests raised by certain participating countries who have questioned whether past PISA task items have provided an adequate indication of the mathematical ability of participating learners due to the inclusion and possible interference of text and contextual elements which have, potentially, served to restrict and inhibit demonstration of mathematical problem-solving abilities. Such protests have argued that past PISA items have relied too heavily on language literacy as a requirement for managing mathematically driven tasks. It is also in line these similar concerns that calls were made for the title 'Mathematical Literacy' (as employed in the framework to describe the mathematical domain of the study) to be modified in the 2012 study to a title that no longer contains reference to the word 'literacy'. This, however, has not come to fruition (c.f. OECD, 2012b). See Stacey (2012) for a discussion of these and other issues pertaining to the 2012 OECD-PISA study. All other comments described here were outlined in Stacey's presentation at the 2012 ICME conference in Seoul, Korea.

[52] This quotation draws attention to an important point, namely that the PISA study is concerned the domain of *school* based knowledge while the ALL study is concerned with the domain of *post-school* adult numeracy. So, while the PISA study is driven by a goal for the assessment of a particular form of mathematical ability, the ALLS study is driven by an alternative goal for measuring the extent to which adults exhibit a form of numerate behaviour in their day-to-day lives. It is this difference in goals that prompts different requirements with respect to the situations and contexts that are constituted as valid and/or appropriate for exploration. It is also worth noticing that a conception of the knowledge domain of mathematical literacy that promotes a life-preparedness orientation is aligned closer to the conception of appropriate mathematically literate behaviour and associated knowledge described in adult numeracy frameworks (such as the ALL study) than to that described in the PISA (and other mathematically oriented) frameworks.

contexts in the context of formal large-scale assessment practices, and of the necessary sacrifice of particular elements of the contextual environment to facilitate increased accessibility of the context and prioritisation of key skills or concepts (PIAAC Numeracy Expert Group, 2009, p. 26). Secondly, the cultural basis of the notion of authenticity is highlighted: "The desire to retain authenticity, however, may at times be at odds with the need to establish cultural appropriateness of tasks and stimuli and reduce context effects." (PIAAC Numeracy Expert Group, 2009, pp. 30-31).

Moving beyond assessment frameworks, a reading of the other literature reveals far less explicit specificity of the types of contexts and the level of authenticity of contexts through which mathematically literate, numerate and/or quantitatively literate behaviour is perceived to develop. Most authors are simply content to use the terms 'real', 'real-world', 'real-life', 'everyday', 'realistic', or 'authentic', without explanation of the precise meanings of these terms or whether they allow for the inclusion of contrived, parable-like and cleaned contexts, or whether such contexts are restricted purely to real contexts. Pugalee et al. (2002, p. 303), for example, stress that "Authentic tasks are a critical tool in developing the level of mathematical understanding and conceptualizing indicative of mathematical literacy.", but fail to explain precisely what is meant by the term 'authentic' in relation to the level of 'realness' of the task situation and whether cleaned, modified or constructed tasks are included in this description. In the majority of the literature on mathematical literacy, numeracy and/or quantitative literacy there is, seemingly, an assumption that the reader understands the scope of reality intended and signified in or through the usage of the words 'realistic', 'authentic' and 'real-world'. However, as Ginsburg et al. (2006, p. 7) problematize, "realistic is not real", and it is primarily through real contexts – with messy numbers, multiple solutions, complex variables and extraneous factors − that genuine understanding of the structure of knowledge and legitimate participation in a context is achieved. Realistic contexts, on the other hand, are designed to resemble real situations, but the design or modification of the context is suited to the promotion of the learning of a particular (commonly mathematical) concept (Ginsburg et al., 2006, p. 7). It is this light that It is this light that Ginsburg et al. (2006) suggest that,

> The contrast between decontextualized, abstracted mathematics (e.g., "What is 23 × 13") and highly contextualized mathematics (e.g., "When can you retire and how do you know?") might be best described as a continuum from abstract to real, with "realistic" somewhere in the middle. (p. 7)

Wiggins (2003) is the only other author in the literature read who also emphasises the importance of authentic contexts, but, confusingly for this discussion, uses the words 'authentic' and 'realistic' interchangeably. For Wiggins (2003),

> How should we define "realistic"? An assessment task, problem, or project is realistic if it is faithful to how mathematics is actually practiced when real people are challenged by problems involving numeracy. The task(s) must reflect the ways in which a person's knowledge and abilities are tested in real-world situations. (p. 127)

In other words, if a task or teaching situation involves a realistic context, then adequate engagement with the task or situation must entail engagement with the forms of knowledge, participation, communication and decision-making practices employed by people who operate in those situations in the real-world on a daily basis. This does not mean that alternative forms of participation (including mathematical forms) cannot be explored. Rather, that before such alternative forms are investigated, acknowledgement and credence must first be given to how the situations would actually be experienced by participants who engage in the situations on a regular basis and the methods, knowledge and decision-making strategies used by those participants to ensure successful participation.

It is my contention that the general lack of specificity in the literature of the types of contexts and the degree of authenticity of those contexts required for the development of mathematically literate, numerate and/or quantitatively behaviour makes it possible for non-real (i.e. modified, cleaned, constructed, parable) contexts to also be included in related problem-solving practices. This is particularly true in assessment items where there is sensitivity for ensuring that all learners are equally able to access questions without being (dis)advantaged by the particular nature of a context. As suggested in the OECD-PISA frameworks:

> It should be noted that there are some made-up elements of the problem – the money involved is fictitious. This fictitious element is introduced to ensure that students from certain countries are not given an unfair advantage. (OECD, 2009, p. 92)

This is not to suggest that the inclusion of non-real contexts is undesirable (in certain types of task situations), but rather, as indicated by Ginsburg et al. (2006, pp. 7-9), that non-real contexts signify a prioritising of mathematical elements rather than structures of knowledge, participation and communication that more appropriately reflect legitimate forms of thought and behaviour in real-world practices. The inclusion of non-real components in a context signifies that it is mathematics and not context that is positioned as the dominant organising principle of an activity.

A final issue relating to the arenas of application specified in the literature is general consensus that the development of mathematically literate, numerate and/or quantitatively literate behaviour must occur across a variety of disciplines and through the integration of mathematics in a variety of subjects or classes. In this regard the responsibility for the development of such behaviour must not fall exclusively on the shoulders of the mathematics teacher. The following quotations show this consensus.

> Only by using the diverse aspects of numeracy in real contexts will students develop the habits of mind of a numerate citizen. Like literacy, numeracy is everyone’s responsibility. (L. A. Steen, 1999, p. 11)

> In life, numbers are everywhere, and the responsibility for fostering quantitative literacy should be spread broadly across the curriculum. Quantitative thought must be regarded as much more than an affair of the mathematics classroom alone. (L. A. Steen, 2001b, p. 58)

> When teaching mathematics is seen as a way of teaching people how to think, it can no longer be isolated. Its implications spread throughout the curriculum and it has a place in every class. (Richards, 2001, p. 36)

> Yes, it does make sense to teach QL across the curriculum. Indeed, I can’t conceive of any other way it could be done effectively without turning it into a discipline instead of a “literacy”. (Ewell, 2001, p. 46)

Despite these sentiments, there is still an underlying assumption in all conceptions of mathematical literacy, numeracy and/or quantitative literacy that it is mathematical knowledge that is positioned as the primary organising principle of such behaviour. As such, the mathematics teacher is perceived to have a key role to play and the mathematics classroom is positioned an important site for the development of such behaviour. This immediately raises the question of whether it is possible to prioritise anything other than mathematical considerations in a mathematical classroom where, inevitably, the primary organising principle is the domain of mathematical knowledge and a mathematically oriented curriculum. In light of this, perhaps Ewell's (2001) suggestion in the last quotation given above is misguided. Perhaps the only way to truly develop a literacy which empowers learners in the way in which they are able to operate in the world is by establishing a discipline that is structured around a context-oriented organising principle and terrain rather than a mathematically based curriculum.

In this sense, the situation in South Africa is distinctly different from the majority of the international conceptions on mathematical literacy, numeracy and/or quantitative literacy in that a specific subject (discipline) – Mathematical Literacy – has been established to achieve the goal of ensuring greater levels of numeracy amongst the schooling population in South Africa (DoE, 2003a, p. 9). In South Africa, the primary responsibility for ensuring the development of mathematically literate behaviour falls squarely and almost exclusively on the shoulders of the Mathematical Literacy teacher and, seemingly, not on the shoulders of the Mathematics teachers. Unfortunately, and as is explored in more detail in Chapter 8 below, this concentration of responsibility in a single subject has not facilitated the prioritising of more than just mathematical goals, knowledge and an associated mathematical gaze.

6.4 Category 4 – Components (and/or features) associated with courses, assessments and/or pedagogic practice

This category of analysis involves identification of specific components or features associated with courses, assessments and/or pedagogic practice that are prioritised as key areas of focus in the development or assessment of mathematically literate, numerate and/or quantitatively literate behaviour and which have potential influence the structure of this behaviour. As is shown in Chapter 8 below, this category of analysis provides particularly insightful information on areas of prioritising in the South African school subject Mathematical Literacy. Specifically, as a fully-fledged subject certain components associated with, for example, assessment requirements or specifications for the subject have a tremendous impact on how the implemented curriculum differs from the intended curriculum and, consequently, on pedagogic practices in the subject at a classroom level. However, this category of analysis reveals far less in relation to international literature on mathematical literacy, numeracy and/or quantitative literacy. This is because the majority of the materials read comprise general descriptions of components or traits of mathematically literate, numerate and/or quantitatively literate behaviour rather than detailed descriptions of curriculum or courses designed to facilitate the development of this behaviour. It is only in certain assessment frameworks (such as the OECD-PISA (1999, 2003, 2006, 2009, 2012b), ALL (Gal et al., 2005) frameworks) that there is evidence of specific assessment-related components which reflect and impact on the expected structure of envisioned forms of mathematically literate, numerate and/or quantitatively literate behaviour. As such, the discussion below is focused on the ways in which assessment-related issues influence and also restrict particular forms of behaviour associated with conceptions of mathematical literacy, numeracy and/or quantitative literacy.

To begin with, Neill (2001, p. 11) suggests that tests or assessments of numerate behaviour must exhibit the following characteristics:

- common real-life situations;
- cross-curricular contexts (e.g. contexts drawn from science, technology, etc.);
- cross-strand uses of mathematics (i.e. integration of content from different strands); and
- competencies of a global nature, including:
    - choosing to use maths,
    - choosing maths appropriate to the context,
    - correctly applying maths chosen,
    - interpreting mathematical findings appropriately for the context,
    - communicating the findings, and
    - critically evaluating statements of a mathematical nature made by others.

A number of discussion points and/or questions can be raised concerning this list. For example, how do we define the scope of 'common' real-life situations and surely situations that are common and/or familiar to one group of learners may be uncommon and/or unfamiliar to another group? This raises concerns regarding the appropriateness of using contexts in an assessment, especially if the usage of particular contexts may advantage some learners and disadvantage others. A further concern can be lobbied in relation to the suggestion that assessment items must include focus on 'global competencies' – especially if one takes seriously Jablonka's (2003) comments regarding the socially or culturally situated nature of mathematical literacy. It is also worth mentioning that the competencies as listed above exhibit tremendous similarity to the process of mathematising identified in the OECD-PISA frameworks (OECD, 1999, 2003, 2006, 2009) and that these competencies clearly prioritise mathematical considerations and concerns. An assessment aligned to these conditions would prioritise mathematical competency and modelling as primary components of mathematically literate, numerate and/or quantitatively literate behaviour.

Shifting focus slightly, Wiggins (2003) makes the following statement regarding the assessment of quantitatively literate behaviour:

> … the challenge is to assess students' abilities to bring to bear a repertoire of ideas and skills to a specific situation, applied with good judgement and high standards. (p. 125)

Now consider the OECD-PISA assessment frameworks (OECD, 1999, 2003, 2006, 2009) viewed in the light of this comment. The OECD-PISA assessment frameworks are designed to provide an assessment of the literacy skills (including reading, scientific and mathematical literacy) of 15-year-olds, and employ open-ended, closed, and multiple choice type questions to achieve this objective. Viewed through the lens of Wiggins' (2003) comment, it is questionable whether it is possible to assess genuine contextual sense-making practices with due consideration given to the types of situational strategies and resources that people might actually employ when solving such problems in real-world settings through multiple-choice or closed question types. However, since the OECD-PISA frameworks offer an explicit statement of intention to assess the learners' abilities to impose a mathematical gaze on the world and to use such a gaze to make sense of problems involving mathematised recontextualisations of real-world practices, this is clearly not a concern for the designers of these assessments. The question remains though: it is possible to assess a form of mathematical literacy that does not prioritise mathematical considerations over contextual sense-making practices through traditional closed-ended questions? And if not, then what is the alternative? The ALL framework (Gal et al., 2005) offers one such alternative approach. In contrast to the OECD-PISA frameworks, the test items in the ALL framework includes only free-response items in which respondents are able to write answers to questions in their own words using any suitable technique (Gal et al., 2005, p. 168). This approach certainly makes provision for respondents to be able to draw on a variety of techniques and considerations in solving real-world problems, hereby drawing the framework closer to the goal associated with contextual sense-making practices aimed at understanding legitimate forms of knowledge and participation in real-world practices. However, the ALL framework also restricts the level of textuality of the assessment tasks, stating that two-thirds of the tasks are designed to include little or no text and citing as a reason the intention to reduce overlap with the separate Document and Prose Literacy components of the assessment framework. In other words, at the level of stated intention in these frameworks, any text employed is reduced or eliminated in two-thirds of the test items to negate the influence of contextual factors and to facilitate the directed assessment of mathematical considerations − a seemingly alternative form of prioritisation of the mathematical terrain over real-world considerations.

The discussion in this section has served to highlight issues relating, particularly, to the assessment of mathematically literate, numerate and/or quantitatively literate behaviour. The discussion has also illustrated particular challenges involved in assessment frameworks that aim to assess and prioritise more than just mathematically legitimised forms of knowledge and participation. Having highlighted these issues, I certainly do not claim to be able to provide solutions to these issues, nor am I able to offer alternatives. However, the point is hopefully clear: the issues surrounding the development of a shared understanding of mathematically literate, numerate and/or quantitatively literate behaviour are complex, and the implications for the incorporation of such notions into the formal structure of the schooling system – and, particularly, formal assessment structures − raise even further questions and fewer answers.

6.5 Category 5 – Dominant orientations in international perspectives of mathematical literacy, numeracy and/or quantitative literacy

In Chapter 5 of this part of the study, I introduced a framework of possible *orientations* associated with particular conceptions of mathematically literate, numerate and/or quantitatively literate behaviour, as informed by the dimensions of the *Agenda(s) and Intentions(s)* promoted within the different conceptions (c.f. page 48 above). As described above, these agendas and intentions represent respectively:

- Agenda → The dominant perspective or agenda in a body of literature in terms of the extent to which the literature prioritises mathematical knowledge and techniques or contextual sense-making practices (and associated legitimate forms of knowledge participation and communication in those contexts) as the dominant organising principle of an activity.
- Intention → The explicit or implicit external impetus for which a particular conception of mathematical literacy, numeracy and/or quantitative literacy is directed − for example, for more effective participation in the workplace, or for critical citizenship, and so on.

The collective of the identified agendas and intentions were then reflected in a framework shown in Figure 7 on page 48 above.

Employing this framework to the literature read for this study facilitates comparison of the dominant orientations within the various international perspectives on mathematical literacy, numeracy and/or quantitative literacy, and also comparison to the dominant orientation in the South African subject-matter domain of Mathematical Literacy[53]. Although it is not feasible, within these pages, to provide an explicit description of my reasoning for the specific categorisation of every piece of literature read in the framework, an illustration is necessary. For purposes of this illustration, consider the OECD-PISA frameworks (OECD, 1999, 2003, 2006, 2009). As regards the dominant ***Agenda(s)*** in the framework, as has been illustrated at various points in the discussion in the pages above, these frameworks contain an explicit prioritising of mathematical structures over real-world forms of knowledge and participation in the solving of problems. Some examples of this include:

- although real-life contexts may be employed, allowance is made for such contexts to be modified to facilitate heightened access to the mathematical components of the problem scenario;
- allowance is also made for the inclusion of purely scientific questions and contexts where no extra-mathematical contextual link is included;
- the process of mathematising encouraged in these framework is explicitly mathematically grounded and promotes the imposition of a mathematical gaze on real-world structures and the recontextualisation of such structures according to mathematical principles;
- despite allowances made for the inclusion of non-contextually based problems, primary focus in the framework is on assessing learners' ability to perform contextually based calculations (i.e. mathematics in use) and to create and analyse mathematically structured models to represent real-world situations.

All of the above suggests an explicit prioritising of mathematical knowledge, structures and applications over considerations involved in contextual sense-making practices. It is mathematical forms of participation used to develop mathematised forms of understanding that dominate and not an impetus for understanding the real-world (and of how people might think, behave and communicate in that world). This clearly eliminates the framework from being associated with the final life-preparedness agenda dimension in the table. And, although there is some mention of un-contextualised calculations, it is clearly the second and third agendas – namely, the *Ability to perform calculation in real-world contexts* and *Modelling* – that dominate in these frameworks. Specifically with respect to Agenda 2, it is the dimension of Applications (Agenda 2 [a]) that dominates and not prioritisation of Numeracy-in-Context (Agenda 2 [b]) type practices. This is evidenced in the expectation in the OECD-PISA frameworks for engagement with scientific and abstract mathematical contents and the use and application of those contents for solving problems encountered in extra-mathematical contexts.

Now consider the dominant ***Intention*** espoused within this framework. The following definition of mathematical literacy is provided in the OECD-PISA frameworks:

[53] See page 120 below for an elaborated discussion of the comparison of the dominant orientation in the subject-matter domain of Mathematical Literacy to those that characterise international perspectives on mathematical literacy, numeracy and/or quantitative literacy.

> *Mathematical literacy* is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2003, p. 24)

This definition suggests that mathematical literacy is seen as something empowering and that possession of the faculty of a mathematical gaze positions the user in a better position to be able to make sense of and cope with the demands of the world. The following quotation illustrates the consequences, from the perspective of the OECD-PISA framework, of not developing this gaze:

> Failure to use mathematical notions can result in confused personal decisions, an increased susceptibility to pseudo-sciences, and poorly informed decision-making in professional and public life.[54] (OECD, 2003, p. 27)

The OECD-PISA conception of mathematical literacy, thus, falls in line with an intention for the development of *Human Capital*. Jablonka (2003, p. 81) herself offers the same classification of the OECD-PISA framework, arguing that "PISA is intended to estimate and compare the stock of 'human capital'."

Through utilisation of this same approach I have categorised the various literature on mathematical literacy, numeracy and/or quantitative literacy read for this study. This categorisation is shown in Figure 10 on page 82 below.[55] From this categorisation, I contend that the largest portion of the literature read embody statements of ***Intention*** promoting the development of mathematically literate behaviour as a form of empowerment and for developing and enhancing worth and value for the workplace and the economy: namely, mathematical literacy for the development of *Human Capital*. A considerably smaller number of articles promote a dominant intention of mathematical literacy for any of the other intentions. This does not mean that such intentions are not present or referenced in some of the literature in which mathematical literacy for Human Capital is prioritised, but, rather, that they do not constitute the dominant intention in such literature. Notice that this dominant intention for the development of Human Capital is also reflected to an exclusive and overwhelming degree in the literature and documents that relate to the South African subject-matter domain of Mathematical Literacy. In South African, participation in the subject is proclaimed as an avenue to enhanced functioning in future career and daily-life practices.

[54] My mother-in-law, who achieved a double-G (yes, a 'GG'!) symbol for mathematics in her matriculation examinations and, yet, who now runs her own business very successfully, would in all likelihood not agree with this statement. She also has no objection to me mentioning her result in this study. She treats it as a point of honour and pride that she achieved the lowest mark ever for mathematics in the history of her school.

[55] Note that this figure also includes categorisation of the literature that relates specifically to the South African school subject Mathematical Literacy – even though discussion of this subject has not yet been provided. Detailed discussion of the subject is provided in Chapter 8 below (c.f. page 91), and during that discussion (c.f. page 120) I refer back to this figure to highlight the dominant orientation in the subject in comparison to the dominant orientations that characterise international conceptions of mathematical literacy, numeracy and/or quantitative literacy. In brief, this later discussion highlights that the dominant orientation in the National Curriculum Statement (DoE, 2003a) is characterised by an intention for the development of Human Capital and by agendas for enhancing application and modelling abilities, while in the CAPS curriculum (DBE, 2011a) the agendas shift to a prioritising of modelling and contextual sense-making practices. In contrast to both of these, the national examinations for the subject (under both curriculum structures) promote an almost exclusive focus on numeracy in context, albeit retaining the intention for the development of Human Capital.

As regards dominant ***Agenda's*** promoted in the literature, much of the discussion in the pages above has highlighted different ways and areas of focus in which mathematical components are prioritised over real-world components. It should come as no surprise, then, that my categorisation of the literature reveals that the second and third agenda's along the spectrum dominate throughout the largest portion of the literature. Namely, focus on *mathematical calculations in real-world and/or mathematised contexts* – with specific prioritisation of application-related practices (Agenda 2 [a]) rather than engagement with elementary contexts in contextual settings (Agenda 2 [b]) − and the *design and analysis of mathematical models to represent real-world situations*. And, although these agendas do not promote a prioritising of mathematical considerations at the exclusion of real-world contexts, they certainly point to a prioritising of a mathematical gaze over contextually based problems. Only a limited number of authors stipulate a direct prioritising of a life-preparedness agenda that is accompanied by acknowledgement and consideration of real-world concerns and considerations, and by emphasis on contextual sense-making practices aimed at enhancing understanding of existing and possible alternative forms of knowledge and participation in contextual situations. Interestingly, notice the wide variation in the dominant agenda prioritised in the South African documentation and literature relating to the subject-matter domain of Mathematical Literacy. In contrast to the dominant agenda prioritised in the majority of international literatures read (i.e. for Applications – Agenda 2 [a]), it is the Agenda 2 dimension of Numeracy-in-Context (Agenda 2 [b]) that dominates the South African documentation (most of which reflects government curriculum and assessment documentation). However, there is also some curriculum or assessment documentation for the subject that prioritise agendas of Application (Agenda 2 [a]), Modelling (Agenda 3) and Contextual Sense-Making Practices (Agenda 4). As is discussed later (c.f. page 106 below), this variation in the dominant agenda prioritised in different official documents has resulted in the development of a "spectrum of pedagogic agendas" (Venkatakrishnan & Graven, 2007) in the subject, each of which comprise different criteria for legitimate participation in the subject and for the structure of knowledge required to facilitate endorsed participation. An elaborated discussion of these and other issues relating specifically to the subject-matter domain of Mathematical Literacy in South Africa is provided in Chapter 8 (starting on page 91) below.

An intervening word of caution is necessary here. I acknowledge that my reading of the literature on mathematical literacy, numeracy and/or quantitative literacy is limited and that a more extensive reading and consequent categorisation of the literature in the framework of agendas and intentions may lead to identification of different trends in the dominant orientations than those that I have alluded to above. I also acknowledge that my selective sourcing of literature may have resulted in a collection of literature which promotes a common perspective and orientation. That said, I have tried to read as widely as possible and to deliberately source literatures that offer differing perspectives on features associated with mathematically literate, numerate and/or quantitatively literate behaviour to facilitate a comprehensive analysis. In sourcing literature I have also chosen to focus particular attention on works that are considered to be pivotal works in the field and, as such, are referenced frequently in discussions and debates (for example, the work of Lynn Arthur Steen). I have also deliberately sourced information on the OECD-PISA and ALL assessment frameworks since these frameworks are used on a global scale and are widely considered to provide an accurate measure of a particular form of mathematically literate behaviour. In light of this, there is a strong case to argue that the dominant orientations (and associated intentions and agendas) that characterise much of the literatures read provide a valid and appropriate reflection of prevailing areas of prioritising in international (and South African) conceptions of mathematically literate, numerate and/or quantitatively literate behaviour.

A possible and likely reason for this prioritisation of agendas characterised by mathematically structured and legitimised forms of participation stems from the fact that the majority of the conceptions of mathematical literacy, numeracy and/or quantitative literacy do not separate the development of associated behaviour from engagement with scientific mathematics practices or from the domain of mathematics. Instead, the development of mathematically literate, numerate and/or quantitatively literate behaviour is seen to stem through participation in and engagement with all forms of mathematical contents, including scientific contents. In sum, the dominant perspective in much of the international literature is that to be mathematically literate, numerate and/or quantitatively literate means to be literate and/or numerate with mathematics and not with something else other than mathematics. The result of this perspective is that participation in such practices is inevitably, understandably and appropriately legitimated according to mathematical knowledge, structures and forms of communication.[56]

This, then, brings to an end the analysis and discussion of the forms of behaviour associated with various conceptions of mathematical literacy, numeracy and/or quantitative literacy described in international literatures. Looking ahead, in the next chapter I move to discuss what I perceive to be key distinctions between facets of behaviour associated with the terms mathematical literacy, numeracy and/or quantitative literacy and my reasons for prioritising the term ‘mathematical literacy’. Thereafter (in Chapter 8), I discuss the South African conception of mathematically literate behaviour as encompassed in the subject-matter domain of Mathematical Literacy and compare this conception to the international perspectives described above.

[56] Although, it must be said that there is a risk in ‘painting all of these different conceptions with the same brush’ since the basis of legitimisation in different forms of mathematical practice will vary. Namely, the basis of legitimisation for mathematical practices that prioritise modelling will be different from, for example, those that prioritise pure mathematics concepts, or from those that engage applied mathematical principles.

Prioritisation of Mathematical knowledge and participation

SPECTRUM

Prioritisation of knowledge / participation that facilitate contextual sense-making practices

Agenda				**Intention** Mathematical Literacy, Numeracy and/or Quantitative Literacy for:				
				Developing Human Capital	Cultural Identity	Environmental Awareness	Social Change	Evaluating Structures
Mathematically dominant goals	**1. Mathematical Competence**	1 [a] Literacy in Mathematics	International (INT)	(Niss, 2003); (OECD, 1999, 2003, 2006, 2009)				
			South Africa (SA)					
		1 [b] Numeracy	INT					
			SA					
	2. Mathematics in Context	2 [a] Application	INT	(Niss, 2003); (OECD, 1999, 2003, 2006, 2009); (L. A. Steen, 1999, 2001a, 2001b, 2003a, 2003b; L. A. Steen, (Ed), 1990; L. A. Steen, et al., 2001);				
			SA	*(DoE, 2003a)*				
		2 [b] Numeracy-in-Context	INT	(Ginsburg et al., 2006); (Neill, 2001)				
			SA	*(DoE, 2005b, 2007, 2008c, 2008d, 2009a, 2009b, 2009c) (DBE, 2010a, 2010b, 2012a, 2012b, 2013b, 2013c, 2014b, 2014d)*				
Contextually dominant goals	**3. Modelling**		INT	(de Lange, 2003, 2006); (Ewell, 2001); (Ginsburg et al., 2006); (Hughes-Hallett, 2001, 2003); (Niss, 2003); (OECD, 1999, 2003, 2006, 2009); (Packer, 2003); (Pugalee, 1999; Pugalee et al., 2002); (Richardson & McCallum, 2003); (Schoenfeld, 2001); (L. A. Steen, 1999, 2001a, 2001b, 2003a, 2003b; L. A. Steen, (Ed), 1990; L. A. Steen, et al., 2001); (Van Groenestijn, 2003); (Venkatakrishnan & Graven, 2007); (Graven & Venkatakrishnan, 2007)	(D'Ambrosio, 2003)	(Fusaro, 1995)	(Skovsmose, 1992, 1994a)	(Jablonka, 2003)
			SA	*(Brombacher, 2007); (DoE, 2006); (DBE, 2011a)*			*(Christiansen, 2006, 2007); (Julie, 2006)*	
	4. Contextual Sense-Making Practices		INT	(C. Hoyles, Noss, & Pozzi, 2001; C Hoyles, Wolf, Molyneux-Hodgson, & Kent, 2002)		(Fusaro, 1995)	(Frankenstein , 2009b)	(Jablonka, 2003);
			SA	*(Brombacher, 2007); (DBE, 2011a);* (Gal et al., 2005- at a conceptual level); *(PIAAC Numeracy Expert Group, 2009 - at a conceptual level)*			*(Christiansen, 2006, 2007)*	*(Julie, 2007)*

Figure 10: Categorisation of the literature on mathematical literacy (including the South African subject Mathematical Literacy), numeracy and/or quantitative literacy according to the dimensions of dominant Agendas and Intention

CHAPTER 7
WHAT'S IN A NAME? MATHEMATICAL LITERACY, NUMERACY OR QUANTITATIVE LITERACY?

As mentioned above, the complexity involved in making sense of the array of literature read for this study stems from the often differential but sometimes synonymous or interchangeable usage of the terms mathematical literacy, numeracy and/or quantitatively literate to describe, primarily, the relationship between school mathematics and out-of-school contexts (and the usage of that mathematics to make sense of problems encountered in such contexts). As suggested by Van Groenestijn (2003),

> The initial problem we encounter is confusion about the definitions of quantitative literacy, numeracy and mathematical literacy. The three terms originally came from different perspectives but today have the same intention and cover almost the same areas. (p. 229)

Furthermore:

> Although there are differences in wording, these definitions [of numeracy, quantitative literacy and mathematical literacy] have a common intention. All three focus on the competencies of individuals to make sensible use in real-life situations of the mathematics they learned in school. … Hence the three labels – quantitative literacy, numeracy, and mathematical literacy – now can be used more or less interchangeably, at least in English speaking countries.[57]
> (Van Groenestijn, 2003, p. 230)

While I agree with Van Groenestijn's contention that all three include focus on the ability to make sense of real-life situations using mathematics learned in school, I argue that there are important distinctions in the knowledge, content, structure of participation, and areas of focus that different authors have associated with conceptions of each. This sentiment is echoed by L. A. Steen, et al. (2001) who suggests that,

> The capacity to deal effectively with the quantitative aspects of life is referred to by many different names, among them quantitative literacy, numeracy, mathematical literacy, quantitative reasoning, or sometimes just plain "mathematics". Different terms, however, convey different nuances and connotations that are not necessarily interpreted in the same way by all listeners. (p. 6)

In the immediate section below I outline some of the differential connotations and areas of emphasis placed on these terms by certain authors and provide reasons for my privileging of the term mathematical literacy in the context of this study.

[57] Despite this statement, Van Groenestijn (2003, p. 230) goes on to distinguish numeracy, mathematical literacy and quantitative literacy and to suggest a privileging of the term numeracy so as to emphasise the relationship between numeracy and literacy as parallels.

7.1 Differing opinions on the meanings associated with the terms mathematical literacy, numeracy and quantitative literacy

In contrast to Van Groenestijn (2003) statement above, others such as L. A. Steen, et al. (2001), Steenken (2003), Hughes-Hallett (2003), and Kirst (2003) make an explicit distinction between mathematical literacy, on the one hand, and numeracy and quantitative literacy on the other. For these authors, while numeracy and quantitative literacy both describe the type of skills, knowledge and attributes associated with using mathematics to solve problems that are based in and which arise from real-world situations (and, so, can be used interchangeably), mathematical literacy refers to something different – namely, competence with mathematical calculations, knowledge and techniques:

> … typical of the distinction between quantitative literacy, which stresses the use of mathematical and logical tools to solve common problems, and what we might call mathematical literacy, which stresses the traditional tools and vocabulary of mathematics. (L. A. Steen, et al., 2001, p. 17)

> There is, therefore, an important distinction between mathematical and quantitative literacy. A mathematically literate person grasps a large number of mathematical concepts and can use them in mathematical contexts, but may or may not be able to apply them in a wide range of everyday contexts. A quantitatively literate person may know many fewer mathematical concepts, but can apply them widely. (Hughes-Hallett, 2003, p. 92)

> I see a significant chasm between mathematical literacy and quantitative literacy. … On the other hand, the foundations of quantitative literacy lie in mathematical literacy. (Steenken, 2003, p. 184 & 182)

Pugalee (1999, p. 19) further confuses matters. While privileging the term mathematical literacy, he frequently uses the terms mathematical literacy and numeracy interchangeably and argues that the term numeracy is the British equivalent of mathematical literacy.[58]

Then there are those who simply privilege one term over the others, sometimes without reason. Neill (2001, pp. 10-11), for example, uses the term numeracy exclusively and proceeds to situate numeracy as a sub-set of the wider field of mathematics. Niss (2003, p. 216), on the other hand, deliberately, privileges the term mathematical literacy, arguing that "The main reason I prefer mathematical literacy is that the broadness of the term "mathematical" captures better than the somewhat narrower term "quantitative" what we actually seem to be after …". De Lange (2003, 2006) offers agreement with this perspective.

Then there are those who choose to reject the more frequently used terms mathematical literacy, numeracy and/or quantitative literacy and replace them with alternative conceptions. Packer (2003), for example, uses the term 'empirical mathematics' to emphasise the necessary relation of mathematics to a specific empirical instance in contextualised mathematics problem-solving activities. Frankenstein (2009a), on the

[58] L. A. Steen, et al. (2001) similarly argues that numeracy is the British expression for quantitative literacy, but, as has already been highlighted above, he does not attach equivalent meaning to the terms mathematical literacy and quantitative literacy.

other hand, refers to 'criticalmathematical literacy'[59] and explains this conception as follows:

> The main goal of a criticalmathematical literacy is not to understand mathematical concepts better, although that is needed to achieve the goal. Rather, it is to understand how to use mathematical ideas in struggles to make the world better. In other words, the question to be investigated about my criticalmathematical literacy curriculum is not "Do the *real* real-life mathematical word problems make the mathematics more clear?" The key research questions are "Do the *real* real-life mathematical word problems make the social justice issues more clear?" and, "Does that clarity lead to actions for social justice?"
> (Frankenstein, 2009a, pp. 1-2, emphasis in original text)

D'Ambrosio (2003, p. 247) similarly diverges from usage of the conventional terms, arguing that there is an imbalance between focus on quantitative and qualitative goals in mathematics education and that 'Ethnomathematics' provides an alternative approach to the elitism of traditional mathematics, restores cultural dignity, and provides equal opportunity for all to develop their knowledge. Enthomathematics comprises three strands which work together to ensure the development of critical citizenship: Literacy (including verbal, written and numerical literacy); Matheracy (which involves the ability to create hypotheses, prove, justify and draw conclusions); and Technoracy (which involves critical familiarity and competence with technology) (D'Ambrosio, 2003, pp. 236-238).

In summary, the discussion above has highlighted the differential usage and meanings attached to the terms mathematical literacy, numeracy and quantitative literacy throughout the literature. Despite each term containing a shared area of focus on the relationship between mathematics and real-world contexts and applications, different authors use the terms differently to emphasise particular aspects of this relationship.

My own preference is associated with the term mathematical literacy. In the section below I provide reasons for my preferencing and privileging of this term and highlight the particular areas of focus and forms of participation or behaviour that I associated with this term.

7.2 My privileging of the term 'mathematical literacy'

My primary reason for privileging the term mathematical literacy over numeracy and/or quantitative literacy is a pragmatic one: in South Africa there is a subject called Mathematical Literacy and, so, it makes sense to use a term that is recognisable and which has a direct correlation to existing curricular and pedagogic practice.

But, how precisely does my conception of mathematical literacy differ from those for numeracy and quantitative literacy (and even mathematical literacy) described in the literature and outlined above? The remainder of this section is devoted to answering this question.

De Lange (2003, p. 75; 2006, p. 13) points out that many of the definitions and expressions of quantitative literacy and numeracy focus primarily on the numerical or quantitative aspects involved in the application of mathematics. This sentiment is echoed

[59] The lack of a space in the word 'criticalmathematical' is deliberate.

by Niss (2003, p. 215), who argues that variation in interpretations of the term quantitative literacy "is mainly a matter of how narrowly the word 'quantitative' is to be understood, vis à vis the involvement of numbers and numerical data.". L. A. Steen, et al. (2001, p. 6), for example, provides as expression of quantitative literacy as "The capacity to deal effectively with the quantitative aspects of life"; Hughes-Hallett (2003, p. 93) suggests that the "cornerstone of quantitative literacy is the ability to apply quantitative ideas in new or unfamiliar contexts."; and Ginsburg et al. (2006, p. 1) focus on quantitative aspects in motivating for the need for increased numeracy amongst adults: "As quantitative and technical aspects of life become more important, adults need higher levels of numeracy to function effectively in their roles as workers, parents, and citizens."

Both de Lange (2003, 2006) and Niss (2003) argue against this narrowing of focus on primarily quantitative aspects and promote, instead, a conception of mathematical literacy (which is the term they both privilege) that encompasses a broader application of mathematical knowledge. This broader application includes the premise that "All kinds of visualizations belong as well to the literacy aspect of mathematics and constitute an absolutely essential component for literacy …" (de Lange, 2003, p. 76), where 'all kinds of visualizations' includes reading maps, understanding plans (de Lange, 2003, p. 76), and, presumably, also other visual or spatial resources such as assembly diagrams, instructions for appliances, models, and so on.

In promotion and defence of this argument, de Lange (2006, p. 14) identifies and differentiates four different literacies − (i) numeracy, (ii) quantitative literacy, (iii) spatial literacy, and (iv) mathematical literacy, each of which reflect a particular relationship to different phenomenological categories. De Lange (2006, p. 15) illustrates the relationship between these literacies and their connection to particular phenomenological categories in the diagram shown in Figure 11 below:

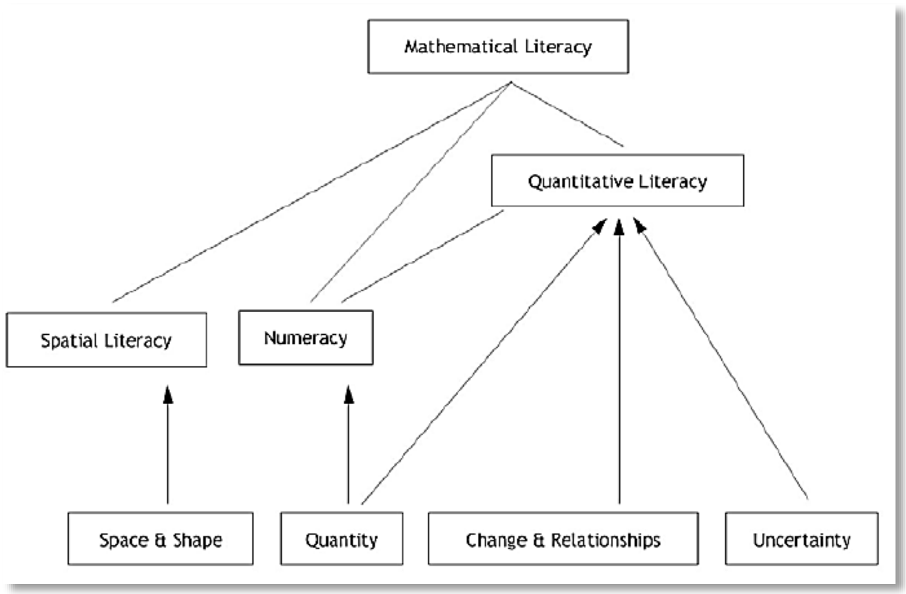

Figure 11: De Lange's (2006) 'Tree structure of mathematical literacy'

As illustrated in the diagram, De Lange views *numeracy* as reflective of the ability to work with and perform calculations involving numbers and data. Numeracy is positioned as a particular sub-set of quantitative literacy. *Quantitative literacy*, by contrast, is seen to involve functionality in a range of categories and, so, includes skills and knowledge associated with more than just numbers and data. *Spatial literacy* is the third literacy and is conceptualised as prioritising the development of skills associated with engagement with spatial objects, including 2-and 3-dimensional representations such as maps, plans, navigational tools, geometric representations, and so on. The final literacy, *Mathematical literacy*, is conceptualised as the overarching literacy comprising all of the other literacies (de Lange, 2003, pp. 80-81; 2006, pp. 14-15).

I find de Lange's distinction between numeracy and quantitative literacy useful and agree with his assertion that mathematical literacy is the overarching category comprising both numeracy and quantitative literary plus the additional spatial literacy. However, I find his association of each literacy type to specific phenomenal categories restrictive. This categorisation suggests that each literacy is determined by the types of contexts or phenomenon that a person is exposed to and is able to show functionality or competence in, rather than the skills and knowledge that they are able to demonstrate in solving problems based in real-life contexts. By contrast, I contend that mathematically literate behaviour is characterised by the ability to decide on and employ appropriate strategies to solve any problem irrespective of the context or structure of the problem. Furthermore, I envision that such behaviour is not defined or bounded by the particular phenomenological categories in which the problems are situated or the specific mathematical skills employed to solve those problems. A further reservation of de Lange's 'Tree structure of mathematical literacy' schematic relates to my view that there are other literacies which do not appear in this diagram but which are a central component of mathematically literate behaviour and, as such, need to be considered in any conception of mathematical literacy. These additional literacies are elaborated on in the immediate discussion below.

As such, and in contrast to de Lange, I suggest an alternative conceptualisation of the distinction between numeracy and quantitative literacy that does not rely on an association with phenomenal categories. Namely, that numeracy involves competence with basic mathematical content, calculations, techniques and knowledge, not limited specifically to numbers and quantity but encompassing all mathematical contents; while quantitative literacy involves the *functional use* of such content, calculations, techniques and knowledge in making sense of problems grounded in real-world situations. Numeracy is thus seen as a pre-requisite for quantitative literacy. The third literacy – that of spatial literacy − remains in line with de Lange's conceptualisation of the term, involving understanding and competence with spatial representations and objects such as maps, plans, 2-and 3-dimesional views of objects, techniques for estimating distances, and so on. Importantly and distinct from de Lange's conceptualisation, quantitative literacy (and so also numeracy) is seen as a perquisite for spatial literacy: for example, the ability to use a plan effectively relies on the ability to measure accurately and to perform necessary conversions and calculations.

I content that three further literacies need be added to this existing categorisation of literacies that comprise the overarching framework of mathematical literacy. To begin with, given the inherently text (both spoken and written) rich nature of contextually based problem-solving interactions, a key literacy required in contextual sense-making practices is that of *text literacy*. Text literacy reflects an individual's capacity for making sense of messages and information conveyed through written and/or oral text. The conception of text literacy as employed in this study is seen to comprise a combination of both prose literacy and document literacy. Given that 'prose' refers to a language form that exhibits a grammatical structure, prose literacy involves "the knowledge and skills needed to understand and use information organised in sentence and paragraph formats" (Kirsch et al., 1998, p. 113). In the context of mathematical literacy related practices, this involves the capacity to interpret and understand information and instructions encountered in textual and language format that specifies facets of the contextual situation under investigation and/or the specific problem-solving requirements required in relation to that situation. In addition to the overwhelming amount of information communicated in prose format, information is also communicated extensively through the use of documents that are organised in 'matrix structures' (i.e. with a clearly identifiable row and column structure) (Mosenthal & Kirsch, 1998, p. 641). Document literacy, then, involves the ability to make sense of information presented in and through tables, signs, indexes, lists, schedules, charts, graphs, maps, and forms (Kirsch & Mosenthal, 1988, p. 2). This form of literacy is particular essential in a format of mathematical literacy oriented towards life-preparedness since, as argued by Kirsch et al. (1998, p. 118), while prose literacy is the dominant form of literacy in schools, documents tend to be the principal form of communication in out-of-school settings. Taken together, prose and document literacy facilitate successful interpretation of and engagement with both quantitative and other information presented in textual formats, including financial documents (bills, invoices, tickets, quotations, payslips), newspaper articles, adverts, tables, graphs, timetables, brochures, and so on. Importantly, within the framework of mathematical literacy textual literacy is seen to interact in an intricate and intertwined way with the other literacies such that successful and enhanced engagement with encountered information is facilitated through a combination of both quantitative and textual literacy knowledge and skills. For example, in making sense of financial documents (document literacy), elements of prose literacy and quantitative skills are employed (e.g. understanding of the meaning of the terminology 'tax' and checking that the tax value on an invoice has been correctly determined); and in working with maps (spatial literacy), both document literacy (e.g. making sense of a distance table) and quantitative skills (e.g. estimating travelling times) may be required.

The next literacy for consideration is that of *statistical literacy*. Statistics pervade real-world practices, and the ability to function effectively in everyday life requires the capacity for critical engagement with encountered statistical information. However, and as illustrated and argued in detail by Gal (2002), statistical literacy is a separate literacy from numeracy (and from quantitative and spatial literacy, and also from the domain of scientific mathematics) and does not develop in a sustainable form through engagement with these literacies. Instead, statistical literacy develops through a complex interaction of knowledge bases (including literacy skills, statistical knowledge, mathematical knowledge, knowledge of context, and skills and knowledge that facilitate critical analysis of statistical information) and dispositional elements (including beliefs, attitudes, and a critical stance). Where the knowledge bases facilitate interpretation and understanding of statistical information, the dispositional elements highlight a necessary and crucial inclination on the part of the individual to active the knowledge bases in the process of critical engagement with statistical contents (Gal, 2002, p. 4). The conception of mathematical literacy adopted in this study, then, shares this perspective that statistical literacy is distinct from numeracy, quantitative literacy and spatial literacy, and, consequently, requires a unique and dedicated site of development. That said, the interconnectedness of these various literacies, together with textual literacy, is again acknowledged and emphasised: successful engagement with statistical information is reliant on an individuals' ability to interpret and understand presented information (in both prose and document format) and employ a variety of quantitative techniques that facilitate critical analysis.[60]

[60] In similar vein to the conception of statistical literacy offered by Gal (2002), *scientific literacy* is another form of literacy that engages mathematical knowledge and structures and, yet, which exists outside of the domain of disciplinary mathematics. As with statistical literacy and also with the

The final literacy to be considered as an essential component of mathematical literacy – what I have termed *real-world literacy*. This component comprises considerations that directly affect functionality in real-world situations − considerations that are often not of a mathematical nature, which commonly override mathematical considerations, which impact on how people think, act and communicate in real-world situations, and which influence decision-making processes. For example, when buying a house there are many considerations other than cost which affect the type of house bought, including, amongst others, the location, features and condition of the house (and even the attitude and personality of the estate agent). These are real-world considerations that are context-specific, and the rules for making sense of these considerations are often only experienced or learned in the specific context in which they are encountered. These considerations work together with the mathematical considerations to allow for informed decision-making practices.

In the context of mathematical literacy related practices, numeracy, quantitative literacy, spatial literacy, statistical literacy and textual literacy all comprise elements of mathematics or have the potential to facilitate mathematical investigation.[61] As such, I refer to these as comprising the *mathematically informed practices* of mathematical literacy. By contrast, real-world literacy exists completely in the domain of the contextual.

The diagram in Figure 12 below – as an adapted version of de Lange's (2006) 'Tree structure of mathematical literacy' schematic − illustrates my view on the relationship between the various literacies that make up the overarching category of mathematical literacy.

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conception of mathematical literacy adopted in this study, both context and mathematical knowledge are required to successfully engage in the practices of science (Watson & Callingham, 2003, p. 5), and yet neither of these terrains alone define the structure of legitimate participation in the discipline. Instead, it is through a complex interaction of particular contents, knowledge, and skills drawn from both of these terrains, together with specialised scientific knowledge, and with beliefs, attitudes and a critical perspective, which facilitates the structure of legitimate participation. See (Holbrook & Rannikmae, 2009, pp. 276-277) for a discussion of different components and attributes associated with scientific literacy, and (Shamos, 1995) for a discussion of different levels of scientific literacy.

[61] Note that, in classifying numeracy, quantitative literacy, spatial literacy and document literacy as comprising predominantly 'mathematically informed practices', it is not my intention to deny the presence of contextual elements in problem-solving scenarios involving these literacies or to deny the necessity for engagement with and understanding of these contextual elements for successful completion of problem-solving processes. Rather, in the context of the specific domain of mathematical literacy, I see these literacies as comprising both mathematical and contextual elements and, as such, that practices accessed through these literacies facilitate mathematical forms of engagement with problems and resources encountered. In each of these literacies there is potential for engagement with mathematical structures and elements, and for understanding how mathematical forms of participation facilitate a particular type of understanding of a problem scenario. By contrast, real-world literacy places emphasis on understanding contextual (and commonly non-mathematical) factors that affect action and behaviour in a contextual situation.

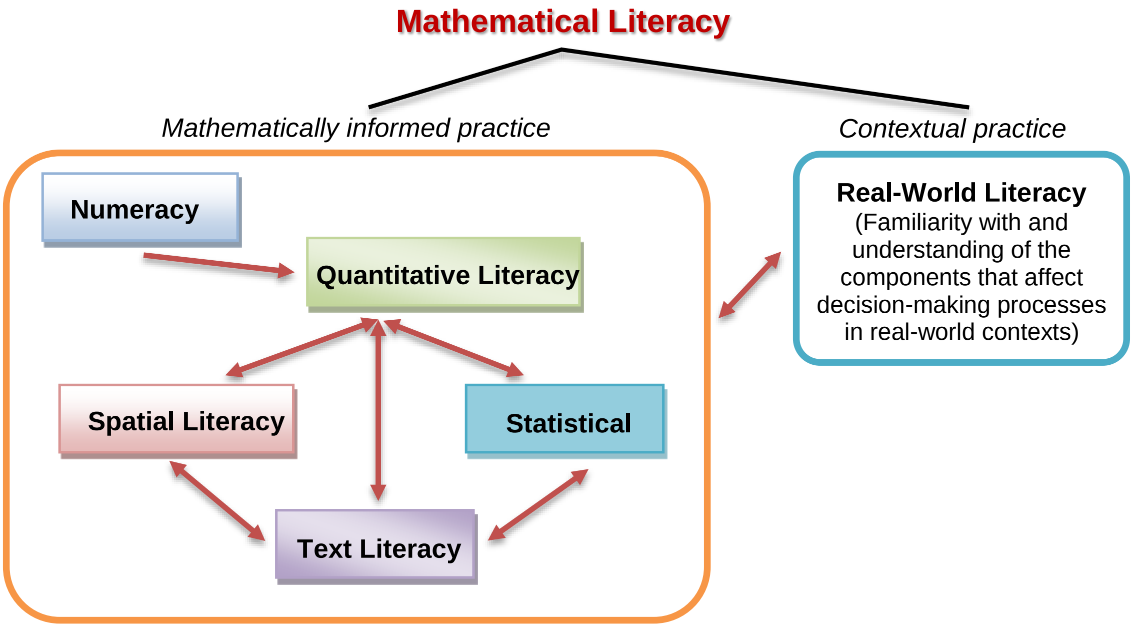

Figure 12: The literacies that characterise mathematical literacy

As a final comment, it is crucial to emphasis the 'literacy' component of the conception of mathematical literacy as described above. The conception of mathematical literacy adopted in this study is perceived to comprise a collection of various literacies such that being mathematically literate is akin to showing proficiency in all of these domains. Proficiency in this context is not to be equated to a minimal level of engagement or a set of basic skills – to the ability to only 'read and write' the contents of the domains. Instead, proficiency is to be equated with full functional competence in the domain, including the ability to interpret and understand the contents of the domain, to engage critically with these contents, and to communicate these contents effectively and in an accessible format. Proficiency is to be equated with functional engagement – specifically, with the use of the contents of the domain to engage and solve problems, and to challenge and critique existing structures. Being mathematically literate involves the capacity to interact with complex real-world scenarios, to engage confidently with the language and resources employed in these scenarios, to employ statistical, quantitative and spatial tools to investigate these resources, and to communicate opinions and results in a critical way through a variety of mediums.

This, then, concludes the discussion on my privileging of the term mathematical literacy and my views on various literacies that characterise mathematical literacy. In the next chapter (Chapter 8) discussion now shifts to a focus on the South African school subject Mathematical Literacy and the structure, intention, and dominant orientation of the conception of mathematically literate behaviour prioritised in the subject.

CHAPTER 8
MATHEMATICAL LITERACY IN SOUTH AFRICA

In this chapter I provide a discussion of the dominant structure of participation and knowledge prioritised in past and current practices in the South African subject-matter domain of Mathematical Literacy. Comparison of this dominant structure is also made to international conceptions of mathematical literacy, numeracy and/or quantitative literacy. A key point that I make in this chapter is that curriculum intentions prioritise forms of participation in the subject characterised by an emphasis on agendas for applications, modelling and contextual sense-making practices. However, it is a dominant and near exclusive emphasis on numeracy in context type practices that characterise the national assessments for the subject which define and dictate the structure of legitimate participation in the subject. As a result, successful participation in the subject Mathematical Literacy is dependent on a degree of mathematical competency and not on understanding of real-world practices.

8.1 Introduction: the unique South African situation

The discussion so far has concentrated on descriptions of mathematical literacy, numeracy and quantitative literacy that frame the terms with respect to a specific type of ability, competence or behaviour. So, to be mathematically literate, numerate or quantitatively literate implies the ability to behave in a particular way, or to exhibit certain traits or skills (Bowie & Frith, 2006, p. 30; Christiansen, 2006, p. 6). Central to this behaviour is the ability to use mathematics to make sense of situations and solve problems based in real-world contexts.

The situation in South Africa is somewhat unique and different in that mathematical literacy is framed as a secondary school *subject* (called Mathematical Literacy) that is available for learners in the Further Education and Training Band (FET) (i.e. Grades 10, 11 and 12) rather than as a behaviour, ability or competence.
As such, not only do the curriculum documents[62] for this 'subject' provide a particular vision of what it means to be mathematically literate[63] through a definition and statement of purpose for the subject. Instead, they also outline an explicit statement of the teaching and learning curriculum (including mathematical content and appropriate contexts), stipulations regarding assessment, a description of the taxonomy levels that determine cognitive demand in the content, indicators of progression, and descriptors of the competencies that learners must be able to exhibit at each Grade (as evidence of the development of their mathematically literate behaviour).

This embodiment of the notion of mathematically literate behaviour in an independent fully fledged school-based subject positions the South African experience of mathematical literacy as distinctly different to many international perspectives. Many of the international perspectives described above emphasise that mathematical literacy (or numeracy or quantitative literacy) is the responsibility of every teacher in every subject and that mathematically literate behaviour must be developed in an interdisciplinary fashion and beyond the exclusive domain of the mathematics classroom. In the South

[62] (DoE, 2003a, 2008d, 2009c); (DBE, 2011a)
[63] For the remainder of this and further discussions in this study, the subject Mathematical Literacy is referenced using title capitalisation, while the competence or behaviour of being mathematically literate is referenced using a lowercase title.

African context, by contrast, the primary responsibility for the development of mathematically literate behaviour rests with the teachers of the *subject* Mathematical Literacy. Furthermore, the curriculum and contexts that learners are exposed in South African schools are determined not by the demands, relevance and applicability of specific subject areas, but, rather, by the authors of the curriculum documents for the subject and, in turn, by the textbook authors and assessment specialists who interpret those curriculum documents. Mathematical Literacy in South Africa then, is a school subject with its own curriculum and is assessed as a fully-fledged discipline independent of any other subject. It is considered to be an equivalent qualification to every other full Grade 12 subject, including Core Mathematics (which is the South African equivalent of a scientific mathematics course).

8.2 A brief history of the secondary school subject Mathematical Literacy in South Africa

The diagram shown in Figure 13 is provided to orient the reader to the contents of the discussion in the pages below. This diagram shows a timeline of the various curriculum changes and associated documents associated with the subject Mathematical Literacy since the inception of the subject in the South African curriculum framework in 2003.

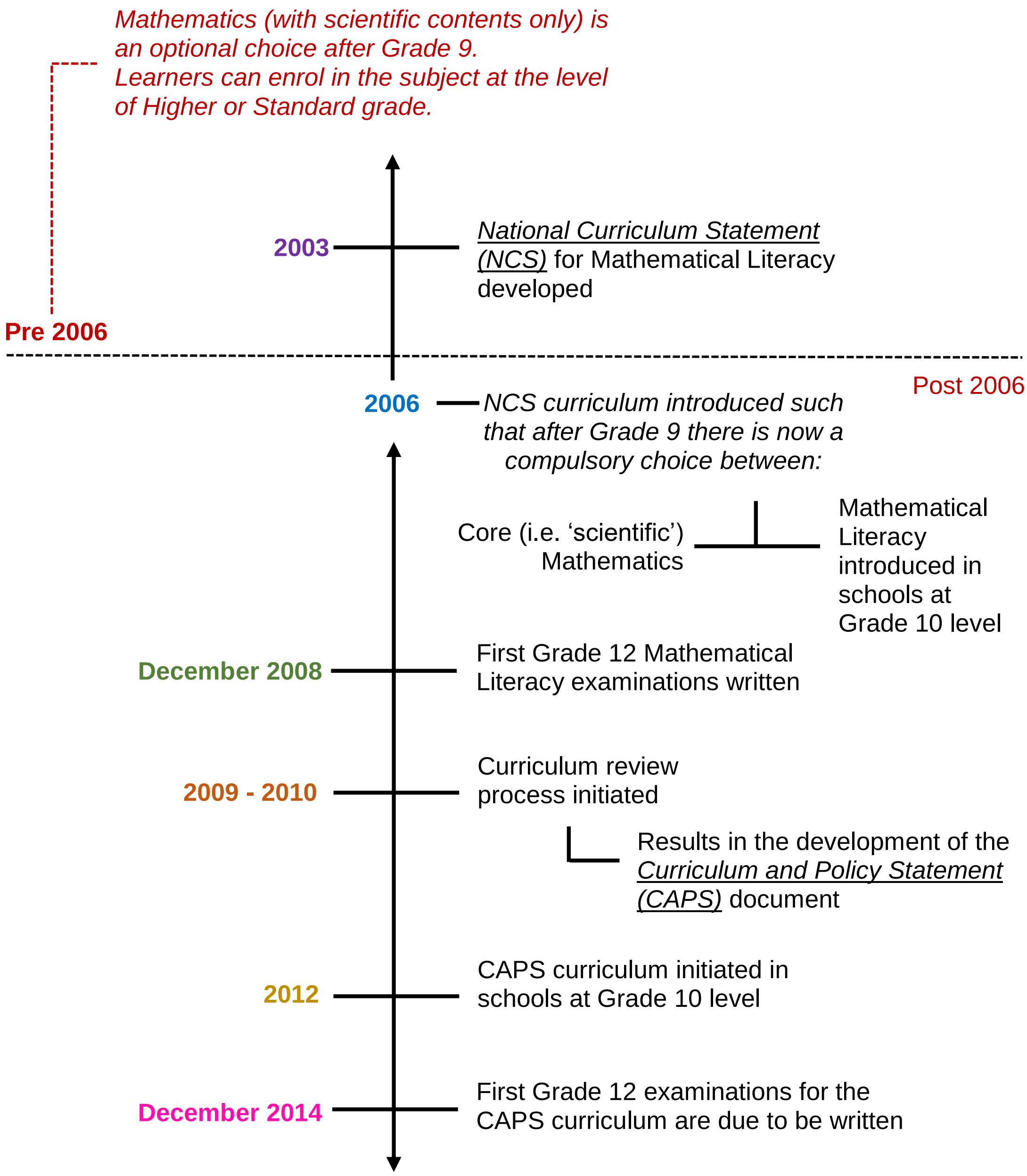

Figure 13: Timeline of curriculum change for the subject Mathematical Literacy

The subject Mathematical Literacy is a relatively new phenomenon in South Africa, having only been initiated as a component of the National Curriculum Statement (NCS)[64] (DoE, 2003a) at Grade 10 level in 2006. This cohort of Grade 10 learners became the first group to write a formal Mathematical Literacy examination at Grade 12 level two years later in 2008 – to apparent great success: a national pass rate of 78,7% (DoE, 2008a, p. 12). This pass rate is particularly impressive when compared with the pass rate of 44,5% for the subject Core Mathematics in the same year (CDE, 2010, p. 2), which is the alternative subject choice to Mathematical Literacy.[65]

[64] The NCS stipulates policy on curriculum and assessment in the schooling sector in South Africa and, as such, stipulates policy with respect to the 'what' and the 'how' of learning and teaching for all grades from Grade R to 12. Each subject in the NCS has a separate curriculum statement outlining specific curricular and assessment stipulations for the subject (DBE, 2011a, p. 3).

[65] A mark of 30% is required to pass in a subject. As such, the 'pass rate' for a subject refers to the number of learners who achieved 30% or more in that subject.

Prior to the introduction of Mathematical Literacy as a school subject in 2006, the only mathematically oriented subject available was a scientific mathematics course titled Mathematics. Mathematics was compulsory for all learners up to the end of Grade 9 and became an optional choice in Grades 10, 11 and 12. Learners could choose Mathematics at either the Higher or Standard Grade levels, with each level providing differentiated access to courses of study at tertiary institutions. The successes of this system were minimal: approximately 40% of the learners who completed Grade 12 had dropped Mathematics at the end of Grade 9 and, so, exited the schooling system with only a minimal and basic level of mathematical competence. Of the remaining 60% of the learners who opted for Mathematics, 50% of these were entered for the Standard Grade examination and under 9% for the Higher Grade examination, with pass rates for both Higher and Standard Grades amounting to 50% of those enrolled (all figures quoted in (Venkatakrishnan & Graven, 2006, p. 15)). In other words, prior to 2006, only 5% of all learners enrolled in Grade 12 successfully completed Mathematics on the Higher Grade.

The introduction of the subject Mathematical Literacy as a school subject in 2006 and the requirement that learners opt for either Mathematical Literacy or Core Mathematics (the equivalent[66] of the previous subject Mathematics) was, thus, a deliberate attempt to rectify this situation in which large numbers of learners exited the schooling system with minimal mathematical knowledge. Mathematics or Mathematical Literacy is now compulsory for all learners across all grades. Current enrolment figures (i.e. from the 2013 November national examinations) posit approximately 54% of Grade 12 learners writing the Mathematical Literacy examinations and the remainder participating in Core Mathematics (DBE, 2014a, p. 159 & 125).

The status and importance of Mathematical Literacy as a school subject has been enhanced by the fact that both Mathematical Literacy and, more especially, Core Mathematics hold an important place as ‘gateway’ subjects − namely, as subjects “for which learner performance is assessed for entry to tertiary institutions.” (UMALUSI, 2009, p. 14). In other words, high marks in Mathematical Literacy and Core Mathematics are pre-requisites for entrance into many of the courses offered at higher education institutions.

Despite the relative newness of the subject, Mathematical Literacy has undergone a ‘facelift’ in recent years with the development and implementation of the Curriculum and Policy Statement (CAPS) curriculum[67] (DBE, 2011a). This ‘revised’ curriculum was implemented in Grades 10, 11 and 12 in January 2012, 2013 and 2014 respectively. These CAPS documents have been introduced to replace the NCS curriculum and the documents developed to support that curriculum[68] (DBE, 2011a, p. 3). My understanding is that the CAPS documents were based on a two-fold intention. Firstly, to consolidate the various and many curriculum and supporting documents for each subject into a single document per subject to enable teachers to more easily come to grips with the curricular and assessment requirements for the subjects that they teach. And, secondly, to unpack the existing statements of curriculum in the NCS documents

[66] The word ‘equivalent’ is being used here to signify equivalence in terms of intended focus in the subject – namely, the learning of *scientific* mathematical content – rather than equivalence of curricular or cognitive demand or difficulty.
[67] For the sake of transparency, I need to point out that I was the primary author of the Mathematical Literacy CAPS document.
[68] For Mathematical Literacy, these supporting documents include: *Subject Assessment Guidelines* (DoE, 2005b, 2007, 2008d); *Learning Programme Guidelines* (DoE, 2005a); *Teacher Guide* (DoE, 2006); and the *Examination Guidelines* (DoE, 2008c; 2009c).

in order to define in a more explicitly way the content that must be taught. This second point is achieved by doing away with the Learning Outcomes and Assessment Standards contained within the NCS documents and replacing those outcomes and standards with explicit statements of content organised around content *topics*. In simplistic terms, the CAPS documents are designed to stipulate for teachers in precise terms what must be taught, how it must be taught, and the means and structures through which the taught work is to be assessed; there is to be no guess-work at all on the part of the teachers as to the teachable and examinable curriculum for each subject.

Importantly, and as per the brief given to me in the development process of the Mathematical Literacy CAPS document, the curriculum outlined in the CAPS documents is not a new curriculum. Rather, the CAPS curriculum draws from and, in some cases and for some subjects, builds or expands on the curriculum outlined in the NCS documents. This is reflected in the Mathematical Literacy CAPS document in the fact that every curriculum topic and section in the CAPS curriculum has a point of reference in a specific learning outcome and assessment standard in the Mathematical Literacy NCS document. And, although certain content topics from the NCS have been excluded from the Mathematical Literacy CAPS document[69], no completely new topics have been introduced.

This, then, concludes the brief sojourn into the history of the subject-matter domain of Mathematical Literacy. In the next section (Section 8.3), the discussion shifts to a focus on identifying the particular agenda and statement of intention for the subject expressed in the curriculum documents, and the implications of this dominant agenda and intention for pedagogic practices in the subject.

8.3 Definitions, statements of intention and curricular agendas for the subject Mathematical Literacy[70]

In this section I outline the particular intention and agenda for the South African conception of the notion of mathematically literate behaviour espoused and prioritised in the definitions and statements of purpose contained in both the NCS and CAPS related documents for the subject-matter domain of Mathematical Literacy. In this regard, the discussion is divided into three sub-sections. In the first sub-section I discuss the

[69] Not long after the Mathematical Literacy NCS curriculum document was implemented, a series of Subject Assessment Guidelines (DoE, 2005b; 2007, 2008d) were developed to specify the core assessable curriculum in the subject. The 'Core Assessment Standards' contained in the Subject Assessment Guidelines excluded certain content topics listed in the NCS that were deemed to be too complex for teachers and/or learners to engage with. This included the content topics of trigonometry, basic linear programming, scientific notation, transformation geometry, angles, financial indices, cumulative frequencies and ogive curves, and standard deviation and variance. Many of these same topics were the topics that were excluded from the CAPS curriculum.

[70] As was discussed above, despite the subject Mathematical Literacy only having been introduced in 2006 as part of the National Curriculum Statement (NCS) framework (DoE, 2003a), the subject was exposed to a curriculum reform process through the Curriculum and Assessment Policy Statement (CAPS) framework (DBE, 2011a) in 2012. At the time of the initial writing of this part of the study, the NCS framework was in operation and the CAPS framework was still in development − as such, much of the discussion in the initial draft of this chapter reflected the structure of legitimised participation in the NCS framework. However, at the time of finalising this study, the NCS framework has now been replaced entirely by the CAPS framework. For this reason it has been necessary to update the chapter by also including commentary on the current structure of legitimised participation in the CAPS conception of the subject. My decision to provide analysis of two different curriculum frameworks is, thus, necessitated by the timing of my study in relation to curriculum change and reform in the country, and not by an aversion to trees or affection for printer cartridges.

definition, statement of purpose or intention and consequent philosophy and agenda of the NCS conception of Mathematical Literacy. In the second sub-section I provide a review of the literature that problematises this curriculum. In the third sub-section I analyse the statement of intention, philosophy and dominant agenda of the CAPS conception of Mathematical Literacy, both in relation to the NCS agenda and philosophy and to areas of concern raised about the NCS.

8.3.1 Definitions, statements of purpose and dominant intention(s) and agenda(s) in the NCS conception of the subject Mathematical Literacy

The NCS document for the subject Mathematical Literacy provides the following definition:

> Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems. (DoE, 2003a, p. 9)

A statement of purpose is also provided, from which the following extract is drawn:

> The inclusion of Mathematical Literacy as a fundamental subject in the Further Education and Training curriculum will ensure that our citizens of the future are highly numerate consumers of mathematics. In the teaching and learning of Mathematical Literacy, learners will be provided with opportunities to engage with real-life problems in different contexts, and so to consolidate and extend basic mathematical skills. Thus, Mathematical Literacy will result in the ability to understand mathematical terminology and to make sense of numerical and spatial information communicated in tables, graphs, diagrams and texts. Furthermore, Mathematical Literacy will develop the use of basic mathematical skills in critically analysing situations and creatively solving everyday problems. (DoE, 2003a, p. 9)

The statement of purpose further specifies instances, relating to the contextual categories of everyday life, the workplace and democratic society, where individuals are required to make use of mathematical skills to solve problems and inform decisions (DoE, 2003a, pp. 9-10).

My intention, now, is to use the same categories that were used earlier in this study (see sub-section 5.2.3 on page 49 above, or footnote 71 below for ease of reference)[71] to decompose this definition and statement of purpose, together with other statements that prioritise particular forms of knowledge and participation in the subject-matter domain of Mathematical Literacy. This helps to identify key areas of intention in the subject in relation and comparison to international perspectives.

8.3.1.1 Category 1 – Considerations of mathematics

Both the definition and statement of purpose make explicit and continuous (i) reference to mathematics and, particularly, to (ii) the use value of mathematics for making sense of the everyday world. The subject Mathematical Literacy, then, is tasked not so much with teaching mathematics as with developing the ability to *apply* mathematical content to solve problems. This emphasis on application is consistent with international perspectives on mathematical literacy, numeracy and/or quantitative literacy.

Since Mathematical Literacy is a subject, the NCS for Mathematical Literacy clearly outlines (iii) strands of mathematical content and organises those strands according to Learning Outcomes: Numbers and operations in context; Functional relationships; Space, shape and measurement; and Data Handling. These strands or groupings of content are largely consistent with the dominant phenomenological strands which are emphasised throughout much of the international literature.

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Category	Sub-category		
Considerations of mathematics	(i) Reference to mathematical content		
	(ii) Reference to the 'use' or 'application' of mathematical content/knowledge/techniques		
	(iii) Strands of mathematical content		
	(iv) Scope of mathematical content		
	(v) Recognition of contextual forms of participation and less formal mathematical techniques and/or non-mathematical (situational) techniques		
	(vi) Distinguished from scientific mathematics		
Interplay of content, contexts and/or competencies	(i) Perceived role of content, contexts and/or competencies		
	(ii) Specified skills, competencies and/or traits		
Arena of application	(i) Location		
	(ii) Context		
Components	Specified components and/or features associated with courses, assessments and/or pedagogic practice		
Orientation	Intention (i.e. ML for)	(i) Human Capital	(ii) Cultural Identity
		(iii) Environmental Awareness	(iv) Social Change
		(iv) Evaluating Structures	
	Agenda	Agenda 1: Mathematical Competence	Literacy in Mathematics
			Numeracy
		Agenda 2: Mathematics in context	Application
			Numeracy-in-Context
		Agenda 3: Modelling	
		Agenda 4: Contextual Sense-Making Practices	

As regards (iv) the scope of mathematical content prescribed for the subject, the statement of purpose restricts mathematical content to 'basic mathematical skills', but no further detailed explanation is provided in either the NCS or in any supporting documents as to what constitutes 'basic mathematics' and/or the scope of this basic mathematics. The only hint provided is contained in the following statement in the NCS (DoE, 2003a):

> The learning achieved in Mathematics in the General Education and Training band provides a base from which to proceed to the demands of Mathematical Literacy in the Further Education and Training band. The essentials of numeracy are taken further by working in contexts which become increasingly relevant. (p. 11)

This statement suggests that primary focus in Mathematical Literacy is on the application of the mathematical content learned in the GET phase (Grades 7, 8 and 9) in making sense of increasingly complex contexts, rather than exposure to increasingly more difficult and abstract content. This notion is supported in the Learning Programme Guidelines for Mathematical Literacy (DoE, 2005a, p. 10) which states that "The mathematical knowledge developed in Grades R-9 is revisited and embedded in authentic contexts." In other words, primary focus should be on application and problem-solving rather than on acquisition of new mathematical knowledge. Hence the confusion at the inclusion of such topics as financial indices, transformation geometry, trigonometry (including sine and cosine rules), standard deviation and variance, as well as reference to the quadratic formula[72]. These distinctly 'scientific' topics were subsequently excluded[73] from what came to be titled the 'Core Assessment Standards' (DoE, 2005b, 2007, 2008c, 2008d, 2009c), which specified the *examinable curriculum* for Grade 12 in 2008. The consequence of this exclusionary process is that the scope of the mathematical content component in the assessment component of the NCS curriculum framework for the subject equates to no more than Grade 9 level mathematics. There is little or no expectation for engagement with scientific content except where such content has a direct application to a common problem encountered frequently in everyday situations. The restructuring of the NCS curriculum in the Core Assessment Standards, thus, seemed to resolve the initial mismatch between the statement of purpose and the curriculum content component for or of the subject.

Other than mention of the need for "estimating efficiently" (DoE, 2003a, p. 15, c.f. Assessment Standard 11.1.1), there is no explicit statement in the NCS or in other supporting documents of an expectation for the (v) recognition of contextual forms of participation and less formal mathematical techniques and/or non-mathematical (situational) techniques. In my opinion, this suggests prioritisation of a dominant agenda for ensuring the correct usage of appropriate mathematical techniques, structures and

[72] A word of explanation is necessary here. Reference to the quadratic formula occurs as part of an *example* to illustrate a statement in Assessment Standard 11.1.1 which refers to the skills that must be developed to enable learners to be able to perform calculations in an appropriate sequence in a complex equation or formulae (DoE, 2003a, p. 15). My understanding and interpretation is that the quadratic equation was listed as a possible example of a complex formula due to the many and varied calculations required within the equation (squaring; multiplication; subtraction; square rooting; division; and understanding of the symbol "±"). This statement was seemingly not listed as a statement of intention for learners to engage with this formula in the process of working with parabolas. Nonetheless, this latter interpretation is certainly how the statement was interpreted by many textbook authors and teachers.

[73] The intention was that the entire curriculum would be phased in over a three year period. As such, only certain 'Core Assessment Standards' would be included in the first Grade 12 examination in 2008. Thereafter, the assessment standards that had been excluded from these Core Standards would be reintroduced on a year by year basis. This has never happened and the more recent CAPS document has continued with this trend by deliberately excluding the distinctly 'scientific' statements and content referenced in the NCS.

knowledge forms in solving problems, rather than for coming to understanding existing and possible alternative forms participation in contextual situations. From my personal interactions with teachers and subject advisors, this observation seems to be playing out in precisely this way in pedagogic practices in the classroom, in the way in which Grade 12 Mathematical Literacy examinations are structured, and in areas of prioritising and focus in the marking of these examinations. This apparent prioritising of mathematical techniques and denial of contextual, less formal mathematical and/or non-mathematical forms of knowledge and participation is largely consistent with international perspectives on mathematical literacy, numeracy and/or quantitative literacy.

On the issue of (vi) differentiation to scientific mathematics – or Core Mathematics as it is called in South Africa – no specific statement is provided in the NCS for Mathematical Literacy. This is somewhat understandable in that Mathematical Literacy and Core Mathematics have been established as separate subjects and, so, the differentiation between these subjects is found in the statements of content, methodology and purpose contained in the curriculum statements for each subject. This is an important point because, as separate subjects, Mathematical Literacy and Core Mathematics were never designed to allow “articulation and portability” (Venkatakrishnan & Graven, 2006, p. 24) between the subjects. Rather, the subjects were designed with differing intentions, agendas, curricular goals and contents, and with differing criteria according to which legitimate participation with the subject-related contents is endorsed. Brombacher (2006) validates this observation by suggesting that “Mathematical literacy is different from mathematics not in level or complexity but rather in kind and purpose.” (Cited in Venkatakrishnan & Graven, 2006, p. 23).

However, the Mathematical Literacy NCS does offer a suggestion of differentiation to Core Mathematics based on the issue of career choice. Namely:

> Mathematical Literacy should not be taken by those learners who intend to study disciplines which are mathematically based, such as the natural sciences or engineering. (DoE, 2003a, p. 11)

A corresponding statement can be found in the NCS for Core Mathematics:

> If a learner does not perceive Mathematics to be necessary for the career path or study direction chosen, the learner will be required to take Mathematical Literacy. (DoE, 2003b, p. 11)

At a school level, then, the choice between Mathematical Literacy and Core Mathematics should be based on a decision regarding career choice and, hence, field of study at tertiary level or field of work when exiting school.

A reading of the curriculum statements for the two subjects immediately reveals core differences in kind and purpose for the two subjects and a possible rationale for the espoused distinction in the criteria for legitimate participation based on career choice. At a level of intended purpose[74], legitimate participation in the subject-matter domain of Mathematical Literacy is focused on engagement with, primarily, elementary mathematical content in solving problems related to increasingly more complex real-life contexts. As such, there is very little increase in the level of abstraction of the content dealt with and, rather, the same or similar content is used repeatedly in different contexts. Importantly, the learning of the mathematical content is not the end goal; rather, the end goal is the use of mathematical content to solve problems situated in real-world contexts. Most of the contexts that are specified for engagement relate to aspects of daily life that may be of relevance to the learners currently or at some point in the future. In Core Mathematics, by contrast, a key agenda at the level of intended purpose is exposure to mathematical content characterised by increasing levels of abstraction and also by emphasis on engagement with the content without necessarily understanding appropriate contexts of application of the content. Furthermore, much of the content contained in the Core Mathematics curriculum has an explicit scientific base. This distinction between the differential nature of the content dealt with in the two subjects is illustrated in the organisation of the Mathematical Literacy curriculum around the content strands of Numbers, Functional Relationships, Space, Shape and Measurement, and Data Handling, rather than the traditional content categories of Algebra, Geometry, and Trigonometry – which, together with the inclusion of Data Handling and Finance, form the structure of the Core Mathematics curriculum.

Another key difference between the two subject-matter domains as described in their respective curriculum documents relates to the type of problem-solving practices that characterise legitimate participation in each domain. In terms of the distinction made on page 40 above between modelling and applications, the NCS document for Mathematical Literacy posits problem-solving practices that are reflective of *modelling* processes as a key component of the subject. These modelling processes are directed towards identification and utilisation of a variety of mathematical contents that facilitate the generation of descriptions (albeit mathematical descriptions) of everyday practices. In Core Mathematics, by contrast, problem-solving processes are prioritised that are intended to illustrate "connections between Mathematics as a discipline and the application of Mathematics in real-world contexts." (DoE, 2003b, p. 10). These processes involve the imposition of scientific mathematical contents in real-world contexts and are, hereby, reflective of the processes involved in mathematical *applications*. Furthermore, where modelling processes are employed in Core Mathematics, many such models are a distinctly intra-mathematical nature: namely, models are generated to facilitate descriptions of scientific mathematics contents. In short, in Mathematical Literacy there is an expectation for participants to use mathematics to describe real-world situations, while in Core Mathematics there is an expectation for participants to come to understand how scientific mathematics contents can be applied in a variety of both intra and extra-mathematical.

The information in Table 3 on the page below shows a concise summary of some of the key differences between Mathematical Literacy and Core Mathematics in South Africa.

[74] I am deliberately restricting the comparison to the level of 'intended purpose' as stated in the curriculum statements. In my opinion, the reality at the level of the classroom is very different, particularly for Mathematical Literacy: what is being taught in many Mathematical Literacy classrooms is a form of mathematical competency with basic mathematical content (i.e. basic 'numeracy' (as per my description of the concept provided earlier)) rather than problem-solving, application and/or contextual sense-making practices. This issue is discussed in more detail in sub-section 8.3.2 on page 122 below.

It is worth mentioning that the comparison shown in this table bears a great deal of resemblance to the differentiation made between components of mathematical literacy, numeracy and/or quantitative literacy and scientific mathematics in analysis of the international literature (c.f. Table 2 on page 62 above).

Table 3: Comparison of Mathematical Literacy and Core Mathematics in South Africa

Mathematical Literacy	Core Mathematics
• Elementary (basic) mathematical content. • Problem-solving in real-world contexts related to daily life experiences. • Mathematical content should always be used in a real-world context. • Problem-solving processes are characterised by modelling processes used to generate descriptions of everyday contexts.	• Scientific mathematical content that increases in complexity and level of abstraction. • Mathematical content is learned often without consideration of the practical application of the content. • Problem-solving processes are characterised by the mathematical applications to intra and extra-mathematical contents.

Despite the construction of Mathematical Literacy as a separate subject to Core Mathematics, the positioning of the two subjects as compulsory alternative choices in the schooling system has prompted an inevitable comparison and measuring of the two subjects against each other.

Mathematical Literacy is, thus, seen as the appropriate subject choice for learners who cannot cope with the demands of Core Mathematics[75] (rather than the appropriate subject choice for those wishing to pursue a particular career). Learners are able to change from Core Mathematics to Mathematical Literacy at any stage during the FET phase (despite the system not being designed to allow articulation and portability and the fact that the Mathematical Literacy qualification is a three-year qualification). Since this allowed movement is unidirectional and a move from Mathematical Literacy to Core Mathematics is seen as virtually impossible[76], this sends a clear message regarding the perceived lower level and lower complexity of Mathematical Literacy with respect to Core Mathematics. This situation is further enhanced by a statement in the NCS for Core Mathematics which,

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[75] As a reminder of the contradictory world in which we live ... Mathematical Literacy was introduced for the learners who were not doing Mathematics and continued to be reserved for learners who are not able to cope with the demands of Core Mathematics. However, primary emphasis in the subject is still placed on mathematical contents and knowledge (!!) and the subject is criticised and ridiculed because the mathematics is so basic (!!!). Haaibo!

[76] In principle it is possible to change subject choice from Mathematical Literacy to Core Mathematics(DoE, 2003c, pp. 48-49). However, my participation in the educational terrain in South Africa has afforded me the opportunity to observe that the move from Core Mathematics to Mathematical Literacy is viewed as acceptable and appropriate in terms of the learners' ability to cope with the level of mathematical knowledge and competence required in the Mathematical Literacy curriculum. The same is not held to be true for a move from Mathematical Literacy to Core Mathematics. Rather, the basic and perceived unspecialised level of mathematical competence required in Mathematical Literacy is seen to be inadequate preparation for the demands of the largely esoteric Core Mathematics curriculum. It is, of course, completely understandable why this situation exists. The mathematical terrain in Mathematical Literacy is prioritised over the contextual and modelling components (see the discussion that continues below for more details). Furthermore, the mathematical component of the Mathematical Literacy curriculum is considerably more elementary and less esoteric than the Core Mathematics curriculum. As a result, it is inevitable that a move from Mathematical Literacy to Core Mathematics is seen as incommensurable. It is my contention that if primary focus in Mathematical Literacy were to be placed on the contextual sense-making elements of the subject, then a move from Core Mathematics to Mathematical Literacy would be equally incommensurable.

Venkatakrishnan and Graven (2006, p. 23) point out, positions Mathematical Literacy as a sub-set of Core Mathematics:

> Being literate in Mathematics is an essential requirement for the development of the responsible citizen, the contributing worker and the self-managing person. Being mathematically literate implies an awareness of the manner in which Mathematics is used to format society. It enables astuteness in the user of the products of Mathematics such as hire-purchase agreements and mathematical arguments in the media, hence the inclusion of Mathematical Literacy as a fundamental requirement in the Further Education and Training curriculum. The development of literacy in Mathematics, in the sense outlined here, is also a fundamental responsibility of the Mathematics teacher and other educators. The requirements of the Assessment Standards in Mathematics ensure this. (DoE, 2003b, p. 62)

By implication, learners in the Mathematical Literacy classroom only become mathematically literate; learners in the Core Mathematics classroom become much more than this. Thus, despite all intentions for Mathematical Literacy and Core Mathematics to be different in ‘kind and purpose’, the reality is that − with both subjects promoting mathematical goals − they are primarily different in level and complexity of mathematical knowledge and competence.

8.3.1.2 Category 2 − Interplay of content, context and/or competencies

Through explicit reference to real-world applications in both the definition and statement of purpose, there is recognition that this subject involves more than just the learning of basic mathematical content, but also the application of such content in the arena of application of the real-world. As emphasised in the NCS,

> Contexts are central to the development of Mathematical Literacy in learners. Mathematical Literacy, by its very nature, requires that the subject be rooted in the lives of the learners. It is through engaging learners in situations of a mathematical nature experienced in their lives that the teacher will bring home to learners the usefulness and importance of mathematical ways of thought in solving problems in such situations. (DoE, 2003a, p. 42)

Importantly, however, there is an inconsistent tension between a focus on the learning of mathematical content and emphasis on problem-solving in real-world contexts in the NCS and other supporting documents. For example, the NCS specifies that,

> The approach that needs to be adopted in developing Mathematical Literacy is to engage with contexts rather than applying Mathematics already learned to the context. (DoE, 2003a, p. 42)

This statement suggests that primary form of legitimate participation in the subject involves engagement with and sense-making of real-world contexts, and that mathematical forms of participation and the particular mathematical techniques required in sense-making practices are of a secondary concern. In other words, primary focus is on contextual sense-making practices and not on learning mathematics. This sentiment is echoed by Christiansen (2007) who argues that,

> This [a section taken from the statement of purpose in the NCS] does indeed give the impression that whatever mathematics will be evoked from working with "real-life problems" will be subordinate to the solving of those problems, as will consolidation and extension of mathematical skills. (p. 96)

The Subject Assessment Guideline (SAG) documents for Mathematical Literacy[77] send a subtly different message, as the rather extensive quotation below illustrates (DoE, 2005b, 2007; 2008d, pp. 7-8):

> Learners must be exposed to both mathematical content and real-life contexts to develop these competencies. On the one hand, mathematical content is needed to make sense of real life contexts; on the other hand, contexts determine the content that is needed.
>
> When teaching and assessing Mathematical Literacy, teachers should avoid teaching and assessing mathematical content in the absence of context. At the same time teachers must also concentrate on identifying in and extracting from the contexts the underlying mathematics or 'content'.
>
> Assessment in Mathematical Literacy needs to reflect this interplay between content and context. Learners should use mathematical content to solve problems that are contextually based.
>
> Assessment tasks should be contextually based, that is, based in real-life contexts and use real-life data, and should require learners to select and use appropriate mathematical content in order to complete the task. Some assessment tasks might more explicitly give learners the opportunity to demonstrate their ability to 'solve equations', 'plot points on the Cartesian plane' or 'calculate statistics such a mean, median and mode for different sets of data' while other assessment tasks might be less focused on specific mathematical content and rather draw on a range of content to solve a single problem.
>
> Teachers need to design assessment tasks that provide learners with the opportunity to demonstrate both competence with mathematical content and the ability to make sense of real-life, everyday meaningful problems.

A similar message is explicated in the Teacher Guide[78] for Mathematical Literacy:

> The challenge for you as the teacher is to use situations or contexts to reveal the underlying mathematics while simultaneously using the mathematics to make sense of the situations or contexts. (DoE, 2006, p. 4)

The message contained in these various documents suggest that legitimate participation in the subject must be characterised by a dual emphasis on the development of

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[77] These documents complement the NCS and focus specifically on the *assessment* requirements for the subject. The three SAG documents (i.e. 2005, 2007 and 2008) differ with respect to the examinable curriculum stipulated for each grade, specifically in relation to the composition of the 'Core Assessment Standards' listed in each of the documents. The 2007 and 2008 documents, thus, refer to a larger portion of the original curriculum contained in the NCS than the 2005 document.

[78] The Teacher Guide document was developed to provide teachers with an illustration of the type of pedagogic approach envisioned in the subject. The teacher guide, thus, contains a collection of units, each of which focus on a particular problem or context, and which illustrate how the curriculum can be covered in an integrated way through those problems and contexts.

mathematical knowledge and content together with the ability to apply such content to make sense of contextualised problems. Is this so different from the message contained in the NCS that focus must be on engaging with contexts? In my opinion, the difference is significant. The NCS is suggesting that primary focus must be on making sense of the context (albeit through engagement with mathematical techniques – hence the deliberate choice of the title 'Mathematical Literacy' for the subject). The supporting documents, on the other hand, suggest that the mathematically appropriate techniques and solutions are of central concern (often at the near exclusion and subordination of contextual entities and elements). The former privileges the contextual terrain, while the latter privileges the mathematical terrain.

The Learning Programme Guideline document for Mathematical Literacy[79] illustrates this distinction more explicitly:

> The teaching and learning of Mathematical Literacy should thus provide opportunities to analyse problems and devise ways to work mathematically in solving them. Opportunities to engage mathematically in this way will also assist learners to become astute consumers of the mathematics reflected in the media.
>
> In summary, Mathematical Literacy aims to develop four important abilities
>
> 1. The ability to use basic mathematics to solve problems encountered in everyday life and in work situations.
> 2. The ability to understand information represented in mathematical ways.
> 3. The ability to engage critically with mathematically based arguments encountered in daily life.
> 4. The ability to communicate mathematically.
>
> (DoE, 2005a, p. 8)

The prioritising of mathematised forms of participation over contextual sense-making practices in the subject is clearly evident in these statements.

The final 'nail in the coffin' for the subordination of contextual sense-making practices to mathematical considerations appears in the Examination Guideline documents which provide the explicit instruction to teachers to "*Be careful that the context doesn't interfere with the mathematics and detract from the mathematics.*" (DoE, 2008c; 2009c, p. 5, my emphasis). And, since the purpose of this document is to specify the criteria according to which the Grade 12 national examinations are to be set (and, hence, according to which teachers must model their teaching and internal assessment), this statement will in all

[79] The Learning Programme Guideline document provides guidance on the sequencing of learning, teaching and assessment across each of the grades (DoE, 2005a, p. 15). As such, this document provides teachers with an example of a year plan and illustrates how to construct work schedules and lesson plans.

likelihood override the intention in the NCS to prioritise contextual sense-making practices over predominantly mathematical forms of participation.[80]

This issue of the mixed messages imprinted in and between different curriculum documents is problematised in various local literatures. Bowie and Frith (2006, pp. 31-32) argue that the Mathematical Literacy curriculum looks too much like scientific mathematics on two fronts. Firstly with regards to the structuring of the curriculum for both Mathematical Literacy and Core Mathematics according to almost identical Learning Outcome categories (broadly: Numbers, Functional Relationships, Measurement, and data Handling). And, secondly, in terms of the inclusion of esoteric mathematical content (such as trigonometry and transformation geometry) in the Mathematical Literacy curriculum, and the structuring of the assessment standards predominantly in terms of mathematical content. Christiansen (2006) offers a related observation:

> … the ML NCS is a political hybrid product. Though it states that "[t]he approach that needs to be adopted in developing Mathematical Literacy is to engage with contexts rather than applying Mathematics already learned to the context" (chapter 3, 'contexts'), it has an obvious focus on mathematical skills and concepts throughout. It is using claims of utility to justify itself, yet its content is distinctly mathematical. (p. 10)

And, in a different paper she offers a similar comment:

> … the formulation of the assessment standards are clearly written with a focus on learning the skill, with the contexts as illustrators of the use of mathematics, rather than mathematics being used as a tool to solve a specific problem. (Christiansen, 2007, p. 97)

Venkatakrishnan and Graven (2006) offer a similar suggestion:

> The examples provided within the Assessment Standards in the curriculum statement, …, do emphasise the use of 'real' problems, but the format tends to stress their use as useful 'vehicles' upon which mathematical content can then be carried and foregrounded. (p. 20)

They also go on to point out how the inclusion of lists of teachable and assessable mathematical *content* at the back of the NCS document (in the 'Content and Contexts' section (DoE, 2003a, pp. 38-43)) sends a clear message regarding the central role of the mathematical content component of the subject in teaching and assessment practices (Venkatakrishnan & Graven, 2006, p. 20).

[80] Through my own interactions with teachers, national examiners, and exam papers, I have borne witness to how this state of affairs is playing out precisely in this way in the teaching, learning and assessment of the subject-matter domain of Mathematical Literacy. A cursory reading of a Grade 12 matriculation examination immediately reveals a dominant emphasis on mathematical content and techniques throughout the question paper. Every question has a distinctly mathematical focus and accurate mathematical knowledge, techniques, solutions and narratives are prioritised over real-world considerations and over contextual sense-making practices. In short, the examinations assess the extent to which learners can do mathematics in context rather than the extent to which they can engage with a context (and make use of mathematics in this process). This issue will be explored and evidenced in more detail in Part 7 (see Chapter 25 starting on page 402) during analysis of a set of Grade 12 exemplar examination papers through the lens of the components of the developed theoretical language of description for the knowledge domain of mathematical literacy.

Mthethwa (2009, p. 111) also problematises such ‘deviations’ amongst the curriculum documents, but from a theoretical perspective. He argues that one of the possible consequences of this discrepancy between the NCS and supporting documents is the development of a gap between the pedagogic recontextualising field (i.e. the work of teachers in the classroom) and the official recontextualising field (i.e. the curriculum and other supporting documents), or between the intended curriculum and the implemented curriculum (c.f. Bernstein, 1996).

While Mthethwa is predicting a possible consequence, Graven and Venkatakrishnan (2007) and Venkatakrishnan and Graven (2007) offer evidence of a ‘spectrum of pedagogic agendas’ that have developed in response to this differential emphasis on mathematical forms of participation and contextual sense-making practices between the curriculum and supporting documents.[81] The spectrum of pedagogic agendas is comprised of the following agendas − (i) context driven; (ii) content and context driven; (iii) mainly content driven; and (iv) content driven − and, as suggested by Venkatakrishnan and Graven (2007, p. 77), these varying pedagogic agendas “traverse across the purpose of contexts and the degree of integration of contexts within pedagogic situations.” As such, within each pedagogic agenda there is a specific prioritising (or not) of contextual sense-making practices over the learning of mathematical content and mathematical forms of participation. Venkatakrishnan and Graven (2007, p. 82) argue that the NCS and supporting documents for Mathematical Literacy prioritise, primarily, the second pedagogic agenda – the content and context driven agenda.

Irrespective of which pedagogic agenda predominates in the curriculum, supporting documents, and classroom practice, the presence of a spectrum of pedagogic agendas provides evidence of the differential interpretation of the relationship between content and contexts in the subject (and associated differential emphasis on the legitimation of contextual and mathematical forms of participation). This differential interpretation is spurred, in all likelihood, by the mixed messages emanating from the curriculum and/or supporting documents. This points to varied and inconsistent opinions amongst the various role-players in the subject (including curriculum developers, examiners and teachers) regarding the primary intention, purpose, and required pedagogy for the subject Mathematical Literacy. This also points to varied and inconsistent opinion regarding the form of legitimate participation in the subject, and the structure of knowledge (including the dominant domain – mathematical or contextual − from which that knowledge is to be drawn) and pedagogic action that is required to facilitate this legitimate participation

Alongside specifying content and contexts, the NCS also makes reference to specific skills (competencies) which learners are expected to develop. Many of these skills are specified within individual assessment standards, but also in the ‘Competence Descriptions’ which appear near the end of the document (DoE, 2003a, pp. 54-65). This emphasis on skills is shared by Brombacher (2007, p. 15) who argues a key characteristic of the subject Mathematical Literacy involves an interplay between mathematical content and real-world contexts that facilitates the development of a set of competencies which are universally applicable across a range of contexts and problem situations. Bowie and Frith (2006, p. 30) offer a similar perspective, arguing that the definition of Mathematical Literacy provided in the NCS makes it clear that the three key elements in the subject include mathematical content, contexts and “the abilities and behaviours that a

[81] This ‘spectrum of pedagogic agendas’ was introduced and discussed in Footnote 19 on page 39 above.

mathematically literate person will exercise." However, they also argue that the NCS is not clear on how the three way interplay between content, contexts and competencies/abilities/behaviours must play out in a classroom situation. Furthermore, despite Bowie and Frith's acknowledgement of the competencies component of the subject and Brombacher's insistence on the centrality this component in the development of mathematically literate behaviour, neither the NCS nor any other supporting documents provide a definitive list of competencies, competency clusters or broad categories of competencies to be developed. Rather, in most instances it is mathematical content, and in some instances specific contexts, that are foregrounded in the Assessment Standards listed in the NCS (and replicated in supporting documents). Again, this allows for a potentially differential interpretation of the structure of legitimate participation in the subject and key areas of focus in the subject and, possibly, accounts for the varied opinion on the intended purpose of the subject and the various 'spectrum of pedagogic agendas' that have resulted.

The emphasis in the South African conception of Mathematical Literacy on a three-way interplay of content-contexts-competencies is consistent with the perspectives of several bodies of international literature.[82] However, the fact that Mathematical Literacy curriculum is not explicitly organised around a clearly defined list, cluster or grouping of skills positions the subject differently to many international perspectives. The implication of this in the South African situation is that the lack of clarity over specified skills has resulted in a prioritising of specialised mathematical knowledge, routines and forms of participation and communication over the development of a general set of widely applicable competencies. This absence of specification of competencies in the subject further elaborates a prioritising of mathematically legitimised forms of participation.

8.3.1.3 Category 3 − Arena of application

Both the definition and statement of purpose of Mathematical Literacy in the NCS are explicit in their emphasis on the usage of mathematics in solving problems and making sense of situations involving *real-world contexts*. This arena of application of the real-world includes (i) ***locations*** ranging from everyday life to the workplace and to the roles and responsibilities of a citizen in national and global issues (DoE, 2003a, pp. 9-10). As I have argued elsewhere (c.f. North, 2008), this differentiation between locations provides a source of progression in the subject. Namely, as learners move from Grades 10 to 11 to 12, so the locations of the contexts that they are required to work with become increasingly unfamiliar and complex – almost as though the world-view of the learner is being expanded from familiar personal issues to increasingly unfamiliar workplace and national and/or global issues (DoE, 2003a, pp. 42-43). Within these various locations, a variety of different (ii) ***contexts*** are specified relating to personal, financial and social situations (DoE, 2003a, pp. 9-10). Such contexts include reference to, for example, shopping, personal and business budgets, planning and costing trips, and statistical data relating to prevalent national issues such as HIV-AIDS. It is inevitable that there is variation in the contexts specified for reference in the subject in comparison to contexts referenced in international literature. This is because, as a situated practice, the contexts deemed appropriate for investigation and engagement are directly informed by the social, cultural, economic and political specificities of the

[82] But also stands in opposition to those such as Ewell (2001), Jablonka (2003), and Frith and Prince (2006, 2009)) who argue that since mathematical literacy, numeracy and/or quantitative literacy is a socially situated practice, it is inconceivable that a universally applicable set of competencies can be identified for solving mathematical based problems situated in real-world contexts.

country in which a particular conception of mathematical literacy, numeracy and/or quantitative literacy is promoted. With respect to the overarching categories of locations, however, the locations of everyday life, workplace and national and/or global issues specified in the subject share stark similarity to the types and scope of locations specified within international perspectives.

A further important consideration with respect to contexts relates to the 'types' of contexts deemed appropriate for investigation and engagement in the subject. By 'types' I am referring not so much to the scope or locations of the contexts (e.g. personal life; workplace; etc.) as to the level of *authenticity* of the contexts. Bowie and Frith (2006) provide an appropriate starting point with the concerns that they raise regarding the types of contexts that are appropriate for the teaching of the subject-matter domain Mathematical Literacy:

> If the Mathematical Literacy curriculum is to have credibility as a preparation for coping with the kinds of poorly-defined problems that make up the real demands of life and work, then inauthentic "applications" must be avoided. (p. 32)

For them, inauthentic contexts imply "pseudo-contextualisations" (Bowie & Frith, 2006, p. 32), namely contexts which bear no relation or resemblance to reality, but which are presented as though they *are* representative of real-life. These pseudo-contextualisations constitute the equivalent of du Feu's (2001) category of 'contrived' contexts.

Frith and Prince (2006, p. 53), citing Usiskin (2001), similarly caution against the use of "contrived 'real-life' examples masquerading as 'reality'", and argue that the teaching and learning of the subject Mathematical Literacy requires the use of contexts that are real for those involved[83] and that require as much in-depth understanding of the contextual components of the context as of the mathematics employed to investigate those contexts. This is an important observation since it highlights that understanding of mathematical content is not sufficient for legitimate and endorsed participation in contextual sense-making practices; rather, understanding of the real-world terrain is equally important if enhanced sense-making is to occur.

Given this general agreement on the need for authentic and realistic contexts that bear a high degree of resemblance and connection to the structure of real-world practices, and given du Feu's (2001) concerns regarding the usage of 'contrived' contexts (c.f. page 70 above), it begs the question of what stipulations the NCS and supporting documents for Mathematical Literacy provide with respect to appropriate contexts in the teaching of the subject.

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[83] This is an interesting and important emphasis in that Frith and Prince are arguing that Mathematical Literacy involves socially situated practice and that the types of contexts that are most real and relevant to a group of learners vary from one community to another (Frith & Prince, 2006, p. 53). This raises questions regarding the viability of using a national examination to assess competency in this subject and the types of contexts that can be included in such an examination without disadvantaging particular groups of learners. And, if an approach is adopted that learners must not be prevented from being able to answer questions in an assessment due to the unfamiliarity of the context and/or the language used in the context – as the statement "Be careful that the context does not interfere with the mathematics and detract from the mathematics" listed in the Subject Assessment Guidelines (DoE, 2008d; 2009c, p. 5) for the subject suggests − then this posits the mathematical and not the contextual components of the subject as the dominant focus of assessment. And for Frith and Prince, this prioritising of mathematical forms of participation negates the type of behaviour that is necessary to develop an in-depth understanding of the contextual situations (see immediately below).

In most of the curriculum documents there is acknowledgement of the need for relevant contexts, specifically relevant to the lives of the learners:

> Each context should be relevant to the learners.
> (DoE, 2008d; 2009c, p. 5)

> Teachers should choose meaningful contexts to embed the content gleaned from the Assessment Standards in clusters across the Learning Outcomes where possible. (DoE, 2005a, p. 13)

> Mathematical Literacy, by its very nature, requires that the subject be rooted in the lives of the learners. (DoE, 2003a, p. 42)

However, alongside this emphasis on relevance there is also an explicit recognition of the central role of the mathematical elements of 'appropriate' contexts and that such appropriate contexts must be comprised of mathematical components and/or be of a mathematical nature:

> It is through engaging learners in *situations of a mathematical nature* experienced in their lives that the teacher will bring home to learners the usefulness and importance of mathematical ways of thought in solving problems in such situations. To this end it is very important for the teacher to incorporate local and topical issues into the Learning Programmes that they design. The practices of the local community, the home environment and local industry provide a wealth of relevant contexts to explore. (DoE, 2003a, p. 42, my emphasis)

> Many local and international studies have shown the existence of a set of attitudes described as 'mathsphobia' in school-going learners and in the population at large. It is the responsibility of the teacher, in implementing this curriculum, to endeavour to win learners to Mathematics. Real-life contexts which *lend themselves to mathematical ways of thought* are ideal for doing this. (DoE, 2003a, p. 43, my emphasis)

The two quotations above suggest that for the authors of the NCS curriculum, a context is only important in as much as they contain mathematical components that can be extracted to illustrate the usefulness or application of mathematics in facilitating sense-making of particular mathematical aspects of the context. This implies that contexts for which formal mathematical techniques do not provide an effective tool for contextual sense-making practices are not considered to be of value or relevance for investigation in the subject. This sentiment is echoed by Julie (2006, pp. 67-68), who argues that the inclusion of qualitative (i.e. non-mathematical and situational techniques) and/or common sense approaches to solving contextualised problems should be discouraged in the teaching of Mathematical Literacy and are 'antithetical' to the goals of the Mathematical Literacy.

Recognition is also provided of the possibility of the use of cleaned and context-free contexts or problems in the teaching of Mathematical Literacy:

> Teachers, naturally, also have the freedom to use well-designed simulated problems as context. (DoE, 2005a, p. 14)

> Question 1 [in the Mathematical Literacy Grade 12 Paper 1 examination] could contain some basic calculations and simple short questions. (DoE, 2008c; 2009c, p. 4)

Despite these references to different sources and/or locations of possible contexts, there is no explicit statement that the contexts must contain a high degree of resemblance or link to reality (i.e. real and/or cleaned contexts) or a warning against the use of fictitious or contrived contexts. It is, perhaps, no wonder, then, why all of the national Grade 12 Mathematical Literacy examinations papers since the inception of the subject contain a predominance of contrived contexts or constructed contexts, with some purely mathematical and context-free or semi-contextualised calculations, and hardly any questions containing what could be classified as 'real' contexts. Two examples of the types of context-free and contrived questions that dominate the Mathematical Literacy national examinations are provided in Figure 14 and Figure 15 on the page below.

In fact, my own analysis of the examination papers since 2008[84] reveals that, apart from street maps and tables of data, none of the examinations contain any other form of authentic, un-cleaned, and/or genuine resources (e.g. adverts, newspaper articles) relating to a real-life scenario. <u>All</u> of the resources employed are contrived or cleaned. For a subject that is supposedly about engaging with and making sense of the real (and complicated and messy) world, this state of affairs presents a particularly narrow and primarily mathematised experience of the world.

In sum, while the curriculum and supporting documents provide explicit statements of a requirement for the mathematical grounding of appropriate contexts, there is no accompanying statement of a requirement for authenticity or link to reality. The consequence is that all national assessments for the subject and, by implication, pedagogic practices in the subject, include a spectrum of context types, ranging from context-free to contrived, but largely absent of authentic and real contexts. Mathematical knowledge and content, and mathematically endorsed forms of participation and communication with or of that knowledge and/or content, are prioritised in all practices involving the teaching, leaning and assessment of Mathematical Literacy in South Africa. And, as is discussed in Part 3 of this study (c.f. Chapter 11 starting on page 167 below), this state of affairs is particularly problematic when viewed through the lens of Dowling's (1998) concerns and criticisms levelled against the inclusion of primarily Public Domain practices and associated mathematised representations of reality in the teaching of mathematics.

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[84] See North (2010) for a detailed analysis of the 2008 Mathematical Literacy National Examinations according to content coverage and cognitive demand, and in relation to whether the examinations mirror the purpose and intention of the subject, including the usage of authentic contexts. In answer to the question posed as the title of the paper *How mathematically literate are the matriculants of 2008?*, a key conclusion presented is that the examinations "do not reflect the underlying intention and purpose of this subject and fail to assess sufficiently the extent to which students can apply mathematical content to solve and make sense of problems encountered in daily life." (North, 2010, p. 229). This issue will again be investigated and elaborated in Part 7 (see Chapter 25 starting on page 402) during analysis of a set of Grade 12 exemplar examination papers through the lens of the components of the developed theoretical language of description.

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QUESTION 1

1.1 1.1.1 Simplify:

(a) $15{,}43 + 46{,}08 \times 15{,}6875$ (2)

(b) $\dfrac{17-5}{3} \times (29{,}35 - 10{,}63)$ (2)

1.1.2 Write $2{,}875$ as a common fraction in its simplest form. (2)

1.1.3 Convert R110,35 (South African rand/ZAR) to Algerian dinar (DZD) if 1 ZAR = 9,48 DZD. (2)

1.1.4 Convert 3 024 cm to metres. (2)

1.1.5 Calculate $6\frac{1}{4}\%$ of 420 000. (2)

Figure 14: Context-free and semi-contextualised questions in the 2010 Mathematical Literacy Grade 12 Paper 1 national examination paper (DBE, 2010a, p. 3)

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2.2 At Freedom High School the basic boys' uniform consists of a pair of pants and a shirt with the option of wearing a tie. The pants may be either long or short, and the shirt may be either long-sleeved or short-sleeved. They are allowed to wear any combination of these three items of clothing when they are on a trip.

2.2.1 Complete the tree diagram on ANNEXURE C to illustrate ALL the possible combinations of these three items of clothing that the boys may wear on a trip. (7)

2.2.2 When the boys are at school, they are only allowed to wear ONE of the following combinations of the uniform:

- Long pants with a long-sleeved shirt and a tie
- Short pants with a short-sleeved shirt and no tie

If ONE of the boys in the bus were randomly selected, use the completed tree diagram on ANNEXURE C to determine the probability (in decimal form) that he would be wearing one of these two combinations. (3)
[28]

Figure 15: A contrived context in the 2010 Mathematical Literacy Grade 12 Paper 2 national examination paper (DBE, 2010b, p. 6)

This lack of specificity of the level of authenticity of the contexts to be dealt with is not limited to the South African situation. Rather, and as was discussed previously, much of the international literature on mathematical literacy, numeracy and/or quantitative literacy remains similarly silent on this issue. This lack of specificity allows for a grey-area to develop in which it is deemed appropriate for non-real (i.e. contrived, cleaned and possibly even context-free) problems to be included in teaching and learning aimed at the development of mathematically literate, numerate and/or quantitatively literate behaviour. And, as the discussion in the previous paragraph has illustrated, this is precisely what has happened in the subject-matter domain of Mathematical Literacy South Africa, with the consequence that contrived and non-real mathematised problems now dominate pedagogic and assessment practices in the subject.

8.3.1.4 Category 4 – Components (and/or features associated with courses, assessments and/or pedagogic practice)

As has been discussed previously, some of the international literature on mathematical literacy, numeracy and quantitative literacy identify specific components associated with courses, pedagogic and, especially, assessment practices aimed at developing mathematically literate behaviour. Since the subject Mathematically Literacy is a fully fledged subject aimed at developing knowledge and traits associated with a particular conception of such behaviour, it is inevitable that the NCS curriculum and supporting documents specify components and/or features of the subject which direct pedagogic and assessment practices for the subject. Some of the components associated with directing pedagogic practice have already been discussed (e.g. dual emphasis on content and context), but several components associated specifically with *assessment* practices are worth mentioning.

Firstly, an assessment taxonomy identifying four levels of cognitive demand informs all assessment practices in the subject (DoE, 2005b, 2007; 2008d, pp. 8; 27-29). The four levels of the taxonomy are: *Level 1: Knowing*; *Level 2: Routine procedures in familiar contexts*; *Level 3: Multi-step procedures in a variety of contexts*; and *Level 4: Reasoning and reflection*. Of particular interest for this study is the observation made by Venkatakrishnan, Graven, Lampen, and Nalube (2009, pp. 47-48) that across all four levels of the taxonomy “there is a recurring reference to calculations and procedures.” A consequence of this is that any aims in the curriculum and/or supporting documents for contextual sense-making practices (including consideration of non-mathematical factors, techniques and considerations in problem-solving situations) are continuously be subjugated to mathematical structures, principles and forms of participation in the context of assessments since the taxonomy does not allow for non-calculation based solutions (Bowie, 2010). This observation is similarly expressed by Venkatakrishnan et al. (2009, p. 49): “the mathematical calculation thread allows for procedural mathematics to dominate in ways that often work against the aims of the curriculum.” In other words, irrespective of the dominant agenda promoted in the curriculum documents, the mathematical basis of the assessment taxonomy ensures that mathematically structured knowledge and associated mathematically endorsed forms of participation and communication are prioritised in the activities of the subject over all other possible forms of participation and associated knowledge structures.

A second and related component is the structuring of examinations for the subject according to a two-paper structure, with the two papers differentiated according to cognitive demand rather than according to content or contexts[85]. The Paper 1 examination is constituted as a "basic knowing and routine applications paper" – a basic skills paper – with the intention of assessing understanding of basic mathematical concepts and the ability to perform basic calculations in context (DoE, 2008c; 2009c, p. 4). This assessment of 'basic skills' is ensured through the stipulation that the Paper 1 examination paper may only include questions from Levels 1 and 2 – the two lowest levels − of the taxonomy. 60% of the examination *must* comprise Level 1 questions[86], with the remaining 40% at Level 2. No Level 3 or 4 questions are to be included (DoE, 2008c; 2009c, p. 4). Paper 2, on the other hand, is defined as an "applications, reasoning and reflecting paper" – a problem-solving paper – with the intention of assessing ability to use mathematics to solve problems based in various contexts. Questions from Levels 3 and 4 of the taxonomy are prioritised (Level 3: 40%; Level 4: 40%), with only a small number of questions from Level 2 (20%) allowed. No Level 1 questions are included in this examination (DoE, 2008c; 2009c, p. 4). The total percentages of marks allocated to questions at each level of the taxonomy across both examination papers are summarised in Table 4 below (DoE, 2008c; 2009c, p. 4):

Table 4: Percentage of marks to be allocated to different taxonomy levels in the Mathematical Literacy examinations

THE FOUR LEVELS OF THE MATHEMATICAL LITERACY ASSESSMENT TAXONOMY	Grade 10	Grades 11 and 12		
		Paper 1	Paper 2	Overall Allocation
Level 1: Knowing	30% ± 5%	60% ± 5%		30% ± 5%
Level 2: applying routine procedures in familiar contexts	30% ± 5%	40% ± 5%	20% ± 5%	30% ± 5%
Level 3: Applying multi-step procedures in a variety of contexts	20% ± 5%		40% ± 5%	20% ± 5%
Level 4: Reasoning and reflecting	20% ± 5%		40% ± 5%	20% ± 5%

The prioritising of the mathematical terrain and mathematical calculation as the basis for legitimate participation in the Paper 1 examination is explicit and obvious. By contrast, since the focus of the Paper 2 examination is application and problem-solving, it would

[85] It is generally acknowledged that differentiating the papers according to content would be antithetical to the type of approach that is needed to solve problems in the real-world and, so, is contrary to the aims of the subject. An 'integrated' approach to teaching and especially to assessment is, thus, promoted: integrated in the sense that learners must be expected to draw on a variety of mathematical contents and techniques when solving problems based in real-world contexts. As suggested in the Learning Programme Guidelines for the subject:

> Teachers should view the learning outcomes as integrated and connected. This will allow learners to develop a more coherent view of mathematics. For example, one may start from a context which focuses on data handling (Learning Outcome 4) and look for opportunities in that context to pose questions or investigations where learners will calculate, estimate and solve problems or procedures which are described in Learning Outcome 1. (DoE, 2005a, p. 13)

It is also in this vain that the Examination Guideline documents (DoE, 2008c; 2009c, p. 4) state that within the setting of an examination, each context used within a question should contain reference to content from "at least two different Learning Outcomes."

[86] The pass mark for Mathematical Literacy is 30%. The percentage of marks allocated to Level 1 questions across both examination papers is 30%. This suggests that in order to pass this subject learners need only be able to successfully complete questions at the *Knowing* level of the taxonomy, such as reading information from a table or resource or adding or subtracting two values. The correlation between the percentage of marks allocated to Level 1 questions and the pass mark is not coincidence. With more than 50% of the FET population of learners opting for Mathematical Literacy, there is clearly motivation to ensure that as many learners as possible are able to pass in the subject.

be reasonable to expect that contextual sense-making practices would form the basis for legitimate participation in this examination. However, this is not the case. Rather, the stipulation that the examination must examine the content specified in each learning outcome equally (i.e. 25% allocated to each learning outcome) (DoE, 2008c; 2009c, p. 5) together with the mathematical base of the taxonomy ensures that the examination is designed in a check-box structure: is every concept specified in the curriculum included in the examination? (check✓); does this question fall under Learning Outcome 1, 2, 3 or 4? (check✓); is 25% of the total marks allocated to each learning outcome? (check✓); and so on. Content coverage in an equally distributed way is a defining feature of both examinations. This is verified by Bowie and Frith (2006) who argue that,

> the grade 12 Mathematical Literacy examination will have a spread of questions that ensure that each of the learning outcomes is allocated 25% of the total marks. This allows the mathematical content to assume the importance of the major organising principle for assessment. (pp. 31-32)

The South African situation is somewhat different to the specific components relating to assessment practices and frameworks identified for the international literature. Large scale assessment frameworks such as PISA (OECD, 1999, 2003, 2006, 2009) and ALL (Gal et al., 2005) are characterised by consistency between the dominant structure of knowledge and legitimate participation prioritised in assessment strategies and in the conception of mathematically literate, numerate and/or quantitatively literate behaviour measured in those strategies. In South Africa, by contrast, assessment structures do not promote the same dominant structure of knowledge and criteria for legitimate participation as the curriculum statement for the subject. Instead, a variety of subject-related components and, most especially, assessment requirements and stipulations (such as the division of the two examination papers according to cognitive demand, and the stipulation that any contexts employed must not inhibit access to mathematical structures) directly inform pedagogic practices in the subject in a way that is contrary to the original curricular intention and philosophy. In international assessment frameworks, it is the conception of mathematical literacy, numeracy and/or quantitative literacy that is prioritised in the framework which informs the structure of the assessment components. By contrast, in South Africa it is the dominant structure of knowledge and criteria for legitimate participation that characterise the national examination papers which define and determine the conception of mathematically literate behaviour (and the structure of knowledge and participation that is seen to propagate this conception) that is prioritised in pedagogic action in the subject.

8.3.1.5 Category 5 − Dominant orientation in the NCS conception of the subject-matter domain of Mathematical Literacy in South Africa

The discussion so far has highlighted a degree of incongruence between the definition and statement of purpose for the subject Mathematical Literacy provided in the NCS curriculum and the structure of legitimate participation promoted and prioritised in other supporting documents. I have suggested that this provides a possible reason for the wide variation in opinion on the purpose and areas of focus in the subject amongst various role-players, including curriculum writers, textbook authors, examiners and teachers, and, hence, for the spectrum of pedagogic agendas identified by Venkatakrishnan and Graven (2007). This difference in opinion revolves primarily around the prioritising of mathematical content, contextual entities or competencies as the basis for legitimate participation in the subject.

Despite this variation in opinion and the sometimes contradictory messages espoused in the various curriculum and supporting documents, it is still possible to identify the dominant agendas and intentions that are prioritised in the South African conception of Mathematical Literacy. To do this I draw once again on the two-dimensional framework of *Agendas* and *Intentions* introduced in sub-section 5.2.2.3 on page 47 above (and represented in Figure 7). Also note that, as part of the discussion of the dominant agendas and intentions in the subject-matter domain of Mathematical Literacy, I make reference to the dominant agendas as intentions in international conceptions of mathematical literacy, numeracy and/or quantitative literacy. These dominant agendas and intentions are categorised in the framework of agendas and intentions in Section 6.5 on page 82 above. As a reminder, this categorisation also includes the documentation that relates and refers to the South African subject-matter domain of Mathematical Literacy (formatted in *italics* in the framework). As such, the reader may wish to refer back to that utilised framework to contextualise the discussions in the sub-sections below that make comparisons of the categorisation of the literature on the subject-matter domain of Mathematical Literacy and the literature on more general conceptions of mathematically literate, numerate and/or quantitatively literate behaviour.

8.3.1.5.1 Dominant Intention(s) for the NCS conception of Mathematical Literacy

As regards the dominant *Intention* that is promoted through and for the South African conception of Mathematical Literacy, various statements in the NCS and supporting documents reference several of the categories of intentions shown in the framework. To begin with, an analysis of Chapter 1 in the NCS (DoE, 2003a, pp. 1-7) – a generic chapter that appears in the NCS documents for all subjects and which provides an introduction to the components of the entire National Curriculum Statement curriculum framework – reveals reference to at least four of the categories:

- the inclusion of words such as "high knowledge and high skills" suggest an intention for the development of *Human Capital*;
- the inclusion of words such as "valuing indigenous knowledge systems" hint at an intention for the promotion of *Cultural Identity*;
- the inclusion of words such as "social transformation", "human rights", "inclusivity", "social justice" suggest an intention for *Social Change*; and
- words such as "environmental justice" suggest that the curriculum is presented as an avenue for *Environmental Awareness*.

Despite reference to four of the categories, the inclusion of the following statement in Chapter 1 – under the heading "The kind of learner that is envisioned" (DoE, 2003a, p. 5) − suggests the dominance of an intention for *Social Change* in the broader NCS framework:

> Of vital importance to our development as people are the values that give meaning to our personal spiritual and intellectual journeys. The Manifesto on Values, Education and Democracy (Department of Education, 2001:9-10) states the following about education and values:

> *Values and morality give meaning to our individual and social relationships. They are the common currencies that help make life more meaningful than might otherwise have been. An education system does not exist to simply serve a market, important as that may be for economic growth and material prosperity. Its primary purpose must be to enrich the individual and, by extension, the broader society.*

> The kind of learner that is envisaged is one who will be imbued with the values and act in the interests of a society based on respect for democracy, equality, human dignity and social justice as promoted in the Constitution.

Shifting to the chapters in the NCS document that deal specifically with the components of the subject Mathematical Literacy, there are, similarly, statements that make reference to a variety of differing intentions for the subject. Numerous statements in the NCS prioritise the *mathematical literacy for Human Capital* intention and the conception that by engaging in the subject Mathematical Literacy learners are better prepared to cope with the demands of daily life and the workplace and, so, increase their value and worth in the economy:

> The inclusion of Mathematical Literacy as a fundamental subject in the Further Education and Training curriculum will ensure that our citizens of the future are highly numerate consumers of mathematics. (DoE, 2003a, p. 9)

> The Further Education and Training subject, Mathematical Literacy, should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy. (DoE, 2003a, p. 10)

> Mathematical Literacy will ensure a broadening of the education of the learner which is suited to the modern world. (DoE, 2003a, p. 10)

> ... realise that mathematical literacy contributes to entrepreneurial success.
> (DoE, 2003a, p. 10)

> Students proceeding to Higher Education institutions will have acquired a mathematical literacy that will enable them to deal effectively with mathematically related requirements in disciplines such as the social and life sciences.
> (DoE, 2003a, p. 11)

These statements are, however, intertwined with statements of intention for *Social Change* and for *Evaluating Structures*:

> To be a participating citizen in a developing democracy, it is essential that the adolescent and adult have acquired a critical stance with regard to mathematical arguments presented in the media and other platforms. (DoE, 2003a, p. 11)

> The concerned citizen needs to be aware that statistics can often be used to support opposing arguments … (DoE, 2003a, p. 11)

> In the information age, the power of numbers and mathematical ways of thinking often shape policy. Unless citizens appreciate this, they will not be in a position to use their vote appropriately. (DoE, 2003a, p. 11)

> Being mathematically literate implies an awareness of the manner in which Mathematics is used to format society and enables astuteness in the user of the products of Mathematics (DoE, 2003a, p. 43)

An explicit statement of intention for the promotion and development of *Cultural Identify* is also included, specifically in reference to the promotion of ethnomathematical principles in the teaching of Mathematical Literacy:

> Another aspect of providing access and affirmation for learners of Mathematics is to look at examples of Mathematics in the variety of cultures and societal practices that exist in our country. (DoE, 2003a, p. 43)

> Ethnomathematics provides a wealth of more recently developed materials, sensitive to the sacredness of culture, for use in the classroom. (DoE, 2003a, p. 43)

> ... use the concepts of rotation, symmetry and reflection in describing decorative Ndebele and Sotho mural designs. (DoE, 2003a, p. 29)

Finally, references are also made to an intention for the development of mathematically literate behaviour to facilitate *Environmental Awareness*. Many of these references are contained in listings of specific contexts for investigation or examples of fields of application that are embedded in the assessment standards in the curriculum component of the NCS rather than in the statement of purpose for the subject (which, perhaps, hints at the secondary importance placed on this intention). For example:

> ... use mathematical literacy in a critical and effective manner to ensure that science and technology are applied responsibly to the environment and to the health of others (DoE, 2003a, p. 10)

> ... investigate the rate of depletion of natural resources (DoE, 2003a, p. 21)

> ... interpret graphs of temperature against time of day during winter over a number of years to investigate claims of global warming (DoE, 2003a, p. 23)

Despite the varied reference to several statements of intention in the Mathematical Literacy component of the NCS, and despite the dominant intention for Social Change in the broader NCS framework, my reading of the NCS leads me to contend that the dominant intention promoted for Mathematical Literacy is for the development of *Human Capital*. Engagement with the NCS presents the reader with an image that a mathematically literate learner is a learner who is empowered to use mathematics to solve problems and inform decisions relating to their daily-life experiences. A mathematically literate person is financially savvy, economically productive, and engaged critically on economic, social and political issues. In short, a mathematically literate individual is a (more?) desirable commodity. This conception of mathematical literacy is consistent with the dominant intention identified within the international literature read for this study.

8.3.1.5.2 Dominant Agenda(s) for the NCS conception of the subject-matter domain of Mathematical Literacy

The second dimension in unpacking the dominant orientation prioritised in the subject-matter domain of Mathematical Literacy involves the issue of *Agenda*, which relates specifically to the extent to which the learning of mathematical content, techniques and mathematised forms of participation are prioritised over contextual sense-making practices (or vice versa). In previous discussion above I have highlighted issues relating to the dual emphasis on content and context in the NCS curriculum and the, sometimes, ambiguous statements in supporting documents regarding the form of legitimate participation – and the structure of knowledge according to which legitimate participation is determined − in the practices of the subject. Drawing from this discussion, it is my contention that at the level of intended purpose the NCS promotes a form of participation in the subject that reflects, primarily, an agenda associated with the second category: namely, developing the ability to perform *calculations in real-world contexts*. More specifically, the NCS promotes the dimension of Agenda 2 that is characterised by Application (i.e. Agenda 2 [a]) (as opposed to mere Numeracy-in-Context − Agenda 2 [b]). Furthermore, the NCS posits as a key component of legitimate participation in the subject the utility of a variety of forms of mathematical content (both complex and elementary, and including esoteric contents such as trigonometry) in engagement with real-world problem-solving contexts. The NCS document does also include rhetoric that alludes to modelling processes (Agenda 3); however, the dominance of explicit statements of mathematical contents and structures in the Assessment Standards in the curriculum document ensures that the direction of movement is always from the mathematics to the extra-mathematical.

It is my opinion that the NCS curriculum has never prioritised a form of participation that has as a primary objective the development of mathematical competence (Agenda 1 in the framework). Rather, the NCS curriculum is built on the assumption that learners entering the subject already have an understanding of some level of mathematical content (having completed Grades 8 and 9 level mathematics) and, so, focus in the subject should be on contextualised applications of the already learned mathematics. Similarly, although the NCS hints at the development of mathematically literate behaviour for contextual sense-making practices (Agenda 4 in the framework), the specific body of content listed for the subject clearly prioritises mathematical agendas over the contextual terrain.

Importantly, however, not all of the documents that accompany and/or support the NCS curriculum prioritise the same agenda or the same component of an agenda. Specifically (and as is discussed in more detail immediately below), all of the documents that specify the structure and focus of assessment-related practices in the subject prioritise forms of participation consistent with the Numeracy-in-Context (and not the Application) component of Agenda 2. By contrast, the Teacher Guide document (DoE, 2006) that provides examples of the types of pedagogic practice expected in the subject prioritises a form of participation that reflects an agenda for modelling processes (Agenda 3). This inconsistency in the dominant agenda prioritised in the different documents made available to teachers is a likely explanation for the spectrum of pedagogic agendas that Venkatakrishnan and Graven (2007) have identified in classroom practices in the subject.

As mentioned directly above, at the level of implementation it is, I contend, the Numeracy-in-Context dimension of Agenda 2 that dominates in national assessment practices (and also in the documents that elaborate on the structure of these assessment practices, such as the Subject Assessment Guidelines (DoE, 2005b, 2007, 2008d) and Examination Guidelines (DoE, 2008c, 2009c)) and, so, also at the level of classroom practice. A quick reading of any of the Grade 12 Mathematical Literacy national examinations since 2008 provides clear evidence of this predominantly mathematical agenda, with each and every question driven by a particular mathematical concept or calculation, and with the structure and organisation of the examination papers around mathematical content topics rather than around contexts or real-world problem situations.

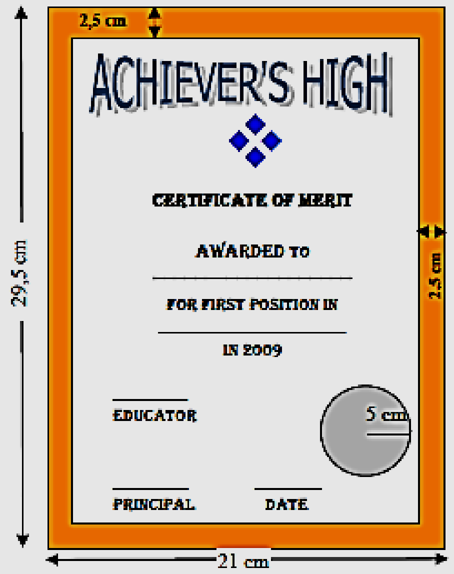
Participation in the examinations is legitimated according to mathematical knowledge and structures, and successful participation in the examinations is determined to a large extent by the ability of the participants to generate mathematically accurate narratives. Contextual sense-making practices (Agenda 4) are largely absent and reference to contextually relevant knowledge and forms of participation and communication are considered inappropriate for use in the examinations. Furthermore, any included contexts serve simply to provide settings in which calculations can be performed, and understanding of contextual elements is neither necessary for successful engagement with the questions or for the generation of endorsable narratives to the problem scenarios. The extract shown in Figure 16 on the page below, taken from the 2009 National Mathematical Literacy Paper 1 examination paper, illustrates clearly and explicitly the irrelevance of the contextual domain and the dominance of mathematical calculations, knowledge structures and considerations as the basis of legitimate participation in the subject (DoE, 2009a, p. 4).

While the dominant agenda in the NCS curriculum for mathematical forms of application shares similarity to trends in the dominant agenda prioritised in international perspectives on mathematical literacy, numeracy and/or quantitative literacy, the same cannot be said for implemented practice in the subject-matter domain of Mathematical Literacy. While much of past and current practice in the subject is characterised by a dominant agenda that promotes and prioritises a structure of participation involving contextualised numeracy-type calculations with elementary mathematical content (i.e. Agenda 2 [b]), only a small number of perspectives on general conceptions of mathematical literacy, numeracy and/or quantitative literacy prioritise the same agenda. Instead, most other perspectives promote an agenda for mathematical literacy that involves more complex and intricate problem-solving processes, characterised either by application of more complex mathematics (i.e. Agenda 2 [a]), or by modelled engagement with more complex contextual environments (i.e. Agenda 3). The implemented reality of Mathematical Literacy in South Africa, thus, promotes a form and agenda of or for mathematically literate behaviour that is largely not prioritised in other parts of the world.

2.1 Ms James, an educator at Achiever's High, is responsible for preparing the prize-giving certificates for the annual academic awards day.

The certificate is rectangular in shape with a uniform 2,5 cm shaded border, as shown in the diagram below.
The outside measurement of the certificate is 21 cm by 29,5 cm.

A gold or silver circle with a radius of 5 cm indicating the performance level of the learner is placed on the certificate.



2.1.1	Write down the length of the diameter of the circle.	(1)
2.1.2	Write down the length of the unshaded part of the certificate.	(2)
2.1.3	Calculate the area of the circle. Use the formula: **Area** = πr^2, where $\pi = 3{,}14$ and r = radius.	(3)
2.1.4	Calculate the perimeter of the outside of the certificate. Use the formula: **Perimeter = 2 (l + b)**, where l = length and b = breadth.	(2)

Figure 16: Extract from the 2009 National Mathematical Literacy Paper 1 Examination

8.3.1.6 A comparative analysis of the South African NCS conception of Mathematical Literacy and international perspectives of mathematical literacy, numeracy and/or quantitative literacy

Throughout the discussion above on the conception of mathematically literate behaviour encompassed in the subject-matter domain of Mathematical Literacy, I have alluded to areas of commonality and divergence with international perspectives. In the discussion below I summarise and consolidate particularly important areas of comparison.

In many respects, the structure of knowledge and participation that is prioritised in the South African conception of Mathematical Literacy shares tremendous similarities with many of the international perspectives on mathematical literacy, numeracy and/or quantitative literacy, specifically with regards to:

- an emphasis on the use-value of mathematics;
- the phenomenological strands according to which the content is organised;

- differentiation to scientific mathematics;
- minimal or no allowance made for informal mathematical or non-mathematical situational considerations and techniques;
- the three way interplay between content-context-competencies;
- the scope of the arena of application – personal life, workplace, national and/or global issues − in which mathematically literate, numerate and/or quantitatively literate behaviour is to be developed; and
- the lack of specificity with respect to the level of authenticity and/or nature of contexts.

Another distinct area of convergence with much of the international literature relates to the dominant intention in subject − namely, primarily for the development of Human Capital.

However, there are also areas of divergence, starting with the dominant emphasis in the subject on a form of participation that prioritises an agenda for numeracy in context. Although this agenda does share some degree of convergence with perspectives that promote a structure of legitimate participation that is characterised by engagement with contextualised mathematics practices (Agenda 2), in most instances these perspectives promote a distinct agenda for applications involving complex mathematical structures (Agenda 2 [a]) and not merely for engagement with elementary mathematical contents (Agenda 2 [b]). This dominant emphasis on an agenda for numeracy in context and an accompanying emphasis in the subject on engagement with basic or elementary mathematical content, leads to a further area of divergence with those perspectives on mathematical literacy, numeracy and/or quantitative literacy that promote and emphasise the use of more sophisticated and abstract contents (for example, the OECD-PISA frameworks). This area of divergence is possibly explained by the fact that most international perspectives do not separate the development of mathematically literate, numerate and/or quantitatively literate behaviour from the teaching of esoteric mathematical contents. In South Africa the reverse is true – namely, the development of Mathematically Literate behaviour is treated as an activity that is separate from engagement with scientific mathematics contents and, as such, is constituted in an entirely separate subject.

A further area of divergence relates to specified assessment requirements stipulated for the subject-matter domain of Mathematical Literacy (for example, the structure of examination papers and the associated taxonomy framework). These additional components, which commonly do not receive reference in the international literature, play a tremendous role in shaping pedagogic practice in the subject and in defining the form of legitimate participation (and the structure of knowledge to be utilised in order to facilitate legitimate participation) in subject-related practices. Crucially, it is these additional components which result in pedagogic and assessment practices that are in contradiction and opposition to the structure of legitimate knowledge and participation prioritised in the NCS curriculum for the subject and also to the structure of envisioned behaviour espoused in many international perspectives on mathematical literacy, numeracy and/or quantitative literacy.

Despite these areas of divergence, there is a key area of similarity that is crucial to highlight. Namely, that mathematically literate behaviour in South Africa and also in most other international perspectives is conceptualised as involving interactions with the everyday world through a distinctive mathematical gaze: a mathematically literate individual is someone who is able to understand and describe the world from a mathematical and mathematised perspective. From this position, mathematical content, knowledge and structures provide an inevitable and appropriate organising principle of any learning process involved in the development of mathematically literate behaviour.

With analysis of the NCS curriculum now completed, I shift attention to the more recent CAPS curriculum structure and to analysis of the dominant intentions and agendas – and associated structure of knowledge and participation –prioritised in that curriculum.

8.3.2 Dominant Agenda(s) and Intention(s) in the CAPS conception of the subject Mathematical Literacy[87 & 88]

There are several differences in the way in which the Mathematical Literacy curriculum is structured and presented in the NCS and the CAPS frameworks which signify a shift in the dominant intention and agenda prioritised for the structure of legitimate participation in the subject. These differences include a restructuring of the curriculum according to Basic Skills and Application Topics in the CAPS framework, an explicit statement of requirement for the prioritising of contextual considerations, knowledge and forms of participation alongside mathematical considerations, and further explicit statements concerning the necessary interplay content, contexts and skills in specified contextual sense-making practices. Each of these is discussed in detail below, together with the significance of these differences for the dominant intention and agenda prioritised in the CAPS framework.

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[87] It is not my intention to provide an analysis of the CAPS document according to the same categories that were used to analyse and organise the literature on mathematical literacy, numeracy and quantitative literacy, and the NCS document. This is because the CAPS document is simply an extension and/or remodelling of the NCS document, and, so, many of the comments made regarding the NCS also apply to the CAPS. The intention of the discussion in this section, then, is to highlight areas of significant difference between the NCS and CAPS and to illustrate how these differences reflect a shift in the dominant intention and agenda in the CAPS.

[88] As mentioned previously, I was directly involved in the development of the CAPS curriculum document for the subject-matter domain of Mathematical Literacy. Amazingly, after having been given the brief that the CAPS curriculum was not a new curriculum and, rather, was intended to provide an 'unpacking' of the NCS curriculum, I was left entirely to my own devices to determine the structure of the curriculum, specific areas of focus in the curriculum, and also to determine the dominant philosophy and orientation of the curriculum. And, so, like a bull in a china shop (or, maybe, a more accurate analogy is 'like a tank in a china shop') I ploughed forth, merrily making decisions (entirely on my own!) that would ultimately dictate the structure of legitimate participation in the subject. It should come as no surprise, then, that the Mathematical Literacy CAPS curriculum is dominated by a white, middle-class perspective that promotes engagement with, primarily, middle-class economic and social contexts and related problem scenarios.
It should also come as no surprise, then, that my own privileging of a dominant agenda for contextual sense-making practices in the subject Mathematical Literacy is contained to some degree in the CAPS curriculum framework and, particularly, in the philosophy for the subject that is promoted in that framework. Evidence of this emerges in the course of the discussion below, and the reader may wish to reflect on my involvement and influence in the development of the CAPS framework when areas of convergence between the CAPS framework and my own position are established.

8.3.2.1 NCS vs. CAPS: from Learning Outcomes to Topics

Where the NCS curriculum is structured around Learning Outcomes and Assessment Standards, the CAPS document presents a curriculum structured around Topics and Sections. This shift has prompted a restructuring of the Mathematical Literacy curriculum, as summarised in Table 5 below:

Table 5: Comparison of the NCS and CAPS curriculum frameworks: Learning Outcomes versus Topics

<table>
<tr><th>NCS</th><th colspan="2">CAPS</th></tr>
<tr><td rowspan="2">LO 1: Numbers and Operations in Context</td><td rowspan="3">Basic Skills Topics</td><td>Interpreting and communicating answers and calculations</td></tr>
<tr><td>Numbers and calculations with numbers</td></tr>
<tr><td>LO 2: Functional Relationships</td><td>Patterns, relationships and representations</td></tr>
<tr><td></td><td rowspan="5">Application Topics</td><td>Finance</td></tr>
<tr><td rowspan="2">LO 3: Space, Shape & Measurement</td><td>Measurement</td></tr>
<tr><td>Maps, plans and other representations of the physical world</td></tr>
<tr><td rowspan="2">LO 4: Data Handling</td><td>Data handling</td></tr>
<tr><td>Probability</td></tr>
</table>

Several explanatory comments are necessary. Firstly, the table has been deliberately constructed to show correlation between the content categories in the NCS and the CAPS frameworks. For example, the CAPS topics of *Interpreting and communicating answers and calculations* and *Numbers and calculations with numbers* are based on the content and skills outlined in *Learning Outcome 1: Numbers and Operations in Context* in the NCS. The topic of *Finance* in the CAPS framework stands out as an anomaly in that it correlates to assessment standards located in Learning Outcome 1 in the NCS. However, in the CAPS framework this topic is positioned as an Application Topic and not as a Basic Skills topic (which is what the contents of Learning Outcomes 1 and 2 are predominantly categorised as), hence the differential positioning of this topic in the table. A significant change from the NCS to the CAPS framework (as demonstrated in the table) includes the division of Learning Outcome 3 on Measurement into the separate topics of *Measurement* and *Maps, plans and other representations of the physical world*. This has been done to enhance emphasis on the spatial literacy component of the Mathematical Literacy curriculum. A similar division has occurred from Learning Outcome 4 to *Data Handling* and *Probability* to enhance the status of the Probability contents in the curriculum.

