Lattice-Structure Based Adaptive MMSE Detectors for DS-CDMA Systems

By

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Abstract

There has been significant interest in the research community on detectors for DS-CDMA systems. The conventional detector, which detects users' data bits, by using a filter matched to the users' spreading codes, has two major drawbacks. These drawbacks are (1) its capacity is limited by multiple access interference (MAI) and (2) it suffers from the near-far problem. The remedy to these problems is to use a multiuser detector, which exploits knowledge of users' transmission and channel parameters to mitigate MAI. Such detectors are called multiuser detectors (MUD). A number of these detectors have been proposed in the literature. The first such detector is the optimal detector proposed by Verdu. Following Verdu's work a number of suboptimal detector were proposed. These detectors offer better computational complexity at the expense of the bit error rate performance. Examples of these detectors are the decorrelating detector, the minimum mean squared error detector (MMSE), the successive interference cancellation and parallel interference cancellation. In this thesis, we consider the adaptive DS-CDMA MMSE detector, where lattice-based filter algorithms are employed to suppress MAI. Most of the work in the literature has considered the implementation of this detector using the Least Mean Square (LMS) algorithm. The disadvantage of using the LMS algorithm to implement the MMSE detector is that the LMS algorithm converges very slowly.

The main aims of this thesis are as follows. A review of the literature on MUD is presented. A lattice based MUD is then proposed and its performance evaluated using both simulation and analytical methods. The results obtained are compared with those of the LMS-MMSE detector. From the results obtained the adaptive Lattice-MMSE detector is shown to offer good performance tradeoff between convergence results and BER results.

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List Of Acronyms

ARIB	-Association for Radio Industry and Business
AWGN	-Additive White Gaussian Noise
BER	-Bit Error Rate
DS-CDMA	- Direct Sequence Code Division Multiple Access
ETSI	-Europe Telecommunications Standards Institude
FDD	-Frequency Division Duplex
FDMA	-Frequency Division Multiple Access
GAL	-Gradient Adaptive Lattice
GSM	- Global System for Mobile communications
ITU	-International Telecommunications Union
LMS	-Least Mean Square
LS	-Least Square
MAI	-Multiple Access Interference
MF	-Matched Filter
MMSE	-Minimum Mean Squared Error
MU	-MU
MUD	-Multiuser Detector
PIC	-Parallel Interference Cancellation
SIC	-Successive Interference Cancellation
SIR	-Signal to interference ratio
SNR	-Signal to noise ratio
TDD	-Time Division Duplex
TIA	-Telecommunications Industry Association
TDMA	-Time Division Multiple Access
W-CDMA	-Wideband Code Division Multiple Access
UMTS	-Universal Mobile Telecommunication System
UWCC	- Universal Wireless Communications Consortium

List Of Symbols

A_k	-The k^{th} user's transmission amplitude
$\mathbf{A}$	-The matrix of the amplitude of all the users
b_k	-The data bit of the k^{th} user
$\hat{b}_k$	-Data bit estimate for the k^{th} user
$\mathbf{b}$	- Vector of the data bits of all the users
$c_{k,l}$	-The channel fading coefficients of the k^{th} user's l^{th} path at
$\mathbf{c}_k$	-The k^{th} user's channel fading vector
$\mathbf{C}$	-The matrix of the samples of the users' channel coefficients
d_i	-The backward prediction error coefficients of the i^{th} stage the lattice filter
$\mathbf{F}_P$	-The crosscorrelation of the samples of either the chip rate sampled chip-matched filter output or the received signal
$\mathbf{F}_S$	-The crosscorrelation of the samples of the over-sampled output of the chip-matched filter
f_c	-The carrier frequency in hertz
f_i	-The forward prediction error coefficients of the ith stage of
$\mathbf{H}$	-The matrix of the crosscorrelation of users' spreading sequence
K	-Number of users
$k_{i,f}$	-The forward reflection coefficient of the i^{th} stage of the LS-Lattice filter
$k_{i,b}$	-The backward reflection coefficient of the i^{th} stage of the LS-Lattice filter
k_i	-The i^{th} stage reflection coefficient
L	-Number of resolvable channel paths
M	-Total number of bits transmitted by each user
$\mathbf{n}$	-Noise samples vector
P_k	-Probability of error for the k^{th} user
$s_k(t)$	-Spreading sequence of the k^{th} user

$\mathbf{s}_k$	-Vector of the samples of the spreading sequence of the k^{th} user
$\mathbf{S}$	-The matrix of the samples of the users' spreading sequence
$r(t)$	-Received signal
σ	-The variance of additive white gaussian noise
w_c	-The carrier frequency in radians per second
θ_k	-The phase offset of the kth user's modulator from the carrier frequency
$\beta_{k,l}$	-The k^{th} user's l^{th} path attenuation factor
τ_k	-The k^{th} user's delay
$\tau_{k,l}$	-The propagation delay experienced by the k^{th} user's signal on the lth path
$\varphi_{k,l}$	-The phase shift introduced by the channel as the kth user's signal travel through the l^{th} path
λ_i	-The ith eigenvalue of the matrix $\mathbf{F}_P$

Chapter 1
Introduction

1.1 Background

The market of wireless communication is one of the fast growing markets today. Important examples of wireless communication systems are cellular mobile communication systems, cordless telephone systems and mobile data networks. Such systems offer voice and/or data services to mobile users. The reason behind the rapid growth in wireless communications was the introduction of digital technology, which made it possible for the following attractive features to be realized.

- A high percentage of the service area to be covered.
- Accommodation of a large number of communicating users.
- Lower power consumption, weight and size of the mobile terminal.

The earliest cellular communication systems such as the Total Access Communication System (TACS) in United Kingdom, Spain and Italy, Nordic Mobile Telephone (NMT) in Scandinavia and the Advanced Mobile Phone Service (AMPS) in the United state (US) employed analog technology and were termed first generation systems. Since these systems employed analog technology their mobile terminal were big in size, and weight and were characterized by high power consumption. In addition to this, they covered a small percentage of service area, resulting in small system capacity. Second generation systems are based on digital technology and have been introduced in major markets of the world. Standards for the second cellular generation system are

- Global System for Mobile Communication (GSM)- developed in Europe
- The IS-54 and IS-95 – developed in the United States
- Personal Digital Cellular (PDC) system- developed in Japan

The major shortcoming of the second generation systems are its incapability to offer a wide range of services on a single system, e.g. most of the second generation cellular telephone only offered voice services and short message service (SMS).

Future mobile communication systems, termed third-generation (3G) mobile communication system will have to meet the following requirements

- High flexibility
- High system capacity
- Low cost and ease of implementation

Flexibility is demanded with respect to offering a wide range of voice and data services with different and variable data rates of up to 2Mbit/s, frequency & radio resource management, system deployment and international roaming.

In a bid to find standards and recommendations, which will ensure that these requirements are met, the International Telecommunication Union (ITU) and other bodies launched a number of research efforts in the 1990s. The air interfaces adopted by the various bodies for the 3G mobile communication systems are summarized below.

At the beginning of 1997, Japan's Association for Radio Industry and Business (ARIB), adopted Wideband Code Division Multiple Access as the air interface for the 3G mobile communication system and began to develop detailed standardization of this interface. Soon, Europe and the US, followed suit, by rolling out their standardization. During 1997, a joint agreement between Japan and Europe was reached as far as the parameters to be used for W-CDMA were concerned. This led the European Telecommunications Standards Institute (ETSI), to select W-CDMA as the Universal Mobile Telecommunication System (UMTS) terrestrial air interface scheme for frequency-division duplex (FDD) frequency bands. In no time, they were backed by the Asian and American GSM operators, who also adopted W-CDMA as their interface. For time-division duplex (TDD), the ETSI proposed what is referred to as Time Division CDMA (TD-CDMA). In December 1997, the Telecommunication Industry Association (TIA) TR45.5 committee in the United States adopted a framework for W-CDMA, which is compatible with the IS-95 standards they adopted for second-generation mobile communications systems. While TR45.3, which was responsible for laying down the IS-136 standards, adopted TDMA as the air interface, based on recommendations from the Universal Wireless Communications Consortium (UWCC) in 1998.

As can be seen from the standards adopted by the various standardization bodies world wide, spread spectrum code-division multiple access (CDMA) scheme is going to play a vital role in 3G mobile communications systems. This scheme is the subject of this thesis, where the focus is on multiuser detection, in particular adaptive minimum mean squared error (MMSE) detectors.

In this Chapter a brief description will be given of direct sequence code division multiple access (DS-CDMA) scheme. In addition to this, the aim, outline and contribution to this project will be presented.

1.2 DS-CDMA System

Spread spectrum is a means of transmission in which a transmitted signal occupies a bandwidth much greater than the minimum bandwidth required to send it. The bandwidth increase, while not necessary for communication overcome the effect of intentional interference (jamming) and also hide the signal from being intercepted by a spy. Spreading of the spectrum is accomplished by using either time hopping (TH), frequency hopping (FH), direct sequence (DS) and hybrid modulation. All these method uses a pseudo-random code sequence (sometimes referred to as signature sequence) but they create the spread spectrum signal differently as explained below

- Frequency hopping. The signal is rapidly switched between different frequencies within the hopping bandwidth pseudo-randomly, and the receiver knows before hand where to find the signal at any given time.
- Time hopping. The signal is transmitted in short bursts pseudo-randomly, and the receiver knows beforehand when to expect the burst.
- Direct sequence. The digital data is directly coded at a much higher frequency. The code is generated pseudo-randomly, the receiver knows how to generate the same code, and correlates the received signal with that code to extract the data.
- Hybrid modulation. This method utilizes two or more of the above-mentioned methods so as to produce a more robust transmission scheme.

CDMA is a multiple access scheme, where a number of users transmit their information using the entire allocated bandwidth simultaneously but their signals are modulated by a unique code sequences using one of the modulation technique mentioned above. The spread spectrum modulation technique mostly preferred for CDMA is direct sequence spread spectrum. This is due to its low cost and ease of implementation and as such is the area of focus in this thesis.

Spread Spectrum CDMA scheme has a number of attractive features when compared to other multiple access schemes. Below is a list of just a few of them

- *Frequency reuse factor of one:* Since all the users in CDMA use the same frequency, the frequency reuse factor of this scheme is one.
- *Low probability of interception:* This property stem from the fact that the spread signal has a very low power level and thus cannot be detected by a casual listener.
- *Immunity to multipath fading:* One advantage of CDMA receiver is that they are able to exploit the multipath fading through the use of Rake combining.
- *Soft handoff*: A handoff occurs in any cellular system when a mobile terminal switches a call from one base station to another as you travel. In CDMA however, every mobile terminal and every base station are on the same frequency. In order to begin listening to a new base station, the mobile terminal only needs to change the pseudo-random sequence it uses to decode the desired data from the jumble of bits sent for everyone else. While a call is in progress, the network chooses two or more alternate base stations that it feels are handoff candidates. It simultaneously broadcasts a copy of a call to each of these base stations. The mobile terminal can then pick and choose between the different sources, and it move between them whenever it feels like it. It can even combine the data received from two different base stations to ease the transition from one to the other.

The two main drawbacks of DS-CDMA are the near-far problem and the multiple access interference (MAI). The near-far problem is the phenomena where a signal from a user with a weak transmission power is overpowered by the strong interfering user(s), while MAI stems from the fact that the cross-correlation of the users' spreading sequence is non-zero. The conventional way to minimize the near-far problem, in CDMA systems is make use of a power control scheme, which attempts to make the power received at the base station from each mobile unit equal. Under this scheme, the base station samples the power levels being received from all users and then transmits a power adjustment command to all users. On the other hand, the conventional way to mitigate MAI is through the design of spreading code with very small cross-correlation property between two different codes.

The gentleman known as Verdu proposed an alternative method that can be used to mitigate the near-far problem and MAI. In his paper published in 1986, he showed that the near-far problem and MAI could be eliminated through the use of a multiuser detector. This had a significant impact in the field of spread spectrum since immediately following his work many multiuser detectors for CDMA system were proposed.

1.3 Aims and Contributions

The focus of this thesis is on the adaptive implementation of the MMSE detector using a lattice structure. In addition to this a comparison of the performance of this detector with that of the LMS-MMSE detector is given. The original contributions made in this project are as follows

- The derivation of a lattice-MMSE detector structure for CDMA systems (Chapter 5).
- Analytical performance model of the lattice-MMSE detector (Chapter 5).

The following publications have resulted from this work

- B.C.D Thakadu & F Takawira, "Lattice Structure Implementation of the Adaptive DS-CDMA detector", Proceeding of the 2nd Annual South African Telecommunications Networks Applications Conference, September 1999.
- B.C.D Thakadu & F Takawira, "Lattice Structure based MMSE detector and its Performance Comparison with the LMS-MMSE detector in a multipath Fading Channel", to be submitted to IEEE Transactions on Vehicular Technology.

1.4 Thesis outline

This Chapter has given an introduction to CDMA and the aim of the thesis. In Chapter 2, the system model used throughout this thesis is presented. In addition to this a review of DS-CDMA detectors is presented. These detectors are: (1) the conventional detector, (2) the optimal detector proposed by Verdu, (3) the minimum mean squared error (MMSE) detector, (4) the decorrelating detector, (5) successive interference cancellation and (6) the parallel interference cancellation.

Chapter 3 looks at the details of the LMS-MMSE detectors. The optimal weighting coefficients of the MMSE detectors are derived and then the adaptive implementations of these detectors are presented. Also covered in this chapter is the convergence and BER analysis models of the LMS-MMSE filter coefficients.

In Chapter 4 a literature review of the lattice equalizer is presented, while Chapter 5 focus on the implementation of the MMSE detectors using a lattice structure and contain original contributions. Chapter 6 presents a review of blind adaptive MMSE detectors and finally in Chapter 7, conclusions are made.

Chapter 2
Multiuser Detection in AWGN and Fading channels

2.1 Introduction

In this Chapter a review of detectors, which mitigate Multiple Access Interference (MAI) is presented. These detectors, which have been shown to outperform the conventional detector, require information of all the users' parameters in order to detect users' transmission. Hence they are called multiuser detectors (MUDs).

This Chapter is organized as follows. Section 2.2 presents the received signal model, which will be used throughout this Chapter. Following this, Section 2.3 looks at the conventional detector in both non-fading and multipath fading channels. Sections 2.4 and 2.5 look at MUD detectors in an additive white Gaussian noise (AWGN) and fading channels, respectively. Finally Section 2.6 presents conclusions.

2.2 DS-CDMA Signal Model

The DS-CDMA system under consideration is modelled as an asynchronous K-user system operating in a channel with L resolvable paths. The modulation scheme used is BPSK with bit duration T and chip duration $T_c=T/N$, where N is an integer and is called the processing gain. Shown in Figure 2.1 is the typical block diagram of a DS-CDMA transmitter. As can be seen from this diagram, at the transmitter side, the data stream form the sources are first spread by multiplying them with a known spreading sequence. Assuming that the k^{th} user's spreading waveform is given by $s_k(t)$ and that the bits are denoted by $b_k(t) = \{+1,-1\}$, then the spread data stream can be mathematically written as:

$$x_k(t) = \sum_{m=0}^{M-1} b_k(m) s_k(t - mT) \qquad \textbf{(2. 1)}$$

where we assumed that each user transmit a total of M bits. The baseband spread data streams are then converted to a higher frequency by multiplying them with a carrier of frequency f_c before being transmitted. Thus the transmitted signal for the k^{th} user is given by

$$z_k(t) = \sum_{m=0}^{M} A_k b_k(m) s_k(t - mT)\cos(2\pi f_c t + \theta_k) \qquad \textbf{(2.2)}$$

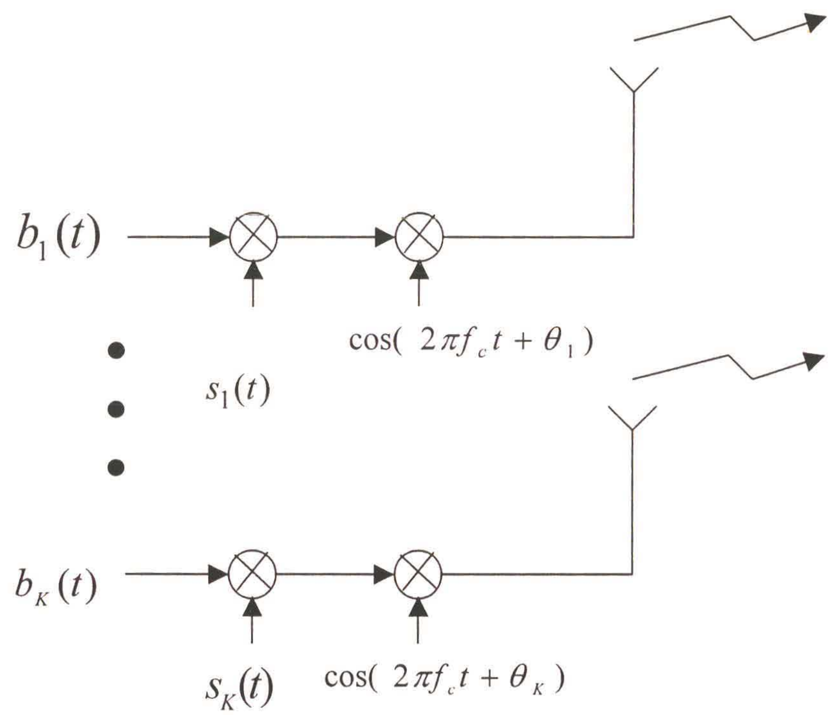


Figure 2.1: Typical Block diagram of Direct Sequence CDMA transmitters.

where A_k is the amplitude of the k^{th} user and θ_k is the phase shift of the k^{th} user's carrier frequency. Due to the time-variation of the communication medium, the transmitted signal is scattered into a number of multipath components and superimposed on this components is additive white Gaussian noise and other user's transmission. Assuming we have L resolvable paths then the received signal at the receiver is given by the following equation [48,63,83,86]:

$$r'(t)=\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=1}^{M}A_k\beta_{k,l}(m)b_k(m)s_k(t-mT-\tau_k-(l-1)T_c)\cos(2\pi f_c t+\varphi_{k,l}(m)+\theta_k)+n'(t)$$

(2. 3)

where $\beta_{k,l}(m)$ and $\varphi_{k,l}(m)$ are the attenuation factor and phase shift associated with the l^{th} path of the k^{th} user's multipath component. τ_k is the delay of the k^{th} user. It should be noted that in the above Equation we made the assumption that the propagation delay associated with the l^{th} path is given by $(l-1)T_c$. In addition to this we assumed that the channel fading coefficients are constant over at least one bit interval. At the receiver the received high frequency signal is converted to baseband by modulating it with a carrier of the same frequency as the one used at the transmitter side. This baseband-received signal is given by the following expression

$$r(t)=\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=1}^{M}A_k\beta_{k,l}(m)b_k(m)s_k(t-mT-\tau_{k,l}-(l-1)T_c)\cos(\varphi_{k,l}(m)+\theta_k)+n(t)$$

(2.4)

The resulting baseband signal is then sampled at a rate, which is greater or equal to the chip rate. Let $\phi_{k,l}(m)=\theta_k+\varphi_{k,l}(m)$ and $c_{k,l}(m)=\beta_{k,l}(m)\cos(\phi_{k,l}(m))$. Then the baseband-received signal can be written in matrix form as follows [48,63,83,86].

$$\mathbf{r}=\mathbf{SCAb}+\mathbf{n}$$ **(2.5)**

where the matrices and vector used above are defined as follows:

- $\mathbf{r}=[\mathbf{r}(0),...,\mathbf{r}(M-1)]\in C^{MNP}$

  $\mathbf{r}(n)=[r(nT),...,r(nT+(NP-1)T_s)]\in C^{NP}$

- $\mathbf{S}=diag[\mathbf{S}(0),...,\mathbf{S}(M-1)]$

  $\mathbf{S}(n)=diag[\mathbf{S}_1(n),...,\mathbf{S}_K(n)]$
  $\mathbf{S}_k(n)=[\mathbf{s}_{k,1}(n),...,\mathbf{s}_{k,L}(n)]$
  $\mathbf{s}_{k,l}=[\mathbf{0}_{mNP+\tau_{k,l}},\mathbf{s}_k,\mathbf{0}_{(M-m-1)NP+\tau_{k,l}}]$
  $\mathbf{s}_k=[s_k(0),...,s_k(NP-1)]$

- $\mathbf{C}=diag[\mathbf{C}(0),...,\mathbf{C}(M-1)]$

$$\mathbf{C}(n) = diag[\mathbf{c}_1(n),...,\mathbf{c}_K(n)]$$
$$\mathbf{c}_k(n) = [c_{k,1}(n),...,c_{k,L}(n)]$$

- $\mathbf{A} = diag[\mathbf{A}(0),..., \mathbf{A}(M-1)]$

  $\mathbf{A}(n) = diag[A_1,..., A_K]$

- $\mathbf{b} = [\mathbf{b}(0), \mathbf{b}(1),..., \mathbf{b}(M-1)]$

  $\mathbf{b}(n) = [b_1(n),..., b_K(n)]$

- $\mathbf{n} = [\mathbf{n}(0),..., \mathbf{n}(M-1)]$

  $\mathbf{n}(i) = [n(0),..., n(NP-1)]$

Please note that $\boldsymbol{r}$ is the vector of all the samples of the received signal over the entire M-symbol duration. As a special case, in a non-fading channel (i.e. **C=I**) the above Equation for the received baseband signal simplifies to:

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n} \qquad \textbf{(2.6)}$$

and the channel has only one propagation path (i.e. L=1).

2.3 Conventional Detector

This Section briefly looks at the conventional way of detecting users' transmission in both non-fading and multipath-fading channel. It starts by first looking at the non-fading conventional detector. Finally it looks at the multipath-fading (Rake) conventional receiver.

In a non-fading channel the conventional detector (see Figure 2.2) correlates the received baseband signal with the spreading sequence of the user of interest and uses the output of the correlator (matched filter) as the decision statistics. Using the baseband received signal (Equation (2.6)) and the spreading sequence matrix, then the decision statistics is given by:

$$\begin{aligned}\mathbf{y} &= \mathbf{S}^T\mathbf{r} \\ &= \mathbf{S}^T\mathbf{SAb} + \mathbf{S}^T\mathbf{n}\end{aligned} \qquad \textbf{(2.7)}$$

and finally the estimate of the transmitted information is given by $\hat{\mathbf{b}} = sign(\mathbf{y})$.

A multipath-fading channel presents us with some form of time-diversity. The conventional rake receiver uses this inherent time-diversity to improve the performance of the conventional detector in a multipath-fading channel. Thus instead of using a single matched filter (MF) to demodulate one user's data, we employ a bank of matched filters, where each matched filter branch is matched to one of the multipath components of the desired user (see Figure 2.3). The outputs of these matched filters are then fed into a gain combiner (preferably maximal ratio combiner), whose output is used as a decision variable. For the case where the gain combining technique used is the maximal ratio, the decision variable for all users' detector is given by

$$\begin{aligned}\mathbf{y} &= \mathbf{C}^H\mathbf{S}^T\mathbf{r} \\ &= \mathbf{C}^H\mathbf{S}^T\mathbf{SCAb} + \mathbf{C}^H\mathbf{S}^T\mathbf{n}\end{aligned} \tag{2.8}$$

One of the disadvantages of the conventional detector is that it treats MAI as additive white Gaussian noise. As illustrated in Figure 2.4, this assumption severely degrade the performance of the conventional detector because truly speaking MAI is not a white Gaussian process. Another disadvantage of the conventional detector is that it suffers from the near-far problem.

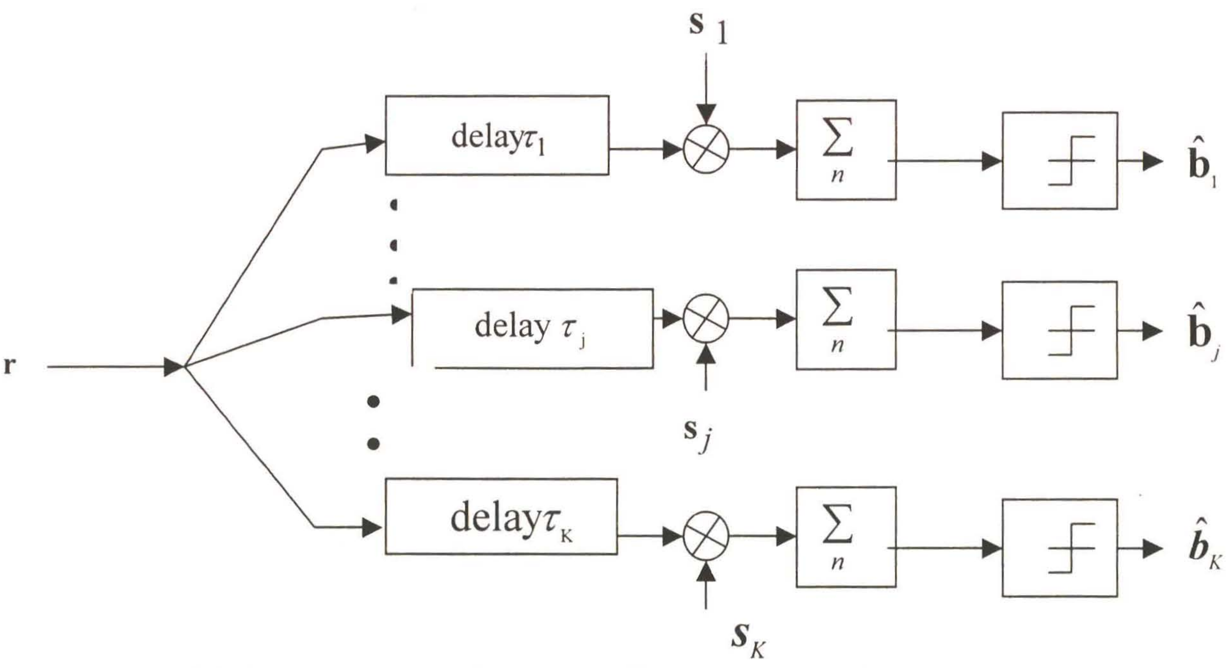


Figure 2.2: Conventional asynchronous DS-CDMA receiver bank.

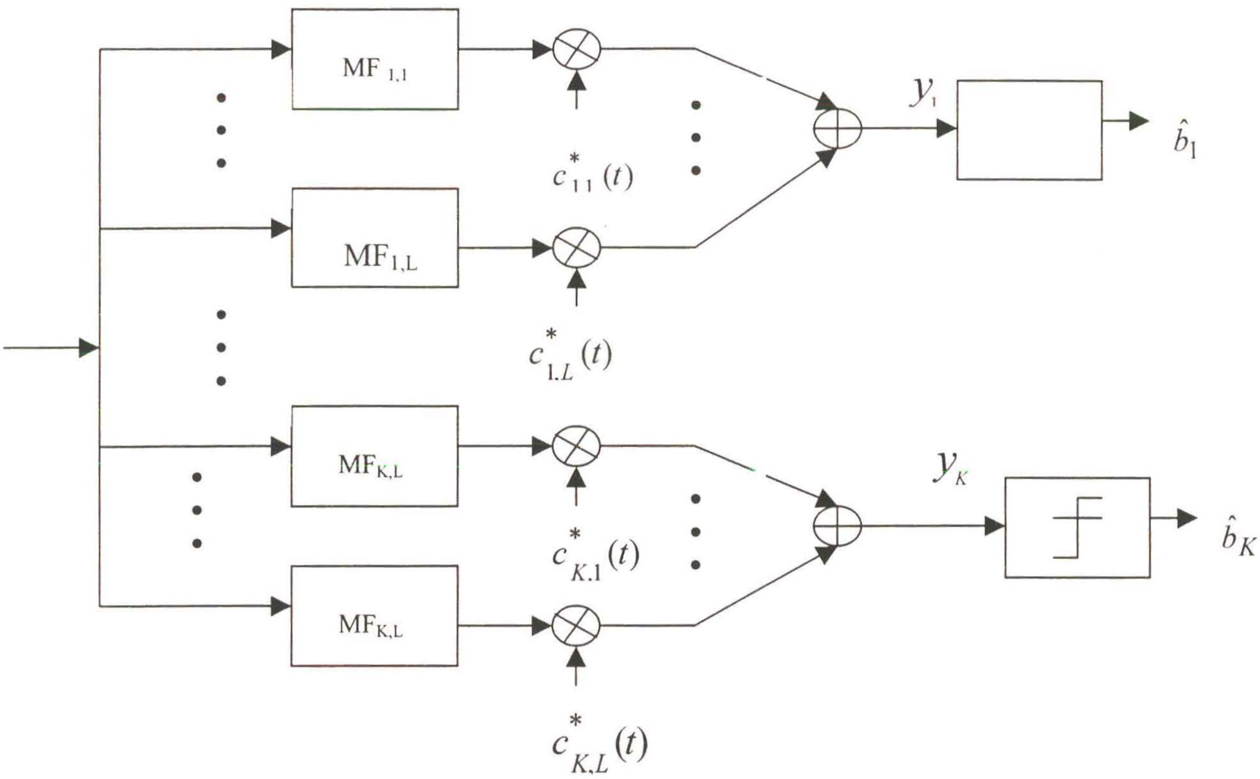


Figure 2.3 Conventional Rake Detector.

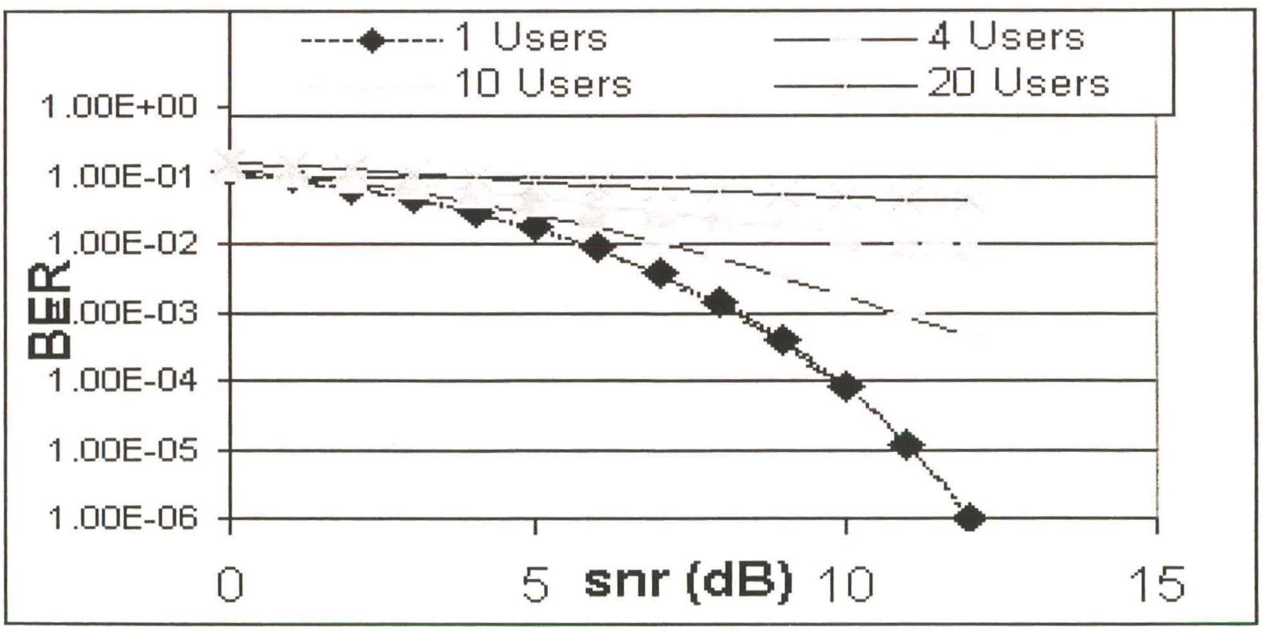


Figure 2.4: BER performance of the conventional detector-simulation results

2.4 Multiuser detection in AWGN Channel

Before Verdu proposed the optimal detectors [114-117] in the early eighties, most of the specialists in the field of spread spectrum were of the opinion that MAI can be modelled as a white Gaussian process. This 'opinion' proved costly, as it hindered any progress being made in developing detectors, which can mitigate MAI. Although the complexity of Verdu's detector makes it practically not viable, it played such a significant role in the development of multiuser detectors, since a number of sub-optimal detectors were proposed immediately after its development. Example of these detectors are: Decorrelating , the Minimum Mean Squared Error (MMSE) , Polynomial Expansion , Successive Interference Cancellation , Parallel Interference Cancellation and Neural Network detector .

