Readiness of learners to study Space, Shape and Measurement in Mathematical Literacy: case studies at two Durban schools

By

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ABSTRACT

This dissertation reports case studies conducted at two secondary schools in Durban. With the introduction of the Further Education and Training (FET) curriculum in 2006, learners who terminate their study of Mathematics at the end of grade nine will have to study Mathematical Literacy, an applications-based mathematics course.

In South Africa the Mathematics results at the grade twelve exit examination are generally poor and learners are known to underachieve in the field of geometry. This study was aimed at determining the readiness of learners in studying geometry under the Learning Outcome, Space, Shape and Measurement in the Mathematical Literacy curriculum.

Questionnaires were administered to grade nine learners who had elected not to study, or were excluded from studying, Mathematics from grade ten onwards. Data was sought to determine what factors influence the decision to discontinue mathematics, and to gauge attitudes to the study of mathematics. Learners were given a test to measure geometry skills and knowledge that they ought to have acquired by the end of the senior phase in geometry. Data was also obtained from focus group interviews with both of learners and educators.

This study indicates that learners do not continue mathematics because of the difficulties they experience. Furthermore they do not have the requisite skills and knowledge in Space, Shape and Measurement to cope with the Mathematical Literacy curriculum. Nevertheless, the attitudes of learners to the study of this new subject are positive. It is suggested that educators need to conduct baseline assessments to determine learners' abilities so as to plan appropriate revision measures before continuing with the teaching of Mathematical Literacy.

PREFACE

The work described in this thesis was carried out in the School of Science, Mathematics and Technology Education, University of Kwa-Zulu Natal, from March 2004 to April 2006, under the supervision of Mrs. Sally Hobden (Supervisor).

This study represents original work by the author and has not otherwise been submitted to any tertiary institution in any form for any degree or diploma. Where use has been made of the work of others, it is duly acknowledged in the text.

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I dedicate this study to my wife, Merle and my sons, Mishlin and Kaylin. Thank you for being so supportive and understanding, especially for the long hours that I spent hibernating in the study. I also thank you for being my inspiration to soldier on during difficult times.

CHAPTER ONE

In this chapter I give a brief motivation for this study. The introduction of Mathematical Literacy into the Further Education and Training curriculum is viewed against the background of achievement in mathematics by South African learners. Matriculation examination results and the Third International Mathematics and Science Study are used to assess the state of mathematics education.

1.1 BACKGROUND TO THIS STUDY

The South African education system is in a state of transition. At present learners are awarded a qualification referred to as a senior certificate once they have successfully completed the grade 12 examination. Candidates must write examinations in at least two languages and four other learning fields called subjects. Except for the language of instruction, none of the subjects are compulsory. Subjects are chosen in grade 10, the beginning of the senior secondary phase. In particular, a learner may discontinue the study of mathematics. However substantive changes are being effected as educational reforms have followed from political renewal.

In the new curriculum, the senior secondary phase is referred to as the Further Education and Training band (FET). The National Senior Certificate (NSC) is the qualification that is awarded upon the completion of grade 12. In order to achieve the NSC learners must offer two languages (one being the language of learning and teaching), Life Orientation, Mathematics or Mathematical Literacy and three other subjects. Mathematical Literacy is a new subject being introduced into the curriculum; and it must be studied by all learners who do not study Mathematics. Statistics reveal that, from 2000 to 2003, a constant 42% of learners countrywide did not include mathematics in their curriculum (Mukwevho, Khosa, & Kgobe, 2003).

In some schools, mathematics is compulsory up to grade 12. In others, learners may choose courses which may or may not include mathematics. Generally mathematics is included in science or commerce-based packages. Learners are usually advised not to continue with mathematics if they have been unsuccessful in the subject Mathematical Literacy, Mathematics, and Mathematical Sciences (MLMMS) in grade nine. The examination results are used as criteria to place learners into courses but the final decision is left to the parents.

The final mark in MLMMS in grade 9 is derived as follows: 75% from informal assessments such as assignments and projects; and 25% of the score comes from supervised Continuous Tasks for Assessment (CTA). Until 2004, 10% of the CTA was tested under examination conditions; subsequently the examination aspect has become optional. Furthermore, in line with national policy, marks are adjusted upwards to allow learners to pass into grade 10. Parents and learners are arguably misled into believing that the latter will cope comfortably with mathematics in grade 10. As mathematics becomes more formalized in grade 10, with the introductory topics being abstract and challenging, performance becomes less satisfactory. A learner who struggles in grade ten is likely to develop a negative perception of the subject, and this in turn may lead to worse results. It must be borne in mind that at present 50% of the total mark in grade 10 comes from the final examination and this proportion increases to 75% in grade 11. From this one may conclude that good results in grade nine are not necessarily maintained in grade 10 and beyond.

Over the years learners have been performing poorly in mathematics in the senior certificate examination. A summary of results in mathematics in the grade 12 examination nationally over an eight year period is provided in Table 1.1. (Mukwevho et al., 2003). Although since 2001 there has been a steady increase in the percentage of candidates that are passing, there is still cause for concern. If one looks at the number of successful candidates in relation to the total writing the senior certificate examination from 2001 to 2003, the respective percentages are as follows: 27.4%; 33.0% and 34.5%. In 1990 when education department were segregated in terms of race the pass rates in mathematics were as follows: Whites 97%; Indian 76%; Coloured 74%; and African 15% (Reddy, 2006a). With migration of learners to schools they were previously excluded from, such racially-based statistics are no longer available.

Table 1.1 Mathematics results in the senior certificate examination from 1996 to 2003.

Year	Total number of candidates	Number who wrote mathematics	Percentage who wrote Mathematics	Number who passed mathematics	Percentage of candidates who passed
1996	518 077	214 720	41.4	106 324	49.5
1997	538 189	252 618	46.9	116 938	46.3
1998	552 862	279 702	50.1	117 827	42.1
1999	511 474	281 304	55.0	122 225	43.4
2000	489 941	284 017	58.0	106 256	37.4
2001	449 371	263 945	58.7	123 149	46.7
2002	443 821	260 898	58.8	146 446	56.1
2003	440 267	258 323	58.7	151 905	58.8

In addition, the Third International Mathematics and Science Study (TIMSS), coordinated in South Africa by the Human Sciences Research Council, showed the dire state of mathematics in this country (further discussion follows in the next chapter). However, what is of consequence is that the citizens of a developing democracy need to be 'mathematically literate' in order to participate in the global community of nations.

Prior to the promulgation of the SA Schools Act (84 of 1996), school attendance was not compulsory for all South African children. Those with no access to formal education, or access only to poor quality education, became further marginalized as a consequence of illiteracy. The inability to read and write in English and /or Afrikaans, languages of communication in commerce and industry, meant that certain segments of the population were disadvantaged in terms of employment. Private sector initiatives were set up to address the issue of literacy.

Advancements in science and technology have impacted strongly on life and living in the 21st century. Interactions in the workplace, at home and generally within society are becoming increasingly mathematically-orientated. Against this background comes the introduction of Mathematical Literacy into the curriculum. This field of study, drawing from real-life experiences, reinforces the role of mathematics in the modern world (Department of Education, 2003).

Curriculum reforms in South Africa have seen the introduction of Curriculum 2005 (C2005), incorporating an outcomes-based approach. The reference to the year in the title meant that the first group of learners experiencing the curriculum would have exited the curriculum in 2005. However, after being put into practice, the curriculum was reviewed and subsequently revised. The Revised National Curriculum Statement (RNCS) was introduced in the foundation phase (grades one to three) in 2001 and will gradually be phased into the other grades, up to grade nine by 2007. In the interim, in the senior secondary phase (grades ten to twelve), subsequently renamed the Further Education and Training band (FET), the decision was taken to maintain the current curriculum. In the latter, learners were given the option of discontinuing Mathematics in grade ten. Table 1.1 shows the percentage of senior certificate candidates who have written the Mathematics examination. Approximately 40% of matriculation candidates do not study mathematics. This raises the question as to why learners have opted out of mathematics, that is, why do learners not continue the study of mathematics when given a choice. Finding answers to this question may help to inform educators teaching Mathematical Literacy.

1.2 MATHEMATICAL LITERACY

Mathematical Literacy, also referred to as numeracy in other countries, is defined as the knowledge and skills that students require to understand and use mathematical ideas and techniques in the real world. In a submission to the Department of Education on the Draft Curriculum Statement on Mathematical Literacy, the Association for Mathematics Education in South Africa (AMESA) defined Mathematical Literacy as "the ability to read, write and engage with information and situations that are numerical in nature and mathematical in structure" (Hallendorff, 2003, p. 11). Numeracy involves more than arithmetic computations. It is the type of mathematics that is required for everyday use. The Tasmanian Department of Education makes the following reference to numeracy: "to be numerate is to have and to be able to use appropriate mathematical knowledge, understanding, skills, intuition and experience whenever they are needed in everyday life" (Stoessiger, 2003, p. 2).

A particular interest of mine is geometry and I chose to investigate how this topic will unfold in Mathematical Literacy. Geometry constitutes part of the third Learning Outcome (LO 3) and therefore the focus is on this learning outcome in this study. A learning outcome (LO) is defined as "...a statement of an intended result of learning and teaching. It describes knowledge, skills and values that learners should acquire by the end of the Further Education and Training band" (Department of Education, 2003, p. 7). In the National Curriculum Statement for Mathematical Literacy, LO 3 entitled "Space, Shape and Measurement" states the following:

> The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.

An assessment standard (AS) is an indication of what a learner is supposed to know at a particular level. "They embody the knowledge, skills and values required to achieve the Learning Outcomes" (Department of Education, 2003, p. 7). Assessment standards ensure that there is progression relating to a particular concept from one grade to the next. For LO3 there are six assessment standards each at grades 11 and 12 and five at grade 10.

The assessments standards for grade 10 are as follows:

1. Solve problems in 2-dimensional and 3-dimensional contexts by
    - Estimating, measuring and calculating
    - Checking values for solutions against the contexts in terms of suitability and degree of accuracy
2. Convert units of measurement within the metric system
3. Draw and interpret scale drawings of plans to represent and identify views
4. Solve real-life problems in 2-dimensional and 3-dimensional situations by the use of geometric diagrams to represent relationships between objects
5. Recognize, visualize, describe and compare properties of geometrical plane figures in natural and cultural forms

(Department of Education, 2003, p. 7).

My experience as an educator informs me that learners have most difficulty in grasping spatial concepts in the senior secondary phase of education. Problems are experienced in geometry, especially from grade 10 onwards. It is found that shortcomings higher up stem from lack of knowledge and skills that ought to have been acquired in the intermediate and senior phases (De Villiers, 1997). Geometry has a highly structured nature; a thorough understanding of basic concepts is required before progression to more advanced study is possible. This is supported by the Van Hiele theory which is discussed in chapter two. In this study I will attempt to assess what skills and knowledge pertaining to geometry the learners may bring to the Mathematical Literacy classroom.

1.3 OVERVIEW OF THIS STUDY

This study attempts to preview the mathematical conceptual knowledge and problems-solving skills that grade nine learners have with respect to space, shape and measurement. It must be borne in mind that the focus is on learners who are not studying mathematics because they have elected not to do so, or as a consequence of their unsatisfactory previous performances. Another challenging factor for both educator and learner is related to the affective aspect – the attitudes to mathematics which learners carry with them. Does the learner have the motivation to assimilate mathematical concepts, more especially geometry, and the inclination to employ them in seeking solutions to real-life problems? In other words, what attitudes do learners have towards geometry?

Mathematical Literacy educators need to have some knowledge of the learners they will be receiving, so that they may be adequately prepared. Can they assume that learners have internalised the basic knowledge and skills taught in previous grades? Do they have to assess learners before embarking on an instruction programme? To what extent do learners need to be motivated to take a keen interest in the study of Mathematical Literacy?

The study was conducted at two secondary schools in Durban. A motivating factor in choosing these schools was that 50% or more of the learners at these schools do not continue mathematics in the senior secondary phase. Grade 9 learners who were certain of not offering mathematics in grade 10 were involved in the survey. They were asked to complete a questionnaire (Appendix A) and also do a test (Appendix B) which assessed understanding and application of grade 9 MLMMS concepts. In their grade ten year, two groups of learners, one from each school, participated in focus group interviews to offer insights into learners' attitudes to mathematics. I also conducted focus group interviews with educators to draw out issues pertaining to mathematics.

In summary this study sought answers to the following questions:

1. Why do learners not continue mathematics in grade 10?
2. (a) What basic geometric and measurement skills do grade 9 learners, who do not continue mathematics in grade 10, have?

   (b) What attitudes do these learners show towards geometry?

I believe that answering these questions will provide insight into why some learners did not choose Mathematics, and now that they have that choice overridden, what knowledge, skills and attitudes in the field of shape, space and measurement they will bring into the Mathematical Literacy classroom. These insights may inform the teaching of Mathematical Literacy from 2006 onwards.

In Chapter 2 I review literature related to this study; while the methods that were used are discussed in chapter 3. The data is analysed in chapter 4, and the findings and suggestions are found in chapter 5.

CHAPTER 2

REVIEW OF THE LITERATURE

In this study an attempt is made to identify factors that deter learners from continuing Mathematics in grade ten where it is optional. More specifically, research is focused on the field of geometry to determine whether it acts as a barrier to the study of mathematics; and to try and infer what implications this may have for the study of Space, Shape and Measurement in the Mathematical Literacy curriculum.

In this chapter the status of mathematics education in South Africa is reviewed to determine if there are problems in terms of learner performance. The TIMSS study is used to inform this point of view. Attitudes to Mathematics are examined to establish whether this is a factor affecting results. There is an overview of geometry education as practised both internationally and within South Africa; and the van Hiele theory is reviewed as a theoretical platform for the study of geometry. Mathematics education is viewed in terms of the out-going and new curricula and finally, an effort is made to place Mathematical Literacy in context.

2.1 MATHEMATICS EDUCATION IN SOUTH AFRICA

One way of evaluating the state of mathematics education in South Africa is by assessing the matriculation (grade 12) examination results. On releasing the 2003 results, the then-minister of education, Professor Asmal reported that the mathematics results had improved from 56.1% to 58.8%. ("Professor Asmal on matric results," 2003). However, at the end of 2004, the new minister of education, Mrs. N. Pandor pointed out that the pass rate in mathematics had decreased to 57.2%, but noted that the number of passes on the higher grade had increased (Pandor, 2004).

A closer look at the statistics of the 2004 national examination results reveals the following. Of the 467 985 candidates, 276 094 wrote mathematics, a percentage of 59%. There were 156 795 successful candidates, a percentage of 56.7%. This implies that only 33.6% of the candidates in 2004 completed matriculation with success in mathematics. The figures for the three preceding years were 34.5%; 33% and 27.4% respectively (Masehela, 2005). It must also be borne in mind that in order to assist candidates to pass, marks are inflated in some cases. Furthermore, these figures also do not indicate that some candidates do not pass the subject on the grade attempted – standard grade candidates may pass on the lower grade.

In 2004, only 24 143 candidates passed mathematics on the higher grade, a percentage of 15.4% (Masehela: 2005). There is no indication of the types of schools these candidates attended but it seems likely, given the history of this country, that they are most commonly from ex model C schools rather than previously and possibly presently disadvantaged ones. Masehela (2005) also draws attention to the fact that although more females attempt mathematics than males, the pass rate for the latter is higher. Reddy (2006a) also notes that African females experience the greatest discrimination as far as mathematics education is concerned. This indicates that males will continue to occupy traditionally male-dominated fields such as engineering. In addition to matriculation data, studies at lower grades reveal a bleak picture. The Quality Learning Project (QLP) results reported by Kanjee (cited in Taylor et. al 2003) indicate that grade 9 and grade 11 learners performed very poorly in mathematics. "Generally, learners demonstrated poor understanding of concepts in all topics of the mathematics syllabus, especially geometry (Taylor et. al, 2003, p. 43). An intervention programme by the National Department of Education was introduced in 1994. Across the nine provinces, 102 schools with the potential to excel were selected for the Dinaledi project. These schools were provided with additional and effective material and human resources as well as support to improve learning and teaching in mathematics and science. There have been problems relating to the selection of the schools, and the project still has to be independently evaluated (Reddy, 2006a), but this intervention is a significant start in addressing the crisis.

In 2000, it is estimated that 20% of South African schools did not offer mathematics and science at the grade 12 level. According to the Centre for Development and Enterprise (CDE), those who could have passed did not enrol, and those who could have passed higher grade attempted it on the standard grade (Blaine, 2004). How it was determined that these learners would have passed is open to debate, but the CDE went on to describe the state of science and mathematics education in South Africa as a national crisis. TIMSS gives a good indication of where SA stands in the international community.

2.2 TIMSS

The Third International Mathematics and Science Study (TIMSS), subsequently renamed Trends in International Mathematics and Science Study, was conducted in 41 countries in 1994/5. The study comprised 5 components.

1. A comparison of curricula and material resources such as written texts and audio visual equipment (Bos & Köller,1998)
2. An achievement study with tests which comprised multiple choice questions, short answer questions, and questions requiring multiple step solutions (Howie, 1998). In the mathematics test there were 13 items on measurement, and 23 items on geometry.
3. Interviews with principals regarding organization of the school and surveys with teachers enquiring about lesson planning and preparation, as well as participation in professional activities
4. Case studies which were conducted in Japan, United States and Germany where contextual factors such as home environment, youth culture and socio-economic status were taken into account
5. Video recording of lessons in the countries referred to above (Bos & Köller, 1998).

The study was confined to three groups: 9 year olds (in grades 3 and 4); 13 year olds (in grades 7 and 8); and those in their final year of study in secondary school (Howie, 1998). Achievement tests were devised by mathematics educators working in conjunction with TIMSS National Research and against the background of curriculum frameworks encompassing content, performance expectations and perspectives. Achievement tests had a duration of 90 minutes, assessing five content areas namely fraction and number sense, measurement, data representation, analysis and probability, geometry and algebra (Howie, 1998).

In South Africa the study was coordinated by the Human Sciences Research Council (HSRC, 2000). The 9 year olds were excluded, as a language problem was foreseen. Altogether 300 schools participated, representing all previous departments of education; and both private and public schools. There was an 85% response rate made up of 9792 13 year olds and 2757 final year learners (Bos & Köller, 1998).

South African learners in grades 7 and 8 performed poorly, being ranked last of 41 countries. They obtained a score of 348 out of 800, compared to an international mean of 484. Grade 12 learners performed no better, being placed last of 21 countries. The latter achieved a mean of 352, 142 points below the international mean. Furthermore the study revealed that the highest-achieving learners in South Africa were on par with the average learners in Singapore, the country which was ranked first; and the average performers in South Africa were below the lowest-achievers in all but four countries. Internationally male learners produced better results than females but in South Africa no significant difference was found.

The study was repeated in 1998. TIMSS-Repeat or TIMSS-R gave countries that had participated previously the opportunity to track trends and other countries to compare mathematics and science education in their countries to international standards. Once again the Asian bloc countries – Singapore, Korea, Hong Kong and Japan performed very well. South Africa on the other hand obtained a mean score of 275 in comparison to the international mean of 487, lower than Morocco and Tunisia, the two other African countries that participated, and other developing countries such as Malaysia and Chile (Howie, 1999). The study was conducted for the third time in 2003. Six African countries participated in TIMSS 2003: Egypt, Tunisia, Morocco, Botswana, Ghana and South Africa. South Africa obtained a national mean of 264 – the lowest score once again – in comparison to the international mean of 467. An analysis done in terms of the previously racially-segregated education departments, shown in Table 2.1 (Reddy, 2006a) clearly illustrates that most difficulties are experienced by African learners.

