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An exploration of the barriers to effective geometric thought in the Further Education and Training phase of selected secondary schools in the Umlazi District.

by

Kalavani Naicker

211557433

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PHD: Mathematics Education

Supervisor: Prof V. Mudaly

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Declaration

I declare that this study, An exploration of the barriers to effective geometric thought in the FET phase of selected secondary schools in the Umlazi District, is my own work and that all sources that I used have been indicated and acknowledged by means of complete reference.

Supervisor: **Prof. Vimolan Mudaly**

Student: **Mrs Kalavani Naicker**

Signature: [redacted]

Date: 29/06/2021

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ABSTRACT

Geometry, a branch of mathematics has a history from the study of practical measurement in ancient Egypt to properties of shapes in Greek geometry. Learning and teaching of geometry in South Africa has posed many challenges for educators and learners. In 2011 Euclidean geometry was reintroduced after the removal to redress the inequalities of apartheid and to provide uniform access to mathematics for all learners. This study was therefore conducted in three secondary schools, conveniently selected, to explore the barriers that hamper the performance of Euclidean geometry in the FET phase and to explore ways of eradicating these barriers. Research questions generated data from educators and learners aimed at identifying factors influencing effective geometric thought.

This study was guided by the two metaphors, namely acquisition and participation and their impact on the teaching and learning of geometry. The study focussed primarily on the cognitive developmental theory of Piaget and social constructivist theory of Vygotsky both of which supported the acquisition metaphor. It further examined the participative metaphor of Sfard's theory of commognition. The study adopted the interpretive paradigm with the qualitative design. Purposive sampling was used since the study was conducted with grade 11 learners and educators teaching in the FET phase. Questionnaires, semi structured interviews and focus group discussions were used to collect the data. Lesson observations were used for triangulation purposes. The research instruments were piloted in order to ensure validity and reliability.

Data was first organized according to the research questions which was then coded and divided into themes. It was found that many of the barriers to learning geometry are primarily attributed to a lack of engagement of learners in meaningful learning situations. Findings also highlighted language, attitude, the classroom environment, and gaps in educators' methodology and content knowledge has being some of the factors that contributed to the poor learner participation. Recommendations are made for stakeholders to develop effective geometric thought. This study proposes that greater emphasis be placed on learner-centred methods of teaching. Communication in the form of verbal, written and visual should be considered when teaching geometry. Allowing learners to express their experiences to think and develop new knowledge is essential. Learners need to become active participants of their learning

and learning needs to take place in more realistic situations to improve understanding.

I-ABSTRACT

I-Jiyomethri, igatsha lezibalo linomlando osuka ekutadisheni kwesilinganiso esisebenzayo Egibhithe lasendulo kuya azakhiweni zomumo weJiyomethri yamaGrikhi. Ukufunda nokufundisa iJiyomethri eNingizimu Afrika kudale izingqinamba eziningi kothisha nabafundi. Ngo-2011 iJiyomethri yaphinde yafakwa ngemuva kokususwa ukuze kulungiswe ukungalingani kobandlululo kanye nokuhlinzeka ngokulingana ukuthola izibalo zabo bonke abafundi. Ngakho-ke lolu cwaningo lwenziwe ezikoleni zamabanga aphakeme ezintathu, ezikhethwe kalula, ukuhlola imigoqo ephazamisa ukusebenza kweJiyomethri esigabeni seFET nokuhlola izindlela zokuqeda lezi zithiyo. Imibuzo yocwaningo yakhiqiza idatha evela kothisha nakubafundi ehlose ukukhomba izinto ezinomthelela ekucabangeni okusebenayo kweJiyomethri.

Lolucwaningo beluqondiswa izingathekiso ezimbili, okungukuthi ukuzuza nokubamba iqhaza kanye nomthelela wazo ekufundisweni nasekufundweni kweJiyomethri. Ucwaningo lugxile kakhulu kumqondo wokuthuthuka wokuqonda kaPiaget kanye nethiyori yezakhi zenhlalo kaVygosky zombili ezazisekela isingathekiso sokutholwa. Iphinde yahlola umbono kaLave noWenger (1991) owawugqamisa ukubaluleka kwesifaniso sokubamba iqhaza futhi okokugcina, umbono kaSfard wokuhlonishwa. Ucwaningo lwamukele ipharadigm yokuhumusha ngomklamo wekhwalithi. Isampuli enenhloso isetshenzisiwe kusukela isifundo senziwa nabafundi bebanga le-11 nothisha abafundisa esigabeni se-FET. Amaphepha emibuzo, izingxoxo ezihleleke kancane kanye nezingxoxo zamaqembu okugxila kuwo kusetshenziselwe ukuqoqa imininingwane. Ukubheka izifundo kusetshenziselwe izinhloso zonxantathu. Amathuluzi zocwaningo ahlolwa ukuze kuqinisekiswe ukuba semthethweni nokwethembeka.

Idatha yayihlelwe ngokuya ngemibuzo yocwaningo eyabe isibhalwa ngamakhodi yahlukaniswa yaba ngamatimu. Kunconyelwa ababambiqhaza ukuthi bathuthukise umcabango weJiyomethri osebenzayo. Lolu cwaningo luphakamisa ukuthi kugxilwe

kakhulu ezindleleni ezigxile kubafundi zokufundisa. Ukuxhumana ngendlela yokukhuluma, okubhaliwe nokubukwayo kufanele kubhekwe lapho kufundiswa iJiyomethri. Ukuvumela abafundi ukuthi baveze ulwazi lwabo ukuze bacabange futhi bathuthukise ulwazi olusha kubalulekile. Abafundi kudingeka babe ngabahlanganyeli abakhuthele ekufundeni nasezidingweni zokufunda kwabo okumele ukwenzeke ezimeni ezingokoqobo ukuthuthukisa ukuqonda.

ABBREVIATIONS

CAPS	Curriculum and Assessment Policy Statement
NSC	National Senior Certificate
TIMSS	Trends in International Mathematics and Science Study
DBE	Department of Basic Education
FET	Further Education and Training
GET	General Education and Training
MNCS	Mathematics National Curriculum Statement
RNCS	Revised National Curriculum Statement
NCTM	National Council for Educators of Mathematics
ICT	Information and Communication Technology
ZPD	Zone of Proximal Development
PBL	Problem Based Learning
NCTM	National Council of Educators of Mathematics

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Chapter: One

1.1 INTRODUCTION TO THE STUDY

Geary (2011) highlighted that one of the main impediments to African headway is the underachievement of learners in mathematics in South Africa. The poor performance in mathematics has been identified as a significant challenge for the country (DBE, 2012). According to Chikiwa (2017), NSC mathematics diagnostic reports have shown low learners' scores in the geometry questions thus reintroduction of Euclidean geometry in 2014 is an added contribution to the poor performance of mathematics in South Africa. As a mathematics educator, it is evident that mathematics education in South Africa is facing many challenges. This challenge is supported by researchers (Fleisch, 2008; DBE, 2017; Reddy, 2015) in their findings that mathematics education is struggling according to a wide range of assessments. It is imperative to discover a way forward to meet the many challenges that contribute to poor learning and achievement in mathematics. The aspect of mathematics relating to geometry is the area in which learners experience great difficulty in understanding. Knowledge of geometry in all phases is fundamental because it is viewed as a tool that empowers humans to tackle various problems and to understand the world with its comprehensive geometric structures (Smith, 2004). This study explores the barriers to effective geometric thought in the Further Education and Training (FET) phase. This study not only identifies the obstacles but also explores strategies to eradicate these barriers. The following chapter introduces this study by presenting the background, the problem statement, the purpose, the research questions, significance, definition of keywords, and the thesis outline.

1.2 BACKGROUND OF THE STUDY

Geometry is a section in mathematics dealing with shapes and space. Dube (2016) highlights the critical role geometry plays in fostering learners' critical thinking and problem-solving abilities. Learners begin understanding and express their general surroundings with the use of geometric shapes. They analyze and solve problems using these geometric shapes. In support, Lampert (2010) states that geometry helps learners develop connections between real life and mathematics.

Many vocations use geometric shapes and objects to make decisions, for example, engineers, gardeners, and carpenters. The importance of including geometry in the curriculum, as outlined by Baykul (2005), permits the advancement of critical thinking and problem-solving in mathematical studies at school. Additionally, geometry assists in teaching other mathematics topics; for instance, rectangles, squares, areas, and circles are mainly used to guide the techniques of operations. He further communicated that geometry is a branch of mathematics that identifies with day-to-day life, for example, the area of a room, a building design, science and art. Baykul (2005) also added that geometry assists with acquiring awareness about the world in which they live and appreciate, for example, the shape of crystals and the orbits of space objects which are geometric in nature. The content of geometry should assist learners in enjoying mathematics and have fun.

It has been six years since geometry returned as an examinable, compulsory learning area of mathematics in the FET phase (DBE, 2012). According to DBE (2011), geometry is written as a section of paper two, with 20% allocated to geometry in grade 10 and 26.7% in both grades 11 and 12. This is indeed a significant challenge for learners and educators. The results in geometry have not improved over five years, as seen in Table 1.1.

Table 1.1 Performance of Euclidean Geometry from 2014-2019

National Average Performance in Euclidean Geometry from 2014- 2019						
	2014	2015	2016	2017	2018	2019
Question 8	59	28	57	46	61	49
Question 9	38	38	44	34	49	44
Question 10	34	29	36	34	31	44

SOURCE: 2014-2019 NSC DIAGNOSTIC REPORTS (DBE, 2019, P. 194)

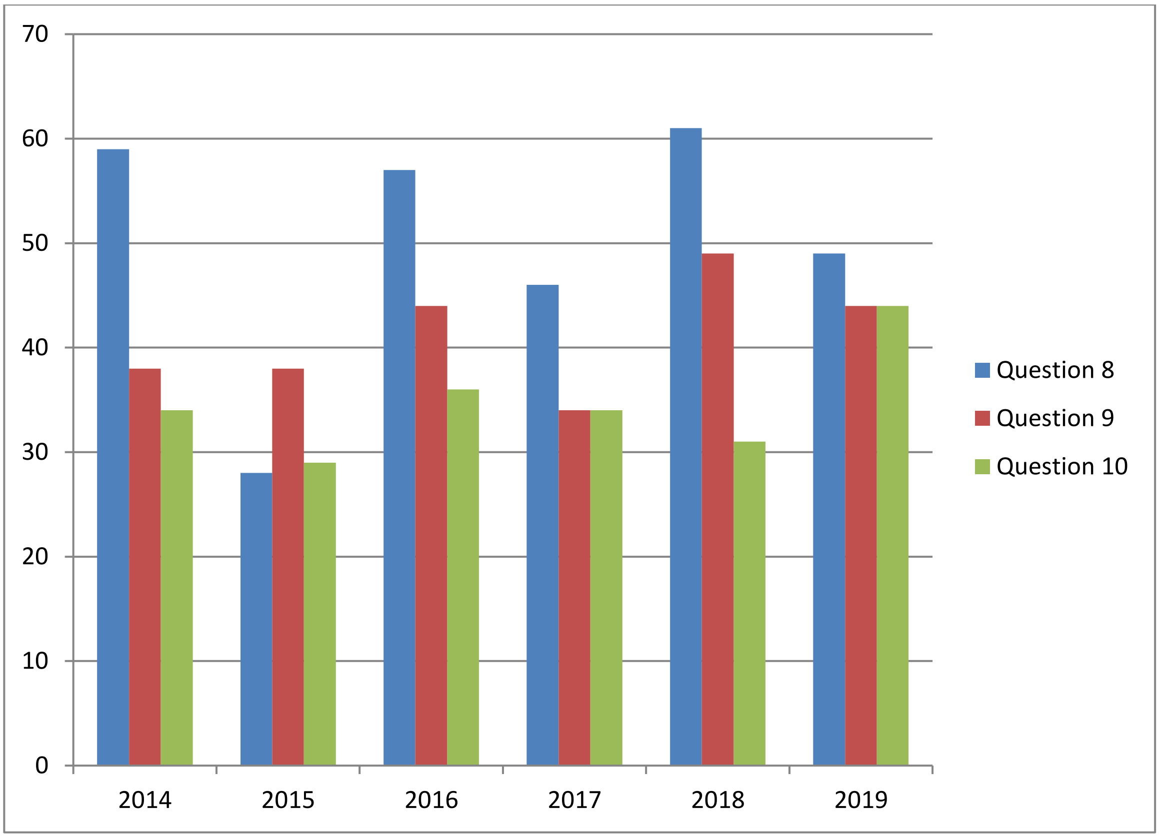

FIGURE 1.1 PERCENTAGE PERFORMANCE OF GEOMETRY (*SOURCE: 2014-2019 NSC DIAGNOSTIC REPORTS (DBE, 2019, P. 194)*

The table 1.1 and the graph in figure 1.1 display the percentages of average national performance in geometry from 2014 to 2019 after its reintroduction into the curriculum. Questions eight to ten are the questions relating to Euclidean geometry in the National Senior Certificate Examination from 2014 to 2019. It is evident that the average performance has been lacking in the sections about Euclidean geometry. Learner performance in question eight is better as the question is based on recall-type questions. The application-type questions were answered poorly. Although there is a slight improvement from 2018 to 2019, it is not significant. The reasons associated with the performance are the language of instruction (DBE, 2019), learner-specific socio-economic circumstances, inability to visualize, the educators' and learners' attitude towards geometry (Crook, 2017), qualification and competence (Khumalo, Molepo & Mji, 2016), and classroom environment (Venkat & Graven, 2017).

Learners' achievement in geometry is one of the major concerns in mathematics. The data above demonstrates the low success level achieved in the teaching and learning of geometry. Mathematics and geometry have been frightening for many learners (Zamkova, Prokop, & Stolin, 2016) because mathematics is viewed as a system on its own. Bowie (2013) identified one reason underlying this failure: the expected geometrical thinking skills of learners are lower. Traditionally high school geometry is taught from an axiomatic structure, much like Euclid's Elements recorded geometric learning in 300 BCE. Many learners battle with geometry content and do not see the more extensive objective for doing geometry. Jones (2007) proposes that geometry assists learners with fostering the abilities of representation, critical thinking, instinct, perspective, problem-solving, conjecturing, deductive reasoning, logical arguments, and proof when taught appropriately. Geometry is an essential school subject since it gives perspective for developing learners' deductive thinking capabilities and obtaining spatial awareness (Zamokova, Prokop & Stolin, 2016).

Educators become disappointed with most learners' poor achievement in geometry, which leaves them frustrated (Chinnappan, 2012). However, the word geometry which translates to measures of space, is a highly relevant subject full of real-world connections. Geometry is one of the least understood and despised branch of mathematics. It appears contradictory that a subject with such direct real-world associations and prompt pertinence can be a source of dissatisfaction to educators and learners.

Learners' performance in mathematics has been a major concern countrywide (Mullis, 2000; Smith, 2004; TIMSS, 2015). Educators, learners, parents, curriculum developers, and the public have all passed the responsibility to one another for the poor performance in mathematics. This performance outcome in mathematics can be ascribed to either external or internal barriers, as cited by Hillage & Aston (2001, p. 67). According to many researchers, factors affecting learners' achievement can be divided into instructional factors, demographics, and individual factors (Campbell, 2000; Epstein, 2004; Fenema and Sherman, 1986). Much of the research completed identifies the factors contributing to the performance in geometry in terms of demographic, instructional, and individual dynamics. However, there are limited studies on the barriers to language and visualization.

The problems of learners' achievement in geometry cannot be isolated from how educators interact with learners of the subject.

In this light, this study will look into systemic, pedagogical, and individual factors, however focussing on reasoning, visualization, and communication in the teaching and learning of geometry.

1.3 STATEMENT OF PROBLEM

As a mathematics educator of 25 years, my experience revealed that learners struggle with mathematics, especially geometry. Geometry, considered a means for comprehending and connecting with the space in which we live, is perhaps the most natural, concrete, and reality-connected part of mathematics (Fauvel and van Maanen, 2006). Idris and Tay (2004) distinguished the three teaching and learning geometry issues as language, visualization, and instruction. There are numerous variables in the classroom that impact the learning of geometry, and each of these components has its effect on the learner, and the thinking developed. Several researchers (Clements and Battista, 1995; Giganti, 2007; Steele, 2013) highlight various challenges learners face while learning geometry. These factors can be divided into learner-related, pedagogical, and school-related aspects.

It is in the language of geometry that individuals portray and discuss visual structures of our physical world. The language of geometry assists learners with thinking deductively and to reason on their own. Fundamental to learners' performance in geometry is the ability to solve problems. The National Curriculum Statement Grades R-12 (DBE, 2013) specifies that the main objective is to deliver learners that can recognize and tackle issues and make decisions using critical and creative thinking. Giganti (2007) argues three parts to learning mathematics: skills, concepts, and solving problems. Skills are the tools of mathematics, such as the expertise to add and manipulate numbers. Concepts are the ideas in mathematics, such as the concept of a triangle. Problem-solving is the skill to apply mathematics in different situations.

Language is the method by which mathematical ideas are conveyed between the educator and learner either through oral or written strategies. Mercer (2000) emphasizes the focal role that language plays in the educating and learning cycle of geometry.

In communicating the theoretical framework for this study, the focus is on three key concepts: experience, reasoning, and language. It is to be guided by the social constructivist theories of Vygotsky's zone of proximal development and Sfard's commognition framework related to discursive learning of geometry. Social constructivist theorists advocate that learners build new knowledge upon the knowledge that they have previously acquired. Kanselaar (2015) stated that social constructivism develops a learner by allowing the learner to encounter an environment first-hand to acquire dependable and trustworthy knowledge.

Much of the teaching of mathematics today is based on the conventional teaching method. In such fixed and rigid teaching, the educator tells or explains the mathematical concepts to learners, and the learners use this to get a correct answer (Tarnovsky, 2016). Teaching created by this method follows the chalk and talk-orientated view of mathematics. The constructivist view shifts away from this conventional teaching method to the educator becoming the facilitator of learning. Vygotsky's (1978) theory of Zone of Proximal Growth (ZPD) and scaffolding shares this view. It describes the significant relationship between educator and learner and between the parent and child (Scrimsher and Tudge, 2003). The Trends in Mathematics and Science Study (2017) demonstrated a positive correlation between educators' high endorsements of chosen classroom practices and learner achievement. Vygotsky (1978) clarified that the core responsibility of the educator and the parent is to arrange for learners to participate in activities that are within their Zone of Proximal Growth. By telling, illustrating, demonstrating, adjusting, pointing, prompting, providing pictures, diagrams and models, clarifying procedures, and asking questions, educators and parents construct scaffolds. These scaffolds increase learners' thinking and affect development positively (Murtagh & Webster, 2010). Culture and social interactions are emphasized in his theory as an important aspect involved in the development of human consciousness. Vygotsky's (1978) framework accentuated forces that are outside the child, that is, the impact of culture (Gredler & Shields, 2008). The significance of Vygotsky's (1978) theory is stressed in the distinction between rudimentary functions and higher mental capabilities (Lefrancois, 2010). Development moves from the essential cognitive functions to higher mental functions such as thinking, problem-solving, and visualizing, mainly through social cooperation that cultures make conceivable. Van Hieles' theory also echoed these various levels of development. Dutch researchers Dina Van Hiele and Piere Van Hiele (1952) identified groups that should not be skipped in the understanding of geometry. These levels are visual, descriptive or insightful, conceptual and social, formal derivation, thoroughness, or meta-mathematics.

Many learners neglect to reach a degree of understanding of geometrical concepts and to develop sound reasoning and visual abilities expected to solve problems. Goos, Stillman and Vale (2007) expressed that perception and thinking processes are essential for mathematical thinking and significant in the learning of geometry. Duval (2006) indicated that geometric reasoning includes the intellectual rationale of visualization and thinking. Visualization and thinking are fundamental mental abilities required for geometry, and these cognitive processes are interlinked, advancing learners' achievement in geometry (Duval, 2006).

The Department of Education (2017) depicted mathematics as a language on its own, a way of thinking and conveying which every learner requires. In such a manner, mathematical skills are essential in creating logical models for comprehension. Sfard's (2008) theory on commognition described mathematics as a discourse i.e. as a tool of communication. She described learning mathematics as a process in which learners extend their dialogue by individualizing mathematics discourse. To individualize, she said, was to communicate through rules. Her theory explains thinking as a form of communicating and solving problems. She views geometric learning as shifting towards a more refined mathematical discourse. Geometry is a type of communication recognizable by its word use, visual mediators, routines, and narratives. The language of geometry, particularly in the understanding of geometry concepts, assumes a significant part in learning advanced geometric ideas.

The performance of mathematics in school is affected by school-related factors (Bowie, 2013). Johnson (2000) highlighted that school climate, such as infrastructure and physical resources, impacts mathematical achievement. Studies on the state of mathematics in South Africa uncovered that Black learners were under- accommodated for and achieved the worse in mathematics (Jojo, 2019). Schools and class sizes impact learners' success in secondary school mathematics (Heyd-Metzuyanim & Graven, 2015). Research revealed that the geometry curriculum over emphasized retention of facts and under-emphasized comprehension and application of these concepts to deduce and make associations (Tsanwani, 2009). Wragg (1998) argued that training educators is essential so they know what is expected of them. An absence of required knowledge on the part of educators also has a significant impact on learners' performance (Wragg, 1998). Teaching strategies need to encompass: deductive reasoning, problem-solving in various contexts using real-world ones, visualization using graphs, pictures, and technology, to teach geometry effectively (Mullis, 2000).

It is against these factors that the researcher decided to investigate the barriers facing teaching and learning of geometry at the secondary school level and to identify strategies to eradicate these barriers to improve learner performance in mathematics.

1.4 OBJECTIVES OF THE STUDY

This study therefore aims to explore the barriers to effective geometric thought and determine strategies to eradicate these barriers in the further education and training (FET) band of secondary schools. In order to achieve this purpose, the following objectives were developed:

1. To identify the barriers in learning geometry.
2. To determine how systemic factors contribute to the learners' performance in geometry.
3. To ascertain the pedagogical factors affecting geometry performance in schools.
4. To establish strategies that can be adopted to improve geometry performance in schools.

1.5 RESEARCH QUESTIONS

The accompanying research questions will be answered to accomplish the goals of this study

1. What are the barriers that contribute to poor performance when learning geometry?
2. How do systemic factors contribute to the performance in geometry?
3. How do pedagogical factors influence learner performance in geometry?
4. How can the identified barriers in geometry at secondary schools be eradicated?

1.6 SIGNIFICANCE OF THE STUDY

This study aims to be significant as it offers insight to educators, policymakers, and people responsible for educational decisions at different instructional levels on measures that can be taken to overcome the existing challenges associated with learning geometry. It adds to the information we have about the barriers and possibilities of learning and teaching mathematics at schools. It further provides explicit outcomes to explore in order to overcome the barriers experienced, which can be utilized as a source to other researchers who want to do their research in this area.

According to the National Curriculum Statement for Mathematics (2011), the learners must develop the skill to effectively use the language of mathematics, to gather, examine and organize quantitative information, to analyze and evaluate conclusions as well as to utilize mathematical process abilities and properties of shapes and objects to distinguish and solve problems critically. It is also specified that learners must express themselves appropriately by utilizing descriptions in words, charts, symbols, tables, and graphs.

Geometry develops logical reasoning and critical thinking (Battista, 2007), which is outlined in the CAPS skills. Usiskin, Peressini, Marchisotto and Stanely (2003) emphasized how the power of deduction is illustrated in geometry, as a fundamental ability that learners require to develop. The language of geometry is a vital skill that develops during the learning process. It is crucial to understand learners' and educators' barriers to geometry to improve students' achievement in mathematics. Teaching becomes irrelevant if it does not meaningfully address learners' barriers. Many educators and schools have attempted various teaching strategies and programs to make learners understand geometry, at times with success and sometimes not. It is thus important to investigate the barriers that hinder effective geometric thought and the possible strategies that can be implemented to overcome these barriers. Geometry assists us to address and describe in an organized manner the world in which we live. By strengthening learners’ knowledge of geometry, educators provide an advantage in this competitive global economy. The objective of this research, to improve learners' performance in geometry is therefore very important. It would guide learners towards the opportunity of pursuing careers in engineering, and other important fields where learners require mathematics, a subject in which most of our learners’ experience difficulties in performing at a satisfactory level.

Atebe and Schafer (2011) supported that the major challenge that we are experiencing in South Africa is that learners are not performing well in geometry. From 2008 to 2014, geometry was optional in the NSC examination in order to increase the grade 12 mathematics performance. The impact of insufficient geometry teaching has been documented in the TIMSS 1999 and 2015 reports (TIMSS, 2015). This report uncovered that South African learners fared inadequately in mathematics and a substantial number of high school learners struggle to comprehend the basic geometry concepts. Many learners underperformed due to the numerous barriers that they encounter in learning geometry. The results of this research could benefit the socio-economic dispensation of the country by adding to an increase in the number of learners entering into the mathematics and science fields at university level and subsequently the quantity of graduates in mathematics and science, as well as an increase in the number of qualified, competent mathematics educators. Information acquired in this study may contribute to reducing the mathematics crisis experienced in the country. The history of underperformance by learners in this subject discourages learners from pursuing careers crucial to the country's development. The lack of fundamental professionals, such as educators, doctors, scientists, and other scientifically orientated professionals, is also responsible for the economy's slow growth. Learners who set their goals on entering fields of study that require mathematics are discouraged because they are failing mathematics at grade 12 level, prohibiting them from pursuing vocations of their choice. These factors validate this study and show those who spearhead change what deprives the learners of good opportunities in learning geometry and what needs to change to improve the learning and teaching conditions for compelling geometric thought.

1.7 ASSUMPTION OF THE STUDY

The following assumptions are made regarding this study:

- The research instruments used will elicit reliable responses;
- The educators and learners will fully understand the questions they will be asked;
- The participants will provide honest expressions of their knowledge considering that anonymity will be maintained;
- The lessons observed will be realistic.
- In South Africa geometry is compulsory at the FET phase, yet learners struggle to understand the concepts and skills under this topic.
- There are mathematics educators that experience challenges in the teaching of proofs and other essential concepts in geometry.

1.8 LIMITATIONS OF THE STUDY

The sample of learners and educators was chosen from only three schools in the Umlazi district. The problem being investigated focuses on learners' and educators' barriers to adequate geometric thought. Only six grade 11 learners from each of the schools were selected for this study due to convenience. Educators teaching mathematics in the FET phase were chosen to take part in this study. The small sample questions the external validity of the findings. The schools selected for this study were convenient for the researcher to access, and the results and findings will therefore not be generalized.

The study was conducted within the qualitative research paradigm. Marshall and Rossman (1989) argue that there is a weakness in qualitative research in transferability of results as each qualitative research approach as its own unique features. Much of the data analysed was based on the researchers' subjective interpretation, it should be noted that this could result in bias of findings. An audio and videotape was used to record the interviews and focus group discussions. The presence of the audio and videotape did at times affect the sincerity of the responses. However, as the discussion continued the participants became more accustomed to the presence of the audio and video and video tape recorder.

All attempts were made to minimize the influence of the limitations on the research process.

1.9 DEFINITION OF TERMS

This section defines the key terms used in the study

Learner – Dean (2019) described a learner as someone who is learning about a particular subject, or how to do something.

Mathematics- is a manner of arranging, expressing, and manipulating information using correct numerical terms that can be represented by means of patterns, symbols, and theorems (Jupri, 2017).

Geometry – Geometry is defined as a science of physical space. Initially the Greeks called it "earth measure," the science of measuring the land (Royal Society, 2011, p. 23). In this study, geometry, a branch of mathematics, is concerned with; properties and relations of points, lines, surfaces, and solids or the shape and relative arrangement of the effectiveness of something. It also involves the study of positions of object movements and the space around the objects to improve activities.

Barriers- According to the DBE (1997), barriers are challenges that arise from the failure of the educational system to accommodate diversity and resources, resulting in learning breakdown or hampering the learner from getting access to the provision of education.

Curriculum – A sequence of possible experiences set up in schools to discipline children and youth in ways of thinking and acting, whether it is carried out in groups or individually, inside or outside the classroom.

Learning – According to Prozesky (2015), learning can be formal or informal experiences. We experience day-to-day things, which happen to make us change the way we think and act.

Teaching – Carter (2003) defined teaching as a practice of experienced educators passing expertise and wisdom to learners faced with many challenges.

Spatial reasoning –Refers to interpreting and understanding the inherently geometric world (Barrows, 2010). Spatial ability or visuospatial ability is the capacity to understand, reason, and remember the spatial relations among objects or space.

Spatial Skills are spatial abilities. It refers to how learners view space and shape and conceptualize and interpret space and shape in terms of two and three-dimensional views. Spatial skills also include how learners react to visual stimuli material and their capability to manipulate visual patterns (Carroll, 1993).

Visualization refers to knowledge representations formulated around physical objects and events (Hamilton and Ghatala, 1994). Visualization is grounded on the construction of a symbolic representation of concepts and the establishment of relations.

1.10 THESIS OUTLINE

The thesis is presented in six chapters. Chapter one provides the background to the study. Chapter two presents a review of some relevant literature from both local and international sources. Chapter three outlines the theoretical framework underpinning this study. Chapter four presents the research methodology that was used in this study. This includes the epistemological perspective of the study, the research design, issues of trustworthiness, the details of the participants, data collection methods, data analysis procedures, and ethical issues that were considered during the study.

Chapter five presents the analysis of the study concerning each research question. The analysis is done after the data is coded into themes based on the research question. Data obtained from the lesson observation is presented separately.

Chapter six presents an in-depth discussion of the findings. The findings are presented in relation to the theoretical framework used to analyze data collection. In this chapter, recommendations obtained from the study are proposed for all stakeholders in education to consider for implementation. Limitations of the research and as well as concluding remarks are also presented in this study.

Chapter Two: Literature Review

2.1 INTRODUCTION

In this fast-paced world and a period of innovation, mathematics assumes a significant part in the improvement of a child's mind. Mathematics, especially geometry, is crucial to comprehending our advanced world and coordinating it with the new developing technological knowledge. Geometry and spatial thinking are essential by themselves and should provide a critical mental foundation for learning other topics in mathematics. Encouraging the teaching of geometry adequately can mean empowering more learners to discover success in mathematics. According to Dell'Oilo and Donk (2007), it has been shown that conventional methods of teaching mathematics have been ineffective in the instruction of geometry.

The first chapter outlines the problem statement, research methodology, and contextual background of the study. The present chapter reviews the literature on the barriers experienced in the teaching and learning of mathematics. The success of teaching geometry relies upon recognizing the factors that hinder the learning and teaching process. This study aims at identifying barriers and looking at the possible strategies that can assist in eradicating the barriers highlighted. It is structured around the research questions. Whilst related literature across the globe is evaluated, local and global viewpoints were not seen independently but uniquely intertwined with understanding the barriers to successful geometric thought.

2.2 BACKGROUND AND HISTORY OF EUCLIDEAN GEOMETRY

Geometry is one of the longest established branches of mathematics which deals with the properties, measurement and relationship of lines, angles, points, co-ordinates, solids and surfaces. The word geometry originally comes from the Greek word "geometrien" where "geo" means place and "metric" means "measure" (Hendricks & Adu, 2016, p. 1). In ancient societies geometry played an important role in surveying, navigation and buildings. The earliest geometers were Egyptian surveyors who built property boundaries and large buildings in shapes like squares, circles, triangles and rectangles which were not easy to do by eye (Van Manen, 2016).

Geometry is divided into Euclidean and non-Euclidean geometry. During the nineteenth and twentieth centuries, geometry went through a period of growth that led to the content of geometry and its internal diversity increasing (Jones, 2000). Much of the development of geometry during the twentieth century was inspired by the work of Felix Klein (1849-1925). He advocated that geometry should be viewed as the study of the properties of space that are invariant under a given group of transformations. With this definition it became possible to classify geometry into various geometries ranging from topology as the most general, through projective and affine geometries, to Euclidean geometry. The different types of geometry are based on the planes, angles and approaches they take. In South African schools' co-ordinate geometry (analytical) and Euclidean geometry is studied.

Euclidean geometry is the study of plane and solid shapes and their properties are based on the theorems and axioms developed by the Greek mathematician Euclid. While Euclidean geometry aims to understand flat, two-dimensional spaces, non-Euclidean geometry studies curved surfaces. Non-Euclidean geometry is also known as hyperbolic geometry. Euclid, who is believed to be the first mathematician who wrote definitions, axioms and postulates based on points and lines, laid the foundation for what we now call Euclidean geometry. According to Fomunyam (2016) Euclid's book of Elements summarised and synthesised most of what was known about geometry in Greece at that time.

The following propositions, which are a good reference to today's Euclidean geometry, are highlighted by King (2018, p. 31) as some of Euclid's Elements:

- Definition 1: "Equal circles have equal diameters or equal radii".
- Proposition 13: "If a straight line is vertically drawn to meet another straight line, then a right angle is formed or two right angles are formed".
- Definition 4: "Straight lines inside a circle are equally distant from the centre if perpendiculars drawn to them from the centre are equal".
- Proposition 6: "If two straight lines are perpendicular to the same plane, then the straight lines are parallel".

Clark and Worger (2016) conclude that these elements are now a central pillar in geometry and are essential within geometry education. Euclidean geometry is studied in schools around the world and the teaching and learning is problematic not only in South Africa but around the world (Novak and Tassel, 2017). Jupri (2017) highlight that geometry is considered an essential aspect of mathematics which helps learners make sense of the world and promotes learner critical thinking skills, deductive reasoning ability and logical arguments. Through geometric knowledge, learners are able to analyse, describe, and understand the world in which they live as well as equip them with necessary tools which can be applied in other areas of mathematics (Kosa, 2016).

Euclidean geometry is the starting point for writing proofs in the mathematics curriculum. The perception that proofs in geometry are too far-fetched from everyday mathematics stems to a large extent from the glaring defect in the present day mathematics education, namely outside of geometry there are essentially no proofs (Novak & Tassell, 2017). For 2500 years, the system of geometry that has appeared most natural to human adults is Euclidean plane geometry. It is a formal system that characterizes two-dimensional shapes in accord with distance, angle, and directional relationships among their parts (Yixuan, Lei, Peng, & Jinhong, 2016). Euclidean geometry continues to be the first system of formal geometry that learners learn, and the system accord best with human intuition. Early philosophers (Plato, Descartes and Kant) have argued that Euclidean geometry comes naturally to human mind, even to the minds of humans who lack all instruction in mathematics.

In recent years the nature of geometry has continued to expand. A number of contemporary developments in mathematics are predominantly geometrical. Geometry, in all its variety, is also rich in application. Whiteley (2004) suggests the following applications that use geometry:

Computer aided design (CAD) and geometric modelling

Robotics

Medical imaging

Computer animation and visual presentations

Further applications where geometric problems arise are in chemistry, material physics, biology and most fields of engineering. In the scientific world the importance of geometry appears naturally in many sectors of the planet, such as the structure of the solar system, in geological information, rocks and crystals, flowers and even in animals for specific purposes. The importance of geometry in the lives of people cannot be overemphasized.

2.3 SOUTH AFRICAN CURRICULUM REVIEWS

It cannot be disputed that mathematics is fundamentally essential to daily living since most human activities rely on mathematical knowledge. With desegregation in education post-1994, the provision of quality education to learners became the primary goal to reform movements in the education system (Spaul, 2013). The majority of the population received inferior education, which prompted the post-apartheid government to arrange quality education for all learners, a flagship goal (Bowie, 2013). The delivery of quality teaching and learning in mathematics presented a vital challenge in the area of education.

Curriculum 2005 (C2005), which was driven by Outcomes-Based Education (OBE), was introduced by the Department of Basic Education (DBE, 2005) due to challenges encountered with OBE. The main characteristic of OBE was teaching by cooperative group instruction. This change in curriculum created difficulties for educators to identify struggling learners in the teaching of geometry. OBE was introduced in 1997 to conquer the curricular divisions of the past and was reviewed in 2000 due to the poor results achieved after its implementation. In 2002, the Revised National Curriculum Statement (RNCS) for grades R-9 and the National Curriculum Statement (NCS) for grades 10-12 was implemented (DBE, 2005). Educators discovered that the RNCS was similar to the C2005, and execution challenges were still present. The RNCS was not favoured by the curriculum implementers because of vague terminology and inadequate educator training. In 2012 the curriculum was again modified and changed to the Curriculum and Assessment Policy Statement (CAPS), which was carried out in all phases of learning. Currently, CAPS is the implemented curriculum in the country whose primary objective is to deliver learners capable of utilizing basic and creative thinking in making choices to recognize and discover solutions to problems and work viably as people or with others as individuals of a group. The CAPS target is also to ensure that learners manage and organize themselves in dealing with their activities effectively and adequately by gathering, analyzing, and critically assessing applicable data. CAPS encourage learners to be exposed to visual, symbolic, and language abilities in different approaches to effectively convey their experiences and identify that problem-solving situations do not exist in isolation. Learners ought to exhibit a comprehension and interpretation of the world as a series have connected systems.

This CAPS curriculum guides learners towards the correct use of mathematics in the use of vocabulary, concepts, and application. It is further expected to skill learners adequately to learn to listen, talk, reason logically and apply mathematical knowledge, explore, break down a problem, represent and decipher information, as well as to pose and tackle problems. Learners build consciousness of the significant part that mathematics plays in real-life contexts, including their development. CAPS was regulatory and demanded uniformity in implementation across the country. Ramatlapana and Makonye (2015) indicated that this solution was authorized in light of the fact that the opportunity with the performance of the changed curricular was counter-productive since learners' continued to underperform in mathematics as displayed in TIMSS (2015). This significant educational strategy is favoured in the country since it is helpful in the teaching of low achieving learners from disadvantaged socio-economic backgrounds as it utilizes a more structured educator-directed instructional method (Ramatlapana and Makonye, 2015).

Major changes noted over the years led to confusion amongst learners and educators (Novak and Tassel, 2017). This depicts that curriculum transformation in South Africa is a continuing process. The above changes in the educational plan prompted confusion amongst educators, further placing a barrier to the learning and teaching of geometry (Fosnot, 2013).

2.4 AN OVERVIEW OF TEACHING AND LEARNING OF GEOMETRY IN SOUTH AFRICA

When the FET curriculum was implemented in 2006 Euclidean geometry was embedded into the mathematics paper three which was optional. Learners who wanted to study Euclidean geometry had to take Mathematics paper three as an additional subject. In 2012 the CAPs replaced the NCS and Euclidean geometry became integrated into Mathematics paper two. From 2012 onwards only two mathematics papers were written in grade 10. By 2014 all learners studying mathematics from grade 10 to grade 12 were tested on Euclidean Geometry. An absence of Euclidean geometry for six years would have implications for both teaching and learning in South African schools. Currently in South Africa it is compulsory for all learners to be taught geometry until grade 9. After grade 9 these learners can choose to take mathematics or mathematical literacy.

Euclidean geometry has been cited as one of the topics that are problematic for the majority of the learners and educators (Novak & Tassell, 2017). CAPs focus is more on content and each topic contributes towards the acquisition of specific skills and Euclidean geometry is one of the ten main content topics taught in mathematics in the FET phase (DBE, 2011). It is generally taught in term three in grades 10 and 11. An overview of the CAPs syllabus is provided below in table 2.1.

Table 2.1 An overview of the grade 10 to 12 Euclidean geometry

Grade	Content Overview
10	I. Revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of triangles II. Investigate line segments joining the midpoint of two sides of a triangle III. Properties of special quadrilaterals: the kite, parallelogram, rectangle, rhombus, square and trapezium IV. Investigate and make conjectures about properties like the sides, angles, diagonals and areas of these quadrilaterals. V. Proves these conjectures VI. Solve problems and prove riders using the properties of parallel line, triangles and quadrilaterals
11	I. Investigate and prove theorems of the geometry of circles assuming

	results for earlier grades as axioms, together with one other result concerning tangents and radii of circles. • The line drawn from the centre of a circle perpendicular to a chord bisects the chord; • The perpendicular of a chord passes through the centre of the circle; • The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); • Angles subtended by the chord of the circle, on the same side of the chord, are equal; • The opposite angles of a cyclic quadrilateral are supplementary; • Two tangents drawn to a circle from the same point outside the circle are equal in length; • The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment; II. Solve circle geometry problems, providing reasons for statements when required. III. Prove riders.
12	I. Revise earlier (grade 9) work on the necessary and sufficient conditions to be similar. II. Prove (accepting results established in earlier grade) • That a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the mid-point theorem as a special case of this theorem); • That equiangular triangles are similar; • That triangles with sides in proportion are similar; • The Pythagorean theorem by similar triangles III. Solve circle geometry problems, providing reasons for statements when required IV. Prove riders

Adapted from CAPs for the FET Grades 10-12. p. 25-48

The above syllabus for the FET phase requires educators to have in-depth content knowledge and pedagogical content knowledge to execute successfully. Each year requires knowledge from the previous years in order to be successful. A gap in any year will create a gap in the learning of geometry and challenges in concept understanding. Practical instruction in geometry teaching and learning requires an educator to develop sound mathematical knowledge as well as instructional skills and strategies making use of useful resources and activities that guide the teaching activities and further assist in the effective delivery of the lesson (Luneta, 2014). Teaching the above geometry curriculum involves knowing to recognize interesting geometrical problems and theorems, appreciating the history and cultural context of geometry, and understanding the many varied uses to which geometry is put (Jojo, 2019). In the FET phase learners are required to have prior knowledge. In earlier grades learners experience geometry through recognizing, distinguishing and writing definitions of all types of 2-D and 3-D shapes in terms of their sides and angles, and properties of congruency and similarity (DBE, 2011). This curriculum content suggests that a gap in the understanding of concepts such as line, parallel, perpendicular, bisect, congruent, similar etc., which should have been mastered in the earlier phases, will result in learners experiencing challenges with the FET content.

Proofs and theorems form an integral part of the Euclidean geometry content taught at schools. According to Brijlall (2017), a proof serves as a convincing demonstration that some mathematical statement is true in all cases without exception. Grade 12 learners are expected to know all theorems and proofs including those done in the previous grades. According to Kolb (2014) understanding and applying theorems is a highly ranked level of proof and reasoning ability. The objectives for teaching geometry were summarised by Jones (2007) as a means to develop geometrical insight, spatial awareness, and visualization. The learning of geometry additionally provided information of geometric properties and the capacity to apply these properties of mathematical shapes and theorems in critical thinking. Jones (2007) added that geometry instruction makes a consciousness of geometry in society and considers contemporary utilization of geometry. Geometry promotes the aptitude of information in displaying and problem-solving for real-world settings by coordinating innovative abilities in geometric settings. This supports the idea of visual teaching of geometry to instil a positive outlook towards mathematics. Adjei

(2020) outlines the following outcomes that are achieved in the learning of geometry, ability to visualize problems; familiarity and facility with a wide range of geometry facts and problem-solving techniques; understanding of logical structure of geometry-axioms, conjectures, theorems and counterexamples, and visual reasoning

Regardless of the geometric abilities that assume a significant part in mathematics in South Africa, research (Alex and Mammen, 2016) illustrates that there are a few impediments in the teaching and learning of geometry. Studies over the years have shown that there has been dissatisfaction regarding high school geometry syllabus and poor performance of learners in geometry (Burger and Shanghnessy, 1986; Alex and Mammen, 2016). Researchers Atebe and Schafer (2011) argue that the teaching of geometry is quite possibly the most overwhelming encounter in most schools across the country, mainly because learners experience challenges in apprehending the instructional methods used by educators. Additionally, Mji and Makogato (2006) revealed from their study that many learners have a great fear of mathematics. Learners cannot comprehend the essential ideas of mathematics due to various reasons. Some of the plausible reasons could be their disposition, the inability to see the meaning behind the learning of geometry, larger class size, insufficient resource materials such as textbooks, reference books, support material, and the inability to comprehend mathematics associations to other fields.

Luneta (2014) contended that inadequate educator knowledge in teaching geometry becomes a challenge for learners to understand essential geometry concepts and thus cannot succeed in developing adequate problem-solving skills. Depper, Sonderegge and Stice (1991) highlighted the following problems experienced in developed countries in the teaching of geometry that needed attention: real world relevance and conceptual foundation of geometric concepts were not given due consideration; oral communication skills were not encouraged by educators; mathematics fell short to connect to the child's environment and thus the child could not perceive the significance and immediate application; geometry is not integrated with other topics and was isolated from other learning areas; learners did not engage in extensive group work in most classes; educators experience problems with the changing curriculum because they are not included in the formulation of the curriculum and, non-accessibility of sufficient resources such as textbooks, models and other media to satisfy academic needs for the new curriculum.

Euclidean geometry is a significant challenge at school level, mostly in the FET phase that is grades 10, 11 and 12. This challenge may stem from the learners' lack of experience as well as geometric knowledge and understanding from the lower grades. Hamzah (2017) recommends an improved primary school geometry syllabus with the van Hiele levels that would foster success in senior secondary schooling. The van Hiele theory was constructed by Dutch researchers Dina van Hiele and Pierre van Hiele in 1957 to describe how learners learn geometry. Geometric thinking is described in five levels. Hamzah (2017) further argued that the general geometry syllabus focuses more on high school instruction and insufficient subject matter is evident in primary schools. Feza and Webb (2005), in their studies, highlighted that one of the reasons for failure in geometry at the higher grades is that learners are moved from one grade to the next in the primary phase even though they may be lacking experience and knowledge. They also concluded that language proficiency remained a barrier to learners achieving the expected requirements since English is a foreign language for most of them. Many countries (Singapore, China, Australia and U.S.A) have placed great emphasis on mathematics teaching and learning.

As such, the performance of learners in mathematics and geometry in a comparative perspective is scrutinized extensively. Several large-scale assessments TIMSS (2015) and the Programme for International Learner Assessment (PISA) rise to debates about the relative performance of different nations for mathematics, science, and literacy (Coughlan, 2016). This has sparked debate around the performance of Asian countries versus countries in the west. Singapore is one of the top-performing countries in mathematics, including geometry, according to TIMSS report (2015). Creemer and Kyriakide (2008) argue that adequacy on learner accomplishment is staggered in nature and educational results are affected by factors at learner level, classroom level, the school level, and public and provincial levels. Therefore, this study explores all the above levels to identify strategies that will assist in reducing poor performance.

2.5 FACTORS THAT CONTRIBUTE TO CHALLENGES IN THE TEACHING AND LEARNING OF GEOMETRY

2.5.1 ATTITUDE TOWARDS GEOMETRY

Farooq and Shah (2008, p. 34) define attitude broadly as "a way of thinking, acting or feeling" which affects development. A comprehensive definition that includes the cognitive and behavioural components is useful in appreciating the development of attitudes towards mathematics. Learners' attitudes are affected by their experiences within their environments as well as the culture existing therein. In the words of Neale (2009, p. 87), attitude towards mathematics is "aggregated measures of liking or disliking of mathematics, a tendency to engage in or avoid mathematics activities, a belief that mathematics is useless or useful". This definition aptly lends credibility to the significant role of an uplifting perspective towards the teaching and learning of mathematics, specifically geometry.

Utley (2004) defined attitude towards geometry as a set of beliefs focussing on geometry that predisposes a person to respond in a certain way. Bindak (2004) expressed that if learners' developed a positive attitude towards geometry they are expected to like geometry, participate in classroom activities and be high achievers in geometry. In most instances, attitudes are marked by strong emotions in favour or against geometry. Attitudes are connected with some feeling that relates to past learning experiences and or the learning environment. Rumanova and Vallo (2012) found that learners had greater confidence about their geometric skills and were aware of its usefulness when they had a positive attitude towards learning geometry. According to Gresham (2018) attitude towards geometry include liking, enjoying and interest in geometry, or the opposite, and at worst geometry phobia. This means that learners' have to like geometry, enjoy the activities performed in geometry and have an interest at heart for geometry. Guzel and Sener (2009) emphasised four components that express a learners' attitude towards geometry, namely, enjoyment, motivation, perception of the importance of geometry and freedom from fear of geometry. According to Sarmah and Puri (2014), attitude refers to the inclination of an individual to react optimistically or adversely towards a circumstance, idea, object, or someone else. Yang (2013) highlights the school, peers, the learner's home environment, and society as factors affecting a learner's attitude towards geometry

Several studies (Mensal, Okyere and Kuranchie, 2013; Philippou, 2003; TIMSS, 2008) found that there is a significant positive connection between attitude and performance in mathematics. Researchers (Henson and Eller, 1999) suggested the following methods to improve learners' attitudes towards mathematics, to deal with the feeling of uneasiness towards mathematics or negative mathematical self-image. Educators must praise learners, assign challenging tasks to learners while aware of their capacity level, and give learners the self-sufficiency needed to finish these tasks. The more learners understand, the less anxious they will feel. Research (Alex and Mammen, 2016; Fiore, 1999) on attitudes towards mathematics has concentrated on two significant features, mathematics self-image or self-worth and mathematical anxiety. According to Fiore (1999), strengthening learners' endeavours in mathematics starts with assisting learners in fostering a positive self-image. Alex and Mammen (2016, p. 32) state that: *"*Mathematics self-concept refers to a person's perception of their ability to learn new topics in mathematics and to perform well in mathematics classes and tests.*"*

Attitudes can transform and develop with time (Syyeda, 2016). A cheerful attitude improves learning (Mutai, 2011), and a negative attitude can obstruct performance (Joseph, 2013). Challenges that learners' experience in their learning can influence the development of an attitude. Feza and Webb (2005) researched grade 7 competency in geometry and found that many of them lacked the skills needed to answer geometry problems. The absence of prior experience and information contribute to the learner's inability to adapt to advanced thinking in the later years of schooling. A similar study was conducted by Alex and Mammen (2016) on grade 10 learners and the results concluded that regardless of the expectations of the CAPs learner's level of knowledge was at level 0 as prescribed by the Van Hiele's levels. This leads to the learners losing interest and developing a negative attitude towards the learning of geometry. Mji and Makgoto (2006) point out that underperformance of learners in mathematics could be accredited to incompetent educators teaching this subject. The South African curriculum change in 2006 removed geometry from the curriculum resulting in colleges producing educators of mathematics without content knowledge for the teaching of geometry. Educators who had not studied this topic in their secondary schooling or their educator education lacked interest and had a negative attitude towards the teaching of geometry (Sanni, 2007).

A study conducted by Tsao (2017) in China found a significant positive relationship exists between usefulness of geometry and attitude towards learning of geometry. Many of the educators interviewed had a positive attitude towards the usefulness of geometry and believed that geometry was a valuable and necessary topic which could help in future careers and education thus learners also liked and enjoyed learning geometry. The educator assumes a vital role in the learning of geometry (Mensal et al., 2013). Therefore, educators and all those responsible for the schooling of learners have an extraordinary duty in assisting to develop a positive attitude towards the learning of geometry. Aiken (1972) and Haladyna (1983) further expounded that it is not only the educator's attitude that is responsible for the learners’ attitude but also the educator’s understanding of geometry is critical in constructing learners’ attitude and their extent of confidence.

The educator or adult, as indicated by Vygotsky’s (1978) Zone of Proximal Development, also assumes a critical role in the learner’s instruction of geometry. If the attitude of the more knowledgeable other is negative, this will affect the learning process and hinder performance. Vygotsky argued that “higher psychological processes are originally social processes shared between people, particularly children and adults” (Brown and Ferrara, 2000, p. 22). Initially, the child encounters dynamic problem-solving with others, then progressively functions on his own and internalizes the idea. Brown and Ferrara (2000) emphasize that a negative attitude at this point from others will affect the child’s learning process. Haladyna (1983) also emphasized that the general attitude of the class towards mathematics relates to the quality of teaching and the psycho-social climate of the class. King (2018) contends that the teaching approach that connects to reality helps learners develop and apply geometry concepts to a problem that makes sense to them.

As learners move from the primary phase to senior phase of schooling, their level of interest and motivation to continue learning geometry is very low (Mji & Makgoto, 2006). In support, Gottfrield (2001) indicated that academic motivation in the subject mathematics dropped drastically from early schooling through to later years. According to Billstein and Williamson (2003), there is a decline in positive interest towards mathematics in middle school years. Research indicates that there are several factors that affect learners' interest and motivation in learning geometry, such as feeling valued, fear of punishment, the classroom surroundings, the level of difficulty of the task, real-life exercises, advantages, and threats from peers/ educators and the actual instruction. Educators have a more significant influence on learners' levels of motivation and interest compared to parents since learners spend most of their day in the school. According to Wentzel (1997), learners who feel motivated and eager to participate in classroom activities obtain support and are valued by their educators. Usiskin (1982) reveals that numerous learners do not succeed in comprehending key ideas in geometry, leading to a lack of interest and the development of a positive attitude towards geometry. Maddock and Maroun (2018) suggests that the usefulness of geometry and how one feels when solving geometric problems (enjoyment or boredom) and one's confidence in geometry will ultimately be useful in learning geometry. For instance, if the learner feels that geometry is important in solving day to day problems both at in and out of school then the learner will be motivated to learn the subject. Also the enjoyment of the subject or being confident in that particular subject could give rise to learning the subject effectively.

2.5.2 LEARNING MATHEMATICS AS A DISCOURSE

2.5.2.1Teaching Mathematics as a Language

The myth that mathematics is a language-free subject existed for a very long time. The reform document presented by the National Council for Educators of Mathematics (NCTM) (2000), Curriculum and Evaluation Standard (1989), articulates communication as one of the five key objectives of procuring mathematical understanding. The importance of this skill is shared by the South African Revised National Curriculum Statement of 2002 (DBE, 2006), which specifies that one of the seven significant goals that learners should acquire is to communicate proficiently. It is further emphasized by National Curriculum Statement grade R-12 (DBE, 2011) that educators of mathematics should develop learners who are competent to communicate successfully utilizing visual, illustrative, or language abilities in different modes (DBE, 2011). These results infer that learners be presented with different interrelated encounters that urge them to read, compose and talk about mathematics.

Italian astronomer and physicist Galileo Galiliei drew attention to the following quote, “mathematics is the language in which God has written the universe.” The inquiry that always emerges is whether mathematics is genuinely a language similar to English or Chinese. Language is an arrangement of words or codes utilized inside a subject and may allude to an arrangement of communication using images or sounds. Researchers in mathematics (Bahr, 2008; Morgan, Craig, Schuette and Wagner, 2014) argue that mathematics satisfies all the requirements for consideration as a language since it contains a glossary of phrases, words, or symbols with association to meaning. One may view mathematics to contain grammar in the form of specified rules that detail how vocabulary is utilized and grammar that coordinates this into linear structures or propositions as well as narratives or discourses consisting of strings of syntactic schemes. The images, their implications, linguistic structure and syntax are similar throughout the world and mathematicians, researchers and others utilize this to communicate ideas.