In this Section a review of these detectors in a non-fading channel is presented, starting with Verdu's optimal detector, while the next Section will look at these detectors in a fading channel.

2.4.1 Optimal Detector

The implementation of the optimal detector requires knowledge of all the users' spreading sequences, signal energies and time delay. Throughout this Section we will assume that the receiver has knowledge of all these quantities. The optimal detector estimates the transmitted information sequence, by selecting the information sequence that optimizes a certain criterion. The optimization criterion used here is the joint a posteriori criterion, which is mathematically stated as [79,115,118]:

$$\begin{aligned}\mathbf{b} &= \arg\ \max P(\mathbf{b} \mid \mathbf{r}) \\ &= \arg\ \max P(\mathbf{b} \mid r(t), t \in [0, MT])\end{aligned} \tag{2.9}$$

It should be noted that this is not the only optimization criterion, which can be used to estimate the transmitted sequences. As an example we can also use the minimum probability error as the optimization criterion [79]. Going back to Equation (2.9), if one assumes that the data streams are equiprobable then one has [79,115,118]

$$P(\mathbf{b} \mid r(t), t \in [0, MT]) = P(r(t), t \in [0, MT] \mid \mathbf{b}) \tag{2.10}$$

Thus we can use the maximum likelihood detector to estimate the transmitted sequences. The probability density function of the event $r(t), t \in [0, MT] \mid \mathbf{b}$ is normal and is given by [118]:

$$f_{r(t),t\in[0,MT)|\mathbf{b}}(r(t),t \in [0,MT] \mid \mathbf{b}) = D\exp(-\frac{1}{2\sigma^2}\int_0^{MT}(r(t)-\mu(t))^2 dt) \quad \textbf{(2.11)}$$

where $D = \frac{1}{(2\pi)^{M/2}\sigma^L}$, σ^2 is the variance of additive white Gaussian noise and the mean $\mu(t)$ is given by:

$$\mu(t) = \sum_{m=0}^{M}\sum_{k=1}^{K} A_k b_k(m) s_k(t - mT - \tau_k) \quad \textbf{(2.12)}$$

From (2.10) and (2.11), it can be deduced that maximization of (2.9) is equivalent to maximizing [115]

$$\begin{aligned}\Omega(\mathbf{b}) &= -\int_0^{MT}(r(t)-\mu(t))^2 dt \\ &= 2\int_0^{MT}\sum_{k=1}^{K}\sum_{m=1}^{M} A_k b_k(m) s_k(t - MT - \tau_k) r(t) dt - \int_0^{MT}\left[\sum_{k=1}^{K}\sum_{m=1}^{M} A_k b_k(t - MT - \tau_k)\right]^2 dt \\ &= 2\sum_{m=1}^{M}\mathbf{b}^T(m)\mathbf{A}(m)\mathbf{r}(m) - \sum_{m=1}^{M}\sum_{j=1}^{M}\mathbf{b}^T(m)\mathbf{A}(m)\mathbf{R}(m-j)\mathbf{A}(j)\mathbf{b}(j)\end{aligned} \quad \textbf{(2.13)}$$

where $\mathbf{R}(i)$ is a KxK crosscorrelation matrix with coefficients given by [115]

$$[\mathbf{R}(i)]_{kj} = \int_{-\infty}^{\infty} s_k(t-\tau_k) s_j(t - iT - \tau_j) dt \qquad \forall i \in [0, M] \ \forall k, j \in [1, \ldots, K]$$

The second term in Equation (2.13) is given by

$$\sum_{i=1}^{M}\sum_{j=1}^{M}\mathbf{b}^T(i)\mathbf{A}(i)\mathbf{R}(i-j)\mathbf{A}(j)\mathbf{b}(j) = \sum_{i}\mathbf{b}^T(i)\mathbf{A}(i)[\mathbf{R}(0)\mathbf{A}(i)\mathbf{b}(i) + 2\mathbf{R}(1)\mathbf{A}(i-1)\mathbf{b}(i-1)] \quad \textbf{(2.14)}$$

Thus Equation (2.13) simplifies to

$$\Omega(\mathbf{b}) = \sum_{m=1}^{M}\mathbf{b}^T(m)\mathbf{A}(m)[2\mathbf{r}(m) - \mathbf{R}(0)\mathbf{A}(m)\mathbf{b}(m) - 2\mathbf{R}(1)\mathbf{b}(m-1)] \quad \textbf{(2.15)}$$

The sequence of bits that maximizes Equation (2.9) can be found by finding the longest path in a 2^{K-1} layered graph. This task is accomplished by using the Viterbi algorithm with states given by S={-1,+1} and branch matrix given by Equation (2.15). In practice the maximum-likelihood detector for DS-CDMA systems can be implemented by following the matched filter bank (whose output is given by Equation (2.7) for the non-fading channel) with a Viterbi algorithm.

The major disadvantage of the maximum-likelihood sequence detector is that it has a computation complexity that is exponential with the number of users, despite offering the best performance when compared to the conventional and/or suboptimal detectors. This makes the practical implementation of the optimal detector impossible, since a typical DS-CDMA system will be required to support many users. In addition to this it requires knowledge of some information the detector might not have prior knowledge of, for example the amplitude of all the users.

2.4.2 Suboptimal Detectors

Inspired by Verdu's major breakthrough in multiuser DS-CDMA detection, a number of suboptimal multiuser detectors, which offers good performance/complexity tradeoffs, were derived. This Section looks at some of these detectors in an AWGN channel.

2.4.2.1 Decorrelating Detector

The decorrelating detector, which was proposed by Kohno et al [54] and Schneider [103], is a linear detector (see Figure 2.5). It processes the output of the matched filter bank (Equation (2.7) for the non-fading channel) by multiplying it with the inverse crosscorrelation of the spreading sequence of all the users. This crosscorrelation matrix is given by

$$\mathbf{R} = \mathbf{S}^T\mathbf{S} \qquad \textbf{(2.16)}$$

Multiplying the output of the correlator with the inverse of Equation (2.16) yield the following results

$$\begin{aligned} \mathbf{b} &= \operatorname{sgn}\left(\mathbf{R}^{-1}\mathbf{y}\right) \\ &= \operatorname{sgn}\left(\mathbf{Ab} + \mathbf{R}^{-1}\mathbf{n}\right) \end{aligned} \qquad \textbf{(2.17)}$$

Note that the linear mapping matrix (L) in Figure 2.5 is given by $L = R^{-1}$. Looking at the expression in Equation (2.17), it should be clear that the output of the decorrelating detector is MAI free.

The decorrelating detector offers a number of attractive features. These include

1. It performs much better than the conventional detector. This was proved by Lupas and Verdu in [66,67], where they analyzed the performance of the decorrelating detector.
2. No knowledge of the received amplitudes is required (see Equation 2.16 and 2.17).
3. Its computational complexity is better than that of the optimal detector.

The major drawback of the decorrelating detector is that it enhances the noise. In addition to this it requires the inversion of a matrix, which increases the computational complexity of the detector. Despite the noise-enhancing problem, the decorrelating detector has been shown to outperform the conventional detector.

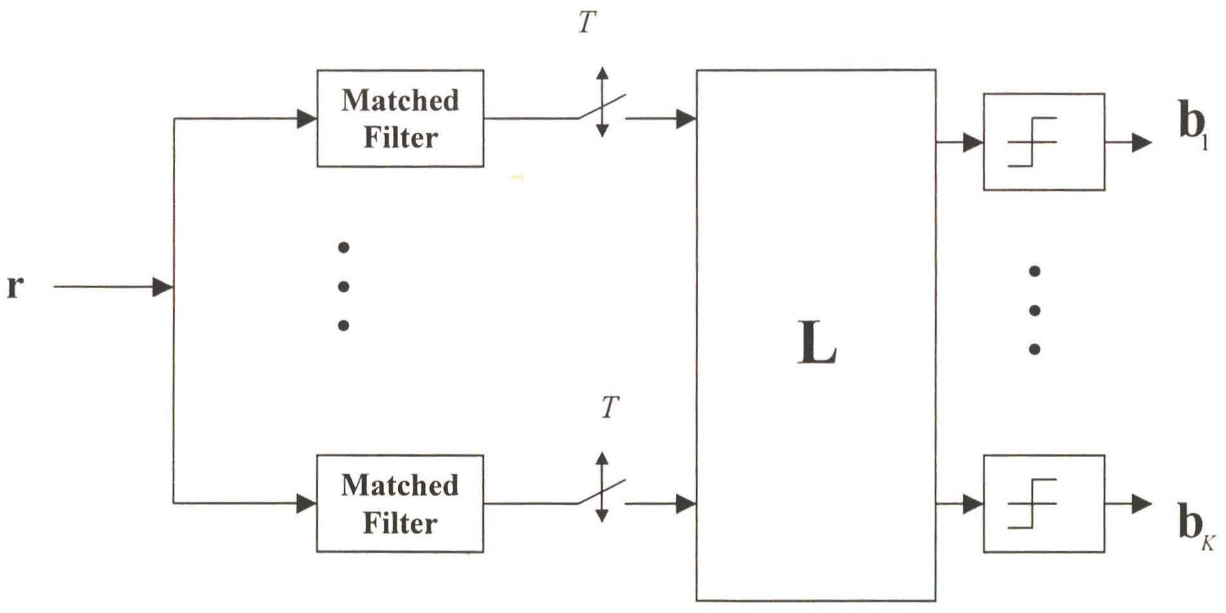


Figure 2.5: DS-CDMA linear detector.

The decorrelating detector has received considerable attention in the literature. As stated above, the performance of this detector was compared with that of the conventional detector in [66,67] by Lupas and Verdu. They went on to show that the decorrelating detector is the solution to the minimax problem in [66].

In a bid to alleviate the problem of having to compute the inverse of a very large matrix, Wijasuriya et al [126,127], Bravo [8], Zheng and Barton [136], and Juntti and Aazhang [46] proposed truncated-window decorrelators, where a sliding observation window that spans several symbol intervals is used. In addition to this Juntti [47], proposed a number of algorithms, which can be used to implement the decorrelating detector without the need to compute the matrix inversion. In [11,75-77,120], the authors have proposed adaptive decorrelating detectors, which avoid the need to compute the inverse of the correlation matrix.

Later in this Chapter we look at the decorrelating detector in a multipath fading channel.

2.4.2.2 Minimum Mean Squared Error Detector

A more detailed study of the MMSE detector will be presented in the next Chapter but for now it suffice to say that the Minimum Mean Squared Error (MMSE) detector, like the decorrelating detector, is a linear detector, which processes the output of the matched filter bank by applying linear mapping. This linear mapping is selected such that the mean squared error between the actual data and the output of the detector is minimized and is given by:

$$\mathbf{L} = [\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]^{-1} \tag{2. 18}$$

where **R** is the matrix of the crosscorrelation and was defined in Equation (2.16) and **A** is the matrix of the users' amplitude and was defined in Equation (2.5).

The MMSE detector was first proposed by Xie et al [131]. Following this Poor and Verdu [91], Oppermann et. al [85] and Honing and Veerakachen [40] performed the BER analysis of this detector.

In a bid to reduce the computational requirement of the MMSE detector Madhow and Honing [68], Miller [72] and Rapajic and Vucetic [94] independently proposed training based adaptive MMSE detectors. Miller in [72] then analyzed the dynamics of these detectors. Techniques for the acceleration of the convergence of the adaptive MMSE detector, were presented by Rapajic and Vucetic [95], Honing [34], Oppermann and Latva-aho [84] and Wang and Poor [123-124]

Honing et. al [38-40], proposed a blind MMSE detector, which requires no training sequence. The convergence of this detector was then examined by Roy in [100]. In [56], Krishnamurty proposed an adaptive step size blind adaptive MMSE detector and analyzed its convergence.

2.4.2.3 Successive Interference Cancellation Detector

The successive interference cancellation (SIC) detector estimates the users' bits by processing the received signal using a number of stages. At each stage one user's bit is estimated and interference due to it is eliminated from the received signal, thus reducing MAI. The simplest successive interference cancellation scheme uses the output of the matched filter to estimate bits of the users as illustrated in Figure 2.6. The order in which users' bits are estimated depends on their relative power. In practice, at each stage the user with the strong energy is the one whose bit is estimated, since it's easier to achieve acquisition and demodulation of such user's transmission. Another reason for demodulating the users in descending order is that, the removal of a strong interferer will benefit the remaining weak users. Assuming $b_{k,j}$ is the bit estimate obtained for user k at stage j, then the decision statistics for user i at stage (j+m) is given by [43]:

$$y_{i,(j+m)} = \mathbf{s}_k^T \mathbf{r} - \sum_{k=1}^{i-1} \mathbf{s}_i^T \mathbf{s}_k b_{k,j} - \sum_{l=i+1}^{K} \mathbf{s}_i^T \mathbf{s}_l b_{k-1,l} \qquad \textbf{(2.19)}$$

A brief description of the successive interference cancellation scheme, which uses the outputs of the matched filters as the decision variables, is as follows:

1. Using the outputs of the bank of matched filters, determine which user has the strongest energy.
2. Using the output of the matched filter of the user detected above as the decision variable, make an estimate of the transmitted bit of the corresponding user.

3. Regenerate an estimate of the transmitted signal for the user detected in (2), using this user's amplitude, spreading sequence and timing information.
4. Subtract the signal generated in (3) from the received signal.
5. Using the results of (4) start at (1), until all the users' transmissions have been detected.

The problem with the SIC detector occurs if the initial data estimates of the detector are wrong. Such an errors have a negative impact on the performance of the system. Thus, it is crucial that the data estimates of at least the strong users that are cancelled first be reliable.

Like other multiuser detectors the SIC detector has received considerable attention in the literature. In [10,17-19,45,88,96,97,98], the linear SIC was presented and it's performance studied using analytical and/or simulation approaches. An improved single stage SIC scheme using signal orthogonalization was studied in [6], while its performance was analyzed in [8,61,80,81,103,107,119] assuming tentative decision. Linear and non-linear SIC schemes were compared in [42,108]. In [12,19,20], the SIC detector was proposed and investigated, which cancels the interference in the spectral domain. Finally, coded systems with SIC were considered in [7,55,119].

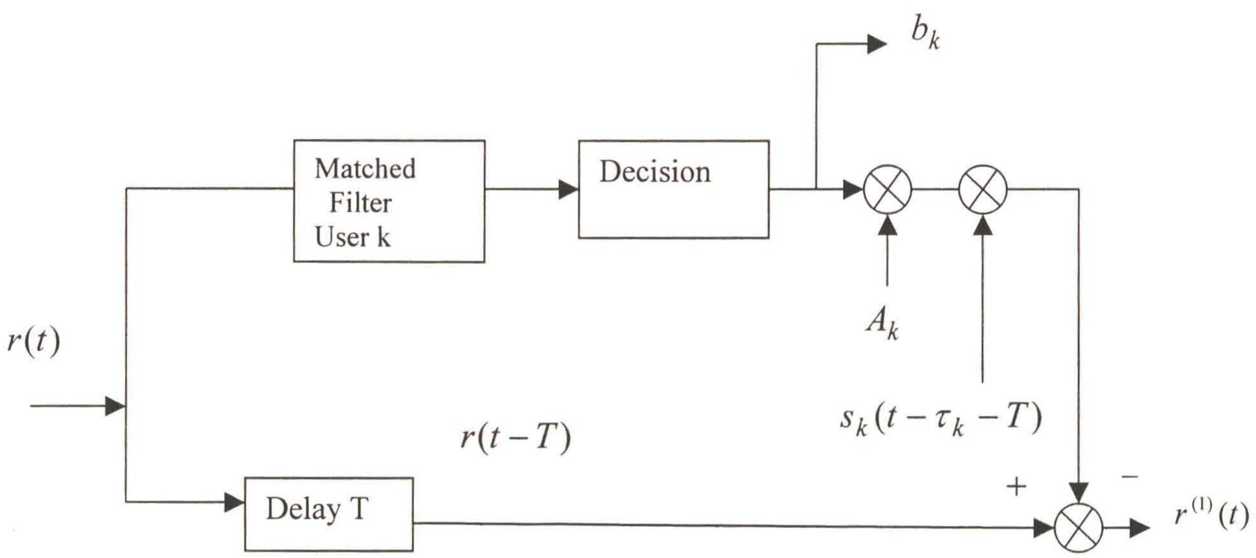


Figure 2.6: Successive Interference Cancellation- first stage

2.4.2.4 Parallel Interference Cancellation Detector

As the name suggests, the Parallel Interference Cancellation (PIC) detector estimates and cancels interference concurrently. This detector is shown in Figure 2.7. The operation of the PIC detector is summarized below (assuming the use of the conventional detector)

- The outputs of the bank of matched filters are scaled by the amplitudes.
- At the second stage, the scaled outputs of the matched filter are then spread again.
- The MAI experienced by, say the k^{th} user is formed by adding the outputs of the spreaders of the interfering user. The resulting signal is then subtracted from the bit delayed received signal. This operation is done for all the users.
- Finally, the vector of signals obtained from (3) is passed through a bank of matched filter, whose outputs are used as the decision statistics.

Assuming the detector has knowledge of the amplitude and delay timing of all the users, the decision statistics of the k^{th} user is given by

$$y_k = A_k \mathbf{s}_k \mathbf{r} - \sum_{\substack{j=1 \\ j \neq k}}^{K} A_j \mathbf{s}_k^T \mathbf{s}_j b_j \qquad \textbf{(2.20)}$$

The PIC detector was first proposed in [54]. In [111] Varanasi and Aazhang looked at another PIC structure, which uses hard decision. Some of the contributions of multistage PIC with soft tentative decisions are found in [2,23,53,61,111,112,131].

A number of authors have proposed adaptive implementation of the PIC detector. In [132] Xue et al proposed an adaptive PIC detector, which does not require explicit amplitude estimation. Ghazi-Moghadam and Mostafa looked at the blind PIC detector in [28]. In [12,14,15,31,99] partial interference cancellation schemes using the PIC detector were studied.

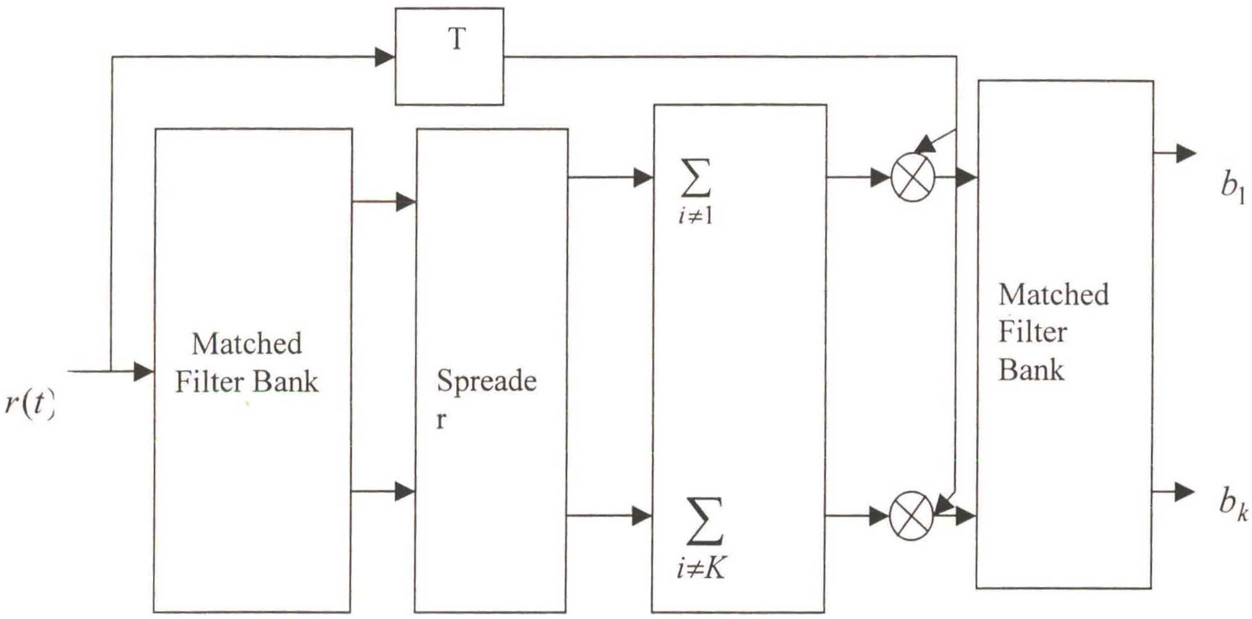


Figure 2.7 PIC detector

2.5 Multiuser detection in Fading Channels

DS-CDMA MUDs in fading channel are designed to take advantage of the inherent time-diversity presented by the channel. In these detectors, diversity combining is either performed prior or after MAI has been suppressed as illustrated in Figure 2.8.

This Section looks at MU detection in a fading channel. For simplicity, throughout this Section the fading complex channel coefficients will be assumed to be Rayleigh distributed.

2.5.1 Optimal detector

The optimal detector presented here is implemented using Maximum Likelihood Sequence Detection (MLSD). As was the case in a non-fading channel, the MLSD makes its decision as follows

$$\mathbf{b} = \arg \max_{\mathbf{b}\in\{-1,+1\}^{MK}} P(\mathbf{r} \mid \mathbf{b}) \qquad \textbf{(2.21)}$$

where $P(\mathbf{r} \mid \mathbf{b})$ is the pdf of the received signal vector ($\mathbf{r}$) conditioned on the data vector ($\mathbf{b}$). This pdf has a normal distribution and is given by

$$P(\mathbf{r} \mid \mathbf{b}) = D \exp(-\mathbf{r}^H \textstyle\sum_{\mathbf{r}|\mathbf{b}}^{-1} \mathbf{r}) \tag{2.22}$$

where the scalar D is given by [47]

$$D = \frac{1}{(2\pi)^{MKL/2} \det(\sum_{\mathbf{r}|\mathbf{b}})}$$

The covariance of the received signal ($\sum_{\mathbf{r}|\mathbf{b}}$) is given by [47]

$$\textstyle\sum_{\mathbf{r}|\mathbf{b}} = \mathbf{S}^T\mathbf{S} + \sigma^2 \sum_{\mathbf{h}|\mathbf{b}}^{-1}$$

where the covariance of the data-amplitude product ($\mathbf{h}$=$\mathbf{CAb}$) conditioned on $\mathbf{b}$ is given by

$$\textstyle\sum_{\mathbf{h}|\mathbf{b}} = \mathbf{B}^H E[\mathbf{C}^H \mathbf{C}]\mathbf{B}$$

$$\mathbf{B} = diag\{A_1 b_1^{(0)} \mathbf{I}_L, A_2 b_2^{(0)} \mathbf{I}_L, \ldots, A_K b_K^{(M-1)} \mathbf{I}_L\}$$

From Equations (2.21) and (2.22) it should be apparent that if constant envelope modulation is applied, the MLSD rule could be expressed as

$$\begin{aligned} \mathbf{b} &= \arg \min_{b \in \{-1,+1\}^{MK}} \mathbf{z}^H \textstyle\sum_{\mathbf{r}|\mathbf{b}}^{-1} \mathbf{r} \\ &= \arg \min_{b \in \{-1,+1\}^{MK}} \left(\left(\mathbf{S}^T\mathbf{S} + \sigma^2 \textstyle\sum_{\mathbf{h}|\mathbf{b}}^{-1} \right)^{-1} \mathbf{r} \right)^H \mathbf{r} \end{aligned} \tag{2.23}$$

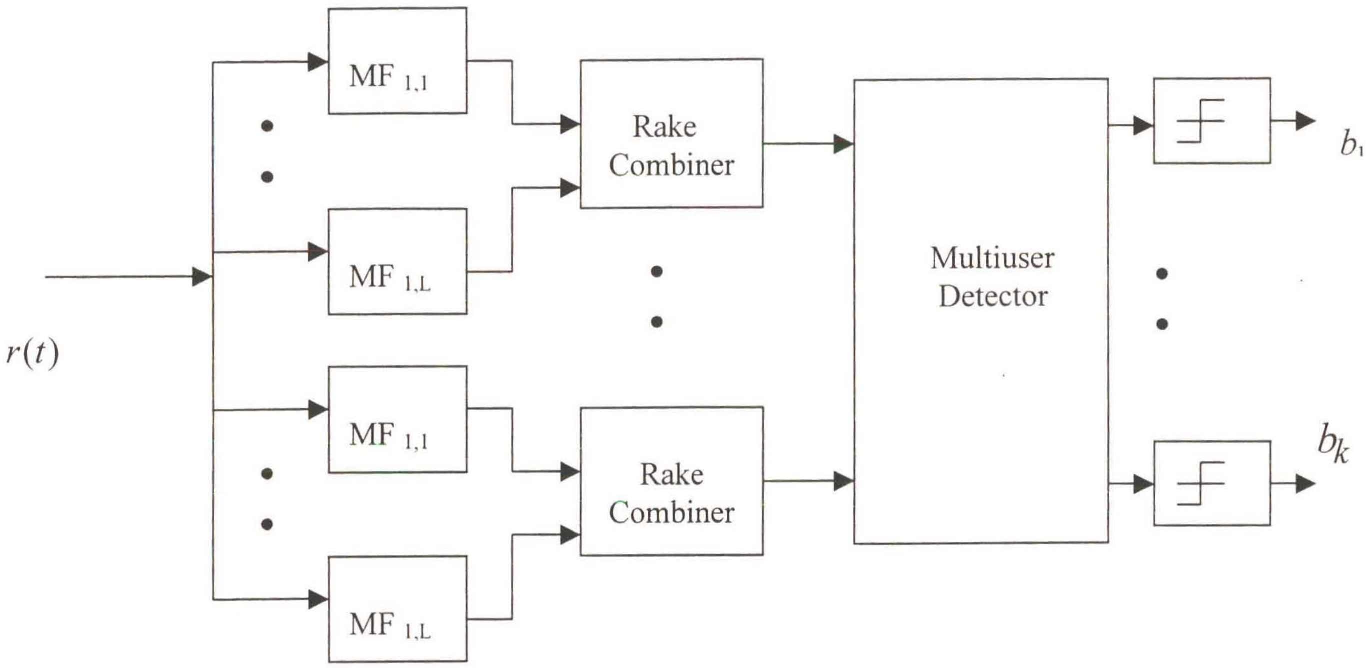
In the above expression the term

$$\hat{\mathbf{h}} = (\mathbf{S}^T\mathbf{S} + \sigma^2 \textstyle\sum_{\mathbf{h}|\mathbf{b}}^{-1})\mathbf{r} \tag{2.24}$$

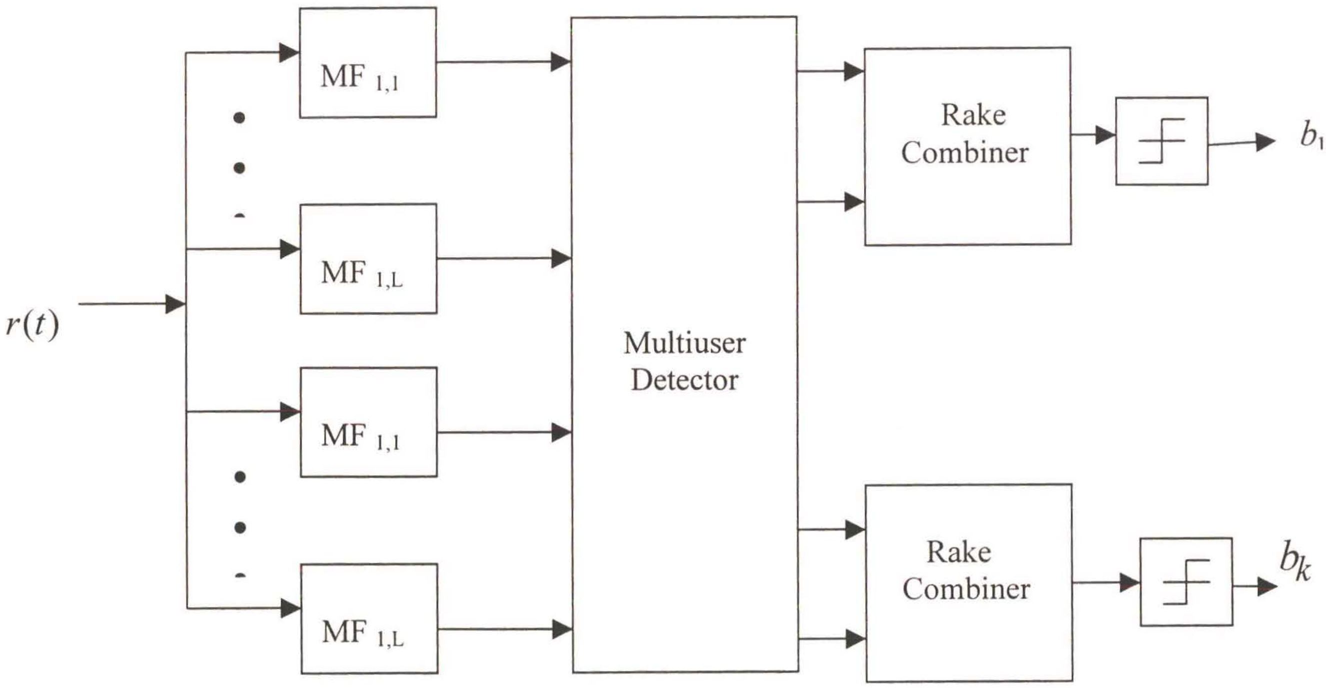
is the MMSE estimate of the data-amplitude product vector. Since this estimation must be performed for all the possible data sequences, the MLSD receiver is prohibitively complex for practical implementation.

2.5.2 Suboptimal detector

This subsection looks at the detectors presented in Section (2.4.2) in a fading multipath channel.



(a) Precombining detector



(b) Postcombining detector

Figure 2.8. Multiuser structures for fading channels.

2.5.2.1 Decorrelating Detector

The decision statistics of the decorrelating detector in a multipath fading channel are formed by multiplying the matched filter outputs by the inverse crosscorrelation of the spreading sequence of the transmitting users for all the paths, followed by diversity combining. Thus the decision statistics of the decorrelating detector is given by

$$\mathbf{y} = \mathbf{C}^H \mathbf{C} \mathbf{A} \mathbf{b} + \mathbf{C}^H \mathbf{R}^{-1} \mathbf{n} \tag{2.25}$$

where $\mathbf{R}$ is the crosscorrelation matrix and was defined in Equation (2.16). Liu and Li [65] analyzed the performance of the decorrelating detector in a fading channel. A blind adaptive implementation of the decorrelating detector in a fading channel was considered by Ulukus and Yates in [120].

2.5.2.2 MMSE Detector

With the MMSE detector, diversity combining can be performed either prior or after MAI suppression and the resulting MMSE detectors are termed Pre-combining and Post-combining MMSE detectors, respectively. The details of the two MMSE detectors will be presented in the next Chapter, but for now it will suffice to just give the linear mapping applied by these detectors and the resulting decision variable. In the case where diversity combining is performed prior to MAI suppression (Precombining MMSE detector) then the linear mapping is given by (see Chapter 3)

$$\mathbf{G} = \mathbf{A} \mathbf{C}^H \mathbf{S} (\mathbf{A} \mathbf{C}^H \mathbf{S}^T \mathbf{S} \mathbf{C} \mathbf{A} + \sigma^2 \mathbf{I})^{-1} \tag{2.26}$$

and the resulting decision statistic variable is given by (see Chapter 3)

$$\mathbf{y} = (\mathbf{A} \mathbf{C}^H \mathbf{S}^T \mathbf{S} \mathbf{C} \mathbf{A} + \sigma^2 \mathbf{I})^{-1} \mathbf{S} \mathbf{C} \mathbf{A} \mathbf{r} \tag{2.27}$$

The linear mapping of a Post-combining MMSE detector is given by (see Chapter 3)

$$\mathbf{W} = \mathbf{S} (\mathbf{S}^T \mathbf{S} + \sigma^2 E[\mathbf{C} \mathbf{A} \mathbf{A} \mathbf{C}]^{-1})^{-1} \tag{2.28}$$

and the resulting decision statistics is given by (see Chapter 3)

$$\mathbf{y} = (\mathbf{S}^T \mathbf{S} + \sigma^2 E[\mathbf{C} \mathbf{A} \mathbf{A} \mathbf{C}]^{-1})^{-1} \mathbf{S} \mathbf{r}$$

The major advantage offered by the MMSE detectors is that they lend themselves to adaptive implementation. The implementation of the MMSE detector using an FIR filter whose weighting coefficients are updated using the LMS algorithm (LMS-MMSE detector) was considered in [60,63,83,84], where simulation results were presented. The performance of the LMS-MMSE detector was then analyzed in [4,130].

