MODELLING AND CONTROL

OF A CO-CURRENT SUGAR DRYER

By

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Submitted in fulfilment of the requirements for the degree of Master of Science in Engineering in the school of Chemical Engineering, University of Natal, Durban

Durban
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À mes parents,

ma soeur
ABSTRACT

The drying of sugar is the last step in the recovery of solid sugar from sugar-cane. To ensure that the sugar can be transported and stored, the final moisture content leaving the sugar mill must be carefully controlled.

Data spanning periods of normal plant operation were collected at the Tongaat-Hulett Ltd Darnall sugar mill. These measurements were reconciled to achieve instantaneous mass and energy balances across the sugar dryer.

Using these measurements, a general model has been developed to simulate the sugar drying. It includes ten compartments through which the sugar and drying air flow, with a mass and energy balance in each compartment. It was assumed that a “film” around the sugar crystal is supersaturated, and that crystallisation is still occurring. A sorption isotherm determining the equilibrium moisture content of the sugar, at which point mass transfer ceases, was included. The model has been matched to process measurements by adjusting the heat and mass transfer coefficients.

A Dynamic Matrix Controller was developed and tested off-line on the model, using the reconciled measurement sequences. The controller manipulated the inlet air temperature in order to control the exit sugar moisture content. The model predictive control format successfully dealt with the large process dead-time (5 minutes).
PREFACE

Tongaat-Hulett Sugar Ltd initiated this project to have access to heat and mass transfer coefficients in the process of drying sugar in a co-current rotary dryer, and to investigate improved control algorithms for exit moisture content.

The investigation required data collected from the Darnall sugar mill, in the North coast of Kwazulu-Natal. Other studies were made in the postgraduate offices of the School of Chemical Engineering at the University of Natal, Durban under the supervision of Professor Michael Mulholland.

The following courses were completed with the corresponding credits and results achieved:
DNC4DC1  Process Dynamics and Control  (16.0)  64%
DNC5RT1  Real Time Process Data Analysis  (16.0)  70%

I hereby declare that this dissertation is my own work, unless stated to the contrary in the text, and that it has not been submitted for a degree to any other university or institution.

B. Lacave

Date

As the candidate's supervisor I have approved this dissertation for submission
Signed: Michael Mulholland
Name: Michael Mulholland
Date: 2002.06.12
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### Alphabet

- \( a \)  
  Interfacial area  
  \( \text{m}^2 \cdot \text{m}^{-3} \)

- \( A \)  
  Sugar dryer section  
  \( \text{m}^2 \)

- \([A]\)  
  Matrix containing coefficients for a linear system of ordinary differential equations

- \([AC]\)  
  Matrix containing coefficients for the Euler integration

- \([B]\)  
  Matrix containing coefficients for a linear system of ordinary differential equations

- \([BC]\)  
  Matrix containing coefficients for the Euler integration

- \( B_{OL} \)  
  Matrix

- \( B_o \)  
  Matrix

- \([CM]\)  
  Matrix containing coefficients to build \([A]\) and \([B]\)

- \([CC]\)  
  Matrix containing coefficients to build \([AC]\) and \([BC]\)

- \( C_{pa} \)  
  Heat capacity for air  
  \( \text{kJ} \cdot \text{kg}^{-1} \cdot \text{C}^{-1} \)

- \( C_{ps} \)  
  Heat capacity for sugar  
  \( \text{kJ} \cdot \text{kg}^{-1} \cdot \text{C}^{-1} \)

- \( C_{pvap} \)  
  Heat capacity for vapor  
  \( \text{kJ} \cdot \text{kg}^{-1} \cdot \text{C}^{-1} \)

- \( D_A \)  
  Diffusivity coefficient for air  
  \( \text{m} \cdot \text{s}^{-1} \)

- \( D_S \)  
  Diffusivity coefficient for sugar  
  \( \text{m} \cdot \text{s}^{-1} \)

- \( e_{CL} \)  
  Closed-loop error

- \( e_{OL} \)  
  Open-loop error

- \( E_a \)  
  Activation energy  
  \( \text{kJ} \cdot \text{mol}^{-1} \)

### Error

- \( f_A \)  
  Air flow  
  \( \text{kg} \cdot \text{s}^{-1} \)

- \( f_s \)  
  Sugar flow  
  \( \text{kg} \cdot \text{s}^{-1} \)

- \( \text{Funct B} \)  
  Function

- \( \text{Funct b} \)  
  Function

- \( F_w \)  
  Flow of water added  
  \( \text{kg} \cdot \text{s}^{-1} \)

- \( G \)  
  Crystallisation growth rate  
  \( \text{m} \cdot \text{s}^{-1} \)

- \( H \)  
  Enthalpy  
  \( \text{kJ} \)

- \( h_s \)  
  Sugar hold up volume fraction  
  \( \text{kg} \cdot \text{kg}^{-1} \)

- \( h_i \)  
  Heat transfer coefficient  
  \( \text{kJ} \cdot \text{m}^{-2} \cdot \text{K}^{-1} \)
<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>I</td>
<td>Impurity</td>
<td>kg.kg⁻¹</td>
</tr>
<tr>
<td>k</td>
<td>Coefficient</td>
<td></td>
</tr>
<tr>
<td>K₀</td>
<td>Crystallisation parameter</td>
<td></td>
</tr>
<tr>
<td>K₁</td>
<td>Crystallisation parameter</td>
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<tr>
<td>K₂</td>
<td>Crystallisation parameter</td>
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<tr>
<td>K₃</td>
<td>Crystallisation parameter</td>
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<tr>
<td>kₛ</td>
<td>Mass transfer coefficient</td>
<td>kg.m⁻².s⁻¹.Pa⁻¹</td>
</tr>
<tr>
<td>Lₜₜ</td>
<td>Latent heat of vaporisation of water</td>
<td>kJ.kg⁻¹</td>
</tr>
<tr>
<td>Mₚ</td>
<td>Mass of crystal</td>
<td>kg</td>
</tr>
<tr>
<td>Mₚᵢ</td>
<td>Mass of impurity</td>
<td>kg</td>
</tr>
<tr>
<td>Mₛ</td>
<td>Mass of sucrose</td>
<td>kg</td>
</tr>
<tr>
<td>Mₜ</td>
<td>Mass total</td>
<td>kg</td>
</tr>
<tr>
<td>Mₚₜ</td>
<td>Mass of water</td>
<td>kg</td>
</tr>
<tr>
<td>Mₚᵢᶠ</td>
<td>Mass of water in the feed</td>
<td>kg</td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>P₀</td>
<td>Pressure reference</td>
<td>Pa</td>
</tr>
<tr>
<td>Pₚₜ</td>
<td>Partial pressure of air at the boiling point</td>
<td>Pa</td>
</tr>
<tr>
<td>pur</td>
<td>Purity</td>
<td>%</td>
</tr>
<tr>
<td>P₁₀₀₀</td>
<td>Partial pressure of air at T = 100 °C</td>
<td>Pa</td>
</tr>
<tr>
<td>RH</td>
<td>Relative humidity</td>
<td>%</td>
</tr>
<tr>
<td>SC</td>
<td>Solubility coefficient</td>
<td></td>
</tr>
<tr>
<td>Sᵣ</td>
<td>kg sucrose dissolved in film per kg of dry sugar crystal</td>
<td>kg.kg⁻¹</td>
</tr>
<tr>
<td>SOL</td>
<td>Mass percent of sucrose</td>
<td>%</td>
</tr>
<tr>
<td>SS</td>
<td>Supersaturation in the film</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
<td>°C</td>
</tr>
<tr>
<td>T₀</td>
<td>Temperature reference</td>
<td>°C</td>
</tr>
<tr>
<td>Tₘₐₓₚᵢₜ</td>
<td>Temperature absolute</td>
<td>°C</td>
</tr>
<tr>
<td>Tₚₜ</td>
<td>Boiling point temperature</td>
<td>°C</td>
</tr>
<tr>
<td>Tₜᵢʳ</td>
<td>Dry bulb temperature</td>
<td>°C</td>
</tr>
<tr>
<td>Tₜₑᵢₜ</td>
<td>Wet bulb temperature</td>
<td>°C</td>
</tr>
<tr>
<td>U</td>
<td>Vector containing the bounding conditions</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>Penalty weight on squared set-point deviation</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>Moisture</td>
<td>kg.kg⁻¹</td>
</tr>
<tr>
<td>Wₛ</td>
<td>Moisture in the sugar at saturation</td>
<td>kg.kg⁻¹</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>-----------------</td>
<td>-------------------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>( W_{\text{nequilibrium}} )</td>
<td>Moisture in the sugar at equilibrium</td>
<td>kg.kg(^{-1})</td>
</tr>
<tr>
<td>( x_{\text{CL}} )</td>
<td>Closed-loop response</td>
<td></td>
</tr>
<tr>
<td>( x_{\text{OMEAS}} )</td>
<td>Present measurement</td>
<td></td>
</tr>
<tr>
<td>( x_{\text{OL}} )</td>
<td>Open-loop response</td>
<td></td>
</tr>
<tr>
<td>( x_{\text{SP}} )</td>
<td>Set point</td>
<td></td>
</tr>
<tr>
<td>( \bar{X} )</td>
<td>Vector containing the unknown</td>
<td></td>
</tr>
<tr>
<td>( y_A )</td>
<td>Fraction of moisture in the air</td>
<td>%</td>
</tr>
<tr>
<td>( % \text{Water} )</td>
<td>Mass percentage of water in the sugar</td>
<td>%</td>
</tr>
</tbody>
</table>

**Subscript**

- \( A \) : Air
- \( AO \) : Inlet air
- \( An \) : Outlet air
- \( s \) : Sugar
- \( s_{\text{equilibrium}} \) : Sugar at equilibrium
- \( s_0 \) : Inlet sugar
- \( s_0 \text{ corrected} \) : Corrected value of inlet sugar
- \( s_0 \text{ fixed} \) : Fixed value of inlet sugar
- \( s_n \) : Outlet sugar
- \( s_n \text{ equilibrium} \) : Outlet sugar at equilibrium

**Greek letters**

- \( \alpha \) : Function
- \( \beta \) : Function
- \( \chi \) : Filter parameter
- \( \gamma \) : Filter parameter
- \( \rho_A \) : Air density \( \text{kg.m}^{-3} \)
- \( \rho_S \) : Sugar density \( \text{kg.m}^{-3} \)
- \( \Delta m \) : Vector move
- \( \Delta m_{\text{UQM}} \) : Vector unbounded quadratic optimum control move
- \( \Delta m_{\text{PAST}} \) : Vector past input
- \( \Delta P \) : Pressure difference \( \text{Pa} \)
- \( \Delta W_s \) : Sugar moisture content difference \( \text{kg.kg}^{-1} \)
- \( \Delta x \) : Interval \( \text{m} \)
\( \Lambda \)  Penalty weight on the control move
\( \xi \)  Function
\( \psi \)  Function
CHAPTER 1
INTRODUCTION

1.1 LAYOUT OF THIS DISSERTATION

In Chapter 1, the background of drying technology and drying control are developed. A brief presentation of the co-current sugar dryer considered is made in Chapter 2. The techniques used to reconcile the data collected from the plant are explained in Chapter 3. Then these data were used to construct a model able to simulate the drying process in a co-current sugar dryer. The theoretical development is described in Chapter 4. Chapter 5 deals with the arrangement and properties of the controller, and how the program for model and controller was built. In Chapter 6, conclusions and recommendations end the thesis.

1.2 DRYING

1.2.1 Process

Van der Poel et al. (1998) defined the drying process as a thermal separation process: the volatile liquid is vaporised from the solid. A drying agent reduces external vapour pressure, and carries the vaporised liquid away. In their book, they describe the drying process. The moisture in the solid is noted as \( X \) and measured in kg of water per kg of dry solid. The drying rate \( m \) is determined by measuring the change in water content with the time,

\[
m = -\frac{m_{S,dry}}{A} \frac{dX}{dt}
\]  

(1.1)

where \( m_{S,dry} \) is the mass of the dry solid

\( A \) is the surface area in contact with the drying agent

The temperature, the pressure and the velocity of the drying agent, and the temperature and the velocity of the solid influence the drying conditions. The drying curve is defined as the relation between the drying rate and the water content under constant drying conditions.

The drying rate is mostly constant in the first part of the drying. The water is taken from the liquid on the surface and carried by the drying agent, say air. (Fig. 1-1, Section I). The surface of the solid dries out when the water content goes down to a value called the critical value \( X_{crit} \).  

1-1
In the second drying period: the drying rate drops with the decreasing water content of solid (Fig. 1-1, Section II).

The residual water is bound to the solid by sorption. The drying rate decreases rapidly with the decreasing water content and tends to zero as the hygroscopic equilibrium water content $X_{eq}$ is approached. The regime between the maximum hygroscopic water content $X_{cr2}$ and the equilibrium value of $X_{eq}$ is designated as the third drying period. (Fig. 1-1, Section III)

Liptak (1998) defined the drying process in four steps, adding a preheating step:

- Preheating: the particle is heated up to the drying temperature.
- Constant drying rate (Fig. 1-1, Section I): the moisture is removed by evaporation from the surface of the solid.
- Falling drying rate (Fig. 1-1, Section II): the particle is dried.
- Falling drying rate (Fig. 1-1, Section III): the water within the film diffuses to the surface and then is evaporated.

1.2.2 Dryers

Different types of dryers exist to dry different materials. Two main types can be distinguished: batch dryer, and continuous dryer. For any drying operation, a dryer must have a source of heat, a way of removing the liquid vaporized from the surface of the solid, a mechanism to mix the solid and the drying agent. In his paper, Lipták (1998) drew up an exhaustive list of batch dryers (atmospheric tray types, tunnel dryers, vacuum and freeze dryers, batch kilns, fluidised-bed...
dryers), and continuous dryers (co-current, counter-current). The sugar dryer studied in the present work is a co-current dryer. It is interesting to note in this case that the sugar itself provides much of the heat for evaporation.

Typical continuous dryer temperature profiles are as in Fig. 1-2. The different zones of the drying process are represented along the dryer. A-B describes the pre-heating zone. In the zone B-C, the material is dried at constant rate. Then in the zone C-D-E, the drying rate drops, the temperature increases. The sugar temperature follows the same profile in the both co-current and counter-current configurations. However, the profile is not the same for the air temperature. Often, with the counter current dryers, it is necessary to add cool air at the exit to decrease the sugar temperature. An interesting phenomenon will be observed in the present work on a co-current dryer where the exit sugar temperature exceeds the exit air temperature.

![Temperature profiles along the dryer](image)

Figure 1-1: Temperature profiles along the dryer (Lipták, 1998)
1.3 SUGAR DRYING

1.3.1 Different types of moisture

Sugar moisture is an issue in the conditioning of sugar. If sugar crystals are not dried enough, caking can occur. Moist crystals in contact go on crystallising, and the crystals agglomerate into a cake.

Marijnissen and de Bruijn (1996) note that there are three different kinds of water in a sugar crystal.

- **Free moisture**: this is considered as the water at the surface of the crystal in contact with the surrounding air.
- **Bound water**: this water is trapped inside a thin sucrose film around the sugar crystal. It results from too quick removal of water at the surface of the crystal.
- **Inner, inherent or internal water**: here the water is trapped inside the sugar crystal lattice.

In the case where raw sugar is being dried, only the free moisture is removed. Raw sugar contains a lot of impurities (ash, fructose and glucose, colour) that do not allow the formation of bound water.

1.3.2 Drying process

Sugar drying follows the general process of drying explained in part (1.2.1). According to Thompson (1998), white sugar is a hygroscopic material that needs to be stored in a place where the relative humidity is below 65%. A high level of impurities and small crystal size increase the moisture content at equilibrium, but a high temperature decreases it. So the sugar is usually dried with hot air. However, Thompson drew attention to the following possibility, assuming that the wet sugar coming from centrifugals had enough energy to dry itself: he suggested then that hot air was not necessary since cold air was needed to cool the sugar at the exit of the dryer.

Savaresi *et al.* (2001) defined two modes of drying sugar. The first is the “standard-mode”. It is assumed that everywhere in the sugar dryer, the moisture content is always reasonably above zero. The second is the “over-dried mode”: a possible moisture content close to zero in the sugar at the exit characterizes this mode.

1.3.3 Drying Model

Tait *et al.* (1994) proposed a model for a counter-current sugar dryer. They based their model on a mass and energy balance with the assumption that crystallisation is still occurring inside the
sugar dryer. When the drying process is looked at from a sugar crystal point of view, the mechanism of evaporating water from the film surrounding the crystal appears more complicated. The film contains sucrose, water and impurity. The partial pressure of the water is reduced due to the presence of sucrose and impurity. As the water is taken from the film, it becomes more and more saturated so that crystallisation can occur. In the model, the sugar dryer is divided into several equal compartments. It is assumed that the sugar and air leaving the compartment are at equilibrium.

Shardlow et al. (1996) proposed another sugar drier model based on the above model of Tait et al. (1994). They made modifications to improve it. The diffusion of water through the film around the sugar crystals is accounted for using Fick's law of diffusion in the calculation of the partial pressure of the water. They decided as well to decrease the crystallisation rate by 40% in the film.

