UNIVERSITY OF KWAZULU-NATAL

HIERARCHICAL MODULATION WITH SIGNAL SPACE AND TRANSMIT DIVERSITY IN NAKAGAMI-M FADING CHANNEL

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June 2013

As the candidate's supervisor I agree to the submission of this dissertation.

Date of Submission:

Supervisor:

Professor Hongjun Xu

Declaration

I, Ayesha Saeed, declare that,

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ii. This dissertation has not been submitted for any degree or examination at any other university.
iii. This dissertation does not contain other persons' data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons.
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Signed:

For Ami and Abu.

With thanks to Prof. Xu and Dr. Quazi.

As you start to walk out on the way, the way appears.

-Rumi.

Abstract

Hierarchical modulation (HM) is a promising scheme for wireless image and video transmission, exploiting the benefits of unequal error protection to ensure enhanced system performance. However, there is a limiting factor to the benefits of using only hierarchy to improve bit error rate (BER) performance of a transmission system. Diversity, namely signal space diversity (SSD) and Alamouti transmit diversity (ATD), can be introduced to improve BER performance results for HM systems. This dissertation presents the BER analysis of hierarchically modulated QAM with SSD and using maximal ratio combining (MRC) to retrieve the transmitted symbol from N receiver antennas. In addition, the study includes the BER analysis of an identical system in an ATD scheme employing two transmit antennas and N receiver antennas with MRC.

SSD comprises of two fundamental stages: constellation rotation and component interleaving. The angle at which the constellation is rotated can affect the performance of the system. In the past, the rotation angle is determined based on a design criterion which maximizes the diversity order by minimizing the Euclidean square product or, alternatively, minimizes an SER expression. In this dissertation, a simple method for determining a rotation angle at which system performance is optimal for hierarchical constellations is presented.

Previously, the BER analysis for HM involves an intricate approach where the probability of an error occurring is determined by considering the probability of a transmitted symbol exceeding past a set decision boundary. This dissertation presents the Nearest Neighbor (NN) union bound approach for determining an accurate approximation of the BER of an HM system with SSD. This method of analysis is later extended for an ATD scheme employing HM with SSD.

Although introducing diversity elevates the system performance constraints on HM, it does so at the cost of detection complexity. To address this issue, a reduced complexity maximum-likelihood (ML) based detector is also proposed. While the conventional ML detector performs an exhaustive search to find the minimum Euclidean distance between the received symbol and all possible modulated symbols, the proposed detector only considers the nearest neighbors of the received symbol. By reducing the number of comparisons, a complexity reduction of 51.43% between the proposed detector and the optimal detector for 16-QAM is found.

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List of Acronyms

ATD	Alamouti Transmit Diversity
AVC	Advanced Video Coding
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CSI	Channel State Information
DVB-T	Digital Video Broadcast – Terrestrial
HM	Hierarchical Modulation
i.i.d	Independent and Identically Distributed
JSCC	Joint-Source Channel Coding
LSB	Least Significant Bit
MIMO	Multiple-Input Multiple-Output
ML	Maximum-Likelihood
MRC	Maximal Ratio Combining
MSB	Most Significant Bits
NN	Nearest Neighbor
PEP	Pair-wise Error Probability
PSK	Pulse Shift Keying
QAM	Quadrature Amplitude Modulation
SER	Symbol Error Rate
SNR	Signal-to-Noise Ratio
SSD	Signal Space Diversity
STBC	Space-Time Block Coding
UEP	Unequal Error Protection

Part I

Introduction

1. Introduction

Next generation wireless multimedia communication requires robust transmission schemes able to support high data throughput without extending bandwidth and increasing channel capacity. Above all, the need for continuous voice and video streaming as well as reliable communication with fewer bit errors has become a fundamental challenge. Although link adaptive techniques improve transmission performance, one cause of degradation in throughput results from the use of a feedback link between the receiver and the transmitter. Power adaptive systems have similar shortcomings as well as the additional cost for the required amplifiers. While encoding schemes have proven to ensure greater data protection, the reduction in the data rate and the added computational complexity are considerable.

Nearly four decades ago, Cover presented a study in [1] on broadcast channels where a single transmitter communicates to multiple receivers under various channel conditions simultaneously. In the study, Cover proposed the classification of data into different categories each assigned a level of protection in accordance to the priority of the data. In effect, it would allow high priority (base) data to be recovered by all receivers regardless of channel conditions while those receivers under fortunate channel conditions would also be able to recover the less important (refinement) data. While Cover introduced the theory of unequal error protection (UEP) in the encoding stage of the transmission system, later studies extended the concept to modulation with hierarchy as a more flexible method of establishing varying levels of error protection as it does not limit bandwidth [2-4].

Hierarchical modulation (HM), also known as embedded or multi-resolution modulation, employs UEP in the form of non-uniformly spaced constellation points. Although HM improves reliability of communication by ensuring greater error performance for high priority data, it does so at the cost of deteriorating error performance of low priority data. Additionally, power consumption increases proportionally with hierarchy. As a result, there is a constraining factor to the benefits of using only hierarchy as a tool for improving system performance. By incorporating diversity with HM, the benefits of UEP can be exploited while at the same time the performance of both the base and refinement bits can be enhanced.

Conventional diversity techniques employ redundant transmission of a signal over identical channels which may be orthogonal in time or frequency. While time diversity coupled with error correction coding allows for system performance improvement, large delays result at the receiver in the case where the channel is slowly varying. Similarly, frequency diversity becomes inefficient when the coherence bandwidth of the channel is greater than the frequency spectrum of the scheme [5].

Signal space diversity (SSD) is a technique which exploits the intrinsic diversity of multi-dimensional constellations to improve error performance. Since the scheme employs uncoded modulation, there is no additional redundancy added to bit stream resulting in SSD consuming no additional transmit power, space or bandwidth [6]. Furthermore, smaller mobile devices can be achieved as the scheme does not employ any additional antennas.

Additionally, the Alamouti transmit diversity (ATD) introduces an alternative approach to antenna diversity [5]. While the conventional schemes implement multiple antennas at the receiver, the schemes are almost exclusively applied to base stations as the additional cost, size and power required make the scheme too expensive to be applied at remote stations. To address this, ATD proposes the use of two antennas at the transmitter. This allows for improved error performance, increase in data throughput and expansion of channel capacity by decreasing the transmission system's susceptibility to channel fading without additional feedback links or limiting bandwidth with redundancies [5].

1.1. Hierarchical Modulation

HM exploits UEP by use of non-uniformly spaced constellations whereby higher priority data is mapped by the most significant bit (MSB) positions, ensuring greater protection, in contrast to lower priority data which is mapped to the least significant bit (LSB) positions. Consequently, all receivers regardless of channel conditions should be able to recover at least the coarse or basic data mapped by the MSB thereby improving transmission quality and facilitating continuous data streaming.

Over the years, HM has been developed further into numerous schemes. In [7], HM has been employed as a technique for upgrading existing digital broadcast systems whereby the base data carries information required by the original system while the refinement data carries additional information required by the upgraded system. Adaptive HM schemes have also been introduced in [8], where voice and multiclass data are transmitted over varying channel conditions. Under poor channel condition, only voice data is transmitted using a binary phase-shift keying (BPSK) signal and as the conditions improve M-ary quadrature amplitude modulation (M-QAM) signal along with data assignment according to priority is used to increase throughput. Two-user opportunistic scheduling using adaptive HM is proposed in [9] where the conventional opportunistic scheduling to transmitting to the first best user in each transmission time slot is challenged. Using rate adaptive HM, the size of the QAM constellation is selected in accordance with the channel conditions of the first best user. However, if the channel conditions of the second best user permit the decoding of the base data in the selected HM-QAM, then information is transmitted to both the first and second best users.

In more recent years, UEP has been implemented in the form of HM and joint-source channel coding (JSCC) to develop a robust wireless progressive multi-media transmission system [10-11]. This research is extended further in a recent publication which introduces multiple levels of UEP using multiplexed HM in order to improve progressive multimedia transmission [12]. Additionally, [12] also proposes a scheme to counter the adverse effect of power consumption in HM systems by reducing peak-to-average power ratio without the any performance loss. Power allocation is also considered in [13] where adaptive HM is applied to a demodulate-and-forward cooperative communication system. The proposed optimization function enables the system to adaptively determine the optimal hierarchical constellation and power allocation parameter that will maximize channel capacity. Moreover, the performance of HM in the DVB-T (Digital Video Broadcasting – Terrestrial) standard has also been presented in [14] in order to preserve a low bit error rate (BER) required for successful signal decoding.

Performance analysis of HM systems is considered in numerous publications. In [15] and [16], the exact BER of HM using QAM and phase-shift keying (PSK) signals, respectively, is determined by considering the probability that a transmitted symbol exceeds a decision boundary and becomes an error. The results support this method of analysis as being extremely accurate despite its complexity. This method for determining the BER is extended in more recent publications by the same authors where using a recursive algorithm is developed in [17] and a closed form expression for generalized HM is derived in [18].

The performance analysis of hierarchical constellations has also been explored in systems with diversity reception under imperfect channel conditions [19] as well as correlated multiple-input multiple-output (MIMO) Nakagami-m fading channels using orthogonal space-time block coding (STBC) [20]. More recently, the performance of HM in cooperative communication systems has also been considered [21]. Furthermore, the performance of UEP H.264/AVC coded video using hierarchical 16-QAM is also investigated in [22].

It is evident from reviewing past literature that there is a high demand and large scope for HM systems. It can also be noted from more recent literature that HM is often combined with alternative techniques in order to enhance system performance or reduce power consumption. This motivates further the need for introducing diversity to HM in order to improve system performance without increasing power and cost or leading to loss of bandwidth. In addition, the need for a more efficient error analysis method is also observed.

1.2. Signal Space Diversity

SSD exploits the inherent property of multi-dimension constellations by performing constellation rotation according to an optimum angle to increase the minimum number of distinct components between any two constellation points. In addition, performing interleaving ensures each component experiences independent fading such that the receiver can still accurately recover a transmitted symbol even if one of its components has experienced deep fading [6]. Consequently, system performance is improved while only requiring additional processing power to perform maximum-likelihood (ML) detection and without consuming additional bandwidth, transmit power or space.

The process of constellation rotation is fundamental to the performance of the system as it results in determining the position of the distinct components as illustrated in Figure 1.1. As the distance between the distinct components increases, the probability of error decreases and system performance improves. Thus determining a suitable angle of rotation which fully exploits the diversity gain is crucial for the performance of SSD. In previous literature, various criterions are applied in determining the optimum angle of rotation. Most commonly, the method of maximizing the minimum Euclidean product distance between unique components is presented [23-25]. Alternatively, in [26] the symbol error probability is determined as an expression dependent on the angle of rotation. By extension, the angle of rotation is determined by minimizing the probability of error. Despite their effectiveness, the methods of determining an optimum rotation angle are lengthy and complicated. A simple and equally effective method of analysis, specifically for hierarchical constellations where the distance between the constellation points may vary, has not yet been presented in literature.

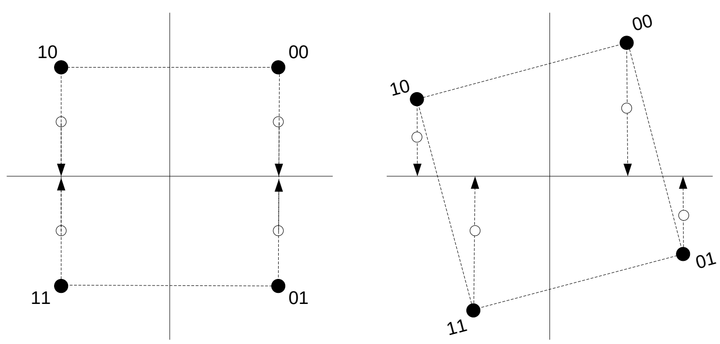

Figure 1.1 Increasing diversity gain by constellation rotation

In the past, very little attention has been placed on combining HM and SSD for enhanced system performance. While conference publications such as [27,28] have briefly considered such a scheme, a detailed error analysis or a method for determining the optimum rotation angle are omitted. From the limited simulation results presented in [27], it is evident that there exists a substantial BER performance gain for HM by incorporating SSD.

1.3. Alamouti Transmit Diversity

Alamouti space-time block coding, first presented in [5], is an alternative approach to introducing antenna diversity. It employs two transmit antennas to enhance transmission quality at the receiver without loss of bandwidth or the need for additional power. The resultant system has a diversity order equivalent to a system employing maximal ratio combing (MRC) with two receiver antennas and a 3 dB loss in signal-to-noise ratio (SNR) as a result of the power of the two transmit antennas being halved. In extension, the Alamouti scheme with two transmit antennas and N receiver antennas has a diversity order of $2N$ [5].

HM with ATD schemes have been considered and proposed in [29] where user cooperative diversity is achieved between mobile terminals. More recently, Hochwald and Stauffer present an extended study in [30,31] where unitary-transform code and direct-sum Alamouti code combined with HM indicated performance gains with single and multiple antenna transmission schemes. On the other hand, the performance of systems combining SSD with Alamouti STBC has been discussed extensively in [32-34]. The results from schemes employing Alamouti STBC to introduce transmit diversity into a conventional SSD systems indicate greater resistance to channel estimation errors. In [34], an exact expression for symbol error rate (SER) performance of a system employing SSD with Alamouti is presented. To date, there is no published literature which considers combining HM and SSD with ATD in order to develop a system with high resistance to channel fading.

While enhancing the performance of HM by incorporating SSD and ATD is the main focus, the effect of detection complexity is also considered. Conventional ML detectors perform exhaustive searches where the received symbol is compared with all possible modulated symbols to find the minimum Euclidean distance. Despite its detection accuracy, the high complexity often limits the data throughput of a system as large delays occur between received symbols being decoded. Thus there is a need for an alternative decoding technique which reduces the complexity of detection of HM signal without compromising accuracy.

2. Motivation and Research Objective

In this section, the motivation of this dissertation and the objectives of the research are highlighted. Enhanced HM systems have been proposed in [10-13] whereby performance is improved by implementing redundancy in the form of encoding or introducing adaptive power allocation techniques. Although effective, these schemes prove to be inefficient in bandwidth usage and vulnerable to deep channel fading. This necessitates for HM systems which employ effective diversity techniques to enhance performance without the addressing the challenges of increasing power, space and cost or loss of bandwidth. SSD exploits the inherent properties of multi-dimensional constellation to introduce diversity whilst only increasing detection complexity. In addition, ATD is an alternative approach to implementing antenna diversity.

Conventional performance analysis of HM systems as presented in [15-18] requires an exhaustive evaluation of each transmitted symbol and its probability to exceed its decision boundaries. While highly accurate, the method of analysis is far too complicated and time consuming. There is a need for an equally accurate yet less complicated method of error analysis for HM systems. The nearest neighbor (NN) union bound approximation approach as presented for a conventional SSD system in [35] is extended to HM systems in order to provide an accurate and simple method of error analysis.

Determining the optimum angle of rotation is crucial to the performance of an SSD system. While this topic is well discussed in literature [23-26], the methods of analysis presented are often high in complexity. A simple but effective way of determining the optimum rotation angle specifically for hierarchical constellations where distances between constellation points vary according to hierarchy is yet to be presented.

Conventionally, optimal detection is performed by exhaustively searching for the minimum Euclidean distance between the received symbol and all possible modulated symbols. The high complexity of such detectors results in deteriorating throughput. Consequently, there is a demand for ML-based reduced complexity detectors which consider only the nearest neighbors of the received symbol resulting in a reduction in decoder complexity without loss of accuracy.

3. Contribution of Included Papers

The contributions of this dissertation are presented in the form of two journal papers submitted in full for peer review to the *Institute of Engineering and Technology (IET) Communications*. The papers are presented in Part II of this dissertation while Part III presented concluding remarks.

3.1. Paper A

A. Saeed, T. Quazi, and H. Xu, "Hierarchical Modulated QAM with Signal Space Diversity and MRC Reception in Nakagami-m fading channels," IET Communications, vol. 7, no. 12, pp. 1296-1303, August 2013.

In Paper A, an innovative scheme employing HM and SSD with MRC reception in a Nakagami-m fading is proposed in order to enhance system performance. The NN approximation is applied to determine the BER performance of an HM system employing SSD. In extension, a novel approach to deriving the optimum rotation angle for HM constellations by means of maximizing the Euclidean distance between the unique components is also proposed. Results are validated against simulations for varying hierarchy, channel conditions, and number of receiver antennas.

3.2. Paper B

A. Saeed, T. Quazi, and H. Xu, "Alamouti Space-Time Block Coded Hierarchical Modulation with Signal Space Diversity and MRC Reception in Nakagami-m Fading Channel," IET Communications, [Awaiting assignment to issue], 2014.

In Paper B, a novel approach to enhancing HM performance by incorporating SSD and Alamouti STBC in Nakagami-m fading channel with MRC reception is presented. The NN approach for BER analysis is proposed along with an ML-based reduced complexity detector which considers only the nearest neighbors of a received symbol. Simulation results indicate enhanced performance of the proposed system as well as confirm the accuracy of the BER analysis and the proposed detector for varying hierarchy, channel fading and receiver antennas.

4. References

[1] Thomas M. Cover, "Broadcast Channels," in *IEEE Transactions on Information Theory*, vol. 18, 1972.

[2] C.-E.W. Sundberg, W.C. Wong, and R. Steele, "Logarithmic PCM weighted QAM transmission over Gaussian and Rayleigh fading channels," *IEEE Proceedings*, vol. 134, no. 6, pp. 557-570, October 1987.

[3] Werner Henkel et al., "UEP Concepts in Modulation and Coding," *Advances in Multimedia*, vol. 2010, no. 1, p. 14, January 2010.

[4] Lee-Fang Wei, "Coded Modulation with Unequal Error Protection," *IEEE Transactions on Communications*, vol. 41, no. 10, pp. 1439-1449, October 1993.

[5] Siavash M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE Journal on Select Areas in Communications*, vol. 16, no. 8, pp. 1451-1458, October 1998.

[6] Joseph Boutros and Emanuele Viterbo, "Signal Space Diversity: A Power- and Bandwidth-Efficient Diversity Technique for the Rayleigh Fading Channel," *IEEE Transactions on Information Theory*, vol. 44, no. 4, pp. 1453-1467, July 1998.

[7] Hong Jiang and Paul A. Wilford, "A Hierarchical Modulation for Upgrading Digital Broadcast Systems," *IEEE Transcactions on Broadcasting*, vol. 51, no. 2, pp. 223-229, June 2005.