2.5.2.3 Successive Interference Cancellation Detector

The simplest implementation of the SIC detector in fading channels, uses the output of the conventional rake detectors as the bit soft estimates and users strength estimation. This detector needs information of the channel coefficients and multipath delays of all users in addition to information stated in Section 2.4.2.3. Assuming that the detector has knowledge of this information then its algorithm is as follows

1. Using the outputs of the conventional detector estimate the strongest user and use the output of that detector to estimate the user's bit.
2. Regenerate the received signal due to the user detected above, using the channel information (fading coefficients and timing information), user's amplitude and spreading code.
3. Subtract the regenerated signal in (2) from the delayed version of the received signal. Repeat (1) until all the users' bits have been detected.

The SIC detector in fading channels were presented in [49,101,102]. The analysis of the SIC detector in fading channels have been reported in [51,87,88,101,102,106].

2.5.2.4 Parallel Interference Cancellation Detector

As was the case with the SIC, the simplest implementation of the PIC detector in a fading channel uses the output of the conventional-rake receiver, as the initial bit estimates. For the MAI to be cancelled the detector needs to know the fading channel coefficients and propagation delays of all the multipath components of all the users, in addition to the requirements specified in Section 2.4.2.4. Assuming the detector has this information and that specified in Section 2.4.2.4 then the PIC detection algorithm can be summarized as follows

1. Using the channel coefficients and the propagation delay of the paths of the interfering user, interfering users timing and amplitude and the outputs of the conventional-rake detector of the interfering users, regenerate the MAI suffered by the user of interest.
2. Subtract the MAI generated above from the received signal.
3. The resulting signal of (2) is then fed into the second conventional-rake receiver, whose output is used as the decision statistics.

The above algorithm is performed concurrently for all the users. The PIC for fading channels was presented in [125]. In [22,102], the performance of the PIC detector in a fading channel was analyzed. Finally, in [133] the adaptive implementation of the PIC detector in a fading chamel was presented.

2.6 Conclusion

The capacity of the conventional detector is severely affected by MAI. In addition to this, the conventional detector suffers from the near-far problem. Verdu made the major break-through in MU detection, when he proposed the optimal detector. The major problem with Verdu's detector is that its computational complexity makes it not practically viable.

Following Verdu's work a number of suboptimal detectors were proposed. These detectors offer a good performance-complexity tradeoff. Examples of these detectors are: (1) the decorrelating detector, (2) MMSE detector, (3) Successive Interference Cancellation and (4) Parallel Interference Cancellation.

In this Chapter a review of the optimal and suboptimal detectors was undertaken.

Chapter 3
LMS-MMSE Multiuser Detector

3.1 Introduction

This Chapter considers the MMSE detector and it's implementation using an LMS-based algorithm and its outline is as follows: Section 3.2 presents the system model to be used throughout this Chapter. Section 3.3 derive the optimal MMSE solution, while Section 3.4 looks at the adaptive implementation of the MMSE detector using the LMS algorithm. Section 3.5 looks at the convergence behavior of the LMS-MMSE detectors while Section 3.6 presents the bit error rate (BER) expression for (1) the conventional detector, (2) Ideal MMSE detector, (3) LMS-based MMSE detector. Finally in Section 3.7 conclusions are presented.

3.2 System Model

The system model used in this Chapter is identical to the one presented in Section 2.2 and will be summarized below for convenience.

The DS-CDMA system under consideration consists of K users asynchronously transmitting data in a fading channel with L resolvable paths. As described in Section 2.2, the received signal is given by

$$\mathbf{r} = \mathbf{SCAb} + \mathbf{n} \tag{3.1}$$

where, **r**, **b** and **n** are the vector of the received signal, data bits and noise samples, respectively while **S**, **C** and **A** are the matrices of the users' spreading code, channel fading coefficients and amplitude, respectively. For a complete definition of these quantities see Section 2.2. One thing, which should be borne in mind, is that the received baseband signal is sampled at a rate higher or equal to the chip rate. In addition to this, it should be noted that if chip-matched filtering is assumed, then the samples in Equation (3.1) are taken at the output of the chip-matched filter. Throughout this Chapter, additive noise and users' bits are assumed to have the following properties:

1. The source bits of one user are uncorrelated with past and future bits of itself and all bits of the other users. Using this property and the fact that the user's binary data bits are equiprobable, then the first and the second moment of the vector **b** can be written as follows

$$\begin{aligned} E[\mathbf{b}] &= 0 \\ E[\mathbf{b}\mathbf{b}^H] &= \mathbf{I} \end{aligned} \tag{3.2}$$

2. Additive noise is a zero mean process and is uncorrelated with the bits of the users. This property gives us the following equations

$$\begin{aligned} E[\mathbf{b}\mathbf{n}^H] &= 0 \\ E[\mathbf{n}] &= 0 \end{aligned} \tag{3.3}$$

In addition to these properties when no chip-matched filtering is employed or when the output of the chip matched filter is sampled at the chip-rate then additive noise is modelled as white Gaussian noise, thus yielding the following property

$$E[\mathbf{n}\mathbf{n}^H] = \sigma^2\mathbf{I} \tag{3.4}$$

where σ^2 is the variance of the noise.

3.3 Optimal MMSE detector

MAI suppression using the MMSE DS-CDMA detector can either be performed prior or after multipath combining as depicted in Figure 2.8. The MMSE detector, which performs multipath combining prior to MAI suppression, is referred to as the precombining MMSE detector, while the one, which first suppresses MAI, is called the postcombining MMSE detector. These two detection strategy are the subject of the this section and we first start with the precombining MMSE detector.

3.3.1 Precombining MMSE detector

The optimal weighting coefficients of the precombining DS-CDMA MMSE detector are selected such that the following cost function is minimized [48,63,83,86]

$$\mathbf{J} = E[|\mathbf{b} - \hat{\mathbf{b}}|^2] \tag{3.5}$$

where $\hat{\mathbf{b}}$ is the vector of the estimate of the transmitted bits for all the users and is defined as follows

$$\hat{\mathbf{b}} = \mathbf{G}^H \mathbf{r} \tag{3.6}$$

where $\mathbf{G}$ is the matrix of the weighting coefficients for all the users. Expanding Equation (3.5) and using properties (3.2) and Equation (3.3) and finally substituting Equation (3.6) one obtains the following results

$$\begin{aligned}\mathbf{J} &= E[(\mathbf{b}-\hat{\mathbf{b}})(\mathbf{b}^H - \hat{\mathbf{b}}^H)] \\ &= E[\mathbf{b}\mathbf{b}^H] - E[\mathbf{b}\hat{\mathbf{b}}^H] - E[\hat{\mathbf{b}}\mathbf{b}] + E[\hat{\mathbf{b}}\hat{\mathbf{b}}] \\ &= \mathbf{I} - \mathbf{A}\mathbf{C}^H\mathbf{S}^T\mathbf{G} - \mathbf{G}^H\mathbf{S}\mathbf{C}\mathbf{A} + \mathbf{G}^H E[\mathbf{r}\mathbf{r}^H]\mathbf{G}\end{aligned} \tag{3.7}$$

Differentiating the above Equation with respect to $\mathbf{G}$ and setting the result to zero yields the following results

$$\mathbf{G} = \mathbf{SCA}\left(\mathbf{SCAAC}^H\mathbf{S}^T + E[\mathbf{nn}^H]\right)^{-1} \tag{3.8}$$

From the above Equation it follows that the soft output of the precombining MMSE detector is given by

$$\hat{\mathbf{b}} = \left(\mathbf{SCAAC}^H\mathbf{S}^T + E[\mathbf{nn}^H]\right)^{-1}\mathbf{AC}^H\mathbf{S}^T\mathbf{r} \tag{3.9}$$

The term $\mathbf{AC}^H\mathbf{S}^T\mathbf{r}$ is the output of the conventional Rake receiver. As can be seen from Equation (3.8), the optimal weighting coefficients depend on the instantaneous channel coefficients. This might pose a problem if the channel coefficients are varying rapidly, since the detector might have problems tracking the channel variations. The remedy for such a problem is to use the post-combining MMSE detector, which will be presented in the next Section.

The advantage of the precombining detector is that it only requires one transversal filter per user, irrespective of the number of multipaths present, thus reducing the computational complexity of the detector to some extent. In a non-fading channel (L=1 and **C**=**I**), Equation (3.8) simplifies to the following results

$$\mathbf{G} = \mathbf{S}(\mathbf{S}\mathbf{S}^T + E[\mathbf{n}[\mathbf{A}\mathbf{A}]^{-1}\mathbf{n}^H])^{-1} \qquad (3.10)$$

3.3.2 Postcombining MMSE detector

The postcombining MMSE detector, unlike the precombining MMSE detector selects its weighting coefficients such that the mean of the square of the absolute value of the difference between the data amplitude product and its estimate is minimized [59,83]. The data amplitude product vector is defined as follows

$$\mathbf{h} = \mathbf{C}\mathbf{A}\mathbf{b} \qquad (3.11)$$

and its estimate denoted as $\hat{\mathbf{h}}$ is given by the following expression

$$\hat{\mathbf{h}} = \mathbf{W}^H\mathbf{r} \qquad (3.12)$$

The cost function minimized by the postcombining detector is thus given by the following expression

$$\mathbf{J} = E[|\mathbf{h} - \hat{\mathbf{h}}|^2] \qquad (3.13)$$

Expanding the above Equation yields the following results

$$\begin{aligned}\mathbf{J} &= E[(\mathbf{h} - \hat{\mathbf{h}})(\mathbf{h}^H - \hat{\mathbf{h}}^H)] \\ &= E[\mathbf{h}\mathbf{h}^H] - E[\mathbf{h}\hat{\mathbf{h}}^H] - E[\hat{\mathbf{h}}\mathbf{h}^H] + E[\hat{\mathbf{h}}\hat{\mathbf{h}}^H]\end{aligned} \qquad (3.14)$$

Substituting for $\mathbf{h}$ and $\hat{\mathbf{h}}$ in the above Equation results in the following expression for the cost function

$$\mathbf{J} = E[\mathbf{CAAC}^H] - E[\mathbf{CAAC}^H]\mathbf{W} - \mathbf{W}^H E[\mathbf{CAAC}H] + \mathbf{W}^H[\mathbf{r}\mathbf{r}^H]\mathbf{W} \qquad (3.15)$$

Differentiating the above Equation with respect to the weighting coefficient matrix and setting the resulting equation to zero and simplifying the results, one obtains the following expression for the optimal weighting coefficients

$$\mathbf{W} = \mathbf{S}\left\{\mathbf{S}\mathbf{S}^T + E\left(\mathbf{n}\left(E[\mathbf{CAAC}^H]\right)^{-1}\mathbf{n}^H\right)\right\}^{-1} \qquad (3.16)$$

From the above Equation for the optimal weighting coefficients of the post-combining MMSE detector, it can be seen that the weighting coefficients depend on the average of the fading channel coefficients. This property makes the post-combining MMSE detector, a good choice in relatively fast fading channel. The only drawback with the post-combining MMSE detector is its computational requirements, since for every multipath component a transversal filter is required.

3.4 LMS based DS-CDMA MMSE detectors

The transversal structure of the MMSE detector makes it easily amenable to implementation using a filter whose coefficients are iteratively computed using an adaptive algorithm. Such implementation greatly reduces the computational requirements of the MMSE detector, since the matrix inversion inherent in the computation of the optimal weighting coefficient can be avoided. This Section looks at the implementation of both the precombining and postcombining DS-CDMA MMSE detectors using the Least Mean Squared (LMS) algorithm. Since each user might transmit infinitely many bits, the adaptive MMSE detector presented here process the received signal in blocks, where each block spans a number of bits. This also reduces the computational complexity of the MMSE detector quite considerably.

3.4.1 LMS precombining MMSE detector

Assume that the detector is to process the following vector of the received signal, which spans P bits

$$\mathbf{r}_P(m) = \left[\mathbf{r}(m),\ldots,\mathbf{r}(m-(P-1))\right] \qquad \textbf{(3.17)}$$

and that the user of interest is the k^{th} user. The vector of this user's weighting coefficients at time m, which is the k^{th} row of **G**, will be denoted as $\mathbf{g}_k(m)$, which initially is set to a null vector. Following the definition of Equation (2.5), the dimension of this vector is PNS - by - 1 where we assumed that within one symbol interval S samples of the received signal are taken.

The ideal method of obtaining the weighting coefficients will be to use the method of steepest decent, where the optimal coefficients of the user's weighting coefficients are obtained iteratively using the following update equation [32,93]

$$\mathbf{g}_k(m+1) = \mathbf{g}_k(m) + \frac{1}{2}\mu[-\nabla \mathbf{J}_k(m)] \quad \textbf{(3.18)}$$

where $\nabla \mathbf{J}_k(m)$ is the k^{th} row of the gradient of the vector $\mathbf{J}(m)$ defined in Equation (3.7) and μ is the step size parameter, which should be small enough to ensure convergence of the coefficients. Differentiation of Equation (3.7) yields the following equation [32,93]

$$\nabla \mathbf{J}_k(m) = (2E[\mathbf{r}_P(m)\mathbf{r}_P^H(m)]\mathbf{g}_k(m) - 2E[\mathbf{r}_P(m)b_k^*(m)]) \quad \textbf{(3.19)}$$

Since the correlation matrix $E[\mathbf{r}_P(m)\mathbf{r}_P^H(m)]$ and the vector $E[\mathbf{r}_P(m)b_k^*]$ are not readily available, instantaneous values are usually used to approximate them. Thus the gradient vector approximation is given by the following expression

$$\nabla \hat{\mathbf{J}}(m) = 2\left(\mathbf{r}_P(m)\mathbf{r}_P^H(m)\mathbf{g}(m) - b_k^*(m)\mathbf{r}_P(m)\right) \quad \textbf{(3.20)}$$

Substitution of Equation (3.20) into Equation (3.18) yield the following update equation for the weighting coefficients vector

$$\mathbf{g}_k(m+1) = \mathbf{g}_k(m) + \mu[\mathbf{r}_P(m)\mathbf{r}_P^H\mathbf{g}(m) - b_k(m)\mathbf{r}_P(m)] \quad \textbf{(3.21)}$$

There is only one thing we need to specify how it is determined. This is the data bit $b_k(m)$. This information depends on the mode the transmitter and receiver are in. These modes are training-based mode and decision-directed mode. During training mode, the transmitter sends data bits, which the receiver has knowledge of. The receiver uses these bits to adapt its weighting coefficients. After the weighting coefficients have converged to their optimal value the receiver switches to decision directed mode, where the previous decisions made by the detector are used to update the weighting coefficients. These decisions are made according to the hard-decision rule

$$\hat{b}_k(m) = \mathrm{sgn}(\mathbf{r}_P^H(m)\mathbf{g}(m)) \quad \textbf{(3.22)}$$

where the hat symbolize that the bit made by the detector is an estimate of the actual transmitted bit and sgn return the sign of the argument to it. In [29,32,93], the condition, which must be met by the step-size parameter to ensure that the weighting coefficients converge to their optimal solution, was determined. This condition is given by

$$0 < \mu < \frac{2}{\lambda_{\max}} \quad \textbf{(3.23)}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the matrix $\mathbf{F}_P = E[\mathbf{r}_P(m)\mathbf{r}_P^H(m)]$.

3.4.2 LMS postcombining MMSE detector

In this subsection we turn our attention to the adaptive implementation of the postcombining detector using the LMS algorithm. For this MMSE detector the number of FIR filter employed to suppress MAI equals the number of resolvable multipath component the channel has. Let $\mathbf{W}_k(m)$ be the matrix of the weighting coefficients for the k^{th} user with elements

$$\mathbf{W}_k(m) = [\mathbf{w}_{k,1}(m),...,\mathbf{w}_{k,L}(m)] \quad \textbf{(3.24)}$$

where the vector $\mathbf{w}_{k,l}(m)$ is a vector of the weighting coefficient of the k^{th} user's l^{th} multipath component and is computed using the following update equation

$$\mathbf{w}_{k,l}(m+1) = \mathbf{w}_{k,l}(m) + \mu[\mathbf{r}_P(m)\mathbf{r}_P^H(m)\mathbf{w}_{k,l}(m) - b_k(m)\mathbf{r}_P(m)] \quad \textbf{(3.25)}$$

Assuming the detector has knowledge of the channel fading coefficients or their estimate, for the user of interest (i.e. $\mathbf{c}_k(m) = [c_{k,1}(m),...,c_{k,L}(m)]$ is known to the detector), then the data estimate for the k^{th} user's transmission are given by [47,48]

$$\hat{b}_k(m) = \text{sgn}\left(\sum_{l=1}^{L} c_{k,l}(m)\mathbf{w}_{k,l}(m)\mathbf{r}_P(m)\right) \quad \textbf{(3.26)}$$

3.5 Convergence Analysis of the LMS-MMSE detector

This Section looks at the convergence analysis of the LMS-MMSE detector. We only consider the convergence analysis of the precombining MMSE detector and we start by first looking at the case where either the output of the chip-matched filter is sampled at the chip-rate or no chip-matched filter is used. Following this, subsection 3.5.2 looks at the case where the chip-matched filter output is sampled at a rate much higher than the chip rate.

3.5.1 CASE 1

Throughout this Section we will assume that the k^{th} user is the user of interest. The update equation for this user's weighting coefficients was presented in Equation (3.21) and is given again here for convenience

$$\mathbf{g}_k(m+1) = \mathbf{g}_k(m) + \mu[\mathbf{r}_P(m)\mathbf{r}_P^H(m)\mathbf{g}(m) - b_k(m)\mathbf{r}_P(m)] \qquad \textbf{(3.27)}$$

Let us denote the difference between the weighting coefficients computed using the above Equation and the optimal weighting coefficients computed using Equation (3.8) for the k^{th} user at time m as $\Delta\mathbf{g}_k(m)$. It can be easily shown that the update Equation for $\Delta\mathbf{g}_k(m)$ is given by [32]

$$\Delta\mathbf{g}_k(m+1) = (\mathbf{I} - \mu\mathbf{r}_P(m)\mathbf{r}_P^H(m))\Delta\mathbf{g}_k(m) \qquad \textbf{(3.28)}$$

Taking the expectation of the above Equation and bearing in mind that $\Delta\mathbf{g}_k(m)$ is independent of $\mathbf{r}_P(m)$ we obtain the following expression

$$E[\Delta\mathbf{g}_k(m+1)] = E[\mathbf{I} - \mu\mathbf{F}_P]E[\Delta\mathbf{g}_k(m)] \qquad \textbf{(3.29)}$$

Using an iterative approach and noting that $\Delta\mathbf{g}_k(0)$ is deterministic, it can be shown that the above Equation can be written as

$$E[\Delta\mathbf{g}_k(m+1)] = [\mathbf{I} - \mu\mathbf{F}_P]^{m+1}\Delta\mathbf{g}_k(0) \qquad \textbf{(3.30)}$$

The matrix $\mathbf{F}_p$, which is Hermitian, can be written as

$$\mathbf{F}_P = \mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Gamma}^H \qquad \textbf{(3.31)}$$

where $\mathbf{\Gamma}$ is a normalized unitary matrix of the eigenvectors of $\mathbf{F}_p$, while $\mathbf{\Lambda}$ is a matrix of the eigenvalues of $\mathbf{F}_p$. Using Equation (3.31), we can rewrite Equation (3.30) as follows

$$\begin{aligned} E[\Delta\mathbf{g}_k(m+1)] &= (\mathbf{I} - \mu\mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Gamma}^H)^{m+1}\Delta\mathbf{g}_k(0) \\ &= \mathbf{\Gamma}(\mathbf{I} - \mu\mathbf{\Lambda})^{m+1}\mathbf{\Gamma}^H\Delta\mathbf{g}_k(0) \end{aligned} \qquad \textbf{(3.32)}$$

The matrix of the eigenvectors $\mathbf{\Gamma}$ can be written as a sum of two orthogonal matrices, which are the signal subspace matrix (denoted as $\mathbf{E}_s$) and the noise subspace matrix (denoted as $\mathbf{E}_n$), respectively. If the optimal weighting coefficients and the initial estimate of the optimal coefficients are in the signal subspace, then the above Equation simplifies to

$$E[\Delta\mathbf{g}_k(m+1)] = \mathbf{E}_s\left(\mathbf{I} - \mu\mathbf{\Lambda}_s - \mu\sigma^2\mathbf{I}\right)^{m+1}\mathbf{E}_s^H\Delta\mathbf{g}_k(0) \qquad \textbf{(3.33)}$$

where $\mathbf{\Lambda}_S$ is a diagonal matrix of the non-zero eigenvalues of the matrix $\mathbf{S}_P(m)\mathbf{C}_P(m)\mathbf{A}_P(m)\mathbf{A}_P(m)\mathbf{C}_P^H(m)\mathbf{S}_P^T(m)$. Equation (3.33) tells us that when the initial weighting coefficients vector is inside the signal subspace, then the convergence of $E[\mathbf{g}_k(m)]$ to the optimal weighting coefficients only depends on those eigenvalues of $\mathbf{F}_p$ that are inside the signal space.

Most of the work in the literature treats the eigenvalue spread as the measure of convergence performance [29,32,93]. Using the same performance measure and assuming that the weighting coefficients are initialized inside the signal subspace, one obtains the following performance index

$$P_{s,eig} = \frac{\lambda_{s,\max} + \sigma^2}{\lambda_{s,\min} + \sigma^2} \quad \textbf{(3.34)}$$

where $\lambda_{s,\max}$ and $\lambda_{s,\min}$ are the maximum and minimum eigenvalues of the matrix $\mathbf{\Lambda}_S$. In contrast to this, if the weighting coefficients are initially inside the noise subspace vector then the eigenvalue spread is given by the following expression

$$P_{n,eig} = \frac{\lambda_{s,\max} + \sigma^2}{\sigma^2} \quad \textbf{(3.35)}$$

From Equations (3.34) and (3.35), it can be seen that initialization of the weighting to within the signal subspace results in reduced eigenvalue spread, leading to faster convergence rate. The problem with the above performance measure is that it only gives us an idea about the convergence rate of the LMS algorithm; it doesn't say anything about the misadjustment noise, which happens to play a vital role on the performance of adaptive algorithms. In [64], the authors proposed another measure, which penalize both misadjustment and the rate of convergence experienced by the LMS algorithm. This measure is given by the following expression

$$P = \zeta \tau_{\max} \quad \textbf{(3.36)}$$

where $\tau_{\max} = (\mu\lambda_{\min})^{-1}$ is the largest convergence time constant and $\zeta = tr(\mathbf{F}_P)$ is the misadjustment noise. Using Equation (3.36), the performance measure of the LMS algorithm when the weighting coefficients are initialized to within the signal subspace is given by [64]

$$P_s = \frac{\sum_{i=1}^{M} \lambda_{s,i} + N_1\sigma^2}{\lambda_{s,\min} + \sigma^2} \tag{3.37}$$

where M is the number of column in Λ_S and N_1 is the number of filter coefficients. On the other hand when the weighting coefficients are initialized to within the noise subspace then Equation 3.36 yields the following performance index

$$P_n = \frac{\sum_{i=1}^{M} \lambda_{s,i} + N_1\sigma^2}{\sigma^2} \tag{3.38}$$

From Equations (3.37) and (3.38) it should be apparent that initialization of the weighting coefficients to within the signal subspace yields a smaller performance index value when compared to noise subspace initialization. Since the performance index is directly proportional to misadjustment noise and inversely proportional to the convergence rate, it follows that signal subspace initialization yield better performance when compared to noise subspace initialization.

3.5.1 CASE 2

The focus of this Section is on the convergence analysis of the LMS-MMSE tap-weight vector when the output of the chip-matched filter is sampled at a rate much higher than the chip rate. Throughout this analysis we will assume that the tap weights are spaced one sample apart. As stated in Section 3.2, additive noise cannot be modelled as white when the chip-matched filter output is sampled at a rate much higher than the chip-rate. This complicates matters, since the over-sampled signal cannot be easily split into the noise and signal subspaces.

The only way around this problem is to assume that the chip-matched filter has a magnitude response that is non-zero inside the range $-(1/2T_c) < f < (1/2T_c)$ and zero otherwise [64]. If this assumption holds then over-sampling the output of the chip-matched filter is equivalent to sampling the chip-matched filter at the chip-rate, followed by interpolation. Figure 3.1 shows how over-sampling the output of the chip-matched filter can be modelled under the assumption that the chip-matched filter is perfectly band limited.

In Appendix A, the signal model used in this Section for the analysis of the convergence behavior of the over-sampled output of the chip-matched filter is developed. From the results obtained there, the autocorrelation of the over-sampled finite input vector $\mathbf{r}_P(m)$ is given by

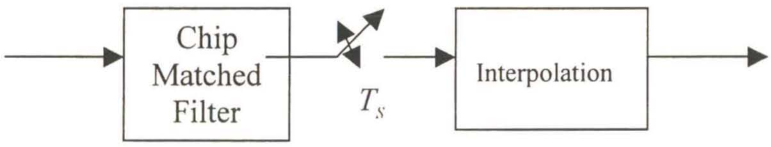


Figure 3.1 Modelling an oversampled signal.

$$\mathbf{F}_S = \begin{bmatrix} S^{-1/2}\mathbf{Y}_S\mathbf{\Gamma} & \mathbf{X} \end{bmatrix} \begin{bmatrix} S\mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} S^{-1/2}\mathbf{\Gamma}^H\mathbf{Y}_S^H \\ \mathbf{X}^H \end{bmatrix} \qquad \mathbf{(3.39)}$$

where S is the number of samples per chip and $\mathbf{F}_S$ is the autocorrelation of the oversampled chip-matched filter output. $\mathbf{Y}_S$ is the interpolating matrix, $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ are the eigenvectors and eigenvalues matrices of the chip-rate sampled output of the chip-matched filter and were defined in Equation (3.31). Finally the matrix $\mathbf{X}$ is any matrix with properties: (1) it is orthogonal to $\mathbf{Y}_S\mathbf{\Gamma}$ and (2) it satisfies $\mathbf{X}^H\mathbf{X} = \mathbf{I}$.

As was done in the last Section, it can be shown that the mean of the difference between the weights computed using the LMS update equation and their optimal value is given by

$$E[\Delta\mathbf{g}_k(m+1)] = (I - \mu\mathbf{F}_S)^{m+1}\Delta\mathbf{g}_k(0) \qquad \mathbf{(3.40)}$$

Substituting Equation (3.39) into Equation (3.40) and simplifying yields

$$E[\Delta\mathbf{g}_k(m+1)] = S^{-1}\mathbf{Y}_S\mathbf{\Gamma}(\mathbf{I} - \mu S\mathbf{\Lambda})^{m+1}\mathbf{\Gamma}\mathbf{Y}_S\Delta\mathbf{g}_k(0) + \mathbf{X}\mathbf{X}^H\Delta\mathbf{g}_k(0) \qquad \mathbf{(3.41)}$$

As we did in the previous Section we can split the matrix $\mathbf{\Gamma}$ into the signal subspace $\mathbf{E}_s$ and the noise subspace $\mathbf{E}_n$. If we assume that the vector $\Delta\mathbf{g}_k$ was initially in the signal subspace then Equation (3.41) can be written as

$$E[\Delta \mathbf{g}_k(m+1)] = S^{-1}\mathbf{Y}_S\mathbf{E}_S(\mathbf{I} - \mu S\mathbf{\Lambda}_s - \mu\sigma^2 S\mathbf{I})^{m+1}\mathbf{E}_S^H\mathbf{Y}_S\Delta \mathbf{g}_k(0) + \mathbf{X}\mathbf{X}^H\Delta \mathbf{g}_k(0)$$

(3.42)

where the vector $\mathbf{\Lambda}_S$ was defined in the previous Section. From Equation (3.41) and (3.42) the following deductions can be made:

1. As long as $|1 - \mu\lambda_j| < 1$ for all j, then the limit as m approaches infinity of Equation (3.41) approaches

$$\lim_{m\to\infty} E[\Delta \mathbf{g}_k(m+1)] = \mathbf{X}\mathbf{X}^H\Delta \mathbf{g}_k(0) \quad \textbf{(3.43)}$$

   The λ_j used here is the j^{th} eigenvalue of the matrix $\mathbf{F}_S$ and should not be confused with that of the matrix $\mathbf{\Lambda}_S$, which is represented as $\lambda_{s,j}$.
2. For the MSE of the adaptive filter to be at it's minimum, $\Delta \mathbf{g}_k(m)$ should be in the subspace spanned by $\mathbf{X}$
3. The eigenvalues of $\mathbf{F}_S$ are S times the eigenvalues of $\mathbf{F}_P$.
4. If the vector $\Delta \mathbf{g}_k(0)$ was initially in the signal subspace then the largest convergence time constant is given by $\tau_{\max} = (S\mu(\lambda_{s,\min} + \sigma^2))^{-1}$. Similarly if the vector $\Delta \mathbf{g}_k(0)$ was initially in the noise subspace then the largest convergence time constant is given by $\tau_{\max} = (S\mu\sigma^2)^{-1}$

From deduction 3, it can be seen that the misadjustment of the adaptive filter when the output of the chip-matched filter is over-sampled is given by

$$\begin{aligned}\zeta_S &= tr(\mathbf{F}_S) \\ &= S\,\mathrm{tr}(\mathbf{F}_P) \\ &= S\sum_{l=1}^{M}\lambda_{s,i} + SN_1\sigma^2\end{aligned} \quad \textbf{(3.44)}$$

where M is the number of column in $\mathbf{\Lambda}_S$ and N_1 is the number of filter coefficients. Using deduction 4 and Equation (3.43) one can write the performance indices for the both the signal subspace and noise subspace initialization as follows

$$P_s = \frac{\sum_{i=1}^{M} \lambda_{s,i} + N_1 \sigma^2}{\lambda_{s,\min} + \sigma^2} \qquad (3.45)$$

$$P_n = \frac{\sum_{i=1}^{M} \lambda_{s,i} + N_1 \sigma^2}{\sigma^2} \qquad (3.46)$$

Thus one can conclude once again that signal subspace initialization yield better performance than noise subspace initialization.

3.6 BER Analysis of the MMSE detectors

This Section is dedicated to the bit error rate (BER) analysis of the MMSE detector and its LMS implementation. First we will look at the bit error rate of the precombining detector. Following this we will look at the bit error rate analysis of the postcombining detector. Throughout this Section we will assume that if a chip-matched filter is used then the sampling rate is the chip-rate so that all assumptions made in Section 3.2 hold.