Table 2.1 South African TIMSS results in terms of previously segregated departments

Education Department	TIMSS mean
House of Assembly (HOA) : Previously exclusively White	468
House of Delegates (HOD) : Previously predominantly Indian	366
House of Representatives (HOR):Previously predominantly Coloured	314
Department of Education and Training (DET) : Previously African	227

It was found that scores in TIMSS and TIMSS-R were low in almost every topic, with the lowest achievement in algebra – a score of 293 out of 800 (Howie, 1999). However, the 2003 study indicated that at both grade 8 and 9 levels, learners performed best in measurement (305 out of 800) and data handling (312 out of 800), and were weakest in geometry with a mean score of 257 out of 800 (Reddy, 2006b).

South African learners are really not conscious of their poor performance as between 74 and 80% of grade 7 and 8 learners felt that they had achieved good results. Grade 12 learners also had an incorrect perception of their performance. An interesting fact is that while learners in countries such as Japan, Korea and Hong Kong felt that hard work is a prerequisite for good performance (Bos & Köller, 1998), 76 % of South African learners believed that good luck is needed to achieve good results (HSRC, 1997).

Researchers at the HSRC (HSRC, 1997) have advanced seven possible reasons for the poor performance of South African learners.

1. Poor socio-economic background: South Africa is slowly emerging from an extremely difficult past and the vestiges of apartheid are still evident. A large part of the population (about 40%) remains unemployed; African citizens are still engaged in unskilled labour.

2. Insufficient or no resource materials: This follows from the fact that in the apartheid era, schools were controlled by different education departments when there were major discrepancies in the allocation of funds. Many rural schools, previously administered by the Department of Education and Training (DET) still do not have electricity or tap-water.
3. Shortage of qualified and motivated teachers: A generation has been "lost" in the pursuit of liberation. The motto "liberation before education" contributed to many marginalized learners exiting the education system prematurely. Furthermore the white apartheid government under the leadership of Dr. Verwoerd believed that Africans should be trained for the labour markets and hence did not require education in mathematics and science.
4. Negative peer influence: As the majority performs poorly, there is no motivation to excel. Learners feel pressurised to keep in line with their peers.
5. Perception that mathematics is difficult: Learners are aware that those who have preceded them, experienced difficulties with mathematics and have failed to produce good results. As a consequence they believe that the subject matter is beyond their capabilities.
6. Learners do less homework in mathematics and science: This is related to the fact that they experience less success in these subjects and hence feel de-motivated to complete homework exercises.
7. Instruction is not done in the learner's home language: South Africa has 11 official languages. According to Statistics South Africa (StatsSA), English is spoken by 13% of the population but is the medium of instruction in the majority of South African secondary schools. This places a tremendous burden on learners whose second or perhaps third language is English. Only 21% of learners wrote the TIMSS test in their home language (HSRC, 1997).

In addition to the factors above, the HSRC (HSRC, 1997) also determined that about 27% of South African teachers have no formal qualifications in mathematics, were trying to cope with average class sizes of 50, and had to do more administrative duties than those in other countries.

A study conducted by Chacko (2003) in the Limpopo Province which had the lowest mean score of 226 in TIMSS-R, revealed that in township schools 75% of mothers of learners and 60% of fathers are unemployed or engaged in unskilled labour such as domestic work, mining or gardening. A systemic evaluation study conducted by the KwaZulu Natal Department found that many learners in primary school were denied access to quality education as a result of unfavourable home circumstances – parents are not able to provide basic resources such as reading materials. In the city, 75% of the fathers and 70% of mothers are in professional careers. Studies have shown that there is a positive correlation between parents' education and job status, and the achievement of their children.

On the basis of the TIMSS results, it may be concluded that mathematics education in South Africa is in a state of crisis. Furthermore, in a survey involving 34 000 grade 6 learners from 1000 public schools, it was found that learners are not achieving the expected outcomes: 50% in natural science, 60% in the language of learning and 80% in mathematics. The study was done by the Department of Education as part of its ongoing systemic evaluation (Pandor, 2006). Learners' attitudes to the study of mathematics are now reviewed to determine to what extent they influence performance.

2.3 LEARNERS' ATTITUDES TO MATHEMATICS

Maree (2006) points out that insufficient attention is paid to learners' interest in mathematics although achievement in secondary school gives a good indication of possible success at the tertiary level. Mathematics requires problem-solving skills that many other school-based learning areas do not inculcate. Hammouri, (2004, p. 241) says that achievement in mathematics involves "a complex and dynamic interaction between cognitive, affective and motivational variables . There are interrelated variables that impact upon performance. Attitudes and perceptions are two of them. Negative attitudes may arise from parents who have had unpleasant experiences with mathematics themselves, peers who are not achieving the results they ought to, and teachers who are underqualified and only partially competent. Early negative experiences shape perceptions that may persist throughout life. On the other hand as confidence improves and learners feel less anxious, performance improves (Geldenhuys, 2000).

One of the key factors related to attitude is mathematics anxiety. Geldenhuys (2000, p. 24) observes that mathematics anxiety "has been used to describe the panic, helplessness, paralysis and mental disorganization that arise among some people when they are required to solve a mathematical problem". Anxiety developed at an early age leads to a decline in achievement and may result in dropout in secondary school (Thijsse, 2004). Mathematics anxiety is a chronic condition that may contribute to an increased heart rate and perspiration, an inability to communicate and failure to recall any mathematical information (Geldenhuys, 2000).

Teachers have an important role to play in allaying students' fears of mathematics. Positive encouragement by educators can change attitudes, but Thijsse (2004) points out that teachers themselves may experience anxiety. Teachers who are forced to teach mathematics because of insufficient qualified personnel at their school may not feel confident and this unease may be detected by the learners.

The culture of learning at schools also has a profound impact on performance in mathematics. The lack of free and compulsory education for African learners and the discrepancies that prevailed between racial groups created much resentment during the apartheid era. Violent resistance by learners and student unions failed to make an impact on the policies and practices of the ruling party. This led to a complete breakdown of the culture of learning and teaching in African schools. Disruptive behaviour has continued into the 1990's and beyond. Despite a decade into democracy, schools are still beset by problems such as irregular attendance, frequent disruptions, neglect of schoolwork, laziness, drug and alcohol abuse, cheating in exams, anti-social behaviour, criminal activities, and mass action (Baloyi, 2004).

In 1997, the State launched a campaign referred to as Culture of Learning, Teaching and Service (COLTS). This initiative was based on the following principles:

1. Provision of basic resources for effective teaching and learning
2. Setting up effective and supportive school governing bodies
3. Making the school free of crime and violence
4. Maintaining discipline and dedication amongst learners, educators, principals and officials of the education department

The education budget for ex-Department of Education and Training schools was increased from R31,8 to R51,1 billion in support of the COLTS campaign. Unfortunately there were shortcomings such as failure of the education department to deliver resources (Baloyi, 2004).

The environment in which educators and learners work in some South African schools has negative implications for the culture of learning and teaching. Some schools lack resources such as libraries and computers and basic services such as electricity and sanitary facilities. In rural and farm areas in Limpopo, Eastern Cape and KwaZulu Natal mud structures, shacks, trees, and old buses are used as classrooms. Noting the poor quality of teaching and learning, poor teacher morale, lack of facilities, ineffective school governance and inequalities within the education system, the then Minister of Education, Professor Asmal launched the "Tirisano" (meaning 'working together') project in 2000 (Baloyi, 2004).

Personal experience led me to believe that the study of geometry in the mathematics curriculum contributes to mathematics anxiety. Persistent failure in this field of study leads to much frustration and ultimately avoidance. The literature on geometry education is looked at in some detail.

2.4 GEOMETRY EDUCATION IN SOUTH AFRICA.

It is common knowledge to those in mathematics education that traditional congruence geometry is a problem area for the majority of learners. De Villiers (1997) also makes reference to the fact that worse results are produced in geometry than in algebra.

High achievers may produce excellent results, but the average and below-average struggle. Questions in algebra, trigonometry and analytical geometry help to boost marks. Difficulties experienced in geometry are international and not peculiar to South Africa (van Niekerk, 1998). Matriculation examiners' reports regularly draw attention to the fact that Euclidean geometry is most problematic

Euclid's book "Elements" has been the main influence on geometry education. It was used from the 14th century in European universities and from the 19th century in schools. France and Germany have deviated from Euclid's work, opting for a more applications-orientated approach, while British secondary schools have continued to teach Euclidean geometry (Human & Nel, 1978). The school geometry curriculum has evolved into a combination of the works of Euclid, Legendre, Hilbert and Birkhoff. In South Africa the approach is more formal than in other countries (Usiskin, 1987). Human and Nel (1978) identify four types of geometry that have made up school curricula: congruency geometry, vector geometry, analytical geometry and transformations. There has been a misunderstanding that the latter is non-Euclidean geometry but as Usiskin (1987) points out it is another approach to teaching Euclidean Geometry. Traditional congruency geometry has been retained in the South African curriculum presently being taught alongside analytical geometry. The latter was taught before being replaced by vector geometry, and then re-introduced in the 1990's (Human & Nel, 1978). Recently attention has been drawn to the non- Euclidean geometries of Lobachevsky-Bolyai (spherical geometry), projective geometry (Usiskin, 1987), tessellations, fractal geometry, knot theory, and coding theory (de Villiers, 1997), some of which have been incorporated into the new curriculum.

Reasons suggested for poor geometry performance include poor teacher qualifications and low teacher morale, language issues and poor foundational geometric preparation in the primary schools. These will be discussed in turn.

Both internationally and within South Africa researchers have voiced concerns about the lack of qualified and competent mathematics educators. Usiskin (1987) states:

"we will not be able to work from problems to solutions in school geometry without knowledgeable teachers" (p. 42). (Van Niekerk, 1998) says that teachers need to upgrade their knowledge and they need to be able to recognize shortcomings and design appropriate intervention strategies. Low teacher morale also contributes to poor performance. Baloyi (2004) points out that as a result of a number of factors – poor working conditions, poor conditions of service, breakdown of discipline, lack of parental commitment and poor school management, amongst others have led to teacher morale being at its lowest. This in turn has manifested in a litany of shortcomings such as high absenteeism, lack of punctuality, apathy, lack of cooperation, and declining confidence.

The language issue is another that merits attention. The lack of language competency acts as a barrier to the study of geometry (Feza & Webb, 2005). Having to acquire technical terminology impedes the advancement of second language English speakers through the van Hiele levels (De Villiers, 1997). African teachers are not sufficiently competent in English and have difficulties with technical terms (Van Niekerk, 1996). The cultural background of the learner also has a bearing on the learning of geometry (Feza & Webb, 2005).

De Villiers (1997) points to the insufficient preparation of learners at primary school; and this point of view is shared by Usiskin (1987). Niven, (1987) feels that too much emphasis is placed on proofs in the early stages, and not enough attention is paid to relevance to the real world. Dreyfus and Hadas (1987) claim that the failure to construct logical arguments and organize thoughts logically has resulted in the decline of geometry.

A study of geometry education is incomplete without viewing it against the background of the van Hiele theory. No other theory has had such a profound impact on evaluating geometry curricula, methods of instruction, and the understanding of the learning difficulties that students experience.

2.5 THE VAN HIELE THEORY

This theory was developed by a Dutch couple, Pierre van Hiele and Dina van Hiele-

Geldof at the University of Utrecht in 1957. After the death of Dina shortly after completing the study, Pierre forwarded the theory for publication (De Villiers, 1997, Crowley, 1987). De Villiers (1997) notes that whilst Pierre's study was descriptive in explaining difficulties experienced by learners and focusing on insight in the learning of geometry, Dina's was more prescriptive, focusing on the order in which geometry is learned and the function of instruction in raising levels of thought.

There are five levels in the van Hiele theory. The van Hieles numbered them from 0 to 4 which some researchers follow. American geometers use 1 to 5, reserving level 0 for the pre-recognition stage of development. The latter which is used by South African researchers (see for example de Villiers, 1997 and van Niekerk, 1998) will be adopted in this study. The levels are described in some detail.

Level one: Visualization. This is called the visual level (van Niekerk, 1998) or recognition level (de Villiers, 1997). At this level shapes are recognized by appearance (van Niekerk, 1998). A rectangle may be recognized as such because it "has a long shape" (Bennie, 1999, p. 4). They are looked at in totality rather than by their properties. Learners can identify shapes and redraw them (Crowley, 1987).

Level two: Analysis. Geometric figures are identified in terms of their properties rather than appearance. However properties are looked at in isolation (Mistretta, 2000). The relationships between properties are not understood (Crowley, 2000). The properties may be established by observing, drawing, measuring and modeling (van Niekerk, 1998). Definitions are uneconomical, for example, a rectangle is defined as a figure with opposite sides equal, opposite sides parallel and all angles equal to 90 degrees.

Level three: Informal deduction. Learners understand and are able to form relationships between properties within a figure and also among different figures (Crowley, 1987). At this level learners are able to reason logically (Mistretta, 2000) and are capable of writing short proofs (de Villiers, 1997). Definitions are now economical, for example, a rectangle is a parallelogram with one angle equal to 90 degrees.

Level four: Deduction. Learners can construct proofs (Mason, n.d.), understand the significance in deducing a result (Crowley, 1987), and the role of axioms, theorems and proofs (de Villiers, 1997).

Level five: Rigour. Students at this level can work in different axiomatic systems, both Euclidean and non-Euclidean. This level is not required for secondary school geometry, as most of the latter is taught at level 3 (van Niekerk, 1998).

There are five properties (Crowley, 1987) or characteristics (de Villiers, 1997) of the van Hiele model.

1. *Sequential*: Learners progress through the levels in a fixed order (de Villiers, 1997). Mastery at one level is necessary before progress to the next level occurs (Crowley,1987)
2. *Advancement*: Progress through the levels depends on the experience with the content and not on age or level of maturity (Crowley, 1987). This implies that there is no certainty that learners engaged with geometry education will pass through the levels.
3. *Adjacency*: What is inherently natural at one level becomes non essential at the next level (de Villiers, 1997)
4. *Language*: Each level has its own language and symbols (Crowley, 1987). De Villiers (1997) refers to this property as distinction. Feza and Webb (2005) note that lack of competency in language acts as a barrier in progressing through the levels.
5. *Mismatch*: No learning takes place if the instruction is not matched to the learner's level of mathematical understanding (Feza and Webb, 2005). De Villiers (1997) calls this concept separation where two people at different levels cannot understand each other.

The Van Hiele model divides the learning process into five phases (Crowley, 1987). The first phase is called the *information* phase in which the teacher gains an insight into learners' pre-knowledge through discussions. This is followed by *directed orientation* where the teacher provides materials so that the learners are exposed to concepts being taught. The next phase, referred to as *explication* allows students to become self-reliant and exchange ideas. In the fourth phase, *free orientation,* learners are given open-ended tasks which have many methods. The last phase, *integration, provides* the opportunity to summarise and review what has been learnt (Mistretta, 2000).

which have many methods. The last phase, *integration, provides* the opportunity to summarise and review what has been learnt (Mistretta, 2000).

The theory was first applied in the Russian education system. In the 1960's Russian education officials expressed concern that learners were performing poorly in geometry while they were doing well in other subjects. It was thought that insufficient attention was being paid to geometry at the primary school level (de Villiers, 1997). The geometry curriculum was revised to conform to the principles of the van Hiele theory (Crowley, 1987). Subsequently researchers found that grade eight learners exposed to the new curriculum had a better understanding than those in grades 11 and 12 who had passed through the old curriculum (de Villiers, 1997).

Van Niekerk (1998) points out that the van Hiele theory applies to the South African context. According to Feza and Webb (2005), learners who have completed primary school should be at level two. However a study by de Villiers and Njisane cited by Feza and Webb (2005) and de Villiers (1997), shows that only 45% of grade 12 African learners acquired level two whereas the curriculum requires mastery of level three and higher. These learners may still be operating at the concrete level rather than at an abstract level (Feza and Webb, 2005). Transition to a higher level is especially difficult for second language learners. Acquiring technical terminology acts as a barrier to learning (Feza and Webb, 2005). De Villiers (1997) says that that African languages have no equivalent terms for some words that are in the English language. Adding to the complexity of the issue is the fact that some African teachers have difficulties with the English language (van Niekerk, 1996).

In order to determine the van Hiele levels of grade 8 learners and in an attempt to raise them, a study was conducted with 23 learners in the United States. After a pretest, a unit of study designed according to van Hiele principles was used to guide learners from level one to level two. A posttest similar to the pretest found that 70% had progressed to level two; and correct responses increased from 26% to 67%. An opinion survey conducted after the test showed that learners found geometry more enjoyable (Mistretta, 2000).

Researchers have made recommendations with respect to the application of the Van Hiele theory in geometry education. Mason (n.d.) states that geometry instruction should be at the appropriate level. This point of view is echoed by Crowley (1987) who says that educators need to identify the van Hiele level at which learners are operating so that there is a match between instruction and the learner's understanding. Van Niekerk (1998) says that educators need to recognize short-comings and plan appropriate intervention strategies. Masehela (2005) also points out that intervention programmes are ineffective if there is a mismatch between the level of teaching and learners' abilities. Mistretta (2000) draws attention to the fact that lessons should be continuous, and built in layers, rather than offering isolated facts. Furthermore she emphasizes the importance of real-life applications of concepts.

Van Niekerk (1998) found from her studies that it is advisable to start geometry education with three dimensional experiences in the real world which then transforms to two dimension in the classroom, gradually leading to the formulation of theorems and definitions comprising formal geometry. De Villiers (1997, p. 43) calls for reform at the primary school level: "It seems that no amount of effort at secondary school will be successful, unless we embark on a major revision of the primary school curriculum along Van Hiele lines. The future of secondary school geometry thus ultimately depends on primary school geometry!"

2.6 MATHEMATICAL LITERACY

Curriculum reforms in the South African education landscape have seen the introduction of an outcomes-based approach. The senior secondary phase in the old curriculum has been renamed the Further Education and Training band (FET). In this phase, which extends from grades 10 to 12, those learners who do not study Mathematics must do Mathematical Literacy. Mathematical Literacy has taken on two meanings in the South African context. Firstly it is a personal competence, and secondly it is a school subject. The competence has been described by many researchers in ways discussed in the following section.

The International Life Skills Survey defines mathematical literacy as "an aggregate of skills, knowledge, beliefs and dispositions, habits of mind, communication capabilities, and problem-solving skills that people need in order to engage effectively in quantitative situations arising in life and work" (Steen, 2000, p. 6). The Program for International Student Assessment, conveys the same theme in its definition: "An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics in ways that meet the needs of the individual's current and future life as a constructive, concerned and reflective citizen" (Alsina, 2002, p. 241). This is also mirrored by the National Curriculum statement in proclaiming that "...mathematical literacy should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy" (Dept. of Education, 2003, p. 10). Alsina (2002, p. 241) extends the ideas further in offering the following definition: Mathematical literacy implies ..."confidence with mathematics, mathematics in context, cultural appreciation, number sense, interpreting data, practical skills, logical thinking, symbol sense, making decisions, prerequisite knowledge". Steen (2000) points out that quantitative literacy does not only differ from mathematics, it requires much more than basic mathematical skills – concepts not usually associated with traditional school mathematics. Unlike mathematics which is embedded in the abstract, numeracy is concerned with specific issues.

The National Adult Literacy Survey (Steen, 2000, p. 5) in focusing only on the arithmetic aspect gives a much narrower definition: "The knowledge and skills required to apply arithmetic operations, either alone or sequentially, using numbers embedded in printed material (e.g. balancing a checkbook, completing an order form). Stoessiger (2003, p. 2), on the other hand, makes reference to the fact that numeracy, although viewed as arithmetic by some, actually refers to mathematical skills "required to function in every-day life – at home, at work, in the community."