Rastikian et al. (1999) designed a model for a counter current cascading rotary sugar dryer. They preferred to work with the convective heat transfer coefficient in the gas film $h_{ga}$ and the convective mass transfer coefficient in the gas film $k_{ya}$ rather than with the overall heat and mass transfer coefficients, because they were able to estimate a good value for the interfacial surface area. They did not estimate the phenomenon of axial diffusion, dispersion or back mixing of the solid. According to them, the resistance to the heat transfer lies only in the gas film. The film of sucrose solution surrounding the sugar crystals is supposed to remain supersaturated for the whole drying process. So this implies that the drying rate depends on the driving force in the gas film not on the water remaining in the film around the crystals.

Savaresi et al. (2001) based their work on a first principles model similar to Tait et al. (1994), optimising the time and spatial discretisation using two separate models of vapour/air and sugar/moisture transport. Working at lower sugar moisture contents, they found that their model could not accurately simulate the moisture contents, but it could predict the temperatures well. They developed guidelines for a controller, including their model as a black box which could be switched from the “standard mode” drying mechanism to the “over-dried mode” drying mechanism.

Table 1-1 illustrates the different values found in the literature for the heat and mass transfer coefficients. These values can be separated into two groups. Tait et al. (1994), Shardlow et al. (1996), and Savaresi et al. (2001) have more or less the same results characterising data coming from Australia. Rastikian et al. (1999) have worked with data coming from France. The origin of the sugar might have an effect on the heat and mass transfer coefficients.
Table 1-1: Comparison of heat and mass transfer coefficient

<table>
<thead>
<tr>
<th></th>
<th>Heat transfer coefficient (kW.m(^{-2}).K(^{-1}))</th>
<th>Mass transfer coefficient (kg water.m(^{-2}).s(^{-1}).Pa(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tait et al. (1994)</td>
<td>0,3000</td>
<td>72 \times 10^9</td>
</tr>
<tr>
<td>Shardlow et al. (1996)</td>
<td>0,0036</td>
<td>7,9 \times 10^9</td>
</tr>
<tr>
<td>Rastikian et al. (1999)</td>
<td>0,0019</td>
<td>27 \times 10^9</td>
</tr>
<tr>
<td>Savaresi et al. (2001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>“Standard mode”</td>
<td>0,0030</td>
<td>2,7 \times 10^9</td>
</tr>
<tr>
<td>“Over-dried mode”</td>
<td>0,0038</td>
<td>4,05 \times 10^9</td>
</tr>
</tbody>
</table>

1.4 CRYSTALLISATION

Tait et al. (1994) worked with the following equations for the crystallisation rate:

\[
G = 2,060 \cdot 10^{-6} (SS - 1,0046) \exp\left(FT - 1,75 \frac{\text{impurity}}{W_s}\right) \tag{1.2}
\]

where

\[
FT = \frac{-E_{\text{act}}}{1,987 \cdot 10^{-3}} \left(\frac{1}{273,16 + T_s} - \frac{1}{333,16}\right)
\]

\[
E_{\text{act}} = 15,0 - 0,2(T_s - 60)
\]

\(SS\) is the supersaturation of the film that is calculated from \(SOL\) the mass percent of sucrose in a saturated film without impurities and \(SC\) the solubility coefficient correcting for the impurity level. \(SOL, SC, SS\) are given by Tait et al. (1994).

\[
SOL = 64,407 + 0,07251 T_s + 0,0020569 T_s^2 - 9,035 \times 10^{-6} T_s^3 \tag{1.3}
\]

\[
SC = 1,0 - 0,088 \frac{I}{W_s} \tag{1.4}
\]

\[
SS = \frac{S}{W_s} \frac{100 - SOL}{SOL \cdot SC} \tag{1.5}
\]


\[
G = K_1 \cdot \left(SS - (1 + K_0)\right) \exp\left(K_2 - K_3 \cdot IW\right) \tag{1.6}
\]
• \( K_0 \) is a constant to take account of the threshold of growth at lower saturation.
• \( K_1 \) is the major proportionality constant relating to the degree of over saturation.
• \( K_2 \) is used to describe the extent to which growth rate varies with temperature.
• \( K_3 \) describes the dependence of growth rate on the level of impurities present.

\( K_2 \) behaves according to an Arrhenius law type:

\[
K_2 = \frac{Ea}{R} \left( \frac{1}{273.16 + T_s} - \frac{1}{333.16} \right)
\]

where \( Ea \) is the activation energy in \( \text{kJ.mol}^{-1} \)

\( R \) is the universal gas constant

The parameters found by Love (2001) result from his study of the modelling and control of a pan boiling crystalliser.

Ben-Yoseph et al. (2000) described some of the influences on the crystal growth of the film. When a crystal grows, sucrose is taken from the film to become sugar crystal so the concentration of sucrose decreases. This phenomenon generates heat that raises the film temperature. So the film temperature can be higher than the air temperature.

1.5 DATA RECONCILIATION

Bazin et al. (1998) worked with data collected from a zinc concentrate rotary dryer. They wanted to illustrate the problems linked to the calculation of non-measured variables arising from error in the data measured. Their paper shows how errors can be propagated from a simple calculated variable such as the gas flow rate, through the mass and energy balance.

1.6 CONTROL

1.6.1 Predictive Control

The problem with control of a sugar crystalliser is dealing with dead time and huge model order. Clarke et al. (1987) explained how a predictive control algorithm was found to improve control in the face of these difficulties. The idea is to use a moving time horizon within which to predict the plant behaviour to determine optimal control actions.

1.6.2 Dynamic Matrix Control

Dynamic Matrix Control is one of the techniques using Predictive Control. García et al. (1989) studied the family of Predictive Control algorithms. They give an overview of DMC and
CHAPTER 1

INTRODUCTION

MAC (Model Algorithmic Control). They concluded that even if it works well, some progress still needed to be made in the field of linear and especially non-linear systems.

In 1997, Aitchison and Mulholland obtained very good results with the control of a concentric tube heat exchanger using an adaptive control based on Dynamic Matrix Control. A similar scheme, based on an accurate non-linear model, is used in the control of this sugar dryer.

1.7 DRYER CONTROL

Shinskey determined the moisture content by (1.8), where any consistent units may be used for any kind of dryable material:

\[ w_p = \frac{w_c F_a C_p}{AH_w \zeta} \ln \left( \frac{T_i - T_w}{T_o - T_w} \right) \]  

(1.8)

- \( A \) = Solid surface area
- \( C_p \) = Specific heat of air
- \( F_a \) = Air flow
- \( H_w \) = Latent heat of water
- \( T_i \) = Air inlet temperature
- \( T_o \) = Air outlet temperature
- \( T_w \) = Air wet bulb temperature
- \( \zeta \) = Mass transfer coefficient
- \( w_c \) = Critical moisture content of the dried product
- \( w_p \) = Moisture point

To control \( w_p \) the two ratios \( \frac{w_c F_a C_p}{AH_w \zeta} \) and \( \frac{T_i - T_w}{T_o - T_w} \) need to be kept constant. The first ratio can be assumed constant because the change of the air flow or the solid surface area that depends on the crystal size are compensated by the mass transfer coefficient. Concerning the second ratio, in an adiabatic dryer, \( T_w \) is supposed constant along the dryer. So the control should be done on the air temperature.

Trelea et al. (1996) developed a strategy to control a mixed flow corn dryer. The moisture content of the corn or the exhaust air is usually the controlled variable. The manipulated variable can be the air temperature, the gas flow in the burner or the residential time of the product in the dryer. They worked with two control strategies: PI (Proportional Integral) control and LQG (Linear Quadratic Gaussian) control. It was found that the PI control furnished almost as good results as the sophisticated LQG control.
Lipták (1998) described in Fig. 1-3 a conventional way of controlling a co-current rotary dryer. In this kind of control, the optimum inlet air temperature is the maximum that the product can tolerate. If high temperature can degrade the product, a high temperature alarm is needed (TAH). If the steam and air valve are fully open, the product moisture cannot be controlled, so alarms are placed to detect these situations.
Pérez-Correa et al. (1998) chose the following manipulated and controlled variables: the inlet gas temperature as a manipulated variable and the solid moisture content measured on-line. They worked with two control schemes: a PID control, and an extended horizon self-tuning regulator. The PID controller was able to control the dryer but it needed periodic tuning, and presented a small overshoot in the response. The self-tuning controller responded with better results: smaller overshoots, and better settling times.

Dryer efficiency is defined as the ratio between the theoretical energy required for evaporation and the actual energy consumed. Clarke (2000) presented figures suggesting that the average dryer efficiency is around 40-60%. It may even go down to 10% for old machines. It is obvious that control is needed. In the UK, most of the dryers use single loop temperature control that gives good results, however, lots of limitations. The ideal solution would be to measure directly the product moisture content but this remains still very expensive. They proposed then a system able to determine the moisture content simply by measuring two temperature. Their system called the “Delta T” algorithm has given good results.

Temple and van Boxtel (2000) developed a strategy to control a fluidised bed tea dryer. Similarities exist between sugar drying and tea drying. They first tried to work with a direct moisture content feed-back. It provided good results but showed some limitations because of dead-time from the heater system. They worked then with an intermediate exhaust temperature that can be converted into moisture value. This gave good control without requiring expensive instrumentation.

Savaresi et al. (2001) proposed a very simple way of controlling the sugar dryer just with the output sugar temperature measurement which is easier than measuring the output sugar moisture content – see Fig. 1-4. The basic idea of the control scheme was to inject an impulse at regular intervals to excite the system (signal $\mathcal{D}$) in order to detect the current operating condition of the system. Signal $\mathcal{D}$ was the output sugar temperature response. It was obtained as the difference between the real temperature and its average value calculated from the low pass filter. Signal $\mathcal{D}$ was the result from the multiplication of delayed signal $\mathcal{D}$ and signal $\mathcal{D}$. The time delay $r$, had to equal the residence time of the sugar in the dryer because the effect of the impulse appeared only $r$, minutes later. Signal $\mathcal{D}$, after being transformed by the non-linear static characteristic,
was re-balanced giving an adjusted moisture content response. The time delay $\Psi$

which was bigger than $r$, was used to avoid interferences between injection and controller output. With signal $\sigma$, the integration provided a permanent adjustment to the amount of sprayed water until the next injection. The signal is then adjusted with the moisture set-point. The original point in this work was to control the sugar drying by adding water to the sugar before it was dried.

**Figure 1-2: Control scheme** (Savaresi et al., 2001)
CHAPTER 2
THE DRYER CONSIDERED IN THIS STUDY

2.1 THE PLANT

2.1.1 Description of the equipment

The Darnall Sugar Mill (Tongatt Hulett Sugar Ltd) uses a rotary drum dryer as the last stage in the raw sugar milling process. The dryer operates in continuous co-current mode.

![Schematic view of the sugar dryer](image)

Figure 2-1: Schematic view of the sugar dryer

It consists of a large drum (12.5 m long and with internal diameter of about 1.75 m), set at a slight angle, about 3°, in order to let the sugar move slowly under gravity from the inlet to the outlet. The average residential time of the sugar in the dryer is about 5 minutes. The drum slowly rotates around its longitudinal axis (about 10 revolutions per minute), in order to expose the sugar to the air continuously, and to mix the sugar. To this end, the internal surface of the drum is entirely covered with concave louvres (usually called “flights”), which lift the sugar in the drum. The inlet wet/hot sugar passes through the dryer, moving co-current with air flow, and out at the lower end. It is dropped onto a conveyor that takes the dried/cooled sugar to the sugar hopper.
2.1.2 Description of the process

After the centrifugation, in which the sugar crystals are separated from the mother liquid, they are then dried in the rotary dryer. They have a size ranging from 0.4 to 0.7 mm. The average value used in the model was obtained from a technician at the Darnall sugar mill as 0.57 mm. At the entrance of the sugar dryer, the sugar moisture content is around 1%, it is dried down to a value of 0.1% (mass/mass). In the general working conditions, the feed sugar flow rate is 10 kg.s\(^{-1}\) at a temperature of 55°C. For the air, the temperature is around 80°C, and the air to sugar ratio is between 0.2 and 0.7 kg air.kg sugar\(^{-1}\).

2.1.3 The instrumentation

Six temperature sensors have been used to collect data from the plant. They are mineral insulated Pt100 sensors Class B with Tci Ptxi-Oop 0-150 °C head mount transmitters.

The temperature sensors were placed at different locations on the sugar dryer to measure the inlet air (T1) and the inlet sugar (T2) temperatures, the outlet air (T3) and the outlet sugar (T4) temperatures, and the dry bulb (T5) and wet bulb (T6) temperatures. The inlet sugar temperature sensor was placed at the bottom of a bucket elevator that carries the sugar to the entrance of the dryer. This sensor was continuously in contact with the sugar crystals. The outlet sugar temperature sensor measures the temperature intermittently because the sensor was placed in a
discharge hopper at the exit of the dryer. This hopper collects the dried sugar up to a set load, then dumps it onto a product conveyor. The period of this cycle varies inversely with production rate.

A 0-4 kPa DP cell was used to gauge the air flow rate at the entrance of the drier. A 4-20 mA driver board drove these circuits and provided an input voltage for an A/D converter scanned by a laptop PC SCADA system.

The temperature sensors were calibrated by the following procedure: the working range of temperature is between 30°C-90°C. Thus the temperature sensors were calibrated using a point at 20°C and a point at 100°C using water in a thermos flask.

2.1.4 Pictures of the dryer

The photographs shown in Fig. 2-3 to 2-5 were taken on 29th June, 2000.

![Figure 2-1: Picture of the dryer](image-url)
Figure 2-2: Picture of the sugar coming into the dryer

Figure 2-3: Picture of the sugar crystals at the end of the dryer
3.1 **ON-LINE DATA COLLECTION**

3.1.1 *Air flow*

The pressure drop across the ducting varies with the square of the flow in turbulent flow, so measuring the differential pressure gives a signal that can be related to the flow rate, as follows:

\[ f_x = k \times \sqrt{\frac{\Delta P}{T_{\text{absolute}}}} \]  

(3.1)

3.1.2 *Sugar flow*

The sugar flow is not directly measurable on the plant. Estimates were made from oscillations in the outlet sugar temperature record.

The sugar temperature sensor was placed in the container that collects the sugar at the exit of the sugar dryer. Every time that this weight of the container reaches 350 kg, it empties. This creates a periodic rise and fall of the sensor temperature. From this, the sugar flow rate can be calculated.

![Diagram](https://via.placeholder.com/150)

*Figure 3-1: Determination of sugar flow*
3.1.3 Moisture in the air at the entrance

The dry bulb and wet bulb temperature sensors had been placed close to the sugar dryer to give a relevant indication of the moisture in the feed air. $W_{A0}$ is calculated by an equation in common use in Tongaat-Hulett Sugar Ltd:

$$W_{A0} = 0.000010286T_{wet}^3 - 0.0004466T_{wet}^2 + 0.001504T_{wet} - 0.0056 - 0.00042(T_{dry} - T_{wet})$$

Due to poor calibration of the temperature sensor (Pt100), the wet bulb temperature sensor gave a higher signal than the dry bulb temperature sensor. Thus, with the help of the South African Weather Bureau, the data for the days studied were compared with the data from three meteorological stations around Darnall: Durban, Mtunzini and Mandini for the temperature and the relative humidity.
3.2 DATA TREATMENT

3.2.1 Relation between $H_A$ and $W_A$

The enthalpy of humid air can be estimated from its water content and temperature as follows:

$$H_A = \left[ W_A C_{Pw} + (1 - W_A) C_{Pw} \right] \left( T_{Air} - T^\circ \right) + W_A L_H$$  \hspace{1cm} (3.3)

where $T^\circ$ is a reference datum temperature.

3.2.2 Balances

At the steady state, the energy balance and mass balance around the dryer are given by:

$$f_A H_{A0} + f_S H_{S0} = f_A H_{An} + f_S H_{Sn}$$  \hspace{1cm} (3.4)

$$f_A W_{A0} + f_S W_{S0} = f_A W_{An} + f_S W_{Sn}$$  \hspace{1cm} (3.5)

assuming that the airflow and the sugar flow remain constant along the dryer.

From (3.4), $H_{An}$ can be found, and using (3.3), $W_{An}$ can be obtained. Then:

$$\Delta W_S = \frac{f_A}{f_S} (W_{An} - W_{A0})$$  \hspace{1cm} (3.6)

Following this, a reverse calculation is done to find the sugar moisture at equilibrium with the moisture at the exit in the air. These inverse calculations are based on the equations from Tait et al. (1994).