3.6.1 BER Analysis for the precombining detector

A. Ideal MMSE

Lets start by assuming that the channel fading coefficients are non-varying. Later in this Section we will amend the expression to include fading channel effects. In a non-fading channel the bit error rate is given by the following expression [47-48,91]

$$P_{e,k} = 2^{1-K} \sum_{\substack{\mathbf{b} \in \{-1,1\}^{PK} \\ b_k = 1}} Q\left(\frac{\Re(\mathbf{g}_k^H \mathbf{SCAb})}{\sqrt{\sigma^2 \mathbf{g}_k^H \mathbf{g}_k}} \right) \qquad (3.47)$$

where Q is the complementary error function and $\Re()$ returns the real part of its input. A simpler approximation for the bit error rate expression can be derived if one assumes that the distribution of the k^{th} user's decision variable has a Gaussian distribution. To derive this BER expression we first start by defining the following correlation function for the user of interest

$$\boldsymbol{\alpha}_k = E[\mathbf{r}_P(m) b_k] \tag{3.48}$$

For simplicity we will assume that $P=1$ and the index P will be dropped out were unnecessary. Using the above definition, the optimal coefficients of the k^{th} user can be written as follows [4]

$$\mathbf{g}_k = \boldsymbol{\alpha}_k \mathbf{F}^{-1} \tag{3.49}$$

and the minimum MSE that can be achieved by the k^{th} user's adaptive filter can be shown to be as follows [73]

$$J_{\min,k} = 1 - \boldsymbol{\alpha}_k^H \mathbf{F}^{-1} \boldsymbol{\alpha}_k \tag{3.50}$$

Let's denoted the decision variable of the k^{th} user's detector as $y_k(m)$, then this variable can be written as follows

$$y_k(m) = \mathbf{g}_k^H \mathbf{r}(m) \tag{3.51}$$

The mean of the decision variable conditioned on $b_k(m) = 1$ is given by the following expression

$$\begin{aligned} \bar{y}_k &= E[y_k(m) \mid b_k = 1] \\ &= \mathbf{g}_k^H \boldsymbol{\alpha}_k \\ &= 1 - J_{\min,k} \end{aligned} \tag{3.52}$$

where $J_{\min,k}$ is defined above. The variance of the decision variable is given by the following expression

$$\begin{aligned} \sigma_{y_k}^2 &= E[(y_k - \bar{y}_k)^2 \mid b_k = 1] \\ &= E[y_k^2] - \bar{y}_k^2 \\ &= \mathbf{g}_k^H E[\mathbf{r}\mathbf{r}^H] \mathbf{g}_k - \mathbf{g}_k^H \boldsymbol{\alpha}_k \boldsymbol{\alpha}_k^H \mathbf{g}_k \\ &= \mathbf{g}_k^H \mathbf{F} \mathbf{g}_k - \mathbf{g}_k^H \boldsymbol{\alpha}_k \boldsymbol{\alpha}_k^H \mathbf{g}_k \\ &= \boldsymbol{\alpha}_k^H \mathbf{g}_k - \mathbf{g}_k^H \boldsymbol{\alpha}_k \boldsymbol{\alpha}_k^H \mathbf{g}_k \\ &= J_{\min,k}(1 - J_{\min,k}) \end{aligned} \tag{3.53}$$

Thus the signal-to-interference ratio is given by the following expression

$$\begin{aligned} SIR_k &= \frac{\bar{y}_k^2}{\sigma_{y_k}^2} \\ &= \frac{1-J_{\min,k}}{J_{\min,k}} \end{aligned} \tag{3.54}$$

Finally the expression for the bit error rate is given by the following expression

$$\begin{aligned} P_{e,k} &\approx Q(SIR) \\ &= Q\left(\frac{1-J_{\min,k}}{J_{\min,k}}\right) \end{aligned} \tag{3.55}$$

In a fading channel expressions (3.47) and (3.55) are the conditional BER expressions, the condition being that the channel coefficients are non-time varying. The BER expression is then obtained from these expressions by averaging them over the probability density function of the fading channel coefficients of all the users' paths. For simplicity we will denote this average using the following notation (using Equation (3.55))

$$P_{e,k} = E\left(Q\left(\frac{1-J_{\min,k}}{J_{\min,k}}\right)\right) \tag{3.56}$$

As an illustrating example the BER expression for an ideal MMSE detector in a case where we have two users transmitting data in a two path fading channel is:

$$P_e = \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty f(c_{1,1})f(c_{1,2})f(c_{2,1})f(c_{2,2})Q\left(\frac{1-J_{\min,k}}{J_{\min,k}}\right)dc_{1,1}dc_{1,2}dc_{2,1}dc_{2,2} \tag{3.57}$$

B. LMS-MMSE detector

For the LMS MMSE detector, the MSE at the output of the k^{th} user's detector at time m is given by the following expression [131] (see also Equation (3.7))

$$J_k = 1 - \mathbf{g}_k^H \boldsymbol{\alpha}_k - \boldsymbol{\alpha}_k^H \mathbf{g}_k + \mathbf{g}_k^H \mathbf{F} \mathbf{g}_k \tag{3.58}$$

When the excess MSE is small compared to the minimum MSE then we can use the following approximation $\boldsymbol{\alpha}_k \mathbf{g}_k \approx \mathbf{g}_k^H \mathbf{F} \mathbf{g}_k$, so that the above expression can be written as follows

$$J_k = 1 - \mathbf{g}_k^H \mathbf{F} \mathbf{g}_k \quad \textbf{(3.59)}$$

Likewise Equation (3.52) and (3.53) can be written as follows

$$\bar{y}_k \approx 1 - J_k \quad \textbf{(3.60)}$$

and

$$\sigma_{y_k}^2 \approx J_k (1 - J_k) \quad \textbf{(3.61)}$$

Thus the BER expression for the LMS-MMSE detector in a non-fading channel is given by the following expression

$$P_{e,k} \approx Q\left(\frac{1 - J_k}{J_k}\right) \quad \textbf{(3.62)}$$

When infinite training symbols are used, J_k can be computed as follows [32,93]:

$$\begin{aligned} J_k &= J_{\min,k} + J_{exc,k} \\ &= J_{\min,k}\left(1 + \sum_{i=1}^{M} \frac{\mu\lambda_i}{2 - \mu\lambda_i}\right) \end{aligned} \quad \textbf{(3.63)}$$

where λ_i are the eigenvalues of the matrix $E[\mathbf{r}_p(m)\mathbf{r}_p^H(m)]$. In a fading channel the BER expression can be written as follows

$$P_k = E\left(Q\left(\frac{1 - J_k}{J_k}\right)\right) \quad \textbf{(3.64)}$$

3.6.2 BER Analysis for the postcombining detector

In this subsection we derive the BER expression of the MMSE detector, which performs post multipath combining. We assume that the combining vector $\mathbf{c}_k(m) = [c_{k,1}(m), \ldots, c_{k,L}(m)]$ for the k^{th} user is complex Gaussian vector with zero mean. The output of the l^{th} path interference-suppressing filter for the k^{th} user is given by

$$z_{k,l}(m) = \mathbf{w}_{k,l}^{H}(m)\mathbf{r}_P(m) \tag{3.65}$$

Lets define the vector $\mathbf{z}_k(m)$ as

$$\mathbf{z}_k(m) = [z_{k,1}(m),..., z_{k,L}(m)]$$

Then the decision statistic for the k^{th} users signal can be written as follows

$$y_k(m) = \mathbf{c}_k^H(m)\mathbf{z}_k(m) \tag{3.66}$$

Let

$$Q = \begin{pmatrix} \mathbf{0}_L & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{0}_L \end{pmatrix} \tag{3.67}$$

and

$$\mathbf{v} = [\mathbf{c}_k^T(m), \mathbf{z}_k^T(m)]^T \in 2^L \tag{3.68}$$

where $\mathbf{0}_L$ and $\mathbf{I}_L$ are the $L-by-L$ zero and identical matrices, respectively. Then the decision variable $y_k(m)$ can be written in terms of (3.67) and (3.68) as follows [47,48,59]

$$y_k(m) = \mathbf{v}^H\mathbf{Q}\mathbf{v} \tag{3.69}$$

The vector $\mathbf{z}_k(m)$ conditioned on the data bits $\mathbf{b}_P(m)$ is a complex Gaussian random vector. Since the combing vector $\mathbf{c}_k(m)$ also has a Gaussian distribution the probability of error for the k^{th} user conditioned on the data bits $\mathbf{b}_P(m)$ is given by the following expression [47,48,59]

$$P(error \mid \mathbf{b}_P) = \sum_{\substack{i=1 \\ \lambda_i<0}}^{2L} \prod_{\substack{j=1 \\ j\neq i}}^{2L} \frac{\lambda_i}{\lambda_i - \lambda_j} \tag{3.70}$$

where λ_i $i = 1,2,...,2L$ are the eigenvalues of the matrix $E[\mathbf{v}\mathbf{v}^H \mid \mathbf{b}]\mathbf{Q}$ and

$$E[\mathbf{v}\mathbf{v}^H \mid \mathbf{b}] = \begin{pmatrix} E[\mathbf{c}_k(m)\mathbf{c}_k^H(m)] & E[\mathbf{c}_k(m)\mathbf{z}_k^H(m) \mid \mathbf{b}_P(m)] \\ E[\mathbf{c}_k(m)\mathbf{z}_k^H(m) \mid \mathbf{b}_P(m)] & E[\mathbf{z}_k(m)\mathbf{z}_k^H(m) \mid \mathbf{b}_P(m)] \end{pmatrix} \tag{3.71}$$

Finally the BER expression for the k^{th} user is given by

$$P_k = \frac{1}{2^{PK-1}} \sum_{\substack{\mathbf{b}_P\in\{-1,1\}^{MK-1} \\ b_k=1}} P(error \mid \mathbf{b}_P(m)) \tag{3.72}$$

The above expression applies to any linear detector. Since the conventional detector is a linear detector it follows that its BER expression can be computed using the above expression, where instead of computing the weighting coefficients adaptively we set them to the spreading sequence vector.

3.7 Conclusion

This Chapter considered the MMSE detector and its adaptive implementations using an LMS algorithm. It started by first looking at the signal model to be used throughout this Chapter. Following this, the optimal coefficients of both the precombining and postcombining MMSE detectors were presented.

Section 3.4 then presented the adaptive implementations of the precombining and postcombining MMSE detector using the LMS algorithm. This was then followed by the analysis of the convergence behavior of the MMSE detector. Finally, the BER analysis of the MMSE detector and its adaptive implementations were presented. Simulation and analytical results of the LMS-MMSE MUDs are presented in Chapter 5, together with those of the conventional and lattice-based MUDs.

Chapter 4
Lattice Filters

4.1 Introduction

Filtering is a signal processing operation whose objective is to process a signal in order to manipulate the information contained in the signal. In other words, a filter is a device that maps its input signal to an output signal facilitating the extraction of the desired information from the input signal. There are two types of filters. The first one is a time-invariant filter, where the filter parameters are non-varying. The second filter type is the time-varying (adaptive) filter. The coefficients of the adaptive filter are time varying.

The operation of an adaptive filtering algorithm involves two basic processes: (1) a filtering process designed to produce an output in response to the input data, and (2) an adaptive process, the purpose of which is to provide an algorithm for adjusting the filter coefficients. The operation of the adaptive filtering algorithm is affected by the choice of the structure. Examples of adaptive filter structure are: Transversal filter, lattice structure and Systolic array. In this Chapter a detailed study of the lattice structure is presented.

This Chapter is organized as follows. In Section 4.2 we looks at linear prediction using the FIR filter. In Section 4.3, the lattice structure is introduced and the order update equation of the lattice filter structure presented. Joint process estimation is then covered in Section 4.4,while in Section 4.5 conclusions are presented.

This Chapter is based on the worked published in [32,35,36,70,93].

4.2 Linear Prediction

Consider the following set of a stationary discrete-time stochastic process:

$$\mathbf{u}(m-1) = \{u(m-1), u(m-1), \ldots, u(m-N)\} \in \mathrm{C}^{\mathrm{N}} \tag{4.1}$$

There are some application where one might be interested in predicting the future sample *u(m)* given $\mathbf{u}(m-1)$. This process can be accomplished by using an FIR filter (see Figure 4.1(a)), whose coefficients are optimized in the mean squared error sense in accordance with Weiner filter theory. From Figure 4.1(a), it should be apparent that the predicted value is given by

$$\hat{u}(m) = \sum_{k=1}^{N} w_{f,k} u(m-k) \tag{4.2}$$

The error between the actual data and the predicted value is called the forward prediction error and is given by

$$\begin{aligned} f_N(m) &= u(m) - \hat{u}(m) \\ &= u(m) - \sum_{k=1}^{N} w_{f,k} u(m-k) \end{aligned} \tag{4.3}$$

where the variable N symbolizes the filter order. Let $a_{N,k}$; $k = 0,1,\ldots,N$ denote the weighting coefficients of a new FIR filter, which are related to the coefficients of Equation (4.3) as follows

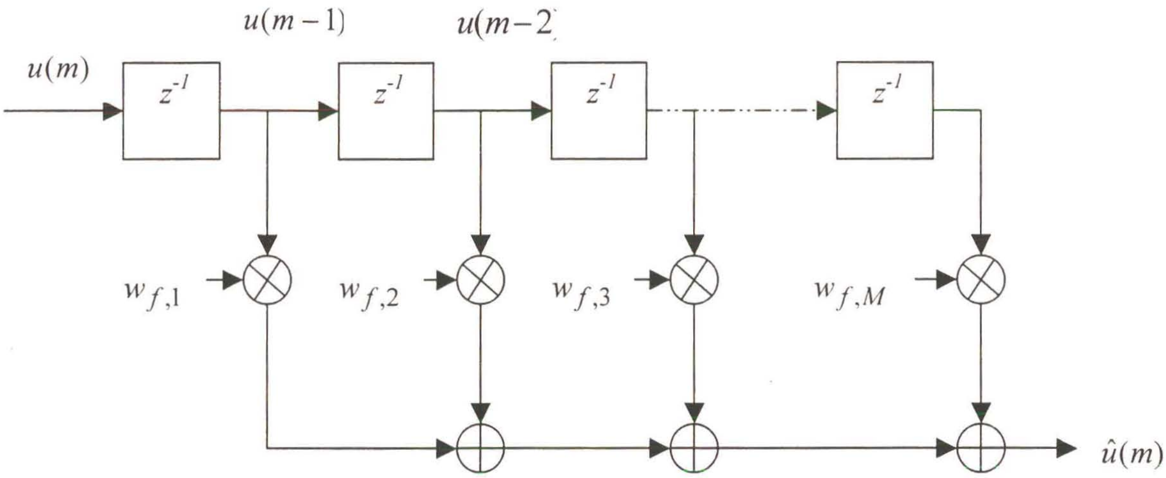
$$a_{N,k} = \begin{cases} 1 & k = 0 \\ -w_{f,k} & k = 1,2,\ldots,N \end{cases} \tag{4.4}$$

Then Equation (4.3) can be written as follows

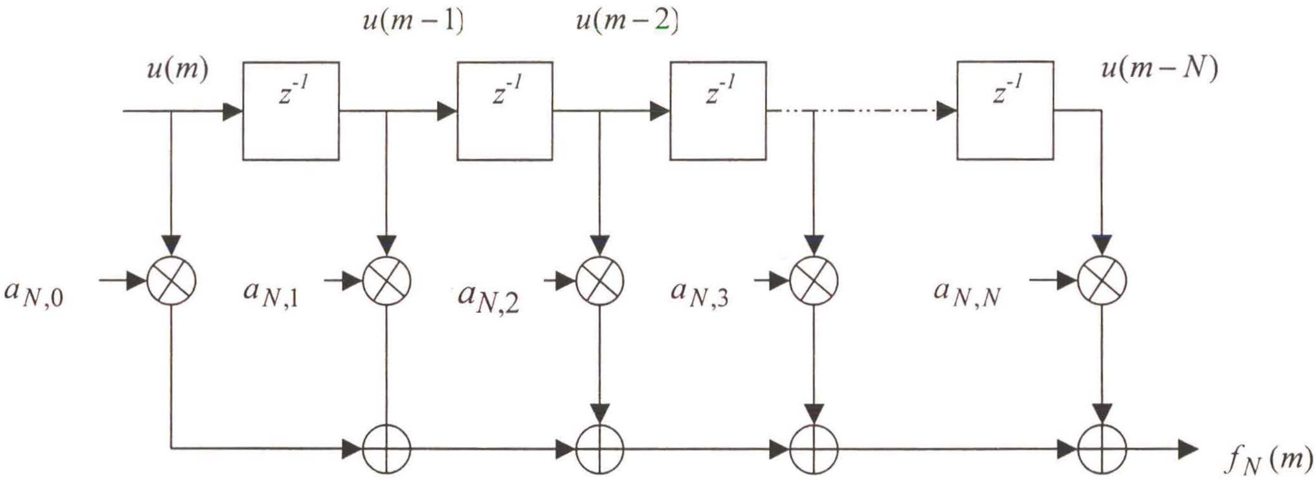
$$f_N(m) = \sum_{k=0}^{N} a_{N,k} u(m-k) \tag{4.5}$$

and the resulting filter is said to be the prediction-error filter and is shown in Figure 4.1(b). The optimal Weiner-Hopf equation of the coefficients given by Equation (4.2) is [32]:

$$\mathbf{R}_N \mathbf{w}_f = \mathbf{r} \tag{4.6}$$



(a)



(b)

Figure 4.1 (a) One-step forward predictor filter; (b) forward prediction-error filter

where $\mathbf{w}_f = [w_{f,1}, \ldots, w_{f,N}]$, and $\mathbf{R}_N$ and $\mathbf{r}$ are the input crosscorrelation and autocorrelation with the desired signal u(m), which are given by :

$$\mathbf{R}_N = E[\mathbf{u}(m-1)\mathbf{u}^H(m-1)]$$
$$= \begin{bmatrix} r(0) & r(1) & \cdots r(N-1) \\ r(-1) & r(0) & \cdots r(N-2) \\ \vdots & & \\ r(N-1) & r(N-2) & \cdots r(0) \end{bmatrix} \quad \textbf{(4.7)}$$

$$\mathbf{r} = E[\mathbf{u}(m-1)u^*(m)]$$
$$= \begin{bmatrix} r^*(1) \\ r^*(2) \\ \vdots \\ r^*(N) \end{bmatrix} \quad \textbf{(4.8)}$$

Note that r should not be confused with the r of Chapter 2 and 3 which refers to the received signal. Using Equation (4.3), the expression of the forward prediction-error power is given by

$$P_N = E[|f_N|^2] = r(0) - \mathbf{r}^H \mathbf{w}_f \quad \textbf{(4.9)}$$

Equation (4.6) and (4.9) can be combined to form one expression, which is

$$\mathbf{R}_{N+1} \begin{bmatrix} 1 \\ -\mathbf{w}_f \end{bmatrix} = \begin{bmatrix} P_N \\ \mathbf{0}_N \end{bmatrix} \quad \textbf{(4.10)}$$

where $\mathbf{0}_N$ is the N-by-1 null matrix and from Equations (4.8) and (4.9) the $\mathbf{R}_{N+1}$ matrix is given by

$$\mathbf{R}_{N+1} = \begin{bmatrix} r(0) & \mathbf{r}^H \\ \mathbf{r} & \mathbf{R}_N \end{bmatrix} \quad \textbf{(4.11)}$$

Using Equation (4.4), with $\mathbf{a}_N = \{a_{N,0},..., a_{N,N}\}$, Equation 4.10 can be written as follows:

$$\mathbf{R}_{N+1}\mathbf{a}_N = \begin{bmatrix} P_N \\ \mathbf{0}_N \end{bmatrix} \quad \textbf{(4.12)}$$

In another form of prediction, one might be interested in predicting the sample $u(m-N)$ given the following samples

$$\mathbf{u}(m) = [u(m), u(m-1),..., u(m-N+1)] \quad \textbf{(4.13)}$$

This second form of prediction is referred to as the backward linear prediction and is illustrated in Figure 4.2(a). From this diagram it follows that the estimate of the backward linear predictor is given by

$$\hat{u}(m-N) = \sum_{k=1}^{N} w_{b,k} u(m-k+1) \quad \textbf{(4.14)}$$

where $\mathbf{w}_d = [w_{d,1}, w_{d,2}, ..., w_{d,M}]$ are the weighting coefficients of the backward linear predictor filter. The difference between the actual data value and the estimated value given by Equation (4.14), is called the backward prediction-error and is given by

$$d_N = u(m-N) - \sum_{k=1}^{N} w_{d,k} u(m-k+1) \quad \textbf{(4.15)}$$

Due to the symmetry of the input autocorrelation function of both the backward and forward prediction-error filters, the optimal backward prediction weighting coefficients are the mirror image of the optimal forward predictor weighting coefficient, i.e.

$$w_{d,k} = w^{*}_{f,N-k+1} \quad \textbf{(4.16)}$$

Using Equation (4.4) and (4.16), the backward prediction-error filter can be expressed as follows (see Figure 4.2(b)):

$$d_M(m) = \sum_{k=0}^{N} a_{N,N-k} u(m-k) \quad \textbf{(4.17)}$$

The optimal Weiner-Hopf equation of the backward predictor coefficients in Equation (4.14) is given by [32]:

$$\mathbf{R}_M \mathbf{w}_d = \mathbf{r}^{B*} \quad \textbf{(4.18)}$$

where the vector $\mathbf{r}^B$ is obtained by rearranging the element of vector $\mathbf{r}$ defined in Equation (4.8), in reverse order, i.e.

$$\mathbf{r}^B = \begin{bmatrix} r^*(N) \\ r^*(N-1) \\ \vdots \\ r^*(1) \end{bmatrix} \quad \textbf{(4.19)}$$

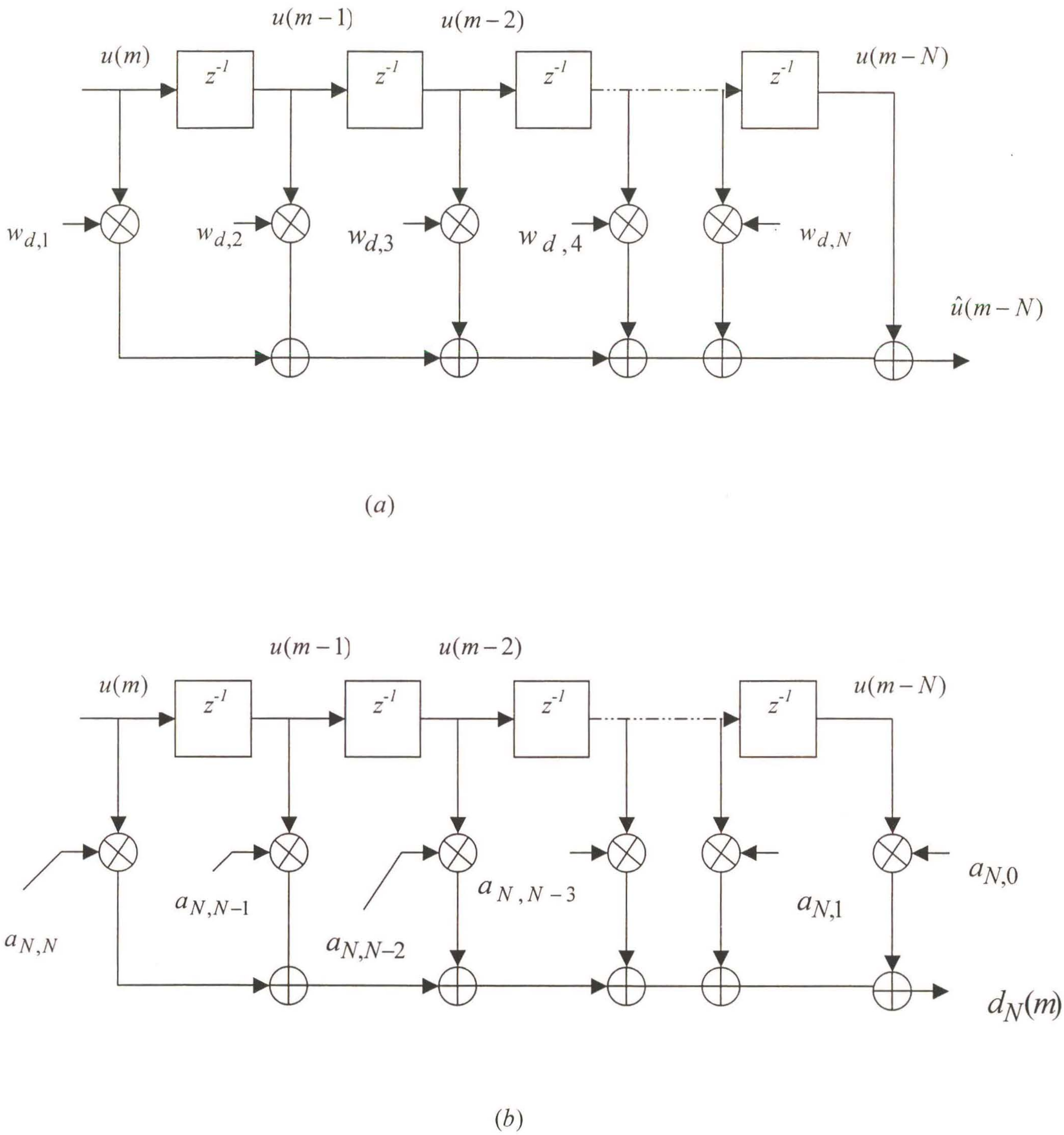


Figure 4.2 (a) One-step backward predictor filter; (b) backward prediction-error filter

By using Equation (4.15), the expression of the backward prediction-error power is given by

$$P_N = E[|b_N|^2] \\ = r(0) - \mathbf{r}^{BT}\mathbf{w}_d \quad \textbf{(4.20)}$$

Combining Equation (4.18) and (4.20) into a single equation yields the following results

$$\mathbf{R}_{N+1}\begin{bmatrix} -\mathbf{w}_d \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0}_N \\ P_N \end{bmatrix} \quad \textbf{(4.21)}$$

where

$$\mathbf{R}_{N+1} = \begin{bmatrix} \mathbf{R} & \mathbf{r}^{B*} \\ \mathbf{r}^{BT} & r(0) \end{bmatrix} \quad \textbf{(4.22)}$$

Similarly, one can write Equation (4.21) in terms of the backward prediction-error filter weighting coefficients as follows:

$$\mathbf{R}_{N+1}\mathbf{a}_N^B = \begin{bmatrix} \mathbf{0}_N \\ P_N \end{bmatrix} \quad \textbf{(4.23)}$$

Equation (4.10) and (4.12) are referred to as the augmented Weiner-Hopf equations to a forward prediction-error filter, while (4.21) and (4.23) are referred to as the augmented Weiner-Hopf equations of the backward prediction-error filter.

We now look at the recursive equations to calculate the prediction-error filter coefficients and the prediction-error power. These equations are obtained by using the augmented Weiner-Hopf equations and the resulting recursive equations are termed the Levinson-Durbin algorithm.

The basic operation of the Levinson-Durbin algorithm is as follows: Given the solution of the augmented Weiner-Hopf equation for a prediction-error filter of order (i-1) we can compute the corresponding solutions for the augmented Weiner-Hopf equation of order i, where i=1,2,3,…,N and N is the final order of the filter.

Let the weighting coefficients vector of the forward prediction-error filter of order i be given by

$$\mathbf{a}_i = [a_{i,0},\ldots,a_{i,i}] \quad \textbf{(4.24)}$$

Then the corresponding weighting coefficients for the backward error filter $\mathbf{a}_i^{B*}$ is obtained by reverse arrangement of the element of the vector $\mathbf{a}_i$ and taking the complex conjugate of the resulting vector. Lets assume that we have the *(i-1)-by-1* vectors $\mathbf{a}_{i-1}$ and $\mathbf{a}_{i-1}^{B*}$, denoting the weighting coefficients of the forward and backward prediction-error filter, respectively. Then the Levinson-Durbin recursive equation for the weighting coefficients is given by:

$$\mathbf{a}_i = \begin{bmatrix} \mathbf{a}_{i-1} \\ 0 \end{bmatrix} + k_i \begin{bmatrix} 0 \\ \mathbf{a}_{i-1}^{B*} \end{bmatrix} \qquad \textbf{(4.25)}$$

where k_i is a constant. Likewise, the Levinson-Durbin recursion equation can be stated as follows

$$\mathbf{a}_i^{B*} = \begin{bmatrix} 0 \\ \mathbf{a}_{i-1}^{B*} \end{bmatrix} + k_i^* \begin{bmatrix} \mathbf{a}_{i-1} \\ 0 \end{bmatrix} \qquad \textbf{(4.26)}$$

Now that we have the recursive update equation for both the forward and backward prediction-error filter, lets turn our attention to the problem of finding the recursive algorithm for the ithstage prediction-error power, P_i. Premultiplying both side of Equation (4.25) by $\mathbf{R}_{i+1}$, the (i+1)-by-($i+1$) autocorrelation of the input, yields

$$\mathbf{R}_{i+1}\mathbf{a}_i = \mathbf{R}_{i+1}\begin{bmatrix} \mathbf{a}_{i-1} \\ 0 \end{bmatrix} + k_i \mathbf{R}_{i+1}\begin{bmatrix} 0 \\ \mathbf{a}_{i-1}^{B*} \end{bmatrix} \qquad \textbf{(4.27)}$$

Using Equation (4.12) yield the following results

$$\begin{bmatrix} P_i \\ \mathbf{0}_i \end{bmatrix} = \mathbf{R}_{i+1}\begin{bmatrix} \mathbf{a}_{i-1} \\ 0 \end{bmatrix} + k_i \mathbf{R}_{i+1}\begin{bmatrix} 0 \\ \mathbf{a}_{i-1}^{B*} \end{bmatrix} \qquad \textbf{(4.28)}$$

Substituting Equation (4.22) into the first term on the right hand side of Equation (4.28) and simplifying yield the following results

$$\mathbf{R}_{i+1}\begin{bmatrix}\mathbf{a}_{i-1}\\0\end{bmatrix}=\begin{bmatrix}\mathbf{R}_i & \mathbf{r}_i^B\\ \mathbf{r}_i^{BT} & r(0)\end{bmatrix}\begin{bmatrix}\mathbf{a}_{i-1}\\0\end{bmatrix}$$
$$=\begin{bmatrix}\mathbf{R}_i\mathbf{a}_i\\ \mathbf{r}_i^{BT}\mathbf{a}_i\end{bmatrix} \quad \textbf{(4.29)}$$
$$=\begin{bmatrix}P_{i-1}\\ \mathbf{0}_{i-1}\\ \mathbf{r}_i^{BT}\mathbf{a}_i\end{bmatrix}$$

Likewise, substituting Equation (4.11) into the second term of Equation (4.28) and simplifying the resulting equations, yield the following results:

$$k_i\mathbf{R}_{i+1}\begin{bmatrix}0\\ \mathbf{a}_{i-1}^{B*}\end{bmatrix}=k_i\begin{bmatrix}\mathbf{r}_i^H\mathbf{a}_{i-1}^{B*}\\ P_{i-1}\\ \mathbf{0}_{i-1}\end{bmatrix} \quad \textbf{(4.30)}$$

Let's define the scalar

$$\chi_{i-1}=\mathbf{r}_i^{BT}\mathbf{a}_{i-1}$$
$$=\sum_{l=1}^{i-1}r(l-i)a_{i-1,l} \quad \textbf{(4.31)}$$

then it can be shown that $\mathbf{r}_i^H\mathbf{a}_{i-1}^{B*}=\chi_{i-1}^*$ [32]. Substitution of Equations (4.29) and (4.30) into Equation (4.28) and using the definition in Equation (4.31) yield:

$$\begin{bmatrix}P_i\\ \mathbf{0}_i\end{bmatrix}=\begin{bmatrix}P_{i-1}\\ \mathbf{0}_{i-1}\\ \chi_{i-1}\end{bmatrix}+k_i\begin{bmatrix}\chi_{i-1}^*\\ \mathbf{0}_{i-1}\\ P_{i-1}\end{bmatrix} \quad \textbf{(4.32)}$$

From the above Equation the following deductions can be made:

1. $P_i=P_{i-1}+k_i\chi_{i-1}^*$ **(4. 33)**

2. $\chi_{i-1}=-k_iP_{i-1}$ **(4.34)**

Substitution of Equation (4.34) into Equation (4.33) yield the following order update equation for the prediction-error power

$$\begin{aligned} P_i &= P_{i-1}(1 - k_i k_i^*) \\ &= P_{i-1}(1 - |k_i|^2) \end{aligned} \tag{4.35}$$

The parameter k_i is termed the reflection coefficients and later in the Chapter we are going to develop its update equation.

4.3 Lattice Filter and Linear Prediction

In the previous Section both the forward and the backward prediction filter were implemented using two different transversal filters. In practice these processes can be implemented using one filter, whose structure is of a lattice form. This filter structure, which is efficient, combines several forwards and backwards prediction-error coefficients into a single structure, giving rise to a lattice structure. Hence this filter is called the lattice predictor. Let's develop the order update equation of this filter.