[8] Md.J. Hossain, P.K Vitthaladevuni, M.S. Alouini, Bhargava, V.K. , and Goldsmith, A.J. , "Adaptive hierarchical modulation for simultaneous voice and multiclass data transmission over fading channels," *IEEE Transactions on Vehicular Technology,* vol. 55, no. 4, pp. 1181-1194 , July 2006.

[9] Md. Jahangir Hossain, Mohamed-Slim Alouini, and Vijay K. Bhargava, "Rate Adaptive Hierarchical Modulation-Assisted Two-User Opportunistic Scheduling," *IEEE Transactions on Wireless Communications*, vol. 6, no. 6, pp. 2076-2085, June 2007.

[10] S.S. Arslan, P.C. Cosman, and L.B. Milstein, "Coded Hierarchical Modulation for Wireless Progressive Image Transmission," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 9, pp. 4299-4313, November 2011.

[11] Suayb S. Arslan, Pamela C. Cosman, and Laurence B. Milstein, "Progressive Source Transmissions using Joint Source-Channel Coding and Hierarchical Modulation in Packetized Networks," in *IEEE Global Telecommunications Conference*, Honolulu, 2009, pp. 1-6.

[12] Soek-Ho Chang, Minjoong Rim, Pamela C. Cosman, and Laurence B. Milstein, "Optimized Unequal Error Protection Using Multiplexed Hierarchical Modulation," *IEEE Transactions on Information Theory*, vol. 58, no. 9, pp. 5816-5840, September 2012.

[13] Lindong Wu, Zesong Fei, Chen Luo, and Jingming Kuang, "Adaptive Hierarchical Modulation and Power Allocation of Cooperative Relay," in *7th International Conference on Wireless Communications, Networking and Mobile Computing*, Wuhan, 2011, pp. 1-5.

[14] R. Stukavec and T. Kratochvil, "Performance of Hiearichcal Modulation in DVB-T," in *20th International Conference Radioelektronika*, 2010, pp. 1-4.

[15] P.K. Vitthaladevuni and M.-S. Alouini, "BER computation of 4/M-QAM hierarchical constellations," *IEEE Transactions on Broadcasting*, vol. 47, no. 3, pp. 228-239, September 2001.

[16] P.K. Vitthaladevuni and M.-S. Alouini, "Exact BER Computation of Generalized Hierarchical PSK Constellations," *IEEE Transactions on Communication*, vol. 51, no. 12, pp. 2030-2037, December 2003.

[17] Pavan K. Vitthaladevuni and Mohamed-Slim Alouini, "A Recursive Algorithm for the Exact BER Computation of Generalized Hierarchal QAM Constellations," *IEEE Transactions on Information Theory*, vol. 49, no. 1, pp. 297-307, January 2003.

[18] P.K. Vitthaladevuni and M.S. Alouini, "A closed-form expression for the exact BER of generalized PAM and QAM constellations," *IEEE Transactions on Communications*, vol. 52, no. 5, pp. 698-700, May 2004.

[19] Nuno M.B. Souto, Francisco A. B. Cercas, Rui Dini, and Joao C. M. Silva, "On the BER Performance of HierarchicalM-QAM Constellations With Diversity and Imperfect Channel Estimation," *IEEE Transactions on Communications* , vol. 55, no. 10, pp. 1852-1856, October 2007.

[20] Guangqiu Li, Zhenyu Zhang, and Jianwu Zhang, "Exact BER Analysis of Hierarchical M-QAM in Spatially Correlated MIMO Nakagami Fading Channels," in *International Confrence on Wireless Communications, Networking and Mobile Computing*, 2007, pp. 389-392.

[21] Min-Kuan Chang and Shi-Yong Lee, "Performance Analysis of Cooperative Communication System with Hierarchical Modulation over Rayleigh Fading Channel," *IEEE Transactions on Wireless Communications*, vol. 8, no. 6, pp. 2848-2852, June 2009.

[22] Ahmed B. Abdurrhman and Michael E. Woodward, "Unequal Error protection of H.264/AVCvideo Using Hierarchical QAM," in *Telekomunikacioni forum*, Srbija, 2008, pp. 593-596.

[23] Seyed Ali Ahmadzadeh, Seyed Abolfazl Motahari, and Amir K. Khandani, "Signal space cooperative communication," *IEEE Transactions on Wireless Communications*, vol. 9, no. 4, pp. 1266-1271, April 2010.

[24] Hongjie Zhao, Hua Wang, Jingming Kuang, Zesong Fei, and Chaoxing Yan, "Improved coded cooperation scheme with signal space diversity in wireless networks," in *International Conference on Wireless Communications & Signal Processing*, 2009, pp. 1-6.

[25] G. Taricco and E. Viterbo, "Performance of component interleaved signal sets for fading channels," *Electronic Letters*, vol. 32, no. 13, pp. 1170-1172, June 1996.

[26] Majid N. Khormuji , Umar H. Rizvi, Gerard J. M. Janssen, and S. Ben Slimane, "Rotation Optimization for MPSK/MQAM Signal Constellations Over Rayleigh Fading Channels," in *10th IEEE International Conference on Communication systems*, Singapore, 2006, pp. 1-5.

[27] Soonki Jo, Jinyong Choi, and Jongsoo Seo, "An Advanced Hierarchical Modulation with Rotated Constellation," in *The 12th International Conference on Advanced Communication Technology*, vol. 1, 2010, pp. 515-518.

[28] Shu Wang, Soonyil Kwon, and Byung K. Yi, "On Enhancing Hierarchical Modulation," in *IEEE International Symposium on Broadband Multimedia Systems and Broadcasting*, Las Vegas, 2008, pp. 1-6.

[29] Thomas Kaiser and Yisheng Xue, "Cooperated TDMA uplink based on hierarchically modulated Alamouti code," in *International Zurich Seminar on Communications*, Zurich, 2004, pp. 200-203.

[30] E. Stauffer and B. Hochwald, "Low-Complexity Multi-Stream Space-Time Codes-Part I: Direct-Sum Codes and Design Criteria," *IEEE Transactions on Communications.*, vol. 60, no. 1, pp. 81-99, January 2012.

[31] E. Stauffer and B. Hochwald, "Low-Complexity Multi-Stream Space-Time Codes-Part I: Direct-Sum Codes and Design Criteria," *IEEE Transaction on Communications*, vol. 60, no. 1, pp. 81-99, January 2012.

[32] Tolga Kurt and Hakan Delic, "Space-time coding and signal space diversity in the presence of channel estimation errors," in *Canadian Confrence on Electrical and Computer Engineering*, Niagara Falls, 2004, pp. 273-276.

[33] Taejin Jung and Kyungwhoon Cheun, "Design of Concatenated Space-Time Block Codes Using Signal Space Diversity and the Alamouti Scheme," *IEEE Comunications Letters*, vol. 7, no. 7, pp. 329-331, July 2003.

[34] Sungho Jeon, Jaekwon Lee, Ilsoo Kyung, and Man-Sik Kim, "Component-interleaved Alamouti coding with rotated constellations for signal space diversity," in *IEEE International Symposium on Broadband Multimedia Systems and Broadcasting*, Shanghai, 2010, pp. 1-6.

[35] Sungho Jeon, Ilsoo Kyung, and Man-Sik Kim, "Component-Interleaved Receive MRC with Rotated Constellation for Signal Space Diversity," in *Vehicular Technology Conference Fall*, Anchorage, 2009, pp. 1-6.

[36] G. Taricco and E. Viterbo, "Performance of component interleaved signal sets for fading channels," *Electronic Letters*, vol. 32, no. 13, pp. 1170- 1172, June 1996.

Part II

Included Papers

Paper A

Hierarchical Modulated QAM with Signal Space Diversity and MRC Reception in Nakagami-m fading channels

Ayesha Saeed, Tahmid Quazi, Hongjun Xu

Published in IET Communications

Abstract

This paper combines hierarchical modulation (HM) with signal space diversity (SSD) to present an unequal error protection (UEP) scheme with improved bit error rate (BER) performance for single as well as multiple antenna reception by employing maximal ratio combing (MRC) in Nakagami-m fading channels. The optimal rotation angles for different constellation priority parameter α are derived. A theoretical BER expression in Nakagami-m fading channels using the nearest neighbor (NN) approximation approach is also derived. This is confirmed as a valid performance approximation for single and multiple antenna reception through simulations. The resultant scheme produces improved BER performance for both important and less important data without consuming any additional power than a non-SSD system at the cost of increased complexity at detection. Specifically, performance gains of up to 18 dB and 17 dB at a BER of 1×10^{-5} for base and refinement bits respectively are observed for single receive antenna with Nakagami $m = 1$. A further improvement of roughly 3 dB at a BER of 1×10^{-6} for base bits at $N = 3$ receiver antennas is found as hierarchy is increased. Though the gain is observed to decrease as m increases, a gain of 14dB at 1×10^{-6} for $\alpha = 4$ is present as m increases from $m = 1$ to $m = 3$.

1. Introduction

The growing demand for multimedia communication over noisy wireless channels poses the constraint on transmission systems to provide enhanced error protection in order to improve the quality of reconstructed data. Unequal error protection (UEP) is a technique which makes use of the fact that not all the information in a multimedia data stream is equally important. Based on the study in [1], data can be divided into different categories based on its importance and assigned varying levels of error protection. Hierarchical modulation (HM) implements UEP by mapping high priority data with the most significant bit (MSB) positions and lower priority data with least significant bit (LSB) positions in the constellation. Since HM constellations comprise of non-uniformly spaced signal points, high priority data is ensured greater protection than low priority data. As a result, all receivers under various channel conditions should be able to recover the basic or coarse data represented by the MSB while those receivers under less hostile channel conditions or better RF receivers would also be able to recover the refinement or detail data represented by the LSB [2].

HM is largely employed in image and video transmission over noisy wireless channels. In [3], a wireless image transmission scheme using hierarchical QAM demonstrates the performance gain. Similar results are presented for video transmission through wireless channels [4] as well as power line communication [5]. In a recent study in [6], cross-layer design employing UEP in the form of joint source channel coding in addition to HM presents further performance improvement in wireless image and video transmission.

HM is also implemented to enable two-user opportunistic scheduling [7], adaptive modulation schemes [8], correlated multiple-input multiple-output (MIMO) systems [9] as well as cooperative communication systems [10] amongst several other applications.

More recently, Hochwald and Stauffer in [11,12] present unitary-transform code and direct-sum Alamouti code combined with HM indicated performance gains with single and multiple antenna transmission schemes. Additionally, Chang *et al* propose a scheme in [13] to attain multiple levels of UEP using multiplexed hierarchical QAM. Results in Rayleigh fading channels indicate a performance improvement for progressive data transmission.

HM aims to improve the quality of communication of a system by ensuring low error probability for high priority data, however, at the cost of the performance of low priority data. Further, as hierarchy is increased, additional power is required to transmit the non-uniformly spaced constellation. As a result, there is a limit to the benefit of using only hierarchy as a method to improve system performance.

Without consuming any additional bandwidth, transmit power or space, signal space diversity (SSD) is a technique which takes advantage of the intrinsic diversity of multi-dimensional constellations to improve BER performance [14]. Combining HM and SSD can further improve bit error rate (BER) performance for both high priority data and low priority data. Combining HM and SSD is briefly examined in [15] for a 16-QAM constellation over a Rayleigh fading channel. However, no derivation for BER performance is documented nor a method for obtaining an optimum angle of rotation for different constellation priority parameter. This paper further looks at HM with SSD for single as well as multiple antenna reception by employing maximal ratio combing (MRC) in a Nakagami-m fading channel. It presents a technique for acquiring a rotation angle for a specific hierarchical constellation at which performance can be optimized. Among various error analysis methods for rotated constellations with component interleaving [16-18], the nearest neighborhood approximation analysis in [17] is extended to hierarchical modulated M-QAM with SSD to attain a closed-formed theoretical BER expression in Nakagami-m fading channels. This method of error analysis for HM proves not only to be an accurate approximation for BER but reduce computational complexity from the previous approaches [2, 19-21] which consider the probability of each symbol succumbing to error for each bit position. In addition, this paper presents a detailed model for a 16-QAM constellation which can easily be extended to larger constellation sizes.

The paper is organized as follows. Section 2 entails a detailed description of the system model and all necessary parameters. In Section 3, the approach to obtaining the optimized rotation angle is outlined before a theoretical BER expression is derived using the nearest neighbor approximation in Section 4. Simulation results for single and multiple antenna reception, in accordance with the DVB-T (Digital Video Broadcast – Terrestrial) standard [22], are presented in Section 5 to verify theoretical expressions under various channel conditions. Section 6 contains concluding remarks.

2. System Model

The system model of 4/M-QAM hierarchical modulated QAM with SSD comprises of an input bit stream which is Gray mapped before hierarchically modulated using relevant distance and energy parameters. A hierarchical modulated symbol pair is then rotated and passed through the component interleaver before transmitted across N independent and identically distributed (i.i.d) Nakagami-m fading channels. At the receiver, MRC is employed to combine the signal and passed through the de-interleaver. Maximum likelihood (ML) detector is then used to decode the de-interleaved signal set.

In the system model, an input bit stream is partitioned into the in-phase and quadrature-phase sub-channels such that the resultant bit streams are of the form $b_1^I, b_2^I, \dots b_{k/2}^I$ and $b_1^Q, b_2^Q, \dots b_{k/2}^Q$, respectively, where $k = \log_2 M$ and M represents the constellation size. The sub-channel bit streams are then independently Gray coded using (A.1).

$$\begin{cases} g_1 = b_1 \\ g_i = b_i \oplus b_{i-1} \end{cases} \tag{A.1}$$

where $\oplus$ represents modulo-2 addition and $2 \leq i \leq k/2$.

It should be noted that the corresponding hierarchical constellation, refer to Figure 2 in [19], is labeled and Gray encoded in the same fashion. The in-phase bits are encoded such that the constellation points are labeled from right to left starting from 0 to $\sqrt{M}$-1. The labels are then converted to binary form and Gray encoded. A similar process is applied to encode the quadrature-phase bits where the constellation points are labeled from top to bottom starting from 0 to $\sqrt{M}$-1. Once independently Gray encoded the in-phase and quadrature-phase bit streams are combined in $i_1 q_1 i_2 q_2 \cdots i_{k/2} q_{k/2}$ manner to form a single bit stream.

In HM, there are two essential distance parameters: the distance between two neighboring symbols within the same quadrant as $2b$ and the minimum distance between two symbols in adjacent quadrants as $2a$. These two distance parameters are shown in Figure 2 in [19].

The constellation priority parameter, as defined for the DVB-T (Digital Video Broadcast – Terrestrial) standard in [22] as $\alpha = a/b$, determines the hierarchy of bit priority. For example, uniform 16-QAM by setting $\alpha = 1$ and alternatively base bits are given higher priority over refinement bits when $\alpha > 1$.

The energy parameter is a key parameter to control bit hierarchy and is given by [20].

$$E_s = 2\left(a + \left(\frac{\sqrt{M}}{2} - 1\right)b\right)^2 + \frac{2}{3}\left(\frac{M}{4} - 1\right)b^2 \tag{A.2}$$

Using the distance and energy parameters, the Gray encoded input bit stream is mapped to its corresponding symbol in the hierarchical constellation. Before transmission is performed, the modulated signal is further processed for SSD.

Similar to the conventional modulation with SSD [16], the pair of rotated signals is given as $\widetilde{x_i} = x_i R^{\theta}, i = 1,2$, where $x_i = \begin{bmatrix} x_i^I & x_i^Q \end{bmatrix}$ consists of the in-phase and quadrature-phase components of the modulated signal and R^{θ} is given by [16]

$$R^{\theta} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{A.3}$$

Pairs of the rotated consecutive signals are formed and passed onto the component interleaver. The in-phase and quadrature-phase components of each symbol in a symbol pair are then interleaved to form a new symbol set $u_1$and u_2 as shown in (A.4).

$$u_1 = \tilde{x}_1^I + j\tilde{x}_2^Q \tag{A.4.1}$$

$$u_2 = \tilde{x}_2^I + j\tilde{x}_1^Q \tag{A.4.2}$$

where u_1 and u_2 are transmitted over an orthogonal channel, assumed to be independent time slots such that u_1 is transmitted in the first time slot and u_2 is transmitted in the subsequent time slot. In addition, the symbols are transmitted over a single transmission antenna over N i.i.d Nakagami-m fading channels with additive white Gaussian noise (AWGN) from the N receiving antennas at the destination. The corresponding received signal is given by

$$r_{i,j} = h_{i,j}u_i + n_{i,j} \qquad i \in \{1,2\}, \qquad j \in \{1,2 \cdots, N\} \tag{A.5}$$

where $n_{i,j}$ have i.i.d entries according to the complex Gaussian distribution $\mathcal{CN}(0,1)$ and $h_{i,j}$ is the fading coefficient in time slot i and at receive antenna j. The fading is modeled as the Nakagami-m distributed random variables according to [9]

$$f(h_{i,j}) = \frac{2m^m}{\Omega^m \Gamma(m)} h_{i,j}^{2m-1} \exp\left(-\frac{m{h_{i,j}}^2}{\Omega}\right) \tag{A.6}$$

where $\mathrm{E}\left[{h_{i,j}}^2\right] = \Omega$ is the average fading power, $\Gamma(\cdot)$ is the Gamma function and m is the Nakagami-m fading parameter. Assume full channel state information (CSI) is known at receiver. Based on MRC, the combined signal is given by

$$r_i = \sum_{c=1}^{N} h_{i,c} r_{i,c} \ , \quad i \in \{1,2\} \tag{A.7}$$

Then the ML detection under MRC reception is given by [23]

$$\hat{y}_1 = \arg\min_{\mathrm{x_k} \in \tilde{S}} \{h_2^2 |r_1^R - h_1^2 x_k^R|^2 + h_1^2 |r_1^I - h_2^2 x_k^I|^2\} \tag{A.8.1}$$

$$\hat{y}_2 = \arg\min\nolimits_{\mathrm{x_k} \in \tilde{S}} \{h_1^2 |r_2^R - h_2^2 x_k^R|^2 + h_2^2 |r_2^I - h_1^2 x_k^I|^2\} \tag{A.8.2}$$

where $\tilde{S}$ represents the signal set, $r_i = r_i^R + j r_i^I$, $i \in [1,2]$, $\hat{y}_i$ is the detected symbol for the respective time slot and h_i^2 is the instantaneous combined fading term based on the sum of individual instantaneous branch fading powers as shown in (A.9).

$$h_i^2 = \sum_{c=1}^{N} h_{i,c}^2 \tag{A.9}$$

The detected signal is then un-rotated using the transpose $\left[R^\theta\right]^T$ of the rotation matrix in (A.3). If the detected rotated symbol is represented as the row vector $\hat{y}_i = \left[\hat{y}_i^I \quad \hat{y}_i^Q\right], i = 1,2,$ consisting of the in-phase and quadrature-phase components, then the corresponding un-rotated symbol is given as $y_i = \hat{y}_i\left[R^\theta\right]^T$. The resultant signal is then separated into its in-phase and quadrature-phase sub-channels. The corresponding k bits are then retrieved and assigned to their respective bit positions (MSB i_1 to LSB i_k). In the case of 4/16-QAM, two base bits, which carry the high priority data, are determined as bit i_1 and bit q_1, while the remaining bits are considered refinement bits.