From $W_{An}$, the moisture mole fraction $y_A$ is calculated:

$$y_A = \frac{29W_{An}}{18 + 11W_{An}}$$  \hspace{1cm} (3.7)

The vapour pressure [Pa] of pure water at temperature $T$ [$^\circ$C] is given from Tait et al. (1994) by:

$$p(T) = 1367,6 - 132,54T + 9,635T^2 - 0,115T^3 + 0,00132T^4$$  \hspace{1cm} (3.8)
The sugar is at temperature $T_s$ but the moisture on the surface of the crystals does not exert a partial pressure $p(T_s)$ for two reasons:

(i) The elevation of boiling point due to the dissolved solids clearly impacts on the vapour-pressure/temperature relationship, and this may be compensated for (following Tait et al., 1994) by scaling the vapour pressure of the water at the operating temperature using the ratio of the pure vapour pressures expected at 100°C to that at pure water at the actual boiling point of the solution. (See (3.9))

(ii) As the solution becomes more saturated, there is simply less and less water present, and a Henry's Law effect is expected in which the vapour pressure will drop in proportion to the "concentration" of water present. Firstly, note the variables:

$W_s$: kg water in film per kg of dry sugar crystal  
$S_F$: kg sucrose dissolved in film per kg of dry sugar crystal

In the present approach to define an isotherm it is assumed that the crystallization from the film onto the crystal surface is a slow process, so $S_F$ is taken as a constant, and different amounts of water associated with it are considered. There will be a particular $W_s$ which the given $S_F$ just saturates ($W_{SAT}$). As further water evaporates, lower values of $W_s$ arise representing various degrees of supersaturation for the fixed $S_F$. Following Tait et al. (1994), from this a further scaled reduction in the exerted water vapour pressure in the ratio of $W_s/W_{SAT}$ is inferred.

Applying the product of the ratios suggested by (i) and (ii) above, the following expression is obtained for the water vapour pressure exerted by the crystal film at a given temperature $T$:

$$p_s(T) = p(T) \cdot \frac{p(100)}{p(T_{bp})} \cdot \frac{W_s(S_F)}{W_{SAT}(S_F)}$$  \hspace{1cm} (3.9)

Following Tait et al. (1994), the boiling point at 1 atm can be predicted by:

$$T_{bp} = 100 + 2 \frac{100 \cdot S_F}{W_s(S_F) \cdot pur}$$  \hspace{1cm} (3.10)

Let

$$\beta = \frac{100 \cdot S_F}{W_s(S_F) \cdot pur}$$  \hspace{1cm} (3.11)
So

\[ p_s(T) = p(T) \cdot \frac{p(100)}{p(100 + 2\beta)} \cdot \frac{100}{\text{pur}} \cdot \frac{S_F}{W_{\text{SAT}}(S_F)} \]  \hspace{1cm} (3.12)

Now it is noted that the partial pressure of water in the air surrounding the drying crystal is:

\[ p_A = y_A \cdot p(100) \]  \hspace{1cm} (3.13)

where \( p(100) \) is simply used to represent the atmospheric pressure. An equilibrium should be reached (no further drying) when:

\[ p_A = p_s \]  \hspace{1cm} (3.14)

Looking for this equilibrium at a given sugar temperature \( T_s \) there is:

\[ y_A = p(T_s) \cdot \frac{1}{\beta \cdot p(100 + 2\beta)} \cdot \frac{100}{\text{pur}} \cdot \frac{S_F}{W_{\text{SAT}}(S_F)} \]

\[ \beta \cdot p(100 + 2\beta) = \frac{100 \cdot p(T_s)}{y_A \cdot \text{pur} \cdot W_{\text{SAT}}(S_F)} \cdot \left( \frac{S_F}{W_{\text{SAT}}(S_F)} \right)_{T_s} \]  \hspace{1cm} (3.15)

It is possible to evaluate the right-hand-side (RHS) of this equation for a given \( y_A, T_s \) and \( \text{pur} \) as follows: the solubility of pure sucrose at temperature \( T_s \) is given by:

\[ SOL = 64,407 + 0,07251 T_s + 0,0000569 T_s^2 - 9,035 \cdot 10^{-4} T_s^3 \]  \hspace{1cm} (3.16)

The solubility coefficient (to correct for impurities in the film) is given by:

\[ SC = 1,0 - 0,088 \frac{I}{W_s} \]  \hspace{1cm} (3.17)

For a given \( S_F \) associated with this amount of water \( W_s \), this correction for impurities is used to calculate the supersaturation as:
CHAPTER 3

\[ SS = \frac{S_F}{W_S(S_F)} \left( \frac{100 - SOL}{SOL \cdot SC} \right) \]  

(3.18)

Noting that

\[ \frac{I}{W_S} = \left( \frac{100}{\text{pur}} - 1 \right) \frac{S_F}{W_S(S_F)} \]  

(3.19)

it follows that,

\[ SC = 1 - \alpha \left( \frac{S_F}{W_S(S_F)} \right) \quad \text{with} \quad \alpha = 0.088 \left( \frac{100}{\text{pur}} - 1 \right) \]  

(3.20)

For a solution which is just super-saturated to the point at which crystallisation begins \((SS=1+K_0)\) with \(K_0 = 0.0046\) according to Tait et al., 1994). It follows that:

\[ \left( \frac{S_F}{W_S(S_F)} \right)_{\text{SAT}} = \frac{(1+K_0)SOL}{(100-SOL)+(1+K_0)\alpha} \quad \text{with} \quad SOL = SOL(T_S), \quad \alpha = \alpha(\text{pur}) \]  

(3.21)

Thus, for a given \(y_{\alpha}, T_S\), and \(\text{pur}\) the right-hand-side (RHS) of equation (3.15) can be evaluated. The left-hand-side (LHS) can be inverted to obtain the appropriate value of \(\beta\) by plotting \(\beta\) vs RHS, and fitting a curve to this plot to obtain:

\[ \beta = -2 \cdot 10^{-12} (\text{RHS})^2 + 8 \cdot 10^{-6} (\text{RHS}) + 0.0962 \]  

(3.22)

allowing evaluation of \(\frac{S_F}{W_S(S_F)}\) from equation (3.11).

Finally, the equilibrium moisture content of the drying sugar is obtained, for the given \(y_{\alpha}, T_S\), and \(\text{pur}\) by

\[ W_{S, \text{equilibrium}} = \frac{(S_F)_{\text{FEED}}}{\left( \frac{S_F}{W_S(S_F)} \right)} \]  

(3.23)

The case for \(\frac{W_S(S_F)}{W_{SAT}(S_F)} > 1\) is similar, except that this ratio does not appear in equation (3.9) (i.e. it is replaced by 1). This results instead of equation (3.22) in
\[ \beta = \text{MAX}(0, -4 \cdot 10^{-10} \text{ (RHS)}^2 + 2 \cdot 10^4 \text{ (RHS)} - 17.171) \quad (3.24) \]

Which of these to use is not known \textit{a priori}, so a test is necessary:

\[
\text{If } \frac{\text{pur}}{100} \{\beta\}_{\text{FROM (3.22)}} > \left( \frac{S_f}{W_s(S_f)} \right)_{\text{SAT}}
\]

\[
\text{then } \left( \frac{S_f}{W_s(S_f)} \right) = \frac{\text{pur} \{\beta\}_{\text{FROM (3.22)}}}{100}
\]

\[
\text{else } \left( \frac{S_f}{W_s(S_f)} \right) = \frac{\text{pur} \{\beta\}_{\text{FROM (3.22)}}}{100}
\quad (3.25)
\]

The feed moisture content \( W_{50} \) is approximately 0,01kg.kg\(^{-1}\). The initial \( S_f \) is estimated as close to saturation. It is noted that if the drying proceeds to equilibrium:

\[ W_{50} = W_{5n\text{equilibrium}} + \Delta W_s \quad (3.26) \]

![Figure 3-1: Profiles of sugar moisture content when \( W_{5n} \) is fixed at equilibrium](image)

The results from the procedure up to equation (3.26) are shown in Fig. 3-3. To verify, the calculations are done forward to eventually compare the \( y_a \). The purpose of the above exercise is as follows: what is known with measurable accuracy is \( \Delta W_s \) (from the mass and energy balance (3.4) and (3.5)). However, the feed sugar moisture content is not continuously monitored. Thus a limiting case of equilibrium at the dryer exit has been considered to determine if this implies a consistent feed moisture in (3.26). Subsequently it has become
clearer in the actual dryer modelling that equilibrium is not generally matched, due to the slow progress of the sugar in the dryer providing enough time for crystallization from the sugar moisture. Thus the current strategy is based on a "more constant $W_{so}$". This is not a serious limitation for estimation of the transfer coefficients, since these are predominantly related to $\Delta W_s$ away from equilibrium.

### 3.2.3 Transformation to obtain suitable $W_{so}$

Bear in mind, that there is some confidence in the estimate of $\Delta W_s$. If $W_{sn}$ is fixed at the equilibrium result of section (3.2.2), an estimation of $W_{so}$ is obtained from:

$$W_{so} = W_{sn} + \Delta W_s$$  \hspace{1cm} (3.27)

This produces an unlikely plot where $W_{so}$ frequently falls below 0.1% moisture, and it is expected that $W_{so}$ should be a little above this. The explanation is that the transfer is rate-limited, and that equilibrium has not been achieved. A method is required to "lift" such $W_{sn}$ values above equilibrium to restore a more constant $W_{so}$. Eq. (3.26) can be expected to hold at the lowest sugar rates measured -- $f_s = 7$ kg.s$^{-1}$. This implies that the feed the sugar has about 1.0% moisture ($W_{so} = 0.010$ kg.kg$^{-1}$). Setting $W_{so\, fixed} = 0.010$, an error is observed:

$$Error = W_{so\, fixed} - W_{so}$$  \hspace{1cm} (3.28)

with $W_{so}$ obtained as in (3.26). This error was fitted as:

$$Error = 0.000044f_s^2 - 0.000556f_s + 0.001711$$  \hspace{1cm} (3.29)

Now this "smooth" error was used to lift $W_{sn}$ above equilibrium, thus resulting in a more constant $W_{so}$:

$$W_{sn} = W_{sn\, equilibrium} + Error$$  \hspace{1cm} (3.30)

and obviously:

$$W_{so\, corrected} = W_{sn} + \Delta W_s$$  \hspace{1cm} (3.31)

Finally, the new estimates of $W_{so}$ and $W_{sn}$ are subjected to the following double filter ($0 < \chi < 1$, $0 < \gamma < 1$) to distribute the process variations more or less equally, at the same time maintaining the exact calculated gap $\Delta W_s$ throughout, which varies in time as the mass and energy balance
varies. Since most of the time there is no equilibrium, $\Delta W_s$ is the key result implying the heat and mass transfer coefficients at any instant.

$$W_{Sn} = \gamma \text{Max} \left[ W_{S0\text{average}} - \left( Sm\Delta W_s + \chi (\Delta W_s - Sm\Delta W_s) \right), 0 \right]$$

$$W_{S0} = W_{S0\text{average}} + \gamma \text{Max} \left[ Sm\Delta W_s + \chi (\Delta W_s - Sm\Delta W_s) - W_{S0\text{average}}, 0 \right]$$

$$+ (1 - \chi)(\Delta W_s - Sm\Delta W_s)$$

$$+ (1 - \gamma)(Sm\Delta W_s + \chi (\Delta W_s - Sm\Delta W_s) + W_{S0\text{average}})$$

$W_{S0\text{average}}$ is the average of $W_{S0}$ on the whole set of data. The value is set at $0.010 \text{ kg.kg}^{-1}$. $Sm\Delta W_s$ is an average value of the $\Delta W_s$. Fig. 3-4 illustrates the result of the whole procedure of reconciliation.

![Graph](image)

**Figure 3-1: Profiles of reconciled sugar moisture content**

3.2.4 Final reconciled data set

From all of these calculations, it is possible to establish a consistent data set for the model.

The data coming into the model are: $W_{S0}, W_{A0}, T_{S0}, T_{A0}, S_{Fa}, f_s, f_A$.

The data coming out are: $W_{Sn}, W_{An}, T_{Sn}, T_{An}, S_{Fa}$.

The following graphs represent for each trial:

- The air and sugar temperature versus time
- The air and sugar moisture versus time
- The air to sugar ratio and sugar flow versus time

Data were collected on four different days. One day is split into two parts because the set of data was too long.
Figure 3-1: Data set from the 29 August 2000

[Graph showing data over time for various variables such as temperature, moisture, and air to sugar ratio]
Figure 3-2: Data set from the 16 October 2000
Figure 3-3: Data set from the 18 October 2000 (part I)
Figure 3-4: Data set from the 18 October 2000 (part II)
Figure 3-5: Data from the 13 December 2000
3.2.5 Discussion

3.2.5.1 Temperature

The temperatures are direct measurements from the plant. No reconciliation treatment was carried out on them. The average values are:

<table>
<thead>
<tr>
<th>Trial Value (°C)</th>
<th>Trial 29/08/00</th>
<th>Trial 16/10/00</th>
<th>Trial 18/10/00 (I)</th>
<th>Trial 18/10/00 (II)</th>
<th>Trial 13/12/00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{Ao}$</td>
<td>81.3</td>
<td>82.4</td>
<td>85.4</td>
<td>94.0</td>
<td>75.9</td>
</tr>
<tr>
<td>$T_{So}$</td>
<td>57.6</td>
<td>52.7</td>
<td>60.5</td>
<td>59.5</td>
<td>60.2</td>
</tr>
<tr>
<td>$T_{An}$</td>
<td>50.9</td>
<td>46.8</td>
<td>51.1</td>
<td>48.8</td>
<td>50.5</td>
</tr>
<tr>
<td>$T_{Sn}$</td>
<td>52.9</td>
<td>48.8</td>
<td>55.8</td>
<td>54.6</td>
<td>52.0</td>
</tr>
</tbody>
</table>

The behaviour of the air and sugar temperatures at the exit of the sugar dryer is interesting. The two curves are quite parallel. They join each other when the sugar flow is low and there is otherwise a temperature gap of a few degrees Celsius when the sugar flow is higher. Clearly when the sugar flow is low, an equilibrium is reached before the end of the dryer between the air and the sugar. Besides it is worth noticing that most of the time the sugar outlet temperature is higher than the air temperature. Firstly, it was thought that the calibration of the sensor might have failed but the agreement between these two temperatures at low sugar rates confirmed the similar behaviour of the sensor. In the drying process, energy is required to evaporate the water from the film around the crystal. The hot air provides this energy. The following two temperature profiles (Fig. 3-10) could explain why the sugar temperature is higher than the air temperature.

The sugar temperature is raised by the heated air at the start of drying. So the air temperature drops. The sugar temperature decreases more slowly so a gap of temperature can occur between the sugar and the air. This gap appears if the sugar flow rate is high. Sugar and air have not spent enough time in the dryer to reach a thermal equilibrium. Even if there is an apparent moisture equilibrium, the sugar temperature is always a little bit higher. A possible explanation is that a gradient of temperature occurs across the crystal, with the surface being slightly cooler. Once it lands in the exit hopper, where temperature is measured, there is no further surface evaporation, so a more uniform higher temperature is conducted through the hopper.
Figure 3-1: Hypothetical temperature profiles along the dryer

This behaviour in the temperatures profiles was also observed by Ben-Yoseph et al. (2000). They suggested that the heat generated by the crystallisation occurring inside the dryer was large enough to raise the crystal temperature.

3.2.5.2 Moisture

Only the moisture of the feed air is independently available (wet and dry bulb temperature). The other moisture contents are obtained from the data reconciliation.