Secondly, the CAPS framework differentiates between Basic Skill Topics and Application Topics. The distinction in structure between these topics is explained by means of a diagram (Figure 17 on the page below) and a discussion (see the selection of quotations below).

> Much of the content in the Basic Skills Topics comprises elementary mathematical content and skills that learners have already been exposed to in Grade 9 … The inclusion of this content in this document provides teachers with the opportunity to revise these important concepts and provide learners with the opportunity to explore these concepts in contexts. It is expected that a firm grasp of the concepts in the *Basic Skills Topics* is necessary for making sense of the content and contexts outlined in the *Application Topics*.
>
> The *Application Topics* contain the contexts related to scenarios involving daily life, workplace and business environments, and wider social, national and global issues that learners are expected to make sense of, and the content and skills needed to make sense of those contexts. It is expected that learners will integrate content and skills from the *Basic Skills Topics* in making sense of the contexts and content outlined in the *Application Topics*. (DBE, 2011a, p. 11, all emphasis in original text)

Furthermore:

> The *Basic Skills Topics* have been included to the left of the other topics [on the diagram shown in Figure 17 above] to indicate that the content and skills outlined in these topics permeate all of the other topics in the curriculum. It is expected that learners will integrate the content and skills from these three topics with confidence in any context and in any other topic in which they have relevance and application. (DBE, 2011a, p. 12, emphasis in original text)

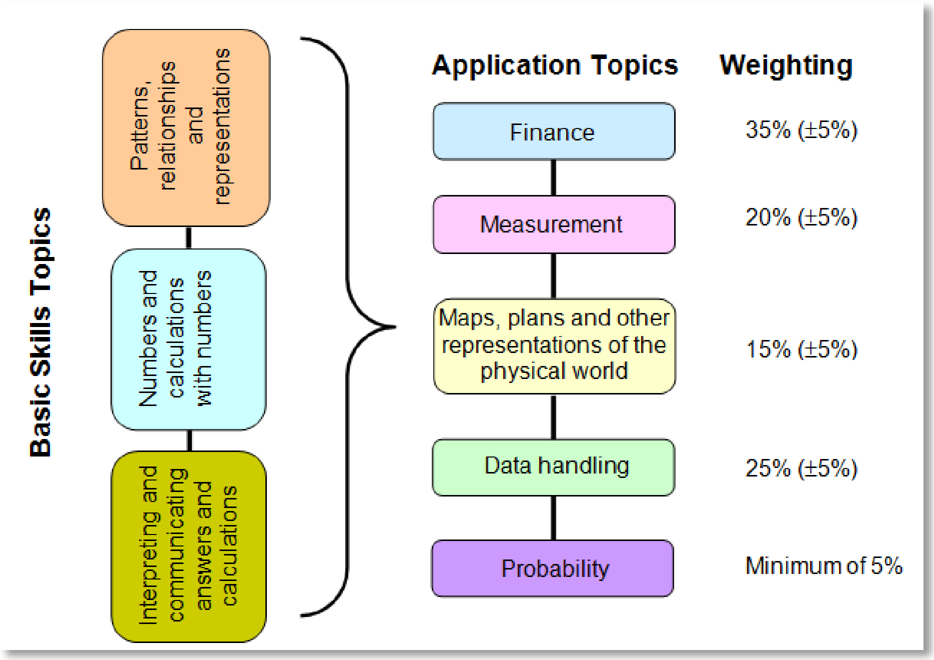


Figure 17: Basic Skills and Application Topics in the CAPS document (DBE, 2011a, p. 12)

The differentiation between the content in the Basic Skills topics and the content and contexts specified in the Application Topics is, perhaps, best summarised as a distinction between elementary mathematical content and skills, on the one hand, and application, contextualised problem-solving and contextual sense-making practices (that make use of this elementary content), on the other.

Importantly, this differentiation also affects the way in which assessments are structured in the CAPS framework, in that primary focus in the examinations is on the content, contexts and problem-solving activities specified in the Application Topics rather than on content specified in the Basic Skills Topics. In fact, as is shown on Figure 17 above, no weighting of marks is allocated for the contents of the Basic Skills Topics. Instead, in theory, assessments are now expected to determine the extent to which learners are able to employ basic skills and utilise elementary content in order to make sense of the contextual situations (and problems encountered in those situations) identified in the Application Topics (DBE, 2011a, pp. 12, 95-100).

The differentiation between Basic Skills and Application Topics signifies an important shift in the dominant agenda that is prioritised for legitimate participation in the CAPS conception of the subject as compared to the NCS framework: the development of mathematical knowledge is being explicitly subordinated to problem-solving practices that prioritise contextual sense-making practices. In the CAPS conception of Mathematical Literacy, the primary agenda for the problem-solving process is the development of a more comprehensive understanding of particular aspects of specified contextual situations, together with the ability to successfully complete tasks encountered in real-world settings. And, in this conception, any mathematical content and knowledge applied and/or employed are seen as in being in service to this goal of contextual sense-making: the learning of mathematics and mathematical applications are not the prioritised goals of the problem-solving process.

8.3.2.2 Consideration of non-mathematical techniques and considerations

A second key shift that characterises the change from the NCS to the CAPS framework is the explicit call in the CAPS framework for recognition of the potential limitations of mathematical techniques and mathematically derived solutions adequately and appropriately representing real-world practices. This call is accompanied by a further statement of expectation of the need to understand and give credence to real-world considerations which are often not mathematical in nature but which may affect the decisions that people make in their daily lives. As suggested in the CAPS document (DBE, 2011a):

> Alongside using mathematical knowledge and skills to explore and solve problems related to authentic real-life contexts, learners should also be expected to draw on non-mathematical skills and considerations in making sense of those contexts. E.g. although calculations may reveal that a 10 kg bag of maize meal is the most cost-effective, consideration of the context may dictate that the 5 kg bag will have to be bought because the 10 kg bag cannot fit inside the taxi and/or the buyer does not have enough money to buy the 10 kg bag and/or the buyer has no use for 10 kg, etc. In other words, mathematical content is simply one of many tools that learners must draw on in order to explore and make sense of appropriate contexts. (p. 7)

This statement is reinforced by additional statements provided in the official presentation slides used in the national CAPS-related training programmes for subject advisors and teachers conducted during 2011:

The point?

To be mathematically literate implies:

- having the capacity to use mathematics and other techniques and considerations to make sense of authentic real-world problems (IF YOU WANT TO)
- having an awareness that daily life is (more often than not) not structured around mathematical principles:
  - this means having an understanding that although we can use mathematics to make sense of a situation, there are often non-mathematical considerations that affect our decisions and actions;
  - this means having an understanding that mathematical models and mathematical solutions have limitations, and do not always present the most appropriate solution;
  - this means recognising the role of informal or less formal techniques used for solving problems.

(DBE, 2011b, Slides 12 & 13)

There are two separate facets to be identified in these statements. The first comprises a call for recognition of the plethora of less formal mathematical techniques (for example, estimation, trial and improvement) that people employ as they go about their daily lives, and an accompanying expectation for such techniques to be endorsed when employed in problem-solving practices in assessments and examinations in the subject. The second facet calls for recognition of and focus on contextual (often non-mathematical) considerations which may affect how people think, act, communicate as they make decisions in particular situations – for example, the colour, make and features of a cell phone versus the cost of the phone.

In essence, the shift indicated in these statements signals a deliberate attempt to initiate a move away from predominantly mathematical and calculation based forms of participation and, instead, towards the inclusion of contextual and real-world considerations, knowledge and participation structures. Importantly, despite this shift, there is certainly no denial in the CAPS framework of the importance of the mathematical component of the subject and consistent emphasis is referenced in the document on the role of mathematical calculations and modelling processes for informing contextual sense-making practices. Rather, the shift is simply a call for recognition of the limitations of an exclusive mathematical gaze approach to engagement with real-world problems and for consideration of the non-mathematical reality that affects legitimate participation in real-world practices.

8.3.2.3 Content, context and skills

The CAPS document attempts to clarify some of the ambiguities that exist between the NCS and supporting documents through explicit specification of the scope of mathematical content, the nature of contexts, and the expected relationship between content, contexts and skills.

With regards to mathematical content, the CAPS document identifies as one of five key elements of the subject the use of "*elementary* mathematical content" (DBE, 2011a, p. 7, emphasis in original text). 'Elementary' is clarified to be imply content that is largely of a non-esoteric or non-abstract nature that individuals might make use of in solving problems encountered in daily life, workplace, and social and political settings. To further emphasise this emphasis on largely non-abstract content, an additional proviso is stipulated:

> As a rule of thumb, if the required calculations cannot be performed using a basic four-function calculator, then the calculation is in all likelihood not appropriate for Mathematical Literacy. (DBE, 2011a, p. 7)

This attempt to clarify more clearly the scope of mathematical content required in the subject is a reaction to the contradictory statements in the NCS that specify “basic mathematical skills” (DoE, 2003a, p. 9) but then list scientific topics in the assessment standards. This deliberate and explicit narrowing of content in the CAPS framework also reflects an attempt to restrict the mathematical component of the subject to allow for increased prioritising of non-mathematical and/or real-world elements and contextual forms of knowledge and participation.

On the issue of contexts, the CAPS document goes to great lengths to point at that the contexts that to be dealt with in the subject must be “authentic (i.e. are drawn from genuine and realistic situations) and relevant”. Furthermore, “Wherever possible, learners must be able to work with actual real-life problems and resources, rather than with problems developed around constructed, semi-real, contrived and/or fictitious scenarios.” (DBE, 2011a, p. 7). There is, thus, a deliberate attempt to ground the content of the subject in the real-world rather than in the mathematised world and to explicitly inhibit emphasis on the use of contrived contexts. Furthermore, the CAPS document also signals the need for learners to be able to make sense of both familiar and unfamiliar contexts and, so, prioritises the development of generalisable skills to facilitate this process:

> a *primary aim* in this subject is to equip learners with a set of skills that transcends both the mathematical content used in solving problems and the context in which the problem is situated. In other words, both the mathematical content and the context are simply tools: the mathematical content provides learners with a means through which to explore contexts; and the contexts add meaning to the mathematical content. But what is more important is that learners develop the ability to devise and apply both mathematical and non-mathematical techniques and considerations in order to explore and make sense of any context, whether the context is familiar or not. (DBE, 2011a, p. 8, emphasis in original text)

This statement posits the CAPS framework in line with the thinking of Brombacher (2007), de Lange (2003, 2006), and others who argue for the central role of competencies in the development of mathematically literate behaviour.[89] In so doing, the CAPS framework again tries to dispel some of the uncertainty and ambiguity expressed in the NCS and supporting documents. This is specifically in relation to the structure of legitimate participation in the subject and the structure of knowledge to be utilised to facilitate that legitimate participation. This is done by presenting a single and explicit conception of the central role of the relationship between content, contexts and competencies in the development of mathematically literate behaviour.

[89] In fact, Brombacher’s (2007) content-context-competencies diagram presented and discussed on page 66 above also features in the CAPS document (DBE, 2011a, p. 8).

8.3.2.4 Sense-making of contexts

Inherent in all discussion of content, contexts and competencies in the CAPS document is an apparent attempt to position contextual sense-making practices as a key goal in the subject. The phrase "explore and make sense of the context", and different variations of this phrase, appear incessantly, repeatedly and somewhat irritatingly in virtually every paragraph contained in the section of the CAPS document (i.e. Section 2) which describes the components of the subject.[90] This appears to be a deliberate attempt to move beyond the emphasis on 'problem-solving' in the NCS – with 'problem' commonly being associated with a mathematical component − and towards a genuine interest in investigating, exploring and understanding contexts and forms of participation that are considered appropriate and legitimate in those contexts. Once again this signifies a deliberate attempted shift away from the dominant agenda in the NCS on contextualised mathematics calculations and practices towards an agenda that prioritised sense-making practices with authentic and complex real-world environments.

8.3.2.5 Emphasis on a form of problem-solving characterised by modelling processes

Although no explicit statement of requirement for modelling-related practices is made in the CAPS document, there are several instances in the document in which learners are directed towards modelling processes in engagement with real-world scenarios. The first such instance appears in a statement of content in the section on Banking, Loans and Investments (in the Application Topic of Finance) where the instruction is given for learners to:

> Model loan and investment scenarios using a pen, paper, basic calculator and tables, spreadsheets, and/or available loan calculators (e.g. *calculators available on bank websites*). (DBE, 2011a, p. 56)

Interestingly, all of the other references to modelling-related practices are made in the context of suggested assessment tasks and are done so implicitly through an expectation for a structure of problem-solving that involves the recreation of a particular real-world practice through use of a variety of mathematical and contextual resources and techniques. As such, suggestions are provided for learners to:

> Plan a trip between two cities or countries, making use of maps, bus/train/taxi/flight timetables, tariff tables, exchange rates (if necessary) and the AA fixed, running and operating cost tables (if necessary). (DBE, 2011a, p. 75)
>
> Calculate the actual housing density for a suburb, settlement or township by comparing the number of people living in a household to the area of land occupied by the household; critique municipal housing density policies in terms of the findings of this project. (DBE, 2011a, p. 69)
>
> - Investigate the considerations involved in the construction of a house

[90] Being as pedantic as I am, I counted twelve such references on the three pages that provide a description of the components of the subject (DBE, 2011a, pp. 7-9). The total count of such references for the whole document is approximately equivalent to the most recent value of Pi.

- After interpreting the plans of a house, build a scale model and perform perimeter, area and volume calculations in the context of fencing, paint, concrete, etc.
- Analyse a budget for the building project
- Analyse inflation figures to predict possible adjustments to building costs. (DBE, 2011a, p. 78)

All of these statement direct participants in the subject towards engagement with a wide range of resources, contents and considerations to facilitate either the completion of a particular real-world task or the recreation of a segment of real-world practice in order to better understand the practice. I contend that this provides evidence of an expectation (albeit implicit) for a structure of legitimate and endorsed participation in the subject that is characterised by the capacity for modelling-related practices and processes.

However, there is also potential for confusion with this prioritised agenda. Firstly, there is no explicit and deliberate statement of expectation for participation in the subject to be characterised by modelling-related processes. Secondly, the majority of the implied statements of modelling-related processes only appear in suggested assessment practices and not in statements of curriculum contents that reflect classroom and pedagogic practice. This leads to the potential for inconsistency in the dominant agenda according to which the structure of endorsed participation is legitimised in assessment and pedagogic practices in the subject.

8.3.2.6 Communication

The CAPS document, unlike the NCS, also provides an explicit statement on the importance of communication as a facet of mathematically literate behaviour. Importantly, this communication involves not only narratives of mathematical working and solutions, but also narratives that reflect decision-making processes after consideration of both mathematical and non-mathematical issues at play in a context (DBE, 2011a, p. 8). In the CAPS framework, decision-making and communication are interrelated processes that work hand-in-hand to facilitate and reflect the structure of appropriate, effective and legitimate forms of functioning in real-world practices. This sentiment is shared by Frith and Prince (2006, pp. 53-54) who similarly emphasise the central role of communication and decision-making as an expression of mathematically literate behaviour. Emphasis on a form of communication that takes cognisance of both mathematical and contextual nuances and considerations points, once again, to a deliberate attempt in the CAPS framework to foreground both mathematical and conceptual components in problem-solving practices aimed at enhancing understanding of contextual situations.

8.3.2.7 Dominant intentions and agendas in the CAPS document

The discussion above has highlighted that within the CAPS framework – and at the level of curriculum intention − there is the promotion and prioritisation of a dominant ***Agenda*** for contextual sense-making practices (Agenda 4) and, to a lesser and more implied degree, the agenda associated with Modelling (Agenda 3). And, although there are elements of the CAPS framework that do emphasise the agenda associated with numeracy-based contextual calculations (Agenda 2 – specifically Agenda 2 [b]), these are prioritised to a far lesser degree than Agenda 3 and, particularly, Agenda 4. In the CAPS framework, any numeracy elements are always in service to a broader goal for contextual sense-making practices. This suggests a shift from the NCS to the CAPS along the spectrum of agendas towards increased prioritising of contextual sense-making of contexts over primarily mathematical considerations and forms of participation.

Interestingly, this shift in the CAPS framework towards a structure of legitimate participation in the subject characterised by increased prioritisation of an agenda for contextualised sense-making practices is contrary to the dominant agendas for modelling and contextualised applications in the majority of the international literatures on mathematical literacy, numeracy and/or quantitative literacy read for this study.

This area of divergence is easily explained. Despite the fact that Mathematical Literacy is positioned as a subject that is entirely separate from scientific mathematics, pedagogic and assessment practices in the NCS framework continued to prioritised participation in the subject primarily according to mathematical structures. As such, a key intention of mine in authoring the CAPS document and in defining a modified philosophy and agenda for the subject in this document was driven by the intention to move the subject further away from the domain of mathematics and from mathematical and mathematised forms of participation. Hence, the increased emphasis in the CAPS framework on the contextual terrain and on associated contextually legitimate forms of knowledge, participation and communication. If you now consider that the majority of international perspectives on mathematically literate, numerate and/or quantitatively literate behaviour do not call for a separation of mathematical literacy from the domain of mathematics, then it is logical, inevitable and appropriate that within those conceptions the envisioned pattern of behaviour is legitimated and organised around mathematical structures and forms of participation. It is the separation of the knowledge domain of mathematical literacy from the domain of mathematics in South Africa that has prompted necessary re-conceptualisations of the basis of legitimate participation in the subject and of the dominant structure of knowledge required to facilitate access that that legitimate participation. With this in mind, my assessment is that the conception of mathematically literate behaviour espoused in the CAPS curriculum is more closely aligned to the type of participation conceptualised in various international perspectives of adult numeracy than to perspectives of school-based mathematical literacy. Conceptions of adult numeracy generally prioritise engagement with mathematical contents as being in service to the solving and sense-making of contextualised problem scenarios – and the CAPS conception of Mathematical Literacy prioritises a similar form of participation.

Despite the attempt in the CAPS to shift the dominant agenda prioritised in the subject closer towards contextual sense-making practices, this attempt is largely thwarted by the continued prioritisation of mathematised practices and mathematically legitimised forms of participation in the national assessments. As discussed above in relation to the NCS framework, the national examinations for Mathematical Literacy – both in the past and also at present – continue to prioritise a near exclusive agenda for numeracy in context type calculations (Agenda 2 [b]). Agendas for modelling (Agenda 3) and, particularly, for contextual sense-making practices (Agenda 4) are virtually excluded from these national assessments such that participation in the assessments is endorsed almost entirely according to appropriate and legitimate mathematical structures, knowledge, routines and forms of communication.[91] Even the contexts employed are predominantly contrived or reconstructed mathematised representations of reality, with every effort made to enhance and facilitate access to the mathematical components of the contexts. In short, participants who understand mathematics will function successfully in these assessments; participants who understand the world will not. Considering that pedagogic practices in the NCS framework followed suit with the prioritising of an agenda for numeracy in context type practices to prepare learners for successful engagement in the assessments, it seems likely that a similar situation will occur in the CAPS framework. As such, it appears that the attempt at curriculum reform in the subject will, thus, be largely undermined by the inflexibility and unchanging agenda that dominates the national examinations.

At the level of ***Intention***, the CAPS follows suit in promoting a conception of mathematical literacy for the development of *Human Capital*, and continues to emphasise the role of the subject in enabling learners to become "a self-managing person, a contributing worker and a participating citizen in a developing democracy." (DBE, 2011a, p. 7). Interestingly, while the NCS framework also makes passing reference to most of the other intentions, the CAPS framework does not reference any other intention beyond an intention for Human Capital. This point is important in that it signifies that an intention for critically Evaluating Structures − which Jablonka (2003) argues is a necessary component of any conception of mathematical literacy that hopes to promote critical citizenship (c.f. page 46 above) – is absent in the CAPS structure. However, despite the absence of an *explicit* statement for this intention, there are a small number of statements of content in the curriculum that indicate a limited degree of the type of critical engagement characteristic of a Critical Evaluation intention. For example, "Evaluate and critique the validity of expressions and interpretations of probability presented in newspapers and other sources of information." (DBE, 2011a, p. 94).

WHERE TO FROM HERE

In Part 3 of this study I investigate the underlying basis for some of the claims and criticisms directed at current practices in the subject Mathematical Literacy. In doing so I bring to light the reasons why the continued legitimisation of participation in the subject according to mathematical structures is inhibiting the development of mathematical knowledge, future career opportunities, and life-preparedness. To do this I draw on the work of Paul Dowling (1998), and focus specifically on the elements of his theoretical language that highlight the forms of mythologising associated with the type of Public Domain practices that characterise the subject Mathematical Literacy.

[91] In Part 7 and Chapter 25 (starting on page 402) I demonstrate this state of affairs through detailed analysis of the 2014 Grade 12 National Exemplar Examinations for the subject Mathematical Literacy. However, in this analysis I argue that the dominant structure of knowledge in these examinations – as evidenced in the structure of the signifiers, routines and endorsed narratives that characterise the examinations − is reflective of a form of participation associated with of the Mathematical Competency domain of practice of the knowledge domain of mathematical literacy. In connecting this current discussion to the later analysis of the examinations, the analysis reveals that it is the agenda of Numeracy-in-Context prioritised in the examinations, which is characterised by and facilitated through engagement with a structure of knowledge that relies on a form of mathematical competency.

PART 3
THEORY-INFORMED ANALYSIS OF CURRENT PRACTICES IN THE SUBJECT MATHEMATICAL LITERACY

INTRODUCTION AND OVERVIEW

In this part of the study, I problematise the current structure of participation in the subject and argue that an alternative structure of participation is required that involves engagement with contextual sense-making practices.

Paul Dowling (1998) has theorised extensively on the relationship between academic (generally) and mathematical (specifically) knowledge and everyday and/or extra-mathematical knowledge and practices, and also problematises particular forms of this relationship. Given that the current structure of participation in the subject-matter domain of embodies a requirement for combined engagement with mathematical and everyday practices, Dowling's theoretical language is of direct relevance to the analysis (and problematising) of the current structure of knowledge and participation in the subject. As such, to facilitate this analysis, a major component of the discussion in the pages below is focused on elaborating key aspects of Dowling's theoretical language. Particular emphasis in this discussion is centred on his contention that particular forms of the relationship between mathematical and everyday practices in the context of the school system both promotes and sustains educational disadvantage in schooling and has implications for the successful (or unsuccessful) apprenticeship of learners into the discipline of school mathematics.

This part of the study is comprised of four chapters (Chapter 9 to Chapter 12). In Chapter 9 I provide a broad overview of Dowling's theory, with specific focus on the classification of practices, strategies and positions within school mathematics, and the implications of this classification for the successful apprenticeship of learners into mathematics. In Chapter 10 I engage with a concept that has particular relevance for the focus of my study, namely the relationship between mathematical and everyday and/or extra-mathematical knowledge and practices. The discussion in this chapter outlines Dowling's argument regarding the incommensurability of academic (including mathematical) and everyday or domestic practices, and the ways in which elements of educational disadvantage are (re)produced as a result of an emphasis on the need for relevance in mathematics education. Areas of coherence and divergence between Dowling's and Basil Bernstein's theories are also presented. In Chapter 11 I make use of Dowling's language of description as a lens through which to describe, analyse and problematise current practices in the subject-matter domain of Mathematical Literacy. Here I argue that Public Domain of mathematics practices dominate, with the consequence that participants in the subject are denied access to Esoteric Domain mathematical structures while at the same time only being exposed to mathematised and mythologised recontextualisations of everyday practices. The value, validity and currency of the learning process in this subject is, then, brought into question. In the final chapter (Chapter 12) I discuss perceived limitations of Dowling's language of description.

The diagram in Figure 18 below illustrates the structure of the contents of this part of the study.

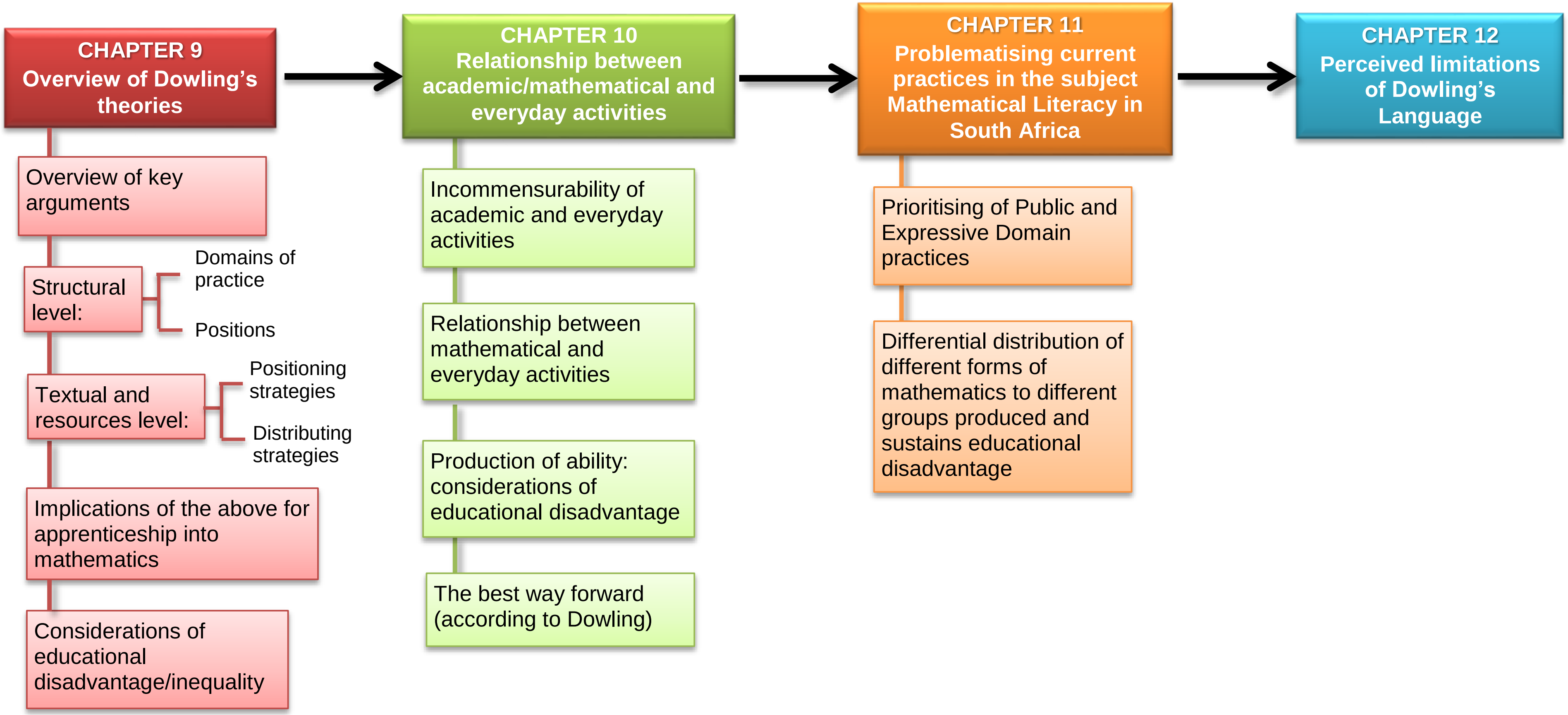


Figure 18: Overview and chapter structure of Part 3 of the study

CHAPTER 9
A BROAD OVERVIEW OF KEY ASPECTS OF DOWLING'S LANGUAGE OF DESCRIPTION

9.1 'Road Map' and broad overview of Dowling's language

In this section of the chapter I begin by presenting a diagrammatic overview of various components of Dowling's language of description, following which I outline and summarise certain key arguments in Dowling's theoretical language regarding the relationship between mathematical and extra-mathematical and/or everyday practices, and particularly regarding the incommensurability of this relationship. Many of these arguments are elaborated in more detail in later sections of the chapter.

Figure 19[92] below provides a 'road map' of the connections between the various components of Dowling's language of description discussed in the pages below. This figure is intended to illustrate visually the structure and direction of the intended discussion and, in so doing, to (hopefully!) help the reader to more easily navigate the contents of the chapter.

[92] It is essential to note that this diagram is by no means exhaustive and certainly does not contain all aspects and categories that Dowling describes in his theory. Dowling provides a more comprehensive diagrammatic representation of his theorising on page 148 of his book *The Sociology of Mathematics Education: Mathematical Myths / Pedagogic Texts* (1998). It is also important to note that although the diagram is presented in a linear way (to highlight the structure of the discussion contained in the pages below), this is in no way intended to suggest that the various components of Dowling's theory form a linear process. Rather, as Dowling consistently points out in his writings, the relationship between the textual and structural levels of his language of description are 'dialectical' (Dowling, 1994, p. 131; 1998, pp. 20, 132): texts influence positions and access to the domains of practice in an activity; but the structure of an activity will, in turn, influence the nature and contents of the texts that are developed for the activity.

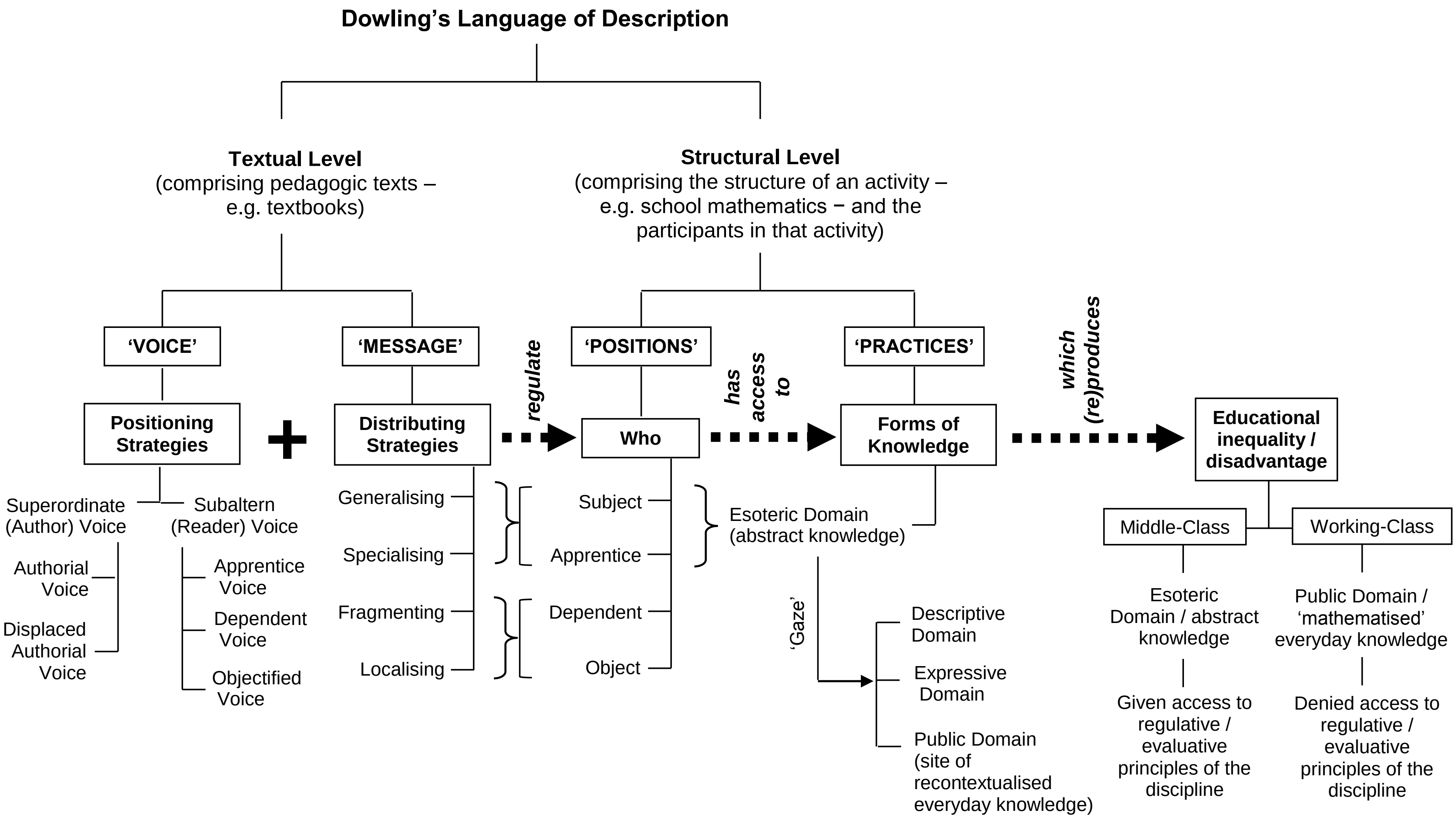


Figure 19: 'Road Map' of the structural and textual levels of Dowling's language of description

Dowling's theory constitutes a "language of description" (Bernstein, 2000, p. 132; Dowling, 1994, p. 124) for describing the transmission (and acquisition) of knowledge and how this transmission promotes or sustains class inequalities and/or educational disadvantage in schooling. In particular, Dowling's theory is focused generally on the relationship between academic and everyday activities[93] and specifically on the relationship between abstract mathematical knowledge transmitted in the context of schooling and the techniques and strategies used to solve problems in the everyday or extra-mathematical world and/or daily life. A central argument in Dowling's language of description is that academic (generally) and mathematical (specifically) activities are incommensurate with everyday activities. Namely, that academic or mathematical knowledge cannot be used as a theory for making sense of everyday practices and that attempts to enhance understanding of academic or mathematical knowledge through the inclusion and incorporation of everyday contexts limits rather than facilitates the development of abstract knowledge (Dowling, 1995a, p. 9; 1995b, p. 209). In alternative terms, there is a disjunction between the knowledge and routines used to solve 'real-life' problems in the classroom and those that are actually used in the real-world (Dowling, 2010c, p. 9). This situation is compounded in the context of the schooling system where mathematics focussing on relevance (and associated texts) are commonly made available to lower ability students (who are often from predominantly working-class backgrounds) while abstract mathematics is made available to higher ability students (often from predominantly middle-class backgrounds) (Dowling, 1994, p. 138; 1998, pp. 236-241; 2010a, Slide 2; Hoadley, 2007, p. 684). The inclusion of relevance in mathematics in the school context, then, produces and promotes educational difference and disadvantage.

In arguing for the incommensurability of mathematical and everyday activities, Dowling is not so much denying the applicability of mathematics for solving real-world problems. Rather, Dowling is problematising the inclusion of everyday and/or extra-mathematical contexts in the *school mathematics classroom* to enhance the view (or what Dowling would call the 'myth' (Dowling, 1998, p. 33)) that mathematics is useful for making sense of the everyday world. The everyday world that students are presented with in the mathematics classroom is not a replica of the real thing; it is a virtual, "mathematised" reality (Dowling, 1998, p. 33). Thus, the students who are exposed to this reality − most commonly students from working-class backgrounds who have been classified as being of lower ability − do not develop an enhanced understanding of the everyday; nor do they develop a comprehensive understanding of the mathematics since the inclusion of the everyday is at the expense of a focus on abstract mathematical concepts (Dowling, 1995b, p. 219). A focus on relevance and the inclusion of the everyday in the school mathematics classroom, then, subjugates certain groups of students to a position of complete dependency on the teacher to interpret both the everyday and the mathematical, and to the specialised role of master-of-none.

In developing a theoretical language of description of the nature of the relationship between mathematical and everyday or extra-mathematical activities, Dowling argues that different forms of knowledge are contained in different *domains* of practice (or action). Furthermore, these forms of knowledge are distributed through different *strategies*, primarily embedded within pedagogic texts (e.g. textbooks), that reproduce features of the different domains and provide or restrict access to the different forms of

[93] Dowling (1995b, p. 214) uses the term 'activity' to encompass both knowledge and practices that may occur within a particular context or problem. The term activity as used in this chapter encompasses both of these concepts.

knowledge associated within each domain (Dowling, 1998, pp. 145-151; Hoadley, 2006b, pp. 8-10; 2007, pp. 683-684). Different strategies thus provide access to different domains of practice and the knowledge contained in those domain. Within the schooling system, these differing domains and strategies are made accessible to groups of students with differing abilities who are commonly from different social-class background, hereby producing and reproducing educational disadvantage. Dowling's language of description, then, provides an account of the conditions under which successful 'apprenticeship' into the academic discipline of mathematical knowledge takes place.

In line with the discussion above, Ensor and Galant (2005) provide the following description of key facets of Dowling's theory:

> Dowling (1998) extended Bernstein's notion of classification to provide a useful model for categorising different kinds of mathematical statements, probing apprenticeship and apprenticing strategies, and providing an account of the relationship between school mathematics and other domains of practice. (p. 291)

The discussion above comprised a broad synopsis of key aspects of Dowling's theoretical language of description of the relationship between mathematical and extra-mathematical and/or everyday knowledge and practices. The remainder of this section provides a more detailed discussion of aspects of this language of description that constitute mathematics as an activity and the practices, positions and strategies contained therein.

9.2 Mathematics as an 'Activity'

Mathematics, according to Dowling (1998), constitutes an 'activity':

> An activity is a structure of relations and practices which, essentially, regulates who can say/do what. It constitutes positions which are always hierarchical (although not necessarily simply hierarchical). The practices of the activity are distributed within this hierarchy. Activities are produced by and reproduced in human subjects – who move, routinely, between activities – and by texts. (p. 120)

An activity such as school mathematics, then, is comprised of *practices* and *positions* that determine who (positions) can say and do what (practices), and *strategies* that distribute the practices and produce and reproduce positions within the activity. The *practices* of an activity refer to the contents of the activity: i.e. what happens in the activity. The practice of school mathematics, for example, involves the teaching and learning of mathematical content. The *positions* of an activity refer to the different positions available to the participants in the activity. These include the position of Subject (in school mathematics this position is commonly filled by the teacher) and Apprentices, Dependents or Objects (these roles are commonly filled by the students) (Dowling, 1998, p. 20 & 122). *Strategies* are the techniques used which provide or deny access to different practices within an activity (Dowling, 1998, p. 20 & 145) – and in Dowling's empirical analysis these strategies were found in the 'texts' (specifically textbooks) used by the teachers and students in the pedagogic process.

With the above in mind, Dowling distinguishes three levels in his language of description. The *structural level* outlines the <u>positions</u> filled by human individuals in an activity and the <u>practices</u> that those individuals participate in within the activity. The *textual level* outlines how texts (e.g. school mathematics textbooks), through employing various positioning and distributing strategies, provide differential access to different practices and positions in the activity (Dowling, 1998, pp. 131-132). And the *resources level* which describes the various textual resources (or what Anna Sfard (2008) refers to as 'discursive resources') which embody and facilitate positioning and distributing strategies to particularise/localise or abstract/generalise situations and problems, hereby providing or restricting access to the different domains and practices within an activity. These textual resources include photographs, pictures, drawings, cartoons, graphs, tables, maps, plans, symbols, terms, and techniques (Dowling, 1998, pp. 150 - 154). The relationship between these three levels is 'dialectical': texts directly influence the structure of an activity, but in turn are also influenced and shaped by the practices and attitudes of the human individuals who fill positions in the activity (Dowling, 1994, p. 131; 1998, p. 132)

It is the structural level of Dowling's language of description that provides a particularly useful means for analysing and problematising existing practices in the subject-matter domain of Mathematical Literacy. For this reason, primary emphasis in the discussion below is placed on elaborating the components of this level and of the relevance of these components to the empirical terrain of the subject-matter domain of Mathematical Literacy.

9.3 The Structural Level of Dowling's language of description − Domains of Practice and Positions

9.3.1 Domains of Practice

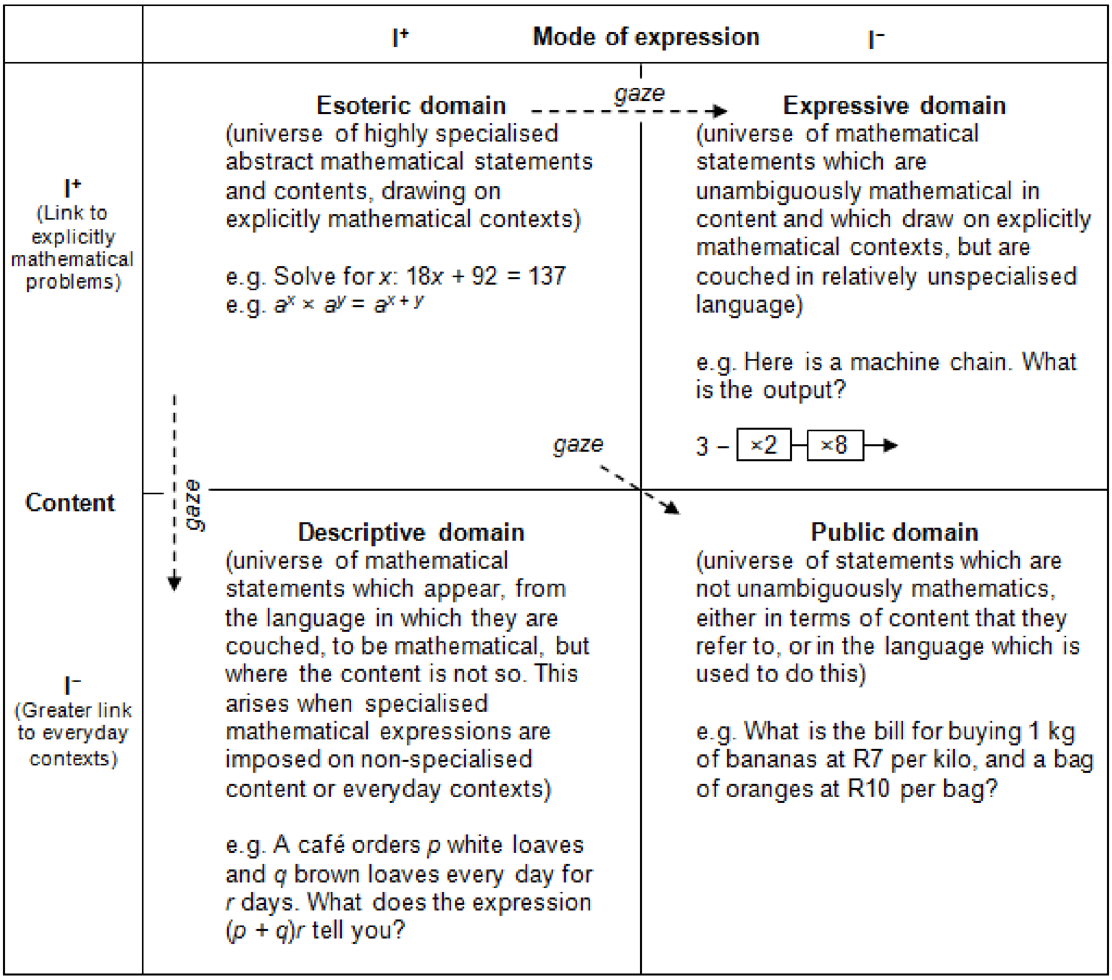
Dowling argues that knowledge in the context of an activity can be either strongly or weakly associated, classified, or institutionalised with specialised practices.[94] In specific relation to school mathematics, knowledge in the mathematics classroom is either strongly or weakly institutionalised according to specialised mathematical principles. This association with mathematics occurs in relation to both: the *content* of a message – including not only knowledge and/or skills but also, according to Sethole, et. al., (2006, p. 119) , to the nature of the context – i.e. mathematical or everyday – from which the content is drawn; and the *expression* of the message – namely, how the message is

[94] Dowling originally used the word 'classification' to refer to association with mathematical knowledge (1995b, 1998), but later altered this to refer instead to the 'institutionalisation' of knowledge with respect to mathematics (2008; Dowling, 2009a, 2009b). This change signifies a break with Bernstein's notion of classification, where classification refers to the relations between different educational contents, contexts or categories and the degree of boundary maintenance between those categories (Hoadley, 2007; Maton & Muller, 2007; Singh, 1997). In contrast, Dowling uses the word 'institutionalisation' to refer to "the extent to which a practice exhibits an empirical regularity that marks it out as recognisably distinct from other practices (or from a specific other practice)." (2009a, p. 13).
Bernstein presents a general theory with a high level of abstraction and which can be taken to refer to a variety of sites, including the curriculum, the structure of the school, and forms of knowledge. Dowling, by contrast, narrows the theoretical description to focus specifically on the relationship and distinction between empirical practices such as school mathematics, and positions, relationships and forms of knowledge within those practices. Furthermore Bernstein employs the term classification to refer specifically to a boundary between categories (e.g. between Mathematics and another subject-matter discipline) – and the implication of that boundary on power relations. Dowling, in turn, argues that classification strength is not a fixed quality of a subject-matter domain such as mathematics and, rather, that different elements of the domain can be differently classified according to both or either of content and expression (Straehler-Pohl & Gellert, 2013, p. 319).

transmitted and the signifiers (including words/method/language) used to transmit the message.

Strong institutionalisation (I^+) of expression and content are, thus, characterised by explicit signification of specialised mathematical language, terminology, visual mediators, contents, skills, routines and explicitly mathematical contexts. Weak institutionalisation (I^-), by contract, is characterised by restricted signification of these components together with signification of extra-mathematical and/or non-mathematical elements and contents (Dowling, 2009b, p. 15) (Sethole et al., 2006, p. 119).

Differing possible combinations of strong and weak institutionalisation of mode of expression and contents gives rise to four possible 'domains of practice' within the terrain of mathematics: Esoteric Domain, Public Domain, Expressive Domain, and Descriptive Domain (see Figure 20).



Adapted from Ensor and Galant (2005, p. 292) and Dowling (1998, p. 135).

Figure 20: Domains of mathematical practice

The *Esoteric Domain* is the region of a mathematical activity that is strongly associated with specialised mathematical knowledge with respect to both content and expression. In all respects, activity within this domain contains explicit mathematics and the use of abstracted and generalisable mathematical principles, signifiers, routines and narratives: "the esoteric domain comprises the specialised forms of expression and content which are unambiguously mathematics." (Dowling, 1994, p. 130). According to Dowling, this is the domain that contains what is seen to be legitimate mathematical content, language, actions and practices (P. Dowling, 2008, p. 4). As such, the mathematical principles that regulate the practices of the activity and against which the practices of the activity – and participation in the activity − are legitimised and endorsed are explicit in this domain (Dowling, 1994, p. 129). Furthermore, Dowling (1998) argues that it is only in this domain that full access to these regulative principles is possible:

> Because ambiguity is minimised in the esoteric domain, specialised denotations and connotations are always prioritised. It is, therefore, only within this domain that the principles which regulate the practices of the activity can attain their full attention. The esoteric domain may be regarded as the regulating domain of an activity in relation to its practices. (p. 135)

As such, it is in this domain of practice that students must engage if they are to be successfully apprenticed into the position of master (Dowling uses the word ‘subject’) (1998, p. 140) of the activity (Dowling, 1998, p. 141; Ensor & Galant, 2005, p. 297).

Although the Esoteric Domain contains the ‘non-negotiable’ aspects of school mathematics, school mathematics contains more than just this strictly mathematical component. Rather, teaching and the role of the teacher (‘pedagogic theory’ in Dowling’s terms) (P. Dowling, 2008, p. 3) has an impact on how mathematical knowledge is transmitted, and it often through the teaching component that attempts are made to cast a gaze outside of the Esoteric Domain and to establish links between the Esoteric Domain and the extra-mathematical and/or everyday world: “The practice [mathematics] must also constitute a more weakly institutionalized region in order to permit entry into it; this is the *Public Domain*.” (Dowling, 2010a, Slide 2, emphasis in original slide). The result of this ‘mathematical gaze’[95], as everyday settings are brought into the mathematics classroom and recontextualised according to mathematical principles[96], is the development of the *Public Domain* of school mathematics as a collection of recontextualised and reformulated or ‘mathematised’ problems (P. Dowling, 2008, p. 4). As Dowling (2010a) suggests,

> In the case of school mathematics, the public domain seems to comprise a collection of everyday activities (such as shopping and other domestic practice) that have been re-shaped, re-contextualised to conform to mathematical principles. (Slide 2)

Practices and problems in the Public Domain are generally weakly institutionalised in terms of both content (I^{-}) and expression (I^{-}) (Dowling, 1998, pp. 135-136). In other words, the problems in this domain appear to be about something other than mathematics, are not overtly mathematical, and, although it may be obvious that there is mathematics in the problem, the focus appears to be on something other than the mathematics (Dowling, 1998, pp. 135-136). Furthermore, despite the fact that the problems in this domain are recontextualised according to mathematical principles, the regulative and evaluative mathematical principles underpinning the activity are not explicit and rather are ‘hidden’ by the everyday and/or extra-mathematical signifiers and elements (Dowling, 1998, pp. 135-136).

[95] A more detailed discussion of the notion of the ‘mathematical gaze’ and associated mathematical myths – the Myths of Reference, Participation, and Emancipation – are provided in sub-section 10.2.2 below starting on page 156.

[96] It is in relation to this notion that Straehler-Pohl and Gellert (2013, p. 320) state that the key concepts inherent in Dowling’s domain of practices model are *gaze* and *recontextualisation*.

It is essential to insert an intervening comment here. In using Dowling's theories, some have misconstrued the locality of this Public Domain and have equated the Public Domain to the everyday or Public Domain knowledge to everyday knowledge (c.f. (Bernstein, 1999, p. 170, Note 1; Hoadley, 2007, p. 683)). Dowling's usage of the term refers to the space where everyday contexts, problems, practices, and knowledge are recontextualised according to abstract esoteric principles, practices and knowledge. The Public Domain is the space where the everyday represents a *virtual* reality of life, a *mathematised* reality of life. The Public Domain is not the everyday; it is a misrepresentation of the everyday.

The remaining two domains – the *Expressive and Descriptive Domains* – are also the result of the imposition of a 'mathematical gaze' from the Esoteric Domain on the everyday world, and both represent an alternative form of recontextualisation than that which occurs in the Public Domain. In the Expressive Domain, non-mathematical language (expression) is embedded and foregrounded within an explicitly mathematical context and is used to signify and give expression to mathematical content (Dowling, 1998, p. 135) – for example, where fractions are equated to pieces of cake and equations to a seesaw, scale or balance. In the Descriptive Domain, the situation is reversed as mathematical language is used to describe non-mathematical content. For Dowling, this is the domain of mathematical modelling as mathematical concepts and tools (e.g. sine curves) are employed in generating descriptions of extra—mathematical concepts (e.g. wave or tidal motion). Importantly, the regulative and evaluative principles of the Esoteric Domain cannot be fully realised in practices that remain in either of these domains due to the 'interference' of the extra-mathematical components: "the esoteric domain must signify differently because of the recruitment of a non-mathematical setting, so that, once again, the principles of the esoteric domain cannot be made fully explicit within [these] domain[s]." (Dowling, 1998, p. 137).

Having described the various components of each of the domains, what remains is to identify the different uses and usefulness of this model. Ensor and Galant (2005, p. 291 & 293) argue that the model is powerful for three reasons. Firstly, the model provides a tool for analysing the classification of the contents and/or discourse of an activity such as school mathematics. For example, the model could be used to determine the extent to which a particular text (e.g. an exam or a textbook) employed in the activity of school mathematics privileges certain domains of practice over other domains. Secondly, the model provides a framework for discussing the relationship between mathematics and out-of-school practices, and particularly between Esoteric Domain mathematical content and everyday problems and situations (P. Dowling, 2008, p. 4). In this regard, the model illustrates how the Esoteric Domain contains the non-negotiable content, language and mathematical knowledge of school mathematics. The three other domains (Public, Expressive and Descriptive) are, then, the result of differing interactions between a 'mathematical gaze' cast from the Esoteric Domain and everyday extra-mathematical practices. The third and final utility of the model is that it illustrates the process for apprenticeship into mathematics. This topic of 'apprenticeship' – and, more specifically, of different 'positions' that exist within the domains of practice − requires more elaboration and is dealt with in detail in sub-section 9.3.2 below.

In brief summary, in the discussion above it was posited that mathematics as an activity is comprised of different domains of practice, with each domain differentiated in terms of the extent to which knowledge in that domain involves engagement with weakly or strongly institutionalised mathematical contents. However, according to Dowling (1998, p. 131), activities are not neutral entities and, rather, construct *positions* in the activity in relation to how knowledge and practices are distributed to different participants in the activity and also in different domains of practice. And it is to the topic of 'positions' in the activity of mathematics that the discussion now shifts.

9.3.2 Positions

Dowling (1998, p. 140) identifies four main positions within an activity: Subject, Apprentice, Dependent and Object. The *Subject* is the most dominant position in an activity: this is the position that has mastered the practices and regulative principles of the activity. Every other position is then, to a greater or lesser extent, subordinated to and/or objectified by the Subject position (Dowling, 1998, p. 140).

The *Apprentice* position: "The activity, in effect, regulates 'who' can say or do or mean 'what'. Clearly, the activity must provide for the generation of new subjects. … the process of subject generation is appropriately referred to as 'apprenticeship'." (Dowling, 1998, p. 140). In other words, participants in the Apprenticeship position engage in actions with the intention, at some point in the future, to become potential Subjects of the activity:

> Successful apprenticeship to an activity is achieved (metaphorically) upon the completion of a one-hundred-and-eighty-degree rotation of the apprentice who thereby 'moves' from 'outside' to 'inside' the activity and becomes its Subject. (Dowling, 1998, p. 123)

Importantly, apprenticeship in mathematics involves successful engagement with the Esoteric Domain contents of the discipline, which, in turn, facilitates the capacity for defining mathematical structures and generative principles according to which a mathematical gaze can be cast over everyday practices. In other words, Apprentices are invited to participate in recontextualising activities that facilitate the development of Public Domain activities.

The third position, the *Dependent* position, is a subordinated position to the Apprentice in respect to the Subject. In this position, the participants are exposed to mathematical and/or mathematised and recontextualised practices, but where the structure and regulating principles of the activity are decided and imposed by the Subject. As a result, participants in this position are not directly exposed and do not have independent access to the regulating principles. Instead, they are 'dependent' on the Subject to make visible these regulating principles – since it is the Subject that determined the criteria for and principles of mathematisation and recontextualisation. Participants in the Dependent position are not construed as potential future Subjects and, as a consequence, the final 'career' outcome of this position is less certain: the Apprentice becomes the subject, but there is no certainty what the Dependent will become (Dowling, 1998, p. 141). This position is different from the Objectified position (see below) in that a Dependent may be fully aware that they are operating outside of the everyday world and that encountered problems are mathematical in nature, but is still reliant on the Subject of the activity to interpret and make explicit the regulating principles of the activity.

The fourth position is the *Objectified* position. This position relates primarily to Public Domain practices that have been recontextualised – through a 'mathematical gaze' − according to the principles of the Esoteric Domain. When practices are recontextualised in this way, positions must be created within the recontextualised practices; and participants in the learning process are invited to recognise themselves in these positions in the problems, as though the problems are their own and relate directly to their lives: students are invited to become *objects* in the problems (Dowling, 1996, p. 402). Participants positioned as Objects in such recontextualised Public Domain practices have no control or independent access to the regulative principles of the practice and are again reliant on the Subject to make these principles visible and explicit. Furthermore, because the recontextualised practice is only able to reflect a mythologised version of the actual practice and because the mathematical principles are hidden or obscured in the practice, within the Objectified position the students neither learn sufficiently about mathematics or about extra-mathematical contents (Dowling, 1998, p. 141).

The discussion above has identified different positions which characterise practices associated with the activity of mathematics. In the immediate discussion below, the way in which these positions are determined and distributed to different groups of participants in the activity is explored in brief.

9.4 Brief discussion of components of the 'Textual' and 'Resources' levels of Dowling's language of description[97]

Dowling argues that it is commonly, but not exclusively, through the pedagogic texts associated with an activity that the practices of an activity are determined and positions within that activity are defined: "All texts (re)produce, in part, the practices of an activity (or activities) of which they are utterances." [98] (Dowling, 1998, p. 131) and "Thus the practices and subject positions in an activity are instantiated in pedagogic texts" (Dowling, 1994, p. 131). Analysis of the texts associated with an activity, thereby, provide insights into the nature and structure of the activity, the structure of knowledge in the activity, and the way in which that knowledge is differentially distributed to different groups of participants in the activity (Dowling, 1998, p. 1 & 132).

The interplay between the positions and practices of an activity is achieved through various *strategies* (positioning and distributing strategies) employed within the text. Strategies are employed through the inclusion in texts of specific textual resources (c.f. the 'Resources Level' of Dowling's theoretical language) (e.g. visual mediators such as photographs, cartoons, pictures, symbols; and words/vocabulary that provide instructions and direction and which index particular routines to be engaged with). These textual resources distribute the message of the text to the reader in way that either generalises, specialises, fragments or localises the message and its contents. This, in turn, positions the reader in a particular voice in the text and locates the reader in a particular position in the activity (apprenticed voice facilitates Apprenticeship; dependent voice facilitates Dependency; and objectified voice facilitates Objectification). In overly simplistic terms, the 'resources' and 'textual' levels of Dowling's language of description

[97] The pre-examination version of this thesis contained a considerably more detailed discussion of the textual and resource levels of Dowling's theoretical language, of the components that characterise these levels, and of how the three levels of the theoretical language intersect. This discussion is available on a document of supplementary materials supplied for download here (and/or on a CD attached to the book version of this thesis). Alternatively, see Dowling (1998, pp. 143-147) for an in-depth discussion of the notions of 'message' and 'voice' in a text, distributing and positioning strategies, and the relation of these to the identified categories of positioning.

[98] Dowling uses the term '(re)produce' to signify "the dialectical nature of production/reproduction" (Dowling, 1998, p. 20). In the context of texts, (re)produce signifies that while texts may influence the positions and practices in an activity, the activity in turn influences the format and structure of the texts that are developed for the activity (Dowling, 1994, p. 131; 1998, p. 143).

intersect in the following way. Resources or signifiers that initiate a strong visual code of presence[99] in a text invite the reader to identify with the problem, hereby initiating a localising or fragmenting of a problem, and the positioning of the reader in the dependent or objectified voice. Resources with no visual code of presence, by contrast, interpellate the reader as subject to the authority of the text, embodying generalising or specialising strategies, and position the reader in the apprenticed voice. For example, the inclusion of a photograph in a text leads the reader to believe that the problem is real and that they are an active participant in solving the problem. The photograph localises the problem to a specific context (i.e. the context shown in the photograph). The inclusion of mathematical symbols and operators or geometric diagrams, on the other hand, immediately generalizes the problem, for the reader, beyond a specific context or location.

In brief summary, the discussion above highlighted Dowling's contention that mathematical texts employ strategies which differentially position participants in a mathematically related activity and, in so doing, afford varying degrees of access to different forms and domains of mathematical knowledge and practice. These strategies are largely embodied in the deliberately employed textual resources that characterise the text. In the next section I elaborate on the interplay between the structural, textual and resources levels of Dowling's theory and highlight the implications of this interplay for successful apprenticeship into the discipline of mathematics.

9.5 Considerations of (successful) apprenticeship in mathematics

In addressing the issue of apprenticeship in the discipline of mathematics, Ensor and Galant (2005) pose and answer the following question:

> How do the structuring of mathematical knowledge, and the relationship between different sites of practice, impact on the nature of apprenticeship, that is, on how we induct learners into mastery of mathematics? Such mastery is achieved in our terms when learners … have grasped the 'generative principles' (Dowling 1998) of whatever discourse they have been induced into, and are able to produce appropriate learning performances. (p. 297)

Furthermore:

> Apprenticeship of students into mathematics, in Dowling's terms, involves the successful move from Public to Esoteric Domain. Interruption of this trajectory inhibits students' ability to master mathematics. (Ensor & Galant, 2005, p. 297)

In other words, both Ensor & Galant and Dowling (1998, p. 140) contend that apprenticeship in school mathematics and, hence, eventual mastery in the subject, is only possible if participants have access to Esoteric Domain contents. Given that in this domain the content and expression of all messages are explicitly mathematical, exhibiting vertical discourse and texts and language containing high discursive saturation[100], students with access to this domain have access to the regulative principles that underpin the content. Here they gain access to generalising strategies that can be applied to a variety

[99] Dowling (1998, p. 153) uses the phrase 'Visual code of presence' to refer to the extent to which a textual resource invites a reader to identify with the problem (or with the context and/or characters in a problem) and into believing that they are an active participant in solving the problem.
[100] See sub-section 10.1.1.1 on page 149 below for an elaborated discussion of the concept of discursive saturation.

of problems and contexts (via the imposition of a 'mathematical gaze'), and here they have the potential to develop understanding of mathematical concepts.

And this, in turn, is only possible if students are exposed to texts that provide access to Esoteric Domain knowledge and the principles underpinning this knowledge. In other words, students only gain access to the position of Apprentice if they are positioned, by the texts that they encounter and by the teachers administering those texts, in an apprenticed voice and to distributing strategies that generalize and specialize mathematical knowledge (Dowling, 1998, p. 149). And, by gaining access to this Esoteric Domain knowledge and the principles that underpin this knowledge, Apprentices are then able to participate in and dictate the criteria for recontextualisation practices through the imposition of a mathematical gaze on everyday practices.

Fragmenting and localising strategies employed in texts have the opposite effect. These strategies restrict access to Esoteric Domain knowledge and, rather, foreground localised, context-dependent techniques that draw on unspecialised knowledge and meanings. Students exposed to these types of strategies are, thus, restricted to the Public, Expressive and Descriptive Domains of practice and to associated texts and discourse exhibiting dependency or objectified voices. Within these domains the criteria of the imposed recontextualising gaze are invisible and the presence of non-mathematical elements render the mathematical principles underpinning the practice less visible or even invisible. As such, participants who are given access to Public, Expressive and/or Descriptive Domain practices through fragmenting and/or localising strategies are more likely to be positioned as Dependents or Objects.

However, this does not mean that the teaching of mathematics should confine itself only to the Esoteric Domain. Rather, Dowling argues that potential subjects for an activity are always attracted to an activity through the Public Domain: "The public domain is, in this sense, the principal arena in which an activity selects its apprentices." (Dowling, 1998, p. 149). As such, if no projection is made from the Esoteric Domain to the Public Domain, then no new apprentices will be "hailed" into the activity (Dowling, 1998, p. 141). The move from the Public Domain to the Esoteric Domain is also not a linear process. Rather, the Expressive Domain of practice provides a bridge from the Public to Esoteric Domain. Conversely, the Descriptive Domain provides a bridge from the Esoteric to the Public Domain:

> There is no natural route into the esoteric domain of mathematics … Nor, of course, can mathematics education begin and remain exclusively in the esoteric domain; there has to be a way in and this will always be via the public domain. Pedagogic action must then construct trajectories that lead into the esoteric domain via the expressive and that lead to the public domain from the esoteric via the descriptive. … in general, in respect of any specialist region of mathematics, the whole of the map should be traversed in one way or another. (Dowling, 2009b, p. 27)

Despite the limitations of practices that remain exclusively within the Expressive, Descriptive and Public Domains, then, these domains are an essential part of apprenticeship in mathematics. Students are attracted to mathematics through the Public Domain; the Expressive Domain provides a bridge from the Public to the Esoteric Domain; and the Descriptive Domain provides a bridge from the Esoteric to the Public Domain. Successful mathematics teaching, thus, involves facilitating a journey that begins in the Public Domain, moves through the Expressive Domain into the Esoteric

Domain, and returns to cast a new gaze on everyday problems in the construction of new Public Domain problems through the use of Descriptive Domain practices.

This described space and movement through the space is illustrated in Figure 21 below.

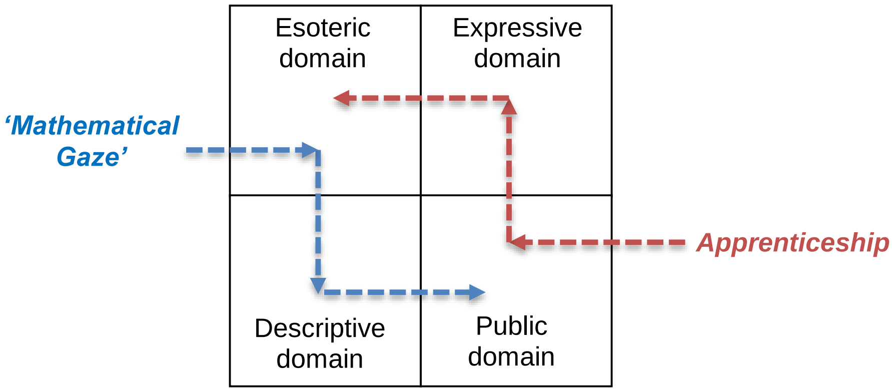


Figure 21: Apprenticeship in mathematics

As summarised by Dowling (1998, p. 141): "In thus establishing an apprenticed position as a limited subjectivity with respect to any region of the esoteric domain, the apprenticed position will have undergone what can, metaphorically, be described as a 180-degree rotation from the public to the esoteric domain."

CHAPTER 10
THE RELATIONSHIP BETWEEN ACADEMIC AND MATHEMATICAL KNOWLEDGE AND EVERYDAY NON-MATHEMATICAL ACTIVITIES

The discussion above provided a detailed overview of the space occupied by certain aspects of Dowling's theoretical language of description. This chapter provides a more focused discussion, primarily concerned with Dowling's thoughts on the relationship between both academic and mathematical knowledge and everyday activities. The discussion in this chapter has particular relevance to my broader study on the conceptualisation of what it means to be mathematically literate in the South African context and the classification of what counts as legitimate knowledge in the subject-matter domain of Mathematical Literacy, where the interplay between mathematical knowledge and everyday practices is a key consideration.

One of Dowling's primary contentions relates to the incommensurability of academic and everyday activities in general terms, with specific focus on the relationship between mathematical and everyday activities.[101] In particular, Dowling argues that it is not possible to use academic (generally) and mathematical (specifically) knowledge, practices and discourse to make sense of everyday practices without affecting the authenticity of the view and experience of the everyday practice that arises out of such an interaction: "the widely held belief that school knowledge can be made relevant to the everyday and to working practices in any direct sense is, I shall argue, mythical" (Dowling, 1995b, p. 209). I present his argument in this regard in two parts. The first focuses on a discussion of the relationship between academic and everyday activities in general terms. The second provides a narrowed discussion with a specific focus on the relationship between mathematics as an academic discipline and everyday activities. In both parts of the discussion I present arguments positioning academic/mathematical and everyday activities as constituting fundamentally different domains of practice and highlight the subordination of everyday activities to the principles of the academic/mathematical in interactions between these two activities.

10.1 The incommensurability of academic and everyday knowledge

10.1.1 Academic and everyday activities as fundamentally different domains of practice

In positing everyday and academic knowledge and practices as fundamentally different domains of practice, Dowling (1995b) makes the following claim:

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[101] Dowling uses the term 'academic knowledge' to refer to 'abstract thought' (Dowling, 1995a; 1998, p. 87), 'esoteric knowledge', 'school(ed) knowledge' or 'specialised knowledge' (Bernstein, 1999, p. 159) – namely, knowledge or practices that contain a high level of abstraction and/or Esoteric Domain contents. He also refers to practices that arise in the context of such knowledge as 'intellectual' practices (Dowling, 2010c, p. 1). The word 'everyday', on the other hand, refers to 'concrete' (Dowling, 1998, p. 87) or 'common-sense' knowledge, "common because it applies to all, and common because it has a common history in the sense of arising out of common problems of the living and dying." (Bernstein, 1999, p. 159). Practices arising from such everyday contexts are described as 'domestic' and/or 'manual' practices (Dowling, 1998, p. 104).

> The proposition that I wish to develop is that the division of labour constitutes two distinctive modes of social relations. These modes generate, respectively, academic and everyday practices and knowledges which are, thereby, mutually incommensurable. (p. 209)

Inherent in the quotation above is the argument that the academic and the everyday constitute distinct regions of practice and fundamentally different domains of practice (Dowling, 2009b, p. 8), and that each domain of practice is a separate system that is comprised of different actors, social relations, structures, practices, and criteria for legitimate participation and communication (Dowling, 1998, p. 24). Knowledge within each of these domains is, therefore, differently acquired and employed (Ensor & Galant, 2005, p. 286). As suggested by Wheelahan (2010, p. 76), while the purpose of academic disciplines is to produce knowledge about the objects that they study, everyday knowledge is developed through the strategies and techniques that people employ while they seek to deal with issues and solve problems that are important to them in their everyday lives.

Importantly, given the distinctive nature of these different domains of practice, transfer of knowledge, practice and language from one domain to another is not a linear process. Rather, knowledge, practice or language must be 'recontextualised' as it moves from one domain of practice to another (Dowling, 1998, p. 24; Ensor & Galant, 2005, p. 286). As P. Dowling (2008) argues,

> For some time, now, I (and I'm certainly not alone in this) have been arguing that the meanings of utterances and other actions do not carry over between different contexts and that what defines a context as such is the nature of the alliance and/or opposition in respect of which an utterance (or re-utterance) or action (or re-action) stands as a tactic. So utterances and actions are recycled within contexts − sometimes achieving status as slogans − and between contexts as resources for different, often quite different, purposes; the result is a recontextualising of the source utterance or action. (p. 1)

This opinion is also shared by both Walkerdine and Ensor & Galant:

> I argue that this may be the same signifier [i.e. the use of the word 'more'] as in the practices of the home, but it is not the same sign. … I argue further that such signifiers are made to signify when united with a signified within a particular practice, from which they take their meaning. Such practices are discursively regulated with the participants positioned in particular ways. The production of mathematical signs within practices renders them at once both socially and historically specific. (Walkerdine, 1990, p. 53)

> It [the paper] has gone on to suggest that we should understand the relationship between different sites of practice not as 'transfer' but as recontextualisation, a process which delocates, relocates and reconfigures forms of knowledge in terms of the social imperatives, identities and internal structuring of different sites. (Ensor & Galant, 2005, p. 297).

Dowling characterises the distinctive nature of academic and everyday knowledge and practices using the concept of 'Discursive Saturation' and links this concept to Basil Bernstein's concepts of 'Vertical and Horizontal discourses'. These concepts are now discussed.

10.1.1.1 Discursive Saturation

Discursive Saturation refers to the extent to which an activity – and, hence, the structure of legitimate and endorsed participation in that activity − is regulated by language, most commonly evidenced in the pedagogic texts used in the teaching and learning process: for example, curriculum documents, textbooks, and assessments. Activities in which the structure of legitimate participation is highly regulated by language use are considered to have strong discursive saturation (DS^+). Academic disciplines, such as high school mathematics, would be classified as exhibiting DS^+. This is because the language and associated word/vocabulary signifiers (or what Dowling refers to as symbolic resources) employed in such disciplines is often highly specialised and generalisable. Furthermore, an understanding of the language (and symbolic resources) is often a pre-determining factor for effective, successful and legitimate and/or endorsed participation and communication in the discipline. (Dowling, 1995b, p. 213; 1998, pp. 32, 103 & 138). Activities in which legitimate and endorsed participation is weakly regulated by language use are considered to have low discursive saturation (DS^-). Everyday practices would be classified as exhibiting DS^- as the language is largely unspecialised and localised and often not generalisable beyond the context in which the language is employed. Furthermore, in everyday practices an understanding of generalisable specialised language is often not pre-determinant for successful and/or endorsed participation in that practice (Dowling, 1995b, p. 213; 1998, pp. 32, 103 & 138).

Dowling (1998) summarises as follows:

> Practices exhibiting high discursive saturation are associated with a degree of context-independence or generalization; practices exhibiting low discursive saturation are associated with comparative context-dependency or localization.
>
> Mathematics is clearly a case of high discursive saturation, a practice which is highly organised at the level of discourse and so produces generalized utterances. … Domestic and manual practices are examples of low discursive saturation, because they are not generally highly organised at the level of discourse and so they produce localized utterances. (pp. 103-104)

Importantly, Dowling (1994, p. 128; 1998, p. 138) argues further that a crucial distinction between activities exhibiting high and low discursive saturation is the extent to which the regulative and evaluative principles are recognizable within the discourse of the practice. As summarised by Jablonka and Bergsten (2010, p. 40), "'Discursive Saturation' is a dimension that describes the extent to which a practice … has explicit principles of regulation." Activities exhibiting high discursive saturation generate descriptions that draw explicitly and overtly on the principles regulating the activity, primarily through engagement with generalising and specialising strategies (Dowling, 1998, p. 138). It is for this reason that participation in such activities provides direct access to the regulative and evaluative principles of the activity and, hence, to apprenticeship in the activity. It is also for this reason that when everyday contexts or problems (characterised by low discursive saturation) are introduced into an academic/mathematical discipline (characterised by high discursive saturation), the everyday context is subordinated to the discourse, principles and criteria of legitimate participation of the high discursive saturation activity. As Dowling (1998, p. 138) suggests, "Indeed, such subordination is to a large extent necessary, because of the relative inflexibility of the grammar of the recontextualizing esoteric domain."

Activities exhibiting low discursive saturation, on the other hand, contain language that is highly localised and dependent on the immediate context in which the language is developed or employed, and characterised primarily by localising and fragmenting strategies (Dowling, 1998, p. 139). As such, descriptions of the regulative and evaluative principles of the activity are only made implicitly in such activities. It is for this reason that Dowling argues that participation in practices that exhibit low discursive saturation restricts or inhibits access to the regulative principles of the activity and, hence, position participants as dependents or objects in the activity.

10.1.1.2 Vertical and Horizontal Discourses

Dowling's theorising on the concept of discursive saturation, by his own admission, relates directly to Basil Bernstein's (1999) notion of *vertical* and *horizontal discourses* (Dowling, 1995b, pp. 219-222; 1996, p. 408; Ensor & Galant, 2005, p. 289). Detailed analysis and discussion of these concepts is beyond the scope of this immediate discussion and I am assuming that given the prevalence of these concepts in education research the researcher is already familiar with the concepts. However, a detailed discussion of these concepts is provided on the supplementary materials supplied for download here and/or on a CD attached to the book version of this thesis.[102] The discussion here, then, is focused particularly on the relevance of these concepts to Dowling's argument regarding the incommensurability of academic and everyday practices is necessary.

The distinction offered by Bernstein (1999) between vertical and horizontal discourses, between hierarchical and horizontal knowledge structures, and between knowledge structures with strong and weak grammars, is of direct relevance to this study in the following way. The to-be-developed language of description of a structure of knowledge for the domain of mathematical literacy that prioritises a life-preparedness orientation is situated as a *horizontal knowledge structure* exhibiting relatively *weak grammar* within a *vertical discourse*. Furthermore, this language of description promotes the view that a link from this horizontal structure to the authentic everyday world (i.e. to a *horizontal discourse*) is both possible and desirable for enhancing understanding of the world, and for expanding and developing the 'mathematical literacy gaze' of the students who participate in the subject. Mathematics, on the other hand, which forms the principle site of analysis for Dowling, is situated as a *horizontal knowledge structure* exhibiting relatively *strong grammar* within a *vertical discourse*. And one of Dowling's key arguments is that attempts at links between this structure and the everyday world result in the imposition of a mathematical gaze and the mythologising of a virtual reality. Bernstein's conception of discourses, knowledge structures and grammars, thus, provides a structure for distinguishing components of the language of description of knowledge in the subject-matter domain of Mathematical Literacy to the structure of knowledge in other school-based academic disciplines (such as mathematics). The positioning of the language of description for the structure of knowledge of the domain of mathematical literacy as a particular discourse and knowledge structure is illustrated in Figure 22 below.

[102] The discussion in the additional materials dealt in detail with the characteristics of vertical and horizontal discourses, the differentiation between horizontal and vertical knowledge structures, and the concepts of verticality and grammaticality.

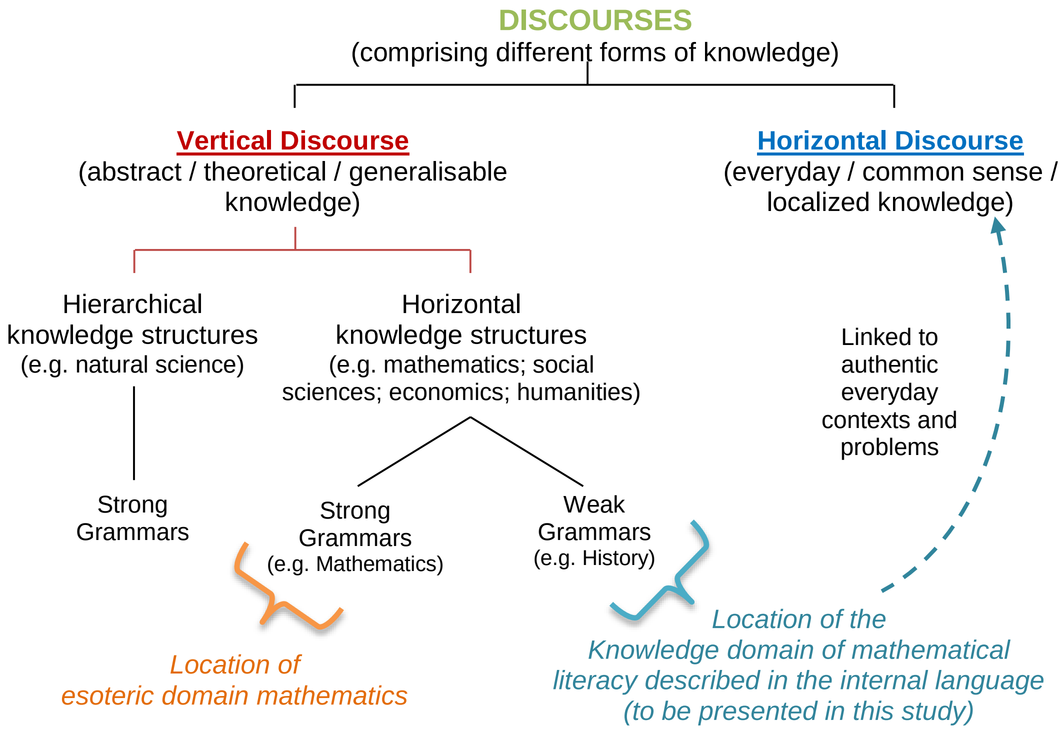

Figure 22: Positioning the developed language of description as a particular form of discourse and knowledge structure

For Dowling, the difference in the structure of the activities that give rise to horizontal (everyday) and vertical (academic) discourses and the categorisation of horizontal and vertical discourses as comprising DS^+ and DS^- practices respectively, posits academic and everyday practices as fundamentally different activities. Movement between these activities is, then, not possible without a radical recontextualisation of the principles and/or structures of one of the activities. Furthermore, every practice and region of practice contain different role players, different alliances between the role players in that practice, and different use of language and discursive resources in the practice. As such, moving from one practice to another does not simply involve transferring skills and concepts from one setting to another. Rather, such movement involves a completely new negotiation of social relations and alliances, the development of a new form of endorsed communication, and a recontextualisation of the particular meaning of language, structures, and principles that define the practice and the structure of legitimate participation in the practice (Dowling, 2010b, p. 5).

This is confirmed by Bernstein (1999) who argues,

> As part of the move to make specialised knowledges more accessible to the young, segments of horizontal discourse are recontextualised and inserted in the contents of school subjects. However, such recontextualisation does not necessarily lead to more effective acquisition … A segmental competence, or segmental literacy, acquired through horizontal discourse, may not be activated in its official recontextualising as part of a vertical discourse, for space, time, disposition, social relation and relevance have all changed. (p. 169)

All of the above, suggests Dowling, points to a mythologizing. Namely, "The notion that schooling can have direct relevance to the everyday, which is to say, non-academic world is thus revealed as itself mythical." (Dowling, 1995b, p. 221).

10.1.2 The subordination of the everyday to the academic

The second part to Dowling's argument concerning the incommensurability of academic and everyday practices relates to what Dowling (1998, p. 24) terms the 'principle of recontextualisation'. Namely,

> … insofar as an activity can be empirically described as exhibiting a particular structure of social relations, then this structure will tend to subordinate to its own principles any practice that is recruited from another activity. (Dowling, 1998, p. 24)

Key to Dowling's argument is the contention that in interactions between *different* activities, one activity tends to cast a 'gaze' over the other activity and to recontextualise the activity according to its own principles, structures, discursive characteristics and knowledge forms, hereby ignoring crucial principles, structures and knowledge forms of the recontextualised activity that make the two activities unique and different (Dowling, 1998, p. 121). Following this line of thinking, in interactions between academic and everyday practices it is commonly the academic practice that is privileged over the everyday, with an 'academic gaze' cast over the everyday practice such that the everyday is recontextualised according to academic principles: "the everyday setting is, after all, no more than a token. The Esoteric Domain can always be prioritised." (Dowling, 1995b, p. 221). The result is that the image of the everyday practice that is presented within the academic practice is only a mythical impression. Namely, the everyday practice is presented not in the way it would actually occur but, rather, in a way that the academic thinks is might or should occur – and so, running becomes about speed, distance and time; painting about surface area; and cooking about ratios and conversions. As Dowling (1998, p. 32) suggests, "There is, of necessity, a dislocation between these academic contexts and the context of evaluation of the practices which are mythologised." And the consequence of this involves "presenting distorted practices to students who may not have had the chance to participate in the relevant activity and so know better." (Dowling, 2010b, p. 5)

10.1.3 Summary (of the discussion on the incommensurability of academic and everyday knowledge and practices)

In this section of the chapter I have presented a discussion of Dowling's view on the incommensurability of academic and everyday activities in reference to two arguments. The first purports academic and everyday activities as fundamentally different domains of practice in terms of social relations, actors, language, discursive resources, structures and knowledge forms. This was done in reference to Dowling's concept of discursive saturation and Bernstein's conceptions of horizontal and vertical discourses. The second argument presents the notion of a 'gaze' and suggests that when everyday activities are incorporated into academic activities, the everyday activity is subordinated to the 'gaze' (i.e. principles, structures, contents, language, discursive characteristics, routines or techniques, knowledge forms) of the academic activity. The consequence is that the everyday activity presented within the academic activity is a mythologizing of the activity rather than an accurate or realistic representation of the structure of legitimate participation and communication in the activity. Taken together, these two arguments suggest that interactions between academic and everyday activities are unequally yoked and that the result of such interactions is always the loss of something in one or other of the activities, most commonly in the everyday activities.

The discussion in this section of the chapter has focused on establishing – at a general level − the distinctive nature of the structure of knowledge and communication (discourse) associated with academic and non-academic or everyday practices and forms of participation. In section 10.2 below, the discussion now shifts to focus on a more specific and directed site of academic activity – namely, on the relationship between *mathematical* knowledge (as a form of academic knowledge) and non-mathematical (everyday) knowledge, practices and forms of communication and participation.

10.2 The relationship between mathematical and non-mathematical knowledge and/or practices

> But mathematics teachers can at least stop pretending that they are teaching mathematics because you need it to do shopping properly. Instead, they can introduce their students to the esoteric domain of mathematics to enable them to become the subjects of its languages. (Dowling, 1995a, p. 17)

The discussion in section 10.1 above focused specifically on the general categories of ‘academic’ and ‘everyday’ knowledge and practices. These general arguments presented regarding the incommensurability of everyday and academic activities can and are easily translated into a specific discussion about the incorporation of everyday contexts in the teaching and learning of the school subject of mathematics.

According to Dowling (1998, p. 3), the current trend in school mathematics education “is orientated more towards the widespread dissemination of mathematical use-values: not more mathematicians, but a more mathematically competent workforce and citizenry.”[103]. In other words, there is a fixation in the teaching of mathematics on the need for relevance, usefulness and utility, accompanied by a need to promote the view that mathematics does have application in the world and is a useful tool for making sense of the world. It is in this vein that every teacher works ardently to find answers to the dreaded question of “where will we ever use this in life, huh?”

For Dowling (1998, p. 2), this emphasis on the need for relevance, utility and applicability in mathematics education has given rise to a mythology that involves the positing of mathematics as “a mythologizing activity to a degree that is probably unparalled on the school curriculum.” The mythology centres on the relationship between mathematics and everyday practices and the belief that mathematics provides a useful lens for analysing and making sense of structure of participation and practices in the everyday (Dowling, 1998, p. 2).

However, for Dowling, mathematics does *not* provide a meaningful tool for making sense of the everyday and of the structure of participation and practices in everyday contexts.

[103] Dowling is not alone in questioning the relationship between everyday and academic practices. Rather, as Ensor and Galant (2005, p. 285) point out, this is an issue that is questioned and challenged by much of the sociological research focussing on mathematics education.

Rather, views of reality seen through an imposed mathematical lens present a distorted and mythical view of the reality. He argues this standpoint on two fronts. The first relates to the distinctive nature of mathematics and everyday practices and, particularly, the distinctive structure and role of language in each of those domains of practice. The second relates to the concept of a ‘mathematical gaze’ and the consequent mythologising of everyday practices through three inter-related myths: the Myth of Reference, the Myth of Participation and the Myth of Emancipation. Both of these arguments are now discussed in detail.

10.2.1 The distinctive nature of mathematics and everyday activities

As was discussed in the previous section of this chapter, Dowling uses the term *discursive saturation* to refer to the extent to which an activity – and, hence, the structure of legitimate participation in that activity − is regulated by language. Mathematics, as an academic discipline, is characterised by esoteric content and practices, and contains language that is (i) highly specialised and has a high degree of linguistic precision (for example, the word ‘circle’ applies to a very specific shape exhibiting specific features and has the same meaning irrespective of the context or practice in which the word is used) (Dowling, 1995a, pp. 5-6). Language in mathematics is also largely (ii) context independent and contains (iii) generalisable strategies that can be applied to a variety of settings and/or problems (Dowling, 1995a, p. 6). Understanding the language used in mathematics is often (iv) a pre-determining factor for successful and legitimate participation in the practice. All of the above suggests that legitimated and endorsed participation in the domain of mathematics is strongly regulated through the use of language and symbols and, as such, is characterised by the use or development of vertical discourse that exhibits high discursive saturation (DS^+).

By contrast, everyday settings and practices that occur in those settings often do not contain (i) highly specialised language and language with a high degree of linguistic precision: for example, you do not need to understand specialised language to be able to successfully complete your shopping. Language in everyday settings is largely (ii) context dependent and words that have meaning in one context may not have meaning in another context (for example, the terms VAT, SITE, PAYE have a particular meaning in the context of ‘tax’ but have no meaning beyond this context). Furthermore, practices and participation in everyday settings commonly contain (iii) localising strategies (Dowling, 1995a, p. 8) (as opposed to the generalising strategies involved in mathematics) (for example, the technique used for calculating personal income tax is specific to the context of income tax and does not have application to other forms of tax or beyond the context of tax). An understanding of language is often not (iv) a pre-condition for the successful participation in an everyday practice and it is usually possible to develop an understanding of appropriate language through participation in the context in which the practice occurs (Dowling, 1995a, p. 6). As Dowling (1995a) suggests,

> In everyday practices, language itself is used much in the same way as physical resources: it is far less a matter of whether or not an utterance represents a true proposition or a linguistically well-formed statement than whether it serves the purpose within the immediate context. (p. 6)

All of the above suggests that participation in everyday practices is weakly regulated through the use of language and symbols and, as such, is largely characterised by the use or development of horizontal discourse that exhibits low discursive saturation (DS^-).

Mathematical activities, and the knowledge and practices contained within those activities, then, are constituted differently to everyday activities and are differently regulated in terms of discourse, knowledge, language, communicative artefacts, and criteria for legitimate participation. By implication, is it problematic when attempts are made to use one type or form of activity or discourse to make sense of a different activity or discourse. When an everyday context such as shopping is placed into the practice or domain of mathematics, a horizontal DS^- practice is subjected to the structures and evaluative principles of a vertical DS^+ activity (Dowling, 1995b, p. 221). The everyday is no longer sequenced according to principles and criteria for endorsed participation of the particular localised practice or context, but according to the principles and participation criteria of abstract Esoteric Domain activity (Dowling, 1995b, p. 221). The result is a mythologising of the everyday activity, a mythologising that voices to students that the everyday practice involves the use of formal mathematical structures and techniques and that an understanding of these mathematical components is necessary for effective participation in the practice. As Dowling (1995b) suggests,

> However, rather than grasping the lived reality of the horizontal domestic practices, the mathematising of domesticity constitutes a mythical plane which occupies a space outside of both mathematics and the quotidian. The students are objectified by the mathematical gaze and recontextualised as homunculi which inhabit not the everyday world, but the mythical plane. Mythologised shopping is now subject to the generalisable evaluative principles of the vertical practice of school mathematics. (p. 221)

To illustrate this argument, consider the everyday practice of shopping. In the mathematics classroom, shopping − and associated price comparisons and considerations of best buy options – is presented as comprising concepts and calculations with routines involving ratios, rates and proportional reasoning. Students are encouraged to convert values to kilograms, grams, litres or millilitres to facilitate using procedural methods and basic calculator techniques as they work towards determining the most cost effective option as the best buy choice. Students are (mis?)led to believe that this is a realistic view of the types of considerations and calculations that a shopper encounters and that being able to do these calculations will ensure a successful (and more effective?) shopping experience. The reality of shopping, however, is very different. How many shoppers whip out a calculator and piece of paper in the middle of a shopping expedition and, while ignoring their three children who are just about to tuck into the shelf containing the chocolates, start to do pen and paper calculations involving ratio, proportion and rate? Rather, the shoppers make use of a range of different facilities and resources available to them in the shops, including shelf labels that show the price per kilogram or litre, or quick mental estimation. They also base their decisions of the most appropriate size or quantity to buy as determined not only on considerations of value or cost, but also on a whole range of other considerations. Some of these include: the most convenient size or quantity for a particular task; the size or quantity that is easier to carry or the size or quantity that fits most comfortably in a taxi or cupboard; and how much money is immediately available at the time of the purchase. The reality of shopping, the principles that define how people think, behave and communicate in the context of shopping, and the techniques that people make use of to inform decisions when shopping are remarkably different to those that define the criteria for successful participation in mathematised shopping practices encountered in the mathematics classroom.

Does this mean that mathematics has no utility in real-world contexts? Dowling (2010b) responds as follows:

> This is not to say that mathematics can be of no use whatsoever. … In engaging these problems, I am quite prepared to fetch whatever resources I may have to hand and, on occasion, these include school mathematical resources. The point, however, is that the setting always determines which resources are fetched and how they are recontextualised.
> … the development of strategies appropriate to any given activity—whether it be political action relating to policing, an academic activity, such as science, or a domestic activity, such as dining out − is effectively achieved only within that activity, because they must reproduce the practices that make them recognisably distinct. (p. 11)

In other words, mathematics and everyday practices are different domains of practice and the structures, language and rules that define successful practice, participation and communication in mathematics are vastly different to those that define successful participation and communication in the world. As such, if the intention is to learn mathematics, then a mathematics classroom focussing primarily on esoteric mathematical content and on making the regulating mathematical structures explicit is the perfect place to achieve this. However, if the intention is to learn about the world, then the best place to achieve this is through participation in the (non-mathematised) world. Believing that the mathematics classroom is an appropriate place to make sense of the world is a misnomer.

10.2.2 The 'Mathematical Gaze' and associated mathematical myths

The second aspect of Dowling's argument regarding the incommensurability of mathematical and everyday knowledge, practice and participation involves the conception of a 'mathematical gaze' and four associated mathematical myths. The concept of the gaze and the associated myths are now discussed in detail.

Dowling (1998, p. 10) argues that when mathematicians look at the world and when they bring problems involving everyday contexts and practices into the classroom, they see the world and associated practices and forms of participation and communication through a mathematical lens or a 'mathematical gaze'. In doing so, they structure the world and daily practices in the world according to mathematical principles and structures and participation criteria: they privilege the world according to a mathematical view (Dowling, 2009b, p. 26). They describe the world and the activities that take place in that world using specific mathematical terms and language and argue that mathematics is an effective tool for making sense of day-to-day practices and for describing the structure of and criteria for legitimate participation in the world:

> It is as if mathematics were casting a gaze on people's lives, reorganising them according to its own structures and then handing them back: you see how much better life would be if we were all mathematicians. (Dowling, 1995a, p. 4).

Mixing concrete becomes about using ratios (or possibly about conversions, or possibly about measuring quantities); painting a room turns into a consideration of surface area and paint conversion factors; running a marathon becomes about calculations involving distance, time and speed (with no mention of training, carbo-loading or steroid use); and shopping is transformed into a practice involving percentages, rates, conversions and a whole multitude of other calculations and concepts (most of which are only calculable with the use of a basic calculator, always readily available and strategically located in a pocket, bag or brassiere). All in the world is calculable and all is able to be described using mathematical terms, symbols and models. As suggested by Ensor and Galant (2005):

> It has been a matter of controversy whether, and to what extent, we can describe the practices embedded in routines of work and everyday life as 'mathematics', or whether these practices – in shopping malls, on building sites, in games, or arrayed in ethnic artefacts – become mathematical only because we cast a mathematical 'gaze' upon them and 'see' them as mathematical. (p. 293)

But herein lies the problem. As was discussed above, commonly in interactions between the academic and the everyday, it is the structure and regulative principles of the academic that is prioritised over the everyday (Dowling, 1995b, p. 221). The interaction between mathematics and everyday activities is no different: mathematical structures, principles, knowledge, and forms of communication and participation are generally prioritised and fore-grounded ahead of everyday considerations, to the extent that commonly used (and efficient) everyday forms of communication and practice are no longer deemed as sufficient or appropriate for describing real-world practices. Instead, deliberately selected mathematical activities, symbols, knowledge and language take priority and participation in the everyday practices are now legitimated according to mathematically structured criteria:

> I do not intend to claim that, in its origins, mathematics has no connection with the empirical; that would be absurd. But, despite all of its referral to the 'real world' − which I take to mean the world beyond mathematics per se − the mathematics curriculum, the non-arbitrary esoteric domain, is primarily constituted as self-referential, self-contained. The 'real world', wherever it appears in a mathematics lesson or test must be made to conform with abstract mathematical structures. (Dowling, 2009b, p. 10)

The result is a 'recontextualisation'[104] of the everyday practice and a reconstituting of the practice as a "virtual reality, a mythical domesticity within which all is rational and all is

[104] The term 'recontextualisation' is used deliberately by Dowling, and others like Ensor and Galant, to emphasise the distinctive transformation that must take place when moving from one domain of practice and/or activity to another. In particular, Dowling uses the term to refer to the process where the practices of a particular activity are subordinated to the principles of another (2009a, p. 19; 2010c, p. 1). This term links directly to Dowling's concept of the 'principle of recontextualization' referred to on page 152 above.
Notice that Dowling uses the term differently to the conceptualisation of 'recontextualisation' used by Bernstein. For Bernstein, recontextualisation refers to the rules or procedures by which a form of educational knowledge is moved from one site to another (Singh, 1997, p. 7): "Through recontextualisation, a discourse is moved from its original site of production to another site, where it is altered as it is related to other discourses." (Singh, 2002, p. 573). For example, a teacher in a classroom will decide on how a particular section of the curriculum must be sequenced, what must be emphasised, and what must be evaluated at the end of the learning process. The teacher has recontextualised the original curriculum according to a particular set of structures to facilitate the teaching and learning process. For Bernstein, recontextualisation of knowledge from one site to another brings with it altered relations of power (classification) and control (framing), which in turn affects the ideological meaning that is attached to the knowledge in that site (Singh, 1997, p. 7).
For a more detailed discussion on the distinction between Dowling and Bernstein's usage of the term 'recontextualisation', refer to Dowling (2009a, pp. 14-16).

calculable." (Dowling, 1998, p. 33). Dowling refers to this virtual or mythical reality as the Public Domain of school mathematics[105], a domain that contains a collection of everyday sites and activities that have been transformed by mathematics (Dowling, 2010c, p. 1). The construction of this Public Domain of mathematics entails the casting of a gaze from the Esoteric Domain of mathematics over an aspect of the (non-mathematical) everyday world and the recontextualisation of that world according to mathematical structures, hierarchies, principles, forms of communication, and participation criteria (Dowling, 2009b, p. 26). As Ensor and Galant (2005, p. 293) suggest, "This process of recontextualising denatures these everyday activities, subordinating them to the pedagogic imperatives and internal structuring of school mathematics." Similarly for Skovsmose (1994a):

> If that thesis is acceptable [regarding the central role of reflective knowing in the development of critical mathemacy[106]] it means that most of the epistemological approaches used in interpreting phenomenon in mathematics education are misguiding or at least biases in concentrating on mathematics, ignoring the conditions for the genesis of reflective knowing. (p. 48)

The dilemma with this imposed mathematical gaze is that the image of the everyday practice that is presented in the Public Domain of the mathematics classroom is not real; it is a mythical view of what would actually happen in that situation and of how people would actually think, behave, participate and communicate in the situation. As Dowling (1998, p. 33) suggests, "But it wouldn't be better, because mathematised solutions always fail to grasp the immediacies of the concrete setting within which …, problems and solutions develop dialectically." Mathematics is not shopping and the techniques, considerations, resources, and forms of communication used and needed to make sense of problems in the classroom compared to in the shops and other everyday activities are often different and unrelated. In short, the criteria according to which successful and endorsed participation in problem situations encountered in the mathematics classroom is legitimised are completely different to the legitimisation criteria for successful participation in everyday practices. Furthermore, while mathematicians may label certain everyday practices as involving mathematics, it is questionable whether the people engaged in everyday practices would constitute their activity in mathematical terms and whether they will make use of formal mathematical techniques and knowledge in solving problems related to those activities. As P. Dowling (2008, p. 4) suggests, "Whilst the esoteric domain objective is mathematically legitimate, the public domain message is suspect, to say the least; … . You might learn mathematics like this, but you're going to get a naïve view of the nonmathematical world that it recontextualised as its public domain." What we are left with, according to Dowling, is a powerful mythologising: namely, that mathematics can be used to generate an accurate and realistic understanding of everyday practices and forms of legitimate participation in such practices. The reality, however, is the reverse: the image of the everyday practice that is presented in the mathematics classroom is not an accurate reflection of how that practice is experienced or engaged in real life or of the knowledge, techniques and considerations that influence the structure of participation in that practice.

[105] And, hence, we have now travelled full circle. Namely, from the discussion at the beginning of this chapter of the different Domains of Mathematical Practice that characterise the structural level of Dowling's language of description, to this discussion of how the Public Domain of mathematical practice represents a particular relationship between mathematical and everyday practices and, particularly, the mythologising of the everyday practices.

[106] See Part 4, Chapter 14 and 14.4.5.1 below (starting on page 230) for a detailed description of the notion of 'reflective knowing'.

It is also important to point out that different individuals employ differing generative principles in the construction of a mathematical gaze for a particular recontextualised practice. For example, while a teacher may legitimise participation in calculations involving concrete quantities through engagement with the mathematical structure of ratios, a different teacher or a textbook author may legitimise participation in the same practice through engagement with the mathematical structures of measurement and volume calculations. This is significant in that it highlights that it is the individual who imposes a mathematical gaze on an everyday practice who controls and defines the criteria, principles and structures on which the mathematisation process is based. As a result, any participants who are expected to engage with a recontextualised and mathematised practice are reliant on the person who conducted the mathematisation process to make explicit and visible the generative principles and structures of the practice. And, as has already been discussed, this has implications for participants who engage exclusively in Public Domain practices: such participants are utterly dependent on a Subject in the practice (e.g. a teacher) to make explicit the regulative principles and criteria of these Public Domain practices.

The imposition of a mathematical gaze on everyday non-mathematical practices gives rise to three 'mathematical myths' about the practice and also about the relationship of the participant (who is imposing the gaze) to the practice. These myths include the Myth of Reference, the Myth of Participation, and the Myth of Emancipation. Each of these myths are now discussed in detail.

10.2.2.1 Myth of Reference

> Mathematics is mythologized as being, at least potentially, about something other than itself. (Dowling, 1998, p. 4).

The Myth of Reference describes the widely held assumption that mathematics can refer to activities and practices other than itself and can be used as a tool for making sense of and describing those activities and the structure of participation in the activities (P. Dowling, 2008, p. 1). In Dowling's (1998, p. 6 & 16) terms, mathematics is seen as a set of 'exchange values', something that can be used to cast a commentary on non-mathematical activities. The myth encourages us to believe that it is possible to move between two spheres, one of which is always mathematical. As Dowling (1998, pp. 6-7) points out, though, it is always the mathematics casting a commentary on something else and seldom the other way around.

Importantly, when mathematics is employed to make sense of other activities, the recontextualised activity is presented as being real, as an accurate and realistic reflection of the actual practice and of participation in the practice. This process denies that the recontextualised activity is something less than real. For example, in the mathematics classroom, shopping is presented as an activity that really does involve ratio and rates.

And herein lies the myth: when mathematical principles are used to make sense of non-mathematical activities in the classroom, the result is always a "colonising of non-mathematical activities" (Dowling, 1998, p. 33), with the non-mathematical setting consumed by the mathematics and leaving behind only a trace that there is something outside of the mathematics (Dowling, 1998, p. 16). As such, the image of the everyday activity that is recontextualised in the mathematics classroom is not authentic or realistic and does not give accurate consideration to how people actually act, think, behave and communicate in that activity in daily life. Rather, the recontextualised image of the everyday is the image as seen from the perspective of the mathematician, seen through the 'mathematical gaze'. As Dowling (1995a, p. 9) argues, "However, they [the mathematical/scientific texts] are no more likely to generate plausible solutions to everyday practical problems, because the everyday is not structured according to mathematical principles." The myth, then, is that "the descriptions resulting from casting a mathematical gaze upon the world are indeed about that which they appear to describe." (Christiansen, 2007, p. 98).

As a final comment, it is worth noting that the Myth of Reference operates through an explicit recognition and privileging of mathematical principles and forms of participation over any real-world considerations: for example, the appropriate use of ratio and proportion is privileged over any real-world considerations in a best-buy shopping scenario. As such, although the Myth of Reference views mathematics as a tool that can be employed to make sense of any real-world situation, the movement is always towards the Esoteric Domain of mathematics and away from the real world (Dowling, 2001, p. 22). The Myth of Reference, then, can be associated with a move beyond the Public Domain towards the Esoteric Domain.

Importantly, the "myth of reference is distributed to the apprenticed voice" (Dowling, 1998, p. 295) and, so, is associated with the Apprentice position. Students who are apprenticed into mathematics are given direct access to the Esoteric Domain content and practices of the subject. This Esoteric Domain content is always prioritised over Public Domain content and/or contexts, and the inclusion of Public Domain contexts serves purely to draw apprentices into the activity. In this way, the 'mathematical gaze' is inculcated into the apprenticed practice, voice and position such that apprentices are given the means and the control to decide and determine the criteria according to which a mathematical gaze is to be cast and the generative mathematical structures and principles that define the gaze (Dowling, 1998, p. 292; 295). In short, it is the Apprentice and Subject of the activity who are able and who believe it appropriate and justifiable to cast a gaze beyond the domain of mathematics on the extra-mathematical world.

10.2.2.2 Myth of Participation

> Nevertheless, school mathematics frequently presents the myth that its 'real world' applications constitute a necessary condition for adequate domestic practice and, indeed, for adequacy in a whole range of other non-mathematical activities. I have referred to this myth as the myth of participation. (Dowling, 2009b, p. 26)

The Myth of Participation of mathematics claims that mathematics is a universal tool and that understanding of mathematical concepts and techniques allows us to understand and control the world (P. Dowling, 2008, p. 1). Mathematics is, thus, presented as a necessary pre-condition for effective participation and functioning in everyday domestic practices: "mathematics is a necessary supplement to what the student already knows if they are to optimise their own lives." (Dowling, 2010b, p. 8).

This view of mathematics promotes a mythologising in two ways. Firstly, this view is simply an extension of the 'mathematical gaze' discussed above. By claiming that mathematics provides a means for making sense of the world and for understanding everyday practices, everyday practices are scrutinised though a mathematical lens, mathematical structures are imposed on those practices, and effective participation in those practices is only deemed possible if the imposed mathematical structure is understood (Dowling, 1998, p. 10). The everyday practice is transformed or recontextualised into a 'virtual reality' which, although it may bear some resemblance to what would actually happen in daily life, is still a mathematised and distorted view of reality. Hence, the everyday practice presented is a mythical image of the reality.

Secondly, this view is a myth precisely because many people are able to function perfectly well in the context of everyday or domestic practices without understanding the mathematical concepts or techniques that the mathematical gaze imposes on such practices. As Dowling (1998, p. 10) suggests, "mathematical skill is neither necessary nor sufficient for optimum participation within these [everyday] practices". You do not need to understand ratios and rates to be able to complete your shopping, nor do you need to have an understanding of surface area to be able to paint a room.

In contrast to the Myth of Reference – where there is an explicit privileging of mathematical principles over real-world considerations and where there is a direct move beyond the Public Domain, the Myth of Participation draws participants into believing that the context is real and they have a direct participatory role in that context (i.e. are 'objectified' by the context (Dowling, 1998, p. 144)). As such, the Esoteric Domain origin of the gaze is hidden and there is no deliberate movement beyond the Public Domain (Dowling, 2001, p. 22). The Myth of Participation facilitates and is in turn facilitated by practices associated exclusively with the Public Domain.

Whereas the Myth of Reference is distributed to Apprentices – who are given control over the generative principles and structure of the mathematical gaze, the Myth of Participation is distributed to participants positioned as Dependents (and associated dependent voice in pedagogic texts) in the domain of mathematical practice (Dowling, 1998, p. 295). Students who are exposed primarily to Public Domain content and contexts and to the fragmenting and localising strategies that characterise practices in this domain are commonly subjugated to the position of Dependent or Object. Students are lead to believe that the problems that they are solving are about real life (rather than mathematics), that the problems provide a realistic map of the everyday world, and that they are active participants in the problem-solving process. They are invited to identify with the problem and the characters in the problem, as though the problems and contexts are their own; and they are invited to provide opinions and to suggest strategies for solving the problems. Students are further led to believe that understanding of mathematics provides the key to solving these problems and for making sense of the contexts in which the problems are situated. This is precisely the Myth of Participation. Participants in Dependent or Objectified positions, then, are directly inculcated into the Myth of Participation (Dowling, 1998, p. 250; 293; 295).

10.2.2.3 Myth of Emancipation

> Revealing the truly mathematical content of what might otherwise be regarded as primitive practices elevates the practices and, ultimately, emancipates the practitioners. This is the myth of emancipation. (Dowling, 1998, p. 15).

The Myth of Emancipation – considered by Dowling (2001, p. 20) to be a globalized version of the Myth of Participation − is commonly associated with the term 'ethnomathematics' and with research that strives to celebrate the already existence of mathematical content and techniques within the practices of different cultural groups (Dowling, 1998, p. 12). In this view, (externally imposed) analysis of the practices of different cultural groups will reveal the existence and usage of mathematical structures and concepts by the practitioners in those practices (Dowling, 1998, pp. 11-12). So the weaving patterns used on baskets constructed by Xhosa women are seen to indicate conceptions of transformation geometry (translations, rotation, reflection, and enlargement). In this view, the universal language of mathematics already exists in these practices, and all that remains is for that mathematics to be extracted and revealed to the practitioners in those practices. Revealing the underlying mathematics elevated the status of the ‘primitive practices’ and, therefore, emancipates the practitioners of these practices from their primitive understanding and/or existence (Dowling, 1998, p. 15).

This view, argues Dowling (1998, p. 33), also presents a mythologised view of mathematics: “the myth of emancipation frequently mythologizes diverse non-industrial cultures.” This myth is revealed in two ways. Firstly, this view is once again simply an extension of the ‘mathematical gaze’ scenario – and, so, is inculcated in and distributed from the Apprentice (and Subject) position. When a practice specific to a certain culture and which takes place in a certain context is analysed and deemed to contain mathematics, it is inevitably European mathematical principles, recognition symbols and participation criteria that are imposed and used to provide a language of description of those practices. And so, Europeans look at the cultural practices with a ‘mathematical gaze’ and through a distinctly European mathematical lens and then claim that the mathematics was always existent in those practices (Dowling, 1998, p. 15). The myth, then, is that rather than celebrating non-European cultural practices and forms of participation, the imposition of the ‘mathematical gaze’ results in a recontextualisation of the practice according to foreign, European structures (Dowling, 1998, p. 17). Non-European practices are re-described and consequently suppressed using European structures. Added to this, Dowling (1998, p. 17) argues that this view is driven not by an intention to promote the cultural practice itself (for example, basket weaving) but rather by a focus on elevating mathematical structures and principles (for example, the learning of transformation geometry).

The second component to the mythologising of this emancipatory perspective relates to the claim within this view that revealing the mathematics in such practices emancipates the practitioners of those practices. By means of challenging this view: will teaching basket weavers about transformation geometry make them more efficient, capable or successful weavers?; is an understanding of transformation geometry essential for being an effective and successful weaver (the Myth of Participation)?; and, crucially, is it only through the internalisation of the *European* view of such practices that emancipation is achieved? Perhaps Dowling’s response to these questions would be that mathematics is neither a pre-condition or necessary component for the successful participation in such practices and, rather, that such cultural practices have existed successfully for centuries without imposed European knowledge structures and interference.

10.3 The ‘production of ability’: considerations of educational disadvantage

In the discussion above I outlined Dowling’s arguments regarding the incommensurability of academic/mathematical and everyday practices. What remains for this section is to link this discussion to Dowling’s key sociological concern with how educational disadvantage is produced and reproduced through the differential distribution of the different domains of mathematical practice to different groups of students in the teaching and learning of mathematics.

Dowling (1998, p. 51) coins the term ‘production of ability’ to highlight the perspective that “the curriculum does work in order to recontextualize these essentially non-educational differences [in class, race and gender] as differences in educational attributes and performances.” In other words, Dowling argues that the notion of ‘ability’ is a constructed notion that has more to do with social difference than with an actual attribute. As a result, differentiation according to ability and, hence, the preservation of social difference and disadvantage is reinforced through curriculum structure and the types of knowledge made available to students from different racial, class and gendered backgrounds (c.f. Dowling, 1998, pp. 49-69).

Dowling presents his argument through an analysis of two mathematics textbooks, one of which is aimed at supposedly higher ability students (who stem from predominantly middle-class backgrounds) and the other at supposedly lower ability students (who stem from predominantly working-class backgrounds). From the results of this analysis, Dowling (1995b) is able to make the following accusation:

> The tendency of the two textbook series to specialise their modes of mythologising constitutes a distributing strategy. ‘Higher ability’ students are apprenticed into descriptive mythologising; ‘lower ability’ students are provided with participative mythologising. (p. 219)

‘Descriptive mythologizing’ in this statement refers to esoteric or academic domain activities comprising vertical discourses and exhibiting high discursive saturation (DS^+), of which school mathematics is one such practice. The word ‘descriptive’ is used here to infer that students participating in such activities are given access to the regulative principles and structures of the activity through engagement in Esoteric Domain practices. This process affords apprenticeship into the domain, inculcating participants into the Myth of Reference. This process further enables participants to cast a gaze on the world and to generate descriptions of the world using these ‘privileged’ principles and structures. ‘Participative mythologizing’, on the other hand, refers to activities comprising horizontal discourses and exhibiting low discursive saturation (DS^-), a feature characterised by everyday domestic activities (Dowling, 1995b, p. 219). Students involved in these activities are invited to ‘participate’ in the activities and are indicted into the Myth of Participation, but are never given access to the regulative principles and structures underpinning the activities. Such students are not given access to the tools that enable them to cast mathematical descriptions on or of the world and, as such, are relegated to positions of Dependent or Object in the learning process. In emphasising this distinction, Dowling (1995a, p. 7) then asserts that within the context of the schooling system, curriculum associated with Esoteric Domain knowledge – imbued with various forms of descriptive mythologizing − are commonly made available to high ability students commonly in better resourced schools located in middle and upper class environments. By contrast, curriculum focussing on relevance – imbued with various forms of participative mythologising − are made available primarily to low ability students commonly in poorer resourced schools located in working-class environments. This sentiment is echoed by Bernstein (1999):

> When segments of horizontal discourse become resources to facilitate access to vertical discourse, such appropriations are likely to be mediated through the distributive rules of the school. … These insertions [of horizontal discourse in vertical discourse] are subject to distributive rules, which allocates these insertions to marginal knowledge and/or social groups. (p. 169)

In the South African context, this situation would translate into Core Mathematics (abstract scientific mathematics) being offered to higher ability students and Mathematical Literacy (mathematics in the world) offered to lower ability students, which is precisely the trend in the majority of schools throughout the country.

> My argument is that everyday, or horizontal, practices constitute us all and that this is unavoidable and inevitable. Academic, or vertical, practices have been systematically distributed on class and racial lines, however. This has entailed the effective exclusion of the majority of the populations of both South Africa and Europe from the academic. This is variously achieved via the non-existence or inadequacy of schooling provision or, more subtly, by the insistence of the inclusion of the everyday and the relevant in terms of participative mythologising. (Dowling, 1995b, p. 222).

The insistence on 'relevance' for optimising participation in everyday (DS^-) practices, then, is only seen as a fundamental criterion for lower ability and/or working-class students: "'Low ability', by contrast, tends to be constructed as demanding residence in the *public domain*." (Dowling, 2009a, p. 31). Dowling (1995b, p. 219) cites the dilemma with this differentiation of curriculum as comprising two components. Firstly, the 'relevance' is actually not real; rather, it is a mythical relevance that has been constructed through the imposed lens of the 'mathematical gaze' and the associated Myth of Participation. Secondly, the regulative or evaluative mathematical principles underpinning these everyday 'relevant' activities are generally hidden or rendered invisible to the students who, instead, are positioned as Dependents that are reliant on the teacher to make these principles explicit. For example, in a question that asks students to calculate the quantity of concrete that a builder needs to make to fill a certain portion of the foundation trench of a house, the underlying mathematical concept that students are expected to make use of is volume. This concept, however, is not explicit and, rather, students have to make the transfer between 'quantity of concrete' and volume. The result of this two-fold dilemma is that students come to experience 'relevant' problems with a skewed, mythologised, mathematised impression of the problem and without developing a realistic or authentic understanding of either the everyday practice or of the mathematical content inherent in the practice. As Dowling (1995b, p. 219) suggests, "'Lower ability'/working class students are, thus, provided with 'relevance' at the expense of either mathematical or everyday use-value.". In short, while drowning in mythical relevance and distorted reality, at no point are the working-class students, supposedly of lesser ability, afforded the opportunity to explore the mathematical concepts that provide access to top-end careers.

By contrast, students engaging with academic or esoteric mathematical content and texts (DS^+ practices), comprising primarily higher ability students drawn primarily from middle-class environments, are given direct access to the regulative principles underpinning mathematical activities: "'High ability' is therefore constructed as meriting entry into mathematical discourse." (Dowling, 2009a, p. 31). For example, consider a question relating to factorisation: students are told the type of category of factorisation into which the specific expression falls and are shown appropriate methods for determining the factors of the expression, and at no stage in working with the problem it is unclear or hidden as to what is required to make sense of the problem. Furthermore, although everyday contexts may be incorporated into lessons involving academic or esoteric content, it is always evident to the student that the context is secondary to the mathematical principle, and at no point are students led to believe that the everyday is something other than imaginary or contrived. Students involved in such academic mathematical practices, then, have both access to the mathematical content and control over the mathematical gaze: they understand that the reality of the everyday is different to the mathematical calculations and contexts that they are concerned with. The result is that "'High ability'/middle class students are thus to be apprenticed into academic mathematics and into the principles of the descriptive gaze." (Dowling, 1995b, p. 219), all of which are seen as essential traits of high-end professions comprising, amongst other, engineering, medicine, architecture, and economics.

This sentiment is shared by Hoadley (2007) who, through analysis of classroom data from four schools situated in middle-class environments and four schools in working-class contexts in Cape Town, makes the following observation:

> This study shows how students in different social-class contexts are given access to different forms of knowledge, that context-dependent meanings and everyday knowledge are privileged in working-class context, and context-independent meanings and school knowledge predominate in the middle-class schooling contexts. (p. 682)

The structure and distributive rules of the schooling system, thus, translate and provide (and limit) access to different types of knowledge, participation, communication, and mythologizing to students from differing class and social backgrounds, hereby producing and reinforcing educational disadvantage and inequality. And all of this is achieved under the guise of differential 'ability'.

In short and in summary, the emphasis on relevance in mathematics, then, has created a differentiated curriculum that reinforces social division and difference by only making certain aspects of the curriculum available to different social groups. And so, according to Dowling (2010a), an emphasis on relevance in mathematics serves to reproduce social class divisions:

> In my analysis of UK junior high school texts, I found that texts directed at 'high ability' students moved from the public domain to the esoteric and back (via the descriptive), apprenticing them into new knowledge. Texts directed at 'low ability' students remained in the public domain, so that these students were confined to a culture that comprised recontextualised versions of what they already knew; the text mythologised their lives. Furthermore, it was apparent, by an analysis of both the content of the public domains in the different books and of the physical form of the books, that social class was a key indicator of 'ability'. We might say, then, that school mathematics functions as a device translating social class into 'ability'. (Slide 2)

10.4 The 'best' way forward (according to Dowling)

> The democratic principles dictate, then, that the schools open up the availability of academic discourses to all. The acquisition of such vertical practices necessarily involves subjugation to the evaluative principles of these discourses. Subjectivity of necessity entails subjugation. Academic subjectivity entails subjugation to a discipline. Only in this way can the DS^+ practices be made available as structured resources for the interrogation of everyday practices by the practitioners themselves: discipline and then and only then mathematise. … Both forms of dialogue [between the academic and the everyday], however, are predicated upon prior apprenticeship into the disciplines of the academic. (Dowling, 1995b, p. 222).

A similar sentiment is shared by Adler, Graven, and Pounara (2000) who state: "Only when students have gained mastery over the key aspects of mathematics can they reach beyond mathematics to 'mathematise' other areas" (cited in (Ensor & Galant, 2005, p. 291)). Young (2009) makes a different yet related appeal for the separation of specialised knowledge and everyday knowledge in schooling:

> For children from disadvantaged homes, active participation in school may be the only opportunity that they have to acquire powerful knowledge and be able to move, intellectually at least, beyond their local and the particular circumstances. It does them no service to construct a curriculum around their experience on the grounds that it needs to be validated, and as a result leave them there. … The concept of knowledge differentiation [between school and non-school knowledge] implies that much knowledge that is important for pupils to acquire will be non-local and counter to their experience. Hence pedagogy will always involve an element of what the French sociologist Pierre Bourdieu refers to … as *symbolic violence*. The curriculum has to take account of the everyday local knowledge that pupils bring to school, but such knowledge can never be a basis for the curriculum. (pp. 15-16, emphasis in original text)

What the authors above are suggesting is that it only becomes possible to use the academic to accurately analyse, evaluate and model the everyday world once there is mastery of the academic. In other words, master the academic first and then use of the academic to make sense of an other. In the eyes of Dowling, anything less than this results in a distorted and incomplete view of the everyday as well as incomplete understanding of the mathematics: a glass only half full.

CHAPTER 11
PROBLEMATISING CURRENT PRACTICES IN THE SUBJECT MATHEMATICAL LITERACY THROUGH THE LENS OF DOWLING'S LANGUAGE[107]

If Dowling's (1998) concerns regarding the incorporation of out-of-school contexts in the teaching of mathematics are deemed to be valid, then this presents a particular challenge for the South African school subject Mathematical Literacy. This is particularly pertinent given that the subject has as a primary purpose an intention to develop in students engaged in the subject "the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems." (DoE, 2003a, p. 9). In this chapter I analyse current practices in the subject through the lens of Dowling's language of description – namely, from a 'Dowlingaling'©™[108] perspective − and highlight the problematic situation that exists as a result of a dominant emphasis in the subject on Public Domain of mathematics practices. I also highlight the extent to which the presence of the subject – as a compulsory alternative to the scientific course Core Mathematics − in the secondary schooling system facilitates a degree of educational disadvantage in this system.

As discussed in Part 2 (specifically Chapter 8 starting on page 91 above), current practices in the subject matter domain of Mathematical Literacy are dominated by an agenda for contextualised mathematics practices and, particularly, by a numeracy in context dimension. The result is that participation in the subject is legitimised primarily according to the appropriate use of mathematical knowledge, mathematical routines, mathematically endorsed forms of communication, and mathematical narratives. Any contextual elements that are encountered are treated largely in service to mathematical structures, such that contextual considerations, knowledge and forms of participation are subordinated or rejected in favour of the mathematical counterparts.

Despite the inclusion of statements in the original NCS curriculum (DoE, 2003a) emphasising a dual focus on both mathematical and real-world contexts and contents, it is agendas associated with the mathematical components of the subject that dominate the focus of national assessments and, as a consequence, pedagogic practices at classroom

[107] It is important for me to acknowledge that the comments made in this chapter regarding current practices in the subject-matter domain of Mathematical Literacy − and the use of Dowling's language of description for analysing those practices – are based to a large extent on observation of these practices rather than through the application of a formal theoretically-informed methodology. And this formal methodology is certainly available, as demonstrated by Dowling (1998) in his analysis of empirical mathematics textbooks. Note that this is a deliberate decision on my part and not an oversight. It would have been possible to focus the major empirical analysis component of this study on the analysis of current practices in the subject through the methodology demonstrated by Dowling. However, I have chosen instead to focus on the development of a theoretical language for describing the structure of knowledge in a conception of the subject-matter domain of Mathematical Literacy that prioritises a life-preparedness orientation. Identifying and describing the problematic space characteristic of current practices in the subject, then, serves as a motivating (albeit crucial) precursor to the central component of this study rather than as the primary focal point.

[108] Pronunciation: *Dow-ling-a-ling*.
In the same way that a disciple of Bernstein is called a Bernsteinian, a disciple of Dowling would then be called a Dowlingaling. Of course, it would have been more appropriate to employ the common convention of adding 'ing' to the end of Dowling's name such that a disciple of Dowling would be called a Dowlingian, but *Dowlingaling* has a much nicer cowbell-type ring to it. ☺
Please note that I have copyrighted © and trademarked (™) the term so that when it becomes an internet hit and sells for millions, then no one else can lay claim to the origins of the term.

level. The recent CAPS curriculum revision process (DBE, 2011a) attempted to rectify this situation by placing greater emphasis on real-world considerations and contextual sense-making practices alongside mathematical considerations. However, the mathematical basis of the taxonomy used in national assessments continues to ensure the continuing dominance of a mathematically driven agenda characterised, primarily, by numeracy in context related practices in the CAPS structure.[109] The reality in South Africa, then, is that Mathematical Literacy has been and continues to be reduced to the teaching and assessment of mathematical calculations in context (often contrived and/or fictitious contexts) rather than contextual sense-making and/or life-preparedness.

Since much of Dowling's work involves criticisms and concerns regarding the inclusion of everyday contexts in the teaching of mathematics in a classroom setting, and since current practices in the subject-matter domain of Mathematical Literacy prioritise mathematics-in-context calculation structures, it becomes possible to analyse current practices in the subject through the lens of Dowling's theoretical language. Interestingly, when Dowling's work is applied to the South African situation one is immediately struck by the extent to which his criticisms and concerns regarding the incommensurable relationship between mathematical and everyday practices and forms of knowledge are being evidenced and enacted.

To begin with, one of Dowling's primary contentions is that interactions between mathematical and everyday practices commonly result in the imposition of a mathematical gaze over the everyday practices and the consequent recontextualisation and subordination of the everyday/non-academic practice to mathematical/academic structures, knowledge and participation criteria. Christiansen's (2007) coding of the statements of curriculum in the National Curriculum Statement (DoE, 2003a) for the subject-matter domain of Mathematical Literacy using Dowling's four domains of mathematical practice provides clear evidence of this (at the level of the intended curriculum at least).[110] Christiansen (2007) concludes:

> Most of the statements in the NCS for ML were coded as belonging to the descriptive domain, which is characterised by the recontextualisation of non-specialised setting but describes the contents of this setting in mathematical forms of expression. (p. 94)

The Descriptive Domain, by Dowling's (1998, p. 4) own admission, is the realm of mathematical modelling – namely, the realm in which a gaze is cast from the Esoteric

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[109] In May 2014 the National Department of Basic Education distributed exemplar Grade 12 Mathematical Literacy CAPS examination papers. As predicted and expected, these examination papers continue to reinforce and prioritise the development of mathematical knowledge and mathematical forms of participation over contextual sense-making practices and elements of life-preparation. See Part 7 and Chapter 25 of this study (starting on page 402) for a more detailed analysis of the dominant domain of prioritising in these examination papers (and also in comparison to other texts, including a textbook, a section from the CAPS curriculum, and course notes for a pre-service teacher education course).

[110] The mere fact that Christiansen was able to code the various assessment standards in the Mathematical Literacy NCS curriculum statement using Dowling's four domains of practice adds credence to the argument that the dominant agenda in the NCS is the development of mathematical knowledge. This is because the scope of Dowling's work is concerned with the learning of mathematical knowledge and issues associated with the inclusion of everyday contexts in the mathematics classroom. As such, the fact that the assessment standards are able to be categorised in the domains of practice schematic suggests the continued and explicit presence and prioritising of mathematical structures in the statements. It is also of interest that Christiansen did not find evidence of assessments standards that did not fall within at least one of Dowling's domains, which would have indicated an attempted reach beyond mathematical structures towards an alternative structure of legitimate participation and knowledge in the subject.

Domain of mathematics to describe practices that exist outside of the domain of scientific mathematics. As such, this dominance of emphasis on Descriptive Domain practices in the NCS suggests a preoccupation with generating mathematical descriptions of everyday practices. Christiansen (2007) interprets this dominance of Descriptive Domain practices to be suggestive of an alternative consideration:

> To me this dominance of the descriptive domain implies that the curriculum is intended to teach the learners something they can either apply in the original setting or which will inform them more generally in those same or similar settings. (p. 94)

In other words, the NCS framework promotes a conception of mathematically literate behaviour as comprising a set of generalizable skills which transcend the specific context of application and which have application across a range of contexts and problem situations. Christiansen (2007, p. 94) problematizes this perspective or conception by arguing that the notion of transfer of knowledge from one context to another is not a straightforward process. Furthermore, she argues that such a perspective ignores the fact that participants in everyday practices employ localised strategies drawn from the immediate context of application to solve problems rather than a set of generalisable and/or transferrable mathematically based skills.

My own experience in working with Mathematical Literacy teachers and in engaging with Mathematical Literacy related textbooks and examinations (see, for example, (North, 2010)) provides evidence of the replication of a similar situation: irrespective of the context being dealt with or the problem being solved, mathematical considerations dominate.[111] However, I contend that despite the dominant agenda in the NCS curriculum for a form of Descriptive Domain type practices, it is actually primarily Public and Expressive Domain practices that dominate at the level of the implemented curriculum. This is most clearly evidenced in the Grade 12 national examinations for the subject[112] (see, for example, (DBE, 2012a, 2012b)) that are inundated with questions that reflect recontextualised and mathematised everyday Public Domain type practices in which only mathematically derived narratives for problem-situations are endorsed (for an example, see Question 1.2.1 in Figure 23 below). Expressive Domain type problems are also employed with fervour, such that many of the questions employ unspecialised and often non-mathematical language despite an explicit expectation for the use of mathematical

[111] Note that this prioritising of mathematical knowledge and structures has continued to dominate despite the change from the NCS to the CAPS curriculum structures, and also despite the heightened emphasis in the CAPS curriculum on the recognition of everyday forms of knowledge and participation in real-world problem-solving practices. The primary reason for this involves the continued prioritisation of mathematical structures in the national Grade 12 examinations, which − through a ‘backwash’ effect − has impacted on the structure of pedagogic practice at the level of the classroom and, in so doing, has effectively negated the philosophical intention of the CAPS curriculum. The situation in the subject-matter domain of Mathematical Literacy in South Africa provides a clear and classic exemplar of the dialectical nature of the fields of knowledge recontextualisation (c.f. Bernstein, 1996). Moreover, the situation in the subject further illustrates how the structuring of knowledge and participation in the field of pedagogic recontextualisation, (influenced primarily through nationally set examinations) is able to completely transform the intended curriculum (developed in the official recontextualising field) such that the implemented curriculum contradicts and negates the intended curriculum.

[112] The Grade 12 National examinations are written by all candidates enrolled in the subject at Grade 12 level. Since these examinations count for 75% of the final Grade 12 year mark, performance in these examinations is important for future career and study opportunities. The significance of these examinations ensures that classroom based pedagogic practices are directed, by and large, towards preparation for these examinations. For this reason, I contend that the structure of legitimised knowledge and participation in these examinations is, to a large degree, also reflective of the structure of classroom-based pedagogic practice.

routines and structures in the generation of narratives that are considered legitimate and endorsable (for an example, see Question 1.2.2 (a) in Figure 23 below).[113]

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1.2 The Nel family (two adults and two children) were on holiday for nearly one week.

- They left home after breakfast on Saturday morning and arrived at the guesthouse in time for supper.
- On Sunday and Wednesday they ate all their meals at the guesthouse.
- On Monday they visited a game park.
- On Tuesday they went on a nature walk.
- On Thursday they went on a boat cruise.
- They left George after breakfast on Friday and returned to Klerksdorp.

TABLE 1: The Nel family's holiday costs

	ITEM	COST*
1	**Accommodation only**	R1 050 per day per family
2	**Meals at the guesthouse:**	
	Breakfast	R60 per person per day
	Lunch	R90 per person per day
	Supper	R120 per person per day
3	**Travelling costs:**	
	Long distance driving (to and from Klerksdorp) and meal costs en route	R1 602,86 for the return trip
	Local driving (in and around George)	R513,60 for the duration of the holiday
4	**Entertainment costs:**	
	Nature walk, including breakfast	R120 per adult and R100 per child
	Visit to the game park, including lunch	R200 per person
	Boat cruise, including supper	R200 per adult and R150 per child
	Other entertainment	R2 000
	***All the costs above include value-added tax (VAT).**	

1.2.1 Determine the total amount that they paid for accommodation. (2)

1.2.2 (a) Write down an equation that could be used to calculate the total cost of meals eaten at the guesthouse in the form:

Total cost (in rand) = ... (3)

(DBE, 2012b, p. 4)

Figure 23: Public and Expressive Domain questions in a Mathematical Literacy examination paper

Irrespective of the specific domain of mathematical practice preferenced in the examinations for the subject, one thing is explicitly obvious and clear: every question in

[113] In Part 7 and Chapter 25 (starting on page 402) I employ a method of textual analysis to demonstrate the predominance of Public and Expressive Domain practices in set of Grade 12 examination papers.

the examinations assesses a particular mathematical concept or technique, such that endorsed and legitimised participation in the examinations is based on the appropriate use of mathematical knowledge, contents, routines and forms of communication. The most important factor is always the mathematical: the correct mathematical layout; the correct mathematical notation; the most accurate mathematical solutions; and so on. And no provision is made for consideration of the structure of knowledge, practice and communication that characterises participation in everyday settings. Instead, students are led to believe (indoctrinated into believing?) that the everyday world is structured according to mathematical structures and, as a result, that mathematically generated narratives provide a legitimate means for describing participation in that world. As highlighted by Venkatakrishnan and Graven (2007, p. 72), "The preamble to the curriculum specification in this document [NCS] and other related policy documentation emphasise the idea of ML involving the development of a 'mathematical gaze' on the world."

However, despite an espoused intention for the apprenticing of students into a domain of practice that facilitates the casting of a mathematical gaze over everyday practices, in reality – through exposure only to Public and Expressive Domain contents and practices – participants in the subject are relegated to positions of Dependent and Object in the pedagogic process. In the name of 'relevance', Esoteric Domain mathematical contents and practices are explicitly downplayed, discouraged and excluded in the subject, such that participants are denied access to such contents. Instead, participants are exposed to problem-solving scenarios involving mathematised and mythologised representations of everyday practices that are characterised by a plethora of localising and fragmenting strategies – such as reference to fictitious people (including 'Didi'− a contestant in a game show (DBE, 2012a, p. 6) – whoopee!) living fictitious (and somewhat boring) lives, photographs, and deliberately constructed (and often contrived) resources. In all of these problem-solving scenarios, the generative mathematical structures – determined by the teachers and/or examiners who have developed the scenarios − are hidden and rendered invisible to the participants. As such, the participants are dependent on their teachers to make visible and explicit the mathematical structures and principles that facilitate legitimate participation in the problem-solving scenarios and the generation of appropriate and endorsable narratives about the problem scenarios. Furthermore, participants are invited to believe that they have a role to play in solving the problems (i.e. Objectification) – through questions such as "which … would be the best option …?" (DBE, 2011d, p. 6), "how much would you …" (DBE, 2011c, p. 7), and "which ONE … you would advise …" (DBE, 2013c, p. 4) − and that successfully solving the mathematised and mythologised problems prepares them for more effective participation in their everyday lives and practices – the 'myth of participation': "... Mathematical Literacy, should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy." (DoE, 2003a, p. 10). In short, participants in the subject-matter domain of Mathematical Literacy are only apprenticed into the myth of the mathematical gaze without being fully apprenticed into the content that makes understanding of the foundations of the gaze possible: Mathematical Literacy students are being positioned as "objects rather than cognising subjects" (Christiansen, 2007, p. 101).

And what might be the consequences of this current state of affairs in the subject? Christiansen (2007) offers an accurate suggestion:

> By claiming that it is about life-related topics, the curriculum renders the underlying (mathematical) organising principles of the content invisible to the learners (and possibly to some teachers, too), who therefore will *not* learn mathematics, unless the teachers is in a position to ensure coherence and progression of mathematical concepts. (p. 99)

Furthermore and secondly, the subject is not empowering students to better cope with the world because the contexts they are exposed to are mythologised, mathematised and commonly contrived contexts. The consequence is that students are being restricted access to both mathematical knowledge and real-world functionality. The issue is, again, problematised by Christiansen (2007):

> If learners learn neither mathematical knowledge nor competencies of relevance to their lives, neither mathematical gaze not livelihood gaze, what do they learn? (p. 100)

And the answer?

> As an implication, the curriculum does not teach what it sets out to do; it ends in a no-mans-land between mathematics and life-related content. … In that respect, the curriculum is likely to contribute to the reproduction of social inequalities. (Christiansen, 2007, p. 91)

While Christiansen is largely making projections based on theoretical analysis rather than observed practice, this precise situation is now playing out in reality with respect to the way in which both participants in the subject and other role-players are experiencing the efficacy and value of the subject. Participants in the subject come to believe (and/or are led to believe) that the subject is preparing them for the world outside of school, for empowered functioning in that world, including for tertiary study and for the world of work. By contrast, role-players outside of the subject (parents, other teachers, universities, employers) – all of whom legitimise participation in the subject according to the level and complexity of the mathematical knowledge and contents that participants are expected to deal with − view the subject as nothing more than a form of second-rate inferior mathematics. It is in this realm that Mathematical Literacy is referred to rather unaffectionately by both teachers and learners as ‘maths zero’, ‘maths lite’, ‘diet maths’, ‘mango maths’[114] and even the ‘second (and long forgotten) cousin to mathematics’. In similar vein, it is not uncommon to hear teachers, parents and other express concern over the fact that students are ‘closing doors for themselves’ by ‘dropping’[115] to Mathematical Literacy[116]. Some schools and bodies also encourage students to ‘stick it out’ on Core

[114] ‘maths zero’, ‘maths lite’ and ‘diet maths’ are all references to ‘Coke Lite’, ‘Coke Zero’ and ‘Diet Coke’, such that each of these drinks represents the watered down version of the pure and unadulterated Coke drink. ‘Mango maths’, on the other hand, refers to ‘Mango airlines’ − which is the low-cost (i.e. cheap!) airline partner (painted in bright orange of all colours – presumably to avoid in-air collisions) to the more prestigious national carrier South African Airways. Clearly the Core Mathematics teachers have so much free time that they can devote large chunks of their days to thinking up derogatory nicknames for the subject ... and, yet, we still wonder why there is a crisis in mathematics education in the country!

[115] In South Africa, the change from Core Mathematics to Mathematical Literacy is viewed not so much as a move or a choice, but as a downwards leap towards the bowels of …

[116] This is particularly relevant for the learner who is at risk of failing Core Mathematics (i.e. achieving less than 30%) but who is adamant that they want to study to become a doctor, engineer or something else with even the slightest hint of a scientific base and, so, does not want to limit their options by changing to Mathematical Literacy! Mmmmmhhhh ... can you also spot the flawed logic here?

Mathematics for as long as possible so as not to limit their future career choices.[117] Still others encourage (even force in some instances) their students to take Mathematical Literacy rather than Core Mathematics in order to boost pass rates and results.[118] Universities and other Tertiary Institutions have responded to this vast distinction in the pass rates for the two subjects by stipulating differentiated faculty-level entrance requirements for Mathematical Literacy and Core Mathematics results[119], and also by limiting access to certain faculties for only those students who have completed Core Mathematics (with a certain minimum result)[120].

And yet the woes of the students do not end here. Instead, Dowling's prediction regarding the differential provision of relevant contextualised mathematics to weaker ability/working-class students and scientific esoteric mathematics to higher ability/middle-class students has also come to fruition in South Africa. Mathematical

[117] Of particular interest in this regard is the Independent Examinations Board (IEB). For the 2013 academic year, the IEB gained permission from UMALUSI* for Grade 12 learners enrolled in Core Mathematics to also enrol to write the national Grade 12 examinations for Mathematical Literacy (IEB, February 2013) − and this despite the Mathematical Literacy qualification being a three-year qualification and the Core Mathematics learners never having been involved in this qualification. This initiative is driven by a motivation and intention to retain learners in Core Mathematics by providing an 'escape route' for those at risk of failing in the run up to the final examinations – with the unstated expectation that although these learners might fail Core Mathematics they should easily be able to pass Mathematical Literacy. As with many other perspectives on mathematical literacy, this initiative is grounded in the belief that the structure of knowledge and participation in the subject is based on mathematical principles. Hence, participation in Core Mathematics (characterised by complex Esoteric Domain contents) is deemed to be more than sufficient preparation for the lower level of mathematical demand required for participation in Mathematical Literacy.
You can see my less-than-polite response to this initiative in (North, 2013b, unpublished work).
** UMALUSI is an external independent quality assurance body responsible for ensuring the standard and standardisation of assessment practices in the General and Further Education and Training bands in the South African education system*.