Let $A_i(z)$ and $G_i(z)$ be the transfer function of the forward and backward prediction-error filter of order i, then the transfer function of the forward predictor filter can be specified in terms of the prediction error coefficients as follows

$$\begin{aligned} A_i(z) &= \sum_{l=1}^{i} a_{i,l}^* z^{-l} \\ &= a_{i,0} + \sum_{l=1}^{i} a_{i,l}^* z^{-l} \\ &= 1 + \sum_{l=1}^{i} a_{i,l}^* z^{-l} \end{aligned} \tag{4.36}$$

Likewise, from Equation (4.17) the equation of the transfer function of the backward prediction-error filter is given by

$$\begin{aligned} G_i(z) &= \sum_{l=1}^{i} a_{i,i-l} z^{-l} \\ &= z^{-i} \sum_{l=0}^{i} a_{i,l} z^{l} \end{aligned} \tag{4.37}$$

From [32], the scalar version of the Levinson-Durbin algorithm for the prediction-error weighting coefficients is given by

$$a_{i,l} = a_{i-1,l} + k_i a^*_{i-1,i-l} \quad \textbf{(4.38)}$$

where $a_{i,l}$, $a_{i-1,l}$ are the l^{th} weighting coefficients of the forward prediction-error filter of order i and i-1, respectively and $a^*_{i-1,i-l}$ is the l^{th} weighting coefficients of the backward prediction-error filter of order i-1. Starting with Equation (4.36) and (4.38), lets determine the order update equation of the forward prediction-error filter. Substitution of (4.38) into (4.36) yields the following results

$$\begin{aligned} A_i(z) &= 1 + \sum_{l=1}^{i} \left(a^*_{i-1,l} + k^*_i a_{i-1,i-1}\right) z^{-l} \\ &= 1 + \sum_{l=1}^{i-1} \left(a^*_{i-1,l} + k^*_i a_{i-1,i-1}\right) z^{-l} + k^*_i z^{-i} \\ &= 1 + \sum_{l=1}^{i-1} a^*_{i-1,l} z^{-l} + k^*_i z^{-i} \left(1 + \sum_{l=1}^{i-1} a^*_{i-1,l} z^{l}\right) \\ &= A_{i-1}(z) + k^*_i z^{-i} A_{i-1}(z^{-1}) \end{aligned} \quad \textbf{(4.39)}$$

From Equation (4.37) it should be clear that the last term on the right hand side of (4.39) is equivalent to $k^*_i z^{-1} G_{i-1}(z)$. Substitution of this term, results in the following equation

$$A_i(z) = A_{i-1}(z) + k^*_i z^{-1} G_{i-1}(z) \quad \textbf{(4.40)}$$

Let $U(z)$, $F_i(z)$ and $B_i(z)$ be the z-transform of the input sequence *u(m)* and the forward and backward prediction-error output ($f_i(m)$ and $\mathrm{d}_i(m)$), respectively. Then the output of the forward prediction-error is given by

$$A_i(z)U(z) = A_{i-1}(z)U(z) + k^*_i z^{-1} G_{i-1}(z)U(z) \quad \textbf{(4.41)}$$

Since $F_i(z) = A_i(z)U(z)$ and $B_i(z) = G_i(z)U(z)$, then the above Equation simplifies to

$$F_i(z) = F_{i-1}(z) + k^*_i z^{-1} B_{i-1}(z) \quad \textbf{(4.42)}$$

The time-domain transform of the above Equation yields the following results

$$f_i(m) = f_{i-1}(m) + k^*_i d_{i-1}(m-1) \quad \textbf{(4.43)}$$

The above Equation is the order update equation of the forward prediction-error filter. To obtain the corresponding order update for the backward prediction filter, we start at $G_i(z) = z^{-i} A_i(z^{-1})$. Substitution of Equation (4.39) into this equation and simplifying yields

$$\begin{aligned} G_i(z) &= z^{-i}\left(A_{i-1}(z^{-1}) + k_i z^i A_{i-1}(z)\right) \\ &= z^{-i} A_{i-1}(z^{-1}) + k_i A_{i-1}(z) \\ &= z^{-1} G_{i-1}(z) + k_i A_{i-1}(z) \end{aligned} \quad \textbf{(4.44)}$$

Multiplying both side of the above Equation by $U(z)$ yields the following results

$$B_i(z) = z^{-1} B_{i-1}(z) + k_i F_{i-1}(z) \quad \textbf{(4.45)}$$

The time-domain transform of the above Equation yield the following results

$$d_i(n) = d_{i-1}(n-1) + k_i f_{i-1}(n) \quad \textbf{(4.46)}$$

The above Equation is the order update equation of the backward prediction-error filter, which completes the order update of the lattice predictor. Figure 4.3 illustrates the lattice filter described by the recursive relationship in Equation (4.43) and (4.46). Now let's turn our attention to the problem of developing the optimal equation of the reflection coefficients. The optimality criterion adopted for the computation of the optimal reflection coefficients is the minimum MSE, where the average of the sum of the powers at the output of the i stage is minimized. Thus the cost function is given by [32,93]

$$J_i = E[|f_i(m)|^2 + |d_i(m)|^2] \quad \textbf{(4.47)}$$

Substitution of Equation (4.43) and (4.46) into the above Equation yield the following results

$$\begin{aligned} J_i = &\left(E[|f_{i-1}(m)|^2] + E[|d_{i-1}(m-1)|^2]\right)\left(1 + k_i k_i^*\right) + \\ &2k_i E[f_{i-1}(n) d_{i-1}^*(m-1)] + 2k_i^* E[d_{i-1}(m-1) f_{i-1}^*(m)] \end{aligned} \quad \textbf{(4.48)}$$

Differentiating the above Equation with respect to the reflection coefficient k_m one obtains

$$\nabla J_i = 2k_i E[|f_{i-1}(m)|^2] + E[|d_{i-1}(m-1)|] + 4E[f_{i-1}^*(m) d_{i-1}(m-1)]$$

(4.49)

Equating the above expression to zero, the following expression for the optimal reflection coefficient is obtained

$$k_{i,opt} = -\frac{2E[d_{i-1}(m-1)f_{i-1}^{*}(m)]}{E[|f_{i-1}(m)|^2 + |d_{i-1}(m-1)|^2]} \quad \textbf{(4.50)}$$

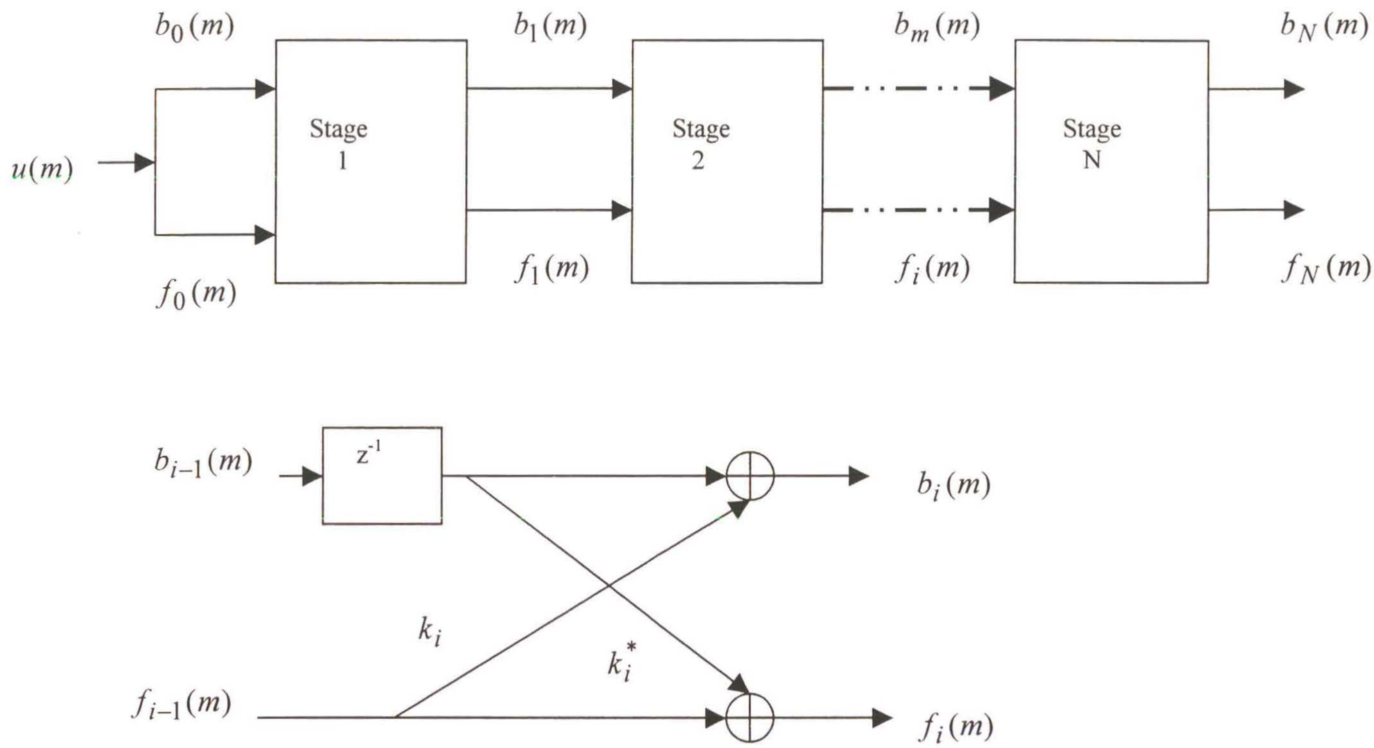


Figure 4.3 **Lattice filter**

The above Equation for the optimal reflection coefficients involves the use of ensemble averages. In practice this information is not readily available. In such cases an adaptive algorithm is used to approximate the reflection coefficients. A number of adaptive algorithms for approximating the reflection coefficients were presented in [29,32,35-37,93]. The one adopted in this research is the one presented by Proakis in [29,93] and is given by

$$k_i(m) = k_i(m-1) - \frac{f_{i-1}(m-1)d_i^{*}(m-1) + d_{i-1}^{*}(m-2)f_i(m-1)}{v_i(m-1)} \quad \textbf{(4.51)}$$

where $v_i(m)$ is the step-size parameter. There exist two version of the above Equation. These are the normalized and the un-normalized step-size equation. In the un-normalized case the step-size parameter is kept constant while in the normalized case the step-size is updated using the following equation

$$v_i(m) = wv_i(m-1) + |f_{i-1}(m)|^2 + |d_{i-1}(m-1)|^2 \quad \textbf{(4.52)}$$

where w is the forgetting factor. The lattice predictor has a number of useful properties. We are going to wrap up this Section by summarizing them

Properties of Lattice Predictor

- $E[d_i(m)d_j(m)] = P_i\delta_{ij}$
- $E[f_i(m+i)f_j(m+j)] = P_i\delta_{ij}$
- $E[f_i(m)d_j(m)] = \begin{cases} k_iP_i & \text{i} \geq \text{j} \\ 0 & \text{otherwise} \end{cases}$

4.4 Stochastic Lattice Filter

In Section (4.2) and (4.3) we looked at linear transversal and lattice prediction filters, respectively. Adaptive versions of these filters are usually employed in applications involving Linear Predictive Coding (LPC) and Adaptive Differential PCM (ADPCM) Coding. Of the two prediction filters, the lattice filter is the one often used because of the speed at which its coefficients converge.

In this Section we look at another lattice based filter, called the Gradient Lattice (GL) filter. A list of application where this filter is usually employed include echo and intersymbol interference cancellation. In the next Chapter, we are going to employ this filter to mitigate MAI in CDMA systems.

The GL filter is obtained by adding additional coefficients and a summation to the lattice predictor of the previous Section, as illustrated in Figure 4.4. Since in the previous Section, the update equations of the lattice predictor were presented, this Section focuses only on the ladder Section. From Figure 4.4, the output of the GL filter is given by

$$\hat{b}(m) = \sum_{i=0}^{N} h_i^* d_i(m) \tag{4.53}$$

where $h_i; i = 0,1,..., N$ are the weighting coefficients of the ladder Section. This equation can be written is matrix form as

$$\hat{b}(m) = \mathbf{h}^H \mathbf{d}(m) \tag{4.54}$$

where **h** and **d** are the (N+1) vectors defined by

$$\begin{aligned} \mathbf{h} &= [h_0, h_1, ..., h_N]^T \\ \mathbf{d} &= [d_0(m), d_1(m), ..., d_N(m)]^T \end{aligned} \tag{4.55}$$

The optimal weighting coefficients of the ladder Section are obtained by minimizing the mean of the difference between the desired signal ($b(n)$) and it's estimate, which is given by Equation (4.54). Thus the cost function to be minimized is given by

$$\begin{aligned} J(m) &= E[| b(m) - \hat{b}(m) |^2] \\ &= E[b(m)b^*(m)] - 2E[b(m)\hat{b}^*(m)] + E[\hat{b}(m)\hat{b}(m)] \\ &= E[b(m)b^*(m)] - 2E[b(m)\mathbf{d}^H(m)]\mathbf{h} + \mathbf{h}^H E[\mathbf{d}(m)\mathbf{d}^H(m)]\mathbf{h} \end{aligned} \tag{4.56}$$

Differentiating the above Equation with respect to weighting coefficients **h** yields the following results

$$\nabla J(m) = -2E[b(m)\mathbf{d}^H(m)] + 2\mathbf{h}^H E[\mathbf{d}(m)\mathbf{d}^H(m)] \tag{4.57}$$

Equating the above Equation to zero and simplifying yields the following expression for the optimal weighting coefficients

$$\mathbf{h} = \{E[\mathbf{d}(m)\mathbf{d}^H(m)]\}^{-1} E[\mathbf{d}(m)b^*(m)] \tag{4.58}$$

Using the orthogonality properties of the lattice filter, the above Equation can be written in scalar form as follows

$$h_i = \{E[b(m)b^*(m)]\}^{-1} E[d_i(m)b^*(m)] \quad \textbf{(4.59)}$$

In practice the weighting coefficients are not computed using the above Equation since the ensemble averages required are not readily available. Instead these coefficients are computed using adaptive algorithms. In [35-37,93], adaptive algorithms for computing these coefficients were presented. In this project, the adaptive algorithm adopted is [93]

$$h_i(m+1) = h_i(m) + \frac{2e_i(m)b_i^*(m)}{v_i(m)} \quad \textbf{(4.60)}$$

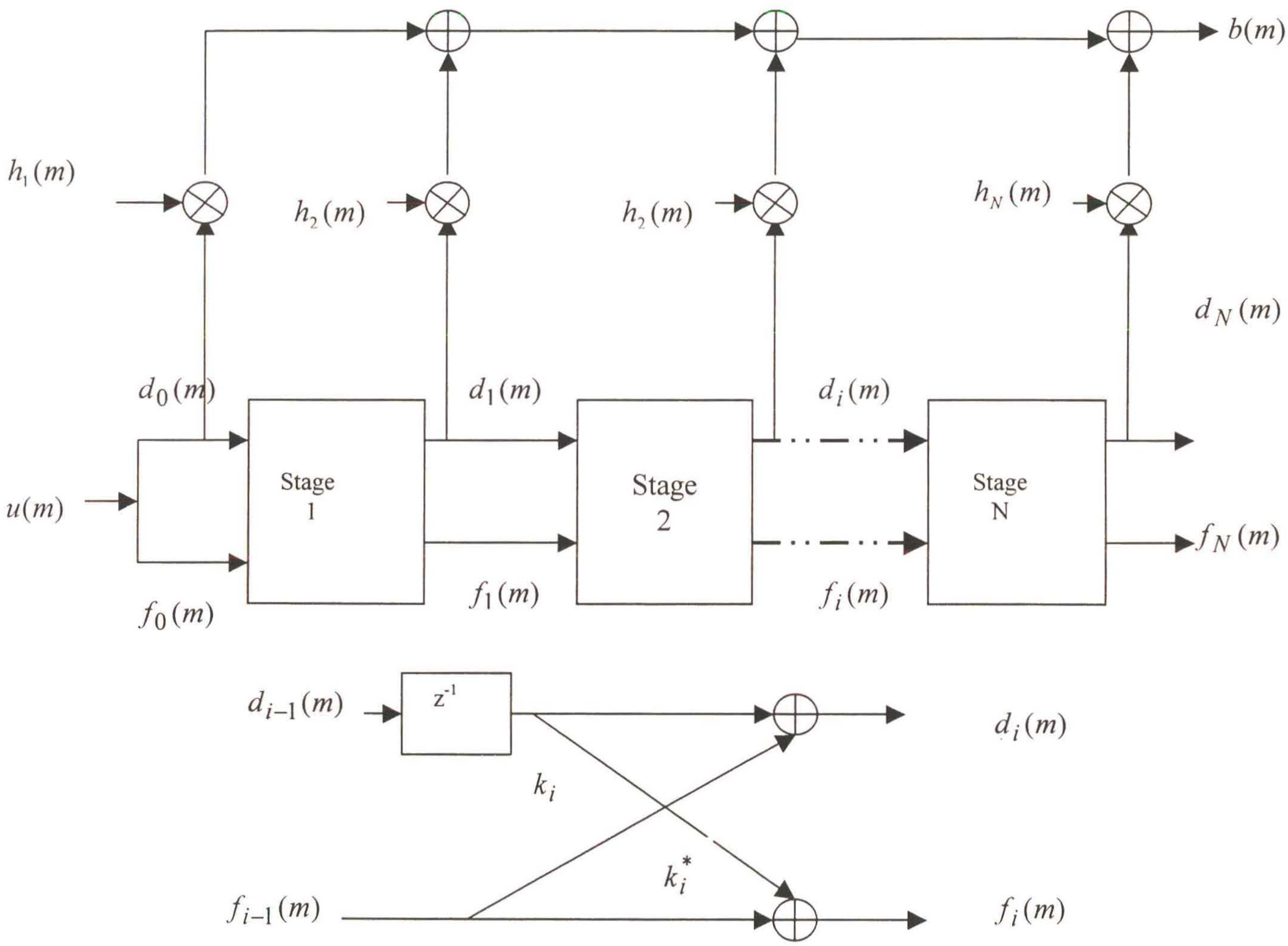


Figure 4.4 Gradient Lattice filter

where $v_m(n)$ is the inverse step-size parameter and was defined in the previous Section, $e_M(n)$ is the error between the desired signal and the filter output and is given by

$$e_i(m) = e_{i-1}(m) - h_i^*(n) d_i(n) \qquad \textbf{(4.61)}$$

where $e_0(m) = b(m)$

4.5 Conclusions

In this Chapter we looked at the linear prediction problem using an FIR filter, where we showed how backward prediction and forward prediction is achieved. In short the forward prediction problem involve the estimation of the future value of a stationary discrete-time stochastic process, given a set of past samples of the process. In contrast to this, backward prediction involve the estimation of a previous sample given the current samples of the stationary discrete-time stochastic process. The optimal coefficients of both these prediction filters were obtained by minimizing the MSE optimality criterion.

In Section 4.3, we then showed how the prediction problem can be solved using the lattice structure, where the issue of interest is determining the reflection coefficients. The lattice predictor offers a number of desirable properties

1. *Order Recursive structure*, the input output relationship of the various stages of the lattice predictor are computed using an order recursive algorithm.
2. *Statistical decoupling of the individual stages*. This property is the one which makes the convergence of the lattice equalizer much faster than the LMS algorithm.
3. *Efficient computation of the forward and the backward prediction error.*

In Section 4.4 we then presented the gradient lattice joint process estimator, where given the vector of the received signal we have to adaptively determine the desired signal.

Chapter 5
Lattice MMSE Multiuser Detector

5.1 Introduction

In Chapter 3 the MMSE detector and its adaptive implementation using a transversal filter whose coefficients are adjusted using the LMS algorithm (LMS-MMSE detector), were presented. The LMS-MMSE detector was shown to reduce the computational requirements of the MMSE detector while not degrading BER performance. The only drawback with the LMS-MMSE detector is its inherent slow convergence rate.

This Chapter presents the lattice structure based implementations of the MMSE detector. As it will be shown, the lattice implementations of the MMSE detector offer good tradeoff between BER and convergence performance. In this Chapter the focus will primarily be on the gradient lattice algorithm (GLA), but a brief study of the LS-lattice MMSE detector will be presented together with its simulation results. This Chapter presents original unpublished work.

In the next Section a brief overview of the system model and the notation adopted in the rest of this Chapter will be presented. Following this, Section 5.3 derives the optimal coefficients of the lattice-MMSE detector. In Section 5.4, adaptive implementations of the lattice MMSE detector will be presented, while Section 5.5 present the derivation of the excess MSE. Section 5.6 discusses the convergence analysis of the gradient lattice MMSE detector. BER expressions are presented in Section 5.7 and finally Section 5.8 concludes the Chapter.

5.2 Signal Model and Notation Used

The received system model to be used throughout this Section was defined in Chapter 2. From that Chapter we have the following result

$$\mathbf{r} = \mathbf{SCAb} + \mathbf{n} \tag{5.1}$$

The n^{th} element of the vector $\mathbf{r}$ in the bit interval m is given by

$$r(mB+n) = \sum_{k=1}^{K}\sum_{l=1}^{L} A_k b_k(m) c_{k,l}(m) s_k(nT_s + mBT_s - \tau_{k,l}) + n(mB+n) \qquad for \quad 0 \le n \le B \tag{5.2}$$

where T_S is the sampling rate and $B = PN$ is the number of samples taken per bit period. The lattice MMSE detectors process the backward prediction error coefficients to estimate users' transmissions. Shown in Figure 5.1 is a diagram of the centralized lattice equalizer, used to suppress MAI. This diagram is identical to the lattice structure presented in the previous Chapter with the exception of one minor adjustment. The modification made to this structure is that all the users' ladder coefficients are fed from one lattice section, making the computational complexity of the LMS and the lattice MMSE detectors almost the same. It should be noted that this structure is used for both the precombining detector, which requires one set of ladder coefficients to be used for every user and a postcombining detector, which uses L sets of the ladder coefficients vectors for every user, to suppress MAI. It should also be noted that with a decentralized lattice MMSE detector there wouldn't be any reduction in computational complexity, since for every user a single lattice section will be required.

In this Chapter, we will use the same notation used in the previous Chapter, where we denoted the forward prediction error coefficients by the symbol f, while the backward prediction error coefficients will be denoted by the symbol d. These symbols will be subscripted with the relevant parameters. For example the i^{th} backward prediction error coefficient will be denoted as d_i. Likewise the vector of the backward prediction error coefficients will be denoted as $\mathbf{d}$.

Using the above notation and the order update equation of the gradient lattice algorithm one has the following order update equation for the lattice coefficients of both the precombining and postcombining gradient lattice-MMSE detector's

$$\begin{aligned} f_i(n) &= f_{i-1}(n) + k_i d_{i-1}(n-1) \\ d_i(n) &= d_{i-1}(n-1) + k_i f_{i-1}(n) \end{aligned} \tag{5.3}$$

where $f_i(n)$ and k_i are the i^{th} stage forward prediction error and reflection coefficient of the lattice filter. The first stage prediction coefficients are updated using

$$f_0(n) = d_0(n) = r(n) \tag{5.4}$$

The order update equations for the LS-lattice filter are identical to those mentioned above with only one minor adjustment. The backwards and forward prediction coefficients are updated using different reflection coefficients (for a detailed study of the LS-lattice filter refer to Appendix B). Following the notation adopted in Appendix B, where the backward and forward reflection coefficients of the i^{th} stage of the lattice filter are denoted as k_i^d and k_i^f, respectively for the LS-MMSE detector one ends up with the following order update equations

$$\begin{aligned} f_i(n) &= f_{i-1}(n) + k_i^f d_{i-1}(n-1) \\ d_i(n) &= d_{i-1}(n-1) + k_i^d f_{i-1}(n) \end{aligned} \tag{5. 5}$$

In the previous Chapter and Appendix B, we showed how the vector of the backward error coefficients $\mathbf{d}$ could be written in terms of the input signal. From the results obtained there, we have the following equation:

$$\mathbf{d} = \mathbf{Lr} \tag{5.6}$$

where $\mathbf{L}$ is the matrix of the backward prediction error coefficients of the lattice structure. To conclude this Section let's look at how the decision statistics are performed. The decision statistics of the precombining lattice-MMSE detector are made according to the following rule

$$y_k = \mathbf{h}_k^H \mathbf{d} \tag{5.7}$$

where $\mathbf{h}_k = [h_{k,1},...,h_{k,N}]$, $k = 1,...,K$ is the vector of the ladder coefficients of the k^{th} user. Similarly the decision statistics of the postcombining lattice-MMSE detector are

$$y_k = \sum_{l=1}^{L} c_{k,l}(m)\mathbf{t}_{k,l}\mathbf{d} \tag{5.8}$$

where $\mathbf{t}_{k,l}$ is the vector of the l^{th} ladder coefficients of the k^{th} user's ladder section of the lattice filter.

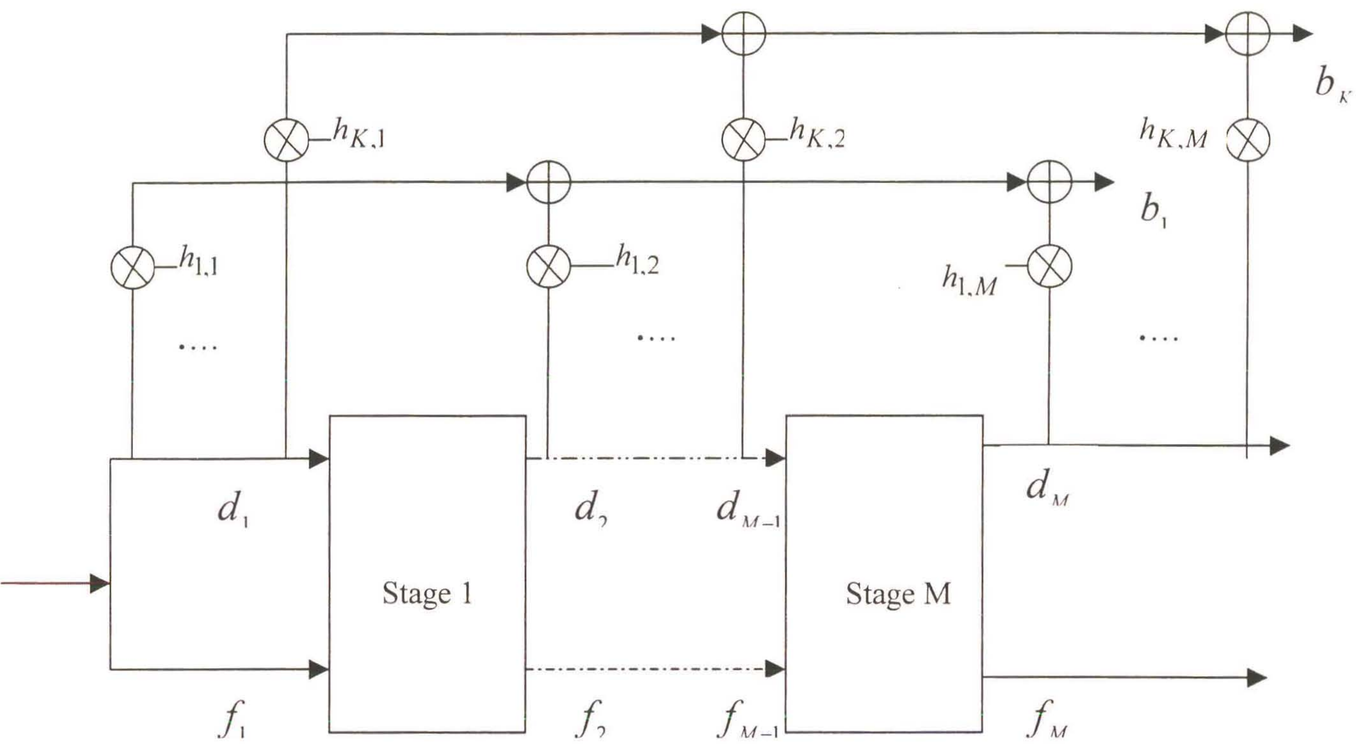


Figure 5.1: Typical diagram of the lattice equalizer used to suppress MAI in a centralized detector

5.3 Optimal Solutions

Now that the backward prediction coefficients have been expressed in terms of the sampled baseband received signal, in this Section the optimal solution of both the precombining and postcombining lattice-MMSE detectors are developed.

Since the ladder coefficients of all the users' detectors are fed from a single lattice section we first start by presenting the optimal solution of the lattice section while subsequent subsections presents the optimal solutions of the ladder coefficients. The optimal reflection coefficients of the gradient lattice MMSE detector are computed as follows [32,93]

$$k_i = \frac{-2E[f_{i-1}(mB+B)d_{i-1}^{*}((m-1)B+B)]}{E[f_{i-1}^{2}(mB+B)]+E[d_{i-1}^{2}((m-1)B+B)]} \quad \textbf{(5.9)}$$

where $B = NP$ is the number of samples taken per bit period. Note that in Equation (5.9), the reflection coefficients are updated once per bit duration. An alternate way of specifying the lattice filter will be to use the tap-weight matrix $\boldsymbol{L}$. The elements of this matrix are arranged as follows

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & \cdots & & 0 & 0 \\ w_{f,1,1} & 1 & 0 & \cdots & 0 & 0 \\ w_{f,2,2} & w_{f,2,1} & 1 & 0 \cdots & 0 & 0 \\ \vdots & \cdots & \ddots & & & \\ w_{f,N,N} & w_{f,N,N-1} & \cdots & w_{f,N,1} & 1 & \end{bmatrix}$$

The element $w_{1,1}$ of the matrix $\boldsymbol{L}$ is computed as follows [32,93]

$$w_{1,1} = \frac{F(1)}{F(0)}$$

while the remaining unknown elements are computed in vector form using the following equation [32,93]

$$\mathbf{w}_{f,i}^{T} = \begin{bmatrix} F(0) & \mathrm{F}(1) & \cdots & \mathrm{F}(i) \\ F(1) & \mathrm{F}(0) & \cdots & \mathrm{F}(i-1) \\ \vdots & & & \\ F(i) & \mathrm{F}(i-1) & \cdots & \mathrm{F}(0) \end{bmatrix}^{-1} \begin{bmatrix} F(1) \\ F(2) \\ \vdots \\ F(i+1) \end{bmatrix} \quad \textbf{(5.10)}$$

where $F(n),\ n = 0,\dots,i$ are element of the matrix $\mathbf{F} = E[\mathbf{r}\mathbf{r}^{H}]$ and $\mathbf{w}_{f,i} = [w_{f,i,1},\dots,w_{f,i,i}]$ for $i = 2,\dots,N$ refers to the vector of the unknown elements of the i^{th} row of the matrix $\boldsymbol{L}$. For the LS-lattice filter, the expressions for computing the optimal backward and forward reflection coefficients were derived in Appendix B. From the results presented there, we have the following expressions

$$k_i^f = -\frac{\sum_{j=1}^{D} w^{D-i} f_{i-1}^*(jB+B) d_{i-1}((j-1)B+B)}{\sum_{j=1}^{D} w^{D-i} |d_{i-1}((j-1)B+B)|^2}$$

$$k_i^d = -\frac{\sum_{j=1}^{D} w^{D-j} d_{i-1}^*((j-1)B+B) f_{i-1}(jB+B)}{\sum_{j=1}^{D} w^{D-j} |f_{i-1}(jB+B)|^2} \qquad \textbf{(5.11)}$$

where $0 < w \leq 1$ is the forgetting factor and D is the processing window size. Now that we have presented the optimal way of computing the lattice coefficients of both the gradient and LS-lattice filter, we now focus our attention on the derivation of the optimal ladder coefficients of the precombining and the postcombining MMSE detectors, in the next subsections.

5.3.1 Precombining MMSE detector

In this subsection the optimal ladder coefficients of both the gradient and LS precombining lattice MMSE detectors are derived, starting with the gradient lattice MMSE detector.

