An exploration of the role of visualisation in the proving process of Euclidean geometry problems.

by

Lola Reddy

Submitted in partial fulfilment of the academic

Requirements for the degree of

Master of Education in the

School of Science, Mathematics, and Technology Education,

Faculty of Education,

University of KwaZulu - Natal

January 2015

ABSTRACT

This study explores the role of visualization in the proving and solving process of Euclidean geometry problems in a grade 11 Mathematics class. The study investigates whether visual representations form an integral part of proof. This study is conducted within the Interpretative approach. Mudaly's (2012) adaptation of Kolb's Experiential Learning Theory underpins this research.

Five randomly selected mathematics grade 11 learners from a class that I did not teach were given two Euclidean Geometry activities to complete. This research was conducted in the second school term but the activities were based on the theorem the angle subtended at the centre of a circle equals twice the angle subtended at the circumference. This theorem according to the grade 11 Mathematics work schedule is planned to be taught in the third school term. This was done to ensure that these participants had no prior knowledge or experience with the theorem because it was used as the foundation for the activities.

The data gathered from this study showed that majority of the learners in this research regarded diagrams as being a significant part in proof. All participants revealed that they felt that diagrams were helpful in efficiently solving geometric problems. One of the advantages of diagrams that they declared was that diagrams give a better understanding of a problem .The learners were able to form meaningful connections between the various concepts shown in the diagram and this lead them to successfully solving the problems. It must however be stated that the diagrams can only be useful to learners if they have the necessary prior knowledge for the question. The study showed that since the learners were lacking in their prior knowledge they were unsuccessful in their proving and solving.

The research also disclosed that although learners seem to value the use of diagrams in proving/solving geometric problems, they have insufficient experience in sketching their own diagrams. There seems to be some merit in the use of diagrams but teachers need to create more exercises that involve learners sketching their own diagrams in order to solve the geometric problems and learners must have a good conceptual understanding of the necessary prior knowledge in order to solve correctly.

PREFACE

The work described in this thesis was carried out in the School of Science, Mathematics and Technology Education, University of KwaZulu – Natal, from July 2013 to January 2015 under the supervision of Dr V Mudaly (Supervisor).

This study represents original work by the author and has not otherwise been submitted in any form for any degree or diploma to any tertiary institution. Where use has been made of the work of others, it is duly acknowledged in the text.

Lola Reddy

January 2015

TABLE OF CONTENTS

CHAPTER FIVE: Data analysis

CHAPTER SIX: Research findings, limitations and recommendations

LIST OF FIGURES

LIST OF TABLES

DEDICATION

To my guru, God and gold, Sri Sathya Sai Baba, beside you my master there is none. I humbly offer this effort at your Divine Lotus Feet. Thank you dear Lord for helping me to complete this thesis.

ACKNOWLEDGEMENT

My heartfelt gratitude and appreciation is extended to the following people:

My parents, my brother , sister, Aunty Ambi, my cousin Shamala and my dearest friend Devan. Thank you all for motivating and encouraging me when I wanted to give up. Thank you for advising, assisting and supporting me unquestioningly throughout the writing of this thesis.

To my supervisor, Dr V .Mudaly for your time, patience and guidance.

CHAPTER ONE

INTRODUCTION TO STUDY

1.1 Introduction to research

The focus of this study was to investigate the role that visualisation plays in the solving and proving of Euclidean geometry problems. I enjoy solving geometric problems but from my experience it seems that mathematics learners struggle with geometry. Van Putten (2008, p.9) claims that solving geometric riders causes fear and anxiety amongst mathematics learners. The results from their responses to geometric problems in tests and exams that I have marked over the years have indicated that there is a distinct problem with the teaching and learning of geometry.

A scrutiny of the learner's responses to geometric questions indicate that these learners memorize proofs of theorems and geometric rules without understanding why certain concepts are true. According to Van Putten (2008, p.10) teachers of geometry in South Africa encourage the learners to learn of the theorems. She further states that about two application riders are discussed and then an exercise is given for homework. The techniques used by teachers to teach geometry could be responsible for the learner's performance in geometry.

Kutama (2002, p.8) declares that learners memorize theorems as a means of escape to prevent themselves from feeling frustrated when they are working with geometric problems. The learners seem to have difficulty with deduction and proof. Van Putten (2008, p.8) declares that most learners find constructing proof more difficult than any other geometric section. In my experience, many learners do not even attempt to answer questions that involve proving.

There is great value in the use of diagrams when solving or working with geometric proofs. According to Kutama (2002, p.10) a learners understanding of a problem increases when they have to draw their own diagram for a geometric problem because they would be able to notice certain concepts that they previously would not have observed. It was my belief on the effectiveness of diagrams in solving geometric problems and the learner's negative responses towards geometry that led me to choose this study. I wanted to explore the role that using diagrams either physically or mentally can play in solving geometric problems. This was done with the aim of trying to improve learner's attitude and their success rate at solving geometric problems.

1.2 Background of Euclidean Geometry in South African curriculum

When the FET (Further Education Training) curriculum was implemented in 2006 Euclidean Geometry was embedded into Mathematics paper three and became optional. According to Bowie (2009) one of the reasons that geometry became optional was that the teachers themselves did not know the content of geometry well enough. Learners who wanted to study Euclidean Geometry had to take Mathematics paper three as an additional subject. In 2012 the CAPS (Curriculum and Assessment Policy Statement) replaced the NCS (National Curriculum Statement). The optional Mathematics Paper 3 became integrated in the compulsory Mathematics paper one and two. This would suggest that from 2006 to 2011 only a few mathematics grade ten to grade twelve learners were exposed to Euclidean geometry. Now that it has become compulsory the very same learners who are becoming mathematics teachers will be involved in the teaching of geometry. An absence of Euclidean geometry for six years would have implications for the future of both the teaching and learning of Euclidean geometry.

The first set of learners who wrote paper three at matric level was in the year 2008. Howie et al (2010, p.1) claim that in 2008 only 3.8% (12 466) of the Grade 12 mathematics learners nationally wrote the optional Paper 3 and of those who wrote Paper 3, almost half (6 155) achieved less than 30%. According to the National Diagnostic Report on learner performance in the 2012 Matric Mathematics paper 3, a larger number of candidates showed greater proficiency in the answering of statistics and probability sections than the Euclidean Geometry section. This indicates that the learners appear to be more competent in statistics and probability than Euclidean geometry. The poor results from the National Diagnostic Report on learner performance in the 2012 Matric

Mathematics paper 3, suggest that the learners have difficultly when working with geometry.

My exploration of visualisation in the solving Euclidean Geometric problems may contribute towards the teaching and learning of Euclidean Geometry. Van Putten (2008, p.16) declares that a teacher's negative attitude towards geometry because of his/her difficulties with geometry can influence the learners attitude towards geometric problems. The use of visualisation could make the teachers and learners geometric experiences more rewarding. The change or the incorporation of visual methods in the teacher's pedagogic practices in geometry may also contribute towards an improvement in the teachers and learners understanding of Euclidean geometry.

1.3. Key research questions

My research questions are:

- Is visualisation an integral part of proof? Why?
- Can learners use diagrams with guidance to find geometric solutions? Why?

Euclidean geometry is the study of geometry based on the assumptions made by the Greek mathematician Euclid. Within the context of my study visualisation would refer to the sketching of diagrams or the use of symbols while solving Euclidean geometry problems. It also extended to the learner envisaging a picture of the problems that were given.

1.4. Structure of the study

The study comprises of six chapters. Chapter one introduces the study. The background to the study and the key research questions are presented here. Chapter two provides the literature review about the research topic. This chapter includes a discussion on visualisation, visual literacy and diagrams. The literature reviewed supports the view that visualisation is an important aid in the understanding of mathematics. The literature discussed the role of visualisation in the teaching and learning of mathematics, problem solving using visualisation, proof and visualisation. I found that there were very few articles which focused on Euclidean geometry proof and visualisation. Majority of the literature focuses on the role of visualisation in the teaching and learning of mathematics. My study will involve visualisation in the form of diagrams. According to Winn (1987) a diagram is an abstract visual representation that exploits spatial layout in a meaningful way.

Chapter three examines the theoretical framework. The researcher has made use of Mudaly's (2012) adaptation of Kolb's Experiential Learning Theory. Experience based learning, Kolb's Experiential Learning Theory, learning styles, the Learning Style Inventory and the adaptation of Kolb's Experiential Learning Theory is deliberated. The focus is on how learner's experiences with the diagrams in the activities lead to learning.

Chapter four explains the research methodology used in this study. How the research was conducted in terms of the methodological approach, research design, research context, the sample, data/collection, analysis and ethical issues is looked at. My study used a mixed method approach since it will consisted of both qualitative and quantitative data analysis methods. I will made use of responses to classroom exercises, semi – structured one to one interviews, structured observation and semi-structured questionnaires for my data collection. The classroom exercise responses and questionnaires were quantitatively analysed and the interviews and observation were qualitatively analysed. The participants for my research were five grade eleven mathematics learners. Their participation was voluntary and they were randomly selected.

The study was conducted within the interpretative approach. According to Cohen, Manion and Morrison (2007) the focus of the interpretive paradigm is to seek to understand from within the subjective world of human experience. Hennie, Rensburg and Smit (2004) state that the interpretative researcher is encouraged to used various methods of data collection and analysis to ensure validity of results. It was appropriate since my aim is not to predict but to comprehend the role of visualisation in the proving process of Euclidean Geometry problems.

Chapter five presents the analysis of the study. This chapter deals with the findings and analysis of the data obtained from the learners responses to the activities, observation, questionnaires and interviews. Figures and tables were used to show the learners responses to the activities. Chapter six includes the discussion of the research findings. It provides a further discussion of the analysis; recommendations and the limitations of this study.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter begins with a brief history of visualisation. This is followed by a definition of visualisation, the value of visualisation, the processes involved in visualisation, underpinning skills of visualisation and the effects of visual literacy in mathematics. I also discuss the history of diagrams, the advantages and difficulties associated with the use of diagrams and Euclidean geometry. This chapter ends with an examination of Euclidean geometry in South Africa and the Van Hiele Theory.

2.2 History of Visualisation

Visualisation is not a new discovery in Mathematics. Stylianou (2002) reveals that visual reasoning in Mathematics can be traced back to Mesopotamia and Greece. This research study focuses on visualisation and Euclidean geometry. Euclidean geometry which is the study of plane geometry is named after Euclid of Alexandria who was born around 330 BC. Euclid wrote a book about geometry called The Elements. Euclid arranged the geometrical ideas into definitions, axioms and theorems. His arguments in the book greatly relied on geometric figures or shapes. This indicates that visualisation has been used in Mathematics for more than 2000 years. The value of visualisation has been explored by numerous researchers (Arcavi (2003); Hanna and Sidoli (2007); Mancosu (2005); Presmeg (2006) and Zimmerman and Cunningham (1991)). According to Presmeg (2006, p.28) research in visualisation started slowly in the late 1970s and early 1980s. During this period research on visualisation in the teaching and learning of mathematics focused on theoretical psychology. She goes on further to state that it was in the 1990s that research in visualisation became a substantial field in mathematics education and that it was only in the 2000s that the research in visualisation began to include semiotic aspects and theories. This infers that visualisation has been an area of interest for researchers for more than 30 years.

2.3. What is visualisation?

To visualise can mean to see something either in your mind or physically. In Euclidean geometry visualisation would refer to diagrams that are either seen physically or mentally. Arcavi (2003,p.217) defines visualisation as the ability, process and product of creation , interpretation, use of and reflection upon pictures, images, diagrams, in our minds , on paper or with technological tools. Zimmermann and Cunningham (1991, p.1) describe visualisation as a process of constructing or using geometrical or graphical representations of mathematical concepts, principles or problems. While Mudaly and Rampersad (2010,p.38) state that visualisation can be a physical or mental process , Lavy (2006,p.25) describes visualisation as a process of construction or use in geometrical or graphical presentations of concepts, or ideas built by means of paper and pencil , computer programs or imagination. These definitions emphasize that visual can be either mental or physical and that visualisation of mathematics problems can take place in our minds, on paper or by using technology. In this research the focus will be on the role of visualisation in solving / proving Euclidean geometry problems.

2.4 Value of Visualisation

Jacques Salomon Hadamard was a French mathematician who contributed towards Mathematics. He surveyed scientists and mathematicians and questioned them about how they solved problems and discovered new concepts. Some of the mathematicians he spoke to were Albert Einstein, Carl Friedrich Gauss and Henri Poincaré. According Thornton (2001) Hadamard found that these prominent mathematicians used pictures to cultivate their thoughts and only used formal algebraic conventions when they had to share their results with others. This would indicate that since leading mathematicians used images to develop their thoughts, images could definitely be considered useful to learners when working with Euclidean geometric problems. This then is something the teachers of mathematics can take into account when planning their daily pedagogic practices.

Wheatley (1997, p.281) argues that "visual reasoning plays a far more important role in the work of today's mathematicians than is general acknowledged". Arcavi (2003, p.235) states that the significance of visualisation is becoming recognized. He further declares that visualisation is being acknowledged as an important component in reasoning, proving and problem solving. Barbosa (2007, p.2) also asserts that the value of visualisation is being recognised by mathematics educators. This escalation of the realisation of the value of visualisation implies that it should be used to improve teaching practices. Konyalioglu, S, Konyalioglu, A.C,. Ipek, S., and Ahmet, (2005, p.2) proclaim that visualisation assists learners by making mathematical concepts concrete and clear to understand. According to Dörfler (2004, p.1) visualisations assist with understanding, insight, invention and detection.

Piggott and Woodham (2008, p.28) identify three purposes for visualising. These are to gain a better understanding of a problem, to model and to plan ahead. Yilmaz, Argün and Keskin (2009, p.131) state that visualising a concept or a problem refers to a mental image of the problem and to visualize means to understand the problem in terms of that image. Zimmermann and Cunningham (1991, p.3) declare that the purpose of visualisation is to assist with mathematical understanding and discovery. They however emphasize that vision is not visualisation and that if one sees it does not automatically mean that one understands. A development of understanding seems to be one of the key advantages of making use of visualisation. Visualisation appears to have an array of advantages which can be of beneficial in improving learners understanding of mathematics.

Hanna and Sidoli (2007, p.74) claim that visualisation is mainly applicable for communication. They further explain that diagrams can be used for explaining, suggesting or attaining higher levels of conviction. Arcavi (2003, p.217) declares that the purpose of visualisation is to show and communicate information, to discover new ideas and to build upon previous knowledge. Communication of information appears to be a common trait of visualisation. Rolka and Röske (2006, p.458) assert that visualisation decreases the complexity when dealing with a significant amount of information. This could be beneficial to learners when working with geometric problems. Zimmermann and Cunningham (1991, p.4) claim that visualisation not only gives depth and meaning to the understanding of mathematical problems but it also encourages creative discoveries. Yilmaz, Argün and Keskin (2009, p. 132) declare that visualisation is an important part of mathematical understanding, vision and thinking. Piggott and Woodham (2008, p.27) state that visualisation aids in the development of ideas towards a solution and in the identification of the central components of a problem and the relationship between them. Since Stylianou and Silver (2004) declare that the role of visualisation in the teaching and learning of mathematics is growing teachers should make more of an effort to encompass visualisation in mathematics lessons. This is re-enforced by Presmeg (1986). She states that a visual method involves a visual image which forms an essential part of a solution.

These declarations highlight that visualisation can be used as a powerful tool in mathematics. There are several reasons such as increasing the understanding of a problem, discovering new concepts from symbols inherent in a diagram and decreasing the difficulty of a problem that support the use of visualisation in mathematics. Although they all stress the importance of visualisation, according to Presmeg (1995) despite the many advantages of visualisation it is still undervalued in the classroom. Elliott (1998, p. 46) agrees with this states that more emphasis is placed on algebraic than visual methods for solving mathematical problems. Arcavi (2003, p.226) declares that in spite of the apparent value of visualisation it is still underappreciated in the practice and the theory of mathematics. According to the above researchers there are various advantages to why visualisation should be used in the classroom. However despite its obvious significance its presence in the practice of mathematics is underestimated.

2.5 Processes involved in visualisation

Mudaly and Rampersad (2010, p.39) describe processes involved in visualisation. In the context of this research the visualisation process begins with the learner seeing an image which is a diagram or a mental image. This image will have a certain meaning for the learner. The learner's meaning of the image will be influenced by the learner's previous knowledge. Meaning of the image is developed through reflection, interaction with the new stimuli and other given data. Through this activity, internalization and externalization of knowledge takes place. New knowledge is created through the learner's reflection and interaction with their previous knowledge. Mudaly and Rampersad (2010, p.39) explain that the internalized new knowledge is used to influence what is seen or added to the new diagram. These new changes / knowledge then impacts on the way the learner views the same image. Mudaly and Rampersad (2010, p.39) state that diagrams allow the learner to change or cultivate new knowledge.

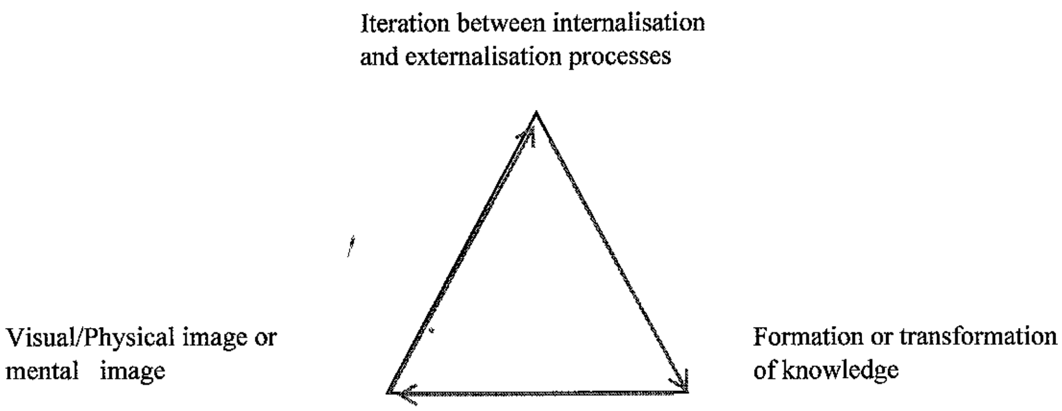


Figure 1: Processes involved in visualisation. Source: Mudaly and Rampersad (2010, p.39)

Siew Yin (2010) made note of five processes and seven roles of visualisation. The five processes that were identified by her were understanding, connecting, constructing, using the visual representation to solve the problem and encoding to answer the problem. The process of understanding refers to the relationships between the components of the problem. The connecting process deals with making links to problems that were solved in the past. The forming of a visual image is what the process of construction consists of. The visual representation is then utilized to solve the problem and encoding is the process whereby the problem is answered.

The seven roles that she documented were understanding the problem, simplifying the problem, connecting to a related problem, catering for individual learning styles, substituting for computation, as a tool to check the solution and to transform the problem into mathematical form. Visualisation in the role of understanding helps learners to understand the relationship between the different components of the problem. In the role of simplifying a problem visualisation allows learners to recognise a less complicated version of the problem. The learners solve, understand and then find an approach that will successfully solve all those type of problems. In the role of connecting to related problems the learners relate the given problem to their past problem solving encounters. With regards to accommodating for individual learning styles, when solving problems each learner has their own way of using visualisation.

Visualisation serves as an alternative for computation. Learners can solve a problem visually without performing computations. Another role visualisation plays is as an instrument to verify a solution. It can be used to check if an answer to a problem is within reason. The seventh role identified was to put the problem into mathematical form. From visual representation, mathematical forms may be identified. She stated that the above seven roles are the functions that visualisation can play when solving mathematical problems.

Diezmann (2000, p.1) explains that writing information on a diagram is a translation process that is made up of decoding linguistic information and encoding visual information. During this process the learner's knowledge is reorganized and new knowledge can be created. This suggests that when learners view a question and the diagram all that they write down on the diagram reflects their understanding of the question.

2.6. Underpinning skills of Visualisation

Zimmermann and Cunningham (1991, p.5) state that the ability to draw a figure, to interpret the figure with understanding and to use the figure to assist in solving problems are fundamental visualisation skills.

Piggott and Woodham (2008, p.29) identify five skills which they feel are necessary for visualisation. These skills are internalizing, identifying, comparing, connecting and sharing. Internalization would refer to the learner being able to concentrate on the problem and then identifying the most relevant characteristics about it. Identifying focuses on identifying an idea that is meaningful to the learner. With comparing, the learner must be able to inspect different images and recognize what is different or the same. In connecting the learner is being able to make associations by recalling processes instead of the individual images. In the last skill which is sharing, the learner should be capable of recounting his/her personal visualisation to the class.

Siew Yin (2010,p.4) suggests that if teachers want to assist learners in developing visualisation skills, the curriculum material developers and the teachers must become aware of the factors that guide learners choices of methods for solving and the processes and roles that visualisation play in mathematical problem solving. Thornton (2001, p.255) makes suggestions for learners to become more efficient visual thinkers. He states that when learners are working with geometric proofs they must draw three diagrams. One must be for a special case, one for a general case and one for a counter example. Thornton (2001,p.255) declares that the learner must question why the result is true in the special case, if it is true in the general case and why it is not true in the counter example. Diezmann (2000b, p.7), states that learners' lack of skills in diagrammatic representation are responsible for their difficulties and errors in creating precise and efficient diagrams. This reinforces the significance of having visualisation skills. By taking into account the above mentioned suggestions made by researchers both teachers and learners may be able to improve upon their visualisation skills.

2.7 The effects of Visual literacy in Mathematics

Literacy refers to the ability to read and write. Visual denotes that which can be seen physically or mentally. Bamford (2003, p.1) declares that the term 'visual literacy' was created by writer John Debes in 1968. Bamford (2003, p.1) refers to the terms visual literacy as what is seen with the eye and mind. Wileman (1993, p.114) defines visual literacy as "the ability to 'read,' interpret, and understand information presented in pictorial or graphic images". Bleed (2005, p.5) gives several definitions of visual literacy. One of Bleed's (2005, p5) definitions of visual literacy is a "group of competencies that an individual can develop by seeing and at the same time having and integrating other sensory experiences." According to Mudaly (2008, p. 4) visual literacy refers to the internal processes that the mind goes through after viewing an image. This image can be mental or physical. He further declares that visual literacy is concerned with the product of the visual stimulus. An example of the visualisation will be a physical or mental picture of a quadrilateral. Visual literacy would refer to many images of quadrilaterals such as a square, rectangle or a parallelogram. Stokes (2002, p.10) defines visual literacy as the learner's proficiency in interpreting and generating images for communication of concepts.

Mudaly (2008, p.4) states that visualisation linked with logical thought is visual literacy. This would imply that if learners are visually literate then they should be able to think rationally and logically when they see an image physically or mentally. This would increase the learner's chances of successfully solving mathematical problems. Sosa (2009) states the biggest problem with developing visual literacy is that teachers do not know how to judge images themselves because they have never been given any formal training. This would imply that in order to make learners visually literate, the teachers need to be visually literate. Bleed (2005, p.10) declares that visual literacy helps teachers connect with learners and the quality of learning is enhanced.

Heinich, Molenda, Russell, and Smaldino (1999) suggest two ways for developing visual literacy skills. In the first method learners are assisted in interpreting and creating meaning from visual stimuli or reading visuals through analysing techniques. The second method is to assist learners to write or encode visuals for communication. Stokes (2002, p.11) states that visualisation assists one in making sense of things that previously would have been considered as incomprehensible. For learners to solve mathematical problems rational and logical thinking is essential. It would then that seem that if learners become visually literate by teachers who are formally trained in visualisation, then their likelihood of solving mathematical problems would increase.

2.8 Diagrams

2.8.1 History of diagrams

Alshwaikh (2009, p.2) declares that diagrams are part and parcel of mathematics. This may imply that mathematical diagrams are of essence in Mathematics According to Robson (2008), diagrams have been used almost four thousand years ago by ancient civilizations such as Old Babylon. Geometric diagrams have been found on Babylonian clay tablets that could be dated back to about 1700 B.C. Geometric diagrams were also discovered in Chinese, Egyptian and Indian mathematics Netz (1999) says that diagrams played a vital role in Greek mathematics. Jamnik (2001, p.2) also reveals that in Ancient Greece and during the time of Aristotle and Euclid, diagrams were used to explain theorems and for proofs in geometry. This indicates that diagrams have played an important role throughout the ages in the history of mathematics.

2.8.2 What is a diagram?

A diagram is described by Mudaly (2012, p.22) as being able to give a physical form for a mental form. He explains that diagrams should allow the person viewing the diagram to see a complete picture in their mind. Mesaroš (2012, p. 321) portrays a diagram as being an illustrative tool of visualisation and Winn (1987) depicts a diagram as a visual representation that uses spatial arrangement in a profound way. Diezmann (1995, p.223) states that diagrams may consist of words and/or abstract pictures. Diezmann and English (2001) assert that everyone should be able to interpret and produce general purpose diagrams. Diagrams in Mathematics can be looked upon as drawings or sketches to represent something that we see physically or in our mind.

2.8.3 Advantages of using diagrams

Diagrams fulfil several roles in mathematics. Alshwaikh (2009, p.2) states that "there is nearly consensus that diagrams are important in doing, learning and teaching mathematics mainly in visualisation, mathematical thinking and problem solving". Agathangelou, Papakosta, and Gagatsis (2008, p.1) declare that visual representations play an essential role since they support reflection and communicate mathematical ideas. Plummer and Bailin (1997, p. 26) state that diagrams play a vital role in in communicating mathematical meaning. Mudaly (2012, p.30) states that self-explanatory diagrams help learners cultivate a better understanding of the mathematical problem. Van Blerk, Christiansen and Anderson (2008, p.1) declare that geometric diagrams are not just simple images because they contain conceptual information. Kutama (2002, p.10) states that when learners draw diagrams by themselves their understanding of Euclidean geometry increases. He explains that by drawing, learners are able to observe some relationships on the diagram that they would have not noted if they had not redrawn or drawn the diagram themselves. According to the above researchers diagrams enhance learners understanding in mathematics and they also aid in conveying mathematical meaning about the problem to the learners. This implies that diagrams may assist learners in making sense of mathematical problems.

Carney and Levin (2002) proposed five functions that pictures serve in text processing. These are decorative, representational, organizational, interpretational and transformational. Diezmann (2004, p. 81) declares that mathematicians have being aware of the value of diagrams as cognitive tools for a long time. Diezmann (1999, p.1) claims that the benefits of creating a diagram are related to its efficacy as a cognitive tool. According to Bertel (2005, p. 1) diagrams and sketches are important for cognitive tasks. Diezmann (2004, p.81) claims that diagrams have three cognitive advantages. She declares that diagrams assist in the identifying of the key concepts in a problem. Diezmann (2004, p.81) explains "diagrams are an inference making knowledge representation system that has the capacity for knowledge generation." Thirdly, Diezmann (2004, p.81) declares diagrams, support visual reasoning. Although diagrams can be effective as cognitive tools Diezmann (1999, p.7) suggests that teachers must find out what support is needed in order for learners to successfully make use of the diagrams as cognitive tools. In the classroom this would imply that the teacher must examine the mathematical problem and then analyses what cognitive skills are necessary in order to solve it before it is given to the learner to work on.