3. Optimization Angle

The angle at which the constellation is rotated is fundamental to the performance gain of the system. Previously, the optimum angle is determined based on a design criterion which either maximizes the diversity order [24,25] or maximizes the minimum Euclidean square product distance between components [24, 26-27]. Alternatively, the optimum rotation angle can be determined by minimizing the SER expression as shown in [18].

For this paper, performance is optimized when a hierarchical 16-QAM constellation is rotated such that the minimum distance between the unique components is maximized. The angle of rotation must ensure that unique components adjacent to one another are optimally spaced apart. As the angle of rotation increases, the performance improves as each unique component is spaced further apart. However, if the angle is too large, the unique components begin to overlap with each other leading to poor performance. Since hierarchical constellations are non-uniformly spaced, the optimization angle varies according to hierarchy.

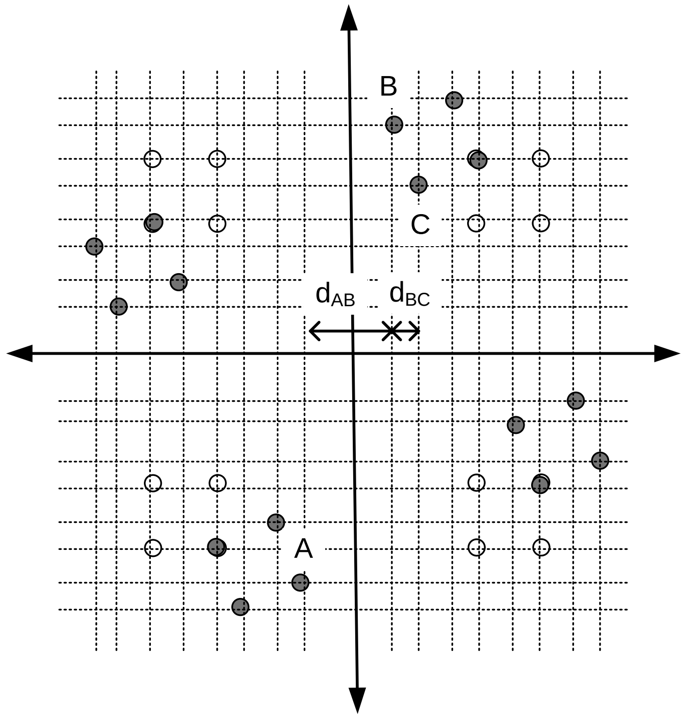

Figure A.1 Optimum rotation angle for 16-QAM Constellation

Consider the Euclidean distance between the real components of symbols A and B with the Euclidean distance between the real components of symbols B and C. As the angle of rotation increases, distance d_{AB} decreases whilst distance d_{BC} increases. Performance is optimized when these distances are equal and the unique components of symbols are equally spaced apart (i.e. $d_{AB} = d_{BC}$).

Using the rotation matrix given in (A.3), the real component of the rotated symbols is determined as below

$$x_A = [-a\cos\theta + (a + 2b)\sin\theta] \tag{A.10.1}$$

$$x_B = [a\cos\theta - (a + 2b)\sin\theta] \tag{A.10.2}$$

$$x_C = [a\cos\theta - \mathrm{a}\sin\theta] \tag{A.10.3}$$

Performance is then optimized when

$$x_B - x_A = x_C - x_B \tag{A.11}$$

Substituting the values from (A.10) and simplifying using the constellation priority parameter defined earlier as $\alpha = a/b$, the optimization angle is determined in (A.12).

$$\tan\theta = \frac{\alpha}{\alpha + 3} \qquad (\mathrm{A}.12)$$

This method of analysis can be easily extended to a hierarchical 64-QAM constellation to determine the optimum rotation angle for the corresponding hierarchical parameter α.

4. Performance Analysis of HM with SSD

Previously, performance analysis for HM [2,19-21] uses an approach where BER is determined by considering the probability that a transmitted symbol exceeds past a set decision boundary and becomes an error. Though extremely accurate, the approach requires each symbol in the constellation to be considered to determine the error for each bit position. Consequently, it is unnecessarily complicated and time consuming.

In this section, the NN union bound approach, as present in [17], is adapted to determine an accurate approximation of the BER for hierarchical 16-QAM with SSD and MRC reception. The conventional union bound method evaluates the probability error of a signal set $\tilde{S}$ by considering the pairwise error probability (PEP) of all possible pairs of transmitted and received symbols. The union bound error probability, as is defined in [26] and given as

$$P_S^U(e) \leq \frac{1}{|\tilde{S}|} \sum_{x \epsilon \tilde{S}} \sum_{\substack{\hat{x} \epsilon \tilde{S} \\ x \neq \hat{x}}} P(x \to \hat{x}) \tag{A.13}$$

where $|\tilde{S}|$ represents the cardinality of the signal set and $P(x \to \hat{x})$ denotes the unconditional PEP that a transmitted symbol x is detected as $\hat{x}$.

The NN union bound approximation simplifies the conventional union bound approach by only considering the symbols with the smallest Euclidean distances to the transmitted symbol. As a result of the implementation of an ML detector at the receiver, an error occurs when a transmitted symbol is detected as either its nearest perpendicular neighbour or its nearest diagonal neighbour.

To determine the error rate for base bits, bits i_1 and q_1 in the case of 4/16-QAM, the PEP that a transmitted symbol is incorrectly detected as a perpendicular or diagonal neighbour in a different quadrant. Without loss of generality, suppose symbol x_B in Figure A.2 is transmitted. First, probability of error due to perpendicular neighbours is considered (such as between symbols x_B and x_C in Figure A.2). The perpendicular error probability for base bits $P_{Base}^{Perpendicular}$ is given by

$$P_{Base}^{Perpendicular} = \frac{1}{2}\left[\frac{8}{16}P_{Perpendicular}^{I} + \frac{8}{16}P_{Perpendicular}^{Q}\right]$$

$$= \frac{1}{2}P(x_B \to x_C) \qquad (A.14)$$

where $P_{Perpendicular}^{I}$ represent the PEP due to perpendicular in-phase neighbours and $P_{Perpendicular}^{Q}$ represent the PEP due to perpendicular quadrature-phase neighbours.

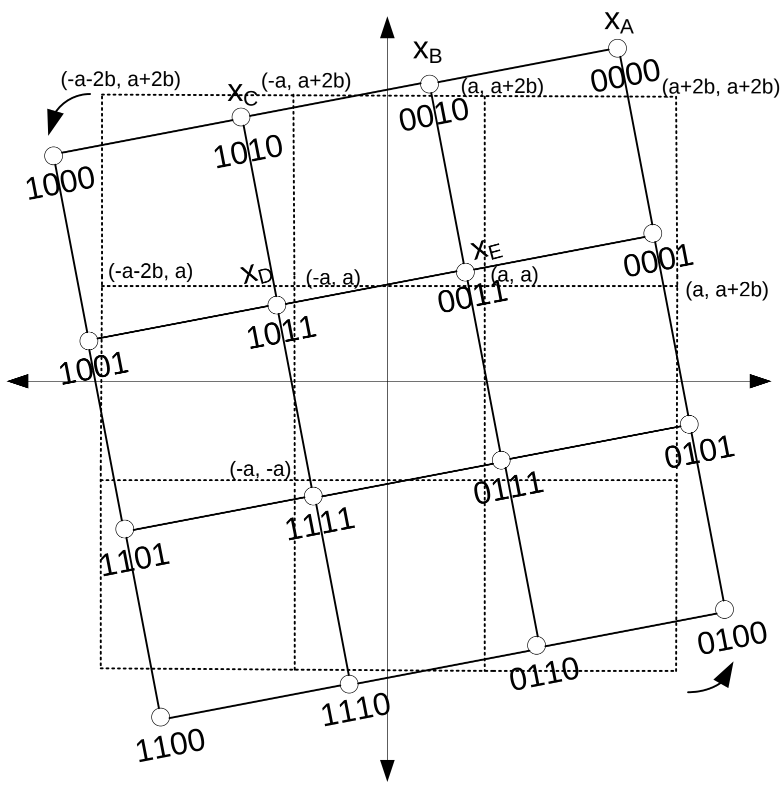

Figure A.2 Rotated hierarchical 16-QAM nearest neighbor error analysis

At the receiver side, taking into account the channel gains h_I and h_Q, the received symbol can be represented as,

$$y_B = [h_I\,(a) \quad h_Q\,(a+2b)] \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= [h_I\,(a\cos\theta - (a+2b)\sin\theta) \quad h_Q(a\sin\theta + (a+2b)\cos\theta)] \qquad (A.15)$$

The error probability between perpendicular neighbours, according to [17], is expressed as

$$P\left(x_B \to x_C | h_I, h_Q\right) = Q\left(\frac{\|y_B - y_C\|}{\sqrt{2}N_o}\right) \tag{A.16}$$

where $\|y_B - y_C\| = \sqrt{(y_B^I - y_C^I)^2 + \left(y_B^Q - y_C^Q\right)^2}$. The error probability in (A.16) can be further simplified as

$$P\left(x_B \to x_C | h_I, h_Q\right) = Q\left(\sqrt{\frac{\alpha^2}{\alpha^2 + 2\alpha + 2} SNR\left(h_I^2 \cos^2 \theta + h_Q^2 \sin^2 \theta\right)}\right) \tag{A.17}$$

where $SNR = \frac{E_s}{N_o} = \frac{(\alpha^2+2\alpha+2)2b^2}{N_o}$ from the average symbol energy of hierarchically modulated 16-QAM, $Q(\delta)$ is the Gaussian Q-function defined as $Q(\delta) = \frac{1}{2\sqrt{\pi}} \int_\delta^\infty e^{\frac{-\delta^2}{2}}$ and the random variable h_i^2, $i = 1,2$, have a central chi-square distribution as described in [17]. Substituting $\gamma_I = h_I^2 SNR$, $\gamma_Q = h_Q^2 SNR$, and $\beta_1 = \frac{\alpha^2}{\alpha^2+2\alpha+2}$ into (A.17),

$$P\left(x_B \to x_C | h_I, h_Q\right) = Q\left(\sqrt{\left(\gamma_I \beta_1 \cos^2 \theta + \gamma_Q \beta_1 \sin^2 \theta\right)}\right) \tag{A.18}$$

Then the error probability can be obtained by averaging the conditional PEP in (A.18) over the independent fading distributions for MRC reception as shown in (A.19) below.

$$P(x_B \to x_C) = \int_0^\infty \int_0^\infty P\left(x_B \to x_C | h_I, h_Q\right) f_\gamma(\gamma_I) f_\gamma\left(\gamma_Q\right) d\gamma_I d\gamma_Q \tag{A.19}$$

where the probability density function for the combined SNR in the presence of MRC in a Nakagami-m fading channel is given in (A.20) for N received antennas [28].

$$f_\gamma(\gamma_i) = \left(\frac{m}{\bar{\gamma}}\right)^{mN} \frac{\gamma_i^{mN-1} \exp\left(-\frac{m\gamma_i}{\bar{\gamma}}\right)}{\Gamma(mN)} \tag{A.20}$$

where m is Nakagami-m fading parameter and $\Gamma(\cdot)$ is the Gamma function.

The trapezoidal approximation to the $Q(\delta)$ function, as shown in [29], is used to simplify the above analysis. The approximation is implemented over a total of p iterations as shown in (A.21)

$$Q(\delta) = \frac{1}{2p}\left(\frac{1}{2}\exp\left(\frac{-\delta^2}{2}\right) + \sum_{k=1}^{p-1} \exp\left(\frac{-\delta^2}{S_k}\right)\right) \tag{A.21}$$

where $S_k = 2\sin^2(k\pi/2p)$.

The final PEP can be written as

$$P(x_B \to x_C) = \frac{1}{4p}\left(\frac{2m}{2m+\bar{\gamma}\beta_1\cos^2\theta}\right)^{mN}\left(\frac{2m}{2m+\bar{\gamma}\beta_1\sin^2\theta}\right)^{mN}$$
$$+\frac{1}{2p}\sum_{k=1}^{p-1}\left(\frac{mS_k}{mS_k+\bar{\gamma}\beta_1\cos^2\theta}\right)^{mN}\left(\frac{mS_k}{mS_k+\bar{\gamma}\beta_1\sin^2\theta}\right)^{mN} \quad (\mathrm{A.22})$$

The error probability between diagonal neighbours is small enough to be negligible for this approximation. However, when considering the performance of the base bits, the error probability between diagonal neighbours (such as x_B and x_D or x_E and x_C in Figure A.2) becomes comparable as hierarchy is increased (specifically, as distance a becomes increasingly larger than distance b). The diagonal error probability $P_{Base}^{Diagonal}$ for base bits is then given as,

$$P_{Base}^{Diagonal} = \frac{1}{2}\left[\frac{4}{16}P_{Diagonal}^{I} + \frac{4}{16}P_{Diagonal}^{Q}\right]$$
$$= \frac{1}{8}P(x_B \to x_D) + \frac{1}{8}P(x_E \to x_C) \quad (\mathrm{A.23})$$

where $P(x_B \to x_D)$ and $P(x_E \to x_C)$ in (A.23) can be derived using the aforementioned method. The final error probability for base bits is then determined as

$$P_{Base} = \frac{1}{2}P(x_B \to x_C) + \frac{1}{8}P(x_B \to x_D) + \frac{1}{8}P(x_E \to x_C) \quad (\mathrm{A.24})$$

Similarly, the error probability for the refinement bits can be defined as the PEP that a transmitted symbol (such as symbol x_A in Figure A.2) is detected as a perpendicular neighbour within the same quadrant (such as symbol x_B in Figure A.2).

$$P_{Refinement} = \frac{1}{2}\left[\frac{16}{16}P_{Refinement}^{I} + \frac{16}{16}P_{Refinement}^{Q}\right] = P(x_A \to x_B) \quad (\mathrm{A.25})$$

Similar to the derivation of $P(x_B \to x_C)$ in (24), $P(x_A \to x_B)$ can be derived as (A.26) with $\beta_2 = \frac{1}{\alpha^2+2\alpha+2}$.

$$P(x_A \to x_B) = \frac{1}{4p}\left(\frac{2m}{2m + \bar{\gamma}\beta_2 \cos^2\theta}\right)^{mN}\left(\frac{2m}{2m + \bar{\gamma}\beta_2 \sin^2\theta}\right)^{mN}$$

$$+ \frac{1}{2p}\sum_{k=1}^{p-1}\left(\frac{mS_k}{mS_k + \bar{\gamma}\beta_2 \cos^2\theta}\right)^{mN}\left(\frac{mS_k}{mS_k + \bar{\gamma}\beta_2 \sin^2\theta}\right)^{mN} \tag{A.26}$$

5. Simulation Results

This section presents simulation results for hierarchically modulated 4/16-QAM with SSD and MRC reception in Nakagami-m fading channels to verify the NN approximation approach for the BER performance. The optimum rotation angle is determined using the method described in Section 3. The angle at which BER performance for base bits, as oppose to refinement bits, is optimum is considered since the intention of HM is to improve system performance by ensuring higher protection for high priority data. Consequently, performance gain at the obtained optimal rotation angle is observed as hierarchy is varied by increasing the ratio between the distances (α). The gain in BER performance under multiple antenna reception (for $N \geq 3$) is also presented. The simulations are performed over i.i.d Nakagami-m fading channels with AWGN. It is also assumed that CSI is fully available at the receiver. The optimum rotation angle derived from theory in Section 3 for $\alpha = 1$ and $\alpha = 4$ as approximately $\theta = 14.036°$ and $\theta = 29.745°$, respectively, is used for simulation.

5.1. Single Antenna Reception with m=1

The BER performance of the system using only a single antenna at the receiver with $m = 1$ is considered in this subsection. Using the determined optimum rotation angle, the NN BER performance analysis is verified against Monte Carlo simulations. Figures A.3-A.4 show the performance gain between hierarchical modulated constellations with and without SSD. As hierarchical priority parameter is increased, the performance gain for base bits increases at the cost of decreasing gain for refinement bits. However, the overall performance of the system is improved by ensuring enhanced protection for high priority data. Furthermore, the simulations indicate that the theoretical BER expressions derived in Section 4 are accurate approximations of system performance.