[118] This motivation is driven by the belief that the Mathematical Literacy examinations are significantly easier to pass than the Core Mathematics, and there certainly appears to be some truth in this perception. In 2013 the pass rate for Mathematical Literacy was 87,1% and 59,1% for Core Mathematics (DBE, 2014a, p. 125 & 159). This is a remarkable distinction, indeed, especially considering that predominantly weaker candidates (i.e. candidates who cannot cope with the demands of Core Mathematics) opt for Mathematical Literacy. This belief has promoted a trend amongst teachers and principals in certain schools to 'force' learners to enrol in Mathematical Literacy (often not even offering Core Mathematics as a subject choice at the school) in order to boost the pass rates for the school.
However, closer analysis of all of the results and not just the pass rates for the two subjects reveals a completely different picture. For example, the same percentage (approximately 5%) of the learners in both subjects achieved 70% and above in the 2013 examinations, while a *smaller* percentage of learners in Mathematical Literacy achieved 80% and above (1,7%) and 90% and above (0,1%) than in Core Mathematics (80% and above = 2,6%; 90% and above = 0,8%) (DBE, 2014a, p. 125 & 159). As such, although Mathematical Literacy is easier to pass, it is a misconception that learners who participate in the subject are more likely to do better than in Core Mathematics. See (North, 2013a) for a more detailed discussion of this and other 'myths' relating to the subject-matter domain of Mathematical Literacy.

[119] For example, at Rhodes University in Grahamstown the minimum entrance requirement for entrance into a degree in the Faculty of Science is: Core Mathematics at Level 5 (60 – 69%) or Mathematical Literacy at Level 6 (70 – 79%) (Rhodes University, 2010).

[120] Of course, what should always be remembered (and what is seemingly always forgotten) is that Mathematical Literacy was introduced in 2006 to accommodate those learners who at Grade 10 level would previously have opted to not do any form of mathematics at all (since Mathematics was an optional choice in the pre-2006 curriculum framework). In this sense, participation in Mathematical Literacy was never intended to provide access to degrees of tertiary study requiring scientific mathematics contents, nor was it intended to 'steal' learners away from Core Mathematics. Instead, Mathematical Literacy was introduced to ensure that all learners would exit the secondary schooling system with a certain degree of mathematical competence. Unfortunately this intention has been overlooked, with the implication that Mathematical Literacy is now labelled as a 'limiting' option because it does not afford access to the same opportunities as Core Mathematics.

Literacy is reserved primarily for those students who cannot cope with the demands of Core Mathematics, and in the South African context the majority of these (supposedly) weaker students stem from poorly resourced schools in poorly resourced working-class communities.[121] By contrast, the majority of the students who are successfully able to participate in the Esoteric Domain contents of Core Mathematics stem from better resourced schools in middle-class communities and environments. And, since successful participation in the Esoteric Domain contents is a necessary requirement for participation in a larger range of study and career options – and particularly for study opportunities that facilitate access and movement to middle and upper-class employment, it is the students in the better resourced middle-class educational environments whose already existing privilege is sustained and protected. By contrast, the students who participate (or who are forced to participate) in Mathematical Literacy are relegated to a significantly more limited future study and career opportunities, and where the majority of these opportunities only provide access to lower paying professions and not to avenues of upward economic mobility. All of the above demonstrates that Dowling's concern regarding the differentiated access of relevant mathematics and esoteric mathematics afforded to weaker ability and higher ability students respectively, and the implications of this for (re)producing educational disadvantage and for future career opportunities, is coming to fruition in a very real way in South Africa through the differential access afforded to the subjects Mathematical Literacy and Core Mathematics. The presence of a format of the subject-matter domain of Mathematical Literacy in which participation is legitimised according to mathematical knowledge and structures, then, is facilitating and contributing to the production of educational disadvantage and the preservation of social and economic disadvantage.

If Dowling is correct, and the way in which the current situation is playing out in South Africa suggests strongly that this is the case, then a major rethink is needed about the primary intention, dominant structure of knowledge and areas of prioritising in the subject-matter domain of Mathematical Literacy. This is particular pertinent if the subject is to contribute in a worthwhile and empowering way to the education, development and career trajectory of the students who participate in the subject. I contend that this is only possible if the structure of participation in the subject is based on a knowledge domain in which participation in practices involving both mathematical and everyday structures is no longer legitimised exclusively or primarily according to mathematical knowledge and structures. This shift would then position the dominant structure of knowledge in the subject outside of the scope of Dowling's domains of practice language of description and the mythologising and disadvantage dimensions highlighted in and through that language. Instead, participation in the subject is to be legitimised according a knowledge domain that facilitates and prioritises a life-preparedness orientation, in which sense-making of real-world environments and practices and explorations of alternative forms of participation in those practices are prioritised. The language of description (of the structure of knowledge and participation for the knowledge domain of mathematical literacy) presented in the next part (Part 4) of this study is an attempt at such a revised conceptualisation.

However, a final word of clarity and/or caution is necessary. It is not my intention to argue that all conceptions of mathematical literacy, numeracy and/or quantitative literacy that prioritise mathematised and mathematically legitimated forms of

[121] And, given that the percentage of the population that live in working-class environments is higher than the percentage living in middle-class environments, it is of little wonder that enrolment figures in Mathematical Literacy (53,7% in 2013) are higher than in Core Mathematics (42,3% in 2013) (DBE, 2014a, p. 125 & 159).

participation in engagement with contextual practices inhibit the development of mathematical knowledge and relegate participants to positions of Dependent or Object in the pedagogic process. Instead, I am arguing that when a conception of mathematically literate behaviour is characterised by a form of participation that is legitimised according to a mathematically biased knowledge base, a mathematised view of real-world problem-solving situations, and a learning process structured around mathematics as the primary organising principle, then it is problematic when claims are made that this behaviour (better) prepares participants for engagement in real-world practices. Instead, when the mathematical terrain is prioritised, then the only world that participants are prepared for is the mathematised world. And, this does not equate to more effective or empowered functioning in the (more commonly non-mathematised) real-world: an understanding of ratio and proportion does not equate to the ability to successfully mix concrete; understanding of financial formula does not equate to effective financial management. Importantly, this issue is not problematic in conceptions of mathematical literacy that are embedded, encountered and facilitated in and through the domain of scientific mathematics. This is because participants who operate in this domain are aware that they are functioning from a mathematical perspective and that a mathematically endorsed form of participation is required. Furthermore, participants who operate in this way engage with esoteric domain contents and, so, gain access to the regulative principles of any generated and imposed mathematical gaze. Rather, this issue becomes problematic when – in a manner analogous to ‘a wolf in sheep’s clothing’ or ‘mutton dressed as lamb’ − participants are led to believe that they are not learning about mathematics (which is precisely the myth propagated in Mathematical Literacy since the subject is separated from the domain of mathematics). Instead, participants are assured (definitely definitely, we promise 👍) that they are learning about the real-world, but are then evaluated according to mathematical non-contextual criteria. This is the Public Domain of mathematics; and it is precisely when practices remain exclusively in this domain that positions of dependency and objectification result as participants are excluded from the decision-making processes that determine the principles according to legitimate participation in a problem-solving practice is endorsed.

In short, my primary argument is that if a conception of mathematical literacy is to truly prepare participants for a potentially more empowered functioning in the real-world, then a life-preparedness orientation must be facilitated. This is to be achieved through engagement with authentic contexts, through acknowledgement of the limited role of formal mathematical structures and techniques in adequately describing real-world situations, and through recognition that any mathematics appropriated and/or used must be in service to a goal for broader contextual sense-making practices. A life-preparedness orientation must be facilitated through the positioning of the contextual domain as the dominant and primary organising principle of the knowledge domain.

CHAPTER 12
PERCEIVED LIMITATIONS OF DOWLING'S LANGUAGE

In the chapter above I employed Dowling's language of description as a means for describing and problematising current practices in the subject-matter domain of Mathematical Literacy in South Africa, with a particular focus on the educational disadvantage that is (re)produced and reinforced through the inclusion and presence of this subject in the South African schooling system. And for this purpose, Dowling's language of description provides a useful and valid theoretical instrument.

Given the current problematic state of affairs in the subject as highlighted in the chapter above, a key intention in the remainder of this study is the presentation of an alternative language of description of the structure of knowledge for the knowledge domain of mathematical literacy. This structure of knowledge is characterised by the prioritisation of a life-preparedness orientation over the learning and development of mathematical knowledge. Despite the fact that this life-preparedness orientation reflects a particular form of relationship between the domains of mathematics and the everyday world, I contend that this language of description demonstrates the complementarity of mathematical and everyday practices, hereby contradicting Dowling's assertion regarding the incommensurability of these different domains of practice. In so doing, the structure of knowledge – and associated forms of legitimate participation and communication in practices that draw on this knowledge − as described in the alternative language of description is positioned beyond the scope of the gaze of Dowling's language of description and of the various forms of mythologising, positioning and elements of educational disadvantage highlighted in that language.

To facilitate the development of this alternative language of description, it is necessary − in this chapter − to highlight what I perceive to be certain limitations of Dowling's theoretical language, particularly with respect to the utility and validity of Dowling's claims regarding the incommensurability of mathematical and everyday practices for a particular structure of relation between mathematical and non-mathematical practices.

To begin with, Dowling is primarily concerned with how the incorporation of non-mathematical elements in classroom practices inhibits access to the learning of mathematical content and knowledge, and with the different domains of practice in which mathematical learning and activity occurs. And his emphasis on the imposition of a mathematical gaze and the associated Myths of Reference, Participation and Emancipation all hinge on a prioritising of mathematically based goals (i.e. the development of esoteric mathematical knowledge and apprenticeship into the domain and discourse of this knowledge). The language, then, proves inadequate if the dominant goal in the learning process does not involve the mathematical terrain, but, rather, involves a different terrain – such as the terrain of the real-world. If the learning process prioritises the development of an understanding of real-world practices as seen from the perspective of real-world participants and not from the perspective of a mathematically orientated learning process, then Dowling's language is unable to provide an adequate description, interpretation and/or analysis of this activity. Similarly, if a learning process recognises the role of mathematics in providing a particular perspective or view of the world but also acknowledges the limitations and restrictions of that view, then this activity once again falls beyond the gaze of Dowling's theoretical language. This is not to suggest that Dowling does not recognise that his language of description is based on a particular area of focus – namely, the learning of mathematical knowledge and apprenticeship into mathematical activity. Rather, that Dowling's language only provides a perspective of the relationship between mathematical and non-mathematical practices when the learning of mathematical knowledge is prioritised as the dominant goal. If mathematical knowledge is no longer prioritised and dominant in an activity – or if the regulative principles that define the structure of legitimate participation and communication in the activity − are no longer mathematically based, then access to Esoteric Domain mathematics is refuted as the defining requirement for apprenticeship into the activity. And when this occurs, then Dowling's language is incapable of providing a valid or sufficient reading of the appropriate domains of practice, possible positions, and the structure of legitimate knowledge, communication and participation within the activity.

In light of the above, a key argument that I present throughout this study is that current conceptions of the subject-matter domain of Mathematical Literacy – espoused primarily through curriculum documents and national assessments – do prioritise mathematical goals and the imposition and inculcation of a mathematical gaze over real-world or life-preparation goals. In so doing, these current conceptions reinforce the mathematical Myths of Participation, Reference and also Emancipation in pedagogic practices in the subject. As such, what is needed is an alternative language of description of the structure of knowledge for the knowledge domain of mathematical literacy which prioritises a life-preparedness orientation over the learning of mathematical knowledge and which recognises that a mathematically based view of the world provides only one view – and often a limited view − of the world. A life-preparedness orientation recognises that although mathematical knowledge can facilitate an alternative understanding of the world, the structures that define action, decision-making, communication and endorsed participation in the world are often not mathematically based. As such, preparation for the world must acknowledge the role of non-mathematical factors and considerations which might affect decision-making and the limitations of a mathematical gaze in reflecting reality. In this orientation, although the role of mathematics in facilitating an alternative view of the world is recognised, mathematics is always seen as in service to a goal for a better understanding of an aspect of reality and of the structure of legitimate forms of participation in that reality. Preparation for life and not the learning of mathematical knowledge is the ultimate goal. I contend that a conception of the knowledge domain of mathematical literacy which prioritises a dominant structure of knowledge and participation that facilitates a life-preparedness orientation no longer falls within the domains of mathematical practice identified by Dowling and, so, is positioned beyond the scope of Dowling's language of description. I also content that this orientation contradicts Dowling's contention that mathematical and real-world practices are incommensurable and, rather, that a life-preparedness orientation embodies a perspective which posits the relationship between mathematical knowledge and real-world practices as 'complementary'[122] and as facilitating access to a broader and more empowered functioning in real-world problem-solving. Apprenticeship into this form of mathematical literacy, thus, requires a structure of knowledge and associated forms of participation and communication that are not regulated by the principles of Esoteric Domain mathematical knowledge. In

[122] The notion of 'complementarity' is employed extensively by Vithal (2002, 2003, 2008). The usage of the term here is taken to suggest that mathematical and real-practices are often in opposition to each other but also serve to provide an alternative view of the other. Most importantly and as suggested by Vithal (2008, p. 48), "both are needed to understand the phenomenon." A more detailed discussion of the concept of 'complementarity' and the relevance of this concept to the language of description for the structure of knowledge in the subject-matter domain of Mathematical Literacy to be presented in this study is given in Part 4 and Chapter 13 below (starting on page 183).

alternative words, participation in this form of mathematical literacy requires access to a different Esoteric Domain (and different Public Domain) of knowledge than that which characterises the domain of mathematics. And the intention of the to-be-presented language of description for the knowledge domain of mathematical literacy is to define and describe the structure of this different Esoteric Domain knowledge and the format of the legitimised communication and participation associated with this structure of knowledge.

A further limitation of Dowling's language – but one which does not have a direct implication for the language of description to be presented in this study − is the dichotomous nature of the domains of practice and, particularly, with no consideration given as to how these different domains might apply for students at different stages of development. For example, a Public Domain activity for a Grade 1 student might be an obviously mathematical activity for a Grade 3 student for whom the context does not pose a barrier to accessing the mathematical components of the problem. The dichotomous nature of the domains and the suggestion that a mathematically based activity must fall within one of the domains does not account for the different ways in which a single problem can be experienced by different groups of students at different stages of development.

A third criticism refers to Dowling's suggestion that primary focus in the mathematics classroom should be on the learning of Esoteric Domain mathematical knowledge and that the application of such knowledge can easily occur once the knowledge is in place. Although I agree with the notion that a comprehensive understanding of mathematical knowledge is required before an application of that knowledge is possible, research on mathematical modelling theories suggests that the skills and competencies required for constructing and/or analysing mathematical models are vastly different from those involved in the development of mathematical knowledge (for example, see Blomhoj & Jensen, 2003; Skovsmose, 1994b). This perspective contradicts Dowling's emphasis on the prioritisation of mathematical knowledge above all else.

A further criticism is Dowling's ardent emphasis on the incommensurability of mathematical and non-mathematical practices, and the related projection that the practices of the world are not based in mathematical activities, knowledge and structures and that it is only through the imposition of a mathematical gaze that such practices are 'discovered'. There certainly is truth that mathematicians tend to see the world through a mathematical gaze and to cast real-world practices as constituting mathematics (even through the participants engaged in those practices might not employ formal mathematical techniques or knowledge). However, employing a mathematical gaze on real-world practices and identifying supposedly mathematical structures at play in these practices, does facilitate the possibility for reflecting back on Esoteric Domain mathematical content and, so, for establishing an alternative and/or potentially broader view and understanding of the situation. This is not to suggest that a mathematical approach empowers the participants, but it may provide a different understanding of the problem and a different way of thinking, acting and talking about the problem. In other words, the mathematical gaze provides an opportunity for identifying aspects of real-world practice that could be developed through Esoteric Domain mathematical means, and for possible alternative forms of functioning in the practice.

Dowling's emphasis on the dichotomous nature of mathematical and everyday practices is also limiting. His language seems to suggest that you are either doing mathematics or you are engaging in everyday practices, but that you certainly cannot do both successfully at the same time. He also suggests that mathematical content cannot be learned in the context of application (i.e. needs driven) but must be learned external to the context (at least if successful apprenticeship into the discipline of mathematics is to occur). This limited and limiting view of the interaction between the mathematical and real-worlds denies how engineers, architects, bankers, builders and others who employ mathematically based calculations, techniques and knowledge in solving problems encountered in real-world settings constantly develop and redefine their base of mathematical knowledge to suit the ever changing constraints, conditions and considerations of real-world contexts (see, for example, Grabiner, 1974/1986; Mousoulides, Pittalis, & Christou, 2006). This dichotomous view denies that mathematical knowledge can be expanded through interaction with real-world problems and that the demands of a context can have an impact on whether or not a person can successfully use mathematics to solve a problem. To imply that problem-solving is a one-way process from mathematics to real-world is to deny the role that real-world problem situations have had on redefining and reconstituting the canon of mathematical knowledge. It is for this reason that I contend that Vithal's (2008) usage of the notion of 'complementarity' provides a more accurate and powerful illustration of the dialectical relationship between the mathematical terrain and the real-world.

A further concern is Dowling's suggestion that mathematical modelling is positioned within the Descriptive Domain of practice and constitutes a movement from the Esoteric Domain to the Public Domain of practice. But what if a mathematical model is constructed to represent a genuine real-world practice and if the construction of the model is accompanied by recognition of the limitations of the model and non-mathematical real-world factors which may affect action and decision-making in the practice? Then the eventual model that is constructed does not provide a description of a Public Domain practice (as suggested by Dowling) but, rather, a reasonably close reflection of the real-world situation (which is not accommodated in Dowling's categorisation of domains of mathematical practice). In other words, there are modelling situations that involve the use of Esoteric Domain mathematical contents which do not fall within the restrictive boundaries of Dowling's language.

[123] The discussion above has sought to highlight certain limitations of Dowling's language of description. In particular, the discussion has shown that although the language has application for a particular conception of practices involving the engagement of mathematical and non-mathematical contents, the language proves inadequate when the learning of mathematical knowledge and techniques no longer dominates the pedagogic process. In other words, the language is only adequate and sufficient when applied to the

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[123] Straehler-Pohl and Gellert (2013, pp. 320-321) raise another concern – but one which is not directly relevant to my use of Dowling's theory and, so, I have chosen to include it in a footnote and not as part of the main discussion. Straehler-Pohl and Gellert's concern relates specifically to Dowling's contention that his language of description can be extended beyond the domain of textual analysis to have applicability for analysis of dialogic text such as that which characterises classroom interactions. For Dowling (1998, p. 112 & 164), each utterance can be treated as a monologic text − i.e. as an individualised event that is separated from preceding or later dialogue and from surrounding context. This is problematic for Straehler-Pohl and Gellert, and they argue that this perspective is also incompatible with Bernstein's theory. For Straehler-Pohl and Gellert, text and context are intricately connected, and it is not possible to accurately analyse an instance of text without consideration of the impact of surrounding contextual factors on the nature and structure of the text – "the meaning of a text depends in its context and simultaneously a context is shaped by the communication of text." (Straehler-Pohl & Gellert, 2013, p. 321). For Straehler-Pohl and Gellert, then, the lack of emphasis on the role of context in shaping dialogue within Dowling's theory does not facilitate the use of the theory for adequately analysing interactions involving direct discursive dialogue between participants.

specific empirical terrain of the development of mathematical knowledge in school-based settings.

WHERE TO FROM HERE

Looking ahead, the current problematic space occupied by the subject-matter domain of Mathematical Literacy demands a rethink of the structure of knowledge and the criteria for legitimate participation in the subject. And this is precisely my intention in the Part 4 of the study. Namely, to provide a language of description of the structure of knowledge for the knowledge domain of mathematical literacy and forms of participation associated with the practices of this domain that are characterised by the prioritisation of a life-preparedness orientation and dominated by an agenda for contextual sense-making practices rather than for the development of mathematical knowledge. As is discussed and demonstrated, a format of the subject-matter domain of Mathematical Literacy that is associated with the conception of knowledge described in the internal language is positioned outside of the reach of Dowling's language of description and, in so doing, addresses his concerns regarding the incommensurability of mathematical and everyday practices and associated elements of educational disadvantage.

PART 4
TOWARDS AN INTERNAL LANGUAGE OF DESCRIPTION OF THE STRUCTURE OF KNOWLEDGE FOR MATHEMATICAL LITERACY

INTRODUCTION AND OVERVIEW

In this part of the study I outline the components and contents of an alternative language of description of the structure of knowledge in a conception of the knowledge domain of mathematical literacy that prioritises a life-preparedness orientation. This language of description positions the Esoteric Domain of the knowledge domain of mathematical literacy as comprising four domains of practice, including Everyday, Mathematical Competency, and Modelling knowledge, accompanied by a domain of practice that facilitates a form of critical engagement through various levels of Reasoning and Reflection, all grounded in and directed towards sense-making of reconstituted real-world practices. These reconstituted practice are posited as the Public Domain of the knowledge domain of mathematical literacy. Throughout this discussion I make a consistent and repetitive argument that engagement in the knowledge domain of mathematical literacy is legitimised according to critical contextual sense-making practices, which, in turn, is facilitated through continuous and integrated participation in all of the domains.

To facilitate presentation of the components of the internal language of description, the following structure is provided for the discussion in this part of the study. In Chapter 13, I introduce two concepts of relevance to the structure of knowledge and associated forms of legitimate participation envisioned for the knowledge domain of mathematical literacy – namely, ‘complementarity’ and ‘forms of learning’ (cumulative and segmented). The notion of complementarity posits mathematical and contextual practices as co-joined elements that facilitate more comprehensive contextual sense-making, and I employ this notion to inform the way in which content and context are seen to interact in the domains of practice that characterise the knowledge domain of mathematical literacy. I make reference to the notions of segmented and cumulative learning to argue that although practices associated with the knowledge domain of mathematical literacy run the risk of imbuing knowledge that are largely context dependent, participation in the domains of practice of the knowledge domain are characterised by cumulative learning processes. These processes facilitate the transfer of skills developed through pedagogic processes to the terrain of the real-world outside of the classroom. In Chapter 14 I then describe, in detail, the structure of knowledge and participation that characterise each of the domains of practice of the knowledge domain of mathematical literacy in which a life-preparedness orientation is prioritised. The ways in which cumulative learning is to be promoted in each domain of practice is also stated. In Chapter 15 I clarify and elaborate on my contention that the knowledge domain of mathematical literacy (as described in the internal language) is positioned beyond the scope of Dowling’s theoretical domains of practice schematic.
The structure of this part of the study is illustrated Figure 24 below.

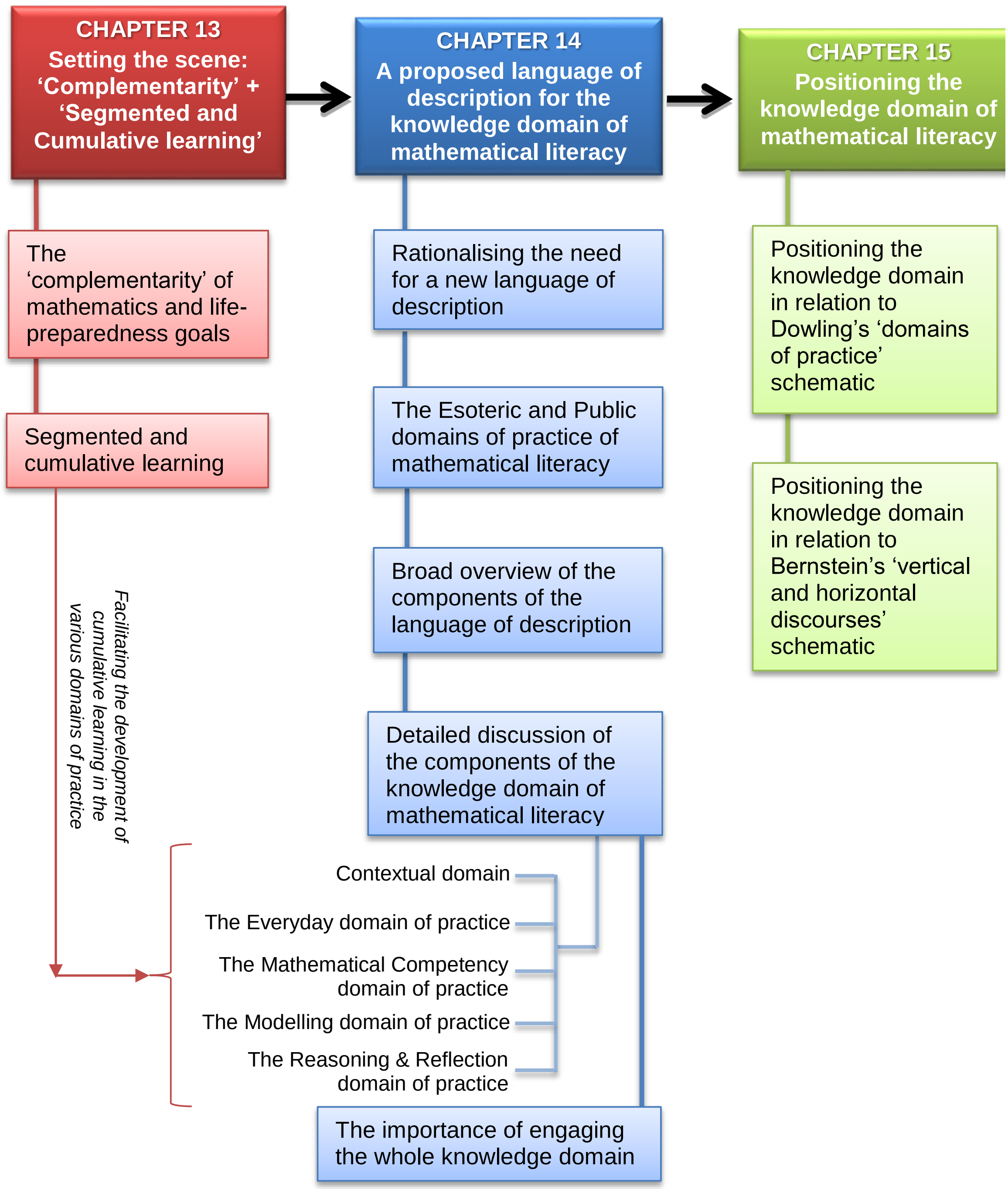

Figure 24: Overview of the contents of Part 4 of the study

CHAPTER 13
SETTING THE SCENE: 'COMPLEMENTARITY' AND 'SEGMENTED' AND CUMULATIVE' LEARNING

In this chapter I introduce two concepts, both of which have direct bearing on the validity of the components of the internal language of description for the structure of knowledge in mathematical literacy and, as such, require consideration. The first concepts relates to the notion of 'complementarity' (Vithal, 2002, 2003, 2008) and, in opposition to Dowling's contention regarding the incommensurability of mathematics and everyday practices, posits the co-joining of mathematical and everyday practices as facilitating a more comprehensive perspective of a practice. This notion of complementarity informs the way in which mathematical content and context are seen to interact in the components of the internal language of description. The second concept relates a distinction between different forms of learning, differentiated between segmented and cumulative learning (Maton, 2009). In discussion of the components of the internal language I argue that each component of the language facilitates a form of cumulative learning that negates the context-dependent nature of certain practices and enables participants to utilise the skills and knowledge developed in school beyond the walls of the classroom.

13.1 The 'complementarity' of mathematical and life-preparedness goals

Venkat (2010) makes the following statement with regards to the subject-matter domain of Mathematical Literacy:

> When combined with the relatively widespread reference to problem-solving across a broad range of real-world contexts in the ASs [Assessment Standards] and the examples attached to them, (e.g. ''The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts which include financial aspects of personal, business and national issues'' p14), there is evidence that, in spite of the apparent mathematical structure, attempts have been made to incorporate a life preparation orientation into the structuring of the curriculum.
> In summary, therefore, a life-preperation orientation, in which contextualisation in everyday-life situations is central, is a prevalent feature of the ML curriculum. (p. 55)

She reiterates this emphasis on the non-mathematical orientation of the subject at a later point in the paper:

> … neither the ML curriculum, nor the enactment of it depicted in the excerpts above are organised around the needs of the 'mathematical terrain' – the focus, instead, in an ML lesson is on the terrain of the situations being investigated, and it is this terrain that grounds discussions. (Venkat, 2010, p. 65)

As the discussion in Part 2 of this study has demonstrated, I certainly disagree with Venkatakrishnan's suggestion that the Mathematical Literacy curriculum and, particularly, the enactment of that curriculum through national assessments, textbooks and pedagogic practices in classrooms throughout South Africa, is driven by non-mathematically orientated goals. However, the conception of Mathematical Literacy that Venkatakrishnan is describing in many ways encapsulates perfectly a vision for the structure of legitimate knowledge and participation that negates the pitfalls associated with an emphasis on mathematised and mythologised forms of participation. A format of the subject-matter domain of Mathematical Literacy is needed in which the needs of the mathematical terrain are of secondary importance and, instead, a life-preparedness orientation characterised by a dominant agenda for contextual sense-making practices and a dominant intention for the critical evaluation of both mathematical and contextual structures are prevalent. The internal language of description of the structure of knowledge for the knowledge domain of mathematical literacy presented in this part of the study foregrounds this type of orientation.

However, foregrounding a life-preparedness orientation by no means denies the crucial role that mathematics can play in helping to describe and represent a situation and in providing a different and potentially broader understanding of a situation: the life-preparedness orientation is not an all-or-nothing agenda. Rather, as suggested by Vithal (2008), mathematics and life-preparation (or, as referred to specifically by Vithal, mathematics and context), are ‘complementary’:

> Complementarity offers a powerful means for dealing with the problem of understanding the development and co-existence of significantly different, even opposing theories, explaining the same phenomenon. … The theories appear to be in opposition to each other and yet complementary. *Most importantly, both are needed to understand the phenomenon fully*. (p. 48, my emphasis)

Complementarity, then, is the term used to encapsulate that the relationship between mathematics and context is “both antagonistic and co-operative” Vithal (2002, p. 34).

As argued by Dowling (1998) and others (e.g. Bernstein (1996), Walkerdine (1990), Maton and Muller (2007)), and as acknowledge by Vithal (2002, 2003, 2008), mathematics and context are in opposition to each other and, importantly, that engagement in one field invariably leads to a suppression or subordination of the other field. However and despite this perceived relationship of opposition, Vithal (2003) argues that dual interaction between mathematical content and real-world contexts provides the opportunity for a different, potentially more in-depth and more complete or comprehensive understanding of both the mathematics and the context: “engagement in one can bring insights into the other, and change how each is experienced.” (Vithal, 2003, p. 321). Masingila, Davidenko, and Prus-Wisniowska (1996, pp. 194-195), express a similar sentiment, suggesting that “reality does not only serve as the application area but also as the source for learning.”

In relation to a conception of the subject-matter domain of Mathematical Literacy that prioritises a life-preparedness orientation, I contend that a view of mathematical content and real-world contexts as ‘complementary’ is essential, albeit with a necessary condition. Namely, that access to the mathematical content provides the opportunity to see the world from a different perspective and, in certain instances, provides for a broader understanding of a situation. However, this recognition of the use-value of mathematics must be underpinned by a dominant focus on contextual sense-making practices and not on the learning of mathematics. In other words, although there is no denial of the usefulness and power of mathematics in facilitating modelled (re)descriptions of segments of reality, this must be accompanied by recognition of the limitations of mathematics in accurately and adequately describing the structure of legitimate participation in real-world practices. Different and often non-mathematical, informal and colloquial forms of knowledge and reasoning that affect and direct action, thought and successful communication in the lived-world must also be consideration. In simple terms, the distinction is one of purpose or end-goal: the end-goal in a life-preparedness orientation is for a deeper understanding of the genuine lived-world and of appropriate and legitimate forms of participation in that world, and not for the development of mathematical knowledge. In a life-preparedness orientation, the development and utilisation of any mathematical knowledge must always be in service to a goal for enhanced contextual understanding, functioning and participation.

13.2 Segmented and Cumulative learning

13.2.1 Rationalising the need for a discussion on forms of learning

A format of the knowledge domain of mathematical literacy driven by an agenda for contextual sense-making practices runs the risk promoting the development of a form of knowledge that is context dependent, where the knowledge and skills developed are only relevant to specific localised problems in particular contexts of application. This is problematic for a conception of mathematical literacy that promotes a life-preparedness orientation, since this orientation ultimately requires that the subject must shift learners beyond the boundary of the classroom and into the real-world. If this shift is not prioritised, and if what happens in the classroom does not facilitate the development of a 'real-world gaze' over real-world practices, then the need for the subject is rendered defunct. This is because the learners, upon leaving school, could simply learn what they need to learn to better function in their lives whilst 'on the job' or 'in the shops'. As such, and employing the words of Maton (2009), what is needed is the promotion of a form of 'cumulative learning' as opposed to the type of 'segmented learning' that commonly accompanies pedagogic practices involving of contextualised mathematics practices.

In light of the above, the immediate discussion below is focused on highlighting the characteristics of cumulative and segmented learning. These notions are then referenced at various points in the discussion of the developed internal language of description to draw attention to the facets of each component of the developed language through which it is envisioned a form of cumulative learning is facilitated.

13.2.2 Segmented and cumulative learning

Maton (2009) extends Bernstein's (1999) (primarily intellectual) work on discourses and knowledge structures to explore how different forms of knowledge might be realised through a curriculum or a pedagogic process (Maton, 2009, p. 45).

By drawing on the distinction made between the development of knowledge in a hierarchical knowledge structure (knowledge builds (integrates) on and subsumes previous knowledge) and in a horizontal knowledge structure (knowledge is more localised and context dependent and, so, develops as a collection of different segments of knowledge) (c.f. Bernstein, 1999), Maton suggest that a similar distinction can be made with respect to how knowledge develops over time. Namely, "according to whether they [learners] build on their previously learned knowledge, and take that understanding forward into future contexts or learn knowledge that is strongly bounded from other knowledges and contexts." (Maton, 2009, p. 45). The former type of learning is attributed as *cumulative learning* – where knowledge is able to be transferred across a variety of contexts and through time, and the latter as *segmented learning* – where knowledge is primarily localised and context dependent and, so, where transfer from one context or time to another is restricted (Maton, 2009, p. 45). Cumulative learning is further associated with the generation of a 'knowledge code', namely where 'legitimate' knowledge is reflected in the possession of a body of explicit contents, principles, techniques, and skills for a particular field[124] (Maton, 2009, p. 46). Segmented learning, on the other hand, is more strongly associated with a 'knower code', where 'legitimate' knowledge is attributed to the "attitudes, aptitudes and dispositions" of the participants in the learning process rather than to a body of explicit principles of structures. In other words, the knowledge rests with the knower – the participant – whose opinion and choice of tactic are drawn on to facilitate sense-making practices (Maton, 2009, p. 46).

Now consider problem-solving activities directed towards engagement with and sense-making practices of contextual situations. Contextual sense-making practices in these situations requires understanding of contextually based and/or localised terminology, strategies, techniques, knowledge and forms of participation for solving problems. Legitimate knowledge in such practices is, then facilitated through a *knower code* and knowledge development through *segmented learning*. Although participants may come to engage with different contextual situations, these situations exist independently (i.e. are 'segmented') from each other, and knowledge of and successful engagement in one situation does not necessitate or preclude successful engagement in a different context. For Maton (2009, p. 51), this situation presents a contradiction and an associated dilemma. Namely, that in contrast to the dominant form of segmented learning facilitated through a knower code, the aim of pedagogic activity in authentic learning practices is to enable the development of higher order principles and skills which can be extended beyond the specific context of investigation to the broader world of work and society. This is the characteristic of *cumulative learning*. As a result, the assessment of authentic learning practices remains primarily at the level of the knowledge code, where it is the explicit principles, techniques and skills that are the primary focus.[125] As suggested by Maton (2009),

> The aim of authentic learning environments is to enable students to derive higher-order principles, but the knower code often characterising such environments means that students will often succeed only if they already possess those principles.[126] (p. 51)

[124] This conception of a knowledge code bears resemblance to Dowling's assertion that it is only through participation in Esoteric domain practices that participants are afforded access to the generative and evaluative principles of an activity. Similarly, the conception of a knower code bears some level of resemblance to Dowling's conception of the Public Domain and his argument regarding the objectification of participants in Public Domain activities by the practices in that domain and the positioning of such participants as either Dependents or Objects. See (Dowling, 1998, pp. 140-141) and/or Part 3, Chapter 9 and section 9.3 above, starting on page 138 for a reminder of these concepts.

[125] Reflecting this statement to current practices in the subject-matter domain of Mathematical Literacy, despite an emphasis in various curriculum documents for engagement with contextual situations and with forms of knowledge and participation that facilitate access to these situations (knower code), it is the assessment of mathematical structures and principles employed in these situations (knowledge code) that is prioritised.

[126] Notice that this statement bears a high degree of resemblance to Dowling's (1998) argument that it is only through access to Esoteric Domain practices that participants develop the capacity to identify the generative principles according to which recontextualised and/or mathematised contextual practices are structured and defined. Furthermore, it is only through engagement with Esoteric Domain contents that participants develop the capacity to cast a mathematical gaze over real-world contents and to constitute such contents in Public Domain of mathematics practices. This argument was described in detail in Part 3, Chapter 9 and section 9.3 above, starting on page 138.

For Maton (2009, p. 55 & 51), this mismatch between "means and ends" not only inhibits the development of cumulative learning but also disadvantages learners since the knowledge that is legitimised through assessment (i.e. knowledge code) is different to the knowledge that is legitimised through the pedagogic process (i.e. knower code).

The discussion in this chapter established the complementary relationship envisioned for mathematical content and real-world contexts, and distinguished two types of learning and associated codes of knowledge. The discussion below now moves to presentation of the specific components of the language of description of the structure of knowledge envisioned for a conception of the knowledge domain of mathematical literacy that prioritises a life-preparedness orientation. The notions of complementarity and segmented and cumulative learning are employed at various points in the presentation of this internal language to evidence and validate features of the language.

CHAPTER 14
A PROPOSED INTERNAL LANGUAGE OF DESCRIPTION OF THE KNOWLEDGE DOMAIN OF MATHEMATICAL LITERACY

In this chapter I present the internal language for describing a particular form and structure of knowledge and participation associated with a conception of the knowledge domain of mathematical literacy that prioritises and facilitates a life-preparedness orientation. This language of description positions the Esoteric Domain of the knowledge domain of mathematical literacy as comprising domains of practice involving Everyday, Mathematical Competency, and Modelling practices and knowledge, accompanied by a domain of practice that facilitates a form of critical engagement through various levels of Reasoning and Reflection, all grounded in and directed towards sense-making of real-world practices. These reconstituted real-world practices are posited as the Public Domain of the knowledge domain of mathematical literacy. I also rationalise the need for this language of description in relation to arguments presented in previous parts of the study regarding the current problematic structure of participation in the subject-matter domain of Mathematical Literacy. Throughout the discussion in the chapter I make a consistent and repetitive argument that engagement in the knowledge domain of mathematical literacy is legitimised according to critical contextual sense-making practices, which, in turn, is facilitated through continuous and integrated participation in all of the domains.

14.1 The need for a reconceptualised internal language of description: bridging the Theoretical Framework and Literature Review components of this study

The discussion above on the 'complementarity' of mathematical and life-preparedness goals for the subject-matter domain of Mathematical Literacy serves to illustrate a duality and tension inherent in any conception of mathematically literate behaviour. Namely, between the development and application of mathematical knowledge, skills and techniques on the one hand, and, on the other hand, engagement with contextual forms of understanding, knowledge, routines and legitimate forms of participation. In Part 2 of this study I demonstrated that current practices in the subject-matter domain of Mathematical Literacy are dominated by an agenda for numeracy in context type practices that involve mathematised and mathematically legitimised forms of knowledge, participation and communication with contextual problem-solving scenarios. A life-preparedness orientation, accompanied by a dominant agenda for contextual sense-making practices, is not evidenced in current practices in the subject. In Part 3 of this study I then outlined Dowling's concerns regarding the incorporation of out-of-school contexts in the mathematics classroom and the resultant mathematical myths facilitated through the imposition of a mathematical gaze on these contexts. By applying components of Dowling's theoretical language to current practices in the subject, I was, thus, able to make the claim that the dominant emphasis on Public Domain type mathematics practices in the subject facilitates the positioning of participants as Dependents and Objects in the pedagogic process. This inhibits successful apprenticeship in the domain of esoteric mathematics and also inhibits successful preparation for engagement in real-world problem-solving experiences. The distribution of the practices of this form of 'relevant' mathematics to learners who are (supposedly) weaker at mathematics (the majority of whom are located in poorly resources schools in working-class backgrounds), thus, facilitates a barrier to social and economic mobility for these learners. The presence of this subject in the schooling framework, then, serves to reinforce an element of educational and social disadvantage.

There is, thus, a dire need for a reconceptualisation of the structure of legitimate participation in the subject and, particularly, of the structure of knowledge required to facilitate this legitimate participation. I contend that one format for this reconceptualisation involves the prioritisation of a life-preparedness orientation that facilitates a form of participation legitimised according to a dominant agenda for contextual sense-making practices and a dominant intention for the critical evaluation of any structures encountered in these contextual sense-making practices.

This does not mean that mathematical components are not considered important and that there is no emphasis on the learning and application of mathematical knowledge and techniques; rather, mathematical components are considered crucial precisely because they facilitate an alternative understanding of a real-world situation. However, it is the understanding of the real-world situation that is the ultimate goal and not the mathematical processes that facilitates this understanding: whether a graph is drawn to represent a situation or whether a table of values is used is irrelevant to the larger goal of understanding and solving the problem. Mathematics and context are seen as 'complementary', but in this orientation it is the contextual terrain and the associated agenda for contextual sense-making practices that is the ultimate goal. Thus, while the importance of mathematical considerations are acknowledged, mathematics is positioned and viewed as one of many tools – including informal mathematical techniques and non-mathematical situational real-world knowledge, considerations and forms of participation − that can be employed to facilitate deeper understanding of a real-world context and/or of a problem situated in such a context. In Dowling related terms, apprenticeship in the subject is no longer defined according to mathematical structures; instead, apprenticeship involves functionality in real-world problem-solving practices.

14.2 Conceptualising the internal language of description in relation to Dowling's theoretical language and other theoretical concepts

The process involved in the development of the internal language of description presented in the pages below is an exercise in *theory development* – more specifically, an exercise in the development of a theoretical language to describe a *structure of knowledge*. This structure of knowledge reflects a particular form of participation and communication associated with a conception of the subject-matter domain of Mathematical Literacy in which a life-preparedness orientation is prioritised (and where this life-preparedness orientation is characterised by a dominant agenda for contextual sense-making practices and a dominant intention for critical evaluation of mathematical and contextual structures encountered and engagement with in the sense-making practices). Several further comments in relation to the statement made above are necessary.

To being with, it is only the *internal* component of the language of description that is presented in this part of the study. This internal component represents a conceptual language operating at a high level of theoretical abstraction and characterised by explicit and entirely theoretical indicators, constructs, grammars and syntax (Jablonka & Bergsten, 2010, p. 39). The external component of the language of description, comprising the means and methodology through which the theoretical components of the internal language are employed to provide a description of empirical practices relating to the subject Mathematical Literacy, is elaborated in Part 5 of the study (c.f. page 253 below).

For the sake of continuity and consistency, it is also possible to reflect on the development of this internal component in relation to Dowling's theoretical constructs. Doing so reveals that the internal component represents a description of the 'Esoteric Domain' of the knowledge domain of mathematical literacy. In the pages below I argue that this Esoteric Domain is comprised of four sub-domains of practice: (i) an Everyday domain; (ii) a Mathematical Competency domain, (iii) a Modelling domain, and (iv) a Reasoning and Reflection domain. Participation in the practices of the collective of all of these domains is posited as a necessary condition for successful apprenticeship in this domain and, consequently, for the development of the life-preparedness orientation. The internal component, then, provides a definition and description of the structure of legitimate participation and communication in relation to the contents of the knowledge domain of mathematical literacy, and of the regulating principles that define the criteria for this legitimised participation and communication. These regulating principles also define the terrain from and/or in which that knowledge is drawn or embedded.

The internal dimension also constitutes a 'Public Domain' of practice of the knowledge domain of mathematical literacy. This Public Domain – referred to in the internal dimension as the 'Contextual Domain' – is the collection of reconstituted real-world contextual situations, characterised by a high degree of authenticity and realism, and identified for investigation and engagement in the subject. Importantly, despite a requirement for a high degree of authenticity of these contextual situations, this Contextual Domain represents a domain of *recontextualisation*: namely, the contexts engaged with in the classroom setting only ever reflect a representation of a segment of reality and can never reflect reality in its entirety, either in terms of scope or complexity. In the pages below (c.f. sub-section 14.4.1.2) I rationalise this issue further. I argue that a contextual environment is comprised of a vast collection of 'focal events' and that in a classroom situation we are only able to focus on specific and selected focal events. This selection process facilitates the generation of a significantly *limited* perspective of the structure of legitimate and endorsed participation in the contextual environment. As with Dowling's Domains of Practice framework, new participants in the subject-matter domain of Mathematical Literacy are hailed into the practices of the subject through the Public (Contextual) Domain contents – namely, through the selection of focal events relating to contextual environments specified for investigation. Successful apprenticeship in the subject, then, involves movement from this Public (Contextual) Domain into the Esoteric Domain. In this way engagement with Everyday, Mathematical Competency, Modelling and Reasoning and Reflection domain knowledge and practices facilitates a gaze to be cast over the contents of the Public (Contextual) Domain and, so, for understanding of the structure of existing and possible alternative forms of legitimate participation in the contents of this Contextual Domain. In turn, understanding of the components of the Contextual Domain facilitates a gaze to be recast over the real-world since, by engaging in contextual sense-making practices, heightened preparedness for engagement in real-world practices is facilitated. The Dowlingaling©™ style Esoteric and Public Domain components of the internal dimension of the language of description are illustrated in Figure 25 below.

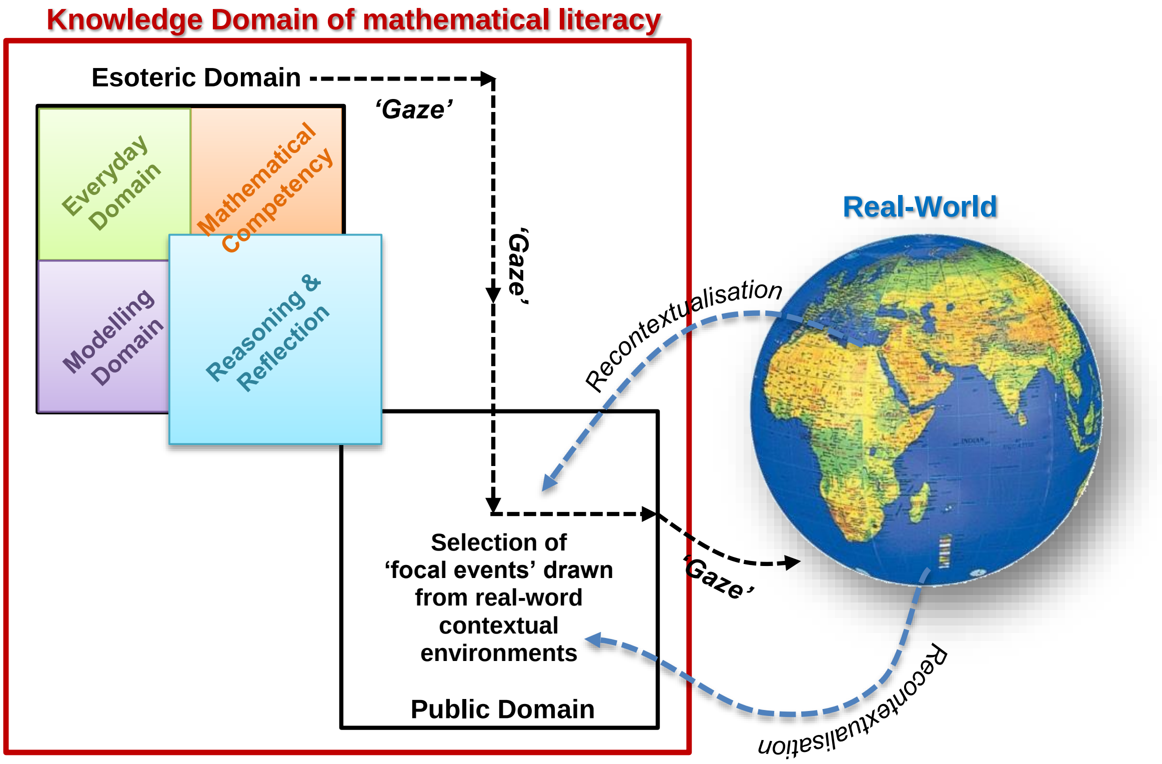

Figure 25: The 'Esoteric' and 'Public' Domains of mathematical literacy

In light of – and in relation to – the discussion above, it is also illustrative to establish the particular 'theoretical level' at which the internal dimension of the language of description is positioned. Namely, the internal dimension is characterised by a description of the structure of legitimate participation associated with practices that prioritise a particular perspective regarding the relationship between mathematical and contextual elements, contents, and knowledge. As such, although the internal language is developed for application in the specific empirical terrain of the subject Mathematical Literacy, the language also represents a general description of knowledge and, so, has relevance to any situation involving contextual sense-making practices that draw on both mathematical and contextual elements. For this reason the internal component is conceived as providing a description of mathematical literacy as a knowledge form or knowledge domain. Importantly, this theoretical language presents an 'ideal-type' schematic of the structure of knowledge associated with a particular conception of mathematical literacy. As such, there is every possibility that the enacted behaviour of participants (in the subject or in any variety of settings and/or practices involving this knowledge domain) may be in complete contrast to this 'ideal-type' schematic – as influenced, affected and determined by the particular environments in which the participants encounter this knowledge and by the structure of the esoteric domain contents selected, recontextualised and prioritised for use in those environments.

14.3 Broad overview of the components of the internal language of description

The intention of this section is to orientate the reader to the key components of the internal language of description before embarking on more detailed discussion and motivation of each component. To this end, the diagram shown in Figure 26 below provides a 3-dimensional representation of the components and levels of the language of description.

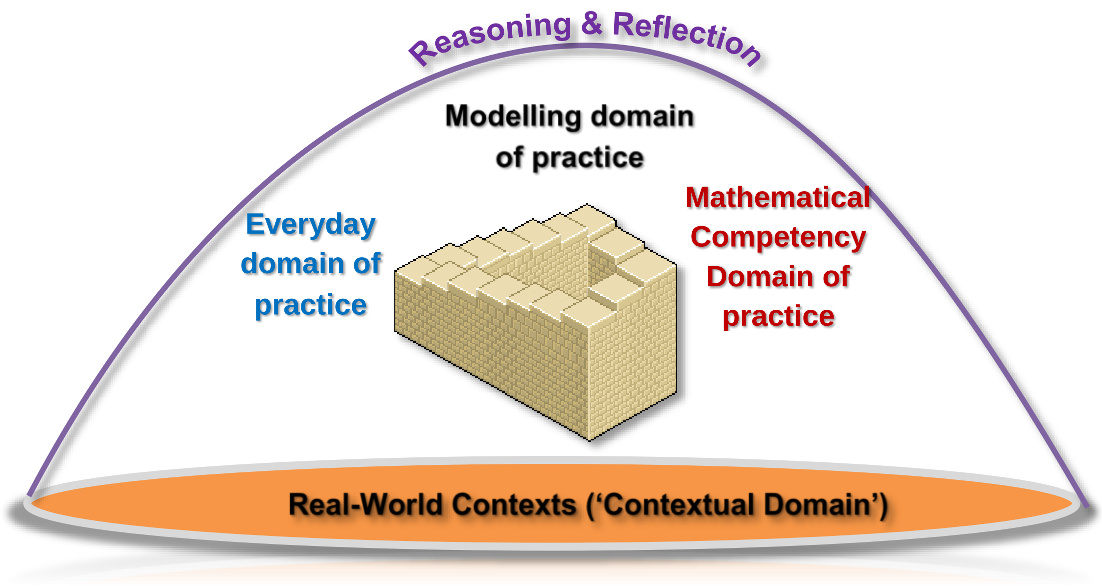

Figure 26: Diagrammatic overview of the language of description of the knowledge domain of mathematical literacy

At the base of the language is the 'Contextual Domain' – namely, the domain of reconstituted *real-world contexts* modelled on authentic practices. Contexts provide the point of entry into and the focus of and motivation for the pedagogic process and, hence, the Contextual Domain is constituted as the 'public domain' of mathematical literacy.

The 'body' of the language is comprised of three domains of practice – *Everyday*, *Mathematical Competency* and *Modelling*. The Everyday domain of practice comprises 'practical intelligence' – namely the ability to discern and employ various localised and contextually based (and often non-mathematical, intuitive and/or informal mathematical) techniques and considerations that inform practice and decision-making in everyday situations. The Mathematical Competency domain of practice emphasises the development of a core of mathematical knowledge, particularly in preparation for the Modelling domain of practice of the language. The Modelling domain involves the development and analysis of models designed to simulate (re)descriptions of real-world situations and of possible forms of participation that facilitate the solving of problems in those situations. This component constitutes the intersection of the Everyday and Mathematical Competency domains of practice as both mathematical and contextual forms of knowledge, participation and communication are considered to facilitate contextual sense-making practices.

A key point of distinction between the Modelling domain of practice and the Everyday and Mathematic Competency domains relates to the way in which engagement with reality is experienced in each domain. Namely, participation in both the Everyday and Mathematical Competency domains is directed towards engagement with and understanding of different elements of an existing reality – the former domain with distinctly contextual elements and the latter domain with distinctly mathematical elements. By contrast, participation in the Modelling domain of practice is directed towards: recreating or reconstructing aspects of reality to enhance understanding of existing (contextual and mathematical) structures in these aspects; to explore possible alternative ways of functioning and participating in the aspects; or to explore how participation in the aspect of reality might change if the structures that define and direct participation in the reality shift. While participation in all of the domains is dominated by an agenda for contextual sense-making, it is engagement in the Modelling domain of practice that facilitates investigation of alternative forms of participation in contextual environments in both existing and possible future formats of those environments.

The Everyday, Mathematical Competency and Modelling domains of practice are joined together using an Escher-style 'staircase' to indicate how it is through collective engagement with all three domains of practice that contextual sense-making practices are engaged, a broader view the structure of legitimate forms of participation in a real-world situation is developed, and a degree of life-preparedness is facilitated. The 'staircase' further indicates that domain of practice is informed by and is empowered through alignment with the other domains. The specific nature of the relationship between each of the three components is discussed in more detail in the pages below.[127]

The final domain of practice is the domain of *Reasoning and Reflection*. This is the domain of practice that brings together the other three domains to facilitate a life-preparation orientation through the subordination of mathematical goals and, instead, prioritisation of a form of participation characterised by contextual sense-making practices and critical engagement and evaluation with or of contextual and mathematical structured encountered in those practices. This is achieved through reasoning and reflection on issues such as, but not limited to: which components of a real-life situation to highlight and which to ignore in defining the precise nature of a problem situation; which mathematical methods to use in modelling a situation; interpreting and validating mathematical solutions with respect to the original problem situation; identifying the limitations of mathematical models and associated mathematical solutions and accounting for contextual and localised techniques and considerations which may serve to override the mathematical; reinventing models to suit changing conditions in the real-world situation; and identifying assumptions, ideals and values which underpin models and the particular perspective of reality generated by and presented through those models.

Taken together, the intersection of the Everyday and Mathematical Competency domains of practice through the Modelling domain and the enactment of Reasoning and Reflection throughout this process, facilitates a language of description for the structure of knowledge in the subject-matter domain of Mathematical Literacy characterised by a form of participation in which life-preparedness is posited as the primary goal of the

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[127] It is, perhaps, worthwhile to point out in a preliminary way that a hierarchical relationship is envisioned between the Everyday, Mathematical Competency and Modelling domains of practice. This hierarchical relationship is reflected in the thinking that understanding of mathematical concepts and techniques is seen as a prerequisite for successful engagement in modelling processes, as is understanding of contextual elements, contents, terminology and notation of the contextual environments (or focal events within those environments) to which the modelled representations relate. However, this is certainly not to suggest that there is a one-to-one correlation between mathematical knowledge (or between everyday knowledge) and the ability to construct models or successfully engage in applications of such knowledge to solve problems. Rather, I concur with Skovsmose's (1994a, p. 47) suggestion that "even if students learn mathematics, no guarantee exists that the developed competence is sufficient when it comes to situations of application. More has to be mastered than pure mathematics in order to apply mathematics." This sentiment is echoed empirically by Blomhoj and Jensen (2003).

pedagogic process. Each component of the knowledge domain of mathematical literacy is now discussed in detail in Section 14.4 below.

14.4 Detailed analysis of the components that characterise the knowledge domain of mathematical literacy

14.4.1 Contextual Domain

In this immediate discussion, a ‘context’ refers to a situation, scenario, and/or experience drawn from real-life or the real-world that provides the setting within which something occurs or something exists. In the knowledge domain of mathematical literacy, a central goal is the investigation of these real-life or real-world experiences or situations to enhance understanding of the situation and of existing and possible alternative forms of legitimate participation in the situations, and, so, to prepare participants for participation in similar and/or other experiences or situations. In this section I discuss in detail the context-types and contextual attributes that are envisioned as appropriate for a conception of mathematical literacy that seeks to facilitate life-preparedness.

14.4.1.1 Context types

As indicated in Part 2 of this study, real-life contexts and problems situations occurring in these contexts are central to all conceptions of mathematical literacy. An area of divergence, however, is the extent to which such situations and contexts must be ‘authentic’. Namely,

> one which is embedded in a true existing practice or subject area outside mathematics, and which deals with objects, phenomena, issues or problems that are genuine to that area and are recognised as such by people working in it.
> (Niss, 1992, p. 353; Referenced in: Palm, 2009, p. 4)

Brown, Collins, and Duguid (1989) argue similarly, but focus more explicit attention instead on the socially situated nature of authentic activities:

> The activities of a domain are framed by its culture. Their meaning and purpose are socially constructed through negotiations among present and past members. … Authentic activities then, are most simply defined as the ordinary practices of the culture. (p. 34)

My interpretation of the two quotations above suggests that a context is to be classified as authentic if the context provides an unaltered view of reality and, so, provides as close an experience as is possible of how that context is experienced by participants who engage in the context in their daily lives. In alternative language, an authentic context is one that reflects the way in which knowledge is actually and ultimately used in that context, in which the complexity of the context is retained, and in which multiple perspectives and solutions paths are recognised (Herrington & Oliver, 2000, pp. 25-26). Furthermore, authentic contexts require authentic activities: namely, activities that have direct relevance and relation to a problem-solving activity situated in an authentic real-world context (Herrington & Oliver, 2000, pp. 25-26) and where the dominant focus of the activity it to facilitate greater understanding of the context or of a problem in that context. A mathematised context – or a context that has been modified, simplified or reconstructed to accentuate or make more accessible the mathematical components of the context, and in which a particular mathematical solution path and solution are prioritised – is, thus, not categorised as 'authentic' according to this conception.

A further comment is necessary with respect to Brown, Collins and Dugid's (1989) specific emphasis on the link between authentic activities and the ordinary practices of a culture. I contend that this emphasis is somewhat problematic since it suggests that engagement with such activities embodies a form of learning which is entirely localised within a specific culture and which does not extend beyond the practices of that culture. This provides an explicitly limited learning perspective, particularly for the subject-matter domain of Mathematical Literacy which aims to provide broader insights and perspectives into the workings of the world (and not just of particularised cultural practices). I suggest that substitution of Brown, Collins and Dugid's usage of the words 'authentic activity' with the words 'relevant activity' provides a more apt description: namely, activities that are directly *relevant* to a group of participants are those which stem from the ordinary practices of the culture in which the activities are based. Authentic activities, on the other hand, refer to those activities which have a strong connection to reality, but these activities are not necessarily tied to a particular cultural setting and, rather, transcend cultural boundaries.

Importantly, however, the process of categorising contexts is not simply a matter of deciding whether a context is authentic. Rather, there are a host of context-types and context-forms that exit on a spectrum from entirely mathematical (and non-authentic) and purely authentic contexts. As has been discussed previously (c.f. Part 2, Chapter 6 and section 6.3 above, starting on page 68), du Feu (2001, p. 2) identifies five context-types ranging on the mathematical-authentic spectrum: (i) context free; (ii) contrived, (iii) parable, (iv) cleaned and (v) real. Sethole et al. (2006, p. 126), offer an extended characterisation by focussing not only on the authenticity/inauthenticity dimension but also on the 'nearness' (familiarity) or 'farness' (novelty or unfamiliarity) of a context in relation to the learners' own experiences of the world. Where du Feu (2001) offers a horizontal dimension for differentiating the extent to which a particular context is linked to reality, Sethole et al. (2006), offer both a horizontal and a vertical differentiation based on the link to reality and the familiarity/unfamiliarity dimensions respectively.

From the descriptions given by Sethole et al. (2006, p. 126) of authentic tasks as "tasks in which the context is used genuinely or without major modifications", it seems appropriate to suggest a concordance between the categories of 'real' and 'cleaned' contexts identified by du Feu (2001) and the categorisation of 'authentic' contexts posited by Sethole et al. (2006). The addition to this conceptualisation is that real and cleaned contexts can be either near (familiar) or far (unfamiliar). Similarly, du Feu's (2001) contextual categories of 'context-free', 'contrived' and 'parable' are easily aligned to the categorisation of near and far 'inauthentic' contexts identified by Sethole et al. (2006). This concordance between the horizontal and vertical dimensions of different context-types is illustrated in the diagram shown in Figure 27 below.

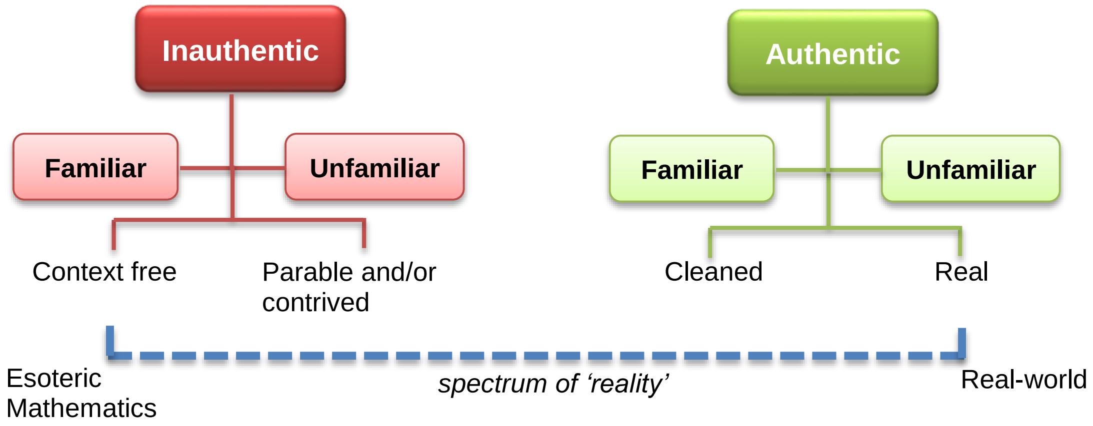

Figure 27: Categories of contexts

I contend that it is only contexts that fall within the 'Authentic' branch of the diagram above – namely, cleaned and real (both familiar and unfamiliar) contexts – which are appropriate for inclusion in practices characterised by a life-preparedness orientation. Inauthentic contexts, by contrast, deliberately sacrifice aspects of contextual reality, authenticity and complexity to promote increased access to mathematical components of a problem situation or solution path. As such, engagement with inauthentic contexts is antithetical to the goals of a conception of mathematical literacy that prioritises a goal for contextual sense-making practices and associated life-preparedness over a goal for the development of mathematical knowledge. Furthermore, by prioritising mathematical components, inauthentic contexts facilitate Public Domain type practices that embody a mathematised gaze over real-world practices and the inculcation of the myth of participation with participants who are ultimately relegated to positions of Dependent or Object in the pedagogic process.

By comparison, engagement with 'real' contexts facilitates a form of participation that is positioned outside of the frame of Dowling's (1998) language of description. This is because Dowling's language is primarily concerned with everyday practices that are constituted through a mathematical gaze and reorganised or recontextualised mathematically within the bounds of the mathematics classroom. Dowling's language does not extend to contexts that remain authentic and deeply embedded in reality, or with practices that facilitate engagement with contextual and non-mathematised forms of participation. Participants engaging with real contexts, then, where the development and deployment of a mathematical gaze is not prioritised, are no longer susceptible to the challenges associated with the mathematically structured Myths of Reference and Participation.

Can the same be said to be true of 'cleaned' contexts? Perhaps the answer to this question lies in the rationale prioritised for cleaning the context. But first, clarification is necessary on precisely what constitutes a 'cleaned' context. du Feu (2001) offers some insight in this regard, suggesting that cleaned contexts are

> essentially real contexts, but where the mathematical model has been simplified in order to make the question accessible to the user or possible in the time constraints of an examination. (p. 2)

In line with this quotation, my own experience in the terrain of mathematics education has led me to observe that contexts are commonly cleaned in a mathematics classroom to facilitate (increased) access to mathematical components of a problem by removing contextual components that may distract or restrict access to the mathematical components. I contend that this form of cleaning intention facilitates the generation of Public Domain tasks and associated practices, with a deliberate prioritising of mathematical components and mathematised forms of participation. This cleaning intention reinforces the imposition of a mathematical gaze over real-world practices, together with the associated myths of Reference and Participation that commonly accompany these mathematisation processes.

Cleaned contexts in a conception of mathematical literacy that prioritises a life-preparedness orientation are contexts where certain aspects of the reality of the contexts have been removed or simplified or restated in alternative text. Also included in this categorisation of cleaned contexts are contexts that have been reconstructed or developed from scratch – as opposed to the utilisation of an existing authentic resource – but which bear a high degree of resemblance to the authentic resource and, so, to the reality of participation in the real-world situation. I contend that there is a two-fold rationale for cleaning, modifying or (re)constructing contexts in a conception of mathematical literacy dominated by a life-preparedness orientation. The first reason is to afford greater access to (and understanding of) particular contextual elements of the context, based on the level of development of learners and the level of 'farness' (unfamiliarity) of the context for the learners. For example, a teacher may choose to reconstruct a payslip for a group of Grade 10 learners rather than to engage them with an original or real payslip in order to reduce the complexity of the payslip and/or the amount of information shown on the payslip. The teacher may then opt to reintroduce a differently reconstructed payslip for Grade 11 learners – one that contains more complicated and detailed information; and, finally, an authentic and unaltered payslip for Grade 12 learners. Cleaning a context in this way provides the opportunity to explore different components of the context in different ways and at different times to facilitate a more comprehensive and in-depth understanding of the context.

Although not referring specifically to the cleaning of contexts, I interpret Palm's (2009) work to suggest an alternative and second rationale for cleaning, modifying or (re)constructing contexts. Namely, that when dealing with real-world situations that are simulated in a classroom setting,

> A restriction of comprehensiveness is always necessary. It is not possible to simulate all aspects involved in a situation in the real world and consequently it is not possible to simulate out-of-school situations in such a way that the conditions for the solving of the task will be exactly the same in the school situation. (Palm, 2009, p. 8)

Brown et al. (1989, p. 34) offer similar commentary when suggesting that, "When authentic activities are transferred to the classroom, their context is inevitably transmuted; they become classroom tasks and part of the school culture." In other words, the function of classroom practice is to facilitate understanding of particular knowledge and/or of a particular concept. This ultimately involves some level of selection and prioritising of certain knowledge components over others. The cleaning of a context facilitates this necessary process. In doing so, the cleaning of a context for this purpose also makes it possible to analyse and identify similarities and commonalities (and differences) across and between different contexts, hereby opening up the potential for generalisations across differing fields of practice.

As hinted at in the quotation above, Palm (2009, p. 8) uses the notion of ‘comprehensiveness’, together with the notion of ‘fidelity’, to position a simulated school-based task in relation to the real-world situation simulated in the task. Comprehensiveness refers to the range of aspects of a real-life situation that are able to be included in a school-based simulation of the scenario. Fidelity refers to the degree to which each aspect in the school based task provides a fair and accurate representation of this aspect in the real-world situation. ‘Representativeness’, then, refers to the combination of comprehensiveness and fidelity and, so, provides an indication of the degree of resemblance between the school-based simulation and its counterpart in authentic real-world practice (Palm, 2009, p. 8). Reflecting this discussion to the language of description of the structure of knowledge for the knowledge domain of mathematical literacy, for a life-preparedness orientation to be facilitated, any context engaged with – whether real, cleaned, modified, or constructed – must exhibit a high degree of representativeness of its real-world counterpart. It is only through exposure to highly representative problem situations that genuine contextual sense-making practices are facilitated; and it is further only through exposure to such problem situations that contextually appropriate and legitimate forms of knowledge, participation and communication are encouraged and validated:

> … a repetitive encounter with word problems[128] that are simulations with a high degree of experienced representativeness and include figurative contexts that are experienced as meaningful affects students so that they increase their engagement in the figurative contexts, and a larger portion of the students will use their knowledge of the real-world situations described in the tasks in their word problem solving. (Palm, 2009, p. 14)

14.4.1.2 Analysis of context as a ‘limited’ perspective

Having established an intention in the knowledge domain of mathematical literacy – and in any practice or terrain associated with this knowledge domain – for an exploration contexts that reflect as accurately as possibly authentic participation in the context, an important acknowledgement is necessary. Namely, that any analysis of a real-world setting or situation in an academic environment can and will always only provide a limited analysis, perspective and understanding of that situation. Goodwin and Duranti’s (1992) work on the ‘attributes’ of educational contexts provides some useful insights in this regard. A key argument in this work is that any analysis of a context always only provides a restricted or limited view of the context. This is because every context is always embedded in a broader contextual environment and, as such, all possible factors and variables that impact on and influence behaviour and engagement in a context cannot be considered or considered in totality. By implication, an analysis of a context constitutes an analysis of a particular ‘focal event’ that represents a specific and limited phenomenon in the contextualised environment (Goodwin & Duranti, 1992, pp. 2-3). Importantly, however, adequate understanding of a focal event cannot be determined without consideration of various extraneous factors – for example, cultural setting, and current socio-historical and/or economic factors. These factors exist beyond the focal event but which have a direct bearing on the nature and structure of the event and on how participants behave and engage in the event at a particular point in time (Goodwin & Duranti, 1992, p. 3). The context is thus a ‘frame’ – a ‘field of action’ −

[128] Although Palm focuses primarily on ‘word problems’ employed in the context of the teaching and learning of mathematics, I believe that the applicability of his work to the contextually based problems inherent in the subject-matter domain of Mathematical Literacy is prevalent and appropriate.

that envelops the focal event and provides the resources needed for adequate and valid interpretation of the event (Goodwin & Duranti, 1992, p. 3).

The discussion in the immediate sub-section below elaborates on the contextual attributes that direct and influence participation and decision-making processes in a field of action.

14.4.1.3 Contextual attributes

Building on the idea that a context comprises "two orders of phenomena" – (i) a focal event and (ii) a field of action – "that mutually inform each other to comprise a larger whole" (Goodwin & Duranti, 1992, p. 3 & 4), Goodwin and Duranti identify four overarching attributes, parameters or dimensions of contextual environments that influence and affect how participants engage with a focal event embedded within a broader field of action. These are: (i) setting; (ii) behavioural environment; (iii) use of language; (iv) extra-situational knowledge (Goodwin & Duranti, 1992, pp. 6-8). Each of these dimensions is discussed below.

(i) *Setting* refers to the social and spatial environment within which a particular interaction takes place (Goodwin & Duranti, 1992, p. 6) – in other words, the specific setting in society and/or the environment in which the focal event is located. For example, the focal event might involve the use of a map in the setting of a city, or a construction project in the setting of a school building, or calculation of income tax in the setting of the personal finances of an employee. Bansilal (2013) argues that in the specific domain of the subject Mathematical Literacy, the 'setting' refers to the particular real-life context under investigation.[129]

(ii) *Behavioural environment* refers to the framing that establishes the conditions for how participants behave and act in relation to the focal event (Bansilal & Debba, 2012, p. 305): using alternative terms, the behavioural environment refers to the <u>site</u> in which an interaction with a contextual environment occurs. Examples of different behavioural environments for the subject-matter domain of Mathematical Literacy include classroom interactions and discussions, assessment (test or examination) settings (Bansilal, 2013), or actual engagement in daily practices. Importantly, the site of interaction between participants and a real-world contextual environment impacts directly on the way in which participants engage with a context, on the types of resources, tools, and constraints that they bring to bear on the context, and on the criteria according to which practice in or with the context is legitimised and/or endorsed. For example, learners in the subject Mathematical Literacy will, in all likelihood, respond to a real-world context encountered in the setting of an examination paper differently to an encounter with that same context as part of a classroom discussion or outside of a classroom setting (i.e. as part of a daily life practice). As a consequence, the behavioural environment plays a significant role in influencing the structure of the real-world perspective and outlook that participants develop as they engage with a contextual event. This environment further influences the structure of participation that is deemed legitimate and appropriate for engagement with the

[129] Note that in Bansilal's (2013) conception, 'setting' refers only to the context being explored and not also to the specific <u>*site*</u> (e.g. pedagogic or assessment) in which learners encounter a particular contextual environment. The issue of site is characterised by Goodwin and Duranti (1992, p. 7) in terms of the 'behavioural environment' of a contextual phenomenon (see contextual attribute (ii) below).

contextual event, and of the types of resources participants are expected and required to draw on to replicate and generate legitimate or endorsed practices.

Goodwin and Duranti (1992, pp. 7-8) identify the third contextual attribute of (iii) *use of language* as a means to emphasise "the way in which talk itself constitutes a main resource for the organization of context" (Goodwin & Duranti, 1992, p. 8). For these authors, language in a context invokes specific meaning – i.e. "invokes context" – and provides the basis for further communication and talk about the context (Goodwin & Duranti, 1992, p. 7). In specific reference to the subject-matter domain of Mathematical Literacy, Bansilal and Debba (2012, p. 305) make use of the phrase 'contextual language' to refer to words or phrases that hold a specific meaning within a context. Included in contextual language are technical terms that serve as descriptors of aspects of a contextual situation, often with a specific meaning that is localised in and bound to the context, and comprehension of which is crucial to understanding of the parameters of the context. For example, sometimes the term 'return trip' is used to describe the entire trip for a journey (i.e. both the trip to a destination and the trip back from the destination) and sometimes it is used to refer only to the trip on the way back. Understanding of the specific intended meaning of this technical term as employed by a particular individual in a particular contextual instance is crucial for being able to successfully communicate with this individual and for being able to successfully engage with any problem-solving situations that make reference to the term. Context-specific language also comprises signifiers[130] that index the focal event(s) under analysis as well as the particular variables and contents to be engaged with in relation to the focal event(s). For example, the term 'income-tax' indexes precisely the focal event under analysis, and, when accompanied by other signifiers such as 'tax bracket formula', draws attention to the specific component of the focal event that must be engaged with (and also to the routines that must be employed in engagement with the focal event). Following on from this, it seems a logical assumption that inability to engage with the language of the context becomes an inhibitor to successful and legitimised engagement and participation in the contextual environment.

The fourth dimension or contextual attribute comprises (iv) *extra-situational context* (Goodwin & Duranti, 1992, pp. 8-9) or *extra-situational background knowledge* (Bansilal & Debba, 2012, p. 305). This attribute relates to how an "appropriate understanding of a conversational exchange requires background knowledge that extends far beyond the local talk [or focal event] and its immediate setting." (Goodwin & Duranti, 1992, p. 8). Within the knowledge domain of mathematical literacy, extra-situational background knowledge refers to information, resources and knowledge that are essential for illuminating not only the focal event but also the background of the wider context and field of action in which the focal event is located. And it is this combination of understanding of both focal event and of the position and significance of that focal event within a broader contextual environment that is of central importance for facilitation of a life-preparedness orientation. By means of illustration, consider the scenario where

[130] In Part 5 (starting on page 253) I operationalise an external dimension of the language of description to facilitate a gaze to be cast from the theoretical constructs of the internal dimension of the language over empirical practices relating to the subject-matter domain of Mathematical Literacy. This external language is grounded in the work of Anna Sfard (2008) and is focused on identification of the unique discursive resources that characterise and regulate the structure of legitimate participation and communication in the knowledge domain of mathematical literacy. *Signifiers* constitute a particularly important discursive resource, since it is the signifiers employed in a segment of discourse that index not only the specific event under investigation, but also the routines required to facilitate the generation of narratives about the event that will be endorsed by other participants in the domain. The role of signifiers is dealt with in detail as part of this later discussion and operationalisation process.

learners in the subject-matter domain of Mathematical Literacy are being shown how to predict prices by performing calculations using given inflation rate values. It would be entirely possible for the learners to engage with this limited focal event [i.e. performing a specific type of inflation related calculation] successfully in isolation of any broader discussion of how inflation in South Africa is calculated, factors that affect and determine domestic inflation, and how inflation rates inform decision-making in a variety of situations. It is in reference to this exact issue that Goodwin and Duranti (1992) lament:

> Generally the focal event is regarded as the official focus of the participants' attention, while features of the context are not highlighted in this way, but instead treated as background phenomena. ... In line with this, the boundaries, outlines, and structure of the focal event are characteristically delimited with far more explicitness and clarity than are contextual phenomena. ... The effect of this is that it becomes easy for analysts to view the focal event as a self-contained entity that can be cut from its surrounding context and analyzed in isolation, a process that effectively treats the context as irrelevant to the organization of the focal event. (p. 10 & 11)

However, if the goal of the pedagogic process in this instance is to help the learners to develop an enhanced and more comprehensive understanding of inflation, of the impact of inflation on people's lives, and of how inflation related considerations can influence decision-making processes, then consideration of broader inflation-related contextual knowledge and considerations (such as those mentioned above) must be brought to bear on the pedagogic process.

14.4.1.4 Contextual characteristics and attributes prioritised for the knowledge domain of mathematical literacy

In light of the discussion above, a three-fold priority with respect to contextual situations is envisioned for the knowledge domain of mathematical literacy. Firstly, the contextual situations appropriated for recontextualisation and engagement in the Public Domain of mathematical literacy must exhibit a high degree of representativeness to the real-world practice and, as such, must reflect an authentic as possible representation of the structure of legitimate real-world participation.

Secondly, despite emphasis on representativeness and authenticity of contexts, any engagement with a contextual situation must be accompanied by recognition that this engagement is only with one (or more) of many possible focal events of the situation and that engagement with the focal event only describes a limited perspective of the contextual environment in which the event is located. This understanding also translates to the Modelling domain of practice of the internal language of description, where it is essential that participants understand that the construction of a single model to represent a situation in no way represents the sum-totality of reality for that situation. In the scope of an academic domain of knowledge such as is mathematical literacy, any attempted reconstruction or analysis of reality is always only partial, limited and/or incomplete.

Thirdly, any attempt at a comprehensive understanding of a contextual situation is only possible if consideration is given to more than just the immediate features of the focal event. Namely, to the variety of contextual resources drawn from the wider field of action in which the focal event is positioned which impact on the way in which engagement with the focal event is legitimised and endorsed, and which signify areas of focus, attention and prioritising in the event. And, this is only possible through consideration of the contextual attributes – including setting, behavioural environment, language use, and extra-situational knowledge – that impact and influence how people think and act and the decisions they make as they engage in situations encountered in their real-world settings.

Contexts constitute the Public Domain and point of entry into the knowledge domain of mathematical literacy, hereby providing the foundation of the knowledge domain and the motivation and impetus for all activity in this domain. In promotion and pursuit of a life-preparedness orientation, it is imperative that these contexts – as reconstituted representations of reality – provide participants with as close to a real-world experience as possible such that engagement with the contexts facilitates some degree of preparation for future engagement in real-world practices. Anything less results in a mathematised world-view and a prioritising of mathematical considerations, knowledge and forms of participation characteristic of Public Domain of mathematics type practices, and the resultant lessening of life-preparation and heightened subordination of participants to positions of dependency in the pedagogic process.

14.4.2 Domain of Practice #1: Everyday domain

Given the centrality of real-world contexts – in promotion of a life-preparedness orientation − in the language of description of the structure of knowledge for the knowledge domain of mathematical literacy, a key facet of the language involves acknowledgement of the various considerations which may affect decision-making in a context. This acknowledgement includes recognition that many of the considerations that affect problem-solving and decision-making in these contexts may not be of a formal mathematical nature or may not involve mathematical elements at all. For example, the decision of which cell phone to buy may be informed by considerations of cost, but also by the colour of the phone, the features of the phone, and whether or not a particular phone is available when standing in the store. Furthermore, although there is the potential to create various mathematical models to determine best buy options based on a comparison of cell phone costs for different packages, there are also ways to do this comparison involving far simpler and less formal mathematical procedures – employing, perhaps, what some authors have referred to as everyday mathematics and/or practical intelligence.

In light of this, a sojourn into the literature on everyday mathematics and practical intelligence is necessary to describe more acutely how the structure of legitimate participation from the perspective of the Everyday domain of practice is characterised in knowledge domain of mathematical literacy.

14.4.2.1 'Everyday mathematics'

There is an array of literature (e.g. Rogoff and Lave (1984); Scribner (1984); de la Rocha (1985); T. N. Carraher, Carraher, and Schliemann (1985); Nunes, Schliemann, and Carraher (1993); Masingila (1994); Dowling (1998)) that positions mathematical practices that occur in everyday situations as different to those occurring in the formal setting of the classroom: the distinction made between 'everyday' mathematics and 'academic' mathematics, between 'informal' and 'formal' mathematics (Greiffenhagen & Sharrock, 2008), or between 'common-sense' and 'educational' knowledge (Painter, 1999).

Painter (1999), although focussing on learning processes in general and, specifically, on the acquisition of language (and not directly on the learning of mathematical knowledge), offers a useful initial distinction between these two forms of knowledge. For Painter (1999), common-sense knowledge is taken to mean

> Knowledge that appertains to the visible material world, that is functional for the routine living of daily life, that is non-specialized, shared by all members of the culture of community and realized through everyday forms of talk. … The learning of commonsense knowledge is informal and gradual, an incidental part of participating in the routines of everyday life. (p. 68)

This suggests that the learning of common-sense knowledge is inherently unstructured (and/or non-hierarchical), un-sequenced, unplanned and distinctly localised in the immediate context of exposure or use (Painter, 1999, p. 70). Painter extends this characterisation to suggest that the development of common-sense knowledge, in the words of Bernstein (1975), comprises weak framing and weak classification (Painter, 1999, p. 70).

Educational knowledge, by contrast, is directly concerned with the development of generalisable abstracted knowledge that has application beyond a localised context of use. This process is facilitated through engagement with content and knowledge that is hierarchically and sequentially organised, and which, "because of its specialized and abstract nature, it is inaccessible to incidental, observational learning and is likely to be accessed through conscious teaching and learning … ." (Painter, 1999, p. 70).

Key to the distinction made between these two 'types' of knowledge is the recognition that people in out-of-school situations commonly employ techniques and strategies for solving problems that bear little or no resemblance to formal procedures taught in the context of the classroom. As suggested by Greiffenhagen and Sharrock (2008),

> Studies of everyday mathematics demonstrated that people who do not perform well in school mathematics (and therefore might think that they can't do mathematics) are frequently efficient and successful in solving everyday and street arithmetic problems (i.e., 'really' can do mathematics). These studies aimed to highlight the alienating effects of classroom instruction and to establish that the acquisition of rationality does not require schooling.[131] (p. 4)

T. N. Carraher et al. (1985) and also D. W. Carraher and Schliemann (2002) offer similar acknowledgement:

> There are reasons for thinking that there may be a difference between solving mathematical problems using algorithms learned in school and solving them in familiar contexts out of school. Reed and Lave (1981) have shown that people who have not been to school often solve such problems in different ways from people who have. This certainly suggests that there are informal ways of doing mathematical calculations which have little to do with the procedures taught in schools.
> (T. N. Carraher et al., 1985, p. 21)

[131] Remember my mother-in-law? The one who got a GG symbol in mathematics at school? I'm fairly sure she would really like this quote.

> One of the ways that everyday mathematics research has helped in this regard has been to document the variety of ways people represent and solve problems through self-invented means or through methods commonly used in special settings. (D. W. Carraher & Schliemann, 2002, p. 273)

de la Rocha (1985, p. 195) describes this phenomenon by arguing that people employed in everyday practices commonly switch between 'normative' and 'non-normative' techniques, the former characterised by accuracy and the latter by simplified, estimated or non-mathematical techniques.[132] Importantly, the decision of which technique to employ is driven entirely by consideration of the method that will most efficiently or easily facilitate the completion of a task or attainment of a goal:

> … the principal features of cooking and dieting that affected shifts in normative and non-normative measurement practice were those that allowed the dieter to resolve the conflicts; i.e., to achieve cooking objectives without compromising weight loss objectives. (de la Rocha, 1985, p. 195)

Similarly for Rogoff (1984):

> what is regarded as logical problem solving in academic settings may not fit with problem-solving in everyday situations, not because people are "illogical" but because practical problem-solving requires efficiency rather than a full and systematic consideration of all alternatives. (p. 7)

Scribner (1986, p. 22) describes this form of activity as "redefining preset problems into 'subjective' problems." This situation is contrasted with the classroom setting where principled, procedural and predominantly normative techniques − together with an emphasis on accuracy − are prioritised.

T. N. Carraher et al. (1985) employ the term 'convenient-group' to explain the difference between the type of mathematically based problem-solving techniques employed in everyday settings compared to the classroom:

> How is it possible that children capable of solving a computational problem in the natural situation will fail to solve the same problem when it is taken out of context? In the present case, a qualitative analysis of the protocols suggested that the problem-solving routines used may have been different in the two situations. In the natural situations children tended to reason by using what can be termed a 'convenient-group' while in the formal test school-taught routines were more frequently, although not exclusively, observed. (p. 25)

The term 'convenient-group' identifies how people deliberately choose strategies – sometimes 'normative' and other times 'non-normative' − based on the specific quantities and values that they are dealing with, and that people change their strategies based of the convenience of the strategy for solving a problem in a particular setting. In other words, in everyday problem situations it is the nature and context of application that

[132] To illustrate by means for an example: de la Rocha (1985) identifies that in a study on the measuring methods employed by a group of women enrolled in a Weight Watchers dieting programme, the women initially resort to accurately measuring out food quantities using scales and measuring instruments (normative techniques). However, they then soon resort to a form of estimation once they have developed a sense for size portions or by remembering how much of a particular glass, cup or bowl must be filled to give a particular portion (non-normative technique).

defines the strategy used. A similar sentiment is shared by Naresh (2009) who, in analysing the mathematical practices employed by bus drivers in India during the course of their normal work duties, suggests that,

> … bus conductors' workplace mathematics has certain unique characteristics that are shaped by the context and the tools specific to the workplace. (p. 112)

Lave, Murtaugh, and de la Rocha (1984) express a similar sentiment by arguing that everyday problem situations and solution strategies employed in those problem situations are 'dialectically constituted' in that the types of solution strategies employed constantly change and adapt in response to changes in the nature, characteristics and demands of the everyday situation.

The exact opposite situation occurs in the context of the classroom. Here learners are encouraged to employ the same formalised procedure irrespective of the context or problem; here, selected or employed strategies are determined by the specific mathematical topic being dealt with in class and not by the contexts in which that content has application. And, even if the regulating mathematical principle which underpins the pedagogic process is not made explicit, the structure and setting of the classroom – and the expectations for learning mathematics associated with the classroom setting – direct learners towards a particular form of (implicitly defined) mathematically legitimised participation (c.f. Wyndhamn, 1993).

de la Rocha (1985, p. 198) argues along similar lines, using the term 'knowledge-in-use' to encapsulate the context-specific situated knowledge and strategies that people employ when solving everyday problems. This knowledge-in-use commonly stands in contrast to the "cultural fund of general knowledge" (de la Rocha, 1985, p. 193 & 194) that reflects the acceptable rules for functioning in a particular setting[133] and certainly stands in contrast to the largely context-independent, highly mathematised strategies that dominate in the setting of the mathematics classroom.

It is the role of these 'knowledge-in-use' and 'convenient-group' strategies, together with the dialectical relationship between everyday settings and the problem-solving techniques employed in those settings, and contrasted with how problem-solving occurs in the classroom, which gives rise to the dilemma that "In many cases attempts to follow school-prescribed routines seemed in fact to interfere with problem solving." (T. N. Carraher et al., 1985, p. 28).

Agreement over the distinctive nature of everyday and academic practices involving calculations is, however, not unanimous. Greiffenhagen and Sharrock's (2008) criticism of Jean Lave's work is a case in point. Lave (e.g. (1988)) and others present everyday mathematics and academic mathematics as two distinct types or forms of mathematical activity, each defined, bounded and guided by different rules and criteria for legitimate and successful participation, and each of which develop and evolve differently and independently of each other. Greiffenhagen and Sharrock (2008), by contrast, argue that the difference is not so much in kind or type as in context of application and/or purpose:

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[133] For example, the techniques and knowledge that a person might employ when assembling a table (the 'knowledge in use') – influenced by the tools they have available and their prior knowledge of assembling furniture items – may differ vastly to the instructions given in the assembly manual for the furniture item (the 'cultural fund of general knowledge').

> … the main difference between the two environments [everyday and formal school-based mathematical/classroom] is the criteria of success (not the relations between numbers). (p. 12)

And further:

> The pivotal role of the case studies of everyday mathematics is to provide a contrast with school mathematics teaching. However, any genuine comparison between everyday applications of arithmetic and school arithmetic will be complicated, since the aims and purposes of teaching mathematics in schools are multiple and complex, and the instructional import of any moment of mathematical activity may have only an indirect relationship to 'everyday problem-solving' of the sort found in everyday activities such as dieting or shopping. (Greiffenhagen & Sharrock, 2008, p. 12)

In other words, the purpose or goal of school mathematics is completely different to the goal of mathematics performed in everyday contexts, and it is this goal that differentiates the types of mathematical principles and knowledge utilised in the two practices. The goal of the everyday context is more localised and immediate, and although limited and localised elements of mathematical calculation may be appropriated for use in everyday situations, the desired outcome in the everyday is for successful real-world functioning and not for the development of mathematical knowledge. The goal of the mathematics classroom, on the other hand, is to equip learners with an ever expanding body of mathematical knowledge – to "expand their computational resources" (Greiffenhagen & Sharrock, 2008, p. 15) – that is generalizable and applicable beyond a single instance. And, although everyday contexts may be appropriated in the teaching of mathematics to facilitate access to mathematical principles, the ultimate goal is for abstraction and generalisation beyond localised contextually-bound instances. D. W. Carraher and Schliemann (2002), acknowledging a turn-around in their previous perspective, now offer a similar viewpoint:

> However, we believe it would be a fundamental mistake to suggest that schools attempt to emulate out-of-school institutions. After all, the goals and purposes of school are not the same as those of other institutions. (pp. 249-250)

D. W. Carraher and Schliemann (2002) associate the difference in goals with a difference in 'meaningfulness'. Namely, "What makes everyday mathematics powerful is not the concreteness of the objects or the realism of the situations dealt with in everyday life, but the meaning attached to the problems under consideration (Schliemann, 1995)." (D. W. Carraher & Schliemann, 2002, p. 249). For Carraher and Schliemann, calculations performed in everyday settings are always done with a deliberate intention for generating meaning and for a specific outcome, and this intention and outcome are ever present and influence the choice of strategy employed. In the mathematics classroom, by contrast, calculations are commonly performed without reference to the meaningfulness of the calculation for a specific problem situation and, also, often in the absence of a larger problem-solving goal:

> The analysis of problem-solving solutions in and out of schools suggests that students commonly learn algorithms for manipulating numerical values without reference to physical quantities, only reestablishing clear links to the problem context in the end when the units of measure are finally attached to the numbers. By contrast, individuals solve problems in the workplace using mathematics as a

> tool to achieve goals that are kept present throughout the solution processes with continuous reference to the situation and the physical quantities involved. As such, the problem solvers in the workplace are normally aware of how the quantities generated in the course of the computations are related to the problem at hand. (D. W. Carraher & Schliemann, 2002, p. 245)

Masingila et al. (1996) similarly position differential goals and meaningfulness as key areas of distinction:

> These differences in mathematics practice appear to be explained by the fact that: (a) problems in everyday situations are embedded in real contexts that are meaningful to the problem solver and this motivates and sustains problem-solving activity (F. Lester, 1989), and (b) 'the mathematics used outside school is a tool in the service of some broader goal, and not an aim in itself as it is in school'. (Nunes, 1993, p. 30; Cited in Masingila et al., p.176)

Despite the difference in goals or meaning attached to calculations performed in particular settings, it is important to note that the goals are not perceived as incommensurable. Rather, the goals of the mathematical classroom provide a view of alternative possible forms of participation in everyday contexts: the mathematics classroom provides the opportunity for learners to extend their knowledge beyond localised contextual instances and to build up a more generalised base of knowledge that reflects possible alternative ways of thinking about and acting in certain situations (Greiffenhagen & Sharrock, 2008, p. 15). Conversely, the everyday world provides a crucial reservoir of problem-solving techniques that can facilitate understanding of particular mathematical structures and applications of those structures (Masingila et al., 1996, p. 177).

The discussion above has highlighted the multitude of variation in opinion regarding everyday forms of participation that employ levels of mathematical engagement and the type of engagement that characterises participation in the discipline of formal mathematics. Of particular importance is the fact that people engaged in everyday problem-solving situations commonly employ a variety of localised, context dependent informal techniques that are commonly not reflected in formal school-taught practices. Thus, if the internal language of description of the structure of knowledge for the knowledge domain of mathematical literacy is to prioritise a life-preparedness orientation that prepares people for empowered functioning in the world, then cognisance must be given of the types of informal techniques that people employ when solving problems in real-world situations. Importantly, this perspective does not denounce the role of formalised mathematical techniques for solving problems – and I acknowledge explicitly my belief that formalised mathematical approaches (and the development of a 'mathematical gaze') provide an alternative view on the world which creates the potential for a broader understanding of certain real-world practices. However, if the dominant agenda is for contextual sense-making practices, then exclusive focus on formal mathematical approaches will not facilitate this agenda and, rather, a voice must be given to contextual considerations and contextual forms of knowledge, participation and communication that reflect the structure of authentic and legitimate real-world practice. However, the reverse is also true. Namely, that it is also necessary to recognise that while sometimes it is appropriate to employ informal methods for solving everyday problems, at other times the nature of the situation or context demands a more formalised approach. For example, while estimation may be appropriate for informing best-buy decisions in a supermarket (as opposed to a formal procedure involving rates and proportion), this informal approach may have disastrous consequences for a nurse administering medicine dosages. In short, the terrains of the mathematical and the contextual must be seen as complementary, with engagement in each terrain facilitating understanding of different components of a real-world practice or of different possible forms of participation in the practice.

The danger of not providing a space for the authentic and unadulterated voice of the real-world to be heard in pedagogic processes is that learners come to see the activities of the Mathematical Literacy classroom and the activities of the real-world as divorced. These same learners also come to believe that the knowledge learned in the classroom has no bearing on the structure legitimate and empowered participation in everyday life. If access to the real-world voice is denied, then mathematical agendas will dominate and the life-preparedness orientation will be subordinated to mathematically dominant intentions.

A final observation is necessary with respect to studies of everyday mathematics (and criticisms thereof). Observed from a Dowlingaling©&™ perspective, studies of everyday mathematics reflect the imposition of a mathematical gaze on everyday practices and the (re)description of those practices according to mathematised structures and principles. The fact that practices in supermarkets, kitchen and workplace are so aptly described as constituting mathematical practices bears testimony to the fervour with which mathematicians lay claim to the mathematical basis of the world in which we live and, hence, the ability to describe the world through a mathematical lens – precisely the Myth of Reference. Perhaps one of the reasons why many people are able to complete everyday tasks so successfully – after having struggled or failed in formal mathematics at school – is because they do not experience these everyday activities as mathematical and, consequently, do not draw on mathematical experiences or knowledge in attempting to solve these problems? Perhaps these activities are not really mathematically based at all and, rather, only become mathematical as mathematicians impose biased and foreign mathematical structures and conceptions on the situations? So, perhaps the distinction between 'everyday' and 'academic' mathematics is not a distinction at all, simply because it is only the latter domain that constitutes a form of mathematics. In reflecting on the common mathematical convention that "you can only compare like with like", perhaps the issue here is that the structures of the everyday and the academic are nothing alike and, so, are not able to be compared.

14.4.2.2 'Practical intelligence'

There is a further component to be added to the distinction of everyday and academic (classroom-based) mathematical practices. Namely, alongside the common use of informal techniques in everyday practices, participants in such practices similarly rely on and are influenced by a whole host of considerations that commonly have nothing to do with mathematics and are not grounded in any form of mathematical basis whatsoever. However, these considerations have a direct impact on the way people think, act, and communicate, and, hence, on their decision-making processes. For example, although consideration of 'best-buy' may influence a decision of which product to buy, other factors such as the amount of money on hand in the purse, or the amount of space available in the kitchen cupboard, or the means of transportation available to transport the goods, may all come to inform and influence decision-making in this particular situation. Irrespective of which option is the best option from a mathematical perspective, non-mathematical contextual considerations may influence the situation differently. In simplistic terms, the real-world is, more often than not, *not* driven by formal mathematical structures and processes. As such, a language of description for the knowledge domain of mathematical literacy that prioritises a life-preparedness orientation through an agenda for contextual sense-making practices must take cognisance of this component of participation in real-world problem-solving practices through recognition and promotion of 'practical intelligence'.

The term 'Practical Intelligence' – also referred to as 'Everyday Cognition' (Vithal & Skovsmose, 1997), 'Everyday Intelligence' (Goodnow, 1986), and plain 'common sense' (Dowling, 1995b) – refers to a form of "mind in action" (Scribner, 1986, p. 15):

> I use the term to refer to thinking that is embedded in the larger purposive activities of daily life and that functions to achieve the goals of those activities. Activity goals may involve mental accomplishments (deciding on the best buy in a supermarket) or manual accomplishments (repairing an engine) but, whatever their nature, practical thinking is instrumental to their achievement. So conceived – as embedded and instrumental – practical thinking stands in contrast to the type of thinking involved in performance of isolated mental tasks undertaken as ends in themselves. (Scribner, 1986, p. 15)

Scribner is suggesting that practical intelligence is tied to the context of its application (in daily life) and that this link to reality or to a specific problem setting distinguishes it from what occurs in an academic environment (which translates into 'academic intelligence'). Goodnow (1986, p. 143) offers a different perspective, suggesting that practical or everyday intelligence refers to "situations where people can use, and have some interest in using, their past knowledge to solve a real-life problem." In this conception, practical intelligence in solving problems is informed by prior experience of a situation or prior knowledge that enables functioning in that situation, as contrasted to the learned knowledge transmitted and acquired in an academic setting. Wagner and Sternberg (1986, p. 54) argue similarly, suggesting that much of the knowledge upon which competence in real-world settings depends is 'tacit knowledge': tacit in the sense that the knowledge is not openly expressed or stated, is often disorganised, unstructured and relatively inaccessible beyond a specific setting or situation, and, so, cannot easily be taught through formal instruction. For example, estimating best buy options in the shops – who teaches us this? And if we do want to teach it, how do we do so when the nature of the estimation depends entirely on the shopping items being dealt with? Based on this analysis, Wagner and Sternberg (1986, p. 54) suggest the following facets of tacit knowledge: (i) practical rather than academic; (ii) informal rather than formal; (iii) usually not directly taught.

Scribner's (1986, pp. 21-28) 'suggested model of practical thinking' provides a useful framework for summarising more precisely some characteristics of practical intelligence. These characteristics include:

- *problem formulation*: namely the ability to not only solve problems but also to define the specific nature and requirements of each problem − influenced directly by the specific context in which the problem occurs – together with the most appropriate technique or strategy for solving the problem;
- *flexible modes of solutions* (or what Masingila et al. (1996, p. 178) refer to as "flexibility in dealing with constraints"): put simply, the 'one shoe fits all' problem-solving strategy does not work in real-world situations, where the conditions imposed by the specific context in which the problem is located have a direct bearing on the most appropriate method required for successful engagement with the problem;
- *incorporation of the environment into the problem–solving system*: this aspect of practical intelligence is accompanied by recognition of the impact of peripheral and contextual influences on the type of strategy that provides the most efficient solution

path in a particular setting and, hence, on the rejection of a single definitive solution strategy for engagement in a contextual problem scenario;

- *dependency on setting-specific knowledge*: "just because I can read and understand the label on a bag of cement does not mean that I can successfully mix the cement into concrete with appropriate consistency for a particular job". Rather, successful and endorsed participation in contextualised problem-solving practices is often reliant on engagement with specialised insight about a context or problem situation that can only be deemed through direct engagement in the problem situation. External and academic analysis of the problem commonly does not lead to the same level of understanding or functionality with the problem. This dependency on setting-specific knowledge also involves the use of "cultural artefacts and conventions" (Masingila et al., 1996, p. 196) developed in the arena of application and specifically suitable to the immediate problem-solving task;
- *effort-saving as a higher order solution strategy*: this component of practical intelligence acknowledges that the development of confidence and skill in solving everyday problems successfully is commonly exhibited through the utilisation of time and effort-saving strategies. Namely, the more skilled a person becomes at solving particular type of problem, the more easily they are able to adopt more efficient solutions strategies. And, since practical problem-solving tends to take place in situations in which the conditions of the situation are constantly changing, the ability to develop efficient solution strategies requires in-depth understanding of both the conditions of the problem and the environment in which the problem is situated. Hence, effort-savings strategies embody higher order thinking patterns (and not laziness!).

Ford (1986) offers further insights into the features of practical intelligence:

> In the case of practical intelligence, there are essentially two unique defining features.
> First, the goals to be accomplished must be transactional: that is, they must refer to an effect outside of the person (e.g., fixing a flat tire, controlling someone else's behaviour) rather than to an effect inside the person (e.g., understanding a concept, experiencing a sunset). … The second defining feature of practical intelligence is that the goals to be accomplished must be important either to the individual being assessed or to the cultural group of which that individual is a part (or both). (pp. 183-184)

In other words, practical intelligence is employed when an individual is faced with a situation in their daily-life in which they need to solve a problem and in which they have a vested interest in solving that problem to inform action or decision-making.

This quotation by Ford (1986) brings the discussion back to the distinction made by Greiffenhagen and Sharrock (2008) and by D. W. Carraher and Schliemann (2002) between everyday and academic mathematics on the grounds of differential 'goals' or 'meanings'. Namely, the goals of practical intelligence, in a similar way to everyday mathematics, revolve around the need to solve a localised and situated problem, the conditions of which – together with techniques required for solving the problem − are defined by the nature of the problem and by the structure of the context in which the problem is located. Furthermore, appropriate knowledge for solving the problem is commonly developed in the context in which that knowledge is required or through a previous experience with the same or a similar situation. Importantly, however, practical intelligence is not limited to the utilisation of everyday mathematical techniques in solving problems. Rather, practical intelligence involves an understanding and consideration of all components of a problem, including awareness of the most suitable technique (mathematical (formal or informal) or not) for solving a problem and a host of other 'qualitative' considerations[134] (Julie, 2006, pp. 67-68) that may affect decision-making in the situation. I contend that it is practical intelligence that enables the CEO of a company to study the economic models and projections supplied by statisticians and then to choose an alternative path than those inferred in the models based on personal experience of the market, the need to sustain relationships with existing clients, and/or concern for the well-being of his or her employees. Qualitative and/or intuitive decisions are as much, if not more, prevalent in real-world decision-making than those founded in mathematics: practical intelligence acknowledges this and utilises a combination of both qualitative and mathematical considerations to inform decision-making practices.

In relation to the language of description for the knowledge domain of mathematical literacy, a mathematically literate individual is one who can not only perform the necessary calculations and develop the mathematical models, but who also has an acute awareness of the limitations of such models when applied to real-world settings. This individual also has awareness of the presence of non-formal mathematical techniques commonly employed in real-world forms of participation and which provide efficient and appropriate solutions, and the impact of other qualitative considerations on decision-making processes. A mathematically literate individual is both mathematically and practically intelligent.

14.4.2.3 Does being practically intelligent (and a practitioner of everyday mathematics) constitute a level of mathematical literacy?

At a recent symposium of the various fellows and their supervisors that form part of the Sasol Inzalo Fellowship Foundation, the director of the programme – Dr. Marietjie Vosloo – asked the following question[135] of me on hearing an update of my research:

> If the everyday or contextual components are to be prioritised over the mathematical components, then why has a title for the subject been chosen [i.e. Mathematical Literacy] that makes explicit reference to mathematics?

This question addresses a key issue for consideration: namely, that exhibiting practical intelligence or skill at performing everyday mathematics calculations does not point to the existence of a degree of mathematical literacy. Most citizens are able to do best buy options to complete their shopping, or are able to estimate how much it will cost to fill petrol into a vehicle, or can determine the most appropriate size shirt to buy. But, as both

[134] In fairness, it is important to point out that Julie (2006) does not agree with the inclusion of such qualitative considerations in pedagogic practices in the subject-matter domain of Mathematical Literacy in South Africa. In this regard he argues that: "those situations in which people use mathematical ways in some instinctive way should not be colonised for classroom use and teaching, no matter how much mathematical elegance might be extracted from such activities." (Julie, 2006, p. 67). His argument in this regard is that there simply is not enough clarity over the extent to which people actually need and make use of formalised mathematical calculations in everyday situations and, as such, it is doubtful whether these contexts provide an appropriate avenue for the development of mathematical understanding. Notice the focus on the acquisition of mathematical knowledge in this perspective; which is why, understandably, the perspective stands in opposition to my own.

[135] Due to various reasons including age, poor attention span and memory capacity equivalent to that of a flea, I have forgotten the precise wording of the question. As such, the quotation here reflects a paraphrasing of the original question.

D. W. Carraher and Schliemann (2002), and Greiffenhagen and Sharrock (2008), point out, these sorts of everyday techniques signify little more than simplistic functionality:

> There seems to be very relatively little mathematical activity in children's out-of-school activities, and when mathematics comes into play, it does not seem to call for a deep understanding of mathematical relations. (D. W. Carraher & Schliemann, 2002, p. 271)

> The examples [from the work of Lave] illustrate that the demands for computation in practical affairs such as supermarket shopping are few, limited and, often, inconsequential … (Greiffenhagen & Sharrock, 2008, p. 18).

Being able to make best buy decisions, to choose the best cell phone based on the features of the phone, or the ability to employ common-sense in a situation does not constitute a level of mathematical literacy in the language of description developed in this chapter. Rather, practical intelligence, together with an understanding of efficient everyday techniques for solving situated problems, is only one component of the language that deserves recognition – but a component that hitherto has been given almost no recognition in conceptions of mathematical literacy (or numeracy and/or quantitative literacy). As such, although this component deserves recognition and attention, this component must be accompanied by the ability to perform formal mathematical calculations (Mathematical Competency domain of practice), construct models to represent real-life situations (Modelling domain of practice), and to critically analyse and challenge existing structures encountered in all problem-solving practices through application of various levels of reasoning and reflection (Reasoning and Reflection domain of practice). It is the collective association of all of these components which, I believe, provides an expanded, holistic and more empowered world-view, and, as such, constitutes the behaviour of a mathematically literate individual.

14.4.2.4 Cumulative and segmented learning in the Everyday domain of practice

The discussion above on the issue of "does being practically intelligent (and a practitioner of everyday mathematics) constitute a level of mathematical literacy?" brings to the fore the need for a discussion of the role of segmented and/or cumulative learning in the Everyday domain of practice component of the internal language of description. Specifically, the Everyday domain of practice comprises the knowledge, skills and practical intelligence required to function in everyday tasks and practices – such as shopping and associated best buy decisions, and estimated petrol and other cost calculations. The knowledge and skills required in such situations is inherently localised and context-dependent, and the knowledge and skills employed are often determined by the context of use rather than through a prior understanding of a concept. Furthermore, the knowledge and skills appropriate in one context may have no use value in a different context or problem situation. All of this suggests that engagement in the Everyday domain of practice component of the language facilitates a form of *segmented learning*, with participants positioned solely in the role of a knower code. This begs the question, as is implied in the query posed by Dr. Vosloo, that if the skills required to function in everyday life can be developed in the context of their use – as is the case in a segmented learning process – then why must these skills, concepts and contexts be included in schooling? In alternative terms, how – if at all – is cumulative learning facilitated through participation in practices associated with the Everyday domain of the language, such that participation in these practices facilitates a degree of life-preparedness for the world outside of the classroom?

One response to this would be to argue that it is through the development of effort-saving techniques (Scribner, 1986, pp. 25-26) that a form of cumulative learning is developed and/or evidenced. Namely, that participants who are able to move beyond a formulaic or procedural approach for solving a problem towards the development of alternative methods, strategies and solution paths based on the need for efficiency, are exhibiting a form of higher order thinking that transcends the localised conditions of the problem. Such participants are able to choose which criteria to employ and which to ignore, and are able to generalise and transform accepted procedures and techniques into methods that allow for an equally accurate but more efficient functioning and form of participation in the context. I contend that this characterises a distinctive form of cumulative learning. It is, perhaps, in this light that T. N. Carraher et al. (1985) make the claim that,

> The results support the thesis proposed by Luria (1976) and by Donaldson (1978) that thinking sustained by daily human sense can be – in the same subject – at higher level thinking than out of context. (p. 27)

Despite this potential argument, my original caution remains. Namely, that although it is crucial in the language of description for the knowledge domain of mathematical literacy – and associated forms of participation that facilitate life-preparedness − to give credence and acknowledgement to the previously ignored and overlooked role of everyday considerations, techniques and forms of participation for solving problems based in real-world contexts, a curriculum or subject or assessment that operates entirely at this level of engagement does not adequately reflect the type of mathematically literate behaviour envisioned for this domain of knowledge. Being mathematically literate is about more than being able to complete shopping successfully and a fully-fledged school subject is certainly not required for this type and level of contextual practice. As is borne testimony by the many people who are able to function in the world efficiently and adequately on a daily basis, direct access to instruction in practical intelligence in schooling is not a prerequisite functioning in everyday life. Even effort-saving techniques – whether or not they provide evidence of cumulative learning – are developed regularly within localised contexts of use by participants who have had no access to direct teaching of such skills within a formal schooling structure. In other words, direct engagement in practices associated with the Everyday domain of the language in a formal schooling structure is not a pre-requisite for successful participation in everyday practices. While the Everyday domain of the language is an essential domain, it is only one component of the language and must function in conjunction – in complementarity − with the other domains of practice. It is only through engagement in Everyday practices in conjunction the forms of participation associated with the other domains of practice that characterise the knowledge domain preparation for life and for engagement with other-than-everyday tasks is facilitated. From the reverse perspective, failure to engage with the practices of this domain facilitates the structuring of legitimate participation in the entire knowledge domain according to primarily mathematical structures, imbued with an impetus for mathematising and Public Domain of mathematics type practices − which is contrary to the life-preparedness orientation of the language. In essence, then, although the Everyday domain of practice is only one component of the knowledge domain of mathematical literacy, it is a necessary component if a life-preparedness orientation is to be prioritised and facilitated. In short, participation in the Everyday domain of practice is characterised by aspects of both segmented and cumulative learning, which is a characteristic that Maton (2009) argues is reflective of all learning practices. Although aspects of cumulative learning are facilitated through participation in this domain, segmental learning will dominate if a reach is not made from this domain into the other domains of practice.

And yet, perhaps this in itself signifies and demonstrates a form of cumulative learning: engagement with knowledge, skills, techniques and forms of participation from a variety of domains of practice – everyday and mathematical, informal and formal? Perhaps the notions of generalisability and cumulative learning are inherent in the processes that are employed as learners develop the ability to know when to employ mathematics and when to ignore the mathematics in favour of a less formal or informal technique. Perhaps these notions are also inherent as learners combine knowledge of the everyday and practical intelligence with knowledge of mathematics in an appropriate way in order to deem a broader view of the world? And as learners develop the ability to switch between a mathematical and real-world gaze depending on which gaze provides a more empowered view of and functioning in the world? In alternative terms, perhaps the facilitation of a cumulative learning process is embodied in the development of an understanding of the complementary nature of the real-world and mathematical content? And, as is discussed in the pages below (c.f. page 230), it is the domain of practice of Reasoning and Reflection that facilitates this understanding.

14.4.3 Domain of Practice #2: Mathematical Competency

14.4.3.1 Overview of the component

The discussion so far has continuously emphasised the need for a structure of knowledge for the knowledge domain of mathematical literacy (and an associated structure of participation with this knowledge structure for the subject-matter domain of Mathematical Literacy) that prioritises and facilitates a life-preparedness orientation. This orientation is driven by a dominant agenda for contextual sense-making practices (and associated intention for the critical evaluation of structures). In this agenda, the terrain of the mathematical − and associated forms of mathematical knowledge, structures, contents, routines and communication – are subordinated to the terrain of the contextual such that engagement with the mathematical terrain is always in service to a broader goal for understanding of a contextual situation and of legitimate and appropriate forms of participation in that situation.

Despite this emphasis on a dominant agenda for contextual sense-making practices and associated life-preparedness orientation, I acknowledge not only the presence of mathematical calculations and mathematically informed decisions that permeate daily life, but also contend that an understanding of mathematical principles and structures is empowering. In this regard, a person who is able to check that the cost values on their bill have been calculated correctly − through application of an understanding of the mathematical structures that regulate calculations of time, tariffs, money and VAT – is more empowered in their daily life decision-making than a person who is not capable of performing the same check. As such, understanding of appropriate mathematical concepts and the development of a level of mathematical competence – what

Skovsmose (1994a, p. 47) refers to as 'mathematical knowing'[136]: "which refers to the competencies normally understood as mathematical skills" − are important aspects of the language of description of the structure of knowledge for the domain of mathematical literacy.

But what precisely constitutes a 'mathematical competency' domain of practice? And what is meant by 'appropriate mathematical concepts'? At an earlier point in this study (c.f. page 87 above), as part of a discussion on my own conceptualisation of the distinction between numeracy, quantitative literacy, and mathematical literacy, I made the following statement with respect to numeracy and quantitative literacy:

> numeracy involves competence with basic mathematical content, calculations, techniques and knowledge, not limited specifically to numbers and quantity but encompassing all mathematical contents; while quantitative literacy involves the *functional use* of such content, calculations, techniques and knowledge in making sense of problems grounded in real-world situations.

My intention in terming this component of the language of description the 'Mathematical Competency' domain of practice, then, is to deliberately emphasise the *mathematical basis* of the structure of participation in this domain and the explicit focus on the development of mathematical knowledge, techniques and skills. The characteristics of this domain of practice are perhaps best encapsulated through the strands of mathematical proficiency (c.f. Kilpatrick, 2001) referred to as *conceptual understanding* and *procedural fluency*: conceptual understanding – "comprehension of mathematical concepts, operations, and relations"; procedural fluency – "skill in carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately." (Kilpatrick, 2001, p. 107). Elements of *adaptive reasoning* are also included here, namely the ability to see connections and relationships between

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[136] Importantly, Skovsmose uses the term 'mathematical knowing' to refer to knowledge of mathematical content in a broad sense that includes competence in working with and reproducing theorems and proof, complex mathematical algorithms, and other components of abstract esoteric domain mathematics. In associating the numeracy component of the language to this notion of mathematical knowing, I am in no way suggesting that being mathematically literate (in the way in which the knowledge domain of mathematical literacy is constructed in the language of description in this study and, by implication, in the South African school subject Mathematical Literacy) requires knowledge of and the ability to work with formalised esoteric domain theorems and proofs or other such abstracted content. Rather, my usage of the term is simply to signify the mathematical basis of the domain of practice and the associated goal for developing a level of competence with certain mathematical contents. The specific scope and boundary of these contents is determined by the social community in which a conception of mathematically literate behaviour is instantiated.

Notice that in making this last statement I am arguing that despite my attempt to describe the knowledge domain of mathematical literacy, the enacted form of this knowledge domain is a *social construct*. As such, the structure of legitimate participation in any pedagogic activity that draws on this knowledge domain is ultimately determined by the demands of the social context in which the activity is embedded. This position is consistent with my commitment to an Interpretivist (Social Constructionism) paradigmatic orientation and the ontological and epistemological that characterise this orientation. This position is also consistent with my sociological stance and my concerns regarding the differential distribution of differently constructed and legitimised forms of mathematical knowledge to groups of learners in the schooling sector. All of these issues were outlined in detail in Part 1 of the study.

mathematical concepts[137] (see also Kilpatrick et al. (2001) for a detailed discussion of these concepts).

It is in this domain of practice that a level of mathematical competence on a wide range of content topics is developed. Although it is beyond the scope of this study to suggest the specific scope of these content topics[138], it is important to emphasise that the usage of the term 'mathematical competency' implies the development of a level of proficiency with mathematically based techniques, concepts and calculations across a variety of content strands. So, for example, for the strand of Data Handling (Statistics), a basic level of mathematical competence includes (but is not limited to) an understanding of the function of mean, median and modal measures of central tendency and how to calculate each of these. Similarly for the strand of Measurement, understanding of the concepts of area and volume, together with familiarity of *relevant* formulae and the ability to use those formulae would be included in the Mathematical Competency domain of practice of this language of description.[139]

It is also important to emphasise that mathematical competency includes a focus on both the development of numeracy skills (namely, competence with numbers, mathematical techniques and skills) and on quantitative literacy (namely, the *functional use* of mathematical content, knowledge and calculations through engagement with real-world contexts).[140] In other words, not only must participants learn about mathematical concepts, but they must also be able to *apply* those concepts in contextualised situations. Note that this does not imply mathematical modelling and the construction of detailed mathematical models to represent real-world situations; rather, it involves the use of mathematics and mathematical calculations in contextualised situations.[141] Reflecting

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[137] Adaptive reasoning, however, is not only limited to this component of the language and, rather, is also considered a crucial part of the modelling process envisioned in the Modelling domain of practice of the language (c.f. page 220 below). Adaptive reasoning is also a component of the type of 'Reflective Knowing' (c.f. page 230 below) that is seen to bring the Everyday, Mathematical Competency and Modelling domains of practice together in this language of description. These issues are elaborated on at various points in the discussion in the remaining pages of this part of the study.

[138] However, as was discussed in detail in Part 2 of this study, many of the expressions of mathematical literacy, numeracy and/or quantitative literacy include similar emphasis on the content strands or domains of Number Concepts, Relationships, Measurement, Space and Shape, Data Handling and Probability/Likelihood. It would, thus, be fair to argue that whatever specific content topics are chosen, these topics will be informed or bounded by these overarching strands.

[139] Contrast these situations with the form of knowledge envisioned for Mathematical Literacy. This knowledge form would involve critical engagement with each of the measures to determine what each measure may say or disguise about the data set. Or, in the context of area and volume calculations, this knowledge form would involve estimation of the quantities of tiles needed to tile a floor taking into account grouting spaces required between the tiles, the position of stress joints and the need to start tiling outwards from these joints, the position of half tiles or oddly shaped tiles that need to be cut, and so on.

[140] The distinction between Numeracy and Quantitative Literacy (as well as Spatial Literacy and Document Literacy) was established on in Part 2 and Chapter 7 (starting on page 83) of the study.

[141] An explicit distinction between 'modelling' and 'application of mathematics' was provided in Part 2, Chapter 5 and sub-section 5.2.2.1 above, starting on page 37. A brief reminder of this distinction is provided on page 220below as part of the discussion of the characteristics of the Modelling domain of practice of the knowledge domain of mathematical literacy. However, to illustrate the distinction as implied in this discussion, consider the context of cell phone costs. Application as described above might involve the use of knowledge of mathematics to determine call costs for given tariff values and/or formulae. Modelling, on the other hand, might involve the development of a series of graphs, calculations and/or equations that enable the participant to make a decision about which cell phone system would be preferable under a certain set of criteria, together with consideration of different qualitative factors that might influence the choice of phone or phone system. The application dimension involves a focus on the functional use of mathematics in context; the modelling dimension involves the development of a representation of a real-world situation to facilitate a broader understanding of the situation.

back briefly on the ‘spectrum of agendas’ identified for characterising the dominant domain of prioritising in mathematical literacy related practices (c.f. Part 2, Chapter 5 and sub-section 5.2.2.1 above, starting on page 37), the descriptions given above posit the Mathematical Competency domain of practice as reflecting a dominant agenda for Mathematics in Context (Agenda 2), with credence given to both the Numeracy in Context (Agenda 2 [b]) and the Applications (Agenda 2 [a]) dimensions of this agenda.

Drawing on the concept of complementarity (c.f. page 183 above), while the complementarity of the terrains of the mathematical and the contextual are acknowledged, in this domain of practice the mathematical terrain takes precedence over the contextual. Thus, although I envision that development of mathematical competency occurs through engagement with mathematics in contextualised problems, I also acknowledgement that engagement with non-contextualised or esoteric domain mathematical problems may be necessary – hereby constituting mathematics as the public domain of this domain of practice. For example, a teacher may choose to explain the mathematical basis of the concept of a ratio and associated notation before exposing learners to contextualised situations in which ratios have application. This perspective does not suggest an alliance with the opinion that mathematical content must be learned first and only then can application occur, or to deny the role of contexts in providing meaning and access to mathematical concepts. However, the perspective is grounded in the position that confidence with mathematical content, techniques and knowledge enables more effective and efficient modelling processes, which in turn adds a further instrument to the repertoire of available strategies for use in contextual sense-making practices. In layman’s terms, it is difficult to use mathematics to make sense of a problem when you are still grappling to understand the mathematics; similarly, it is also sometimes difficult to learn mathematics embedded in a context when the complexity of the context inhibits access to the mathematics.

As regards the scope of ‘appropriate mathematical concepts’ envisioned for engagement in the Mathematical Competency domain of practice, I have deliberately avoided framing this content as basic or elementary – as is the case in some of the literature on mathematical literacy, numeracy and/or quantitative literacy. I have also avoided making an explicit statement of the distinction between what constitutes elementary/basic content versus complex/abstract content, simply because what is considered elementary or localised in one community may be deemed advanced or abstract in a different community. What is, perhaps, more useful is to define the scope of the ‘appropriate’ content in relation to the specific types of contexts which are deemed appropriate, relevant and necessary for exploration in a particular community. This suggests that rather than a canon of mathematical knowledge and concepts which have applicability across a range of topics, cultures and contexts (namely, the ‘Myth of Reference’ (c.f. Dowling, 1998)), the specific mathematical concepts identified for engagement are related directly to specific cultural practices and contexts that are relevant to specific communities at a particular point in time. Thus, although the primary goal of this Mathematical Competency domain of practice is the development of a basic level of competency with mathematical concepts, the mathematical concepts are still ultimately in service to a contextualised real-world goal. Further, this definition of appropriate content in relation to specific contexts also implies that the content need not be restricted to basic or elementary content – namely, the type of basic arithmetic commonly utilised in everyday situations (Greiffenhagen & Sharrock, 2008, p. 17) – and, rather, can include abstracted or esoteric content as required by a specific problem or contextual situation. In other words, being mathematically literate (in relation to the knowledge domain of mathematical literacy described in the internal language of description in this study) implies being able to make sense of a variety of real-world situations and problems encountered in those situations and not just a limited array of situations that involve only basic or elementary mathematical concepts. Furthermore, the scope of the problem-solving (and the content required in that problem-solving) should be determined not by a predetermined canon of mathematical content but the problem situations and contexts that are deemed necessary for investigation in a particular culture or setting. This conception overlaps with de Lange's (2003, p. 81) conception of 'Advanced Mathematical Literacy'[142], which is defined by the needs of participants to fit into a specific community of practice both within workplace and societal roles, and, so, promotes a vision of different curricula for different groups of learners based specifically on the intended career paths to be pursued by these learners. It is assumed that these curricula must also contain a general dimension that facilitates interaction with aspects of wider society which are not specifically defined or constrained to a particular career path.

14.4.3.2 The relationship of the Mathematical Competency domain of practice to the other components of the knowledge domain of mathematical literacy

A comment is necessary on the relationship of the Mathematical Competency domain of practice to the other domains that comprise and characterise the knowledge domain of mathematical literacy. Firstly, there is a sense in which the Mathematical Competency domain of practice stands somewhat in isolation to the Everyday domain: you do not need to have practical intelligence or an understanding of the rules that dictate practice in everyday settings to develop mathematical competence. Similarly, mathematical competence does not facilitate understanding of the techniques and rules that dictate legitimate forms of participation in everyday contextual practices. However, consider that legitimate practices in the knowledge domain of mathematical literacy are characterised by engagement with authentic contextual environments and the resources of those environments. It stands to reason, then, that initial successful engagement with everyday or contextual elements of the environments and related resources serves as a precursor to effective and successful engagement with further components of the activity, including mathematical calculation. In other words, successful engagement with the mathematical components of a context or contextual resource is often dependent on prior understanding of the context-specific elements, terminology, contents and notation of the context or resource. As such, in the context practices and activities associated with the knowledge domain of mathematical literacy, there is an element of hierarch between the Everyday and the Mathematical Competency domains of practice. However, in remaining committed to a perspective that views the mathematical and contextual terrains as complimentary, I contend that interplay between the Everyday and Mathematical Competency domains of practice provides a broader view of a real-world situation and of existing and possible alternative forms of legitimate participation in that situation. By means of illustration consider a shopper presented with a best-buy decision. The shopper does not need to understand ratio and rates to make a best buy option. However, understanding that ratio and rates is the principle that underpins this sort of comparison equips the shopper to be able to use formal calculation to make informed decisions in similar situations but where no shelf label is available or where estimation is too complicated, or in a different situation where a greater level of precision or accuracy is necessary. The person who simply

[142] de Lange (2003, p. 81) distinguishes Advanced Mathematical Literacy from Basic Mathematical Literacy, with this latter literacy constituted as a level of mathematical literacy expected of all 15-year old learners irrespective of their prospective career path.

gives up because the values are too difficult or because the localised technique they commonly employ is no longer adequate, is not as empowered as the person who can (if they want to)[143] resort to a more formal approach for solving a problem.

Secondly and with respect to the component of the language that involves the development and analysis of mathematical models of real-world situations (the Modelling domain of practice). The mathematical competence that is the goal of this domain is seen an important precursor for successful participation and functioning in the Modelling domain, since the ability to develop mathematical models lies, in part, on a prior understanding of mathematical knowledge of the concepts and contents that underpin the models. However, as has been mentioned previously (c.f. footnote 127 on page 127 above), a one-to-one correlation between mathematical knowledge and the ability to construct models is also not envisioned. Rather, there is recognition that the skills involved in constructing and analysing models are not equivalent to those involved in doing mathematics. In this sense, the relationship between this Mathematical Competency domain of practice and the Modelling domain of practice is hierarchical. Similarly, there is also an element of hierarchy in the relationship between the Everyday and Modelling domains of practice in that the ability to successfully construct models to accurately reflect and describe real-world practises is dependent on a comprehensive understanding of the contextual elements of the practices and the contextual environments in which the practices are embedded.

14.4.3.3 Cumulative and segmented learning in the Mathematical Competency domain of practice

The notion of cumulative learning is considerably easier to define in the Mathematical Competency domain than in the Everyday domain of practice. Primary focus in this component of the knowledge domain of mathematical literacy is the development of a basis of mathematical knowledge and skills – a level of mathematical competency, accompanied by conceptual and procedural understanding – that provides the capacity to deal with quantitative information and processes encountered in real-world settings, and a foundation for participation in modelling processes. This development of mathematical competency involves generalisable mathematical knowledge and skills, developed as part of a hierarchical mathematical knowledge structure, able to be applied across a range of situations (albeit through the imposition of a mathematical gaze on those situations). In this sense, the mathematical knowledge and skills developed in this domain of practice are not specific to a localised context and have general application across a range of problem situations. Furthermore, successful participation in this domain of practice is not dependent on the characteristics of the participants (i.e. the knower code) but, rather, on engagement with definitive body of knowledge considered to be the legitimate source of knowledge (i.e. the knowledge code). All of the above suggests that participation in the Mathematical Competency domain of practice is characterised by a cumulative learning process.

[143] This condition of 'if they want to' is an important one. This is because it suggests that a person not only has the capacity to use formal calculation to solve problems where an informal technique proves inadequate, but also that they are able to discern the situations in which the use of a formal technique is appropriate and/or provides a more efficient solution path. This ability to discern the appropriateness of a particular technique for a particular situation is a key trait of a mathematically literate individual with *productive disposition* (Kilpatrick, 2001, p. 107).

14.4.4 Domain of Practice #3: Modelling

The third domain of practice in the knowledge domain of mathematical literacy – the Modelling domain − involves the processes and competencies associated with the development and analysis of mathematically structured models to describe and reflect possible forms of participation in real-world situations and to influence decision-making processes in relation to those situations. This domain of practice bears resemblance to what Skovsmose (1994a, p. 47) calls 'Technological knowing' − "which refers to abilities in applying mathematics, and to the competences of model building".

Importantly, and as already mentioned (c.f. Part 2 and sub-section 5.2.2.1 starting on page 37, as well as footnote 141 on page 216 above) usage of the term 'modelling' in this study denotes the process involved in moving from a particular problem situation based in an authentic real-life context (as opposed to a mathematised situation) to a mathematical reconstruction of that context. The direction of movement, then, is from the real-world to the mathematical reconstruction (and back again to the real-world).[144] This mathematical reconstruction is seen to provide an alternative view of the situation and to facilitate a different or broader understanding of possible forms of participation in the situation. Modelling is distinguished from applications: in the context of this study, applications are construed as involving a movement from the mathematical to the real-world (and back to the mathematical again), with the ultimate goal being for forms of functional engagement and problem-solving that employ mathematised and mathematically legitimised forms of participation.

In the immediate sub-section below I now outline various theoretical orientations and perspectives on modelling and explain my rationale for privileging a *realistic* perspective for the Modelling domain of practice in knowledge domain of mathematical literacy.[145]

14.4.4.1 Modelling perspectives and orientations

In deliberately promoting a particular conception of modelling and associated modelling processes for the knowledge domain of mathematical literacy, it is necessary to align this deliberately selected conception in a particular orientation and/or perspective. This is necessary because, as W. Blum, et al. (2002) point out,

[144] The specific components or stages envisioned in this reconstruction process are discussed in sub-section 14.4.4.2 below (c.f. page 223).

[145] Importantly, this 'realistic perspective' is not to be equated with the paradigmatic orientation of Realism (haaibo!). As discussed in Part 1 of this study, the paradigmatic orientation of the study is that of Interpretivism (or Social Constructionism). As is discussed below, the 'realistic' modelling perspective directs the modelling process towards understanding of real-world contexts and forms of participation in those contexts (which is consistent with the focus on a life-preparedness orientation prioritised in the language of description of the knowledge domain of mathematical literacy). Although this modelling perspective aims to develop representations and (re)descriptions of real-world practices, this orientation acknowledges the limited perspective and gaze afforded by a model and recognises that a multitude of models can be developed to describe the same practice (with the structure of each model determined by the differential perspectives and areas of prioritising of the model developer). In other words, this modelling perspective acknowledges the subjective nature of the modelling process and that a model represents only one of many possible *constructions* of a segment of reality. This modelling perspective does not perceive a single or ultimate reality or truth that is best able to be described by a particular model. As such, this modelling orientation is entirely consistent with the Interpretivist perspective of the constructed nature of knowledge.

> There are a number of different elements and characterisations of modelling and applications; some of these are posing and solving open-ended questions, creating, refining and validating models, mathematising situations, designing and conducting simulations, solving word problems and engaging in applied problem solving. All of these link the field of mathematics and the world. (p. 159)

Niss, Blum, and Galbraith (2007) – cited in Venkat (2010, p. 56) – argue that mathematical modelling in the context of mathematics education is driven by one of two orientations: as "applications and modelling *for the learning of mathematics*" or "learning mathematics so as to *develop competency in applying mathematics and building mathematical models*." (Niss et al., 2007, p. 5, emphasis in original text). The difference is again one of intention or goals. The former positions the learning of mathematical content as priority and makes use of the modelling process to facilitate this learning; while the latter posits the modelling process as a priority, working from the assumption the mathematical knowledge is already known and that the primary activity is the construction of the model. Importantly and as suggested by Venkat (2010, p. 56), this latter orientation "acknowledges to a greater extent the importance of the 'extra-mathematical' realm". And, for a language of description of the knowledge domain of mathematical literacy that prioritises contextual sense-making practices (and an associated life-preparedness orientation) over the mathematical terrain, this consideration of the extra-mathematical realm is crucial. As such, it is this latter orientation of modelling that is envisioned for the Modelling domain of practice.

Blomhøj (2008) extends this categorisation of different research orientations of modelling by suggesting that within the research on modelling it is possible to identify six main research perspectives. These include: (i) realistic; (ii) contextual; (iii) educational; (iv) epistemological; (v) cognitive; and (vi) socio-critical. Although it is beyond the scope of this discussion to explore each perspective in detail, it is of relevance to explore briefly the realistic, educational and socio-critical perspectives.

According to Blomhøj (2008), the dominant goal in the *realistic perspective* is sense-making practices of real-life situations, and mathematical models provide a useful tool for achieving this goal:

> Therefore, in order to teach mathematical modelling in a form that will really be helpfully for students in their subsequent professions, one needs to study carefully authentic real life modelling, and on the basis thereof to design situations where students work with authentic modelling supported by relevant technology, and assess the model and its results against the reality. (p. 2)

For the *educational perspective*, on the other hand, "The main idea of this perspective is to integrate mathematical modelling in the teaching of mathematics." (Blomhøj, 2008, p. 4). In other words, the construction of mathematical models is in service to the development of mathematical knowledge.

The *socio-critical perspective* promotes the vision that mathematics has a 'formatting power' (Skovsmose, 1994a, 1994b). In this perspective, mathematical models form the basis for many of the structures that underpin economic, social and political life. As such, the capacity to critically interpret and engage with these models is an essential component for critical citizenship and for "developing and maintaining societies based on equality and democracy." (Blomhøj, 2008, p. 8).

The necessity for outlining these three perspectives in particular is to position the Modelling domain of practice in the knowledge domain of mathematical literacy predominantly in the realistic perspective[146], albeit with characteristics and areas of emphasis that reflect modelling practices associated with both the educational and socio-critical perspectives. The traditional educational perspective explicitly prioritises an agenda for mathematical and mathematised forms of participation and for the use of modelling processes to demonstrate and enhance mathematical understanding and ability. From a Dowlingaling©™ position, this perspective on modelling is characterized by the casting of a mathematical gaze from the Esoteric Domain of mathematics to provide descriptions of non-mathematical practices – giving rise to what Dowling (1998, p. 136) refers to as the Descriptive domain of mathematical practice. This traditional perspective, characterised as it is by the Myth of Reference, is, thus, the antithesis of the life-preparedness orientation prioritised in the internal language of description of the knowledge domain of mathematical literacy. However, the perspective on modelling prioritised in this study also contains an, albeit different, educational component – namely, for the development of modelling-related competencies that facilitate the generation of context-specific models for any real-world situation in which a generated model provides insight into existing and possible alternative forms of participation. In other words, the perspective on modelling prioritised for the language of description of the knowledge domain of mathematical literacy is dominated by an agenda for understanding real-world forms of participation and practices, but also by an agenda for developing widely-applicable modelling skills and strategies.

The realistic (and in part educational) perspective on modelling is also characterised by elements of the socio-critical perspective. In particular, given that many real-world situations and practices are structured and characterised by pre-existing models, comprehensive contextual sense-making must be accompanied by an element of critical analysis of and engagement with these existing models. This is to determine how the models have been constructed, and the assumptions that underpin the models and which have informed the structure of participation facilitated by the model. In the internal

[146] A potential noticeable absence in the discussions presented in this study so far is reference to the Realistic Mathematics Education (RME) theory. And, by positioning the Modelling domain of practice of the knowledge domain of mathematical literacy in this 'realistic' perspective, it is now worth considering if there are overlaps between the structure of knowledge prioritised for this knowledge domain and the structure of knowledge and participation characteristic of RME practices.
A primary emphasis in RME theory is the notion that the learning of mathematics should take place through the application of mathematical contents, concepts and tools to 'realistic' problem situations. In this regard, 'realistic' refers not so much to the authenticity of the real-world base of the problem as to whether learners have a connection to the problem and are able to experience them as real in their own minds (van den Heuvel-Panhuizen, 2003, pp. 9-10). The description above immediately positions the primary agenda in RME differently to the primary agenda in the language of description for the knowledge domain of mathematical literacy in two ways. Firstly, primary focus in RME is on the development of mathematical knowledge and the usage contexts and modelling processes to facilitate this development: "Although these criteria already give a good indication of what is necessary to have a model emerge, the most important is that the problem situations and activities bring the students to identify mathematical structures and concepts." (van den Heuvel-Panhuizen, 2003, p. 116). This is clearly the antithesis of the life-preparedness orientation goal in the language of description for mathematical literacy and orients RME theory more towards the educational perspective on modelling with a dominant agenda for applications (Agenda 2 [a]) and modelling (Agenda 3) for the learning of mathematics. Secondly, as has been discussed in the Context Domain component of the language of description (c.f. sub-section 14.4.1 starting on page 194 above), authenticity of context and link to reality is an essential component of a life-preparedness orientation; this same emphasis on authenticity of context is not foregrounded to the same extent in the RME theory. In light of the above, I contend that the language of description for the structure of knowledge in the knowledge domain of mathematical literacy is differentially positioned from the goals in the RME theory: not quite on opposite ends of a spectrum, but distanced nonetheless.

language of description of the knowledge domain of mathematical literacy, this socio-critical perspective is facilitated through a dominant intention for critical evaluation of (both mathematical and contextual) structures[147] which, in turn, is facilitated through the Reasoning and Reflection dimension of the knowledge domain.[148]

Establishing the dominant perspective of realism for the Modelling domain of practice facilitates further discussion on the purpose of the modelling process envisioned for the knowledge domain of mathematical literacy. In particular, the modelling process is not to be viewed as an end in itself: participants are not to be taught to build models for the sake of building models. Rather, the building of models is in service to a larger goal of contextual sense-making and associated decision-making practices, and, so, preparation for life. In the real-world, people seldom (if ever) construct models of loan situations, cell phone comparisons, or of the implications of strike action because they like the way the model looks[149]; rather, they build models to inform decision-making practices. Furthermore, although the importance of the mathematical components and structure of models is recognised, these components are viewed as only one dimension of the modelling and associated sense-making process. Equally, each model is seen as only possible representation of a problem situation. Other representations (and other models) of the same problem may exist, and other tools may be utilised in the sense-making process that do not have a mathematical basis. As expressed eloquently by Pollak (1969, p. 400), “In the course of practicing mathematisation from real life, students will, incidentally, discover that for some situations mathematics is quite irrelevant.” This perspective does not deny the power of mathematically based models in providing and facilitating broader descriptions of the world; but, rather, acknowledges that the mathematised view of the world is only one of many possible views. Denial of this reality constitutes an imposition and prioritising of a mathematically imposed gaze on the world.