If diagrams are used successfully the diagrams can be utilized to show the validity of the proof. Bardelle (2005, p.252) proclaims that diagrams support proof processes and are pertinent to visual proofs. Giaquinto (2007) maintains that visualisation can be used for discovery, justification and proof." Lomas (2002, p.209) argues that diagrams are the focus in many geometric proofs. Plummer and Bailin (1997, p. 25) comment that diagrams play a fundamental role in the communicating and understanding of mathematical proof. Carter (2009, p. 2) states that using pictures and diagrams in proofs has played a vast role in mathematics. Since it seems that diagrams are useful in proving, learners can be encouraged to make use of them when working with geometric riders and when trying to prove concepts. Mudaly (2010b, p.176) comments that diagrams are remarkable tools for making sense in mathematics.

Dossena and Magnani (2005, p. 765) describe two roles that diagrams play. They state that diagrams can help one to understand things that are difficult to grasp and that they can help produce new knowledge. In the classroom this would imply that learners can gain a better understanding of concepts and that they can create new concepts that were previously unknown to them. Mudaly (2010b, p.176) declares that self-explanatory diagrams assist learners to develop an understanding of the mathematical problem. He further states that it is this understanding which aids in the solving of the problem. This implies that that if the diagrams assist learners in understanding the key concepts in the given problem the learners should be able to attain the necessary answers for a correct solution or proof.

Wheatley (1997, p.285) declares that when mental images or diagrams are created more mental space is made available for new images and relationships to be constructed. Dossena and Magnani (2005, p.764) declare "diagrams allow us to overcome the difficulty in constructing representations of mathematical critical situations and objects". Mudaly (2008, p.6) concluded from his research that the use of dynamic diagrams assisted learners in making conclusions for conjectures Bardelle (2009, p.257) states from the findings of her research that figures are tools which help to find results. This infers that by learners making use of diagrams they can arrive at more accurate conclusions about the relationships between the components in the diagrams. Kutama (2002, p.10) declares that diagrams allow learners to notice relationships between concepts that they would have not noticed if they had not redrawn the diagram. Mudaly (2012, p.29) pronounced from his research that when the diagrams sketched were realistic, the chances of getting a correct solution was greater. This draws attention to the quality of the diagram that the learner sketches. It suggests that a correct diagram may lead towards a more accurate solution whereas an incorrect diagram maybe misleading. Bertel (2005, p.4) claims that actions carried out on external diagrams reflect actions carried out mentally. This implies that the teacher can use the learner's actions on diagrams to gain a better understanding of the learner's mental reasoning. This in turn may be used by the teacher to correct any misconceptions or misunderstandings that the learner may have. It can help improve upon learners understanding of certain problematic concepts or prevent the teacher from wasting time on re teaching concepts which the teacher assumes to be problematic. The National Diagnostic Report on learner performance in the 2012 Matric Mathematics paper 3 suggests that teachers teach learners to write information given onto the diagram. It goes on further to say this will contribute towards the learner's ability to solve geometric riders.

The literature has indicated that there are different functions of diagrams such as the promotion of cognition and understanding about mathematical problem, arriving at conclusions and supporting the proving process. Diezmann (1999, p.7) proposes that teachers develop exemplars of fundamental problem components and recognize the levels of learners diagram production. She claims that in doing this, teachers will be able to comprehend and attend to concerns of learner diagram quality. Diezmann (1999) asserts that this will also help teachers to improve the effectiveness of the use of diagrams as cognitive tools. These advantages support Dreyfus's (1994, p.233) argument that visualisation is not just an aid but a vital tool in mathematics.

Schönborn and Anderson (2008) identified three factors which they feel affect the learners' ability to interpret diagrams. The first one is learner's previous conceptual knowledge of significance to the diagram. This is followed by the learners reasoning skills that they use to envisage the diagram and the mode in which the concepts are displayed on the diagram. Teacher's consideration of the above factors may result in an improvement of learner's interpretations of diagrams. Diezmann (2004, p.82) suggest that learners must be diagram literate if diagrams are to be used in teaching, assessment and learning.

2.8.4 Difficulties when using Diagrams

According to Stylianou (2002, p.305) although the use of diagrams in the 18th century was a well -accepted practice in mathematics but in the 19th century it lost its credibility. The reason Stylianou (2002, p.305) claims for this is that in many cases using diagrams became misleading. Presmeg (1986, p.44) noted three problems with the use of diagrams. The first one was that the diagram may draw attention to unnecessary details. The next reason was that diagrams may prevent one from recognizing certain concepts and the last reason was that a complex diagram may prevent one from developing profounder thoughts. Kutama (2002, p.10) declares that if the learner sketches an incorrect diagram their ability to solve the problem may be impeded. Teachers need to consider these possible challenges that learners may encounter when they providing diagrams for learners to solve mathematical problems.

Deriving from Eisenberg and Dreyfus's (1991) research, Arcavi (2003, p.235) classifies difficulties associated with visualisation into three main categories. These are 'cultural', cognitive and sociological. He explains that a 'cultural 'difficulty has to do with the values and beliefs that the learners have about mathematics. This would refer to what the learners feel is acceptable and unacceptable in mathematics and what doing mathematics is all about. The cognitive difficulties make reference to whether thinking visually is difficult or easy. One of the cognitive difficulties he describes is whether the learner is able to make a proficient translation to and fro between visual representations and analytical representations. The sociological difficulties make reference to matters of teaching. Arcavi (2003, p.236) declares, "...many teachers may feel that analytic representations, which are sequential in nature, seem to be more pedagogically appropriate and efficient."

Mumma (2010, p.1) states "the worry is that once diagrams with their rich array of spatial properties are allowed to represent geometric objects, the ability to isolate the standing of each claim in a geometric argument is compromised." He elaborates that noticing something in a diagram may become too complex. Lomas (2002, p.208) says that diagrammatic proofs are not given much credit by logicians and mathematicians. He explains further that they are thought of as just being aids to proof because of the generalization that can occur from unplanned characteristics of a diagram. This draws attention to Presmeg (1986, p.44) declares that a diagram may draw attention to unimportant details. This could distract the learners and lead to them being unable to solve the mathematical problems. Once again teachers need to be mindful of the possible challenges learners could face if when they are providing a diagram to solve / prove a geometric problem. The teachers need to ensure that the diagram is not too complex or distracting to prevent learners from being misled.

Dreyfus and Eisenburg (1991) state that generating and processing visual representations is challenging for most learners. They put forward three reasons why analytic processing is simpler for the learners than visual processing. The reasons they put forward were beliefs, information processing and cognitive efficiency. With regards to belief they stated that the teacher gives the learner the impression that visual methods in mathematics are subordinate to analytic methods. They explained that this takes place when it is emphasized that an image is not proof. With regards to the processing of information they explain that mathematicians use visualisation in their own work but when it comes to conveying that same work these mathematicians make use of analytical means. Cognitive processing deals with the complexity of diagrams. They declare that since diagrams can be so multifaceted it needs cognitive processing to make to understand it.

Lavy (2006, p.25) discusses difficulties with diagrams with regards to visual thinking. She points out that a diagram or image may draw thoughts to unimportant details or lead the viewer to wrong data. Lavy (2006, p.25) argues that a standard diagram or image may encourage rigid thinking and in the future this could prevent the learner from recognizing the same concept in a non-standard diagram. Learners therefore need to be exposed to various diagrams based on the necessary concepts. From the literature reviewed it becomes evident that researcher's greatest concern with regards to the use of diagrams in mathematics is that it can distract the learner and lure their attention on to that which is irrelevant. Teachers need to take all this into consideration if they wish to make use of diagrams in mathematical problems.

2.9 Euclidean Geometry

2.9.1 What is Euclidean Geometry?

Geometry is a part of Mathematics that deals with spatial relationships. Atebe (2008, p.13) defines geometry as a study of the "properties of spatial objects and the relations between those properties". It is a branch of mathematics that deals with relationships, properties and measurement of angles, lines, points, solids and surfaces. Nixon (1892, p.1) states that "geometry is the Science which treats shape, size and position of figures: it is based on definitions, axioms and postulates: these granted, all the rest follows by reasoning." Howie, Putten and Stols (2010, p.1) argue that geometry teaches us life skills such as logic and reasoning. Howie et al (2010, p.1) declare that the core skills that one learns in geometry are logical thinking and reasoning. They go further to state that these skills are not just important in geometry but for everyday life. This emphasizes the importance of geometry in the curriculum. According to Van Putten (2008, p.15) the life skills honed by geometric reasoning is significant.

Euclid of Alexandria was born around 300 BC. He was a Greek mathematician who was called the "father of geometry". He was the author of the Elements. Euclidean Geometry is based on his work called The Elements. The Elements is made up of thirteen books. The Elements is made up of definitions, theorems, axioms, constructions, mathematical proofs. Book one is about the fundamentals of plane geometry involving straight lines. The other books explore topics such as fundamentals of plane geometry involving the circle, proportion, similar figures, constructions of rectilinear figures and platonic solids. The definitions, theorems, axioms, constructions, mathematical proofs and other fundamentals of plane geometry that are taught in schools can be found in The Elements. Atebe (2008, p.30) declares that geometry is significant because it helps develop learners deductive reasoning and logically skills

2.9.2 Euclidean Geometry in South Africa

When the FET (Further Education Training) curriculum was implemented in 2006 Euclidean Geometry was embedded into Mathematics Paper three and became optional. Learners who wanted to study Euclidean Geometry had to take Mathematics paper three as an additional subject. According to the Department of Education (2009), in 2008 3, 8% of the Grade 12 mathematics learners nationally chose to write paper 3. The results indicated that of those learners almost half of them attained less than 30%. According to the National Diagnostic Report on learner performance in the 2012 Matric Mathematics Paper three, a larger number of candidates showed greater proficiency in the answering of statistics and probability sections than the Euclidean Geometry section. This indicates that the learners appear to be more competent in statistics and probability than Euclidean geometry. The report suggests that teachers teach learners to write information given onto the diagram. It goes on further to say this will contribute towards the learner's ability to solve geometric riders.

According to Bowie (2009) one of the reasons that Euclidean Geometry was made optional was the understanding that teachers were unfamiliar with its content. De Villiers and Dhlamini (2013, p.101) are of the view that a mathematics teachers proficiency in mathematics will determine how the teacher will teach it.

In 2012 the CAPS (Curriculum and Assessment Policy Statement) replaced the NCS (National Curriculum Statement). The optional Mathematics Paper three is now integrated in the compulsory Mathematics Paper one and two. From 2012 onwards only two mathematics papers will be written in grade 10. By 2014 all learners studying mathematics from Grade 10 to Grade 12 will be tested on Euclidean geometry in their second mathematics paper .This would suggest that from 2006 to 2011 only a few mathematics teachers and Grade ten to Grade twelve learners participated in the teaching and learning of Euclidean geometry. Now that it has become compulsory all mathematics teachers and learners will be involved with this area of mathematics. An absence of Euclidean geometry for six years would have implications for both the teaching and learning of Euclidean geometry. Currently in South African schools it is compulsory for all learners to be taught geometry until Grade 9. After Grade 9 these learners can choose to take Mathematics or Mathematical Literacy.

2.9.3 The van Hiele Theory

The van Hiele theory was developed by Pierre van Hiele and his wife Dina van Hiele-Geldof in separate doctoral dissertations in 1957. The van Hiele theory describes how learners learn geometry. It helps us to understand why learners experience difficulties in geometry. Ndlovu (2013,p.277) comments that the reintroduction of Euclidean Geometry into the curriculum requires that both teachers and curriculum material writers reacquaint their understanding of the van Hiele ideas on the teaching and learning of geometry.

2.9.3.1The van Hiele Levels

The van Hiele theory states that there are five Levels to describe how learners learn to reason geometry. The five levels are sequential and hierarchical. The progress from one level to another is more dependent on mathematical experiences than on chronological age. In the original works the levels were numbered from zero to four. The Americans started numbering the levels from one to five. In this study it will be labelled from one to five. They are (adapted and summarised from Mason (1998) and Usiskin (1982) :

2.9.3.1.1 Level One: Visualisation

At this level learners recognise basic figures by appearance without paying attention to parts, attributes or properties. The properties of the figures / shapes are not perceived. Learners make decisions based on perception and not reasoning. The learner can learn the names of figures and can recognise a shape as a whole. E.g. squares and rectangles seem to be different.

2.9.3.1.2 Level Two: Analysis

Learners view figures as collections of properties. They can identify the properties of figures. They can recognise and name the properties of the geometric figures, but they cannot see the relationships between the figures. They cannot discern between the necessary and sufficient properties of an object. They have an inability to consider an infinite variety of shapes. E.g. squares and rectangles seem to be different.

2.9.3.1.3 Level Three: Abstraction

At this level children recognize relationships between types of shapes. They can create meaningful definitions and give informal arguments to justify their reasoning, but the role of formal deduction is not understood. This means that simple deduction can be followed but proof is not understood. Kutama (2002,p.3) declares that in order to solve Euclidean geometry problems learners must have deductive skills, an accumulated knowledge of theorems and be at van Hiele level 3. He explains further that it is at this level where learners should be able to write proofs of theorems using deduction.

2.9.3.1.4 Level Four: Deduction

The learner understands the significance of deduction and roles of postulates, theorems and proof. They can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. They should be able to construct proofs such as those found at high school level.

2.9.3.1.5 Level Five: Rigor

Learners understand the necessity for rigor and are able to make abstract deductions. They understand how geometry proofs and concepts fit together to create the structure geometry. They understand the formal aspects of deduction such as establishing and comparing mathematical systems. They can understand indirect proof and proof by contrapositive and non-Euclidean systems.

2.9.3.2 Properties of the Levels

The van Hiele Levels have five properties (adapted and summarised from Mason (1998) and Usiskin (1982) :

2.9.3.2.1 Fixed sequence

The levels are hierarchical. This means that they cannot miss out a level. The learner cannot be at a certain level without having gone through the previous level.

2.9.3.2.2 Adjacency

Properties which are intrinsic at one level become extrinsic at the next.

2.9.3.2.3 Distinction

Each level has its own linguistic symbols and network of relationships. If a learner is simply given the definition and its properties, without being allowed to develop meaningful experiences with the concept, the learner will not be able to apply this knowledge beyond situations used in the classroom.

2.9.3.2.4 Separation

Two persons who reason at different levels cannot understand each other. The van Hiele's believed that this was one of the main reasons for failure in geometry. Teachers may believe that they are expressing themselves clearly and logically, but their Level 3 or 4 reasoning may not be understandable to learners at lower levels. Teachers will not be able to understand the learners thought processes.

2.9.3.2.5 Attainment

The van Hiele's recommend five phases for guiding learners from one level to another. The phases are:

2.9.3.2.5.1 Information

Through discussion, the teacher identifies what learners already know about a topic and the learners become oriented to the new topic.

2.9.3.2.5.2 Guided orientation

Learners do tasks that enable them to explore implicit relationships. They explore the objects of instruction in carefully structured tasks such as folding, measuring or constructing. The teacher ensures that learners explore specific concepts.

2.9.3.2.5.3 Explication

Learners express what they have discovered in their own words and relevant mathematical vocabulary is introduced.

2.9.3.2.5.4 Free orientation

Learners do more complex tasks enabling them to master the network of relationships in the material. They apply the relationships they are learning to solve problems and investigate more open – ended tasks.

2.9.3.2.5.5 Integration

Learners summarize and integrate what they have learned, developing a new network of objects and relations. The teacher may give the learners an overview of everything that they have learned. The teacher may give the learners an assignment to remember the principles and vocabulary. Usually traditional instruction only involves this last phase.

The van Hiele properties state that the levels are hierarchical. This implies that a learner cannot reach Level 3 without having gone through Level 2. On examining the levels, one would notice that visualisation is a prerequisite for the other levels. According to de Villiers (2010) the attainment of Level 2 involves the acquisition of the technical language by which the properties of the concept can be described. He further declares that transition from Level 1 to Level 2 involves recognizing certain new relationships between concepts and the refinement and renewal of existing concepts. Level 3 involves the logical relationships between the properties of the figures. According to the theory learners who are below level 3 can only do proofs by memorisation. De Villiers (2010) translates Van Hielie (1973: 94) assertions that learners are only ready for Level 3 when their network of relations in Level 2 is adequately established.

The van Hiele theory declares that instruction developed according to the five sequential phases of learning: inquiry, directed orientation, explication, free- orientation and integration promotes the acquisition of a level. Crowley (1987) states that the van Hieles writings encourage that children are presented with a wide variety of geometric experiences. The van Hiele Levels has five properties and according to the property of distinction each level has its own linguistic symbols and network of relationships. This property asserts that the learners must be allowed to develop meaningful experiences with definitions and properties of shapes for advancement through the levels. This infers that learners need to be given van Hiele Level appropriate activities to allow them to progress from their current level to the next.

This theory provides insight into why learners experience difficulties in geometry, I think that every mathematics teacher should be made aware of it. If teachers identify the van Hiele levels of their learners and then utilise the van Hiele's five phases, learners can be guided from one level to the next. By teachers changing their pedagogic practices to cater for the learner's inadequacies, the learners will be able to ascend to higher levels. Van Blerk et al (2008, p.1) state that if learners want to be able to be successful in proof in geometry then they must be able to decode diagrams in the questions that are given. The visual level is the first van Hiele level and according to the theory, failure at this level prevents the learners from being able to prove mathematical problems successfully. The theory suggests that if the learners are able to successfully function on the visualisation level then they will be able to progress to the next level. This reasserts the importance of visualisation in geometry.

Usiskin (1982) states that the van Hiele theory's ability to describe and predict behaviour and to prescribe procedures for the attainment of levels (of thinking) are important attributes. De Villiers and Dhlamini (2013, p.116) states that there is a need to investigate how geometry is taught and to see how the learners level of development is considered in terms of the van Hiele theory. Atebe (2008, p.60) declares that the van Hiele theory provides insight into why learners have difficulties in formal proving and in geometry. It would therefore seem that by making use of the van Hiele theory, teachers can contribute towards learner's comprehension of geometric concepts.

2.10. Conclusion

This chapter discussed from a literature perspective visualisation in Mathematics. Using diagrams in Euclidean geometry can be both advantageous and challenging. The chapter concludes with a look at the van Hiele theory and how knowledge of this can benefit the learner's performance in geometry.

CHAPTER THREE

THEORETICAL FRAMEWORK

3.1. Introduction

In this chapter the researcher looks at the theory that underpins this study. The researcher has made use of Mudaly's (2012) adaptation of Kolb's Experiential Learning Theory. Experience based learning, Kolb's Experiential Learning Theory, learning styles, the Learning Style Inventory and the adaptation of Kolb's Experiential Learning Theory is discussed. The focus is on how learner's experiences with the diagrams in the activities lead to learning.

3.2. Experience - Based Learning

Experience based learning refers to learners gaining knowledge or skills through their experiences. It can be viewed as a learner – centred approach of learning. Andresen, Boud & Cohen, (2000, p.225) declare that the most prominent feature of experience based learning is that the teaching and learning focuses on the learner and their experiences. These experiences can be from the learners past and/or present. It is the experiences of the learner that lead the learner towards the creation of new knowledge. Andresen et al (2000, p.225) declare that a significant trait of experience based learning is that the learner's examine their experiences they undergo through reflecting, evaluating and reconstructing. Learners construct their own meaning about a context via their personal experience. This type of learning can take place individually or in groups. Some features of experience based learning that differentiates it from other learning approaches are discussed by Andresen et al (2000, p.225). They assert that the learning takes place through the involvement of the learner's intelligence, senses and feelings. Another feature of experience based learning is that learning encompasses the learner's identification and use of their significant life experiences. Andresen et al (2000, p.225) also declare that the learners reflection upon their previous experiences is responsible for their understanding. Knowledge is therefore created through the learner's transformation from experiences that they have. Experience based learning draws on experiences and stimulates reflection about these experiences. The result is continual learning.

Kolb (1984, p.20) states that the learning is called experiential for two reasons. The first he says is to connect it to its intellectual roots in the compositions of Piaget, Lewin and Dewey. The second he declares is to highlight the chief function that experience plays in the process of learning. Boud, Cohen & Walker (1993, p.8) made five propositions about learning based on experience. The first one states that experience underpins and provokes the learning process. They go on to claim that the learners are actively involved in building their own experiences. Learning is viewed as a holistic process which is constructed both socially and culturally. The fifth proposition proclaims that socio-emotional contexts influence learning.

Kolb & Passarelli (2012, p.9) declare that the practices of experience based learning are most successful when educating is holistic, learning-orientated and learner- centred. Andresen et al (2000, p226) identified some vital criteria of experiential based learning. They proclaim that experience based learning should involve something personally important to the learners and that the focus should be on the learner's personal involvement with the experience. Reflection is viewed as a vital stage in this type of learning and prior knowledge must be considered. They also recognise that learning is a holistic process and that the teachers must create a feeling of respect, trust, transparency and concern for the welfare of the learner. Andresen et al (2000, p226) further state that in their opinion these attributes are required together before an experiential based learning activity can take place.

Kolb (1984, p.25) declares that there are several characteristics of experience based learning. He states that learning is best looked upon as a process. According to him the focus on the process of learning makes this type of learning different from the idealist and behavioural approaches to learning. Learning takes place through related experiences in which knowledge is changed and recreated.

Kolb(1984,p27) views learning as a continuous process based on experience whereby the learner continually draws out knowledge and tests out that knowledge from their experiences. Kolb & Kolb (2005, p.2) state that the learning process can be expedited by examining, testing and integrating learners ideas to form more developed ideas.

They proclaim that the learning process requires the resolving of encounters between dialectically opposed ways of adjustment to the world. It is disagreements and differences that steer the learning process. Kolb (1984, p.36) describes learning as an active, self - directed process that can be applied in everyday life. Piaget as cited in Kolb & Passarelli (2012, p.3) explains "that learning occurs through equilibration of the dialectic processes of assimilating new experiences into existing concepts and accommodating existing concepts to new experience". In the learning process learners are required to reflect about the different ways of thinking. Learning is an all rounded process of adapting to the world. Kolb & Kolb (2005, p.1) say that learning involves the thinking, comprehending and behaving of the learner.

Kolb (1984, p.34) states that learning involves communications between the learner and the environment. Kolb (1984, p.41) further states that "learning is the process whereby knowledge is created through the transformation of experience". Kolb (1984) declares that this definition highlights many significant traits of the experiential learning process. In contrast to content and outcomes, adaptation and learning is stressed. Knowledge is viewed as a process that causes change. Knowledge is being produced and is not just attained or transmitted. Kolb (1984, p.28) comments that learning changes experience in its subjective and objective forms. He declares that in order for us to make sense of learning we must comprehend the nature of knowledge and in order for us to understand the nature of knowledge we must understand learning. "Over this century the experiential learning movement has evolved in an eclectic fashion, making its presence felt at all level of education" Andresen et al (2000, p228).

In experiential based learning, learning centres on the learner adapting and learning. Knowledge is viewed as a process that changes because it is been always constructed and re constructed.

3.3 Kolb's Experiential Learning Theory (ELT)

David A Kolb is an American Educational Theorist who was born in 1939. He is currently a professor of Organizational Behavior at the Weatherhead School of Management, Case Western Reserve University. He has received four honorary degrees in recognition of his contribution to experiential learning in higher education. Kolb is best known for his research in experiential learning. Kolb published a book entitled Experiential Learning: experience is the source of learning and development in 1984. This book spoke of the theory called "Experiential Learning" that emphasized the key role that experience plays in the process of learning. The experiential learning theory is based on the works of some well- known researchers such as William James, John Dewey, Kurt Lewin, Jean Piaget and Paulo Freire Kolb and Passarelli(2012, p.3). Kolb (1984) combined these works to create the experiential learning theory. Kolb (1984, p.21) describes the experiential learning theory as being holistic since it brings together experience, perception, cognition and behaviour.

Andresen et al (2000, p.230) claim that is it David Kolb who laid much of the basis for the modern experiential education theory. Kolb (1984) comments that experience in the experiential learning theory is not enough. He says that something must be done with the experience. Kolb & Passarelli (2012, p.3) describe the experiential learning theory as a "dynamic view of learning based on a learning cycle driven by the resolution of the dual dialectics of action/reflection and experience/abstraction". (Kolb, 1984, p.41) declares that "knowledge results from the combination of grasping and transforming experience". Kolb and Passarelli (2012, p.3) declare that grasping experience is when information is absorbed and transforming experience is how the learners interpret and react to the information.

Kolb (1984) created a model for experiential learning that is based on the experiential learning theory. This model is made up of four stages. These four stages are displayed in figure 2. These stages are concrete experience (CE), reflective observation (RO), abstract conceptualization (AC) and active experimentation (AE). Concrete experience and abstract conceptualization are portrayed as modes of grasping and reflective observation and abstract conceptualization are portrayed as modes of transforming experience declare Kolb, Boyatzis & Mainemelis (2001, p.2). Concrete experience forms the foundation for observations and reflections.

The Experiential Learning Model can begin with any of the stages but usually it starts with concrete experience and follows through in the sequence. According to Kolb (1984) effective learning only takes place when the learner completes all four stages of the model. In the Concrete experience stage the learner receives knowledge which then leads them to reflect about the experience. From this reflection or thinking about the experience they adjust the knowledge that they already have and create abstract concepts. They then build new ideas and actively test these ideas by experimenting in the world. From their testing of the deductions the learners collect new ideas and the process continues with concrete experience.

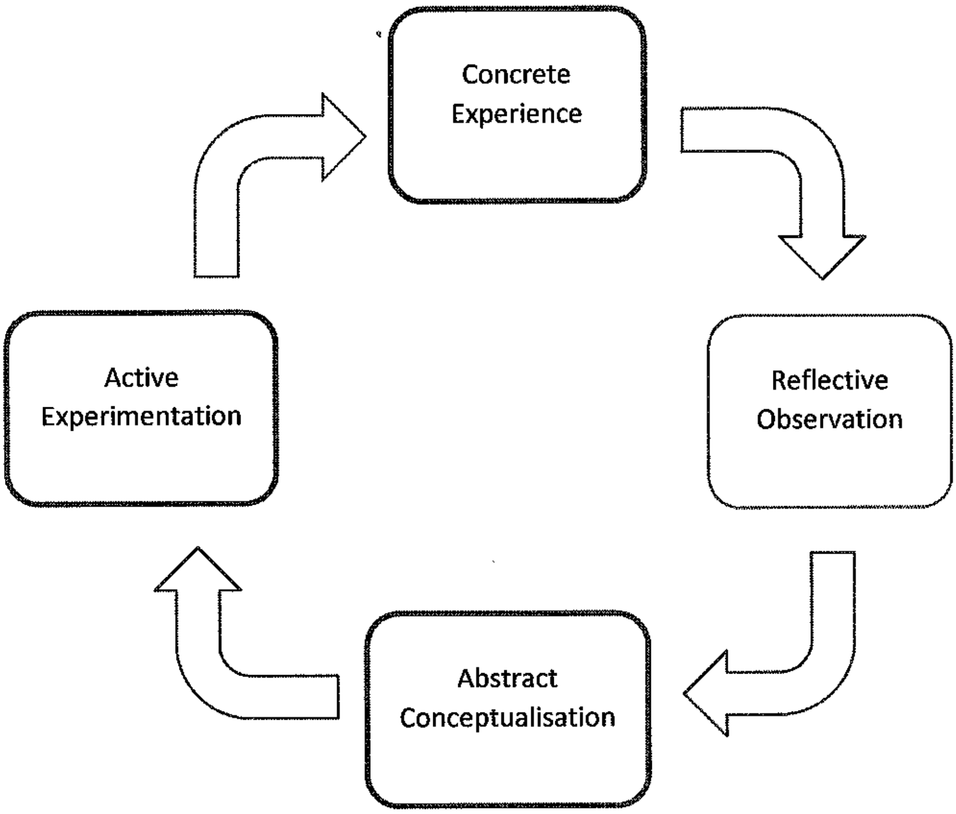


Figure 2: The Experiential Learning Cycle (Kolb, 1984)

3.3.1 Concrete Experience

This stage consists of having or undergoing an experience. It relates to the learner actively going through an activity or doing something. The learner recalls an existing experience or the learner is subjected to a new experience. This involves performing an activity or experiencing an activity. This stage begins with the learner being actively involved. At this stage the teacher can choose an activity (experience) to display a concept. This activity (experience) should allow the learners to work with the concept in many ways.