The Cockroft Report (Cockroft, 1982), released after a study commissioned by the

British Government, advanced two attributes of numeracy: "The first of these is an 'at homeness' of numbers and an ability to make use of mathematical skills which enable an individual to cope with the practical demands of everyday life. The second is the ability to have some appreciation and understanding of information which is presented in mathematical terms" (Steen, 2000, p. 5). As can be seen, the notion of assigning a 'value' component to numeracy, in addition to skills, has been expounded.

In the South African context, from 2006, Mathematical Literacy has become a school subject. Mathematical Literacy is defined in the National Curriculum Statement (Department of Education, 2003, p. 9) as an area of study that "provides learners with an awareness and understanding of the role that mathematics plays in the modern world." Noting that Mathematical Literacy is contextually-based, the definition further explains that the intention of including it in the curriculum is to allow "learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyze everyday situations to solve problems" (Department of Education, 2003, p. 9).

The scope of Mathematical Literacy in the National Curriculum Statement sheds light on the justification of the inclusion of ML into the curriculum. It is envisaged that ML would equip learners to:

1. Employ numbers in solving problems relating to social, financial and personal matters
2. Perform calculations relating to budgets
3. Use functions and graphs to solve problems
4. Address issues pertaining to space and shape
5. Handle, and critically interact with, data
6. Use computational tools with confidence (Department of Education, 2003).

In citing the increased use of quantitative thinking in virtually every field of human activity, Steen (2000), draws attention to the fact that many adults in American society, although having been to high school, remain "functionally innumerate" in that educated adults, although consistently having to interact with numerical data, remain numerically incompetent. In a similar vein, Wallace (2000) points out that economic development is dependent on having a reasonable percentage of the population technically and mathematically literate. Steen (2000) expresses concern in that, although college graduates are able to handle concepts relating to algebra and calculus, they lack computer, technical and statistical skills demanded in the workplace.

In the South African context, Hobden (2003, p. 1) states that "advocacy of Mathematical Literacy is based firstly on the practical value of being able to deal with quantitative situations in personal and work situations." Hobden, (2003) also alludes to the fact that Mathematical Literacy is essential for responsible citizenship in a democracy. In the Mathematical Literacy curriculum, geometry is included in Learning Outcome 3: Space, Shape and Measurement. This learning outcome is stated as follows:

> The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions
> (Department of Education, 2003, p. 9).

A study of the Assessment Standards (Appendix M) shows that geometry knowledge and skills acquired in the GET phase are prerequisites for Learning Outcome 3. Furthermore, learners should be at least on Van Hiele level 3 (Informal deduction) in order to cope with the demands of studying Space and Shape.

2.7 CONCLUSION

Analysis of the grade twelve examination results shows there is a serious problem with mathematics education in South African. This is confirmed by the TIMSS study which illustrates that problems arise as early as grade eight. Furthermore, geometry seems to be a factor affecting performance – this has been the finding of the latest TIMSS study in 2003. In assessing performance in geometry, the van Hiele theory provides a basis from which to do so. The literature indicates that mathematics anxiety may also contribute to negative attitudes, and hence lead to poor performance in mathematics and more specifically in geometry. I elected to carry out these case studies at two schools at which, I know from personal experience, produce unacceptable mathematics results in the senior certificate examination partly due to the fact that learners' conceptual understanding of geometry is poor. In doing these studies, I hoped that educators may benefit in the teaching of Mathematical Literacy – both at these institutions and others with similar demographics.

In the next chapter I describe the participants in the case studies, what data I collected, and I also discuss the instruments I used in gathering the data.

CHAPTER THREE

METHODOLOGY

The objective was to acquire information that would allow me to answer the research questions, namely

1. Why do learners not continue mathematics in grade 10? (RQ1)

2 (a) What basic geometric and measurement skills do grade 9 learners, who do not continue mathematics in grade 10, have? (RQ2a)

(b) What attitude do these learners show towards geometry? (RQ2b)

3.1 RESEARCH DESIGN

I chose to obtain answers to my questions from all the learners at the two schools, who had elected not to continue mathematics in grade 10 so I adopted a quantitative approach in this regard. Quantitative methods, in determining responses to a fixed set of questions, can be administered to a larger group of participants, and the findings may be generalized to other similar situations (Patton, 2002). However, I also wanted to engage both learners and educators in discussion to elicit personal points of view, so I included a qualitative aspect. A qualitative approach is concerned with the qualities of a situation being investigated, seeking to find relationships between entities. The researcher is closely associated with the situation and the participants, studying and interpreting phenomena in their natural settings. Quantitative methods place emphasis on measurement and are directed towards analyzing relationships between variables, generally adopting an experimental approach. In seeking data, the researcher may have no contact with the participants in the study (Denzin & Lincoln, 2000). The advantage of a qualitative study is that it allows an in-depth, detailed study but is restricted in terms of the number of participants that may be involved. In adopting both methods, I quote Albert Einstein who said "Not everything that can be counted counts, and not everything that counts can be counted" (Patton, 2002, p. 12).

In this investigation, considering time and financial constraints, I have elected to do two descriptive, and to a certain extent, evaluative case studies. According to Soy (1996, p. 1), case studies "emphasize detailed contextual analysis of a limited number of events or conditions and their relationships." Critics of case studies point to the following limitations: (a) The number of cases is generally small and this does not allow for generalizing, (b) an intensive study may not be objective, and (c) issues may be explored but no conclusions may be made. However generalizations may be extended to similar cases and the richness of data obtained may lead to further studies.

I have based my research design on a similar type of study by Hobden (2003). In an attempt to document the attitudes and abilities of pre-service teachers with respect to Mathematical Literacy at the University of Natal (now University of KwaZulu Natal), Hobden carried out a case study which included both those who had studied mathematics up to grade 12 and those who had not. Pre-service teachers who had the intention of specializing in subjects other than mathematics were required to complete a compulsory module in Mathematical Literacy if they had not studied mathematics up to grade 12. Each group was asked to complete a questionnaire assessing their experiences in mathematics, to determine what influenced the decision to continue or terminate the study of mathematics in grade 10. The second part of the questionnaire consisted of a mathematics test to assess levels of thinking in measurement, statistics and mathematics. This was followed by three focus group sessions with groups of students selected on the basis of the number of years since they had finished school.

Hobden (2006) also used a case study to determine the mathematical proficiency of grade 10 learners to preview the type of learners that educators of Mathematical Literacy can expect to receive into their classrooms. A total of 1405 learners from 5 rural secondary schools were tested on the grade 9 learning outcomes over a period of three years. The test items based on TIMSS, and matched with grade nine learning outcomes were administered to grade 10 learners irrespective of whether they studying were mathematics or not.

Two schools that afforded easy accessibility were chosen for this study. Staff management teams at these schools consented to the study after consulting with their Governing Bodies. The case study at another school that was chosen at the outset had to be abandoned when permission was refused. The other motivating factor in singling out these schools is that 50% to 75% of learners do not study mathematics in the senior secondary phase. It was felt that learners at these schools could provide an answer to the question as to why learners do not continue mathematics beyond grade nine.

Valeview Secondary School is an ex-House of Delegates institution based in the south of Durban. At present approximately 40% of the learners are from the community while the rest travel from semi-rural areas such as Hammarsdale and Cliffdale. The former speak English, the language of instruction at school; the latter have isiZulu as home-language. The socio-economic status of the learners of this school ranges from below-average to average. A substantial number of learners come from single-parent homes. Although the school fees payable is modest in comparison to neighbouring schools, parents have difficulties in meeting this commitment. Learners from more favourable social circumstances enroll at other schools in the area. At the time of the research being conducted there were four class units in grade 10, two of which had mathematics included in the curriculum. Mathematics was taught in one of the four grade 11 classes and one of the three classes in grade 12.

Railside Secondary School, an ex-House of Representatives institution, in the Durban Central district has some learners residing in the area, but a large proportion travel from neighbouring suburbs and the township of KwaMashu. Many learners have attended primary schools in the area and, although having a mother-tongue of isiZulu are fairly proficient in English. The socio-economic status of these learners may be described as below-average to slightly above-average. Railside learners differ from those at Valeview in the sense that the former experience difficulties that are more social than economic. From grades 10 to 12, the respective number of class units studying mathematics was two in six;

two in five and two in four. The number of units per grade decreases because a substantial number of learners who do not progress to the next grade exit the system, and these are largely from non-mathematics classes.

3.2 RESEARCH INSTRUMENTS

I used a questionnaire to determine why learners do not continue the study of mathematics when it becomes optional, and also to determine attitudes to the study of geometry. A questionnaire allows a researcher to collect large amounts of data quickly and cost-effectively. The advantages that it has over interviews are that respondents may remain anonymous and are under no pressure to give an immediate response. Analysis of data, especially of closed questions is fairly easy (Gillham, 2000). With respect to the questionnaire, Cohen et.al, (2000) state the following: "The questionnaire is a widely used and useful instrument for collecting survey information, providing structured, often numerical data, being able to be administered without the presence of the researcher, and often being comparatively straightforward to analyze" (p. 245). Gillham (2000) also draws the researcher's attention to pitfalls in using the questionnaire: it is difficult to motivate the respondents and this leads to incompleteness and inaccuracies. Furthermore, literacy problems and the fact that many people talk more easily than they write may lead to incorrect or inappropriate responses which cannot be corrected once the questionnaire has been collected.

I chose to conduct focus group interview to acquire first-hand information from both learners and educators. Noting that questionnaires have limitations, they may be supported by interviews. Stewart and Shamsadani (1990) define the focus group interview as a number of people interacting over an issue of interest in the presence of a moderator. Focus groups provide the situation where the synergy of the group adds to insight. They provide in-depth qualitative data similar to brain-storming sessions. Flick (1998) describes it as a group interaction to produce data and insights. The strength of focus group interviews is that they use open-ended questions to which individuals are given the opportunity to respond. Unlike the directed interview, participants are not forced to say something because a question is posed. The open-ended approach gives the opportunity to explain points of view and to share attitudes and experiences (Krueger, 1994). A limitation of the focus group interview is that interaction between group members could mean that responses are not independent of each other. Participants may be reluctant to express views that are contrary to those held by the majority, and especially those that are more vociferous. There is also the likelihood that those who agree to participate in focus groups may hold different views from the general population. Therefore care must be taken in extrapolating.

I administered a geometry test and collected the learners' grade 9 mathematics examination marks to determine their mathematical abilities. The relationships between the research questions and the instruments used in this study are summarized in Table 3.1

Table 3.1 Relationship between research questions and type of data collected.

Method	Research Question 1	Research Question 2(a)	Research Question (2b)
Questionnaire (n = 94)	*		*
Geometry Test (n = 94)		*	
Interview with Learners (n = 2)	*		*
Interview with Educators (n = 3)	*		*
Documentary Evidence		*	

In collecting data, an attempt was made to find out:

With respect to RQ1

1. Possible reasons why learners do not continue mathematics after grade 9
2. Whether learners are concerned about not continuing the study of mathematics
3. Whether the study of geometry influences the termination of mathematics in grade ten

With respect to RQ2a

1. If geometry or algebra is perceived to be more difficult
2. Learners' abilities in geometry and their Van Hiele levels of thinking
3. Educators' views on learners' abilities
4. Learners' marks in the grade 9 examination

With respect to RQ2b

1. Whether learners preferred to study algebra, geometry or numbers
2. Whether learners like or dislike activities related to geometry
3. If learners would be keen in studying Mathematical Literacy
4. Learners' thoughts on the study of mathematics, and geometry in particular
5. Educators' views on learners' attitudes

I now discuss each research instrument in some detail.

3.2.1 Questionnaire administered to learners

Bearing in mind that a substantial number of learners would be responding in their second language, I designed a questionnaire using language that was easy to understand (Refer to Appendix B). As noted by Fink (2003, p.11), "a straightforward question asks for information in an unambiguous way and extracts accurate and consistent information". I intended to administer them so that explanations to each question could be given before they were answered. The questionnaire, including both quantitative and qualitative type questions was designed to determine attitudes to mathematics. The structure of the questionnaire is discussed below.

In question one, respondents were asked to indicate why they were not continuing mathematics in grade 10. Possible reasons were advanced and each had to be rated on a four-point scale.

Question two was included to assess whether learners preferred algebra or geometry. Grade 9 mathematics topics were listed in random order; and respondents were asked to indicate their feelings on a 5-point scale, ranging from strong like to strong dislike. The intention was to determine whether the study of geometry is a factor preventing learners from choosing mathematics in grade ten.

In question three respondents were asked to indicate whether they found algebra or geometry more difficult to comprehend. The objective was to either confirm or refute my belief that difficulties experienced in geometry surpass those experienced in algebra.

Drawing from assessment criteria documented in the Mathematical Literacy curriculum, and geometry skills which learners had been exposed to in MLMMS, the participants were asked in question four to rate activities encountered in geometry. A five-point scale, ranging from strong like to strong dislike, was used.

In question 5, the respondents' perceptions of the value of mathematics in terms of career options were questioned. They were asked whether they felt concerned that they may experience constraints in terms of career choices on account of not studying mathematics.

Based on the assumption that the learners under review were not going to continue mathematics, question 6 asked whether they were keen on studying Mathematical Literacy. The responses would probably give some indication of what attitudes to mathematics such learners may present in the Mathematical Literacy class.

Although the questions were directed at geometry with the focus of study being "Space, Shape and Measurement", this was not made apparent to learners in the questionnaire. The intention was that learners' responses should give some indication of whether geometry is a barrier in the study of mathematics. The qualitatively-based questions asked respondents to make statements in support of choices they had made. There were also questions that attempted to draw out attitudes to mathematics. Open questions allow for unexpected answers and let respondents describe their feelings and experiences (Fink, 2003). Although a communication problem was anticipated, with some learner's written skills in English being limited, it was felt that the essence of their thoughts could be interpreted.

The questionnaire was directed at learners who had elected not to study, or were excluded from studying, mathematics in grade 10. Schools offer subject packages (also referred to as "bundles") to learners. Performance in mathematics in the grade 9 year determines whether a learner continues in grade 10 or not. The cohorts of learners involved in the study represent the last group to pass through senior secondary education with the option of not offering any mathematics in their study. As from 2006, learners who do not study mathematics are compelled to do Mathematical Literacy. Initially the instrument was piloted with a group of learners at a school not included in the study, the responses analysed and refinements made. Instead of asking learners to merely indicate the reason for discontinuing the study of mathematics, I asked them to rate each reason on a four-point scale. Question 2 was changed from asking learners to rate the different topics in mathematics in terms of difficulty to indicate their like or dislike on a five-point scale. Question 4 relating to activities in mathematics was added in.

The questionnaires were then administered to grade nine learners towards the end of the year. Prior to commencement, learners were briefed once again about the study and given the opportunity to decline participation. Assistance was sought from educators based at these schools. The target of reaching at least one hundred learners was not achieved as learners were busy completing CTA-related activities. Having learners complete the questionnaire any earlier would have been less meaningful as course selection for grade ten had not been completed. When I arrived at Valeview Secondary School, I found that some learners were away on a Careers Expo. This was an unfortunate oversight on the part of the coordinator. Forty questionnaires were administered by an educator at the school during the course of the following week. The data was coded, entered on Excel, and then transferred to SPSS for analysis. The code book for data capture may be found in Appendix I.

3.2.2 Geometry Test

Learners were requested to complete a test. Refer to Appendix C. The objective of this content-assessment aspect was to determine what mathematical skills, pertaining to geometry specifically, learners are expected to bring to the Mathematical Literacy class. Questions were drawn up based on the NCS with reference to learning outcomes identified in the Mathematical Literacy curriculum. Some of the tasks that learners are expected to carry out are: calculate area and perimeter of polygons, estimate measurements, measure accurately using appropriate instruments, identify 2- and 3- dimensional shapes, describe observations, and solve contextually-based problems. Educators of the participating learners were consulted in order to confirm that learners had prior exposure to the skills and knowledge being tested.

Question 1: Recognition and estimation of angles

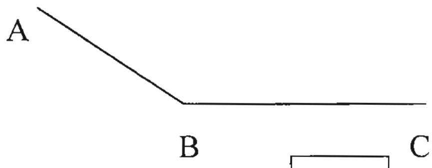
1.1 Choose the correct option. Ring the letter of the correct answer.

Angle ABC is

A Acute : between 0° and 90°

B Obtuse : between 90° and 180°

C Reflex : between 180° and 360°



1.2 Give the approximate measurement of angle ABC to the nearest 10°

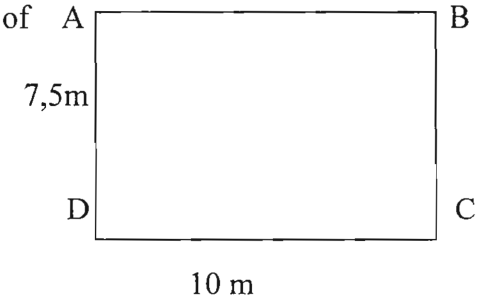
Figure 3.1 Geometry test question 1

The first question asked respondents to classify an angle and give its approximate measurement. The objective was to determine if learners could identify and estimate the measurement of an angle. Identification of angles is done in the intermediate phase (grades 4 to 7). The knowledge that is required to classify the angle corresponds to the first Van Hiele level (VH1), namely, visualization, as this is done by its appearance. Approximating the measurement corresponds to VH2, because the boundaries of the obtuse angle must be known. With respect to estimation; it is known from experience that learners are lacking in this ability as they fail to check the validity of their answers. In the Mathematical Literacy curriculum, Assessment Standard 3 (grade 10) refers to estimating, measuring and calculating angles from 0° to 360°, which confirms that this competence will be required.

Question 2: Calculation of area and the use of the theorem of Pythagoras

The diagram on the right shows a rectangular garden of dimension 7,5 metres by 10 metres.

Calculate the area of the garden in square metres



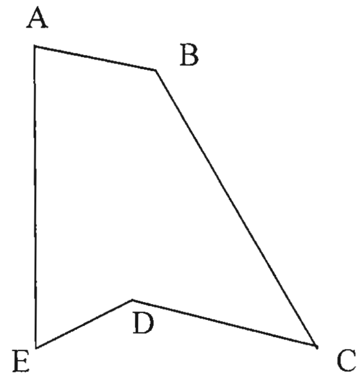
If a tap is situated at point B, what is the shortest length of watering hose (hosepipe) that is required to reach point D?

Figure 3.2 Geometry test question 2

The second question tested the calculation of area of a rectangle; and the use of the theorem of Pythagoras. The concept of area is introduced in the intermediate phase, with the square and rectangle being the polygons initially dealt with. In grade 8, rectangular area is used as a basis to introduce formulae for the areas of parallelograms and triangles (Johnson, Davidson, Jaffer, & Galant, 2000; Human, Olivier, & le Roux, 2002). The Theorem of Pythagoras is covered fairly extensively in grade 9 (Johnson, Liebenberg, Davidson, & Jaffer, 2001). In order to identify and apply the correct area formula learners should be performing at least at Van Hiele level 2: analysis. In the second part of the question, application of Pythagoras' theorem was implied and knowledge at VH3 is necessary to effect the calculation. In the Mathematical Literacy curriculum, this aspect is covered under assessment standard 3, which mentions that the learner should be able to estimate, measure and calculate lengths and distances using the Theorem of Pythagoras. Learners should also be able to determine the perimeter and area of common polygons.

