



DURBAN CAMPUS

Title

Real Options Analysis in Strategic Decision Making

By

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CONFIDENTIALITY CLAUSE

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TO WHOM IT MAY CONCERN

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Due to the importance of this research it would be appreciated if the contents remain confidential and not be circulated for a period of ten years.

Sincerely

P GOVENDER

DECLARATION

This research has not been previously accepted for any degree and is not being currently submitted in candidature for any degree.

Signed.....

096479

Date.....

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ABSTRACT

The research addresses the management dilemma of a decrease in the number of capital project investments, due to the current methods of capital budgeting (i.e. net present value analysis using discounted cash flows) being ineffective, because it does not effectively deal with uncertainty in the investment, and also does not take management's flexibility into account.

It has been determined that a strategic options framework can be used to provide a more meaningful assessment of future business opportunities under uncertainty. The options approach complements the conventional net present value criterion in evaluating risky investment. The options approach provides an immediate and important perspective on value creation because the options approach takes into consideration that management has the choice of deferring the investment to a later date when circumstances are more certain, and there is less risk involved, or the choice of completely abandoning the investment.

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CHAPTER 1: THE PROBLEM

1.1 INTRODUCTION

Investment in research and development in new products and marketing is becoming increasingly important. Companies need to upgrade existing products and develop new products. Taking risk is a constant in business today through globalisation. The business frontier has become an even smaller place to trade in. Through trading within this front, exchanges, ideas and competitive work fronts have become the norm. It is for this reason that capital budgeting is needed. Companies embark on capital-intensive projects, they foresee major returns and want their resources to add value to their bottom line.

Capital budgeting is an extremely important aspect of a firm's financial management. It is for this reason that strategy is formed by important resource-committing actions that influence the business development of the firm. This is embodied in corporate capital allocations. The proper evaluation of investment projects is critical for the success of this implementation. Therefore a firm that makes mistakes in its capital budgeting process will take a number of years to recover.

Managers (decision makers) usually tend to ignore recommendations derived from capital budgeting exercises and instead base their decisions on intuitive judgement, because traditional planning approaches fail to capture the full value of opportunities and adaptability under uncertainty i.e. traditional planning approaches do not take into consideration the flexibility that management have to alter the course of a project in response to changing market conditions.

This dissertation appraises a new approach – the application of the Real Options Approach in investment analysis. The Real Options Approach offers the possibility of improving strategic decisions. This analysis takes into consideration

that the investment can be abandoned or deferred at any stage during the project life, thereby releasing more value to the project. The reason why this new investment analysis is called the "Real Options Approach" is because it is similar to a financial option on an asset or share, where the holder has the right to buy or sell an asset at some date in the future at a predetermined price. As in financial engineering, this right has value, and should be considered in the valuation of the project.

The use of the Real Options Approach in valuing capital projects captures more value associated with strategic operating and investment flexibility available to management than would be achieved by the existing valuation methods i.e. the discounted cash flow approach.

1.2 BACKGROUND OF THE RESEARCH

"Companies benefit by keeping their options open."

The analogy between financial options and corporate investments that create future opportunities are both intuitively appealing and increasingly well accepted. Executives readily see that today's investment in Research and Development (R&D), or in a new marketing program, or even in a multi-phased capital expenditure can generate the possibility of new products or markets tomorrow. But for many, the leap from the puts and calls of financial options to actual investment decisions has been difficult and deeply frustrating.

Conventional cash flow analysis has failed to capture the essence of strategic decision-making, which is adaptability. This is when management reviews circumstances that were uncertain at the project evaluation stage, during the project life cycle and reacts to it. The discounted cash flow approach, which calculates the net present value of a project, does not take this into account and

therefore this has introduced another technique that is the focus of this dissertation i.e. the application of the Real Options Approach in investment analysis.

There have been two stages in the evolution of the valuation models used in project analysis i.e.

- (1) Static models, in which an investment project is completely described by a specified stream of cash flows whose characteristics are given.
- (2) Controllable cash flow models, in which projects can be managed actively in response to the resolution of external uncertainties.

The first stage, static, mechanistic, models treat projects as inert machines that produce specified streams of cash flows over time. The analyst must value these projects by discounting the cash flows at an appropriately determined risk adjusted discount rate. This is the well-known “discounted cash-flow (DCF)” approach to valuation.

In dealing with strategic projects most companies manipulate hurdle rates in order to mask the tendency of under-valuation. Miller and Upton¹ also provide evidence that DCF valuations are downwardly biased and poorly correlated with revealed market value. The past two decades have seen the evolution of academic research aimed at valuing managerial flexibility.

An option is defined as the right to buy or sell an asset at some date in the future at a predetermined price. This right has value, because on the exercise date (the date on which the option is to be exercised), the holder can decide whether to buy the asset or not. If the asset has increased in value beyond the exercise price, the holder would be able to buy it at a bargain price. On the other hand, if

¹ Miller and Upton 1985

the price of the asset has fallen below the exercise price the holder would simply not exercise the option and so would lose nothing.

The following terms are generally used:

- call option – the right to buy an asset at a predetermined price.
- put option – the right to sell an asset at a predetermined price.
- American option – sell up to a specific date.
- European option – sell on a specific date.

Luehrman² draws an analogy between an investment opportunity and a call option because management have the right, but not the obligation to acquire operating assets. Operating flexibility like options to defer investments, abandon the project, contract the scale, expand the scale and the option to switch have been valued extensively in an attempt to explain the value of capital investments.

1.3 MOTIVATION FOR THE RESEARCH

Capital budgeting is an essential activity in any company and through the capital budgeting process long-term investment decisions are made. The existing investment valuation models (i.e. the discounted cash flow approach) under-values certain projects because it does not take into account flexibility arising from uncertainties in cash flows. This under-valuation of projects leads to the project not being pursued, and thereby leads to a decrease in income received by the company.

The most amount of value is added to the company if at the initial stage in the process of a project, the correct valuation method is chosen. This research clearly indicates the shortcomings of the discounting cash flow approach, and

² Luehrman 1998

presents the real options approach for capital budgeting. This approach is seen as a major breakthrough that has the potential to yield a large amount of value.

The real options approach stems out of the financial option on a share price, however the step from the puts and calls of financial options to actual investment decisions has been difficult and deeply frustrating. The calculations required to value real options are seen as dauntingly complex, and practical how-to advice on the subject has been scarce and mostly aimed at specialists.

This research however will create and evaluate the real options model, and present to executives emphasising that this is a model that should be used to value capital projects. This research does not aim to make assumptions, thereby simplifying the model, it aims to present to management a model that can be used to analyse capital budgeting and to make the decision of whether to invest into the project.

1.4 VALUE OF THE RESEARCH

This research will have two major contributions in strategic decision analysis. Research and development investments can usually be abandoned, if required. The investments constitute sequential premiums paid to establish strategic options that eventually can establish alternative routes to future business expansion. Irreversible investment decisions refer to the subsequent resource commitments on real and intangible assets associated with exercise of existing strategic options.

The consideration of an abandonment option arrangement makes the firm engage in an opportunistic project that otherwise would be rejected. The option to abandon the investment has value, because it provides an opportunity for future gains while limiting the investment commitment if the project develops

unfavourably. The abandonment option perspective applies particularly well to retractable investments.

When the conventional net present value approach is extended to include the alternative of deferring the investment, the investment might yield a positive net value. Therefore it will be advantageous to postpone the investment until more is known about the future payoffs from the investment. The option to defer the investment has value, because it provides the potential for a higher return, while limiting the downside risk of the investment proposition. The deferral option perspective is particularly suited to the analysis of irreversible investment commitments.

The options analysis also looks at the case where a project has a positive net present value, however there is a possibility of yielding positive cash flows and a possibility of yielding negative cash flows, therefore the project is risky, and although there is a positive net present value, this might not be an acceptable proposition.

The consideration of abandonment opportunities adds flexibility to the firm's initial development of strategic options. It allows the firm to opt out of the project if circumstances develop unfavourably. By making relatively small resource commitments at the initial stages of strategic option developments, the firm reduces the sunk cost incurred in case of project abandonment. Similarly, the inclusion of deferral opportunities when arranging irreversible investments in a strategic options exercise adds flexibility to the firm's resource commitments. It allows postponement of commitments until times when circumstances are considered most opportune.

1.5 PROBLEM STATEMENT

The research topic “Real options analysis in Strategic Decision Making” deals with using real options to determine the value of capital projects. The management research question hierarchy (as shown below) was used to convey the dilemma.

1.5.1 Management Dilemma

The management dilemma faced is that there are a decreasing number of capital projects that are being pursued. This dilemma is usually a symptom of an actual problem. Also with the projects that are being pursued, the conventional techniques of capital budgeting do not take into consideration the risk involved in the probability that there will be a certain yield. This risk can be sufficient to make the project unacceptable.

1.5.2 Management Questions

- The management question is can we increase the number of projects being pursued and what should be done to improve the number of capital projects being pursued?
- Which is the most suitable technique that can be incorporated in the analysis of capital projects that will provide management with the clearest representation of the real life situation?
- How can we motivate the importance of this technique, and does this technique replace or does it complement the conventional capital budgeting techniques of net present values and future cash flow analysis? This will address the problem of whether the real options analysis technique of capital budgeting should be used in conjunction with the other techniques or does the circumstance dictate.

1.5.3 Research Questions

- Should management discontinue the use of current techniques of capital budgeting, and how credible are these techniques?
- Can a real options analysis model be formed that can be used to analyse the current and all future capital projects?

What would be the most appropriate method/s, which can be used (present and looking to the future) for capital budgeting with consideration to: -

- The capital investment at the start of the project, and the cash flows during the project lifecycle.
- Fast pace, turbulent business environments
- Sensitivity analysis (cost consciousness) of the real options model, in the aim of guaranteeing sustained cash flows.

1.5.4 Investigative Questions

There are two major investigative questions:

1. What are the disadvantages of the current methods of capital budgeting?
 - Can the current models (net present value method) be adjusted so that it can accommodate for changes specific to the project?
 - Is there a need to find a replacement to the current methods of capital budgeting?
2. What are the advantages of the real options analysis of capital budgeting?
 - Does this new model replace or does it supplement the current models?
 - What are the limitations for the real options model?
 - How easily can the model be incorporated into current practices?

In this research the real options analysis model will be generated for engineering projects. This model will then be applied to current and future projects.

1.6 OBJECTIVES OF THE RESEARCH

The objectives of this research are to:

- Determine the shortcomings of the Discounted Cash Flow based technique in valuing capital investments.

A traditional calculation of net present value (NPV) examines the project as a whole and concludes it is a go or a no-go. NPV calculates the value of a project by predicating its payouts, adjusting them for risk, and subtracting the investment outlays. But by boiling down all the possibilities for the future into a single scenario, NPV does not account for the ability of executives to react to new circumstances - such as “spend a little up front, see how things develop, then either cancel or go full speed ahead”, i.e. NPV does not consider management flexibility.

- Establish a model to value investments using the real options technique.

This model through adaptations will be able to be used in various industries, but for the purpose of this research, it will be evaluated and tested on engineering economic decisions.

- Bridge the conceptual gap and promote wider use of the real options approach.

Real options theory has shown potential for analytical applications in strategic management particularly to evaluate flexibility and timing issues. Yet the options approach has not been widely incorporated to analyse business opportunities and adaptability in strategic investment decisions.

This research aims to bridge this conceptual gap and promote wider use of an options analytical approach. This method distinguishes between abandonment and deferral option scenarios, which are presented to analyse different strategic investment situations. The real options analysis explicates how firms invest in business development and explains the frequent deferral of strategic investments.

1.7 RESEARCH METHODOLOGY

Looking at the degree of research question crystallisation, the research process adopted is of an exploratory nature whereby information was gathered on an ongoing basis. The exploratory study was used to investigate scenarios affecting the real options analysis.

The research adopted a case study approach, designed for depth rather than width. There was a focus on a full contextual analysis and an emphasis on detail, allowing evidence to be verified and avoiding missing data.

A suitable model for the real options analysis had to be developed. Once the model was developed, it was evaluated and tested to current and future engineering capital projects. This method as compared to its counterpart (monitoring) is less time consuming. The advantage of monitoring is that the survey is more accurate, but subject to manipulation.

The relevant population is the engineering industries in South Africa. In order to limit the study in terms of the risk associated with each country's environment, foreign industries were not considered.

The variables of interest (the attitude towards using the real options method to value capital projects, the circumstances dictating this and the validity of the model) are ordinal in nature.

The research design is a “causal study” as it is concerned with learning “why”. Why is there a decline in the number of capital projects, and is there a method of analysing the feasibility of a project that is more realistic than the conventional net present value methods of capital budgeting?

The research undertaken utilises secondary data as its main source of information. Various reports have been compared, bringing validation to the research model that was established.

This study is neither a cross sectional study (which is carried out once only and is representative of a snapshot of one point in time) nor a longitudinal study (which allows for the researcher to study the same events over time), because once the valuation model is developed it can be used by numerous projects in the future.

1.8 LIMITATIONS OF THE RESEARCH

The dissertation is quite extensive in its subject content, however I believe that the following factors need to be highlighted in order to give the reader an idea of some of the limitations the writer was exposed to. This would further enhance the readers’ understanding of the subject matter.

The study is limited because once the real options valuation model has been established, there has not been a “real life” project that it has been tested on. The model is however validated by case studies, considering different scenarios. With the application of a “real life” project the limitations of the model can be highlighted, and the required changes can be noted.

The valuation model itself also had limitations due to it assuming that the amount and timing of a project's capital expenditures are certain. However in most business opportunities they are not. Therefore the model will have to be adapted to handle those circumstances, but the adaptation helps only if we can describe the uncertainty.

If the uncertainty associated with a project changes over time, which in real life projects is fairly common, the model will have to be adjusted to take this into consideration.

The model will be able to add value associated with deferring an investment decision, however there may be costs associated with deferral. In most cases when there are predictable costs to deferring, the option to defer an investment is less valuable, and it would be foolish to ignore those costs. This can be incorporated into the model, however the decision when to invest or not to invest must be known.

1.9 STRUCTURE OF THE RESEARCH

The dissertation first goes through the literature review, to give the reader a background to previous methods of capital budgeting namely the discounted cash flow approach that calculates the net present value of an investment. Before progressing on to the real options approach, the dissertation will highlight the limitations of the discounted cash flow approach.

Part B of the literature study concentrates on the development of the Options Valuation model framework for basing current and future forms of capital budgeting.