Mercer (2000) engaged in broad research into classroom discourse and emphasized the responsibility of educator-learner talk in the learning of mathematics. Once a chatty space is set up, the educators can expand learners’ knowledge and critical thinking in the course of more excellent dialogue (Gibbons, 2003). Active discussion on mathematics between peers and educators is essential. Bishop (2012) highlighted “the ways in which we talk and interact with each other are powerful because they affect who we are and who we can become with respect to mathematics” (p. 70). Robust discussions within a classroom will allow learners to develop confidence and consider themselves mathematicians, thus improving their performance in mathematics.

Researcher (Ezrailson, Karnon, Loving and McIntyre, 2006) established that when educators centre around communication and association in the classroom, learners preserve 90% of what they execute and say as they take part in conversations. Interestingly, these researchers stated that learners would only remember 20% of what they hear, 30% of what they see and 50% of what they see and hear (Ezrailson et al.., 2006, p. 56). It is evident that when learners are engaged in communication in a mathematics classroom it plays a significant part in improving the achievement of redundant geometry learning and teaching. In perceiving the significance of communication as an approach to consolidate learners' information and understanding, many countries (Ontario, U.S.A., Singapore and Japan) have included it as a critical ability that must be taught in the mathematics curriculum (Mecoli, 2013). Bahr (2008, p. 21) described three parts of language applicable to mathematics, "the language of the learner, the language of the educator and the language of the subject." Language includes "external operations as aids to the solution of internal problems" (Bahr, 2008, p. 43). Haliday and Matthiessen (2004) further portrayed language in mathematics as a planned semiotic framework involving architecture beyond natural and verbal language. Mathematical thinking, therefore, encompasses the utilization of discourse, images, drawings, signals and activities as characteristics of language. Mckinney (2010) contends that language should be viewed as the driver of expression rather than as a vehicle since what we see and are able to learn is an action of our language process. In their research, Wu and Lin (2014) found four main topics related to communication in mathematics in the Taiwanese's Mathematical Curriculum. Similarly, in Singapore (Ginsburg, Leinwood, Anstrom and Pollock, 2005) and China (Wang, 2001), research into the mathematics curricula emphasized communication as a skill needed to be developed in both educators and learners for effective outcomes. In South Africa, the 2005 mathematics education curriculum emphasized language as playing a significant part in learning mathematics, stating that the learner should use mathematical language to converse mathematical concepts, speculations, and thought procedures (DBE, 2006).

Despite the many researchers emphasizing the importance of communication as an important tool in building understanding for learners in the mathematics classroom, Battista (2007) found that in United States, these interactions are not typically found in a mathematics classroom.

2.5.2.2 Teaching geometry as a Language

The essential capacity of language in geometry instruction is to empower both the educator and learner to express mathematical knowledge with precision. Words are incredible in developing concepts, procurement of forthcoming capacities, and the exchange of understanding concepts. Patkin (2015) indicates that language has three fundamental purposes viz permitting individuals to communicate with each other, directing the learning cycle, and allowing learners to remember information beyond memory. This confirms the important role that language plays not only in communicating ideas but in addition it assists in developing thinking. Haliday (2003, p. 121) contends that learning language incorporates acknowledging "how to mean"; thus, the language of geometry includes figuring out how to make and share mathematical meaning utilizing language relevant to the specific situation, which is more than seeing and responding to words separately.

Sfard (2008) stress that analyzing the learners' and educators' discourse in a mathematics classroom is relevant to better understand learning and teaching phenomena in geometry. There must be an active dialogue between the educator and learners for accomplishing the outcomes of geometry teaching as espoused by Mercer and Littleton (2007) when they said that "for an educator to teach and for a learner to learn, they must use talk and engage in joint activity to create and negotiate a shared communicative space" (p. 21).

Brown and Renshaw (2007) state that effective questioning techniques can bring forth this dialogue approach. Questioning strategies should elicit learners' thoughts, reasons, experiences and opinions. Small (2013) stated that peer-peer communication, questioning and reflection all assume a significant part in mathematics classroom and provide opportunities for effective learning of geometry. The ability to communicate at the various levels, as indicated by the Van Hiele model, is of paramount importance for the success of geometry learning. Clements and Battista (1992) added that as learners advance in geometric thought, they display a distinct degree of language and communication than that acquired previously. The language utilized by educators likewise assumes a vital part in the improvement of comprehending two-dimensional shapes and their associations to other shapes, which is foundational to the advancement of geometry. Chinnappan and Lawson (2005, p. 213), in support, stated that language affords a degree of “representational fluency” when learning about two-dimensional shapes. Mercer (2000) concluded that the symbolic language of mathematics, in both the teaching and learning of geometry, satisfies a double role as a medium of communication and as a channel of thought by making it conceivable to address mathematical ideas, structures and connections. A lot of Vygotsky’s (1978) research of cognitive development centres around the importance of language. According to Atebe (2008), to teach mathematics meaningfully, especially geometry, language plays a significant role in developing a learner’s conceptual understanding. Kilpatrick, Swafford and Findell (2001) point out that significant measures of conceptual understanding are the capacity to address mathematical situations distinctly and realize how various representations can be valuable for their indicated purpose. The conceptual understanding of geometry lies within the meaning that learners associate with geometric concepts. Feza and Webb (2005) highlight that the key to successful accomplishment in learning geometry lies in the procurement of specialized terminology. This resonates with Atebe’s (2008) conclusion that students should secure the right technical concepts and utilize them accurately, to impart their thoughts regarding ideas in geometry. The difficulty of learners in understanding geometrical concepts is due to many concepts which have different meanings in daily life and in different contexts (Patkin, 2015). For example, solid object in daily life means having particle coherence. In mathematics a solid object means three-dimensional geometric image like polyhedron or cone. The word “side” in daily life means any party, teams, interest or opinion that contradicts with another party. In geometry a “side” refers to one of the lines connecting the points of polygon (Patkin, 2015). The educator therefore needs to provide clear definitions and explanations of concepts to prevent misconception and difficulties.

Usiskin (2012) claimed that mathematics is both a written and spoken language and that the spoken aspect is necessary for the understanding of mathematical concepts. He further articulated that mathematics is a representational or pictorial language since the study of geometry considers the properties of pictures, where these pictures establish themselves as part of the language of mathematics. While participating in meaningful mathematical discourse, learners figure out how to foster clarifications and rationalizations, which results in insightful thinking (Jansen, 2012). Supporting this, Ramsussen and Yackel (2002, p. 65) stated that “an emphasis on explanation and justification in a mathematics classroom leads to mathematics learning that emphasizes reasoning.” This skill is therefore essential for effective geometric thought. Jansen (2012) added that discourse presents learners with opportunities to discuss, explain and justify their thoughts, pose questions and answer problems.

It is important for educators constantly adjust their own training to develop useful and significant geometric talk in their classrooms (Brodie, 2007). Sheldon and Epstein (2005) support that when learners communicate their understanding of ideas, rules, theorems and rationale involved in the steps of solving a rider, they have the chance to build upon and expand their comprehension of information structures. Communication in mathematics necessitates that learners become familiar with the language related to the mathematical concepts they are being taught. This knowledge then empowers learners to construct a comprehension of ideas by having to clarify them (Brodie, 2007). For learners to become proficient in geometry, knowledge and understanding of vocabulary is essential. Good teaching practice includes instilling in learners the applicable mathematical vocabulary. In studying geometry educators must encourage learners to communicate their understanding of geometrical concepts and expressions using their own words, diagrams and the relationships between symbols and diagrams that form the basic geoemtric knowledge.

Recognizing the relevance of language assuming a significant part in the instruction of geometry drives one to rethink the teaching of geometry. The motivation behind learning geometry and achieving its outcomes infers that exposing learners to a number of different encounters that support them in, amongst other things, to peruse, compose and talk about mathematics will achieve these goals. Clement (2003) concurs with the important role that language assumes in the teaching of geometry, advocating that comprehending the correct terminology will assist learners in gaining greater confidence in geometry. According to Atebe (2008), our regular speech and writing contain rich vocabulary in a number of geometric concepts such as line, parallel, plane, point, rectangle, circle, square and triangle and this vocabulary assists us to convey our ideas to others in accurate forms.

Leiken (2015) point out that the van Hiele levels highlight the significant part language plays in the instruction of geometry as it progresses through the levels from actual to abstract formations. In focussing on the importance of language, van Hiele (1999) mentions that the many disappointments in the teaching of geometry are a consequence of language hurdles, for example, the educator utilizing language of an elevated level than is perceived by that of a learner. Learners who have finished primary schooling ought to be at van Hiele thinking level two, which recommends that learners must explain and address the attributes and relationships between two-dimensional shapes and three-dimensional objects in an assortment of orientations (DBE, 2010). Van Hiele (1999) focussed on the significance of the presentation, use and attainment of language at every stage of deduction, in recognition of language serving specific significance inside the learning phases. It is therefore fundamental for educators to use appropriate words and symbols when introducing new concepts and to make certain that there is decent comprehension of foundational ideas. The correct usage of the language of geometry establishes the path for future work and empowers learners to tackle geometrical problems and improve communication thoughts.

Pugalee (2004) emphasized the significant association between language and mathematical learning. He affirms that writing helps learners internalize the features of viable communication, which encourages thinking and problem-solving abilities. He further recommends that educators carefully analyse learners' writing in mathematics to understand their thought processes and how they utilize facts. In the early years Emig (1983) claimed that writing in content areas, such as mathematics, could cause learners to analyse, compare facts, and synthesise relevant information. Further claims are made that, in this manner, the act of writing serves as a means for learners to reflect on their thinking and on what they are learning (Machaba, 2013). In the event that writing is considered as a way of communication, in particular between the learner and the educator, then it should allow learners to convey their feelings, fears, attitudes, experiences, and ideas about mathematics and how well they understand the content. This can lead to higher cognitive functions such as basic reasoning and critical thinking, which in turn will lead to better learning of geometry. These practices closely relate to Vygotsky's (1978) theory of the ZPD, where the learner is able to use experiences to develop solutions. The writing of mathematics is supported by NCTM (2000) were they recommend that writing in mathematics must be encouraged across all grades. NCTM (2000) additionally suggests that learners are urged to given written clarification on how they arrived at a solution for a geometric problem, as they would appear in a textbook. Writing as a form of communication in geometry lessons assists learners to merge their reasoning and ponder over their activities with the aim of refining their thoughts and concepts. It, therefore, becomes fundamental for the improvement of teaching that all forms of communication in the language of instruction be utilized for learners to reflect and express their thoughts.

2.5.2.3 Teaching Mathematics and Geometry to English Second Language Users

The mother tongue language of the learner is of significance to build a clear structure of mathematical ideas since thoughts communicated between the instructor and the learner either through oral or written material is important (Cocking and Mestre, 2004). There has been debate on language proficiency and learning ability in mathematics for a long time and a lot of this discussion focussed on the achievement of disadvantaged learners with restricted expertise in English. Imparting mathematical thoughts for understanding the meaning is not easy even when the educator and learner have a specific first language; however, the issue is more intense when the language of instruction is different. Learners using a foreign language in understanding geometry further present problems as illustrated by Orton (1992, p. 39) when he states that:

"Language is important not only for communicating but also because it facilitates thinking. The language used for thinking is most likely to be the first language; thus mathematics communicated in one language might need to be translated into

another to allow thinking and would need to be translated back in order to converse with the educator".

The ability to acquire mathematics in a second language requires a learner to be skilled at two new languages: ordinary English and mathematical English. Howie (2003) found that fluency in English is a significant factor in learning science and mathematics at secondary schools. A study conducted by Lim and Presmeg (2011) found significant challenges when teaching mathematics to learners for whom English isn't their first language. Their study found that there is a significant deferral in covering the expected curriculum because the educators needed to interpret the mathematical concepts for learners. National and international overviews of mathematical achievement uncovered that many secondary school learners were unable to distinguish and name shapes like triangle, trapezium, parallelogram, and rhombus and kite (Triadafillidis, 1995). Howie (2003) concluded that learner achievement is even mediocre with regards to tasks involving comprehension of properties of shapes. Geometry taught in South African schools is mainly in English. Since English isn't the mother tongue language of most learners and educators, this necessitates code-switching to explain concepts that learners have difficulty with. Code switching is a strategy utilized by educators to disseminate knowledge to learners in multilingual and bilingual contexts. Munoz and Mora (2006), found that code-switching allow learners to be active participants, bridge the gap between educators 'and learners as well as build confidence in learners' concepts of geometry.

The challenge in assisting learners becomes a dual task of teaching mathematics and English for mathematics educators. Adler, Reed and Bapoo (2002, p. 45) highlighted this in the following statement "Mathematics educators, in contrast, have a dual-task. They face the major demand of continuously needing to teach both mathematics and English at the same time". The problem associated with translating terminology in geometry and retaining meaning presents a major challenge for educators of mathematics. Learners are unable to communicate effectively with educators when they are unable to comprehend the instruction of the educator, which is further stressed by Cooney's (1975, p. 97) words "communication breaks down when people do not have certain concepts." A good understanding of basic concepts like transversal, parallel lines, angle and side will assist the learners when reasoning in proofs. Concepts empower learners to build information and talk to each other about geometry. The importance of discussion about geometric concepts is extremely important in the teaching of geometry, yet in a study conducted by Ndaba (1997) on the utilization of language in geometry lessons in KwaZulu-Natal schools, discussion as a method of teaching is hardly used. The absence of using language to explain ideas adequately infers a deficiency of comprehension of mathematical concepts and this was evident in secondary school learners' inability to be consistent in their use of words. In a study conducted by Kutama (2009) learners' poor performance in geometry, was attributed to learners' inability to communicate thought using either talking, writing or drawing in the activities given. He found that learners only recognized a few shapes, made meaningless sentences and mathematical statements that lacked cohesion, and could not explain in words what they observed during the activities which could lead to construction of geometric concepts. Kutuma (2009) suggested that learners must communicate their ideas in their mother tongue and the language of instruction interchangeably. The idea is that through talking and sharing their experiences with geometric concepts, might help learners understand and operate at a higher level of geometric thinking.

2.5.3 VISUALIZATION IN GEOMETRY TEACHING AND LEARNING

This segment of the literature portrays the significance of visualization as a method for creating spatial capabilities and geometric thinking. Visualization serves the purpose of being both an inner and outer course for delivering and manipulating images that form a fundamental part of geometric and spatial thinking. Visualization can therefore be perceived as a critical component for creating spatial capabilities. The idea of visualization in geometric thinking is described and the part that visualization plays for constructing and communicating geometric concepts is discussed as a necessary component. In Euclidean geometry visualization would refer to diagrams that are either seen physically or mentally. Moyer (2001) found that leading mathematicians, Albert Einstein, Carl Friedrich Gauss, used pictures to cultivate their thoughts and only used algebraic conventions when they had to share their results with others. This study aims to ascertain if visualization of geometry problems is a barrier to effective geometric thought since leading mathematicians used images to develop their thoughts, images could definitely be considered useful to learners when working with Euclidean geometric problems. Visualization assists learners by making geometric concepts concrete and clear to understand.

Couto and Vale (2014) depicted geometry as containing visualization and understanding of geometric figures. Visualization is also a key aspect of spatial reasoning (Clements, 1999; Liben, 2006; Maier, 1996). Learners foster the capacity to visualize, problem-solving, critical thinking, deductive reasoning, clarifying and proving, as they develop and advance in school (Jones, 2007). The capacity to reason about geometric figures starts by seeing with non-verbal reasoning (Van Hiele, 1999), where learners depict mathematical shapes by their appearances without the words required for characterizing what they perceive (Levenson, Tirosh & Tsamir, 2011). Concrete experiences provide tangible experiences to help learners develop concepts. Clements (2003) and Moyer (2001) claimed that what learners identify about specific properties of figures may be evaluated through classroom discussion. Both concrete and virtual schemes have appeared to enhance the visual skills of learners (Gutierrez, Leder and Boero, 2016). Visualization is pertinent for conveying both verbally and non-verbally in this manner, creating theoretical comprehension in geometry (Arcavi, 2003; Clements, 2003). Researchers (Zamkova, Prokop and Stolin, 2016) describe visualization not merely as the capacity to view and portray an object but to address, change, produce, record and consider visual data. A study conducted by Drezmann and Lowne (2012) revealed that the absence of prior experience and poor construction of mental ideas results in learners encountering challenges in visualizing and assessing their thinking. Solving geometric activities frequently requires spatial thinking and clear guidance (Gillies & Haynes, 2011). Ancavi (2003) described visualization as an ability that assists learners with identifying shapes to make more intricate shapes or figures, and to uncover connections between them. Geometry topics, as indicated in table 2.1 above, expect learners to identify objects visually and recognize their characteristics by matching them with their prior encounters involving related entities. According to Tay (2003, p. 41), learners should have the ability to “touch-see-and do” and interrelate with objects of their content to be successful at geometry in a more inventive and successful manner. While developing methods to teaching and learning geometry, it is imperative to make certain that the arrangement in the early high school stage motivates learners to foster an interest in geometry by making available opportunities to explore concepts visually and tackle real-life issues.

Visualization is a critical tool for enhancing learners' understanding of a variety of concepts in different learning areas. Visualization in mathematics is the process of forming images and the effective implementation of these images improves mathematical discovery and understanding of mathematical problems. The use of visualization to support learner exploration of mathematical concepts is not new since it always encouraged higher levels of engagement (Sue & Griffin, 2016). Arici and Aslan-Tukak, 2015) likewise asserted that the utilization of visual illustrations to ascertain, depict and rationalize geometric outcomes was fundamental for finding out about shapes. In contrast, Bishop (2012) described visualization as an intricate process that elaborated symbolism with or without a chart to coordinate information into significant arrangements that were critical in directing the advancement of an answer. The first form of learning a child experiences is the ability to identify objects based on their visual images. Heeralal and Dhurumraj (2016) indicated that the capacity to make mental pictures was inborn in all of us. In spite of this statement, Presmeg (2006) discovered that not all individuals preferred using visualization to solve a problem since some individuals preferred rote-type learning experiences, while others preferred knowledge in particular learning situations.

Popular systems for visualization have brought many benefits for the successful teaching of geometry in particular. English (2001) asserts that diagrams are a physical forms of a mental image and it allows learners to associate a word with a pictures. Hendricks, Botha and Adu (2016) state that diagrams play a vital role in communicating mathematical meaning. Kutuma (2002) expresses that when learners draw diagrams by themselves their understanding of Euclidean geometry increases. In the South African classroom, English is not the mother tongue of many learners, diagrams will assist learners to associate words with physical pictures. Learners can observe relationships on the diagrams that they may not note if they had not have drawn a diagram. The above researchers encourage the use of diagrams as visual tools to enhance the teaching of geometry. Diagrams can play a significant role in teaching geometric proofs. Lavy (2006) asserts that diagrams support proof processes and are important to visual proofs. In support Carter (2009) state that pictures and diagrams in geometric proofs are fundamental in communicating and making sense of concepts. A significant part of the geometry content taught at the FET phase focuses on proofs and theorems as indicated in table 2.1, diagrams and visual images may assist learners to develop a better understanding of theorems and proofs.

Farajallah (2016) stated that visual media contributes to learners' geometry achievement and facilitates their active involvement. The incorporation of geometry software such as GeoGebra into the learning of Euclidean geometry has been suggested by researchers (Williams, Charles-Organ & Adesope, 2017; Kosa, 2016), to inspire learners and encourage geometry learning. The advancement of technology has allowed learners to experience visually many figures that they may not have physically experienced before. The use of computers can create narrative animations and interactive animations, allowing the viewer to make their explorations. Moodley (2013) stated that a computer-generated programme can assist in Euclidean geometry learning because it facilitates an active interactive manipulation of geometric figures. A learner can move, relate or stretch the figure, and observe what properties change. Drawing accurate 3-D figures lead to new insights into Euclidean geometry. This suggests that the school curriculum must place greater emphasis on visualization to develop reasoning skills since the learner is able to communicate and understand the problem better when geometric problems are presented visually, using computers and other media. This study aims to investigate the use of visual tools in geometry lessons.

2.5.4 GEOMETRY CURRICULUM

There has been much apprehension with regard to the present implemented curriculum across the globe. In many countries, literature displays a great emphasis on the remembering of facts and does not give much focus on understanding and the utilization of these facts (Yakkel, Cobb, & Wood, 1999). Tsanwani (2009) proposes that an educational programme that does not view learners as capable of higher-order reasoning and thinking should be replaced and supported with essential and applicable information and activities. According to Venkat and Graven (2017), a curriculum that supports the growth of learners' skills and information, significantly associated with their learning and consequently their attainment.

According to Dube (2016), Euclidean geometry, consisting of statements justified by proofs, which depends on mathematical axioms and logic, requires educators to assist learners in linking new knowledge to existing knowledge. Most educators in South Africa perceive geometry as an especially troublesome part of the mathematics curriculum and intentionally stay away from teaching it (Wentzel, 2016). According to Adler and Pillay (2016), Euclidean geometry is among the new topics introduced into the school mathematics curriculum in South Africa that seems to be a great challenge to most educators and learners. Adolphus (2011) stated that one of the problematic ideas that educators struggled to teach in mathematics is Euclidean geometry, because of the technicalities related to language, visualization of objects for better identification of properties, and inadequate conceptual understanding. According to Aldridge, Fraser and Ntuli (2009), the ongoing curriculum changes since 1994 have resulted in educators being insufficiently prepared, especially for the instruction of mathematics in the rural areas.

Studies in South Africa highlights that both learners’ and educators’ geometry progression does not meet the levels of advancement as indicated by the van Hiele’s levels of geometric thought (Remillard, 2005; Feza & Webb, 2005). The NSC diagnostic report (2015 – 2019) repeatedly highlights the difficulties experienced by educators with the teaching of geometry. The Subject Assessment Guidelines for Mathematics (DBE, 2005), an adjacent report to the Mathematics National Curriculum Statement (MNCS, 2003) for the FET band, designated Euclidean geometry's evaluation standards as optional. The assessment standards for Euclidean geometry were excluded in the main mathematics assessment of the CAPs curriculum in 2014. As specified by the subject assessment guideline, the justification behind making geometry optional was to give educators time to capacitate themselves to teach the content of Euclidean geometry. The introduction and implementation of the MNCS happened during a period of political reform where the key focus was on nation-building. Since the democratic elections in 1994, there were major changes made to the mathematics curriculum. The eighteen departments separated along racial lines collapsed to form a single ministry to create a centrally controlled education o that was non-discriminatory and cohesive. The curriculum has undergone three phases of development that is Curriculum 2005 (C2005), an outcomes-based curriculum, the National Curriculum Statement (NCS) and the Curriculum Assessment Policy Statements (CAPS). These ongoing curriculum changes have been a barrier in that they have de-motivated educators and learners (Bowie, 2015). The justification for the inclusion of geometry in the school mathematics curriculum is to furnish learners with opportunities to foster spatial appreciation and geometrical instinct with the capability to visualize and apply geometrical properties and theorems (Jones, 2000). It is, therefore, necessary that teaching approaches encourage and develop the utilization of conjecturing, deductive thinking, and proof. There is a need for fostering the proficiency of applying geometry through demonstrating and critical thinking in a range of situations.

Goldenberg (1996, p. 23) recommends the re-organization of the geometry curriculum on what he refers to as “habits of mind” that alludes to a changed manner in which learners think about mathematics and adds that the curriculum should include the ability to interpret images and diagrams. He further identified the following skills as significant habits to be included in the geometry curriculum, the ability to visualize, decipher between seeing and discussing presented information, searching for differences, combining discovery with deductions, and developing systemic clarification with proof. Despite these habits being included in the South African planning schedules for mathematics, it is imperative that the training of educators for implementation for the successful teaching of geometry be given due consideration.

Phaeton (2016) differentiated between the expected programme and the programme executed in that the expected programme is the curriculum as planned by the educational system, and the executed programme is the one educator's practice in the classroom. The implemented curriculum relies largely on educators themselves as well as educator preparedness and re-skilling while in-service. Getting the intended curriculum delivered becomes challenging when it necessitates convincing educators to change teaching methodologies after years of practice in a certain way. This is the reason the traditional “chalk and talk” method is dominant in South African schools. Researchers (Bantwini, 2009; Rogan and Grayson, 2003), in support of this view, expressed that an all-around well-planned curriculum drive will fail if there are drawbacks in the execution process. Accordingly, Parker (2006) suggested that educators must be included in planning the intended curriculum for its implementation with conviction, leading to effective learning.

Although the mathematics curriculum in the USA gives emphasis to learners being able to apply the knowledge that they obtain in the geometry classroom, there is insufficient active participation of learners to acquire such knowledge (NCTM, 2000). In the USA, geometry is taught in one year, which lies between two years of algebra, and thus teaching is not very effective as learners are unable to attain the necessary skills and abilities for problem-solving (Stylianides, 2007). This curriculum cramps many geometry topics in one year, and the curriculum is not very integrated. The objective of teaching geometry, according to the NCTM (2000), is for it to empower learners to become problem solvers in the work environment; geometry ought to set up the most capable learners to become scientists, statisticians, and engineers.

The educational programme in Australia is coordinated into content strands, and geometry is joined with measurement, implying that the two content areas are connected. The curriculum is divided into "strands" to refer to the various divisions of geometry, namely shapes, transformation, and location (Seah, 2015, p. 23). Learning of material ought to be created from four capability strands of comprehension, familiarity, critical thinking, and reasoning. There is minimal emphasis on visualization in the Australian curriculum, and any change towards defining or proving geometric concepts is lacking. Regardless of Seah's (2015) claim that the Australians' curriculum assists learners 'to form an advanced understanding of concepts, it lacks to build effective geometric arguments. Lowrie, Logan and Scriven (2012) found that most learning in secondary and primary school contexts revolves around numbers and algebra. The accentuation on geometry and especially spatial thinking is absent or is limited (Wares, 2014). He further claims the need for reasoning skills to conclude using non-routine problems was not present in geometry lessons.

As indicated by NCTM (2000), thinking and verification should be granted the most extreme significance at all levels. Critical thinking can be encouraged through all branches of mathematics by using various settings. Thinking and proof are taught inductively in the USA mathematics curriculum (NCTM, 2000). Singapore, one of the top-performing countries in the world, has a mathematics curriculum that depends on mastery guidelines. Communication and the language of geometry form an important part of the teaching process (Dindyal, 2015). The central focus of their curriculum is the development of problem-solving competency, and technology is integrated into the teaching of geometry (Ministry of Education Singapore, 2019).

Jayaprakash (2005) looked at the teaching of geometry in England and Germany and found that they focus on proper planning and applicable methodology for evaluation and recording of learners' advancement and achievement. Educators utilize these innovative techniques and methodologies in creating an active, participative environment for teaching geometry. The NCTM (2000) suggests that instructors at each level assist learners in exploring a variety of possibilities and using them to develop efficient reasoning skills and proofs to affirm or disprove these possibilities. The importance of proof in the geometry curriculum is highlighted by Stylianides (2007) for doing, knowing, and communicating geometry. Wu (2018, p. 67) portrayed geometric proof as the "guts of mathematics" and further argued that learners should know how to write and understand the proof. This is supported the Brousseau (1997) that learners should be given a chance to prove or refute conjectures depending on their encounters. Preferably when learners complete secondary schooling, their clarifications ought to be more mathematically thorough, and they should be able to provide the properties they used in proofs (NCTM, 2000). It is significant for the development of effective geometric thought that learners be given adequate opportunities to become critical thinkers by listening and communicating their claims (NCTM, 2000).

Globally., proof and theorems form part of secondary school Euclidean geometry. Kolb (2014) highlighted that educators in South African schools experience great difficulty in teaching proofs and theorems. The application of theorems in riders poses a major challenge to many educators. Educators ‘content knowledge and their background learning experiences of proof and theorems may have an impact on the teaching and learning of proof and theorems (Fung et al.., 2017).

2.5.5 EDUCATOR QUALIFICATIONS

Researchers Mostafa, Javad and Feza (2017) emphasized that educators are the deciding factor of flourishing reform within an instructive training framework, so progression of educators through training is a strategy to get maximum and effective change within the schooling system. Kanjee and Moloi (2012) likewise argued that there must be effective educator training for South Africa to develop quality mathematics education that rejuvenates educators’ expertise. Thus, it is also important for educators to attempt to understand how their students think and learn and identify the learning encounters of their learners so they can re-skill themselves. Kanjee and Moloi (2012) further added that necessary support must be given to educators by relevant authorities. It is significant for educators to have the essential requisite geometry content knowledge and skills and be able to apply a variety of strategies that emphasize problem-solving for learners to master the concepts of geometry. Studies conducted on educators’ knowledge of geometry have consistently shown that geometry knowledge is problematic in terms of subject content (Jones, 2000; Ponte and Chapman, 2006; Telima, 2011). In South Africa, a large number of practicing educators have not been exposed to geometry, or some have only one year of university-level geometry, making the teaching of geometry for them very difficult.

Due to scarcity, in South Africa, of qualified educators in mathematics, many unqualified educators for the subject are teaching mathematics (Hendricks, Botha & Adu, 2016). The quality of mathematics educators and the quality of teaching are among the most important factors that shape learning and growth of learners (Ajibade, 2016). Kukla (2000) argued that it is only through constant associations that individuals construct their reality. Social interaction of the educator is practical to this study. Vygotsky emphasized in his ZPD the important role that the educator plays in assisting the learner. It is of great significance to note that educators' encounters occur in their specific circumstances as well as in the curriculum execution process. The ZPD is the gap between what learners know and what they accomplish given suitable and instructively strong direction and educational assistance. Since the ZPD includes connecting material that has not been acquired to material that learners' can recognize, educators assume an essential part in the implementation of the ZPD in the classroom. The responsibility of the educator is to build suitable scaffolds, key social associations, and guidelines that direct effective learning developments.

According to Grobler, Moloi, and Thakhordas (2017) mathematics educators have poor content knowledge and outdated teaching practices are some of the reasons for poor learner performance in mathematics. Marmlok-Naamam (2017) concur that most South African educators still lack fundamental understanding of mathematics and pedagogical practices. Chan and Yung (2018) emphasise the need for high quality mathematics educators to have a positive and significant impact on learner performance. Genz (2006) claimed that based on a number of sources, it is evident that learners at lower levels are not adequately developing the necessary skills in geometry to equip them for the content of more advanced geometry. Bantwini (2009) supports this argument in his report by saying that educators avoid teaching geometry in the primary phase due to insufficient pedagogical knowledge, poor lesson planning abilities and not enough instructional and appraisal techniques. It is a common practice in South Africa that educators in primary schools must teach all learning areas (DBE, 2013) since there is no requirement of specialization, which negates the effective teaching of geometry. Researchers (Arends, Winnaar and Mosimenge, 2017) state that there is a positive relationship between educator content knowledge and learner attainment in mathematics as learners taught by educators with a higher content and pedagogical knowledge will perform better.

The process of scaffolding in the learning of geometric concepts becomes difficult when the educator lacks content knowledge (Mji and Makgato, 2006). Learner's knowledge of the content, how educators should engage with learners mathematically, being perceptive of how learners deliberate about geometry and the challenges that they may encounter with newly introduced content, are significant academic realities for thought. According to Singh (2002, p. 121), “pedagogical content knowledge is the capacity of the educator to transform subject knowledge into forms that are pedagogically powerful”. This highlights the educator's fundamental role in adapting the content to suit the learner's background, which can only be achieved if the educator has solid content knowledge. Educators with insufficient geometric knowledge exacerbate the problems that students experience in the subject. Kutama (2012) concurs that the main source of poor performance in geometry originates in the detachment between mathematics pedagogical courses and the requirement of mathematical learner outcomes in schools. Adjusted educator training is therefore important to develop innovative ways in the teaching of geometry. Geometry requires expert subject matter that educators need to specialize in to impart to the learners. This typically necessitates an in-depth comprehension of concepts so that educators can extract and understand the learners’ mathematical thoughts (Brodie, 2007). Most mathematics educators do not have a thorough knowledge of the subject material and teaching abilities needed to clarify concepts, but their teachings also comprise rigid algorithms that learners’ are told to follow (DBE, 2010; Luneta, 2015). Arends, Winnaar and Mosimege (2017) recommend that for effective teaching of geometry, secondary school mathematics educators should have a solid understanding of the geometry subject material, an optimistic outlook towards mathematical instruction, as well as an arrangement with appropriate pedagogical principles.

According to Alfifi and Abed (2017), mathematics educator’s qualifications and teaching experiences may result in learners performing poorly in mathematics. According to Pournara and Barnby (2019) several countries have made attempts to improve mathematics educators’ content knowledge in order to improve learner performance. However, in South Africa there is little evidence that these interventions have had much impact on learner performance in geometry (Heeralal & Dhurumraj, 2016). Shortfalls in geometric teaching knowledge produce diagnosed challenges for learners. Couto and Vale (2014, p. 59), in support, stated that “In order to be a good professional, capable of teaching maths, it is crucial to deeply know mathematics and therefore it’s crucial to have the ability to put to work strategies which are capable of making the learners learn.” Colfland and

Strickland (2004) additionally recognized the necessity for improved secondary educator training in order to coordinate innovative strategies for geometry instruction for educators who do not identify the purpose or usefulness of learning geometry utilizing technology. In United Kingdom, Nathan and Knuth (2003) determined that geometry was the learning area in which educators in training performed inadequately because of the absence of confidence in the subject material. Their assessment of educators in the primary phase indicated that their knowledge of geometric vocabulary was weak and they showed low self-esteem to teach geometry. Similarly, Van Klinken (2010) claimed from his study that Australian pre-service educators' knowledge of geometry is scarce, leading to Australian primary school learners underperforming in geometry. Poor concept knowledge by educators is a problem in many schools and leads to poor geometric thought. In support, Browning, Edson, Kimani and Aslan-Tutak (2014) tracked down that the battle that pre-service educators' had with classification of geometric theory have been an ongoing problem. The issue of pre-service educators' inability to define or know the properties of shapes impacts on learner attainment when they enter the teaching profession. Their lack of confidence and negativity influences their teaching of geometry (Marchis, 2012).

Studies conducted in South Africa (Horn, 2009, Jojo, 2019) have established a positive relationship between educator knowledge and learner achievement. Educators' ability and qualification in the area of geometry have been under discussion as being fundamental in deciding learner accomplishment (Anstey & Clark, 2010). Educators must have ongoing access to programmes, workshops, and seminars to generate an opportunity for them to get hold of updated knowledge and abilities on the best way to deal with and assist learners in the classroom. According to Reys, Reys, Lapana and Haliday (2003), inexperienced educators are not provided with satisfactory opportunities to advance their insight and skills, and training for these educators is inferior. A good performance in geometry will result in a decent achievement in mathematics at grade twelve NSC level and will permit learners to register for and follow professions of high ranks (Bansilal, 2015). It is subsequently significant that extensive quality preparation of mathematics educators happens so that they can furnish learners with skills to tackle problems using critical thinking. Prior to the re-dressing of the education system in South

Africa, educator training took place in specialized colleges of education. Many colleges have since closed and or changed to be centres for Further Education and Training (FET) institutions or Technical and Vocational Education and Training (TVET) in the post-apartheid era. Mathematics educator training now takes place at universities. The **b**achelor of **e**ducation (B. Ed) degree with mathematics at second-year level qualifies students to teach mathematics. It is also a phenomenon in South African schools to have academic graduates without any professional teaching qualifications tasked to teach mathematics on account of both the high unemployment rate and the scarcity of educators for the subject. Some of these educators display a lack of necessary pedagogical and content knowledge for the effective specialization in the teaching of geometry. According to DBE (2011, p. 34), content knowledge of one of the grade 11 Euclidean geometry theorems states that: “a line segment drawn from the centre of the circle to the midpoint of the chord is perpendicular to the chord”. To be able to teach this theorem an educator needs to have a good understanding of “congruency”. Geometry concepts such as perpendicular and chord are specialized concepts and educators with a lack of content will experience difficulties to get them across to learners. According to Loewenberg Ball, Thames and Phelps (2008), rather than just knowing geometry, the educator must know exactly how to get geometry concepts across to learners. Adolphus (2011) stated that that one of the problematic ideas that educators struggled to teach mathematics is Euclidean geometry, because of technicalities related to language, visualization of objects for better identification of properties, and inadequate conceptual understanding. Proving theorems and application of these theorems in problems have always been a struggle for many educators in South Africa who lack the necessary content knowledge (Adjei, 2020). For an educator with little experience and content knowledge of geometry making conjectures, identifying the correct theorems to apply to proof and identifying key properties on a diagram can be challenging for the educator to get across to learners.

Kaino (2015, p. 225) describes the existing mathematics educators in school as an amalgamation of a group of educators with differing school subject knowledge and different strategies of classroom practice together with “proactive, reactive and over-reactive educators”. According to Kaino (2015), these educators with differing subjects require ongoing expert support to address and modify their practice to accommodate an environment characterized by severe challenges such as resource restrictions, infrastructure limitations and inadequate provision of learner support materials. Jojo (2019) suggested that mathematics educators ought to go through mandatory professional training before being permanently employed. The gaps in content knowledge can be addressed during this training and re-skilling programme, and educators will be more adequately equipped with the capabilities of managing the teaching of geometry even in challenging situations. This training is evident in Asian nations like South Korea and Singapore, where learners are excelling in mathematics.

2.5.6 EDUCATOR CLASSROOM PRACTICE

According to Doody and Noonan (2013), pedagogical knowledge is the educators 'deep knowledge about the processes and practices or methods of teaching and learning, i.e., how to teach it. The source of pedagogical knowledge (how) is content knowledge (what). A shortcoming is either one will negatively impact on learners' performance.

Teaching strategies determine learner outcomes in the subject as they determine the content acquired and the extent of contact and communication between learners and educators. According to Adler and Pillay (2016), many countries have engaged in greater learner participation to acquire knowledge, resulting in better mathematical competence. Fraser (2012) further echoed these sentiments using the social constructivist perspective of shifting from the conventional educator controlled strategy of learning to a more dynamic methodology that encourages learners to take part in obtaining information through ongoing discussions, projects, practical work, activities, and alternate approaches to assist them with reflecting and clarifying their mathematics. The instruction of Euclidean geometry necessitates initially that learners should be urged to experiment, discover and explore into space and shape, and then support the results by applying the discovered facts. A number of studies in South Africa have revealed that teaching of geometry is still administered using the lecture method or conventional axiomatic method (DellÖlio and Donk, 2007, 2014; Howie, 2003; Siyyepu and Mtonjeni, 2014). It has furthermore been discovered that this methodology achieves learning by rules and memorisation. Traditional axiomatic methods of teaching emphasizing memorization were argued by researchers Ramsussen and Marrongelle (2006) as not allowing for learners to foster a reasonable understanding of mathematics. These educator-centered methods have been criticised for not encouraging problem-solving skills. Various methods must be implemented to improve learners' thinking abilities and make the instruction of information more proficient. Within this situation, geometry requires a strong educational methodology over and above in-depth knowledge to attain a pleasant and stimulating environment for learners. The responsibility of the geometry educator is to guide learners to have a conducive and relaxed thinking space that allows for an understanding of geometric concepts.

The level of effectiveness within the geometry classroom is dependent on the implementation of good educator practices (Leighwood and Jantzi, 2006). Horn (2009) characterizes successful educating as a task that prompts improved learner accomplishment, utilizing results that make a difference to the future achievement of the learners. Important considerations toward improving the teaching of geometry were outlined by Horn (2009) as being aware of how educators use classroom guidance to connect learners, how they adjust their communication and instructing methodologies, how certain they are in explaining their aims to learners and whether they employ classroom conversation. Reyes (2012) argues that educators should adjust their teaching practices to suit learners and are obligated to teach mathematical topics different from the way they acquired instruction. Educators should identify the shortfall from their own personal instruction and adjust their teaching to benefit their learners. According to Van de Walle (2004) the systems and ideas of encouraging geometry teaching are currently more advanced and involve more than previously acknowledged conventional abilities. Most of the current educators got their schooling in the conventional methodology where rote learning and memorization were the critical skills with limited emphasis on understanding. Reyes (2012) maintains that the focus has now shifted to learners' dynamic engagement in the educating and learning cycle i.e., the educator adopts a learner-centred approach by engaging the learners in classroom activities and discussion. The false notion among mathematics educators and learners is that

Euclidean geometry can be memorized for assessment or promotion; therefore, it does not need applications to real-life situations.

Following Belbase's (2010) study, it became apparent that the following general conceptions of learners' attitudes towards mathematics hinder learner outcomes in geometry; namely that there is only one right answer to a mathematics problem and that learning facts and processes by heart will permit learners to find a solution to problems easily. Additionally, learners' belief that geometry is an individual action and the subject content does not need to relate to learners' lives, or the real world adds to the negative attitude that learners have towards learning geometry. Researchers (Muijis and Reynolds, 2001) ascribe all these assertions to the lecture strategy, a teaching method that places emphasis on facts, ideas and procedures as information to be memorized. As noted here, both educators and learners are of the belief that the practice of teaching and learning geometry by rote learning and without understanding, is acceptable.

Morris (2001) articulated the fundamental need for mathematics to incorporate opportunities for clarification by the educator, conversation between the educator and learner and among learners, as well as to provide realistic applications for consolidation. The participatory teaching method is still not utilized by many educators teaching geometry in spite of the declining learner achievement over the years with the conventional teaching strategy. This study wishes to explore teaching methods that have resulted in barriers to effective geometric thought. To provide instruction in geometry successfully, it is crucial to ensure that learners comprehend the concepts they are learning and the procedures included in a particular problem instead of essentially learning rules.

Pashler et. al (2008) described four types of learners that have four different learning styles, which educators must identify to improve their teaching of geometry. The first type of learner, referred to as the semiotic learner, perceives knowledge and information through concrete experiences and then processes it using reflection and observation. They distinguish their practical encounters by themselves and learn to build ideas by observing and sharing. These learners are confident that their encounters will result in deep thinking and are exceptionally fruitful in dissecting direct encounters from various perspectives. They need individual attention. The second type of learner is the analytic learner who perceives information through abstract experiences and processes them through the reflective observation technique. They arrange systems by joining perception with existing information. They offer significance to efficient learning. They can resolve the problem through rational thinking and examination and they re-look at the problem when unexpected situations emerge. The third type of learner is the one that learns through common sense. This type of learner sees the information by means of a theoretical conceptualization strategy and processes it through dynamic participation. The key focus for this type of learner is the outcome. They bind together the theory and practice and get familiar with the speculations by testing. They are perfect in problem-solving. They pay attention to strategic thinking. Lastly, the fourth type of learner, referred to as the dynamic learner, perceives the information through actual encounters and processes it by means of active engagement. They bring together insight and practice. They learn through the process of an experimentation strategy. They believe in self-revelation. They are energized by new items and enjoy exploring. Identifying the specific type of learner will allow the educator to adapt the content to suit that learner and thus become more successful in their teaching of geometry.

It is essential to assess singular learning styles for the way forward in the teaching and learning of geometry (Hein and Budny, 2000). Babagdogan (2000) stated that if the learner's learning style is identified, then it would be clearer how that individual absorbs information and the kind of teaching strategy that should best be applied. The educator can therefore use this information to build proper teaching conditions for both teaching and learning. The expectation is that educators give cautious consideration to the instructing procedures that will assist with learners' learning styles, resulting in more effective geometry learners (Tsanwani, 2009). Singh (2002), in support of recognizing learning styles, states that each learner has a unique learning style, and subsequently, no single method for the imparting of geometric material may give adequate alternatives, commitments, social interactions, and settings expected to work with fruitful learning and achievement.

The level of correlation between educators' conviction that supports educator change and classroom practice is a significant factor in a development's success (Alex & Mammen, 2016, p. 54). In the event that these two are not viable, it might prompt low educator morale and even rejection of the new idea. Therefore, including mathematics educators in curriculum reform will decide if they support the attempt to implement the curriculum in the classroom (Alex & Mammen, 2016). Gresham (2018) indicated that contrary beliefs by educators to curriculum changes might lead to challenges and poor classroom practices. The manner in which educators engage learners in the classroom is an important indicator for the success of geometry. Cochran-Smith and Lytle (2015) profess that allowing learners to participate in classroom tasks is a fundamental part of educator strategy. According to Nur and Nurvitasari (2017), the educators' methodology in teaching geometry is important in order to make geometry more interesting and challenging. A challenging classroom ambiance will allow learners to express their views without being afraid of any reprisals. Educators must possess effective communicative abilities to develop a positive environment to provide quality teaching in geometry. Carr and Skinner (2009) added that educators ought to have successful teaching methodologies, strategies, and skills to inform their teaching plans and lessons. The success of geometry lessons also depends on educators evaluating their lessons to meet the different educational needs of learners.

The mathematics education system in Japan teaches Euclidean geometry through inquiry and recommends that learning begins with concrete, realistic activities (Kariyana and Sonn, 2016; Spaul, 2013). Stigler and Herbert (2004) found that Japanese educators adopted a typical model of good practice by using problem-solving techniques in which learners engage to solve the problem on their own, which is then followed by discussion. Learners should be engaged with measures that foster critical thinking and conceptual reasoning of shapes (Battista, 2007). Memorizing concepts isn't as significant as building meaning, utilizing learners' own insights and methods of thinking to carry out tangible and mental manoeuvring, deduction and reflection. Geometry teaching should be directed by detailed information on the progression of meaning and mental images that learners' may develop as they advance from instinctive to formal reasoning.

The mind-set educator method used in the USA and UK while the mastery approach is embraced by the top six achieving countries, which are Asian countries. The mastery approach allows all learners to learn a specific concept before moving on to more complex concepts, whilst the mind-set approach moves from broader concepts before breaking down a problem (Vernitski, 2016). Singapore, one of the best-performing countries globally, allows educators to discuss and evaluate their methods of teaching mathematics daily to improve their standards of teaching in the classroom (Vernitski, 2016). The Netherlands utilizes calculators and realistic problems that learners frequently encounter, while Japanese educators do not use either but still have good performance (Stigler and Herbert, 2004). Spaul (2013) advocates that instructional practices should include a variety of methods to develop an understanding of geometry. In the Netherlands, the educational system recognizes Realistic Maths Education (RME) as an essential teaching approach. RME is based on the technique of teaching mathematics by relating it to realistic situations. Couglon (2016) found that Asian countries outperformed western countries in the international assessments carried out by TIMSS (2017). Mullis et. al. (2016) provided the following table on the performance of geometry by these countries according to the TIMSS study. The table presents the mean geometry scores obtained by the various countries. It also displays the mathematics performance of each of the countries.

TABLE 2.2: Geometry scores obtained by the top six achieving countries for TIMSS(overall mathematics scores in brackets)

Mullis et al. (2016)

	England	Japan	Hong Kong	Korea	Singapore	U.S.A
TIMSS 2015	514 (518)	598 (586)	602 (594)	612 (606)	617 (621)	500 (518)
2011	498 (507)	586 (570)	597 (586)	612 (613)	609 (611)	485 (509)
2007	513 (513)	584 (570)	580 (572)	600 (597)	590 (593)	480 (508)

It is clear from the above table that Singapore, Hong Kong, Korea, and Japan out-performed England and the U.S.A. According to Couglan (2016), teaching is a respected profession in Asian countries, and educator development is a significant component of mathematics instruction. In Japan, educators observe each other's teaching and then draw lesson points from these observations to develop themselves professionally. Singapore focuses on concrete, pictorial, and abstract models in mathematical learning. Hands-on experiences of concepts, language, and visual representation assume a fundamental role in teaching and learning (Boylan, 2016). The assumption in Asian countries is that mathematics is a universal language acquired roughly the same way through different languages (Leung, 2014). According to research conducted by Reynolds, Sammons, De Fraine and Van Damme (2014), in Asian countries, the focus on classroom practice and educator development is intensified.

Learner activity in the classroom should co-relate in accordance with learners' levels of understanding. Battista (2007) agrees that learners need numerous and diverse exercises to empower them to advance through the levels of reflection. Educators must be cognizant of and work according to their learners' degree of comprehension. Van Hiele (1986) theorized that the failure of a lot of educators to coordinate the learners' degree of geometric thinking with instruction is a barrier in promoting meaningful understanding of the topics. Researchers (Atebe, 2008; Ramatlanapana and Makonye, 2015) believe that educators should pay attention to learner's perspectives as there are sometimes differences between what educators want learners to acquire and what educators actually accomplish from their lessons. Developing a good idea of how learners think and reason while engaging in solving a problem improves the manner educators become familiar with understanding the learner.

Major misconceptions are created in geometry teaching when the communication of information between the educator and learner is at various degrees of thinking. In the event that the educator functions and communicates geometric thought at a level that is different from those of the learners', the outcome is that concepts will not be fully understood or acquired. In support, Luneta (2015) pointed out the importance of educators and learners operating at the same level for compelling geometric thought. Mason (1997) highlighted the following levels of understanding prescribed by the Van Hiele researcher, to assist educators with the teaching of geometry. Level one refers to the visual level where geometric shapes are recognized depending on their manifestation and without considering their properties. Level two looks at the descriptive or analytic level where learners identify properties. At this level, learners 'are unable to categorize figures hierarchically and cannot comprehend, for example, why a square is also referred to as a rectangle. At level three, which is the abstract or relational level, the learner is now ready to comprehend and develop abstract definitions that recognize fundamental and adequate conditions of an idea and formulate relationships between various shapes. A learner should be at this level when taking geometry in the senior secondary phase. Level four refers to the formal deductive level where learners can reason using descriptions, theorems and axioms. Learners at this level have the ability to build deductive proofs starting from the given and moving to what is supposed to be proved. Therefore, one can expect to accomplish successful learner outcomes in the instruction of geometry when learners reach this level of understanding, especially in secondary schooling. Level five is the final level of understanding of geometric thought, which refers to the rigor or meta-mathematical level. At this level, learners are able to think formally and evaluate various axiomatic frameworks. This level of understanding must be achieved at the teaching of tertiary mathematics.

The above levels determine the shape and design of mathematics curricula throughout the world and influence the standards of geometry in the United States. Kosa (2016) support the Van Hiele theory's role for effective teaching of geometry and emphasize the significance of progressive learning as communicated by Van Hiele's theory. This is supported by the NCTM (2000, p. 16), "A school mathematics curriculum should provide a road map that helps educators guide learners to increasing levels of sophistication and depth of knowledge."

Learners acquire a greater comprehension of geometric concepts by the sort of assignments and learning encounters they engage in. According to the NCTM (2000, p. 98), "Learners learn mathematics through the experiences that educators provide. Learners' understanding of mathematics, their ability to use it to solve problems, their confidence and their disposition towards mathematics is shaped by the teaching they encounter in school."

The important role the educator plays in using appropriate instructional methods in the teaching and learning process is highlighted by van Hiele (1986, p. 50) as "the transition from one level to the next does not occur naturally, but it takes place under the influence of a teaching and learning program." Learners need to be actively involved in suitable exercises as espoused by NCTM (2000), that comprehension mathematical ideas can be built throughout learners' school years if they are actively engaged in tasks and experiences designed to deepen and connect their knowledge. The NCTM (2000) reports that within this teaching and learning program, learners move from one level to the next, and their knowledge and comprehension of geometric ideas grows and builds in a progressive manner. A crucial component of the van Hiele's theory is that students at a lower level of reasoning cannot comprehend information and teaching introduced to them at a more elevated level, and this is the reason why learners experience barriers with learning mathematics, especially geometry. Learners, who grow through the instructional learning stages where the correct sequence of teaching is utilized, build a thorough understanding of geometric concepts at each level.

Learners start geometric reasoning as soon as they start processing information in their early schooling. Each learner moves through the various levels at different ages, depending on their experiences. Leiken (2015) highlighted that learners' progress through the Van Hiele's levels in accordance with instruction that is organized into five phases. Phase one is where information is gathered by the educator, by first ascertaining what the learner already knows about the topic and using this information to introduce the new topic. Phase two refers to guided orientation, where the educator guides learners to explore specific concepts in meticulously organized assignments such as folding, estimating or developing. Phase three is the explication phase, where learners express their feelings in their own words after the educator has introduced the relevant mathematical concepts. Phase four is the free orientation level in which learners apply the knowledge learnt to other problems and open-ended tasks. The final phase is called the integration phase, where learners sum up and coordinate what they have realized, prompting them to developing new objects and relations.

Learners must progress through each level sequentially, acquiring new knowledge along the way, taking into consideration that each level has its own phrasing, images, ideas, and thinking methodologies. Learners at different levels find it difficult to understand each other. Reece and Walker (2016) emphasise the need for educators to identify the functioning level of their learners so they can prepare appropriate geometric activities for understanding. According to Armah, Cofie, and Okpoti (2017), an in-dept conceptual understanding of geometrical properties at the primary level is not only important for learners in learning geometry, it will also determine whether they can cope when they proceed to secondary school. Alfifi and Abed (2017) stated that once a learner has reached an advanced level, it is difficult for them to recall what it resembles to think at a lower level. In light of van Hiele's level, it is hence significant for educators to teach at the suitable level of the learners in the geometry class. Teaching dependent on driving learners to acquire information by inquiry, for them to use the inductive method to develop information and comprehension at the lower levels, with the goal of applying the deductive inquiry to achieve a higher understanding, is an advised sequential pattern to follow for the success of geometric thought (Luneta, 2015).

The inductive method is a technique for building up rules, speculation and inferring formulae, while the deductive method is that of applying the reasoned outcomes and improving ability and proficiency in solving problems (Varghese, 2009). The methodology that educators fuse into their teaching is critical for the learning of mathematics. Inductive and deductive methodologies refer to activities attempted by educators in the geometry classroom so that learners can adjust teaching material to achieve the essential results (Nejla, 2005). Hamzah (2017) supports this in her contention that in learning, learners need to be dynamic intellectually and to construct information based on cognitive maturity by moving from the inductive to the deductive methodology. With the inductive approach, the educator leads from the concrete to abstract, particular to general, and from example to formula. According to Hamzah (2017), the procedure in mathematics is to initially do lots of examples and afterward generalize; for instance, when teaching parallel lines, learners should first measure the alternate angles, corresponding angles, etc., and then generalize based on their measurements.

Nejla (2005) highlighted the accompanying benefits of utilizing the inductive approach; it assists in fostering the logical viewpoint among the learners. Educators can develop characteristics of critical thinking and create a habit of sharp perception among learners appropriately and precisely. This promotes a logical and mental way of teaching mathematics, and by utilizing this technique, learners get different opportunities to assume a functioning part in the learning process. There are, however, disadvantages concerning this approach. The following are some of the disadvantages identified by Hamzah (2011); in the situation where the amount of information is enormous, the outcomes or conclusions drawn from such a strategy are not found to be conclusive, all topics in mathematics cannot be taught with properly using this strategy, and it can only be utilized when educators have adequate time at their disposal to engage in interactive teaching.