14.4.4.2 Modelling processes envisioned in the Modelling domain of practice in the knowledge domain of mathematical literacy

Having established the dominant perspective that characterises the structure of participation in the Modelling domain of practice, it is also necessary to highlight a description of the particular processes envisioned for the modelling practices of this domain. The *mathematisation cycle* described in the OECD-PISA frameworks (1999, 2003, 2006, 2009) provides a useful starting point simply because of the prevalence of reference to this process in the literature on mathematical literacy, numeracy and/or quantitative literacy and modelling in general. The OECD-PISA frameworks position this cycle as a “fundamental process that students use to solve real-life problems” (OECD, 2003, p. 38) and a process which involves the “organisation of perceived reality through the use of mathematical ideas and concepts” (OECD, 1999, p. 46). A reformatted version of the OECD-PISA mathematisation cycle is illustrated in the Figure 28 below (OECD, 2003, p. 38):

[147] Possible intentions for mathematical literacy related practices were dealt with in Part 2 and sub-section 5.2.2.2 above (starting on page 44).
[148] The Reasoning and Reflection domain of practice of the knowledge domain of mathematical literacy is dealt with sub-section 14.4.5 on page 230 below.
[149] Unless you are a mathematics teacher with an ardent mathematical gaze and who believes completely and utterly (but most probably unknowingly) in the Myth of Reference.

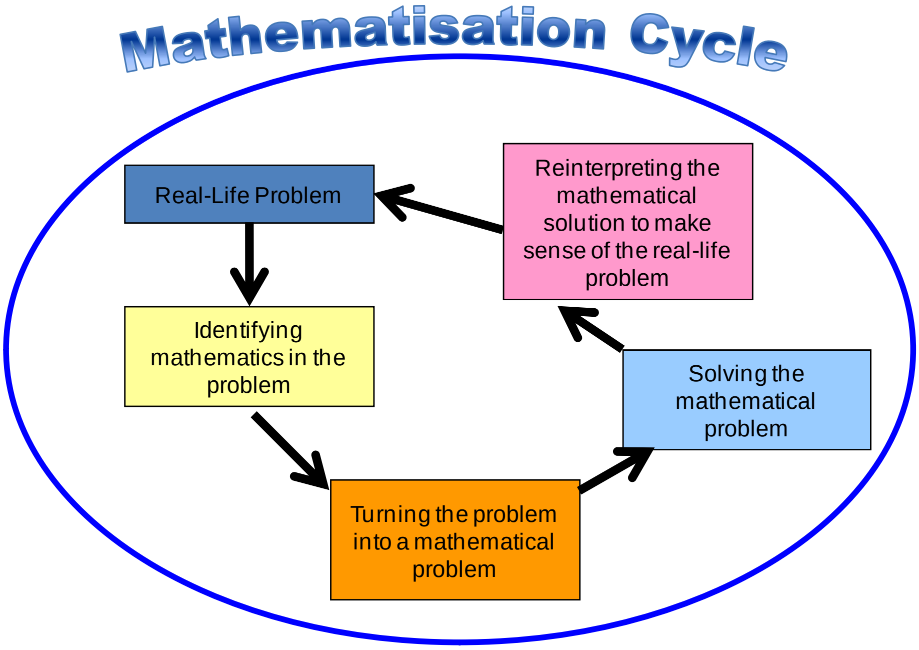

Figure 28: OECD-PISA Mathematisation Cycle

Drawing on the work of Treffers (1987), OECD-PISA (1999) identify two levels of the mathematisation process: *horizontal mathematisation* occurs as a real-life problem is transformed into a mathematical problem; *vertical mathematisation* involves working with the problem in the mathematical world and the process involved in selecting and utilising a variety of mathematical techniques, operations and contents (OECD, 1999, pp. 46-47). To summarise, "to mathematize horizontally means to go from the world of life to the world of symbols; and to mathematise vertically means to move within the world of symbols." (van den Heuvel-Panhuizen, 2003, p. 12). The test items in the OECD-PISA frameworks serve to assess both levels of the mathematisation process.

The mathematical basis of this mathematisation process in the context of the OECD-PISA frameworks – driven primarily by mathematical goals and an assessment of competencies associated with the application of mathematics – is clear. Any contextual components encountered in the mathematisation process are only relevant in that they provide a setting for the mathematics to play out. Contextual sense-making is not the primary goal; rather, the process involved in employing appropriate mathematical structures to arrive at a mathematically derived solution is the primary area of focus. This is most clearly evidenced in the included emphasis on the process of vertical mathematisation, which, ultimately, embodies a move towards further abstraction and generalisation within the mathematical system (van den Heuvel-Panhuizen, 2003, pp. 12-13 & 14). Furthermore, the structure of the cycle assumes that a mathematical solution path to a problem is always possible. No credence is given to the possibility that the mathematised solution path might not be the most efficient way to solve the problem or whether this mathematised approach accurately reflects the structure of endorsed participation employed by those who engage with the same or similar problem situation in the real-world. Or, perhaps the problem situations have been so mathematised that the mathematical solution path provides the only sensible solution to the problem? This framework, then, is simply too restrictive for a language of description of the structure of knowledge for the domain of mathematical literacy that prioritises an orientation that is not driven exclusively by mathematical considerations.

W. Blum and Ferri (2009, p. 46)[150] provide an alternative characterisation of the modelling process and one which I believe more suitably reflects the process envisioned for the Modelling domain of practice of the knowledge domain of mathematical literacy. A modified version[151] of this characterisation is illustrated in Figure 29 below:[152]

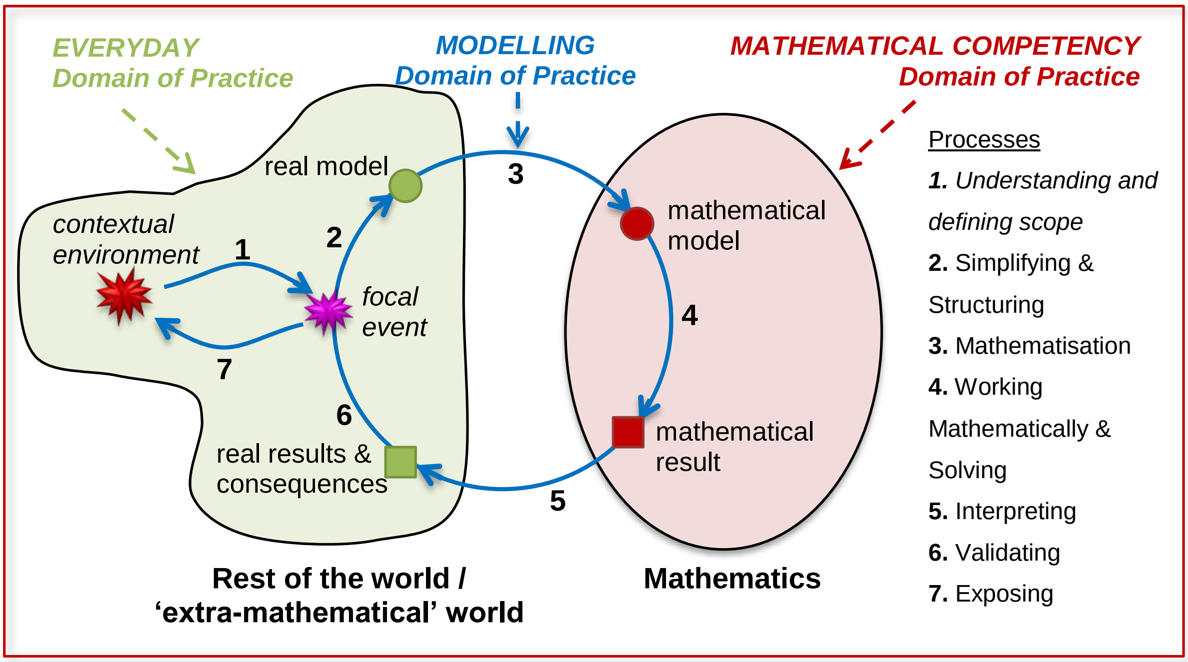

Figure 29: The modelling process envisioned for the Modelling domain of practice

By way of explanation, the modelling cycle reflected in Figure 29 begins in a particular situation – a *focal event* – that is embedded in a wider real-world *contextual environment*. It is within this contextual environment and in relation to the specific focal

[150] The original conception of this model was presented in (W Blum & Leiß, 2007).
It is worth pointing out that Blomhøj (2008, p. 6) categorises the modelling process employed by W. Blum and Ferri (2009) as reflecting an *educational modelling perspective* that promotes a dual educational goal for challenging and developing mathematical knowledge and for developing modelling competency. My use of this modelling framework is consistent with my earlier comments that aspects of the modelling perspective promoted for the Modelling domain of practice comprise an educational perspective. However, I have also modified Blum and Ferri's (2009) original framework (see footnote 152 immediately below for a discussion of the modifications) to reflect the prioritised realistic modelling perspective in the Modelling domain of practice. I have raised this point to illustrate that I am aware of the educational perspective that underpins this modelling framework and that I have equally recognised the need to modify the framework to reflect consistency with the realistic perspective that characterises the structure of legitimated participation attributed to the Modelling domain.
[151] All modifications/additions to the original characterisation are formatted in italics in Figure 29.
[152] Note that I have modified the diagram slightly from its original format to suit the intentions of this study. Firstly, the star currently labelled 'contextual environment' was originally represented as 'real situation and problem' and the star currently labelled 'focal event' was represented as 'situation model'. These labels have been changed to facilitated consistency with the terms used in the discussion of the components and characteristics of the Contextual Domain envisioned for the knowledge domain of mathematical literacy (presented in sub-section 14.4.1 on page 194 above).
Secondly, the labels 'Everyday domain of practice', 'Modelling domain of practice' and 'Mathematical Competency domain of practice' have been added and do not appear in the original construction of the diagram. These additions to the diagram are intended to illustrate how various domains of practice envisioned for the knowledge domain of mathematical literacy are incorporated and facilitate successful engagement in the Modelling domain. Further discussion of individual aspects of the modelling cycle is provided below.

event that a problem situation is encountered and for which the construction of a model is deemed necessary and appropriate. This model is seen to provide a means for describing facets of the problem situation, for generating understanding of the situation, for investigating possible ways – and associated forms of participation in the situation − in which the problem situation can be successfully engaged, and, ultimately, for informing decision-making practices in the situation.[153] A key and initial process involved in modelling for sense-making practices is (1) Understanding the precise nature, structure, variables and constrains of the situation. This process also involves understanding the embedded nature of the focal event within the wider contextual environment – i.e. the scope of the focal event in relation to the scope of the wider contextual environment. This requires consideration of the facets of the wider environment that may need to be considered in order to appropriately and adequately deal with the problem situation encountered in the focal event. Understanding of the problem is followed by (2) simplification and structuring of the problem situation, possibly through a process of 'cleaning' the context or focal event in which the situation is embedded. This will ensure that the problem is more accessible to the problem solver based on their current knowledge or experience base of the situation, and to define more clearly the criterion of the problem situation. This process transforms the real situation into a *real-world model*. This real-world model is then subjected to the process of (3) mathematisation, and is (re)produced and (re)structured according to mathematical principles and knowledge to produce a *mathematical model*. During this process, certain real-world elements may be side-lined (but not completely forgotten!) to facilitate greater access to the mathematical components of the model. This mathematical model is then (4) solved through mathematical working to produce a *mathematical result* to the problem. However, since the primary intention in constructing the model is to facilitate greater understanding of the real-world situation and to inform real-world decision-making practices, the mathematical result must be "re-translated into the real-world" (W. Blum, et al., 2002, p. 153): that is, it must be (5) interpreted with respect to the types of *consequences*, criteria, constraints, and everyday considerations that affect decision-making in that real-world situation. The problem solver then (6) validates the entire modelling process. This process of validation includes consideration of: whether the mathematical model and the solution to this model reflect a form of legitimate participation and narrative of real-world situation; whether the model and solution provide valid and appropriate information about the real-world model and the original focal event on which the real-world model is based; and any other factors that exist outside of the mathematically based modelling process which might affect decision-making differently from the types of conclusions drawn out of the modelling process. It is also through the process of validation that a decision may be reached to repeat the modelling process for the same problem situation in order to include or exclude certain variables and/or constraints so as to improve the readability and applicability of the

[153] This statement is important in that it posits that a primary goal of the Modelling domain of practice is engagement with real-world problem situations that *cannot be understood and/or solved more effectively through a different means*. However, despite this priority of modelling for facilitating contextual sense-making and decision-making practices, I also acknowledge that it may prove necessary for participants to learn to model through simplified and/or mathematised situations in order to develop a competency with the modelling process. In other words, learners cannot be expected to generate complex models to facilitate understanding of real-world situations if they have not first developed the ability to construct models: the skill involved in constructing a model precedes the ability to use a model to make sense of a situation. In other words, the modelling process is not seen as an ends in itself. Ultimately, learners must not be expected to model so that they can learn how to model. Rather, learners must construct models because they facilitate a broader understanding of a situation. As such, it may prove necessary to teach the modelling process through simplified situations that do not resemble reality in all aspects but which effectively demonstrate the modelling process.

model (W. Blum & Ferri, 2009, p. 47). Finally, the model is (7) exposed, compared and reflected back against the broader contextual environment in which the focal event (for which the model is representative) is positioned. This is done to see whether the structure of and outcomes for the model are consistent and compatible with the variables and constraints of the broader contextual environment – in other words, whether the model provides an adequate and accurate means for making statements and conclusions about more than just the isolated focal event. In simpler terms, the process of exposure is driven by the premise that even though a model may provide a reasonable representation of a limited and deliberately and selectively constrained focal event, this does not mean that the model is adequate when broader variables and constraints from the wider contextual environment are considered. The model is also exposed to find inconsistencies or inaccuracies in the model in relation to the structure of legitimate and endorsed forms of real-world participation in the broader environment, and to verify if the original variables and constraints in existence at the time of the model development still apply. All of this is done with the intention of redefining and restructuring the model to improve the consistency and accuracy of the model as a representation of a segment of reality. And, since every model represents only a particular (and limited) perspective of a situation, to generate a broader perspective and understanding of a situation "If need be (and more often than not this is the case in 'really real' problem solving processes), the whole process has to be repeated with a modified or a totally different model." (W. Blum, et al., 2002, p. 153). Repeating the process allows for different variables and constraints to be considered, and, hence, for the generation of a potentially different perspective of the problem situation.

Further comments on certain features of the modelling framework shown in Figure 29 are necessary. Firstly, notice the deliberate 'messiness' of the space representing Reality compared to the uniformity of the space occupied by the mathematical world. This is done deliberately to reflect that the real-world is messy, with calculations, solutions paths and solutions seldom comprising of easy-to-use values, or clear guidelines (if any), or of simplistic and easily observed variables. The domain of mathematical practice, by contrast, is more ordered and hierarchical, with knowledge and tools drawn from a common corpus to facilitate structured and accurate outcomes.

Secondly, by clearly distinguishing between real-world and mathematical components employed in modelling practices involving real-world contexts, the framework facilitates allowance for consideration of everyday knowledge, practical intelligence, and other practices characteristic of the Everyday domain which may affect decision-making in reality. Furthermore, the framework also recognises the potential for generating mathematised descriptions of reality as an alternative view or representation of the structure of legitimate participation in a real-world practice, which is facilitated through engagement with components of the Mathematical Competency domain of practice. In other words, the structure of the modelling process embodied in the framework facilitates allowance for both a mathematical gaze and an everyday gaze over real-world practices – the complementarity of the mathematical and the contextual terrain. Finally, the separation of reality from mathematics sends an important message: reality is not mathematics; they are different domains and terrains of practice, and a further component – that of modelling – is needed to facilitate interaction between these two domains.

Thirdly, as demonstrated in the modelling framework schematic, the Modelling domain of practice provides an interaction – a bridge – between the Everyday and Mathematical domains of practice. Whereas the Everyday and Mathematical Competency domains can function independently of each other, the Modelling component requires an interaction and/or integration of facets of both of these domains.

Fourth, when mathematisation occurs, this mathematisation is positioned primarily at the *horizontal level*. There is no intentional goal for the utilisation of vertical mathematisation as a means for developing and facilitating access to increasingly abstracted and generalisable mathematical knowledge.[154] Importantly, this does not imply that there is no progression or advancement envisioned for mathematical literacy, but, rather, that this progression is not associated with the hierarchical development of mathematical knowledge. Instead, progression is deemed to be based on issues such as familiarity of context, complexity of context, and the ability to solve problems independently and without guidance.

Fifth, although the modelling process is presented as a cycle with sequentially numbered processes, the modelling process is not a linear process and there is certainly no expectation that a person engaged in a modelling activity must follow every process in the order or sequence shown in the framework. Rather, the modelling process is a complex process that involves continuous movement backwards and forwards throughout the different processes and between the terrains of the contextual and the mathematical, with the stage of validation occurring continuously, together with consideration of appropriate methods and suitability of choice of initial constraints and variables. What is most important in this regard is that there is an attempt made for consideration of and engagement in all of the processes and not whether the processes are followed in a particular order.

Sixth, notice that the modelling process always starts and ends back in the real-world and concludes with consideration of real-world consequences and considerations alongside any mathematical solutions. Any modelling process within the Modelling domain of practice of the knowledge domain of mathematical literacy must return to the real-world and must allow for consideration of contextual and qualitative factors that affect decision-making processes, but which may lie outside of the modelling process and/or have nothing to do with mathematical considerations or mathematisation processes. For example, the decision on which cell phone to buy may be informed by a cost factor – and a mathematical model may provide insights into which cell phone system is the more affordable under a given set of conditions. But, the ultimate choice of cell phone is, in all likelihood, influenced *more* by considerations of the brand, features, colour and availability of the phone, not to mention whether or not the phone is ugly: very few people choose to buy an ugly phone no matter how cheap the phone may be. In an orientation for life-preparedness, it is consideration of both mathematical and contextually legitimate and endorsed forms of participation that are deemed essential for facilitating comprehensive and enhanced understanding of real-world practices.

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[154] However, it could be argued that vertical mathematisation is very much a part of the Mathematical Competency domain of practice of the language of description and that this vertical mathematisation is a key source of cumulative learning in this domain. This is certainly true. However, this does not deter from my primary argument that the learning of mathematics must always be in service to or appropriated by a life-preparedness agenda. As such, if vertical mathematisation is needed to facilitate access to a context or problem situation in a context (i.e. abstraction outside of mathematics (Stillman, 2012)) then such mathematisation is deemed a necessary part of the problem-solving process. But, vertical mathematisation with the intent of developing mathematical knowledge purely for the sake of developing such knowledge and with no intention for the utilisation of that knowledge in preparation for the world beyond the classroom (i.e. abstraction within mathematics (Stillman, 2012)) does not fall within the scope of the internal language of description for the knowledge domain of mathematical literacy.

14.4.4.3 The relationship of the Modelling domain of practice to the other components of the knowledge domain of mathematical literacy

The relationship of the Modelling domain of practice to the Everyday and Mathematical Competency domains has already been established. Namely, the Modelling domain provides a bridge between these other two components; and a Modelling domain (as part of the domain of knowledge of Mathematical Literacy) that facilitates a life-preparedness orientation *must* interact with both the Everyday and the Mathematical Competency domains if an agenda for contextual sense-making practices is to be prioritised over numerically and mathematically based goals.

As mentioned previously, the Mathematical Competency and Modelling domains exist in a hierarchical relationship: competency with mathematical concepts and techniques is seen as an important precursor to successful modelling capability. An equally hierarchical relationship exists between the Modelling and Everyday domains of practice. Understanding of real-world contextual knowledge, considerations and forms of communication that influence decision-making practices in a focal event are essential in a modelling process that seeks to re-describe a segment of reality in order to facilitate understanding of existing forms of participation and also of possible alternative (and more mathematically informed) forms of participation in the focal event. Of course, modelling can occur successfully without appropriate and comprehensive understanding of a contextual situation; but the end result of this process will be a wholly mathematised representation of reality that does not adequately reflect the structure of legitimate and endorsed forms of participation in the context. If an agenda for contextual sense-making practices is to be successfully prioritised in modelling processes, then comprehensive understanding of the contextual terrain is essential. In short and summary, successful modelling is facilitated through *complementary* engagement with both the mathematical and contextual terrains and with both the Everyday and the Mathematical Competency domains of practice. Where the Everyday domain provides access to forms of participation that reflect how people successfully engage in real-world problem-solving situations in everyday settings, the Mathematical Competency domain provides access to possible alternative forms of participation in those contexts and to an alternative world-view. Each domain of practice adds a further level of comprehension and empowerment to the situation and each provides the participant with a broader world-view.[155]

But a gap remains: What enables a participant to effectively move navigate between the real and mathematical worlds, constantly shifting roles, and drawing on different structures, principles and considerations, in order to successfully solve problems? What

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[155] A word of caution is necessary here. I am certainly not suggesting that it is the collective of Everyday and Mathematical Competency practices that constitute Modelling, and, as a consequence, that the capacity for modelling develops directly through interaction in the practices associated with these two domains of practice. Rather, and as is illustrated in the modelling framework captured in Figure 29 on page 225, modelling involves a complex interaction between the mathematical and contextual terrains (and, so, between the Everyday and Mathematical Competency domains of practice) that is facilitated through a multitude of processes, including – to mention but a few – structuring, mathematisation, validation and exposure. The capacity to successfully engage in these processes in a consistent and integrated way as part of a consolidated modelling activity is not developed through interaction in Everyday and Mathematical Competency practices. The capacity to construct models to reflect and describe real-world situations is developed through continuous and repetitive engagement in (all of the) modelling processes for a multitude of different problem situations. Blomhoj and Jensen (2003) argue that modelling is learned better when smaller parts of a process are practiced away from the broader context. As such, while Modelling practices require elements from the Everyday and Mathematical Competency domains, Modelling certainly cannot be reduced to these two domains.

enables a participant to decide whether a modelling process is necessary and appropriate? What enables this participant to decide which components of the mathematical solution to utilise and which to reject? What enables this participant to employ a particular mathematical technique as the basis for a model? And, what enables this participant to balance Mathematical and Everyday domains of practice and terrains and to choose the most effective component for engaging in a situation? In other words, what is it that enables a participant to bring together all other components of the knowledge domain of mathematical literacy – the Everyday, the Mathematical Competency, and the combination of these two in the Modelling domain – to successfully solve problems and/or interact with real-world situations? This missing component – as the component which envelops, permeates and brings together all of the other domains − constitutes the final domain of practice of the knowledge domain of mathematical literacy: the *Reasoning and Reflection* domain.[156]

14.4.5 <u>Domain of Practice #4: Reasoning and Reflection</u>

This component of the language of description is based on a combination of Skovsmose's (1994a, 1994b) work on 'Reflective Knowing' and Bansilal's (2013) work on 'Contextual Reasoning'. Each of these concepts is reviewed in detail and then collated into a single category titled 'Reasoning and Reflection'. Following this, a discussion of levels of Reasoning and Reflection is provided, and as part of this discussion I highlight that it is this domain of practice that facilitates a dominant intention in the knowledge domain of mathematical literacy for the critical evaluation of structures encountered in problem-solving processes. I conclude the section by positioning the Reasoning and Reflection domain as envisioned for the language of description for Mathematical Literacy in relation to the other domains of practice discussed thus far.

14.4.5.1 Overview of Skovsmose's conception of 'Reflective Knowing'

To understand Skovsmose's (1994a, 1994b) intention for the derivation of the term 'Reflective Knowing'[157], it is necessary to position the term within his broader theory on the *formatting power of mathematics* and the role of *mathemacy* in responding to this formatting power. Skovsmose (1992) argues that in a highly technological society mathematics 'intervenes' in reality as a tool that can be used for describing and interpreting phenomenon but also for building models on which the foundations of societal structures and decision-making processes are based. Issues of economics, of politics, of social welfare, are all steeped in mathematically based arguments, built around mathematical models, and informed by mathematically structured considerations. Mathematics formats reality, colonises aspects of reality and reorganises reality according to mathematically based structures (Skovsmose, 1992, p. 6): "perhaps God did not organise the world according to mathematics, but the thesis suggests that humanity has now embarked on just such a project." (Skovsmose, 1994b, p. 43). Vithal (2003, p. 317) extends this idea to the classroom setting to suggest that learners in mathematics

[156] The domain of Reasoning and Reflection has an integral role to play in facilitating the development of cumulative learning through participation in the Modelling domain of practice. For this reason, a discussion of this topic is delayed until sub-section 14.4.5.5 below (starting on page 246).
[157] Skovsmose deliberately uses the word 'knowing' rather than 'knowledge' to emphasise that reflection does not stem from an already existing authoritative or definitive body of knowledge. Rather, reflection is more closely associated with a competency, a disposition, and an ability that develops over time through constant engagement in problem-solving situations (Skovsmose, 1994b, pp. 101-102) − but which, nonetheless, can be abstracted across problem situations and contexts.

classrooms who are imbued with the skills to model mathematically are thus inducted into this formatting power of mathematics and, so, become ‘formatters’. For Skovsmose, this situation is problematic (in a highly technological society) from the perspective of democratic participation in societal practices, because only those who have access to the technological and highly mathematical knowledge and assumptions that underpin societal, economic and political structures are in a position to challenge these structures. And, if the majority of the population do not possess this capacity – what Skovmose refers to as democratic competence (1990, 1992, 1994a, 1994b) – then does a democratic society really exist? As suggested by Skovsmose (1990) in this illustrative quotation:

> A society based on advanced technology faces a specific problem of democracy. If a society is based on manual tools the idealistic interpretation of democratic competence becomes plausible; no specific technological knowledge is needed to evaluate the acts and decisions of the people in charge. Quite the contrary occurs in a highly technological society. The content of democratic competence is rapidly changing towards a tremendous complexity. On the face of it only a limited group of people seem to be able to manage this complexity. In fact this competence seems to presuppose a certain amount of technological knowledge including mathematics. The consequences seem to be that only a limited group of people can obtain a democratic competence and then become able to evaluate the actions of the people in charge. This is the problem of democracy in a highly technology society. (p. 123)

A possible solution is to develop in citizens a democratic competence: “the basis of knowledge and understanding which is necessary if the delegation of sovereignty is to be subjected to any sort of control” (Skovsmose, 1992, p. 4). And, in the context of educational practice, to position learners in the role of ‘critical formatters’ who are able to analyse and critique the formatting power of mathematics and the assumptions underpinning that formatting in societal structures:

> If people are to be not only receivers of information and instructions but also able to criticise, evaluate, understand, i.e. to provide input to the democratic institutions, then they must get an understanding of some of basic structuring principles in society. (Skovsmose, 1992, p. 10)

This democratic competence is to be achieved through a critical mathematics education agenda and through the development of a level of *mathemacy*, comprising mathematical, technological and reflective knowing. In particular, it is the component of reflective knowing that orients mathemacy towards a critical competency (Skovsmose, 1990, p. 124; Vithal, 2003, p. 317).

For Skovsmose (1994a, p. 47), mathematical, technological and reflective knowing constitute three different types of knowing towards which mathematics education can be orientated. These three types of knowing are, thus, perceived as a crucial component of a critical mathematics education that equips participants to respond to and challenge existing social conditions, structures and inequalities in promotion of a (more) democratic and/or equal society.[158] Mathematical knowing − as has already been discussed briefly previously (c.f. page 215 above) – refers to knowledge and skills associated with mathematical content, structures and techniques. Technological knowing – which has been associated with the 'Modelling component' of this language of description (c.f. page 220 above) – refers to the competence involved in applying mathematics and in developing mathematical models to represent real-world situations. The distinction between mathematical knowing and technological knowing, then, is one of relation to the real-world: mathematical knowing makes limited or no attempt at connection of mathematics to the world, while technological knowledge does. The third type of knowing – that of reflective knowing – refers to the skill and competence "in reflecting upon and evaluating the use of mathematics" (Skovsmose, 1994a, p. 47) and, particularly, on the assumptions underpinning mathematical structures and the consequences of the use of mathematics in the formatting of social structures:

> Reflective knowledge, to be interpreted as a more general conceptual framework, or metaknowledge, for discussing the nature of models and the criteria used in their constructions, applications and evaluations. (Skovsmose, 1990, p. 124)

For Skovsmose (1994a, p. 47), reflection is not concerned with how learners come to grasp mathematical ideas but rather with an understanding "about mathematics" and the power of mathematics in formatting the world. The learning of mathematics is not the focus; the role of mathematics in shaping society – and preparing learners to respond critically to this role – is the primary concern:

> The focus must be on the *functions* of the applications of mathematics in society – and not just on modelling as such.
> (Skovsmose, 1992, p. 8, emhpasis shown in original text)

Crucially, the type of knowledge utilised in constructing models (technological knowledge) is different to the knowledge required for analysing and evaluating such models (reflective knowledge) (Skovsmose, 1990, p. 124). For this reason, if the construction of models is not accompanied by reflective knowledge, then the assumptions underpinning those models remain unchallenged and critical democratic competence remains stifled.

Taken together, the three forms of knowing – mathematical, technological, and reflective – constitute *mathemacy,* a form of literacy in mathematics, and it is this mathemacy that provides a source of empowerment in critical education "because it can be a means to organize and reorganize interpretations of social institutions, traditions, and proposals for political reforms." (Skovsmose, 1994a, p. 52). Mathemacy, and the interaction of the three forms of knowing, provide a basis from which the formatting power of mathematics and the use of mathematics to inform structures and decision-making processes in society can be analysed, challenged and critiqued.

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[158] "The most general and uniting idea [of critical education] is: *If education, as both a practice and a research, should be critical it must discuss basic conditions for obtaining knowledge, it must be aware of social problems, inequalities, suppression, etc., and it must try to make education an active progressive force.* A critical education cannot be a simple prolongation of existing social relationships. It cannot be an apparatus for prevailing inequalities in society. To be critical, education must react to social contradictions." (Skovsmose, 1994a, pp. 37-38, emphasis in original text).

Returning to the notion of Reflective Knowing[159] as the key to providing mathemacy with a critical component, Skovsmose (1994b, p. 99) necessitates a relationship between Reflective Knowing and technological knowing as "two different types of knowledge, but not two independent types" on two fronts. Firstly, Skovsmose (1994a, p. 49) argues that a modelling process – a process involving technological knowing – involves the transition or movement between 'different language games'. Namely, from a *natural language* (i.e. the language of the real-world) to a *systemic language* as a particular situation in the real-world is condensed, simplified, 'cleaned' to make the problem situation clearer and more accessible (i.e. Process 2 in the modelling cycle for the Modelling domain of practice – the structuring process (c.f. page 226 above)). The next transition is from the *systemic language* to a *mathematical language* as the problem situation is rearranged according to mathematical principles and structures (i.e. Process 3 in the modelling cycle for the Modelling domain of practice – the mathematising process (c.f. page 226 above)) (Skovsmose, 1994a, pp. 49-50). Although Reflective Knowing is crucial in facilitating identification of the role of the values and norms that are brought to bear on problem-solving situations encountered in authentic situations, I contend that Reflective Knowing is further required to effect the transition between these different languages successfully. Reflective Knowing is also required to enable participants to shift from one language to the other and to manage the processes and structures which dictate and guide action in the different languages.[160] In this regard, it is Reflective Knowing which enables the participant to make a distinction between "what is seen as worthwhile and what looks workable" (Skovsmose, 1994a, p. 50), including consideration of the need for adoption or rejection of particular mathematical or contextual elements in a problem-solving process.

The second aspect of the relationship between Reflective Knowing and the modelling process contained in technological knowing relates to the thesis that "technological knowing itself is unable to predict and analyse the results of its own production; reflections are needed." (Skovsmose, 1992, p. 7). This is because "Technical knowing is born shortsighted" (Skovsmose, 1994a, p. 48): the language game of mathematics orients the purpose of the engagement towards the prioritisation of accuracy, efficiency, and mathematically measurable and validated outcomes and forms of communication; whereas a different language game (for example, a language game oriented toward 'morality') would yield a different interpretation and engagement of the problem-solving experience. For this reason, when a mathematically oriented perspective is intertwined with a different terrain comprising different language and different criteria for legitimate and endorsed participation and communication, some form of reflection is needed. This reflection facilitates mediation not only of the language of the new terrain but, more especially, the use of mathematical language and mathematically structured forms of participation in that terrain: "Reflections must be based on a wider horizon of interpretations and pre-understandings. It has to grasp the situation in which technological knowing is at work." (Skovsmose, 1994a, p. 48). Furthermore, those who operate in a technological knowing mode and those who construct models or who use models to make

[159] Please note that I am now deliberately switching to a capitalised form of the term 'reflective knowing' for specific use and reference as a component of the internal language of description of the structure of knowledge for the knowledge domain of mathematical literacy. This is to ensure consistency with the way in which I have referenced similar concept-titles throughout the study.

[160] It could always be argued that this transition between different languages or different worlds – the real and the mathematical – is part of the modelling process and, as such, is a skill that must be developed as part of the modelling component of the language. I contend that it is Reflective Knowing that facilitates this transition successfully and which enables a participant to know when and how to effect the transition. In this sense, this aspect of the Reflective Knowing component develops as part of the model development learning process rather than independently of the modelling process.

decisions are often not capable of criticising the models themselves or the assumptions made in the construction of the models, simply because the assumptions made are their own. It is in this vein that Vithal (2003) suggests that,

> When pupils are inside the one, they seem unable to seriously engage with the other. This means that the process of formatting located inside in mathematics is in complementarity with critically reacting to that formatting located outside mathematics. (p. 318)

A critical mathematics education, then, requires the ability to not only build models and to mediate the transition between different languages engaged in the construction of the models, but also the ability to critically reflect on existing models and the assumptions underpinning those models – the defining feature of a critical mathematics education. As suggested again by Vithal (2003):

> This means that reflective knowing includes two necessary but opposite forms of knowing – one inside mathematics, and the other outside mathematics, reflecting from some context back onto the mathematics. (p. 318)

In summary, Reflective Knowing, thus, serves the following functions in a framework for critical mathematics education:[161]

> A first task in reflective knowing is to make explicit the preconditions which become hidden when mathematical language gives a neutral cover to a mathematisation process such as modelling; its second task is to address the uncertainties and problems connected to the transition between the different language games involved in the mathematisation process; and its third task is to provide mathemacy with an element of empowerment. (Vithal, 2003, pp. 9-10)

14.4.5.2 Overview of Bansilal's notion of 'Contextual Reasoning'[162]

Skovsmose's (1994a, 1994b) notion of Reflective Knowing is directed, primarily, towards critical analysis of the use and formatting power of mathematics in modelling processes that make a reach towards an extra-mathematical terrain. Although this notion also encompasses the terrain of Mathematical Literacy, I contend that a further dimension is necessary for consideration, specifically, of the contextual dimension of the problem-solving process. This additional dimension serves to accommodate the

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[161] See also (Skovsmose, 1994b, pp. 106-114) for a more detailed theoretical discussion of the functions or tasks of Reflective Knowing in a critical mathematics education framework.

[162] In Part 5, Chapter 17and sub-section 17.2.2 above (starting on page 276), I again make reference to Bansilal's (2013) notion of 'Contextual Reasoning'. In that discussion, I draw on the work of Sfard (2008) to argue that the knowledge domain of mathematical literacy can be distinguished from other knowledge domains (such as mathematics) according to the various discursive resources that characterise the domain. As a contextually based domain, mathematical literacy comprises a combination of mathematical and contextual discursive resources, of which Contextual Reasoning is one such resource. Other resources characteristic of the knowledge domain of mathematical literacy include both mathematical and contextual words/vocabulary, visual mediators, routines, and endorsed narratives. Importantly, that discussion forms part of the presentation of the external language of description for the study, an external language that provides the structure and means by which the internal language of description for the knowledge domain of mathematical literacy can be used as a lens through which to analyse empirical phenomenon associated with the school subject Mathematical Literacy. The linkage of the component of Contextual Reasoning in both the internal and external languages of description is a deliberate attempt to ensure consistency between the two languages.

ardent emphasis in the developed internal component of the language of description on the subordination of mathematical structures to the status of tool – as a 'means to an end' – in the problem-solving process. This dimension also facilitates recognition of the vast array of contextually defined variables, influences, and considerations that direct and inform the structure of legitimate participation and decision-making processes in contextual problem-solving encounters. Bansilal's (2013) work on 'Contextual Reasoning' provides the basis for this additional dimension.

According to Bansilal (2013), Contextual Reasoning is the "reasoning, arguments, assumptions, and justifications about issues arising in the context." From this description, it could easily be argued that such reasoning shares overlap with the notion of Reflective Knowing in that reasoning about a context must surely also consider the impact of mathematical calculations and consequent a mathematical gaze on the perspective generated about that context. For me, however, Contextual Reasoning involves more than this. It further involves awareness that analysis of a focal event embedded in a broader contextual environment only provides a particular and limited perspective of the environment. It involves awareness that any selection of a focal event for analysis inherently involves selective and subjective choices regarding variables, constraints and considerations for inclusion (and for exclusion). It involves interrogating the impact of participants' own historical, economic and social perspectives on their analysis of a focal event. And it involves awareness and recognition of the multitude of (often non-mathematical) considerations that come to bear on how people act and the decisions that people make in contextualised situations. What is important to note about all of these elements of Contextual Reasoning is that they embody considerations that relate directly to contextual issues but not necessarily to the use of mathematics in context.

In summary, where Reflective Knowing embodies continual critical analysis of the use and formatting power of mathematics in (re)describing extra-mathematical contextual situations, Contextual Reasoning embodies awareness and engagement with aspects of the contextual situations (and of the participants' own backgrounds) which impact on how participants think and behave in those situations, and which inform and direct decision-making processes. For me, it is only through the combination of both Reflective Knowing and Contextual Reasoning that a more encompassing or complete understanding of participation in a contextual situation is possible. And for the knowledge domain of mathematical literacy that embodies a life-preparedness orientation as is envisioned in this study, reasoning and reflection of both mathematical and contextual dimensions is essential.

It is, thus, as a means of embodying the two notions of Reflective Knowing and Contextual Reasoning that I have chosen to use the term *Reasoning and Reflection* to describe the final domain of practice of the language of description for the knowledge domain of mathematical literacy.

14.4.5.3 Levels of Reasoning and Reflection

Skovsmose (1992, 1994b) describes six entry points or steps – or what Gellert, Jablonka, and Keitel (2001) refer to as levels[163] – in the development and/or application of Reflective Knowing. These are summarised as follows:

[163] Gellert et al. (2001) only identify the first five levels and do not engage in a discussion of the sixth level.

Level 1: reflecting on whether mathematical methods have been used correctly;
Level 2: reflecting on the consistency and appropriateness of the mathematical methods used (from a mathematical perspective);
Level 3: reflecting on the reliability of the solution for the purpose;
Level 4: reflecting on the appropriateness of using mathematics in a specific context;
Level 5: reflecting on the broader consequences of the use of mathematics in a specific context;
Level 6: reflecting on the reflection of the use of mathematics in a particular context.

To facilitate the use of these levels in relation to the subject-matter domain of Mathematical Literacy, a redefinition of the levels is necessary. These redefined levels are presented alongside the original levels in

Table 6 on the page below.

The redefinition of these levels is driven by a two-fold motivation.[164] Firstly, the original levels are concerned with *critical* reflection of mathematisation processes – namely, questioning the assumptions that are made or have been made when calculations are performed and models constructed to recognise and understand the value-laden nature of these constructions. However, for my own purposes of analysis of textual resources relating to the subject-matter domain of Mathematical Literacy, I employ these levels as a means for both sense-making of situations that employ the use of mathematics in (re)describing contextualised settings and for critical reflection of the power of calculations and models as an instrument of formatting.

[164] Note that the redefinition of these levels is a direct result of a dialectical interaction of theory and empirical resources: it is through engagement with the levels of Reflective Knowing with specific empirical textual resources relating to the subject-matter domain of Mathematical Literacy that the need for redefinition was identified.

Table 6: Redefined levels of Reflective Knowing for use in this study

	Original Levels of Reflective Knowing	Redefined levels that incorporate elements of both Mathematical Reflection and Contextual Reasoning
Level 1	reflecting on whether mathematical methods have been used correctly;	reflecting on the meaning of contextual and/or mathematical elements (including symbols, signifiers, notation) and terminology in a problem situation, the relationship between these elements, and their relevance for the problem situation;
Level 2	reflecting on the consistency and appropriateness of the mathematical methods used (from a mathematical perspective);	reasoning about: • the scope of the specific focal event (or sub-event of the focal event) under analysis; • the embedded nature of a focal event within a broader contextual environment; • and the inclusion and exclusion of variables and constrains in defining the scope of the focal event to be analysed;
Level 3	reflecting on the reliability of the solution for the purpose;	reasoning about the most appropriate method (contextual or mathematical) for use in a specific context, or reflecting on the consistency and appropriateness of the methods already used;
Level 4	reflecting on the appropriateness of using mathematics in a specific context;	reasoning about how to employ chosen methods accurately and with the appropriate structure or working, or reflecting on whether methods have been used correctly and accurately;
Level 5	reflecting on the broader consequences of the use of mathematics in a specific context;	reflecting on the reliability of the solution for the purpose, including reasoning about possible extra-mathematical factors and methods that influence decision-making processes in contextual situations;
Level 6	reflecting on the reflection of the use of mathematics in a particular context.	reflecting on the broader consequences of the use of mathematics and/or contextual strategies in a specific context;
Level 7	---	reflecting on the reflection of the use of chosen methods in a particular context.

In other words, for me, Reflective Knowing provides the means for the following. Firstly, understanding a problem situation – which includes selecting appropriate techniques and contents that can be used to develop an appropriate solution strategy for the problem. Then executing chosen techniques to develop a particular solution strategy. Finally, critically analysing those techniques and solution strategy (and the decisions made in selecting and executing those techniques) to understand the value-laden perspective that underpins and embodies the particular (re)description or (re)construction of reality. It is this motivation that has facilitated the addition of the a redefined 'Level 1' level of reflection that involves *reflecting on the meaning of elements (including symbols, signifiers, notation) and terminology in a problem situation, the relationship between these elements, and their relevance for the problem situation*. It is also this same motivation that has inspired the rephrasing of text in the descriptors of several of the levels to embody reflection on both methods that have already been used in a previously constructed models or calculation as well as reasoning on the types of methods, techniques, contents and strategies appropriate for use in the development of a model or in a calculation that is yet to be performed. I have also reordered several of the levels (for example, the original Level 1 is now repositioned as Level 4 in the redefined list) to reflect a more common sequence of reflection/reasoning that I encountered while engaging with empirical resources relating to the subject-matter domain of Mathematical Literacy. And although the levels are not prescriptively hierarchical (see page 241 below for an elaborated discussion of this point), the following loose structure now applies to the order of the redefined list of levels: (1) understanding the elements that make up a problem situation and (2) the precise scope of the problem situation; (3) choosing the best method and then (4) conducting the method in an appropriate way; (5) deciding if the solution to the problem adequately describes the problem situation or if there are other issues that need to be taken into consideration; (6) understanding that the perspective of the situation that is presented through the solution is only one of many possible perspectives; (7) thinking about all of the decisions have been made in the problem-solving process and how these decisions have impacted on the structure of the solution and on the viewpoint embodied in the solution.

Secondly, in light of the central agenda for sense-making practices in the language of description of the structure of knowledge for the knowledge domain of mathematical literacy, additional attention is needed on, particularly, *contextual elements* – many of which may be non-mathematical in structure – that come to bear on how participants behave and make decisions in contextual situations. In other words, recognition of the importance of Contextual Reasoning in contextualised problem-solving practices is necessary to facilitate contextual sense-making and associated life-preparedness. Hence the focus in several of the levels on reasoning about contextual signifiers, routines, knowledge and forms of participation that influence problem-solving strategies and decision-making processes in contextualised problem situations.[165]

By way of elaboration of each of the levels: *Reasoning/Reflection at the first level* (R/R[1]) involves examining signifiers present in the problem scenario and/or the text provided to describe the scenario (in the case of a textual activity). This level also involves identifying appropriate routines, methods and contents that are indexed by these signifiers and, consequently, which facilitate the generation of a legitimate solution strategy for the problem. This level of reasoning is essential for coming to understand precisely what the problem situation is about and what is required to generate a solution strategy for the problem situation that is ultimately endorsed.

The *second level of Reasoning/Reflection* (R/R[2]) involves defining the precise scope of the event under analysis and the variables essential for consideration in dealing with that event. This level of R/R also entails recognition of the embedded nature of a focal event within a broader contextual environment and the consequent need to consider facets and variables from the broader environment to facilitate a more complete understanding of the reasons why participants act in particular ways in the focal event. Furthermore, this level embodies awareness that conscious and/or unconscious decisions are made to include and exclude or ignore certain variables and constraints when defining the scope of a focal to be analysed. Consequently, there is a need for reflection on the consequences of this deliberate/indirect inclusion/exclusion process and recognition that changing the criteria of inclusion/exclusion might well generate a different impression of the criteria for endorsed participation in the contextual setting. This level (together with R/R[5])

[165] The elements of Contextual Reasoning included in the levels shown in

Table 6 are by no means an exhaustive description of the type of reasoning involved in contextual situations, especially given that engagement with contextual situations may yield a multitude of variables and considerations which come to bear on how participants act and make decisions in those contexts. Nonetheless, I have tried to capture what I believe to be certain general facets of Contextual Reasoning that could be considered in any interaction in a contextual environment.

further involves awareness that every contextual situation facilitates engagement with a different variety of problem-solving techniques and considerations that influence decision-making.

R/R[3] involves asking questions about the most appropriate technique(s) and method(s) required to develop an appropriate solution strategy for the problem situation. This includes questioning whether mathematics is actually required to solve the problem – which is why Vithal (2003, p. 318) refers to this as the level of "selecting mathematics"[166] − or whether there are another, more initiative, less formal, or 'qualitative' (Julie, 2006) techniques that can be employed and which may yield a different or more efficiently-arrived-at solution. As suggested by Skovsmose (1992),

> [These] questions attacks that variant of the true-false ideology which tells that formal methods must be preferred[167]. Formal methods may reach further in some situations, but they do not always work to give an appropriate answer. By contrasting formal techniques with intuitive ones it becomes possible to see formalization as only one possible way of handling a problem, and this experience is important in developing Reflective Knowing. (p. 8)

Importantly, the technique(s) and method(s) identified for use are done so through interpretation of the signifiers present in the problem scenario or in the text that describes the problem scenario. These signifiers index not only the specific scope of the event under analysis, but also the contents – and consequent techniques and methods –required to deal with problems posed about that event.[168]

The *fourth level of Reasoning/Reflection* (R/R[4]) then involves asking questions about the way in which identified methods of working must be employed to ensure accuracy and consistency, and to ensure that the structure of working reflects practices that are endorsed by other participants who engage in the same practices. For example, participants who perform calculations involving money must understand not only the need for accuracy of calculation, but also the relevance of including currency symbols so that the specific nature of the currency being dealt with is obvious. The need for appropriate rounding in different financial contexts (e.g. to two decimal places for day-to-day calculations, but to five or six decimal places in banking institutions) must also be considered. This level of reasoning/reflection further involves reflection on the structure and working of method(s) that have already been used in the construction of models and the validation of the accuracy of those methods.

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[166] Note that Vithal (2003) cites the ***original*** points of entry of (1) and (2) as 'selecting the mathematics' and 'executing the mathematics correctly', which is in the opposite order to that cited by Skovsmose. Although the order is in itself not important, I feel that it is necessary to highlight this distinction so that the reader does not assume that I have misread Vithal's interpretation of the levels of Reflective Knowing.

[167] And, in so doing, this fourth level of reflection directly challenges Dowling's (1998) Myth of reference and the associated notion that mathematics can be used to describe any situation.

[168] The notion of 'signifiers' in text and the role of these signifiers – as a particular type of discursive resource − in indexing particular routines and contents (both contextual and mathematical) required for the generation of endorsed narratives for problem scenarios is dealt with in detail in Part 5 of the study (c.f. Chapter 17 and sub-section 17.1.2 below, starting on page 269).

The *fifth level of Reasoning/Reflection* (R/R[5]):

> is necessary to address the notions of appropriateness and reliability in a specific context in a way that explicitly accommodates the nonmathematical constituents. Even when calculations are done correctly and the techniques have been checked for consistency, the result may not be useful for the purpose in hand. (Gellert et al., 2001, p. 71)

The principle of calculating accurately the surface area of the walls of a building in square metres (m^2) springs to mind, especially since paint is sold in litres – and sometimes only in multiples of a specific quantity of litres (e.g. 2-litres or 5-litres or 10-litres or 20-litres). This level is what the OECD-PISA frameworks refer to as "Making sense of the mathematical solution in terms of the real situation." (OECD, 2003, p. 27). An essential component of this level of R/R is recognition of the role of contextual (and potentially non-mathematical) considerations and factors that might influence decision-making in a particular context and which may even contradict or negate a mathematically derived solution.

The *sixth level* of *Reasoning/Reflection* (R/R[6]) considers the formatting power of mathematics by questioning how a particular mathematical (re)construction or (re)description of a segment of reality is perceived and experienced by others, and the values and opinions that underpin this (re)construction and (re)description, and which, consequently, are imposed on the world. This level also involves reasoning about the participants' historical, cultural and economic backgrounds as a source of formatting and positioning. Reasoning on the impact of this formatting and positioning on the way in which participants engage in a focal event and on the variables and constraints they may choose to include and exclude or ignore in dealing with and defining the scope of the focal event is also included. Further, at this level of R/R questions are posed about the ways in which participants choose to solve problems and make decisions in contextual situations and the context-specific factors that impact on decision-making, and how these methods and factors might align or differ from formal and/or taught mathematical methods. This level of reasoning recognises that a mathematical gaze cast on a situation provides a potentially different perspective of the structure of legitimate or endorsed participation in the context in relation to how practitioners actually engage in the context in real-life.

The *seventh level of Reasoning/Reflection* (R/R[7]) involves reflection on the decisions, rationales and thought processes that have informed how the previous six levels of reasoning/reflection have been employed. In other words, this seventh level is a self-reflection "upon the way we have reflected." (Skovsmose, 1992, p. 9). This level is a level of self-criticism and self-challenge of the way in which the various levels of critical reasoning and reflection have been conducted and whether critical reasoning has in fact taken place. Furthermore, engagement in this level of reasoning/reflection involves awareness and questioning of whether the critical analysis process conducted is itself not a formatting process.

From the elaborated discussion of the various levels of Reasoning and Reflection above, it is now possible to reflect that it is, thus, the domain of Reasoning and Reflection that facilitates a dominant *intention* in the knowledge domain of mathematical literacy for a form of participation in contextual sense-making practices that involves the critical evaluation of both mathematical and contextual structures encountered in these making practices. The promotion of this dominant intention, together with the prioritised dominant agenda for contextual sense-making practices, facilitates the promotion of a (particular form) of life-preparedness orientation for the knowledge domain (and for associated practices in a format of the subject-matter domain of Mathematical Literacy that draws its esoteric domain of practice from this knowledge domain). In making this connection to the previously discussed agendas and intentions framework, the discussion has now swung full circle to demonstrate the presence of both the dominant agenda and dominant intention required for the promotion of a life-preparedness orientation in the components and structure of the internal language of description of the knowledge domain of mathematical literacy.

To conclude the discussion on the levels of Contextual Reasoning and Mathematical Reflection that comprise the domain of Reasoning and Reflection in the language of description for the knowledge domain of mathematical literacy, three final comments are necessary. Firstly, although the order of the levels does indicate an expectation for the type of reasoning and/or reflection that occurs at different stages in a contextual sense-making problem-solving process, the steps are not prescriptively hierarchical in the sense of having to reason/reflect at Level 1 first before moving on to Level 2, and so on. Furthermore, the labels "Level 1", "Level 2", etc., are given primarily as labels rather than as prescriptive indicators of order. As suggested by Skovsmose (1992),

> We are only talking about steps put in some analytical order, not about the steps which actually may be taken by children and students (when I talk about the first, the second, etc., step, it just has to be read as different steps). (p. 8)

It is also to be expected that in a problem situation driven by a dominant agenda for contextual sense-making practices, certain elements of Contextual Reasoning precede and/or accompany any considerations of mathematics. This is because a genuine understanding of the focal event, of the variables that characterise this event, and of the boundaries of the event with respect to a broader contextual environment, are all necessary before engagement with mathematical components of the event or with a mathematised representation and reconstruction of the event is possible. In sum, the levels of Reasoning and Reflection are not sequential or linear, and the particular level and form of reasoning/reflection required depends on the nature of the context and of the required structure of participation and engagement with that context.

Secondly, in an attempt at an overarching categorisation of the different levels of Reasoning/Reflection, one possibility is to draw and build on Vithal's (2003, pp. 317-318) categorisation of the original levels of Reflective Knowing. In this regard it becomes possible to argue that the first four levels of Reasoning/Reflection are concerned, primarily, with reflections of the processes involved in coming to understand and solve the problem. Levels 5, 6 and 7, by contrast, are concerned primarily with reflections on *contextual issues* that arise as a result of the use of mathematics in scenarios involving real-world context elements. Additionally, where Levels 1 to 4 place emphasis on the actual formatting process involved in (re)describing an aspect of reality, Levels 5, 6 and 7 are more aligned to critical analysis and reflection of this formatting process and of the perspective of reality that results through this formatting process – namely the critical education aspect.[169]

Thirdly, and finally. To reiterate and emphasise a comment already made, for me it is the collective of both Mathematical Reflection and Contextual Reasoning that provides the grounds for critical analysis of both mathematical and contextual elements in a problem-solving scenario that involves the use of mathematics in representing, describing and sense-making of a contextual situation. Emphasis only on reflection of mathematical elements facilitates the potential for the development of a restrictive mathematical gaze of a situation, while emphasis only on contextual elements (and reasoning associated with such elements) negates the usefulness of mathematics in providing an alternative representation and perspective of possible legitimate and useful forms of real-world participation.

14.4.5.4 The role of Reasoning and Reflection in the language of description for mathematical literacy

It has already been argued that the language of description for the knowledge domain of mathematical literacy is comprised of an Everyday domain of practice, a Mathematical Competency domain of practice, and a Modelling domain of practice that brings together the Everyday and Mathematical Competency domains. The domains are all rooted in genuine and authentic real-world contexts and driven by an orientation towards life-preparation. What role, then, does the Reasoning and Reflection domain play in the interaction between these different domains and in facilitating a life-preparedness orientation for mathematical literacy? This question is addressed in the discussion below.

The discussion of Skovsmose's (1994a) notion of Reflective Knowing in sub-section 14.4.5.1 (c.f. page 230) above posited the perspective that, at a general level, the move from a real-world situation to a mathematical model of the situation and then back again to the real-world situation is seen as a shift between different language games, each with their own structure, rules, principles and procedures. Furthermore, as emphasised by Skovsmose (1994b),

> A model can never be a model of reality. We have to identify elements of reality which are to be conceived as being the important ones; we also have to decide which relationships among these elements are essential. In this way we create a *system*; and this is not part of reality. The system is conceptual and created by means of certain interpretations of reality, i.e. by means of a certain theoretical framework for looking at reality, and by having certain knowledge-constituting interests in mind. (p. 103)

The skills and competencies associated with the Reasoning and Reflection domain as described above facilitate this transition and enable participants to know when to switch from real-world mode to mathematical mode, and back again. It is the skills and competencies associated with Reasoning and Reflection domain which equip participants

[169] Skovsmose (1994b, p. 120) and by Gellert et al. (2001, p. 71) also provide a categorisation of the original six levels of Reflective Knowing that differs slightly from the categorisation provided by Vithal (2003) . For these authors, the first two (original) levels focus on *mathematical tools*; Levels 3 and 4 on the relationship between the mathematical tools and the real-life context represented in the model – between the "mean and ends" (Gellert et al., 2001, p. 71); Level 5 on the consequences or effect of using a mathematically based model to make sense of a specific context or situation; and Level 6 on analysis of the whole modelling process and on how Levels 1 to 5 have been conducted.

with the ability to: select appropriate mathematics; exact that mathematics; to make sense of the solution to the mathematical problem in relation to the original real-world problem; and to consider other situational, qualitative and non-mathematical factors and considerations which may affect decision-making in a real-world situation. Furthermore, these skills also equip participants with the ability to understand how their perspective or (re)description of the real-world situation is only one of many possible perspectives and descriptions that is affected and determined by their own backgrounds and by the variables, resources and constraints that have been used to define the scope of the focal event under analysis. Reasoning and Reflection are the overarching competencies that enable a mathematically literate individual to engage with Everyday contents, to choose to employ specific Mathematical Competency contents, and to facilitate integration of Everyday and Mathematical Competency contents in the process of Modelling. And all of this in search of a more comprehensive, broader perspective and (re)description of a problem situation relating to an aspect of reality. It is the domain of Reasoning and Reflection that enables a mathematically literate participant to decide when it is easier and more appropriate to use an intuitive, localised problem-solving approach to a problem and when such an approach will prove inadequate and, so, a mathematically structure approach is more suitable. It is Reasoning and Reflection that enables the mathematically literate participant to realise the limitations of mathematically based models in particular contextual situations and to realise that a mathematically based (re)description of an aspect of reality is only one of many possible such (re)descriptions. It is through Reasoning and Reflection that participants come to understand that, more often than not, the rules and structures that dictate action, participation and legitimate communication in the classroom are very different to those that dictate the counterparts of these in the real-world. Reasoning and Reflection is the domain that brings together all others in the knowledge domain of mathematical literacy, it is the domain that promotes equal consideration of contextual and mathematical considerations, and facilitates a movement away from mathematical goals and towards enhanced understanding of a contextual situation. *Reasoning and Reflection is the domain that directly facilitates the prioritisation of and access to a life-preparedness orientation for the knowledge domain of mathematical literacy*.

The discussion above presents Reasoning and Reflection in very general terms with respect to the knowledge domain of mathematical literacy. What is now necessary is to relate specific aspects or levels of Reasoning and Reflection to the modelling process envisioned in the Modelling domain of practice, particularly since it is this Modelling domain which brings together the Contextual, Everyday and Mathematical Competency domains and which ultimately orients this modelling process away from mathematical goals and towards a life-preparation orientation. The diagram shown in Figure 30 on the page below illustrates the levels of Reasoning and Reflection (e.g. *R/R[1]&[2]*) expected for the different processes of the modelling cycle in the activities of the Modelling domain of practice.

With reference to the points of entry or levels of Reasoning and Reflection, it becomes possible to identify or associate the different levels with different processes of the modelling cycle. To begin with, a relevant focal event must be identified as a site for analysis, accompanied by acknowledgement of the limited and embedded nature of the focal event within a broader contextual environment (Process 1: <u>Understanding and Defining Scope</u>). This involves determining which elements of the focal event must be considered for analysis, together with the variables and constraints that must be included and those that must be excluded for purposes of the analysis – and the implication of decisions regarding elements for inclusion and exclusion on the description of the focal event that is constructed. This process necessitates *Reasoning/Reflection at Levels 1 and 2.*

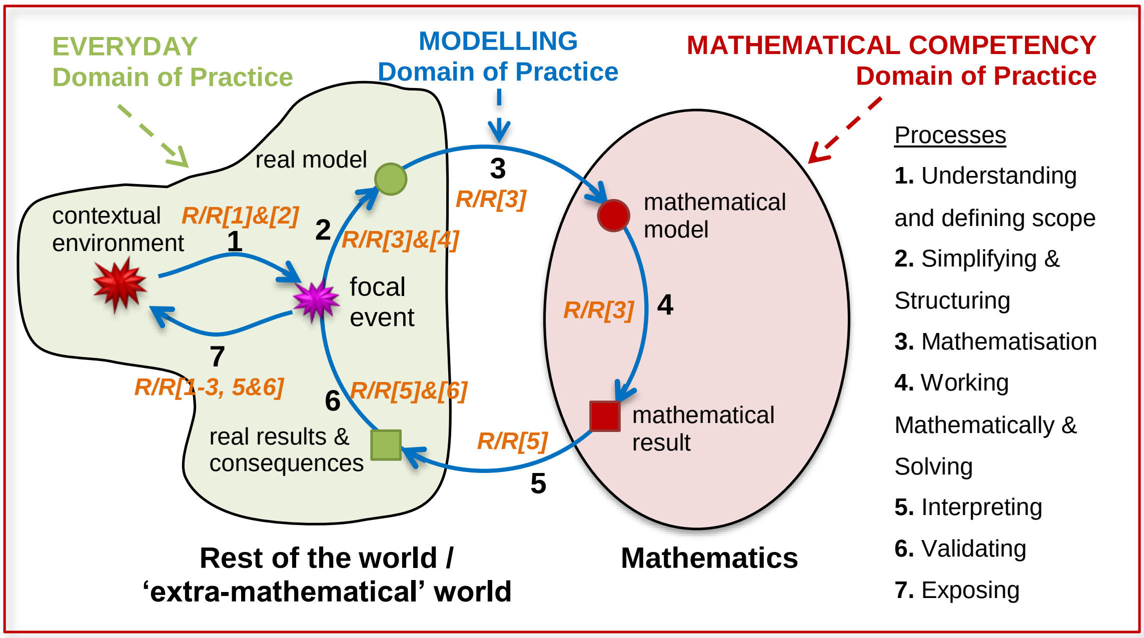


Figure 30: Envisioned levels of Reasoning and Reflection in the Modelling domain of practice

Once the scope of the focal event has been established, a real model must be created to represent a problem situation encountered in the focal event under analysis (Process 2: Simplifying and Structuring). This involves determining which elements of the focal event are important for consideration in dealing with the problem situation, which elements can be removed and which elements need to be 'cleaned' to make the specific demands of the problem situation more accessible. Importantly, it is also in this process where it is determined whether a mathematically based approach to solving the problem is appropriate or whether a localised, intuitive solution strategy can be employed. This process necessitates the *second and third levels of Reasoning/Reflection* which facilitates analysis of the specific structure of the problem situation and reflection on the need for a particular form and type of solution path – whether that be through a mathematically based model or through an alternative means.

If, through reflection on the problem situation and the structure of that problem in Process 2, it is determined that a mathematically based modelling approach is appropriate for describing the practices associated with a particular situation, a necessary shift must then be made from the real-world to the mathematical world through the process of Mathematisation (Process 3). This transition again necessitates the *third level of Reasoning/Reflection* in which questions are posed regarding the appropriate form of the content required for the modelling process.

Process 4: Working Mathematically and Solving necessitates the *fourth level of Reasoning/Reflection* as methods and solutions are evaluated to ensure the accuracy, validity and consistency of these methods and associated working. During this process, Reasoning/Reflection again takes place regarding the efficiency and appropriateness of the contents and methods employed during the mathematisation process (i.e. R/R Level 3).

Process 5: Interpreting again involves a transition between the worlds of the mathematical and the contextual, but this time from the direction of the mathematical to the real. As such, the *fifth level of Reasoning/Reflection* − which facilitates interpretation and judgement of the appropriateness of the determined solution in relation to the original structure and demands of the problem situation based in reality − is necessitated during this process. It is during this process that questions are posed regarding the reliability of the developed (re)description of reality in relation to the demands and criteria of endorsed legitimate participation in contextual scenario.

Process 6: Validating once again necessitates the *fifth level of Reasoning/Reflection* as the determined solution − if deemed appropriate and valid during the previous process – is again considered alongside other context based situational factors and considerations which may affect decision-making in the real situation. *Reasoning/Reflection at Level 6* is also necessitated to consider the 'baggage' (historical, cultural, economic, political) that the participants themselves bring to bear on the problem situation and whether this 'baggage' has resulted in the construction of a representation of reality that is inconsistent with the structure of existing forms of legitimate and endorsed participation in that reality. As suggested by Skovsmose (1994b),

> A main problem related to the mathematical modelling process has to do with the concealment of pre-understandings. This is the phenomenon of disguising the complexity of the construction of the conceptual system which constitutes the very foundation of the model itself. Therefore, the indication of pre-understandings becomes a major task for reflection. (p. 105)

Process 7: Exposing involves exposing the model, associated solutions and any extra-mathematical considerations back to the broader contextual environment in which the focal event and associated problem situation is located. This process of exposing is crucial for assessing whether the model and associated solutions adequately and validly reflect a structure of participation that is endorsable in relation to forms of participation that characterise the broader environment beyond the limited and deliberately constrained focal event. Once again, *Reasoning/Reflection at Levels 5 and 6* are crucial in this process. *Reasoning/Reflection at Levels 1, 2 and 3* may also be employed during this process, particularly if questions are raised regarding the appropriateness and validity of the methods employed and the consequent need to consider possible alternative methods to generate an alternative and potentially more appropriate (re)description of the segment of reality contained in the problem scenario.

The role of Reasoning and Reflection during Processes 5 and 6 in particular are crucial for a conception of mathematical literacy orientated towards life-preparation, since it is in the move from the mathematical to the reality that a mathematical gaze is commonly imposed on the world and the Myth of Reference dominates. It is the role of Reasoning and Reflection in each of these processes to bring to light the limitations of the mathematical model in representing aspects of reality. Furthermore, Reasoning and Reflection illuminates the limitations of the mathematical methods in reference to other situational and intuitive techniques which may be employed in the reality, and the role of Everyday components and practical intelligence in informing decision-making alongside (or instead of) mathematically based reasoning. It is also during these two processes that Reasoning and Reflection ***must*** necessitate movement back into the reality and out of the mathematical. It is during these two processes that Reasoning and Reflection facilitates the complementarity of the Mathematical Competency and Everyday domains of practice, but, by allowing a voice for both components, ensures that it is not only mathematical concerns that are prioritised. Access to the terrains of both worlds is granted, with contextual sense-making practices facilitated.

So far the discussion has excluded reference to the seventh level of Reasoning/Reflection in connection to the knowledge domain of mathematical literacy. This level − together with the sixth level of Reasoning/Reflection − has direct implications for facilitating access to a form of cumulative learning process for engagement with the contents of the knowledge domain. This level also facilitates cumulative learning in the type of practices and behaviour that are envisioned will result through participation in a format of the subject-matter domain of Mathematical Literacy that draws its esoteric domain of practice from this knowledge domain. This discussion is dealt with in the next section.

14.4.5.5 The role of the Reasoning and Reflection domain of practice for facilitating a cumulative learning process

The internal language of description presented in this study promotes a structure of knowledge for the knowledge domain of mathematical literacy formulated around a dominant goal for engagement with authentic real-life contexts and problems encountered in those contexts in promotion of a life-preparedness orientation. This orientation is characterised by a dominant agenda for contextual sense-making practices and dominant intention for the critical evaluation of structures encountered in those practices. However, and as has already been alluded to, when viewed through the lens of Maton's (2009) work on segmented and cumulative learning this life-preparedness goal presents a challenge. Namely, the knowledge associated with the domain of mathematical literacy is primarily based in a segmented learning framework (i.e. contextually based, localised practices and knowledge) where knowledge is legitimised through a knower code. As such, is it really possible to claim that participation in a format of the subject that defines the esoteric domain contents of the subject according to the contents of this knowledge domain, prepares learners for the world outside of the classroom or for the world beyond those contexts directly dealt with in the classroom? Surely this requires a process of cumulative learning, accompanied by the development of hierarchical and generalisable knowledge − which is in apparent contradiction with the knowledge base of the authentic problem-solving practices associated with mathematical literacy? But maybe the question is what it is that is being decontextualized or generalised in mathematical literacy?

I have already argued that participation in the practices of the Everyday and the Mathematical Competency domains of practice are characterised by elements of cumulative learning (c.f. sub-sections 14.4.2.4 and 14.4.3.3 on pages 212 and 75 above respectively), albeit with the former directed towards engagement with primarily contextual elements and the latter with mathematical structures. However, the reverse situation is true with respect to the practices associated with the Modelling domain, and a form of segmented learning clearly characterises the envisioned modelling process. Namely, the construction of different models to represent various (separate) authentic situations based in real-world practices, together with consideration of various localised and highly situational techniques, considerations and factors which may influence decision-making, illustrates an instance of segmented learning. Each model constructed, and the integration of the Everyday and Mathematical Competency components in the construction and analysis of the model, provides a piece-meal, apportioned and/or segmented view of reality. When models of different segments of reality are developed, there are commonly no connections made between these models and between the understanding or perspectives of reality generated through and by the models. Furthermore, knowledge developed in one modelling context is commonly not seen to have application in a different situation. In essence, then, deployment of Reasoning/Reflection Levels 1 to 4 in the modelling process does not extend participants beyond the bounds of specific, localised and contextually based models and, as such, does not provide an avenue for cumulative learning. I contend, however, that it is through engagement in and/or at the fifth and, primarily, the sixth and seventh levels of Reasoning and Reflection that a move towards cumulative learning is achieved. For this reason, incorporation of these levels is crucial in the pedagogic practices of the subject-matter domain of Mathematical Literacy (and in any practice associated with the development of mathematically literate behaviour in general) if the goal of a life-preparedness orientation is to be facilitated.

Starting with the seventh level of Reasoning and Reflection – namely, reflecting on the way in which various forms of reasoning/reflection have been employed throughout the modelling process. It is this level of reflection that provides a gaze over the whole modelling cycle and which facilitates not only analysis of the individual processes in the modelling cycle but also enables the gaze to be cast beyond a single model to look for similarities and differences between and across models. It is this form of reflection that facilitates the adaptation of existing models to suit changing conditions in the reality, or the construction of new and more representative or efficient models to describe the situation. It is this form of reflection that facilitates recognition of the need for additional models that stand alongside existing models to provide an alternative view of legitimate forms of participation in the situation or an additional view of a different component of the situation. It is this form of reflection that enables participants to make connections between different models of the same situation or even different models across contexts. In doing so, this form of reasoning/reflection moves the structure of legitimate knowledge from a knower code based in a localised situation to a knowledge code involving techniques, contents and knowledge embedded across a range of contexts. It is this form of reflection that enables participants to see the similarities and differences between the contents, techniques and knowledge employed in different contexts and across different models. In short, it is this form of reflection that enables a gaze to be cast beyond the boundaries of individual segmented contexts towards a more connected, integrated view of reality and, so, brings participants closer to preparation for more effective and empowered functioning in life.

The sixth level of reflection involves reflecting on the broader consequences of the way in which the modelling process reconstructs and (re)presents reality, and the values and assumptions underpinning this reconstruction and (re)presentation. As with the seventh level, this level requires a movement beyond a single model to consider the position of this model in relation to the broader world environment and to other similar (or different) models which may already be in existence. This level of reflection acknowledges that analysis of a focal event provides a limited and limiting perspective of the broader contextual environment in which the event is embedded. This level of reflection recognises that multiple models are required for a more complete perspective of a contextual environment and facilitates the search for connections between models constructed for different focal events within the same contextual environment to generate a more complete picture of the environment. And, so, in a way similar to building a puzzle, this level of reasoning seeks to build a more complete perspective and understanding of a contextual environment through accumulation of models constructed for isolated and individual focal events. This form of reflection also requires knowledge of not only contextual factors – including economic, political and social − but also of a vast array of mathematical contents, techniques and skills that may underpin the models developed within these contexts, as well as experience with more than one model. This form of reflection requires an integrated understanding of the role of a model in a broader environment and the ability to assess the underlying assumptions, ideals and values that underpin the model. This requires knowledge that extends beyond a localised, contextually based knower code to knowledge that transcends isolated contexts and models – namely, a knowledge code. In short, this level of reasoning combines models in a cumulative way, seeking connections and disconnections in an attempt to construct a more realistic and complete perspective and understanding of a segment of reality.

In summary, participation with the different components of the knowledge domain of mathematical literacy through the modelling process runs the risk of engaging participants in the development of localised, contextually based knowledge that has no or little application beyond the immediate context(s) in which it is encountered and utilised. This signifies the potential for a form of segmented learning with legitimate knowledge positioned at the level of the knower code. This is problematic for a subject that promotes and prioritises a life-preparation orientation in which the situations explored in the classroom *must*, ultimately, provide preparation for life outside of the classroom. In the discussion above I have argued that it is through engagement with the sixth and seventh levels of Reasoning and Reflection that a movement beyond the segmented learning process and beyond localised and segmented understandings is achieved. Reasoning and Reflection at the sixth and seventh levels, thus, facilitates a move from the legitimisation of participation (in the subject or in any practice associated with the knowledge domain of mathematical literacy) from the knower to a knowledge code.

14.5 The importance of 'moving and shaking' through the knowledge domain

As a final comment, it is essential to understand that the life-preparedness orientation envisioned for the knowledge domain of mathematical literacy is facilitated through movement through **ALL** of the domains of practice that characterise the knowledge domain. In alternative terms, what characterises mathematical literacy as a distinctive knowledge domain is *all* of the domains of practice that characterise the domain; and, as such, legitimate participation in the knowledge domain is determined by the ability successfully navigate all of the domains and to move between the domains as determined by the demands of a contextual sense-making practice. And, as discussed in Section 14.2 on page 189 above, successful apprenticeship in the domain of mathematical literacy relies on movement through the Public Domain of recontextualised real-world contexts (i.e. the Contextual Domain of the knowledge domain), through all of the components of the Esoteric Domain (comprised of Everyday, Mathematical Competency, Modelling and Reflective Knowing sub-domains) to facilitate sense-making of the components of the Contextual Domain, and then back to the real-world beyond the boundaries of the classroom to engage in enhanced real-world functioning. Apprenticeship in the knowledge domain of mathematical literacy involves the development of the capacity for enhanced real-world functioning, and this capacity is facilitated through pedagogic processes that promote life-preparation through exposure to the Contextual, Everyday, Mathematical Competency, Modelling and Reasoning and Reflection domains of practice.

In the form of the knowledge domain of mathematical literacy as described in the internal language of description, mathematically literate behaviour, then, is characterised by the following components. Namely, the ability to employ modelling processes to generate descriptions of authentic real-world practices, to consider both everyday and mathematical knowledge, understandings, structures and contents, and to employ various forms of critical reasoning and reflection on problem-solving processes, all in service to the development of more comprehensive and enhanced contextual sense-making and real-world functioning.

In this regard, it is envisioned that classroom-based problem-solving activities that reflect the structure of participation associated with the knowledge domain of mathematical literacy will be characterised by a variety of questions or tasks that move participants through the different domains of knowledge to facilitate enhanced understanding of a contextual situation. Engagement in the practices of the Everyday domain will facilitate understanding of the contextual elements of the problem-scenarios. Engagement with mathematical elements of the contexts or resources will be facilitated through the practices of the Mathematical Competency domain. While Modelling domain related practices will facilitate elaborated investigation of more complex aspect of the contextual environments. Finally, Reasoning and Reflection will ensure an integrated and critical problem-solving process.

Importantly, however, this expectation for engagement with all of the domains of practice does not denounce the potential for engagement with only particular domains at specific points in a contextual sense-making practice or in a pedagogic process. It would be ludicrous to suggest that teachers in the subject-matter domain of Mathematical Literacy must engage in modelling practices day-in and day-out, or that every activity prepared by a teacher must involve forms of engagement that characterise practices and forms of participation associated with each domain of practice. Instead, teachers will make deliberate pedagogic choices to engage different situations through particular participation structures characteristic of the different domains of practice depending on the demands of the contextual situations under investigation, the specific area of the curriculum being dealt with, and the skills prioritised at a particular stage in the pedagogic process. However, the crucial and key point is that it is problematic – and the knowledge domain of mathematical literacy is misunderstood and misrepresented – when pedagogic practices and participation remain predominantly or exclusively in only one or some of the available domains of practice. Thus, while teachers may choose to deliberately focus on a particular dimension of the knowledge domain of mathematical literacy at a particular point in time, this decision must be accompanied by recognition and realisation of the need to ensure engagement with the other domains during other components of pedagogic practice to ensure more complete and comprehensive contextual sense-making. From the perspective of the internal language of description of the knowledge domain of mathematical literacy presented in this study, comprehensive and critical sense-making and associated life-preparation is facilitated through engagement in all of the domains of practice. Participation in only some of the domains, by contrast, is seen to lead to limited or restricted understanding and stunted life-preparation.

CHAPTER 15
LOCATING THE KNOWLEDGE DOMAIN OF MATHEMATICAL LITERACY

Consider the diagram shown in Figure 31.

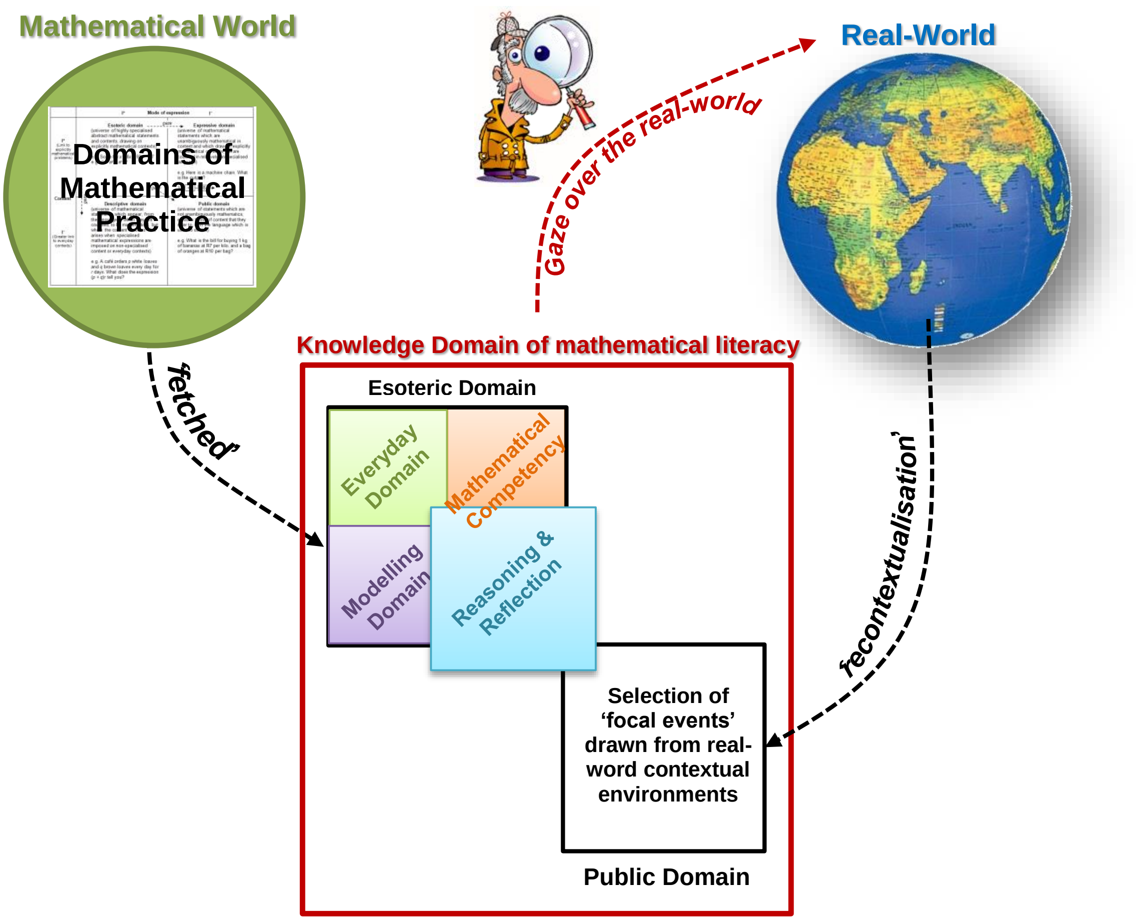
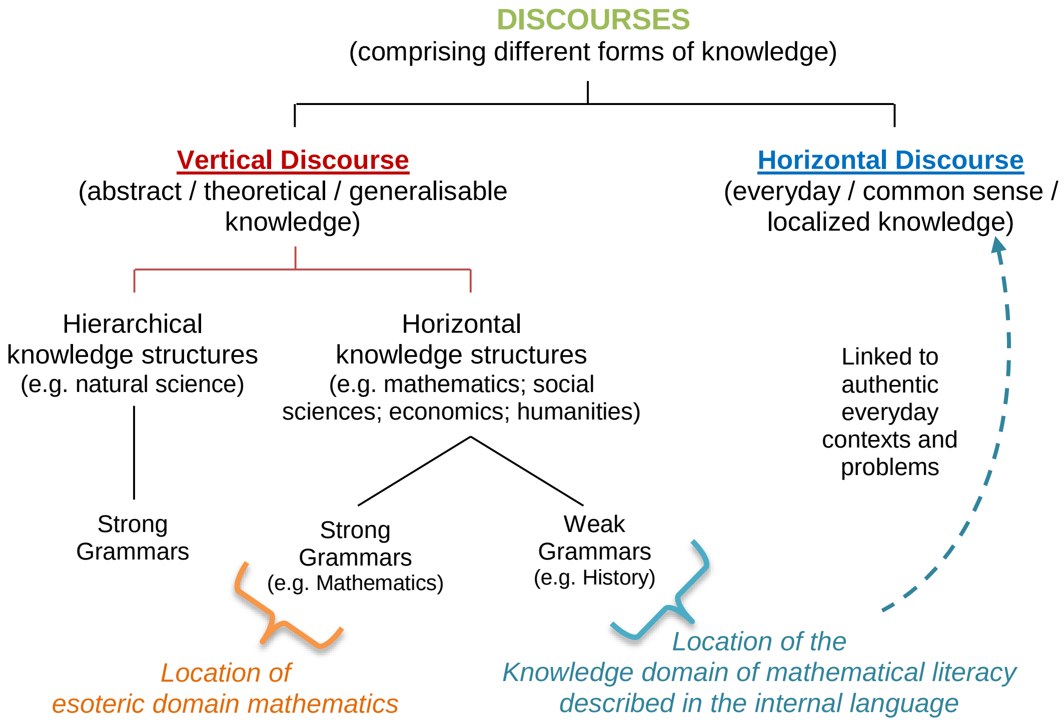


Figure 31: The knowledge domain of mathematical literacy in relation to the mathematical and the real-world terrains

The diagram illustrates that a format of the knowledge domain of mathematical literacy characterised by the prioritisation of a life-preparedness orientation exists outside of the exclusive territory of the mathematical world. This is ensured through a dominant agenda for contextual sense-making practices that includes consideration of everyday forms of knowledge, practices, techniques, participation and communication (facilitated through engagement in the Everyday domain of practice). Equally, the knowledge domain is not located exclusively in the real-world, since – as has been argued above – the contextual situations and focal events dealt with in the domain represent apportioned, limited and piece-meal representations of complex, intricate and vast real-world practices – namely, the Public (Contextual) Domain of practice. Despite a degree of deliberate separation from the exclusive terrain of the mathematical world, the knowledge domain draws on elements of content, routines, knowledge and forms of participation from the mathematical world to facilitate a level of mathematical engagement with the contents of the Contextual Domain. This is achieved through engagement in the Mathematical Competency and Modelling domains of practice. Crucially, however, contents are 'fetched' from the mathematical terrain to facilitate sense-making practices in contextual environments. Mathematical elements are not fetched to facilitate the development of mathematical knowledge, and the contents to be fetched are determined by the demands of the contextual problem situation and not by a prescribed mathematical curriculum or pedagogic sequence. In short, the direction of movement is always from the contextual to the mathematical and back to the contextual, such that apprenticeship in the domain is characterised by enhanced real-world functioning and by the ability to discern the role (and the when and the where) of mathematical and contextual forms of participation in facilitating this enhanced functioning.

The knowledge domain of mathematical literacy is, thus, positioned outside of the mathematical world and also somewhat outside of the real-world (albeit to a lesser degree). It is in direct reference to this perspective that I continuously make the claim that the practices associated with the knowledge domain of mathematical literacy are positioned outside of the exclusive terrain of the mathematical world. It is for this reason that these practices present a viable alternative to the problematic mathematically mythologised practices (theorised through Dowling's (1998) language) which currently characterise participation in the subject Mathematical Literacy. Furthermore, the positioning of the knowledge domain outside of the real-world also ensures that the practices of the domain are not entirely segmented and context dependent and cannot be judged purely from the perspective of everyday understandings. Instead, the domain of mathematical literacy is classified as a form of disciplinary knowledge that can be taught, developed, and transferred beyond the immediate site of pedagogic engagement. This disciplinary knowledge is comprised of two dimensions – a mathematical dimension and a contextual equivalent, each of which is essential for successful and endorsed participation with the contents of the domain.

As discussed briefly in sub-section 10.1.1.2 on page 150 above (and illustrated in Figure 22 on page 151 – repeated below), this distinction between the knowledge domain of mathematical literacy and the mathematical and real-world terrains can also be represented in relation to Bernstein's (1999) work on vertical and horizontal discourses.



The domain of esoteric mathematics – the specific site of analysis of Dowling's theoretical language – is characterised by vertical forms of discourse, an associated hierarchical knowledge structure, with strong grammaticality and high levels of discursive saturation (DS+). The knowledge domain of mathematical literacy, by contrast, is characterised by elements of both vertical and horizontal discourse and traverses a space between academic/abstract/theoretical/generalisable and everyday/common-sense/localised knowledge. The disciplinary elements of the knowledge domain of mathematical literacy primarily reflect characteristics of a horizontal knowledge structure with weak grammaticality and low levels of discursive saturation (DS^{-}). However, when mathematical contents are appropriated for use in contextual sense-making practices, then such practices reflect a combination of strong and weak grammatically as both mathematical and contextual elements are engaged in a complementary way. These disciplinary elements are further accompanied by a deliberate and explicit connection to elements characterised by horizontal forms of discourse (specifically, everyday forms of understanding, knowledge and participation).

WHERE TO FROM HERE

This, then, concludes the discussion of the components of the knowledge domain of mathematical literacy characterised by an orientation for life-preparedness. However, the language of description of the knowledge domain of mathematical literacy presented in this part of the study is characterised entirely by theoretical constructs. As such, in order for a gaze to be cast from this theoretical language over empirical practices that drawn on the knowledge domain, it has been necessary to develop an external language of description characterised by an explicit framework that demonstrates the operationalisation of the theoretical constructs in relation to empirical practices. This external language is presented in Part 5 of the study.

PART 5
THEORISING AND OPERATIONALISING AN EXTERNAL DIMENSION FOR THE DEVELOPED LANGUAGE OF DESCRIPTION

INTRODUCTION, OVERVIEW AND RATIONALE

In the previous part of the study I presented a theoretical language for describing the structure of knowledge and practices associated with a format of the knowledge domain of mathematical literacy in which a life-preparedness orientation is prioritised. This internal language of description presented the knowledge domain of mathematical literacy as comprising of four Esoteric domains of practice. Namely, the Everyday, Mathematical Competency, Modelling, and Reasoning and Reflection domains. The collective of these domains that facilitates engagement with apportioned and recontextualised versions of real-world practices (the collection of which is constituted as the Public Domain of the knowledge domain of mathematical literacy and is referenced in the internal language of description as the Contextual Domain). Importantly, the presented language only comprises the internal and/or theoretical dimensions of the language of description and remains entirely in the realm of the theoretical. The presented internal language does not include a clear, explicit, or validated connection (or means of connection) to an empirical space. As such, the intention for this part of the study is to present an external language of description (c.f. Bernstein, 2000) to facilitate the casting of a gaze from the internal language (the theory) over empirical phenomena related to the specific empirical site of the subject-matter domain of Mathematical Literacy to provide descriptions and analysis of those phenomena.[170] In essence, then, the primary motivation behind the development of an external language of description is to operationalise the components of the internal language. This is to see if the theory can in fact provide a valid reading, analysis and differentiation of empirical phenomenon relating to the subject-matter domain of Mathematical Literacy. Of lesser concern is what the theory is able to tell us about those phenomena.

A key intention for this external language of description is to provide a means for distinguishing – theoretically and from a research-informed perspective – practices within the subject-matter domain of Mathematical Literacy that promote a life-preparedness orientation from those that promote a different (perhaps primarily mathematical) orientation. One way to achieve this is by identifying and analysing the nature and structure of knowledge and legitimate participation that characterises a version of the subject-matter domain of Mathematical Literacy that draws on the format of the knowledge domain of mathematical literacy in which a life-preparedness orientation is prioritised. This process is facilitated through a methodology for making visible the *Dominant Domain of Prioritising* – Everyday, Mathematical Competency, Modelling,

[170] See Straehler-Pohl and Gellert (2013) for an example of a similar attempt to develop an external language of description that is consistent with particular theoretical constructs and which can be operationalised in an empirical setting. Straehler-Pohl and Gellert's attempt involves the development of an external language of description that draws on, is consistent with, and operationalises Bernstein's (1996) conceptualisation of classification (as an indicator of power relations) in order to more closely analyse and render visible structural elements of mathematics classroom discourse. For (2013), operationalisation of the notion of classification through this developed external language of description provides a means for furthering investigation into why particular groups of learners are either not given access to or engage unsuccessfully with the contents of school-based mathematical knowledge.

Reasoning and Reflection, or a combination of these, or something else entirely − within a particular component/resource/practice relating to the subject. In alternative terms, the external language facilitates analysis of the characteristics and/or structure of knowledge and legitimate participation in an activity by immediately bringing into question which domain of practice is prioritised through the promotion and legitimisation of a particular practice or segment of knowledge in and/or through the activity.

To achieve this intention, a two-fold approach is adopted in this part of the study: firstly, the external language is theorised and the contents and structure of the language are presented; secondly, the external language is operationalised in reference to an empirical task drawn from the subject-matter domain of Mathematical Literacy.

Theorisation of the external language involves utilisation of the works of Sfard (2007, 2008) and Bansilal (2013) to describe the specific nature of the tools and discursive resources – comprising signifiers (words/vocabulary and visual mediators), routines and endorsed narratives − that characterise the structure of legitimate participation in a contextually-oriented knowledge domain such as mathematical literacy. In short, I argue that the practices of the knowledge domain of mathematical literacy are discursively mediated and that this domain is characterised by unique contextual discursive resources (including contextual signifiers [words and visual mediators], routines, reasoning, and endorsed narratives) that distinguishes it from discursively-mediated practices in other domains (such as mathematics). As such, a focus on the discursive resources employed in a practice or activity, and analysis of the nature of the signifiers (words and visual mediators) and routines, together with the structure of the narratives that are endorsed, provides a means for identifying more clearly and explicitly the dominant domain of practice prioritised in an activity. Furthermore, this process also provides a means for differentiating and distinguishing practices and segments of knowledge which, on the surface, may appear to promote identical or similar goals.

Operationalisation of the external language occurs on two levels. At the first level, the contextually domain-specific discursive resources identified in the theorising phase of the external language are used to analyse and differentiate the three domains of practice – Everyday, Mathematical Competency and Modelling − of the knowledge domain of mathematical literacy. The role of Reasoning and Reflection in facilitating the generation of endorsed narratives in each domain is also explored. This processes results in the development of an explicit framework for identifying and differentiating practices associated with each of the four domains of practice and, hence, of the characteristics of the structure of knowledge and endorsed forms of participation associated with the practices of each domain. At the second level of analysis, the same framework is then employed to identify and analyse the nature and structure of the discursive resources (words/vocabulary, visual mediators, routines and narratives) in the questions in an exemplar text-based task drawn from the terrain of the subject-matter domain of Mathematical Literacy. Based on the characteristics of the discursive resources identified for each question, the dominant domain of practice and associated forms of knowledge and participation prioritised in each question (and also the activity as a whole) are identified. This process facilitates correlation and categorisation of each question (and also the activity as a whole) according to the domains of practice that characterise the knowledge domain of mathematical literacy. I conclude this discussion by arguing that the presented framework and operationalisation process demonstrate the utility and validity of the framework for externalising the internal components of the language of description, and, hence, for facilitating analysis of empirical phenomenon through the gaze or lens of the theoretical internal language.

The following structure applies to discussion in this part of the study. In Chapter 16 I revisit the discussion on internal and external languages of description to clarify the need for an external dimension – most particularly for facilitating the use of the theoretical internal dimension of the language in analysis of empirical resources. In Chapter 17 I then describe and theorise the components of the external dimension. This is achieved largely in reference to Sfard's (2008) work on the characteristics of a discourse, and I argue that the practices associated with the knowledge domain of mathematical literacy are discursively mediated and are characterised by uniquely identifiable discursive resources (including words/vocabulary, visual mediators, routines and endorsed narratives). These unique discursive resources differentiate the knowledge domain and its associated practices from other knowledge domains. In Chapter 18 I operationalise these theoretical components of the external dimension of the language of description in two ways. Firstly, in relation to the components of the internal language: namely, I identify and differentiate the discursive resources that characterise each of the domains of practice that comprise the knowledge domain of mathematical literacy. Secondly, in relation to an empirical activity: namely, I identify the discursive resources in the questions in a specially-designed exemplar task and then compare and differentiate the questions in relation to the dominant domain of practice through which participation in each question is legitimised. This level of operationalisation demonstrates the utility of the external dimension for facilitating analysis of empirical practices through the lens of the theoretical components of the internal dimension of the language of description. I conclude this chapter by reflecting on challenges faced in the operationalisation process and possible consequent limitations of the external dimension of the language of description.

Figure 32 and Figure 33 (on pages 256 and 257 respectively) are provided to orient the reader to the contents and structure of this part of the study.

As a final introductory comment, it is essential to note that the operationalisation of the external dimension of the language of description of facilitates a shift in the level of analysis. Namely, the discussion of the theoretical internal dimension of the language in Part 4 operates at the level of knowledge production or development (i.e. the field of knowledge production (c.f. Bernstein, 1996)) and references theoretical constructs relating to terrain of the *knowledge domain of mathematical literacy*. The first level of operationalisation of the external dimension of the language in relation to the discursive resources that characterise forms of participation associated with each of the domains of practice that comprise the knowledge domain of mathematical literacy (c.f. Chapter 18 and sub-section 18.1 on page 295 below) remains at this level of knowledge production. However, the second level of operationalisation reflects a shift in the level of analysis from the level of knowledge production to the level of pedagogic interaction (i.e. the field of pedagogic recontextualisation (c.f. Bernstein, 1996)). This level involves analysis of the discursive resources (and consequent dominant domains of prioritising) in the questions in an empirical activity relating to the subject-matter domain of Mathematical Literacy (c.f. Chapter 18 and section 18.2 on page 313 below). This shift is necessary and appropriate since it reflects the role of the external dimension of the language of description in facilitating a connection between empirical practices and the domain of knowledge (as described in the internal dimension of the language) on which those practices are based.

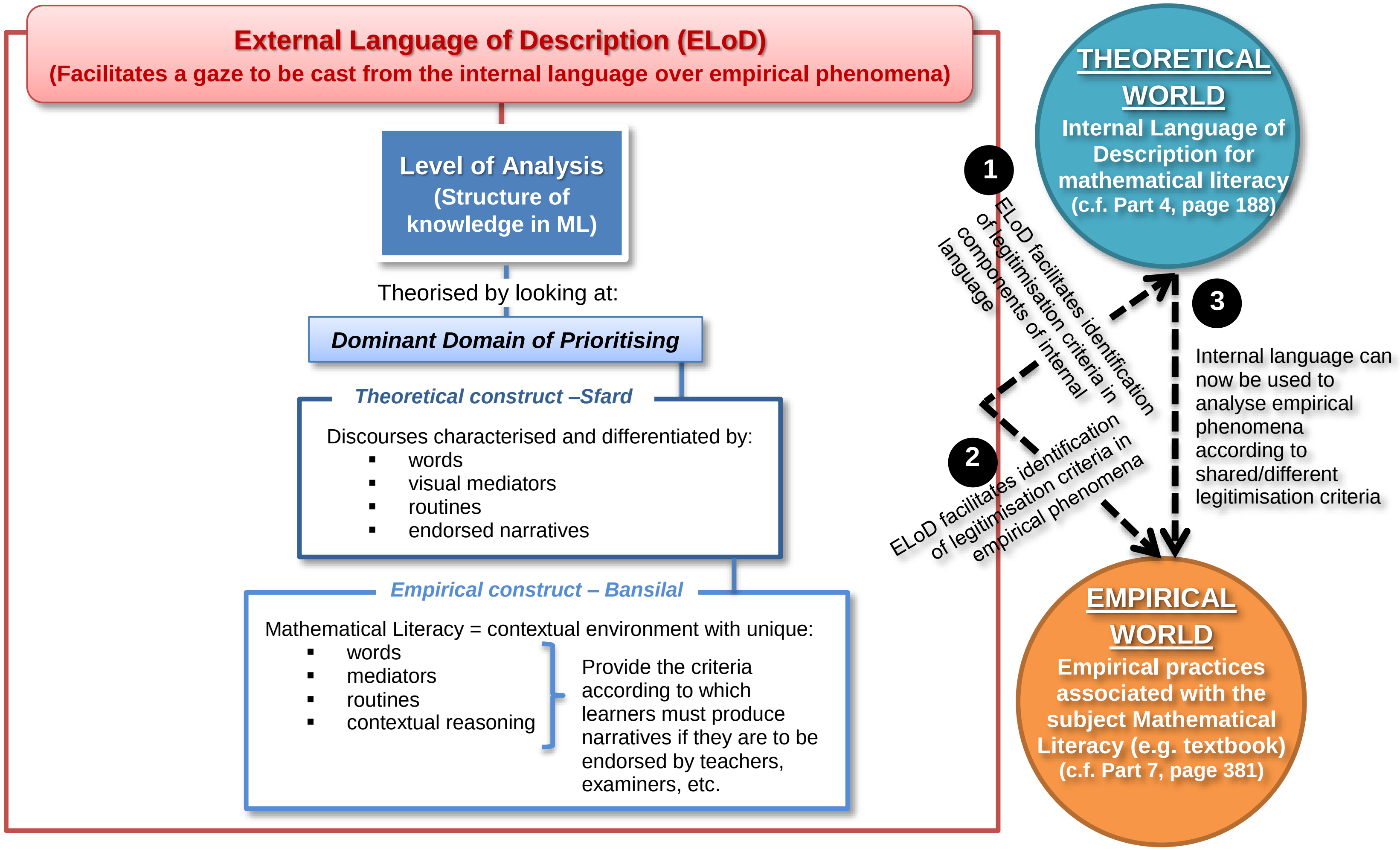


Figure 32: Overview of the contents of Part 5 of the study

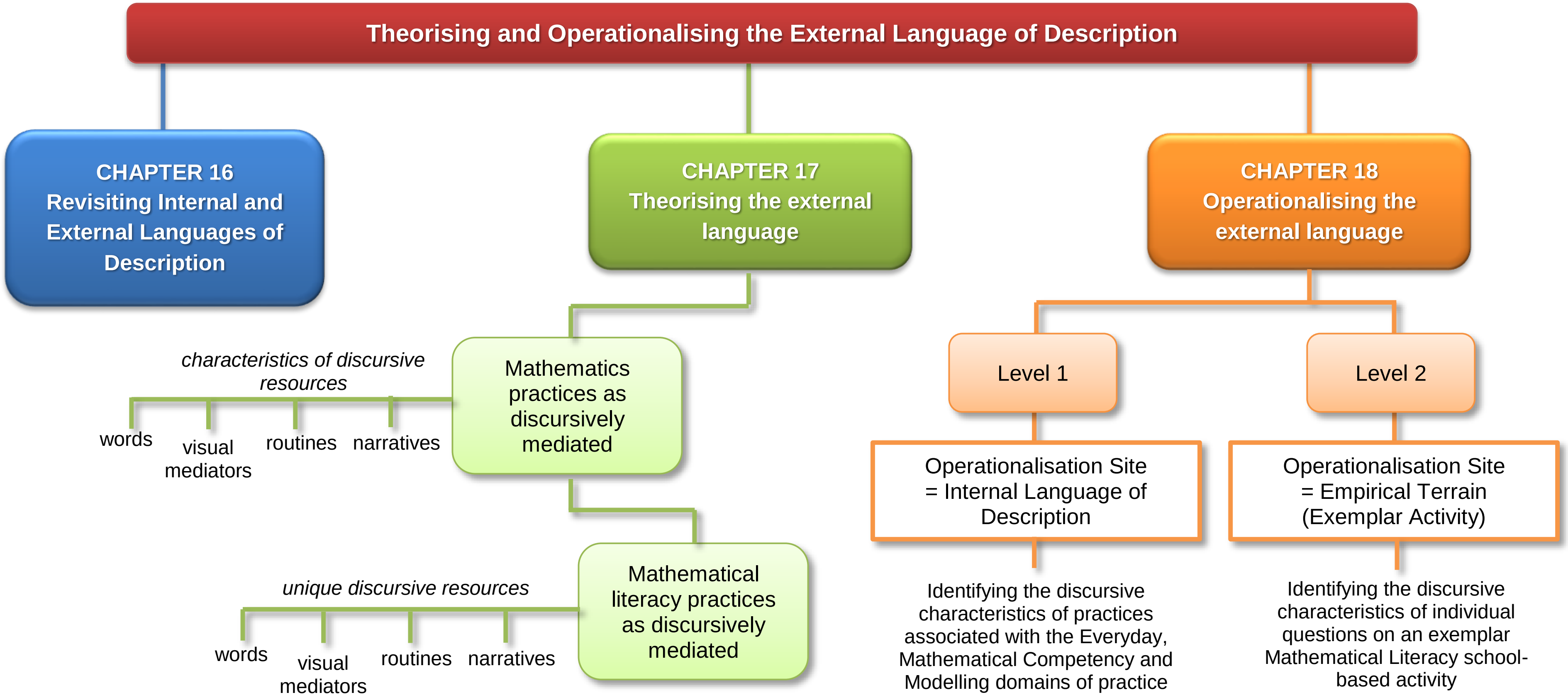


Figure 33: Chapter structure of Part 5 of the study

CHAPTER 16
REVISITING INTERNAL AND EXTERNAL LANGUAGES OF DESCRIPTION

As discussed previously (c.f. Part 1 and Chapter 2, starting on page 16), a language of description is a means for describing something or for translating or transforming one language into another (Bernstein, 2000, p. 132). In distinguishing between internal and external components of a language of description, the internal language refers to the concepts, syntax, relations and components that comprise a theoretical description or language – namely, that which is internal to the language; while the external language "describes something other than itself" (Muller, 2012, p. 11). This internal language represents the means through which a gaze is able to be cast from the internal language to provide a description of an empirical space (Bernstein, 2000, p. 132) (the 'relations to' component of the language) (Bernstein, 1999, p. 170). In combination, the internal and external languages, thus,

> describe the track from the internal symbol structure – the conceptual pile – to the concrete case, the empirical instance, to an instantiation of *context*. (Muller, 2012, p. 11, emphasis in original text)

Importantly, the relationship between internal and external languages of description is dialogic in nature (Dowling, 1998, p. 124): the internal language of description directs the structure and syntax of the external language of description; inversely, empirical analysis through the external language (re)informs the composition of the internal language to facilitate a more accurate and precise description of the empirical (Jablonka & Bergsten, 2010, p. 39; Morais, 2002, p. 564).[171] Furthermore, as suggested by Muller (2012, p. 17) (citing Bernstein (2000, p. 170), it is the combination of the internal and external languages that provides a more holistic, detailed and richer picture of an analysed space:

> The analysis … reveals the inter-dependence between properties internal to the discourse and the social context, field/arena, in which they are enacted and constituted. Briefly, 'relations within' and 'relations to' should be integrated in the analysis. (Bernstein, 1999, p. 170)

The internal dimension − of the language of description of the structure of knowledge associated with a format of the knowledge domain of mathematical literacy in which a life-preparedness orientation is prioritised − was presented in Part 4 of the study. My primary intention in this part of this study, then, is to present and operationalise an external dimension to facilitate a gaze to be cast from the presented internal language to provide a description, reading and analysis of empirical phenomenon comprising various

[171] I can certainly attest to an experience of the dialogic relationship between the internal and external languages of description while developing the language of description for the knowledge domain of mathematical literacy in this study. The internal language was developed first and in isolation of the external language. The external language was then developed to facilitate the use of the components of the internal language for analysing and describing empirical phenomenon. In this way, the internal language directly affected the structure of the external language. However, while developing and employing the external language in relation to empirical resources, it became obvious that revision of the internal language was needed to facilitate engagement with the empirical phenomenon in a more comprehensive and accurate way. In this way, attempted utilisation of the external language directly re-informed a revision of the components of the internal language.

textual resources relating to the subject Mathematical Literacy. The combination of these external and internal components, thus, constitutes a complete, coherent and consistent language of description. Specifically, this language provides a description of the structure of knowledge that characterises a format of the knowledge domain of mathematical literacy in which a life-preparedness orientation is prioritised. The language further describes the structure of participation that is endorsed in a form of the subject-matter domain of Mathematical Literacy that draws on this knowledge domain and orientation as the source of legitimate knowledge and practice.

To facilitate this interplay of the internal and external components of the language of description I draw on the notion of 'legitimisation' - namely, the means through which knowledge and practices are legitimised and/or endorsed in a domain. To achieve this end I identify, firstly (in this part of the study), the legitimisation criteria of knowledge and participation in each components of the internal language. Secondly (in Part 7), I employ the same technique to identify the dominant legitimisation criteria employed in various textual resources − a curriculum extract, a textbook chapter, national assessments, and course notes for a teacher education course − related to the empirical terrain of the subject-matter domain of Mathematical Literacy. Employing the same technique to determine the criteria according to which knowledge and participation are legitimised in both the internal language and an empirical space facilitates analysis of the empirical space through the lens and gaze of the components of the internal language. The external language described and operationalised in the pages below makes explicit how the legitimisation criteria are to be identified.

To facilitate the development of this external language of description I draw primarily on Anna Sfard's (1991, 2001, 2007) work on *characteristics of a discourse* and argue that Sfard's work provides a means for identifying differences in the characteristics of the knowledge and practices that characterise an activity in an empirical space. I also draw on Bansilal's (2011, 2013) work on *contextual discursive resources characterising contextual discourses* as a means of operationalising Sfard's theory in the specific terrain of the subject-matter domain of Mathematical Literacy. The mind-map shown in Figure 34 illustrates a summary of this intended process.

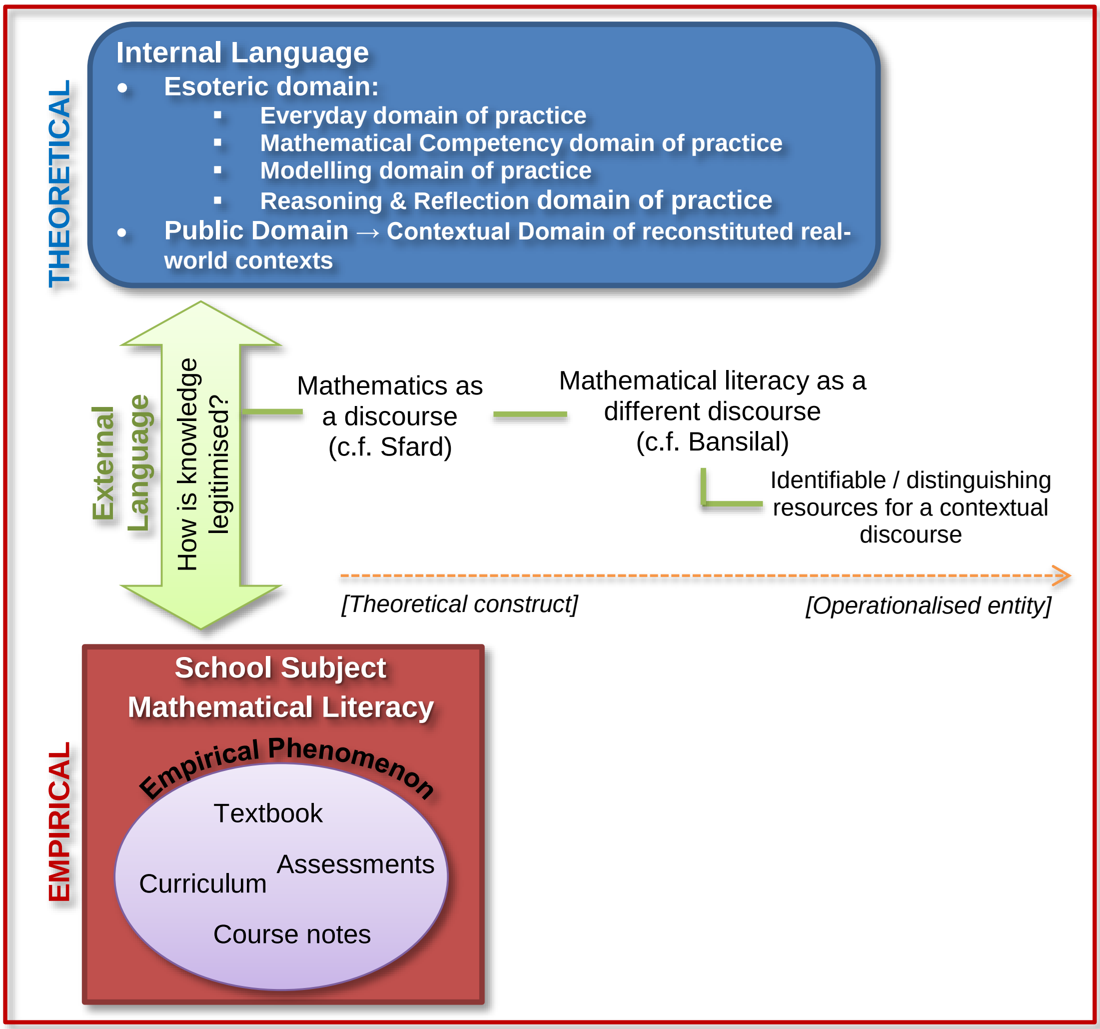


Figure 34: Relationship between the internal/external languages and empirical phenomenon for this study

Importantly, the external language of description enables an analysis of empirical activities at the level of the *structure of knowledge* that characterises the activities. The external language does not make provision for identification and analysis of the criteria according to which the participants who engage with the officially legitimised and endorsed knowledge further (re)legitimise and (re)endorse that knowledge or a modified version of that knowledge.

This further level of analysis is beyond the scope of this study.[172]

A final note is necessary, specifically related to my intention to steer away from utilisation of Dowling's (1998) work in the development of an external language. This is particularly pertinent given how instrumental Dowling's theory has been in providing a framework for problematising current practices in the subject-matter domain of Mathematical Literacy and in providing the impetus for the development of an alternative language of description for the structure of legitimate knowledge in the knowledge domain of mathematical literacy. Dowling's theory proves appropriate for problematising current practices in the subject because current practices position the subject primarily within a *mathematical* frame – which is the domain in which Dowling's theory is grounded. The internal language of description for the knowledge domain of mathematical literacy developed in Part 4 in this study facilitates a shift in the structure of legitimate knowledge outside of an exclusively mathematical frame and one in which the dominant terrain involves a life-preparedness orientation. And, since the esoteric domain for mathematical literacy now comprises something other than mathematics and, as such, Dowling's theory no longer provides a legitimate language or describing practices associated with this revised esoteric domain. This being the case, an alternative means is necessary for identifying the generative principles of the structure of knowledge and participation that characterise the varying domains of practice which constitute the revised esoteric domain for mathematical literacy. And, it is the notion of legitimisation, as embodied in the concept of dominant mode of knowledge prioritising and as evidenced in the specifically and deliberately employed discursive resources that characterise the practices of each domain, which I believe provides a suitable means for this facilitating this intention.

Having clarified and differentiated the scope and intention of the internal and external languages of this study, I move now to further development of the external language.

[172] However, as a preliminary vision for possible future research in this regard, Karl Maton's (2000, 2007, 2009) work on *modes of legitimisation* provides a potential means for theorising the way in which participants (re)legitimise officially legitimised knowledge. In short, Maton (2007, p. 88) employs a theory titled *Legitimation Code Theory* to argue that domains of knowledge and practice comprise not only knowledge but also participants – called 'Knowers' – who influence and shape how that knowledge and associated skills, activities, and practices are legitimised and endorsed. For example, while a curriculum writer may choose to emphasise particular knowledge, a textbook author and examiner may interpret that knowledge differently from each other and also from the curriculum writer and, hence, legitimise that knowledge through prioritisation and endorsement of different variables, constraints and areas of emphasis. For Maton (2007, p. 104), it is, thus, only through consideration of both the structure of the knowledge embedded in a domain of practice and of the influence of the participants who engage with the knowledge that a more complete analysis and understanding of the domain is possible. To facilitate an emphasis on the role of participants in influencing the structure of what is considered legitimate knowledge, Maton characterises two dominant modes of legitimisation – Knowledge Mode (ER+; SR−) and Knower Mode (ER−; SR+). Each of these modes reflect differing emphasis on the means by which knowledge and practices are legitimised, and Maton identifies two primary means – namely, either through an Epistemic Relationship (ER) to the knowledge of the field or through Social Relationship (SR) to other knowers in the field. Identification and analysis of emphasis placed by participants on a relationship to the epistemic or the social, then, provides a means for distinguishing the criteria according to which different participants engage with and (re)legitimise the officially endorsed knowledge in a knowledge domain such as mathematical literacy. Importantly, however, Maton's legitimisation mode characterisations are limited specifically with respect to the lack of detail over how to distinguish between practices that fall within the same mode of legitimisation but which exhibit different claims and criteria to legitimacy of both knowledge and practice. To account for this limitation, Parker and Adler's (2012) concept of 'legitimating appeals' provides a suitable empirically grounded method for identifying specific criteria according to which knowledge and practices are legitimised and, hence, for providing a further level of comparison and differentiation of practices associated with the subject-matter domain of Mathematical Literacy.

CHAPTER 17
THEORISING THE EXTERNAL LANGUAGE OF DESCRIPTION

In this chapter I theorise the external dimension of the language of description of the knowledge domain of mathematical literacy. This external dimension provides the means through which the theoretical internal language is operationalised to facilitate analysis and description of empirical activities through the lens of the theoretical components of the internal language.

The external dimension of the language of description is grounded largely in the work of Anna Sfard (2007, 2008). Drawing on this work I argue that in the same way that the practices of mathematics are discursively mediated, the practices associated with the knowledge domain of mathematical literacy are similarly discursively mediated, albeit through engagement with differently constituted and unique discursive resources (including words/vocabulary, visual mediators, routines and endorsed narratives). I then employ the work of Sarah Bansilal (2013) to identify in an explicit way the discursive resources associated with the practices of the knowledge domain of mathematical literacy. This provides a concrete means for differentiating the knowledge domain of mathematical literacy from other knowledge domains and for establishing the criteria according to which participation in practices associated with the knowledge domain is legitimised. In Chapter 18 (c.f. page 295 below) I employ this same process to identify the characteristics of the discursive resources that define the structure of legitimate participation in each of the domains of practice that constitute this knowledge domain. This process provides a means for distinguishing and differentiating different practices in relation to the dominant domain according to which participation in a practice is legitimised.

A road map of the structure of the contents of this chapter is given in Figure 35 below.

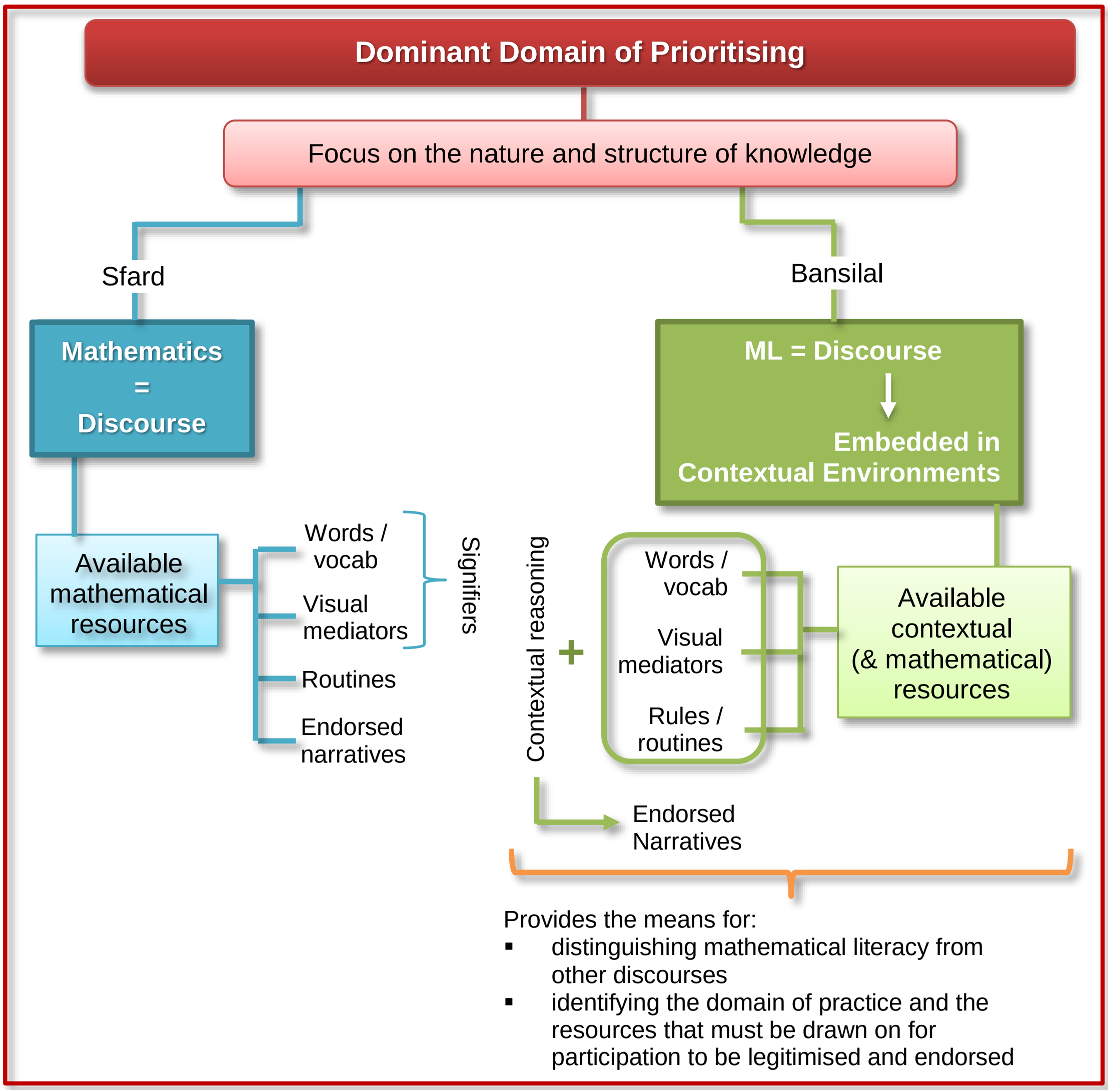


Figure 35: Road Map and overview of the External Language of Description

17.1 Mathematics as characterised by discursively-mediated practices[173]

Anna Sfard (2007, 2008) argues that the domain of mathematics constitutes a discourse[174] such that the practices of the domain are discursively mediated, and, as such, the learning of mathematics involves the gradual initiation into and development and expansion of this discourse by and within[175] the individual (Sfard, 2002, p. 28). From this perspective, learning mathematics does not involve individualised or impersonal acquisition of mathematical concepts, entities and contexts (Sfard, 2007, p. 569); rather, learning mathematics involves a transformation and development in the way in which a person thinks and communicates[176] about mathematics – the process of "changing one's discursive ways in a well-defined manner" (Sfard, 2001, p. 4). Furthermore, this transformation forms part of a process of "collectively performed patterned activity"

[173] Sfard's (2007, 2008) work focuses, generally, on the notion that engagement with the contents of a discipline is tantamount to learning a new and particular discourse. She then uses the more narrowly-defined domain of mathematics to apply and illustrate the more general theory. It is for this reason that the discussion below makes reference to the more general theory as well as to the application of the theory in relation to the domain of mathematics.

[174] Sfard (2007, p. 573) defines a discourse as: "The different types of communication that bring some people together while excluding some others". In other words, discourse is a type of communication characterised by specific actions and reactions (Berger, 2013, p. 2) that are differently rendered permissible according to the specific context in which the communication occurs. From this perspective, any society can be perceived to comprise of partially overlapping communities of discourse that are set apart and are able to be distinguished by the objects on which communication occurs, the types of mediators used in the communicative process, and the rules followed by the participants as they communicate with other members of the discourse (Sfard, 2008, pp. 91, 93 & 129). Applying this perspective to the domain of mathematics suggests that mathematics as a discourse constitutes a particular form of communication comprising uniquely mathematical-based characteristics and structure (i.e. communication that draws on symbols and resources that are uniquely mathematical in nature). Participation (or exclusion) from the discourse is, then, dependent on a participants' ability to access and utilise this mathematical communication and the symbols, notation and resources that make up the communication, and to communicate not only with themselves but also with others who share the same discourse (Sfard, 2007, p. 575).

[175] Deliberate usage is made of the words 'by' and 'within' here to signify that the development of a discourse involves both internal and external transformation in ways of thinking and communicating – both with oneself and with other participants in the discourse.

[176] Sfard argues that thinking and communicating are irrefutably connected, with thinking viewed as a "*special case of the activity of communicating*." (Sfard, 2001, p. 5, emphasis in original text). Specifically, thinking reflects the individualised form of communicating – namely, of communicating with oneself (Sfard, 2002, p. 26). As a consequence, thinking is not seen as a self-contained process that is separate from the process of communication and, rather, represents an act of communication in itself – albeit not of an interpersonal nature (Sfard, 2007, p. 572). To emphasise this dialogic relationship of thinking and communicating, Sfard (2007, p. 572) introduces the term *Commognition* as representative of the processes of cognition and communication (see also (Sfard, 2008, p. 83)). Sfard then goes on to argue that the learning of new concepts and contents are facilitated through the process of 'commognitive conflict' (also described as 'discursive conflict' (Sfard, 2002, p. 48)). Namely, the situation in which an individual encounters a scenario or concept which cannot be adequately or sufficiently described with their existing discourse and, as such, the individual's capacity for communication about that scenario or concept is restricted or, perhaps, impossible. For Sfard (2007, pp. 575-578), these moments of commognitive conflict between an individual's existing discourse and a broader or alternative discourse provide the motivation and need for the expansion of an existing discourse. And, it is this need for expansion of the discourse which prompts learning and expansion or modification of engagement with certain 'meta-discursive rules' (see footnote 179 below) of the discourse.

(Sfard, 2008, p. 86) – namely, participation in conversations with other participants in the discourse.[177 & 178] As summarised by Sfard (2007),

> Learning mathematics may now be defined as the individualizing of mathematical discourse, that is, as the process of becoming able to have mathematics conversations not only with others, but also with oneself. (p. 575)

And, in the localised context of the schooling system, learning in general – and learning mathematics specifically – involves becoming adept in a historically established discourse (Ben-Zvi & Sfard, 2007, p. 119).[179]

Importantly, Brodie and Berger (2010, p. 171) further explain that in Sfard's theory a discursively-mediated activity such as mathematics comprises sub-discourses which relate to each other in different ways – "some are isomorphic, some subsume others, while some are incommensurable" (Brodie & Berger, 2010, p. 171) − and which collectively constitute the discourse as a whole. Endorsed participation in a discourse and/or the development and acquisition of a discourse, thus, entails both the ability to communicate using the referents of the discourse, flexible movement between the sub-discourses, and recognition of how the sub-discourses relate and interact to produce the discourse as a whole (Brodie & Berger, 2010, p. 171).

17.1.1 Resources of a discourse

According to Sfard (Ben-Zvi & Sfard, 2007; Sfard, 2007; Sfard & Lavie, 2005), discourses are made distinct by a variety of interrelated 'discursive or communicational characteristics'[180] (Sfard, 2007, p. 575). These comprise (i) vocabularies

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[177] As suggested by Sfard (2001, p. 4), "*learning is nothing more than a special kind of social interaction aimed at modification of other social interactions*." (emphasis in original text)

[178] In promoting this perspective of patterned collective activity, Sfard (2008, pp. 76-80 & 92) is privileging a 'participationist' view of learning over an 'acquisitionist' view. See also (Sfard, 2002, pp. 14-32) for a comprehensive discussion of the distinction between participationist and acquisitionist views of learning.

[179] Note that Sfard distinguishes between two different levels of learning – object-level and meta-level, and argues that both are necessary for mastery of a discourse (Ben-Zvi & Sfard, 2007, p. 119). *Object-level learning* leads to an expansion in the objects that the participants are exposed to, understand and are able to reproduce in communication: "object level learning leads simply to an extension of the discourse – it increases the set of "known facts" (endorsed narratives) about the investigated objects." (Ben-Zvi & Sfard, 2007, p. 119) (see also Sfard, 2007, p. 575). Meta-level learning, on the other hand, involves a disruption and reconstruction or rejection of existing meta-rules in favour of different or extended meta-rules. As such, meta-level learning results in a transformation of the discourse and the development of a new vocabulary to describe the altered and/or extended discourse (Ben-Zvi & Sfard, 2007, p. 119) (see also Sfard, 2007, pp. 575-576). And it is this learning at the meta-level that is facilitated through *commognitive conflict* (see footnote 176 above).

[180] Different authors use different terms to refer to the four discursive characteristics identified by Sfard. Sfard herself uses the terms 'characteristics' and 'features' interchangeably (see, for example, (Sfard, 2007, p. 575 & 573)), while Brodie and Berger (2010, p. 172) refer to 'elements' and Bansilal (2013) to 'constructs'. Without wanting to further confuse matters, I utilise the term 'resources' when referring to the discursive characteristics, features, elements or constructs of a discourse, and use this term interchangeably with the term 'characteristics'. The primary reason for this is to provide consistency with Greeno's (1991, p. 174) usage of the term 'resources', which he employs while characterising mathematics as a domain and then identifying the structure of the domain as comprising various resources which facilitate the activities of knowing, understanding and reasoning in the domain. In the discussion below (c.f. page 273) I establish a clearer connection between the concepts employed by Sfard and Greeno − hence my intention to preference a term that is consistent to both works.

(words/keywords), (ii) visual mediators, (iii) routines, and (iv) narratives[181]. What distinguishes Mathematics from History is the way in which participants in each of these discourses talk, the objects, symbols, notation and other resources that they utilise in this talk, and the narratives that are endorsed in the discourse. It is, thus, through consideration of the specific discursive resources that discourses can be distinguished and development with and within a discourse can be traced:

> Discursive development of individuals or of entire classes can then be studied by identifying transformations in each of the four discursive characteristics: the use of words characteristic of the discourse, the use of mediators, endorsed narratives, and routines. (Sfard, 2007, p. 575)

Successful engagement with and utilisation of a discourse is then reflected in the ability to identify, locate and make use of these discursive resources in communication with others (and with oneself in the development of thought). For newcomers entering the discourse, endorsed participation involves learning the appropriate use of the discursive resources through and in communication with the other members of the discourse (Bansilal, 2013).

With the above statement in mind, in the pages below I argue that, from the perspective of Sfard's theory, the knowledge domain of mathematical literacy is characterised by a different form and structure of *discourse* to scientific mathematics and, as such, the discursive resources that characterise the discursively-mediated practices in mathematical literacy are differently constituted to those in mathematics. To facilitate this later discussion a detailed description of the structure of each discursive resource or characteristic is now provided.

(i) *Vocabularies (words/keywords)*: "A discourse counts as mathematical if it features mathematical words" (Sfard & Lavie, 2005, p. 245). In other words, successful participation in a discourse involves internalisation and communicative utilisation of *vocabulary* and *keywords* that are (historically) endorsed by the wider community of participants who are engaged in the discourse. And, in the case of the discourse of mathematics, this involves words and vocabulary that have specific and often exclusive[182] meaning in relation to mathematical objects, calculations, techniques and reasoning.

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[181] Sfard (2008, p. 161) categorises *vocabulary* and *visual mediators* as the tools of the discourse and the *routines* and *narratives* as reflecting the substance, form and resulting outcome of the discursive process.

[182] Sfard (2008, pp. 128-129 & 161) argues that mathematical discourse is unique in that the objects of the discourse are almost exclusively intra-mathematical: namely, that many of the objects are created by the discourse itself and, as such, do not exist outside or in isolation of the discourse − for example, the word 'factorise' has no meaning outside of the context of mathematics. Sfard (2008, p. 161), thus, refers to mathematics as an 'autopoietic' system: i.e. a system that produces the things it talks about. This is contrasted with the discourse characteristics of, for example, biology, where the objects of the discourse pre-exist the discourse and exist outside and independently of the discourse. In such instances, the discourse then provides a means for describing the objects, but not as source of creation of the objects. So, where many such discourses are created for the sake of communication about a physical reality, mathematical discursive objects are created primarily for the sake of communication about ... well ... mathematical objects (Sfard, 2008, p. 193).
Sfard (2008, p. 161; see also p.194) argues further that the uniqueness of mathematical discourse is a key factor that contributes to the difficulty faced by many participants in successfully developing, replicating and engaging in and with the discourse. This is because this intra-discourse characteristic leads to a circular process in the development of the discourse. Namely, knowledge of the objects of the discourse is necessary to facilitate engagement with the discourse (because the objects do not exist in the physical reality); but to be aware of the existence of the objects and to have knowledge of those objects means first understanding the discourse.

Examples of mathematics specific vocabulary include words such as ‘factorise’, ‘product’, ‘denominator’ and ‘numerator’, and ‘tangent’. Importantly, certain words used in the mathematical discourse do exist beyond the specific boundaries of the discourse – for example, ‘difference’, or ‘half’ (Berger, 2013, p. 3). However, even these words commonly have different meaning when employed as part of mathematical discourse to the colloquial use of the words. For example, ‘difference’ in mathematical discourse refers primarily to the operation of subtraction, whereas the same term used in everyday language might refer to a range of possible meanings about how two or more items are not the same. Sfard (2008, p. 133) argues that word usage by participants is particularly indicative of the structure of the discourse since word usage reflects the externalised or vocalised form of the internal world-view. In other words, the way in which participants communicate about a discourse reflects their own internal thinking and level of development with or in the discourse.

(ii) *Visual mediators* are visual and visible (Berger, 2013, p. 3) objects or images that participants utilise and operate on as they seek to identify the objects of the discourse and to communicate with each other (Sfard, 2008; Sfard & Lavie, 2005, p. 133 & 245). In relation to mathematical discourse, these mediators include symbols (e.g. x^2), operators (e.g. +, −, ×, ÷), notation (e.g. ‘m’ represents gradient – but can also represent a unit of measurement), graphs (e.g. algebraic and statistical), diagrams and drawings (e.g. drawings for Euclidean Geometry or Analytical Geometry problems), and physical tools (e.g. shapes or counters)[183]. Many of these mediators have been created specifically to facilitate communication in the domain of mathematics and, as such, refer to symbolic entities that only exist within mathematical discourse and do not reference concrete entities that exist outside of the discourse (Sfard, 2008, pp. 133-134 & 148).[184] For Sfard (2008, p. 147), all communication is mediated visually even if visual mediators are not explicitly present in the discourse: “in spite of the famous “intangibility” of mathematical objects, mathematical communication depends on what we see no less than do other, less abstract types of talk.” (Sfard, 2008, p. 146). From this perspective, participants who engage with abstract mathematical entities first attempt to create a mental visual image of a concept to facilitate engagement, operation on and communication about the concept. Visual mediation and the ability to successfully engage with and (re)create the visual mediators of a discourse are, thus, key facets to successful inculcation into and communication with the discourse.

(iii) “*Narrative* is any text, spoken or written, that is framed as a description of objects, or of relations between objects and activities with or by objects, and that is subject to *endorsement* or rejection, that is, to being labelled as *true* or *false*.” (Sfard, 2007, p. 574, emphasis in original text). In the domain of mathematics, narratives include proofs, theorems and definitions, all of which constitute mathematical theories about the nature of and/or relationship between mathematical objects. Importantly, with many of the objects of mathematical discourse being distinctly intra-mathematical, endorsement of a mathematical narrative is subject to the consistency, deductive relations, and validity of the narrative in relation to the internal structure and rules of the discourse. No validation of the coherence of the narrative in providing a description of an object that exists in the

[183] These different forms of mediators can be categorised according to three categories of visual mediators identified by Sfard (2008, p. 162): namely, − iconic mediators (such as graphs and diagrams), symbolic mediators (such as notation), and concrete mediators (such as marbles or counters used for counting).

[184] Following on from the discussion in footnote 182 above (c.f. page 266): visual mediators in many other discourses are able to refer to material or concrete objects that exist in the world and, as such, can either actually be seen, handled or imagined. Visual mediators in the domain of mathematics, by contrast, commonly refer to non-concrete or abstract entities that cannot be physically accessed or handled.

concrete or physical world (i.e. outside of the mathematical discourse) is necessary (Sfard & Lavie, 2005, p. 246). It is also important to note that the site of learning or use of mathematical contents (e.g. a school mathematics course, compared to a university mathematics course, compared to the use of mathematics in an engineering setting) can impact on the rules and/or criteria of endorsement (Berger, 2013, p. 3).

The fourth characteristic of a discourse – (iv) *routines* – is described by Sfard (2007, p. 574) as well-defined repetitive patterns or regularities in the actions of the participants of the discourse that are characteristic of the structure and contents of the discourse. Importantly, specifically in relation to mathematical discourse, routines are not to be equated solely to mathematical procedures. Rather, as suggested by Brodie and Berger (2010, p. 172), mathematical procedures form a sub-set of mathematical routines and the wider set of routines includes categorising mathematical objects, identifying similarities and differences, and justifying, generalising, and/or endorsing and rejecting existing or constructed narratives (Berger, 2013, p. 3).[185 & 186] For this reason, Sfard (2007, p. 574) argues that routines characterise almost every aspect of mathematical discourse and can be noticed through observation of how participants use words and visual mediators, and in how narratives about mathematical objects are created: "thus, the routine is an all-encompassing category that partially overlaps with the three former characteristics (word use, mediator use, and endorsing narratives)" (Sfard, 2007, p. 574). In the context of a discourse, routines are important because it is the regularity and often predictability of people's behaviour that makes it possible for one person to interpret, understand and relate to what another person is saying (Sfard & Lavie, 2005, p. 247). Extending this idea to the domain of mathematics, the inability of a participant to identify and/or construct routines impacts on their ability (or inability) to generate narratives that are endorsed by the wider community and on their ability (or inability) to connect or differentiate objects

[185] Sfard (2007, pp. 574-575) further distinguishes between routines that function at an object-level and those that generate rules at a meta-level. Routines that operate at an object-level result in manipulation of the properties of the mathematical objects dealt with − for example, the routine for factorisation of a trinomial expression. Sfard argues that the principles that regulate these routines are commonly explicit to participants. Routines that operate at a meta-level, on the other hand, involve narratives about the nature of the discourse itself rather than about the objects of the discourse. Such narratives are generally not explicit and are evidenced through the actions of individuals rather than through a formally recognisable set of principles. For example, the rules that govern how a definition for a concept is arrived at.
Although not explicitly stated by Sfard (at least not in the readings I have completed for this study), this division between object-level and meta-level routines seemingly has a connection to the different forms of learning − object-level and meta-level – identified by Sfard (c.f. Ben-Zvi & Sfard, 2007) and as discussed in footnote 179 (c.f. page 265 above). Namely, that meta-level learning – which involves a disruption and consequence expansion of an existing discourse – will, presumably, not occur if routines are only constructed and engaged with at an object-level. Or, alternatively and in the opposite direction, that meta-level learning facilitates the construction of and engagement with routines at a meta-level. However, more inquiry is necessary to validate this assumption.
[186] Sfard (2008, pp. 222-258) also distinguishes between three types of routines. *Explorations* are routines whose purpose is the development of endorsed narratives or the substantiation of narratives as endorsable (Sfard, 2008, p. 224). *Deeds* are routines that produce a change in the objects in a discourse and not just in the narratives about those objects (Sfard, 2008, p. 237). *Rituals* are routines whose purpose is the creation and sustainment of a bond with other people (Sfard, 2008, p. 224) – what Berger (2013, p. 3) refers to as a goal for social approval. Ritualistic routines involve working with other people in harmony in the act of mimicking their actions. Where explorations and deeds involve a degree of agency on the part of the participants in the strategies used in the selection and utilisation of the routines, rituals are considerably more restricted in this regard. In the context of this discussion and study, of primary concern are explorative routines.
In this study, primary focus is on routines as explorations, particularly since − as suggested by Sfard (2008, p. 225) − it is this type of routine that embodies the "gist of discourses cultivated in schools."

in the discourse. This, in turn, limits their participation in the discourse and their ability to internalise and extend their understanding and use of the discourse.

An important point made by Sfard (2007, p. 575) is that the narratives and routines that are endorsed can differ from one context to another as influenced by the meta-rules that are negotiated and accepted by particular groups of participants. Hence, what might be considered an appropriate justification in the context of a school-based mathematics classroom may well be rejected in a university or workplace setting.

Although the four discursive resources of a discourse have now been described in some detail, an important gap remains – namely, a discussion of how these discursive resources influence and structure communication in a discourse. To facilitate this discussion, consideration must now be given to the role of *signifiers* in a discourse.

17.1.2 Signifiers in a discourse

Sfard (2008, p. 154) argues that "... mathematical communication involves incessant transitions from signifiers to other entities ...". From this statement it follows that signifiers play a central role in communication and, as such, deserve additional attention. To facilitate this, brief exploration into the realm of *semiotics* is necessary.[187 & 188]

People make use of a variety of signs or gestures on a daily basis in the act of communication with other people (Culler, 1976, p. 90) − for example, shaking hands as a form of greeting; rubbing thumb and forefinger together to signify money; or raising a middle finger to express 'mild' dissatisfaction or irritation (Seiler, 2013). From a semiological perspective, anything can constitute a sign (Barthes, 1964, p. 9) – a word, a sound, an image, a smell, an act, an object – but such things only become signs when meaning is attached to them such that they *signify* – both for the person using the sign and for the person interpreting the sign – a specific meaning or understanding (Chandler, 2013). Semiotics[189 & 190] is the field of study concerned with the study of these various types of signs (Perakyla, 2005, p. 870) – the "science of signs" (Culler, 1976, p. 90) − and, particularly, with how signs are used to generate and convey specific (and inherently

[187] The field of Semiotics is vast and includes an entire tradition of research stemming back to the early 1900's. In the discussion below I focus specifically on the components of this theory that relate to *signs*, *signifiers* and *signifieds*, since these are the components that have direct relevance to the dimensions of the external language of description and associated instrument used to analyse empirical texts.

