

**A PRODUCTION FUNCTION ANALYSIS OF FRESH MILK
PRODUCTION IN THE HIGHLANDS OF ERITREA**

BY

WOLDEAB KIBREAB GHEBREMARIAM

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DECLARATION

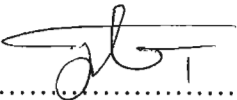
I hereby certify that this thesis is the result of my own original work, unless specifically stated in the text and/or specific acknowledgement is made.

WOLDEAB. K. GHEBREMARIAM
UNIVERSITY OF KWAZULU-NATAL

Signed..........

Date...30-04-04.....

I, Dr. Nsahlai, I. V. Chairperson of the supervisory committee, approve the release of this thesis for examination.

Signed..........

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ABSTRACT

This study presents a production function analysis of fresh milk producers in the Highlands of Eritrea for the year 2002, dealing with the most important factors of production. Most dairy farmers are located in the Central Zone and Southern Zone (Mendefera and Dekemhare) areas of the Highlands of Eritrea. To ensure representative production functions, the Highlands of Eritrea were divided into three respectively homogenous study areas, namely Central Zone, Mendefera and Dekemhare. Most data for this study were collected by survey using a questionnaire, as dairy farms' recorded data were scarce. The annual milk yield record and purchased concentrates per farmer were obtained from their respective milk collecting centres and Dairy Associations belonging to each study area.

Basically, an attempt was made to pool the data of the three study areas, using dummy variables to test if the three study areas' regressions have a common intercept and a common slope. However, from the analysis, the intercept and slope dummy coefficients for the pooled data were found to be statistically significant at the 1% and 5% levels of probability. Thus, it was not economically as well as statistically practical to pool the three areas' data to determine a common function that represents the sample dairy farmers of the Highlands of Eritrea as a whole. For this reason, a separate analysis was conducted for each study area.

The analysis used the Cobb-Douglas function (double-log) form using multiple regressions. However, while analysing the data using ordinary least squares (OLS) regressions, strong intercorrelations were encountered among some factors of production. These intercorrelations resulted in some of the parameters having negative production coefficients where, a priori, all such coefficients are assumed to be non-negative. Thus, to tackle the multicollinearity problem, a ridge regression technique was used at different levels of the biasing constant, c , where the regression coefficients in the ridge trace start to stabilize and the variance inflation factor (VIF) of each parameter and the average of the VIFs are close to one.

The final fitted model includes those variables, which were significant at the 1% and 5% levels of probability. However, for the Mendefera study area those variables significant at 10% level of probability were included as their t-statistic values were considerably greater

than one and nearly significant at the 5% level of probability. From the regression coefficients of the final fitted model for each study area, the elasticities of production with respect to the factors of production, *ceteris paribus*, were estimated. The highest response in production to a one percent change, *ceteris paribus*, is due to milking cows followed by concentrates and labour for the Central Zone Dairy farmers. However, for the Southern Zone (Mendefera and Dekemhare) the highest response next to milking cows came from forage and labour. The regression coefficients of all the factors of production in each study area were greater than zero and less than one, implying rational use of the resources. However, the sum of the elasticities of production was found to be greater than one for each area of production, indicating increasing returns to scale.

Components of the production function and cost calculations including marginal product (MP), values of marginal product (VMP), marginal rate of substitution (MRS), least-cost combinations of inputs, profit maximizing combinations of inputs and the short-run cost functions for each category within the sample of dairy farmers in each study area were estimated. All the VMP's of the resources for the Central Zone dairy farmers were found to be greater than the corresponding unit price of the resources. This implies that the resources are utilized inadequately. However, for the Southern Zone (Mendefera and Dekemhare) the variable concentrates is over-utilized, as the VMP is less than the unit price of the input.

The marginal rate of substitution of concentrates for forage, *ceteris paribus*, showed that the Central Zone sample dairy farmers were utilizing the two resources almost equally. But for the Southern Zone sample dairy farmers the MRS of the mentioned resources showed a higher dependence on concentrates than forage. From the least-cost combination of concentrates and forage analysis it was found that none of the sample of dairy farmers was allocating resources on a least-cost basis.

The profit maximizing combination of inputs showed generally a considerable improvement of milk yield and margins for all the sample of dairy farmers relative to the present situations. However, the profit maximizing criteria (i.e. $VMP_x = P_x$), assumes perfect knowledge, a risk free environment and competitive marketing systems. This has to be considered when advising

sample farmers as to the optimal combination of concentrates and forage.

The short-run cost function also indicates use of resources at below optimum levels. When the average variable cost of the resources is less than the unit price of output, then use of the resources is in the rational area of production. Based on the analysis of the three study areas, the average variable cost of the lower one-third group of sample dairy farmers of the Southern Zone was found to be greater than the unit price of output. This means that the farmers were not covering the short-run costs of production. The MC of concentrates for the lower one-third group of sample dairy farmers was found to be greater than the price per litre of fresh milk in the Southern Zone. This implies more than optimum use of the input (i.e. where $MC = P_y$).

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INTRODUCTION

Eritrea is the latest independent African country, which obtained independence in 1991. The country is geographically located in the North-eastern part of the continent, lying between latitude $12^{\circ}40'$ and $18^{\circ}02'$ north and longitudes $36^{\circ}30'$ and $43^{\circ}20'$ east. It shares boundaries with Sudan to the north and west, with Ethiopia to the south and with Djibouti to the southeast. On the eastern side, it has a coastline of about 1000 kilometres along the Red Sea. The land surface area is 124320 km^2 (Trevaskis, 1975). Figure I shows a general map of Eritrea.

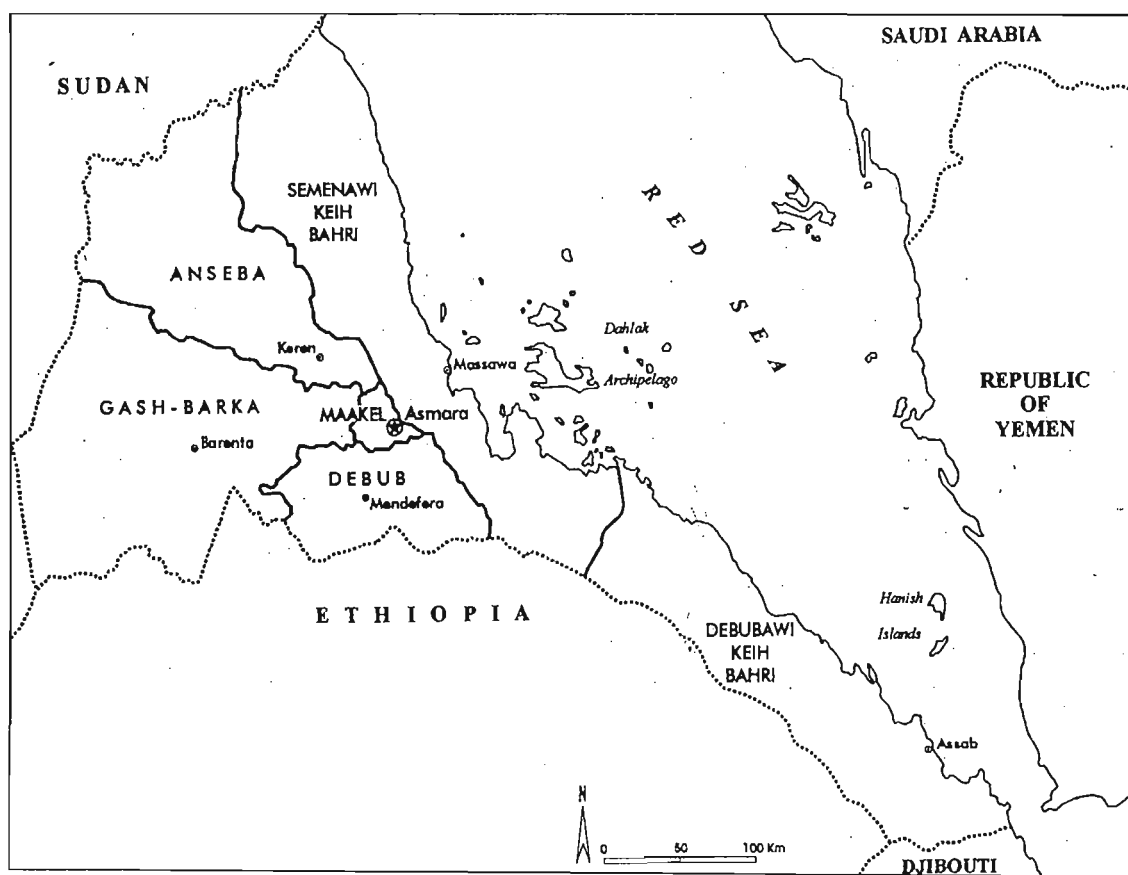


Fig. I: General map of Eritrea.
Source: The Government of Eritrea

POPULATION: There are no reliable population census data completed in Eritrea to date. However, there are varied estimates of the population at different times. The Ministry of Agriculture (MOA) (1996) has published some of the estimates of Eritrea's population size given by different authors at different times. For example, Longrigg (1945) stated that the population size of Eritrea in 1943 was 757,000 out of which 565,000 were sedentary and 192,000 were nomads. Trevaskis (1975) put the population size at 1,031,000 in 1952. Based on the 1999 estimates, the population size of Eritrea is 3,719,000 with a growth rate of 4.57% per year (FAO/GIEWS, 2001). The population is composed of nine ethnic groups.

CLIMATE: The climate of Eritrea ranges from hot arid along the Red Sea, to temperate sub-humid in isolated micro-catchments in the eastern escarpments and the mean annual temperature ranges from less than 19⁰C in the Highlands to more than 30⁰C in the coastal areas. The main factor determining temperature is altitude where temperature falls by 1⁰C for every 200 mm rise in elevation (FAO, 1994). The climate over 70% of the country is hot arid with mean annual temperature of more than 26⁰C and mean annual rainfall of less than 400 mm, even in its favourable times. The total annual rainfall tends to increase from north to south. It varies from less than 200 mm in the northwestern lowlands to more than 700 mm in the southwestern lowlands bordering Ethiopia. The greenbelt zone receives relatively higher rainfall with some areas such as Filfil and Solomuna receiving in excess of 1000 mm mean annual rainfall. The coastal plains receive very little rain, usually between 50-150 mm/year (MOA, 1996).

VEGETATION: It is generally accepted that the vegetation cover of Eritrea has been reduced greatly during the last century. The estimated forest cover has declined from 30% (beginning of the last century) to 11% by 1952 (NEMP – E, 1995). According to the most recent estimates by FAO (1994), the forest cover has now reduced to less than 0.4%.

AGRICULTURE: About 80% of the Eritrean people live in rural areas and primarily depend on agriculture or pastoralism for their means of living. Based on climate and soil parameters the country is divided into six agro-ecological zones, namely the Central Highland Zone (CHZ), Western Escarpment Zone (WEZ), South Western Lowland Zone (SWLZ), Green Belt

Zone (GBZ), Coastal Plain Zone (CPZ) and the North Western Lowland Zone (NWLZ). These were identified in the agricultural sector review of FAO (1994).

Eritrea is only able to meet 60% of its food requirements in a good year, due to various factors including erratic rainfall, shortage of agricultural land, soil degradation and disruption of agriculture during the long war (30 years) for independence. This is well below the potential that could be achieved with better land management. Grazing land, browsing and barren land is estimated to constitute more than 90% of the total area (MOA, 2000). Table I summarises the land use of Eritrea.

Table I: Land use system in Eritrea.

Land Use	Hectares	Percentage of total
Cultivated land: Rain fed	417,000	3.42
Irrigated land	22,000	0.18
Disturbed forest	53,000	0.43
Forest plantations	10,000	0.08
Wood land and Scrubland	673,000	5.52
Browsing and grazing land	6,967,000	57.16
Barren land	4,047,000	33.21
Potential irrigable land	600,000	4.92
Potential rain fed land	1,050,000	8.61
Total	12,189,000	100.00

Source: FAO (1994)

Despite the population's heavy dependence on agriculture, it contributes only 26% to Eritrea's gross domestic product (GDP). Of that value, livestock accounts for 15% (MOA, 2002).

LIVESTOCK: Livestock is an important component of the agricultural production system in the country. FAO (1994) estimated the indigenous livestock resources in Eritrea at 1.65 million Tropical Livestock Units (TLU = 250 kg). The latest survey of the Ministry of Agriculture, Department of Animal Resources (DOAR, 1997) unpublished working document,

indicates that, Eritrea has the following estimated number of livestock: 1,927,000 cattle, 4,662,000 goats, 2,129,000 sheep, 504,000 donkeys, 9,000 mules, 5,000 horses, 319,000 camels and 3,200 pigs, which are Large White derivatives. The livestock resources in Eritrea have been seriously affected by 30 years of war, consecutive drought, shortage of feed, endemic disease and erroneous government policies such as (nationalizing private sectors, breakdown of institutions and supporting services etc).

Commercial dairy farming was started by the Italian settlers during the 19th century when Eritrea was under Italian rule. The growing demand for milk and milk products, especially in the urban areas, stimulated the development of small dairy farms. On the other hand, Italian settlers were given large areas of fertile land in different parts of the country (mainly in the Highlands) in which they established large modern dairy farms with improved dairy breeds, mainly Holstein Friesian. The intensively managed Italian farms clearly demonstrated the progress that can be achieved by proper management and upgrading practices (Teclu, 1995).

When American military bases were established in Eritrea in 1950s, the demand for milk and milk products increased substantially. This increased milk demand encouraged many small-scale Eritrean farmers to enter the enterprise and produce milk for marketing. Until 1960, only small-scale milk processing was carried out by a small milk pasteurisation plant at Asmara known as *Ente Cooperativa dille Consumo Eritreo (ECCE)*. This plant was established and owned by private Italian owners until it was taken over by the Eritrean Dairy Farmers Association in 1969 (Teclu, 1995). Further, Teclu noted that, even after the British replaced Italian rule and the Imperial rule of Ethiopia replaced the British, the Italian owned dairy farms were operating normally under their original owners. Hence, milk production was at a continuous growth phase up to the mid-1970s, i.e. before the nationalization of the industry by the former military government of Ethiopia. Consequently, this growth was geared to the development of small and medium scale commercial farms. However, the breakdown of the institutions and supporting services, the prolonged war for independence, and the severe shortage of feed resulting from continuing droughts led to reduced dairy activities and production.

According to MOA reports, the livestock population in Eritrea has been reduced by 50-70% in the last two decades. Consequently, since 1991 the government of the state of Eritrea has taken considerable measures to rehabilitate the livestock sector, by providing extensive annual vaccinations against major infectious disease and undertaking intensive training programs for livestock technicians and farmers (MOA, 1996).

Nevertheless, the dairy farmers are still faced with three fundamental problems: how to produce, how much to produce and what to produce. The latter may not be a pertinent problem as milk production is their primary activity. Thus, this study focuses mainly on how to produce and how much to produce for the dairy farmers of the Highlands of Eritrea, using the available factors of production. The most important cost items for the fresh milk producers of the dairy farmers in the Highlands of Eritrea are feed and labour. The proper or improper use of these factors of production will result in profits (losses). Hence, the main objective of this study is to conduct economic analyses of fresh milk production using the most important factors of production in the Highlands of Eritrea.

From the production function analysis, the elasticities of output with respect to the resources used, the marginal product and value of marginal product, profit-maximizing combination of inputs, the marginal rate of substitution and least-cost combination of inputs, and short-run cost function can be estimated. Thus, it is hoped that the results derived from such analysis will enable agricultural economic advisors and dairy extension agents to advise farmers on how to improve the allocation of their scarce factors of production. The results will also provide useful information for further study on similar farming systems.

A review of production functions and their use in estimating resource allocation on farms are presented in the next Chapter. The problems encountered in the analysis of production functions and proposed solutions are reviewed as well. Chapter 2 deals with the research methodology and a description of the study areas. In Chapter 3 to 6 various production function analyses are conducted. The dissertation concludes with a discussion of the implications of the results.

CHAPTER 1

Production Function Analysis and Related Problems

Production function studies have the potential to yield answers to many economic questions (what to produce, how to produce and how much to produce). The results of production function analysis are also suited to aid an individual entrepreneur or society in making the best decisions concerning the use of his or society's scarce resources. The basic concepts, techniques involved and problems encountered in production function analysis are reviewed in this chapter.

1.1 Basic Theoretical Concepts of Production Functions

A production function is the relationship between the quantities of various inputs per period of time and maximum quantity of commodity that can be produced per period of time. More specifically, a production function is a table, a graph, or an equation showing the maximum output rate that can be achieved from any specified set of inputs (Mansfield, 1982).

Symbolically, a production function can be written as:

$$Y = f(X_1, X_2, X_3, \dots, X_k) \quad (1.1)$$

Where: Y = output

$X_1, X_2, X_3 \dots X_k$ = are different inputs that take part in the production of Y.

f = functional symbol that signifies the form of the relationship that transforms inputs into output.

However, this symbolic relationship does not specify which inputs are variable and which are fixed. A resource is said to be variable, if its quantity can vary during the production period. Conversely, a resource that cannot vary in its quantity is fixed or often called a technical unit. Doll and Orazem (1984) stated that technical units have varying capacities to absorb and transform variable inputs into outputs. A sandy soil, for example, can absorb less water than a clay soil; the capacity of an average Holstein cow is greater than that of a Jersey cow. Therefore the above notion can be rewritten as:

$$Y = f(X_1, X_2, X_3, \dots, X_{k-1} / X_k) \quad (1.2)$$

Where: X_k = the fixed input (technical unit) while all other inputs are Variable.