5.3.1.1. Gradient Lattice MMSE detector

The optimal ladder coefficients of the gradient lattice MMSE detector are selected such that the following cost function is minimized [32,93]

$$\mathbf{J} = E[|\mathbf{b} - \hat{\mathbf{b}}|^2] \qquad \textbf{(5.12)}$$

where the data estimates are made according to the following equation

$$\hat{\mathbf{b}} = \mathbf{H}^H \mathbf{d} \qquad \textbf{(5.13)}$$

where $\mathbf{H} = [\mathbf{h}_1, ..., \mathbf{h}_K]^T$ is the matrix of the ladder coefficients for all the users. Expanding Equation (5.12) and substituting Equation (5.13) yields the following results

$$\begin{aligned}\mathbf{J} &= E[(\mathbf{b}-\hat{\mathbf{b}})(\mathbf{b}^H-\hat{\mathbf{b}}^H)] \\ &= E[\mathbf{b}\mathbf{b}^H]-E[\mathbf{b}\hat{\mathbf{b}}^H]-E[\hat{\mathbf{b}}\mathbf{b}^H]+E[\hat{\mathbf{b}}\hat{\mathbf{b}}^H] \\ &= \mathbf{I}-2E[\mathbf{b}\mathbf{d}^H]\mathbf{H}+\mathbf{H}^H E[\mathbf{d}\mathbf{d}^H]\mathbf{H}\end{aligned} \tag{5.14}$$

Differentiating the above Equation with respect to $\mathbf{H}$ and equating the results to zero yield the following expression

$$\mathbf{H}^H E[\mathbf{d}\mathbf{d}^H] = E[\mathbf{b}\mathbf{d}^H] \tag{5.15}$$

Substitution of Equation (5.6) and (5.1) into the above Equation and simplifying the results yield the following equation

$$\mathbf{H} = \mathbf{SCA}\left\{\mathbf{SCAAC}^H\mathbf{S}^T\mathbf{L}^H + E[\mathbf{n}\mathbf{n}^H]\mathbf{L}^H\right\}^{-1} \tag{5.16}$$

which is the expression of the optimal ladder coefficients of the gradient lattice MMSE detector. We round-off this Section by deriving an expression for the minimum MSE of the gradient lattice equalizer. Substitution of Equation (5.16) into Equation (5.14) and simplifying the results yields the following expression for the minimum MSE.

$$\mathbf{J}_{\min} = \mathbf{I} - \mathbf{AC}^H\mathbf{S}^T\left(\mathbf{SCAAC}^H\mathbf{S}^T + E[\mathbf{n}\mathbf{n}^H]\right)\mathbf{SCA} \tag{5.17}$$

This minimum error vector is identical to that of the transversal filter obtained in Chapter 3. Thus it follows that the LMS and GAL algorithm have the same minimum MSE.

5.3.1.2 Least Square Lattice MMSE Detector

The LS-lattice MMSE detector unlike the gradient lattice MMSE detector, selects its optimal ladder coefficients by minimizing the least square cost function [32,93]

$$\begin{aligned}\mathbf{J} &= \sum_{i=1}^{D} w^{D-i}\,|\mathbf{b}(i)-\hat{\mathbf{b}}(i)|^2 \\ &= \sum_{i=1}^{D} w^{D-i}[\mathbf{b}(i)\mathbf{b}^H(i)-2\mathbf{b}(i)\mathbf{d}^H(i)\mathbf{H}+\mathbf{H}^H\mathbf{d}(i)\mathbf{d}^H(i)\mathbf{H}]\end{aligned} \tag{5.18}$$

where $\mathbf{b}(i)$ and $\mathbf{d}(i)$ are the i^{th} elements of the vectors $\mathbf{b}$ and $\mathbf{d}$, respectively, while $\hat{\mathbf{b}}(i)$ is the estimate of the data vector and finally D is the processing window size. Differentiating the above Equation with respect to the ladder coefficients $\mathbf{H}$ and equating the results to zero yield the following results

$$\mathbf{H} = \frac{\sum_{i=1}^{D} w^{D-i}\mathbf{d}(i)\mathbf{b}^{H}(i)}{\sum_{i=1}^{D} w^{D-i}\mathbf{d}(i)\mathbf{d}^{H}(i)}
= \frac{\sum_{i=1}^{D} w^{D-i}\left(\mathbf{S}(i)\mathbf{C}(i)\mathbf{A}(i)\mathbf{b}(i)\mathbf{b}^{H}(i) + \mathbf{b}(i)\mathbf{n}^{H}(i)\right)}{\sum_{i=1}^{D} w^{D-i}\mathbf{r}(i)\mathbf{r}^{H}(i)\mathbf{L}^{H}(i)} \qquad \textbf{(5.19)}$$

For large value of D the above Equation simplifies to the following results, which is the expression for the optimal ladder coefficients of the LS-lattice precombining MMSE detector.

$$\mathbf{H} = \frac{\sum_{i=1}^{D} w^{D-i}\mathbf{S}(i)\mathbf{C}(i)\mathbf{A}(i)}{\sum_{i=1}^{D} w^{D-i}\left(\mathbf{S}(i)\mathbf{C}(i)\mathbf{A}(i)\mathbf{A}(i)\mathbf{C}^{H}(i)\mathbf{S}^{T}(i) + \mathbf{n}(i)\mathbf{n}^{H}(i)\right)\mathbf{L}^{H}(i)}$$

5.3.2 Post-combining MMSE detector

The previous subsection derived the expressions for the optimal ladder coefficients of the precombining lattice MMSE detectors. In this subsection the derivation of the optimal ladder coefficients of the postcombining lattice-MMSE detectors starting first with the gradient lattice-MMSE detector, is presented.

5.3.2.1 Gradient Lattice-MMSE detector

The postcombining gradient lattice MMSE detector chooses the optimal ladder coefficients such that the mean of the square of the error between the data-amplitude product vector and its estimate is minimized. This can be written in vector form as follows

$$\mathbf{J} = E[|\mathbf{p} - \hat{\mathbf{p}}|^{2}] \qquad \textbf{(5.20)}$$

where $\mathbf{p} = \mathbf{CAb}$ and $\hat{\mathbf{p}} = \mathbf{T}^{H}\mathbf{d}$ are the data amplitude product vector and its estimate, respectively. Expanding the above Equation and substituting for both $\mathbf{p}$ and $\hat{\mathbf{p}}$, one obtains

$$\mathbf{J} = E[\mathbf{p}\mathbf{p}^H] - 2E[\mathbf{p}\hat{\mathbf{p}}^H] + E[\hat{\mathbf{p}}\hat{\mathbf{p}}^H] \\ = E[\mathbf{CAAC}^H] - 2E[\mathbf{CAd}^H]\mathbf{T} + \mathbf{T}^H E[\mathbf{dd}]\mathbf{T} \tag{5.21}$$

Differentiating the above Equation with respect to the ladder coefficients and simplifying the results one obtains the following results

$$\mathbf{T} = E[\mathbf{dAC}^H]\{E[\mathbf{dd}^H]\}^{-1} \tag{5.22}$$

substituting for $\mathbf{d}$ yields the following results

$$\mathbf{T} = \mathbf{S}\left(\mathbf{SS}^T\mathbf{L}^H + E[\mathbf{n}(\mathbf{CAAC}^H)^{-1}\mathbf{n}^H]\mathbf{L}^H\right)^{-1} \tag{5.23}$$

which is the required optimal coefficients of the postcombining gradient lattice MMSE detector.

5.3.2.2. LS-Lattice Detector

The optimization criterion for the LS-lattice postcombining detector is as follow

$$\mathbf{J} = \sum_{i=0}^{D} w^{D-i} \,|\, \mathbf{p}(i) - \hat{\mathbf{p}}(i) \,|^2 \tag{5.24}$$

where the vector $\mathbf{p}$ and $\hat{\mathbf{p}}$ were defined previously. By expanding the above Equation, differentiating the resulting equation with respect to $\mathbf{T}$, and equating the derivative to zero and finally simplifying the results one obtains the following expression for the optimal ladder coefficients

$$\mathbf{T} = \frac{\sum_{i=0}^{D} w^{D-i}\mathbf{L}(i)\left(\mathbf{S}(i)\mathbf{C}(i)\mathbf{A}(i)\mathbf{b}(i)\mathbf{b}^H(i)\mathbf{A}(i)\mathbf{C}^H(i) + \mathbf{S}(i)\mathbf{C}(i)\mathbf{b}(i)\mathbf{n}(i)\right)}{\sum_{i=0}^{D} w^{D-i}\mathbf{L}(i)\left(\mathbf{r}(i)\mathbf{r}^H(i)\right)\mathbf{L}^H(i)} \tag{5.25}$$

For large values of D the above Equation simplifies to

$$\mathbf{T} = \frac{\sum_{i=0}^{D} w^{D-i}\mathbf{L}(i)\left(\mathbf{S}(i)\mathbf{C}(i)\mathbf{A}(i)\mathbf{AC}^H(i)\right)}{\sum_{i=0}^{D} w^{D-i}\mathbf{L}(i)\left(\mathbf{S}(i)\mathbf{C}(i)\mathbf{AAC}^H(i)\mathbf{S}^T(i) + \mathbf{n}(i)\mathbf{n}^H(i)\right)\mathbf{L}^H(i)} \\ = \frac{\sum_{i=0}^{D} w^{D-i}\mathbf{L}(i)\left(\mathbf{S}(i)\right)}{\sum_{i=0}^{D} w^{D-i}\mathbf{L}(i)\left(\mathbf{S}(i)\mathbf{S}^T(i) + \mathbf{n}\left(\mathbf{C}(i)\mathbf{A}(i)\mathbf{A}(i)\mathbf{C}^H(i)\right)^{-1}\mathbf{n}^H\right)\mathbf{L}^H(i)} \tag{5.26}$$

5.4 Adaptive Lattice-MMSE detectors

In this Section the focus is on adaptive implementation of the DS-CDMA MMSE detector using the lattice structure. Unlike the LMS algorithm, which only adaptively computes the weighting coefficients, the lattice equalization algorithms have two sets of coefficients to compute adaptively. These are the reflection and the ladder coefficients. Since for both the precombining and the postcombining detector a single lattice filter is used, it makes sense to specify the adaptive equations of the reflection coefficients here and not in subsequent subsections. We first start with the adaptive equation of the gradient lattice structure. Since the chanrel is assumed to be constant over one bit interval it suffices to adaptively compute the reflection coefficients at the bit rate thus giving us the following adaptive algorithm for the gradient lattice MMSE detector's reflection coefficients

$$\hat{k}_i(m) = \hat{k}_i(m-1) + \frac{\left(\hat{f}_i((m-1)B+B)\hat{d}_i^*((m-1)B+B) + \hat{d}_i^*((m-2)B+B)\hat{f}_i((m-1)B+B)\right)}{v_i(m-1)} \tag{5.27}$$

where $i = 1,\ldots,PSN$ is the stage number. The variable $v_i(m)$ is the inverse step-size parameter. There exist two versions of the above expression [29]. The first version keeps the inverse step-size parameter constant $(v_i(m) = v_i)$ and is subsequently called the non-normalized step-size algorithm. The second one, called the normalized step-size version computes the inverse step-size using the following equation

$$v_i(m) = wv_i(m-1) + |f_i(mB+B)|^2 + |d_{i-1}((m-1)B+B)|^2 \tag{5.28}$$

It should be noted that if the channel is varying at a rate much higher than the symbol rate then updating the reflection coefficients at the bit rate could lead to the reflection coefficients not converging to their optimal solution. As a remedy, the reflection coefficients would have to be adaptively computed at a higher rate. For the LS-lattice filter, the reflection coefficients are updated as follows:

$$k_i^f(m) = -\frac{\psi_i(m)}{r_{i-1}^b(m)}$$

$$k_i^d(mP+n) = -\frac{\psi_i^*(m)}{r_{i-1}^f(m)} \quad \textbf{(5.29)}$$

where ψ_i, r_i^b and r_i^f are computed as follows

$$\psi_{i+1}(m) = w\psi_{i+1}(m) + \alpha_i(m) f_i(mB+B) d_i^*((m-1)B+B)$$

$$r_{i+1}^f(m) = r_i^f(m) - \frac{|\psi_{i+1}(m)|^2}{r_i^b(m)}$$

$$r_{i+1}^b(m) = r_i^b(m) - \frac{|\psi_{i+1}(m)|^2}{r_i^f(m)} \quad \textbf{(5.30)}$$

$$\alpha_{i+1}(m) = \alpha_i(m) - \frac{\alpha_i^2(m)|d_i(mB+i)|}{r_i^*(m)}$$

where the variables in equation (5.30) are initialized as follows

$$\alpha_0(m) = 1$$

$$r_0^f(m) = r_0^b(m) = wr_0^f(m) + |f_0(mB+B)|^2$$

$$r_i^b(-1) = r_i^f(0) = 1/\mu \quad \textbf{(5.31)}$$

$$\alpha_i(-1) = 1$$

$$\psi_i(-1) = 0$$

There are few points worth mentioning at this point in connection with what was observed when the lattice-MMSE detector simulation software developed as part of the project, was ran using different parameters.

It was observed that when the time-varying step sizes are initialized according to the same criteria used when the lattice algorithms are used to mitigate intersymbol interference (i.e. when $v_i(-1)$ for the GAL, and $r_i^b(-1)$ and $r_i^f(0)$ for the LS, are set to a value greater than zero but less than one) the coefficients of the detector do not converge. As a remedy to this situation one had to initialize these coefficients to a higher value, preferably greater than 1000.

Now that the update equations of the reflection coefficient, the rate at which they are updatedand the initialization of parameters used to compute them have. been specified, we now turn our attention to the adaptive equations of the ladder coefficients of both the precombining and post-combining lattice-MMSE detector.

5.4.1 Adaptive Precombining Lattice MMSE detector

In this Section the adaptive equation of the ladder coefficients of the precombining lattice MMSE detector are specified, starting with the gradient lattice algorithm. These coefficients are updated at the bit rate. For simplicity we will assume that the pre-windowed vector of the backward error coefficients to be processed is denoted by $\hat{\mathbf{d}}(m)$.

5.4.1.1 Gradient Lattice MMSE detector

The ladder coefficients of the gradient lattice MMSE detector are adaptively computed using the following expression

$$\hat{h}_{k,i}(m+1) = \hat{h}_{k,i}(m) + \frac{2(\hat{b}_k(m) - \hat{\mathbf{h}}^H(m)\hat{\mathbf{d}}(m))\hat{d}_i^*(mB+B)}{v_i(m)} \tag{5.32}$$

where the hat over the above variables indicate that they are estimates of the optimal values. The data symbol $\hat{b}_k(m)$, is known to the detector during training mode and is the previous estimate of the symbol made by the detector during decision directed mode. From Equation (5.7), the data estimates are made according to the following rule

$$\hat{b}_k = \operatorname{sgn}\left(\mathbf{h}_k^H(m)\mathbf{d}(m)\right) \tag{5.33}$$

5.5.1.2 LS Lattice MMSE detector

For the LS-lattice MMSE detector, the update equation for the ladder coefficients is

$$\hat{h}_{k,i}(m+1) = \hat{h}_{k,i}(m) + \frac{\alpha_i(m)d_i^*(mB+B)(\hat{b}_k(m) - \hat{\mathbf{h}}^H(m)\hat{\mathbf{d}}(m))}{r_i^b(m)} \tag{5.34}$$

where the variable $b_k(m), r_i^b$ and $\alpha_i(m)$ are obtained as specified before. Finally the data estimate is made according to Equation (5.33).

Shown in Figure 5.2 and 5.3 are the BER simulation results for the adaptive lattice-MMSE detectors considered in this Section, the precombining LMS-MMSE detector and the conventional detector, where for adaptive MUDs each detector was given 20000 iteration to converge. In both diagrams the number of users and processing gain used were 8 and 31, respectively. The results shown in Figure 5.2 are for a non-fading channel, while those shown in Figure 5.3 are for a 3 paths correlated fading channel. The correlated fading channel coefficients were generated using the method presented in [134], where the Doppler frequency was set at 0.0001.

As can be seen from this diagrams in a non-fading channel, the LS lattice-MMSE, GAL-MMSE and the LMS-MMSE detectors have the same BER performance, while in a fading channel the lattice-MMSE detectors perform slightly worse than the LMS-MMSE detector, as the signal to noise ratio increases above approximately 15dB.

5.4.2 Adaptive Post-combining Lattice MMSE detector

As was the case with the LMS-MMSE detector, the lattice postcombining MMSE detector uses L sets of the ladder coefficients vector to suppress MAI, where L is the number of multipath components. The update equations for both the gradient and the LS-lattice MMSE detectors' ladder coefficients of the l^{th} component of the k^{th} user, respectively are

$$\hat{t}_{k,i,l}(m+1) = \hat{t}_{k,i,l}(m) + \frac{2(A_k c_{k,l}(m)\hat{b}_k(m) - \hat{\mathbf{t}}^H(m)\hat{\mathbf{d}}(m))\hat{d}_i^*(mB+B)}{v_i(m)} \quad \textbf{(5.35)}$$

$$\hat{t}_{k,i,l}(m+1) = \hat{t}_{k,i,l}(m) + \frac{\alpha_i(m)\hat{d}_i^*(mB+B)(A_k c_{k,l}(m)\hat{b}_k(m) - \hat{\mathbf{t}}^H(m)\hat{\mathbf{d}}(m))}{r_i^b(m)} \quad \textbf{(5.36)}$$

where the data estimates are made according to

$$b_k = \mathrm{sgn}\left(\sum_{l=1}^{L} c_{k,l}(m)\mathbf{t}_{k,l}^H(m)\mathbf{d}(m)\right) \quad \textbf{(5.37)}$$

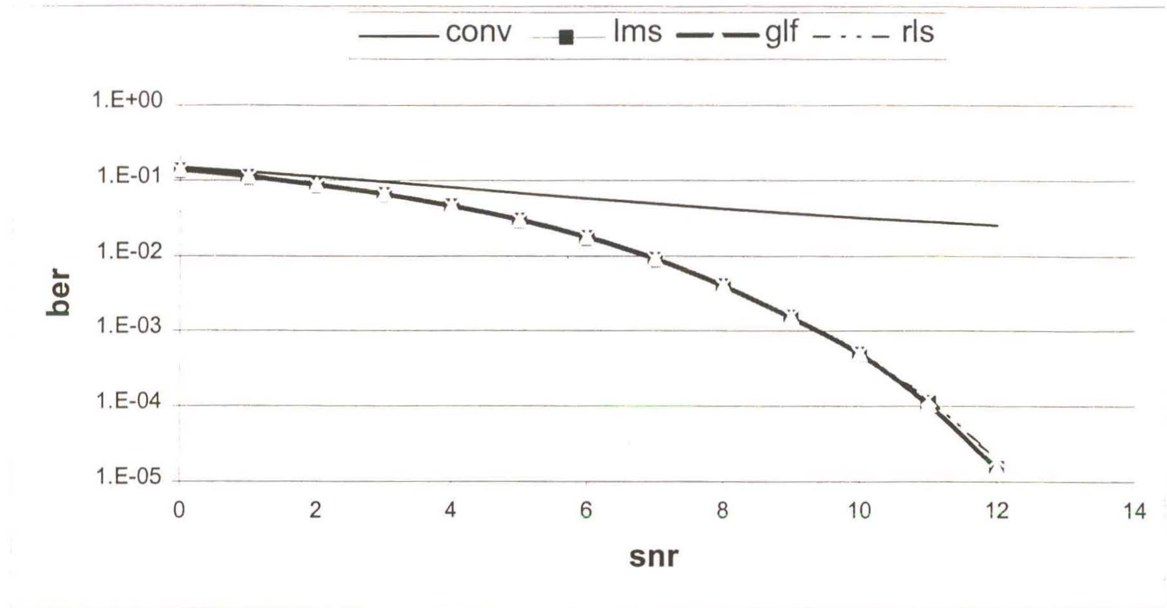


Figure 5.2 BER results for the conventional, LMS-MMSE, GLF-MMSE and RLS-MMSE Detectors in a non-fading channel.

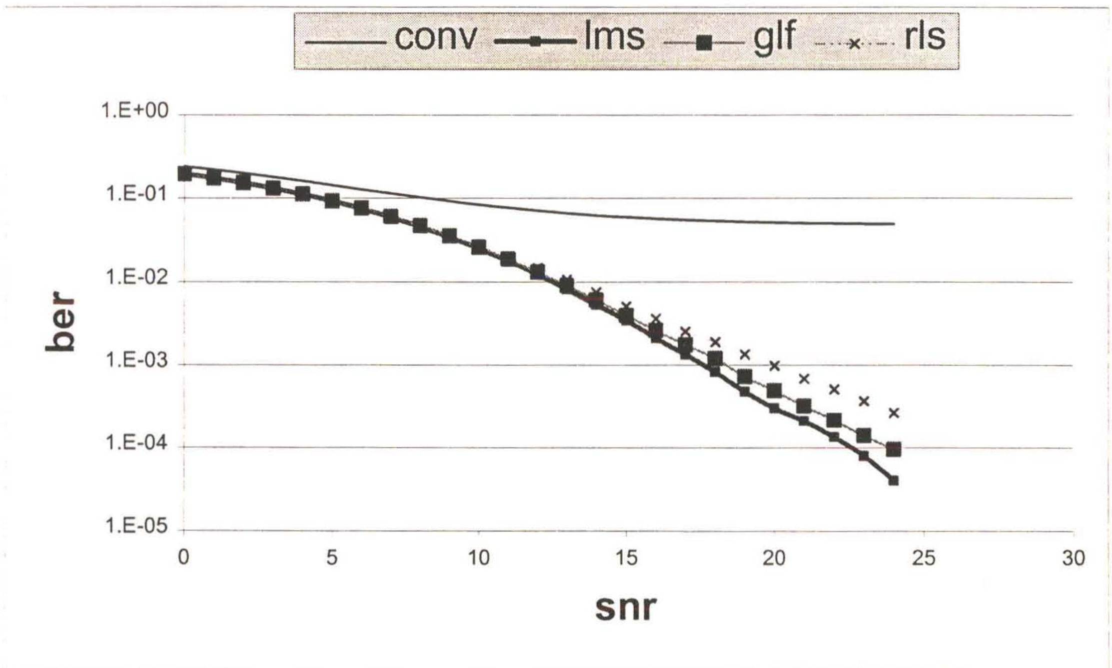


Figure 5.3 BER results for the conventional, LMS-MMSE, GLF-MMSE and RLS-MMSE Detectors in a 3 path fading channel. The MMSE detectors in this case were implemented using a precombining detector structure.

Shown in Figure 5.4 are the simulation BER results of all the postcombining MMSE detectors presented in this dissertation together with the results of the conventional detector, in a three paths fading channel. As can be seen again around the regions of high signal to noise ratio, the BER results of the lattice-MMSE detectors are slightly worse. This is further confirmed by Figure (5.5), which compares the output SNR of the three detectors considered in Figure (5.4). The explanation of why this happens will be presented in the next Section.

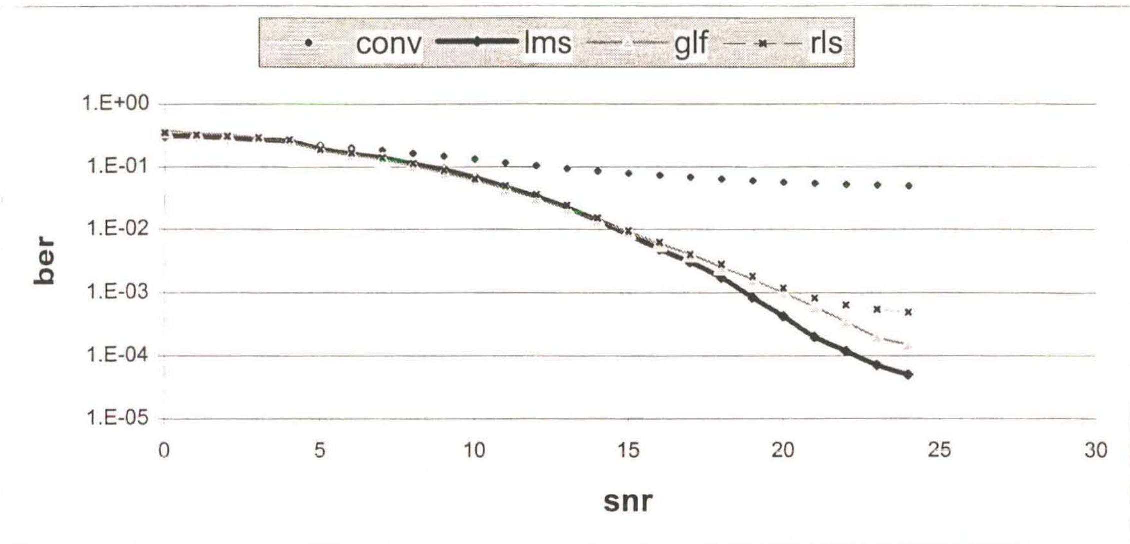


Figure 5.4 BER results for the conventional, LMS-MMSE, GLF-MMSE and RLS-MMSE Detectors in a 3 path fading channel. The MMSE detectors in this case where implemented using a postcombining detector structure.

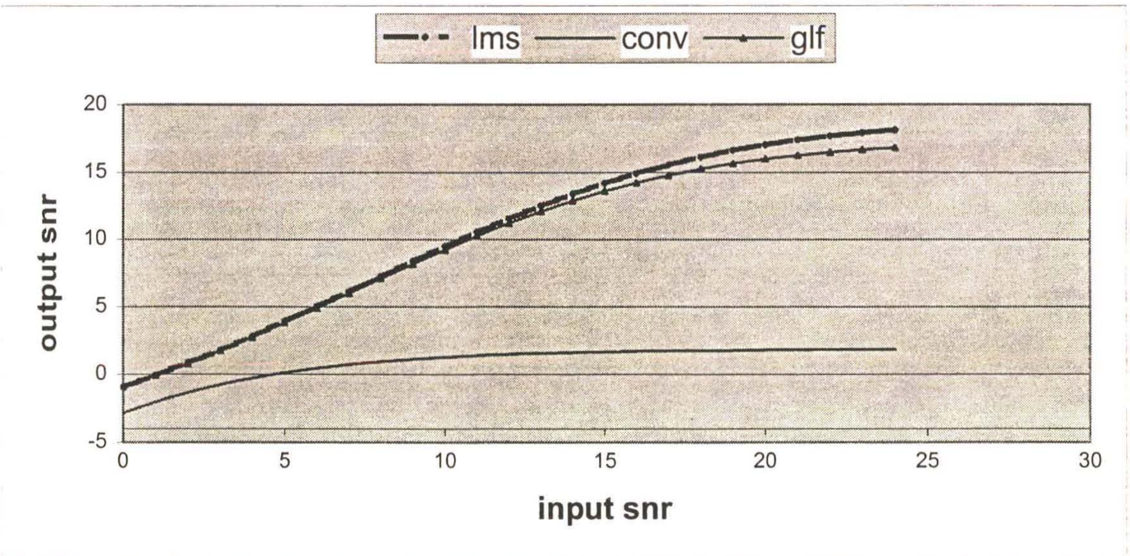


Figure 5.5 Signal to Noise ratio at the output of one the filters used in both of the LMS-MMSE detector and the gradient lattice-MMSE detector in a single paths fading channels.

5.5 Determination of the Excess MSE

In this Section the excess MSE exhibited by the gradient lattice MMSE detector is quantified. We will only focus on the non-normalized step-size update algorithm, avoiding the complexity presented by normalized step-size algorithm.

From Equation (5.32) it can be deduced that the error at the output of the GAL filter is given by the following expression (all subscripts are dropped for ease of notation)

$$e_N(m) = b(m) - \hat{\mathbf{h}}^H(m)\hat{\mathbf{d}}(m) \tag{5. 38}$$

where $\hat{\mathbf{h}}(m)$ and $\hat{\mathbf{d}}(m)$ are noisy estimates of the ladder and backward prediction coefficients vectors, respectively. The two vectors in the above expression can be written as:

$$\begin{aligned} \hat{\mathbf{h}}(m) &= \mathbf{h} + \boldsymbol{\varepsilon}_h(m) \\ \hat{\mathbf{d}}(m) &= \mathbf{d}(m) + \boldsymbol{\varepsilon}_d(m) \end{aligned} \tag{5.39}$$

where $\mathbf{h}$ and $\boldsymbol{\varepsilon}_h(n)$ are the optimal ladder coefficients and weight-error vectors, respectively, $\mathbf{d}(n)$ is a vector of the backward prediction coefficients obtained if the optimal reflection coefficients are used and lastly $\boldsymbol{\varepsilon}_d(n)$ is the error in the backward prediction coefficients vector due to noisy reflection coefficients. Substitution of Equation (5.39) into Equation (5.38) and expanding, gives:

$$\begin{aligned} e_N(m) &= b(m) - \mathbf{h}^H\mathbf{d}(m) - \mathbf{h}^H\boldsymbol{\varepsilon}_d(m) - \boldsymbol{\varepsilon}_h(m)\mathbf{b}(m) - \boldsymbol{\varepsilon}_h^H(m)\boldsymbol{\varepsilon}_d(m) \\ &= e_{N|opt}(m) - \mathbf{h}^H\boldsymbol{\varepsilon}_d(m) - \boldsymbol{\varepsilon}_h(m)\mathbf{d}(m) - \boldsymbol{\varepsilon}_h^H(m)\boldsymbol{\varepsilon}_d(m) \\ &= e_{N|opt}(m) - z(m) \end{aligned} \tag{5.40}$$

where $e_{N|opt}(m)$ is the optimal error at the output of the lattice equalizer at time$m$ and $z(m)$ is given by

$$z(m) = \mathbf{h}^H\boldsymbol{\varepsilon}_d(m) + \boldsymbol{\varepsilon}_h^H(m)\mathbf{d}(m) + \boldsymbol{\varepsilon}_h^H(m)\boldsymbol{\varepsilon}_d(m) \tag{5.41}$$

Evaluation of $E[e_N(m)e_N^*(m)]$ yields:

$$E[e_N(m)e_N^*(m)] = E[e_{N|opt}(m)e_{N|opt}^*(m)] + E[z(m)z^*(m)]$$
$$= J_{\min} + E[z(m)z^*(m)] \quad \textbf{(5.42)}$$

The quantity $J_{\min}$ in Equation (5.42) is the minimum MSE and is given by the k^{th} element of Equation (5.17), while the term $E[z(m)z^*(m)]$ is the excess MSE and the derivation of its expression follows.