Chapter 3 does a breakdown of the analysis of the Valuation model. Here three case-by-case scenarios are explored. Each scenario, i.e. the option to abandon an investment, the option to delay an investment and the option to sample, is explored and the findings are reached for the different circumstances in chapter 4.

The recommendations for future research are then discussed in chapter 5. The limitations of the real options approach are discussed. The real options approach carefully looks at the organisation and changes in the market. Taking it one step further would be to have a capital budgeting tool that would also consider the effects of competitors on the business.

Finally the dissertation concludes and summarises the topic. In evaluating a project one needs to estimate future benefits and costs. Risk control, debt and return are hence major considerations.

1.10 SUMMARY

This dissertation looks at valuating the feasibility of a project, and the primary focus is capital budgeting. The management dilemma is that there are a decreasing number of capital projects that are being pursued and the management questions asked are what should be done to improve the number of capital projects being pursued, and which is the most suitable technique that can be incorporated in the analysis of capital projects, that will provide management with the clearest representation of the real life situation?

From this the objectives of the research were established i.e. determining the limitations of the present valuation method, which is the discounting cash flow approach, and establishing a new model, which is termed the real options approach, that will provide answers to the management questions.

The research process was of an exploratory nature and was used to investigate scenarios affecting the real options analysis. The research adopted a case study approach, designed for depth rather than width. There was a focus on a full contextual analysis and an emphasis on detail, allowing evidence to be verified and avoiding missing data.

The limitations of this research is that once the real options valuation model has been established, there has not been a “real life” project that it has been tested on. The model is however validated by case studies, considering different scenarios.

CHAPTER 2: LITERATURE SURVEY

2.1 PART A: REVIEW OF CAPITAL BUDGETING MODELS

Capital budgeting is a vital activity. It is the process by which organizations make long-term investment decisions. Textbooks in accounting and finance discuss numerous evaluation criteria, including payback period, accounting rate of return, internal rate of return, and Net Present Value.

2.1.1 THE DISCOUNTED CASH FLOW APPROACH

In financial economics, capital budgeting decisions are seen to embody the essence of strategic management³. Capital budgeting typically uses cash flow analysis to evaluate individual projects, although investment decisions ideally should be considered from a corporate perspective. The past three decades have seen the impressive advances in the theory of finance, however practical procedures for capital budgeting have evolved only slowly⁴.

2.1.1.1 NPV Approach

The standard technique used in capital budgeting derives from the original proposition by Fisher in 1907, with a model of valuation under certainty. The net present value model takes into account the time value of money. It involves estimating a project's future cash flows, discounting these cash flows at the company's required rate of return (cost of capital) and subtracting the cost of the investment from the present value. If the result is positive, this indicates that the project results in an increase in shareholders' wealth as the project more than earns the required rate of return.

³ Brealey & Myers, 1996

⁴ Brennan & Schwartz, 1985

The formula is

$$NPV = \sum_{t=1}^n \frac{C}{(1+k)^t} - I$$

Where: C_t = Net cash flow at time t

I = cost of the investment

k = cost of capital

The cost of capital is determined from an asset-pricing model such as the Capital Asset Pricing Model (CAPM).

Example

Project X has the following cash flow stream:

Year	0	1	2
Cash flow	-10000	8000	6000

The cost of the investment is R10 000 and this will result in cash flows of R8 000 in year one and R6 000 in year two. The firm's cost of capital is 20%.

To compute the projects net present value (NPV) we perform the following calculation:

	Cash flow	PV Factor	Present Value
Cost	-10 000	1.000	-10 000
Year 1	8000	0.833	6 664
Year 2	6000	0.694	4164
Net Present Value			<u>828</u>

Project X has a positive NPV, and should therefore be accepted. The shareholders' wealth will be increased by the net present value amount.

The problem with this approach is that it treats the project as an inert machine that produces specified streams of cash-flow over time. When the risk does not remain constant over the project life, the use of this approach is questionable, as determining the correct discount rate (shown as k above) becomes difficult. The NPV approach is not able to handle complexity and uncertainty, therefore a number of variations of this approach have been proposed, i.e. the sensitivity analysis, simulation and decision-tree analysis, which is well-known.

2.1.1.2 Sensitivity Analysis

This analysis uses a base case scenario estimate of the key primary variables from which the base case NPV is evaluated. The key primary variables are then systematically varied and changes in the NPV are noted. It is important to identify the crucial variables that contribute to the risk of the investment⁵ but the further limitations are that this approach considers the effect of NPV of only one error at a time and therefore cannot accommodate a combination of errors. Another limitation is that it cannot be used if the variables are interdependent.

2.1.1.3 Simulation and Decision Tree Analysis

Recognition of the importance of project analysis in the ability to manage or control project cash flows is not new. Hertz in 1964 had shown that the Monte Carlo analysis could be used to value a project when the future decisions could be related to a set of stochastic state variables whose values could be simulated, while in the same year Magee had shown that decision trees could be used in more complex cases in which the optimal future decisions must be determined by taking into account subsequent stochastic events and decisions.

In this stage of the development of project valuation, project cash flows are treated as endogenous because they are under the control of the decision

⁵ Brealey and Myers, 1996

maker. Therefore this stage is often called the “controllable cash-flow stage” of project analysis development.

Although decision tree analysis and the Monte Carlo simulation are useful techniques for estimating the probability distribution of future cash flows, they offer little guidance as to how future possibilities affect project risk and therefore project discount rates. Therefore as long as the problem of risk and the appropriate discount rate remain unsolved, it is not possible to integrate the Monte Carlo analysis and decision trees into the value maximization framework.

2.1.1.4 Limitations of the Discounted Cash Flow Approach

The limitations of the Discounted Cash Flow Approach are widely acknowledged.⁶ Its main deficiency lies in its total neglect of the stochastic nature of output prices and of possible managerial responses to price variations. Dixit & Pindyck⁷ also acknowledge that the NPV rule is easy, but it makes assumptions that the investment is either reversible (i.e. it can be undone and the expenditures recovered should market conditions turn out to be worse than anticipated) or that, if the investment is irreversible then it cannot be delayed (i.e. if the company does not make the investment now it will lose the opportunity forever).

Shortcomings of conventional capital budgeting have been recognised, as cash flow analysis fails to express important organisational effects. That is, capital budgeting should support investment initiatives emerging in different parts of the firm, while resource commitments with overall corporate effects should be considered at the executive level. Ideally, capital budgeting integrates strategic planning, individual incentives, and corporate control.

⁶ Davis, 1996 and Brennan & Schwartz, 1985

⁷ Dixit & Pindyck, 1995

Conventional strategic analysis determines an optimal strategic fit between external business opportunities and internal organisational capabilities from an overall corporate perspective. Some strategic planning models even incorporate capital budgeting to evaluate strategic alternatives⁸.

2.1.2 RISK

This research would not be complete without a brief overview of risk, and its importance to the company when evaluating projects. One of the situations when risk is most crucial is termed the Achilles' heel of the perfect markets model of capital budgeting. Consider a corporation that has equity with a market value of R120m and cash holdings of R110m. Suppose that this corporation is offered a gamble that takes the following form. By accepting the gamble, the corporation agrees that a coin is flipped immediately. If the coin comes up heads, the corporation receives a cheque for R102m. The certified cheque is already written, so that there is no uncertainty about whether the corporation will receive this amount of cash. If the coin comes up tails, the corporation has to write a cheque for R100m, so that most of its cash holdings disappear. Assuming that cash was valued dollar for dollar in the firm's equity, if the coin comes up tails, the value of equity falls to R20m. Since the gamble takes one second to reveal its payoff, its payoff is unsystematic risk. Hence, its net present value is simply the expected payoff, which here is R1m. Therefore this gamble is worth taking, but in reality there is no such company that takes that gamble. This means companies do not apply modern capital budgeting as it is usually taught in business schools.

This is clear evidence that companies do not take gambles that have large volatility for small expected gains. The reason for this is that managers know that

⁸ Ansoff, 1988

generally volatility matters. In the example of a gamble that can lose R100m, managers know that the firm with equity of R20m is not the same firm as the one with equity of R120m. This is because, as the value of the firm falls, the firm becomes unable to take advantage of valuable opportunities that it could take advantage of if it had more equity capital. Finance theory states that a firm that gets close to financial distress finds it difficult to enter contracts that require financial commitments on the part of the counter parties. There are four reasons why this is so:

- 1) If a firm's probability of financial distress is not trivial, it becomes difficult to raise equity because of the under investment problem. If the firm has debt outstanding, when the value of equity is low, funds provided by shareholders benefit mostly the debt holders by making the debt safer. Since the new shareholders expect a fair rate of return on their equity and the debt holders get the benefits from the new equity, the old shareholders lose in that they must be the ones who provide the fair return to the new shareholders. As a result, the old shareholders will be reluctant to let the firm raise new equity and the firm will often not be able to fund new projects with equity.
- 2) Jensen and Meckling emphasized that when the firm's probability of financial distress is not trivial, it becomes profitable for the shareholders to increase the risk of the firm even if it is costly to do so. The reason for this is that when a firm is performing poorly, default makes the shares worthless. By taking large risk, shareholders have a chance to restore value to their equity if the risk works out. If the risk results in large losses, shareholders are not worse off since in that case equity would have had little or no value anyway. Because of the incentives for firms to increase risk close to distress, a firm that is close to distress finds it difficult to raise funds from banks and from the debt markets. Potential investors will assume that the firm will want to raise risk, so that new funds might be

prohibitively expensive or might be encumbered by restrictions that rob the firm of the flexibility it needs to take advantage of new opportunities.

- 3) As the probability of financial distress increases, stakeholders no longer find it worthwhile to invest in their relationship with the firm. For any firm to be successful, there must be individuals and corporations who find it worthwhile to invest in firm-specific capital. Such investment is worthwhile if the firm is healthy and growing, but becomes less so when the firm is distressed and cannot take advantage of its growth opportunities. As a firm becomes closer to distress, workers feel that they are better off working on improving their value to other firms than on increasing their firm-specific knowledge. Suppliers will not expand their capacity to deliver products whose use is specific to the firm because they do not know whether the firm will be able to grow and use these products. Customers may be reluctant to buy products from the firm because they cannot be sure that after-sale services and warranties will correspond to what was promised. As a result of these difficulties, firms close to distress have to pay more for some services or have to offer lower prices to customers to offset the impact of their financial fragility.

- 4) A firm with a weak financial situation finds it more difficult to raise funds when managers pursue their own objectives and value firm growth. This is because capital markets find it difficult to distinguish between good projects that increase firm value and bad projects that management want to undertake to increase firm size. As a result of the fact that management pursues its own objectives and has information about projects that the capital markets do not have, firms with large cash flow shortfalls are often unable to finance valuable projects.

It follows from this that a firm with a non-trivial probability of financial distress may not be able to invest in projects that it would find valuable if its probability of

distress was zero. This is because it may not be able to raise the funds to invest in such projects or because the costs of such projects may be too high because of the firm's financial fragility. Increases in total risk makes it more likely that a firm will end in a situation where it cannot take advantage of valuable projects and hence such increases are costly to the firm. Capital budgeting techniques developed under the assumption that capital markets are perfect allow no role for the cost of increases in total risk associated with new projects because they assume that contracting is costless and perfect. With such an assumption, the problems that crop up when a firm becomes close to financial distress disappear because the firm can always costlessly recapitalize itself so that it is no longer close to financial distress. In the real world, such costless recapitalisation is a dream. As a result, total risk matters has to be taken into account when a firm evaluates a project.

When total risk matters, the capital budgeting techniques are simply not correct and have to be changed. Each project has a cost that impacts on the firm's total risk. To take this cost into account, firms have to quantify their total risk, have to understand the cost of increasing total risk, and have to understand how a new project impacts the total risk of the firm. The appropriate measure of total risk is not the firm's cash flow or equity return volatility. Increasing the risk of the firm when the probability of distress is not affected has no obvious cost to a firm. However, any increase in risk that increases the probability of distress is costly and should be accounted for when evaluating the costs and benefits of a project. Because the risk that is costly is the risk associated with large losses, the appropriate measures of risk are lower-tail measures of risk such as Value-at-Risk or Cash-flow-at-Risk rather than measures such as volatility of stock returns or volatility of cash flows.

For a firm that understands its total risk and the costs associated with an increase in total risk, project valuation consists of decreasing the return by the cost of its impact on the firm's total risk. Consider again the previous example.

With the traditional capital budgeting techniques, the gamble paying R102m with probability 0.5 and losing R100m with the same probability, has a value of R1m for the firm that agrees to it. This project has a considerable lower tail risk since the firm could lose five-sixths of its equity. If the firm measures its lower tail risk and can estimate its cost, it can assess the cost of taking the gamble. One would expect this gamble to have a cost due to its impact on the firm's risk that exceeds R1m and consequently the firm would not take the gamble. Hence, this simple change in how capital budgeting is undertaken will generally lead to a situation where projects with small positive expected returns and large volatility will not be undertaken. Though currently such projects are ignored by firms, managers ignore them because they feel that it is right to do so rather than because modern finance theory tells them to do so. Having capital budgeting rules that lead to correct decisions in the presence of capital market imperfections would lead to a situation where modern finance theory can be applied consistently in firms and in a way that increases their value.

2.1.3 THE REAL OPTIONS APPROACH

It is commonly acknowledged that organisations must assume risk to create new business opportunities while risk is perceived manageable. On the other hand, assuming excessive risk can jeopardise the future viability of the firm. The assessment of risk is critical in strategic analysis. Hence, there is a need to develop approaches that better relate the dynamics of business risk to strategic decision making, i.e. there is a need to develop better analytical techniques to guide firms strategic decisions under circumstances of increasingly dynamic competition. The application of option pricing theory provides an opportunity to make strategic decision analysis more effective.

Real options analysis is a new way of thinking about corporate investment decisions. It is based on the premise that any corporate decision to invest or

divest real assets is simply an option. It gives the option holder a right to make an investment without any obligation to act. The decision-maker, therefore, has more flexibility, and the value of this operating flexibility should be taken into consideration. Therefore one can take advantage of the options approach in modelling capital decision problems.

2.1.3.1 Early work in Option Valuation

Financial options are known to have existed for centuries however economists struggled with the unpredictability of stock and commodity prices. Independent earlier work by Bachelier in 1900 and Kendall in 1953 showed that price movements were independent of one another and therefore rigorous valuation tools could not be used.