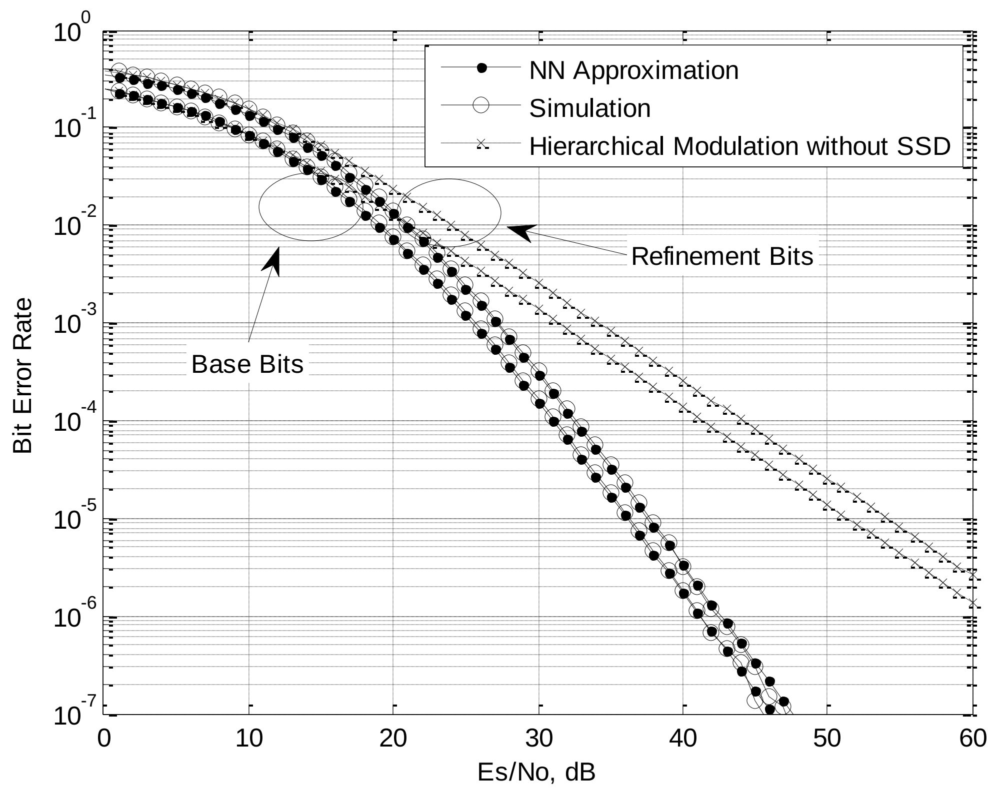

Figure A.3 4/16-QAM HM with SSD at $\alpha = 1$ and $m = 1$

In Figure A.3, the hierarchical constellation is uniformly spaced with the hierarchical priority parameter set to $\alpha = 1$. As a result, the base bits have a minor performance improvement to refinement bits. However, when the performance between hierarchically modulated signal with and without SSD is compared, a performance improvement of approximately 15 dB and 16 dB is observed at a BER of 1×10^{-5} for base and refinement bits, respectively.

As the ratio between the distances is increased and base bits are given higher priority, the BER performance for base bits improves further while the performance for refinement bits degrades. This is expect, as increasing hierarchy results in greater protection for base bits at the cost of lower protection for the refinement bits. In Figure A.4, for $\alpha = 4$, a performance gain between hierarchically modulated signal with and without SSD of roughly 18 dB and 17 dB at a BER of $1 \times 10^{-5}$is observed for base and refinement bits respectively.

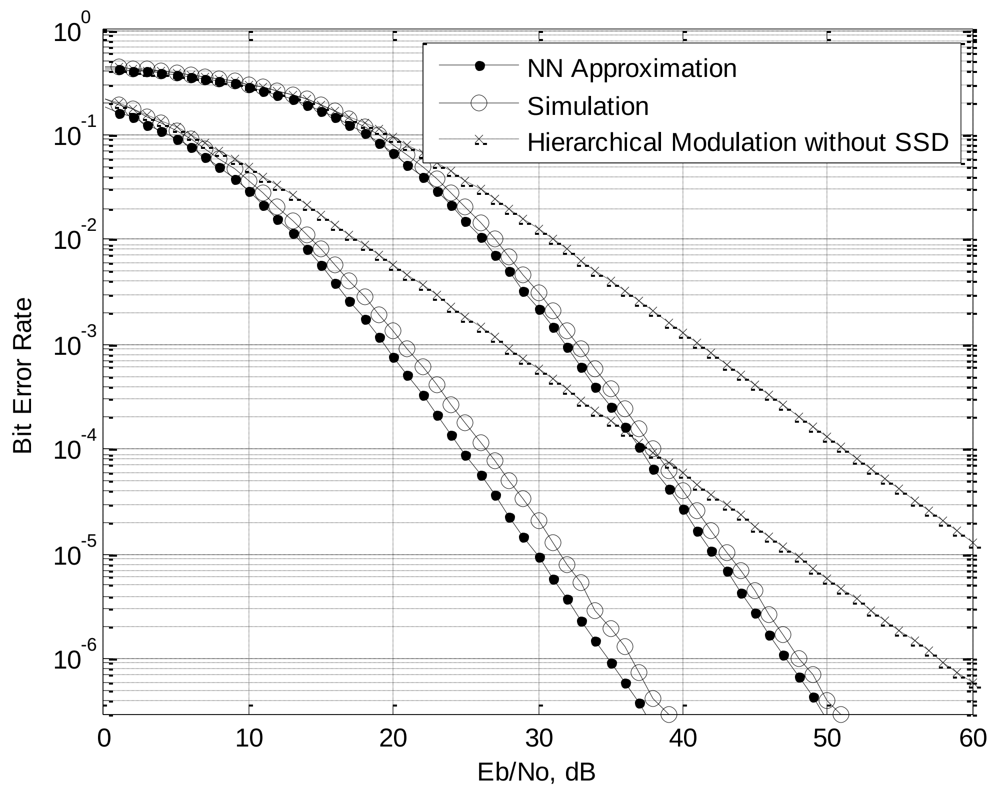

Figure A.4 4/16-QAM HM with SSD at $\alpha = 4$ and $m = 1$

The observed performance gains between hierarchically modulated 4/16-QAM with and without SSD for $\alpha = 1$ and $\alpha = 4$, in Figure A.3 and Figure A.4 respectively, demonstrate the decrease in the proposed scheme's susceptibility to channel fading. As a result of rotating the hierarchical constellation, each symbol has two distinct components which reduces the possibility of a symbol being incorrectly detected. In addition, component interleaving ensures independent fading is experienced by the unique components as a result of which the original symbol can still be correctly recovered by the detector if one component, in-phase or quadrature-phase, is corrupted by channel fading.

5.2. Multiple Antenna Reception with m=1

In this section, the system performance for multiple receiver antennas with $m = 1$ is analysed in the form of BER performance for increasing hierarchy. Specifically, the system performance is observed for N=3 and N=4 receive antennas.

Using the aforementioned rotation angles for the respective hierarchical constellation, the NN approximation for BER performance for N=3 and N=4 receiver antennas is verified against simulation results with $m = 1$. Figures A.5- A.6 illustrate the improvement in BER performance with increasing receiver antennas for various values of hierarchy. The results further confirm the NN approximation as an accurate estimation of system performance.

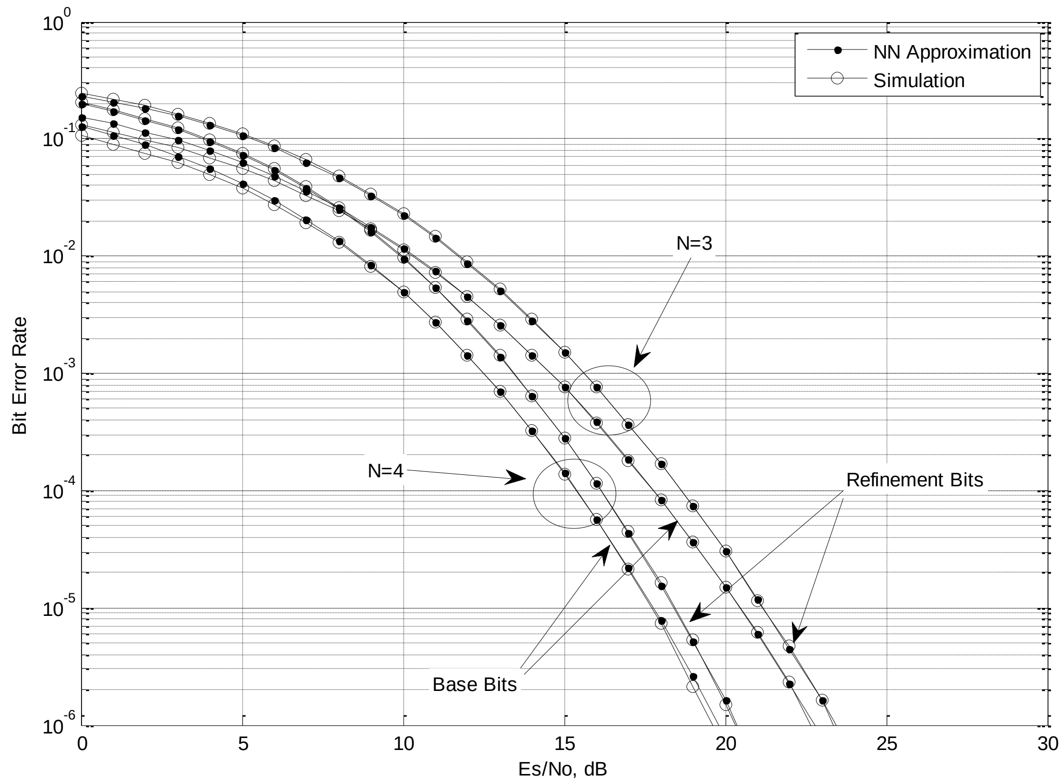

Figure A.5 4/16-QAM HM with SSD and MRC for N=3 and N=4 receive antennas at $\alpha = 1$ and $m = 1$

In Figure A.5, the hierarchical constellation is uniformly spaced with the hierarchical priority parameter set to $\alpha = 1$. The performance improvement from N=3 antennas and N=4 antennas is approximately 4 dB at a BER of 1×10^{-6} for both base and refinement bits.

In Figure A.6, hierarchy is increased to $\alpha = 4$ while the performance improves with the increase in number of receivers. The performance improvement from N=3 antennas and N=4 antennas remains at approximately 4 dB at a BER of 1×10^{-6} for both base and refinement bits. However, the overall performance of the base bits improves while the performance of the refinement bits degrades as hierarchy is increased. Specifically, at N=3 antennas, the base bits improve by roughly 3 dB at a BER of 1×10^{-6} while the refinement bits deteriorate by about 3 dB at a BER of 1×10^{-6}. Similarly, at N=4 antennas, the base bits improve by approximately 4 dB at a BER of 1×10^{-6} while the refinement bits degrade by nearly 4 dB at a BER of 1×10^{-6}.

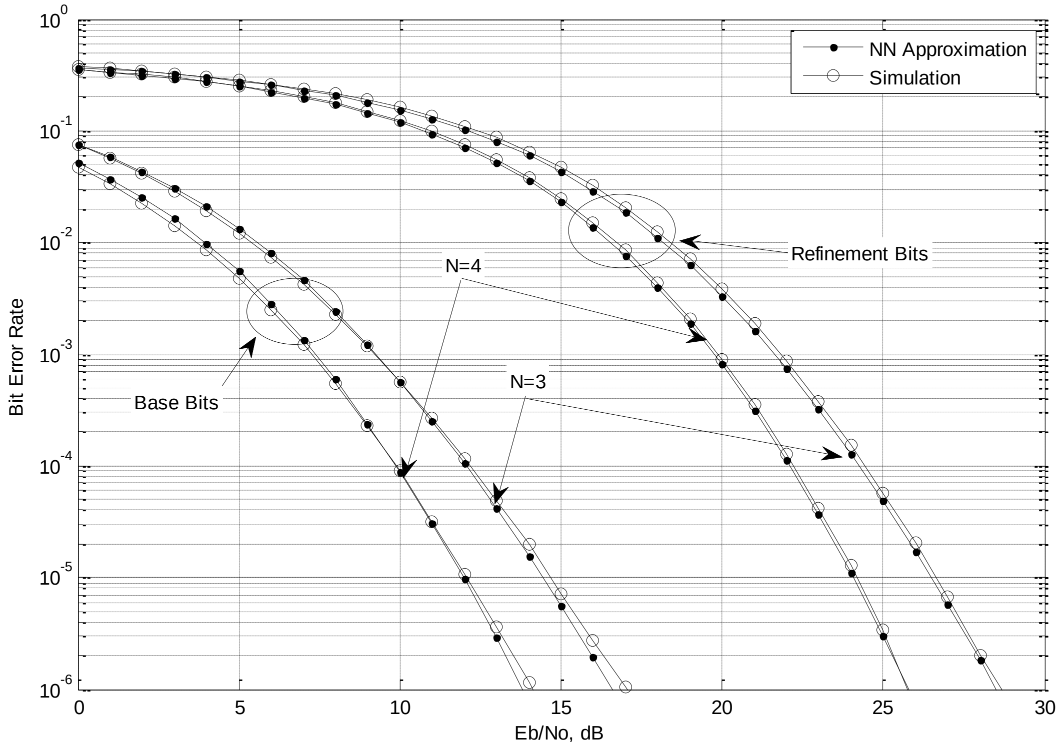

Figure A.6 4/16-QAM HM with SSD and MRC for N=3 and N=4 receive antennas at $\alpha = 4$ and $m = 1$

Performance gains observed in Figures A.5- A.6 exhibit the effect of additional receiver antennas in a transmission system. The deterioration of performance for the refinement bits in Figure A.6 once again demonstrates the effect of increasing hierarchy.

5.3. Further Results in Nakagami-m fading

Lastly, the performance of HM with SSD is analysed for various Nakagami-m fading parameters (m) in a single receiver antenna system. These results can easily be extended for multiple antennas, however, are not included in this paper due to space constraints.

In this section, the accuracy of the NN approximation is further confirmed where the constellations are rotated at the aforementioned optimum angles for the respective hierarchy. Specifically, performance for $m = 1$, $m = 3$, and $m = 5$ is compared.

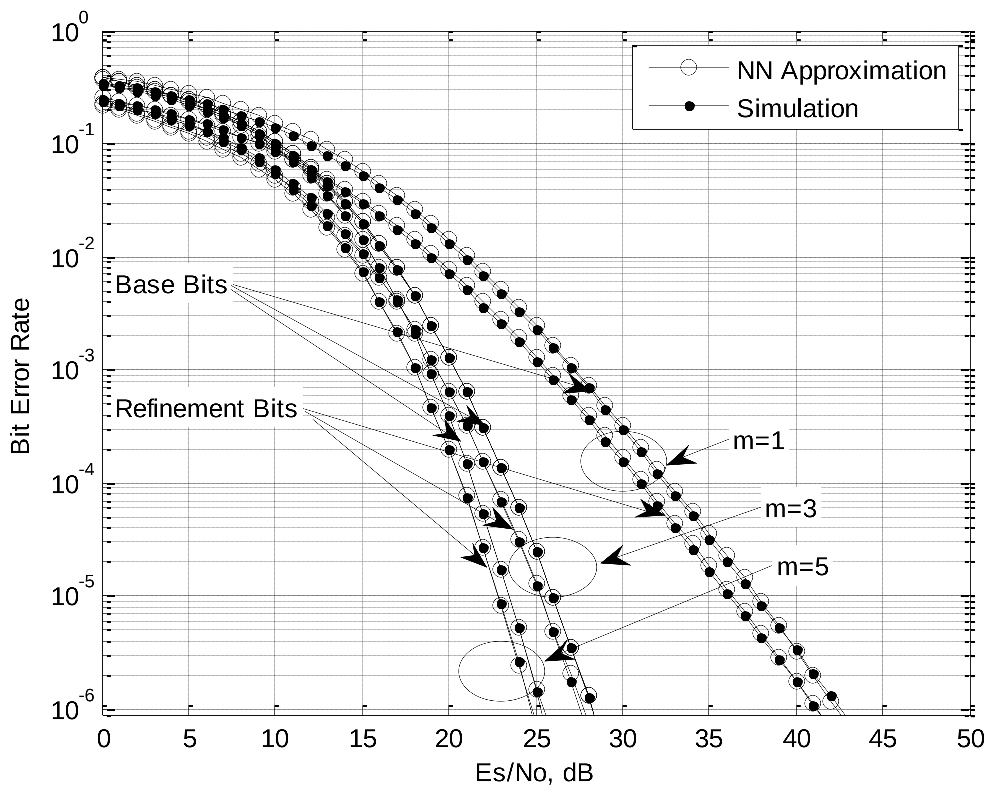

Figure A.7 4/16-QAM HM with SSD at $\alpha = 1$

As the Nakagami-m parameter m is increased the BER performance of the proposed scheme improves accordingly. This can be observed in Figures A.7- A.8 for various values of hierarchy. In Figure A.7, with a uniformly spaced constellation, the BER performance improves by nearly 12dB at a BER of 1×10^{-6} for both base and refinement bits as the channel conditions improve from

$m = 1$ to $m = 3$. However, an improvement of only 3dB at a BER of 1×10^{-6} for both base and refinement bits is observed as m increases from $m = 3$ to $m = 5$.

For non-uniformly spaced constellations, the BER performance for base bits is observed to improve while the performance of the refinement bits degrades, as expected. In Figure A.8, where $\alpha = 4$, the BER performance is seen to improve by 14dB at a BER of 1×10^{-6} for both base and refinement bits as m increases from $m = 1$ to $m = 3$. A modest improvement of 2dB is observed at a BER of 1×10^{-6} for both base and refinement bits as m increases from $m = 3$ to $m = 5$.

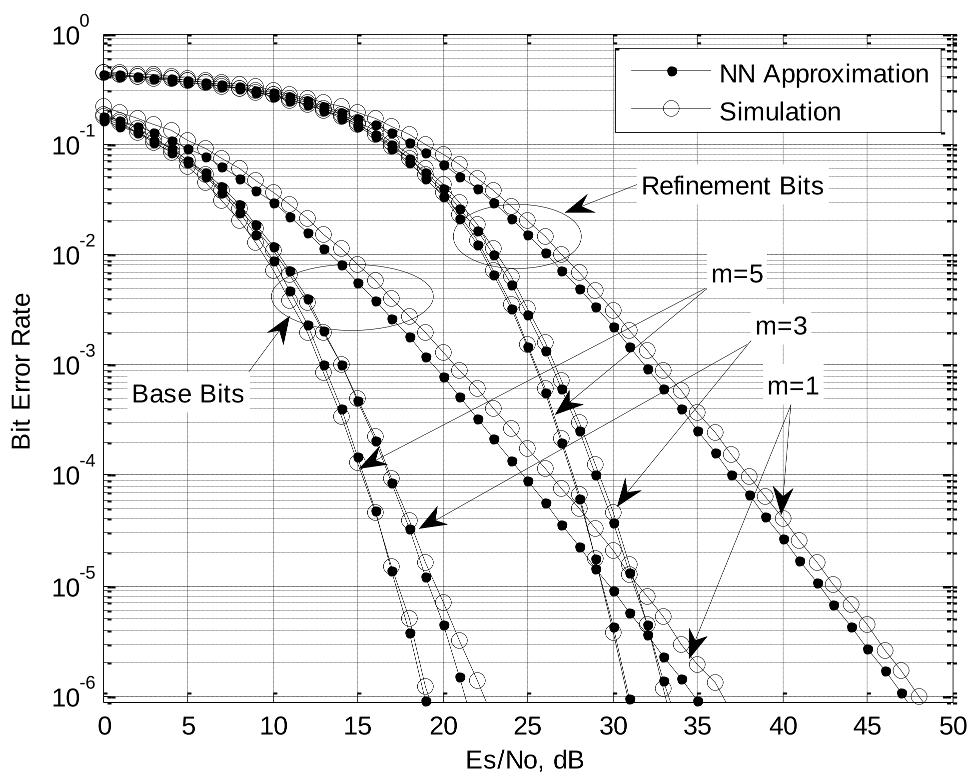

Figure A.8 4/16-QAM HM with SSD at $\alpha = 4$

It can be concluded that the performance gain is decreasing as channel conditions improve for higher values of hierarchy. This is expected as the performance of an SSD system does not differ from a non-SSD system in the absence of deep fading.