[188] The methodology of textual analysis employed in Part 7 of this study (c.f. page 381 below) to facilitate analysis of empirical texts relating to the subject-matter domain of Mathematical Literacy also draws on methods grounded in the field of semiotics. Specific focus on the notion of signifiers in the development of the external language of description is, thus, a deliberate attempt to ensure consistency between the external language developed in this part of the study and the methodology employed in the analysis of empirical texts in Part 7.

[189] The Swiss linguist Ferdinand de Saussure (see, for example, (Saussure, 1966 [1916])) and American philosopher Charles Sanders Pierce (see, for example, (Peirce, 1931-1958)) are considered to be the founding fathers of Semiotics. Arguably the most influential writer in the field of recent times has been Roland Barthes (see, for example, (Barthes, 1964, 1972)).

[190] Some authors claim that Semiotics is based on the analysis of signs in *language* (see, for example, (Manning & Caullum-Swan, 1994, p. 466)). However, Moore (2004, p. 162) argues that Semiotics actually has its origins in the terrain of mathematical logic and that the shift to linguistics involved a "series of shifts in which the semiotic exemplar migrates across disciplinary boundaries, first from (positivist) mathematical logic and scientific method, to (structuralist) linguistics and cultural anthropology, to (post-structuralist) discourse theory in the humanities and literary criticism."

culturally-embedded) meanings (Scott, 2006) (and/or with the meanings that are embedded within different signs (Chandler, 2013)).

In line with this perspective, Manning and Caullum-Swan (1994, p. 466) argue that semioticians view "social life, group structure, beliefs, practices, and the content of social relations as functionally analogous to the units that structure language." As such, messages, texts and any other communicative act can then be thought of as systems of signs (Seiler, 2013) that can be "read" (Manning & Caullum-Swan, 1994, p. 466), and that interpretation of these signs, thus, provides a window into a particular social and cultural world.[191]

Specifically with respect to the concept of a 'sign', a sign is something (anything) that represents something else and/or a particular meaning in another person's mind (Manning & Caullum-Swan, 1994, p. 466). Each individual sign entity comprises two facets – an *expression* and a *content* (Barthes, 1964, p. 39; Dowling, 1998, p. 107), with the expression characterised as a word, symbol or sound, and the content as the thing that the word, symbol, or sound refers to, or constructs and describes meaning of (Manning & Caullum-Swan, 1994, p. 466). For example, the word 'inflation' is a sign that is comprised of the word (i.e. *expression*) inflation, and at the same time this word triggers for the person reading or engaging with the word a particular *concept* relating to an aspect of their daily-life or lived experience. This expression-content relationship can also be characterised as a relationship of *signifier* (the form that the sign takes) and *signified* (the concept or idea that the sign expresses or represents and describes meaning of) (Barthes, 1972, pp. 112-113; Culler, 1976, p. 19). The signifier is referred to as the material or physical form of the sign – i.e. a word or act or symbol or visual mediator which triggers or indexes a mental construction of the signified concept (Chandler, 2013; Seiler, 2013). A sign is, thus, the whole – the "associative total" (Barthes, 1972, p. 113) − that results from the association of signifier and signified, and every sign always comprises both signifier and signified: it is impossible for a sign to include a signifier without that signifier signifying something (Culler, 1976, p. 19). Importantly, however, the same signifier can stand for many signifieds. For example, the expression 'bank' can refer to a financial institution when used in the context of financial situations, but also to a sloped piece of soil. The expression 'return' can be taken to signify 'coming back' (as in 'returning home') or taking something back (as in 'returning something at the shops') or 'there and back' (as in a 'return trip'). The expression 'plan' can be taken to signify an architectural drawing (as in a 'house or floor plan') or an action of organisation (as in 'planning a journey'). Similarly, many signifiers could stand for a particular signified (Chandler, 2013). Culler (1976, p. 19 & 23) clarifies this issue by referring to the 'arbitrary' nature of the sign – namely, that there is no inevitable link between a signifier and a specific signified:

> Not only does each language produce a different set of signifiers, articulating and dividing the continuum of sound in a distinctive way; each language produces a different set of signifieds; it has a distinctive and thus 'arbitrary' way of organizing the world in concepts and categories.

[191] For Saussure, analysis of language equates to analysis of social facts and of how material objects are used in a social setting (Culler, 1976, p. 51). It is for this reason that interpretation of the signs that make up the language in a text provides a view into the social base from which those signs have developed and in which the signs are embedded.

It is for this reason that the same word in two different languages can refer to different meanings (Culler, 1976, p. 23).[192]

Returning to Sfard's work, in the context of a discourse *signifiers* are words or symbols – i.e. vocabulary or visual mediators[193] − that are positioned and operate as nouns in the utterances of the participants as they communicate, and which index or signal the unique objects and resources (signifieds) that constitute and characterise the discourse[194] (Sfard, 2008, p. 154). Signifiers, thus, index the specific objects of the discourse on which communication is focused at a particular point in time by particular participants. Furthermore, the signifiers – together with the particular form of the realisation of the signifier[195] − also index and direct which routines must be engaged with to deal with specific objects of the discourse and the types of narratives to be generated about those objects. As such, by identifying the signifiers in a segment of communication and how participants respond to those signifiers with respect to routines employed and narratives that are constructed and endorsed, we can then start to analyse the nature and structure of the communication between participants in a domain. We can also analyse how that communication differs from other domains and, consequently, of the criteria according to which participation in the discursively-regulated practices of the domain is legitimised.

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[192] Further discussion of three ranges of possible meanings associated with or conveyed through the relationship between signifier and signified entities is provided in Part 6 (c.f. Chapter 21 and sub-section 21.2 below, starting on page 373) where the methodology of textual analysis is presented. Briefly, the three meanings are: denotative, connotative and mythical. Denotative meaning represents the literal meaning of a sign; connotative meaning is the range of possible meanings that the sign triggers in the mind of the reader or audience; and a mythical meaning occurs when the connotative meaning of a sign triggers a pre-determined or deliberate meaning (i.e. an ideology). In the analysis of empirical texts I focus predominantly on the connotative meanings that a particular combination of signifier-signified(s) in the text index in the mind of the reader/learner/teacher, specifically in relation to mathematical and/or contextual routines and endorsed narratives for the subject-matter domain of Mathematical Literacy.

[193] It is important to specify that in the context of this discussion and study, signifiers do not exist as an independent characteristic of a discourse and, rather, are embedded within *both* word and mediator use. For example, the inclusion of the *word* 'factorise' in a mathematical question or statement signifies to a participant the nature of the mathematical object being dealt with and the required procedures, routines or operations needed. Similarly, the *visual mediator* $2x^3$ also constitutes a signifier, this time indexing a mathematical object relating to cubic functions (as well as a host of other objects relating to the combination of a constant value with a variable that includes an exponent). The need to clarify this point is necessitated by the fact that some authors (c.f. Bansilal, 2013; Brodie & Berger, 2010) have substituted the discourse characteristic 'word use/vocabulary' with 'signifier/object', with the implication that in these works signifiers and visual mediators are constituted as separated and independent entities.

[194] In relation to the specific domain of mathematics, Sfard (2008, p. 172) characterises mathematical objects as *discursive objects* that are *abstract in nature* and which have *distinctly mathematical signifiers*.

[195] Sfard (2008, pp. 154-160) uses the term 'realization of a signifier' to refer to the way in which a particular signifier is realised by a participant as they engage with routines and attempt to generate narratives that are ultimately endorsed. For example, the signifier "Solve $2x + 3 = 0$"* may be realised through a graph drawn to represent the function $f(x) = 2x + 3$ and identification of the value for which this function equals 0. In this instance, the graph represents the realisation of the signifier $2x + 3 = 0$ that was utilised to develop an endorsed narrative about the solution to the equation. Notice that there are numerous other realisations which could have been employed in attempting to find a solution to the equation, and that the graph represents only one of many possible realisations. Sfard (2008, p. 165) elaborates on this idea by suggesting that each realisation can be used as a signifier that can be realised even further; as such, each signifier can then been seen as the base for a 'tree of realizations'. Importantly, it is the *collective* realisation tree for a particular signifier which constitutes the mathematical object that stands as a referent for the signifier: i.e. a single realisation of a signifier does not constitute the object indexed by that signifier.

**Note that this collective signifier comprises embedded signifiers comprising both words ("solve") and visual mediators (2x; +; 2x + 3). Thus, although here I have considered the signifier as a whole, it would also be possible to consider each of the component parts of the collective signifier as signifiers themselves.*

17.2 Mathematical literacy as characterised by discursively-mediated practices

The discussion above has drawn on Sfard's (2007, 2008) theoretical language to promote the perspective that the domain of mathematics is characterised by discursively-mediated practices. As a result, this domain can be differentiated from other domains according to the uniquely mathematical nature of the words, visual mediators and routines employed by participants as they seek to generate mathematical narratives that are ultimately endorsed by other participants in the domain. Following this line of thinking, it is then a logical step to also constitute the knowledge domain of mathematical literacy as characterised by discursively-mediated practices. Importantly, however, the discursive resources – specifically, signifiers (i.e. words/vocabulary and visual mediators), routines and narratives − that facilitate successful, legitimate and endorsed engagement and communication in practices associated with the knowledge domain of mathematical literacy domain are differently constituted from those in the domain of mathematics (and also from other knowledge domains). My intention in this section of the chapter is to identify more clearly and explicitly the nature and structure of the discourse that characterises the knowledge domain of mathematical literacy and the specific and unique nature of the signifiers, routines and endorsed narratives that characterise this discourse. To facilitate this discussion, a diversion into Greeno's (1991) work on *conceptual domains* is necessary.

17.2.1 Conceptual and contextual domains

Bansilal (2013), drawing on the work of Greeno (1991), argues that mathematics is characterised as a collection of 'conceptual domains', with each conceptual domain representing an 'environment' that comprises a collection of resources and tools (Greeno, 1991, p. 175). Successful engagement in a domain is then characterised by the ability to navigate the domain and to locate and engage the relevant and appropriate tools and resources available in the domain. "Learning the domain, in this view, is analogous to learning to live in an environment: learning your way around, learning what resources are available, and learning how to use those resources in conducting your activities productively and enjoyably." (Greeno, 1991, p. 175).[196] In other words, learning is characterised by the increasingly skilled use of the tools and resources of the domain (Bansilal, Mkhwanazi, & Mahlabela, 2012, p. 100). Bansilal (2013) then succinctly connects the notion of conceptual domains (Greeno, 1991) and discourses (Sfard, 2008)[197] by arguing that if each conceptual domain in mathematics is thought of as an environment, then the participants who engage in that environment will employ a specific and uniquely mathematical discourse. The tools and discursive resources of the domain

[196] Greeno (1991, pp. 174-175) argues that this perspective of learning stands in contrast to more traditional perspectives where a subject-matter domain is seen as a collection of external facts, rules and procedures that must be internalised to facilitate reasoning, understanding and knowing in the domain. In the 'environmental' view, learning is not about knowing concepts, but about the ability to find and use concepts: "The person's knowledge, however, is in his or her ability to find and use resources, not in having mental versions of maps and instructions as the basis for all reasoning and action." (Greeno, 1991, p. 175).

[197] Amongst other things, the work of Greeno (1991) and Sfard (2007, 2008) share a social-learning perspective (Lave, 1988) in which learning is viewed as a social activity that involves interaction, collaboration and competition with other participants (c.f. Greeno, 1991, pp. 175-176) (c.f. Sfard, 2007, pp. 569-571).

are then the words and visual mediators (i.e. signifiers)[198], routines and narratives that structure and define the discourse of the domain.[199] It may be further feasible to conceive that each domain in the environment is, thus, characterised by a sub-discourse (c.f. page 265 above) of the collective discourse that specifies the unique and appropriate tools and resources required for endorsed participation in the domain. This relationship between conceptual domains and discourses is illustrated in the diagram shown in Figure 36.

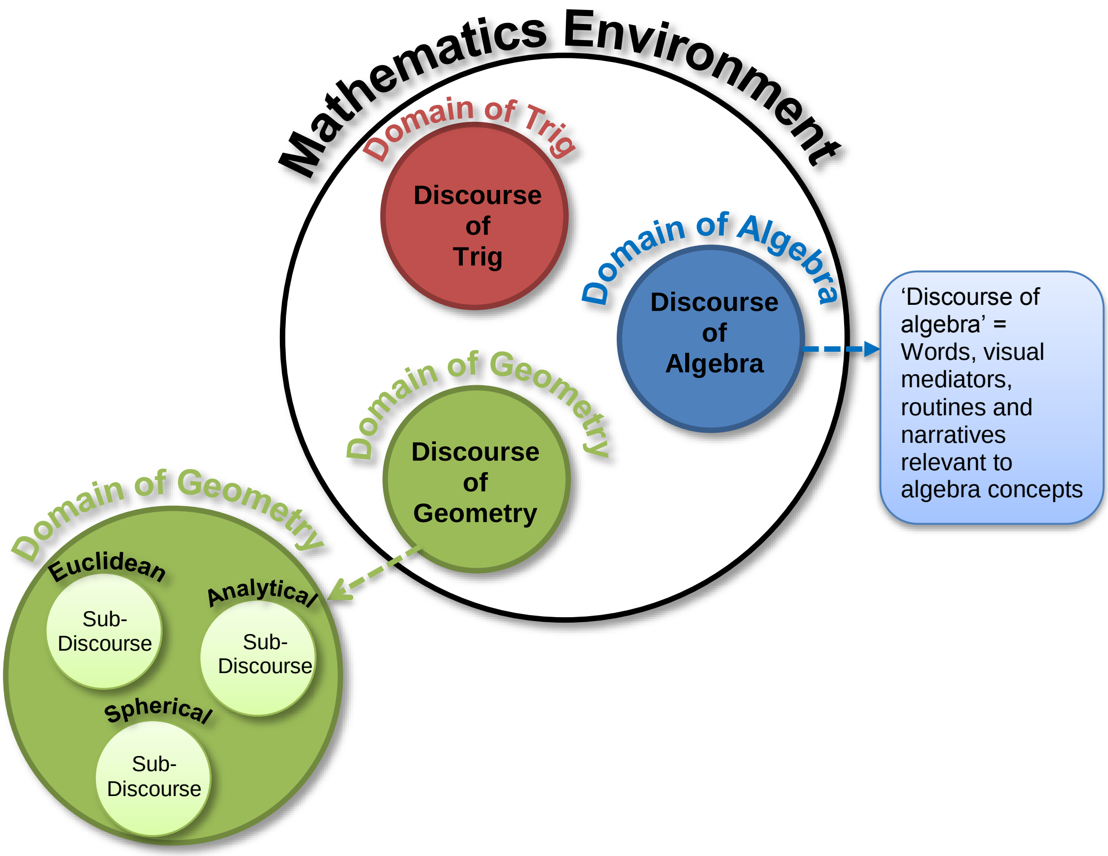

Figure 36: Conceptual domains and discourses in mathematics

If mathematics is conceived of as an environment with available tools and resources, then the knowledge domain of mathematical literacy can also be conceived of in a similar way, albeit with differing tools and resources. Furthermore and crucially, where mathematics draws from one conceptual domain strand (i.e. scientific/abstract mathematics content),

[198] Reiterating a comment made in footnote 193 above (c.f. page 271), Bansilal (2013) substitutes the discourse characteristic of 'words/vocabulary' with 'signifiers', such that her interpretation of Sfard's (2008) work results in a characterisation of the four components of a discourse as: signifiers, visual mediators, routines and narratives. My own reading of Sfard's work does not comply with this interpretation. Rather, I conceive that both words/vocabulary and visual mediators serve as signifiers in the discourse. For this reason, I continue to make reference to the characteristic 'word/vocabulary' as the first of the four characteristics of a discourse, I do not dichotomise signifiers as a separate or independent characteristic, and, rather, I contend that both words and visual mediators serve as signifiers when positioned as nouns in utterances that index objects.

[199] In footnote 180 above (c.f. page 265) I stated my preference for the term 'resources' when referring to the four discursive characteristics of a discourse, and cited the need for consistency with the term used by Greeno as the motivating factor. Now that an explicit connection has been made in the paragraph of text preceding this footnote between the discursive 'characteristics' of a discourse (as identified by Sfard) and the 'resources' of an environment (as identified by Greeno), I hope that my intention for preferencing the term 'resources' is now clear and justified to the reader.

mathematical literacy entails the complementary use and negotiation of tools and discursive resources from two different terrains – the mathematical terrain and a contextual terrain[200] (Bansilal & Debba, 2012, p. 304) (see also Bansilal et al., 2012, p. 100). However, a caution must be raised. Although participants engaging in mathematical literacy practices draw on both mathematical and contextual tools and resources to make sense of real-world problem situations, it is somewhat misleading to conceive that they draw separately from two distinct terrains. Instead, it is more appropriate to conceive of the space engaged in by participants in mathematical literacy related practices as constituting a 'blended domain' positioned at the intersection of the mathematical and contextual terrains (see Figure 37 on the page below). To use a metaphor, mathematical literacy participants are constantly performing a balancing act, with one foot (or maybe only three toes) positioned in the mathematical terrain and the other foot (or remaining six ... just kidding ... seven toes) positioned in the contextual terrain.[201]

By implication, engagement in problems that relate to the knowledge domain of mathematical literacy (and, by implication, problems encountered in a form of the subject-matter domain of Mathematical Literacy that draws on this domain as the basis of legitimate knowledge and participation) ultimately always involves a degree of real-world contextualisation together with a certain degree of mathematical structure and process. As such, and reiterating the thinking of Greeno (1991), successful engagement in this blended domain is dependent not only on the ability to identify which resources and tools are necessary to solve problems, but also which domain – mathematical or contextual − these tools and resources must be drawn from.[202]

[200] The use of the word 'a' instead of 'the' when referring to contextual terrain is deliberate. This is because every context brings with it a new set of tools, resources and constraints with respect to what counts as valid and acceptable knowledge and practice in that context. As such, it becomes tricky to talk about 'the' contextual terrain since this implies that there is a collective understanding of the contents of this terrain and that that the contents of the terrain can be adequately and comprehensively described. The situation is different in mathematics where there is relatively consistent understanding of what constitutes mathematical content and of what belongs in the terrain of mathematics.

[201] My emphasis on an unequal number of toes must not be interpreted as suggesting that Mathematical Literacy participants are somewhat awkward looking. Rather, it is simply an attempt to emphasise, yet again, the increased emphasis that I place on contextual constructs versus mathematical structures in my conception of practices associated with the knowledge domain of mathematical literacy.

[202] It is my own belief that this constant negotiation and requirement for decision-making around which terrain and knowledge domain tools and resources must be drawn from is one of the reasons why many learners find it difficult to engage in real-world problem-solving activities. It is also my belief that some learners who perform poorly in such activities do so because they misinterpret the discourse or intention of the task developer. For example, by providing what would be an acceptable real-world solution or method rather than a formal mathematical technique and solution − rather than due to poor understanding of content or concepts. In other words, a clash of discourse between the learner and, for example, an examiner. However, my statements here are based on pure intuition and observation, and more research is needed before an evidenced statement can be provided in this regard.

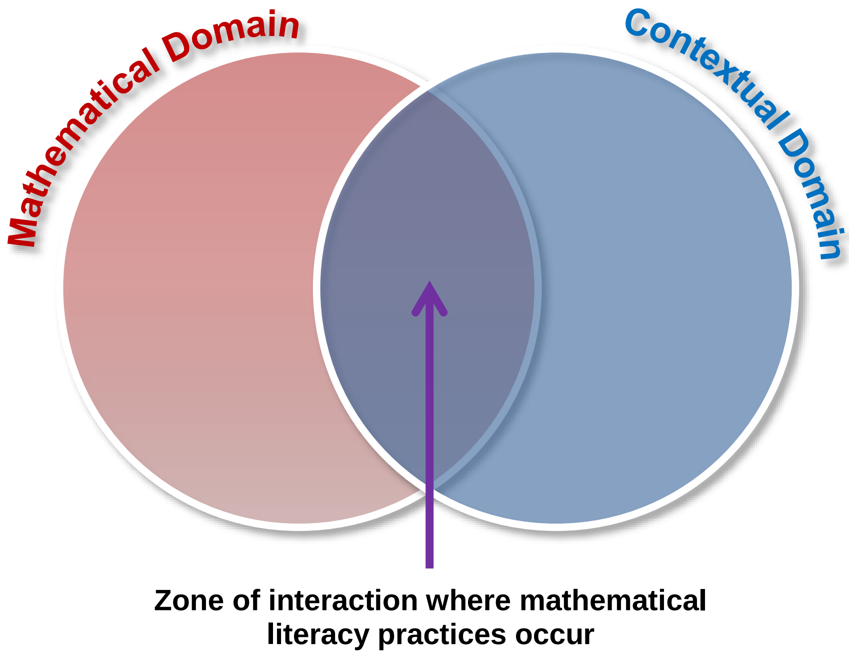

Figure 37: Mathematical literacy as a blended-domain

If the knowledge domain of mathematical literacy is conceived of as an environment characterised by the blending of the terrains of the mathematical world and the contextual world, then the following line of thinking emerges. Namely, that it becomes possible to visualise (as illustrated in Figure 38 below) the environment of mathematical literacy as comprised of a variety of broad contextual categories within which there exists a multitude of problem situations. Furthermore, each problem situation is then characterised by a specific sub-discourse (of the overarching discourse that characterises the practices of an entire contextual category) comprising a variety of both mathematical and contextual discursive elements, resources, tools, structures, and terminology. These discursive resources facilitate successful engagement with the problem situation and successful communication with other members who engage in that problem situation on a daily basis.

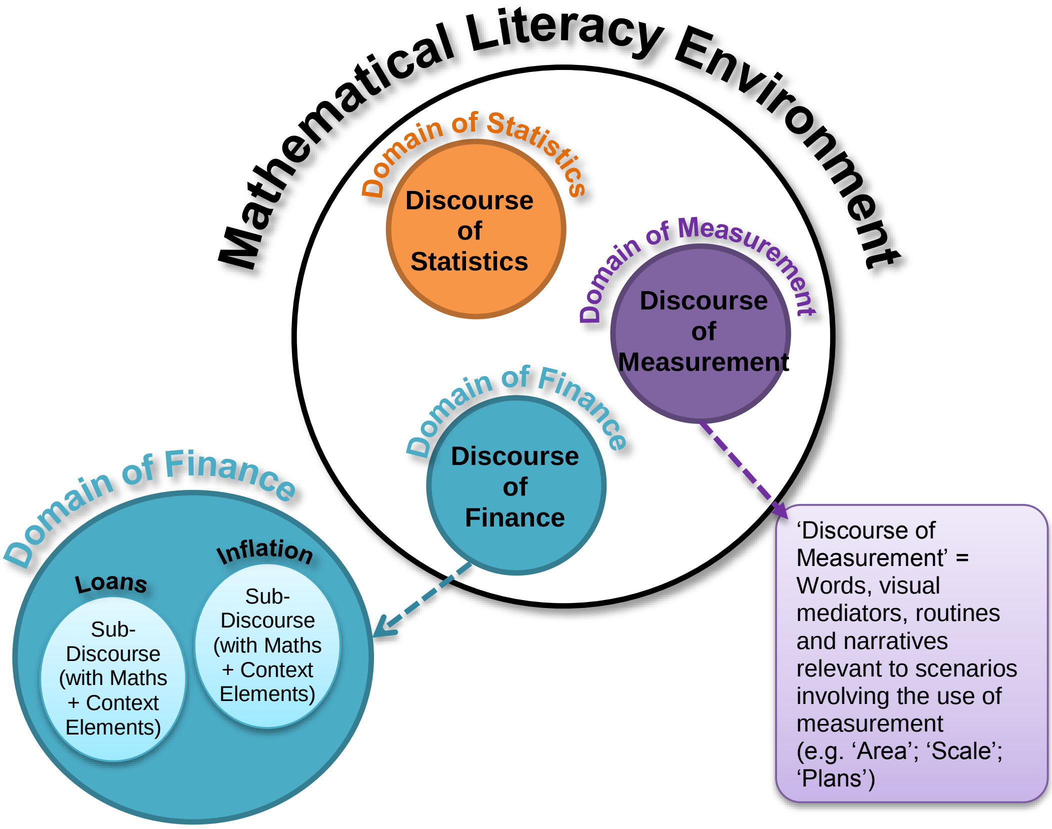

Figure 38: The environment of mathematical literacy

What remains now is to describe the constitutive features of this blended environment and, particularly, of the discursive resources – comprising signifiers (i.e. words/vocabulary and visual mediators), narratives and routines – that make it possible to identify practices and knowledge that are endorsed and legitimised within this environment.

17.2.2 Discursive resources of the knowledge domain of mathematical literacy

It has already been established that the discursive resources that characterise the terrain of scientific mathematics contents are generally easy to identify given their uniquely and obviously mathematical esoteric nature and structure. In this terrain, the words/vocabulary used, together with the visual mediators (such as diagrams, graphs, symbols and notation), are, for the most part, uniquely and distinctly intra-mathematical.[203] As a consequence, the routines that employ these signifiers and the narratives that are endorsed through the deployment of appropriate routines, are also similarly uniquely intra-mathematical and, hence, easily identifiable and easily able to be classified as belonging to the unique knowledge domain of scientific mathematics.

[203] And even when some of the signifiers used are not exclusively intra-mathematical – such as in the case of word problems that include reference to extra-mathematical contents – the contents of the problems are still easily associated with the domain of mathematics due to the obviously mathematical slant of the problems and the limited extent of the reach to reality and authenticity. In short, mathematical structures provide the organising principle and generative structures of all activities in this domain, and the participants in this domain are generally aware of this.

The scenario is not as clear cut in the knowledge domain of mathematical literacy, where every problem-solving scenario inevitably involves a blending of discursive resources from both mathematical and contextual terrains as mathematical structures are appropriated to engage with problem situations relating to real-life contextual settings. In particular, a key issue which reinforces the distinction between contextually engaged knowledge domains (like mathematical literacy) from knowledge domains that do not emphasise contextual engagement (like mathematics), and of the resources employed in each of these domains, relates to the "dynamic, socially constitutive, ... interactively sustained, time bound" (Goodwin & Duranti, 1992, p. 5 & 6) nature of contextual phenomenon. The primary argument presented in this regard is that humans do not simply engage passively with the contents of a contextual environment but, rather, have capacity to reshape the contexts in which their actions occur: in other words, their relationship to a context is *reflexive* in nature (Goodwin & Duranti, 1992, pp. 4-5 & 7). A key marker here is that different participants engaged in the same problem situations (i.e. same focal events) may draw from differing fields of action[204] (comprising different variables, considerations and constraints), with the consequence that their engagement with the same context and problem situation may yield differing perspectives, outcomes and/or results. Often this difference is influenced by the particular social, economic and/or historical perspectives that the participants bring to bear on the contexts that they encounter, which in turn affects which aspects of a context are deemed relevant (or irrelevant) and, consequently, are prioritised (or ignored) in engagement with the focal event (Goodwin & Duranti, 1992, pp. 4-5). Furthermore, in as much as participants themselves influence the nature of interaction and engagement with contextual environments, contextual situations are also not static entities and, rather, are dynamically mutable (Goodwin & Duranti, 1992, pp. 4-5). As such, as participants move from one context to another they have to constantly adapt to differing fields of action, variables and constraints within those fields of action.

The implication of this from the perspective of problem-solving practices associated with the knowledge domain of mathematical literacy is the following. Namely, even though the same mathematical structure may appear in two different contexts, as participants move from one context to the other they may respond differently to the mathematical structure based on what they deem relevant to the specific focal event under investigation and what they deem to be an appropriate field of action for that event. The situation is very different in the domain of mathematics − where only limited (or no) engagement

[204] As a reminder, in Part 4 in this study (c.f. Chapter 14 and sub-section 14.4.1.2 above, starting on page 198), I referenced the work of Goodwin and Duranti (1992) in arguing that within the location of an academic knowledge domain such as mathematical literacy, analysis of a 'context' actually involves analysis of one or more 'focal events' of a wider contextual environment, where the focal events represent a directed and limited area of focus of the wider environment. Furthermore, analysis of the entirety of a contextual environment (and all of the factors that impact on participation in that contextual environment) is not deemed possible in such an academic setting: in other words, the scope, complexity and intricacy of a real-world environment cannot be captured in an investigation or discussion or activity conducted in a classroom setting. However, given that various focal event(s) are embedded within a much broader contextual environment (i.e. a 'field of action'), more comprehensive understanding of the focal event (and of how participants actually engage with and experience that focal event in real or lived experience) is only possibly through consideration of a variety of attributes of the broader contextual environment in which the focal event is embedded, including: the *setting* (specific social and/or spatial location) of the focal event; the *behavioural environment* or site in which participants are exposed to the focal event; context-specific technical *language* of which understanding is essential for successful engagement with and/or in the focal event; and *extra-situational background knowledge* which refers to information, resources and other sources of knowledge which exist outside of the immediate frame of reference of the focal event but which provide insight into how the structure of the focal event – and participation in that focal event – is influenced and determined by factors that exist in the broader contextual environment

with contextual environments is expected – in several ways. Firstly, there is a consensually agreed upon body of mathematical knowledge that is not influenced as succinctly by the social, economic and/or social perspectives of the participants.[205] Secondly, in the domain of mathematics the tools and resources needed for engagement with the focal event are commonly available in the focal event itself, thus negating the impact of resources drawn from outside of the event on the interpretation of the event. Thirdly, the agreed-upon knowledge generally does not shift as participants move from one context to another: the term 'factorise' and the types of factorisation do not change. In this sense, the content is more static, enabling greater opportunity for generalisation. These three points of differentiation indicate that the contextual resources that participants are expected to engage with as they seek to establish and replicate legitimised and endorsed practices in a contextually oriented knowledge domain differ significantly from those in a non-contextually orientated domain[206]. Furthermore, legitimised practices in a contextually oriented knowledge domain require the use of discursive resources that shift from context to context, that draw from both the context and also from the mathematical domain, and which exhibit unique and context-specific characteristics.

Given that Sfard's (2007, 2008) theoretical language is empirically grounded in the non-contextually orientated domain of scientific mathematics, a reformulation of this work is necessary to facilitate application of the work to the contextually oriented terrain occupied by the knowledge domain of mathematical literacy. To achieve this I draw on, extend and reconceptualise Bansilal's (2013) work on *context-specific resources*. And it is to a discussion of this work and related concepts that the discussion now shifts.

By also drawing on the distinction between mathematical literacy as a contextually oriented domain and mathematics as a non-contextually oriented domain, Bansilal (2013) identifies four categories of *contextually specific discursive resources*. These are: (i) contextual words/vocabulary; (ii) contextual visual mediators; (iii) contextual rules

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[205] This is certainly not to deny the influence (interference?) of political, economic, social and cultural perspectives on the selection of mathematical contents prioritised in different settings and on the differential distribution of contents to different groups of participants. Rather, the comment is simply inferring that any selection draws from a body of consensually agreed upon and recognised contents.

[206] Of course, it could be argued that in the domain of mathematics the mathematical content provides the context – namely, that mathematics is its own context. To clarify, my usage here of the term 'context' in the reference 'contextually oriented domain' refers to knowledge domains such as mathematical literacy that pursue engagement with authentic real-life situations in an attempt to better describe and/or understand those situations through a dominant orientation for critical contextual sense-making practice. I contrast this with non-contextually oriented domains (which is how I classify disciplinary mathematics) where the reach to and understanding of authentic reality is not a primary goal. So, what does this mean for mathematical modelling and/or application activities? In this conceptualisation, mathematical modelling aimed at generating representations of reality would be classified as part of a contextually oriented knowledge domain, while modelling and application activities of exclusively intra-mathematical contents (e.g. for the development of more intricate knowledge of an esoteric mathematical concept) would be classified as non-contextually oriented. Importantly, these comments on modelling and application also need to be considered in relation to the distinction made in Part 2 (c.f. sub-section 5.2.2.1 starting on page 37) of this study between modelling and applications. In that regard I made the argument that while a particular agenda for modelling-related practices is appropriate for the knowledge domain of mathematical literacy, an agenda for application (the practice of which is viewed as dominated by mathematically structured goals) is not.

(and routines); and (iv) contextual reasoning[207] − and distinguishes these from the discursive resources associated with a non-contextually oriented domain such as mathematics. For Bansilal, these contextual discursive resources serve to illuminate the focal event in a context and draw attention to features of the surrounding and broader contextual field of action which structure and influence participation in the focal event.[208] Notice that these contextual discursive resources reflect almost direct correlation to Sfard's conception of the four discursive characteristics or resources of a discourse[209], albeit reconceptualised in terms of a knowledge domain that is characterised by the blending of the mathematical and contextual terrains and, as such, which requires both contextual and mathematical forms of discourse. A detailed discussion of each of the contextual discursive resources is provided below.

(i) *Contextual words/vocabularies* are words or terms that signify a particular focal object within a real-world context. Furthermore, and as suggested by Bansilal (2013), these signifiers are bounded by the parameters of the experiential context.[210 &] For me, a clear example of a contextual word that functions as a signifier is the word 'inflation'. For participants who have already been exposed to the concept of inflation, a subsequent reading of this word indexes a variety of potential focal events relating to the broader context of inflation, including price change, and/or reduced buying power, and/or average price change of a whole basket versus the price change of individual items, and so on. From a calculation perspective, the term inflation might also signify the need to consider

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[207] Note that Bansilal's (2013) original conception of the aspects of contextual background knowledge includes: (a) contextual signifiers; (b) contextual visual mediators; (c) contextual rules; and (d) contextual reasoning. Once again I have deliberately substituted the category (a) contextual signifiers with (i) contextual words/vocabulary. As already discussed in footnotes 193 and 198, my reason for doing this is based on an interpretation of Sfard's (2007, 2008) work that suggests that *both* words/vocabulary *and* visual mediators serve as signifiers and that it is not appropriate to delineate signifiers as a separate category. Hence my consistent reference to the four categories that Sfard continually references in her own writings and reiterative reconceptualisation of Bansilal's (and Brodie and Berger's (2010)) interpretation of these original four categories.

[208] As has been discussed previously (c.f. footnote 204 on page 277 above, as well as in Chapter 14 and sub-section 14.4.1.2 in Part 4 above, starting on page 198), Goodwin and Duranti (1992) argue that every contextual environment (comprising various 'focal events' embedded within a broader 'field of action) is comprised of four contextual attributes. Successful analysis of or engagement with the context requires awareness and consideration of these contextual attributes. The attributes are: setting, behavioural environment, language use, and extra-situational background knowledge. It is in direct relation to the last mentioned of these attributes – namely, extra-situational background knowledge − that Bansilal (2013) introduces the four contextual resources, arguing that the resources exist outside of the immediate focal event but draw attention to facets of the broader field of action which must be engaged with to ensure a more complete understanding of the focal event. I am employing Bansilal's work on contextual resources in a different manner in that I conceive that these resources have the potential to permeate any attribute of a context and do not exist exclusively within the terrain of the extra-situational background knowledge that is brought to bear on the focal event. In other words, for me, contextual signifiers (words/vocabulary and visual mediators) and routines are embedded in every aspect of the setting of the context, are similarly embedded within and influenced by the structure and site of the behavioural environment. Contextual signifiers also form part of the technical language specific to the focal event, and appear in the extra-situational background knowledge that positions the focal event in a broader field of action. This perspective is supported by my further contention that it is only through engagement with the contextual resources in an appropriate way in *all* aspects of the context that participants are able to produce narratives that are endorsed by the wider community of participants who participate in the context.

[209] A notable omission here is the characteristic of 'endorsed narratives'. This characteristic is dealt with in the immediate discussion below.

[210] Although, as has already been discussed, different participants establish different constraints and boundaries for a field of action and draw on different variables as they navigate through a context and engage with a specific focal event embedded within the context. This means that the delineated context boundary may differ from participant to participant, with the consequence that different participants may index more or less of a contextual environment through the same signifier.

whether the appropriate calculation required involves a percentage increase, or the rate of price change, or compounding price change over multiple periods. To expand on Bansilal's conception, it is important to consider that not all words included in a segment of text or discourse serve to index a focal object. Rather, certain words also serve to signify the specific nature and structure of the routine(s) required to generate an endorsable narrative (in the form of a solution) for the problem scenario which facilitates legitimate participation in the scenario[211] − for example, words such as 'calculate' or 'determine' or 'what do you think' or 'estimate' or 'other than mathematics'. In relation to practices associated with the knowledge domain of mathematical literacy, then, words/vocabularies in a segment of text or an activity can serve to signify not only the specific context or focal object under investigation, but also the type and structure of the routine(s) required for the generation of an endorsed narrative.

By way of illustration of the function of signifiers in a segment of text, consider the following question (shown in Figure 39 below) drawn from a Grade 12 Mathematical Literacy examination paper (DBE, 2013b, p. 6):[212]

[211] As a preliminary statement for purposes of clarity, an endorsed narrative in the context of a Mathematical Literacy activity can take the form of a description of the elements of a contextual problem scenario, together with description of the relations between both contextual and mathematical elements of that scenario and of the routines indexed by the signifiers in the text and required for the development of a solution to the problem scenario. Alternatively, a narrative can also take the form of a response or solution (in the form of a calculation, or a statement, or an opinion) to a problem scenario that reflects a legitimate response or solution in the eyes of the task developer. So, if a participant offers an opinion to a question where the task developer expects a calculation, then that narrative may not be endorsed by the task developed (even though the opinion narrative may be endorsed by people who engage in the real-life scenario on a daily basis). The concept of endorsed narratives specifically in relation to the knowledge domain of mathematical literacy is dealt with in more detail on page 291 below.

[212] It is essential to point out that although the discussion immediately above is focused on elaboration and illustration of the characteristics of the discursive resources associated with the *knowledge domain* of mathematical literacy, I am aware that this exemplar text is drawn from a different and potentially dislocated site of practice – namely, the *empirical terrain* of the subject Mathematical Literacy. Importantly, this empirical terrain represents a site of pedagogic recontextualisation of the components of the knowledge domaain and, so, does not necessarily reflect the form of legitimised practices associated with and prioritised for that domain. While I acknowledge this potential dislocation, I have deliberately opted to engage with a resource from this empirical terrain for two reasons. Firstly, the resources facilitates the opportunity to illustrate at a general level the role of signifiers in a segment of text. Secondly, engagement with this specific resource further and specifically facilitates illustration of the form, structure and level of analysis of textual resources relating to the subject-matter domain of Mathematical Literacy that I engage in both during the operationalisation of this external language in Chapter 18 (see page 295 below) and in the empirical analysis component of this study (see Part 7 starting on page 381 below).

2.3 Rodney decides to deposit a fixed amount into his bank account at the end of each month. The bank offers an interest rate of 9% per annum, compounded monthly.

At the end of two years, the final amount in his account was R104 753,89.

Calculate the fixed amount that was regularly deposited at the end of each month.

The following formula may be used:

$$x = \frac{A \times \frac{i}{12}}{\left[\left(1 + \frac{i}{12}\right)^n - 1\right]}$$

where x = fixed monthly deposited amount; A = final amount
i = annual interest rate; n = number of deposits (4)

Figure 39: Question extract from the 2013 Paper 2 examination

The inclusion of vocabulary signifiers in the text such as ‘deposit’, ‘bank account’, and ‘interest rate’ index the specific focal event under analysis – namely, the workings of a banking account (as embedded within a broader contextual environment on banking and finance) and, specifically, the notion (and calculation) of a future value amount. Other signifiers, such as the words ‘calculate’ and ‘the following formula may be used’, signify the nature and structure of the routine(s) required in the generation of a solution for the scenario. Namely, the use of the given formula (which, by the way, is not specified in the Mathematical Literacy curriculum and, so, is an unfamiliar and/or un-encountered formula for most learners writing the exam[213]) and substitution into and manipulation of the formula to generate a calculation-derived solution. Importantly, notice that the inclusion of the signifier ‘calculate’ together with the formula (and the various visual mediators that make up the different components of the formula) focus attention directly on the need for a *mathematical calculation approach* to the problem. These signifiers also signify the need to engage with the contents of the problem by drawing on knowledge and contents primarily from the mathematical knowledge domain.[214] In relation to the required calculation, yet other word signifiers (such as ‘interest rate’, ‘compounded monthly’, ‘end of two years’ and ‘final amount’), together with various visual mediators in the text (such as 9% and the stated money value) and in the formula (such as the variables ‘A’, ‘i’ and ‘n’ and the various mathematical operators), index possible entities for consideration and possible inclusion in the calculation.

The inclusion of the word ‘may’ in the instruction “The following formula may be used” deserves special attention since it signifies (mythically?) for participants that there are

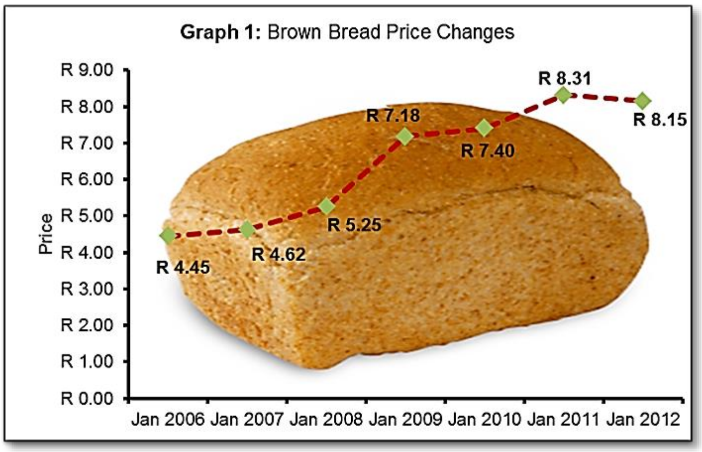
[213] The NCS curriculum for Mathematical Literacy (DoE, 2003a) (on which this examination is based) includes specification for learners to engage with situations involving compound interest calculations and specifically those that require the use of the compound interest formula. However, there is no specification for learners to work with investments involving a fixed monthly deposit or with any financial formulae other than the simple and compound interest formulae.

[214] I have used the word ‘primarily’ deliberately here because although the question is prompting participants for a calculation using a given formula, successful engagement with the requirements of this calculation is not possible without understanding of various contextual elements of the problem scenario, including knowledge of what an interest rate and a deposit is, and of the relevance of these entities for the required calculation. However, this question has been structured to assess competency with the given formula and not understanding of the contextual elements of the scenario under investigation.

potential alternative methods for solving this problem. And, yes, there are alternative methods for solving the problem, with the two primary being: (1) constructing an algebraic equation to represent the growth of an investment with a fixed monthly deposit over time and then substituting into and manipulating this formula to solve for the deposit variable (which would ultimately give the same formula as is provided in the question text); (2) using the method of trial-and-improvement by selecting possible monthly deposit values and performing manual calculations involving sequential reasoning and compounding calculations (for 24 periods!) to check which monthly deposit value gives the desired total amount value. However, neither of these methods is as straightforward or as accessible as using the formula provided (although both of these methods require and demonstrate significantly more understanding of the problem situation and of the contextual entities and complexity of the environment in which this problem situation is embedded). Thus, although a seeming 'choice' is provided, this is somewhat of a mythical choice with respect to simplicity and accessibility of solution; and a higher possibility of success seems likely for those who make use of the provided mathematically structured method rather than an alternate method.

(ii) *Contextual visual mediators* refer to information presented in visual form – as diagrams (for example, drawings of 3-dimensional objects to be manufactured), pictures (including plans and photographs), and graphs (Bansilal, 2013). I also include in this categorisation symbols (e.g. currency symbols; mathematical operators) and notation (e.g. the percentage sign (%); or units (e.g. m^2) used for signifying quantities). Note that Bansilal (2013) categorises visual mediators as representations of a "non-textual" form, which, presumably, implies that visual mediators are visual (e.g. graphical/diagrammatic/pictorial) in nature. This is a categorisation that I do not agree with. For me, the value 10% − even when cited within a contextual environment – is a visual mediator (even though it is a type of textual representation) that includes both a numerical (10) and a notational (%) component. However, whereas in a knowledge domain like mathematics the inclusion of the signifier 10% might index a particular type of calculation, when encountered in a contextually oriented practice the percentage value signifies both a mathematical and an other-than-mathematical component. For example, 10% may signify a specific *change* in relation to a real-world object − perhaps price, or size, or some other quantity. This, in turn, signifies either the type of calculation that must be performed using the signifier, or the type of calculation that gave rise to the signifier, or the type of interpretation and understanding that must be applied to the scenario. This example illustrates a key component of contextual visual mediators in a contextually oriented knowledge domain like mathematical literacy that draws on both mathematical and contextual discursive elements. Namely, that the visual mediators commonly signify both a mathematical object and an other-than-mathematical object, and the participants have to consider and negotiate how these objects drawn from different domains interact, relate or perhaps negate and contradict each other. Furthermore, participants also need to consider the nature and structure of the routine(s) required − as signified through the visual mediator – to generate a narrative for the problem situation that will ultimately be endorsed. As with words/vocabularies, visual mediators in the context of practices associated with the knowledge domain of mathematical literacy serve a dual function: to highlight the particular context and embedded focal object(s) or event(s) under investigation, and to signify the structure of the specific routine(s) required to generate an endorsed narrative for the problem situation.

As a further example of the dual function of visual mediators, consider the two graphs shown in Figure 40 below: *Graph 1* shows the trend in the changing price of bread over a period of time; *Graph 2* shows the trend in the change in the inflation rate for the bread price over the same period of time.



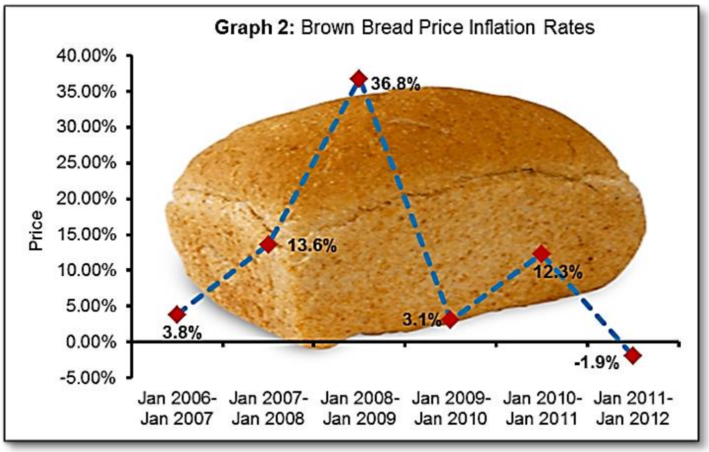


Figure 40: Graphs showing changing prices and inflation rates for Brown Bread

Engagement with the information on the graphs requires consideration of the significance of changes in the graphs, such as increasing or decreasing segments and differences in the steepness of the segments – all of which represent a type of mathematical engagement with and understanding of graphs. However, more comprehensive understanding of the information shown in the graphs requires consideration of the significance of these changes with respect to the event represented in the graphs and the contextual environment in which the event is positioned – namely, the changing prices of bread. Furthermore, understanding of and the ability to replicate how the inflation rate values on Graph 2 have been arrived at requires recognition of the appropriate routine involved for the calculation of an inflation rate – namely, the routine for a calculation involving percentage change between two consecutive values.

Now notice the following. If a purely mathematical understanding of graph work is applied to interpretation of both graphs, then it could be argued that a decreasing segment indicates a decrease in price. While this would certainly be accurate in relation to the information shown on *Graph 1*, when this feature is considered in the context of the inflation rates shown on *Graph 2*, then the purely mathematical interpretation is incorrect: a decrease in the inflation rate (as indicated by a decreasing segment of the graph) indicates a *lower rate* of inflation – the price has still increased, just as a smaller rate (or by a smaller percentage). In the context of inflation, a graph that drops below the horizontal axis to a negative rate would indicate a decrease in price. The point (pardon the pun ☺) is that successful engagement with this graphical contextual visual mediator for inflation rates requires understanding of components from both the mathematical discourse on graphs and the contextual discourse on inflation, together with the ability to negotiate how these discourses relate in a complementary way.

To further illustrate the role of contextual visual mediators in indexing the focal event under analysis and appropriate routines required for the generation of a solution to a problem that is potentially endorsable, consider once again the example of the examination question (DBE, 2013b, p. 6) (given in Figure 39 on page 281 above). Visual mediators in this segment of text include reference to a monetary figure (that is identifiable only through the inclusion of a currency symbol and two decimal places to signify cents) and a percentage value (i.e. ‘9%). Importantly, the visual mediator of the percentage value must be considered in conjunction with the vocabulary signifiers of ‘interest rate’, ‘per annum’ and ‘compounded monthly’ to give meaning to the significance or relevance of this percentage value and to further index how the value is to be interpreted and/or used. Specifically, the percentage value represents a rate of increase of money and, so, reflects a corresponding calculation involving percentage increase together with an exponential (compounding) calculation (and not another type of representation or calculation). The deliberate inclusion of a formulaic visual mediator constructed with various mathematical entities (operators, brackets, an equal sign, and so on) serves to index explicitly the requirement for a calculation technique for the solving of the problem and, specifically, for the utilisation of a routine that involves substitution and manipulation of the entities in a formula. Also notice that a deliberate inclusion of variables in the formula (as opposed to actual descriptors of contextual entities) further reinforces the requirement for a mathematically dominant problem-solving structure.

As a final comment on contextually oriented visual mediators, it is important to note that it is often through the combination of contextual visual mediators and contextual words/vocabulary that clear(er) signification is established as to the object that constitutes the focal event in a contextual environment, to the constraints and variables to be considered (or ignored or excluded) in that environment, and the routine(s) that must be employed to generate a narrative that is ultimately endorsed. For example, as has been mentioned above, the combination of contextual words/vocabulary and contextual visual mediator that gives rise to the signifier ‘interest rate of 9%, compounded annually’ delineates clearly the specific contextual entity represented by the percentage value (namely, the rate of increase of a monetary value) and, further, the requirement for a compounding-type calculation if the rate is to be used in a calculation.

The third contextual resource identified by Bansilal (2013) is that of (iii) *contextual rules* – and, although not stated explicitly by Bansilal, I argue that this resource category (as it applies to the form of discourse characteristic of a contextually orienteted knwoledge domain) is to be equated to the discursive characteristic of a (non-contextually oriented) domain that Sfard (2007, 2008) refers to as ‘routines’.[215] Contextual rules are described by Bansilal as rules and procedures that are bound to a context and, in the case of a contextually oriented knowledge domain like mathematical literacy, need to be interpreted in relation to other contextual information, words, and visual mediators to facilitate contextual sense-making practices with the focal event under investigation. Appropriate interpretation of these contextual rules is particularly important for facilitating the generation of potentially endorsable narratives for problem situations

[215] Namely, well-defined repetitive patterns or regularities in the actions of the participants of the discourse that are characteristic of the structure and contents of the discourse (Sfard, 2007, p. 574).

encountered in that focal event. Bansilal (2013) explains further that “These rules are used for calculations in the context” and cites examples such as the rule for calculating the transfer duty for a house (e.g. *For a purchase price of R500 001 to R1 000 000, the transfer duty is 5% on the value above R500 000*). However, I contend that limiting contextual rules to those involving calculations within a contextully-oriented knowledge domain is restrictive and does not provide adequate scope for inclusion of meta-rules (for example, the instruction ‘read and make sense before calculating’) or rules and procedures that are non-calculation based. For example, the rule or instruction given on the order or format in which to read and interpret a table or values (see Figure 41 below), or on how to interpret a graph (e.g. a growth chart) (see Figure 42 below). Similarly, these calculation-based rules do not allow for interpretation of the procedure to be followed for completing an online application form, where specific information must be entered in a specific order and format before necessary calculations can be generated (see Figure 43 below).

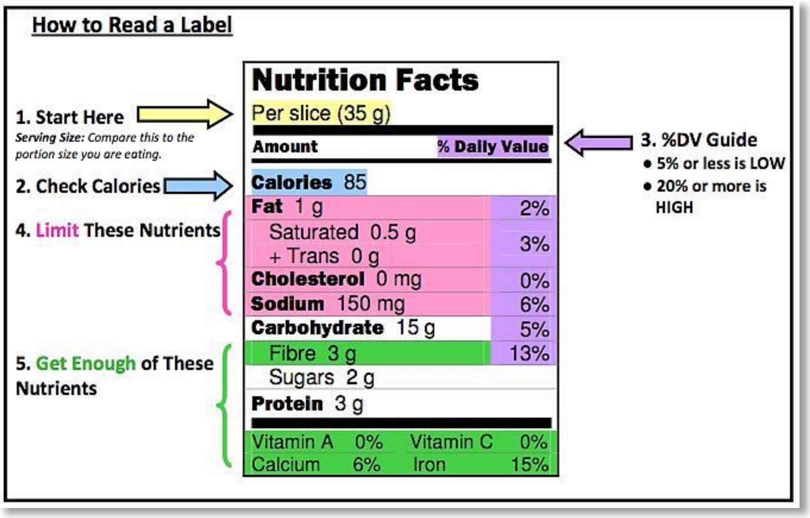


Figure 41: Instructions for reading a label (Forever Active Website, 2013)

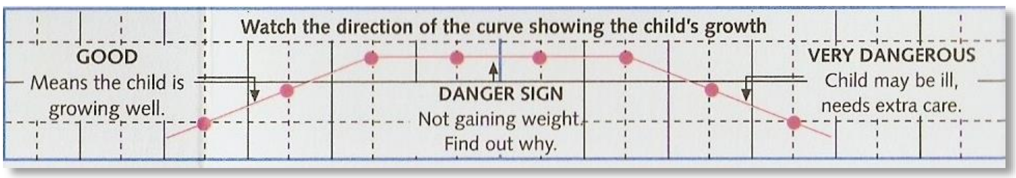


Figure 42: Instructions for interpreting a growth pattern on the Road to Health Chart (DoH, 2005)

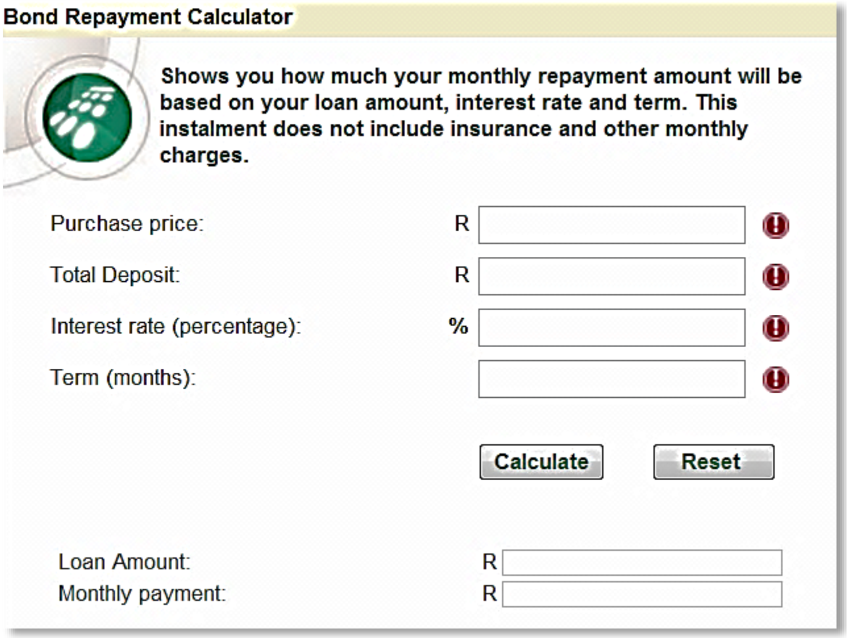


Figure 43: Bond Repayment Calculator (Nedbank, 2013)

Furthermore and essentially, it is the routines that are selected and employed – as a consequence of the particular interpretation of the signifiers (words/vocabularies and visual mediators) in a text or activity – in presenting solutions to contextualised sense-making practices which, ultimately, give rise to narratives about the problem situation that are either endorsed or rejected. The selection and deployment of appropriate routines in contextual sense-making practices is, thus, a key facet in the generation of endorsed narratives for those practices.

To illustrate this point more clearly, consider once again the example of the examination question (DBE, 2013b, p. 6) (given in Figure 39 on page 281 above). The various signifiers in the text of this question − and particularly the inclusion of a formula with prescribed variables, operations and entities – index the requirement for specific mathematically structured routines involving substitution and calculations (including multiplication, addition, powers, subtraction, and division). And, the ability to generate a narrative in the form of a single calculated solution value for this problem that will be endorsed by the examiner is directly dependent on recognition of the need to employ this routine of substitution. The ability to utilise the routine and the consequent mathematical calculations with accuracy and appropriate mathematical sequencing and structure is also required. This emphasis on a specific calculation-based routine and the accompanying expectation for the generation of a mathematically structured narrative is clearly evidenced through the way in which the solution for this question is presented in the memorandum for the examination paper (DBE, 2013c, p. 9) (see Figure 44 below).

Question	Solution	Explanation
2.3	i = 9% pa n = 24 months A = R104 753,89 $x = \dfrac{\text{R}104753{,}89 \times \dfrac{9\%}{12}}{\left[\left(1+\dfrac{9\%}{12}\right)^{24}-1\right]}$ ✓A ✓SF ✓A = R4 000 ✓CA **OR** $x = \dfrac{\text{R}104753{,}89 \times \dfrac{0{,}09}{12}}{\left[\left(1+\dfrac{0{,}09}{12}\right)^{24}-1\right]}$ ✓A ✓SF ✓A = R4 000 ✓CA **OR** $x = \dfrac{\text{R}104753{,}89 \times 0{,}0075}{\left[\left(1+\dfrac{0{,}09}{12}\right)^{24}-1\right]}$ ✓A ✓SF ✓A x = R4 000 ✓CA **OR** $x = \dfrac{\text{R}104753{,}89 \times 0{,}01}{\left[(1+0{,}01)^{24}-1\right]}$ ✓A ✓SF ✓A x = R3 883,59 ✓CA	1A interest rate per month [**Note: do not penalise if % sign is omitted but calculation is done correctly**] 1SF substitution 1A number of months 1CA simplification **OR** 1A interest rate per month 1SF substitution 1A number of months 1CA simplification **OR** 1A interest rate per month 1SF substitution 1A number of months 1CA simplification **OR** 1A interest rate per month (**NPR**) 1SF substitution 1A number of months 1CA simplification **NPR** **Correct answer only: full marks** (4)

Figure 44: Memorandum for a Mathematical Literacy examination question

Analysis of the memorandum clearly reveals an explicit requirement for the utilisation of mathematical routines, rules, techniques, contents and associated skills for the generation of an endorsable solution narrative. And, given that most of the mark allocations shown in the margin on the right hand side of the memorandum demonstrate the association of marks to a calculation component (e.g. simplification) or a mathematical process (e.g. substitution), a mistake in mathematical technique will, in all likelihood, result in only a partially endorsed narrative (which, in turn, is evidenced by the non-awarding of a mark).

The statements above point to a direct relationship between: the signifiers present in a text or activity; the routines and narratives employed in the generation and employment of a solution strategy for a problem scenario; and the structure of the narratives about the problem scenario (commonly in the form of a stated solution) that are ultimately endorsed and legitimated as reflecting a valid solution or (re)construction of the aspect of reality represented in the problem scenario. By way of elaboration, the signifiers (words/vocabularies and visual mediators) in a segment of text index not only the object or focal event under investigation, but also the type and structure of the routine(s) required for engagement in relation to that event. Furthermore, the nature and structure of these signifiers – and particulary whether the signifiers index objects and routines embedded primairly within the mathematical domain or a contextual domain or from both domains – also influence the nature and structure of the narratives that must be accessed and employed about the problem scenario, about the routines that are utilised in the generation of a solution for the problem scenario, and also about the solution itself. It is, thus, the combination of the selected and utilised routine(s) together with the nature and structure of the narrative(s) employed which influences and impacts on whether a particular narrative (and associated solution strategy) to a contextual sense-making practice is endorsed or rejected[216]: appropriate interpretation of signifiers, with consequent identification and utilisation of consistent narrative(s) and routine(s), leads to a generated narrative for the problem scenario about the object or focal event that is endorsed; and selection and employment of the incorrect or inconsistent routine and narrative may lead to a solution narrative that is rejected. This relationship of signifiers – routines – narratives is illustrated in Figure 45 below.

[216] Notice that I identifying two different sites in which 'narratives' are employed. Firstly, narratives are employed throughout the problem-solving process as communication occurs about the contextual environment under analysis, about the problem scenario to be dealt with, and about the routines and strategies employed through engagement with the problem scenario. The second site involves the use of narrative in presenting and describing the determined solution for the problem scenario or (re)construction of an aspect of reality. And, it is in this second site that endorsement or rejection of the narrative that describes the solution or (re)construction of reality occurs. This endorsement or rejection, in turn, is affected by whether the determined solution or (re)construction is consistent with the routines and narratives indexed through the signifiers in the original text – as interpreted by the person with enough power to endorse or reject. This issue is revisited on page 291 below when the contextually oriented discourse characteristic of endorsed narratives is dealt with.

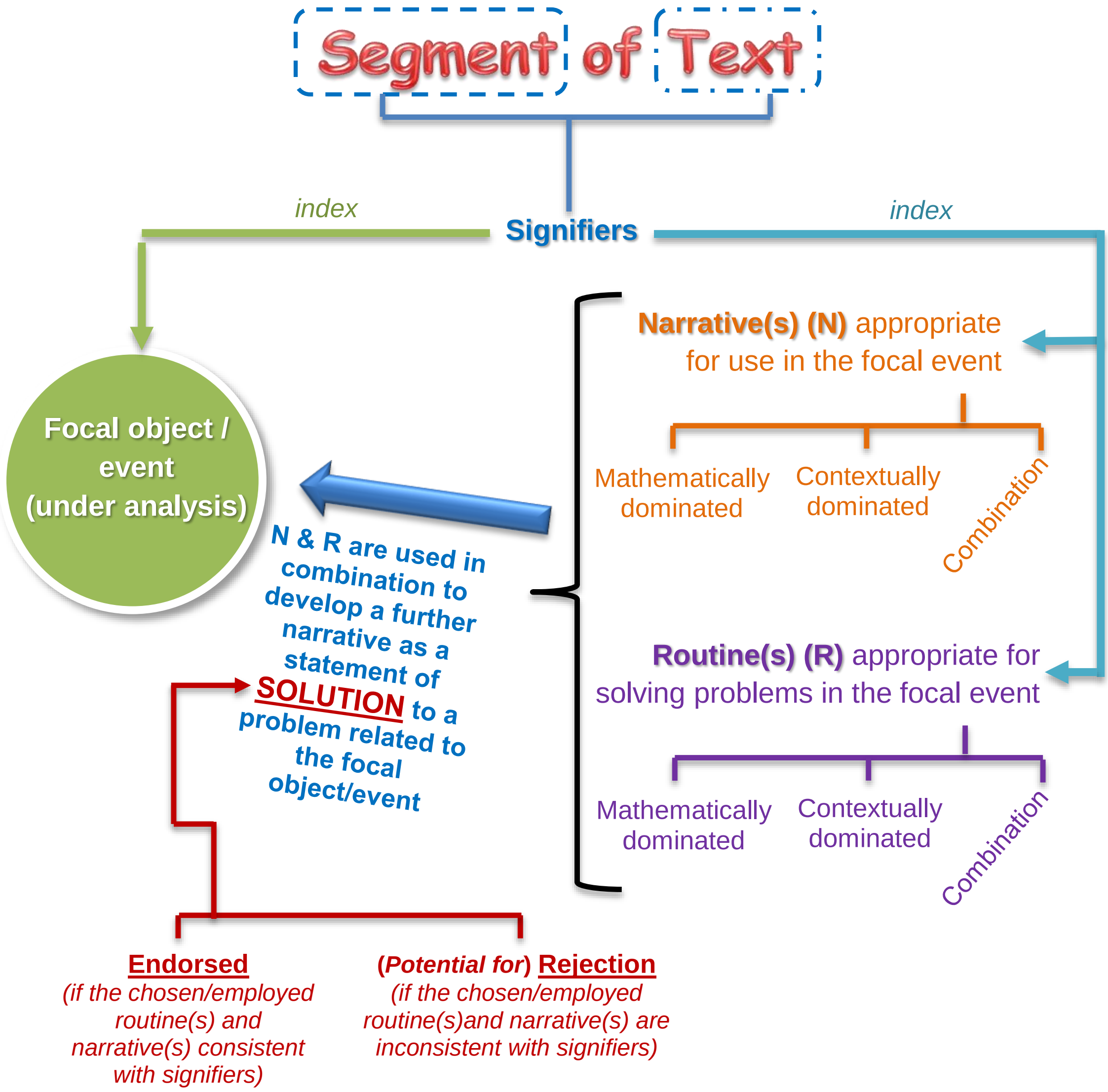


Figure 45: Relationship of signifiers - routines – narratives

Notice from the diagram that signifiers in a segment of text index routines and required narratives that comprise either predominantly mathematical structures, or predominantly contextual structures, or a combination of both. For example, a question may ask for a calculation to be performed, or for the provision of an opinion (based on analysis of contextual elements of a problem scenario), or a calculation followed by an opinion (or vice versa). In this sense, the signifiers in the text index for participants whether an endorsed narrative and solution strategy for the problem is generated through access to routines embedded primairly in the mathematical terrain, the contextual, or in both terrains. And, so, the use of a routine from a terrain that is not indexed through the signifiers − and/or the use of narratives to describe and discuss the context, problem scenario and associated solution strategy that is inconsistent with the signifiers − opens the potential for a rejection of the developed narrative and associated solution. Reflecting this comment back on the example of the question drawn from the Mathematical Literacy examination calculation (given in Figure 39 on page 281 above), an attempt to answer this question through estimation rather or with a description of the process rather than with an actual calculation will, in all likelihood, result in a solution narrative that is rejected either in part or in whole.

As a final comment in relation to contextual routines, it is also necessary to point out that, specifically in the case of mathematical discourse, Sfard's conception of routines also includes routines involving such things as categorising mathematical objects, identifying similarities and differences, justifying, and generalising (Brodie & Berger, 2010, p. 172). However, in the case of contextual routines engaged in a contextually orientated knowledge domain, Bansilal (2013) separates out these types of routines as part of what she refers to as *contextual reasoning* (see immediately below). As such, in relation to the discursive resources embedded in a contextually orientated knowledge domain, contextual rules or routines refer primarily to physical routines, tasks and/or procedures that facilitate engagement with the focal event of a contextual environment and the generation of narratives for problem scenarios encountered in that focal event.

The fourth contextual resource identified by Bansilal (2013) – that of (iv) *Contextual Reasoning* – is the "reasoning, arguments, assumptions, and justifications about issues arising in the context." (Bansilal, 2013, p. 7). As discussed previously, Contextual Reasoning involves critical analysis of a variety of issues relating to the site of analysis (focal event), including questioning such things as: the meaning and significance of certain signifiers present in a text and the relevance of these signifiers for the problem scenario and for the types of routines and narratives required to generate a further narrative (about the solution) that is endorsed; the impact of the deliberate selection of a focal event for analysis on the view generated about the wider contextual environment in which the focal event is embedded; the rationale for the inclusion and/or exclusion of certain elements of the focal event and/or wider contextual environment in the contextual sense-making process; the impact of the participants' historical, cultural and economic backgrounds on the way in which they think about, behave and make decisions in the contextual situation; and the role of other extraneous qualitative factors which impact on actions and decision-making processes in contextual situations. However, as also argued previously in Part 4 of the study, Contextual Reasoning alone is not enough given that activities that draw on the knowledge domain of mathematical literacy draw on processes and knowledge from both contextual and mathematical terrains. It is for this reason that I made the argument for Contextual Reasoning to be accompanied by various levels of Reflective Knowing which facilitate specific reflection on mathematical elements of contextual sense-making practices and, particularly, on the possible implications of the formatting power of mathematics in generating particular descriptions of reality. The combination of Contextual Reasoning and Reflective Knowing give rise of the Reasoning and Reflection domain of practice of the knowledge domain of mathematical literacy.

Importantly, the combination of Contextual Reasoning and Reflective Knowing is the glue that brings together the other three discursive resources (words/vocabulary, visual mediators, contextual routines) and which facilitates comprehensive engagement with a specific focal event embedded in a wider contextual environment. Namely, it is Reasoning and Reflection that facilitates appropriate identification of the focal event under investigation, recognition of the constraints and/or boundary of the contextual environment in which the focal event is embedded, identification of variables to be considered or rejected, successful and appropriate interpretation and engagement with contextual and mathematical signifiers (words/vocabulary and visual mediators) and routines, and analysis of the appropriateness of a particular solution or answer in relation to contextual constraints.

Returning to consider the example of the examination question (DBE, 2013b, p. 6) (given in Figure 39 on page 281 above). Given that this question is dominated by a requirement for the utilisation of mathematical routines (involving substitution in a formula and engagement with various calculation operations), several levels of reflection on the mathematical elements of the problem situation are required. These include: identifying which values and entities are of relevance for substitution in the formula; how and the order in which the various calculations in the formula are to be conducted; and whether the calculated answer provides an appropriate answer with respect to the focal event under investigation. However, engagement with the question text also requires reasoning on various contextual elements of the text and focal event under analysis, including understanding of the meaning of contextual entities such as interest rate and deposit. Successful engagement with this problem situation is, thus, dependent on reasoning and reflection of both contextual and mathematical entities of the problem scenario.

In sum, it is the domain of Reasoning and Reflection – and the associated elements of Contextual Reasoning and Reflective Knowing − that makes possible appropriate and successful engagement with a contextual environment and the available contextual discursive resources present in that environment. It is for this reason that Bansilal (2013) claims that the process of contextual reasoning (Reasoning and Reflection in this study) incorporating, as it does, the other three discursive resources, facilitates the construction of *endorsed narratives*[217] in a contextual environment.[218]

Drawing on the discussion above, the fifth discursive resource characteristic of the discourse form employed in a contextually oriented domain is that of (v) *narrative*.[219] In Sfard's (2008, p. 134) original conception, narrative is "any sequence of utterances framed as a description of objects, of relations between objects, or of processes with or by objects ...". In the specific site of the knowledge domain of mathematical literacy – characterised as it is by a blending of contents and knowledge from contextual and mathematical terrains – one form of narrative is as follows. Namely, as descriptions of the elements of a contextual problem scenario, together with descriptions of the relations between both contextual and mathematical elements of that scenario and of the routines indexed by the signifiers in the text and required for the development of a solution to the problem scenario. In other words, narrative is about the way in which participants talk about the problem scenario, and the descriptions they offer of that scenario and of the methods and techniques to be employed in contextual sense-making practices. I refer to this narrative as 'narrative-of-process'. Another form of narrative is as description or statement of the outcome of a particular problem-solving process – in other words, the solution(s) that participants offer to a problem scenario and the way in which they justify

[217] Note that Bansilal (2013) refers specifically to 'endorsed contextual narratives'. I, in turn, have chosen to exclude the specific criteria of 'contextual' and, rather, to refer simply to 'endorsed narratives'. This is because in practices associated with the described knowledge domain of mathematical literacy, narratives based on knowledge and structures from both mathematical and contextual knowledge terrains are possible. In the context of this study and this immediate discussion, specification of 'contextual' only would restrict narratives to those that draw on knowledge and techniques embedded in contextual terrains only. In adopting this position I am certainly not suggesting that Bansilal is indicating that mathematically structured narratives are not possible or endorsable in the knowledge domain of mathematical literacy. Rather, in this study I have created a more bounded distinction between mathematical and contextual knowledge structures than has Bansilal in her own work. As such, a more bounded description of possible formats of endorsed narratives is also necessary here.

[218] These 'endorsed contextual narratives' − specifically related to discourses based in contextual environments − are akin to the characteristic of endorsed narratives identified by Sfard (2007, 2008) in her general discussion of the characteristics of discourses. In making this connection between contextual reasoning and endorsed contextual narratives, Bansilal (2013), thus, completes the translation of Sfard's work on characteristics of general discourse to the specific site occupied by contextual discourses based in contextually oriented knowledge domains.

[219] Note that the characterisation of narratives as the fifth discursive resource is my own contention and is not reflective of Bansilal's (2013) work on this topic.

and legitimise this solution(s) (i.e. 'narrative-as-solution'). This statement of outcome can be characterised by the solution to calculations, or by the voiced analysis and/or interpretation of a resource, or through the offering of an opinion or description. And, in the case of a modelling activity, this narrative-as-solution reflects the (re)described or (re)constructed segment of reality that results from the modelling process. Narrative, then, permeates not only the entire problem-solving process, but is also reflected in the 'story' that is presented as a description of the solution to the problem scenario encountered in the contextual scenario under investigation.

Reiterating an earlier comment, a narrative is the *outcome* of a discursive process that begins with interpretation of signifiers (mathematical and contextual words/vocabulary and visual mediators) to facilitate identification (and also utilisation) of an appropriate routine(s) that enables the generation of a suitable and legitimate narrative. This narrative is then presented as a solution to a particular problem scenario, a description of a problem-solving process, or a description of a (re)constructed segment of reality. Furthermore, as highlighted in the discussion and in Figure 45 on page 289 above, there is the potential for this generated narrative (in part or as a whole) to be endorsed or rejected. A narrative-as-solution for a text-based activity is likely to be endorsed if there is correlation between the way in which the task developer intends for the signifiers to be interpreted and the way in which a participant actually interprets the signifiers. This is evidenced in the narratives that they develop and employ about the problem scenario and the routines utilised in the generation of these narratives. In the situation where the narrative-as-solution represents a description or (re)construction of a segment of reality (through, for example, a modelling process), an endorsed narrative reflects consistency with a form of participation that is seen as legitimate or appropriate in the eyes of other participants who engage in the real-life setting on a regular basis.

By way of illustration, consider again the example of the examination question (DBE, 2013b, p. 6) (given in Figure 39 on page 281 above). If we consider this question in conjunction with the associated memorandum for the question (shown in Figure 44 on page 287 above), then it is obvious that there is an explicit expectation for participants who engage with this question to generate a narrative-as-solution that is based on a mathematical structure driven by mathematical routines involving substitution and various calculation operations. Furthermore, every aspect of the narrative-of-process that ultimately leads to the narrative-as-solution is similarly legitimised according to mathematical structures and knowledge. An endorsed narrative for this question is, thus, dependent on a learner being able to successfully identify the values of relevance for substitution into the formula, followed by accurate substitution, then by correct mathematical operations on the substituted values. Finally, perhaps, recognition is required of the need to present the answer as a monetary value with a currency symbol and rounded off two decimal places (although there is not specific indication given for this requirement in the stated mark allocation descriptors). An attempt to generate a solution through a technique that does not involve a formal mathematical calculation (e.g. estimation) will in all likelihood result in the presented narrative-as-solution being rejected either in part or in whole by the examiner.

There are two further issues of particular interest in the memorandum that serve to highlight the requirement for the generation of an endorsed narrative based on the utilisation of almost entirely mathematically structured routines. First is the instruction given in the right hand column of the memo: "[Note: do not penalise if % sign is omitted but calculation is done correctly]". In essence, this instruction negates the need for learners to consider the importance of the inclusion of context-specific notation in the act of communicating their generated narrative – which, in the context of an everyday or workplace practice, could lead to disastrous consequences. Instead, in the context of this examination question, primary (and arguably exclusive) concern is whether learners are able to perform the calculation with accuracy. This instruction, thus, provides a further clear statement of the prioritising of mathematical structures and processes, and the relegation of considerations of the significance and importance of contextual entities for the problem scenario. Secondly, if the inserted currency symbols and text 'pa' and 'months' were excluded from the memorandum, then there would be nothing to distinguish this solution from a solution that might be found in a scientific mathematics activity. In fact, this very same formula and calculation are part of the curriculum for the scientific Core Mathematics subject. The endorsed narrative and associated routines shown in the memorandum are so dominated by mathematical structures that it becomes difficult in this instance to discern how the generation of an endorsed narrative for this this question reflects a form of mathematically literate behaviour that is different to the type of skills required for successful engagement in a scientific mathematics course.

It is also worth mentioning that despite the inclusion of various contextual elements in this question, it is possible to answer this question without having any understanding of the contextual environment in which this problem situation is embedded. Furthermore, engagement with this question does not assess or assist in the development of in-depth understanding of this contextual environment. This is because the question is driven almost entirely by an agenda for the assessment of mathematical skills and contents (c.f. Agenda 2 [b] in the spectrum of agendas given on page 39 in Part 2 of the study) and not by an agenda for contextual sense-making practices to facilitate enhanced understanding of the contextual environment in which the problem is embedded.[220] Even the signifier 'compounded monthly' is of little significance here because the formula has been constructed in such a way that understanding of this signifier is not essential for successful engagement with the formula. In short, it is possible to answer this question with little or even no understanding of the context of investments and interest calculations simply by recognising the values to be substituted into the formula, the correct positions of those values in the formula, and by accurately performing the various operations and calculations in the formula. The endorsed narrative-as-solution for this question is based on the correct utilisation of the given formula and on the appropriate and accurate demonstration of mathematical skills involved in substitution and calculations involving multiplication, addition, powers, subtraction, and division (in that order). There is little or no requirement for understanding of or engagement with the contextual environment in which the problem scenario is embedded.

This, then, brings to an end the discussion and overview of the discursive resources – words/vocabulary, visual mediators, routines and narratives − that characterise the type of discourse commonly associated with a contextually orientated knowledge domain like the knowledge domain of mathematical literacy. In the next subsection I now present a further contention that it is the discursive resources of routines and endorsed narratives, specifically, which determine the dominant domain of practice that is prioritised in an activity relating to this knowledge domain.

[220] There are no further questions attached to this question or associated with the contextual environment under investigation in this question in the examination paper. In other words, this question − dominated as it is by an agenda for assessment of mathematically structures contents, knowledge and skills − represents the sum total and entirety of analysis and investigation of this specific focal event and contextual environment.

17.2.3 The importance of routines and endorsed narratives for identifying the dominant domain of practice that is prioritised

Crucially, in the context of this study, it is the discursive characteristic of the endorsed narrative for an activity that is analysed as a particular focal point for identifying and characterising the domain of practice (and associated structure of knowledge) that is prioritised in an activity and through which participation in the activity is legitimised. By way of explanation. The signifiers direct attention towards the need for engagement with contents embedded in a particular terrain (and, hence, towards a particular knowledge construction about a problem situation). The signifiers also index particular routines/rules/procedures to be followed. However, it is the structure of the endorsed narrative(s) that reflects precisely how legitimate knowledge and participation are perceived in an activity. It is also the structure of the endorsed narrative that reflects whether the knowledge and associated routines required are to be drawn and legitimised primarily from the perspective of a mathematical gaze (i.e. the Mathematical Competency domain of practice), or from the perspective of the participants who engage in real-world practices (i.e. the Everyday domain of practice), or through an attempted reconstruction of a segment of reality (i.e. the Modelling domain of practice). From this perspective, it is possible that the routines that are endorsed and the narratives that are legitimised might draw from a different domain than the statement or question in an activity. For this reason I contend that it is not enough to simply look at the signifiers (words/vocabulary and visual mediators) present in a text and then to state from these signifiers which domain of practice is prioritised in an activity. Rather, the signifiers must be analysed in conjunction with and in comparison to the routines and narratives that are encouraged and ultimately endorsed through the narrative-as-solution.

CHAPTER 18
OPERATIONALISING THE EXTERNAL DIMENSION

The operationalisation of the external dimension of the language of description for the knowledge domain of mathematical literacy occurs on two levels. To begin with, the elements of the external language are used to identify and differentiate the discursive resources – including words/vocabulary, visual mediators, routines and endorsed narratives − that characterise the domains of practice that comprise the knowledge domain of mathematical literacy. The role of Reasoning and Reflection in facilitating the generation of endorsed narratives in each domain of practice is also be explored. This level of operationalisation, thus, provides an explicit framework for distinguishing and categorising different practices and/or activities according to the characteristics that the activities share with the discursive resources of each of the domains of practice.

At a second level of operationalisation, an especially developed exemplar text-based Mathematical Literacy task is introduced and utilised to demonstrate how the components of the external language can be used to identify and analyse the discursive resources in the questions in the activity. Furthermore, based on the characteristics of these discursive resources, individual questions are categorised, located and distinguished with the Everyday, Mathematical Competency, and Modelling domains of practice of the knowledge domain of mathematical literacy. Specific elements of the activity are also analysed through the lens of the Contextual Domain, and the role of Reasoning and Reflection in facilitating successful engagement with the demands of the activity is highlighted. Following this, analysis of the spread of questions associated with each domain of practice is provided to facilitate postulation on the dominant domain prioritised for the activity as a whole.

Crucially, a key goal in the operationalisation of the external language at these two levels is to demonstrate the validity and utility of the external language of description for facilitating a gaze to be cast from the internal language of description over empirical phenomenon (in this case, textual phenomenon). It is my belief that this correlation between the external and internal languages of description is evident in the operationalisation process.

18.1 Level 1 Operationalisation of the external dimension of the language of description (in relation to the domains of practice of the knowledge domain of mathematical literacy)

As a precursor to employing the external language of description to identify the structure or knowledge in the Everyday, Mathematical Competency and Modelling domains of practice, it is necessary to reiterate a key point. Namely, that the knowledge domain of mathematical literacy is characterised as a blended domain such that engagement in the practices of the domain entails engagement with both contextual and mathematical forms of knowledge and discourse (including signifiers, routines and narratives). This is an essential point for consideration since it highlights that any contextual sense-making activity that draws on this knowledge domain inherently always involves both contextual and mathematical elements. For this reason, it is envisioned that the structure of knowledge in each domain of practice of the knowledge domain of mathematical literacy inherently involves the use of discourse that includes both contextual and mathematical discursive characteristics. However, it is also envisioned that these ever-present contextual and mathematical resources are differently prioritised in each domain of practice, with participation in certain domains endorsed and legitimised through heightened emphasis on the generation of narratives that draw primarily on contextual knowledge, and participation in others on access to more mathematically structured narratives. In light of the above, my primary objective in this section of the chapter is to analyse how different activities differently prioritise the need for contextual or mathematical forms of engagement and communication, and how this differential prioritising affects the criteria according to which successful participation in an activity is endorsed and legitimised.

I return now to application of the external language to identify the structure of the discursive resources that characterise practices and forms of participation associated with each of the domains of practice of the knowledge domain of mathematical literacy.

18.1.1 The characteristics of discursive resources in the Everyday domain of practice

As a brief reminder, the Everyday domain of practice of the knowledge domain of mathematical literacy is characterised by the use of knowledge and associated practices that reflect and resemble how participants engaged in real-life situations might think and act as they solve problems and make decisions in those situations. Much of the problem-solving techniques that characterise this domain of practice rely on localised understanding of the real-life contexts in which the problem situations are embedded and of legitimate and appropriate forms of thought, action and communication in those contexts. Some of the practices associated with this domain may draw on formalised discipline-specific knowledge (such as formal mathematics). However, such practices need to be accompanied by the ability to successfully navigate and engage with a variety of (often qualitative) factors which impact on successful and legitimate participation in a context, some of which are extraneous and unrelated to the formalised discipline-specific knowledge employed to solve problems. When operating in this domain of practice, of primary importance is establishing a way of working in a real-life context that reflects the behaviour of people who are engaged in that real-life context on a regular basis.

Importantly for this immediate discussion, discourse in this Everyday domain of practice is largely characterised by the use of colloquial discourse based on how people who actually engage in the real-life contexts under investigation actually communicate and converse as they go about their daily business. Furthermore, where more specialised language is used, this language is specialised largely in relation to context-specific entities or contents and is not necessarily specialised in the sense of being generalisable beyond the immediate context of use. Being able to communicate effectively in a real-life situation requires intricate understanding of the context and of the type of words, language, symbols, techniques and practices that are endorsed within the context and which enable successful communication with other participants in the context. For a participant to participate in a context in a way that is endorsed and considered legitimate, they must act, behave and talk in the way that the other participants who are already engaged in the context act, behave and talk. Successful and endorsed participation in a contextual problem situation from the perspective of the Everyday domain of practice involves thinking, acting and communicating in a way that reflects a high degree of correlation and authenticity to real-life practice.

Following this line of thinking, the discursive resources that characterise discursive practices in the Everyday domain similarly reflect the way people talk, think, and act in real-life contextual practices. Practices associated with this domain of practice require and involve the interpretation and use of context-specific ***words/vocabulary*** and ***visual mediators***. Successful engagement in a problem situation then requires the ability to communicate using the context-specific words/vocabulary, context-specific notation and symbols, and context-specific representations – all of which enable successful communication and engagement with the other participants in the context. Importantly, this certainly does not exclude the presence of possible mathematical words/vocabulary and visual mediators in practices and texts associated with this domain of practice. However, when encountered within this domain of practice, any mathematical elements are directly tied to specific components of the context situation under investigation and facilitate access to and/or understanding of the context. In other words, the mathematical elements do not reflect an imposition of mathematics on the contextual situation (a 'mathematical gaze'), but, rather, reflect the genuine presence of mathematical entities in daily practices in the context. In such situations, legitimate and endorsed participation in the context is reflected through the ability to successfully interpret, navigate and engage with the relationship between contextual and mathematical elements of the context.

As an example of the nature of the words/vocabulary and visual mediators that characterise practices in this Everyday domain of practice, consider a classroom-based activity that explores a DIY project of painting a room where the painter needs to work out the quantity of paint needed. From the perspective of the Everyday domain of practice, legitimate participation in the activity must include the need for recognition that the person involved in the painting process may make use of terms such as 'amount of paint' (as opposed to a more formal mathematical notion of 'spread rate') and 'approximately' (to indicate understanding that paint quantities are estimated quantities and that painting is not always an exact science). Further recognition is required that the painter might not be particularly concerned over the need to include mathematically appropriate notation (such as indicating area values in units^2) in any calculations performed to determine paint quantities. This is because, ultimately, the primary goal for the painter in the painting process is the successful completion of the painting project; and of, perhaps, secondary or lesser concern is whether the correct words and notation have been used in this process. Importantly, however, recognition is also necessary that the person doing the painting must have at least some knowledge of the appropriate way in which to communicate about their painting requirements and of the words/vocabulary appropriate to this context – or else they might never be able to explain to someone else what materials they need. Furthermore, there must also be recognition that the painter must have some knowledge of the painting process and of how to interpret words/vocabulary and visual mediators that are presented to them in relation to the painting process. For example, to successfully complete the painting job, the painter must be able to read and understanding the instructions given on the back of a paint tin, including information on how to prepare the walls, the possible need for a primer, and possibly also the given spread rates. The point I am making here is that although this practice is able to be completed without access to specialised painting language, some understanding of the context of painting and of the notation and terminology employed in this context is necessary for successful and endorsed participation in the context (and also for happy marriages!).

Practices that reflect characteristics of this domain of practice also require and involve the use of ***routines*** (as indexed through the presence of context-specific words/vocabulary and visual mediators) that have context-specific application and structure. For example, the collective signifiers 'inflation rate of 10%' index the possible requirement for a routine involving a calculation with a percentage value. However, the specific nature and structure of this calculation and of how the percentage value is to be used and interpreted is dependent on the specific context in which the percentage is employed and/or encountered and on the forms of legitimate engagement with percentage values that are endorsed in that context. So, even though practices in the Everyday domain of practice may be characterised by formal mathematically structured routines, these routines are directly influenced by the specific and localised requirements of the context. Furthermore, many of the routines employed in this domain of practice draw on less formal, less generalisable and more context-dependent techniques for solving problems – such as estimation and problem-specific rounding. In sum, in this domain of practice, successful engagement with a routine is dependent on the ability to identify the necessary and relevant structure of the routine to be employed in a specific contextual environment, to employ that routine with a structure of working that is appropriate and endorsable in the contextual environment, and to communicate the results of that routine with contextually legitimate and endorsable narrative(s).

Reflecting again on the example of a classroom-based activity that explores a DIY painting project, from the perspective of the Everyday domain of practice legitimate participation in the activity must include recognition that the person doing the painting may employ a routine for some sort of informal measurement of the dimensions of the room – perhaps with a ruler, or with one arm stretched out as an estimation of one meter, or perhaps by taking strides across the room − so that they are able to communicate to a shop assistant at the building supply store about their paint requirements. Importantly, measurement of these types of dimensions often involves a relative degree of inaccuracy and it is not common in the context of this type of DIY project for either the painter or the person who sells the painting supplies to insist on a high degree of accuracy. Further recognition is required that the painter may also decide to employ a calculation routine (for the surface area of the walls) in order to more accurately communicate their paint requirements to the building supply store. However, this type of calculation in this type of context commonly also does not include a demand for a high degree of accuracy, either in terms of measurement of dimensions, or in terms accurate calculation, or in terms of issues involving rounding (e.g. to whole units verses working to a certain number of decimal places). Such calculations might also only give cursory consideration to the presence of windows and doors in the room and the implication of these open spaces for the quantity of paint required. Then there is the need for recognition that when buying the paint the painter might employ a routine that involves selecting combinations of paint tin sizes based on the cost effectiveness of certain tin size combinations. Or the painter might deliberately buy more paint than is required to account for possible wastage issues, or may buy less to save money and with the intention of thinning the paint. There must also be recognition that although the painter might think they are buying the right amount of paint, if they run out they can always return to the shop to buy more; and, if they have paint left over, then this is also not an issue because the additional paint can be used for future painting jobs. In other words, from the perspective of participation in the Everyday domain of practice, accurate calculation of paint quantity is not a necessary requirement for endorsed participation in this context. Instead, in this context, the difficulty with accurate measurement is acknowledged and, as a result, estimation is positioned as an appropriate and legitimate means of working. This does not mean that accuracy is not valued, but, rather that for this specific activity estimation provides an adequate, appropriate and possibly more efficient means of working.

Practices associated with the Everyday domain of practice are further characterised by ***narratives*** that reflect the nature and structure of authentic participation and practice in a real-life context. For a narrative to be endorsed in this domain of practice it must reflect genuine understanding of the context in which the problem situation is embedded and of the factors which affect and determine the nature and structure of participation and decision-making in the context. This involves the appropriate interpretation and use of words/vocabulary and visual mediators, together with identification and utilisation of routines that are appropriate and relevant to the context. An Everyday endorsed narrative to a problem embedded in the Everyday domain of practice must reflect consistency between the signifiers embedded in the setting and any routines employed in relation to the problem. The endorsed narrative must also provide a solution, description or explanation of the problem situation that provides an accurate and authentic view of the structure and nature of real-world participation in that context.

Reflecting back now on the example of a classroom-based activity that explores a DIY painting project. From the perspective of legitimate participation in the Everyday domain of practice an endorsed narrative to the problem of how much paint is needed must take into account that: paint is sold in litres (and not in square units – which is the outcome that a formal calculation provides) and also only in full litres in certain tin size combinations; paint quantities are always rounded up; additional paint is commonly bought to account for the fact that painting is not an exact science and commonly involves spillage and wastage. From the perspective of the Everyday domain of practice, an endorsed Everyday narrative for a painting activity must comprise a solution to the activity and communication of this solution that reflects how a person who is actually engaged in a painting activity might act and speak about the activity. The endorsed narrative must also recognise the types of considerations and factors which might influence their actions and decision-making processes.

The domain of *Reasoning and Reflection* also takes on specific characteristics in practices associated with the Everyday domain of practice. Given that participation in Everyday domain practices involves replicating the nature and structure of legitimate everyday real-world forms of participation, reasoning and reflection, particularly, on contextual elements plays a prominent role in facilitating understanding of the context and of the elements and variables of the context that are necessary for consideration and inclusion in any contextual sense-making and decision-making processes. Contextual Reasoning facilitates identification and interpretation of the signifiers (words/vocabulary and visual mediators) that are relevant to the context, and of the contextual entities and objects indexed by those signifiers. Contextual Reasoning is also essential for facilitating identification of appropriate routines that allow for the generation of narratives that will be endorsed in the context and which provide an indication of the nature and structure of legitimate and authentic participation in the context. Where some degree of mathematical structure, knowledge and routine is required, this must be accompanied by a level of reflection on mathematical elements to ensure that the employed methods reflect consistency with the demands of the mathematical terrain and also with the structure of appropriate, endorsed and legitimate participation in the context. Importantly, in this Everyday domain of practice, successful mathematical forms of participation and associated reflection is facilitated through prior reasoning on and understanding of the contextual elements, demands, and constraints of the problem-scenario.

In summary, participation in practices associated with the Everyday domain of practice of the knowledge domain of mathematical literacy is characterised by the generation of narratives that reflect the nature and structure of authentic participation in real-life contexts as experienced by participants who engage in those contexts on a regular basis. The generation of such narratives is further characterised by the interpretation and use of words/vocabulary and visual mediators that reflect localised and context-specific meanings, and which signify specific context-dependent contents and objects. The generation of endorsed Everyday knowledge narratives also involves identification and use of specific routines whose structure and means of deployment are similarly influenced by the structure of the contexts in which they are engaged. The generation of endorsed narratives within the Everyday domain of practice and consequent legitimised participation with practices in this domain requires stated evidence of understanding of the contents of the context and, particularly, of the nature of authentic, realistic and context-appropriate forms of participation and communication.

18.1.2 The characteristics of discursive resources in the Mathematical Competency domain of practice

Practices in the Everyday domain of practice are characterised by an appeal for understanding and replicating the knowledge and techniques used by people who are actively engaged in real-world problem situations. Practices in the Mathematical Competency domain of practice, by contrast, are characterised by distinctive attempts to draw on technical and specialised mathematical knowledge and to employ mathematically structured techniques to make sense of and find solutions for problems encountered in real-world settings. Problem-solving from the perspective of the Mathematical Competency domain of practice involves the imposition of a mathematical gaze over real-world practices, and directed focus on mathematical elements of a problem situation and associated mathematically structured techniques that facilitate successful engagement with these elements. Practices associated with this domain of practice are founded on the belief that mathematics provides a necessary and appropriate tool for (re)describing real-world practices and for enabling sense-making of problem situations encountered in those practices: those who are capable of using use mathematics are empowered to better understand and engage in the world. Practices in this domain also place a high priority on the accuracy of mathematical working and solutions, and of the appropriate use of mathematical notation, language and routines. This does not mean that contextual elements of a problem scenario are not recognised, considered and valued in this domain of practice; rather, that mathematical considerations and structures are generally prioritised over contextual equivalents, and formal mathematical knowledge, solutions and workings are seen as essential and necessary tools for engagement in real-world problem scenarios. Narratives for problem situations developed in this domain are only endorsed if they embody appropriate mathematical words/language and mediators, and demonstrate understanding of mathematically structured routines. Furthermore, practices in this domain are accompanied by a dual agenda: alongside (and sometimes overshadowing) an agenda for contextual sense-making is a further (often dominant) agenda for the development or enhancement of mathematical knowledge (c.f. Agenda 2 [b] in the spectrum of agendas presented in Part 2 of the study on page 39 above).

Importantly for this immediate discussion, discourse in this Mathematical Competency domain of practice is commonly characterised by the use of and reference to distinctly mathematical contents alongside reference to contextual entities relevant to the context under investigation. Effective and successful communication in practices associated with this domain requires access to mathematical knowledge and contents, together with the ability to engage with mathematical elements of contextual situations and to generate mathematically structured narratives-as-solutions for problem situations encountered in those situations. In this domain of practice it is not enough to simply understand the context and how real-life practitioners might think, act, talk and behave in that context.

Rather, successful participation in practices associated with this domain − and the ability to communication successfully with other participants in the domain − involves being able to solve problems and communicate in a way that reflects the legitimate practices of a mathematician. Successful and endorsed participation in a contextual problem situation from the perspective of the Mathematical Competency domain of practice involves thinking, acting and talking in a way that reflects a mathematised (and, possibly, a mythologised) representation and understanding of a contextual situation.

Following this line of thinking, the discursive resources that characterise discursive practices in the Mathematical Competency domain of practice reflect an emphasis on primarily mathematical knowledge, contents and structures. However, this emphasis on primarily mathematical structures is also accompanied by a requirement for engagement with contextual discursive resources that serve to describe and index elements of the contextual situation under investigation. For this reason, practices embedded in this domain of practice require interpretation and use of both context-specific and mathematically structured <u>*words/vocabulary*</u> and <u>*visual mediators*</u>. Importantly, however, successful communication in this domain of practice – and the generation of endorsed narratives for problem situations that are considered to be legitimate − requires specific utilisation of mathematically appropriate words/vocabulary and visual mediators. Communication that only makes use of contextually bound colloquial words/vocabulary and visual mediators and which does not reflect a mathematically structured (or mathematised) understanding of the context will, in all likelihood, not be endorsed by other participants in the domain. Put another way, the presence of contextual signifiers serve to index the specific contextual object or focal event under investigation and, so, direct attention on the type of problem-solving approach required. However, it is the mathematical signifiers in particular which serve to index the specific mathematical contents and associated (mathematical) knowledge and (mathematically structured) routines required to generate (mathematically structured) narratives about the context that are endorsed and legitimated.[221] In this domain of practice, the inability to successfully interpret and engage with the mathematical signifiers will, in all likelihood, result in the utilisation of inappropriate routines and the consequent generation of a rejected narrative.

The example employed above of a classroom-based activity that explores a DIY painting project again provides a suitable context for illustrating the characteristics of the words/vocabulary that embody practices associated with this domain of practice (and, so, for comparing the characteristics of discursive resources in this domain of practice with those from the Everyday domain of practice). From the perspective of the Mathematical Competency domain of practice, legitimate participation in this activity must include emphasis on an accurate calculation of the quantity of paint required. As such, a format of this activity designed for use in the Mathematical Literacy classroom from the perspective of participation in the Mathematical Competency domain will include

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[221] As a case in point, if you refer back to the Mathematical Literacy examination question presented in Figure 39 on page 281 above, I made the statement that even though the question was based on a real-life context of finance (specifically, loans and annuities), it is possible to engage with the demands of the contents of this question with little or no understanding of the context in which the problem is embedded. This is because the question is driven almost exclusively by mathematical structures and processes. As such, successful engagement with the question requires the appropriate interpretation of the mathematical signifiers in the text (specifically the given equation and associated variable descriptors) in order to correctly identify which mathematically structured routines (namely, substitution and simplification involving various mathematical operations) must be employed so that the generated narrative-as-solution is endorsed. In this question – which is a clear example of a question embedded within the Mathematical Competency domain of practice – the generation of an endorsed narrative is directly reliant on the ability to interpret and engage with mathematical signifiers, elements and routines.

mathematically directing words/vocabulary such as ‘calculate’, ‘round answers to two decimal places’, and/or ‘spread rate’. The activity might also include direct reference to a visual mediator in the form of a formal spread rate for converting accurately from square units to litres. There may also be an expectation for participants who engage in the activity to make use of appropriate notation – such as square units (e.g. m^2) when referencing calculated surface area values. From the perspective of legitimate participation in the Mathematical Competency domain of practice, participants engaged in this classroom-based activity who do not employ appropriate mathematically structured words/vocabulary, notation, and visual mediators will not generate a narrative for the activity that is endorsed in entirety.

Given the requirement and expectation in the Mathematical Competency domain of practice for mathematically derived and structured narratives about contextual situations, the ***routines*** employed in the generation of these narratives similarly embody primarily mathematical processes and structures. When operating from this domain of practice, participants draw on their mathematical knowledge and understanding as they read into problem situations the need to ‘calculate’, or ‘work out’, or ‘draw a graph’ while seeking the most appropriate mathematically structured routines and accompanying representations to generate endorsable narratives. Importantly, legitimate participation in the utilisation of these mathematically structured routines in this domain of practice must also be accompanied by emphasis on the accuracy of working and the appropriateness of the structure of the mathematical working according to mathematically defined norms.

Reflecting again on the example of a classroom-based activity that explores a DIY painting project, from the perspective of the Mathematical Competency domain of practice, legitimate participation in this activity requires an accurate calculation of the quantity of paint required. And, this accurate calculation, in turn, involves a specific requirement for the utilisation of certain mathematically structured routines – possibly calculations involving surface area (with the added requirement for as-accurate-as-possible measurement, together with additional calculations that account for the effect of window and door spaces) and calculations involving formal conversions (possibly using a particular routine for simplifying rates). Rounding routines that facilitate consideration of appropriate ways to round values in a calculation to ensure accuracy, together with consideration of the most appropriate way in which to round the final calculated paint quantity (e.g. up or down; to the nearest litre or nearest 5-litre), also need to be employed. And, throughout all routines employed there is an expectation for accuracy of working, appropriate structure of working, and inclusion of relevant and necessary notation. From the perspective of the Mathematical Competency domain of practice, participation in this activity is considered legitimate if the required mathematically structured routines for calculating area, converting between different systems, and rounding are employed with accuracy and appropriate mathematical structure.

Practices associated with the Mathematical Competency domain of practice are further characterised by ***narratives*** that accurately reflect the nature and structure of a particular mathematical gaze over a real-world situation. For a narrative to be endorsed in this domain of practice it must reflect a mathematised perspective of a real-world practice. An endorsed narrative reflects consistency and coherence between identified mathematical signifiers and employed mathematically structured routines, and embodies the accurate and appropriate use of mathematical language, mediators, and routines in communication with other members of the discourse. From the perspective of the Mathematical Competency domain of practice, the problems of the world are able to be solved using mathematics – and an endorsed narrative for this domain achieves just that: namely, sense-making practices through the use of mathematical words/vocabulary and visual mediators, and through demonstration of the utility of mathematical routines for facilitating engagement with real-world problems. Importantly, however, given the necessary presence of contextual elements in all activities relating to the knowledge domain of mathematical literacy, there is every possibility that an endorsed narrative drawn from this domain of practice may also comprise a requirement for recognition of the presence and role of these contextual entities in any mathematical working or statements. For example, context-specific notation (e.g. a currency symbol) to signify the meaning attached to specific calculated mathematical entities in relation to the context. There is also the possibility that an endorsed narrative drawn from this domain of practice may comprise a requirement for consideration of coherence or divergence between mathematical and real-world (Everyday domain) practices.

Reflecting now on the example of a classroom-based activity that explores a DIY painting project. From the perspective of the Mathematical Competency domain of practice an endorsed narrative for the activity requires a statement of a calculated solution that seems realistic from a mathematical calculation perspective (i.e. in relation to the dimensions and other values used in the calculations). This endorsed narrative must reflect the accurate interpretation of mathematical elements in the problem scenario and accurate engagement with the identified mathematical routines involving the calculation of surface area and conversions. The endorsed narrative for this situation may also be characterised by explicit recognition of the need for accuracy in all calculations and of the implication of rounding at different points in the calculation procedure on the accuracy of the calculations. For this specific situation, the endorsed narrative may also require rejection of the formal mathematical rules for rounding in favour of the use of context-based logic about quantities of paint that are able to be bought in the shops. For example, in contravention of formal mathematical conventions, a calculated answer of 4,2 litres might be rounded *up* to 5 litres to account for the fact that more than 4 litres are needed or that the paint is only sold in 5 litre tins. Depending on the task developer, the endorsed narrative may also comprise a specific requirement for the inclusion of context-based notation, such as the 'R' symbol to indicate a monetary value or the 'ℓ' symbol to indicate paint volume.

As with the Everyday domain of practice, components of the domain of *Reasoning and Reflection* also takes on specific characteristics in relation to the discursive resources of practices associated with the Mathematical Competency domain of practice. Given that participation in Mathematical Competency domain practices involves the interpretation and use of specific mathematical signifiers and routines in the generation of a mathematised perspective of a real-world situation, the dimension of reflection on mathematical elements, particularly, plays a prominent role in facilitating successful engagement with the mathematical elements of the problem situation. Mathematical Reflection facilitates appropriate identification and interpretation of mathematical words/vocabulary and visual mediators present in a contextual scenario and of the specific mathematical contents and associated routines indexed by those signifiers. Reflection specifically on mathematical elements is also essential for ensuring that routines are employed with accuracy and in an appropriate structure of working so that any narratives that are generated reflect consistency with mathematical norms. Importantly, a degree of reflection and reasoning on contextual elements is also required, particularly for identifying the specific focal event under analysis and the broader contextual environment in which that event is embedded – which, in turn, can impact on the types of resources, contents and routines that are considered legitimate for use in that event. This form of reasoning/reflection on contextual elements is further required for determining the extent to which a mathematically structured narrative provides a valid and legitimate representation of an appropriate form of participation in the context. However, although an element of Contextual Reasoning is required, in practices associated with the Mathematical Competency domain of practice it is ultimately reflection on mathematical elements that plays a dominant role in facilitating the generation engagement with mathematical signifiers and routines in the generations of mathematically structured endorsable narratives.

In summary, participation in practices associated with the Mathematical Competency domain of practice of the knowledge domain of mathematical literacy is characterised by discursive resources that reflect mathematised forms of engagement and communication in real-world problem-solving scenarios. Legitimate participation in activities and practices drawn from this domain is defined by criteria, primarily, for successful engagement with and utilisation of mathematical words/vocabulary, mathematical visual mediators, and mathematically structured routines and processes. Although engagement with contextual entities is required in the Mathematical Competency domain of practice, these entities commonly serve simply to signify the specific contextual environment in which mathematical routines must be employed and about which mathematically structured narrative must be generated. The contextual entities, in this sense, provide access to the mathematical elements, but it is appropriate engagement with the mathematical elements which ultimately lead to the generation of endorsable narratives. The generation of endorsed narratives within the Mathematical Competency domain of practice and consequent legitimised participation with practices in this domain, thus, requires demonstrated evidence of understanding of and utility with specific mathematical contents, knowledge and structures.

18.1.3 The characteristics of discursive resources in the Modelling domain of practice

The Everyday and Mathematical Competency domains of practice legitimise participation according to coherence with everyday practices and knowledge and mathematised practices and knowledge respectively. The Modelling domain of practice of the knowledge domain of mathematical literacy, by contrast, prioritises an agenda for the (re)description and/or (re)construction of a segment of reality to develop a more detailed and broader understanding of existing and possible alternative forms of participation in that segment. Activities in the Modelling domain of practice are directed towards consideration of the way in which people think, behave and act as they solve problems in everyday settings, together with consideration of the possible utility of mathematical approaches and understanding in providing alternative perspectives for solving those same problems. The reach to the mathematical terrain in modelling activities is an attempt to provide alternative methods for understanding real-world problem situations and, particularly, for finding methods which may provide access to a more varied form of participation in a contextual situation. Activities in the Modelling domain of practice are also directed towards critically analysing existing problem-solving techniques (everyday and/or mathematical) and for providing possible alternative techniques which allow for a different or more varied understanding of a real-world situation. Legitimate participation in the Modelling domain of practice is, thus, demonstrated by the ability to develop models of a real-world practice in order to better understand how people act and behave in that practice, possible alternative ways of thinking and acting, and possible alternative formats for that practice given a change in existing social, economic, and political conditions. And, reiterating an earlier discussion point, whereas legitimate participation in the Everyday and Mathematical Competency domains is defined by different forms of engagement with element of an existing practice, participation in the Modelling domain of practice is directed towards a further goal. Namely, towards generating descriptions existing practices (and different forms of participation in those practices) and understanding how participation in the practice can change if the structures that define and direct participation shift.

To achieve this objective, modelling activities drawn from the Modelling domain of practice are characterised by various processes[222] that facilitate constant interplay between contextual and mathematical elements and terrains in order to describe existing practices and, also, possible alternative forms of participation in those same practices. Contextual elements are crucial in this domain of practice since it is an agenda for a contextual sense-making that drives the modelling process. However, mathematical elements and mathematisation processes are also essential since they provide an alternative means for engaging with the contents of the context. Crucially, it is at the intersection of contextual and mathematical elements and terrains that activities associated with the Modelling domain of practice differ from activities associated with the Everyday and Mathematical Competency domains. Modelling does not involve prioritisation of everyday practices and knowledge over mathematical equivalents (or vice versa). Rather, modelling strives to find ways in which mathematical practices and knowledge serve to compliment everyday practices and knowledge. Modelling rejects mathematised versions of reality that do not give credence to the central role of everyday factors in influencing action, behaviour and though, and mathematised versions of reality that downplay or contradict everyday practices.

By way of illustration of the discursive resources that characterise practices associated with the Modelling domain of practice, a possibly more suitable context is necessary to the painting scenario discussed previously. Consider, instead, the scenario of prolonged strike action involving civil service employees (including teachers) demanding a higher salary increase than has been offered by the State departments (see Figure 46 on page 307 below). In this instance, the civil service employees embarked on strike action for almost three weeks, during which a 'no-work, no-pay' policy was enforced by the employer. In relation to the Modelling domain of practice, one possible modelling activity for this context involves analysis of the loss of income incurred by the striking workers during the strike action and the amount of time that it takes the workers to recoup this lost pay if the employer agrees to their demands (and/or if the employer does not agree at all, or only agrees in part). It is important to highlight certain facets of the modelling process for this scenario. To begin with, when viewed from the perspective of participation in the Modelling domain of practice, the intention for the model is to provide a means for developing a broader understanding of the strike scenario and, particularly, of the wider financial consequences of strike action for those involved. This is not to suggest that the model can accurately reflect all components of participation, action and thought in the scenario. In fact, the model is only ever able to provide a particular and limiting perspective of the strike action scenario, as influenced by the necessary selection process around what information to include and what to exclude and by what elements of the focal event to focus on and which elements to ignore. This is also not to suggest that the strike action is driven entirely by financial considerations; rather, the modelling processes recognises the role and presence of other non-financial factors (e.g. political motivation) which affect action and decision-making processes in this scenario. As such, the function

[222] The processes involved in the modelling cycle envisioned for the Modelling domain were dealt with in detail in Part 4, Chapter 14 and sub-section 14.4.4.2 (starting on page 223).

of this particular model in this scenario is to provide an alternative understanding of the scenario that offers a perspective of a possible alternative form of participation in that scenario – and a form of participation that facilitates the potential for different decision-making choices. This alternative form of participation is not posited as the only alternative or as a better form of participation – simply as one possible alternative form that provides a different understanding of the scenario and that facilitates the potential for a different form of decision-making in the scenario. It is also necessary to stress that the goal for the construction of the model is for the development of a particular perspective of understanding of the context, and any methods (everyday or mathematical) employed are in service to and facilitate the attainment of this goal.

In relation to the modelling processes envisioned for the strike action activity described above, a (re)constructed representation of this scenario that is considered legitimate from the perspective of the Modelling domain of practice must demonstrate a number of understandings. These include understanding of why people might choose to act in the way that they do (i.e. possible reasons for going on strike), together with a perspective of how they might act differently (or might still choose to act in the same way) if imbued with a different understanding of the scenario or of particular components of the scenario (i.e. specifically relating to the financial implications of strike action). To achieve this objective, movement must be made and consideration given to elements of both the contextual and mathematical terrains that are of relevance to this situation.

CIVIL SERVANTS GO ON STRIKE

Mfundekelwa Mkhulisi and Sne Masuku

Delivery comes to halt today

PUBLIC service delivery, including education and the courts, is expected to grind to a halt today as millions of workers march on the Union Buildings and other centres around the country this morning.

Teacher unions say public schools will be closed and pupils have been advised to stay home. Government offices are also expected to be closed.

The public servants are demanding an 8,6 percent salary increase and a R1 000 housing subsidy. Government is offering a 7 percent increase and a housing allowance of R630.

"We have put our march on hold for a long time hoping that the government will come up with a reasonable offer," said Cosatu's Sifiso Khumalo.

He said the march by about 1,3 million workers was to put pressure on government.

"But this does not mean our doors are closed to negotiations. The ball is in their court," he said.

Khumalo said after the march union leaders would meet the employers at the bargaining council.

"If they come up with something better we may call off the strike," he said.

Khumalo said the marches would take place in all provinces except Mpumalanga and KwaZulu-Natal.

"They refused us permission to march in those provinces but there will be demonstrations," he said.

He said the unions will demand a response within 24 hours to their demands.

Schools are expected to be the hardest hit, with only a few weeks before the start of the trial examinations.

KZN education department spokesperson Muntu Lukhozi said: "Pupils can hardly afford to lose a minute as they prepare for their final examinations. This situation could bring about disillusionment among learners."

About 210 000 Public Servants Association members have been protesting since last month.

Cosatu general secretary Zwelinzima Vavi will address the Cape Town marchers before they proceed to parliament.

In Pretoria the marchers will converge on Schubart Park and proceed to the Union Buildings.

Tshwane Metro Police spokesperson Melvin Vosloo said: "We are expecting about 30 000 marchers but we are ready for any eventuality."

FLASHBACK: Sadtu members converged on Orlando Community Hall in Soweto recently to get a report on wage negotiations with the government. The teachers and other public servants will march for better pay across the country today. PHOTO: BAFANA MAHLANGU

Figure 46: Sowetan article on civil servant strike action (Mkhulisi & Masuku, 2010)

Reflecting now on the structure of the discursive resources that characterise discursive practices associated with the Modelling domain of practice[223], clearly modelling activities comprise both contextual and mathematical *signifiers*, together with signifiers that index a requirement for engagement with modelling processes. These signifiers include context-specific words/vocabulary and visual mediators that index specific elements of the focal event under analysis to be either considered for inclusion or exclusion in any modelling processes. These contextual signifiers are combined with words/vocabulary and visual mediators that index the requirement for specific mathematical objects, contents and

[223] The discussion below is structured with a separation of discussion of the nature of the discursive resources that characterise modelling-related practices and illustration of the nature of these discursive resources in relation to the strike action scenario. This is a deliberate move and decision, and is motivated by my belief that illustration of the characteristics of the discursive resources in relation to the strike action scenario is best served through discussion and demonstration of the way in which these discursive resources function in an integrated way to facilitate a particular form of discourse about the scenario.

routines in the development of a suitable model designed to reflect a particular vision of participation in the context indexed through the contextual signifiers. And, both the contextual and mathematical signifiers are further accompanied by additional signifiers that index the specific and directed requirement for engagement in modelling-related practices (as opposed to participation in the practices from an exclusively Everyday or Mathematical Competency perspective). Importantly, it is the combination of both contextual and mathematical signifiers that index the specific structure of the alternative representation or reconstruction of reality that is to be developed through the modelling process, and of specific areas of scrutiny and focus in the model. A misreading or misinterpretation of the signifiers might result in a constructed model that inadequately reflects the nature of legitimate participation in the context and, consequently, of rejection of the model.

Modelling activities also involve a multitude of *routines* that are indexed by both the contextual and mathematical signifiers present in the activity. Importantly, both contextually and mathematically structured routines, together with a variety of modelling meta-routines, are necessary for successful participation in modelling activities. The contextually structured routines are particularly prevalent during the first two and last three stages of the modelling process. It is during these stages that negotiation ensues regarding which features of the context must be included and/or excluded, whether the developed model provides a legitimate perspective of endorsed participation in the context, and of the most appropriate and accessible means to expose and communicate the contents of the model to other participants in the environment. It is during these processes that consideration of and engagement with the types of routines employed by people who are actively engaged in real-life situations must occur, together with consideration, comparison and critical analysis of any developed alternative mathematical routines and processes to existing forms of legitimised and endorsed practices. The mathematically structured routines, on the other hand, play a crucial role during the mathematisation process and during the construction and development of the model as a mathematised representation of reality. The specific structure of the mathematical routines required and ultimately employed depends entirely on the features and structure of the model to be developed as indexed through the signifiers in the activity. What is important to notice is that both contextual and mathematical routines are required for the model to be considered as a legitimate and endorsable representation of reality and of participation in that reality. In other words, a model-as-narrative will only be endorsed if sufficient consideration is given to how problems are actually and already solved in everyday contexts together with added consideration the utility of mathematical structures in providing alternative methods of solution. Furthermore, it is also essential to recognise that the specific routines employed in the development of a model are dependent on the structure and format of the model, with the possibility that different models may represent the same situation in different ways and through the use of different routines. This facet of the modelling process has significant implications for the way in which narratives to modelling activities are endorsed, since evaluation of the legitimacy of a model and of the techniques employed therein depends on analysis and evaluation of the coherence between the routines employed and the intention, area of focus and structure of the model. This issue is discussed in more detail in the paragraph below.

Although a requirement for dual emphasis on both contextual and mathematical signifiers and routines distinguishes, to a limited extent, the Modelling domain of practice from the Everyday and Mathematical Competency domains, it is through the characteristic of *endorsed narratives* that the basis of legitimate participation in the Modelling domain is more clearly and explicitly defined. Bearing in mind that the primary goal of the modelling process envisioned for the Modelling domain of practice is the (re)description or reconstruction of a segment of reality (or of a practice in that reality) to facilitate understanding of existing structures of participation and/or possible alternative ways of functioning in that reality. For a developed narrative-as-solution for an activity associated with the Modelling domain to be endorsed, the narrative must demonstrate deliberate and explicit movement between both the mathematical and the contextual terrains in which the focal event under analysis is embedded. In alternative terms, an endorsed narrative in this domain of practice must demonstrate recognition and understanding of the knowledge and practices associated with both the Everyday and Mathematical Competency components of a problem situation, since it is through the accessing of both domains that understanding of the structure of current and possible alternative forms of participation is made possible. And, for this to be achieved, the endorsed narrative-as-solution must demonstrate explicit and deliberate movement through the various processes that characterise the cycle of modelling envisioned for the Modelling domain of practice. Importantly, and as indicated by the previous sentence above, this in no way implies that endorsed and legitimised participation in modelling activities simply involves the combining of practices and knowledge from the Everyday and Mathematical Competency domains of practice: the Modelling domain of practice cannot simply be reduced to a combination of the other two domains of practice.[224] Successful engagement in activities associated with the Modelling domain of practice involves considerably more than just consideration of mathematical and contextual elements of a problem scenario. Instead, successful engagement requires application of the processes of modelling, characterised by explicit movement between the real and mathematical worlds, and by consideration of the both existing and possible alternative ways of action, though and behaviour. Notice, however, that this does not mean that there is a requirement for every modelling activity to engage in every process of the modelling cycle, and in some modelling activities greater emphasis will be placed on certain processes and some processes may even be excluded. However, an endorsed narrative for a modelling activity comprises an explicit attempted demonstration of how elements from both the contextual and the mathematical terrains have been considered and negotiated to develop a particular understanding and/or perspective of the practice or context under investigation. Furthermore, it is possible for participants engaged in modelling activities to develop varying models that employ different routines to (re)describe the same aspect of reality. For this reason, the endorsement of a model also involves analysis of the coherence of the routines to the demands and requirements of the problem scenario. The suitability of those routines in helping to develop a perspective of reality that represents a legitimate and

[224] It is necessary to mention here that participation in both the Everyday and the Mathematical Competency domain of practice involves utilisation of certain (but different) of the processes that characterise the modelling cycle envisioned for the Modelling domain of practice. For example, participation in an activity from the perspective of the Everyday domain of practice may involve utilisation of the processes involved in (1) understanding and defining the scope of the focal event under investigation and (2) simplifying and structuring the event to make it more manageable to deal with. By contrast, participation in an activity from the perspective of the Mathematical Competency domain of practice may involve utilisation of the processes of (3) mathematisation, (4) working mathematically and solving the problem, and (5) interpreting the calculated solution in relation to the requirements of the context. Importantly, notice from the above that participation in the Everyday and Mathematical Competency domains of practice entails emphasis on only certain processes and, particularly, those processes that facilitate understanding and exploration of either everyday practices or mathematised representations of those practices. Participation in activities drawn from the Modelling domain of practice, by contrast, entails constant negotiation of both the contextual and mathematical terrain – and constant movement between both of these terrains through the various processes of the modelling cycle – in search of a broadened understanding of a situation.

feasible view of a possible forms of participation in the scenario must also be considered.[225]

By way of illustration of the characteristics of discursive resources associated with practices drawn from the Modelling domain of practice, consider the example of the strike action scenario. Presentation of the strike action scenario for which the model must be constructed (specifically in the context of a classroom activity) includes both contextual and mathematical *signifiers*. These include words/vocabulary (e.g. strike lasted for *three weeks*; strike demands included an appeal for an increase in *salary* and *housing allowance)* and signifiers (e.g. salary increase demand of *8%*), that refer to various elements and conditions of the strike. Crucially, this combination of contextual and mathematical signifiers serve to index the specific elements of the strike which must be included for consideration in the model. In terms of the strike action scenario, this includes consideration of the financial effect of the loss of pay as a result of the no-work-no-pay policy, and, so, the what the loss of pay will be and how long (in months or years) it will take for the agreed increase amount to offset the loss of pay. Participants also need decide on the structure and format of the model and how these elements must be engaged with in the model. And, by indexing specific elements of the scenario for consideration and inclusion in the model, the signifiers also indirectly draw attention to possible *routines* for use in the construction of the model.[226] These routines include, for example: the calculation routines for determining the loss of pay during the strike action (simple addition of salary values) and the amount of time it takes to recoup this lost pay once a rate of increase has been determined (percentage calculation of salary increase amount; division of lost pay amount by increase amount); and/or the routine for structuring and presenting the information in a particular format to make the information more accessible and understandable (see Figure 47 on page 311 below); and/or the routine for comparing the calculation-based model to other factors which might influence participation in the strike but which have not been included for consideration in the model. Clearly there is potential for different participants to construct models for the strike action scenario that are differently formatted and structured and, so, which employ different routines. This facet of the modelling process further differentiates this type of modelling activity from an activity embedded in the Mathematical Competency domain of practice. This is because rather than prescribing specific contents and methods (as is commonly the case in the Mathematical Competency domain of practice), here participants are expected to use any appropriate technique and to draw on any appropriate contents that facilitate specific understanding of aspects of the scenario.

[225] In alternative terms, there is no single correct approach to the development of a model of a particular situation (although some developed models may provide a more complete or enhanced understanding of a situation than others). As a result, the suitability of the model must be evaluated in terms of the extent to which it provides an appropriate and valid understanding and representation of both existing and possible alternative forms of participation in a scenario.

[226] I say ‘indirectly’ here since a key requirement of the modelling process involves the independent identification, selection and utilisation of routines, and, so, the activity and/or text will not be structured in such a way as to directly stipulate or prescribe necessary routines.

Current pay	R 7 000,00	Strike Pay		**Pay Loss**
New Salary offered by Government (@ 7%)	R 7 490,00	Pay after 1 week of strike	R5 250	**R 1 750,00**
New salary demanded by unions (@ 8,6%)	R 7 602,00	Pay after 2 weeks of strike	R3500,00	**R 3 500,00**
Salary difference	**R 112,00**	Pay after 3 weeks of strike	R1 750,00	**R 5 250,00**

Months worked after strike	Total pay recovered	Months worked after strike	Total pay recovered	Months worked after strike	Total pay recovered	Months worked after strike	Total pay recovered
1	R 112,00	13	R 1 456,00	25	R 2 800,00	37	R 4 144,00
2	R 224,00	14	R 1 568,00	26	R 2 912,00	38	R 4 256,00
3	R 336,00	15	R 1 680,00	27	R 3 024,00	39	R 4 368,00
4	R 448,00	**16**	**R 1 792,00**	28	R 3 136,00	40	R 4 480,00
5	R 560,00	17	R 1 904,00	29	R 3 248,00	41	R 4 592,00
6	R 672,00	18	R 2 016,00	30	R 3 360,00	42	R 4 704,00
7	R 784,00	19	R 2 128,00	31	R 3 472,00	43	R 4 816,00
8	R 896,00	20	R 2 240,00	**32**	**R 3 584,00**	44	R 4 928,00
9	R 1 008,00	21	R 2 352,00	33	R 3 696,00	45	R 5 040,00
10	R 1 120,00	22	R 2 464,00	34	R 3 808,00	46	R 5 152,00
11	R 1 232,00	23	R 2 576,00	35	R 3 920,00	**47**	**R 5 264,00**
12	R 1 344,00	24	R 2 688,00	36	R 4 032,00	48	R 5 376,00

Figure 47: Strike action model - summary of recouped pay over time

From the perspective of participation in the Modelling domain of practice, a model that constitutes an endorsed narrative for this strike action scenario must provide a perspective of the scenario that facilitates understanding of the financial implications of the strike action for those involved in the strike. An endorsed narrative must also recognise the possibility for multiple model representations or formats of this problem scenario (e.g. tables, graphs, discussion, calculations, etc.) and of the possibility that different routines may be used, all of which may lead to a common or similar understanding of the financial implications of prolonged strike action. Furthermore, the narrative must also offer recognition that the model only provides a limited perspective of the strike action scenario and that even if participants are aware of the financial implications reflected in the model they may still choose to embark on strike action for a variety of other reasons and influenced by a variety of other factors. In summary, for a model of this strike action scenario to be endorsed as offering a legitimate representation of a possible form of understanding of the scenario, the model must accurately reflect the financial implications of prolonged strike action together with recognition of why participants might still choose to participate despite awareness of these financial implications.

As with the Everyday and Mathematical Competency domains of practice, components of the domain of Reasoning and Reflection also comes to play a crucial role in interaction and negotiation of the discursive resources in activities associated with the Modelling domain of practice.[227] Contextual Reasoning is essential for facilitating understanding of the contextual elements of the focal event under investigation, and, particularly, for analysis of signifiers in the activity that index elements from the context that are necessary for consideration and inclusion in the constructed model. For example, in relation to the strike action scenario, Contextual Reasoning facilitates understanding of contextual components of the scenario such as the agreed upon salary increase amount, the meaning of a no-work-no-pay policy and the association of this no-work-no-pay policy to a loss of income. Contextual Reasoning is also essential in latter stages of the modelling process

[227] In Part 4, Chapter 14 and sub-section 14.4.5.4 (starting on page 242) I associated different levels of Contextual Reasoning and Mathematical Reflection to each processes that characterises the modelling cycle. I do this to illustrate the crucial role of both Reasoning and Reflection in facilitating the successful generation of a model that provides a legitimate perspective of possible forms of participation in a contextual scenario.

as the results of the model are validated in relation to the structure of the focal event under investigation and as the results are communicated in the form of a narrative. For example, in relation to the strike action scenario, Contextual Reasoning facilitates comparison of the modelled narrative of the financial implications of prolonged strike action to other factors which may influence participants' reasons for striking. This is done to determine whether the results of the developed model provide the opportunity for an alternative way of thinking about future strike action, or whether the results of the model are simply negated by existing political or other economic factors and/or considerations. Mathematical Reflection, on the other hand, plays a crucial role in the mathematisation process and in the construction of the model as mathematical routines are employed in the attempted generation an alternative perspective of possible forms of participation in or understanding of the context. For example and in relation to the strike action scenario. Mathematical Reflection is essential for facilitating identification of the particular calculation routines that enable comparison of the salary increase amount (calculated as a percentage of a chosen existing salary amount) and the total lost pay amount. Further Mathematical Reflection is also required for determining the most appropriate way in which to represent and structure the model so as to make the results of the model accessible to a particular target audience (and so, to ensure the endorsability of the model). From the above, it is essential to notice that for practices associated with the Modelling domain of practice, reasoning and reflection on both contextual and mathematical elements are crucial for facilitating successful movement between the contextual and mathematical terrains. Engagement with both of these terrains is necessary for facilitating movement through the various modelling processes, for facilitating successful engagement with the discursive resources from each terrain, and for coming to understanding both existing and possible alternative forms of participation as reflected in and through the developed model. Reasoning and reflection on both contextual and mathematical components are, thus, crucial for coming to understand how and why people talk, behave and think in a particular way in a contextual situation. These forms of reasoning and reflection are also necessary to understand how people might talk, behave and think under different conditions and with a different understanding of the situation, and of the possible advantages and restrictions of both current and alternative forms of participation.

This, then, concludes the differentiation of the structure of knowledge and participation in the domains of practice according to the structure of the discursive resources that characterise each domain. With this process complete, it now becomes possible to analyse, compare and differentiate empirical textual resources by determining whether the discursive characteristics of those resources reflect practices and knowledge associated with the Everyday, Mathematical Competency or Modelling domains of practice. In other words, it is now possible to use the framework of the external language to facilitate a gaze to be cast from the theoretical components of the internal language over empirical resources. It is to demonstration of this process that I now turn in the second level of operationalisation of the external dimension of the language of description for the knowledge domain of mathematical literacy.

18.2 Level 2 Operationalisation of the external dimension of the language of description (in relation to an exemplar text-based empirical resource)

This second level of operationalisation involves application of the external language of description to a specifically designed text-based task that reflects problem-solving practices envisioned for the subject-matter domain of Mathematical Literacy. This level of operationalisation is intended to facilitate analysis of the primary domain of practice (of the knowledge domain of mathematical literacy) through which participation in and with both individual questions and the activity as whole is endorsed (i.e. the 'Dominant Domain of Prioritising'). Importantly, notice that this second level of operationalisation involves a shift in the level of engagement from the field of knowledge production (c.f. Bernstein, 1996) (i.e. the knowledge domain of mathematical literacy) to the field of pedagogic recontextualisation (c.f. Bernstein, 1996) (i.e. the empirical terrain of the subject-matter domain of Mathematical Literacy).

Before this level of operationalisation is possible, certain preliminary comments are necessary, specifically in relation to the *level* and *site* of analysis in this operationalisation process.

18.2.1 A note on the 'site' of analysis

Both the operationalisation of the external language of description and the application of the external language to empirical phenomenon relating to the subject-matter domain of Mathematical Literacy involve analysis of the discursive resources embedded in the specific and limited site of *text* – specifically, *written text*. In particular, a key area of focus is analysis of the endorsed narrative(s) transmitted through the text and the structure of knowledge that must be accessed and engaged with to generate and replicate this endorsed narrative(s). And, in the context of written text specifically related to the subject-matter domain of Mathematical Literacy, an endorsed narrative is evidenced either through the structure and contents of a solution or in the written commentary provided by the task developer (recorded in, for example, an associated teacher's guide document). The endorsed narrative can also be witnessed through a written statement of intention for a pedagogic process (for example, in a curriculum document or course notes for a teacher education course). The scope of the empirical analysis process in this study does not extend to the site of dialogue between participants, or analysis of pedagogic practices, or any other element relating to the teaching-learning process in the subject Mathematical Literacy that does not involve written text.

For me, identification and analysis of a recorded written endorsed narrative provides a window into the way in which the task developer legitimises participation with the problem-solving scenario presented in the text. This, in turn, illuminates the structure of the knowledge (and the terrain from which the knowledge must be drawn) that the task developer deems as essential for successful and endorsed participation in the problem scenario. However, even in the analysis of written text there is potential for alternative endorsed narratives to those that are recorded in the text − namely, the narrative(s) which is actually endorsed by the person who uses and enacts the text in pedagogic practice. In other words, even though a segment of written text might provide a particular endorsed narrative for a problem scenario, the person using the text (who may be the task developer or someone else who had nothing to do with the development of the task[228]) may ratify several alternative versions of the recorded endorsed narrative. The person may also endorse the same problem in a different way to that stipulated in the text by the task developer. The key point here is that although my usage of the external language provides a lens through which to analyse the dominant domain of practice and associated structure of knowledge prioritised in a particular activity, the actual enactment of the activity by a practitioner may yield lesser or different emphasis on this identified domain of prioritising.

As a final comment on the site of analysis, it is also important to recognise that the stated endorsed narrative within a segment of text (i.e. the narrative that is endorsed by the person who has constructed the text) may be completely different and, perhaps, even contradictory to the endorsed criteria for legitimate participation stipulated by the people who actually engage in the real-world context under investigation. This is because, as has been stated previously, any context that is brought under investigation in the knowledge domain of mathematical literacy is only able to provide a recontextualised perspective of reality, with consideration of only certain and limited contextual entities, variables and constraints. This issue is further exacerbated by the necessary blending of mathematical and contextual elements in problem-solving processes in all activities relating to the knowledge domain.

18.2.2 A note on 'levels' of analysis of a text

In analysis of a text, different levels of analysis can be applied to identify how narratives are endorsed in the text. For example, an activity can be analysed at a question-by-question (i.e. micro) level to give a detailed indication of the discursive resources and dominant domain of practice (and consequent structure of knowledge and criteria for endorsed participation) legitimised for each question. The limitation of this level of analysis is that focus on individual entities results in a loss of detail about how these entities fit together to generate a bigger picture.

A different approach would be to identify and analyse the endorsed narrative for an entire activity (i.e. from a more global or macro-level perspective) to develop a sense of how participation is legitimised through the whole activity and the dominant domain of practice according to which the endorsement of narratives is linked. Bearing in mind that a key goal for the conception of the knowledge domain of mathematical literacy (characterised by the prioritisation of a life-preparedness orientation) presented through the internal language of description in this study is to generate endorsed narratives that navigate through all of the domains. In light of this goal, this macro-level analysis facilitates a view of whether or not there is attempted movement between the domains of practice and/or inclusion of discursive resources from all or only some of the domains of practice.

Of course, it would then be possible to increase the macro level view even more by considering an entire text rather than just a single activity or segment of the text. For example, through analysis of a chapter in a textbook instead of just a single activity contained in the chapter, or analysis of an entire textbook instead of just one or several

[228] For example, a teacher who is using an examination that has been set by the National Department of Education or a textbook author who interprets the curriculum differently from the way the curriculum developer intended.

chapters in the book, and so on. While the benefit of this approach is the ability to generate a picture of the dominant domain of prioritising for an entire text, this process is accompanied by the loss of detail of how collective sections or individual questions might differ or relate to the trend for the global text.

In the operationalisation process below, I demonstrate, firstly, the application of the external language at a micro question-by-question level. This serves to illustrate precisely how the components of the external language are able to be used to identify the structure of the discursive resources embedded in a segment of text and, consequently, how identification of the structure of these discursive resources facilitates categorisation of the questions according to the domains of practice that characterise the knowledge domain of mathematical literacy. Secondly, I then demonstrate how through consideration of the collective analysis of the individual questions it becomes possible to generate a macro-level view of the domain of practice that is prioritised predominantly for the whole activity, together with consideration of whether and/or how an attempt is made for movement through all of the domains of practice. The approach that is adopted is, thus, intended to facilitate both a micro-level and a more macro-level analysis of the empirical activity. The micro-level analysis makes possible consideration of issues of cohesion or anomaly within the activity. And, the macro-level analysis facilitates a means for comparison of the whole activity to other activities in the same space (e.g. in the same textbook, examination or curriculum document) or in a different space (e.g. another textbook or examination).[229]

18.2.3 Operationalising the external dimension of the language of description in relation to an exemplar activity

Operationalisation of the external language in relation to a specifically designed Mathematical Literacy exemplar activity occurs in a four stage process:

1. To begin with, I present the complete empirical activity. This activity has been designed in the form of an assignment or worksheet for participants to engage with either for homework or during class time. Although an indication is provided of the categorisation of each question in the activity according to the domains of practice, no explanations are provided on the activity for the criteria according to which these categorisations were determined. Such explanations are reserved for the second and third stages of operationalisation.
2. In the second stage I then show how five individual questions drawn from the larger activity can be analysed in an in-depth way using the components of the external languages. In particular, I demonstrate how analysis of the signifiers (words/vocabulary and visual mediators) in each question text provides an indication of the routines required to generate endorsed narratives for the questions and the dominant domain of practice prioritised in each narratives. This, in turn, is then used to differentiate questions and, particularly, to categorise the questions according to the different domains of practice of the knowledge domain of mathematical literacy.

[229] Note that this approach is demonstrated initially in the operationalisation process below (where analysis of a specifically designed exemplar task is provided), but is also employed in Part 7 of the study in the analysis of specific empirical texts that relate to different sites of practice in the subject-matter domain of Mathematical Literacy. As such, the comments made here relate not only to the operationalisation process demonstrated in this chapter, but also to all further efforts at analysis of empirical textual resources in this study.

3. In the third stage I provide a table showing a summary of the categorisation of each of the remaining questions in the activity according to the domains of practice. As part of this third stage I consolidate the categorisation processes of the first three stages to discuss how participation in the activity as a whole is legitimised in relation to the possible domains of practice of the knowledge domain of mathematical literacy.

In summary of the first three stages, where the first stage of operationalisation demonstrates the type and spread of questions characteristic of an activity for the subject-matter domain of Mathematical Literacy, this third stage demonstrates the spread of the categorisation of the questions for the whole activity according to the different domains of practice. The second stage, by contrast, demonstrates in a detailed and in-depth way how, and the criteria according to which, the categorisation of each question is determined.

4. In the fourth stage of the operationalisation process I reflect on the operationalisation process, and highlight challenges encountered and consequent potential limitations.

18.2.4 Operationalisation Stage 1: Presenting the complete activity

In this stage I present an activity developed on a topic that is of relevance to the subject-matter domain of Mathematical Literacy – specifically, the topic of vehicle finance, which is positioned in the broader contextual environment of Loans and Investments (as specified in the curriculum for the subject). The activity is 'complete' in the sense that it includes a variety of questions that assist participants in navigating through various aspects of the context on vehicle finance. Importantly, I have deliberately chosen to present and demonstrate the analysis of an entire activity rather than a shortened activity that only includes a selection of questions from a larger activity. My reasons for this include: firstly, to demonstrate the necessity for movement between the different domains of practice in order to generate a more comprehensive and complete understanding of the context under investigation; and, secondly, to facilitate analysis and discussion of the complete activity (which would not be possible if only a limited number of questions were included for analysis). The activity is characterised by a question-answer format that is commonplace in the subject-matter domain of Mathematical Literacy. The questions in the activity are developed on a resource drawn from a real-life publication – specifically a web-page based advert showing the costs associated with different vehicle-finance options for the same vehicle.

Importantly, I have designed the activity (and the questions contained therein) specifically to illustrate applicability of the external language of description for analysing individual and collective questions in relation to the components and domains of practice that make up the knowledge domain of mathematical literacy. This point necessitates two further comments. Firstly, this activity, thus, in some ways represents an 'ideal-type' characterisation of what I believe an activity associated with a conception of the knowledge domain of mathematical literacy that is driven by a life-preparedness orientation should look like. The implication of this is that there is a high degree of cohesion between the contents, structure and areas of focus in the activity and the form of participation envisioned for contextualised sense-making practices as described in the internal language of description. This is further accompanied by a deliberate attempt to demonstrate the necessity for equivocal engagement in the Everyday, Mathematical Competency and Modelling domains in the activity (see page 356 for further discussion of this point). Expected engagement with virtually all of the levels of Reasoning and Refection in the activity is a further implication, as is the deliberate structuring of the activity around a real-world resource that bears a high degree of representativity to an authentic context. Secondly, this activity is 'original' in the sense that it is not drawn from a previously developed or published source; nor has this activity been explicitly trialled with a group of learners in a classroom setting. Nonetheless, the activity reflects consistency with similar activities involving the same contextual scenario and equivocal real-world resource that I have developed and engaged with learners in a classroom setting. As such, although this specific activity is original in its development, the type, structure and contents of the activity are not untested.