Question 3: Calculation of perimeter

ABCDE is a pentagon (5-sided figure) shown below. Measure each side and enter the values in the table drawn below.



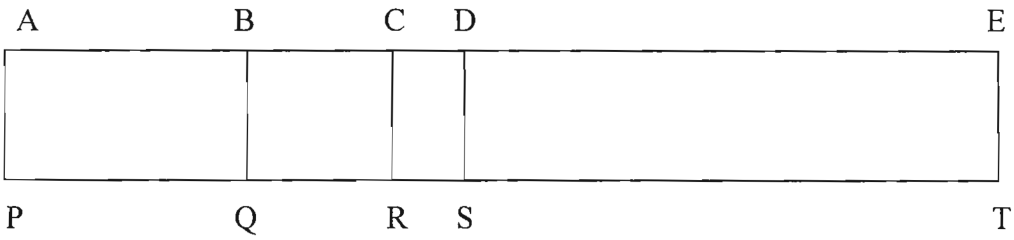
SIDE	MEASUREMENT (mm)
A B	
B C	
C D	
D E	
A E	

Now calculate the perimeter of the pentagon.

Figure 3.3 Geometry test question 3

Learners' ability to measure, using a ruler; and the meaning of perimeter were assessed in question three. Using a ruler to measure lengths is a basic skill initially taught in the foundation phase (grades one to three). The concept of perimeter is studied in the senior phase. Perimeter is usually associated with a formula (mainly with respect to a rectangle), and the intention was partly to determine whether learners will try to apply a formula in this case. Measuring the lengths requires VH1 knowledge; applying the concept of perimeter requires analysis which is at VH2.

Question 4: Identification of rectangles



One of the rectangles in the figure has been named in the table. Name all the other rectangles in the figure shown above. Fill your answers in the table.

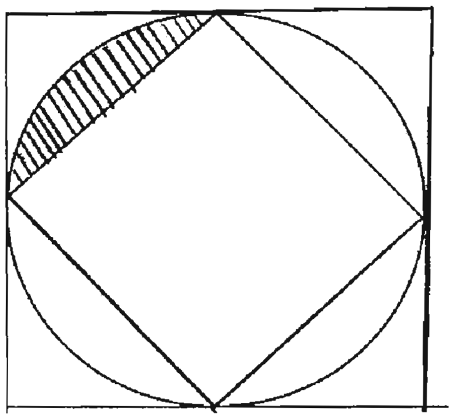
ABQP						

Figure 3.4 Geometry test question 4

In question four respondents were asked to name all the rectangles in a sketch. Identification of rectangles is a basic skill that is studied in the intermediate phase. Learners also had to realize that a square is a special type of rectangle and that some rectangles are embedded. Identification of distinct rectangles such as DETS requires VH1 knowledge (visualization), whilst identifying those that are made up of two or more rectangles, such as CETR, and recognizing a square as a rectangle (BCRQ) needs VH2 level operation.

Question 5: Description of a sketch

Describe in words what you see in the diagram below.



Explain how you would calculate the area of the shaded part.

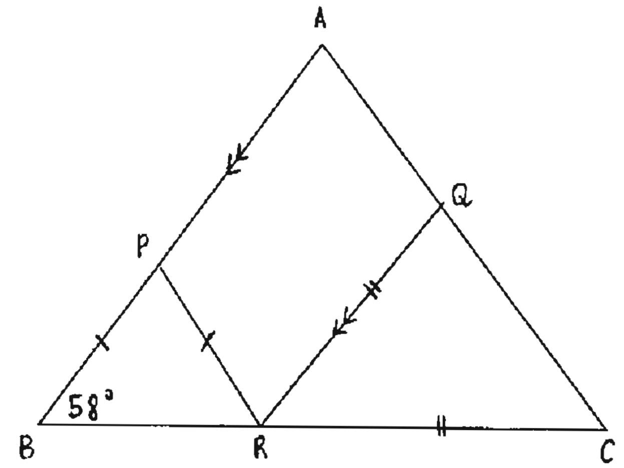
Do not do any calculations.

Figure 3.5 Geometry test question 5

The skill in describing what is observed; and the ability to explain how a procedure is carried out was brought out in question five. Mathematically-challenged learners generally write the first formula that comes to mind and substitute values into them, without knowing exactly what is required. This question sought to assess how learners interpret what they see and verbalise the method they propose to use in solving the problem. Identifying the shapes is an application of Van Hiele level two because the geometric shapes are embedded and furthermore the rectangle within the circle is deliberately not orientated in the traditional way; describing how to calculate the shaded area, is also a task at the second level. A similar type of question, asking respondents to explain how to calculate the area of a figure comprising a rectangle and a triangle was used by Mistretta (2000) in her studies.

Question 6: Calculation involving parallel lines and triangles

In the diagram below PB = PR; RQ = RC and QR || AB. B = 58°.



Calculate the measurement of A.

Figure 3.6 Geometry test question 6

Question six presented a Euclidean geometry calculation problem. Although this type of problem is not specifically included in the Mathematical Literacy curriculum, it constitutes part of the geometry covered in grades 8 and 9. The component parts of the question are foundational; and the symbolism such as that of parallel lines is required in the Mathematical Literacy curriculum. See, for example Assessment Standard 10.3.6 in Appendix M. This question may be categorized as one on Van Hiele level three, as learners are required to work with properties within a figure and make logical deductions.

Learners were asked to answer the test upon completing the questionnaire. Calculators and rulers were provided. No time constraints were imposed, but it was noted that more time was required than anticipated. It was difficult to motivate learners to make a concerted effort to answer the test. The common belief was that all association with mathematics was being terminated, and it was unreasonable to expect learners to complete tasks in geometry. On the positive side, some learners were very co-operative and trusting. Learners at Valeview seemed more willing to participate than those at Railside.

Answers to the test were drawn up by myself and moderated by two educators of mathematics. The expected answers are found in Appendix N. Answers were marked accordingly and the marks converted to percentages. These were rounded off to the nearest integer and the data analysed using SPSS. The test was also analysed according to the quality of answers given. Common errors and misconceptions were recorded for each question. Responses were analysed to determine the Van Hiele levels at which learners are operating.

3.2.3 Focus Group Interviews with Learners

The focus group at Railside comprised of 6 learners (3 males; 3 females): Kevin, Philip, Adam, Rianna, Felicity, and Justine. These learners were selected by their class teachers. At Valeview there were 5 females and two males: Pamela, Dianne, Nancy, Stacy, Natalie, Valerie, Cyril and Steven. One male who had initially agreed to participate withdrew just before commencement while another was absent from school. Two sessions were conducted with learners. Volunteers were called for after the mechanics of the group interview were explained to learners. It was pointed out that learners should be able to articulate their thoughts verbally. Interestingly, mainly girls volunteered to participate; the objective of composing gender-balanced groups was not achieved. Nevertheless, those that were involved in the process shared invaluable information. A list of questions was compiled (see Appendix C). Copies of the list were given to the learners prior to the interview. A time interval of between ten and fifteen minutes was allocated for learners to make notes. This gave learners an opportunity to gather their thoughts to ensure a free-flowing discussion. While the interview was in progress, learners could refer to their notes. The interviews were recorded on an audio recorder.

3.2.4 Group Interviews with Educators

Two of the interviews were with pairs of educators from Valeview and Railside Secondary Schools; a further with two former colleagues who are seasoned educators. Henry who teaches mathematics and computer studies has 19 years of teaching experience. He has been involved with grades 9 and 10 in recent years. John is a mathematics specialist and has been teaching for 17 years. He is noted for articulating his thoughts without reservation and there was no hesitation in inviting him to participate. Betty (14 years experience) and Martin (24 years in the profession) are educators at Railside Secondary. Betty teaches mathematics only while Martin also does geography. Both have many years experience teaching grades 8, 9 and 10. Andy has been teaching for 24 years and Joe for 23. Both educators are exclusively teachers of mathematics and they have taught from grades 7 to 12. They teach at different schools but the schools are somewhat similar to Railside and Valeview in terms of demographics.

Three interviews were conducted. Two of the interviews were conducted during the school holiday and the third at the end of a school day. As in the case of the learners, a list of questions was drawn up (Refer to Appendix D) and presented to the educators about 15 minutes prior to the interview. In jotting down salient points, educators were requested not to liaise with each other. The interviews were fairly structured in that the schedule was followed but educators were free to digress. In keeping with trends in focus group interviews, other questions followed from the discussion.

The focus group interviews were recorded on an audio recorder. Information was transcribed, using learners' and educators' written responses as guides. Responses were then analysed thematically: difficulties in mathematics, attitudes to mathematics, study of geometry, and Mathematical Literacy.

3.2.5 Documentary Evidence

Learners' mathematics marks for the grade nine year, inclusive of continuous assessment and examination were obtained. These were entered into SPSS and the means for the school were compared. A comparison in terms of gender was also done. Schools could not provide marks specifically for geometry as these were not recorded separately.

3.3 ETHICAL CLEARANCE

Permission was sought from Principals and Governing Bodies of the participating schools for the following: (a) administration of the questionnaire and geometry test, (b) focus group interviews with learners, (c) focus group interviews with educators, and (d) access to learners' grade 9 MLMMS marks. Consent was sought from parents of learners participating in the focus group interviews. Educators were given letters of invitation to participate in the interviews. Copies of the relevant documentation relating to the above, together with the copies of the instruments used were submitted to the UKZN ethical clearance committee for approval. Evidence of acceptance is found in Appendix A.

3.4 CONCLUSION

With the exception of a small group at Railside who refused to participate, the learners were very cooperative in these studies. Educators were extremely helpful, both in providing insights and assisting with the administration of the questionnaire. Finding suitable times to conduct the focus group interviews presented a challenge, but educators were prepared to make time available during the school vacation. In the next chapter the data that was obtained is analysed in terms of the research questions.

CHAPTER 4

ANALYSIS OF THE DATA

A total of 94 learners participated in the survey. There were 41 respondents at Railside Secondary School (12 male; 29 female) and 53 at Valeview (15 male; 38 females). A substantial number of males adopted an indifferent attitude and refused to participate. One of the reasons given for declining involvement was the uncertainty of continuing or terminating mathematics in grade 10. At the time of the survey, course selection at both schools had not been finalized; the final grade 9 mark is ultimately used to determine whether learners continue mathematics or not. Ages ranged from 13 to 18; the males having a mean of 15.2 and the females 15.1. The mean age of the learners at both schools was the same, namely 15.1 years.

Analysis of the data leads to the following claims:

1. Difficulty with mathematics has the most influence on the decision whether to continue mathematics in grade 10
2. Difficulties are experienced in both branches of mathematics: algebra and geometry
3. Learners are ill-equipped to study space, shape and measurement concepts in Mathematical Literacy
4. Learners do not have a negative attitude to the study of geometry

4.1 WHY DO LEARNERS DISCONTINUE MATHEMATICS AT THE GRADE NINE LEVEL?

Assertion 1.

The perceived level of difficulty in mathematics is a major factor that determines whether learners continue with it in grade 10.

Firstly I will present evidence from the questionnaire and focus group interviews with learners and educators to support this statement, and then discuss some of the reasons given by learners and educators for the difficulties.

Table 4.1 gives the means for the various reasons advanced by learners for not continuing mathematics in grade 10. Scores ranged from 0 (of no importance) to 3 (very important). A higher score implies that the reason is more influential in the decision that is made. The reasons have been ranked according to the total of the means for both schools.

Table 4.1 Ranking of reasons advanced for not continuing mathematics in grade 10.

Reason	Railside Mean	Ranking	Valeview Mean	Ranking
Mathematics is difficult	2.11	1	2.08	1
I don't like Mathematics	1.66	4	1.20	2
I was not allowed to because of my results	2.00	2	0.98	4
I think that mathematics is not interesting	1.80	3	0.96	5
I do not require it for my future career	1.37	5	1.00	3
I was advised not to take Mathematics	1.03	6	0.77	6
My friends did not choose it	0.83	7	0.31	7

Figure 4.1 compares the means obtained by each school for each category; Figure 4.2 compares scores of males and females. The figures are found on the next page.

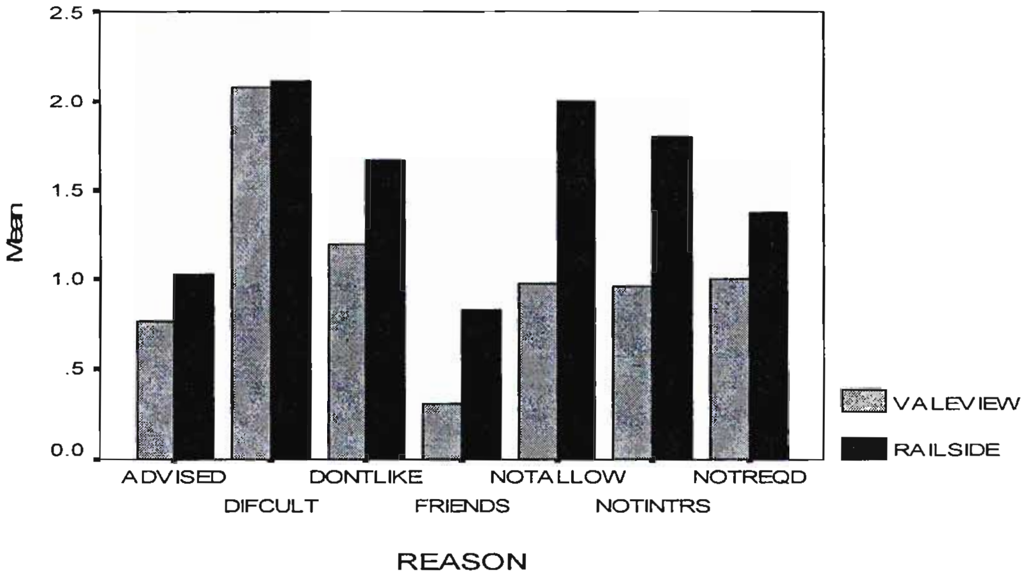


Figure 4.1 Reason for not continuing mathematics: comparison by school

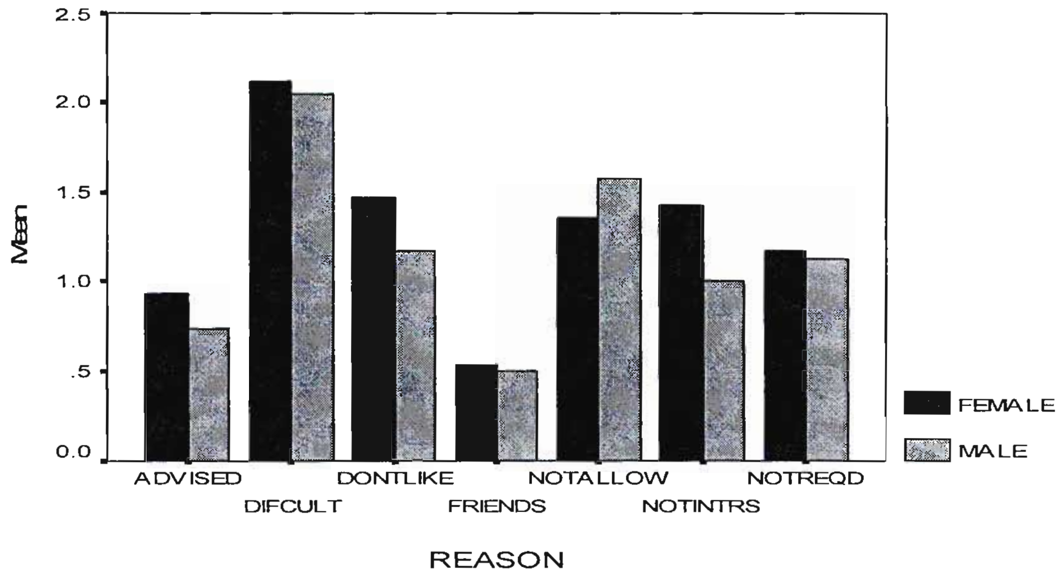


Figure 4.2 Reason for not continuing mathematics: comparison by gender

The abbreviations used above are explained: ADVISED: I was advised not to take mathematics; DIFCULT: Mathematics is difficult; DONTLIKE: I don't like Mathematics; FRIENDS: My friends did not choose it; NOTALLOW: I was not allowed to because of my results; NOTINTRS: I think that mathematics is not interesting; NOTREQD: I do not require it for my future career

Table 4.1 and figures 4.1 and 4.2 show that difficulty experienced with mathematics was advanced as the main reason by learners at both schools. The means for difficulty surpasses the second most important reason by 30% in the case of girls and 22.5% in the case of boys.

In the focus group interview, learners at Valeview Secondary were unanimous in associating difficulty with mathematics. That was the reason advanced for not choosing it. Dianne expressed the thought: "What goes in my mind is that it is difficult. I cannot understand it" while Stacey stated that "In my mind the first thing that came to me, I like math but math is difficult". Natalie also claimed that she liked the subject but did not choose it because it felt difficult. Cyril was aware of the importance of studying mathematics in that it offered more lucrative job opportunities "but because I don't understand it, I can't to it." Pamela felt that had she been more competent, she would not have "thrown away the chance."

Railside learners believed that mathematics is difficult. Stress was mentioned by one learner while another found it difficult and time-consuming. Kevin thought that mathematics means "someone asking me questions I don't know the answers to..." Rianna pointed out that "It's a difficult subject, all the fractions and hard sums – and it's time-consuming" while Felicity expressed the view that had she been performing as well in grade 9 as she had in earlier grades, she would have chosen mathematics. Philip was concerned about failing mathematics: "Failing sir, now that's the big problem. When you are in grade 10, fail two subjects and you are gone. Sometimes you are... not so sure about your answers. You get all confused"

Commenting on the CTA, Justine observed: "You know what, sir? We were lost. We don't know what we were doing. My maths teacher last year told me I shouldn't do maths this year because it will bring my whole grade down. I wanted to do accounting, but I needed maths." Rianna noted that she had always experienced difficulties with mathematics: "Maths is difficult. You must be a fast learner – a quick achiever, a person that catches on very fast. And I had a problem with maths from grade 1. In grade 8, I was doing well, but still I was having difficulties."

Philip felt that there was a problem with methodology. He felt that teachers should use "easier methods" but could not elaborate any further. Justine thought that educators should focus more on the weaker learners: "Some children catch on faster than other children and some don't really get to understand. I think the teachers should spend more time with those that are not understanding the work." Rianna was of the opinion that group work could help: "Maybe you should put children in groups. If you catch on more intelligently – then you should help- not to say like others are stupid. And the teacher should repeat her lesson, in case you don't catch on what she said."

Educators thought that learners experienced problems with mathematics because they believed that it is difficult. Andy felt that "There's a stigma attached to maths – that it's difficult". Joe concurred with this view. Henry also thought that the common perception that mathematics is difficult is a deterrent, drawing attention to the fact that learners produce significantly better results in other subjects: "So basically kids have that perception now and it's difficult to change that perception." Joe commented: "They think maths is difficult so therefore we shouldn't take mathematics and some of them think 'maybe I'm not going to a tertiary institution so the science course should not be part of my curriculum".

In the focus group interviews, reasons advanced by educators for poor performance included poor arithmetic background, poor work ethic and a compartmentalised curriculum. Martin felt that teaching learners with insufficient background knowledge presented a challenge; and also pointed to the fact that learners were not prepared "to exert themselves". Betty drew attention to large class numbers; the lack of resources; and the absence of innovative materials. She thought that computer-aided instruction would make the teaching and learning of mathematics more exciting. Joe noted that learners do not complete assignments and as a consequence performed poorly in tests. He identified the teaching of fractions as the most challenging. Andy thought that learners relied heavily on calculators; and that they could not recall work done previously.

