The pedagogical content knowledge (PCK) of Rwandan grade six mathematics teachers and its relationship to student learning

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Declaration

I, Jean Francois MANIRAHO, declare that

1. The research reported in this thesis, except where otherwise indicated, is my original research.

2. This thesis has not been submitted for any degree or examination at any other university.

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Signature of student

Date: 17th/ July/2017

Dedication

To my loving and prayerful dad, Maniraho, and my late mum, Mukayuhi, who gave me the gifts of love and knowledge.

To my lovely wife, Ernestine, who has been my inspiration throughout this challenging journey, for continuously and singly supporting the family while I was away, and to our children, Neza and Shulman, whose curiosity, always asking ‘Why is dad away?’ encouraged me to work even harder so that I could go back to the family.

My deepest thanks.

This could not have been achieved without you.

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Abstract

This thesis documents a study of the pedagogical content knowledge (PCK) of Rwandan grade six mathematics teachers, as demonstrated through tests and through their teaching, and its relationship to learning, as indicated by improvement in learners' test scores. This study is the first exploration of its kind into the pedagogical content knowledge of Rwandan mathematics teachers and its relationship to their content knowledge, teaching and their learners' learning.

Five research questions guided the research:

- How do Rwandan grade six learners perform on a standardized mathematics test, and what learning gains do they achieve over the course of grade six?
- What is the level of declarative knowledge, in particular content knowledge and PCK, of Rwandan grade six teachers?
- What is the nature and extent of the practical PCK (see section 2.3.2) of grade six teachers?
- How do teachers' content knowledge, declarative PCK and practical PCK relate to each other, and to background factors such as education, socio-economic status and teaching experience?
- How do learners' background factors and teachers' declarative and practical knowledge relate to learners' achievement gains over the course of grade six?

The study was positioned in the context of teacher knowledge. As PCK has not been clearly defined in the literature, the notion of *Mathematical Knowledge for Teaching* was utilized.

To a large extent, the study replicated three previous studies carried out in South Africa and Botswana, which enabled comparison of the results across these studies. This study included a detailed analysis of the teaching practices which were documented, which was not included in the other studies. For this purpose, a framework of descriptors was developed. This framework represents a theoretical contribution to this field of study.

In terms of the methodology of the study, the research tools included a teacher test, a teacher questionnaire, video recording of lessons, learner questionnaires and learner pre- and post-tests. The sample was chosen through stratified random sampling, and included 20 teachers from different schools in Rwanda, and 638 learners. The data were collected during 2013.

The analysis of the learner pre-test indicated that the Rwandan grade six learners performed well, in particular on the SACMEQ numeracy level designated as *basic numeracy*. In addition, they demonstrated significantly greater improvements in their test scores by the end of grade six than their counterparts in the South African studies.

The teachers' PCK test scores were positively correlated with their content knowledge scores. The results suggest that Rwandan teachers are more skilled in unpacking mathematics, whereas

South African teachers are more skilled in recognising learners' mathematics thinking. Both groups of teachers displayed content knowledge difficulties within some areas.

The analysis of video-recorded lessons indicated that most of the participating teachers accessed learners' prior knowledge but did not use it to inform their teaching, and that self-feedback dominated, potentially negatively affecting learners' self-esteem. Practices which have been found, in research, to be effective for facilitating learning were observed during some lessons, such as sharing of seat work and giving process feedback. Other effective practices, such as making connections and linking content, were observed infrequently during the lessons, which may highlight an area where intervention would be beneficial.

Completion of on-the-job training was positively correlated to some aspect of teachers' demonstrated practical PCK, such as mathematical content construction. The teachers' level of education was only significant in terms of its correlation to the types of feedback teachers provide: teachers who had completed some tertiary education before their teacher training never used task/product feedback.

Only two background factors in learners' lives were found to have a significant correlation to their learning gain: learners who were roughly the expected age for grade six, and learners who attended private schools, achieved greater learning gains. Learning gains did not correlate to teachers' declarative knowledge scores. They did correlate to two aspects of practical PCK observed during the lessons: learning gains were lower in classes where teachers were observed less frequently engaging content connections ($p<0.01$), and higher in classes where teachers were observed more frequently engaging tasks ($p<0.1$). It appeared that teachers addressing learners' misconceptions individually might have a slight negative correlate with learning gains ($p=0.048$).

The main contributions which this study has made to this area of research are as follows: the development and testing of a descriptive instrument for PCK as demonstrated in teaching; documentation of teaching practices in Rwandan mathematics classes, which suggests variation in practical PCK across teachers; the finding that Rwandan learners have good mastery of *basic numeracy* by grade six and achieve substantial learning gains in mathematics during grade six; and the tentative finding that PCK as demonstrated in teaching, with the few exceptions mentioned above, does not correlated with learning.

The study does not claim to have developed the ultimate language of description for practical PCK in mathematics education, and further refinement of the descriptive instrument developed in this study is recommended. The study also raises questions about the reasons for the differences in teaching and learner performance noted across different African countries, which could be a valuable area for further research.

Acronyms

ANOVA	Analysis of Variance
CK	Content Knowledge
CKTM	Content Knowledge for Teaching Mathematics
HCK	Horizon Content Knowledge
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
KZN	KwaZulu-Natal
MfT	Mathematics for Teaching
MINEDUC	Ministry of Education of Rwanda
MKT	Mathematical Knowledge for Teaching
PCK	Pedagogical Content Knowledge
SA	South Africa
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SCK	Specialized Content Knowledge
SES	Socio-Economic Status
SFAR	Students Financing Agency of Rwanda
SPSS	Statistical Package for the Social Sciences
TIMSS	Trends in International Mathematics and Science Study
UKZN	University of KwaZulu-Natal

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1 BACKGROUND, PROBLEM FORMULATION AND AIMS

1.1 Introduction

After completing my undergraduate degree and being appointed as a tutorial assistant for mathematics at the former Kigali Institute of Education, currently known as the University of Rwanda (College of Education), I became aware that we had relatively few students who had chosen to study mathematics. The prevalent explanation was that mathematics is difficult compared to other subjects. This challenged me to explore whether the subject itself was really more difficult than other subjects, or whether their perception of it had something to do with how it is taught in Rwanda. I found that there was little data on this topic, and ultimately I decided to pursue a PhD program in Mathematics Education in order to interrogate the nature and quality of mathematics teaching in the country.

Fortunately, I obtained a scholarship from the Government of Rwanda/SFAR to undertake doctoral studies in South Africa, which helped me to pursue my goal of studying the pedagogical content knowledge of Rwandan grade six mathematics teachers, its manifestations in teaching and its relationship to learning.

I believe that the topic I have chosen is significant not only for Rwanda but also for the region and even, more generally, at the international level. This is due in part to the fact that the socio-economic and contextual factors relating to this study differ from those in most of the existing research conducted in this domain. These include cultural aspects such as the impact of culture on teachers' behaviour in the classroom, on their attitudes towards learning and on the relationships between the learners themselves (Broadfoot, Alexander, & Phillips, 1999). On a personal level, as a teacher myself, although the study was focussed on Rwandan grade six mathematics teachers specifically, it has contributed to my existing knowledge, specifically with regard to the role of pedagogical content knowledge (PCK), also called knowledge of content and students (KCS), in classroom teaching situations.

During my literature review I could find no previous study of this kind which had been conducted in Rwanda. This study thus contributes new insights into the knowledge and practices of Rwandan grade six mathematics teachers, which may be of value not only to Rwanda but to the region in general. The Rwandan education sector could benefit from the findings and recommendations of this study, should it wish to review its current initiatives to educate and develop mathematics teachers.

1.2 Context of the study

Formal education is believed to have been introduced in Rwanda around 1900. In Rwanda, the term 'formal education' refers to preschools, primary schools (grades 1-6), secondary schools (grades 7-12) and universities. While nursery schools are generally managed through parents' initiatives, the latter three are controlled by the national Ministry of Education. The first three levels of the education system listed above fall into three categories, namely: private, state owned and semi-independent schools – such as religious schools which receive public funding. In this study, the categories of state owned and semi-independent schools are considered to be public schools.

To pass from primary to secondary school, grade six learners are required to sit for a compulsory national examination. In grade nine all learners must sit for another national exam to determine their subject specialisations. State schools are free for all children up to grade twelve in line with Rwanda's commitment to 'education for all'.

Although mathematics has been given status through Rwanda's proclaimed vision of constructing a knowledge-based economy, fewer students are accepted for mathematics or science programmes at university than for programmes in the humanities and arts because of the high rate of failure in national mathematics examinations at all levels (MINEDUC, 2003).

African countries, in general, face a challenge regarding the language of instruction in their schools. Frequently the language of instruction is not the learners' mother tongue. This can have a negative impact on learners' understanding of the subject matter, and thus on their performance. This has been well documented in the case of South Africa (Christiansen & Aungamuthu, 2012; Gerber, Engelbrecht, Harding, & Rogan, 2005; Setati, Chitera, & Essien, 2009). In Rwanda, Kinyarwanda is the language used for teaching during the first three years of primary education and then from grade four this changes to a European language. From the end of Rwanda's colonization by Germany and Belgium until 2008 this was French, but in 2008 the language of instruction for the higher grades was changed to English (Gahigi, 2008; MINEDUC, 2013). This implies that when grade six learners sit for the national examination that paves the way to high school, which is given in English, they have only had three years of instruction in English. This applies to mathematics as well.

The international Millennium Development Goals for education are another factor influencing the Rwandan education system. Rwanda is committed to the international development targets for education such as *education for all* (EFA) (MINEDUC, 2003), which commits to compulsory and free primary education for each and every child. Although the objectives are commendable, practice lags behind because of insufficient infrastructure and other basic needs, as well as inadequate training to enable teachers to handle the challenge of introducing

English as the medium of instruction (MINEDUC, 2010). Due to this fact, some parents have been concerned that their children may have been promoted to the next grade without sufficient knowledge or adequate preparation because the teachers found it impossible to cover all of the required content. This issue has been highlighted by Schollar (2008), who argues that if learners are routinely promoted from one grade to the next without having mastered the content and foundational competences of preceding grades they will face an increasing cognitive backlog that progressively inhibits their acquisition of more complex competencies. In Rwanda the extent of this practice, the type of teaching used in schools and learners' learning before the national examination has not been interrogated before the present study.

1.3 Statement of the research problem

The main research question which this study explores is:

What types and levels of pedagogical content knowledge (PCK) are used by Rwandan mathematics grade six teachers, and how are these related to their own content knowledge, their teaching and their learners' achievements?

The hypothesis was that the PCK of Rwandan mathematics grade six teachers is positively correlated to their content knowledge, their teaching, and their learners' achievement.

The main research question has been subdivided into the following research questions:

1. How do Rwandan grade six learners perform on a standardized mathematics test and what learning gains[1] are achieved over the course of grade six?
2. What is the level of declarative knowledge, in particular content knowledge and PCK, of Rwandan grade six teachers?
3. What is the nature and extent of the practical PCK of the grade six teachers?
4. How do teachers' content knowledge, declarative PCK and practical PCK relate to each other, and to background factors such as education, socio-economic status and teaching experience?
5. How do learners' background factors and teachers' declarative and practical knowledge relate to learners' learning gains over the course of grade six?

I have focused on grade six mathematics lessons in order to find out about the ways in which teaching and other factors influence learners' performance. As discussed in Section 4.3, my overall choice to focus on PCK, as well as my choice of research approach, was informed by a constructivist perspective on learning. The choice of grade level was made in order to enable

[1] 'Learning gain' is used throughout this thesis to refer to the difference in a learner's performance on the post-test compared to the pre-test. I am aware of the problem of labelling this as a learning gain, as it is based on two test performances only, but found no better term, hence this explanatory footnote. See also Section 5.5.

comparison with previous studies conducted in South Africa and Botswana (Aungamuthu, Bertram, Christiansen, & Mthiyane, 2010; Carnoy, Chisholm, & Chilisa, 2012), as well as with the SACMEQ studies (Hungi et al., 2010) – all of which have focused on grade six. Hence, this study attempts, in part, to replicate these studies, in the sense that I have used the same tests and research approaches, but with the addition of a more research-informed instrument for interrogating the PCK of teachers.

1.4 **Aims**

The major aim of this study was to determine the types and levels of PCK of Rwandan mathematics grade six teachers and to examine how this relates to their content knowledge, their teaching and their learners' achievements.

The sub-aims of the study were to:

- Provide a critical commentary on mathematics teaching in Rwanda, with the potential to inform practices around teacher education and development,
- Enable a comparison with equivalent studies from Southern Africa (Aungamuthu, Bertram, Christiansen, & Mthiyane, 2010; Carnoy, Chisholm, & Chilisa, 2012) and elsewhere (Sorto, Marshall, Luschei, & Carnoy, 2009), and
- Contribute to the research on the role of PCK in teaching and learning.

While no study of this kind has been done in Rwanda previously, researchers have pointed to the need for studies on teachers' PCK and its links to learning. The reasons are multiple, as will be unpacked further in later chapters of this thesis and below. One reason is that there is little empirical analysis to help policy makers understand the low level of learners' performance in schools or how to improve it (Carnoy & Chisholm, 2008).

On the one hand, positive connections have been found between mathematics teachers' performance on tests of PCK and their learners' performance in USA and Germany, two developed contexts. On the other hand, in developing countries the links between mathematics teachers' performance on tests of PCK and their learners' performance (e.g. North West Province of South Africa and Botswana) remain weak (see literature review in Chapter 3). As I will discuss later, many of these studies do not engage the practical PCK of teachers in detail, making the practices of teaching a 'black box' in the understanding of links between teachers' knowledge and learners' learning. Thus, this study improves our informed understanding of the role of teachers' knowledge in facilitating learning in the Rwandan context by comparing its results with the outputs of other studies done in different contexts and with different instruments of measurement, and by unpacking the link between knowledge and classroom practice.

1.5 Outline of the research process

The study commenced in January 2013. First, I piloted my learner test to interrogate if the test would be able to capture a spread in performances, i.e., not be consistently too easy or too hard for the majority of learners. Based on a simple analysis of the piloted learner test, I revised some of the test questions as the analysis showed that some questions were above the learners' level of skill and competencies. The final version of the test was almost identical to the test used in the previous studies conducted in South African and Botswana.

Around the end of January 2013, I obtained authorization to collect data for my research from the Rwandan Ministry of Education. After getting this authorization, I visited the Rwandan National Primary Schools' Inspection Office to obtain information on the locations and socio-economic classifications of the schools in my target areas. During the same period, I travelled around the country to establish first contact with potential research sites. Some schools rejected my request; teachers at these schools seemed uncomfortable with having their lessons video recorded. The fact that I had to negotiate access with school representatives, who were not always available, required me to make more than two visits to some schools. During these visits I also had to meet the learners' parents in order to inform them about my research and obtain their consent for their children's participation in the research. Thus, I learned first-hand how time consuming gaining access to data collection sites tends to be.

Data collection started in the first week of February 2013. I started with the rural schools first. Collecting data was not a simple task because at each school I had to oversee the completion of learner questionnaires, learner tests, teacher questionnaires and teacher tests, as well as collect parents' consent forms and video record a lesson – all on the same day. In some schools it went well, but in most cases I had to come back the following day to complete the video recording of a lesson.

Rwanda's transition from the colonial language of French to English has been a difficult process not only for learners but also for teachers. I was often requested to translate some questions for respondents on both the tests and questionnaires. It may also have proven difficult for some teachers to teach a lesson in English in front of a video camera, and at least one teacher cut the lesson short because of this problem. This same teacher struggled substantially to complete the questionnaire and test, with the result that it was impossible to gauge his actual knowledge. In the end, this teacher was excluded from the sample, bringing the number of teachers tested down from 20 to 19.

I used the period between April and September to code learners' and teachers' test responses. At the same time I captured the data from the responses to both learners' and teachers' questionnaires. During October and November 2013 I conducted my second phase of data collection, during which I gave the learner post-test. During December 2013 I coded the learner post-test and captured the data from my second phase of data collection.

After completing data collection and capture in mid-February 2014, I started to interrogate the data. A considerable amount of time was taken up by analysing video recorded lessons as there were few instruments available (Ball, Thames, & Phelps, 2008; Ramdhany, 2010) to measure mathematics teachers' PCK. I found that the existing instruments dealt with theoretical PCK and looked at teaching in ways that I did not feel were satisfactorily connected to PCK. (I explain my reasons for developing my own instrument in greater detail in section 4.7.) Hence, my own instrument was developed through an iterative process of applying the categories I had generated to the video data I had recorded, adjusting the categories, and so on, until no further adjustments were deemed necessary.

In April 2015 I worked with a statistician to complete the final level of analysis and I completed the first draft of this thesis. The findings included in this thesis were obtained through a deep analysis which involved making judgments while analysing the video recorded lessons and making comparison and evaluations while analysing my quantitative data. While I was manipulating the data and doing some preliminary analysis in 2014, the outputs I obtained from my data enabled me to produce several research papers. To date I have produced four papers from my data analysis, in collaboration with my supervisor. Two of them have been published (Maniraho & Christiansen, 2015; 2016) and two others are under review with different journals.

In the subsequent section, I will explain how the chapters of this thesis are organised.

1.6 Structure of thesis chapters

This thesis is structured as follows:

Chapter 1 deals mainly with the background to the study, the problem statement, aims of this research and structure of the document. It also provides a timeline of the study.

Chapter 2 describes the conceptual frameworks used in this study. The frameworks described are essentially engaging mappings of teachers' knowledge and their pedagogical content knowledge in particular. Discrepancies between different frameworks are engaged, and clarification of terms used in this study presented.

Chapter 3 provides a literature review which was done based on the existing theories of pedagogical content knowledge, linking their significance to this study.

Chapter 4 explains the methodology used in the study, including methods for data sampling, data gathering, checking validity and the instrument which was developed to measure teachers' pedagogical content knowledge. The methods used for the statistical analysis of the data are also detailed here.

Chapter 5 presents the analysis of data collected from learners' tests, designed to measure learning gain. Thus, it provides the background for answering the first research question.

Chapters 6 and 7 both deal with teachers' knowledge by trying to characterize their declarative and practical knowledge respectively. This provides the background for answering the second, third and fourth research questions.

Chapter 8 explores the correlations between 'learning gain' and the learners' background variables, teachers' declarative knowledge and the teachers' demonstrated practical PCK. Thus, it provides the background for answering the fifth research question.

Chapter 9, the final chapter, presents the core conclusions of this study, based on the findings. It also discusses the contributions this thesis makes to the existing body of knowledge on this subject, limitations of the study and recommendations for future research.

2 CONCEPTUAL FRAMEWORK

2.1 Introduction

In this chapter I present the conceptual framework which has informed the concepts and instruments used in this study. The framework described in this chapter is the one I used to conduct my analysis of the selected PCK subcategories using the video-recorded lessons and other kinds of data gathered for this aim. In this respect, the conceptual framework I used has helped me to arrive at answers to my five research questions, mentioned in Section 1.3.

This chapter covers four main sections. After this short introduction, I will explore factors affecting learning (Section 2.2) by focusing on teachers' work in classroom activities. In this study, the analysis of data highlighted how some of those factors influenced learning in grade six mathematics lessons in Rwanda. When a teacher is teaching, it is his/her role to work with the intention to create a classroom environment which is conducive to learning. This requires him/her to take into consideration all the elements which make up the classroom learning environment and ensure that they work together harmoniously to facilitate learning.

In the next section (2.2), I review the factors affecting learning/learner performance and PCK research lines that have been developed by different theorists. I discuss these in relation to other conceptualizations and explain why I chose to use some of the teachers' knowledge categories and some PCK elements while leaving others out. The details on PCK have been highlighted in Section 2.4. The final section of this chapter (2.5) highlights how I drew on the work of other scholars to inform my choice of PCK sub-categories, specifically KCS and KCT (Knowledge of Content and Teaching), with links to SCK (Specialized Content Knowledge) as well as the development of indicators for these categorizations.

2.2 Factors affecting learning

Scholars such as Tikly (2011) and Carnoy et al. (2012) argue that learner learning is a function of the human and cultural capital that learners bring to school, the teacher's capacity to teach the subject matter (including their use of teacher content knowledge, which includes content knowledge, pedagogical knowledge and pedagogical content knowledge), the cognitive demands teachers make of learners in the classroom, the amount of time spent on the subject matter that is to be taught (curriculum), the quality of the teacher's pedagogy in the classroom and peer conditions in the classroom – such as learners' socio-economic background and the number of learners in the class (Carnoy et al., 2012).

Based on this understanding, it is obvious that for a learner to learn a number of factors come into play (Figure 2-1). Some of these are relevant to this study while others are not. Those which I consider to have relevance have been incorporated into my PCK instrument (Table 4-1) and include process factors such as teachers' practical PCK (cf. Section 4.3). The factors that have been excluded will be discussed at the end of this section. The framework which I have developed has been derived from the existing theories on teacher knowledge and on the effects of teacher knowledge observed during learning activity. The theories that have been drawn on most heavily are those put forward by Shulman, Ball, Grossman, Kanyongo, Schreiber and Hattie. All of them agree on the point that Content Knowledge and Pedagogical Knowledge intersect during teaching activity, which then gives rise to the concept of Pedagogical Content Knowledge as a descriptor for this category of 'knowledge overlap'. The PCK that teachers are supposed to possess should enable them to transform content knowledge into forms that are easier for learners to access. Figure 2-1 summarizes the overarching framework of factors influencing teaching learning outcomes; some of these, such as the home environments of learners and teachers, have been taken into account in this study.

The diagram presented in Figure 2-1 is based on ideas from Tikly's (2011) work. Its significance to this study is that it contributes to a better understanding of the range of factors which must be considered in order to be able to say anything about the effect of PCK on learning. Research indicates that there is a strong correlation between learner performance and socio-economic factors (Bayat, Louw, & Rena, 2014; Okioga, 2013). Accordingly, the socio-economic context has been taken into account in this study.

When considering factors, it is easy to make an assumption of causality; i.e., that the presence of certain factors causes certain effects. However, it would be naïve to claim that low socio-economic conditions *cause* low performance among learners – the mechanisms of causality are substantially more complex. Thus, these factors are interrogated in the study to determine their correlation to learner performance, but the study does not attempt to make a causal link.

As can be seen in Figure 2-1, theoretical pedagogical content knowledge and practical content knowledge are both assumed to impact learning outcomes through the practice of teaching. Keeping in mind that the study examines the influence of pedagogical content knowledge on learners' achievement, and learners' learning is a targeted outcome, I could not ignore the reality that learners can have misconceptions, and hence some ways of supporting their learning are preferable for assisting learners in constructing knowledge.

Both types of PCK are aimed at the acquisition of new knowledge or deepening existing knowledge. PCK thus assumes a perspective on learning which is more in line with constructivism than behaviorism. This is discussed further in Section 4.2.

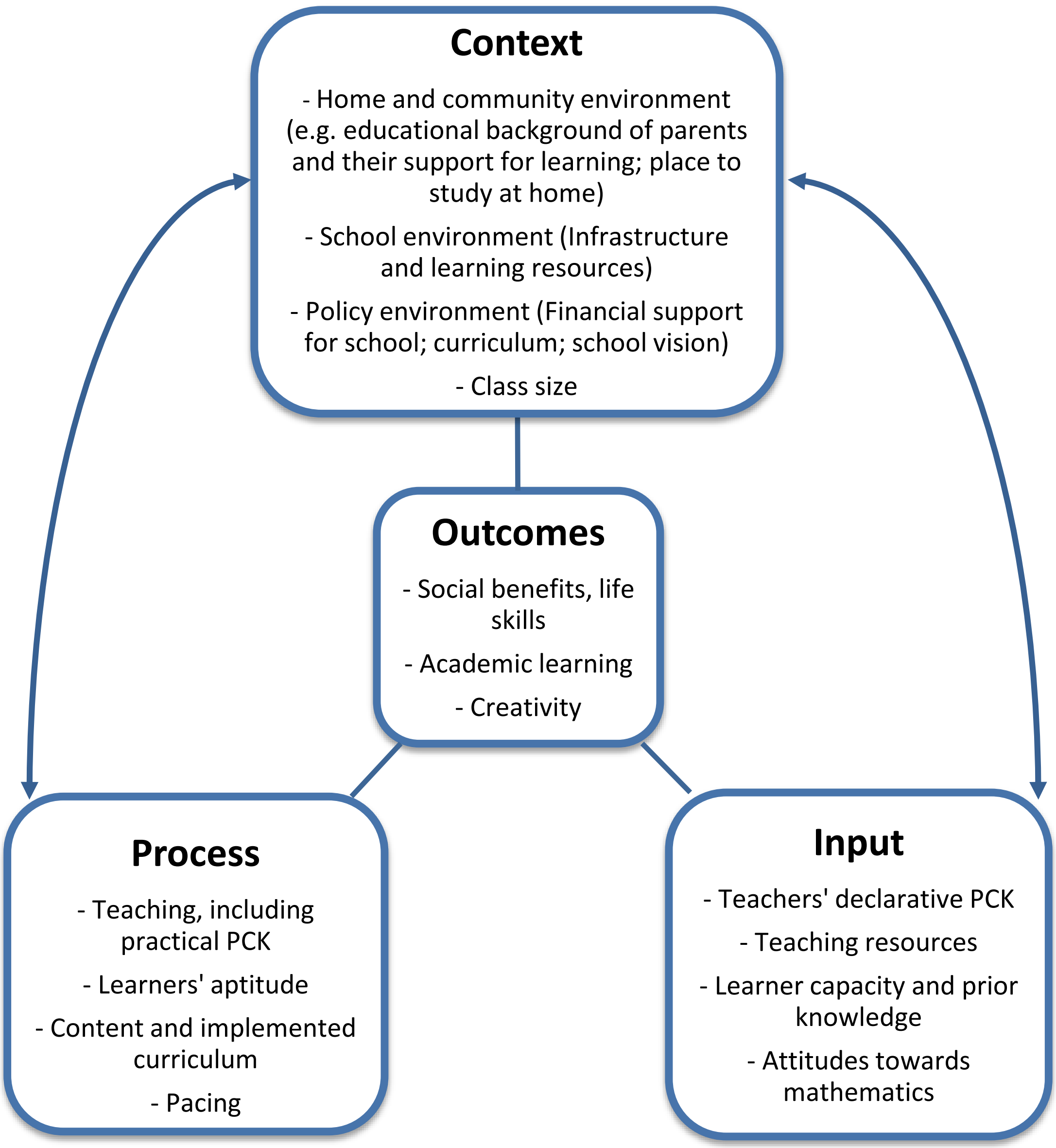


Figure 2-1: Factors which affect teaching learning outcomes

Figure 2-1 is also based on the assumption that it is next to impossible for learners to acquire new knowledge without taking into consideration the context of learning. In this study, the contextual aspect includes the educational background of parents and the support they give to their children (for example providing books to read or a place to study at home), classroom learning and teaching resources. However, there are some other factors which scholars have identified which can influence teaching and learning outcomes which have not been considered in this study. These include the attitude of both teachers and learners toward mathematics. To have investigated these would have required additional instruments which are notoriously difficult to adapt to different cultural contexts (cf. Andrews & Diego-

Mantecon, 2015). Others factors were not considered as important in the Rwandan context, such as teachers' access to health care, as in Rwanda all teachers receive medical coverage from the government.

Before ending this section, I would like to suggest that while there has been much focus on making teaching more 'participatory' or 'learner-centred' – a notion which, in my view, does little to assist with developing an understanding of what makes learning happen – existing research points to other factors as being more important. In particular, curriculum coverage and high cognitive demand appears to make a difference in mathematics education (Mewborn, 2001; Reeves, 2005; Spaull, 2011; Van der Berg et al., 2011).

The factors mentioned above all influence the methods that teachers use while performing the work of teaching in one way or another. In the next section, I focus in particular on the aspect of teacher knowledge.

2.3 Teacher knowledge

This section will review the main theories which have informed this study and the relationship between them, with a focus on pedagogical content knowledge.

2.3.1 Categorizations of teaching knowledge

Normally, teaching is seen as the act of helping a learner to learn and progressively function more independently; this means that a teacher is, in some way, a facilitator.

People who do not teach often state that good mathematics teachers should be competent in the mathematical computations which they are teaching (Bransford, Brown, & Cocking, 2000). However, many teachers who are good at performing mathematical procedures are unable to provide conceptual explanations for the procedures they perform. Studies in both North America and South Africa have demonstrated that many primary school teachers lack conceptual understanding of the mathematics they are expected to teach (Mewborn, 2001). SACMEQ has also explored this issue across Southern Africa (Makuwa, 2011). With the exception of Mozambique, it has been argued that the content knowledge of mathematics teachers is low across SACMEQ countries (Spaull, 2011). In Rwanda, before the establishment of teacher training centres, primary school teachers were trained to teach all primary school subjects and the content knowledge of these practicing mathematics teachers has not been studied.

Originally, seven categories of teacher knowledge were proposed by Shulman (1987, p. 8), namely: pedagogical content knowledge, content knowledge, general pedagogical knowledge, curriculum knowledge, knowledge of learners and their characteristic, knowledge of educational context and knowledge of educational ends.

By delineating these knowledge categories, Shulman attempted to identify all of the different types of knowledge which teachers are required to be equipped with professionally, based on observations of teachers. These categories have been used both normatively and descriptively. In practice, it is challenging, if not impossible, to separate Shulman's teachers' knowledge categories because they are so closely inter-related. His category dealing with general pedagogical content knowledge takes into consideration strategies and principles for how classroom activities are organized and managed based on the content considered at that particular moment. This is not unrelated to his knowledge category about teachers' knowledge of learners and their characteristics, because classrooms are managed in relation to the learners present (Krause, Bochner, & Duchesne, 2006). When it comes to Shulman's teacher knowledge category dealing with educational contexts, he suggested that teachers are expected to know the cultural community in which the school is situated (Shulman, 1986). This is in line with his teacher knowledge category about educational ends, purposes and the values governing it. To include this knowledge category reflects Shulman's view of professionalism meaning the ability to engage with the goals and values reflected in the classroom, not simply the mastery of teaching as a technical skill.

The last three categories – namely content knowledge, curriculum knowledge and pedagogical content knowledge – are more connected than the other four of Shulman's teacher knowledge categories. From my understanding, content is taught using pedagogical content knowledge, which Shulman defines as a special combination of content and pedagogy that teachers employ in order to make content more accessible to their learners (Shulman, 1987, p. 8). In selecting and organising content, teachers draw on curriculum knowledge about links between subjects and topics and materials which facilitate such linkage (Shulman, 1987, p. 10).

Of the types of teacher knowledge identified by Shulman in 1986, content knowledge, pedagogical knowledge and pedagogical content knowledge seem to come out top in influencing learners' outcomes, judging from the amount of attention they receive in the literature. However, the literature also shows that contextual/situational empirical research is still needed in order to interrogate how significant a role the different types of knowledge play in different contexts (Baumert et al., 2010; Hill, Rowan, & Ball, 2005). This suggests a need for a cross-cultural instrument which can be used to interrogate teachers' knowledge – especially in how it manifests in classrooms and relates to the learning opportunities provided.

Some researchers have suggested that at least four of the different kinds of knowledge identified by Shulman are essential for effective teaching (cf. Eggen & Kauchak, 2001). With regard to mathematics in particular, researchers such as Ball and Adler have gone further and engaged both theoretically and empirically with what they have called *mathematics/mathematical knowledge for teaching* (MfT).

Other authors have engaged the same elements in different ways. However, I have chosen to use the categorization provided by MfT in my work. Below, I have used these categories to generate an overview of the elements identified by different authors (see Table 2-1). What cannot be seen from the table is that even if the theorists agree on the subcategories of PCK, they may still disagree on which categories of knowledge are the most important. An example is the work done by Ball et al (2005), in which they measured mathematical knowledge for teaching with consideration of common knowledge of mathematics and specialized mathematical knowledge, but without consideration of horizon content knowledge, which is considered also to be an element of content knowledge in the work of Hill et al (2008).

Table 2-1 provides a summary of the different models of teacher knowledge developed by various scholars. With the exception of horizon content knowledge, all of the types of teacher knowledge are found in Shulman's work and have been adapted by other scholars in different ways. Two of the types of teacher knowledge, namely common content knowledge and specialized content knowledge, appear in nearly all of the models, while Shulman's knowledge of education ends and knowledge of educational context have not been taken up in any of them.

The challenge arises when one attempts to categorise the types of teacher knowledge shown above as declarative or practical knowledge. I will explore this distinction below, and then discuss the knowledge categories 'content knowledge' and 'pedagogical knowledge' in more detail. I will then go into a deeper engagement with 'pedagogical content knowledge' (PCK) and 'mathematical knowledge for teaching' (MfT), discussing how different authors have characterized and sub-divided these postulated knowledge domains.

Table 2-1: Types of teacher's knowledge appearing in various scholars' models

'P' stands for 'present in work of …'

SCHOLARS	TYPES OF TEACHERS' KNOWLEDGE							
	CK (Subject Matter Knowledge)			PCK				
	CCK	HCK	SCK	KCT	KCS	KCC		
	Common content of math	Knowledge of math 'horizontally'	Specialized mathematics knowledge	Knowledge of content and teaching	Knowledge of learners and their characteristics	Curriculum knowledge	Knowledge of educational ends	Knowledge of educational context
Shulman (1987)	P		P	P	P	P	P	P
Grossman (1990)	P		P	P		P		
Rowan et al, (2001)	P		P					
Ball, Hill, Schilling (2005)	P		P					
Adler (2006)			P					
Hill, Ball, Schilling (2008)	P	P	P	P	P	P		
Baumert (2010)	P	P	P					
Hurrel (2013)	P	P	P	P	P	P		

2.3.2 Declarative versus practical knowledge

A number of studies have been conducted on what teachers are required to know in order to teach effectively (Depaepe, Verschaffel, & Kelchtermans, 2013). Teacher knowledge, and in particular teachers' mathematics knowledge, is still an attractive area of research, as different scholars have proposed different criteria for teachers of mathematics, with variation between those teaching at primary school or high school levels (Adler & Davis, 2006). Bertram and Christiansen (2012) propose three key aspects of teacher knowledge as illustrated in Figure 2-2.

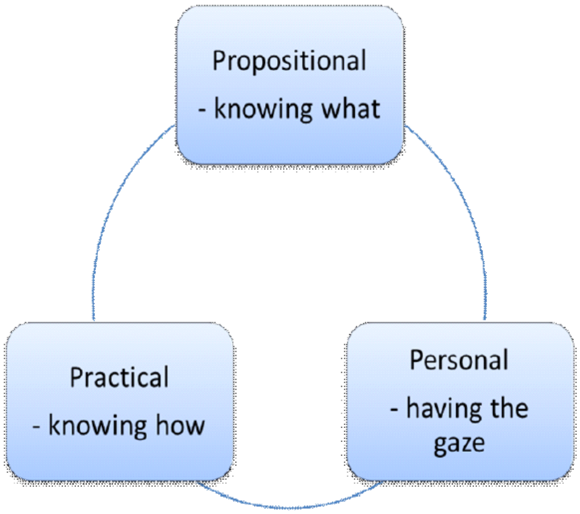


Figure 2-2. Three aspects of teacher knowledge

Replicated from Bertram & Christiansen (2012, p. 3).

Bertram and Christiansen (ibid) have classified teacher knowledge analytically. But while these knowledge types may be separated during analysis, they cannot be in practice, which raises the question of how they relate to each other (Bertram & Christiansen, 2012).

Based on their priorities in education, different countries set up different education policies which in turn influence their education systems. Some scholars (cf. Tatto et al., 2008) have gone further and proposed that the types of knowledge which should be emphasized in teachers' training depend on their programme.[2] That is professional knowledge, professional practice and professional engagement, which in my understanding are compatible with the three aspects of teacher knowledge described by Bertram and Christiansen.

Declarative knowledge is seen as more theoretical (knowing *that*) where the teachers consider their orientation towards their lessons, the curriculum and their knowledge of

[2]The team has used this framework to interrogate the teacher education systems in several countries, with Botswana as the sole representative of the African continent. Amongst other things, they found that there is a distinctive national character to each system (Tatto et al., 2008; Van den Bergh, Ros, & Beijaard, 2014).

learners' misconceptions and difficulties, and can both verbalize their decisions and justify them. On the other hand, practical (knowing *how*) is determined by the teacher's practice of teaching in the class and includes actions of responding to learners' questions and mistakes (Star, 2000).

While teacher knowledge is generally considered declarative knowledge, applying a more practice-based concept of knowledge would imply seeing PCK and general pedagogical knowledge as integrated with teachers' practice in the classroom. This has motivated some authors to distinguish between declarative and practical teacher knowledge, and thus also to engage the issue of the connection between these and how they develop. For instance, Star (2000) states that they are assumed to be distinct but related knowledge forms, and that the answer to which of the two comes first is: "it depends".

Other researchers have engaged teaching from the perspective of which teacher actions in the classroom are most strongly correlated with learner performance. The most comprehensive meta-study of quantitative research on teacher effectiveness is, to the best of my knowledge, the one overseen by Hattie (Hattie, 2013). His work suggests – unsurprisingly – that high levels of teacher engagement are more effective. This includes the teacher being credible, using formative assessment, giving feedback, being clear, and having a good relationship with the learners (Hattie, 2013). While their small sample size prevents their results from being generalised, Baker and Chick (2007) found clear differences between the PCK of two teachers with different experience: while they often suggested similar ideas when discussing the same topics, the one with less experience performed better. This highlights the difference between knowledge and practice or, further, between declarative and practical knowledge. However, it does not pinpoint what a teacher needs to know in order to be able to do this effectively.

This is not the place to go into a discussion of the nature of knowledge or whether the knowledge that informs the practice of a practitioner is applied declarative knowledge, knowledge developed through reflection in practice, or a combination of these (see Schön (1983). However, for the purpose of engaging teachers' knowledge, it is important to be aware of a potential distinction between what teachers make explicit when asked by researchers and what is reflected in their practice, and not to assume a direct and simple connection between the two. For instance, one teacher may be able to list common learner conceptions in algebra based on literature but tend not to make such distinctions in actual teaching situations, while another teacher may battle to construct such a list but teaches in a way which counters or challenges the most common learner conceptions.

For this purpose, I work with two PCK constructs in this work. 'Declarative PCK' will be used to refer to PCK as it is revealed in 'declarations' in response to questions, where even an answer to a multiple choice question is considered a 'declaration'. 'Practical PCK' will be used to refer to what I interpret as manifestations of PCK in the practice of teaching.

What this implies is discussed in greater detail in Section 2.5. The analysis of declarative and practical types of PCK in the data is presented in Chapters 7 and 8 of this thesis, respectively.

In the sections below, CK and PK (pedagogical knowledge) have been discussed briefly because they are related to PCK and according to some researchers CK is one possible route to PCK (Krauss & Blum, 2012) or a prerequisite for teachers' PCK (Ball et al., 2008). After that, I engage in more detail with PCK.

2.3.3 Content knowledge

Content knowledge is understood as the knowledge teachers have of the subject matter they are teaching (Shulman, 1987). Teachers' content knowledge must represent a deep or profound understanding of the material in order to facilitate deep conceptual learning (Jordan et al., 2008). Shulman (1986) argues that teachers need not only to understand that something is so, but also *why* it is so.

Content knowledge is almost always included in the models of the fundamental knowledge sets which a teacher should have (Shulman, 1987; Mustafa, 2008) as it is next to impossible for someone to teach without a sufficient content knowledge of the subject matter s/he is supposed to deliver. Content knowledge has a positive influence on pedagogical content knowledge according to studies conducted in Germany and Costa Rica (Krauss, Neubrand, Blum, & Baumert, 2008; Sorto et al., 2009) while a study in Turkey found that content knowledge had a positive influence on effective teaching practice (Mustafa, 2008). However, studies in Southern Africa have not been able to confirm this (Hungi et al., 2010).

There is a substantial body of work in mathematics education which points to the importance of conceptual understanding in mathematics learning in particular, and this would generally imply that the teacher needs to have conceptual understanding him/herself (Kilpatrick, Swafford, & Findell, 2001; Ma, 1999). This implies what Ma (1999) refers to as deep and broad knowledge, so that concepts can be connected and given different representations. Content knowledge is a significant aspect of teaching because it affects planning, explaining, task setting, questioning and finally feedback and assessment (McNamara, 1991).

Even though my particular interest is not to examine what could affect teachers' knowledge, it is important for my study to consider CK in relation to the PCK of teachers, so as to 'factor out' content knowledge if indeed it can be considered separate from PCK. To do so, I considered the content knowledge subcategories identified by Ball and her colleagues in their work, where three types of content knowledge: common content knowledge (CCK), specialized content knowledge (SCK) and knowledge at the mathematical horizon (HCK) have been considered (Ball et al., 2008).

The common content knowledge is common to everyone who has studied a given subject. In this view, mathematics teachers have mathematics knowledge in common with other professionals who have studied mathematics. For example, any mathematics teacher would know what a fraction is and how to convert it to decimals. Specific teaching situations, however, may require additional specialised content knowledge (SCK). For instance, a mathematics teacher would need to understand why the long division algorithm works, be able to recognize variations of it, not simply carry out long division. That is also in line with knowledge at the mathematical horizon requiring teachers to be aware of the relationship between topics in mathematics they are teaching with consideration to both former and future topics in the curriculum – similar to Shulman's curriculum knowledge (Hill, et al., 2008).

In this study, I have used CK to refer to all of these aspects. However, as will be discussed in the methodology, not all aspects were represented in the research tool design. The teachers' test, for example, focussed mostly on SCK, while the observational data rarely enabled a distinction of level or aspect of CK.

2.3.4 Pedagogical knowledge

Chapuis (2003) notes that *pedagogy* can be a somewhat nebulous concept, as it is essentially the combination of knowledge and skills required for effective teaching without being specific to a particular school subject or discipline. It includes strategies to manage and organize a classroom (Shulman, 1987). To determine whether a teacher has such competency is however not a simple task as it requires that 'effective teaching' be defined.

It is easier to collect data on teachers' qualifications, experience, or training than to get a precise idea of their command of subject matter or their classroom behaviour (Gabrielle, 2009). Even if content knowledge (CK) was considered crucial by Shulman (1987) and prioritized at the top of his list of types of teacher knowledge, PK is often regarded as more fundamental to primary school teaching (Gess-Newsome, 1999). Pedagogical knowledge may be seen as implying an understanding of cognitive, social, and developmental theories of learning and how they apply to learners in the classroom (Rowan et, al; 2001). Thus, understanding how learners construct knowledge, acquire skills and develop habits of mind becomes easier if a teacher is equipped with deep pedagogical knowledge (Mishra & Koehler, 2006).

In this study, pedagogical knowledge was not interrogated on the teacher test, and the observational data did not provide any clear differences in practical pedagogical knowledge displayed.

As my study focused more strongly on PCK, I will now discuss this in more detail.

2.4 Pedagogical content knowledge

Pedagogical content knowledge has been seen as a complex type of teacher knowledge, and as such is not easy to measure. However, while theorists share a general idea of what PCK is, many disagree about what should actually be included in, or excluded from, PCK. Within mathematics education, specifically, there are discrepancies between the different conceptualisations of PCK, as Kaarstein's comparison of three PCK frameworks demonstrates (Kaarstein, 2014). What seems to be widely accepted is that the way to both clarify the notion of PCK and to make it easier interrogate empirically is to work with the 'components' which make up PCK. Furthermore, while arguments are often made as to the importance of PCK to teaching and ultimately to learning, scholarly evidence of how PCK relates to learners' mathematical outcomes is actually quite thin (Krauss, Neubrand, Blum, & Baumert, 2006). This is not helped by the difficulty of instrumentalizing the PCK notion so that it can be 'measured'. For instance, Hill, Ball and Schilling (2008) found that the tools which they used to measure one aspect of PCK (knowledge of content and students) were so imperfect that they advised the community to not rely heavily on their conclusions.

I will discuss these different aspects of PCK over the following pages, starting with a discussion of the origin and evolution of the concept.

2.4.1 Origin and evolution of the concept of PCK

An educational psychologist, Lee Shulman, coined the term *pedagogical content knowledge* (PCK). In his view, PCK should include the knowledge in practice which helps teachers to direct what is done in classrooms related to the organization of the content for pedagogical purposes. Shulman emphasized that the teacher is supposed to know *how* (pedagogy) and *what* (content) to teach (Shulman, 1987). After its introduction in 1986, PCK became, and has remained, a useful notion to practitioners and an interesting research topic. The notion of PCK in relation to mathematics education has been explored by a substantial number of researchers (Ball & Bass, 2000; Ball, Thames, & Phelps, 2008; Grossman, 1990; Krauss & Blum, 2012; Shulman, 1986). This, however, has also meant that Shulman's conception of PCK has received different criticisms from different researchers at different times.

The most influential criticisms of Shulman's conception posit that Shulman has not explained how PCK could be distinguished from other teacher knowledge types empirically (Bromme, 1995, Ball, Thames, & Phelps, 2008). Grossman, a colleague of Shulman's, suggested that curriculum knowledge, which Shulman took as an independent category of teacher knowledge, should have not been separated from PCK.

Four years after Shulman's publication, Grossman (1990), with a focus on examining the qualifications which should be required of those entering the teaching profession, suggested four components which needed to be taken into consideration when considering PCK from Shulman's perspective, namely: knowledge about the purposes of teaching; knowledge of students' understanding and potential misunderstanding; knowledge of curriculum and curricular materials; and knowledge of instructional strategies and representations for teaching particular topics. In the same year, Mark (1990) restructured Grossman's categories by including the knowledge of the subject matter as a PCK component.

To overcome those previous limitations, new models of PCK have been proposed that do not place pedagogical content knowledge in a separate knowledge category (Baumert et al., 2010; Gess-Newsome, 1999; Marks, 1990). Pedagogical content knowledge is seen rather as the bridge connecting content knowledge and the practice of teaching and as a highly specialised knowledge that teachers possess which combines subject-specific content knowledge with a pedagogical focus. Context also has to be considered while teaching a particular content (Bednarz & Proulx, 2009; Gess-Newsome, 1999; Marks, 1990).

Nonetheless, a considerable number of researchers in mathematics education have continued to use Shulman's notion of PCK as a starting point for their own work. In their early paper, Rowan et al (2001) assessed PCK based on two dimensions: teachers' knowledge of subject matter and teachers' knowledge of effective teaching practices in a given content area, thus rejoining two of Shulman's categories. While it may appear that the fewer the sub-aspects considered the simpler the task will be, Rowan et al. (ibid) found it challenging to write items and scenarios that provided clear and complete information to respondents using only two categories.

In mathematics education, the work of Ball et al. (2008) introduced *mathematical knowledge for teaching* (MKT or MfT) also known as *content knowledge for teaching mathematics* (CKTM), drawing on Shulman's ideas on PCK, but, as I will discuss below, also overlapping with Shulman's notion of content knowledge. In their view, *mathematics for teaching* involves an 'unpacking' of the mathematical concepts taught, which they measured through teachers' knowledge of content and students. It is a kind of special mathematics which is different from the mathematics that mathematicians or engineers need (Adler & Davis, 2006).

Apart from judging Shulman's PCK as being theoretical, the most obvious way in which MfT differs from PCK is that CK and PCK together form MfT (Ball et al., 2008), whereas they are dissimilar knowledge categories in Shulman's model. In addition, curriculum knowledge, which is an independent knowledge category in Shulman's view, is a component of PCK in MfT (Grossman, 1990; Ball, et al., 2008). The extensive work with MfT done by Ball et al. has the potential to provide the empirical evidence for a positive relation between teachers' PCK and learner learning outcomes (Deapepe, Verschaffel, & Kelchtermans, 2013). This has pushed me to consider their perspective on PCK in this research. However, there are competing perspectives on PCK, as I will discuss below.

2.4.2 Characterizations and components of PCK

PCK has usually been defined either in general or in specific terms but tested as declarative knowledge. Different views of which aspects of PCK are most important have led to academic debates and have prompted different authors to put forward divergent ideas about which types of knowledge should be included within PCK. Disagreement on what should be considered the key aspects or sub-categories of PCK continues to present a challenge to this day. Some authors, in their study of PCK, have focused on the role played by the content, even if their perception of PCK also included pedagogy (Sorto, Marshall, Luschei, & Carnoy, 2009). I will discuss two different attempts at characterizing PCK, and then position this work based on the notion of PCK used in other work.

Shulman placed greater emphasis on PCK by putting forward the knowledge of students' (mis)conceptions, knowledge of instructional strategies and representations as the pillars of PCK (Shulman, 1986). Others in mathematics education, such as Ramdhany (2010) agree with Shulman that the key aspects of PCK are teacher knowledge (sound content and curricular knowledge); an understanding of how learners think and the ways in which they learn; an ability to use representations and examples to make the subject matter comprehensible to learners; an ability to identify and address learner errors and misconceptions; and an ability to teach in a way that makes connections between the learners' prior, current and future knowledge.