Resources may be fixed for several reasons in the short run.

- The manager may be using exactly the right amount of resources, meaning that an increase or decrease in the quantity used would lower his profit.
- The time period involved in the production process may be so short that the farmer is unable to change the amount of resource he has, e.g. land and building.
- The farmer may not want to vary the amount of input. For example, a dairy farmer may change the rations fed to a cow in milk to determine the effect on the quantity of milk produced by the cow. For this evaluation, a cow is a fixed resource and the feed is a variable resource. He could change the size of his herd, but such a change is not relevant to the question he is investigating.

In the long run, a manager has the opportunity to change the level of usage of all inputs. Therefore, production functions are usually estimated for the short run, because at least one resource is variable. The actual estimation of a production function is an empirical task, which uses time – series or cross -sectional data gathered from farms or research stations. Data of these types provide knowledge of the production functions that enable producers to improve decisions concerning resource allocation.

From practical experience, farmers develop a subjective knowledge of their production functions, one for ‘good’ years and one for ‘bad’ years. Reenen and Davel (1991) suggested that, to gain a better understanding of the typical production function in agriculture, it is necessary to consider the total physical product (TPP), average physical product (APP) and marginal physical product (MPP). The TPP is the same as the total yield and indicates the total output obtained with a certain quantity of inputs. The APP is the quantity of output per unit input. In other words it represents the average rate at which an input is transformed into a product. So it measures the efficiency of the variable input used in the production process.

$$APP = \frac{\text{total product}}{\text{total input}} = \frac{Y}{X}$$

The MPP is the extra output for one unit increase in input, i.e., the addition to the total product as a result of the addition of an extra unit of input.

$$\text{MPP} = \frac{\text{change in total product}}{\text{change in total input}} \text{ or } \frac{\Delta Y}{\Delta X_i}$$

Geometrically, MPP is given by the slope of the TPP curve. For a classical production function, the APP and MPP curves appear as follows in figure 1.1.

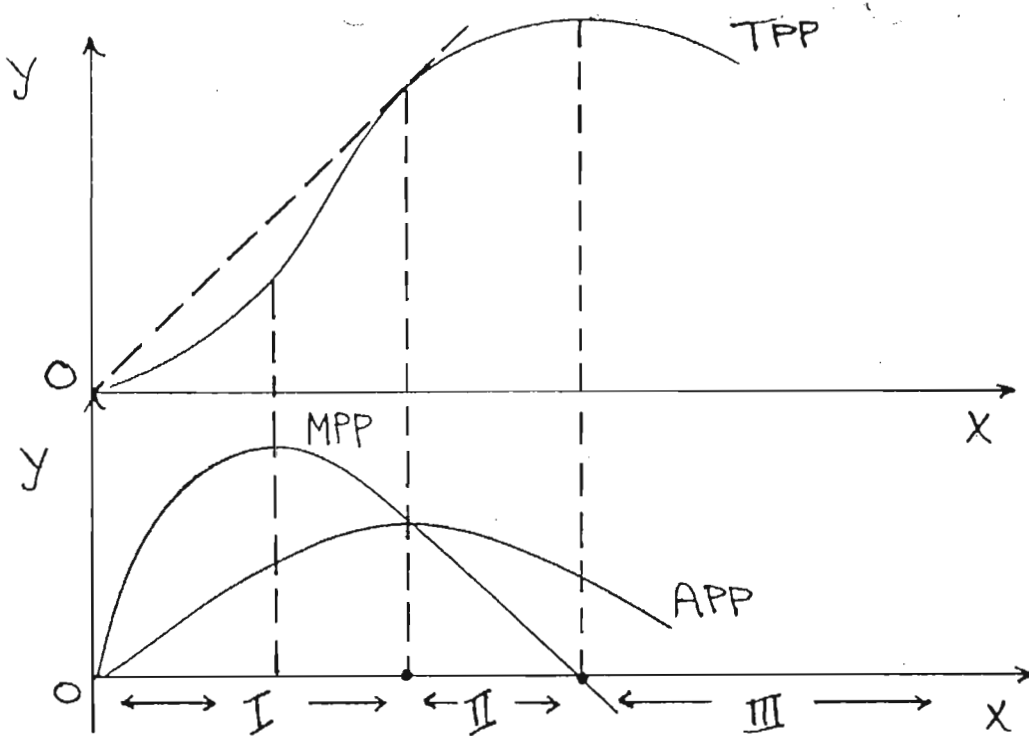


Figure 1.1: The classical production function and the three stages of production (Doll and Orazem, 1984, pp.37 – 39).

The classical production can be separated into three stages of production as shown in figure 1.1.

- Stage I occurs where APP is rising (i.e. $\text{MPP} > \text{APP}$)
- Stage II occurs where $\text{MPP} \leq \text{APP}$ but $\text{MPP} \geq 0$ (rational zone)
- Stage III occurs where $\text{MPP} \leq 0$

Under this assumption of a production function, a general recommendation regarding input use can be made even when prices of inputs and products are unknown.

- ❖ Provided that the output has some value, inputs should be added at least until stage II is reached, as the efficiency of the variable inputs is rising throughout stage I.

- ❖ Even if the input is free, it should not be used in stage III, as further input increment decreases output.

In the classical production function, elasticity of production (E_p) is defined as:

$$E_p = \frac{\% \text{ change in output}}{\% \text{ change in input}}$$

In other forms it is determined by:

$$E_p = \frac{\Delta Y / Y}{\Delta X / X} = \frac{X \cdot \Delta Y}{Y \cdot \Delta X} = \frac{MPP}{APP}$$

Therefore, E_p is greater than one in stage I, and less than one but greater than zero in stage II. MPP is negative in stage III. The point of diminishing returns can be defined to occur where $MPP = APP$, or E_p is one. This is the minimum amount of variable input that should be used, and it occurs where the efficiency of the variable input is at a maximum. However, the exact amount of input use can be determined only when choice indicators, such as input and output prices are known (Doll and Orazem, 1984, pp. 40 – 41). In practice, it is seldom possible to estimate a classical production function, as input and output levels are seldom observed outside of the rational zone (stage II of production). Instead, other functional forms that provide better approximations of observed data should be used. Robert (1956, pp. 156) suggested that, for reliable estimates of farms included in the sample which are homogeneous in respect to soil types, techniques of farming, and crop and livestock enterprises, the most commonly used function is the “Cobb – Douglas” function.

1.2 The Cobb – Douglas Production Function

Of possible algebraic forms, the Cobb – Douglas has been the most popular in farm - firm analyses. This algebraic model provides a compromise among

- Adequate fit of the data,
- Computational feasibility and
- Sufficient unused degrees of freedom to allow for statistical testing. (Heady and Dillon, 1961)

Further he pointed out that, the Cobb–Douglas function is a relatively “efficient user” of degrees of freedom. Such efficiency is important where research resources are limited and

collection of farm - firm data is expensive. Moreover, Plaxico (1955, pp.664 – 665) and Heady (1946, pp. 991 – 992) similarly stated that the function postulates complementarities among inputs and allows diminishing marginal productivity and diminishing marginal substitution among factors as well as increasing or decreasing returns to scale. However, when a Cobb – Douglas type function is to be used, certain procedures (which will be discussed later) should be followed in assembling and analysing the data, in order to draw reliable conclusions.

The Cobb – Douglas or power function, in the form generally used is:

$$Y = aX^b \tag{1.3}$$

Where: Y = output

X = the variable resource

a = constant

b = defines the transformation ratio when X is at different magnitude (quantity).

The exponent or b coefficient is the elasticity of production, which can be used directly. The equation is estimated in logarithmic form as:

$$\ln Y = \ln a + b \ln X \tag{1.4}$$

This allows either constant, increasing or decreasing marginal productivity. It does not allow an input-output curve embracing all three. The power function for k resources in equation (1.5) has the same mathematical characteristics discussed for equation (1.3), when input – output curves are derived for one resource with the others held constant.

$$Y = aX_1^{b_1} X_2^{b_2} X_3^{b_3} \dots \dots \dots X_k^{b_k} \tag{1.5}$$

Thus, it can also be rewritten in a form that is linear in its parameters as:

$$\ln Y = \ln a + b_1 \ln X_1 + b_2 \ln X_2 + \dots + b_k \ln X_k \tag{1.6}$$

The assumptions of constant elasticity and marginal product with only a plus or minus sign, regardless of input or output magnitudes, are retained.

1.2.1 Multiple regression analysis with linear parameters

Suppose the production relationship between an output Y and inputs X₁, X₂... X_k has, on a priori grounds of economic, biological, or physical logic, been postulated as the following single equation model:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k + \epsilon_i \tag{1.7}$$

Where: Y = the dependent variable

X_1, X_2, \dots, X_k = the independent (explanatory) variables

ϵ_i = the error due to the fact that the postulated independent variables do not completely explain Y (i.e. some input factors of minor importance have not been taken into account).

$\beta_1, \beta_2, \dots, \beta_k$ = are the population regression coefficients.

In such terms Y is the effect variable and X_1, X_2, \dots, X_k the causal factors. It is desired to specify equation (1.7) empirically, i.e. to determine the value of the parameters, if these values are known. Thus, interest is on the causal relations as a whole, especially the β 's, and not just in the value of Y that may be estimated by using the equation. Provided that some basic assumptions hold true, the fact that equation (1.7) is linear in the parameters makes estimation of these parameters by multiple regression rather straightforward. All that is required is a sample of $n \geq k + 1$ sets of data showing the value of Y for various levels of each of the independent variables (causal factors). Suppose such data are available:

Y_1	X_{11}	X_{21}	X_{k1}
Y_2	X_{12}	X_{22}	X_{k2}
.				.
.				.
.				.
Y_n	X_{1n}	X_{2n}	X_{kn}

Where Y_j ($j = 1, 2, \dots, n$) is the level of Y attained when the i^{th} of the k inputs is at the level X_{ij} . The estimate of equation (1.7) to be derived from the given data will be of the form:

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + \dots + b_k X_k + \epsilon_i \tag{1.8}$$

Where b_k , termed a sample regression coefficient, is an estimate of β_k , and \hat{Y} estimates Y . Since equation (1.8) is an estimate of equation (1.7), the relation between Y and \hat{Y} is often of the form:

$$Y_j = \hat{Y}_j + \epsilon_j \tag{1.9}$$

In other words, it is not to be expected that equation (1.8) will exactly predict the level of output, Y_j , forthcoming from a given set of input quantities, $X_{1j}, X_{2j}, \dots, X_{kj}$. There will generally be some discrepancy or residual between the observed and predicted value of Y .

This residual is the term e of equation (1.9). It is an estimate of the theoretical error, ϵ , of equation (1.7). For convenience in algebraic manipulation, equation (1.8) is often written as:

$$\hat{Y} = b_0X_0 + b_1X_1 + b_2X_2 + \dots + b_k X_k \quad (1.10)$$

Where X_0 is a dummy variable, which is always equal to one (Heady and Dillon, 1961, pp. 110).

1.2.1.1 Interpreting estimated parameters

The Cobb - Douglas regression equation (1.5) obtained by ordinary least square (OLS) regression is linear in its logarithmic form.

$$\ln Y = \ln a + b_1 \ln X_1 + b_2 \ln X_2 + \dots + b_k \ln X_k \quad (1.11)$$

The estimated partial regression coefficients of the double log function are direct estimates of the elasticity of production of the explanatory factors as they indicate the percentage change in output which will result from a one percent increase in the input of various factors. The elasticities are independent of the unit of measurement (Heady, 1946, pp. 991). The estimated partial regression coefficients (b_1, \dots, b_k), of equation (1.7), computed by multiple linear regressions, are estimates of the net relationship between the respective X_k and Y . Each parameter is an estimate of the change in the dependent variable in response to a one-unit change in the particular X_k when the other independent variables are held constant, regardless of how Y and X_k are measured. However, Furtan and Gray (1981, pp.82), appreciated the double-log (Cobb-Douglas) function to be more useful in estimating production relationships because of its flexibility.

1.2.1.1.1 Coefficient of determination

The coefficient of multiple determination, R^2 , is a measure of the degree of linear association between the dependent variable and the collective independent variables. Strictly speaking, the coefficient is applicable only for the least squares estimator, since it forces the co-variances between the explanatory variables and the error term to be zero. Thus, the total variability of Y can be divided unambiguously between the variability of the X_k 's and ϵ_i .

$$R^2 = \frac{\text{variability in } Y \text{ associated with } X\text{'s}}{\text{total variability in } Y}$$

$$= \frac{\text{sum of squares due to regression}}{\text{total sum of squares}}$$

R^2 is a ratio with a range from zero to one. For example, if $R^2 = 0.75$, then 75% of the variability in Y is estimated as being associated with the variability of the X_k . Adding independent variables increases R^2 at least slightly even if there is no true relationship between the added independent variable and the dependent variable. Also, as the number of independent variables increases, the degrees of freedom of the equation decrease. Thus, for instance, two observations exactly determine the regression line for a simple two – variable equation; there are no degrees of freedom, and $R^2 = 1$. In general $R^2 = 1$ in a multiple regression when the model is a perfect fit. Intuitively, R^2 near one with very few degrees of freedom suggests a misleading overestimate of the actual degree of association. This idea has lead to the use of a corrected coefficient of determination,

$$R^2_{\text{adjusted}} = 1 - (1 - R^2) \frac{(N - 1)}{(N - K - 1)}$$

Where: N = number observations

k = number of independent variables

The interpretation of R^2_{adjusted} is the same as the interpretation of $R^2_{\text{unadjusted}}$, i.e. the proportion of the variation in Y associated with the variation in the X_k 's. It is mathematically possible, however, for R^2_{adjusted} to be negative. A negative coefficient should be treated as a zero level of association.

If the dependent variable is transformed in the process of equation specification, the coefficient of determination is no longer comparable between the transformed and untransformed versions. In particular, R^2 for an equation transformed to logarithms cannot be directly compared with the R^2 for the untransformed equation. In the logarithmic equation, R^2 estimates the proportion of the variation of the logarithm of Y , which is associated with the variation of the logarithms of the X_k . This is clearly not the same as measuring the proportion of the variance of the observed Y associated with the variation of the observed X_k . The antilog of the calculated values of $\log Y$ must be obtained and these values used with the observed Y to compute the R^2 value for a direct comparison with the untransformed equations. To re-emphasise the point, a large R^2 for an equation transformed to logarithms does not necessarily

mean that transformation is the preferred alternative (Tomek and Robinson, 1990).

1.2.1.1.2 Statistical tests

Heady and Dillon (1961) and Tomek and Robinson (1990) clearly stated that, in evaluating estimated variables, the most common statistical procedure is the t – test. The hypothesis usually tested is the null hypothesis of no relationship between an independent variable and a dependent variable. For example,

$$H_0: \beta_k = 0 \text{ versus } H_A: \beta_k \neq 0$$

Assuming that the errors are normally distributed with a mean of zero and variance one, the value of t to test whether the sample regression coefficient b_i is significantly different from zero at some probability level α is given by

$$t = \frac{b_k - \beta_k}{\sqrt{\text{var}(b_k)}}$$

Given the hypothesis $\beta_k = 0$, the computed statistic simply becomes

$$t = \frac{b_k}{S.E.(b_k)} \quad \text{where, } S.E.(b_k) = \sqrt{\text{var}(b_k)}$$

If the computed t statistic is larger than the tabled level of t_α with $(N - k - 1)$ degrees of freedom, then the b_k , is significantly different from zero at the α level of probability. Therefore, the null hypothesis is rejected and the alternative hypothesis is accepted (implying that there is a relationship between the dependent variable and the independent variable). In empirical research, a common statement is that the coefficient is statistically significant. But if the computed t is less than the table value, then the opposite is true.

1.3 Quantifiable Concepts related to Production Functions

1.3.1 Returns to scale and elasticity of production

Assuming no relevant inputs have been excluded in equation (1.12)

$$Y = f(X_1, X_2, \dots, X_k) \tag{1.12}$$

The estimates of the parameters of the Cobb – Douglas form of a production function provide the direct estimation of the elasticity of output with respect to inputs, and these elasticities are used to draw economic inferences regarding the allocation of resources. If the sum of

elasticities is greater than, equal to, or less than one, then correspondingly, returns to scale are increasing, constant, or decreasing (Hoch, 1962). However, Heady and Dillon (1961, pp.230) stated that practical statements about returns to scale can be made only if the entrepreneur can actually make a proportionate change in all the inputs considered. Where inputs are not in his control, he cannot make a proportionate change in every input factor. There is then little point in telling him that more profit can be attained by increasing or decreasing his scale of operation if uncontrollable factors are included in the recommendation.