There was a revolutionary change during the early 1970's with the seminal work on option pricing by Black and Scholes⁹ and Merton¹⁰ and the creation of the Chicago Board Options Exchange in 1973. The discount rate problem was eventually solved in the context of claims on financial assets, in showing how to value a claim whose payoff is contingent on the value of another asset, Black and Scholes developed the technique of risk-neutral, or equivalent martingale pricing. The methodology they employed was based on viewing an option as a combination of stock investment and borrowing. The fundamental basis for such an approach lied in the assumption of "no arbitrage" conditions in the market in which the asset and the borrowing took place. The standard assumptions on which option valuation relies on are frictionless markets, a constant risk-free interest rate and the underlying asset pays no dividends.

⁹ Black F and M Scholes, The Pricing of Options and Corporate Liabilities

¹⁰ Merton RC, Theory of Rational Option Pricing

Therefore C_t , the instantaneous cash-flow rate from an irreversible investment project follows the diffusion process:

$$\frac{dC_t}{C_t} = \alpha dt + \sigma dZ(t)$$

Where α is the expected growth rate of cash flows
 σ is the standard deviation of the cash-flow process.

This model was later formalised by Cox and Roxx in 1976, Constantinides in 1978, Harrison and Pliska in 1981, and others. The far-reaching implication for project analysis is that if the expected rates of change in the underlying cash-flow drivers are risk adjusted, the resulting "expected" (risk-adjusted) cash flows can be discounted at the risk-free interest rate, regardless of the types of future decision contingencies inherent in the project.

Brennan & Swartz and McDonald & Siegel in 1985 were the first to actually employ these insights in the valuation of real assets, thus helping to complete this stage in the development of project valuation, which has become known as "real options" analysis. The term "real options" recognise both the similarities and the differences between the valuation of rights to controllable cash flows and the valuation of financial options. The similarities arise not only because the ability to control or manage a cash-flow stream represents an option, but more importantly, because equivalent martingale pricing techniques are appropriate to both real and financial options. The major difference is that although financial options are almost always options on traded assets¹¹, the rights to controllable cash flows typically cannot be reduced to claims on traded assets.

¹¹ This is not always the case, as an option on a foreign stock index converted to another currency at a fixed exchange rate is an example of a financial option on non-traded assets.

The option theory has gained wider acceptance in financial markets since Scholes and Merton won the 1997 Nobel Prize for Economics for their work on option-pricing theory.

The option framework of decision-making is based on the opportunity to make a decision after it is seen how events unfold. The various types of options considered are postponement, abandoning the investment, contracting or expanding the investment and the option to switch the use of an asset to the next best alternative.

2.1.3.2 The option to postpone

The traditional NPV rule implicitly assumes that the opportunity to undertake a project will be lost if a project is not undertaken immediately. However in most situations this is not a plausible assumption. Under most circumstances there is some relevant time period, typically a few years, over which the company owns the right to undertake the project. Such rights may be associated with the ownership of land, natural resources, technical expertise, market share, or patents, for example.

Studies by Ingersoll & Ross in 1992 and Ross in 1995 focus on interest rate uncertainty. These studies point out that the right to undertake a project in the future has value even if the project has a negative NPV if undertaken today. There may exist some probability that future interest rates will decline resulting in a positive NPV. If the company undertakes the project today, it may forego the option to undertake the investment in the future, and the NPV must be adjusted accordingly. Therefore there may be an opportunity cost of undertaking the project today versus waiting for interest rate to improve. If so, the value of the project must also reflect the option value to the right to delay investment.

Ingersoll & Ross compute postponement option values by simulating future interest rate paths assuming a simple diffusion process for interest rates.

Ehrhardt & Daves have also developed a model that extends the work of Ingersoll & Ross by using a mean-reverting interest rate process that allows both the level and the slope of the yield curve to change in a manner that is consistent with historically observed yield curves and with past interest rate behaviour. It is also demonstrated that by holding the volatility of interest rates constant, the functional form of the interest rate process has a significant impact on option values.

The option to expand also corresponds to a deferral option. Both the deferral and expansion options constitute call options providing opportunities to commence or extend business activity. The deferral option perspective is particularly suited to the analysis of irreversible investment commitments.

2.1.3.3 The option to abandon for salvage value

This option is considered if the market conditions decline severely forcing management to abandon current operations permanently and realise the salvage value of capital equipment and other assets.

The abandonment option perspective applies particularly well to retractable investments. Abandonment and contracting options are put options that provide opportunities to withdraw from or scale down business activity. Abandonment options can also be construed as options on options to expand future activity levels, where abandonment corresponds to lapsing the option to expand. When applying these options to practice, the realism of the underlying flexibilities should be carefully evaluated. Realised salvage values rarely match up to the expected values, and subcontractors are usually less than willing to live up to prior indications when environmental conditions are in their disfavour. Therefore, using option models to evaluate this type of capacity adjustment should be treated with caution, because theoretically derived option values might not reflect their true worth at the time of exercise.

2.1.3.4 The option to switch

Margrabe in 1978 valued the option to exchange one non-dividend paying risky asset for another. Both Assets are assumed to be following a stochastic process and are also correlated. Margrabe shows that in the absence of dividends the option to exchange two risky assets is worth more alive than exercised. Future price uncertainty creates a valuable switching option that benefits short-lived projects.

Kulatilaka and Trigeorgis in 1994 study the general flexibility to switch operating modes. They consider manufacturing systems where there are choices on the type of technology and thus varying output levels. They show that the option to switch between technologies has higher worth than operating in one technology.

2.1.3.5 Growth options

Myers in 1977 draws an analogy of future investment opportunities to ordinary call options on financial assets. Kester in 1984 developed this analogy further and surveyed a number of American businesses and concluded that the investment decisions today can create the basis for investment decisions tomorrow.

In capital budgeting and strategic management alike, risk is traditionally perceived as a negative factor. High risk is reflected in high discount rates, which reduce the net present value of expected future cash flows from new business projects. Within the industrial economics tradition that has inspired the predominant strategy paradigm, risk is mirrored in the intensity of the industry's competitive forces. Return is determined by competitive forces arising from industry structure, entry barriers, power of suppliers and buyers, product substitutability, and adherence to generic strategies of cost leadership or

differentiation¹². In this analysis, risk is associated with the level of competitive rivalry among firms in the industry that influences the level of economic returns and profit variability over time. The strategy adopted is that firms should seek to reduce risk by neutralising the competitive forces in the industry. However, risk-induced procedures might play a much more proactive and dynamic role in the firm's strategy process than is generally recognised. This is accentuated by evidence that competition is becoming increasingly dynamic across industries.

2.2 PART B: DEVELOPING THE VALUATION MODEL

The traditional NPV criterion is well suited to analyse immediate deterministic investment opportunities since NPV is the intrinsic value of a real option. However, under conditions of uncertainty, the ability to delay and wait for additional information before making a terminal decision has value. The options approach treats a capital investment decision as an opportunity or a discretionary right to invest.

We will briefly review some of the fundamental concepts used in valuing financial options. Two important analytical models are the Black and Scholes and the Binomial models for pricing standard call and put options on common stock. Even though our ultimate interest lies in the real options, it is important to understand these two analytical models from which the real options are developed.

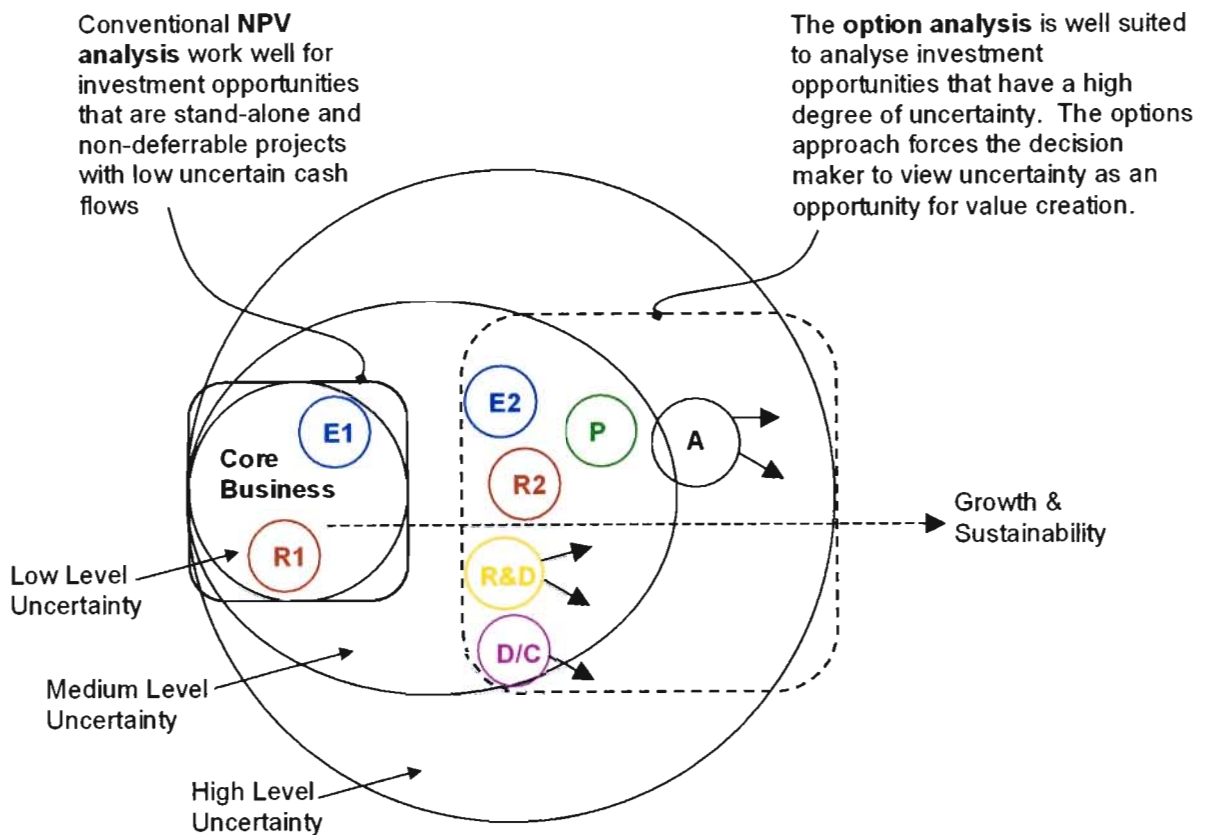
FIGURE 2.1 conceptualises the degrees of uncertainty associated with various investment opportunities. In real options terms, managers are making contingent decisions, decisions to invest and disinvest that depend on unfolding future

¹² Porter, 1980

events. Therefore, the options approach would be most useful for investment situations with medium to high levels of uncertainty.

2.2.1 Black And Scholes Model

In 1973, Black and Scholes changed the study of option-pricing theory with their development of the first equilibrium model based on risk-free arbitrage. With the Black and Scholes model, over a short time interval an investor can replicate the future payoff of the stock option by constructing a portfolio involving the stock and a risk-free asset. The model provides a trading strategy in which the investor cannot profit with a portfolio return equal to the risk-free rate. The Black and Scholes model is a continuous-time model and assumes no transaction costs or differential taxes, a known constant risk-free interest rate, a stochastic diffusion process for stock price and no stock dividends.



R1 - Stand-alone, non-deferrable equipment replacement decisions under low uncertainty.

E1 - Stand-alone, non-deferrable expansion decisions under low uncertainty.

R2 - Equipment replacement decisions under high uncertainty (flexible manufacturing systems).

E2 - Expansion decisions under high uncertainty (international investment opportunities).

A - Mergers and acquisitions.

R&D - Research and development.

D/C - Divest / Abandonment / Contraction.

P - Projects that can be deferred.

Figure 2.1: Investment categories where uncertainty creates most opportunities.

A standard call option gives its holder the right but not the obligation to buy a fixed number of shares at the exercise price (K) on the maturity date.

If stock price is S, the Black and Scholes formula for the price of the call (C) is:

$$C = SN(d_1) - Ke^{-r_f T} N(d_2)$$

where

$$d_1 = \{ \ln(S/K) + [r_f + (s^2/2)]T \} / (s \sqrt{T})$$

$$d_2 = \{ \ln(S/K) + [r_f - (s^2/2)]T \} / (s \sqrt{T}) = d_1 - (s \sqrt{T})$$

$N(\cdot)$ is the standard cumulative normal distribution function;

T is the time to maturity;

r_f is the risk-free rate of return;

σ^2 is the volatility of the stock return.

The model is independent of the expected rate of return or the risk preference of investors. The advantage of the model is that all the input variables are observable except the variance of return, which can easily be estimated from historical stock price data.

2.2.2 Binomial Option-Pricing Model

The binomial model for pricing stock options is a discrete time model; it clearly explains the fundamental economic principle of option valuation by the risk-less arbitrage method. The binomial model provides a good analytical approximation for the movement of the stochastic variable and can be used to value derivative securities when exact formulas for the stochastic process are not readily available. A single-period binomial model to price a call option is shown, which illustrates the risk-free arbitrage principle of valuation. The basic idea is to develop an appropriate hedge portfolio to replicate the future returns on the call.

It will also be shown that the binomial framework is useful for modelling and pricing real options.

In the single-period model, an investor assumes that the stock price S at the end of the period will take one of two values:

uS with probability q or dS with probability $1 - q$.

Let C be the current value of the call option;

C_u and C_d the value of the call at the end of period one if the stock price goes to S_u and dS , respectively.

In the single period model, the call expires one period away, and hence the payoff of the call at the expiration date is $C_u = \max [0, uS - K]$ with probability q and $C_d = \max [0, dS - K]$ with probability $1 - q$.

The single-period binomial lattice for stock price, call and the hedge portfolio is presented in FIGURE 2.2.