6. Conclusion

In this paper, HM with SSD and MRC under Nakagami-m fading is presented. The optimum rotation angle is derived for each respective value of α. The theoretical BER expression is derived using the NN approximation and is validated against Monte-Carlo simulation. For single antenna reception, the performance gain between HM with SSD and without SSD is observed at up to 18 dB and 17 dB at a BER of 1×10^{-5} for base and refinement bits, respectively. For multiple antennas, the BER performance improvement is observed as the number of receiver antennas is increased. Specifically, a performance improvement from $N = 3$ and $N = 4$ antennas is observed to be roughly 4 dB at a BER of 1×10^{-6} for both base and refinement bits. BER performance is also observed as m is increased and a gain of up to 14dB at 1×10^{-6} is observed for $\alpha = 4$ as channel conditions improved from $m = 1$ to $m = 3$.

References

[1] Thomas M. Cover, "Broadcast Channels," *IEEE Tranactions on Information Theory*, vol. 18, no. 1, pp. 2-14, January 1972.

[2] P.K. Vitthaladevuni and M.-S. Alouin, "A Recursive Algorithm for the Exact BER Computation of Generalized Hierarchical QAM Constellations," *IEEE Transactions on Information Theory*, vol. 49, no. 1, pp. 297-307, January 2003.

[3] M.A. Kader, F. Ghani, and R. Badlishah, "Development and Performance Evaluation of Hierarchical Quadrature Amplitude Modulation (HQAM) for Image Transmission over Wireless Channels," in *Third International Conference on Computational Intelligence, Modelling and Simulation*, 2011, pp. 227-232.

[4] Yoong Choon Chang, Sze Wei Lee, and R Komiya, "A low complexity hierarchical QAM symbol bits allocation algorithm for unequal error protection of wireless video transmission," *IEEE Transactions on Consumer Electronics*, vol. 55, no. 3, pp. 1089-1097, August 2009.

[5] F.X. Coudoux, P. Corlay, M.G. Gazalet, D. Bueche, and M. Slachciak, "On the Use of Hierarchical Modulations for Robust Video Transmission over Power Line Medium," in *IEEE International Symposium on Power Line Communications and Its Applications*, 2007, pp. 482-486.

[6] S.S. Arslan, P.C. Cosman, and L.B. Milstein, "Coded Hierarchical Modulation for Wireless Progressive Image Transmission," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 9, pp. 4299-4313, November 2011.

[7] Md. Jahangir Hossain, Mohamed-Slim Alouini, and Vijay K. Bhargava, "Rate Adaptive Hierarchical Modulation-Assisted Two-User Opportunistic Scheduling," *IEEE Transactions on Wireless Communications*, vol. 6, no. 6, pp. 2076-2085, June 2007.

[8] Md. Jahangir Hossain, Pavan Kumar Vitthaladevuni, Mohamed-Slim Alouini, Vijay K. Bhargava, and Andrea J. Goldsmith, "Adaptive Hierarchical Modulation for Simultaneous Voice and Multiclass Data Transmission Over Fading Channels," *IEEE Transactions on Vehicular Technology*, vol. 55, no. 4, pp. 1181-1194, July 2006.

[9] Guangqiu Li, Zhenyu Zhang, and Jianwu Zhang, "Exact BER Analysis of Hierarchical M-QAM in Spatially Correlated MIMO Nakagami Fading Channels," in *International Confrence on Wireless Communications, Networking and Mobile Computing*, 2007, pp. 389-392.

[10] Min-Kuan Chang and Shi-Yong Lee, "Performance Analysis of Cooperative Communication System with Hierarchical Modulation over Rayleigh Fading Channel," *IEEE Transactions on Wireless Communications*, vol. 8, no. 6, pp. 2848-2852, June 2009.

[11] B. Hochwald and E. Stauffer, "Low-Complexity Multi-Stream Space-Time Codes-Part II: Unitary-Transform Codes," *IEEE Transactions on Communications*, vol. 60, no. 2, pp. 375-383, February 2012 2012.

[12] E. Stauffer and B. Hochwald, "Low-Complexity Multi-Stream Space-Time Codes-Part I: Direct-Sum Codes and Design Criteria," *IEEE Transaction on Communications*, vol. 60, no. 1, pp. 81-99, January 2012.

[13] Seok-Ho Chang, Minjoong Rim, Pamela C. Cosman, and Laurence B. Milstein, "Optimized Unequal Error Protection Using Multiplexed Hierarchical Modulation," *IEEE Transaction on Information Theory*, vol. 58, no. 9, pp. 5816-5840, September 2012.

[14] Joseph Boutros and Emanuele Viterbo, "Signal Space Diversity: A Power- and Bandwidth-Efficient Diversity Technique for the Rayleigh Fading Channel," *IEEE Transactions on Information Theory*, vol. 44, no. 4, pp. 1453-1467, July 1998.

[15] Soonki Jo, Jinyong Choi, and Jongsoo Seo, "An Advanced Hierarchical Modulation with Rotated Constellation," in *The 12th International Conference on Advanced Communication Technology*, vol. 1, 2010, pp. 515-518.

[16] Zaid Paruk, "Signal Space Cooperative Communication with Partial Relay Selection," Electrical, Electronic and Computer Engineering, University of Kwa-Zulu Natal, Durban, MSc Thesis 2012.

[17] Sungho Jeon, Ilsoo Kyung, and Man-Sik Kim, "Component-Interleaved Receive MRC with Rotated Constellation for Signal Space Diversity," in *Vehicular Technology Conference Fall*, 2009, pp. 1-6.

[18] Jihoon Kim and Inkyu Lee, "Analysis of Symbol Error Rates for Signal Space Diversity in Rayleigh Fading Channels," in *IEEE International Conference on Communications*, 2008, pp. 4621-4625.

[19] P.K. Vitthaladevuni and M.-S. Alouini, "BER computation of 4/M-QAM hierarchical constellations," in *12th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 1, 2001, pp. B-85- B-89.

[20] P.K. Vitthaladevuni and M.-S. Alouini, "A closed-form expression for the exact BER of generalized PAM and QAM constellations," *IEEE Transactions on Communications*, vol. 52, no. 5, pp. 698-700 , May 2004.

[21] P.K. Vitthaladevuni and M.-S. Alouini, "Exact BER Computation of Generalized Hierarchical PSK Constellations," *IEEE Transactions on Communication*, vol. 51, no. 12, pp. 2030-2037, December 2003.

[22] R. Stukavec and T. Kratochvil, "Performance of Hiearichcal Modulation in DVB-T," in *20th International Conference Radioelektronika*, 2010, pp. 1-4.

[23] M.Z.A. Khan and B.S. Rajan, "Single-symbol maximum likelihood decodable linear STBCs," *IEEE Transactions on Information Theory*, vol. 52, no. 5, pp. 2062–2091, May 2006.

[24] Seyed Ali Ahmadzadeh, Seyed Abolfazl Motahari, and Amir K. Khandani, "Signal space cooperative communication," *IEEE Transactions on Wireless Communications*, vol. 9, no. 4, pp. 1266-1271, April 2010.

[25] Majid N. Khormuji , Umar H. Rizvi, Gerard J. M. Janssen, and S. Ben Slimane, "Rotation Optimization for MPSK/MQAM Signal Constellations Over Rayleigh Fading Channels," in *10th IEEE International Conference on Communication systems*, Singapore, 2006, pp. 1-5.

[26] G. Taricco and E. Viterbo, "Performance of component interleaved signal sets for fading channels," *Electronic Letters*, vol. 32, no. 13, pp. 1170-1172, June 1996.

[27] Hongjie Zhao, Hua Wang, Jingming Kuang, Zesong Fei, and Chaoxing Yan, "Improved coded cooperation scheme with signal space diversity in wireless networks," in *International Conference on Wireless Communications & Signal Processing*, 2009, pp. 1-6.

[28] I.A. Falujah and V.K. Prabhu, "Performance analysis of MQAM with MRC over Nakagami-m fading channels," *Electronics Letters*, vol. 42, no. 4, pp. 231-233 , February 2006.

[29] M. Salehi and J.G. Proakis, *Digital Communications*, 5th ed. Boston: McGraw-Hill, 2008.

Paper B

Alamouti Space-Time Block Coded Hierarchical Modulation with Signal Space Diversity and MRC Reception in Nakagami-m Fading Channel

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Abstract

This paper presents a novel approach to an Alamouti transmission scheme for hierarchical modulation (HM) with signal space diversity (SSD) in a Nakagami-m fading channel. The BER performance of the proposed system is determined using the nearest neighborhood (NN) approximation approach and is briefly discussed. In addition, a reduced complexity maximum likelihood (ML) based decoder is proposed. Simulation results verify a significant BER improvement of the proposed system when compared to previously implemented schemes as well as the accuracy of the NN approximation and the proposed detector. Specifically, gains of up to 8 dB at a BER of 1×10^{-5} for both base and refinement bits have been observed when comparing the proposed HM SSD Alamouti scheme with an HM SSD scheme without Alamouti transmit diversity (ATD) at a hierarchy of $\alpha = 4$, channel conditions at $m = 1$ and $N = 1$ receiver antennas. Additional results for varying hierarchy at increasing number of receiver antennas and improving channel conditions is also presented. The performance of the proposed reduced complexity decoder is also presented as well as its complexity reduction analyzed.

1. Introduction

The growing demand for multimedia communication over noisy wireless channels poses the requirement on transmission systems to provide enhanced error protection on certain classes of transmitted data in order to improve the quality of data reconstruction at the receiver. Unequal error protection (UEP), introduced in [1], is a technique which exploits the fact that not all data in a multimedia transmission is equally important. The scheme categorizes the data according to importance and assigns varied levels of error protection accordingly. Hierarchical modulation (HM) employs UEP by use of non-uniformly spaced constellations whereby higher priority data is mapped by the most significant bit (MSB) positions, ensuring greater protection, in contrast to lower priority data which is mapped to the least significant bit (LSB) positions. Consequently, all receivers regardless of channel conditions should be able to recover at least the coarse or basic data mapped by the MSB. This will thereby improve overall transmission quality and facilitate continuous data streaming.

HM is largely employed in image and video transmission over noisy wireless channels [2,3]. Chang et al propose a scheme in [4] to attain multiple levels of UEP using multiplexed hierarchical quadrature amplitude modulation (QAM). Recently, Hochwald and Stauffer in [5,6] presented unitary-transform code and direct-sum Alamouti code combined with HM and indicated performance gains with single and multiple antenna transmission schemes. Alamouti STBC coded HM in [5,6] provides transmit diversity. On the other hand, signal space diversity (SSD) is alternative technique which exploits the intrinsic diversity of multi-dimensional constellations to improve error performance without consuming additional transmit power, space or bandwidth [7]. The performance of systems combining conventional modulation with SSD and Alamouti space-time-block coded (STBC) has been discussed in various conference publications including [8]. Transmission system employing HM with SSD has also been investigated in [9], however with the omission of performance analysis and a formal approach to selecting an optimized rotation angle. In this paper we propose a new Alamouti STBC coded HM scheme with SSD. In the new modulation scheme, we will take into account single and multiple antenna reception in a Nakagami-m fading channel.

Among various error analysis methods for rotated constellations with component interleaving [10,11], the nearest neighborhood (NN) approximation analysis in [8,10] is extended to Alamouti transmit diversity (ATD) for HM QAM with SSD to attain a closed-formed theoretical BER expression in Nakagami fading channels. This method of error analysis for HM proves not only to be an accurate approximation for BER but reduces computational complexity from the previous approaches [12,13] which consider the probability of each symbol succumbing to error for each bit position. In addition, a reduced complexity maximum likelihood (ML) based detection scheme is presented. Lastly, it should be noted that this paper presents a detailed model for a 16-QAM constellation which can easily be extended to larger constellation sizes.

The paper is organized as follows. Section 2 entails the system model of the proposed transmission scheme. The NN approximation approach is presented in Section 3 while a reduced complexity ML-based decoder is proposed in Section 4. Finally, Section 5 presents results verifying theoretical performance analysis against simulations.

2. System Model

The system model consists of two pre-processing blocks where HM and SSD are applied prior to the implementation of the Alamouti transmission diversity technique. Applying HM comprises of phase partitioning an input bit stream into the relative in-phase and quadrature-phase bits before the data is Gray mapped according to the relevant distance and energy parameters. SSD is implemented by rotating four consecutive hierarchically modulated symbols by an optimum rotation angle θ, as elaborated in Section 2.2. The system model is illustrated in Fig B.1. HM and SSD will be briefly discussed in the following subsections.

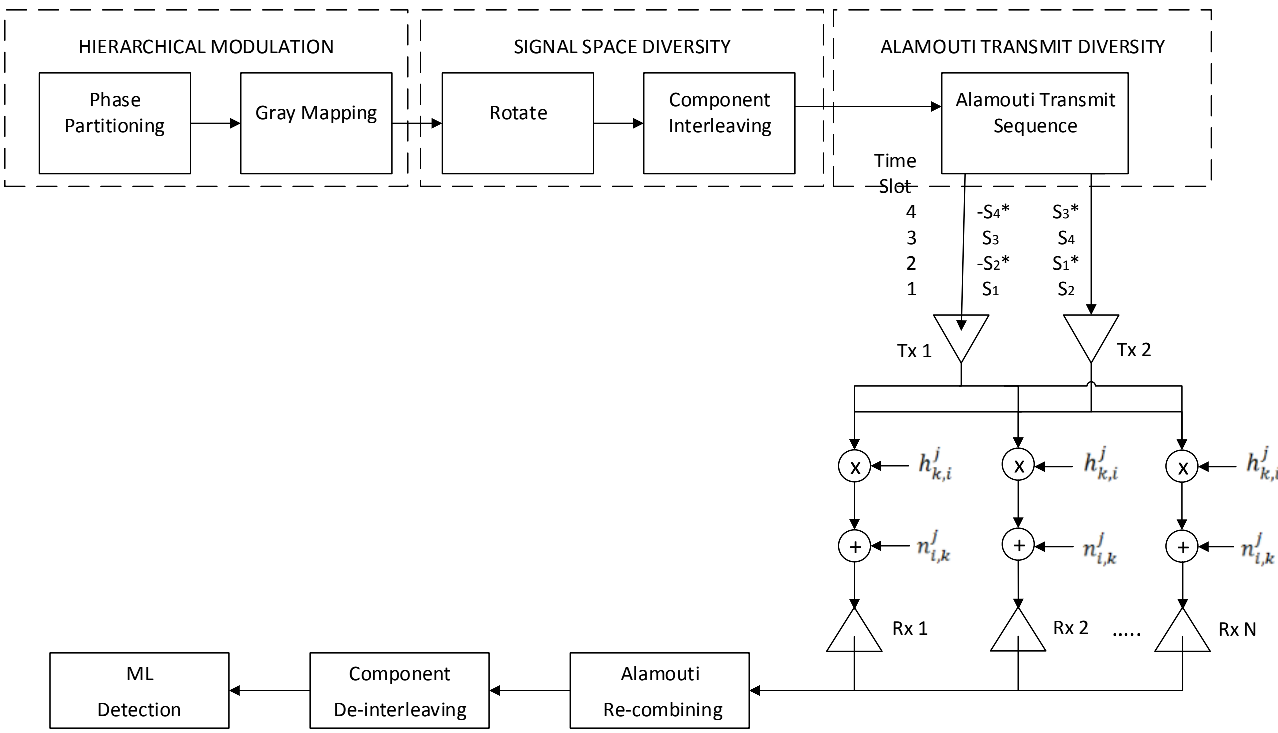

Fig. B.1 System Model

2.1. Hierarchical Modulation

For a hierarchical constellation, the symbols are labeled according to a Gray mapping procedure which enables UEP. The constellation points are labeled right to left (or vice versa) from 0 to $\sqrt{M}-1$, where M represents the constellation size, and converted to binary. The corresponding Gray code is then given by (1) forming the in-phase bits. Similarly, to obtain the quadrature-phase bits, the constellation points are labeled from top to bottom (or vice versa) from 0 to $\sqrt{M}-1$, converted to binary and then Gray coded. Finally, the two phases are combined in $i_1q_1i_2q_2 \ldots i_{k/2}q_{k/2}$ fashion, where $k = \log_2 M$.

Similarly, an input bit stream of the form $b_1^I b_1^Q b_2^I b_2^Q \ldots b_{k/2}^I b_{k/2}^Q$, is phase partitioned such that the in-phase and quadrature-phase sub-channels are separated to form two distinct bit streams. Gray coding is then applied to each bit stream independently using (1):

$$\begin{cases} g_1 = b_1 \\ g_i = b_i \oplus b_{i-1} \end{cases} \tag{1}$$

where $\oplus$ represents modulo-2 addition and $2 \le i \le k/2$.

These Gray coded bits are then hierarchically modulated using two distance parameters. The distance between two adjacent symbols in the same quadrant is represented as $2b$ and the distance between two adjacent symbols in neighbouring quadrants is represented as $2a$. Hierarchy is determined using a constellation priority parameter defined according to the DVB-T standard [14] as $\alpha = \frac{a}{b}$. Thus, a uniform constellation is achieved when $\alpha = 1$ while hierarchy is achieved when $\alpha > 1$.