Other theorists argue that PCK should be divided into three types of knowledge as follows: knowledge of the multiple solution paths of mathematical tasks; knowledge of learner misconceptions and difficulties; and knowledge of mathematics-specific instructional strategies (Jordan et al., 2008; Krauss, Neubrand, Blum, & Baumert, 2006). Krauss et al. (2006) recommend that as teacher training may impact PCK, information about training should be included when designing questionnaires for assessing PCK.

Hill, Ball, & Schilling (2008) focus on only the KCS aspect of PCK, using the four major categories of common learner errors; learners' understanding of content; student developmental sequences; and common learner computational strategies. In my opinion one cannot assume that an evaluation of teachers' 'theoretical' KCS will correspond to classroom practice, as teachers may well exercise more contextualized versions of KCS where, for instance, a learner's understanding of a specific concept or process is seen against her/his history of understanding.

Hill, et al. (2008) have put forward a model of PCK which uses the categories of *knowledge of content and students* (KCS), *knowledge of content and teaching* (KCT), and *knowledge of curriculum* (KC). They put emphasis on KCS as a subset of PCK, which itself is a subset of the larger construct of what they call *mathematical knowledge for teaching* (MfT). Based on their analysis of the mathematical demands of teaching, Ball et al., (2008) hypothesize that Shulman's categories of content knowledge and pedagogical content knowledge can be subdivided in these respects, as illustrated in Figure 2-3.

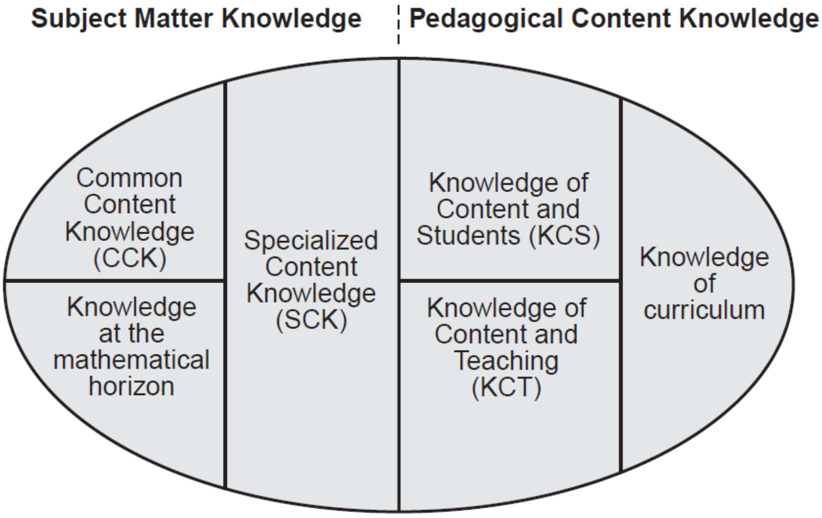


Figure 2-3. Domain map for mathematical knowledge for teaching

(Replicated from Ball, Thames, & Phelps, 2008, p. 377)

For Ball et al. (2008), KCS, KCT and *knowledge of content and curriculum* (KCC) are sub-categories of PCK, while other aspects such as *common content knowledge* (CCK), *horizon content knowledge* (HCK) and *specialized content knowledge* (SCK) are sub-categories of *content knowledge*, which form MfT when added to PCK.

This model can be critiqued for its strong separation of CK and PCK. Perhaps the work of the German COACTIV team implies a stronger connection of PCK to CK. They work with three subscales to measure PCK, namely: knowledge of mathematical tasks (task), knowledge of learner misconceptions and difficulties (learner) and knowledge of mathematics-specific instructional strategies (instruction), based on the assumption that the potential of tasks for learners' learning can be exploited by considering various solution paths (Krauss, Neubrand, Blum, & Baumert, 2006).

COACTIV's approach deals primarily with the way teachers explain and represent mathematical content, their knowledge of how they relate mathematics and learner cognitions, and teachers' knowledge of the importance, purpose and nature of mathematics tasks. For them to do such investigations, lesson scenarios presenting knowledge about typical errors and difficulties of learners and knowledge about several possibilities to solve mathematical tasks were used. Some of their test items were more related to Ball et al.'s (2008) conception of SCK, which might reflect different views of PCK between the German study and the U.S. study (cf. Bertram & Christiansen, 2012; Kaarstein, 2014).

In Australia, Beswick, Brown, Wiright, and Watson (2001) identified the components of PCK as: identifying learners' errors or misconceptions; constructing or using tasks and tools for developing learners' understanding; knowledge of a range of representations of a particular mathematical idea; and the way ideas are explained to learners.

In Table 2-2, the components of pedagogical content knowledge are presented according to different scholars' conceptions of PCK. Some models have more elements in common than others.

Table 2-2: Components of Pedagogical Content Knowledge

Extended from Soonhy (2008) and Depaepe (2013). 'P' indicates that the category was present in the referenced text.

SCHOLARS	Student understanding	Teaching purposes for a subject matter	Curriculum	Instructional strategies and representation	Assessment	Context	Pedagogy	Subject matter	Math tasks & cognitive demand
Shulman (1987)	P			P	P				
Tamir (1988)	P		P	P					
Smith and Naele (1989)	P	P		P					
Grossman (1990)	P	P	P						
Cochran et al. (1993)	P					P	P	P	
Geddis et al. (1993)	P		P	P					
Even (1993)	P			P					
Fernandez-Balboa &Stiehl (1995)	P	P		P		P		P	
Magnusson et al. (1999)	P	P	P	P	P				
Rowan, et al. (2001)	P		P				P		
Hasweh (2005)	P	P	P	P	P	P	P	P	
Ball, Hill, Schilling (2005)	P								P
Loughran et al. (2006)	P	P		P		P	P	P	
Adler (2006)	P								P
Ball et al. (2008)	P		P	P					
Krauss et al. (2008)	P			P					P
Hill, Ball, Schilling (2008)	P			P					P
Baumert et al. (2010)	P			P					P
Nilssen (2010)	P					P	P		
Watson and Nathan (2010)	P			P					P

An example of a component embraced by different scholars is the inclusion of learner understanding as a component of PCK. This shows how powerful knowledge of learners' understanding is considered to be, and must be seen in the light of the widespread research on this in the wave of constructivism becoming a widely-held theory.

While there are overlaps between the views of teachers' knowledge as shown in the overview above, there are also discrepancies. For example, the work of Grossman (1990) suggests that pedagogical content knowledge should include curriculum knowledge rather than considering it as a separate type of teacher knowledge. From Table 2-2, it can be seen that since 2005, scholars have included mathematics tasks and cognitive demand as PCK components. An additional issue is that the boundaries of the different categories may be perceived differently by different researchers. This is illustrated in the comparison done by Kaarstein (2014) which shows that some test items were classified as mathematics CK by Krauss and Blum (2012) and as mathematics PCK by Ball et al. (2008). An overview of the different authors' assessments of PCK as either declarative or practical knowledge is presented in Table 2-3.

Table 2-3: Overview of authors' orientations and their main methods

Extended from Depaepe (2013).

Authors with practical orientations and their main methods		**Authors with theoretical orientations and their main methods**	
Foss and Kleinsasser (1996)	Observations, video tapes focusing on instructional actions of teachers.	**Balboa and Stiehl (1995)**	Focusing on generic nature of PCK among college professors.
Stump (2001)	Transcription of video lessons, focusing on conceptual and procedural representation.	**Rowan et al; (2001)**	Multiple choice tests focusing on theoretical classroom scenarios.
Blanco (2004)	Classrooms observations on problem solving.	**Hill, Rowan, Ball (2005)**	Teacher questionnaires aimed to measure mathematical knowledge needed by teachers to teach mathematics.
Escudero and Sanchez (2007)	Video lessons and observation notes focusing on task instructions.	**Adler (2006)**	Test items on mathematical practices.
Koellner et al. (2007)	Videos of group teachers' interactions. Tasks were given to them and they were requested to think like learners to come up with lesson plans.	**Hill, Ball, Schilling (2008)**	Test items and cognitive interviews with focus on teachers' knowledge used in classroom teachings.
Ball et al. (2008)	Videos and audio tapes focusing on tasks and their mathematical demand in teaching.	**Watson and Nathan (2010)**	Interviews aimed at teachers' PCK investigation.
Tirosh et al. (2011)	Task-based observations.	**Baumert (2010)**	Test items to assess conceptual understanding on CK and on PCK.

The discrepancies between the views of what constitutes PCK have obviously led to differences in how the presence and extent of PCK are measured. As I will show below, the focus has been mostly on PCK as declarative knowledge, and this, I argue, necessitates looking at other research in mathematics education for what constitutes practical PCK. I will engage that in the last part of this chapter.

Interlude

My choice to evaluate the PCK levels of Rwandan grade six mathematics teachers using Balls' conceptualization of PCK (Ball et al., 2008) is detailed in the next section. Using video analysis I was able to construct an image of what is happening during classroom teaching based on the PCK criteria mentioned below. While some relate to KCT, others relate to KCS, KCC or SCK. However, it is possible to have one criterion which could reflect two types of PCK subcategories at the same time. While Ball et al (2008) place those criteria under MfT, I believe that PCK is the best framework for this study because SCK and KCC do not play a significant role in this study. In addition, based on the literature, I decided to exclude common content knowledge (CCK), which is included in the MfT map proposed by Ball et al. (2008), as CCK is not considered to be part of PCK.

2.4.3 Research instruments for measuring PCK

Pedagogical content knowledge has been interrogated empirically in different ways since it was first introduced as a defined concept in 1986 by Shulman. As shown in Table 2-3, some researchers have studied the practical manifestations of PCK, others only declarative PCK, while a few studies have engaged both. Elements of PCK are also targeted by some of the instruments designed for observing teachers' classroom practices, such as the schedule for categorizing *mathematical discourses in instruction* (MID) (Adler & Ronda, 2014).

Some of the researchers who have worked extensively on PCK (e.g. Neubrand, 2006; Stump, 2001) have focused on tests that were administered to teachers or to both learners and teachers which helped to map declarative PCK. The following example is extracted from the work of Ball and colleagues (2008).

$$\begin{array}{r} 307 \\ -\ 168 \\ \hline 261 \end{array}$$

This example illustrates of one of the most common errors in subtractions among grade three learners. According to Ball et al (ibid) the item was placed on the test with the purpose of identifying whether grade three teachers were able to recognize that the answer is incorrect and identify the most likely cause of the error.

Test items and/or interviews with teachers are the main instruments that have been used by a number of researchers (Adler & Davis, 2006; Ball, Hill, & Bass, 2005; Baumert et al., 2010; Rowan et al., 2001). These involve asking teachers to explain learners' errors, discuss how they would engage learners, consider learners' thinking, describe how they unpack methods for their learners, and teachers' ability to use a range of meaningful representation. These all focus on declarative PCK.

In my review of the literature on teacher knowledge (cf. section 3.3), as detailed in the following paragraphs, I found that most empirical studies of teachers' PCK assessed it using tests or interviews, while few engaged the PCK of teachers as demonstrated in their classroom practices. Test, questionnaire, interview, lesson observation, observation of meetings, document analysis (lesson plans, log books etc.) and concept mapping are the instruments most commonly used to measure pedagogical content knowledge (Depaepe et al., 2013). However, all these instruments are not used with the same frequency. The most widely used instruments, according to Depaepe et al., (2013, p. 19) have been document analysis (lesson plans, log books, etc.) and interview and meeting observations, followed by lesson observations and tests respectively. The favouring of instruments of this type highlights the lack of research on the relationship between teachers' PCK and learner learning outcomes (Deapepe, Verschaffel, & Kelchtermans, 2013), because it is virtually impossible to correlate teachers' PCK and learner learning outcomes using, for example, document analysis and interviews only, as they cannot properly reflect the classroom situation. This further demonstrates the need for this study, which has used tests, questionnaires and lesson observations to obtain measures of teachers' pedagogical content knowledge.

It is hoped that the instrument developed for this study enables new knowledge to be generated about declarative and practical PCK. Sorto et al. (2009) admit that due to the limitations of their instruments the empirical basis for understanding PCK and its influence on teacher effectiveness and learner achievement was very limited.

I would like to note that most of the studies done on PCK in mathematics education were conducted in the USA (Depaepe et al., 2013). In my view, this could be associated with the fact that the PCK model originated in the USA and Shulman's colleagues, such as Grossman and Marks, quickly took his ideas forward. However other factors must also play a part, such as the epistemological underpinnings of the PCK concept. Depaepe et al. (2013) found that 90% of the American studies did not define PCK, while the European studies used Shulman's original concepts. Of the 811 studies reviewed by Depaepe's team, only 6% were conducted in Asia and 5% in Australia (Depaepe et al., 2013). However, the 811 studies were identified through a search of only three databases, namely ERIC, Web of Science, and PsycInfo, and Depaepe et al. only reviewed studies done in English – both factors which could have biased the review. This still suggests, however, that more studies of PCK, especially in the developing world, are needed. Depaepe et al., (2013) found only one study from the African continent – a study that had been conducted in Botswana. While they did not appear in the Depaepe's search, studies have been conducted in South Africa, such as Ramdhany's study (2010) on teachers' practical PCK.

In the following sections I will discuss the way measurement of PCK has been conceived by different scholars.

2.4.3.1 Measuring declarative PCK

The declarative aspects of PCK have been studied more comprehensively than the practical ones. This is likely due in part to the fact that PCK has been understood as a form of knowledge, which is often considered declarative, but also because it is easier to use tests than to conduct in-depth observations in order to infer the presence or absence of PCK (c.f. Deapepe et al., 2013).

A variety of instruments have been used to measure declarative PCK. In this study I had participating teachers complete a multiple choice mathematics test to establish their declarative PCK. This approach is supported by Olszewski, Neumann, & Fischer (2010), who suggest that measuring declarative PCK using multiple choice tests has a number of advantages, including that the data that it generates is easy to code, but a better understanding can be gained if tests are complemented with interviews, observations or other qualitative methods.

As mentioned before, there are a number of instruments which have been used by different scholars to measure teachers' declarative PCK and it was not easy to develop an instrument which can reliably capture the full range of PCK. Rowan et al. (2001) report that one of the major difficulties they faced in their study was to develop items which could provide a clear indication of the teachers' pedagogical content knowledge, as some of their items were either too easy or difficulty for the teachers in the study. Ball et al (2005) used an instrument in which a participating teacher identified the instructional practices used to teach a particular content, along with a questionnaire which captured the teacher's educational background. This reflects the objective of many of the scholars to develop their instruments with Shulman's original conception of PCK in mind, necessitating questions on how teachers might deal with learners' common misconceptions and difficulties in learning a particular content and also the strategies which they use while teaching.

Researchers have developed tools to measure PCK based on the way they have defined it. For example, Baumert et al. (2010) opted for a one-dimensional instrument which combined CK and specialized knowledge items. The tests included tasks to assess: teachers' capacity to recognize various solutions which a learner can give and to recognize learners' misconceptions and difficulties; the strategies they use to identify learners' errors and comprehension complexities; and teachers' knowledge of different representations.

While the many variations between the declarative PCK measurements outlined by different scholars complicate matters, the complexity increases when trying to compare the different instruments and measurements. This problem arose in Kaarstein's work in which she compared three frameworks for measuring knowledge for teaching mathematics, namely the Professional Competence of Teachers, Cognitively Activating Instruction and the Development of Students' Mathematical Literacy (COACTIV), the Learning Mathematics for Teaching (LMT) and the Teacher Education and Development Study in Mathematics (TEDS-M) (Kaarstein, 2014). For example, the COACTIV project used questions which encouraged the teachers to give as many responses as possible, meaning that the scores for declarative PCK measurements were theoretically unlimited. Kaarstein (ibid) argues that one of the questions which the COACTIV project used to measure PCK could have been considered a CK question in another researcher's framework. Similarly, the LMT project, a CK question was judged by Kaarstein to be a PCK question as it captured the way teachers react in teaching situations (Kaarstein, 2014). So not only do researchers disagree about the relationship of sub-categories within PCK – in the COACTIV project, curricular knowledge was located within the other knowledge category, which was not the case in the LMT project – but the operationalizations do not always coincide. As Kaarstein argues, making declarative PCK operational in its basic categories is still problematic (Kaarstein, 2014).

More recently, since different studies have found evidence that content knowledge is positively correlated to pedagogical content knowledge (Deapepe et al., 2013), researchers have opted to include questions related to content knowledge in tests designed for measuring pedagogical content knowledge. That was the case in the teacher test used in this study, where some questions were related to content knowledge whereas other questions aimed to assess teachers' pedagogical content knowledge. The relationship of the two types of knowledge is explored in Chapter 8 of this study, based on data from the teachers' questionnaire.

2.4.3.2 PCK classroom indicators

Instruments to measure both declarative and practical PCK have been conceived of differently by different scholars (Kaarstein, 2014; Ramdhany, 2010). This is exacerbated by the characteristics of practical PCK, as it requires judgments to be made about what is happening in a classroom situation where different observers may judge the same event differently. However, as discussed in Section 2.3.2, some common elements – such as making connections, 'unpacking' the concepts and algorithms, addressing learners' misconceptions, the usage of representations, and assessing prior learner knowledge – have been put forward by a considerable number of PCK researchers. It is then the role of teachers to create classroom environments which incorporate these elements.

Classroom teaching is complex as it involves many different elements working together. Scholars such as Carnoy and Chisholm (2008) have treated PCK as a single concept, without clarifying how it can be recognized, making it impossible for their results to be compared with those of others or analysed. Other scholars, such as Ramdhany (cf. 2010, p. 24), have attempted to develop the concept further; Ramdhany, however, noted that his instrument would need further refinement in order to capture his selected PCK subdomains. For the same reason a more refined instrument was needed for this study which would reflect my selected PCK categories as well as the considerations which will be discussed below.

There are researchers such as Boston and Smith (2009); Hugo (2013); Mhlolo, Venkat and Schäfer, (2012) who posit that connections are important while teaching mathematics. Their ideas about the types of connections that should be made in a classroom situation differ, however. Barmby, Harries, Higgins and Suggate (2009) propose that representations related to the way people associate mathematics and the real world, association between mathematics and other school subjects, and the connections within mathematics itself have been observed frequently in what is considered good teaching. This is also in line with the work of Ma (1999), who posits that connectedness in teaching helps learners to learn a cohesive body of knowledge rather than fragmented parts.

The connections which teachers make in relation to a given concept while they are teaching are primordial to learners as they help them to acquire new knowledge. Yet they depend on the target which the teacher has in mind, as s/he may engage connections to motivate new content, for linking to applications, or for moving between abstract and concrete engagement with the content. Nuancing connection in this way may help researchers to examine this aspect of teachers' knowledge and practice.

A model has been developed which demonstrates mathematical connections, including different representations, implications, part-whole relationships, procedures, and instruction-oriented connections (Businskas, 2008). The question is how teachers represent these connections in their classroom teachings in order to introduce and clearly characterize the mathematical ideas which they want learners to learn. Different possibilities exist and can be used together or separately within the same lesson depending on the cognitive levels on which teachers wish to engage their learners. Different representations allow a concept to be presented in two or more ways and linked – as in algebraic connections and graphs; whereas with part-whole connections, one concept is linked to another either by inclusion or by generalization (Businskas, 2008). Using implications as logical connections provides opportunities to explain to learners how one concept leads to another; while instruction-oriented connections engage some concepts as pre-requisites for other concepts.

Mhlolo (2013) argues that teaching mathematics with variation has merits in terms of facilitating learners' understanding of mathematical concepts. However, as mentioned in the examples given in the work of Venkat and Adler, (2012) special attention is needed as stated problem/representation might not connect with the topic to be covered.

Mhlolo refers to four kinds of variations which may be engaged, often in this order: contrast, separation, generalization and fusion (cf. Mhlolo, 2013). These particular types of variations may be used in particular across examples, so as to make concepts stand out in increasingly specific ways.

That idea of using different **representations** of mathematical concepts was – to the best of my knowledge – first introduced by Lesh, Post and Behr in 1987. In their paper, they stress that it is in the connections between different representations that concept images are directed towards the scientific concept, so it is clearly one aspect of 'connections', though one that has received much attention on its own (Lesh, Post, and Behr, 1987).

"*The reason that one problem can be solved in multiple ways is that mathematics does not consists of isolated rules, but connected ideas,*" Ma explains in her seminal book (Ma, 1999, p.112). Engaging different ways of approaching the same problem helps to highlight mathematical structures and connections, and helps learners to engage in mathematical judgments, in particular if the teacher engages the learners in **unpacking** the methods. One way to do so is to show learners different methods/approaches and then compare or analyse them; another is to invite learners to devise their own methods and then compare or analyse these.

Ball (1988) argues that there are three ways in which teachers might respond to learners' claims during classroom teachings: direct the learner to pursue their ideas outside of the scheduled curriculum; evaluate the truth of their claim; or engage the learners in exploring the truth of their claim. All of these constitute forms of feedback. In active learning, the feedback which teachers give might be confirmative, critical, constructive or a combination of these (Van den Bergh et al., 2014). Whatever type of **feedback** is given, the main aim is to further learning, whether by correcting temporary or impartial conceptions which learners might have developed, helping learners to reflect upon their own work; or directing learners' attention to particular features of the content. During times when corrective feedback is used, it is useful for the teacher to be able to **identify learners' thinking**, about which a lot is known from previous research (Batanero, Estepa, Godino, and Green, 1996; Erlwanger, 1973; Liu, Lin, and Tsai, 2009). There are different strategies which teachers can use to do this. They may identify the error/misconception and provide the correct form, or indicate that the error/misconception has been made but not provide a correction, allowing the learner to correct it; Bitchener, Young and Cameron (2005) term these as direct (explicit) or indirect feedback strategies, respectively. While these are alternative forms of feedback, the focus of feedback may also vary: it can relate to the task and the correctness of the answer (product); it can relate to the process which the learner has used, directing the learner to more correct or more efficient ways of working; it can direct the learner to see patterns in her/his way of working and take responsibility for monitoring his/her own processes (self-regulation); or it can engage the personal (self) (Hattie & Timperley, 2007). Hattie and Timperley argue that feedback focusing on the learner's 'self' generally is relatively unproductive: it does not facilitate learning of specific content to be told that you are smart or stupid or a hard worker. Task feedback on its own is also not the most productive, they argue, but it improves if process feedback is added. They add that self-regulation feedback requires having correct information which is considered as a base on which it is efficiently constructed.

These ideas have helped me to identify categories which could assist me in capturing the ways in which teachers interact with their learners during classroom teaching. This was taken into consideration while designing the instrument used for analysing teachers' practical PCK in this study, which will be discussed in Chapter 4.

2.5 Indicators of PCK components related to this study

As has been demonstrated in the discussion thus far, it is challenging to determine meaningful indicators of the inter-related characteristics of the practical PCK of teachers, due to their intrinsic nature. Much of the difficulty results from the lack of clear definitions and concept boundaries, not for lack of trying, but due to the nature of the concept (as discussed in the previous section). Below, I attempt to sketch the PCK indicators which were used in this study. The instrumentalization of these (i.e. the development of my practical PCK instrument for analysis of classroom observations), is detailed in Section 4.7.1.

First, let me explore a well-known instrument designed to document what is happening in classroom teaching through structured classroom observation, the Mathematical Quality of Instruction (MQI) developed by Hill, et al. (2012). This instrument is designed primarily to measure the mathematical work which takes place in classrooms, specifically in middle and elementally schools (ibid). The instrument targets the quality of mathematical instruction through four main dimensions, namely: richness of the mathematics; errors and imprecision; working with students and mathematics; and student participation in meaning-making and reasoning (Hill et al., 2012). Each of these dimensions has sub-dimensions. For example, *working with students and mathematics* has the sub-categories *remediation of student errors and difficulties* and *responding to student mathematical productions in instruction* (ibid).

Despite the usefulness of this instrument, I found it necessary to develop my own tool, involving video analysis, for this study rather than using the MQI instrument, for the following reasons. Firstly, it is assumed that teaching actions reflect the overall pedagogical approach used by a teacher in various ways (cf. Naiditch, 2010). However, the MQI instrument is more theoretical, distinct from a pedagogical approach (Hill, et al., 2008), which then, once used, could diverge from KCT (one of the PCK sub-category), therefore being unsuitable for this study. Secondly, as Cohen, Raudenbush and Ball (2003) point out, the MQI instrument implies, to some extent, instructions focusing on classroom resources and the ways they are used. However, this could vary across different topics in mathematics and may not be the main approach in the Rwandan schools. I therefore need a more inclusive instrument.

Thirdly, I found that some of the sub-dimensions of the MQI instrument are not detailed enough to exclude the potential for confusion and hence yield results which may not be harmonious across contexts. For example, the MQI instrument works with linking and connecting mathematical representations, ideas and procedures but does not distinguish between the different kinds of connections and links (cf. Venkat and Adler, 2012) that teachers may make during classroom teachings. This is also the case for the category on working with students and mathematics, where the instrument measures whether teachers can understand and respond to students' mathematically substantive productions like errors (cf. Legutko, 2008) but does not consider the *way(s)* in which teachers react to or address these substantive productions. Another example is the MQI's sub-dimension for measuring the lack of clarity in teachers' presentations of tasks or content, which does not reflect the types of tasks (cf. Neubrand, 2006) likely to be given.

A final reason for my decision not to use the MQI instrument is that there are some classroom practices which are considered, in the literature, to be important in learning but which are not reflected in the instrument. Examples of these classroom practices are content construction (cf. Mhlolo, 2013), prior knowledge assessment (cf. Furner, Yahya, and Duffy, 2005) and the different types of feedback (cf. Hattie and Timperley, 2007) given to learners.

While I did not use the MQI instrument, and based my instrument primarily on an instrument used in a similar study done in South Africa, I did draw on the ideas of Ball et al. (2008) in terms of their interest in refining and empirically validating PCK (Deapepe et al., 2013).

As will be discussed in greater detail in Chapter 3, I worked primarily with two types of teacher knowledge, namely: Knowledge of Content and Students (KCS) and the Knowledge of Content and Teaching (KCT), as these reflect teachers' pedagogical content knowledge (PCK) both theoretically and practically, which was the focus of this study. Another concern was the limited time available for observations, which meant that I had to restrict myself to what could be identified within a single lesson.

2.5.1 Indicators of KCS

Knowledge of content and students (KCS) implies interpreting learners' thinking and reasoning while they are performing a task. In this study, the teacher test provided a measure of teachers' ability to do this, albeit decontextualized from the classroom, as teachers were asked to respond to examples of common misconceptions which learners exhibit while solving tasks.

The fact that teachers could identifying learners' misconceptions and errors on the test does not mean, however, that they can correctly explain the concepts that have been misunderstood during classroom activities. This is one reason I chose to use video recording of classroom teachings in this study. Ball et al. (2008) posit that teachers need to be able to predict what learners will find interesting and motivating when choosing examples to use during teaching. They argue that when assigning a task teachers need to have a sense of the approach learners are likely to use and whether they will find the task easy or hard. Teachers should also be able to interpret learners' incomplete thinking related to their language use (Ball et al., 2008), recognize their misconceptions and identify their difficulties and the strategies they use to solve a particular problem (Baumert et al., 2010).

In learning, it is generally assumed that some learners will attempt to use methods that are different than what they have been taught. In doing so, misconceptions might arise before teaching or during teaching and are likely to persist if not proved incorrect (Brophy, 1987). Swan (2001) argues that misconceptions may be related to the normal process of conceptual development or be a result of previous teaching. For example, in an earlier stage of conceptual development learners understand multiplication as producing a bigger number, but as concepts of multiplication are developed further the multiplication of two fractions produces a smaller number. The use of representation in teaching becomes important at this point. Assessing learners' prior knowledge can also be useful. Swan (2001) posits that an evaluation of learners' prior knowledge should involve a task on a well-known topic so that learners recognize their own interpretations, errors and misconceptions. Teachers then need to have the capability to recognize the errors and misconceptions as well and address them.

From the PCK perspective, having a knowledge of learners' understanding involves teachers knowing what the learners already know about the new topic so they can anticipate areas of difficulty. For a teacher to have a clear picture of this, s/he needs to know how learners conceive the topic s/he is teaching as well as understand their interests, their abilities and hence their needs, in order to be able to motivate them to learn the new concepts (Park & Oliver, 2008), which again requires effective use of teaching strategies..

These elements, as well as others related to classroom teaching, have been incorporated into my instrument for analysis of video-recorded lessons, as shown in Table 4-1.

2.5.2 Indicators of KCT

Knowledge of content and teaching connects in several ways with knowledge of content and students. On one hand, as mentioned in Section 2.4.1 above, a teacher needs to have knowledge of the learners who are to do a particular classroom task. This will allow the teacher to select an appropriate task for the topic which is being taught. On the other hand, Ball et al. (2008) state that many mathematical tasks in teaching require a mathematical knowledge of instruction in order to be able to select examples leading the learners into a deeper understanding of the content. They also argue that teachers need to recognize the advantages and disadvantages of the instructional methods they use to teach a given idea for them to come up with suitable representations to use.

The elements discussed above, where content and pedagogy interface, are at the core of the practical analysis of teachers' actions during teaching that is conducted in this study. The instructional decision is also an important aspect to consider. It is accepted that during teaching activities it is a teacher's role to determine which learners' input to consider and which to ignore. A teacher also has to decide when to break for more explanation or for the introduction of a new task and where to make a mathematical point (Ball et al., 2008).

Special emphasis has been made in this study of the fact that, theoretically, KCT captures what should happen in classroom teachings more clearly than the other PCK subcategories considered for this study. Some of the items on my teachers' test capture its relevance in classrooms. With those items, it was possible to model theoretically the instruction level of teachers in their classrooms as they capture the ways teachers support learners during teaching activities. However, I would like to note that practically, the manner in which teachers react to or respond to the learners' misconceptions on a particular task may also reflect his/her special content knowledge. This concurs with Ball et al. (2005) who posit that teachers need to know how to explain, listen and examine learners' work in order to be able to identify the source of learners' errors and choose constructive examples. Special content knowledge can therefore enhance teachers' knowledge of their learners' understanding on a given topic based on the types of errors and misconception learners are presenting either during classroom teachings or in their assignments, tests and homework.

This study interrogates teachers' knowledge of content and teaching (KCT) in terms of: their capability to make content connections in order to engage new knowledge within the lesson; their methods for constructing mathematical content through practices/variations; the types of feedback they give to learners; their methods for unpacking the methods⁄concepts to make the content more accessible to learners; their ability to engage learners on tasks which clarify key concepts.

In this work, in order to be able to map my sampled teachers' KCT, I took into consideration the way teachers explained standard mathematics problems and the different representations used, as well as the connections made between them (Baumert et al., 2010) when the topic required this. This was true also for observations of the nature of the mathematical tasks that were given and the cognitive demand placed on learners; together with the teacher's ability to identify multiple ways of solving a given mathematical problem and unpacking mathematics.

Before concluding this chapter, I would like to note that one of the KCC indicators included in my instrument was the way in which teachers relate the topics they teach to other topics covered in previous grades or to be covered in later grades. In my understanding, it becomes easier for a teacher to give applications of the topic s/he is teaching if s/he knows what the learners have been taught previously on the topic and will be taught in the future in order to be able to connect the current teaching to this.

I also would like to note that the subject material to be taught is presented in curricula. Ball et al. include Shulman's notion of curriculum knowledge under KCT, including both vertical and horizontal curriculum knowledge. From my understanding, being familiar with these may enable teachers to make appropriate links between content taught at different grades. It is then understandable that those above mentioned researchers support teachers having knowledge of content and curriculum during classroom activities.

2.6 Conclusion

This chapter has explored the various conceptualizations of PCK and its possible sub-categories as discussed in the literature. I have chosen to work with the notion of MfT due to its merits (cf. Deapepe et al., 2013) and its division of PCK into KCS, KCT and KCC. My focus is on KCS and KCT in particular and a framework has been outlined based on, among other things, the importance highlighted in the research of understanding learner thinking, providing constructive feedback, using representations and making connections. The third PCK subcategory in the MfT model (c.f. Ball et al., 2008), i.e. KCC, has only been engaged through observations of the connections teachers make, derived mainly from the teachers' questionnaire. The observation categories that were developed also have components of specialized content knowledge (SCK) – for more on this, see Section 4.7.

As one of the issues considered in this study is the correlation between CK and PCK and their relationship to learner learning outcomes, content knowledge is acknowledged as an important additional factor to consider, together with the various background variables shown in Figure 2-1. The differences between the MTK model first put forward by Ball et al. (2008) and the new conceptualization of MTK (Hurrell, 2013) which attempts to connect subcategories, particularly CK and PCK, have been noted. This conflict does not affect this study as it focusses on the correlation between CK and PCK, rather than on their direct relationship.

The theoretical framework allows for engaging both declarative and practical PCK components. My indicators of declarative PCK rely more on the work of Ball et al. (2008), whereas indicators of practical PCK draw on studies done on classroom practices.

In Chapter 3, which follows, I will review different factors that influence learning. Furthermore, I will highlight the major PCK research lines which have been followed by different scholars. The following chapter, then, summarizes the literature review which framed the study.

3 LITERATURE REVIEW

3.1 Introduction

In the previous chapter, I engaged the different factors that influence learning, moving from a very inclusive perspective to teacher knowledge then on to different characterisations of teacher knowledge and, in particular, the construct of PCK. Before that, I reviewed literature related to teaching and learning in general. My reading led me into literature that explored different types of teacher knowledge (Adediwura & Bada, 2007; Baaumert et al., 2010; Ball et al., 2008; Bertram & Christiansen, 2012; Gabrielle, 2009; Kanyongo & Brown, 2013; Sorto et al., 2009) and, of particular interest, pedagogical content knowledge (PCK) (Deapepe et al., 2013; Jordan et al., 2008; Mishra & Koehler, 2011; Marks, 1990; Punya Mishra & Koehler, 2006; Veal, 1999). Some of the literature which proved particularly helpful explored the ways in which researchers have attempted to measure teachers' PCK (Baker & Chick, 2006; Baxter & Lederman, 1999; Deapepe et al., 2013; Rowan et al., 2001).

Of special interest to me were studies which attempted to use video analysis as a tool for measurement (Neubrand, 2006; Ramdhany, 2010). I studied the indicators for both declarative and practical PCK that were used in previous research. As I found these often not to be well defined, I used further exploration of the literature to develop my own indicators.

This chapter summarises my review of the literature relating to the topics mentioned above.

3.2 Findings on factors affecting learning/learner performance

A number of factors have been found to impact learner achievement across contexts, including African countries; these include curriculum and policy, characteristics of individual schools such as leadership and culture; teachers' knowledge and teaching strategies, and learner backgrounds such as what they bring to the task and their home background (Carnoy et al., 2012; Hattie, 2003; Kanyongo et al., 2007). In a review of literature on this aspect (which, it should be noted, related mostly to studies conducted in the so-called 'developed' world), the factors which appeared to have the greatest impact on learning were learners' attitudes, background and aptitude (Carnoy et al., 2012; Hattie, 2013). Hattie estimates these factors account for 50% of learner performance, the teachers' role accounts for an addition 30%, and other aspects such as peers and the general school environment account for the remaining 20% (Hattie, 2003). Hattie argues that "*It is what students 'bring to the table' that predicts achievement more than any other variable; and it is what teachers know, do, and care about which is very powerful within schools*".

Studies from other contexts reflect some of the same relationships between factors, but also some deviations. For instance, while Hattie argues that a teacher's knowledge, actions and concerns matter in learner performance, in studies conducted across several of the SACMEQ countries the impact of teachers' subject knowledge on grade six learners' performance was found to be insignificant (Spaull, 2011). Van der Berg and Louw (2006, p. 12) take a similar line, arguing that "*The impact of having a good teacher is largely restricted to children of a higher SES – i.e. those with a family background that supports learning better are likely to reap substantially better benefits from good quality education than those that do not."* It appears that the variation in learners' performance in these contexts has very little to do with teaching. However, a recent study in Kenya found that *"quality of mathematics instruction is more critical in improving learning gains among low-performing students.*" (Ngware, Ciera, Musyoka, & Oketch, 2015, p. 111).

Spaull (2011) argues that in South Africa socio-economic factors are by far the strongest predictor of learner performance, reflected in greater variation in performance between schools than within schools. Other major factors, including language (Christiansen & Aungamuthu, 2012; Ouane & Glanz, 2011; Setati et al., 2009) and the educational background of learners' primary caregivers (Spaull, 2011) are not to be ignored, but are strongly linked to the socio-economic context, making it virtually impossible to treat them as separate factors.

In order to evaluate the role of teaching in learner performance in this study it was necessary to collect data on some of the factors mentioned above. However, as this was a replication study (see Chapter 4), the potential to do this was limited to some extent. I have engaged this in more detail in Section 4.6.

If teaching is the focus, then teacher's knowledge must be investigated.

3.3 Teacher knowledge

To date, no studies have been conducted on the mathematics content knowledge and PCK of Rwandan teachers, except for the work of Habineza (2015) which found that student teachers developed their concept images of the definite integral over the course of a semester's teaching, but to varying degrees. Studies of the CK of teachers in Southern Africa indicate that teachers' CK was relatively low; this was indicated by the fact that some teachers were not able to answer questions relating to the curriculum they were teaching (Carnoy et al., 2012). In a South African study, 40 teachers were asked short answer or multiple choice questions about the content they were teaching and were found to have inadequate PCK (Kaino & Moalosi, 2013). In international studies as well, teachers have been found to have limited PCK, to focus more on procedure than concepts, and to have different PCK for different topics (cf. Depaepe et al., 2013).

As discussed in the previous chapter, various approaches have been used to interrogate teacher knowledge as well as student teacher learning. Some studies have asked teacher education graduates or students in their final year whether they feel they have acquired various competencies or types of knowledge (cf. Schmidt et al., 2008). Others have used tests and interviews to identify teachers' declarative knowledge (Beswick, Callingham, & Watson, 2012; Hill, Ball, & Schilling, 2008; Krauss & Blum, 2004). And some have used classroom observations to infer the knowledge of teachers (Hill, Ball et al., 2008; Stump, 2001).

One example of a study which combined both approaches was conducted in Panama and Costa Rica (Sorto et al. 2009). The researchers used videotaped lessons and questionnaires to document teacher knowledge. Their results showed that both teachers' level of content knowledge and their level of specialized knowledge for teaching mathematics were questionable (Sorto et al., 2009).

A study conducted in the USA which also used multiple instruments and approaches investigated teachers' mathematical knowledge for teaching (MfT) and mathematical quality of instruction (MQI) (Blunk, et al. 2008). While the instruments used in this study were more specific and detailed with respect to PCK, the researchers recognized after conducting the study that they could not measure the impact of teachers' knowledge on learners' achievement without data measuring that achievement. However, a link was found between teachers' knowledge and the mathematical quality of what they did in class and differences among the teachers, such as their use of curriculum materials and their beliefs, were noted.

In their review of PCK in the literature, Depaepe et al. (2013) summarized the research into six major lines, namely:

- The nature of teachers' PCK;
- The relationship between PCK and CK;
- The relationship between PCK and instructional practice;
- The relationship between PCK and student learning outcomes;
- The relationship between PCK and personal characteristic; and finally
- The development of teachers' PCK.

I follow this structure in my review of the literature below, with the exception that the first item, i.e., the nature of teachers' PCK, is covered in Chapter 2 and the last item is excluded as it is beyond the scope of this work.

3.4 Findings on the relationship between PCK and CK

Internationally, the most significant large scale study of PCK was conducted as part of the German COACTIV project. The researchers claim that it was possible to make an analytical distinction between CK and PCK, and found a correlation of 60% between PCK and CK scores on their knowledge tests (Krauss et al., 2006). It is important to note that assessing PCK without consideration for CK could therefore lead to skewed results. Interestingly, they found that the correlation was stronger for teachers who had had more education in mathematics (ibidem), which supports the claim that content knowledge is one possible route to PCK (Krauss & Blum, 2012). However, this may differ between teachers at academically oriented German high schools and teachers of lower grades.

It is important not to assume that these results transfer to other educational systems, as Germany has very high content knowledge requirements for their teachers. The study of grade six teachers conducted in KwaZulu-Natal, South Africa, followed suit and separated teacher test items into content knowledge and PCK and also found a correlation, though the limited number of teachers in the study limits the strength of this result (Christiansen, pers. comm., 2016).

3.5 Findings on the relationship between teachers' declarative PCK and instructional practice

Depaepe et al (2013) in a review of a range of studies which used interviews, observations, tests, questionnaires and interventions, identified three key results: that instructional quality is correlated to PCK; that this is more so for PCK than for CK; and that coursework may improve CK, PCK and instructional quality. However, the review does not explore the ways in which the instructional practices were categorized in this study. The COACTIV study (Baumert et al., 2010) found that instructional quality was categorised using three dimensions, namely (a) "the provision of cognitively activating learning opportunities" (p. 149), interrogated through collected tasks and homework activities, (b) individual learning support, identified through "the degree to which teachers provided adaptive explanations, responded constructively and patiently to errors, whether learners perceived the pacing as adequate, and whether the teacher-learner interaction was respectful and caring" (p. 150), and (c) effective classroom management. This is very different from the method used to determine the quality of instruction in a recent study conducted in Kenya (Ngware et al., 2015), which looked at the teachers' demonstration of the strands of mathematical proficiency, the cognitive demand of tasks, and the mathematical knowledge demonstrated by the teachers. Hill, et al. (2008) found that teachers with higher MfT made fewer mathematics errors, link concepts and procedures more, chose more helpful examples, and responded better to learners. These different systems of categorization make it impossible to generalize the findings from the different studies.

While the studies mentioned above have at least clarified their notion of quality in instructional practice, the same cannot be said for all studies. For instance, the study conducted by Sorto et al. in Panama and Costa Rica did not specify any criteria, but still made claims about observable PCK (2009). The same was the case in the study undertaken by Carnoy and Chisholm in South Africa and Botswana, which does not specify how manifestations of PCK were identified (Carnoy et al., 2012; Sorto et al., 2009). It appears that it was left to the judgment of the individual researchers who were coding the videos to determine the extent to which PCK was manifested in the recorded lesson (personal communication with participants in coding workshop). Ramdhany (2010) used the same data collection instruments as Carnoy and Chisholm in his study in KwaZulu-Natal, South Africa. However, he worked with very broad categories without clear indicators for recognizing types of PCK in the observations (Ramdhany, 2010). He mentioned that this resulted from inadequate piloting of the research instrument. It is partially against this backdrop that I decided to develop my own instrument. Consequently, while the data collection tools used in this study were replicated from the previous studies conducted in South Africa, analysis of the video recordings was substantially different.

The importance of the balance between conceptual understanding and procedural knowledge is often emphasised in teacher education programmes (Bossé & Bahr, 2008). It is not clear to what extent this was foregrounded in the COACTIV study mentioned above, but it was addressed in the Kenyan study by engaging with the strands of mathematical proficiency. It was also explicitly engaged in the KwaZulu-Natal study, where it was found that teachers foregrounded procedural knowledge or memorization, and that teachers' offering opportunities to develop strategic competency may be correlated to learning (Ally & Christiansen, 2013).

3.6 Findings on the relationship between teachers' declarative PCK and learner performance

Some scholars have considered it inadequate to look only at PCK and instructional practice without considering their effect on learning. As a result, they have investigated the relationship between teachers' declarative PCK, instructional practice/practical PCK and learner performance. It should be noted that such studies are still limited in the context of developing countries' (Deapepe et al., 2013). However, one recent study in South Africa reported that providing teachers with a year of in-service training focused on improving teachers' knowledge of mathematics was linked to small but significant improvements in learner performance, when compared to a control group (Pournara, Hodgen, Adler, & Pillay, 2015).

In the German COACTIV study, Baumert et al. (2010) used interviews and learners' mathematics tests scores to interrogate the connection between PCK and learning. Their findings revealed that teachers' declarative PCK was correlated with their learners' outcomes. A study conducted by American researchers found that, in terms of pedagogical knowledge, between two groups of teachers in Costa Rica and Panama, the Costa Rican teachers applied better pedagogical techniques in their teaching (Sorto et al., 2009).

Adedoyin (2011) reports that in his research in Botswana, teachers' PCK was correlated with their learners' performance. A link between PCK and learner performance could not be established in the grade six study conducted in KwaZulu-Natal, however (Ramdhany, 2010). This illustrates the importance of contextual considerations when interpreting research results.

3.7 Findings on the relationship between teachers' declarative PCK and teachers' personal characteristic

Considering the relationship between teachers' declarative PCK and their personal characteristics, Blömeke, Suhl, and Kaiser (2011) posit that gender has little if any effect on, or correlation with, PCK. They state that language proficiency has more impact on CK than PCK, but that teachers' teaching experience has a positive influence on his/her PCK, as measured in tests.

The findings from studies which explore the link between teachers' level of education and their PCK vary substantially, probably because there are substantial differences between the education systems in the various countries represented (Depaepe et al., 2013). On one hand, Zhou, Peverly, and Xin (2006) found that American teachers' PCK was different to that of Chinese' teachers, for example. This could be related to differences in the teacher training programmes in these two different contexts. In Panama and Costa Rica, Sorto et al. (2009) found that the differences between the PCK of teachers Panama and Costa Rica was small. The range of findings need to be interrogated much more intensely as many factors can impact the development of teachers' PCK, including things like training, mentoring (Nilssen, 2010) and group discussions (Barnett, 1991).

The work done to date does not provide a solid basis for drawing conclusions about what constitute the major elements of PCK, because each study relates to a different context. It is therefore reasonable to expect that studies in other contexts, such as Rwanda, could show new understandings of the key aspects of PCK.

I now explore existing gaps in the literature.

3.8 Gaps to fill

This study makes a valuable contribution in terms of attempting to close some of the gaps identified in this field by challenging some of the weaknesses of earlier studies. As discussed previously, these include the use of poorly defined criteria to determine PCK in mathematics classroom teaching situations, the dearth of empirical research studies which explore both declarative and practical PCK in various contexts, and the relative absence of detailed studies of PCK and its relation to teaching on the African continent. This final issue was highlighted by Broadfoot, Alexander, and Phillips, (1999) in their research on learning across countries; they argue that comparative studies need to be contextualized within an analysis of the national culture, pedagogic traditions and educational priorities of a particular country.

In terms of the development of defined criteria for interrogating PCK as it manifests during classroom teaching, this study will potentially contribute not only to the understanding of PCK in Rwanda but also to the field more broadly. In many studies (cf. Rowan et al., 2001), the impact of teachers' PCK on learning has been overlooked. Hill, et al. (2008) agree that without measuring learner gains in learning, it is impossible to determine whether variation in teachers' declarative or practical PCK is correlated to variation in learner performance. As a result, the findings in these studies with regard to effective teaching practice are related to normative influences. Another problematic approach is illustrated by a medium scale study conducted by Carnoy and Chisholm (2008), in which PCK was simply rated as 1, 2 or 3 in quality, without differentiating the various types of PCK or defining clear criteria for the ratings. Thus, though the study engages a full range of data, it is unable to generate accurate results with regard to the role of PCK in facilitating learning. Ramdhany's (2010) attempt to improve this led him to the conclusion that his classroom analysis of video recorded lessons was not strongly enough informed by the concept of PCK as it has been unpacked in other research.

What have I set out to do, therefore, is to work from a categorization of the sub-categories of PCK to identify relevant research-based categories, such as the connections made by the teacher, and feedback to learners (see Chapter 2). Within these categories, I specify possible variations (eg. different types of connections, different types of feedback) based on the literature. This allows me to interrogate correlations between, say, one type of connection, and learning, as well as the interplay between these factors.

Internationally, a limited number of investigations have been made into what 'average' teachers know about learners' mathematical thinking and other aspects of PCK (Hill, et al., 2008). This study addresses this gap by contributing empirically-based insights into the practical and declarative PCK of Rwandan mathematics teachers to this research field. In addition, as this study replicates the data collection methods used in several previous South African studies, it enables comparison of results between this study and those.

This study will also contribute to the body of knowledge on teaching and learning at the primary school level; complementing the work of Krauss et al. (2008) conducted at secondary school level which found that the degree of cognitive connectedness between the PCK and CK of mathematics teachers was a function of the degree of their mathematical expertise.

Positive corrections have been found in American and German studies between the performance of mathematics teachers on tests of their PCK or mathematics for teaching (MfT) the quality of their teaching, and their learners' performance (Ball et al., 2008; Carnoy et al., 2012; Hill, Rowan, & Ball, 2005). The body of knowledge related to this in the developing context is limited, however. A replication study conducted in Rwanda would add to the body of knowledge on the role of PCK in learning.

While this study attempts to address a number of gaps in the research on PCK, I would like to note that teachers' PCK differs across mathematics sub-domains and from topic to topic rather than being a general competency (Hadfield, Littleton, Steiner, & Woods, 1998). This will act as a limitation in this research due to the methods of data collection. It would have been time consuming to try to obtain data for different topics, and it would have been disruptive to ask the teachers to teach on the topics I requested at the times I was present at their school.

3.9 Chapter Summary

This chapter has summarised the literature on the research topic, highlighting the major factors which might influence teachers' practical knowledge in classroom situations. Correlations that have been found in previous studies between CK and PCK, PCK and teaching, PCK and teachers' background variables, and PCK and learner performance were discussed. Exploration of the literature revealed how limited the range of empirical work on PCK really is and helped me to identify some gaps to target in this study.