Henry and John both felt that poor conceptual knowledge, inability to use formulae correctly, and manipulation of numbers were impediments in teaching mathematics. Learners had the perception that the branches must be dealt with separately: "See they don't see the relationship between algebra and geometry – they separate both" and "Yeah. I think that the problem there is they are looked at in compartments. We teach geometry, then algebra – so they can't relate geometry and algebra. Instead of integrating it, they in fact separate it – trying to figure out why is there geometry in algebra." Martin thought that learners should be grouped into class units according to their abilities. When pointed out that this was the practice at his school, he agreed, but stated that the process was flawed. He found that no feasible reason could be advanced as to why such diversity existed within one class. Martin also made the moot point that the entire senior phase (grades 7 to 9) should be accommodated in the secondary school.

Betty attributed poor performance to a number of factors: "They have not grasped basic concepts and you know when you don't have a good grounding – it affects everything else around that. Um... you know they don't have much help. Students don't have much help from their peers because their peers de-motivate them eh... from their parents – because their parents are unable to assist, from schools like ours... I'm talking about resources; and even teachers because we are so busy trying – you know we got so much to do that we can't focus on the important things." Andy said that in the case of his learners, no work was done beyond the classroom; the assignments that are produced are copied. Joe also had the same problem with his learners: "Lack of discipline on the part of pupils in terms of doing their homework. As mentioned, very little or no extra effort is put into the subject". Interestingly John voiced exactly the same sentiments: "... maths ends when the maths lesson ends. The students don't go and put that extra effort – not any more. Because we finish our maths lesson but then maths is over for the day." Henry found that his learners spent considerably less time on Mathematics than on other subjects.

Mathematics is associated with higher order thinking skills and concerned with abstract concepts. In comparison to other learning areas, mathematics assessment involves more than recall of knowledge; it's the application of skills and knowledge in problem-solving that learners find most challenging. The abstract nature of algebra seems to present many difficulties for learners. It is likely that problem solving is not related to contextual situations as suggested by the outcomes based approach. Educators are probably still adopting traditional approaches in their teaching , bearing in mind that learners have to revert to the old curriculum in grade 10. In comparing mathematics with other learning areas, learners express discontent in that the former requires more application as opposed to recall of facts.

In some schools learners do not continue mathematics because they are not allowed to, as a result of poor previous performance. This was the second most important reason given by learners at Railside (mean: 2.00). However this was a less important reason at Valeview (mean: 0.98). At Railside Secondary, learners have the option of choosing from packages. One course offers Physical Science with Mathematics; another is a commerce course with mathematics and. a third course which includes mathematics is a general one. The remaining 50% of the learners do not study mathematics. Performance in grade 9 is used as a guide in determining which learners are offered the aforementioned science and commerce courses. At Valeview Secondary, learners are also offered packages. The English, NS and MLMMS results are taken into consideration. If these are average or above then learners are allowed to choose the science course. Parents are advised accordingly, but the final decision is left to them. In recent years only one class in four has studied mathematics at the senior secondary level.

4.2 IS GEOMETRY A REASON FOR THE PERCEIVED DIFFICULTY IN MATHEMATICS?

Assertion 2

The perceived difficulty learners experience with mathematics is multifaceted but skewed marginally towards geometry.

In the questionnaire learners were asked to indicate if they found algebra or geometry more difficult to understand. Refer to appendix B (question 3). The results are summarised in Table 4.2.

At Railside, nine out of the 32 positive responses indicated that geometry is more difficult, constituting 28.1% of the total. However 14 of the learners (43.8%) were uncertain. One can therefore conclude that geometry does not stand out as a problem area at this school. The situation was different at Valeview where 27 out of 51 participants (52.9%) felt that geometry presents more difficulties, almost four times as many who opted for the algebra option. This analysis indicates that there is a leaning towards geometry being more difficult, but the evidence is not substantial.

Table 4.2 Learners' beliefs about the relative difficulty levels of algebra and geometry

	RAILSIDE	VALEVIEW	TOTAL
Algebra is more difficult	5	7	12
Geometry is more difficult	9	27	36
Neither is more difficult	4	3	7
Not sure which is more difficult	14	14	28
Total number of responses	32	51	83
Inappropriate or no response	9	2	11

It is to be expected that algebra should present more problems as it is a branch in mathematics that is only introduced in grade 8, whilst geometry concepts are studied from grade 1. Perhaps geometry becomes problematic as the approach becomes more Euclidean, with a shift to theory. However what is clear is that a large percentage cannot decide where the problem lies. From grade 10 onwards geometry is a major problem area.

In the group interview, two of the seven learners at Valeview thought that algebra was a major problem with 'solving for x' expressed as an area of concern. Three learners had difficulties with geometry with one mentioning angles associated with parallel lines in particular; and the other two felt that "everything is difficult." Railside learners thought that both aspects of mathematics, at the grade nine level are difficult. Philip's concern was that a lot of thinking is required in that "...if you are going to be studying for the exam, you won't find those exact answers in your book and in the paper". In comparing mathematics with other learning areas, learners express discontent in that the former requires more application as opposed to recall of facts. This was also evident in Justine's comment: "...and another thing too about maths – you have to sit and figure out all the numbers and problem-solve all those numbers.

All the educators were of the view that learners' performance in geometry is worse than that in algebra, but pointed out that this was more so at the senior secondary level. Betty was of the belief that algebra is more problematic in the junior classes, and learners had more difficulties in geometry from grades 10 to 12. She believed that less attention should be focused on the theorems themselves; more time should be spent on developing problem-solving skills. Martin felt children had a dislike for geometry: "You find most children saying, "I'll take algebra anytime, and yet it's a foreign concept compared to geometry which they have been doing all the time." Andy also thought that geometry at the senior secondary level is more difficult. He believed that learners failed to see relationships between concepts.

Joe said that his learners failed to apply the theorems in solving riders and even if they arrived at a solution they failed to communicate it effectively in writing. Henry found that even at the grades 8 and 9 levels, learners experienced difficulties in geometry: "I think the biggest problem is to logically solve a problem, in other words failure to find a logical starting point. Give any kid a problem, he struggles to conceptualize – this is x and therefore the other angle has to be $2x$ – he doesn't have a starting point." Henry thought that more time should be devoted to teaching problem-solving strategies. John was emphatic in naming geometry as the problem area. His experience is that even if learners understand concepts they fail to write correct statements: "The other thing is they understand the concepts, say like alternate angles are equal but when they write it down they say that plus that equals 180 degrees." Sometimes long and drawn-out statements are written because learners are not sure of their solutions: "... they try to write...you know when they write an essay – I mean that's how I feel sometimes...they are trying because they don't know and they put anything in the question, just to ..."

Whilst the learners differed in their views about their abilities in geometry, all the educators were convinced that geometry contributed to poor performance. Strangely, learners seem to be unaware of their lack of understanding of geometric concepts and their inability to solve problems. This was also evident in the geometry test where learners rated the question 'very easy', having given an incorrect answer.

4.3 ARE LEARNERS READY FOR MATHEMATICAL LITERACY?

Assertion 3

Learners are not ready to cope with Space, Shape and Measurement in Mathematical Literacy.

In analyzing the responses to the test items, attention is drawn to the high frequency of errors and common misconceptions. This is supported by quoting some of the answers that were given. Reference is also made to grade ten Mathematical Literacy texts where similar types of examples may be found. It must be borne in mind that concepts that have been tested have been taught in previous grades.

Table 4.3 shows the means of the scores obtained by the participants in the test that was given. Scores were converted to percentages.

Table 4.3 Mean percentage scores obtained by learners in the geometry test

Question	Railside		Valeview	
	Male	Female	Male	Female
1	47	22	25	25
2	19	12	22	29
3	38	31	63	49
4	47	22	35	21
5	13	15	9	14
6	7	0	0	1

As can be seen, the scores are remarkably low. Learners' understanding of concepts relating to space, shape and measurement is poor. Furthermore, 11 of the 41 learners at Railside did not attempt any of the questions, with four indicating that they did not know any answers. The responses to each question are looked at in some detail.

Question one: Recognition and estimation of angles

Question one is found in Appendix C. Learners were required to identify an angle according to its shape (three alternatives were given) and then give an approximate measurement. The intention was to direct attention to the obtuse angle, having a measurement of 130°. Unfortunately, due to an oversight, the angle was not marked in the diagram. Eleven learners chose the first option, that is, acute. Answers for the measurement ranged from 7 degrees to 170 degrees. Eight learners thought that the angle is reflex, which is a correct answer, but gave obtuse measurements. Of those that chose the obtuse option, six gave no estimation of the measurement, whilst many others gave acute measurements. Only nine learners (9.6%) gave a reasonable estimation. There was difficulty in recognizing angles although these were defined in the question. This is a similar finding to that obtained in TIMSS-R in 1998 (question N15) where 32.7% of South African learners were able to estimate an angle closest to 30°, indicating a persistent problem in angle estimation.

In question L8 where learners had to estimate the height of a tree, given the height of a boy standing next to it, 21.8% gave the correct answer. (Refer to Appendix K.) This indicates that learners do not have the ability to approximate values. In the Mathematical Literacy curriculum Assessment Standard one of the Shape, Space and Measurement Learning Outcome requires that learners have the ability to estimate and identify angles and questions of this nature are found in the Mathematical Literacy textbooks. (See, for example, Aungamuthu et.al (2005, pp. 119 – 122)).

Question Two: Calculation of area and the use of the theorem of Pythagoras

Learners had to calculate the area of a rectangle; and were required to calculate the length of the diagonal. The application of the theorem of Pythagoras was not explicitly stated. (Refer to Appendix C). There was difficulty in choosing the correct area formula and as a result the perimeter was calculated instead of the area. Forty two learners, comprising a percentage of 44.7% were able to calculate the area correctly. There were errors in calculation and the feasibility of the answer were not assessed e.g. hosepipe length given as 35 mm or 5 cm. Learners cannot distinguish between such basic geometric constructs such as points, line segments and angles; they are not operating even at the lowest Van Hiele level. The majority of the learners did not answer the second part of the question in which the Theorem of Pythagoras had to be used, most probably because they did not recognize it as such. Some answers that were given include: (a) A + B + C + D = 7,5m; (b) ABCD = 1750°; (c) $10^2 + 7{,}5^2 = 1600m^2$;

(d) 75m or 17m and (e) L = ½ A – b

In TIMSS-R, (Appendix K) only 12.4% gave the correct answer to J10 (calculation of the area of rectangles) while 49.7% chose the option in which the lengths and breadths of the two rectangles were added. In K5 (calculation of the area of a rectangle, given the perimeter and length) only 3.4% respondents were successful with 31.2% choosing the option giving the sum of the perimeter and length; and 14.7% choosing the option giving the product of the two. In the Mathematical Literacy curriculum all the concepts covered in this question are found in the first assessment standard in grade 10. With respect to the Mathematical Literacy textbooks, questions on area are to be found in Long et.al. (2005, pp. 58 -63). See also Aungamuthu et.al (2005, p. 115); Goba et.al. (2005, pp. 146 – 147); and Roberts et.al (2005, pp. 203 – 210). Applications on the Theorem of Pythagoras are made reference to as follows: Vermeulen (2005, p. 143); Aungamuthu (2005, pp. 111 – 113) ; Long (2005, p. 56 and Goba (2005, p. 163).

Question Three : Calculation of perimeter

Learners had to measure the sides of a pentagon and determine the perimeter. (Refer to Appendix C). This was a simple question which was relatively well answered. Twenty of the 94 respondents measured incorrectly. There was failure to approximate and hence decide whether the answer is realistic. Twenty five learners did not know what perimeter means, seven confusing it with area. Tanner et.al. (2002) draw attention to the fact that many British learners have difficulty making a distinction between area and perimeter. Eleven learners tried to find a formula to determine the perimeter. Some answers that were given include:

P = 2(length + breadth); P = length x breadth; $P^2 = h^2 \times w^2$.

Measuring of lengths and the concept of perimeter are found in the first Assessment Standard in grade 10 so these skills will be required in Mathematical Literacy. Textbooks that were referred to, discuss the perimeter of regular figures such as triangles, rectangles, circles or a combination of these: See, for example, Goba (2005, pp. 145 – 147); Aungamuthu et.al. (2005, p. 109); Vermeulen et.al.(2005, p. 141); Roberts et.al (2005, p. 201). As in the previous question, it is found that learners are not operating at the second Van Hiele level as they would be expected to.

Question Four : Identification of rectangles

Learners were required to name all the rectangles in a diagram; one was named as an example. Whilst four were distinct, one of which was a square, six were embedded. The most common error encountered was the naming of the vertices of the rectangle in the incorrect order e.g. AEPT instead of AETP. Table 4.4 shows the number of rectangles identified by learners.

Table 4.4 Number of rectangles identified in the geometry test

	Distinct rectangles				Embedded rectangles					
Number of rectangles	1	2	3	4	1	2	3	4	5	6
Number correctly identified	7	11	34	28	1	4	7	2	4	4

Learners did not consider all the possibilities. Twenty eight of the 83 learners who answered the question, were able to identify the four distinct rectangles (including the square), but as can be seen there was an inability to recognize those being made up of two or more rectangles such as ADSP. It is also found in British schools that learners have difficulties in finding embedded shapes (Tanner, Jones, & Davies, 2002). At the grade ten level, the identification of rectangles is implied in problem solving, so learners are apt to experience difficulties if they cannot identify them.

Question Five : Description of a sketch.

There was failure in expressing observations in a correct manner as a result of language difficulties. The square within the circle was referred to as a "diamond", showing that orientation of shapes is important in identifying them. Tanner et.al. (2002) also mention that shapes are identified by orientation and that this arises in misconceptions such as defining straight lines as horizontal lines. The circle was identified as a "round shape". At the grade 10 level in the Mathematical Literacy curriculum, learners are expected to describe and compare properties of plane figures. The educator will most likely assume that the learners are able to identify these figures. Again it is found that some learners are not even operating at the lowest Van Hiele level. The learners were unable to explain how to calculate the area of the shaded section; again language problems are evident. Some explanations make no sense. Some answers that were given include: (a) Multiply the area of the square and the circle, (b) Measure all the sides and subtract the shaded part and (c) I will add the area of the circle with the square and divide it with the triangle.

None of the grade 10 Mathematical Literacy texts referred to ask learners to describe observations although this is one of the requirements in assessment standard 3 (Department of Education, 2003). This is unfortunate since I think that verbalizing observations will assist in problem solving as the educator will have an insight into the learner's thought processes.

Question Six: Calculation involving parallel lines and triangles

Learners were required to calculate an angle of a triangle using the concepts of parallel lines and isosceles triangles. Sixty learners answered this question; 34 offered no response. Of the 60 respondents, 57 obtained a mark of zero; three attained one mark each. Many indicated that it is too difficult. Although it can be seen that the angle is acute, answers such as 180^0 and 418^0 were given, again indicating that learners cannot even cope with VH level one questions. This is in agreement with responses given in question one where learners failed to estimate the angle.

Some answers that were given are as follows (a) $116^0 - 180^0 = -64°$, (b) A + B + C = 580,

(c) B = 58 x 100 = 5800, (d) 58° – 180° = 122° (e) Area of the triangle is 180°,

(f) $360^0 - 114^0 = 244^0$; $244^0 + 58^0 = 302^0$

Questions of this type are found in the Mathematics curriculum rather than in Mathematical Literacy. However, taking into account the concepts of parallel lines and isosceles triangles taught in grades eight and nine, learners should have been able arrive at the correct answer. The fact that learners arrive at answers that are unrealistic, and do not have the capacity to evaluate the validity of the answers, shows that skills and knowledge in geometry are lacking. This problem has long been evident in South Africa as the TIMSS results indicate. In TIMSS-R (question O3), related to identifying a pair of supplementary angles involving parallel lines, 20.6% were successful with 33.4% choosing the option of two obtuse angles. In Q10 which was a more challenging question involving adjacent angles, only 6.8% of the respondents gave the correct answer. (Refer to Appendix K).

I obtained the marks attained by learners in the grade 9 examination. These are marks of learners who had participated in the survey, that is, completed the questionnaire and attempted the geometry test. The raw scores out of 100 that were obtained by continuous assessment (CA: 75%) and Continuous Tasks for Assessment (CTA: 25%). Table 4.5 gives a summary of these marks. The actual marks are found in Appendix J.

Table 4.5 An analysis of marks obtained by learners in the grade 9 examination (expressed as percentages)

School	Gender	No of respondents	Minimum mark	Maximum mark	Mean	Standard deviation
Railside	Males	12	14	38	29.5	7.882
	Females	29	12	41	29.2	7.464
Valeview	Males	15	16	41	31.1	6.696
	Females	38	17	42	29.6	6.421

4.4 DO LEARNERS DISLIKE GEOMETRY?

Assertion 4

Learners do not have a negative attitude to the study of geometry.

I asked learners to indicate their feelings about 12 mathematics topics. This item in the questionnaire may be found in Appendix B. The responses of learners were coded with the score ranging from -2 for strong dislike to 2 for strong like. For further details refer to the codebook in Appendix I. The results are shown in Table 4.6, and further illustrated in Figure 4.3.

Table 4.6 Table of means showing attitude to mathematics topics

TOPIC	RAILSIDE MEAN	VALEVIEW MEAN
Ratio and proportion	0.15	0.52
Statistics	-0.21	0.86
Substitution	0.15	.050
Products	0.65	0.76
Factorization	-0.03	0.73
Solving equations	0.03	0.91
Operations on polynomials	-0.26	0.26
Parallel lines	0.15	0.52
Triangles	0.68	0.79
Congruence	-0.46	0.57
Quadrilaterals	0.00	-0.11
Solving problems in geometry	-0.13	0.24

A negative mean shows that there is an overall dislike for the topic. The results show that Railside learners dislike congruence the most and this is followed by polynomials, statistics, factorization and geometry problem-solving. Congruence is probably liked the least because it involves abstract geometry. Perhaps teachers still favour the traditional theoretical approach and it is unlikely that transformation geometry is being used. The work on polynomials also has a degree of abstractness and the type of questioning is probably not related to context. Interestingly parallel lines are quite popular and the section on triangles comes out as the most popular. At Valeview, on the other hand, learners expressed a more positive attitude to the various section listed. Only quadrilaterals had a negative mean. The least-liked sections were geometry, problem solving; polynomials; and substitution. The section on parallel lines was rated as the most popular. At both schools neither algebra nor geometry showed up as more popular.