<table>
<thead>
<tr>
<th>Table 3-1: Average moisture values from the trials</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Value (kg·kg⁻¹)</strong></td>
</tr>
<tr>
<td>W_{AD}</td>
</tr>
<tr>
<td>W_{SO}</td>
</tr>
<tr>
<td>W_{An}</td>
</tr>
<tr>
<td>W_{Sn}</td>
</tr>
</tbody>
</table>
$W_{Sn}$ varies between 0 and 0.003. In some trials [16/10/00, 18/10/00 (II), 13/12/00], $W_{Sn}$ reaches as low as zero. This is similar to what Savarese et al. (2001) found in their work. This phenomenon, occasionally occurring when the sugar rate is low, appears to be an "over-dried" mode.
CHAPTER 4
MODEL

4.1 MODEL OF THE DRYER

4.1.1 Theory

The model is based on the usual equations for convective heat transfer and mass transfer between the liquid around the sugar crystal and the stream of air. However, as the water is evaporated and the sugar cools, the liquid film becomes supersaturated and sucrose may crystallize, leaving the film with a lower purity. The rate of this crystallization is determined according to Love (2001) and Tait et al. (1994) who used equations developed by Broadfoot et al. (1980). For convenience, the relevant nomenclature is repeated in Table 4-1.

| $a$ | Interfacial area ($m^2.m^3$) |
| $A$ | Sugar dryer section ($m^2$) |
| $C_{ph}$ | Heat capacity for air ($kJ.kg^{-1}.C^{-1}$) |
| $C_{ps}$ | Heat capacity for sugar ($kJ.kg^{-1}.C^{-1}$) |
| $C_{pvp}$ | Heat capacity for vapor ($kJ.kg^{-1}.C^{-1}$) |
| $D_A$ | Diffusivity coefficient for air ($m.s^{-1}$) |
| $D_S$ | Diffusivity coefficient for sugar ($m.s^{-1}$) |
| $f_A$ | Air flow ($kg.s^{-1}$) |
| $f_S$ | Sugar flow ($kg.s^{-1}$) |
| $G$ | Crystallisation growth rate ($m.s^{-1}$) |
| $h_S$ | Sugar hold up volume fraction ($kg.kg^{-1}$) |
| $h_t$ | Heat transfer coefficient ($kW.m^{-2}.K^{-1}$) |

Table 4-1: Relevant nomenclature for dryer model

The equations for dryer modelling are:

Water mass balance in the sugar, i.e. mass moisture in the sugar per unit mass of sugar:

$$\frac{\partial W_s}{\partial t} = -k_s \cdot \frac{a}{\rho_s} \cdot (p_s - p_A) + \frac{f_S}{A h_S \rho_s} \cdot \frac{\partial W_s}{\partial x} + D_s \cdot \frac{\partial^2 W_s}{\partial x^2} \tag{4.1}$$

Water mass balance in the air, i.e. mass moisture in the air per unit of mass air:

$$\frac{\partial W_A}{\partial t} = f_A - f_S - D_A \cdot \frac{\partial W_A}{\partial x} \tag{4.1}$$
\[
\frac{\partial W_A}{\partial t} = +k_g \cdot \frac{a}{\rho_A} \cdot \frac{h_s}{(1-h_s)} \cdot (p_s - p_A) + \frac{f_a}{A(1-h_s) \rho_A} \cdot \frac{\partial W_A}{\partial x} + D_A \cdot \frac{\partial^2 W_A}{\partial x^2} \tag{4.2}
\]

Heat balance in the sugar:

\[
\frac{\partial T_s}{\partial t} = -k_g \cdot \frac{a \lambda}{\rho_s C_p} \cdot (p_s - p_A) + h_i \cdot \frac{a}{\rho_s C_p} \cdot (T_A - T_s) + \frac{f_s C_p s}{A h_s \rho_s C_p} \cdot \frac{\partial T_s}{\partial x} + D_s \cdot \frac{\partial^2 T_s}{\partial x^2} \tag{4.3}
\]

Heat balance in the air:

\[
\frac{\partial T_A}{\partial t} = +k_g \cdot \frac{a LH}{\rho_A C_p} \cdot \frac{h_s}{(1-h_s)} \cdot (p_s - p_A) - h_i \cdot \frac{a}{\rho_A C_p} \cdot \frac{h_s}{(1-h_s)} \cdot (T_A - T_s)
\]

\[
+ \frac{f_a C_p_A}{A \rho_A C_p} \cdot \frac{1}{(1-h_s)} \cdot \frac{\partial T_A}{\partial x} + D_A \cdot \frac{\partial^2 T_A}{\partial x^2} \tag{4.4}
\]

Concentration of sugar in the film, i.e. mass dissolved sugar per unit mass of dry sugar:

\[
\frac{\partial S_F}{\partial t} = -G a + \frac{f_s}{A h_s \rho_s} \cdot \frac{\partial S_F}{\partial x} + D_s \cdot \frac{\partial^2 S_F}{\partial x^2} \tag{4.5}
\]

These five equations have two unknowns: \( h_i \) and \( k_g \), respectively the heat transfer coefficient and the mass transfer coefficient.

4.1.2 The model

The dryer is simulated using a multi-compartment model which is solved over a time sequence using the computer language MATLAB®. The dryer is divided into ten compartments (all of the same size). Certain assumptions are made:

- The air and sugar temperature at the compartment exit represent their respective temperatures throughout the compartment.
- The air and sugar flow are supposed constant in the compartment.
- The heat loss through the cylindrical shell is neglected. Although the estimated loss could be as high as 10%, the drying is treated as an adiabatic process (See Appendix A).
4.1.2.1 Partial pressure drying force from moisture content

To simplify the calculations, the partial pressure of the air moisture $p_A$ and the partial pressure of the sugar moisture $p_S$ should be related to the respective moisture contents on each step. Two parameters are created to give a value to $p_A$ and $p_S$, respectively $\alpha$ and $\beta$:

\[
p_A = \xi W_A \tag{4.6}
\]
\[
p_S = \psi W_S \tag{4.7}
\]

$p_A$ is evaluated as in section 3.2.2 from Tait et al. (1994). Rearranging,

\[
p_A = 101325 \frac{W_A}{W_A + \frac{1}{18} + \frac{1}{29}} \tag{4.8}
\]

$p_S$, the partial pressure expected by the super saturated film, is estimated as in section 3.2.2 from Tait et al. (1994) by:

\[
p_S = \frac{p(T_S) p(100)}{p(T_{sp})} \frac{W_A}{W_{sat}} \tag{4.9}
\]

The boiling point temperature is calculated from the following formula suggested by Tait et al. (1994):
\[ T_{bp} = 100 + 2 \cdot \frac{\%\text{Dry}}{100 - \%\text{Dry}} \]  

(4.10)

where:

\[ \%\text{Dry} = 100 \cdot \frac{S_f + I}{S_f + I + W_s} \]  

(4.11)

\( p(T_s), \ p(100), \ p(T_{bp}) \) are calculated from the equation (3.8).

\( \xi \) and \( \psi \) need to be re-evaluated in every compartment after each iteration.

### 4.1.2.2 Sugar hold up volume fraction \( h_s \)

\( h_s \) is a parameter defined as

\[ h_s = \frac{\text{Volume of Sugar}}{\text{Volume of Air + Volume of Sugar}} \]  

(4.12)

On average the sugar flow rate is around 10 kg.s\(^{-1}\), and the residence time 5 minutes. A 5 minute residence time corresponds to a hold-up of 3 tons of sugar within the dryer. This has a volume of 2 m\(^3\). The volume of the dryer is around 30 m\(^3\).

So, \( h_s \) is set at 0.0625.

### 4.1.2.3 Crystallization

To complete the model equations, the crystallisation rate \( G \) in equation (4.5) is required. It is assumed that even if the sugar has left the crystallisation pan, and has gone through the centrifugation step, this crystallisation will continue, allowing the moisture partial pressure to increase. \( S_f \) is a parameter defined as the mass of dissolved sucrose in the moisture film per unit mass of sugar.

The calculation of the growth rate is done following Love (2001).

\[ G = K_1 \left( SS - (1 + K_0) \right) \exp \left( K_2 - K_3 \frac{I}{W_s} \right) \]  

(4.13)

The parameters have been set according to the results of Love (2001)

- \( K_0 = 0.005 \)
- \( K_1 = 0.000 \ 000 \ 87 \)
- \( K_2 = \frac{E_u}{8.314 \cdot 10^{-3}} \left( \frac{1}{273,16 + T_s} - \frac{1}{333,16} \right) \)  

(4.14)

- \( K_3 = 1.75 \)
Love (2001) defines $E_a$ as:

$$E_a = 62.86 - 0.84(T_s - 60) \quad (4.15)$$

### 4.1.2.4 Air flow

With a view to including both co-current and counter-current, and more complex arrangements, a facility is provided to specify inflows and outflows of air at every compartment.

**Figure 4-1: Illustration of the airflow design in the sugar dryer**

$f_{AI}$ is for the external air flow coming into each compartment. So the air flow supplied at the entrance with the sugar is $f_{AI}$. $f_{AXi}$ represents the airflow going out of the sugar dryer at the compartment $i$. So the main airflow going out in the modelled case is $f_{AXN}$.

Air mass balances for the sugar dryer are thus represented:

$$
\begin{align*}
  f_{A1} &= 0 + f_{AF1} - f_{AX1} \\
  f_{A2} &= f_{A1} + f_{AF2} - f_{AX2} \\
  f_{A3} &= f_{A2} + f_{AF3} - f_{AX3} \\
  \vdots \\
  0 &= f_{AN-1} + f_{AFN} - f_{AXN}
\end{align*}
$$

This can be re-arranged to give:
Thus,

\[
\begin{pmatrix}
    f_{A1} \\
    f_{A2} \\
    f_{A3} \\
    \vdots \\
    f_{AXN}
\end{pmatrix}
= 
\begin{pmatrix}
    0 & 0 & 0 & \cdots & 0 \\
    1 & 0 & 0 & \cdots & 0 \\
    0 & 1 & 0 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
    f_{AI} \\
    f_{A2} \\
    f_{A3} \\
    \vdots \\
    f_{AXN}
\end{pmatrix}
+ 
\begin{pmatrix}
    1 & -1 & 0 & 0 & 0 & \cdots & 0 \\
    0 & 0 & 1 & -1 & 0 & \cdots & 0 \\
    0 & 0 & 0 & 0 & 1 & -1 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    f_{AF1} \\
    f_{AF2} \\
    f_{AF3} \\
    \vdots \\
    f_{AFN}
\end{pmatrix}
\]

In the case of this study, the extra air feeds and off-takes are not used.

4.1.2.5 Discretisation of spatial derivatives

The whole model is constituted by different equations, mostly non linear. As the model is written in the MATLAB® language, it is easier to transform all the equations into a discrete form following these principles:

\[
\begin{align*}
\frac{\partial^2 C}{\partial x^2} &= \frac{C_{i+\Delta x} - 2C_i + C_{i-\Delta x}}{\Delta x^2} = \frac{C_{i+1} - 2C_i + C_{i-1}}{\Delta x^2} \\
\frac{\partial C}{\partial x} &= \frac{C_i - C_{i+\Delta x}}{\Delta x} = \frac{C_i - C_{i+1}}{\Delta x}
\end{align*}
\]

All of the discretised equation coefficients can be regrouped in one matrix called the \([CM]\) matrix. See appendix B.

The model can be represented by the single system of equations:

\[
\frac{d\bar{X}}{dt} = \bar{A}\bar{X} + B\bar{U}
\]
The $[A]$ and $[B]$ matrix are built from elements of the $[CM]$ matrix.

$\vec{U}$ is a vector that contains all of the exogenous inputs (feed: flow and conditions).

### 4.2 AXIAL DISPERSION

#### 4.2.1 Theory

In this section, a theoretical description of the axial dispersion is developed for comparison with modelled dispersion in section 4.2.2. An instantaneous unit mass point source expanding from $x = 0$ at $t = 0$, with Fick’s diffusion

$$flux = -D_a \frac{\partial C}{\partial x} \quad (4.19)$$

can be shown (Seinfeld, 1975) to have the 1-dimensional "puff" distribution:

$$C(x,t) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ \frac{-x^2}{2\sigma^2} \right\} \quad \text{with} \quad \sigma = \sqrt{2D_a t} \quad (4.20)$$

It is easily shown that the total mass of this distribution is:

$$M = \int_{-\infty}^{\infty} C(x,t) \, dx = 1 \quad (4.21)$$

Displacing the "puff" at velocity $v$ will clearly result in:

$$C(x,t) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ \frac{-(x-vt)^2}{2\sigma^2} \right\} \quad (4.22)$$
If the distribution is integrated along a duct, the integral results from a concentration step at the duct entrance from 1 to 0 at $t = 0$. The following curve in Fig. 4-3 is obtained:

\begin{align*}
C(x,t) &= 1 - \int_{-\infty}^{x} \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x' - vt)^2}{2\sigma^2} \right) dx' \\
C(x,t) &= 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{x} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x' - vt)^2}{2\sigma^2}} dx' \\
&= 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{x} e^{-\frac{(x' - vt)^2}{2\sigma^2}} dx' \\
&= 1 - \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-y^2} dy
\end{align*}

Figure 4-1: Axial dispersion in a duct

The formula for the incoming step $(t=0)$, from $C(0,0)=0$ to $C(0,0)=1$, is found by integrating as follows:

Let $y = \frac{x' - vt}{\sigma \sqrt{2}}$, so $dy = \frac{1}{\sigma \sqrt{2}} dx'$. And

\begin{align*}
C(x,t) &= 1 - \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(x' - vt)^2}{2\sigma^2}} dx' \\
&= 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{x} e^{-\frac{(x' - vt)^2}{2\sigma^2}} dx' \\
&= 1 - \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-y^2} dy
\end{align*}

Now, $\frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-y^2} dy = \text{erf}(x)$ and $\text{erf}(\infty) = 1$

$\frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-y^2} dy = \text{erfc}(x)$ and $\text{erfc}(\infty) = 0$

So, for $x \leq vt$,  

4-8
\[ C(x,t) = 1 - \frac{1}{2} \text{erfc} \left( \frac{vt - x}{\sigma \sqrt{2}} \right) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{vt - x}{\sigma \sqrt{2}} \right) \right] \] (4.24)

And, \( x \geq vt \),

\[ C(x,t) = \frac{1}{2} \left[ 1 - \text{erf} \left( \frac{x - vt}{\sigma \sqrt{2}} \right) \right] = \frac{1}{2} \text{erfc} \left( \frac{x - vt}{\sigma \sqrt{2}} \right) \] (4.25)

4.2.2 Comparison of theoretical and modelled axial dispersion

The first step when the model was created in MATLAB® was to ensure that the axial dispersion was well predicted. Several runs were done with the following profiles:

- The heat and mass transfer coefficients were set to zero.
- Air and sugar steps were considered separately.
- An increase of temperature occurs during the modelled period time: step of 50°C.

It was chosen to work with this temperature for convenience, but it was equally possible to deal with the moisture in the sugar or the air.

The modelled data captured as a result were analysed to be sure that the axial dispersion modelled matched the theory. The theory was represented as follows:

If \( x \leq f_i \cdot t \), \( T = T_{\text{min}} + (T_{\text{max}} - T_{\text{min}}) \cdot 0.5 \cdot \left( 1 + \text{erf} \left( \frac{f_i \cdot t - x}{\sigma \sqrt{2}} \right) \right) \) (4.26)

If \( x \geq f_i \cdot t \), \( T = T_{\text{max}} + (T_{\text{max}} - T_{\text{min}}) \cdot 0.5 \cdot \left( 1 - \text{erf} \left( \frac{x - f_i \cdot t}{\sigma \sqrt{2}} \right) \right) \) (4.27)

\( f_i \) can be either the sugar flow or the air flow.

4.2.2.1 Improved convection

Early tests suggested that the convection and diffusion aspects of the model should be separated, to reduce “numerical diffusion”. Indeed, if the discretisation of the model is shown on a grid analysis (Fig. 4-4):
Figure 4-1: Grid analysis for the discrete model

On the y-axis the temperatures are on a scale that can represent the temperature inside the dryer. The x-axis represents the length along the dryer but predictions are only done at multiples of $\Delta x$. The "movement" inside the dryer is not smooth as in reality but a succession of small jumps. When a change in temperature occurs, the temperature step moves along the dryer as in Fig. 4-5:

Figure 4-2: Increase of temperature

This situation cannot be represented in the grid analysis. The movement must agree with the predefined crossing points to maintain accuracy. So a new $[CC]$ matrix was built similar to the $[CM]$ matrix containing only the convective coefficients. The integration process of (4.18) was split into two steps. Firstly, an integration of the Euler type is done for the convective aspect of the model only. This needs two new matrices $[AC]$ and $[BC]$.

$$\bar{X}_{i+1} = \bar{X}_i + dt \cdot (AC \cdot \bar{X}_i + BC \cdot \bar{U}_i)$$ (4.28)
Then the integration represented (4.18) can be done, using the new $\bar{X}$. A “stop-start” convection mode was used such that the total distance travelled was represented by the nearest whole $\Delta x$ step. This “operator splitting” gave only a small ripple on the model output.

4.2.2.2 Results

In order to validate the model, the theoretical diffusion profile has been developed for comparison with the numerical solution of diffusion by the model. One example of the test is shown in the following figure. A step of +50°C was chosen to increase the air temperature from 40°C to 90°C. These graphs show the front of the step-change moving forward through the dryer. Initially, the temperature inside the dryer is 40°C, and this rises to 90°C as the step passes through. Different values for $D_A$ were tried from $10^{-2}$ to $10^{-7}$ m$^2$.s$^{-1}$, the results are shown in Fig. 4-6. Similar results are observed with $D_S$. (Note, for the model of the actual process, $D_A = D_S = 10^{-6}$ m$^2$.s$^{-1}$ was used).
As the value of $D_A$ decreases, the model plot tends to join the theoretical plot. The theoretical axial dispersion prediction at low axial diffusivity, $D_A < 10^{-3} \text{ m}^2\text{s}^{-1}$, gives acceptable agreement. The modelled air temperature along the dryer follows the forced temperature step. When $D_A \geq 10^{-3} \text{ m}^2\text{s}^{-1}$, significant deviations are observed - especially for high values of axial diffusivity, the prediction tends to diverge. Similar results were obtained with the sugar for the temperature and the moisture content.
4.3 MOISTURE CONTENT ISOTHERM

Sugar crystals left for a while in contact with air will achieve a state of equilibrium. The moisture content in the film surrounding the sugar crystal is in equilibrium with the moisture content in the air. The model equations should be able to recreate this phenomenon.

Two situations are relevant. Firstly, the sugar crystal is considered at a temperature $T_0$, where it is in equilibrium with the surrounding environment. This situation can represent an initial equilibrium state. Secondly, the sugar crystal is moved to a temperature $T_1$ that is higher than $T_0$. The water contained in the film is going to leave it to reach a new equilibrium. As the crystallisation has a slight effect, it is neglected in the following analysis.

The sorption isotherm of white sugar for different temperatures from Schindler and Juncker (1993) has been used as reference. (Fig. 4-7)

![Figure 4-1: Sorption isotherms of white sugar for different temperatures (Schindler and Juncker, 1993)](image)

The model equations proposed by Tait et al. (1994) are solved inversely in an EXCEL spreadsheet to simulate an equivalent isotherm.
Before entering the sugar dryer, parameters are fixed to calculate the mass percent of water in the sugar feed.