3.3.2 Reflective Observation

This stage is made up of reviewing or reflecting on the experience. The recalled or new experience is reflected upon to make meaning. The learner alters or adds to their thoughts based on their previous experiences. It is at this stage that learners reflect on what, how and why they learnt. It can also include evaluation of an experience. The learner would use their own thoughts and feelings in creating opinions.

3.3.3 Abstract Conceptualization

This stage is about concluding or learning from the experience. The reflection results in an idea being developed or being transformed into an abstract concept that already exists. The learner can create a generality or concepts if the experience that they undergo fits a pattern. It is at this stage that the learner tries to make sense of what transpired. The learner compares what they have done, reflected upon and what they already know. Abstract conceptualization involves the interpretation of the events that occurred and the understanding of the relationships between them. Learners should be able to examine what they learnt and theorise what they are going to do next. At this stage the questions that the learners may be asking is how and what if?

3.3.4 Active Experimentation

In this stage the learner plans and attempts what they learnt. The learner applies the concept to see what the outcome is. This application is a follow up from stage three. The learner will want to enquire about their ideas and testing is needed. This experimentation restarts the cycle.

Kolb & Yeganeh (2012, p.3) state that learning from experience is a process of forming knowledge that involves "a creative tension among the four learning modes". Concrete experience creates the basis for reflections and observations. Kolb et al (2001, p.3) declare that the reflections are assimilated and distilled to form abstract concepts. They further explain that new implications that can be actively tested are drawn. This testing results in the creation of new experiences. When the cycle is followed, the learner has a concrete experience (stage one) which is followed by reflection and observation of that experience (stage two). This reflection and observation steers the learner towards the developing of abstract concepts (stage three) .This is then used to test out the ideas. The investigating of the hypothesis results in new experiences. Kolb et al (2001, p. 3) state that the Experiential Learning Model suggests that "learning requires abilities that are polar opposites, and that the learner must continually choose which set of learning abilities he or she will use in a specific learning situation ".

Teaching activities that support concrete experience are readings, fieldwork and problem sets. Brainstorming, journal entries and discussion can be used for reflective observation. For abstract conceptualization learners can be engaged in model building, lectures, projects and papers. Teaching activities that can be used to aid active experimentation are case study, simulation, fieldwork and projects. The Experiential Learning Model can assist teachers in helping learners to foster their critical – thinking skills. The learners' abilities to generalize can also be improved upon. Kolb & Kolb (2005, p.4) state that

Experiential Learning Model postulates that learning is a key element of human development and that how learners learn influences their personal development.

3.4 Learning Styles

Kolb's identified four learning styles which he based on his four – stage learning cycle. Kolb & Passarelli (2012, p.4) explain that a learning style portrays the distinctive way a learner goes through the learning cycle. The way that the learner goes through this cycle is based on their inclination towards the four different learning modes. They explain that factors such as certain life experiences, one's genetic material and one's environment affects the learners learning style. Kolb(1984) states that learning depends on how we handle a task and how we respond to and adjust to the experience .The four learning styles that he recognized are diverging, assimilating, converging and accommodating. Kolb (1984) proposed that the learner's learning style was made up of two pairs of preferences that learners have in how they approach learning. He explained that in approaching a task the learner will have a preference for either watching or doing and in responding to a task the learner will have a preference for either thinking or feeling.

Concrete experience which is about having the experience, is associated with feeling. Reflective observation is associated with watching and Abstract conceptualisation involves thinking. Active Experimentation which is about trying out what has been learnt is associated with doing. The four learning styles are created from the combining of these preferences. If the teacher is aware of the learning styles of the learners then the learning can be adjusted according to the preferred method. Kolb (1984) has revealed that educational specialization, career choice, personality type, learning styles, job role and tasks influence ones learning style.

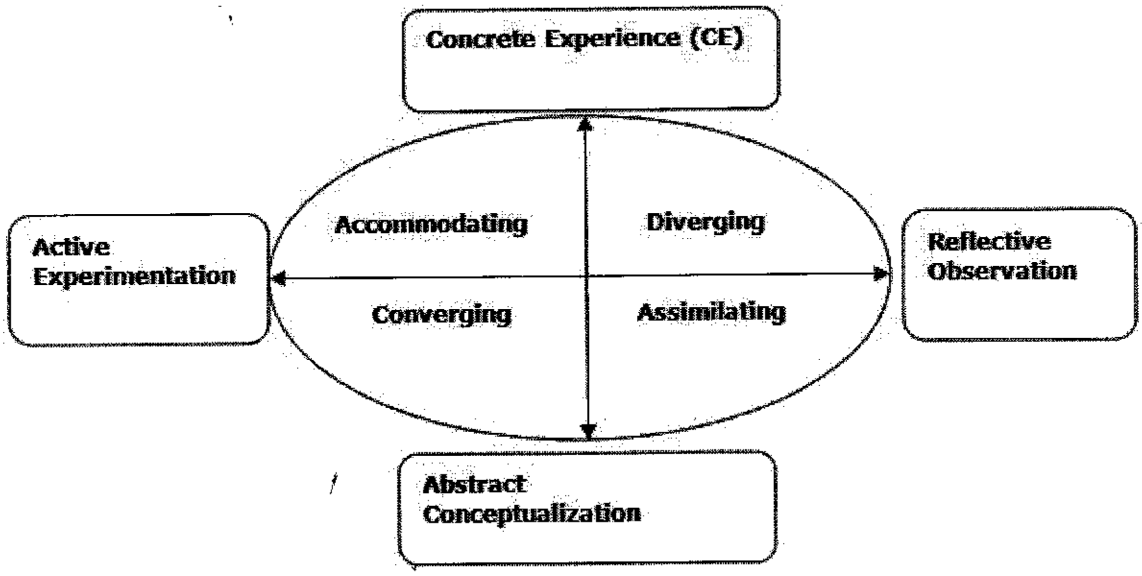


Figure 3: The Experiential Learning Cycle and Learning Styles. Kolb (1984)

3.4.1 Diverging

In this learning style the leading learning abilities are Concrete Experience and Reflective Observation. These learners have a preference for feeling and watching. Learners with this learning style are good at observing concrete situations from several perspectives. They prefer to observe rather than do something. These learners perform well under conditions where they need to come up with ideas. They enjoy collecting information, favor group work, have wide cultural interests and are open minded.

3.4.2 Assimilating

Abstract Conceptualization and Reflective Observation are the main learning abilities in this learning style. These learners have a preference for thinking and doing. Learners who have this learning style are good at comprehending an extensive range of information. They are less focused on people. They are also able to organize the information in a coherent, concise way. Learners of this learning style have a great interest in abstract concepts and ideas. They enjoy reasoning inductively and appreciate logical value more than practical value.

3.4.3 Converging

In this learning style the chief learning abilities are Abstract Conceptualization and Active Experimentation. The preference of these learners are thinking and doing. These learners are able to easily develop concrete functions for ideas and philosophies. These learners are good at to solving problems. The emphasis in this learning style is on practical solutions to problems. These learners enjoy working with technical tasks, problem solving and decision making.

3.4.4 Accommodating

The Accommodating learning style most prevalent learning abilities are Concrete Experience and Active Experimentation. These learners have a preference for feeling and doing. Learners with this learning style learn easily from being actively involved in the situation. These learners enjoy challenges and new experiences. They rely more on people for information than on their own technical analysis. They make use of trial and error rather than logic. These learners easily adapt to changing situations.

By teachers being aware of a learner's learning style the teacher can then create lessons to be adjusted towards the preferred style of learning.

3.5. The Learning Style Inventory

According to Kolb & Kolb (2005, p.2) the Experiential Learning Model will vary according to the learners learning style and the context of learning. Kolb (1984) developed a system based on the experiential learning theory to understand learners learning. He named it the "Learning Styles Inventory" (LSI). The Learning Styles Inventory is based on the experiential learning theory. It was designed to fulfil two roles. One of the roles is to help as an educational tool with one's comprehension of learning from experience and to also aid one in one's learning approach. The other role it performs is to act as a research tool for examining the experiential learning theory and the traits of individual learning styles. Over the past 35 years five versions of the Learning Styles Inventory have been published.

Table 1: Kolb's Learning Style adapted from
http://www.businessballs.com/kolblearningstyles.htm

Experiential Learning Theory Stage	Doing (Active Experimentation - AE)	Watching (Reflective Observation - RO)
Feeling (Concrete Experience - CE)	Accommodating (CE/AE)	Diverging (CE/RO)
Thinking (Abstract Conceptualization - AC)	Converging (AC/AE)	Assimilating (AC/RO)

3.6 Experiential learning and the teacher

According to Kolb & Passarelli (2012, p.9) there are four propositions that capture the basic philosophy of experiential learning. The first one is that educating is a relationship. Teacher's attitudes and actions can impact negatively or positively on the learners. The next proposition is that educating is about developing the learner in terms of cognitive, social and emotional knowledge. The third proposition is that educating focuses on the learner. They declare that the spotlight should be on how the learners attain their solutions. They suggest that this can be achieved by concentrating on core concepts, the course of inquiry and critical thinking. The last proposition is that educating is learner centred.

Kolb & Passarelli (2012, p.12) further declare that there are four teaching roles. These are facilitator, expert, evaluator and coach. They state that these roles provide a holistic structure for applying experiential learning. Kolb's (1984) Experiential Learning Model and learning styles can be used by teachers to critically evaluate the learners learning and to create more suitable learning opportunities. Teachers can plan their activities in a fashion that will allow each child to participate in a way that is more suitable. The learning areas that learners have difficultly can be pinpointed and improved upon. This practice can aid the learners in the learning process and teachers in teaching more effectively. It can be used to create more appropriate teaching and learning opportunities.

3.7 An adaptation of Kolb's Experiential Learning Model

For this research I will be making use of Mudaly's (2012) adaptation of Kolb's experiential learning model. This adaptation of the model consists of concrete experience, reflection, abstract conceptualisation and active engagement with the diagram.

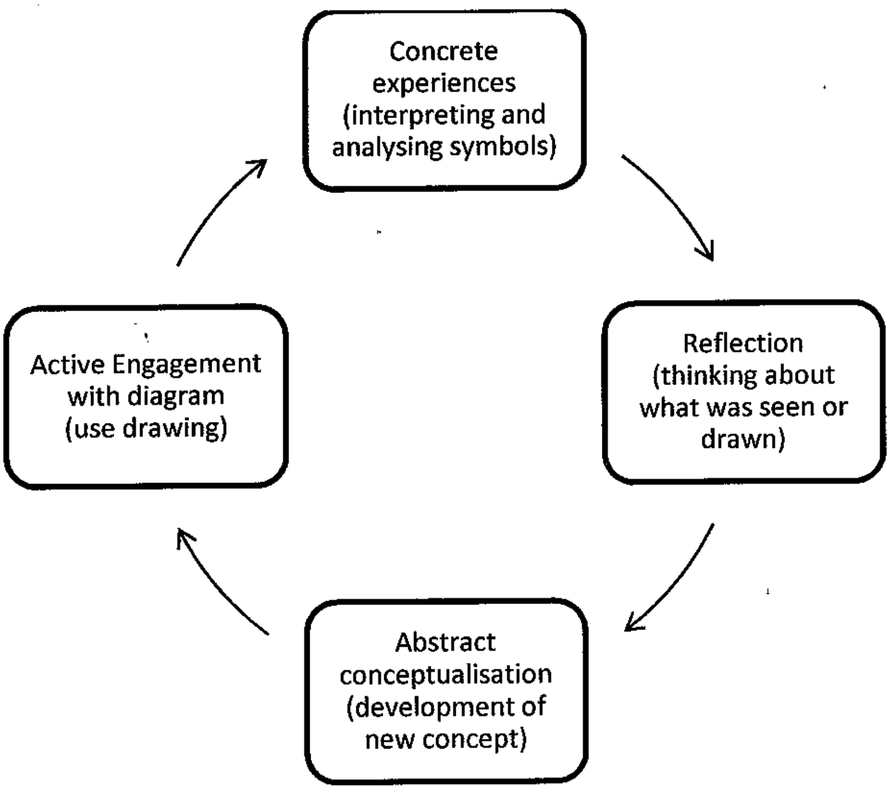


Figure 4: An adaptation of Kolb's Experiential Learning Model from Mudaly (2012)

Kolb's experiential learning model focuses on learner's general experiences relating to learning. Mudaly's (2012) adaptation focuses on learners experiences with the drawing of diagrams. His adaptation begins with the learner seeing a diagram or drawing a diagram.

The adaptation of Kolb's Learning Model supports activity one of the research study. Activity one was made up of three questions and an exercise. In question one learners were given eight diagrams. In this question they were required to measure and record the values of the ‹AO_1C and ‹ABC. This finding out of the values of the angles formed part of the concrete experience stage. The next part of the activity involved learners having to reflect on their measurements of the angles found from question one and their prior geometric knowledge. In questions two and three the learners were required to write down and prove the conjecture that they had realised. Their interpretation of the relationship between the angle at the centre of the circle and the circumference would have helped the learner to realise/attempt to prove the conjecture. This forms part of the abstract conceptualisation stage. In the next stage of active engagement learners had to complete an exercise made up of four questions. This exercise was an application of the conjecture. It involved questions whereby the learner had to solve for the unknown by applying the conjecture.

In activity two, question one, the learners are provided with three diagrams. This formed the concrete experience part of the model. They were provided with the following information: O is the centre of the centre of the circle in each diagram. A, B and C are points on the circumference of the circle. OA, OB and OC are radii. Figure four shows this activity.

1. O is the centre of the circle in each diagram. A,B and C are points on the circumference of the circle. OA , OB and OC radii. Study the figures below and answer the questions that follow:

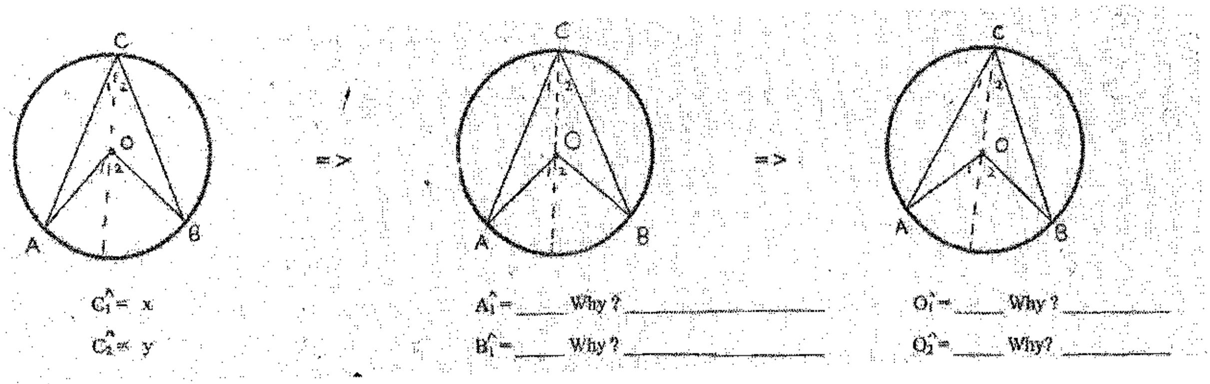


Figure 5: Activity two, question one

The learners were involved in examining the information in the three diagrams. According to Mudaly (2012) when the learners understand/examine the symbols within the diagram meaning construction begins.

In activity two, question one, the learners had to interpret and analyze the information given within the diagrams. This would have resulted in them building their own understanding about the symbols inherent in the diagram. The information within the diagrams such as OA, OC and OB are radii would have resulted in them been able to fill in the blanks below diagram two and three.

Mudaly (2012) states that in the process of reflection and interaction with prior knowledge new knowledge is created. In activity two the new knowledge created by the learners about the conjecture would depend on the symbols inherent in the diagrams and the learner's prior geometric knowledge. The concrete experience of the diagrams would have resulted in them thinking about the symbols and information within the diagrams. The reflection process would have begun when they started thinking about what they saw in the diagrams together with their former geometric knowledge.

The reflection stage is followed by the abstract conceptualisation stage. The learner's reflections are linked with their previous knowledge and converted into abstract concepts. It is here that whatever the learners comprehended in the diagram becomes more significant to them resulting in new knowledge being created. In this stage the learners would have tried to make sense and interpret what they had noticed in the diagrams. With regards to activity two, it is here that the learners should have realised that the angle at the centre was twice the angle at the circumference of the circle. At this stage in the activity the learners would have been able to fill in the blanks to questions three and four.

3. Is there a relationship between AOB and ACB? If so, state this relationship?____

Figure 6: question three, activity two

4. Can you make any deductions? If so, please state it.

Figure 7: question four, activity three

In activity two the new knowledge was created by the learner reflecting on their prior geometric knowledge and interacting with the symbols in question one. This new knowledge influenced the learner's responses to questions two to four. The active experimentation stage follows abstract conceptualisation. Learner A was unsuccessful in completing most of the questions in activity one. After having completed the guided proof she attempted the exercise from activity one once more. It was here that she put into practice that of which she had learnt through her experience in activity two. In her second attempt of this exercise her responses were more successful.

Mudaly (2012, p.24) declares that the new knowledge created becomes internalized and is used to affect what is added on or seen in an existing diagram. The learner can enter the learning cycle at any stage and go through the sequence. In order for successful learning to take place all stages must be fulfilled. The model is a cycle so the process continues.

3.8 Conclusion

In this chapter the researcher explored how Mudaly's (2012) adaptation of Kolb's Experiential Learning Theory supports this study. The researcher looked at the different stages of learners doing / having an experience with a diagram, reviewing/reflecting on the experience with the diagram, the concluding/learning from the experience and the learners then trying out of what was learnt. Experience based learning, Kolb's Experiential Learning Theory, learning styles, the Learning Style Inventory and the adaptation of Kolb's model were examined. Learner's experiences with the diagrams was discussed.

CHAPTER FOUR

RESEARCH DESIGN AND METHODOLOGY

4.1. Introduction

This chapter deals with the methodological approach, research design, research context, the sample, data collection, analysis and ethical issues. According to Budram (2007,p.42) research methodology involves " identifying of the research problem, formulating hypotheses, review of literature, designing of methodology, identification and designing of research instruments, sampling procedures, data collection, data analysis, drawing conclusions and making recommendations and preparation of the final report."

In this study five grade eleven learners were asked to complete a number of geometric tasks. These activities were designed to investigate if visualisation forms an integral part of the learners proving process and to examine whether diagrams on their own could be considered as proof. The completion of these activities was followed by individual interviews and the completion of a questionnaire. These activities took approximately 3 weeks to complete.

4.2 Methodological Approach

Cohen, Manion and Morrison (2000) declare that the underlying principle of research helps in choosing the methodology, research design and data collecting methods. For this study a mixed method approach was used. It is made up of elements of quantitative and qualitative methods that were used to explore learner's use of visualisation in solving geometry problems. While qualitative research involves the gathering of verbal or textual data, quantitative research involves the collecting of numerical data. Hatch (1998) declares that qualitative research is a thorough analysis of a problem in order to comprehend human behaviour. This type of research focuses on the experiences of individuals, what they feel and the causes of these viewpoints. With quantitative data individuals views and experiences can be measured. The data collected from quantitative methods can be represented graphically or statistically.

Creswell cited in Maree (2007,p.260) declares that a mixed method approach is a procedure for collecting, analysing and 'mixing' both quantitative and qualitative data during the research process in order to gain a better understanding of a research problem. According to Spicer (2004) the advantage of using mixed methods allows the researcher to attain the best of both approaches. Since I wanted to gain an in-depth understanding about learner's perceptions, experiences and beliefs in the solving of geometry problems, I decided to adopt a mixed method approach.

Vos (2006) states that triangulation is the process when quantitative and qualitative methods are brought together to obtain multiple measures of the same phenomena by the application of several research instruments. Maree (2007) defines triangulation as the process of using multiple methods to gain knowledge of the same phenomenon using different methods. Budram (2009, p.43) states that triangulation involves the union of quantitative and qualitative methods when an area under enquiry is explored from different angles in order to gain a greater understanding. The order of my data collection for this study was: learner's evaluation activities, classroom observation, interviews and questionnaire.

The study was conducted within the Interpretivist paradigm. Cohen, Manion and Morrison (2007, p.21) claim that in the context of the interpretive paradigm, understanding the subjective world of human experience is key. In this paradigm concern for the learner is utmost. The world is seen as changeable and it is the people who define the meaning of circumstances. This paradigm allows the researcher to understand how people make sense of the contexts in which they exist. My study made use of this paradigm because it was concerned with learner's use of visualisation in geometry. I wanted to interpret how the learners created visual images and the strategies they used in proving and solving geometric problems. I began with the learners and tried to understand their interpretations of the world. The focus was on their understanding of and reasoning for the use of diagrams in the proving and solving of geometric problems. After the interviews, observation and filling in of the questionnaires I attempted to interpret their actions.

4.3 Research Design

I used both qualitative and quantitative approaches for this study. Two evaluation activities, an interview, observation and a questionnaire was used to gather data. The first part of activity one consisted of eight diagrams whereby the learners had to measure the angles. The next part of activity one was questions based on the measurement of the angles from the first part. This was followed by an exercise. Activity two was a guided proof of the theorem that they may have deduced in activity one. These instruments will be discussed later on in this chapter.

4.4 Research Context

The research was conducted at a school in Shallcross. Learners who attend the school are mainly from Marianhill, Shallcross, Pinetown and Northdene. The school has an enrolment of approximately 1172 learners. The number learners in a senior mathematics class is about 32. It is a well-resourced school in terms of electronic resources and stationery.

4.5 Sampling

Sampling is defined as a process whereby a small number of persons or events is chosen and analysed to find out something about the entire population from which the persons were selected Koul (1988).

Cohen et al (2007, p.100) state that there are several factors one needs to consider when considering a sample. These are the size, representativeness and parameters of the sample. They also suggest that access to the sample and the sampling strategy to be used also need to be considered.

My research study was conducted on a small scale and it included five grade eleven learners. Maree (2007, p.291) claims that when an appropriate sample is applied it will have the potential to be generalised to a larger population. The learners chosen as the sample were from a grade 11 mathematics class that was not taught by me. They were taught by my colleague who was aware of my research study. This made accessibility to these learners easy. These learners were randomly selected. According to Cohen et al (2007, p110) in a random sample the chances of learners of the wider population being selected are known. I wrote all the learners names on pieces of paper and placed it into a box. Five names were then drawn at random. Cohen et al (2007, p.110) state that with random sampling each member of the population has an equal chance of being chosen. The participation of the learner's whose names that were drawn was voluntary.

4.6 Data Collection

4.6.1 Interview

Cohen et al (2007) state that an interview involves the collecting of information through direct verbal interaction between the persons involved. According to Neuman (2006, p.304) an interview is a short-term secondary social interaction between two strangers with the aim of one person's obtaining certain information from the other. Interviews were one of the methods I used to gather the data and all five learners took part in the interview process. There are many reasons why I chose to interview the learners. One of the reasons was that I wanted to find out about their attitudes and beliefs about diagrams in the proving and solving of geometric problems. Secondly, in an interview if the interviewer is unclear about the interviewee's responses, the interviewer can clear any misunderstandings then and there. This assists in the interviewer gathering sufficient and meaningful information. Another reason was that, since the questions were based on the learner's responses to the activities, I needed the learners to recall their responses. In order for them to do this, I had to present to them the activities that they answered. This would have been impossible to have done using a questionnaire. The interview was also an appropriate method for gathering rich data since I only had to interview five learners.

Durrheim and Wassenaar (2002) declare that participants should be assured about the confidentiality of the information they provide during an interview. Before the interviews were conducted I told the learners that the interview will be recorded and that whatever was said by them will be anonymously used in the research. The interviews were conducted over a few days after the learners had completed activity one and activity two. The reason for this was that I needed to analyse each learners response to the activities and then design appropriate questions to gain insight as to why they responded the way they did.

The interview was semi-structured and was made up of open ended questions. According to Fontana and Frey (2000) an advantage of making use of semi-structured interviews is that the person interviewing can notice any non - verbal signs and can thereafter investigate the interviewees suggested views. Nieuwenhuis (2007) states that semi-structured interviews can be defined as a 'line of inquiry '. He explains further that this is so because it allows the researcher to explore the developing lines of inquiry. The interviews with the learners were face - to - face. Maree (2007, p.87) states that a semi – structured interview is used "to *corroborate data emerging from other data sources*". I chose open ended questions because it made discussion and exploration of the concerns possible. All the learners were asked similar questions in the same order. This increased the comparability of the learner's responses. This assisted in the organizing and analysing of the data. The learners were questioned about their responses to activities one and two. In order to understand the learner's responses to the interview questions, the learner's replies were probed. The open ended questions allowed me to gain a greater insight into learners experiences with regards to determining if diagrams on their own can be considered as proof and whether visualisation is an integral part of proving and solving geometric problems .

All interviews were recorded and transcribed. Interviews were recorded to ensure that essential information was not left. Koul (1988, p.176) states that interviews have advantages such as : it allows for a chance to extensively probe certain areas of inquiry ,permits greater depth of response, which is not possible through other means of inquiry and enables the interviewer to be able to gather information concerning attitudes to certain questions.

4.6.2 Questionnaire

A questionnaire can be defined as a list of questions which the respondent has to answer. Baro cited in Koul (1988, p.142) defines a questionnaire as "a systematic compilation of questions that are administered to a sample of population from which information is desired". Questionnaires are common ways of obtaining all types of information since it is through this method that data is generated Budram (2009, p. 43). Kumar (2005) defines a questionnaire as a written list of questions where participants are required to read and interpret what is expected of them and then to answer in writing. I tried to ensure that my questionnaire was brief, readable and simple to answer so that the learner's answers will be beneficial to the investigation. A properly designed questionnaire expedites the analysis process which can be made even easier if the researcher is involved in the design Cohen et al (2000). The learners completed the questionnaires a few days after the interviews were completed. Walker (1985) declares that a questionnaire is quick and easy to fill and is immediately and directly accessible to the researcher.

