

Using artefacts to support an embodied approach to learning trigonometry

A case study of Grade 10 learners

By
Caresse Niranjana
(212 599 567)

In partial fulfilment of the requirement for the

DEGREE OF MASTER OF EDUCATION

In the School of Mathematics Education

(Edgewood Campus)

Faculty of Humanities

University of KwaZulu-Natal

Supervisor: Professor Deonarain Brijlall

2013

DEDICATION

This work is dedicated to my husband Ivan
Niranjan and daughters Kiara and Alka
Niranjan

ACKNOWLEDGEMENTS

I wish to thank the following persons:

- Prof D. Brijlall for his mentoring, facilitation and guidance throughout my research.
- My dear husband Ivan Niranjan for his motivation, support, inspiration and financial support.
- My daughters Kiara and Alka for being understanding, patient and bearing with my busy schedule during the course of the study.
- Ms L. Gething for editing my dissertation.
- William Dansoh from the library for all his assistance.
- University of Kwa-Zulu Natal for financial support .

DECLARATION

I, Caresse Niranjana declare that:

1. This research report in this dissertation, except where otherwise stated, is my original work.
2. This dissertation has not been submitted for any examination or degree at any other university.
3. This dissertation does not contain other person's data, pictures, graphs or any other information, unless acknowledged as being sourced from other persons.

Signed.....(Student)

As the candidate's Supervisor I agree to the submission of this dissertation.

Signed..... (Supervisor)

Table of Contents

Chapter One Overview and rationale of study

Introduction

1.1	Overview	1
1.2	Background	1
1.3	Purpose of the study	2
1.4	Motivation	3
1.4.1	Researcher's personal experience and interest	3
1.4.2	Learners' difficulties in understanding trigonometric ratios	4
1.4.3	Learners' inclination towards procedural methods rather than conceptual understanding in trigonometry.	4
1.5	Research questions and focus of Enquiry	6
1.6	Terminology	6
1.6.1	Artefact	6
1.6.2	Concrete Model	7
1.6.3	Manipulatives	7
1.6.4	Embodied approach	7
1.7	Importance of using artefacts in mathematics instruction	8
1.8	How do artefacts help students learn mathematics?	9
1.9	Shortcomings of the use of artefacts	9

1.10	Artefacts in pedagogy	10
1.11	Definition of trigonometric ratios	10
1.12	Overview of this study	11
1.13	Synopsis	12

Chapter Two

Literature review

2.1	Introduction	13
2.2	The process of utilising manipulatives in a mathematics classroom	16
2.3	Comprehensive literature review and findings	18
2.4	A critique of the use of manipulatives	19
	2.4.1 Potential drawbacks and mistakes	20
	2.4.2 Benefits and advantages	22
2.5	Piagetian theory	23
2.6	Manipulatives in present-day classrooms	26
2.7	Closing the gap between abstract and concrete thinking	27
2.8	Illustrations and physical models	29
2.9	Translation model	30
2.10	Conclusion	31

Chapter Three

Theoretical framework

3.1	Introduction	32
3.2	Conceptual framework	32
3.3	Constructivism: a learning theory	33
3.3.1	Core tenets of constructivism	33
3.4	Characteristics of a constructivist teacher	37
3.5	Creating a constructivist learning environment	38
3.6	Social constructivism	38
3.7	Constructivism	39
3.8	Vygotsky and socio cultural theory	40
3.9	An embodied approach	40
3.10	Conclusion	41

Chapter Four

Research methodology

4.1	Introduction	42
4.2	Critical research questions	42
4.3	Qualitative research methodology	42
4.3.1	Theoretical perspective	43
4.3.2	Actual methods used for data collection	43