Before entering the sugar dryer:

\[ M_{Total} = M_{Crystal} + \left( M_{Sucrose} + M_{Water} + M_{Impurity} \right)_{in\ the\ film} \]  

(4.29)

Where \( M_{Sucrose} \) refers to the dissolved sucrose in the film. The parameters fixed are: \( M_{Crystal} \), \( M_{Sucroses} \), \( T_{30h} \), and \( pur \).

\[ pur = 100 \cdot \frac{M_{Sucrose}}{M_{Sucrose} + M_{Impurity}} \]  

(4.30)

From (4.30), \( M_{Impurity} \) is calculated.

It is assumed that the water in the film at the feed \( (M_{Waterf}) \) is supersaturated to the point of crystallisation. From (3.16), (3.20), and 4.1.2.3,

\[ M_{Waterf} = M_{Sucrose} \frac{100 - SOL + (1 + K_o) \cdot \alpha \cdot SOL}{(1 + K_o) \cdot SOL} \]  

(4.31)

The mass percentage of water in the sugar can be calculated:

\[ \% \text{Water} = 100 \cdot \frac{M_{Waterf}}{M_{Waterf} + M_{Crystal} + M_{Sucrose} + M_{Impurity}} \]  

(4.32)

Inside the dryer in the present isotherm analysis, it is assumed that there is no crystallisation. The water content of sugar is fixed at different points lower than \( \% \text{Water} \) to determine the isotherm. The calculation process is then similar to the one in paragraph 4.1.2.1 to find \( \rho_s \). The equilibrium is reached when \( \rho_s = \rho_A \). From there the relative humidity is found.

With the Tait et al. (1994) equations used to determine the vapour pressure of the crystal film, it was resolved to run the main simulation model to equilibrium to check whether it was able to simulate the Schindler and Juncker (1993) isotherm. The inherent difficulty of this is that it is very dependent on the original amount of dissolved sucrose in the crystal film, as supplied to the dryer. Ultimately this sucrose, or the portion left after partial crystallisation during the drying, determines the reduction of the film vapour pressure, and thus determines the equilibrium air moisture content. Finally, this initial film sucrose content was "tuned" to match a single point \( (W_s = 0.008 \text{ and RH} = 45 \% \text{ with the } 30^\circ\text{C sorption isotherm}) \), giving the complete isotherm in Fig. 4-8.

In a normal dynamic situation there will be a change of moisture content with time until an equilibrium is reached, and that final equilibrium moisture will, on the basis of the theory used, have no dependence on the mass of dry sugar. It is rather dependent on the mass of dissolved sucrose. So the attempts to "tune" \( S_m \) were merely a means to "second-guess" the typical conditions under which the isotherm might have been generated.
Resemblance/Difference:
- The graphs are located in the correct range of values
- The shape is similar: when the water content of sugar tends to go up, the relative humidity increases in a hyperbolic way. The gap between 0.01 and 0.02 for $W_s$ is observed.
- The slope of the curves is not so pronounced for the low range of $W_s$, especially for high temperatures.

The difficulty in fitting exactly the Schindler and Juncker (1993) isotherm remains in that the model depends on too many parameters that are not exploited here. Besides, the sorption isotherms from the literature are for white sugar, while the model is arranged for raw sugar.

The Tait et al. (1994) equations could be solved to give similar behaviour to the Schindler and Juncker (1993) isotherm for equilibrium sugar moisture content. However, the match was found to be strongly dependent on the initial dissolved sucrose in the water film, for which a single value was eventually chosen to fit the entire isotherm. The double equilibrium of zero net crystallisation and zero net evaporation was not achieved, and even if it were, it would be independent of the amount of dry sugar present.
4.4 THE HEAT AND MASS TRANSFER COEFFICIENTS

4.4.1 Results

Using the reconciled data, the model written in the MATLAB® language has been run over the associated time sequences. The results are shown in Fig. 4-9 for the 29/08/2000, Fig. 4-10 for the 16/10/2000, Fig. 4-11 for the 18/10/2000 (part I), Fig. 4-12 for the 18/10/2000 (part II), Fig. 4-13 for the 13/12/2000.

After a few runs of the program, trying to fit the model to the plant data, it appeared that the heat and mass transfer coefficient varied together. For each trial, a best tuning was found. The results are in Table 4-1.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Heat transfer coefficient (kW·m⁻²·K⁻¹)</th>
<th>Mass transfer coefficient (kg water·m⁻²·s⁻¹·Pa⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29/08/2000</td>
<td>0.013</td>
<td>1.3×10⁻⁹</td>
</tr>
<tr>
<td>16/10/2000</td>
<td>0.011</td>
<td>1.1×10⁻⁹</td>
</tr>
<tr>
<td>18/10/2000 (part I)</td>
<td>0.015</td>
<td>1.5×10⁻⁹</td>
</tr>
<tr>
<td>18/10/2000 (part II)</td>
<td>0.015</td>
<td>1.5×10⁻⁹</td>
</tr>
<tr>
<td>13/12/2000</td>
<td>0.014</td>
<td>1.4×10⁻⁹</td>
</tr>
</tbody>
</table>

An average value is determined for the model.

\[
h_t = 0.0136 \text{ kW·m}^{-2} \text{·K}^{-1}
\]
\[
k_g = 1.36 \times 10^{-9} \text{ kg water·m}^{-2} \text{·s}^{-1} \text{·Pa}^{-1}
\]

The ranges of values for the transfer coefficients from the literature are 0.3 to 0.0019 kW·m⁻¹·K⁻¹ for the heat transfer coefficient and 2.7×10⁻⁹ to 72×10⁻⁹ kg·m²·s⁻¹·Pa⁻¹ for the mass transfer coefficient (Table 1-1). The estimated values found from the reconciled data are within the literature heat transfer range, and just below the literature mass transfer range. The sugar quality in terms of origin and location might explain this difference. The indicated average values are used in subsequent modelling.
Figure 4-1: Comparison between data plant and data model for 29 August 2000
Figure 4-2: Comparison between data plant and data model for 16 October 2000
Figure 4-3: Comparison between data plant and data model for 18 October 2000 (I)
Figure 4-4: Comparison between data plant and data model for 18 October 2000 (II)
Figure 4-5: Comparison between data plant and data model for 13 December 2000
4.4.2 Discussion

4.4.2.1 Moisture content

The model response concerning the exit sugar moisture is acceptable in most of the trials. The model manages to remain around the plant value with particularly good matches in Fig. 4-10 and Fig. 4-12. The changes in $W_{Sn}$ are simulated very nicely. The model is able to simulate the "over-dried" mode described by Savarei et al. (2001). The close tracking of $W_{Sn}$ in most cases is largely the result of the reduced pressure differential driving the mass transfer as the equilibrium is approached. Recall that the reconciled data in section 3.2 was arranged to achieve equilibrium when the plant was operating at its lowest sugar rates. This was the only way of fixing the absolute levels of $W_{Sn}$ and $W_{So}$ (though the gap $\Delta W_s$ was more accurately known). Thus the dryer model is simply using the same available sugar moisture until it meets mass transfer resistance near the dryer exit. Figures 3-3 / 4-9, 3-4 / 4-10, 3-5 / 4-11, 3-6 / 4-12 and 3-7 / 4-13 need to be compared in these given pairs. This is the same evidence, as in 3-5 / 4-11, that lower sugar rates $f_s$ often result in better comparisons of measured and modelled $W_{Sn}$. This is clearly because the equilibrium determines the final transferable amount of moisture. Conversely, it is in the periods of higher sugar rates where matches are achieved, that it is hoped there is support for the fitted mass transfer coefficients in Table 4-1.

The air moisture from the model gives a very smooth curve. It follows the plant data quite correctly being able to simulate the changes. The response is better than for the sugar moisture.

4.4.2.2 Temperature

Again, because of the process of data reconciliation, the predicted and measured temperature of the sugar ($T_{Sn}$) and the air ($T_{An}$) follow each other well, with the $T_{Sn}$ measurement and prediction being particularly close. The fact that the modelled exit air temperature $T_{An}$ is usually 2-3 °C higher than the reconciled measurement is related to the modelled $W_{Sn}$ being slightly higher than the measured $W_{Sn}$ in similar periods, arising as in section 4.4.2.1. In the reconciled measurement, the latent heat required to evaporate this residual moisture would bring $T_{An}$ down to its reconciled measurement.

4.4.2.3 Dissolved sucrose $S_{Fn}$

It is noted that the $S_{Fn}$ arising from the model, differs somewhat. This is because the reconciled data were based on a steady-state mass and energy balance, and thus could not recognize the continuing crystallization of sucrose from the film onto the crystal. The initial sucrose content for reconciliation was merely set at saturation, and not supersaturation. It is noted that $W_{Sn}$ and $S_{Fn}$ generally move together in Fig. 4-9 to 4-13, showing that a particular concentration is being
maintained in the crystal film, and this no doubt is the concentration at which the vapour pressure is significantly reduced, restricting the mass transfer. In Fig. 4-11, this prevents $W_{Sn}$ from matching the measurements, but this is not the case in Fig. 4-12.
CHAPTER 5
ADAPTIVE CONTROL

5.1 DYNAMIC MATRIX CONTROL THEORY

5.1.1 Definition

Dynamic Matrix Control is a form of Model Predictive Control (MPC) which uses a step-response convolution model for prediction of the effect of possible control actions. Since the early work of Cutler and Ramaker, (1979) and Garcia and Morshedi (1984), these controllers, particularly DMC, have proved their worth in many industrial applications.

5.1.2 Theory

Consider a 2-input, 2-output system, for example a rotary sugar dryer (Fig. 5-1) in which air temperature ($T_{ao}$) and the flow of water added ($F_w$) cause variations in the sugar exit temperature ($T_{sn}$) and its moisture content ($W_{sn}$). If the system is steady and a step is made in $T_{ao}$, two separate responses for $T_{sn}$ and $W_{sn}$ are expected. Likewise, distinct responses for $T_{sn}$ and $W_{sn}$ for a step in $F_w$ would be expected. This is shown graphically in Fig. 5-2, for unit

Figure 5-1: Rotary dryer with exit moisture and temperature determined by air temperature and water addition
positive steps in $T_{AO}$ and $F_w$. Note that only changes in $T_{Sn}$ and $W_{Sn}$ from their original steady values are considered.

For the input vector $m$ ($T_{AO}$, $F_w$), now consider not just one step but a series of control vector moves $\Delta m_1, \Delta m_2, ..., \Delta m_M$, over a sequence of $M$ time steps. If the system is linear, the resultant sequence in $x (T_{Sn}, W_{Sn})$ over $P$ intervals can be built by shifting, scaling and superposing the above step responses:

\[
\begin{pmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  \vdots \\
  x_M \\
  x_{M+1} \\
  x_P
\end{pmatrix} =
\begin{pmatrix}
  B_1 & 0 & 0 & 0 & 0 & \cdots & 0 \\
  B_2 & B_1 & 0 & 0 & 0 & \cdots & 0 \\
  B_3 & B_2 & B_1 & 0 & 0 & \cdots & 0 \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
  B_M & B_{M-1} & \cdots & B_1 & 0 & \cdots & 0 \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
  B_M & B_M & B_M & B_M & B_M & \cdots & B_1
\end{pmatrix}
\begin{pmatrix}
  \Delta m_1 \\
  \Delta m_2 \\
  \Delta m_3 \\
  \vdots \\
  \Delta m_M \\
  \Delta m_{M+1} \\
  \Delta m_P
\end{pmatrix}
\]

This represents the convolution model for future outputs as $x = B\Delta m$, where the "matrix of matrices" $B$ is generally known as the "Dynamic Matrix". Now defining the $P \times M$ matrices:
and the present measurements \((P)\) and past inputs \((M)\):

\[
x_{\text{OMEAS}} = \begin{pmatrix} x_{\text{OMEAS}} \\ x_{\text{OMEAS}} \\ x_{\text{OMEAS}} \\ \vdots \\ x_{\text{OMEAS}} \end{pmatrix}
\]

and

\[
\Delta m_{\text{PAST}} = \begin{pmatrix} \Delta m_{-M+1} \\ \Delta m_{-M+2} \\ \Delta m_{-M+3} \\ \Delta m_{-M+4} \\ \Delta m_0 \end{pmatrix}
\]

then the “open-loop” response, corrected for present model offset \(x_{\text{OMEAS}} - B_0 \Delta m_{\text{PAST}}\), is

\[
x_{\text{OL}} = x_{\text{OMEAS}} + [B_{\text{OL}} - B_0] \Delta m_{\text{PAST}}
\]

(5.2)

Notice that \(B_{\text{OL}} \Delta m_{\text{PAST}}\) gives the contribution of past control input steps (moves) to the future output, whereas \(B_0 \Delta m_{\text{PAST}}\) merely predicts what the \textit{present} output should be according to the past moves. The “closed loop” response up to the \(P\)-step horizon is obtained by including the contribution of the future control moves \(\Delta m\) :

\[
x_{\text{CL}} = x_{\text{OL}} + B \Delta m
\]

(5.3)

On each time-step it is possible to compute the future open-loop response \(x_{\text{OL}}\) based on past inputs and the present output. Thus the control problem to achieve the desired trajectory \(x_{\text{CL}}\) amounts to finding suitable \(\Delta m\) as in Fig. 5-3.
A constrained multivariable Linear Dynamic Matrix Controller (LDMC), based on the linear programming solution of Chang and Seborg (1983), and the formulation of Morshedi et al. (1985), has been developed as follows.

Define $x_{sp}$ to contain a sequence of set-points for the outputs up to the time horizon $P$ steps ahead, so that the open loop error may be calculated in advance as $e_{OL} = x_{OL} - x_{sp}$. Then the closed-loop error for a control move sequence $\Delta m$ will be, using equation (5.3):

$$e_{CL} = x_{CL} - x_{SP} = e_{OL} + B\Delta m$$

Generally only a limited sequence of $N$ moves ($\Delta m^*$) are optimised ($N << P$). This is equivalent to setting $\Delta m_k = 0$ for $k > N$, or alternately replacing $B$ with the non-square $P \times N$ matrix:
\[
A = \begin{bmatrix}
B_1 & 0 \\
B_2 & B_1 \\
\vdots & \vdots \\
B_M & B_{N-1} & \cdots & B_1 \\
\vdots & \vdots & \ddots & \vdots \\
B_M & B_M & \cdots & B_{M-N+1}
\end{bmatrix}
\]

Then

\[
e_{\text{CL}} = e_{\text{OL}} + A\Delta m^*
\] (5.5)

Now a quadratic objective function \( J \) is defined dependent only on the strategy \( \Delta m^* \) with \( W \) and \( A \) matrices that are usually diagonal \( W = \begin{bmatrix} w_1 & 0 \\
\vdots & \ddots \\
0 & w_M \end{bmatrix} \) and \( A = \begin{bmatrix} \lambda_1 & 0 \\
\vdots & \ddots \\
0 & \lambda_M \end{bmatrix} \).

\[
J(\Delta m^*) = (e_{\text{CL}})^T W (e_{\text{CL}}) + (\Delta m^*)^T A (\Delta m^*)
\]

\[
= (e_{\text{OL}} + A\Delta m^*)^T W (x_{\text{OL}} - x_{\text{SP}} + A\Delta m^*) + (\Delta m^*)^T A (\Delta m^*)
\] (5.6)

By minimising \( J \) with respect to \( \Delta m^* \), it is possible to find an optimal sequence of control moves, \( \Delta m^* \), which achieve minimum square deviation from the set-point trajectory up to the time horizon \( P \), for minimum square control move effort. It is the weights in the matrices \( W \) and \( A \), generally diagonal, which determine the extents to which deviations of either parameter are discouraged. Higher “gains” will generally be associated with higher values in \( W \) than in \( A \). The values in \( A \) cause “move suppression”. It is easily shown that differentiation of \( J \) with respect to the elements of \( \Delta m^* \), and setting the result to the zero vector, yields the unbounded quadratic optimum control move strategy

\[
\Delta m_{UQO} = - [A^TWA + A]^T A^T W e_{\text{OL}}
\] (5.7)

The sequence of actual control settings is clearly obtained by adding the successive moves:

\[
m_{\text{OPT}} = L \Delta m_{UQO} + m_{\text{INIT}}
\] (5.8)
A global method which will seek the minimum of $J$ within defined constraints for both the inputs $m$ and the outputs $x$ requires Quadratic Programming, and is quite computation-intensive. In a less demanding approximation, Linear Dynamic Matrix Control (LDMC), Morshed et al (1985) seek that combination of control moves which will get us as close as possible to $\Delta m_{opt}$, yet keep us within the constraints. This re-definition of the problem then allows us to use Linear Programming to handle the constraints. Although it does not guarantee us the quadratic optimum, it is expected to be close (and identical within the constraints).

Ultimately, the solution for the optimal $\Delta m$ found, whether constrained or otherwise, contains optimal values for the limited sequence of steps $\Delta m_1, \Delta m_2, \ldots, \Delta m_p$, but it is only the first step $\Delta m_1$ which is actually implemented, before the entire optimisation process is repeated on the next time-step. The effect of optimising more than one step is that the first step can be more severe (overshooting), with subsequent steps correcting the steady-state response.

5.2 APPLICATION TO EXIT MOISTURE CONTROL

For the dryer considered in this study, the control model is single-input, single-output, where only the inlet air temperature is varied to achieve the desired exit moisture. An adaptive DMC technique is proposed. The model simulates the drying process in real time with the same flow and temperature used on the plant, using the collected data.