Questionnaires can consist of open-ended or close-ended questions. In close-ended questions the respondents are given a various possible choices which they can choose from to respond to the question. Kumar (2005) states that in closed questions the respondents are given a choice of alternatives. In my questionnaire the learners were required to tick/cross the appropriate box. The disadvantage with the closed questions is that the learners are forced to choose from the alternatives that are given. In open-ended questions the respondent may answer the question in any way they feel appropriate.

Open-ended questions allow the participants to qualify and explain their responses Cohen et al (2000). Maree and Pieterson (2007) reveal that open ended questions may disclose the participants thinking processes and other information of significance. With open - ended questions the learners are able to disclose their own opinion without being constrained. However a disadvantage of open questions is that if the responses from the learners are varied then me as a researcher will have difficulty in analysing and coding them.

In the designing of the questionnaire I tried to ensure that the questions were easy to understand and unambiguous. To ensure the suitability of the questionnaire it was discussed with two of my colleagues who were familiar with my study. I was unable to pilot the questionnaire because my research participants were the only learners who participated in the activities involving visualisation. An advantage of making use of this data collection method is that I was able to standardise the questions that were asked and to control the amount of data that the learners supplied. The questionnaires were administered by the researcher. The advantage to this was that if the learners did not understand a question I was able to clarify it for them. The questionnaire was based on the learner's responses to the activities and aimed to provide answers to the proposed research questions.

4.6.3 Observation

Cohen et al (2000) refer to observation as the gathering of fresh data as it takes place . They further state that access is obtained to personal knowledge and occurrences at the site. Observation is a method of obtaining data by watching behaviour or noting physical attributes in their natural surroundings. This study involved unstructured observation. Cohen et al (2000) proclaim that unstructured observation means that the researcher records everything that he / she sees taking place in the classroom. The learners were given the activities to complete and how they went about completing the activities was observed.

Koul (1988, p.172) explains that the observation method of data collection has advantages. He states that observation is an effective way to gain information about human behaviour especially in a specific situation. He also explains that it allows the researcher to code and document activities at the time that it takes place.

The only observation that took place in this study was of the learners completing the activities took place once in the classroom. Cohen et al (2007, p.396) assert that distinctive feature of observation is that it allows the researcher the chance to obtain 'live data' from a natural setting. I noted which parts of the activity took longer to complete, which they chose to leave out and which they were are unsure about how to answer. The observation process gave me credible information that the learner may not have been able to present in an interview or questionnaire.

4.6.4 Evaluation Activity

The evaluation activity was made up of two activities. All the participants were learners from a colleagues grade eleven mathematics class. This was done because I did not want to pressurise participants into feeling that since I was their mathematics teacher they had to respond correctly. The five learners completed these activities in the classroom and they were given as much time as they required to complete them. There were two activities and both were based on Euclidean geometry. The activities centred on a theorem from the grade eleven third term mathematics work schedule. This activity was planned so that it was completed before the actual theorem was taught in the classroom. To ensure that there were no misunderstandings the participants completed the activities under my supervision.

The first part of activity one was made up of eight diagrams whereby the learners had to measure the values of the angles. All learners were provided with protractors. The next part of activity one was questions based on the first part. Here the learners had to answer questions based on their observations from the measuring of the angles. This was followed by an application exercise which was made up of four questions. Activity two was a guided proof of the theorem that the learners may have deduced in activity one. It consisted of diagrams and text that was meant to guide the learner towards a proof. It was made up of three diagrams and incomplete statements. Learners had to fill in the blanks.

The aim of the activities was to evaluate the role that diagrams played when learners were proving and solving geometric problems. The purpose of the first part of activity one was for the learners to notice the conjecture from their measuring. The function of the next part of activity one was to investigate whether the learners could prove the conjecture by just being aware of the conjecture, meaning only using text. The aim of the last part of activity one was to determine if learners could prove the conjecture by making use of a given diagram Activity two aimed to investigate the role of diagrams and guidelines in the proving of the conjecture.

4.7 Analysis

The analysis of data consists of the arranging complex data into themes, patterns and relationships. Mayan as cited in Maree(2007,p.295) states that data analysis is a procedure of observing patterns, questioning those patterns, asking additional questions, pursuing more data, furthering the analysis by sorting, questioning ,thinking, constructing and testing conjectures.

Miles and Huberman (1994, p.10-11) state that data analysis is made up of three activities that take place at the same time. These activities are data reduction, data display and conclusion drawing and verification. Data reduction is made up of choosing, simplifying and changing the transcripts and field notes. The data is then arranged and sorted into codes. In data display the data is organised so that the researcher can make deductions from it. These conclusions could be represented in forms of tables, charts or graphs. For conclusion drawing and verification the researchers draw conclusions and look for patterns and explanations.

According to Creswell cited in Maree (2007) a mixed method approach consists of the analysing of qualitative and quantitative data. Data was collected using evaluation activities, interview, questionnaire and observation. All the data that was collected was read and topics emerging from the data were identified. This was examined to look for duplication. The similar topics were categorised and I looked for patterns and relationships between them. Maree (2007) asserts that data must be collected, processed, condensed and interpreted using triangulation to make the research study trustworthy, reliable and valid.

4.8 Ethical Issues

The precautions, steps and efforts that researchers put into practice to protect the research participant while working with them for the data production is referred to as ethical issues, McMillan and Schumacher (2006).

The possibility that the result of the research will be misleading must be minimised. The design, conduct and research report must be in agreement with the accepted standards of scientific aptitude and ethical research. The appropriate permission must be obtained from the participants and the rights and interests of all those involved must be protected. The confidentiality of the information given to the researcher must be guaranteed and the university ethics committee must be consulted about unclear ethical issues. All these guidelines were taken into account for this study.

All research studies should follow certain ethical principles such as autonomy; non-maleficence and beneficence, Durrheim and Wassenaar (2002, p.66). Autonomy refers to gaining consent from every learner participating in the research. This should be voluntary and the learners should be able to withdraw at any time. Consent for this research was obtained from the learners parents/guardians. Ali and Kelly (2004) say that receiving informed consent is a procedure that supports individual autonomy and assists in safeguarding the rights of the participants by letting them decide for themselves what are in their best interest and what risks they are prepared to take. During the planning of this research study much consideration was given to the ethical issues. Since the learners were minors consent had to be obtained from the parents (see Appendix A). The parents received a letter clearly explaining to them, the nature of the study, when the interviews will be conducted and that participation is voluntary. Durrheim and Wassenaar (2002) assert that consent should be informed and voluntary. They also declare that the participants should be given an explanation of the research so that they can make knowledgeable decisions whether to be part of the research study.

Non-maleficence means not to do any harm. At no point during the research study did the learners come to be any harm. There was no physical, emotional, and social or any other type of harm inflicted on the participants. Beneficence refers to the benefit of the study.

The study was done with the aim of trying to improve learner's attitude and their success rate at solving geometric problems.

Since Euclidean geometry has been recently reintroduced into the mathematics curriculum it would cause past and new challenges to surface. Based on the outcome of the research study, teachers can adjust their classroom practises in terms Euclidean geometry lessons accordingly. All the participants were assured of confidentiality and they were informed about how the information will be used. To make sure that my questions where not framed in a bias way I asked a peer to examine the interview schedule.

I applied for ethical clearance from the University of Kwa-Zulu Natal in order to carry out the research. In the application I explained how the ethical issues regarding the participants will be attended to. After the ethical clearance was issued, the data collection process commenced. A letter was written to the school principal requesting permission to conduct the research study in his school. Written consent was attained from the principal of the school and the department of Education to use the school as the site for the research study.

4.9 Conclusion

This chapter gives an overview of how this research study was conducted. It explains the research methodology, research design, the context, sample, data collection methods and ethics.

CHAPTER FIVE

DATA ANALYSIS

5.1 Introduction

In the previous chapter I discussed how the research was conducted in terms of the methodological approach, research design, research context, the sample, data collection, analysis and ethical issues. In this chapter I analyse the evaluation activities, the interviews, the questionnaires and the classroom observations. I will discuss in detail the learners measuring of angles within the circle in activity one, question one; the conjectures that the learners made, the proving of the conjectures, the proving of the conjectures with a diagram; the exercise involving the application of the conjecture, the guided proof and the learners perceptions about diagrams. This study focused on the role that visualisation plays in the solving of geometric problems and riders. For the purpose of this analysis I will refer to the five learners who participated in the research as learner A, B, C, D and E. A worksheet made up of two activities, Annexure A, was administered to the 5 grade eleven learners. The next chapter will look at the research questions, discuss my findings, make recommendations and list the limitations of my study.

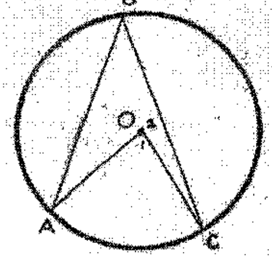
5.2 Measuring of angles within the given circle

Activity one was made up of three questions and an exercise. In question one learners were given eight diagrams. In this question they were required to measure and record the values of the $\angle AO_1C$ and $\angle ABC$. All learners were provided with protractors. The focus of the question one was the measuring of the angles in the different diagrams to see if learners were able to notice a relationship between $\angle AO_1C$ and $\angle ABC$. Each of the sub questions in question one had a diagram and below each diagram was a space for the learner to record the value for $\angle AO_1C$ and $\angle ABC$. The position for entering the values of the measurements of the angles was one below each other so that learners could to easily compare these values for each question.

Activity One

1. O is the centre of the circle in each diagram. Using the protractor measure the value of angles $A\hat{O}_1C$ and $A\hat{B}C$.

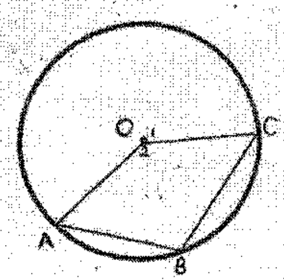
1.1.



$A\hat{O}_1C =$ ________________

$A\hat{B}C =$ ________________

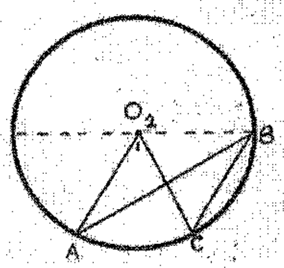
1.2.



$A\hat{O}_1C =$ ________________

$A\hat{B}C =$ ________________

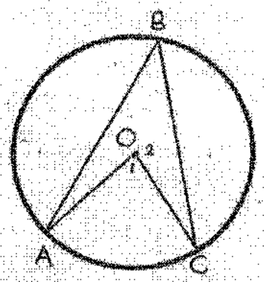
1.3.



$A\hat{O}_1C =$ ________________

$A\hat{B}C =$ ________________

1.4.



$A\hat{O}_1C =$ ________________

$A\hat{B}C =$ ________________

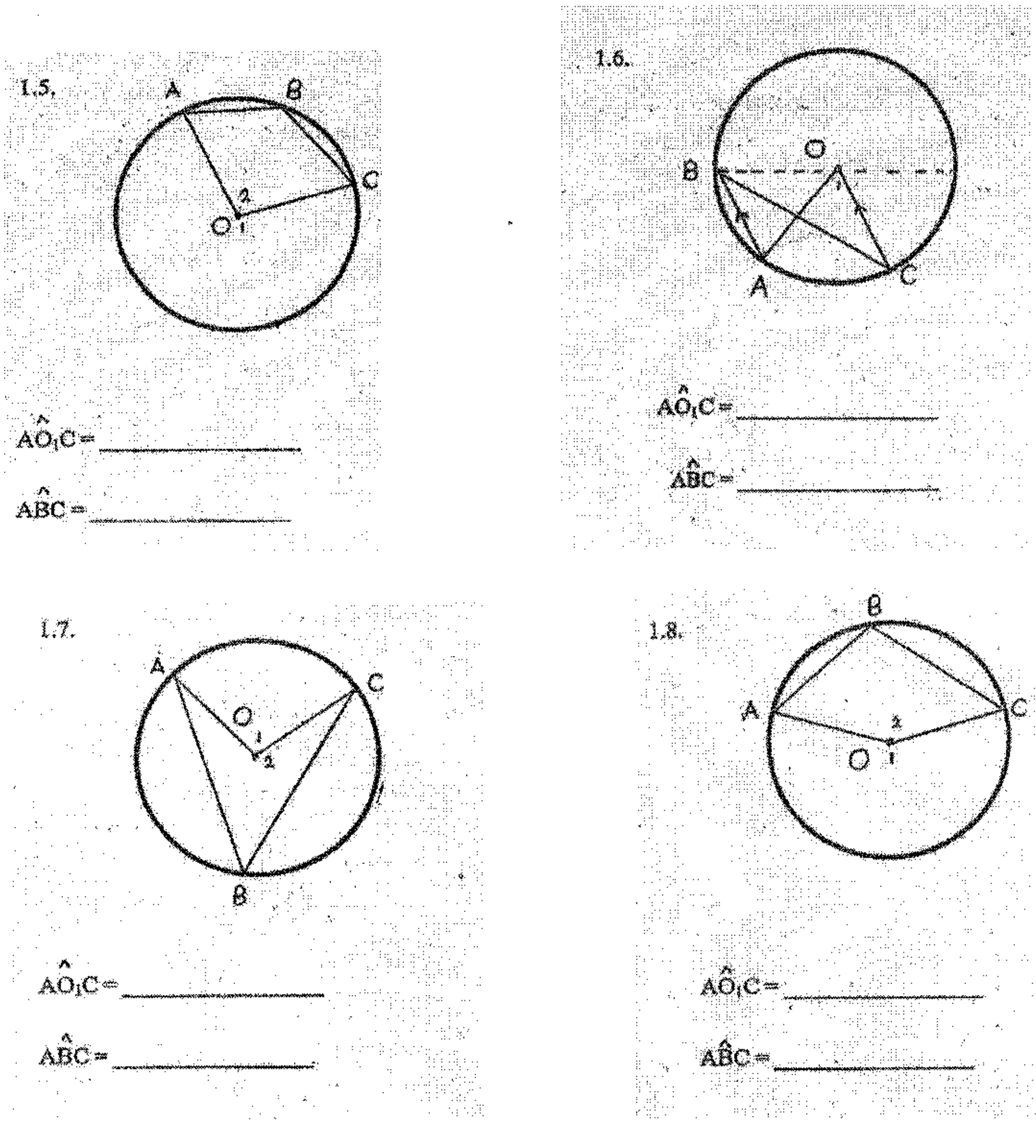


Figure 8: Activity one, question one

This part of the activity took a long time for the learners to complete because they had to position the protractor in a particular way and then record the measurement. There was some evidence that these learners were not au fait with the use of protractors. Some learners had to be guided in the use of the protractor. They had difficulty in locating the correct value of the angle from the protractor and they did not know which value to record as the value of the angle from the protractor. Although the instruction in the activity required the learners to measure $\angle AO_1C$, three learners measured the $\angle AO_2C$ which was in fact the obtuse angle of ∠AOC. I did not influence their choices of angles and merely observed what they did.

All learners responded to all the sub questions in question one, activity one. Of the five learners only two learners measured all the angles correctly in all the diagrams in each question. On examining the responses of the other three learners it was noticed that knowing how to measure the angle was not the problem. All three of these learners who had answered incorrectly, had measured the wrong angle. Learner A measured the wrong angle in question 1.2 and question 1.5. Learner C and Learner E measured the wrong angle in question 1.8. Learners A, C and E all measured $\angle AO_2C$ instead of $\angle AO_1C$.

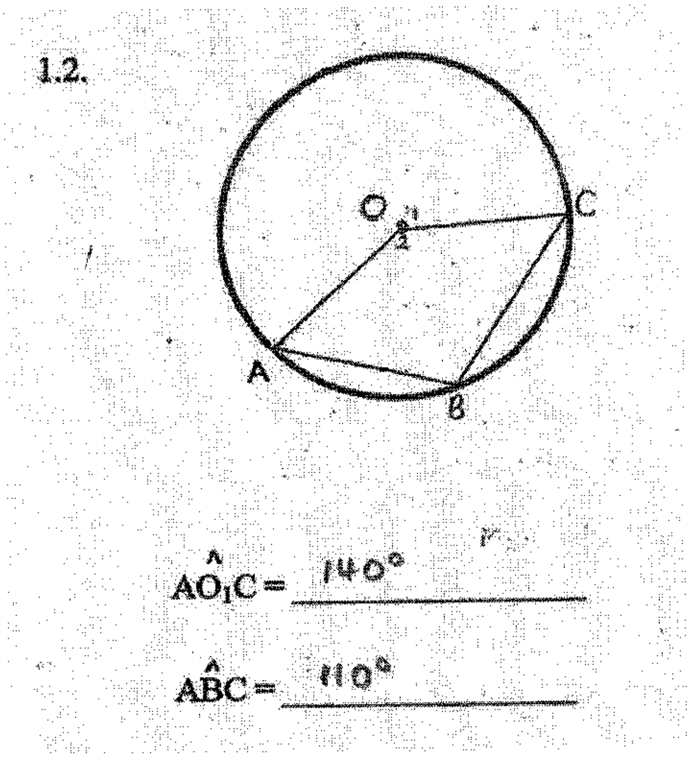
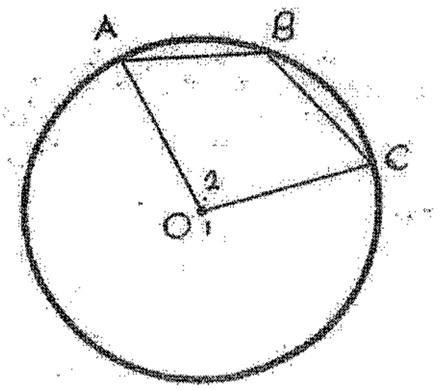


Figure 9: Learner A's measurement of $\angle AO_1C$ in question 1.2

1.5.



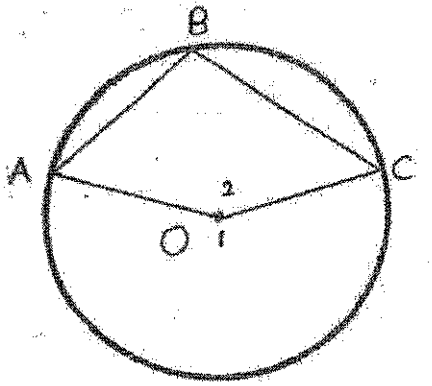
$\hat{AO_1C}$ = 100°

$\hat{ABC}$ = 130°

Figure 10: Learner A's measurement of angle AO_1C in question 1.5

Figure 9 and figure 10 show how learner A measured the obtuse angle of AOC.

1.8.

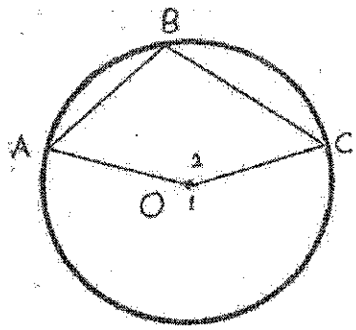


$A\hat{O}_1C =$ 150

$A\hat{B}C =$ 105

Figure 11: Learner C's measurement of $\angle AO_1C$ in question 1.8

1.8.



$A\hat{O}_1C =$ 150°

$A\hat{B}C =$ 105°

Figure 12: Learner E's measuring of the incorrect angle

Figures 2 to 5 show that although these angles were labelled as $\angle AO_1C$ and $\angle AO_2C$ these learners ignored this and instead of measuring reflex $\angle AOC$, they measured the obtuse $\angle AOC$. This problem can be attributed to the way the question was framed and the manner in which the diagram was provided. It could have been clearly shown on the diagram which $\angle AOC$ ought to have been measured or the question should have asked for both the reflex and obtuse angles of $\angle AOC$ to be measured. This is indeed a limitation. In hindsight it was realised that the question could have been framed differently.

The purpose of question one in this activity was to get the learners to recognise a pattern between the values of the angles that they were measuring. In each question they were asked to measure the angle at the centre and the angle at the circumference of the circle. This activity was designed with intention of getting learners to realise that there was a relationship between the angle at the centre of the circle and the angle at the circumference. The learner's responses to question one indicates that although learners know how to measure the angles using the protractor, some of them had difficulty in identifying the correct angle to measure. It could also imply that learners A, C and E just did not follow instructions in the measuring of the angles. They did not identify the correct angle to measure before they started measuring. This would imply that reading instructions and understanding the question may be an important first step. The table below gives a summary of learner's responses to question one, activity one. Table one lists the names of the learners and their responses to question one. The tick indicates that they measured correctly and if they answered incorrectly an explanation is given about their error. The most common error that the learners made in this question was to measure the incorrect angle.

Table 2: Learner's responses to question one, activity one

Learner	Q1.1	Q1.2	Q1.3	Q1.4	Q1.5	Q1.6	Q1.7	Q1.8
A	✓	Measured wrong angle	✓	✓	Measured wrong angle	✓	✓	✓
B	✓	✓	✓	✓	✓	✓	✓	✓
C	✓	✓	✓	✓	✓	✓	✓	Measured wrong angle
D	✓	✓	✓	✓	✓	✓	✓	✓
E	✓	✓	✓	✓	✓	✓	✓	Measured wrong angle

5.3 Learner Conjectures

Below each question in the worksheet, a large amount of space without lines was left for the learner's responses. This was done so that learners could answer the questions without restraint and also to prevent lines or restricted space from influencing their responses. Question 2 .1 of activity one was dependent on question one. The learners were asked to write down a conjecture drawn from what they observed in question one. All learners answered this question.

Three of the five learners, learner B, D and E, correctly responded by stating that the ∠AOC is twice the size of ∠ABC. Learners A, C and E where learners who did not accurately measure the angles in question one. The realisation of the conjecture was dependent on the learner's measurements in question one. Learner A responded that ∠AOC *"is increasing or decreasing by 30° or more"*. Her conjecture was unexpected. On examining her response I noticed that the difference between the values of the angles that she had measured was really 30° or more. In spite of this learner measuring the angles incorrectly for question 1.2 and question 1.5, her observation from question one was partially correct according to her measurements. The manner in which the diagrams were sketched contributed towards her deduction. Learner A's observation was partially correct but it had nothing to do with the expected result. The observation was in fact a coincidental result and implies that teachers ought to be cautious about the diagrams that they provide. Distractions like these may have the effect of redirecting the learners thought and may render the activity ineffective. Figure 13 displays learner A's observation.

2. 1.Examine angle $A\hat{O}C$ and angle $A\hat{B}C$ from 1.1. to 1.8 and write down a conjecture about what you observe?

$A\hat{O}C$ is increasing or decreasing by 30° or more.

Figure 13: Learner A's response to question 2.1

During the interview learner A was asked to examine her response to this question. Although her response in the worksheet was incorrect, her response to this question during the interview was positive. She was able to successfully deduce the relationship between∠ AO_1C and ∠ABC. When asked about whether she would be able to prove this she said she felt that she needed a diagram or something to show it.

Below is a part of the transcript of the interview.

INTERVIEWER: I can see from your answer to 2.1.that you have noticed that $\angle$ AO_1C is increasing or decreasing by 30° or more. Now according to your measurements this is correct, but if you look at all these angles here, do you notice anything?

LEARNER A: Not really.

INTERVIEWER: I want you to, just to look at the values of these angles, look at these values and see if there is anything there, is there any relationship between the values?

LEARNER A: Okay. (Pause) I see that $\angle AO_1C$ is two times bigger than $\angle ABC$. Ja, in most cases it's just two times bigger.

INTERVIEWER: Okay. Now if you look at the circle. Here's the circle here. If you look at $\angle AO_1C$, in which position on the circle does it lie?

LEARNER A: It is in the centre.

INTERVIEWER: And if you look at ‹ABC where about in the circle, where about is this angle?

LEARNER A: On the circle.

INTERVIEWER: Alright. Can you be more specific? Where about on the circle is it?

LEARNER A: On the circumference.

INTERVIEWER: So can you see any special relationship between $\angle ABC$ which is on the circumference and $\angle AO_1C$ which is in the centre?

LEARNER A: Well $\angle AO_1C$ is two times the size of $\angle ABC$ then maybe the angle at the centre is two times the angle on the circle, I mean circumference.

INTERVIEWER: Okay. Do you think that you can prove this?

LEARNER A: Well I am not sure, I feel like I need a diagram or something to show it. I don't think I can.

Although during the completion of activity one learner A was unable to deduce the conjecture, in the interview under guidance she was able to successfully state the conjecture. She also revealed a need for a diagram if she had to prove this conjecture. Although she took longer than the others to arrive at the conjecture, it does show that with carefully planned experiences learners can see and understand particular concepts. It is also noted that in this research, the learners were not given more examples when they encountered difficulties. It might be necessary to increase their experiences and get them actively engaged with more examples. An additional point to be noted is the fact that the learner felt that a diagram was necessary for the completion of a proof. This indicates an intrinsic recognition of the value of a diagram in the proving process.

Learner B was able to correctly deduce that $\angle AO_1C$ is twice the size of $\angle ABC$. Learner C responded that if one angle was determined one would be able to determine the value of the other angle. This learner stated that $\angle AO_1C$ and $\angle ABC$ have a relationship. Learner C went on to declare that if one finds the value of one side then it will be easy to calculate the value of the other side since $\angle ABC$ is twice $\angle AO_1C$ .This would imply that if the value of the angle at the centre is known then the angle at the centre of the circle can be determined. In this case if one has the value of $\angle ABC$ one can find the value of $\angle AO_1C$. Although this learner was able to successfully deduce a relationship between the angles, the second part of the observation was incorrect because she became confused about which angle was twice the other angle. Her mistake could stem from the learners being given eight different diagrams to measure and that $\angle AOC$ was made up $\angle AO_1C$ and $\angle AO_2C$. In the future, in order for learners to correctly identify angles and the relationships between them, we can encourage them to shade or mark the relevant angles with different colours or to even write down their values on the diagram. In this instance the learner could have coloured the relevant angles or even wrote down the value of each measured angle on the diagram. She would then have been able at glance to easily identify and compare the values of the relevant angles. These proceedings may assist learners in making the key components in a geometric problem more visible. This learner also referred to the values of these angles as sides instead of measurements. Figure 14 displays Learner C's response to question 2.1.

2. 1.Examine angle $A\hat{O}C$ and angle $A\hat{B}C$ from 1.1. to 1.8 and write down a conjecture about what you observe?

$A\hat{O}C$ and $A\hat{B}C$ have a relationship becaus if you get one side then it will be easy to calculate the other side becaus $A\hat{B}C$ is twice $A\hat{O}C$

Figure 14: Learner C's response to question 2.1

Learner D correctly responded that ∠AOC was two times the value of ∠ABC.

2. 1.Examine angle $A\hat{O}C$ and angle $A\hat{B}C$ from 1.1. to 1.8 and write down a conjecture about what you observe?