Equation (5.41) can be simplified to the following equation

$$z(m) = \mathbf{h}^H \boldsymbol{\varepsilon}_d(m) + \boldsymbol{\varepsilon}_h^H(m)\hat{\mathbf{d}}(m) \quad \textbf{(5.43)}$$

where we have used Equation (5.39) to simplify the right hand side of Equation (5.41) to two terms. Evaluating $J_{ex}(m) = E[z(m)z^*(m)]$ gives:

$$J_{ex}(m) = E[\mathbf{h}^H \boldsymbol{\varepsilon}_d(m)\boldsymbol{\varepsilon}_d^H(m)\mathbf{h}\ ] + 2E[\mathbf{h}^H \boldsymbol{\varepsilon}_d(m)\hat{\mathbf{d}}^H(m)\boldsymbol{\varepsilon}_h(m)]$$
$$+ E[\boldsymbol{\varepsilon}_h^H \hat{\mathbf{d}}(m)\hat{\mathbf{d}}^H(m)\boldsymbol{\varepsilon}_h] \quad \textbf{(5.44)}$$

On assuming that $E[\boldsymbol{\varepsilon}_h(m)] \to 0$ as $m \to \infty$ then the middle term in the above Equation goes to zero as m approaches infinity, thereby simplifying the equation to:

$$J_{ex}(m) = tr\{\mathbf{h}\ \mathbf{h}^H E[\boldsymbol{\varepsilon}_d(m)\boldsymbol{\varepsilon}_d^H(m)]\} + tr\{E[\boldsymbol{\varepsilon}_h\boldsymbol{\varepsilon}_h^H]\left(E[\mathbf{d}(m)\mathbf{d}^H(m)] + E[\boldsymbol{\varepsilon}_d(m)\boldsymbol{\varepsilon}_d^H(m)]\right)\}$$
$$\textbf{(5.45)}$$

It should be noted that the first term in the above Equation is the excess MSE, which arises due to the ladder coefficients not being optimal, while the second one arises due to the error in the reflection coefficients. Expression (5.45) explains why the BER performance of the lattice-MMSE detectors is slightly worse when compared to the LMS-MMSE algorithm. As can be seen from this expression, the excess MSE at the output of the lattice algorithms exceed that of the LMS algorithm. Thus it is expected that the BER and output signal to noise ratio of the lattice-MMSE detectors will be slightly worse than those of the LMS-MMSE detector. To complete the derivation of the excess MSE we need to find the expressions of the four quantities on the right hand side of Equation (5.45). In Section 5.3 the expression used to compute the variable $\mathbf{h}$ was obtained and according to [70], the term $E[\mathbf{d}(m)\mathbf{d}^H(m)]$ is computed as follows

$$E[\mathbf{d}(m)\mathbf{d}^H(m)] = \left[\mathbf{L}E[\mathbf{r}^H\mathbf{r}]\mathbf{L}^H\right]$$

It should be noted that the above matrix is a diagonal matrix. Now we need to determine the expressions for $E[\boldsymbol{\varepsilon}_h\boldsymbol{\varepsilon}_h^H]$ and $E[\boldsymbol{\varepsilon}_d\boldsymbol{\varepsilon}_d^H]$, starting first with the expression for $E[\boldsymbol{\varepsilon}_h\boldsymbol{\varepsilon}_h^H]$. To determine this expression we use the method used by Proakis in [93]. First we rewrite the expression of the ladder update equation of the gradient lattice algorithm given by Equation (5.32) in vector form, as follows

$$\begin{aligned}\hat{\mathbf{h}}(m+1) &= \hat{\mathbf{h}}(m) + \mathbf{V}\hat{\mathbf{d}}(m)e_N^*(m) \\ &= \hat{\mathbf{h}}(m) + \mathbf{V}\left(b_k^*(m)\hat{\mathbf{d}}(m) - \hat{\mathbf{d}}(m)\hat{\mathbf{d}}^H(m)\hat{\mathbf{h}}(m)\right) \\ &= \left(\mathbf{I} - \mathbf{V}\hat{\mathbf{d}}(m)\hat{\mathbf{d}}^H(m)\right)\hat{\mathbf{h}}(m) + \mathbf{V}b_k^*(m)\hat{\mathbf{d}}(m)\end{aligned} \quad \textbf{(5.46)}$$

where $\mathbf{V} = diag\left[2/v_0,\ldots,2/v_M\right]$ is the matrix of the step size parameters. Assuming the availability of the correlation, $E[\mathbf{d}(m)\mathbf{d}^H(m)]$ and $E[b_k^*(m)\mathbf{d}(m)]$, then the above Equation can be written as follows

$$\hat{\mathbf{h}}(m+1) = (\mathbf{I} - \mathbf{V}E[\mathbf{d}(m)\mathbf{d}^H(m)])\hat{\mathbf{h}}(m) + \mathbf{V}E[b^*(m)\mathbf{d}(m)] + \boldsymbol{\varepsilon}_h(m)$$

(5.47)

where $\boldsymbol{\varepsilon}_h(m)$ is the error between the optimal ladder coefficient vector and its adaptive version and is assumed to have a mean of zero. If we assume that $\hat{\mathbf{h}}(m)$ has converged to its optimal value then according to Equation (5.46) and (5.47), the error term $\boldsymbol{\varepsilon}_h(m)$ is equivalent to $\mathbf{V}\hat{\mathbf{d}}(m)e_N^*(m)$. From this, it follows that the covariance of $\boldsymbol{\varepsilon}_h$ is given by:

$$\begin{aligned}\operatorname{cov}[\boldsymbol{\varepsilon}_h(m)] &= \operatorname{cov}[\mathbf{V}e_N(m)\hat{\mathbf{d}}(m)] \\ &= \mathbf{V}\mathbf{V}^H\begin{bmatrix} E[e_N(m)e_N^*(m)\mathbf{d}(m)\mathbf{d}^H(m)] + E[e_N(m)e_N^*(m)\mathbf{d}(m)\boldsymbol{\varepsilon}_d^H(m)] \\ + E[e_N(m)e_N^*(m)\boldsymbol{\varepsilon}_d(m)\boldsymbol{\varepsilon}_d^H(m)] \end{bmatrix}\end{aligned} \quad \textbf{(5.48)}$$

To simplify the above Equation we note that if the mean of $\hat{\mathbf{h}}(m)$ has converged to its optimal value, then $e_N(m)$ should have converged to its minimum value and is uncorrelated with both $\mathbf{d}(m)$ and $\boldsymbol{\varepsilon}_d$. In addition to this we assume that $E[\boldsymbol{\varepsilon}_d(m)] \to 0$ as $m \to \infty$. Using these assumptions one ends up with the following simplified equation for the covariance given in Equation (5.48)

$$\operatorname{cov}[\boldsymbol{\varepsilon}_h(m)] = \mathbf{V}\mathbf{V}^H J_{\min}\left(E[\mathbf{d}(m)\mathbf{d}^H(m)] + E[\boldsymbol{\varepsilon}_d(m)\boldsymbol{\varepsilon}_d^H(m)]\right) \quad \textbf{(5.49)}$$

The components of $\boldsymbol{\varepsilon}_h(m)$ are uncorrelated with each other since $E[\mathbf{d}(m)\mathbf{d}^H(m)]$ and $E[\boldsymbol{\varepsilon}_d(m)\boldsymbol{\varepsilon}_d^H(m)]$ are diagonal matrices. This further implies that the components of $\hat{\mathbf{h}}(m+1)$ are decoupled and may be considered separately. Each of these decoupled components is a first order difference equation of a filter with impulse response [93]:

$$a_i(m) = (1 - \frac{2}{v_i} E[d_i(mP+i)d_i^*(mP+i)])^m \tag{5.50}$$

As m tends to infinity the above Equation can be written as:

$$a_i(m) = \frac{1}{1-(1-\frac{2}{v_i} E[d_i(mP+i)d_i^*(mP+i)])^2} \tag{5.51}$$

Assuming that this filter is excited by a zero mean white Gaussian noise process with variance given by the i^{th} component of the covariance of the vector $\boldsymbol{\varepsilon}_h(m)$, which according to Equation (5.49) is given by

$$\sigma_i = J_{\min}\left(\frac{2}{v_i}\right)^2 E[d_i(mP+i)d_i(mP+i)] + J_{\min}\left(\frac{2}{v_i}\right)^2 E[\varepsilon_{d|i}(mP+i)\varepsilon_{d|i}^*(mP+i)] \tag{5.52}$$

Then the variance of the noise at the output of the filter is given by:

$$E[\boldsymbol{\varepsilon}_h\boldsymbol{\varepsilon}_h^H]_{i,i} = J_{\min}\left(\frac{2}{v_i}\right)^2 \frac{E[d_i(mP+i)d_i^*(mP+i)] + E[\varepsilon_{d|i}(mP+i)\varepsilon_{d|i}^*(mP+i)]}{(1-(1-\frac{2}{v_i} E[d_i(mP+i)d_i^*(mP+i)])^2)} \tag{5.53}$$

Now lets derive the expression for $E[\boldsymbol{\varepsilon}_d\boldsymbol{\varepsilon}_d^H]$. We start by first writing the adaptive lattice coefficients as follows (for simplicity of notation we will assume that $n = mB + B$)

$$\begin{aligned} \hat{f}_i(n) &= f_i(n) + \varepsilon_{f,i}(n) \\ \hat{d}_i(n) &= d_i(n) + \varepsilon_{d,i}(n) \\ \hat{k}_i(n) &= k_i + \varepsilon_{k,i}(n) \end{aligned} \tag{5. 54}$$

where $\varepsilon_{f,i}$, $\varepsilon_{d,i}$ and $\varepsilon_{k,i}$ are the error terms in the forward prediction error, backward prediction error and the reflection coefficients, respectively, which are assumed to have the mean of zero. Using the definition above the order update equation for the forward prediction error coefficients can be written as follows:

$$\hat{f}_i(n) = f_i(n) + \varepsilon_{f,i}(n)$$
$$= f_{i-1}(n) + \varepsilon_{f,i-1}(n) - k_i d_{i-1}(n-1) - \varepsilon_{k,i}(n) d_{i-1}(n-1) - k_i \varepsilon_{d,i-1}(n-1) - \varepsilon_{k,i}(n)\varepsilon_{d,i-1}(n) \quad \textbf{(5. 55)}$$

Squaring both side of the above Equation and taking the mean one obtains the following expression

$$E[f_i^2(n)] + E[\varepsilon_i^2(n)] = E[f_{i-1}^2(n)] + E[\varepsilon_{f,i-1}^2(n)] - 2k_i E[f_{i-1}(n) d_{i-1}(n-1)]$$
$$- 2k_i E[\varepsilon_{f,i-1}(n)\varepsilon_{d,i-1}(n-1)] + (k_i^2 + E[\varepsilon_{k,i}^2(n)])(E[d_{i-1}^2(n)] + E[\varepsilon_{d,i-1}^2(n)])$$

In the above Equation we assumed that the lattice coefficients are real. The resulting Equation can be amended to cater for the case where these coefficients are complex. From the above Equation the following deductions can be made:

$$E[f_i^2(n)] = E[f_{i-1}^2(n)] - 2k_i E[f_{i-1}(n) d_{i-1}(n-1)] + k_i^2 E[d_{i-1}^2(n)]$$
$$E[\varepsilon_{f,i}^2(n)] = E[\varepsilon_{f,i-1}^2(n)] - 2k_i E[\varepsilon_{f,i-1}(n)\varepsilon_{d,i-1}(n-1)] + E[\varepsilon_{k,i}^2] E[d_{i-1}^2(n-1)]$$
$$+ (k_i^2 + E[\varepsilon_{k,i}^2]) E[\varepsilon_{d,i-1}^2(n)] \quad \textbf{(5. 56)}$$

The term $E[f_{i-1}(n) d_{i-1}(n-1)]$ in the first observation in Equation (5.56) is equivalent to [32,93]

$$E[f_{i-1}(n) d_{i-1}(n-1)] = k_i E[f_{i-1}^2(n)] \quad \textbf{(5. 57)}$$

Using the above relationship and the fact that $E[f_i^2(n)] = E[d_i^2(n)]$, one ends up with the following expression

$$E[d_i^2(n)] = E[d_{i-1}^2(n)](1 - k_i^2) \quad \textbf{(5. 58)}$$

If we assume that $\varepsilon_{f,i}$ and $\varepsilon_{d,i}$ are uncorrelated with one another and that $E[\varepsilon_{f,i}^2(n)] = E[\varepsilon_{d,i}^2(n)]$ then the second observation in Equation (5.56) simplifies to the following expression

$$E[\varepsilon_{d,i}^2(n)] = E[\varepsilon_{d,i-1}^2(n)] + E[\varepsilon_{k,i}^2(n)] E[d_{i-1}^2(n)] + (k_i^2 + E[\varepsilon_{k,i}^2]) E[\varepsilon_{d,i-1}^2(n)] \quad \textbf{(5. 59)}$$

From these results it follows that the diagonal elements of the matrix$E[\boldsymbol{\varepsilon}_d \boldsymbol{\varepsilon}_d^H]$ are computed using Equation (5.59). The other elements of this matrix are all zero if we assume that the errors at two different stages are uncorrelated with one another. The quantity $E[\varepsilon_{k,i}^2(m)]$ in Equation (5.59) is computed as follows [36]:

$$E[\varepsilon_{k,i}^2] = \mu_i^2 \frac{\left(1-k_i^2\right)E[d_i^2]}{2-3\mu_i E[d_i^2]} \quad \textbf{(5. 60)}$$

where $\mu_i = \frac{2}{v_i}$. To conclude this Section lets summarize how the excess MSE of the gradient lattice algorithm is obtained.

Summary of the excess MSE of the Gradient Lattice algorithm

The major result of the derivation presented in this thesis is that the equation for the excess MSE is given by

$$J_{ex}(m) = tr\{\mathbf{h}\ \mathbf{h}^H E[\boldsymbol{\varepsilon}_d(m)\boldsymbol{\varepsilon}_d^H(m)]\} + tr\{E[\boldsymbol{\varepsilon}_h\boldsymbol{\varepsilon}_h^H]\left(E[\mathbf{d}(m)\mathbf{d}^H(m)]\} + E[\boldsymbol{\varepsilon}_d(m)\boldsymbol{\varepsilon}_d^H(m)]\right)$$

where

$$E[\mathbf{d}(m)\mathbf{d}(m)] = \left[\mathbf{L}E[\mathbf{r}^H\mathbf{r}]\mathbf{L}^H\right]$$

$$\mathbf{h} = \left[\mathbf{SCA}\left\{\mathbf{SCAAC}^H\mathbf{S}^T\mathbf{L}^H + E[\mathbf{nn}^H]\mathbf{L}^H\right\}^{-1}\right]_i$$

$$E[\boldsymbol{\varepsilon}_h\boldsymbol{\varepsilon}_h^H]_{i,j} = \begin{cases} J_{\min}\dfrac{E[\mathbf{dd}]_{i,i} + E[\boldsymbol{\varepsilon}_d\boldsymbol{\varepsilon}_d^H]_{i,i}}{(1-(1-\frac{2}{v_i}(E[\mathbf{dd}^H]_{i,i})^2)} & i = j \\ 0 \qquad otherwise \end{cases}$$

$$E[\boldsymbol{\varepsilon}_d\boldsymbol{\varepsilon}_d^H]_{i,j} = \begin{cases} E[\boldsymbol{\varepsilon}_d\boldsymbol{\varepsilon}_d^H]_{i-1,i-1} + E[\varepsilon_{k,i}\varepsilon_{k,i}^*]E[\mathbf{dd}^H]_{i-1,i-1} + \left(k_i^2 + E[\varepsilon_i\varepsilon_i^*]\right)E[\boldsymbol{\varepsilon}_d\boldsymbol{\varepsilon}_d^H]_{i-1,i-1} & \mathrm{i} = j \\ 0 \quad \text{otherwise} \end{cases}$$

$$E[\varepsilon_{k,i}\varepsilon_{k,i}^*] = \mu_i^2 \frac{\left(1-k_ik_i^*\right)E[\mathbf{dd}]_{i,i}}{2-3\mu_i E[\mathbf{dd}]_{i,i}}$$

Our results in Section 5.7 show that this derivation is accurate.

5.6 Convergence Analysis of the Gradient Lattice MMSE detector

The condition, which must be met by the step size parameter to ensure the convergence of the adapted reflection coefficients to their optimal value, was obtained in [29]. From the results presented there one has the following condition

$$0 < \frac{1}{v_i} < \frac{2}{E[| f_{i-1}(mB+B)|^2] + E[| d_{i-1}(mB+B)|^2]} \qquad \textbf{(5.61)}$$

Let's now look at the condition, which has to be met for the ladder coefficients of the user of interest to converge. The scalar representation of Equation (5.47) is given by

$$h_{k,i}(m+1) = \left(1 - \frac{2}{v_i} E[\mathbf{d}(m)\mathbf{d}^H(m)]_{i,i}\right) h_{k,i}(m) + \mu_i E[b_k(m)\mathbf{d}_i(m)] + \varepsilon_{h,i}(m) \qquad \textbf{(5.62)}$$

For the ladder coefficients to converge to their optimal value the step size must satisfy the following equation

$$\begin{aligned} &|1 - \frac{2}{v_i} E[\mathbf{d}(m)\mathbf{d}^H(m)]_{i,i}| \leq 1 \\ &0 < \frac{2}{v_i} \leq \frac{1}{E[\mathbf{d}\mathbf{d}^H]_{i,i}} \end{aligned} \qquad \textbf{(5.63)}$$

Figure 5.6 shows the convergence results of the lattice-MMSE detectors and the LMS-MMSE detector obtained using a custom built simulator where the step size for the LMS algorithm was initialized to $7*10^{-4}$. As can be seen from these results, the lattice MMSE detectors converge far quicker than the LMS MMSE detector. From these results and the BER results obtained in Section 5.4 it can be said that the lattice-MMSE detector offers superior convergence-BER performance trade-off when compared to the LMS-MMSE detector.

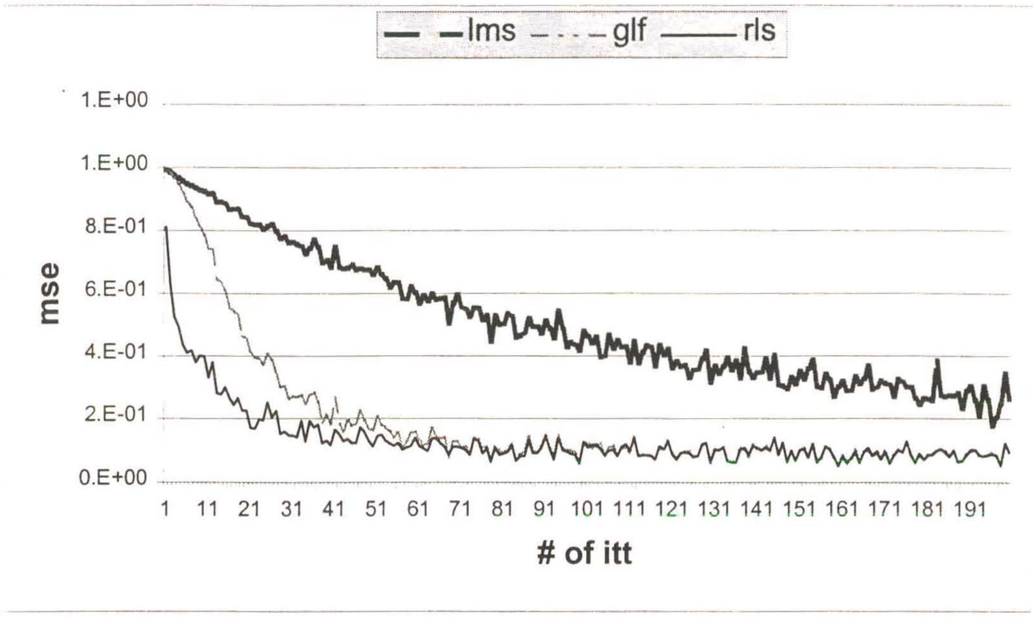


Figure 5.6. Convergence results of the gradient adaptive lattice and LMS-MMSE detector with the step sizes set at $\mu = 7*10^{-4}$.

5.7 BER Analysis of the Lattice Structure

In this Section the BER expression of the gradient lattice MMSE detectors are summarized since they are similar to the ones presented in Chapter 3, starting with the precombining detector.

5.7.1 Precombining Detector

The MSE at the output of the k^{th} user's precombining gradient lattice MMSE detector assuming infinite training sequence is given by the following expression

$$J_k(\infty) = J_{k,\min} + J_{k,exc}(\infty) \quad \textbf{(5.64)}$$

where $J_{k,\min}$ is the minimum mean squared error and is given by the k^{th} element of Equation (5.17) and $J_{k,exc}(\infty)$ is the excess MSE which is computed according to Equation (5.45). If we assume that the channel is non-fading then the BER expression is given by

$$P_{k,e} = Q\left(\frac{1 - J_k(\infty)}{J_k(\infty)}\right) \quad \textbf{(5.65)}$$

As was mentioned in Chapter 3, in a fading channel the above expression is the conditional probability and the BER expression is obtained by averaging it over the pdf of all the users' fading channel coefficients and using the notation adopted in Chapter 3, this is given by

$$P_k = E\left(Q\left(\frac{1 - J_k(\infty)}{J_k(\infty)} \right) \right) \tag{5.66}$$

Shown in Figure 5.7 are the simulation and analytical BER results of the adaptive LMS-MMSE detector and gradient lattice MMSE detector in a non-fading channel. Figure 5.8, 5.9 and 5.10 shows (1) the simulation results of the conventional, and MMSE detectors discussed in this dissertation, (2) analytical and simulation results of the LMS-MMSE detector and (3) analytical and simulation results of the GAL-MMSE detector in a single path fading channel, respectively.

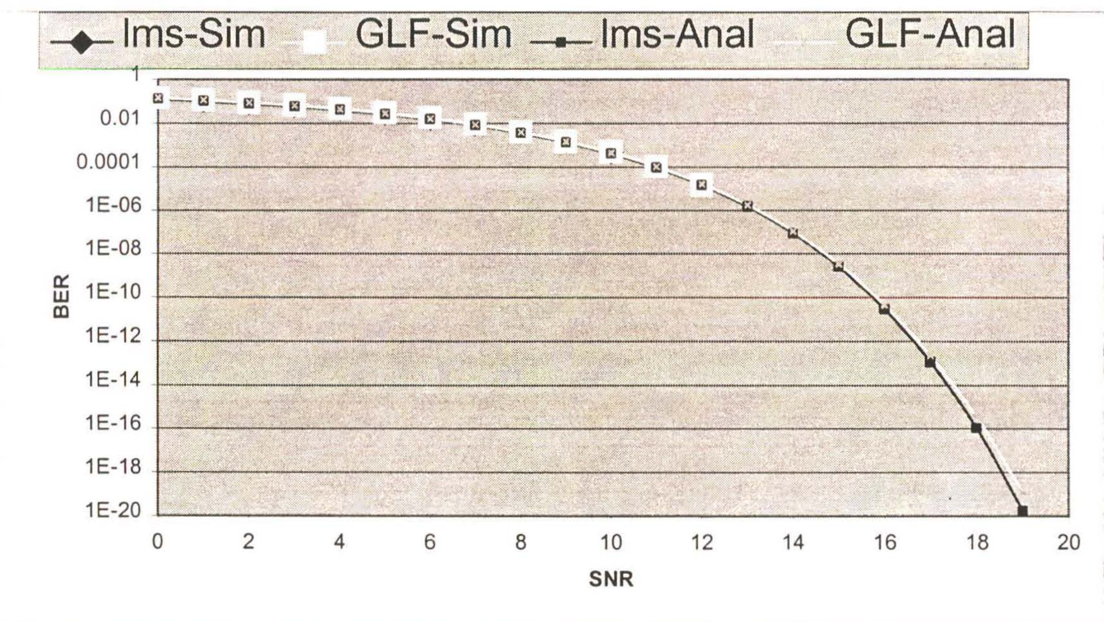


Figure 5.7. BER results of the LMS and gradient lattice MMSE detectors in a non-fading channel obtained using both simulation and analytical method.

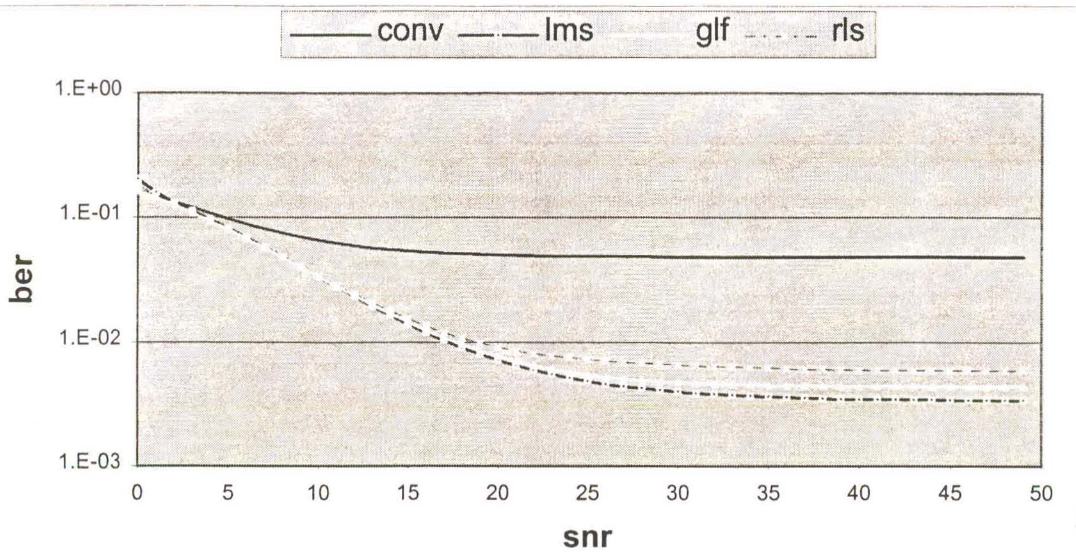


Figure 5.8. BER simulation results of the MMSE detectors considered in this Chapter together with those of the conventional detector in a single path fading uncorrelated channel.

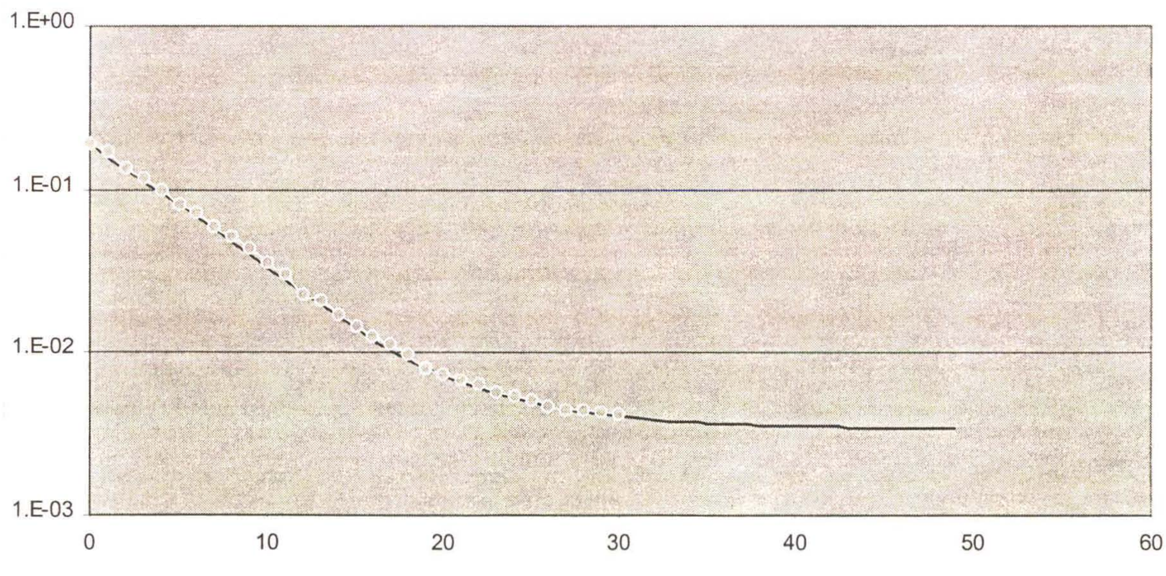


Figure 5.9. BER results of the LMS-MMSE detectors. White solid line is for the analytical expression while the solid line is for the simulation.

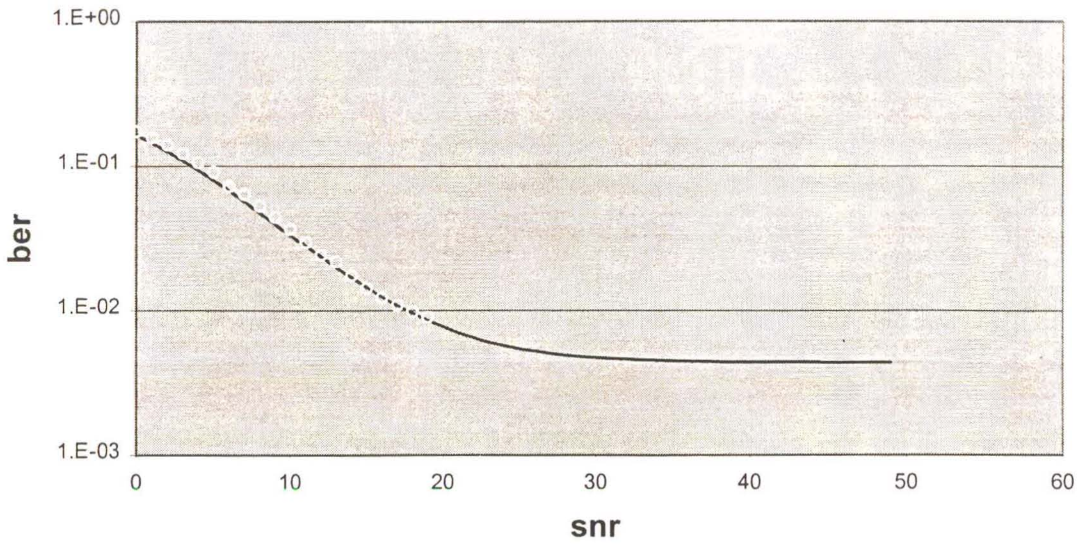


Figure 5.10. BER results of the lattice-MMSE detectors. Dots are for analytical results while the solid line is for the simulation.

Clearly from Figure 5.7 and 5.10, the analytical results, which are obtained using our derivation closely, matches the simulation results.

5.7.2 Post-combining Detector

The output of the l^{th} filter branch of the k^{th} user's postcombining Detector is given by the following expression

$$z_{k,l}(m) = \mathbf{t}_{k,l}(m)\mathbf{d}(m) \tag{5.67}$$

The vector $\mathbf{z}_k(m)$ can be defined as

$$\mathbf{z}_k = [z_{k,1},\dots,z_{k,L}]^T \tag{5.68}$$

Then the decision statistics of the k^{th} user is given by the following expression

$$y_k(m) = \mathbf{c}_k^H(m)\mathbf{z}_k \tag{5.69}$$

where the channel coefficient vector for the k^{th} user is assumed to be known. Using the same notation adopted in Chapter 3, the decision statistics can be written as follows

$$y_k(m) = \mathbf{v}^H\mathbf{Q}\mathbf{v} \tag{5.70}$$

where the matrix $\mathbf{Q}$ is as given by Equation (3.65) and the vector $\mathbf{v}$ is given by

$$\mathbf{v} = [\mathbf{c}_k^T(m), \mathbf{z}_k^T(m)] \quad \textbf{(5.71)}$$

The vector $\mathbf{z}_k$ conditioned on the data bits $\mathbf{b}_P$ is a complex Gaussian random vector. Since the combing vector $\mathbf{c}_k$ also has a Gaussian distribution the probability of error for the k^{th} user conditioned on the data bits $\mathbf{b}_P$ is given

$$P(error \mid \mathbf{b}_P) = \sum_{\substack{i=1 \\ \lambda_i<0}}^{2L} \prod_{\substack{j=1 \\ j\neq i}}^{2L} \frac{\lambda_i}{\lambda_i - \lambda_j} \quad \textbf{(5.72)}$$

where λ_i $i = 1,2,\ldots,2L$ are the eigenvalues of the matrix $E[\mathbf{v}\mathbf{v}^H \mid \mathbf{b}]\mathbf{Q}$ and

$$E[\mathbf{v}\mathbf{v}^H \mid \mathbf{b}] = \begin{pmatrix} E[\mathbf{c}_k(m)\mathbf{c}_k^H(m)] & E[\mathbf{c}_k\mathbf{z}_k^H \mid \mathbf{b}_P] \\ E[\mathbf{c}_k\mathbf{z}_k^H \mid \mathbf{b}_P] & E[\mathbf{z}_k\mathbf{z}_k^H \mid \mathbf{b}_P] \end{pmatrix} \quad \textbf{(5.73)}$$

Finally the BER expression for the k^{th} user is given by

$$P_k = \frac{1}{2^{PK-1}} \sum_{\substack{\mathbf{b}_P\in\{-1,1\}^{MK-1} \\ b_k=1}} P(error \mid \mathbf{b}_P) \quad \textbf{(5.74)}$$

5.8 Conclusion

This Chapter looked at the lattice-MMSE detector, where the focus was mainly on the gradient lattice MMSE detector, even though a brief study of the LS-lattice MMSE detector was presented. It first started by looking at the received signal model and notation adopted in Section 5.2. Following this, Section 5.3 presented the optimal coefficients of the lattice-MMSEdetectors. Section 5.4 then presented the adaptive implementation of the lattice-MMSE detectors and simulation BER results. Section 5.5 derived the excess MSE suffered by the gradient lattice algorithm, while Section 5.6 presented the convergence performance of the gradient lattice-MMSE detector. Finally Section 5.7 presented the analytical BER expressions for the gradient lattice MMSE detector.

From the results presented in this Chapter it was observed that the lattice-MMSE detectors' BER performance are slightly worse in the region of high input SNR when compared to the LMS-MMSE detector. In contrast to this it was observed that the convergence of the lattice-MMSE detectors far outperforms the convergence of the LMS-MMSE detector. Thus it can be concluded that the lattice-MMSE detector offers a better BER-Convergence tradeoff when compared to the LMS-MMSE detector. A comparison between simulation and analytical results shows that our analytical derivation is accurate.

Chapter 6
Blind MMSE Multiuser Detector

6.1 Introduction

The adaptive detectors presented in Chapters 3 and 5 require the adaptive filters to be adapted using training data information before the required data is sent through. The major drawback of these detectors is that when the system parameters changes, a training sequence will have to be re-sent to allow the weighting coefficients of the adaptive filters to converge otherwise the detector will ill-perform. These changes might be due to the number of users in the system changing and/or due to the changes in the channel.

In this Chapter we present another MMSE detection scheme, which does not require the need for training sequence. This detector is called the blind MMSE detector and requires only information of the users' spreading sequences and timing information. The outline of this Chapter is as follows:

Section 6.2 presents the blind MMSE detector. Following this the adaptive version of the simple blind MMSE detector is presented in Section 6.3. Section 6.4, then presents the adaptive step–size adaptive MMSE detectors, while Section 6.5 will look at the convergence of these detectors. Finally conclusions are presented in Section 6.6.