The model is used to generate updated step responses in real time as follows. For a $P$-step horizon, $P$ separate solutions are continuously being updated on each step. These solutions have exactly the same time-varying inputs as the actual process model, except that the manipulated variable ($T_A$) is given a small fixed offset from the actual input (in this case, $+1^\circ C$ is adequate). As each control step is executed, one of the solutions (in sequence) is reset to the present modelled output. Between this time and the next "resetting" of this particular solution, its output trajectory will progressively deviate from the model output trajectory, on account of the $+1^\circ C$ offset in input. This difference will clearly be a good representation of a local step response, and at any time, each of the $P$ solutions gives a different point on this step response (Fig. 5-4).
Note that the model parameters change as the operating point moves, ensuring a continuous updating of the step response.

Figure 5-1: Continuously updated step response from parallel solutions of real-time model

The difficulty in this control strategy is that the variable measured is the exit moisture sugar content. It is not easy to have this measurement on-line because it involves complicated analysis and it is expensive. One solution would be, as Temple and van Boxtel (2000) suggest, to have access to the exit moisture sugar content through another measurement. The outlet air temperature looks like a good candidate because it is reliable and easy to measure. The idea would be to do step in the sugar moisture and to analyse the response at the exit in the air temperature. From there a function would be created that would transform air temperature into sugar moisture content.
5.3 THE STRUCTURE OF THE PROGRAM

The program written in MATLAB® language is shown in Appendix E. The program is divided into three main parts. Plant data are read from a separate file.

![Program Diagram]

**Figure 5-1: Program diagram**

5.3.1 **Initialisation**

Initialisation is done at the beginning of the program prior to the main time-step cycle. This part of the program is divided into smaller ones where the initialisation is done for:

- The control
- The sugar and air properties. They are also shown in Appendix B.
- The matrices and the vector, especially for the calculation of α, β and G.
- The plotting information

5.3.2 **Model**

5.2.3.1 **Present operating conditions**

The data are read from another file. Every five seconds, the values of time, $W_{So}$, $W_{Ao}$, $T_{So}$, $T_{Ao}$, $S_{fo}$, $f_s$, $f_A$, $W_{Sn}$, $W_{An}$, $T_{Sn}$, $T_{An}$, $S_{fn}$ are obtained from a single file record.

5.2.3.2 **Heat and mass transfer coefficients**

These values are set from the comparison between the reconciled data and the model.
5.2.3.3  Determination of the vector flow \( f_A \)
This represents in the MATLAB language what is developed in section 4.1.2.4.

5.2.3.4  Initialisation of \( X \) and \( U \)
This initialisation is set at the first iteration. It is assumed that the situation inside the dryer is
same along the dryer at \( t = 0 \).

5.2.3.5  Control computations
The small perturbation on the inlet air temperature +1°C gives a small offset – see section 5.2.

5.2.3.6  Axial Convection
The axial convection developed here represents the theory developed in § 4.2.2.1. The idea of
moving in one step to avoid numerical dispersion is implemented. Besides, a test has been
added “dx/dt too small for sugar flow” and “dx/dt too small for air flow” if the movement is
greater than one division.

5.2.3.7  Tuning of A and B matrices
To build the \([CM]\) matrix described in § 4.1.2.5 and shown in Appendix B, the vectors \( \alpha \), \( \beta \) and
\( G \) need to be calculated. The matrices \([A], [AC], [B] \) and \([BC]\) are then built.

5.2.3.8  Integration
Following the discrete model, the single system of equations (4.18) becomes:
\[
\bar{X}_{i+1} = e^{A\Delta t} \bar{X}_i + \left[ e^{A\Delta t} - I \right] A^{-1} B \bar{U}_i
\]
(5.9)
The matrix exponential is defined as:
\[
e^{A\Delta t} = \mathcal{L}^{-1} \left\{ \left[ sI - A \right]^{-1} \right\}
\]
(5.10)
where \( \mathcal{L} \) is the Laplace’s transform and \( I \) is the identity matrix.
\( A_x = e^{A\Delta t} \) and \( B_x = \left[ e^{A\Delta t} - I \right] A^{-1} B \) are defined for the computation.
For the convection, the integration is done with an Euler model using the matrix \([AC]\) and \([BC]\).
Then the integration is done with \([A_x]\) and \([B_x]\).

5.3.3  Control

5.3.3.1  Dynamic matrix control
The theory is translated into the MATLAB® language. To facilitate the reading of the program,
the Table 5-1 translates the symbols used in the theory to their equivalents in the program. It is
possible to vary the set point to different values to see how the controller behaves.
Table 5-1: Translator from theory to program

<table>
<thead>
<tr>
<th>Theory</th>
<th>MATLAB® language</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta m )</td>
<td>DM</td>
</tr>
<tr>
<td>( \Delta m_{UQO} )</td>
<td>dmopt</td>
</tr>
<tr>
<td>( \Delta m_{PAST} )</td>
<td>dmpast</td>
</tr>
<tr>
<td>( e_{OL} )</td>
<td>eol</td>
</tr>
<tr>
<td>( B_{OL} )</td>
<td>DMol</td>
</tr>
<tr>
<td>( B_{0} )</td>
<td>DM0</td>
</tr>
<tr>
<td>( W )</td>
<td>WW</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>Lam</td>
</tr>
</tbody>
</table>

5.3.4 Plotting

All of the information concerning the temperatures, the moistures and the flows during the time period are kept in a results.dat file that can be opened with EXCEL. From the MATLAB®, three kinds of graphs are plotted. An example is shown in Appendix D.

- The temperature and moisture content profile along the dryer at \( t \)
- The comparison between the data from the plant and the data from the model
- The control response

5.4 RESULTS AND DISCUSSION

5.4.1 Off-line

The controller was tuned with the following values:

\[
\begin{align*}
W &= 2000000 \\
\Lambda &= 1
\end{align*}
\]

\( W \) is the penalty weight on squared set-point deviation \( W_{S_0} \).

\( \Lambda \) is the penalty weight on the control move \( \Delta T_{AD} \).

Bigger values for \( W \) would tend make the controller unstable.
The results are shown in Fig. 5-6 and Fig. 5-7 for fixed plant conditions (i.e. no other disturbances). The set point was given different values to see how the controller responded.

Figure 5-1: Response from the controller for $W_{Sn}$

Figure 5-2: Controller effect on $W_{An}$, $T_{An}$, $T_{Sn}$, $T_{An}$

The effect is very significant on the inlet air temperature. When the set point is increased, the air temperature drops: indeed, less heat is needed to evaporate water from the moist sugar. Consequently, the outlet air and sugar temperature drop but less.
5.4.2 As on-line

The results are shown in Fig. 5-9 and Fig. 5-10. The controller seems to have some difficulties in following the set point when the natural process variations are fed through on the other variables, providing disturbance loading.
Figure 5-2: Controller effect on $W_{Am}$, $T_{Ao}$, $T_{Sm}$, $T_{An}$ compared with the data

Note that the “data” values in Fig. 5-10 refer to the original outputs recorded in the plant data, whereas the “exit” values refer to the changed outputs arising from the new variation of $T_{Ao}$. 
5.4.3 Effect of $W$ the penalty weight on squared deviation $W_{Sn}$ from set-point

Different values of the penalty weight on squared set-point deviation were tried to see how it affects the controller. As the penalty weight is increased, the closed-loop becomes oscillatory and less stable, eventually becoming a limit cycle with bang-bang control action.

5.4.4 Advantages

The DMC technique has a considerable advantage for the control of the dryer, because the algorithm explicitly handles dead-time, which will result from the near plug-flow of sugar.
assumed. This is a good indicator that automatic control is required, as operators find it difficult to make corrections with these long responses.

The controller is seen to react to noise on the process measurement sequences, but still gives acceptable control.
CHAPTER 6
CONCLUSIONS

6.1 CONCLUSIONS

The drying process in the sugar industry is not the best understood. This is probably because the mechanism is not simple: it involves several fields from chemical engineering science. More than a simple process of evaporation, sugar drying includes convective heat and mass transfer, equilibrium, and the kinetics of crystallisation.

Data were collected from the Darnall sugar mill belonging to Tongaat-Hulett Sugar Ltd. The co-current rotary sugar dryer was of the concave louvre type. As it was not possible to have access to the whole set of process variables, only the temperatures and the air flow were recorded. A technique was developed to reconcile the data. It was based on a strong confidence in the calculated difference between the sugar moisture content at the entrance and at the exit of the dryer. Assuming that the exit sugar moisture content was at equilibrium at low sugar rates, a plausible reconciled data set was found.

The model built to simulate this sugar dryer was directly inspired by Tait et al. (1994). Heat and mass balances for the moisture of both sugar and air, and the tracking of the dissolved sucrose in a residual "film" along the dryer, with simultaneous crystallisation from the film, were the main features of the model. To avoid any numerical diffusion, an effective convection scheme was developed, based on the integration of the total sugar and air movement.

The model was set up to be able to recreate the sorption isotherm determined by Schindler and Juncker (1993). The Tait et al. (1994) equations could be solved to give similar behaviour to this isotherm for equilibrium sugar moisture content. However, the match was found to be strongly dependent on the initial dissolved sucrose in the water film, for which a single value was eventually chosen to fit the entire isotherm. The double equilibrium of zero net crystallisation and zero net evaporation was not achieved, and even if it were, it would be independent of the amount of dry sugar present.

The tuning to match the collected data was done by varying the heat and mass transfer coefficients. It was found that better results were achieved when the sugar rate was low, especially for the exit sugar moisture content, though the best periods to determine the transfer coefficients would in fact have been under conditions far from equilibrium (high sugar rates).
The ranges of values for the transfer coefficients from the literature are 0.3 to 0.0019 kW.m\(^{-1}\).K\(^{-1}\) for the heat transfer coefficient and \(2.7 \times 10^{-9}\) to \(72 \times 10^{-9}\) kg.m\(^{-2}\).s\(^{-1}\).Pa\(^{-1}\) for the mass transfer coefficient (Table 1-1). The estimated values found from the reconciled plant data were within the literature heat transfer range \((h_c = 0.0136\ kW.m^{-2}.K^{-1})\), and just below the literature mass transfer range \((k_x = 1.36 \times 10^{-9}\ kg\ water.m^{-2}.s.Pa)\). The sugar quality in terms of origin and location might explain this difference.

An application of adaptive control theory was done on the sugar dryer employing unconstrained Dynamic Matrix Control (DMC). The aim was to control the exit sugar moisture content by manipulating the inlet air temperature. A MATLAB\textsuperscript{®} program was written to simulate closed-loop control with otherwise steady process conditions, and then using actual process variations recorded on the plant. It gave acceptable performance, handling the system dead-time without difficulties.

The DMC technique has a considerable advantage for the control of the dryer, because the algorithm explicitly handles dead-time, resulting from the near plug-flow of sugar. This is a good indication that automatic control is required, as operators find it difficult to make corrections with these long time-responses. The controller was seen to react to noise on the process measurement sequences, but still gave acceptable control.

Implementation of closed-loop control will rely on a feedback measurement for the exit moisture content. Methods have been proposed in this work for estimation of this parameter using temperature and flow measurements.
REFERENCES


APPENDIX A  HEAT LOSS

The heat provided by the air is used by the sugar to evaporate the moisture from the sugar. In the following calculation, following a personal communication with Prof. M. Mulholland, we determine whether a significant portion of the supplied heat might be lost to the surroundings, rather than being entirely used in the drying process as assumed in the model.

Coulson and Richardson (1965) provide the following relations for the convective heat transfer coefficient acting on the surface of heated pipes in the air:

\[
\text{STREAMLINE} \\
\quad h = 0.28 \left( \frac{\Delta T}{d} \right) \frac{1}{\Delta T} \\
\text{TURBULENT} \\
\quad h = 0.29 \left( \frac{\Delta T}{d} \right) \frac{1}{\Delta T}
\]

with \( h \) = heat transfer coefficient in [lb-cal.ft\(^{-2}\).h\(^{-1}\).ø\(^{-1}\)]
\( \Delta T \) = temperature difference in [øC]
\( d \) = pipe diameter in [ft]

Converting to SI Units:

\[
\text{STREAMLINE} \\
\quad h = 1.18 \left( \frac{\Delta T}{d} \right) \frac{1}{\Delta T} \\
\text{TURBULENT} \\
\quad h = 1.22 \left( \frac{\Delta T}{d} \right) \frac{1}{\Delta T}
\]

with \( h \) = heat transfer coefficient in [W.m\(^{-2}\).øC\(^{-1}\)]
\( \Delta T \) = temperature difference in [øC]
\( d \) = pipe diameter in [m]

Taking an average dryer temperature of 60 øC, and an ambient temperature of 20 øC, we could then expect a maximum convective heat loss from our 1.75 m diameter by 12.5 m long dryer as follows:

\[
\text{STREAMLINE} \\
Q = 1.18 \left( \frac{40}{1.75} \right) \frac{1}{\Delta T} \times 40 \times \pi \times 1.75 \times 12.5 \\
= 7092 \text{W} = 7.092 \text{ kW} \rightarrow
\]

\[
\text{TURBULENT} \\
Q = 1.22 \left( \frac{40}{1.75} \right) \frac{1}{\Delta T} \times 40 \times \pi \times 1.75 \times 12.5 \\
= 8434 \text{W} = 8.434 \text{ kW} \rightarrow
\]
Now considering the radiation loss (Coulson and Richardson (1965), pp223-224), we have

\[ Q = e \sigma (T^4 - T_{\text{ambient}}^4) \times \pi \times 1.75 \times 12.5 \]

where

e is the emissivity (assume 0.9)
\( \sigma \) is the Stefan-Boltzmann constant $= 1.01 \times 10^{-8} \text{ lb-cal.ft}^{-2} .\text{h}^{-1} .\text{K}^{-4}$

\[ = 5.73 \times 10^{-8} \text{ W.m}^{-2} .\text{K}^{-4} \]

so
\[ Q = 0.9 \times 5.73 \times 10^{-8} \times \left( (40 + 273,15)^4 - (20 + 273,15)^4 \right) \times \pi \times 1.75 \times 12.5 \]
\[ = 7907 \text{ W} = 7,907 \text{ kW} \rightarrow \]

So we consider in this dryer a heat loss to the surroundings of approximately 16 kW through convection and radiation. For a sugar flow of 30 tons per hour, losing 1% by mass of moisture, the latent heat required is approximately 188 kW. Clearly this heat loss of nearly 10% of the evaporation heat would affect the various balances which have been performed in the calculations. Nevertheless, it has been considered that the effect would not greatly alter the results presented, and thus the calculations have not been repeated with this heat loss taken into account.
\[ \begin{array}{|c|c|c|c|c|}
\hline
\frac{\partial W_i}{\partial t} & \frac{\partial W_j}{\partial t} & \frac{\partial (c_{ij} T_j)}{\partial t} & \frac{\partial (c_{ij} T_j)}{\partial t} & \frac{\partial S_j}{\partial t} \\
\hline
W_{3,1} & \frac{D_3}{(\Delta t)^3} + f_s \frac{1}{2Ah_j \rho_j \Delta x} & & & \\
\hline
W_i & -2 \frac{D_i}{(\Delta t)^3} - k_s h_j \beta_i \rho_i h_i & k_s h_j \beta_i \rho_i h_i & -k_s \rho_i \Delta \beta_i & \\
\hline
W_{i,1} & \frac{D_i}{(\Delta t)^3} \frac{f_1}{2Ah_j \rho_j \Delta x} & & & \\
\hline
W_{i,1} & k_s \alpha_i \frac{-2}{(\Delta t)^3} \rho_i h_i & \frac{f_{sat}}{A h_j \rho_j \Delta x} & k_s \rho_i \lambda \alpha_i & \\
\hline
W_{i,1} & \frac{D_i}{(\Delta t)^3} \frac{f_{sat}}{2Ah_j \rho_j \Delta x} & & & \\
\hline
\end{array} \]

**Table B.1: Overview of CM matrix**

**APPENDIX B**

**CM MATRIX**
APPENDIX C  PARAMETER VALUES

C1  GENERAL

Different parameters are used to model the sugar dryer. Some of these are physical, and were found in the technical description of the dryer, or obtained from the Darnall sugar mill. The physical properties of sugar are taken from Peacock (1995), Tait et al. (1994), Love (2001).

C11  Dryer design

Table C-1: Dryer design values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$2.4$ m²</td>
</tr>
<tr>
<td>$L$</td>
<td>$12.5$ m</td>
</tr>
</tbody>
</table>

C12  Sugar crystal properties

Table C-1: Sugar crystal properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{ps}$</td>
<td>$1.25$ kJ.kg⁻¹.K⁻¹</td>
</tr>
<tr>
<td>$D_s$</td>
<td>$10^{-6}$ m².s⁻¹</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>$1500$ kg.m⁻³</td>
</tr>
<tr>
<td>Length of crystal edge</td>
<td>$57.10^{-5}$ m</td>
</tr>
</tbody>
</table>

The model uses not the crystal size but a parameter $a$ that represent the surface offered to the environment over the volume. It is assumed that the crystal model for the sugar crystal is cubic so,

\[
a = \frac{6 \cdot (57 \cdot 10^{-5})^2}{(57 \cdot 10^{-5})^3}
\]  

(C.1)
## C13 Air properties

Table C-1: Air properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{pA}$</td>
<td>1.045 kJ kg$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$D_A$</td>
<td>$10^6$ m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.9 kg m$^{-3}$</td>
</tr>
</tbody>
</table>
APPENDIX D GRAPHS

D1 Graphs from the MATLAB® program

The data used in this appendix come from the 18 October 2000 (part II).