1.3.2 Marginal product and value of marginal product

Considering the Cobb – Douglas production function (Debertin, 1986),

$$Y = aX_1^{b_1} X_2^{b_2} X_3^{b_3} \dots X_k^{b_k}, \quad (1.13)$$

the marginal product of X_k (MPP_{X_k}) can be obtained by the partial derivative of the production function with respect to the X_k , keeping other resources constant. For example

$$MPP_{X_1} = \frac{\partial Y}{\partial X_1} \Big|_{X_2, \dots, X_k}$$

The value of marginal product (VMP) is the value of the incremental unit of output resulting from an additional unit of input (X), when the output (Y) sells for a market price P_y . Thus,

$$VMP_{X_k} = \frac{P_y \cdot \Delta TPP}{\Delta X_k}$$

$$VMP_{X_k} = P_y \cdot MPP_{X_k}$$

Assuming the Cobb – Douglas approach, if the sample consists of firms, profit maximization by the k^{th} firm would yield,

$$VMP_{X_k} = \frac{P_y \cdot \partial Y}{\partial X_k} = P_{X_k} \quad (1.14)$$

Where, P_y and P_{X_k} are prices of output and input respectively, assumed to be constant to the firm under competition. Hence, when production involves k variable inputs the least cost (profit maximizing) rule can be generalized as:

$$\frac{VMP_{X_1}}{P_{X_1}} = \frac{VMP_{X_2}}{P_{X_2}} = \dots = \frac{VMP_{X_k}}{P_{X_k}} \quad (1.15)$$

However, Hoch (1962, pp. 35 -36) justified that equation (1.14) does not necessarily hold in

general but is rather a particular form of a more general case, which can be expressed by replacing equation (1.14) with equation (1.16):

$$\frac{P_y \cdot \partial Y}{\partial X_k} = P_{X_k} R_{X_k} \quad (1.16)$$

Where R_{X_k} is some constant not necessarily equal to one. The equation states that firm equilibrium has been attained where the value of marginal product equals the price of the factor times some constant. The constant R_{X_k} has been interpreted as reflecting institutional or entrepreneurial restrictions on farm behaviour.

1.3.3 Marginal rate of input substitution

The marginal rate of input substitution (MRIS) is the amount at which one input (X_2) must be substituted for another (X_1) to maintain a constant level of output (Y), when X_2 is increased by one unit, holding other inputs constant at their respective geometric means.

$$\text{MRIS of } X_2 \text{ for } X_1 = \left| \frac{dX_1}{dX_2} \right|$$

For the power function equation, the MRIS is given by Heady and Dillon (1961, pp.84) as:

$$\frac{\Delta X_1}{\Delta X_2} = \frac{-b_2 X_1}{b_1 X_2}$$

If inputs X_1 and X_2 are changed in constant proportions, the MRIS remains constant at b_2/b_1 , even though the level of output changes. However, this condition is unrealistic for two classes of inputs such as carbohydrate and protein feeds for a growing and fattening animal. It is known that the MRIS of high protein feeds for high carbohydrate feeds is highest when the animal is young, and declines as the animal ages and the growing stage merges into the fattening stage. Hence, the marginal rate of substitution will not remain constant. Nevertheless, it is possible that rates of factor substitution may remain constant as all inputs of the production process are increased in the same proportion for a firm (Heady and Dillon, 1961, pp. 84).

1.3.4 Least-cost combination of inputs

According to Doll and Orazem (1984, pp. 112 – 117), the least cost combination of inputs occur when:

$$\text{MRIS of } X_2 \text{ for } X_1 = -\frac{P_{X_2}}{P_{X_1}} \quad (1.17)$$

This can be rewritten as:

$$\frac{\Delta X_1}{\Delta X_2} = -\frac{P_{X_2}}{P_{X_1}} \quad (1.18)$$

Economic efficiency in the factor – factor relationship is attained when the necessary and sufficient MRIS is equal to or less than zero, as either ΔX_1 or ΔX_2 will be negative for movements along the isoquant in the economically relevant range. Clearly, the least–cost combination of X_1 and X_2 occurs when the isocost line just touches the isoquant. However, for some special cases, such as lumpy inputs where the isoquant has no slope, and for corner solutions, the least cost criterion given by equation (1.17) generally does not hold.

The criterion in equation (1.18) for the least–cost combination of inputs can be rewritten again as:

$$\frac{MPP_{X_2}}{MPP_{X_1}} = \frac{P_{X_2}}{P_{X_1}} \quad (1.19)$$

Substituting the derivatives of the production function into this expression, and equating to the given price ratio, enables the quantity of one input to be determined in terms of the other (i.e. X_1 in terms of X_2 and vice versa). Hence, for profit maximizing firm(s) from k resources, equation (1.14) of the previous section (1.3.2) is applied.

1.3.5 The cost function

A firm's cost function shows various relationships between its costs and its output rate. The firm's production function and the price it pays for inputs determine the firm's cost functions. Since the firm's production function can pertain to the short run or long run, it follows that the cost functions can also pertain to the short run or long run. The short run is a time period, which is so brief that the firm cannot change the quantity of some of its inputs (firm's plant

and equipment). These are the firm's fixed inputs. Inputs like labour, fertilizer and feed, which the firm can vary in quantity in the short run, are the firm's variable inputs. Given the optimal input combination, it is a simple matter to determine the profit maximizing firm's cost of producing any level of output, since this cost is the sum of each input used by the firm multiplied by the price of the input. Hence, given the cost of producing each level of output the firm's cost function can be defined (Mansfield, 1982, pp.169).

Doll and Orazem (1984, pp.52 – 59) further state that in practice, cost functions can be estimated directly from observed costs and output data records on farms or indirectly from estimated production functions.

Total Cost (TC) = Total variable costs (TVC) + Fixed Costs (FC)

$$TC = (P_{X_1}X_1 + P_{X_2}X_2 + \dots + P_{X_k}X_k) + FC \quad (1.20)$$

Variable costs are those that vary with the amount of output produced. Fixed costs are independent of the amount of output produced.

Average total cost (ATC) = $\frac{TVC + FC}{Y}$, where Y is the total output.

The marginal cost (MC) measures the change in total cost resulting from a unit change in output, i.e.

$$MC = \frac{dTC}{dY} = \frac{dTVC}{dY}, \text{ as fixed costs remain constant.}$$

Hence, the profit maximizing level of output can also be computed by applying the output rule:

$$MC = P_y$$

Where: P_y = price of output

1.4 Problems Encountered in the Analysis of Production Functions Based on Farm

Cross-sectional Survey Data

Doll and Orazem (1984, pp.22) point out that agricultural production is a complex biological process. But production functions relating output to inputs are real. Because of the complexity of agricultural process, the true mathematical forms of production functions are not known. However, Heady and Dillon (1961, pp.102) suggested that knowledge of biological, economic or other environmental factors might exist to provide some guide or basis for selecting a production function that best fits or describes the data. Frequently, however, previous knowledge as a basis for selection may not exist, so several algebraic forms may be used initially, along with various empirical criteria for selecting among production functions..

Rasmussen and Sandilands (1962, pp. 1) sorted out the principal problems encountered in the calculations of production functions based on farm accounts as follows:

1. Choice of function;
2. Choice of variables and difficulties arising from the high degree of inter-correlation between them;
3. Difficulties arising from possible errors of measurement of the input variables; and
4. The problem of inter-farm and intra-farm relationships;

1.4.1 Choice of function (algebraic form)

In choosing an algebraic form for a production function due consideration must be given to whatever is known of the logic or basic mechanics of the production process. Also the selected function must be computationally manageable both during estimation and validation. Different approaches or basis for selection of the 'best fit' algebraic form are discussed by Heady and Dillon (1961, pp. 203).

1.4.2 Choice of variables

Whatever type of model is used to depict the production process, the researcher has to decide on the variables to be used. The omission of relevant variables or the inclusion of unwarranted variables would bias the coefficients of the fitted model in an economic sense; and it would not be expected to truly depict the production process, either structurally or predictively.

Although, the choice of variable inputs is largely an arbitrary one, but it should necessarily give an empirical function. Regarding the inputs, Rasmussen and Sandilands (1962, pp.3) stated that the general principle has been followed of including all inputs that can be considered to determine the costs of achieving the given output. At the same time, it was thought preferable to group the inputs in such a way that they represented the main cost items to which the farmers (firms) are accustomed. Furthermore, Rasmussen and Sandilands (1962, pp.66 – 67) noted in their study that, the use of four independent variables gave a better fit and a smaller residual than the use of seven independent variables. Thus, they concluded in their research work that, some where between four and seven was the optimum number of variables for the farms production function analysis. However, further they recommended that, if one is interested in information about more variables, then a method of alternative further breakdown of some of the groups of inputs into many more variables might be preferable.

1.4.3 Aggregation of variables

The problems associated with aggregation over inputs are most likely to arise in the estimates of production functions for firms. In such cases the number of input categories is large and quality differences in inputs between and within firms are to be expected. At any point in time, a given farm firm employs a unique number of inputs combined in some fashion. In order to reduce the number of input variables to a manageable size, it usually becomes necessary to aggregate the different inputs into a smaller number of categories. The manner in which these inputs are aggregated may influence the estimated parameters, which are the basis of the total and marginal income (productivity) estimates. To minimize specification bias due to aggregation in such cases, two working rules should be used (Heady and Dillon 1961, pp.216 – 217; Plaxico 1955, pp. 665).

1. Perfect complements, i.e. resource categories that have to be used in fixed proportions, should be treated as a single input. The use of one such resource implies the use of its complements. To include each of the complementary categories would lead to multicollinearity because of the perfect correlation between levels of the complementary inputs.
2. Perfect substitutes should also be aggregated into a single input category.

Failure to aggregate perfect complements and perfect substitutes will bias estimates just as

will the aggregation of two inputs which are imperfect substitutes or complements.

Further more, Heady and Dillon (1961, pp.228 – 229) suggested that the usual procedure has been to simply add the money value of such micro – inputs to provide an aggregative input. However, the arithmetic sum of the micro – inputs introduces bias in the resultant estimates, except when the micro – inputs that are summed are always used in fixed proportions. Bias can be reduced by using as the aggregated input, not the arithmetic sum of the micro – inputs, but their geometric sum, i.e., their product. Thus, if micro – inputs x_1, x_2, \dots, x_n are to be aggregated into a single category X for use in a Cobb – Douglas function, aggregation as specified by equation (1.21) must be regarded as less satisfactory than aggregation as in equation (1.22). It is obviously anomalous to use equation (1.24) when the basic estimation feature of the Cobb – Douglas function is a logarithmic transformation into a linear form.

$$X = x_1 + x_2 + \dots + x_n \quad (1.21)$$

$$X = x_1 x_2 \dots x_n \quad (1.22)$$

A perfect method would be to aggregate the micro – input multiplicatively with each micro-input aggregated proportionately to its (unknown) elasticity of production. Similarly, Griliches (1957) stated that to aggregate several inputs into one is to use geometric indices with weights proportional to the elasticities of the respective inputs (Of course, if elasticities were known we would not be trying to estimate them).

1.4.4 Multicollinearity

Multicollinearity is a problem that arises if some or all of the explanatory variables are highly correlated with one another. If multicollinearity is present, the regression model would experience difficulty telling which explanatory variable(s) is influencing the dependent variable. A multicollinearity problem reveals itself through low t – statistics (i.e. large standard deviation). Cases of this nature could result to an erroneous conclusion that the coefficients are insignificant and hence should be dropped from the regression, while the R^2 is quite large and significant. Intuitively, this means that the explanatory variables together provide a great deal of explanatory power, but that multicollinearity makes it impossible for the regression to decide which particular explanatory variable(s) is providing the explanation (Koop, 2000, pp.88 – 89).

Dielman (1991, pp. 281 – 283) suggested some possible detecting methods for multicollinearity problems, which are by:

1. Computing the pairwise correlations between the explanatory variables. One rule of thumb suggested by some researchers is that multicollinearity may be a serious problem if any pairwise correlation is bigger than 0.5.
2. A large overall F statistic but small t statistics. Unfortunately, this method of detecting multicollinearity will not always be effective, as multicollinearity may result in some of the t values being small, but not all of them.
3. Computing variance inflation factors (VIFs). If X_1, X_2, \dots, X_k represent the k explanatory variables in a regression, then by regressing X_j on the remaining $k - 1$ explanatory variables, we can obtain a coefficient of determination R_j^2 , which is then used to compute the VIF for the variable X_j as follows:

$$VIF_j = 1/(1 - R_j^2).$$

A variance inflation factor is a measure of the strength of the relationship between each explanatory variable and all other explanatory variables in the regression. The value of R_j^2 measures the strength of the relationship between X_j and the other $k - 1$ explanatory variables. Although, it is not completely certain how large the VIFs have to be to suggest a serious problem of multicollinearity, below are some suggested guidelines:

- VIF_j larger than 10 indicates that multicollinearity may be influencing the least square estimates of the regression coefficients.
- If the average of the $VIF_j = \sum VIF_j / k$ is considerably larger than 1, then serious problems may exist, which indicates how many times larger the error sum of square for the regression is due to multicollinearity than it would be if the variables were uncorrelated.

Some of the possible solutions to multicollinearity problems suggested by different authors, even though they have some drawbacks, are:

1. To remove those variables that are highly correlated with others, but no information will be obtained on the omitted variables. Heady and Dillon (1961, pp.136) stated that if the absolute correlation coefficient of the variable inputs is close to or greater than 0.8, the regression analysis should be carried through with the highly correlated variables omitted. Which variable(s) to omit and

which to retain should be decided on the basis of the logic – physical, biological or economic relevance of the production process being examined.

2. A priori information from previous empirical work in which the collinearity problem happened to be less serious or from the relevant theory underlying the field of study; for example, in the Cobb-Douglas production function, if constant returns to scale is expected to prevail.
3. Adding more data can break the pattern of multicollinearity.
4. Other techniques like principal components and ridge regression are often used to solve the problems of multicollinearity.

According to Neter *et al.* (1990), ridge regression remedies multicollinearity by modifying the method of least squares to allow biased estimators of the regression coefficients. They also stated that when an estimator has only a small bias and is substantially more precise than an unbiased estimator, it may well be the preferred estimator since it will have a large probability of being close to the true parameter value. One of the limitations of ridge regression is that the optimum value of the biasing constant, c , to be used varies from one application to another and the choice is thus a judgmental one (usually between 0 and 1). However, a commonly used method for determining the value of c is based on the ridge trace and the VIF value of the explanatory variables. Explicit discussion on the ridge regression coefficients, VIF, and R^2 is given in Neter *et al.* (1990), as the biasing constant c is changed gradually from zero.

1.4.5 Omission of variables

Suppose all the X 's, the true inputs, are measurable except one, say X_k . As long as this variable is uncorrelated with any other X 's, its omission will not bias the estimates of $\beta_0, \beta_1, \dots, \beta_{k-1}$. However, still the assumption of zero correlation between the excluded factor and any other input is not likely to hold in the real world. In general, a certain degree of correlation is to be expected. Thus, the result will be a tendency to overestimate one or more of the coefficients of the included variables (Heady and Dillon 1961, pp.214; Koop, 2000, pp. 87). However, it is not clear if this tendency is enough to compensate for the downward bias in the estimate of E_p , and the returns to scale due to the omission of the coefficient of the excluded variable (Griliches, 1957, pp. 10 – 14).

1.4.6 Zero input levels

When using the Cobb – Douglas function, at least some quantity of each input must be used in order for the output not to be zero. But in the real world, such a condition does not hold true; indeed, most sample data, either real world or experimental, will contain observations with one or more of the inputs at zero level. The practical difficulty arises in the conversion of the raw data to logarithmic form, as the logarithm of zero is minus infinity. To overcome this problem, the zero observations may be replaced by some figure of arbitrary small size or a constant can be added to all observations for the particular input category (Heady and Dillon, 1961, pp.229; and Battes, 1997, pp.251). Battes (1997) further noted that if the number of ‘zero–observation cases’ is a significant proportion of the total number of sample observations, then excluding the ‘zero cases’ might result in seriously biased estimators of the parameters of the production function. The problem can be solved by using a dummy variable such that estimators are obtained with the full data set without introducing bias.

1.4.7 Inter-farm versus intra-farm interpretation

The Cobb-Douglas function applied to observations of different firms during a particular point or period of time describes inter-farm production relationships. Nevertheless, if the different firms are on essentially the same function, such inter-farm relationships could be useful in individual farm planning. In general, if the different farms use essentially similar techniques of production and produce essentially the same combination of products, it is not unreasonable to expect the production function of individual farms to closely resemble the derived inter-farm function (Plaxico, 1955, pp.672). Furthermore Plaxico (1955) suggested that the heterogeneity of functions can be eliminated by careful selection of sample farms, systematic aggregation, and judicious use of derived data to see that it is applied to similar farms. However, despite these precautions, estimates may be subjected to sizable bias due to:

- Exclusion of certain inputs;
- Managers likely seek to maximize returns over a time period other than the period considered;
- Farmers use a mixture of old and new techniques;
- Managers plan on the basis of expected price and technical relationships but the analysis is based on realized ratios.

Heady (1946) suggested that an inter-farm production function derived for a large geographical location at a specific point or period of time may result in biased estimates of the returns to specific resources used by individual farms. To minimize such biasedness, Heady divided the farms in his study into groups on the basis of location within the state and on the basis of scale of operations (total capital) to test the hypothesis that better managers are found on large farms and also that there might be a range of increasing as well as decreasing returns to scale. However, despite these strategic adjustments and assumptions, a certain percentage of unexplained variance in total product resulted from variation between farms in respect to techniques employed, weather conditions and perhaps, to some extent, prices received for products.