In the next chapter, I describe the methodology used in this study.

4 METHODOLOGY AND METHODS

4.1 Introduction

This chapter presents the methodology used in this study. Research requires that the methods used to sample and collect data must be carefully chosen. However, the choices in this study were limited due to it being a replication study, as previously mentioned.

The data collection methods used in this study are discussed in this chapter, along with the research design, the context, validity and reliability issues and ethical considerations. In addition, I present the instrument I developed for the purpose of measuring teachers' practical PCK in the subcategories that were selected for this study. The chapter also describes the methods used for data analysis, with more detail provided in Chapter 5. To frame the study, I begin by discussing my choice of paradigm.

4.2 Research paradigm

Paradigms in research have been a focus of engagement and characterization for various scholars (Guba & Lincoln, 1994; Louis, Lawrence, & Keith, 2011; Merriam, 2002; Patton, 1990). For example, Patton (1990) considers a paradigm a world view, a general perspective, and a way of breaking down the complexity of the real world, whereas Guba and Lincoln (1994) consider paradigms as basic belief systems which have roots in epistemology, ontology and methodological suppositions. While discussing commonly used paradigms in research such as positivism, post-positivism, constructivism and critical theory, these scholars have taken different views on the ontology, epistemology and methodological issues within each paradigm.

With respect to the positivism paradigm, researchers like Guba and Lincoln (1994) posit it as having a realist ontology or, as Creswell et al. (2008) put it, it assumes an objective reality. The epistemological assumption of positivism is that the researcher and the object of research are independent of each other (Guba & Lincoln, 1994) and people can come to know reality through observation and induction from observation (Creswell et al., 2008). Methodologically, positivism works with observations, experiments and other manipulative methods (Guba and Lincoln, 1994; Creswell et al., 2008). Positivism has often been criticized for assuming that complete knowledge can be obtained (Antwi & Hamza, 2015), whereas later epistemologies see knowledge as fragile (always containing an element of uncertainty) and adapt their methodologies accordingly.

This applies to the post-positivism paradigm which works from the assumption that through research someone can at best state that there is a high probability that truth has been approached (Guba & Lincoln, 1994). Like positivism, post-positivism is mainly justified by its general goal of discovering cause and effect relationships and predicting and controlling future behaviour on the basis of present behaviour. Ontologically, according to

Guba and Lincoln, post-positivists do not doubt that reality exists and epistemologically they argue that objectivity remains a controlling ideal usually represented using qualitative methods. Thus, the main way in which post-positivists differ from positivists is their assumption of the fragility of knowledge, which results methodologically in the attempt to reject hypotheses rather than prove them (ibid).

A more substantial difference exists between these paradigms and idealist constructivism[3]. The constructivists believe that there can be multiple realities and that none is more privileged than the other (Merriam, 2002). Ontologically, Creswell et al. (2008) argue that constructivists see reality as constructed, socially developed and accordingly multiple realities can exist based on how people construct them.

The epistemological position of constructivists is that people cannot separate themselves from what they know and cannot observe the world without being affected by their knowledge (or beliefs). Guba and Lincoln (1994) posit that for constructivists reality arises through continuous interaction between the research and the object of research, thus appearing as a collapsing of the ontological and epistemological positions. As a result of this context-dependency of any data construction and analysis, constructivists are more likely to engage qualitative methods (Cupchik, 2001). They argue that the interpretation and analysis of qualitative data should take into account the particular moment and context in which observations were conducted as they are constructed through context dependence (Hennig, 2002). This is the reason I have included socio-economic characteristics of my respondents in my data collection, which is in keeping with the replication aspect of the present study and the awareness that socio-economic status (SES) varies through context. When research involves human respondents, the social dynamics between the researcher and the respondents are also important (Guba and Lincoln, 1994).

Finally, the critical paradigm is a realist paradigm which supports an objective reality but, just like post-positivism, emphasizes the fragility of knowledge (Guba & Lincoln, 1994). It emphasizes, however, that central to any understanding of reality is the process of unpacking factors related to power such as social, cultural, political, gender and economic aspects. Researcher and the object of research are understood as dependent on each other in this paradigm as well, and so the context in which data are to be collected is emphasized. The reliability of data is rooted in the interaction between the researcher and the research object, with careful consideration of power dynamics (Guba & Lincoln, 1994).

In some ways, this study uses mixed methodology as it involves both quantitative and qualitative methods. Mixed research recognises and works with the fact that the world is not exclusively quantitative or qualitative; it is not an either-or world but a mixed world, even though the researcher may find that the research has a predominant disposition (Louis et al., 2011). In this work, however, I have quantified the qualitative analysis of the

[3] As a research paradigm, not to be confused with constructivism as a learning theory.

observations for the purpose of investigating correlations to other factors. A more qualitative analysis of observational classroom data is planned for later research. This could be viewed as an opportunity missed in the current study, but singlehandedly undertaking the first medium scale study of this type has been time consuming and did not allow for further qualitative analysis.

I will discuss the choice of my research design (Section 4.4) in relation to the research paradigm.

My choice of research paradigm was inspired mainly by the ontology and epistemology of the constructivist theorists explained above. Epistemologically, it assumes the existence of different realities, reflected in the acknowledgement that my interpretation of PCK is only one of many that are possible. Ontologically, it is in line with the constructivist argument that none of the various realities is more real than another. Hence, any declaration I made about what something 'is', could be taken as 'within the constraints of the present research'.

Various other factors may be responsible for the occurrence of a certain thing and, as noted by Guba and Lincoln (1994), those factors might be beyond people's control. For example, in this study the use of questionnaire and test results helped to explore the factors which could be behind learners' poor performance and their relationship to the socio-economic backgrounds of learners. However, my approach to considering the study context (Guba & Lincoln, 1994) is only one of many other possible approaches which could be used by other researchers. I cannot claim that my results cannot be challenged by others but only that they provide one understanding generated from my collected data (Maxwell & Mittapalli, 2010).

Epistemologically, this research assumes that the reality that is presented was constructed through different constructions of what happens in the classroom which have been recognized by analysing my video-recorded classroom lessons based on my particular PCK perspective reflected through my developed PCK analysis instrument, with the intention of making this as transparent as possible to enable the scrutiny of others.

Methodologically, as the constructivists' theories suggest, this research used observations as one of its methods of data collection. I did this by video-recording teaching lessons. However, I have not ignored the fact that my presence could have influenced both the teaching and the learners' knowledge construction in one way or another, at a given moment in time. The triangulation which I used attempts to balance this to some extent.

4.3 Theory of learning

In the previous paragraphs, I have discussed constructivism as a research paradigm. However, in Section 1.3, I reflected that this study has also evaluated learners' learning. Having that in mind, and knowing that different learning theories exist, I found it relevant to also take a position in relation to theories of learning. Most current approaches assume learning to involve mental work in interaction with the surroundings, engaging cognition, social relations, affect and experiences with discourses. These include constructivist, cultural-historical and sociological approaches to understanding learning and knowing mathematics (cf. Lerman, 2013).

Central to constructivism, as inspired by Piaget, is the belief that learners construct their own knowledge based on their interactions with the world around them, including the social world (ibid). This relates to mathematical content construction through mathematical practices/variations -- one of the teachers' classroom actions which this study addresses. In other words, learners make sense of their experiences by constructing schemas which are then applied to situations which appear to the learners to be similar. This also means that learners' mental concept images are not directly accessible but must be approached through interrogating their choices and actions in specific situations. This perspective has been widely used in researching learners' so-called misconceptions (Christiansen & Aungamuthu, 2012), and the idea of constructing a multiple choice test where the incorrect options are informed by such research suggests that the replicated studies, at least to some degree, were informed by constructivism. This is consistent with the considerations in this study, where the manner in which teachers engaged learners' errors and unpacked the content they were teaching has been noted.

Piaget claimed that there were always some elements of assimilation and accommodation in play at the same time (Lerman, 2013). Wittrock (1992) argues that meaningful learning occurs when the learner creates relationships between new concepts and prior knowledge, experience and new information. This highlighted to me the value of observing the methods teachers use to access their learners' prior knowledge. The theory is that each learner forms his/her own representation of knowledge (Dalgarno, 2001) when he/she actively explores the surrounding environment, which enables the learner to recognize inconsistencies between his/her current knowledge representation and (new) experiences. To reflect this, in this study I observed teachers' techniques for illustrating and representing content without ignoring their methods for engaging learners in classroom learning tasks.

As indicated in Section 3.2, I work from the position that learners come to school with different perceptions about the world as they come from different backgrounds and different relationships to decontextualized knowledge. It is in this line that Dalgarno (2001) posits that learning occurs within a given social context and that such interaction is a necessary part of the learning process. That view has informed this study in terms of assessing teachers' methods of doing progression in their lessons and linking content to other content. This view also reflects ideas inspired by Vygotsky's socio-cultural learning perspectives, which assume that people first learn on a social plan and then, over time, internalize it (Vygotsky, (1980). This is not far away from illustration and representations together with teachers' ways of task engagement shown through my video analysis tool. Vygotsky and others from this school of thought also distinguish between everyday concepts and scientific concepts, where the latter requires some sort of systematic instruction (ibid).

In my view, the notion of PCK is compatible with both of these theories of learning. Whether the learning happens through presenting learners with new experiences from which they generate schema or from other forms of instruction, teaching requires the teacher to transform the content so as to make it accessible to the learners, and this utilizes PCK (Ball et al; 2005). In the present study, that could be mirrored through teachers' ways of unpacking the content they have to teach.

With his Marxist background, Vygotsky was no stranger to the influence of social contexts on learning. In his work, Lerman (2013) also mentioned that children from different class backgrounds (middle and working classes) do not pass through school in the same way. In one way or another, this view influenced this study (cf. Sections 2.2 and 8.2). He added that while it may come as a surprise to teachers and researchers, questions set in everyday contexts are likely to be misrecognised by working class children. From my understanding, setting such questions requires careful attention. In this study, this alerted me to the value of noting teachers' use of leaning tasks in their classroom practices.

Before ending this section, I would like to note that the learning theory discussed in this section should not be confused with the theory of instruction (cf. Moshman, 1982) which is much related to teaching.

Section 4.4 below discusses the research design of this study.

4.4 Research design

The overall design of this study was determined, to a large extent, by the choice to conduct a replication study, with the advantages of regional comparisons that have been mentioned (see Section 1.3). Thus, the study is a correlation study which combines qualitative and quantitative approaches. One of the important benefits of this is the opportunity it presents to gain a deeper understanding of the phenomena under study, as the two approaches complement each other by permitting the stability of the results gained through the contrasting approaches to be assessed.

In the study, quantitative data were obtained mainly from mathematics tests that were given to learners, mathematics and PCK tests that were given to their teachers, and questionnaires that were given to both learners and teachers, while qualitative data were obtained from the video recordings of classroom lessons. In the case of the classroom videos, the interactions between teachers and learners could be observed, creating a basis for engaging some of my research questions. However, in order to investigate questions such as how teachers' practical PCK relates to learners' performance, it was necessary to quantify the coding of the practical PCK in some way.

In a study which aims to interrogate the effect of one factor – in this case PCK – on learning, it is important to collect information which allows the researcher to take into account the effect of any other potential factors. I have previously discussed the many factors which may influence the teaching-learning situation and the outcomes in the form of learners' learning and test performances. The questionnaires were an important instrument for trying to capture such information. However, as previously mentioned, the data which would provide the best measure of socio-economic status varies across contexts, both between countries and within countries. In a country where many families rely on subsistence farming[4], measures from other contexts cannot be applied uncritically. While I am aware that the statistical analysis may show a correlation between variables, this does not imply a causal relationship; this is clarified in Chapter 8.

The problem or issue that a researcher is studying determines not only the research design but notably the research techniques used (Kane & O'Reilly-deBrun, 2001). While the study which was replicated here did not characterize its research design, I have considered the six common types of mixed method research designs proposed by Creswell et al. (2008) namely: sequential explanatory design, sequential exploratory design, sequential transformative design, concurrent triangulation design, concurrent nested design and concurrent transformative design.

As the qualitative and quantitative data were collected at the same time, this study used a concurrent design. It could be argued that the study has elements of what Creswell et al. (2008) refer to as a concurrent transformative study, because it may provide a different perspective on PCK, or of what they call a concurrent nested approach, because the analysis of the video observations is used to interrogate the correlation between PCK and learner performance. However, the purpose of the study was as much to get a sense of the classroom practices and knowledge of Rwandan teachers. I therefore argue that this research falls into the concurrent triangulation design, which allows the use of two different data collection methods in one study in order to corroborate findings (see Figure 4-1).

[4] As high as 90% according to Ministry of Education, Republic of Rwanda (2011).

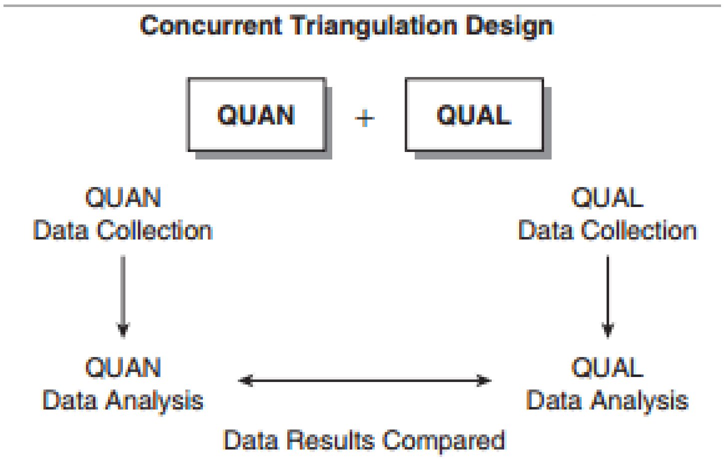


Figure 4-1. Concurrent triangulation design

Source: Creswell et al, 2008, p. 181

One of the advantages of this design is that the limitations of using one method are minimised by using multiple methods. Besides, it accepts the integration of the output from both types of data during data interpretation (Creswell et al., 2008) as it has been done in this thesis by investigating the correlation between teachers' declarative PCK and practical PCK by linking these to the results from learner tests and questionnaires.

4.5 Research site and participants

4.5.1 Site description and context

This study was conducted in Rwanda, a small east African country characterized by volcanoes, mountains, forests and lakes. Agriculture plays an important role and constitutes roughly a third of the national GDP (Abbott, Sapsford, & Rwirahira, 2015). Though one million of Rwanda's eleven million citizens live in the capital, official estimates indicate that 80-90% of the population is engaged in farming (ibid). This is mostly subsistence farming which uses traditional tools and methods, in part because of the hilly nature of the land. The human development index for 2015 was 0.483 – a doubling over the last 25 years, but still low (ibid), reflecting that more than 60% of the population live below the income poverty line (ibid). Since 2012, education has been free up to grade 12, but in practice parents are expected to contribute towards materials, upkeep of the school and even teacher development (ibid). Almost all children are in school, but many have to repeat grades, resulting in a current gross enrolment ratio of 134%. Less than 10% of the adult population has some secondary or higher education, but almost all primary school teachers are trained to teach. This is the context within which the data for this study were collected.

Data were collected for this study during 2013. As detailed in Section 4.5.2, twenty schools were selected in Rwanda for the purpose of this study. The schools in which data were collected fell in two categories, namely ten less-resourced schools and ten fairly well-resourced schools, selected from both public and non-public schools, within my sampled districts (as detailed in Section 4.5.2).

The government of Rwanda is putting a particular focus on science and mathematics education which is seen as the core of socio-economic development. Although mathematics has been prioritized by Rwanda's vision of constructing a knowledge-based economy, fewer students study mathematics than fields related to the humanities and arts, because of the high rate of failure in national mathematics examinations at all levels[5] (MINEDUC, 2003).

The majority, if not all, of the African countries face a language problem in their education systems because the languages of instruction generally are additional languages to the learners' mother tongue. It is obvious that this has a negative impact on learners' understanding of the subject matter which then negatively affects their performance (Niesche, 2009).

The majority of Rwandans (90.8%) speak Kinyarwanda at home (as their mother tongue), while 5.6% speak English, 2.6% speak French and the remaining 0.8% communicate in other languages. Kinyarwanda is the language used for teaching in the first three years of primary education and then English is introduced as the language of instruction (MINEDUC, 2013). This implies that a Rwandan grade six learner has been instructed in English for only three years before sitting for the national examination test, which is given in English.

The international millennium goals for education may also be considered to be a factor which impacts the education system. Rwanda is committed to international development targets in education such as education for all (EFA) (MINEDUC, 2003), whereby primary education is compulsory and free for each and every child. Another international influence on the Rwandan education system is the *Global Partnership for Education*. While the policies of this programme are quite good, factors such as inadequate infrastructure and inadequate quantity or quality of teacher training to address the challenges involved in introducing English as the medium of instruction, may reduce teaching quality (MINEDUC, 2010). Thus, learners are likely to be promoted without sufficient knowledge and prerequisites.

The Ministry of Education, (Ministry of Education, 2006) has stated that Rwanda has a ten year *Long Term Strategy and Financing Framework* (LTSFF 2006-2015) and a five year *Education Sector Strategic Plan* (ESSP 2006-2010). Key 2015 targets in the ten year strategy include increasing the number of learners who complete their primary education from 51% (2004) to 112% by 2015 (not yet updated). In order to achieve these targets, the

[5] Failing the mathematics examination does not, in the current system, exclude applications from studying mathematics at university level.

government of Rwanda introduced strategies to address the deficit in training for primary school teachers and fund the purchase of teaching and learning materials, especially for the sciences, including mathematics.

4.5.2 Sampling

For this study, I selected a sample of twenty primary schools from seven of the thirty districts in Rwanda. Stratified sampling was used to identify the districts and schools.

The districts were chosen from three of Rwanda's five provinces. They were selected on the basis of accessibility in order to reduce the costs of travelling during data collection. Of the provinces, one is constituted by a city. I chose this one because I expected to find well-resourced schools in it. I chose the two other provinces using simple random sampling. I then used stratified sampling to select the districts within these three provinces. To select the individual schools, I used stratified random sampling where the strata were based on the socio-economic status of schools. As the strata had to be mutually exclusive, in this case, the school resources were taken into consideration in order to get a sense of school categories. Schools in which basic learning materials were available, such as geometry kits for both learners and teachers, and where learners did not have to share books, were categorized as well-resourced, whereas schools without these facilities were considered non-resourced schools. I was able to obtain this information from the National Schools Inspectorate Division of the Rwandan Ministry of Education. The accuracy of this information was attested to by the official who supplied it to me and who had worked for more than 15 years in the inspectorate. My own observations at the schools corroborated this information as well.

4.5.3 Research Participants

The participants in this study were Rwandan grade six learners and their mathematics teachers at the selected schools. In both phases of data collection twenty teachers participated, while 713 learners completed the pre-test during the first phase and 638 learners completed the post-test during the second phase. Overall, 638 learners completed both tests.

The participants in this study had a direct relationship with each other. This relationship was observed primarily between the major research participants, namely teachers and their learners, through their everyday interactions not only in classroom situations but also outside of the classrooms. In addition to this, I began interacting with the teachers and learners immediately after receiving permission from the principals of schools. This primarily consisted of me explaining the project to the teachers and negotiating their consent, followed by an open conversation with research participants to provide further clarification about the project wherever required.

4.5.3.1 Access/gatekeeper issues

I personally collected all of the data used in this study and captured them in a database for analysis. This required me to have contact with all of the participants in the study. To gain access to them, I first had to negotiate with school heads to allow me to work with the teachers. At some schools I had to meet with the learners' parents to explain the purpose of my research before they would consent to their children participating. While this was time consuming, no issues arose in the process.

4.5.3.2 Rwandan grade six mathematics' teachers in the sample

For the study, 20 grade six Rwandan mathematics teachers were initially selected and agreed to participate; 10 were women and 10 were men. (This was a coincidence; I did not consider gender in my sampling.) Their average age was 42; while this seems to be high in the Rwandan context, the youngest in the group was 22 years old whereas the oldest was 59 years old. Even if I have not asked such information during my data collection period, this did not surprise me because usually experienced teachers are assigned to teach grade six mathematics in Rwanda. Among the 20 teachers, 15 held a diploma for having completed six years of secondary school studies. Three had obtained a D7 qualification, which is a qualification between senior six secondary and university studies which was offered in the earlier Rwandan school system but is no longer offered. Two of the teachers held bachelors' degrees; one in education and another in accounting.

Of the 20 teachers, one showed unwillingness to complete the questionnaire and test, partially because of language issues. I was concerned that the test result would not reflect the teacher's knowledge, which would skew the results, and hence I decided to not use his test results in my data analysis, which means that data from only 19 teachers and their learners have been considered in the analysis of the teachers' declarative knowledge. However, I did observe his lesson and have included this in my analysis of the teachers' practical PCK.

The test and questionnaire were written in English, although all of the teachers who wrote it had completed their studies in French, as this was the language of instruction formerly (Gahigi, 2008) under colonialism.

For more on the background of the teachers, see Section 6.2.

4.5.3.3 Rwandan grade six learners in the sample

The Rwandan grade six learners involved in this research were from the participating schools as detailed in paragraph 4.5.2 above. I note that only learners in classes taught by participating teachers and who completed both tests were included in my analysis.

Both female and male learners participated in the study. Girls were a slight majority (53.8%) over boys (46.2%).

Rwandan education policy has inclusive education as a concern. However, because of economic issues, the numbers of classes reserved for learners with disabilities who are supposed to have special equipment are limited. Most of such schools are situated in towns which are a considerable distance away for many people. A challenge I observed during my school visits was that because of the insufficient number of such schools, learners with disabilities were included in the regular classrooms, especially in the rural areas. On one hand, some of the teachers told me that they have not been trained to deal with such cases, and on the other hand, teachers confirmed that learners are not prepared to accept those with disabilities, leading to stigmatisation.

In some classes, there was large number of learners, requiring three or four learners shared a seat intended for two. Though grade six learners in Rwanda should be around 13 years of age, in some classes learners ranged from 11 to 19 in age.

The mobility of the learners between schools was relatively high as only 6% of the learners had attended the same school from grade one to grade six. Only 6.5% of the learners reported that they were looked after by both parents at home. However, my experience is that it is quite common for learners to live with both parents, so it is possible that the learners misunderstood the question, perhaps taking it to refer to whom in the home attended to their needs.

The vast majority of the learners – around three quarters – reported getting homework every day, and only about 5% said they had homework once a week or never. About a third of the learners could not answer whether they had attended preschool, but of those who did answer, more than 90% said yes. This high attendance at pre-primary school might reflect the Rwandan government's efforts to encourage parents to send their children to pre-primary schools.

4.6 Data collection

4.6.1 Collection procedures

The data were collected in two phases, involving two visits to each school.

In phase one, I administered a learner questionnaire and test as early as possible in the year as a baseline; this took two months due to the travelling required. The learners' questionnaire included learners' biographical details, indicators of their family socio-economic status, home language, and their perception about school violence. The test included some items from the grade five and six curricula (see description of questions in Appendix E). I also administered questionnaires to teachers covering general items such as their background (education levels, teaching experience, etc.), and mathematical knowledge items, i.e. a test on mathematics knowledge and on PCK (see Appendix F). One lesson with each teacher was also video-recorded.

The second phase, as the first one, took approximately two months. I conducted the learners' post-test to check if there had been any gains in performance compared to the pre-test. This allowed me to correlate learning gains with both learner and teacher data.[6] One more lesson was video-recorded for some of the teachers who consented to this.

Originally, the target was to video-record at least two lessons for each teacher, but several of them did not wished to participate in this. As this inconvenience came at a later stage of the study, I decided to consider one video-recorded lesson for each teacher, which makes a total of 19 video recorded lessons. The fact that all 19 teachers completed the teacher questionnaire and wrote the teacher test thus allowed for a complete set of data for 19 teachers.

All of the learners who wrote the pre-test also completed the learner questionnaire. However, because of various issues such as absenteeism and drop out, a number of learners missed the second learner test, resulting in a drop of 11%, from 713 respondents to 638.

As mentioned before, my thesis is part of a larger project on grade six mathematics teaching and learning. For this reason, both learners and teachers' tests together with the used questionnaires have not been included anywhere in this thesis as they may be used in other future studies, related to the project.

4.6.2 Methods of data collection

4.6.2.1 Teacher questionnaire

This questionnaire was aimed at capturing as much information from teachers as possible. Key issues included: teachers' level of schooling; years of teaching experience; training they had received; how they gained knowledge on curriculum; their socio-economic status; and the most common problems they faced in their classrooms. The teachers' questionnaire was completed immediately after they finished the mathematics test. This process took a substantial amount of time (around 4 hours) and personally I felt that the questionnaire was unnecessarily long, although the additional information added depth.

4.6.2.2 Teacher test

This test was composed of 24 different items, some of which had more than one sub-question, making a total of 63 items. Some items dealt with content knowledge and others with pedagogical content knowledge (see Table 6-2). As previously noted (see page 30), the items on PCK reflected through MfT were aimed to mainly capture the way teachers unpack the algorithm while teaching a particular topic (KCT); how they identify errors made by learners (KCS); the way they identify correct or incorrect solutions given by learners and how they identify the reasons which could be behind their learners' choice of

[6]In this phase, I also examined learners' note books, to get a sense of curriculum coverage and sequencing. However, these data were not analysed as part of the doctoral work.

answers (KCS). The teacher test was given the same day and at the same time as the learners' pre-test at each school. While learners were busy doing their test, so were their teachers. This helped me to ensure that learners were not influenced by their teachers as the latter were also busy. Details regarding the teacher test content areas and knowledge types which I assessed are provided in Chapter 6. A description of the questions has been included in Appendix F, but the actual test is not to be published.

4.6.2.3 Learner test

The learner test was composed of 40 multiple choice questions. Each question had four answers to choose from, one of which was correct. There were items on numbers/arithmetic, statistics, geometry, algebra/number patterns and measurement. The test was given as a pre-test and later as a post-test in each of the participating classes. A description of the questions from the learner test has been included in Appendix E, but the actual test is not to be published.

4.6.2.4 Learner questionnaire

That questionnaire was also important in this study as it collected background information on the learners. As I discussed in Chapter 3 of this thesis, there are different factors which may influence learning and have a positive or negative impact on learners' performances. To tap that information, my questionnaire included questions on learners' home language, the level of education of their guardians at home and the basic items which they possess at home, as indicators of their socio-economic status. Other questions addressed, for example, whether or not they had attended pre-school, their views on learning and how often they were given homework. They completed this questionnaire immediately after completing the mathematics test. I collected all of the completed mathematics tests and then distributed the questionnaires to learners and teachers. This became quite a lengthy process, and I am grateful to both the learners and the teachers for taking the time to participate in this study.

4.6.2.5 Lesson observations

At each of the schools in the sample, the observation of lessons took place on a separate day, after both learners and teachers had completed the tests and questionnaires. During my first school visits, some of the teachers expressed reluctance to be video recorded. They were only in favour of taking the test and filling in the questionnaire. However, after I explained to them the usefulness of this research to mathematics education in Rwanda, they agreed to be video-recorded.

4.6.3 Trustworthiness of the data collected

I have already described the sampling process, and I think it had sufficiently random elements to avoid a biased sample, while also ensuring that the ranges of schools in Rwanda were represented. Through randomization of samples, their representativity/transferability was increased, i.e., increasing external validity. Still, some teachers who did not wish to be video-recorded – saying that they were not sufficiently confident because of the language barrier – were replaced. There is a possibility that this skewed the sample. For instance, it could be possible that teachers within a certain age group would be more likely to feel confident teaching in English.

Retrospectively, I regret that I asked the teachers to teach in English, even though that was the required language of instruction at the time, as it could also have affected the choices of teachers which was what I wanted to interrogate through the observations. I consider this the single greatest threat to the validity of the study. On the other hand, it did enable my supervisor and others to compare my coding to the video recordings, which increased inter-coder reliability.

The test and questionnaire were written in English, although all of the teachers who wrote it had completed their studies in French, as this was the former language of instruction. In hindsight I believe this was an error: there is no policy dictating which language must be used in research data collection and so I had the option of translating the test from English to Kinyarwanda or French in order to avoid any misunderstanding of the questions on the part of the teachers.

In terms of the trustworthiness of the data used in this study, respondents were free to consent or not. This suggests honesty on the part of the respondents as they could have refused to participate if they had preferred to.

In three of the observations, I failed to record the entire lesson because I had to change the cassette, or because I was inexperienced in the use of the camera. Limited observation time was lost, and I used my notes from the observation to recall the content lost, but it remains a source of error.

When I was video recording lessons, I sometimes observed changes in the classrooms from the previous day on which I had been administering tests and questionnaires. This most often involved the learners' seating arrangements. In most of the cases this reflected the way teachers grouped learners for them to do activities together. I observed that when a teacher wanted to give group work activities in his/her teaching, classroom seating was pre-arranged, which reflected that they were actually using group work.

This suggests that some teachers may have strived to adjust their teaching in other ways. It is a standard concern in all classroom observations, in particular those of short duration. It is hard to say if it is possible for teachers to change the way they engage connections, representations, and other aspects and give feedback based on one lesson observation, but it remains a potential source of error across all video recordings.

This also illustrates that there may be differences between what teachers believe are desired practices and what they see as desired practices in discourses about mathematics education or teaching in general. Alternatively, they may agree with the discourse but not always find themselves in a position to realise it in practice. In both cases, it demonstrates the saturation of education by normative discourses.

Among the things which increase the reliability of this research is its replicative nature and the fact that the period between the learners' tests was not so long that situational factors may have changed substantially but not so short that the participants could remember the first test.

When it comes to the tests, language problems were observed in most of the schools, for both teachers and learners, as I had to translate some questions from the questionnaires and test into Kinyarwanda for them to answer. However, some teachers who had more language difficulties than others were unable to finish answering the questionnaire on the same day, which pushed me to allow them to keep it and return it to me the following day when I returned to video record their lessons. As the information in the questionnaire was personal, I firmly believe this could only have affected validity positively.

While I recognise that the teacher test did not reflect all aspects of PCK, as has been discussed previously, I elected to make no major changes in order to provide a stronger basis for comparison with the previous studies, thus de facto using the same test as in the earlier studies. As a methodological choice this had short-comings, just as its alternative would have had. For the test to have covered all aspects of PCK, it would have had to cover all of the PCK subdomains as described by Deapepe et al. (2013), making it an excessively long test.

Assessing learning is not a simple task. The Oxford Dictionary defines learning as the acquisition of knowledge or skills through study, experience, or being taught, but this does not address the connectedness of the knowledge and skills ('understanding'), changes in learners' attitudes, changes in the ways learners perceive the world, and other aspects. Another factor is that learners may tend to improve their educational outcomes over time, simply due to their increasing maturity (Marsden & Torgerson, 2012). This also needs to be taken into account. Thus, when this study used changes in test performance as a measure of learning, which can again be correlated with measures of teaching, it was done with recognition of how crude this measure is. However, it is the best option available when a study involves a large number of respondents.

It is common knowledge that when multiple choice tests are administered some learners will guess to some extent. To determine whether the majority of learners have guessed for a particular question, it is common to look at the distribution of answers for the provided options. If the frequency is close to 25% for all four options, it is likely that learners simply guessed. However, as an analysis of learner responses to science questions on the TIMSS test has indicated (Dempster, 2007), learners use various strategies to narrow down their choices, in particular when answering a test in a language other than their mother tongue. These strategies include rejecting answers containing unfamiliar words, selecting answers that contain words that are also in the question, or selecting an answer based on the pattern of choices in preceding items. Thus, even for situations where two or three answer options have equal response frequencies, the possibility that the majority of learners have guessed cannot be excluded. However, this method is quite crude and does not help to determine whether only some learners were guessing for some questions.

One way to interrogate this is to determine whether learners answered some of the multiple choice questions correctly on the pre-test but failed to answer the same questions correctly on the post-test, or changed from one incorrect to another incorrect option. As shown by Christiansen & Aungamuthu (2013), this may reveal a high likelihood of guessing which could not be determined otherwise. Of course, it is possible that learners have 'unlearned' or forgotten previous content, but when a substantial number of learners 'change' a substantial number of their answers, including substituting one incorrect answer for another, other reasons become more likely.

While this may appear to relate to the method used by the examiners to set the pre- and post-tests, the modification of multiple choice items done by Kettler et al. (2011) in their work showed that there was no meaningful difference in reliability between tests in the original item condition and the modified item condition. In any case, this was not a factor in the present study, as the pre- and post-test questions were exactly the same.

One concern with collecting the information from learners which they were asked on the questionnaire is the extent to which learners are capable of providing accurate information. For instance, it rests on shared understandings of what it means to have piped water or what constitutes a brick house. It also assumes that learners have accurate knowledge and understanding of their parents' level of education, the number of books in their home, and so on. In my view, this is another challenge to the validity of the data collected– one I inherited with the instrument.

During this course of this study, I have presented some of findings of this research at various conferences. Suggestions from different scholars have been taken into consideration in refining my discussion of results in this document. However, despite the triangulation design I used in order to strengthen my data, as I will discuss in Chapter 8, my results cannot be generalised to other contexts due to the small sample size used. However, the multi-site data collection suggests that there is a reasonable likelihood that the data provide a snapshot of Rwandan teaching and learning at the time of the study.

With regard to the validity of the instruments used in this research, the tools I used for data collection were the same as those used for a South African grade six project conducted under the University of KwaZulu-Natal. However, there were issues in the South African study in that the learner test was found to not be well suited to the learners' actual level of performance (personal communication with project researcher). For this reason, I piloted the learner test in Rwanda before starting the data collection process.

See Section 4.8.4 for reflections on the trustworthiness of the coding and analysis of the data.

4.6.4 Ethical considerations

Before starting data collection, I received ethical clearance from the University of KwaZulu-Natal (protocol reference number HSS/0064/013D: see Appendix A). As my data were collected in Rwandan schools, I also received permission from the Ministry of Education in Rwanda for collecting data in Rwandan primary schools (research permission no: MINEDUC/S&T/0115/2013, reference number 0079/12.00/2013: see appendix C).

The identity of respondents in the study was protected by using number codes for the names of respondents and of schools to maintain their anonymity. With regard to the security of data, my data will be stored under lock and key for a period of not less than five years, per institutional requirements.

As already mentioned, the participants had the right to withdraw from the study at any time. None exercised this right. I would like to note, however, that I have included one video-recorded lesson for a teacher who did not allow me to record a second lesson.

I was concerned that my presence could influence respondents in some way during data collection, and wanted to protect against them feeling coerced in any way. One of the ways that some researchers address this is to train someone else to collect the data, so that participants feel that if they elected to not participate, for example, the researcher wouldn't know. However, I opted to collect the data myself not only because I wanted to own this research but also in order get more experience with the process of data collection.

On the other hand, I could not ignore the possibility that teachers who had participated in the study could read or hear about the results and could feel that it had exposed their inadequacies. I concluded that this was a possibility that I could not avoid, as some of my results have been published in papers. However, during my analysis of data and my writing of this thesis, I have kept in mind the fact that I have guaranteed the anonymity and confidentiality of all information about both the respondents and the participating schools. In this way I could prevent someone reading my research from being able to recognise an individual or school that was mentioned. In addition, as I only had one video from each teacher, there was a chance that their general teaching approaches and cross-lesson elements were not fairly represented, which could add to their embarrassment should they read the study and see their weaknesses pointed out. I have thus tried very hard not to be judgmental but descriptive and analytical in my results.

I informed the participating teachers that all of the data and everything emanating from the data would be used only for research purposes and that it would not affect their jobs in any way. However, after I had completed the data collection, some participants phoned me wishing to know my assessment of their classroom teaching practices. It is of course positive that the teachers sought out feedback on their teaching, but I felt it was premature to share any views before the research was complete. I therefore informed them that the research results would be made available online as soon as the thesis has been accepted. In retrospect, I worry that the research results will not provide sufficient input for those teachers wishing for specific feedback on their own practice, and hence I need to consider ways to engage with these teachers.

The process of obtaining consent was done some time before the data collection process started. I read the introductory letter to the learners and informed them that the test results would not affect their grades and that they were free to withdraw from the study at any point. Learners were also given consent letters for parents/guardians to sign, in English. However, the school principals informed me that they normally communicate with parents/guardians using Kinyarwanda. To overcome this problem I arranged a meeting at each school with parents/guardians in order to explain to them what the study was about and discuss the contents of the consent letter. This of course did not eliminate the possibility that parents or guardians felt intimidated by my presence and in that way coerced to participate in the study. This power dynamic is very real and requires that participants and guardians be approached very respectfully; even so, it can never be eliminated.

4.7 Video analysis instrument

As mentioned in Chapter 3, this study has used the analysis of video-recorded lessons as one of its key elements. From my knowledge of the research literature, few studies have attempted to develop an instrument to analyse observed PCK. Some exceptions are Ball et al. (2008) and Sorto et al. (2008). Other studies (e.g., An, Kulm, & Wu, 2004; Bayazit & Gray, 2006; Kleickmann et al., 2013) which also involved video-recording the teaching of mathematics in the classroom, only mentioned general concerns regarding classroom observations. Adler and Ronda (2014) developed an analytical framework for describing teachers' discourse during mathematics instruction, but this was published only after I had started my analysis.

Earlier, I explained why I chose to develop my own instrument and not utilise the existing MQI instrument (see discussion on pp. 32ff). Before I describe how I developed my video analysis instrument in Section 4.7.1, I would like to discuss some of the reasons informing my decisions, particularly with regard to the MfT sub-categories put forward by Ball et al. (2008).

One of the reasons for developing my own practical PCK analysis instrument was that practical PCK, by its very nature, can only be interrogated indirectly: by interpreting someone's knowledge in action. Further, as PCK is a concept which has been conceptualised and reconceptualised differently by various authors, I wanted an instrument which reflected my own understanding of PCK.

What takes place during classroom teachings is complex in nature. The professional judgments teachers make about how to construct mathematical content, or the way content is connected progressively through the use of illustrative examples and problems designed clarify the concepts, are not easily described. This is also the case for the methods which teachers use to deal with learners' misconceptions and their thinking during a given task, which might require unpacking the methods used to complete the task in order for the teacher to provide constructive and targeted feedback. These subtle processes are difficult to infer from observable actions.

I could have chosen to code for the cognitive demand of tasks, using existing categories (Boston & Smith, 2009; Stein et al., 2000). Because of my interest in teachers' PCK, however, I chose instead to observe the way teachers engage their learners in tasks, in particular in terms of the nature of connections made. Furthermore, doing mathematics tasks which require complex and non-algorithmic thinking (Boston & Smith, 2009) often takes substantial classroom time, even extending over several periods, making it difficult to code for cognitive demand on the basis of an isolated lesson, which was the conditions of this study due to its replicative character.

To recognize someone's KCC during classroom teaching is not straightforward. In this case, I opted to include sequencing actions in my video analysis instrument, as it reflects KCC. Hugo (2013, p. 89) details different types of sequencing in lessons, and I have drawn on his work in the development of my instrument (see criterion 2 on p. 66).

The TIMSS video studies have explored connections between classroom practices and learners' test performance (Neubrand, 2006). One way to characterise the lessons was according to how tasks were engaged: worked in seatwork (SW), but not shared in classwork (CW); posed or only checked in CW; worked on and solved in SW, and shared in CW; worked on and solved in both SW and CW; or worked on, solved and shared entirely in CW. This was another element to take into account when designing the instrument; including considering the extent to which this should be considered related to PCK or to pedagogical knowledge and strategies only.

I believe that the discussion above highlights the immense challenges of trying to infer practical PCK from teachers' actions in a systematic way. There are even those – such as Baxter and Lederman (1999) – who think that it is a contradiction in terms to try to infer PCK from the practice of teaching. Adler and Zain (2006) note that when a teacher is teaching, he or she needs to interpret the specific mathematical thinking and reasoning in which each learner has engaged, and in doing so he/she will draw on some form of PCK. However, according to the SACMEQ studies (SACMEQ, 2011), the extent to which this is included in teacher education varies; some teacher education programmes seem to put emphasis on teacher content knowledge while other programmes seem to put more emphasis on pedagogical training. It is necessary to investigate what is done in teaching in order to know if emphasis on both teacher subject matter knowledge and pedagogical training are demonstrated and how it links to learning. It is as has been suggested by McNamara (1991) that PCK is not CK added to PK, but that a teacher reflecting on classroom practice may create his or her own PCK.

I now turn to the detailed development of my video analysis tool in the next section.

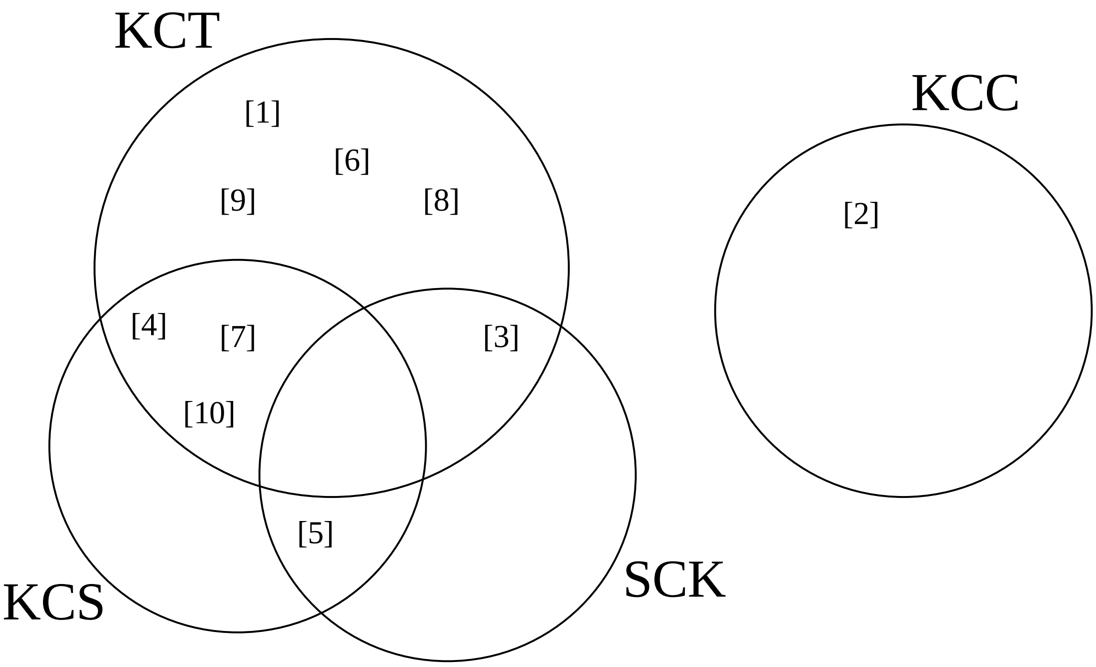
4.7.1 The development of my video analysis instrument

As previously stated, my PCK subcategories, namely KCT, KCC, KCS, are drawn from the work of Ball et al. (2008) who placed them under MfT. In the development of my instrument, I generated criteria for each subcategory based on the work of different scholars. Due to the complex nature of classroom teaching, and in order to increase content and construct validity, more than one criterion was required for each subcategory. Each criterion was then assigned at least three options which a teacher could take while teaching in the classroom. Ten criteria are discussed below together with their respective options of variation within classroom situations.

Figure 4-2 shows the criteria in relation to the sub-categories of PCK. Following that, I discuss the literature which informed the criteria and their options of variation. As can be expected from constructed distinctions, there is some overlap at the level of option of variation.

Most criteria fall under KCT. That is because my video analysis instrument is an observation-based instrument which involves observing both teachers and their learners' actions during classroom teaching activities. KCC and KCS often play a stronger part in planning, assessing and evaluating, which unfortunately were beyond the scope of this study to engage.

Below, I outline the criteria, their relation to the sub-categories of PCK, and the options included in each criterion based on the research literature. I do not go into how these were operationalized; that is discussed in Section 4.8.3.



[1] Content connections
[2] Progression and linkage to other content
[3] Content construction through variation/practices
[4] Use of illustrations and representations
[5] Engaging learners' (systematic) errors
[6] Nature of feedback
[7] Focus of feedback
[8] Unpacking of content
[9] Problem solving engagement activities
[10] Engagement of learners' prior knowledge

Figure 4-2: The ten criteria in relation to the four sub-categories of PCK/MfT

4.7.1.1 Criterion 1: Content connections

The content connections engaged by the teacher in order to create new knowledge within the lesson constitute the first of the criteria I used to clarify KCT. As discussed previously, strong arguments have been presented for conceptually focused mathematics education, and what Hattie (2003) refers to as "connected representations". This illustrates the need for learners to see different mathematical concepts, ideas, algorithms and processes not as independent from each other but as connected parts of the discipline. Thus, I felt that awareness of content connections and ability to engage these in teaching would be a central component of KCT, and a way to try to explicate the above mentioned perspectives.

The framework for the six possible options mentioned in my instrument under this criterion has roots in the work of Mhlolo, Venkat and Schäfer (2012) in which five types of connections in practice have been drawn from Businskas (2008). These are: different representation of the concept, part-whole connections, implication connections, instructional oriented connections (called *prerequisites connections* in my instrument), and procedural connections; the definitions of these types are taken from Businskas' (2008) work. I have also added one option applicable to a situation in which teachers do not show any kind of content connection during their teaching.

Different representations are subdivided further into two types: *alternate* and *equivalent representations*. A representation is considered as an alternate of the other if they come from two different forms of representations: symbolic (algebraic), graphic (geometric), pictorial (diagrammatic), manipulative (physical object), verbal or written description; whereas an equivalent representation represents the concept in a different way but using the same mode – for instance, rephrasing a verbal representation (Mhlolo et al., 2012, p.3).

Mhlolo et al (2012) define the implication connection as an "if … then… representation", suggesting that one concept leads to another. Procedure connections are understood as those in which a procedure relates to concepts or other procedures. Finally, instruction-oriented connections are understood as those related to the fact that some concepts are pre-requisites to understanding related concepts, whereas within part-whole connections one object is part of a more complex whole by inclusion or generalization (Mhlolo et al., 2012, p. 2).

4.7.1.2 Criterion 2: Progression and linkage to other content

My second criterion is the only one which relates directly to KCC through engaging the progression/sequencing of the lesson and linkages to other sessions. Unless the curriculum is strongly determined by national or regional authorities, describing not only the desired outcomes but also what must be taught and when, teachers will always have to engage in some degree of selection and sequencing of content.

To do so, five different options were taken from Hugo's recent work (2013) which relate specifically to the character of the linkage with other sessions and to how the progression is made: (i) simple to complex, (ii) particular to general or vice versa, (iii) theoretical to practical, (iv) concrete to abstract or (v) from everyday to specialized. To understand the above progressions, Hugo gives an example of the Montessori 'golden bead system' as an illustration of moving from everyday to specialized with the purpose of introducing learners to the concept of working with base ten. He argues that the illustration might also provide an example of moving from concrete to abstract and from simple to complex (Hugo, 2013, p.25), which suggests that careful attention may be needed during my coding process as one action could combine different options for progression.

Hugo discusses how hierarchical knowledge structures demand that specific content and skills are covered at early stages as these are needed as a basis for higher levels. However, within shorter teaching episodes, such as a lesson, different forms of progression may be used pedagogically. He notes that "Sequentiality moves up and down as much as it moves connectively across, working with part to whole, concrete to abstract, simple to complex, and particular to generalization" (Hugo, 2013, p. 89).

Hugo argues that when a learner makes a mistake in mathematics, a good teacher should work backwards in the sequence in order to identify which preceding operations and concepts need to be clarified to the learner. However, while doing so, the teacher should keep in mind that each learner may need clarification at a different point.

4.7.1.3 Criterion 3: Content construction through variation/mathematical practices

The third criterion intends to tap into the KCT/SCK subcategories. Depending on the content that a teacher is engaging at a particular time – for example, geometry-related topics, the content may need to be (re-)constructed to make it accessible to the learners. One way to do so is for the teacher to present the constructs to the learners. But in order for the learners to grasp the content, the teacher needs to provide examples which vary in some way. Using variation theory, Marton, Tsui, Chik, Ko and Lo (2004) suggest that this can be done in a number of ways, such as looking at what remains the same across different examples, or comparing examples and non-examples. From this, generalizations can be made, or variations can be combined. The merits of teaching mathematics with variation has been pointed out by Mhlolo (2013) who discusses the variations contrast, separation, generalization and fusion, which have been taken into account in the formulation of this criterion's options. As Marton et al. (2004) describe the *contrast* approach from variation theory, for learners to know what something is, they must know what it is not. They add that as any one thing/concept may have a large number of characteristics which may generate diverse understandings of the thing in question, separation is crucial to single out the defining or critical characteristics. In teaching situations, examples of this abound. In algebra, for example, when teaching what a conic section *is*, it makes sense for the teacher to also engage learners about what a conic section is *not*. *Separation* (cf. Marton et al., 2004) is understood to be the way an aspect of something is experienced and separated from other aspects when it varies while other aspects stay invariant (cf. Marton et al., 2004, p. 16).