Given the nature of this activity as an 'ideal-type' construction, in developing the activity I have deliberately incorporated questions that contain a variety of signifiers (including words, symbols, equations, and graphs). This intentional act is done to facilitate discussion on the ways in which different signifiers potentially index different objects – sometimes contextual, sometimes mathematical – for those who encounter and engage with them. The act also facilitates illustration of when and how particular signifiers – which may even appear to be mathematical in structure (e.g. an equation or a graph) – index contextual elements, discursive resources and narratives rather than (expected) mathematical ones.

For each question in the activity, an indication is given of the dominant domain of practice through which legitimate participation in the question is legitimised. The specific method used to determine this categorisation is illustrated and discussed in detail (for selected questions only) in Stage 2 of the operationalisation process (c.f. page 327 below).

Also note that a mark allocation is indicated for each question. For purposes of the analysis of this textual resource, the mark allocations have been included to provide a means for comparing the 'weighting' of the questions associated with each of domain of practice. This weighting also provides an important indication of the amount of time needed to answer each question as well as differences in the levels of cognitive demand of the questions. There is clearly potential for disagreement over the indicated mark allocations, but the primary principle that has informed the way in which marks have been allocated in this activity is that a single mark is allocated for each necessary step of working. However, this does not happen in all cases, particularly when the same calculation is repeated several times. Worked solutions are provided in Stage 3 of the operationalisation process, and, as such, it is here that more information is provided on the specific criteria according to which marks are allocated.

With these preliminary comments complete, it is now possible to present the exemplar text-based activity.

Car Loan Activity

The picture below is an advert showing various vehicle finance options for a car. Study the advert carefully and then answer the questions that follow.

(Source: www.nissanspecialoffers.co.za, sourced 15 October 2013)

Model	Months	Interest Rate	Retail Price	Deposit	Initiation Fee	Balloon	Monthly Payment	Total Payment
MICRA 1.2 VISIA+ MT	72	7.31%	R134 300	0%	R1 140	35%	R1 882	R182 509
MICRA 1.2 VISIA+ MT	36	3.50%	R134 300	7%	R1 140	0%	R3 673	R132 228
MICRA 1.2 VISIA+ MT	60	6.81%	R134 300	10%	R1 140	35%	R1 745	R151 705
MICRA 1.2 VISIA+ MT	60	6.50%	R134 300	20%	R1 140	35%	R1 459	R134 545
MICRA 1.2 VISIA+ MT	60	8.03%	R134 300	0%	R1 140	35%	R2 148	R175 885
MICRA 1.2 VISIA+ MT	60	10.89%	R134 300	0%	R1 140	35%	R2 385	R190 105

Monthly instalment and total cost of credit includes Bank Initiation fee of R1140 incl VAT but excludes monthly service fees of R57 and mandatory insurances. Interest rates are linked to prime (currently 8.5%) and accordingly are subject to change in the event that prime changes. Finance offers subject to approval from Nissan Finance, a division of Wesbank, a division of FirstRand Bank Ltd, an authorized financial services and credit provider, NCRCP20. Offer valid until 30 September 2013 or while dealer stocks last. Prices and specifications subject to change without prior notification. *Recommended retail price excludes PDI, all on road costs (license & registration), metallic paint & accessories. Derivative shown as per advertised recommended price

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Selection of questions on the Car Loan Activity based on developing an understanding of this context and resource[230]

				E	MC	M
1.						
	1.1.	Why do you think a deposit needs to be paid when buying something on credit (like when buying this car)?	(1)	✓		
	1.2.	Consider the finance option payable over 60 months at an interest rate of 6,50%. Calculate the Deposit amount payable on this option.	(3)		✓	
2.						
	2.1.	Explain what a Balloon Payment is and how the balloon payment is determined?	(2)	✓		
	2.2.	What is the disadvantage of a Balloon Payment for the buyer?	(1)	✓		
	2.3.	What is the advantage of the Balloon Payment for the buyer?	(1)	✓		
	2.4.	Why do you think dealers offer Balloon Payments on car purchases?	(1)	✓		
	2.5.	Consider the finance option payable over **60 months** at an **interest rate of 6,50%**: Calculate the Balloon Payment amount due at the end of the loan period for this option.	(3)		✓	
3.						
	3.1.	Explain the purpose of a Monthly Payment value in a loan scenario.	(1)	✓		
	3.2.	What is the difference between the Monthly Payment value and the Total Payment value?	(2)	✓		
	3.3.	What types of factors might impact on the size of a Monthly Payment value'?	(3)	✓		

Domain of practice: E, MC, M

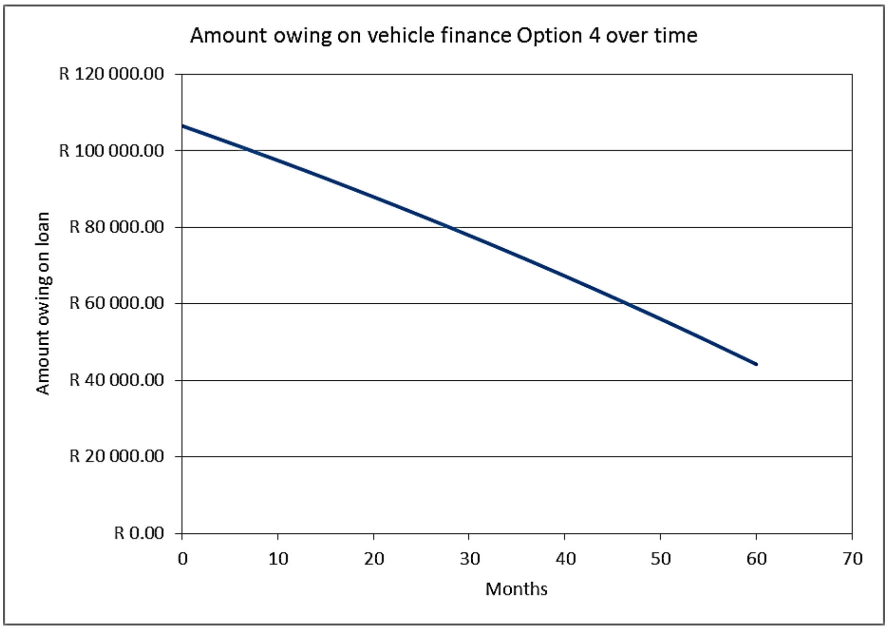
[230] Note: Every question based on the resource includes some form of Reasoning and/or Reflection. However, it simply is not possible to indicate the precise level of Reflection and/or Reasoning in the limited space available in the table above, which is why a column for Reasoning and Reflection is not included. A more comprehensive description of the different levels of Reflection and/or Reasoning associated with each question is provided in the pages below in Stage 3 of the operationalisation process (c.f. page 354 below).

No.	Question	Marks	E	MC	M
3.4.	Consider the finance option payable over **60 months** at an **interest rate of 6,50%**: The total amount paid in monthly payments over the whole loan period can be calculated using the formula: *Total repayment amount =* *monthly payment* × *number of months for which the monthly payment is made* Use this formula to determine the total amount that will be paid back in <u>Monthly Payments</u> for this loan.	(3)		✓	
4.	Now use a calculation to show how the Total Payment amount shown on the advert has been determined.	(2)		✓	
5.	Now use another calculation to work out the grand total cost of this loan (60 months; 6,50% interest rate) that includes ALL costs (i.e. deposit, monthly payments, balloon payment, once off initiation fee and monthly service fee).	(4)		✓	
6.	Compare your answer from (5) above to the <u>Total Payment</u> value for this loan option (60 months; interest rate of 6,50%) shown on the advert. Why do you think the advert has been designed so that the <u>Total Payment</u> value that is advertised only shows the total of the monthly payment values and the balloon payment but does not show the grand total cost of the loan?	(1)	✓		
7.					
7.1.	Work out how much more a person will have to pay to buy this vehicle on this car finance option (60 months; interest rate of 6,50%) than if they were to buy the car cash (at the cash Retail Price).	(2)		✓	
7.2.	Why do many people choose to buy a car through vehicle finance rather than to buy the car with a cash payment even though the vehicle finance options costs more?	(2)	✓		
8.	Use a calculation to compare the total amounts that will be paid back – including *deposit*, *total monthly payments*, and *balloon payment* – for Option 3 and Option 4 and state which option is the more cost effective option in terms the total amount paid for the car.	(13)		✓	

No.	Question	Marks	E	MC	M
9.	Look at the second loan option shown on the table (36 months at 3,5% interest rate). According to the information in the table, it appears as though it will actually cost less to buy the car through vehicle finance (i.e. R132 228) than buying it cash (i.e. at the Retail Price of R134 300). Is this really true? Explain.	(7)		✓	
10.					
10.1.	Finance Option 5 has a higher monthly payment amount than Option 1. How is it possible, then, that the Total Payment on Option 5 is lower than on Option 1?	(4)	✓	✓	
10.2.	Option 3 has a lower monthly payment than Option 2. How is it possible, then, that the Total Payment on Option 3 is higher than on Option 2?	(5)	✓	✓	
10.3.	Use the information given in the resource to describe how the length of a loan can impact on the total amount paid back for the loan, and give a possible reason why this impact occurs.	(4)	✓		
10.4.	Use the information given in the resource to describe how the interest rate on a loan can impact on the total amount paid back for the loan, and give a possible reason for why this impact occurs.	(4)	✓		
10.5.	Describe the relationship between loan length – interest rate – deposit and how this relationship affects the monthly payment on a loan.	(5)	✓		

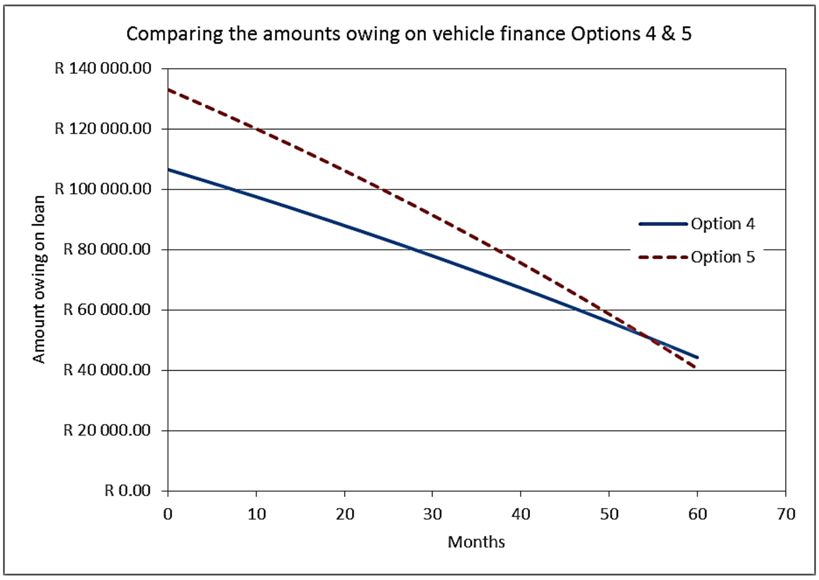
11. Consider the finance option payable over **60 months** at an **interest rate of 6,50%**:

The graph below shows the amount of money owing on this loan option at the end of every month for the whole length of the loan period.



Question		Marks	E	MC	M
11.1.	Approximately how much is still owed on the loan after 20 months?	(1)		✓	
11.2.	Approximately how much is still owed on the loan after 4 years?	(1)		✓	
11.3.	After approximately how many months will R55 000 still be owed on the loan?	(1)		✓	
11.4.	The price of the vehicle is R134 300,00. But this graph starts at just above R100 000,00. Explain why this is the case and give the accurate value at which this graph starts on the vertical axis.	(4)		✓	
11.5.					
11.5.1.	Approximately how much of the loan will be paid off in the first half of the loan period (i.e. after 30 months)?	(3)		✓	
11.5.2.	Approximately how much of the loan will be paid back in the second half of the loan period (i.e. between the 30th and 60th months)?	(3)		✓	
11.5.3.	Explain why a greater amount of the loan is paid back during the last 30 months than during the first 30 months.	(4)		✓	

			E	MC	M
11.6.	Explain – in relation to what happens in the loan − why this graph is not linear but, rather, can be described as *decreasing at an increasing rate*.	(3)		✓	
11.7.	Explain – in terms of what happens in the loan − why the graph does not reach R0,00 on the vertical axis after 60 months (i.e. why is the loan not paid back completely after 60 months?).	(2)		✓	
11.8.	The graph below shows a comparison of the amount of time taken to pay back the loan for the Nissan Micra on finance Option 4 and Option 5.				



			E	MC	M
11.8.1.	Why does the graph for Option 5 start higher on the vertical axis than the graph for Option 4, and give the value on the vertical axis that the graph for Option 5 will start at?	(4)		✓	
11.8.2.	Explain why the graph that represents Option 5 decreases at a faster rate than the graph for Option 4.	(3)		✓	
11.8.3.	Explain why a graph provides an effective way for comparing the amount owing over time on these two vehicle finance options.	(3)		✓	

			E	MC	M
12.					
12.1.	The costs involved in buying a car – some of which are shown on the advert − are only one dimension of the costs involved in owning and using a car. Investigate some of the other types of costs and estimated amounts for those costs which a person who buys and uses a car will have to deal with and manage.	(3)	✓		
12.2.	So far we have only investigated the costs involved in buying a car. However, costs are not the only factors that affect the type of car that a person may choose to buy. What other factors − other than cost − might affect the choice of car that a person might decide to buy?	(2)	✓		
13.	Do you think that spending a large amount of money on a car or changing cars often is a good way to spend your money? Explain.	(2)	✓		
14.	Describe the type of people that this advert might appeal to or might be designed for (i.e. what is the economic/cultural/age status of the type of people who might be interested in engaging with this advert.[231]	(3)	✓		
15.	Imagine that you are a car salesperson dealing with a potential customer who is considering buying this car. Develop a presentation that shows how you would use the information shown on the car advert resource to help the customer to better understand the loan process.	(14)[232]			✓

[231] Although this question is categorised in the Everyday domain of practice due to the requirement for (non-mathematical) understanding of the characteristics of the participants who might engage in the context under exploration, the question also contains an explicit requirement for a specific form of reasoning in the generation of an endorsed narrative. The type of reasoning required here reflects that associated with Level 6 − namely, reasoning about the effect of the participants backgrounds on the structure of their participation − from the Reasoning and Reflection domain of practice.

[232] The mark allocations for the three ‘modelling-type’ questions (Questions 15, 16 and 17) are substantially higher than for most of the other questions in the activity. This is due to the fact that these questions are considerably more open-ended than most of the other questions and also require significantly more work on the part of the participants in determining what strategy to employ, and also in the employment of the chosen strategy. Importantly, although a fixed mark allocation is provided for each of these questions, these questions would actually be assessed using descriptions of modelling processes envisioned for the generated narratives for the questions (see, for example, the mark allocation strategy employed for Question 16 as shown in the analysis table on 346 below). The mark allocations shown here, thus, indicate the maximum possible scores.

			E	MC	M
16.	Consider the finance option payable over 60 months at an interest rate of 6,50%: Develop a presentation that could be used to educate potential buyers about the effect and benefit of paying more than you have to when taking out a loan on the amount of time taken to repay the loan and the total amount paid back for the loan. This presentation must include a variety of resources and/or tools (e.g. tables, graphs, diagrams, etc.) that demonstrate this effect visually.	(20)			✓
17.	Two tables are given on the page below. • Table 1 shows the changes in the prime lending interest rate over a period of time. • Table 2 shows the values and method used to determine the monthly payment on a loan amount. Use both tables to explore the impact of changes in the prime interest rate on the amount paid back in total payments on a loan. You can decide on what loan amount to use and the loan length, but in your calculations you must make use of every interest rate value shown in Table 1. Assume that the loan is given at the prime lending rate.	(20)			✓
18.	The presentation and calculation models that you constructed in (15) and (16) only provide a limited or restrictive perspective of factors that affect the loan and the amount paid for the loan. Explain why this is the case.	(2)	✓		

TABLE 1 – Historical Prime Interest Rates	
Date Changed	**Prime Lending Rate**
2012/07/19	8.50%
2010/11/1	9.00%
2010/09/09	9.50%
2010/03/25	10.00%
2009/08/ 3	10.50%
2009/05/28	11.00%
2009/05/04	12.00%
2009/03/25	13.00%
2009/02/06	14.00%
2008/12/12	15.00%
008/06/13	15.50%
2008/04/11	15.00%
2007/12/07	14.50%
2007/10/12	14.00%
2007/08/17	13.50%
2007/06/08	13.00%

TABLE 2 – Repayment Factors			
Interest Rate	**Loan Length**		
	3 years	5 years	10 years
8.0%	31.33	20.28	12.13
8.5%	31.57	20.52	12.40
9.0%	31.80	20.76	12.67
9.5%	32.03	21.00	12.94
10.0%	32.27	21.25	13.22
10.5%	32.50	21.49	13.49
11.0%	32.74	21.74	13.78
11.5%	32.98	21.99	14.06
12.0%	33.21	22.24	14.35
12.5%	33.45	22.50	14.64
13.0%	33.69	22.75	14.93
13.5%	33.94	23.01	15.23
14.0%	34.18	23.27	15.53
14.5%	34.42	23.53	15.83
15.0%	34.67	23.79	16.13
15.5%	34.91	24.05	16.44
16.0%	35.16	24.32	16.75
Repayment amount = Loan amount ÷ 1 000 × factor value			

18.2.5 Operationalisation Stage 2: Detailed analysis of *five questions*

Each question in the Car Loan activity above is categorised as reflecting a form of participation associated with a particular domain of practice of the knowledge domain of mathematical literacy. In the discussion below I now demonstrate the method used to arrive at this categorisation. This method involves identification and analysis of specific signifiers (words and visual mediators) in the questions, and also of the routines that must be employed to facilitate the generation of the stated endorsed narratives contained in the worked solutions for those questions. In other words, this stage of operationalisation demonstrates the utility of the external language for identifying the structure of legitimate participation in each question in relation to and through the lens of the components of the internal language.

Importantly, the format of this detailed analysis facilitates specification and discussion of elements of each question that relate to Everyday, Mathematical Competency, and Modelling forms of participation, and also to a specific Contextual Domain and/or to a specific focal event(s) in the Contextual Domain. Identification of different levels of Reasoning and Reflection engaged in each question is also provided. A point for consideration is that the categorisation of a question is made according to the *dominant* domain of practice according to which a narrative for the question is endorsed. It is entirely possible that a question (as a collection of sub-questions) may require access to more than one domain of practice to facilitate successful generation of an endorsed narrative. However, the ultimate categorisation of a question is achieved through concentration on the dominant domain of practice that underpins the structure of legitimate participation in the question. Added to this, although a separate category of analysis is provided in the instrument for the discursive characteristic of 'Endorsed Narratives', in the discussion in the pages above it was stated that for both Sfard and Bansilal this resource represents a product of all of the other discursive resources. For example, generation of a narrative about a contextual environment that is endorsed by the community of participants who engage in that environment is only possible if the narrative comprises the appropriate use of contextual words, contextual mediators, and contextual routines. Following this line of thinking, when employing the instrument it is a logical assumption that an indication of a *mathematically endorsed narrative* should be accompanied by indicators of the presence or use of *mathematical* words, mediators, and routines; and an *endorsed contextual narrative* by *contextual* words, mediators, routines and reasoning. It would be an inconsistency (but one perhaps worth exploring) for an activity comprising mathematical words, mediators, and routines to be characterised by an endorsed contextual narrative (and vice versa). Finally, note that an empty cell in the table indicates that there are no features in the question text that relate to the contents of that cell.

In choosing specific questions for analysis, I have deliberately selected questions from the consolidated activity that demonstrate practices and legitimate forms of participation associated with each of the domains of practice, together with questions that are able to be solved through access to knowledge and routines drawn from more than one domain of practice:

- Everyday domain of practice → Question 1.1
- Mathematical Competency domain of practice → Question 1.2
- Potential for a question to be solved from forms of participation associated with either the Everyday or Mathematical Competency domains of practice → 10.1
- Analysis of a question that includes introductory text and commentary → Question 11
- Modelling domain of practice → Question 16

18.2.5.1 Analysis of Question 1.1 in the original activity

1.1	***Why do you think a deposit needs to be paid when buying something on credit (like when buying this car)?***
	Official Endorsed Narrative: [233] ***A deposit represents a pledge that the person who is buying an item on credit will pay back the outstanding amount.***

Contextual Domain				
	Setting	**Behavioural Environment**	**Use of language**	**Extra-situational background knowledge**
Sub-Event = Deposit	Buying on credit	Classroom activity or assessment	'Deposit'; 'Credit'	What a deposit is and why it is payable ⟶ i.e. way in which credit purchases work.

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
Question Text	**Words/Vocab**	'*Why do you think*' ⟶ signifies that the learners must apply their own understanding of the contextual situation. '*deposit*' ⟶ signifies the specific concept being dealt with and also directs attention (possibly in a distracting way) to a percentage entity in the table (i.e. indexing the % in the table might lead participants to believe that they need to use the % in a calculation). '*buying on credit*' ⟶ signifies the context that the learners must draw their knowledge on or from.			R/R Levels 1, 2 and 3 ⟶ (1) reasoning about the meaning of specific contextual terminology and their relevance for the problem situation; (2) reasoning about the scope of the specific event under analysis and which elements and variables to work with and (3) about the most appropriate method and associated routines requires to generate a solution strategy that will be endorsed. *(i.e. The generation of a successful endorsed narrative for this question requires: understanding that the question involves an explanation of the notion of a deposit in general terms – and not specifically in relation to this context; reasoning that the focal event under exploration involves the concept of 'deposit', which is embedded within and related to a variety of other variables in the bigger context of loans or credit; and reasoning about issues such as: must the percentage value in the table be used? Must a calculation be performed?)*

[233] I am deliberately using the word 'Official' here to indicate that this is the solution that is endorsed in the mind of the task developer and recorded in the text in terms of what the ideal-type answer or narrative would be. There is, of course, every possibility that the task developer (or another practitioner who is using the task for pedagogic or assessment purposes) – while engaging with a participant's responses to the question – might also accept other narratives which vary from the official narrative but which still reflect aspects of the official narrative.

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
	Visual mediators	None in the question; but if they refer to the table then the visual mediator becomes the % values that reflect the deposit percentage.	From alongside, if the word 'deposit' signifies for participants the need to consider the % value in the table then participants may respond through a mathematical structure/interpretation/approach.		
Solution Text	**Routines**	The routine to be accessed and applied here involves recall and communication of specific and technical understanding of the contextual entity of a 'deposit' as it relates specifically to the localised context of a credit purchase or a hire-purchase agreement.	The points below only apply if the endorsed narrative accepts a mathematically based structure as a solution (see alongside and below): • % notation; • understanding that 7% translates to 7/100 or 0,07; • routine or structure for calculating a percentage of a value.		R/R Level 3, 4 & 5 → (3) reasoning about the need to provide a routine in the form of an explanation, (4) that this explanation includes reference to specific contextual entities relating to the notion of a deposit, and then (5) reflecting on the accuracy and completeness of the explanation. If a mathematically based endorsed narrative – and, by consequence, mathematical routines – is accepted, then some level of mathematical reasoning/reflection is required, specifically: R/R Levels 1 to 5 specifically on mathematical elements → (1) reasoning about the meaning of mathematical elements in the question and (2) on the specific scope of the event or context under analysis that signifies the need for an appropriate routine for solving the problem *(i.e. routine involving the calculation of a percentage of a value)*; then (3) reasoning about whether a mathematical calculation does in fact provide an appropriate method for answering the question and what method would be suitable *(i.e. a percentage calculation)*; (4) reflecting on whether this calculation has been used correctly and (5) on the reliability of the calculated solution in terms of the providing an appropriate description of the aspect of reality under analysis in the focal event.
Question Text	**Endorsed narratives**	Driven by *contextual understanding* → i.e. Participants need to understand what a deposit is and the function of a deposit with respect to loan scenarios.			In the theoretical discussion of the components of the internal language it was pointed out that the Reasoning and Reflection component is an overarching component that occurs throughout engagement with the problem situation/solution, and without which generation of an endorsed narrative is impossible. In other words, the Reasoning and Reflection dimension facilitates the use and interpretation of signifiers and routines in the production and development of endorsed narratives for a problem situation.
Solution Text		If the accepted answer is: *a deposit is a percentage of the purchase price that must be paid to secure the purchase and almost as a promise that the purchase is completed and honoured.*			

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
		→ Then the endorsed narrative for the solution is the same as for the question = i.e. *specialised or localised contextual understanding*. However, if the following is accepted as an answer: *the deposit is calculated as a percentage of the purchase price – for example, for Option 2 the deposit is 7% of R134 300 = R9 401* → then the endorsed narrative recognises a mathematical structure for the solution. And if this solution is accepted, then a very specific mathematically and/or calculation based *routine* must be applied.			It is precisely for this reason that the levels of the Reasoning and Reflection component that have been utilised in this specific problem situation relating to understanding of the notion of a 'deposit' have been listed at the point of use – i.e. when interpreting the words/vocabulary, or when deciding on whether a particular routine is appropriate – rather than in this cell alongside the description for the characteristics of possible endorsed narratives. The same principle is applied in all subsequent analysis of individual questions.

18.2.5.2 Analysis of Question 1.2 in the original activity

1.2 Consider the finance option payable over 60 months at an interest rate of 6,50%. Calculate the deposit payable on this option.

***Official Endorsed Narrative*: Deposit rate = 20% → Deposit amount = 20% × retail price = 20% × R134 300,00 = R26 860,00**

Contextual Domain

	Setting	Behavioural Environment	Use of language	Extra-situational background knowledge
Sub-Event = Deposit	Buying on credit.	Classroom activity or assessment	'deposit'; 'interest rate'; 'finance option'.	• That the retail price refers to the cash price of the car. • That a deposit is calculated as a percentage of the retail price.

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
Question text	**Words/Vocab**	'*deposit payable*' → signifies a specific and localised contextual concept that the learners must work with. Importantly, within this question, inclusion of these words is supposed to signal to participants the specific type of calculation that is required. However, this will only happen if participants first understand what the terms 'deposit payable' mean. i.e. If they don't understand what a deposit is (i.e. that it is a monetary value determined as a percentage of the original price) then they might just write down the percentage value from the table. Also, they need to understand what the terms mean in order to index that they have to work with the retail price of the vehicle – since the need to work with this value is not signified anywhere. In other words, the mathematical calculation component is dependent on contextual understanding of the notion of a deposit that is not signified through either words or visual mediators. My sense, then, is that this question cannot be answered without a combination of knowledge drawn from both contextual and mathematical domains → hence the reason why the phrase 'deposit payable' is positioned within the contextual domain above.	Introduction: '60 months' → directs attention to a specific location in the table, but must be combined with the 2nd signifier of '6,50%' to accurately establish position. Question: '*Calculate*' → signifies mathematical solution or method is required or is viewed as acceptable. '*deposit payable*' → these terms serve as a dual signifier: i.e. signifying not only a specific localised contextual entity (of which understanding is necessary to know what values to work with in performing a calculation), but also the type of calculation that is required to produce a correct response to the question (i.e. an endorsed narrative). '*option*' → directs attention as to where to find the relevant information about the deposit in the table.		R/R Levels 1, 2, 3 → (1) reasoning on the meaning of both contextual and mathematical elements in the problem situation, (2) the significance of these elements for identifying the precise scope of the event under analysis and the information and variables relevant to that event, and, consequently, for (3) determining the most appropriate methods required to engage with the problem situation. *(i.e. It is clear that a calculation is required, but the precise structure of the calculation required − and the variables to be included in that calculation − is dependent on the appropriate interpretation of the type of calculation routine that applies to the contextual entity of a deposit in the specific context of a car loan scenario.)*

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
Introduction	**Visual Mediators**	'60 months' and '6,50%' ⟶ notice here that these mediators are being used here to signify position in the table and not value. They are also not being used here to signify a particular type of calculation required. In this sense these visual mediator serve to locate the problem situation in a specific space or entity within the focal event. This is why they are recorded as contextual entities and not mathematical ones.			R/R Level 2 ⟶ reasoning about the inclusion and exclusion of variables and constrains in defining the scope of the focal event to be analysed. *(i.e. Reasoning about which information in the resource is relevant to the specific problem situation based on the provide constraints or guidelines.)*
Question Text			None in the question; but when they refer to the table they access the percentage value of 7% which signifies not only the value to be used in the calculation (i.e. 7% or 7/100 or 0,07) but also indirectly the type of calculation required. I say indirectly here because the participants are not being directly instructed on what type of calculation to do – their understanding of what a deposit is and how to calculate a deposit is required for this (i.e. localised contextual understanding – which is why comments have been recorded in the 'everyday' domain column alongside).		Same as for the 'Words/Vocabulary' characteristic above ⟶ i.e. even though the visual mediators are exclusively mathematically based, understanding of and further engagement with the visual mediators is not possible without some level of reasoning about the contextual elements or focal event to which the visual mediators relate and which contextual elements must be included in engaging with the visual mediators – namely, what a deposit value is and the fact that it is calculated as a percentage of the retail value.
Solution Text	**Routines**	Contextual rules for understanding that a deposit is calculated as a percentage of the retail value of the car. In other words, the contextual rule must first be accessed before the mathematical calculation structure or approach can be applied.	• % notation and associated % formats as 7%, 7/100 and/or 0,07. • Correct application of percentage calculation (i.e. % of value). • Stating that the answer is a monetary value and not a percentage value.		R/R Levels 4 & 5 specifically on mathematical elements ⟶ (4) employing the chosen method and associated routine(s) with accuracy and consistency of structure and working; (5) reflecting on the reliability of the solution in presenting a suitable description of reality of the context/problem scenario under investigation.
Question Text **Solution Text**	**Endorsed Narratives**	There is the possibility that the narrative will only be endorsed in its entirety if the mathematical solution comprises certain contextual elements such as a currency symbol and correct number of decimal places indicating that the values involve money.	The official endorsed narrative for this question is dominated by knowledge and routines drawn from the Mathematical Competency domain of practice. And for both the question and the solution, the supplied narrative will only be legitimised and endorsed if it involve: • a correct answer to a mathematically based calculation; • the correct structure of working according to mathematical principles or criteria.		

18.2.5.3 Analysis of Question 10.1 in the original activity

10.1 Finance Option 5 has a higher monthly payment amount than Option 1. How is it possible, then, that the Total Payment on Option 5 is lower than on Option 1?

Official Everyday Endorsed Narrative:	Official Mathematical Competency Endorsed Narrative:
Even though the monthly repayment on Option 5 is higher, the loan length on this option is lower (60 months on Option 5 vs. 72 months on Option 1).This means that on Option 5 a buyer will pay more every month but for a shorter period of time – which results in the Total Payment value on Option 5 being lower than Option 1. *Notice that on both options there is no deposit and there is the same balloon payment amount, so neither of these variables impact on the difference in the Total Payment amount.*	*Since the balloon payment is the same for both options we do not need to consider or include these costs to be able to compare the total cost of Options 1 and 5 and only need to focus on the Total Repayment values. We also do not need to consider the deposit values because these values are not included in the calculation of the Total Payment value:* *Option 1: Total repayments = R1 882,00 × 72* *= R135 504,00* *Option 5: Total repayments = R2 148,00 × 60* *= R128 880,00* *So, the Total Repayment value for Option 5 works out to be lower for Option 1. This is because even though the monthly repayment value on Option 5 is higher, Option 5 has a shorter loan length.*

<table>
<tr><th colspan="5">Contextual Domain</th></tr>
<tr><td rowspan="2">Sub-Event = understanding the relationship between Loan Length – Monthly Payment – Total Payment.</td><th>Setting</th><th>Behavioural Environment</th><th>Use of language</th><th>Extra-situational background knowledge</th></tr>
<tr><td>Car loan (vehicle finance).</td><td>Classroom activity or assessment</td><td>‘monthly payment’; ‘Total Payment’</td><td>How to calculate/determine Total Payment values using Loan Length and Monthly Payment values.</td></tr>
</table>

<table>
<tr><th></th><th></th><th>Everyday domain of practice</th><th>Mathematical Competency domain of practice</th><th>Modelling domain</th><th>Domain of practice of Contextual Reasoning & Mathematical Reflection</th></tr>
<tr><td rowspan="2">Question text</td><td rowspan="2">Words/Vocab</td><td>‘Option 1’ and ‘Option 5’ ⟶ signify specific locations on the resource that contains the contents that need to be engaged with in order to generate an endorsed narrative. Engagement with contents from any of the other options will potentially lead to a non-endorsed narrative. Notice that the resource does not include the labels ‘Option 1’, ‘Option 2’, etc. As such, there is an expectation on the part of the task developer that participants understand that the finance conditions recorded in the first row of the table represents Option 1, the second row represents Option 2, and so on.

‘monthly payment’ and ‘Total Payment’ ⟶ signify specific entities in the loan scenario that participants need to consider and engage with in order to generate a narrative that will be endorsed. These two words (combined with specification of the two options that must be considered) define the specific and localised event that is brought into focus in the question.

‘How is it possible’ ⟶ this phrase signifies the possible expectation for participants to generate an endorsed narrative in the form of a context orientated description or opinion. And, in the absence of a specific instruction for a calculation, some participants may interpret the inclusion of this phrase as signifying that a calculation will not be accepted as an endorsed narrative.</td><td>‘higher’ and ‘lower’ ⟶ these words potentially signify the need for participants to compare and explain differences in the sizes of certain quantities, and perhaps also to perform a mathematical calculation (i.e. a subtraction) to determine the difference.</td><td rowspan="2"></td><td rowspan="2">R/R Levels 1, 2 & 3 ⟶ (1) reasoning about the meaning and relevance of specific contextual elements or terminology for the problem situation; (2) reasoning about the scope of the event under analysis, the embedded nature of that event within a broader contextual environment, and about the inclusion and exclusion of variables and constrains in defining the scope of the focal event; and (3) reasoning about the most appropriate method that must be employed to generate a narrative about the problem scenario that is endorsed.
(i.e. Reasoning about which Options to consider, what elements or components of each option to work with (specifically Loan Length, Monthly Payment and Total Payment values), what aspects (if any) of the broader resource or context must be utilised, and what routines must be employed (i.e. whether a calculation, a description, or a combination of both is required) in order to generate a solution strategy for the problem scenario that is ultimately endorsed.)

R/R Level 1 specifically on mathematical elements ⟶ (1) reasoning about the meaning of the signifiers ‘higher’ and ‘lower’ in the question and whether these signifiers index the need for a particular type of calculation (i.e. a subtraction).</td></tr>
<tr><td colspan="2">Notice that for the most part the text in the question directs participant’s attention to contents primarily drawn from the Everyday domain of practice. Based on this it would be reasonable to expect that the endorsed narrative for the question would similarly draw on routines and contents also drawn from the Everyday domain of practice. However, as is demonstrated below, it is also possible to generate a mathematically structured narrative for this question that provides the same level of endorsement as a contextually structured narrative. The reason for this is that even though the text in the question does not signify any need to calculation, the entities under consideration in the question are inherently related through calculation: i.e. every Total Payment value is dependent on and determined by a relationship between a specific Loan Length and Monthly Payment amount.</td></tr>
</table>

<table>
<tr><th></th><th></th><th>Everyday domain of practice</th><th>Mathematical Competency domain of practice</th><th>Modelling domain</th><th>Domain of practice of
Contextual Reasoning & Mathematical Reflection</th></tr>
<tr><td>Question Text</td><td>Visual Mediators</td><td>The numbers ‘1’ and ‘5’ could also be categorised at visual mediators that signify a specific location or specific contents on the resource. Importantly, the numbers ‘1’ and ‘5’ as employed in this text are descriptors and do not carry the same meaning or characteristics as numbers employed in a mathematical structure.</td><td></td><td></td><td>R/R Level 1 ⟶ reasoning about the meaning and relevance of specific contextual and/or mathematical elements and, specifically, recognising that the inclusion of numbers in the question serve as descriptors and not at numerical entities.</td></tr>
<tr><td rowspan="2">Solution Text</td><td rowspan="2">Routines</td><td colspan="2">As discussed above and also as suggested by the ‘Official Endorsed Narrative’ supplied with or after the question, both narratives structured on purely contextual routines and knowledge or on purely mathematical routines and knowledge are endorsed. And the particular domain of practice that is preferenced determines the routines that are employed in the generation of the narrative.</td><td rowspan="2"></td><td rowspan="2"><u>If contextually structured routines are employed:</u>
R/R Levels 1 to 5 ⟶ (1) reasoning on the meaning of contextual signifiers in the question text and their relevance to the type of routine required in generating an endorsed narrative (i.e. recognising that a descriptive explanation would provide an appropriate for of a narrative); then (2) identifying the relevant elements of the loan scenario that must be engaged with (i.e. Loan Length, Monthly Payments and Total Payments) and the specific nature of the relationship that exists between those elements; (3) recognising the need to provide a narrative in the form of a description/explanation and then (4) providing the narrative so that it meets all of the criteria of the problem scenario; (5) finally, reflecting on the reliability and accuracy of the description.

<u>If mathematically structured routines are employed:</u>
R/R Levels 1 to 5 ⟶ (1) reasoning about the meaning of mathematical signifiers in the question text and their relevance to (2) not only the scope of the specific event under analysis (i.e. recognising the need to work with entities relating to Loan Length, Monthly Payment and Total Payment values), but also to (3) the type of routine required for generating an endorsed narrative (i.e. recognising the need for multiplication of Loan Length values by Monthly Payment values, together with interpretation and comparison of the values that result from these calculations); (4) reasoning on how to employ the identified routines in an accurate and appropriately structured way; and, finally, (5) reflecting on the reliability of the calculated solution in relation to the requirements of the question and also the context of which the solution presents a particular description.</td></tr>
<tr><td>Contextual routines employed in the generation of an endorsed narrative that is legitimised according to contextual criteria:
• Correct navigation of the resource to locate the appropriate options as well as the appropriate information for each option (i.e. Loan Length, Monthly Payment and Total Payment values).
• Effective communication and explanation of the relationship between Loan Length and Monthly Payment as determinants of the Total Payment value.</td><td>Mathematical routines employed in the generation of an endorsed narrative that is legitimised according to mathematical structures:
• Firstly the correct navigation of the resource to locate the appropriate options as well as the appropriate information for each option (which is actually a contextually structured routine – see alongside).
• Recognition (and operationalization) of the relationship between Loan Length, Monthly Payment and Total Payment and particularly the mathematically based process that Total Payment = Loan Length (months) × Monthly Payment.
• Interpretation and communication of the results of the calculations.</td></tr>
</table>

<table>
<tr><th colspan="2"></th><th>Everyday domain of practice</th><th>Mathematical Competency domain of practice</th><th>Modelling domain</th><th>Domain of practice of Contextual Reasoning & Mathematical Reflection</th></tr>
<tr><td>Question Text</td><td rowspan="2">Endorsed Narratives</td><td colspan="2">As discussed above, for this question it is possible for a narrative based on structures drawn from either the Everyday or Mathematical Competency domain of practice to be endorsed and legitimised. As such, the ultimate structure of the narrative that is endorsed depends on whether the routines employed in the construction of the narrative are drawn exclusively from the Everyday or the Mathematical Competency domain of practice. Below are descriptions of the criteria according to which the narratives embedded within each of these domains of practice are endorsed.</td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>Solution Text</td><td>The contextually structured routines lead to a narrative that is endorsed in the Everyday domain of practice if the narrative includes recognition and understanding:
• of the interestedness between the length of a loan and the Monthly Payments associated with each loan option, and how the Loan Length and Monthly Payments interact to determine and influence the Total Payment amount;
• that a higher monthly payment does not necessarily lead to a higher Total Payment amount.</td><td>The mathematically structured routines lead to a narrative that is endorsed in the Mathematical Competency domain of practice if:
• suitable calculations are provided to demonstrate the relationship between Loan Length and Monthly Payment on the Total Payment amount for each option;
• an accurate interpretation of these calculations is provided that demonstrates understanding of the dependency of the Total Payment value on the Loan Length and Monthly payment values, and that a higher monthly payment does not directly translate to a higher Total Payment amount.</td></tr>
</table>

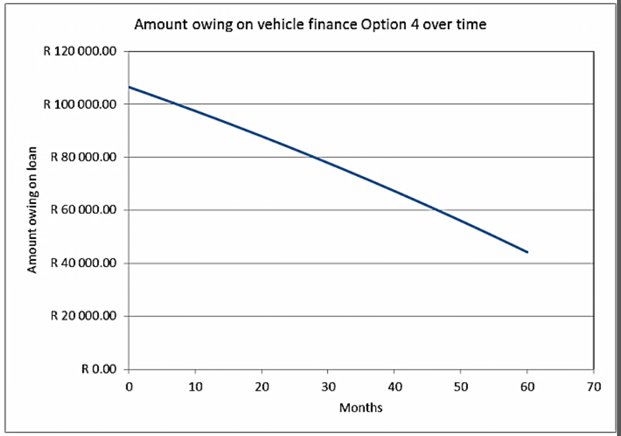
18.2.5.4 Analysis of Question 11 in the original activity, including specific focus on sub-questions 11.1 and 11.4

11. Consider the finance option payable over 60 months at an interest rate of 6,50%:
The graph below shows the amount of money owing on this loan option at the end of every month for the whole length of the loan period.

11.1 Approximately how much is still owed on the loan after 20 months?

...

11.4 The price of the vehicle is R134 300,00. But this graph starts at just above R100 000,00.
Explain why this is the case and give the accurate value at which this graph starts on the vertical axis.



It is important to notice here that it is not only the questions that contain signifiers that direct attention towards particular routines and structures, but also the introductory text and the graph itself. As such, any analysis of this series of questions must also include analysis of the role of the introductory text and associated graph in directing focus towards accessing particular forms of knowledge, tools and strategies. This question, then, has been deliberately selected for analysis to illustrate how a scenario (and associated sub-questions) involving a graphical visual mediator is analysed and the structure of legitimate participation determined. The analysis starts with an interrogation of the introductory text and of the graph, and, thereafter, shifts to analysis of questions 11.1 and 11.4.

- Question 11.1 has been included for analysis to illustrate how and why a question that contains numerous contextual references is categorised in the Mathematical Competency domain of practice.
- Question 11.4 has been included for analysis to illustrate how (equal) access to both the Everyday and the Mathematical Competency domains of practice is required to successfully generate an endorsable narrative for the question.

18.2.5.4.1 Analysis of introductory text and graph for Question 11

11.	*Consider the finance option payable over 60 months at an interest rate of 6,50%:* *The graph below shows the amount of money owing on this loan option at the end of every month for the whole length of the loan period.*

Contextual Domain				
Sub-Event = graphical/visual illustration of how the amount owing on a loan changes over time → i.e. of how a loan is paid back.	**Setting**	**Behavioural Environment**	**Use of language**	**Extra-situational background knowledge**
	Car loan (vehicle finance).	Classroom activity or assessment.	'finance option'; 'payable over 60 months'; 'interest rate'; 'whole length'; 'loan period'.	How to interpret and analyse a graph.

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
Question text	**Words/Vocab**	'amount of money owing on this loan at the end of every month' → this segment of text includes various contextually specific terms that relate directly to the context of loans and which direct participants' attention to the localised concept under investigation − namely, how the amount owing on a loan changes over time. An important point is that participants who do not understand the contextual meaning of the words 'amount owing on the loan at the end of every month' and the significance of these words in relation to how a loan works might not be able to engage successfully with the graph. This may affect their ability to generate legitimate endorsed narratives for the questions that are based on the graph. Notice that the text does not include terms such as 'increase' or 'decrease' which would further direct participant's attention towards how the amount owing on the loan actually changes. The exclusion or absence of these words might signal the intention of the task developed for the participants to have to identify the trend directly from the shape of the graph without any guidance.	'Graph' → there is every possibility that the inclusion of this word (as opposed to 'picture', or 'photograph', or even 'diagram') signifies for participants a form of mathematical engagement in this question. i.e. For some participants encountering the word 'graph' may signify the need to look for dependent and independent variables, *x* and *y* axes, gradients, and a host of other graphing concepts related to interpreting and analysing mathematically structured graphs. 'End of every month' → if judged from a mathematical perspective, the inclusion of this text signifies for participants that the scenario being dealt with involves *discrete* values. However, this is in contradiction with the graph which shows a continuous curve. If judged or experienced from a contextual or everyday knowledge perspective this would be unproblematic; but if judged/experienced from a mathematical perspective then this issue may cause confusion.		R/R Level 1 specifically on contextual elements → reasoning about the meaning of specific contextual elements or terms and their relevance for the situation. *(i.e. Reasoning about the meaning of certain contextual terms, such as 'amount of money owing', and what these terms signify with respect to the elements of the wider contextual field of action and the specific focal event that must be dealt with.)* R/R Level 1 specifically on mathematical elements → reasoning about the meaning of specific mathematical elements or terms and their relevance for the situation. *(i.e. Reflecting on the significance of the term 'graph' and what the inclusion of this term in the question signifies with respect to the criteria according to which an endorsed narrative is to be generated – i.e. clearly the endorsed narrative must involve the use of the graph, as opposed to a purely calculation approach.)*

<table>
<tr><th></th><th></th><th>Everyday domain of practice</th><th>Mathematical Competency domain of practice</th><th>Modelling domain</th><th>Domain of practice of Contextual Reasoning & Mathematical Reflection</th></tr>
<tr><td>Question Text</td><td>Visual Mediators</td><td colspan="2">This graph is categorised as a visual mediator signifying both mathematical and contextual structures.
A graph is an inherently mathematical tool that provides the means for representing relationships between different entities or variables. As such, many of the skills associated with utilising a graph draw on a mathematical structure or knowledge base – for example, reading information from the graph in two different directions (i.e. horizontal to vertical axis; and vice versa), or describing what the shape of the graph signifies about the relationship between the entities or variables represented on the graph. There is thus every possibility that on seeing a graph some participants will immediately access mathematical skills and contents associated with graph work in order to make sense of the graph.
Despite the inherently mathematical basis of the graph, the graph and all associated elements (e.g. labels on axes, values on axes) are presented to reflect the relationship between specific contextual entities relating to the loan scenario. From the way in which the graph is presented – including the descriptive nature of the labels on the axes and even the inclusion of symbols (i.e. Rand symbol) for the values on the vertical axis − participants experience the graph as a sense-making tool for a particular aspect of the loan scenario and not as an isolated mathematical entity. In other words, participants are left with no doubt that the inclusion of the graph provides a perspective of an aspect of a real-life scenario and not an aspect of an exclusively mathematical relationship (like an esoteric mathematical function such as a quadratic function would do).
As mentioned above, further evidence of the emphasis on the contextually embedded nature of the graph over purely mathematical structure is the fact that the discrete nature of the data is ignored in the construction of a continuous curve. If mathematical correctness was the primary concern here then the graph would be incorrect; however, for the purposes of showing the trend in the changing amount of money owing on the loan over time, the mathematical incorrectness of the graph is overlooked in favour of a representation that best illustrates the relationship between specific contextual entities.
Despite the strongly embedded nature of the graph within contextual elements, questions could be asked about certain mathematical elements or considerations that have informed the construction of the graph:
• The scale on the vertical axis increasing by R20 000 instead of by a smaller amount (which would make the graph easier to work with from a non-mathematical perspective). This may have been done deliberately so that participants can be tested on their ability to estimate values from a graph.
• The fact that the horizontal axis ends at 70 even though the loan period is only 60 months → this may have been done deliberately in an attempt to see if the participants might try to (incorrectly) extend the graph.
As a final comment, notice that the inclusion of the graph as a visual mediator is clearly intended to signify for participants the need to use this graph in the answering of any questions that follow. However, as is discussed below in the analysis of Question 11.1, there is the possibility that some participants may not respond to the signifier in this intended way and, rather, may respond to the absence of the term ‘graph’ in a question by choosing a calculation routine over a routine that references the graph.</td><td></td><td>R/R Levels 1 and 2 → (1) reasoning about the meaning of specific contextual and mathematical elements included in the graph and their relevance for the focal event under analysis; (2) understanding of the embedded nature of the limited focal event depicted in the graph within a broader contextual environment on loans, and on the implication of the inclusion of only certain elements about the loan scenario in the graph (and the exclusion of other elements).
(i.e. In order to successfully relate the information depicted in the graph to particular elements about the loan scenario, participants need to come to understand that the graph is illustrating the change in the money owing on the loan over time, and that this change is determined through consideration of a monthly interest calculation followed by deduction of a monthly repayment value. They also need to understand that the non-linear shape of the graph further depicts the way in which interest is calculated on the outstanding amount.)

R/R Levels 4 & 3 → (4) reasoning about whether the graph is correct and on why the graph has been constructed in the way that is has (e.g. reflecting on why the scale on the vertical axis increased in units of 20 000 and what this means for being able to use the graph for purposes of accuracy).
Participants may also want to reflect on
(3) whether a graph provides an appropriate and useful tool for making sense of this particular aspect of the loan.</td></tr>
<tr><td>Solution</td><td>Routines</td><td colspan="4" rowspan="3">The Routines that are required to generate Endorsed Narratives about the graph and particularly about aspects of the loan scenario represented in the graph depends on the specific questions – and signifiers included in those questions − that are posed about the graph and associated loan scenario.</td></tr>
<tr><td>Question</td><td rowspan="2">Endorsed Narratives</td></tr>
<tr><td>Solution</td></tr>
</table>

18.2.5.4.2 *Analysis of Question 11.1 in the original activity*

11.1 Approximately how much is still owed on the loan after 20 months?

Official Endorsed Narrative: By reading from the graph: ≈ R85 000,00 (with leeway of R2 000,00 in either direction).
Note that a reading of R90 000,00 is not an acceptable reading since the corresponding value on the vertical axis to 20 months on the horizontal is not positioned directly in the middle between the R80 000,00 and the R100 000,00 marker.

Contextual Domain

Sub-Event	Setting	Behavioural Environment	Use of language	Extra-situational background knowledge
Sub-Event = graphical/visual illustration of how the amount owing on a loan changes over time → i.e. of how a loan is paid back.	Car loan (vehicle finance).	Classroom activity or assessment.	'Approximately'; 'still owed'.	How to read information from a graph, and particularly to estimate between given values on the axes.

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
Question text	**Words/Vocab**		'Approximately' → the inclusion of this word signifies the need to estimate a value from the graph, which in turn signifies that the value is not able to be read off directly using the given values on the axes but, rather, in all likelihood appears in-between two given values. '20 months' → in the context of this question this combination of number and descriptor signify a specific location on the graph. Importantly, notice that the question does not include explicit specification of the need to use the graph to determine an answer to the question (although the Official Endorsed Narrative does specify the use of the graph). There is, thus, an expectation that participants will understand that the inclusion of the graph in the introduction serves as a signifier that the questions below the graph are based on – and must be answered by reference to – the graph. However, there is still the possibility that due to the absence of the textual signifier 'graph' from the question, some participants may attempt to solve this problem through an alternative method. In such cases a decision must to be made about whether the narrative provided is to be endorsed, and there is every possibility that		R/R Level 1 → (1) reasoning about the meaning of specific contextual terms (such as 'still owed') and their relevance for the situation, as well as the meaning and relevance of mathematical elements or terms and on what these terms signify about the way in which an endorsed narrative is to be generated. *(i.e. Reasoning about the significance of the term 'approximately' as signifying the likely need to estimate a value from the graph by working between two given values rather than to work with a given value. Also, reasoning about the inclusion of the signifier '20 months' not as a time value but as an indicator of position on the graph.)*

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
			endorsement or non-endorsement will depend on the specific nature of the behavioural environment[234].		
Question	**Visual Mediators**				
Solution Text	**Routines**		The routine required to generate an endorsed narrative involves recognising that a specific value on the horizontal axis is given. Furthermore, the corresponding value on the vertical axis must be found by mapping the horizontal axis value to the graph and across to a corresponding vertical axis value (or by identifying an appropriate point on the graph that comprises the given horizontal axis value). However, since the question does not specify the need to make use of the graph signifier in generating an endorsed narrative, there is the possibility that a calculation based routine might be employed. However, the specific format of this calculation based routine is unclear and would rely on participants making use of some format of the Annuity Formula (which is not given in the resource or any of the questions, but which some participants may have been exposed to in the classroom teaching process).		R/R Levels 1 to 5 specifically on mathematical elements → (1) reasoning about the meaning and relevance of mathematical elements of the graph (*such as the fact that 20 months is located on the horizontal axis*) and (2) on what specific aspects of the broader scenario on loans is represented in this graph; (3) reasoning about whether a mathematical or a contextual method is required for answering the question in an appropriate way *(i.e. identifying the need to employ the mathematically based skill associated with reading a value from the graph)* and then (4) employing this method in an accurate way; (5) reflecting on whether the determined solution appropriately reflects a viable solution for the problem scenario and also on whether the use of this method is actually necessary (since the question does not specify the need to use the graph).

[234] From personal experience of being involved in the examining and marking process of the secondary school-based Mathematical Literacy examinations for the Independent Examinations Board (IEB), in behavioural environment of the marking of national examinations, a generated narrative for this question that did not make use of the graph would still be endorsed. However, in the context of a classroom discussion or activity, the teacher might have a particular interest in determining whether learners are able to utilise the graph in a particular way and, so, may reject any narratives that do not make use of the graph or may further instruct participants that use of the graph is essential.

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
Question Text Solution Text	Endorsed Narratives		The official mathematically structured narrative for this question stipulates three main criteria for endorsement and legitimisation: The solution must comprise (1) a monetary value that is (2) located on the vertical axis. Furthermore, this value must (3) fall within a specific range to be considered to be appropriately accurate. In light of this, the question is classified primarily within the Mathematical Competency domain since the primary skills utilised in the generation of an endorsed narrative include estimation and an understanding of the interrelatedness of horizontal and vertical axis values that form the graph. Further, notice that even though (as was discussed above) the graph comprises numerous contextual elements, this question is structured in such a way that the endorsed narrative is generated in the Mathematical Competency domain. In other words, the mathematical direction of this question – and the mathematical basis of the skills that need to be accessed to generate an endorsed narrative for the question – offset the significance of the contextual entities present in the graph. And continuing the discussion initiated above, given the absence of explicit reference of the vocabulary signifier ‘graph’ in the question, there is also the possibility that in certain behavioural environments a narrative that employs a calculation based routine rather than a routine that draws on graphical analysis skills will be endorsed.		

18.2.5.4.3 Analysis of Question 11.4 in the original activity

11.4	***The price of the vehicle is R134 300,00. But this graph starts at just above R100 000,00. Explain why this is the case and give the accurate value at which this graph starts on the vertical axis.***
	<u>Official Endorsed Narrative</u>: The graph shows the amount owing on the loan or financed amount – and this loan/financed amount is not the Retail Price of the car but, rather, the Retail Price minus the Deposit. i.e. The loan amount is R134 300,00 – (20% × R134 300,00) = R134 300,00 – R26 680,00 = R107 440,00. So the graph actually starts at R107 440,00 on the vertical axis.

Contextual Domain				
	Setting	**Behavioural Environment**	**Use of language**	**Extra-situational background knowledge**
Sub-Event = graphical/visual illustration of how the amount owing on a loan changes over time → i.e. of how a loan is paid back	Car loan (vehicle finance).	Classroom activity or assessment.	'Vertical Axis'.	Understanding the difference between the retail Price of a vehicle and the loan amount taken out to finance the vehicle (where the loan amount excludes the deposit paid).

		Everyday domain of practice	**Mathematical Competency domain of practice**	**Modelling domain**	**Domain of practice of Contextual Reasoning & Mathematical Reflection**
Question text	**Words/Vocab**	'Price of the vehicle' → signifies the specific contextually embedded loan related entity and associated value of this entity (i.e. R134 000) that participants might have to deal with in this question. I use the word 'might' since although this value may be of relevance, this will only be confirmed through engagement with the remaining text and mediators in the question. Notice that the question refers to 'Price of the vehicle' and not 'Retail Price' – which is how the vehicle sale price is referenced on the original resource. As such, to be able to engage successfully with this questions participants need to make the connection between these two different contextual constructs.	'graph' → directs the participants attention to the need to focus on a particular element(s) of the graph in order to generate narrative that will be endorsed. 'starts at just above R100 000,00' → signifies a specific location on the graph that participants must analyse and interpret. 'accurate value' → may signify the need for participants to perform a calculation rather than to simply estimate a value from the graph. In other words, the inclusion of this text signifies a possible need to consider a calculation routine alongside a routine for analysing a graph.		R/R Levels 1, 2 & 3 → (1) Reasoning about the meaning of specific terms, whether the terms signify contextual or mathematical elements, and what these terms signify with respect to the elements of the loan scenario that are relevant for this question. There must also be reasoning about which features of the graph are of direct relevance to the generation of an endorsed narrative *(i.e. the intercept with the vertical axis)*. (2) Recognising that the focal event under analysis only deals with limited aspects of the wider contextual environment on the vehicle loan and identifying the specific contextual entities from the broader environment that are of specific relevance in this question *(i.e. to successfully engage with this question participants need to relate recognise that the signifiers in the text indicate the need to engage*

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain	Domain of practice of Contextual Reasoning & Mathematical Reflection
		'Explain' ⟶ the inclusion of this word on its own (as opposed to 'explain with calculations' or 'show') has the potential to signify the need for an endorsed narrative that is comprised of a description or explanation of some sort. This has the potential to cause some level of confusion given the later inclusion of the textual signifiers 'accurate value' which seems to signify a possible demand for a calculation.	'starts' and 'vertical axis' ⟶ reiterates a specific location on the graph that participants must engage with, and – when considered in conjunction with the preceding terms 'accurate value' – the possible need to use a calculation to determine the value at this location.		*with the Retail Price of the vehicle compared to the Vehicle Finance amount (which is a result of the difference of the Retail Price and the Deposit).They also need to recognise that other aspects of the broader contextual environment on the loan scenario − such as monthly payments or loan length − are not of relevance in this question.).* (3) Reasoning about the most appropriate method that will generate an endorsed narrative and whether a mathematically structured narrative to this problem *(i.e. calculation and/or engaging with the graph)* is in fact what is required and what will ultimately be endorsed.
Question Text	**Visual Mediators**	'R134 300' ⟶ this mediator signifies the need to engage with a specific loan entity (i.e. car retail price) shown on the resource. As mentioned in the cell alongside, the visual mediator of R100 000 explicitly signifies a location on the graph rather than a specific loan entity or value. However, this visual mediator also *implicitly* signifies the need to consider and reflect on a value that is lower than the Retail Price, and, in doing so, indirectly directs attention towards the need to consider the Vehicle Loan amount in comparison to the Retail Value.	'R100 000' ⟶ signifies a location on the graph.		R/R Level 1 specifically on contextual elements ⟶ reasoning about the meaning of specific contextual elements and what these elements signify with respect to the elements of the loan scenario that are relevant for this question *(i.e. recognising that R134 300 represents the Retail Price and that R100 000 represents the Vehicle Finance amount.)* R/R Level 1 specifically on mathematical elements ⟶ (1) reasoning about which feature of the graph is of direct relevance to the generation of an endorsed narrative *(i.e. the intercept with the vertical axis)*.

<table>
<tr><th></th><th></th><th>Everyday domain of practice</th><th>Mathematical Competency domain of practice</th><th>Modelling domain</th><th>Domain of practice of Contextual Reasoning & Mathematical Reflection</th></tr>
<tr><td>Solution Text</td><td>Routines</td><td>Understanding of the relationship between the contextual elements Retail Price – Deposit – Loan Amount is essential for being able to demonstrate understanding of the mathematical components of the question (i.e. for identifying the accurate value at which the graph starts on the vertical axis).</td><td>The recorded endorsed narrative stipulates two mathematically based routines:
1. either a calculation or a statement of the deposit amount and/or the finance amount (as the difference between the retail price and deposit);
2. an explicit connection between this calculated or stated finance amount and the value at which the graph starts on the vertical axis.</td><td></td><td>R/R Levels 3 to 5 ⟶ (3) reasoning about the most appropriate method and associated routines required to generate an endorsed narrative, and particularly whether the method and routines embody a contextual or mathematical structure (i.e. deciding on whether a description of contextual elements is appropriate or whether a calculation is required; (4) then employing the chosen method and associated routines with appropriate structure of working, and checking that all aspects of the problem situation have been dealt with (i.e. ensuring that the developed narrative includes both a comparison of the Retail Value and the Loan Amount, as well as making a connection between these values and the intercept of the graph with the vertical axis); (5) reflecting on the reliability of the calculated solution with respect to the existing values shown on the graph.</td></tr>
<tr><td>Question Text</td><td rowspan="2">Endorsed Narratives</td><td colspan="2" rowspan="2">In this instance, the ability to successfully engage with this question – and, hence, to generate an endorsed narrative for this question – relies on a distinctive interplay between knowledge and interpretation drawn from both the Everyday and Mathematical Competency domains of practice. In other words, this question requires that participants make an explicit connection between contextual elements of the loan scenario (the price of the car) and specific mathematically based features of the graph (the value at which the graph starts on the vertical axis); and it is the connection between these two domains that is of central concern in the question. As such, this question is dominated by the requirement for a three-way relational interplay between: (i) calculation; (ii) a specific loan entity (determined through the calculation and through understanding of the relation between retail price, deposit and finance amount); (iii) and a position on a graph. And it is for this reason why the endorsed narrative for this question is categorised as spanning both the Everyday and Mathematical Competency domains of practice.

Also note that it is the requirement for this relational interplay between domains of practice that distinguishes this question from Question 1.2 (see above). In that question the contextual elements of the question simply serve to provide access to the mathematically based structure that is required in the generation of an endorsed narrative. In this Question 11.4, by contrast, the movement is not from one domain providing access to another but, rather, a constant interplay between domains.</td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>Solution Text</td></tr>
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18.2.5.5 Analysis of Question 16 in the original activity

16. ***Consider the finance option payable over 60 months at an interest rate of 6,50%:***
Develop a presentation[235] that could be used to educate potential buyers about the benefit of paying more than you have to when taking out a loan on the amount of time taken to repay the loan and the total amount paid back for the loan. This presentation must include a variety of resources and/or tools (e.g. tables, graphs, diagrams, etc.) that demonstrate this effect visually.

Official Endorsed Narrative:

- ***The contents of the presentation must demonstrate understanding of the impact of an increased repayment amount (i.e. a higher amount than is actually required) on the amount of time taken to pay back a loan and the total cost of a loan, and must also demonstrate that the participant has correctly understood and is able to communicate effectively and clearly about this aspect of the loan scenario under analysis.***
- ***The presentation must also demonstrate that the participant has carefully considered which elements of the loan scenario must be included in the model in order to most clearly and comprehensively describe this relationship (Modelling processes 1 and 2), plus which mathematical elements and calculations provide a useful and appropriate means for making sense of the situation (Modelling process 3).These mathematical elements must also be employed with accuracy and appropriate structure (Modelling process 4). In other words, the presentation must demonstrate understanding of the interplay of contextual elements of the problem situation together with the utility of mathematical techniques in describing and making sense the relationship.***
- ***The specific contents that are included in the presentation and the way in which they are presented and represented should provide evidence as to which components of the model are deemed legitimate for providing a valid representation of the relationship between repayment amount and total cost (Modelling processes 5 and 6).***
- ***The presentation must also demonstrate that the participant has thought carefully about the most effective and appropriate way in which to present the contents of the model to make these contents accessible and understandable to the target audience (Modelling process 7). This must include consideration of when it is appropriate and useful to use mathematical representations (e.g. graphs and calculations) and when it is necessary to make use of contextual language and descriptions that relate directly to the loan scenario.***

[235] In the curriculum statement for Mathematical Literacy (DBE, 2011a, p. 9) there is an explicit statement of focus on ‘communication’ as a key element in the development of mathematically literate behaviour. This communication is to include the use of terminology, notation and instruments that are appropriate for use in the mathematical terrain and relevant to the contextual situation under analysis. I have interpreted this statement to suggest that a key facet of the subject involves exposure to different ways and techniques of/for communicating results to ensure that the results are accessible to other participants in the subject but also to participants who are actively engaged in the real-world situation under investigation. The ability to communicate effectively is a skill that is essential for effective functioning in classroom and in real-world daily life and work place practices. For me, the ability to decide on the best way in which to present, structure, format and communicate the information that is reflected in a model is a key component of the modelling cycle, specifically as part of the modelling process that deals with ‘Exposing’ the model (see Part 4 and sub-section 14.4.4.2 above, starting on page 223) for a reminder of the characteristics of the modelling process of ‘Exposing’).

Contextual Domain				
	Setting	Behavioural Environment	Use of language	Extra-situational background knowledge
Sub-Event = loan or credit purchase	Car loan (vehicle finance).	Classroom activity or assessment.	'money owed on the loan at the end of every month'; 'total amount paid for the loan'.	• How a loan works (i.e. when interest is calculated, when repayments are made, etc.). • How to use a table or graph to model a loan scenario. • How to develop a presentation and to communicate effectively to a particular target audience.

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain of practice	Domain of Contextual Reasoning & Mathematical Reflection
Question text	Words/Vocab	Question: '*Develop a presentation*' ⟶ signifies that an element of communication is required and although the communication may involve mathematical elements (as specified later in the question), the communication also involves non-mathematical considerations (e.g. how to put the presentation together – sequencing, structure, etc.; what presentation instrument to use; what content to include and exclude in the presentation). 'educate' ⟶ the presentation is for a non-specialist. As such, the presentation and its contents must be accessible and the level of technicality must be reduced. 'potential buyers' ⟶ signifies an element of the focal event under investigation (i.e. not dealing with selling component, which would require access to different information/variables/resources). 'benefit' ⟶ signifies that one of the options must be better than the other.	Introduction: '60 months' ⟶ directs attention to a specific location in the table, but must be combined with the 2nd signifier of '6,50%' to accurately establish position. Question: 'resources and/or tools (e.g. tables, graphs, diagrams, etc.)' ⟶ notice here that the signifiers 'resources and/or tools' index the requirement for the inclusion of a collection of objects that assist in the communication process. In some circumstances, these objects might include non-mathematically structures resources. However, the addition of the signifiers 'e.g. tables, graphs, diagrams, etc.' now indexes a specific requirement for the use and inclusion of mathematical tools and/or representations to be employed in the solving of the problem scenario and communication about the scenario. Also notice that although the inclusion of the text 'e.g.' signifies that the 'tables, graphs and diagrams' are only	Notice that in this question there is a direct contextual emphasis − i.e. developing an understanding or awareness of a particular component related to loan repayments − and the use of mathematics is presented as a tool for assisting in generating this understanding and awareness. This differentiates this question from Question 1.2 that was categorised in the Mathematical Competency domain of practice where the primary concern was with a calculation – albeit, calculation of a contextual entity. The presence of a mathematical calculation entity also distinguishes it from a purely contextual problem (i.e. a problem embedded within the Everyday domain of practice – such as Question 1.1). So, although the question includes words that indexes both contextual and mathematical entities, when the question is considered as a whole there is a *process* required that involves describing, representing and/or reconstructing a segment of reality through the use of mathematical tools. It is this process of attempted (re)construction and description that positions this problem in the 'modelling' domain.	R/R Levels 1, 2, 3, 5 & 6 ⟶ (1) reasoning about the meaning of specific elements and terminology, whether the elements signify contextual or mathematical routines, and the relevance of these elements for the approach to be adopted to generate an endorsed narrative; (2) reasoning about the specific scope of the event under and the variables from the broader contextual environment that must be included in any chosen solution strategy; (3) then reasoning about the method and associated routines that must be employed to generate a narrative for the problem scenario that will be endorsed, and particularly whether this method must comprise a mathematical or contextual structure – or a combination of both; (5) reasoning about the need for consideration of non-mathematical issues and factors for inclusion in the presentation; (6) reasoning about the specific target audience that this presentation is aimed at and how characteristics of this target audience influence the contents of the presentation.

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<tr><th></th><th></th><th>Everyday domain of practice</th><th>Mathematical Competency domain of practice</th><th>Modelling domain of practice</th><th>Domain of Contextual Reasoning & Mathematical Reflection</th></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td><td>‘benefit of paying more’ → signifies that the bigger option must be the better option: so, if participants end up with a solution that is contradictory to this then they should realise that there is a problem with the narrative that they have constructed.

‘loan’ → signifies the specific focal event under investigation.</td><td>examples of possible options, the given examples do not include non-mathematical objects, hereby reinforcing a requirement for mathematically structured resources.</td><td rowspan="2">This ‘process’ is also signified through the fact that the method required to solve the problem is not explicit, and some level of reasoning is required on the part of the participants in determining what variables must be considered and what structures or processes must be followed in order to solve the problem. In other words, the way in which the question is phrased – and the lack of explicit solution-path structure – prompts participants into the modelling process where they have to consider what information is relevant, how to engage with the relevant information, what mathematical structures to use, how to interpret and communicate any results, etc.

The phrase ‘develop a presentation’ could perhaps also be taken to signify a process in that it is asking participants to make decisions about what information is relevant for inclusion and exclusion.

Perhaps the key point here is that it is not the presence of specific words or visual mediators that signify the need for modelling. Rather, it is a combination of:
(1) the way in which the activity directs attentions towards developing an understanding of a real-world situation through the (re)construction and representation of an aspect of the loan scenario that involves the use of mathematical tools;
(2) the way in which the activity engages participants in a process in which they have to make decisions about how to approach the problem, what information and strategies to use, and so on. In other words, the activity pushes the participants into the cycle or process of modelling.</td><td rowspan="2">(i.e. Reasoning about the meaning of the different vocabulary in the question and what contextual or mathematical entities are signified through these vocabulary; reasoning about what focal event is being dealt with, the scope or boundary of that focal event, and which specific variables must be considered, understood and employed in order to engage with the focal event in an appropriate way; reasoning about what contents must be included in the presentation and the best way to present these contents; reasoning about a variety of extra-mathematical factors which affects the type of information that must be included in the presentation in order to make the presentation accessible to the relevant audience; reasoning about the backgrounds of the audience to ensure that the contents of the presentation are suitable and so that a form of communication is used that is relevant and accessible.)</td></tr>
<tr><td colspan="2">‘amount of time’ and ‘total amount paid back’ → these two phrases serve a dual purpose:
(1) signify the two main contextual events that must be explored in the context of the focal event;
(2) But, these two phrases also signify mathematical entities → i.e. they signify for participants the specific variables that must be engaged with, which in turn signifies the type of calculation and/or mathematical structure and/or mathematical approach needed
(e.g. total paid back = repayment × number of times the repayment is made → and the number of times the repayment is made is dependent on the amount of time needed to pay the loan [i.e. the length of the loan]).

‘demonstrate this effect visually’ → this series of words serves to signify a specific requirement for communication to include visual components. However, these visual components must include mathematically structured representations (as indexed by the signifiers ‘e.g. table, graph, diagrams, etc.’. By implication, there is the possibility that the presentation of a narrative that only includes text and which does not include visual resources will not be endorsed.</td></tr>
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<tr><th></th><th></th><th>Everyday domain of practice</th><th>Mathematical Competency domain of practice</th><th>Modelling domain of practice</th><th>Domain of Contextual Reasoning & Mathematical Reflection</th></tr>
<tr><td rowspan="2">Question Text</td><td rowspan="2">Visual Mediators</td><td></td><td><u>Introduction</u>:
'60 months' and '6,50%' → notice here that these mediators are being used here to signify position in the table and not value. They are also not being used to signify a particular type of calculation.</td><td rowspan="2"></td><td rowspan="2">R/R Level 1 & 3 → (1) reasoning about the meaning of specific elements and terminology, whether the elements signify contextual or mathematical routines, and the relevance of these elements for the approach to be adopted to generate an endorsed narrative (i.e. Reflecting about what information in the table is relevant to the problem situation for this specific loan scenario); (3) reasoning about what the most appropriate method and associated routines must be employed, and the structure of these routines.</td></tr>
<tr><td colspan="2"><u>Question</u>:
None in the question. But, the inclusion of the vocabulary signifiers of 'total amount paid back', 'length of the loan' and especially 'repayment' direct attention to use of interest rate, loan periods and possibly also factor values from a factor table → all of which include a variety of visual mediators − some of which are contextually based and others mathematically structured − involving percentage values, time values (e.g. years), formulae with symbols (e.g. repayment = loan amount ÷ 1000 × factor).
In other words, although the question may not include direct reference to visual mediators, the participants still need to make extensive use of visual mediators as they move through the modelling process. The use of these signifiers (and associated levels of Reasoning and Reflection) is recorded and reflected in the 'Routines' dimension below.</td></tr>
<tr><td>Solution Text</td><td>Routines</td><td>Constructing the presentation → consideration of what type of presentation to use, how to structure the presentation, what arguments to present, and so on.</td><td>Use of various mathematical structures, methods or contents, including:
Interest calculations using a given interest rate value → i.e. calculating a % of a value.
Various basic operations (+, −, ×) when constructing a model of the loan scenario.
Use of formulae (e.g. perhaps the formula for calculating the monthly</td><td>A possible description of the modelling process for this activity:

<u>Process 1</u> → Understanding & Defining Scope:
Understanding and defining the scope of the problem situation – and contents and information relevant to the problem situation – in relation to the broader focal event) (i.e. this problem deals exclusively with the impact of different repayment</td><td>As described when presenting the internal language of description[236], various levels of Reasoning and Reflection can be associated with the different processes in the modelling cycle:
Process 1 → R/R Levels 1 & 2
(i.e. Thinking about the specific requirements of the problem situation and which aspects of the focal event must be</td></tr>
</table>

[236] When discussing the role of the domain of Reasoning and Reflection in the language of description for the knowledge domain of mathematical literacy (c.f. Part 4 starting on page 230), I theorised how the different levels of Reasoning/Reflection can be associated with different processes in the modelling cycle. Namely: the first modelling process of Understanding and Defining Scope involves the *first and second levels of Reasoning/Reflection*; modelling process 2 of Simplifying and Structuring necessitates the *second and third levels of Reasoning/Reflection*; process 3 of Mathematisation necessitates the *third level of Reasoning/Reflection*, while process 4 of Working Mathematically and Solving necessitates the *fourth level*; process 5 of Interpreting involves the *fifth level of Reasoning/Reflection*; and both process 6 of Validating and process 7 of Exposing necessitate *Reasoning/Reflection at levels 5 and 6*.

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain of practice	Domain of Contextual Reasoning & Mathematical Reflection
			repayment value; or for calculating the total paid for the loan). • Use of different notations and visual mediators → e.g. % symbol, currency symbol and formats, rounding. • Mathematical structures, skills and contents associated with drawing an appropriate graph.	values on the real cost and length of a loan, and not with any other aspects relating to the focal event of vehicle finance). • Locating the relevant and applicable section or information in the given resource and ensuring that all necessary contextual terms are understood (such as the meaning of the term 'repayment' and 'total cost') in relation to the focal event of vehicle finance. <u>Process 2</u> → Simplifying and Structuring: • Deciding what variables must be considered and which can be excluded (*e.g. Must the deposit be included or excluded? Must the balloon payment be included?*) • Deciding on what an appropriate model of the problem situation might look like (i.e. tabular?; or graphical?) and what variables and information would be relevant, appropriate and necessary for inclusion. <u>Processes 3 & 4</u> → Mathematisation & Working Mathematically: Constructing the model of the loan scenario and making use of appropriate and legitimate representations to illustrate the results of the mathematical working. <u>Process 5</u> → Interpreting: Making sense of the information show in the model and associated representations to analyse the impact of increased repayment value on the time taken to pay back a loan (and the consequent real cost value).	*included or excluded for this problem situation.)* Process 2 → R/R Levels 2 & 3 *(i.e. Analysing the variables required and an appropriate structure for a solution path, as well as considering the appropriateness of using mathematics as a tool for making sense of the problem situation.)* Processes 3 & 4 → R/R Levels 3 and 4 *(i.e. (3) Reasoning about the most appropriate method and associated routines to employ and the structure of those routines; and then (4) employing the routines with accuracy and appropriate structure of working as influenced by the scope and structure of the context.)* Process 5 → R/R Levels 5 & 3 *(i.e. (5) Reflecting on the reliability of the calculated and/or modelled solution as providing a viable (re)description of the segment of reality under analysis, and (3) again reflecting on the suitability of using the chosen methods in this particular context or problem.)* Process 6 → R/R Level 5 & 6 *(i.e. (5) Reflecting once again on the validity of the determined solution in relation to the structure of the real-life situation. Furthermore, reasoning is required as to other extra-mathematical factors which may contradict the*

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain of practice	Domain of Contextual Reasoning & Mathematical Reflection
				Process 6 → Validating: Checking that the model and associated calculated/represented/stated solutions accurately and validly reflect the information know about the focal event *(e.g. comparing the calculated real cost value to the real cost value shown in the advert to see if they compare adequately; or if the calculated solution states that the standard repayment is better – which is in contradiction to the signifier provided in the question – then questions must be asked the validity of developed model).* Process 7 → Exposing: Recording the calculated and represented information in a presentation in a way that is appropriate and accessible for the target audience.	*mathematical solution and/or which may influence how participants respond in the problem situation. Validation also requires (6) consideration of the how the world perspective and position of the perspectives themselves may have influenced the structure of the model and consequence solution and perspective generated about the problem scenario.)* Process 7 → R/R Levels 3, 5 & 6 *(i.e. In exposing the solution to a wider audience through the development of a presentation, participants have to consider (6) what aspects of the model to present to best illustrate a solution to the real-world problem, (2) what aspects of the focal event to focus attention on and which to exclude, (6) how to make the content accessible by reasoning about the characteristics of the target audience, and (5) whether there are any other factors that are not included in the model which may impact on decision-making in the problem situation which the audience need to be aware of (e.g. insurance costs, admin fees.)*
Solution Text	**Endorsed Narrative**	From an everyday or purely contextual perspective, an endorsed narrative involves analysis of the model and valid or contextually sensible statements about the problem situation without awareness or cognisance of mathematical methods or structures that have been used to develop the model → i.e. when a person is given the model and they are able to make statements about the problem situation	From a mathematical competency perspective, an endorsed narrative for this activity involves the correct use of mathematical structures, methods and routines involving calculation of differing repayment values and total cost amounts. In other words, the primary criteria for the generation of endorsed narratives in this domain are for any and all mathematical components and	A narrative that endorses the Modelling domain as the dominant domain of practice for this activity is characterised by emphasis on the ability to make sense of the relationship between repayment amount, repayment period and total cost (and particularly on recognition that a higher-than-required repayment amount significantly reduces both the repayment period and the total cost) through the development and utilisation of a set of mathematically based tools or resources that provide a reliable	

		Everyday domain of practice	Mathematical Competency domain of practice	Modelling domain of practice	Domain of Contextual Reasoning & Mathematical Reflection
		from the model, but are not able to explain how the model has been constructed. Or, an endorsed narrative for this domain of practice involves a purely intuitive response to the situation based on knowledge of the workings of loans and without access or a reach to any formal mathematical structures or processes. The accessibility, structure and design of the presentation – i.e. the 'communication' dimension of the problem scenario − are also key characteristics of a contextually embedded endorsed narrative.	calculations and workings to be correct and accurate. In this form of the endorsed narrative, the correctness of the mathematics is considered more important than the structure and accessibility of the presentation. Furthermore, the 'communicative' dimension in this endorsed narrative has more to do with other participants' abilities to make sense of the mathematics, rather than with responsibility being placed on the developer of the presentation to make the contents easily accessible to the target audience.	representation of the problem situation which can be used to provide a more in-depth understand the problem situation. For this specific activity, the endorsed narrative is also characterised by the use of an appropriate form of communication that is able to associate the mathematically modelled results with contextual elements of the scenario to make the results accessible to a particular target audience. In other words, an endorsed narrative for this activity in this domain of practice is characterised by the successful development of an appropriate and valid model and associated resources and representations that reflect a legitimate view of participation in the context. There must also be demonstration of how the model can be utilised to better understand the problem situation. The model must further be communicated in a format that reflects the structure of appropriate and endorsed participation of the people who engage in the context on a regular basis. This suggests that an endorsed narrative for modelling for this activity is characterised by continual movement between the contextual terrain of loans and the mathematical terrain. This occurs as an attempt is made to reconstruct a segment of the focal event on loans to better understand the particular relationship between repayment amount and real cost of the loan. This also occurs as an attempt is made to communicate about this relationship in a way that demonstrates understanding of the relationship and understanding of the characteristics of the participants who engage in the context on a regular basis.	

18.2.5.6 Reiterating the distinction between activities drawn from the Everyday, Mathematical Competency, and Modelling domains of practice

Having demonstrated analysis of individual questions in the exemplar activity, it is possible to once again illustrate the differential nature of the discursive resources that characterise forms of participation associated with the practices of each of the domains of practice, albeit this time in relation to specific empirical task-based questions.

To begin with, consider *Question 1.1* that is categorised as reflecting a form of participation associated with Everyday practices (c.f. page 328 above). A primary objective in this question is gauging understanding of a particular aspect of the loan scenario – namely, the notion of a deposit and the function of a deposit payment. This question, thus, requires an understanding of context-specific elements of the real-world context of loans, and the development of an endorsed narrative for this question is only possible through intricate understanding of this context-specific entity. It is for this reason that participation in this question is legitimised through access to knowledge and practices associated with the Everyday domain of practice. This is because there is a dominant priority in this domain for understanding how people think, act and communicate in contextual settings and the types of knowledge and techniques that are required to ensure successful engagement, participation and communication in those settings.

Question 1.2, on the other hand, contains an explicit requirement for the generation of a narrative-as-solution through engagement with a specific mathematical calculation (c.f. page 331 above). Successful engagement with this question is reflected, primarily, in the extent to which the calculation has been performed accurately and with appropriate mathematical structure of working. This certainly does not mean that consideration of contextual elements is not necessary for successful engagement with the question – and, as was pointed out above, successful engagement with the mathematical calculation component of the question is facilitated through initial engagement with specific contextual entities and contextual understanding of aspects of the loans scenario. However, the presence of contextual entities does not defer from the primary agenda for the use of a calculation approach in the generation of a solution narrative that is considered legitimate. The dominant domain of prioritising in this question is different to in Question 1.1. This is because in Question 1.2 primary focus is on the generation of a mathematical gaze of the problem scenario. In Question 1.1, by contrast, dominant focus is on the development of a shared understanding (with those who actually engage in this scenario as part of their everyday experience and practice) of the specific meaning of contextual elements of the loan scenario.

Now consider *Question 11.4.* In the analysis of this question in the pages above, I argue that successful engagement with the question is only possible through necessary engagement with knowledge and contents drawn from both the Everyday and Mathematical Competency domains of practice (c.f. page 343 above). For this question, the generation of an endorsed narrative requires utilisation of mathematical skills associated with reading a graph, together with specific understanding of contextual elements of the scenario to facilitate a connection to be made from the graph readings with specific entities in the loan. Utilisation of the skills of reading a graph in isolation of understanding of specific elements of the loan scenario would inhibit the generation of an endorsed narrative; similarly, understanding of the elements of the loan scenario without the ability to engage with the graph would similarly inhibit the generation of an endorsed narrative. For this question, engagement with both terrains is tantamount to successful engagement with the problem scenario.

Crucially, Question 11.4 does not constitute a modelling activity that would characterise a form of participation associated with the Modelling domain of practice. This is because – and as was demonstrated through analysis of Question 16 (c.f. page 346 above) – modelling does not simply involve engagement with contextual and mathematical entities. Rather, modelling involves an attempted (re)construction or (re)description of an aspect of reality to better understand the relationship between certain entities in that reality, to generate an understanding of a particular form of participation in that reality, or to generate an alternative understanding of an aspect of or participation in that reality. As such, unlike in Question 11.4, in a modelling process the solution path is not set, nor is there is single solution path. Rather, crucial decisions must be made about how the aspect of reality is to be (re)described and what elements (contextual and mathematical) are necessary for inclusion or exclusion in the construction of that particular (re)description. Further consideration is also required on whether the chosen elements (mathematical and contextual) do in fact provide a useful perspective on the nature of existing and/or alternative forms of participation in the context.

18.2.6 Operationalisation Stage 3: Categorisation of each question

In this third stage of operationalisation of the external language I state the categorisation of the structure of legitimate participation in each question in relation to the domains of practice of the knowledge domain of mathematical literacy. Importantly, the detailed analysis presented in Stage 2 above has been used to inform the categorisation of each of the questions below.

Note that I have chosen to present the analysis deemed from this level of operationalisation in the form of a tabular summary (see Table 7 below) in order to conserve space. However, a more detailed analysis document that provides specific explanations and discussion per question is available for download here (and/or is available on the CD supplied at the back of the book version of the thesis). An example of this more detailed analysis format is given in Appendix A on page 484 below. This Stage 3 level of operationalisation provides a 'zoomed out' perspective from the Stage 2 format so that it is now possible to see, in a more global way, the spread of questions across the different domains of practice. The disadvantage of this format is the loss of capacity for detailed explanation of the structure of the discursive resources that characterise each question and, consequently, of criteria according to which the structure of legitimate participation in each question is determined.

Table 7: Summary of the categorisation of the questions in the Car Loan activity according to the domains of practice of the developed language of description

<table>
<tr><th rowspan="2"></th><th rowspan="2">Everyday (E)</th><th rowspan="2">Mathematical Competency (MC)</th><th rowspan="2">E or MC</th><th rowspan="2">E & MC</th><th rowspan="2">Modelling</th><th colspan="7">Reasoning & Reflection</th></tr>
<tr><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th></tr>
<tr><td>**Question (Marks)**</td><td>1.1 (1)</td><td></td><td></td><td></td><td></td><td colspan="5">✓(CE)*</td><td></td><td></td></tr>
<tr><td></td><td></td><td>1.2 (3)</td><td></td><td></td><td></td><td colspan="3">✓</td><td colspan="2">✓(ME)*</td><td></td><td></td></tr>
<tr><td></td><td>2.1 (2)</td><td></td><td></td><td></td><td></td><td colspan="5">✓(CE)</td><td></td><td></td></tr>
<tr><td></td><td>2.2 (1)</td><td></td><td></td><td></td><td></td><td colspan="5">✓(CE)</td><td></td><td></td></tr>
<tr><td></td><td>2.3 (1)</td><td></td><td></td><td></td><td></td><td colspan="5">✓(CE)</td><td></td><td></td></tr>
<tr><td></td><td>2.4 (1)</td><td></td><td></td><td></td><td></td><td colspan="5">✓(CE)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>2.5 (3)</td><td></td><td></td><td></td><td colspan="3">✓</td><td colspan="2">✓(ME)</td><td></td><td></td></tr>
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<tr><td></td><td>3.2 (2)</td><td></td><td></td><td></td><td></td><td colspan="5">✓(CE or ME)</td><td></td><td></td></tr>
<tr><td></td><td>3.3 (3)</td><td></td><td></td><td></td><td></td><td colspan="5">✓(CE)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>3.4 (3)</td><td></td><td></td><td></td><td colspan="3">✓</td><td colspan="2">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>4 (2)</td><td></td><td></td><td></td><td colspan="3">✓</td><td colspan="2">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>5 (4)</td><td></td><td></td><td></td><td colspan="3">✓</td><td colspan="2">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td>6 (1)</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>✓ (CE)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>7.1 (2)</td><td></td><td></td><td></td><td colspan="3">✓</td><td colspan="2">✓(ME)</td><td></td><td></td></tr>
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<tr><td></td><td></td><td>8 (13)</td><td></td><td></td><td></td><td colspan="3">✓</td><td colspan="2">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>9 (7)</td><td></td><td></td><td></td><td colspan="3">✓</td><td colspan="2">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>10.1 (4)</td><td></td><td></td><td colspan="5">✓</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>10.2 (4)</td><td></td><td></td><td colspan="5">✓</td><td></td><td></td></tr>
<tr><td></td><td>10.3 (4)</td><td></td><td></td><td></td><td></td><td colspan="5">✓(CE)</td><td></td><td></td></tr>
<tr><td></td><td>10.4 (4)</td><td></td><td></td><td></td><td></td><td colspan="5">✓(CE)</td><td></td><td></td></tr>
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<tr><td></td><td></td><td></td><td></td><td>11 (Graph and Intro)</td><td></td><td colspan="4">✓</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>11.1 (1)</td><td></td><td></td><td></td><td colspan="5">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>11.2 (1)</td><td></td><td></td><td></td><td colspan="5">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>11.3 (1)</td><td></td><td></td><td></td><td colspan="5">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>11.4 (4)</td><td></td><td colspan="5">✓(ME or CE)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>11.5.1 (3)</td><td></td><td></td><td></td><td colspan="5">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>11.5.2 (3)</td><td></td><td></td><td></td><td colspan="5">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td>11.5.3 (4)</td><td></td><td></td><td></td><td colspan="3">✓</td><td colspan="2">✓(ME)</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>11.6 (3)</td><td></td><td colspan="5">✓</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>11.7 (2)</td><td></td><td colspan="5">✓</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>11.8 (Graph and Intro)</td><td></td><td colspan="3">✓</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>11.8.1 (4)</td><td></td><td colspan="5">✓</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>11.8.2 (3)</td><td></td><td colspan="5">✓</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>11.8.3 (3)</td><td></td><td></td><td></td><td>✓</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>12.1 (3)</td><td></td><td></td><td></td><td></td><td colspan="4">✓</td><td>✓(CE)</td><td></td><td></td></tr>
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<tr><td></td><td>13 (2)</td><td></td><td></td><td></td><td></td><td colspan="4">✓</td><td>✓(CE)</td><td></td><td></td></tr>
<tr><td></td><td>14 (3)</td><td></td><td></td><td></td><td></td><td colspan="4">✓</td><td>✓(CE)</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>15 (14)</td><td colspan="6">✓(CE)</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>16 (20)</td><td colspan="6">✓(ME)</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>17 (20)</td><td colspan="6">✓(ME)</td><td></td></tr>
<tr><td></td><td>18 (2)</td><td></td><td></td><td></td><td></td><td colspan="7">✓</td></tr>
<tr><td></td><td colspan="5"></td><td colspan="7"></td></tr>
<tr><td>**Question count**</td><td>17</td><td>15</td><td>2</td><td>8</td><td>3</td><td colspan="7"></td></tr>
<tr><td>**Marks allocated**</td><td>(38)</td><td>(52)</td><td>(8)</td><td>(19)</td><td>(54)</td><td colspan="7"></td></tr>
</table>

Three trends evident in the information shown in the table are worth consideration. Firstly, the questions in the activity facilitate movement through all of the domains of practice of the knowledge domain of mathematical literacy. Engagement with the activity exposes participants to Everyday forms of knowledge, practice, reasoning, and skills relating to the context of vehicle finance, while at the same time imbuing an expectation for engagement with mathematically-legitimised components of this contextual environment. Several of the questions in the activity also facilitate more in-depth and elaborated engagement with elements of the contextual environment on loans through specification of complex and open-ended modelling processes. The endorsed narratives for these processes acknowledge the possibility of multiple solutions paths and strategies, hereby promoting contextual sense-making practices over mathematically-legitimised forms of participation. Secondly, the relative equivalence in the total marks allocated to questions associated with each of the domains of practice signifies equal emphasis placed on each of domains in terms of engagement with the contextual environment. In other words, participation in each domain is equally valued and considered of equal importance for the development a broadened and potentially more enhanced understanding of the contextual environment on vehicle finance. Thirdly, notice the dominance of questions that engage levels 1 to 5 of the Reasoning and Reflection domain of practice, but the relative absence of questions that engage levels 6 and 7 of the domain. As will be discussed in more detail below, this suggests the prioritisation of engagement with functional and structural components of contextual sense-making practices rather than with critical analysis processes.

18.2.7 Operationalisation Stage 4: Some reflections on the operationalisation process of the external dimension of the language of description

Having demonstrated the operationalisation process of the external dimension of the language of description, the discussion in this sub-section is focused on a degree of reflection on this operationalisation process, on challenges encountered during this operationalisation process, and on possible consequent limitations of the external dimension.

The first challenge encountered is that my consistent attempts to categorise the structure of participation required in each question in relation to a specific domain of practice of the knowledge domain of mathematical literacy were thwarted by the inevitable and necessary presence of both contextual and mathematical signifiers and elements in almost every Mathematical Literacy related activity or question. As such, what I had initially envisioned would be a simple categorisation process – driven through the identification of either everyday or mathematical discursive resources − turned out to be considerably more complex. This is because the process shifted towards identifying the dominant terrain that is prioritised (specifically through the endorsed narratives and the routines specified in the generation of those narratives) rather than on identifying the terrain that is included and the terrain that is excluded.

This initial challenge was exacerbated by a further significant challenge encountered while attempting to distinguish and categorise activities, practices and forms of participation associated with the Modelling domain of practice from those associated with the other two domains. Phrased differently, the challenge and associated question I constantly faced in the operationalisation processes is that if every activity involves some sort of interplay between mathematical elements and contextual elements, then how are activities drawn from the Modelling domain of practice to be distinguished from forms of participation associated with the Everyday and Mathematical Competency domains? I resolved this issue by adopting the stance that modelling activities involve a deliberate attempt at the reconstruction of a real-world practice or scenario to explore possible alternative forms of understanding of components of the practice or scenario, or of possible alternative forms of participation in that practice or scenario. Modelling involves coming to understand how people behave, think and act in a particular setting, together with how they might behave, think and act differently if alternative variables and components are considered. Forms of participation associated with the Mathematical Competency domain of practice, by contrast, are more particularly concerned with finding a solution to a problem through a mathematical or calculation approach; while practices associated with Everyday forms of participation are primarily directed towards understanding how people act, think and behave in everyday practices.

Linked to this is a limitation in the external language of description for being able to adequately distinguish practices which draw on knowledge and contents from primarily the same domain of practice, but which differently prioritise elements of that domain. For example, a word problem in mathematics contains both mathematical and contextual elements, even though many of the contextual elements (and understanding thereof) are superfluous to successful engagement in the problem scenario. By contrast, a question that asks learners to calculate the inflation rate from two given price values requires some level of understanding the contextual entity of inflation, of what an inflation rate is, and of how an inflation rate is to be calculated. This understanding of the contextual elements is essential for successful engagement with the calculation components of the question. And, yet, if both of these question types were encountered in the subject-matter domain of Mathematical Literacy, then they would both be categorised as reflecting practices and forms of participation associated with the Mathematical Competency domain of practice – even though the questions differently prioritise the level of interaction required between mathematical and contextual elements. What is ultimately needed is a spectrum of forms of participation in each domain of practice. However, the development of this spectrum is beyond the scope of this study and presents opportunity for future research.

Yet a further challenge in the operationalisation process involved encountering elements that do not explicitly or obviously reflect practices associated with the Everyday, Mathematical Competency, or Modelling domains of practice. For example, the requirement in a task for participants to develop a presentation to illustrate the results of an endorsed narrative that has been developed through engagement with the practices associated with a particular domain of practice (see Questions 15 and 16 in the Car Loan activity above). I resolved this issue by reflecting that in the realm of scientific mathematics, effective communication with appropriate mathematical terminology, structures and notation is essential for effective communication with other participants in the knowledge domain. In mathematical literacy the situation is somewhat different since communication with other members in the domain requires utilisation not only of appropriate mathematical structures, but also of tools, techniques, notation and vocabulary that reflect the nature and structure of endorsed forms of communication in everyday and workplace settings. In mathematical literacy, communication involves significantly more than just presenting a result; it also involves the crucial element of communicating the result in a way that is understandable and accessible to other people who participate in the subject as well to people who participate regularly in the contextual scenario under investigation. As such, where there is a requirement in an empirical activity relating to the subject (and to the knowledge domain of mathematical literacy) for a particular structure of communication, I now consider that requirement as a directive for recognition of and reflection on the structure of appropriate and accessible communication from an Everyday knowledge perspective.

Although not encountered in the Car Loan activity, a potential challenge to be faced in analysis of other empirical activities relating to the subject-matter domain of Mathematical Literacy is the presence of activities and/or questions which do not fit within any of the domains of practice that characterise the knowledge domain of mathematical literacy. For example, there is always the possibility that an empirical activity will be characterised by un-contextualised scientific mathematics problems. It is my intention to resolve this issue by considering that a category of exclusion for an activity or question item signifies the possibility of inconsistency with the primary goal for the development of a life-preparedness orientation that characterises the knowledge domain of mathematical literacy described in the internal language of description presented in this study. This is certainly not to imply that forms of participation that exist outside of the domains of practice schematic of the knowledge domain of mathematical literacy must be rejected and excluded from all conceptions of mathematical literacy. However, in the context of South Africa where a distinctive subject exists that positions the knowledge domain of mathematical literacy as separate from the domain of mathematics, then the inclusion of such activities in this terrain subverts this intention. The inclusion of such activities also subverts the attempts to overcome the forms of mythologising commonly associated with contextualised mathematics practices through the promotion of a structure of participation dominated by a life-preparedness orientation.

As a final comment on the operationalisation process, it is important to also consider the Reasoning and Reflection domain of practice. In particular, although the first five levels of Reasoning and Reflection appear in almost all of the questions, it is significant that only one question (Question 14) deals explicitly with the sixth level of reasoning/reflection. Namely, reasoning and reflecting on the broader consequences of the use of mathematics in contextual situations (although this level of reasoning/reflection is expected in an implicit way in the modelling processes associated with Questions 15, 16 and 17). Furthermore, none of the questions deal explicitly with the seventh level of reasoning/reflection. Namely, with questioning the way in which reasoning or reflection about specific (mathematical or contextual) discursive resources, knowledge and techniques employed in the problem-solving process has been conducted, and/or reflecting in a critical way on the models constructed by others and on the assumptions made in the construction of those models. This suggests that although a level of reasoning/reflection is expected in every question, this reasoning/reflection is limited primarily to analysis of signifiers in the problem situation and the selection and deployment of appropriate routines and methods required to generate acceptable and/or endorsed solutions. The 'critical education' dimension of the Reasoning and Reflection domain of practice is, thus, relegated to heightened priority on reasoning and reflection about structural and functional aspects of the contextual sense-making process. In short, the exemplar activity is primarily concerned with whether participants are able to engage with the given contextual resource and contextual environment in a functional way and are able to answer questions relating to the resource and environment. Of significantly lesser concern is the ability of the participants to critically analyse the problem-solving and sense-making process and the assumptions and potential biases that influence and direct this process.

WHERE TO FROM HERE

The development process of the internal and external dimensions of the language of description for the knowledge domain of mathematical literacy is now complete. As such, it is now possible to employ this consolidated language to analyse and compare textual phenomenon relating to the subject-matter domain of Mathematical Literacy. These textual phenomenon include a section of the curriculum statement for the subject, a textbook chapter, national exemplar examination papers, and course materials for a teacher education course. This empirical analysis process through the lens of the language of description provides a further means for evaluating the extent to which the developed theoretical language is able to facilitate valid descriptions and comparisons of empirical phenomenon (and of the structure of legitimate participation in these empirical phenomenon). And, this is done specifically in relation to the empirical terrain of the subject-matter domain of Mathematical Literacy.

However, prior to engagement with the language of description in a formal empirical analysis process, in Part 6 a discussion is provided on methodological considerations involving the process of textual analysis. This discussion provides the necessary groundwork and contextualisation for the formal empirical analysis process of the textual phenomenon conducted in Part 7.

PART 6
METHODOLOGY − TEXTUAL ANALYSIS

INTRODUCTION AND OVERVIEW

In the previous part (Part 5) of the study I theorised and operationalised the external dimension of the language of description mathematical literacy. A significant part of the operationalisation process involved analysis of an exemplar textual activity to demonstrate a strategy for identifying discursive resources in a text and the use of the characteristics of these discursive resources to further identify the dominant structure of knowledge and legitimate participation in and with the text. In Part 7, I use this same method to analyse and compare various empirical textual resources drawn from the empirical terrain of the subject-matter domain of Mathematical Literacy, both in relation to each other and in relation to the characteristics of the domains of practice of the knowledge domain of mathematical literacy. This operationalisation process has already involved the methodology of textual analysis (with a specific focus on discourse analysis) and methods associated with the field of semiotics (through specific focus on identifying and analysing signifiers in textual discourse). And, all of the empirical phenomena analysed in this study are *textual* in nature. For this reason, the primary methodology employed in the analysis of these empirical phenomena is that of *textual analysis*.

My intention in this part of the study is to discuss in a more explicit way various considerations and perspectives on the methodology of textual analysis and associated methods, together with discussion of the merits and limitations of this form of analysis. To facilitate this intention, the discussion in the pages below is divided into four chapters (Chapter 19 to Chapter 22). In Chapter 19 I identify how my specific paradigmatic commitment to Interpretivism in this study directly impacts on the structure, form, coherence and appropriateness of the intended methodology of textual analysis and associated methods. In Chapter 20 I discuss various preliminary issues for consideration in relation to the methodology of textual analysis, including specifying the process and object of the analysis process (namely, textual analysis and the structuring of knowledge in textual resources respectively). I then discuss various interpretations of the notion of a text, together with specification of the features of the texts to be analysed in this study. I extend this discussion to outline interpretations on the process of textual analysis and identify the specific process employed in the analysis of empirical textual resources in this study. In Chapter 21 I outline different methodological approaches to textual analysis and argue that the approach adopted in this study is embedded in the sociological tradition with a humanistic approach. Furthermore, the adopted approach is driven by a form of discourse analysis, at a lexical level of the text, which includes the use of certain methods drawn from the field of semiotics, and with an intention for identifying the connotative meaning embedded in a text. By the conclusion of the chapter I will have made clear the features of both the methodology and associated methods employed in the analysis of empirical textual resources relating to the subject-matter domain of Mathematical Literacy. Finally, in Chapter 22 I discuss limitations and criticism of the methodology of textual analysis – with particular focus on the possibilities for projection and counter-transference, and highlight the necessity for reflexivity to offset these potential issues.

Figure 48 below provides a roadmap of the structure of the discussion in this part.

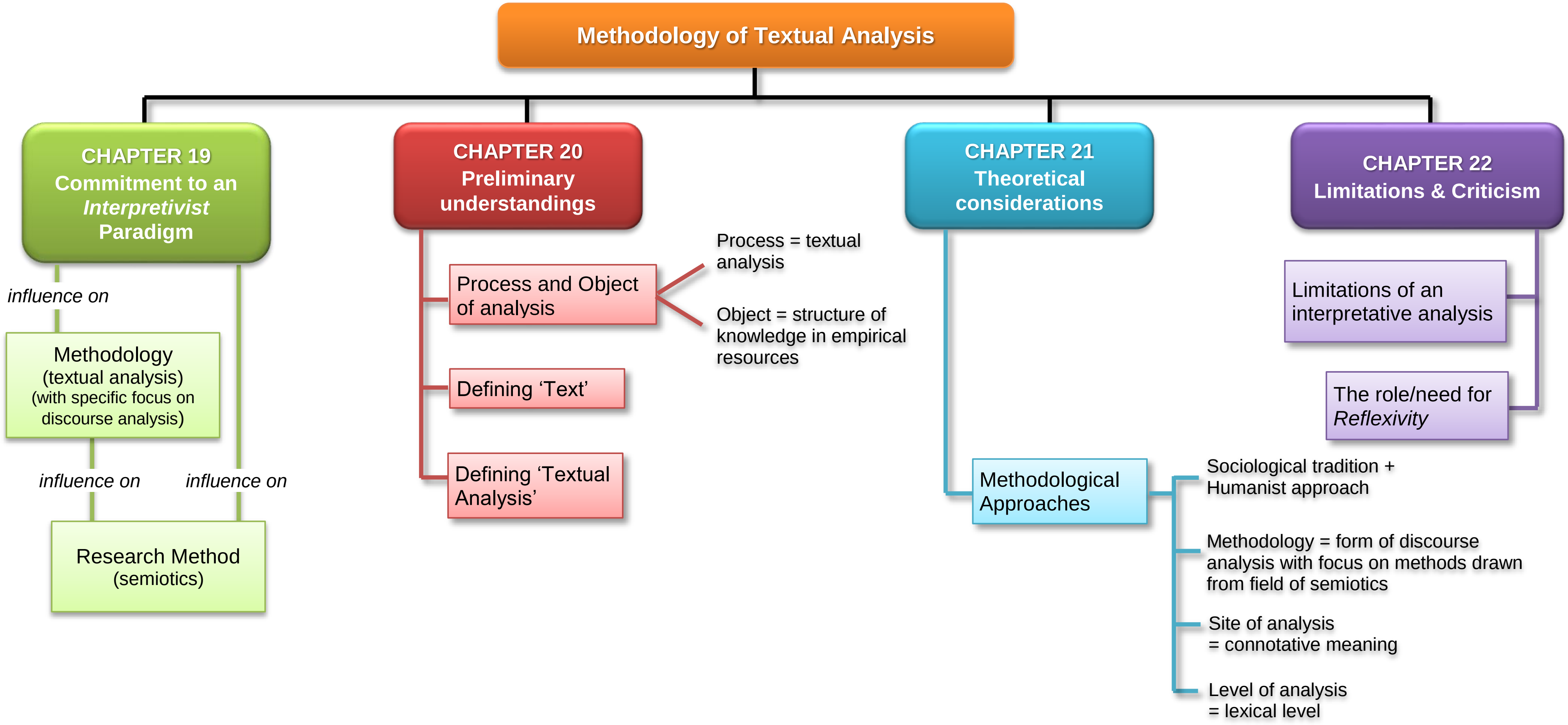

Figure 48: Overview and chapter structure of Part 6 of the study

CHAPTER 19
SETTING THE SCENE – TRACING METHODOLOGY AND METHODS TO THE PARADIGMATIC ORIENTATION OF THE STUDY

The need to revisit the dominant paradigmatic commitment in this study is necessitated by the fact that the commitment to a particular paradigmatic 'world-view' directly influences the ontological and epistemological perspectives that structure the research process and, consequently, the form and structure of the methodology and associated research methods employed (Cohen, Manion, & Morrison, 2011, p. 6).

As has been argued previously (c.f. Part 1 and Chapter 1, starting on page 10), this study is positioned within an Interpretivist paradigm. *Ontologically*, the study is dominated by a post-structuralist relativist position in which all cultures are seen to make sense of the world differently and, as such, there is no single correct reality; rather multiple and varied social realities are possible as dependent on the culture and context in which a person finds themselves at a particular point in time (McKee, 2003, pp. 9-10). *Epistemologically*, knowledge is perceived as socially constructed and socially situated, and is seen not to exist independently of our interpretations of it (Scotland, 2012, pp. 11-12). From this position, the intention or aim of the research process is that of interpretation, reconstruction and understanding of the way in which people have constructed perceptions of reality, with awareness that the way in which the researcher interprets and responds to those perceptions of reality is influenced by their own subjective consciousness and experience of the world (Scotland, 2012, pp. 11-12) (Guba & Lincoln, 1994, p. 113). *Methodologically*, the type of methodologies compatible with this interpretative intention are those that involve direct interaction between researcher and respondent (or empirical resource) (Guba & Lincoln, 1994, p. 111). These methodologies provide the means for understanding phenomenon from an individual's perspective, for understanding how groups of individuals interact, and for understanding how the socially embedded nature of interactions impact on the perceptions of reality that are constructed and experienced (Scotland, 2012, p. 12). As such, the methodologies are characterised by hermeneutics (interpretation of text and/or language) and dialogue (Guba & Lincoln, 1994, p. 111). To facilitate this methodological intention, any employed research *methods* must provide the means for understanding behaviour and, particularly, for understanding respondents own perspectives of their behaviour. The methods employed must allow for the voice of the participants to be heard and must not dominate or subvert this voice (Scotland, 2012, p. 12). Importantly, and as a further reminder, although this study is dominated by an interpretive paradigmatic world-view, 'Advocacy and activism' (Guba & Lincoln, 1994, p. 113), together with a critical dimension, are also key elements of the research process.[237] This critical dimension is driven by a sociological impetus[238] to overcome the current state of disadvantaged study and career opportunity afforded through participation in the subject-matter domain of Mathematical Literacy. It is this sociological impetus which, in large part, provides the motivation for my developing an alternative conception of knowledge on which the practices of the subject are based in an attempt to afford increased social and career opportunity for those who participate in the activities of the subject. In sum, the primary intention of this study involves an attempted interpretation and description of the structure of knowledge and participation in practices associated with the knowledge domain of mathematical literacy (and, by implication, the practices associated with the subject-matter domain of Mathematical Literacy that draws on this knowledge domain). However, this intention is grounded on an underlying emphasis for the reconstruction of existing conceptions of reality for the knowledge domain (and, by implication, also for the subject).

Reflecting now on the significance of this discussion for the analysis of empirical textual resources relating to the subject-matter domain of Mathematical Literacy in this study. O*ntologically*, the research process in this study is driven by an intention to understand current practices associated with the subject-matter domain of Mathematical Literacy, to problematise those practices, and to describe an alternative conception of knowledge for the knowledge domain of mathematical literacy that serves to negate some of the existing challenges in the subject. *Epistemologically*, the research process views knowledge in practices associated with the knowledge domain of mathematical literacy (specifically where a life-preparedness orientation is prioritised) as characterised by a particular form of relationship between mathematical and contextual terrains (and also of relationship of knowers in the domain to this knowledge). This view of knowledge for the domain has direct implications for the way in which knowledge in practices associated with the domain are structured and participation in relation to that knowledge is legitimised[239]. This ontological and epistemological position necessitates a methodology that facilitates understanding of how the practices of participants (specifically in the empirical terrain of the subject-matter domain of Mathematical Literacy) either reflect coherence or divergence with this alternative conception of knowledge and participation. To achieve this end, the primary methodology selected for utilisation is drawn from the realm of

[237] According to Guba and Lincoln (1994, p. 113), an aim or intention for ‘advocacy and activism’ is entirely consistent with the interpretative agenda of the Constructivist paradigm.

[238] This does not mean that I am not somewhat swayed by more the recently resurrected realist movement and associated assertions that there is a body of powerful knowledge that *all* learners should have access to irrespective of their ‘power’ (class, status, social/economic/political standing) (Moore, 2013; Wheelahan, 2010). In particular, if we take seriously that what counts as knowledge is in part socially shaped, then my attempt to (re)construct knowledge for the knowledge/subject domain of Mathematical Literacy presents a challenge to the existing status quo of practice and the structure of legitimate participation in the subject (the sociological impetus of the study). However, from the critical realist perspective, participation in the subject only represents one site of knowledge contestation and positioning. As such, although changes in the structure of knowledge may affect the structure of power relations in the subject, it is only through understanding of the underlying structures and mechanisms in broader society which direct, influence and determine the structure of legitimised knowledge and participation at a broader level, that access to the ‘valued’ knowledge is possible. For purposes of effecting change through the findings of this study, the critical realist perspective – combined with a sociological impetus – thus, offers some measure of attraction.

[239] The process of outlining the domains of practice that collectively comprise the knowledge domain of mathematical literacy was demonstrated in the theorising of the internal language of description for the knowledge domain in Part 4 of the study. The process of identifying the structure of knowledge and participation in each of these domains of practice through identification of the discursive resources in each domain was demonstrated in the theorising and operationalisation of the external language of description in Part 5 of the study. It is through the combination of these two processes that the structuring of knowledge and participation has been defined.

hermeneutics and involves analysis of the discursive resources embedded in (and associated discourse communicated through) textual resources for practices associated with the knowledge domain – in other words, *the methodology* of textual analysis with a particular focus on analysis of discourse in the text. This methodology provides the means for identification and interpretation of the 'world-view' of the authors of these texts and, specifically, of how these authors perceive the structure of legitimate knowledge and the criteria for legitimate participation in practices involving the use of mathematics in contextualised problem situations. This methodology is accompanied by specific *methods* involving analysis of the discursive resources embedded in empirical texts through techniques associated with the field of semiotics (namely, identification and interpretation of the signifiers [words/vocabulary and visual mediators] in a text and the associated routines that facilitate the generation of endorsed narratives).[240]

In keeping with both the relativist world-view and sociological impetus for this study, the textual analysis process employed is accompanied by recognition that any interpretation of textual phenomena only provides a limited perspective and a particular viewpoint (and only one of many viewpoints) of the phenomena. Furthermore, rather than being driven by a search for absolute 'truth' or 'accuracy', the research process is instead directed by an attempt to identify, interpret and understand the 'world-views' of the author(s) of various texts with respect to the way in which legitimate knowledge and participation is structured and prioritised in the texts. This interpretation is accompanied by recognition of the 'situated nature' of the texts. Situated in this regard refers to the impact of the social, cultural, economic, and political background and positioning of the author(s) of the texts on the structure of the meaning embedded within the texts and on areas of prioritising in the texts. Situated also refers to the positioning of the texts themselves within broader contextual (social, cultural, economic and political) environments[241]. In doing so, the methodology facilitates recognition of the subjective nature of the analysis process and of the influence of the researcher's own values and social positioning on choices and decisions made in the analysis process. The methodology also facilitates recognition of the possibility for varied and multiple interpretations of the text, and of the embedded nature of texts as a socially, economically and politically constructed entity.

In light of the above, I contend that the employed methodology in this study is consistent with the stated epistemological and ontological underpinnings of the research process.

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[240] Note that the specific role of semiotics in the textual analysis process in this study is discussed in more detail in the pages below (c.f. the discussion starting on page 373).

[241] A warning to the reader here: please do not interpret this statement as reflecting commitment to a critical research paradigm. My intention in this study and in the chosen methodology is predominantly Interpretivist, albeit with a sociological swagger. However, my own position is that understanding of the role of a text as an element of positioning and as a site of recontextualisation of knowledge is impossible without consideration of the social, political and economic environment and motivation in and for which the text was developed. For me, interpretation of these environmental structures gives a further level of meaning for the structure and contents of the text. However, in the context of this study, identification and interpretation of the structures serve a goal for facilitating understanding; and a critical element or orientation for effecting change is largely lacking. It is in this regard that an orientation to an Interpretivist paradigmatic orientation dominates over any critical ontological persuasions.

CHAPTER 20
INTENDED METHODOLOGY: *TEXTUAL ANALYSIS* – PRELIMINARY UNDERSTANDINGS

The discussion in this section presents certain preliminary understandings of the methodology of textual analysis. These include specification of the process and object of analysis in the textual analysis process, definition of what constitutes a text in the context of the empirical analysis in this study, and discussion of the intention of the textual analysis process.

20.1 The process and object of analysis

Before embarking on a detailed discussion of the concept of a text and associated theoretical and methodological considerations for the analysis of texts, it is necessary at the outset to make a distinction between the 'process' and the 'object' of analysis.

At a surface level my intention is to analyse various empirical texts associated with the subject-matter domain of Mathematical Literacy. As such, the 'process' employed here is that of *textual analysis* – namely, the analysis and interpretation of these empirical texts as a sense-making activity. This 'process' is driven by the desire to gauge the potentially differing fields of knowledge production that have been drawn on by the authors of these texts in the construction of the texts. This process also provides the potential to illuminate the consequent prioritising of differing intentions (e.g. the learning of mathematical knowledge; the development of a life-preparedness orientation) for the teaching and learning of the contents of subject that are reflected in and transmitted through the texts. As such, the 'object' of analysis is the *structuring of knowledge* in these texts and how this structuring of knowledge relates to the different components of the knowledge domain of mathematical literacy (as described in the internal language of description). The 'process' of textual analysis thus facilitates a methodology for identifying, interpreting and comparing differences in the ways in which the texts structure knowledge.

It is particularly important to make this point upfront given the sociological perspective or intention that underpins this study. Much of the literature read in relation to the process of textual analysis – and especially literature that supports a sociological perspective − includes a focus on the text as an instrument of positioning, power and control (for example, see (Apple, 2004) and/or (Dowling, 1998)). The object of analysis in such approaches is, in essence, a focus on 'Knowers' (Maton, 2007) and the role of texts in legitimising and/or disenfranchising the cultural capital of different groups of knowers who engage with the texts (Palli, Tienari, & Vaara, 2010). By contrast, the object of the textual analysis process for this study is 'Knowledge Structures' (Maton, 2007) – namely, on the form of the knowledge represented in the different texts and on the different criteria according to which knowledge (and consequent participation with that knowledge) is legitimised. A concentrated focus on 'Knowledge' does not, however, deter from my sociological concern with the educational disadvantage that results from the legitimisation of practices associated with the subject-matter domain of Mathematical Literacy exclusively or primarily according to mathematical principles. Rather, focus on knowledge structures and on the fields of knowledge production from which the content in the texts is drawn provides the means for identifying the extent to which practices in the subject prioritise mathematical or life-preparedness principles. This, in turn, facilitates analysis of the extent to which such practices reinforce the positioning of the subject within a restrictive or disadvantaging 'public domain of mathematics' frame or within an alternative frame aimed at enhancing and elaborating participation in real-world practices.

Having specified the distinct difference in the process and object of the empirical analysis for this study, discussion now shifts to elaboration of the notion of a text and to different perspectives on the process of textual analysis.

20.2 Defining 'text'

Textual analysis inherently involves *analysis* of *text*. As such, a clear statement is necessary regarding the meaning of the word 'text' as it is understood and employed in the context of this study.

A possible appropriate starting point is the assertion that a text constitutes a duality as both a physical and a semiotic object. The text is physical in the sense that it occupies a physical presence (e.g. a book contains pages and a cover). And the text is semiotic in the sense that it includes a variety of forms of symbols and signs (including words and/or language and/or pictures) that describe and represent a particular view of an object (Lehtonen, 2000, p. 72).

At a further basic level, a text is a form of communication or message system – a 'communicative artefact' (Lehtonen, 2000, p. 72) – that represents a particular interpretation of the *meaning* of something that exists in reality (McKee, 2003, p. 4). This 'meaning' is contained and expressed in the words and other signifiers (e.g. pictures) that make up a text and represent a level of discourse between the author(s) of the text and the intended audience. Note that 'discourse' in this context refers to a group of statements that reflect the way in specific meaning is attached to particular structures of knowledge and behaviour, that define what counts as valuable knowledge, and that structure the basis of legitimate action and thought in relation to that knowledge (Cohen et al., 2011, p. 574 & 589). Following this line of thinking, a text represents a 'material trace' (McKee, 2003, p. 15) of the way in which an individual or group (i.e. the author(s) of the text) have interpreted a situation and the meaning they have attached to that situation – the text is evidence of the 'production of meaning' (McKee, 2003, p. 25) by the authors. It is for this reason that Halliday (1978, p. 136) and Ifversen (2003, p. 61) describe text as a 'semantic unit' (i.e. a unit that conveys a meaning) and a 'unity of meaning' respectively. It is also in this vein that McKee (2003) suggests that,

> ... whenever we produce an *interpretation* of something's *meaning* – a book, television programme, film, magazine, T-shirt or kilt, piece of furniture or ornament – we treat it as a text. A text is something that we make meaning from. (p. 4, emphasis in original text)

Importantly, notice that the notion of a text as employed by McKee is not restricted to paper formats. Rather, a text includes a variety of mediums, including auditory (e.g. music), visual (e.g. television and film), as well as various physical objects (e.g. furniture). In fact, for Lehtonen (2000, p. 72), a text can be expressed in any symbolic form, including direct interaction between agents. In this sense, it is not the structure or form of an object that constitutes it as a text, but, rather, that the text represents the human interpretation and description of the object. Also notice that a text need not be restricted to descriptions in linguistic form (i.e. descriptions that employ words and language).

Rather, as suggested by Dowling (1998, p. 131), non-linguistic expressions and utterances (e.g. diagrams and pictures) are also included in the definition of text.

Whatever the form of the text, what is generally agreed upon is that a text – whether it is constituted by a single utterance or by a sequence of utterances – is characterised by a level of internal organisation, cohesion, logic and structure, with clearly defined symbolic entities, and which comprises a totality (Aamotsbakken, 2008, p. 25; Halliday, 1978, p. 136; Ifversen, 2003, p. 61; Lehtonen, 2000, pp. 72 - 73)[242].

In relation to the empirical textual phenomena (i.e. curriculum document, textbook section, examinations, and course notes) for the subject-matter domain of Mathematical Literacy analysed in this study, all of the phenomena are *paper and/or printed* texts and all include a combination of linguistic (words and language) and non-linguistic (pictures and diagrams) elements. Textual entities such as movies, music and formal art (e.g. paintings) do not characterise these texts. All of the texts are also 'monologic texts' (Dowling, 1998, p. 131) in that they present a singular voice (namely, the voice of the author of the text[243]) that can be read in relation to a single activity – namely the subject-matter domain of Mathematical Literacy. The texts also all constitute 'factual texts' (as opposed to literary texts) that inform or instruct by giving information and facts.

There are, however, distinctions between the empirical texts that must be recognised, particularly with respect to the <u>intention</u> of the texts. Both the textbook and course notes characterise *pedagogic texts,* which means that the texts are characterised by a joint intention for "informing and producing learning" (Aamotsbakken, 2008, p. 24). In alternative terms, these texts facilitate access to recontextualised knowledge. However, while both constitute pedagogic texts, the textbook and course notes still serve differing agendas. Whereas the textbook focuses predominantly on presentation of content topics and (largely) implicitly on pedagogy, the course notes focus to a much larger extent on pedagogy and methodology and to a lesser extent on content. The course notes are also directed at teachers (i.e. adults) and their teaching methodology, while the textbook is directed at school-going learners and at providing illustrations of methodology which may provide increased access to and understanding of content topics.

In contrast to the textbook and course notes, the national examinations constitute *evaluative texts*, with a dominant intention for measuring understanding (or perhaps knowledge retention) of a prescribed knowledge base or skill set (that has, possibly, been transmitted through a pedagogic text) – namely, for measuring 'worth' (Abma, 1998, p. 434). Finally, the curriculum document is a mixed bag, comprising a statement of the goals, philosophy and/or intention of the subject (both at a school level and for post-school personal, workplace and/or societal purposes), together with a statement of the

[242] See also Cohen et al. (2011, p. 574) who, by drawing on the work of Renkema (2004), present seven main criteria for the classification of a text as a discourse: cohesion, coherence, intentionality, acceptability, informativeness, situationality, and intertextuality.

[243] Clearly the texts might be used by people other than the authors who then add an additional 'voice' to the text. This is done through the way in which they engage with the text (i.e. perhaps agree with the contents of the text, or modify the text to suit their own or a different vision than that of the author) and/or through the way in which they engage others with the text. My analysis, however, is concerned with the 'voice' embedded in the text itself (i.e. the 'enactment' of the discourse in and through the text (Fairclough, 2001, p. 3)) rather than with how that voice is engaged with by others (i.e. the 'enactment' and possible 'inculcation' of the discourse by those who engage with the text (Fairclough, 2001, p. 3)). Clearly, in analysing the voice of a text I make assumptions and predictions about how those who use the text might relate to this voice, but further research would be necessary – possibly in interview format – to determine precisely how people experience, relate to, or negate the dominant voice in the texts.

contents, skills, and resources that make up the core knowledge base for the subject. The criteria according to which participation in the subject is to be legitimated and validated are also included. In difference to the pedagogic texts, the curriculum document is more concerned with providing an explicit statement of the official or endorsed knowledge base and less so with how that knowledge is to be made accessible. Importantly, all of the other texts – the textbook, course notes and national examinations – use the curriculum document as a point of reference for deciding what knowledge to include in the pedagogic, instructional or evaluative process. However, each text interprets, engages with, and presents the content of the curriculum document in different ways and with differing intentions or areas of focus (as discussed above).

A further distinction between the empirical texts analysed in this study relates to the position or site in which the texts are encountered. Although all of the texts relate to the subject-matter domain of Mathematical Literacy, the curriculum, textbook and examinations comprise contents relating specifically to the Grade10, 11 and 12 school-based curriculum. The course notes for the teacher education course, on the other hand, are aimed at teachers who are broadening their understanding of the subject, which includes understanding of curriculum content together with methodology and pedagogic strategies for enhanced teaching. As such, while the textbook and examinations are more strongly bounded to the contents of the curriculum for the subject, the course notes make a reach beyond the school based curriculum to include aspects relating to more general and theoretical issues of pedagogy and methodology that are not bounded within the subject's curriculum document. Furthermore, while the textbook and examinations are designed primarily for learners, the course notes are designed for teachers who want to expand their knowledge of their subject, and the curriculum document for teachers and a variety of other practitioners (such as subject advisors and textbook authors) who are involved in both direct and peripheral participation in the subject. Reflecting back on Bernstein's (1996) notion of the *Pedagogic Device*[244], one way in which to theorise and/or summarise the differences in location of the various texts is by locating the texts within the structure of that device. In this respect, while all of the texts are located within the *Field of Recontextualisation* (and none within the Fields of Production or Reproduction[245]), the curriculum document characterises a text positioned within the *Official Recontextualising Field* and the textbook, examinations, and course notes within the *Pedagogic Recontextualising Field*.

In brief summary, while the texts analysed are all similar in form and in the knowledge domain to which they relate there are significant differences with respect to the intended audiences of the texts (i.e. who it is that is expected to engage with the texts) and the way in which the texts are engaged with.

[244] Refer to Part 1 and Chapter 3 above (starting on page 23) for a more detailed discussion of the Pedagogic Device and of how the different components or parts of this study are located within that device.
[245] The internal language of description for the knowledge domain of mathematical literacy developed in this study constitutes a component of the Field of Production. There is no part of this study that deals directly with empirical instances drawn from the Field of Reproduction.

20.3 Defining ‘textual analysis’

Consider the following statement from Apple (2004):

> Yet, [texts] [246] are surely important in and of themselves. They signify, through their content *and* form, particular constructions of reality, particular ways of selecting and organising that vast universe of possible knowledge. They embody what Raymond Williams calls the *selective tradition*: someone’s selection, someone’s vision of legitimate knowledge and culture ... (p. 182, emphasis in original text)

If a text is, thus, considered as a particular interpretation and representation of reality – a particular world-view, then at a simplest level the process of textual analysis is a *sense-making activity* – a methodology. This methodology is aimed at identifying and understanding the way in which those who constructed the text have attached meaning to the objects represented in the text and have legitimised knowledge and participation in relation to these objects, together with possible reasons for this. This methodology is also aimed at identifying and understanding meanings and legitimisation criteria which have been instantiated implicitly and of which the authors are unaware. As suggested by Lockyer (2008),

> Textual analysis is a method of data analysis that closely examines either the content and meaning of texts or their structure and discourse. Texts ... are deconstructed to examine how they operate, the manner in which they are constructed, the ways in which meanings are produced, and the nature of those meanings.

One possible form of textual analysis, then, involves scrutiny of a text to identify and analyse the structure and contents of possible discourses that make up the text and which are communicated through the text. In this sense, textual analysis provides a means for analysis of the discourse transmitted in and through a text (Cohen et al., 2011, p. 574).

Four further comments are necessary here. Firstly, textual analysis is not an exact science and is an inherently restricted process. This is because as much as we may try to understand the particular meaning that the author(s) of the text have attached to a particular object, that understanding is always based on an explicit interpretation – perhaps even somewhat of a guess – as to the original reasoning and meaning of the author(s): “When we perform textual analysis on a text, we make an educated guess at some of the most likely interpretations that might be made of that text.” (McKee, 2001, p. 3). It is for this reason that McKee (2003, p. 15), drawing on the work of Hartley (1992, pp. 29-30), argues that the process of textual analysis is akin to the process of ‘forensics’. As such, by engaging in textual analysis we are literally looking for clues and evidence that can be “caused to ‘talk’ as mute witnesses, ... coaxed into *telling a story*”. This forensic analysis helps us to reconstruct a limited perspective of the original understanding, interpretation, reasoning, and sense-making employed by the author(s) in

[246] In the original quotation Apple refers specifically to ‘textbooks’ and not to the more general statement of ‘text’. This is because much of his discussion involves analysis of high school textbooks and the way in which these texts position different participants in differential relations of power and control. However, I believe that his statements in this quotation can easily be transferred beyond the specific domain of school-based textbooks to reliably reflect the nature of embedded meaning in texts more generally. It is for this reason that I have replaced the word ‘textbook’ with the word ‘text’ in the quotation.

the production of a text, as well as of the tacit, implicit and often unaware assumptions and values which have informed the author(s) writing.

> This is how textual analysis also works. We can never know for certain how people interpreted a particular text but we can look at the clues, gather evidence about similar sense-making practices ... and make educated guesses. (McKee, 2003, p. 15)

Secondly, texts are always developed by people who are positioned within particular social and cultural (as well as economic and political) settings. As such, analysis of texts provides insight not only into the sense-making practices of the authors but also, particularly, of how those author(s) view the world (their 'world-view') and their place in the world: "We interpret texts ... to try to obtain a sense of the ways in which, in particular cultures at particular times, people make sense of the world around them." (McKee, 2003, p. 1). Is it in similar vein that Perakyla (2005, p. 870) refers to textual material as a 'specimen' of the 'cultural world'. Dowling (1998) extends this idea to argue that since texts reflect the perspectives and thinking of those who construct them, texts contain not only an indication of the practices of an activity, but also position participants (e.g. author and reader) in particular ways (i.e. positioning strategies). As such, analysis of the texts for an activity such as school mathematics provides insight into both the practices of the activity and the subjectivities (and, consequent, differential positioning) of those who participate in the activities:

> If texts are to (re)produce activities, then they must establish textual positions (voices) with respect to each other and distribute practices (messages) in relation to the structure of positions. Texts must constitute positioning and distributing strategies. (Dowling, 1998, p. 20)

The key point here is that texts are not neutral entities and the messages embedded within and conveyed by a text are not free from subjectivity and positioning – "All writing will therefore contain the ambiguity of an object which is both language and coercion." (Barthes, 1967, p. 26). Lockyer (2008) refers to this as the 'preferred meaning' of a text – namely, that texts are constructed with a specific purpose and intention in mind and include the social and cultural perspectives and 'persuasive qualities' of those who construct the texts. In this regard, the goal of the process of textual analysis is to determine the preferred meaning in the text and what that meaning is able to tell us about those who constructed the text and the activity or environment in which the text is embedded. This issue can also be understood from the perspective – such as is expressed by Hodder (1994) and Derrida (1978) – that meaning does not actually reside in the text itself but in the interpretation of the text, and that as a text is re-read (or re-written) in a different context the meaning in the text shifts and is interpreted in different and new ways. As suggested by Hodder (1994, p. 394), "Thus there is no 'original' or 'true' meaning of a text outside specific historical contexts."

Thirdly, and in light of the two comments above, any interpretation of the meaning of a text represents only one – and certainly not the only – possible interpretation of that text: "there is no such thing as a single, 'correct' interpretation of any text. There are large numbers of possible interpretations, some of which will be more likely than others in particular circumstances." (McKee, 2001, p. 4). From this perspective, texts are polysemic (Lockyer, 2008) in that there is always the possibility for multiple and varied interpretation of the meaning of a text. The role of the textual analyst is then to identify various possible and likely interpretations of the text. The role of the textual analyst is *not* to claim whether or not the text is accurate or inaccurate or true or false, because a claim like this merely reflects that "*what they are really saying is 'I agree with what this text is saying about the world'.*" (McKee, 2001, p. 7, emphasis in original text).

Fourth, in as much as a text is open to multiple interpretations, a text can also comprise several discourses and, so, can be deconstructed into several meanings (Cohen et al., 2011, p. 574). Different elements of a text can, thus, differently prioritise or prioritise different forms or structures of knowledge and participation, and can embody different social, economic and/or political perspectives. This is particularly pertinent if a text is authored by several role-players. This issue is an important consideration for my own analysis of empirical phenomenon relating to the subject-matter domain of Mathematical Literacy. Specifically, this issue highlights that analysis of a single component of a text (e.g. analysis of a single chapter in a textbook) potentially only provides a limited perspective of the dominant structure of knowledge and participation or of other possible structures prioritised in the text.

Consider the empirical textual phenomena for the subject-matter domain of Mathematical Literacy that form the focus of the empirical analysis process in this study. My intention for employing the methodology of textual analysis to these textual phenomena is to investigate and generate a description of the 'world-view' of the author(s) of these texts in relation and comparison to the structure of knowledge and participation associated win the various domains of practice of the knowledge domain of mathematical literacy. By 'world-view' here I am referring specifically to the way in which the author(s) of the texts perceive (and prioritise) the roles of contextual and/or mathematical knowledge and practices in the subject and the criteria according to which participation is endorsed and legitimated – in other words, the Dominant Domain of Prioritising in the texts. For example, for me the questions in the national examinations – and particularly the way in which these questions deliberately prioritise specific (mathematical or contextual) elements and processes – provide a glimpse of the 'world-view' of the examiners who set the examination papers. Specifically, these examination papers shed light on the way in which these examiners perceive and prioritise the structure of legitimate knowledge and participation for the subject. The 'world-view' of the author(s) of a text is identified through a method of analysis of the specific discursive resources (words/vocabulary, visual mediators, routines and endorsed narratives) employed in the construction of the texts. By further investigating coherence between these 'world-views' and the discursive characteristics of the domains of practice of the knowledge domain of mathematical literacy, I will then have a common and structured language through which to compare and differentiate the perspectives of the authors and also, by implication, the structure of knowledge and practices prioritised through these perspectives.[247]

[247] In alternative terms, the methodology of textual analysis is driven by an intention to investigate the coherence or divergence between the dominant discourses that characterise each of the empirical phenomena and the structure of the differing discourses that characterise the domains of practice of the knowledge domain of mathematical literacy. Crucially, though, although the textual analysis process adopted in this study draws on discourse analysis, the process is not characterised by discourse analysis alone. Rather, as is discussed on page 373 below, the textual analysis process involves a focus on both the content and form of the text, thus rendering the approach closer to a form of semiotic analysis.

CHAPTER 21
INTENDED METHODOLOGY: *TEXTUAL ANALYSIS* – METHODOLOGICAL APPROACHES

There are a vast number of approaches to textual analysis[248] that differ with respect to both external and internal dimensions. Certain elements of these internal and external dimensions – together with the preferenced choices employed in this study – are discussed below.

21.1 External levels of methodological differentiation

At an external level, Bernard and Ryan (1998, p. 595) identify two broad fields of textual analysis – the *linguistic tradition*, which treats the text itself as an object of analysis; and the *sociological tradition*, which treats the text as a "window into human experience". Different methodologies and approaches are then associated with each of these traditions as influenced by the intention to either interpret meaning(s) embedded within the text or to use the text as an instrument for identifying different possible perspectives of reality. Bernard and Ryan (1998, p. 595 & 596) argue further that within the two fields of textual analysis a distinction exists between *humanist* and *positivist* approaches. Humanist approaches are interested in interpretative approaches directed at the search for meaning, while positive approaches engage "the reduction of texts to codes that represent themes or concepts and the application of quantitative methods to find patterns in the relations amongst the codes."

Given the positioning of my study within the sociological tradition, a key intention of the textual analysis process is a search for meaning (i.e. a humanist approach). Namely, to gauge and come to understand the 'world-view' of the author(s) of the texts and the way in which they construct knowledge and meaning in the texts with respect to a prioritising of mathematical goals or a life-preparedness orientation. Using the texts as a 'window into the human experiences of the authors' provides a means for identifying the dominant domain of practice that is drawn on in the production of the texts, which in turn provides insight into the ways in which different texts present varying intentions for the knowledge and subject domains of Mathematical Literacy.[249]

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[248] For example, Tracy and Munoz (2001, pp. 71-78) identify nine different qualitative approaches to textual analysis, including Conversation Analysis, Discursive Psychology, Critical Discourse Analysis, Action-Implicative Discourse Analysis, Micro-ethnography, Ethnography and Ethnography of Communication, Grounded Theory, Narrative Analysis and Auto-ethnography. Cohen et al. (2011, pp. 574-603) also identify several different types of textual analysis. However, they first distinguish between analysis of 'discourse' and of 'visual media' and identify appropriate methodologies for each of these, including conversational and narrative analysis of discourse and content analysis and grounded theory of visual media. Ryan and Bernard (2000), on the other hand, provide a detailed discussion of approaches to textual analysis that relate specifically to the *sociological tradition*, focussing both on methods that analyse raw text (e.g. word counts and semantic network analysis) and on methods that reduce text to codes (e.g. content analysis and grounded theory).

[249] In this sense, the approach to textual analysis employed in this study bears resemblance to the fourth type of discourse analysis method identified by Wetherell, Taylor, and Yates (2001) (and cited in Cohen et al. (2011, p. 575)). Namely, analysis of the links between language and the "constitution, structure and nature of society". However, in this study the constitution, structure and nature of society is substituted with the structure of knowledge reflected and prioritised in various empirical practices associated with the subject-matter domain of Mathematical Literacy. The other three forms of discourse analysis identified by Wetherell et al. (2001) are:

21.2 Internal levels of methodological differentiation – preferencing a 'method' drawn from the field of Semiotics

At an internal level, Palli et al. (2010) argue that the major theoretical and methodological difference between approaches to textual analysis reside in the notion of *meaning*: namely, in the characteristics of the meaning that is expressed in and through the text and where that meaning is to be found:

> Content analysis sees text (and the choices upon which it is based) as expressions of *content*. In contrast ... linguistically oriented textual analysis ... treats text (and the choices upon which it is based) as *meaning potential* out of which actual meanings in context arise. (Palli et al., 2010, emphasis in original text)

Following a similar line of thinking, Schwandt (2007) argues that approaches to textual analysis range on a spectrum. On the one extreme of the spectrum are those that focus exclusively on analysis of *content* (i.e. what was said in the text) (e.g. content analysis). And, on the other extreme are those approaches to those focussing exclusively on the *form* of text (i.e. how something was said) (e.g. discourse analysis). Those approaches that focus on both content and form are positioned between these two extremes (as illustrated in Figure 49 below).

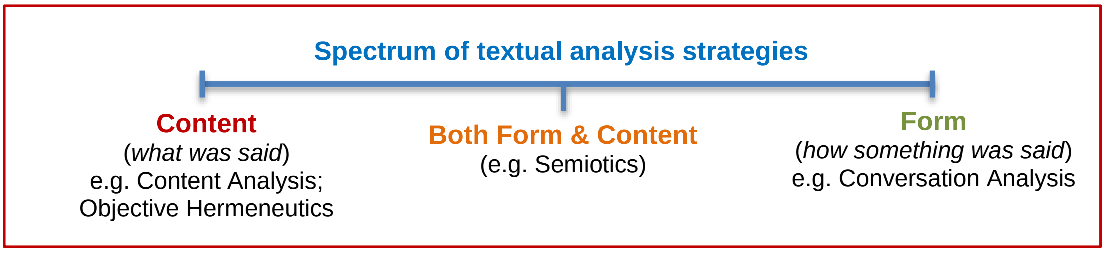

Figure 49: Spectrum of textual analysis approaches

With respect to the analysis of empirical texts in this study, it is a focus on the combination of *both the form and content* of a text that is prioritised. As demonstrated in the discussion and operationalisation of the external language of description, focus on the *discursive resources* embedded in a textual activity relating to practices associated with the domain of mathematical literacy provides a means for identifying the structure of knowledge and participation prioritised in the activity and, hence, for identifying the dominant discourse transmitted through the activity. These discursive resources are in large part signalled through the signifiers (words/vocabulary and visual mediators) embedded in the text and which collectively constitute the construction and presentation of meaning in the text. Focus on identifying and analysing the signifiers in a text represents direct focus on the content of the text since the signifiers serve to index the focal event under analysis and the routines that must be employed in engaging with the focal event. However, the deliberate use of specific signifiers, and particularly the way in which the signifiers are

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- analysing words in context: namely, analysing the ways in which people express themselves and the words used are seen as representative of contextual influences (social, cultural, economic situations) on the people who use the words (in other words, looking at how context influences language used and meanings constructed);
- analysing interactions conducted through language; and
- analysing patterns of language use: namely, analysing the form and structure of the language as a conveyer of emotion, or meaning, or positioning.

positioned in the text and linked to other textual elements – namely, the form of the text (i.e. how something is said) − also provides an indication of the structure of knowledge that is prioritised. As such, focus on both the content and form of a text is necessary to identify the structure of knowledge prioritised in and through the text.