In the initial stage of the inductive technique, the problem-solving is dependent on previous knowledge, thinking, reasoning, and insight of the learner (Singh, 2012). At this stage, the learner does not know about any formula, standard, or procedure for tackling the given problem. Singh (2012) further stated that learners arrive at a generalization or determine a formula through an encouraging interaction of thinking and finding solutions to different comparable problems. The educator using this technique does not furnish learners with rules and formulas but instead allows the learners to develop the rules and formulas through discovery. The inductive method leads from the known to the unknown.

The deductive strategy is the reverse of the inductive technique. With this method, the learner advances from the broad to the specific, theoretical to the substantial, and from rules to examples. The educator in the class gives the relevant formulas and then moves on to examples. For example, the educator provides the rule that the sum of the angles of a triangle adds up to 180^0 then requests the learners to use this rule in application exercises. Merits of this method, according to Nejla (2005), are; that it is short and time-saving, it promotes learning by memorization, this method is advantageous at the review stage, it improves speed and productivity in solving problems, and it eliminates incompleteness and inadequacy of the inductive method. The demerits of this method are identified by Nejla (2005) as follows; the novices find it difficult to understand abstract formulas if they are not familiar with various concrete instances, blind memorization of a large number of formulas causes an unnecessary and significant weight on the mind of learners. In this method, emphasis is placed on remembering formulas and procedures, underplaying the importance of understanding which is educationally unsound. Blind cramming often leads to failing to remember the formulas, and learners are at a loss to recall, which ultimately leads to no learning. This strategy does not support the advancement of discovery, reasoning, and thinking.

Silas (2012) argued that the inductive approach to teaching mathematics is more compelling for teaching circle geometry and trigonometry than merely transmitting knowledge to learners. College (2011) upheld this contention by saying that the deductive method of teaching is significantly less valuable and presents profoundly organized content. In comparison, the inductive method creates optimal opportunities for learning and is, therefore, more suitable in teaching and learning geometry. According to Shoji (2010), the deductive technique of teaching is useful when used in a classroom with a large number of learners, while the inductive technique is powerful when utilized on smaller units or fewer numbers of learners. The deductive strategy is conventional, organized, and predictable, while the inductive technique is customized, supports individual learning, and concepts can easily be recalled and understood. Silas (2012) further argued that the deductive strategy is a technique of confirmation and comes from a basis, while the inductive strategy is an approach exploratory and relies on a learner's point of view or understanding. Nicole (2007) endorsed this view that the deductive method of teaching cannot develop the thinking of learners,' but with use of the inductive method, we can develop the thinking and curiosity of learners. The use of the inductive method gives the educator an opportunity to develop the strength of interest in learners. The inductive strategy supports independent mental activities and active engagement that promotes learner discovery (Zdravko, 2008).

Vinson (2001) contends that powerful mathematics educators realize that they should follow the strategy of giving learners substantial experiences that structure the reason for visual and symbolic mathematical learning. In order to manage the diverse necessities of learners ‘in the mathematics classroom, educators should procure the applicable expert abilities to construct situations that are relevant to the learners. Johnson (2006) suggests that advanced endeavours in classroom practice must take place, so educators are trained to handle various learners’ abilities in the classroom. Educators need to embrace the latest educational methods that respect and consider individual learner requirements (Johnson, 2006), which are aptly supported by the constructivist methodology to learning. This approach depends on the notion that learners must be included in the acquisition of their own content. The following strategies are greatly needed; familiarizing the learners with the contents of instruction, empowering them to work independently, giving them an adequate degree of mathematical capability, expanding learners’ level of motivation, and improving their participation in the geometry, and usefully improving their collaboration with the environment. The learner-centered teaching methods has been emphasised has an important approach to equip learners with skills of creativity and critical thinking. The next section therefore provides some common strategies in learning geometry through the learner-centred methods of teaching.

2.5.6.1 Problem Based Learning (PBL)

Chen (2019) defined Problem Based Learning (PBL) as an instructional practice where learners are the main drivers of learning, by working in groups or individually to solve open-ended problems. Learning begins with a problem to be solved, and the problem is posed in such a way that learners need to gain new knowledge before they can solve the problem. PBL encourages learners to acquire information and develop abilities to build self-assurance when solving geometry exercises. This is supported by Barrows (2010, p. 127) in his clarification that "PBL is an instructional method that challenges learners to learn to learn working cooperatively in groups to seek solutions to the real-world problems." The problems that are provided to learners ought to empower learners' interest and motivate learning geometry content. PBL compels learners to develop critical thinking capabilities and work logically to discover and utilize appropriate resources.

PBL is a methodology that can enhance the viability of geometry teaching (Schettino, 2016). In the traditional geometry lessons, the educator is inclined to provide learners with all the information related to geometrical shapes and their properties, anticipating that learners should utilize this knowledge to complete the given tasks, to determine the extent to which the topic is understood or are learners fundamentally responding from retaining facts. Not many endeavours are made to urge the learners to talk about their thinking and make coherent associations. According to Henderson (2001), fostering the learners' reasoning and geometrical logic is critical and learners need this for developing a sound comprehension of the concepts in geometry. This goal can be achieved by utilizing different teaching methods, one of them being PBL. PBL supports learners in making fundamental associations utilizing the various representations of geometrical and interdisciplinary thoughts in the area of mathematics (Schettino, 2016). Learners will then develop a full awareness of information and processes, not just repeating rules.

By employing a learner-centered teaching method, the educator facilitates learning by identifying learners' endeavours, thoughts and prior knowledge (Zabot, 2010). The PBL approach allows maximum opportunities for learners to be occupied in joint tasks to determine possible solutions. In contrast to traditional methodology, the PBL method permits learners to operate collaboratively and build their education through social interaction (Barrows, 2010). Schettino (2016) describes the classroom climate as one that centres on the problems and the learners' thoughts and not only the educator and his or her power. Learners have prior knowledge and experience, which is not always readily accessible until set off by a problem in a manner that relates to it (Zabot, 2010). This perspective is likewise supported by Vygotsky's (1978) theory, where he advocates that learners have prior knowledge that needs to be developed by the skilled other. Schettino (2016) advocated that the problem in a PBL classroom requires scaffolds so that learners are prepared for what is coming next and if this does not happen, they will be unable to access their prior knowledge. Many educators avoid this type of teaching and attempting to persuade educators of its worth is often difficult. Educators may face many impediments while moving towards this type of methodology in that they need to consider a great deal of appraisal and other classroom strategies simultaneously.

With PBL, learners are guided to discover solutions to problems on their own using geometric concepts learned in the mathematics curricula. This is supported by Barrows (2010) in his definition that PBL is a process in which learners acquire knowledge by understanding or solving specific problems. PBL provides many benefits to learners' in a geometry classroom and supports the participative strategy to learning. Chen (2019) highlights these benefits as:

1. The teaching process, which begins by utilizing an ill-structured problem as the focal point of organizing a curriculum or scenario of learning, inspires the learner to learn, investigate and develop fundamental expert knowledge and problem-solving tools required in the future. During the investigation of unstructured problems, learners foster skills of meta-cognition that permit them to monitor, critique and direct their own inference abilities through problem-solving.

2. Learners become partners in their learning cycle and develop a special role in pursuing meaning and comprehension of matters through self-coordinated learning Barrows (2010). This role contributes to the learners' relationship of old and new information and the understanding the significance of critical thinking techniques and how to re-apply these strategies.

3. The educators' role is that of a facilitator of thinking and meta-cognition, a curriculum designer, question-solving partner, supporter and director of learning. The educator manages the process of PBL by facilitating and monitoring learners' participation.

4. PBL supports group cooperation and learning in which each learner in the group works together to help each other, as a learner and question solver, by giving

and seeing different viewpoints that are imparted among the learner group. These are valuable for explanations of intricate matters, accordingly improving the proficiency of collaboration.

5. PBL supports numerous assessments strategies. It is a constructivist approach that advances inference skills, question-solving intelligence, group work and communication skills and learners' abilities can be displayed and provided as feedback for effectiveness.

In South Africa where the majority of learners are second language English learners, PBL will lend itself well for learners to make associations to the learning of geometry. Chen (2019) describes PBL as a self-orientated model where learning in smaller groups is better. He likewise views it as an association between thinking and content knowledge of learners. He further highlights that the educator who is the expert, skilled person in the classroom be considered as a helper and not a leader. He finally advocates that learning tasks should be based on real-life problems. An example of this is where the teaching of the topic on quadrilaterals can be accomplished by using the PBL instructional approach. Learns can investigate the properties by real life engagement with four sided figures brought into the classroom such as boxes, toys and others. Relationships can be drawn and listed for example “why is a square a rectangle but a rectangle is not a square” or “why are four sided objects different from cylindrical objects”.

2.5.6.2 Learning by Inquiry

Inquiry Based Learning (IBL) is defined by Dorier and Maas (2020) as a student-centered approach where learners are the centre of the learning experience and take ownership of their learning by posing, investigating and answering questions. According to Spronken-Smith and Walker, 2010) IBL is a form of self-directed learning where the students take responsibility for their learning. Inquiry as a learner focussed approach is identified to have a solid, verifiable place in Ancient Greek philosophy since it was used by Socrates while engaging in discourse with interlocutors, employing a questioning approach (Friesen, 2014). Through inquiry, learners look for material and make revelations towards ultimately tackling problems. In support of implementing the IBL Hester (2004), indicates that this technique employed by the constructivist gives learners the freedom to deliver contemplations, confront problems, evaluate thoughts and utilize them in various situations. Educator-learner in addition to learner-learner discussions is highlighted all through the inquiry learning technique. Communicating and discussing collectively using the inquiry approach is alluded to as a joint effort of helpful learning. This technique guided by the educator presents the learners an opportunity to acquire knowledge (DellÓlio and Donk, 2007). This method helps the learners in their own learning and learners experience maximum engagement during the inquiry interaction while the educator is the helper who provides the information as the foundation of the inquiry. Cotton and Dustova (2015) caution that if the educator gives too much information, then the session declines into the axiomatic style.

Bennet (2015) suggests that the educators motivate the learning through inquiry by urging learners to associate to geometry through recognizing invigorating topics and applying meta-cognition by bringing learners into the real world. He also recommends that educators encourage learners to respond by posing insightful questions and utilizing prompts to help learners progress to develop knowledge. Many researchers (Carr, 2010; Hamilton and Ghatala, 1994; Tarricon, 2011) support the idea of meta-cognition. Tarricon (2011) also observed that meta-cognition connects to reflection and meta-cognition is dependent on self-knowledge. Meta-cognition through reflection remains essential for the learning of geometry. Än appropriate example of inquiry learning is the grade 11 teaching of theorems on cyclic quadrilaterals. Diagrams of cyclic quadrilaterals can be given to learners to measure the angles to determine the relationships between the opposite angles and the exterior angles. As suggested by Bennet (2005) they can be further guided by the educator, using prior knowledge of circle theorems, to formulate the proof of the theorems that state that “the opposite angles of cyclic quadrilaterals are supplementary” as well as “the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle”. Pesek and Kirshner (2000), indicate that when direct instruction precedes investigation, learners learn less and overlook important conceptual connections. When investigation opportunities come before direct instruction, learners show better ability to conceptualize the mathematics they are taught. In this regard, the exploration of the opposite angles before the direct instruction will allow learners to construct their own understandings and make connections between what they know from experience and what they are taught by the educator.

2.5.6.3 Co-Operative Learning

Johnson and Johnson (2009, p. 84) defined co-operative learning as a "*relationship of a group of learners that requires position, interdependence, individual accountability and interpersonal skills communication*". Co-operative learning is essential for developing communication skills and learners' abilities to work together in a geometry class (Stuart, 2000). With co-operative learning, learners work together as a team by combining their experiences, knowledge and social skills that are needed in the geometry class. Johnson and Johnson (2009) are supportive of cooperative learning, adding that it allows for learners to work together as a team to maximize the success of all team members. Macpherson (2007) stated that cooperative learning is desirable whenever problem-solving is necessary, divergent thinking or creativity is required, quality performance is expected, tasks are complex, learning goals are deemed highly important, and the social development of learners becomes a major instructional goal.

The co-operative learning technique is one of the primary successful teaching strategies utilized in Singaporean and Japanese mathematics classrooms (Slavin, 2010). Co-operative learning is a learner focussed instructional method that utilizes different techniques to get learners to think and learn, which according to McCracken (2005), is recommended across all grade levels. Co-operative learning is diagnosed to advance active learning and is established within Vygotsky's (1978) theory that advocates social learning. This sort of learning is supported by the NCTM (2000), where it is expressed that "learners who have opportunities, encouragement and support for speaking, writing, reading and listening in mathematics classes reap the benefits: they communicate to learn mathematics and they learn to communicate mathematically" (NCTM, 2000, p. 60). The idea behind co-operative learning is to encourage learning within an effective social group and a conducive learning environment. The key objective behind this learning approach is to improve learners' critical thinking abilities while developing social skills. In order for group learning of geometry to be effective, the educator must play an important role in guiding the process of learning for the successful teaching of geometry.

McCracken (2005) advocates that educators perform the following two fundamental tasks for cooperative learning to yield the desired outcomes. Educators must initially define appropriate and safe objectives for every group of learners to accomplish. This task of objective setting must be worthwhile to learners. Furthermore, educators must teach social abilities by paying attention to the skills of socializing and teamwork so that no single learner in a group dominates to mitigate against the team effort.

Vygotsky (1978) placed a lot of emphasis on social interaction and was of the belief that it is through social interaction that learning takes place. The building of new thoughts and concepts is most valuable when the learners associate and communicate in smaller groups. Co-operative learning supports the idea of the ZPD, where a child's advancement is accomplished through self-directed learning to achieve his expectation for elevated levels of development through co-operative engagement with capable peers and a more skilled person (Slavin, 2010). In addition, utilizing cooperative learning with technology such as the internet and geometry software leads to effective learning of geometry.

McCracken (2005) highlighted that cooperative learning benefits the learner in that the learner experiences less classroom anxiety and stress. Stuart (2000) further stated that many learners think of mathematics as something that causes tension and unpleasantness. Such learners have high anxiety about learning mathematics and trying to succeed. Secondly, McCracken (2005) expressed that learners have difficulty expressing their thoughts on paper or in front of the mathematics class. The cooperative approach is a method that is advocated to assist learners to understand and enjoy mathematics. Thirdly, learners experience increased retention and develop higher self-esteem when engaged in cooperative learning. Fourthly, this method improves social skills, critical thinking and communication, leading to higher academic achievement. Since each learner is responsible for his/her learning and participates within the group, he/she achieves more inherent inspiration to do well in the classroom. Methodologies and procedures can be combined to develop a more extensive cooperative learning environment.

On the contrary, the cooperative learning method does have disadvantages in that it needs a lot of planning, adjusting to and monitoring for effective implementation. It is also true that learners who struggle in social settings have difficulty working in groups. Many researchers (Couglan, 2016; Johnson and Johnson, 2009; Leiken and Zaslavsky, 1999) state that successful cooperative learning requires that learners have both group goals and accountability for solving the problem in mind. Educators minimize problems encountered in implementing this method when a reward system is structured so that learners are individually accountable for their work while they collaborate with their peers for the success of the group (Lindauer and Petrie, 1997). In order to maximize the opportunities to learn in a crowded classroom, learners should help one another. Slavin (2003, p. 45) concluded that "co-operative learning is one of the greatest success stories in the history of modern research."

William (2011) directed this success of utilizing cooperative learning to motivation, social cohesion, personalization and cognitive collaboration. Motivation is promoted when learners help one another and learn because it is in their own interest to do so. This motivation encourages them to tackle complex problems in geometry. Social cohesion occurs when learners assist their contemporaries because they belong to the same group and it has a bearing on their success. Personalization happens when a specific learner is experiencing difficulty and the likelihood that another learner in the group can assist. Where groups are well structured, it is not always the same learner receiving and helping. Cognitive collaboration forces those who contribute to discussions to think through ideas and clarify them for themselves and others in the group. Teaching a subject like geometry using cooperative learning will be beneficial as learners will interact with each other to solve problems. Collaborative learning can be effectively used in solving geometry riders. Learners can build confidence in their understanding of geometric concepts by working in groups and discussing the problem. For example, in the rider below learners can work together to identify key concepts such as diameter, tangent and perpendicular from the problem statement and discuss it and related information about these concepts to assist in solving the problem. They can engage in explaining:

1. How to prove two lines //
2. How to prove a quadrilateral is cyclic?

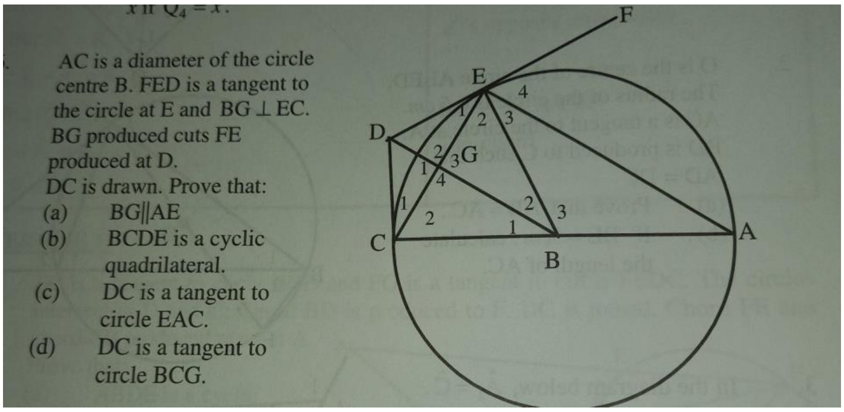

FIGURE 2.1 AN EXAMPLE FOR COOPERATIVE LEARNING

Learning in this manner promotes advanced levels of thinking since the engagement with peers encourages explaining, modelling, discourse, support and combined participation

2.5.6.4 Questioning

Using effective questioning techniques to invigorate thinking is a fundamental strategy that ought to be utilized by educators. The questioning technique that educators use in the geometry classroom will impact the sort of information that learners build and impart (Moyer and Milewicz, 2002). Educators who are able to construct questions successfully in-corporating the various cognitive levels, like knowledge, perception, investigation, synthesis, and evaluation (Bloom, 1956), are more effective in measuring the level of learners' thinking. Educators who utilize good questioning can reduce constrained thinking to promote novel thoughts and connect the recall of trivial facts with constructive meaning (Kamii and Warrington, 1999). To develop successful geometry thinking and reasoning, good questioning techniques have been regarded as an essential tool (Dains, 1986). Despite the many advantages of using the questioning methodology for enhancing learner learning and self-evaluation of the lesson being delivered, it can contrarily affect the knowledge if not communicated properly. The preparation for the development of effective questioning techniques or abilities could be central to the delivery of good geometry lessons in our present-day classroom discussion. The NCTM (2000, p. 176) states that "The educators of mathematics should orchestrate discourse by posing questions for specific tasks that elicit, engage and challenge each learner's thinking, by listening to learners' ideas and requesting learners to clarify these ideas orally or in writing.". The educator's responsibility is to be dynamic in an alternate manner from that in conventional classroom communication, by not being the central figure doing all the talking, presentation, and clarifying alone but should encourage and anticipate learners to do so instead. The educator should rather engage in more listening and allow learners to do greater thinking and reasoning. NCTM (1991) advocates that educators must construct mathematical discourse using effective questioning. It is of great significance for mathematics educators to pose questions that assist them to cooperate with learners towards understanding geometry, to learn to think mathematically, and to learn to surmise, create and solve problems. These are some of the aims identified in the CAPs document (2008) for effective learning of mathematics. Ashlock (2002) states that questioning and explaining is important in diagnosing learners' misconceptions and error patterns in geometry. There is great dependence on verbal communication as an important technique for inspiring learner knowledge which can be achieved through educators' effective questioning and learners' clarification.

Buschman (2001), in his research, demonstrated that asking learners,' the correct questions gives rise to educators being more mindful of what learners understand, and these assist educators in assessing how learners acquire geometric knowledge. For the questioning technique to be viable, it requires well-developed oral questioning skills, including preparing ahead of time, phrasing questions clearly and concisely that invigorates thinking, and giving learners sufficient opportunity to consider and prepare their responses (Buschman, 2001). Moyer and Milewicz (2002) emphasize the significant role the educator plays in utilizing questions by encouraging probing and follow-up questions instead of educators assessing and distancing themselves from the learners' views and aligning with outside guidelines verifiably depreciate the thoughts that the learner has developed. The scaffold questioning technique involves asking stepwise questions in a sequential manner that directs the learner from essential attainment to self-reliant mastery of mathematical ideas (Ashlock, 2002). Questioning can be classified as open or closed. In a geometry classroom the use of open questions to elicit deeper level thinking can be effective for understanding concepts. For example, “what is a triangle with three equal sides called? is a closed question because what has to be done is specified and there is a single right answer. However, a question like “Explore the relationship between a polygon being equiangular and equilateral”, is open ended because the learners have to decide what they are actually going to work on and there is no specific answer to be found. According to Mason (2010) engaging learners in thinking and classroom discourse in a geometry lesson requires educators to be aware of the questioning strategy they are utilizing.

2.5.7 TEACHING AND LEARNING ENVIRONMENT

The school climate, viz its infrastructure facilities, is an important factor contributing to learner's achievement in mathematics. Saritas (2004) suggests from his study that learner achievement is closely associated with a safe, organized, and positive classroom culture. Seah (2015) highlighted that the surroundings of the school, such as infrastructure and physical resources also have an impact on learner attainment. Quimbo (2010) recommends that the availability of resources such as books, models, and visual guides directly contribute to improving learner accomplishment in geometry. Smith and Ragan (1999) define the school learning setting as the educator, the current educational programme, educational resources, and the instructional and more extensive learner community. Several studies (Attwood, 2001; Brodie, 2004; Maree, 1999; Murray, 2010) have shown that there is a positive relationship between schools that have challenging environments and the underperformance of learners'. The physical space is also a contributing factor to the effective learning of geometry. In many South African schools, disorganized and disorderly conditions were discovered to be de-motivating to teaching and learning mathematics (Adler & Pillay, 2016). Researchers (Frazer, 2012, Helm, Beneke and Steinheimer, 2007) all advocate that in developing learners' independent and rigorous thought, a positive learning surrounding that encourages revelation and reflection, active discussion, and sharing of thoughts for both educators and learners must be developed. Such an environment stimulates learners' awareness, gives freedom for learners' visual reasoning, and energizes lesson commitment. According to the DBE (1997), KwaZulu-Natal is one of the provinces that are worst off in terms of physical resources, service equipment, and teaching resources. KwaZulu-Natal is also one of the provinces that experience many challenges in mathematics performance.

In many schools, the only resource in the classroom environment is a chalkboard and chalk (Botha, 2016). In spite of additional consideration given to educational needs by the government, the finances allocated to schools are not sufficient; thus, they are unable to fulfil all the requisitions of making the schools more effective. Botha (2016) articulated that some schools do not have sufficient teaching resources, and educators are compelled to educate learners under these problematic circumstances yet are expected to accomplish learner outcomes. The reason the "chalk and talk" traditional method is utilized is because simple materials like rulers, protractors, and solid figures are hard to find in these schools. Schools that rely fully on government financial allocation experience many problems with infrastructure and the purchase of learner-educator support materials (Fraser, 2012). Textbooks are also scarce and may not be available to every learner. As such, the learning and practice of geometry problems become difficult. Numerous studies have revealed the connection between teaching resources and learner achievement in developing countries (Botha, 2016; Chepkurui, 2004; Fraser, 2012). The textbook is considered the main instructional resource that has appeared to have a crucial impact on the teaching and learning. Chepkuri (2004) highlights the significance of the availability of adequate resources, especially the textbook, as having a positive effect on learners' attainment in mathematics since learners utilize it as a source to understand geometric concepts. Textbooks offer relevant exercises for further practice of the learned concept. Mathematics requires constant practice at home as well as the completion of assignments. Not having a textbook deprives learners of the opportunity to learn, which then impacts negatively on the child's education. Meaningful and effective teaching and learning are extremely challenging in an environment that lacks adequate resource material.

Corpuz and Lucido (2008) emphasize the need for learners' exposure to various media to have distinctive learning encounters for all-encompassing improvement. When resources or multi-media combines with learners' exercises, they do not just fill in as backing up systems in the instruction of geometry but also give value in moulding the learners' activities and subsequently their learning (Trigueross and Lozano, 2007). More than one medium of teaching media is expected to assist learners in acquiring diverse learning experiences. As much as the textbook is an important resource material tool, the Learning and Teaching Support Material (LTSM) must move away from the traditional use of textbooks, reading and writing to various resources with the goal for learners to advance to more extraordinary learning encounters. It is valuable to think about the teaching of geometry as a subject that lends itself to practical applications and gives freedom for learners to utilize a wide range of resources to investigate constantly properties, figures, and facts. To teach learners various shapes, the educator can incorporate devices such as the television, books, models, and computer games.

According to Lajorie (2000), mathematical tools advance learning at various levels and provide for the learning of ideas, rules and facts. Mathematical tools enable learners to manipulate and think about abstract ideas using concrete models. Lesgold (2000) described mathematical tools as traditional, technological, or social. As such, traditional tools are characterized as physical objects such as cubes, cans and bottles and technological tools refers to computers, smartphones and calculators, which can expand learning in unexpected ways. Learners' divided into small units for discussion where they are given a chance to interact, share and challenge ideas are referred to as social tools. The NCTM (2000) recommends the use of these types of tools individually or together based upon the kind of learning that is expected.

Traditional devices are most appropriate for developing learners' basic knowledge and abilities. Combining visual tools with physical tools increases a learner's conceptual knowledge (NCTM, 2000). Such tools support learners' development of foundational knowledge, which is fundamental for learning concepts that are more complex. Innovative devices are more powerful in guiding learners' comprehension of complex ideas and standards. Lajorie (2000) explains that the support that tools provide helps learners to think more deeply about mathematical concepts. The computer software that assists with learning geometry removes the over-learning of routine techniques as it allows learners to execute a task in their own stride. Video recorded lessons form part of a dynamic learning tool for learning geometry. A significant benefit of video-recorded lessons is that complicated problems can be broken down into more reasonable representations in comparison to regular word problems (Lesgold, 2000). The utilization of a data projector enhances video presentations. The smartboard is a successful apparatus used in the geometry classroom to manipulate figures to develop understanding. Social tools are recent considerations and these tools facilitate the learner's ability to solve problems and understand concepts by interacting with their peers (NCTM, 2000). Peers and the use of computers can provide feedback, which allows for learners' awareness of inconsistencies in their reasoning. Along these lines, social devices can help to change understanding.

The expansion of technology over the years has given educators creative ways to involve learners in the learning process. Perkins (2000) indicated that technology empowers both learners and educators to retrieve a broad range of tools that can be utilized in mathematics. He advocates the subsequent three stages for the process of understanding with regards to information and communication technology. These are that technology offers learners' clarifications, it permits relational knowledge to become available and, learners can possess revisable and extensive web explanations.

The NCTM (2000) expressed that technology is a fundamental part of high-quality mathematics education. Learners no longer need to be passive recipients of information as Brown (2004) expressed, since technology as a teaching tool permits for learners' interactivity. Gregg (2001) asserts that advanced innovations have an immense influence on the instruction of geometry. Visualization software and good graphing programs make the teaching of geometry more general and investigative than the conventional focus on remembering rules, proofs, and theorems. This motivating methodology supports studies that allow learners to advance to higher levels of geometric reasoning, perception, analysis, and derivation (Clements and

Batishta, 1995; Van Hiele, 1986). Technology also assists learners to move from observational to sensible reasoning, urges learners to make and test conjectures, facilitates precision in geometric thinking, and empowers the development of autonomy in learning. Research from various investigations has displayed that essential utilization of innovative instruments can maintain both the learning of mathematical method and skills for the improvement of increased mathematical abilities such as critical thinking, rationalizing, and reasoning (Kastberg and Leatham, 2005; Nelson, Christopher and Mims, 2009; Pierce and Stacey, 2010).

Educators upgrade the teaching environment when they incorporate technology satisfactorily while planning lessons, developing learning exercises and both putting together and controlling assessments. A few studies have also revealed that the utilization of technology additionally advances educator confidence with the content aspect of geometry and narrows the content gap (Cassim, 2010; Cox, 2003; Ellis and Steyn, 2003; Mumtaz, 2000). ICT is identified in the CAPs document to enhance the classroom environment whilst also developing the increased thinking skills of the learner (DBE, 2017). Technological tools provide an opportunity to access a wide assortment of critical thinking strategies other than those restricted to pencil and paper methods (Bansilal, 2015). Mathematical software can add to a more profound comprehension of geometry concepts by giving access to a range of representations that assist visualization of the geometrical objects for example Sketchpad. Technology enables the visualization of statistical concepts by making the associations clear. Huang, Liao and Cheng (2014) encourages the following possible solution to promote geometry learning when faced with challenges, viz the use of technology and forming situations to ingrain dynamic cooperation in tasks and advance inspiration in the learning of geometry. Additionally, they suggested that the educator apply the strategy of engaging learners and giving them the opportunity to respond from the lesson digitally presented through reflective and active learning methods, which will unquestionably aid in the learning of geometry.

Advanced computer software can help with geometrical instruction since it permits active participation and interactive manipulation of a figure. A learner has the ability to shift, rotate or draw figures and visualize the effect on the properties. Specially designed geometry software exists to teach geometry in an interactive and investigative way. Many advanced countries take advantage of commercial programmes that are available as it encourages deep-rooted development in learners (Jayaprakash, 2005). He further states that the utilization of technology and geometric software furnishes learners with independent learning skills and reasoning abilities to tackle complex problems in geometry.

Hannafin (2001) found that utilizing an open-ended learning environment where learners have more control over their own learning, in conjunction with dynamic geometry software, prompted positive attitudes of learners towards learning geometry. Research additionally shows that educators take more time to acknowledge and incorporate innovation into their lessons (Hannafin, 2001; Joseph, 2013). Dynamic geometry software includes programs such as Geometers Sketchpad and Geogebra, where learners can create figures and constructions easily. They can adjust figures and move them once created, with the initial properties of the figure remaining intact. The use of computer software allows learners to experiment with different shapes on a computer screen by rotating, sliding, or tossing them over to analyze them from different angles. These experiments are advantageous as it draws out various properties of concepts such as congruence and similarity. Hannafin (2001) advocated that this type of observation regarding two- and three-dimensional shapes in various positions will foster learners developing spatial sense and reasoning skills.

Stols (2012) indicated that appropriate use of technological tools, such as dynamic software, can enhance the teaching of geometry in South Africa, since Euclidean geometry is a visual subject. In some provinces in South Africa (like the Free State) ICT, such as Heymath! And Geogebra has been incorporated into the teaching of mathematics and geometry (Noss n.d). Jezdimirovic (2014) argues that the use of technology comes with various challenges such as educators' lack of content understanding, lack of adequate technological resources, internet connectivity, etc. The advantages of using dynamic software was supported by Chimuka (2017) even in high poverty, rural schools in South Africa, since it had a strong impact on learners 'learning in the context of circle geometry theorems. The use of geometry software was incorporated as a measure to remedy the poor mathematics performance (Bosse and Williams, 2019), but in most cases, schools are under-resourced and educators are not proficient in using geometry software for teaching.

2.5.8. CLASS SIZE

The size of the school and the number of learners allocated to a mathematics class has an impact on achievement. Attwood (2001) demonstrated that class sizes of greater than fifty learners negatively impact scholastic accomplishment in high school mathematics and science. Wood (2000) furthermore discovered when the class sizes were below twenty learners for the teaching of mathematics, the achievement was far more successful (Wood, 2000). The average educator-learner ratio for mathematics in KwaZulu–Natal is 1:35 (DBE, 2019), which does not allow for individual attention of learners. Rutter (2000) explained that when the number of learners in a mathematics class is reduced, it facilitates social relations and fosters educator attention. Smaller classes will assist when using Van Hiele's levels of identifying the ability level of learners since it allows for personalized instruction (Watson, Handal, Maher and McGinty, 2013).

The educator-learner ratio is determined by the post-provisioning norm across the province (DBE, 2010). The educator-learner ratio determines the number of educators allocated to a school. The concept of “weighted” learner is used instead of actual learner. Some learning areas require smaller units therefore each learner is allocated a weighting. For example, in Computer Studies the ideal class size is 27 learners so the weighting is higher. Subjects that have a practical component have a higher weighting. Mathematics is given a similar weighting as the languages and content-based subjects, which directly impacts the class size in mathematics. A study conducted in secondary and primary schools by Bassett and Brown (2011) tracked that learner collaboration with educators dropped significantly in bigger class sizes, which brought about a lower degree of learner engagement. Englehart (2007) also emphasized that smaller classes allowed educators to acquaint themselves with the individual needs of their learners and monitor learners' performance. Consequently, learners become more connected and motivated in their learning due to the increased customized learning environment. Moreover, Fan (2013) announced that educators invest lesser energy in classroom discipline with smaller class sizes, thus increasing instructional time for learners in the mathematics classroom. Dindyal (2015) likewise affirmed that in smaller class units, individualized assistance from the educator could be provided to learners, thus identifying their difficulties resulting in an improvement in achievement. A positive and effective learning environment is one that will promote learner achievement in geometry. Gilles and Haynes (2011) confirmed that learner-centered classroom methodology was expanded in smaller class units, which promoted a more viable teaching environment. Smaller class units provided educators with additional time, sufficient resources, and motivators to develop better lesson plans with enhanced degrees of differentiation (DiBiaise & Mcdonald, 2015). This, in turn, increases educator curriculum development.

2.5.9. SCHOOL POLICIES AND LEGISLATIONS

A focal branch of mathematics is geometry, and geometrical reasoning is a principal manner to engage learners with mathematics. Therefore, it is important for learners to master the skills at each level before moving onto the next level. South African learners that obtain as little as 20% in mathematics in grades 7, 8 and 9, are still allowed to advance to the following year of learning (DBE, 2017). This rule has been touted by numerous individuals as proof of a supposed unyielding decrease in instructive norms (Muller et al..., 2016). This might be misinterpreted as supporting poor accomplishment; however, this decision is undeniably more mind-boggling. Some of the reasons behind this move, as articulated by Muller et al. (2016), are age cohorts and poverty. Age cohorts are the major factor behind this decision. Learners between the age of 13 and 15 years should be in grades 7, 8, or 9. To avoid a bottleneck of overage learners in grade 9, it is inevitable to "push" them to relieve the pressure. Many of these learners have not reached the Van Hiele level of being competent to move to the next phase of learning. Jogo (2019) articulated that the pressure from the higher powers to move these learners through the system is colossal. In the previous twenty years, there has been a significant shift globally in perceiving the teaching of mathematics simply in financial terms (Jogo, 2019).

There is a direct correlation between mathematics and poverty, and this factor affects more than 75% of the schools in the country. A major concern that emanated from the results of the TIMES report (2015) with respect to South African learners was that three out of every five learners do not achieve the basic levels of competencies in mathematical skills needed at their grade. When learner achievement in geometry is broken down, it was found that 84% of the learners from independent schools achieve minimum competency levels while only 67% of the learners attending public fee-paying schools and 25% of learners from no-fee paying schools achieved the minimum competency levels.

A second policy affecting mathematics is the maximum number of years that a learner can remain in a phase (DBE, 2017). According to this policy, a learner may only be retained once in a three-year phase of schooling (DBE, 2017). These phases are referred to as the foundation phase, which includes grades one ending at grade three, the intermediate phase starts at grades four and ends at grade six, the general education and training phase begins at grade seven and exits at grade nine, and the final phase of schooling is the further education and training phase which is from grades ten to twelve. A learner is promoted into the next grade if he/she has repeated a grade in that phase despite the learner not meeting the necessary promotions criteria. This allowed learners to get to the end of the GET phase with a General Education and Training Certificate (GETC), required by the technical colleges (Muller et al., 2016). Muller et al. (2016) clarified that previously authorities and school leaders did not implement "the maximum number of years in a phase" criteria to allow learners an opportunity to attempt to achieve the prescribed school qualification on their own, yet with learners performing so poorly in mathematics this did not work. Too many learners were failing at each grade, and therefore the need to promote learners despite them not achieving the skills and abilities in mathematics.

A third policy is that as per the Curriculum and Assessment Policy Statement (CAPs), mathematics moves at a very high speed, ten packed content topics per term, despite the fact that there are generally only ten weeks of genuine teaching time. The implemented teaching plan is not always the intended teaching plan in the school, according to Johnson (2000). Educators place greater emphasis on the completion of the work plan and not on attaining the required skills as set out by policy documents. The fourth policy is that a mathematics specialist is only assigned in high schools. Primary school educators are required to teach all learning areas, thus allowing a non-specialist to teach learners the foundational skills as are necessary for building their mathematics. Muller et al. (2016) emphasize that during this phase of schooling, the backlogs in mathematics are initiated. Jojo (2019) supports this view in expressing that the lower grades battle to give vital foundations for mathematics, and the impediment is inherited in the secondary schools. Johnson further highlights that due to the lack of specialization, many educators neglect to teach geometry.

2.6 COMMON MISCONCEPTIONS IN THE TEACHING AND LEARNING OF GEOMETRY

According to Dikgomo (1994) a misconception is an impediment that learners experience, which may hinder their understanding of mathematical concepts. According to Smith, DiSessa and Roschelle (1993), a misconception is a learner's conception that produces a systematic pattern of errors. According to Confrey (1987, p.96), constructivists identified a misconception "when relatively stable and functional set of beliefs held by an individual comes into conflict with an alternate position held by the community of scholars, experts and educators as a whole".

A number of researchers have shown that learners have problems in fathoming basic geometric skills, which is significant to advance in the learning of mathematics (Mitchelmore, 1997; Prescott, Mitchelmore and White, 2002; Thirumurthy, 2003). Ozerem (2012) stated that most learners learn geometry based on the rote learning approach. Browning (2014) observed several misconceptions amongst learners and educators when learning geometry. Cunningham and Roberts (2010) found that many of the misconceptions experienced by learners are common to both pre-service and in-service educators in primary and secondary schools. Foundational knowledge of geometry was lacking in senior primary learners who did not have sufficient essential geometric skills and definitions and were thinking at lower than the required level (Adler & Pillay, 2016). Regardless of whether credulous or flawed, learners' prior understanding is the fundamental foundation in constructivist learning. In order to advance in conceptual comprehension, it is vital that learners' misconceptions be given due attention. It is, therefore, significant for educators to use learners' misunderstandings as a building tool for learning mathematics (Thirumathy, 2003).

Researchers (Cunningham and Roberts, 2010; Ozerem 2012) have made available some of the reasons for learners' misunderstandings in geometry. The explanations provided by these researchers are that learners can recognize figures only on their physical appearance, inability to formulate relationships between geometric properties, dependence on rote learning, and oversimplification. In addition, Clement and Battista (1992) listed reasons for learners' incorrect geometric concept development as, insufficient understanding of geometry concepts, oversimplification of explicit rules, learning by repetition, and failure to fathom geometrical ideas. The above justification is comparable as the focal point is on the absence of conceptual understanding because of the rote learning method. Marchis (2012), in addition, expressed that learners develop confusion in geometry on account of concept meaning because formal concept meaning produces an individual concept image. This concept picture may not develop in many learners, and it may not be identified with the correct concept meaning. Archavsky and Goldenberg (2005) discovered that frequently there is a disagreement between the mental picture of the mathematical figure and the verbal meaning associated with it. Subsequently, it is necessary to tackle these misunderstandings when teaching geometry so learners can identify the confusion between the verbal meaning and the mental picture that they possess. Educators must have a good knowledge of concepts and procedures of a topic that they are teaching to learners. They should also have an idea of misconceptions that learners may have developed (Carr, 2010; Carr & Skinner, 2009) to bring these misconceptions to the learners' attention. This supports the need for educator competence to be based on sound knowledge of their learners, including their capacity to recognize learners' basis of misconceptions and foresee their reasoning abilities.

Hill, Chin and Blazar (2008) revealed that mathematics educators who are unable to understand and identify their learners' mathematical ideas and thinking are not successful in their teaching**.** However, research shows that many mathematics educators experience problems recognizing learners' misconceptions and envisaging learners' thinking (Asquith, Stephens, Knuth & Alibali, 2007; Zuya, 2014). A significant barrier that arises is that pre-service educators who possess incorrect understandings of geometrical concepts will probably not give correct learning experiences for learners to develop conceptual reasoning. It is a major stumbling block to ensure that pre-service educators are adequately prepared to enter the geometry classroom without any misconceptions. Zuya (2014) recommends ongoing training of in-service educators to correct misconceptions that they may have developed in their learning. Educators' misconceptions are passed onto learners' therefore effective geometric thought is compelling the educator's geometric knowledge (Couto & Vale, 2014).

Learners experience difficulties with systematically constructing images since they are unwilling to connect shapes with visual reasoning (Presmeg, 2006). However, it must be noted that geometric figures are an association of concepts and spatial illustrations that includes properties like shape, size, and location (Gagatsis, Panoura & Dan Elia, 2010. The study conducted by Ozerem (2012) shows that learners solve geometric exercises by depending on the visual observation of a particular geometrical figure instead of a geometrical deduction based on applied knowledge. This occurrence is identified as the learner's challenge in working with geometrical figures as figural ideas. Gagatsis, Panoura & Dan Elia (2010) suggests that powerful geometrical thinking is described by an association and the congruity among figural and theoretical features of geometrical elements.

Some mathematics resources display poor geometry definitions and a confined number of practice exercises (Cunningham and Roberts, 2010) for learners 'to develop an accurate understanding of concepts. Usiskin, Peressini, Marchisetto and Stanely (2003) conducted an analysis of textbooks prescribed for secondary school learners and discovered that a crucial issue with learners developing incorrect understanding came from model representations of figures. This resulted in learners not shifting from the lower levels of the van Hiele levels of comprehension because learners "do not realize the generality of geometric figures" (Usiskin, Peressini, Marchisetto & Stanely, 2003, p. 34). Frequently, educators themselves are unable to supplement the model representations as they too do not have the necessary knowledge (Usiskin, Griffin and Wilmore, 2008), which results in learners experiencing barriers to visualizing three-dimensional figures beyond the recognizable shapes. Kutama's (2012) and Kitta's (2004) examination of secondary school learners' executing tasks involving three-dimensional figures demonstrated that many learner struggles arose from a failure to develop, retain and manoeuvre a mental picture of a solid in three-dimensional space from its two-dimensional portrayal. Learners' misconceptions must be identified and tackled immediately in order to achieve effective geometric thought.

Researchers (Villers, 1997; Siyepu, 2005; and Roux, 2003) identified that learner’s poor performance in geometry, in South African schools, can be attributed to their understanding of features and properties of shapes- the very fundamentals of geometric understanding. Atebe (2008) concurred from his research involving South African and Nigerian learners. The research found that both South African and Nigerian learners had problems in naming shapes with reasons. The research done by Howie (2001) and Brombacher (2001) also confirmed that many South African learners were able to identify the shape but were unable to provide reasons for naming the shape. The learners’ didnot know the properties of the shapes. Knowing the associations and relationships between shapes may help learners with solving riders and improve their proof skills and competency. Atebe (2008) study also revealed that learners struggled to perceive class inclusions of shapes, for example, they were unable to see that a square is a rectangle.

Kolb (2014) and Kosa (2016) discovered that when figures are not represented in the standard form, learners' experience difficulty in identifying the geometric figure presented in the non-standard form; for example, they are unable to recognize a square which is not drawn on a horizontal base to be a square. Many learners have challenges in seeing class incorporation of figures; for instance, they are unable to perceive that a square is a rectangle, rhombus, and a parallelogram (Feza & Webb, 2005; Marchis, 2012).

According Feza and Webb (2005), geometry language incompetence impedes progress in geometric understanding. The following concepts were identified by Feza and Webb (2005) has confusing for the South African learner: midpoint and bisect; intersects and bisects; complementary and supplementary; and similarity and congruence. Lac of basic geometry terminology may be the result of incorrect learning or memorization of the concepts. It is therefore important for educators to frequently make use of diagnostic and reflective tests as guides to identify learners' alternate geometric vocabulary.

2.7 CONCLUSION

Geometry forms an essential part of the South African school curriculum. It studies objects, points, lines, shapes, spaces, properties, and the interrelated relations between them. This chapter discussed the literature that is associated with the barriers to learning geometry. The literature identifies different factors affecting learners and educators from the perspective of both local and international studies. It also looked at the systemic issues that are accountable for learners' inadequate performance in geometry. The perspective of researchers' views on possible strategies to eradicate these barriers and develop effective geometric thought were also discussed in this chapter.

Chapter Three: Theoretical Framework

3.1 INTRODUCTION

The previous chapter provided a detailed outline of the literature explored, to illustrate the different perspectives on aspects that pose challenges in the teaching and learning of geometry. Strategies that have been advocated for effective geometric thought were also presented in the previous chapter.

The primary objective of CAPS was to produce learners capable of utilizing basic and creative thinking in making choices to recognize and discover solutions to problems and work viably as people or with others as individuals of a group. CAPS encourage learners to be exposed to visual, symbolic, and language abilities in different approaches to effectively convey their experiences and identify that problem-solving situations do not exist in isolation. To "communicate effectively using visual, symbolic, and/or language skills in various modes" (DBE, 2011, p. 3) is an important skill in CAPs. Communication in the teaching and learning of geometry is an important outcome of the mathematics curriculum (DBE, 2011). Problem-solving and cognitive development is therefore central to all mathematics teaching. Collaboration and working together are recommended by researchers for learning geometry. Learners, need to spend time with each other and engage in their learning.

Maxwell (2013) advocated that a theoretical framework is a model or plan of what one intends studying. Baxter and Jack (2008) referred to the theoretical framework as being a firm contingent for a study that guides the interpretation data. The complex nature of learning has led to many arguments as to whether learning is best achieved as individuals or as part of a community (Anderson, Reder & Simon, 2000). Finding a "one fit" theoretical perspective for this study was therefore overlapping and conflicting because of the complex nature of how humans learn and how optimum learning can be achieved. Using one theory would restrict the study to a particular view of learning (Cobb, 2007) and in my view, this will not provide trustworthy conclusions to identifying the barriers to learning geometry or establish strategies to eradicate these barriers. The principles of CAPs and aims of the South African curriculum to produce learners that are able, identify and solve problems and solve and make decisions using critical and creative thinking, made me decide to underpin this study on how the two metaphors, namely acquisition and participation (Sfard, 1991) impact on the teaching and learning of geometry. It presents the merits of theories grouped under the two metaphors and considers their contributions to the teaching and learning for effective geometric thought. The idea of the metaphors, acquisition and participation informing learning, is particularly pertinent to this study considering the complex nature of learning geometry. Sfard (2000) advocated the incorporation of both metaphors as learning is not a purely constructivist or purely participative event. She accentuated the need to consider both metaphors, for effective learning to take place.

This section will show how the two metaphors can link together as part of a greater whole. The learner is not viewed as a passive receiver of knowledge, an empty vessel into which the facilitator must pour knowledge. Rather, the learner is viewed as an active participant who constructs his/her own knowledge. As learners engage in learning and new discoveries are found, thinking develops and new ideas are propagated but they are built on the ideas and developments of the past. Acquisition metaphor is a primary assumption in various learning frameworks such as cognition, constructivism and social constructivism. The focus of this study is on the social cultural theory of Vygotsky (1978). The primary assumption of the participation metaphor is learning frameworks such as cognitive apprenticeship, situativity theory and communities of practice. The participation metaphor will be viewed from Sfard's theory of commognition and the role of language in the geometry classroom.

The difference between acquisition and participation is the way in which knowledge is transferred. With acquisition, transfer means that someone has applied knowledge learned in one context to another context. With the participation metaphor knowing is indivisible from context and the context does not have finite boundaries. Participation envisages learning as undertaking activities within a social context. Learning takes place through learners engaging in communication and discourses, in activities. Much learning is informal, occurring as an integral part of everyday life. Focus with the participation metaphor is on social -cultural practices of living in particular situations and not on cognitive processes within a person.

3.2 ACQUISITION METAPHOR

The acquisition metaphor depicts learning as the acquisition and accumulation of knowledge. It encompasses the constructivist theory of learning, emphasizing ideas of construction of meaning. The acquisition metaphor suggests that knowledge is a commodity to be acquired by individual learners. In acquisition learning is seen as the transmission or construction of knowledge. While it may seem that learning theories can be described as acquisition orientated or participation orientated, in most cases both metaphors are used (Sfard, 2008). The complexity of learning suggests that both metaphors complement each other in theories of learning.

3.2.1 A Social Constructivist Perspective

Social constructivism emanated from the constructivist theory. Machaba (2013) describes the theory of constructivism as placing a child in a functioning role in the learning cycle in which learning does not happen in one piece but it is changed and adjusted according to mental designs, earlier learning, environmental settings and the social interaction of the learner. The hypothesis of social constructivism is that educators cooperate with learners through questioning and conversation, skilfully reacting to learners' thoughts and permitting them to discover connections and anticipate future occasions (Vygotsky, 1987). Learners will not change their ideas simply because someone furnishes them with answers. The National Council of Educators of Mathematics (2000, p. 3) described the perfect educational system as follows:

Imagine a classroom, a school or a school district where all learners have access to high quality engaging mathematics instructions. The curriculum is mathematically rich offering learner opportunities to learn important mathematical concepts and procedures with understanding. Alone or in groups and with access to technology; they work productively and reflectively, with the skilled guidance of their educators. Orally and in writing, learners communicate and engage actively in learning.

This quote supports the idea of constructivism and the type of classroom envisaged. The success of the teaching and learning of mathematics is to understand and communicate mathematically and think critically, which poses challenges to learners and educators. The idea that information can be passed on, prompting understanding, should be dismissed while applying constructivism to teaching. The educator should form a "model of learner's methods of viewing an idea and help the learner in restricting those views to be more satisfactory" from both learner and educator points of view (Confrey, 1990, p.109). The inspiration and commitment of learners in the learning cycle has expanded with the utilization of dynamic learning methodologies. According to George (2017), the constructivist classroom can best be described as one that is full of energy. The dynamic interest of the learner in the teaching and learning cycle and learner orientated teaching, form part of the current paradigm shift, supported for educational practice. The dynamic engagement of learners in the learning process is fundamental, as espoused by Polya (2004), who argues that this requires a classroom different from the authoritative and educator-centred traditional classroom, in which the educator remains in front of the classroom coordinating content that is conveyed to learners. In such traditional classrooms, the educator's role is to pass on information to the learners. From a constructivist point of view, learners are continually building and understanding from life encounters, which may be weak, lacking in internal consistency and may have a restricted scope of wonders. Mathematics educators must take cognisance of these constraints in an endeavour to foster more remarkable and viable developments. Thought must be given to what such developments might be and how their advancements can be accomplished (Confrey, 1990). Classroom encounters that focus on geometric connections to develop learners' spatial sense should inspect direction, orientation, and perspective of items in space, the overall shapes and sizes of figures or objects, and how a change in shape relates to an adjustment in size (Machaba, 2013). In a constructivist class, definitions ought to develop from visualizing and drawing, measuring, contrasting and, classifying figures according to their properties. Learners who memorize a definition or a textbook example are less likely to contextualise or apply the definition in context.

Social constructivism underlines that all cognitive capacities including learning, are reliant upon cooperation with educators, parents and peers (Schunk, 2012). This viewpoint features that cognitive development in a mathematics classroom is socially built from learners' encounters and from educators, through communication. Mcherney and Mcherney (2002) highlight that in social constructivism nothing is gained without preparation, instead, existing knowledge is coordinated with new information to grow the network of understanding. Schunk (2012) further claims that in a social constructivist classroom, knowledge is neither attached to the external world nor entirely to the working of the mind, but it exists as the result of mental inconsistencies that arise from the learners' communication with other people in the environment. Vygotsky (1978) was a social constructivist who fostered the socio-cultural theory.

The work of Lev Vygotsky (1934), particularly his social cultural theory, has become the foundation of much research in cognitive development. The core tenets underlying Vygotsky's social cultural theory, as expressed by Walqui (2006, p.160) are:

- Human development and learning originate in social, historical, and cultural Interactions,
- Use of psychological tools, particularly language, mediate development of higher mental functions, and
- The notion of the zone of proximal development as being the optimal space in which learning occurs

Falchikov (2001) attests that learning as a mediated cycle is social initially and becomes individual because of semantically interceded cooperation of the child and significant others that is the parent, educator and experienced peers. The essential hypothesis of Vygotsky (1978) was based on a child's association with culture which he accepts shapes human mental working, right from the beginning on a social level and later on, the individual level. At the social level, mental functioning occurs between people referred to as inter-psychological, then within the child which is intra-psychological. The importance of Vygotsky's (1978) theory is emphasized when he recognizes rudimentary mental capacities and higher mental capacities. Both these mental functions assume a significant part in the learning of geometry. Existing schema created mental functions, change to higher mental functions such as reasoning, problem-solving and envisioning, are accomplished through social interactions which culture makes possible (Zaretskii, 2009). Vygotsky's theory encourages learners' social experiences to be brought into the geometry classroom. Social interaction plays an important role in the learning of geometry. It is through working together on a variety of tasks that a learner internalizes or adopts socially shared experiences and associated effects and acquires useful strategies and knowledge (Scott & Palincsar, 2013). Learners come into the geometry classroom with daily experiences and ideas from their interactions in society. This experience must be utilized. For example, learners would have experienced different shapes in their interactions and allowing learners to bring this knowledge into the classroom will allow them to develop associations and relationships. This study is based on South African learners that have individual and cross-cultural differences. This diversity can be brought into the geometry classroom when concepts are discussed. For example, when teaching shapes, the cylindrical hut, the traditional weaved basket, the cattle kraal and the traditional drum can be used as examples. Learners can draw association since they can visualize these shapes from their personal experiences. A cattle kraal is a circular shaped object and can be used to bring in concepts of radius, diameter, circumference etc. Using examples that learners have socially experienced will motivate and change the attitude of learners towards the learning of geometry. Allowing learners to engage in the learning of geometry by working in groups will result in cognitive development in a mathematics classroom built from learners' encounters and from educators, through communication. Vygotsky's social cultural theory is based on "knowledge represents a permanent construction, reconstruction and deconstruction of reality", depending on experiences of each individual (Alexandru, 2012, p. 19). This theory places great importance on societal role on individual development. Fuberg and Arnseth (2009) indicate that collaborative learning interactions are therefore encouraged in teaching in geometry.