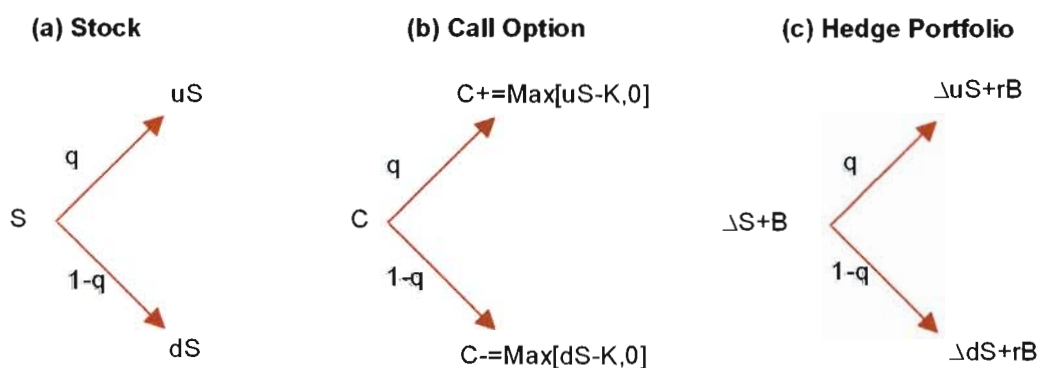


Figure 2.2: One-period binomial lattice for stock, call and hedge portfolio

Assume that an investor can construct a hedge portfolio of stocks and risk-free bonds. For instance one can buy stocks and borrow against them in a proportion

that replicates the future payoff of the call option. Suppose Δ is the number of stocks which the investor needs to buy at price S , B is the amount of funds that can be borrowed at the risk-free rate r_f ; and $r = 1 + r_f$. Notice that for an investor not to make any arbitrage profits, it should be $(u > r > d)$. If $u, d > r$, the investor could make a profit by borrowing and investing in the stock. On the other hand if $u, d < r$, the investor would profit by investing in bonds.

The cost of constructing the hedge at the current time is $\Delta S + B$. The value of this portfolio at the end of one period would be either $\Delta uS + Br$ with probability q or $\Delta dS + Br$ with probability $1 - q$. Since the hedge was selected to replicate the call value at the end of one period: $C_u = \Delta uS + Br$ and $C_d = \Delta dS + Br$. From these two equations, we obtain the following expressions for the values of Δ and B :

$$\Delta = (C_u - C_d) / (u - d)$$

$$B = (uC_d - dC_u) / [(u - d)r]$$

Here Δ is called the hedge ratio because it is the number of shares required to balance the portfolio to exactly replicate the future payoff of the call. The current value of the call cannot be less than the portfolio under the no-arbitrage principle. If $C < \Delta S + B$, then the investor can profit by buying the call and selling the portfolio. Similarly, the current value cannot be greater than the portfolio. The reason is that if $C > \Delta S + B$, the investor would sell the call and buy the portfolio. Thus, in equilibrium, the current value of the option should be exactly equal to the portfolio (i.e., $C = \Delta S + B$). Substituting the values for A and B yields the following exact formula for the price of the call:

$$C = [(C_u - C_d) / (u - d)] + [(uC_d - dC_u) / (r(u - d))]$$

This formula is independent of the probability q , and this value was never used in the risk-free arbitrage pricing method to value the call. Therefore, it does not

really matter what risk preference an investor has. In the binomial approach, the investor can always construct a hedge portfolio and use it with the replication argument to price a call under equilibrium conditions. Another interesting feature is that the model does not indicate how to value the stock, only how to value the option given the value of the stock. Rearranging the formula for the value of the call option, we obtain:

$$C = [p C_u + (1 - p) C_d] / r$$

$$\text{Where } p = (r - d) / (u - d)$$

Because $0 < p < 1$, p can be viewed as a probability, and call value (C) can be interpreted as the expectation taken with respect to risk-neutral probabilities. When the binomial model is used to derive a value for a call option on a stock, the time to maturity is divided into small time intervals Δt to get a better approximation to the Black and Scholes model. The following values are used to develop the multi-period binomial lattice:

$$u = e^{\sigma(\sqrt{\Delta t})}$$

$$d = e^{-\sigma(\sqrt{\Delta t})}$$

$$p = (e^{r\Delta t} - d) / (u - d)$$

Similarly, the pricing formula for a put option using the risk-free arbitrage principle can be obtained. One of the key properties of an option value is that it can never be negative. TABLE 2.1 shows how these variables affect the value of a call and a put option.

Table 2.1: Option Value Variable and their effects on calls and puts

Variable	Effect on Call Option	Effect on Put Option
Increase in stock price (S).	Increases the payoff of the call and hence the call price.	Reduces the payoff of the put and therefore the value of the put.
Increase in exercise price (K).	Reduces the payoff and reduces the call option value.	Increases the payoff and thus the put value.
Greater the time to maturity (T)	Increase the call value due to greater chance of the stock reaching high values	Increases the put value due to greater chance of the stock reaching low values.
Greater volatility of the stock (σ).	Increases the call value due to greater chance of the stock reaching high values	Increases the put value due to greater chance of the stock reaching low values.
Higher interest rate (r).	Higher interest rates lower the present value of the exercise price and hence increases the call option.	Higher interest rate lowers the present value of the exercise price and hence decreases the put value.

2.2.3 A Conceptual Option Framework For Capital Budgeting

It is important to conceptualise how the financial options approach can be used to value flexibility associated with a real investment opportunity. A decision maker with an opportunity to invest in real assets can be viewed as having a right but not an obligation to invest. He or she therefore owns a real option similar to a simple call option on a stock. An investment opportunity can be compared to a call option on the present value of the cash flows arising from that investment (V). The investment outlay that is required to acquire the assets is the exercise price (I). The time to maturity is the time it takes to make the investment decision or time until the opportunity disappears. The analogy between a call option on

stocks and a real option in capital budgeting is shown in FIGURE 2.3. The value of the real option at expiration depends on the asset value and would influence the decision whether to exercise the option. The decision maker would only exercise the real option if to do so were favourable¹³.

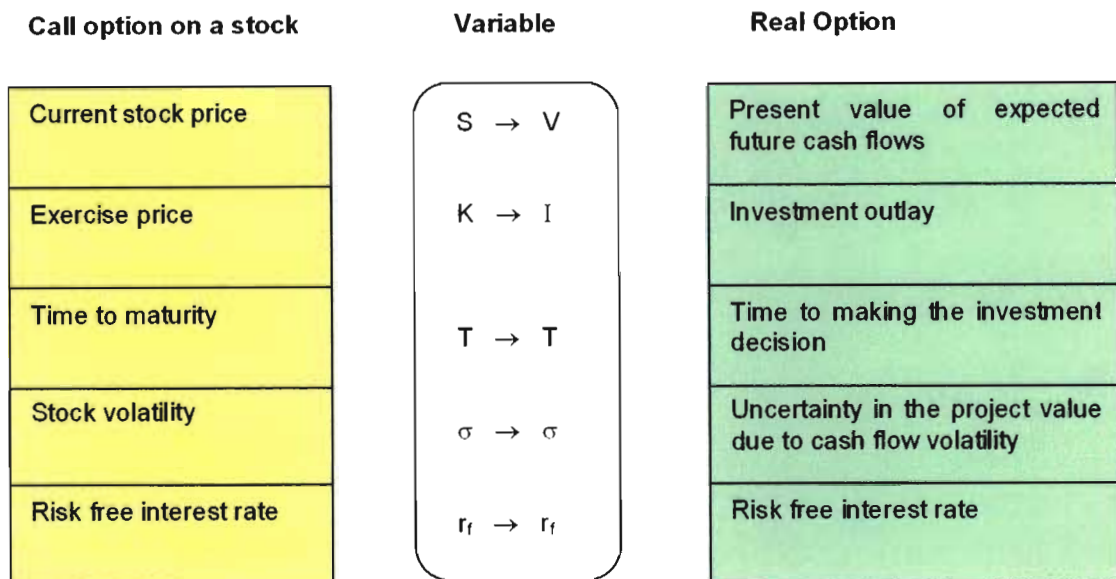


Figure 2.3: Mapping a call option on a stock to a real option

2.2.4 Real Options Analysis: Real Calls And Real Puts

The value of the right to undertake the investment now is (R_{\min}), which is the payoff if the option is exercised immediately. If $V > I$, the payoff is $V - I$, and if $V < I$, the payoff is 0. The true value of the option (R) that one needs to find is the SNPV of the real option. Because one would undertake the investment later only if the outcome were favourable, SNPV is greater than the conventional NPV. The value of flexibility associated with the option to postpone the investment is the difference between the SNPV and the conventional NPV. This is the real option premium (ROP) or the value of flexibility; it is defined as follows:

$$\text{ROP} = \text{SNPV} - \text{Conventional NPV}.$$

¹³ This section is based on Brealey and Myers 1992, Bierman and Smidt 1992, and Trigeorgis 1996.

The concept of a real put option is important from a strategic perspective. A put option provides its owner the right to dispose of an asset when it is favourable to do so. The put works like a guarantee or insurance when things go bad. An early abandonment decision can be viewed as a simple put option. The option to abandon a project early may have value, if an asset has a higher resale value in a secondary market than its use value. The put guarantees that the use value (V) of an asset does not fall below its market value (S). If it does, the option holder would exercise the put. In most instances, it is not possible to make an exact comparison between a standard put option on a stock and a real put option in capital budgeting.

2.2.5 Real Option Valuation Strategies

Suppose a great degree of cash flow uncertainty is associated with an investment. As a result, there may be a good chance that the asset value in a year's time could be much greater than initially estimated. On the other hand, even if the asset value is expected to improve, there is a chance that it could turn out to be much lower. To manage the project's cash flow risk, a decision maker is more likely to consider the following two mutually exclusive investment strategies:

Strategy 1: Invest Now - A decision maker would invest a sum of I today to obtain an asset worth V .

Strategy 2: Invest a Year Later - The decision maker would be willing to pay R for the right to undertake the investment a year later. Assume the investment cost of I is fixed as a result of locking in the price under a contract.

The important question regarding the real option is how much is it worth? Using basic option valuation principles, one can value these two investment strategies and compare which is better. A real option value has two value components.

First, the intrinsic value, which is the minimum payoff that one could obtain if an option is exercised immediately.

Second, a time value that is the extra value over an option's intrinsic value.

The time value relates to the possibility of a favourable movement in asset value as well as discounting. Because the intrinsic value pertains to an immediate action, from a risk management perspective, time value has far more strategic importance in managing risky investments. For this reason, real option valuation is mainly concerned with determining the time value component. The components of an option value are shown in FIGURE 2.4.

Real Option Value = Intrinsic Value + Time Value

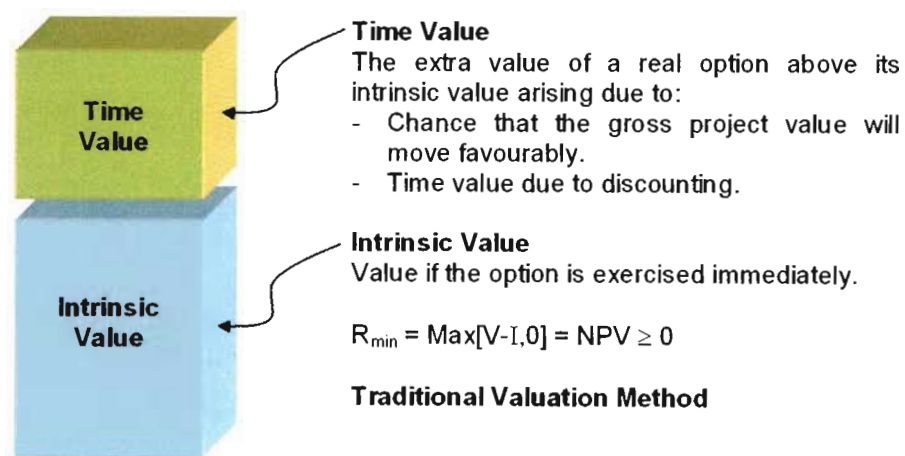


Figure 2.4: Real options value components

The Value of Strategy 1: The value of the right to undertake the investment now is (R_{\min}), which is the payoff if the option is exercised immediately. If $V > I$, the payoff is $V - I$, and if $V < I$, the payoff is 0. Therefore, the intrinsic or theoretical value of the real option is $R_{\min} = \max[0, V - I]$. Under the conventional NPV criterion, one accepts a project if the $\text{NPV} \geq 0$. The intrinsic value of a real option

is therefore simply the conventional NPV criterion that recommends immediate action. The relationship between the conventional NPV criterion and real option value is shown in FIGURE 2.5.

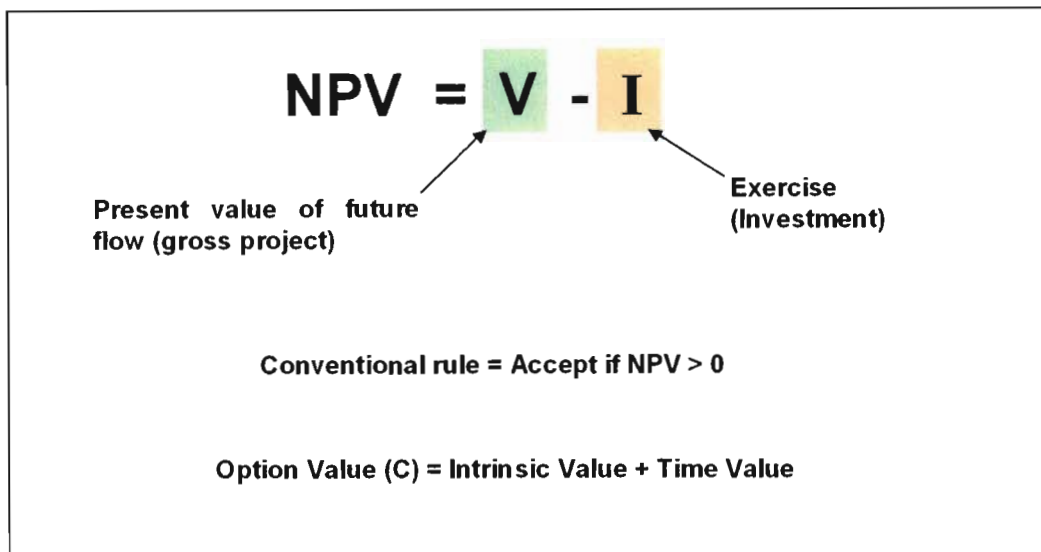


Figure 2.5: The relationship between NPV criterion and Real Option Value.

The Value of Strategy 2: Under the delayed investment strategy, the decision maker would buy a real call option at a price R and deposit the present equivalent amount of I , $PW(I)$ in a risk-free account at r interest per annum. Suppose this strategy would guarantee the investor assets worth V_1 in present value terms.