The average symbol energy E_s as given by [15], shown in (2), is used along with the distance parameters to map the Gray coded input bit stream to its corresponding symbol in the hierarchical constellation.

$$E_s = 2\left(a + \left(\frac{\sqrt{M}}{2} - 1\right)b\right)^2 + \frac{2}{3}\left(\frac{M}{4} - 1\right)b^2 \tag{2}$$

2.2. Signal Space Diversity

To achieve SSD in an Alamouti scheme, four consecutive hierarchically modulated symbols comprising of in-phase and quadrature-phase components $x_i = \begin{bmatrix} x_i^I & x_i^Q \end{bmatrix}$ where $i \in [1:4]$, are rotated using the rotation matrix in (3) to give the corresponding rotated symbols $\tilde{x}_i = x_i R^\theta$.

$$R^\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

The rotated symbols are component interleaved, as shown in (4), to ensure that the in-phase and quadrature-phase components of a single symbol experience independent fading.

$$s_i = \tilde{x}_i^I + j\tilde{x}_{i+2}^Q \qquad i \in [1:2] \tag{4.1}$$

$$s_i = \tilde{x}_i^I + j\tilde{x}_{i-2}^Q \qquad i \in [3:4] \tag{4.2}$$

Symbols s_1 and s_3 are transmitted through antenna 1 (Tx 1 in Fig. B.1) over an orthogonal channel, while symbols s_2 and s_4 are transmitted over an alternate orthogonal channel through antenna 2 (Tx 2 in Fig. B.1).

2.3. Alamouti Transmit Diversity

In this paper we consider a $2 \times N$ ATD scheme for HM with SSD, where N is the number of receiver antennas. The two modulated symbol pairs (s_1, s_2) and (s_3, s_4) are simultaneously transmitted in time slot 1 and slot 3 respectively, while $(-s_2^*, s_1^*)$ and $(-s_4^*, s_3^*)$ are simultaneously transmitted in time slot 2 and slot 4, where $*$ denotes the complex conjugate. It is assumed that the above symbol pairs are transmitted over an $N \times 2$ fading channel $\boldsymbol{H}^j$ $j \in [1:2]$ with N dimensional additive white Gaussian noise (AWGN) $\boldsymbol{n}_i^j$ $i \in [1:2]$. It is also assumed that $\boldsymbol{H}^j$ remains constant during every pair of time slots, time slots j and $(j+1)$, and takes independent values from one pair of time slots to another. The received signal vectors are given as

$$\boldsymbol{r}_i^j = \sqrt{\rho/2}\boldsymbol{H}^j\boldsymbol{S}_i^j + \boldsymbol{n}_i^j \tag{5}$$

where $\boldsymbol{r}_i^j = \left[r_{1,i}^j \, r_{2,i}^j \cdots r_{N,i}^j\right]^{\boldsymbol{T}}$ is the $N \times 1$ dimensional received signal vector and ρ is the the average SNR at each receive antenna $i \in [1:2]$ normalized by a factor of 2. Additionally, $\boldsymbol{S}_1^j = [s_{2j-1} \, s_{2j}]^T$and $\boldsymbol{S}_2^j = [-s_{2j}^* \, s_{2j-1}^*]^T$, $j \in [1:2]$, $\boldsymbol{n}_i^j$ have independent and identically distributed (i.i.d) entries according to the complex Gaussian distribution $\mathcal{CN}(0,1)$ and $\boldsymbol{n}_i^j = [n_{i,1}^j, n_{i,2}^j, \cdots, n_{i,N}^j]^T$. The channel $\boldsymbol{H}^j$ is modeled as the Nakagami-m distributed random variables with $\boldsymbol{H}^j = [\boldsymbol{h}_1^j \, \boldsymbol{h}_2^j]$ where $\boldsymbol{h}_i^j = \left[h_{1,i}^j \, h_{2,i}^j \cdots h_{N,i}^j\right]^{\boldsymbol{T}}$ $i \in [1:2]$**.** For each element in the fading channel matrix $\boldsymbol{h}_i^j$, the complex element $h_{k,i}^j = \psi_{k,i}^j \exp\left(\jmath\varphi_{k,i}^j\right)$ denotes the channel gain from the i^{th} $i \in [1{:}\,2]$ transmit antenna, to the k^{th} $k \in [1{:}\,N]$ receive antenna in the j^{th} $j \in [1{:}\,2]$ time slot where the amplitude $\psi_{k,i}^j$ is modeled as independent Nakagami random variable according to the distribution in (6) given by [16]. (Note: $\jmath$ referes to the imaginary component of the complex element not to be confused with j referring to the time slot).

$$f\left(\psi_{k,i}^j\right) = \frac{2m^m}{\Omega^m \Gamma(m)} \left(\psi_{k,i}^j\right)^{2m-1} \exp\left(-\frac{m\left(\psi_{k,i}^j\right)^2}{\Omega}\right) \tag{6}$$

where $\mathrm{E}\left[\left(\psi_{k,i}^j\right)^2\right] = \Omega$ is the average fading power, $\Gamma(\cdot)$ is the Gamma function and m is the Nakagami fading parameter. The phase $\varphi_{k,i}^j$ is uniformly distributed over $[0, 2\pi]$.

The received signals from all N receiver antennas are then combined according to the Alamouti scheme given in [17] using (7) where $j \in [1{:}\,2]$.

$$y_1^j = \sum_{k=1}^{N} \left(h_{k,1}^j\right)^* r_{k,1}^j + h_{k,2}^j \left(r_{k,2}^j\right)^* \tag{7.1}$$

$$y_2^j = \sum_{k=1}^{N} \left(h_{k,2}^j\right)^* r_{k,1}^j - h_{k,1}^j \left(r_{k,2}^j\right)^* \tag{7.2}$$

The recombined signal is passed through the ML detector using the decision rule given in [17]. Note the combined signals are de-interleaved within the detection.

$$\hat{y}_i = \arg\min_{\mathrm{x_k} \in \hat{S}} \left\{\mu_2 \left|y_i^{1^R} - \mu_1 x_k^R\right|^2 + \mu_1 \left|y_i^{2^I} - \mu_2 x_k^I\right|^2\right\}, \quad i \in [1{:}\,2] \tag{8.1}$$

$$\hat{y}_{i+2} = \arg\min_{\mathrm{x_k} \in \hat{S}} \left\{\mu_1 \left|y_i^{2^R} - \mu_2 x_k^R\right|^2 + \mu_2 \left|y_i^{1^I} - \mu_1 x_k^I\right|^2\right\}, \quad i \in [1{:}\,2] \tag{8.2}$$

where $\tilde{S}$ represents rotated constellation set, $\hat{y}_i$ and $\hat{y}_{i+2}$ represent the detected symbol for the respective time slot and $\mu_i = \mathrm{A}_1^i + \mathrm{A}_2^i$ with A_j^i is defined below.

$$A_j^i = \sum_{k=1}^{N} \left(\psi_{k,j}^i\right)^2 \tag{9}$$

3. Performance Analysis

Previously, performance analysis for HM [12,13] uses an approach where BER is determined by considering the probability that a transmitted symbol exceeds past a set decision boundary and leads to an error. Though extremely accurate, the approach requires each symbol in the constellation to be considered to determine the error for each bit position. Consequently, it is unnecessarily complicated and time consuming. In [18], the NN BER union bound for hierarchical 16-QAM with SSD and maximal ratio combining (MRC) reception has been derived and its accuracy validated. This section uses the NN union bound approach presented in [10,8] in order to extend the results in [18] to determine an accurate approximation of the BER for Alamouti transmitted hierarchical 16-QAM with SSD and MRC reception.

To determine the error rate for base bits (bits i_1 and q_1 in the case of 4/16-QAM) the pairwise error probability (PEP) that a transmitted symbol is incorrectly detected as a perpendicular or diagonal neighbour in a different quadrant is considered. Without loss of generality, suppose symbol x_B in Fig. B.2 in [18] is transmitted. First, probability of error due to perpendicular neighbours is considered. The probability density function for the combined SNR in the presence of MRC in a Nakagami fading channel is given in (10) for N receive antennas [19].

$$f_\gamma(\gamma_i) = \left(\frac{m}{\bar{\gamma}}\right)^{mN} \frac{\gamma_i^{mN-1}\exp\left(-\frac{m\gamma_i}{\bar{\gamma}}\right)}{\Gamma(mN)} \tag{10}$$

The trapezoidal approximation to the $Q(\sigma)$ function, as shown in [20], is used to simplify the above analysis. The approximation is implemented over a total of p iterations as shown in (11)

$$Q(\sigma) = \frac{1}{2p}\left(\frac{1}{2}\exp\left(\frac{-\sigma^2}{2}\right) + \sum_{k=1}^{p-1}\exp\left(\frac{-\sigma^2}{S_k}\right)\right) \tag{11}$$

where $S_k = 2\sin^2(k\pi/2p)$.

The final PEP due to perpendicular neighbours can be written as (12) with $\beta_1 = \frac{\alpha^2}{\alpha^2+2\alpha+2}$.

$$P(x_B \to x_C) = \frac{1}{4p}\left(\frac{2m}{2m+\bar{\gamma}\beta_1\cos^2\theta}\right)^{mN}\left(\frac{2m}{2m+\bar{\gamma}\beta_1\sin^2\theta}\right)^{mN}$$

$$+\frac{1}{2p}\sum_{k=1}^{p-1}\left(\frac{mS_k}{mS_k+\bar{\gamma}\beta_1\cos^2\theta}\right)^{mN}\left(\frac{mS_k}{mS_k+\bar{\gamma}\beta_1\sin^2\theta}\right)^{mN} \tag{12}$$

The error probability between diagonal neighbours is small enough to be negligible for this approximation. However, when considering the performance of the base bits, the error probability between diagonal neighbours (such as x_B and x_D or x_E and x_C in Fig. B.2 in [18]) becomes comparable as hierarchy is increased (specifically, as distance a becomes increasingly larger than distance b). The diagonal error probability $P_{Base}^{Diagonal}$ for base bits is then given as,

$$\begin{aligned} P_{Base}^{Diagonal} &= \frac{1}{2}\left[\frac{4}{16}P_{Diagonal}^{I}+\frac{4}{16}P_{Diagonal}^{Q}\right] \\ &= \frac{1}{8}P(x_B \rightarrow x_D)+\frac{1}{8}P(x_E \rightarrow x_C) \end{aligned} \tag{13}$$

$P(x_B \rightarrow x_D)$ and $P(x_E \rightarrow x_C)$ in (13) can be derived using the method outline in [18].

The final error probability for base bits is then determined as

$$P_{Base} = \frac{1}{2}P(x_B \rightarrow x_C)+\frac{1}{8}P(x_B \rightarrow x_D)+\frac{1}{8}P(x_E \rightarrow x_C) \tag{14}$$

Similarly, the error probability for the refinement bits is determined as (15) where $\beta_2 = \frac{1}{\alpha^2+2\alpha+2}$.

$$\begin{aligned} P_{Refinement} &= P(x_A \rightarrow x_B) = \\ &\frac{1}{4p}\left(\frac{2m}{2m+\bar{\gamma}\beta_2\cos^2\theta}\right)^{mN}\left(\frac{2m}{2m+\bar{\gamma}\beta_2\sin^2\theta}\right)^{mN} \\ &+\frac{1}{2p}\sum_{k=1}^{p-1}\left(\frac{mS_k}{mS_k+\bar{\gamma}\beta_2\cos^2\theta}\right)^{mN}\left(\frac{mS_k}{mS_k+\bar{\gamma}\beta_2\sin^2\theta}\right)^{mN} \end{aligned} \tag{15}$$

According to [17], a $2 \times N$ Alamouti transmission system can be regarded as a $1 \times 2N$ receive diversity system with 3 dB loss. Thus, a $2 \times N$ Alamouti transmitted hierarchical QAM with SSD can be regarded as a $1 \times 2N$ hierarchical QAM with SSD. The results in (12) and (15) can be adjusted by replacing N with $2N$ and introducing a 3 dB loss in SNR to determine the NN approximation for a $2 \times N$ Alamouti transmission system with HM and SSD.

4. Reduced Complexity Detection

Traditionally, optimal detection is offered by ML-based detection schemes which perform an exhaustive search to find the minimum Euclidean distance between all possible modulated symbols and the received symbol. As a result, the ML-based optimal detector also has the highest computational complexity. In order to address this, a reduced complexity detector is proposed where only the nearest neighbors of a received symbol are considered in determining the transmitted symbol. In previous sub-optimal detection schemes, boundary check algorithms are used to determine the nearest neighbors of the equalized received symbol [11]. However, in hierarchical constellations, as hierarchy increases adjacent symbols within the same quadrant grow closer together. As a result, the boundaries separating the symbols grow smaller and the method of detection becomes inaccurate.

In order to overcome this inaccuracy, the proposed scheme comprises of two detection stages. By reducing the number of comparisons made during detection by only considering the most likely symbols, the overall complexity of detection can be reduced. This section presents the proposed sub-optimal detector which is followed by a brief complexity analysis.

4.1. Nearest Neighbor Reduced Complexity Detector

Once all four symbols in a transmission sequence have been received and recombined, $y_1^j, y_2^j\ j \in [1:2]$ according to (7) shown in Section 2, they are then de-interleaved in (16) where $i \in [1:2]$.

$$\tilde{y}_i = (y_i^1)^R + (y_i^2)^I \tag{16.1}$$

$$\tilde{y}_{i+2} = (y_{i+2}^2)^R + (y_{i+2}^1)^I \tag{16.2}$$

The signal can now be processed by the reduced complexity detector as outlined in the steps below. Note that by eliminating a pre-processing stage where the received symbol maybe equalized and un-rotated, as suggested in previous schemes [11], the incorporation of additional complexity is avoided.

Detection Stage 1

1. The nearest neighbor signal set $\tilde{P}$ is determined using a simplified boundary check algorithm where the real and imaginary components of the received recombined and de-interleaved signal $\tilde{y}_i, \tilde{y}_{i+2}$ $i \in [1:2]$ are determined to be positive or negative. In other words, the nearest neighbor signal set consists of all the modulated symbols that lie within the same quadrant as the signal $\tilde{y}_i, \tilde{y}_{i+2}$ $i \in [1:2]$. For this study, 16-QAM is considered resulting in a signal set consisting of four symbols.
2. The determined signal set $\tilde{P}$, which is essentially a subset of the signal set $\tilde{S}$ defined in Section 2, is then rotated using equation (3) and the aforementioned method in Section 2.2.
3. ML detection is then performed using (17) to determine $\bar{y}_i$, $\bar{y}_{i+2}$ $i \in [1:2]$ the detected symbol from stage 1. It should be noted that the ML rule remains identical to (8) except that the nearest neighborhood signal set $\tilde{P}$ is used such that $x_k \in \tilde{P}$.In addition, note that the received and combined symbol y_j^i $i \in [1:2], j \in [1:2]$, as determined in Section 2, is used in the ML detector.

$$\bar{y}_i = \arg\min_{\mathrm{x_k} \in \tilde{P}} \left\{ \mu_2 \left| {y_i^1}^R - \mu_1 x_k^R \right|^2 + \mu_1 \left| {y_i^2}^I - \mu_2 x_k^I \right|^2 \right\}, \quad i \in [1:2] \tag{17.1}$$

$$\bar{y}_{i+2} = \arg\min_{\mathrm{x_k} \in \tilde{P}} \left\{ \mu_1 \left| {y_i^2}^R - \mu_2 x_k^R \right|^2 + \mu_2 \left| {y_i^1}^I - \mu_1 x_k^I \right|^2 \right\}, \quad i \in [1:2] \tag{17.2}$$

Detection Stage 2

Detection stage 1 results in an accurate performance for the detection of refinement bits, but leads to large performance deviations for base bits between the optimal and proposed detectors. To enhance the performance for the detection of base bits, a second stage of detection is implemented using the following steps.

1. The new signal set used for detection in this stage consists of the detected symbol from detection stage 1, $\bar{y}_i$, $\bar{y}_{i+2}$ $i \in [1:2]$, and any perpendicular or diagonal neighbors in an adjacent quadrant as illustrated in Fig. B.2.
2. The determined signal set $\tilde{Q}$, is then rotated using equation (3) and the aforementioned method in Section 2.2.

3. Once more, the new ML detection rule remains the same as (8), however, $x_k \in \tilde{Q}$ where $\tilde{Q}$ represents new rotated nearest neighbor signal set. Yet again, it should be noted that the received and combined symbol y_j^i $i \in [1:2], j \in [1:2]$ is used in the ML detector.

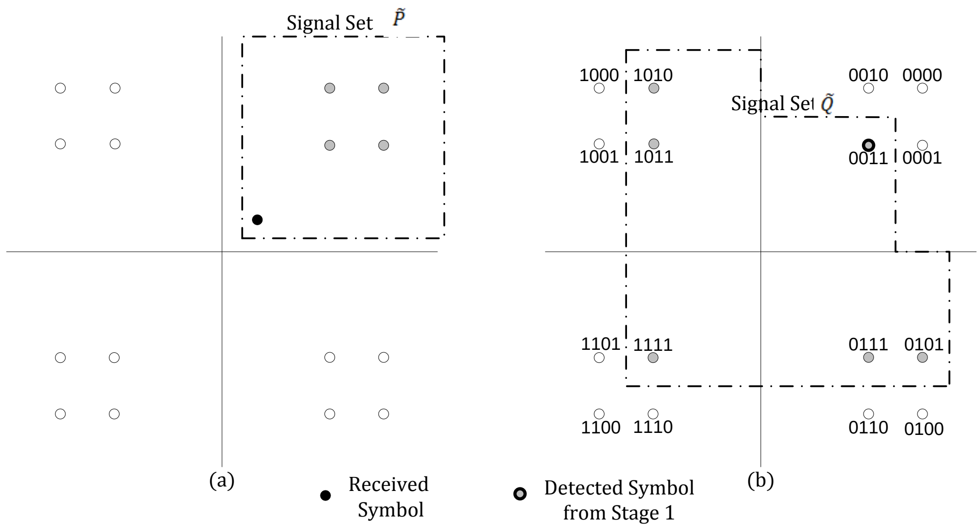

Fig. B.2 ML-based sub-optimal detector (a) Detection Stage 1 (b) Detection Stage 2

4.2. Complexity Analysis

To verify that the objective of reducing the detection complexity, the complexity of the optimal and the reduced complexity detector is considered in this section.

Similar to [21], computational complexity is analyzed by considering the number of real multiplication and additions required for the detection of a received symbol by considering one complex multiplication equivalent to 4 real multiplication and 2 real addition operations. Similarly, one complex addition is considered as 2 real additions. By analyzing equations (7), (8) (9), (16) and (17), Table B.1 lists the complexity, δ, of the various steps of detection in terms of real multiplication and addition operations where N represents the number of receiver antennas and Ω is the number of comparisons made per received symbol.