Berger (2012) expresses that a significant contribution of Vygotsky's theory relates to the inter-relationship of language development and thought. In Vygotsky's book, "Thought and Language", he builds up an association between inner as well as external speech. He believed that inner discourse developed from external discourse whereas thought itself developed socially. A new-born child learns the importance of signs such as pointing, crying and gurgling through communication with parental figures to communicate their requests. The child learns to use these verbal sounds effectively for social interaction. The child begins to utilize, build and develop this aspect of language that starts as a tool that is external to the child, for social interaction. As such, Vygotsky postulated that the child uses a word for communication purposes before the child has a fully developed understanding of that word (Umzadze, cited in Vygotsky, 1987). Learners internalise language patterns as they connect with each one another, hear the words around them and notice the communications of others. These language patterns gradually develop into thought perspectives or examples.

Standards of Vygotsky's (1978) theory perceive that "full cognitive development requires social interaction" (Falchikov, 2002, p. 35) and that to do this, learners need to adjust to new circumstances without losing their own social character simultaneously. Crook (1994) set up that language is a significant coordinator of behaviour that assumes a pivotal role in cognitive development. The significance of language in the improvement of the child's mind is recognized by the social cultural theorist and forms an important component of Vygotsky's theory. Vygotsky contended that the most important social tool a child can develop is language. Berger (2012) in support of Vygotsky, considered speech as a tool empowering people to get things by speaking with one another. Vygotsky (1978) argues that the learner's capacity of insight, memory and thinking in childhood do not act independently and that an interrelationship exists among them. An illustration of interrelationship is memory and thinking, which prompts to logical memory. Vygotsky saw learning more like an apprenticeship. Self- talk is utilized by the youngster for communicating and is logically utilized more as an instrument for self-coordinating and self-regulating conduct. Self-talk stops when the child begins schooling because speaking becomes appropriated and internalized. This self-talk then becomes inner speech which develops through its separation from social speech. The line of communication and the line of inward speech assist with the advancement of speech, by which the child mediates and regulates their tasks through their thoughts. These in turn are mediated by the meaning of indications of inner speech. As indicated by Berger (2012), thinking is in this way mediated by language and consequently develops to a much higher level of complexity. Inner speech is not equivalent in structure to outer speech. Outer speech is the way of transforming thoughts into words whilst inner speech is the inverse in that it changes speech into inward thought (Lantolf, 2001). Problem-solving encounters such as reasoning, memorizing, attending and classifying are coordinated within social experience and supported by speech (Crook, 1994).

Participation in the social life of the community opens learners to a set of interpretive practices that become appropriate. In this way, learners become problem solvers as deciphered by their social group, within socio- cultural settings. This is significant in the learning of geometry, as learners need to impart their thoughts in solving geometric problems. Geometric ideas will become more meaningful for the child when the mathematics educator applies this hypothesis of inner speech to develop the learners thinking. Vygotsky (1978) portrays cognitive advancement as a component of verbal associations between a child and a grown-up. It is through these associations that a child develops language as well as logical thinking. “Tools of the mind” is a unique curriculum that is based on his theory that assists learners learn how to become “masters of their own behaviour” (Vygotsky, 1978). This curriculum teaches language and mathematical skills through associations with peers and educators, frequently in the context of play (Barnett, 2008), which is upheld by the cornerstones of Vygotsky’s theories namely, zone of proximal growth and that of scaffolding.

Educators must encourage learners to communicate their understanding of geometrical concepts and expressions using their own words, diagrams and relationships between symbols and diagrams that form basic geometrical knowledge. The use of different resources to develop creative and critical thinking for example the use of dynamic software will provide for visual understanding of geometric concepts. In this study the learners are multi-lingual and communicating in their mother tongue lends itself to better understanding. The need for code switching therefore becomes important.

Vygotsky's theoretical framework referred to as the Zone of Proximal Development (ZPD) relates to the description of the relationship between learner and educator or between parent and child. This relationship includes learning on both sides as depicted by Ellis (2000) in that the educator learns from and about the child as the child learns on account of the of the educator's activities. The idea of the More Knowledgeable Other (MKO) is an integral part of the ZPD. The MKO as affirmed by Vygotsky is someone who has a superior understanding or higher capacity level than the learner with respect to a task, process or concept. Mcleod (2018) claims that whilst one may expect that the MKO is the educator or older adult this is not really the situation as the learner's peer may be the person with more information or experience. The task of the educator and the parent is to arrange for learners to engage in activities that lie within the ZPD. Learners need a lot of help in their early stages so that as they get older they can expand on this learning and strategies. The primary development of geometry in learners is therefore critical for successful geometry thinking in that it allows for the learner to use this experience to progress in geometric thought. When teaching circle geometry learners must have a good understanding of previous concepts for example radius, circumference, midpoint, bisects, perpendicular etc. Vygotsky's insight that scaffolding builds the intricacy of learners thinking and influences both learning and development positively, is supported by researchers (Barnett, 2008; Gredler and Shields, 2008). Educators and parents build scaffolds by telling, illustrating, showing, correcting, pointing, asking, giving graphs and models, clarifying procedures, posing questions and distinguishing objects. The use of diagram in the teaching of proofs can be used as a scaffold to learning.

Educators may utilize the ZPD to shape an association between what a learner can manage without assistance and what a learner can manage with assistance. Shayer (2002), in support of the ZPD states that this model gives insight into how the child's mind develops. Firstly, Vygotsky showed that there was a gap between the actual developmental level as determined by the learner's problem-solving capacity and, the expected improvement as evolved through problem-solving under adult guidance or in joint effort with more competent peers (Vygotsky, 1978). Crook (1994) accepts that following through with the above concept will lead to appraisal that highlights on potential to learn and teach on ability. Secondly, the connection of the ZPD to issues of instruction, comes from how best to arrange useful coordinated effort either with adults or skilled others. As a child reaches maturity within the ZPD, the child's ability to manipulate knowledge independently displays his skill to be able to solve problems independently of assistance (Vygotsky, 1978). According to this model, learning is a dynamic and an on-going process throughout life of a person as s/he moves in and out of the ZPD, as depicted in figure 3.1. This idea is significant in the learning of geometry because learners with equal levels of mental ability develop at different rates. A valid criticism expressed by Chaiklin (2003) on this theory is that the knowledge level of the most knowledgeable person in the room, will probably limit the level of learning expected. This is often the educator in the classroom. It is therefore important to ensure that the educator teaching mathematics has a higher level of knowledge than the learners do. Understanding the concept of the ZPD has significance for teaching and learning of geometry. Educators of mathematics will be better equipped to teach because they understand the individual learner's knowledge base and what prior knowledge exists, to build on.

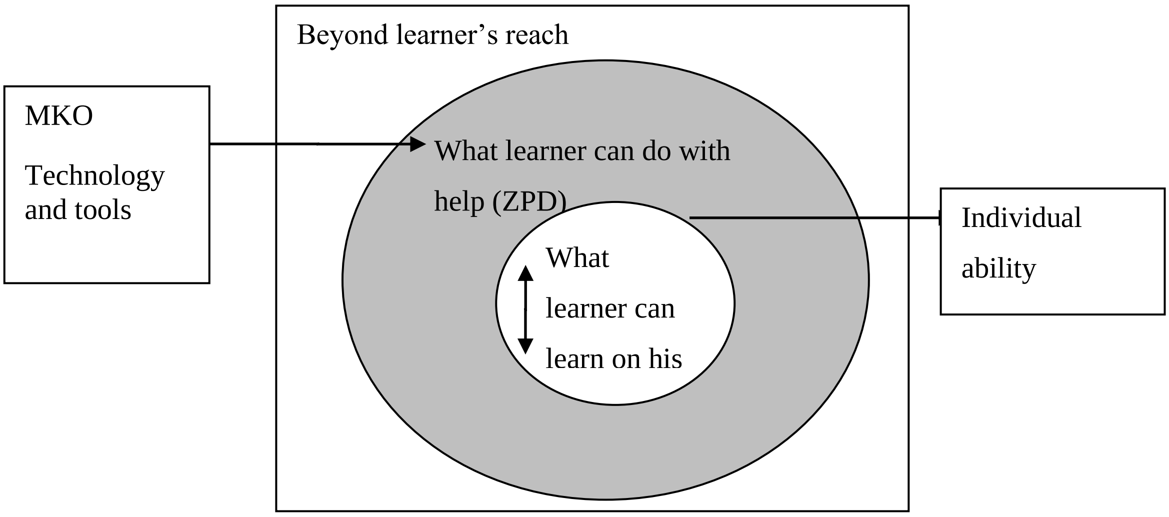

FIGURE 3.1. THE ZONE OF PROXIMAL DEVELOPMENT (CROOK, 1994, P. 12)

Vygotsky (1978) argued that learners' thinking and problem-solving ability fell into three categories, namely those performed independently, those performed with assistance, and those that cannot be performed even with assistance (i.e. those that lie outside the Zone of Proximal Growth). Learners' understanding is facilitated by means of activities such as classroom discussions and exercises both inside and outside classrooms. The learner's activities must start from what the learner can do independently i.e. simple geometric concepts that the learners have encountered previously, to link the already existing knowledge with new knowledge that they can acquire with assistance. Learners develop the skill of completing tasks on their own which they previously were only capable of doing with assistance through continued practice.

Gallimore and Tharp's (1990, p. 85) diagram on the Zone of Proximal Development clearly depicts the recursive nature of the learning process.

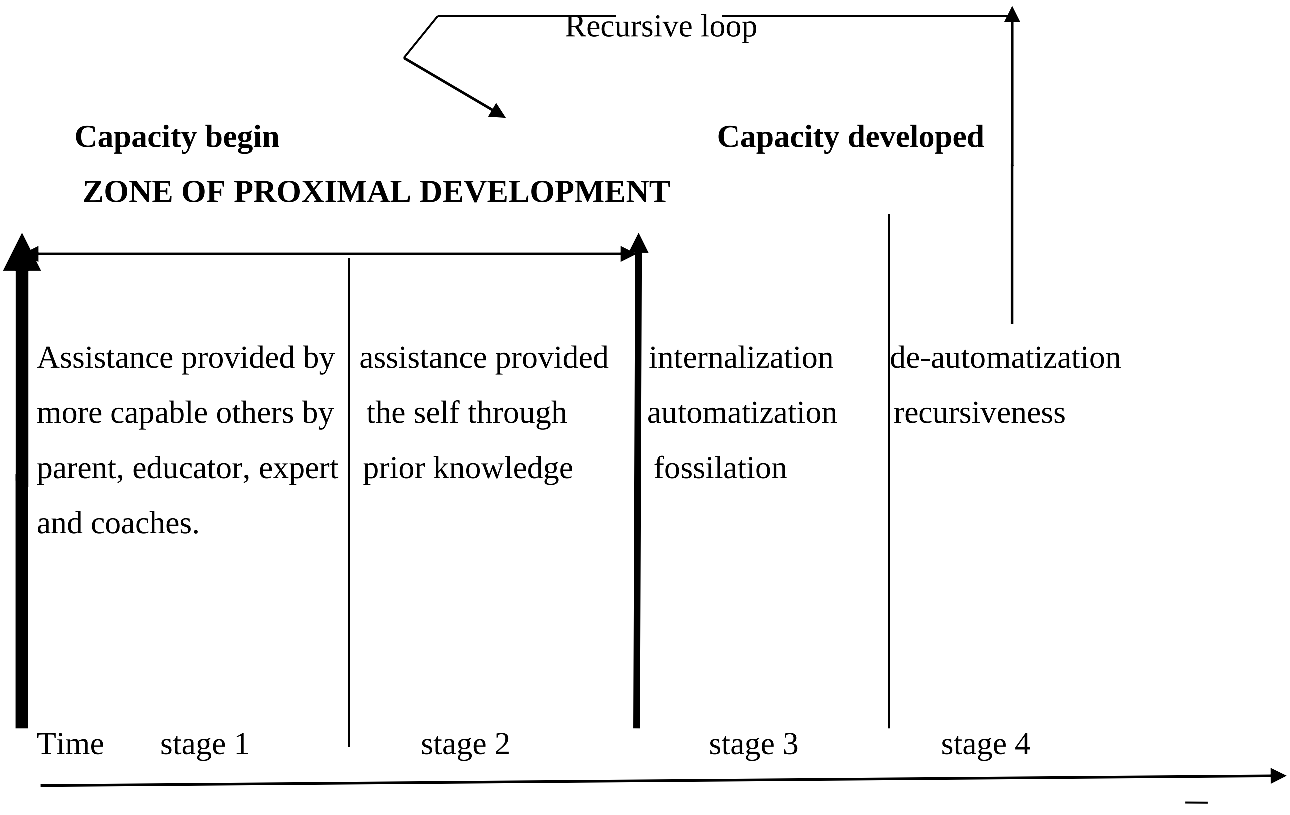

FIGURE 3.2 MODEL OF FOUR STAGES IN THE ZONE OF PROXIMAL DEVELOPMENT (GALIMORE AND Tarp, 1990, p.85)

The four stages, as described by Gallimore and Tarp (1990), will be looked at based on the following theorem "the line drawn from the centre of a circle perpendicular to the chord bisects the chord"

Stage 1: Learning at this stage is assisted by the MKO. This is the stage where the teaching of concepts starts. At this stage learner develop an understanding of language that is appropriate to geometry while relying on expert other to perform the task. Concepts such as line, centre, circle, perpendicular, chord, bisects must be developed. The learner must also understand what is required to be carried out i.e. the learner must show that the line bisects the chord. Modelling, coaching and other methods of scaffolding takes place at this stage.

Stage 2: This is called the self-assisted stage of learning. Learners use prior knowledge to carry out the task without guidance. At this stage control and direction of performance is passed onto learners. Learners practice alone which implies that they can perform certain tasks but are not perfect and require assistance at times. The learner has been taught congruency and definition of radius. The learner uses the previous knowledge to formulate the proof on his own.

Stage 3 – learners reach a stage of independence. Knowledge is fixed. The learners do not need assistance. The learner is able to complete the task on his/her own. The learner can write out the proof of the theorem. The task is achievable without assistance from the MKO.

Stage 4- In this stage learners are at the de-automatisation of performance that leads to going back through the ZPD. Educators' should understand that learners will not necessarily move through these stages the same way or at the same time. It is the educator's responsibility to assist the learner's understanding through the ZPD into the next phase (Vygotsky, 1978)

Long lasting learning is comprised of the same regulated ZPD sequences in which the learner moves from help by most capable other to self-assistance, in a recurring way. The ZPD infers that learners can solve problems at a specific stage only from interaction with educators and co-operation with peers. Further, it implies that once a learner internalises the problem that once was solved with assistance, the learner is able to handle the problem independently. Gallimore and Tarp (1990) conclude that what the learner can do today with assistance, the learner will be able to do alone tomorrow.

The National Council of Educators of Mathematics (NCTM) (2000), reports that the information that learners gain from their own encounters while investigating and interacting with concrete objects in the classroom, have a benefit in acquiring new knowledge based on the theory of the ZPD. According to Von Glasersfeld (1995) this knowledge is the result of learners' active engagement rather than the learners inactively receiving lesson instructions. Simister (2004) identifies the importance of the learner's own voice and claims that emphasis on the regurgitation of facts and repetition of ideas will only produce dull and insipid learners.

Scaffolding is an idea closely linked to the idea of ZPD and changes the degree of support with dialogue as being an important tool of this cycle (Mcleod, 2003). A skilled person changes the amount of help to fit the child's current performance, over the course of a teaching session. Chaiklin (2003) argues that disorderly, disorganized and spontaneous dialogue concepts of a child, are met with the more methodical, logical and rational concepts by the skilled helper. The activities provided in scaffolding instruction are just beyond the level of what the learner can do alone (Olson and Platt, 2000). Brown and Cocking (2000) suggests that the more capable other provides the scaffolds, so that the learner can accomplish the tasks that he or she could not complete, thus helping the learner through the ZPD Vygotsky (1978) defined scaffolding instruction as the "role of educators and other in supporting the learners' development and providing support structures to progress to that next stage or level" (Raymond, 2000, p. 176). An important aspect of scaffolding instruction is that scaffolds are temporary in that as the learner's ability expands, the scaffolding provided by the more knowledgeable other is progressively removed. According to Chang, Sung and Chen (2002) the learner is, finally able to complete the task or master the concepts on his own when scaffolding is removed. Therefore, the objective of the educator utilizing scaffolding teaching strategy, is for the learner to become an independent and self-regulating learner and problem solver (Hartman, 2002). As the learner's knowledge and skill advances, the educator gradually reduces the support provided (Ellis, Larkin and Worthington, 2000). The educator may then take part in co-operative learning following the use of scaffolding guidance as a teaching strategy.

McKenzie (2000) states that scaffolding gives clear guidance and decreases confusion when the educator predicts anticipated problems which learners may experience and gives them stepped guidelines appropriately. Moreover, he argues that it assists to giving structure for tasks, clarifies expectations and incorporates assessment and feedback, thus diminishing confusion, uncertainty, any element of shock and dissatisfaction. Finally, he highlights the importance of scaffolded guidance in a problem-based environment in that in breaks the problem into manageable tasks. This is supported by Ngeow and Yoon (2001, p. 1) who describe PBL conditions as an educational approach that challenges learners to "learn to learn".

Adding credibility to scaffolding guidance and its part in the teaching of geometry Albert (2000) conceptualises scaffolded help that a specialist gives to the beginner as enlisting the learners' interest in the task by first recruiting the learner and simplifying the task. He further considers scaffolded assistance as providing guidance to keep learners inspired in quest for the objective, highlighting certain applicable critical features and, pointing out discrepancies between what has been produced and the ideal solution. Lastly, he views scaffolded assistance as being imperative to lessen pressure and disappointment during problem-solving. Although researchers (Anton and DiCamilla, 2000) concur with the attributes of scaffolding, they argue that the utilization of language and other semiotic frameworks such as gestures, are critical devices for mediating cognitive development. They further maintain, "it is within the ZPD that scaffolding occurs and that semiotically mediated interactions lead to development" (Anton and DiCamilla, 2000, p. 34).

Feza (2013) summarises Vygotsky's perspective to stimulate mathematical thinking in learners to include purposeful play; scaffolding learners' mathematics learning; extending learners thinking through questions; developing mathematics from learners' activities, creating balance between child-initiated activity and adult imitated activity and, connecting learner's informal knowledge with formal mathematics to the explicit purpose of mathematics learning.

Vygotsky's theory of socio cultural development and its components of scaffolding, language and zone of proximal development are relevant to this study as it provides strategies that the educator can utilise in the classroom for improving the teaching and learning of geometry. The essence of social constructivism has been captured through the development of active learning, which is learning by doing, learning by experience, learning through action, learner-centred learning, peer collaboration and cooperative learning. The data analysis will be guided by Vygotsky's theory in terms of acquisition of knowledge by looking at the following aspects, the learners' experiences and encounters being used in the classroom, the use of resources, the understanding of language and geometric concepts, the scaffolding of topics, the use of the learner centred approach, educators content and pedagogical knowledge for teaching geometry and qualifications.

3.3 PARTICIPATION METAPHOR

The participation metaphor is portrayed as giving significance and meaning for the learner by facilitating effective participation in the learning environment. This study involves learners and educators and their barriers to learning geometry. The manner that learners engage in communication to develop geometric thought is important to this study. Discourse using appropriate terminology is an important factor in learner attainment. Expressions such as practice, discourse and communication suggest that the learner must be viewed as an individual needing to take part in the learning activities rather than accumulating knowledge. The complexities involved in the teaching of geometry, would indicate that educators discourse is equally important. Sfard (2000) advocated that for effective learning to take place both metaphors, of participation and acquisition, must be considered. Considering, the multi-lingual classes that South Africa presents, active participation in terms of discourses are important to this study.

3.3.1 Commognition

Sfard's approach is based on the participationist (Lave and Wenger, 1991) vision of learning, in which learning mathematics for instance, is viewed as an initiation into the well–defined practice or discourse of the mathematical community. However, the strength of Sfard's theory is in analysing communication, so it lacks the strong manner presented by communities of practice as advocated by Lave and Wenger (1991). Consequently, changes in thinking results in learning, and learning in a school subject such as geometry, is characterized as a cycle through which a learner adjusts her ways of talking about geometric objects and relationships in a certain, well-defined manner. Further, teaching a subject like geometry involves changing a learners' discourse. Although Sfard's theory focuses on the modification and extension of learners' discourse, her framework is also relevant to the discourse of educators. Educators are learners, they can learn more about their own mathematics just as well as that of learners', through their teaching (Leikin and Rota, 2006). They are thus engaged in mathematical thinking in the classroom. The discursive features of educators' communication in the classroom will be exceptionally influential in learner learning, since their own ways of thinking and communicating will change.

Much of Sfard's theory is consistent with the assumptions of the socio-cultural theory of Vygotsky (1978). Sfard (2007) dismissed the partition that constructivist theorists make between thought and language, as well as between the individual and social aspects of learning. Sfard (2008) used a discursive approach inspired by Vygotsky to make a distinction between language and discourse. Language is a tool while discourse is an action that utilizes this tool or mediates. This viewpoint provides an understanding of mathematics as a social and discursive accomplishment in which talk, diagrams, representations and mathematical objects, play an important role. Consequently, mathematical learning requires several modes of communication (Sfard, 2012). Moschkovich (2012) argued that the word "language" does not mean a list of vocabulary words or grammar rules, but rather the communicative competence fundamental and adequate for competent participation in mathematical discourse.

Sfard (2008) portrayed mathematics as a discourse i.e. as a tool of communication. The commognitive strategy portrays thinking as an individualised communication with oneself. To individualise she said, was to communicate through rules. This form of communication is not only in discussion but also in "talking" to oneself and solving problems (Morris, 2014). In casual talk it can be expressed as 'communicating one's thoughts' or 'putting thoughts in words' (Sfard, 2007, p. 9). The various forms of communication include spoken language, written language, physical objects and artefacts utilized for discursive purposes.

The process of thought and expression is inseparable. Sfard (2008) views learning as moving towards a more sophisticated discourse through participants, and conceptualises mathematics as discourse, a form of communication that is distinguishable by its word use, visual mediators, routines and narratives (Wang, 2015). Sfard's (2008) commognitive framework is established in the claim that thinking is a form of communication and that mathematics is defined as a discourse where mathematical objects are abstract discursive objects. Discourse is referred to as "different types of communication set apart by their objects, the kinds of mediators used and the rules followed by participants and thus defining communities of communicating actors" (Sfard, 2008, p. 93). In other words, a discourse is characterised by the following, key words or vocabulary and the way these key words are used, mediating tools that are visual devices that people use to help themselves while communicating; and a form of repetitive actions which are self-regulated in terms of that discourse.

Word use refers to the utilization of mathematical vocabulary in participative talks. It forms a significant part of communication. In mathematics, vocabulary refers to words that pertain to quantities or shapes. Words in mathematics are profoundly specific. Participants within a mathematics discourse share words that are generally used and have common meanings (Sfard, 2007). Word use is an all-important matter because "being tantamount to what others call word meaning, it is responsible for how the user sees the world, and it is one of the distinctive characteristics of discourses" (Sfard, 2007, p. 245). In particular, a learner's word use distinguishes discourses, which is important in the study of geometry. An important feature of word use in mathematical discourse is objectification, which occurs through reification i.e. replacing the talk about processes and actions with talk about objects, and alienation i.e. using discursive forms that present phenomena in an impersonal way (Sfard, 2008, p. 295). Sfard (1991) elaborated on reification as the transition from operational to structural modes of thinking. Our conceptions are comprised of structural and operational thoughts. A structural conception treats a mathematical concept as an abstract object as if it is real, and an operational concept sees it as involving processes, algorithms and actions. Many visualizations help treat concepts structurally while verbal descriptions often yield themselves to operational kinds of thinking. According to Sfard (2007), the operational conception comes before the structural conception and views word use as more abstract and advanced. In geometry, this is an exception as the static object seems simpler than the algorithm or construction required in creating it.

Visual mediators refer to all the visual objects that are created and represented for example graphs, symbols, diagrams and written words. Visual mediators are "the providers of the images with which discussants distinguish the object of their discussion and coordinate their communication" (Sfard, 2008, p. 147). In order words, visual mediators empower participants in a discourse to identify visually, the objects of their discussion and enhance communication. Therefore, the need for visualization of geometric figures is significant to this study. Sfard (2008) further suggests that visual mediators are important in establishing effective communication since they help to create a common focal point. Tabach and Nachlieli (2011) in support, claim that visual mediators used in communication often influence one's ideas about what is discussed, as well as the chosen discursive actions. There are two types of mediators that Sfard (2008) proposes that serves as important to this study, that is, iconic mediators and concrete mediators. Iconic mediators are visual objects (e.g. diagrams and pictures), that can be scanned with our eyes. Furthermore, these can be constructed by drawing a diagram to represent the words. Concrete mediators are objects that can be physically seen, and manipulated, such as rules or figures. According to Sfard (2008), these concrete mediators do not have to be physical objects; they can be imagined within the learning situation.

Routines are set rules that depict repetitive discursive activity (Sfard, 2008). Routines are generally implicit and they regulate when participants perform a particular action and how well the action is performed (Sfard, 2008). Some examples of routines managing the aims of participants are, responding to questions to please the educator and raising hands before speaking in the classroom. Routines gives valuable data about what the learners do and say as a course of action, to justify their thinking in geometry. Discursive routines are associated with learners' creativities when dealing with a geometric problem, with respect to when and how they apply rules and procedures to solve geometric problems.

Narratives are the set of spoken or composed expressions about mathematical objects that can be subject to endorsement. Sfard (2008) clarifies that communities and individuals accept certain rules, proofs, definitions, axioms and theorems as true or false in mathematical discourse which Wang (2015), refers to as enclosed narratives. The usage of these components by learners can differ from the expectations of the experts of mathematical discourse. Sfard (2008) points out that one of the objectives of school learning is to assist learners change these components of their discourses so they can better engage in activities of mathematics (Wang, 2015).

Sfard (2008) rejects, doing mathematics by following rules set down by the educator and, the view that knowing mathematics means remembering and applying the correct rule when the educator asks a question. She classes this memorized symbolic manipulation type of activity as ritualized guidance that is described as performance, for the sake of connecting with or satisfying others (Heyd-Metzuyanim and Graven, 2015). Lampert (2010) in support, states that learners must participate in a discourse in which they strive to learn more about mathematics, in contrast to ritual participation. Sfard (2007) describes distinctions between ritual and explorative mathematical discourses when she talks about historically established mathematical discourses educators aim for in schools. In explorative discourses, one strives to use definitions as criteria to identify geometric figures instead of direct visual recognition or just checking some partial properties. In ritualized discourses, there is often no need for substantiation routines because claims seem to be self-evident. This transition from ritualised to explorative geometrical discourse is what Sfard (2008) referred to as the meta-level development which present challenges to both learners and educators. According to

Sfard (2007) there are two ways in which the educator can support such transitions. Firstly, the educator can take leadership in appropriate teaching and learning situations and model how words are used and what routines count as acceptable, within the new discourse. By way of example, Sfard (2008) proposes that the educator must demonstrate the use of definitions of geometric figures in a way that enhances direct visual recognition in appropriate situations. In order to succeed, learners need to show confidence in the expert, be willing to take the role of the learner and make changes that bring them closer to explorative geometrical discourses.

The second approach is to support explicitly the desired discourse, by utilizing proper educator classroom exercises and conduct. The educator needs to extract contributions from the learners to recognize and dissect their geometrical discourses against ritualised and explorative discourses. Educator should likewise react in a manner that allows for learners to become mindful of potential differences in the use of words and routines. Sinclair and Moss, (2012) in addition, suggests that the educator may open learners to circumstances in which their discourses would prove insufficient, and support them in the process of understanding the advantages of the new way of doing or saying instead of the method with which they have been familiar.

Sfard's study embraces knowledge develops together through social collaboration with other learners, educators and the use of textbooks to create a fully-fledged understanding of mathematical concepts. It therefore becomes necessary to acquaint learners to language or language symbols before understanding what an expression or symbol means. Sfard's (2007) analogy of introducing new language to learners and the acquisition of this new language with understanding is "like two legs making forward movement possible by not being in the same place at the same time and that one is always ahead of the other at any given moment" (p. 56). Sfard believes that the introduction and the use of language and language symbols when teaching for understanding of algebraic mathematical concepts, is imperative for learners' conceptual development. Similarly, the introduction and the use of language and language symbols are equally important in the teaching and understanding of geometric concepts. For example, without having a good understanding of the concept of a right angled triangle, the learner will experience challenges with the theorem of pythagaros.

Sfard (2000) argues that the "learners of today are usually thrown straight into a pre-determined conversation governed by a set of ready-made rules" (p. 55). Researchers (Burger and Shaughnessy,1986; Fuys et al., 1988; Usiskin, 1982) found that that in most high school geometry courses, the teaching of geometry starts at a deductive level, which is an unknown dialect to learners. The language behind fundamental concepts is inadequate in the teaching of geometry which results in poor performance.

This section described key concepts of Sfard's theory of commognition, particularly those that are pertinent to this study. I focussed on special features of mathematical discourse, word use, routines, visual mediators and narratives. The National Diagnostic Report (DBE, 2017) identified that words and its use was a barrier to learner performance in geometry. Sfard further expressed that communication with use of visual mediators and routines will enhance the understanding of geometry.

3.4 CONCLUSION

The evidence gathered in this study clearly emphasizes that successful academic programmes need to be underpinned by theories supporting acquisition and participation metaphors. The theories discussed that fall under the viz acquisition and participation metaphors i.e. socio- cultural constructivism (Vygotsky, 1978) and comognition (Sfard, 2007), ought to be considered in the teaching and learning of geometry for effective thought. The core tenants underlying Vygotsky's theory of interaction, the use of language and ZPD will be used as the guide to analyse the data gathered. Sfard's theory of commognition and teaching of geometry as a language will also guide this study. Terminology is the key component of success in geometry, therefore the language of geometry is important for its success. The following chapter describes the design and methodology of this interpretive study.

Chapter Four: Research Design and Methodology

4.1 INTRODUCTION

The previous chapter served to provide a comprehensive theoretical framework that guided this study. This chapter outlines the research design and methodology that will satisfy the aims of the research. The presentation in cooperates a description of the selection of participants, how they were selected, data collection techniques and the methods of data analysis. I have also explained my role as a researcher and the ethical issues that I encountered during the data collection and analysis.

4.2 RESEARCH PARADIGM

Paradigms are an accumulation of assumptions, thoughts, insights and perspectives about the idea of the real world. They have been created to understand human phenomena as a logical subject, and as such give direction, direct reasoning and action corresponding to perceived reality. Examples include the positivist, interpretive and critical paradigms. Positivist paradigm is an approach that falls under natural sciences. Interpretive paradigm is related to hermeneutics a theory that originated in the nineteenth century and is principally connected with social science.

Cohen, Manion and Morrison (2007) describe the interpretive paradigm as a focal undertaking set out to comprehend the subjective universe of human experience. Interpretivism is profoundly subjective and researchers put forth the attempt to understand the setting from the inside. This paradigm relates well to qualitative research methodology chosen in this study. I conducted this study from an interpretive epistemological position, viz that what we know is always negotiated in cultural and social contexts. This implies that individuals build implications of reality as indicated by their social settings. Interpretivism is chosen in this study because epistemology emerging out of this study is relativistic and selective, to the actors directly engaged in the social activities. As such, researchers guided by the interpretive paradigm are required to employ naturalistic systems like perception and interviewing. These techniques can guarantee a satisfactory dialogue between the researchers and those with whom they interact, to develop cooperatively a significant reality to avoid misinterpreting the original information (Schwandt 2003). This type of paradigm permits the participants to communicate freely their perspectives in response to the questions posed. The epistemological perspectives of the interpretive paradigm led me to conduct this research using social constructivist theories of research methodology. The assumption of social constructivism is that human beings construct knowledge through their interactions with each other, as well as through their individual processes (Ernest, 2010). Social constructivism accentuates that research on teaching and learning of mathematics must focus on two aspects, viz the educator and the learner. Interpersonal relations, educator and learner or learner and learner, are fundamental in social contexts. The roles of language, text and symbols in teaching and learning mathematics are also important (Ernest, 2010). These ideas suggest that that the study of teaching and learning mathematics is better perceived through observation of classroom practices. The interpretive paradigm has restrictions in that human partiality can never be underestimated as already pointed out, as it is highly subjective (Cohen, Manion and Morrison, 2007). According to Maree (2007), the interpretive paradigm stresses the associations of people as individuals, who structure a social order among themselves in their regular day-to-day life and create routines.

4.3 RESEARCH DESIGN

Research methodological approaches in any study are generally influenced by the nature of the study and the accompanying objectives. Decisions often include selecting either qualitative, quantitative or a mixture of both research methodologies (De Vos et. al, 2011). For the purposes of this study, qualitative research methodology was selected. It is seen as a relevant methodological approach to identify the barriers to effective geometric thought and the strategies needed to eradicate these barriers. According to Flick (2009), qualitative research is expected to approach the world "out there" and to comprehend, describe and in some cases clarify social phenomena from the inside, in a number in various ways:

- By investigating encounters of people or gathering,
- By dissecting cooperation and communications in the making,
- By breaking down reports and documents of experiences and interactions.

Cohen et al. (2018) depict qualitative research in a similar manner when they point out that it may very well be characterized as a specific circumstance in social science that fundamentally relies on watching individuals in their own domain, and associating with them on their own terms. Qualitative research intends to understand the significance that people associate to everyday life. This type of understanding can be described as the effort to acquire a sense of meaning that others give to their situations through an interpretive understanding of their language, art and policies. For Mason (2017) qualitative research is exciting and important. The following four characteristics described by Bryman (2008) influenced the use of a qualitative research design. Firstly, qualitative research places emphasis on words as opposed to on numbers in data collection and analysis, consequently the objective is to get a deep descriptive understanding of the issue being studied. Secondly, qualitative research uses inductive approaches to connect the relationship between theory and research, thus the purpose is to generate theories or hypotheses. Thirdly, qualitative research emphasizes people's translation of the social world. Fourthly, qualitative research mostly involves collection of data from very small samples, using data collected from interviews, observations and documentation. The qualitative research design was suited to my study because I needed to form an in-depth understanding of educators and learners' views of the barriers to effective geometric thought. Krauss (2005) explains that qualitative research recognizes that peoples' experiences differ, as does their reality. The qualitative research employed in this investigation helps to facilitate the understanding of educators' and learners' perceptions regarding the barriers they experience and the strategies needed for effective geometric thought.

Qualitative research design, involves the study, use and collection of a variety of empirical materials that in cooperate case studies, personal experiences, life stories, interviews, observational methods, historical methods, interactive methods and visual texts that describe routines, problematic moments and meanings of individual lives (Denzil and Lincoln, 2005). This study lends itself to qualitative research in that the research is based on educators' and learners' experiences in the classroom. Qualitative research is a field of inquiry in its own right. It is a multi-method involving the interpretive, naturalistic approach. Denzil and Lincoln (2005) characterize qualitative research as a situated activity that locates the observer in the world. For them this methodology comprises of a set of interpretive material practices that make the world visible. They transform the world into a progression of representations including field notes, interviews, conversations, photographs, recordings and memos to the self. It is by these means that qualitative research involves an interpretive, naturalistic approach to the world. In this study, the researcher will be observing things in their natural setting, attempting to make sense of, or interpret phenomena in terms of the meanings people bring to them. The qualitative approach enabled the researcher to immerse herself in the situation or circumstances, present in the context of the study. This study was specifically carried out in three schools in Umlazi with the aim of exploring the barriers within the schooling system in terms of teaching and learning and, identifying possible strategies to overcome these barriers.

4.4 RESEARCH STRATEGY

Creswell and Miller (2000) distinguished the following five strategies that can be used to design qualitative research; namely biography, phenomenology, grounded theory, ethnography and case study.

Biography is utilized to report on and document an individual's life and his encounters as communicated to the researcher. Phenomenology intends to understand and decipher the meaning that subjects give to their everyday life. It ascribes meaning to encounters, topics or ideas. Grounded theory is based more on observation than on deduction. According to Rubin and Barbie (2001), the researcher begins with collection and analysis of data with the aim of developing a theory. Ethnography is the study of an intact social or cultural group and depends on observations over a prolonged period. According to De Vos et al. (2011), the end result of the study is a descriptive, interpretive and holistic, cultural portrait of the group. Finally, case study can be regarded as an exploration or an in-depth analysis of a bounded system or a single or multiple cases over a period of time, Creswell and Miller (2000). The case contemplated can allude to an interaction, action, event, programme, individual or various people. According to Rubin and

Barbie (2001), the researcher enters the field with information of the relevant literature before conducting the field research.

This study used the case study strategy, to explore the barriers for effective geometric thought in secondary schools and to establish strategies to eradicate these barriers. This is, because the case study was a unique way of observing any natural phenomenon, which exists within a set of data (Yin, 2003). Cohen et al. (2018) states that with the case study, the researcher typically observes the characteristics of an individual unit. In this study the unit of study grade 11 geometry. A case study is a descriptive and exploratory analysis of a person, group or event. The following advantages were taken into consideration when the case study was selected.

4.4.1 Advantages of Case Study

According to Yin (2003) one of the benefits of case study method is that it is often finalized within the context of its use, that is, within the situation in which the activity takes place. The researcher must observe the participants within an environment, such as reading in the classroom or solving mathematics problems. In this study, the researcher will be observing the learning and teaching of mathematics. This would contrast with experimental study for instance, which deliberately isolates a phenomenon from its context, focussing on a limited number of variables (Lowe, 2007). Yin (2003) goes on to explain that variations in terms of intrinsic, instrumental and collective approaches to case studies, allows for both qualitative and quantitative analysis of the data. The detailed qualitative accounts often produced in case studies not only help to explore or describe the data in a real life environment, but also helps to explain the complexities of real life situations, which may not be captured through experimental or survey research. This study involved participants in the classroom and their experiences.

4.4.2 Disadvantages of Case study

The researcher also kept the following disadvantages in mind during the study. Yin (2003) describes the three types of disadvantages with respect to the use of case studies. Firstly, case studies are often accused of lacking in rigor. The researcher becomes “sloppy” in the use of the strategy and there may be bias attached to the research. Secondly, case studies provide very little basis for scientific generalization since they use a small number of participants, some concluded with only one subject. The question commonly raised is “How can you generalize from a single case?” (Yin, 2003, p. 98). Thirdly, case studies are often labelled as too long, difficult to conduct and produce a massive amount of documentation. The above disadvantages were taken into consideration when the study was conducted so that it was avoided.

4.5 THE RESEARCH SITE

The constitution of South Africa provides the legal framework for a unified system of education based on cooperative governance at three levels (Lemmer and Van Wyk, 2010). The three levels are categorised as General Education and Training (GET), Further Education and Training (FET), and Higher Education (HE) (Lemmer et al., 2010). This study focuses on the FET band in three schools residing in the Umlazi district. Creswell (2009) points out that the national department of education in accordance with the South African constitution, has given authority to the provincial legislature and government to run the affairs of education, subject to the national policy framework. The provincial department is in charge of general administration and management of educational institutions, in line with policy determined by members of the executive council for education. Educational provisions take place from provincial head offices, districts and circuit offices (Lemmer et al., 2010). The KZN department of education has 12 districts under its administration with several circuit offices. District offices play a pivotal role in ensuring that all learners have access to education of progressively high quality. Based on provincial plans their task is to work collaboratively with principals and educators in schools, with the assistance of circuit offices to improve educational access and retention, and help schools achieve excellence in learning and teaching.

This study was conducted in the Umlazi District and the three secondary schools were under the administration of the Phumelela circuit office. These schools were selected as they are convenient for the researcher to conduct the research since they are within a 10km radius. A detailed description of the research sites will follow so that other researchers and readers will understand the findings.

School A is a school in a semi-urban area with 100% black enrolment. School A caters for learners from grade 8 to grade 12. The school has an average population of 1200 learners. Many of the learners come from low-income homes. There are 37 educators on staff. The language of teaching and learning at the school is English. The majority of educators at the school communicate in IsiZulu.

School B has a mixed population in terms of racial composition and is in an urban area. School B caters for learners from low to middle income. School B is a combined school and has learners from grade R to grade 12. The school has an average population of 1600 learners with 48 educators. The school has about 75% black learners, 20% Indian learners and 5% coloured learners. There is balanced representation of gender and race amongst the staff. The learner body is multilingual but the language of teaching and learning is English.

School C has a mixed population and is in an urban area. School C caters for learners from middle income to high income. School C has learners from grade 8 to grade 12. The school has an average population of 940 learners with 34 educators. The school has about 75% Indian learners and 25% black learners. The language of teaching and learning is English.

4.6 SAMPLING

Sampling refers to decisions taken on where to conduct the research and whom to involve (Creswell and Miller, 2000). According to De Vos et al. (2011) a sample comprises the elements of the population considered for actual inclusion in the study. Stofile (2005) describes sampling as a process which is employed in gathering data from potential participants. It can be used when one gathers data from only a fraction of the population of a group or phenomena, which one intends to study. Purposive sampling means that the respondents are chosen based on their knowledge of the information desired (Cantrell, 2003). In this study, the population group comprises learners from grade 11 and educators teaching mathematics in the further education and training phase in all three schools. The total sample size from the three participating schools for this study is 18 learners for the focus group discussion and 8 educators in all three schools. This number is adequate for the study because the logic and power in qualitative sample selection is that cases that are few are studied in-depth and yield more insight regarding the topic (Mcmillan and Schumacher, 2010). The insight rests on the richness of the information from the cases and the researcher's analytical ability rather than the sample size.

Examples of sampling techniques include purposive and random. Purposive sampling as the name suggests is used when the researcher has already identified the context and study participants with the potential to yield data amiable to the purpose of the study. While random sampling techniques give the most reliable representation of the whole population, non-random techniques, relying on the judgement of the researcher, or an accident, cannot generally be used to make generalizations of the whole population. For the purpose of this study, purposive sampling was used to select the educators and random sampling was used to select the learners. The researcher "handpicked" the cases that were included in the sample based on judgement of typicality and on experience of the control phenomenon (Cohen et al., 2018, p. 34). With purposive sampling, the researcher recognizes that there may be inherent variations in the population of interest (Wolcott, 2008). The researcher attempts to control this by using subjective judgement to select the sample that s/he believes to be a representation of the population. Purposive sampling can lead to very good samples but there is no guarantee that it will be successful (Cohen et al., 2018). Its success depends on two assumptions:

- The research can identify in advance the characteristics that collectively capture all variations. align
- The chosen sample will correctly reflect the distribution of these characteristics.

In each of the three schools' learners were either taught Mathematics and Mathematical Literacy. School A had 124 grade 11 learners of which 55 were mathematics learners and 69 learner mathematical literacy learners. School B had 98 grade 11 learners of which 48 were mathematics learners and 50 were mathematical literacy learners. School C had 125 learners of which 62 learners were mathematics and 63 were mathematical literacy learners. Only School C had two units of mathematics classes. Six grade 11 mathematics learners were selected from each of the schools. This gives a total of eighteen grade 11 mathematics learners randomly selected from a total of 165 mathematics learners from the three schools, for the focus group discussion. Stratified random sampling was used to obatin equal percentage of male and female learners as well as to take care of the different race groups.

The educators that were selected were those that were teaching grade 11 mathematics currently, or taught in the further education and training band. The schools selected had a maximum of three educators in the further education and training phase therefore all of those educators were selected for the study. In one school there was only two educators that were teaching mathematics in the FET phase. The number of educators selected for this study was therefore eight.

4.7 GAINING PERMISSION TO ENTER THE RESEARCH SITE

Permission into the field was gained from the relevant authorities (Appendix 1). Silverman (2016) mentions that although permission by the relevant authorities is important, it is also imperative that all people directly involved in the research, be consulted in the process of gaining access. The researcher consulted and gained permission from the principals of the three selected schools and the schools' governing bodies, before commencing with the research (Appendix 5). The aim of the study was explained to the parents of the learners who participated. Permission from the parents was obtained since the learners were all minors (Appendix 3 Part B). Consent forms were also given to the mathematics educators involved in the study (Appendix 3).

Permission granted at the beginning of the research does not mean that the researcher is entitled to all necessary information and should therefore gain further permission from time to time (De Vos et al., 2011). Gaining further permission to the research site can be a challenging task and it depends on the imagination of the researcher as well as his interpersonal and decision making skills. Silverman (2016) advises that the researcher should treat the community with tact and openness to achieve more rich data and to obtain permission readily. The following factors were considered in this research study availability of the participants, viz the time of the interviews, transport costs, a conducive environment and language barriers.

4.8 ROLE OF THE RESEARCHER

LeCrompt (2000) explains that one of the key points to a successful research attempt is the identity of the researcher. The researcher must become involved in the participants' lives in order to gain an understanding of the participants as expressed by Bogdan, Devault and Taylor (2015). Oancea and Punch (2009) explain that the researcher is expected to become involved in order to discern the participant's habits and thoughts as well as to decipher the social structure that binds them together. The researcher cannot occupy the role of an aloof observer but instead must try to build a relationship, to gain confidence and trust (Schurink, 2001) The researcher's biasness should not influence establishing and building intimate relationships with participants that will foster trust and confidence (Wolcott, 2008).

Berg (2009) however, argues that although the researcher must enter the world of the participants to gain trust and confidence, he or she must also remain aware of what the participants are saying and doing. The researcher, being a mathematics educator, did not relate her opinions and suggestions to the participants. The researcher must sustain a certain degree of aloofness from the participants and their views so that the researcher can go beyond his perception to see what the participants see (Berg, 2009). Bogdan, Devault and Taylor (2015) advise that the researcher must clarify his or her processes and their importance, to the participants. The aims and the objectives of this study, how it was undertaken and visualized, purpose of the results, and whom it will benefit, was clearly set out by the researcher before the commencement of the research. The same information was discussed with all participants. The researcher was honest, patient and made the participants feel comfortable.

In qualitative research, the researcher is the instrument himself or herself. The researcher gathers and analyzes the data, it is therefore possible that he or she will have an effect on the data gathered therefore affecting the validity (Bogdan, Devault and Taylor (2015). In order to decrease the researcher effect on the data, the researcher should strive to blend in with the setting by not disrupting the normal flow of events during the data collection process (Schurink, 2001). Berg (2009) advises that in order to avoid the researcher's influence on the data, more than one data collection method should be used so that symbolic reality and more valid facts can be obtained. The researcher did not express her opinions to the participants but elicited responses using probing questions.

4.9 METHODS OF DATA COLLECTION

Data collection was conducted in three phases

4.9.1 Phase 1: Pilot study

A pilot study was conducted to ensure that the relevant data could be obtained from the respondents. This allowed the researcher to focus on specific areas that may not have been too clear in the initial drawing up of the questions. The pilot study enabled the researcher to test the questions for the semi-structured interviews and focus group discussion to ensure that it provided relevant data. The pilot study also assisted in discovering possible weaknesses, inadequacies, ambiguities and problems in all aspects of the research; so that they could be corrected before the actual data collection took place.

4.9.1.1. Challenges that emanated from the pilot study

In this study the questionnaire, semi-structured interview questions for educators and focus group discussions for learners were piloted. It was important to determine how the design could be improved and to identify flaws in the measuring instrument and whether the phrasing of words required cross checking (Kumar, 2019). One school was selected for the pilot study. There was only one unit of mathematics in each of the grades. The educator was not always available despite pre- arranged meetings. To avoid this problem, the researcher allocated additional dates thus giving the educator enough time if problems were experienced. The semi-structured interview was conducted at the end of the school day. Questions five and eight required more clarity and this was re-worded in the main study. One problem experienced was getting the learners together for the focus group discussion due to their class timetable. The best time was during their break. The researcher therefore allocated more time for the data collection process, to accommodate all participants. Five learners from the FET phase were involved in the focus group discussion. The questions of focus group discussion was well understood and learners were able to express themselves freely. The questionnaire for both learners and educators was pilot tested as well. All questions were answered. No serious challenges were experienced in this instrument.

Findings from the pilot study indicated that more time must be allocated to the data collection process due to unforeseen circumstances. There was a need to have a translator present so that questions can be clearly understood by learner participants. Some parents of the learners needed a detailed explanation about the study before they gave permission for the child/ward to participate in the study. The confidentiality of the process was discussed and this helped to alleviate any fears that they may have had. Parents also felt that participating would affect their child/ward's results. Once the criteria were explained it alleviated their fears. This was also done in the main study.

4.9.2 Phase 2: Questionnaires

A questionnaire is a set of questions on a form, which is completed by participants in respect to a research project. The questionnaire is a widely used and useful instrument for data collection. Qualitative questionnaires generate rich material that is highly informative of various aspects of everyday life, including the past and present of the participants (Eckerdal and Hagstrom, 2016). The questionnaire was used to collect data from the educators (Appendix 4) and learners (appendix 7). It provided rich data that could be easily tabulated and analysed.

4.9.3 Phase 3: Interviews

Interviews were used as one of the data collection methods. An interview is an interaction between the interviewer and the interviewee. It is also referred to as an exchange of views between two or more people on a topic of mutual interest, for the production of knowledge (Cohen et al., 2018). A research interview is a two-person conversation initiated by the interviewer for the specific purpose of obtaining information relevant to the research. According to De Vos et al. (2011), interviews are conducted because people are interested in other people's stories. Oancea and Punch (2009) define qualitative interviews as attempts to understand the world from the participants' points of view and to unfold the meaning of people's experiences. Challenges that face the researcher when conducting qualitative research interviewing are; establishing a rapport in order to gain information from the participants, coping with unanticipated problems and rewards of interviewing in the field, recording and managing large volumes of data generated even by short interviews (Wolcott, 2008).

During the interview process with both learners and educators, the researcher followed the guidelines outlined by De Vos et al. (2011) to ensure that the interviews were effective and provided the data needed to achieve the aims of the study.

- The educator was allowed to speak freely and did about 90 % of the talking.
- The questions were clear and brief
- Questions were asked one at a time
- Questions were semi- structured, which allowed for flexibility and for the participants to respond in their own terms.
- The questions were funnelled from a general to specific…
- Key questions were repeated
- The environment of the interview was comfortable and conducive to the…
- The participant was allowed to express him or herself freely without any disturbance.
- Focus was maintained, as the researcher was in control despite allowing for freedom of speech.

Effective communication during the interview process is also very important to gather information. Listening to and encouraging the participants, are also important during the interview process. Wolcott (2008) advises that clarifying answers and doing a reflective summary is important. The interview that was carried out with the educators was a semi- structured one- one interview. This type of interview is organized around areas of interest while still allowing for considerable flexibility.

4.9.3.1 Semi- Structured Interviews

The semi-structured, one-one interview technique was used where educators were the respondents (Appendix 6). One of the reasons for the selection of this method is that semi-structured interviews help to gain a detailed picture of the participants' perceptions, beliefs and on their experiences on the teaching geometry. This method allowed for greater flexibility and allowed the researcher to follow up on new avenues/ as they emerged in the interview. The researcher was able to follow up on particular, interesting avenues that emerged, and gained a fuller picture of the participants' experiences in the classroom. The interview was approximately 30 minutes long. De Vos et al. (2011) state that, although a set of predetermined questions must be set by the researcher, these questions are to guide that interview process rather than dictate the flow of the interview. The participants were able to share more closely because of the direction that the interview took and were able to introduce issues that the researcher did not consider. The interview schedule is important as it forces the researcher to think explicitly about what she wants the interview to cover. Semi- structured interviews can last for a considerable amount of time and can become involved and intense. The schedule therefore provides direction to the interview process.

One of the strengths of semi-structured interviews is that it is a useful tool for getting large volumes of data quickly. It is also effective in obtaining detailed data. A weakness of this technique is that it involves personal interaction and co-operation. Participants sometimes may be unwilling to share information. The researcher may not ask relevant questions to obtain the desired results. The responses can be constructed. The researcher therefore used a well thought out schedule to obtain the desired result, that is, to explore the barriers to effective geometric thought and establish the strategies to eradicate these barriers. The educators were interviewed in their respective schools during their non-teaching periods or afterschool hours so that the interview process does not affect their teaching time. Clarity of the responses was sought out to ensure that the researcher clearly understood the responses.

4.9.4 Phase 4: Focus Group Discussion

The next method that was used for the data collection was the focus group discussion (Appendix 8). This was used for the grade 11 learners of each school. It is group interviews with participants who have certain characteristics, which are common. What the participants in the group say during the discussion constitutes essential data in the focus group (Andrew and Johnathan, 2006). Focus groups are a powerful means of expressing reality and investigating complex behaviours and experiences. Some of the reasons that led to the researcher selecting focus group discussions are that focus group discussions are a self-contained method, which serves as a principle source of data, making it useful to this study. The data obtained in the focus group discussion can be used as supplementary data to the questionnaires. The learners were at ease during the focus group discussion which lasted for approximately forty minutes. They felt more comfortable to be able to express their views on the barriers to effective geometric thought. The learners carried themselves in a mature manner and the focus group discussion was orderly.

Strengths of the focus group discussion

It has the ability to produce a concentrated amount of data on any topic. Participants make comparisons among each other’s experiences and opinions and this provides valuable insight into complex behaviours. The group provides a stimulating and secure setting for participants to express ideas without fear of criticism. There is great potential to uncover important constructs. The focus group allows for a, deeper understanding of the phenomenon being studied (Yin, 2003).

Weaknesses of focus group discussion

The researcher took into consideration the following weaknesses that may crop up during the focus group discussion

- Biasness
- Since only certain people may participate, their voices and opinions may overpower the others.
- Passive participants may be un-duly influenced.