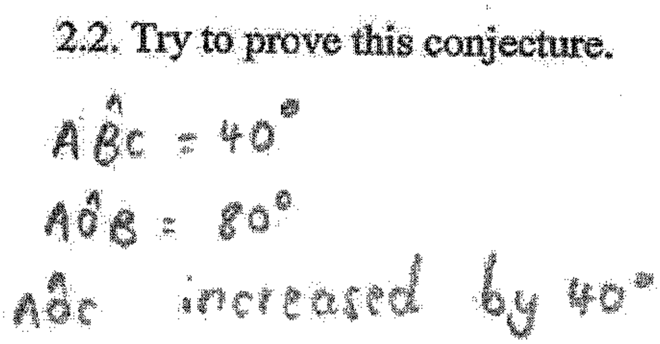
Angle $A\hat{O}C$ is 2 tims angle $A\hat{B}C$.

Figure 15: Learner D's response to question 2.1

Figure 15 shows how learner D correctly responded to the observation. Although learner E measured incorrectly in question 1.8 she still answered question 2.1 correctly. Her inaccurate measurements did not affect her finding. During the interview this learner declared that she arrived at this deduction by her measurement of the angles. Her incorrect measurement of the angle in question 1.8 did not impact on her observation of the relationship between the angles. This learner was the only one who mentioned at this stage that $\angle AO_1C$ was at the middle of the circle and that $\angle ABC$ was along the circle. After having measured and compared the values of angles in question one, four of the five learners were able to correctly deduce a relationship between the value of the angle at the centre of the circle and the angle at the circumference.

5.4 Proving of conjectures

In the second part of question two, the learners were asked to prove the conjecture that they had derived. They were not given any diagrams and the only thing they had to work with was their deduction from their observations. All learners responded to this question. Learner A is the learner who measured two angles incorrectly and wrote down a correct conjecture according to her responses to question one. This learner just chose her own values for $\angle ABC$ and $\angle AOB$. She equated $\angle ABC$ to 40° and $\angle AOB$ to 80°. She then subtracted the values and declared that $\angle AOC$ increased by 40°. This learner also chose the wrong angles for the proof of the conjecture. Figure 16 displays learner A's response.


2.2. Try to prove this conjecture.

$A\hat{B}C = 40^\circ$

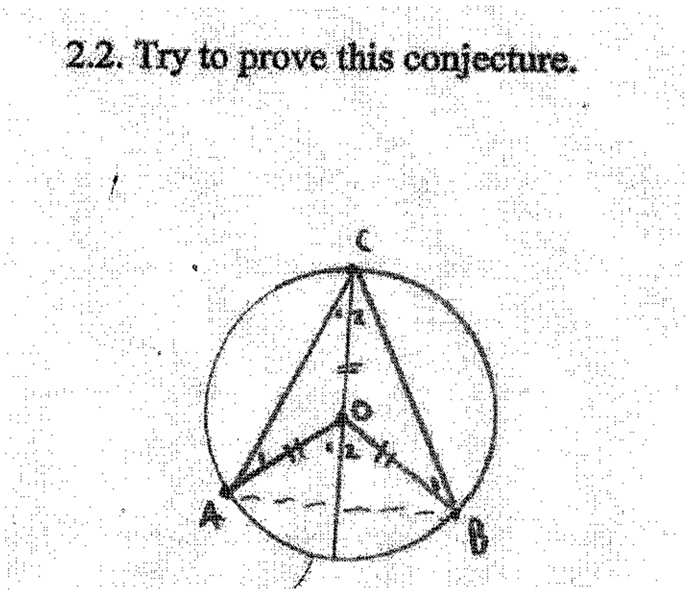
$A\hat{O}B = 80^\circ$

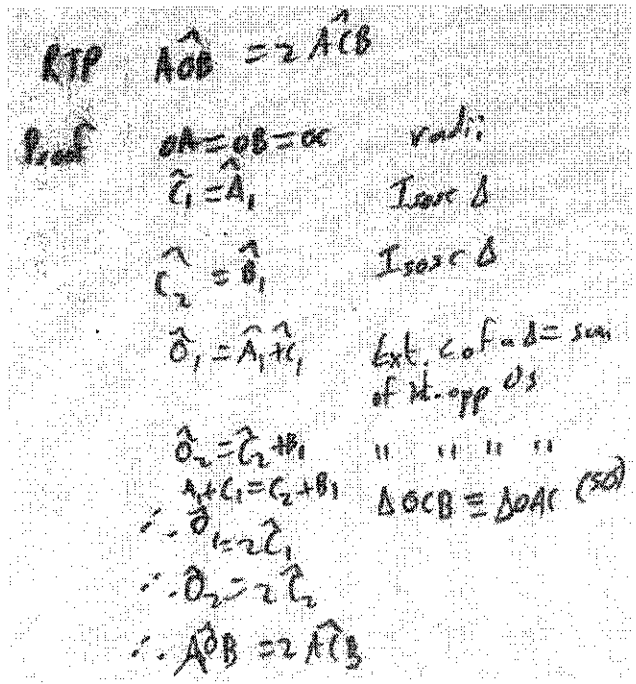
$A\hat{O}C$ increased by 40°

Figure 16: Learner A's response to question 2.2

Many learners may do exactly as Learner A did. It is therefore necessary for teachers to probe the responses given by learners. A further aspect to note is the fact that the learner did not really understand what was expected when asked to prove. It may have been a confirmation of the conjecture but this did not constitute a proof. It may be that these learners have not been exposed to the proving process. There are specific steps required and Learner A used none of them. Specific steps would include the following. Identifying what is given and what is actually required and then perhaps making constructions. Eventually, they would need to engage in a series of logical statements (with reasons) to arrive at a proof. This learner did none of these.

Learner D sketched his own diagram and was the only one who came very close to proving the conjecture. He made use of a diagram to create a logical argument as to why the angle at the centre of the circle is twice the angle at the circumference.





RTP $\hat{AOB} = 2\hat{ACB}$

Proof $OA = OB = OC$ radii

$\hat{C}_1 = \hat{A}_1$ Isos Δ

$\hat{C}_2 = \hat{B}_1$ Isos Δ

$\hat{O}_1 = \hat{A}_1 + \hat{C}_1$ Ext. ∠ of Δ = sum of int. opp ∠s

$\hat{O}_2 = \hat{C}_2 + \hat{B}_1$ " " " "

$\hat{A}_1 + \hat{C}_1 = \hat{C}_2 + \hat{B}_1$ ΔOCB ≡ ΔOAC (SAS)

∴ $\hat{O}_1 = 2\hat{C}_1$

∴ $\hat{O}_2 = 2\hat{C}_2$

∴ $\hat{AOB} = 2\hat{ACB}$

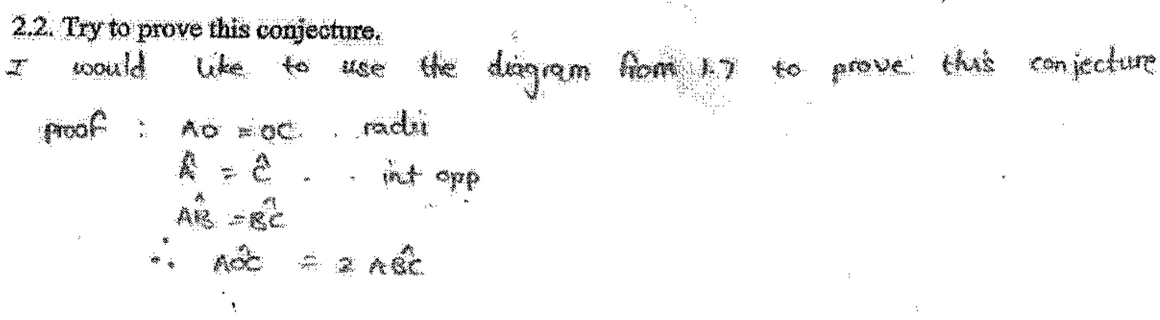
Figure 17: Learner D's response to question 2.2.

Figure 17 indicates learner D's response to the observation in 2.2. This learner sketched his own circle with centre O and points A, B and C that were of the circumference of the circle. Learner D attempted prove with reasons the conjecture. At a glance it seemed that this learner had successfully proved the conjecture but his argument was hindered by the assumption that triangle OCB was congruent to triangle OCA. This might be a common inference and can be attributed to the fact that the radii were all equal. Somehow, learners only look at the equal sides (as denoted by the double strokes on the sides), ignoring the third aspect that needs to be considered (in this case it would be the third side or an included angle). Herein lies the danger of a visual kind. Learners must have a good conceptual understanding of all prior knowledge. The learner's interpretation or analysis of a diagram is dependent on their prior geometric knowledge. It must also be mentioned that teachers must carefully examine learner's proofs because it seemed that Learner D's argument was correct. It was only on close examination that it was discovered that the learner had made a false assumption. This might have been a useful teaching point.

When asked during the interview why a diagram was drawn, this learner declared that the diagram helped to prove that the two triangles will be isosceles and to prove the observation in 2.1.During the interview learner D also declared that diagrams were helpful in proving. He declared that the diagrams helped him to prove that the two triangles were isosceles in question 2.1 Whilst the diagram would have been useful for the proof, the learner still needs to know his/her mathematics. Diagrams are useful only to the extent that the learner knows prior mathematics.

Learner B responded to question 2.2 by stating that she was unsure and that more was needed. During the interview she was asked what she meant by this. She replied that she needed a diagram to prove what she had stated. I then asked her why she felt that she needed a diagram. She replied that it is easier to work when one has a diagram. When asked why she didn't draw a diagram she replied that she did not know how to. This is important to note. Learners are probably not given opportunities to draw their own diagrams and are therefore not able to use these opportunities to solve problems or write out proofs. According to Mudaly (2012, p.22) learners use of diagrams in the solving of problems helps them in their own comprehension of the problem. If learners are given more activities involving the sketching of diagrams then maybe their interpretation of these problems can be enriched.

Learner C responded with *"∠AOC = 2X∠ABC. In order to prove this I need to have a diagram."* Although Learner C's response to question 2.1 was incorrect she correctly stated in question 2.2 that ∠AOC = 2X∠ABC. This incorrect response may be attributed to the learner getting confused about how to write down the relationship between these angles. Learner C's response is that a diagram is needed for the proof. In the interview this learner was asked why she made this comment. The response was that in question 2.2. there was nothing given and that since all that was known was that ∠AOC was 2 times bigger than ∠ABC, a diagram was required to show this. Inevitably, Learner C alludes to the fact that diagrams may be a useful tool for understanding and proving. Learner's is quite similar to the thoughts of Learner B.



2.2. Try to prove this conjecture.

I would like to use the diagram from 1.7 to prove this conjecture

proof : AO = OC radii

$\hat{A} = \hat{C}$ int opp

$A\hat{B} = B\hat{C}$

$\therefore A\hat{O}C = 2 A\hat{B}C$

Figure: 18 – Learner E's response to question 2.2.

Figure 18 shows learner E's response to question 2.2. This learner stated that she would like to make use of the diagram from 1.7. to prove the conjecture. During the interview learner E was asked why she responded in this manner. She replied that she needed something to show how her answer worked. This learner believes that by looking at a diagram she will be able to see the solution. She intimates that the 'something' is probably a bridge that would link what she needs to prove to her understanding of the concept. Her use of the word 'show' would imply that this is related to vision because she would like to 'see' something.

From all the learners' responses, three claimed that diagrams were needed, one was unsure and wanted more and one learner just gave her own values. Majority of the learners felt that a diagram was necessary for the proving process. They felt that the proof could not be completed without the use of one.

5.5 Proving of conjectures with a diagram

In question three the learners were provided with a diagram. They were given a circle with centre O and arc AB subtending ∠AOB at the centre and ∠ACB at the circumference.

3. Given a circle with centre O and arc AB subtending angle AOB at the centre and angle $A\hat{C}B$ at the circumference. Prove that the angle at the centre of the circle is double the size of the angle subtended at the circumference. You may use the reverse side of this page if space is insufficient.

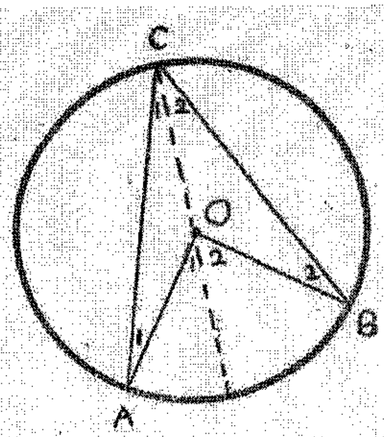


Figure 19: Question 3 from activity 1

The different points and angles on the diagram were labelled. The learners were asked to prove that the angle at the centre of that circle was twice the angle at the circumference. Learners were given the remaining space on the page for their response and additional space if required was available on the reverse side of the page. None of the learners made use of the additional space that was made available. Figure 19 shows question three.

Although all learners attempted to prove that the angle at the centre was twice the size of the angle at the circumference, only one learner D was successful. Learner A substituted her own values for $\angle B_2$, $\angle AOB$ and side BC on the diagram. This learner just wrote down these values on the given diagram. When asked about how she arrived at those particular values for the angles she answered that she had just guessed them because she did not know what to put there. Figure 20 gives learner A's response to question 3.

3. Given a circle with centre O and arc AB subtending angle $A\hat{O}B$ at the centre and angle $A\hat{C}B$ at the circumference. Prove that the angle at the centre of the circle is double the size of the angle subtended at the circumference. You may use the reverse side of this page if space is insufficient.

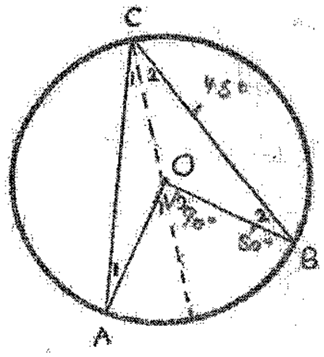
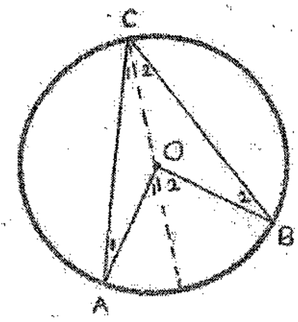


Figure 20: Learner A's response to question three from activity one

Learner B drew her own lines for the proof and she did not place any values on the diagram. She attempted a proof by making use of the sum of the exterior angles of a triangle. By using the diagram she was able to show a relationship between $\angle O_1$ and $\angle A_1 + \angle C_1$. She also indicated that the radii where equal .Unfortunately she was unable to successfully complete the entire proof. In question 2.2 this learner stated that she was unsure about how to prove the conjecture , but in question three where a diagram was provided she made an attempt. The diagram assisted this learner as a tool in writing down some sort of proof. Although her proof was incomplete she was able to observe and state some information that was valuable for the proof.

3. Given a circle with centre O and arc AB subtending angle $A\hat{O}B$ at the centre and angle $A\hat{C}B$ at the circumference. Prove that the angle at the centre of the circle is double the size of the angle subtended at the circumference. You may use the reverse side of this page if space is insufficient.



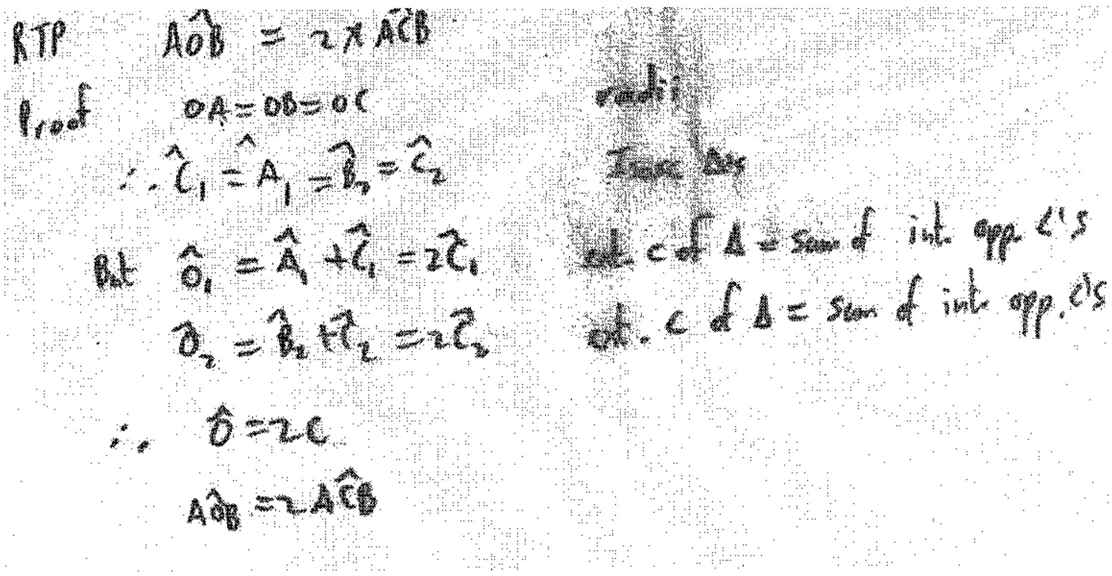
Proof: $\hat{O}_1 = \hat{A}_1 + \hat{C}_1$	ext. ∠'s = sum of opp. ∠'s
AO = BO	radii
∴ $\hat{O}_2 = 2\hat{C}$	

Figure 21: Learner B's response to question 3 from activity 1

For this question learner C also began the proof by equating the radii in the diagram. She went on to state that two times ∠C was equal to ∠O. Although she attempted the proof, it was incomplete and incorrect. When asked during the interview if the diagram in question three was helpful, this learner replied that it was because it made the angle at the centre and the circumference visible. This may imply that being able to actually see the angles on the diagram did make a difference and contributed towards some understanding of the question and hence towards the actual proof.

Learner D attempted to prove this conjecture by equating the radii and the angles of the isosceles triangles. Learner D made use of the diagram in his attempts to prove that ∠AOB = 2∠ACB. His proof for the conjecture in this activity was clearer and more concise in comparison to his proof in the previous question. Unfortunately, inherent in his proof was an incorrect assumption when he stated that the angles of the two triangles are equal.

Again it implies that good prior knowledge is necessary when proving. Figure number 22 shows learner D's response to question three.


RTP $A\hat{O}B = 2 \times A\hat{C}B$

Proof $OA = OB = OC$ radii

$\therefore \hat{C}_1 = \hat{A}_1 = \hat{B}_2 = \hat{C}_2$

But $\hat{O}_1 = \hat{A}_1 + \hat{C}_1 = 2\hat{C}_1$ ext. $\angle$ of Δ = sum of int. opp. $\angle$'s

$\hat{O}_2 = \hat{B}_2 + \hat{C}_2 = 2\hat{C}_2$ ext. $\angle$ of Δ = sum of int. opp. $\angle$'s

$\therefore \hat{O} = 2\hat{C}$

$A\hat{O}B = 2A\hat{C}B$

Figure 22: Learner D's response to question three from activity 1

Learner E attempted the proof with reasons. When questioned about the difference in terms of proving the conjecture in question 2.2. and question three, learner E stated that in question three she had something to start with and that all the important parts on the circle that was needed for the proof could be seen. This learner further stated that she found it better to have a diagram. Although all learners attempted to write the proof by making use of the geometric concepts that they were familiar with such as the exterior angle of a triangle is equal to the sum of the interior opposite angles, only one was able to successfully write part of a proof. This would imply that seeing the important components on the diagram does not necessarily mean understanding the problem. On comparing the learner's responses to question 2.2. and question three, these learners responded more favourably in question three. All learners except learner A, composed statements with reasons in an attempt to prove the conjecture. It would seem that viewing a diagram aided them in their response .Below is a table showing learners responses to question two and question three.

Table 3: Learner's responses to questions two and three

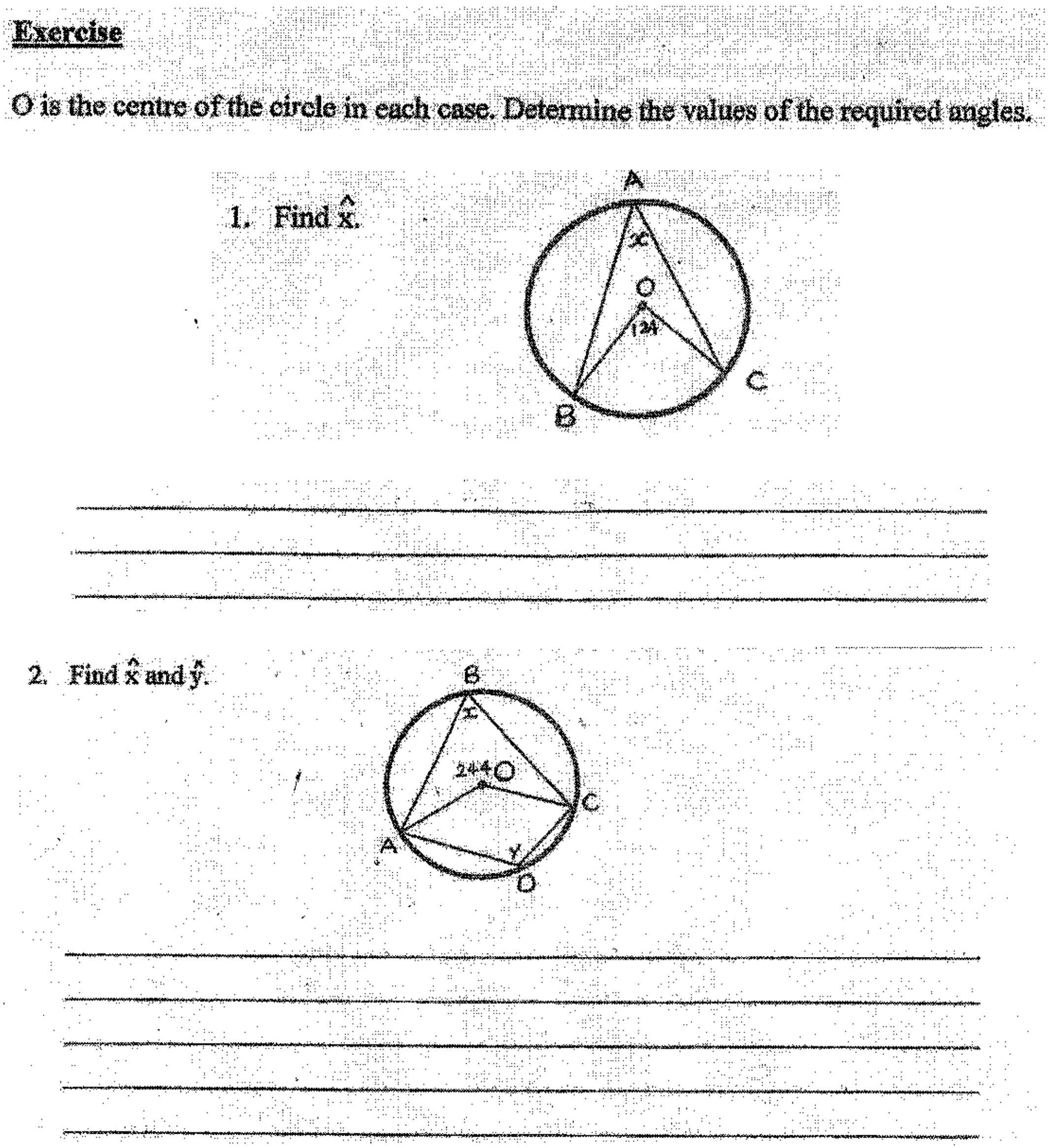
Learner	Q2.1	Q2.2	Q3
A	Incorrect	Incorrect	Incorrect
B	Correct	Incorrect	Incorrect
C	Incorrect	Incorrect - commented that diagram was needed	Incorrect
D	Correct	Correct drew own diagram	Correct
E	Correct	Incorrect – chose diagram from q1.7	Incorrect

A few important aspects emanate from the responses of these learners. Firstly, it seems that having a diagram is important if learners are expected to prove mathematically. Second, teachers must ensure that prior knowledge is understood well before attempting a proof. If prior knowledge is not dealt with then the learners make many incorrect assumptions. It was evident that these learners were not completely used to the structure of a proof. Perhaps if they had some knowledge of the structure of the proving process, then they would have had some direction. In other words, if they are trained to recognise the given information in the question and then identify what they are expected to prove, it might give them a greater opportunity to find the actual proof.

Nevertheless, evidence shows that these learners had very little experience with practical activities, conjecturing and the process of proving. For new knowledge to be created or for old knowledge to be transformed, learners have to have had some experimentation. But this is only possible if they are exposed to these types of procedures. This research engaged them in a rather small scale experiment and it seems like these learners, given time and further guidance, could have arrived at the expected proof.

5.6 Exercise involving application of conjecture

The learners were required to complete an exercise that followed question three. This exercise consisted of four questions. This exercise was based on the conjecture that learners worked with in activity one. In questions one and two from the exercise the learners were just required to calculate the values for the unknown.



Exercise

O is the centre of the circle in each case. Determine the values of the required angles.

1. Find $\hat{x}$.

2. Find $\hat{x}$ and $\hat{y}$.

Figure 23: Question one and two from the exercise

In both these questions the learners had to find the value of $\angle x$ and/or $\angle y$ by using the conjecture that the angle at the centre of the circle is twice the angle at the circumference. Question number two in the exercise demanded a little more application than question one since the values of two angles needed to be found. Of all learners that responded to question one, four answered correctly and learner A did not respond. Of all the learners that answered question two, two answered correctly, two were partially correct and learner A did not respond. Of the two that answered incorrectly one did not divide accurately and the other one's previous knowledge failed her. Although learner A did not answer both these questions when she first attempted the exercise, on her second attempt, after the completion of the guided proof she responded correctly to both questions. It would seem that the guided proof assisted her in developing an understanding of the relationship between the angle at the centre of the circle and the angle at the circumference. In question one and question two, learners were able to easily apply the conjecture. Questions three and four were higher level questions and involved more calculations and reasoning.

3. O is the centre of a circle. $A\hat{O}B$ forms a diameter of the circle. C is a point on the circumference between points A and B. Prove that $A\hat{C}B = 90°$.

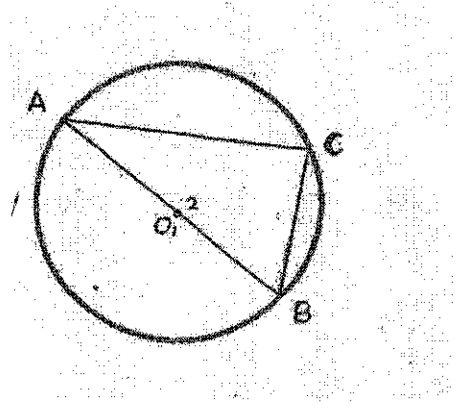


Figure 24: Question three from the exercise

For this question learners were supposed to make use of the value for the angle of the diameter AOB in order to prove that ∠ACB was 90°.All learners answered this question but only three learners were able to prove that the value for ∠ACB was 90°. Learner A just guessed her own incorrect values for the different angles in her first attempt of answering this question. When she answered this question after having completed the guided proof , she stated that the value of the angle was 90° . It seems that the guided proof for the conjecture assisted her in her understanding about the relationship between the relevant angles. Learner C was the other learner that responded incorrectly to this question. She stated that ∠C was 90° but her reasoning was erroneous. All the other learners stated that since ∠AOB was a straight line the value of angle was 180°. They then used this value to conclude that ∠ACB was 90°.