Similarly Hallam *et al.* (1999, pp. 441) confirmed that there are substantial farm-to-farm variations in relationships between inputs and outputs as a result of farm specific factors such as land quality and managerial ability as well as year-to-year variations in weather. Erington (1989, pp.52) also maintained that a central problem in analysing production relations in farm businesses is that most farms produce a mixture of outputs. It is therefore difficult to identify input-output coefficients relating to individual enterprises. Gordijn (1985), citing Bronfenbrenner (1944, pp.35 – 44) wrote that the use of inter-farm functions as a means of determining coefficients and applying these to the individual farm resulted in the coefficients being biased upwards. Explicitly, he summarized that if the inter-farm function is regarded as an envelope of the intra-farm ones, then the second derivatives of the envelope and the individual farm function differ in value, even though the first derivative is equal at the point of contact between inter-farm and intra-farm regressions.

1.4.8 Error of measurement in farm inputs

When the independent variables in regression analysis are subjected to errors of measurement, it follows that there is a systematic downward bias in the size of the regression coefficients. In multiple regression analysis this means that the individual coefficients, and therefore their sum, tend to be smaller than would have been the case had there been no errors of measurement (Rasmussen and Sandilands, 1962, pp.5).

Gujarati (1978, pp.324) showed that the errors of measurement pose a serious problem when they are present in the explanatory variable(s) because they make consistent estimation of the parameters impossible. But if they are present only in the dependent variable, the parameters remain unbiased and hence they are consistent too. As an alternative he suggested that, if errors are suspected in the explanatory variable(s), it is better to find other variables that he called “instrumental variables” which are highly correlated with the original variable(s). This is however, difficult to do in practice.

Gordijn (1985, pp.37), referring to the possible solutions proposed by Galpin (1981), suggested that errors of measurement should be detected and treated as outliers because their existence can make nonsense of the regression line obtained. Thus, the function should be recalculated with the outliers removed. If such errors are due to recording or copying they should be corrected and the regression rerun. But if no reason can be found for their incorrectness, then they can be retained, deleted or down-weighted.

1.4.9 Dummy variables

In regression analysis the dependent variable is frequently influenced not only by variables that can be readily quantified on some well defined scale, but also by variables that are essentially qualitative in nature. Hence, such qualitative variables should be included among the explanatory variables as dummies by constructing artificial variables, that take on values of 1 or 0, to indicate the presence or absence of that attribute, respectively (Gujarati, 1995).

Mayes (1981) pointed out three main uses of dummy variables in econometrics:

- To take account of structural changes in parameters;
- To take account of special events which would otherwise distort the rest of the estimates; and
- To represent categorical variables.

In this study the first form is used to test the effect of regional variation on structural changes in intercept and slope, when data from the three study areas are pooled. The second form is employed to avoid the zero input level problem as it existed frequently in the sample survey data for the farm feed input variable. The dummy variable used to analyse the zero input level problem for farm feed inputs (seed, fertilizers, herbicide/pesticide) costs was assigned a value

of 1 for those who have and/or rent in plots of land to grow green feed or a value of 0 otherwise. Two other dummy variables were used to pool the data of the three study areas to obtain a single production function for the entire Highland of Eritrea as follows:

	<u>Dummy 1</u>	<u>Dummy 2</u>
Central Zone	1	0
Mendefera	0	1
Dekemhare	0	0

The next Chapter deals with the research method used in this study and a description of the study areas.

Chapter 2

Research Methodology and Description of Study Areas

This chapter deals with the research methodology and general description of the dairy farmers in the Highlands of Eritrea. The general description gives some factual information about the study areas. Thus, it will be easy to place the results of this study in context and will allow agricultural advisors and farmers working with similar farming practices to use the production function results.

2.1 Research Methodology

2.1.1 Objectives

The overall objective of the project is to conduct a production function analysis of fresh milk production in the Highlands of Eritrea. The specific objectives are to estimate the elasticities of output with respect to various factors of production employed for the production year (2002) and to make economic calculations for each study area based on the results obtained from the Cobb-Douglas production function analysis. The calculations include the marginal product and value of marginal product of inputs, marginal rate of substitution and least-cost combination of inputs, profit-maximizing combination of inputs and short-run cost functions. The results of the analysis can help to identify the optimal levels and set of inputs that determine the yield of fresh milk for the sample of farmers. Furthermore, it can also serve as base information for agricultural economic advisors and dairy farm extension agents to advise farmers on how to improve the allocation of their scarce factors of production. The results can also be used as prior information for further research on similar farming systems.

2.1.2 Data collection method

The population of commercial dairy farms in the Highlands of Eritrea is mainly concentrated in the Central-Zone “Asmara” and Southern-Zone “Mendefera”, “Dekemhare” and their surroundings. According to Zeggu (1997), the distribution of the dairy cattle population is 75%, 8%, and 4% in the Central Zone, Mendefera and Dekemhare areas, respectively, with pure and grade Friesian breed compositions of 93%, 89% and 100%, respectively. Farmers in these areas are encouraged to deliver and sell their daily milk output to the milk collecting,

cooling and processing centres, where milk is processed and traded to retailers and consumers. Farmers benefit from this arrangement by having access to the milk market and access to concentrate feeds and industrial by-products (wheat middling, wheat bran, oilseed cake, brewery by-products, etc). The quantity of concentrates supplied to each farm is based on the number of registered dairy animals in the herd and on the quantity of milk the farm delivers, as the prices of inputs are lower than they are sold in the free market.

Thus, the samples of farmers included in this study were selected in three steps.

1. Farmers were regionally stratified to avoid qualitative differences (farming practice, biodiversity, and management skills) due to location.
2. Farmers were sorted according to the quantity of annual milk output in litres obtained from the milk collecting and cooling centres in each study area. Thus, farmers producing at least 12500 litres/year in the Central Zone and farmers producing more than 4300 litres/year in both study areas of the Southern Zone were selected.
3. Finally, based on the above criteria 48 farmers from the Central Zone "Asmara and its vicinity", and 72 farmers from the Southern Zone (42 and 30 farmers from Mendefera and Dekemhare, respectively) were randomly selected to avoid sampling bias.

Based on the questionnaire prepared (see Appendix A), targeted farmers were interviewed on their land utilization, herd structure, annual income, and annual expenses on variable inputs and fixed assets for the year 2002. However, since farmers in Eritrea are not experienced in financial-record keeping systems, and given the limited survey study period (of only three months), it was not possible to ascertain their capital investment at market value and to get an estimate of their fixed costs.

2.1.3 Methods of data analysis

The methods used for data analysis in this study include:

1. The Cobb-Douglas production function analysis using
 - 1.1 Multiple Regression Analysis methods.
 - 1.2 Quantification of economic concepts using the fitted production function equation.

The Cobb-Douglas production function is used because of the advantages it offers. For instance, the regression coefficients are the direct elasticities of production with respect to the factors of production, and the elasticities are independent of the unit of measurement. Others advantages were as discussed in Chapter 1. However, during the analysis using ordinary least square (OLS) multiple regressions of the Cobb-Douglas form, strong intercorrelations amongst explanatory variables were encountered. Thus, to avoid the multicollinearity problems a ridge regression technique is used.

The economic concepts quantified from the fitted model include:

- Marginal product (MP) and value of marginal product (VMP);
- Marginal rate of substitution (MRS) and least-cost combination of inputs;
- Profit maximizing combination of inputs; and
- Cost function.

2.2 Description of the Study Areas

The study areas comprise the Central zone, Southern Zone “Mendefera”, “Dekemhare” and their surroundings. However, before applying a production function analysis to the specific study areas analysed in Eritrea, it is important to give a general brief description of the existing situation of dairy farming in Eritrea. Teclu (1995), citing the FAO Livestock Expert Mission report (1992), mentioned that the population of exotic dairy cows showed a 66% reduction from 9,000 before 1975 to an estimated 3,000 in 1992. Thus, the current dairy farms are mainly the remnants of previous farms, which survived the difficult period of war, diseases, severe drought and harsh political administration, and a few farms established after independence.

According to an estimate of the MOA, there are currently about 10,000 dairy cattle in Eritrea, including lactating, dry cows and followers, which are kept for commercial milk production at small and medium scale level, with the exception of a few large-scale commercial dairy farms. Out of these, 8,600 cattle are believed to be pure Holstein Friesian breeds and their crosses, while the balance are local cows, mainly Barka breeds. Based on Zeggu (1997) during an ideal year, a dairy cow yielded 3803, 3485 and 3373 litres during a lactation of 323, 317 and 337

days, in the Central Zone, Mendefera and Dekemhare, respectively. This translates to an average daily milk yield of 11.8, 11 and 10 litres per cow, respectively, for the three study areas taken sequentially.

The average maximum and minimum temperatures and rainfall figures for the last ten years for each study area are shown in Table 2.1. The annual rainfall for Mendefera is slightly higher, followed by Asmara and Dekemhare. The lowest annual rainfall recorded for Asmara and its surroundings was in 1993, but for the Mendefera and Dekemhare study areas it was for the year 2002 (see Appendix B Table 1). The maximum and minimum temperatures for Mendefera seem to be slightly higher than for Asmara and Dekemhare.

Table 2.1: Average annual maximum and minimum temperatures and rainfall for the study Areas in Eritrea, 1992-2002.

AREA	TEMPERATURE		RAINFALL	
	MIN (°C)	MAX (°C)	MIN (mm)	MAX (mm)
Central Zone "Asmara"	9.0	23.3	303.0	688.5
Mendefera	10.1	28.4	390.0	750.0
Dekemhare	8.4	23.9	367.0	579.0

Source: Meteorology service of Eritrea (Civil aviation Eritrea).

2.2.1 Land utilization

It is believed that, for any farmer, the availability of own land for crop and fodder production is important, as it is the cheapest source of feed. In Eritrea most dairy farms are concentrated in and/or very close to cities and towns and over half of the dairy farmers have no irrigable land for forage and pasture production. Based on the sample of dairy farmers interviewed in the Central Zone, only 40% have, on average, 1.987 ha of own irrigable land. The remainder have no irrigable land at all or rent in land tentatively, but are not secured as: (1) the owner may take the land at the end of the contract (i.e. discontinuity of the lease), (2) no official land market, as land belongs to the state. The major crops grown by local farmers in this study area, in order of importance, are wheat, barely, maize, beans and chickpea. Some farmers also use the river and underground water to grow vegetables.

The Southern Zone Highland of Eritrea comprises two study areas, namely “Mendefera”, “Dekemhare” and their surroundings. The “Mendefera” study area includes those dairy farmers around “Dubaruwa” and “Tera-Emni” too. Based on the 42 dairy farmers interviewed in this area 41% use their own irrigable land (averaging 1.5ha) to grow green feed. The remainder uses either purchased green and cereal straw feeds or rent in land to grow fodder. The main crops grown by local farmers, in order of importance, are taff, wheat, barely, maize, sorghum, beans, and vegetables using mainly underground water.

The Dekemhare dairy farmers integrate dairy farming with their orchard and horticulture cultivation, with the exception of a few farmers. Out of the 30 farmers interviewed, 70% owned land (averaging greater than 2.4 ha) particularly around the “Ala” and “Gaden” areas, though the distribution and proportion of land use vary among farmers. Based on the sample data, 69% of the available irrigated land is used for orchard and horticulture cultivation and only 31% is used for green feed production. However, most farmers in this area interplant green feeds under orchard trees and fodder along water channels and border lines of vegetable plots. Wheat, barely, maize and beans are also grown by the local farmers.

Most dairy farmers in the three study areas depend mainly on purchased concentrates, cut and carry green feeds, as well as dried grass/hay and cereal straws to feed their herd. Moreover, as the available industrial by-products (wheat bran, wheat middling and oilseed cake) and concentrate feeds rationed monthly are not enough, farmers purchase whole grains like sorghum and maize at high cost. Some farmers also rent in land (conventional agreement) for green feed production, since there is no official land market, as land belongs to the state by the proclamation no. 58/1994 of 24th August 1994, proclamation no. 95/1997 of 19th May 1997 and legal notice no. 31/1997 of 19th May 1997. The law recognizes three main types of land rights: usufruct on farmland, housing land in rural areas, and leasehold. The law confirms that rights cannot be transferred unless where expressly provided by law. Illegal transactions are null, void and punishable as crime (Government of Eritrea, 1994).

2.2.2 Capital investment

To estimate the capital investment of the dairy farmers, an inventory should be made of the size, quantity and money value of each physical asset (land, fixed improvements, machinery, livestock on hand, stocks, supplies, etc.) compiled at the end of each consecutive financial year. However, in Eritrea, besides that physical assets owned by most dairy farmers are too old for valuations at market value and for calculating the depreciation, dairy farmers are not experienced in record keeping. Moreover, as the farmers are not fairly homogeneous in their asset type and structure the estimated capital investment may lead to bias. Despite this, most farmers use their own capital investment (cash, power machineries, dairy equipments etc.).

2.2.3 Annual farm incomes and expenses

The main annual incomes and variable expenditures of the sample dairy farmers in the three study areas for the year 2002 were collected via a survey conducted during April to June 2003, and include the following items:

Annual Incomes: The main source of income of the dairy farmers comprise: Sales of fresh milk (the annual formal and estimated informal milk sales), cattle sales (including calves and culled cattle) and other items such as manure sales.

Annual Expenses: This category includes the annual farm expenses on variable inputs and comprising the following items:

1. Annual purchased concentrate feeds, including licks;
2. Annual purchased forage, fodder, hay/straw and others;
3. Annual labour cost (including regular, casual, contract and family labour);
4. Annual AI, veterinary service and medicine costs incurred for dairy livestock;
5. Annual farm feed input expenses, which include fertilizer, seed, herbicides/pesticides, etc.; and
6. Annual operating and mechanical costs comprising electricity, water, fuel, oil, grease, repairs and spares.

All the above expenditures are measured in monetary value (Nfa). However, fixed costs such

as depreciation are not included as there is no recorded information and it is difficult to estimate the expenses on fixed assets. The average proportions for the above listed expenses for each study area are given in Table 2.2. However, the proportion may vary from year to year depending on the availability of resources, weather conditions, and price of resources.

From Table 2.2 the expenditure proportion on purchased concentrates including licks is the highest, followed by labour and purchased forage. Especially for the Mendefera study area, purchased concentrates including licks are the highest, not because farmers are using more of the specified input but because they are paying a higher price for the item due to transport distance from the source of the input. However, the proportion of labour cost is lowest in Mendefera relative to the other study areas.

Table 2.2. Proportions of annual expenditures on variable inputs in three study areas of the Highlands of Eritrea, 2002.

Inputs	Study Areas		
	Central Zone (n = 48)	Mendefera (n = 42)	Dekemhare (n = 30)
Concentrates & licks (%)	42.5	53.1	41.7
Forage (%)	21.3	17.7	21.6
Labour (%)	24.1	20.0	29.1
Veterinary & Medicine (%)	1.8	1.6	1.4
Farm feed inputs (%)	8.8	6.3	5.1
Operating & mechanical (%)	1.5	1.2	1.1

Thus, the proceeding Chapters (3 to 6) display the production function analysis of fresh milk production in the Highlands of Eritrea, using the above factors of production including milking cow as explanatory variable.

CHAPTER 3

A Production Function Analysis of Pooled Data of Dairy Farms in the Highlands of Eritrea

To construct a function that fits the sample of dairy farms of the Highlands of Eritrea as a whole, two intercept dummy variables (D1 and D2) were used to account for the differences in the three study areas. The dummy variable D1 takes the value of one if the area is Central Zone and zero otherwise, and D2 takes the value of one if the area is Mendefera and zero otherwise. Thus, the Dekemhare area takes the value of zero in both dummies.

The main objective of using dummy variables is to test if variation in geographic location is statistically a significant factor in milk production. Also, by introducing the multiplicative form (D_i multiplied by X_i), a test of significance may be performed among slope coefficients of the three study areas' data. Thus, if the intercept dummy variables' coefficients are statistically significant (i.e. different from zero), then the hypothesis that the three study areas have a common intercept can be rejected. Similarly, if the slope dummy coefficients are statistically significant (i.e. different from zero), then the hypothesis that the three study areas have a common slope can be rejected. Implicitly, there is variation in milk production among the three study areas.

The intercept and slope dummy coefficients for the pooled data estimated using a ridge regression of the Cobb-Douglas function at the biasing constant, $c = 0.174$, displayed in Table 3.1 show that D1 and D2 are statistically significant, indicating that the three study areas' regression equations have different intercepts. Furthermore, the coefficients of most slope dummy variables were statistically different from zero at the 1%, 5%, and 10% levels of probability. Thus, it would not be statistically and economically advisable to pool the data of the three study areas and fit a production function model that represents the sample of dairy farmers in the Highlands of Eritrea.

Table 3.1: Use of intercept and slope dummy variables to test whether the data of three study areas in the Highlands of Eritrea can be pooled.

Variables	Estimates (b_i)	SE	t-statistic
Concentrates	0.2692	0.0351	7.67
Forage	0.3040	0.0270	11.26
Labour	0.2585	0.0301	8.59
Veterinary & medicine	0.0476	0.0250	1.90
Operating & mechanical	0.0448	0.0156	2.87
Farm feed input (dummy)	0.1466	0.0503	2.92
Milking cows	0.3743	0.0251	14.90
D_1	0.0788	0.0136	5.78
D_2	-0.0546	0.0136	-4.02
D_1 -concentrates	0.0062	0.0011	5.64
D_1 - forage	0.0060	0.0012	5.00
D_1 -labour	0.0055	0.0011	5.00
D_1 -veterinary & medicine	0.0014	0.0018	0.78
D_1 -operating & mechanical	0.0033	0.0017	1.94
D_1 - farm feed input	0.0424	0.0581	0.73
D_1 -milking cows	0.0051	0.0069	0.74
D_2 -concentrates	-0.0029	0.0011	-2.64
D_2 -forage	-0.0011	0.0012	-0.92
D_2 -labour	-0.0014	0.0012	-1.17
D_2 -veterinary & medicine	-0.0053	0.0020	-2.65
D_2 -operating & mechanical	-0.0042	0.0024	-1.75
D_2 -farm feed input	0.0236	0.0626	0.38
D_2 -milking cows	0.0965	0.0143	6.75
Constant	0.1340	0.3753	0.36
F - test	121.354		
R^2_{adj}	0.959		

Hence, a separate production function analysis was conducted for each study area as long as there were enough degrees of freedom for each area's data. Therefore, the production function analysis conducted for each study area (Chapters 4, 5 and 6) would be preferable and advisable in order to make production function inferences and future planning decisions on dairy farming in the three study areas of the Highlands of Eritrea.