4.3.3	The interpretative paradigm	44
4.3.4	How the paradigm fits the present study	45
4.4	Data sources	45
4.5	Data collection procedures	45
4.5.1	Observations	45
4.5.2	Instruments for data collection	46
4.5.3	Semi-structured Interviews	47
4.6	Challenges of conducting classroom-based research	47
4.6.1	Validity and reliability	47
4.6.2	Triangulation	48
4.6.3	Participation	48
4.7	Trustworthiness and ethical issues	49
4.8	Limitations of the study	51
4.9	Conclusion	51

Chapter Five

Results and analysis of written responses and interviews

5.1	Introduction	52
5.2	Analysis of the four tasks on the activity sheet	52
5.2.1	Analysis of task 1 on the activity sheet	53

5.2.2	Analysis of task 2 on the activity sheet	56
5.2.3	Analysis of task 3 on the activity sheet	59
5.2.4	Analysis of task 4 on the activity sheet	64
5.3	The structure and analysis of the interviews	68
5.4	Analysis and discussion of interviews	69
5.5.1	Interview question 1	69
5.5.2	Interview question 2	70
5.5.3	Interview question 3	71
5.5.4	Interview question 4	72
5.5.5	Interview question 5	73
5.5.6	Interview question 6	73
5.5.7	Interview question 7	74
5.5.8	Interview question 8	75
5.5.9	Interview question 9	75
5.5.10	Interview question 10	76
5.5.11	Interview question 11	77
5.5.12	Interview question 12	78

5.5.13 Interview question 13	78
5.6 How models/manipulatives assisted participants to learn during the lesson, and analysis of question 14	80
5.7 Themes that emerged form the analysis of Question 13	82
5.7.1 Fun ,enjoyable, interesting, exciting	82
5.7.2 Visualization/Imagine	83
5.7.3 Improved understanding	83
5.7.4 Encourages teamwork	83
5.8 Conclusion	84

Chapter Six

Conclusions and recommendations

6.1 Introduction	85
6.2 Findings and Conclusions	85
6.2.1 Learners' understanding of trigonometric ratios	86
6.2.2 Difficulties experienced by learners	86
6.2.3 Impact manipulatives had on learners' understanding of trigonometry	87

6.3	Recommendations	88
6.4	Sampling	89
6.5	Limitations	89
6.6	Themes for further research	89
6.7	Conclusion	90

References	91
-------------------	----

Appendices	101
-------------------	-----

FIGURES USED IN THE STUDY

Figure	Description	Page number
Figure 2.1	A diagrammatical representation of the Lesh (1979) model.	31
Figure 5.1	Question 1 of the activity sheet	53
Figure 5.2	Manipulative designed by researcher	53
Figure 5.3	Learners interacting with the manipulative	54
Figure 5.4	Written response of L3 to task 1	55
Figure 5.5	Question 2 on the activity sheet	56
Figure 5.6	Manipulative designed by researcher	56
Figure 5.7	Learners interacting with the manipulative	57
Figure 5.8	Learner 1: Written response to task 2 on activity sheet	58
Figure 5.9	Question 3 on the activity sheet	59

Figure 5.10	Manipulative designed by researcher	60
Figure 5.11	Learners interacting with the manipulative	61
Figure 5.12	Written response of L1	61
Figure 5.13	L4's written response to task 3 on activity sheet	62
Figure 5.14	L5's written response to Task 3 on activity sheet	63
Figure 5.15	Question 4 on the activity sheet	64
Figure 5.16	Manipulative designed by researcher	65
Figure 5.17	Learners interacting with manipulative	66
Figure 5.18	L1's written response	66
Figure 5. 19	L2's written response	67

LIST OF APPENDICES

Appendices	Description
Appendix 1	Grade 10 Activity trigonometry sheet
Appendix 2	Interview Schedule (Semi-Structured)
Appendix 3	Ethical Clearance certificate
Appendix 4	Informed consent for participants
Appendix 5	Letter from editor
Appendix 6	Turnitin Report
Appendix 7	Letter from school principal

Abstract

The purpose of this study was to explore the role of artefacts (manipulatives) in the teaching and learning of trigonometric ratios in grade 10. The study focused on how the use of manipulatives aided learners' mathematical proficiency in the use of trigonometric ratios. The foundation of this research was a case study contained in the interpretative paradigm involving five grade 10 mathematics learners at a secondary school in South Africa.