When *generalizing*, learners do verifications of the wide-ranging validity of a separated-out pattern (Mun Ling & Marton, 2011) on the basis of engaging the learning object. Finally, *fusion* refers to an action in which learners incorporate several critical appearances of variation into a whole (Mhlolo et al., 2012).

Another way in which to (re-)produce content in the classroom is for learners to engage in investigative activities, or mathematical practices, of some sort which then – if the activities work as intended – give rise to the construction of mathematical content. This is the idea behind the theory of didactical situations (cf Brousseau, 2006). Without going into detail, it must be recognized how much careful planning this approach requires. Ideally, such activities should allow learners to approach them without any awareness of what they are expected to construct, but should also be designed so that learners can only complete the activity by constructing the desired content. This requires knowledge of both learners and content. One could argue that the process would follow the same steps as variation theory: from noticing sameness across situations; to separating out an idea, method or concept; to generalizing; and ultimately to combining with other variations; but that would not be accurate to the theory of didactical situations – and furthermore has not been empirically verified.

Thus, the options for this criterion had to be varied enough to capture the range of possibilities for (re-)construction of content, but with a focus on the relationship to the content, not on 'who' did 'what'. This gave rise to these options: (i) investigation by observation of the object/image through continuous variation/contrast, (ii) separation/discussion on mathematical terms, (iii) verifications done by teachers to clarify areas in which learners exhibit doubts by expressing themselves through their mathematics vocabulary, (iv) generalization of the concept, (v) encouraging learners to communicate mathematically while performing a task (Marton et al., 2004; Mhlolo, 2013; Mhlolo et al., 2012).

4.7.1.4 Criterion 4: Illustrations and representations

The fourth criterion of my video analysis instrument investigates teachers' effort to use illustrative examples and teaching aids for lesson concretization or concept representation.

The use of representations and the connections between them has been considered in relation to mathematics lessons by various scholars such as Cuoco (2001). However valuable, as one could imagine, the use of representations in teaching are likely to be influenced by the content to be taught and the context in which that content is taught. Both content and context play a role when a given teacher is considering which appropriate representation to use in his/her lesson. As illustrations and representations have to reflect the essence of the content taught but also be suitable for the learners, I consider this criterion indicative of both KCS and KCT. It is appropriate to mention here that I have only focused on external representations, both because they are a form of manifestation of the teachers' pedagogical content choices, and because internal representations which imply the creation of images in our minds (Barmby et al., 2009; Cuoco, 2001) are not accessible.

Teachers who possess pedagogical content knowledge recognise when topics are hard to understand and therefore engage representations that make them meaningful. For example, for beginners in mathematics, it may not make sense that multiplying two numbers (eg. $\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$) results in a small number until presented in concrete form. This is why teachers' abilities to create effective presentations are so essential. This is linked to conceptual understanding as well, as mathematical concepts may only be recognisable as that which is the same across different representations.

Based on previous work (Businskas, 2008; Cuoco, 2001; de Villiers, 2004; Mhlolo et al., 2012), four options were developed for this criterion: for the teacher (i) to provide verbal concretizations or representations only, (ii) to use drawn teaching aids / representations such as charts, tables, graphs, diagrams, (iii) to use manipulative teaching aids / representations, or (iv) to combine both drawn and manipulative teaching aids / representations.

4.7.1.5 Criterion 5: Engaging learners' errors

Errors and misconceptions are to be expected when learning something new. This is due in part to the fact that, in most cases, learners come to new content with different perceptions or different schemas which are activated by their encounters with the new content. Some learners develop perceptions about the new content which are in agreement with the mathematically accepted ones, while others develop concept images (Tall & Vinner, 1981) at odds with the accepted ones. The teacher's role at that point is to facilitate convergence of those learners' ideas towards the accepted constructs. Identifying and addressing their errors and misconceptions is one way of achieving this.

Criterion five was aimed at investigating KCS but also with consideration of SCK. This was done by noting the teachers' propensity to engage learners' errors and 'misconceptions' and, importantly, the ways in which they address those errors and misconceptions. Recognizing errors and misconceptions which arise in classroom teaching is not a simple task as it requires special content knowledge, as discussed in Section 2.5. One possibility is that the teacher does not recognise the errors, or simply interprets them as lack of effort from the learners. Again, depending on the teachers' level of specialized content knowledge and on the learners, errors and misconceptions might be recognized but be followed by simply correcting incorrect answers, challenging learners individually or sharing and discussing the question with all of the learners in the classroom.

4.7.1.6 Criteria 6 and 7: Nature and focus of feedback

Feedback to learners and, importantly, the kind of feedback given, is the focus of the six and seventh criteria used in this study, again as indicators of teachers' KCS/KCT. Based on available research on the influence of feedback on learning, I decided to include two criteria here: one emphasizing how the feedback is given and another on the content of the feedback.

I identified four options for *how* feedback may be given, namely (i) to give direct feedback, (ii) to give indirect feedback, (iii) to give a cognitive conflict type of feedback, i.e., try to put the learner in a situation which creates a cognitive conflict, and (iv) to give feedback by facilitating debate within the class. The direct and indirect feedback approaches are defined in this study using the work of Bitchener et al. (2005), who state that direct feedback is providing learners with the correct form when an error is identified while indirect feedback involves only pointing out the error without providing the correct form. On the other hand, Jehn (1997) argues that disagreement about the content and issues involved in the task – which may arise from interpreting the content differently - creates cognitive conflict. Such a situation may occur on an individual level or be facilitated during classroom teaching thus, in practice, functioning as a form of feedback to all the learners.

In terms of the focus of the feedback, four options have been identified: feedback on (i) the product/result and the task, (ii) the process, (iii) self-regulation; where the learner learns to ask meta-questions about the process and result and thus locate possible problems, and (iv) self; where the feedback concerns the learner, not the work. These options are defined by Hattie and Timperley (2007, p. 90). They argue that feedback about either task or product indicates to learners whether their work is correct or not in relation to the task. Feedback on the process aims at guiding the learner's way of working on the task; for example: "If you cannot compare the fractions as they are, can you write them in another way that would enable you to do so?" Feedback which focuses on self-regulation tends to help learners to develop the confidence to engage more on a given task and to encourage them to detect errors and fix them themselves (Hattie & Timperley, 2007). It may consist of common meta-questions such as "How can you check your answer?" which learners may then internalize. Praising a learner individually, for example by tell him/her that s/he did well, is considered feedback as well, and is also potentially observable in classroom teaching situations.

4.7.1.7 Criterion 8: 'Unpacking' of content

Criterion eight of my video analysis instrument deals with the strategies which teachers use to 'unpack'[7] methods or concepts to make the content more accessible to learners. It falls within KCT as it is about adjusting content in teaching. Neubrand (2006) identifies five methods or strategies considered to be factors in knowledge acquisition which teachers might use to involve learners in mathematics classroom problem solving. He also notes that the working environment – including variables such as the number of learners within a classroom or the availability of classroom resources (books and other materials) – can play a role in teaching approaches teachers select. I have incorporated four of them in my instrument – but preceded these with the option of not 'unpacking' at all. The four options taken from Neubrand are as follows: teachers use only rules/procedural descriptions to unpack content; teachers engage learners with more than one method to unpack the content but do not follow this with a comparison/analysis of the methods; the teacher demonstrates more than one method to unpack the content and engage the learners with comparison or analysis of the methods; and, lastly, the teacher only uses definitions / conceptual

[7] Lacking a better term, I have used 'unpacking' to refer to this, but this is clearly not the same as 'unpacking' of content in teacher education as previously discussed.

descriptions to unpack the concepts. The fifth option proposed by Neubrand concerns metacognitive actions and I have excluded this option from my instrument because the length of the lessons I have recorded is inadequate for teachers to be able to utilize this option.

4.7.1.8 Criterion 9: Task engagement

Criterion nine, while different from criterion eight, complements it as it deals with the *capability* of teachers to unpack the methods ⁄ concepts they are teaching in order to make them more accessible to learners; this is also intended to measure teachers' KCT. This capability is reflected through three options in my instrument. The first option is that a teacher is not observed to use tasks and alternative strategies to clarify the concept. The second one is that a teacher engages more than one method to unpack the methods ⁄ concepts but does not follow through with a comparison / analysis. The third option is to observe whether or not the tasks given have been worked on as individual seatwork or in a working group but not shared with the rest of the class. These seem to give learners room to express themselves when tasks are worked on individually or in groups, checked and shared in class.

4.7.1.9 Criterion 10: Engagement of learners' prior knowledge

The interactions that take place through communication between teachers and learners both inside and outside of the classroom may not only enhance their social relationships with each other but also help teachers to understand their learners better, both morally and intellectually. The last criterion used in my instrument aims to assess teachers' KCS/KCT through the way they engage their learners' prior knowledge. This criterion allows for three options. The first option is that a teacher chooses to start teaching a new topic/concept without assessing the learners' prior knowledge. The second option is that the teacher does assess the learners' prior knowledge but does not build on it when introducing the new topic. The third option is that the teacher assesses the learners' prior knowledge and does build on it when introducing the new topic. A teacher referring back to previous content that has been taught (covered in the first criterion as making connections) was not considered to be a form of assessing prior knowledge. This is because the teacher is making a connection by referring to what has been taught previously, whereas assessing learners' prior knowledge involves the teacher trying to find out how the learners think by listening and observing. For the same reason I have excluded assessments of what learners have acquired or retained from previous lessons from this category.

If more opportunities are given to learners to connect new knowledge to existing knowledge, learners' generalization potential is increased and that, along with honouring and recognizing learners' knowledge, boosts their self-esteem as they feel that they are contributing to the learning process (Furner et al., 2005).

4.7.1.10 The final observation analysis instrument

The ideas discussed above, as well as other concepts related to PCK which were discussed earlier, provide the basis for the instrument I developed to analyse the video recorded classroom observations. The instrument is summarized in Table 4-1.

4.8 Data analysis

4.8.1 Analysis of learner test results – and questionnaires

The analysis of learner test results was done primarily using Microsoft Excel. As this study was a correlational study in design, the pre- and post-tests had the aim of providing a measure of the difference in learner performance from the beginning to the end of grade six, as an indication of any learning gain. This would then be correlated with other various background factors based on learners' responses on the questionnaire, and eventually with the results from the teacher tests and observations.

As a first step, I entered each learner's response to each and every question from the pre- and post- tests, as well as their answers to the questionnaires, into an Excel spreadsheet. Missing answers were recorded with one code, and unclear answers with another code.

The first step of the analysis was to analyse the test results on their own. I determined the relative frequency of correct answers in the pre- and post-tests. The learner responses were also grouped according to the primary mathematics content domain to enable me to identify in which mathematics areas learners were experiencing difficulties. To provide a broader sense of learners' performance, I opted to use the SACMEQ numeracy level, which range from level 1 to level 8 (see Appendix B). However, in the test there was no question on level eight and only two questions were on level one and seven, respectively.

I then compared the data from the pre- and post-tests. As an in-depth comparison during a previous study found that South African learners changed their answers from the pre- to the post-test (Christiansen & Aungamuthu, 2013), I analysed how 'stable' the learners' answers were between the two tests, ie. the frequency of learners choosing the correct answer both times or changing from an incorrect answer to the correct answer, from the correct answer to an incorrect answer, or from one incorrect answer to another.

The final stage was to correlate test results and 'learning gains' with the other data sets.

Table 4-1. The classroom video analysis instrument

PCK sub-domains	Criterion	Option one	Option two	Option three	Option four	Option five	Option six
KCT (Knowledge of Content and Teaching)	**1: Content connections**	No kind of connections observed.	Different representations are used (equivalent or alternate).	Implication connections.	Procedure connections are used.	Prerequisite connections are observed.	Part-whole relationships are observed.
KCC (Knowledge of Content and Curriculum)	**2:Progression of the lesson and linkage to other content**	No progression or linkage observed.	Progression is from simple to complex.	Progression is from particular to general or vice versa.	Progression is from theoretical to practical.	Progression is from concrete to abstract.	Progression is from everyday to specialized.
KCT/SCK (Special Content Knowledge)	**3:Mathematical content construction through variation/ mathematical practices**	No kind of mathematical content construction through practices/ variations is observed.	Investigation by observation of the object/image through continuous variation/contrast is observed.	Mathematical terms are used by learners to explain why the conjecture is true or false through discussions/ separation.	Verifications are done to clarify areas in which learners exhibit doubts by expressing themselves within their math vocabulary.	Generalization of the concept by leaving or adding properties from complex tasks under organization is observed.	Learners are encouraged to communicate mathematically while performing a task.
KCS (Knowledge of Content and Students)/ KCT	**4: Illustration and representations**	No examples or teaching aids used.	Verbal representations used.	Visual illustrations or representations used.	Manipulative teaching aids used.	Combination of visual and manipulative teaching aids/ representation used.	
KCS/SCK	**5: Engaging learners' errors**	Errors and mis-conceptions are not observed.	Errors or misconceptions are observed by researcher but not recognized by teacher.	Errors or misconceptions are recognized but ignored and incorrect answers are simply corrected.	Incorrect answers arising from misconceptions are challenged individually.	Errors and misconceptions are engaged with learners in groups or in the class.	

PCK sub-domains	Criterion	Option one	Option two	Option three	Option four	Option five	Option six
KCT	**6: Form of feedback given**	No feedback observed.	Direct feedback given.	Indirect feedback is given.	Cognitive conflict type of feedback is given.	Feedback is given through creating debate within the class.	
KCS/KCT	**7: Focus of feedback given**	No feedback observed.	The feedback given is about task or product.	The feedback given is about process.	The given feedback is on the level of self-regulation.	Personal feedback (self) is given.	
KCT	**8: Unpacking of content**	No attempt to unpack the methods⁄concepts is observed.	Only rules/ procedural descriptions are used to unpack content.	More than one method is used to unpack the content but this is not followed by comparison/ analysis.	More than one method is used to unpack the content and comparison or analysis is provided.	Only definitions/ conceptual descriptions are used to unpack the concepts.	
KCT	**9: Task engagement**	The use of tasks to clarify the concept and alternative strategies is not observed.	Posed problems have been worked on through direct teacher-learner interaction.	Tasks are worked on as individual seatwork or in a working groups but not shared in the class.	Tasks are worked on individually or in groups, checked and shared in class.		
KCS/ KCT	**10: Engagement of learners' prior knowledge**	Prior knowledge not engaged.	Learners' prior knowledge noted but not used as foundation for new topic.	Learners' prior knowledge noted and used as foundation for new topic.			

4.8.2 Analysis of teacher test results

Data from the teachers' test was captured on an Excel spreadsheet and then analysed. Missing responses and unclear responses were also noted, with different codes.

As previously mentioned, some test questions focussed on content knowledge and others on pedagogical content knowledge. The test questions included different mathematics subdomains such as numbers, measurement, statistics, probability, etc. However, there were no PCK questions for algebra. The PCK questions included things like unpacking mathematics (KCT) and learning thinking / error analysis (KCS). For example, the category of 'unpacking' included questions which required teachers to unpack the algorithm of learners while they were doing a multi-digit whole number multiplication, whereas the questions on 'learner thinking' required teachers to predict learners' errors and identify misconceptions that learners might have (for example, with regrouping in addition) or else to explain the reasons behind learners' constructing a false identity for a given fraction.

To obtain answers to some of my research questions, I worked out the correlation between CK scores and PCK scores within the different mathematics subdomains considered in this study. This was done in order to determine how teachers' content knowledge in different domains fit with learners' performance gain. For the same purpose, I looked at how content knowledge correlated to each of the above-mentioned PCK categories. This analysis allowed me to make some comparisons on an international level, specifically between Rwandan and South African teachers (Maniraho & Christiansen, 2016).

The test results were also correlated with teachers' responses on the questionnaire (eg. teachers' years of experience, training), and finally with the other data sets.

Though I have raised this elsewhere, I would like to note again here the limited external validity of the results due to the low number of participants.

4.8.3 Analysis of lesson observations

As detailed in Section 4.7.1 and in particular in Table 4-5, my video coding was based on ten criteria related to the PCK subdomains used in this study. Each criterion was linked to different options for behaviour a teacher could demonstrate while teaching. The number of options varied from three to six across the different criteria, including the one common to all criteria, namely *no presence of any option* (cf. Table 4-1).

I followed the coding approach used in previous studies, namely to code the videos at intervals of five minutes for the presence of any of the options provided in the instrument. This was done both to enable comparison with the results from the previous studies, but also for pragmatic reasons. To determine a unit of analysis which could have worked across all the criteria may or may not have been possible, but it would certainly have added another layer of complexity to the coding process. Thus, I divided each lesson into five minutes clips. I then noted my observations of what happened within that time period in terms of teachers' actions, learners' reactions to their teachers' actions and also teachers' reactions to their learners' actions. This helped me to determine which coding categories suited which actions. Each video was watched at least three times, and often more, in order to ensure that the codes were applied consistently. The early coding of lessons was done in consultation with my PhD-cohort group until high inter-coder reliability had been established, after which I coded the remainder of the data myself, albeit with consultation with my supervisor in cases where I had doubt.

It was, obviously, possible to assign several criteria and options within a five minute clip. However, there were also times where I coded for the same options of a criterion within two successive five minute intervals. This occurred when, for example, a teacher gave a set of exercises to learners as individual seatwork and then took time to move around the classroom observing what and how they were performing the given tasks.

For each lesson, I noted its duration, which varied between 45 and 60 minutes despite the official allocation of 50 minutes for a lesson in Rwandan primary school classrooms.

In order to investigate correlations within the data, I changed these codes into quantifiable results. One cannot assume that 'more of a good thing' makes for better teaching (cf. Doyle, 1977). Doyle argues that the occasional use of a particular approach may highlight the content as something important, compared to the frequent use of a particular approach, which makes the content appear as 'more of the same'. Nonetheless, I worked out the frequencies of codes in relation to the total number of 5 minute intervals in a lesson. If, for example, there were 10 intervals in the lesson, and content connections occurred in two of these, I would note that 20% of the lesson included content connections. This is of course a rather crude measure, as it is possible that both the intervals were focused entirely on content connections, or that very little of the 5 minute interval was spent on this.

The time differences spent by teachers on the various options of the various criteria is discussed in Chapter 6 of the thesis, especially by investigating if they relate to differences in their learners' 'learning gains'.

The operationalisation of the coding using the instrument, with examples, is presented in Chapter 7.

4.8.4 Trustworthiness issues of data analysis

Every research study should consider validity / trustworthiness a key concern, because if the study proves to be invalid, the results offer no useful insights. However it is impossible for research to achieve 100 per cent validity (Louis et al., 2011). In this study I attempted, as other researchers have, to maximize validity/trustworthiness. As indicated in Section

4.6.3, one way of doing this was to use both qualitative and quantitative approaches, which are not mutually exclusive (Hammersley, 1992).

To avoid confusion between the selected schools, a numeric code of six digits was given to each learner. From left to right, the first two digits stood for schools, the next digit represented the school teacher, and the last three digits the learners. This facilitated analysis of the data across these three levels.

The confirmability of my study is enhanced by the fact that all my findings emerge from the data collected through my informants and not from any overt predilection (Guba, 1981), although I recognise that analysis is always influenced to some degree by the researcher. To counter this, I have promoted transparency by explaining my reasons for each coding decision and for my analytic process, as described in the previous section.

The operationalisation of the theoretical concepts is in itself an attempt to promote construct validity through clarifying the boundaries of each category used in the analysis. As a substantial part of this thesis concerns the development of an instrument for analysis of practical PCK, I would argue that construct validity is fairly high. One can always argue that a learner test only captures certain aspects of learners' knowledge and competencies, and I concur. However, as has been shown earlier (pp. 57), the test had a fair distribution of questions covering different relevant content areas, when compared to the Rwandan curriculum, and a fair distribution on the SACMEQ levels 3-7.

By including all aspects of PCK in as systematic fashion to the greatest extent that I could, I have attempted to increase the content validity of the lesson observation component of the study. However, as previously recognised, the written teacher test did not cover all aspects of PCK, as it was inherited from the previous studies. In retrospect, it may have been feasible to add a few questions to increase the content validity of the test.

I tested the consistency of my coding by watching lessons repeatedly and coding them again and again on different days to check if I coded the same instances in the same way. I noticed that while coding the teachers who were teaching the same topic they tended to perform the same actions in their classroom teaching, which helped me to ensure uniformity in coding.

I spent substantial time at the onset of the coding of the observations coding and re-coding after discussing with fellow doctoral students and my supervisor, in particular. This lead to some refinement of coding categories, and as this process continued until a high inter-coder-reliability was established I feel attempts have been made to ensure validity in this respect. However, it was not feasible for someone else also to code all observations, and thus there is some room for improvement of the confirmability of the analysis.

The quantitative data analysis in this work was done using Microsoft Excel and SPSS to increase the certainty of error-free calculations.

Each learner's tests one and two were compared in order to determine the learners' gain, which was correlated to their socio-economic status and to their teachers' test scores, in order to investigate possible correlations which could suggest an explanatory cause behind the differences in performance.

Based on the way my results are discussed all types of data, both quantitative and qualitative, have been combined to justify some of my arguments. Figure 4-3, also served as a framework for my data analysis.

In this study, questionnaires were analysed using SPSS descriptive and inferential statistical tools, linking the results to those produced by the video coding as well as the learners' learning gain. This helped me determine the group means, standard deviations and correlation coefficients. I have represented my findings using tables and graphics as well as statistical reporting.

Before ending, I would like to note that a challenge which I had to overcome in my data analysis was the problem of converting qualitative data (recorded videos) into quantitative data. This required me to code qualitative data numerically in order to establish their frequency.

4.9 Chapter summary

Chapter 4 has described the methods and methodology used in this thesis. It has presented in detail the research paradigm and the study design. My research site and participants have been elucidated by describing the research site, the context under which this research was done and the demographic characteristics of the research participants. The sampling method was presented and the data collection procedures described. In this chapter, I have also discussed the trustworthiness of the data collection and data analysis methods and how ethical issues were addressed.

5 ANALYSIS OF LEARNER TEST PERFORMANCE AND PRESUMED LEARNING GAINS

5.1 Introduction

This is the first chapter to address results from the study. It deals with test performance of the Rwandan grade six learners in the study, and their 'learning gains'. Furthermore, I compare these results to those from similar studies conducted in Botswana and two provinces of South Africa (Ally & Christiansen, 2013; Aungamuthu & Christiansen, 2013; M. Carnoy, Chisholm, & Chilisa, 2012; Noubouth, forthcoming; Ramdhany, 2010). South Africa and Botswana share the common challenge of low learner performance in mathematics has compared to performance in other subjects and has compared with other SACMEQ countries (Maree, Aldous, Hattingh, Swanepoel, & Linde, 2006; MINEDUC, 2006; Schollar, 2008). Since Rwanda has not participated in any regional or international mathematics tests, it has not been possible to compare Rwandan learners' mathematics performance with that of learners in other countries (Uworwabayeho, 2009), and so this comparison with South Africa and Botswana is a first. With the help of my supervisor, it was possible to compare these data with the data from a study conducted in KwaZulu-Natal directly, while the results from the studies conducted in North West province and Botswana had to be estimated from the results reported in Carnoy et al. (2012).

This chapter looks at the learners' test performance at the onset of grade six (Section 5.3) and at the end of grade six (Section 5.4), and at the learning gains presumed to have been made during this interval (also Section 5.4). In each Section, the results are compared to the results from the previous studies by topic and across the eight numeracy levels used in the SACMEC studies (See appendix B). When it comes to the learning gains, the "stability" of the learners' answers between the two tests is also considered (Section 5.5), cf. the discussion of the trustworthiness of such measures on p. 60. After concluding that the "learning gain" for the Rwandan learners indeed appears to reflect actual learning, I interrogate the extent to which this is related to the learner background variables (Section 8.2). The chapter ends with a short summary. As a background to the presentation of the results, I first provide some information about the three contexts of the three studies that will be used for comparison with this study.

5.2 The three contexts

Seventy-nine percent of all learners from the SACMEQ countries[8] which took part in SACMEQ III were provided with basic learning materials, and the majority of grade six teachers had appropriate qualifications and attended continuing professional development courses with "satisfactory" regularity (Moloi & Chetty, 2011). Botswana lies near this average with regard to basic learning materials, but around a third of Botswana's learners

[8] This includes Botswana and South Africa.

did not have individual mathematics textbooks (Monyaku & Mmereki, 2011). In South Africa, 82% of the learners were provided with the three basic learning materials (note books, exercise books[9], and geometric tools) required for classroom activities, which is higher than the SACMEQ average, and 36% of the learners were provided with individual mathematics textbooks (Moloi & Chetty, 2011).

The situation in Rwanda with regard to these variables is unknown. However, in the case of the 20 classes which I observed, the situation varied based on the location of the schools. Learners in schools located in urban areas were well equipped compared to those in rural areas. The majority of learners in urban areas had the basic learning materials such as mathematics note books, mathematics exercise books and mathematics kits containing geometric tools, which they needed in order to participate in classroom activities. The situation was vastly different in rural area schools in which, apart from mathematics notebooks, a considerable number of learners did not have mathematics exercise books, and had to combine notes and exercises. Besides, mathematics textbooks were often shared between two or three learners.

The performance of South African grade nine learners in the 2011 TIMSS study was very poor compared to that of other participating countries (Mullis, Martin, Foy, & Arora, 2012), and the same was found for grade six learners in the SACMEQ studies (Howie, 2004). Although Botswana was ranked just above South Africa in both the TIMSS 2011 study and in SACMEQ III (Spaull, 2011), the performance of grade six learners in these two countries was similar in terms of overall test scores. This was also confirmed by the studies conducted in South Africa's North West province and in Botswana (Carnoy, et al., 2012).

The low mathematics achievement in South Africa has been explored in several studies, both large scale correlational studies and smaller scale studies (for an overview, see Hoadley, 2012). The variation between schools is greater in South Africa than elsewhere (Case & Deaton, 1999) due to the apartheid legacy. This is strongly linked to socio-economic factors in the home situation, which is a stronger factor in the performance of South African learners than it is in the other SACMEQ countries (Van der Berg et al., 2011).[10]

[9]In Rwanda, it is not uncommon for learners to have a notebook to use for classwork and an exercise book for homework.

[10] Note that in the South African education system two types of schools are in place, namely: government schools and independent schools. In more affluent areas, government schools can charge substantial fees from parents which enable these schools to provide more materials and hire additional teachers. This typically happens in schools which were reserved for learners of European heritage ("White") during apartheid, which are often referred to as Ex-Model-C schools. Because of the legacy of the deliberately unequal schooling system under apartheid, public schools in South Africa are divided into quintiles on the basis of the affluence of the school.

Some important factors related to achievement within schools have been highlighted (cf. Ally, 2012; Ramdhany, 2010; Reeves, 2005; Spaull, 2011; van der Berg, et al., 2011). However, some might be more significant for certain subject areas. This is the case with regard to the availability of textbooks, which are more essential to language studies than to mathematics (Spaull, 2011).

In a study of grade six teaching and learning in KwaZulu-Natal, 15% of the differences in learner performance were accounted for by the number of days the teacher was absent and by the language of instruction (Christiansen & Aungamuthu, 2012). Looking at the results from SACMEQ III, Moloi & Chetty (2011) found that important factors were teaching practice, teacher development, the use of resources and the (un)availability of teachers with the ability to expose learners to extensive applications and high order questions involving both concrete and abstract problem solving skills. The availability of teachers with this ability was so limited in the KwaZulu-Natal study that it was not possible to assess if they would indeed make a difference (Ally & Christiansen, 2013), but this does highlight the lack of opportunity the majority of South African learners experience to develop proficiency in mathematics.

5.3 Learners' test performance at the start of grade six

Before presenting the analysis of the learners' test performance results I would like to discuss its format and how it was conducted.

The learner test was made up of forty multiple choice questions related to five main topic categories, namely: number/arithmetic; measurement; algebra; geometry; and statistics/data handling and probability. Table 5-1 shows the distribution of questions on the various topics.

Table 5-1. Number of learner test items per topic

<table>
<tr><th colspan="4">Number</th><th>Algebra</th><th>Geometry</th><th>Measurement</th><th>Data handling</th></tr>
<tr><td colspan="4">17</td><td>6</td><td>5</td><td>7</td><td>5</td></tr>
<tr><th>Fractions</th><th>Number sense</th><th>Basic operations</th><th>Word problems</th><td></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td>6</td><td>7</td><td>3</td><td></td><td></td><td></td><td></td></tr>
</table>

Each test question had four answer options of which only one was correct; learners were asked to select the one which they thought was correct. Some of the questions required a basic knowledge of the English language as they were given as word problems whereas others did not, such as questions directly involving the addition, multiplication or division of two numbers.

The test was originally designed by a team of mathematics education researchers from South Africa, Botswana and the USA who used some existing test items but mostly developed new ones (Carnoy, Chisholm, & et al., 2008). Distractors were constructed around common misconceptions from the literature. The test was used in Botswana and the North West province of South Africa; the same test was used in the KwaZulu-Natal study and with very minor and non-significant adaptions again in this study.

In all cases, the learners completed the tests individually and without calculators. For various reasons, I have been requested not to reproduce any of the questions in this thesis. However, the questions included non-verbal questions involving basic operations with whole numbers such as 150×20, questions with limited text (similar to the one in Figure 5-1), questions on finding the next value in a visual or number pattern as well as multi-step word problems.

Ngobile packs eggs. There are 6 eggs in a box. Ngobile packs 9 boxes. How many eggs has she packed all together?	[Accompanied by illustration showing 6 eggs in a box and an empty egg box]

Figure 5-1: A learner test question example on whole number operation with limited text.

The main issue with using this test in Rwanda was a question on map reading using a coordinate grid, as this is not taught to learners in Rwanda in grade six mathematics.

The answer options for the question in Figure 5-1 from which learners could select were: 54 (the correct answer), 15 (a common incorrect answer where learners add numbers without considering the context), 12 (what learners would get if they worked from the picture) and 3 (learners subtracting).

To get a sense of the level of difficulty of the questions, they were categorized according to the SACMEQ levels of numeracy (see appendix B). The frequency of questions for each level is shown in Table 5-2. One question which had been reproduced badly in the photocopied test has been excluded from the count.

Table 5-2. Frequency of questions on each SACMEQ level

Level	1	2	3	4	5	6	7	8
Frequency	1	3	11	12	8	3	1	0

Before administering the test, I piloted a test with more questions that fit the higher SACMEQ levels, but the pilot indicated that this test was too difficult for learners, and thus would not provide relevant information about what learners can and do learn over the duration of grade six. I therefore retained the original test, only changing superficial elements such as the names of persons or places used in the questions to be more ‘Rwandan’.

The post-test was the same as the pre-test, but with the questions in a different order. This enabled me to compare the learners’ responses to each question across the two tests and determine the presumed learning gain.

In order to get a sense of the variations in learner performance across the four data sets, I ordered the questions according to content domains, cf. Table 5-1. The results are shown in Figure 5-2. The graph shows a good correspondence between the performance of the learners in the two South African provinces, some differences between learners in South Africa and Botswana on a few items, and a number of differences between learners in Rwanda compared to the other three data sets.

In this and the following graphs, the learner performance is given as the relative frequency of correct answers in the cohort; how many learners got this answer correct as a percentage of the number of learners in the respective study. Thus, it is an indication of the performance of the *group* of the participating learners, not of individual learners.

At the onset of grade six, the Rwandan learners performed worse than their counterparts on a question concerning order of operations, a question on reading off a grid, a question on recognizing circles amongst other geometrical shapes, and a question on recognizing a two-dimensional side-representation of a three-dimensional figure. The differences in favour of the Rwandan learners showed up mostly in the content domains *number* and *measurement*.[11] Figure 5-3 shows a further breakdown of the learner tests results within *number*.

[11]In considering the comparisons, it must be noted that the Rwandan data include results from private schools as well, unlike the two studies with which it is compared here.

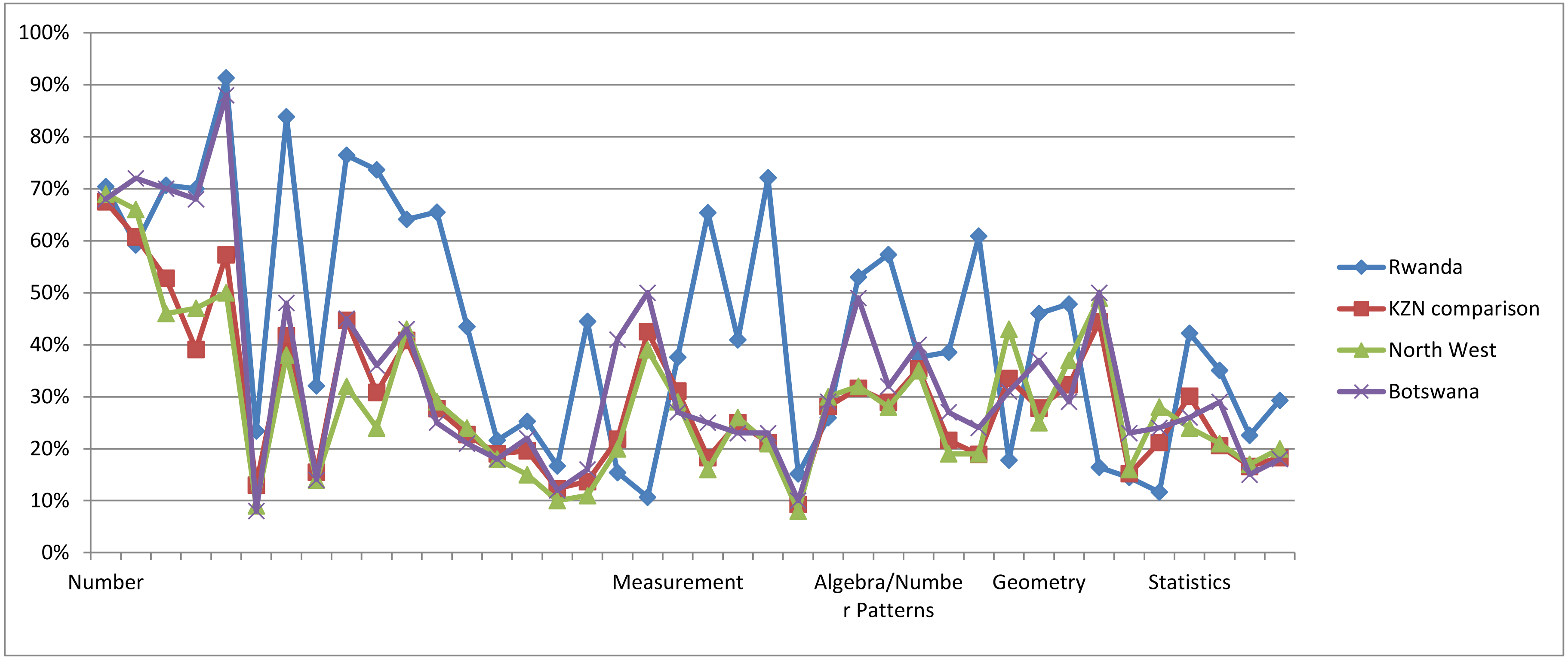


Figure 5-2. Mean test scores % correct for each test item, grouped according to content domain

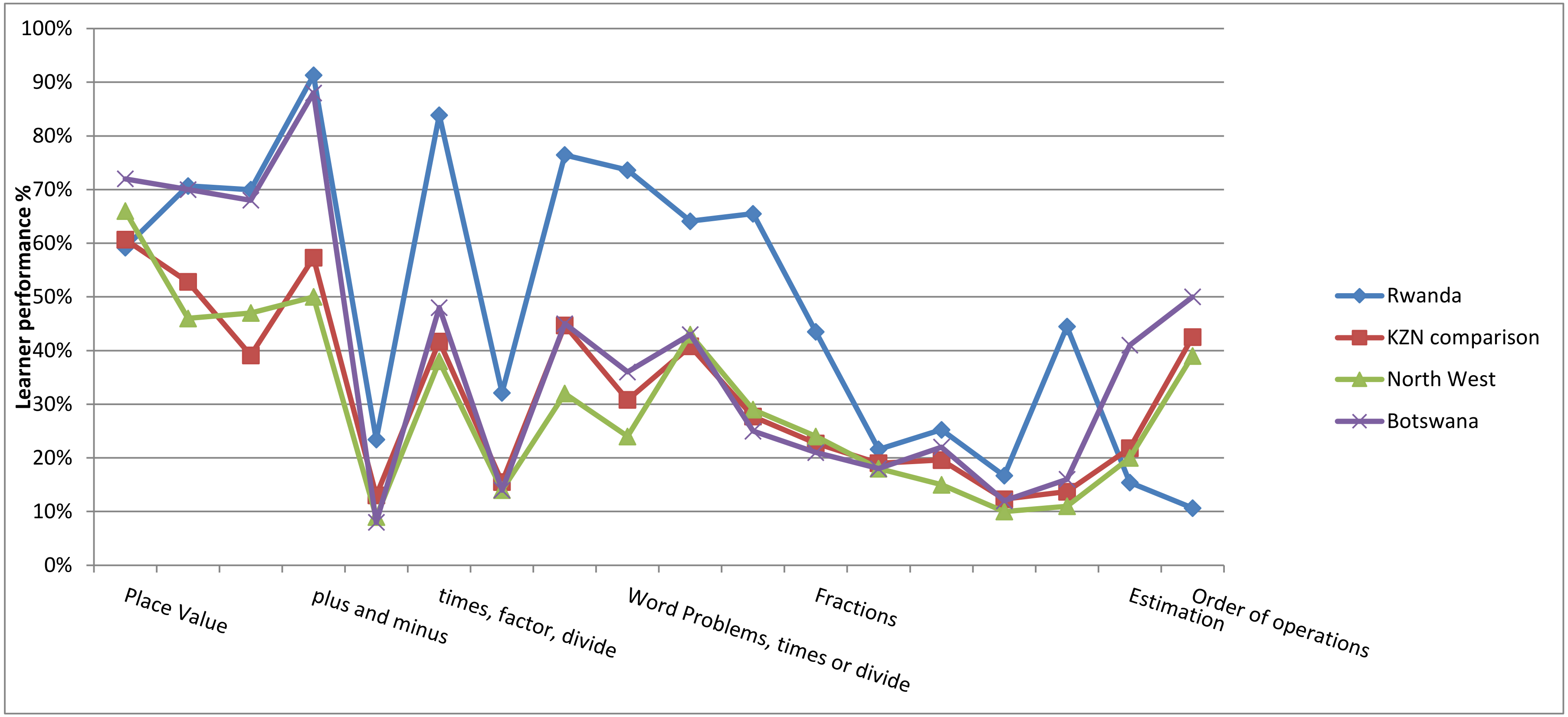


Figure 5-3. Learner performance % correct on items within 'number'

There was only one question on *estimation* and one on *order of operations*, and generally the learners performed badly on these questions, with the Rwandan learners fairing very badly on the order of operations question. Learners across the four studies appear to battle with fractions, except that the Rwandan learners did much better on a word sum involving finding a fraction of a whole number and a word sum involving finding a multiple of a half. The Rwandan learners also did much better on word problems involving multiplication or division of whole numbers, as well as on other basic operations though the learners across all four cohorts did not perform well on a question asking them to find differences between values, read off a table and list the largest difference (a two-step problem). Both the Rwandan and the Botswanan learners did better on place value questions as well as on the only other question, which involved simple addition and subtraction.

Figure 5-4 shows the variation in learners' performance on the seven items classified as *measurement*. The learners across the four cohorts did not do well on a multi-step problem of finding an area of a shape in a grid where the unit square was not 1, while a question on determining time in a different time zone and a question of ordering volumes according to size appear to have been equally difficult for all the learners. (The latter question showed containers with written measurements, and asked learners to order them. The size of the images did not correspond to the written measurements, and thus the question appears to me as much a question of recognising what is legitimate in the context of a mathematics test). However, the Rwandan and Botswana learners did better on a question on recognizing the correct unit for measuring mass, and the Rwandan learners did better on the remaining three questions on converting units and deciding on the most appropriate unit for a particular task.

Next, I coded the questions on the test using the levels of numeracy developed from the SACMEQ studies (Hungi et al., 2010). There was only one question on level 1, one question on level 7, and there were no questions on level 8 (see Table 5-2). For each of the levels 2-6, I calculated the mean frequencies of correct answers. The results are summarized in Figure 5-5.

As can be seen, Botswana and Rwandan learners score somewhat higher than the South African learners on the level 2 questions, and the Rwandan learners scored better than the other three cohorts on level 3 questions. After that, the groups are rather comparable.

This suggests that Rwandan learners perform better on the basic numeracy levels, with level 2 being emergent numeracy and level 3 being basic numeracy (cf. Hungi, et al., 2010, p. 8, detailed in Appendix B).

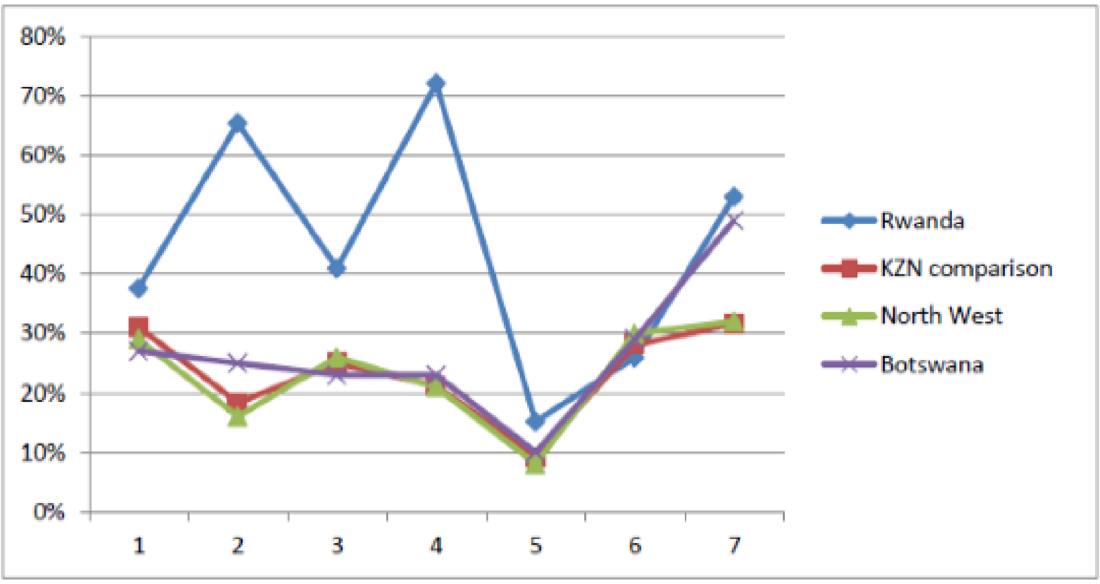


Figure 5-4. Learner performance (% correct) on measurement items

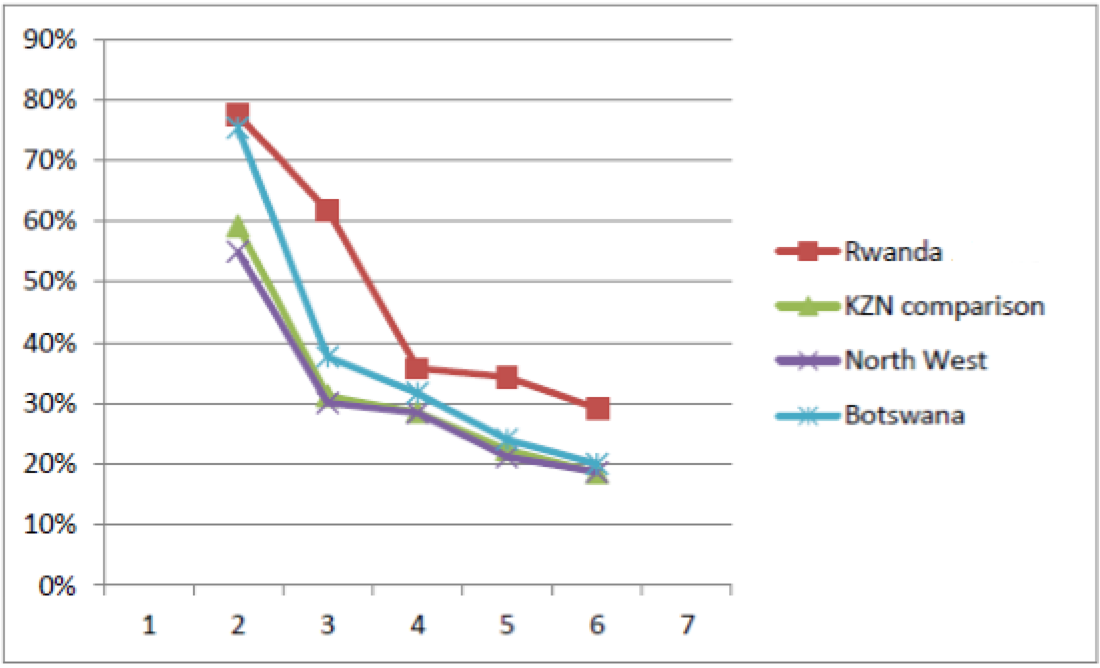


Figure 5-5. Mean performance (% correct) on the pre-test of learners in the four cohorts for SACMEQ numeracy levels 2-6

Looking at the areas where the Rwandan learners performed better than their counterparts in the other three cohorts, it appears that they have a stronger foundation in the most basic level of numeracy; ie numeracy questions using the four basic operations. Otherwise the learners across the four sets of data performed similarly.

The mean test score in the Rwandan sample was 40.3% with a standard deviation of 13 percentage points whereas the mean test score for the KwaZulu-Natal sample was 29.6% with a standard deviation of 13.3 (see Table 5-3). For the Botswanan sample the mean was 33.6%, and for the sample from North West it was 28.6% (Carnoy, et al., 2012, p. 87), with standard deviations of 12.4 and 12.2 percentage points, respectively (Carnoy & Arends, 2012).

These overall results support the hypothesis above, with Botswanan and South African grade six learners performing very similarly, though Botswanan learners performed slightly better than South African learners, and Rwandan learners performing somewhat better overall. Despite the Rwandan sample being smaller than the KwaZulu-Natal sample, the standard deviation was very similar, suggesting less spread in learner performance in Rwanda.

Table 5-3. Test scores on pre-test across the four samples

Mean test score is in percentages of maximum score possible. While I have included all the Rwandan learners who wrote the pre-test, I did not have access to this information for the other cohorts.

	Number of learners in sample	Mean test score	Standard deviation
Rwanda	713	44.2	13.0
KwaZulu-Natal	1276	29.6	13.3
North West	3800	28.6	12.2
Botswana	1750	34.6	12.5

5.3.1 Questions which presented a challenge to Rwandan learners

Here, I discuss in more detail some areas in which Rwandan learners frequently experienced difficulties, identified by selecting incorrect answers to the test questions, and explore what this may mean about their mathematical comprehension.

Around 90% of my respondents use their mother tongue at home, and during the testing I observed that some learners experienced difficulties with language when attempting to answer some of the questions. One clear example of this was question 6. This question was about the determination of the greatest difference between the numbers of raffle tickets sold by a football team. Only 25% of the learners chose the right option in test one, while 47% preferred the option reflecting that they were not taking into account the word "difference". That was also the case during the second test, though the number dropped slightly to 45%. The same matter arose with question 12, in which learners were supposed to choose a fraction equivalent to 2/8. The answer selected by most learners suggested that they did not understand the meaning of "equivalent". No more than 26% picked the correct choice, while 44% chose 1/8 as equivalent to 2/8. This mirrors the analysis of learner responses in relation to language difficulties in South Africa (Christiansen & Aungamuthu, 2012), suggesting that when learners do not understand a question formulation fully, they are likely to opt for an answer which contains something from the question or something with which they are familiar (cf. Dempster, 2007).

The favoured answers to question 40 showed that learners knew the units of weight but that some confused *g* and *kg* or had not considered the context. The question was to determine the weight of a boy; 30% of the learners chose the option with the unit *g* and 53% the (correct) option with *kg*.

I expected a high frequency of correct answers for question 3 in which learners were supposed to write a given word as a number. In the Rwandan mathematics curriculum, place value topic is introduced in grade one where learners learn to recognise units, tens and hundreds. However, for this question, 13% of the learners simply wrote digits without consideration for place value. It is possible they didn't understand the question accurately because of the language problem. For question 7, which also dealt with place values (respondents were asked to determine the value of 6 in 7625) 17% chose an incorrect answer, even though the answer options were written as numerals.

One geometry question also posed problems to many of the learners. More than two thirds of the learners (69%) could not differentiate circles from ovals. This suggests to me that they were not aware of or did not apply the properties of a circle. It was as if to them every 'round' geometric figure which is closed could be considered as a circle. It makes me wonder if this is addressed in schools, or if it was perhaps a language issue.