CHAPTER 4

A Production function analysis of Dairy farms in the Central Zone “Zoba-Maakel” Area

Data for the Central Zone “Zoba-Maakel” area of Eritrea were derived from 48 randomly selected farms in 2002. The sample of farmers included in this study are, however, only those:

1. Having exotic breeds of Holstein Friesians or upgrades of their cross breeds, which are fed indoor for milk producing purposes.
2. Who deliver their daily milk output to the milk collecting and cooling centres and produce at least 12,500 litres per annum, with few exceptions operating mixed farming systems and having own processing and marketing systems.

Annual milk output (Y) was plotted against annual expenditures on factors of production (purchased concentrates, purchased forage, labour, veterinary and medicine expenses, operating and mechanical costs, farm feed inputs, milking cows). The scatter plot revealed that the observations are randomly distributed with no severe violation of normality. Within this context, the Cobb-Douglas regression analysis was conducted with Y regressed on the seven independent variables. The results of the analysis showed strong correlations among some explanatory variables with the effect that some of the coefficients of the parameters had signs that did not make economic sense. The intercorrelations and respective VIF values for each explanatory variable obtained from the Cobb-Douglas function using ordinary least square (OLS) are displayed in Appendix C, Table C. 1. To avoid the impact of multicollinearity a ridge regression technique was used. For this analysis the biasing constant, c , is taken at 0.157 where the regression coefficients in the ridge trace start to get stable and VIF values are very close to one.

4.1 Results and Discussion

The final fitted Cobb-Douglas function using the ridge regression includes only those variables that are significant at the 5% and 1% levels of probability. Thus, the final fitted model of the Cobb-Douglas function is:

$$Y = 10.71X_1^{0.311} X_2^{0.19} X_3^{0.221} X_4^{0.402}$$

This is linear in its logarithmic form as:

$$\ln Y = 2.371 + 0.311 \ln X_1 + 0.19 \ln X_2 + 0.221 \ln X_3 + 0.402 \ln X_4$$

SE	0.050	0.044	0.049	0.058
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t – statistic	6.220	4.318	4.510	6.931
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R ² = 0.913	R ² _{adj.} = 0.897	Σb _i = 1.124
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Where, Y = annual milk output (litres);

X₁ = annual purchased concentrates including licks;

X₂ = annual purchased forage;

X₃ = annual labour cost;

X₄ = number of milking cows in 2002; and

SE = standard error.

The dependent variable (Y) is measured in litres, but all the independent variables used in the production process are measured in terms of value except that of “milking cows”, which is measured in physical terms.

The regression equation is a function linear in the logarithms. The Cobb-Douglas production function has been used since the regression coefficients are the elasticities of production. They indicate the percentage change in output that will, on average, result from a one percent increase in the input of various factors, other factors held constant. The elasticities are independent of the unit of measurement.

The coefficients indicate that the highest output response to 1% increase in input is that of cows (0.402). This indicates that, on average, an increase in the number of milking cows, X₄, by 1%, holding other factors constant, is associated with an increase in annual milk output, Y, of 0.402%. This would mean an average increase of 175 litres per annum for the sample farmers at the geometric mean of annual milk yield, and an increase of 104 and 317 litres per annum for the bottom and top one-third of sample farmers respectively, at their respective geometric means of annual milk yield. The standard errors of the regression coefficients are interpreted as follows: A one percent increase in milking cows, X₄, will, on average, increase annual milk output, Y, from 0.285 to 0.519 percent, holding other factors constant. The

elasticity of production for purchased concentrates is next highest followed by labour and purchased forage respectively.

All the elasticities (regression coefficients) are less than unity and therefore indicate diminishing marginal returns to each production factor, i.e. holding each of the other factors constant, the marginal return of each factor will decrease the more the factor is used. The elasticities of production also indicate the stage of the production function where the farmer is producing. Thus, all the regression coefficients of the function indicate use of resources within the rational area of production (stage II in Figure 1.1). The $\sum b_i$ shows increasing returns to scale for the sample of farms studied. Thus an increase in all factors of production proportionately by 1% will increase annual milk yield by $\sum b_i$ percent. However, this may not hold true at the farm level as some inputs could be out of the farmer's control. The adjusted coefficient of multiple determination indicates that 89.7% of the variation in output is explained by the factors included in the analysis.

4.1.1 Marginal product and value of marginal product

The marginal product is the addition to the total product associated with a small change in total input and the value of marginal product (VMP) indicates the value of the incremental unit of output resulting from an additional unit of input (X), when the output (Y) sells for a constant market price, P_y .

The milk price per litre in this study area for the year 2002 was 4.85 Nfa. The marginal products and value of marginal products for the variable inputs at their respective geometric mean values are shown in Table 4.1. However, as Gordijn (1985) stated in his study when analysing the VMP's, the economic theories of perfect knowledge, risk free and unlimited capital assumptions must be taken into consideration. Thus, with these assumptions, the optimum use of an input is where the VMP_x equals the input price (P_x) or where the ratio, $VMP_x/P_x = 1$. But this ratio seldom equals unity in the real world, since farmers operate in an imperfect competitive market. Thus, they may not allocate their resources optimally according to the $VMP_x/P_x = 1$ criteria because of lack of information, risk averseness and constraints on inputs and output.

Table 4.1: Marginal products and value of marginal products for various inputs, sample commercial dairy farms, Central Zone of Eritrea, 2002.

Inputs	Overall mean		Bottom one-third		Top one-third	
	MP	VMP (Nfa)	MP	VMP (Nfa)	MP	VMP (Nfa)
Concentrates	0.496	2.406	0.482	2.338	0.554	2.687
Forage	0.565	2.740	0.438	2.124	0.744	3.608
Labour	0.404	1.959	0.34	1.649	0.478	2.321
Milking cow	886	4297	799	3875	1068	5180

$VMP_{cow} = (MP_{cow} \times P_y)$. Cost of cow per annum = 2074 Nfa. Price of milk/litre = 4.85 Nfa

Table 4.1 shows that the VMP's for concentrates, forage, labour and milking cows are greater than their respective unit costs. Further, it shows that the MP's and VMP's for most of the factors of production were generally increasing from the bottom one-third of farmers to the top one-third. This implies that the resources are better utilized in the bottom category of farmers than by the top one-third of farmers despite still being under-utilised in all the categories. It appears therefore, that these inputs should be used more extensively up to the point where the VMP of the factor is equal to the input price. However, factors such as lack of information, risk averseness and other constraints may influence the optimum level of input use.

The annual cost of a milking cow is estimated using the capital recovery formula given by Monke and Pearson (1989). The formula calculates the annual payment that will repay the cost of a fixed input over the useful life of the input and will provide an economic rate of return on the investment.

$$A = Z [i (1 + i)^n / ((1 + i)^n - 1)]$$

Where, A = annual payment sufficient to pay the cost;

Z = cost of the fixed input;

n = useful life of the input; and

i = rate of return from the investment.

Thus annual cost of a productive milking cow, purchased for 12000 Nfa, having seven years of useful life and a 6000 Nfa salvage value, and generating a rate of return of 5% per annum, is on average estimated to be 2074 Nfa.

4.1.2 Marginal rates of substitution and least cost combination of inputs

Production function estimates can be used to predict substitution rates between resources. Thus, from the fitted Cobb-Douglas function, the marginal rate of substitution (MRS) between purchased concentrates (X_1) and purchased forage (X_2) at their respective geometric means, *ceteris paribus*, is calculated using the formulas discussed in section 1.3.3.

$$\begin{aligned} \text{MRS of } X_1 \text{ for } X_2 &= \frac{dX_2}{dX_1} = \frac{\partial Y / \partial X_1}{\partial Y / \partial X_2} = \frac{MPP_{X_1}}{MPP_{X_2}} = -\frac{b_1 X_2}{b_2 X_1} \\ &= -\frac{0.311 X_2}{0.19 X_1} \\ &= -0.88 \end{aligned}$$

By similar calculation, the MRS_{X_1, X_2} is calculated and found to be -1.1, and -0.74 for the bottom, and the top one-third categories of sample farmers, respectively.

The least cost combination of concentrates and forage, *ceteris paribus*, occurs when the MRS_{X_1, X_2} is equal to the inverse price ratio of concentrates and forage. Since the variables concentrates and forage are both measured in terms of value, i.e. costs per annum, the cost to purchase a one Nfa worth is one Nfa.

$$\begin{aligned} \text{Therefore, } MRS_{X_1, X_2} &= \frac{\Delta X_2}{\Delta X_1} = \frac{MPP_{X_1}}{MPP_{X_2}} = \frac{P_{X_1}}{P_{X_2}} \\ \Delta X_1 P_{X_1} &= \Delta X_2 P_{X_2} \\ (\text{added cost}) &= (\text{saved cost}) \\ \frac{\Delta X_2}{\Delta X_1} &= \frac{1}{1} \\ \text{Hence, } MRS_{X_1, X_2} &= \frac{P_{X_1}}{P_{X_2}} \\ \frac{MPP_{X_1}}{MPP_{X_2}} &= 1 \\ \frac{0.311 X_2}{0.19 X_1} &= 1 \\ X_1 &= 1.637 X_2 \\ X_2 &= 0.611 X_1 \end{aligned}$$

Thus, the least-cost combination of concentrates (X_1) and forage (X_2) for the year 2002, at the above rate of substitution occurs, *ceteris paribus*, when 20421 Nfa and 12476 Nfa, respectively, are spent on the inputs at the geometric mean of 43529 litres of milk yield per annum. The result reflects that there is a large difference in resource allocation when compared with the actual costs of 27318 Nfa and 14647 Nfa for concentrates and forage, respectively, that farmers spent at the same geometric mean of annual milk yield. The least-cost combination of concentrates and forage is calculated for the bottom one-third and top one-third of sample farmers to obtain more realistic and representative figures, which are displayed in Table 4.2.

Table 4.2: Least-cost combinations of concentrates and forage at different milk yield categories for sample dairy farmers, Central Zone of Eritrea, 2002.

Milk yield level catagory	Geometric mean of milk yield (litres)	Actual geometric means (Nfa)		Least-cost combinations (Nfa)	
		Concentrates	Forage	Concentrates	Forage
Overall mean	43529	27318	14647	20421	12476
Bottom one-third	25859	16689	11222	11904	7273
Top one-third	78809	44263	20139	40002	24438

Table 4.2 shows that for all the categories in the study, the actual cost of concentrates is higher than the least-cost combination of inputs. This implies that farmers are not allocating their resources on a least-cost basis. The most probable reason for the non-optimal allocation of these resources could be due to a lack of knowledge, the unstable price of the resources resulting from drought and shortage of industrial by-products, while the variable quality of feed mix and the existing breed may not give a high milk yield. Farmers were thus spending more than was required to be at the least-cost combination of resources at the particular yield level. However, the farmers are getting returns to the resources since the VMP's of the resources are exceeding their unit price, although the elasticities of production suggest that the farmers operate in the rational zone of production.

4.1.3 Profit maximizing combination of inputs

To produce an output from k inputs, at a minimum cost of inputs to maximize profit, provided that capital is not limited, the ratio of VMP's to the corresponding input price must be equal to one,

$$\text{i.e. } \frac{VMP_{x_1}}{P_{x_1}} = \frac{VMP_{x_2}}{P_{x_2}} = \dots = \frac{VMP_{x_k}}{P_{x_k}} = 1.$$

Under this assumption, the farmer should use inputs up to the point where the last unit cost spent on the input returns back a unit of revenue. However, if the farmer faces a limitation or constraints on the availability of capital to purchase the inputs, the next best alternative is to apply the equimarginal return principle, where costs are minimized for the level of output that can be produced. In other words, the above expression criterion has to be equated to be greater than one. Conversely, if the above expression is equated to less than unity, then the added cost exceeds the added returns, i.e. input use is above the optimum. The cost per unit input and price of output used in the determination of the optimum use of the resources to maximize profit for each category within the study area is presented in Table 4.3.

Table 4.3: Geometric means, unit of input/output, and cost per unit of input/output for the sample dairy farms in Central Zone of Eritrea, 2002.

Input/Output	Geometric means (Nfa)			Unit of Input/output (Nfa)	Cost/Price of unit (Nfa)
	Overall	Bottom one-third	Top one-third		
Concentrates	27318	16689	44263	1.00	1.00
Forage	14647	11222	20139	1.00	1.00
Labour	23816	16792	36402	1.00	1.00
Cows	20	13	30	-	2074
Milk	43529	25859	78810	1 litre	4.85

The profit maximizing combinations of inputs for each category within the sample at their respective geometric means, *ceteris paribus*, is summarized in Table 4.4. The figures show that, holding other factors constant at their geometric means, at the profit maximizing combination of concentrates and forage, annual milk yield can be improved from 43529 litres to 78143 litres on average for the sample of farmers, which is an increase of 79.5%. However, in maximizing profit, costs also increased by 59.5%, with an increment in margin of 100.2%.

The same analysis was conducted for each category within the sample studied, and results show an improvement of milk yield by 81.2% and 72.6% for the bottom one-third and top one-third respectively. The total expenditure increased by 48.7% and 78% and the margins by 124.6% and 68.6%, respectively. The increase in total expenditure in maximizing profit and the increase in margin makes sense for the bottom one-third of sample farmers. The increase in margin of the top one-third of sample farmers is much less when compared with the overall mean and bottom one-third sample of farmers. This is an indication that the top one-third of sample farmers are closer to the optimum level of production. However, the changes in improvements in the other categories appear to be too large and it leads to doubt whether the sample farmers can really achieve the optimal level of resource allocation to maximize profit in the short run, with their existing knowledge, breed quality, low quality feed mixtures and unstable prices of inputs and outputs.

Table 4.4: Profit maximizing combinations of inputs at different categories of sample dairy farms in the Central Zone of Eritrea, 2002.

Input/output	Overall mean		Bottom one-third		Top one-third	
	Actual	Profit max.	Actual	Profit max.	Actual	Profit max.
Cows	20	20	13	13	30	30
Litres/annum	43529	78143	25859	46863	78810	136006
Concentrates (Nfa)	27318	65657	16689	39004	44263	118873
Forage (Nfa)	14647	40112	11222	23829	20139	72623
Labour (Nfa)	23816	23816	16792	16792	36402	36402
Cows (Nfa)	41480	41480	26962	26962	62220	62220
Total cost (Nfa)	107261	171065	71665	106587	163024	290118
Total income (Nfa)	211116	378994	125416	227286	382229	659629
Margin (Nfa)	103855	207929	53751	120699	219205	369511
Margin/cow (Nfa)	5193	10396	4135	9285	7307	12317
Margin/litre (Nfa)	2.39	2.66	2.08	2.58	2.78	2.72
Litres/cow	2176	3907	1989	3605	2627	4534

4.1.4 Cost function

According to Doll and Orazem (1984, pp. 52 – 59), cost functions can be estimated directly from observed costs and output data recorded on farms or indirectly from estimated

production functions. For this study area, the cost function is estimated directly from the data recorded at their geometric mean and indirectly from the fitted Cobb-Douglas function. The cost function is:

$$TC = TFC + TVC$$

Since TFC remains constant in the short run, the TC is the summation of the annual cost of variable inputs at their geometric means. Thus, the total cost for the bottom one-third and top one-third of the sample dairy farms at their respective geometric means of annual milk yield (25859 and 78810 litres) were 71665 and 163024 Nfa, respectively. The short run marginal cost (MC) function is also calculated from the total cost for each variable annual cost, as the MC is the first derivative of the total cost with respect to output.

Hence,

$$\begin{aligned} MC &= \frac{dTVC}{dY} \\ &= d X_i \cdot \frac{P_{x_i}}{dY} \\ &= \frac{P_{x_i}}{MP_{x_i}}, \text{ Where, } i = 1, 2, 3 \dots k \end{aligned}$$

The total, average and marginal costs for each variable in each category of the geometric means of milk yield are calculated using the formulas discussed in section 1.3.5 and the above derivatives and are summarized in Table 4.5.

Table 4.5: The cost structure for different categories of fresh milk producers in the Central Zone area of Eritrea, 2002.