D11 Temperature and moisture content profiles along the dryer

Figure D-1: Temperature profile along the dryer at different times
Figure D-2: Moisture content profile along the dryer at different times

Figure D-3: Temperatures and moisture contents profiles along the dryer for $t = t_{end}$
D12 Comparison between the data from the plant and the data from the model

Figure D-1: Air and sugar temperature from the model

Figure D-2: Air and sugar moisture content from the model
Figure D-3: Comparison of sugar temperature from the plant and the model

Figure D-4: Comparison of air temperature from the plant and the model
**Appendix D**

**Figure D-5**: Comparison of air moisture content from the plant and the model

**D13** Closed loop response

**Figure D-1**: Control influence on the inlet air temperature
Figure D-2: Control response on the exit sugar moisture content
APPENDIX E  PROGRAM

%*******************************************************************
%**
%**  SUGAR DRYER MODEL with DYNAMIX MATRIX CONTROL
%**
%*******************************************************************

clear all

t=0;

%D discretisation
L = 12.5;  % [ m ]
n = 10;
dx = L/n;  % [ m ]
dt = 0.25;  % [ s ]
nsubstep = 1;
tend = 75000;  % [ s ]
datastep = 5;
recompute_gap = 40;

%*******************************************************************
%** INIALISATION **
%*******************************************************************

% FOR CONTROL
mnew = -9999;
mpresent = -9999;
WsnSP = -9999;
ndt = 200;  % no. of integration steps per control step
nopt = 10;  % Number of controller steps to horizon:
time = nopt*ndt*dt
scounter = ndt;
ocounter = nopt;
startflag = 0;
Aso = zeros (5*n, 5*n, nopt);
Bso = zeros (5*n, 3+3*n, nopt);

% FOR SUGAR AND AIR PROPERTIES
a = (6*(57e-5)^2)/(57e-5)^3;  % [m²/m³ ]
Ar = 2.4;  % [ m² ]
roa = 0.9;  % [ kg/m³ of air ]
ros = 1500;  % [ kg/m³ of dry sugar ]
Cpa = 1.045;  % [ kJ/kg air.K ]
Cps = 1.25;  % [ kJ/kg dry sugar. K ]
lambdaH20 = 2.385*10^3;  % [ kJ/kg water ]
hs = 0.0625;  % [ m³ dry sugar / m³ of dryer space ]
Ds = 1e-6;  % [ m²/s ]
Da = 1e-6;  % [ m²/s ]
MBS1dtowed = 0;  % Initialise air travel owed (in No. of dx's)
MBA1dtowed = 0;
% FOR MATRIX AND VECTOR
X = zeros (5*n,1);
U = zeros (3+3*n,1);
Xo = zeros (5*n, nopt);
Uo = zeros (3+3*n,1);
I = eye(5*n,5*n);
CM = le-30*ones(16,5);
CC = 0.0*ones(16,5); % convection coefficients
z = zeros (n,1); % distance along dryer
for i = 1:n
    z(i) = i*dx;
end
XTa = zeros(1, n);
XTs = zeros(1, n);
fa = zeros(n,1); % [ kg/s ]
faf = zeros(n,1);
fax = zeros(n,1);
Waf = zeros(n,1);
Taf = zeros(n,1);

% FOR VECTORS FOR CALCULATION OF ALPHA, BETA and G
W = zeros(n,1);
S = zeros(n,1);
Imp = zeros(n,1);
TTs = zeros(n,1);
SOL = zeros(n,1);
SC = zeros(n,1);
SS = zeros(n,1);
PCDryS = zeros(n,1);
Bp = zeros(n,1);
pTs = zeros(n,1);
pBp = zeros(n,1);
ps = zeros(n,1);
alpha = zeros(n,1);
Ea = zeros(n,1);
G = zeros(n,1);
G0 = zeros(n,1);
K2 = zeros(n,1);
K4 = zeros(n,1);
TaRH = zeros(n,1);
WaRH = zeros(n,1);
RH = zeros(n,1);

% FOR PLOTTING INFORMATION
dtplot = 50*dt; % every 100 th point only
ntplot = round(tend/dtplot);
tplot = zeros(ntplot,1);
results = zeros(ntplot,19); % for storage
tlastplot = 0;
iprofile = 0;
nprofilemax = 6;
dtprofile = round(tend/nprofilemax);
tlastprofile = 0;
iprofile = 0;
Wminplot = 0;
Wmaxplot = 1;
tplot=zeros(ntplot,1); Tsexit=zeros(ntplot,1); Taexit=zeros(ntplot,1); TaOP=zeros(ntplot,1); TaOP_old=zeros(ntplot,1); Tainlet=zeros(ntplot,1); Wsexit=zeros(ntplot,1); WsexitSP=zeros(ntplot,1); Wsn_measP=zeros(ntplot,1); Waexit=zeros(ntplot,1); Gexit=zeros(ntplot,1); Ts=zeros(n,1); Ta=zeros(n,1); Ws=zeros(n,1); Wa=zeros(n,1); Sf=zeros(n,1); Ts_n=zeros(n,1); Ta_n=zeros(n,1); Ws_n=zeros(n,1); Wa_n=zeros(n,1); Sf_n=zeros(n,1);


%*******************************************************************
%** MODEL **
%*******************************************************************

figure(1); clf; figure(2); clf; index=0; datatime=0; modeltime=0; MBA1=0; MBS1=0; fa_factor=0; fs_factor=0; recompute_counter=recompute_gap;

Matrix_final_set_001213; % that opens the file with the data
index_max=4943; % NB! MUST CHANGE FOR EACH DATA FILE

while t < tend
    recompute=0;

    t = t+dt;
    if t < tend
        modeltime=modeltime+dt;
        recompute_counter=recompute_counter+1;
    end

if modeltime > datatime
    index=index+1;

E-3
if index>index_max
    index=index_max;
end
if recompute_counter>recompute_gap
    recompute=1;
    recompute_counter=0;
end
    datatime=datatime+datastep;
end

%% BLOCK INPUT
Ws0 =data_set(index,2);
Wa0 =data_set(index,3);
Ts0 =data_set(index,4);
Ta0 =data_set(index,5);
Sf0 =data_set(index,6);
fs  =data_set(index,7);
fa_in =data_set(index,8);

%% BLOCK OUTPUT from reconciliation
Wsn_meas =data_set(index,9);
Wan   =data_set(index,10);
Tsn   =data_set(index,11);
Tan   =data_set(index,12);
Sfn   =data_set(index,13);

%================================================================
%== Heat and Mass Transfer --
%================================================================
ht= 1.35e-2; % [kW / m2 K]
kg= 1.35e-9; % [kg water / m2 s Pa]
%================================================================

RT=ros*Ar*L*hs/fs; %residential time
% force Super-saturation in crystal film to crystallization point
pur =98; %purity [kg sugar/ (kg sugar + kg impurity)]
If0=((100-pur)/pur)*Sf0; % kg impurity in film per kg dry sugar in crystal

% condition of air feeds and exhausts
for i=1:n
    faf(i)=0.0;
    Waf(i)=0.0;
    Taf(i)=0.0;
    fax(i)=0.0;
end
faf(1)=fa_in;
Waf(1)=Wa0;
Taf(1)=Ta0;
% (actually, fax(n) is reset to take care of the remaining air)

%================================================================

E-4
\% Determination of the Vector flow "fa"
\%===================================================================
\% fain = zeros(2*n-1,1); \% vector type Fa1 Fax1 ... Fafn-1 Faxn-1
\fain
\textbf{for} i=1:n-1
\t\textbf{fain}(2*i-1)=faf(i);
\t\textbf{fain}(2*i)=fax(i);
\textbf{end}
\textbf{fain}(2*n-1)=faf(n);
\\textbf{Fmat1} = \textbf{zeros} (n,n);
\textbf{Fmat2} = \textbf{zeros}(n,2*n-1);
\\textbf{for} i=1:n-1
\t\textbf{Fmat1}(i+1,i)=-1;
\textbf{end}
\textbf{for} i=1:n
\t\textbf{Fmat1}(i,i)=1;
\textbf{end}
\\textbf{for} i=1:n-1
\t\textbf{Fmat2}(i,2*i)=-1;
\t\textbf{Fmat2}(i,2*i-1)=1;
\textbf{end}
\textbf{Fmat2}(n,2*n-1) = 1;
\textbf{fa} = \textbf{inv(Fmat1)}*\textbf{Fmat2}*\textbf{fain}; \% vector type Fa1 ... Fan-1 Faxn
\\textbf{fax}(n,1) = \textbf{fa}(n,1);
\textbf{fa}(n,1)=0; \% to be safe
\% Average of \textbf{fa} for MBA1
\textbf{fsum} = 0;
\\textbf{for} i=1:n-1
\t\textbf{fsum} = \textbf{fsum} + \textbf{fa}(i);
\textbf{end}
\textbf{faverage}=(\textbf{fsum}+ \textbf{fax}(n))/n;
\%===================================================================
\%===================================================================
\% Initialisation of X and U
\%===================================================================
\textbf{if} t == dt
\\textbf{for} i=1:n
\t\textbf{X}(i) = Ws0;
\t\textbf{X}(i+n) = Wa0;
\t\textbf{X}(i+2*n) = Cps*Ts0;
\t\textbf{X}(i+3*n) = Cpa*Ta0;
\t\textbf{X}(i+4*n) = Sf0;
\textbf{end}
\\textbf{for} j=1:nopt
\t\textbf{X0}(:,:,j)=\textbf{X}(:,:,j);
\textbf{end}
\textbf{end}
\% set input vector
\textbf{U}(1) = Ws0;
\textbf{U}(2) = Cps*Ts0;
\textbf{U}(3) = Sf0;
\\textbf{for} i=1:n
\t\textbf{U}(3+i)=Waf(i);
U(3+i+2*n)==1;
if i==1
    U(3+i+n)== Cpa*Taf(i);     % from data file
else
    if ((t==dt)&(mnew==9999))
        U(3+i+n)== mnew;          % computed on last control
        calc step late
    else
        U(3+i+n)== Cpa*Taf(1);    % from data file
    end
end

%===================================================================================================================================
%==== Control computations
%===================================================================================================================================
% Uo=U;
Uo(3+i+n)==U(3+i+n)+1; % Unit perturbation (only about 1C)
scounter=scounter+1;
if scounter>ndt
    scounter=1;
ocounter=ocounter+1;
    if ocounter>nopt
        ocounter=1;
    end
    Xo(:,ocounter)=X(:,); % reset to present output;
end

%=============================================================================%============================================================================================
%== Axial Convection ==
%=============================================================================
MBA1last=MBA1;
MBS1last=MBS1;
fa_factorlast=fa_factor;
fs_factorlast=fs_factor;

MBS2 = kg*a/roa;
MBA2 = kg*a*hs/(roa*(1-hs));
HBS1 = kg*a*lambdaH20/roa;
HBS2 = ht*a/(roa*Cpa);
HBA1 = 1/(Ar*(1-hs)*roa*dx);
HBA2 = a*ht*hs/(roa*Cps*(1-hs));
MDs = Ds/(dx*dx);
MDa = Da/(dx*dx);

%MBS1dtrequired = fs*dt/(Ar*hs*roa*dx);
MBS1dtowed=MBS1dtowed+MBS1dtrequired;
MBS1dt=max(0,round(MBS1dtowed)); % only convect in whole steps
if MBS1dt>1
    error = 'dx/dt too small for sugar flow !!!!!'
    halt;
end
MBS1dtowed=MBS1dtowed-MBS1dt;
MBS1=MBS1dt/dt;
if MBS1dt == 0
    fs_factor=0;
else
    fs_factor = MBS1dt / MBS1dtrequired;
end

MBA1dtrequired = faverage*dt/((l-hs)*Ar*roa*dx);
MBA1dtowed=MBA1dtowed+MBA1dtrequired;

MBA1dt=max(0,round(MBA1dtowed)); % only convect in whole steps
if MBA1dt>1
    error = 'dx/dt too small for air flow !!!!!'
    halt;
end
MBA1dtowed=MBA1dtowed-MBA1dt;
if MBA1dt == 1
    MBA1 = MBA1dt/dt;
    fa_factor = MBA1dt / MBA1dtrequired;
else
    MBA1=0;
    fa_factor=0;
end

if ((MBA1 -= MBA1last) | (fa_factor -= fa_factorlast))
    recompute=1;
end
if ((MBS1 -= MBS1last) | (fs_factor -= fs_factorlast))
    recompute=1;
end
if t==dt
    recompute=1;
end

%-----------------------------------------------------------
% Tuning of A & B matrices
%-----------------------------------------------------------
for o=1:(nopt+1)
    A = zeros (5*n,5*n); % reset all matrices to zero
    AC= zeros (5*n,5*n); % convection matrix
    B = zeros (5*n, 3+3*n); % convection matrix
    BC = zeros (5*n, 3+3*n);
    if o<=nopt
        Xt= XO (: ,0) ;
    else
        % reset all matrices to zero
        % convection matrix
        % temporary 'X' versions with various delays
    end
    % do the actual solution last so solution values stay in arrays
    if recompute % only do this if necessary
        %----------------------------------------
        % Tuning of Vectors alpha , beta and G
        for i=1:n
            W(i)= Xt(i)*fs;
            S(i)= Xt(4*n+i)*fs;
            Imp(i)=If0;
            TTs(i)=Xt(2*n+i)/Cps;
    end
end
\[
\text{SOL}(i) = 64.407 + 0.07251 \times \text{TTs}(i) + 0.0020569 \times \text{TTs}(i)^2 - 9.035 \times 10^{-6} \times \text{TTs}(i)^3;
\]
\[
\text{Ea}(i) = 62.86 - 0.84 \times (\text{TTs}(i) - 60);
\]
\[
\text{K2}(i) = -\left(\frac{\text{Ea}(i)}{8.314e-3}\right) \times \left(1/(273.16 + \text{TTs}(i)) - 1/333.16\right);
\]
\[
\text{K0} = 0.005;
\]
\[
\text{K5} = 0.08;
\]
\[
\text{SC}(i) = 1 - K5 \times \text{Imp}(i)/W(i);
\]
\[
\text{SS}(i) = \left(\frac{S(i)}{W(i)}\right)\left(\frac{100 - \text{SOL}(i)}{\text{SOL}(i) \times \text{SC}(i)}\right);
\]
\[
\text{K1} = 0.00000087;
\]
\[
\text{K3} = 1.75;
\]
\[
\text{PCDryS}(i) = 100 \times \frac{(S(i) + \text{Imp}(i))}{(S(i) + \text{Imp}(i) + W(i))};
\]
\[
\text{Bp}(i) = 100 + 2 \times \frac{\text{PCDryS}(i)}{100 - \text{PCDryS}(i)};
\]
\[
\text{pTs}(i) = 1367.6 - 132.54 \times \text{TTs}(i) + 9.635 \times \text{TTs}(i)^2 - 0.115 \times \text{TTs}(i)^3 + 0.0132 \times \text{TTs}(i)^4;
\]
\[
\text{T100} = 100;
\]
\[
\text{p100} = 1367.6 - 132.54 \times \text{T100} + 9.635 \times \text{T100}^2 - 0.115 \times \text{T100}^3 + 0.00132 \times \text{T100}^4;
\]
\[
\text{pBp}(i) = 1367.6 - 132.54 \times \text{Bp}(i) + 9.635 \times \text{Bp}(i)^2 - 0.115 \times \text{Bp}(i)^3 + 0.00132 \times \text{Bp}(i)^4;
\]
\[
\text{ps}(i) = \text{pTs}(i)/\text{p100}/\text{pBp}(i) \times \min(1, 1/\text{SS}(i));
\]
\[
\text{pa}(i) = 101325 \times \left(\frac{\text{Xt}(n+i)}{18}\right) / \left(\frac{\text{Xt}(n+i)}{18} + 1/29\right);
\]
\[
\text{WaRH}(i) = \frac{\text{Xt}(n+i)}{\text{Cpa}};
\]
\[
\text{TaRH}(i) = (\text{Xt}(n+i) + \text{Imp}(i))/\text{Cpa};
\]
\[
\text{RH}(i) = 100 \times 1.01(10 \times 5.083 - 1665.6/(\text{TaRH}(i) + 228)) \times (0.62/\text{WaRH}(i)) + 1);\]
\[
\text{G}(i) = K1 \times (\text{SS}(i) - (1 + K0)) \times \exp(K2(i) - K3 \times \text{Imp}(i)/W(i));
\]
\[
\text{beta}(i) = \frac{\text{ps}(i)}{\text{Xt}(i)};
\]
\[
\text{alpha}(i) = \frac{\text{pa}(i)}{\text{Xt}(n+i)};
\]

end

%------------------------------------

for i = 1:n
  if i == 1
    ffaim1 = 0;
  else
    ffaim1 = fa_factor * fa(i-1);
  end
  if i == n
    ffaip1 = 0;
  else
    ffaip1 = fa_factor * fa(i+1);
  end
  ffax = fa_factor * fax(i);
  ffaf = fa_factor * faf(i);
  ffa = fa_factor * fa(i);
end