What is hopefully clear from the discussion above is that the key methodology employed in the textual analysis process in this study involves a form of discourse analysis – but with a focus on both the content and form of the text − driven specifically by a method that facilitates identification and analysis of the signifiers in a text. And this method of analysis of the signifiers in a text draws directly from the field of *Semiotics*.

Discussion of the process of semiotics was dealt with in detail in Part 5 (c.f. sub-section 17.1.2 above starting on page 269) and, as such, will not be repeated here. However, by way of extending that discussion, it is useful to consider the work of Manning and Caullum-Swan (1994, p. 466) who, by drawing on the work of Barthes (1964, 1972), identify three different ranges of meanings that can be conveyed through particular relationships of signifier and signified entities: (i) *denotative*, (ii) *connotative*, and (iii) *mythical*. The (i) denotative meaning refers to the literal, common-sense, or obvious meaning of a sign that is a result of the sign referring to an entity that exists in reality (Scott, 2006) – the primary order of signification (Currie, 1999, p. 70). For example, consider the photograph of a learner shown in Figure 50 below. The photograph appears on the back cover of the Via Afrika Grade 10 Mathematical Literacy textbook[250] (Bali et al., 2011).

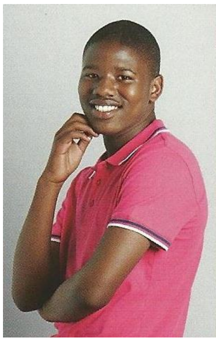

Figure 50: Photograph of a learner shown on the back cover of the Via Afrika Grade 10 Mathematical Literacy textbook

This photograph of a learner *denotes* that a learner was photographed; and it does not matter if the photograph is in black and white, or colour, or sepia, or is taken with a digital or a positive or negative camera – the photograph shows a learner. What you see is what you get – a learner. In this sense the denotative meaning represents a one-to-one correlation between what the sign signifies and the signified object.

The (ii) connotative meaning of a sign, on the other hand, represents the range of possible meanings that the sign triggers in the mind of the reader or audience (Scott, 2006). This range of meanings (and the consequent way in which the sign is utilised by the reader or audience) is affected by the social, cultural, economic, and various other associations to

[250] This book was approved for inclusion – along with seven other books from other publishers for Mathematical Literacy − on the national catalogue that was issued to all schools by the National Department of Basic Education. Schools were required to order one of these eight titles for the subject Mathematical Literacy and could not order a book that was not included on this list. As such, several thousand teachers and thousands of learners will be or have been exposed to this picture − which is precisely why I consider it of value for reference in this discussion.

and in which the reader or audience is positioned and/or embedded – what from a semiotic perspective would be called *codes*[251] (Chandler, 2013). It is for this reason that Barthes (1964, p. 94) suggests that society "holds the plane of connotation" and that this society "speaks the signifiers of the system". This can be conceived of as the secondary order of signification (Currie, 1999, p. 70). Barthes (1964, pp. 89-90) describes connotative meaning as the plane of expression that is itself constituted by a signifying system – i.e. the meaning expressed in the connotative plane stands as a signifier for another object. As suggested by Manning and Caullum-Swan (1994, p. 466), "the process of linking or connecting expressions and content is social and depends upon the perspective of the observer. A sign is essentially incomplete because it requires an *interpretant* ..." (emphasis in original text). Reflecting back on the photograph of the learner, a person looking at the photograph might be moved to appreciation or dislike of the image depending on their personal preference for particular types of photographs, or particular styles of photographs, or based on their own experiences with taking photographs, or based on their bias towards particular race groups, or particular fashion trends (i.e. a pink shirt), and so on. A viewer's response to the photograph is influenced by a variety of personal attributes and different viewers respond differently as influenced by their differential experiential base in the world.

The final possible meaning generated through a sign is that of (iii) myth. A myth occurs when the sign generates a connotative meaning that conveys a particular (perhaps pre-determined or deliberate) meaning – an *ideology* (Chandler, 2013): "A myth is a form of communication in which cultural meanings are structured in such a way as to convey a particular message to those who see, hear or read it." (Scott, 2006). Or, as explained by Barthes (1972, p. 109): "Myth is not defined by the object of its message, but by the way in which it utters this message." Currie (1999, p. 70), thus, describes a myth as something that is a distinct and deliberate mode of signification and not simply a mistaken concept or idea. Manning and Caullum-Swan (1994, p. 467) expand on this idea by referring to a myth as the "unexamined nonempirical or belief-based connections drawn between denotative and conative meanings." – in other words, where there is a disjuncture between the common sense or literate interpretation of a sign and the way in which the sign is used

[251] In the context of this discussion, a 'code' refers to the conventions for communication that make the production and interpretation of texts in a particular social/cultural/economic/political setting possible. A shift in the social/cultural/economic/political environment would be accompanied by a shift in the conventions that make communication possible. The codes, thus, provide the framework within which signs make sense – with the consequence that any attempt at interpretation of a sign (or reading of a text that includes signs) must be accompanied by awareness of the codes in which the sign was employed (Chandler, 2013). Attempted communication through non-dominant codes or through codes that relate to a different social/cultural/economic/political environment can lead to miscommunication and/or confusion (and/or offence – for example, kissing a man or woman on two cheeks is considered a greeting in some cultures, but in other cultures would earn a tight smack from the man's or woman's partner). Chandler (2013) further identifies three different types of codes – social, textual and interpretive. He argues that these three types of codes are essential to a reliable interpretation of a text. Recognition of the social codes provides knowledge of the social world in which the text was developed. Recognition of the textual codes provides knowledge of specific choices for why a particular medium of the text was chosen and the genre in which that text is positioned. And, recognition of the interpretative codes provides the means for identifying the way in which the author(s) of the text perceives the relationship between the social and textual codes, and how that relationship has informed the way in which meaning is presented in the text.

to convey meaning about an object.[252] Awareness of this aspect of meaning signification is important since it signals the existence of power relations through the intentional distortion of meaning in the promotion of a particular perspective (Scott, 2006). It is, thus, only through recognition and demystification of the mythical components of a text that the underlying relations of power and positioning can be revealed. As an example of a myth, consider any of the James Bond films in which '007' always has the latest watch, cell phone, car and a beautiful woman on his arm (or, more accurately, in his bed!). The underlying mythology embedded in these signs is the message that to be successful and powerful and macho (and liked by women) requires access to these items. The signifiers in the movie thus point the viewer towards a particular way of seeing the world. As another example that relates more directly to the subject-matter domain of Mathematical Literacy, consider the picture below (Figure 51) taken from the back cover of the Via Afrika Grade 10 Mathematical Literacy textbook (Bali et al., 2011):

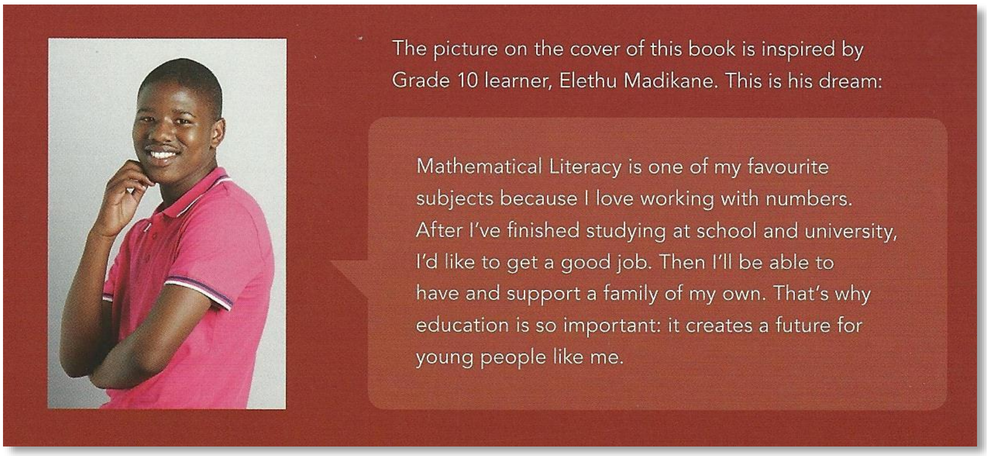

Figure 51: Back cover of the Via Afrika Grade 10 Mathematical Literacy Learner's Book

This single instance of photograph and text generate a plethora of myths, including:

- this book is suitable for black learners (possibly particularly for black learners who are male);
- if you go to University you are more likely to get access to a job; but not just any job – a 'good' job (by implication, does this mean that by not going to University you will only get a 'bad' job?);
- the primary motivations for studying and getting a 'good' job should be unselfish – i.e. for caring for others and not for oneself;
- education is the key to a positive future;

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[252] A more theoretical explanation of a myth is built on the understanding that every sign serves as a signifier for meaning at a higher level – this is represented by Barthes (1972, p. 115) in the following diagram:

<table>
<tr><td>1. Signifier</td><td>2. Signified</td><td></td></tr>
<tr><td colspan="2">3. Sign
I SIGNIFIER</td><td>II SIGNIFIED</td></tr>
<tr><td colspan="3">III SIGN</td></tr>
</table>

A myth then occurs when the message expressed through a sign is taken as representing the meaning of an object rather than as a form of communication that is referring to something else (Gaines, 2001) – as expressed by Barthes (1972, p. 137): "The writer's language is not expected to *represent* reality, but to signify it." (emphasis in original). By its very nature, a signifier signifies another entity; and, so, a myth occurs when a deliberate attempt is made to fix specifc connotative meanings to the signifier rather than to allow recognition that the signifier only represents something else and does not signify meaning in itself (Currie, 1999, p. 70).

But perhaps the biggest mythical message of all expressed through this picture and text is that all of the myths above are facilitated through the use of this particular textbook − this brand of textbook makes all of these things possible. This message is mythical precisely because it is signifying a pre-conceived, deliberate and constructed meaning. And it is mythical because there is no guarantee that the book (or even the subject for which the book was written, or the knowledge domain from which the contents of the book are drawn), is able to provide all of these things for all (or even any) learners.

With respect to the different types of meanings associated with signs, the textual analysis process employed in this study focuses specifically on the *connotative* meanings indexed by signifiers embedded in empirical texts relating to the subject-matter domain of Mathematical Literacy. In particular, the textual analysis process focuses on identifying the possible meanings that the authors of the texts attach to particular concepts – as identified in the signifier-signified relationship. This process is specifically and particular focussed on identifying whether the nature of the signifier-signified relationship directs activity towards mathematically structured and legitimised knowledge and practices, or towards a form of knowledge and participation that is more concerned with establishing a comprehensive and broadened understanding of contextual situations – namely, a life-preparedness orientation. This, for me, is representative of the connotative meaning of the sign (signifier-signified combination) since it reflects an interpretation of possible ways in which meaning has been attached to the sign with respect to a connection to a real-life or a mathematical object. Thus, despite an underlying sociological impetus for this study, my intention is not to engage with mythical meanings embedded within the texts, with relations of power and positioning, or with the social, economic and/or political commentary produced, sustained and/or promoted in and through the text. Rather, my own sociological concern relates to the structuring of knowledge, the ways in which that knowledge is legitimised, and the terrain that is appealed to and prioritised in this legitimisation process. This, in turn, facilitates analysis and identification of the extent to which the prioritised structure of knowledge promotes enhanced preparation for life or access to a form of mathematics that embodies and substantiates Myths of Reference, Participation and Emancipation (Dowling, 1998).[253]

As a final comment, it is perhaps useful to point out that the textual analysis employed in this study occurs at the *lexical level* of the text – namely, at the level of analysis of individual words and other signifiers that make up a segment of text (Palli et al., 2010). The textual analysis in this study is not overly concerned with the structure or syntax of the text – what Palli et al. (2010) refer to as the *semantic-syntactic interface* – and the significance of that structure and syntax with respect to the positioning properties and beliefs and attitudes embedded in the text.

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[253] Although, it could always be argued that the differing structure of knowledge in different texts for the subject-matter domain of Mathematical Literacy gives rise to different forms of mythologizing of the content matter and of the intention of the subject. For example, where the prioritised knowledge structure in a text is dominated by a focus on mathematical principles, the associated mythologising involves a perception of the utility and use-value of mathematics for describing any aspect of the 'real-world' (which is actually now the mathematised world) and for solving problems in that world. By contrast, where the prioritised knowledge structure is dominated by a focus on a life-preparedness orientation, then the associated mythologising involves the expectation that participation in the subject leads to an enhanced and improved life – which is clearly not a given, since quality of life is influenced by considerably more than mere participation in a single subject. The key point from this is that focus on the structure of knowledge is in some way intricately linked with a focus on the mythical meaning transmitted through a text. However, whilst acknowledging this, for purposes of this immediate discussion and for the analysis of empirical texts in this study, my primary focus is on the structure of knowledge itself and not directly on the mythologising power of that knowledge. Investigation of this latter construct certainly makes for interesting exploration in future research endeavours.

CHAPTER 22
INTENDED METHODOLOGY: *TEXTUAL ANALYSIS* − LIMITATIONS AND CRITICISMS

The usefulness, appropriateness and suitability of the methodology of textual analysis for the analysis of empirical texts relating to the subject-matter domain of Mathematical Literacy has been discussed in detail in the pages above. However, some commentary is necessary on both the limitations of this methodology and prevalent criticisms of the methodology.

Perhaps the greatest strength of the methodology of textual analysis also gives rise to its greatest limitation. Namely, in as much as textual analysis provides a means for identifying, interpreting and coming to understand possible world-views of the author(s) of a text, the utterly interpretative nature of the process means that every interpretation is simply one of many possible interpretations. Furthermore, the specific characteristics of the privileged interpretation is directly influenced by the decisions that the analyst makes in choosing how to engage with the text, which elements of the text to focus on, and the specific method(s) to employ in analysis of the text. By embarking on a textual analysis process, the analyst attempts to make an ‘informed guess’ of the meaning that an author has attached to a segment of text, of what they may have been thinking at a particular point in the text production process, and possible reasons or motivations on the part of the author(s) for this. But in the absence of direct conversation with the author(s), this process remains one of subjective interpretation of what the analyst thinks the author might have intended through the clues that they have left behind in the text. As suggested by Cohen et al. (2011, p. 587), “... there is no single privileged, definitive way of analysing discourse nor of the meanings that surface from it.” Furthermore, while the interpretative process has the potential to accurately reflect the world-view of the author(s) of the text, this same process also has the potential to reflect instead the world-view of the analyst. This is because the textual analysis analyst makes specific decisions and choices about how to engage with a segment of text, what methods to employ in the analysis process, and what criteria to use against which to measure the meaning and message transmitted in and through the text. These choices and decisions impact directly on the specific interpretation of the text that is able to be made, and influence the specific lens through which the analyst comes to identify and understand the embedded world-view. Cohen et al. (2011, p. 575) refer to these limitations as issues of ‘projection’ and ‘counter-transference’: namely, “the researcher’s analysis may say as much about the researcher as about the text being analysed, both in the selection of the levels of analysis, the actual analysis, and the imputation of intention and function of discourses in the text, with their corollary in the key issue of reflexivity.”

These various limitations give rise to a prevalent criticism of the methodology of textual analysis. Namely, “If audiences can read a text in a number of ways, then what is the validity and relevance of one textual interpretation?” (Creeber, 2006, p. 82)[254]. This criticism of the restrictive and limited potential of textual analysis is further accompanied by the assertion that although most textual analysts recognise the futility of trying to identify a ‘universal reader’ of a text, these same analysts invariably albeit unknowingly imply that the text is read and interpreted in a certain way by a certain group of

[254] Note that in making this statement Creeber is simply reflecting on one of the major criticisms of the textual analysis process. Creeber does not agree with this criticism and goes on in the article to discuss in detail the merits of textual analysis.

individuals, hereby defining an 'ideal-type' reader[255]. The 'guesswork' component of the textual analysis process is also not looked upon favourably by critics of this methodology, particularly since this guesswork is at the expense of the actual voice of both the author(s) and the audience of the text who are seldom allowed to speak directly for themselves (Creeber, 2006, p. 82).

In light of these limitations, a key issue for consideration in the textual analysis methodology process is that of *reflexivity*:

> As discourse analysis and the interpretation of images involve a large element of subjectivity as intrinsic to the activity, it is incumbent on the researcher to be highly reflexive in the account given, indeed to regard his or her own interpretation as itself as discourse. (Cohen et al., 2011, p. 589)

Reflexivity is the process of examining both the research process and the role that the researcher has on this process (Hsiung, 2010). Examining the research process involves constantly reflecting on whether the research methods chosen are appropriate and adequate for their intended field of application, on whether the theoretical constructs used to describe a particular area of focus are similarly appropriate and valid, and whether there is consistency between the methods employed and the theoretical constructs that enable a description of a research space. Examining the role of the researcher on the research process involves recognition and awareness of how the researcher has influenced the research process (M. Dowling, 2008), including reflecting on the preconceptions that the researcher brings to the research process and on the assumptions that the researcher makes in the way in which they choose to interpret and construct meaning (Hsiung, 2010). In other words, reflexivity involves serious consideration of how "our subjectivity becomes entangled in the lives of others" (Denzin, 1997, p. 27) (quoted in: Mauthner & Doucet, 2003, p. 416) and involves making transparent in an explicit way how the research process has been directed and determined by the decisions and choices made by the researcher (so that these decisions and choices can be accessed and, if need be, challenged by others). Reflexivity involves making the research process itself the object of enquiry and involves recognition that in as much as the goal of the research process is for sense-making, the research process itself is a meaning-making activity which influences the way in which further meanings are interpreted and constructed (Hsiung, 2010).

At various points in this study I have attempted to demonstrate reflexivity and a rigorous degree of transparency. This has been done in relation to the development of the internal language of description, the development of an external language of description to facilitate the use of the internal language as a lens for analysing empirical resources relating to the subject, and in the choice of the methodology of textual analysis to facilitate analysis of these empirical resources. In particular, reflexivity as employed in this study has involved me as the researcher and analyst asking direct questions about my reasons for the choice of the specific methodological approach of textual analysis and about how

[255] Ulriksen (2009) extends this idea by presenting the notion of the 'implied student', where implied denotes the particular pre-conceived perception that lecturers or teachers have about how students should behave, engage and learn and the form and structure of participation that is considered legitimate:

> "the implied student could be understood as *the study practice, the attitudes, interpretations and behaviour of the student, that is presupposed by the way the study is organised, the mode of teaching and assessment, by the teachers and in the relations between the students, enabling the students to actualise the study in a meaningful way.*" (Ulriksen, 2009, p. 522, emphasis in original text)

this choice is directly influenced by my own epistemological perspectives on the research process. Furthermore, I have consistently reflected on and questioned my motivation and intended goals for the research process.256 Reflexivity was employed in thinking about necessary components for inclusion in the internal language to offset the current problematic space in which pedagogic and assessment practices in the subject are positioned. And, particularly, reflexivity was employed to ensure that the components that were included in the language were based on sound research of the field of mathematical literacy and not on my own preferences for a particular format for the knowledge and subject domains. Reflexivity was employed in developing an external language of description that was consistent with the components of the internal language and which provides a means through which to bridge the reach from the internal language to the empirical world. Reflexivity was employed in choosing the particular field of semiotics and in employing components of that field as part of the external language to ensure consistency between the external language and the proposed methodology of textual analysis of empirical texts. Reflexivity was employed in the operationalisation process of the external language where I constantly questioned my choice of method, the suitability of this method, and whether the method would serve to highlight valuable insights or simply reflect my own thinking about practices associated with the knowledge/subject domain of Mathematical Literacy.[257] And in all of this, reflexivity has been reinforced by a concentrated and arduous attempt to ensure transparency of choice and decision. Finally, reflexivity continues to be employed as I shift to analysis of empirical texts relating to the subject-matter domain of Mathematical Literacy, in particular for ensuring that the analysis is grounded in the developed theory and not in my own interpretation of the empirical resources.

WHERE TO FROM HERE

Having operationalised the methodology of textual analysis in Part 5 of the study, and having now outlined and theorised this methodology and associated methods in this part of the study, in Part 7 I turn to utilisation of this methodology and associated methods in directed analysis of various empirical textual resources relating to the subject-matter domain of Mathematical Literacy.

[256] M. Dowling (2008) suggests four types of reflexivity, and the form of reflexivity just described characterises the second of these four types. The first type involves researchers keeping a diary of the thoughts and feelings that influenced their methodological decision-making. This type of reflexivity is commonly employed in feminist research, action research, and ethnographies. The third type of reflexivity – commonly employed in critical ethnography and critical hermeneutics – involves direct examination of the political and social issues that influence and inform the research process. The fourth type of reflexivity – directly espoused by feminist researchers – involves the researchers drawing directly on their own experiences to illuminate meaning from other participants.

[257] Evidence of this form of reflexivity is recorded explicitly in Part 5, Chapter 18 and sub-section 18.2.7 (starting on page 356) where I reflect on some of the challenges experienced in the development and operationalisation of the external language.

PART 7
ANALYSIS OF EMPIRICAL TEXTUAL RESOURCES

INTRODUCTION AND OVERVIEW

In this part of the study I demonstrate utilisation of the internal language of description, through the components of the external language of description, to investigate, interpret and compare various empirical textual resources associated with the subject-matter domain of Mathematical Literacy. In particular, I show how the internal language is able to highlight similarities and differences in the dominant domain of practice and structure of knowledge through and according to which participation in the subject is legitimised. To facilitate this process, the following texts have been selected for analysis:

- the CAPS (Curriculum and Assessment Policy Statement) curriculum document for the subject developed by the National Department of Basic Education (DBE, 2011a), with specific focus on the curriculum topic of Probability;
- exemplar national Grade 12 Mathematical Literacy examination papers (Paper 1 and Paper 2) set by the National Department of Basic Education (DBE, 2014b, 2014c, 2014d, 2014e), also with specific focus on probability-related questions which correspond to the selected or analysed topic in the CAPS curriculum document;
- the Platinum Series Mathematical Literacy Grade 12 textbook published by Maskew Miller Longman educational publishers (Frith et al., 2013a; Frith, Jakins, Winfield, & Yeo, 2013b), with specific focus on Chapter 10 (Probability);
- course notes for a Mathematical Literacy Post Graduate Certificate in Education (PGCE) education course for pre-service teachers (Webb, 2013d) conducted at the Nelson Mandela Metropolitan University (NMMU) in Port Elizabeth, South Africa.

Prior to presenting analysis of each text, I begin in Chapter 23 by outlining my rationale for the selection of these specific texts and intended areas of focus in the analysis process. Brief mention of the specific sampling techniques employed in the selection process is also provided. In Chapter 24 to Chapter 27 I then discuss the analysis of each textual resource. For each text analysed, I begin by first providing background information about the text. Thereafter, two levels of analysis are provided. Analysis at a macro level is focused on the general characteristics of the text as a whole that indicate a particular structuring of knowledge (for example, the types and structure of the contexts included throughout the examinations provide an indication of the extent to which the authors prioritise the need for engagement with authentic contexts). The micro-level analysis, by contrast, is focused on specific segments of the text that deal with contents relating to the topic of Probability and of the way in which participation with these contents is legitimised. Where appropriate, cross-comparison of texts is provided, specifically in reference to areas of similarity and/or divergence of the dominant domain of practice prioritised, the consequent way in which knowledge is structured in the texts, and the criteria according to which endorsed narratives about that knowledge is to be generated.

In brief, the analysis process (summarised in Table 8 on page 383 below) reveals a prioritising of components associated with the Everyday domain of practice in the curriculum section on Probability, and, by contrast and in contradiction, an overwhelming and near exclusive emphasis on Public Domain of mathematics type practices in the examinations. This suggests a high degree of inconsistency between the intentions of the curriculum and the examinations in relation to Probability contents. Analysis of the textbook section reveals a relatively high degree of coherence in the dominant domain of prioritising to the curriculum; however, the absence of Modelling domain components in the textbook chapter limits the opportunity for the development of a life-preparedness orientation through engagement with the textbook. By contrast, the course notes for the PGCE explicitly denounce mathematised forms of engagement in favour of legitimised forms of knowledge and participation – consistent with the components of the Everyday domain − that reflect authentic real-world practice. Modelling processes also dominate demonstrated pedagogic practices in the course notes. This facilitates the positioning of the PGCE texts and the examinations are on opposite ends of a spectrum of differing forms of legitimised participation, with the CAPS curriculum situated on that spectrum closer to the PGCE texts and the textbook marginally closer to the examinations (as illustrated in Figure 52 below):

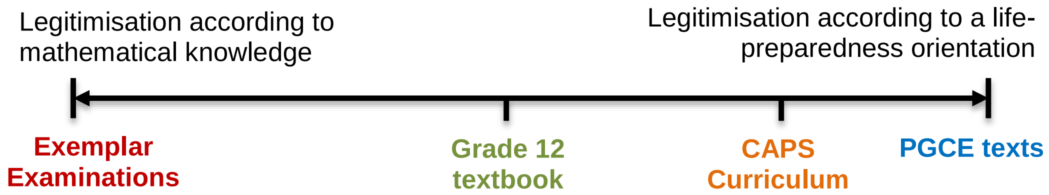


Figure 52: Positioning of the analysed texts on a spectrum of different forms of legitimised participation in the subject-matter domain of Mathematical Literacy

In light of these findings, I argue that it is the conception of legitimate knowledge, participation and communication espoused in the PGCE course notes that is most closely aligned to the life-preparedness orientation envisioned for the knowledge domain of mathematical literacy espoused through the internal language of description. To conclude the analysis process, in Chapter 28 I argue that the discussion and findings presented in this part of the study demonstrate some level of utility and validity of the internal language of description as a means for identifying and describing the dominant structure of practice and knowledge prioritised in empirical practices associated with the subject Mathematical Literacy (and, also, more generally with practices that draw on the knowledge domain of mathematical literacy). However, I also highlight certain limitations and challenges faced when applying the language to empirical settings.

To assist the reader in navigating the contents of this part of the study, a table and a diagram are provided immediately below. Table 8 shows a summary of the conducted analysis of each text in relation to the domains of practice of the internal language of description for the knowledge domain of mathematical literacy; while the diagram in Figure 53 illustrates an overview of the contents and structure of the chapter. [258]

[258] Note the absence of statistical count values in the table for the teacher education course notes. By way of explanation, the curriculum document, examinations and textbook are characterised by distinctive, contained and easily separated segments of text that reflect statements of intention, philosophy, methodology and content. By contrast, the course notes and, particularly, the course reader that is used as the official text for the course read more as a conversation and general discussion of methodological and pedagogical issues relating to the knowledge and subject domains of Mathematical Literacy. A ‘count’ approach for this text is, thus, simply unmanageable and will not necessarily yield insightful data. It is for this reason that analysis of the teacher education course notes and associated texts is conducted through identification, focus and analysis of trends and particular areas of emphasis in the discussion/conversation propagated in the text.

Table 8: Summary of the conducted analysis of each text in relation to the various domains of practice of the internal language of description

<table>
<tr><th colspan="2"></th><th colspan="6">Components of the Internal Language of Description for the knowledge domain of mathematical literacy</th></tr>
<tr><th colspan="2">Text Type</th><th>Everyday Practice (E)</th><th>Mathematical Competency Practice (MC)</th><th>E & MC</th><th>E or MC</th><th>Modelling Practice</th><th>Context Domain, Reasoning & Reflection, and General Comments</th></tr>
<tr><td colspan="2">Curriculum (section on Probability contents only)</td><td>41%</td><td>17,6%</td><td>11,8%</td><td>---</td><td>11,8% (modelling practices are expected in assessment components only)</td><td>Strong commitment to engagement with authentic real-world contexts.
Dominant emphasis on everyday forms of participation and knowledge.
Limited expectation for modelling.
Some expectation for a critical dimension of reasoning and reflection.</td></tr>
<tr><td rowspan="2">Examinations</td><td>Whole exam</td><td>16%</td><td>70%</td><td>10%</td><td>4%</td><td>---</td><td rowspan="2">Dominant emphasis on assessment of mathematical competency.
Mostly contrived contexts employed.
No expectation for modelling or for a critical dimension of reasoning and reflection.</td></tr>
<tr><td>Probability contents only</td><td>---</td><td>100%</td><td>---</td><td>---</td><td>---</td></tr>
<tr><td rowspan="3">Textbook (Probability contents only)</td><td>Explanations</td><td>55,6%</td><td>27,7%</td><td>16,7%</td><td>---</td><td>---</td><td rowspan="3">Some attempt to engage authentic real-world contexts, but primary engagement with cleaned contexts.
Consistent emphasis on both mathematical and everyday forms of participation.
No expectation for modelling.
Limited expectation for a critical dimension of reasoning and reflection.</td></tr>
<tr><td>Examples</td><td>11,1%</td><td>33,3%</td><td>55,6%</td><td>---</td><td>---</td></tr>
<tr><td>Exercises</td><td>33,8%</td><td>46,3%</td><td>18,8%</td><td>1,3%</td><td>---</td></tr>
<tr><td colspan="2">Teacher Education Course Notes</td><td colspan="6">• Ardent commitment to engagement with authentic real-world contexts.
• Dominant emphasis on Everyday forms of understanding of participation in real-world contexts.
• Deliberate downplaying of formal mathematical structures.
• Demonstrated pedagogic or methodological approach reflects Modelling practices.
• Consistent emphasis on reasoning and reflecting critically on existing everyday forms of participation.</td></tr>
</table>

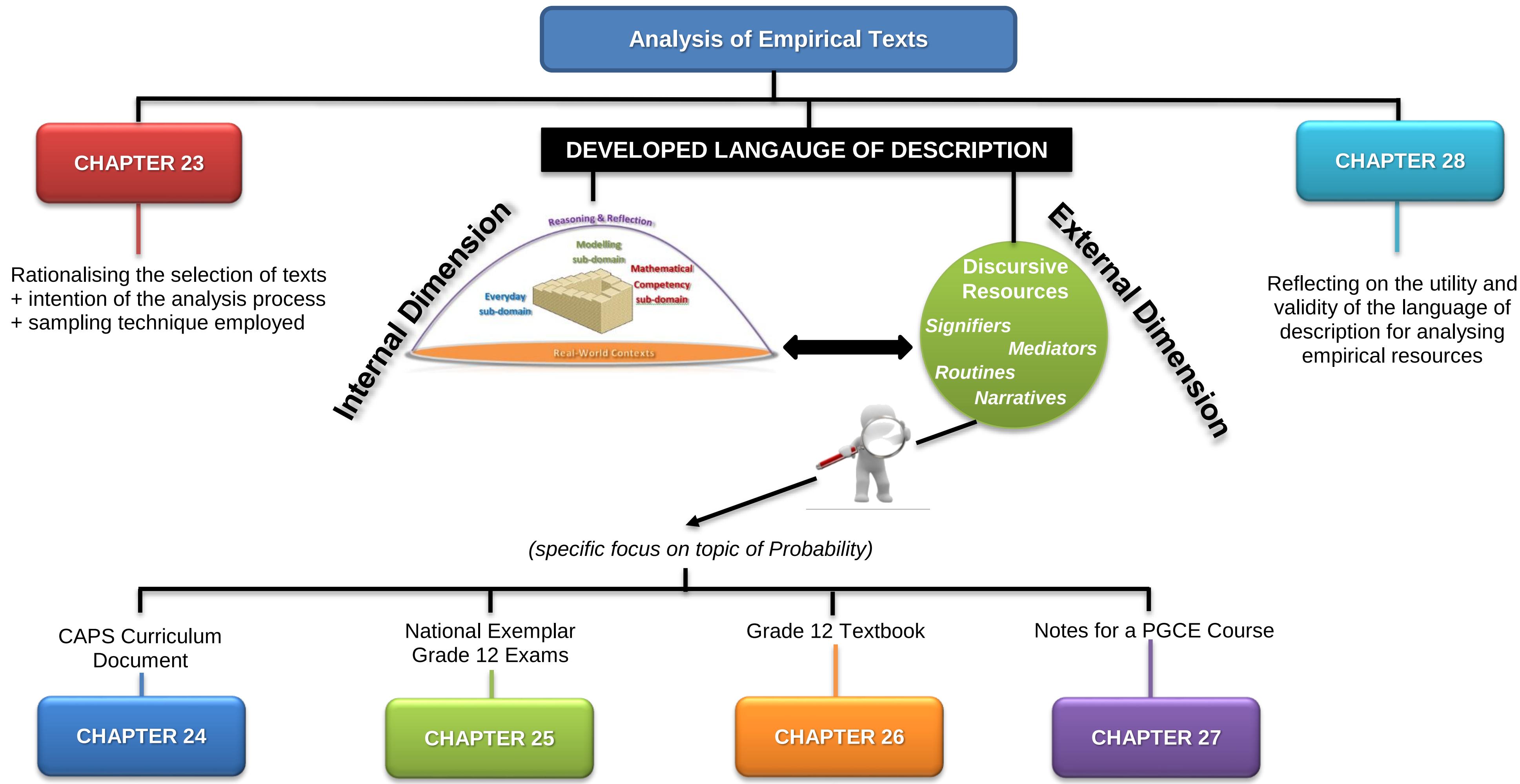


Figure 53: Overview and chapter structure of <u>Part 7</u> of the study

CHAPTER 23
RATIONALISING THE SELECTION OF TEXTUAL RESOURCES

23.1 Rationale for the selection of specific texts for analysis and the intention of the analysis process

In the immediate discussion below I explain the intention of the analysis process of the selected empirical textual resources and provide justification of my rationale for the deliberate selection of each text and of specific areas of focus in the texts.

23.1.1 The intention of the analysis process

To begin with, the primary intention in the analysis of each text is to identify, categorise and compare the way in which the author(s) of the texts legitimise participation in the subject and the dominant structure of knowledge and communication that it prioritised to facilitate these legitimised forms of participation. This is done through analysis of the discursive resources in selected segments of text in each textual resource (specifically, the segments of text that refer to the contents of Probability and primarily at Grade 12 level) to gauge the extent to which these discursive resources reflect the structure of knowledge and participation that characterise the various domains of practice of the knowledge domain of mathematical literacy. Reflecting the discursive resources embedded in each text to those that characterise the domains of practice provides a means for identifying the *dominant domain of prioritising* in each segment of text. This, in turn, involves identification of whether the structure of knowledge in each segment of text is characterised by: an expectation for employed practices involving probability-related contents to reflect how people engage with probability-related issues on a daily basis (Everyday forms of practice, participation and knowledge); or with a form of mathematical engagement with Probability contents (Mathematical Competency forms of practice, participation and knowledge); or with an expectation for modelling or problem-solving processes (Modelling-related forms of practice, participation and knowledge) that investigate and describe existing and possible alternative forms of participation with the Probability-related contents; or with another expectation that is not captured in the internal language. This process of identifying the dominant domain of prioritising in each text segment provides an indication of the dominant structure of knowledge, participation and communication for each segment, and, also, a framework for facilitating comparisons between texts. Crucially, this process further facilitates identification and analysis of the extent to which each of the texts, through engagement with Probability-related contents, comprise an orientation towards life-preparation or towards a different intended outcome.

23.1.2 Rationale for the selection of specific texts

In terms of the rationale for the selection of specific texts, consider first the <u>CAPS curriculum document</u> (DBE, 2011a). For me, the CAPS curriculum document is a necessary stating point in any analysis process for the subject-matter domain of Mathematical Literacy. This is because it is this document that embodies the ‘official’ (i.e. state regulated and endorsed) commentary for both pedagogic and assessment practices. This commentary encapsulates the structure of legitimate knowledge for the subject, the criteria for endorsed participation and communication in the subject, the intention and philosophy of the subject, and, consequently, the dominant areas of prioritising that facilitate endorsed and legitimate participation in the subject.

Another set of texts that also embody a form of 'official' commentary on the structure of legitimate knowledge, participation and communication for the subject – albeit, now only with respect to assessment practices in the subject − are the nationally set Grade 12 exemplar examinations (DBE, 2014b, 2014c, 2014d, 2014e). The decision to focus on nationally set examination papers is a deliberate selection on my part. This decision is motivated by prior observation and experience of the extent to which the structure of the national examinations in the subject-matter domain of Mathematical Literacy have a 'backwash effect' (Allais, 2007) on the form, structure and areas of prioritising of/in pedagogic practices in the subject at classroom level. And, as was the case with the NCS curriculum (DoE, 2003a) and NCS examinations (see, for example, (DoE, 2009a, 2009b)), sometimes the structure of legitimate knowledge and criteria for endorsed participation and communication required for successful engagement in the examinations is different or in contradiction to that espoused in the curriculum. Consequent differences and contradictions to textbooks that have been developed around the contents of the curriculum and, often, prior to the development or writing of examinations, are also common. This issue is particularly pertinent in the South African context where much of what happens in the classroom is influenced by and in preparation for national assessment practices (as evidenced by my observations while participating in this field of practice). As such, it is my position that in as much as the curriculum provides evidence of the 'official' world-view − at the level of the Official Recontextualising Field (c.f. Bernstein, 1996) − of the expected endorsed structure of knowledge, participation and communication in the subject, the national examination papers provide an equally important albeit sometimes contrasting perspective of the structure and criteria of legitimate knowledge, participation and communication − this time at the level of the Pedagogic Recontextualising Field (c.f. Bernstein, 1996). Analysis of the way in which the structure of legitimate knowledge, participation and communication are presented through the examinations, thus, provides a possible indication of the types of practices, routines and narratives that teachers are required to engage their learners in to prepare them for successful and endorsed engagement with and participation in the evaluative dimension of the subject. Also note that I have deliberately chosen to focus analysis on a set of *exemplar examinations* and not on the final end-of-year nationally set examinations. This decision or choice is driven entirely by logistical constraints with respect to the availability of examinations based on the recently implemented CAPS curriculum process. Although I would ultimately have preferred to focus on the official final Grade 12 national examinations to be written at the end of 2014, this study will (hopefully! ☺) already have been handed in for examination by then. As such, the only currently (i.e. July 2014) available examination papers which directly reflect the structure and form of the final Grade 12 examinations are the recently distributed exemplar Grade 12 examinations. These exemplar examinations serve as an example of and precursor to the final examinations and are set by the same national examining panel. As such, it is my belief that the structure of knowledge and the criteria according to which participation with and communication of this knowledge is legitimised in or through engagement with these examinations can be taken to be reflective of 'world-view' of the national examiners. Furthermore, I contend that this 'world-view' can also be taken to evidence the structure of knowledge which will ultimately characterise the final examinations.

The third text selected for analysis is that of a Grade 12 Mathematical Literacy textbook (comprising both a learner's book and a teacher's guide) published by Maskew Miller Longman educational publishers (Frith et al., 2013a, 2013b). My rationale for the selection of a textbook as a source of empirical analysis is driven by an intention to gauge and describe the extent to which other participants operating in the Pedagogic Recontextualising Field (c.f. Bernstein, 1996) – who are, potentially, not driven by an exclusive assessment agenda – legitimise participation in the subject. Furthermore, the structure of knowledge and communication that characterise this legitimised form of participation will also come under scrutiny. Identification of the dominant domain of practice prioritised by the authors of the textbook provides a means for identifying and describing areas of convergence and/or divergence between curricular and assessment expectations and pedagogic practices in the subject. This process also highlights potential implications of these areas of convergence and/or divergence for those who engage with the textbook in terms of their successful participation in practices associated with the subject. The specific selection of the textbook published by Maskew Miller Longman is again a deliberate selection on my part, and there are two primary motivations for this selection. Firstly, and as is discussed in more detail below (c.f. page 425), the text is included on the national catalogue and, so, was exposed to a screening process that supposedly ensured coherence of the text to the contents and philosophy espoused in the CAPS curriculum. For all intents and purposes, then, the contents of the textbook and the approach adopted therein should reflect consistency with the CAPS curriculum. Secondly, this textbook is the highest selling textbook for the subject-matter domain of Mathematical Literacy in South Africa.[259] This fact is of critical importance to me since it signifies that this is the textbook that is being used by more Mathematical Literacy learners than any other textbook. And, this fact, in turn, signifies that this textbook has more potential than any other textbook to influence teacher/learner/parent perception regarding the structure of legitimate practice in the subject, and the associated form(s) of prioritised knowledge and criteria for endorsed participation and communication for the subject.

The final text selected for analysis comprises the course notes for a Mathematical Literacy method course in a PGCE programme for pre-service teachers (Webb, 2013a, 2013b, 2013c, 2013d). By way of rationalising this deliberate selection, consider that so far all of the texts selected for analysis (curriculum, textbook and examinations) are characterised, primarily, by a directed focus on contents to be engaged with in pedagogic or assessment practices in the subject-matter domain of Mathematical Literacy. For all of these texts, any specification of specific methodology for the subject most commonly occurs at an implicit level and as part of and/or in service to the process of engagement with the subject curriculum. As such, the application of the internal language of description to these texts provides a means for identifying the dominant domain of practice and associated structure of knowledge and criteria for legitimate participation and communication transmitted in and through the texts primarily in relation to curriculum (i.e. teachable and/or assessable) contents. By contrast, the teacher education course – and associated course notes or texts – embodies a primary intention for the development of teaching methodology aimed at facilitating engagement with the contents of the subject in a way that reflects the philosophy or intention of the subject-specific curriculum. This intention/philosophy convergence is also evidenced in relation to broader and more general conceptions of mathematically literate behaviour as recorded in research. In the teacher education course, then, methodology is elevated to a status of higher or at least equal priority to content. In light of this, analysis of the PGCE course notes provides a glimpse into how future teachers in the subject are being

[259] I sourced this information from a publisher at Via Afrika educational publishers in Cape Town. The publisher, in turn, sourced the information from national sales reports provided by the Publishers Association of South Africa (PASA). These sales reports reflect the sales per subject for all publishing houses in South Africa.

inculcated into a particular form of participation in and engagement with the contents of the subject (and, by implication, with the contents of the knowledge domain of mathematical literacy) and with particular methods and habits of teaching for the subject. Furthermore, analysis of these notes also highlights how the philosophy and/or intention of the subject as espoused in the curriculum and also in more general literature is reflected, reinterpreted and recontextualised in this inculcation process. The course notes, thus, embody a particular world-view of the structure of practice and knowledge in the subject, on the form of legitimate participation and communication in the subject, and, particularly, on the structure of pedagogic action required to facilitate access to the legitimised form of participation and communication with or of that knowledge. For me, it is of value and interest to describe the extent to which this inculcated world-view of the pre-service teachers is convergent with or divergent from the dominant domain of prioritising espoused in the curriculum (the contents of which they will have to ensure are engaged with appropriately), the textbook (the contents of which they will have to mediate for their learners), and the examinations (the contents of which they will have to prepare their learners for successful engagement).

23.1.3 Rationale for the selection of Grade 12 contents on the topic of Probability

For each of the texts selected for analysis, there is specific focus on identifying the discursive resources associated with *Probability-related contents*. In relation to these contents, there is directed focus on describing whether the characteristics of these discursive resources signify a dominant emphasis on engagement in a way that reflects the reality of everyday participation (Everyday domain of practice), or through mathematical structures, routines and narratives (Mathematical Competency domain of practice), or through modelling-related processes (Modelling domain of practice).

The selection of the topic of Probability for analysis is a further deliberate selection on my part, motivated by four reasons. Firstly, all four of the texts contain a focus on probability-related contents. Secondly, the topic of Probability is categorised as an ‘application’ topic in the curriculum (see page 391 below for an elaborated discussion of the structure of the curriculum). Thirdly, the contents of the topic bear a relatively strong link to authentic real-life practices. Namely, decision-making involving predictions is something that many people do and/or encounter on a regular basis as they engage in real-world scenarios, and the skills associated with this activity are vital to successful engagement in certain real-life practices (such as understanding the implications of investing in a pyramid scheme type investment). From these two reasons given above, it is then reasonable to expect that the contents and espoused or endorsed approach propagated for this topic in the various texts promotes a goal for understanding and sense-making of appropriate and suitable contextual environments. It is also reasonable to expect that these texts further promote and prioritise a form of engagement with the specified contextual environments in a way that reflects real-world practice, rather than for the learning of mathematical knowledge or application of mathematised practices. In short, the topic of Probability has been deliberately selected for analysis to determine the way in which participation with the contents of a topic which naturally lends itself to the investigation of real-world type practices, knowledge and skills is legitimised in texts (and by the authors of those texts) that serve different intentions and agendas. Fourth, the topic of Probability in the CAPS curriculum document is accompanied by an explicit statement of intention and expectation that pedagogic practices in this topic in the subject must prioritise “interpreting situations involving probability [rather] than on the mathematical calculation of probability." (DBE, 2011a, p. 90). This statement demonstrates a commitment in the curriculum to a form of participation with the contents of this topic that is reflective of a type of primarily real-world (and non-mathematised) practice, which in turn reflects a degree of commitment to an orientation referred to in this study as a life-preparedness orientation. As such, a further motivation for my deliberate selection of this topic for analysis is related to an interest in identifying how the authors of other texts (such as the Grade 12 national exemplar examinations and the textbook) respond to this curricular expectation. The structure of these responses, in turn, provides evidence of the positioning of these authors with respect to the structure of legitimate knowledge, the criteria for endorsed participation and communication with or of that knowledge, and the consequent commitment to or rejection of a life-preparedness orientation in this topic and, perhaps, for the subject as a whole.

The decision to focus specifically on *Grade 12 content* in all of the texts is driven in large part by a logistical consideration relating to the availability of nationally set examination papers. Namely, examinations in the Further Education and Training (FET) schooling band (i.e. Grades 10, 11 and 12) that are set or moderated by the state and made nationally available are commonly only available in Grade 12.[260] And, since my intention is to analyse both a section of the CAPS curriculum document and sections from nationally set examinations for the subject, this logistical consideration regarding the grade-specific availability of nationally set examinations has retrospectively impacted on my choice of grade-level focus in the curriculum and also on the grade-specific textbook selected for analysis. However, attempted exclusive focus on Grade 12 contents only is not a simple process, particularly given the way in which the Mathematical Literacy curriculum document is constructed to illustrate and facilitate progression of content and skills from one grade to the next. This progression structure is commonly characterised by a single statement of content/skills/application for Grades 11 and 12 to indicate that a concept must be dealt with in Grade 11 and that there is no new content for that concept in Grade 12, but the concept must be revised and may be examined in Grade 12. Furthermore, sometimes the same content is specified for Grade 10, 11 and 12, but then different applications of that content are expected at each grade level. For these reasons, analysis of the Grade 12 content on probability-related contents in all texts also includes consideration of content/skills/applications specified, particularly, for Grade 11 and sometimes also for Grade 10.

Before shifting focus to specific analysis of each of the individual texts, a short commentary on the characteristics of the particular sampling techniques employed in the selection of the texts is necessary – and is supplied in section 23.2 below.

[260] The situation is somewhat different for the CAPS curriculum structure in that nationally set exemplar examination papers have been set at Grade 10, 11 and 12 levels to demonstrate the intended form, structure and intention of assessment for the CAPS curriculum. My decision to select the Grade 12 exemplar examinations for analysis as opposed to the Grade 10 and 11 exemplar examinations is deliberate. This decision is motivated by the fact that it is this examination which most closely resembles the form and structure of the examination papers that Grade 12 learners will encounter at the end of 2014 when they write the official nationally set examinations for the subject. Furthermore, neither the Grade 10 nor 11 exemplar examinations are out of the same number of maximum marks or same time duration as the Grade 12 examinations.

23.2 A note on the sampling techniques employed in the selection of textual resources

Despite the availability of numerous texts for possible analysis, the texts selected have been chosen by me as the researcher for specific reasons and with particular intentions or motivations in mind. As such, a technique of *purposive sampling* – namely, sampling that relies on the "judgement of the researcher when it comes to selecting the units ... to be studied." (Lund Research Ltd., 2012) − has been employed in the selection of these texts. This sampling technique is consistent with non-probabilistic sampling techniques characterised by subjective judgements and selections of particular units of analysis, and accompanied by recognition that the selected units are in no way representative of a wider population (Cohen et al., 2011, p. 155).

The selection of the curriculum, examinations, and textbook all reflect a form of *typical case purposive sampling* ((Teddlie & Tashakkori, 2009, p. 174), cited in (Cohen et al., 2011, p. 157)). This is because the selection of each of these texts is based on the selection of typical case texts which receive wide exposure in relation to the population of participants in the subject. As a result, I contend that these texts have the potential to influence teacher, learner and public perception about the dominant structure of knowledge and criteria for legitimate participation and endorsed communication in the subject.

By contrast, the selection of the course notes for the teacher education course is motivated by a different selection approach (despite still characterising a form of purposive sampling). Specifically, the course notes were selected on the basis of 'ease of accessibility': put simply, I chose to engage with course notes from a program held at the Nelson Mandela Metropolitan University in Port Elizabeth simply because my association with one of the facilitators of the course ensured that I was easily able to access the course notes. This selection process is, thus, characterised not only by a form of purposive sampling, but also by an element of convenience sampling – namely, the selection of a unit of analysis based on the most easily accessible, readily available, and/or convenient sample (Cohen et al., 2011, pp. 155-156; Lund Research Ltd., 2012).

Clearly the choice of a purposive sampling technique has significant implications for the extent to which any claims made about the textual resources can be generalised and/or extended to describe other practices and texts not specifically analysed in this study. However, the primary intention of the textual analysis process in this study is to verify the utility and validity of the developed language of description of the knowledge domain of mathematical literacy for describing specific empirical resources relating to the empirical terrain of the subject-matter domain of Mathematical Literacy. For this reason there is no intention or expectation for claims to be made that are generalisable beyond the specific localised sites or resources to which they relate. As such, the limiting impact of the employed purposive sampling techniques does not defer from the validity of the employed research process.

CHAPTER 24
ANALYSIS OF EMPIRICAL TEXTUAL RESOURCE #1: CAPS CURRICULUM DOCUMENT

24.1 Background information

As mentioned in the Part 2 of the study, the CAPS curriculum process for Mathematical Literacy was initiated/instantiated at Grade 10 level in 2012, Grade 11 in 2013 and Grade 12 in 2014. As a result, this year (2014) is the first year in which Grade 12 learners will engage with CAPS aligned Mathematical Literacy examinations. In general terms, this curriculum change signifies a shift away from an open-ended teacher-controlled curriculum (as was the case for the previous National Curriculum Statement (NCS) curriculum which was grounded in an Outcomes Based Educational philosophy) towards a more rigidly structured and defined content-driven curriculum. The CAPS curriculum documents are characterised by an explicit structuring and explication of teaching content, pedagogic and methodological expectations, and assessment exemplars, suggestions and requirements. These curriculum documents are intended to be an 'all-in-one' package for teachers, specifying a totality of information and instruction on what to teach, how to teach, and what and how to assess. In specific terms for the subject-matter domain of Mathematical Literacy, the Mathematical Literacy CAPS curriculum statement contains specification of content for teaching and assessment, and also includes a deliberate and explicitly espoused attempted shift in the intention and philosophy of the subject and, consequently, in the expected methodology and teaching approach in or for the subject. This shift is characterised by an intention for decreased emphasis on mathematisation practices and the learning of mathematical content or knowledge, towards increased recognition and prioritising of knowledge and techniques (both contextual and mathematical) that facilitate broadened understanding of specified contextual environments (DBE, 2011a, pp. 8-9). Sense-making of contextual situations and the solving of problems in those situations is elevated to a key status above the learning of mathematical content. At the level of intention in this curriculum framework, assessment is directed towards gauging whether participants are able to demonstrate understanding of facets of specific contextual environments and are able to employ and engage with a variety of techniques and considerations to solve problems in those environments; assessment is not concerned with the level of mathematical competency of the learners (DBE, 2011a, pp. 8-9 and 13-14).

The specific curriculum topic of Probability selected for analysis (c.f. DBE, 2011a, pp. 90-95) or Appendix B on page 484 below for the curriculum extracts that deal with probability-related contents) is classified as an 'application topic' (see page 124 above for a reminder of the distinction between application and basic skills topics in the CAPS curriculum). Importantly, the application topics include specification of contextual situations in which problems must be solved and the content that must be employed in the solving of these problems, and these topics are posited as the primary focus of assessment and as a key intention for the pedagogic process (DBE, 2011a, pp. 13-14). Each topic is further comprised of 'sections' and for each section there is specification of relevant contexts and contents for per grade (i.e. Grades 10, 11 and 12). The specific topic chosen for analysis in this study is titled 'Probability' in the curriculum document (DBE, 2011a, pp. 90-95) and is comprised of four sections: 'Expressions of Probability' (p. 91), 'Prediction' (p. 92), 'Representations for Determining Possible Outcomes' (p. 93), and 'Evaluating Expressions Involving Probability' (p. 94). Suggestions of possible assessment tasks for the contents of Probability are also provided (p. 95). The various sections in the curricular topic of Probability include specification of the types of scenarios or contexts in which dealings with probability-related contents must be engaged – for example, tests where there is the possibility for an inaccurate reading (such as a pregnancy test), gambling scenarios, and risk assessments (such as those that inform car insurance rates). The sections also include specification of specific probability-related content to be dealt with – for example, the difference between an event, the outcome of an event, and the result associated with a particular outcome. Finally, the applications to be completed through the use of the content in the specified contexts are also described – for example, "evaluate and critique statements involving probability shown on adverts and in the press". (DBE, 2011a, pp. 91-94).

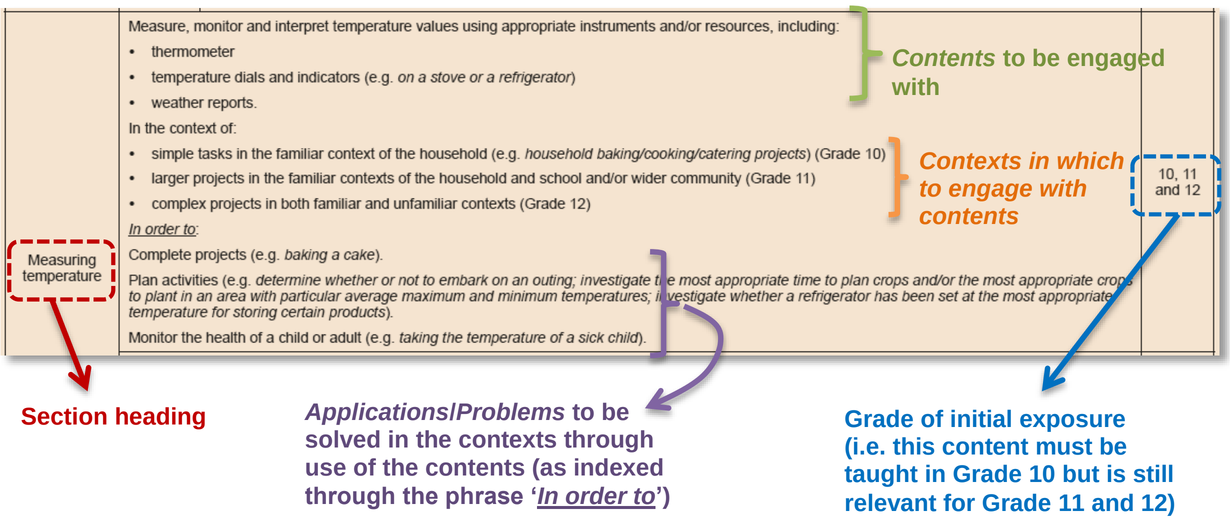
24.2 Analysis: findings and discussion

24.2.1 Macro-level analysis: characteristics of general curriculum structure

Before embarking on specific analysis of the contents of the topic of Probability in the curriculum, it is important to consider the following issue. Namely, that the way in which the contents of the curriculum is structured or specified and the way in which progression is envisioned or intended immediately directs attention towards a particular 'world-view' of the dominant domain of practice and associated structure of knowledge according to which legitimate participation and communication in the subject is endorsed. Namely, and as regards the structure of the contents of the curriculum: almost every description of an aspect of the teachable curriculum includes three components – appropriate (i) *contexts* in which certain (ii) *contents* are to be explored and specific (iii) *applications* of that content or *problems* that must be engaged with.[261] Figure 54 on the page below provides an illustration of this implemented structure (note that the indicators and descriptors shown on the curriculum extract have been included by me and do not appear in the original curriculum document).

This structure immediately signifies two key issues. Firstly, every curriculum section contains explicit specification of real-world contextual situations for exploration. This specification of context provides an immediate degree of correlation between the intention of curriculum and the *Contextual Domain* of the knowledge domain of mathematical literacy: in both the curriculum and the Contextual Domain there is an explicit intention for engagement with authentic real-world contextual environments. On this front, at least, the curriculum reflects some level of coherence to the orientation of the internal language, which, in turn, infers that the internal language provides a suitable (and valid) lens through which to analyse components of the curriculum.

[261] Sometimes, however, the categories of 'contexts' and 'contents' are collapsed into a single category, or the categories of 'contents' and 'applications' are similarly collapsed. This occurs when the contexts themselves constitute the contents to be dealt with or when the applications include specification of the types of contents to be engaged with in a particular problem-solving scenario. The topic of Probability is characterised by the phenomenon where the categories of 'contents' and 'applications' are collapsed, such that specification of necessary applications includes specification of the contents to be engaged with in those applications.



Measuring temperature

Measure, monitor and interpret temperature values using appropriate instruments and/or resources, including:
- thermometer
- temperature dials and indicators (e.g. *on a stove or a refrigerator*)
- weather reports.

In the context of:
- simple tasks in the familiar context of the household (e.g. *household baking/cooking/catering projects*) (Grade 10)
- larger projects in the familiar contexts of the household and school and/or wider community (Grade 11)
- complex projects in both familiar and unfamiliar contexts (Grade 12)

In order to:

Complete projects (e.g. *baking a cake*).

Plan activities (e.g. *determine whether or not to embark on an outing; investigate the most appropriate time to plan crops and/or the most appropriate crops to plant in an area with particular average maximum and minimum temperatures; investigate whether a refrigerator has been set at the most appropriate temperature for storing certain products*).

Monitor the health of a child or adult (e.g. *taking the temperature of a sick child*).

10, 11 and 12

Figure 54: Section of the CAPS curriculum on *Measuring Temperature* in the Application Topic of *Measurement* showing the structuring of teachable curriculum according to *contexts*, *contents* and *applications* (DBE, 2011a, p. 67)

Secondly, there are differing sites of progression specified for the subject in the curriculum – namely, progression from one grade to the next can occur with respect to increased level of cognitive demand of content, increased complexity (and unfamiliarity) of contextual environments under investigation, and increased independence in problem-solving processes[262] (DBE, 2011a, pp. 11-12). This progression structure has a direct influence on the way in which participants in the subject are expected to engage with certain curriculum contents, and, consequently, on the way in which teachers and examiners are expected to teach or assess the curriculum. In particular, in many sections in the curriculum the teachable content is exhausted in Grades 10 and 11, such that there is very little new content in Grade 12. Instead, Grade 12 is posited as a grade in which learners must employ skills and content encountered in previous grades in an integrated way to make sense of complex contextual environments and to solve problems relating to those environments. Both pedagogic and assessment practices in Grade 12 should, thus, ideally prioritise engagement with complex contextual environments and encourage independent problem-solving in relation to those environments. One possible interpretation of this is that there is a dominant intention in the curriculum for the prioritising of *modelling* processes and activities as the outcome of the learning process in the subject. And, if this interpretation is adopted, then, at the level of intention, the curriculum (and specifically the curriculum intention for Grade 12) prioritises a form of participation with the officially endorsed knowledge that is consistent with the structure of legitimate participation characteristic of the Modelling domain of practice of the knowledge domain of mathematical literacy. Whether this consistency is reflected in the individual statements of content and applications that comprise the body of the curriculum remains to be seen, and it is to that issue that the discussion now shifts.

[262] In other words, in Grade 10 there is an expectation for largely directed learning relating to single content strands or concepts. In Grade 11 there is an expectation for learners to engage with problem situations that involve limited direction and the use of contents integrated from multiple curriculum strands. And, in Grade 12 there is an explicit expectation for engagement with numerous complex contextual environments that require utilisation of contents integrated from a variety of curriculum topics and/or sections and which are solved through independent and/or undirected problem-solving processes (DBE, 2011a, pp. 11-12).

24.2.2 Micro-level analysis: characteristics of specific statements in the curriculum that reflect the contents of the topic of Probability

The statements in the CAPS curriculum that describe the contents/contexts/applications for the topic of Probability are given in Appendix B (c.f. page 484 below). For purposes of reference I have numbered the statements in this curriculum extract that indicate a requirement for a particular methodological approach and/or that specify contexts/contents/applications to be engaged with.

Table 9 on the page below provides an overview of some of the general characteristics of these statements.

To begin with (and at a general level of analysis), for every section in the topic of Probability, specification of content and/or application with content is always preceded by a statement of necessary and appropriate contextual situations (see statements 2.1 (a. to i.), 3.1 (a. to i.), 4.1 (a. to i.) and 5.1 (a. to h.)) in which the applications must be embedded, to which the applications must relate, and for which the applications must provide a means of sense-making. Although this is reflective of a general characteristic of the way in which the entire curriculum is structured, the continued presence of this structure in the topic of Probability serves to reinforce a dominant agenda for the investigation and sense-making of contextual situations in any problem-solving scenario (and, seemingly, in any topic of application). This is entirely consistent with the characteristics of the Contextual Domain of the knowledge domain of mathematical literacy in which a life-preparedness orientation is prioritised. At level of intention, then, this topic in the curriculum seemingly promotes elements of a life-preparedness orientation.

Table 9: Overview of the general characteristics of the statements in the CAPS curriculum topic of Probability

Item	Count
Number of different contexts specified for exploration (specified in 2.1 (a. to i.), 3.1 (a. to i.), 4.1 (a. to i.) and 5.1 (a. to h.))	9
Number of statements that index a requirement for a particular application with probability-related contents (2.1.1, 2.1.2, 2.1.3, 3.1.1, 3.1.2, 3.1.3, 3.1.4, 3.1.5, 3.1.6, 4.1.1, 5.1.1) *(Note: Where a statement contains a leader statement with two bulleted [numbered in Appendix B] sub-statements, then the bulleted statements are counted as two separate statements.*	11
Number of additional explanatory statements or statements of guidance (specifically, statements 1, 2.2, and 4.3) *(Note: Statements 2.2 and 4.2 are identical and, as such, are recorded as a single statement.)*	3
Number of statements that specify examples of possible assessment tasks (6.1, 6.2, 6.3, 6.4.1, 6.4.2, 6.4.3)	6

Continuing at a general level, the text is comprised almost entirely of word/vocabulary signifiers, and with only a smattering of visual mediators reflecting possible probability notation and/or formats (e.g. statement 3.1.2 makes reference to the visual mediator of ½ and equates this to 50%). Furthermore, much of the text is constructed in such a way as to signify the nature and structure of appropriate routines to be employed in the generation of endorsable narratives for each statement of application. This is done through the continuous inclusion of signifier verbs such as ‘Recognise’, ‘Understand’, ‘Identify’, ‘Evaluate’ and ‘Develop’, all of which serve a dual function: firstly, to index the requirement for engagement with specific routines in the generation of narratives; and, secondly, to index the required contents of those narratives to secure endorsement. For example, consider the following curriculum statement (DBE, 2011a, p. 93):

> 4.1.1 Identify possible outcomes for compound events by making use of: tree diagrams and two-way tables

Included in the statement is signification of the probability-related resources to be dealt with (namely, tree diagrams and two-way tables) and specific areas of focus and/or contents for consideration in engagement with those resources (namely, identifying outcomes when multiple events are involved). Signification of these resources and contents indexes the requirement for engagement with routines which facilitate interpretation and possible construction of the resources, and with routines which facilitate the use of the resources to identify outcomes. Furthermore, specification of the contents to be engaged indexes a requirement for a particular structure of any generated narratives. Namely, recognising that tree diagrams and two-way tables must be dealt with, and further recognising that these resources must be used to identify outcomes, directly influences and directs the way in which the resources must be engaged with, how the resources must be used. This process also influences and directs what outcomes must be achieved through use of the resources and, hence, what narratives must be communicated through the resources about the context under investigation. Crucially, the deliberate usage or inclusion of the word 'Identify' further signifies a specific endorsability criterion for a generated narrative: namely, endorsed or legitimate engagement with the resources involves demonstration of the ability to 'identify' outcomes – and to communicate the identified outcomes in an appropriate manner − and not specifically to calculate outcomes.

Given that each statement of content or application begins with a verb, it is significant that <u>none of the verbs refer directly to a routine involving a formal calculation</u>. Instead, all of the verbs signify a form of engagement with the application contents that requires demonstration of varying elements of *contextual reasoning* of the way in which people might encounter, engage with, interpret, and communicate about situations in everyday life that involve elements of prediction. The statements seemingly draw focus away from calculation techniques for solving probability-related problems and, instead, focus attention directly on forms of engagement with probability-related concepts and contents which broaden understanding of and enhance engagement with real-world forms of participation and communication involving prediction.[263] This intention is made explicitly clear through the inclusion of a statement at the start of the Probability topic (see Appendix B on page 484 below) denouncing a calculation-based approach in favour of an interpretive approach:

[263] A possible implication of this is that participants who engage with this topic in the curriculum document (e.g. textbook authors and/or teachers) could interpret the statements as indicating a non-requirement for engagement with routines involving the construction of probability values to describe likelihood and, rather, that all probability values must be provided and only interpretation of these given values is required. If this interpretation is adopted, then associated pedagogic practices could significantly limit the extent to which learners are able to develop the ability to describe and represent – as well as interpret – real-world problem scenarios involving probability contents, which, in turn, could inhibit their everyday participation in certain situations. This issue is particularly pertinent in relation to the topic of Probability since, at a foundational level, descriptions involving probabilistic predictions are grounded in a certain level of mathematical structure. The almost complete subordination and/or rejection of mathematical structure for this topic in the curriculum could, thus, impact on the extent to which participation with the contents of this topic results in enhanced real-world functioning and life-preparedness.

1. Calculations involving probability are often confined to *mathematical calculations* primarily in the context of dice, coins and games. Although we may encounter situations involving probability and chance on a regular basis in daily life, it is very seldom that mathematical calculations are needed in order to make sense of those situations. ... What is more important is having an understanding of the concept of probability, together with a sense of whether an event is more or less likely to take place.
   In light of the above, the descriptions given below encourage teachers to focus more on *interpreting* situations involving probability than on the mathematical calculation of probability. This involves developing an understanding of the concept of probability, familiarity with the different notations used in expressions of probability and developing a sense of whether a situation is more or less likely to occur. Alternative contexts outside of the realm of dice, coins and games have also been suggested to reinforce this focus.

(DBE, 2011a, p. 90, all emphases provided in original quotation)

The structuring of each statement of application through use of non-calculation signifying verbs, thus, serves to reflect and reinforce this intention. In so doing, both the individual statements and also the topic as a whole prioritise and legitimise engagement with the contents of the topic according to a form of participation that reflects real-world and non-mathematised practices. And this, in turn, reflects a form of endorsed participation that is consistent with the structure of practice and knowledge which characterise participation in the Everyday domain of the knowledge domain of mathematical literacy.

With the above in mind, a shift from a macro to a micro-level of analysis is now possible through consideration and classification (through the process demonstrated during the operationalisation of the external language of description in Part 5 of the study) of individual statements of application according to the components of the internal language of description. This classification is summarised in Table 10 on the page below.

The information in the table reveals that, when viewed through the lens of the components that comprise the internal language of description for knowledge domain of mathematical literacy, the majority of the statements (41%) require engagement with probability contents through utilisation of knowledge and skills that reflect practices associated with the Everyday domain of practice. A smaller percentage of the statements signify an expectation for a form of engagement that reflects access to and utilisation of knowledge and practices associated with the Mathematical Competency domain of practice (17,6%); and only two statements (11,8%) index a requirement for utilisation of knowledge, practices and forms of participation drawn from both the Everyday and the Mathematical Competency domains.[264] Seemingly a high percentage of the statements index processes associated with the Modelling domain of practice, but this is misleading since all of the 'Modelling statements' refer to descriptions of possible assessment tasks involving probability contents. This is significant in that it suggests that there is an explicit expectation from the curriculum author(s) for assessment practices for this topic to reflect

[264] While analysing the curriculum statements, the ever-present temptation was to analyse how a statement might be interpreted by a third party based on my own experience of the contents referenced in the statement and also of the mathematical routines required to provide access to those contents. If this approach were to be applied to the analysis process, then significantly more of the applications described in the statements would be classified as requiring both mathematical and everyday forms of practice and knowledge for successful engagement. However, the approach ultimately adopted (and which reflects consistency with the demonstrated operationalisation of the external language) involves analysis of the contents and routines indexed through the signifiers present in the text.

modelling activities; however, none of the statements of content or application which reflect pedagogic processes signify a similar requirement for modelling. As such, there is a degree of inconsistency in the dominant domain of prioritising between the pedagogic and assessment requirements in this topic.[265]

The dominance of individual statements which reflect consistency with characteristics of the Everyday domain of practice is significant in that it reinforces the already stated general deduction regarding the dominant domain prioritising for this topic in the curriculum. Namely, the topic of Probability in the curriculum is dominated by an expectation for teachers and learners to engage with the specified contexts and contents in a way that reflects real-world (non-mathematised) practices. For this topic, the author(s) of the curriculum explicitly prioritises the development of enhanced real-world functioning through specification of the requirement for participants in the subject to engage with probability scenarios and contents in a way that directly reflects real-world experiences of prediction and which will enhance their ability to engage with prediction in such real-world experiences.

[265] Although, the requirement and expectation for modelling processes is consistent with a broader statement of intention for independent problem-solving in the curriculum. The issue here, however, is that different routines are prioritised in the pedagogic and assessment processes, with the consequence that the criteria for legitimate participation differ in each domain, as do the structures of the narratives that are endorsed. This could lead to confusion on the part of any participants who engage with the contents of this topic regarding the dominant domain of practice according to which legitimate forms of participation and communication are endorsed.

Table 10: Classification of the statements in the curriculum topic of Probability according to the components of the developed internal language of description for mathematical literacy

	Component of the knowledge domain of mathematical literacy		Classified Question	Count
Curriculum statements (in the topic of Probability)	Primarily Everyday Domain Practices		2.1.1; 3.1.1; 3.1.3; 3.1.4; 3.1.5; 3.1.6; 5.1.1	7
	Primarily Mathematical Competency Domain Practices		2.1.2; 2.1.3; *6.2* [266]	3
	Either Everyday or Mathematical Competency Domain Practices		---	0
	Both Everyday and Mathematical Competency Domain Practices		3.1.2; 4.1.1	2
	Modelling Domain Practices		*6.1*; *6.3*; *6.4.1*; *6.4.2*; *6.4.3*	5
	Reasoning/Reflection	Level 1	3.1.2; *6.1; 6.3; 6.4.1; 6.4.2; 6.4.3* 2.1.1 (C)[267]; 3.1.1 (C); 3.1.2 (C); 3.1.3 (C); 3.1.4 (C); 3.1.5 (C); 3.1.6 (C); 4.1.1 (C) 2.1.2 (M); 2.1.3 (M); *6.2 (M)*	
		Level 2	2.1.1 (C); 3.1.1 (C); 3.1.3 (C); 3.1.4 (C); 3.1.5 (C); 3.1.6 (C); 4.1.1 (C); *6.2. (M)* *6.1*; *6.3*; *6.4.1*; *6.4.2*; *6.4.3*	
		Level 3	3.1.1 (C); 3.1.3 (C); 3.1.4 (C); 3.1.5 (C); 3.1.6 (C); 3.1.2 (M); 4.1.1 (M); *6.2 (M)* *6.1*; *6.3*; *6.4.1*; *6.4.2*; *6.4.3*	
		Level 4	3.1.1 (C); 3.1.3 (C); 3.1.4 (C); 3.1.5 (C); 3.1.6 (C); 3.1.2 (M); 4.1.1 (M); *6.2 (M)* *6.3*; *6.4.1*; *6.4.2*; *6.4.3*	
		Level 5	3.1.3 (C); 3.1.4 (C); 3.1.5 (C); 3.1.6 (C); 3.1.2 (M); *6.2* (M) 4.1.1; *6.1*; *6.3*; *6.4.1*; *6.4.2*; *6.4.3*	
		Level 6	5.1.1 (C)	
		Level 7	---	

However, and as already mentioned, this dominant domain of prioritising in this topic is potentially problematic since the underlying structure of probabilistic notation and application involves mathematical structure and, hence, non-explicit recognition of this structure could result in a limited learning experience of the topic and limited preparation for real-world engagement. Furthermore, emphasis on the Everyday domain as the dominant domain of prioritising reinforces the inconsistency between the dominant domain of prioritising in pedagogic (i.e. Everyday knowledge and participation) and assessment expectations (i.e. Modelling knowledge participation) for the contents of this topic. Finally, despite an emphasis on real-world forms of engagement with probability scenarios, a life-preparedness orientation is facilitated through engagement with more than just the Everyday domain of practice. Overemphasis on this domain at the expense

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[266] Statements that refer specifically to assessment are formatted in *italics* in the table.

[267] The denotations (C) and (M) in the table reflects those questions in which elements of reasoning and/or reflection are focused specifically and/or primarily on contextual or mathematical elements respectively of the problem scenario and of the problem-solving process. Questions for which no such detonation is provided encompass reasoning and/or reflection on a combination of both contextual and mathematical elements. I have done this deliberately to facilitate identification of the heightened emphasis on contextual components of the problem-solving process in the curriculum segment.

of the other domains suggests that a limited degree of life-preparedness will result from engagement with the contents of this curriculum topic in the way encouraged by the document. My intention in analysing other texts in comparison to the curriculum document is, thus, partly driven by a motivation for determining the extent to which the authors of these other texts reflect this dominant domain of prioritising in their dealings with probability contents and how or if they negotiate and/or negate the potential issues highlighted above.

With respect to the domain of Reasoning and Reflection of the knowledge domain of mathematical literacy, the information in the table evidences a dominant expectation for various forms or levels of reasoning on contextual elements − which is to be expected given the dominant prioritising of forms of knowledge, communication and participation associated with Everyday domain of practice. And, although some level of reflection on mathematical elements is expected, much of this reflection is associated with modelling processes characterised in the suggested assessment tasks. Two further particular observations are particularly significant with respect to the levels of Reasoning and Reflection. Firstly, the curriculum statements on the topic of Probability contain no explicit expectation for participants to consider whether and/or how they have reflected on the problem-solving process and on any decisions made during that process (i.e. R/R Level 7). Secondly, only one statement (statement 5.1.1) contains an explicit expectation for participants to critically analyse the way in which probability contents (as a mathematical construct) are used to influence and/or direct decision-making practices (i.e. R/R Level 6). This limited (or absent) expectation for reasoning and reflection at these two levels can be interpreted as signifying a dominant focus in application processes involving probability contents on structural concerns with the problem-solving process and not on critical aspects of the problem-solving process. In alternative terms, primary focus is on understanding the problem and finding a solution to the problem through interpretation of the signifiers in the problem and consequent utilisation of particular forms of knowledge and associated routines that facilitate generation of narratives which serve as reflecting legitimate participation in real-world settings. By contrast, there is only limited (or no) focus on questioning why narratives have been constructed in a certain way about a problem scenario and why these narratives reflect criteria for legitimate participation in the context. There is also no questioning about how deliberate decisions regarding the inclusion and exclusion of variables and routines have affected the structure of the narratives and the ‘world-view’ propagated through the narratives, or about how legitimate participation could be differently construed through the development of narratives according to different criteria of inclusion and exclusion. In short and in summary, the statements relating to the contents of Probability in the curriculum promote a form of engagement with real-world situations involving probability and prediction that reflect the reality of real-world everyday knowledge, communication and participation. This prioritised form of engagement includes a dominant agenda for sense-making of contextual situations and problems, but lacking in an orientation for the development of a critical competency.

To shed further light on my rationale for the categorisation of certain statements in relation to the structure of knowledge and participation associated with the components of the knowledge domain of mathematical literacy, in Table 11 on the pages below I offer elaborated explanations and justifications for particular categorisations.

Crucially, movement through and/or exposure to the various domains of practice of the knowledge domain of mathematical literacy is a key intention for a conception of Mathematical Literacy in which a life-preparedness orientation is prioritised. In the topic of Probability, however, exposure to everyday forms of participation is prioritised while essential mathematical structures are restricted and even excluded. Furthermore, although modelling processes are recognised, these processes are only given priority in assessment contexts and no formal development of these processes is prioritised in statements referring to pedagogic practice. The consequence is that, in reference to the contents of the topic of Probability, the curriculum statement restricts rather than facilitates movement through the various domains of practice, hereby – from my perspective – limiting the possibility for the development of a comprehensive or more enhanced life-preparedness orientation with probability-related contexts/contents/applications.

Table 11: Rationale for the categorisation of selected statements of content/assessment in the CAPS Probability topic according to the components of the knowledge domain of mathematical literacy

Statement	Categorisation	Rationale/explanation
2.1.2 & 2.1.3	Mathematical Competency	Both of these content statements index a requirement for understanding of different probability formats and notation, the rationale for these formats, and consequent routines (involving ratio, proportion, percentages, decimals, division) that facilitate understanding of, engagement with, and possible generation of probability notation, and translation between different formats of this notation. Successful communication about probability concepts through the (re)production of narratives that are endorsed is dependent on the successful utilisation of probability notation. And, this notation is based on a mathematical understanding of the probability scale and of the significance of fractional notation on this scale.
3.1.2	Both Everyday and Mathematical Competency	My own perspective is that understanding of relative frequency and of the difference between relative frequency and theoretical probability is not possible without reference to a specific contextual entity (e.g. such as a coin that is tossed). Equally, understanding of these concepts is impossible without access to understanding of the mathematical structure of a probability value and the proportional relationship represented in that value.
4.1.1	Both Everyday and Mathematical Competency	Tree Diagrams and Two-Way Tables provide a particular mathematically structured representation for sense-making practices of probabilistic situations involving two or more events, and the routines involved in the development of these representations are, by-and-large, mathematically structured. Furthermore, for developed representations to be endorsed, the representations need to follow a particular format which traditionally is determined by mathematical structure. However, the structure of these representations and the ability to construct these representations is equally reliant on the ability to identify specific contextual elements relevant to the problem situation, the nature of the relationship between those elements, and the ability to reflect those elements and relationship appropriately on the resources. In simple terms, successful construction of a tree diagram or two-way table to reflect outcomes for a combination of events is rendered unlikely if understanding of the events or the contexts to which the events relate is not in place. An endorsed narrative for these representations is reliant on both appropriate interpretation and engagement with contextual elements of a problem situation and the ability to reflect those elements on an accurately structured representation.

Statement	Categorisation	Rationale/explanation
The additional explanatory statement attached to statement 4.1.1 (and reflected as statement 4.3 in the curriculum section in Appendix B) is worth discussing (DBE, 2011a, p. 93): + The sections on tree diagrams and two-way tables have been included to provide learners with exposure to different tools and representations that can be used to represent events involving probability in a graphical/pictorial way. The focus in these sections should be on using these representations to identify all of the possible outcomes of an event, especially in situations where the outcomes are not immediately obvious. Learners are not expected to have to use tree diagrams and two-way tables to perform mathematical calculations of probability (e.g. multiplying probabilities along the branches of tree diagrams).		
4.3	N/A	This statement signifies further explicit reinforcement of an intention in this topic for prioritisation of an agenda for contextual sense-making practices over mathematical structure and application. The statement comprises recognition that these two representations offer a particular mathematical resource for identifying, describing and representing outcomes for combined events, but reinforces an expectation for primary emphasis on the utilisation of the resource for sense-making practices rather than for facilitating mathematically structured understanding and/or processes of mathematisation.
6.1, 6.3, 6.4	Modelling	These three suggested *assessment* tasks are all categorised as reflecting practices and forms of communication associated with the Modelling domain. This categorisation is determined on the basis that each task includes a requirement for an attempted (re)description or (re)construction of a segment of reality to better understand an aspect of that reality, and that any narrative that is generated is to be evaluated on the extent to which it provides reasonable and feasible access to understanding of the situation. Finally, for each question participants are required to engage in modelling processes as they navigate and negotiate which contextual elements to focus on, how to deal with those elements, which routines to employ to represent those elements, and the significance of any mathematisation processes on the perspective of reality that is generated. Furthermore, the form and structure of the dialogue through which communication about the generated narratives occurs must also be determined.

This, then, brings to an end the analysis of the examination papers. In the next chapter (0), discussion shifts to analysis of the corresponding topic of Probability in a set of exemplar national Grade 12 Mathematical Literacy examination papers. Reference to the findings of the curriculum analysis process described above will again be cited in this forthcoming chapter to facilitate comparison of the dominant domain of prioritising between the examinations and the curriculum.

CHAPTER 25
ANALYSIS OF EMPIRICAL TEXTUAL RESOURCE #2: NATIONAL GRADE 12 EXEMPLAR EXAMINATIONS

25.1 Background information

With the CAPS curriculum being examined for the first time at Grade 12 level in November 2014, the Department of Basic Education distributed nationally set exemplar examinations for Grade 10, 11 and 12 in 2012, 2013 and 2014 respectively. The purpose of these exemplar examinations was/is to demonstrate the focus and structure of assessment in the CAPS curriculum process (and, consequently, of how assessment practices have changed from the NCS curriculum structure).

All of the exemplar examinations papers for Mathematical Literacy have been set by the same national examining panel that will set the national Grade 12 final examination papers at the end of 2014. As discussed in Chapter 8 and sub-section 8.3.1.4 (starting on page 112 above), the national Grade 12 final examination papers – and, by implication, these exemplar papers –are comprised of two separate examinations titled ‘Paper 1’ (focused on assessing ‘basic skills’) and ‘Paper 2’ (focused on assessing ‘applications). The distinction between the level of cognitive demand of these examinations is reinforced through a requirement for a differential distribution of questions from a four-level assessment taxonomy (see Table 4 on page 113 above). At Grade 12 level, the entire curriculum is assessed in both Paper 1 and Paper 2, together with some concepts from Grades 10 and 11 for which there is no new content in Grade 12 or for which understanding of the Grade 10 and 11 concepts is deemed essential for reflecting understanding of a Grade 12 concept. The Grade 12 exemplar Paper 1 and Paper 2 examinations – which are the focus of analysis in this study – individually total a maximum of 150 marks with a stipulated duration of 3 hours.

It is important to note that the topic of Probability is allocated the lowest weighting of all of the application topics in assessment practices in the subject[268] (DBE, 2011a, p. 13). For this reason, only a small number of questions in both of the examination papers deal with probability-related contents. The consequence of this is that the analysis of these questions only provides a significantly limited perspective of the way in which the author(s) of the examinations legitimise knowledge and participation in the practices of the subject. Nonetheless, this analysis occurs in reference and comparison to the explicit statement or requirement in the curriculum for an interpretive and non-calculation approach in the topic. For this reason, analysis of this small number of questions still provides a suitable and adequate representation of areas of commonality and divergence of the way in which knowledge and participation are legitimised in the examinations in comparison to the curriculum.

[268] The CAPS curriculum document specifies a minimum requirement for an allocation of 5% of the total marks allocated in each examination paper to probability-related contents. The inclusion of this ‘minimum’ requirement was motivated by the intention to overcome historical practices in which many teachers simply did not engage with the contents of this topic in their teaching.

25.2 Analysis: findings and discussion

25.2.1 Macro-level analysis: characteristics of general examination structure and approach

Macro-level observation of the entire examination for both papers immediately reveals certain characteristics regarding the dominant domain of practice and associated structure of knowledge and participation prioritised in these texts. To begin with, although each global question in the examination papers includes focus on a specific contextual situation, many of these situations do not demonstrate a high degree of representation and/or authenticity with respect to real-world practices. Rather, I contend that many of the contexts represent 'inauthentic' and 'contrived' representations, having been deliberately constructed to facilitate access to particular mathematical components of the contexts. This is exemplified through continuous reference to fictitious (and somewhat boring!) personalities and entities in the contextual scenarios (such as 'Siyabonga Bank' (Paper 1: DBE, 2014b, p. 3) and a landline telephone contract named 'Scamtho 250' (2014b, p. 4) – none of which exist in South Africa (or anywhere else in the world), together with a whole host of personalities such as *Marieka* [who owns a coffee shop and who, seemingly in her spare time, is also building a vegetable shade tunnel] (2014b, pp. 6-7), *Jan* [who is randomly studying religious denominations and population distributions in South Africa, but with no apparent interest in the connections between these two data groupings] (2014b, pp. 8-9), *Mrs. Van der Linda* [who lives in Kimberly and whose house plans are somehow of relevance to us – although no reason is given as to why we might want to learn about her house (perhaps we intend to stalk her?)] (2014b, p. 10), *Kevin* [who is 45 years old and works in the tourism industry, but unfortunately no information is given about whether or not he is single and available – or else a date could have been organised with Marieka] (2014b, pp. 11-12), and *Mr. Reddy* [who is a teacher at Kevin's son's school and who, for whatever odd reason, has an interest in building a model of computer "to help create a subject atmosphere in the computer room"? (2014b, pp. 12-13) – won't actual computers already do that?]).[269] Furthermore, both examination papers are characterised by only limited representations of authentic and original real-world signifiers (particularly visual mediators) drawn directly from everyday situations and texts. In examination papers which, supposedly, deal with investigations of real-world scenarios, one would expect to find authentic visual mediators such as newspaper clippings, authentic and original adverts, scanned copies of financial and other documents, and so on. Instead, many of the scenarios presented for investigation are represented through deliberately constructed mediators that have little to no link to authentic and original practice. For example:

[269] And, all of this is only in Paper 1. In Paper 2 there is considerably less socialising involved, although we are presented with the wonderful opportunity to engage with a different host of other interesting (albeit fake) people and organisations, including:

- the Grade 12 class of XYZ High School (do you really think the person who named this school is invited back for prize-giving every year?) (DBE, 2014d, pp. 3-4);
- Megan, who is investigating the prices of pre-owned Smart cars (presumably because she want to buy one of these cars [has she ever tried to sit in one of these? If she had, she would run away, quickly!] and who feels the need to draw box-and-whisker plots to help her to make a decision [at which point the car salesman would probably have given her a discount just to get her out of the dealership!] (DBE, 2014d, pp. 7-8); and
- Ms Springbok, who runs a tuck-shop from her house, who uses probability calculations (and presumably also distribution and ogive curves in her spare time?) to make predictions about customer preferences, and who draws floor plans to determine where in the tuck-shop to put her fridge so that she can minimise walking distance in the shop (seemingly Ms Springbok is not interested in exercise or in using her eyes and brain to make such decisions) (DBE, 2014d, pp. 9-10).

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1.2 Pantsula has a landline contract known as Scamtho 250, which consists of the following monthly tariff system:

- A fixed monthly fee of R299,00
- 150 minutes free per month for landline-to-landline calls
- 100 minutes free per month for landline-to-cellphone calls
- 80 cents per minute (billed per second) for all calls outside the free minutes.

(Paper 1: DBE, 2014b, p. 4)

QUESTION 2

2.1 Marieka owns a coffee shop. She serves a mixed berry and almond polenta cake that is baked in espresso cups at her coffee shop. She uses the recipe below to make the cake.

Mixed Berry and Almond Polenta Cake

Makes 15 espresso cups

Ingredients
6 eggs separated (keep the yolks for mayonnaise or scrambled egg)
140 g butter
140 g castor sugar
140 g ground almonds
250 g fat-free cottage cheese
75 g mixed frozen berries
25 g polenta

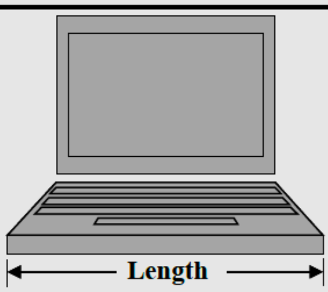
Bake at 356 °F until light brown, 30 to 40 minutes.

(Paper 1: DBE, 2014b, p. 6)

5.2 Mr Reddy is a teacher at Kevin's son's school. Mr Reddy is the computer and Mathematics teacher.

5.2.1 Mr Reddy decides to build a model of a computer laptop to help create a subject atmosphere in the computer room.

He drew a scale diagram of the laptop as shown alongside.



(Paper 1: DBE, 2014b, p. 12)

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QUESTION 1

1.1 The Grade 12 class of XYZ High School is planning a Dinner and Dance evening to raise funds which they will use towards their matric farewell. The organising committee is divided into two groups. One group will investigate the use of the school hall and the other group will try to find another possible venue.

Tickets for the Dinner and Dance evening will be printed at the school at a cost of R2,20 per ticket.

(Paper 2: DBE,2014d, p. 3)

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QUESTION 3

3.1 Megan investigated the price of pre-owned Smart cars. She summarised the data in a table with the age of the car and the selling price.

Table 1: Age and price of pre-owned Smart car

Age in years	3	7	8	4	1	2	5	2	2	1	1	4
Price in thousand rand	115	68	64,9	100	130	120	88	110	130	135	170	110

(Paper 2: DBE, 2014d, p. 7)

Figure 55: Extracts from the Grade 12 Mathematical Literacy Paper 1 and Paper 2 exemplar examinations that demonstrated the contrived nature of many of the context used in the exams

Importantly, notice that some attempt has been made to include visual mediators in the form of actual photographs of real-world items (such as the telephone, coffee cup, and car) to ground the situations in reality. However, I argue that these visual mediators serve simply to generate a perceived and mythical link to reality rather than reflecting authentic and everyday contexts. For example, consider the picture of the coffee cup in Paper 1: Question 2.1 (DBE, 2014b, p. 6). This picture is, presumably, intended to signify that the situation under investigation is a genuine and authentic real-world situation – specifically in this instance, a situation involving a recipe for a cake. However, the given resource does not look at all like an authentic recipe; instead, it looks like a resource that has been deliberately constructed to serve a particular purpose in the examinations.[270] Furthermore, the picture shown on the examination is not the same picture show on the original recipe found on the website, thus hinting at the possibility that the picture shown in the examination is not actually for this cake. So, even though the ingredients and quantities are realistic, the resource itself has been reconstructed in such a way that the experience of authenticity is diminished. If this is a real-recipe that reflects an authentic baking activity, and if the authors of the examinations want to provide the opportunity for engagement with this real-world resource and related activity, why, then, has an actual recipe not been scanned and included for analysis?[271] Why is no indication given that this cake is actually made up of two parts – the cake component and a berry coulis (which I assume, from my limited time in the kitchen, is some sort of berry sauce), and that the ingredients shown on the examination version are for the cake component only? Why are no instructions given as to how the ingredients must be mixed? Why is an actual extract of a banking conversion table for ingredients not supplied (as would be in a recipe book)? And, why is the baking temperature given in imperial units while the other measurement values are given in metric units?[272] The key point for consideration here is that, for me, the questions associated with this context of baking and the associated contrived contextual visual mediator of the recipe are not directed towards assessment of understanding of baking practices or of the utility of certain forms of mathematical thinking in enhancing understanding of this practice. This is exemplified in the fact that it will be impossible to use this recipe to successfully complete the task of baking this cake without further research or help. Rather, the questions are directed towards assessing specific mathematical contents (specifically, conversions, ratio and proportion[273]) and the context of baking simply provides window-dressing for this mathematisation process. In

[270] Interestingly, the recipe (but not the picture) is drawn directly from an authentic original source, uncovered by the super-sleuth skills of my supervisor! The recipe is, seemingly, sourced from: www.hartford.co.za/recipes/brookdales-mixed-berry-almond-polenta-cake-recipe. Notice, however, that no reference is made in the examinations to the sourcing of the recipe from this website, which certainly would have heightened the impression and experience of authenticity when encountering the resource.

[271] Given that this is a national examination paper there is every possibility that the absence of authentic resources is affected and determined by copyright concerns on the part of the Department of Basic Education. While I acknowledge the complexity involved in the use of authentic resources in such settings, it is also important to point out that this issues is resolved in the examinations for other subjects (for example, through the inclusion of and reference to historical resources in History and literary resources – such as newspaper cartoons and articles – in English). Furthermore, I contend that if contextual sense-making practice were prioritised in the examinations, then engagement with authentic contextual resources would be a necessary requirement for successful participation in the examinations and, so, heightened effort would be made to source permission for engagement with such resources. At present, however, this does not seem to be a priority, as evidenced by the complete absence of authentic resources in the examinations.

[272] On the original recipe found on the above mentioned website, the baking temperature is given in metric units (degrees Celsius). This has seemingly been modified on the examination so that students (in the first question based on this resource – Question 2.1.1) can now be assessed on their ability to use a given temperature conversion formula to convert from degrees Fahrenheit to degrees Celsius.

[273] Can you see the potential for a ratio question in this context? Well, the examiners thought it appropriate to ask learners to write down the ratio *polenta* : *mixed frozen berries*, which is, clearly, an essential skill for being able to successfully make this cake!

other words, the context in this question is irrelevant – wherefore it's actually worse that an authentic recipe was used, thus, representing precisely an instance of the 'myth of participation' (Dowling, 1998) – and any number of different contexts (e.g. shopping, mixing concrete, weather reports) could have been employed to assess the same mathematical concepts. Or, put differently yet again, participation in this question is legitimised according to whether the participants are able to generate mathematically structured narratives that reflect appropriate forms of mathematical notation and communicative artefacts. Participation is not legitimised according to whether the generated narratives reflect understanding of the context and, specifically, of the practices and appropriate forms of participation required for successful engagement in the task(s) encountered in that context.

In Part 4, Chapter 14 and sub-section 14.4.1 (starting on page 194), I argued that in practices associated with a conception of the knowledge/subject domain of Mathematical Literacy characterised by a life-preparedness orientation, both actual ('real') and 'cleaned' contexts are appropriate for investigation. In this regard, 'Cleaned' refers to a real-world setting that has been deliberately modified and/or simplified to provide increased access to the context but while still retaining a strong degree of reflectiveness of the authenticity of the context and of real-world practices in that context. What the discussion above has highlighted is that most of the contextual visual mediators employed in the examinations papers do not constitute either 'real' or 'cleaned' resources: rather, they are mediators that have been deliberate constructed to provide access to particular mathematical knowledge and practices, and which bear little or no resemblance to authentic everyday practice. In other words, the mediators are contrived.

However, not all of the contexts are contrived, with a small number of contexts reflecting characteristics of cleaned contexts and at least two instances (Paper 1: Questions 3.1 and 5.1.2) which reflect real and/or authentic contextual information and/or resources. Table 12 below shows a summary of the frequency of the different context types in the examination papers.

Table 12: Frequency of different context types in the Grade 12 Mathematical Literacy Paper 1 and Paper 2 exemplar examination papers

	Paper 1		Paper 2		Combined
Context Type	**Questions**	**Frequency**	**Questions**	**Frequency**	**Total Frequency**
Context free	---	0	---	0	0
Contrived	1.2; 1.3; 2.1; 5.1; 5.2.1; 5.2.2	6	1.1; 1.2; 3.1; 3.2; 4	5	11
Cleaned	1.1; 2.2; 3.2; 4.1; 4.2	5	2.1; 2.2	2	7
Real/authentic	3.1; 5.1.5	2	---	0	2

Despite the presence of cleaned and real/authentic context types, it is significant (as highlighted by the information in the table) that the majority of the contexts utilised in the examination papers are of a contrived nature, with a relatively high degree of inauthenticity, and constructed deliberately for purposes of elaborating mathematical structures and knowledge. Furthermore, even when cleaned and/or real/authentic contexts or contextual visual mediators are employed (such as in Paper 2: Question 2.1 shown in Figure 56 below), the questions developed about or around the contexts immediately delve into a form of mathematical engagement with the contexts driven by a focus on the assessment of calculation techniques, rather than by an orientation to facilitate increased contextual sense-making and/or understanding of the context. And, as is discussed shortly, this has implications for the extent to which the internal component of the language of description is able to provide a valid framework for analysis of the examinations and, also, for the extent to which the examinations are able to adequately prepare learners for participation in real-world practices.

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QUESTION 2

2.1 The Southern African Large Telescope (SALT) is the largest single optical telescope in the Southern Hemisphere and among the largest in the world.

It has a hexagonal primary mirror array of 11 metres across, comprising 91 individual 1,2 m hexagonal mirrors. Each mirror weighs about 100 kg.
SALT can detect the light from faint or distant objects in the universe, a billion times too faint to be seen with the unaided eye – as faint as a candle flame would appear at the distance of the moon.

Picture of the SALT

SALT is situated at Sutherland, a small town in the Northern Cape, South Africa.

2.1.1 A scientist from the United Kingdom claims that the total weight of the mirrors is 1 450 stone.

Verify this claim.

You may use the following conversions: **1 stone = 14 pounds**
1 pound = 0,45359 kg (6)

2.1.2 The ring wall of the telescope on which the dome is supported, is 17 m high with a diameter of 26 m. This ring wall has a steel structure covered with insulation panels, with 61 rectangular louvers that open at night. The louvers are each 2,25 m wide and 98 cm high.

(a) Calculate the surface area of the ring wall that is covered with insulation panels.

You may use the following formula:

Surface area of a cylinder = $2 \times \pi \times \textbf{radius} \times \textbf{height}$
where $\pi = 3{,}142$ (7)

(b) If each insulation panel is 5,1 m wide, how many sides does the polygon, which is formed by the steel structure and the insulation panels, have?

You may use the following formula:

Circumference of a cylinder = $2 \times \pi \times \textbf{radius}$
where $\pi = 3{,}142$ (4)

Figure 56: An extract of Question 2.1 from the Grade 12 Mathematical Literacy Paper 2 exemplar examination (DBE, 2014d, p. 5)

Shifting focus away temporarily from the structure of the contexts and associated contextual resources that characterise the examinations, cursory analysis of the stated instructions and questions in the examinations provides further evidence of the prioritising of mathematical structures and forms of practice and participation. To being with, it is significant that for both examinations the following instruction is given on the papers: “Round off ALL final answers to TWO decimal places, unless stated otherwise.”

(DBE, 2014b, p. 2; DBE, 2014d, p. 2). Inclusion of this statement immediately signifies that consideration of specific contextual restrictions/conventions/requirements regarding rounding are unimportant and unnecessary, and that a specific mathematical routine and convention for rounding dominates. Furthermore, the statement also signifies that learners are not being assessed on their understanding of contextual nuances and constraints, nor on their ability to engage with contextual contents in a way that reflects authentic real-world practice: contextual sense-making practices are not the main priority. As a result, generated narratives which do not following this mathematical convention for rounding and, instead, apply contextual logic, routines and conventions will, in all likelihood, not be endorsed by the examiners. This statement, positioned as it is at the very beginning of the examination, sends a clear signal regarding the dominance of prioritisation of mathematical structures, knowledge, contents, routines and forms of participation in the generations of narratives for or about the questions in the examinations.

As regards question structure, it is significant that the signifier 'calculate' appears frequently in the examination questions[274], as does the signifier 'determine'[275]. And, although the signifier 'determine' could be interpreted to index a variety of routines for generating a narrative, this signifier is often accompanied by other signifiers that direct attention towards specific mathematical entities, contents, techniques and routines required for use in the generation of endorsable narratives. For example: "1.1.4 Determine the service fee as a percentage of the deposited amount." (Paper 1: DBE, 2014b, p. 3). The same principle applies to the way in which other verb signifiers are employed to direct the learners' attention towards particular contents and routines with those contents:

> 5.1.3 ... Describe how Kevin's taxable income was calculated.
> (Paper 1: DBE, 2014b, p. 11)
>
> 2.1.1 ... Verify this claim.
> You may use the following conversions: 1 stone = 14 pounds
> 1 pound = 0,4535 kg
> (Paper 2: DBE, 2014d, p. 5)
>
> 3.2.3 Explain, with reasons, whether the data represented in these box-and-whisker plots are DISCRETE or CONTINUOUS.
> (Paper 2: DBE, 2014d, p. 8, emphasis in original text)

Cursory analysis of the examination papers also reveals the extent to which there is direct and explicit signification of a requirement for learners to make use of prescribed and given formulae to solve problems.[276] This formulaic signification or expectation restricts possible forms of participation with the contents of the questions and limits the structure

[274] See, for example:
- Paper 1 → 1.1.3, 1.2.1, 1.2.2 (a), 1.3.1, 1.3.2, 2.1.2, 2.2.2, 2.2.4, 3.1.6, 3.2.2, 5.1.1, 5.1.2, 5.2.2 (c), and 5.5.5 (e) (DBE, 2014b).
- Paper 2 → 1.1.1, 1.2.1, 1.3, 2.1.2 (a), 2.1.3 (b), 2.2.2, 2.2.3, 3.1.1, 4.2.2, 4.2.3, and 4.3.3 (DBE, 2014d).

[275] See, for example:
- Paper 1 → Questions 1.1.2, 1.2.2 (c), 2.1.5, 2.2.2, 2.2.3, 3.1.2, 3.2.5, 4.1.4, 4.1.5, 5.1.5 (a), 5.2.1 (b), 5.2.2 (d) (DBE, 2014b).
- Paper 2 → Questions 3.1.3 (a) (DBE, 2014d).

[276] See, for example:
- Paper 1 → Questions 1.2.2 (c), 1.3.2, 2.1.1, 2.2.1, 2.2.2, 2.2.3, 2.2.4, and 3.2.5 (DBE, 2014b).
- Paper 2 → Questions 1.1.2, 2.1.1, 2.1.2 (a), 2.1.2 (b), 2.2.3, and 4.2.3 (DBE, 2014d).

of endorsed narratives to those generated through utilisation of the prescribed routine: in other words, participation in such questions is legitimised primarily through appropriate utilisation of the given formula. And, even though narratives generated through the utilisation of alternative routines may be endorsed, since the given formulae provide a form of direct signification of the examiners expectation for the 'most appropriate' method for solving the problem, learners who attempt to employ alternative (and non/indirectly signified) routines face a potentially more complex problem-solving experience.

It is further significant that for every instance where learners are asked to draw graphs to represent certain scenarios and/or data, completed axes and grids for the graphs that already include numerical scales and/or categorical or textual labels for the axes are given (in annexures). The provision of visual mediators in this format transforms the process of graph development to describe relationships to a simple exercise in plotting points. Learners are expected to demonstrate little or no understanding of: the specific and unique structure of the relationship between particular contextual elements; of the significance of this relationship for the way in which the information must be represented and reflected on the graph; and of the impact of this relationship on defining and determining the structure of legitimate participation in a context. Yet again, mathematical comprehensive, knowledge and techniques are prioritised over contextual sense-making practices.

Shifting focus briefly from the examination question papers to the memorandums, it is immediately noticeable that the memorandums are characterised by narratives presented as continuous streams of calculations – as illustrated in Figure 57 below.

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KEY TO TOPIC SYMBOL:

F = Finance; M = Measurement; P = Scale, Maps, Plans and other representations
DH = Data Handling; L = Likelihood and Probability

QUESTION 1 [36]

Ques	**Solution**	**Explanation**	**Topic**
1.1.1	R28 955,47 ✓A	1A answer (1)	F L1
1.1.2	Amount (in rand) = 2 39,10 + 3 100,00 + 110,00 + 500,00 ✓M = 5 949,10 ✓A	1M adding correct amounts 1A answer (2)	F L1
1.1.3	A = R31 194,57 – R850,00 ✓M = R30 344,57 ✓A B = R33 798,11 – R33 540,64 ✓M = R257,47 ✓A	1M subtracting correct amounts 1A value of A 1M subtracting correct amounts 1A value of B (4)	F L1
1.1.4	$\text{Percentage} = \dfrac{\overset{\checkmark M}{\text{R}31{,}74}}{\text{R}2\,239{,}10} \times 100\%$ ✓M $= 1{,}42\%$ ✓A	1M using correct values 1M calculating percentage 1A answer (3)	F L1
1.1.5	2 weeks ✓✓A	2A answer (2)	M L1

Figure 57: An extract from the memorandum for the Grade 12 Mathematical Literacy Paper 1 exemplar examination which demonstrates the prevalent expectation for calculation based solutions (DBE, 2014c, p. 2)

Apart from the inclusion of specific context-signifying visual mediators, such as currency symbols (e.g. “R”) and words (e.g. “Amount (in Rand)”), the narratives presented here can easily be taken to reflect the types of narratives associated with certain types of public domain mathematics practices found in a junior-grade scientific mathematics classroom. It is also significant that the descriptors or explanations given for mark allocations almost always indicate a requirement for mathematical structure, notation and/or calculation. A reading of the memorandums for the examination papers should leave no doubt that for the authors of the examinations, participation with the contents of the examinations (and, by implication, participation with the contents of the knowledge/subject domain of Mathematical Literacy) is legitimised according to mathematical proficiency. Any generated narratives-as-solutions are evaluated and endorsed according to whether specific and signified or stipulated mathematical routines have been employed with appropriate accuracy and whether communication of the narrative-as-solution includes appropriate mathematical notation, symbolism and artefacts.

From the perspective of the developed language of description for a conception of the knowledge domain of mathematical literacy in which a life-preparedness orientation is prioritised, the explicit prioritising of mathematical structures, routines and narratives in the examinations − through the way in which contrived contexts are employed, questions are structured, and solutions are presented − is enormously problematic. Specifically, and as was argued in Part 4 and sub-section 14.4.1 above, prioritisation of a life-preparedness orientation – as a means of overcoming the challenges identified by Dowling (1998) with primarily public domain mathematical practices – is only possible through participation in and with genuine and authentic real and/or cleaned contexts that bear a high degree of resemblance to authentic everyday real-world forms of participation and practice. Since the majority of the contexts in the examinations prioritise engagement with contrived scenarios, this immediately indicates subordination of life-preparedness principles to a heightened prioritising of mathematical structures, routines, knowledge and forms of participation. By consequence, this brings into question whether the internal component of the language of description is able to provide a valid assessment of the structure of knowledge and criteria for legitimate communication and participation in the examinations. Namely, the language of description for the knowledge domain of mathematical literacy was developed as a means for describing the structure of knowledge in practices (e.g. in the subject-matter domain of Mathematical Literacy) where participation is not legitimised according to mathematical structures. Consider if the examinations primarily characterise a particular domain of mathematical practice (hereby falling within Dowling's (1998) domains of mathematical practice schematic) and, so, legitimise participation according to mathematical knowledge, structures and communicative artefacts. Then, the examinations are effectively located outside of the scope and reach of the internal language and cannot be validly analysed, interpreted or described through the lens of this language. The consequence of this is two-fold. Firstly, any attempted characterisation of the examinations as prioritising a life-preparedness orientation is negated. Secondly, the examinations – through the promotion of a particular mathematised form of the relationship between mathematics and extra-mathematical practices (namely, practices associated with the Public and Expressive Domains of mathematics) – serve as an instrument for the forms of mythologising as identified by Dowling (1998). This second consequence is discussed in more detail on page 419 below.

Despite my assertion regarding the unsuitability of the internal component of the language of description as an instrument for analysis of the examinations, I will continue to demonstrate analysis of the questions in the examinations that assess contents on the topic of Probability. This will facilitate comparison of the structure of legitimate participation in the examinations in relation to the other textual resources analysed.

25.2.2 <u>Micro-level analysis: characteristics of specific segments of text in the examinations that reflect the contents of the topic of Probability</u>

Having considered how the general characteristics of the structure, contents and approach of the examinations index a prioritising of mathematical routines and mathematically endorsed narratives, analysis of the specific questions in the examinations that index contents on the topic of Probability provides further evidence of this dominant domain of prioritising.

The specific questions in the Paper 1 (3.1.7, 5.2.2 (f) & (g)) and Paper 2 (4.3.1 and 4.3.2) examinations that assess the contents/contexts/applications for the curriculum topic of Probability are given in Appendix C (c.f. page 491 below), together with memorandums for these questions. Reference to specific questions is made according to the numbering of the questions as shown in the question extracts on Appendix C.

To begin with, it is worth noting upfront that the examinations only include very limited emphasis on probability-related contents, with only five questions − comprising a total of 13 marks (out of a possible 300 marks, thus equalling 4,3% of the total marks) − included across both examination papers (6 marks in Paper 1 and 7 marks in Paper 2).[277] This very limited emphasis on probability-related contents has significant implications for the extent to which analysis of the dominant domain of prioritising in these questions can be used to provide legitimate commentary about the examinations as a whole. As such, to generate a more valid interpretation of the dominant structure of the routines and knowledge according to which generated narratives in the examinations are endorsed, in the discussion presented below I demonstrate categorisation of both the questions relating to probability contents in relation to the domains of practice of the knowledge domain of mathematical literacy. I repeat this same process for all of the other questions in the examinations.

Before doing this, however, consider that the all of the contexts employed in the probability-related questions are either contrived (as in Paper 2: Question 4.3, where learners are asked to consider the types of probability calculations Ms Springbok might perform while running her tuck-shop) or are 'forced'. This is in the sense that, although the contexts might reflect an aspect of reality, the contexts are not characterised by real-world relevant or appropriate probability-related considerations. For example, in Paper 1: Question 3.1.7, and particularly Paper 1: 5.2.2 (f) and (g), learners are asked to perform probability calculations relating to money spent at the tuck-shop – no, not Ms Springbok's tuck shop, another one – which is not a context in everyday life that is commonly associated with prediction and likelihood. The consequence is that in none of the probability-related questions are learners apprenticed into engagement with the contexts (through the use of relevant probability contents) in a way that tests their understanding of the context or exposes the structure of authentic participation in that context. In short, an orientation for contextual sense-making of the contexts under investigation is entirely absent; in fact, investigation of the contexts is also entirely absent, with the contexts seemingly only providing 'window-dressing' for contextualised forms of mathematical calculation and expression.

This non-engagement with contextual sense-making practices is further reinforced through the structure of the questions and the routines that are indexed in and through the signifiers included in those questions. Almost all questions index the requirement for learners to employ a routine involving writing down a probability value in an appropriate mathematically endorsed format; and this requirement is indexed through the deliberate inclusion of instructions (in the form of combinations of vocabulary/word signifiers) such as: "what is the probability" (Paper 1: DBE, 2014b, p. 8 & 13, Questions 3.1.7 & 5.2.2 (f)); "Express the likelihood ..." (Paper 1: DBE, 2014b, p. 13, Question 5.2.2 (g)); and "Write, in simplified form, the probability ..." (Paper 2: DBE, 2014d, p. 9, Question 4.3.2). This requirement is further explicitly reflected in the structure of the officially endorsed narratives-as-solutions presented in the memorandums, where every narrative-as-solution is represented entirely and exclusively as an expressed probability value in the mathematical format of a percentage, decimal or fraction (DBE, 2014c, p. 7 & 11; DBE, 2014e, p. 10). In all instances, mark allocations in the memorandum reflect an explicit requirement for the appropriate structure of the probability notation

[277] Interestingly, this is below the minimum required mark weighting of 5% for this topic specified in the CAPS curriculum document (c.f. DBE, 2011a, p. 13).

(e.g. "1A number of events [with the mark indicator positioned at the numerator of the probability fraction]; 1A number of outcomes [with the mark indicator positioned at the denominator of the probability fraction]" (see, for example, (Paper 2 Memo: DBE, 2014e, p. 11, Questions 5.2.2 (f) & (g))). And, in only one question (Paper 2: 4.3.1) is there some association made between probability and prediction; but, even in this instance, the endorsed narrative-as-solution in the memorandum is characterised by an explicit expectation for engagement with a specific proportion-based calculation routine as the basis of legitimate participation in the question (Paper 2 Memo: DBE, 2014e, p. 10). Interestingly, the memorandum for Question 4.3.3 specifies a probability routine as an element of the endorsed narrative-as-process that gives rise to the official narrative-as-solution (Paper 2 Memo: DBE, 2014e, p. 11) (see the 'Explanation' column in the memorandum for Question 4.3.3 shown in Figure 68 on page 495 in Appendix C for reference to an expectation for the probability routine). However, this specification is incorrect, since the elements of the narrative-as-process that are explicated as reflecting a probability-related routine are, in fact, nothing more than an expression of a ratio of two quantities (and where this ratio has nothing to do with likelihood or prediction). For this narrative, then, there is the possibility for incoherence between the examiners' (incorrect) understanding of the required or validated routine and the learners' understanding of that same routine. Although, in this instance this incoherence will not affect the legitimacy of the final generated narrative, but may affect how teachers who use the memorandum for future pedagogic purposes come to understand and engage with such ratio-related calculations).[278]

From the discussion above, it becomes possible to associate that successful and endorsed participation with the examination questions on the topic of Probability requires utilisation of knowledge and routines and the generation of narratives based almost exclusively in mathematical terrain contents – specifically, with forms of participation associated with the Public, Expressive and Descriptive Domains of mathematical type practices identified by Dowling (1998). Importantly, this form of engagement not only entirely dismisses Everyday domain and Modelling domain considerations, but is also inconsistent with the structure of knowledge, participation and communication associated with the Mathematical Competency domain of practice. This is because participation in this latter domain of practice is characterised and underwritten by engagement with primarily mathematical components of *authentic and realistic contextual situations*. Significantly, this exclusive emphasis in the examinations on mathematical forms of knowledge, participation and communication in pseudo-realistic contexts signifies that the examination questions reflect contents and/or applications only loosely associated with three statements (out of eleven possible statements) in the CAPS curriculum document. And, given that the probability-related statements in the curriculum reflect a dominant prioritising of practices and associated forms of knowledge, participation and

[278] For means of mere comic interlude – and for no other intellectual reason whatsoever – it is, perhaps, also worth noticing that a specific mark is allocated in the final step of the solution for Question 4.3.3 for rounding of the answer to a whole number (because, after all, you cannot deal with a decimal portion of a person – even if the person is seriously tall). But wasn't there an instruction on the front cover for all answers to be rounded off to two decimal places unless otherwise stated (and there was no such otherwise stating of anything in the question)? Moreover, what is even more confusing (comical?) is that it does not matter if the answer is rounded up or down – both forms are endorsed. So, there seems to be some sort of expectation for learners to engage with the fact that rounding must take place here because the answer involves numbers of people and this context cannot involve decimals. But, this expectation is not really taken too seriously because it is ok to either add an extra person or take one away without considering the significance of this on the constraints of the contextual environment under investigation. And, nor is it considered in any way problematic that the endorsability criteria in the question is in contradiction with a previously stated endorsability criteria. Haaibo!

communication characteristic of the Everyday domain, there is complete divergence between the dominant domain of prioritising of the author(s) of the curriculum and the examinations. The examinations, thus, only assess a small component of the stipulated curriculum and in no way engage learners with the interpretative-type philosophy or intention that is explicitly stated and prioritised in the curriculum. Furthermore, it is also significant that none of the contexts listed in the CAPS curriculum as suitable contextual environments for investigation of probability-related contents are reflected in the examinations. Again, this illustrates complete divergence of the dominant domain of prioritising between these two texts. As with Everyday domain practices, Modelling components are also entirely absent in the examinations. This is, perhaps, not surprising given the absence of modelling components in the curriculum statements that reflect probability-related contents. However, it is somewhat amusing that although the curriculum document does explicitly prioritise modelling components in all suggested *assessment* tasks for the topic of probability, the examinations – which constitute the official national assessment task for the subject at Grade 12 level − do not include this same expectation.

Of course, it could be argued that this dominant world-view of the structure of legitimate knowledge, participation and communication in the examinations is only characteristic of the questions that deal with probability contents. And, since these questions constitute only a small minority of the total questions and possible marks for the examinations, it is impossible to infer anything from analysis of these questions about the dominant domain of prioritising for other questions (based on other sections or topics) or for the examination as a whole. However, further analysis of all of the questions in the examinations highlights a similar trend throughout – as summarised in Table 13 below.

Table 13: Classification of the questions in the Grade 12 Mathematical Literacy exemplar examinations according to the components of the developed internal language of description

Paper 1						
	Components of the Internal Language of Description					Domains of Mathematical Practice
	Everyday (E)	E & MC	E or MC	Mathematical Competency (MC)	Modelling	
Count	8	3	4	11	0	32
% Count	13,8%	5,2%	6,9%	19,0%	0,0%	55,2%
Total Marks	19	10	8	25	0	88
% of Marks	12,7%	6,7%	5,3%	16,7%	0,0%	58,7%

Paper 2						
	Components of the Internal Language of Description					Domains of Mathematical Practice
	Everyday (E)	E & MC	E or MC	Mathematical Competency (MC)	Modelling	
Count	8	3	0	4	0	25
% Count	20,0%	7,5%	0,0%	10,0%	0,0%	62,5%
Total Marks	24	7	0	15	0	104
% of Marks	16,0%	4,7%	0,0%	10,0%	0,0%	69,3%

Both Papers Combined						
	Components of the Internal Language of Description					Domains of Mathematical Practice
	Everyday (E)	E & MC	E or MC	Mathematical Competency (MC)	Modelling	
Count	16	6	4	15	0	57
% Count	16,3%	6,1%	4,1%	15,3%	0,0%	58,2%
Total Marks	43	17	8	40	0	192
% of Marks	14,3%	5,7%	2,7%	13,3%	0,0%	64,0%

Notice that in the table I have indicated the count of questions and marks associated not only with the domains of practice identified domains of practice of the knowledge domain of mathematical literacy but also those associated with domains of mathematical practice (as identified by Dowling (1998)). It is only possible to do this for the examinations due to the utilisation of, primarily, contrived contexts in the examinations and the explicit emphasis on mathematically forms of engagement with those contexts.

From the information in the table it is immediately observable that characteristics of knowledge and communication associated with mathematically legitimised forms of participation (such as those that characterise the domains of mathematical practice identified by Dowling (1998)) dominate the examinations questions.[279] And, although some opportunity is given to consideration of practices characteristic of the Everyday and

[279] In North and Christiansen (Forthcoming - 2015) we employ Dowling's (1998) domain of mathematical practices schematic (illustrated in Figure 20 on page 139 above) to demonstrate the predominance of Public and Expressive (and, to a lesser extent, Descriptive) Domain of mathematics type practices in these examinations papers.

Mathematical Competency domains, there are no questions in the examinations that provide the opportunity for engagement with Modelling components. For the examiners – both in the questions set on probability contents and for the examination as a whole – participation in the subject-matter domain of Mathematical Literacy is legitimised primarily and almost exclusively according to the mathematical competencies of the learners and according to their ability to generate mathematically structured and mathematically endorsed narratives. The contexts, understanding of those contexts, understanding of the realistic ways in which mathematics has application in those contexts, and understanding of the ways in which people participate in the contexts on a daily basis, are all, seemingly, irrelevant in the examination process − other than as a means for contextualising mathematical structures.

Crucially, *none* of the questions in the examinations embody an expectation for learners to demonstrate knowledge, practices and forms of communication associated with the Modelling domain of practice. And, given that this is a central tenet of the progression structure in the CAPS curriculum, which, in turn, signifies an explicit expectation on the part of the curriculum author(s) for Grade 12 learners in particular to develop and demonstrate modelling-related competencies, the complete absence of Modelling domain practices in the examinations signifies significant incoherence between the intention of the author(s) of the curriculum and the authors of the official (and widely publicised) assessment structure.

As regards the domain of Reasoning and Reflection of the internal language, the dominant emphasis in the examinations on the legitimisation of participation according to the demonstration of mathematical competency ensures a requirement for varying levels of reflection on mathematical elements in particular in almost all questions in both examinations. Some degree of reasoning on contextual elements is required to facilitate engagement with the given contextual environments[280]; however, since the majority of the contextual environments are contrived and do not reflect a high degree of representation of authentic practice, it is reflection on mathematical elements that is mostly necessary to facilitate identification of the required mathematical routines to be used in the generation of appropriate and endorsable narratives. The form of this mathematical reflection involves identifying mathematical elements and appropriate mathematical routines through signifiers in the given questions (R/R Levels 1 and 3), and reflecting on how best to employ those routines to ensure accuracy of working and the generation of narratives that comprise appropriate mathematical communicative symbols and notation (for example, ensuring that all answers are rounded off to two decimal places) (R/R Level 4). Reflecting on the suitability of the generated solution-as-narrative in relation to the features of the contextual environment under investigation (R/R Level 5) is not highly prioritised since the dominant emphasis on mathematised forms of

[280] In fact, the only noticeable attempt to engage learners in any form of direct reasoning specifically on contextual elements appears in Question 1.4 (DBE, 2014d, p. 4) in the Paper 2 examination, where learners are asked to give a non-financial or non-calculation based reason as to why a particular venue might be chosen to host a dinner and dance event. The question seems to draw attention to the presence of non-mathematical qualitative factors that influence decision-making processes in real-world practices, and there is an explicit expectation in the CAPS curriculum for such factors to be engaged with in the subject (c.f. DBE, 2011a, pp. 8-9). However, the provided official endorsed narratives trivialise this attempt somewhat by providing endorsed narratives such as “They will not be responsible to tidy up the venue.” (DBE, 2014e, p. 4). The question and associated endorsed narratives do not engage learners in any form of complex or intricate reasoning about the sorts of factors that might actually influence decision-making processes regarding venue selection in authentic daily-life practice. Instead, the expectation is merely that the participants must deduce from the given text and from references to particular contextual characteristics made in the text which venue the examiners have deliberately constructed as the most suitable choice.

participation embodies an expectation that mathematical and/or calculated solutions represent the endorsed and favourable form of the narrative-as-solution. It is also noticeable that there are no questions that comprise an expectation for any form of critical reflection (R/R Levels 6 and 7) on the consequences of the generation of mathematically structured narratives in the solving of contextualised problems or, in fact, on whether mathematical narratives offer suitable and appropriate representations of existing forms of communication and participation in the given contextual environments. Instead, the examinations seemingly comprise an underlying assumption that it is appropriate to generate mathematical descriptions and narratives for any context, irrespective of the context, and also irrespective of whether mathematics is actually employed in that context in reality. This issue is particularly evident in the questions in the examinations that refer directly to probability-related contents. In these questions, there is no expectation whatsoever for learners to engage with any form of contextual reasoning or with any form of critical analysis of the generated narratives or of the suitability of those narratives for reflecting authentic practice in the given context. In fact, had some form of critical reasoning been encouraged, then it would have been immediately possible for learners to question the validity of employing probability-related narratives in the given contextual environments, since these environments are clearly not representative of authentic situations in daily-life practice in which understanding of probability-related contents facilitates successful and endorsed participation. Furthermore, there is also only minimal mathematical reflection required in the probability-related questions, particularly since almost all of the questions include signifiers that directly index the need for probability-related routines. In fact, the main (only?) form of reflection on mathematical elements required in these questions involves deciding which values are to be included in the expressed probability ratios. All in all, only minimal utilisation of the processes associated with the domain of Reasoning and Reflection is required for successful engagement in the probability-related questions.

As has been argued previously, the envisioned life-preparedness orientation for Mathematical Literacy is facilitated through deliberate recognition of and attempted movement through the various components that comprise the knowledge domain of mathematical literacy. This movement is accompanied by recognition of how people function in daily-life, how people might function if they were to employ mathematics in daily-life practices, and how daily-life practices are able to be reconstructed through modelling processes to facilitate understanding of alternative possible forms of participation in the practice. This process does not occur adequately either in the questions that deal with probability contents or in the examination as a whole. When viewed through the lens of the developed language of description, this suggests that the current structure of the examinations does not engender engagement with the contents of the subject (and, by association, with the knowledge domain of mathematical literacy) or with the real-world contexts specified in the curriculum in a way that facilitates the development of a life-preparedness orientation.

This point then begs the question that if the examinations do not facilitate a life-preparedness orientation, what is facilitated through the examinations? Is it, instead, mathematical learning and development? Reflecting back on the contents of the discussion in Part 3 of this study and as mentioned above, I contend that the dominant form and structure of the questions in the examinations reflect practices associated, primarily, with Public and Expressive Domains of mathematical practices (Dowling, 1998). And, as has already been argued, near exclusive emphasis on such mathematics practices without explicit and deliberate attempted movement to Esoteric Domain contents or practices, results in the positioning of participants as dependents and objects in the learning process (rather than as apprentices of the domain of scientific mathematics). This process also indoctrinates participants in various forms of mythologising (particularly the myth of Participation regarding the necessity of mathematics for enhancing real-world participation). Furthermore, the process ensures the sustainment and reproduction of social or educational disadvantage. This is because the form of mathematics that characterises the examinations, and which only affords limited study and career opportunity, is most commonly made available to learners of 'weaker' mathematical ability who stem predominantly from poorer and less resources schools and working-class or sub-stratum family backgrounds. The language of description of the knowledge domain of mathematical literacy was developed in a direct attempt to counter the power of this mythologising by redefining the structure of knowledge in practices associated with this knowledge domain (specifically in the site of the subject-matter domain of Mathematical Literacy) through the prioritisation of a life-preparedness orientation. The absence of this life-preparedness orientation in the examinations and the prioritising, instead, of Public and Expressive Domain mathematics practices, thus, render the examinations fallible to the forms of mythologising identified by Dowling and position the examinations as instruments of educational disadvantage. It is of little wonder, then, why people like Jansen (2012) are so critical of the examinations and continuously and ardently discourage participation in the subject (c.f. page 1 above).

This, then, brings to an end the analysis of the examination papers. In the next chapter (Chapter 26), discussion shifts to analysis of a Grade 12 Mathematical Literacy textbook and, particularly, to comparison of the dominant domain of prioritising in that textbook in relation to the examinations and CAPS curriculum.

CHAPTER 26
ANALYSIS OF EMPIRICAL TEXTUAL RESOURCE #3: A GRADE 12 MATHEMATICAL LITERACY TEXTBOOK

26.1 Background information

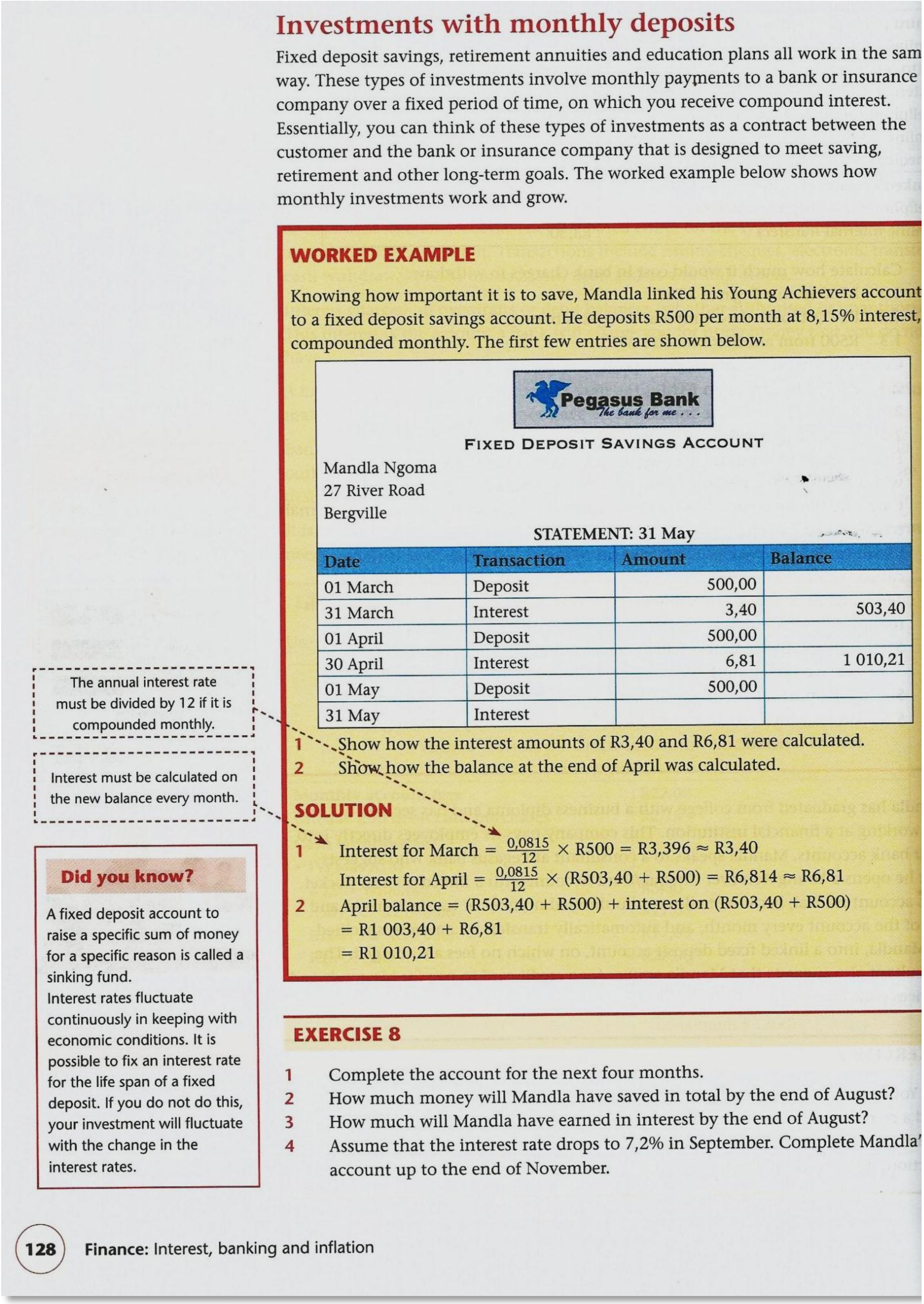
The Grade 12 Mathematical Literacy textbook selected for analysis is published by *Maskew Miller Longman* educational publishers and comprises both a learner's book (Frith et al., 2013a) and a teacher's guide (Frith et al., 2013b). The *learner's book* is structured according to the school terms (Terms 1 to 4)[281], with each term divided into chapters and with each chapter including content drawn from particular CAPS curriculum topics and sections (c.f. Frith et al., 2013a, pp. 2-3). The chapter headings largely reflect the section headings given in the CAPS document. A 'revision test' is provided at the end of every chapter, together with a variety of assessment items (including assignments, investigations, and exemplar or practice examination papers) at the end of every term. The form and structure of these assessment items directly reflects the stipulated assessment requirements as given in the CAPS curriculum (c.f. Frith et al., 2013a, pp. 2-3). Importantly, all of the required curriculum content for Grade 12 is covered in the pages allocated for Terms 1 to 3 in the textbook. The pages for Term 4, on the other hand, are dedicated entirely to preparation for the final national examinations through provision of exemplar practice examinations (c.f. Frith et al., 2013a, pp. 2-3). This characteristic is consistent with the regulated scheme of work provided in the CAPS curriculum (DBE, 2011a, p. 19).

Each chapter in the learner's book includes focus on various contextual environments and focal events in those environments as stated in the curriculum, together with explanations of content concepts relevant to those contextual environments or to problems encountered in those environments. These explanations generally include various mediums of communication, including words or descriptions, pictures (e.g. photographs) and diagrams (e.g. graphs and tables). Sometimes the explanations include discussions of the problem scenarios or the contexts and/or worked examples of particular formats of methodology (e.g. calculation or other solution strategies) for solving problems. Exercises containing practice questions are commonly included after explanations to consolidate and extend concepts dealt with in the 'teaching' components of each section. These exercises, similarly, include various mediums of communication. Solutions for the exercises are provided at the back of the learner's book; however these

[281] Note that this structuring of the textbook according to 'terms' rather than according to context or contents topics immediately positions the textbook both as a source of content and pedagogy, and as an instrument of regulation (i.e. framing – (c.f. Bernstein, 1975)), particularly of the sequencing of content. In other words, teachers who use this textbook are given no opportunity to decide on the most appropriate sequencing of the content; instead, they are encouraged and instructed to work through the book in a linear fashion, starting at the beginning and working towards the end. Importantly, this issue is not specific to this textbook. Rather, during the official (i.e. state conducted) textbook screening process, a key 'compliance' requirement was for the textbooks to reflect the sequence of working as stipulated in a scheme of work supplied in the CAPS curriculum, and, as such, all textbooks selected for inclusion on the National Catalogue follow the same sequencing of content. The CAPS curriculum process has, thus, served to significantly weaken the internal framing (for the teacher) of pedagogic practice in the Mathematical Literacy classroom, at least with respect to sequencing of content. And, since the 'suggested' scheme of work in the CAPS curriculum also includes indication of suggested time allocations for each section of teaching, the curriculum, seemingly, has also weakened control (for the teacher) over the pacing of the pedagogic process.

solutions comprise mainly answers and not full worked solutions. An extract showing the structure of a typical page in the learner's book is given in Figure 58 (on page 423) below.

The *teacher's guide*, by contrast, includes full worked solutions for all exercises and assessment items provided in the learner's book, together with a degree of commentary and advice regarding lesson preparation, lesson structure and assessment guidance for each section (see, for example, (Frith et al., 2013b, p. 14)). A variety of additional assessment and consolidation resources (tests, examinations, and consolidation exercises) are also included at the back of the teacher's guide (c.f. Frith et al., 2013b, pp. 219-277). An extract showing the structure of a typical page in the teacher's book is given in Figure 59 below (c.f. page 424).



Investments with monthly deposits

Fixed deposit savings, retirement annuities and education plans all work in the sam way. These types of investments involve monthly payments to a bank or insurance company over a fixed period of time, on which you receive compound interest. Essentially, you can think of these types of investments as a contract between the customer and the bank or insurance company that is designed to meet saving, retirement and other long-term goals. The worked example below shows how monthly investments work and grow.

WORKED EXAMPLE

Knowing how important it is to save, Mandla linked his Young Achievers account to a fixed deposit savings account. He deposits R500 per month at 8,15% interest, compounded monthly. The first few entries are shown below.

Pegasus Bank
The bank for me . . .

FIXED DEPOSIT SAVINGS ACCOUNT

Mandla Ngoma
27 River Road
Bergville

STATEMENT: 31 May

Date	Transaction	Amount	Balance
01 March	Deposit	500,00	
31 March	Interest	3,40	503,40
01 April	Deposit	500,00	
30 April	Interest	6,81	1 010,21
01 May	Deposit	500,00	
31 May	Interest		

The annual interest rate must be divided by 12 if it is compounded monthly.

Interest must be calculated on the new balance every month.

1 Show how the interest amounts of R3,40 and R6,81 were calculated.
2 Show how the balance at the end of April was calculated.

SOLUTION

1 Interest for March $= \frac{0,0815}{12} \times \text{R}500 = \text{R}3,396 \approx \text{R}3,40$

Interest for April $= \frac{0,0815}{12} \times (\text{R}503,40 + \text{R}500) = \text{R}6,814 \approx \text{R}6,81$

2 April balance $= (\text{R}503,40 + \text{R}500) + \text{interest on } (\text{R}503,40 + \text{R}500)$

$= \text{R}1\,003,40 + \text{R}6,81$

$= \text{R}1\,010,21$

Did you know?

A fixed deposit account to raise a specific sum of money for a specific reason is called a sinking fund.
Interest rates fluctuate continuously in keeping with economic conditions. It is possible to fix an interest rate for the life span of a fixed deposit. If you do not do this, your investment will fluctuate with the change in the interest rates.

EXERCISE 8

1 Complete the account for the next four months.
2 How much money will Mandla have saved in total by the end of August?
3 How much will Mandla have earned in interest by the end of August?
4 Assume that the interest rate drops to 7,2% in September. Complete Mandla' account up to the end of November.

128 **Finance:** Interest, banking and inflation

Figure 58: Extract showing the structure of a typical page in the Grade 12 Mathematical Literacy *learner's textbook* (Frith et al., 2013a, p. 128)

Unit 3 Inflation

Resources: A basic calculator

Teaching guidelines

Time allocation: 2 hours

Key concepts that the learners should understand by the end of this unit include the following:

Inflation is a continuous, average, annual increase in the price of goods and services in an economy over a period of time, expressed as a percentage.

The most well-known types of inflation measures are the CPI, which measures consumer prices, and the GDP deflator, which measures inflation in the entire domestic economy. However, financial indicators like the CPI and the GDP will not be examined, but can be studied.

By the end of this unit, learners must be able to:

investigate the changes in the prices of goods and services
work with and calculate rates of inflation
investigate the effect of inflation on the purchasing power of money and the value of goods and services over time.

New to Grade 12 in this unit is interpreting and analysing graphs showing changes in the inflation rate over time and understanding that a decreasing graph does not necessarily indicate negative inflation (deflation) in price.

Also new to Grade 12 in this unit is the evaluation of situations involving proposed price increases (salary negotiations, fee increases, and so on).

STEP-BY-STEP SOLUTIONS **Exercise 16**

1 Average inflation rate = $\frac{(3{,}7 + 3{,}7 + 4{,}1 + 4{,}2 + 4{,}6 + 5 + 5{,}3 + 5{,}3 + 5{,}7 + 6 + 6{,}1 + 6{,}1)}{12}$ = 4,98%

2 New price = R189 000 × 1,0498 = R198 412,20

3 R189 000 × $\frac{100}{104{,}29}$ = R181 225,43

4 Rate of inflation for February = 3,7% p.a.. Therefore, the inflation rate per month = $\frac{3{,}7}{12}$.

Price at the end of February (i.e. beginning of March) = R189 000(1 + $\frac{0{,}037}{12}$) = R189 582,75

5 Price at the end of March = R189 582,75(1 + $\frac{0{,}041}{12}$) = R190 230,49

6 Price in 1 year's time = R189 000 × 1,0498 = R198 412,20

Price in 2 years' time = R198 421,20 × 1,0498 = R208 293,13

Price in 3 years' time = R208 302,58 × 1,0498 = R218 666,13

Price in 4 years' time = R218 676,05 × 1,0498 = R229 555,70

Price in 5 years' time = R229 566,12 × 1,0498 = R240 987,58

7 Learners' answers will vary. Encourage discussion. Here are some pointers:

All prices do not increase at the same percentage. The price of food stuffs, and so on, are quick to change because of increases in fuel, droughts, and so on. Food prices and oil prices can change quickly due to changes in supply and demand in the food and oil markets. It is possible for some prices to decrease. Some electronic goods, like computers and digital cameras, actually decrease in price as the supply increases or as newer technology appears.