Question four required the learners to calculate the value of ∠C. Joining point O to point B and point O to point D would have assisted learners in the proving process since it would have created two angles at the the centre of the circle. The value of ∠A could be used to calculate the the reflex angle ∠BOD. From here onwards one can find obtuse angle ∠BOD and use this to find ∠C.

4. The circle with centre O passes through the vertices of quadrilateral ABCD. OB and OD are drawn. If A = 60°, calculate the value of C.

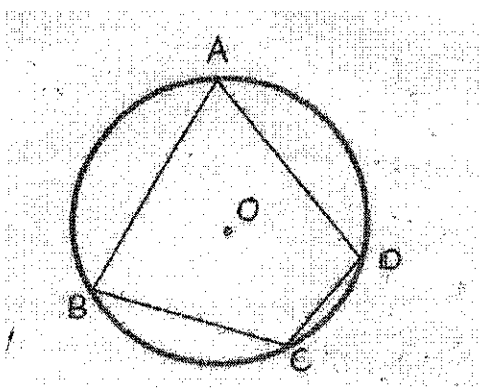


Figure 25 : Question four from the exercise

In this question Learner A and Learner B did not respond. Learner A re-attempted this question after she had completed the guided proof. In her response she placed the values of the ∠BOD and ∠BAD on the diagram. She immediately states that ∠BOD is 120°. She goes on to correctly determine the value of the reflex ∠BOD. Her calculations thereafter of ∠BCD are incorrect because she multiplies the value of the angle at the centre by two instead of dividing it by two. It would seem that although the guided proof assisted her in correctly calculating the value of some of the angles in this question she became confused about which angle was twice the other angle. The values for ∠BOD and ∠BAD was written down on the diagram. Maybe if she wrote down the value of reflex ∠BOD of 240° on the diagram she would been able to highlight what she had. This may have assisted her in the calculating of ∠BCD. Also by drawing her attention to ∠BCD she would have noticed that it is smaller than reflex ∠BOD.

Learner C made an attempt to solve for the value for angle. She joined point O to point B and point O to point C. Learner C wrote down the value for ‹A on the diagram. Using all of this information she was able to find obtuse ∠BOC. She thereafter found the value for reflex ∠BOC but labelled it as ∠C. Her calculations stop there. Learner C was questioned about why she chose to join point O to B and point O to point D. She replied that if you join OD and OB you get the centre and if you have this it then becomes easier to find the value of other angles. I think that she tried to get this diagram to resemble the previous ones in the conjecture and exercise. I went on to as her about why she wrote down the value of 60 ° at ∠A. Learner C then used an example to state that if ∠A is 50° then ∠O is twice this. She explained that by writing down the values on the diagram it made the problem easier to understand.

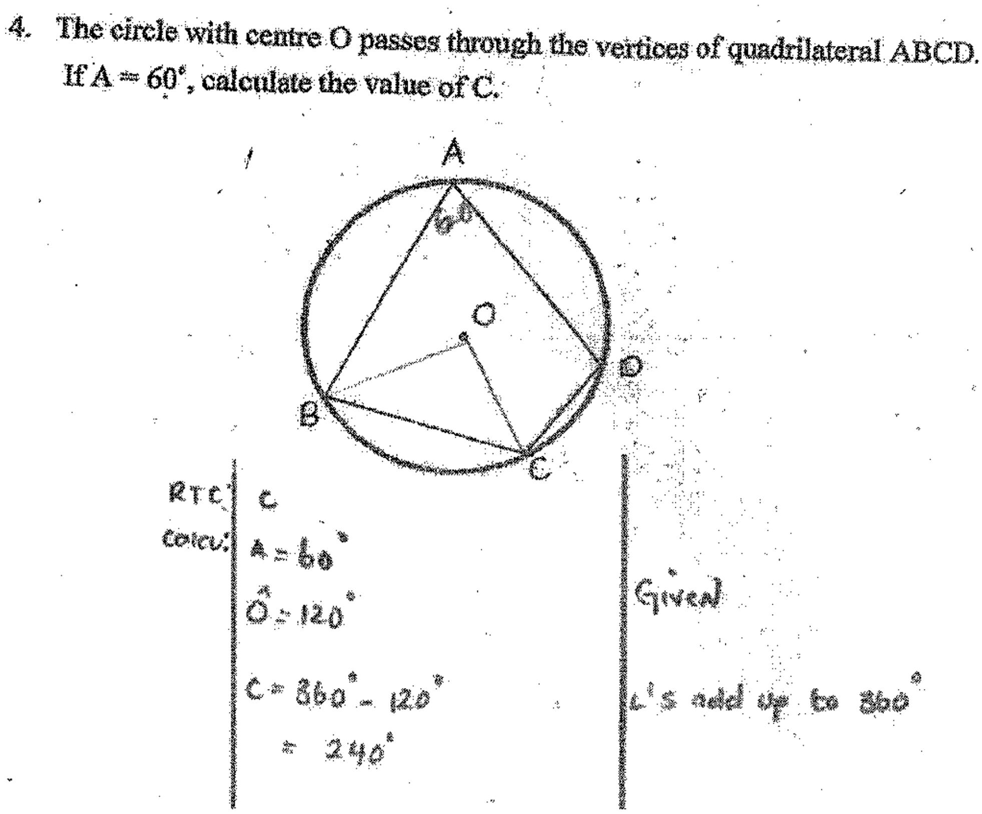
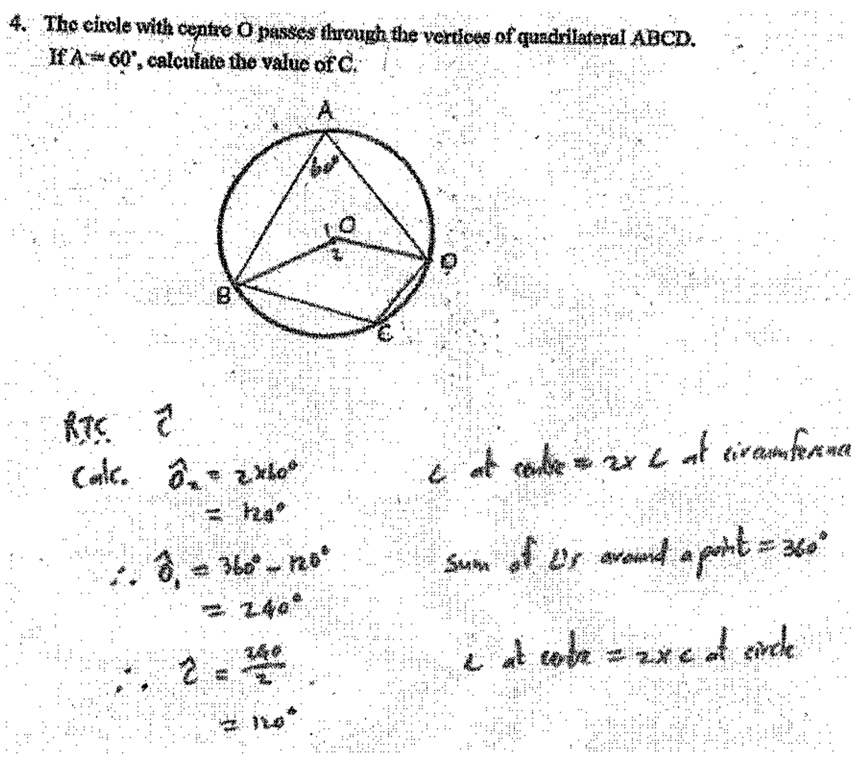


Figure 26: Learner C's response to question four

Learner E joined points OB and OD in the diagram. During the interview she stated that this was done so that she could see the angles in the middle clearly. She went on further to assert that she needed to see these angles in order to find the answer. Learner E added the value of ∠A to itself and multiplied it by two. This is incorrect. This learner declared that ∠BOD = 2∠BCD. This was followed by her dividing the value of ∠BOD by 2 and saying that the value for ∠C was 120°. In determining the values for these angles she applied the conjecture. Learner D wrote down the value for ∠A on the diagram. He joined points O to B and O to D. He also relabeled ∠O as ∠O_1 and ∠O_2 on diagram. This labelling helped when he referred to the different angles that made up ∠O. He then calculated ∠O_2 by applying the conjecture. ∠O_1 was found by using the sum of angles of a revolution. The value of ∠BCD was determined by halving the value of ∠O_1. Both learners C and E joined points OB and OD in the diagram but neither of them labelled them so that one could differentiate between the obtuse and reflex angle of ∠BOD. This may imply that learners need to be made aware that when working with angles of this nature one, has to be wary that one has to be more specific with the labelling. It may also mean that learners are not using diagrams often enough and therefore overlook the labelling.


4. The circle with centre O passes through the vertices of quadrilateral ABCD. If A = 60°, calculate the value of C.

RTC $\hat{C}$

Calc. $\hat{O}_2 = 2 \times 60^\circ = 120^\circ$ ∠ at centre = 2x ∠ at circumference

$\therefore \hat{O}_1 = 360^\circ - 120^\circ = 240^\circ$ Sum of ∠s around a point = 360°

$\therefore \hat{C} = \frac{240}{2} = 120^\circ$ ∠ at centre = 2x ∠ at circle

Figure 27: Learner D's answer to question four

The table below shows the learners responses to the questions in the exercise.

Table 4: Learner's responses to exercise

Learner	Q 1	Q 2	Q 3	Q 4
A	No response	No response	Incorrect	No Response
B	Correct	Correct	Correct	No Response
C	Correct	Incorrect	Incorrect	Incorrect
D	Correct	Correct	Correct	Correct
E	Correct	Correct	Correct	Correct

All the learners who answered question one were correct. Learner A did not respond to this question. For question two, Learner A chose not to respond and Learner C answered incorrectly. With regards to question three, only three learners were able to correctly respond. I think that question four was most challenging for the learners. Two learners chose not to respond, two got the correct answer and one got the incorrect answer. Learner A did not respond to any of the questions in this exercise except question three but after having completed the guided proof she attempted all questions in the exercise. Learner D and learner E responded correctly to all the questions. Perhaps as a last point here, it must be noted that it seems that some learners are in the habit of not even attempting a solution. It is necessary for teachers to encourage learners to try a solution, even at the expense of being incorrect. This would assist the learners to make the necessary connections later when the work is being corrected by the teacher.

5.7 Guided Proof

After completing activity one learners were required to complete activity two. Activity two was a guided proof of the conjecture from activity one. It was made up of four questions. In activity one, question 2.2. the learners were given an application where they had to prove the conjecture that they stated. In activity one, question three they were required to prove the conjecture after a diagram was given.

In activity two, diagrams and guidelines are provided. It is a guided proof to assist the learner in the proving of the conjecture from activity one. It was made up of three diagrams and incomplete statements. Learners had to fill in the blanks. This activity was intended to take learners through the proof for the theorem that the angle at the centre is twice the angle at the circumference.

Question one consisted of three diagrams of circle with centre O. A, B and C where points on the circumference of the circle and OA, OB and OC where given as radii. Learners were required to fill in the blanks for this question and the reasons as to why they answered the way they did. Figure 28 is question one from the activity that learners had to complete.

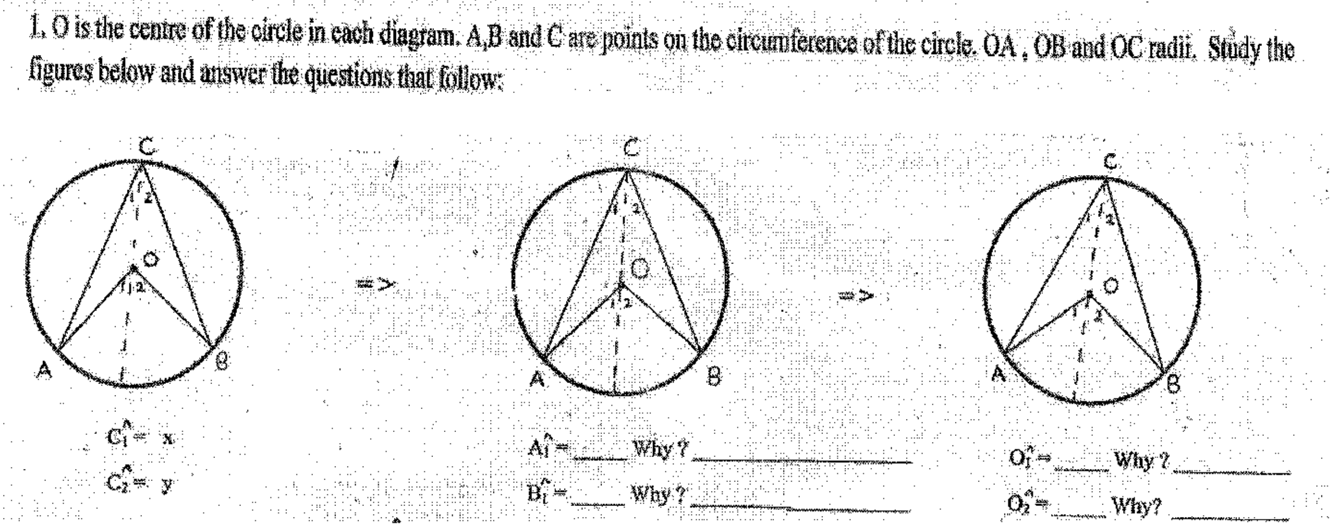


Figure 28: Activity two, question one

Learners were given that $\angle C_1 = x$ and that $\angle C_2 = y$ below the first diagram. These statements and the diagram were intended to guide the learners towards filling in the blanks below the second diagram. Here learners were asked to list with reasons which angles where equal to $\angle A_1$ and $\angle B_1$. Below the third diagram they were asked to fill in with reasons the angles that could be equated to $\angle O_1$ and $\angle O_2$.

The second part of this activity asked the learners to look at the diagrams and fill in the angles that where equal to $\angle AOB$ and $\angle ACB$.

2. From the above diagrams: $A\hat{O}B$ = ______

$A\hat{C}B$ =

Figure 29: Question two from activity two

Question three asked learners whether they noticed a relationship between $\angle AOB$ and $\angle ACB$. They were then asked to state the relationship that they noticed.

3. Is there a relationship between AOB and ACB? If so, state this relationship?______________________________

Figure 30: Activity two , question three

4. Can you make any deductions? If so, please state it.

____

Figure 31: Activity two, question four

Every learner that attempted the activity was able to successfully complete this activity. Learner A is the only learner who re-attempted this activity after her interview. She too, was able to complete it correctly. She had initially left the entire activity blank after completing activity one. In her second attempt I observed that she had written down the letters x and y in the position of $\angle C_1$ and $\angle C_2$ on the first diagram. In the second diagram she wrote down x in the position of $\angle C_1$ and $\angle A_1$ and y in the position of $\angle C_2$ and $\angle A_2$. She wrote down 2x and 2y in the positions of $\angle O_1$ and $\angle O_2$ in the third diagram. Learner A's response to question one is seen in figure 32.

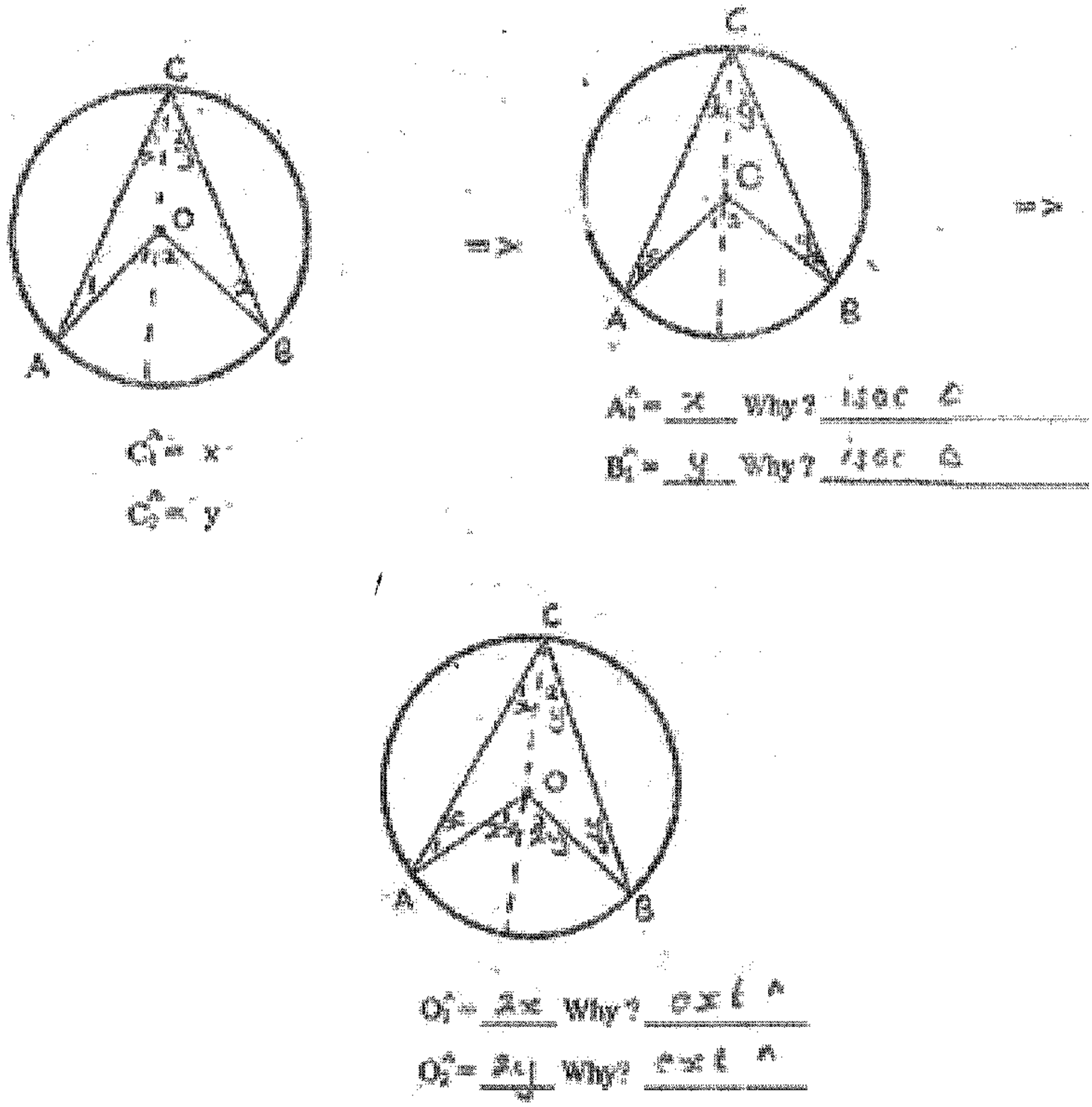


Figure 32: Learner A's response to activity two, question one

While she was completing this activity I asked her why she had done this. She said that it helped her to see which values were equal to each other. She further explained that by doing this she had everything about the question in one place and that this helped her to see clearly what the problem was all about.

She was able to correctly fill in with reasons all the blanks in question one. Figure 26 is Learner A's response to question two. In this question she was able to deduce that ∠AOB was equal to two times x + y and conclude that ∠AOB was two times ∠ACB.

2. From the above diagrams: $A\hat{O}B$ = $2x + 2y$

$A\hat{C}B$ = $x + y$

Figure 33: Learner A's response to activity two, question two

She responded to question three by declaring that ∠AOB is twice ∠ACB. This is seen in figure 34.

3. Is there a relationship between $A\hat{O}B$ and $A\hat{C}B$? If so, state this relationship? $A\hat{O}B$ is two times $A\hat{C}B$

Figure 34: Learner A's response to activity two, question three

In question four she deduced that the angle at the centre was two times the angle at the circle. Although her terminology was incorrect she was able to make a correct deduction.

4. Can you make any deductions? If so, please state it.

Angle at centre is two times angle at circle.

Figure 35: Learner A's response to activity two, question four

This is the same learner who did not answer the exercise and responded incorrectly to question 2.1, 2.2 and 3 in activity one. She did not respond to most of the questions in the exercise, measured the incorrect angles in question one and was unable to write down the conjecture from question 1. Learner A is the learner who just guessed her own values when she was completing the activity. It would seem that despite all of this, the guided proof which was made up of diagrams and directions assisted her in proving the conjecture that she initially she could not even observe.

After the interview and completing the guided proof, learner A attempted the exercise from activity one again. I observed that this time she was writing down information on the diagram. She was questioned about why she wrote everything down on the diagram. She explained that this helped her to *see* clearly what information the question provided her with. Again, the idea that she needs to see the information in one composite diagram is an important one. Perhaps this is important for teachers when presenting problems and their solutions. Further, while completing question four, I noticed that she joined points B to O and O to D. When I enquired about why she did this she replied that she wanted it to look like the other diagrams. By making the diagram in question four resemble the ones that she was familiar with, she would be able to easily apply the conjecture that was written. After the interview and having completed the guided proof this learner was able to complete all the questions in the exercise and she even got some of them correct.

In activity two, Learner B also placed the variables x, y, 2x and 2y on the diagrams in the positions of $\angle C_1$, $\angle A_1$, $\angle C_2$, $\angle A_2$, $\angle O_1$ and $\angle O_2$. It would seem that that this was done to show the relationship between the angles on the diagrams. Below the second diagram she stated that $\angle A_1$ was equal to $\angle C_1$ and in brackets she wrote down x. She then equated $\angle B_1$ to $\angle C_2$ and wrote down y in brackets. She was really asserting that $\angle A_1 = \angle C_1 = x$ and that $\angle B_1 = \angle C_2 = y$. In the second diagram she went on to declare that $\angle O_1 = 2x$ and that $\angle O_2 = 2y$. This was also indicated on the diagram. Figure 36 displays Learner B's response to activity two, question one.

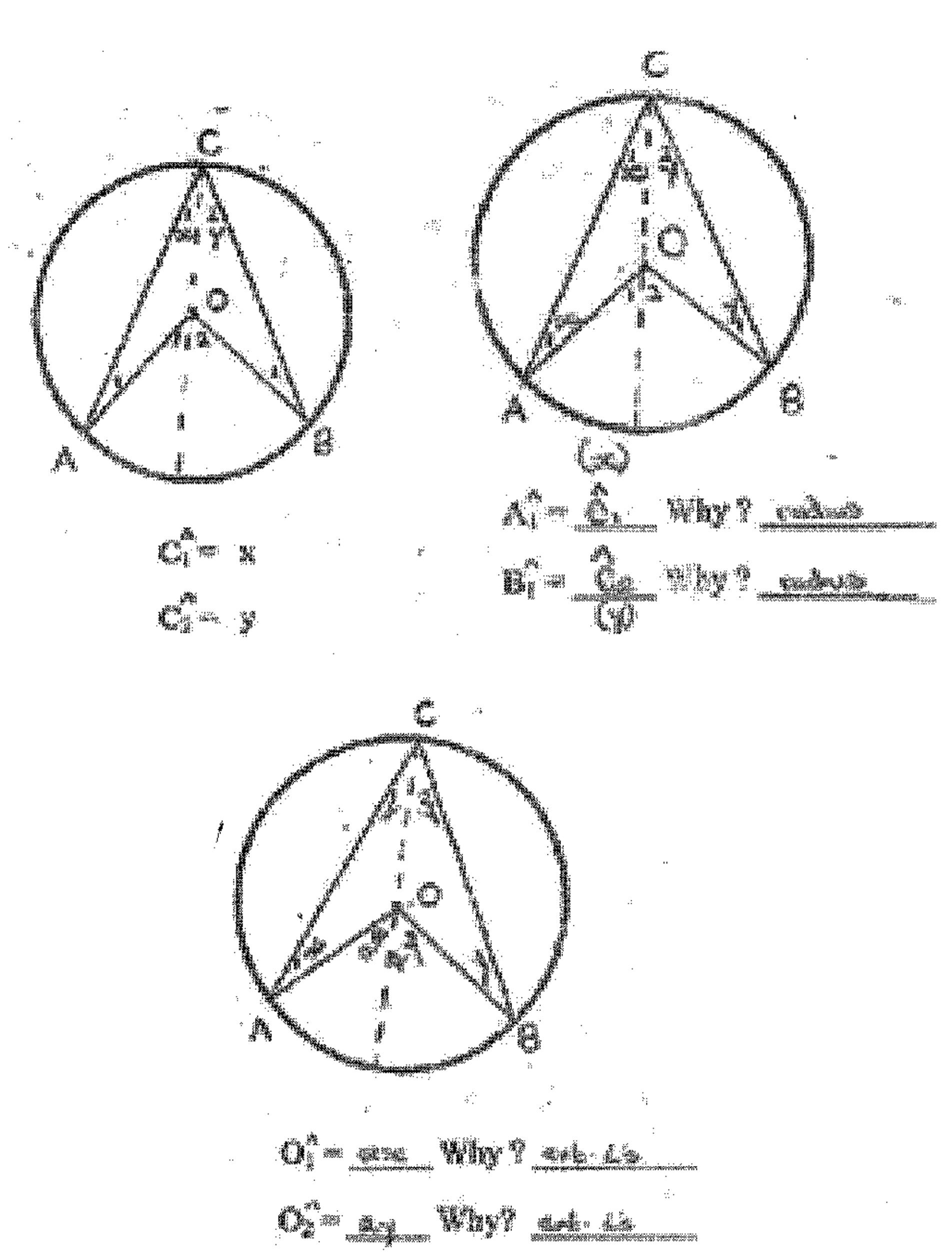


Figure 36: Learner B's response to activity two, question one

Figure 36 displays Learner B's response to question two from activity two. In this question she indicated that ∠AOB was equal to two times ∠ACB and that ∠ACB was equal to half ∠AOB. In question two learner B chose to respond in terms of the given angles ∠AOB and ∠ACB whereas learner A equated the angles in terms of x's and y's.

2. From the above diagrams: $\hat{AOB}$ = $2 A\hat{C}B$

$A\hat{C}B$ = $\frac{1}{2} A\hat{O}B$

Figure 37: Learner B's response to activity two, question two.

This learner stated that ∠O was twice the size of ∠C although what she really meant was that ∠AOB = 2X∠ACB. This once again draws our attention to the labelling of angles. The learners in this research, on several occasions, did not correctly name the angles that they were referring to. Figure 28 displays Learner B's acknowledgement that there is a relationship a between angle ∠AOB and ∠ACB. She however stated that she was unsure if there were any deductions.

3. Is there a relationship between AOB and ACB? If so, state this relationship? Yes. $\hat{O}$ is always twice the size of $\hat{C}$.

4. Can you make any deductions? If so, please state it.

I am not sure if there are any deductions.

Figure 38: Learner B's response to activity two

During the interview learner B was asked to compare the different activities that she was asked to complete and comment about which aided her the most in proving the conjecture. She claimed that the guided proof was most helpful since it provided all the information in terms of the circle and the lines that it subtended. To her these were the core elements that were required for the proof. This would imply that this learner found that having a diagram and being guided along the proving process of a conjecture assists one in a successful proof.