4.9.5 Phase 5: Lesson Observation

Observational research is a valuable method for studying classroom contexts because it allows researchers to collect detailed information about environmental factors and educator behaviours, within a natural and authentic setting. Observation lies at the centre of all case study research because it allows the researcher to capture information within situations and enables the case to be seen through the eyes of the participants (Cohen et al., 2018). It has been widely used to collect data on learner-educator interactions (Smith, 2004), technology integration (Morrison and Lowther, 2010), instructional quality (Hesse-Biber and Leavy, 2006) and specific teaching and learning behaviours. The observational method was selected to better answer question three of the research questions (Appendix 4). My intention was to maintain objectivity as much as possible by keeping my distance. The lesson observation satisfied the aim of this study to shed more light on the instructional practices that are happening in the classrooms at the three schools and to observe the educator-learner interaction. The learners were not considered as active participants since they were not interviewed. It provided an opportunity to be able to determine the level that metaphors of acquisition and participation to engage learners in effective geometric thought, is utilized. It was important to determine if educators used activities and language appropriate to encourage geometric thought. Furthermore, the observations were used to gain insight as to whether prior knowledge and experiences of learners was considered, and to what extent dialogue was entertained. The observation for each participant lasted for about fifty minutes. Lessons were videotaped to allow the researcher to replay them while translating and analysing the data. The manner in which educators encourage geometric thought in learners were observed i.e. either through the acquisition metaphor where the influence of constructive models of learning is seen as a transmission and construction of knowledge or the participative metaphor where learning is seen through learners engaging in discourses and communication or activities. The lessons observed were lessons on circle geometry. The lessons were observed on consecutive days to allow for continuity. This allowed for stages, as indicated in figure 3.2 of Vygotskys theory, to be observed. The topic on circle geometry requires learners to have a rich vocabulary of concepts such as, congruency, parallel lines, perpendicular lines, chord, diameter, subtends, sector, circumference. The learners understanding of these concepts are extremely important for the proofs and application to riders. Prior knowledge is an integral part of the social constructivist theory. In earlier grades learners were taught many concepts that are needed for the success of circle geometry. For example, angles that are formed when lines are parallel, supplementary and complementary angles and the relationship of the exterior angle of a triangle and the interior opposite angles. Sfard’s theory on commognition speaks about the importance of word use in the geometry classroom.

Participant observation was only possible during class time therefore proper permission and arrangements needed to be made not to disrupt the lesson. The observational schedule was also utilised to verify the answers that were obtained in the interview with the educators. The data collected from observations directly inform the improvement of teaching practices (Hill and Grossman, 2013). Three geometry lessons in one educator class in each of the three schools were observed. The educators selected was one of the educators that were interviewed. The observation was to get an in-depth insight into the way educators deliver a geometry lesson in a natural setting. Bryman (2001) points out that the researcher should strive at all times towards gaining feelings and impressions and experiencing the circumstances of the real world of the participants by living alongside them, and by interpreting and sharing their activities. For qualitative research it is only by getting close to their subjects and becoming an insider that the researcher can view the world of the participant in that setting (De Vos et al., 2011). Neumann (2001) emphasized that to be able to listen, to see, to inquire, to observe and to write up notes, is of special significance in participant observation.

All the senses are used in participant observation and the researcher should become an instrument that absorbs all sources of information.

Some of the educators did invite me to speak to the learners at the start of the observation, which I briefly did. I assured them of anonymity and that I was not there to criticise or assess them and that I was only there to observe the lesson. I placed myself in an inconspicuous space, where I could observe without being a distraction. Notes were made of the interaction between learner and learner and learner and educator. The observation schedule was also completed. As soon as the observation of lessons and transcription of field notes were concluded, the individual educator interview was conducted, to allow the educator to give greater clarity to interactions that were perhaps not clear in the observation or recording. The table below summarises the lessons observed and the topics.

Table 4.1 : Summary of Lessons observed

Educator	Lesson Observed
Educator AE1 (Educator from school A)	Lesson 1 : Proof of theorem: The angle between the tangent and chord is equal to the angle in the alternate segment. Lesson 2: Simple application of the theorem. Lesson 3: Application of theorem- More advanced examples.
Educator BE1 (Educator from school B)	Lesson 1: Proof of theorems on cyclic quadrilateral. Lesson 2: Application of the theorems Lesson 3: Proof of the theorem on tangent- chord.
Educator CE2 (Educator from school C)	Lesson 1: Proof of theorem: The angle that an arc subtends at the centre of a circle is twice the angle it subtends at the circumference. Lesson 2: Application of the theorem Lesson 3: Solutions of the application worksheet

4.10 DATA ANALYSIS PROCEDURES

De Vos et. al (2011) provided a detailed explanation of activities to be carried out during qualitative data analysis: collection and recording of data; managing the data; reading and memoing; describing, classifying, interpreting; representing and visualizing. The data to be analysed was gathered from the following sources:

Table 4.2 Summary of Data Collection

Participant: Educators	No Received	Participant: Learners	No. Received
1. Questionnaires	8	1. Questionnaires	18
2. Interviews– audio recorded	8	2. Focus group discussion	3 groups of 6 participants
3. Lesson observation – video recorded	3 lessons per school from one educator total 9		

The data was recorded in a systematic manner, video recording of the lessons observed, audio recording of the interviews and written records were obtained, that was appropriate and would facilitate analysis. Ritchie and Lewis (2003) proposed that qualitative data analysis, be conducted in two key stages, managing the data and making sense of the data through either descriptive or explanatory accounts. Qualitative data has to go through the process of data management because it is produced in large volumes (Ritchie and Lewis, 2003). One of the methods of managing and describing qualitative data is applying a thematic framework. This is a framework that is used to classify, organize and describe data according to key themes, concepts and emergent categories. There are two ways of deriving themes for data analysis, viz inductive and deductive theme development (Freeday and Muir- Cochrane, 2006). Inductive theme development involves developing of initial themes from the data. It involves reading and re-reading the data several times to identify patterns from the data and to allow themes to emerge directly from the data. The emerging themes become the categories for data analysis (Freeday and Muir-Cochrane, 2006). Deductive approach involves the use of a priori template of codes to be applied as a means of organizing data for subsequent interpretations (Fereday and Muir-Cochrane, 2006). The template might be drawn from the data or as a priori themes from either research questions or an analytical framework.

Cohen et al.. (2018) presented five methods of arranging and describing qualitative data during thematic analysis. These are; by group, individual, an issue, research question and instrument. In this study, I used the research question method to organize and present the data analysis.
Cohen et al. (2018) state that organizing data analysis using research questions is a useful method for several reasons. Firstly, it draws together relevant data from all sources and collates it to the research question. Secondly, it links research questions to the data thereby closing the “loop on" the research questions that were typically raised in the early part of the inquiry. Thirdly, the method contains a degree of systematisation in the sense that it is possible to draw patterns, relationships and comparisons across data types. This implies that analysis by question allows convenient and clear explorations, as data from different sources can be organized consecutively.

Using the research questions for organizing and analysing data allowed me to connect research question and results clearly, hence ensuring that the data analysis is geared towards answering the research questions and achieving the goals of this study. This method also allowed me to compare results from different sources for consistencies and inconsistencies. This created an opportunity to separate data from different sources and look for meanings of either consistencies or inconsistencies. The researcher ensured that data analysis began immediately after each interview session, this helped to ease the burden of voluminous data collected. Below is a detailed description of the data analysis process followed in this study:

- Firstly, the researcher organized the data into topics that were fixed in the research questions; each category of topic was further divided into sub-categories as the data analysis proceeded.

- The data collected from the interviews was then transcribed into a format that enhanced analysis. The researcher carefully read all data and listened to tape recordings to ensure that no valuable information was missed. Segments, words, phrases, sentences or paragraphs that contained meaningful ideas were used to break data into meaningful units. The questionnaires were also used to enhance and support the data obtained.
- The observation schedules were analysed for commonality in terms of pedagogical content obtained. This was also used to support the data obtained from the interviews. Video recordings were viewed repeatedly to identify the barriers and challenges in the teaching of the necessary skills. Strengths were also identified in the teaching.
- The researcher read the data several times to get a sense of the whole, in order to begin coding. Coding is the name, phrase or symbolic meaning assigned to the data segments. Codes can be activities, events, strategies, participant perspectives or context.
- After coding all the data obtained from the instruments, the researcher sought to find categories from the codes that were generated. Categories are groups of codes. These symbolise the main ides that are used in describing related coded data that as a single category provides meaning to a combination of codes.
- Finally, patterns/themes were derived. After codes had been categorised, themes were discovered from the categories.

Table 4.3 Theoretical Framework and Data Analysis

Theoretical framework Acquisition /Participation	Instruments	Theme	Sub-theme
• Human development and learning originate in social, historical, and cultural interactions	Interviews with educators Lesson Observation Focus Group discussion with learners	Engagement of Learners	Class size Attitude Resources- visual, technology, written
• Use of psychological tools, particularly language, mediate development of higher mental functions And Commognition	Interviews with Educators Focus Group discussion with learners Observation of lessons	Using Language for thinking	Language of Educators Language of educators
• The notion of the zone of proximal development as being the optimal space in which learning occurs	Interviews with Educators Focus Group discussion with learners	Exploring the teaching of Euclidean Geometry. Educator content	Educator Methods Demand of the curriculum Educators Content language

	Observation of lessons	knowledge	

4.11 ETHICS IN QUALITATIVE RESEARCH

The following basic principles as proposed by Cantrell (2003, p.134) were considered for the research to be ethically sound:

- No one was involved in the research without informed consent. All participants were fully aware of the objectives of the research and were given the opportunity of refusing to participate in the research.
- Deception of research participants should be avoided.
- Participants' privacy should be respected and confidentiality should be guaranteed and maintained.
- Accuracy of the data and the interpretation should be the leading principle, which means no fraud or omission with the collection or analysis of data should occur.
- The participants should be respected.
- The well-being of the participants must be considered.

The principle of informed consent arises from the subject's right to freedom and self- determination. Gaining consent will protect and respect the right of self-determination, and places responsibility on the participants should anything go wrong in the research (Cohen et al., 2018). The participants were interviewed in a safe environment that was convenient for them. Consent was obtained from the respondents and the principal of the school that was used. The respondents were informed of the purpose of the research and its intention. The participants were not forced to be part of the study but were willing participants and they could have withdrawn from participation at any time that they wanted to. Their identity remained anonymous. Pseudonyms were used for both the schools and the participants. McMillan and Schumacher (2014) stated that the information about participants must be regarded as confidential unless otherwise agreed through informed consent.

McMillan and Schumacher (2014) advocated that when any research is conducted, approval must be obtained before data can be collected. Data collection was only done after permission was granted from the department of education (see Appendix 1). The researcher also requested permission from the principal of the school to conduct the research and permission was granted (see Appendix 4). Parents were given consent forms in their preferred language, containing details of the research topic, purpose, and the role that their learners would l play in the research, to obtain written consent. The researcher assured parents that names of the learners would be kept confidential and that the learners work would in no way be affected by participating in the research.

Cohen et al. (2018, p. 108) defines deception as a “kind of experimental situation where the researcher knowingly conceals the true purpose and conditions of the research or positively misinforms the subjects or exposes them to painful and embarrassing experiences without their knowledge”. In this study, the researcher ensured that the participants were always fully aware of the information pertaining to the research, so that they were comfortable with and had sufficient knowledge of the research project. A tape recorder was used during the interview process and learners were informed that they would be recorded and that they could decline to answer if they felt uncomfortable.

After the interviews were conducted, a debriefing session was held to rectify any misunderstandings that could have arisen, as suggested by Cohen et al. (2018). The participants were informed of the results of the project as well as the purpose of the results. The participants were also given an opportunity to get clarity by asking questions on the research project. The participants were thanked and letters of appreciated for their assistance and contributions made was made available.

4.12 VALIDITY AND RELIABILITY

Devlin (2006) refers to validity as establishing the “truth value” of the study. Internal and external validity must be maintained to ensure validity of the study. Validity is the key to effective research. Invalid research is worthless. In order to avoid invalid data, the researcher must have confidence in the elements of the research plan, data acquisition, data processing analysis, interpretation and judgement (Cohen et al., 2018). Validity refers to how close the data represents the actual participative experience of the participants.
The following four alternative constructs, as proposed by De Vos et al. (2011), that more accurately reflect the assumptions of the qualitative paradigm used in this study to ensure validity and reliability of the data:

Credibility – Denzil and Lincoln (2005) describe credibility as the goal to demonstrate that the inquiry was conducted in such a manner as to ensure that the participants were accurately identified and described. The researcher needs to adequately state the settings, processes, social group and patterns of interaction, for the research to be credible. In this research, the selected schools were identified and the participants were learners and educators from each of the schools learning or teaching geometry. Credibility for the study was maintained by using different forms of instruments namely questionnaires, semi-structured interviews, lesson observations, and focus group discussions. The use of triangulation in the study was used to ensure authenticity of the data generated.

Transferability- This is the degree to which the findings can be applied to other contexts. The generalisation of qualitative findings to other settings can be problematic. Designing research to include multiple cases, multiple informants or gathering data using more than one method, can strengthen the study’s usefulness. The reason that the study included three different schools was to strengthen the studies usefulness and transferability to a certain degree. The sample was not big enough to generalise to the greater population.

Dependability- All the data collected was recorded. A tape recorder was used in the individual interviews with the educators and the focus group discussions with the learners. This assisted in accurately capturing the data preventing the loss of any information. After lesson observation, the interview was conducted to clarify any confusion or misunderstandings. The lesson was video recorded and verified against the data collected from the interviews and questionnaires. Participants were also given an opportunity to read the findings before submission to see that the data was not interpreted incorrectly. The recordings allowed for direct quotations to be captured correctly.

Confirmability – This refers to the degree of objectivity that exists in the research. The researcher interpreted the responses of the participants as articulated by them. The personal opinion of the researcher was kept out of the report.

All necessary steps were taken into consideration to ensure that the research is valid and to minimize any forms of invalidity that may have crept in.

According to Bogdan and Biken (2006, p.88) reliability in qualitative research is viewed as the fit between what is recorded as data and what has actually occurred in the setting under research quote. Reliability refers to the consistency, replicability over time, over instruments and over groups of participants, of the study.

4.12.1 Validity and Reliability in Interviews

One of the main methods that had been employed to collect data in this study, was interviews. It was therefore necessary for the researcher to adhere to the principles of reliability and validity as set out by Cohen et al. (2018). One of the most practical ways of achieving validity is to minimize the amount of bias. Some of the sources of bias that can arise are from the attitudes, opinions, and expectations of the interviewer, the tendency for the interviewer to see the participants in her/his own image, the tendency to direct the interview to support the researcher's preconceived notions, misinterpretations of what the participants are saying and misinterpretations on the part of the participants as to what was asked.

One way of controlling reliability is to have a highly structured interview with the same format and sequence of words and questions, for each participant (Silverman, 2016 cited in Cohen et al., 2018). The questions were clearly explained to the participants so that all understood exactly what was being asked. A pilot study was conducted to ensure reliability of the data. A tape recorder was used and data was transcribed from this.

4.13 CONCLUSION

The research design and methods used to achieve the aims of the study were discussed in detail in this chapter. The reasons for the use of the selected schools were explained. The manner in which reliability and validity of data was achieved, was also explained thoroughly. All known ethical issues were considered for the research to be successful. At the end of the research all data collected was safely stored. The following chapter focuses on the data analysed.

Chapter Five: Data Analysis

5.1 INTRODUCTION

The preceding chapter focussed primarily on the methodology used for this study. This chapter deals with both the presentation and interpretation of the data obtained. The steps discussed in Chapter 4 were used to collect, present and analyse the data against the research questions. The next chapter discusses the findings of the research in greater, detail making specific reference to the literature review and theoretical framework as discussed in chapters 2 and 3.

The study focussed on the learners and educators doing mathematics in the FET phase. The main aim of the study was to explore the barriers experienced for effective geometric thought. The data collected from the questionnaires and interviews for both learners and educators will be presented based on the research questions, followed by a detailed analysis of the data obtained from the observation of the six lessons.

The barriers experienced by learners are discussed below. These barriers impede the active interaction and participation of learners that would lead to effective geometric thought. The way information is imparted to learners and the acquisition of this knowledge plays a role towards learner performance.

The questionnaire, lesson observation and focus group discussion provided the data to answer the research question dealing with barriers experienced by learners for effective geometric thought. The following four themes emerged from the data collected by the participants:

- Engaging learners in Euclidean Geometry
- Language and communication
- Pedagogical factors
- Educator Content Knowledge

5.2. ENGAGING LEARNERS IN EUCLIDEAN GEOMETRY

5.2.1 Profile of Participants

The profile of the participants was obtained from section A of the questionnaire. This provided the researcher with an understanding of the learner participants whom the researcher has worked with. The significance of this information is to provide readers with the generic context and experience of the participants involved in the research. For the purpose of confidentiality, learners were identified by the letter allocated to their school and number for example A1 represented school A, learner 1. Eighteen learners were involved in this study with six learners from each school. The information gathered from section A of the learner questionnaire is presented below in table 5.1.

Table 5.1 Profile of learners

Participant	Gender	Home language	Do you live with your parents?	Do you Enjoy studying geometry?	Is geometry needed at school?
A1	M	I	OP	N	N
A2	M	I	OP	N	N
A3	M	X	BP	Y	N
A4	F	I	BP	N	N
A5	F	I	A	N	N
A6	F	I	A	N	Y
B1	F	E	BP	Y	N
B2	F	E	OP	Y	Y
B3	F	I	BP	N	N
B4	F	E	OP	Y	Y
B5	M	I	BP	N	Y
B6	M	I	A	N	N
C1	M	E	BP	Y	Y
C2	F	E	BP	N	N
C3	F	I	OP	N	Y
C4	M	E	BP	Y	N
C5	M	E	BP	N	N

C6	F	E	BP	Y	Y

F – FEMALE, M- MALE

E- ENGLISH, I – ISIZULU, X –XHOSA

BP- BOTH PARENTS, OP-ONE PARENT – A- ALONE

Y – YES N- NO

There were eight males and ten female participants. Nine of the learners' home language was IsiZulu, one isiXhosa and eight were English-speaking learners. Ten of the eighteen learners lived with both parents, while five lived with one parent and three lived alone. Eight learners stated that they enjoyed working with geometry at school. Only seven believed that there is a need for geometry at school level and eleven felt that there was no need to do geometry at school level. The data revealed that the language of instruction in comparison to the home language was different in ten out of eighteen of the learners. From their responses it was evident that most learners disliked or found geometry insignificant.

5.2.2 Challenges experienced for Effective Engagement

5.2.2.1 Class Size

Learners that become independent within a classroom develop ways to learn and become lifelong learners. The classroom environment is a key factor in developing independence within a learner. The setup of the classroom was noted as a factor which learners highlighted as an impediment for their success in geometry. Learners alluded to insufficient involvement due to large class sizes. These factors had a direct effect on the manner in which the lesson was delivered and the learners' interaction with their educator.

Most of the learners indicated that the classroom environment was not conducive to the teaching of geometry. Learners from some of the schools stated that their mathematics class size consisted of more than fifty learners and that this provided a major challenge to get individual assistance from the educator.

The table below displays the class size in each of the lessons observed

Table 5.2 Class Size in School

School	No in class
School A	55
School B	48
School C	38

By the figures displayed in the table above, the mathematics class is overcrowded, taking into account that the recommended educator pupil ratio is 1:37. In school A some of the learners share a desk with barely any space on the desk for movement. In school B learners were seated but there was very little space for the educator to move around in the classroom to provide guidance.

In the focus group discussion one of the challenges that learners expressed was the lack of individual attention due to there being to many learners in the class.

Learner A4 stated that *"My maths class has 52 learners, I am not sure that my educator even knows that I exist in the class."*

Learner B1 stated that *"the number of learners in our maths class is 38 but this prevents the educator from providing us with individual assistance. We sometimes are unable to keep up with the educator during the lesson but do not get the chance to speak to the educator individually. We are unable to ask questions to clarify our understanding."*

Educator AE2 stated, "*my major challenge is I have 50 learners. I can am unable to see to all their individual needs."*

Educator CE1 stated that; "*The more capable learners manage comfortably in the large class. It is the learners that are less capable that need more one-on-one time who are disadvantaged in the larger classes.*"

Educator BE2 "*....my learners struggle with proving riders and I am unable to interact with each one*"

Learners and educators were of the belief that smaller classes will allow the learners to get more attention. Educators stated that huge class sizes prevented them from teaching in a learner-centred method. Educators conveyed that it is not possible to implement any interactive lessons with the huge class sizes.

Educator CE2 stated that *"if my class size is small I will be able to spend more time on identifying learner problems when marking as I will be marking a smaller number of scripts."*
Educator BE1 "*A smaller class size will also help with classroom management and more time can be utilised on curriculum.*"

A larger number of learners in a class prevent educators from identifying weaker or slower learners. These educators expressed that they were expected to improve their maths results and geometry was an area where learners needed the individual attention but they could not provide this. All educators believed that smaller classes in geometry would result in higher achievement among learners, especially those with language barriers that need the extra time. Active engagement in lessons to acquire knowledge becomes a challenge in bigger classes. A class size of fifty learners, attempting to express their views while understanding the concepts within an hour, becomes a challenge even for the most experienced and knowledgeable educator. To achieve the goal of getting learners to work independently and to supervise them, was observed in lessons, to be demanding on the educator with large class sizes. Learners mentioned that with such large numbers, it was almost impossible for the educator to provide the ideal support that individual learners actually needed. This scenario presents as a barrier to educators wanting to engage learners actively in lesson participation. Some learners articulated that even group work presents as a challenge owing to classes that are too large.

The large class size affected the methodology utilised by the educators in their teaching of geometry. We are not able to involve learners in our lessons as it becomes too chaotic with large classes. Educators expressed that the class size is determined by the post provisioning norm as set out by the department of basic education.
Educator CE3 stated that "*If the educator pupil ratio is reduced the financial implications are too huge for the department.*"
Some educators implied that a reduced educator pupil ratio would require more educators and the department of education cannot cope with this situation financially. The implication for appointing mathematics educators is even greater considering that there is a shortage of skilled educators. A lack of educators leads to

overcrowded classes. Educators in schools A and B pointed out that schools that have sufficient funds tend to employ more educators from their school funds and this allows them to keep smaller class sizes. However, the schools that rely solely on department funding are unable to reduce their class sizes. This insinuates that the richer schools are more advantaged. Class sizes are also affected by the fact that some schools combine classes in order to ensure that educators have a higher number of free periods. In school A it was noted that the educators did not teach for the allocated time as specified in the Personal Administrative Measures (PAM) document. This could also be the reason for the larger class sizes due to the increased number of non-teaching periods.

Learners stated that a greater emphasis should be placed on them becoming independent learners so they can apply their knowledge outside of the classroom, which will ultimately improve their results in geometry.
Learner C2 stated that "*if we are able to work on our own we would be able to solve problems without the assistance of the educator*".
Learner A3 stated that "we *would be able to form ways to enhance learning on our own*".
Learner A2 "if *we are able to explore on our own we would get a better understanding of geometric concepts"*

Learners conveyed that if the classroom set up was more conducive to independent learning then they would develop a better understanding of geometric concepts and this will lead to greater success in geometry. Getting learners to become independent and being able to learn, so that they are able to apply themselves in and out of the classroom, can only happen in smaller class sizes with adequate resources so that the educator can monitor the learners progress. Class size was indicated by participants as a barrier to teaching geometry. According to Peddler (2006) individual attention in smaller classes are beneficial to the achievement of learners. Identifying learner experiences as specified by Vygotsky and using these experiences to build new geometric will be a challenge in large classes. Getting learners to communicate in large classes requires time and educators are guided by notional times in the classroom. Geometry requires the educator to look at previously acquired knowledge of learners and scaffold their lessons accordingly.

In larger classes this will become difficult. Drawings, models and other devices are important in the geometry class. This will not be effectively implemented in large classes. According to Kohler (2020) in the South African context, most studies conclude that greater class sizes are associated with poorer educational outcomes. With the multi-lingual classes and code switching, larger classes pose a problem for teaching specialized geometry concepts that must be learnt.

5.2.2.2 Resources

From the lessons observed it was noted that there were little or no charts, models or geometry diagrams in the classroom. Teaching aids form a vital part of learning geometry in terms of reinforcing learning and emphasising certain concepts. A classroom that supports visual media builds an implicit understanding of knowledge and the context in which the knowledge can be used. Learners in the focus groups in response to question one on challenges also expressed that there were very little stimulating resources in the classroom.

Learner A2 " *....we have no diagrams, models, figures in our class and we share textbooks*"

Learner C4 stated that, *"It would be nice to see the things we are learning."*

Learner C2 stated that, *"Yes, if we can see what we are learning we will understand it better."*.

Learner A1 *"....... if there was a diagram of a circle labelled in the classroom, I would be able to refer to it for concepts that I forget"*

Learner B2 " *.... my educator draws diagrams on the board. We do not have any models, or diagrams of shapes in our class*"

Visual communication forms an integral tool in the learning of geometry. Learners form mental images during their learning. These images can be formed using pencil and paper or technology. It helps learners to simplify the problem and understanding becomes easier. Learners in school A shared that educators used no teaching aids in the classroom except for the chalkboard. This was also noted in the lessons observed. Learners suggested that seeing what they are learning or how this learning relates to life, will increase their understanding and retention of concepts.

In the lessons observed educators even drew the circle freehand and sometimes it looked more oval than round. Coloured chalk was not available in some schools to show learners the different angles. Educator CE1 indicated in the interview that that he uses different coloured chalk, instruments and cut out diagrams in his class, in the absence of concrete images.

Educator CE1 “I *use different colours to show equal angles and also draw the parallel line in the same colour so learners can see the relationships”*

Educator BE2 stated that*, “Geometry is a visual subject where learners need to develop mental pictures. Most of my learners can only see things in the concrete sense before they internalize and understand*”.

Educators communicated that the use of different colours in a diagram helps learners visualize, and reasoning arouses their thinking abilities. It was also stated that a stimulating classroom with lots of charts and mathematical objects creates interest and motivates learners. Educators communicated that for learners to think about solutions they must visualize the figures and express what they see in words and symbols. They are able to reason better based on what they can see instead of what they are told. Seven out of the eight educator participants articulated that learners understand geometry when they use mathematical tools to enhance

Educator BE1 “*geometry is a visual subject that allows learners to develop mental pictures for better understanding*”

Educator AE2 stated that, *“my learners also are able to think about solutions when they draw the figures themselves. When drawing they develop a better understanding of the concepts done in the classroom*”.

Educator CE3 “*many learners only see the usefulness of geometry when they start working. We can bring these work situations into the classroom so geometry becomes more interesting and worth learning*”.

A lack of adequate was indicated as a barrier to get learners engaged in the lesson.

Educator AE1 “geometry *drawings need instruments which most of our learners do not have. I was teaching measurement of angles. Of my class of 48 only 12 of had a protractor and compass*”

The availability of resources in the respective schools was identified as a contentious issue. Learners in school C believed that they were adequately resourced in terms of textbooks and instruments for geometry. Learners in schools A and B disclosed that they shared textbooks and that the school could not afford to purchase basic instruments for them. In the focus group discussion learner

Learner A2 stated that *"the department provides us with maths sets and calculators and we have all the basic instruments in them".*

Learner A3 responded quickly to this by saying that "*the calculators and maths sets are of a very poor quality and cannot be used for long periods*".

Learner B1 "we *share textbooks and sometimes if I forget my one then we are unable to see what the educator is talking about cos she does not draw it on the board*"

Educator AE3 emphasized *"…due to the number of learners physical resources is a challenge*".

This challenge was agreed by the other educators who highlighted the shortages of textbooks and instruments Some schools are forced to procure learner support material through the department and the material provided is not always of the best quality, which resonates with the sentiments of learners. Learners in schools A and B belong to such schools that procure through the department of education.

The learners in school A also complained of having shortages with the stock of desks and chairs.

Learner A2 *"Currently some of us are standing in the maths class. How are we supposed to learn mathematics or geometry? How do we draw our shapes? Our educator is doing circle geometry and makes us draw the figures in our books."*

During the discussion it was also revealed that there was inadequate provision of duplicating paper in school A and as a result, work sheets were not available to supplement lessons for independent work. Worksheets form an integral resource in a geometry class as it allows for lessons to cater for different levels of learner abilities, scaffolding of examples and learners can work at their own pace. It also serves as a time saving measure as it would prevent learners from rewriting and drawing of diagrams. The educator can also utilise this time to provide more support to learners that are experiencing challenges.

Whilst this might seem to be removed from the idea of learning geometry, the learners' comfort is important. They would need to construct, draw, engage and generally work with what they have in front of them. If they have no basic provisions such as chairs, then learning is certainly going to be negatively affected. According to Drews (2007) the use of diagrams, drawings and other visual resources enhance knowledge of geometric concepts. Geometry is a subject that requires hands on interaction if learners are expected to engage with the lesson. This means that they would have to draw, measure and hypothesize. A geometry classroom without the essential tools that allows learners to engage in the lesson will be challenging for educators.

5.2.2.3 Use of Technology

Learners and educators expressed the need to incorporate technology into the teaching of geometry.

Educator CE1 stated that "*Technology can help us create real situations within the classroom. These situations bring meaning to the geometry."*

Educator AE1 "*Learner are able to form associations between geometric concepts and actual figures the use of technology*"

Educators expressed that many learners experience the 'light bulb' moment after school when they realize that geometry can be used in the real world. We need to make these light bulb moments happen in the classroom. The use of technology and computer software allows us to create this situation. Educator BE2 also expressed that choosing an activity from a real situation and staging it in the classroom using technology will get learners to become more involved in the activity and thus learning with meaning and understanding will take place.

Educator AE2 "*many of our learners come from poorer homes and have not experienced much outside their homes, sometimes we talk about certain figures for example the trapezoid and learners look all confused. Yet technology will allow us to show our learners different types of trapezoids and its properties"*

Educators alluded to learners not being exposed to many situations that use geometry and by using technology; they are able to bring these situations into the classroom.

Educator CE1 "*technology used in the classroom stimulates our learners thinking*"

Educator CE2 "I *use my laptop to teach certain geometry concepts like quadrilaterals and learners enjoy my lessons*"

The use of technology and other visual aids in the classroom stimulates learners' thinking abilities and allows realistic situations to be cultivated in the classroom. Technology also promotes participation in the learning environment. Knowledge is therefore acquired through active participation and these geometry concepts are understood better.

Educator BE2 "*many jobs incorporate geometric figures and technology*"

Educators BE2 stated that jobs such as computer animation, computer graphics, video gaming, mapping and aeronautics all require the skills of geometry and bringing real examples from these fields will arouse enthusiasm and enhance the teaching of geometry. Educator CE1 also expressed that digital technology allows us to bring learning from the outside into the classroom. Educators conveyed that digital tools support visualization and support versatile thinking. In the absence of technology, educators use tools that are available to them to allow learners to use their senses in learning.

"These visual aids allow learners to visualize what I am talking about and allow them to understand concepts better. They are able to solve problems an ease."

Educators argued that digital tools support visualization and this will foster versatile thinking in the geometry class. Educator BE1 claimed that the use of digital tools will bring the "outside to inside" the classroom.

Educator CE1: *"Learning with teaching aids like technology helps learning at different levels*"

It was expressed that facts, procedures and concepts can be understood better with technological tools.

Educator CE1 stated that "*I use different methodologies like construction, paper cutting and material development, ICT and collaborative approaches. Different methods help learners understand concepts more clearly. I sometimes use problem-solving method.*"

Educator CE2 stated that "*Learners are much more interested in technology and I know a bit about integrating ICT in mathematics, but need additional assistance.*"

Educator BE2 pointed out, *"as much as the use of computers in our teaching is a wonderful idea, we cannot even manage the teaching of computers with fewer than 25 learners in the classes. How are we to implement the curriculum with more than 40 learners in a maths class?"*

The use of technology was supported by all educators but the challenges in using it was also mentioned by educators. Lack of sufficient computers, educator training and large class sizes were some of the challenges experienced

However, the educator did agree that every classroom should have a white board and data projector. She also stated that the department of education should consider investing in laptops for educators.

Educator BE1 "*almost every learner has a cell phone and this can be sued in the classroom. Geogebra is online software that can be loaded on learners' phones and utilized in the classroom*"

Educators spoke about the use of the smart-phone in making data bundles available to learners. Some educators expressed concerns about learner responsibility in the use of smart-phones.

Educator CE1 stated "*We can find ways to include smart phones in school responsibly*"

Two educators shared an interesting idea where tablets without SIM cards can be used at schools. Unlimited WIFI data is available to most schools. These tablets can be programmed with restricted access to specific applications involving curriculum and school related matters.

Educator AE3 indicated, *"A mixture of resources used in the classroom will arouse interest and create motivation in the learners. We need to move with our learners and introducing technology to improve geometry is the way to go."*

Educator CE2 articulated similar sentiments "*I did a course with "Geometer Sketchpad. Geometry proofs which is a section that troubles our learners was made easier"*

The incorporation of geometry software in the teaching was suggested as a tool to make teaching of geometry proofs and other concepts interesting and manageable for learners.

Learners said that the use of technology will allow them to get a clearer picture of e theoretical concepts and thus expand their knowledge. Technology will also allow for interactive visual representation of abstract information, to amplify cognition. It was evident from the learners' responses that if they were able to form mental images of the geometry words that are used in the classroom their understanding of geometry will be better. Mental images play a vital role in memory retention.

Learner B1 stated that "*maybe we can use the computer to learn geometry*".

Learner B4 stated that "*Yes, similar to the way we do EGD. We understand EGD better because we draw on CAD and can see what we are doing.*"

EGD is engineering graphics and design and it contains many geometric figures. CAD- computer aided design is a software package that is utilized to draw and visualize drawings. Learners confirmed that they would be able to make more sense of their learning if they are able to see pictorially what they are learning. Making use of computer aided design will bring reality into the classroom and allow learners to 'experience' realistic situations to relate their learning.

Learner A4 disclosed, *"Many of us do not know what the educator is talking about because we have not experienced or seen certain shapes that the educators talking about."*

Learner A5 stated that, *"Yes therefore seeing it on the computer will allow us to also experience the concept. We will become more interested."*

Most learners were of the belief that visualizing a figure using technology and other teaching aids will assist them in understanding and solving problems. They also stated that this will motivate and create interest in the subject content of geometry. Some learners in school C stated that interacting with visual media that links theory with practice, will allow learners to find meaning- learners implied that incorporating teaching aids into the geometry classroom is fundamental to creating effective understanding and interest in the subject.

Learners also said that technology will allow them to associate the concepts to real life, making the geometry more meaningful.

Almost 90% of the learners echoed similar feelings by saying that *"We love technology. We are the generation born in a technological world" and "No one taught us to use the cell phone or the laptop or the iPod but we do it at ease."*

Learners alluded to the use of technology as increasing their interest and allowing them to experiment and find solutions on their own. They conveyed that this will help better their performance in geometry. Technology is a vital teaching aid as it allows for learners to work independently and at their own pace. This generation loves technology and including it in the geometry lessons will arouse learners' attention. Learners implied that some of them do not get the opportunity to be exposed to the world out there and that the use of technology will create this kind of experience.

Learner C3 articulated "V*irtual reality can also be brought into the classroom. 3D figures can be used in the explaining of geometric concepts."*

Virtual reality is a technique that can be in cooperated into the geometry lesson which would allow learners to experience the geometry of the world in fun and interesting ways. The learner's suggestion to use virtual reality may sound far-fetched considering the financial constraints of the education department, however this will be appropriate looking at the vision of the 2030 curriculum.

Learners C2 articulated that "*our learning will not be divorced from reality*".

Some of these learners are information technology learners. They strongly believe that learning can be integrated between the learning areas and this will allow them to see objects that they until now had to normally calculate and construct using traditional pen and paper.

Learners C6 "*there are so many places that geometry is used for example in construction, in buildings ...... With technology we can experience how geometry concepts and figures were used. This will make learning geometry more meaningful for us"*

They felt that the situations where geometry is used can be brought into the classroom, e.g. building bridges, construction of roads. Learner B3 argued that video games also have a lot of geometry application.

Learners expressed their love for playing video games. Video games would enhance the teaching of geometry as it contains many concepts of geometry such as angles, shapes, parallel lines, symmetry, and etcetera. This type of learning will engage learners in the content through interesting and fun ways.

Learner BE5 "... *there are so many geometric shapes and angles in our video games. Even when we are playing we need to move through arcs and angles*"

The learners argued that the use of technology will make geometry applications more interesting and will motivate learners to want to learn. Some of the learners claimed that this kind of participatory learning will improve geometry learning. Again learners emphasized the need for greater engagement in lessons, to improve understanding of geometry.

The use of technology was expressed by both learners and educators have an important tool in the teaching of geometric concepts and proofs. As much as educators expressed that it will come with challenges, they also acknowledged the benefits of incorporating technology into the geometry classroom. According to Jones (2002) using dynamic geometry software provides an understanding of points, circles, and parallel and perpendicular lines. Learners expressed that the use of technology and computer software will motivate and create interest in the learning of geometry.

5.2.2.4 Attitude of educators and Learners

The attitude towards learning and teaching of geometry was indicated has a barrier by both learners and educators. The focus group discussions and interview with educators all expressed that the mind-set of learners will affect their learning of geometry. The data generated from the questionnaire indicated that fifty-eight percent of the learners disliked and had no interest in doing geometry. This negative outlook may be a challenge that educators experience in getting learners involved in the geometry lesson. From the questionnaires, eight learners indicated that they did not like to do geometry and eleven stated that geometry was not needed as a school subject as indicated in table 5.1. Not being able to find meaning in learning of geometry created disinterest and a negative attitude towards geometry**.**

During the focus group discussion, the following comments were made by the learners:

Learner A1 stated, "Where do we use geometry? Why do we do geometry?"

Learner B3 stated, *"Geometry is boring. It has no relevance to the real world."*

Another important point uncovered from the focus group discussion as well as the questionnaire is that a large number of the learners do not find any relevance between the geometry learned in the classroom and day-to-day activities. It was found that educators did not relate the practical life examples to the topic.

Learner C2 "*We learnt to proof congruent triangles but when will we use what we learnt*"

Not being able to see the relevance of congruency in real life examples like engineering, architecture and forestry and its use makes learners dislike and not want to learn geometry. Learners indicated that if educators used real life examples, it will allow them to see the relevance and link their learning, thus developing a better understanding of the concepts. Learners conveyed that interest and motivation to want to learn geometry will be developed when learners can see the connection between real life examples and geometry. The relevance of geometry in real life applications is important, to develop interest in the subject content.

Learner A7 "*it would be exciting if we can go on field trips and do geometry on the outside of the classroom since our educator always talks about geometry being everywhere"*

Learner A3 "*....we see the shapes in our classroom and school about what about why we need to proof all the other stuff*"

Learners mentioned that once they are able to see that geometry can be used in the outside world, they would become more motivated and will want to learn geometry which will ultimately lead to them performing better. Educators can incorporate nature when teaching geometry. Many shapes and patterns are found in flowers, leaves, stems and roots. This can also develop links with other learning areas. The organisation of the digestive system as a tube within a tube can be meaningfully connected with geometry. Other examples that a resourceful educator can use to illustrate the relevance of geometry are; the honeycomb, video games, robotics, architecture and many more. An interesting use of circle geometry that can inspire learners is the GPS system. Meaningful learning takes place with real life tasks. Learners' questioning the use of geometry in real situations is important to their success in learning of geometry It is evident from the learners' responses that learning of geometry takes place in very formal situations and this learning becomes boring for them. More informal settings should be created within the classroom to increase motivation and interest in learning. Learners come into the classroom with a lot experience from their interactions outside of the classroom. This needs to be incorporated into the classroom in order to allow learners to see related meaning. Learners also expressed the need to go out of the classroom to real situations that would allow them to link the work done in class to the real world.

LearnerB3 suggested "*learning geometry will become interesting if we visit job situations where we are able to see the use of geometry*".

Learner B6 expressed, *"It is so difficult and we all fail. It should be removed from the syllabus."*

Learner C2 stated, *"Some sections are nice but we are de-motivated by the results that we get. "*.

Learner A3, "*this discourages us from even trying to do the geometry"*

Learners expressed great disappointment and were disturbed that some of their educators categorize them as being unintelligent or lazy and that is the reason they believe; they perform poorly in geometry. This creates negative feelings and makes them perform worse in geometry. There was a feeling amongst some learners that educators de-motivate learners and this leads to poor performance. Their negative attitude and low levels of motivations were clearly observed. The attitude of educators makes learners feel disinterested and does not stimulate their thinking. Learners communicated that if they feel disinterested and de-motivated it is impossible for them to concentrate on the learning of geometry. They need to want to work and solve problems, to be able to succeed in this aspect of mathematics. Educator's attitudes toward geometry affects the ways learners relate to the learning of geometry. Educators 'attitudes toward learners also determine learners' interest in the subject. Making connections with their experiences in life will create interest and motivate them to want to learn.

Learner C5 stated that *"we feel even more de-motivated when we are doing homework alone. The problems that the educator does in class are the simple ones and then there is no one to guide us at home".*
Learner B3 stated that, *"Yes educators should give us time in class to work in groups and help us when we reach problems."*

The grading of problems, from simple recall to more advanced type, will allow learners to complete homework tasks successfully. If the educator only tackles the simple recall type questions in the classroom this will inevitably lead to learners questioning their abilities when they are working alone with more difficult problems. Allowing learners to interact and work with different level geometric problems will develop learners 'competency in thinking effectively. As learners acquire and master skills through engagement in groups or by themselves, educators should introduce them to new knowledge. This will maintain their interest and learning then becomes an ongoing cyclic process.

Most of the learners divulged that they found geometry too difficult and it needed full concentration and practice. They mentioned that despite the amount of work they put in, they do not do well in geometry. They stated that these poor results create a lot of anxiety and loss of interest in the subject.
Learner A3 stated that *"some of my friends decided not to do maths in grade 10 because of the fear of geometry*."
Learner B1 made it known that their class started with 51 learners in grade 10 but now have fewer than 40 learners doing mathematics. Most learners failed Paper 2

where geometry features prominently. Anxiety and fear is created due to learners being expected to listen and learn the unknown. If learners are experimenting and successfully finding answers on their own, then this fear is eliminated. Learning takes place while participating, communicating, questioning and finding solutions.

Another important point that was raised during the discussion was the lack of foundational knowledge that also created a disinterest in mathematics. The learners from all three schools said that some educators teaching in the primary phase as well as the GET phase, that is in grade eight and nine were not very passionate about teaching geometry. Some indicated that the educator did not like the subject and the educator's negativity rubbed off onto them. Good foundational knowledge allows learners to advance in the learning of geometry. Some learners only did geometry for the first time in grade 10.

Learner A2 stated that "*we were never tested on geometry in primary school. Our educator spent one lesson on geometry concepts. He wrote on the board terms of definitions like line, line segment, triangle, etc. That's all we did*."

Learner B3 stated that sometimes the educator says, "Do *you'll remember doing this last year* and most of us just have a blank look on our face."

Learner C5: "*the educator asks if we studied geometry in earlier classes. In primary school we did not study geometry well and our educator did not encourage us to learn geometry. They did not tell us the importance of learning geometry so we are weak in geometry. Some of us have problems with basic geometry problems*."

Learner B4 stated that "*We find geometry to be a very hard subject. Only a few learners in our class understand and are able to solve the problems*."

Learner A3 "*we started circle geometry this week and our educator started teaching a new theorem. He said we should have known the about exterior angle and interior angles of triangles but many of us had no idea what he was talking about*"

Learners expressed that they are not able to grasp new concepts due to a lack of previous knowledge. It is evident from the discussion with learners that many of them have advanced into the senior grade without foundational knowledge of geometry. This could be the reason for the poor attitude towards learning of geometry and the poor results in the subject. The views of the learners indicate that their pre-knowledge played an important role in the learning of new concepts. New learning depends on current understanding.

Learner A3: "*We feel that geometry is a hard subject cause of our pre knowledge and the educator does not care about the weaker learners. He moves on with the syllabus based on the few talented learners.*"

Some learners stated that the fast pace of the syllabus makes them lose interest in trying to understand the concepts of geometry. The educator caters for learners that understand what is going on in the geometry class and the rest of them are ignored. This again leads to de-motivation and disinterest in geometry. It leads to poor performance. Without a strong foundation learners will experience barriers to learning of geometry. Geometry is a learning area that relies heavily on prior knowledge. On the question of girls performing better than boys, all learners indicated that they did not agree. They felt that it depended on the individual's understanding and gender had nothing to do with it. It was evident that gender does not affect how learners think or communicate.

As expressed by learners and educators the attitude of the educator teaching geometry affects learners' attitude towards the teaching and learning of geometry.

Educators AE2 stated that, "*Our attitude affects the way learners see geometry." A positive attitude by the educator will influence learners to see geometry in a positive light.*"

Educator CE1 concurred by saying, *"I had new learners that came into my class this year, and their educator last year told them that geometry is very difficult. She had a dislike for teaching geometry and these feelings were displayed by learners."*

Educators implied that if the educator has a negative attitude towards the teaching of geometry then learners' performance will be affected. Many of the educators interviewed alluded to this poor attitude amongst maths educators in general. They conveyed that educators especially those in the lower grades, disliked teaching geometry and learners emulate this dislike which affects their performance. Educators that lack exciting and enjoyable experiences with geometry develop a negative attitude. This negative attitude influences learner's performance in geometry.

Educator CE2 "*an attitude is based on a person's experiences and interactions. Some of my learners this year had an educator that rushed through the teaching of geometry*"
Educator BE1 stated that "*a learner with a negative attitude performs poorly as he does not enjoy the learning of geometry*".

Some of the educators expressed that a negative attitude affects a learners' behaviour as they are likely to achieve better in a subject that they enjoy, have confidence in, or find useful. Educators also revealed that an educator that is supportive in the classroom will create a positive attitude amongst learners. Educators CE2 and CE1 displayed a very passionate and positive attitude towards the teaching of geometry. They showed a great interest in creating a learning environment that was exciting and meaningful for the learners to learn geometry. They promote cooperative learning environments that stimulate and motivate their learners. They engage their learners and get them to participate in their learning. Both educators have displayed success in their teaching of geometry.

The data indicated that learners' attitude towards the learning of geometry impacted on how their performance. The educator's attitude also affected how learners related to the learning of geometry. Learners expressed that they needed to see the relevance of geometry in real examples to feel motivated and interested in the subject. Engaging learners' in activities outside the classroom may create a love for geometry and improve their attainment in geometry.

Attitude, class size and resources are important consideration when creating a classroom for engaging learners in the content. A learner with a positive attitude will be susceptible to the geometry content and this may influence the learners' performance. There are concepts in geometry that require individual attention and a large class size does not allow the educator to provide this kind of support. Concept of cyclic quadrilaterals which are more difficult to comprehend requires educators to engage with their learners. Resources provide visual stimulation for learners. Technological devices can be effectively used to stimulate learners' visual senses.

5.3. THE USE OF LANGUAGE AND COMMUNICATION

5.3.1 Language as barrier for concept development

Language and communication was conveyed by both learners and educators have a challenge in teaching and learning of geometry. Data from the questionnaire indicated that learners believed that there should be greater communication in the geometry classroom.

The IsiZulu and isiXhosa speaking learners made it known that with English being their second language; it is difficult for them to understand some concepts of geometry. Many learners indicated that their greatest challenge in geometry was the vocabulary. Eighty-two percent of the learners were of the belief that if there was greater communication in geometry lessons and they were able to express themselves verbally and in writing, it would help them to get more clarity and the educator could correct them immediately. Only five percent of the learners disagreed that there was a need for communication in the geometry lessons and thirteen percent were not sure as to the effect that communication will have on geometry performance improving.

Learner A3 stated, *"As much as IsiZulu is my home language, if the educator talks to us about geometry and we are able to talk back we will master the concepts better. We do well in English first additional and Business Studies."*

Learner A2 stated, *"We need more time to understand the language of mathematics because sometimes our educator speaks in IsiZulu and then explains in English. "*

Learners in school B expressed similar feelings

Learner B6 stated, *"Yes if we can do well in Business Studies by listening and talking and discussing the concepts why can't we talk geometry?".*

Most of the learners in school A and B expressed that they were able to remember and grasp concepts far better in other learning areas like Business Studies and English as they are able to discuss and debate concepts. They understand and learn the content easier and faster. These learning areas also require application and yet they perform better. Learners expressed that geometry teaching can also be made interesting if the educator motivates and inspires learners. Learning geometry is more fun and inspiring when learners are able to express their experiences and can relate them to geometric problems. Some learners indicated that they felt more challenged and interested in the learning when they are involved and do not have to sit back and just listen, write or calculate.

Learner C3 stated "*it is easier to understand geometric concepts when we talk about it and are able to relate it to our experiences*.".

Learner B2 stated "*geometry is more interesting when I am able to express concepts in my own words*".

Learner C3: "*When we discuss and talk about geometry it sparks our minds and makes us think about how to solve the problem. We are also able to remember previous concepts.".*

Learner C4 "*Yes true, when the educator does all the talking and we sit and just listen, and write the note we get bored and we don't do any thinking. For me this is the reason we perform so poorly.".*

Learner B1 stated, *"We should have a vocabulary of geometry concepts and rules and we should be tested on this before we write our P2."*

Learner A6 disclosed, *"My educator has a problem with teaching in English. Most of the learners in our class cannot understand her pronunciation of the geometry concepts and this creates confusion and misunderstandings. My educator used to say similarly instead of similarity. "*

Learner B2 "*My educator is also an IsiZulu home language educator. She teaches all the concepts and proof but she does not do any problems with us*"

Learners alluded to the incapacity of the educator to communicate effectively in the language of geometry. According to Vygotsky (1978) the educator is expected to be the more knowledgeable other in the classroom and her lack of being able to provide the necessary support and knowledge to learners would create frustration and possibly lead to poor performance. Language, in terms of verbal communication and written communication was reported as a problem by the learners. Learners expressed that there were certain concepts that they should have mastered in earlier grades, but did not; this affected their understanding abilities in grade 11.

Learner A2 "*I read my textbook to get the definition of certain concepts as my educator is unable explain in the class. We are doing circle geometry and in the statement the term circumcircle appeared. She could provide a definition*"

Learner B3 " *.............my educator had a problem explaining to us what a secant means"*

It was mentioned that they are doing circle geometry and concepts like circumference, tangent, inscribe, secant, cyclic etc. were terms that they never understood, due to the language barrier. These concepts are important for learners to understand when they are solving problems. Some educators were identified by learners as being unable to explain and define these concepts clearly. Communication in terms of discussions was also expressed as a means to thinking and understanding the work better. Learners expressed that they felt that when they talk about geometry in the classroom, they will be able to associate to other concepts learnt previously and this will help them to solve problems more easily. Learners also implied that learning with their peers by talking, made them become aware of other cultures and allowed them to see the relevance of geometry in life.

Learner B6: "*In technology we were working on a project to build a traditional house. My friend looked a traditional house as hut and I looked at it as a cottage. We used our geometry knowledge of squares, rectangles and triangles to construct the house. We did traditional paintwork on the hut using angles and drawing triangular shapes*"

Language is an integral part of learning geometry. Learners need to understand the language that the educator is using in the classroom. If learners are inactive and not participating in lessons, they become bored and disinterested in the geometry lesson. Learners claimed that discussion allowed for them to think about concepts and share their experiences and prior knowledge. It is evident from the learner responses that it may not only be about understanding the language alone. It seems that educators just do not engage enough with their learners. Learners conveyed that they want to be involved in the lesson. Getting learners to partake in the lesson would allow for them to communicate their understanding and gain clarity of misconceptions that they may have encountered in the lesson. A learner that is idle will not be thinking and developing geometric ideas on his own. If an educator spends 90% of the teaching time speaking, learners do get the opportunity to resolve their problems/questions.

Learner A7 “we do not do much talking in the classroom. My educator does not ask questions or lets us explain anything.”

This was supported by learners A6 and A3.

Clear communication is essential for learning and lack thereof may shed light on why learners have difficulty in geometry. Learners in school C expressed that their educator allows time for them to communicate sometimes and that it assists them to understand the concepts better.

Learner C4 stated, “*my educator allows us to talk about how we arrived at a solution and it helps us to find our mistakes. For example, we were solving a rider and I said that the corresponding angles were equal. In the discussion another learner asked which lines are parallel and I realized I could not use corresponding angles are since the lines were not parallel*.”

Learner B2 indicated that “*Learning takes place by doing instead of merely listening to the educator imparting knowledge*.”

Interviewer: “what does doing mean”

Learners B2 “writing, solving, drawing, talking, asking questions, calculating........”

Talking is part of the “doing” process. To improve their language skills in geometry learners alluded that if they are given greater opportunities to express themselves in the classroom, their understanding of the language of geometry will improve. Learners conveyed that continuous use of geometry language would achieve the goal of remembering concepts without enforcing memorization.

Learner C3 stated, *“Our maths educator sometimes makes us write down the rules and we discuss it with our peers in groups and we then talk about it in class, explaining each rule. This helps us to understand the geometry better.*”

Learner B3 stated “*we remember concepts and rules better when we discuss it in groups”.*

Learners verbalized that when they are put into groups for discussions, they are able to openly discuss concepts without feelings of anxiety or tension. Retention of concepts and rules is greater when it is achieved by learners working in a co-operative environment. In groups, learners are able to think critically, support each other and accept the differences of their peers.

Learner C4 also expressed how it helps when learners go to the board and explain what they understand by a theorem. For example, he stated,

"*We were doing circle geometry and the educator said that the angle at the centre is twice the angle subtended by a chord at the circumference of the circle. I went to the board and had to explain where the circumference was and show the chord and the centre and then the angles.*".

Learners conveyed that pointing out the different concepts on the board allowed them to feel they are part of the lesson. It also provided the opportunity for peer teaching. As expressed by learners, this type of interactive lesson created a sense of wanting to learn and allowed learners to understand and acquire knowledge that they were expected to learn.

This is now not just doing the maths but talking about the maths and understanding in the process. "*Other learners ask questions and we all learn in the process,*", stated learner C4.

Learner B5 stated, *"we do answer questions but they are generally simple questions"*

Open-ended questions are essential are learners to think and explain their thoughts about geometry concepts. It is also important for learners to explain the reasons why they proved a problem in a certain way. Questioning is a significant part of learning geometry. It allows for slower learners to get clarity on concepts and develop a better understanding of the lesson. Learners will become participants in the lesson when they answer questions that make them think about geometric concepts. Learners are able to identify their misconceptions when they are talk and debate geometric concepts. The words and symbols of geometry are used to describe specific shapes, figures and relationships. For many students this language is either new or the words are familiar but are used in unfamiliar ways. The way language is used in the classroom plays an essential role in the learning of geometry. The use of correct vocabulary, syntax and expression was revealed by learners, as a factor that affects their success in geometry. Many learners expressed that greater communication will improve their geometric performance. Learners mentioned that it is easier if the language used in the communication is understood by both the educator and learner, for effective teaching to take place. As much there were some educators that allowed for communication in their classroom, there was a need for greater learner participation in the language of instruction. Learners expressed that geometry terminology needs to be explained and discussed for them to gain a better understanding. This suggestion is relevant considering that geometry has a huge vocabulary of specialised words and for learners to increase this vocabulary talking about the concepts will help.

5.3.2 Language as a barrier for the educator

From the lesson observation it was noted that many of the educators spoke in English which was the medium of instruction. The lessons observed had only one educator that engaged in code switching. Educator AE1 translated her teaching of certain words into IsiZulu for learners to get an understanding of the concept.