The cost of this investment strategy is cost of the real option plus the present value of investment deposited in the risk-free account, $R + PW(I)$, and its payoff today is V_1 . The real option would only have value if its cost is at least equal to or greater than the payoff, $R + PW(I) \geq V_1$ when $V_1 > I$.

$$R \geq V_1 - PW(I)$$

To analyse the intrinsic and time value components of a real option, we consider the following three cases:

Case 1: $V_1 = V$

$$R \geq V - PW(I)$$

$$R \geq V - I + I(1 - e^{-rt})$$

$$R \geq R_{\min} + I(1 - e^{-rt})$$

Case II: $V_1 > V^* > V$

$$R \geq V - PW(I)$$

$$R \geq V^* - PW(I)$$

$$R \geq V + \Delta V - I + I(1 - e^{-rt})$$

$$R \geq R_{\min} + \Delta V + I(1 - e^{-rt})$$

Because $R \geq R_{\min}$, we know that both terms $I(1 - e^{-rt})$ and $\Delta V + I(1 - e^{-rt})$ represent the time value elements, respectively. Under either Case I or Case II, a decision maker would exercise the real option at maturity because it is deep in the money.

Case III: $V_1 < V < PW(I)$

In this instance, a decision maker would not exercise the real option because it is not in the money; thus the option would expire worthless, and the loss would be equal to the cost of the option.

As demonstrated above, a delayed investment strategy is better than a now-or-never investment strategy because a delayed investment strategy guarantees the amount I from the risk-free investment. On the other hand, one would incur a loss if Strategy 1 were implemented and the asset value also turned out to be unfavourable. The conventional NPV method is suited for investment opportunities when full information is available. It is appropriate for rational, well-informed immediate investment decisions. However, under conditions of

uncertainty, the real options framework is a better risk-management tool because arrival of any new information over time changes the intrinsic value.

2.2.6 Merging Decision Analysis And Real Options

Certainly, it may be incorrect to view risk-free arbitrage valuation as the only criterion to value real options in all capital budgeting situations. The opportunity loss function approach is an alternative valuation method when such a risk-free arbitrage assumption does not hold. To decide whether to delay an investment under uncertainty, it is assumed that the random variable of interest is the present value of future cash flows (V). The two acts are defined as (a_1 : do not delay, a_2 : delay for now). The payoff $R(a, \theta)$ for each of the two acts is linear in terms of the state of the world (θ). The investment cost I , is assumed to be constant. Because we have assumed a linear payoff, it is equivalent to assuming a risk-neutral utility function, and hence we use the risk-free rate for discounting cash flows.

From an option perspective, we see that the two acts of a typical investment problem under uncertainty are simply a call and a put option (or similar to the put-call parity concept of stock option pricing¹⁴. For example, the loss function for act a_1 is similar to a call option to delay a_2 when information can be obtained by delaying the investment. The decision to invest (no delay) in this case is analogous to a call option with an exercise price of I on the present value of the future cash flows V with a maturity of one year. The theoretical value of a real option is given by $R_{\min} = \max [V - I, 0]$.

This is exactly the opportunity loss of act a_1 . On the other hand, the loss function of act a_2 is simply the current optimal decision not to invest (delay) and can be viewed as a put. The price of a real option on one investment under uncertainty

¹⁴ Merton and Mason 1985

can be computed by finding the present value of the expected loss functions using the risk-free discount rate as in the risk-neutral valuation method¹⁵.

For example, one can write an expression of the two loss functions of the two acts (a_1 : exercise, a_2 : do not exercise). These loss function formulas correspond exactly to the expressions for the value of the real option and a put option arising from the two terminal acts if risk-neutral valuation were used. The expected opportunity loss of act a_1 is as follows and its expression can be viewed as the value of the call option to invest $C(a_2) = L(a_1, \theta)$:

$$L(a_1, \theta) = \int_{-\infty}^I (0) f(V) dV + \int_{-\infty}^I (V-I) f(V) dV$$

On the other hand, the expected opportunity loss of act a_2 is:

$$L(a_2, \theta) = \int_{-\infty}^I (I-V) f(V) dV + \int_{-\infty}^I (0) f(V) dV$$

Which can be compared to the value of a put option not to invest $P(a_1) = L(a_2, \theta)$. The expected value of perfect information (EVPI) is the smaller of the expected losses of the two actions or the expected loss of the optimal terminal act. Therefore, the value of the investment opportunity including the real option to invest is the EVPI given by:

$$OV = \int_{-\infty}^I (V-I) f(V) dV$$

Which is identical in expression to the pricing formula of a call option under the risk-neutral valuation method when all values are expressed in present values.

¹⁵ Herath and Park

2.2.7 The Basic Option Valuation Model: Opportunity To Replicate

When there is an opportunity to replicate, the basic option valuation model is based on the assumption that a firm can sample a unit of investment to obtain information that will resolve uncertainty regarding the other units. The option value is interpreted as the benefit from taking advantage of more information before making decisions. We use the expected opportunity loss to determine the option value that includes the flexibility to invest sequentially by delaying investments and undertaking other investments only after sampling.

The value of the option and the ROP in this approach is derived from the value of perfect information. An option's value increases with greater uncertainty and decreases when uncertainty is resolved. We use this idea to develop the sequential option revision concept and combine it with the decision-making process. In traditional decision theory methods, including Bayesian methods, uncertainty reduction has been considered to be important. However, from an options approach, both reduction and greater uncertainty are important. The two situations that are likely to arise with respect to these classes of problems are analysed separately. The first occurs when undertaking a single unit provides perfect information regarding all other identical units. The second situation is more interesting and important from both the decision theory and an options approach. It occurs when undertaking a single unit provides imperfect information.

The basic model in the options approach is based on the decision model for investment problems with opportunity to replicate¹⁶. It is presented in FIGURE 2.5. To analyse the relationship between information and real option value when there is the opportunity to replicate, the following assumptions are made. The opportunity to invest in a single unit, or the real option, is compared to a call option on a non-dividend-paying common stock. The variable of interest or the

¹⁶ Bierman and Rao 1978.

asset on which the option value is derived is the value of V (gross project value of a single investment). We also assume V to be a continuous distribution with mean (m') and standard deviation (σ'). The exercise price is assumed to be the fixed investment cost of a single unit I . Notice I is the breakeven value of V , $V_b = I$. The time to maturity of the real option is the time until making the investment decision for all units. Assume that c identical units are to be undertaken.

The opportunity loss from not undertaking the c units from an options approach is similar to the single investment situation, the only difference being the slope of the payoff and loss functions. Assuming that after undertaking one unit one plans to undertake the remaining $c - 1$ identical units, then the slope of the loss function for all units is c . FIGURE 2.6 shows the conditional loss function for undertaking all c units. Certainly, with $E(V) = m' < I$, conventional DCF wisdom tells us not to invest.

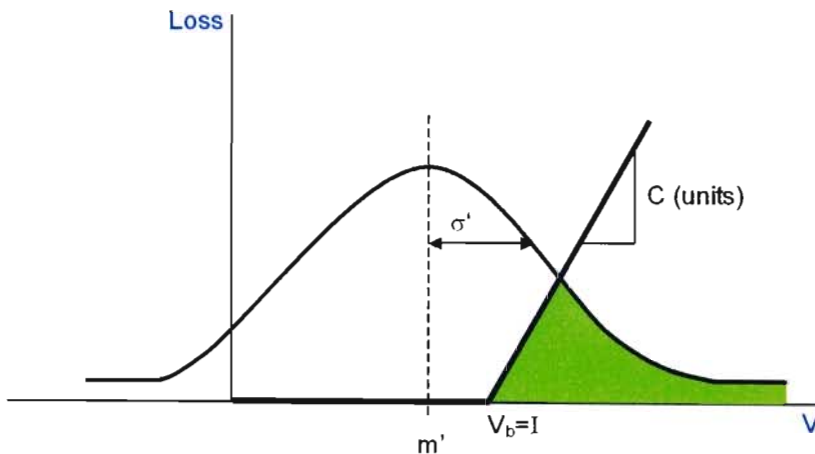


Figure 2.6: Opportunity loss for all units of investment

Undertaking a Single Unit Provides Perfect Information

Suppose investing in a single unit provides perfect information about the outcome of all identical units.

Then the real option value that includes the opportunity to invest in all c units, or OV , is given for any assumed distribution of V by:

$$OV = \int_I^{\infty} C(V-I) f(V) dV$$

Because the NPV of a single unit is negative, it is also equal to the SNPV. The value of c is the feasible number of identical units that can be undertaken. The cost of the option is equivalent to the cost associated with obtaining perfect information. It is the expected loss of one unit when after undertaking the single unit, the outcome V is less than I , and is evaluated as

$$E(L_0) = \int_{-\infty}^I (I-V) f(V) dV$$

In the real options analysis, the ROP is computed as the difference between the SNPV, which includes the value of flexibility (to invest gradually), and the NPV based on a conventional analysis. In this case, because the NPV of a single unit is negative, the ROP is greater than the OV . If we define the expected value of profit with perfect information as $EVPP$, then we can say $OV = EVPP = EVPI = SNPV$. Therefore,

$$ROP = SNPV - NPV.$$

The schematic representation of OV , $SNPV$, NPV and the ROP are shown in FIGURE 2.7. The ROP before taking the sample includes

- (2) The option value associated with the flexibility to delay investments and to only undertake them after resolving uncertainty.
- (3) The loss of a single unit if an immediate decision is made.

The prior option value is large because the uncertainty associated with investing in c units is greater for an incorrect decision. Thus, holding an option now to

invest later is more valuable than not undertaking the investment at all on an expected value basis.

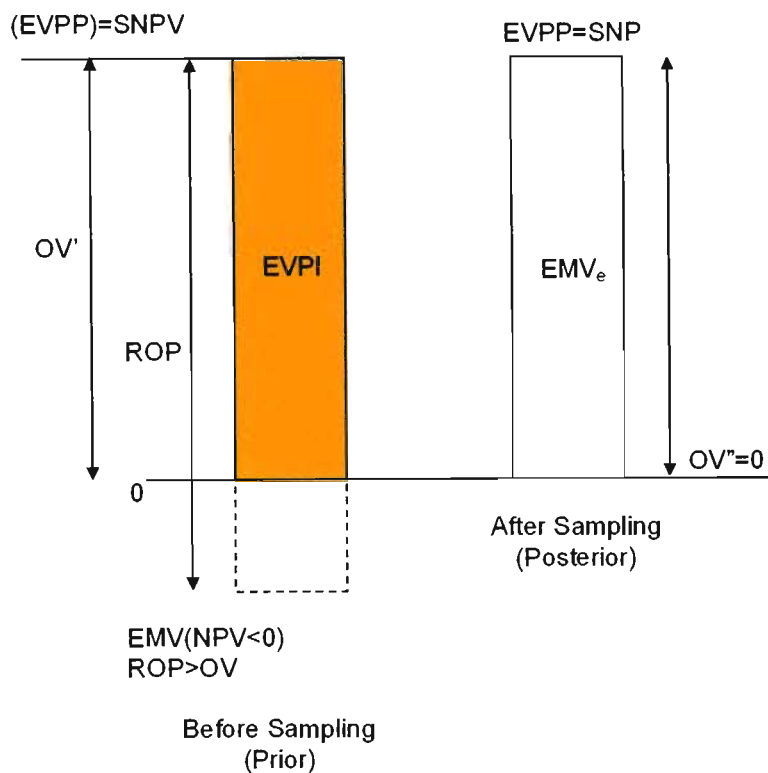


Figure 2.7: Prior and Posterior Option Values - Situation 1.

Suppose that one unit can be sampled and perfect information can be obtained about the outcome of all other units. Then the value of the real option posterior to the sample is zero because all uncertainty is completely resolved. In this case, a decision maker must evaluate whether the (OV') is greater than the cost of obtaining that information, which is the expected loss of one unit $E(L_0)$. If so, a decision maker should sample a single unit, obtain perfect information that will resolve uncertainty completely, and convert the remaining $c - 1$ units, because the posterior option value (OV'') is zero.

Undertaking a Single Unit Provides Imperfect Information

The more interesting and important situation occurs when undertaking a single unit does not provide perfect information. This section will discuss how the options approach can be combined within a Bayesian framework when there is the opportunity to resolve uncertainty via sampling. The option value is directly proportional to the level of uncertainty that needs to be resolved. In other words, the option value will be higher whenever more uncertainty remains to be resolved. When much of the uncertainty is not resolved, a decision maker has greater flexibility by holding an option rather than exercising it. The schematic representation when investing in a single unit provides imperfect information is shown in FIGURE 2.8. The posterior option value is equivalent to the EVPI after taking a sample. In other words,

$$OV'' = SNPV = EMV_e = EVPI_e$$

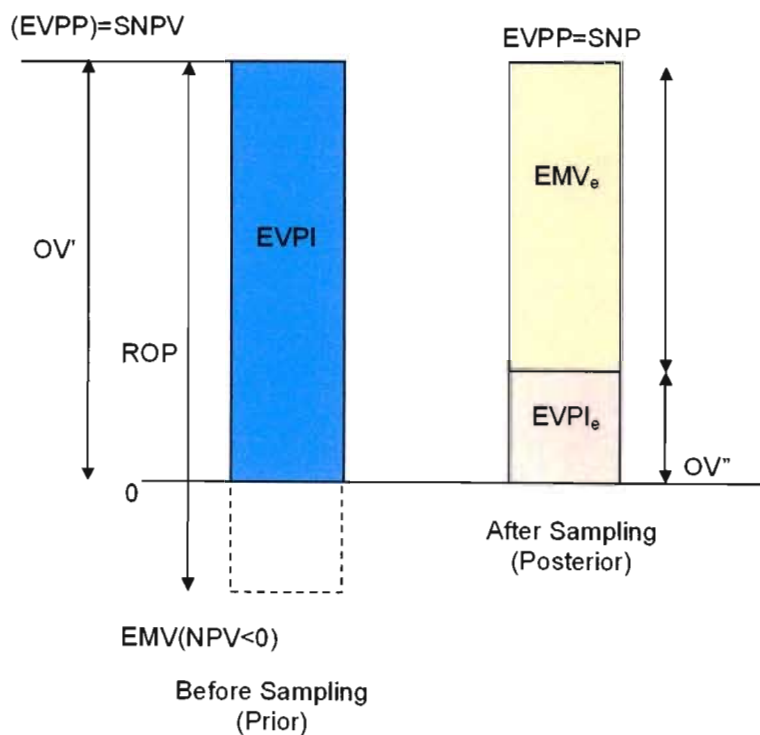


Figure 2.8: Prior and Posterior Option Values - Situation 2.

2.2.8 Real Options Approach To Engineering Economic Decisions

Engineering economic problems can be broadly categorized into five groups¹⁷:

- (1) Equipment and process selection.
- (2) Equipment replacement.
- (3) New product and product expansion.
- (4) Cost reduction.
- (5) Service improvement.