Learner C correctly placed the variables x, y, 2x and 2y on the corresponding angles in the first and second diagrams. This learner's response to question two was correct. She also gave a very detailed explanation about the relationship between ∠AOB and ∠ACB and asserted that there is a relationship between the two angles at the centre and angle at the circle. Learner C did not respond to question four.

Of all the participants' learner D was the only learner who made use of diagrams throughout activities one and two. In activity two he placed variables and markings on the radii to indicate that they were equal in length. He also provided full reasons for his responses to the questions below each diagram. He responded correctly to all the other questions. During the interview learner D indicated that he found guided proof most helpful to prove the conjecture because it gave so much of information and that it lead him towards the actual proof. Learner E correctly answered all of the questions in this activity.

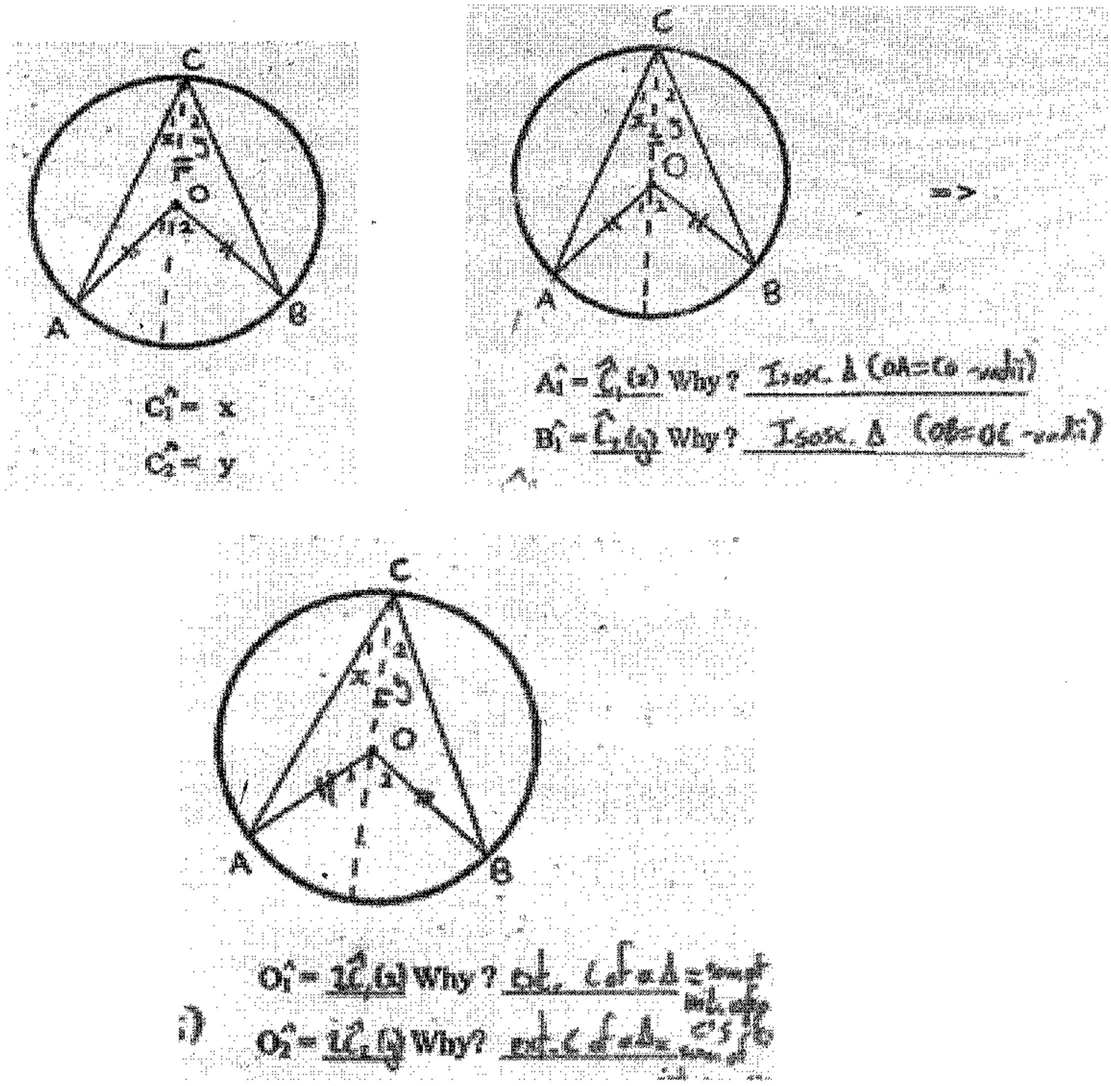


Figure 39: Learner D's response to question one, activity two

Table 5: Summary of learner's responses to activity two

Learner	Q1	Q2	Q3	Q4
A	Correct	Correct	Correct	Correct
B	Correct	Correct	Correct	Unsure, but made correct deduction.
C	Correct	Correct	Correct	No response
D	Correct	Correct	Correct	Correct
E	Correct	Correct	Correct	Correct

Table four shows that most learners responded correctly to the questions in the guided proof. Two learners, Learner B and Learner C had a problem with question four. Question four required that the learners make a deduction about their findings from the guided proof. Question four is in a way a repetition of question three. Question four is a general statement about the relationship between the angle at the centre of a circle and the angle at the circumference in any circle. If the learner has correctly answered question three one would expect them to be able to correctly answer question four. Although Learner B indicated that she was unsure about the deduction in her response to question four, her answer in question three was correct. Learner C successfully answered all questions but left out question four. Most learners correctly answered all the questions in activity two. The responses to this activity was positive. Most of the learners wrote down the letters x and y on the diagrams. Writing down the information on the diagrams may have been beneficial in the re – organising of the learners thoughts and thereby leading to the creation of new knowledge. The diagrams and guidelines seem to have simplified the proving of the conjecture for the learners.

5.8 Learners perceptions about diagrams

The learners were asked to complete a questionnaire after the interview was conducted. The questionnaire was prepared to gather more information about their thoughts on the use of diagrams when solving geometric problems. The questionnaire was made up of four questions. It consisted of open and closed questions.

All learners answered all the questions. Three learners stated that they make use of diagrams when solving geometric problems, one said that they don't use diagrams and the other stated that they only use it on occasions. Question 2.1 asked the learners whether they felt that diagrams were helpful in efficiently solving /proving geometric problems. In spite of their various responses to question one all of the learners responded yes to this question. In activity one question 2.2.the learners were asked to prove the conjecture. Only Learner D was able to sketch his own diagram in order to attempt the proof. Although all the learners acknowledged the effectiveness of the use of diagrams for proof, four of the five learners were unable to sketch an appropriate diagram for the conjecture.

Question 2.2. was a follow up from question 2.1 .The learners were asked to give a reason for their answer in question 2.1. There were many reasons for this. Some of the reasons that they gave were:

- " It helps to see the given and what I need to find";
- "When solving a geometric problem it is necessary for a diagram to be used so that you can see all the information clearly and it becomes easy to understand and answer";
- "Diagrams allow for an illustration of the problem at hand. This assists in the solving of that geometric problem by offering an illustration of what is given and what has to be solved. Diagrams often allow me to solve problems I would not have been able to solve from text alone" ;
- "I think that diagrams can help us to understand things and to see everything";
- "Having a diagram is better to work with because, firstly you can see what you working with and it helps when you can mark off your deductions and also writing your final answer on the diagram so you don't have to search in your answer sheet."

Although all the learners disclosed that they valued the use of diagrams in the solving of geometric problems Learner A stated that she did not use them and Learner E said that she only used it sometimes. A common reason that the learners gave for using diagrams was that it helped them gain a better understanding of the problem. It was stated by Learners A, B, C and E that by having a diagram important symbols inherent in the question became more observable. Learners claimed that information that they felt were of relevance could be easily marked off on the diagram thus making the important components in the question more noticeable. This suggests that diagrams can be used as making sense instruments. Although learners seem to value the use of diagrams in solving geometric problems, it would seem that they have not had sufficient experience in sketching their own diagrams. Teachers need to create more exercises that involve learners sketching their own diagrams in order to prove/solve the geometric problems. It may be that the act of looking at the diagram in junction with the actual question, may assist the learner to understand the problem. Further, it may be that these learners actually form useful connections between the various concepts depicted in the diagram, thus making the solving process easier. This is also dependent on the learner's *a priori* knowledge.

Diagrams may assist learners by making the essential components in the problem and the relationships between these components more observable. New ideas can be built upon from previous ideas. Question three asked the learners whether their teacher made use of diagrams when solving geometric problems. Two learners said '*sometimes*' and the other three learners said '*yes*'. This implies that the teachers of these learners find merit in the use of diagrams. The table below gives a summary of the learner's responses to questions one to three.

Table 6: Summary of learner's responses for question1 to 3

Learner	Q1	Q2.1	Q2.2	Q3
A	No	Yes	Helps to see what is given	Sometimes
B	Yes	Yes	See what you doing, mark off deductions	Yes
C	Yes	Yes	Can see information, easy to understand	Yes
D	Yes	Yes	Diagrams help to solve, couldn't have solved if text alone.	Yes
E	Sometimes	Yes	Diagrams help to understand and see things.	Sometimes

Question four of the questionnaire was based on the learner's responses to question two and question three from activity one and activity two. Question four was made up of eight sub questions. Question 4.1. asked the learners if they were able to prove the conjecture in question 2.2. Although two learners responded that they were able to prove the conjecture, only one was able to partially prove this. Question 4.2 asked the learners to give reasons as to why they were able to prove or not prove the conjecture. Their responses were as follows:

- "I was not sure what to do";
- "Since a diagram is needed to solve a geometric problem in question 2.2 a diagram was not given so it was impossible to see what was needed and it was hard to understand so that why I was not able to prove the conjecture in question 2.2.";
- "I didn't have a diagram to work with. I find it hard and confusing to work without a diagram";
- "I was able to prove that conjecture by using a diagram and simple geometry. When trying to prove that a particular theorem, a diagram is essential in order to think logically";
- "I tried to prove it by using the diagram from 1.7.I could see the angles in the centre and the angle at the circumference. "

Learner A stated that she was unsure how to prove and therefore didn't do anything. Learner B conveyed that she found it problematic and too complicated without a diagram.

Learner C declared that the solving of the problem was dependent on a diagram and since this was not provided, the problem could not be solved. The only person to draw his own diagram was Learner D. He was able to partially prove the conjecture. By constructing an appropriate diagram he was able to the reveal essential elements in the problem. Learner E felt that she needed a diagram and therefore chose her own diagram from the previous question. Four the five of the learners felt that the proof for the conjecture was reliant on a diagram. The remaining learner was just uncertain about what to do. Majority of the learners in this research regarded diagrams as being a significant part in proof. This illustrates that the learners feel that diagrams are required for solving in geometry.

Question 4.3 asked the learners if they were able to prove the conjecture in question three. All learners except learner A replied to this question with a "*yes*". Learner A declared that she was unable to prove the conjecture because she was unsure what to do and that she had just guessed the values. Although all the other learners felt that they had successfully completed the proof, only learner D was able to partially prove the conjecture. This may suggest that the mere presence of the diagram for the question lead these learners to believe that their proofs were successful. All these learners were confident about having proved the conjecture. The reasons that the learners provided for being able to prove the conjecture were:

- "Because the information was given and the diagram was given so it was much easy to understand what was needed to be done and how must it be done so that why I was able to prove it.";
- "The diagram was given to me";
- "A diagram was supplied/given in this question. As stated above, a diagram is essential in order to prove that particular theorem";
- "Well we were given a diagram and this has helped me to have a full idea of what I was working with."

All the learners responses indicate that they were convinced that since a diagram was provided they would be able to effectively write out a proof. Their previous responses displayed uncertainty in being able to prove the conjecture but just the provision of the diagram impacted on their conviction in being able to write a successful proof. Question 4.5 asked the learners if they were able to prove the conjecture in activity two. All replied yes and all learners were able to successfully complete this activity. Their reasons for being able to complete this proof were:

- "It was easier than the 1st two. Here there was a diagram and I was asked questions that helped me prove";
- "Even though a clear diagram was not given but I was able to prove it because of the rules that need to be used";
- "I had a diagram and the information that was given helped me to prove";
- "Activity 2 featured several diagrams and information pertaining to those specific diagrams. Due to the fact that Activity 2 had so much of information given in an easy to understand layout, I was able to prove the theorem much faster";
- "The guidelines were given and to help me to what needed to be proved."

The learner's responses to this question show that the diagrams and guidelines in this activity assisted them. The given diagrams and questions for this activity were designed to help them prove the conjecture. All of the learners were able to successfully complete this activity. The guided proof was the only activity in which every learner was able to successfully prove the conjecture. This infers that the diagrams together with the statements steered the learners towards proving the conjecture. Even learners such as learner A, who was unsuccessful with most of the questions in activity one was able to complete the guided proof correctly. Since she had left so many questions unanswered in activity one, she was asked to re attempt the exercise at the end of activity one after her completion of the guided proof. Her second attempt of completing the application exercise at the end of activity one had very fruitful results. She was able to solve most of the problems in the exercise. Question 4.7 asked the learners how question 2.2, question three and activity two differed. Responses from the learners were as follows:

- "No diagram just a statement in question 2.2 and just a diagram in question3";
- "Question 3 a diagram was given and in question 2.2 there was no diagram. Activity 2 had a diagram and questions. It made me answer questions along the way";
- "In 2.2. I did not have a diagram so it was hard for me to answer. In question 3 I had a diagram and it was easy for me to work with. In activity 2, I had a diagram and it guided me in answering the question. The diagram made it easier and more simple to understand";
- "Question 2.2 had asked for the stated conjecture to be proved, no diagrams or extra information was given. Question 3 had asked for the same conjecture to be proved, but this time, a diagram was given. Activity 2 required me to prove the same theorem, however, this time diagrams and questions that lead up to the proof was given";
- Q 2.2. – Nothing was given. Q3- Diagram was given. Activity 2 There were diagrams and also instructions to help prove the conjecture."

All learners were able distinguish the differences between question 2.2. , question three and activity two. Overall they observed that in question 2.2 they were required to proof the conjecture without a diagram, in question three they were required to prove the conjecture with a diagram and in activity two they were required to prove the conjecture while been provided with a diagram and questions that guided them towards the proof.

The last question asked the learners about which question/activity assisted them the most in proving the conjecture and why. The responses were as follows:

- "Doing activity 2 made it easier, because it helped me towards finding a relationship between angle AOB and angle ACB";
- "Question 2 because you were given diagrams and instructions to follow";

- "Question 3. I was able to work with a diagram in question 3 and I find having a diagram makes it easier in proving the conjecture. Also activity 2 was good because it helped me in proving ";

- "Activity 2, the large amount of given information as well as "flow "of questions which lead to the proof, made proving this conjecture fairly simple. A diagram was given, which is essential in proving the conjecture, was given. Several questions were given which lead me to proving the conjecture, they provided a "flow" of steps";

- "I think Activity 2 because I had to full in the blanks which helped me to come towards the proof. It helped me prove the conjecture."

All learners except learner C stated that activity two was most beneficial for proving the conjecture. Learner C declared that question three and activity two was valuable for proving. In both question three and activity two diagrams were provided. None of the learners found the statement of the conjecture helpful towards a proof. The table below is a summary of the learner's responses to questions 4.1. to 4.8. from the questionnaire.

Table 7: Summary of learner's responses to question 4

Learner	Q4.1	Q4.2	Q4.3	Q4.4	Q4.5	Q4.6	Q4.7	Q4.8
A	No	Unsure what to do	No	Unsure what to go, so guessed values	Yes	Easier than first 2. Questions helped me proof.	Q2 no diagram just a statement, q3 just a diagram	Act 2 helped find a relationship between angles.
B	No	No diagram to work with, so hard and confusing.	Yes	Diagram was given	Yes	Had diagram and information helped in proof	Q2 no diagram so hard, q3 diagram easier. Act 2 simpler and easier.	Q3 diagram helped, Act2 helped with proof.
C	No	Unable to prove because no diagram, can't see what's given	yes	Diagram given, easier to understand, what to do	Yes	Used rules to prove it.	Activity had diagrams and questions, so made it easier.	Activity 2 because given diagrams and instructions to follow.
D	Yes	Diagram helps to think logically that's why able to prove.	Yes	Diagram essential for proof.	Yes	Many specific diagrams, plenty info given, easy to understand, proof was faster.	Q1 no diagram, Q2 only diagram, Q3 diagram and questions that lead to proof.	Act2, large amount of info given, flow of questions lead to proof, made it easier.
E	Yes	Tried to prove using diagram from 1.7.	Yes	Diagram gave full idea of what she was working with	Yes	Guidelines helped me to see what needed to be proved.	Q2.2 nothing given, Q3 diagram given, act2 diagram and instructions.	Act2, had to fill blanks, helped with proof.

From Table 7 we can see that two learners were confident that they had successfully proved the conjecture in question 2.2 because they had both made use of their own diagrams. Four of the five learners believed that they had proved the conjecture in question three. Learner A was the learner who stated that she did not prove the conjecture in question three. Her reason for this was that she was unsure what to do. The responses to question 4.5 indicate that all learners felt that they had proved the conjecture in their responses to activity two. The common reasoning behind this was that the diagrams and guidelines provided, lead them towards the proof. These learners seemed to be convinced that the presence of a diagram will almost guarantee a proof. From their responses it seems that they find diagrams most beneficial because it helped them develop a better understanding of the conjecture.

5.9. Kolb's experiential learning theory

This research is underpinned by Mudaly (2012) adaptation of Kolb's Experiential Learning Theory. This theory centres on the experiences that learners have during the learning process. Mudaly's (2012) adaptation of Kolb's model shows the process that steers the learners in the direction of gaining new geometric knowledge or changing their old geometric knowledge. This model consists of concrete experience, reflection, abstract conceptualisation and active engagement with the diagram.

In this research, activity two supports the adaptation of Kolb's Model. In terms of the concrete experience in the model, learners were provided with three diagrams in question one of activity two. The following information for these diagrams was provided: O is the centre of the centre of the circle in each diagram. A, B and C are points on the circumference of the circle. OA, OB and OC are radii. They were asked to study the figures and answer the questions below the diagrams. The learners were provided with the concrete experience of the diagrams which would have resulted in them thinking about the symbols and information within the diagrams. The reflection process would have begun when they started pondering over what they saw in the diagrams together with their former geometric knowledge. This stage involves the learner thinking about what they saw in the diagrams. The reflection stage is followed by the abstract conceptualisation stage. It is here that whatever the learners comprehended in the diagram becomes more significant to them resulting in new knowledge being created. The new knowledge that the learner created can then be added to the given diagram.

In activity two, the new knowledge was created by the learner reflecting on their prior knowledge and interacting with the symbols in the three diagrams in question one. The learner's internal thoughts and previous knowledge together with that which is inherent in the diagram creates new knowledge. The new knowledge influenced the learner's responses to questions two to four. This model is a cycle so the process continues. In activity two, knowledge about the proof for the conjecture is created. The learners have experiences with the diagrams that are provided. The experiences that these learners have with the diagrams depends on their reflection of their prior knowledge and the diagrams. New knowledge or insight about the concepts are then created.

5.10. Conclusion

In this chapter I have provided data from the participants of my study on the role of visualisation in the solving of geometric problems. The data gained from the evaluation activities, interview, questionnaire and observation validates that diagrams have an important role to play in the solving of geometric problems.

CHAPTER SIX

DISCUSSIONS, LIMITATIONS AND RECOMMENDATIONS

6.1 Introduction

The objective of this study was to investigate the role of visualisation in the solving of Euclidean geometric problems. The research attempted to explore whether visual representations play an integral part of proof. It also looked at the use of diagrams with guidance when solving geometric problems.

6.2 Research Findings

6.2.1 Overall impression of the research

The study revealed that although the participants in this study were grade 11 mathematics learners they were relatively unfamiliar with the use of a protractor. A common problem was that they had difficulty in choosing which value to write down from the protractor for the measurement of each angle. The difficulty was that they did not know whether to read the value of the inner or outer semi - circle.

Three of the five learners measured the incorrect angles. Although the angles were clearly labelled in the given diagrams these learners still measured the incorrect angles. They were unable to differentiate between $\angle AO_1C$ and $\angle AO_2C$. This could imply that whilst reading, understanding and following instructions are important steps in completing questions, these learners had little experience in using these processes. In the future, in order to help learners to correctly recognize angles and the relationships between them, teachers should encourage them to mark or shade the necessary angles. The learners can even make use of different colours or write down values in the relevant positions. This may aid the learners by making the chief components in a geometric problem more noticeable. Also learners not being able to differentiate between different angles could suggest that learners are not using diagrams often enough and are therefore overlooking the labelling.

Learner A's response to activity one was correct due to the coincidental relationship discovered between the diagrams provided. This draws attention to the fact that teachers need to be cautious when planning activities because distractions like a coincidental result may lead to an activity becoming ineffective. After her interview Learner A was able to successfully state the conjecture which she previously got wrong. This shows that well planned experiences can assist learners to understand and notice certain concepts.

It was observed that when Learner B was questioned about why she did not draw a diagram she replied that she did not know how to. Teachers ought to ensure that they create sufficient opportunities for diagram sketching. This could lead to an improvement in the learners understanding and solving of a problem.

It appears that some learners have a tendency to not even attempt a solution. Teachers should urge these learners to attempt a solution even if it may be incorrect. This practice can help learners to make the necessary connections when work is being corrected.

6.2.2 Visualisation and proof

The literature reviewed discussed several advantages of using diagrams when solving geometric problems. Some of the advantages are that they help the learner to understand a problem, to identify key components and relationships between them.

In activity one, question 2.2.the learners were asked to prove the conjecture that they had derived. The only thing that the learners had for the proof was their deduction from their observation. While learner A just wrote down her own values for the proof, learner B stated that she needed a diagram but did not know how to sketch one. Learner C also declared that a diagram was needed and learner E chose her own diagram from the previous question. Learner D was the only one who sketched his own diagram. His proof was only partially correct due to his lack of previous knowledge. All learners except learner A felt the need for a diagram to prove the conjecture. This shows that these learners value the use of a diagram for proof. The success of the proof depends on their prior mathematical knowledge and for this question their ability to sketch an appropriate diagram.

In question three of activity one, learners were provided with a diagram and asked to prove the conjecture that they realized. All learners attempted this question. While learner A just guessed her own values for the angles and learner B was able to show relationships between the angles that were necessary for the proof. Learner C stated during the interview that the diagram helped her in making the important parts of the proof noticeable. This implies that the provision of the diagram assisted her in her understanding of the question. Learner D's incorrect assumption prevented him from proving the conjecture. His lack of mathematical concepts necessary for the proof of this conjecture hindered his attempt. During the interview learner E claimed that since the diagram for the proof was given, all the important parts on the circle necessary for proof became easily noticeable. Their previous responses displayed uncertainty in being able to prove the conjecture but just the provision of the diagram impacted on their conviction in being able to write a successful proof. Four of the five learners responded more favorably to question three than question 2.2.

All learners revealed in the questionnaire that they valued the use of diagrams when proving and solving geometric problems. Some of the reasons they gave for this was that diagrams assisted them to gain a better understanding of the problem and that it made essential symbols in the diagram and the relationship between each more noticeable .The learners also stated that diagrams helped them to make sense of the question. This shows that learners are of the view that diagrams form an important part in solving geometric problems. Part three of question four in the questionnaire asked the learners if they were able to prove the conjecture when the diagram was provided. Although all learners except learner A replied that they were able to write the proof, nobody was really able to do so. This suggests that just the presence of a diagram led the learners to believe that their proofs were correct. Their responses show that learners regard diagrams as a significant part of proof. Whilst these diagrams may increase their conviction they still need to develop a formal proof.

The provision of the diagram seemed to have been beneficial to the learners attempt at proving the conjecture. There are however certain factors that influenced their writing of the proof for the conjecture. Firstly the learners need to understand the prior mathematical knowledge required for a proof. If the learners are unfamiliar with the necessary prior knowledge they may make incorrect assumptions. The learner's prior knowledge affects their responses to the activities. A learner's interpretation or analysis of a diagram is dependent on their geometric knowledge. Although diagrams may be beneficial for proof the learner still needs to know their mathematical content. This implies that the value of diagrams is entirely dependent on the extent to which learners know their geometric knowledge. An example of this was Learner D's response to question three, activity one. His incorrect assumptions contributed towards his incorrect proof.

Secondly from the learner's responses it was evident that these learners were unfamiliar with sketching of their own diagrams and the structure of a proof. Their responses did not display knowledge of how to write a proof. This may imply that these learners have not been exposed to the proving process. The steps necessary for proof such as identifying the given, that which is required, sometimes making constructions and logical statements with reasoning was found to be lacking in the learners responses.

It was also evident that the learners were unfamiliar with the structure of proof. Maybe if these learners were taught how to recognize the given information in a question and identify what they are expected to prove, then they may have a greater chance of proving successfully. It appears that these learners have had very little experience with proving and conjecturing. For new knowledge to be created or for old knowledge to be transformed, learners have to have had engaged in some experimentation.

Within this study visualisation seems to form an integral part of proof. Although the learners in this study feel that diagrams form an important part of proof none of them were able to successfully complete the entire proof for the conjecture. In order for them to be more successful when proving geometric problems knowledge of mathematical content, how to sketch diagrams and techniques of proof needs to be addressed.

6.2.3 Diagrams with guidelines and proof

Activity two was a guided proof of the conjecture in activity one. The guided proof was made up of three diagrams and incomplete statements. The learners were required to answer the questions by filling in the blanks. The intention of the activity was to guide the learners through the theorem the angle at the centre of the circle is twice the angle at the circumference.

Every learner was able to complete this activity successfully. Learner A who was unsuccessful with most of the questions in activity one was also able to complete this proof correctly. All learners stated in the questionnaire that they found activity two most helpful in the proving process. The responses to questionnaire indicate that all learners felt that they had proved the conjecture in their responses to activity two. The common reasoning behind this was that the diagrams and guidelines provided, led them towards the proof. Learner A stated that activity two helped in identifying a relationship between the relevant angles. Learner D indicated that he found guided proof most helpful to prove the conjecture because it gave so much of information and that it lead him towards the actual proof. Most of the learners wrote down the variables on the diagrams. Writing down the information on the diagrams may have been beneficial in the re – organising of their thoughts and thereby leading them to the creation of new knowledge. The diagrams and guidelines seem to have simplified the proving of the conjecture for the learners.

It seems that learners had difficulty with proof when they only had the statement of the conjecture and when they were given just the diagram. Within this research study it seems that diagrams alone were inadequate to complete the proof. However, the diagrams together with the guidelines steered the learners towards proving the conjecture. This shows that diagrams together with the guidelines form an important role in the solving of geometric problems.

6.3 Limitations

This was a small research study since the number of participants were just five. A greater number of participants may help to gain more insight about the research questions. The research was also done in the part of the year when learners had not had that much contact time with geometry. If it were done later on in the year the learners may have had a chance to do more geometric problems and thus become more familiar in the processes of solving and proving.

6.4 Recommendations

The study may be conducted over a longer period involving more learners in different schools. A similar study may be conducted using dynamic geometric software like Geogebra.