Lesson: Theorem: An angle between the tangent and chord is equal to the angle in the alternate segment.

Duration: 55minutes

Words such as chord, line segment, and tangent were translated. She gave certain explanations in IsiZulu. However, learners were not given an opportunity to communicate their thoughts or ideas on the proof.

In the lesson observations certain words were mispronounced.

For example, instead of “tangent” she said “tengen” and instead of “circumference” she said “cumferance”

The mispronunciation of certain words could lead to challenges when learners are working with problems.

The constant need to code switch took up a lot of time during the lesson. Lengthy explanations were given in IsiZulu. Code switching did assist the learners in understanding the proof of the tangent and chord theorem. Code switching also bridged the gap between the formal geometry concepts of chord, circumference, and alternate segment to the language that is understood by the entire class. This kind of code switching will assist learners to understand the concept before it can be used in applications. There were times that the educator paused trying to get the appropriate translated word for the geometry concept.

Educator AE1 *"I code switch to get my learner to understand the concepts. I also am able to explain better in IsiZulu but it takes up a lot of time*"

The data from the questionnaire revealed that 50% of the educators are English second language users. The educators revealed that they all speak English fairly well. The one educator, AE2, stated that she speaks English well but teaches in both IsiZulu and English. She is far more confident to express herself in IsiZulu.

Educator AE2*" I know it is wrong but I already have a problem with geometry and feel far more confident to speak in IsiZulu."*

Educator AE1 "all assessments are in English and if our learners only learn the IsiZulu concept for geometry. They understand the meaning of the concept but are unable to read to and interpret a geometry problem properly"

According to Webb (2008) code switching can be effectively used in mathematics teaching however it must be used cautiously. Code switching is an important tool that allows learners that are more fluent in their mother tongue to understand mathematical concepts, delivered in English. However, many of the educators were of the opinion that code switching can cause more harm than good as some learners become confused. Educators from both schools A and B where the majority of learners are second language users, mentioned that learners also become lazy to learn the English concepts if the educator continues using code switching. For code switching to be effective educators must be fluent in all languages that learners in the classroom speak. With the diverse language speakers that South African schools have, this is a challenge. The educators agreed that geometry is a language on its own and expressed in the language of English. Most of the educators accepted that those learners having problems speaking in English, experience problems with the geometric concepts. More problems are encountered by learners taught by educators who do not speak the same language as them.

Educator C3 stated that *"I have learners from different language backgrounds. It is difficult for them to grasp certain geometric concepts due to the language barrier for example chord, intersect."*

Educator BE1 emphasized, *"All mathematics papers are set in English or Afrikaans so they need to understand the English."*
Educator AE2 stated that, *"I speak both English and IsiZulu and it is not easy to code switch with the correct word translation from English to IsiZulu and maintain the meaning."*

Educators indicated that there is no exact translation to maintain the meaning of some words and this creates even more confusion. From the lesson observation it was noted that it took longer for the educator to give the IsiZulu equivalent of the English geometric proof.

In the interview, one of the educators in school B stated that geometry is a language on its own and for learners that do not come from an English background it poses an immense problem. These learners are now struggling with both English and the language of geometry. Four of the eight educators indicated that often learners with language barriers are misinterpreted as having gaps in geometry content, however it is their struggle with understanding the language of geometry that poses the major challenge. Engaging learners to communicate from the foundational phase, will assist learners better with their understanding of geometry language.
AE2 stated that "*I use simple calculation problems so my learners don't need to read and interpret problems in geometry.*"
Educator CE2 mentioned "*these simple problems do not allow for learners to think and move to higher levels of reasoning*"
The presence of the simple exercises was noted in the lessons observed for both AE1 and educator BE1.
Educators said they used simple calculation problems only, in their exercises, because these tasks lack language dependency for example calculation of angles and sides. No proofs or intricate solving is given in the classroom for practice. Thinking and scaffolding of problems cannot be achieved with the use of calculation problems. Allowing the learner to move to new levels of thinking and problem-solving, requires questions with problem-solving capabilities. The educators also revealed that they selected less cognitively demanding tasks because the learners were limited in their linguistic abilities. These less demanding tasks will not provide learners with stimulation and thinking opportunities thus

entrenching barriers to geometry. Educators may believe that giving learner's simple exercises in the classroom and for homework will encourage learners; however, these learners are unable to complete tasks that require problem-solving. They also are unable to think independently.
Educator BE1 "*This results in rote learning skills and procedures which does not help learners to apply the concepts to problems."*

According to these educators learning of rules without understanding did not allow learners to use brainpower and engage in mathematical discussions with peers in order to explain their understanding. Educators also expressed that it is often through verbalisation that learners get a true and clear picture of their own understandings. Rote learning is learning without understanding and this type of learning cannot be applied to different situations. Educators expressed that this results in lower achievement and that they have reinforced low expectations of learners' performance in geometry. Learners are unable to apply their knowledge to different situations. Their application is limited to a specific situation. Educators stated that learners with language difficulties need more time to grasp the concepts.
Educator BE2 stated that, *"Writing and reading the problem is also a problem in geometry."*
Educator CE1 "*Geometry problems involving proofs are like answering a comprehension passage whereby the answers are right there in the statement and the diagram that you give. It is important to read and understand the statement given to be able to solve the problem. It is a lack of proper language ability that creates a problem.*"
Educator BE1: "*Words in a geometry problem must be represented diagrammatically. It is therefore important to understand the words and meanings to see the words represented in a diagram."*

According to three of the educators, learners are unable to comprehend the requirements of the problems. Identifying key geometric words in a problem and associated meaning will assist learners in solving problems.
Educator BE2 "*active communication is fundamental to learning. Geometry learning is no different. Getting learners to communicate their ideas and thoughts in the classroom keeps them interested and thinking*."

Educator CE2 involved the learners in active communication. Learners were encouraged to use the correct terminology.

From lesson observed

CE2 “which *arc subtends* O_1”

CE2 “......*intersect O*”

Educators articulated that creating a participative environment in the classroom is through active communication and this can only be achieved if both educator and learner speak the same language. The use of correct terminology will encourage learners to speak in the same manner. Understanding the learners’ explanations and providing guidance is fundamental to getting learners to think and make connections. Educators are also required to evaluate the extent of learners’ understanding and this can only be achieved if they understand each other’s language. Active participation encourages dialogue which must be understood by learners to prevent misconceptions.

One of the educators, CE3, stated that, “*The English second language users that come from the primary schools that do English as the home language from foundation phase, perform better in geometry than learners that come from IsiZulu medium schools.*”

Educator BE1 stated for question 4 of the interview, that one of the strategies that would help to change the situation is that learners must be exposed to an English medium of instruction from early ages. The educator stated that schools had a choice to teach in English or IsiZulu from grades 1 to 3. It was only in grade 4 that the medium of instruction is English. This, according to the educator CE2 was the root of the problem as many educators teach in the mother tongue and this creates a problem with learners developing misconceptions that manifest in the secondary phase. Learners are unable to communicate in the language of geometry as they are exposed to incorrect terminology.

Educator AE1 stated “---l*earners that discuss geometry concepts in the classroom, develop better language abilities and understanding*.

He further added that if learners are constantly exposed to the language of geometry and are given many opportunities to verbalize their understandings, they will be able to build their vocabulary and increasingly be able to engage in conversations with peers about geometry and its relationship to life.

Educator CE1 stated that "*I sometimes embed the mathematics problem in a story and learners are able to visualize the problem and it becomes easier for them to understand the problem.*"

Educators CE1, AE1 and BE1 all believed that if learners discuss the problem and are able to relate it to real life, they will be able to think about mathematical solutions to the problem.

Educator CE1 expressed that she would have liked to have had more time with learners while they were engaging in the mathematics discussions in order to point out and guide them during their discussions.

Educator BE2 and educator BE1 stated that educators must leave opportunities for learners to explore concepts and problems on their own, without educators providing step-by-step explanations on how to go about solving problems.

Educator AE2 "*learners gain a better understanding when they are encouraged to think critically about problems they are solving, and consolidate their knowledge as they verbalize their own ideas and or frustrations*".

The vocabulary is not easy. Some words are used only in geometry and therefore are unfamiliar until learners have been taught about them for example concepts like hypotenuse, parallelogram and sphere. There are other words that have different meanings in mathematical English and ordinary English and this confuses learners.

One educator spoke about the syntax of mathematics being a problem for many learners.

Educator CE1 stated that, *"It is not only the vocabulary of mathematics that is difficult but the syntax. The syntax in which geometric ideas are expressed is often more complex than learners is accustomed to in other areas of the curriculum for example parallel // and similarity ≡"*

Educator CE2 stated that, *"learners become slow in writing and reading because they are trying to grasp the language and at the same time have to learn the geometry"*.

Six of the educators articulated that if geometry was taught as a language with the correct vocabulary, syntax, rules and procedures, then learners would become more skilled at it.

Educators believed that there is a confusion between knowing what to do and why and trying to use the correct rules and formulas one idea at this point, consider separating without understanding the reasons. This increases anxiety in learners and creates further miscommunication.

Educators BE1 expressed *"…if you break the problem into small parts then learners are able to find the solution. Sometimes when I read and explain the requirements then they understand as I am more deliberate in what I want them to hear.*"

Educator AE2 indicated, "*educators should first use simple language for their better understanding of mathematical figures and concepts e.g. similarities where -in learners can compare".*

Educator CE3 communicated*, "Social communication within the classroom is important as the learners talk to each other they break a problem into parts that they can understand. What we think they know or do not know may not necessary? be correct so it is important to allow them to decipher a problem and then correct their mistakes within their own social groups. They are comfortable to talk and find solutions.*"

Many educators verbalized that learners experienced difficulties reading the problem fully before tackling it, while some do not act on what they are reading. Educators pointed out a need to provide learners with opportunities to discuss what they have read in the problem statement. They also implied that if learners work collaboratively, then they will be able to remove their misconceptions before they tackle the questions in a problem.

Educator CE2 stated that, "The answer to a geometric rider lies in reading the question clearly and underlining the key words. Many of the learners do not read the given information and what is expected before tackling the problem.".

Many of the educators mentioned that language has an important role to play in the teaching of geometry but there are challenges that they experience, and this is one barrier to the learning of geometry. All participants agreed that communication and active discussion within the geometry classroom is very important. Learners that verbalize their thoughts are able to get a better understanding of concepts and are able to consolidate their knowledge and learning. As much as code switching has an important role to play in getting second language learners to understand the geometric concept, it takes up too much of time. Educators also noted that all assessments are done in English and learners need to communicate more in the language of instruction to perform better. Educators articulated that reading mathematics and understanding the language of geometry especially for second language users, is challenging. Educators in all schools clearly articulated that greater emphasis should be placed on communication and removing misconceptions that learners may have developed.

Communication is a significant tool in the geometry classroom, which drives thinking. If the educator and learner are unable to communicate clearly in the same language, this is a major challenge to understanding concepts. Terminology and use of concepts can be developed when there is active communication between learner and learner and between learner and educator. Regularly talking will allow learners to use these concepts in application activities.

5.4 EXPLORING THE TEACHING OF EUCLIDEAN GEOMETRY

5.4.1 Profile of Participants

The profile of the educator participants obtained from section A of the questionnaire, that is their biographical details, is presented in table 5.3 to provide an understanding of the participants that the researcher was working with. The significance of this information is to provide readers with the generic context and experience of the participants involved in the research. Eight educators were involved in the study. The study initially sampled for nine educators, i.e. three per school however, one of the schools only had two maths educators in the FET phase. The data collected were from the questionnaires, one-one interviews and the lesson observation.

Table 5.3 Profile of Educators

P	**REQV**	**In**	**Years of exp**	**G**	**Home language**	**Average Performance in geometry**
AE1	REQV 14	Y	6-10	M	ISIZULU	30-39
AE2	REQV 14	N	11-20	F	ISIZULU	20-29
AE3	REQV 15	N	>21	M	ISIZULU	30-39
BE1	REQV 14	Y	6-10	F	ENGLISH	30-39
BE2	REQV 15	N	>21	M	ISIZULU	30-39
CE1	REQV 15	Y	>21	F	ENGLISH	40-49
CE2	REQV 15	N	6-10	M	ENGLISH	50-69
CE3	REQV 14	Y	11-20	M	ENGLISH	40-49

P- Participants**, REQV** – Relative Education Qualification Value, **In** – Geometry included in the qualification, **G - Gender**

The educators all stated that the performance of geometry in their classes was very low. The above table provides an indication of the learners' average performance in grade 11 geometry. All educators had more than 5 years of teaching experience. Four of the eight educators that participated in the semi-structured interviews were IsiZulu home language and two of three educators whose lessons were observed were IsiZulu Home language speakers.

5.4.2 Curriculum Demands

The interview with the educators revealed that the curriculum for geometry placed much stress and demands on them in the classroom. All educators indicated that the curriculum for mathematics is long. The work schedule is set out by the department of education and each section is given a fixed amount of time.

Educator CE2 stated that, *"The grade 10 Euclidean **g**eometry must be completed in 11 days; the grade 11 Euclidean geometry is given 14 days and the grade 12 Euclidean Geometry in 11 days. The time allocated per day is under 1 hour, as* *maths is given 4.5 hours per week. This includes revision of grade 10 work and assessments."*

Educator BE1 concurred, *"the time allocated to teaching of geometry is set out in the work schedule. The time does not take into consideration exams or assessments"*.

Educator AE1 indicated, "*It is difficult to complete the syllabus within the designated time. Even if you drive yourself, you sometimes cannot complete the term's work. It must then be carried over to the next term.*"

All educators were of the opinion that geometry needed more time in the curriculum if learners were to master the skills needed to perform well. Educators conveyed that there are certain parts of the curriculum that required more engagement and activity, for learners to grasp and understand the concepts better but they are unable to do so due to time constraints. The issue of not having sufficient time was noted by many participants to be the most significant impediment for implementing learner-based activities.

Educator AE2 "*time is the biggest challenge for me"*

Educator BE1" *I wish we had more time to teach geometry"*

Educator CE1" *more time is needed for teaching of geometry with understanding"*

Educator CE3" *with the short amount of time allocated to the teaching of geometry it is impossible to sit down with learners to discuss geometry problems on a more individual level to develop learners thinking. The completion of the syllabus takes priority.*"

Educator BE2 communicated, *"It is necessary to revise grade 11 work in grade 12 as learners do not remember what was done in the previous year."*

The geometry content as displayed in table 2.1 contains seven theorems and solving of problems that must be completed in term three as well as the revising the concepts from the previous years. Educators indicated that the time allocated does not allow them to cater for learners that are struggling with individual concepts.

Educator CE2 stated, "*Grade eleven circle geometry is a complex topic and has extensive concepts that require deeper level understanding such as cyclic quadrilaterals, and tangents*"

Educator BE2 “Grade *11 geometry is needed assessed extensively in the grade 12 NSC examination and learners perform very poorly in this aspect*”

Educator AE1 “*Learners find learning the theorems with understanding very difficult and therefore are unable to apply them to riders*”

Educator CE1 “*We need to allow our learners time to grasp the theorems and work with them in application exercises to develop an in-dept understanding and use critical thinking to solve these problems*”

Educators indicated that they did not have sufficient time to develop the skills of critical thinking to be used in the application of the theorems.

Educator BE2 “*... educators should be part of the curriculum development.”*

Educator CE2 “*... geometry develops skills for many other topics in mathematics and therefore educators should be included in the them allocation for each topic*”

Educators articulated the need for educators to be part of the curriculum development. Educators understand and know what is going on at ground level. They have the expert knowledge but they are not consulted when the geometry curriculum is developed.

Educator BE1 mentioned,” *There is a difference between the implemented curriculum and the intended curriculum. We implement a curriculum based on our individual circumstances, learners’ experiences and our personal experiences.*”

Many educators shared this view and stated that they want to improve the results of geometry in their schools however; they are unable to meet the demands of the intended curriculum.

Educator AE3 indicated, “*The gap between the curriculums from grade 9 geometry to grade 10 geometries is great. These gaps are too big and de-motivate learners*”.

Educator CE2 “*The geometry curriculum is a carryover from the previous years, it is therefore necessary that some form of monitoring be done to ensure that the curriculum is properly completed at each year.*”

The geometry syllabus is a continuous curriculum and each year’s work is based on the previous year. A gap in the curriculum creates shortcomings in the learners understanding. It is therefore important that when educators teach a section it be completed with thorough understanding. When educators ‘rush through topics to complete syllabus without understanding learners develop barriers. Teaching geometry merely with the intention to complete the syllabus is an attempt in futility. The teaching of geometry cannot be rushed if understanding is to be expected. To achieve deeper thinking and application of concepts, patience and determination must be considered in the teaching of geometry.

Educator CE3 expressed "*there is a need for learners to see meaning in their learning and we only do this by allowing them to participate in activities that show the use of geometry in life*".

The need for learners to take part in their learning was pointed out. Changing the classroom to make it conducive to learning from life situations and experiences takes time, and educators alluded to not having sufficient time to create these scenarios.

Learners pointed out that there was too much in the curriculum to be mastered each year. Most learners claimed that educators teach, based on the curriculum and rushed through the syllabus to meet curriculum demands. Learners expressed in dismay that educators did not care about whether they understood a section, but just moved from one topic to the next.

Learner A3 stated *that, "The aspect that I experience the greatest difficulty in is geometry yet it is the topic that is done in the shortest space of time. I am slow at grasping geometric concepts, but when I ask my educator to repeat she says 'if I keep repeating I will finish this syllabus next year' so I do not ask questions.".*

Learner A2 stated that *"Most of the time I do not understand my educator as she goes very fast. I copy the theorems from the board and try to do the application at home. I experience a lot of problems when it comes to proving something"*

Learner B2 *"I can do the calculation examples; however, I am not able to do proving of two angles equal or proving that a quadrilateral is cyclic*"

Educator CE1 "many *of my learners tackle the questions where they need to calculate an angle but experience problems providing the correct reason*."

Educator BE1 "*my learners experience problems proving. They lack understanding of the concepts and properties.*"

Educators and learners indicated that the aspect that they have the difficulty with is proving and not the simple calculations. There was also the indication that learners experience problems providing the correct reasons for their answers. This is possible due to a lack of proper understanding of the properties and theorems.

Learners and educators indicated that the geometry curriculum has too many packed content areas that must be completed in a short space of time. This affects teaching with understanding. Learners are unable to develop a deeper understanding of the concepts and thus experience problems in the application of these concepts. Educators tend to rush through the curriculum in order to need the demands of examinations and syllabus coverage. It was also noted that previous years' content must be revised and sometimes need to be re-taught for understanding of the current content. Critical thinking can only be developed when learner have a good understanding of all concepts needed for the application of problems.

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Learners and educators indicated that the geometry curriculum has too many packed content areas that must be completed in a short space of time. This, impacts on teaching with understanding. Learners are unable to develop a deeper understanding of the concepts and thus experience problems in the application of these concepts. Educators tend to rush through the curriculum in order to need the demands of examinations and syllabus coverage. It was also noted that previous years' content must be revised and sometimes need to be re-taught for understanding of the current content. Critical thinking can only be developed when learner have a good understanding of all concepts needed for the application of problems. Curriculum demands is a challenge for many educators due to the great emphasis that is placed on assessments. Although assessment is a way of gauging learners' outcomes, rushing through the curriculum to complete the syllabus will lead to poor performance. Geometry requires learners to develop knowledge by exploring, measuring and investigating and this requires patience and time.

5.4.2 Analysis of Lesson Observation

The following section provides a detailed presentation of the lesson observation. I found that this was needed to get a full picture of what actually transpires in the classroom. The lesson observation unveiled educator practices and the level of engagement and interaction that takes place in the classroom. This section clearly shows how educators implement the curriculum of geometry and some of the misconceptions that are created in the classroom. The impact that educators' classroom practices have on the teaching of geometry is fundamental to learners' performance. The extent to which educators are able to bring real situations into the classroom was observed since learners articulated the need for geometry to be significant.

Nine lessons were observed from 3 educator participants that is three lessons from one educator in each of the schools. The lessons were observed in each school on consecutive days. The lessons were observed to identify the barriers created by educator practices in the classroom and identify strategies that help in the learning of geometry.

Table 5.4 Summary of lessons observed Educator AE1

School A Aspects observed	Lesson 1: Tangents 02/08/2019 **Proof of theorem: The angle between the tangent and chord at the point of contact is equal to the angle in the alternate segment**	Lesson 2: 03/08/2019 **Application of the theorem**	Lesson 3: 04/08/2019 **Application of theorems**
Mathematical tools	1. Learners were passive participants. 2. No tools used –technological or social 3. The educator did ask questions but they were limited to one word responses. Did not elicit thinking 4. Learners were required to write the theorem in the books for later reference. Learners were not engaged. No active communication	1. Learners were involved in completion of the exercise. 2. Diagrams were used on the board to explain concepts. No other tools used to create interest or develop thinking. 3. Very little interaction between learner and learner or learner and educator.	1. Simple questions based on the problem on the board were asked 2. Coloured chalk was used to draw the diagram and emphasize certain angles. 3. Communication was noted. Explanations and articulation of geometric concepts very poor. Learners did not use correct terms

			for angles for parallel lines. They spoke about bow shape for angles in the alternate angles and F shape for corresponding angles. Educator accepted these responses and did not correct them.
Learners role	1. Learners wrote down the theorem. 2. Simple exercise was given to apply the theorem. 3. Learners were not requested to use their experiences in finding solutions to prove the theorem. 4. No form of group discussion. Thinking was not encouraged or guided.	1. Learner's experience was not a factor considered in the lesson. 2. Learners worked individually.	1. Very little discussion on prior knowledge or learners' experiences. 2. Active interaction and communication between learners not given attention. 3. Learners were given an opportunity to work by themselves.
Educator role	1. There was no pre-knowledge	1. Basic skills of parallel lines	1. Visual diagrams were

	discussion or revision of skills. 2. The entire lesson was educator-centred with the educator talking for 45 min. of the 55 min. lesson. 3. Learners were only engaged in writing down the theorem and completing the activity at the end. 4. Memorisation of the theorem was encouraged. The educator's definition of the theorem missed key words. The underlined words were missing. The angle between the tangent to a circle and a chord drawn from the point of contact is equal to the angle subtended in the alternate segment by the chord. 5. Educator did not request	and angles were used in the problem. 2. Exercise was not linked to any real life examples. Cognitive level 1 and level 2 examples were completed. 3. Educator used the chalkboard to represent the diagrams. 4. Diagrams were drawn. 5. Solutions were given on the board however very little discussion of learners' solutions. Misconceptions not addressed. 6. Learners' ideas were not given sufficient time.	used to apply the theorems of the problem on the board. Coloured chalk used. 2. No other technology or teaching aids used. 3. Different ability levels of learners not considered. 4. Educator asked questions. 5. Educator wrote out the final solutions on the board. Learners copied into their books and were given an exercise to work with. 6. Exercises given were not stimulating their thinking.

	discussion from the learners. 6. Application exercise given. Cognitive level 1 and level 2 tested Merely recall of facts. No thinking required		

Day 1: educator AE1 lesson – Extract from the lesson:

AE1: Today we are going to do a new theorem.

The following was written: The angle between the tangent and the chord is equal to the angle in the alternate segment.

Learners started writing in their books.

AE1 stated that, “BCD is a tangent. BC is the chord. Can you see it?”

Learner chorused “yes”

AE1 “if you place your fingers on BC and move to the circumference you will find the angle in the alternate segment”

He then went on to prove the theorem.

AE1 “do you understand? “

Learners together said “yes”. Some nodded.

Learners were requested to copy the theorem.

The following geometry problem applying the theorem …put up on the board

Key aspects noted

1. Educator asked simple questions to elicit responses.
2. Learners were not given the opportunity to explore, investigate and work by themselves. Learners were not engaged in the lesson. align
3. The educator had only 8 minutes left in the lesson and gave an exercise from the textbook. Simple cognitive levels 1 and 2 were tested in the exercise. No connection or links to real life examples were made.

4. The solution to the problem was put on the board after getting responses from a few learners.
5. Lesson followed a very formal structure.

Lesson Two

The following is an extract from the lesson:

AE1: “Yesterday we did a theorem. Who remembers the theorem?”
Learner 1: “Angle between tan and chord”
AE1: “Yes, but isn’t there more”
Learner 2 “Yes sir, the angle between the tangent and chord is equal to the angle at the circumference”
AE1 “Is it equal to any angle at the circumference? “
Learner 3 “No, the angle between the tangent and the chord is equal to the angle in the alternate segment”
Learner 4 “Which is the alternate segment?”
AE1” the angle subtended by the chord”

Educator redraws a rough diagram on the board and shows them the angle between the tangent and the chord and the subtended angle at the circumference.
This misconception of the position of the angle at the circumference was established and carried over from lesson one.
Not mentioning that the angle between the tangent and chord at the **point of contact ...** can also result in misconceptions formed, since there can be other angles between the tangent and chord.

The following problem was done by the educator

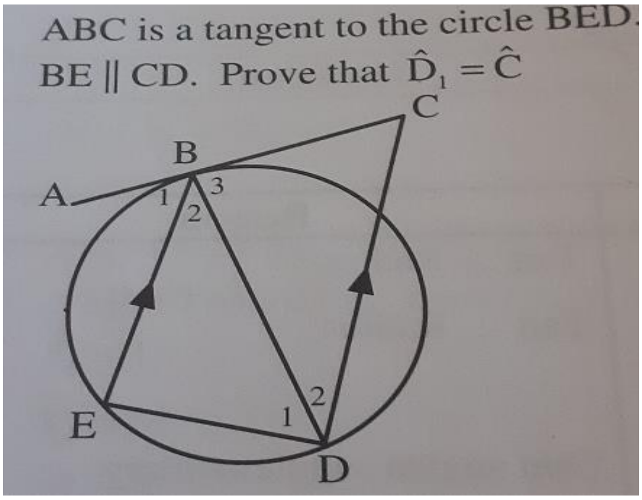

FIGURE 5.1 QUESTION FROM LESSON BY AE1

Educator AE1 “what is the reason for B2= D2 “

Learner “*it is bow shape*”

AE1 “ah yes”

The educator accepted the response but wrote the reason “alternate angles” on the board.

Educator AE1 “now why is B1= C “

Learner “it is F shape”

Educator AE1 “wrote out the solution”

Learners copied.

Educator AE 1 employed traditional, direct methods of teaching. Questioning was not stimulating but solutions were given and reasons required. Very little communication was noted between learner and learner or learners and educator. Solutions were given to learners without learners getting the opportunity to think or communicate with each other. The educator accepted the incorrect terminology without correcting it immediately.

The reasons given by learners, seems to be acceptable to the educator. Most learners also stated the same in chorus. One or two of the learners did use the correct terminology.

Learners’ prior knowledge of parallel lines and angles formed by parallel lines was evident, but without the correct use terminology. Educator did not allow learners to use the knowledge independently to solve, but directed them to the solution. Learner interaction with the diagram was minimal. Learners that did not understand the problem would not have been identified due to the chorusing of answers and the learners not interacting with the problem independently.

Extract from the beginning of lesson 3

Educator AE1: "*Learners today we are going to apply the theorems we learnt over the last two weeks. This is the difficult part of geometry, to integrate all the theorems.*"

The educator's assumption that learners will experience difficulties with solving the problem creates a negative attitude towards solving in geometry, before they can try. This will prevent them from attempting the problem confidently.

The following questions was done on the board

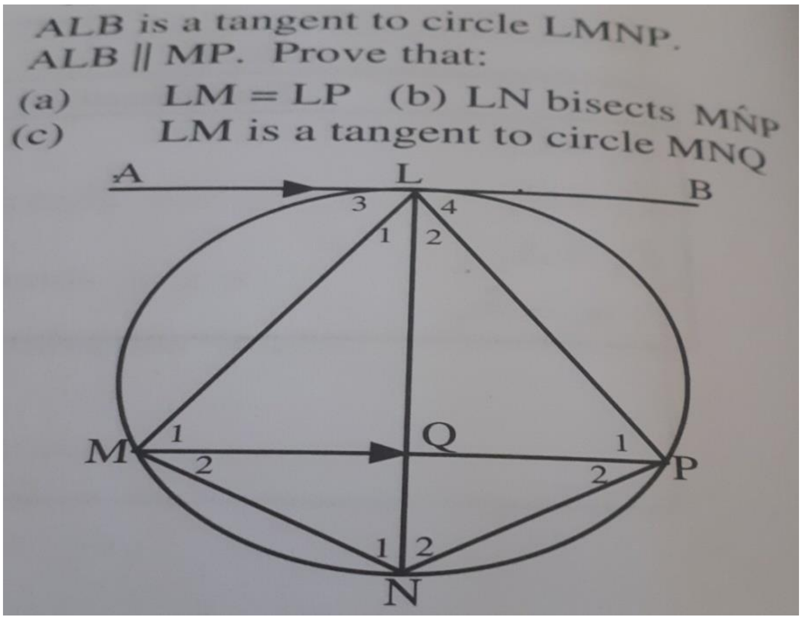

FIGURE 5.2 QUESTION FROM LESSON2 AE1

Extract from lesson–

Educator AE1: "Why is L3= M1"

Learner: "Z shape"

Educator AE1: "correct alternate angles"

Educator AE1: "Name one other angle that M1 is equal to"

Many learners indicated that M1 is equal to P1. Their reason was that it was an isosceles triangle.

Educator AE1: "They are equal but the reason is not isosceles"

The sequence and logic of concepts were absent in learners' thinking.

Educator AE1: "L3=P1 angle between tan and chord"

The statement of the question, and key concepts were not discussed. Concepts like tangent, bisects or parallel were not explained to learners The educator went straight to answering the questions. The educator quickly gave the solution without allowing learners to think. Active discussion and communication which is an integral part of learning, was missing. Opportunities to internalise their thoughts and develop solutions sequentially is not noted. The level of problems given did not include challenging, stimulating geometric problems allowing for higher level of thinking skills.

In the discussion with the educator after lesson three, he made it known that this was the last geometry lesson for the term. The educator also articulated that learners are weak in geometry and therefore he does not attempt difficult questions.

AE1: "*learners cannot tackle questions of higher order thinking. Their language ability is also very poor*".

The educator's negative attitude towards the learners' abilities negates them the opportunity to experience higher order questions. Learners will only experience level 4 questions in the examination. The educator did not cater for all learners in the classroom. He did not provide opportunities for weak learners. It was also noted that the educators command over the content knowledge was poor. The educator indicated that he teaches to cover the examination content. He did not encourage a participative approach to his teaching. Learners were not actively engaged in the lessons.

"*I am not using any fixed teaching method for geometry teaching but my aim is to ensure that I complete the syllabus for the examination*"

Table 5.5 Summary of lessons observed: educator BE1

School B **Aspects observed**	Lesson 1: 29/07/2019 **Theorem of cyclic quadrilateral**	Lesson 2: 30/07/2019 **Application of the theorem**	Lesson 3: 31/07/2019 **Theorem on tangents**
Mathematical tools	1. Learners were active participants for part of the lesson. 2. Instruments were used in the lesson in the form of protractors 3. Questions were limited to one word responses. Did not elicit thinking 4. Active communication was not encouraged in the class.	1. Mathematical tools in the form of social groups were used to create interest or develop thinking. 2. Learners were given the opportunity to express their answers on the board. 3. Lesson was very rushed to complete the syllabus; Syllabus took precedence over understanding.	1. Worksheet with answers of previous exercise handed out. This hampered learners finding their own solution. 2. 2. The urgency to move on with the syllabus was noted. Learners were asked simple questions based on the problem on the board. 3. 3. Use of different colour chalk was used to show the angle between the tangent and the chord and the angle in the alternate segment. 4. Very little communication

			and discussion took place.
Learners' role	1. Most learners were able to measure angles. Those that had difficulty were given help. 2. The concept of supplementary was understood. 3. No form of group discussion. Thinking was not encouraged or guided. 4. Proof was given to learners to copy into books,	1. Active participation of learners in the form of measuring and problem solving. 2. Learners engaged in group activity. Self-discovery of opposite angles of a cyclic quadrilateral are supplementary. 3. Not all learners were interacting with the activity. Large class size made it difficult to control group activity. 4, Learners' answers were discussed by the class.	1. Questions to determine prior knowledge was asked. 2. Concept of tangent, subtended, angle between.... was discussed. Learners experienced difficulty understanding these concepts. 3. Very little learner participation noted. 4. Learners were passive participants in the lesson.
Educator role	1. Pre-knowledge of theorems done. 2. The exterior angle of cyclic quadrilateral not clearly identified. 3. Partial learner-centred and partial	1. Exercise was not linked to any real life examples. 2. Educator completed the level one example on the board. 3. Questions were asked to guide learners to answers,	4. 1. No other technology or teaching aids used, except chalkboard. 2 Educator did most of the talking, learners listened. 3. No

	educator-centred method adopted. Memorisation of the theorem was encouraged. 4. There was a degree of learner engagement directed by the educator. 5. Exercise given contained a mixture of cognitive levels. Learners were given an opportunity to work individually at home but no guidance will be provided.	but not thought provoking questions. 4. Knowledge of circle geometry of educator was good 6. Activity in groups encouraged and educator provided individual groups with guidance.	discussion on the theorems took place in the lesson. Learners copied the proof from the board after the educator completed the lesson. 4. Exercises given did not stimulate thinking. 5. Higher level thinking not encouraged in the lesson.

Educator BE1

Day 1: Educator assigned learners an investigation to measure angles and write down conclusions. The lesson was based on the theorem of cyclic quadrilaterals namely the opposite angles of a cyclic quadrilateral are supplementary.

There were 48 learners in the class. Some learners were sharing desks. More than 50% of the learners did not have protractors. Therefore, they were sharing those as well. The educator walked around to monitor. It was also discovered that some learners did not know how to measure angles. Since the educator was providing assistance, this hindered the progress of the lesson.

Lack of resources was noted. Prior foundational knowledge of measuring was noted as being a challenge.

Educator BE1: "*Learners you would have learnt measuring angles in grade 8.*"

Most learners did complete the exercise however there were a few that were struggling to use the protractor properly. The educator stated that she needed to go on with the lesson.

Educator BE1: "*Learners some of you are taking too long to measure the angles. I am going to give you the answers so we can move on.*"

Educator BE1: "*We have a syllabus that needs to be completed. Your March* ***C****ontrol testing is in two weeks*"

Demands of the syllabus force educators to complete the syllabus despite learners not getting to understand the work.

About 10 learners raised their hands in response to the deduction that opposite angles add up to $180^{0.}$

Educator BE1 "*so we say the opposite angles of a cyclic quadrilateral are supplementary*".

The proofs of the two theorems of cyclic quadrilaterals were put on the board and learners copied them.

Learners were not given the opportunity to derive the proof independently. The emphasis on memorisation of proofs was evident.

Day 2

The educator was doing application of the theorems on cyclic quadrilaterals.

Revision of previous day's work was observed.

The following questions were asked.

"*What is a cyclic quadrilateral?"*

Diagram on the board

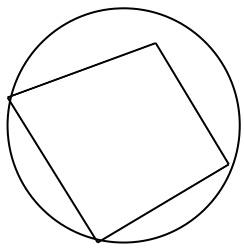

FIGURE 5.3 QUESTION FROM LESSON BE1

"*is this quadrilateral cyclic?" "Why not?* "

"*what do you know about the opposite angles of a cyclic quadrilateral?*"

The educator also revised previous theorems on circles to be used for proving the theorem on tangents.

Learners were given a worksheet with an exercise. The educator did the first one on the board while asking questions and learners were required to complete the others in groups of three. Different cognitive levels were covered in the problems. The educator had done the simplest one on the board and learners copied. Learners were given the opportunity to come to the board and do the answers. Due to the large class size, the group work was not very effective. There was a lot of noise and some learners were not engaging in the work given.

Educator BE1: "*The large number of learners in the class, the short teaching time to finish the syllabus makes it difficult to use the learner-centred method in the classroom*"

Day 3

The work given the previous day was not marked with the learners.

The educator completed all the answers on a worksheet and duplicated these, for the learners.

Educator BE1: "*You can use the worksheet to check your answers. Today we are doing the next theorem which is the related to tangents. Can someone tell me what a tangent is?*"

Giving the answers emphasizes rote learning. Active communication on the problems was absent in the lesson. Learners were not given opportunities to express and discuss their solutions. Stimulating learners' thinking was lacking in the lesson. Some learners would not have attempted the work due to challenges that they may have faced, however no discussion with regard to this was considered.

No learner in the class could define the concept 'tangent'. There was a learner that stated that it is a straight line. Poor foundational knowledge was evident. Questions were of low cognitive level thinking. Questions did not evoke any form of deep thinking. The educator gave most of the answers to the questions asked. Discussion and communication with learners was not encouraged by the educator BE1. The lesson proceeded with educator doing all the talking and learners remaining passive participants. Learners were required to copy everything from the board. The educator's content knowledge was very good.

Table 5.6 Summary of Lesson observation for educator CE2

School C Aspects observed	Lesson 1: 10/08/2019 Theorem: The angle which an arc subtends at the centre of a circle is twice the angle it subtends at the circumference.	Lesson 2: 11/08/2019 Application of the theorem	Lesson 3: 12/08/2019 Learners solutions on application Discussed
Mathematical tools	1. Learners were active participants for most of the lesson. 2. Instruments were needed in the lesson in the form of protractors. 3. Questions were used. Some were simple questions 4.Active communication was encouraged in the class.	1. Mathematical tools in the form of social groups were used to create interest and develop thinking. 2. Learners were given the opportunity to work in groups - communicate with each other, discuss the problem and arrive at a solution. They presented their solutions on the board.	1. Educator selected a learner from the group to present the solution on their own. 2. Many questions were asked and discussion was meaningful. 4. 3. Use of different colour chalk was used. 4. Educator used technology to display solutions.
Learners 'role	1. Learners were requested to use their experiences to deduce the	1. Active participation of learners. 2. Learners engaged in Group	1. Learners engaged in communication within their groups and with

	theorem such as measuring, deducing centre, arc, chord etc. 2. Group work encouraged peer interaction and guidance for slower learners. Thinking was encouraged. 3. Proof was deduced by learners guided by the educator.	activity. 3. Learner's answers were discussed by the class.	the educator and class. 2. Learners asked questions and this reinforced their understanding and removed confusion. 3. Peer teaching was noted throughout the lesson.
Educator role	1. Concepts such as exterior angle of a triangle...., radii equal etc. were revised. 2. Learner-centred method adopted. Communication was noted. 3. Exercises contained cognitive level 1 and 2 problems.	1. Exercises were not linked to any real life examples. 2. Educator guided learners while they worked in groups. 3. Questions were asked to guide learners to answers. Thought provoking questions were encouraged. 4. Content knowledge of educator was very good. 5. Activity in groups encouraged and educator provided individual	3 1. Technology and teaching aids used. 4 2. Educator encouraged learners to talk, question and discuss the problem. 3.Exercises given stimulated thinking. 4. Learners experiencing challenges or struggling proving were not identified.

		groups with guidance. 6. Educator did not rush through the lesson.	

Lesson observation of CE2

Day 1: Educator was teaching the theorem: The angle at the centre of a circle is twice the angle subtended at the circumference of the circle. There were 38 learners in the class in groups of three or four and given a worksheet with circles and angles at the circumference and at the centre of the circle.

Learners were required to measure angles in their groups and write out their findings. Some angles were at the circumference while others were not.

Learners were actively involved in the lesson. Learners showed immense interest in the lesson. There was a positive mood in the classroom amongst the learners and the educator. The educator had sufficient protractors that he brought into the classroom and distributed. The classroom environment was very stimulating with lots of mathematical charts and models.

The class had 38 learners but they were well spaced with sufficient desks for each learner.

Extract from the lesson -

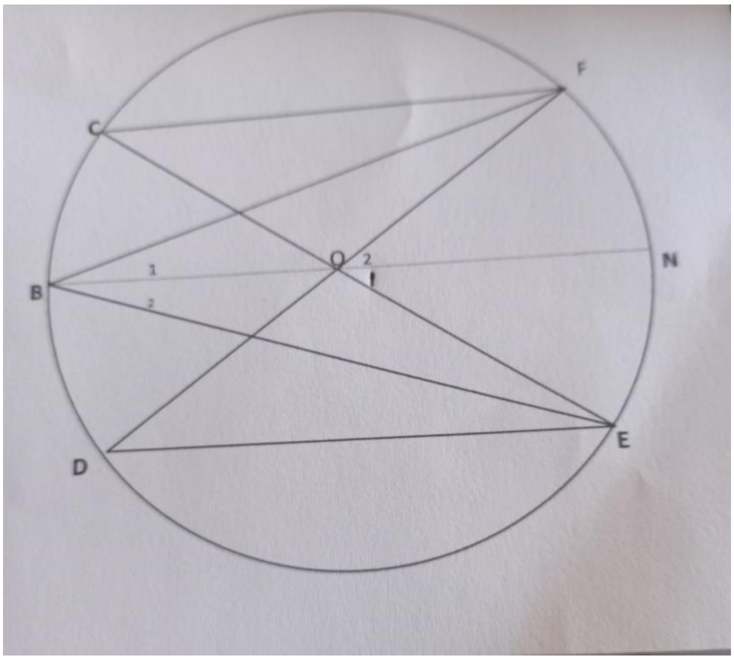

FIGURE 5.4 QUESTION FROM LESSON OF CE2

EducatorCE2: "*What can we deduce*?"

Learner "*angle O_1 is two times more than angle B_2*"

Educator CE2 "*let's write this using the mathematical symbols*"

Learner "*equal to......... so O_1=2 B2*"

Educator CE2 "*very good. Which arc subtends O_1? Does it subtend any other angle?"*

Use of correct terminology in the class ensured that the learners familiarise themselves with the vocabulary.

Learner "*Yes it subtends B1*"

CE2 "*Lets write it in a statement. Who would like to try?*"

Learners were given the opportunity to develop the theorem and write out the statement.

CE2 "*We have shown this by measuring. Now let us work on proving it mathematically. Does anyone have any ideas?*"

Learners tried to analyse from the diagram. Thinking and communicating was encouraged.

CE2 "*now I am going to construct a line from B to N Intersecting O*"

CE2 "*What do you call this line*?"

Learners "*Diameter*"

CE2" now you solve the problem. Show that O1=2B1"

Many learners were able to formulate the proof. It was encouraging to see that learners wanted to find the solution on their own. They did not wait for the educator's solution. The learners seemed to be familiar with working independently to find solutions. Learners were also required to prove the other cases by themselves.

A simple exercise with cognitive levels 1 and 2 were given to learners.

At the end of the lesson the educator stated that the questions given were to encourage the learners and to ensure that they understand the theorem well.

Day 2: More advanced problems.

Learners were in their groups of 3 and a new worksheet was given.

Each group was given an opportunity to provide solutions to the previous work.

CE2 "*we are looking at question one of the exercises*" Learner was requested to read aloud.

CE2 "*let us underline all the key concepts*"

Learners provided responses to the key concepts to be underlined. Discussion and communication was present throughout the lesson.

The learners worked independently in their groups and the educator guided them along. Learners' previous experience was used to determine the solutions. Learners that were struggling were guided by the educator and their peers.

Day 3

The educator gave learners an opportunity to present their solutions. Different learners from the group were asked to present their solutions. The educator displayed each of his solutions on a whiteboard after learners presented theirs. Discussion took place on the differences between the learners' solutions and the educator's. The sequencing of the statements was discussed. The lesson was learner-centred. Some learners explained the reasoning behind their solutions. Learners listened and contributed to the lesson.

Concluding remarks on lesson observation

It was evident from the observations that many of the barriers discussed by learners and educators existed in the classroom. All three educators did not relate geometry problems to real situations. Integrating and developing connections were missing in all lessons observed. Two of the three educators placed more emphasis on completion of the syllabus and not on learners' understanding of the concepts. Two of the three educators emphasized memorisation of the theorems. The classroom environment was not conducive to the learning of geometry as the classroom environment was not very stimulating. Learners were not active participants in the learning process. Resources to assist in understanding of geometric concepts were lacking. Two of the educators isolated the proof from the classroom activities. Questioning used by two of the educations was simple and did not evoke thought. A response pattern where an educator poses a question with a specific response in mind is common in the geometry class. This was clearly seen by two of the educators. This pattern provides limited opportunities for learners who are at varying stages of learning. The problems given to learners in two of the classes were of simple cognitive levels that did not require much thinking. Giving learners solutions in geometry does not help them to answer questions in assessments, as they do not understand the concepts and this leads to poor results. Incorrect use of terminology exacerbates the learners' challenges in geometry as they carry this into the next grade. Communication and active discussion of concepts was absent in some of the lessons. Misconceptions were created by the use of incorrect terminology. Educators CE2 and BE1 made use of the inductive method of teaching by requesting learners to measure angles. Educator BE1 was unable to follow through with the methodology. Educator CE2 however moved from the inductive method to the deductive method. The use of the inductive method allowed learners to move from the known to the unknown. It must be noted that the performance of learners in the class of educator CE2 was much better than that of other educators. It was also noted that learners enjoyed the geometry lessons in this class and they displayed a positive attitude.

5.4.3 Educators Lack of Geometry knowledge

The data from the questionnaires indicate that although all the educators have an educational qualification that is REQV-14 and above, four (50%) of them do not have geometry in their qualifications. In the interviews, it was further revealed AE2 and CE3 did not select mathematics as their major subject for teaching. They were forced to teach mathematics due to the shortage of mathematics educators. They started at the GET phase and moved on to the FET phase. Three of them indicated that they last did geometry at school level which was at the GET level.

Educator AE2 stated that, "*I decided to do mathematics as a major at University because I had no other choice. I was not given my first choice.*"

Educator BE1 shared the similar statement "*I disliked geometry in school but managed the other topics in mathematics but had no choice at university as that was the only area that had space."*

The educators also revealed that it is not easy for them to teach geometry at grade 12 level due to their own lack of foundational knowledge and the complexity of the syllabus.

Educator AE2 stated that "…*we attend workshops sometimes. These workshops are not continuous and do not do in-depth content knowledge which many of us need*".

Educators revealed that although the department of education has provided them with workshops, these are not continuous. The educators stated that they have gained knowledge from these workshops to assist them but still do not feel confident in the teaching of geometry. Educators conveyed that they needed more in-depth workshops that will concentrate on geometry content. One of the educators stated in the interview that she needs a bridging course on school geometry to feel more confident. Having an in-depth knowledge of the geometry curriculum allows the educator to be confident when standing in front of the class. The educator is expected to be the more knowledgeable other in the classroom. The educator is expected to guide learner's discussions for them to acquire knowledge and to engage learners in the lesson. A lack of content knowledge will create a gap in the flow of this process. This major challenge could be a contributing factor for learners' performance in geometry.

Educator CE3 stated, *"I request for help from more experienced educators that have the skill in geometry to help to solve problems."*

Educator BE1 stated that, *"I do not give my learners problems in geometry that I cannot solve."*

Educators alluded to not providing learners with problems that are at a more advanced level due to them not being able to solve these problems themselves. This would imply that learner development of geometric knowledge becomes retarded due to the shortcomings of the educator. The educators also stated that it takes up a lot of time to prepare their lessons for geometry teaching and sometimes they still cannot answer all learners' questions. The lack of content knowledge would prevent the educator from effectively engaging learners in the geometry classroom and from building an implicit understanding of the knowledge and the context in which the knowledge is used. The educator's content knowledge is fundamental for effective transmission of geometry content.

Four educators of the eight educators interviewed stated that due to their poor content knowledge, they themselves sometimes use the incorrect concepts in the classroom. This was also noted in the lesson observation where some educators struggled with the interpretation of geometry problems. On day 3, lesson three Figure 5.2 was given by educator AE1 to learners to complete. With Question (c) educator AE1 struggled to explain to learners that M is the point of contact and angle M_1 will be the angle between the tangent and the chord which will be MQ. Educator AE1 highlighted chord MN and this confused the learners as they could not see any angle in the alternate segment.

Being the more knowledgeable other it is fundamental for the educator to have a thorough knowledge of the geometry curriculum. Teaching of mathematics and especially geometry is unlike other learning areas whereby content can be memorised after reading and then presented in the classroom. Geometry teaching requires a deep understanding of concepts and the ability to explain and apply these concepts in various situations.

Educator CE3: "*Some maths educators create misconceptions by providing incorrect answers to learners' questions*".

Educator CE2 "*thinking develops from the educators' responses. Misconceptions are very difficult to correct. Learners internalize these concepts*".

Educator BE2 stated "*educators that do not understand the concepts properly, confuse learners and this creates misconceptions*".

Educators that guide learners in the wrong direction due to their incorrect definition of concepts or wrong use of concepts, create long term damage. Learner's thinking is guided by questions and responses to these questions. These interactions form a vital aspect of learning.

Educator AE1 was teaching circle geometry and the following problem was being discussed:

O is the centre of circle, SAT, which is inscribed in triangle PQR. ....

A learner requested an explanation of the term inscribed and the educator could not explain.

Learner “*What does inscribe mean?*”

Educator AE1 “*A figure inside a circle*.”

This incorrect definition would lead to misconceptions in the mind of the learner. The learner would assume that any figure inside a circle is inscribed instead of the points being on the circumference of the circle.

From the lesson observation educator BE1 was teaching the theorem on cyclic quadrilateral and provided the following explanation for exterior angle of a cyclic quadrilateral.

BE1 “The *angle outside of the cyclic quadrilateral is equal to the angle on the opposite side*”

Although the educator was showing the correct angle the definition is not correct.

Even the concepts of subtended chord were observed to have posed challenges to the learners and the some of the educators could not provide proper discussion and explanations on these. Not being able to engage in communication due to a lack of knowledge will discourage learners from thinking.

Educator BE1 stated *“our learners come into the secondary school with some never doing geometry in the primary school while others have very little knowledge of geometry”*.

This was also supported by participants AE2, CE1, CE2 and CE3. They all expressed that due to the lack of qualified mathematics educators and no specialization in the primary phase, many educators with no mathematics qualifications teach mathematics up to grade 7 level. Educators articulated that learners have misconceptions, incorrect concepts and some have no geometry knowledge. Educators expressed that many of their learners cannot differentiate between an angle and a side. These challenges create confusion, disinterest and dislike for geometry.

Learner B2 stated that, "*My educator spends more time on algebra and very little time on geometry. She says that if we pass algebra well we don't have to worry about geometry."*

Some educators prefer teaching other sections of mathematics instead of geometry as they do not feel very confident in the teaching of this section. If too much of emphasis is placed on other sections, then insufficient work will be done in geometry and this will hamper the process to develop effective geometric thought.

Learners in both schools A and B also made it known that most of the time homework is given, but no feedback is provided the next day. The educator generally states that due to time constraints s/he is unable to mark the work. Again, learners alluded to working with their peers to correct homework which would be advantageous to getting clarity on their misunderstandings.

Learners in school A did not have much confidence in their educator's content knowledge of geometry. They expressed that the educator constantly made mistakes and sometimes gave incorrect answers. The educator also looked to the "brighter" learners in the class to provide some of the answers.

Learners in schools' B and C were confident with their educators' content knowledge, however indicated that there were topics that the educators struggled to teach.

Learner C2 stated that, *"My educator keeps referring to the textbook when he is teaching geometry proofs."*.

Learner A2 supported this by saying "*My educator does not have a clue as to what she is doing in geometry. For two years I have the same educator. She would start writing on the board, then run to her book check, and return to the board to continue. There are times when she could not answer a question and she would say 'do not worry it won't come out in the exam' or 'I will come back to you with the answer tomorrow."*"

Learners indicated that these responses made them lack confidence in the educator and they question the ability of the educator in the classroom. This aggravates the anxiety levels amongst the learners. If learners are able to notice that their educator lacks content knowledge on topics in geometry, then this creates disinterest in their learning. The educator is expected to be the knowledgeable other in the classroom and a lack thereof will limit the learning to take place. The role model in the classroom is the educator and a display of not being competent in the subject material, is discouraging to the learners.

Learner B4: "*Our educators always emphasize their own methods of teaching based on their interests*".
Learner A4 "*My educator places a lot of importance on book knowledge and gives us lots of exercises from the textbook but does not provide us with the answers. Most of the time we don't understand how to do it so we copy from each other to avoid getting scolded at.".*

Many learners were of the similar view that too much emphases are placed on textbooks and giving exercises from them The textbooks do not cater for real life examples and this makes learners lose interest in geometry. They also indicated that because they do not understand the work, they copy it. This causes the poor performance in assessments.

It was identified that educators may not have a thorough understanding of all topics in geometry. Educators rely on the textbooks to assist them in the classroom. More workshops that can assist with content of geometry was found to be necessary for educators that didnot come into teaching with geometric skills and knowledge. It was noted that many learners come into the secondary school with a gap in basic concepts and foundational skills needed to advance in geometry. This could be attributed to the non-specialization in primary school. The role of the mathematics educator is to facilitate learners 'thinking and learning, and should attempt to motivate learners to learn (Kosa, 2016). Educators' content knowledge of geometry is fundamental for teaching specialized concepts and proofs.

5.4.4 Methods of Teaching

Observation of the lessons, provided evidence that most educators followed the traditional method of lecture or question and answer, instead of the more modern methods. Three geometry lessons in one educator's class in each of the three schools were observed. Of the nine lessons observed only one lesson involved active participation. All other lessons observed involved the educator talking and the learners listening and copying notes from the chalkboard.