Table B.1 Complexity Analysis

	δ_{Mult}	δ_{Real}
δ_{μ_1}	$2N$	1
δ_{μ_2}	$2N$	1

$\delta_{y_i^1}$	$i \in [1:2]$	$2 \times 8N$	$2 \times 4N$
$\delta_{y_i^2}$	$i \in [1:2]$	$2 \times 8N$	$2 \times 4N$
$\delta_{\hat{y}_i}$	$i \in [1:2]$	$2 \times 6\Omega$	$2 \times 3\Omega$
$\delta_{\hat{y}_{i+2}}$	$i \in [1:2]$	$2 \times 6\Omega$	$2 \times 3\Omega$

Thus, the average number of real multiplication and addition operations per received symbol is determined in (18).

$$\Lambda_{Mult} = \frac{36N + 24\Omega}{4} \tag{18.1}$$

$$\Lambda_{Add} = \frac{16N + 12\Omega + 2}{4} \tag{18.2}$$

For the optimal detector, the average number of comparisons made per received symbol is $\Omega = 16$ while for the proposed detection scheme an average of 4 comparison are made during detection stage 1 and an average of 3 during detection stage 2 resulting in a total of $\Omega = 7$ comparisons. Table B.2 compares the average number of real multiplication and addition operations per received symbol required by the optimal ML detector and the proposed reduced complexity ML detector. The efficiency of the proposed detector is then determined using (19).

$$Efficiency = \frac{(Optimal\ ML - Proposed\ ML)}{Optimal\ ML} * 100\% \tag{19}$$

Table B.2 Comparison of average complexity for optimal and proposed detection schemes

	Optimal ML Detector ($\Omega = 16$)	Proposed Detector ($\Omega = 7$)	Efficiency
Λ_{Mult}			
$N = 1$	105	51	51.43%
$N = 4$	52.5	25.5	51.43%
Λ_{Add}			
$N = 1$	132	78	40.91%
$N = 4$	64.5	37.5	41.86%

The significant reduction in number of comparisons required leads to the minimization of computational complexity. As a result, higher data rates can be achieved as the transmission of larger constellations is made feasible. The method of complexity analysis can easily be extended to higher constellations.

5. Simulation Results

The enhanced BER performance of the proposed ATD system for hierarchically modulated 4/16-QAM with SSD simulated and is presented in this section. The accuracy of the NN approximation for BER performance of the proposed scheme is validated against Monte Carlo simulations using the optimal ML detector in a single receiver antenna as well as multiple receiver antennas (for $N \geq 3$) under varying channel conditions (for $m \geq 1$).

The simulations are conducted under i.i.d Nakagami-m fading channel conditions with AWGN as defined in Section 2. In addition, channel state information (CSI) is fully available at the receiver and the modulated signal is Gray coded using (1) as described in Section 2. It is assumed that the transmit and receive antennas are spaced widely enough to eliminate cross-correlation. MRC is employed at the receivers where multiple receiver antennas are simulated. In addition, the total transmit power for the respective hierarchical constellation is constant over the various transmission schemes.

In order to achieve optimum performance, the angle of rotation is determined using (20) as derived in [18] for $\alpha = 1$ and $\alpha = 4$ as approximately $\theta = 14.036°$ and $\theta = 29.745°$, respectively, and used for simulation.

$$\tan\theta = \frac{\alpha}{\alpha + 3} \tag{20}$$

The effect of increasing hierarchy is also observed such that the performance of base bits improves at the cost of the performance of the refinement bits. In addition, the results indicate an enhanced BER performance of the proposed system when compared to previously implemented systems. Lastly, the performance of the proposed reduced complexity ML detector is analyzed against the NN approximation.

5.1. Single Antenna Reception with $m = 1$

In this section, the NN approximation for BER performance is verified against simulations from the implementation of proposed system with single antenna reception in a Nakagami $m = 1$ fading channel. The effect of hierarchy on the proposed system is observed by varying the value of α from a uniform constellation at $\alpha = 1$ to a hierarchical constellation at $\alpha = 4$ in Fig. B.3 and Fig. B.4, respectively. The results validate the accuracy of the NN approximation approach as well as confirm the improved BER performance of the proposed system.

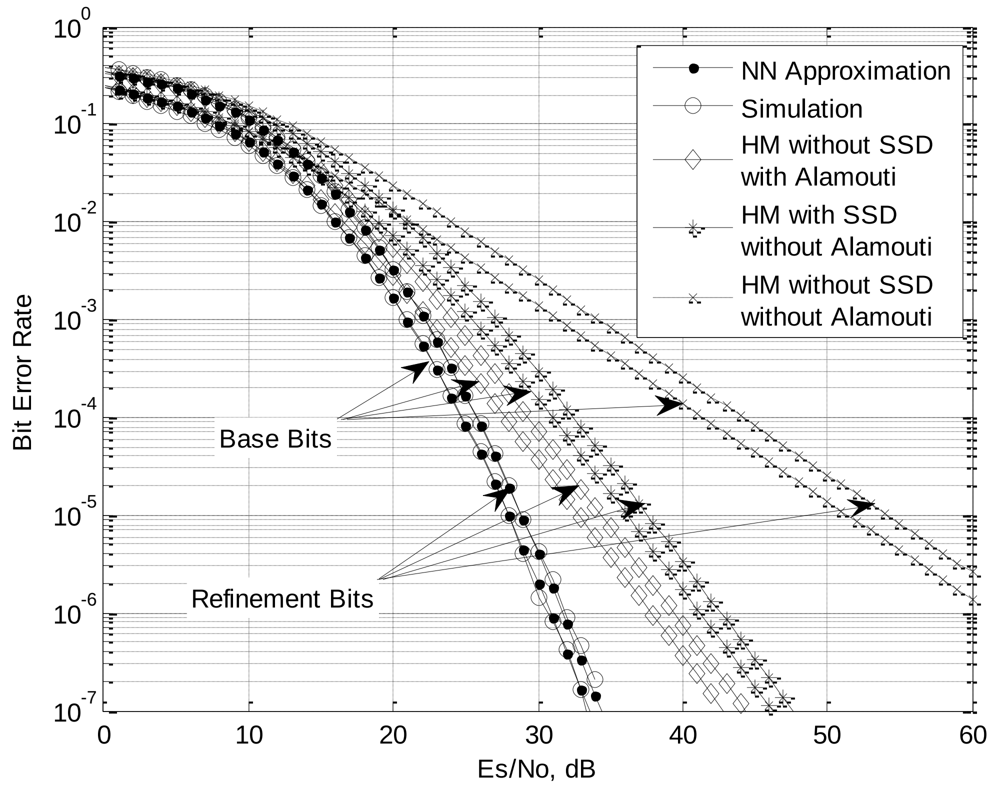

Fig. B.3 ATD scheme for 4/16-QAM HM with SSD at $\alpha = 1$ and $m = 1$

The performance of the ATD is observed for uniformly spaced 16-QAM at $\alpha = 1$ with SSD in Fig. B.3. Expectedly, the difference in BER performance between the base and refinement bits is small. However, a significant performance gain can be observed between the proposed system and previously implemented systems in Fig. B.3. Comparing the performance of the Alamouti transmitted hierarchical signal with SSD against an HM SSD system without Alamouti, a gain of about 8 dB can be observed at a BER of 1×10^{-5} for both base and refinement bits. Similarly, when the proposed system is compared to an Alamouti HM system without SSD, a gain of approximately 5 dB can be observed at a BER of 1×10^{-5} for both base and refinement bits. In addition, when the proposed scheme is compared to a simple hierarchical system without SSD or

ATD, performance improvement of up to 22 dB can be observed at a BER of 1×10^{-5} for both base and refinement bits.

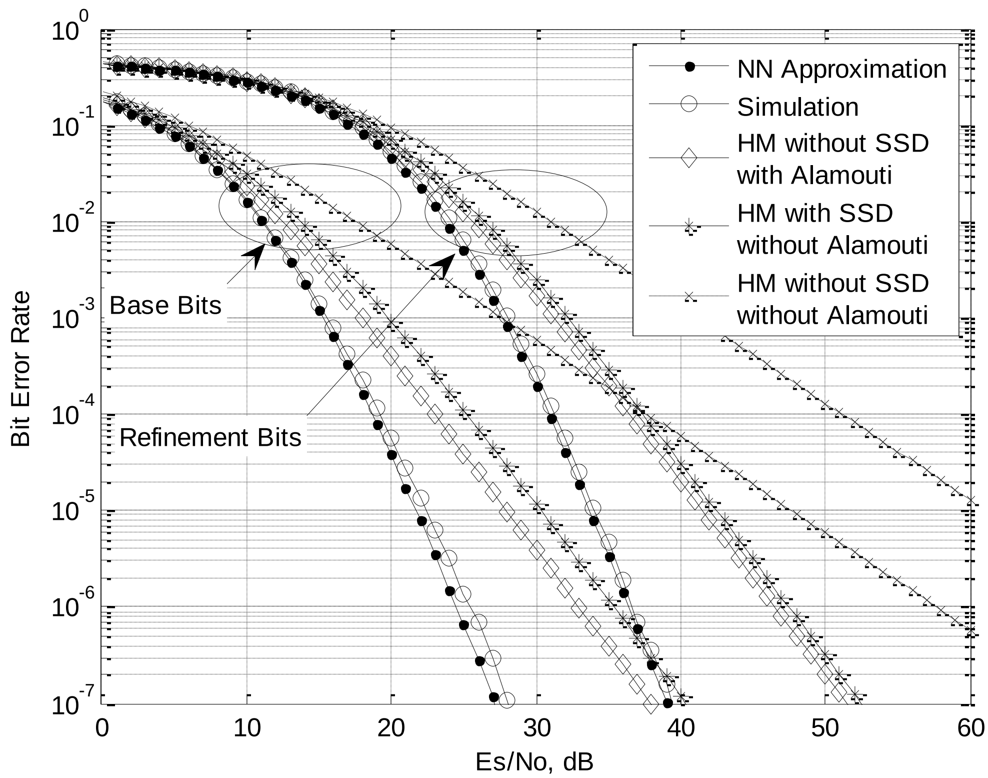

Fig. B.4 ATD scheme for 4/16-QAM HM with SSD at $\alpha = 4$ and $m = 1$

As hierarchy is introduced by increasing the constellation priority parameter to $\alpha = 4$ in Fig. B.4, base bits are observed to perform better than refinement bits. This is anticipated, as increasing hierarchy results in greater protection for base bits at the cost of lower protection for the refinement bits. Further, the performance of the proposed scheme, when compared to a similar scheme without ATD, is found to improve by approximately 8 dB and 7 dB at a BER of 1×10^{-5} for base and refinement bits, respectively. Similarly, when the proposed scheme is compared to an Alamouti HM system without SSD, a gain of nearly 5 dB and 7 dB is observed at a BER of 1×10^{-5} for base and refinement bits, respectively. In addition, a performance gain of up to 26 dB and 25 dB at a BER of 1×10^{-5} for base and refinement bits, respectively, is found between the proposed scheme and a simple hierarchical system without SSD or ATD.

The observed performance gains between the proposed hierarchical 4/16-QAM system and previously implemented equivalent systems for $\alpha = 1$ and $\alpha = 4$, in Fig. B.3 and Fig. B.4 respectively, demonstrates the proposed scheme's reduced susceptibility to channel fading due to the introduction of an additional transmit antenna by implementation of ATD as well as the incorporation of additional diversity in the form of SSD. Rotating the hierarchical constellation results in each symbol obtaining two distinct components while component interleaving ensures independent fading is experienced by the unique components. As a result, the original symbol can still be correctly recovered by the detector if one component, in-phase or quadrature-phase, is corrupted by channel fading.

5.2. Multiple Antenna Reception with Nakagami fading

The effect of multiple receiver antennas and varying channel conditions on the performance of the proposed system is considered in this section. Specifically, the system performance for $N = 3$ and $N = 4$ receiver antennas with Nakagami $m = 1$ is presented. Furthermore, the system performance as the channel conditions vary from $m = 1$ to $m = 3$ with $N = 3$ receiver antennas is also presented. Using the previously determined rotation angles, Fig. B.5-B.8 confirm the accuracy of the NN approximation as well demonstrate the expected performance improvement of the system as additional receiver antennas are included or as channel conditions improve.

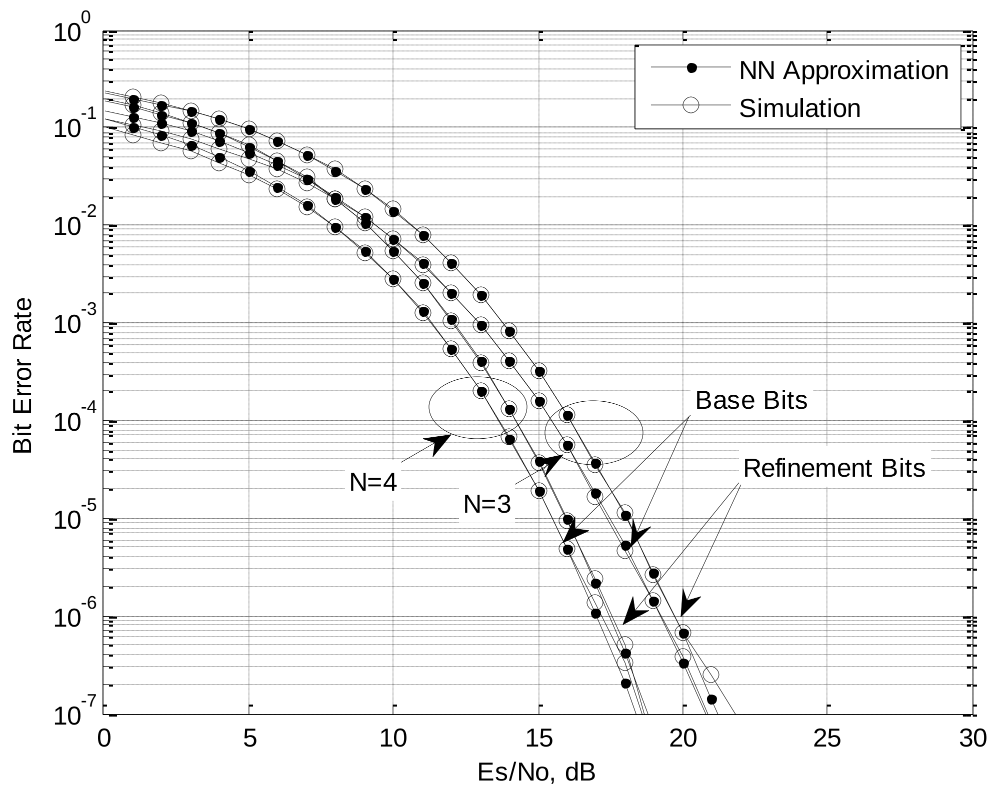

Fig. B.5 ATD scheme for 4/16-QAM with SSD at $\alpha = 1$ and varying values for N with $m = 1$

The effect of additional receiver antennas on system performance is presented in Fig. B.5 and Fig. B.6 for uniformly spaced constellations ($\alpha = 1$) and non-uniformly spaced constellations ($\alpha = 4$), respectively. In Fig. B.5, at $\alpha = 1$, a performance gain of approximately 3 dB is observed at a BER of $1 \times 10^{-6}$for both base bits and refinement bit as the number of receiver antennas increases from $N = 3$ to $N = 4$. Similarly, as hierarchy is increased to $\alpha = 4$ in Fig. B.6, a performance gain of about 2 dB is found at a BER of $1 \times 10^{-6}$for both base bits and refinement bit as the receiver antennas increase from $N = 3$ to $N = 4$.

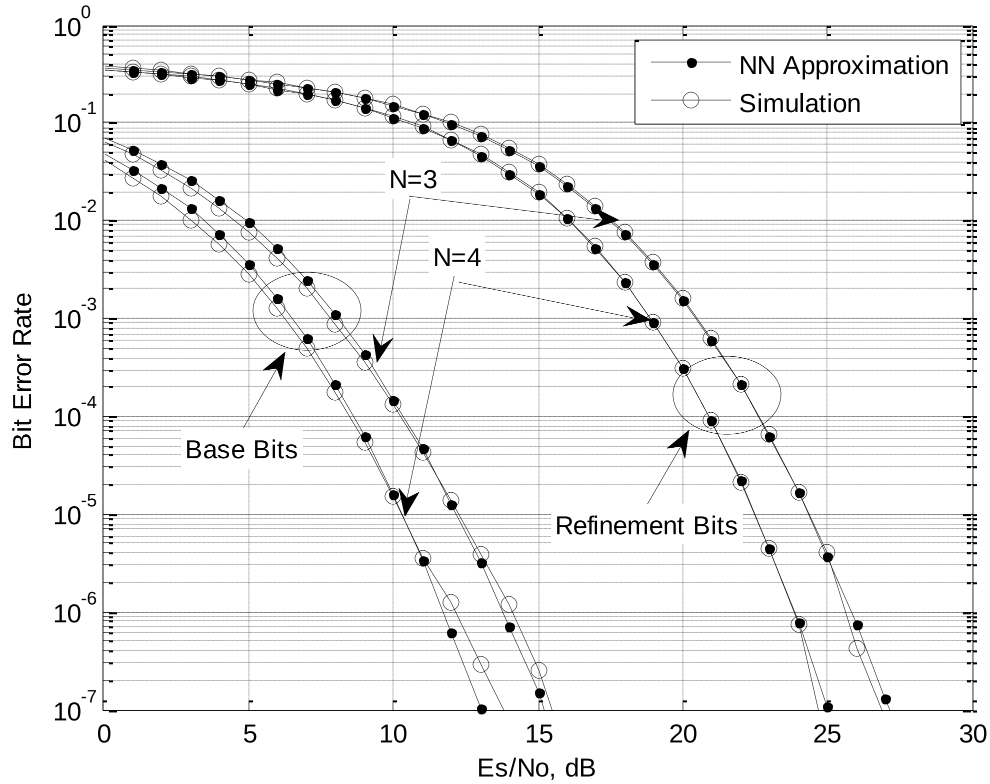

Fig. B.6 ATD scheme for 4/16-QAM with SSD at $\alpha = 4$ and varying values for N with $m = 1$

Once again, the performance improvement of the base bits at the cost of the performance deterioration of the refinement bits is observed as hierarchy is increased regardless of the number of receiver antennas. This behaviour is expected as hierarchy is responsible for enabling greater protection of high priority data mapped to base bits.