REFERENCES

Agathangelou, S., Papakosta, V., & Gagatsis, A. (2008). The impact of iconic representations in solving mathematical one-step problems of the additive structure by primary second grade pupils. *Proceedings of the 11th International Congress of Mathematics Education*. Monterrey, Mexico: ICME.

Ali, S., & Kelly, M. (2004).Ethics and social research. In C.Seale (Ed.), *Researching Society and Culture* (pp.116-126). New York: Sage Publications.

Alshwaikh, J. (2009).*Diagrams as interaction: The Interpersonal (meta) function of geometrical diagrams.* Paper presented at the Proceedings of the British Society for Research into Learning Mathematics, March, 2009.

Arcavi, A. (2003). The Role of Visual Representations in the Learning of Mathematics. *Educational Studies in Mathematics, 52,* 215-241.

Atebe, U.H. (2008).*Students Van Hiele Levels of Geometric Thought and Conception in Plane Geometry: A Collective Case Study of Nigeria and South Africa.* A thesis submitted for the degree of Doctor of Philosophy, Rhodes University, Grahamstown.

Andresen, L., Boud, D., & Cohen, R. (2000). Experience-Based Learning. In G.Foley (Ed.), *Understanding Adult Education and Training* (pp.225-239). Sydney: Allen & Unwin.

Bamford, A. (2003). The Visual Literacy White Paper, Retrieved 15 August 2013, from http://www.adobe.com/uk/education/pdf/adobe_visual_literacy_paper.pdf

Barbosa, A., Palhares, P., & Vale, I. (2007). *Patterns and generalization: the influence of visual strategies.* Paper presented at the Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education.

Bertel, S. (2005). *Show Me How You Act on a Diagram and I'll Tell You What You Think (or : Spatial Structures as Organising Schemes in Collaborative Human – Computer Reasoning),* Paper presented at the American Association for Artificial Intelligence Spring Symposium, Menlo Park, CA.

Bleed, R. (2005). Visual literacy in higher education. *Educause Learning Initiative Explorations, 1*, 1-11. Retrieved from http://net.educause.edu/ir/library/pdf/eli4001.pdf

Boud, D., Cohen, R. & Walker, D. (1993). *Using Experience for Learning* (1ST ed.) Buckingham, SRHE : Open University Press

Bowie, L. (2009). *What is Mathematics Paper 3 for?* Marang Centre for Mathematics and Science Education, Marang News, Issue 5, June 2009.

Boyatzis, R., Kolb, D.A., & Mainemelis, (2001).Experiential learning theory: previous research and new directions. In R.Sternberg and L.Zhang (Eds.), *Perspectives on cognitive, learning and thinking styles.*Mahwah, NJ: Lawerence Erlbaum Associates.

Budram, R. (2009). *The effects of using Visual Literacy and Visualisation in the Teaching and Learning of Mathematics Problem Solving on Grade6 and Grade7 learners.* University of Kwa-Zulu Natal, Durban.

Carney, N.R., & Levin, R.J. (2002). Pictorial illustrations still improve students' learning from text. *Educational Psychology Review, 14*(1), 5-26.

Carter, J. (2009). Visualisation and Understanding in Mathematics. Retrieved 7 June, 2013, from http://www.pantaneto.co.uk/issue33/carter.htm

Clements, D. (2002). Teaching and learning geometry. *A research companion to principles and standards for school mathematics, 11*, 151 -178.

Cohen, L., Manion, L. & Morrison, K. (2007). *Research methods in education.* (6th ed.).London: Routledge.

Diezmann, C.M. (1995). Evaluating the effectiveness of the strategy 'Draw a diagram' as a cognitive tool for problem solving. In *Proceedings of the 18th Annual Conference of Mathematics Education Research Group of Australasia.* (pp223-228). Darwin.

Diezmann, C M. (1999). *Assessing diagram quality: Making a difference to representation.* Paper presented at proceedings of the 22nd Annual Conference of Mathematics Education Research Group of Australasia (pp. 185-191), Adelaide.

Diezmann, C.M. (2000a). The difficulties students experience in generating diagrams for novel problems. In T Nakahara & M. Koyama (Eds.), *Proceedings of the 24th conference of the International Group for the Psychology of Mathematics Education, 2,248-277.*

Diezmann, C. M. (2000b). Making sense with diagrams: Students' difficulties with feature-similar problems. In J.Bana & A. Chapman (Eds.), *Mathematics education beyond 2000: Proceedings of the Twenty-third Annual Conference of the Mathematics Education Research Group of Australasia (MERGA-23)* (pp. 228–234). Fremantle, WA: MERGA.

Diezmann, C. M. (2002). Enhancing students' problem solving through diagram use. *Australian Primary Mathematics Classroom*, 7(3), 4-8.

Diezmann, C. M. (2004). Matrix Network Hierarchy. *epiSTEME-1*, 81.

Diezmann, C. M. (2005). The impact of primary students' informal experiences on their diagrammatic knowledge.

Diezmann, C. M. & English, L. D. (2001) Promoting the use of diagrams as tools for thinking. In A. A. Cuoco (Ed.), *2001 National Council of Teachers of Mathematics Yearbook: The role of representation in school mathematics* (pp.77-89). Reston, VA: National Council of Teachers of Mathematics.

Department of Education, (2009). *National Senior Certificate.* Seminar, Unisa, Pretoria, 6 May 2009.

De Villiers, M.D. (1987). *Research evidence on hierarchical thinking, teaching strategies and the Van Hiele Theory: Some critical comments.* University of Stellenbosch: RUMEUS.

De Villiers, M.D. (2010).*Some Reflections on the Van Hiele theory.* Paper presented at the 4th Congress of teachers of mathematics of the Croatian Mathematical Society, Zagreb, on the 30 June – 2 July 2010.

Dhlamini, S.S., & De Villiers, M. (2013). An investigation into grade 12 teachers' understanding of Euclidean geometry. *Global Research Journal on Mathematics and Science Education ISSN2278-0769*, 2(1), 97-118.

Dörfler, W. (2004). Mathematical Reasoning: Mental Activity or Practice with Diagrams.Retrived 10 July, 2013, from http://rfdz.ph-noe.ac.at/fileadmin/Mathematik_Uploads/ACDCA/DESTIME2006/DES_contribs/Doerfler/doerfler.pdf

Dossena, R., & Magnani, L. (2005). Perceiving the infinite and the infinitesimal world: Unveiling and optical diagrams in mathematics. *Foundations of Science*, *10*(1), 7-23.

Dreyfus, T. (1994). *Imagery and Reasoning in Mathematics and Mathematics Education*, Selected Lectures from the 7th International congress on Mathematics Education, Les Presses Laval, 107-122.

Durrheim, K., & Wassenaar, D. (2002). Putting design into practice : writing and evaluating research proposals. In M.T.Blanche & K.Durrheim(Eds.), *Research in*

Practice: Applied Methods for the Social Sciences. Cape Town: Umiversity of Cape Town Press.

Elliott, S. (1998). Visualisation and using technology in A Level mathematics. *British Society for Research into Learning Mathematics*, 45.

Giaquinto, M. (2007).*Visual thinking in mathematics.* NY: Oxford University Press.

Hanna, G. (1998). Proof as Explanation in Geometry. *Focus on Learning Problems in Mathematics*, 20 (2 & 3), 4-13.

Hanna, G., & Sidoli, N. (2007). Visualisation and proof: A brief survey of philosophical perspectives. *ZDM Mathematics Education, 39*, 73-78.

Heinich, R., Molenda, M., Russell, J.D. and Smaldino, S.E. (1999). *Instructional media and technologies for learning* (6th ed.). Upper Saddle River, NJ: Prentice-Hall.

Howie, S., Van Putten, S., & Stols, G. (2010). Making Euclidean Geometry compulsory: Are we prepared? *Perspectives in Education*, 28(4), 22-31.

Hennie, E., Rensburg, W.V., & Smit, B. (2004).*Finding your way in qualitative research.*Pretoria: Van Shaik.

Ichikawa, S., Manalo, E., & Uesaka, Y. (2007). What kinds of perceptions and daily learning behaviours promote students' use of diagrams in mathematics problem solving? *Learning and Instruction, 17,* (322-335).

Jamnik, J. (2001).*Mathematical Reasoning with Diagrams*. Stanford, CA: CSLI Publications.

Kolb, A.Y., & Kolb, D.A. (2005).*The Kolb Learning Style Inventory 3.1: Technical Specifications.*Boston, MA: Hay Resources Direct.

Kolb, D.A. (1984).*Experiential Learning: Experience as the source of learning and development*. Englewood Cliffs, NJ: Prentice.

Kolb, D.A., & Passarelli, A.M (2012). Using Experiential Learning Theory to Promote Student Learning and Development in Programs of Education Abroad. In M.Vande Berg, M.Page & K.Lou (Eds.), *Student Learning Abroad.* Sterling, VA: Stylus Publishing.

Kolb, D.A., & Yeganeh, B. (2012).Deliberate Experiential Learning: Mastering the Art of Learning from Experience. In K.Elsbach, D.C. Kayes & A Kayeš (Eds.), *Contemporary Organizational Behaviour in Action* (1st) ed., Upper Saddle River, NJ: Pearson Education.

Konyalıoglu, S., Konyalıoglu, A.C., Ipek, S., and Isık, A. (2005). The Role of Visualisation Approach on Student is Conceptual Learning, *International Journal for Mathematics Teaching and Learning*, Sept. 21.

Kutama, M.E. (2002).*An Investigation into process-based instruction in the teaching of grade 8 and 9 Euclidean geometry, a thesis submitted in fulfilment of the requirements for the degree of Master of Education*, University of South Africa.

Lavy, I. (2006). Dynamic Visualisation and the Case of 'Stars in Cages'. *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, 4,* 25-32.

Lomas, D. (2002). What Perception Is Doing, and What it Is Not Doing, in Mathematical Reasoning. *British Society for the Philosophy of Science*, 53, 205-223.

Maree, K (Ed.). (2007).*First steps in research.* Pretoria: van Schaik.

Mason, M. (1998). The van Hiele Levels of Geometric Understanding, *Professional Handbook for teachers, Geometry: Explorations and Applications*, (pp4-8).

McMillan, H. J., & Schumacher, S. (2006). *Research in Education: Evidence-Based Inquiry (Ed.).* New York: Pearson.

Mesaroš, M. (2012). Forms of Visualisation as a solving approach in teaching Mathematics. Retrieved 1 October, 2013, from http://conferences.ukf.sk/index.php/phdconf/phdconf2012/paper/download/960/280

Mudaly, V. (2008). Visual Literacy and Visualisation in mathematics education. (Unpublished).

Mudaly, V. (2010a). Thinking With Diagrams Whilst Writing With Words.*Pythagoras, 71*, 65-75.

Mudaly, V. (2010b). *Can diagrams help in solving problems?* Paper presented at the Proceedings of the 16th Annual Conference of the Association for Mathematics Education of South Africa, George Campbell School of Technology, Durban, Kwa-Zulu Natal, on the 28 March -1 April, 2010.

Mudaly, V. (2012). Diagrams in Mathematics: To draw or not to draw? *Perspectives in Education, 30*(2), 22-31.

Mudaly, V and Rampersad, R. (2010). The role of visualisation in students' conceptual understanding of graphical functional relationships. African Journal of Research in Mathematics, Science and Technology Education. Volume 14 Issue1. UNISA (36-48).

Mumma, J. (2010).Geometric proof checking with diagrams. Retrieved 27 July, 2013, from http://www-logic.stanford.edu/lmh/diagrams/mumma.pdf

National Diagnostic Report on Learner Performance 2012.

Netz, R. (1999). *The shaping of deduction in Greek mathematics: A study in cognitive history*. Cambridge, UK: Cambridge University Press.

Niekerk, R. (2010). *The state of geometry education in South Africa?,*Paper presented at the Proceedings of the 16th Annual Conference of the Association for Mathematics Education of South Africa, George Campbell School of Technology , Durban, Kwa-Zulu Natal, on the 28 March -1 April , 2010.

Nixon, R. C. J. (1892). *Euclid revised*. Oxford: Clarendon Press.

Piggot, J., & Woodham, L. (2008). Thinking through, and by, visualising.*Mathematics Teaching Incorporating Micromath,* 207, 27-30.

Plummer, D.B., & Bailin, S.C. (1997). The Role Of Diagrams In Mathematical Proofs.*Machine Graphics and Vision*, *6*(1), 25-56.

Presmeg.N. (1995).Preference for Visual Methods: An International Study. *Proceedings of PME*, 19(3), 58-65.

Presmeg, N. (1986). Visualisation in high school mathematics. *For the Learning of Mathematics, 6*(3), 42-46.

Rahim, M.H., & Siddo, R. (2009). *The use of visualisation for learning and teaching mathematics*. Paper presenting at Proceedings of the 10th International Conference: Models in Developing Mathematics Education, Dresdon, Saxony, Germany, on 11-17 September, 2009.

Robson, E. (2008). *Mathematics in ancient Iraq: A social history*. (1st ed).New Jersey: Princeton University Press.

Rolka, K., & Rösken, B. (2006). A picture is worth a 1000 words–The role of visualisation in mathematics learning. *International Group for the Psychology of Mathematics Education*, *4*, 457.

Schönborn, K. J., & Anderson T. R. (2008). Development of a model of factors determining student's ability to interpret external representations in biochemistry, *International Journal of Science Education*. (In Press)

Siew Yin, H. (2010, Jan/Feb). Seeing the value of visualisation. *Sing Teach*, *22*, pp. (2-5).

Smith, E. & De Villiers, M. (1989). A comparative study of two Van Hiele testing instruments. Poster presented at the 13th International Conference of PME, Paris.

Sosa, T. (2009). Visual Literacy: The Missing Piece of Your Technology Integration Course. *Techtrends: Linking Research and Practice to Improve Learning*, 53(2), 55-58.

Spicer, N.(2004). Combining qualitative and quantitative methods. In C. Seale (Ed.), *Researching Society and Culture* (pp.294-302). New Delhi: Sage Publications.

Stokes, S. (2002). Visual literacy in teaching and learning: A literature perspective. *Electronic Journal for the Integration of Technology in Education*, *1*(1), 10-19. Retrieved from http://ejite.isu.edu/Volume1No1/pdfs/stokes.pdf

Stylianou, D. A. (2002). On the interaction of visualisation and analysis: the negotiation of a visual representation in expert problem solving. *The Journal of Mathematical Behaviour*, *21*(3), 303-317.

Thornton, S. (2001). A Picture is Worth a Thousand Words. Retrieved 15 August, 2013, from http://math.unipa.it/~grim/AThornton251.PDF.

Van Blerk, A., Christiansen, I. M., & Anderson, T. R. (2008). *Learners' Visual Recognition of Geometry Theorems*. Paper presented at the 11th International Congress on Mathematical Education, Mexico, on the 7-12 July, 2008.

Van Putten,S.(2008).*Levels of Thought in Geeometry of Pre-service Mathematics Educators according to the van Hiele Model, a thesis submitted for the degree Magister Educationis in Assessment and Quality Assurance*.University of Pretoria,Pretoria.

Usiskin, Z. (1982). *Van Hiele levels and Achievement in Secondary School Geometry.* Final report of the CDASSG Project. Chicago: University of Chicago.

Vos, A.S.D. (2006). Combined quantitative and qualitative approach. In A.D. Vos & H.Strydom & C.Fouche & C.Delport(Eds.), *Research at Grass roots*. Pretoria: Van Schaik Publishers.

Wheatley, G.H. (1997). Reasoning with images in mathematical activity. In L.D.English (Ed).), *Mathematical reasoning: Analogies, metaphors and images* (pp.281-298). Mahwah, NJ: Lawrence Erlbaum.

Wileman, R.E. (1993). *Visual communicating.* Englewood Cliffs, NJ.: Educational Technology Publications.

Winn, B. (1987). Charts, graphs, and diagrams in educational materials. In D.M. Willows & H. A. Houghton (Eds.), *The psychology of illustration, Vol. 1.*New York: Springer-Verlag.

Yilmaz, R., Argün and Keskin, M.O. (2009). What is the Role OF Visualisation in Generalization Processes: The Case of Preservice Secondary Mathematics Teachers. *Humanity & Social Sciences Journal*, *4*(2), 130-137?

Zimmermann, W., & Cunningham, S. (1991).Editors' Introduction: What is Mathematical Visualisation? *Visualisation in Teaching and Learning Mathematics,* 1-8.

APPENDICES

APPENDIX A LETTERS OF CONSENT

Letter of Consent: Parents

University of Kwa-Zulu Natal
Edgewood Campus
Pinetown

Dear Parent

I, Miss Lola Reddy an educator at Wingen Heights Secondary, am currently undertaking a Masters course in Mathematics Education at the above University. I am undertaking research of an approach to working with geometric problems in Mathematics to see whether it is beneficial to students understanding of geometry. Part of my study is to assess work completed by students, have them answer a questionnaire and have them interviewed.

I request permission to include your child in my research project. All work undertaken by students is a part of the grade 11 work schedule. The interviews and questionnaires will be conducted during breaks or after school, whichever is convenient for your child. Please note that participation in the study is voluntary and if your child wishes to withdraw from the study at any time, he or she may do so.

Thank you

Yours sincerely

Lola Reddy

Parental Consent

I ____, parent/guardian of ________________ grant / do not grant permission for my child / ward to participate in the above research study. I have read and understand the contents of the above letter.

________________________ _______________

Signature Date

Letter from participating school

University of Kwa-Zulu Natal
Edgewood Campus
Pinetown
7 August 2013

The Principal

Wingen Heights Secondary

1 Wingen Walk

Shallcross

4093

Sir,

I am a Masters of Education Student at the University of Kwa-Zulu Natal. My research title is "***An exploration of the role of visualisation in the proving process of Euclidean geometry problems".*** The purpose of my study is to investigate the role that visualisation plays in the proving of Euclidean geometry problems.

The outcome of the research should provide valuable information which will contribute to the use of visualisation in the mathematics classroom. Learners will have to complete a questionnaire and participate in an interview as part of this research study.

As an educator at Wingen Heights Secondary, I seek your permission to conduct this research study at our school. Confidentiality and anonymity is assured and all ethical considerations will be strictly adhered to.

Thanking you in anticipation of your favourable response.

Yours Sincerely

RESEARCHER: L REDDY DATE

CONTACT NUMBER: CELL: 0837942323

HOME: 031 4018575

Consent from school

I, Mr GM Govender principal of Wingen Heights Secondary grant / do not grant permission for participation in the above research study. I have read and understand the contents of the above letter.

Principal Date

APPENDIX B **TRANSCRIPT OF INTERVIEWS**

Transcript of interview with Learner A

Interview done while looking at students responses to questions in worksheet.

INTERVIEWER: I can see from your answer to 2.1. that you have noticed that angle AO_1C is increasing or decreasing by 30° or more. Now according to your measurements this is correct, but if you look at all these angles here, do you notice anything?

LEARNER A: Not really.

INTERVIEWER: I want you to, just to look at the values of these angles, look at these values and see if there is anything there, if there is any relationship between the values.

LEARNER A: Okay. (Pause) I see that angle AO_1C is two times bigger than angle ABC. Ja, in most cases it's just two times bigger.

INTERVIEWER: Okay, okay good. Now in terms of if you look at the circle. Here's the circle here. If you look at this angle here AO_1C where about in the circle is it?

LEARNER A: It is in the centre.

INTERVIEWER: Okay, and if you look at the angle ABC where about in the circle, where about is this angle?

LEARNER A: On the circle.

INTERVIEWER: Alright. Can you be more specific? Where about on the circle is it?

LEARNER A: On the circumference.

INTERVIEWER: So can you see any special relationship between angle ABC which is on the circumference and angle AO_1C which is in the centre? Can you see anything between it?

LEARNER A: Well angle AO_1C is two times the size of angle ABC then maybe the angle at the centre is two times the angle on the circle, I mean circumference.

INTERVIEWER: Okay. Do you think that you can prove this?

LEARNER A: Well I am not sure, I feel like I need a diagram or something to show it. I don't think I can.

INTERVIEWER: What if we look, let's look at the next question here, question three? What about the diagram here? Here there is a diagram. How did you get all these values for this diagram?

LEARNER A: I just guessed. I didn't know what to put there.

INTERVIEWER: Do you think by this question where there is a diagram and by using this diagram you can maybe have proved what you said before, because we measured everything and we can see what is happening. Do you think that maybe we can prove this?

LEARNER A: I don't know. I think that having this diagram is a starting point. I don't know.

INTERVIEWER: Thank you.

Student now completes guided proof and then redoes exercise.

Completion of exercise and guided proof

- She was able to successfully complete the guided proof.
- I noticed that while completing the guided proof she placed given information on the diagram
- When completing answers to questions one to four I observed her writing down "the given" information on the diagram.
- When I asked her why she did this, she said that by putting everything on the diagram it helped her to think better because she could see what was given.

In completing question four she joined BO and OD, and when I asked her why she did this, she replied that she wanted to make this question look like the other diagrams so that there is an angle at the centre

Transcript of interview with Learner B

INTERVIEWER: Looking at your response to question 2.2. I can see that you said I am unsure and that I need more. What do you mean by this?

LEARNER B: I needed a diagram to prove what I stated in the question.

INTERVIEWER: Why do you think you need a diagram?

LEARNER B: It's easier to work with when I have a diagram?

INTERVIEWER: And why didn't you draw a diagram?

LEARNER B: I wasn't sure how to.

INTERVIEWER: I am now looking at, in this question 2.2. you were asked to prove your conclusion and in question 3 you were given a diagram and asked to prove your conclusion. In the following guided proof you were given diagrams and you were guided towards the actually proof. Which one did you find most helpful in helping you to come to the conclusion?

LEARNER B: The last one.

INTERVIEWER: Why do you say this?

LEARNER B: It gave me all the information that I needed in terms of centre of the circle and which line it subtends.

INTERVIEWER: Anything else you would like to add to that? (shaking head to saying no) Thank you very much.

Transcript of interview with Learner C

INTERVIEWER: I noticed in your activity in 2.2. you stated that that a diagram must be drawn to prove what you came up with, whatever you observed in question 2.1. which is here, why did you say this?

LEARNER C: Well in 2.2. there was nothing given, all we knew was that AOC is 2 times ABC so we needed a diagram to show this.

INTERVIEWER: OK, and let's look at question 3. Question 3 you were given a diagram, do you think that this helped you to show that this angle here AOB is twice the angle ACB?

LEARNER C: Yeah,'cos now I can see the angle at the centre and the angle at the circumference.

INTERVIEWER: I want you now to look at question 4. This one here where you are given the circle with the centre O and you have ABCD the vertices on the circumference and they tell you that. If angle A is 60 degrees calculate the value of C. What I would like to know is why did you join OB and OD?

LEARNER C: O, 'cos , if you join OB and OD you get the centre and if you get the centre it is easier to get other values, other angles.

INTERVIEWER: And I also see that you wrote 60 degrees here by A, why did you do that?

LEARNER C : Because , it makes it easier to get other values 'cos like A is 50 degrees, so if A is 50 degrees you know that angle O is twice angle A.

INTERVIEWER: So in other words you mean when you put it on the diagram you can see what's there, what you have.

LEARNER C: Much more easier to understand.

INTERVIEWER: Okay, thank you.

Transcript of interview with Learner D

INTERVIEWER: Looking at your response in question 2.2. I can see that you chose to draw a diagram to prove what you discovered in question 2.1. Why did you do this?

LEARNER D: The diagram helped me to prove that the 2 triangles will be isosceles, helped me to prove what I proved in 2.1. What I wrote in 2.1.

INTERVIEWER: Okay, so do you feel that diagrams are helpful in proving?

LEARNER D: Yes.

INTERVIEWER: I want to compare what you had. In 2.2. you drew a diagram in number 3/you were given a diagram and in your guided proof you had a diagram and statements that helped you towards proving what you observed. Which one you think helped you the most in coming to your conclusion?

LEARNER D: This, the one with the diagrams and the information.

INTERVIEWER: Why do you think it was helpful?

LEARNER D: Because it gives more information on what is effective.

INTERVIEWER: In which way? How did it help you?

LEARNER D: There is more information given here than the previous one.

INTERVIEWER: Do you mean you can see what you need to do and is that leading you.

LEARNER D: Yes it's leading me towards .the actual proof.

INTERVIEWER: Thank you.

Transcript of interview of Learner E

INTERVIEWER: Looking at your activity, your response to question 2.1. , I can see that you came to the conclusion that the angle OC, AOC is 2 times the size of angle ABC. How did you come to this conclusion?

LEARNER E: I came to the conclusion from all my measuring.

INTERVIEWER: In 2.2. why did you, in 2.2. they asked you to prove this conjecture. But I see in this one you chose a diagram from the questions that you were given. Why did you do this?

A G: I needed something to show how my answer works, to prove it.

INTERVIEWER: If you look at question 2.2 here in this one you were asked to prove the conjecture and in question 3 you were given a diagram and asked to prove the conjecture. Do you think that this makes any difference to your proof that you did in 2.2. and the one you did in number 3?

LEARNER E: Yes, definitely. In question 3 I had something to start with, I could see all the important parts on the circle that I needed for the proof and I found it better to have a diagram.

INTERVIEWER: While looking at your responses to exercise I want to draw your attention to question 4. I can see in this question here you joined O to B and then you joined O to D. Why did you do this?

LEARNER E: I joined OB and OD because it helped me find the answer and to see the angles in the middle clearly.

INTERVIEWER: Why did you need to see the angles in the middle?

LEARNER E: To help me find an answer.

INTERVIEWER: Okay, thank you.

APPENDIX C QUESTIONNAIRE

Name: ______________________________

Please answer the following questions.

1. Do you usually use diagrams when solving/proving geometric problems?

Yes	No	Sometimes

2. 1. Do you think that the use of diagrams is helpful in efficiently solving/proving geometric problems?

Yes	No	Sometimes

2.2. Give reasons for your choice above.

____

____

____

____

____

3. Does your mathematics teacher make use of diagrams to prove/solve geometric problems?

Yes	No	Sometimes

4. Refer to your competed activities when answering the following questions.

4.1. In question 2.2. Where you able to prove the conjecture that you observed from 2.1?

Yes	No

4.2. Give reasons for your choice above.

____

____

4.2. In question 3 where you able to prove the conjecture you observed from 2.1. ?

Yes	No

4.4. Give reasons for your choice above.

4.5. Where you able to prove the conjecture you observed in 2.1. in activity 2.

Yes	No

4.6. Give reasons for your choice above.

4.7. How did question 2.2, question 3 and activity 2 differ?

4.8. Which question 2.2, question3 or activity 2 assisted you the most in proving the conjecture?

Why do you think so?

UNIVERSITY OF
KWAZULU-NATAL
INYUVESI
YAKWAZULU-NATALI

30 October 2013

Ms Lola Reddy (207523449)
School of Education
Edgewood Campus

Protocol reference number: HSS/0112/013
Project title: An investigation into the role and use of visual skills and strategies in mathematics education by trainee and practicing teachers as well as staff at UKZN and schools in Durban

Dear Ms Reddy,

Class Approval

I wish to inform you that your application has been granted Full Approval.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. Please note: Research data should be securely stored in the discipline/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....
Dr Shenuka Singh (Acting Chair)