The questionnaires from the educators indicated the following: introduce the graph first

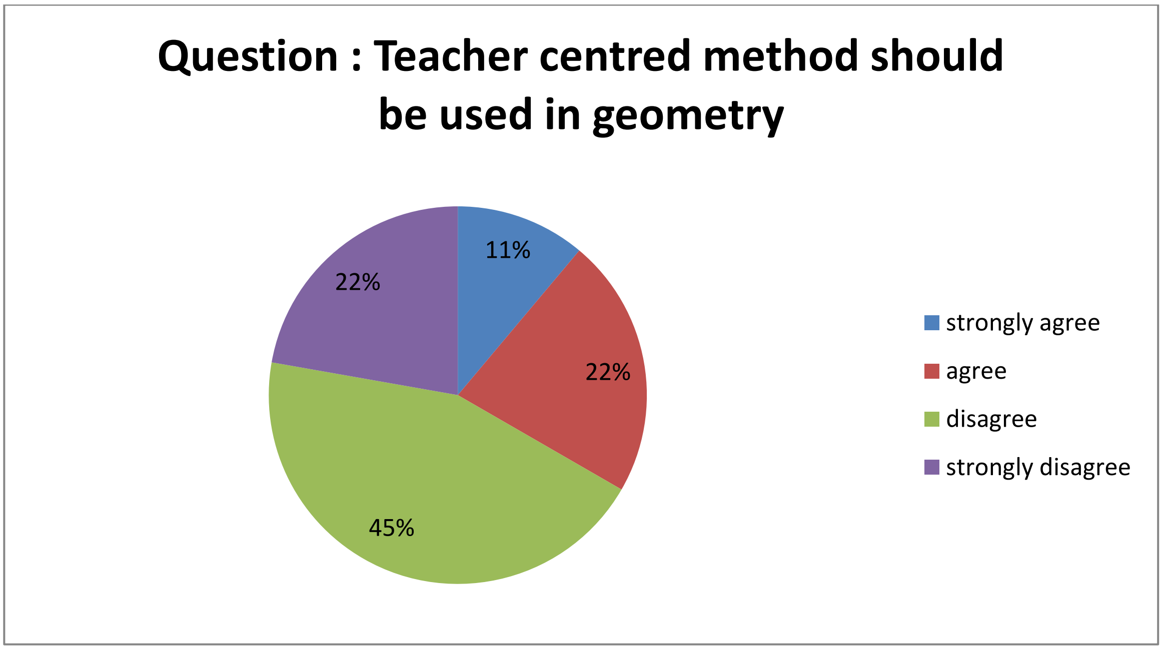

FIGURE 5.5: RESPONSE FROM EDUCATORS ON THE USE OF THE EDUCATOR-CENTRED METHOD

The questionnaire showed that 22% of the educators strongly disagreed and 45% disagreed that the educator-centred method is an appropriate method to be used in teaching geometry. However, this contradicted what was observed in the classroom (refer to the detailed lesson observation above). Most educators utilised the educator-centred method.

In the interview questions on educator methods, the following was revealed:

Educator AE1 indicated, "*I more often use the chalk and talk method. That is how I was instructed when I was in school. There is more order in the classroom.*"

Educator BE2 *"I prefer allowing my learners to explore and find solution themselves but time does not allow me do this, so I teach and they listen."*

Educator CE2 stated, *"My learners enjoy finding solutions to geometry problems and discussing their solutions. Learners sometimes come up with unique solutions and argue their solutions. For me this is learning."*
Educator CE2 further articulated, *"Passive learners struggle more than active learners. They do not actively look for and make connections between what they already know and what they are presently learning."*

In the focus group when learners were questioned on teaching strategies, a few learners articulated that some of their educators teach them well, but not all of them. They added that most educators teach everything first and lastly give them some questions as an exercise.

Learners A3 stated, "*My educator teaches for the entire hour and then when the siren sounds she gives us work on the exercise quickly before we go to the next class.*"

The learners shared that this affects their performance because they are not given practice problems in the classroom. They are also not given the opportunity to ask questions or clarify their understanding.
The three educators in school C believed that allowing learners to be passive recipients of knowledge creates problems when learners are presented with problem-solving situations, as they do not implement required strategies to solve problems. Some educators indicated that they make use of a combination of methods; however, it was not seen in the classroom interaction with learners.

Knowing and being trained as to what is the correct method of teaching, is different from implementing it in the classroom. The educator is a significant factor in the teaching and learning of geometry and if s/he is unable to utilize methods that will encourage thinking skills for geometric thought, it becomes a challenge.
When questioned about time, Educator CE2 reported,
"I don't have a problem since the time that they develop solutions they also learn new concepts on their own. Teaching with understanding helps in the long term and learners become more independent workers."

Some educators stated that they give learners investigation tasks before the introduction of the lesson. Two educators expressed that learners enjoy enquiring. They have very inquisitive minds. So inquiry based learning is useful to get learners motivated and interested in the learning of geometry.

Educator CE1 "*We must develop problem-solving skills in our learners if we want them to become thinkers.*"

Educator CE2 "*it is better to spend the extra time allowing learners to develop a good understanding of the concepts instead of rushing through the syllabus.*"

Four of the eight educators expressed that geometry allows for problem-solving skills to develop and that we need to nurture these skills. They also shared that communication and discussion allow these skills to develop.

Educator BE1 stated that, "*My learners did an investigation on cyclic quadrilaterals at home and when I did the teaching, it made understanding the concepts very easy.*"

Educators expressed frustration that not all learners completed their homework so they were not prepared for the lessons. Parents do not support us. They do not monitor what learners do at home. Most educators shared the sentiment that they understand that many parents do not have the capability to assist their learners in geometry but that should not be an excuse not to supervise the learners.

Five of the educators were of the opinion that learners possess a unique set of experiences and abilities that they bring into the learning environment. As maths educators, we must "tap into these experiences" and find creative ways to use them in the geometry class.

Educator AE3 stated that, *"Ignoring a learner's experience and cultural experiences in the geometry class will alienate the learner from the curriculum and reduce motivation."*

Many educators were of the opinion that this will make it difficult for them to interpret, comprehend and articulate their cultural backgrounds. They believed that maths educators become so caught up in curriculum matters that they forget that these learners come from a background with rich knowledge that can be utilised in the geometry class.

Educator BE1 stated that, "*the best way of teaching geometry is to show learners"*. Three educators agreed that the best way of teaching geometry is to bring shapes into the classroom and give learners the opportunity to discover the properties from experience. Educators also insinuated that bringing in the social settings and allowing learners to participate in these settings using their experiences, will allow learners to think and communicate, resulting in more effective and meaningful learning.

Educators articulated the need to ask leading questions. Three of the educators expressed that sometimes they force their learners to express themselves by asking them questions. Questioning is an important tool in teaching geometry and this technique allows educators to gauge if their learners understand what is being taught in the classroom.
Educator BE1 mentioned that, *"through interpreting my learners' comments, prompting and asking probing questions, I have an opportunity to identify my learners' misconceptions and misunderstandings in geometry thinking and learning*".

This was evident in the lesson observation where she asked learners to explain the properties of a cyclic quadrilateral. These questions allowed the learners to think and name the angles. A good question piques learners' interest and makes them wonder why, gets them to think and it motivates them to make connections with their previous knowledge.

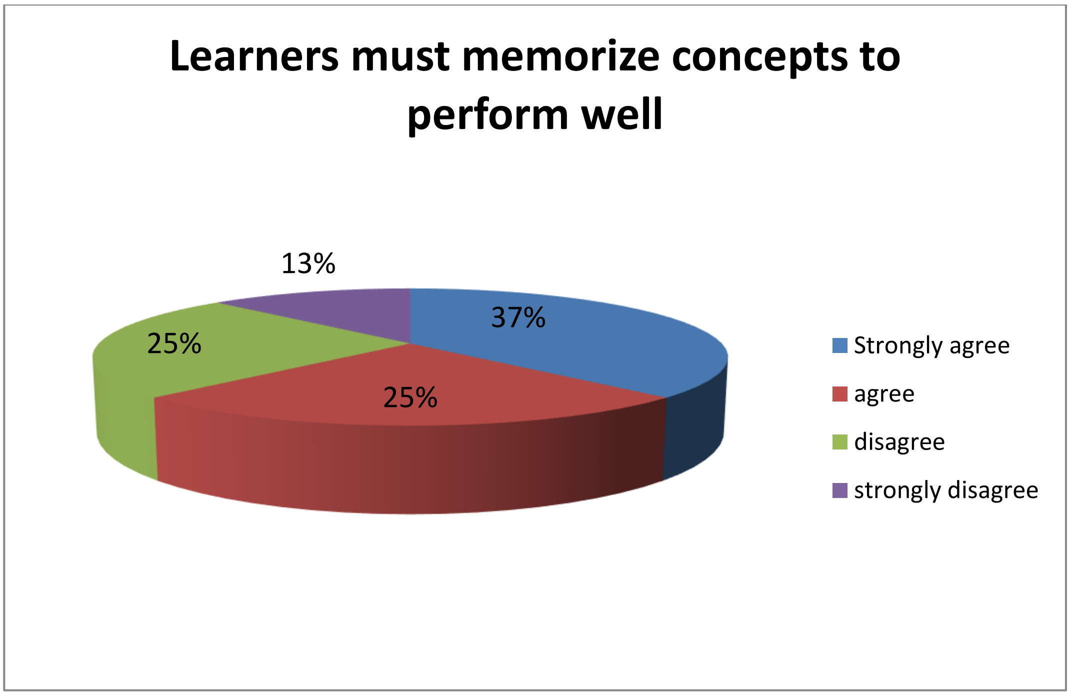

FIGURE 5.6 DIAGRAM TO REPRESENT VIEWS ON MEMORISATION

Five of the educators agreed that if learners memorize skills then they would perform better in geometry. In interviews, most of the educators highlighted the importance of memorization.

Educator BE1 "*my learners don't by- heart the concepts before the exams*"

Educator AE2 "*my learners need to memorize the concepts in geometry to do better*"

This was supported by the learners in the focus group discussion and questionnaire where they indicated that educators place great emphasis on memorization, as a tool of learning.

Learner A6 stated that, *"My educator encourages us to memorize the concepts in geometry."*

Learner B4 stated that, *"Our educator says we perform poorly because we do not learn off the proofs in geometry."*

Learner C2 stated that, *"Yes our educator also scolds us for not learning by heart."*.

Most learners stated that even if they learned by heart the concepts, they found that they did not perform well in the application examples. The learners indicated that when they answered the questions in the questionnaire, they based their answers on what the educator stressed in class. Most learners believed that memorisation does not help to improve their results in geometry. Learners alluded to the educator spending more time talking and giving those notes to memorize, then allowing them to solve problems independently so that they could identify their shortcomings and discuss it in the classroom.

Educators in school C stated that although memorization is important, understanding the concepts of geometry is far more important. They indicated that geometry is a follow-up from previous years and if learners understand the concepts well, they do not need to memorize them.

Educator CE1 said the following, *"It is important to know the learners in your class so you can adapt your teaching style accordingly."*
This was supported by educators CE2 and BE2.
Educator CE2 *"Learners are excited when they work in groups because they can talk about different ways to solve a problem. This encourages learners to find new solutions"*

Educator CE3 indicated, "*When separating learners into groups it is useful to place them in groups such that they can learn from each other. If all learners in the group have the same problems, then they will be not able to learn from other's experiences.*"

Educator CE1 stated that, "*if I do group work I keep the groups to a maximum of 4 learners. This prevents learners relaxing while others are working. "*
Educator AE3 indicated "*Ay, I don't do group work because the discipline in the class becomes unruly and no learning takes place."*

Educator CE2 indicated that he does not choose direct teaching strategies or the traditional approach to teaching geometry, but incorporated other strategies to help learners understand geometry.

CE2 stated that “when *I first started teaching I wanted my class to be quiet. I did not really entertain any form of discussion in my class. My learners worked passively. They came into the class, listened to me teaching and did the problems on their own. A year or two into my teaching of mathematics I started implementing more learner talk sessions and I could see the interest and excitement in my learners. I now see my learners arguing about solutions to problems and I know that they understand and can apply concepts. I enjoy watching my learners engaging and talking geometry.*”

According to educator CE1 “*Participatory learning is more beneficial to learners which involve actively taking part in discussions and collaboratively with one’s peers to achieve the same goal.”*

It was observed during the lesson presentation that educators AE1 and BE1 used only the traditional method of teaching such as lecturing when teaching Euclidean geometry. They focussed more on basic skills and procedure to prove the theorems rather than deeper understanding of the new concepts like tangent and cyclic quadrilateral. Jupri (2017) stated that in a traditional geometry class, learners do not experience the discovery of geometric theorems, nor invent any knowledge, but they are told definitions and theorems are assigned geometric problems and proofs.
Although CE2 did incorporate some group work, he didnot allow much discovery to take place. To improve the teaching and learning of Euclidean geometry mathematics educators are advised to create meaning classroom environment by using learner centred methods (In’am & Hajar, 2017). Through the learner observations of the three participants the learners in the class of educator CE2 displayed a better understanding of the concepts of geometry.

Effective classroom environment is critical for promoting learners ‘conceptual understanding of Euclidean geometry (Copur-Gencturk & Papakonstantinou, 2016). However, it must be noted that high school educators have a challenge to create an engaging mathematics learning environment (Ghousseini & Herbst, 2016). This was noted by the participants in the lesson observation and interview where geometry learning is not collaborative bit more educator-centred. Geometry lessons lend itself to a learner-centred environment where learners are actively involved in the construction of their knowledge.

5.5 STRATEGIES IDENTIFIED BY EDUCATORS TO OVERCOME BARRIERS

a) Motivation

In the semi-structured interviews, educators indicated the need for motivating learners to do geometry. The educators interviewed agreed that geometry must be made more interesting and fun for learners to be motivated and enjoy the subject. Comparisons of results from the past should not be brought into the classroom to de-motivate learners, as stated by three of the educators. Some statements expressed by educators:

Educator AE2 stated that "*sometimes we need to do teaching outside the confinement of the classroom walls"*.

Educators BE1," *it is necessary to take our learners on mathematics excursions and show them real examples of geometry use".*

Educator AE2 stated "*base line testing is very useful when learners enter the FET phase to gauge their foundational ability*."

Base line testing provides educators with an accurate assessment of the learner's competency levels in geometry. This will allow educator to plan, prepare and develop effective lessons for geometry. A need for motivating learners by including real-life example into the geometry curriculum.

b) Concept development and Vocabulary

From the focus group discussion educator expressed a need to develop geometric concepts. A need for building vocabulary in geometry was indicated as a measure to improve learner skills in proving.

Educator AE3 "*Geometry contain a rich list of words which learners need to know before they can prove riders*"

Educators CE4 "*There is a need to develop concepts from foundation phase"*

Educator BE3 "*concepts such as line and line segment, parallel and perpendicular, bisect and intersect are important for the development of geometry*."
In addition, Educator BE1 "*expressed the concept of interior and exterior angle, midpoint and bisect."*

Concepts form an integral part for proving theorems and solving riders. Misunderstandings of concepts will lead to serious challenges when proving riders and this can lead to frustration and anxiety among learners. It is therefore fundamental that learners get a good understanding of these concepts in the early stages of learning geometry.

c) Communication

Educators and learner interviewed indicated the need for communication both verbally and visually. Communication and the use of language between the educator and the learner are very important. Educators must be caring, friendly and create an environment for active participation in the geometry lessons.
Educator AE1 stated that "*A conducive learning environment that makes the learners comfortable and want to engage in the geometry lesson, must be created*."
Educator AE3 shared the same sentiments, "*learners must not be afraid to communicate in the classroom*".
Educator BE2 stated that "*the two- way communication between learner and educator develops thinking skills which are necessary for learning geometry*".
Educators CE2 indicated "*When learners ask questions do not scold them*."
Educator CE1 also stated that "*always use the appropriate language of geometry when communicating with learners*".
Educator BE1 supported this "L*earners model the educator. We must speak constantly in the language of instruction so that learners know and use the correct concepts*."
Educator AE2 "*Having a chart in the front of the class with geometry vocabulary is useful. Learners can use these concepts when communicating*."
Educator BE2 also stated that "*Having charts that contain geometry terminology with meanings will allow learners to master words for communication*."

Educators (CE1, CE2, and BE2) were of the opinion that the discourse in the classroom should be learner-centred so that the focus remains on the learners discussing geometry with each other rather than with the educator.

Educator CE2 "*Keep the communication learner-centred more than educator-centred.*"

By keeping the communication learner-centred the educator is able to observe much greater engagement and learning from the side of learners.

Communicating by using visual media was identified as a strategy that educators mentioned in the interviews. This was also expressed by learners in the focus group discussions. Self- access learning using technology or the internet can also be used to enhance the teaching of geometry. Four of the educators pointed out that there are interesting and effective geometry software that can be used in the classroom viz Sketchpad, Geogebra and Geometry Solver.

Educator BE1, "*learners now days are in the techno generation. They like to do things and explore on their own. Educators need to guide learners and ask them to explore and the learner will find the solution*".

Educator CE1 stated that "*we used LOGO for a while and it was very interesting and learners enjoyed it. It was a free maths tutor that was user friendly*."

LOGO was a programming language that used a turtle to draw geometric figures. It was a very enjoyable programme for learners.

Educator BE2 indicated, "*the use of technology supports learner-centred learning which is what we want to achieve in the classroom*".

The older educators expressed the need to be trained in the use of technology as they were not techno savvy.

CE2, CE1 and BE1 also expressed the need to include the use of many senses in the teaching of geometry, which will improve learners' thought. The ability to see, touch, feel and hear is important in learning geometry.

Educator CE2 "*Visualising, listening and feeling are all senses that I try to incorporate in my teaching by using technology.*"

Learner BE6: "*using technology will help us see and move figures to see different things happening to the figure*"

Communication in geometry is recognized as an important aspect of geometry learning. Geometry communication skills include the ability to express an idea orally, written and in the form of images. Communication in geometry will encourage learners' conceptual abilities. According to Lee (2015) communication helps learners develop mathematical thinking and problem-solving skills, and helps learners' correct misconceptions about concepts.

d) **Problem based learning and Inquiry based learning**

Although problem based learning was not noted in the lesson observation, focus group discussions and interviews indicated a need to involve the learners in the lessons. Educators indicated a need to create problem solving situations within the classroom to get learners to participate in their learning independently.

All educators placed a lot of importance on the need for more problem-based activities in the classroom and engaging investigative activities. Five of the eight educators highlighted that we must not place too much of emphasis on memorising concepts.

Educator AE2 stated that, "*I believe that learning by investigating and problems is better. I do not practice this all the time due class size.*"

Educator BE1 indicated, "*Learners like to be challenged by being given problem-solving questions. They enjoy the competitive spirit that is created to solve problems.*"

Educator CE3 "*We must give them problems that encourage thinking and communication and guide them through it*. "

Most educators believed that active participation in the content of geometry encourages better understanding and the ability for learners to remember these concepts without memorisation.

Educator CE1 stated that "*we stress that learners must memorise concepts because we want to save on time to complete the syllabus but learners do not understand and apply the concepts appropriately therefore they are unable to solve problems*".

Problem based learning helps learners explore and formulate their own knowledge. Chen (2019) describes PBL as a self-orientated model where learning in smaller groups is better to develop learners thinking and geometrical senses. Although problem based learning has its challenges in the implementation, learners understanding of geometry concepts can be achieved.

e) Questioning

The use of questioning was identified by educators in the interviews as an effective strategy to get learners actively involved in lessons and evoke effective geometric thought.

Educator CE2 stated that, "*questioning helps gauge a learner's prior knowledge*".

Educator AE3 indicated, "*I use questioning to measure if my learners understand the work.*"

Educator AE1 pointed out "*Questions help us guide learners towards a solution. When problems are given, instead of giving answers, questions can be used to guide learners*."

Educator BE1 indicated "*simple language must be used in the questions when relating to learning with language barriers*".

Some educators stated that learners do not understand the requirements of a question. Educator AE3 suggested the creation of a table with the meaning of words used in questions for example, define, describe, solve, hence calculate, show by proving, state.

These definitions will help the learner to understand the requirements.

Questions that evoke deeper thinking and critical analysis are key to improving geometry understanding. Appropriate questions will help learners and educators learn from each other. Educators use of effective questioning techniques will recognize learners misunderstanding and help them develop better understanding of definitions and concepts.

f) Real Life Application

All eight educators interviewed agreed that relating teaching and learning with real life applications could enhance in-depth learning. Three of the educators clarified that real life applications must be based on the learners' demographic background to attract learner attention.

Educator CE3 indicated "…*geometry is an area of mathematics that can be related to many careers like engineering, graphics designer, landscaper and aeroplane pilot. Geometry can even be related to the sports field. We must create examples to use these career interests to develop motivation and interest in the subject content.*" Educator BE1 stated that, "T*his requires a lot of planning as the textbooks that we use do not integrate real life experiences with topics in mathematics. Our textbooks still follow the traditional way doing a few examples and then providing the exercises.*"

Educator CE2 suggested, "...at *times learners can research how a topic applies to their career choice. This will also develop interest in wanting to learn".*

Some educators (CE2, CE1, and BE2) expressed that real life examples allow learners to relate their learning to the world and makes them want to learn more.

" *...learners can relate to a geometry problem better when it is based on a real life situations and this also makes their learning more interesting*".

To develop meaning and interest in the geometry lessons, educators 'should incorporate real life examples into their teaching and the exercises that they present to learners. Real-life examples assist learners to solidify their concept understanding and may assist them to apply these concepts with ease.

5.6 CONCLUSION

The data collected from the questionnaires, interviews and lesson observations was intended to explore barriers to effective geometric thought in the FET phase of selected secondary schools in the Umlazi District. The data was gathered through a qualitative method, which was carried out in three schools, with eight educators and eighteen learners. The data collected provided rich information to answer the research questions. The main themes that were identified from the data collected were engaging learners in the geometry class, language and teaching of geometry. Geometry concepts and terminology are extremely important to develop learners to solve theorem and prove riders. Basic terminology and foundational skills direct geometry learning and the success of effective geometric thought.

Chapter Six: Findings and Recommendations

6.1 INTRODUCTION

Chapter five provided a detailed presentation of data. The research questions were used to sift the data, a sense making process of the participant's words. Major themes for each question were identified based on the data collected. The theoretical framework of Sfard's (2018) theory based on the two metaphors of learning which are acquisition and participation was employed as the broader theoretical backdrop against which interpretation was conducted. Theories under constructivism that is, Vygotsky (1978), and commognition theory of Sfard (2017) was used. Vygotsky argued that social learning preceded cognitive development. Sfard (2018) emphasized the need for both metaphors of learning to be considered in teaching and learning for effective thinking to take place. This study considered the above theories in the analysing the data and in the discussion of the findings.

In this chapter, a discussion of the key findings will be presented in relation to the themes of the research questions. The literature review, theoretical framework and responses are considered in the findings.

6.2 DISCUSSION OF THE FINDINGS

Barriers are the challenges that are experienced in the learning and teaching process that affect the effective geometric thought. Chapter two provided different philosophies on barriers that are experienced by educators and learners. This study investigated learners, educators and systemic barriers. The lesson observation was used as the main source of data which was supported by the interviews and focus group discussions. Twelve of the eighteen learners endorsed that geometry was a difficult subject and they experienced challenges which limit effective geometric learning. Table 5.1 displayed that many learners disliked learning geometry. The pedagogical findings were investigated in terms of the educators and their challenges in the teaching of geometry. This study considered the barriers that educators experienced in achieving effective geometric thought. The educator is the key agent in implementing the curriculum at school and therefore the educator's subject knowledge, training, motivation, beliefs, language and expectations influence the effectiveness of implementing the curriculum in the classroom. Findings of this study provided answers to the research questions.

6.2.1 An Engaging Classroom Environment

A constructivist classroom is one that allows for social interaction. It encourages verbal interactions between the learner and the educator. It was evident from the responses of the participants that the learning environment in some of the schools is not conducive, with respect to the class size, facilities and infrastructure. A classroom that is overcrowded, with a shortage of resources will not support the view proposed by the constructivist theory. The class size is determined by the post-provisioning norm set by the DBE. Although the learner-educator ratio is averaged at 1:37 this is not always possible to implement in schools. All schools participating in this study had mathematics class sizes of more than 37 learners. Some schools had the advantage of employing additional educators using the school fund and this reduced the class sizes. In the FET phase weighting of subjects are used to determine the number of educators allocated to schools. Mathematics is not given the same weighting like other practical subjects yet geometry requires a lot of practical work. This study found that smaller classes offer the following advantages:

- Allows learners more individual time with the educator;
- Enables more learner-centred teaching methods to be implemented in the classroom
- Increases learning time as there are fewer discipline issues;
- Reduces non-productive contact time.
- Allows for greater communication and participation.
- Allows for effective engagement of learners.

As articulated by one of the educators, the learners that are capable and are grasping concepts manage in the larger classes. There are topics in geometry that lends itself teaching in larger classes like identification of shapes, supplementary angles, complementary angles and vertically opposite angles. However, when it comes to aspects of proving riders, theorems where learners need to work collaboratively to reason and deduce with the assistance of the educator (MKO)

then it becomes challenging to take the learners individual needs and abilities in large class sizes. The average results as indicated by the educators show that a majority of the learners are not performing well in geometry, so smaller class sizes will be an advantage to them. Learners that are coming into the class with barriers like language will benefit from smaller classes. It was evident that the advantage of smaller classes provides the ability to engage in participative methods of teaching. The use of technology that allows for teaching involving realistic situations with visualization will be more effective in smaller class sizes. To allow for individual use of computer equipment the class size must be small. According to Rockoff (2004) communication in small classes is effective as learners are able to listen to each other and greater opportunities are created for thinking and solving problems.
Class size has implications for physical resources and marking of tasks. As communicated by some of the educators, they have large classes and insufficient desks and textbooks to accommodate all learners in the classroom. The larger classes have also made the learners uncomfortable in terms of physical space. The learner: educator ratio has added much pressure on educators to create opportunities for effective teaching and learning. These ratios encourage the use of teaching strategies that are not desirable for learning with understanding.

In this study educators and learners clearly stated that smaller classes will help in identifying learners' barriers and that this will improve geometry performance. The learning environment may not be directly related to how learners think however the learners' comfort has an impact on their performance in geometry and the opportunities that are created to effect thought processing activities. Verbal, visual and interactive communication, as articulated by learners, is certainly a way of encouraging effective thinking and clearly presents as a challenge in larger classes. According to Leung, Baccaglini-Frank and Mariotti (2003) an unfavourable learning environment is a barrier to learners achieving the desired results in geometry. Some classrooms lack basic facilities such as desks and chairs. This has a negative impact on the learning process. Geometry requires a classroom with space for learners to investigate, draw and make inferences. From the lesson observation where learners were required to measure angles, the educator could not provide much support to learners that were struggling due to space and the number of learners in the classroom. A crowded classroom with very little space creates chaos and unruliness and prevents learning. A congested classroom lacks discipline and prevents educators from catering for diversity. Individual needs are not given attention. This has serious implications in the learning process of geometry. The classroom size also dictated the methodology that educators employed in their teaching. From the data, it was found that due to the large number of learners per class, educators employed the educator-centred approach. Lessons on circle geometry require learners to work independently to form deductions. Educators were of the belief that the learner-centred method will take up too much time and create a chaotic atmosphere. The lesson observation also confirmed that when the learner-centred method was used in a large class size, it created unruliness. Educators experienced problems maintaining supervision and control of the class.

Participation in lessons became limited due to adverse classroom conditions. Engaging learners in activities is restricted as educators are unable to support learners while they are working with these large numbers. A constructivist classroom should be one where learners are actively involved in discussion, communication, drawing, measuring, contrasting, visualising and classifying objects. This cannot be achieved in large classes. Space in the classroom is necessary for learners to engage in the above activities. To involve learners in communication and discussion in large classes becomes very challenging as all learners will not get opportunities to participate effectively in the learning process. Since not all learners achieve outcomes at the same time rate, to provide individual attention to get learners to acquire new knowledge, is difficult in large classes.

A lack of adequate resources like textbooks has a negative influence on learning geometry. Learners need textbooks to complete homework and to gain more practice with problem-solving. There is also a need for learners to read and develop prior knowledge before they enter the classroom. This is certainly difficult to do without a textbook. Every learner should have a mathematics textbook to practice geometry problems. Some schools were very disadvantaged in terms of resources and this created a deprived learning environment for the learners in the geometry class. In some classrooms learners shared desks, which seriously hampered the learning process in the geometry class where learners are expected to measure angles, draw figures and interpret. A lack of essential resources such as desks, chairs and textbooks does not allow for effective acquisition of knowledge. An ideal learning classroom is one that supports the principles of acquisition and participation. It is a classroom that allows for laughter, imagination, engagement, visualization and creativity, to discover new information. A large classroom presents the challenge of organizing learners for thought processing activities, like communication and discussion.

An important finding from the geometry lessons observed was the need for learners to visualize figures, shapes and diagrams. For example, the teaching of circle geometry, tangents and cyclic quadrilaterals can be effectively done using Sketchpad and GeoGebra software where the properties can be demonstrated. Visual mediators are an important artefact in the theory of commognition which supports a participative learning environment Sfard (2008). Learners emphasized the need to visualize what they were learning. Some learners have limited resources available to them and have never encountered some of the figures and shapes that are discussed in the classroom. The use of only the chalkboard and textbook was noted as limited resources in terms of teaching aids. As seen in school C, where the educator improvised and brought real life experiences into the classroom to discuss circle geometry, the learners understood. Learners were able to link words such as angle at the centre, angle at the circumference, tangent at the point of contact and identifying angle in the alternate segment, to the visual images and to talk about what they were able to see. This led to them being able to tackle the problems with ease. Visualizing allows for mental images to develop and becomes a vital tool for communicating both verbally and non-verbally (Leung, 2008). Communication is essential for thinking and learning. A lack of being able to visualize will therefore create barriers to learning geometry. This study found that when learners are able to see and visualize what they are learning, they are able to make connections with their own experiences and link prior knowledge, thus leading to more meaningful learning. The concept of visualization supports higher levels of engagement by learners. Reasoning and visualization go together. What the learner can see he can also cognitively reason about, therefore visualization is an important tool for spatial ability. Visualization also provides the opportunity for learners to communicate what they see based on the mental images they have developed. Thinking is encouraged when learners link mental and visual images. This form of communication allowed learners to develop problem-solving skills. Concepts are developed from these concrete examples. Learners are able to communicate based on the mental images that they have developed, however not all learners get an opportunity to have sufficient experiences. It was evident in this study from the learners' responses that there was a need for visual images to be brought into the classroom so they can see what they are learning. Using a data projector and whiteboard proved very successful in teaching geometry and getting important concepts across to the learners. The use of the smartborad is also a useful tool to visualize concepts, interior and exterior angles of triangle as well as cyclic quadrilateral, of geometry. Participants also identified the use of technology as an important tool that supports visualization of objects. Technology allows learners to see examples in real life situations and brings forth the relevance of geometry in the real world. The use of computers can create interactive animations allowing learners to make their own explorations. An under- resourced, large classroom will not support the effective use of tools that allow visualization.

In the lesson observed, the use of different colours on the diagram also sparked learners' interesting the lesson. There are various cost-effective ways that can be utilized to spark interest and motivate learners in the geometry class. These could be but not limited to, wall paper, animation, whiteboard and prepared worksheets with diagrams. Creating an engaging and participative environment was noted as a strategy to pique learners' thinking and interest in the geometry lesson.

6.2.2 Meaningful Learning

Wilson (2012) encourages creating meaningful situations for learning geometry to allow learners to see the relevance of their learning A lack thereof will create disinterest and a negative attitude towards the learning of geometry content. A learner that is disinterested and de-motivated in geometry will not be able to sufficiently acquire the content of geometry. Developing a positive attitude should be the starting point in developing effective geometric thought. It was noted in this study that many learners move away from mathematics into mathematical literacy due to feelings of disinterest and de-motivation. Researchers (Lui and Bonner, 2016) articulated that there is significant relationship between interest and achievement in mathematics. Learners' responses supported the above view. It was found from the educators' comments that a positive attitude improves learners' performance in geometry while a negative attitude hampers their performance.

It was evident from the collected data that many learners had a negative attitude towards the learning of geometry. Their negative attitude towards the subject is intensified by the poor results in mathematics, especially geometry, and the level of difficulty of the content of geometry. Learners used their past experiences and results to judge the difficulty of geometry content. This attitude was reinforced when after learning; they still do not perform well in geometry. Learners with a positive attitude and interest in geometry perform better Verschaffel & Greer, 2013). As evident from the data collected many of the learners had a disinterested, negative attitude which contributed to the poor results. A negative attitude by the educator heightens learners' negative or disinterested feelings on the subject. Educators highlighting the difficulty of the subject or professing that geometry is only for the talented learners, creates lack of enthusiasm to want to learn and engage in the subject content. Learners must be able to see the purpose behind doing the subject for them to change their attitude towards the subject. Real life examples and experiences must be brought into the classroom to increase the levels of interest and motivation in the content of geometry. It was evident from the educator's' responses that learners must see the connection between geometry and real-life situations, to become motivated. Knowing the properties of shapes and applying it to real situations will allow learners to see the meaning of geometry and develop a positive attitude. According to Van Oers (2019) the ultimate goal of any subject content lies in the relevance it has when the learner exits the schooling system. Learners will be able to enjoy learning geometry when they are able to connect the formal knowledge that is gained in the classroom to activities that are outside the classroom. Learners responses of "*where do we use geometry*?" or "*why do we need to learn geometry*?" shows their need to want to see the relevance of geometry. The concepts of geometry appear everywhere however many educators do not bring it into the classroom. Some simple examples named by educators are the patterns on wallpaper, computer games, animation, architecture and landscaping. These are examples that include geometric concepts that can be brought into the classroom so that learners can see the relevance and become more interested in the content. Allowing learners to communicate their experiences from real situations will bring relevance to content being taught in the classroom and this will motivate and create interest in learners. Figures, shapes are noticed by learners daily, however learners experience challenges when concluding properties of these figures. It is the properties that are needed to solve geometric riders and theorems. Using the basic geometric skills of measuring objects in real life will allow learners to develop a better understanding of these concepts. Learners will also see the relevance of geometry in doing this type of activities. Proofs require reasoning and critical thinking therefore a positive outlook towards geometry will encourage these skills.

The participation metaphor emphasizes the need to provide meaning and relevance for the learner by effective participation in the learning process (Sfard, 2008). A learner's attitude towards geometry will change if s/he is able to draw a link between the outside world and the classroom environment. A learner that is participating by communicating his experiences in the classroom will be able to think at a more advanced level and develop new knowledge in the process. The situated learning theory (Lave and Wenger, 1991) speaks about learning within situations. The relevance of learning geometry will be seen by learners when they link their experiences from outside the classroom to inside the classroom. This will create a sense of excitement, fun and motivation to learn geometry and thus effective thinking will ensue from this practice. With the advancement of technology almost every situation can be brought into the classroom to expose learners to real situations and bring foster relevance and interest into the classroom.
Verschaffel and Greer (2013) express that learning in isolation for the purpose of syllabus coverage solely, is meaningless learning. This study has shown that many of the educators rushed through their teaching to complete the syllabus without getting learners to understand and think about the content. The instability that arose from changes in the curriculum also led to anxiety and de-motivation amongst educators and learners. The continuous changes in the curriculum prevented learners from getting assistance from family members and friends outside of school. The curriculum must be meaningful to learners for them to interact with it. There must be a link between the curriculum and real life experiences. These experiences must be specific to the learners so that they are able to relate to them.

The geometry curriculum is so compartmentalised that it does not allow for situations that include many concepts to be brought into the classroom. As such, learners are unable to see the relevance of learning geometry. The current curriculum and resources available do not provide sufficient opportunities for learners to visualize their learning so that they are able to communicate about what they are seeing and in effect, start thinking about new ideas. It is through engaging learners in a mathematical discourse that allows for them to link different topics of geometry and situations from life (Sfard, 2008).

Educators' attitudes toward the subject affects the learners' performance in geometry. A positive attitude, i.e. expressing a liking for the subject, influences the way the learners look at the content of geometry. According to Beilock and Willingham (2014) when educator's' attitudes provide greater motivation and interest in the subject, this will improve the performance of geometry. An educator's positive attitude will encourage effective thinking in the learning of geometry. Grading problems based on levels of difficulty will help learners to answer simple questions and create the interest in wanting to answer more questions. Learners' comments that, "the educator does the simple ones in the class and then there is no one to guide us at home", result in the learners becoming frustrated. Learners become de- motivated when they are unable to complete their homework and in most cases do not have a more knowledgeable other to provide assistance. It is important for educators to grade the exercises that learners are doing on their own (Beilock & Willingham, 2014). The fear of failing to solve geometry problems forces learners not to do homework. Educators must allocate time in the classroom to work on problems and guide them through these problems. Creating situations for effective learning and thinking in the geometry classroom brings meaningful experiences into the lives of learners and keeps them interested and motivated to want to continue learning.

6.2.3 Language and Communication

The language of instruction and effective communication was identified by learners and educators as a barrier to effective teaching and learning of geometry. Effective communication and thinking according to the participationists view go hand in hand. In this study it is evident from the learners' view that when given the opportunity to communicate it allows them to think and develop new ideas and knowledge. It was evident from table 5.1 that the home language of many of the learners is not English however; the medium of instruction for geometry is English. In South Africa, we have eleven official languages. Although it is acceptable for learners to be taught in their mother tongue in the foundation phase, when they get to the senior phase the language of instruction is either English or Afrikaans. Mathematics is examined in English or Afrikaans. Gafoor and Kurukkan (2015) state that the teaching of mathematics, especially geometry in the lower grades creates many challenges in the senior phase as learners are unable to map their understanding of concepts correctly.

Warren and Miller (2015) emphasised the vital role language plays in the acquisition of geometric knowledge. Active communication in geometric lessons encouraged learners to become active participants and these motivated learners to think, to apply themselves and very importantly, enjoy their lessons. Engaging learners by allowing them to use language to communicate in the geometry classroom provoked thinking. In the focus group learners articulated that their participation in the lessons was limited and that educators did most of the talking. It was clear that it was not only the barrier to English usage, but the limited communication in the classroom that contributed to their poor results. As stated by Vygotsky (1978) it is through interactions that a child develops language as well as logical thinking. Geometry is a language on its own as expressed by educators, and a barrier in the language of instruction would indicate that the learner is now facing a double challenge. Vygotsky (1978) highlighted the important role that language and symbols play in concept development. In this study, both learners and educators also emphasized the role language plays in teaching and learning. Language both in its written and verbal form is of critical importance to the learning of geometry. Language is a fundamental tool for educators to assess levels of learners' thinking by the key words they use to describe geometric concepts.

Incorrect use of words, as seen in the lesson observation, leads to misconceptions when solving geometric problems. Bloor & Bloor (2007) advocate that progression in geometric reasoning requires appropriate instruction, involves assigning key words to visual mediators and developing narratives to define shapes. Since learners view the prospect of proving through the lens of their existing geometrical knowledge, educators' attempts at communication would result in sense- making and allow for different meaning to emerge. For example, questions such as "Which lines are parallel", which straights line intersect? and What facts do we know when lines that are parallel? These questions will essentially allow learners to refresh their knowledge of geometrical properties and relations. A rich repertoire of geometric vocabulary, concepts, definitions, and theorems will assist learners understand and carry out proving activities with ease. (parallel lines, transversal, alternate angles, corresponding angles, complementary, supplementary, vertically opposite, exterior, interior, chord, diameter, radius, cyclic and tangent are some of the geometric concepts that are commonly used in grade 11 circle geometry)

The framework for mathematical discourse developed by Sfard (2008) identified interrelated elements of key words, visual mediators, narratives and routines. A break in the links between each of the elements creates misconceptions. As noted by educators and learners, teaching rules and proofs only, without understanding, makes the learning meaningless. An over-emphasis of the rote learning method was evident by most of the educators in this study and this could be a contributing factor to the poor results. Sfard (2008) considered cognitive processes and interpersonal communication as facets of the same phenomenon and viewed thinking as communication. The lack of getting learners actively involved in communicating their understanding of geometric concepts discouraged learners from thinking. Sfard (2008) described mathematics as a discourse i.e. as a tool of communication. She spoke of "talking" to oneself and solving problems. In this study the lack of key words and of developing narratives presented a barrier to learners and educators.

Talking about geometric objects and relationships, changes a learner's thinking. This change in mathematical discourse develops new knowledge. This study found that language plays a significant role in how learners understand and learn geometry. Communication using the appropriate language to match geometric concepts, certainly allows for thinking and learning taking place.

If a learner is experiencing this barrier in the medium of instruction, then the barrier is already created for learning geometry. It may not be the content or the inability to solve problems, but the inability to comprehend, communicate and understand the vocabulary of the language of instruction, which is English. It was evident from the learners' responses that vocabulary and understanding concepts was also a problem. Educators did not always speak in the language of instruction and some taught in IsiZulu, which added to the challenges that learners were experiencing. Greater engagement in the language of instruction will enhance the learners' ability to understand geometric concepts.

As expressed by Sfard (2008), word use refers to mathematical vocabulary, which forms an important part of communication. In this study learners expressed that they only saw certain concepts and words in the examination which presented challenges since they did not understand the meaning. It must be noted that in secondary schools, researchers found that teaching of geometry starts at a deductive level, which is foreign to learners. At this level learners are expected to apply reasoning skills according to the Van Hiele's theory. This study found that many learners enter the secondary phase not functioning at the appropriate level which could be attributed to their poor reasoning and thinking skills. It is therefore important for educators to identify the level that learners are functioning at, and to use the necessary interventions to raise them to the expected level of functioning. It is with the use of appropriate language that this can be achieved. It was shared by an educator that a base line assessment is useful in ascertaining the level of competency of the learners. Language does not mean a list of vocabulary words but rather the communicative participation in a mathematical discourse. When learners express themselves educators will be able to assess their level of understanding. This important role of active communication is summed up by one of the participant's responses;

It was noted in the lesson observation and responses from both learners and educators that there is insufficient engagement of learners in the form of communication. Learners articulated that if they are given more opportunity to engage in their learning by communicating in the language of geometry they will be able to master geometric concepts and get a better understanding of geometry. Rote learning of concepts led to many learners not grasping concepts properly. There is too much emphasis on narratives in isolation. Sfard (2008) highlighted the need to form associations with key words, routines and visual mediators or it will have meaning to the learners. As seen in the lesson observation, the emphasis was on the completion of the syllabus of set rules, proofs and theorems with very little communication to link the learning.

As articulated by both constructivist theorists, Piaget and Vygotsky, acquisition of knowledge is the responsibility of the learner. A learner that is actively involved in acquiring knowledge will not need to memorise concepts as this learner will connect their own experiences to acquiring new knowledge. The learner's acquisition of knowledge will scaffold from what he knows to what he does not know. It was expressed by the constructivist theory that interacting with learners by using questioning and discussion techniques, learners are able to formulate relationships and connect their learning to previous knowledge. This type of learning is more meaningful and relevant. It was evident that questioning piqued learners' thinking and communication. Questioning led to active discussion as noted in the lesson observation and this resulted in learners gaining a greater understanding of the concepts.

It is a reality that in South Africa a large number of learners receive their teaching in a language, which is not their mother tongue. A subject like mathematics poses a major challenge with its highly specialised terminology. Geometry information is available in English, which is considered a language of the wider community. It is only through regular practice of the terminology associated with geometry can learner's master geometric terminology and improves their performance. Vygotsky's theory also expresses the importance of language in the learning process. When learners engage in their learning by talking geometry, it changes their thinking and through these changes in learners' discourse they modify and extend their geometric discourse.

It was found from this study that many primary schools are still using the mother tongue as the language of teaching and learning. These learners come into the senior phase with poor foundational terminology and vocabulary in English and thus experience challenges when communicating and expressing themselves in the classroom. This barrier was highlighted by learners when reading and comprehending geometric problems. As indicated by educators, the solution to geometry problems lies in reading the problem carefully and interpreting this information. Without adequate language skills, this poses a major challenge to the learners. When a leaner comes into the senior phase without a proper language background it creates a barrier to scaffolding from what the child knows, to grasping new information. According to the Van Hiele's levels of understanding, a learner needs to enter the senior phase at a certain level and if the language barrier exists then this learner will not be at a level to comprehend and interpret geometric problems. Being able to communicate at different levels is one of the key principles underpinning Van Hiele's theory. Vygotsky described cognitive development as a function of verbal interactions that exists between the learner and the adult. A challenge in language will mean a break in communication between the educator and learner, and will inhibit the learner's cognitive development. This results in impeding the learning of geometry and therefore poorer results. As learners progress in geometric thought they exhibit a different level of language and communication than previously. From the data collected it was evident that educators did not provide adequate opportunities for learners to communicate in the classroom and this impacted negatively on their ability to think and understand certain geometric concepts.

It was also identified that the barrier to learning in the appropriate language of instruction is also created by systemic barriers. The following are some of the systemic factors that have been identified from the learners' and educators' responses with regard to language as a barrier to learning geometry. Learners are taught in their mother tongue in the lower grades and come into the senior phase with a lack of proper use of English. Proper geometric vocabulary is not mastered in the lower grades resulting in ineffective communication which retards thinking and solving of problems. Educators articulated that not all geometric words can be directly translated into IsiZulu and retain the exact meaning. This was noted in the lesson observation where learners experienced problems while explaining simple geometric terms. According to Kishindo and Kazima (2004) geometry is regarded as a language on its own and trying to master two languages in the senior phase poses major challenges. The learner in the secondary phase should be at level three as stated by Van Hiele. The learners should be able to understand and form abstract definitions that distinguish between necessary and sufficient conditions for a concept and understand relationships between different shapes. This might be the reason that learners grapple with thinking at a higher level as they have not been given the opportunity to engage in effective discourse and to think and operate at an advanced level. If the learner is grappling with language and vocabulary, then this level will not develop because the levels prior to this level would not have been developed. According to Planas and Setati (2009) educators often tend to do examples that are simple which do not provide enough cognitive stimulation. This does not expose learners to problem-solving examples and their thinking skills are not aroused, since problem-solving questions require more comprehension and understanding of geometric vocabulary. The syntax of geometry is more complex than other learning areas and requires greater language skills. Grappling with language and geometry simultaneously requires more time allocation for the teaching. There is a clear need for English to be the medium of instruction in the lower grades when teaching mathematics so that the necessary foundational terminology will be mastered with the correct language usage and understanding.

Communication is the key to solving geometry problems and improving the results. There is therefore a need for this practice to start from the lower grades. Communication allows for in-depth thinking to take place so that learners develop an understanding of geometric skills in the process. It was evident that for effective thinking both the learner and educator must be able to understand each other. Communication brings relevance and meaning to the learning process and is a key factor in thinking and making sense of geometry. As such, a language barrier presents as a challenge to many learners. It is evident from learners’ responses that a way to overcome this challenge is seeing and for them to experience meaning to their learning. Using the appropriate language of instruction and with constant communication in the classroom based on what they have experienced and can see, will allow learners to become more proficient in the language of geometry.

Engaging learners in a discourse does not necessarily mean getting learners to express themselves verbally. It can also be in writing or engaging with objects and other artefacts. This study revealed that allowing learners to engage in visual artefacts using technology encourages them to communicate on what they are able to see. Being able to participate in the learning process in the form of seeing, communicating and thinking will make learning geometry more effective. Written communication is important for the understanding of geometric concepts. Learners are able to express their inner thoughts on paper, see their mistakes, and learn from them. Communication in groups and participation in the lessons allows learners to learn from each other. Thinking and communicating are activities that the learner engages in to promote learning.

It was evident from the educators' responses that the time allocated to teaching of geometry, was not sufficient. The geometry syllabus is given approximately 12 hours in each grade that is grades 10, 11 and 12 and this is not sufficient time to teach with understanding. Learners, especially those experiencing language barriers need more time to grasp concepts and understand the language of geometry to tackle problems adequately. It was found that educators select "less cognitively demanding tasks" for learners due to language barriers. This disadvantages learners as they are not exposed to problem-solving examples that are more challenging and are often examined. Learners therefore encounter these examples only in the examinations and therefore fail them or do not attempt these problems.

6.2.4 Experience and Foundational Knowledge

Learners who had a lack of prior knowledge did not want to learn and could not get success in the further level. It is one of the factors that could contribute to the failure in mathematics at the secondary school level. Learners are unable to relate previously learnt mathematical knowledge to new geometry concepts. Educators indicated that when questioned on previous knowledge, learners were generally unaware of the geometry concepts discussed. The foundational knowledge that learners acquire in the lower grades in geometry is a key factor that determines the good performance of high school learners. Theorems are foundation of geometry. However, learners are encouraged to memorize these theorems rather than identify what theorem fits into a particular situation. Geometry knowledge learnt in this way is limited and cannot be applied effectively. Simple foundational concept can be drawn from previous knowledge but some more complicated theorems require learners to explore and be creative. According to this study learners have misconceptions on some foundational concepts such as parallel lines, exterior angle of a triangle, use of congruency, the difference between similar triangles and congruent triangles. Some learners sometimes confused the concept of angle and side. Learners were requested a measure an angle and a response was “angle A = 5cm”. This is a simple concept that is learnt in primary schools yet learners come into the secondary school with this misconception. In circle geometry basic foundational concepts of radius, chord, quadrilateral, lines, circumference and diameter are some of the essential concepts to build on to develop mastery in the theorem. For example, teaching the theorem on “line drawn from the centre..............perpendicular to the chord..........................”, simple concepts of perpendicular, radii are equal, congruency must be master this theorem and apply it riders.

As stated by Vygotsky (1978), previous knowledge must be used to gain new knowledge which will not be possible in the absence of previous knowledge. A learner must progress through the Van Hiele levels sequentially to achieve success in geometry. Not having the necessary foundational knowledge prevents learners from participating in the classroom discussion out of fear, and this exacerbates the negative attitude that learners develop towards learning geometry. There must be active dialogue between the educator and learner for the successful accomplishment of teaching of geometry. Creating a positive attitude and developing interest in geometry will enhance the levels of motivation in learners to want to learn geometry. Vygotsky’s (1978) theory talks about the relevance of foundational development for effective geometry. This foundational knowledge is what learners acquire through their daily experiences. Many learners expressed “I just don’t get it”, showing frustration due to poor concept formation in the lower grades. This creates fear, anxiety and dislike for the subject matter. These emotions de-motivate learners and inhibit acquisition and understanding of geometric concepts. Learners are unable to think and develop new concepts based on previously learnt concepts if they did not acquire these concepts in the lower grades. Proofs are fundamental to learning geometry and proofing is perceived to be complex so educators are advised to assist learners to develop theses processes in their early grades (Hannafin et al., 2009). The concept of the zone of proximal development can only be implemented effectively if the learner has the foundational knowledge, so he is able to acquire higher mental functions through interaction with the more knowledgeable other.

6.2.5 Acquiring Knowledge using Participative Methods

It was clear from the observation of the lessons that many educators used the educator-centred methods for teaching geometry. Most educators spent 80% of the time teaching by talking. The only form of participation from the learners was written communication that is, through the exercises given at the end of the lesson. When work is given, there is very little guidance from educators as it is generally given/done at the end of class time. The examples done on the board are also very simple examples and are generally the first ones from the textbook exercise. Memorization is encouraged without understanding concepts. This presents challenges as learners are not given opportunities to think and develop their own knowledge. Slower learners tend to be forgotten in some classes as educators connect with those learners that are keeping up with them. This again creates disinterest in geometry and a block to effective thinking. The use of the learner-centred method where learners participate in their learning is more meaningful and encourages thinking. Communication of one's thoughts, experiences and understandings is important in the geometry classroom. Thinking is a form of communication and communication is active participation in the learning process.

From the lesson observation most lessons were conducted in using ritualized instruction techniques and encouraged memorization. This form of instruction does not support learning with meaning, while a geometric discourse will support active participation, thinking and development of new knowledge. For example, the educator can take leadership in a mathematical discourse by demonstrating the use of appropriate geometric definitions in a way that enhances direct visual recognition in appropriate situations. The utilization of a mathematical discourse in the teaching of geometry allows the educator to modify and change learners' thinking. Educators can expose learners to situations in which their discourses will prove insufficient for example when learner states that the angle subtended at the centre of a circle is equal to twice the angle in the alternate segment and support them in a process of understanding the advantages of new ways of doing things for example clarifying the statement by communicating that the angle at the centre of the circle is twice the angle at the circumference of the circle (the alternate segment is not necessarily at the circumference).

The method that the educator adopts in the geometry classroom affects the way learners’ think and perform in the classroom. Developing skills and maintaining the learners’ interest is very important for effective thinking of geometric problems and for improving results (Adler & Pillay, 2016). The use of different educator methods to stimulate thinking and understanding is encouraged by researchers. In classrooms where educators utilised different methods of teaching, it was noted that learners were motivated and interested in the content as seen in lesson two of educator C2. The principles of both acquisition and participation are important in the method employed by the educator, to create an ideal classroom for effective geometric thought.

The kind of interaction between the learners and educators, and learners and learners, are strongly influenced by the methodology used in the classroom. The learner-centred method emerged as an area that educators needed support with. As much as educators are aware that the learner-centred method is a method that will help to improve the performance in geometry, they do not implement it in their teaching. In this study it is noted that the method of memorisation and educator-centred methods are not providing much success in the classroom. It was articulated by many learners that despite them studying for geometry, they are unable to perform well. This is due to the methods of teaching where learning takes place without understanding. Mathematics educators were accustomed to the chalk and talk method and belief that drilling concepts in geometry will yield the best results. Teaching should not be limited to this method since it often leads to learning without meaning. Hands on activities, problem-solving methods and inquiry-based learning, are some strategies that are required for motivating and developing interest in geometry. The theory advocated by Vygotskys (1978), emphasizes the need to allow learners to use their experiences and current knowledge to develop new knowledge, with the assistance of the educator. Providing opportunities for learners to use their previous knowledge will stimulate them to think and integrate the previous knowledge into developing new knowledge. This is learning with understanding and not just memorisation.

In attempting to solve problems on their own learners communicate with one another by sharing experiences and learning in the process. Learners become problem solvers as interpreted by their social group within socio cultural settings (Zamkova, Prokop & Stolin, 2016). Communication is key to effective thinking and leaning in geometry especially since cognitive skills are developed. The educator-centred method certainly does not allow for effective communication. Sfard (2008) spoke about changes in thinking leading to learning and this is achieved when learners talk about geometric objects and their relationships. The best performance was seen by learners being taught by educator CE2. This educator utilized problem-solving methods and inquiry based learning in his methodology. Problem based learning allows learners to use their prior experiences and knowledge to solve a problem and to develop new ideas. Learners have the necessary knowledge but it needs to be tapped by problems that require self- study. Problem based learning encourages learners to discuss their reasoning and to make logical connections. Learners are curious and enjoy working with problems, but very seldom given the opportunities to stimulate their thinking in solving problems by themselves. Learners are able to retain their knowledge and skills and create self-confidence when solving problems.

Findings also revealed that visualization enhances communication when teaching geometry. Linking geometric concepts to real life situations has proven to make understanding of geometry easier for learners. Smaller groups are better as they promote discipline and learners getting “lost” in the group. The use of co-operative groups is therefore encouraged in the teaching of geometry.

Time allocated to the teaching of geometry determines the method of teaching and learning. More time is needed when implementing the learner-centred method. During lessons, more time must be dedicated to learners’ active participation and lesser time to educators' talking in the classroom. A single advanced geometric rider can sometimes take learners thirty minutes to solve. Learners must read the statement, identify key concepts form the statement, determine the properties or associations they can draw from these statements and then associate it to the diagram before answering the questions. Active discussions must be a priority in the geometry classroom during the identification of key concepts and the properties and associations that can be inferred. Time allocation is therefore a factor that affects the teaching and learning of geometry.