In question 29, learners were shown a line of pictures of containers, with the amount of liquid in each container written on its label. It was the learners' task to read the labels in order to find out how much each container holds, and order them by quantity. About a third (34%) of the learners made their choice on the basis of visual cues rather than labels. They simply looked at how big or how tall the container appeared, without reading the labels. This could mirror another language problem: a difficulty in understanding the instructions. It could also indicate a lack of understanding of what is legitimised in the mathematics classroom and what is not.

Many learners also failed to correctly answer question 11 in which they had to select the fraction greater than $\frac{1}{2}$. On that question, 39% of the learners favoured the option of taking $\frac{1}{3}$ as greater than $\frac{1}{2}$ whereas 21% chose $\frac{3}{8}$. These two choices suggest that for some learners the consideration was the magnitude of the denominator and for others the consideration was the magnitude of both the denominator and numerator. This reflects a common misconception around fractions before they have been reified as rational numbers. Another difficulty was the word ‘greater,’ which some learners appeared to understand as meaning ‘better’, in which case $\frac{1}{3}$ is ‘greater’ than $\frac{1}{2}$ because it allows more people a piece. In another fraction question, learners were supposed to find the age of a child whose age is $\frac{1}{5}$ of his/her grandmother, whose age is 55. This involved learners knowing how to multiplying a fraction by a whole number. However, only 45% selected the correct answer, suggesting that many learners had difficulties multiplying the fraction by the whole number or could not interpret the question, perhaps again due to the unfamiliarity of the language.

I had not expected the problems I encountered to be so common, as the content was taken from the curriculum for lower grades. However, the results suggest that learners in grade six may well still be struggling with this level of content. This finding is mirrored in countless other studies internationally.

5.4 Learners’ knowledge at the end of grade six, and “learning gain”

It is not easy to determine exactly what the performance of learners on a standardized test indicates about their proficiency in numeracy and mathematics. However, the results from the Rwandan study, compared to the results from the previous studies in South Africa and Botswana, indicated that the Rwandan learners performed better than the South African and Botswanan learners on the test at the start of grade six, and even more so at the end of grade six, having improved their performance on the medium numeracy levels (4-6) more than on the lower numeracy level 2, where it was already reasonably strong. A boxplot of the distribution of the Rwandan scores for the two tests is shown in Figure 5-6, and the distributions of scores are shown in Figure 5-7.

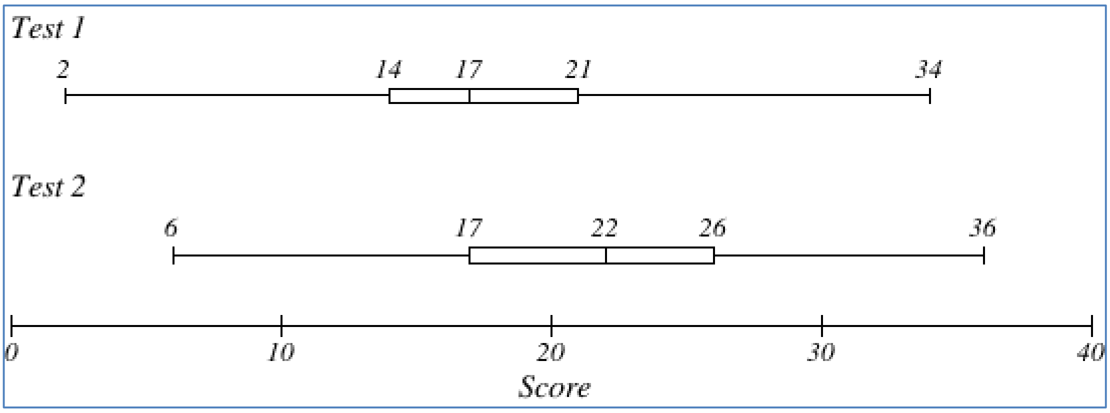


Figure 5-6: Box plot of the learner test scores for the pre- and post-tests

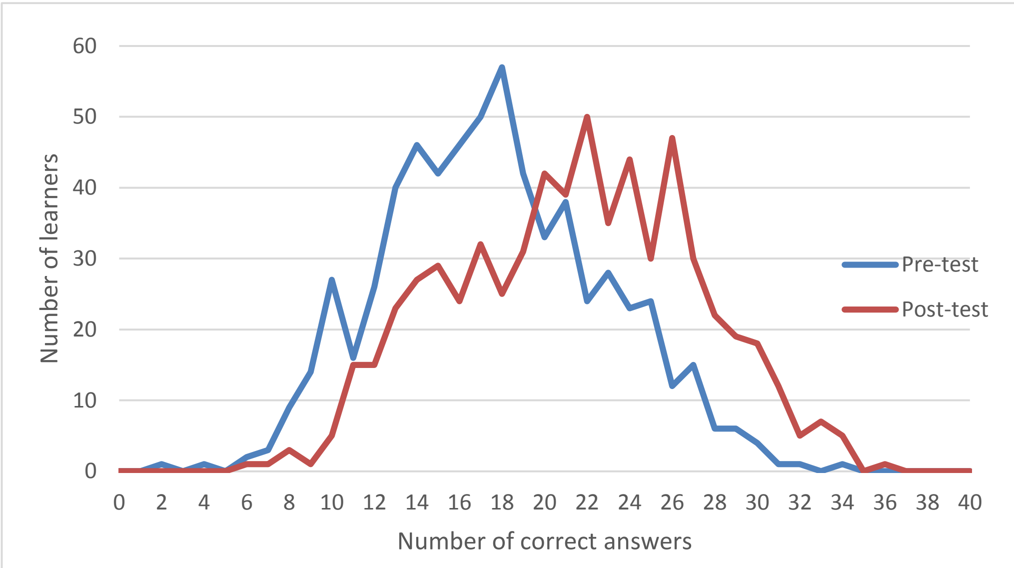


Figure 5-7. Distribution of number of correct answers in the Rwandan pre- and post-tests. Only learners who completed both tests have been included.

In both tests, the Rwandan learners scored 30 percentage points or more above the KZN learners on a substantial number of questions. Table 5-4 shows the content of these questions.

Table 5-4: Questions with large difference in mean scores between KZN and Rwandan learners

Questions where difference in mean scores (in %) between the KZN and the Rwandan learners was 30 percentage points or more on pre- and/or post-test. (*) indicates a question requiring basic operations on whole numbers.

Question content	Pre-test		Post-test	
	KZN	Rw	KZN	Rw
Place value; being able to determine the value of the '7' in 4175. (*)	38	71	40	80
Order of operations.	45	11	43	85
Addition of a four digit and a three digit number with regrouping from units to tens. (*)	58	92	63	91
Multiplication of a two digit number by a three digit multiple of 100. (*)	42	84	47	89
Division of a three digit number by a two digit number. (*)	46	77	46	86
Numerical equation; 16 x ____ = 32 x 2	16	34	17	50
A word problem on multiplication of a two digit by a one digit number. (*)	29	65	25	74
A word problem on division of a three digit number by a one digit number. (*)	31	74	32	82
A word problem on finding the unit fraction of a whole, two digit number.	14	45	16	62
Determining the next number of squares in a pattern sequence.	27	58	38	73
Number pattern on division by one digit number requiring finding input value.	41	64	46	76
Finding the next number in a number pattern of a more visual nature.	20	62	23	65
Visual shape recognition; Identifying number of circles in collection of shapes.	45	17	50	17
A contextual problem involving picking the right unit of measurement for a given task.	18	65	22	65
A conversion problem changing a mass in *kg*, with one decimal, to *g*.	21	73	24	78
Conversion problem changing minutes to hours and minutes.	25	41	26	56

I note in particular the substantial improvement of the Rwandan learners on the 'order of operations' question - this is grade six content in Rwanda, and the results suggest that many learners have indeed learned this. Overall, the Rwandan learners appear to have done substantially better on questions involving basic operations on whole number (the ones marked with an asterisk in Table 5-4). This indicated to me that it would be worthwhile to explore whether the Rwandan learners had better 'basic numeracy' overall. I interrogated this using the SACMEQ numeracy levels (Hungi et al., 2010) described in Appendix B. These questions also include topics overall than numeracy. For example, level 5 includes questions on converting basic measurement units from one level of measurement to another (i.e., meters to centimetres). Thus, the second to last question in Table 5-4, involving conversion from *kg* to *g*, was coded as a level 5 question. The description for level 3 included questions like interpreting place value of whole numbers up to thousands, and so the first question in Table 5-4 was coded as level 3.

The Rwandan learners' performance improved on level 4 questions in particular, whereas the Botswanan learners' improvement was more uniform across SACMEQ levels, as was that of the KZN learners, to a more limited extend (Figure 5-8).

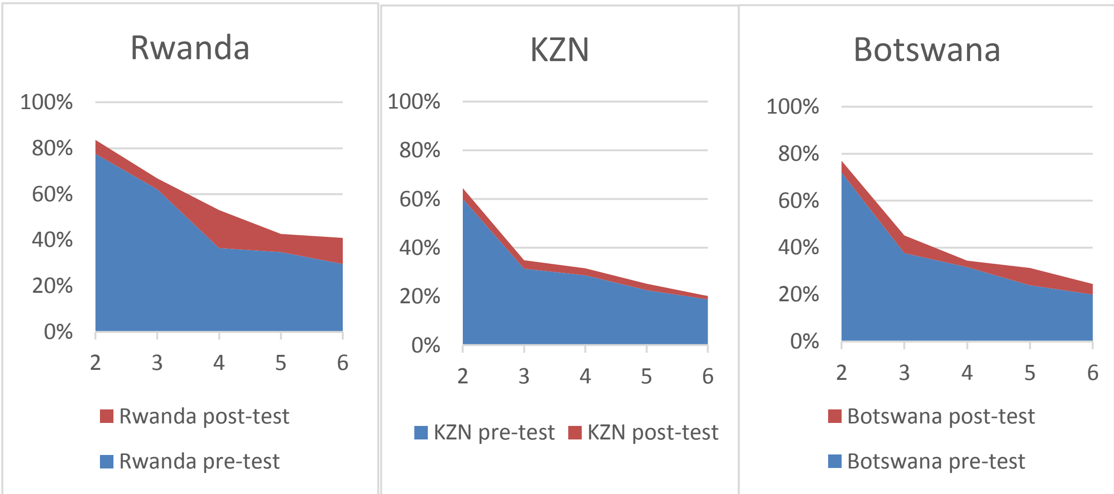


Figure 5-8. The pre- and post-test results for the three samples, for SACMEQ levels 2-6.

In terms of the SACMEQ reference points, most of the Rwandan learners appeared to be operating on the *basic numeracy* level (level 3) and by the end of grade six, more than half of them demonstrated *beginning numeracy* (level 4). Table 5-5 shows the number of learners, mean scores and standard deviations for the post-test results of the four studies. Also included is the 'learning gain,' determined as the mean pre-test score subtracted from the post-test score. For the samples from North West and Botswana, the information is based on the work of Martin Carnoy, Linda Chisholm, & Bagele Chilisa, (2012b, pp. 74, 76 and 87).[12]

[12]Due to poor readability on the photocopied tests, two questions were excluded from the Botswanan results (Carnoy et al., 2012, p. 74). This may have affected the results to some extent.

Table 5-5: The mean and standard deviations for the post-test and the mean 'learning gain' in the four studies

The scores are in percentages of maximum score. The learning gain is in percentage points, and includes only those learners who wrote both the pre- and the post-tests.

	Number of learners in sample	Post-test mean score	Post-test standard deviation	Mean "learning gain"
Rwanda	638	53.3	14.2	9.2
KwaZulu-Natal	1276	32.5	15.3	2.9
North-West	3800	31.6	12.4	3.0
Botswana	1750	38.6	14.4	4.0

While this seems to indicate that the Rwandan learners did indeed learn more, it is important to examine the extent to which these results reflect actual learning.

5.5 Presumed or actual learning gains

A substantial body of literature exists on learning and how it is related to learners' background and the teaching they experience. Best known is perhaps the meta-study of Hattie and colleagues (Hattie, 2008). However, most studies compare the learners' test results before and after an intervention or a period of teaching and use the difference as indicative of a 'learning gain'. Few consider to what extent learners changed their answers for individual questions. However, if an overall 'learning gain' is to be taken as an indication of learning, it seems reasonable to expect that most learners would have a fair share of the same correct answers on both the pre- and post-test, but with more correct answers on the post-test. The results presented and discussed in this section are significant. They give an image of consistency in the thinking of grade six learners of my sample, one element which reflects learning.

When questions were selected out for which 75% or more of the learners selected the correct answer in both the pre-test and the post-test, I found that there were 16 such questions for the Rwandan cohort, but only 2 for the KwaZulu-Natal cohort. In addition, 89% of the Rwandan learners who selected the correct answer to a basic operations question on the pre-test chose the same answer on the post-test, compared to only 57% of the South African learners.

Overall, the Rwandan learners improved their scores substantially more than did the KwaZulu-Natal learners (Table 5-6) even though they started from a higher mean score.

Table 5-6: Learners' relative improvement in the KZN and the Rwanda studies

	Number of learners to do both tests	Mean in test 1	Mean in test 2	Percentage point improvement	Relative improvement
KZN	1276	29.6	32.5	2.9	9.8%
Rwanda	638	44.1	53.3	9.2	20.8%

In the KwaZulu-Natal study, only 45% of the learners improved their score by more than one mark between the two tests, while this was the case for 68% of the Rwandan learners in the study. More than a quarter of the KZN learners lost marks between the two tests, while that was true for 'only' 14% of the Rwandan learners. This implies that more Rwandan learners learned more even if some still appeared to have 'unlearned;' this may also indicate that some learners guessed the answer for some questions.

One question dealing with the order of operations, on which the Rwandan learners showed remarkable improvement, was particularly interesting. On the pre-test, most Rwandan learners picked the answer that would have been correct if they had simply worked from left to right. In the post-test, almost all of the learners provided the correct answer. This topic is only covered in grade six in Rwanda, and so the results indicate that it was taught across all of the schools in the study and was learned by most of the learners.

Looking at the questions individually, the South African learners generally did not change their score significantly between the two tests. The exceptions are two questions, namely question 21 and 25, where the mean score improved by 11 and 10 percentage points, respectively. Comparing this to the Rwandan learners, on 19 of the 40 questions the private school learners improved their score by more than 10 percentage points, and on 3 of these by more than 20 percentage points. While the results for the public schools were slightly less impressive, they still improved their score by more than 10 percentage points on 13 questions, and on one of these by more than 20 percentage points.

In the study of grade six mathematics learning in KwaZulu-Natal, it was found that a substantial number of learners actually changed their answers between the pre- and the post-test (Aungamuthu & Christiansen, 2013). This means that overall differences in scores cannot be understood without also considering what Aungamuthu and Christiansen refer to as the 'stability' of their answers (ibid). This measure indicates the extent to which learners were retaining learning or perhaps relying partially on guessing.

One way to look into this is to interrogate the changes in test scores for individual learners. The results are shown in Table 5-7.

Table 5-7: Learner performance changes

A negative score change indicates that the learners scored less on the post-test than on the pre-test.

Score change between two tests		-5 or more	[-2,-4]	[-1,+1]	Improved [+2,+4]	Improved 5 or more
South Africa	**Number of learners**	104	226	339	323	219
	Percentage of learners	8.6%	18.7%	28%	26.7%	18.1%
Rwanda	**Number of learners**	23	57	121	173	264
	Percentage of learners	3.6%	8.9%	18.9%	27.1%	41.3%

The inconsistency shown by some of the learners might be linked to language: learners whose home language was not English showed more misconceptions in their answers (Christiansen and Aungamuthu, 2012).[13] It is possible that learners misunderstood the questions in some cases. This could be one explanation for the fact that scores decreased by 2 or more percentage points between the two tests for 27.3% of the South African learners and 12.5% of the Rwandan learners. Another viable explanation is of course that some guessing occurred.

I note that when it comes to basic operations at the lower SACMEQ levels, 72% - 86% of the Rwandan learners selected the same answer on both tests, indicating substantial learning retention, whereas for the South Africa cohort only 40% -53% of the learners did the same. In one of the questions, there was a correlation of 79.1% between the learner score on the pre-test and the share of learners who stayed with a correct answer on the post-test. This basically means that the more learners selected the correct answer on the pre-test, the more likely they were to select the same answer on the post-test. There was only one outlier, namely a question where only 10.5% of the learners selected the correct answer on the pre-test, but 94.0% of these selected the same answer on the post-test.

Overall, these findings indicate a fair degree of consistency in the performance of the grade six learners in the Rwandan cohort (in contrast to those in the KwaZulu-Natal study). The consistency in the performance of the Rwandan learners reflects learning gain within that particular school year. The difference in test scores can therefore reasonably be assumed to represent a true learning gain (no longer needing to be referred to as 'learning gain').

[13] As previously mentioned, the majority of Rwandans (90.8%) speak Kinyarwanda at home (as their mother tongue), and 5.6% speak English. In KwaZulu-Natal only 7.4% of the population speak English as their home language whereas the majority i.e. 90.5% speak isiZulu at home (Christiansen & Aungamuthu, 2012).

Table 5-8: Pearson correlations between learner test results and learning gain in Rwanda

** signifies that the correlation is significant at the 0.01 level (2-tailed).

		Learner pre-test	**Learner post-test**
Learner post-test	Pearson Correlation	0.657**	
	p-value	0.000	
	N	638	
Learning gain	Pearson Correlation	-0.301**	.503**
	p-value	0.000	0.000
	N	638	638

Learners' pre-/post- test results are correlated with a correlation coefficient of 0.657, p-value=0.000. The learning gain is negatively correlated to the results on the pre-test with a correlation coefficient of -0.301, p-value = 0.000. This suggests that learners who did well on the pre-test tended to have lower improvements in scores. Learning gain was positively correlated with the results on the post-test with a correlation coefficient of 0.503, p-value = 0.000. This makes sense since learning gain is more likely to be correlated to a good post-test result. However I note that Table 5-6 is based on the results of the 638 learners who completed both pre- and post-tests.

5.6 Chapter summary

Chapter 5 explored first the learners' pre-test performance results at the start of grade six and compared them with results from the similar studies that have been conducted in Botswana and South Africa. I then discussed the learners' test performance at the end of grade six and again compared this with previous studies. I interrogated the 'stability' of the learners' answers by taking into consideration the responses each learner gave on two tests for similar questions. Some stability differences were observed when compared with the cohorts. Overall, the Rwandan learners scored better on average on the pre-test, they appeared to have improved their test performance substantially more than the other cohorts, and their answers were more 'stable' than those of the KwaZulu-Natal learners.

The correlations between test scores and learning gain suggested that the learners who scored better on the pre-test were more likely to improve their scores.

When considering the difference between the results from the four cohorts, I note the differences in the colonial histories of the countries. Whereas Botswana and South Africa were heavily influenced by colonisation by the British Empire, Rwanda was under the colonial influence of Germany and, to an even greater extent, Belgium, which established French as the language of instruction and developed a number of educational initiatives. Thus, it is likely that the education systems in these countries still reflect their very different histories and the respective cultural-educational differences in attitudes to education (cf. Broadfoot, 1999), including value-laden choices about what is considered best practice or what the purpose of education is (cf. Boltanski & Thévenot, 2000; Christiansen, 2014).This makes it particularly interesting to have established this current snapshot of Rwandan learners' test performance, as the change to English medium instruction may or may not lead to changes in the influences on the regulative and instructional discourses of Rwandan schooling.

In the following chapter, I analyse teacher knowledge as it was expressed in the teacher test, and its relation to learning gain.

6 ANALYSIS OF TEACHERS' DECLARATIVE KNOWLEDGE

6.1 Introduction

In this chapter, I present the results of my analysis of the declarative knowledge of the Rwandan grade six mathematics teachers based on the results of the teacher test, which included questions on CK and on PCK. The fact that Rwandan learners generally perform poorly in mathematics compared to other subjects (MINEDUC, 2006) suggests the relevance of investigating the knowledge of mathematics teachers, so that researchers and teacher educators can better understand the type of mathematics knowledge which they possess. Besides, in the Kinyarwanda language, there is a saying that *"Ntawe utanga icyo adafite*". This means that "none can give or offer something which s/he does not have". In this context, it implies that teachers need to be equipped with sufficient knowledge to enable them to facilitate learning.

Apart from this short introduction, the chapter summarises the participating Rwandan grade six mathematics teachers' CK and declarative PCK test results, after first summarising the background data from the teacher questionnaire. The chapter further interrogates correlations between these test results and the background variables. The chapter ends with a short conclusion.

6.2 The background of the teachers

Exposure to pre-service professional teacher training was limited amongst the 19 teachers who participated in the study. Two reported that they had not received any pre-service professional teacher training, while 7 had received training or a short course of less than one year's duration. This implies that the rest, more than half (10), of the teachers never got a pre-service professional teacher training of at least a year.

The teachers reported that in-service teacher training was uncommon. Of the 8 teachers who responded to this item, 3 had never received an in-service in mathematics training and the 5 who reported that they had indicated that it had only involved one or two courses which addressed numbers, geometry, data handling measurement and algebra; pedagogy or PCK content were not mentioned.

While 9 of the respondents indicated that their school principals rarely observed their teaching, 10 said that they were sometimes observed.

When asked how they felt about teaching the mathematics curriculum, 17 said that they felt adequately prepared, while two were unsure.

In order to obtain some form of measure of the teachers' socio-economic status, they were asked about which items they possessed. This has been summarized in Table 6-1.

Table 6-1. Number of teachers who reported that they possessed specified items

Percentages rounded to whole numbers

Item	Number	Percentage	Item	Number	Percentage
Daily newspaper	15	79	**Refrigerator**	1	5
Weekly or monthly magazine	17	89	**Car**	Not answered	Not answered
Radio	16	84	**Electricity**	11	58
TV set	10	53	**Cattle**	2	11
Computer / laptop	8	42	**No cattle**	3	16
Piped cold water	3	16	**Brick house**	7	37
Piped hot water	1	5	**Clay or wood house**	5	26
Internet access	4	21	**Less than 50 books**	9	47
Video/DVD player	5	26	**More than 50 books**	2	1

From a Rwandan perspective, it appears strange that 16% of the teachers did not possess a radio in their homes: my personal experience is that radios are widespread throughout Africa, in particular in the rural areas. However, it is likely that these are being replaced by TV sets. On the other hand, I was not surprised by the result that only 1 (5%) of the teachers had a refrigerator, because they are expensive relative to a Rwandan primary school teachers' salary. Similarly, it was not surprising that just 21% had internet, while twice as many had a personal computer. None of the teachers responded to a question related to possession of a car and this made it impossible to know if they did or did not have cars. From my personal observation, personal ownership of a car is still rare in Rwanda.

The general lack of access to piped water and electricity was reported and a large percentage reported that they read a daily newspaper (79%) or a weekly or monthly magazine (89%).

6.3 Overall teacher test performance

Table 6-2 shows the content areas and knowledge types evaluated in the teacher test. Counting each sub-question as a question, there was a total of 63 questions on the test.

Table 6-2. Teacher test content areas and knowledge types

Question	Content Area	Knowledge Type	MfT category for PCK
1	Numbers	CK	
2	Numbers	PCK	KCS
3	Numbers	PCK	KCS
4	Numbers	PCK	KCS
5	Numbers	CK (3 sub-questions) PCK (3 sub-questions)	KCS/KCT/(SCK)
6	Numbers	PCK	KCT/KCS/(SCK)
7	Numbers	CK	
8	Numbers/Algebra	CK	
9	Numbers	PCK	KCS
10	Algebra	CK	
11	Geometry	CK	
12	Geometry	CK	
13	Geometry	CK	
14	Geometry	PCK	KCT
15	Geometry	CK	
16	Measurement	CK	
17	Measurement	CK	
18	Measurement	PCK (3 sub-questions) CK (1 sub-question)	KCS/KCT/(SCK)
19	Statistics/Probability	CK	
20	Statistics	CK	
21	Statistics	CK	
22	Statistics	PCK	KCT/(SCK)
23	Statistics	PCK	KCS
24	Statistics	CK	

None of the teachers answered all of the questions correctly. The one with the highest score got 46 out of 63 (73%) correct, while only one teacher scored less than 50%, and that was a very low score of 8 out of 63 (13%), which was likely due to language difficulties. Figure 6-1 shows the distribution of scores on the teacher test.

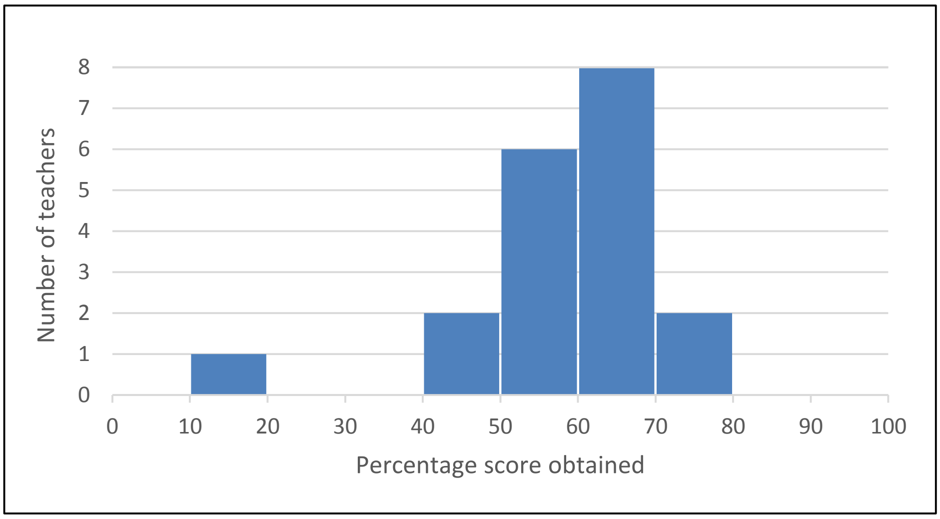


Figure 6-1. The performance of the 19 teachers on the teacher test

There were no questions for which all 19 teachers got the correct answer, but also no questions for which none or only one got the correct answer. However, there was a significant spread between questions, from only two teachers choosing the correct answer to all but one doing so. On just over half of the 63 questions/sub-questions, more than half the teachers chose the correct answer, and there were 22 questions/sub-questions on which 15 or more of the teachers chose the correct answer.

Combining all the questions relating to both content knowledge and PCK, the teachers' performance clearly varied from one topic area to another, as shown in Figure 6-2.

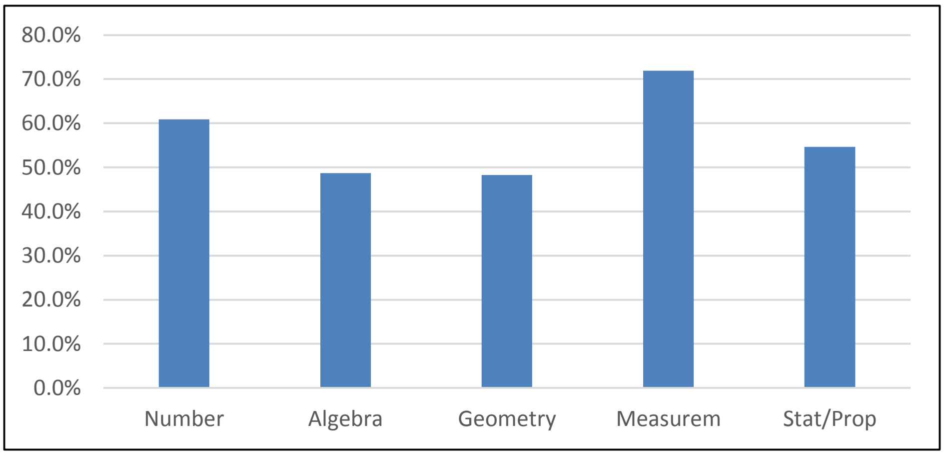


Figure 6-2: The mean teacher test results per topic area, across CK and PCK

However, there were substantial variations in how well the teachers performed on the topic areas, as indicated in the box plots in Figure 6-3.

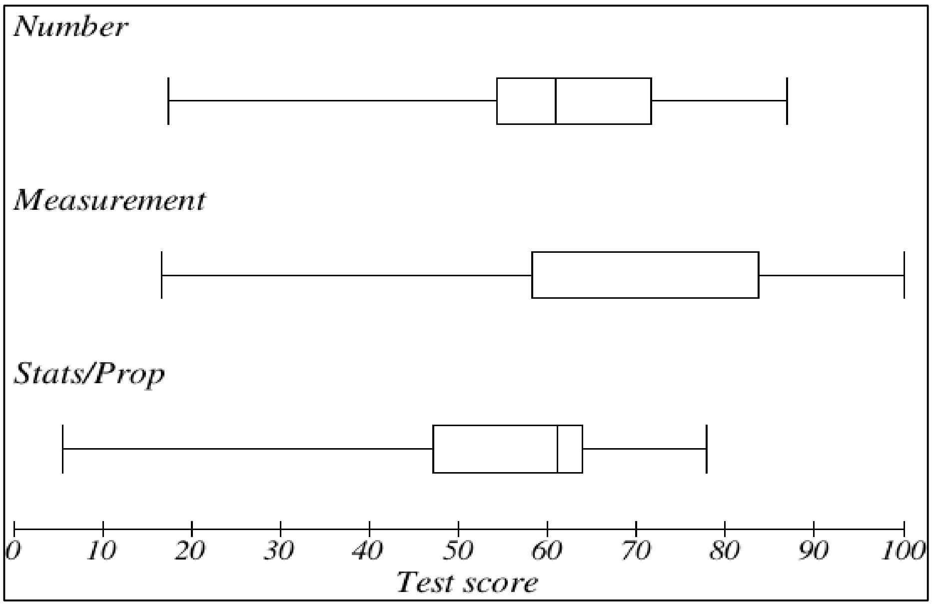


Figure 6-3: Box plots for teacher test scores on three topic areas

For all categories, the teacher with the overall low score had the minimum score, but in algebra three other teachers also scored zero. The sample is, however, too small to draw any conclusions from this.

Surprising to me is the correlation (0.708) between the scores of the Rwandan and KZN teachers on each question (Figure 6-4), though the distribution around the diagonal shows the Rwandan teachers generally performing better than the South African teachers.

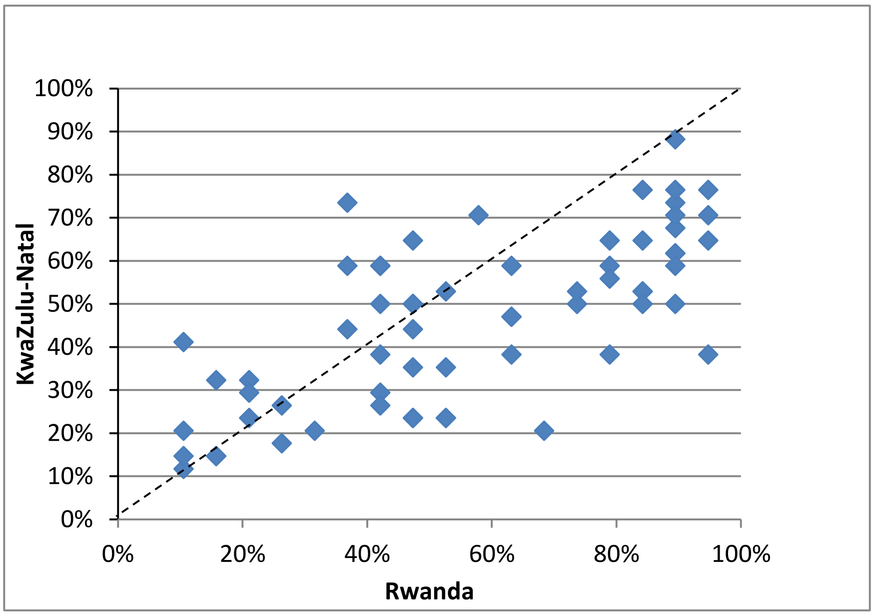


Figure 6-4: KZN teachers' score on test questions compared to Rwandan teachers' score

There were ten questions where the KZN teachers outscored the Rwandan teachers by more than 10%, but 35 questions where the reverse was true. The KZN teachers scored better on 7 CK questions and 3 PCK questions, better on 5 geometry questions and on 5 other content area questions. On the other hand, the Rwandan teachers scored better on 20 CK questions and 15 PCK questions and also better on 12 number questions and 11 statistics/probability questions.

In the discussion above, I have simply worked with each sub-question as an independent question. However, there were times where a more nuanced interpretation of the teachers' knowledge emerged from comparing answers across sub-questions. One such case was question 20.

Question 20 dealt with statistics and presented a case where an investigation was done to find out the type of music which was most popular among learners at school. The question contained a frequency table of favourite musical genres. For each of four graphical representations, respondents were to indicate if the representation was appropriate. While eight of the teachers got the correct answer to two of these questions and seven of the teachers got the correct answer to three of these questions, only one teacher got all four correct, and another one got all four incorrect. So while it is possible to look at the performance of the teachers on each of the four sub-questions and see this as a relatively strong area of knowledge, looking across the results reveals that most of the respondents did not have mastery of the content.

It is partially for this reason that I have included two additional sections to engage teachers' answers more qualitatively, highlighting what stood out as particular areas of difficulty and ease, respectively.

In addition, this points to some methodological issues. Simply analysing results based on number of correct answers to different types of questions may indeed be misleading. However, the test would have had to have been constructed differently in order relate the answers to each other in the analysis. This is something I would like to engage further in my future research.

6.4 Teachers' difficulties on the test

Teachers exhibited difficulties with both pedagogical content knowledge and content knowledge questions.

6.4.1 Teachers' difficulties with PCK questions on the test

In terms of PCK, one aspect which teachers appeared to struggle with was unpacking learners' thinking (which is related to MfT under the *Knowledge of Content and Students* (KCS) category). The second question on the test probed whether or not teachers could recognise the logic of a learner's alternative algorithm. The question involved multiplication of natural numbers, and respondents were asked to select the correct description of the learner's process. The results showed that 16 of the 19 (89.5%) teachers could not correctly identify what the learner was doing; of these, 12 selected the option that the learner had just been lucky to obtain the correct answer. Question 6 also aimed at assessing KCS. Teachers were asked to identify the error in a learner's reasoning demonstrated in the representation of a mixed number (fraction) using small blocks. Only 2 (10.5%) of the respondents selected the correct answer (compared to 20.6% of the teachers in the KwaZulu-Natal study). The majority of respondents selected the option which did not engage the learner's representation or reasoning, only the incorrectness of the answer. Question 14 also was about recognising learner thinking; respondents were presented with an example of a learner's work dealing with right angles and asked to predict the learner's responses to five questions, based on his error pattern. While 15 of the teachers selected the correct answer to two of these questions, only two teachers got all five correct, and none got four correct, suggesting that most of the respondents struggled to identify the learner's error pattern.

6.4.2 Teacher' difficulties with content knowledge questions

6.4.2.1 Number

Only one number question caused the teachers some difficulty. In question 1, teachers were asked to indicate the number of decimal numbers between 0.30 and 0.40. More than half of the respondents (11/19 or 57.9%) selected an incorrect answer (while in the KZN study 70.6% of the respondents selected an incorrect answer). Given that their preferred choice was a finite number, it brings into question their ability to shift from natural numbers to real numbers in their classroom teachings, and the accuracy of their teaching of real numbers.

6.4.2.2 Algebra

With regard to algebra, the respondents appeared to find one question particularly challenging. A situation was presented where learners' were tasked with developing a rule to predict the number of cubes in a pattern as the pattern changed. Four solutions were proposed, and for each, the teachers were asked to indicate if it was correct or not. As this was entirely about the mathematical correctness of the answers, I classified this as a CK question. Two of the answers were correct, namely one that listed the sum which could be done to find the answer (an inductive method), and one that stated a formula. Two of the answers were incorrect, providing incorrect formulae. While 12 (63.2%) of the teachers correctly identified both the latter options as incorrect (compared to 58.8% and 47.1%, respectively, of the KwaZulu-Natal teachers), 11 (57.8%) of the teachers picked the inductive answer as correct (compared to 70.6% of the KwaZulu-Natal teachers), while only 2 (10.5%) recognised the formula as actually being correct (compared to 14.7% of the KwaZulu-Natal teachers). However, I am not sure it is fair to consider this as evidence that the teachers experienced difficulty; it is possible that they assumed that only one option could be correct, or preferred the inductive version, or believed it was more effective with their learners.

6.4.2.3 Geometry

Difficulties were also observed for some geometry questions. One example involved question 15, a CK question with four sub-questions, each of which consisted of a description of a shape. Respondents were asked to indicate if the shape was possible or not. Here, 13 (68.4%) maintained that it is possible to construct a rectangle which is not a parallelogram (compared to 41.2% of the KwaZulu-Natal teachers). Twelve (63.1%) said it is impossible to have a parallelogram with diagonals of equal lengths (compared to 26.5% of the KwaZulu-Natal teachers). If the teachers had recognised that a parallelogram with diagonals of equal lengths is a rectangle, they should also have said that it is impossible to construct a rectangle which is not a parallelogram (or the opposite), but 11 (57.8%) of the teachers did not exhibit such consistency in their answers. Furthermore, 68.4% claimed that it is impossible to have a square which is also a rectangle (compared to 19.4% of the KwaZulu-Natal teachers). These answers reflect an exclusionary view of common shapes, rather than understanding some shapes to be subsets of others – eg. any square is also a rectangle, and any rectangle also a parallelogram. The only statement for which the responses had a high frequency of accuracy was the last one where 17 (89.5%) respondents confirmed that it is impossible to have an equilateral right triangle (compared to 61.8% of the KwaZulu-Natal teachers). Some of the incorrect choices brought into question how the teachers introduce and define basic geometry shapes to their learners.

These were not the only questions where scores were substantially below the mean, however. An example is question 12 which requested teachers to identify which of a series of shapes was symmetrical. The same question was asked on the learner test. I expected high scores from the teachers, however only 3 (15.8%) selected the correct answer (compared to 32.4% of the KwaZulu-Natal teachers). This suggests that the other 16 (84.2%) of the teachers did not have an accurate understanding of this concept, which again raises the question of how some of the teachers are able to impart concepts to their learners which they themselves don't fully understand. Comparing the teachers' scores to those of the learners, I found that 14% of the learners answered the question correctly, and 71.4% of these were taught by the three teachers who had answered correctly.

The opposite situation arose around question 13 which investigated whether teachers and learners could recognise a three dimensional figure when presented from a side view. Only 2 (10.5%) of the teachers were able to identify the correct answer (compared to 41.2% of the KwaZulu-Natal teachers), while 12.9% of the learners answered this correctly on the post-test. What is peculiar is that only 4 of these were taught by the two teachers who answered the question correctly. This was a learner question with low 'stability,' possibly as a result of guessing.

6.4.2.4 Measurement

All of the teachers answered all of the measurement questions correctly.

6.4.2.5 Statistics and probability

On a question about ratio in relation to probability, only 4 (21.1%) of the teachers selected the correct answer (compared to 32.4% of the KwaZulu-Natal teachers); 34.8% of the learners answered this question correctly in the pre-test and 40.1% in the post-test. I have no explanation for this result.

The teacher test included two questions dealing with statistical representations and the teachers struggled with both. One question present a situation in which two grade two learners from the same class created different representations to illustrate the number of teeth lost by their classmates. The teachers were asked to identify which representation best illustrated the concepts of centre and spread. Sixteen (84.2%) of the respondents were unable to identify the best representation (compared to 85.3% of the KwaZulu-Natal teachers). The second question dealt with choosing the most appropriate graphical representation of data for learners' travel time. Four different graphs were provided and teachers were asked to choose the most appropriate of the four. Only three (15.8%) made the appropriate choice (compared to 14.7% of the KwaZulu-Natal teachers).

Together, these problems suggest that the teacher may have difficulty recognizing suitable representations for mathematical concepts. As using representations is a skill which can facilitate learning, this is worth exploring further.

In summary, most of the questions on which the teachers did not perform well were directly related to the content taught in grade six, which is cause for concern. This finding should inform the approach to content knowledge in Rwandan teacher education programs.

6.5 Teachers' ease with certain test questions

Despite the difficulties noted in the previous section, a number of the test questions were answered correctly by the teachers. All but one of the teachers correctly identified an error in addition; calculated a percentage from a number; calculated the area of a path around a rectangular swimming pool; and identified the incorrect answer to a question dealing with the area of a composite figure. All but two correctly answered most of the questions dealing with interpreting ratios; understood that a right angled triangle cannot be equilateral; and recognised a rectangular prism as having six faces. Below, I comment on a few examples.

6.5.1.1 Number

Question 4 addressed content knowledge concerning fractions and decimals, although it was formulated as a task to evaluate learner responses. In the scenario that was presented, learners had to order the numbers 0.003, 0.35, 0.3 and 0.035. Two of the learners' responses in the example were incorrect, and around four-fifths of the teachers correctly identified this, compared to half or fewer of the KwaZulu-Natal teachers. Roughly 20% of the teachers incorrectly indicated that 0.3 was the largest of the four numbers. For the two other learner responses given in the example, the answer was correct but the reasons behind their answers were not. Providing good reasons, 16 (84.2%) of the teachers could see that the learner's use of fractions was correct (compared to 76.5% of the KwaZulu-Natal teachers).

Question 7 dealt with order of operations[14]. A scenario was given in which a teacher asked the learners to write expressions that, when evaluated, gave an answer of 10. The teachers were asked to select which expressions had a value equal to 10; this was classified as a CK question. Sixteen (84.2%) of the teachers were able to identity both a correct and an incorrect numerical answer to an expression in the format $a + b \times c$.[15] However, that alone does not demonstrate that the teachers had a sound understanding of the principle of order of operation. Two of the teachers answered all of the expressions incorrectly, while three teachers answered three correctly. Eight teachers (42.1%) answered all but one correctly, and only 6 (31.6%) teachers answered all five expressions correctly. Less than half of the teachers were able to answer the question with $100 \div 5 \times 2$ correctly, while the second hardest was one that contained addition and subtraction only. It is possible that this is due to a misunderstanding of the BODMAS sequence (see footnote 14).

Question 9 explored teachers' ability to recognise the reasoning used by learners. A scenario was presented in which all of the learners had selected the correct answer for a question comparing the strength of two mixtures, but three different lines of reasoning had been used and only one was correct. The respondents were asked to indicate which lines of reasoning were correct. Twelve (63.2%) of the teachers were able to recognise that a line of reasoning which did not use ratios was incorrect. However, they had more difficulty evaluating the two lines of reasoning which used ratios: only 4 (12%) teachers correctly identified one argument as incorrect, and only 8 (42%) correctly identified the correct argument as such.

[14] In some countries sometimes referred to as BODMAS (Brackets first, Orders [exponents, logarithms and roots], Division and Multiplication [left to right], Addition and Subtraction [left to right]). In South Africa, the 'O' is sometimes taken to refer to 'of', which may lead to confusion around multiplication. Another problem is that the ordering is often taken to mean that division always precedes multiplication and addition always precedes subtraction, which is incorrect.

[15] I do not have this information for KwaZulu-Natal at hand, but around half of the teachers got each of these questions wrong.

6.5.1.2 Algebra

Nothing to note.

6.5.1.3 Geometry

Question 11 explored whether teachers could determine the number of faces in a drawing of a 3D geometrical shape. This was one of the questions which many teachers answered correctly: 17 (89.5%) selected the correct answer. Similar results were found for question 15, where respondents were asked whether geometric figures could be drawn from properties given in statements. However, results were not always consistent, as has been discussed elsewhere.

6.5.1.4 Measurement

Question 18 interrogated whether teachers could recognise the thinking patterns learners demonstrated and explored the methods teachers used to identify learners' errors (using KCS, KCT and, to a limited extent, SCK). It also indirectly investigated whether they could calculate the perimeter of a composite figure. Four options were provided; teachers were asked to identify whether each option was *correct* or *incorrect*. One option simply stated a numerical value for the perimeter, but was actually the numerical value for the area of the figure. All but one of the teachers, 18 (94.7%), recognised this as incorrect (compared to 70.6% of the KwaZulu-Natal teachers). Another option divided the figure into smaller parts and add the perimeters of each together; 14 (72.7%) of the teachers recognized that this was incorrect (compared to 50% of the KwaZulu-Natal teachers).

6.5.1.5 Statistics and probability

Question 20 investigated the teachers' use of graphical representations in teaching. Five sub-questions were given, with two options to choose from for each. The average number of teachers who selected correct solutions was 11 (56.5%). However, of greater interest than this was their consistency across sub-questions.

I will now discuss what the results indicated with regard to the teachers' content knowledge.

6.6 Teachers' content knowledge

The results presented here are all derived from the teacher test. I should note that during all of the lessons I observed, I did not detect a single error in the content that was taught.

The test focused on five mathematics content domains, namely: numbers, algebra/patterns, geometry, measurement and statistics/probability. These questions covered both CK and PCK questions (with the exception of the algebra questions related to PCK). An overview of the distribution of questions across categories is provided in Table 6-2. The results per mathematics content domain are shown in Table 6-3, where the number of correct answers for each question was calculated as a percentage of 19[16] and then averaged. The mean scores presented in the table therefore indicate the percentage of teachers who selected the correct answer.

Table 6-3: Content Knowledge scores by learning area

		Content knowledge scores per content area (number of questions in brackets)				
		Numbers, operations and relation-ships (10)	**Patterns, functions, and algebra (4)**	**Space and shape (geometry) (7)**	**Measure-ment (3)**	**Data handling and probability (11)**
Rwanda	**Mean**	72.1%	48.7%	44.4%	78.9%	43.1%
	St.Dev.	0.16	0.32	0.16	0.23	0.20
KZN	**Mean**	56.8%	47.8%	51.7%	44.1%	33.7%

The results indicate, to the extent that the test provides an accurate representation of the teachers' knowledge, that their knowledge was greatest in the content areas of *measurement* (78.9%) and *numbers* (72.1%). These are also the two areas in which the Rwandan teachers performed much better than their counterparts in the KwaZulu-Natal cohort. The Rwandan teachers scored very low (between 43% and 49%) for the remaining three content areas i.e. algebra, geometry and probability/statistics. As discussed previously, many teachers displayed common misconceptions with regard to geometry and had difficulty with graphical representations in statistics.

6.7 Teachers' declarative PCK

The PCK questions on the teacher test also fell into different sub-domains of mathematics. There were no PCK questions for algebra. Most of the questions dealt with unpacking concepts or identifying thinking patterns and analysing errors (see Table 6-4).

The teachers did not demonstrate the same weaknesses or strengths in their PCK across the different mathematics content areas coved within the test. Their best mean score was for questions dealing with statistics/probability (72.9%). Table 6-4 shows the results for both the Rwandan and KwaZulu-Natal cohorts. What stands out is the stronger PCK of the Rwandan teachers on *measurement* and *statistics*.

[16] Thus *de facto* reflecting unanswered questions as incorrect.

Table 6-4: PCK test scores by learning area

Number of questions per category in brackets.

	Numbers, operations and relationships (13)	Space and shape (5)	Measurement (4)	Data handling and probability (7)
Rwanda	52.2%	53.7%	**64.9%**	**72.9%**
KZN	44.8%	48.8%	42.2%	49.6%

As previously mentioned, most of the PCK questions dealt with *unpacking mathematical concepts*. I found that the teachers' content knowledge was strongly correlated to *unpacking mathematics* with a correlation coefficient of 0.703 (though the limited number of questions is a potential source of error). This is a stronger correlation than that in the KwaZulu-Natal study (0.59). The standard deviations were 0.29 and 0.03 for Rwanda and KZN respectively.

In contrast, content knowledge among teachers in the KwaZulu-Natal study was strongly linked to *analysing learner thinking/errors* with a correlation of 0.768, while the correlation in Rwanda was weak: 0.418. That could suggest that it is not so much that these knowledge areas are related, but that teacher education and practice in Rwanda focuses more on unpacking mathematics while in South Africa it focuses more on being able to identify learners' thinking patterns.

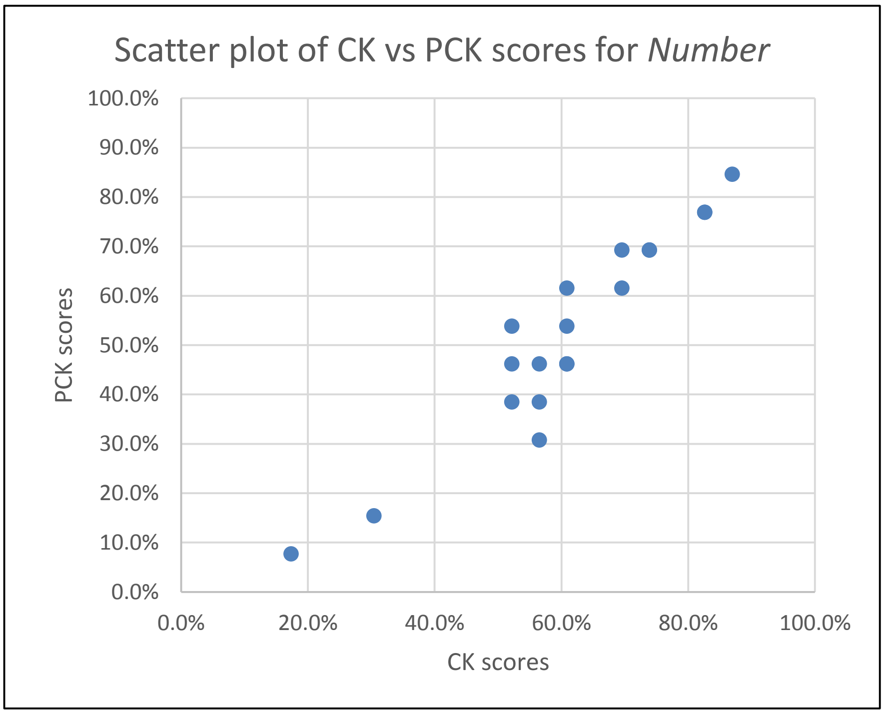


Figure 6-5. Correlation between CK scores and PCK scores on 'number'

CK scores are abscissae and PCK scores ordinates. As several teachers obtained the same scores, three of the points in the plot represent more than one teacher.