6.2 Blind MMSE Detector

The system considered in this Chapter, comprises of K users asynchronously transmitting data in a white Gaussian channel. From Chapter 2, the received signal model can be written as

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n} \tag{6.1}$$

The weighting coefficients of the blind MMSE detector are chosen such that the mean output energy of the detector is minimized. Assuming user k is the user of interest and that $\mathbf{g}_k$ denote the vector of his/her weighting coefficients. Then the optimality criterion used to obtain the blind MMSE weighting coefficients is given by [38-39,68]

$$J = E[|\mathbf{g}_k^H \mathbf{r}|^2] \qquad \text{subjected to the constraint } \mathbf{g}_k^H \mathbf{s}_k = 1 \tag{6.2}$$

In [38-39,68], the authors showed that the above optimality criterion, called the Minimum Output Energy (MOE), is equivalent to MSE optimality criterion. The constraint in (6.2) ensures that the signal of the desired user is not nulled out. To ensure that this constraint holds the weighting coefficients are decomposed into two orthogonal components. These are (1) the vector $\mathbf{s}_k$, which is the spreading sequence of the user of interest and (2) a vector $\mathbf{x}_k$ that is orthogonal to the user's spreading sequence. This kind of detector representation is termed canonical representation and mathematically is written as follows

$$\mathbf{g}_k = \mathbf{s}_k + \mathbf{x}_k \tag{6.3}$$

From [92], the optimal weighting coefficients of the blind MMSE detector for the desired user are given by

$$\mathbf{g}_k = \frac{\mathbf{F}^{-1}\mathbf{s}_1}{\mathbf{s}_1^T \mathbf{F}^{-1}\mathbf{s}_1} \tag{6.4}$$

6.3 Adaptive Blind MMSE Detector

Like the MSE criterion, the MOE function lends itself to adaptive implementation using the simple LMS algorithm. Since the vector of the spreading sequence is fixed, the only component to be computed adaptively is $\mathbf{x}_k$. The vector of the received signal in the interval $[mT,(m+1)T)$, is given by $\mathbf{r}(m)$. The unconstrained estimate of the gradient of the cost function in (6.2) is given by $2\mathbf{r}(m)^T \mathbf{g}_k(m)\mathbf{r}(m)$. Using this vector, the vector $\mathbf{x}_k$ can be computed adaptively as

$$\mathbf{x}_k(m+1) = \mathbf{x}_k(m) - \mu\mathbf{r}^T(m)\mathbf{g}_k(m)\mathbf{r}(m) \tag{6.5}$$

To ensure that the vectors $\mathbf{s}_k$ and $\mathbf{x}_k$ are orthogonal, the received signal vector in the above Equation is replaced by a component orthogonal to $\mathbf{s}_k$, which is given by

$$\mathbf{r}(m) - \mathbf{r}^T(m)\mathbf{s}_k\mathbf{s}_k \tag{6.6}$$

Substituting the above Equation into (6.5), results in the following update Equation for $\mathbf{x}_k$

$$\mathbf{x}_k(m+1) = \mathbf{x}_k(m) - \mu\mathbf{r}^T(m)\mathbf{g}_k(m)\left(\mathbf{r}(m) - \mathbf{r}^T(m)\mathbf{s}_k\mathbf{s}_k\right) \tag{6.7}$$

The decision statistics of the blind MMSE detector is then given by the following expression

$$y_k = \mathbf{g}_k^T(m)\mathbf{r}(m) \tag{6.8}$$

where $\mathbf{g}_k(m) = \mathbf{s}_k + \mathbf{x}_k(m)$

6.4 Adaptive Step-size Adaptive MMSE Detector

The step size parameter of the adaptive MMSE detector presented in Equation (6.5) is kept constant. The problem with such a step size parameter is that either convergence speed is traded for excess MSE or vice versa. In this Section we present an adaptive step size algorithm that can be used in the adaptive MMSE detector. The algorithm for obtaining such a step size uses information about the weighting coefficients to ensure that both convergence speed and excess MSE are taken into consideration. Below, a description of the adaptive step size algorithm for both the traditional adaptive and blind MMSE detector is given, starting first with the traditional MMSE detector. It should be noted that the MMSE detector presented in this Section requires two adaptive filters. One filter is used to suppress MAI and the other to adaptively compute the step size parameter.

6.4.1 Decision Directed Adaptive Step size Algorithm

The weighting coefficients of a decision directed MMSE detector are chosen such that the cost function

$$\begin{aligned} J &= E\left\{(b_k(m) - \mathbf{g}_k^T \mathbf{r}(m))^2\right\} \\ &= 1 - 2\mathbf{r}^T(m)\mathbf{g}_k + \mathbf{g}_k^T \mathbf{r}(m)\mathbf{r}^T(m)\mathbf{g}_k(m) \end{aligned} \qquad \textbf{(6.9)}$$

is minimized. From Chapter 3, we know that the adaptive algorithm for computing such weighting coefficients is given by

$$\mathbf{g}_k(m) = \mathbf{g}_k(m-1) - \mu(b_k(m) - \mathbf{g}_k^T(m-1)\mathbf{r}(m-1))\mathbf{r}(m) \qquad \textbf{(6.10)}$$

The adaptive step size algorithm is obtained by minimizing (6.9) with respect to the step size parameter μ. Such a minimization yields the following expression for the update of the step size parameter [56]

$$\mu(m) = \left[\mu(m-1) - \alpha\left[b_k(m) - \mathbf{r}^T(m)\mathbf{g}_k(m)\right]\mathbf{r}^T(m)\mathbf{Y}(m)\right]_{u-}^{u+} \qquad \textbf{(6.11)}$$

where $u-$ and $u+$ are the bounds and if the step size happens to be outside this bound it is truncated. The parameter $\alpha > 0$ is called the learning rate and the vector $\mathbf{Y}(m)$ is the estimate of the derivative of the weighting coefficients with respect to the step size parameter (i.e. $d\mathbf{g}_k(m)/d\mu|_{\mu=\mu(m)}$). Differentiating (6.10) with respect to the step size yields the following expression for $\mathbf{Y}(m)$

$$\mathbf{Y}(m) = \mathbf{Y}(m-1) - \mu(m-1)\mathbf{r}(m-1)\mathbf{r}^T(m-1)\mathbf{Y}(m-1) + \mathbf{r}(m-1)\left[b_k(m-1) - \mathbf{g}_k^T \mathbf{r}(m-1)\right] \qquad \textbf{(6.12)}$$

6.4.2 Blind Adaptive Step Size MMSE detector

The optimality criterion and the adaptive algorithm for the blind MMSE detector were given in (6.2) and (6.7). Minimization of (6.2) with respect to the step size parameter yields the following update Equation for the step size

$$\mu(m) = \left[\mu(m-1) - \alpha\mathbf{r}^T(m-1)\mathbf{g}_k(m-1)\mathbf{r}^T(m-1)\mathbf{Y}(m-1)\right]_{\mu-}^{\mu+} \qquad \textbf{(6.13)}$$

The update of $\mathbf{Y}(m)$ is obtained by differentiating (6.7) with respect to the step size parameter and is given by the following expression

$$\mathbf{Y}(m) = \left[\mathbf{I} - \mu(m-1)\mathbf{r}\mathbf{r}^T\right]\mathbf{Y}(m-1) + \mu(m-1)\mathbf{r}(m-1)\mathbf{Y}(m-1)\mathbf{r}^T(m-1)\mathbf{s}_k\mathbf{s}_k \\ -\mathbf{r}^T(m-1)\mathbf{g}_k(m-1)\left(\mathbf{r}(m-1) - \mathbf{r}^T(m-1)\mathbf{s}_k\mathbf{s}_k\right) \quad \textbf{(6.14)}$$

6.5 Convergence Analysis of Adaptive Step Size Algorithm

In this Section we look at the convergence analysis of the step size algorithm for both the decision directed and blind MMSE detectors. We first start by first reformulating the optimization problem of (6.2) into an unconstrained problem.

A. Reformulation of the Constraint Optimization

Let the n^{th} component of $\mathbf{g}_k$ be $g_{k,n}, n = 1,..., M$. The constrained optimization of (6.2) can be eliminated by computing the elements of $\mathbf{g}_k$ independently. These components can be solved via

$$g_{k,n}(m) = \frac{1}{s_{k,n}}\left(1 - \sum_{\substack{i=1 \\ i \neq n}}^{N} s_{k,i}g_{k,i}\right) \quad \textbf{(6.15)}$$

Let the vector $\mathbf{r}_n$ and $\mathbf{g}_{k,n}$ be the vector of the received signal orthogonal to $s_{k,n}$ and weighting coefficients without the n^{th} component. The j^{th} component of $\mathbf{r}_n$ is given by $r(j) - r(n)s_{k,j} / s_{k,n}$, where $r(j), j \neq n,$ is the j[th] component of $\mathbf{r}$. Using these vectors the unconstrained optimization criterion is given by

$$J_n = E\left\{\left(\frac{r(n)}{s_{k,n}} + \mathbf{g}_{k,n}^T \mathbf{r}_n\right)^2\right\} \quad \textbf{(6.16)}$$

Defining $y_n = -r(n) / s_{k,n}$, then the optimization in (6.16) can be written as follows

$$J_n = E\left\{\left(y_n - \mathbf{g}_{k,n}^T \mathbf{r}_n\right)^2\right\} \quad \textbf{(6.17)}$$

From the above Equation it can be seen that the new unconstrained optimization criterion is the MSE criterion, thus the analysis presented below holds for both the blind and decision directed MMSE detector. The LMS algorithm for the computation of the weighting coefficients of the above Equation assuming an adaptive step size is summarized below

$$\begin{aligned}
\mathbf{x}_k(m) &= \mathbf{x}_k(m-1) + \mu(m-1)\mathbf{r}_n(m-1)(y_{k,n}(m-1) - \mathbf{r}_p^T(m-1)\mathbf{g}_{k,n}(m-1)) \\
\mu(m) &= \left[\mu(m-1) + \alpha\left[y_{k,n}(m-1) - \mathbf{r}_p^T(m-1)\mathbf{g}_{k,n}(m-1)\right]\mathbf{r}_n^T\mathbf{Y}(m-1)\right] \\
\mathbf{Y}(m) &= \left[\mathbf{I} - \mu(m-1)\mathbf{r}_n\mathbf{r}_n^T\right]\mathbf{Y}(m-1) + \mathbf{r}_n\left[y_{k,n}(m-1) - \mathbf{r}_p^T(m-1)\mathbf{g}_{k,n}(m-1)\right]
\end{aligned} \quad \textbf{(6.18)}$$

B. Convergence Analysis

The adaptive step size update Equation in (6.18) can be rewritten as follows

$$\mu(m+1) = \mu(m) + \alpha e(m)\mathbf{r}_n^T(m)\mathbf{Y}(m) + \alpha Z(m) \quad \textbf{(6.19)}$$

where $e(m) = y_n(m) - \mathbf{r}_n^T(m)\mathbf{g}_{k,n}$ and $Z(m)$ is a scalar having the shortest distance required to bring $\mu(m) + \alpha e(m)\mathbf{r}_n^T(m)\mathbf{Y}(m)$ inside the region $[\mu-, \mu+]$. The convergence analysis presented here uses the ODE approach and a weak convergence method presented in [57] and is accomplished by proving that the continuous-time interpolation of $\mu(m)$ and Z(m) converges. The continuous-time interpolation of $\mu(m)$ and Z(m) are given by [56]

$$\begin{aligned}
\mu^\alpha(t) &= \begin{cases} \mu(0) & \mathrm{t} \le 0 \\ \mu(n) & 0 < \mathrm{t} \in [\mathrm{n}\alpha, (\mathrm{n}+1)\alpha) \end{cases} \\
Z^\alpha(t) &= \begin{cases} 0 & \mathrm{t} < 0 \\ \alpha \displaystyle\sum_{n=0}^{t/\alpha-1} Z(n) & \mathrm{t} \ge 0 \end{cases}
\end{aligned} \quad \textbf{(6.20)}$$

The weak convergence is the generalization of the convergence in distribution and is stated as follows: We say the vector $\mathbf{x}(m)$ converges weakly to the vector $\mathbf{x}$ if and only if for any bounded and continuous function $f(.)$, $E[f(\mathbf{x}(m))] \to E[f(\mathbf{x})]$. $\mathbf{x}(m)$ is said to be tight if and only if for each $\eta > 0$, there is a compact set K_η such that $P\left(x(m) \in K_\eta\right) \ge 1 - \eta$ for all m.

Theorem 6.1: Assume that $\{\mathbf{r}_n, y_n\}$ is bounded with probability one and that there is a symmetric positive definite matrix **H** such that

$$\sum_{j=m}^{\infty} | \left[\mathbf{E}(m)\mathbf{r}_n^T(j)\mathbf{r}_n(j) - \mathbf{H} \right] | = O(1) \quad \text{in mean} \tag{6.21}$$

and that there is a continuous function $g(.)$ such that as $j \to \infty$ for each $\mu \in [\mu-, \mu+]$ and $m \geq 0$ we have

$$\begin{aligned} \frac{1}{i}\sum_{j=m}^{i+m} \mathbf{E}(m)e(m)\mathbf{r}_n(j)\mathbf{Y}(j) &\to g(\mu) \text{ in probability} \\ &= \mathbf{E}e(m)\mathbf{r}_n(m)\mathbf{Y}(m) \\ &= -\frac{1}{2}\frac{\partial \mathbf{E}e^2(m)}{\partial \mu} \end{aligned} \tag{6.22}$$

then, $\{\mu^{\alpha}(.), Z^{\alpha}(.)\}$ is tight in $D^2(-\infty, \infty)$ and any weakly convergent subsequence has the limit $\{\mu(.), Z(.)\}$, which is a solution of

$$\frac{d\mu}{dt} = g(u(t)) + z(t)$$

Note that with the Gaussian noise in the DS-CDMA system signal model of Equation (6.2), the boundedness assumption of Equation (6.21) is not satisfied. However if the noise is truncated to some finite value, then the above theorem applies. Theorem 6.1 states that the adaptive step size will spend most of its time in an arbitrarily small neighborhood of the local minimum of Equation (6.17). The above theorem dealt with a large but still bounded time interval. Another case of interest is what happens when $\alpha \to 0, i \to \infty$ and $i\alpha \to \infty$. The following theorem tells us what happens.

Theorem 6.2. Assume that the conditions in theorem 6.1 are met. Let $\{q_{\alpha}\}$ denote a sequence of integers such that $q_{\alpha} \to \infty$ as $\alpha \to \infty$ and $\alpha q_{\alpha} \to \infty$. Suppose that there is a unique local minimum μ of (6.17) such that $\mu \in [\mu-, \mu+]$. Then, $(\mu^{\alpha}(\alpha q_{\alpha} + .)$ converges weakly to μ.

The reader is referred to [56] for proof of these two theorems.

6.6. Conclusion

In this Chapter we presented a survey of the blind MMSE detector. The advantage of this detector over the decision directed MMSE detector is that the blind algorithm requires no training sequence for the weighting coefficients of the detector's adaptive filter to converge. The information required by the blind MMSE detector are the spreading sequence and the timing of the desired user.

Also presented in this Chapter is an adaptive step size adaptive MMSE detector and its convergence analysis. Such MMSE detector requires two adaptive filters. One adaptive filter computes the weighting coefficients of the detector while the second one computes the adaptive step size. The adaptive step size algorithm was motivated by the fact that as the weighting coefficients get closer to their optimal value one would want the step size parameter to approach zero, thus reducing excess MSE. In contrast, when the weighting coefficients of the user of interest are not close to their optimal coefficients we want the step size parameter to grow to some upper bound so that the weighting coefficients can converge quickly.

Chapter 7
Conclusions

7.1 Summary

In this thesis we considered multiuser detectors for DS-CDMA systems, with special focus on the adaptive MMSE detector. We presented an implementation of the DS-CDMA MMSE detector based on the lattice algorithms. The performance of these detectors then had to be compared with that of the LMS-MMSE detector. Chapter 1, which is an introduction briefly, described DS-CDMA systems. It then summarized the thesis layout and contributions made in this project.

Chapter 2 presented a review of DS-CDMA multiuser detectors. Firstly the system model used for DS-CDMA systems was presented. Following this we looked at the conventional detector in both non-fading and fading channels and the disadvantages of this detector where highlighted. We then went on and looked at multiuser detectors, starting with Verdu's optimal detector in non-fading and fading channels. The suboptimal detectors presented were the decorrelating, MMSE, Successive Interference Cancellation and Parallel Interference detectors.

Chapter 3 focused on the MMSE detector. The topics covered in this Chapter are: (1) Optimal solutions for the LMS-MMSE detector, (2) Adaptive implementation of the MMSE detector using the LMS algorithm, (3) Convergence analysis of the LMS-MMSE detector filter coefficients and (4) Analytical BER expression for the MMSE detectors.

Chapter 4 then presented a literature review of lattice filters where the focus was on the gradient lattice algorithm. The LS-lattice algorithm is treated in Appendix B. This Chapter first looked at linear prediction using an FIR filter. Following this, the lattice structure was introduced and it was shown how it could be used to perform linear prediction. Finally we looked at the Stochastic lattice Equalizer.

Chapter 5 presented the lattice-Based MMSE detectors, where firstly optimal coefficients of both the stochastic and the LS-lattice MMSE detectors were presented. Following this the adaptive lattice-MMSE detectors were presented. BER and convergence simulation results were presented. From these results, the lattice-MMSE detectors where seen to offer a good trade-off between convergence and BER results when compared to the LMS-MMSE detectors. The expression of the excess mean squared error was then derived. Using this expression the analytical BER results for the gradient lattice-MMSE Precombining detector was then obtained and shown to be accurate.

In Chapter 6 blind MMSE detectors were reviewed. Firstly the optimal solution for the blind MMSE detector was presented. Following this, the adaptive implementation of the blind MMSE detector was presented. We then went on and looked at the adaptive step size adaptive MMSE detectors and analyzed their convergence.

7.2 Conclusions and Future Work

In this thesis, lattice-based MUDs were proposed. The performance of these detectors were then compared to that of the LMS-MMSE MUDs, where the following metric were used as the measures of performance:

- Bit Error Rate (BER).
- Convergence Rate.
- Output signal to noise ratio.

From the results presented, it was observed that the convergence rate of the lattice-based CDMA MUDs proposed in this thesis is far superior to that of the LMS-based MMSE MUDs. In contrast to this, the LMS-based MMSE MUDs where seen to offer slightly better BER results. From these results it can be concluded that the lattice-based MUDs offer superior convergence-BER trade-off when compared to the LMS-based MMSE detectors. This superiority in performance trade-off of the lattice-based MUDs is beneficial in fast fading channels. In such channels, the optimal solution of the coefficients of the adaptive MUDs varies rapidly and the LMS-MMSE MUDs might have problem tracking the optimal solution due to its inherent slow convergence rate and this will lead to vast performance deterioration of the detector. On the other hand, the rapid rate of convergence exhibited by the adaptive lattice-based MUDs means that these detectors would, to some degree be able to track the time-varying optimal solution of their coefficients leading only to a slight deterioration in performance.

Research in lattice-based MUDs can continue in the following direction

- Analysis of the performance of the lattice-based MUDs when quantization error is taken into account.
- Incorporating coding techniques for improved BER performance of the detector.
- Joint adaptive channel estimation and symbol estimation to improve the accuracy of the conventional channel estimation algorithm [63].

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Appendix

Appendix A : Signal Model for Fractionally Spaced Chip Matched Filter Output.

In this Appendix the signal model used in Section (3.4.2) is developed. As was mentioned in that Section, when an ideal chip-matched filter is assumed then sampling the output of the chip-matched filter at a rate much higher than the chip rate can be modeled by sampling at the chip rate followed by interpolation.

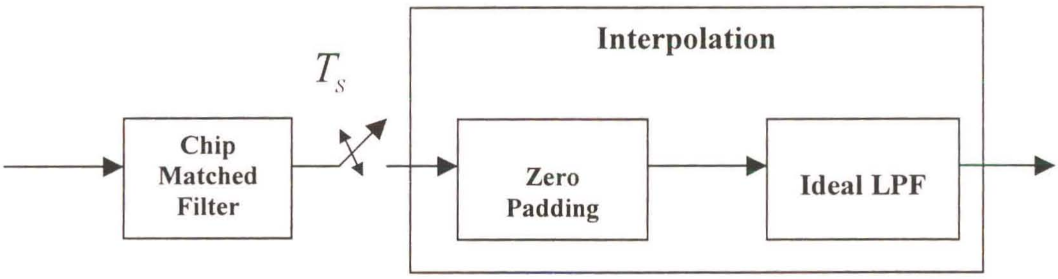

Figure A.1 Block diagram of a how the Fractionally Spaced (FS) signal model can be achieved.

Figure A.1 shows how interpolation can be modeled. As can be seen from this diagram, the output of the ideal chip-matched filter is fed into an expander. The function of the expander is to insert zeros between consecutive pair of samples. The number of zeros padded between the samples is $S-1$, where S is the number of samples to be taken per chip. The output of the expander can be written as follows [63]

$$\mathbf{r}_{\exp} = \mathbf{D}_S \mathbf{r} \qquad \textbf{(A.1)}$$

where $\mathbf{r}_{\exp}$ is a length-$N_{\exp}$ vector of the expander output and $\mathbf{r}$ is the input vector of length $N_{\exp} / S$. As an example when S=2 the Matrix $\mathbf{D}_S$ is defined as follows [63]

$$\mathbf{D}_S = \begin{bmatrix} & & \vdots & & \\ & 1 & 0 & 0 & \\ & 0 & 0 & 0 & \\ \cdots & 0 & 1 & 0 & \cdots \\ & 0 & 0 & 0 & \\ & 0 & 0 & 1 & \\ & & \vdots & & \end{bmatrix} \quad \textbf{(A.2)}$$

For now lets assume that the matrix $\mathbf{D}_S$ is infinite in both directions, later we will look at the finite dimension. The output vector of the expander is then passed through a lowpass filter, which is also assumed to be ideal. This filter is assumed to have the following magnitude response [81]

$$|Y(e^{jw})| = \begin{cases} S, & -\frac{\pi}{S} < w < \frac{\pi}{S} \\ 0, & \text{otherwise} \end{cases} \quad \textbf{(A.3)}$$

and the resulting output of the lowpass filter, which is the over-sampled signal, is written as follows

$$\begin{aligned} \mathbf{r}_S &= \mathbf{Y}_S \mathbf{r}_{\exp} \\ &= \mathbf{Y}_S \mathbf{D}_S \mathbf{r}_{\exp} \\ &= \mathbf{B}_S \mathbf{r}_{\exp} \end{aligned} \quad \textbf{(A.4)}$$

where $\mathbf{B}_S = \mathbf{Y}_S \mathbf{D}_S \mathbf{r}_S$ is a symmetric Toeplitz matrix. From the above Equation it should be clear that the interpolation operation is given by the matrix $\mathbf{B}_S$. The correlation of $\mathbf{r}_S$ is therefore given by

$$\begin{aligned} \mathbf{F}_S &= E[\mathbf{r}_S \mathbf{r}_S] \\ &= \mathbf{B}_S \mathbf{F}_P \mathbf{B}_S \\ &= \mathbf{B}_S \mathbf{\Gamma} \mathbf{\Lambda} \mathbf{\Gamma} \mathbf{B}_S \end{aligned} \quad \textbf{(A.5)}$$

where $\mathbf{F}_P$ was defined in chapter 3. To complete the derivation the following two lemmas will be needed.

Lemma 1: If two vectors $\mathbf{h}_1$ and $\mathbf{h}_2$ are orthogonal (i.e. $\mathbf{h}_1^H \mathbf{h}_2 = 0$), then their interpolated versions will also be orthogonal (i.e. $(\mathbf{B}_S \mathbf{h}_1)^H \mathbf{B}_S \mathbf{h}_2 = 0$).

Proof: Let the n^{th} element of $\mathbf{h}_1$ and $\mathbf{h}_2$ be $h_1(n)$ and $h_2(n)$, respectively. By Parseval's theorem and the orthogonality of $\mathbf{h}_1$ and $\mathbf{h}_2$

$$\begin{aligned}\mathbf{h}_1^H\mathbf{h}_2 &= \sum_n h_1^*(n)h_2(n) \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_1(e^{jw})H_2(e^{jw})dw \\ &= 0\end{aligned} \quad \textbf{(A.6)}$$

where $H_1(e^{jw})$ and $H_2(e^{jw})$ are the Fourier transform of $h_1(n)$ and $h_2(n)$. Now, the Fourier transform of $\mathbf{B}_s\mathbf{h}_1$ is $Y(e^{jw})H_1(e^{Ljw})$ and similarly for $\mathbf{B}_s\mathbf{h}_2$. Therefore again by Parseval's theorem we have the following

$$\begin{aligned}(\mathbf{B}_S\mathbf{h}_1)^H\mathbf{B}_S\mathbf{h}_2 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y(e^{jw})|^2 H_1(e^{jSw})H_2(e^{jSw})dw \\ &= \frac{S^2}{2\pi}\int_{-\pi/S}^{\pi/S} H_1(e^{jSw})H_2(e^{jSw})dw \\ &= 0\end{aligned} \quad \textbf{(A.7)}$$

Lemma 2: If the ideal LPF $Y(e^{jw})$ has the magnitude response given in (A.3), then $\|\mathbf{B}_S\mathbf{h}\|^2 = S\|\mathbf{h}\|^2$

Proof: By Parseval's relation, we have

$$\begin{aligned}\|\mathbf{B}_S\mathbf{h}\|^2 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y(e^{jw})|^2 H(e^{jSw})H(e^{jSw})dw \\ &= \frac{S^2}{2\pi}\int_{-\pi/S}^{\pi/S} H_1(e^{jSw})H_2(e^{jSw})dw \\ &= \frac{S^2}{2\pi S}\int_{-\pi}^{\pi} H_1(e^{jSw})H_2(e^{jSw})dw \\ &= S\|\mathbf{h}\|^2\end{aligned} \quad \textbf{(A.8)}$$

The columns of $\mathbf{\Gamma}$ are orthogonal and have unit norm since they are normalized eigenvectors of a Hermitian matrix. Thus by Lemma 1, the columns of $\mathbf{B}_S\mathbf{\Gamma}$ are also orthogonal, whereas Lemma

2 reveals that the squared norm of each column of $\mathbf{B}_S\boldsymbol{\Gamma}$ is S. If we choose $\mathbf{X}$ to be orthogonal to $\mathbf{B}_S\boldsymbol{\Gamma}$ and satisfy $\mathbf{X}^H\mathbf{X}=\mathbf{I}$, then the unitary similarity transform of $\mathbf{R}_S$ is given by

$$\mathbf{F}_S=\begin{bmatrix} S^{-1/2}\mathbf{B}_S\boldsymbol{\Gamma} & \mathbf{X}\end{bmatrix}\begin{bmatrix} S\Lambda & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix} S^{-1/2}\boldsymbol{\Gamma}^H\mathbf{B}_S^H \\ \mathbf{X}^H\end{bmatrix} \quad \textbf{(A.9)}$$

In deriving the above expression we assumed that the whole received signal is processed at the same time at the receiver. In a practical windowed implementation, where we use a subvector $\mathbf{r}_P(m)$ of $\mathbf{r}$, we have the following relationship

$$\mathbf{r}_{P,S}\approx\hat{\mathbf{B}}_S\mathbf{r}_P(m)$$

where $\hat{\mathbf{B}}_S$ is a symmetric Toeplitz matrix with finite dimension.

Appendix B : Least Square (LS) Lattice Algorithm

In Chapter 4 we considered the gradient lattice filter. One property of this lattice equalizer is that it uses only one set of reflection coefficients to compute both the backward and forward error prediction coefficients. In this Appendix, we consider another lattice equalizer, which uses two different sets of reflection coefficients to compute both the forward and backward error prediction coefficients. This lattice equalizer is called the least square (LS) lattice filter.

Lets assume that the input vector is given by $\mathbf{u}(m)$. Then according to [30,33,94], the LS forward and backward prediction errors are given by (using the same notation adopted in Chapter 4 to denote the forward and prediction error coefficients)

$$\begin{aligned} f_i(m) &= \mathbf{a}_{f,i}^T(m)\mathbf{u}(m) \\ d_i(m) &= \mathbf{a}_{d,i}^T(m)\mathbf{u}(m) \end{aligned} \qquad \textbf{(B.1)}$$

where $\mathbf{a}_{f,i}(m)$ and $\mathbf{a}_{d,i}(m)$ are the LS forward and backward predictors for the i^{th} stage at time m. Using a method which is similar to the one used in Section (6.3), the order updates Equations for the backward and forward prediction error filter can be shown to be [94]

$$\begin{aligned} f_i(m) &= f_{i-1}(m) + k_i^f d_i(m-1) \\ d_i(m) &= d_{i-1}(m-1) + k_i^d f_i(m) \end{aligned} \qquad \textbf{(B.2)}$$

where k_i^f and k_i^d are the forward and backward reflection coefficients for the i^{th} stage, respectively. The optimal forward reflection coefficients is obtained by minimizing the following cost function [30,94]

$$J = \sum_{j=1}^{m} w^{m-j} \,|\, f_i^2(j) \,| \qquad \textbf{(B.3)}$$

Minimization of the above Equation yields the following results [30,94]

$$k_i^f = -\frac{\sum_{j=1}^{m} w^{n-j} f_{i-1}^*(j) d_{i-1}(j-1)}{\sum_{j=1}^{m} w^{n-j} \,|\, d_{i-1}(j-1) \,|^2} \qquad \textbf{(B.4)}$$

Likewise, the optimal solution of the backward reflection coefficient of the ith stage is obtained by minimizing $J = \sum_{j=1}^{m} w^{m-j} \mid d_i(j) \mid^2$ and the corresponding optimal solution is given by

$$k_i^d = -\frac{\sum_{j=1}^{m} w^{n-j} d_{i-1}^*(i-1) f_{i-1}(i)}{\sum_{j=1}^{n} w^{n-j} \mid f_{i-1}(i) \mid^2} \tag{B.5}$$

In practice, the reflection coefficients are computed adaptively, where the numerator and denominator of Equation (B.4) and (B.5) are computed recursively [30,94]. Lets denote the numerator of Equation (B.4) and (B.5) as ψ_i and denominators of Equation (B.4) and (B.5), respectively as r_i^f and r_i^d. Then the reflection coefficients are computed using the following Equations

$$\begin{aligned} k_i^f(m) &= -\frac{\psi_i(m)}{r_{i-1}^b(m-1)} \\ k_i^d(m) &= -\frac{\psi_i^*(m)}{r_{i-1}^f(m-1)} \end{aligned} \tag{B.6}$$

where parameters used to compute these coefficients are computed using the following expression [30,94]

$$\begin{aligned} \psi_{i+1}(m) &= w\psi_{i+1}(m-1) + \alpha_i(m-1) f_i(m) d_i^*(m-1) \\ r_{i+1}^f(m) &= r_i^f(m) - \frac{\mid \psi_{i+1}(m) \mid^2}{r_i^d(m-1)} \\ r_{i+1}^d(m) &= r_i^d(m) - \frac{\mid \psi_{i+1}(m) \mid^2}{r_i^f(m)} \\ \alpha_{i+1}(m) &= \alpha_i(m) - \frac{\alpha_i^2(m) \mid d_i(m) \mid^2}{r_i^*(m)} \end{aligned} \tag{B.7}$$

The above Equations are initialized as follows

$$\begin{aligned}
&\alpha_0(m) = 1 \\
&r_0^f(m) = r_0^b(m) = w r_0^f(m-1) + |f_0(0)|^2 \\
&r_i^b(-1) = r_i^f(0) = 1/\mu \\
&\alpha_i(-1) = 1 \\
&\psi_i(-1) = 0
\end{aligned} \tag{B.8}$$

So far we have look at the lattice section, without mentioning anything about the ladder section. To wrap-up this appendix we give the adaptive algorithm for computing the ladder coefficients [94]

$$\begin{aligned}
&h_i(m) = h_i(m-1) - \frac{\alpha_i(m) d_i^*(m) e_{i+1}(m)}{r_i^d(m)} \\
&e_{i+1}(m) = e_i(m) + d_i(m) h_i(m)
\end{aligned} \tag{B.9}$$

where $e_i(\mathrm{m})$ and $h_i(m)$ are the error at the output of the filter and ladder coefficients, respectively.