Methods of teaching should be selected based on the type of learners in the classroom. Educators must know their learners well. Cooperative groups should include learners with diverse abilities and characteristics. This will encourage learners to work together and learn from one another to accomplish assigned learning goals and tasks. Communication within the group will also allow others to learn new words and syntax of geometry concepts. Reasoning and thinking skills develop when learners discuss and work together to solve geometric problems.
Vygotsky's (1978) theory on the capitalising of knowledge that learners possess to guide them towards understanding new knowledge, supports educators CE2's view of teaching geometry practically, by inquiry or group work. Learners are given a task and use their experiences and own knowledge, guided by the educator to develop new knowledge. This was seen by educator BE2, who first gave the learners a task to complete an investigation on properties of a cyclic quadrilateral, and then guided the learning towards formulating the proof on cyclic quadrilaterals. The investigation required learners to measure opposite angles and write their findings. It then requested learners to measure the interior angles and opposite exterior angle and to write their findings. This allowed learners to use their knowledge of measurement to make conclusions. The educator then guided them towards the two theorems of cyclic quadrilaterals. There was active discussion and communication in the classroom. This type of inquiry-based learning, gives learners the opportunity to search for material and make their own discoveries toward solving problems. Communication and discussion follow from the discoveries. The educator afforded the learners an opportunity to participate in the learning process. This links Sfard's (2008) commognition theory where thinking takes place by communication. By both communicating in writing and then

expressing their findings learners were able to develop new geometric skills. The challenge is that the learners must be at the level where they can complete the task of measuring accurately. The use of Van Hiele's levels to ensure that we understand the level of our learners plays an important role in identifying barriers. This type of learning is effective when all learners are able to think at the same level. Grouping of learners according to their level of understanding to prove riders will beneficial to the learning process. Educators will get the opportunities to provide support and identify misconception within the group.

6.2.6 Policies and Regulations

Some schools force learners to do mathematics in the FET phase even though they are unable to cope with it in the GET phase. The following findings were identified from the literature, the interviews and questionnaires:

- Many learners go into the FET phase (grades 10-12) without passing mathematics in the GET phase (grades 7-9);
- These learners are condoned to be promoted due the relaxed promotion requirements in the GET phase. The relaxed promotion strategy was added to address the inequalities in education. These learners are progressed to the next grade by having mathematics results condoned, if mathematics is the only subject preventing the learner from being promoted into the next grade.
- Learners are permitted to choose mathematics in the FET phase with a mark of 20%. This is a major challenge for both learners and educators.
- A learner can only remain in a phase for a maximum of four years. For example, in the GET (grades 7-9) phase, a learner can only be retained once. Some of the learners are slower and need more time to grasp concepts, but are moved to the next grade before they reach their cognitive level of understanding the concepts of mathematics more especially geometry.

Learners that have not mastered the skills effectively from the previous grades will experience barriers and will not be able to operate at the same level of thinking as other learners in the classroom. It was found from the interviews with educators that many learners in their geometry classes have not mastered previous skills but were promoted to the next grade. Learners that are at different levels of thinking must be identified and the necessary intervention needs to be provided. Educators can assess learners' thinking abilities, when they communicate their understanding in the classroom. Engaging learners in the classroom lesson was also articulated as a means to identify misconceptions and to correct the

6.2.7 Educator Qualification to Provide Guidance

Many of the educators have an educational qualification however; it is not necessarily in mathematics. Most schools reported that the average performance in geometry is below 40%. One of the key tenants of Vygotsky's (1978) theory was the Zone of Proximal Development (ZPD). In this theory the ZPD involves connecting what the learner can do without help to what the learner can do with help. In this regard it proposes that a more knowledgeable other (MKO) who is the educator plays a pivotal role in the classroom. The educator is required to build scaffolds to link previous knowledge to new knowledge. Educators that are experiencing barriers themselves with the content will not be in a position to identify individual learner's needs and allow them to move out of the zone of proximal development.

In this study six of the eighteen learners indicated a lack of confidence in their educator's content knowledge. Some learners indicated that the educator dislikes teaching geometry and rushes through it. The educator is expected to be the skilled person in the classroom, both in content and methodology, to guide and provide necessary support. In this regard an unskilled educator will be challenged in offering guidance to a learner in the ZPD. It is only with guidance that the learner will move out of the ZPD and link previous knowledge to new knowledge. The educator needs to encourage participation in activities to allow this development. Scaffolding of instructions is the task of a skilled individual that adjusts the guidance required by the learner to reach beyond their current understanding. It was noted in this study that this is a challenge for an unskilled person.

Qualification in mathematics education is a key factor in the teaching of geometry. Experience and mathematical qualification contributed to the way educators went about teaching geometry. Due to the lack of qualified mathematics educators, educators without a qualification are forced to teach the subject. Some of these educators last did geometry at grade 9 level and are now expected to teach mathematics to grade 12 learners. Four of the educators in this study stated that they were not very confident with the content that they are teaching to the learners. These educators are unable to provide adequate scaffolding, learning experiences or give proper instructions, based on their own lack of experience and knowledge of the content. It was noted in this study that learners in the lower grades are not learning geometric concepts appropriately to prepare them for the secondary school curriculum. It is expected that learners will be at level 3 of the Van Hiele model when they enter secondary school; however, this is not possible if educators in the lower grades are unskilled and inexperienced educators. It is a common practice that educators in the primary phase are expected to teach all learning areas. Geometry is a specialised learning area with its own terminology and if not mastered correctly, it poses many challenges. Educators with a lack of adequate content knowledge therefore exacerbate the problems that learners experience in geometry.

This study found that some of the educators that choose to teach mathematics at secondary school level did not have the in-depth understanding of geometric concepts so they themselves were operating at a lower level with respect to geometric knowledge. Learners below level 3 can only do proof and geometry problems through memorisation. The over- emphasis on memorisation was noted by both learners and educators at the expense of in-depth understanding and abstract thinking. Educators need to possess sound pedagogical knowledge and skills to deal efficiently with the difficulties geometry learners' experience. Educators' expertise is acknowledged by educators as being an important factor in determining learner achievement.

Workshops provided by the Department of Education do not sufficiently support content knowledge. These workshops do assist with methodology and educators gain some assistance with teaching in the classroom however these workshops are not continuous. Educators without formal training in geometry, experience problems with vocabulary and are unable to provide proper explanations to learners. Educator's' lack of content knowledge creates their own dislike and anxiety for the subject. There was a consensus among educators interviewed that the root cause of poor performance in geometry relates to educators with insufficient content knowledge. Deficits in teaching knowledge create difficulties for learners to grasp skills in relevant thinking and learning. Educator's' lack of content knowledge will hamper effective communication and discussion in the classroom. Educators expressed that an educator that is experienced and skilled is able to encourage effective geometric thought.

Not all educators teaching mathematics have a formal qualification in geometry, in light of the curriculum change that took place in 2006. This change resulted in many educators not having sufficient content knowledge to teach geometry. Educators whose school curriculum did not include geometry would effectively need the additional training in content, to skill them adequately to be in classroom. Educator training of mathematics educators must be given careful attention by education authorities to assist those educators that lack specific content knowledge and methodological skills. There is an urgent need for ongoing training of educators. The training must include school content in geometry and not only methodology. Mathematics educators must have a deep understanding of geometry that is appropriate for school mathematics that they are going to teach. The success of geometry depends on educators knowing and understanding geometry themselves, to be able to teach it effectively. It is therefore fundamental that educators be given continuous support and access to in-service programmes, short seminars and workshops.

6.2.8 Curriculum

The reason to include geometry in the school curriculum is to provide learners with opportunities to develop spatial awareness and the ability to visualize and apply geometrical properties. The need for developing skills and providing problem-solving opportunities forms part of the geometry syllabus. In this study it was noted that inadequate opportunities for development of spatial awareness or problem-solving skills was practiced. Educators claimed that the curriculum is very demanding. However, even a well-designed curriculum will fail if there are weaknesses in the implementation process. Many of the topics in geometry depend heavily on previous knowledge. If there are gaps in learning from the previous year, then learners are unable to move forward in their development of new material. Thinking is a major part of geometry learning and educators need to develop this skill from early years with cognitively challenging problems.

The mathematics syllabus does not take into consideration time used for tests and examinations. The work schedules start at the beginning of the term and end at the end of each term. The time allocated to teaching of geometry is not sufficient for all the necessary skills to be mastered. Educators expressed "*the time allocated to teaching of geometry is set out in the work schedule. The time does not take into consideration exams or assessments".* Educators suggested that there was a need to increase the amount of time allocated to teaching of geometry, so that greater participation of learners in the lessons can be encouraged.

It was noted in this study that the implemented curriculum is different from the intended curriculum. Phaeton (2016) differentiated between the intended and implemented curriculum and made known that the implemented curriculum is the one that depends on the educators' education and in-service training. A lack of adequate training results in educators teaching based on their idea of what is expected of them. Educators tend to rush through the syllabus to meet deadlines instead of giving importance to the actual learning of skills. The tracking of the curriculum by supervisors, forces educators to complete the syllabus as a paper record keeping exercise at the expense of learning with understanding, in geometry. The reason for the teaching of geometry, as outlined in the literature review, is compromised as the annual teaching plan is prioritized.

6.2.9 Visual Communication

Mathematical tools must be considered to allow for understanding of geometric concepts. Learners are able to understand geometry better when they are able to experience it in real life situations. These real life situations must be brought into the classroom by concrete tools. Pictures, diagrams, objects and videos are some of the tools that can be used to enhance the teaching of geometry. Effective geometric thought can be achieved by allowing the learner to communicate what they are able to see. The "see- touch- do "idea is very important for geometry learning. As stated by Tay (2003) learners must be able to "touch-see-and do "and interact with objects of their learning to learn geometry in a more imaginative and successful way. Learners develop effective geometric thought when they communicate in the classroom. They communicate based on their experiences and on what they have seen. The use of technology in the classroom will allow learners to better experience objects and shapes. It was observed that there was insufficient use of mathematical tools to enhance the teaching of geometry in the classroom.

Mathematical tools foster learning at different levels. Educators pointed out that tools will assist then to provide concrete models of abstract ideas that will allow learners to manipulate and think about ideas. This type of teaching allows learners to engage with the learning material. Complex problems are made easier with these tools. Different tools can be utilised like the whiteboard, data projector, charts, objects, pictures, diagrams, calculators, mathematical instruments and computer software. These tools can be traditional, technological or social. The use of social tools, such as small group discussions where learners interact with each other to share and challenge ideas, enhances thinking in the geometry classroom. These mathematical tools can be used independently or together to ensure that understanding and effective thinking takes place. The presence of traditional tools like the chalkboard, charts and pictures was noted in the classes. The use of the chalkboard was a tool that was commonly used by educators.

Technological tools are very effective in facilitating learners' understanding of complex problems. Technological tools allow for visualization which will lead to effective communication that encourages thinking. Most learners enjoyed the use of computers and technological devices when learning. It created interest and allowed them to experience meaningful geometric situations. Learners expressed that it is easier to "talk about what you can see". In the process of talking to others or to oneself thinking takes place. The use of computers in the learning of geometry equips learners with critical thinking skills required to solve problems. With the use of technological devices learners are able to learn new knowledge while the educator provides guidance. When the educator plays the role of a facilitator the learning of geometry becomes more meaningful as learners arrive at the solutions by themselves. Learners shared that they were able to remember concepts better when they are able to relate it to a diagram or picture. Technological devices support the display of different visual images to enhance the teaching of geometry.

Smart phones are accessible to most learners and they are confident in using them. These devices should be incorporated into the teaching of geometry. It was evident from the responses of both educators and learners that the use of technology will enhance the teaching of geometry and develop effective geometric thought. Although educators did articulate that smart phones have their disadvantages in school, the benefits that can be derived if used responsibly outweigh the disadvantages. As stated by two educator schools need to develop a policy on the use of the smart phones in the classrooms. The suggestion provided was to allow learners the use of a smart phone without a SIM card and access only to curriculum related material through the use of the schools WIFI.

6.3 INTEGRATION OF ACQUISITION AND PARTICIPATION METAPHORS

In this study the barriers to effective geometric thought were identified. The metaphors of acquisition and participation were used to guide this study to identify learning barriers. It was evident from the participants' responses that both metaphors are needed for effective geometric thought. The acquisition metaphor is the most established metaphor for learning but must be complimented with the new metaphor of participation. It was established in this study that the use of both metaphors should be encouraged to improve the learning of geometry. The acquisition metaphor suggests that knowledge is a commodity acquired by learners through facts and content while the participation metaphor emphasizes 'learning by doing'. Many educators over emphasized the acquisition metaphor in this study and this was not effective in geometry learning. The mere transmission of knowledge by the educator using the chalk and talk method was ineffective in geometric thought. Allowing learners to engage in a discourse and communicate their learning is important for effective geometric thought.

Communication was considered a significant tool by all participants, in the learning of geometry. Second language users indicated that increased communication in the classroom will allow them to master the language of geometry. It was also evident in this study that learners will communicate effectively based on their experiences.

This form of communication will pique learners' cognitive skills and get them to develop new knowledge based on their prior knowledge. All participants strongly supported a need for visual representations which allows learners to develop mental images that can be used in their communication in the geometry classroom. It was noted in this study that communication without experiences and mental images will be ineffective. This study found that the use of technology in the form of animation, computer software and virtual reality will create real situations that learners can use and interact with, to link prior knowledge and to develop new information. Learners are able to understand concepts better when they are able to see them and associate them to real situations. As stated by Sfard (2007) communication is the key to thinking, however the visual representation that brings real situations into the classroom will make this communication more valuable and successful. The model below a diagrammatic representation of the cycle to learning geometry as found in this study. It incorporates both acquisition and participation metaphors of learning.

Diagrammatic Model of Geometric learning as found in this study

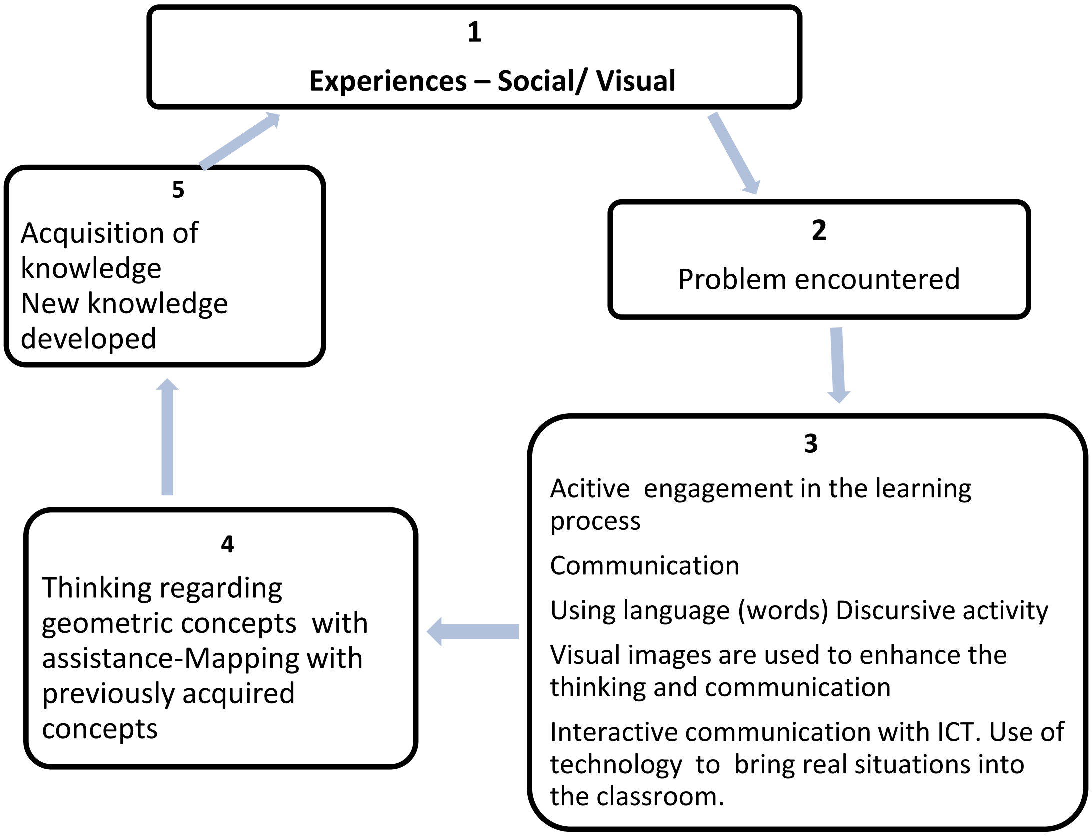

FIGURE 6.1 MODEL ON ACQUISITION AND PARTICIPATION TO DEVELOP LEARNING GEOMETRY

Cyclic Steps in the Acquisition of new knowledge

1. Learners' experiences are accumulated from their encounters through social interactions in communities. New knowledge is developed from the learners' existing experiences and prior knowledge.

2. Problems are encountered regularly in the learners' interactions.

3. The problem lends itself to active communication. Communication allows the learner to activate prior knowledge and experiences. Engaging in a discourse on geometric concepts using words, vocabulary and visual mediators enables the learner to make connections with the problem and with existing knowledge. The use of ICT and other tools allows for the development of mental images that links learning to real and meaningful situations. This makes the learning more interesting and enjoyable.

4. These interactions allow the learner to start thinking and map previously

acquired skills and knowledge to the current problem. The educator serves as a facilitator during this process of acquisition of knowledge. The learner is an active participant in the acquisition of knowledge.

5. This allows for the development of new knowledge and experiences.

It is during step 3 and step 4 that the learner is in the zone of proximal development. At step 4 the learners are able to assimilate the thinking, together with the more knowledgeable other to develop new concepts or knowledge. Vygotsky (1978) referred to this thinking as complex thinking. This knowledge now becomes part of previous experiences and the cycle will continue. This is a recursive cycle. Considering this recursive cycle, the teaching of geometry will be more meaningful and it will be understood better by learners since the learner is actively involved in the learning. Learning takes place from an inductive approach to a deductive approach. This allows the content to be understood and remembered for longer periods without memorisation.

Communication allows learners to become active participants in their learning. The participation metaphor emphasizes 'learning by doing' for effective geometric thought. Through engaging in discourse learners will be learning. Learners will bring in their own real felt experiences and this will be more meaningful which will lend to other applications. Educators should be facilitators in the learning process and allow learners to engage in geometric discourse to pique their prior knowledge and experiences. It is important for educators to have good content and pedagogical knowledge to guide and offer support to learners. ICT and other learning tools form an integral part of the learning process and should be incorporated frequently in the classroom.

6.4 SUMMARY OF STRATEGIES TO ERADICATE THE BARRIERS

The following strategies were identified in this study to develop effective geometric thought: align all

- Classroom environment must support the suggestions of the constructivist view. A classroom that is full of laughter, fun and interest, a classroom that supports physical and visual resources will allow for active participation.

- Educator should motivate and create interest by engaging learners in the lesson.
- More time must be allocated to the learning of geometry
- The use of modern technology allows for the teaching of geometry to be more meaningful since realistic situations can be created.
- Communication is the key to thinking. Greater emphasis must be placed on the language of instruction.
- Learners need to be more active in the lesson. They must be given time to explore and find solutions on their own.
- Smaller classes allow for greater activities that support participation of learners in the lesson.
- Resources such as a data projector, laptop and whiteboard in each class will in-co-operate real life examples into the lesson, thus making it more meaningful.
- Educators should communicate in the language of instruction from the lower grades so correct terminology in learnt. The learner's; vocabulary of geometric concepts will be developed.
- Geometry must be taught as a language. Constant discussion, written and verbal communication should be used to encourage learners thinking.
- Active learner participation is needed. Group teaching and problem based learning activities will allow learners to enjoy geometry
- The class must be visually stimulating, that is, it should have charts, mathematical objects, and mathematical instruments. Computer exploration of topics provides powerful visual images and intuitions that can contribute to a person's growing understanding in geometry. By visualising problems, learners are able to have a global picture of the problem to be solved.
- The use of technology should be responsibly encouraged however it must be strictly monitored. In line with seeking workable alternatives to the rigid axiomatic methods to teaching Euclidean Geometry, the focus on computer programs such as Geometers Sketchpad, is to facilitate and enhance learners' ability in making and testing conjectures.

The strategies identified all highlight communication has been a key to effective geometric thought. Verbal communication was highlighted as a fundamental tool of encouraging learners to talk and engage in a discourse of geometry, by using questioning techniques. Visual communication is the second form of communication that allows learners to see what they are learning in geometry. Tools used to visually stimulate, bring about interest and motivate learners to think about geometric concepts, allow for effective thought. Lastly, the use of interactive communication using technology was identified to bring real situations into the classroom. Participants in this study expressed that using a combination of communication techniques encourages effective geometric thought.

6.5 RECOMMENDATIONS

The following section presents recommendations which are useful for educators, learners, curriculum implementers and the Department of Education for effective geometric thought. These recommendations have been developed based on the major findings presented above in chapter 5.

- **Communication**

Communication has proven to play a significant role in getting learners to think and develop their own knowledge. The constant practicing geometric concepts enhance the language of geometry. Learners must be given adequate opportunities to express their thinking both verbally and in writing. Communication encourages learners to engage with the content of geometry and evokes deeper levels of thinking. Communication in the form of verbal, written, visual and interactive use of ICT should be encouraged in the geometry classroom.

Communication plays a pivotal role in the teaching and learning of geometry as seen in the literature review and the data collected from educators and learners in this study. In teaching geometry there should be connections i.e. learners must be able to make links between the language of geometry, symbols used in geometry, pictures and the way geometry is used in solving problems. It is through active communication that learners are able to make these connections. The following are recommendations pertaining to communication within the geometry classroom:

o The role of language in communication requires greater attention for effective communication. Geometric terminology comprises a set of linguistic symbols for communicating geometric ideas inside and outside the classroom. Mathematics educators will have to give special attention to the general use of appropriate language as well as the specific language of geometry. This includes correct use of the language by the educator, practising geometric terminology as well as monitoring the use of language by the learners. When learners come in direct contact with a figure the learner should develop the appropriate technical term or language to communicate ideas about the figure.

- Attention should also be given to learners' reading comprehension and this responsibility lies not only with the language educator, but with every other subject educator. This applies especially to learners experiencing barriers in understanding English. Constant practice of terminology in the form of communication was expressed by learners and educators as a means to improve geometric language.

- More opportunities must be given to learners to communicate geometrical concepts with one another during classroom lessons since this is one way in which learners can express their thoughts and be able to conceptualize these concepts. Discussion and communication of geometric concepts in the language of teaching and learning (LOLT) plays a fundamental role in developing the thinking and reasoning skills in learners.

- Modelling of open questions and problem-solving through group work activities encourages the use of verbal descriptions with visual forms of reasoning. Focussing attention on the meaning and derivations of key words helps learners to develop effective ways of communicating their thinking.

- Considerations must be given to include a section in paper two, to allow learners to express themselves in words about their understanding of geometry concepts before the application.

- **Method of teaching**

Teaching using the traditional methods of defining shapes and formulating conjectures does not promote effective geometric thought. The implication of this is that learners need preliminary explorations of properties of geometric shapes before they can attempt to respond. Learners need to be actively involved in their learning.

- Educator methodology should focus on greater learner interaction and reduce educator-centred learning. Rote learning without understanding must be discouraged as this is not learning to develop thinking. As much as rote learning has its place in learning of geometry, it does not allow for learners to apply the concepts or to solve problems.

Educators need to re-assess the manner in which they teach definitions of geometric concepts. Educators should also be cognisant that learners enter the senior phase with differences in background knowledge, confidence and abilities. Geometry teaching should therefore incorporate different methods to make the subject more enjoyable and meaningful. The direct teaching method should be replaced with the participative method. Learners must be actively engaged with the content, as professed by the constructivist approach. The following should be considered as part of the teaching practice in the classroom:

- Prior knowledge and experience must be given absolute importance and every effort must be made by educators to bring it to the fore, to develop geometric thought. This will provide an opportunity to develop questioning routines with learners and thus promote mathematical discourse and mathematical argument, encouraging thinking. The use of visualization, questioning and logical statements need to be developed within the context of mathematical discourse to provide more detailed information about learners' geometric thinking.

- Educators need to play a pivotal role in identifying the source of learners 'misconceptions and incorporate the correct methodology in their teaching to close the gap posed by these difficulties.

- To enhance learners' understanding of geometric definitions, mathematics educators must encourage learners to engage in realistic situations that will provide them the opportunity to develop the required skills of the intended curriculum, with meaning.

- Visual communication is a significant factor that can influence the development of spatial and geometric reasoning. Educators need to be cognisant of the Van Hiele's levels when teaching geometry, as many learners do not reach cognitive level three when they enter into the FET phase, so visual representation of geometric figures is necessary. Communication of descriptors and definitions of geometric concepts necessitates visual mediation. Visualization should be encouraged in proving theorems and statements in mathematical analysis. The use visual images bring the discipline of geometry very close to reality.

- Making an effort to educate themselves on new trends in teaching geometry, using modern literature and incorporating different strategies in their teaching like the use of dynamic software, promote interest in the learning of geometry.

- Interactive communication with the use of technology is effective in the teaching of geometry. The use of effective training models of ICT tools in teaching should help learners to grasp geometrical concepts easier. The interactive nature of ICT suggests that the role of the educator changes more to that of a facilitator and monitor of knowledge acquired. A policy should be developed on the responsible use of smart phones in the classroom.

- Learners should not be provided with ready- made definitions of geometric concepts such as tangent, cyclic quadrilateral, secant etc. By providing learners with such ready-made definitions, misconceptions can be created where learners believe that there exists "only one correct definition for each concept". Learners are thus denied the opportunity to search for alternative definitions in cases where definitions are presented as items 'cast in stone'.

- The traditional approach to teaching geometry does not correlate with the theories of constructivism which advocate that knowledge is not passively received but actively built up by cognizing the object. This theory should be implemented to help improve the results in geometry.

- Educators must engage in professional workshops. These workshops will benefit the educators through sharing of ideas with colleagues in an attempt to revise out-dated strategies of teaching mathematics and improve their content knowledge.

Relating the content of geometry to real life gives learners purpose to the subject and motivates them to want to learn. Isolated facts on worksheets reinforce the fact that geometry is irrelevant and need not be studied. Geometry teaching should link the school and the community that learners reside in, to make learning more meaningful.

- **Learning environment and class size**

For effective geometric thought learners need to be engaged and participate in the learning process. Active engagement evokes thinking. The learning environment must support activities and stimulate learners' thinking abilities. This process will be more successful if the learning environment and class size can be re-assessed.

The Department of Education should re-look at the learner educator ratio for mathematics. Geometry, which is a specialised learning area, should accordingly be allocated the similar weighting as skilled-based subjects. This will reduce the class size and allow learner-centred methods to be more effective. The implementation of the theory of Vygotsky (1978) to utilize the learner's previous experience to develop new concepts will be more effective in a smaller class size. Further, Sfard's (2018) theory to allow for discussion and active communication in the classroom will be effective and meaningful in a smaller classroom. More funds need to be allocated to improve mathematics classrooms. A good environment will encourage participation, engage learners and offer a sense of success. A more conducive mathematics classroom will motivate both learners and educators. A positive attitude towards geometry can improve the overall results. A smaller number of learners in the classroom will assist educators to identify learners that have not reached the appropriate Van Hiele's levels and educators can provide individual attention to these learners.

- **Well-resourced school**

One finding from the study revealed that there is a need for learners to see what they are learning. A well-resourced school will encourage interest and thinking. The shortage of teaching and learning materials in schools' influences learners' attitudes and interest in learning geometry. It is therefore the responsibility of the Department of Basic Education to provide schools with relevant learning resources in order to promote the effective implementation of the geometry content, for effective thought. A data projector, smart board, laptop or white board, are some of the facilities that must be considered for the geometry classroom by both school administrators, Schools' Governing Bodies and the Department of Basic Education, when planning and budgeting are being done. Modern technology is part of the life of today's learner and thus must be incorporated into the geometry lesson if we intend to sustain their interest and improve their attitude towards the learning of geometry. Visual representation encourages communication and gets learners to think.

- **Educator Qualification and Experience**

Pedagogical knowledge and content knowledge of educators' in geometry is a vital ingredient in learning and achievement. Greater educator training workshops are needed to assist educators in the teaching of geometry. Educators that did not have geometry in their qualifications must be re-skilled to teach it with confidence. This will create a positive teaching and learning environment and motivate learners to want to learn geometry.

Specialist mathematics educators should be used in the lower grades. There is a need for learners' foundational knowledge to be good so that further studies in geometry are not hampered by the lack of foundational knowledge. Constant monitoring, supervision and guidance must be provided to educators so that educators implement the intended curriculum across all grades. The teaching profession should be marketed to attract learners with high mathematical skills.

- **Curriculum Changes**

Changes to curriculum should be minimized. Education is constantly evolving and changes will be necessary. However, when these changes are considered, educators should be included in the development of the model being devised. As implementers of the curriculum, it is necessary that they have a say in these changes for them to teach effectively in the classroom. This will also prevent resistance to changes.

6.6 RECOMMENDATIONS FOR FUTURE STUDY

This study achieved its aim, namely, to explore the barriers to effective geometric thought in the FET phase. However, it has opened the following avenues for further research:

1. This was a small-scale study conducted in three schools in the Umlazi District within KwaZulu-Natal. Further research needs to be done on a broader scale to include more schools and more educators. If more schools were to be added to this kind of research, one would gain a more representative sample. A larger scale study will be more reliable and the results from such a study would have greater implications for educators on a broader level.
2. The researcher recommends further research throughout South Africa in order to gather more conclusive results.
3. A longer, comparative study on the teaching of geometry as a language with communication and interactive communication with the use of ICT, incorporating visual mediators and routines, would reveal further implications that language has on the teaching and learning of geometry.
4. Reading and comprehending the given information specific to geometry, posed barriers and serious challenges for learners. A qualitative study would determine whether a causal relationship exists between mother tongue language and academic performance in geometry.
5. This study focussed on the learners' barriers in understanding geometry in secondary schools, therefore further studies should investigate the learners' barriers to effective geometric thought in primary schools.

6.7 CONCLUDING REMARKS

To improve the performance of geometry in the FET phase, all stakeholders must play a significant role. The purpose of this study met its goals by identifying the barriers to effective geometry thought. The major findings of this study show that there is a myriad of barriers that make effective geometric thought a challenge for learners. The research questions provided data on various barriers from the perspectives of, learner related factors, pedagogical factors and school related factors. Furthermore, data was collected on how these barriers can be eradicated. It was evident that both metaphors namely, acquisition and participation plays an essential role in developing effective thought in geometry learning.

For effective geometric this study suggests the use of a learner centred teaching approach. The learners' senses must be evoked using communication in form of verbal, visual, written and technological resources. The use of visual representation and technology will allow for meaningful experiences in the geometry classroom. This will encourage interactive communication and in turn promote thinking. The recommendations are provided to various stakeholders with the aim of putting measures in place for effective geometric thought. If learners are unable to learn the way, we teach then we must teach the way they learn.

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8. APPENDICES

APPENDIX 1 PERMISSION FROM DEPARTMENT OF BASIC EDUCATION

education

Department:
Education
PROVINCE OF KWAZULU-NATAL

Enquiries: Phindile Duma Tel: 033 392 1063 Ref.:2/4/8/1853

Mrs K Naicker
28 Tensing Road
Umkomaas
4170

Dear Mrs Naicker

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"AN EXPLORATION OF THE BARRIERS TO EFFECTIVE GEOMETRIC THOUGHT IN THE FET PHASE OF SELECTED SECONDARY SCHOOLS IN THE UMLAZI DISTRICT",** in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the
Intended research and interviews are to be conducted.

6. The period of investigation is limited to the period from 24 July 2019 to 10 January 2022.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma at the contact numbers below.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.

10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

____

Dr. EV Nzama

Head of Department: Education

Date: 24 July 2019

KWAZULU-NATAL DEPARTMENT OF EDUCATION Postal Address: Private Bag X9137 • Pietermaritzburg • 3200 • Republic of South Africa **Physical Address:** 247 Burger Street • Anton Lembede Building • Pietermaritzburg • 3201 **Tel.:** +27 33 392 1063 • **Fax.:** +27 033 392 1203• **Email:**Phindile.Duma@kzndoe.gov.za •**Web:**www.kzneducation.gov.za Facebook: KZNDOE….Twitter: @DBE_KZN….Instagram: kzn_education….Youtube:kzndoe

APPENDIX 2 ETHICAL CLEARANCE

UNIVERSITY OF
KWAZULU-NATAL
INYUVESI
YAKWAZULU-NATALI

16 October 2019

Mrs Kalavani Naicker (211557433)
School of Education
Edgewood Campus

Dear Mrs Naicker,

Protocol reference number: HSSREC/00000267/2019
Project title: An exploration of the barriers to effective Geometric thought in the FET Phase of selected secondary schools in the Umlazi District

Approval Notification – Expedited Application

This letter serves to notify you that your application received on 16 August 2019 in connection with the above, was reviewed by the Humanities and Social Sciences Research Ethics Committee (HSSREC) and the protocol has been granted **FULL APPROVAL**

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

This approval is valid for one year from 16 October 2019.
To ensure uninterrupted approval of this study beyond the approval expiry date, a progress report must be submitted to the Research Office on the appropriate form 2 - 3 months before the expiry date. A close-out report to be submitted when study is finished.

Yours sincerely,

[redacted]

Professor Urmilla Bob
University Dean of Research

/ms

Humanities & Social Sciences Research Ethics Committee
Dr Rosemary Sibanda (Chair)
UKZN Research Ethics Office Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Website: http://research.ukzn.ac.za/Research-Ethics/

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

INSPIRING GREATNESS

APPENDIX 3(A) CONSENT FORM TO PARTICIPANTS

Part A

Date:

Greetings to you

My name is Kalavani Naicker. I am a PhD learner at UKZN department of Mathematics Education. My contact details are as follows 0842277705 and email is kalay@telkomsa.net.

You are being invited to consider participating in a study that involves research on **AN EXPLORATION OF THE BARRIERS TO EFFECTIVE GEOMETRIC THOUGHT IN THE FET PHASE OF SELECTED SECONDARY SCHOOLS IN THE UMLAZI DISTRICT.**

The study is expected to enrol 12 learners and 3 educators in 3 Secondary Schools in the sp Amaze District. The learners will be given a questionnaire that needs to be completed and returned to the researcher and to be part of a focus group discussion. The educators will also need to complete a questionnaire and will be subjected to an interview that will be recorded. The duration of your participation if you choose to enrol and remain in the study, is expected to be about an hour for the learners and two hours for the educators.

There are no risks associated with this study. We hope that the study will identify the barriers to geometric learning and teaching and establish the factors that will eradicate these barriers.

This study has been ethically reviewed and approved by the UKZN Humanities and Social Sciences Research Ethics Committee (approval number (HSSREC/00000267/2019).

In the event of any problems or concerns/questions you may have, contact the researcher at the above details provided or the UKZN Humanities & Social Sciences Research Ethics Committee, contact details are as follows:

HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS ADMINISTRATION

Research Office, Westville Campus

Govan Mbeki Building
Private Bag X 54001
Durban
4000
KwaZulu-Natal, SOUTH AFRICA
Tel: 27 31 2604557- Fax: 27 31 2604609
Email: HSSREC@ukzn.ac.za

Your participation in this study is voluntary and you may withdraw your participation at any time without any consequences. It would however be appreciated that you stay for the duration of the data collection as your information is important for the study. There are no incentives or reimbursements that will be provided for your participation. Your name will remain anonymous. All data obtained will be safely stored in the strong room.

CONSENT

I ________________________________(Name) have been informed about the study entitled, An exploration of the barriers to effective geometric thought in the FET phase of selected schools in the Umlazi District, by Kalavani Naicker, a learner at the University of Kwa-Zulu Natal.

I understand the purpose and procedures of the study.

I have been given an opportunity to answer questions about the study and have had answers to my satisfaction.

I declare that my participation in this study is entirely voluntary and that I may withdraw at any time without affecting any of the benefits that I usually am entitled to.

I have been informed that there is no compensation or risks associated with this study.

If I have any further questions/concerns or queries related to the study I understand that I may contact the researcher at 0842277705.

If I have any questions or concerns about my rights as a study participant, or if I am concerned about any aspect of the study or the researchers then I may contact:

HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS ADMINISTRATION

Research Office, Westville Campus
Govan Mbeki Building
Private Bag X 54001
Durban
4000
KwaZulu-Natal, SOUTH AFRICA
Tel: 27 31 2604557 - Fax: 27 31 2604609
Email: HSSREC@ukzn.ac.za

Additional consent, where applicable

I hereby provide consent to:

Audio and video-record my interview / focus group discussion YES / NO

________________________ ________________________

Signature of Participant **Date**

________________________ ________________________

Signature of Witness **Date**

(Where applicable)

Part B

Consent of parent of learner

I ___________________parent of _________________________________(Name) have been informed about the study entitled, An exploration of the barriers to effective geometric thought in the FET phase of selected schools in the Umlazi District, by Kalavani Naicker a learner at University of Kwa-Zulu Natal.

I understand the purpose and procedures of the study.

I have been given an opportunity to answer questions about the study and have had answers to my satisfaction.

I declare that my child/ward's participation in this study is entirely voluntary and that he/she may withdraw at any time.

I have been informed that there is no compensation or risks associated with this study.

If I have any further questions/concerns or queries related to the study I understand that I may contact the researcher at 0842277705.

If I have any questions or concerns about any aspect of the study or the researchers then I may contact:

HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS ADMINISTRATION

Research Office, Westville Campus

Govan Mbeki Building

Private Bag X 54001

Durban

4000

KwaZulu-Natal, SOUTH AFRICA

Tel: 27 31 2604557 - Fax: 27 31 2604609

Email: HSSREC@ukzn.ac.za

________________________________ __________

Signature of Parent **date**

APPENDIX 3(B) CONSENT FORM TRANSLATED TO ISIZULU FOR PARTICIPANTS

Isithasiselo sesibili

Ifomu lesivumelwano kubahlanganyeli.

Usuku:

Sanibonani

Igama lami nginguKalavani Naicker. Ngingumfundi we-PHD eMnyangweni wezeziBalo e-UKZN. Imininingwane yami yokuxhumana 0842277705 imeyili ithi kalay@telkomsa.net.

Umenyelwa ukuthi ucubungule ubamba iqhaza ocwaningweni olubandakanya ucwaningo lokuthola LOKUFINYELELA KWAMABHARAYA KUPHELA I-GEOMETRIC NGESIKHATHI ESIFUNDISWENI SESIFUND SEZIKOLO SE-FUNDI ESEKUFUNDIWE E-UMLAZI DISTRICT.

Lolu cwaningo kulindeleke ukuthi lusebenzise abafundi abayi-12 nothisha aba-3 ezikoleni ezintathu ezingamasekhondari eMlazi District. Abafundi bazonikezwa uhlu lwemibuzo okudingeka ukuthi baiufunde baluphendule bese lubuyiselwa kumcwaningi futhi babe yingxenye yengxoxo yeqembu eligxile kakhulu ekudingideni isihloko. Othisha nabo kuzodingeka ukuthi bagcwalise uhlu lwemibuzo futhi bayobamba iqhaza engxoxweni ezoqoshwa. Isikhathi sokubamba kwakho iqhaza uma ukhetha ukubhalisa nokuhlala esifundweni kulindeleke ukuthi kothisha kube ihora kubafundi kuzoba ngamahora amabili.

Azikho izingozi ezihambisana nalolu cwaningo. Siyethemba ukuthi lolu cwaningo luzokhomba izithiyo ekufundeni nasekufundisweni kwejometri futhi luthole izinto ezizosusa lezi zingqinamba.

Lolu cwaningo lubuyekeziwe futhi lwavunywa yiKomiti Yezokuziphatha Yezenhlalo Yezenhlalo Yezizwe Zase-UKZN (nenombolo yokuvunyelwa_ HSSREC/00000267/2019).

Uma kwenzeka kuba nezinkinga noma nofisa ukukusho/ukukubeka/noma unemibuzo ungaxhumana nomcwaningi kule mininingwane engenhla noma iKomiti Yezindinganiso Zokuziphatha Zabantu Base-UKZN, imininingwane yokuxhumana ngale ndlela elandelayo:

IMISEBENZI YAMABANDLA NOKUQHUBEKELA ISAYENSI YOKUFUNDA KWEZOBUCHWEPHESHE
IHhovisi Lokucwaninga, Ikhampasi laseWestville
Isakhiwo seGovan Mbeki
Isikhwama sangasese X X00001
EThekwini
4000

IKwaZulu-Natali, ENINGIZIMU AFRIKA
Ucingo: 27 31 2604557- Ifeksi: 27 31 2604609
I-imeyili: HSSREC@ukzn.ac.za

ISIVUMELWANO

Mina-________________________________ (Igama) ngazisiwe ngocwaningo olunesihloko esithi, Ukuhlolwa kwezithiyo ekusetshenzisweni komqondo wejometri okusebenzayo esigabeni se-FET sezikole ezikhethiwe esifundazweni saseMlazi, nguKalavani Naicker umfundi eNyuvesi yaKwaZulu-Natali.

Ngiyayiqonda inhloso nezinqubo zocwaningo.

Nginikezwe ithuba lokuphendula imibuzo mayelana nalolo cwaningo futhi ngithole izimpendulo ngokweneliseka kwami.

Nukubamba iqhaza kwami kulolu cwaningo kungokuzithandela ngokuphelele futhi ngingahoxisa nganoma yisiphi isikhathi ngaphandle noma yiziphi izinzuzo engivame ukuba nazo.

Ngatshelwa ukuthi akukho sinxephezelo noma ubungozi obuhambisana nalolu cwaningo.

Uma ngineminye imibuzo / ukukhathazeka noma imibuzo ephathelene nalolu cwaningo ngiyaqonda ukuthi ngingaxhumana nomcwaningi kule nombolo 0842277705.

Uma nginemibuzo noma ukukhathazeka ngamalungelo ami njengomhlanganyeli ocwaningweni, noma uma ngikhathazekile ngesici socwaningo noma abacwaningi ngingaxhumana:

IMISEBENZI YAMABANDLA NOKUQHUBEKELA ISAYENSI YOKUFUNDA KWEZOBUCHWEPHESHE
IHhovisi Lokucwaninga, Ikhampasi laseWestville
Isakhiwo seGovan Mbeki
Isikhwama sangasese X X00001
Ethekwini
4000
IKwaZulu-Natali, ENINGIZIMU AFRIKA
Ucingo: 27 31 2604557 - Ifeksi: 27 31 2604609
I-imeyili: HSSREC@ukzn.ac.za

Imvumo eyengeziwe, lapho kuvlgaba nesidingo
Nginikeza imvumo yokuba:
Kuqoshwe iphimbo le-intavyu yami/ingxoxo yeqoqo yebo/cha

_________________________ ________________

Ukusayina kobambe iqhaza usuku

_________________________ _________________

Ukusayina kukakakazi usuku

Mina-______________ (mzali ka)_________________(igama) ngitshelwe ngalolu cwaningo olunesihloko esithi, Ukuhlolwa kwezithiyo ekucabangeni kwezimpawu zejometri okusebenzayo esigabeni se-FET sezikole ezikhethiwe esifundazweni sase Mlazi, nguKalavani Naicker umfundi eNyuvesi yaKwaZulu-Natali.
Ngiyayiqonda inhloso nezinqubo zocwaningo.

Nginikezwe ithuba lokuphendula imibuzo mayelana nalolo cwaningo futhi ngithole izimpendulo ngokweneliseka kwami.

Nukubamba iqhaza kwengane yami kulolu cwaningo kungokuzithandela ngokuphelele futhi angahoxa noma yinini.

Ngatshelwa ukuthi akukho sinxephezelo noma ubungozi obuhambisana nalolu cwaningo.

Uma ngineminye imibuzo / ukukhathazeka noma imibuzo ephathelene nalolu cwaningo ngiyaqonda ukuthi ngingaxhumana nomcwaningi kule nombolo 0842277705.

Uma nginemibuzo noma ukukhathazeka mayelana nanoma iyiphi ingxenye yocwaningo noma abacwaningi ngingaxhumana:

IMISEBENZI YAMABANDLA NOKUQHUBEKELA ISAYENSI YOKUFUNDA KWEZOBUCHWEPHESHE

IHhovisi Lokucwaninga, Ikhampasi laseWestville

Isakhiwo seGovan Mbeki

Isikhwama sangasese X X00001

EThekwini

4000

IKwaZulu-Natali, ENINGIZIMU AFRIKA

Ucingo: 27 31 2604557 - Ifeksi: 27 31 2604609

I-imeyili: HSSREC@ukzn.ac.za

_____________________ ________________

Isiginesha yosuku lomzali

APPENDIX 4 OBSERVATION SCHEDULE FOR EDUCATORS LESSON

School : ____________________

Educator's name: ______________________

Topic: ____________________Date: ___________________

VO – Very Often, O- Often, SM – Sometimes, S –Seldom, N- Never

Aspects observed	VO	O	SM	S	N
Worthwhile Mathematical tools					
1. Learners are engaged					
2. Learners use a variety of mathematical tools					
3. Conjectures, generalisations and 'what if' questions abound.					
4. Learners communicate about the math tasks at hand.					
Learners' Role in the lesson					
1. Learners present solutions.					
2. Learners question one another.					
3. Learners use a variety of tools to reason, make conjectures, solve problems and communicate their					

thinking.					
4. Learners are actively involved in the lesson					
Educator's Role in the lesson					
1. Educator provides real life situations for the introduction to the lesson.					
2. Educator uses a learner-centred method to teach.					
3. Educator has good content knowledge.					
4. Educator enquires about learners' understandings of the concepts before progressing with lesson.					
5. Educator uses technology to encourage interest.					
6. Educator has a positive attitude towards teaching.					
7. Educator caters for learners with different learning abilities.					
8. Educator listens to learners' ideas in order to develop their skills.					

9. Educator relates classroom discussions to social matters**.**					
Classroom environment					
1. Class size is appropriate.					
2. Good infrastructure is noted.					
3. Classroom has visual media to encourage interest.					

APPENDIX 5 LETTER TO SCHOOL PRINCIPALS.

28 Tensing Road

Craigieburn

Umkomaas

4170

Secondary School

Umlazi District

Phumelela Circuit

Chatsworth

4092

The Principal

RE: PERMISSION TO CONDUCT RESEARCH

I hereby wish to conduct research towards the completion of my PHD in Education at the University of KwaZulu-Natal (Egdewood Campus). The topic for the research is **AN EXPLORATION OF THE BARRIERS TO EFFECTIVE GEOMETRIC THOUGHT IN THE FET write out in full PHASE OF SELECTED SECONDARY SCHOOLS IN THE UMLAZI DISTRICT.**

The purpose of the study is:

1. To identify the barriers in learning geometry.
2. To determine how systemic factors contribute to the learners' performance in geometry.
3. To ascertain the pedagogical factors affecting geometry performance in schools.
4. To establish strategies that can be adopted to improve learner performance in schools.

The research will be conducted in three schools in the Umlazi District, Phumelela Circuit. The research will involve questionnaires and interviews with learners and mathematics educators. Mathematics educator lessons will be observed and results of the schools for the past three years in the NSC examination will be analyzed. Parents will be given consent forms to sign to allow their children to participate in the study. Only learners whose forms are signed will be allowed to participate in the research. Educator interviews will not be conducted during teaching and learning time. The focus group discussion for learners will be conducted during their breaks.

Participants will be informed that participation is on a voluntary basis. I do assure you that confidentiality will be maintained of the school name, learners and educators. Participation is voluntary and participants can withdraw their participation. A report on the findings and recommendations will be forwarded to your office after the research is completed.

I trust that my request will be favourably considered.

Thanking You.

____

Mrs K.Naicker [Principal]

Persal 13585908

Newhaven Secondary School

APPENDIX 6 QUESTIONNAIRE FOR EDUCATORS

Section A: Personal Particulars (Place a X in the box)

1. Educational Qualifications

1.1 REQV 14 ☐ 1.2 REQV15 ☐

1.3 REQV 16 ☐ 1.4 REQV 17 or greater ☐

2. Does your qualification have geometry in it?

2.1 Yes ☐ 2.2 NO ☐

3. Years of teaching experience.

3.1 0-5years ☐ 3.2 6-10 years ☐

3.3 11-20yrs ☐ 3.4 > 21years ☐

4. Gender

4.1 Male ☐ 4.2 Female ☐

5. Home language

5.1 English ☐ 5.2 Afrikaans ☐

5.3 IsiZulu ☐ 5.4 Other ☐

6. What is the average performance of your grade 11 learners in geometry?

6.1 0-29% ☐ 6.2 30-39% ☐

6.3 40-59% ☐ 6.4 > 60% ☐

7. Do you offer private tuition?

7.1 Yes ☐ 7.2 No ☐

8. Do you encourage outside school mathematics tuition?

8.1 Yes ☐ 8.2 No ☐

Section B:

In each of the following questions please place an x according to your choice

1. Strongly agree 2. Agree 3. Disagree 4. Strongly disagree

Statements	**1**	**2**	**3**	**4**
1. Educator centered approach should be used in the teaching of geometry.				
2. Learners need to communicate their own mathematical thinking during geometry lessons.				
3. Learners must memorize concepts to perform well.				
4. Educators' qualifications in geometry affects the teaching of geometry.				
5. Educators' attitudes toward geometry affects teaching.				
6. Educators need to use teaching aids to improve learner? performance in geometry.				
7. More time should be allocated to the teaching of geometry.				
8. Geometry should be taught as a module on its own.				
9. Geometry is not for all learners.				
10. Learners are not interested in mathematics.				
11. My school does not have sufficient resources for geometry teaching.				
12. Class sizes are too large.				
13. Poor Infrastructure affects the teaching of geometry.				
14. Learners with a language barrier perform poorly in geometry.				
15. Time allocation for geometry is appropriate.				

16. Foundational knowledge is lacking.				
17. The educator –learner ratio supports teaching of geometry.				
18. Curriculum changes in mathematics have created confusion.				
19. Parents play little or no role in assisting with acquisition of geometry skills.				
20. Specialist mathematics educators should be teaching mathematics in the foundation and intermediate phases.				
21. Poor visualization skills impact on learners' performance in geometry.				
22. Technology will help to better geometry performance.				
23. Greater emphasis on problem-solving is needed.				

APPENDIX 7 SEMI- STRUCTURED INTERVIEW QUESTIONS FOR EDUCATORS

1. What teaching methods do you use in your geometry lessons?
2. How would you rate the learner performance of geometry compared to other branches of mathematics?
3. How can learners be motivated to enjoy geometry lessons?
4. State some of the challenges that you experience in your teaching of geometry.
5. Weakness in the language of geometry is a problem that many mathematics learners face. What can be done to improve the situation?
6. What strategies can be used to assist learners to visualize a geometric rider that has many different figures like triangles, circles, rectangles etc.?
7. What strategies will help learners think about geometrical concepts on their own?
8. Discuss some support materials that will assist in the teaching of geometry.
9. Do you encourage active discussion of geometry problems within your classroom?
10. Is communication important in the learning of geometry?
11. What role do you place on the use of technology in your classroom?

APPENDIX 8 QUESTIONNAIRE FOR LEARNERS

Section A: Place an X in the appropriate box.

1. What is your gender?

1.1 Male ☐ 1.2 Female ☐

2. What is your home language?

2.1 English ☐ 2.2 IsiZulu ☐

2.3 Afrikaans ☐ 2.4 Other ☐

3. Do you live with?

3.1 both parents ☐ 3.2 One parent ☐

3.3 Guardian ☐ 3.4 Alone ☐

4. Do you enjoy studying geometry?

4.1 Yes ☐ 4.2 No ☐

5. Is geometry needed in school mathematics?

5.1 Yes ☐ 5.2 No ☐

Section B:

In each of the following questions please place an x according to your choice

1. Strongly agree 2. Agree 3. Disagree 4. Strongly disagree

Statements	**1**	**2**	**3**	**4**
1. The learning environment in the geometry class is conducive.				
2. Class sizes contribute to learner performance in geometry.				
3. My school is well resourced physically i.e. desks, chairs, etc.				
4. My school has enough instructional material for the learning of geometry i.e. textbooks; instruments etc.				
5. Curriculum changes affect learning of geometry.				
6. Many learners lack foundational knowledge needed for the learning of geometry.				
7. The use of technology and media will assist in the learning of geometry				
8. Learners have a negative attitude towards geometry.				
9. Girls perform better than boys in geometry.				
10. Geometry is the most interesting branch of mathematics				
11. Memorization is the key to good performance in geometry.				
12. Extra classes are needed for good performance in geometry				
13. Learners have the necessary instruments for solving				

geometry problems.				
14. Educator's attitude towards geometry affects learners' performance.				
15. My educator is very competent in the teaching of geometry.				
16. My educator provides support when solving geometry problems.				
17. Geometry topics are related to real- life situations				
18. Frequent monitoring and feedback can help learners improve geometry skills.				
19. Spending more time on a concept in geometry can help learners' master geometrical problems.				
20. Learners should be more involved in practical work than theoretical learning.				
21. Learners should be willing to learn on their own,				
22. Greater emphasis on problem-solving is needed.				
23. Geometry should be learnt as a language on its own.				
24. Communicating about geometric concepts will improve learner thinking.				
25. Identification of properties of geometric figures helps to solve geometry problems.				
26. Computers and interactive boards can make geometry easier and interesting.				
27. Visualizing different figures will improve understanding in				

geometry.				
28. Constructing of simple geometrical figures using straight edges, compasses, protractors and setsquares will improve geometry learning.				

APPENDIX 9 SEMI- STRUCTURED QUESTIONS

Questions for learners' focus group discussion

1. What are the challenges that you experience in the learning of geometry?

2. Do educators' attitudes affect the learning of geometry? How?

3. Explain what strategies your educator can use to encourage thinking in the geometry class.

4. How do learners communicate their issues when solving problems?

5. Do you get the opportunity to express your thinking in the classroom?

6. Does your educator provide feedback on homework?

7. Does your educator allow you to work on your own to solve geometry riders?

8. Will technology assist in learning geometry?

9. Will visualization assist in in-depth thinking of geometric shapes?

APPENDIX 10 TURNITIN REPORT

An exploration of the barriers to effective geometric thought in the Further Education and Training phase of selected secondary schools in the Umlazi District

ORIGINALITY REPORT

14%	12%	2%	5%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	uir.unisa.ac.za Internet Source	2%
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