In my data, there was a correlation of only 0.650 between the overall CK and PCK scores. However, the correlation between CK scores and PCK scores on *numbers* was very strong with a correlation coefficient of 0.948 (Figure 6-5). For *geometry*, there was no apparent link as the correlation coefficient was -0.086.

There was a weak correlation coefficient between CK and PCK for *measurement* (0.449) and an even weaker correlation for *statistics/probability* (0.352).

Again, there was substantial variation in the performance of individual teachers (Figure 6-6).

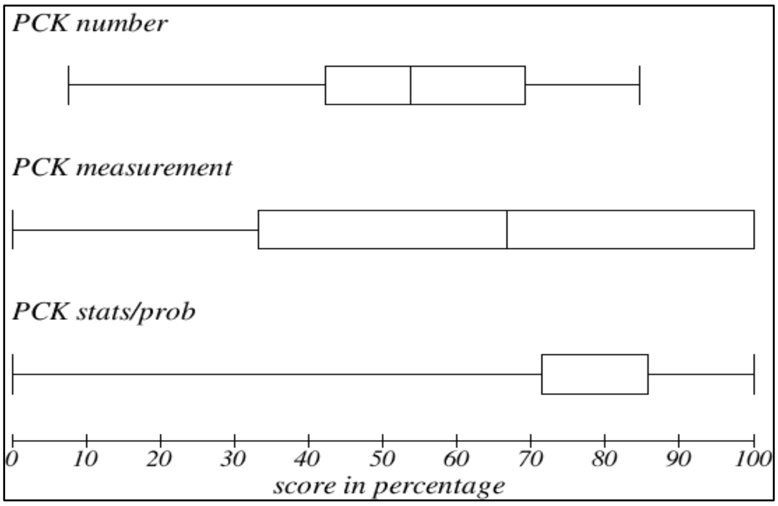


Figure 6-6: Box plot for the teachers' score on three topic areas of PCK questions

I now turn my attention to the links between teacher test results and background variables.

6.8 Links between teacher test results and teacher background variables

I used the teacher test results and the information from the teacher questionnaire to interrogate possible links between their knowledge (to the extent that it could be inferred from the test) and the background factors that were investigated using the questionnaire.

As there was a considerable range of ages among the teachers – spanning 37 years (which is the same span as in the SACMEQ study (Hungi et al., 2010)), I interrogated teachers' performance by age. In addition, the Rwandan teachers had an average of 11 years of experience, while teachers in the SACMEQ study had an average of 12 years of experience. Unsurprisingly, there was a weak negative correlation of -0.23 between teachers' performances and their ages as they had similar backgrounds in terms of their education.

While I had expected higher test scores from teachers with higher levels of education, the results showed no correlation (-0.08) between level of education and performance on the test. This could be related to the fact that the differences between the teachers' educational backgrounds was small, as was detailed in Chapter 4. I investigated whether teachers with more experience, particularly as mathematics teachers, did better on the test than other teachers in the sample. The correlation was 0.15, which was too low to claim any link.

Some researchers (c.f. Beswick et al., 2012) argue that teachers with good PCK are able to present mathematics concepts in a way which is more meaningful and results in fewer misconceptions. Krauss et al. (2006) argue, however, that PCK levels can explain learners' gain in a non-trivial way. In present day Rwanda, primary school teachers are trained in special schools known as Teacher Training Centers (TTC) whereas secondary school teachers usually train at the University of Rwanda. It would be relevant to consider my findings in the light of the teaching at the Teacher Training Centers, and I hope to do so in my future work.

The results showed that the data from the teachers' test results on PCK questions are negatively skewed (Skewness statistic=-2.374) which means that the test marks are concentrated on higher rather than lower marks. The mean mark is 57.82 and there are fewer teachers below this mark than there are above the mark. I note that there is one teacher with a low mark (12.9) who seems to be an outlier, as discussed earlier.

Table 6-5: Descriptors of teachers' declarative PCK scores

Descriptors			**Statistic**	**Std. Error**
Teacher test score	**Mean**		57.820	2.8285
	95% Confidence Interval for Mean	Lower Bound	51.900	
		Upper Bound	63.740	
	5% Trimmed Mean		59.406	
	Median		60.484	
	Variance		160.004	
	Std. Deviation		12.6493	
	Minimum		12.9	
	Maximum		74.2	
	Range		61.3	
	Interquartile Range		10.5	
	Skewness		-2.374	0.512
	Kurtosis		8.388	0.992

My motivations for using an ANOVA test are explained in Section 7.4.1. The results are presented in Table 6-6.

Table 6-6: ANOVA test results on teachers' declarative PCK and background variables

Factor affecting teachers' test score	Category	Descriptors of TEST MARK			ANOVA Tests			
		N	Mean	St. Dev.	F	df1, df2	p-value	Comment
Gender	Male	10	53.71	15.32	2.356	1, 17	0.143	No significant difference
	Female	9	62.54	8.33				
Teacher's age	<30 yrs	4	60.89	3.58	0.130	2, 16	0.879	No significant difference
	31-50 yrs	10	57.42	17.59				
	>50 yrs	5	56.45	6.45				
Teacher's training	No Training	3	58.06	5.59	0.067	4, 14	0.991	No significant difference
	<1 year	7	58.06	21.48				
	1 year	3	54.30	5.66				
	3 years	3	58.60	2.46				
	>3 years	3	60.22	6.11				
Teacher's Experience	2-5 years	3	62.37	2.46	0.297	2, 13	0.748	No significant difference
	6-10 years	1	56.45					
	11+ years	12	55.38	15.24				
How often does the principal watch you teach?	often	4	60.48	1.61	0.171	2, 14	0.845	No significant difference
	sometimes	12	56.05	15.53				
	rarely	1	59.68					
Language usage	1	2	51.61	2.28	0.664	3, 13	0.589	No significant difference
	3	8	53.83	17.71				
	4	3	57.53	7.96				
	6	4	64.52	3.95				

As indicated, I interrogated whether teachers' declarative PCK was associated with background variables such as teaching experience, training, monitoring by the principal, gender or age. The ANOVA test did not show any significant differences linked to any of these background characteristics.

The central question of this thesis, namely: the connection between the teachers' test results and the learners' learning gain, is discussed in Chapter 8.

6.9 Chapter summary

Chapter 6 has presented the Rwandan teacher' test results which relate to their declarative knowledge in terms of CK and PCK. First, results reflecting their content knowledge and pedagogical content knowledge were presented. These results indicated that the teachers'

CK was highest for numbers and measurements, while their PCK was much better in ‘unpacking mathematics’ compared to other PCK sub-categories considered in this study. The links between teachers’ declarative knowledge and background variables was then explored and no correlations were found. The declarative knowledge of the Rwandan teachers’, as determined from the results, was then compared with that of teachers from the KwaZulu-Natal cohort. There was a strong correlation between the performances of the teachers in both cohorts. While this finding cannot be generalised because the sample is small, there is a reasonable likelihood that this finding does reflect the situation on a more general level.

7 ANALYSIS OF TEACHERS' PRACTICAL PCK

7.1 Introduction

This chapter presents an overview of the practices used in mathematics classes in Rwanda based on the twenty lessons that were video recorded in this study. While it is only possible to provide 'snapshots' of these practices, they provide some insight into teachers' daily practices in their classroom. I have interpreted these practices using a coding scheme which I have developed, linking them to background variables captured from the teacher questionnaire and the teachers' declarative knowledge captured from the teacher test.

In the sections which follow, I discuss the teachers' demonstrated practical PCK and its relationship to background variables. I describe the instrument which I used to analyse the video recorded lessons in detail. I also provide detailed examples of all codes used in order to facilitate transparency. Finally, I investigate whether teachers' theoretical and practical PCK correlate.

7.2 Analysis of lesson observations

The following examples of the video coding, provided criterion by criterion and option by option, highlight the operationalization of the coding. Some options included in my observation analysis instrument have not been discussed here, due to their absence in all the observations. I have included examples of coded situations and discuss border cases.

7.2.1 Content connections

Multiple types of content connections can manifest during a lesson at the same time, since they are not mutually exclusive. Below are two examples of teachers using different representations, ie. manipulatives (example 1) and narratives (example 2).

Example 1

One of the lessons was on *set theory*. The teacher had brought a variety of materials such as avocadoes, tomatoes, bananas, books and chalk. After showing these to the learners, one learner was asked to group them into *foods* and *school materials*. The learners appeared curious but confused about what the teacher wanted them to learn. When the teacher then asked a second learner to group the items in the *foods* group into *vegetables* and *fruits*, the learners began to grasp the general concept of sets that was being taught.

Example 2

At the start of the lesson, the teacher placed different objects such as pens and chalk on her desk. She started by asking the learners where these items could be found in bulk. The learners gave various responses, including shops; this appeared to be the response the teacher was looking for. She asked them for the appropriate term to use when they find different objects in shops, and one of them came up with the term '*mixture*'. She then announced that the topic of the day was *price of mixture*.

Both examples have been coded as usage of alternate representations.

The lesson described in example 3 provided me with an example of what I coded as implication connections, here in the form of a definition, which highlights the way one concept leads to another. That is to say that the conclusion is reached/obtained based on prior evidences.

Example 3

Halfway through the lesson the teacher explained to learners a kind of set constructed by using if…then… statement. She told the learners that *if* they find any set containing precisely one element, *then* it will be called a singleton set.

In order to determine whether to code for procedural connections, I focused on how the teacher explained a concept to learners or how they demonstrated how to solve a problem. In the example below, a procedure (ie. determining the surface area of a cube) explicitly engaged a concept which previously had been introduced (ie. area of a square); on this basis I coded this as a procedural connection.

Example 4

One of the geometry lessons was about finding the surface area of a cube. The teacher told the learners that as they knew the characteristics of a cube and how to draw it, they would be able to calculate its surface area.

During the lesson the teacher said: To find the surface area of a cube, you first find out the area of one face which is a square ("*you know that you multiply side by side*") and then … now for its volume, you need to first calculate … then …

In the next example, the teacher appears to have wanted learners to know how to work out two measures before applying them, and I therefore coded it as an example of prerequisite connections.

Example 5

A teacher ended his lesson on statistics by telling learners that as they had seen how to work out mode and mean, the next topic would deal with the areas of application for those statistical measures.

Example 4 on p. 117 is one case where the teacher indicated that he could not teach learners how to find the surface area of a cube until they knew what a cube is, its properties and so on. Another example of prerequisite connection was a lesson on *proportion*s, where the teacher said that to understand inverse proportions, learners had to first understand direct proportions. Another teacher told the learners that it would make it easier to simplifying numbers if they remembered the divisibility test. Yet another teacher told the learners that in order to determine the number of factors for any number one must know how to list those factors. All of these were coded as prerequisite connections.

The last option under content connections was part-whole relationship connections. In one instance which was coded as a part-whole connection, a teacher explained to the learners that mathematics has different sub-parts including geometry, statistics, algebra etc. and added that their topic that day fell under geometry. The following example has a somewhat implicit part-whole connection, as the teacher engaged the learners with different expressions which could be derived from the same formula:

Example 6

The teacher was teaching how to work out simple interest ($SI = \Pr T$) and showed learners how to deduce the formula for any other of the four components once three of them were known i.e. $T = \frac{SI}{\Pr}$; $P = \frac{SI}{rT}$ and $r = \frac{SI}{PT}$.

7.2.2 Progression of the lesson and linkage to other sessions

As previously discussed, I coding for the following situations:

- Progression from everyday to specialized
- Progression from concrete to abstract
- Progression from theoretical to practical
- Progression from particular to general or *vice versa*
- Progression from simple to complex.

Example 4, discussed on p. 117, appeared to move from simple to complex, as it began with one side of a cube and then worked with the entire surface area. Another example occurred in a lesson in which a teacher deduced a formula for the sum of interior angles of a regular polygon = *180 (n-2)* where *n* is the number of sides. The teacher started working through one example of dividing a polygon into triangles but then generalised and abstracted.

In this study, the coding of progression from everyday to specialized was applied to the real life examples teachers gave to learners as ways to introduce mathematical concepts. Some of these examples are presented below.

Example 7

A teacher began the new topic for the day by asking learners what kind of business they would be interested in starting in the areas where they lived. After they gave their responses she asked them what the purpose of running a business would be. From the answers she was given she focussed in on the notion of interest, then explained that they would be learning about the *calculation of simple interest*.

Other teachers related their topics to everyday situations in order to introduce the relevant mathematical terms.

Example 8

One teacher used the Rwandan census of the previous year for learners to get sense of the term *data collection* in statistics.

The following situation was somewhat problematic to code as an example of relating the topic to everyday situations, as the teacher referred to a negative mark in the example, which is not used in the Rwandan system. Nonetheless, the example had sufficient relation to situations known to learners that I included it.

Example 9

The teacher used marks in the teaching of whole number addition. She asked learners to imagine that they had written two tests for which they received the marks negative eight and positive ten. She then asked them what they could say in terms of gaining or losing and the sum of the marks of both tests, positive two.

Coding progression from theory to practice did not occur often in the data. One of the exceptions was the following lesson:

Example 10

After learning how to calculate the average speed of a body in motion the teacher informed the learners that the next session would take place in the school laboratory so that they could conduct an experiment to test practically what they had learnt in theory.

Even if the time allocated to the lesson did not allowed the class to proceed with practice, if the learners were told they would be putting into practice what they had learnt in theory, I still coded this as progression from theory to practice, occurring in one instance of the lesson.

Example 1 on p. 116 – where the teacher introduced sets through an exercise involving sorting physical objects – constitutes, I believe, an example of progression from the particular to the general, because the sets of the everyday objects were later generalised to an decontextualized set concept, as well as from concrete to abstract.

I also used the particular to general code for a situation in which a teacher used the particular formula for calculating the average velocity of a body in motion, i.e. $s = \frac{d}{t}$, to refer to car, and then generalized the usage of that formula to other bodies in motion.

7.2.3 Mathematical content construction through practices/variations

The following example does not concern learners communicating with each other, but because the teacher encouraged the learner to express her thinking in both words and writing in front of the whole class, it was coded as encouragement to communicate mathematically.

Example 11

A learner was solving a geometrical problem on the blackboard in front of the class. At first she wrote her solution down correctly step by step without talking. After approximately one minute, the teacher asked to explain her solution. The learner indicated that the area of a square is equal to $line \times line$ (instead of $side \times side$). The teacher corrected the learner, who then continued solving the problem correctly.

A similar situation occurred during a lesson on polygons:

Example 12

The teacher asked the learners to name the different types of polygons, encouraging them to discuss this with each other. Some learners said that a polygon with nine sides was called a *ninegon*. Other learners corrected them.

This criterion also includes the coding of the way teachers asked learners to investigate an object/image by observation and through variation or contrast. A key example of this occurred during the lesson on the cube:

Example 13

The teacher had brought a large box containing various manipulative geometric objects of different shapes and asked the learners to pick from the box only cube shaped objects. Learners were encouraged to say why they thought something was a cube or not.

During this exercise learners were given an opportunity to explore how a cube compared to the other manipulative objects in the box and to verbalise how they were the same or different.

To code if mathematical terms were used by learners to explain whether a conjecture was true or false through discussions/separation, I included situations where it was clear that the teacher had asked learners to present arguments for their answers, as in example 14, keeping in mind that separation suggests that unlike figures may have a right angle yet appear dissimilar.

Example 14

A teacher had stuck drawings of different types of triangles on manila paper to one of the blackboard corners. He called learners up one by one to select the right angled triangles and give reasons for their choices, using the particular characteristics of right angled triangles.

Example 15 was coded as content construction because of the communal construction of content through reasoning and discussion.

Example 15

The teacher gave the learners group exercises. During task time, the teacher did not provide any assistance. After about six minutes, she stopped the learners and requested group representatives to write their group solutions on the blackboard. The next step was to determine through class discussion if each solution was correct or not. Each learner was free to defend his/her position.

Occasions where such engagement led to focusing on particular, generalizable properties from the complex tasks done I coded as generalization. One of the examples was from the lesson on set theory (see also example 1, p. 116).

Example 16

During the lesson on sets, after the teacher showed the learners what a set could be using different representations and manipulative objects, she asked them to define a set. Using the various answers they gave, which the teacher noted on the blackboard, a definition of a set was developed collaboratively.

7.2.4 Illustrations, representations and teaching aids for lesson concretization

It was more straight-forward to recognize this criterion during coding because its focus is more practical than theoretical. I watched for any visual or physical representations presented to learners and listened carefully to what the teachers said during the lessons. This allowed me to code examples such as this one as the use of representation for lesson concretization:

Example 17

In a lesson on bodies in motion, the teacher gave a verbal example of a car travelling from Kigali to Muhanga. The example was only provided verbally and learners would have had to generate this representation visually in their minds.

To code if the teacher used drawings to concretize the lesson was also straightforward, as teachers used blackboards to draw objects or figures depending on the topic, or used pre-prepared manila papers drawings (as in Example 14).

Coding the usage of manipulative teaching aids was similar. Some of the teachers used homemade materials while others used aids provided by the school. In most of the lessons teachers opted to use either drawings *or* manipulative objects, but in a few lessons teachers combined drawings and manipulative objects. Some teachers began with drawings and then later introduced manipulatives, while others did it the other way around. However, I only coded that a combination of both occurred if a given teacher used both during the same five minute interval.

7.2.5 Recognizing and addressing errors and misconceptions

There were a few occasions on which I observed what I considered to be an error or a misconception but the teacher did not address it.

Some of the teachers asked learners to raise their hands if they had done well on a given task, and then counted the hands. On most of these occasions the teacher did not respond to the show of hands with a further action, even when the majority did not raise their hands. On these occasions I assumed that the teachers had recognized the presence of errors, even though they had not identified the specific errors, so I coded these situations as *ignoring learner errors*.

Sometimes a teacher asked learners to read the instructions for task aloud before completing it. Some learners experienced problems pronouncing words while reading and the teacher demonstrated the correct pronunciation. I coded such situations as *recognizing and addressing errors* if the words were mathematical terms or if it involved the incorrect reading of numbers.

A difficult situation arose in terms of coding when the teachers moved around during seat work. They would stop and interact with learners through discussion in small groups. In these situations I coded that *errors and misconceptions were identified and addressed*. I recognize that this is a potential source of error, as in some cases I was not able to hear the interaction, and thus it is possible that the teacher addressed things other than content issues (for instance, praising a learner for their work), engaged disciplinary issues (such as asking learners to focus on the task), or even discussed something unrelated to the situation (such as inquiring about a learner’s health).

For the last three options of this criterion, I had to consider how incorrect or incomplete learner responses were engaged by the teachers. This was generally easily coded. However, I was uncertain about how to code one type of situation. This was the case of teacher-learner question-answer situations, which took place usually when teachers were about to introduce the topic of a new lesson. In those cases, incorrect answers were almost always ignored by the teachers who simply called on other learners. I decided to code these situations as e*rrors and misconceptions are recognized but ignored and incorrect answers are simply corrected* (option 3 under this criterion), even if the correction of the answer happened through other learners’ responses.

A similar situation arose more frequently in large classes. One example was the following:

Example 18

During a lesson with a class of approximately 55 learners, a learner was sent to the blackboard to find the answer to 3/2 x 3/4 and calculated 3/8 as the answer. During the same lesson, another learner was asked to simplify the fraction 10/24 but she instead divided the number, and got the answer 0.4. For each question the teacher asked learners to raise their hands so he could see how many had arrived at the correct answer, but he did not appear to act on this information in any way, even when the number of learners who raised their hands was quite small.

I decided to code such cases as *errors/misconceptions recognized but simply corrected*.

The last two options of the criterion dealt with how incorrect answers which had arisen from misconception/errors were challenged by the teacher, either individually or by engaging the class. Cases coded as the former were situations in which a learner or group of learners were working on a task at their desks and the teacher stopped to engage them in how to progress with the task. While it appeared that he was challenging the learners this instance may have fallen more accurately under the option *nature of feedback,* explained in Section 0. The second code was generally used for a situation in which one learner completed a task in front of the class. An example is given below.

Example 19

A learner was asked to find out the surface area of a cube and got confused about the formula that was needed. Earlier in that lesson, the teacher had told the learners that in order to find the surface area of a cube they first had to find the area of one of its square faces, and then multiply the result by six. Instead of doing so, the learner took the length of one edge of the cube and multiplied it by six. This situation was then discussed by the whole class in order to help the learner in front, who then managed to come up with the correct solution.

7.2.6 Nature of feedback given to learners

In this option, feedback is investigated through two separate criteria, namely the nature and the focus of feedback (the *how* and the *what*).

The first of these criteria includes four different options. I coded a response from the teacher as *direct feedback* when there was a teacher-learner interaction where the teacher responded with a clear direction or instruction on how to correct or improve a response. This occurred when learners were directly asking questions of their teachers, as well as when learners responded to questions or tasks from the teacher.

Inexplicit feedback occurred most often in individual or group work situations. These include cases in which a teacher pointed out where in the learners' work the error had occurred, but left them to correct the error for themselves.

A challenge arose with coding *cognitive conflict* feedback. It was not easy for me to recognize if learners had contradictory ideas about what they were learning unless they were given the opportunity to ask questions or were asked to solve problems on the blackboard. One instance which I coded under this option was a question raised by a learner during a lesson on geometry.

Example 20

During the lesson, the teacher told the learners that to find out the surface area of one face of a cube they had to square the length of one edge. A learner raised her hand (looking confused) and asked why they could not continue to use $side \times side$ to find out the surface area of a square.

It was apparent that the shift from one term to another for the concept (square area) created cognitive conflict for this learner and that the feedback she was given was a clear example of *cognitive conflict* feedback.

Feedback through *class debate* was not often observed during my coding. One example of this kind of feedback, however, was as follows:

Example 21

During a lesson, learners working in groups arrived at different solutions to the same problem. The teacher asked each group to justify its answer and refute the other groups' solutions. This generated a class debate, which resolved when one group's solution was accepted as the correct one.

7.2.7 Focus of feedback given to learners

In some lessons, the teacher appeared to attempt to motivate learners by praising them when they answered correctly. For instance, a teacher would write "very good" in a learner's exercise book or ask the class to applaud when a learner gave a correct answer. I coded such situations as *personal* (self) *feedback*, though retrospectively I have become aware that I should perhaps have made a stronger demarcation between this and *task/product feedback*.

Feedback about *task/product* occurred in situations where correct or incorrect answers were simply accepted or acknowledged as such. For instance, to inform a learner that his/her answer was incorrect some teachers gave responses such as "*Your answer is wrong. Does someone else have the correct answer?*"

However, I also observed instances in which teachers helped learners arrive at the correct solution to their given task. An example of this occurred during a lesson in which the teacher asked directive questions of a learner who was at the board working on a task, in order to prompt her to remember the procedure for solving that particular type of problem. On one occasion the teacher asked the learner what needed to find out before she could calculate the total surface area of a cube. This question reminded the learner that she had to start by finding out the surface area of one face of the cube. This was coded as *process feedback*.

The last option under this criterion was *self-regulatory feedback*. I coded for this option when teachers directed learners to correct their errors themselves, thereby demonstrating methods for checking solutions – as in the following example.

Example 22

A learner was calculating the speed of a motorcycle, and wrote his final answer as 32km. The teacher asked him if 32km expressed the speed and, after about 30 seconds, the learner added /h and rewrote the answer as 32km/h. The teacher had thus demonstrated the practice of checking the answer against the question.

7.2.8 Unpacking the methods/concept to make the content more approachable

While I was coding this criterion, I noticed teachers who informed learners that various methods exist, without demonstrating to their learners more than one method. I could not code such cases under this criterion.

In unpacking content some teachers limited themselves to a procedural focus, only describing rules/procedures which might be used. An example of this was the following:

Example 23

During a lesson on place value the teacher told the learners that to determine the place value of each digit in a given number they needed to work from right to left using the place value chart.

To me this represented a rule without justification and I coded it as *only rules/ procedural descriptions are used to unpack content*.

On other occasions, teachers defined concepts in reference to other concepts. An instance of this occurred during a lesson on bodies in motion in which a teacher defined and explained each term in the formula s = d / t, where *s* stands for speed, d for distance and *t* stands for time. This was coded as *engaging different methods to unpack the content but not followed by their comparison/ analysis*.

Only if more than one method was demonstrated did I code it as *unpacking the methods*. One option was that teachers *demonstrated the methods without comparing them*; another that *demonstrations were followed by comparison*. The former occurred in a case where a teacher demonstrated two ways of determining the number of faces of a cube, firstly by opening a homemade manipulative and secondly by using a drawing of a cube, but did not compare the methods in terms of their efficacy or any other aspect. The latter occurred in a lesson on polygons, where the teacher showed learners two ways of determining the numbers of sides of a given polygon: counting them or calculating them when the sum of the interior angles is known. Next, she asked the learners which method they preferred. When the learners expressed different preferences, she explained that the best method to use will depend on the way in which a problem is posed.

7.2.9 Putting into place problems to clarify the concept and alternative strategies

Engaging learners in activities/problems can be done in various ways in order to enhance their understanding of the subject matter. Many theorists, including Dewey, have claimed that learning is enhanced if learners are engaged in some work during learning sessions (Novack, 2005).

Some teachers wrote tasks on the blackboard, and then solved them together with learners in a question-answer form. The role of the teacher in the activity was to ask directive questions in order to draw out the answers which s/he then noted on the board. I coded this as *problems worked on through direct teacher-learner interaction*.

Another option under this criterion was for teachers to *ask learners to work on tasks individually or in groups*. When the answers were not shared with the whole class, I coded it as *tasks have been worked on as individual seatwork or in a working group but not shared with the class*. Sometime, however, the teacher would ask those who got the correct answers to put up their hands and be counted. When seatwork was following by sharing answers, often by inviting a learner to solve the given activity/problem on the board, I coded the situation as *tasks are worked on individually or in groups, checked and shared in class*. During one observation, each group of learners had its own question, whereas generally all groups would work on the same or very similar tasks.

7.2.10 Engagement of learners' prior knowledge

Some teachers chose to start a new topic without any attempt to engage their learners' prior knowledge. In such a situation, a teacher might start with a statement such as: "Today, we are going to look at …together ...." and then mention the title of the topic, perhaps write it on the blackboard, followed in most cases by providing definitions of the key words within the title. This was coded as *not engaging learners' prior knowledge*.

In other cases, teachers would start the lesson by asking learners about what they had seen or done the previous day or even just two hours earlier. For instance, in a lesson on statistics, a teacher started by asking what event had taken place in the country the year before (referring to a census). Other teachers started their lessons by asking learners to remind them what they had covered during the previous mathematics lesson. While these situations could be interpreted as engaging learners' prior knowledge, there was no further engagement with the learners' responses, no further questions were asked, and after the interaction, the lesson moved on to something new. Even when such activities took between 3 to 8 minutes, if it did not lead to further engagement with the responses, I coded it as *absence of assessment of learners' prior knowledge*.

To try to determine if teachers were assessing learners' prior knowledge, I had to compare the activities in which learners were engaged at that particular time and the topic of the day as announced by their teachers. When the teachers engaged them in an activity or asked questions about content which they had not previously covered, I coded that as *assessing their prior knowledge*. Here is an example in which a teacher asked learners what they thought or knew about something before proceeding:

Example 24

In a lesson on geometry, a teacher was halfway through writing down the formula for finding the total surface area of a cylinder, but then stopped himself. Instead, he asked if there was any learner who could work out the surface area. One learner deduced the formula for the surface area of a hollow cylinder from his prior knowledge about cylinders.

As it appeared that the teacher had intentionally prompted the learners to access their prior knowledge, I coded this instance as *assessment of prior knowledge to engage the topic of the day*.

Having explained how the coding categories were operationalized, I now turn to the results of the analysis of the lessons.

7.3 Teachers' practical PCK as demonstrated in observed lessons

Apart from a few PCK subcomponents which were demonstrated by almost all of the teachers, most PCK subcomponents were demonstrated by only a few teachers. Painting with a broad brush, I would say that one tendency stood out: the teachers tended to not make any attempt to access learners' prior knowledge. Although there were a few instances in which teachers did attempt to access learners' prior knowledge, they did not then use it as foundation to teach new content. In Table 7-1, I present the PCK-related practices that I observed most frequently during the twenty lessons.

Table 7-1: PCK-related practices frequently observed during lessons

PCK criteria (cf Table 4-1)	Commonly observed practices
1. Content connections made to create new knowledge	- Different representations (equivalent or alternate) - Procedural connections
2. Progression of the lesson and linkage to other sessions, to allow learners to assimilate the concept	- The progression was from simple to complex -The progression was from every day to specialize.
3. Mathematical content construction through practices/variations	- Learners were encouraged to communicate mathematically while performing a task - Mathematical terms were used by learners to explain why the conjecture was true or false through discussions/separation - Investigation by observation of the object/image through continuous variation/contrast
4. The use of illustrative material, representations and teaching aids	-Drawn teaching aids/representations - Manipulative teaching aids/representations

PCK criteria (cf Table 4-1)	Commonly observed practices
5. Teacher engagement with errors and misconceptions	-Errors and misconceptions were recognized but ignored and incorrect answers were simply interpreted/corrected - Incorrect answers from risen misconception/ errors were individually challenged - Errors and misconceptions were shared and discussed with learners
6. Form of feedback given	- Explicit direct feedback
7. Focus of feedback given	- Personal feedback (self) - The given feedback was about task or product - The feedback given was about process to create product
8. Teacher's approach to unpacking the methods/ concept to make the content accessible to learners	- Only definitions/conceptual were used to unpack the concepts - Only rules/procedural descriptions were used to unpack the methods - More than one method/way was shown to unpack the methods /concepts but this was not followed by comparison/analysis
9. Teacher's approach to engaging tasks in teaching.	- Posed problems were worked on in teacher-learner direct interaction - Posed problems as seatwork individually or in working group were worked on, checked and shared.
10. Teacher's approach to engaging learners' prior knowledge.	- Prior knowledge was not engaged

On average, I coded for 39.30 occurrences of practical PCK per lesson, but as the duration of the lessons varied the number of occurrences had to be standardized for the purpose of comparing lessons. I therefore worked with the number of occurrences per hour rather than per lesson. The mean total number of occurrences per hour of contact time was found to be 45.13, with a standard deviation of 8.15 codes (Table 7-2).

Table 7-2: Summary statistics for total number of occurrences of practical PCK

	N	Min	Max	Mean	Std. Dev.	Skewness		Kurtosis	
						Statistic	Std. Error	Statistic	Std. Error
Total number of coded teacher actions	20	20	61	39.30	10.27	-0.052	0.512	-0.146	0.992
Total number of coded teacher actions per hour	20	31.2	62.4	45.13	8.15	0.389	0.512	0.218	0.992

The total number of occurrences is skewed slightly to the right but not significantly as Skewness = 0.389 is within the -1 to +1 range. Based on the number of classroom practices reflected by my videos analysis tool, the majority of the teachers demonstrated more than 7 practices of the 14 practices for which I have coded (cf. Section 7.3.1).

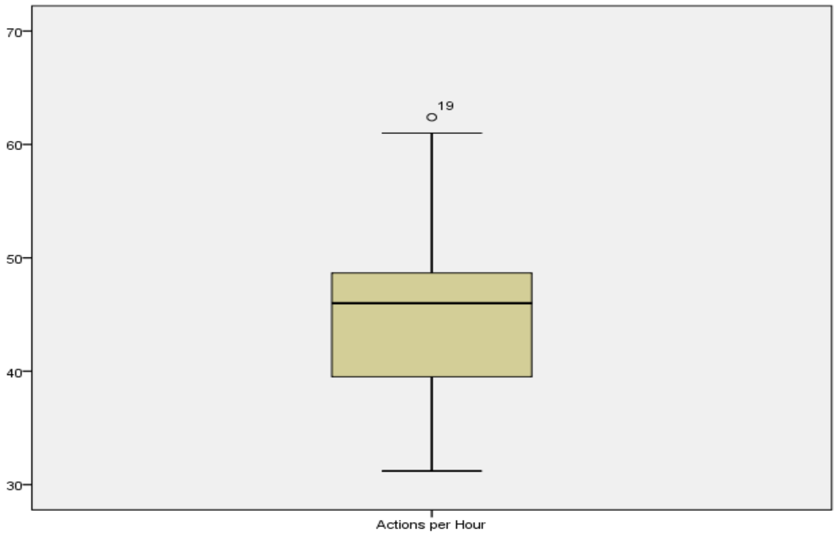


Figure 7-1: Box plot of the total number of teacher actions coded, per hour

Given this skewness, I set out to see if it was possible and meaningful to construct a "typology" of lessons based on the analysis of the twenty videos.

7.3.1 Attempting to construct a typology

In order to create a typology of lessons, I decided to minimise the number of categories with which I was working. First, I removed the categories which had not been used.[17] For instance, I had coded no occurrences of teachers working from concrete to abstract when linking new material to previously taught material. Next, and perhaps more questionably, I decided to exclude any categories which had been used for only one or two teachers for 20% of the time or less. This included the occurrences of *implicit connections* and *comparison of different methods/ways to unpack the concept*. Naturally, I excluded the category of 'no evidence' (the third column in Table 4-1) as non-occurrence during a particular interval implied the occurrence of one of the other PCK sub-categories during that interval. Finally, to reduce the data set, I combined similar categories based on meaning and frequency. For instance, *engaging learners' errors more fully than simply correcting them* could be done with *learners individually* or *a class as whole.* As these categories had not been coded for very frequently, I merged them. The result was 14

[17] This does not mean that the options that were removed are not useful for measuring classroom practices -- and indeed, they could be relevant in a different data set. But without any teachers having demonstrated them they were not useful in constructing a typology – or in the correlational analyses, for that matter.

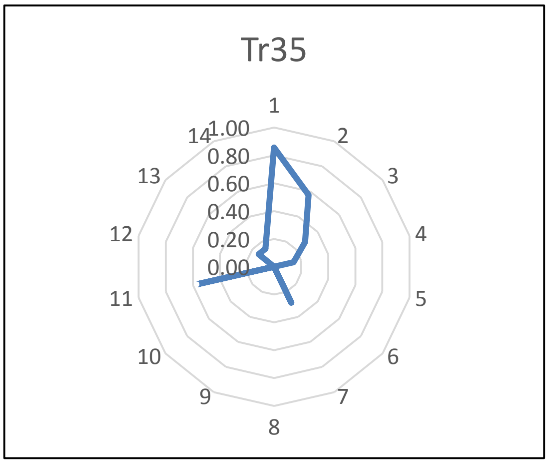
categories, which are listed below with the PCK categories from Table 4-1 inserted in brackets.[18]

1. Teacher engaged tasks in the teaching [I2-4]
2. Direct feedback is given [F2]
3. Feedback is given, but not as direct feedback [F3-5]
4. The teacher uses illustrative material, representations and teaching aids [D2-5]
5. Incorrect answers are challenged either with learners individually, in smaller groups or on the class [E4-5]
6. Some form of mathematical content construction through practices/ variations is observed [C2-6]
7. There is evidence of progression of the lesson and/or linkage to other sessions [B2-6]
8. Content is ‘unpacked’ conceptually or in more than one way [H3-5]
9. Process feedback and or self-regulation feedback is given [G3-4]
10. Procedure connections are drawn][A4]
11. Task/product feedback is given [G2]
12. Different representations are connected, implication connections are made, and/or or part/whole connections are made [A2, A3, A6]
13. Only rules/ procedural descriptions are used to unpack content [H2]
14. Pre-requisite connections are observed [A5]

I then looked for patterns inductively and, in particular, for similarities across teachers.

Taking into account the similarities across teachers, I realised that there were teachers who showed little evidence of the PCK criteria I had constructed (see Figure 7-2 for an illustration of two such cases, the lesson of the teachers coded as Tr35 and Tr46). However, Tr35 used some PCK categories for more than half of the lesson, while this was not the case for Tr46.

[18] In the statistical analysis, I decided to simply collapse all the connections into one category. This was the approach used in chapter 8. Other choices were of course possible, but with this transparency, I hope that the choices I made and my findings can be better critically engaged.



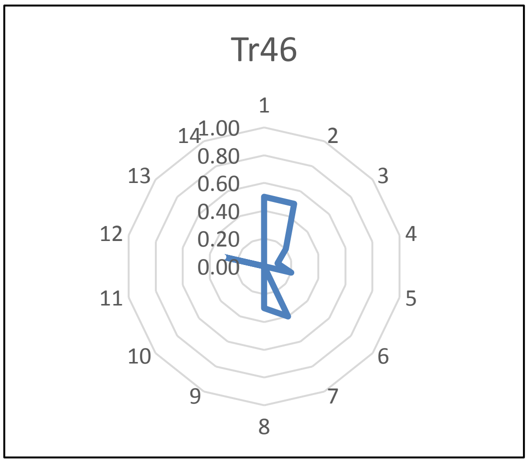
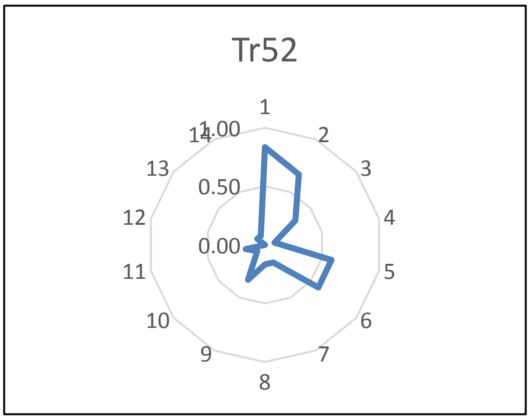


Figure 7-2. An example of 2 lessons, Tr 35 and Tr 46, which yielded limited evidence of practical PCK

At the other end of the spectrum, there were teachers who demonstrated almost all the PCK categories listed above. Some demonstrated several categories frequently, such as Tr52 and Tr53 (see Figure 7-3)



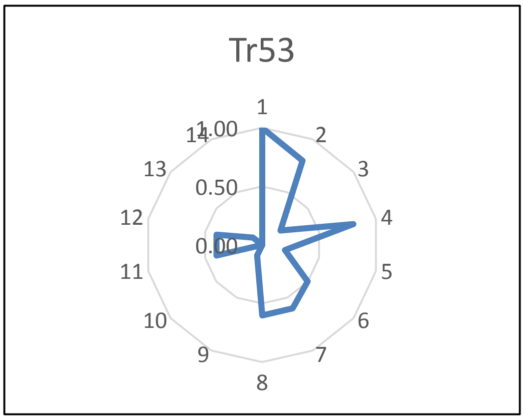


Figure 7-3: Examples of lessons during which multiple PCK categories were coded

However, there were also teachers who demonstrated a substantial number of PCK categories but only a few of them frequently. Figure 7-4 shows the coding of two such lessons, taught by Tr43 and Tr37.

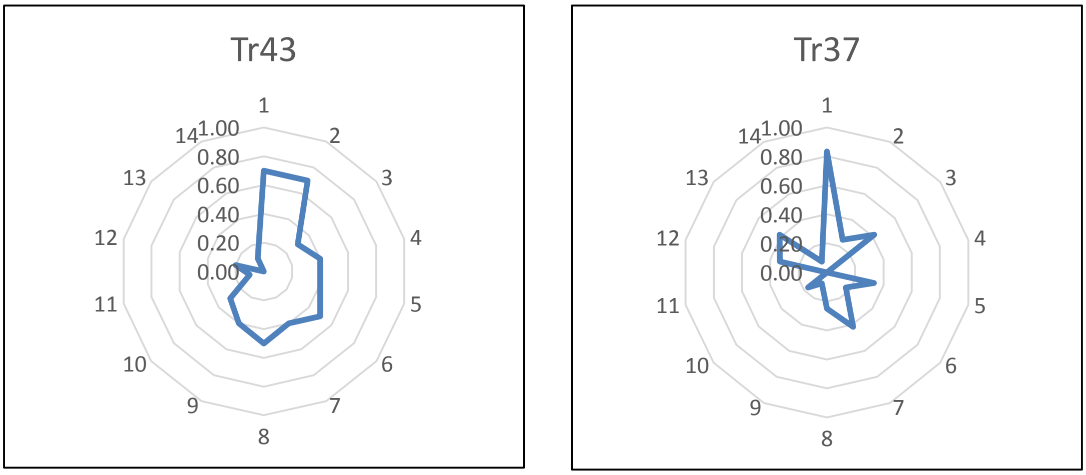


Figure 7-4. Examples of lessons during which several PCK categories were exhibited, but the majority of these during less than half of the lesson

Finally, I looked at the extent to which teachers engaged some of the more ‘progressive’ or so-called ‘learner-centred’ forms of teaching. For this purpose, I looked at the extent to which the teacher used approaches other than correction to address learners’ misconceptions and errors [E4-E5], the extent of content construction through practices [C2-C6], and the extent to which feedback was process or self-regulation focused [G3-G4]. This varied substantially. Figure 7-5 contrasts two extremes, namely Tr50, the teacher who used these practices the most, and Tr42 who never used any of them. The two spider diagrams show how little overlap there was.

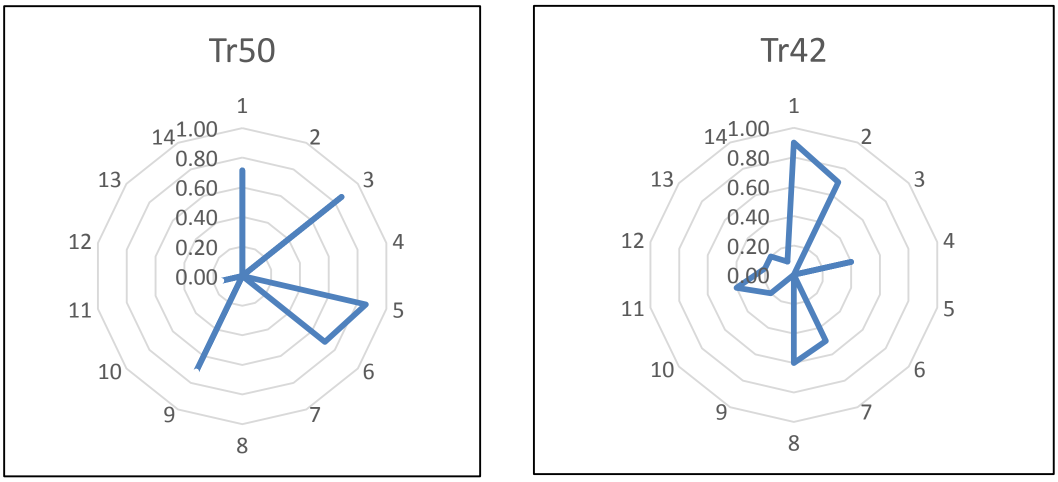


Figure 7-5. Examples of lessons with a high number of instances of ‘progressive’ PCK (left) versus no instances (right)

I again remind the reader that my investigation of teachers' practical PCK did not employ a topic-specific perspective. This means that it was possible for a certain teacher to demonstrate no instances of certain of the elements of practical PCK targeted in this study, not because s/he did not have the knowledge and skill to do so, but because these were not appropriate for the mathematical content domain of the day. However, the most common strategies demonstrated were *indirect feedback* and *individual challenging of misconceptions*, where a teacher approached a learner demonstrating a misconception and tried to assist him/her individually. Figure 7-6 shows how often that strategy was used by the teachers, as a percentage of the time intervals during the observed lesson. Besides these, encouraging discussion about the problems after individual work in order to further clarify concepts was considered a *constructive strategy* and was demonstrated by 18 of the 20 teachers.

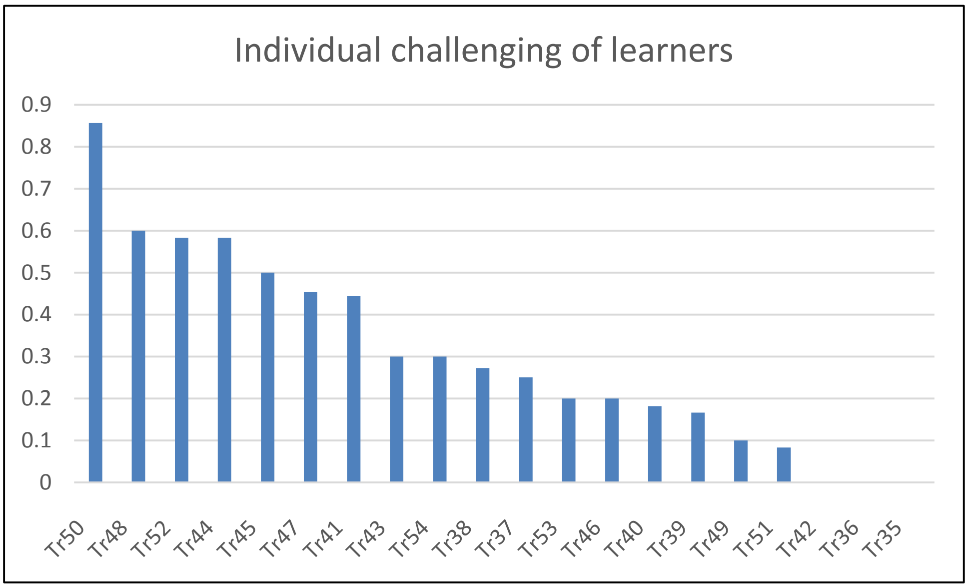


Figure 7-6. Frequency of teachers challenging learners' misconceptions individually in % of time intervals during observed lessons

As perhaps is to be expected with 14 different categories in play, I found that it was not possible to create clear 'types' of lessons on the basis of my coding for practical PCK, as the lessons varied in so many ways. This made it harder to look for relationships between the teachers' demonstrated practical PCK and other data.

7.4 Relationships between teachers' demonstrated practical PCK and background variables

According to the literature (cf. Section **Error! Reference source not found.**), there are everal variables which may affect teachers' practical PCK. Some of those variables are discussed below, in reference to the Rwandan context. However, before the dependence of

PCK on other variables can be identified, basic model assumptions need to be tested on the PCK variables.

7.4.1 Testing for normality of the data

These assumptions pertain to the homogeneity of variance; the normality of each response variable was tested using ANOVA. The PCK categories used in the test are taken from my classroom video observation tool (see Table 4-1). Those which offer a more comprehensive view of a teacher's PCK were analysed together, as discussed in Section 7.3.1.

Some of the response variables for my PCK categories were found to have normality constraints, as shown in Table 7-3 and Appendix D1. These include the use of illustrative materials/representations (p-value=0.005); not recognising or ignoring errors/misconceptions (p-value=0.000)*;* no feedback observed (p-value=0.000), only procedural unpacking of content (p-value=0.000), and engagement of learner's prior knowledge (p-value=0.000). As ANOVA could not be used for this category, non-parametric tests were used.

Table 7-3: Tests of normality assumptions

	Kolmogorov-Smirnova			**Shapiro-Wilk**		
	Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
Content connections	0.135	19	0.200*	0.959	19	0.558
Progression and linkage to other sessions	0.114	19	0.200*	0.942	19	0.287
Mathematical content construction	0.105	19	0.200*	0.957	19	0.519
Use of illustrative materials-representations	0.216	19	0.020	0.84	19	0.005
Errors and mis-conceptions are not observable	0.107	19	0.200*	0.979	19	0.933
Errors/ misconceptions not recognised, or ignored	0.331	19	0.000	0.598	19	0.000
Incorrect answers challenged individually or on class	0.093	19	0.200*	0.961	19	0.601
No feedback observed.	0.345	19	0.000	0.73	19	0.000
Direct feedback.	0.135	19	0.200*	0.969	19	0.752
Process or self-regulation feedback	0.11	19	0.200*	0.968	19	0.746
The given feedback is about task or product	0.222	19	0.014	0.794	19	0.001
Personal feedback (self)	0.154	19	0.200*	0.942	19	0.284
Feedback, excl direct	0.142	19	0.200*	0.927	19	0.155

	Kolmogorov-Smirnova			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
No attempt to unpack	0.114	19	0.200*	0.931	19	0.179
Only rules/ procedural descriptions are used to unpack content.	0.423	19	0.000	0.572	19	0.000
Unpacking methods/concepts	0.128	19	0.200*	0.918	19	0.104
Engaging Tasks	0.139	19	0.200*	0.953	19	0.449
Engagement of learners' prior knowledge	0.495	19	0.000	0.463	19	0.000
a. Lilliefors Significance Correction						
*. This is a lower bound of the true significance.						