Figure 59: Extract showing the structure of a typical page in the Grade 12 Mathematical Literacy *teacher's guide* (Frith et al., 2013b, p. 77)

Importantly, there is <u>*no*</u> general commentary provided in either the learner's book or the teacher's guide regarding the author(s) interpretation of the overarching philosophical intention or orientation for the subject-matter domain of Mathematical Literacy, of the underlying methodological approach adopted in the textbook, or of suggested possible alternative methodological and/or pedagogic approaches for pedagogic practices with the contents of the textbook in the subject. In this sense, there is no explicit statement provided by the authors of their 'world-view' with respect to the structure of legitimate practice, knowledge, communication and participation in the subject-matter domain of Mathematical Literacy as a whole. However, specifically for the chapter on Probability, the textbook authors do provide some commentary (in the teacher's guide only – (c.f. Frith et al., 2013b, pp. 137-145)) on their interpretation of the methodological approach preferenced in the curriculum for this topic. As such, inference of the world-view of the authors is achieved through subjective analysis of these statements of intention, together with analysis of the ways in which these statements of intention are enacted in the contents in the learner's book and endorsed through the narratives-as-solutions provided in the teacher's guide.

This textbook is one of the eight textbooks for the subject-matter domain of Mathematical Literacy selected for inclusion on the national catalogue of textbooks. This catalogue is distributed to all public schools in the country and the schools are required to select books shown on the catalogue. The selection of eight textbook titles for inclusion on the catalogue occurred after a rigorous screening process in which the textbooks were evaluated by officials acting on behalf of the National Department of Basic Education. This screening process evaluated, amongst other things, comprehensive correlation of content to the CAPS curriculum and adherence to the pedagogic approach espoused in that curriculum (for example, the requirement for the inclusion of authentic contexts and a focus on problem-solving in those contexts). The inclusion of this textbook on the national catalogue and, by implication, validation of the textbook in the screening process, is important. This signifies a supposed high degree of correlation between the textbook and the CAPS curriculum with respect: to content coverage, methodology and approach; the structuring of knowledge and the criteria for successful access to and engagement with this knowledge; and the way in which the relationship between mathematical knowledge and the extra-mathematical world is presented and engaged. A key intention of the analysis of the textbook through the lens of language of description is to determine whether these various levels of correlation do, in fact, occur.

Analysis of the textbook is focused exclusively on the components of the book that deal with the curricular topic of Probability. Analysis in this regard includes focus on explanations (and associated descriptions, pictures and diagrams) and consolidation exercises in the learner's book, together with any assessment items based on this section. Specific questions in additional tests or examinations in the learner's book and teacher's guide are also scrutinised. Corresponding worked solutions and commentary supplied in the teacher's guide for all probability-related contents in the learner's book and for additional assessment items in the teacher's guide are also scrutinised.

26.2 Analysis: findings and discussion

26.2.1 Macro-level analysis: general characteristics of textbook structure and approach

In contrast to the examinations, initial contact with the learner's book immediately generates the impression that for the authors of this textbook participation in the subject-matter domain of Mathematical Literacy involves engagement with authentic real-world practices. This impression is achieved through the inclusion of a plethora of contextual resources and visual mediators (photographs, pictures, maps, plans, scanned resources, graphs, and tables – all in full colour that directly index real-world practice and situations. The vast majority of these visual mediators characterise 'cleaned' resources (with only a very small number of instances in which actual documents − e.g. bills, statements, newspaper articles, etc. − have been scanned and inserted). However, most of these cleaned resources reflect a relatively high degree of representivity of real-world practice and serve to generate the experience that engagement with the resources is akin to engagement in authentic practice.

The strong link to reality is further reinforced through boxes of text placed in the margin (an example of which is illustrated in Figure 60 alongside) which provide specific additional information, particularly about contextual elements of the scenario under investigation. For me, the inclusion of a segment of text like that shown alongside immediately draws attention to certain elements of complexity that may be encountered when performing calculations in reality and of how this complexity is currently managed in that reality.

Figure 60: Illustration of the textboxes commonly found in the margins of the textbook

> **Note:**
> When calculating the amount of tax that is owed, small differences might be found between manual tables and computer based programs based on the statutory rates of tax. Both methods are acceptable in terms of the Income Tax Act, as long as the results are within the rules provided for in the Act.
> (Frith, Jakins, Winfield, & Yeo, 2013a, p. 219)

The inclusion of these additional statements of explanatory text signifies that for the authors of the textbook it is important that learners come to understand the complexity and intricate nature of calculations performed in real-world environments, and, further, that the ability to calculate is not sufficient for effective and successful engagement in the world. For me, this provides direct evidence of the prioritisation of a life-preparedness orientation for the subject.

Despite this seeming promotion of a life-preparedness orientation, there are characteristics of the text that contradict this intention. In particular, and as with the examinations, the authors of the textbook employ the strategy of referencing fictitious people in the text[282], sometimes even including photographs of these people (although I seriously doubt whether the people shown in the photographs are actually the persons referenced in the text) (see, for example, (Frith et al., 2013a, pp. 48, 49, 51, 118, 121, 133, 219 & 244)). This strategy seemingly reflects an attempt to make the situations under investigation appear more real and authentic. For me, the opposite is achieved: if learners begin to believe that the names are imaginary, then they might also start to believe that the contexts and contents under investigation are equally fictitious, and that participation in the subject is concerned with the reconstructed or imagined world and not the real-world. And, this serves to undermine an intention for a life-preparedness orientation. Importantly, and as discussed previously, some degree of recontextualisation of the contextual environment is always required to facilitate engagement with the environment in a pedagogic process. However, the type of recontextualisation employed in the textbook – through the introduction of fictitious characters and images – serves to de-authenticate the contexts, making them seem less plausible and less real.

In terms of the dominant domain of prioritising indexed through the deliberate inclusion of specific signifiers in the text, although there are instances of text that tap into a form of contextual understanding and reasoning[283], these instances are significantly

[282] Including, amongst many many others, Mandla (who is making provision for his future retirement), Ashwin (who runs a Spaza shop), Nadia (who is baking a chocolate dessert), Bongiwe (who is having a house built), and Mr. and Mrs. Bezuidenhout (who recently got married).

[283] For example:

- "3. Why do you think it is necessary to calculate [interest] to six decimal places?" (Frith et al., 2013a, p. 120) outweighed by an emphasis on mathematical structures and calculation-based routines for the generation of narratives. For example, cursory analysis of the sections on Loans and Maps[284] (Frith et al., 2013a, pp. 128-137 & 146-165) reveals a dominance of focus on calculation-based solutions to problem scenarios and only minimal consideration of how people actually engage in these scenarios. This analysis also reveals the presence of significant non-mathematical influences which may direct thought and behaviour in these scenarios, the forms of contextual reasoning that people engage with in as they make decisions and communicate in these scenarios, or of how calculation-based narratives could be used by participants in the contexts to inform and adapt their behaviour. Furthermore, it is also significant that cursory analysis of the contents page of the learner's book and also of the text throughout the book does not reveal an explicit and significant prioritisation of modelling components or of opportunities for independent and elaborated problem-solving. This is an explicit specification in the CAPS curriculum, particularly for Grade 12 learners.[285] For me, the discussion above suggests that for the author(s) of these sections, legitimate participation in the investigated contexts is characterised primarily by the ability to perform calculations to describe and solve problems and by the ability to reflect mathematically (namely, through demonstration of Mathematical Competency). Demonstration of the ability to reflect on existing forms of participation in those contexts and on the limitations of using calculation-based narratives to describe these existing forms of participation does not constitute a primary criteria for legitimate participation. However, more detailed analysis of all segments of the text in the textbook would be necessary to validate this claim.

In summary, macro-level analysis of the learner's book reveals an explicit emphasis on engagement with contextual situations that bear a high degree of resemblance to real-world practice. However, there are also elements in the text that signify a possible prioritising of calculation-based routines and mathematically legitimised generated narratives in engagement with these contextual situations. This issue is now investigated in more detail through analysis of the contents in the textbook that deal specifically with probability-related concepts.

- "3. 'Rewards' are often offered on credit cards. But do not be fooled – you always end up paying for them in one way or another. Investigate the costs involved in earning credit rewards from three different banks." (Frith et al., 2013a, p. 125)
- "3. Create a sample of an annual IRP5 form for Juanita's records." (Frith et al., 2013a, p. 219)
- "3.3 Critique Mr. Mtshali's travel route, commenting on the distance, estimated travel times, traffic lights, and so on, and suggest a possible alternative travel route." (Frith et al., 2013a, p. 299)

[284] I contend that these two sections deal with two contextual situations in which understanding of non-mathematical real-world issues is essential for successful participation in the situations. As such, analysis of these sections provides an immediate opportunity to gauge the extent to which the authors of a text prioritise calculation or non-calculation based routines and considerations in the generation of endorsed narratives about the structure of knowledge and participation in the context under investigation.

[285] Although there are some instances involving modelling processes (albeit very directed modelling) in the content components of the learner's book (see, for example, (Frith et al., 2013a, pp. 190-191)), it is in the context of the provided assessment tasks that the greatest expectation for modelling exists. Specifically, <u>all</u> of the investigations and assignments embody an expectation for learners to engage in problem-solving processes involving the investigation and solving of a problem or the completion of a specific task through utilisation of a variety of contents and applications (c.f. Frith et al., 2013a, pp. 112, 114, 196, 198, 262 & 266). This is significant in that it reflects an expectation on the part of the authors for learners to engage with modelling-type processes in the context of assessment tasks despite the complete absence of explicit preparation for such processes in the course of ordinary teaching. This feature and approach is somewhat consistent with the approach espoused in and through the CAPS curriculum document, where statements of possible assessment tasks also largely reflect an expectation for independent problem-solving and modelling-type processes.

26.2.2 Micro-level analysis: characteristics of specific segments of the textbook that reflect contents of the topic of Probability

Bearing in mind that the CAPS curriculum prioritises an interpretative approach to probability contents and explicitly downplays a calculation-driven pedagogy, a key intention of the analysis process of the probability contents in the textbook is to determine whether the authors of the textbook reflect consistency with this approach in their writings.

To begin with, analysis of Probability chapter (i.e. Chapter 10) in the learner's book (c.f. Frith et al., 2013a, pp. 238-251) through the lens of the Contextual Domain component of the internal language reveals a deliberate attempt on the part of the authors to engage with contextual situations that reflect some degree of authentic and/or realistic practice. To this end, contexts employed for investigation include weather reports, newspaper articles, scenarios involving HIV/Aids, gambling and the national Lottery, and various forms of risk assessments – and, thus, reflect consistency with the types of contexts specified in the CAPS curriculum. However, apart from two small sub-sections that deal with the Lottery and with gambling scratch cards (Frith et al., 2013a, p. 246 & 247), it is my perception that most of the other contextual resources and associated visual mediators have been deliberately constructed – including the newspaper article and various data sources used for exploration of contents relating to HIV/Aids and risk assessments. As such, rather than being confronted with authentic adverts showing, for example, differences in car insurance rates for men and women or with predicted success rates for beauty products or drug tests, instead many of the given scenarios are described by means of words/vocabulary, with relevant statistical and probabilistic information included in these descriptions. Nonetheless, most of the constructed contexts provide suitable contexts for the exploration and investigation of probability-related practices and, so, reflect practices consistent with the Contextual Domain of the knowledge domain of mathematical literacy.[286] However, it is also worth mentioning that despite this attempt

[286] Unfortunately, users of this textbook will not escape the curse of having to investigate (on two occasions!) the probability of drawing out matching underwear (sorry, I meant to say 'socks') from a drawer (Frith et al., 2013a, p. 239 & 251), or of having to use probabilistic tools (i.e. tree diagrams) to predict the gender of a baby (Frith et al., 2013a, p. 290). Yet another generation of learners will exit the schooling system believing that choosing clothes and having babies involves combinatorics ... or, perhaps and possibly more likely, maybe another generation of learners will exit the schooling system thinking how silly their teachers are to try to con then into believing that putting on clothes and having babies is all about probability and combinatorics?
Of course, it could be argued that this is merely a pedagogic move and that the authors of the textbook are employing this form of questioning deliberately as a way of providing learners with initial and oversimplified exposure to probability concepts to facilitate a point of entry into more complicated applications and modelling. In this sense, this type of activity could be construed as a type of 'public domain' activity for mathematical literacy that facilitates access (at a later stage) to the other domains of practice. For me, this position is problematic: exposing learners to contrived and nonsensical scenarios does not facilitate understanding of either contextual or mathematical components. Instead, it generates the impression that mathematics is not useful and that applications of mathematics have to be invented to demonstrate the utility of mathematics. Rather, I contend that it would be more useful for the authors of this textbook to avoid contrived applications and to focus instead on presenting the basic esoteric principles of probability-related contents in an un-contextualised way. By promoting this approach I am conceding that it is possible to consider that certain elements of elementary esoteric mathematics can be constituted as a 'public domain' of mathematical literacy, particularly in the context of schooling and/or pedagogic action. In other words, in order to facilitate complex modelling processes it may be necessary to first expose learners to particular esoteric mathematics principles which will be employed in the modelling. Similarly, it is unrealistic to expect younger learners to engage in complex real-world problem-solving and sense-making if they have never been exposed to some of the mathematically-defined principles which define and structure elements of participation in those contexts. This is a dimension of the developed language of description which requires further research and elaboration and

on the part of the authors to engage with authentic environments in which probability-related practices have relevance, this engagement occurs largely at a superficial level, directed towards understanding general aspects of the contextual environments and of certain forms of probabilistic calculations and considerations in those environments. In-depth engagement with elements of the contexts, with various forms of participation in the contexts, and, particularly, with the use of probability-related contents and applications to inform and enhance participation and decision-making in the contexts commonly does not occur. A possible implication of this is that teachers and learners who engage with the textbook may only develop a superficial understanding of the utility of probabilistic-related contents and actions for informing decision-making and communication practices in particular real-world environments.

Importantly, the issue highlighted immediately above signals a flaw in the developed internal dimension of the language of description, and, particularly, in the components of the Contextual Domain of that language. The domain provides clear specification of the types and structure of the contexts deemed appropriate for investigation in a life-preparedness oriented format of the knowledge domain of mathematical literacy. However, insufficient provision is made for consideration of the level (or depth) of the engagement with those contexts or of the potential of that level of engagement for facilitating comprehensive (or only surface-level) understanding of how certain contents shape, direct and facilitate participation and decision-making in the contexts. Furthermore, the internal language does not account for the difference in the experience of reality – and, hence, the degree of life-preparedness − that results through engagement with cleaned contexts in comparison to real contexts, or with differences in the degree to which contexts are cleaned and the implication of this for the experience of authentic reality that results. Nor does the internal language make provision for deliberate pedagogical moves beyond the domain of the authentic/real-world contextual on the part of the teacher. These limitations of the language of description for the knowledge domain of mathematical literacy are discussed in more detail in Chapter 28 on page 455 below. Acknowledgement of the need for further research into the characteristics of the (reconstituted) real-world contexts that populate the public domain of mathematical literacy is given in the discussion on 'Possibilities for further research' on page 465 below.

Operating at a global-level, cursory analysis of the chapter structure reveals an immediate attempt to reflect the contents of the curriculum in the chapter. Specifically, the different headings in the chapter reflect precisely and exactly the section headings in the CAPS curriculum, and all content and context components specified in the curriculum appear to have been included in some form or another in the chapter. Thus, at the level of structure, at least, there is consistency between the textbook chapter on Probability and the curriculum contents of the same topic.

Continuing at a global-level, but this time in relation to the approach adopted in the textbook chapter, various statements of explanation and guidance in the teacher's guide suggest recognition on the part of the textbook authors of the curriculum specification or requirement for an interpretive pedagogic approach for the contents of this topic. For example:

which has been earmarked as a possible future research objective (see the discussion of 'Possibilities for further research' on page 465 below).

> When studying probability, focus more on interpreting situations that involve probability, rather than on mathematical calculation of probability. (Frith et al., 2013b, p. 137)
>
> ... the focus is not on finding the exact probability of winning the lottery, but rather on the likelihood of the event occurring and clarifying beliefs that are false (such as if I play the same numbers in the lottery every week then I am more likely to win). (Frith et al., 2013b, p. 137)

A commitment to a life-preparedness orientation, with a critical intention, is similarly espoused in the supplied teacher's guide commentary:

> People lose money playing the lottery and other gambling games ... In this unit, learners explore whether it is really worth playing the Lotto or Power Ball. (Frith et al., 2013b, p. 143)
>
> Many people have come up with different 'systems' to increase their chances of winning [when playing the Lottery]. In this unit, learners will critique and analyse these 'systems'. (Frith et al., 2013b, p. 143)

As such, there is a level of consistency between the approach specified for this topic in the curriculum and that specified and demonstrated by the textbook authors – at least at the level of the stated intention of the authors.

However, analysis of individual statements of text in the chapter and in other elements of the textbook that reference probability-related contents reveals a different reality – this time at the level of implemented intention. To begin with, consider the contents of Table 14 on page 431 below which shows a summary of the classification of various statement of text in the chapter according to characteristics associated with the domains of practice of the knowledge domain of mathematical literacy.[287] Note that that the contents of Table 14 only reflect categorisation of the statements of text in the Probability chapter that relate to pedagogic activity (including explanations, worked examples and exercise questions), and does not include categorisation of the questions in any of the provided assessment tasks (e.g. chapter revision text and exemplar examination papers). The categorisation of

[287] By way of explanation of the methodology employed to achieve this summary:

- Classification of the statements in the <u>*exercises*</u> is done on a question-by-question basis, except in Exercises 2 and 4 where the exercises are presented as class activities rather than in question-and-answer format. For these two exercises I analysed the extent to which the class activity involved learners in everyday, mathematical, or modelling practices, and classified the various aspects of the activity accordingly. Also note that classifications of questions in the exercises are done through consideration of both the question and the supplied solution in the teacher's guide.
- Classification of the statements in the <u>*worked examples*</u>: in instances where only a single activity/question/problem characterised the example, then both the description of the example and the statement of solution are treated as a single description and characterised accordingly; in instances where several questions and solutions are posed to investigate different facets of engagement with a problem situation, then each question and associated solution is categorised individually.
- Classification of the statements that comprise the <u>*explanations or descriptions of content and/or context*</u> is done by paragraph. However, there are instances where both words/vocabulary and a visual mediator are used to explain and/or represent the same concept. In such situations, both forms of signifier are treated as a single entity.

Clearly this method of classification has significant limitations, particularly since the methodology does not take into consideration the difference in the lengths of the paragraphs of explanations – which could be taken as an indication of heightened emphasis on particular aspects of knowledge. Nonetheless, for purposes of the analysis provided in this chapter, I contend that this methodology provides an appropriate indication of the dominant way in which participation with the probability contents is legitimised.

questions that reference probability-related contents in assessment tasks is discussed shortly.

Table 14: Classification of the statements in the textbook chapter on Probability (Frith et al., 2013a, pp. 238-249; 2013b, pp. 138-143) according to the components of the developed internal language

Statements that reflect:		Component of the Internal Language: Mathematical Competency	Everyday	E & MC	E or MC	Modelling
Explanations of context/content	Count	5	10	3	0	0
	% of Count	27,7%	55,6%	16,7%	0,0%	0,0%
Worked examples	Count	6	2	10	0	0
	% of Count	33,3%	11,1%	55,6%	0,0%	0,0%
Exercise questions	Count	37	27	15	1	0
	% of Count	46,3%	33,8%	18,8%	1,3%	0,0%

The information in the table reveals an inconsistency between the narratives of explanations, examples and exercises. Specifically, where *explanations* of contexts and contents prioritise Everyday domain descriptions, considerations and forms of participation, the *examples* that illustrate possible ways of working with probability-related contents downplay Everyday components and prioritise, instead, either mathematical forms of working (i.e. Mathematical Competency components) or, primarily, engagement with problem scenarios through consideration of both everyday and mathematical components, routines and narratives. This inconsistency is easily explained through consideration that many of the explanations of contexts and content comprise signifiers which index general narratives of the contextual elements that characterise the contexts under investigation and of the relevance of probability in those contexts. The *worked examples*, on the other hand, provide more focused and directed statements regarding specific routines to be employed to engage with certain types of probability-related scenarios. And, given that a probabilistic description is ultimately a mathematically structured description, it is, thus, appropriate that a reach is made into the terrain of mathematics to demonstrate the workings of certain probability-related routines.

By contrast, what is harder to explain is the further inconsistency between the dominant domain of prioritising in exercise questions and the examples and explanations. As discussed above, some of the provided explanations prioritise a form of everyday engagement and examples that emphasise the necessity of both mathematical and everyday knowledge and routines in generating appropriate and endorsed narratives. However, the exercise questions comprise an expectation for engagement with probability-related contents primarily through mathematical routines. Furthermore, these exercises also prioritise the generation of narratives based on the utilisation of these mathematical routines – and a survey of the structure of many of the solutions for the exercise questions provided in the teacher's guide provides immediate evidence of this[288]. And, although the exercises do prioritise a degree of limited engagement with everyday components and everyday forms of participation[289], this inconsistency remains

[288] See, for example, the structure of all of the solutions associated with Exercise 1 (Frith et al., 2013b, p. 138) or with Questions 1 and 2 of the Revision Test (Frith et al., 2013b, p. 144).
[289] This is evidenced through the relatively common use of descriptive narratives (rather than calculation-based narratives) offered as solutions for exercise-related questions in the teacher's guide. See, for example, the narratives provided as solutions for Exercise 8 (Frith et al., 2013b, p. 143).

significant. Specifically, this inconsistency highlights a contradiction (albeit to a limited degree) between the stated intention of the authors for the prioritisation of an interpretative and everyday-type of engagement with the contents of this topic and the enacted intention which involves prioritisation of mathematically structured forms of engagement. This inconsistency also highlights a contradiction between the explicitly stated or envisioned structure of participation for this topic in the curriculum and the structure of participation legitimised by the textbook authors. Crucially, this trend in the prioritisation of Mathematical Competency components is further evidenced (and to an even greater degree) in all assessment-related questions that reference probability contents – as shown in
Table 15 and Table 16 below.[290]

Table 15: Classification of the questions in the *Revision Test* in the textbook chapter on Probability (Frith et al., 2013a, pp. 250-251; 2013b, pp. 144-145) according to the components of the developed internal language[291]

	Component of the Internal Language				
	Mathematical Competency	**Everyday**	**E & MC**	**E or MC**	**Modelling**
Count	22	10	4	0	0
% of total count	61,1%	27,8%	11,1%	0,0%	0,0%
Marks allocated	58	14	13	0	0
% of total marks	68,2%	16,5%	15,3%	0,0%	0,0%

Table 16: Classification of the probability-related questions in the *assessment tasks* in the learner's book and teacher's guide according to the components of the developed internal language[292]

	Component of the Internal Language				
	Mathematical Competency	**Everyday**	**E & MC**	**E or MC**	**Modelling**
Count	39	12	10	7	0
% of total count	57,4%	17,6%	14,7%	10,3%	0,0%
Marks allocated	105	16	24	15	0
% of total marks	65,6%	10,0%	15,0%	9,4%	0,0%

[290] There is always the possibility that this focus on mathematical competency and calculation components is pedagogically informed – namely, that the textbook authors are deliberately engaging mathematical structures to provide learners with necessary grounding that will facilitate access to more complex contextual sense-making and problem-solving process. However, the textbook authors never make this latter move; instead, they remain consistently in the domain of mathematical competency and calculation. For me, this suggests a deliberately predominance and emphasis on this form of participation and practice over other forms of participation, and not as a deliberate pedagogic act or as a medium to an alternative form of practice.

[291] The Revision Test appears on the last two pages of the Probability chapter and serves to consolidate and assess all of the contents covered in the chapter. As such, the test provides direct signification of the way in which the textbook author(s) legitimise participation (and the dominant domain of practice prioritised in that participation) with probability-related contents in assessment practices in the subject.

[292] The list below gives the assessments tasks and the specific questions in each task that were analysed to determine the information summarised in the Table 16:

- Preliminary Examination Paper 1 (learner's book [LB]): Questions 4.2.3, 4.3.1, 4.3.2 & 4.3.3 (Frith et al., 2013a, p. 273);

As mentioned previously, the textbook authors make a deliberate attempt – through the contexts referenced and the explanations and worked examples provided − to reflect the intention of the CAPS curriculum for the promotion of a form of engagement with probability contents that reflects a high degree of authenticity with respect to everyday real-world practice. However, the information shown in the tables above highlights that, in most question-based scenarios, the authors diverge from this intention by legitimising engagement with probability-related contents through utilisation of mathematically structured knowledge, routines and forms of communication – namely, through a focus on characteristics associated with the Mathematical Competency domain of practice.

Crucially, however, despite a dominant emphasis on mathematically structured forms of participation, the practices that characterise the textbook contents do not constitute the same type of public domain practices that characterise the examinations (see page 419 above). This is due to the fact that the textbook authors have ensured that most of the contexts engaged with share a high degree of representation of the authentic real-world. As such, even though much of the participation in those contexts involves mathematically oriented forms of practice, there is an underlying agenda for contextual sense-making in and of those contexts – albeit from a mathematically derived perspective. This distinguishes the practices in the textbook from those in the examinations, since the questions in the examinations are almost exclusively concerned with the assessment of mathematical knowledge and routines, and any contexts included are largely irrelevant to the problem-solving process.

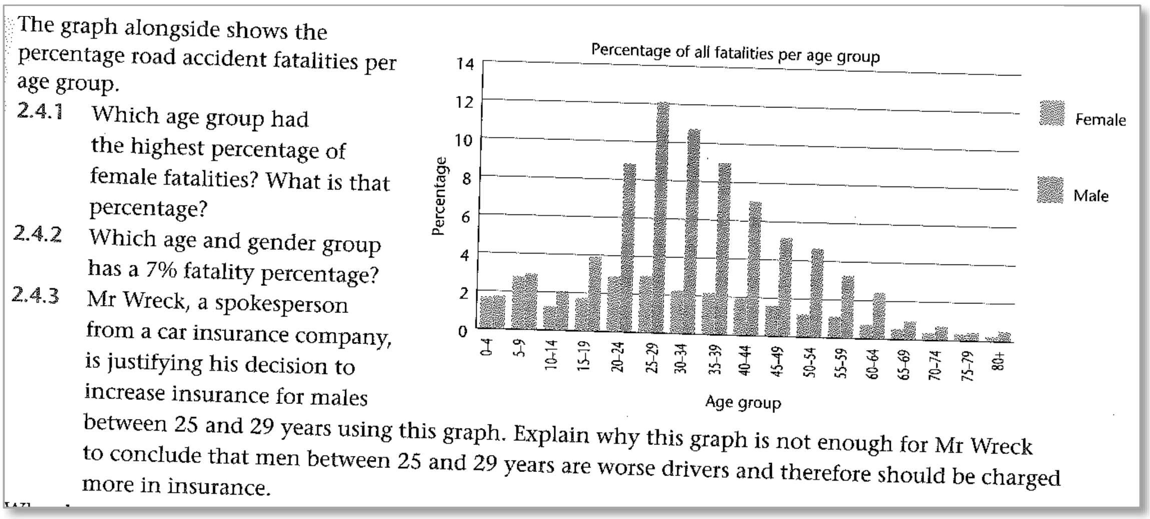
Reflecting again on the contents of Table 14, Table 15 and Table 16, a further significant finding is the complete absence in the chapter contents (including in explanations, worked examples, exercises, and all assessment tasks) of a form of engagement with probability-related contents that reflects characteristics associated with the Modelling domain of practice of the knowledge domain of mathematical literacy. While the absence of Modelling domain components in the explanations, worked examples and exercises can be explained through reference to the absence of any form of Modelling domain components in the descriptions of content and applications in the CAPS curriculum, the same justification cannot be applied to explain the absence of Modelling domain components in assessment practices in the textbook. This is because all of the statements of suggested assessments in the curriculum embody an expectation for some form of

- Preliminary Examination Paper 2 (LB): Questions 4.1.1, 4.1.2, & 4.2.1 to 4.2.4 (Frith et al., 2013a, pp. 280-281);
- Practice End-of-Year Examination A Paper 1 (LB): Questions 3.1.1, 3.1.2 & 4.1.3 (Frith et al., 2013a, pp. 290, 292);
- Practice End-of-Year Examination A Paper 2 (LB): Questions 2.1.3, 2.2.3, 3.4, 4.1.2 & 4.3.3 (Frith et al., 2013a, pp. 297, 299 & 300);
- Practice End-of-Year Examination B Paper 1 (LB): Questions 5.4.1, 5.4.2 & 5.5 (Frith et al., 2013a, pp. 308-309);
- Practice End-of-Year Examination B Paper 2 (LB): Questions 3.3, 3.4, 3.5 & 5.6 (Frith et al., 2013a, p. 314 & 317);
- Term 3 Test (teacher's guide only [TG]): Questions 5.2 to 5.5 and 6.1 to 6.4 (Frith et al., 2013b, p. 225 & 226);
- Preliminary Examination Paper 1 (TG only): Questions 4.6.2, 4.7.1 & 4.7.2 (Frith et al., 2013b, p. 245);
- Preliminary Examination Paper 2 (TG only): There were no probability-related questions in this exam (which, by the way, signifies that this examination paper does not conform to official Education Department requirements regarding the assessment of curriculum contents in this subject) … naughty, naughty! (Frith et al., 2013b, pp. 248-255);
- Target Worksheet 10A (TG only): All questions (Frith et al., 2013b, p. 274);
- Target Worksheet 10B (TG only): All questions (Frith et al., 2013b, p. 275).

modelling and, so, it is reasonable to expect this expectation to be reflected in certain forms of practice in the textbook.

The absence of an expectation for modelling processes for the topic of Probability in the textbook is significant in at least two respects. Firstly, this absence stands in contradiction to the stipulation in the curriculum for Grade 12 learners (in particular) to engage in independent problem-solving and modelling practices as they participate in practices that involve the generation of (re)descriptions of practices encountered in the real-world. Secondly, engagement with modelling processes, together with an understanding of both everyday and mathematical elements, is necessary for the development of more comprehensive understanding of existing and possible alternative forms of participation in particular real-world environments. Furthermore, and as argued previously, it is through the utilisation of components of the all of the domains of practice that a broadened and more complete understanding of a contextual situation is rendered possible and preparation for life is facilitated. As such, this lack of expectation for engagement with Modelling domain practices and components restricts the type of understanding and participation with real-world applications of probability-related contents experienced through engagement with the textbook, hereby limiting the potential for the facilitation of a high degree of life-preparedness.

With respect to the domain of Reasoning and Reflection of the knowledge domain of mathematical literacy, the continued presence of contextual elements and structures in all scenarios presented in the textbook ensures that regular engagement with various levels of reasoning on contextual elements is necessitated alongside reflection on mathematical components in all problem-solving processes. What is particularly significant, however, is that there are three instances in the textbook[293] characterised by an expectation for a form of critical reasoning (i.e. Level 6) regarding the implication of the use of mathematical representations and/or understandings of probability-related contents in the analysis of real-world contents (for an example, see Question 2.4.3 in Figure 61 below).


The graph alongside shows the percentage road accident fatalities per age group.

2.4.1 Which age group had the highest percentage of female fatalities? What is that percentage?

2.4.2 Which age and gender group has a 7% fatality percentage?

2.4.3 Mr Wreck, a spokesperson from a car insurance company, is justifying his decision to increase insurance for males between 25 and 29 years using this graph. Explain why this graph is not enough for Mr Wreck to conclude that men between 25 and 29 years are worse drivers and therefore should be charged more in insurance.

Figure 61: Example of a textbook question associated with the sixth level of the Reasoning and Reflection domain of practice

[293] Instance 1: LB, Chapter 10 (Probability) − Exercise 8, Question 2.4.3 (Frith et al., 2013a, p. 249). Instance 2: LB, Chapter 10 (Probability) – Revision Test, Question 5.4 (Frith et al., 2013a, p. 251). Instance 3: LB, Preliminary Examination Paper 1, Question 4.2.4 (Frith et al., 2013a, p. 281).

In this regard, the textbook authors have attempted to enact the curriculum expectation for a level of critical engagement with probability contents encountered in everyday life, but have also extended this to consider (in a small way) possible limitations of mathematical structures for adequately describing real-world practices. This effort, potentially, signifies limited recognition of the limitations of a 'mathematical gaze' for describing real-world practices and of the need for an expanded gaze to facilitate enhanced understanding of such practices.

This, then, brings to an end the analysis of the textbook chapter. In the next chapter (Chapter 27), discussion shifts to identification of the dominant domain of prioritising in the course notes for a Post Graduate Certificate in Education Mathematical Literacy method course for pre-service teachers.

CHAPTER 27
ANALYSIS OF EMPIRICAL TEXTUAL RESOURCE #4: COURSE NOTES FOR A TEACHER EDUCATION COURSE[294]

27.1 Background information

The teacher education course notes analysed in this study were developed for use in a Post Graduate Certificate in Education (PGCE) Mathematical Literacy teaching methods course held at and by the Nelson Mandela Metropolitan University (NMMU) in Port Elizabeth (South Africa). The PGCE is a one-year full-time post-graduate diploma course for university graduates who have completed an undergraduate university degree (or equivalent qualification) and who intend to enter the teaching profession (at either primary or secondary school levels).

The course is comprised of three main strands: (i) lectures on general teaching theory, methodology and classroom management; (ii) subject-specific specialisations (i.e. 'methods' courses) that deal with subject-specific content, methodology, pedagogic practices and classroom management practices; and (iii) 'teaching practice' which affords participants the opportunity to spend time at selected or allocated schools to observe and participate in formal teaching activities. The course notes analysed in this study are the materials engaged with by PGCE students during their participation in the Mathematical Literacy method specialisation lectures.

The course notes analysed reflect the notes and materials utilised during the 2013 academic year. There were a total of 66 students who enrolled in the PGCE for this year, of which six participated in the Mathematical Literacy methods course. Students who participate successfully in this methods course qualify to teach Mathematical Literacy at Grades 10, 11 and 12 levels at secondary school. Successful participation in this methods course does not equip the student-teachers to teach Core Mathematics at secondary school level; for this, students need to also participate in the Core Mathematics method specialisation course. However, successful participation in the Core Mathematics method specialisation course enables participants to teach both Mathematics and Mathematical Literacy.

The Mathematical Literacy Methods course in 2013 involved 18 contact sessions with participants during the year, with each session lasting 1 hour 30 minutes. The method course included focus on the intended philosophy of the subject, teaching methodology, content (as specified in the CAPS curriculum statements), and general principles of classroom practice.

Given that the Mathematical Literacy method course ran for an entire year, there is simply too much material available for analysis for purposes of this current discussion. For this reason, analysis of the course notes includes specific focus on only three textual forms utilised in the course: (i) an official study guide booklet (Webb, 2013d) which contains, amongst other things, a description of the intended outcomes for the methods course, discussion of general curriculum and assessment issues for the subject, discussion of philosophical orientations for the subject, content development, and descriptions of

[294] The course coordinator for the PGCE programme at the Nelson Mandela Metropolitan University has granted me written permission (via e-mail correspondence) to analyse the PGCE Mathematical Literacy Method course materials. This e-mail correspondence is available on request.

course-related assessment tasks; (ii) an official course reader (Brombacher, 2007) which contains a discussion of different conceptions of mathematical literacy and activities that demonstrate a particular form of pedagogy for the subject in relation to specific content topics (of which Probability is one such topic); and (iii) an examination paper (Webb, 2013b) written by the participants in the Mathematical Literacy methods course. Analysis of the study guide and course reader provide an indication of the ways in which participants in the course were encouraged to think about the structure of legitimate practice and associated forms of knowledge and participation in the subject. By contrast, analysis of the examinations highlights particular areas of content/pedagogy/methodology legitimised by the course convener(s). This, in turn, provides further evidence of a particular prioritised structuring of participation and knowledge for the subject. Furthermore, analysis of the course reader also provides evidence of the way in which the course convener(s) legitimises participation specifically in relation to probability-related contents, hereby providing a direct point of comparison to the other texts analysed thus far.

As with the other textual resources already discussed, analysis of the study guide, course reader and examination involves identification, interpretation and engagement with the signifiers employed in the text. Analysis also includes identification of the format and structure of the methodological or pedagogic and content routines indexed by those signifiers. Finally, the analysis process aims to reveal the knowledge structure required in the utilisation of these routines in order to generate narratives about forms of teaching or learning which will be endorsed by the course convener(s). The various ways in which teachers are inculcated into a particular form of practice, participation and pedagogy in the subject is then compared to the form of participation and prioritised domains of practices and associated knowledge structures which characterise the curriculum document section, textbook chapter, and national examination questions. Possible implications for classroom practice of commonality and/or divergence between these textual resources is also discussed.

27.2 Analysis: findings and discussion

27.2.1 Macro-level analysis: general characteristics of the structure and legitimised approach in the course notes

> Becoming a mathematical literacy teacher is not an easy task. You cannot merely follow a text book, and in class start tomorrow where you left off today. The subject content is important, but as a graduate with one year of tertiary mathematics successfully completed, there will be little emphasis on your own content knowledge but rather on how relevant contexts from our daily lives, and world affairs, can be woven together with the requirements of the content prescribed in the CAPS document to make ML a meaningful experience to your learners when you go into the schools. (Webb, 2013a)

As the quotation above illustrates, from the very first letter distributed to the PGCE candidates there is a clear and explicit statement of orientation for the subject-matter domain of Mathematical Literacy. Namely, that participation in and with the contents of this subject (for both teachers and learners) involves investigation and engagement with real-world situations that reflect authentic daily-life practice. Furthermore, knowledge and understanding of mathematical content is deemed insufficient for engagement with such situations (and, hence, for participation in the subject). This statement of orientation is reinforced by further instructions positing the development of pedagogic strategies based on engagement with real-world resources as a central component of the subject-specialisation course:

2) You are required to bring an article from the newspaper or other media resource, electronic or material, each week STARTING FROM THE FIRST TIME WE MEET IN FEBRUARY and each of you will be required to give a short description of how you could use the article in a lesson in order to teach a particular section of the ML curriculum.
3) You will be required to work in groups of 2-3 to research and investigate, on the internet, websites that can be used as a platform for an ML lesson. ... Possible topics could include:
   - The effect of the transport drivers' strike;
   - Our carbon footprint;
   - Electricity or petrol hikes – causes and effects;
   - Democracy and voting.

   (Webb, 2013a, all emphasis in original text)

This emphasis on engagement with real-world contexts as the starting and focal point in the problem-solving process is also reinforced in the *study guide booklet*, either through reference to authentic real-world resources (such as an advert for a car loan (Webb, 2013d, p. 4)), or through the posing of questions relating to problems or scenarios encountered in real-world settings:

> In what ways can you, as a teacher, use mathematics to sensitise your learners to the magnitude of the HIV/AIDS pandemic in Africa? (Webb, 2013d, p. 5)
>
> We fill in forms for chain stores in order to receive cards and participate in information dissemination about products – but what are the underlying reasons for these practices? (Webb, 2013d, p. 16)
>
> Interest plays a large role in an economy that spends rather than saves. How does interest affect us? (Webb, 2013d, p. 19)

In short, engagement with the contents of the study guide sends a clear message: namely, participation in the subject-matter domain of Mathematical Literacy involves (or should involve) engagement with real-world contexts and problems encountered in such contexts.

Equally, engagement with the contents of the study guide also reinforced the importance of mathematics in problem-solving processes in contextualised environments. To this end, references of the need for the prospective teachers to "Include mathematical activities and exercises" (Webb, 2013d, p. 5) in tasks developed for learners are commonplace, as are references to mathematical terminology (such as 'graphs', 'formulae', and 'geometry') (Webb, 2013d, pp. 12, 15, & 21).

Commitment to a form of participation in the subject that involves engagement with authentic real-world problem-solving scenarios (and the use of mathematics in those scenarios) is, however, most obvious in the contents of the *course reader* that the pre-service teachers are required to engage with in detail. The reader is inundated with visual mediators of a photographic nature showing specific real-world problem-solving contexts. A variety of personalities are also introduced in those contexts, such as two informal traders Elsie and Thabo (who are real people and not contrived or mythical characters – as evidenced photographs of these personalities] who sell chicken food and food snacks respectively). Finally, also littering the pages of the reader are a plethora of visual resources (newspaper articles, data sources, growth charts for children, tariff tables), all of which reflect real-world practices and scenarios. In short, engagement with the signifiers and contents of the course reader immediately generates the impression that the subject-matter domain of Mathematical Literacy involves participation in authentic real-world problem-solving practices.

However, it is more than the inclusion of signifiers that index authentic real-world contexts which prompt an experience of real-world engagement when working through the reader. Rather, the form and structure of the discourse employed in the reader and the way in which participants are directed – through that discourse − in problem-solving processes also embody an explicit agenda and statement of orientation for contextual sense-making practices. Crucially, these problem-solving practices are posited as characterising a form of participation that reflects components associated with ALL of the domains of practice – Everyday, Mathematical Competency and Modelling – of the knowledge domain of mathematical literacy. For the author of the course reader, engagement in Mathematical Literacy related activities involves understanding how and why people act and think in particular ways in everyday settings (Everyday knowledge), but also of how mathematics provides a means for informing existing or alternative forms of participation in those settings (Mathematical Competency knowledge).[295] This dual commitment is reflected in the following segment of text drawn from the reader:

> The more general challenge is: how do individuals in their day-to-day lives learn how to interpret these tables (formulae) differently? And, for us as teachers: how do we develop in individuals that knowledge?
> On the one hand, contextual knowledge is an important factor. ...
> On the other hand experience [of mathematical calculation] is crucial. ...
> (Brombacher, 2007, p. 52)

Furthermore, this dual commitment is also accompanied by a rejection of mathematisation practices that do not give adequate and appropriate consideration to authentic (and useful or valuable) practice:

> Of course there are also questions that neither experience nor contextual knowledge can answer. For example, how much would you pay at the airport if you had parked for exactly 2 hours? Not one second more and not one second less! The table does not give us any clarity in this regard and 2 hours seems to be included in two bands: 1 − 2 hours and 2 − 4 hours. Frankly, I don't think this is important – I believe this is the kind of question asked by students who have spent their lives in classes where they expect to be asked "trick questions". And this is a trick

[295] As a classic example of this intention, in one section in the reader (Brombacher, 2007, pp. 28-36) the author demonstrates how the future and present value formulae (which involve largely esoteric-type mathematical calculations and manipulations) can be used to investigate the process through which a furniture company who sell goods on a hire-purchase basis determine the monthly payment amount for different hire-purchase agreement options. However, immediately on completion of this form of Mathematical Competency engagement with the problem-scenario, the author then provides a visual mediator print-screen snapshot of an online loan repayment calculator and demonstrates and explains how engagement with this Everyday resource results in the same outcome as the future and present value formulae. For the author of this reader, understanding of how people function in the world and of how mathematics informs and sometimes underpins (often unknowingly) that functioning are both central to pedagogic practices in the subject.

> question. It is a trick question because (a) the likelihood of that happening is probably quite remote, and (b) the parking garage will have a policy (programmed in their tills) and more likely than not will charge the person using the garage R18,00 to avoid any arguments arising. The parking garage has a far more difficult decision to make and isn't concerned with the "exactly two hours problem" (and by the way, neither are people in their day-to-day lives). The more difficult question facing the parking garage is: how long do they give the clients to get out of the garage once they have paid ... (Brombacher, 2007, p. 52)

This dual commitment is also accompanied by an explicit intention for a critical dimension – namely, a dimension in which existing practices (both contextual and mathematical) are questioned and challenged, and where alternative forms of participation are considered:[296]

> The Johannesburg Municipality tried to avoid any confusion and in the process got themselves into more trouble with their table. (Brombacher, 2007, p. 52)

> Both the airport and the municipality could have been more precise in communicating what they meant. (Brombacher, 2007, p. 53)

This critical dimension and intention elevates participation in the subject towards engagement with a level of Reasoning and Reflection that involves analysis and questioning of the role of contextual and/or mathematical strategies in describing real-world practices (i.e. Level 6 of the Reasoning and Reflection domain of the knowledge domain of mathematical literacy). Given that the course reader is utilised by potential future teachers in the subject, the presence of this dimension is significant in that it directs pedagogic practice towards a form and structure of participation reflected in the CAPS curriculum but somewhat absent in the other texts (specifically the national examinations and the textbook). Teachers who engage in this PGCE course, therefore, will hopefully come to see the importance of the critical reasoning/reflection intention in practices relating to the knowledge/subject domain of Mathematical Literacy, despite the absence of this intention in other subject-related resources.

Given that the course reader is directed towards pre-service and practicing teachers, it is also worth noting that the reader is characterised by a considerable amount of discourse that engages different levels and forms of reasoning and reflection. This discourse offers reflection/reasoning specifically on the meaning of particular signifiers in textual resources, on the structure and methodology for different routines, and on the format of generated narratives which reflect legitimate forms of participation in a contextual environment. The reader engages PGCE participants in reasoning and reflecting practices to a much greater extent than any of the other texts analysed. The author of the reader, and, by implication, the course convener who has chosen to use this reader as the primary text in the PGCE course, thus, place high priority on the need for Mathematical Literacy

[296] In Part 2, Chapter 5 and sub-section 5.2.2.1 (starting on page 37) I referenced different identified agendas and intentions for varying conceptions of mathematical literacy, numeracy and/or quantitative literacy. This intention reflects characteristics of the identified fifth intention for the critical evaluation of structures (both contextual and mathematical) encountered in contextual sense-making practices. This intention – together with a dominant agenda for contextual sense-making practices – is considered a key component of the life-preparedness orientation espoused for the knowledge domain of mathematical literacy (as described in the presented language of description). And, as discussed in Part 4 (c.f. sub-section 14.4.5 above, starting on page 230), this intention is further associated with, particularly, the sixth (and also the seventh) level of Reasoning and Reflection that characterise the Reasoning and Reflection domain of practice of the knowledge domain of mathematical literacy.

teachers (and associated learners) to reason and reflect about their practice. In particular, the course reader encourages participants to reason and reflect about the way in which people think and act in everyday settings and the suitability of the use of mathematical routines for describing, complementing and/or modifying existing practices in those settings.

A further significant aspect of the course reader is the extent to which employed and/or demonstrated routines incorporate reasoning/reflection of both contextual and mathematical elements. Furthermore, the course reader provides deliberate and explicit acknowledgement that successful engagement with specific routines requires interpretation of both mathematical and context-specific signifiers: "Although one situation or context may seem like another, there could be nuances that actually make the situations different and because of that require formulae that are appropriate to one situation to either be modified for the other or completely replaced." (Brombacher, 2007, p. 56). For the author of the reader, endorsed narratives in the subject-matter domain of Mathematical Literacy must give consideration to the role of contextual elements for influencing and shaping the way in which people think and act as they solve problems. It is for this reason that engagement with the contents of the reader generates an experience that although mathematics provides an important tool for solving problems encountered in real-world settings, the ultimate goal in the subject is for engagement with and sense-making of real-world problem situations and contextual environments, and not exclusively or predominantly for the learning of mathematical knowledge.

While the author of the course reader explicitly demonstrates a commitment to a dual emphasis on forms of participation that reflect elements of practice associated with both Everyday and Mathematical Competency domains of practice, demonstration of Modelling processes are central to the pedagogic approach espoused in the text. In Chapters 2, 3, 4 and 5 of the reader (Brombacher, 2007, pp. 22-110), the author engages with the topics 'People living with Mathematics', 'Freaked out on Formulae', 'So what's the use of all this data handling?', and 'Why probability?', and for each of these topics the author engages various problem scenarios through a modelling-related approach. In every problem-solving instances the author demonstrates how a segment of reality can be recreated and re-described through utilisation of mathematical structures and routines. The author also demonstrates the necessity for the interpretation and critical evaluation of generated narratives in relation to the constraints of the contextual environment and, particularly, in relation to the structure of existing forms of legitimate and endorsed participation in those environments. Although no explicit statement of orientation or commitment is provided, the author of the reader demonstrates time and again the utility of practices and processes associated with the Modelling domain of practice for facilitating understanding and generating descriptions of existing and possible alternative forms of participation in real-world problem-solving scenarios.

The course reader is characterised by Everyday, Mathematical Competency and Modelling components, together with a commitment to components of the Contextual and Reasoning/Reflection domains. As such, when viewed through the lens of the presented language of description for the knowledge domain of mathematical literacy, the course reader is able to be directly categorised as promoting a form of pedagogic practice, participation, and communication in the subject-matter domain of Mathematical Literacy that is characterised by a life-preparedness orientation. The presence, particularly, of an expectation for comprehensive Modelling and critical Reasoning and Reflection practices distinguishes the PGCE course reader definitively from all of the other texts, and solidifies a vision for the subject that involves in-depth and critical engagement with real-world contextual situations in order to better understand both existing and alternative forms of participation in those situations. The author of the course reader is intent on ensuring that those who engage with the reader develop the necessary knowledge and skills to be able to engage successfully and in an effective, enhanced and critical way in real-world problem situations – in other words, life-preparation.

Of course, it could always be argued that despite this dominant orientation in the reader, there is no guarantee that the course convenor will emphasise or express the same orientation in the contact sessions, in the study guide text, or in any assessment requirements in the course. By implication, this would mean that there is no guarantee that the PGCE participants will share in this life-preparedness orientation either in the course or in their future teaching in the subject. Without access to empirical evidence, it is only possible to posit that this argument is *likely* negated for two primary reasons. Firstly, the study guide notes developed by the course convenor rely heavily on engagement with the course reader, to the extent that nine out of the eleven units in the study guide require direct engagement with the contents of the reader. Furthermore, the structure of the study guide itself is in large part directly correlated to the structure of the reader. Secondly, the examination set by the course convenor for the PGCE candidates (Webb, 2013b, 2013c) is dominated by discourse that closely reflects the discourse employed by the author of the course reader. This is most particularly evidenced in the emphasis placed on the centrality of engagement with authentic real-world problem scenarios and of the need for dual consideration of both mathematical and contextual structures in problem-solving processes. Narratives provided in the examination memorandum include a requirement for participants to provide statements such as "Skills needed in real life situations dictate competencies that must be taught and learned" and "The context must be intertwined throughout with the content" (Webb, 2013c, p. 2 & 4). These expected and endorsed forms of narrative reinforce consistency with the structure of legitimate practice, knowledge and participation prioritised in the course reader. The examination paper also includes an explicit requirement for participants to reflect on this prioritised form of knowledge and participation in relation to the CAPS curriculum contents and in relation to pedagogic action modelled on the interplay of knowledge-participation-contents (see, for example, (Webb, 2013c, pp. 3-4, Questions 3, 4 & 5)). In short, the messages about the subject, dominant areas of focus in the subject, and the promoted structure of pedagogic action in the subject are consistent between the study guide, the course reader and the examination for the course. For this reason, there is every expectation that participants in the PGCE course who engage with the study guide and, by implication, the course reader and the examination, will be exposed to an orientation for the subject that promotes life-preparedness. Furthermore, these is also every expectation that such participants will engage with and propagate a form of knowledge and participation in the subject that embodies equal emphasis on Everyday, Mathematical Competency and Modelling components. However, as with any discipline/field, recontextualisation will happen.

The distinct orientation of the course reader towards life-preparation immediately distinguishes the reader from both the national examinations (where structures of knowledge and participation drawn from the domain of mathematical practices are prioritised) and the textbook (where Modelling elements are largely not prioritised). What is less clear, however, is the degree of coherence or divergence between the course reader and the CAPS curriculum. With respect to the official study guide text, various appendices are provided in the study guide which are drawn directly from the CAPS document (Webb, 2013d, pp. 22-63). The contents of these pages cover key facets of the intended curriculum philosophy for the subject, the structure of the contents of the curriculum, and issues relating to assessment specifications in the subject (including taxonomy level descriptors and examples). Significantly, there are two units in the study guide that directly reference the components of the appendices that deal with CAPS curriculum-related *assessment* requirements. However, the discussion in the study guide on 'What is Mathematical Literacy' and the traits of a Mathematical Literacy learner (Webb, 2013d, p. 8) does not direct participants towards engagement with the CAPS philosophy but, rather, to a section of the course reader that deals with these issues and also to other prescribed research papers. And, as is discussed below, the contents of this section in the course reader and the prescribed research papers are in some part divergent from the pedagogic intention prioritised in the CAPS. As such, despite explicit links and dealings with components of the CAPS curriculum, there is also a degree of divergence between the PGCE course and the curriculum in relation to the espoused intention for the subject.

Expanding on the discussion above, it is worth mentioning that the course reader was developed before the CAPS curriculum and, as such, there is no specific focus in the reader on a form of engagement in the subject to reflect specific CAPS related practices or the CAPS philosophy. Instead, the reader rationalises an intention for a particular form and structure of practice, participation and knowledge for the subject through reference to various international conceptions of mathematical literacy, numeracy and/or quantitative literacy, specifically drawing on and making reference to the works of (Gal et al., 2005; OECD, 2003; NCED, 2001; SCANS, 1991). The study guide that the course reader accompanies extends this 'international exposure' by providing a required reading list of the works of (de Lange, 2003; Ewell, 2001; Hughes-Hallett, 2003; Packer, 2003; Richardson & McCallum, 2003; Schoenfeld, 2001; L. A. Steen, 2003a). Reflecting back now on the categorisation of literature relating to conceptions of mathematical literacy, numeracy and/or quantitative literacy read for this study (c.f. Part 2, Chapter 6 and sub-section 6.5, starting on page 77). It is significant that virtually all of the works above (bar the work of Gal et al. (2005), which focuses specifically on adult numeracy rather than on school-based or general conceptions of mathematically literate behaviour) were categorised as reflecting a dominant agenda for modelling and/or the ability to perform calculations in context. By contrast, the CAPS curriculum is categorised as reflecting a dominant agenda for sense-making of real-world contextual situations. This situation is rationalised by considering that in a significant number of these internationally derived conceptions of mathematical literacy, numeracy and/or quantitative literacy, the development a particular form of mathematically literate behaviour is seen to occur in conjunction with the teaching of scientific mathematics contents and not as something that develops in isolation of such contents. As such, knowledge and practices in these conceptions continue to be (appropriately) legitimised according to mathematical structures. The CAPS curriculum, on the other hand, is developed for a conception of mathematical literacy that is separated from the domain scientific mathematics and, as such, requires legitimation of knowledge and participation in a different terrain – namely, in the terrain of life-preparation, driven by an orientation for critical contextual sense-making and not for the development of mathematical knowledge and skills. The study guide and associated reader draw primarily on conceptions of mathematical literacy, numeracy and/or quantitative literacy that stand in difference to the CAPS curriculum. However, the author of the course reader is able to formulate, describe and promote a conception of legitimate participation in the subject Mathematical Literacy that facilitates a form of life-preparedness engagement. This is achieved through prioritisation of Everyday, Mathematical Competency and Modelling practices, and is consistent with the orientation of the CAPS curriculum. This promoted life-preparedness orientation is further consistent with a conception of legitimate participation in the practices of the subject in which participation is no longer legitimised according to primarily mathematical structures. Instead, endorsed participation is now legitimised through sense-making practices that reflect a form of participation which can be understood, validated and utilised by people who engage in real-world practices. However, although the author of the course reader is able to do this successfully, the fact that the study guide and course reader direct the PGCE participants towards engagement with conceptions of mathematical literacy, numeracy and/or quantitative literacy that legitimise knowledge and participation primarily according to mathematical structures creates the potential for confusion. In particular and as a form of rhetorical questioning and reflection; are these participants aware and/or are made aware of the divergence between these conceptions and those that ultimately dominant the pedagogic approach espoused in course reader and also in the CAPS curriculum? And, if this awareness is not made explicit, could this lead to divergence in the way in which the participants demonstrate pedagogic practice in the subject from the approach demonstrated in the reader and intended in the curriculum?

In summary, a general level of analysis of the primary texts engaged with during the PGCE course reveals a genuine and dominant intention for the promotion of a life-preparedness orientation in these texts. This is evidenced through the continued expectation for engagement with authentic real-world contexts and continued emphasis on the utilisation of Everyday, Mathematical Competency and Modelling forms of practice in those contexts to facilitate comprehensive contextual sense-making practices. In comparison to the examinations and textbook, the texts associated with the PGCE course are the only texts that closely and (more than) adequately reflect a life-preparedness orientation. In so doing, the PGCE course notes – and the course reader in particular − also reflect a greater level of consistency with the pedagogic intention of the CAPS curriculum than do the other two texts.

27.2.2 Micro-level analysis: characteristics of specific segments of the PGCE course notes that reflect contents of the topic of Probability

Two units (Units 9 and 10) in the study guide text reference the topic of Probability (Webb, 2013d, pp. 17-20). In both of these units participants are immediately directed to Chapter 5 in the course reader (Brombacher, 2007, pp. 87-110) to engage with the probability-related contents of that chapter. For this reason, analysis of the contents of the PGCE course notes that deal with the topic of Probability is focused primarily on the contents of Chapter 5 in the course reader.

Starting with the study guide text, what is immediately significant is the way in which the employed discourse in the text legitimises participation with probability-related contents in relation to authentic everyday applications and utility rather than according to mathematical structures and applications:

- Reflect on the activities [i.e. probability-related activities in the course reader] to identify:
  - How the understanding of the likelihood of an outcome of an event can influence decision making;
  - The role of other factors in decision making;
  - The power of modelling events to help develop an understanding of them vs. theoretical analysis and calculation.[297] (Webb, 2013d, p. 17)

[297] Notice here, too, the direct and explicit legitimisation of modelling-related practices for the subject. This is discussed further below.

- Use a dictionary to clarify any unfamiliar English words and read the relevant chapter of the Study Material [i.e. course reader] in order to:
    - Consolidate understanding of and analyse pyramid schemes;
    - Identify examples of actual pyramid schemes; and
    - Analyse what motivates people to participate in schemes (that must fail). (Webb, 2013d, p. 19)

As indicated in both of the extracted selections of text above, legitimate participation with the probability-related contents in the PGCE course is based on the utilisation of a form of knowledge that reflects understanding of real-world issues involving probability-related considerations. Nowhere is there mention of a requirement for mathematical knowledge as informing the basis or organising principle of legitimate practice. For the convenor of the PGCE course, generated endorsable narratives (by the PGCE candidates) on probability-related contents are to be characterised by discursive resources that reflect understanding of the ways in which probability-related contents inform decision-making in everyday real-world practices. These narratives are also to be characterised by demonstrated and communicated understanding of how such contents come to be overshadowed by other factors that (re)direct and influence participation and decision making processes. Furthermore, this implies an expectation for communication about probability-related contents to employ discursive resources that are accessible and understandable to those who participate in these contexts on a daily basis. From the perspective of the presented language of description, this reflects a form of participation that is, predominantly, consistent with practices associated with the Everyday domain of practice. This same structure of participation is further evidenced in probability-related revision questions posed at the end of the study guide (Webb, 2013d, pp. 66-67). Here, questions such as "Explain how a pyramid scheme or chain letter works.", "How would you explain to a friend, who is interested in participating in a pyramid scheme, why pyramid schemes HAVE to fail?", and "Discuss the advantages and disadvantages of the Lotto and multi-level marketing.", all comprise an expectation for participants to draw on understanding of probability-related contents from an everyday (non-mathematised) perspective. These questions comprise an expectation for participants to discuss, through the use of everyday forms of discursive resources as well as any mathematical means deems relevant, the influence of probability-related contents and understandings on decision-making processes in authentic real-world problem scenarios. Also notice that the questions comprise an explicit expectation for participants to employ characteristics associated with the domain of Reasoning and Reflection to critically engage with probability-related decision-making processes in everyday settings.

Alongside this dominant domain of prioritising of everyday forms of participation, in the first quotation given above, there is also an explicit statement of expectation for participants to recognise and, by implication, engage with *modelling* processes as a means to interpret, describe and understand aspects of reality and/or forms of participation in that reality. This expectation is again reflected in the revision questions at the end of the study guide, specifically through the inclusion of the question:

> If the originator (starter) of a pyramid scheme wanted to make more money, would it be more advantageous to increase the number of people or increase the amount of money each person pays? Give a mathematical reason for your answer. (Webb, 2013d, p. 66)

Importantly, this question is the first indication given in the study-guide of a requirement for a mathematical form of engagement with probability-related contents; previously, everyday considerations and everyday forms of knowledge and communication have dominated. However, given that this question appears as part of the revision questions, which, in turn, is only engaged with by participants once Chapter 5 in the course reader has been dealt with, it is to that chapter that attention must now shift.

Three pages before the end of Chapter 5 (and twelve pages into the chapter), the author of the course reader – in discussing the "Role of probability (likelihood) in a Mathematical Literacy curriculum" (Brombacher, 2007, p. 100) − makes an explicit assertion regarding his position on the structure of legitimate participation with probability-related contents in the subject-matter domain of Mathematical Literacy:

> Mathematically literate people need an informal understanding of probability or likelihood. By informal I do not mean intuitive only, but rather an understanding that is based on analysis of situations rather than the use of formal axioms, definitions and theorems of probability (the science thereof). ...
> I am not suggesting that there is no place for formal probability theory – of course there is. What I am asking is whether or not there is a place for it in the Mathematical Literacy classroom. I am arguing that probability in the Mathematical Literacy classroom should focus on how these notions play out in day-to-day events by analysing the events from first principles.

At the level of stated intention, for the author of the course reader, engagement with real-world problem-solving practices in the subject-matter domain of Mathematical Literacy must involve utilisation of forms of knowledge, communication and participation that closely reflect authentic and legitimate real-world and everyday practices. When viewed through the lens of the internal language of description, this reflects a dominant prioritising of practices and discursive resources associated with the Everyday domain of practice. What remains to be seen is whether the author reflects coherence with this stated domain of prioritising in the pedagogic approach and discourse employed in the earlier pages of the chapter.

At the start of Chapter 5 the audience is presented with the heading "Why Probability?" and then immediately confronted with two newspaper articles discussing two different pyramid schemes, the arrest of the founders of the schemes, and the impact of the schemes − in terms of financial loss − for those who were conned into investing in the schemes (Brombacher, 2007, pp. 88-89). The inclusion of these visual mediators at the very start of the chapter sends an explicit and immediate message. Namely, in this chapter you will deal with the *real-world* (and not the contrived or mathematised or some other version of the world), with how people act and think in this real-world, and with the types of factors that influence decision-making in this world.

Following the presentation of the newspaper extracts, a commentary is provided in which certain aspects of the pyramid schemes mentioned in the newspapers are discussed. During this discussion, the author of the reader makes the following statement:

> What is the role of Mathematical Literacy in this situation? In this chapter I will explore how pyramid schemes work and demonstrate why they must fail. I will also argue that the investors in these schemes, rich and poor, educated and uneducated alike, invest because they are mathematically illiterate! (Brombacher, 2007, p. 90)

This statement is significant for several reasons. Firstly, if the participants in the course were left in any doubt after engaging with the newspaper extracts that this chapter involves engagement with participation in a real-world setting, this statement should now alleviate or eliminate that doubt. For the author of the course reader, the intention of this chapter is to investigate pyramid schemes and not probability-related mathematical structures, knowledge and techniques. Clearly probability has some bearing on the context (or else why would we be investigating the context as part of the chapter on Probability?), but the primary intention is to investigate, engage with, and come to some deeper level of understanding of pyramid schemes. Secondly, for the author of the reader, being mathematically literate involves being able to make informed decisions about issues which affect daily-life practice and decision-making. As such, the orientation of the chapter (at least at the level of stated intention) is directed towards preparation for more informed and effective engagement in life – namely, life-preparation. Thirdly, this statement indexes an expectation for a dimension of critical reasoning and reflection in the subject-matter domain of Mathematical Literacy, at least with respect to pyramid schemes and other probability-related contexts. Namely, mathematically literate individuals are critical of the world, of factors that impact and influence decision-making practices in the world, and of the structure of their participation in that world. It is not enough to simply understand and participate in the world: to be mathematically literate involves being aware and critical of the implications of particular forms of practice.

Moving beyond the first two pages and into the rest of the chapter, it is immediately significant that there is an overwhelming presence of visual mediators, accompanied by narratives, which demonstrate and engage participants in the *modelling* of possible scenarios involving pyramid schemes (for example, see Figure 62, Figure 63 and Figure 64 below).

List of names received by Siyabonga	List of names received by Lebohang	List of names received by Asanda	List of names received by Graeme	List of names received by Mark	List of names received by Boeta	List of names received by Justin	List of names received by Charl
1. Benson	1. Benson	1. Joseph	1. Joseph	1. Papi	1. Papi	1. Sibusiso	1. Sibusiso
2. Tonic	2. Tonic	2. Pierre	2. Pierre	2. Jimmy	2. Jimmy	2. Vuyo	2. Vuyo
3. Siboniso	3. Siboniso	3. Siboniso	3. Siboniso	3. Mbulelo	3. Mbulelo	3. Mbulelo	3. Mbulelo
4. Calvin	4. Calvin	4. Calvin	4. Calvin	4. Calvin	4. Calvin	4. Calvin	4. Calvin
5. Emille	5. Emille	5. Emille	5. Emille	5. Emille	5. Emille	5. Emille	5. Emille

List of names received by Nkosinathi	List of names received by Katlego	List of names received by Loots	List of names received by Jacques	List of names received by Hershelle	List of names received by Andrew	List of names received by Makhaya	List of names received by Andre
1. Benedict	1. Benedict	1. Gift	1. Gift	1. Elroy	1. Elroy	1. Daine	1. Daine
2. Tonic	2. Tonic	2. Pierre	2. Pierre	2. Jimmy	2. Jimmy	2. Vuyo	2. Vuyo
3. Siboniso	3. Siboniso	3. Siboniso	3. Siboniso	3. Mbulelo	3. Mbulelo	3. Mbulelo	3. Mbulelo
4. Calvin	4. Calvin	4. Calvin	4. Calvin	4. Calvin	4. Calvin	4. Calvin	4. Calvin
5. Emille	5. Emille	5. Emille	5. Emille	5. Emille	5. Emille	5. Emille	5. Emille

Figure 62: Tabular representation of a chain letter-type pyramid scheme shown in the PGCE Course Reader (Brombacher, 2007, p. 91)

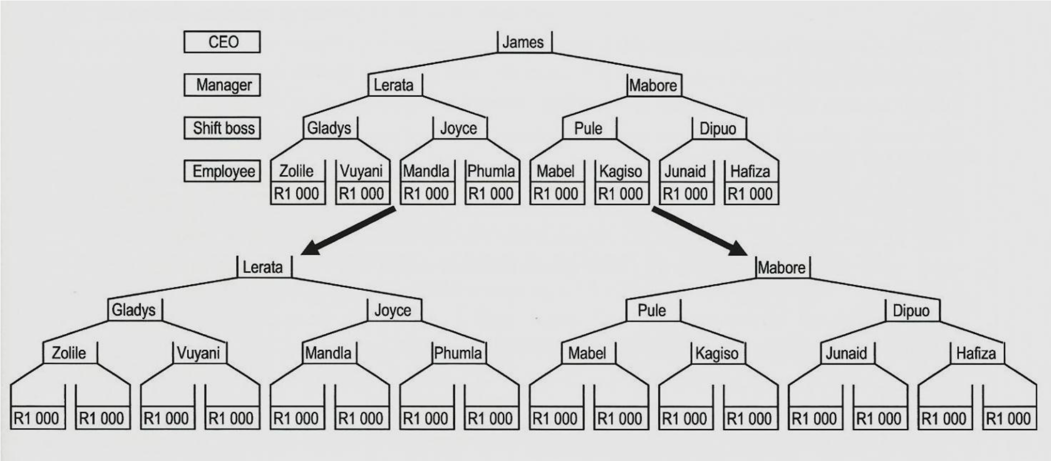


Figure 63: Diagrammatic representation of a company- type pyramid scheme shown in the PGCE Course Reader (Brombacher, 2007, p. 97)

Stage	Number of companies	People who have lost all of their money			People who have made money from the scheme		Total		
		Number of Employees	Number of Shift bosses	Number of Managers	Number of CEOs	Number of retired CEOs	Number of participants	Number of winners	Number of losers
1	1	8					8	0	8
2	2	16	8				24	0	24
3	4	32	16	8			56	0	56
4	8	64	32	16	8		120	8	112
5	16	128	64	32	16	8	248	24	224
6	32	256	128	64	32	24	504	56	448
7	64	512	256	128	64	56	1 016	120	896
8	128	1 024	512	256	128	120	2 040	248	1 792
9	256	2 048	1 024	512	256	248	4 088	504	3 584
10	512	4 096	2 048	1 024	512	504	8 184	1 016	7 168
11	1 024	8 192	4 096	2 048	1 024	1 016	16 376	2 040	14 336
12	2 048	16 384	8 192	4 096	2 048	2 040	32 760	4 088	28 672
13	4 096	32 768	16 384	8 192	4 096	4 088	65 528	8 184	57 344
14	8 192	65 536	32 768	16 384	8 192	8 184	131 064	16 376	114 688
15	16 384	131 072	65 536	32 768	16 384	16 376	262 136	32 760	229 376

(Table 4: Analysis of participation in the first 15 stages of the company pyramid scheme.)

Figure 64: Tabular representation of a company-type pyramid scheme shown in the PGCE Course Reader (Brombacher, 2007, p. 98)

The word 'modelling' in the previous paragraph is deliberately emphasised to draw attention to the fact that the entire pedagogic approach adopted in this chapter in the course reader (as well as in the other chapters that deal with content-related topics [i.e. Chapters 2, 3 and 4]) involves the use of various forms of modelling processes to investigate, describe and understand how pyramid-schemes work and, particularly, the inherent risks in such schemes for those who invest in them. As such, despite the absence of an explicit statement in the chapter of a requirement or expectation for modelling processes in engagement with probability-related contents, analysis of the pedagogic approach employed and demonstrated in the reader through the lens of the language of description reveals a high degree of prioritising of problem-solving techniques associated with the Modelling domain of practice. Through utilisation of various contextually relevant visual mediators, routines and narratives, together with a limited number of mathematically structured representations and basic calculations, the author of the course reader generates descriptions of different aspects of the workings of pyramid schemes that reflect practices and experiences associated with these schemes. In doing so, the author provides participants with a means to understand pyramid schemes and, also, to critically engage with and challenge existing conceptions of participation in this contextual environment. In doing so, the author of the reader also, albeit implicitly, demonstrates a particular form of pedagogic engagement envisioned for the subject – specifically, a form of engagement characterised by modelling practices in which employed mathematical techniques, notation and routines are in service to a broader goal for contextual sense-making practices.

Despite an emphasis on modelling processes, the level of mathematical engagement with pyramid scheme related contents is minimal, and where some level of calculation and/or mathematical structure is demonstrated, this is done at a basic calculation level[298] rather than through reference to complex and /or formal probability-related routines. An illustrative example of this is the 'tree-diagram' included in the reader (see Figure 63 above) to demonstrate, in a visual way, the workings of a particular type of pyramid scheme. In many Mathematical Literacy classrooms this diagram would be formally introduced as a tree-diagram and the workings of the diagram will be explained using formal probability-related narratives. In the reader, by contrast, the signifier 'tree-diagram' is never employed. Instead, this visual mediator is presented as a useful tool for visually demonstrating the workings of a particular real-world process. For the author of the reader, the utility of the diagram is of more significance than whether the diagram has been appropriately drawn and labelled according to conventional mathematical probability-related structures. It is for this reason that engagement with this diagram and with all other visual mediators in the chapter signify routines that require engagement with and communication of everyday forms of contextual knowledge and understanding of the workings of pyramid schemes. Equally, provided narratives employ discursive resources that reference everyday and contextual colloquiums, phrases, notation, and routines, such that the supplied discourse is accessible to all who have an interest in understanding the workings of pyramid schemes and not simply to those who have been exposed to formal probability-related contents. And, even where some hint of signification of a requirement for more mathematically based engagement is provided – for example, "Let us try to illustrate these two points numerically." (Brombacher, 2007, p. 97), the employed format of engagement and the routines and narratives employed in that engagement continue to draw heavily on contextually based discursive resources and narratives[299]:

Stage 1:

- There is a company with 8 vacancies for employees
- 8 new employees (in the diagram above: Zolile, Vuyani, Mandla, Phumla, Mabel, Kagiso, Junaid and Hafiza) join the company

[298] i.e. At the level of Numeracy in Context (Agenda 2 [b]) – refer to the discussion of the 'spectrum of agendas' for varying conceptions of mathematical literacy, numeracy and quantitative literacy given on page 39 above in Part 2 and sub-section 5.2.2.1.

[299] Although, after demonstrating and discussing three stages in the method shown below, the final representation of this process is the provision of a table of numerical values (see Figure 64 above). Importantly, the table is not developed to promote a particular form of mathematical working or to enhance understanding of a particular mathematical structure. Rather, the function of this table is to highlight, summarise and/or compliment understanding of the working of the pyramid scheme. As such, despite the dominance of numerical components in the table, this visual mediator serves to reinforce and broaden contextual and everyday understanding of the structure of the scheme.

- 1 CEO retires
- NOTE – we will ignore the people who joined the company before Zolile et al.

(Brombacher, 2007, p. 97)

Interestingly, it is only after 11 pages of discussion (which draws primarily on everyday knowledge and understanding) on the workings of pyramid schemes that the notion of 'prediction' (as a central notion in school-based probability-related curricular contents) is referenced, albeit only in an implicit and indirect way:

> The mathematically literate person knows that for every "satisfied" customer there are also likely to be some dissatisfied ones. For every person who lost weigh on a particular programme there are many who did not. For every person who met his/her love match via the internet there are hundreds who did not. And for every person who made a greater than expected profit in an investment scheme, there are many people who have lost their money. (Brombacher, 2007, p. 99)

Furthermore, it is only on the 12th and 13th pages of the chapter (which has fourteen pages in total) that very limited signification of vocabulary and notation associated with more formal mathematically based probability structures are referenced:

> I am confident that it can be shown, using the theory of probability, that:
> P(success on the company scheme) = 0.125
>
> And because success and failure are complimentary events:
> P(failure on the company scheme) = 1 – 0.125 = 0.875.
> ...
> P(winning the Lotto) = $\frac{1}{13\,983\,816}$ i.e. 1 in 13,9 million
> (Brombacher, 2007, p. 100 & 102)

The relative absence of emphasis on mathematising practices and formal mathematical routines and narratives is significant in that it highlights consistency between the author's articulated intention and their employed practice involving the prioritisation of everyday forms of participation and communication with probability-related contents. The author of the course reader is adamant – both through statement of intention and employed practice − that the subject (as well as future teachers of the subject who graduate from the PGCE course) should equip participants in the subject with the skills and knowledge to be able to act, think and communicate effectively. Furthermore, participants should also be able to act, think and communicate critically in everyday situations in a way that is consistent with the reality of existing forms of legitimate participation in those situations. In other words, a continued commitment to a life-preparedness orientation, with dominant emphasis on knowledge associated with the Everyday domain, practices associated with the Modelling domain, and a limited degree of Mathematical Competency to facilitate effective functioning with the components of the other two domains of practice.

This emphasis on modelling practices and everyday forms of knowledge, communication and participation is consistent with the discourse espoused in the study guide text. A further element of consistency between these two texts is the degree of expectation and/or demonstration of elements of reasoning and reflection on the use of mathematical and contextual techniques, routines and narratives in problem-solving processes. The probability chapter in the course reader is inundated with constant and consistent commentary and questioning of existing forms of participation in probability-related practices, including situations involving pyramid schemes, the lottery, and multi-level marketing practices. Through this commentary the author of the reader demonstrates the utility of modelling processes for describing forms of participation, and, also the importance of questioning existing structures (i.e. Level 6 of the Reasoning and Reflection domain of practice). More than any of the other texts analysed, the course reader constantly challenges existing forms of participation and demonstrates possible alternative forms of participation that draw on informed decision-making practices. Where all of the other texts are mostly content with forms of reasoning and reflection that facilitate successful engagement in problem-solving processes, the course notes demonstrate an ardent commitment to forms of reasoning and reflection that challenge existing structures in search of more informed forms of participation in the world. As such, where most of the other texts (bar the examinations) promote some degree of orientation for life-preparation, the course reader prioritises an orientation not only for life-preparation but also for critical real-world participation. In other words, it is the only one of the empirical textual resources which gives more than lip service to both domains of Modelling and Reasoning and Reflection.

In bringing the discussion to a close, consider now a comparison of the dominant structure and form of knowledge and legitimate participation prioritised for the subject in the PGCE texts in relation to the CAPS curriculum, the textbook and the national examinations. As an initial comment, comparison of PGCE texts to the other texts is complicated somewhat by the fact that the PGCE course reader was developed prior to the development of the CAPS curriculum (to which the textbook and national examinations are aligned). As a result, direct comparison at the level of topic-specific content is not entirely possible. What is possible, however, is comparison of the way in which participation is legitimised in each text and the dominant and prioritised structure of practice that is endorsed in this legitimised form of participation.

Both at a general level and specifically in reference to probability-related contents, the PGCE texts reflect consistency with the stated expectation in the CAPS curriculum for consideration of everyday forms of practice, for modelling processes, for a level of critical reasoning and reflection, and for a form of engagement with probability contents that prioritises a non-calculation interpretative approach. However, divergence between these texts does occur in relation to the extent to which a commitment to modelling practices is prioritised: while the course reader explicitly prioritises modelling as the dominant form of pedagogic engagement with probability-related contents, the curriculum only specifies an expectation for modelling processes in assessment-related tasks for these contents.

In comparison to the PGCE texts, the Grade 12 textbook prioritises a form of engagement that is more consistent with the structure of knowledge and participation associated with the Mathematical Competency domain of practice and, where contrived contexts are employed, with the Public Domain of mathematical practice. Despite a commitment in the textbook to engagement with real-world contexts (albeit, most contexts are ‘cleaned’), participation with probability-related contents is legitimised primarily according to competency with mathematical knowledge and routines, and endorsed narratives are characterised largely by mathematically based discursive resources. This distinguishes the textbook from the PGCE texts, particularly given the ardent commitment in these latter texts to a structure of participation in probability-related scenarios that is legitimised according to whether generated narratives adequately, closely and validly reflect everyday forms of knowledge, communication and practice. In the PGCE texts, more formal elements of Mathematical Competency are deliberately downplayed in favour of everyday forms of participation. The ardent commitment to practices associated with the Modelling domain in the PGCE texts is another point of divergence from the textbook, specifically since an expectation and/or requirement for modelling is completely absent in the probability chapter in the textbook. Finally, the textbook chapter only contains minor emphasis on forms of reasoning and reflection that comprise an expectation for critical engagement with existing forms of participation in real-world practices, whereas this is a dominant element in the PGCE texts. All of the above reflects, potentially, the prioritisation of different agendas and intentions for participation in the subject in these texts. A possible implication of these areas of divergence is that graduates from the PGCE course who use the textbook in their teaching may need to adapt their pedagogic practice to suit the practice demonstrated and prioritised in the textbook. Alternatively, graduates may seek a different textbook which is more closely aligned to the structure of Mathematical Literacy practice encountered in the PGCE course. My own sense and prediction is that, amidst the busyness (chaos?) and demands of classroom and school-based practice, it is the teacher's conception of legitimate participation and knowledge that will shift and align to that espoused and demonstrated in the textbook, hereby limiting the degree of life-preparedness with probability-related contents facilitated through the pedagogic process.

It is, however, in the National Exemplar examination papers that the greatest divergence from the dominant domain of prioritising in the PGCE texts is to be found. To begin with, the authors of the examinations prioritise near exclusive focus on a structure of knowledge and form of participation drawn from the domain of mathematical practices (and not from the knowledge domain of mathematical literacy). Furthermore, the contexts engaged with in the examinations are largely contrived (both for the probability-specific questions and also at a more general level), thus contradicting not only the stipulation in the CAPS curriculum for engagement with authentic real-world contexts but also the ardent emphasis in the PGCE texts on sense-making practices associated with real-world situations. Where the PGCE course notes demonstrate a definitive commitment to a life-preparedness orientation accompanied by a dimension of critical engagement, the examinations prioritise only mathematical efficiency and mathematisation practices. In this sense, the PGCE texts and the examinations are on opposite ends of a spectrum of differing forms of legitimised participation, with the CAPS curriculum situated on the spectrum closer to the PGCE texts and the textbook marginally closer to the examinations. This vast divergence in the dominant domain of prioritising between the PGCE texts and the national exemplar examinations has potential serious implications for classroom practice and, particularly, for the extent to which the critical life-preparedness orientation espoused in the PGCE texts is able to be successfully modelled and realised in classroom practice. As has already been discussed, the structure of the national examinations and areas of prioritising in those examinations have a significant impact on pedagogic and classroom practices in the subject. As such, if the graduates from the PGCE course attempt to reflect practices consistent with the dominant domain of prioritising in the PGCE texts there is every possibility that their pedagogic practice will not adequately prepare learners for successful and legitimate engagement in the examinations. In alternative terms, learners who attempt to engage with examination contents from an everyday knowledge perspective – as is prioritised in the PGCE texts − will, in all likelihood, generate narratives that will not be endorsed. A potential and likely consequence is that PGCE graduates will have to modify their pedagogic practice to converge with a conception of Mathematical Literacy in which participation and knowledge are legitimised according to mathematical structures. On the other hand, by abandoning the dominant domain of prioritising espoused in the PGCE course, participants will be relegating their learners to a form of engagement in the subject that offers only limited and restrictive life-preparedness. Instead, participants will, in all likelihood, be relegated to positions of objectification and dependency as they engage with mathematised and largely public domain practices, and, in so doing, be indoctrinated in the Myths of Reference, Emancipation and (particularly) Participation associated with such practices. Whatever the outcome, one thing remains clear: the continuing dominant emphasis on examination preparation as the driving force of pedagogic practice in the South African schooling system will, in all likelihood, preserve the subjugation of a critical life-preparedness orientation in favour of whatever dominant orientation is preferenced by the national examiners.

This, then, brings to an end the analysis of the notes for the PGCE course and the discussion of the textual analysis process as a whole. In the next chapter (Chapter 28) I consolidate this discussion by evaluating the extent to which the language of description for the knowledge domain of mathematical literacy has facilitated a valid means for describing and comparing the structure of legitimate participation and knowledge espoused in different empirical texts associated with the subject-matter domain of Mathematical Literacy. Limitations and opportunities for modifications of the language of description are also discussed.

CHAPTER 28
REFLECTING ON THE UTILITY AND VALIDITY OF THE LANGUAGE OF DESCRIPTION

In this chapter I reflect on the extent to which the internal language of description provides a valid means for interpreting, analysing and comparing differences in the structure of knowledge and criteria for legitimate participation prioritised in empirical textual practices associated with the knowledge domain of mathematical literacy. I also reflect on challenges encountered in the use of the internal language in this analysis process and on consequent limitations of the current format of the language for this purpose.

The primary intention of the language of description is to describe the structure of legitimised participation and knowledge associated with a conception of the knowledge domain (and associated practices in the subject-matter domain) of mathematical literacy in which a life-preparedness orientation is prioritised. In this conception, effective engagement with a real-world problem-solving experience is facilitated through: understanding of how people think, act and communicate on an everyday basis and of the factors they have to consider and engage with in that (or in similar) problem situations; understanding of how people might think, act and communicate differently if they employ a mathematical gaze (i.e. mathematical practices, knowledge and routines) on the situation; and the ability to employ a variety of techniques and considerations (contextual and mathematical) to generate (re)descriptions of problem-situations in order to facilitate investigation and understanding of current and possible alternative forms of participation in the problem situations. These facets are categorised in the internal language as reflecting Everyday, Mathematical Competency and Modelling forms of practice, knowledge, and participation. A commitment to engagement with authentic real-world contexts (Contextual Domain) and with various levels of reasoning/reflection about the implications of particular decisions made in the problem-solving process (domain of Reasoning and Reflection) are also central components of the internal language. With respect to these various components of the internal language, the envisioned life-preparedness orientation for critical contextual sense-making is facilitated through deliberate engagement with all components.

In light of this intention for the internal language, what has hopefully become obvious from the discussions in the chapters in this part of the study is that different agents in the subject-matter domain of Mathematical Literacy – through texts and practices – legitimise participation in the subject differently and comprise differing expectations for engagement with contents and practices through utilisation of varying forms of knowledge structures. And, it is through the application of the internal language of description as a lens for analysis of empirical instances that these differing domains of practice and knowledge prioritisation have been revealed. It is the internal language of description which has facilitated identification and classification of varying and sometimes competing and/or incommensurable discourses in different sites of practice in the knowledge and subject domain of Mathematical Literacy. It is the internal language of description which has provided appropriate terminology for describing differences in the way in which varying discursively-mediated domains of practice legitimise participation and communication in the knowledge domain and subject, and the structure of knowledge that is appealed to in these legitimised forms of participation and communication. And, it is the internal language of description which has facilitated investigation of the extent to which different discourses and related practices in the subject either reinforce existing relations of social and educational disadvantage (through the prioritisation of engagement with public domain mathematics practices), or seek to subvert and alleviate existing structures of disadvantage through promotion of forms of participation that facilitate a life-preparedness orientation. These are the successes of the internal language.

Despite these perceived successes, engagement with the internal language in the terrain of the empirical has not been without challenges. Firstly, although emphasis is placed in the internal language – through the Contextual Domain – on engagement with either authentic/real or cleaned contexts to facilitate life-preparedness, the language does not make sufficient provision for consideration of the extent to which continuous and/or predominant engagement with cleaned contexts reduces the level of authenticity experienced in the learning process and, hence, the degree of life-preparedness which may result from this process. The language also does not differentiate between varying levels of authenticity of cleaned contexts (which can also impact on the experience of 'reality' deemed in the learning process) or facilitate consideration of whether resources are cleaned to highlight particular contextual or mathematical elements of the context. For example, consider the Grade 12 textbook in comparison to the PGCE course reader. The textbook employs resources that have been significantly cleaned so as to provide access to (primarily) mathematically based probability-related contents, while the course reader engages significantly more with authentic resources/contexts and contexts that have been cleaned to exemplify or provide access to elements of contextual understanding of probability-related contents. The result is that PGCE participants who engage with the course reader are exposed to a more realistic and authentic life-preparation experience of probability-related applications than the learners who engage with the textbook. The internal language of description is not adequately able to describe and/or accommodate this difference in experience.

Secondly, it is relatively easy to identify whether a form of Everyday or Mathematical Competency practice, knowledge, participation and/or communication is required through the characteristics of the discursive resources contained in a segment of text. However, the same is not true of practices and knowledge associated with the Modelling domain, especially since modelling activities are posited as embodying both Everyday and Mathematical Competency components. In the internal language of description, the domain of Modelling is theorised as characterised by various modelling processes or levels which facilitate the reconstruction or (re)description of a segment of reality through a deliberate movement from reality, through a mathematisation process, and back to reality. These levels are deemed to be identifiable through discursive resources (theorised in the External Language) that index a requirement for engagement with both Everyday and Mathematical Competency elements. However, the attempted utilisation of the internal and external languages to identify modelling-related activities in the empirical texts revealed a lack of specificity in the external language with respect to the characteristics of the discursive resources that characterise such activities. This issue was, potentially, influenced and exacerbated by my engagement with textual resources which largely only revealed a limited commitment to modelling processes, and engagement with alternative resources with a stronger modelling focus may have facilitated identification of more explicit modelling processes and discursive resources employed in those processes. Nonetheless, the result is that modelling activities generally came to be identified as constituting questions or descriptions that comprise an expectation for elaborated problem-solving processes rather than through the presence of particularly-characterised discursive resources in the text. And, this method of characterising text is then reliant on subjective interpretation of the text rather than on a coherent, consistent and theoretically informed methodology, which, in turn, facilitates potential challenge to the validity of the analysis process. Furthermore, neither the internal or external languages provide a means for identifying and differentiating different types of modelling activities, different extents of modelling (for example, some activities may require elaborated modelling processes while other may only require a very limited form of modelling activity), or the level of mathematisation required in modelling activities encountered in different sites of practice. In short, a heightened degree of specificity of the characteristics of modelling-related activities – and the distinctive nature of the discursive resources that characterise and reference such activities – is required in both the internal and external languages of description.

The third issue involves specification of the seventh level of the domain of Reasoning and Reflection (namely, 'reflecting on the reflection of the use of chosen methods in a particular context') in the internal language of description. However, there is a complete absence of this level of reasoning/reflection in any of the empirical texts analysed (including in the specifically-designed activity utilised in the operationalisation of the internal language through the components of the external language). For me, this signifies two possibilities. Firstly, this dimension of the domain of Reasoning and Reflection is drawn from Skovsmose's (1994b) notion of levels of Reflective Knowing – which refers, particularly, to applications of mathematics in which participation is legitimised according to mathematical structures. As such, there is the possibility that this component of the internal language is not representative of the type of reasoning/reflection associated with a conception of the knowledge domain of mathematical literacy that is separated from the domain of scientific mathematics contents and in which a life-preparedness orientation is prioritised. As such, perhaps this form of reasoning/reflection should be removed from the internal language? Or, secondly, this element is a key component of practices associated with the knowledge domain of mathematical literacy, and the absence of this element of reasoning/reflection in all of the empirical texts analysed demonstrates shortcomings in these texts and the inhibition of life-preparedness in the practices embodied in or through these texts. However, more theorising of the relevance and/or necessity of this level of reasoning/reflection for the promoted conception of the knowledge domain of mathematical literacy is necessary before a definitive decision can be reached on the implication of the absence of this level for the development of a life-preparedness orientation.

In summary and conclusion: it has been demonstrated that the internal language and accompanying external language provide a useful means for analysing, interpreting and differentiating the structure of legitimate knowledge and participation associated with practices relating to the knowledge domain of mathematical literacy. However, there are also shortcomings in the languages which give rise to concerns regarding the validity of elements of the analysis process and also of the languages themselves. The goal and intention of future research work is to refine and restructure both the internal and external components of the language of description to overcome these shortcomings and to ensure heightened levels of internal and external coherence and validity.

CONCLUSION

In concluding this study I begin by providing a broad overview of the components of the study and discuss the ways in which the guiding research questions for the study have been engaged and answered. I then discuss and reinforce certain limitations of the study. Hereafter I highlight how the arguments presented in the study signify a problematic form of currently legitimised practices in the subject Mathematical Literacy and signify the ways in which the developed theoretical language of description might be employed by policy makers to inform policy and curriculum decisions and reforms for/in the subject. I conclude by presenting possible opportunities for future research, and share some parting words before a joyful goodbye.

BROAD OVERVIEW OF THE STUDY – REVISITING THE TITLE, RESEARCH QUESTIONS, MOTIVATION AND INTENTION OF THE STUDY

This study sought to identify and distinguish in a clear, explicit and theoretically informed way the basis of varying legitimised forms of participation associated with the subject-matter domain of Mathematical Literacy in South Africa. In direct relation to this topic, the motivation for the study stems from an attempt to understand prevailing concerns and criticisms of the current structure of mathematically legitimised forms of participation in the subject. Furthermore, of particular concerns and interest is the way in which the structuring and positioning of the subject in the schooling curriculum framework contributes to a degree of educational and social disadvantage. It is in response to this state of affairs that the key component of this study has involved the development and presentation of a sociologically-oriented alternative conception of the structure of legitimate participation in the subject that seeks and serves to negate and alleviate these concerns and to provide a more worthwhile and empowering experience for participants in the subject. These components of the study were directed by the first and second research questions:

1. *In what ways does Dowling's (1998) language of description provide a means for problematising current teaching and learning practices associated with the school subject Mathematical Literacy?*

2. *What characterises mathematical literacy as a knowledge domain? What might the regulating principles of mathematical literacy be when taking into account previous work on the nature of mathematical literacy?*

The first research question facilitated investigation of the current structure of mathematised forms of participation in the subject (c.f. Part's 2 and 3 of the study). In answering this question, I drew on the work of Dowling (1998) (in Part 3) to argue that current practices in the subject are characterised by, primarily, public and expressive domain of mathematics practices. Exclusive engagement with these forms of practice limits access to esoteric mathematical contents and, hereby, stunts successful apprenticeship in the domain of mathematics. These forms of practice are also characterised by engagement with mathematised and mythologised representations of reality, such that engagement in the practices of these domains does not facilitated more empowered functioning in real-world practices. Current practices in the subject-matter domain of Mathematical Literacy, thus, restrict and limit the development and future opportunities of the participants in the subject. Furthermore and crucially, the affording of participation in this subject to, primarily, Black learners located in working-class environments, contributes to the preservation of a degree of educational and social disadvantage. It is in response to this current state of affairs that I sought to answer the second research question through the development of an alternative language of description of the structure of legitimate practice, participation and knowledge in activities relating to the knowledge domain of mathematical literacy (theorised and presented in Part 4 of the study). In this alternative language, it is a form of participation characterised by a life-preparedness orientation (accompanied by an agenda for contextual sense-making practices and an intention for the critical evaluation of contextual and mathematical structures encountered in the sense-making process) that is prioritised, instead of mathematically legitimised forms of participation, structures, practices, and knowledge. To facilitate this orientation, the re-described knowledge domain of mathematical literacy is conceptualised as comprising four esoteric domains of practice – Everyday, Mathematical Competency, Modelling, and Reasoning and Reflection, all founded in engagement with authentic real-world practices and resources (described as the Contextual Domain of practice). This reconceptualised knowledge domain is perceived to be positioned outside of the domain of mathematics and, hence, outside of the domains of practice identified in Dowling's (1998) theoretical language. As such, the recontextualised knowledge domain facilitates a form of participation in the subject-matter domain of Mathematical Literacy that addresses and alleviates criticisms and concerns with current practices in the subject.

Since the developed theoretical language is an entirely theoretical construct, a further element was necessary to facilitate use of the theory in analysis of empirical practices. Both the development of this 'external' dimension and the analysis of empirical practices were in response to the third and fourth research questions:

3. *What would constitute an external language of description that would enable a (re)description of an empirical space within the terrain of the subject Mathematical Literacy in terms of reference of mathematical literacy as a knowledge domain?*

4.
    a. *How can the external language of description, in conjunction with the internal language of description for mathematical literacy, be used to determine the dominant basis of legitimisation in a segment of the Mathematical Literacy curriculum, a textbook, course notes for a teacher education course, and national assessment items?*
    b. *How can identification of the dominant basis of legitimisation in these empirical spaces be used to determine coherence or disjunction within and between practices and discourse associated with the subject Mathematical Literacy?*

In answering the third research question, I focused on the development of an external dimension of the language of description. This external language is aimed at identifying and distinguishing the discursive resources in a segment of discourse in an empirical text and/or practice in relation to the uniquely identifiable discursive resources that characterise the domains of practice of the described knowledge domain of mathematical literacy (c.f. Part 5 of the study). This process facilitated analysis of empirical texts and practices through the lens of the components of the internal dimension of the developed language of description. Testing this process in relation to four empirical texts drawn from different sites of practice in the terrain of the subject Mathematical Literacy (Research Question 4. above – and Parts 6 and 7 in the study) illustrated the utility and validity of the theoretical language for describing differences in the structure of practice and associated forms of legitimised knowledge and participation in empirical activities in the subject. In particular, application of the external dimension of the language of description illustrated how engagement with and analysis of differently constituted and employed discursive resources in practices involving contextual-mathematical engagement provides an effective means for identifying areas of prioritising in the practices. In turn, the components of the internal dimension of the language of description provide a means for describing and categorising these varying forms of practice (Research Question 4. a.). Engagement with the discursive resources in varying textual resources, thus, provided the means for identifying and comparing areas of coherence and/or divergence between the structure of practice and participation promoted, legitimised and endorsed in the texts (Research Question 4. b.).

This is the key finding of this study: namely, that the developed language of description provides an appropriate and valid means for describing and comparing the structure of legitimised participation, communication and knowledge in practices that draw on the knowledge domain of mathematical literacy. However, there are also challenges with the language of description, specifically relating to the lack of specificity for identifying the precise characteristics of the discursive resources that reflect Modelling domain practices and for distinguishing the differential degree of life-preparedness facilitated through engagement with authentic and cleaned contexts. These limitations and complications were discussed in Chapter 18 and Chapter 28 in Part 5 and Part 7 respectively.

LIMITATIONS OF THIS STUDY

Several limitations of this study were highlighted briefly in the introduction of the study (c.f. page 5 above). Limitations of the developed internal and external dimensions of the language of description were also highlighted in Chapter 18 and Chapter 28 in Part 5 and Part 7 respectively, together with limitations of the highly subjective interpretive textual analysis process in Chapter 22 in Part 6. In the discussion in this part of the conclusion I revisit and consolidate the discussion of these limitations and the implication of this for the validity of the research process conducted in this study.

To begin with, the developed and presented language of description for the knowledge domain of mathematical literacy reflects a particular characterisation of the relationship between mathematical and contextual practices and of legitimised forms of participation with those practices. Specifically, the language provides a description of forms of practice and participation in a conception of mathematical literacy that is separated from the domain of scientific mathematics contents. Thus, the theoretical language is unable to provide a valid description of practices involving the development of mathematically literate behaviour which occur through and in engagement with scientific mathematics contents or in the context of a scientific mathematics discipline. Since South Africa is unique in the separation of mathematical literacy from the domain of engagement with scientific mathematics contents, this significantly restricts the applicability of the language of description and of the potential for generalising deductions and observations made in the development of the language beyond the South African environment. That said, if we shift beyond the formal schooling sector there is the potential for the application of the developed language in a variety of domains (e.g. adult education; vocational training) that focus on the development of context and/or work specific content and skills and where engagement with scientific mathematics contents is subordinated to goals for enhanced life and/or work-place functioning.

A further limitation relates to the methodological empirical analysis facilitated in the study. Namely, the empirical analysis process focused exclusively on the analysis of *textual* resources and, specifically, on the discursive characteristics of those resources. As such, although this analysis process revealed the utility and validity of the theoretical language of description for identifying the dominant domain of practice and associated legitimised structures of participation and knowledge prioritised in different texts, the language remains untested in relation to classroom practice and also in relation to training directed at pre- and in-service teachers. In this regard, the language has only been shown to provide a valid description for a particular aspect of pedagogic activity for the subject-matter domain of Mathematical Literacy, and further testing is required to determine if the language is more widely and generally applicable to other areas and sites of pedagogic practice.

As regards the employed methodology of textual analysis, this methodology is highly interpretive and subjective, and the conclusions made about the analysed textual resources are influenced, in large part by, my own epistemological and ontological world-view, by the specific decisions and selections made by me during the research process, and also by my academic, social, economic and political characteristics. Furthermore, the employed textual analysis process only focused on the limited domain of discourse in the texts as evidenced through identification and engagement with the signifiers in the text. Analysis of a different component and/or characteristic of the text (e.g. grammatical structure; conversation structure; positioning strategies) through a different textual analysis method (e.g. word count; schema analysis) will likely yield a significantly different interpretation of the meaning conveyed in and through the texts. Commitment to a different tradition of analysis (e.g. to a positivist tradition instead of to a sociological tradition) is likely to have the same effect. The textual analysis process of the empirical textual resources for the subject-matter domain of Mathematical Literacy also only focussed on the limited content topic of Probability, hereby, again restricting the analysis of the structure of legitimate participation in the subject to a limited site of practice. All of these issues highlighted above further limit the extent to which any deductions and conclusions made about the structure of legitimate participation in analysed textual resources relating to the subject Mathematical Literacy can be used to define and describe forms of practice and participation in other sites of practice. These include other texts in the subject, other curriculum or content topics in the subject, in other texts associated with conceptions of mathematical literacy that draw on a differently constituted knowledge domain of mathematical literacy, or other practices in the subject that do not involve engagement with textual resources.

A further issue for consideration is concern over the extent to which the developed language of description for the knowledge domain of mathematical literacy simply highlights perceived ‘gaps’ or ‘inadequacies’ in current areas of focus and practice in the subject Mathematical Literacy. It has already been established that the language provides an alternative conception of mathematically literate behaviour to that currently envisioned and enacted in the subject. As such, application of the language to current pedagogic or assessment practice in the subject may simply highlight and reflect deficiencies in the current system in relation to imposed and largely untested components of the theoretical language, and in relation to my own preferenced perspective on the required and appropriate structure of legitimate participation in the subject. Consequently, such application of the language runs the risk of not adequately reflecting deficiencies and unsuitable and/or unrealistic components of the theoretical language. In this sense there is an inherent risk that the language is self-referential, is positioned outside of the scope of current practices associated with the subject Mathematical Literacy and, so, cannot be adequately trialled and tested within this empirical space.

In short and summary, the language of description for the knowledge domain of mathematical literacy presented in this study has been developed in relation: to a specific empirical terrain; a particular form of the relationship between mathematical and contextual practices; a particular theoretical perspective of the problematic nature of current practices in the subject; a deliberately selected and theorised internal dimension and a deliberately selected and theorised external dimension; and a highly interpretive methodology employed in analysis of deliberately selected textual resources. This study and the conclusions and deductions made therein, are, thus only valid under these specified conditions and constraints. Analysis of the same empirical practices and resources through a differently selected and constructed theoretical language will, in all likelihood, yield a differing perspective on the structure of existing forms of practice, participation and knowledge in the subject. This study, then, is a limited and subjective Interpretivist inspired analysis of practices associated with the knowledge and subject domain of Mathematical Literacy.

IMPLICATIONS OF THIS STUDY FOR THE SUBJECT MATHEMATICAL LITERACY

As theorised in various components of this study and as evidenced directly in the textual analysis of the national examination papers, current conceptualisations of the subject Mathematical Literacy prioritise explicitly mathematically legitimised forms of knowledge and participation in the subject. And, as argued throughout, it is this form of mathematically legitimised participation that is contributing to a reduction in the degree of life-preparedness facilitated in the subject. Furthermore, the rendering of participants in the subject as Dependents and/or Objects through a lack of explication of the generative mathematical principles and structures of the problems encountered ensures only limited development of mathematical knowledge and skills. And, it is response to this issue that many of the criticisms about Mathematical Literacy that were highlighted at the start of this thesis are directed (c.f. page 1 above). In light of and in response to this current state of affairs, a key intention in this study has been to theorise a reconceptualised structure of knowledge. This structure of knowledge is specifically focussed on a format of the knowledge domain of Mathematical Literacy that is separated from the domain of scientific mathematics (as is evidenced in the South African curriculum framework through the separation of the subject Mathematical Literacy from the subject Core Mathematics). This reconceptualised structure of knowledge is dominated by the prioritisation of a life-preparedness orientation.

Given my contention that the current format of participation in the subject facilitates a degree of educational disadvantage in the curriculum framework, a pertinent question worth considering now is the following. Namely, how the arguments presented in this study and, particularly, how the presented reconceptualised structure of knowledge for the knowledge domain of Mathematical Literacy might be interpreted by various role-players in the education community to inform and/or reform practices in the subject Mathematical Literacy to provide a more empowering learning experience? This is the issue that is addressed in the immediate discussion below.

If policy makers take seriously the argument that the prioritisation of a life-preparedness orientation is a necessary alternative to the current structure of mathematically legitimised form of participation in the subject, then a reformulated curriculum (and associated national examination – and, consequently, textbooks) intention and structure is necessary. Both the previous NCS curriculum and the current CAPS curriculum are structured primarily around familiar historical mathematical fields or topics: Numbers and calculations with numbers; Patterns, relationships and representations; Measurement; Data Handling; Probability. However, the CAPS curriculum does also include the topics of 'Interpreting and communicating answers', 'Finance', and 'Maps, plans and other representations of the physical world', all of which signify a break from the traditional and historical mathematical groupings and an emphasis on topics more directly related to real-world environments. To facilitate a life-preparedness orientation this trend in the CAPS curriculum needs to be extended such that the entire curriculum be reformulated around particular contextual groupings, fields and/or topics. Possible fields in this regard might include: Finance (including personal and business finances – e.g. budgets, investments, taxation, pensions schemes); Citizenship (including proportional representation in voting procedures, the implications of prolonged strike action, the use of statistics for informing and influencing public opinion); Measurement at home and in life (including a focus on measurement tools and topics that inform DIY maintenance projects and which also facilitate a degree of preparation for technical employment opportunities); Planning (including engagement with various tools and resources that facilitate more efficient functioning in daily-life practices – e.g. timetables [bus, train], production and other schedules); Visualisation (including engaging with maps, plans, and various 2-and 3-diemsnional representations); Communication (including engagement with a variety of strategies and tools for presenting and communicating mathematical and other information and arguments in an accessible and socially responsible manner); and, where possible, Technology (including engaging with a variety of technological resources that facilitate more empowered functioning in the other fields/topics listed above – e.g. plans drawing software, use of spread sheets for generating budgets). Importantly, a life-preparedness orientation is facilitated through the prioritisation of contextual environments and associated problem-scenarios as the focal point of the learning process – hence why the fields suggested above encompass broad contextually-relevant contents rather than specified well-known mathematical topics. And, within each of these contextually oriented fields, the mathematical content specified for use will be determined by and in service to the contextual environments and problem scenarios under investigation. Contextual practices and contextually-legitimised and acceptable forms of participation will form the structuring principle of the learning process, and mathematical forms of participation will be embraced as a means of exploring alternative forms of participation in those fields but not as a means of legitimising participation in those fields. Furthermore, a life-preparedness orientation is best served through the promotion of an investigative problem-solving approach (akin to an inquiry-based learning approach) since it is this type of engagement that commonly characterises problem-solving practices in out-of-school settings. This signifies a requirement for the complete reformulation of the current teacher-led practices encouraged in the curriculum for the subject (although, the CAPS curriculum does promote a heightened degree of modelling-related practices – and this would need to be encouraged and extended in any future curriculum reform).

However, curriculum reform is not enough, and as has been evidenced time and again in the South African context, often it is the structure and form of participation legitimised in the national examinations which has a more direct bearing on classroom practice. For this reason, the prioritisation of a life-preparedness orientation in the subject Mathematical Literacy requires a complete reformulation of the national examination papers. To begin with, it is questionable whether a written closed-book examination is the best format for evaluating the skills associated with a life-preparedness orientation. Many real-world practices involve collaboration between different participants and seldom are such participants expected to solve problems without being able to avail of a host of physical and technological resources. Furthermore, in many real-world practices evaluation is based on the quality of a completed product or project rather than evidenced through the ability to engage in a paper-based problem solving process. This facet highlights a potential need for a practical component for the subject (which supports the suggestion above for the promotion of an inquiry-based learning approach in the subject). Assuming a small likelihood for the possibility of support for a complete restructuring of the form of evaluation offered in the subject and a higher likelihood for the continuation of text-based examinations in the subject, a higher degree of life preparedness is still possible through a restructuring of the examination papers. In short, the examination papers should read like a newspaper, inundated with newspaper articles, advertisements, and a variety of other contextually relevant and authentic resources which centralise specific contextual environments as the focus of the educational experience. Questions based on these resources should aim to investigate the benefits and limitations of existing forms of participation in those environments, and to explore possible alternative and more mathematically informed forms of participation relevant to those environments. Questions should prompt students to investigate how and why people act and think in particular ways in such environments, and should provide the opportunity to engage alternative forms of behaviour. The measure of mathematically literate behaviour afforded by the examinations should be evidenced in the abilities of the participants to engage in contextual sense-making practices through utilisation and consideration of a variety of tools and resources, some of which relate directly to the contexts themselves and others which draw on a distinctly mathematical basis.

As evidenced in the textual analysis process described in Part 7 of this study, although some degree of contextual sense-making is facilitated in the practices espoused within textbooks, participation in these practices remains grounded primarily in mathematical structures. In pursuit of a life-preparedness orientation, textbooks should read like (intelligent?) magazines, structured around broad contextual fields of practice, populated by a vast array of authentic real-world resources, and characterised by activities and forms of questioning that engage students in sense-making practices (of both contextual and mathematical elements) of complex and intricate real-world scenarios. Textbooks need to be organised around contextual problem scenarios and not around mathematical topics or structures.

Similarly, classroom practices should be characterised by in-depth discussions on prevalent and topical issues (e.g. load shedding and the implications thereof on production and economy), and by practical and project-based investigations that facilitate open-ended in-depth investigations of real-world scenarios and practices. Participants in the subject should be inculcated and apprenticed into engaging with real-world resources with a critical stance, and should be encouraged to source their own topics, resources and problems of interest for investigation in class. Participants should be invited to participate in investigations and to disseminate findings to their classmates and also to wider audiences. And the ultimate measure of developed mathematically literate behaviour within a classroom setting should be the ability of participants, both individually and in collaboration with others, to engage efficiently, with confidence, and in an appropriate and legitimate manner in the practices of a contextual environment.

The successful promotion and prioritisation of a life-preparedness orientation in Mathematical Literacy also requires a revised perception of the value and currency of the subject amongst those who elevate scientific mathematics to a status of critical importance. Mathematical Literacy is currently looked down upon by some who promote and argue the currency of Mathematics for future economic and career outcome (and also by those who cherish the inherently esoteric and complex nature of mathematical contents), in part because Mathematical Literacy and Core Mathematics are pitted against each other in an either/or selection process. The revised conception of the structure of legitimate knowledge in Mathematical Literacy reinforces this distinction further, and in so doing runs the risk of facilitating even greater disdain at the lack of scientific mathematical structures in the subject. It is essential that those who currently hold this positon are enlightened as to the problematic structure of current mathematically legitimised forms of participation in the subject, of the degree of educational disadvantage that is facilitated through this current structure of participation, and of the enhanced functioning offered by the revised conception of mathematically literate behaviour. It is essential that those who hold this position recognise that a version of the subject oriented towards life-preparedness offers the potential for an enhanced educational experience and for more empowered functioning in real-life practices. This is a state of affairs to be celebrated and not negated by those who have invested fully in the mythologising practices of the mathematical gaze.

Despite my hope in this future for the subject and, particularly, for the participants who engage in the subject, my participation in various fields of practice relating to the subject (teaching, teacher education, curriculum development, assessment) leads me to believe that this hope will not be achieved. The prioritisation of a life-preparedness orientation embodies a requirement for a comprehensive and intricate understanding of real-world practices and of legitimate forms of participation in those practices. Furthermore, the ability to investigate alternative and mathematically-legitimised forms of participation in those practices, particularly as envisioned in the Modelling domain of practices of the developed internal language of description, requires a high degree of confidence in personal mathematical ability and in the processes of critical reasoning and reflection. Unfortunately, many of the teachers allocated to this subject do not share a high degree of confidence in their own mathematical ability, in their ability to participate in modelling processes, or in their capacity to engage critically with contextual resources. As a result, many teachers in the subject are not able to move beyond basic forms of mathematical engagement towards in-depth investigations of complex contextual practices. My prediction, then, is that irrespective of any curriculum and/or examination reform initiatives, participation in the subject will remain at the level of engagement with basic mathematical concepts in (contrived) contextual scenarios, at least in part due to the lack of confidence of the teachers to participate in contextual sense-making practices. Only time will tell the true merit of this prediction.

POSSIBILITIES FOR FURTHER RESEARCH

Given the various limitations described above, a first intention in future research projects is to eliminate existing deficiencies in the theoretical language. This is particularly with respect to the lack of specificity in the nature and structure of the discursive resources that characterise the Modelling domain of practice. Additional focus is also required on the differential experience of life-preparedness deemed through engagement with real versus cleaned contextual problem-solving situations (see Part 5 and Chapter 18 on page 356 for an elaborated discussion of these limitations of the external dimension of the language).

A second possibility is to further test the validity of language of description in relation to classroom based pedagogic interactions between teachers and learners in the subject-matter domain of Mathematical Literacy. Interactions in pre- and in-service teacher professional development contexts also provide a further site of analysis and application. This process would serve a two-fold purpose. Firstly, use of the language as a lens for analysis of pedagogic interactions would facilitate a means for comparison of the degree of coherence and/or divergence between the structure of participation and practice legitimised in observed classroom and/or professional development interactions in relation to the curriculum and national examinations. This, in turn, provides a further instrument for analysis of the degree of coherence between the intended, implemented and assessed curriculum. This project could be extended to comparison of the ways in which teachers (including pre-service teachers) and learners differently legitimise participation in practices in the subject, possible reasons for this, and of implications of any identified differences for the learners' successful and endorsed (or unsuccessful) participation in the subject. Similarly, this project could be extended to facilitate analysis of the differential ways in which examiners, learners and markers legitimise participation in the questions in the national examinations. Additional areas of focus could include the means through which any differences in legitimisation criteria are managed by the examiners and markers, and the extent to which particular areas of poor achievement by learners in the examinations can be attributed to a clash or contradiction in the employed legitimisation criteria rather than to a lack of understanding or skill. Secondly, the process can be used to investigate whether, in fact, the promotion of a life-preparedness orientation is possible through engagement with limited recontextualised representations of real-world practices in a classroom setting (as opposed to in the terrain of the real-world). Furthermore, investigation is also required into whether cumulative learning with largely context-dependent knowledge is, in fact, able to be facilitated (as was argued during the presentation of the components of the language of description – see Part 4). Furthermore, this process would provide the opportunity to investigate the extent to which an espoused life-preparedness orientation – if achievable in the context of classroom practice – is able to negate the mythologising practices identified by (Dowling, 1998) (as was argued in Chapter 11 in Part 3 and Chapter 14 – specifically section 14.1 – in Part 4).

A third possibility is to employ the theoretical language for analysis and comparison of more general conceptions of mathematical literacy and adult numeracy that exist beyond the South African environment. Doing this would provide a means for comparing and differentiating, theoretically, the structure of envisioned practice in the subject-matter domain of Mathematical Literacy to other practices focused on the development of mathematically literate behaviour. This would provide a means for describing in a more explicit way differences in the form of behaviour and participation envisioned for the South African context in relation to those envisioned in other environments.

A fourth possibility is research focussed on specification and elaboration of the structure and characteristics of the potential 'public' domains that facilitate entry into the knowledge domain of mathematical literacy. This will involve analysis of the recontextualisation process of real-world environments necessitated by pedagogic processes, and specification of the characteristics of recontextualised environments that are considered appropriate (and those considered inappropriate) for engagement in the knowledge domain. Related to this branch of research is investigation into a potential public domain for mathematical literacy that is comprised of elementary forms of esoteric mathematics, where this public domain facilitates entry into and is a necessary pre-cursor to more complex forms of real-world problem-solving and modelling processes.

In short and summary, this study has presented a theoretical language with a high degree of internal consistency and coherence, but with only limited exposure to empirical settings and practices. The scope of opportunity for further testing of the validity and utility of the language in empirical practices relating both to the South African subject-matter domain of Mathematical Literacy and to other more general conceptions of mathematically literate behaviour is varied and extensive.

PARTING WORDS

In seeking to find a solution to existing problematic practices in the subject-matter domain of Mathematical Literacy, I conceptualised an alternative form of participation for the subject. This alternative form of participation is directed towards a goal for more enhanced and empowered preparation for and functioning in the practices of the real-world beyond the boundary of the classroom. It is my sincere belief that the structure of participation envisioned in the re-described conception of the knowledge domain of mathematical literacy will facilitate this enhanced form of participation and practice, and that participants who engage with this knowledge form will be imbued with a more valuable and empowering educational experience. This is my hope … this is my vision.

And on one final, final, final note (I promise!), thank you very much for reading my work.

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APPENDIX A
EXEMPLAR STRUCTURE: STAGE 3 EXTERNAL LANGUAGE OPERATIONALISATION

Everyday Domain	Mathematical Competency Domain	Modelling Domain	Domain of Reasoning & Reflection
2.4 Why do you think dealers offer balloon payments on car purchases? (1) *<u>Official Endorsed Narrative</u>:* *The inclusion of the balloon payment option creates the impression that the loan is more affordable than it actually it due to the offering of lower repayments. As such, this option makes the loan look more attractive to potential buyers.* ✓			Primarily R/R Levels 1 to 5 specifically on contextual elements.
	2.5 Consider the finance option payable over 60 months at an interest rate of 6,50%. Calculate the *Balloon* payment amount due at the end of the period. (3) *<u>Official Endorsed Narrative</u>:* *Balloon payment = 35%* ✓ *of <u>Retail Price</u>* ✓ *= 35% × R134 300,00* *= R47 3005,00* ✓		R/R Levels 1, 2 and 3. R/R Levels 4 & 5 specifically on mathematical elements.
[300] 11.6 Explain – in relation to what happens in the loan − why this graph is not linear but, rather, can be described as *decreasing at an increasing rate*. (3) *Official <u>Everyday</u> plus <u>Mathematical Competency</u> Endorsed Narrative: From the discussion in 11.5 above, as the amount owing on the loan decreases every month, so the interest each month is calculated on a different and smaller value. This means that every month a different and smaller amount of interest is added to the money still owing on the loan. The fact that every months a different amount of interest is added results in the graph being <u>non-linear</u>.* ✓ *The graph <u>decreases</u> because the amount owing on the loan every month gets less and less as the loan is paid off. This occurs because the monthly repayment amount is always bigger than the changing interest amount that is added every month.* ✓ *The graph <u>decreases at an increasing rate</u> because as more of the loan is paid off, so the interest is calculated every month on a decreasing value, which means that every month a smaller and smaller amount of interest is added. The result is that the amount still owing on the loan decreases at a faster and faster rate as the interest added drops.* ✓			Given that the officially endorsed narrative comprised both contextual and mathematical components, at every level of R/R consideration needs to be given to both contextual and mathematical elements and considerations. As such, for these questions R/R Levels 1 to 5 on both mathematical and contextual elements are required.

[300] The generation of an endorsed narrative for both questions 11.6 and 11.7 requires that connection be made between explicit mathematical entities on the graph (referenced through technical mathematical language such as ‘not linear’ and ‘decreasing at and increasing rate’) and the relationship of these mathematical entities to contextual trends associated with the payment of the loan. In other words, an endorsed narrative for these questions requires the successful integration of both mathematical and contextual understanding and representations. It is for this reason that the questions have been allocated to reflect the requirement that both domains be accessed, connected and referenced in the narrative.

APPENDIX B
CURRICULUM EXTRACT ON THE TOPIC OF PROBABILITY (DBE, 2011A, PP. 90-95)

Topic: Probability — Grades 10, 11 and 12

Suggested teaching time:	Recommended texts and/or resources:
Grade 10: 1−2 weeks Grade 11: 1−2 weeks Grade 12: 1−2 weeks	• Textbooks • Coins and dice • Games involving coins and dice; weather reports; newspaper articles referring to probability; cosmetic and other products making statements regarding probability (*e.g. 80% of the women who used this product …*); products showing success and failure rates for their usage (*e.g. pregnancy tests; drug tests*); information on a lottery; etc.

1. Calculations involving probability are often confined to *mathematical calculations* primarily in the context of dice, coins and games. Although we may encounter situations involving probability and chance on a regular basis in daily life, it is very seldom that mathematical calculations are needed in order to make sense of those situations. E.g. *you don't need to be able to calculate the probability of winning a lottery to know that even though there is a chance of winning, that chance is very small.* What is more important is having an understanding of the concept of probability, together with a sense of whether an event is more or less likely to take place.
   In light of the above, the descriptions given below encourage teachers to focus more on *interpreting* situations involving probability than on the mathematical calculation of probability. This involves developing an understanding of the concept of probability, familiarity with the different notations used in expressions of probability and developing a sense of whether a situation is more or less likely to occur. Alternative contexts outside of the realm of dice, coins and games have also been suggested to reinforce this focus.

Scope of contexts and/or content per section and grade:

Section	Grade 10	Grade 11	Grade 12
Expressions of probability Prediction Representations for determining possible outcomes	Explore probability in scenarios involving: • games using coins and dice • weather predictions	Explore probability in scenarios involving: • games using coins and dice • weather predictions • tests where there is the chance of inaccurate results • cosmetic and other products making statements regarding probability	Explore probability in scenarios involving: • games using coins and dice • weather predictions • tests where there is the chance of inaccurate results • cosmetic and other products making statements regarding probability • lottery and other gambling games • risk assessments • newspaper articles containing references to probability
Evaluating expressions involving probability	---	---	

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Topic: Probability		**Grades 10, 11 and 12**
Section	**Content/skills to be developed in appropriate contexts**	**Grade**
2. Expressions of probability	2.1. Work with situations involving probability, including: a. games that make use of coins and dice b. weather predictions	10, 11 and 12
	c. tests where there is the chance of inaccurate results (e.g. *pregnancy test; drug test*) d. products making statements regarding probability (e.g. *a cosmetic product that claims that 80% of the women who used the product now have less visible wrinkles*) e. tables and graphs containing data and statistics*	11 and 12
	f. national lotteries (e.g. *PowerBall*) g. gambling scenarios (e.g. *slot machines*) h. risk assessments (e.g. *in applications for car insurance*) i. newspaper articles that refer to "likelihood", "chance" and/or "probability"	12
	<u>*In order to*</u>: 2.1.1. Recognise the difference between the following terms: Event and outcome/result 2.1.2. Recognise that probability is expressed using a scale that ranges between: 0 (events that cannot take place – impossible events) and 1 or 100% (events that are certain to take place) 2.1.3. Recognise that the probability of an event is expressed using fractions, percentages and decimal notation.	10, 11 and 12
	Additional comments: 2.2. * In Grade 11, the scope of the data that learners are expected to work with relates to the personal lives of learners and the wider community. In Grade 12, the scope of the data relates to the personal lives of learners, the wider community, and national and global issues. For more specific examples of the types of data that learners are expected to explore in each grade, refer to the topic *Data handling* above.	

Topic: Probability		**Grades 10, 11 and 12**
Section	**Content/skills to be developed in appropriate contexts**	**Grade**
3. Prediction	3.1. Work with situations involving probability, including: a. games that make use of coins and dice b. weather predictions	10, 11 and 12
	c. tests where there is the chance of inaccurate results (e.g. *pregnancy test; drug test*) d. products making statements regarding probability (e.g. *a cosmetic product that claims 80% of the women who used the product have less visible wrinkles*)	11 and 12
	e. national lotteries (e.g. *PowerBall*) f. gambling scenarios (e.g. *slot machines*) g. risk assessments (e.g. *in applications for car insurance*) h. newspaper articles that refer to "likelihood", "chance" and/or "probability"	12
	<u>*In order to*</u>:	
	3.1.1. Recognise that expressions of probability are only predictions about the outcome of an event (*e.g. Although there is always a chance that someone may win a lottery, this does not mean that there will always be a winner every time the lottery is played.)* 3.1.2. Understand the difference between the relative frequency and the theoretical probability of an event (e.g. *The theoretical probability of a tossed coin landing on heads is ½ (50%). However, it is possible to toss a coin 10 times and for the coin to land on heads all 10 times − this is the relative frequency of that event.)*	10, 11 and 12
	3.1.3. Recognise that expressions of probability are predictions about the future based on events of the past (*e.g. Car insurance rates for people between the ages of 18 and 25 years are generally higher than those for people between the ages of 30 and 55 years. This is because historically there have been more motor vehicle accidents involving 18 to 25 year olds than 30 to 55 year olds.)* 3.1.4. Recognise that expressions of probability can only predict the trend of an outcome over a long period of time (for a very large number of trials) and cannot accurately predict the outcome of single events (*e.g. Even though people aged 18 to 25 years are deemed more likely to be involved in a motor vehicle accident than any other age group, this does not necessarily mean that it is not possible that another age group might experience a higher number of accidents during the course of a year. However, based on trends in the past, it is more likely that people aged 18 to 25 years will be involved in an accident.*)	11 and 12
PAGE 92	3.1.5. Recognise the difference between situations where the outcome of one event impacts on the outcome of another and situations where the two outcomes do not impact on each other. (*e.g. If a person buys more than one lottery ticket, does this increase the chance of winning? And if a person plays a slot machine, does his or her chance of winning increase the more times he or she plays?*) 3.1.6. Recognise the difference between predictions that are based on knowledge and intuition about a situation (e.g. *the outcome of a sports match or horse race*) and expressions of probability that are based on long-term trends in data. (e.g. *Even though we can use the historical win-lose record of two soccer teams to get a sense of who we believe might win in an upcoming match, there are simply too many other factors that impact on the performance of the teams (injuries of players; performance of the teams on the day) to be able to predict with certainty what the outcome of the match will be. Our "prediction" of who the winning team will be is based on personal preference or knowledge about the two teams rather than on long-term historical trends.*)	12

Topic: Probability		**Grades 10, 11 and 12**
Section	**Content/skills to be developed in appropriate contexts**	**Grade**
4. Representations for determining possible outcomes	4.1. Work with situations involving probability, including: a. games that make use of coins and dice b. weather predictions	10, 11 and 12
	c. tests where there is the chance of inaccurate results (e.g. *pregnancy test; drug test*) d. products making statements regarding probability (e.g. *a cosmetic product that claims that 80% of the women who used the product now have less visible wrinkles*) e. tables and graphs containing data and statistics*	11 and 12
	f. national lotteries (e.g. *PowerBall*) g. gambling scenarios (e.g. *slot machines*) h. risk assessments (e.g. *in applications for car insurance*) i. newspaper articles that refer to "likelihood", "chance" and/or "probability"	12
	<u>*In order to*</u>: 4.1.1. Identify possible outcomes for compound events by making use of: tree diagrams and two-way tables.+	10, 11 and 12
	Additional comments: 4.2. * In Grade 11, the scope of the data that learners are expected to work with relates to the personal lives of learners and the wider community. In Grade 12, the scope of the data relates to the personal lives of learners, the wider community, and national and global issues. For more specific examples of the types of data that learners are expected to explore in each grade, refer to the topic *Data handling* above. 4.3. + The sections on tree diagrams and two-way tables have been included to provide learners with exposure to different tools and representations that can be used to represent events involving probability in a graphical/pictorial way. The focus in these sections should be on using these representations to identify all of the possible outcomes of an event, especially in situations where the outcomes are not immediately obvious. Learners are <u>not</u> expected to have to use tree diagrams and two-way tables to perform mathematical calculations of probability (*e.g. multiplying probabilities along the branches of tree diagrams*).	

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Topic: Probability		**Grades 10, 11 and 12**
Section	**Content/skills to be developed in appropriate contexts**	**Grade**
5. Evaluating expressions involving probability	5.1. Work with situations involving probability, including: a. games that make use of coins and dice b. weather predictions c. tests where there is the chance of inaccurate results (e.g. *pregnancy test; drug test*) d. products making statements regarding probability (e.g. *a cosmetic product that claims that 80% of the women who used the product now have less visible wrinkles*) e. national lotteries (*e.g. PowerBall*) f. gambling scenarios (e.g. *slot machines*) g. risk assessments (e.g. *in applications for car insurance*) h. newspaper articles that refer to "likelihood", "chance" and/or "probability" <u>*In order to*</u>: 5.1.1. Evaluate and critique the validity of expressions and interpretations of probability presented in newspapers and other sources of information. e.g. *Discuss the validity of statements such as:* • *"If you choose the same numbers every week for the lottery, then this will increase your chances of winning"* • *"The more tickets you buy, the higher your chances of winning"* • *"This team has a higher chance of winning the match than the other team".*	12

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Topic: Probability — Grades 10, 11 and 12

<table>
<tr><th>Section</th><th>Content/skills to be developed in appropriate contexts</th><th>Grade</th></tr>
<tr><td rowspan="3">6.
A</td><td>Possible assessment:
6.1. <u>Assignment</u>: Unfair play
• Develop a game using coins and/or dice and make the game unfair (that is, there is a higher probability of losing)
• Give the game to your classmates and ask them to determine (without doing any calculations) whether the game is fair and if not, why not.</td><td>10</td></tr>
<tr><td>Possible assessment (incorporating all probability concepts):
6.2. <u>Investigation</u>: Tossing coins
• Toss a coin a small number of trials and then determine the probability of the tossed coin landing on heads for this experiment
• Toss the same coin for a very large number of trials and then determine the probability of the tossed coin landing on heads for this larger experiment
• Compare the probability values for the two experiments, discuss why they are different and explain how the notion that “there is a 50% chance that a tossed coin will land on either heads or tails” has been determined.
OR
6.3. <u>Investigation</u>: Pregnancy tests
Investigate the concepts of “false positives” and “false negatives” for a pregnancy test.</td><td>11</td></tr>
<tr><td>Possible assessment:
6.4. <u>Investigation</u>: Probability in the world
6.4.1. Investigate how betting odds are determined for a sports event and evaluate the reliability of these odds
OR
6.4.2. Investigate the following statements in the context of the national lottery and/or gambling:
• “If you choose the same numbers every week for the lottery, then this will increase your chances of winning”
• “The more tickets you buy, the higher your chances of winning.”
• “The probability of winning a game improves if there has not been a winner for some time”
OR
6.4.3. Investigate the use of probability in determining “risk” in applications for car, household and life insurance.</td><td>12</td></tr>
</table>

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APPENDIX C
QUESTIONS ON PROBABILITY IN THE MATHEMATICAL LITERACY GRADE 12 EXEMPLAR EXAMINATIONS

QUESTION 3

3.1 Jan studied the different religious denominations to which people belong in South Africa. TABLE 2 below shows the information from the 2012 population profile of South Africa.

TABLE 2: Percentage of people in South Africa that belonged to religious denominations in 2012

	RELIGIOUS DENOMINATION	SYMBOL	PERCENTAGE MEMBERS
Christian	Zion Christian Church	Z	11,1
	Charismatic/Pentecostal churches	CP	8,2
	Methodist Church	MC	6,8
	Uniting/Dutch Reformed Church	UD	6,7
	Anglican Church	A	3,8
	Catholic Church	C	7,1
	Other Christian churches	OC	36
Non-Christian	Muslim	M	1,5
	Unspecified religion	U	1,4
	Other	O	2,3
	None	N	15,1

[Source: www.indexmundi.com]

3.1.1 Which religious denomination has the highest percentage of people that belong to it? (2)

3.1.2 Determine the total percentage of people that belong to Christian denominations. (2)

3.1.3 Determine the range of the data above. (2)

3.1.4 Arrange the religious denominations in ascending order of their percentage members. Use the given symbols. (2)

3.1.5 Use ANNEXURE B to complete the bar graph representing the percentage of people belonging to the religious denominations in TABLE 2 above. (5)

3.1.6 In 2012, the population of South Africa was 48 810 427.

Calculate how many people belonged to none of the religious denominations in 2012. (2)

3.1.7 If a person were chosen at random in South Africa, what is the probability that the person would be Catholic? (2)

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5.2.2 Mr Reddy gave his Mathematics learners an assignment to conduct a small survey on how much pocket money the boys and girls in the class spent during the lunch break at school on a particular day. The results of the survey (in rand) were as follows (arranged in ascending order):

The amount of money spent by the boys surveyed:

9	10	10	12	12	12	12	12
14	15	15	16	18	20	25	

The amount of money spent by the girls surveyed:

0	6	6	9	9	10	10	10
11	11	11	11	12	20	25	30

(a) Write down the total number of learners surveyed. (1)

(b) Write down the modal amount spent by the boys. (1)

(c) Calculate the mean amount of money spent by the girls. (4)

(d) Determine the median amount of money spent by the girls. (3)

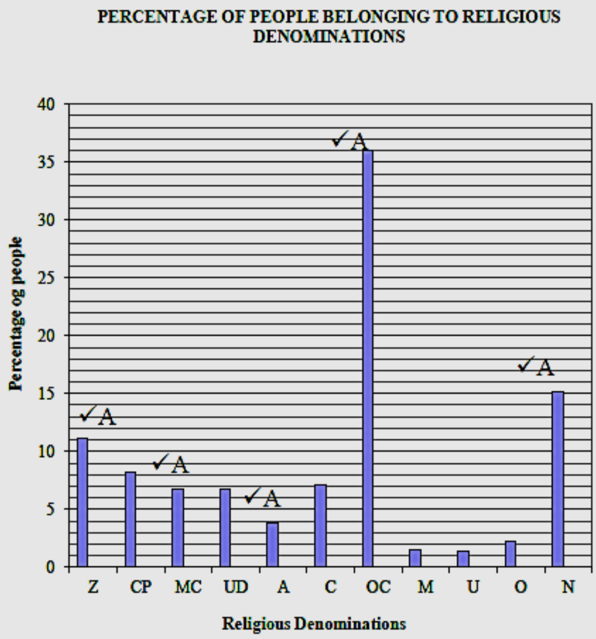
(e) Calculate the difference between the maximum amount spent by a girl and the minimum amount spent by a boy. (2)

(f) What is the probability that a boy selected at random from those boys surveyed would have spent R10,00? (2)

(g) Express the likelihood that a learner surveyed would have spent exactly R30,00 during lunch break. (2)

[38]

Figure 65: Questions (3.1.7 and 5.2.2 f & g) that deal with Probability contents in the Grade 12 Mathematical Literacy Paper 1 Exemplar Examination Paper (DBE, 2014b, p. 8 & 13)

QUESTION 3 [29]			
Ques	**Solution**	**Explanation**	**Topic**
3.1.1	Other Christian churches ✓✓A	2A answer (2)	**DH** L1 (2)
3.1.2	Total = 11,1 + 8,2 + 6,8 + 6,7 + 3,8 + 7,1 + 36 ✓M = 79,7 ✓A	1M adding correct values 1A answer (2)	**DH** L1 (2)
3.1.3	Range = 36 – 1,4✓M = 34,6 ✓A	1M subtracting correct values 1A answer (2)	**DH** L2 (2)
3.1.4	U; M; O; A; UD; MC; C; CP; Z; N; OC ✓✓A	2A answer (2)	**DH** L1 (2)
3.1.5		1A point Z 1A point MC 1A point A 1A point OC 1A point N (5)	**DH** L1 (5)
3.1.6	N = 15,1% of 48 810 427 ✓M $= \frac{15,1}{100} \times 48\ 810\ 427$ = 7 370 374,477 ✓A ≈ 7 370 374	1M using correct percentage 1A answer (2)	**DH** L1 (2)
3.1.7	P(Catholic) = 7,1% ✓✓A = 0,071	2A correct probability (2)	**L** L2 (2)

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Ques	Solution	Explanation	Topic
5.2.2(c)	Mean = $\dfrac{0+6+6+9+9+10+10+10+11+11+11+11+12+20+25+30}{16}$ ✓M ✓A $= R\dfrac{191}{16}$ ✓S ✓CA $= R11{,}9375$ $\approx R11{,}94$ ✓A	1M adding values 1A number of girls 1S simplifying 1CA answer (4)	DH L1 (2) L2 (2)
5.2.2(d)	Median = $\dfrac{10+11}{2}$ ✓M $= R\dfrac{21}{2}$ ✓CA $= R10{,}5$	1A identifying central values 1M finding mean 1CA answer (3)	DH L1 (1) L1 (2)
5.2.2(e)	Difference = R30 – R25 ✓M = R5 ✓A	1M subtracting 1A answer (2)	DH L1 (2)
5.2.2(f)	P(R10, boy) = $\dfrac{2}{15}$ ✓A ✓A	1A numerator 1A denominator (2)	L L2 (2)
5.2.2(g)	P(R30) = $\dfrac{1}{31}$ ✓A ✓A	1A numerator 1A denominator (2)	L L2 (2)

Figure 66: Memorandum for Questions 3.1.7 and 5.2.2 f & g that deal with Probability contents in the Grade 12 Mathematical Literacy Paper 1 Exemplar Examination (DBE, 2014c, p. 7 & 11)

4.3 Ms Springbok surveyed a group of her customers to find out what they bought at the tuckshop. She summarised the results in TABLE 2 alongside.

She decided to use the data in TABLE 2 to make predictions.

TABLE 2: Survey results	
Product	**Number**
Vetkoek	6
Chips	3
Sweets	4
Cooldrink	5

4.3.1 If she surveys 12 more customers, how many customers could she expect to pick vetkoek, based on the data from TABLE 2? (4)

4.3.2 Write, in simplified form, the probability that the next customer coming to her tuckshop will buy sweets or cooldrink. (3)

4.3.3 Based on the number of cooldrinks sold per week, use the survey and calculate how many customers buy at the tuckshop per week. (4)

Figure 67: Questions (4.3.1, 4.3.2 and 4.3.3) that deal with Probability contents in the Grade 12 Mathematical Literacy Paper 2 Exemplar Examination (DBE, 2014d, p. 9)

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Ques	Solution	Explanation	
4.2.3	Percentage increase of sales $= \frac{\text{Increased number sold per week}}{\text{Original number sold per week}} \times 100\%$ $= \frac{409 - 144}{144} \times 100\%$ ✓M ✓SF $\approx 184{,}03\%$ ✓CA	1M subtracting 1SF substituting 1CA percentage (3)	L2
4.2.4	The number of cooldrinks increase from 144 per week to 409 per week. $\therefore$ the percentage increase is 184% ✓O This means it is nearly 3 times more than what she sold before. The decrease in the price is from R8,00 to R5,00. A person knowing the price is R8,00 would not have enough money to buy a second bottle, but persons coming with R10 might buy 2 bottles. This will only double her sales. ✓O The increase is just too much. ✓O	1O recognising how much more she needs to sell 1O reasoning about the decreased price and its effects 1O conclusion (3)	L4
4.3.1	P(vetkoek) $= \frac{6}{18} = \frac{1}{3}$ ✓A ✓A Predicted number $= \frac{1}{3} \times 12$ ✓M $= 4$ ✓CA	1A number of events 1A number of outcomes 1M multiplying probability with 12 1CA predicted number (4)	L3
4.3.2	P(sweets or cooldrink) $= \frac{9}{18}$ ✓A ✓A $= \frac{1}{2}$ ✓CA	1A number of events 1A number of outcomes 1CA simplification (3)	L3

Figure 68: Memorandum for Questions 4.3.1 and 4.3.2 that deal with Probability contents in the Grade 12 Mathematical Literacy Paper 2 Exemplar Examination Paper (DBE, 2014e, p. 10 & 11)

APPENDIX D
LETTER OF ETHICAL CLEARANCE

UNIVERSITY OF KWAZULU-NATAL™
INYUVESI YAKWAZULU-NATALI

15 February 2013

Mr Marc North 981196084
School of Education
Pietermaritzburg Campus

Protocol reference number: HSS/0066/013D
Project title: The basis of legitimisation of mathematical literacy in South Africa.

Dear Mr North

Expedited Approval

I wish to inform you that your application has been granted Full Approval through an expedited review process.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. Please note: Research data should be securely stored in the school/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Professor Steven Collings (Chair)
/px

cc Supervisor Professor IM Christiansen
cc Academic leader Dr MN Davids and Dr R Mudaly
cc School Administrator Ms Bongekile Bhengu

Professor D Wassenaar (Chair)
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