While both time and risk are fundamental to all investments in these five categories, the strategic nature and the degree of uncertainty can vary considerably. For example, cost reduction and service improvement decisions have low uncertainty, because cost and revenue information can be estimated with relative ease. The traditional DCF approach would work well in these situations. Engineering projects in the other three categories have greater strategic values, because they commonly include research and development (R&D) investments for new product design and development, expansion into new geographical areas, strategic decisions that would change the nature of business, capacity expansion decisions under highly uncertain markets, investment in flexible manufacturing systems (product and process flexibility), ability to delay and wait for more information, and sequential investment decisions with/without replication.

When a decision can be delayed, or there is the possibility to obtain additional information, or there is an opportunity to replicate, we may use a Bayesian decision framework¹⁸. While the traditional tools (such as decision tree analysis or Monte Carlo simulation) are useful in valuing investment decisions under uncertainty, the options approach is better suited for several reasons. First, it is associated with a great deal of uncertainty. Examples in these three categories

¹⁷ Park 1997

¹⁸ Bierman and Rao 1978, Prueitt and Park 1990

enables a decision maker to capture future opportunities by lowering the chance of overlooking them. Second, options involve choices rather than immediate commitments. Also, they enable decision makers to view uncertainty as an opportunity to create value. This options mindset is critical for both strategic reasons and shareholder value creation.

2.3 SUMMARY

The standard technique that is presently used in capital budgeting is the net present value model, which involves discounting a project's cash flows and subtracting the cost of the investment from the present value. If the result is positive, this indicates that the project is feasible.

Its main limitation of this model is its total neglect of the stochastic nature of output prices and of possible managerial responses to price variations. Also capital budgeting should support investment initiatives emerging in different parts of the firm, while resource commitments with overall corporate effects should be considered at the executive level.

Real options analysis is a new way of thinking about corporate investment decisions. It is based on the premise that any corporate decision to invest or divest real assets is simply an option. It gives the option holder a right to make an investment without any obligation to act. The decision-maker, therefore, has more flexibility, and the value of this operating flexibility should be taken into consideration. Therefore one can take advantage of the options approach in modelling capital decision problems.

This chapter goes on to develop the real options model. The Black and Scholes model for option pricing sets the foundation for this model. It starts off by mapping a call option of a stock to a real option on a project, established the real options

model and finally looks at the different real option strategies that are available to management.

CHAPTER 3: ANALYSIS OF THE VALUATION MODEL

3.1 INTRODUCTION

In understanding the various valuation models as discussed in Chapter 2, it adds value that the research relate the models to functional examples in order for the framework to be set. To this end, Chapter 3 states some of the case studies as compared to conventional approaches to provide the reader with a better understanding of the Real Options approach.

3.2 CASE STUDY 1: AN OPTION TO ABANDON EARLY

A manufacturer is considering purchasing a general-purpose machine costing R102,000 to expand its current metal fabrication operations. The market for metal products is highly volatile. A year from now, one of the following outcomes is anticipated in the marketplace: (1) A highly favourable market with a probability of 40% generating additional cash flows of R150,000; and (2) a very unfavourable market with a probability of 60% with cash flows of R90,000. If the market turns out to be unfavourable, the manufacturer has an option to dispose of the general-purpose machine for a resale value of R95,000 to another manufacturer. Assume that the discount rate is 14% and the risk-free rate is 8%.

3.2.1 Conventional Approach

If the conventional analysis is done, the decision analyst would obtain an

$$\begin{aligned} \text{NPV} &= -R102 + [R150(0.4) + R90(0.6)]/1.14 \\ &= -R2 \text{ (in R000)} \end{aligned}$$

With a negative NPV the decision would be no-go. A more careful decision analyst would in fact consider a strategy of abandoning the machine early, thereby including this possibility as a part of the analysis. The strategic NPV including the abandonment opportunity would be

$$\begin{aligned}\text{SNPV} &= -R102 + [R150(0.4) + R95(0.6)]/1.14 \\ &= R0.64\end{aligned}$$

The value of the abandonment opportunity therefore would be
 $\text{SNPV} - \text{NPV} = \2.64 .

Certainly, we could also view the value of the abandonment opportunity to be
 $(R95 - R90)(0.6)/1.14 = R2.63$.

In the above calculations, we simply used a decision-tree approach to layout the problem and to consider all the complexities in a single strategic decision.

3.2.2 Real Options Approach

Now consider how different this analysis would be if one applies option-pricing theory and risk-free arbitrage methods to value the early abandonment decision. The opportunity to abandon one machine is a simple European put option in which the use value is defined as an exercise price equal to the market value. Because there are only two future outcomes for the use value in Year one, one can use the single-period binomial model and value the put option. The probabilities associated with the future market outcomes are not required, because one can develop a perfect hedge to replicate the future payoff of the put option. The data (in R000) for the binomial model are:

PW of the future cash flows $V = R100$;
 use value if the market is favourable $= V^+ = R150$;
 use value if the market is unfavourable $= V^- = R90$;
 exercise price $I = R95$;
 $1 + \text{risk-free rate} = 1 + 0.08 = 1.08$.

Using the binomial model, we compute the following:

$$P^+ = \max[0, I - V^+] = \max [0, 95 - 150] = 0$$

$$P^- = \max[0, I - V^-] = \max [0, 95 - 90] = 5$$

$$u = 150/100 = 1.5,$$

$$d = 90/100 = 0.9,$$

$$p = (1.08 - 0.9)/(1.5) - 0.9 = 0.3,$$

$$1 - p = 0.7$$

$$\begin{aligned} P &= [pP^+ + (1 - p)P^-]/(1+r) \\ &= [(0.3)(0) + (0.7)(5)]/1.08 \\ &= 3.24. \end{aligned}$$

Based upon the binomial model, the put option value to abandon the machine in Year 1 is R3.24. Therefore, the strategic NPV with the option to abandon in Year 1 based on the risk-free arbitrage pricing is

$$\begin{aligned} \text{SNPV} &= \text{NPV} + \text{abandonment option value:} \\ &= -R2 + R3.24 \\ &= R1.24 \end{aligned}$$

Based on this valuation method, the decision would be to purchase the milling machine. The binomial lattice for Example 1 is shown below.

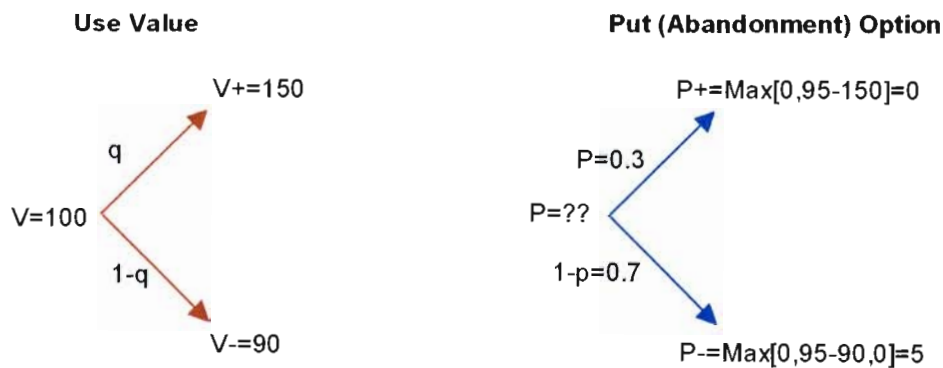


Figure 3.1: The binomial lattice for Example 1.

3.2.3 Summary

Notice that the value obtained by the risk-free arbitrage method is higher than the one obtained with the decision-tree approach. The reason is that the risk-free rate was used in the binomial approach. In this example, the risk-free arbitrage valuation was possible because one could construct a perfect hedge. In practice, however, one may not be easily able to construct a perfect hedge to replicate the future payoff. When risk-free arbitrage is possible, a decision maker does not need to determine the appropriate risk-adjusted discount rate and does not need to estimate subjective probabilities. The problem of selecting the appropriate discount rate disappears. Notice that if certainty equivalent payoffs were found and the strategic NPV was recomputed based on the decision-tree approach using the risk-free rate, economic results would be equivalent to the binomial model.

3.3 CASE STUDY 2: OPTION TO DELAY AN INVESTMENT

A company is planning to undertake an investment of R2 million to upgrade an existing product for an emerging market. The market is very volatile, but the company owns a product patent that will protect it from competitive entry until the next year. Due to the uncertainty of the demand for the upgraded product, there is a chance that the market would be in favour of the company. The present value of expected future cash flows is estimated to be R1.9 million. Assume a risk-free interest rate of 8% and a standard deviation of 40% per annum for the PW of future cash flows.

3.3.1 Conventional Approach

If the traditional NPV criteria were used, the decision would be not to undertake the investment because it has a negative NPV of R0.1 million.

3.3.2 Real Options Approach

From an options approach, the investment will be treated as an opportunity to wait, and then to undertake the investment a year later if events are favourable. Assume that one can use the Black and Scholes formula to value the opportunity to undertake this investment in Year 1. The opportunity to wait can be considered as a European call option on the present value of the future cash flows $V = R1.9$ million, with an exercise price of $I = R2$ million expiring one year from now. Because $r_f = 8\%$ and $\sigma = 40\%$, using the Black and Scholes formula one obtains:

$$d_1 = 0.4718, d_2 = 0.0718, N(d_1) = 0.68082, N(d_2) = 0.5279.$$

The value of the real option is therefore:

$$\begin{aligned} \text{SNPV} &= 1.9(0.68082) - 2e^{-(0.08)}(0.5279) \\ &= R0.318 \text{ million} \end{aligned}$$

$$\begin{aligned} \text{ROP} &= 0.318 - (-0.1) \\ &= R0.418 \text{ million.} \end{aligned}$$

3.3.3 Summary

The ability to wait gives a decision maker a strategic NPV of R0.318 million instead of a negative NPV of R0.1 million. If the market for the product turns out to be favourable, the company will exercise the real option in the money and undertake the investment in Year 1. On the other hand, if the market a year later turns out to be unfavourable, the decision would be not to undertake the investment. The opportunity cost is only R0.318 million compared to the actual investment cost of R1.9 million.

3.4 CASE STUDY 3: INVESTMENT WITH SAMPLING

In converting a job shop operation to a flexible cell manufacturing operation, we often must decide whether to convert the whole factory or to convert gradually. Normally, it costs less to convert the whole factory at one time than to convert

partially over time. However, the gradual conversion is a less risky investment because we can dictate the level of automation as we see fit. Moreover, the gradual automation can serve as an investment sampling process; the investment (automating the whole factory), although initially desirable, may appear unacceptable after obtaining additional information and being able to make sequential decisions.

To structure this decision problem, consider a situation in which six identical manufacturing cells need to be converted. Assume that we can first convert one cell and assess how economical it is before converting the remaining cells. After observing the initial results of the single-cell conversion, we could pursue the following investment strategies:

- (1) Stop all together any further conversions.
- (2) Convert the remaining five cells.

In fact, there could be a lot more options as we continue to try more cells before converting the remaining cells. For this illustration, only the first two options will be considered.

3.4.1 Case 1: Undertaking a single cell provides perfect information

3.4.1.1 Conventional Decision- Tree Analysis

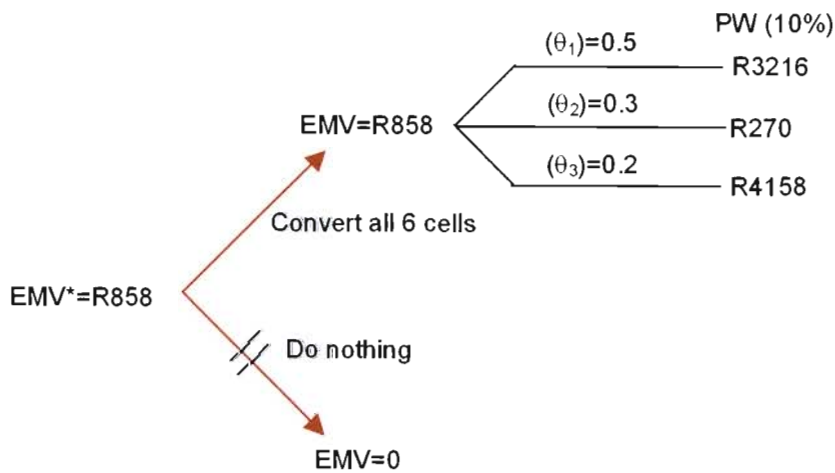
A preliminary analysis indicates the estimated incremental revenues over a 10-year planning horizon affected by each of the three assumed classes (θ_1 , θ_2 , and θ_3) of cell performance as shown in TABLE 3.1. The cell conversion costs are estimated to be R1000 per cell, and the assumed discount rate is 10%.

Table 3.1: Financial data for just converting one cell.

Outcome	Probability	Annual Returns	NPV
Excellent (θ_1)	0.5	R250	$-R1000+R250(P/A,10\%,10) = R536$
Fair (θ_2)	0.3	R170	$-R1000+R170(P/A,10\%,10) = R45$
Poor (θ_3)	0.2	R50	$-R1000+R50(P/A,10\%,10) = R693$

The expected NPV and the variance of NPV at an interest rate of 10% over the planned economic life of 10 years are given by, $E[\text{NPV}] = R143 > 0$ and $\text{Var}[\text{NPV}] = 469$.

With $E[\text{NPV}] > 0$ for a single unit, we may attempt to convert all 6 cells immediately. The resulting total expected NPV of converting all 6 units amounts to R858, which is the expected monetary value (EMV) of the optimal decision, as shown in FIGURE 3.2.

**Figure 3.2: Decision tree for conventional analysis**

The decision branch that is not crossed out refers to the optimal decision. Notice, however, that the total variance associated with the conversion of all 6 cells is $\text{Var}(6X) = 36\text{Var}(\text{NPV}) = 36(496) = 2814$. Although the $E[\text{NPV}]$ of a single cell is positive, there is still a 20% probability that cell performance will be poor, with each unit losing R693 and totalling to R4158 if all 6 cells are converted. On the other hand, there is a 30% probability that each cell would only yield a marginal

NPV of R45, or a total of R270 for the fair case, and a 50% probability that each cell would yield an NPV of R536 or a total of R3216 for the best case.