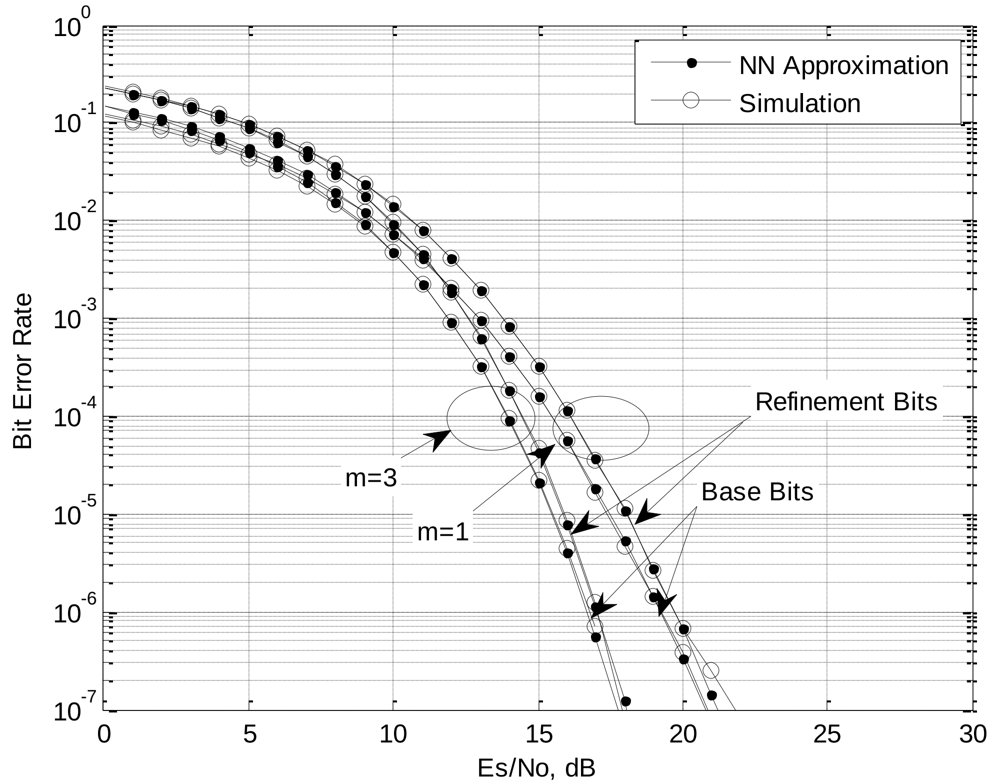

Fig. B.7 ATD scheme for 4/16-QAM with SSD at $\alpha = 1$ and varying values for m with $N = 3$

The effect of improving channel conditions is shown in Fig. B.7 and Fig. B.8 for $\alpha = 1$ and $\alpha = 4$, respectively. For uniformly spaced constellation in Fig. B.7, a performance improvement of about 3 dB is observed at a BER of 1×10^{-6} for both base and refinement bits as m increases from $m = 1$ to $m = 3$. Similarly, as hierarchy is introduced in Fig. B.8 with $\alpha = 4$, gains of roughly 3 dB and 2 dB are seen at a BER of 1×10^{-6} for base and refinement bits, respectively.

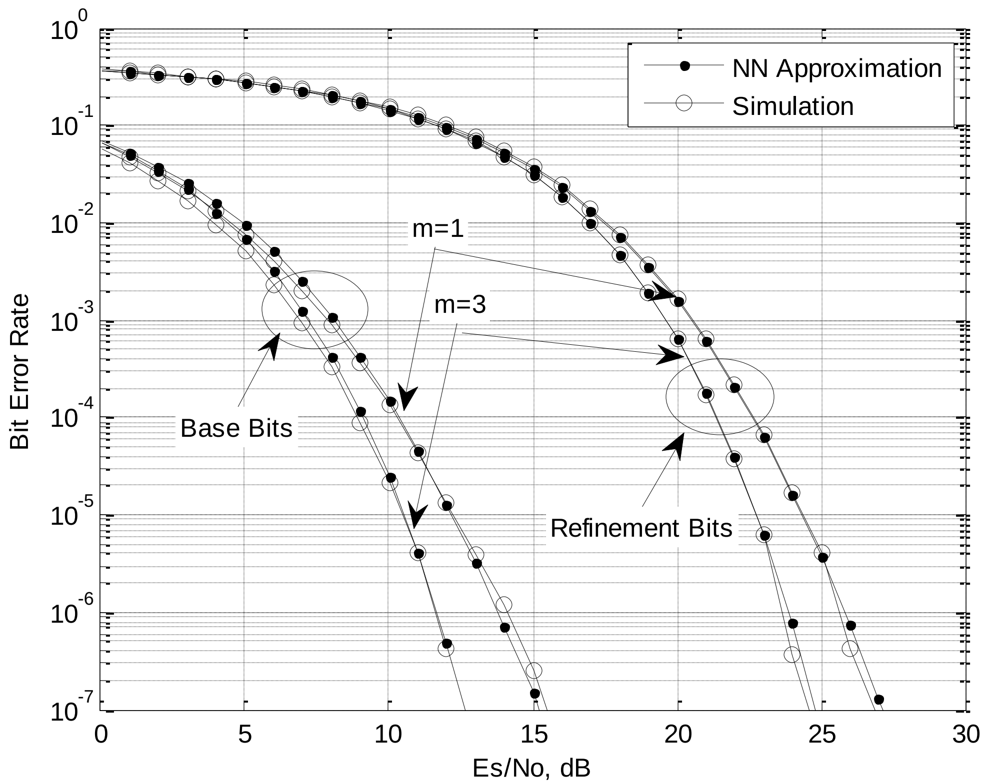

Fig. B.8 ATD scheme for 4/16-QAM with SSD at $\alpha = 4$ and varying values for m with $N = 3$

It can be noted that the gains observed as the number of receiver antennas increases from $N = 3$ to $N = 4$ while $m = 1$ and the improvements found for improving channel conditions from $m = 1$ to $m = 3$ while $N = 3$, for the respective hierarchical constellation, are parallel. Thus, increasing the number of receiver antennas results in similar performance gains as improving channel conditions for the proposed system. It should also be noted that the performance gain is decreasing, approximately by 1 dB, as N or m increases for higher values of hierarchy. This is due to the fact that the performance of an SSD system does not differ from a non-SSD system in the absence of deep fading.

5.3. Reduced Complexity Detection

The performance accuracy of the reduced complexity ML detection scheme is considered in this section. Consider the performance of the NN approximation for BER and the proposed detection schemes in Fig. B.9 where an i.i.d Nakagami $m = 1$ channel is simulated with a $N = 1$ single receiver antenna. For both uniformly spaced constellations with $\alpha = 1$ as well as with increasing hierarchy at $\alpha = 4$, performance of the proposed reduced complexity ML detector closely matches the NN approximation for the BER. The simulations verify the proposed detection scheme as an accurate method of decoding HM symbols with SSD and ATD resulting in a substantial decrease in computational complexity.

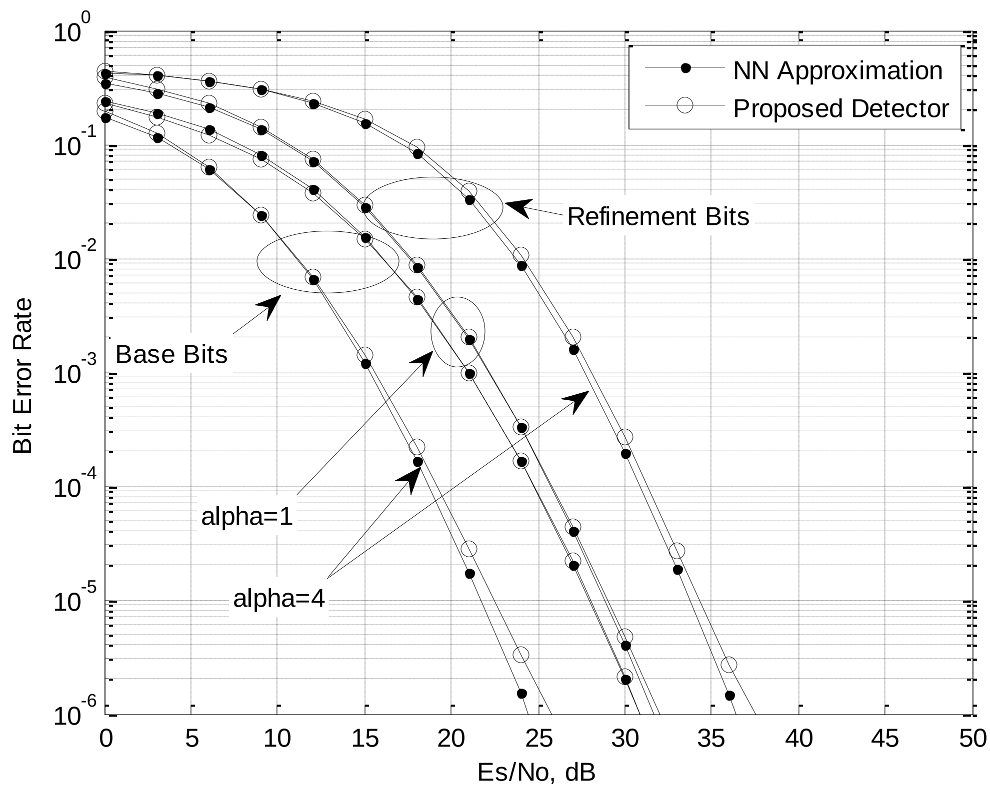

Fig. B.9 BER performance of proposed reduced complexity ML detector for $N = 1$, $m = 1$ with $\alpha = 1$ and $\alpha = 4$

6. Conclusion

In this paper, an ATD scheme for HM QAM with SSD in Nakagami fading channels is presented. A new approach to approximating BER performance for Alamouti transmitted HM with SSD and MRC using the NN approximation is briefly presented. Further, a reduced complexity ML-based detector is proposed. Simulation results validate the accuracy of the NN approximation as well as the enhanced performance of the proposed system, with gains of up to 26 dB at a BER of 1×10^{-5} for both base and refinement bits at $\alpha = 4$, $m = 1$ and $N = 1$ observed between the proposed scheme and a simple HM system lacking SSD and ATD. The performance of the system with varying hierarchy, number of receiver antennas and improving channel conditions is also presented. The proposed reduce complexity ML-based detector shows complexity reductions of up to 51.43% and is shown to perform as accurately as the optimal detector.

References

[1] Thomas M. Cover, "Broadcast Channels," *IEEE Tranactions on Information Theory*, vol. 18, no. 1, pp. 2-14, January 1972.

[2] Yoong Choon Chang, Sze Wei Lee, and R Komiya, "A low complexity hierarchical QAM symbol bits allocation algorithm for unequal error protection of wireless video transmission," *IEEE Transactions on Consumer Electronics*, vol. 55, no. 3, pp. 1089-1097, August 2009.

[3] S.S. Arslan, P.C. Cosman, and L.B. Milstein, "Coded Hierarchical Modulation for Wireless Progressive Image Transmission," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 9, pp. 4299-4313, November 2011.

[4] Seok-Ho Chang, Minjoong Rim, Pamela C. Cosman, and Laurence B. Milstein, "Optimized Unequal Error Protection Using Multiplexed Hierarchical Modulation," *IEEE Transaction on Information Theory*, vol. 58, no. 9, pp. 5816-5840, September 2012.

[5] B. Hochwald and E. Stauffer, "Low-Complexity Multi-Stream Space-Time Codes-Part II: Unitary-Transform Codes," *IEEE Transactions on Communications*, vol. 60, no. 2, pp. 375-383, February 2012 2012.

[6] E. Stauffer and B. Hochwald, "Low-Complexity Multi-Stream Space-Time Codes-Part I: Direct-Sum Codes and Design Criteria," *IEEE Transaction on Communications*, vol. 60, no. 1, pp. 81-99, January 2012.

[7] Joseph Boutros and Emanuele Viterbo, "Signal Space Diversity: A Power- and Bandwidth-Efficient Diversity Technique for the Rayleigh Fading Channel," *IEEE Transactions on Information Theory*, vol. 44, no. 4, pp. 1453-1467, July 1998.

[8] Sungho Jeon, Ilsoo Kyung, and Man-Sik Kim, "Component-Interleaved Receive MRC with Rotated Constellation for Signal Space Diversity," in *Vehicular Technology Conference Fall*, 2009, pp. 1-6.

[9] Soonki Jo, Jinyong Choi, and Jongsoo Seo, "An Advanced Hierarchical Modulation with Rotated Constellation," in *The 12th International Conference on Advanced Communication Technology*, vol. 1, 2010, pp. 515-518.

[10] Sungho Jeon, Jaekwon Lee, Ilsoo Kyung, and Man-Sik Kim, "Component-interleaved Alamouti coding with rotated constellations for signal space diversity," in *IEEE International Symposium on Broadband Multimedia Systems and Broadcasting*, Shanghai, 2010, pp. 1-6.

[11] Zaid Paruk and Hongjun Xu, "Performance Analysis and Simplified Detection for Two-Dimensional Signal Space Diversity with MRC Reception," *South African Institute of Electrical Engineers*, vol. 104, no. 3, pp. 97-106, September 2013.

[12] P.K. Vitthaladevuni and M.-S. Alouin, "A Recursive Algorithm for the Exact BER Computation of Generalized Hierarchical QAM Constellations," *IEEE Transactions on Information Theory*, vol. 49, no. 1, pp. 297-307, January 2003.

[13] P.K. Vitthaladevuni and M.-S. Alouini, "BER computation of 4/M-QAM hierarchical constellations," in *12th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 1, 2001, pp. B-85- B-89.

[14] R. Stukavec and T. Kratochvil, "Performance of Hiearichcal Modulation in DVB-T," in *20th International Conference Radioelektronika*, 2010, pp. 1-4.

[15] P.K. Vitthaladevuni and M.-S. Alouini, "A closed-form expression for the exact BER of generalized PAM and QAM constellations," *IEEE Transactions on Communications*, vol. 52, no. 5, pp. 698-700 , May 2004.

[16] Guangqiu Li, Zhenyu Zhang, and Jianwu Zhang, "Exact BER Analysis of Hierarchical M-QAM in Spatially Correlated MIMO Nakagami Fading Channels," in *International Confrence on Wireless Communications, Networking and Mobile Computing*, 2007, pp. 389-392.

[17] Siavash M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE Journal on Select Areas in Communications*, vol. 16, no. 8, pp. 1451-1458, October 1998.

[18] Ayesha Saeed, Tahmid Quazi, and Hongjun Xu, "Hierarchically Modulated QAM with Signal Space Diversity and MRC Reception in Nakagami-m fading channels," *IET Communications*, vol. 7, no. 12, pp. 1296-1303, August 2013.

[19] I.A. Falujah and V.K. Prabhu, "Performance analysis of MQAM with MRC over Nakagami-m fading channels," *Electronics Letters*, vol. 42, no. 4, pp. 231-233 , February 2006.

[20] M. Salehi and J.G. Proakis, *Digital Communications*, 5th ed. Boston: McGraw-Hill, 2008.

[21] Jin-Sung Kim, Sung-Hyun Moon, and Inkyu Lee, "A New Reduced Complexity ML Detection Scheme for MIMO Systems," *IEEE Transactions on Communications*, vol. 58, no. 4, pp. 1302-1310, April 2010.

Part III

Conclusion

1. Conclusion and Future Work

Hierarchical modulation ensures greater reliability of wireless multimedia transmission by using UEP and allocating greater error protection for important data than less important data. As a result, high priority data is ensured greater error performance at the cost of deteriorating error performance for low priority data. From the literature review, it is evident that there is a high demand for HM systems in progressive image and video transmission over wireless channels. This motivates for research enhancing HM systems to ensure improved error performance for both high and low priority data without consuming additional space, power and cost or leading to loss of bandwidth. Signal space and Alamouti transmit diversity are two schemes incorporated with HM in this study in order to address this problem. The NN approximation approach is employed in determining the BER analysis of the proposed schemes while a method for obtaining the optimized angle of rotation in also presented. A ML-based reduced complexity detector is also introduced. This dissertation incorporates two journal papers accounting for the results this research.

In Paper A, HM incorporating SSD with multiple antenna reception using MRC under Nakagami-m fading channels is considered. The optimum angle of rotation is determined according to the specific constellation hierarchy while a theoretical BER expression is derived using the NN approximation approach. The theoretical analysis is then verified against Monte-Carlo simulations for single and multiple antenna reception with varying channel conditions and increasing constellation hierarchy. Performance gains between HM with SSD and a simple HM scheme is observed at up to 18 dB and 17 dB at a BER of 1×10^{-5} for base and refinement bits, respectively, for single antenna reception at $m = 1$ and $\alpha = 4$. Performance gains for both base and refinement bits are also presented for multiple antenna reception and as channel conditions improve. Specifically, gains of about 4 dB at a BER of 1×10^{-6}, for both base and refinement bits, are observed as number of receiver antennas increases from $N = 3$ to $N = 4$ while gains of up to 14 dB at 1×10^{-6}, for both base and refinement bits, is observed as channel conditions improved from $m = 1$ to $m = 3$.

In Paper B, a scheme encompassing HM with SSD and ATD with multiple receiver antennas using MRC under Nakagami-m fading is presented. The NN approximation approach to determining BER is extended for Alamouti dual transmit antennas and is discussed briefly. In addition, a ML-based reduced complexity detector is introduced in order to address the problem of high decoding complexity of the received symbol. Simulation results verify the method of performance analysis as accurate as well as the enhanced system performance of the proposed scheme for varying number of receiver antennas, channel conditions and constellation hierarchy. Specifically, performance gains of up to 26 dB at a BER of 1×10^{-5} for both base and refinement bits at $\alpha = 4$, $m = 1$ and $N = 1$ observed between the proposed scheme and a simple HM scheme. The performance of the proposed ML-based detector is also determined as accurate when compared to the NN approximation while a complexity reduction of 51.43% is attained between the proposed and optimal ML detectors.

While the NN approximation presented for the BER analysis of 16-QAM can easily be extended for higher constellations, a generic formula for M-QAM constellation remains to be determined. Similarly, for the method of determining the optimized rotation angle, a generic formula for M-QAM constellations is a topic for future research.

In extension, the proposed schemes in this dissertation can also be investigated in multiplicative or cascaded fading channels. In urban environments, the likelihood of multiple diffractions of a wireless signal motivates for the study of schemes under cascaded fading channels.

Additionally, increasing the diversity order by means of double rotating HM constellations can be investigated further.