Inputs	Overall mean			Bottom one-third			Top one-third		
	TVC (Nfa)	AVC (Nfa)	MC (Nfa)	TVC (Nfa)	AVC (Nfa)	MC (Nfa)	TVC (Nfa)	AVC (Nfa)	MC (Nfa)
Concentrates	27318	0.63	2.02	16689	0.65	2.08	44263	0.56	1.81
Forages	14647	0.34	1.77	11222	0.43	2.28	20139	0.26	1.34
Labour	23816	0.55	2.48	16792	0.65	2.94	36402	0.46	2.09
Cows	41480	0.95	1.22	26962	1.04	2.60	62220	0.79	1.96
Total	107261	2.47	-	71665	2.77	-	163024	2.07	-

Table 4.5 shows that the MC for all farm categories is less than the selling price of fresh milk (4.85 Nfa/litre) and this implies that the resources are not used optimally. The MC for the bottom one-third is greater than the MC of the overall mean and top one-third of sample farmers and this coincides with the results obtained under the MP and VMP analysis conducted in the previous sections. So, in the short run, under competitive markets, the sample of fresh milk producers in the Central Zone area can use more inputs as long as the selling price of fresh milk per litre is greater than AVC and optimise their fresh milk yield up to the point where the MC is equal to the selling price of the output. This point lies on or above the AVC curve and it is the portion of MC that is in stage II of the production function (Doll and Orazem, 1984, pp. 64-72).

CHAPTER 5

A Production Function Analysis of Dairy Farms in the Southern Zone “Mendefera” Area

Data for the dairy production function analysis of the Mendefera and its surrounding area are taken from a random sample of 42 dairy farms for the year 2002. Dairy farms in this area are smaller in herd size as well as production scale (litres/annum) relative to the Central Zone dairy farms. A majority of the sample dairy farmers are local cereal growers while the remaining few farms were established after independence. Some farmers produce green feed through irrigation. The sampling criteria used is similar to that applied to the Central Zone except that the selected sample of farms included only those farmers producing at least 4300 litres of fresh milk per annum.

The annual milk yield was plotted against the values of six explanatory variables (purchased concentrates, purchased forage, annual labour cost, veterinary and medicine expenses, operating and mechanical costs, milking cows) and one dummy variable. From the scatter plot it was apparent that one of the farmer's annual milk yield far exceeded that of other sample farmers and his herd size and expenditure on dairy inputs were much higher than recorded in other farms. Based on a test for outliers this farmer was excluded before conducting a multiple regression analysis of annual milk yield (Y) on the explanatory variables. The dummy variable was assigned a value of one for those farmers who had land for growing green feed and/or spend on farm feed inputs (seed, fertilizer and weed/pest control) and zero otherwise. However, from the output of ordinary least square (OLS) regression, strong multicollinearity was encountered among some variables (see Appendix C, Table 2). To avoid the multicollinearity effect on the regression coefficients, a ridge regression analysis was used by modifying the ordinary least square (OLS) to allow a biasing constant, c , at 0.167. At this level of c , the ridge trace of the regression coefficients of most explanatory variables started to stabilise, and the VIF values of each explanatory variable and the average of the VIF values were close to one as well.

5.1 Results and Discussion

From the regression analysis, the explanatory variables included in the model are those which are statistically significant at the 1%, 5% and 10% level of probability, as the t-statistics for annual operating and mechanical cost and the dummy variable were considerably greater than one and nearly statistically significant at the 5% level of probability. Thus, the final estimated ridge regression model in Cobb-Douglas form is:

$$Y = 0.46X_1^{0.156} X_2^{0.410} X_3^{0.376} X_4^{0.029} X_5^{0.664} + 0.124D$$

This is linear in its logarithmic form as:

$$\ln Y = -0.777 + 0.156 \ln X_1 + 0.410 \ln X_2 + 0.376 \ln X_3 + 0.029 \ln X_4 + 0.664 \ln X_5 + 0.124D$$

SE	0.069	0.071	0.063	0.017	0.059	0.063
t – statistic	2.249	5.755	5.965	1.700	11.275	1.973
$R^2 = 0.954$	$R^2_{adj.} = 0.944$	$\Sigma b_i = 1.635$				

Where, Y = annual milk yield (litres);

X_1 = annual purchased concentrates including licks;

X_2 = annual purchased forage;

X_3 = annual labour cost;

X_4 = annual operating and mechanical costs;

X_5 = number of milking cows in 2002;

D = dummy variable for annual farm feed input expenses; and

SE = standard error.

All the variable inputs are measured in terms of value except for the annual milk yield (in litres), and the number of milking cows is measured in physical units.

From the estimated model the highest response in milk yield is due to a change in milking cow (X_5); that is, an increase of 1% in milking cows (X_5) is associated with an increase of 0.664% in annual milk yield, keeping other factors constant at their respective geometric means. This implies an increase of 57 litres for the entire sample of farms. The change in milk yield to a 1% change in milking cows is also calculated for the bottom one-third and top one-third of sample farmers at their respective geometric mean milk yield, so that the estimates will not be unrealistic. Hence, the changes in milk yield in response to a 1% change in milking cows at their respective geometric means of 5230 and 16655 litres are 35 and 111 litres per annum,

respectively. The elasticity of production with respect to forage (0.410) is the next highest followed by labour, concentrates, farm feed expenses and annual operating and mechanical costs. Thus, a 1% increase in the use of forage will increase milk yield by 0.410%, which is an increase of 35 litres per annum. A 1% increase in labour, concentrates, and annual operating and mechanical cost one at a time, *ceteris paribus*, is associated with an increase of 32, 13, and 3 litres per annum respectively. The regression coefficient of the dummy variable indicates that those farmers who have land for growing green feed tend to be relatively better off than those having no land and their annual milk yield is on average 0.124% higher, *ceteris paribus*.

The adjusted coefficient of multiple determination indicated, for the entire sample, that 94.4% of the variance in output is associated with the quantities of resources used. The elasticity of each factor of production is less than one, which indicates use of the resources in the rational area of production (stage II). However, the sum of the elasticities is greater than one, indicating increasing returns to scale when all the factors of production are increased proportionately by 1%.

5.1.1 Marginal product and value of marginal product

The marginal product (MP) indicates approximately the average return that might be expected from the addition of one unit worth of the various productive agents; and the value of marginal product (VMP) is the marginal product multiplied by the unit price of the product. The MP's and VMP's estimated at the geometric means of output are given in Table 5.1.

The milk price per litre for this study area for 2002 was 5.00 Nfa, and the annual cost of a productive cow purchased for 12000 Nfa, having an average useful life of nine years, a 5000 Nfa salvage value, and generating a rate of return of 5% per annum, is estimated to be 1688 Nfa using the capital recovery formula derived by Monke and Pearson (1989).

Table 5.1: Marginal products and value of marginal products for sample commercial dairy farms, Southern Zone “Mendefera” area of Eritrea, 2002.

Inputs	Sample mean		Bottom one-third		Top one-third	
	MP	VMP (Nfa)	MP	VMP (Nfa)	MP	VMP (Nfa)
Concentrates	0.110	0.549	0.083	0.415	0.166	0.830
Forage	0.570	2.850	0.411	2.055	0.840	4.200
Labour	0.423	2.114	0.308	1.540	0.614	3.070
Operating & Mechanical	0.168	0.840	0.134	0.670	0.220	1.100
Milking cow	819	4095	656	3280	1030	5150

$VMP_{cow} = (MP_{cow} \times P_y)$. Cost of cow per annum = 1688 Nfa. Price of milk/litre = 5.0 Nfa.

From Table 5.1 the VMP’s for purchased concentrates for the three categories of sample farmers is less than the unit price of the input. Similarly, the VMP’s for the annual operating and mechanical expenditures are less than their unit cost for the entire sample and bottom one-third category at their respective geometric means. This indicates use of inputs at more than their optimum level and a need to reduce them. However, the variables milking cow, forage and labour have VMP’s considerably greater than their unit costs. So, these resources need to be used more extensively, *ceteris paribus*. For the bottom category of sample farmers, the VMP of concentrates is well below the unit price of the resource, and it is the lowest VMP when compared with the VMP’s of concentrates of the other categories of sample farmers. This implies that these farmers were more dependent on the use of concentrates than others.

5.1.2 Marginal rate of substitution and least-cost combination of inputs

From the previous section, for all categories of sample farmers, it is clear that farmers in and around Mendefera are utilizing concentrates beyond the optimum level and under-utilizing forage. To assess how the farmers can improve their resource allocation, the marginal rate of substitution (MRS) and least-cost combination of the two resources are calculated. The marginal rate of substitution for the entire sample at the geometric mean of annual milk yield, *ceteris paribus*, was -0.193, and for the bottom and top one-third categories of sample dairy farmers the MRS was -0.202 and -0.198 respectively.

The least-cost combination, *ceteris paribus*, is when the marginal rate of substitution of concentrates for forage equals the inverse price ratio of the inputs.

$$\text{Therefore, } MRS_{x_1, x_2} = \frac{dX_2}{dX_1} = \frac{MPP_{x_1}}{MPP_{x_2}} = \frac{P_{x_1}}{P_{x_2}}$$

$$\frac{MPP_{x_1}}{MPP_{x_2}} = \frac{1}{1}$$

Since MPP_{x_1} and MPP_{x_2} are the first derivatives of concentrates (X_1) and forage (X_2) with respect to output, *ceteris paribus*, the least cost combination of the resources is when:

$$X_1 = 0.38X_2$$

$$X_2 = 2.63X_1$$

Thus, substituting the above expressions in the estimated model, the least cost combination of concentrates and forage is calculated for the entire sample and the bottom and top one-third of sample farmers at their respective mean milk yields (Table 5.2).

Table 5.2 Least-cost combinations of concentrate and forage at different milk yield categories for sample dairy farmers, Southern Zone “Mendefer” area of Eritrea, 2002.

Category	Geometric mean of milk yield (litres)	Actual geometric means (Nfa)		Least-cost combinations (Nfa)	
		Concentrates	Forage	Concentrates	Forage
		Sample mean	8609	12237	6192
Bottom one-third	5230	9810	5220	2280	5993
Top one-third	16655	15657	8131	4955	13022

From Table 5.2, at the geometric mean milk yield of the entire sample, the optimum combination of concentrates and forage, *ceteris paribus*, occurs when 3238 Nfa is spent on concentrates and 8510 Nfa on forage. This means a reduction of 8999 Nfa on concentrates and an increase of 2318 Nfa on forage compared to the actual use, which is a net gain (lower cost) of 6681 Nfa. A similar analysis for the bottom and top one-third of sample farmers indicated that, expenditure on concentrates could be reduced by 7530 and 10702 Nfa, and that on forage increase by 773 and 4891 Nfa, respectively. The margins of the least-cost combinations were reductions of cost of 6757 and 5811 Nfa for the bottom and top one-third of sample dairy farmers, respectively (see Table 5.2).

5.1.3 Profit maximizing combination of inputs

Provided that capital is not limited and the assumptions of perfect knowledge, free risk and competitive markets of inputs and outputs are not violated, the Southern Zone “Mendefera” area dairy farmers can optimise their resource use to maximize profit up to the point where, *ceteris paribus*, the value of marginal product of an input is equal to the corresponding resource’s unit price:

$$VMP_{x_1} = P_{x_1}$$

The cost per unit input and price of output used in determining the optimum use of concentrates and forage to maximize profit, keeping other factors constant at their geometric means for each category within the sample, is presented in Table 5.3.

Table 5.3: Geometric means, unit of input/output, and cost per unit of input/output for dairy farmers in the Southern Zone “Mendefera” areas of Eritrea, 2002.

Input/Output	Geometric Means (Nfa)			Unit of Input/Output (Nfa)	Cost/Price of unit (Nfa)
	Sample mean	Bottom one-third	Top one-third		
Concentrates	12237	9810	15657	1.00	1.00
Forage	6192	5220	8131	1.00	1.00
Labour	7656	6379	10195	1.00	1.00
Operating & Mechanical	1486	1131	2194	1.00	1.00
Cows	7	5	11	-	1688
Milk	8609	5230	16655	1 litre	5.00

The profit maximizing level of concentrates and forage are calculated for the entire sample, as well as for the bottom and top one-third of sample farmers at their respective geometric means of milk yield. The results are shown in Table 5.4.

Table 5.4 shows, *ceteris paribus*, that at the profit maximizing combination of concentrates and forage, annual milk yield can be improved from 8609 litres to 13009 litres on average for the entire sample of farmers, which is an increase of 51.1%. However, while maximizing profit, cost also increased by 15.1% with an increase in margin of 439.2%. By a similar analysis for the bottom one-third of farmers, milk yield can be improved by 39%, costs

decreased by 0.7% and the margin increased by 215.8%. For the top one-third of sample farmers milk yield and margin improved by 72.6% and 129.9%, respectively, with the cost increasing by 42.6%. For the bottom category of sample farmers the margin at the actual level was negative. However, at the optimum (profit maximizing) level, a gain in margin is revealed. This is because the sample farmers were already spending too much on concentrates and too little on forage. Thus, the farmers in all categories can improve their profit margin through proper allocation of resources at the optimum level.

Table 5.4: Profit maximizing combinations of inputs at different categories of sample dairy farms in the Southern Zone “Mendefera” area of Eritrea, 2002.

Input/Output	Entire sample mean		Bottom one-third		Top one-third	
	Actual	Profit max.	Actual	Profit max.	Actual	Profit max.
Cows	7	7	5	5	11	11
Litres/annum	8609	13009	5230	7269	16655	28739
Concentrates (Nfa)	12237	6715	9810	4079	15657	12991
Forage ((Nfa)	6192	17648	5220	10722	8131	34143
Labour (Nfa)	7656	7656	6379	6379	10195	10195
Oper. & Mechanical (Nfa)	1486	1486	1131	1131	2194	2194
Cows (Nfa)	11816	11816	8440	8440	18568	18568
Total cost (Nfa)	39387	45321	30980	30751	54745	78091
Total income (Nfa)	43045	65045	26150	35345	83275	143695
Margin (Nfa)	3658	19724	-4830	5594	28530	65604
Margin/cow (Nfa)	523	2818	-966	1119	2594	5964
Margin/litre (Nfa)	0.425	1.516	-0.924	0.770	1.713	2.283
Litres/cow	1230	1858	1046	1454	1514	2613

5.1.4 Cost function

The cost function for dairy farms of the Southern Zone “Mendefera” area is estimated directly from the observed costs and output data collected at their geometric means and indirectly from the fitted production function equation. The cost function is:

$$TC = TFC + TVC$$

Since TFC remains constant in the short run, the TC is defined as the summation of the annual cost of variable inputs at their geometric means. Thus, the TC for the entire sample, bottom

and top one-third of sample farmers at their geometric means of milk yield (8609, 5230 and 16655 litres) were 39029, 30980 and 54300 Nfa, respectively. The short-run marginal cost (MC) function is also calculated from the TC for each annual variable cost, as the MC is the first derivative of the TC with respect to output.

Hence,

$$\begin{aligned} MC &= \frac{dTVC}{dY} \\ &= d X_i \cdot \frac{P_{X_i}}{dY} \\ &= \frac{P_{X_i}}{MP_{X_i}}, \text{ Where, } i = 1,2,3\dots k \end{aligned}$$

and the optimum level of production is where MC = price of output (P_y).

The total, average and marginal costs for each variable in each category are calculated using the formulas discussed in section 2.3.5 and the above derivatives, and are given in Table 5.5.

Table 5.5: The cost structure for different categories of sample fresh milk producers in the Southern Zone “Mendefera” area of Eritrea, 2002.

Inputs	Entire sample mean			Bottom one-third			Top one-third		
	TVC (Nfa)	AVC (Nfa)	MC (Nfa)	TVC (Nfa)	AVC (Nfa)	MC (Nfa)	TVC (Nfa)	AVC (Nfa)	MC (Nfa)
Concentrates	12237	1.421	9.111	9810	1.876	12.023	15657	0.940	6.026
Forage	6192	0.719	1.754	5220	0.998	2.434	8131	0.488	1.191
Labour	7656	0.889	2.365	6379	1.220	3.244	10195	0.612	1.628
Oper. & Mech.	1486	0.173	5.950	1131	0.216	7.460	2194	0.132	4.543
Cows	11816	1.373	2.532	8440	1.614	3.162	18123	1.088	2.014
Total	39387	4.575	-	30980	5.924	-	54745	3.260	-

From Table 5.5 the MC for concentrates for the entire sample, bottom and top one-third of dairy farmers is greater than the selling price of fresh milk (5.00 Nfa). Similarly, the MC of operating and mechanical cost for the entire sample and bottom one-third of dairy farmers is greater than the unit price of output. This implies that the resources are over-utilized perhaps because the severe drought encountered during the year could have caused shortages of

alternative feeds, and secondly, the distance of the study area (56 km) from the source of the purchased feed could result in the sample farmers spending more to acquire these resources. However, the other resources are used inadequately since the MC for each resource is less than the product price. This agrees with the VMP's discussed in section 5.1.1. Therefore, in the short run, under competitive markets, unlimited capital and a risk free environment the sample of fresh milk producers in the Southern Zone "Mendefera" area can use more inputs as long as the selling price of fresh milk per litre is equal to or greater than AVC and optimise their fresh milk up to the point where the MC is equal to the selling price of fresh milk per litre.

CHAPTER 6

A Production Function Analysis of Dairy Farms in the Southern Zone “Dekemhare” Area

The data for a production function analysis of dairy farms in the Dekemhare area was extracted from a sample of 30 dairy farmers in 2002. Most of the sample farmers grow horticulture products with the exception of a few farmers who grow local cereals as well. The dairy farms' size, milk yield, and marketing systems are similar to the “Mendefera” area discussed in the previous chapter. Hence, the sampling criteria used were the same as in the previous chapter.