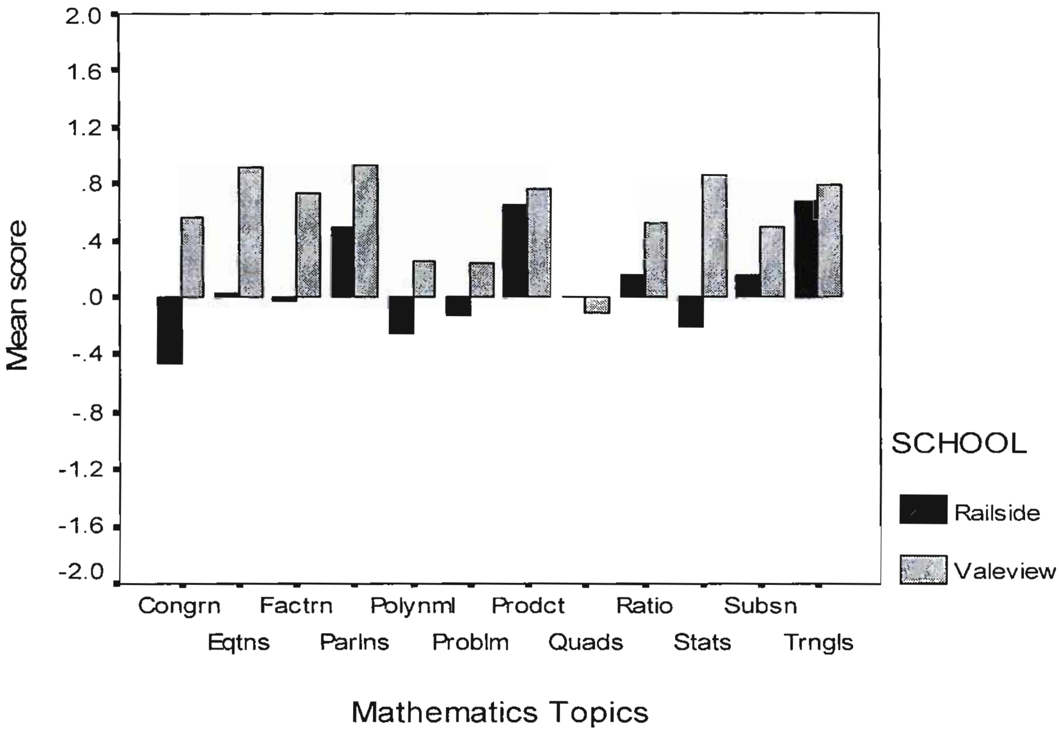


Figure 4.3 Learners' attitudes to mathematics topics: comparison by schools

In order to further evaluate attitudes to geometry, I asked learners to rate, on a 5-point scale, their feelings on activities they may engage in when working with space, shape and measurement. The questionnaire is in Appendix B. In the analysis, the scores were again coded from -2 (strong dislike) to 2 (strong like). Figure 4.4 compares responses at the two schools, while Figure 4.5 compares male and female responses. The activities, as they are represented in the graph, are as follows: working with a calculator, drawing shapes like rectangles and circles, drawing a plan of a house, working with mathematical instruments, measuring lines and angles, working with numbers, calculating perimeter, area and volume, drawing house plans. However these differences are not substantial. The only negative mean is that of calculating perimeter, area and volume as expressed by males, but again this is negligible.

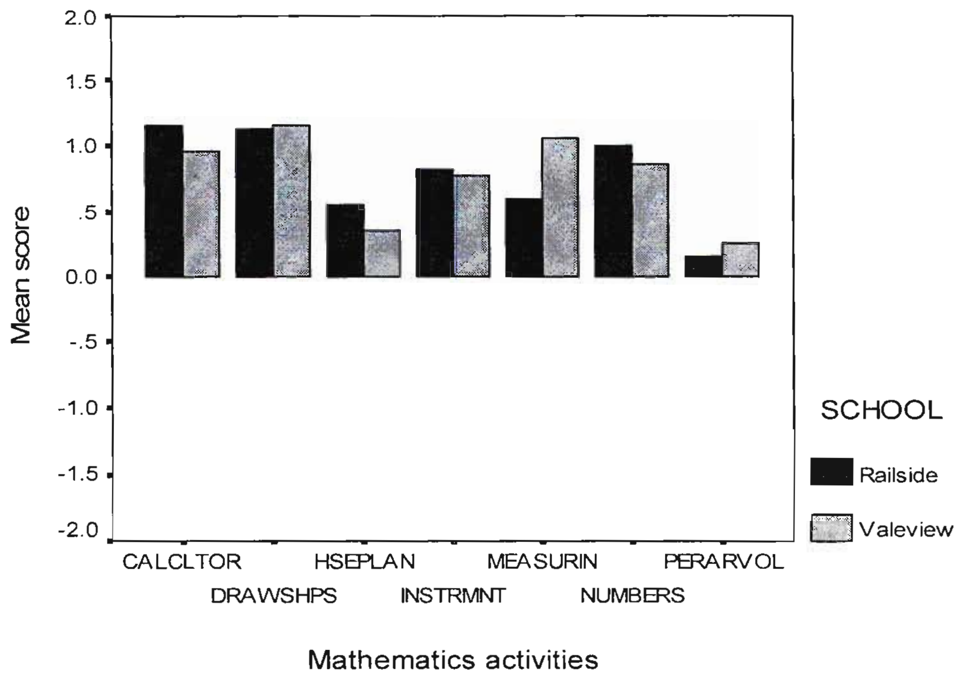


Figure 4.4 Attitudes to mathematics activities: comparison by school

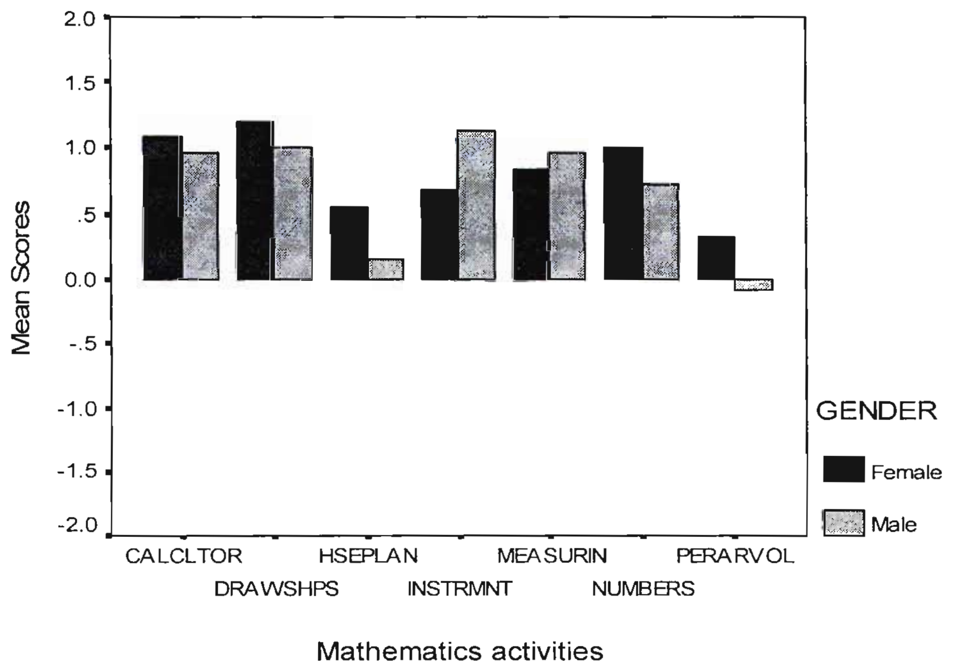


Figure 4.5 Attitudes to mathematics activities: comparison by gender

Educators were asked whether they preferred teaching algebra or geometry. The intention was to determine if they were inclined to focus on one at the expense of the other, or if they inadvertently or otherwise conveyed negative perceptions to the learners. Martin and Betty did not prefer one over the other, noting that whether teaching algebra or geometry, they found introducing new concepts to learners very enjoyable. Andy expressed a leaning to algebra while the Joe felt geometry more interesting : "Well, I am very confident in teaching both these subjects and I really enjoy the geometry aspect – you know when you have a really lovely problem and you see you can arrive at a solution in three or four different ways. It is like a very good story – arriving at a conclusion, and I hope kids can see it like that also."

John also indicated a preference for geometry: "To me geometry is much more interesting. I like geometry because it involves a lot of thinking, it's not straight-forward – with a lot of rules you can apply – so many different variations that you can use to get to the answer. Now that's the reason why I like geometry. In terms of algebra the reason why it becomes monotonous…" But John emphasized once again that learners prefer to do algebra; and Henry concurred: "… although geometry is much more interesting than algebra, but from the kid's point of view, when you say you are doing geometry, you hear the buzz in the class – and if they can't solve they don't try, they simply leave it alone but in algebra you see some attempt being made each time." These educators are enthusiastic about teaching geometry but concede that learners still have a preference for algebra. The learners' diminished interest in geometry is probably related to their poor performance; and not to the attitude conveyed by educators.

4.5 LEARNERS' VIEWS ON THE CONTINUED STUDY OF MATHEMATICS

Learners felt that they should study mathematics to enhance career opportunities. At Railside 90.9% of males and 64.3% of females were concerned that they are not continuing mathematics beyond grade 9. The percentages at Valeview were 86.7 and 71.4 for males and females respectively. Fewer females are worried about not continuing mathematics. This is consistent with the historical trend of more males pursuing scientifically-orientated careers, with many females either entering the humanities field or not studying beyond secondary level. This is unfortunate as gender equity has been declared a priority in the new democratic South Africa.

Learners were asked whether they would like to study real-world mathematics. At Railside 72.7% of the males and 73.1% of females indicated that they would study mathematical literacy. The learners at Valeview showed more enthusiasm with males returning a positive response of 86.7% and females 81.1%. These learners are keen to study Mathematics but find it beyond their capabilities; and possibly see Mathematical Literacy as a compromise.

4.6 CONCLUSION

Although learners have an encouraging, positive attitude to the study of mathematics concepts and development of skills, they are severely lacking in basic prerequisites. Now that learners do not have the option of terminating the study of mathematics at the end of grade 9, concerted efforts are required to by all stakeholders to ensure that learners entering the Mathematical Literacy classroom are sufficiently prepared to meet the challenges. In the final chapter which follows, I offer some explanations for the observations made at the two schools.

CHAPTER 5

CONCLUSIONS AND IMPLICATIONS

Studies of examination statistics show that about 42% of South African learners do not continue the study of Mathematics in grade 10 when it becomes optional (Mukwevho, 2003). With the introduction of a new curriculum in 2006, those learners who do not study Mathematics must study Mathematical Literacy. These case studies were carried out to determine what factors influence the decision to continue or terminate the study of mathematics and what influence the study of geometry has on the decision. These studies also tried to find out the abilities and attitudes to mathematics in general, and geometry in particular, that learners are expected to bring into the Mathematical Literacy class. Information was gathered from questionnaires administered to learners, learner focus groups, focus group interviews with educators and analysis of grade 9 examination results.

In this chapter I offer some possible explanations for the poor geometry ability found at the two schools, presenting them from of the learners' perspective, educators' perspective and the curriculum. Finally I comment on possible shortcomings and constraints within this study; and conclude by making some recommendations.

5.1 LEARNERS' ABILITIES, ATTITUDES AND BELIEFS

Secondary school educators claim that learners advance through the intermediate phase in primary school without mastering basic concepts. As a result they have to teach learners with poor pre-knowledge and skills in geometry. In discussing geometry education, de Villiers (1997) is in agreement with this claim. Van Niekerk (1998) also questions the kind of work done at the primary school level. Learners, however, in claiming that mathematics is difficult, make little reference to primary school experiences. They do admit that mathematics is considerably easier in grade seven compared to grades eight and nine.

With respect to geometry, there is very little or no progress through the Van Hiele levels. When learners are placed in grade 8 in the secondary school, the latter should have passed through levels 1 and 2, that is, visualization and analysis. However, the educators who participated in this study point out that the learners cannot see relationships between concepts, and thus have a tendency to write long and drawn-out meaningless statements in geometry. De Villiers (1997) notes that learners only reach a particular level once they have passed through the preceding ones; age and level of maturity do not have a bearing. The best intentions and interventions by secondary school educators are of no consequence if there is mismatch between the learner's ability and the demands of the curriculum (Feza & Webb, 2005). As the results of this study show, learners who are completing grade 9 have still not mastered Van Hiele level one – they are, for example, unable to recognize rectangles.

Analysis of the results in the TIMSS-R study, with respect to geometry items, also reveals that learners do not understand concepts they should have mastered in earlier years. There is confusion over concepts such as length, breadth, area and perimeter: when learners were given the length and perimeter of a rectangle and asked to calculate the area, they simply found the product of the given quantities. Furthermore there is a lack of ability to estimate, and as a result, inability to assess the feasibility of answers. Multi-step calculations seem to present even more challenges.

Contrary to expectations, this study reveals no conclusive evidence to indicate that learners have a negative perception of geometry. When I started this study I assumed that learners do not have favourable experiences in geometry, and this area of study will be cited as the main concern. Even though they underachieve and they acknowledge that it is more difficult than algebra, learners indicate a liking for geometry topics. Forty seven learners listed geometry topics when they were asked to write about their favourite section in mathematics, while 33 listed geometry topics under the least favourite. The survey by questionnaire showed that geometry involving congruent triangles is liked the least, probably because the approach is more formal, leading on to more structured geometry in grade 10. Educators believe that where there are negative perceptions, these could be shaped by senior students who have experienced geometry beyond grade 9.

The language of mathematics presents a challenge to learners. Justine, a learner at Railside Secondary School, made the following comment in the focus group interview: "I don't understand the teacher even if I listen to her so many times". Mathematics is a language of symbols and if a learner has not internalized these symbols and acquired the necessary technical vocabulary, teaching and learning are meaningless. In geometry each van Hiele level has its own language and symbols (referred to as distinction). Feza and Webb (2005) found in their studies that teachers use terminology that could be understood by learners who are on the third Van Hiele level. Failure to master the language at any level will impede a learners progress at the next level.

Furthermore, in South Africa, with eleven official languages, the vast majority of learners learn mathematics in their second language. At the schools where the case studies were done, English is the medium of instruction, but the majority of learners speak isiZulu at home. These learners face a double challenge – to understand the language of instruction and to understand the language of mathematics. In their study, Feza and Webb (2005) found that lack of competency in English (the language of instruction) is a barrier to learning geometry. Hobden (2003) notes that pre-service teachers also struggle to understand mathematics taught through the medium of English. Barnes (2005) draws attention to the fact that learning mathematics in a second language is one of the causes of low achievement. Tobias (2003) points out that it is more difficult to communicate and understand mathematics than an ordinary language. She goes on to say that mathematical language has levels of technical terms that are structured as a hierarchy. The teacher needs to determine the linguistic level at which the student is operating, so that there is no discrepancy between 'teacher-talk' and 'student-talk' (Tobias, 2003, p. 25).

Learners inexplicably believe that achievement is attainable without hard work. Although learners at the schools surveyed expressed a keen interest in producing good results, educators note that they are not prepared to work diligently. Homework assignments and even classroom-based tasks are not done. Educators express the concern that the study of mathematics ends with the termination of the mathematics lesson. The TIMSS study has revealed that learners believe that achieving good results in mathematics is beyond their control and luck is needed to do well (HSRC, 1997). If such a misconception cannot be dispelled, then the best efforts of educators are in vain. At Valeview and Railside secondary schools, there is an ongoing erosion of the culture of learning. This process has the most impact on the study of mathematics and science, subjects that usually require more application of knowledge than others. As marks are adjusted at schools to allow learners to pass grade 9, something that learners are aware of, there is no concerted effort to do well. This attitude also persists in grades ten to twelve.

Poor attendance is also a factor that has a bearing on results. A HSRC (2000) media release reported that according to TIMSS-R, learners in South Africa spent less time on learning activities in the classroom, in comparison to learners elsewhere. It was found that in some schools as few as 120 days per year were spent on teaching and learning. This was compounded by the fact that the attendance of grade 8 learners in 69% of the schools was poor; they arrived late at school, and absconded from classes (HSRC, 2000). TIMSS-R focused on grade 8 learners; the attendance problem probably applies to other grades also. At the schools surveyed, educators reported that when learners have completed a term test, they do not attend school for the rest of the term.

The Culture of Learning, Teaching and Services (COLTS) initiative, driven by the National Department of Education, seems to have made little impact in the long term – it has not been a sustained effort. While there are many schools of excellence, both urban and rural and from previously segregated departments, there are many more that are not achieving the results they ought to.

5.2 EDUCATORS OF MATHEMATICS

At the schools where this study was conducted, educators are qualified specialists as is the case in most urban schools (Howie, 1998). Learners had no complaints about the quality of instruction, or the confidence and ability of the educator. However with the introduction of Mathematical Literacy, non-mathematics educators will be drawn into the mathematics department. Educators are already being re-skilled to teach in the mathematics field. In the short term educators with limited or no formal training in mathematics education, face the challenging task of facilitating the study of mathematical knowledge and skills. Furthermore they will be interacting with learners who would otherwise have terminated the formal study of mathematics in grade 9, learners with limited basic knowledge and skills and perhaps an unfavourable attitude to mathematics. Hobden (2003) found that learners who discontinued mathematics in grade ten were influenced by the personality of the mathematics teacher and expresses the concern that educators "recycled" from redundant subjects or novices may not have the empathy and patience to teach Mathematical Literacy.

At the time that the TIMSS study was done it was found that 27% of mathematics educators have no formal qualification in the subject and they are found predominantly in the rural areas (Howie, 1998). Furthermore TIMSS-R found that only 16% of mathematics educators were older than 40 years, an indication that they are leaving the profession early (HSRC, 2000). Primary schools also do not have sufficient teachers of mathematics and others are asked to assume this position. In a systemic evaluation study conducted by the KwaZulu Natal Department of Education, it was found that 30,2% of primary school educators were either unqualified or under-qualified (Sithole, Nkosi, Gudazi, Zungu, & Biyela, 2003). This has serious implications for the development of basic concepts. Personal experience informs me that learners are even taught incorrectly, and it is very difficult to remove these misconceptions. It is important for mathematics educators to determine the pre-knowledge of learners. What is required is "...an inquiry approach where teachers and learners engage in conversation that allow teachers to ascertain prior knowledge, language use, cultural frameworks and levels of understanding of learners in terms of a particular topic..." (Feza & Webb, 2005, p.45). Hobden (2003) recommends that educators develop creative strategies to facilitate learning in mathematics; and suggests that effort must be made to give attention to learners in small groups.

At Valeview and Railside educators stated that although they are reasonably satisfied with their working environment, they find large classes difficult to work with.

Many South African educators try to deliver under challenging working conditions which have an impact on results. This is more so among the African (formerly DET) schools, and to some extent ex-HOD and ex-HOR schools. Some schools lack the basic infrastructure, have over-crowded classrooms and offer a service to learners from dismal socio-economic circumstances. With over 50 learners accommodated in a small room, extremely limited material resources, and exposure to the elements in some cases, it is remarkable that some mathematics learning takes place. The systemic evaluation study in KwaZulu Natal revealed that up to 23% of educators were unhappy about their jobs (Sithole et al., 2003). TIMSS has also revealed that South African educators are required to perform more administrative tasks than their counterparts in other countries (HSRC, 1997). This is an unsatisfactory situation as educators should be concentrating their efforts on facilitating the learning process. At Valeview Secondary School, for example, educators are required to collect school fees from learners. Record-keeping is also a tedious task, and being required to maintain a large (often unnecessary) database consumes much of the educator's time.

Resource materials were found to be lacking in the schools that were surveyed. Educators expressed the view that lack of sufficient and appropriate learner support materials (LSM) is a deterrent in promoting learning. This is a problem that presents itself in most ex-HOD, ex-HOR and virtually all ex-DET schools in the province of Kwa-Zulu Natal. Schools are allocated funding according to their quintile (previously decile) ranking. Those in urban and peri-urban areas are ranked highly, having amongst other facilities, tapped water, electricity and tarred access roads. The limited funding they receive is spent on paying for essential services; there is not enough to purchase resource materials. These schools also have relatively low fees to cater for learners from deprived socio-economic backgrounds.

With the introduction of the new curricula, NCS and FET, the state is providing learner support materials in the form of textbooks. While this endeavour is welcomed, provision needs to extend to other resources. The computer, for example, has become an essential learning tool. If learners have access to such support materials as mathematics texts and video presentations of lessons; and hands-on experience in working with software; their interest would be aroused and their understanding would most likely be enhanced. Perhaps the perception that mathematics is difficult may change.