7.4.2 Dependency of teachers' practical PCK on their education

The teacher questionnaire contained several questions regarding the teachers' educational background. One question dealt with the highest level of education the teacher obtained before beginning training to become a teacher; another dealt with the length of their teacher training. The results found that 14 of the teachers had completed high school only (grade 12) and 5 teachers had further education; 2 of these had bachelor's degrees.[19] As the groups were rather small, I simply compared these two groups using t-test with unequal variance. The table below shows the results which were found to be significant; Appendix D2 contains the results for all variables.

Table 7-3: T-Tests of practical PCK observed in lessons and level of education

This concerns education other than teacher education, and the table includes significant result only.

Practical PCK (variables meet ANOVA assumptions)	Level of education	Descriptive Statistics			T-Tests			
		N	Mean	Std. Deviation	t	df	p-value	Comment
The given feedback is about task or product	D6	14	0.167	0.146	2.505	17	0.023	Significant
	D7	5	0.000	0.000				

Analysis of variance (ANOVA) was used to evaluate the correlation of teachers' practical PCK with their level of teacher education. The results are presented in Appendix D2. The results show that only one of the PCK categories differed significantly with the time spent

[19] Note that since one teacher did not complete the questionnaire, as previously mentioned, the following results are based on 19 lessons only. In some cases, not all teachers had answered all questions, wherefore the number of lessons used in the analysis is further reduced.

in teacher training, namely the extent to which the feedback focused on task or product (see Figure 7-3).

The teachers who had further education after completing high school (cf. Section 4.5.3) and before they started teacher training or started to work as a teacher were not observed simply giving task/product feedback, whereas teachers who had only completed high school before training as a teacher used this type of feedback at times. The difference between the two groups was significant (p-value 0.023), as previously mentioned.

Table 7-4: ANOVA tests of the dependency of teachers' practical PCK on their teacher education (variables meet normality assumptions)

Practical PCK	Teacher Training	Descriptive Statistics			ANOVA-Tests			
		N	Mean	Std. Deviation	F	df1, df2	p-value	Comment
Mathematical content construction	No Training	3	0.566	0.158	2.91	4, 14	0.06	Significant at 10% sign level
	Less than 1 year	7	0.392	0.16				
	1 year	3	0.183	0.161				
	3 years	3	0.139	0.241				
	>3 years	3	0.267	0.208				

On the other hand (see Table 7-4), taking into consideration teachers' training, as detailed in Appendix D3, an interesting finding here was that teachers who had no teacher training or less than one year of teacher training were significantly more likely to engage learners in mathematical content construction (p-value 0.060).

7.4.3 Dependency of teachers' practical PCK on their level of experience

If level of education does not affect the use of different categories of practical PCK, it seems likely that years of teaching experience would. The results of the ANOVA test of this is shown in Appendix D4, and the significant results are summed up in Table 7-5. Unfortunately, this question was not answered by all of the teachers, which further challenges the validity of the results.

On one hand, teachers who had taught for 11 years or more were more likely to engage tasks in their teaching (p-value 0.058). On the other hand, the three teachers who had only taught for 2-5 years used representations much more frequently than the other teachers (p-value 0.086). It should be kept in mind that there is little spread in the years of experience amongst the teachers.

Table 7-5: ANOVA tests of the dependency of teachers' observable practical PCK on their teaching experience

Significant results only.

Practical PCK (variables meet normality assumptions)	Teacher experience	Descriptive Statistics			ANOVA Tests		
		N	Mean	Std. Deviation	F	df1, df2	p-value
Engaging tasks	2-5 years	3	0.770	0.067	3.569	2, 13	0.058
	6-10 years	1	0.500	-			
	11+ years	12	0.813	0.120			

Practical PCK (variables do not meet normality assumptions)	Teacher experience	Rank summary		Kruskal Wallis Tests			
		N	Mean Rank	Test Statistic	df	p-value	Comment
Use of illustrative materials - representations	2-5 years	3	13.83	4.904	2	0.086	Significant at 10% sig level

Table 7-6: T-Tests of practical PCK observed in lessons and gender of teacher

Significant results only.

Practical PCK (variables meet normality assumptions)	Gender	Descriptive Statistics			T-Tests			
		N	Mean	Std. Deviation	t	df	p-value	Comment
Mathematical content construction	Male	10	0.202	0.217	-3.331	17	0.004	significant at 10% sig level
	Female	9	0.466	0.101				
Personal feedback (self)	Male	10	0.361	0.272	-1.916	17	0.072	significant at 10% sig level
	Female	9	0.617	0.310				

Practical PCK (variables DO not meet Normality assumption)	Gender	Rank summary		Mann-Whitney U Tests		
		N	Mean Rank	Test Statistic	p-value	Comment
No feedback observed.	Male	10	12.95	15.50	0.007271	Significant at 5% sig level
	Female	9	6.72			
Engagement of learner's prior knowledge	Male	10	11.35	31.50	0.082697	Significant at 10% sig level
	Female	9	5.58			

7.4.4 Dependency of practical PCK on gender

A t-test was used to interrogate variance in the PCK categories in relation to gender. Table 7-6 shows the only three significant results.

Female teachers in the sample engaged learners in mathematical content construction more often than the male teachers, with a very strong level of significance (p-value 0.004). On the other hand, male teachers more frequently engaged learners' prior knowledge (p-value 0.083).

There were three significant differences between male and female teachers when it came to feedback and engaging learners' incorrect responses. Firstly, the male teachers were found during more intervals of their lessons to give no feedback at all (p-value 0.007). Secondly, there were no situations where female teachers ignored an incorrect answer or simply corrected it, while 7 of the 10 male teachers did so at times. Thirdly, the female teachers more often gave self-feedback (p-value 0.072). I have not evaluated this as positive or negative, as the literature suggests this is less of a factor than the focus of the feedback itself (cf. Hattie & Timperley, 2007).

7.4.5 Dependency of practical PCK on school leadership

School leadership is one of the factors which frame teachers' practice. The teacher questionnaire contained questions regarding how often the principal came to observe lessons.

The number of times which the teachers were visited by their school principals seems to have little influence on their demonstrated practical PCK. Table 7-7 shows the significant results from my data. The only classroom practice which is associated with the school principals' visits is the form of the feedback given to learners.

Table 7-7: ANOVA Tests of practical PCK on observations of teaching by principal

Significant results only.

Practical PCK (variables meet normality assumptions)	Principal visits	Descriptive Statistics			ANOVA Tests			
		N	Mean	Std. Deviation	F	df1, df2	p-value	Comment
Frequency of observation	often	4	0.569	0.273	3.265	2, 14	0.069	Significant at 10% sig level
	sometimes	12	0.268	0.181				
	rarely	1	0.333	-				

The four teachers who were visited regularly by their principal where more than double as likely to engage in process or self-regulation feedback, compared to the teachers who were only visited sometimes (p-value 0.069).

7.4.6 Links between teachers' knowledge demonstrated in teaching and SES indicators

In this section, I analyse whether the indicators used for teachers' socio-economic status (SES) might correlate in some way to their classroom practices. In the original study, the various indicators were intended to be combined into one measure of SES, but I found that approach problematic as combining SES information would quantify social inequality in a way that was incorrectly weighted, potentially resulting in inaccurate conclusions. Using ANOVA, most of the variables were not found to be significant as their p-values were greater than 5% or even 10% (Table 7-8). The PCK aspects were documented by observation using my PCK instrument (Table 4-1), and the PCK totals reflected in this chapter were based on the number of times a particular PCK indicator was observed (see Table 4-1).

As there are no obvious explanations for the significant relationships that were found, they appear largely to be curious coincidences. For instance, lessons taught with more connections were more likely to be taught by teachers with less access to commodities such as TV, a weekly magazine, hot water or a refrigerator. Teachers who read a weekly magazine were also less likely to have linkage to other lessons, but more likely to access learners' prior knowledge. Teachers with no radio engaged learners in more constructions of mathematics and more feedback, but fewer engaged their learners in work on tasks as part of their teaching. More errors were observed in the lessons of teachers who possessed a TV, and teachers with piped water in the house gave more feedback.

What this indicates more than anything is perhaps that choice of teaching style depends more on other factors than on SES – such as the topic of the lesson or the teacher's gender and experience, as already discussed.

From the results on a 5% level highlighted in the preceding tables, the relationship teachers' home possessions and the following six classroom practices was found to be significant; making mathematical connections while teaching; progression of the lesson and linkage to other sessions; mathematical content construction through practices/variations; recognizing and addressing learners' errors and misconceptions; giving both process and product feedback; and assessing learners' prior knowledge.

Table 7-8: Part A of the ANOVA Test for dependency of practical PCKs on teachers' possessions

**Significant results on a 5% level. *Results which are significant on a 10% level.

PCKs observed	Possession of item	Daily Newspaper		Weekly Magazine		Radio		TV set		Computer/laptop		Piped cold water	
		Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value
AT: Total connections observed	Do not have	3.93	0.33 (1,17)	4.06	3.04 (1,17)	5.00	0.75 (1,17)	4.88	4.55 (1,17)	4.40	1,93 (1,17)	3.94	0.50 (1,17)
	Have	3.25	0.572	1.50	0.099*	3.65	0.397	3.00	0.048**	3.11	0.183	3.00	0.487
BT: Total linkages	Do not have	2.87	0.25 (1,17)	3.06	4.25 (1,17)	5.00	2.69 (1,17)	3.38	1.23 (1,17)	3.30	1.48 (1, 17)	2.75	0.00 (1,17)
	Have	2.25	0.652	0.00	0.055**	2.47	0.119	2.27	0.284	2.11	0.241	2.67	0.953
CT: Total mathematical construction	Do not have	3.00	2.75 (1,17)	3.29	.50 (1,17)	7.00	7.85 (1,17)	3.75	0.29 (1,17)	2.90	1.15 (1, 17)	3.56	0.39 (1,17)
	Have	5.00	0.115	4.50	0.488	3.00	0.012**	3.18	0.600	4.00	0.299	2.67	0.541
DT: Total aids used	Do not have	3.27	1.11 (1,17)	3.71	0.01 (1,17)	4.50	0.13 (1,17)	5.00	2.28 (1,17)	2.80	1.51 (1, 17)	3.63	0.03 (1,17)
	Have	5.25	0.306	3.50	0.937	3.59	0.727	2.73	0.149	4.67	0.236	4.00	0.865
ET: Total errors observed	Do not have	4.40	0.01 (1,17)	4.24	1.06 (1,17)	4.00	0.07 (1,17)	3.25	4.26 (1,17)	4.20	0.19 (1, 17)	4.56	0.37 (1,17)
	Have	4.50	0.941	6.00	0.317	4.47	0.792	5.27	0.055**	4.67	0.671	3.67	0.550
FT: Total form of feedback	Do not have	4.73	0.29 (1,17)	4.88	0.004(1,17)	7.00	1.65 (1,17)	5.38	0.50 (1,17)	4.40	0.84 (1, 17)	4.38	5.51 (1,17)
	Have	5.50	0.599	5.00	0.952	4.65	0.216	4.55	0.490	5.44	0.377	7.67	0.031**
GT: Total focus of feedback	Do not have	9.13	0.11 (1,17)	9.29	2.60 (1,17)	12.00	6.41 (1,17)	9.88	2.58 (1,17)	8.90	0.12 (1, 17)	9.00	0.07 (1,17)
	Have	8.75	0.742	7.00	0.125	8.71	0.022**	8.45	0.127	9.22	0.735	9.33	0.798
HT: Total Unpack	Do not have	3.20	0.12 (1,17)	3.29	0.37 (1,17)	4.50	0.50 (1,17)	4.38	2.85 (1,17)	4.00	1.20 (1, 17)	3.00	0.31 (1,17)
	Have	3.00	0.904	2.00	0.554	3.00	0.492	2.27	0.110	2.22	0.176	4.00	0.587
IT: Total engaged tasks in teaching	Do not have	8.40	0.02 (1,17)	8.53	1.27 (1,17)	11.00	5.86 (1,17)	9.00	1.71 (1,17)	8.20	0.17 (1, 17)	8.31	0.09 (1,17)
	Have	8.25	0.889	7.00	0.276	8.06	0.027**	7.91	0.209	8.56	0.685	8.67	0.768
J3: Learners' prior knowledge accessed	Do not have	0.13	1.53 (1,17)	0.12	6.29 (1,17)	0.00	0.33 (1,17)	0.13	0.34 (1,17)	0.10	0.90 (1, 17)	0.25	0.54 (1,17)
	Have	0.50	0.234	1.00	0.023**	0.24	0.572	0.27	0.568	0.33	0.357	0.00	0.474

Table 7-8: Part B of the ANOVA Tests of the dependency of practical PCKs on teachers' possessions

PCKs observed	Possession Of item	Piped HOT water		Internet access		Video/DVD player		Refrigerator		Electricity		Cattle	
		Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value
AT: Total connections observed	Do not have	4.00	4.16 (1,17)	3.87	0.09 (1,17)	4.14	1.60 (1,17)	4.00	4.16 (1,17)	4.00	0.11 (1,17)	3.82	0.04 (1,17)
	Have	0.00	0.057**	3.50	0.763	2.80	0.223	0.00	0.057**	3.67	0.745	3.50	0.841
BT: Total linkages	Do not have	2.89	1.77 (1,17)	2.60	0,28 (1,17)	2.79	0.03 (1,17)	2.89	1.77 (1,17)	3.00	0.16 (1,17)	2.47	2.69 (1,17)
	Have	0.00	0.201	3.25	0.606	2.60	0.874	0.00	0.201	2.58	0.697	5.00	0.119
CT: Total mathematical construction	Do not have	3.44	0.04 (1,17)	3.60	0.44 (1,17)	3.29	0.19 (1,17)	3.44	0.04 (1,17)	4.29	1.71 (1,17)	3.53	0.36 (1,17)
	Have	3.00	0.853	2.75	0.516	3.80	0.673	3.00	0.853	2.92	0.208	2.50	0.555
DT: Total aids used	Do not have	3.89	1.30 (1,17)	3.80	0.08 (1,17)	3.93	0.27 (1,17)	3.89	1.30 (1,17)	4.71	1.05 (1,17)	3.65	0.02 (1,17)
	Have	0.00	0.271	3.25	0.780	3.00	0.609	0.00	0.271	3.08	0.320	4.00	0.893
ET: Total errors observed	Do not have	4.33	0,49 (1,17)	4.87	2.99 (1,17)	4.36	0.04 (1,17)	4.33	0.49 (1,17)	3.57	1.57 (1,17)	4.71	2.73 (1,17)
	Have	6.00	0.495	2.75	0.102	4.60	0.846	6.00	0.495	4.92	0.227	2.00	0.117
FT: Total form of feedback	Do not have	4.83	0.20 (1,17)	4.93	0.02 (1,17)	5.00	0.09 (1,17)	4.83	0.20 (1,17)	5.14	0.10 (1,17)	5.00	0.28 (1,17)
	Have	6.00	0.662	4.75	0.900	4.60	0.768	6.00	0.662	4.75	0.751	4.00	0.606
GT: Total focus of feedback	Do not have	9.06	0.00 (1,17)	9.27	0.82(1,17)	9.14	0.10 (1,17)	9.06	0.00 (1,17)	9.43	0.38 (1,17)	9.18	0.62 (1,17)
	Have	9.00	0.979	8.25	0.378	8.80	0.751	9.00	0.979	8.83	0.544	8.00	0.444
HT: Total Unpack	Do not have	3.33	1.36 (1,17)	3.13	0.01 (1,17)	3.71	2.22 (1,17)	3.33	1.36 (1,17)	3.57	0.23 (1,17)	2.94	0.96 (1,17)
	Have	0.00	0.260	3.25	0.944	1.60	0.154	0.00	0.260	2.92	0.638	5.00	0.342
IT: Total engaged tasks in teaching	Do not have	8.39	0.04 (1,17)	8.47	0.20 (1,17)	8.57	0.64 (1,17)	8.39	0.04 (1,17)	8.86	0.78 (1,17)	8.47	0.49 (1,17)
	Have	8.00	0.843	8.00	0.664	7.80	0.435	8.00	0.843	8.08	0.390	7.50	0.494
J3:Learners' prior knowledge accessed	Do not have	0.22	0.16 (1,17)	0.27	0.77 (1,17)	0.21	0.00 (1,17)	0.22	0.16 (1,17)	0.14	0.17 (1,17)	0.24	0.57 (1,17)
	Have	0.00	0.698	0.00	0.391	0.20	0.961	0.00	0.698	0.25	0.686	0.00	0.572

Table 7-8: Part C of the ANOVA tests on dependency of the teachers' practical PCKs on their possessions

PCKs Observed	Possession of item	Brick house		More than 50 books	
		Mean	F (df1, df2) / p-value	Mean	F (df1, df2) / p-value
AT: Total connections observed	Do not have	3.75	0.01 (1,17)	3.56	0.21 (1,17)
	Have	3.86	0.917	4.00	0.654
BT: Total linkages	Do not have	3.25	1.94 (1,17)	3.22	0.86 (1,17)
	Have	1.86	0.181	2.30	0.367
CT: Total mathematical constructions	Do not have	3.50	0.04 (1,17)	4.22	2.34 (1,17)
	Have	3.29	0.847	2.70	0.144
DT: Total aids used	Do not have	3.00	1.39 (1,17)	3.89	0.06 (1,17)
	Have	4.86	0.255	3.50	0.809
ET: Total errors observed	Do not have	4.50	0.04 (1,17)	4.11	0.30 (1,17)
	Have	4.29	0.851	4.70	0.591
FT: Total form of feedback	Do not have	4.50	0.81 (1,17)	5.00	0.03 (1,17)
	Have	5.57	0.381	4.80	0.867
GT: Total focus of feedback	Do not have	8.92	0.15 (1,17)	9.33	0.31(1,17)
	Have	9.29	0.708	8.80	0.574
HT: Total unpacking	Do not have	3.17	0.00 (1,17)	3.56	0.33 (1,17)
	Have	3.14	0.986	2.80	0.574
IT: Total engaged tasks in teaching	Do not have	8.42	0.02 (1,17)	8.89	0.33 (1,17)
	Have	8.29	0.886	7.90	0.251
J3: Learners' prior knowledge accessed	Do not have	0.08	1.93 (1,17)	0.11	0.58 (1,17)
	Have	0.43	0.182	0.30	0.458

7.5 Correlation between teachers' declarative and practical PCK

As the analysis described in the previous section did not produce an overall measure of the teachers' practical PCK, I have explored all possible correlations between categories of declarative knowledge represented on the teacher test (CK, PCK, topic areas and PCK categories) and the adjusted categories of practical PCK, as discussed in Section 7.3.1. Given that I had both test results and observation results for 19 teachers, I only looked at correlations outside of the interval]-0.5;0.5[. The results are presented in Table 7-9. I had no observations coded for J1 and J2 (cf. Table 4-1), so only J3 is included in the table. Also, the two test questions on *representations* and only three test questions on *measurement* were treated as the same categories.

Table 7-9: Significant correlations between declarative and practical PCK categories

Declarative PCK category	Number of questions	Practical PCK category	Pearson correlation value
Test score	63	Errors ignored or answer simply corrected	-0.57
Test score	63	Errors engaged	0.54
Test score	63	No feedback given	-0.59
Test score	63	Learners' prior knowledge noted and used	-0.67
CK	35	Errors engaged	0.65
CK	35	Unpacking of content more than procedurally	-0.52
CK	35	No feedback given	-0.58
CK	35	Learners' prior knowledge noted and used	-0.66
PCK	28	Errors ignored or answer simply corrected	-0.56
PCK	28	No feedback given	-0.51
PCK	28	Learners' prior knowledge noted and used	-0.57
Number	23	Errors ignored or answer simply corrected	-0.55
Number	23	No feedback given	-0.52
Geometry	12	Errors ignored or answer simply corrected	-0.60
Measurement/ representations	6	Learners' prior knowledge noted and used	-0.74
Data handling	18	Errors engaged	0.61
Data handling	18	Unpacking of content more than procedurally	-0.59
Data handling	18	Learners' prior knowledge noted and used	-0.51
CK number	10	Errors ignored or answer simply corrected	-0.57
CK number	10	Errors engaged	0.60
CK number	10	No feedback given	-0.54
CK geometry	7	Connections through representations, implication or part-whole	0.60
CK geometry	7	Some representation is engaged	0.55
CK geometry	7	No errors observed	0.53
CK stats	11	Connections through representations, implication or part-whole	-0.57
CK stats	11	No errors observed	-0.52
CK stats	11	Errors engaged	0.54
CK stats	11	Unpacking of content more than procedurally	-0.58
Unpacking	13	Errors engaged	0.54
Unpacking	13	No feedback given	-0.53

Surprisingly, the teachers who used connections through representations, implication or part-whole with their learners more frequently tended to score better on CK questions on geometry but worse on CK questions on statistics. Using representation of some kind was however only positively correlated with the score on CK geometry questions. Perhaps even more surprising was that teachers who more frequently used methods other than procedure to unpack content tended to perform worse on CK questions and on statistics questions in general, including CK questions on statistics.

Classrooms where no errors were observed were also more likely to be taught by teachers who scored better on CK questions on geometry, and *vice versa* for CK questions on statistics. There were negative correlations between several of the test scores and the frequency with which teachers ignored observed errors or simply corrected the answer without any other engagements; this was the case for the total test score, the score on PCK questions, the questions on number and geometry, and the CK questions on number. On the other hand, engaging learners' errors was positively correlated to several of the declarative knowledge categories, namely the total test score, the CK score, the scores on CK for number and statistics, the score for all statistics questions and the PCK category of *unpacking*.

Teachers who demonstrated giving feedback in fewer intervals during a lesson also tended to score worse on the test as a whole, on CK questions, on PCK questions, on questions on number, on CK questions on number and on PCK questions on *unpacking*. There were no other noticeable correlations for the use of feedback, despite the central role it appears to play according to international studies (cf. Hattie & Timperley, 2007).

Finally, noting and using learners' prior knowledge was negatively correlated with overall test score, CK score, PCK score and scores on measurement and statistics questions.

Overall, these correlations raise again the question of the level of relationship between CK and practical PCK and between declarative PCK and practical PCK. On one hand, it points out that the practice of engaging learners' errors in teaching is positively correlated to all of the types of knowledge represented on the teacher test which have been considered here (CK, PCK, topic areas and PCK categories). On the other hand, teachers' answers to questions about declarative PCK were not positively correlated to practical PCK, which highlights the need for further research on teachers' content knowledge and practical knowledge.

7.6 Chapter summary

Chapter 7 demonstrated the range of Rwandan grade six mathematics teachers' practical PCK. Differences were found in the extent to which practical PCK was engaged; some teachers were found more frequently to use teaching strategies that have been found to be effective in previous studies, and used variations of these strategies. In other words, most teachers engaged in some of the practices often mentioned in the literature as desirable, and hence it seems fair to suggest that Rwandan mathematics teaching is not a calamity.

This chapter also highlighted that consistent links were not found between the teachers' knowledge, as demonstrated during teaching, and their socio-economic status. It also suggests that it is problematic to consider sub-components of PCK in isolation. The generalization limitations were once again noted. More attention needs to be given methodologies with regard to classroom teaching practices and their relations to learning in different contexts.

8 CORRELATIONAL ANALYSIS OF FACTORS AND LEARNING GAIN

8.1 Introduction

In this chapter, I present the results of my investigation into several factors which could impact learning gain. The first section explores the relationship between learning gain and background variables in the learners' lives, including socio-economic status. The next section explores the relationship between learning gain and teachers' declarative knowledge. Finally, I explore the relationship between learning gain and the results of my analysis of teachers' practical PCK.

8.2 Learning gain in relation to learners' background variables

With reference to his analysis of the SACMEQ III data, Spaull (2013) claims that the inconsistencies in the results he obtained for factors potentially influencing learning in South Africa were caused by modelling two school systems in one. When he looked at the wealthiest 25% of schools against the other 75%, Spaull found that different factors were significant for the two sets. Teacher's education was found to only have an impact in the wealthier group of schools (ibid).

To obtain an idea of the Rwandan learners' socio-economic status and other background variables, I extracted the information from the learner questionnaire regarding possessions in the learners' homes which is presented in Table 8-1. The results reflect the responses of the 713 learners who completed the questionnaire and the first test, although some learners left some questions unanswered. I made the assumption that if a learner had not ticked a given item, the family did not possess that particular item. This assumption is however somewhat problematic, I have realised retrospectively, as unmarked items may also reflect learners' uncertainty. For the question on the number of reading materials found in the home, learners had to pick one of the listed options, and I did not include blank responses for this question.

In addition to the questions represented here, learners were asked about piped cold and hot water in the house; however, I decided that there were too many issues around how learners had answered these questions for them to carry sufficient validity, so I have not included them.

Table 8-1: Summary of SES variables for learners

Item	Quantity possessed (where applicable)	Number of learners indicating possession	Percentage of learners indicating possession
Family owned house (not rented)		309	43.5
Books, newspapers and magazines	None	300	49.8
	About 10	228	37.8
	About 20	72	11.9
	About 50	3	0.5

Item	Quantity possessed (where applicable)	Number of learners indicating possession	Percentage of learners indicating possession
Electricity		528	74.0
Radio		538	75.5
TV set		504	70.7
Computer / laptop		228	31.9
Internet access		174	24.5
Cattle		291	40.8

Concerning the number of books (which might include even the adult books) and magazines which the learners had in their families, around half of the learners reported that did not have books in the home; the majority of the other half estimated that they had around 10 books or magazines at home.[20] Around a quarter of the learners (26%) indicated that they did not have electricity in their homes. The majority of the learners indicated that there was a radio (75.5%) and a TV (70.7%) in their home, but less than a third (31.9%) reported a computer, and only a quarter internet access.[21]

In terms of the objective of this analysis to explore the relationship of these variables to learning gain, learning gain was higher for the learners from private schools, with a mean improvement of 11.5 percentage points, than for those at public schools, who had a mean improvement of 8.2 percentage points (Table 8-2).

Table 8-2: Comparison of private and public schools in the Rwandan sample

	Number of learners to do both tests	Mean in test 1 (%)	Mean in test 2 (%)	Percentage point improvement	Relative improvement
Private	191	50.4	61.8	11.5	22.8%
Public	447	41.4	49.6	8.2	19.7%
Total	638	44.1	54.3	9.2	20.8%

The sample of private schools was too small and the teachers' educational backgrounds too similar in this study to allow me to investigate the extent to which Spaull's finding held true in Rwanda. Instead, I investigated whether there was any relationship between the Rwandan grade

[20] These data were self-reported by children, and therefore should be treated with some caution. From my personal experience about Rwandan parents buying books, the most reliable answer for this question is probably "none". In one way or another, this might be related to both Rwandan culture with its narrative tradition, and to the limited financial capacity of some families.

[21] Cell phones with internet access are widespread across Africa, and it must be considered a flaw in the questionnaire that this was not taken into account. I see this as another example of how the questionnaire was not sufficiently adjusted to the regional context.

six learners' social economic status, or other background variables, their learning gain (Table 8-3) and

The ANOVA results show that the age of a learner is significant at a 5% level (p-value of 0.036). The learners who were at the expected age for grade six performed better, while learners who were older displayed a smaller learning gain. The results found no significant relationships between learning gains and any of the other variables, which surprised me.

I had expected that factors such as having electricity, a computer, or internet access at home would be correlated to learning gain. Learners who had books at home, or had both parents living at home, also did not demonstrate a greater learning gain than learners who did not.

Table 8-4 present the results of this investigation.

Table 8-3: Part A of analysis of correlation between learner gain and background factors

Learner SE Factors	Category	Descriptors of Learning Gain			ANOVA Tests			
		N	Mean	Std. Dev.	F	df1, df2	p-value	Comment
Gender of learner	Male	355	3.64	4.453	0.306	1, 637	0.580	No significant difference
	Female	284	3.44	4.648				
Age group of learner	up to 11 yrs	70	4.17	4.249	2.587	4, 613	0.036	Significant grouping variable on 5% level
	12 yrs	138	4.36	4.605				
	13 yrs	161	3.24	4.785				
	14 yrs	131	3.62	4.703				
	15yrs and above	118	2.74	3.848				
Number of years at same school	1 to 2 yrs	487	3.42	4.337	0.801	3, 634	0.494	No significant difference
	3 to 4 yrs	106	3.76	5.194				
	5 to 6 yrs	40	4.18	5.148				
	7+ yrs	5	5.60	4.159				
Language of learner	Kinya-rwanda	579	3.52	4.545	0.095	2, 637	0.909	No significant difference
	English	33	3.79	5.110				
	French	28	3.79	3.665				
Caregiver of learner	Mother	139	3.63	4.561	1.628	2, 637	0.637	No significant difference
	Father	454	3.62	4.545				
	Both	47	2.53	4.333				
Education of caregiver	None	38	4.24	4.529	1.074	3,636	0.360	No significant difference
	Primary	185	3.09	4.315				
	Part secondary	140	3.62	4.812				
	Full secondary	277	3.72	4.533				
Number of books in house	none	266	3.51	4.564	1.036	3, 593	0.376	No significant difference
	About 10	266	3.73	4.487				
	About 20	62	3.40	4.252				
	About 50	3	0.67	8.145				

The ANOVA results show that the age of a learner is significant at a 5% level (p-value of 0.036). The learners who were at the expected age for grade six performed better, while learners who were older displayed a smaller learning gain. The results found no significant relationships between learning gains and any of the other variables, which surprised me.

I had expected that factors such as having electricity, a computer, or internet access at home would be correlated to learning gain. Learners who had books at home, or had both parents living at home, also did not demonstrate a greater learning gain than learners who did not.

Table 8-4: Part B of analysis of correlation between learner gain and background factors

Items at home reported by learners	Category	Descriptors			ANOVA Tests			
		N	Mean	Std. Dev.	F	df1, df2	p-value	Comment
Radio	Don't have	153	3.80	4.773	0.361	1, 637	0.697	No significant difference
	Have	485	3.47	4.468				
TV	Don't have	177	3.45	4.220	0.122	1, 637	0.727	No significant difference
	Have	462	3.59	4.658				
Computer	Don't have	433	3.54	4.416	0.002	1, 637	0.968	No significant difference
	Have	206	3.56	4.796				
Internet	Don't have	482	3.48	4.535	0.446	1, 637	0.504	No significant difference
	Have	157	3.76	4.555				
Electricity	Don't have	166	3.60	4.707	0.026	1, 637	0.873	No significant difference
	Have	473	3.53	4.483				
Cattle	Don't have	375	3.57	4.457	0.023	1, 637	0.879	No significant difference
	Have	264	3.52	4.660				
House	Rent	349	3.31	4.182	2.175	1, 637	0.141	No significant difference
	Own	290	3.84	4.925				
Did someone read to you when you were young?	Yes	448	3.46	4.516	0.412	1, 569	0.662	No significant difference
	No	123	3.88	4.596				
How often do you have home-work?	Never	12	3.00	2.629	1.589	3, 626	0.175	No significant difference
	Once/week	22	1.59	3.621				
	2-3 times/ week	112	3.21	4.282				
	Every day	484	3.75	4.680				
Did you attend pre-school?	Yes	403	3.69	4.582	0.599	1, 430	0.550	No significant difference
	No	29	3.48	5.103				
How often do you read at home?	Never	27	2.41	4.822	1.293	2, 637	0.275	No significant difference
	Once/week	66	3.12	4.123				
	3 times a week	547	3.65	4.565				
Are teachers ever absent?	No	312	3.84	4.832	2.530	1, 637	0.112	No significant difference
	Yes	327	3.27	4.228				

It seems reasonable to consider the extent to which there is a synergetic effect of various factors, and I am aware for example that in the studies conducted in Botswana and North West Province, several of the questions regarding type of home, number of books, etc. have been used together to make an index of socio-economic status. However, this is not something to be done lightly (cf. Kanyongo et al, 2007). After much deliberation, I have decided not to include socio-economic data in this thesis, despite the weight that it can carry in relation to learning (see discussion in Chapters 2 and 3).

8.3 Links between teachers' declarative knowledge and learning gain

8.3.1 Correlation between teachers' CK test results and learning gain

There was no correlation between teachers' overall score for the CK questions on the test and the learning gain of their learners. However, there was a moderate negative correlation of -0.472 between the scores on the ten CK questions on *number* and learning gain. I can think of no explanation for this peculiar fact.

8.3.2 Correlation between teachers' PCK test results and learning gain

The declarative PCK component of the teacher test was also investigated for correlation with learning gains, as shown in Table 8-5.

Table 8-5: Correlations of teachers' declarative PCK to learning gain

		Teacher's score on PCK questions	**Learner Test 1**	**Learner Test 2**
Learner Test 1	Pearson Correlation	0.106**		
	p-value	0.005		
	N	713		
Learner Test 2	Pearson Correlation	0.075	0.657**	
	p-value	0.058	0.000	
	N	638	638	
Learning Gain	Pearson Correlation	-0.034	-0.301**	.503**
	p-value	0.391	0.000	0.000
	N	640	640	638
****. Correlation is significant at the 0.01 level (2-tailed).**				

In general, taking into consideration both pre- and post-tests, the correlation between teachers' declarative PCK and learning gain is not significant (correlation=-0.034, p-value=0.391). There is a very weak but significant correlation between teachers' declarative PCK and learners' scores on the pre-test (correlation=0.106, p-value=0.005), but learner performance on the post-test is not significantly correlated to teachers' declarative PCK. I found no correlation between the PCK scores for *unpacking mathematics* and *analysing learner errors* with learning gain.

8.4 Correlational analysis of teaching and learning

The effect of teachers' practical PCK on learning gain was assessed using Pearson correlation coefficients because the data did not meet some of the factor analysis requirements, such as having a sample size of more than 200 individuals for sub-groups. In the analysis presented below, I measured the correlation of teaching practice represented on the practical PCK instrument (see Table 4-1) with learning gains. Some of the practices appear to have positive correlations to learning gains, whereas other practices are negatively correlated to learning gains. Two results shown in Table 8-6 were significant at a 5% level. The detailed results are presented in Appendix D8.

As the results shown in Table 8-6 indicate, where teachers were not observed to engage any kind of connections there was a negative correlation to learning gain (correlation = -0.486, p-value=0.030). Due to the small sample size, I cannot generalize this finding, but the results support the idea that making connections in mathematics teaching is a good practice.

Table 8-6: Correlation between teachers' practical PCK and learning gain.

Correlations at a 5% significance level are highlighted. As previously mentioned, one teacher has been excluded from the declarative PCK analysis but observation results for that teacher are included here.

PCK item	Learning Gain (Using class averages)		
	N	Pearson Correlation	p-value
A1: No kind of connections observed	20	-0.486*	0.030
C6: Learners are encouraged to communicate mathematically while performing a task.	20	-0.402	0.079
E4: Incorrect answers from risen misconception/ errors have been individually challenged	20	-0.448*	0.048
G3: The feedback given is about process to create product.	20	-0.389	0.090
I1: Tasks to clarify the concept and alternative strategies were not in place	20	-0.409	0.073
****. Correlation is significant at the 0.01 level (2 tailed).**			
***. Correlation is significant at the 0.05 level (2-tailed).**			

The other significant result (at 5% significance level) was that individually challenging incorrect learner responses was negatively correlated to learning gain (correlation -0.448, p-value 0.048). I could speculate that this is because this practice consumes teaching time or discourages learners from learning from each other's mistakes, but I have not interrogated this further.

Three results were significant on a 10% level. One relates to the last result mentioned above, as it indicates that the frequency with which errors are noted/observed/engaged by teachers is negatively correlated to learning gain. Surprisingly, and counter to what Hattie and Timperley (2007) suggest, process feedback was also negatively correlated to learning gain (correlation = -0.389, p-value=0.090). This finding certainly warrants further interrogation. Equally surprising to me was the result that encouraging learners to communicate mathematically while working on tasks was also negatively correlated to learning gain (correlation = -0.402, p-value=0.079). Less surprising was that not having in place tasks to clarify the concept with alternative strategies was negatively correlated to learning gain (correlation = -0.409, p-value=0.073).

Despite the fact that the small sample size used in this study is a barrier to generalising the results, what the results suggest is that some of the classroom practices examined need more careful consideration when used as they seem to impact negatively on learners' learning. These include, for example, challenging incorrect learner responses individually; not engaging any kinds of connection; and not recognising learners' errors/ misconceptions, all of which were negatively correlated to learning gain in my sample.

Looking at the correlation between categories of teachers' practical PCK and learning gain, the significant results are presented in Table 8-7; details for the grouped categories of teachers' practical PCK and learning gain can be found in Appendix D7. There was a moderate, but highly significant, correlation between the extent to which teachers engaged content connections and learning gain (p-value 0.006). There was a weaker correlation between the extent to which teachers engaged tasks in their teaching and learning gain (p-value 0.080).

Table 8-7. Correlation between categories of teachers' practical PCK and learning gain

Significant results only. **. Correlation is significant at the 0.01 level (2-tailed). *. Correlation is significant at the 0.05 level (2-tailed).

Pearson's correlations		Learning Gain	Comment
Content Connections	Correlation	0.604**	Significant at 5% level
	p-value	0.006	
	N	19	
Engaging Tasks	Correlation	0.412	Significant at 10% level
	p-value	0.080	
	N	19	

8.5 Chapter summary

In this chapter, various factors which could influence learning gain were explored, including learners' background, teachers' declarative and practical PCK and teachers' CK.

The results showed that learners who attended private schools made greater learning gains during the interval between the learner tests than those in public schools, as did learners who were at the expected age for their grade level. However, no correlations were found between teachers' CK and learning gain. This was also the case for declarative PCK, where the correlation with learning gain was not significant. On the other hand, some aspect of practical PCK did show a correlation with learning gain; these include making connections and putting into place tasks to clarify the content in teaching.

Again, due to the small sample size in this study, these results cannot be generalised.

9 CONCLUSION

9.1 Introduction

In this final chapter, the results which have been presented will be brought together into conclusions in response to the research questions that have been investigated in this study. The first section of this chapter deals with the contribution of this thesis to the existing body of knowledge; this includes answering the research questions and discussing these in relation to previous research and the Rwandan context. The second section summarises and discusses the main limitations of this study. The third section presents recommendations for future research, and the final section presents recommendations for the research community in general and for researchers conducting research in the Rwandan context in particular.

9.2 Contribution of the thesis to existing knowledge

Few empirical research studies have explored the practical and declarative PCK of mathematics teachers, and fewer still the relationship it could have to learners' gains in learning over time. This empirical study therefore makes a contribution to this field of investigation. The classroom observation instrument which was developed in this study to document mathematics teachers' classroom practices is a further contribution to the field as it allowed a deeper investigation of PCK than some earlier studies have been able to achieve. For example, Carnoy and Chisholm (2008) only ranked PCK in the lessons used, presumably according to some 'expert judgement', and did not differentiate the different kinds of PCK, while Rowan, et al; (2001) did not consider the learners' learning gain over time. This study contributes toward closing that gap; an achievement which is especially significant in a developing context.

In this study I succeeded in developing an instrument to analyse video recorded lessons which focused on mathematics teachers' practical PCK. The existing literature on teacher knowledge and, specifically, how teachers teach, as discussed in Chapter 4, guided the design of this instrument, which was then tested on the empirical data and adjusted through successive revisions. I by no means claim that this is 'the' ultimate instrument, as new instruments or revisions to existing instruments, will undoubtedly arise in the future, but it represents an attempt to sharpen some of the existing analytical tools.

Answers to the research questions have been presented across Chapters 5, 6, 7 and 8, and will be summarised and discussed here.

9.2.1 How do Rwandan grade six learners perform on a standardized mathematics test, and what learning gains are achieved over the course of grade six?

The 'stability' of learners' answers was analysed by looking at how many learners chose the same answer in both the pre-test and the post-test. I found a strong correlation between the selection of the correct answer on the post-test when it had been selected on the pre-test, indicating that learned knowledge was, to a large extent, retained. The stability of answers for the Rwandan learners was stronger than for the learners in the KwaZulu-Natal cohort, and I concluded that it was fair to consider the learning gain to be indicative of an actual improvement in performance. (See however the discussion in Section 9.3.3).

The 638 Rwandan grade six learners who completed both the pre- and the post-test achieved a mean pre-test score of 44.1% and a mean post-test score of 53.3%, demonstrating a mean increase of 9.2 percentage points or 20.8 percent. This was a more substantial learning gain than those achieved in the earlier studies conducted in KwaZulu-Natal and the North West province of South Africa and in Botswana, in which learning gains of 3 to 4 percentage points were achieved.

Using the terminology from SACMEQ, the Rwandan learners appeared to be operating at the *basic numeracy* level when they took the pre-test; by the time they took the post-test at the end of grade six, more than half had reached *beginning numeracy*. The Rwandan learners' performance improved on SACMEQ level 4 questions in particular, whereas the Botswanan learners' improvement was more uniform across SACMEQ levels, as was the case with the learners in KwaZulu-Natal, to a more limited extent.

Taken together, these results suggest that the Rwandan learners developed better mastery of numeracy and built on it more gradually. However, this needs further exploration, which could be done through analysis of learners' workbooks, textbooks and the curriculum.

Overall, the Rwandan learners scored better on average on the pre-test, as well as on the post-test; their answers were more 'stable' than those of the KwaZulu-Natal learners, they had a stronger basic numeracy than the KwaZulu-Natal learners in particular, and they appeared to have improved their test performance substantially more than the other cohorts, most notably on SACMEQ level 4.

The correlations between test scores and learning gain suggested that the learners who scored better on the pre-test were less likely to improve their scores. Normally, learners who have strong foundations obtain the greatest increase in marks over time, but since this was the same test, there could have been limited scope for improvement for these learners. This is, however, only speculation; I have tried to identify additional patterns to explain this, but without success.

9.2.2 What is the level of declarative knowledge, in particular content knowledge and PCK, of Rwandan grade six teachers?

The data from the teachers' test results on PCK questions were negatively skewed (skewness statistic=-2.374) which means that the test marks are concentrated on marks above the mean mark of 57.82.

The results show that teachers' content knowledge was best in the areas of *number,* where they obtained a mean score of 72.1%, and in *measurement,* where they obtained a mean score of 78.9%. These are also the two areas in which the Rwandan teachers performed much better than their counterparts in the KwaZulu-Natal study. However, I note the disappointing scores of between 43% and 49% for the remaining three content areas i.e. algebra, geometry, probability/statistics. This must be considered in the light of the limited number of questions within each content area, and thus can only be indications of the content knowledge of Rwandan teachers.

The best mean score for PCK was for the *stats/probability* questions (72.9%). It is not possible to compare this in any substantial way to international results, due to the very different approaches to 'measuring' declarative PCK. It can, however, be compared directly to the results from the KwaZulu-Natal study, and there were differences in the scores these two cohorts of teachers obtained on the PCK questions on the test. As previously mentioned, this is likely a result of national differences in teacher education, something I am currently starting to engage through a collaborative international project.

9.2.3 What is the nature and extent of the practical PCK of the grade six teachers?

The findings presented in Chapter 6 showed that the Rwandan grade six teachers demonstrated a number of practical PCK practices which are considered valuable for teachers the literature. They used procedure connections, progression from simple to complex, encouraged learners to communicate mathematically while performing a task, used drawn teaching aids/representations, and engaged in sharing and discussing errors and misconceptions with learners. On the other hand, they were rarely observed to use implication connections, progress from theoretical to practical, generalise a concept by leaving or adding properties from complex task self-regulation, access learners' prior knowledge, and provide cognitive conflict feedback.

Given the limited data and the large number of categories, it was not possible to create any form of 'typology' based on observed practical PCK. This also meant that it was not possible to construct a single measure – or a few summative measures – of practical PCK to use in statistical inferences.

Hill, et al. (2008) found that teachers with higher MfT made fewer mathematics errors, linked concepts and procedures more often, chose more helpful examples and corresponded to their learners' questions. Even if I cannot generalize the findings from the different studies, examining my results in light of the findings of Hill et al (ibid), it is notable that none of the teachers in this study made any mathematical errors while teaching. Furthermore, I did not find that declarative PCK correlated with any of the practices listed by Hill and her colleagues.

9.2.4 How do teachers' content knowledge, declarative PCK and practical PCK relate to each other, and to background factors such as education, SES and teaching

My results suggest that Rwandan mathematics grade six teachers' declarative PCK test results are positively correlated with their CK test results. This result is in line with existing findings (Baumert et al., 2010; Krauss et al., 2006; Ramdhany, 2010). The correlation between the overall CK and PCK scores was 0.650, which is not far from the value of around 0.600 found by Krauss et al. (2006).

There was a stronger correlation between my respondents' content knowledge and the PCK component *unpacking mathematics* with a correlation coefficient of 0.703. This is a stronger correlation than in the KwaZulu-Natal study (0.59). On the other hand, the correlation between teachers' CK and the PCK component of *analysing learner thinking/errors* had a weaker correlation in the Rwandan sample (0.418) than in the KwaZulu-Natal study (0.768).

For the Rwandan teachers, the correlation between CK scores and PCK scores on *number* was very strong with a correlation coefficient of 0.948, but lower for other content areas, with a correlation coefficient as low as -0.086 for CK and PCK for *geometry*.

I found no link between teachers' declarative PCK test performance and their level of education, gender, or years of experience. Concerning the link to experience, this is unlike findings from studies conducted in more developed contexts, such as the study conducted by Krauss et al. (2008). Blömeke et al. (2011) also found that teaching experience was positively linked to declarative PCK. However, my result could have had to do with the limited variation of teaching experience within the sample, as well as the small sample size.

Pertaining to practical PCK, some significant correlations were found between components of practical PCK and education or teaching experience. These included that experienced teachers more frequently engaged their learners in tasks and that teachers who had only obtained a high school certificate before beginning their teacher education engaged in more task/product feedback than their colleagues with further education after high school. In fact, the latter never used task/product feedback in any of the observed lessons. In relation to gender, my analysis of the classroom observations suggested that female teachers in the current study were much more likely than male teachers to construct mathematical content through practices/variations ($t=3.331$, $df=17$, $p\text{-value}=0.004$). A Germany study, on the other hand, found that gender and home language had no effect on PCK (cf. Blömeke et al., 2011).

With regard to socio-economic status and teaching, my study yielded surprising results. For instance, teachers with fewer possessions linked to socio-economic status made more content connections in their lessons. Similarly, those who did not own radios more frequently engaged their learners in construction of mathematics.

9.2.5 How do learners' background factors and teachers' declarative and practical knowledge relate to learners' achievement over the course of grade six?

My fifth research question looked at how these various factors correlated with learners' achievement as 'measured' by their learning gain.

The learners who attended private schools had a better average on the pre-test with a slightly higher learning gain. This was no different from results from other contexts and countries, but the difference between the performances of the two groups in the Rwandan sample was perhaps smaller than some would have expected. The relative improvement only varied slightly, but together with the higher scores on the pre-test, this means that the learners at private schools increased their lead over the course of grade six. In the Rwandan context, this means that more attention must be given to course delivery in public schools, in particular.

The ANOVA results showed that the age of the learner was significantly correlated with learning gain at a 5% confidence level (p-value of 0.036). Learners of the expected age for grade six performed better; while learners who were much older achieved a smaller learning gain. No other background variables showed a significant correlation with learning gain.

There was no correlation between teachers' overall score for the CK questions on the test and the learning gain of their learners. However, there was a moderate negative correlation of -0.472 between the scores for the ten CK questions on *number* and learning gain.

In this study, no significant correlation was observed between the Rwandan grade six mathematics teachers' declarative PCK and their learners' learning gain (correlation=-0.034, p-value=0.391). That is similar to the results from other African countries (cf. Carnoy and Chisholm, 2008), where there were no significant correlation between teachers' knowledge and their learners' learning gain. However, in the German COACTIV study (Baumert et al., 2010) teachers' declarative PCK was positively correlated to their learners' learning gain. It has been proposed that this is because other factors related to socio-economic status play a more important role, so that PCK may only matter in the more affluent school contexts (cf. Spaull, (2011)). The scope of my study was too limited to explore this connection.

Some of the practices of the teachers were correlated to learning gains. Some of these were in line with expectations based on earlier research. For instance, lessons where the teacher did not have in place task and alternative strategies to clarify content was negatively correlated to learning gain (correlation = -0.402, p-value=0.079). Equally expected, lessons not engaging any kind of connections appeared to be related to a negative effect on learning gain (correlation = -0.486, p-value=0.030). Making connections in mathematics teaching is a good practice.

More surprising to me was that individually challenging incorrect learner responses was negatively correlated to learning gain (correlation = -0.448, p-value=0.048). This needs further exploration.

Counter to what Hattie and Timperley (2007) suggest, process feedback was also negatively correlated to learning gains (correlation = -0.389, p-value=0.090). This would certainly need to be followed up by further research in developing contexts.