3.4.1.2 Real Options Approach

Under the suggested options approach, a more plausible decision would be to delay the immediate cell conversion, even though $E[\text{NPV}] > 0$. The decision to invest should be made after resolving uncertainty rather than first investing and then finding out what happens. Certainly, we would undertake the investment if we could resolve most of the uncertainty with the sample. Therefore, we would exercise the option, if the real option were sufficiently in the money. If the real option were not in the money, the option would go unexercised. This creates an asymmetry in the payoffs resulting in a strategic NPV (SNPV) value that includes a premium over the conventional NPV for the value of flexibility.

The gross project value can be viewed as the present value of future cash flows V . The value for V is shown in TABLE 3.2 for each of the three levels of cell performance outcome. The exercise price is the investment cost of a single cell $I = R1,000$. In the options approach the 6-cell investment is undertaken only after resolving uncertainty.

Table 3.2: Gross Project Values.

Outcome	Probability	V
Excellent (θ_1)	0.5	R1536
Fair (θ_2)	0.3	R1045
Poor (θ_3)	0.2	R307

In other words, the option will be exercised only if the real option to invest is sufficiently in the money. This happens only if the sample cell performance turns out to be either excellent or fair.

If cell performance is excellent, the real option value at the exercise date is

$$\begin{aligned}\max[6(V - I), 0] &= \max[6(R1536 - R1000), 0] \\ &= R3216.\end{aligned}$$

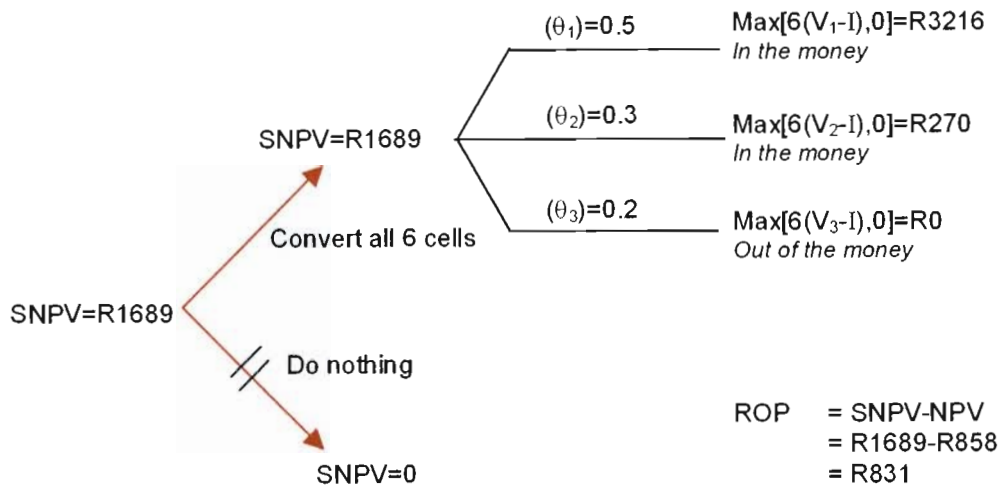
Similarly with fair performance, the real option value is

$$\begin{aligned}\max[6(V - I), 0] &= \max[6(R1045 - R1000), 0] \\ &= R270.\end{aligned}$$

With poor performance, the real option has no value and we do not exercise the option,

$$\begin{aligned}\max[6(V - I), 0] &= \max[6(R307 - R1000), 0] \\ &= 0.\end{aligned}$$

This is presented in FIGURE 3.3. The SNPV, which includes the option to delay the investment, is $\text{SNPV} = R1689$. The SNPV includes an ROP, or $\text{ROP} = \text{SNPV} - \text{NPV} = R832$. Because the NPV of a single unit is positive, it is equal to OV. This is also the value associated with the option to delay the conversion of 6 cells and to undertake the investment only when cell performance outcomes are favourable.



SNPV is the NPV that includes the opportunity to invest in 6 cells after resolving uncertainty. Real option to invest in 6 cells is exercised only if the option is in the money (when payoffs are either positive or zero).

Figure 3.3: The options approach

3.4.2 Case 2: Undertaking a single cell provides imperfect information

Most of the information we can obtain is imperfect in the sense that it will not tell us exactly which event will occur. Such imperfect information may have value if it improves the chances of making a correct decision, that is, it improves the expected monetary value. The problem is whether the reduced uncertainty is valuable enough to offset its cost.

Suppose that a single cell conversion is equivalent to bringing in a technical expert as a consultant on the cell conversion described above. The expert will charge a fee to provide the company with a report that the cell performance is excellent, fair and poor. The expert is not infallible but can provide a valuable technical opinion. From experience, management estimates that, when the actual state of cell performance is excellent, the expert predicts an excellent condition with probability 0.80, a fair condition with probability 0.15, and a poor condition with probability 0.05. Such probabilities would reflect the expert's experience,

modified perhaps by management's judgment. The probabilities shown in TABLE 3.3 express the reliability or accuracy of an expert opinion of this kind.

Table 3.3: Conditional Probabilities.

Observed event from sample	Actual State		
	(θ_1)	(θ_2)	(θ_3)
Conversion seems to be excellent (θ_1)	0.80	0.20	0.10
Conversion seems to be fair (θ_2)	0.15	0.60	0.20
Conversion seems to be poor (θ_3)	0.05	0.20	0.70

To construct the revised probabilities, we need to construct a joint probability table. In TABLE 3.1, we have the original probabilities assessed by management before taking any sample. From these and the conditional probabilities in TABLE 3.3, we can calculate the joint probabilities of TABLE 3.4. Then we obtain the marginal probabilities of future cell performance conditions in TABLE 3.4 by summing the values across the columns.

Table 3.4: Marginal Probabilities.

Joint Probabilities	Sample Prediction Outcome			Marginal Probabilities
	Excellent	Fair	Poor	
(θ_1)	0.400	0.075	0.025	0.50
(θ_2)	0.060	0.180	0.060	0.30
(θ_3)	0.480	0.040	0.140	0.20
Marginal Probabilities	0.480	0.295	0.225	1.00

After receiving the sample information, we need to calculate the probabilities for the branches labelled "Superior," "Fair," and "Poor" in TABLE 3.5.

Table 3.5: Revised Probabilities.

Conditional Outcome	Posterior Probabilities for a given survey outcome		
	Superior	Fair	Poor
(θ_1)	0.833	0.254	0.111
(θ_2)	0.125	0.610	0.267
(θ_3)	0.042	0.136	0.622

3.4.2.1 Conventional Decision Tree Analysis

As shown in FIGURE 3.4, the expected monetary value for the optimal decision after sampling one cell is $EMV_e = R1162$. The expected value corresponding to these two favourable outcomes is $(0.50)(2972) + (0.30)(250) = R1561$. If the cell conversion were to be poor, we would limit the loss to a single cell. The expected cost of information is $(0.20)(693) = R139$. Because the expected value of $R1561$ exceeds the expected cost of $R139$, the Bayesian approach implies undertaking a single-cell conversion to obtain perfect information. Now consider the situation when sampling leads to imperfect information. The delayed investment strategy with gradual conversion results in an expected value of $R1162$, which is $R304$ more than when all units are immediately converted as recommended by the conventional NPV criterion. Hence, even when perfect information is not available, the gradual conversion strategy turns out to be a better investment strategy under uncertainty.

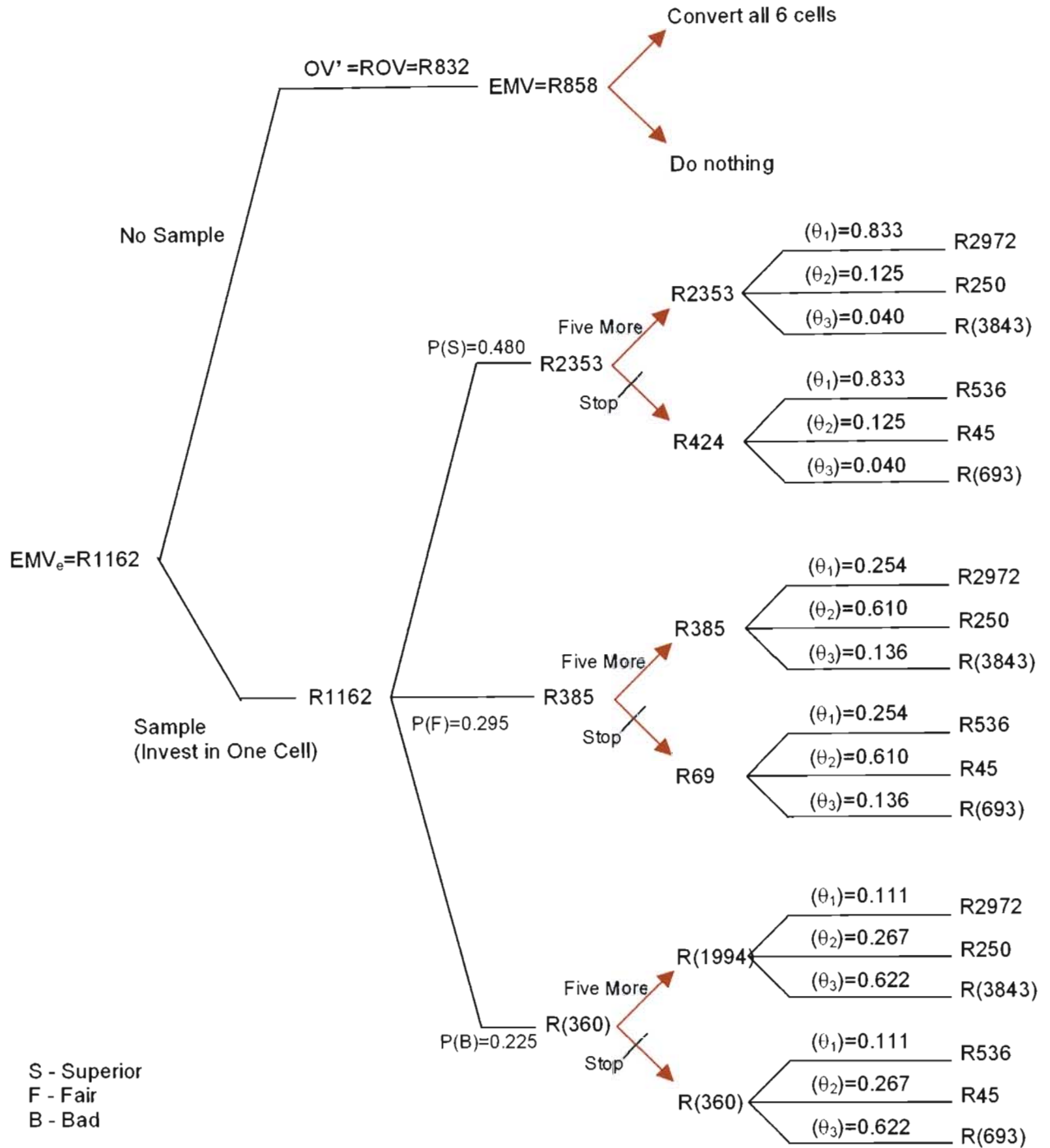


Figure 3.4: The decision tree - Imperfect information

3.4.2.2 Real Options Approach

With the EVPI being R832 and the maximum cost of converting a cell being R693, one would convert one cell first to obtain perfect information about other cells. A firm would convert the other five cells a year later only if cell performance were excellent or fair.

The posterior option value is

$$\begin{aligned} OV'' &= SNPV - EMV_e \\ &= R1689 - R1162 \\ &= R527 \end{aligned}$$

This value has now reduced due to sampling one unit. With the change in the posterior option value, the decision maker can exercise the real option at this time if the decision maker believes that uncertainty has been adequately resolved. If not, further sampling is required. This concept is illustrated in FIGURE 3.5.

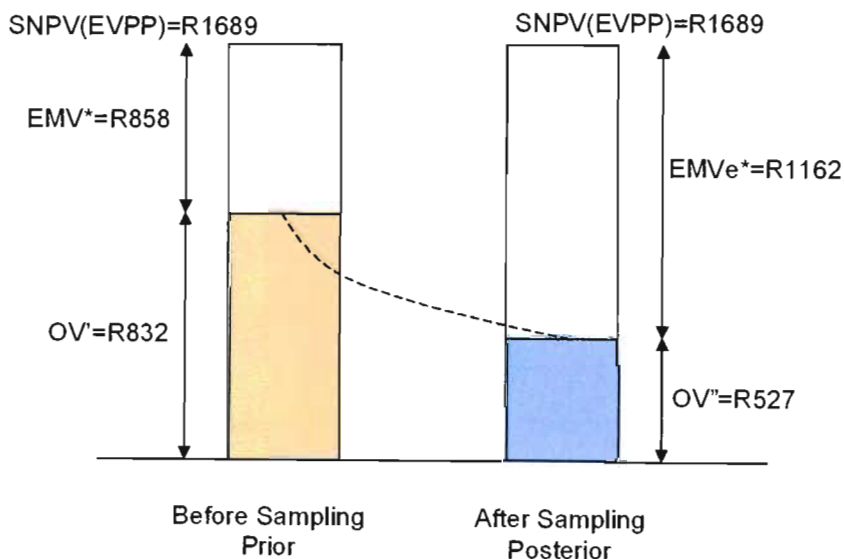


Figure 3.5: Option values, NPV and SNPV - Imperfect information

Suppose, after obtaining the sample, a decision maker finds that the posterior option value has in fact increased. Then it is better to hold an option to invest later. Therefore, under the options approach, sampling one cell before converting the remaining cells is a better investment strategy than the conventional all or nothing approaches.

CHAPTER 4: RESEARCH FINDINGS

4.1 INTRODUCTION

This research started with the management dilemma that there were too few projects that when evaluated yielded positive net present values.

Options perspectives in the strategy-related literature have typically focused on initial investments through joint ventures¹⁹, entry through collaborative venturing²⁰, and sequential market entries²¹. The options perspective is deemed particularly helpful to consider investments in research ventures or new markets as ways to obtain cheap options on new business activities. Collaborative joint ventures and incremental market investments represent ways to limit the resource commitments to activities that might develop future business opportunities.

4.2 REVIEW OF CASE STUDY 1

The first case study analysed in chapter 3 looked at a project that had uncertain cash flows, therefore probabilities were associated with the cash flows. Using the conventional net present value method a negative return was calculated. The option to abandon the asset during the project life cycle was then considered and it was determined that this added value to the investment, and would yield a positive return.

The NPV of an investment obtained using the options framework to capture strategic concerns is called the strategic net present value (SNPV). In the first case study, the SNPV was calculated to be positive. The value obtained

¹⁹ Kogut, 1991; Hurry et al., 1992

²⁰ Chi & McGuire, 1996

²¹ Chang, 1995, 1996

however by the real options approach was higher than the one obtained with the decision-tree approach.