5.3 THE MATHEMATICS CURRICULUM

In South Africa emphasis has been placed on congruence geometry, with the influence of Euclid being most pronounced (Human & Nel, 1978). As matriculation examiners' reports show, learners have tremendous difficulties in drawing up proofs. A common error that is encountered is that learners assume a result in order to prove it. In the curriculum that is being phased out, the approach to geometry has been formal. Usiskin (1987) points out that learners struggle with formal geometry; and Niven (1987) draws attention to the fact that geometry is not sufficiently related to the real world. The abstract nature of mathematics in the outgoing curriculum, and the fact that it is taught separately from other space and shape concepts, contribute to persistent poor results.

The FET mathematics and Mathematical Literacy curricula are based on the principles of real-world mathematics. Presumably it is believed that learners will be more interested and stimulated to perform well. Learners in the focus group interviews, more especially those at Railside Secondary School expressed enthusiasm in engaging with real-world mathematics. If learners can see mathematics at work; not restricted to numerous artificial textbook examples, and confined to the domain of the classroom; then perhaps they will be more willing to acquire the requisite skills and knowledge.

5.4 SHORTCOMINGS AND CONSTRAINTS WITHIN THIS STUDY

The investigation was restricted to two schools: one previously under the former House of Delegates (HOD) serving the Indian community, and the other previously under the House of Representatives (HOR) serving the Coloured community. These schools are similar in that since the lifting of apartheid, large numbers of African learners have been attending them. The findings may be extended to schools with similar demographics. However the study would have been strengthened by the inclusion of an ex- Department of

Education and Training (DET) school with predominantly African learners. This was not possible because of problems in accessibility and time constraints.

A larger sample of learners at each of the schools would have enhanced the study. Unfortunately some of the learners who were eligible to participate refused to do so. The fact that a larger proportion of male learners declined the invitation could mean that the findings do not accurately reflect the situation in terms of gender.

5.5 RECOMMENDATIONS

In order for South Africa to move up the TIMSS rankings and, more importantly, produce more acceptable mathematics results at the grade 12 level, I think that the culture of learning and teaching needs to be addressed. There has to be more accountability on the part of learners, parents and educators where this is lacking. Whilst underachieving schools may lament the lack of resources and suitably qualified personnel, schools of excellence are not classified as such because of resources. Learners at these schools are supported by parents and driven by high expectations and the will to excel. Newspaper reports have shown that even under-resourced schools can do well. Learners, parents, educators and school management personnel need to forge a partnership to uplift the culture of learning and teaching in the community. The study of mathematics and science at the FET level needs to be encouraged, together with a healthy work ethic.

Schools need resources to function. Whilst the chalkboard has served generations of South Africans, it is inadequate as a teaching tool in a rapidly growing technological world. Schools need to be equipped with printed material, audio-visual aids, computers and software, and access to the internet. These should be provided by the state as part of the obligation it owes to its citizens; an investment for the future. The roll-out may take many years, but it ought to be started soon.

Whilst this study looks at readiness for Mathematical Literacy, the study of mathematics needs to be encouraged. Learners claim that mathematics is difficult and thus discontinue its study in grade 10. If universities allocate more points to mathematics in their entrance qualifications, perhaps learners may persevere in its study. There is a tendency for learners to opt for so-called easy subjects, which again relates to the culture of learning found in this country. There is also a belief that it is acceptable to have poor results in mathematics because of the perceived difficulty. In Britain it is traditional to accept that a proportion of learners will be low attainers due to low intelligence or poor background (Tanner et. al., 2002). We must guard against expecting and accepting too little of our learners and instead follow the trend set in high-achieving Pacific Rim countries where 'catch-up' programmes are offered to low attainers (Tanner et al., 2002).

The time spent on learning and teaching in South African schools needs to increase. Although there are 200 school days a year a substantial number are used for examinations and tests. As pointed out earlier learners in many schools do not return after the completion of the examination, having an extended holiday. A minimum number of 220 school days is recommended. It is not feasible to extend the school day as some learners travel some distance to school and back.

In the Mathematical Literacy classroom it is predicted that the educator will have to re-teach concepts that were previously covered. Baseline testing on shape, space and measurement early in the year will help to assess learners' pre-knowledge, and will guide the educator in planning a revision programme. Educators need to be aware that Mathematical Literacy textbooks assume that learners have all the skills and knowledge pertaining to the previous work covered.

5.6 CONCLUSION

I believe that the learners at the two schools, who elect not to continue Mathematics, are not ready to study Mathematical Literacy; and this is going to pose a major challenge to educators. Perhaps learners have been justified in dropping mathematics in grade 10, seeing as how they have been performing so poorly. If the study of Mathematical Literacy was optional, I think many learners might choose not to do it. While learners do not show any negative thoughts to the study of geometry in particular, their general attitude to the study of mathematics is one of apathy. Learners are not prepared to work harder and smarter; and noting that the study of Mathematical Literacy requires insight and logical reasoning, they are going to struggle in the Mathematical Literacy class. Educators are concerned about the situation, and feel frustrated with the persistently poor performance, and lack of commitment.

The performance in Mathematical Literacy seems destined to be mediocre. Considering the fact that a mark of 35%, according to the National Qualifications Framework (NQF), constitutes a pass, shows that poor results are anticipated. Furthermore adjustments will be made to ensure that learners pass. This is the present situation with standard grade mathematics where the results are extremely poor. Under these circumstances, the extent to which the objectives of producing "a self-managing person, a contributing worker and a participating citizen in a developing democracy" (Department of Education, 2003, p. 10) will be achieved, is a matter for speculation.

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APPENDICES

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APPENDIX A ETHICAL CLEARANCE LETTER

UNIVERSITY OF
KWAZULU-NATAL

RESEARCH OFFICE (GOVAN MBEKI CENTRE)
WESTVILLE CAMPUS
TELEPHONE NO.: 031 – 2603587
EMAIL: ximbap@ukzn.ac..za

12 MAY 2006

MR. KS PILLAY (203512712)
EDUCATION

Dear Mr. Pillay

ETHICAL CLEARANCE APPROVAL NUMBER: HSS/06060A

I wish to confirm that ethical clearance has been granted for the following project:

"Space, shape and measurement in mathematics literacy: Implications for the mathematically-challenged learners"

Yours faithfully

....
MS. PHUMELELE XIMBA
RESEARCH OFFICE

PS: The following general condition is applicable to all projects that have been granted ethical clearance:

THE RELEVANT AUTHORITIES SHOULD BE CONTACTED IN ORDER TO OBTAIN THE NECESSARY APPROVAL SHOULD THE RESEARCH INVOLVE UTILIZATION OF SPACE AND/OR FACILITIES AT OTHER INSTITUTIONS/ORGANISATIONS. WHERE QUESTIONNAIRES ARE USED IN THE PROJECT, THE RESEARCHER SHOULD ENSURE THAT THE QUESTIONNAIRE INCLUDES A SECTION AT THE END WHICH SHOULD BE COMPLETED BY THE PARTICIPANT (PRIOR TO THE COMPLETION OF THE QUESTIONNAIRE) INDICATING THAT HE/SHE WAS INFORMED OF THE NATURE AND PURPOSE OF THE PROJECT AND THAT THE INFORMATION GIVEN WILL BE KEPT CONFIDENTIAL.

cc. Faculty Research Office (Derek Buchler)
cc. Supervisor (Sally Hobden)

APPENDIX B QUESTIONNAIRE

MATHEMATICS SURVEY : QUESTIONNAIRE

Dear learner
I thank you for participating in this survey. My aim is to try to find out why learners do not study mathematics from grades 10 to 12. As you may know those learners who do not continue mathematics from 2006 onwards must study Mathematical Literacy. This survey may help to find out whether learners will be prepared to cope with Mathematical Literacy. Please answer the questions to the best of your ability.

To start please fill in the details in the table below.

NAME				
AGE	13	14	15	16
GENDER	MALE		FEMALE	

1. Given in the table below are some possible reasons why you are not going to study mathematics in 2005. Please rate each reason by placing a tick (√) in the correct column.

		1	2	3	4
		Very important	Important	Of little importance	Of no importance
A	I don't like mathematics				
B	Mathematics is difficult				
C	I was not allowed to because of my results in this year				
D	I do not require it for my future career				
E	I was advised not to take mathematics				
F	My friends did not choose it				
G	I think that mathematics is not interesting				
H	Other reason/s Please say what they are				

(a) Place a tick in the column that best describes how you feel about the mathematics topics listed below.

	1 I like it much	2 I like it	3 I neither like nor dislike it	4 I dislike it	5 I dislike it much
A Substitution					
B Products					
C Factorization					
D Solving equations					
E Parallel lines					
F Triangles					
G Congruency					
H Operations on polynomials					
I Ratio and proportion					
J Statistics					
K Quadrilaterals					
L Solving problems in geometry					

(b) Write a sentence about your favourite section in mathematics

____

____

(c) Write a sentence about your least favourite section in mathematics.

____

____

3.Place a tick in the box that shows your point of view.

In my grade 9 mathematics :

I find Algebra more difficult to understand	
I find Geometry more difficult to understand	
I find neither of the two more difficult	
I am not sure which is more difficult	

4. Indicate how you feel about the mathematical activities listed by placing a tick in the appropriate box.

ACTIVITY	1 I like it much	2 I like it a little	3 I neither like nor dislike it	4 I dislike it	5 I dislike it much
Drawing shapes like rectangles and circles					
Working with mathematical instruments					
Working with a calculator					
Measuring lines and angles					
Calculating perimeter, area , and volume					
Working with numbers					
Drawing a plan of a house					

Please state if there any other activities in mathematics that really interest you.

5 When you have completed your matriculation examination you would find that certain career paths are closed to you because you did not study mathematics. Is this a matter of concern to you – does it worry you ?

YES			NO	

Please explain

6. If you were given the opportunity to study Mathematics as it affects you in your daily life e.g. how to make a decision about a cellphone contract or to find out how much it would cost to tile a house, would you be interested?

YES			NO	

Please give a brief reason for your answer.

7. Is there any other comment that you would like to make about mathematics?

Thank you for your cooperation.

OCTOBER 2004

K.S.PILLAY

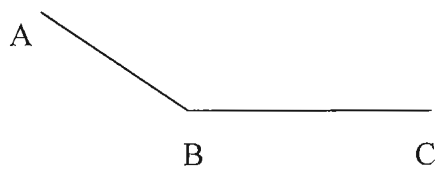
APPENDIX C GEOMETRY TEST

CONTENT ASSESSMENT

1.Please answer the following questions as best as you can.
2. Rate each question by circling the appropriate number.
For example
1 = very easy 2 = easy 3 = a little difficult
4 = difficult 5 = very difficult

NAME	

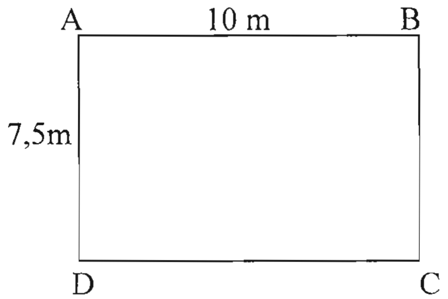
1.1 Choose the correct option. Ring the letter of the correct answer.
Angle ABC is
A Acute : between 0° and 90°
B Obtuse : between 90° and 180°
C Reflex : between 180° and 360°



1.2 Give the approximate measurement of angle ABC to the nearest 10°

1 = very easy 2 = easy 3 = a little difficult
4 = difficult 5 = very difficult

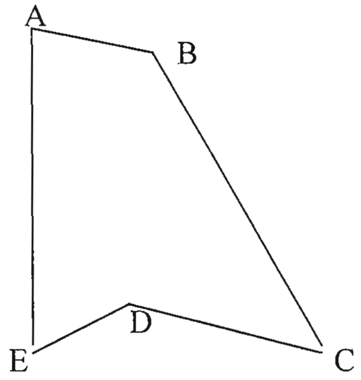
2. The diagram on the right shows a rectangular garden of dimension 7,5 metres by 10 metres.
2.1 Calculate the area of the garden in square metres



2.2 If a tap is situated at point B, what is the shortest length of watering hose (hosepipe) that is required to reach point D?

1 = very easy 2 = easy 3 = a little difficult
4 = difficult 5 = very difficult

3. ABCDE is a pentagon (5-sided figure) shown below. Measure each side and enter the values in the table drawn below.

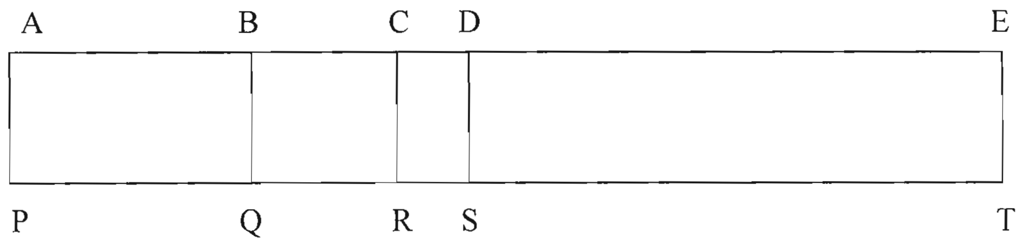


SIDE	MEASUREMENT (mm)
A B	
B C	
C D	
D E	
A E	

Now calculate the perimeter of the pentagon.

1 = very easy 2 = easy 3 = a little difficult
4 = difficult 5 = very difficult

4.



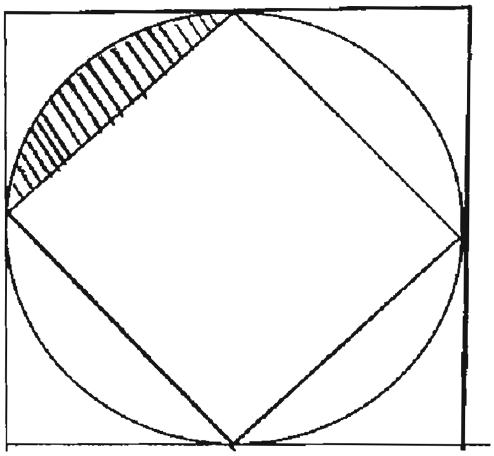
One of the rectangles in the figure has been named in the table. Name all the other rectangles in the figure shown above. Fill your answers in the table.

ABQP						

1 = very easy 2 = easy 3 = a little difficult
4 = difficult 5 = very difficult

---

5. Describe in words what you see in the diagram below.



____

____

____

____

____

Explain how you would calculate the area of the shaded part.

Do not do any calculations.

____

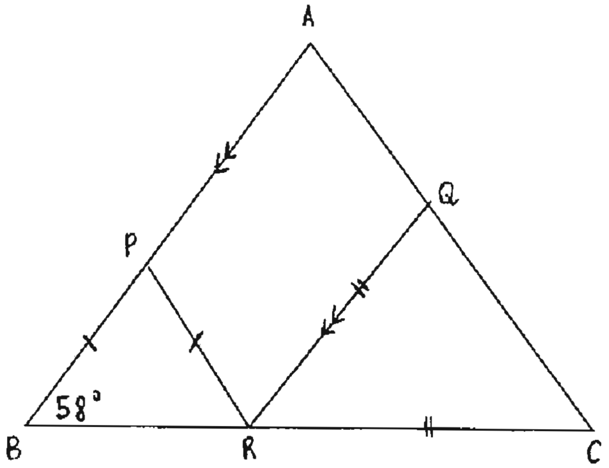
____

____

____

1 = very easy 2 = easy 3 = a little difficult
4 = difficult 5 = very difficult

6. In the diagram below PB = PR ; RQ = RC and QR || AB. B = 58 °.



Calculate the measurement of A.

Thank you for your assistance. Your participation in this survey could assist learners who study Mathematical Literacy from 2006 onwards.

APPENDIX D INTERVIEW SCHEDULE: EDUCATORS

EDUCATOR PROFILE
NAME: ______________________________
SCHOOL: ____________________________
TEACHING EXPERIENCE: GRADE 9 ______________

GRADE 10 ______________

TOTAL (ALL GRADES) ______________

CURRENT GRADES BEING TAUGHT ______________________

1. In your opinion, why do learners not study mathematics in the FET phase i.e. grades 10 to 12?
2. What are the difficulties encountered in teaching mathematics?
3. Given you knowledge of learners who do not choose mathematics, or are not allowed to choose it, do you think that these learners will cope with "Space, Shape and Measurement" in the Mathematical Literacy curriculum?
4. Will you briefly explain how course selection is done at your school?
5. Do you think that learners are sufficiently motivated to study mathematics?
6. Why do you suppose that learners, in general, perform poorly at mathematics?
7. Do you think that learners have more difficulty in algebra or geometry? Why do think that this is so?
8. Do you prefer teaching algebra or geometry? Why?
9. What approaches do you use in teaching geometry?
10. Please comment on the CTA that you administered last year.
11. What are you thoughts on the assessment tasks (content assessment) given to grade 9 learners?
12. What does it mean to be "mathematically literate"? Do you think that there is a need for all South African citizens to be mathematically literate?

APPENDIX E INTERVIEW SCHEDULE : LEARNERS

1. When the word "mathematics" is mentioned to you, tell me what goes through you mind – what do you feel?
2. If you performance was significantly better than it has been, what would your thoughts on mathematics be?
3. What is it about mathematics that you find most difficult?
4. Were you satisfied with the way mathematics was taught to you? If there is anything that can be changed about the way mathematics is taught, what would it be?
5. Do you think that the mathematics you have studied is important? Why?
6. Do you think that it is important to do mental calculations? Explain briefly.
7. What would you say are the challenges that we face in our daily lives where we have to use numbers? Will you say that the mathematics that you study at school prepares you to meet these challenges?
8. Please comment on the MLMMS CTA that you have done.

APPENDIX F LETTER TO PRINCIPAL

THE PRINCIPAL

____

____

____

I am presently a Masters of Education student at the University of KwaZulu Natal. The focus of my study is Mathematical Literacy. My intention is to determine whether learners who do not continue mathematics in the FET band (grades 10 – 12) will be ready to study Mathematical Literacy, which will be introduced in 2006.

I have adopted the following approach in obtaining data :

- A questionnaire (including a test) to be administered to present grade 9 learners who are not studying mathematics from 2005
- a focus group interview with a small group of learners (between 6 and 8)
- a focus group interview with a group of educators who are teaching or have taught grade 9 learners.

I have chosen your school, adopting a case study approach; but I hasten to assure you that neither the name of the school nor those of participating learners or educators will be disclosed.

The questionnaire will be administered on 20 October 2004 and the interviews conducted on 27 October 2004.

I trust you will grant me this opportunity.

Yours faithfully

....

K.S.PILLAY

APPENDIX G LETTER TO PARENTS

…………………………..
…………………………..
…………………………

DEAR PARENT

I am an M.Ed student in Mathematics at the University of KwaZulu – Natal. I am presently engaged in collecting data for my mini thesis. My area of interest is Mathematical Literacy which is to be introduced into the FET curriculum, from 2006.As part of my project I intend conducting a group interview. Your child has agreed to participate in the discussions.

The group session will last approximately 50 minutes and will be conducted on ……………………………

Participation is entirely voluntary and should you have any objections, your child is free to withdraw. In my write-up of the interview no real names (of participants or the school) will be used and all information will be kept confidential. An audio- tape will be used to record the interview, be stored in a safe and, upon completion of the study, be destroyed

Thank you for your co-operation. If you have any questions you may also contact my course supervisor, Sally Hobden at ________________.

Yours sincerely

…………………………

K.S.PILLAY

……………………………………………………………………………………………

I, ……………………the parent/guardian of ……………………..of grade ……….. hereby grant / refuse permission for my child / ward to participate in the aforementioned project.