%------------------------------------

% Building of CM matrix

\[
\text{CM}(1,1) = \text{MDs}; \quad \text{CC}(1,1) = \text{MBS1}; \quad \%Wsi-1
\]
\[
\text{CM}(2,1) = -2 \times \text{MDs} - \text{MBS2} \times \text{beta}(i); \quad \text{CC}(2,1) = - \text{MBS1}; \quad \%Wsi
\]
\[
\text{CM}(3,1) = \text{MDs}; \quad \text{CC}(3,1) = \%Wsi+1
\]
\[
\text{CM}(5,1) = \text{MBS2} \times \text{alpha}(i); \quad \text{CC}(5,1) = \%Wai
\]
\[
\text{CM}(16,2) = 0; \quad \text{CC}(16,2) = \text{ffaf} \times \text{HBA1}; \quad \%Wai
\]

\[
\text{CM}(2,2) = \text{MBA2} \times \text{beta}(i); \quad \%Wsi
\]
\[
\text{CM}(4,2) = \text{MDa}; \quad \text{CC}(4,2) = \text{ffaim1} \times \text{HBA1}; \quad \%Wai-1
\]
\[
\text{CM}(5,2) = -2 \times \text{MDa} - \text{MBA2} \times \text{alpha}(i); \quad \text{CC}(5,2) = - \text{ffaf} \times \text{HBA1} - \text{ffaf} \times \text{HBA1}; \quad \%Wai
\]
\[
\text{CM}(6,2) = \text{MDa}; \quad \%Wai+1
\]
\begin{verbatim}
% d(CpTs)/dt
CM(2,3) = -HBS1*beta(i); %Wsi
CM(5,3) = HBS1*alpha(i); %Wai
CM(7,3) = MDs; CC(7,3) = MBS1; %CpsTs-i-1
CM(8,3) = -2*MDs - HBS2*Cpa/Cps; CC(8,3) = - MBS1; %CpsTs
CM(9,3) = MDs; %CpsTs+i-1
CM(11,3) = HBS2; %CpaTai
CM(16,3) = 0; CC(16,3) = ffaf*HBA1; %force

% d(CpaTa)/dt
CM(8,4) = HBA2; %CpsTs
CM(10,4) = MDa; CC(10,4) = ffaiml*HBA1; %CpaTai-1
CM(11,4) = -2*MDa - (Cps/Cpa)*HBA2; CC(11,4) = - ffax*HBA1 - ffa*HBA1; %CpaTai
CM(12,4) = MDa; %CpaTai+1
CM(16,4) = 0; CC(16,4) = ffaf*HBA1; %force

% d(Sf)/dt
CM(13,5) = MDs; CC(13,5) = MBS1; %Sfi-1
CM(14,5) = -2*MDs; CC(14,5) = - MBS1; %Sfi
CM(15,5) = MDs; %Sfi+1
CM(16,5) = -a*G(i); %force

for j=1:5
    ii=(j-1)*n+i; % row in full vector
    for k=1:5 % block order to be Ws,Wa,CpTsTa,CpaTa,Sf
        jjoff = (k-1)*n;
        % A & AC; B & BC matrix additions
        for m=1:3
            coeff = CM((k-1)*3+m,j);
            coeffC = CC((k-1)*3+m,j);
            ij=i+m-2;
            if ij<l
                else
            end
            if ((k=2)&(k=4))
                kk=(k+1)/2;
                B(ii,kk) = B(ii,kk) + coeff; % forcing boundary conditions
            end;
        end;
        if ij>n
            jj = jjoff+ij;
            A(ii,jj)= A(ii,jj) + coeff; % sets flat profile at 'n'
            AC(ii,jj)= AC(ii,jj) + coeffC; % sets flat profile at 'n'
        end
    end
end

% B matrix additions
if j=2
    B(ii,3+i) = B(ii,3+i) + CM(16,j);
    BC(ii,3+i) = BC(ii,3+i) + CC(16,j);
else
    if j=4
        B(ii,3+n+i) = B(ii,3+n+i) + CM(16,j);
        BC(ii,3+n+i) = BC(ii,3+n+i) + CC(16,j);
end
end
\end{verbatim}
else
    if j==5
        B(ii,3+2*n+i) = B(ii,3+2*n+i) + CM(16,j);
        BC(ii,3+2*n+i) = BC(ii,3+2*n+i) + CC(16,j);
    end
end
end
end

end

%===================================================================
%== Integration
%===================================================================
% for singular A use series to find \( \expm(\text{Adt}_I/\text{I}) \) = \( \left( \expm(\text{Adt}) - \text{I} \right)^{-1} \)
subdt = dt/nsubstep;
tolerance=1e-10;
change=99;
expmAdt_IdivA=subdt*I;

if o<=nopt
    Aso(:, :, o)=expmAdt_IdivA*Adt+I;
    Bso(:, :, o)=expmAdt_IdivA*B;
else
    As=expmAdt_IdivA*Adt+I;
    Bs=expmAdt_IdivA*B;
end

% Now integrate rest using matrix exponential over substeps

% Operator - splitting : First do convection step by Euler
if o<=nopt
    Xo(:,o) = Xo(:,o) + dt*(AC*Xo(:,o) + BC*Uo);
else
    X = X + dt*(AC*X + BC*U);
end
for i=1:nsubstep
    if o<=nopt
        Xo(:,o) = Aso(:, :, o)*Xo(:,o) + Bso(:, :, o)*Uo;
    else
        X = As*X + Bs*U;
    end
end

%**************************************************************************
%** DYNAMIC MATRIX CONTROL ALGORITHM **
%**************************************************************************
if $t=dt$  % first step  
DM=zeros(nopt,nopt);  % Dynamic Matrix (use a simple square system)  
DMol=zeros(nopt,nopt);  % Openloop Matrix  
DM0=zeros(nopt,nopt);  % Measurement Offset Matrix  
ncount=0;  % Initialise counter for Control Time Steps  
% Initialise vector of previous control moves  
dmpast=zeros(nopt,1);  
% Set up Tuning Matrices WW & Lam  
WW=zeros(nopt,nopt);  
for $i=1$:nopt  
  WW(i,i)=6000000;  
end  
Lam=1;  % Limits for Output  
mmax=Cpa*90;  % allow a max air temperature of 90°C  
mmin=Cpa*30;  % allow a min air temperature of 30°C  
%mflag value for control action as warning that it is not yet available  
mnew=-9999;  
end  

if scounter==ndt  % Time for a control step  
  if ocounter==nopt  % have done ndt steps since the nopt perturbation now allow control  
    startflag=1;  
  end  
  ControlSwitch=1;  
  if ((ControlSwitch==1) & (startflag==1))  % use this to switch the controller on at a particular time  
    % Make Dynamic Matrix DM  
    for $i=1$:nopt  
      for $jj=1$:i  
        kk=ocounter-jj+1;  
        if kk<1  
          kk=kk+nopt;  % wrap around  
        end  
        j=i-jj+1;  
        DM(i,j)=Xo(n;kk)-X(n);  % only the difference caused by the unit shift in input  
      end  
    end  
    % Make Openloop Matrix DMol & Offset Measurement Matrix DM0  
    for $i=1$:nopt  
      for $j=1$:nopt  
        jj=min(nopt,nopt+i-j+1);  
        DMol(i,j)=DM(nopt,nopt+1-jj);  % pick off backwards along bottom line of DM  
        jjj=nopt-j+1;  
        DM0(i,j)=DM(nopt,jjj);  
      end  
    end

if $t>0$  % Set point  
  WsnSP=0.0015;  
end
if $t>4000$
  \hspace{1em} WsnSP=0.0025;
end
if $t>7000$
  \hspace{1em} WsnSP=0.0015;
end
if $t>8000$
  \hspace{1em} WsnSP=0.0030;
end
if $t>10000$
  \hspace{1em} WsnSP=0.0020;
end
if $t>12000$
  \hspace{1em} WsnSP=0.0010;
end
if $t>16000$
  \hspace{1em} WsnSP=0.0015;
end
if $t>17000$
  \hspace{1em} WsnSP=0.0025;
end
if $t>22000$
  \hspace{1em} WsnSP=0.0015;
end
if $t>28000$
  \hspace{1em} WsnSP=0.0070;
end
if $t>30000$
  \hspace{1em} WsnSP=0.0020;
end
if $t>38000$
  \hspace{1em} WsnSP=0.0010;
end
if $t>40000$
  \hspace{1em} WsnSP=0.0015;
end
if $t>46000$
  \hspace{1em} WsnSP=0.006;
end
if $t>47000$
  \hspace{1em} WsnSP=0.0015;
end
if $t>52000$
  \hspace{1em} WsnSP=0.0030;
end
if $t>56000$
  \hspace{1em} WsnSP=0.0020;
end
if $t>58000$
  \hspace{1em} WsnSP=0.0010;
end
if $t>62000$
  \hspace{1em} WsnSP=0.0015;
end
if $t>64000$
  \hspace{1em} WsnSP=0.0025;
end
if $t>67000$
  \hspace{1em} WsnSP=0.0015;
end
if $t>68000$
\[
\text{WsnSP} = 0.0030; \\
\text{end} \\
\text{if } t > 70000 \\
\quad \text{WsnSP} = 0.0070; \\
\text{end} \\
\text{if } t > 72000 \\
\quad \text{WsnSP} = 0.0010; \\
\text{end}
\]

\[
\text{Wsn=X(n); } \% \text{ Actual value}
\]
\[
eol = \text{ones(nopt,1)} \ast (\text{Wsn-WsnSP}) + (\text{DMol-DMO}) \ast \text{dmpast}; \\
\% \text{ only one move, so only 1st col of DM}
\]
\[
\text{DMs=DM(:,1);} \\
\% \text{only do least squares part, not constrained search}
\]
\[
dmopt = -\text{inv(DMs' * WW * DMs + Lam)} \ast \text{DMs' * WW * eol}; \\
\text{mpresent=U(3+n+1);} \\
\]
\[
\text{mnew}=\text{mpresent}+\text{dmopt}; \\
\% \text{ Clip externally to limits}
\]
\[
\text{mnew} = \min(\text{mmax}, \max(\text{mmin}, \text{mnew})); \% \text{ actual new absolute}
\]
\[
\text{control action to be used}
\]
\[
\text{dmused}=\text{mnew}-\text{mpresent}; \\
\text{else}
\]
\[
\text{mnew} = -9999; \% \text{ back off line}
\]
\[
\text{WsnSP} = -9999; \\
\text{if } \text{mpresent} = -9999 \\
\quad \text{dmused}=\text{U(3+n+1)}-\text{mpresent}; \% \text{ keep the moves that are}
\]
\[
\text{going on "manually" will help control}
\]
\[
\text{mpresent=U(3,n,1);} \\
\%
\]
\[
\text{else}
\]
\[
\text{dmused}=0; \\
\%
\]
\[
\text{end}
\]
\[
\% \text{ Update past moves vector}
\]
\[
\text{for } i=1:(\text{nopt}-1) \\
\quad \text{dmpast}(i) = \text{dmpast}(i+1); \\
\text{end}
\]
\[
\text{dmpast(nopt)=dmused; } \% \text{ newest move at bottom of vector}
\]
\[
\%**************************************************************************
\%** \text{ Store for plotting and plotting } **
\%**************************************************************************
\%
\]
\[
\% \text{ store points for time-plot if time is right}
\]
\[
\text{if } (t-tlastplot) \ast \text{dtplot}
\]
\[
\text{iplot=iplot+1;}
\]
\[
\text{tplot(iplot)=t;}
\]
\[
\text{Wsexit(iplot)=X(n);} \\
\text{Waexit(iplot)=X(2*n);} \\
\text{Tsexit(iplot)=X(3*n)/Cpsi;} \\
\text{Taexit(iplot)=X(4*n)/Cpa;} \\
\text{Tainlet(iplot)=X(3*n+1)/Cpai;} \\
\text{Gexit(iplot) = G(n);} \\
\%
\]
\[
\text{W}_{a_n}(iplot)=W_{an}; \\
\text{T}_{s_n}(iplot)=T_{sn}; \\
\text{T}_{a_n}(iplot)=T_{an}; \\
\text{S}_{f_n}(iplot)=S_{fn};
\]
if mnew==-9999
    TaOP(iplot)=Ta0;
    TaOP_old(iplot)=Ta0;
else
    TaOP(iplot)=mnew/Cpa;
    TaOP_old(iplot)=Ta0;
end;
WsexitSP(iplot)=WsnSP;
Wsn_measP(iplot)=Wsn_meas;

% store results
results(iplot,1)=t;
results(iplot,2)=fs;
results(iplot,3)=fa_in;
results(iplot,4)=Ws0;
results(iplot,5)=Wa0;
results(iplot,6)=Ts0;
if mnew==-9999
    results(iplot,7)=Ta0;
else
    results(iplot,7)=mnew/Cpa;
end;
results(iplot,8)=Sf0;
results(iplot,9)=X(n);
results(iplot,10)=Wan;
results(iplot,11)=Tsn;
results(iplot,12)=Tan;
results(iplot,13)=Sfn;
results(iplot,14)=X(n);
results(iplot,15)=X(2*n);
results(iplot,16)=X(3*n)/Cps;
results(iplot,17)=X(4*n)/Cpa;
results(iplot,18)=X(5*n);
results(iplot,19)=WsnSP;
tlastplot=t;
end;
for i=1:n
    Ws(i)=X(i);
    Wa(i)=X(n+i);
    Ts(i)=X(2*n+i)/Cps;
    Ta(i)=X(3*n+i)/Cpa;
    Sf(i)=X(4*n+i);
end

% plot chosen profiles if time is right
if (t-tlastprofile»=dtprofile
    iprofile=iprofile+1;
    if iprofile<=nprofilemax
        figure(1);
        subplot(2,nprofilemax/2,iprofile);
        plot(z,Ts,'b',z,Ta,'r');
        axis ([0 z(n) 0 100]);
        legend('Ts','Ta');
        title(t)
figure(2);
subplot(2,nprofilemax/2,iprofile);
plot(z,Ws,'b',z,Wa,'r');
axis ([0 z(n) 0 0.1]);
legend('Ws', 'Wa');
title(t)

end
tlastprofile=t;
end

RHN = 100*1.01/(10^5.083-1665.6/(Ta(n)+228))*(0.62/Wa(n)+1));

end % of time loop

% Save Results
save results.dat results -ascii -tabs;

% Final profiles
figure (3);
cf;
subplot(2,2,1);
plot(z,Ws,'r');
axis ([0 z(n) 0 0.02]);
legend('Ws');

subplot(2,2,2);
plot(z,Wa,'r');
axis ([0 z(n) 0 0.05]);
legend('Wa');

subplot(2,2,3);
plot(z,Ts,'b');
axis ([0 z(n) 0 100]);
legend('Ts');

subplot(2,2,4);
plot(z,Ta,'k');
axis ([0 z(n) 0 100]);
legend('Ta');

% Time variations
figure (4);
cf;
plot
(tplot(1:iplot),Texit(1:iplot), 'b', tplot(1:iplot),Taexit(1:iplot), 'g' );
axis ([0 tplot(iplot) 0 100]);
legend('Texit', 'Taexit');

figure (5);
cf;
plot
(tplot(1:iplot),Wsexit(1:iplot), 'r', tplot(1:iplot),Waexit(1:iplot), 'y' )
axis ([0 tplot(iplot) 0 0.04]);
legend('Wsexit', 'Waexit');
figure (6);
clf;
plot
tplot(1:iplot),Wsexit(1:iplot), 'r', tplot(1:iplot), Wsn_measP(1:iplot), 'y', tplot(1:iplot), WsexitSP(1:iplot), 'b')
axis ([0 tplot(iplot) 0 0.01]);
legend('Wsexit', 'Wsexit_meas', 'WsexitSP');

figure (7);
clf;
plot
tplot(1:iplot),Waexit(1:iplot), 'r', tplot(1:iplot), Wa_n(1:iplot), 'y')
axis ([0 tplot(iplot) 0 0.05]);
legend('Waexit', 'Wa data');

figure (8);
clf;
plot
tplot(1:iplot), Tsexit(1:iplot), 'b', tplot(1:iplot), Ts_n(1:iplot), 'g')
axis ([0 tplot(iplot) 0 100]);
legend('Tsexit', 'Ts data');

figure (9);
clf;
plot
tplot(1:iplot), Taexit(1:iplot), 'b', tplot(1:iplot), Ta_n(1:iplot), 'g')
axis ([0 tplot(iplot) 0 100]);
legend('Taexit', 'Ta data');

figure (10);
clf;
plot
tplot(1:iplot), TaOP(1:iplot), 'b', tplot(1:iplot), TaOP_old(1:iplot), 'g')
axis ([0 tplot(iplot) 0 100]);
legend('TaO', 'TaO_old');

eq_Ts = Ts(n)
eq_Ta = Ta(n)
eq_Wa = Wa(n)
eq_Ws = Ws(n)
eq_RH = 100*1.01/(10^(5.083-1665.6/(Ts(n)+228))*(0.62/Wa(n)+1))
eq_Sf = Sf(n)
save LASTRUN;