The scatter plot of annual milk yield (Y) versus the independent variables (purchased concentrates, purchased forage, annual labour cost, annual veterinary and medicine expenses, annual operating and mechanical costs, farm feed inputs (dummy variable), and number of milking cows) indicated that one farmer was an outlier in terms of his annual milk yield and costs of dairy farm inputs. Having excluded this outlier, a multiple regression analysis of annual milk yield (Y) on the independent variables was conducted. The dummy variable was assigned a value of one for farmers having land for growing green feed and zero otherwise. Incidentally, the farmers who had land were also those who purchase farm feed inputs (fertilizer, seed, and herbicides/pesticides).

From the results of ordinary least square multiple regression analysis, strong intercorrelations among the explanatory variables were evident (see Appendix C, Table 3). To remedy the multicollinearity impact on the estimates of parameters, a ridge regression technique was employed. The ridge regression coefficients started to stabilize at the 0.167 level of the biasing constant, c , when the variance inflation factors (VIF) and the average VIF were close to one.

6.1 Results and Discussion

From the ridge regression of the Cobb-Douglas function, all the explanatory variables were found to be statistically significant at the 1% and 5% levels of probability (Table 6.1). Table 6.1 shows that the elasticities of production for all coefficients are less than unity, indicating

use of the inputs within the rational area of production (stage II). The highest response comes from milking cows (0.417), followed by forage and labour, the elasticities of which are 0.291 and 0.247, respectively.

Thus, a 1% increase in milking cows, *ceteris paribus*, is associated with an annual increase in milk yield of 40 litres for the sample mean, and of 25 and 76 litres for the bottom and top one-third of sample farmers, respectively, at their respective geometric means of annual milk yield. However, farmers having land for growing green feed were found to achieve better milk yields than those having no land, as the dummy variable for farm feed input was statistically significant at the 5% level of probability. Hence, the annual milk yield of dairy farmers having land is higher than those having no land.

The variable veterinary was not statistically significant in both the Central Zone and Mendefera study areas, which is an indication of good health care management in the herd. However, for the Dekemhare dairy farms the variable veterinary is found to be statistically significant, implying a need to invest in herd health care. Thus, an increase in the veterinary and medicine cost of 1% will result to an increase annual milk yield of 0.112%, *ceteris paribus*.

The function explains 96.1% of the variation in milk yield and shows increasing returns to scale, as the sum of the coefficients of the parameters is greater than unity. Thus, increasing all explanatory variables proportionately by 1% will increase annual milk yield by 1.595%. This may, however, not always hold true for all farmers, as some of the variable inputs may be out of the farmer's control.

Table 6.1: The Cobb-Douglas (ridge regression) function for sample dairy farms, Southern Zone “Dekemhare” area of Eritrea, 2002.

Variables	Estimates (b _i)	SE	t-statistic
Concentrates	0.190	0.090	2.11
Forage	0.291	0.053	5.49
Labour	0.247	0.064	3.89
Veterinary and medicine	0.112	0.042	2.67
Operating and mechanical	0.205	0.047	4.36
Milking cows	0.417	0.054	7.72
Farm feed input (dummy)	0.133	0.060	2.22
Constant	-0.578	0.737	-0.784
F - test	99.339		
Σb_i	1.595		
R ² adj.	0.961		

Σb_i = sum of the coefficients.

R²adj. = adjusted multiple correlation coefficient.

6.1.2 Marginal product and value of marginal product

The estimated annual cost of a milking cow and price of fresh milk received by the Dekemhare sample of dairy farmers are the same as for the Mendefera area, namely 1688 Nfa and 5.00 Nfa/litre, respectively. The marginal products and value of marginal products are given in Table 6.2. Most of the variable inputs are under-utilized, as the VMP's are greater than their corresponding unit price. The VMP of the veterinary and medicine expenses is the highest relative to the unit price of the variable, followed by operating and mechanical, forage, and milking cow, in this order, for all the categories of sample dairy farmers. However, the variable concentrates is over-utilized in all groups of sample dairy farmers, while labour is over-utilized by the bottom one-third of sample farmers only.

Table 6.2: Marginal products and value of marginal products for sample dairy farms, Southern Zone “Dekemhare” area of Eritrea, 2002.

Inputs	Sample mean		Bottom one-third		Top one-third	
	MP	VMP (Nfa)	MP	VMP (Nfa)	MP	VMP (Nfa)
Concentrates	0.114	0.570	0.088	0.440	0.172	0.860
Forage	0.528	2.640	0.447	2.235	0.648	3.240
Labour	0.219	1.095	0.189	0.945	0.286	1.430
Veterinary & medicine	2.257	11.285	2.098	10.490	2.776	13.880
Operating & mechanical	1.014	5.070	1.013	5.065	1.135	5.675
Milking cow	564	2820	480	2440	693	3465

$VMP_{cow} = (MP_{cow} \times P_y)$. Cost of cow per annum = 1688 Nfa. Price of milk/litre = 5.0 Nfa

Thus, an additional unit increase of veterinary and medicine cost returns back revenue of nearly 11.285, 10.490 and 13.880 Nfa for the overall sample, bottom and top one-third of dairy farmers. An additional milking cow increases annual milk yield by 471.6 litres/annum for the entire sample and by 456.5 and 461.7 litres/annum for the bottom and top one-third of sample farmers, respectively. Therefore, the sample dairy farmers can improve their annual milk yield and returns by utilizing the resources at optimum level.

6.1.2 Marginal rate of substitution and least-cost combination of inputs

The marginal rate of substitution (MRS) of concentrates for forage are calculated for the whole sample and the bottom and top one-third of sample farmers at their respective geometric means of milk yield. Keeping the other variables constant at their geometric means, the estimates of MRS of concentrates for forage were -0.215 for the entire sample, and -0.197 and -0.265 for the bottom and top one-third of sample dairy farms, respectively.

The least-cost combination, ceteris paribus, is where the MRS of concentrates (X_1) for forage (X_2) equals the inverse input price ratio, i. e. where $MRS = 1$.

$$MRS_{X_1, X_2} = \frac{dX_2}{dX_1} = \frac{MPP_{X_1}}{MPP_{X_2}} = \frac{P_{X_1}}{P_{X_2}}$$

$$\frac{MPP_{X_1}}{MPP_{X_2}} = 1$$

Since MPP_{X_1} and MPP_{X_2} are the first derivatives of concentrates (X_1) and forage (X_2) with respect to output, *ceteris paribus*, the least cost combination of the resources is when:

$$X_1 = 0.653X_2$$

$$X_2 = 1.531X_1$$

Thus, substituting the above expressions in the estimated model, the least cost combination of concentrates and forage is calculated for the entire sample and for the bottom and top one-third of sample farmers (Table 6.3). Therefore, at the geometric mean of milk yield (9460 litres), the optimum combination of concentrates and forage, *ceteris paribus*, is when 7141 Nfa is spent on concentrates and 10936 Nfa on forage. This means a reduction of 8669 Nfa on concentrates and an increase of 5720 Nfa on forage or a gain (lower cost) of 2949 Nfa compared to the actual use of inputs.

Table 6.3: Least-cost combinations of concentrates and forage at different milk yield levels, sample dairy farmers, Southern zone “Dekemhare” area of Eritrea, 2002.

Category	Geometric means of milk yield (litres)	Actual geometric means (Nfa)		Least-cost combinations (Nfa)	
		Concentrates	Forage	Concentrates	Forage
		Sample mean	9460	15810	5216
Bottom one-third	5924	12793	3860	5564	8521
Top one-third	18271	20211	8205	11078	16967

For the bottom one-third of sample farmers the reduction on concentrates is 7229 Nfa and an increase of 4661 Nfa on forage or a gain of 2568 Nfa, which is a higher net gain (lower cost) than for the top one-third category of sample farmers. The expenditure on concentrates for the top one-third category reduced by 9133 Nfa, but increased by 8762 Nfa on forage resulting to a net gain (lower cost) of 371 Nfa. This is due to the fact that the bottom one-third farmers were highly over-utilizing concentrates compared to the top one-third of sample dairy farmers.

6.1.3 Profit maximizing combination of inputs

The VMP's estimated from the Cobb-Douglas function have shown that most of the factors of production included in the analysis were considerably under-utilized. The variable concentrates is over-utilized and forage is under-utilized by all three categories of sample

farmers at their respective geometric means of milk yield. Hence, to establish the optimum use of concentrates and forage, *ceteris paribus*, using the figures given in Table 6.4, the profit maximizing use of the resources occurs when:

$$\frac{VMP_{x_1}}{P_{x_1}} = \dots = \frac{VMP_{x_k}}{P_{x_k}} \text{ is satisfied.}$$

Table 6.4: Geometric means, unit of input/output, and cost per unit of input/output for dairy farmers in the Southern Zone “Dekemhare” area of Eritrea, 2002.

Input/Output	Geometric means (Nfa)			Unit of input/output (Nfa)	Cost/price of unit (Nfa)
	Entire sample	Bottom one-third	Top one-third		
Concentrates	15810	12793	20211	1.00	1.00
Forage	5216	3860	8205	1.00	1.00
Labour	10681	7740	15757	1.00	1.00
Veterinary & medicine	469	316	737	1.00	1.00
Operating & mechanical	1912	1198	3301	1.00	1.00
Cows	7	5	11	-	1688
Milk	9460	5924	18271	1 litre	5.00

Thus, the optimum use of concentrates and forage at the profit maximizing criteria, *ceteris paribus*, is solved for the three categories given in Table 6.5. At profit maximizing level of resource utilization, milk yield increases by 12%, cost by 4% and margin by 273% for the entire sample of farmers. Similarly for the top category of sample dairy farmers, milk yield increases by 24.1%, and cost and net margin by 24% and 24.4%, respectively. However, for the bottom one-third of sample farmers’ milk yield can be improved very slightly (0.56%), with a decrease in total cost by 7% and an increase in net margin of 54%. Nevertheless, the bottom one-third farmers would still be making a loss at the optimum level, albeit a much reduced one. This needs to be further investigated as the analysis is only based on one-year data. However, this may indicate that small-scale farmers experience great difficulty in surviving financially.

Table 6.5: Profit maximizing combinations of inputs at different categories of the sample dairy farms in the Southern Zone “Dekemhare” area of Eritrea, 2002.

Input/Output	Entire sample mean		Bottom one-third		Top one-third	
	Actual	Profit max.	Actual	Profit max.	Actual	Profit max.
Cows	7	7	5	5	11	11
Litres/annum	9460	10567	5924	5957	18271	22676
Concentrates (Nfa)	15810	8987	12793	5628	20211	17357
Forage (Nfa)	5216	13764	3860	8619	8205	26584
Labour (Nfa)	10681	10681	7740	7740	15757	15757
Vet. & Medicine (Nfa)	469	469	316	316	737	737
Oper. & Mechanical (Nfa)	1912	1912	1198	1198	1260	1260
Cows (Nfa)	11816	11816	8440	8440	18568	18568
Total cost (Nfa)	45904	47629	34347	31942	64738	80263
Total income (Nfa)	47300	52835	29620	29785	91355	113380
Margin (Nfa)	1396	5206	-4727	-2156	26617	33117
Margin/cow (Nfa)	199	744	-945	-431	2420	3011
Margin/litre (Nfa)	0.148	0.493	-0.798	-0.362	1.457	1.460
Litres/cow	1351	1510	1185	1191	1661	2061

6.1.4 Cost function

The cost function for dairy farms of the Southern Zone “Dekemhare” area is estimated directly from the observed costs and output data collected at their geometric means and indirectly from the fitted Cobb-Douglas equation. Thus, the cost function is:

$$TC = TFC + TVC$$

Since TFC remains constant in the short run production period, the TC is defined as the summation of the annual cost of variable inputs at their geometric means. Thus, the TCs for the entire sample, and the bottom and top one-third of sample farmers’ geometric mean milk yields (9460, 5924 and 18271 litres/annum) were 45904, 34347 and 64738 Nfa, respectively. The short run marginal cost (MC) is calculated from the TC for each annual variable cost, as the MC is the first derivative of TC with respect to output.

Hence:

$$\begin{aligned}
 MC &= \frac{dTVC}{dY} \\
 &= d X_i \cdot \frac{P_{X_i}}{dY} \\
 &= \frac{P_{X_i}}{MP_{X_i}}, \text{ Where, } i = 1,2,3\dots k
 \end{aligned}$$

The optimum level of production occurs when the MC of the variable input is equal to the unit price of output (P_y). Thus, to determine whether the Dekemhare area sample dairy farmers are producing rationally, i.e. in region II, the average variable cost (AVC) and MC for the overall sample, bottom and top one-third of sample dairy farmers were calculated using the above derivatives and formulas discussed in section 2.3.5. The results are displayed in Table 6.6.

Table 6.6: The cost structure for different categories of sample dairy farmers of the Southern Zone “Dekemhare” area of Eritrea, 2002.

Input	Entire sample mean			Bottom one-third			Top one-third		
	TVC (Nfa)	AVC (Nfa)	MC (Nfa)	TVC (Nfa)	AVC (Nfa)	MC (Nfa)	TVC (Nfa)	AVC (Nfa)	MC (Nfa)
Concentrates	15810	1.671	8.796	12793	2.160	11.365	20211	1.106	5.822
Forage	5216	0.551	1.895	3860	0.652	2.239	8205	0.449	1.543
Labour	10681	1.129	4.571	7740	1.307	5.289	15757	0.862	3.492
Veterinary and medicine	469	0.050	0.443	316	0.053	0.363	737	0.040	0.360
Operating and mechanical	1912	0.202	0.986	1198	0.202	1.544	3301	0.181	0.881
Cows	11835	1.251	3.000	8440	1.425	4.002	18568	1.016	2.437
Total	45904	4.854	-	34347	5.799	-	64738	3.654	-

From Table 6.6 it is evident that the sample fresh milk producers in the Dekemhare area are operating in region II of the production function, as the $MC > AVC$ for all factors of production analysed. Thus, if the objective of the dairy farmers is to generate profit, then they should use inputs where the unit price of milk (P_y) is greater than the AVC. For the overall sample and top one-third of sample dairy farmers, P_y (5 Nfa) $>$ AVC's (4.854 and 3.654) which implies that the farmers are covering all their variable costs and part or all of their fixed costs. However, for the bottom one-third of sample dairy farmers the $AVC > P_y$, thus they are not covering the total variable costs of production.

The MC of concentrates is greater than P_y for all the groups of sample dairy farmers. Similarly, the MC of labour for the bottom one-third of sample dairy farmers is greater than the unit price of output. This indicates that these resources are over-utilized and their use needs to be reduced to optimum (where $MC = P_y$).

DISCUSSION AND CONCLUSIONS

The production function analysis using the Cobb-Douglas function of ordinary least square (OLS) multiple regression showed strong intercorrelations among the explanatory variables. Thus, to avoid the impact of multicollinearity on the estimated parameters, a ridge regression technique was used.

An attempt to pool all the data of the three study areas using dummy variables was also made to estimate a function that fits for the dairy farms in the Highlands of Eritrea as a whole. However, the coefficients of the dummy variables for testing intercepts and slopes differed from zero showing that the three study areas differed from each other. It was thus neither statistically nor economically advisable to implement an analysis based on pooled data since the areas are significantly different from each other. Therefore, if the objective of sample dairy farmers is to maximize profit from the available resources and management, it will be more preferable to base future planning decisions on how to produce and how much to produce on the separate analysis for each study area.

Central Zone “Zoba Maakel”

For the Central Zone sample of dairy farmers, the regression coefficients are less than unity but greater than zero, indicating rational use of the resources. The highest response in milk production to a 1% increase, *ceteris paribus*, is due to milking cows, followed by concentrates, labour and forage. The summation of the regression coefficients (i.e. elasticities) is greater than unity, implying increasing returns to scale. Thus, if the farmer can increase all the factors of production proportionately by 1%, milk yield will increase by more than 1%. The adjusted coefficient of multiple determination (89.7%) indicates a very good fit of the model.

The VMP's indicated that all the resources in the categories of the sample dairy farmers considered for 2002 were under-utilized since the VMP's were found to be greater than the respective unit price of the inputs. Thus, under assumptions of a risk free environment, unlimited capital and competitive markets of inputs and output, the sample farmers can use these resources more extensively, up to the point where the VMP's of the resources are equal

to the corresponding unit price of the inputs. The VMP's also indicate the highest return to a unit increase of an input relative to the unit price/cost of the input. For the bottom one-third of sample dairy farmers the highest return of an additional input comes from concentrates (2.338 Nfa) followed by forage, milking cows and labour, which had VMP's of 2.124, 3880 and 1.649 Nfa, respectively. However, for the top one-third of sample dairy farmers the highest return generated is from a unit addition of forage (3.606 Nfa), followed by concentrates, labour and milking cow. Thus, the sample of farmers can maximize their returns to resources and management accordingly.

For all categories of sample dairy farmers, the marginal rate of substitution of concentrates for forage, *ceteris paribus*, showed a near-unity rate of reduction of forage when concentrates use is increased by 1%, to maintain the same level of output. This implies that the sample farmers are spending the two inputs nearly equally, except for the top one-third of sample farmers who were utilizing slightly more concentrates. The reason could be due to the high quantity of fresh milk they deliver to the milk-collecting centre and as a result receive more concentrates. However, an analysis of the least cost combination of the two resources, *ceteris paribus*, showed none of the categories of sample dairy farmers was allocating them optimally.