……………………… ……………………..

Signature Date

……………………………………………………………………………………………

APPENDIX H LETTER TO EDUCATORS

____

____

____

I am presently a Masters of Education student at the University of KwaZulu Natal. The focus of my study is Mathematical Literacy. My intention is to determine whether learners who do not continue mathematics in the FET band (grades 10 – 12) will be ready to study Mathematical Literacy, which will be introduced in 2006.

I have adopted the following approach in obtaining data :

- A questionnaire (including a test) to be administered to present grade 9 learners who are not studying mathematics from 2005
- a focus group interview with a small group of learners (between 6 and 8)
- a focus group interview with a group of educators who are teaching or have taught grade 9 learners.

I request your participation in the group interview. I appreciate the fact that with your busy schedule, time is at a premium. However I am certain that the discussion will not exceed 45 minutes.

Please find enclosed a copy of an extraction from the Mathematical Literacy curriculum document. The discussion will revolve around the learning outcome "Space, Shape and Measurement". I trust that the interaction will be of benefit to you.

Thank you for your valued assistance.
Yours faithfully

....

K. S. Pillay

APPENDIX I CODEBOOK

NAME	LABEL	VALUES
Identity	SCHOOL AND NUMBER	R=RAILSIDE ; V=VALEVIEW
Name	NAME OF RESPONDENT	
age	AGE OF RESPONDENT	
gender	GENDER	1 = MALE
		2 = FEMALE
REASON FOT NOT CHOOSING MATHEMATICS		
reason1	I DO NOT LIKE IT	1 = OF NO IMPORTANCE
		2 = OF LITTLE IMPORTANCE
		3 = IMPORTANT
		4 = VERY IMPORTANT
reason2	IT IS DIFFICULT	1 = OF NO IMPORTANCE
		2 = OF LITTLE IMPORTANCE
		3 = IMPORTANT
		4 = VERY IMPORTANT
reason3	I WAS NOT ALLOWED TO	1 = OF NO IMPORTANCE
		2 = OF LITTLE IMPORTANCE
		3 = IMPORTANT
		4 = VERY IMPORTANT
reason3	I WAS NOT ALLOWED TO	1 = OF NO IMPORTANCE
		2 = OF LITTLE IMPORTANCE
		3 = IMPORTANT
		4 = VERY IMPORTANT
reason4	IT IS NOT REQUIRED	1 = OF NO IMPORTANCE
		2 = OF LITTLE IMPORTANCE
		3 = IMPORTANT
		4 = VERY IMPORTANT
reason5	I WAS ADVISED NOT TO	1 = OF NO IMPORTANCE
		2 = OF LITTLE IMPORTANCE
		3 = IMPORTANT
		4 = VERY IMPORTANT
reason6	FRIENDS DID NOT CHOOSE IT	1 = OF NO IMPORTANCE
		2 = OF LITTLE IMPORTANCE
		3 = IMPORTANT
		4 = VERY IMPORTANT
reason7	IT IS NOT INTERESTING	1 = OF NO IMPORTANCE
		2 = OF LITTLE IMPORTANCE
		3 = IMPORTANT
		4 = VERY IMPORTANT

LIKE/DISLIKE FOR MATHEMATICS TOPICS		
TOPIC 1	SUBSTITUTION	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 2	PRODUCTS	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 3	FACTORIZATION	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 4	SOLVING EQUATIONS	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 5	PARALLEL LINES	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 6	TRIANGLES	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 7	CONGRUENCY	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 8	OPERATIONS ON POLYNOMIALS	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH

TOPIC 9	RATIO AND PROPORTION	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 10	STATISTICS	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 11	QUADRILATERALS	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
TOPIC 12	SOLVING PROBLEMS IN GEOMETRY	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
AREA OF DIFFICULTY		
ALGEBRA	Algebra is more difficult	1
GEOMETRY	Geometry is more difficult	2
NEITHER	Neither is more difficult	3
NOT SURE	Not sure which is more difficult	4
MATHEMATICAL ACTIVITIES		
ACTIVITY 1	DRAWING SHAPES	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
ACTIVITY 2	WORKING WITH INSTRUMENTS	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
ACTIVITY 3	WORKING WITH A CALCULATOR	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH

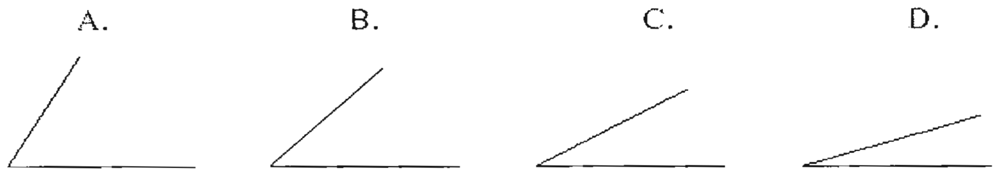
ACTIVITY 4	MEASURING LINES AND ANGLES	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
ACTIVITY 5	CALCULATING AREA , PERIMETER	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
ACTIVITY 6	WORKING WITH NUMBERS	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
ACTIVITY 7	DRAWING A HOUSE PLAN	"-2 = I DISLIKE IT MUCH"
		"-1 = I DISLIKE IT"
		0 = I NEITHER LIKE IT NOR DISLIKE IT
		1 = I LIKE IT
		2 = I LIKE IT MUCH
	RESTRICTED IN CAREER CHOICE	1 = YES
		2 = NO
	WOULD YOU LIKE TO DO MATH LIT?	1 = YES
		2 = NO

APPENDIX J LEARNERS' GRADE NINE EXAMINATION MARKS

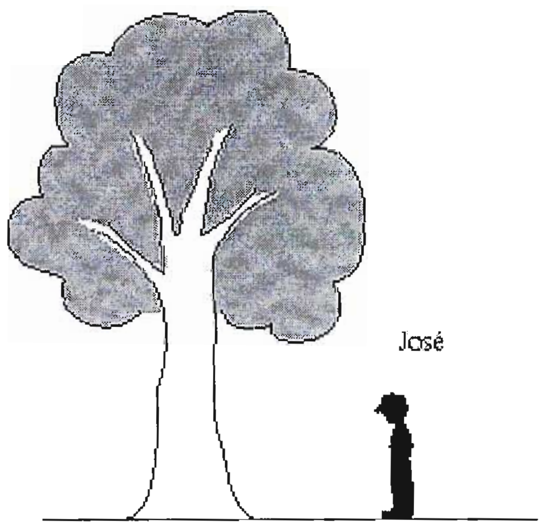
30	26	22	34	28	36	23	33
25	19	34	35	33	25	23	41
39	32	31	23	38	19	36	16
41	25	15	37	35	25	31	35
32	28	29	39	24	27	32	26
33	28	34	24	26	37	34	25
14	38	17	33	31	24	36	37
26	30	28	29	22	36	28	35
16	30	30	22	25	17	21	28
37	27	36	35	27	35	42	36
12	37	35	36	31	34	39	
22	37	38	34	32	18	36	

APPENDIX K TIMSS GEOMETRY ITEMS

N15. Which of these angles has a measure closest to 30° ?



L8.



José is 1.5 m tall. About how tall is the tree?

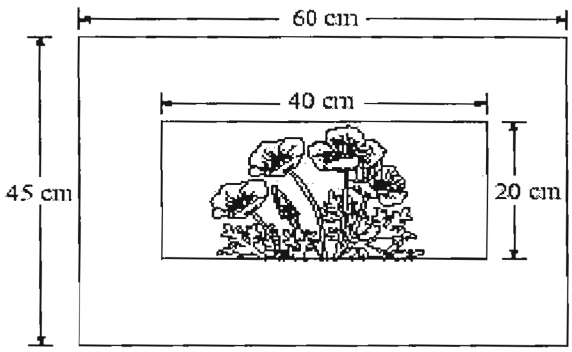
A. 4 m

B. 6 m

C. 8 m

D. 10 m

J10. A rectangular picture is pasted to a sheet of white paper as shown.



What is the area of the white paper not covered by the picture?

A. 165 cm^2

B. 500 cm^2

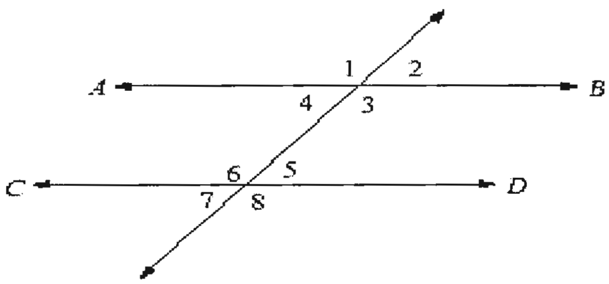
C. 1900 cm^2

D. 2700 cm^2

K5. The length of a rectangle is 6 cm, and its perimeter is 16 cm. What is the area of the rectangle in square centimeters?

Answer:________________________________

O3. In this figure, lines *AB* and *CD* are parallel.



Two angles whose measures must add up to 180° are

A. $\angle 1$ and $\angle 3$

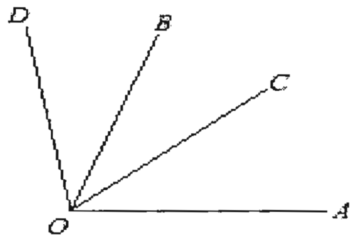
B. $\angle 4$ and $\angle 6$

C. $\angle 2$ and $\angle 5$

D. $\angle 2$ and $\angle 7$

E. $\angle 1$ and $\angle 8$

Q10. In the figure, the measure of $\angle AOB$ is $70°$, the measure of $\angle COD$ is $60°$, and the measure of $\angle AOD$ is $100°$.



What is the measure of $\angle COB$?

Answer:____________________________________

APPENDIX L SOME REMARKS FROM THE QUESTIONNAIRE

Statements indicating that learners like geometry when questioned about their favourite section

I like triangles because it is easy to do it and I always get good result

I like solving problems in Geometry but if I can understand it more than

I like solving problems in geometry because it is interesting

Triangle because I understand it

I like triangles so much but theres that section which is difficult for me.

I like triangles a lot because I always get the best marks and it is not much difficults.It easy to learn it.

I like solving problems in geometry and solving equation it nice doing it.

I like to solve problem geometry because some time I get correct answers.

The section I do likely most is triangle. I do like it much

I like the jomatr because I understand

I like triangles because easy

I like triangles because it is easy to do it and I always get good result.

I like triangles because I am good to do triangles

My favourite section is solving problem in geometry because it easy

Triangles I understand and I know how to lebel them and calculate them

Triangles because it is much better than the other it not difficult

I like working with triangles because I can get the answers very quickly and get everything right.

The most topic that I like in maths is congruency. It is easy to understand. The teacher explains it very nicely

Triangles very interesting be it is very easy to understand them.

Theorem of Pythagoras is my most favourite section
I like products and solving problems in geometry

My favourite section in maths is solving problems in geometry

Triangle is not difficult and the formula is not long at all so I do not forget them

I like parallel lines it much more easy to do.

I just like triangles because I understand

The FUN is my favourite section in maths because it is easy

My favourite section in Maths is the triangles and shapes because I understand it.

Triangles is exciting

In maths I like geometry

My favourite section is triangles because it is fun

Parallel lines, triangles, polynomials, geometry and quadrilaterals

It to draw shapes like rectangle and drawing a plain of a house.

Measuring lines and angles it easy

I realy appriciate doing congruency it is very easy to understand and my teacher explains it very nicely

Only the geometry part interested me.

There are shapes and drawing of things

I like the part about measuring angles

I was really intrest with triangulation and problem solving in geometry and also in quadrilaterals.

I think that geometry is of more fun to me

Measuring angles

Statements indicating that learners dislike geometry

Parallel lines. I don't like parallel lines because I just don't know it

I like parallel lines but not much sometime times I use to understand it sometimes I don't.

Geometry and solving equation because I just don't understand, and I don't understand my teacher

Solving problems in geometry

Parallel lines. I don't like parallel line because it is not easy to do it

My least favourite section in math is parallel lines.

My least in maths is geometry

Quadrilaterals cos I don't even understand it

I don't like triangle because is so difficult

I don't like tyrangles Because I don't think if you do a job which needs maths you will be required to work with them

I don't like mathematics becos of parallel lines

I hate solving problems in geometry because I always get my answers wrong

I dislike parallel lines because I cant understand what the teacher is teaching and I get lost

Solving problems in geometry is very hard and very hard to understand

I do not like geometry because it is very hard

Solving problems in geometry is very difficult

Geometry it is kind of difficult

Solving problems in geometry is very hard. I don't like it

My least favourite section solving problems in geometry

I least like solving problems in geometry and factorization

My least favourite section in maths is geometry

My least favourite section in mathematics is solving problem in geometry

I hate geometry , I don't understand anything from it

I don't like ratio and solving problem in geometry.

My least favourite is geometry. It requires a lot of instruments and measuring

My least section is doing geometry. I think it is very difficult for me cause I don't understand

Congruency because I don't understand it even if I been concentration

The section I dislike in maths is geometry

Parallel lines I don't like it because it got many things to learn and I'm forgetting easy

Solving problems in geometry is the least favourite section because it difficult

APPENDIX M MATHEMATICAL LITERACY ASSESSMENT STANDARDS FOR LEARNING OUTCOME 3

Learning Outcome 3

Space Shape and Measurement

The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.

Assessment Standards

We know this when the learner is able to:

10.3.1 Solve problems in 2-dimensional and 3- dimensional contexts by:

- estimating, measuring and calculating (*e.g. by the use of the Theorem of Pythagoras)* values which involve:
  * lengths and distances
  * perimeters and areas of common polygons and circles,
  * volumes of rights,
  * angle sizes (0 - 360)
- checking values for solutions against the contexts in terms of suitability and degree of accuracy

10.3.2 Convert units of measurement within the metric system

For example

* *convert km to m, mm3 to litres, km2 to m2, cm3 to m3*

10.3.3 Draw and interpret scale drawings of plans to represent and identify views.

For example

* *draw and interpret top, front and side views or elevations on a plan*

10.3.4 Solve real-life problems in 2-dimensional and 3-dimensional situations by the use of geometric diagrams to represent relationships between objects.

For example:

* *draw floor plans and use symbols to indicate areas and positions taken up by furniture in different arrangements.*

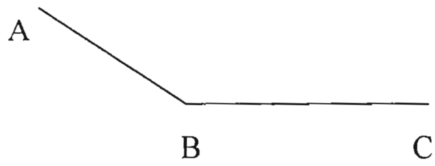
10.3.6 Recognise, visualize, describe and compare properties of geometrical plane figures in natural and cultural forms.

For example:

* *use the concept of tessellation and symmetry in describing tilings, Zulu beadwork and other artifacts.*

APPENDIX N EXPECTED ANSWERS

1.1 Choose the correct option. Ring the letter of the correct answer.
Angle ABC is
A Acute : between 0° and 90°
B Obtuse : between 90° and 180°
C Reflex : between 180° and 360°

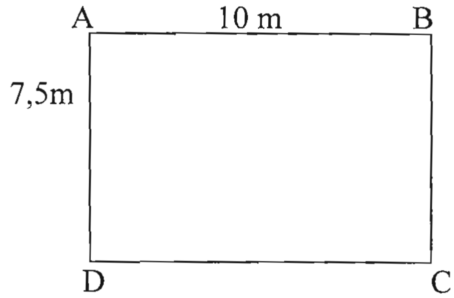


Obtuse or reflex

1.2 Give the approximate measurement of angle ABC to the nearest 10°

130/230

2. The diagram on the right shows a rectangular garden of dimension 7,5 metres by 10 metres.
   2.1 Calculate the area of the garden in square metres



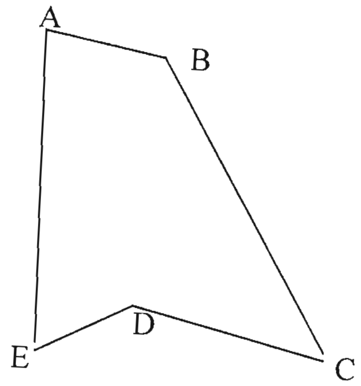
Area = length x breadth
= 10m x 7,5m
= $75m^2$

2.3 If a tap is situated at point B, what is the shortest length of watering hose (hosepipe) that is required to reach point D?

$DB^2 = DA^2 + AB^2$ ***Theorem of Pythagoras***
$= 7{,}5^2 + 10^2$
$= 156{,}25$
$DB = 12{,}5$

Shortest length required is 12,5m.

3. ABCDE is a pentagon (5-sided figure) shown below. Measure each side and enter the values in the table drawn below.



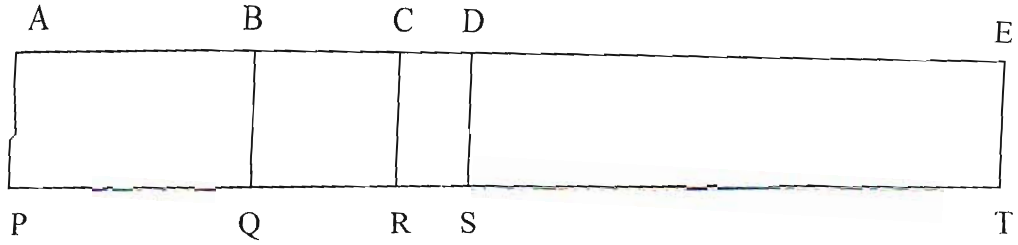
SIDE	MEASUREMENT (mm)
A B	***15***
B C	***40***
C D	***24***
D E	***13***
A E	***38***

** Error margin of 1mm.

Now calculate the perimeter of the pentagon.

***Perimeter** = 15 + 40 + 24 + 13 + 38 mm*

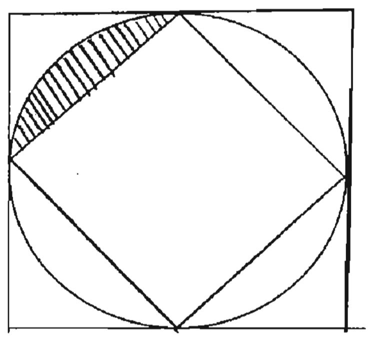
= 130 mm



One of the rectangles in the figure has been named in the table. Name all the other rectangles in the figure shown above. Fill your answers in the table.

ABQP	***ACRP***	***ADSP***	***AETP***	***BCQR***	***BDSQ***	***BETQ***
CDSR	***CETR***	***DETS***				

5. Describe in words what you see in the diagram below.



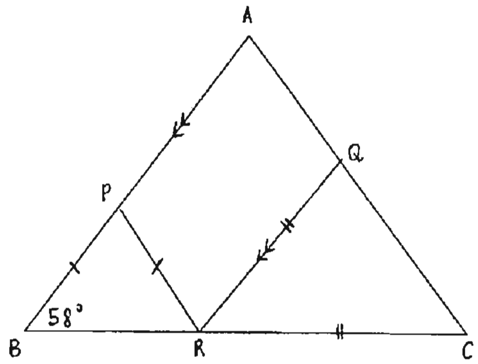
A square; a circle inside the square; a second square inside the circle; a shaded area bounded by the circle and the smaller square

Explain how you would calculate the area of the shaded part.

Do not do any calculations.

Calculate the area of the circle. Calculate the area of the smaller square. Find the difference. Divide by 4.

6. In the diagram below PB = PR ; RQ = RC and QR || AB. B = 58 °.



Calculate the measurement of A.

QRC	*= 58°*	*corresponding angles: AB // QR*
RQC	*= ½ (180° – 58°)*	*base angle of isosceles triangle*
	= 61°	
Therefore A	*= 61°*	*corresponding angles: AB // QR*