4.3 REVIEW OF CASE STUDY 2

The second case study looks at the option to defer an investment for one year. The ability to wait gives the decision maker a positive return instead of the negative return calculated by the NPV method. The option to delay the investment allows the company to assess the market in a year's time. If the market for the product turns out to be favourable, the company will exercise the real option in the money and undertake the investment. On the other hand, if the market a year later turns out to be unfavourable, the decision would be not to undertake the investment. As most investments in specific business activities represent non-recoverable costs, the opportunity cost consideration applies to investments associated with the exercise of strategic options.

The investment abandonment and deferral perspectives have two major applications in strategic decision analysis. The abandonment option perspective applies particularly well to retractable investments. The deferral option perspective is particularly suited to the analysis of irreversible investment commitments. Research and development investments can usually be abandoned. The investments constitute sequential premiums paid to establish strategic options that eventually can establish alternative routes to future business expansion. Irreversible investment decisions refer to the subsequent resource commitments on real and intangible assets, e.g. production plants, sales outlets, training, market promotion, etc. associated with exercise of existing strategic options.

As shown by the case studies, the consideration of abandonment opportunities adds flexibility to the firm's initial development of strategic options. It allows the firm to opt out of the project if circumstances develop unfavourably. By making

relatively small resource commitments at the initial stages of strategic option developments, the firm reduces the sunk cost incurred in case of project abandonment. Similarly, the inclusion of deferral opportunities when arranging irreversible investments in a strategic options exercise adds flexibility to the firm's resource commitments. It allows postponement of commitments until times when circumstances are considered most opportune.

Application of abandonment options during strategic option development, and deferral options during final investments in strategic options exercise, provides a systematic approach to analyse the dynamic evolution of strategic options.

Abandonment options are typically construed as sequential or staged investment paths, so the firm has the opportunity to abandon the project at different points in time during the development period. Initial strategic option development investments cannot be considered perpetual, because they are supposed to lead to investment projects within foreseeable time. An initial development investment might lead to several business opportunities, each representing different potential irreversible resource commitments. Owing to the finite nature of initial staged abandonment options, the option value can be estimated on the basis of single option or simple compound option analysis. A simple compound option provides the opportunity to acquire another option at a later time. The compound option can be seen, for example, as an initial research and development investment, which if it turns out to be successful, provides the opportunity to subsequently test the research results. If the test option has a positive outcome it in turn can lead to one or more irreversible investment commitments as the option to introduce new products or services is being exercised.

The cash outlays in the initial research and development period are often relatively small compared with resource commitments at later development stages. In the subsequent testing period resource commitments increase progressively while the chance of success increases. The value effect of the

abandonment option on smaller initial development investments is significant, so in most instances the consideration of one or two initial investment periods is sufficient to provide a qualitative assessment of the project potential. The imposition of such a relatively simple situation lends itself to computational methods based on analytical solutions. Alternatively, the inclusion of multiple investment stages requires the application of more complex numerical methods.

4.4 REVIEW OF CASE STUDY 3

The third case looks at converting a job shop operation to a flexible cell manufacturing operation. The decision problem is whether to convert the whole factory or to convert gradually. Usually it costs less to convert the whole factory at one time than to convert partially over time. However, the gradual conversion is a less risky investment because one can dictate the level of automation as we see fit. Moreover the automating the whole factory, although initially desirable, may appear unacceptable after obtaining additional information and being able to make sequential decisions.

Under the options approach, it was determined that a more plausible decision would be to delay the immediate cell conversion. The decision to invest should be made after resolving uncertainty rather than first investing and then finding out what happens. Certainly, one would undertake the investment if most of the uncertainty with the sample could be resolved. Therefore the option would be exercised, if the real option were sufficiently in the money. If the real option were not in the money, the option would go unexercised. This creates an asymmetry in the payoffs resulting in a strategic NPV (SNPV) value that includes a premium over the conventional NPV for the value of flexibility.

Real Options Analysis is based on an assessment of the development of a central parameter (state variable) that influences the value of the investment. If more than one state variable is considered, the analysis is more complex, and an

analytical solution might not exist. Analytical solutions are sufficient to effectively analyse most investment considerations, in which case the value of the investment itself appears the obvious state variable to consider. The investment value in turn depends on a number of economic variables, e.g. prices, demand, competition, etc., that are not explicitly considered in the analysis. Instead, these factors can be assessed indirectly through prior assumptions about their influence on the investment value. Real Options Analysis assumes that the value of the state variable, i.e. the investment value, follows the particular stochastic process described by a geometric Brownian motion with drift²². This simplifying assumption allows derivation of numerical formulae based on replication of the project cash flows by a portfolio of traded securities.

Therefore, Real Options Analysis can be applied to determine the value of the perpetual deferral option in a complete capital market, because the development in the investment value can be replicated by a portfolio of securities traded in the market.

4.5 SENSITIVITY OF OPTION VALUES

The option pricing models devise reasonable descriptions of the stochastic nature of future value-creating opportunities. The basic assumption is that a central state variable influencing the investment value of a new activity follows a stochastic process of some type of Brownian motion. Under this simplifying assumption analytical solutions can be derived to determine the value of simple compound options and continuous deferral options. However, in devising these solutions the analyst makes fundamental assumptions about a number of central parameters in the derived formulas. These parameters include the risk-free rate, the future variance of the investment value, and the length of the initial investment period in staged abandonment options. In the case of continuous

²² Hull, 1993; Dixit & Pindyck, 1994

deferral options, other parameters are the investment value appreciation, the discount rate, and the implied dividend payout rate. It is not possible to provide general guidelines on how to determine the parameters correctly. Such analysis can appropriately involve people engaged in the specific business environment.

The model parameters reflect the characteristics of the environmental uncertainty surrounding the projected investment commitments. For example, the variance of the stochastic development in the investment value reflects the uncertainties associated with future demand, input prices, technological capabilities, etc. In continuous deferral options, assumptions about the investment value appreciation and the inversely related dividend payout rate reflect the intensity of competition and technological imitation in the industry. Therefore, sensitivity analysis based on different parameter assumptions can enhance the understanding of environmental effects on strategic decisions.

The options value sensitivity shows that option-pricing models do not provide fixed and ready answers, but establish an analytical framework that allows evaluation of the investment's value in different environmental scenarios. Expecting minute precision from the calculation is missing the point of the analytical mission. Instead, the analysis enables the decision maker to evaluate the stakes and opportunities associated with the investment decision.

The abandonment option approach encourages investment in opportunistic projects that otherwise would be rejected. New research-and-development-related projects are recommended provided they represent sufficiently high opportunistic profit potentials. Conversely, the deferral option approach ensures that new strategic opportunities only are pursued when the present value of the investment exceeds the value of the deferral option, so the likelihood of premature pursuit of new business ventures is vastly reduced.

The option model can be made considerably more complex by including more sequential option stages. Similarly, the continuous deferral option model can be extended in various ways, e.g. by including mean reverting and value jump processes to reflect moves towards assumed equilibrium prices and abrupt changes²³. However, little is known about “normal” returns from new strategic initiatives, and parameters in the basic models already incorporate effects of major environmental characteristics. Hence, it is not obvious that these refinements add very much. Extensive model refinements increase the complexity of finding solutions and can easily obscure the clarity of the analysis to an extent where it simply does not pay off to pursue them. Therefore, the model of simple compound options and continuous deferral options appear sufficient for most practical investment considerations.

4.6 SUMMARY

In contrast to the abandonment option, the deferral option in principle can be pursued indefinitely. Innovations are often introduced long after they have been developed. In the continuous investment deferral option situation the analysis determines the time at which it is optimal to make the irreversible investment commitment, i.e. when investment deferral should be stopped. Hence, a continuous deferral option can be applied to determine an appropriate option value.

The analysis suggests that two fundamental options perspectives can guide strategic decision-making. The abandonment option perspective relates to initial investment in strategic options development, and the deferral option perspective relates to subsequent exercise of already developed strategic options. Existing strategy-related options theoretical frameworks do not distinguish between these two fundamental option perspectives. It is indisputable that the establishment of

²³ Dixit & Pindyck, 1994

appropriate real options can exert a positive influence on a firm's business development, competitive adaptability, and organisational performance. However, financial performance is eventually determined by the premiums paid for the firm's strategic options set, and how well the strategic options portfolio is executed. The application of initial abandonment and subsequent deferral option perspectives provides useful analytical frameworks to guide strategic investment decisions on strategic options acquisition and exercise. The options theoretical frameworks developed in strategic management offer little advice on investment in strategic options development. Similarly, there is limited advice on optimal options exercise. However, the analytical tools of the abandonment and deferral options approaches provide useful support to evaluate specific strategic option investments.

Applying an abandonment options approach to evaluate strategic options development attaches higher opportunistic value to future uncertainty compared with conventional capital budgeting, which punishes risky projects. The use of a deferral options approach to evaluate strategic options exercise considers additional opportunity costs that must be covered by cash flows from the business project. Hence, the application of the Real Options Approach supports assessments of strategic options development and exercise.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

Capital budgeting plays an important role in financial economics. After the strategic direction of the company has been determined, the results of capital budgeting decisions indicate whether this direction is feasible. Capital budgeting typically uses cash flow analysis to evaluate individual projects. The net present value model (which is a capital budgeting tool) takes into account the time value of money. It involves estimating a project's future cash flows, discounting these cash flows at the company's required rate of return and subtracting the cost of the investment from the present value. If the result is positive, this indicates that the project is feasible.

The limitation of this approach is that it does not take into account management flexibility to alter the course of a project in response to changing market conditions. The net present value of the project does not take into account the volatility of the project's cash flows. More volatile cash flows can be more or less valuable than a project with non-volatile cash flows. It is also evident that companies do not take on projects that have large volatility for small expected returns, even though the net present value is positive.

5.2 CONCLUSION

The Real options approach is based on the premise that any corporate decision to invest or divest real assets is simply an option. It gives the option holder a right to make an investment without any obligation to act. The decision-maker therefore has more flexibility, and the value of this operating flexibility should be taken into consideration.

Shortcomings of conventional capital budgeting have been recognised, as cash flow analysis fails to express important organisational effects. That is, capital budgeting should support investment initiatives emerging in different parts of the firm, while resource commitments with overall corporate effects should be considered at the executive level. Ideally, capital budgeting integrates strategic planning, individual incentives, and corporate control.

Conventional strategic analysis determines an optimal strategic fit between external business opportunities and internal organisational capabilities from an overall corporate perspective.

Real options relate to the flexibility created around an organisation's use of both tangible and intangible assets. Hence, a portfolio of real options determines the extent to which a firm is 'physically' capable of adapting within reasonable time spans. Use of quantitative evaluation methods does not exclude intangible assets from the analysis, but limits their inclusion to assets that can be explicated. However, a real options approach does not explicitly consider all important tacit aspects of the firm's strategic options development, and therefore does not pretend to be universal. Nonetheless, it can support analysis of real option structures that improve the firm's ability to adapt and respond to changing environmental circumstances.

The real options approach can help assess the opportunistic potential of strategic options development, and the opportunity costs associated with exercise of strategic options. Real Options Analysis provides useful analytical techniques that support both quantitative and qualitative strategic analyses based on assessments of the environmental risks associated with future business opportunities. A real options approach supports analysis of initial investments in real option development, and subsequent irreversible investments in real options exercises, which are crucial to the creation of firm value.

Firm value is determined by the premiums the firm pays to acquire its strategic options, and how well management executes its strategic options portfolio. Performance depends on the extent to which the right strategic options are developed at appropriate costs, and those strategic options are exercised in the most opportune business situations. This implies that management must develop relevant strategic options in view of the firm's perceived business potential. There is limited theoretical support to the process of creating strategic options, but the abandonment option approach supports analysis of specific investments in strategic options development. Similarly, no theories ensure that existing strategic options are exercised at the most opportune moments, but the deferral option approach supports analysis of specific investments in strategic options exercise. The implications of the options analytical framework based on staged abandonment and continuous deferral option perspectives are two-fold: it makes firms more willing to consider initial development investments in support of new strategic options creation, and it makes premature commitments to strategic ventures less likely.

Perhaps one of the most important aspects of the real options approach is that it can foster fruitful discussions about the strategic consequences of future business opportunities in uncertain environments, where uncertainty itself is opportunistic and not just a negative risk parameter. Real options analysis can provide quantitative evidence of the effects of different environmental conditions. This makes a compelling argument for the use of the analytical methods as proposed by this research. It is more important that the strategic decision makers understand the methodological principles and assumptions, than that they get a marginally more 'correct' option valuation from a complex analytical method. With hindsight, no option valuations hold true value. It is the underlying discussions among managers that matter.

5.3 RECOMMENDATIONS FOR FUTURE RESEARCH

The novelty of the real options approach and the limitations of the DCF method have led to a growing amount of finance literature focusing on valuation and strategic decision making. Although the conceptual foundation for real options is well established, there is scope for further research extensions to some basic theories as well as applications.

Future research in the development of the theory of project analysis should take into account the fact that the cash flows from an investment project are influenced not only by agents within the firm who can react as new information becomes available, but also by the actions of agents outside the firm, such as competitors and suppliers, and that these actions can in turn be influenced by, as well as influence, the actions of the agents within the firm. From this perspective, the cash flows from a project (and therefore the value of the project) can be seen as the outcome of a game, not just between the inside agent and nature, as per the real options approach, but among the inside agent, outside agents and nature.

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APPENDIX 1: DEFINITIONS

Black & Scholes Model

C	Price of the call.	
K	Exercise price on the maturity date.	
S	Current price of the share.	
N()	The standard cumulative normal distribution function.	
T	Time to maturity.	
r_f	The risk free rate of return.	
σ^2	The volatility of the stock return.	

Binomial Option-Pricing Model

S	Stock price.	
uS	Stock price with be a value of uS with probability q .	
dS	Stock price with be a value of dS with probability $1-q$.	
C	Current value of the call option.	
C_u	Call option value at the end of period one if stock price goes to uS .	
C_d	Call option value at the end of period one if stock price goes to dS .	
K	Exercise price on the maturity date.	
Δ	Hedge Ratio – the number of stocks which the investor needs to buy at price S.	
B	Amount of funds that can be borrowed at the risk –free rate r_f	
N()	The standard cumulative normal distribution function.	
T	Time to maturity.	
r_f	The risk free rate of return.	

σ^2	The volatility of the stock return.	
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