Equally surprising to me was the result that encouraging learners to communicate mathematically while working on tasks was negatively correlated to learning gain (correlation = -0.402, p-value=0.048). It is possible there were other factors at play such as poor usage of language of beliefs about the mathematics curriculum (cf. Hill, et al., 2008).

The fact that some of the conclusions are in discord with earlier research findings perhaps emphasizes the importance of considering the context in which studies are undertaken. In my study, it is interesting that there were relatively few differences between the teachers in terms of their teaching, that the difference between the learning gains of the learners at private and public schools was not substantial and that almost all learners appeared to learn (in contrast to the findings from South Africa). This might indicate that education in Rwanda is comparatively ‘equal’ and homogenous. I would expect that a system with already a fairly equal basis could more easily be improved as a whole than a system where extreme inequality must first be addressed. Perhaps this explains why the performance of the Rwandan learners in this study was better than that of learners in the South African cohort – although, of course, this is just speculation.

This study, then, was not only correlational, as discussed in Chapter 3, but also brought to light teachers’ practices in a different, and hitherto under-researched, context.

9.3 Validity and limitations of this study

As with any study, this study is constrained by some limitations. As these have been discussed in Sections 4.6.3 and 4.8.4, the discussion below is of a more reflective nature.

9.3.1 Selection of data collection methods

Data were collected in this study using questionnaires and tests for learners and teachers; video recordings were also made of classroom lessons. I have been challenged by the fact that I was unable to determine the motivations driving teachers’ choices to use a particular teaching approach. Had I interviewed them after the lessons about their reasons for their approaches, not only would I have strengthened my analysis, it would also have been more likely to reflect the lived reality of the teachers, in line with my paradigmatic orientation.

This study was conducted in a particular context. The Rwandan teachers have their own historical-cultural experience which may have influenced their methods of teaching. Had I interviewed them after their lessons, it would have provided me with a stronger foundation for exploring these particular issues. Instead, the study objectifies its subjects in a slightly colonial way by disregarding the cultural aspect of the study and failing to explore the teachers’ own perspectives on their teaching.

There are those, such as Baxter and Lederman (1999), who feel that PCK is an internal construct which is held unconsciously. It is therefore problematic both to try to capture teachers’ PCK through tests and to deduce PCK from observations of teaching. An alternative is to infer PCK from observation-prompted interviews where teachers reflect on their teaching. This is a valid consideration, and worth reflecting on. My approach was instead to assume that PCK has both an internal component which can be inferred from responses to test questions – here referred to as declarative PCK for the sake of simplicity – and a practical component. But this could be challenged.

As I have mentioned earlier, the data could have been strengthened had I also interrogated aspects such as curriculum coverage, classroom management, learners' previous schooling and general pedagogical knowledge (cf. Gordon, Dwayne, & Melvyn, 1994). By far the most significant limitation to the quality of the data is that only one lesson was observed for each teacher. In some cases, I did manage to record a second video, but not only did the data collection itself take almost a full year, the analysis of the twenty videos also took substantial time. There is no doubt that this contributes to the uncertainty and fragility of the findings. However, it would have required either more time or more staff to expand the data collection, and this would have required more substantial funding.

9.3.2 Content validity and the development of my instrument

I am concerned that the teacher test, which was inherited from the previous studies, did not reflect the full range of types of PCK across mathematics topics. I would go so far as to say that the construct validity was reasonable for certain aspects of PCK such as KCS, but that it was problematic that KCC was not represented. The test also reflected the difficulty of drawing a strong distinction between CK and PCK, as discussed by Kaarstein (2014), which could have informed the construction of the SCK category. This challenges some of the conclusions as others may argue for a somewhat different categorisation of the test questions. Any test of this nature will always reflect one particular conception of the underlying concept(s), in this case PCK.

Another issue is that simply analysing results based on the number of correct answers to different types of questions can indeed be misleading. However, the test would have had to be designed differently for it to have been possible to analyse the relationship between the responses.

In my view, my video analysis instrument needs to be refined further so that each video can be analysed for topic specific PCK. However, aspects of this study such as the data collection methods and limitations on time and finances made it impossible to video record teachers' classroom practices when teaching specific mathematical topics, as recommended by Lee (2010).

I have worked from the fundamental assumption that PCK has both a declarative and a practical dimension. As mentioned in the previous Section, this could be challenged.

9.3.3 Validity of conclusions

All learners' test answers were included in the correlation analysis. This obscures the fact that there were answers that had a lower 'stability', indicating that learners may have guessed in these cases. It is interesting to speculate whether different correlations would have been found if the learning gain for only the more 'stable' questions had been considered. I explored this to a small degree by checking whether there was a positive correlation between the pre-test scores and the learning gains for the 30 most 'stable' questions, but there was not, so I have not pursued this idea further at this point.

I found varying correlations between teachers' test scores for CK and PCK questions, depending on the mathematical topic area. However, it would be highly problematic to conclude from this that CK and PCK for geometry are not related, for instance. The questions on the text only represent selected topics, and the content of the CK and the PCK questions was not the same. This is, as previously mentioned, a serious drawback when conducting a replication study, and retrospectively I feel that I should have altered the test. I may conduct a follow-up study involving teachers only, in order to explore this issue further.

An important question with regard to the correlational analysis of practical PCK and learning gain is whether it is indeed fair to assume that "more of a good thing" is better. It may be that certain teaching practices are effective when used sparingly. This is indeed the danger of trying to quantify the mystery and art of teaching, and points to the necessity of supplementing studies such as this with more qualitative work.

I also found that there was no positive correlation between the teachers' content knowledge (as measured in the test), their declarative PCK, and the learners' learning gain. Yet in the lessons observed, no teacher content errors were noted. This could be explored further.

9.3.4 Generalisability or transferability

Rwanda is not South Africa. As Broadfoot's comparison of preferred teaching practices in France and England has shown (Broadfoot, 1999), context matters. It would therefore be suspect to suggest that my findings can in any way be generalised to other contexts.

The random sampling, the small sample size and the limited number of videos I analysed all limit the results from being generalised within Rwanda either. However, I have attempted to make the analysis as transparent as possible so that readers can access my process as well as my findings and determine for themselves to what extent they provide insight into existing classroom practices, teacher knowledge in general and PCK in particular.

9.4 Suggestion for future research

To the best of my knowledge, this is the first study of this kind done in Rwanda, and therefore it is only natural that further research is encouraged, in order to interrogate and extend, if not challenge, these results. In earlier chapters I have hinted at possible avenues for further study. I pick up on these and other outstanding issues now.

First, the answer to my first research question suggests that Rwandan learners establish a relatively firm basis of numeracy and build on it gradually. However, this needs further exploration, which could be done through analysis of learners' workbooks, textbooks and the curriculum, using longitudinal data. I hope to undertake, or participate in, such a study in the future, following learners over at least a three year period to observe the changes in their numeracy.

Alternatively, I suggest that Rwanda participate in some of the regional or international comparison studies.

In Section 9.3, I mentioned some of the problems with the design of the teacher test. It would add to our understanding of Rwandan teachers' content knowledge and declarative PCK to develop a test that better reflects the inclusive concept of PCK or MfT and analyse the results.

Although I believe that the analysis instrument which I developed during this study has been useful, it is certainly an area for further research to refine and expand on this tool, informed by the literature. This instrument was designed specifically to interrogate mathematics teachers' practical PCK. However, I have not found many significant correlations between the elements of PCK targeted in the instrument and other variables which the existing research literature suggests are connected to quality teaching. I therefore would like to invite other researchers in mathematics education to apply this instrument to different contexts, as well as to extend it with topic-specific components.

As already mentioned, a limitation in this study was that only one video was analysed for each teacher. From a methodological perspective, it would be useful to explore the constancy of the various components of practical PCK across lessons taught by the same teacher. I suggest doing so by analysing more classroom videos for each teacher, as well as comparing lessons taught in different contexts.

To gain more substantial insight into the relationship between declarative and practical PCK, I would also suggest using observations as the basis for follow-up interviews with teachers to explore their rationale for the choices that they were observed making.

Some practices that were coded as practical PCK were correlated to variables such as teachers' gender. Future researchers may interrogate further the correlations between such variables and teachers' teaching, as well as learners' learning gains. However, larger samples would be required in order to generate statistical support for any claims along these lines.

Based on my results, it is my view that education in Rwanda is not in a failed state. However, teachers' practices could bear improved, specifically through activities which could help them to recognize their learners' thinking. A natural extension of this project could therefore be to look further into teacher education and teacher learning in Rwanda.

In terms of my personal engagement with this topic, I plan in the near future to be involved in research on the teacher education practices used by the University of Rwanda's College of Education, in collaboration with researchers from Sweden, Greece and South Africa. It is hoped that the diversity of the research team will generate insights across national and regional contexts as well as contribute to theory development. The project will study the notion of good mathematics teaching legitimised in assessment criteria, materials, etc. It will involve conducting interviews with students and novice teachers about the ways in which they adapt the knowledge from their teacher education. And the project will follow student teachers in their last year of the teacher education programme and into the first years of teaching, documenting and analysing changes to their teaching practices over time.

Finally, the fact that Rwandan television broadcasts programmes on education with an emphasis on teaching methods could be a factor influencing what teachers view as acceptable or good practices. I plan to undertake a small scale analysis of this in the near future. This is particularly interesting to me, given that my results suggest that teachers who own a TV are less likely to make connections in their teaching than those who do not.

9.5 Recommendation

Based on the findings of this study, I would like to make the following recommendations.

It was found that the learners' age group was significantly correlated to their learning gains. Learners fifteen years or older - which is above the expected age of a grade six learner in Rwanda – achieved smaller learning gains than learners of the expected age for their grade. In response to this, I would like to recommend to leaders in education that classes be set up for 'adult' and teenage learners who have not completed their primary education, instead of placing them in primary school classes.

Given that significant gaps were identified in the knowledge of the teachers in this study, I recommend that MINEDUC, which offers teacher training, study these findings and take them into consideration when developing courses and exercises for teachers.

There appear to be opposing findings on teachers' PCK in studies from different countries. To explore the reasons for this, I recommend that further research include interviews with teachers about the methods they employ and decisions they make while teaching. In addition, I recommend that further research be conducted to explore the ways in which PCK is presented in teacher education, taken up by students, and recontextualized by teachers.

POST-SCRIPT

Before closing, I would like to let the readers know that this study has been valuable to me. My knowledge has grown and my research competencies have developed throughout the stages of this study, including the writing of this thesis. As a postgraduate student with a background in applied mathematics, I began with the idea that I would investigate the application of mathematics in education. As I read the literature, however, my perspective changed and ultimately I decided to investigate how teachers teach mathematics in their classrooms. My focus thus shifted from mathematics *in* education to *mathematics education*. This shift from one discipline to another required a corresponding shift in my understanding of research and in my use of research skills, which has proven demanding, tiring and eye opening all at the same time. It has required strong dedication to complete this journey, and I am thankful to all who have travelled all or part of the distance with me.

S/he who (re-)searches will find,
and her/his will is the ticket
to the beginning of the journey.

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Appendix A

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

14 February 2013

Mr Jean Francois Maniraho 212558730
School of Education
Edgewood/Pietermaritzburg Campus

Protocol reference number: HSS/0064/013D
Project title: The pedagogical content knowledge of Rwandan Grade 6 Mathematics teachers and its relations to learning

Dear Mr Maniraho

Expedited Approval

I wish to inform you that your application has been granted Full Approval through an expedited review process.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. Please note: Research data should be securely stored in the school/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

Professor Steven Collings (Chair)
/px

cc Supervisor Professor Iben Maj Christiansen
cc Academic leader Dr MN Davids and Dr R Mudaly
cc School Administrator Ms Bongekile Shengu

Professor D Wassenaar (Chair)
Biomedical Research Ethics Committee
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban, 4000, South Africa
Telephone: 27 (0)31 260 2384 **Facsimile:** +27 (0)31 260 4609 **Email:** brec@ukzn.ac.za
Website: http://research.ukzn.ac.za/Research-Ethics/Biomedical-Research-Ethics.aspx
Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

INSPIRING GREATNESS

Appendix B

SACMEQ Mathematics levels

Level 1	Pre Numeracy	Applies single step addition or subtraction operations, recognizes simple shapes, Matches numbers and pictures, count in whole numbers.
Level 2	Emergent Numeracy	Applies a two-step addition or subtraction operation involving carrying, checking (through very basic estimation), or conversion of pictures to numbers. Estimates the length of familiar objects. Recognizes common two-dimensional
Level 3	Basic Numeracy	Translates verbal information (presented in a sentence, simple graph or table using one arithmetic operation in several repeated steps. Translates graphical information into fractions. Interprets place values of whole numbers up to thousands. Interprets simple common everyday units of measurement
Level 4	Beginning Numeracy	Translates verbal information into simple arithmetic problems. Uses multiple different arithmetic operation (in the correct order) on whole numbers, fractions, and/or decimals
Level 5	Competent Numeracy	Translate verbal, graphic, or tabular information into an arithmetic form in order to solve a given problem. Solve multiple operation problems (using the correct order of arithmetic operation) involving everyday units of measurement and/or whole and mixed numbers. Convert basic measurement units from one level of measurement to another (for example meters to centimeters)
Level 6	Mathematically Skilled	Solve multiple- operation problems (using the correct order of arithmetic operation problems (using the correct order of arithmetic operations) involving fractions, ratios, and decimals. Translates verbal and graphical representation into symbolic, algebraic, and equation form in order to solve a given mathematical problem. Checks and estimates answers using external knowledge (not provided within the problem)
Level 7	Concrete Problem Solving	Extracts and converts (for example, with respect to measurement units) information from tables, chart, visual and symbolic presentation in order to identify, and then solve multi-step problems
Level 8	Abstract Problem Solving	Identifies the nature of an unstated mathematical problem embedded within verbal or graphic information and then translate this into symbolic, algebraic, or equation form in order to solve the problem.

Appendix C

REPUBLIC OF RWANDA

Kigali, 2nd January, 2013
Ref: 0079../12.00/2013

MINISTRY OF EDUCATION
P.O BOX 622 KIGALI

Re: Permission to carry out research in Rwanda - No: MINEDUC/S&T/0115/2013

Permission is hereby granted to Mr. Jean Francois MANIRAHO, a student at the University of KwaZulu-Natal, Edgewood Campus, and South Africa to carry out research on: **"The pedagogical content knowledge of Rwandan grade 6mathematics teachers and its relations to learning "**. The research will be carried out in 20 different Schools from Kigali City, Southern and Eastern Provinces depending on the selected sample of teachers and students.

The period of research for which this permission is granted is one year, **from 2nd January, 2013 to 2nd December, 2013**. It may be renewed if necessary, in which case a new permission will be sought by the researcher.

This permission shall be cited in the final research report as follows: "Research conducted under permission No: MINEDUC/S&T/0115/2013".

Please provide Mr. Jean Francois MANIRAHO any support he may require in the course of conducting this research.

Yours sincerely,

Dr. Marie Christine GASINGIRWA
Director General,
Science, Technology and Research
Ministry of Education

Appendix D: Results of statistical tests

D1: Tests of Normality

	Kolmogorov-Smirnova			Shapiro-Wilk		
	Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
Content Connections	0.135	19	0.200*	0.959	19	0.558
Progression and linkage to other sessions	0.114	19	0.200*	0.942	19	0.287
Mathematical content construction	0.105	19	0.200*	0.957	19	0.519
Use of illustrative materials-representations	0.216	19	0.020	0.84	19	0.005
Errors and mis-conceptions are not observable	0.107	19	0.200*	0.979	19	0.933
Errors/ misconceptions not recognised, or ignored	0.331	19	0.000	0.598	19	0.000
Incorrect answers challenged individually or on class	0.093	19	0.200*	0.961	19	0.601
No feedback observed.	0.345	19	0.000	0.73	19	0.000
Direct feedback.	0.135	19	0.200*	0.969	19	0.752
Process or self-regulation feedback	0.11	19	0.200*	0.968	19	0.746
The given feedback is about task or product	0.222	19	0.014	0.794	19	0.001
Personal feedback (self)	0.154	19	0.200*	0.942	19	0.284
Feedback, excl direct	0.142	19	0.200*	0.927	19	0.155
No attempt of unpacking	0.114	19	0.200*	0.931	19	0.179
Only rules/ procedural descriptions are used to unpack content.	0.423	19	0.000	0.572	19	0.000
Unpacking methods/concepts	0.128	19	0.200*	0.918	19	0.104
Engaging Tasks	0.139	19	0.200*	0.953	19	0.449
Engagement of learners' prior knowledge	0.495	19	0.000	0.463	19	0.000

a. Lilliefors Significance Correction
*. This is a lower bound of the true significance.

D2: T-tests for practical PCK categories versus level of education (other than teacher education), variables meet ANOVA assumptions

Practical PCK	Level of education	Descriptive Statistics			T-Tests			
		N	Mean	Std. Dev.	t	df	p-value	Comment
Content Connections	D6	14	0.365	0.199	0.182	17	0.857	Not significant
	D7	5	0.346	0.235				
Progression and linkage to other sessions	D6	14	0.261	0.215	0.026	17	0.980	Not significant
	D7	5	0.258	0.170				
Mathematical content construction	D6	14	0.364	0.212	1.284	17	0.217	Not significant
	D7	5	0.222	0.211				
Errors and mis-conceptions are not observable	D6	14	0.569	0.261	0.037	17	0.971	Not significant
	D7	5	0.564	0.170				
Incorrect answers challenged individually or on class	D6	14	0.330	0.253	-0.266	17	0.793	Not significant
	D7	5	0.363	0.184				
Direct feedback.	D6	14	0.570	0.210	0.325	17	0.749	Not significant
	D7	5	0.530	0.299				
Process or self-regulation feedback	D6	14	0.367	0.226	0.545	17	0.593	Not significant
	D7	5	0.303	0.226				
The given feedback is about task or product	D6	14	0.167	0.146	**2.505**	**17**	**0.023**	**Significant**
	D7	5	0.000	0.000				
Personal feedback (self)	D6	14	0.439	0.340	-1.029	17	0.318	Not significant
	D7	5	0.605	0.186				
Feedback, excl. direct	D6	14	0.290	0.225	1.042	17	0.312	Not significant
	D7	5	0.175	0.168				
No attempt of unpacking	D6	14	0.678	0.256	-0.527	17	0.605	Not significant
	D7	5	0.748	0.258				
Unpacking methods/concepts	D6	14	0.275	0.220	1.021	17	0.321	Not significant
	D7	5	0.168	0.122				
Engaging Tasks	D6	14	0.827	0.121	1.470	17	0.160	Not significant
	D7	5	0.730	0.139				

D2a: Mann-Whitney U tests for practical PCK categories versus level of education (other than teacher education), variables do not meet Normality assumptions

Practical PCK	Level of education	Rank summary		Mann-Whitney U Tests		
		N	Mean Rank	Test Statistic	p-value	Comment
Use of illustrative materials-representations	D6	14	10.14	33.00	0.851	Not significant
	D7	5	9.60			
Errors/ misconceptions not recognised, or ignored	D6	14	9.46	27.500	0.422	Not significant
	D7	5	11.50			
No feedback observed.	D6	14	8.75	17.500	0.071	Not significant
	D7	5	13.50			
Only rules/ procedural descriptions are used to unpack content.	D6	14	10.07	34.000	0.905	Not significant
	D7	5	9.80			
Engagement of learners' prior knowledge	D6	14	9.86	33.000	0.771	Not significant
	D7	5	10.40			

D3: ANOVA Tests of the dependency of teachers' practical PCK on their teacher education (variables meet Normality assumptions)

Practical PCK	Teacher Training	Descriptive Statistics			ANOVA-Tests			
		N	Mean	Std. Deviation	F	df1, df2	p-value	Comment
Content Connections	No Training	3	0.411	0.417	0.209	4, 14	0.929	Not Significant
	Less than 1 year	7	0.396	0.176				
	1 year	3	0.278	0.255				
	3 years	3	0.335	0.107				
	>3 years	3	0.333	0.058				
Progression and linkage to other sessions	No Training	3	0.289	0.342	0.288	4, 14	0.881	Not Significant
	Less than 1 year	7	0.234	0.152				
	1 year	3	0.250	0.250				
	3 years	3	0.194	0.200				
	>3 years	3	0.367	0.208				
Mathematical content construction	No Training	3	0.566	0.158	2.910	4, 14	0.060	Significant at 10% sign level
	Less than 1 year	7	0.392	0.160				
	1 year	3	0.183	0.161				
	3 years	3	0.139	0.241				
	>3 years	3	0.267	0.208				
Errors and mis-conceptions are not observable	No Training	3	0.587	0.399	0.500	4, 14	0.736	Not Significant
	Less than 1 year	7	0.516	0.104				
	1 year	3	0.717	0.301				
	3 years	3	0.620	0.251				
	>3 years	3	0.467	0.306				
Incorrect answers challenged individually or on class	No Training	3	0.347	0.442	0.044	4, 14	0.996	Not Significant
	Less than 1 year	7	0.355	0.206				
	1 year	3	0.283	0.301				
	3 years	3	0.352	0.204				
	>3 years	3	0.333	0.153				
Direct feedback.	No Training	3	0.433	0.404	1.010	4, 14	0.435	Not Significant
	Less than 1 year	7	0.635	0.221				
	1 year	3	0.500	0.200				
	3 years	3	0.427	0.069				
	>3 years	3	0.700	0.100				
Process or self-regulation feedback	No Training	3	0.519	0.329	0.980	4, 14	0.450	Not Significant
	Less than 1 year	7	0.278	0.168				
	1 year	3	0.311	0.301				
	3 years	3	0.473	0.238				
	>3 years	3	0.267	0.058				

Practical PCK	Teacher Training	Descriptive Statistics			ANOVA-Tests			
		N	Mean	Std. Deviation	F	df1, df2	p-value	Comment
The given feedback is about task or product	No Training	3	0.114	0.103	2.015	4, 14	0.147	Not Significant
	Less than 1 year	7	0.064	0.065				
	1 year	3	0.217	0.202				
	3 years	3	0.030	0.052				
	>3 years	3	0.267	0.231				
Personal feedback (self)	No Training	3	0.306	0.529	1.374	4, 14	0.293	Not Significant
	Less than 1 year	7	0.586	0.225				
	1 year	3	0.267	0.252				
	3 years	3	0.715	0.248				
	>3 years	3	0.400	0.265				
Feedback, excl direct	No Training	3	0.466	0.336	1.680	4, 14	0.210	Not Significant
	Less than 1 year	7	0.239	0.129				
	1 year	3	0.328	0.298				
	3 years	3	0.061	0.105				
	>3 years	3	0.233	0.115				
No attempt of unpacking	No Training	3	0.800	0.265	0.277	4, 14	0.888	Not Significant
	Less than 1 year	7	0.701	0.231				
	1 year	3	0.594	0.400				
	3 years	3	0.748	0.231				
	>3 years	3	0.633	0.289				
Unpacking methods/concepts	No Training	3	0.200	0.265	0.428	4, 14	0.786	Not Significant
	Less than 1 year	7	0.228	0.167				
	1 year	3	0.339	0.307				
	3 years	3	0.161	0.151				
	>3 years	3	0.333	0.231				
Engaging Tasks	No Training	3	0.905	0.165	0.918	4, 14	0.481	Not Significant
	Less than 1 year	7	0.774	0.077				
	1 year	3	0.844	0.051				
	3 years	3	0.720	0.206				
	>3 years	3	0.800	0.173				

D3a: Kruskal Wallis Tests of the dependency of teachers' practical PCK on their teacher education (variables do not meet Normality assumptions)

Practical PCK	Teacher Training	Rank summary		Kruskal Wallis Tests			
		N	Mean Rank	Test Statistic	df	p-value	Comment
Use of illustrative materials-representations	No Training	3	7.83	1.850	4	0.763	Not Significant
	Less than 1 year	7	9.21				
	1 year	3	9.83				
	3 years	3	10.67				
	>3 years	3	13.50				
Errors/ misconceptions not recognised, or ignored	No Training	3	10.00	3.115	4	0.539	Not Significant
	Less than 1 year	7	10.64				
	1 year	3	6.50				
	3 years	3	8.83				
	>3 years	3	13.17				
No feedback observed.	No Training	3	8.67	3.689	4	0.450	Not Significant
	Less than 1 year	7	9.29				
	1 year	3	15.00				
	3 years	3	9.83				
	>3 years	3	8.17				
Only rules/ procedural descriptions are used to unpack content.	No Training	3	7.50	1.248	4	0.870	Not Significant
	Less than 1 year	7	10.21				
	1 year	3	10.67				
	3 years	3	11.00				
	>3 years	3	10.33				
Engagement of learners' prior knowledge	No Training	3	8.50	2.126	4	0.713	Not Significant
	Less than 1 year	7	10.00				
	1 year	3	11.33				
	3 years	3	8.50				
	>3 years	3	11.67				

D4: ANOVA Tests of the dependency of teachers' observable practical PCK on their teaching experience (variables meet Normality assumptions)

Practical PCK	Teacher Experience	Descriptive Statistics			ANOVA-Tests			
		N	Mean	Std. Dev.	F	df1, df2	p-value	Comment
Content Connections	2-5 years 6-10 years 11+ years	3 1 12	0.322 0.300 0.372	0.019 - 0.243	0.096	2, 13	0.909	Not Significant
Progression and linkage to other sessions	2-5 years 6-10 years 11+ years	3 1 12	0.187 0.400 0.245	0.070 - 0.223	0.398	2, 13	0.679	Not Significant
Mathematical content construction	2-5 years 6-10 years 11+ years	3 1 12	0.191 0.000 0.334	0.080 - 0.217	1.676	2, 13	0.225	Not Significant
Errors and mis-conceptions are not observable	2-5 years 6-10 years 11+ years	3 1 12	0.569 0.800 0.535	0.175 - 0.269	0.495	2, 13	0.620	Not Significant
Incorrect answers challenged individually or on class	2-5 years 6-10 years 11+ years	3 1 12	0.398 0.200 0.340	0.131 - 0.270	0.231	2, 13	0.797	Not Significant
Direct feedback.	2-5 years 6-10 years 11+ years	3 1 12	0.552 0.500 0.500	0.050 - 0.238	0.067	2, 13	0.936	Not Significant
Process or self-regulation feedback	2-5 years 6-10 years 11+ years	3 1 12	0.359 0.200 0.404	0.076 - 0.246	0.389	2, 13	0.686	Not Significant
The given feedback is about task or product	2-5 years 6-10 years 11+ years	3 1 12	0.083 0.000 0.126	0.144 - 0.143	0.427	2, 13	0.662	Not Significant
Personal feedback (self)	2-5 years 6-10 years 11+ years	3 1 12	0.459 0.600 0.440	0.408 - 0.329	0.101	2, 13	0.904	Not Significant
Feedback, excl direct	2-5 years 6-10 years 11+ years	3 1 12	0.369 0.000 0.276	0.190 - 0.230	1.014	2, 13	0.390	Not Significant
No attempt of unpacking	2-5 years 6-10 years 11+ years	3 1 12	0.720 0.700 0.701	0.119 - 0.286	0.006	2, 13	0.994	Not Significant
Unpacking methods/concepts	2-5 years 6-10 years 11+ years	3 1 12	0.280 0.300 0.225	0.119 - 0.220	0.129	2, 13	0.880	Not Significant
Engaging Tasks	2-5 years 6-10 years 11+ years	3 1 12	0.770 0.500 0.813	0.067 - 0.120	3.569	2, 13	0.058	Significant at 10% sig level

D4a: Kruskal Wallis Tests of the dependency of teachers' observable practical PCK on their teaching experience (variables do not meet Normality assumptions)

Practical PCK	Teacher Experience	Rank summary		Kruskal Wallis Tests			
		N	Mean Rank	Test Statistic	df	p-value	Comment
Use of illustrative materials-representations	2-5 years 6-10 years 11+ years	3 1 12	13.83 7.00 7.29	4.904	2	0.086	Significant at 10% sig level
Errors/ misconceptions not recognised, or ignored	2-5 years 6-10 years 11+ years	3 1 12	8.00 5.50 8.88	0.668	2	0.716	Not Significant
No feedback observed.	2-5 years 6-10 years 11+ years	3 1 12	8.67 14.50 7.96	2.000	2	0.368	Not Significant
Only rules/ procedural descriptions are used to unpack content.	2-5 years 6-10 years 11+ years	3 1 12	7.00 7.00 9.00	1.139	2	0.566	Not Significant
Engagement of learners' prior knowledge	2-5 years 6-10 years 11+ years	3 1 12	12.00 7.00 7.75	4.343	2	0.114	Not Significant

D5: T-tests for frequency of practical PCK observed in lessons, depending on gender of teacher (variables meet Normality assumptions)

Practical PCK	Gender	Descriptive Statistics			T-Tests			
		N	Mean	Std. Deviation	t	df	p-value	Comment
Content Connections	Male Female	10 9	0.338 0.385	0.157 0.250	-0.503	17	0.622	Not significant
Progression and linkage to other sessions	Male Female	10 9	0.236 0.287	0.172 0.235	-0.540	17	0.596	Not significant
Mathematical content construction	Male Female	10 9	0.202 0.466	0.217 0.101	-3.331	17	0.004	significant (1% level)
Errors and mis-conceptions are not observable	Male Female	10 9	0.552 0.584	0.275 0.199	-0.287	17	0.778	Not significant
Incorrect answers challenged individually or on class	Male Female	10 9	0.316 0.364	0.250 0.222	-0.444	17	0.663	Not significant
Direct feedback.	Male Female	10 9	0.481 0.647	0.221 0.215	-1.657	17	0.116	Not significant
Process or self-regulation feedback	Male Female	10 9	0.358 0.342	0.252 0.197	0.150	17	0.883	Not significant
The given feedback is about task or product	Male Female	10 9	0.137 0.106	0.160 0.134	0.456	17	0.654	Not significant
Personal feedback (self)	Male Female	10 9	0.361 0.617	0.272 0.310	-1.916	17	0.072	significant (10% level)
Feedback, excl direct	Male Female	10 9	0.275 0.243	0.235 0.199	0.314	17	0.757	Not significant
No attempt of unpacking	Male Female	10 9	0.638 0.762	0.236 0.265	-1.083	17	0.294	Not significant
Unpacking methods/concepts	Male Female	10 9	0.273 0.218	0.161 0.245	0.594	17	0.560	Not significant
Engaging Tasks	Male Female	10 9	0.769 0.838	0.121 0.136	-1.168	17	0.259	Not significant

D5a: Mann-Whitney U Tests for frequency of practical PCK observed in lessons, depending on gender of teacher (variables do not meet Normality assumptions)

Practical PCK	Gender	Rank summary		Mann-Whitney U Tests		
		N	Mean Rank	Test Statistic	p-value	Comment
Use of illustrative materials-representations	Male Female	10 9	10.05 9.94	44.50	0.967	Not significant
Errors/ misconceptions not recognised, or ignored	Male Female	10 9	10.45 9.50	40.50	0.671	Not significant
No feedback observed.	Male Female	10 9	12.95 6.72	15.50	0.007	Significant at 1% sig level
Only rules/ procedural descriptions are used to unpack content.	Male Female	10 9	10.65 9.28	38.50	0.494	Not significant
Engagement of learners' prior knowledge	Male Female	10 9	11.35 8.50	31.50	0.083	Significant at 10% sig level

D6: ANOVA Tests of practical PCK on school principal's lesson observations (Variables meet Normality assumptions)

Practical PCK	Principals Visits	Descriptive Statistics			ANOVA-Tests			
		N	Mean	Std. Deviation	F	df1, df2	p-value	Comment
Content Connections	often sometimes rarely	4 12 1	0.276 0.367 0.333	0.203 0.170 -	0.396	2, 14	0.680	Not significant
Progression and linkage to other sessions	often sometimes rarely	4 12 1	0.095 0.289 0.250	0.110 0.188 -	1.856	2, 14	0.193	Not significant
Mathematical content construction	often sometimes rarely	4 12 1	0.383 0.272 0.250	0.293 0.189 -	0.422	2, 14	0.664	Not significant
Errors and mis-conceptions are not observable	often sometimes rarely	4 12 1	0.476 0.566 0.750	0.286 0.223 -	0.571	2, 14	0.578	Not significant
Incorrect answers challenged individually or on class	often sometimes rarely	4 12 1	0.453 0.309 0.250	0.335 0.199 -	0.645	2, 14	0.540	Not significant
Direct feedback.	often sometimes rarely	4 12 1	0.395 0.615 0.500	0.327 0.203 -	1.338	2, 14	0.294	Not significant
Process or self-regulation feedback	often sometimes rarely	4 12 1	0.569 0.268 0.333	0.273 0.181 -	3.265	2, 14	0.069	Significant at 10% sig level
The given feedback is about task or product	often sometimes rarely	4 12 1	0.108 0.123 0.250	0.085 0.172 -	0.338	2, 14	0.719	Not significant
Personal feedback (self)	often sometimes rarely	4 12 1	0.386 0.517 0.000	0.483 0.228 -	1.497	2, 14	0.258	Not significant
Feedback, direct	often sometimes rarely	4 12 1	0.374 0.203 0.583	0.338 0.149 -	2.318	2, 14	0.135	Not significant
No attempt of unpacking	often sometimes rarely	4 12 1	0.861 0.663 0.583	0.216 0.273 -	0.973	2, 14	0.402	Not significant
Unpacking methods/ concepts	often sometimes rarely	4 12 1	0.070 0.277 0.417	0.088 0.207 -	2.308	2, 14	0.136	Not significant
Engaging Tasks	often sometimes rarely	4 12 1	0.843 0.765 0.833	0.134 0.130 -	0.600	2, 14	0.562	Not significant

D6a: Kruskal Wallis Tests of practical PCK on school principal's lesson observations (Variables do not meet Normality assumptions)

Practical PCK	Principals Visits	Rank summary		Kruskal Wallis Tests			
		N	Mean Rank	Test Statistic	df	p-value	Comment
Use of illustrative materials-representations	often sometimes rarely	4 12 1	7.25 9.08 15.00	1.988	2	0.370	Not Significant
Errors/ misconceptions not recognised, or ignored	often sometimes rarely	4 12 1	9.38 9.17 5.50	0.647	2	0.724	Not Significant
No feedback observed.	often sometimes rarely	4 12 1	6.75 9.42 13.00	1.768	2	0.413	Not Significant
Only rules/ procedural descriptions are used to unpack content.	often sometimes rarely	4 12 1	9.25 9.08 7.00	0.307	2	0.858	Not Significant
Engagement of learners' prior knowledge	often sometimes rarely	4 12 1	7.50 9.00 15.00	3.989	2	0.136	Not Significant

D7: Pearson correlations between grouped categories of teachers' practical PCK, and learning gain

Pearson's correlations		Learning Gain	Comment
Content Connections	Correlation	0.604**	Significant at 1% level
	p-value	0.006	
	N	19	
Progression and linkage to other sessions	Correlation	0.179	Not Significant
	p-value	0.464	
	N	19	
Mathematical content construction	Correlation	-0.205	Not Significant
	p-value	0.400	
	N	19	
Use of illustrative materials-representations	Correlation	0.388	Not Significant
	p-value	0.101	
	N	19	
Errors/ misconceptions not recognised, or ignored	Correlation	0.002	Not Significant
	p-value	0.992	
	N	19	
Incorrect answers challenged individually or on class	Correlation	-0.337	Not Significant
	p-value	0.158	
	N	19	
Process or self-regulation feedback	Correlation	0.065	Not Significant
	p-value	0.790	
	N	19	
Feedback, excl direct	Correlation	-0.267	Not Significant
	p-value	0.269	
	N	19	
Unpacking methods/concepts	Correlation	0.172	Not Significant
	p-value	0.480	
	N	19	
Engaging Tasks	Correlation	0.412	Significant at 10% level
	p-value	0.080	
	N	19	
Engagement of learners' prior knowledge	Correlation	-0.049	Not Significant
	p-value	0.844	
	N	19	

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

D8: Correlation between teachers' practical PCK and learning gain.

D8a

PCK item	Learning Gain		
	N	Pearson Correlation	p-value
A1: No kind of connections observed	20	-0.486*	0.03
A2: Different representations are used	20	0.148	0.533
A4: Procedure connections are used	20	0.182	0.442
A5: Prerequisites connections are observed	20	0.169	0.475
A6: Part-whole relationships are observed	20	-0.096	0.688
AT: Total connections observed	20	0.287	0.219

D8b

PCK item	Learning Gain		
	N	Pearson Correlation	p-value
B1: No linkage observed.	20	-0.237	0.314
B2: Linkage with other sessions is shown from simple to complex	20	-0.002	0.993
B3: Linkage with other sessions is shown from particular to general or vice versa	20	0.152	0.521
B6: Linkage with other sessions is shown from every day to specialize	20	0.061	0.797
BT: Total Linkage	20	0.108	0.651

D8c

PCK item	Learning Gain		
	N	Pearson Correlation	p-value
C1: No mathematical content construction through practices is observed	20	0.21	0.375
C2: Investigation by observation of the object/image through continuous variation/contrast is observed.	20	0.019	0.937
C3: Mathematical terms are used by learners to explain why the conjecture is true or false through discussions/separation.	20	0.104	0.662
C4: Verifications are done to clarify areas in which learners exhibit doubts by expressing themselves within their math vocabulary	20	-0.016	0.947
C6: Learners are encouraged to communicate mathematically while performing a task.	20	-0.402	0.079
CT: Total - Mathematics	20	-0.328	0.158

D8d

PCK item	Learning Gain		
	N	Pearson Correlation	p-value
D1: No examples and teaching aids used both verbally and practically.	20	-0.292	0.212
D2: Examples and teaching aids for lesson concretization/representation are verbally cited	20	0.305	0.191
D3: Visual aids/representations are used	20	0.017	0.944
D4: Manipulative teaching aids/representations are used	20	0.092	0.7
DT: Total aids used	20	0.209	0.377

D8e

PCK item	Learning Gain		
	N	Pearson Correlation	p-value
E1: Errors and misconceptions are not observable	20	0.283	0.227
E3: Errors and misconceptions are recognized but ignored and incorrect answers are simply interpreted/corrected	20	0.016	0.946
E4: Incorrect answers from risen misconception/ errors have been individually challenged	20	-0.448*	0.048
E5: Errors and misconceptions are shared and discussed with learners.	20	-0.015	0.949
ET: Total errors observed	20	-0.422	0.064

D8f

PCK item	Learning Gain		
	N	Pearson Correlation	p-value
F1: No feedback is observed	20	-0.15	0.529
F2: Direct feedback is given	20	0.015	0.95
F3: Indirect feedback is given	20	0.038	0.872
FT: Total focus feedback	20	0.045	0.852

D8g

PCK item	Learning Gain		
	N	Pearson Correlation	p-value
G1: No feedback is observed	20	0.107	0.653
G2: The given feedback is about task or product.	20	0.205	0.385
G3: Feedback given is about process.	20	-0.389	0.09
G5: The personal feedback (self) is given.	20	0.029	0.904
GT: Total "What" feedback	20	-0.171	0.472

D8h

PCK item	Learning Gain		
	N	Pearson Correlation	p-value
H1: Any attempt to unpacking the methods/concept is observed	20	-0.272	0.246
H2: Only rules/procedural descriptions are used to unpack the methods.	20	0.356	0.123
H3: More than one methods/ways are shown to unpack the methods but not followed by their comparison/analysis	20	-0.248	0.291
H5: Only definitions/conceptual are used to unpack the concepts.	20	0.059	0.805
HT: Total Unpack	20	0.162	0.494

D8i

PCK item	Learning Gain		
	N	Pearson Correlation	p-value
I1: Tasks to clarify the concept and alternative strategies were not in place	20	-0.409	0.073
I2: Posed problems have been worked on by teacher-learner direct interaction.	20	0.208	0.379
I4: Tasks are worked on individually or in groups, checked and shared	20	-0.248	0.292
IT: Total engaged tasks	20	0.053	0.823
J1: Prior knowledge has not been engaged.	20	-0.161	0.497
J3: Learners' prior knowledge noted and used as foundation of the new topic to learn.	20	-0.091	0.704
**. Correlation is significant at the 0.01 level (2 tailed).			
*. Correlation is significant at the 0.05 level (2-tailed).			

Appendix E: Description of tasks from learner test

The test consisted of 40 questions and learners were supposed to answer to all questions in the tests. The questions were in multiple choice format, meaning that from the choice of four answers, a learner had to select the only correct answer. Two answered examples were given at the beginning of the test. Below, a short description of each question in the test is provided.

Question one: This task showed a section of a standard number grid with missing numbers. Learners were asked to fill in one particular missing number.

Question two: A situation in which someone was filling a given number of crates and learners were asked to find the number of bottles packed, given the number of bottles filling a crate.

Question three: This question was about writing numbers given in words in numerical form.

Question four: Learners were required to add two numbers, one of four digits and one of three digits. The task involved regrouping.

Question five: A situation in which someone bought a given number of ½ kg packets of sugar, and learners were asked to give the total mass of sugar bought.

Question six: A situation where members of a football team sold different number of tickets and learners were supposed to indicate the greatest difference between the number of tickets sold.

Question seven: A seven digits number was given and learners were asked to name the position of one of the digits.

Question eight: Two expressions involving multiplication of two numbers were given as equal. On one side of the equal sign, the two numbers were given, and on the other side only one number was given and learners were asked to fill in the missing number for the equality to hold true.

Question nine: A situation in which a number of children were seated in a given number of equal rows. The learners had to determine the number of children in each row.

Question ten: A teacher asked each learner in his/her class to name his/her favorite sport. The information was represented in a bar graph, and learners were asked to determine the number of learners in the class.

Question eleven: Learners had to choose the one fraction greater than $\frac{1}{2}$.

Question twelve: Learners had to choose the fraction equivalent to $\frac{2}{8}$.

Question thirteen: Learners were given an equality. On one side, a six digits number was given, and on the other side the numbers to add up were given in which an empty space was left to fill in the missing number to make the equality true.

Question fourteen: The question was about the subtraction of two decimal numbers.

Question fifteen: Multiplication of a three digit and a four digit number.

Question sixteen: Addition of two fractions.

Question seventeen: Division of a six digit by a two digit number.

Question eighteen: A world problem question in which learners were asked to find out the age of a child whose age was a given unit fraction of his/her grandmother's age.

Question nineteen: A numerical expression with brackets had to be simplified to one number. The answer would however have been the same without the brackets.

Question twenty: Different geometrical shapes were given and learners were asked to name one of these.

Question twenty one: A visual pattern of squares was given. The learners were asked to predict the number of squares at a particular later step.

Question twenty two: The question illustrated a function using set representation of domain and range. The given pairs of numerical values suggested a function of dividing by a single digit number. Learners had to fill in a missing value in the domain.

Question twenty three: In this question, a pattern of matchsticks figures was given. Learners were asked to predict the number of matchsticks at a particular later step.

Question twenty four: A drawing of a 3D geometrical shape was given, and learners had to determine the numbers of faces.

Question twenty five: Different geometrical shapes were given and learners were asked to indicate the number of circles amongst them.

Question twenty six: Different geometrical shapes were given and learners were asked to identify the one with a specified type of symmetry.

Question twenty seven: Part of an algebraic sequence was given, and learners had to provide the missing item.

Question twenty eight: Learners were given a sketch representing some cubes that had been stacked together. Sketches showing different side views of such an object. Learners had to identify which of the sketches showed the left side view of the given object.

Question twenty nine: Learners were presented with images of different containers with volume indicated on the labels. The relative size of the images did not correspond to the labels. Learners were asked to order the containers according to size.

Question Thirty: Learners had to find the highest common factor of two three digit numbers.

Question Thirty one: The mode of a small data set had to be determined.

Question Thirty two: Different units of measurement were given and learners were asked to choose a unit of measurement s/he should you use to measure the length of a school enclosure.

Question Thirty three: This question was about units of time. Learners were asked to work out the numbers of hours which are equivalent to a three digits number of minutes.

Question Thirty four: This question was about units of mass. Learners were asked to convert a given one decimal number from kilograms into grams.

Question Thirty five: A square shape was divided into smaller squares, some inside squares were shaded, and learners were asked to work out the area of the shaded region, given the side length of the smaller square.

Question Thirty six: Learners were given six test results recorded by a particular teacher in his/her class. They were asked to work out the mean.

Question Thirty seven: Learners were asked to determine the time in a city 2 hours behind.

Question Thirty eight: A drawing showed a scale with a boy standing on it. The learners were asked to determine the boy's mass display on the scale. The answer options which were given included units of measurements different from mass measurements.

Question Thirty nine: Learners were given a data set and were asked to work out the median.

Question Forty: This question presented a situation in which a person had a bag with a given number of two types of sweets in it. Learners had to determine the number of one type of sweets in that bag, based on the chances given to choose such sweets from the bag.

Appendix F: Description of tasks from teacher test

The test consisted of 24 questions with a varying number of sub-questions, and teachers were supposed to answer to all questions in the tests. The questions were in multiple choice format and there was a possibility for one question to have more than one answer. Below, a short description of each question in the test is provided.

Question one: Determine the number of decimal numbers which are between two given decimal numbers.

Question two: A situation where a teacher was assessing his/her learners' work from the day's lesson on multiplication and noticed that one of the learners invented an algorithm that was different from the one taught in class. Teachers were required to identify if the algorithm worked or not.

Question three: This question illustrated a situation where someone was planning mini-lessons for learners focusing on particular difficulties that they were having with adding columns of numbers. To target her/his instruction more effectively, that particular person worked with groups of learners who were making the same kind of error and looked at what learners tended to do. Three learner mistakes were presented to teachers who had to determine which learners had the same kind of error.

Question four: A situation in which four boys were working together on a problem of arranging the decimal numbers from smallest to the largest. The answers were given to teachers together with each learner's statement of reasoning. Teachers were asked to indicate which statements were true and which ones were false.

Question five: Teachers were presented with a learner's work reflecting difficulty on solving percent tasks. Teachers were asked to indicate which exercises were correct and which exercise were not. Then, teachers were given three additional percentage tasks and were asked to indicate the ones which that particular learner was likely to get correct using his procedure.

Question six: Teachers were presented with a learner's work, using to demonstrate his/ her assertion on equivalent fractions. The teachers were given explanation of that particular learner's reasoning and were asked to choose the best explanation why that learner's reasoning was incorrect.

Question seven: This question illustrated a situation in which someone asked his/her learners to write expressions that, when evaluated, give an answer of 10. Some expressions were incorrect due to errors in order of operations. Teachers were asked to determine if each expression, as it was written, could be equals to 10.

Question eight: In this question, a pattern of products was given. Teachers were asked to find the answer to a product later in the pattern.

Question nine: A situation where learners compared strength of lemonade made by two people different proportions of lemon juice and water. Different reasoning statements reflecting learners' thinking were given to teachers, who were asked to determine the ones which were correct.

Question ten: Concerned number patterns using sketches of stacked cubes, increasing with steps. Learners were asked to develop a rule to predict the number of cubes at any step and came up with different statements. Teachers were asked to determine which statements were correct and incorrect.

Question eleven: A drawing of a 3D geometrical shape was given, and teachers had to determine the numbers of faces.

Question twelve: Different geometrical shapes were given and teachers were asked to identify the one with a specified type of symmetry.

Question thirteen: Teachers were given a sketch representing some cubes that have been stacked together. Sketches showing different side views of such an object. Teachers had to identify which of the sketches showed the left side view of the given object.

Question fourteen: Teachers were presented with a learner's work about identification of right angles. They were then presented with geometrical figures and asked to identify that learner's most likely answers to that exercise using his/her error pattern.

Question fifteen: The teachers were given different statements reflecting properties of geometric figures and were asked to confirm if the construction of these figures would be possible or not.

Question sixteen: A square shape was divided into smaller squares, some inside squares were shaded, and teachers were asked to work out the area of the shaded region, given the side length of the smaller squares.

Question seventeen: Teachers were presented with a sketch of a paved rectangular-shaped swimming pool which a walkway all the way around. Teachers were to determine the area of the walkway.

Question eighteen: Teachers were asked to identify if some grade six learners' calculations of perimeter were correct or not.

Question nineteen: A situation in which a person had a bag with a given number of two types of sweets in it. Teachers had to determine the number of one type of sweets in that bag, based on the chance of choosing such sweets type from the bag.

Question twenty: Learners graphical representation and statistical summary of data on favorite music was given among the graphical representations, the teachers were asked to indicate the appropriate graphical representations for a learner to answer the question under investigation.

Question twenty one: Teachers were again given some statistical representations, and asked to indicate which one(s) was/were appropriate for a learner to answer the question under investigation.

Question twenty two: This requested teachers to imagine a situation in which two second-grade learners created the given representations of the number of teeth lost by their classmates. Teachers were asked to choose the representation which could be preferable in case of teaching the concepts of center and spread.

Question twenty three: Teachers were presented with a table resulting from learners conducted survey. Learners had written statements based on the information in the table and teachers were to indicate if those learners' statements were accurate or not.

Question twenty four: Learner had collected data related to travel time and displayed it graphically to show how much time the majority of learners travel. Teachers were asked to indicate the representation most appropriate for answering that question.