Keeping other factors constant at their respective geometric means, milk yield at the profit maximizing use of concentrates and forage is improved on average by 1731, 1616, and 1907 litres/cow per annum for the entire sample, the bottom and top one-third of sample dairy farmers, respectively. However, while doing so, the expenditures per cow per annum also increased on average by 3190, 2686, and 4237 Nfa, respectively, with an increase in margin/cow per annum of 5203, 5150, and 5010 Nfa for the entire sample, the bottom, and top one-third sample of dairy farmers, respectively. Thus, the sample of farmers of the Central Zone can maximize profit through optimum use of their resources.

The short run cost function for the Central Zone sample dairy farmers indicated that the total average cost for all the categories of sample dairy farmers is less than the unit price of fresh milk. This implies that the resources are utilized in the rational area of production. The marginal costs of factors of production are also less than the unit price of fresh milk. Thus, in

the short run under competitive markets the sample dairy farmers can use the factors of production up to the point where the marginal cost is equal to the unit price of output.

Southern Zone “Mendefera” Area

The elasticities of production of inputs are all less than unity and greater than zero. This indicates use of the resources in the rational area of the production function. From the estimated model, the highest response in milk yield, *ceteris paribus*, is due to a change in milking cows, followed by forage and labour.

The farm feed input (fertilizer, seed, herbicides/pesticides) that is entered as a dummy variable is found to be nearly significant at the 5% level of probability. However, since only those farmers having land to grow green feed incur costs on farm feed inputs, the dummy variable represents those farmers having land. Thus, for these farmers, milk yield is higher than for those having no land by as much as the magnitude of the dummy coefficient.

The value of marginal product of concentrates, and operating and mechanical cost for the sample dairy farmers of this study area are less than their respective unit costs, despite the operating and mechanical cost for the top one-third of sample dairy farmers being slightly under-utilized. Thus, to improve the profitability of dairy farmers in the sample, a reduction in the use or reallocation of these resources would be advisable. The over-utilization of concentrates may be attributed to the high dependence of these dairy farmers on concentrates resulting from the severe drought during the production period. The distance (50 - 65 kms) from the source of the input is an additional factor contributing to the high cost of the resource. The remaining resources are under-utilized and the sample farmers could increase use up to the point where $VMP_x = P_x$.

The addition of a milking cow resulted in an increase of annual milk yield of 819 litres/annum for the entire sample dairy farmers, and 656 and 1030 litres/annum for the bottom and the top one-third of the sample of dairy farmers, respectively. However, the highest return from a unit increase of an input, relative to the unit price of an input, comes from forage, nearly 2.85 Nfa for the entire sample mean, and 2.055 and 4.20 Nfa for the bottom and top one-third of sample

dairy farmers, respectively. The addition of a milking cow and labour are the next after forage in terms of high returns relative to their respective unit prices.

The marginal rate of substitution of concentrates for forage (-0.193, -0.202, and -0.198) for the entire sample, the bottom and top one-third of sample dairy farmers, respectively, indicated that the farmers are utilizing more concentrates than forage. The least cost combination of the two inputs, *ceteris paribus*, also indicated that none of the categories of sample dairy farmers were allocating the resources on a least-cost basis. From the least cost estimates, a reduction of cost (an increase of margin) of 6681 Nfa for the entire sample of dairy farmers, and 6757 and 5811 Nfa for the bottom and top one-third of sample dairy farmers, respectively, is obtained. Therefore, if the samples of farmers are to make greater profits on the resources and management, they need information on how to better allocate the available resources.

At the profit maximizing use of concentrates and forage milk yield is improved by 51.1%, 39%, and 72.6% for the entire sample, and the bottom and top one-third of sample dairy farmers, respectively. However, while maximizing profit (optimum use of concentrates and forage), *ceteris paribus*, costs also increased, except for the bottom one-third of sample farmers for whom costs decreased by 0.7%. But at the profit maximizing use of the resources they can make a profit of 5594 Nfa compared to the profit at the actual production level, after covering the costs of the factors of production. Thus, under perfect knowledge, risk free and unlimited capital assumptions, the sample of dairy farmers can improve their gross margins by 439.2%, 215.8% and 129.9% for the entire sample, the bottom and top one-third of sample dairy farmers, respectively.

Similarly, the cost function showed that the sum of the average total variable cost for the bottom one-third of dairy farmers is greater than the unit price of output. This indicates that this group of sample dairy farmers are not covering the short run costs of production. The MC of concentrates for all the sample dairy farmers is greater than the unit price of output too, implying that they used to reduce the need of concentrates until $MC = P_y$.

Southern Zone “Dekemhare” Area

The production function of fresh milk production of sample dairy farmers of the Dekemhare area is similar with the one of Mendefera area. The only differences were that the variable veterinary and medicine expense was statistically significant, and the VMP was also the highest relative to the unit cost of the input. This implies a need to invest more in veterinary services and medicines to improve herd health. The bottom one-third of sample dairy farmers were not covering the input costs even at the profit maximizing use of resources. So this group of sample dairy farmers are suffering financially. The most probable reason for this problem could be either a possible error of measurement, i.e. some dairy expenditures are over-estimated, as the dairy farmers used to integrate dairy farming with other agricultural activities (orchard and horticulture products) on a small scale, or it could be due to a lack of knowledge and information on how to allocate their resources optimally.

However, all the elasticities of production are greater than zero and less than unity showing use of the resources in the rational area of production (stage II). The highest response to milk yield after milking cow comes from the production factors of forage, labour and operating and mechanical costs in this order. Dairy farmers having land for growing green feed were producing more milk than dairy farmers having no land.

As for the Mendefera dairy farmers, the sample of dairy farmers of the Dekemhare area are also over-utilizing concentrates and under-utilizing forage. Thus, these dairy farmers need to reallocate their resources. The marginal rate of substitution (-0.215, -0.197 and -0.265) relative to the inverse input price ratio for the entire sample, and the bottom and top one-third of sample dairy farmers, respectively, is evidence of more use of concentrates than forage.

The cost function for the Dekemhare area dairy farmers shows that for the bottom one-third of sample dairy farmers the sum of the average variable cost is greater than the unit price of fresh milk. Thus, the dairy farmers of this group are not able to cover the short run costs of production. The MC of concentrates is also greater than the unit price of fresh milk for all the sample of dairy farmers in this study, which implies that the resource is over-utilized.

Generally speaking, it can be concluded that the sample of dairy farmers in the three study areas are using their resources in the rational area of production. Most of the resources are highly under-utilized with the exception of a few resources, notably concentrates, which are over-utilized by the sample of dairy farmers in the Southern Zone study areas. Besides, the bottom one-third sample of dairy farmers of the Southern Zone were not covering the short run costs for the study period. The reason could be due to a lack of knowledge, lack of information, uncompetitive markets and limited capital, as well as a low quality of feed mix and breed quality.

Thus, according Doll and Orazem (1984), in the short run production process, if the primary concern of the farmer is to stay in business, the farmer should organize his productive resources in such a way that satisfactory profits can be made. If he is unable to do this, he will eventually go out of business. Hence, in general, the sample of dairy farmers studied in the Highlands of Eritrea need to use the available resources in such a way that they can make satisfactory returns to capital and management.

Although, it may be difficult to achieve satisfactory returns in the short-run with the existing knowledge of farmers, feed mix and breed quality, the optimum allocation of resources by dairy farmers in the long term can be attained through research and regular publications of research results and agricultural reviews to provide continuous information to agricultural agents and farmers. Moreover, farmers need greater awareness of livestock insurance to avoid risk of loss and undergo training, especially on financial record keeping systems. Thus, accurate analyses can be obtained from accurate farm records. This may help to promote institutional reform and appropriate policies. The government should also promote competitive markets for inputs and outputs, and improve infrastructures that would help to reduce transaction costs.

Since this study is based on one year's (2002) information on dairy input expenditures and annual milk output, to generalize the production function based on the results of this study alone may not be satisfactory to make future planning decisions on dairy farming in the Highlands of Eritrea; however, it could serve as good prior information for further study. Thus

to concretise the production function of dairy farmers in the Highlands of Eritrea, and to promote future planning decisions and policy reforms, a further study based on recorded time-series data is recommended.

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APPENDICES

Appendix A

Questionnaire for Commercial Dairy Farms in Eritrea

Name / code _____

Study area _____

1. Land utilization (ha)

Land status	Dry land (ha)	Irrigated (ha)	Veld grazing (ha)	Farm yard and waste (ha)
Owned (1)				
Rented (2)				
Total land operated (1+2)				

2. Herd structure (physical measurement)

Breed type	Milking cows	Dry cows	Heifers	Calves	Bulls	Total animals in the herd
Holsteins						
Barka						
Hybrid						
Other exotic breeds						

3. Milk yield (litres)

Breed type	Average of milk produced/cow/day (litres)	Total milk produced/year (litres)
Holstein		
Barka		
Hybrid		
Other Exotic		

Appendix A, continued...

4. Marketing and Consumption (litres)

Milk delivered to market/day (litres)		Milk consumed at home/day (litres)	Milk fed to calves/day (litres)
Formally	Informally		

5. Annual Farm Incomes for 2002 (Nfa)

Milk prices/litre		Milk sales/day (Nfa)		Total milk sales/year (Nfa)	Value of milk consumed on farm (Nfa)	Sale of calves (Nfa)	Cattle sales (Nfa)	Other dairy income (Nfa)	Total dairy income/year (Nfa)
Formal	Informal	Formal	Informal						

6. Annual Farm Expenses for 2002 (Nfa)

Labour cost/year (Nfa/year)				Purchased concentrate feed/year (Nfa)		Purchased forage and fodder feeds & Other costs/year (Nfa)				
Regular	Casual	Contract	Family labour	Concentrates	Licks	Alfalfa	Oats	Maize fodder	Hay	Other

Continued.....

Farm feed (Nfa/year)				AI (Nfa)	Vet & Medicine (Nfa)	Milk transporting (Nfa)	Livestock Insurance (Nfa)	License (Nfa)
Fertilizer	Seed	Weed/pest	Other					

7. Annual Operating and Mechanical Costs (Nfa)

Electricity (Nfa)	Water (Nfa)	Fuel (Nfa)	Oil (Nfa)	Grease (Nfa)	Repairs & spares (Nfa)

Appendix A, continued...

8. Annual farm fixed costs for 2002 (Nfa)

Vehicles (Nfa)		Tractors (Nfa)		Irrigation Equipment (Nfa)		Power Machinery (Nfa)		Improvements (Nfa)		Other (Nfa)
Deprn.	I & L	Deprn.	I & L	Deprn.	I & L	Deprn.	I & L	Deprn.	R & I	

9. Annual farm expenses for land (Nfa)

Owned land tax fees (Nfa)	Rented land (Nfa)	Total land expense (Nfa)

10. Asset values (Nfa)

Asset type	Purchase price (Nfa)	Economic life time (years)	Current value of the asset (Nfa)	Replacement value of the asset (Nfa)
Vehicles				
Tractor				
Irrigation Equipment				
Power machinery				
Other				

Appendix B

Table B.1. Ten years rainfall (mm) trend of the three studied areas of Eritrea.

Area	Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual rainfall
Asmara	1992	0.0	0.0	4.8	5.8	14.0	38.6	127.8	158.7	1.3	15.8	5.3	3.2	375.3
	1993	0.0	0.0	0.0	0.0	10.0	10.0	156.0	79.0	22.0	26.0	0.0	0.0	303.0
	1994	0.0	0.0	3.0	17.0	46.0	16.0	104.0	135.0	61.0	10.0	0.0	15.0	407.0
	1995	0.0	0.0	0.0	59.0	31.0	0.0	165.0	111.0	34.0	0.0	0.0	0.0	400.0
	1996	0.0	0.0	16.0	26.0	28.0	40.0	167.0	80.0	0.0	0.0	44.0	0.0	401.0
	1997	0.0	0.0	9.6	6.4	101.4	44.4	242.7	98.0	0.0	117.0	69.0	0.0	688.5
	1998	0.0	0.0	68.4	40.9	30.8	12.6	141.7	255.2	5.8	7.0	0.0	0.0	562.4
	1999	27.7	0.0	0.0	28.2	8.2	21.6	199.3	199.5	6.2	1.4	2.2	0.0	466.6
	2000	0.0	0.0	3.9	97.0	15.8	32.2	259.4	106.6	19.0	26.8	12.2	0.0	572.9
	2001	0.0	0.0	9.2	38.4	18.8	74.2	205.7	258.0	4.8	0.0	0.0	6.5	615.6
2002	0.0	0.0	0.0	11.8	13.5	28.6	88.7	179.0	44.6	0.0	8.7	0.0	374.9	
Mendefera	1992	0.0	0.0	19.0	61.0	6.0	12.0	181.0	201.0	59.0	40.0	22.0	0.0	601.0
	1993	0.0	0.0	0.0	69.0	162.0	81.0	173.0	57.0	69.0	28.0	0.0	0.0	639.0
	1994	0.0	0.0	0.0	26.0	42.0	94.0	98.0	249.0	68.0	0.0	4.0	0.0	581.0
	1995	0.0	0.0	39.0	33.0	25.0	5.0	178.0	136.0	62.0	0.0	0.0	0.0	478.0
	1996	0.0	0.0	35.0	85.0	86.0	104.0	161.0	191.0	20.0	0.0	68.0	0.0	750.0
	1997	0.0	0.0	0.0	2.0	93.8	53.1	225.3	79.0	2.0	209.0	47.0	0.0	711.2
	1998	0.0	0.0	42.1	3.5	58.5	38.6	160.3	362.7	26.7	0.0	2.5	0.0	694.9
	1999	22.8	0.0	0.0	43.2	0.0	16.1	344.7	198.5	64.9	44.8	5.0	0.0	740.0
	2000	0.0	0.0	13.3	69.3	67.0	40.5	99.4	135.9	22.7	37.5	19.3	0.0	504.9
	2001	0.0	0.0	6.3	19.7	9.4	123.8	224.0	186.0	27.9	17.0	0.0	8.4	622.5
2002	0.0	0.0	28.1	11.5	0.0	51.2	99.4	181.0	8.2	0.0	3.2	8.2	390.8	
Dekemhare	1992	0.0	0.0	0.0	1.0	15.0	0.0	135.0	178.0	38.0	24.0	14.0	0.0	405.0
	1993	0.0	9.0	0.0	61.0	27.0	25.0	174.0	73.0	0.0	13.0	0.0	0.0	382.0
	1994	0.0	0.0	0.0	7.0	12.0	52.0	284.0	128.0	94.0	0.0	2.0	0.0	579.0
	1995	0.0	1.0	2.0	35.0	87.0	0.0	131.0	113.0	2.0	0.0	0.0	0.0	371.0
	1996	0.0	0.0	77.0	25.0	118.0	69.0	168.0	80.0	0.0	0.0	0.0	0.0	537.0
	1997	0.0	0.0	0.0	0.0	18.0	16.5	174.3	67.0	0.0	157.0	12.0	0.0	444.8
	1998	0.0	0.0	16.0	17.3	73.0	0.0	124.3	208.7	0.0	0.0	0.0	0.0	439.3
	1999	17.5	0.0	0.0	0.0	16.0	14.0	167.3	257.7	7.0	0.0	0.0	0.0	479.5
	2000	0.0	0.0	0.0	22.0	9.0	48.0	118.5	177.3	44.5	0.0	0.0	0.0	419.3
	2001	0.0	0.0	0.0	0.0	0.0	126.5	238.5	196.0	13.0	0.0	0.0	5.0	579.0
2002	0.0	0.0	3.0	13.5	4.0	26.4	136.0	176.0	8.5	0.0	0.0	0.0	367.4	

Source: Meteorology Service of Eritrea (Civil Aviation Eritrea).

Appendix C

Table C. 1: Correlation Matrix and VIF values of explanatory variables using OLS regression analysis for the Central Zone of Eritrea 2002.

	Concentrates	Forage	Labour	Vet. & Med.	Ope. & Mech.	Cow	VIF
Concentrates	1.000	0.493	0.645	0.428	0.340	0.803	3.452
Forage		1.000	0.206	0.457	-0.125	0.666	4.535
Labour			1.000	0.406	0.512	0.566	2.170
Vet. & Med.				1.000	0.266	0.606	1.705
Ope. & Mech.					1.000	0.311	1.656
Cow						1.000	5.299

Table C. 2: Correlation Matrix and VIF values of explanatory variables using OLS regression analysis for the Southern Zone “Mendefera” of Eritrea, 2002.

	Concentrates	Forage	Labour	Vet. & Med.	Ope. & Mech.	Cow	VIF
Concentrates	1.000	0.686	0.653	0.502	0.198	0.757	3.149
Forage		1.000	0.624	0.483	-0.080	0.694	3.581
Labour			1.000	0.452	0.240	0.740	2.593
Vet. & Med.				1.000	-0.002	0.372	1.593
Ope. & Mech.					1.000	0.384	3.221
Cow						1.000	5.049

Appendix C

Table C. 3: Correlation Matrix and VIF values of explanatory variables using OLS regression analysis for the Southern Zone “Dekemhare” area of Eritrea, 2002.

	Concentrates	Forage	Labour	Vet. & Med.	Ope. & Mech.	Cow	VIF
Concentrates	1.000	0.822	0.795	0.884	0.867	0.892	8.733
Forage		1.000	0.871	0.766	0.820	0.898	8.776
Labour			1.000	0.728	0.867	0.923	8.513
Vet. & Med.				1.000	0.821	0.849	5.653
Ope. & Mech.					1.000	0.908	7.716
Cow						1.000	18.578