The data collected included a range of methods such as:

- ❖ Activity sheet containing written responses of learners
- ❖ Observations
- ❖ Semi-structured interviews.

The results in this research was analysed qualitatively.

The research findings in this case study indicated that the learners were interested and motivated and that the use of manipulatives assisted learners in understanding the concept of trigonometric ratios. In addition the results showed that the use of manipulatives in teaching and learning mathematics played a positive role in learners understanding of trigonometric ratios at grade 10 level.

The findings of my case study were similar to other research studies regarding the significance of using artefacts (manipulatives) in classrooms in teaching and learning of mathematics. The findings support other research findings that confirm that manipulatives were important mediating tools in the development of conceptual and procedural understanding of mathematical concepts.

Chapter One

Overview and rationale of the study

1.1 Overview

In this chapter the researcher establishes the research process as it developed and provides an overview of the study. The background and purpose of the research project is first discussed, followed by the motivation for conducting this research and relevance of the use of artefacts in mathematical instruction. Research questions are presented, followed by an outline of the successive chapters.

1.2 Background

Mathematics educators are constantly reminded of the poor mathematics results obtained by learners. According to the South African Minister of Education, serious problems in the South African education system are a result of inadequacies in teacher knowledge (Motsheka, 2012). In my teaching experience and in discussions with other mathematics educators I observed that several learners experience difficulty in selecting the correct trigonometric ratio in solving three-dimensional problems in mathematics.

“The National Curriculum Statement (NCS) (Department of Basic Education (DoBE), 2003b) downplayed the importance of Euclidean geometry, as there was no formal, compulsory matriculation assessment of this topic. Trigonometry, in particular, relies heavily on learners’ knowledge of geometry. The lack of emphasis on geometric knowledge and skills in the NCS policy document (DoBE, 2003b) probably influenced learners’ understanding and achievement in trigonometry” (Van Laren, 2012, p. 205). Maor (1998) reiterates that the achievement in trigonometry is powerfully dependent on geometric

concepts, as learners need to relate and identify measurements in drawings of shapes to numerical ratios.

1.3 Purpose of the study

The purpose of the study was to determine how mathematical artefacts enhance the learning of trigonometry among Grade 10 learners and how the use of such artefacts could improve teaching and learning of mathematics. Merrill, Devine, Brown and Brown (2010) state that improving and enhancing content knowledge requires mathematics teachers to implement three-dimensional (3D) solid modelling in mathematics classrooms to improve learners' understanding of mathematical concepts and principles.

Brijlall, Maharaj and Jojo (2006) established that several advantages exist in the use of models (artefacts), which include contributing to a learner's visualisation of space and shape. These visualisations can be explored to guide and deepen learner understanding. Learners are able to use any language to communicate and discuss mathematical ideas. This variety of language aids in identifying conceptions and misconceptions. Teachers are able to present geometry in interesting and exciting ways.

Artefacts are tools used in mathematics instruction that, when used effectively, can positively assist learners to grasp mathematical concepts taught at secondary schools. This research is aimed at understanding how the use of artefacts could enhance the learning process in trigonometry and thus eventually to raise learners' understanding to an abstract level.

1.4 Motivation

This study was motivated by:

- The researcher's personal experience and interest.
- The learners' difficulties in understanding trigonometric ratios.
- The learners' preference for procedural methods rather than for conceptual understanding in trigonometry.

1.4.1 Researcher's personal experience and interest

The use of artefacts in teaching mathematics has become a passionate focus in the researcher's teaching career. The researcher has always felt that learners possessed the ability to learn the mathematical topics current in the school curriculum. The obstacle that presented itself was how the researcher was to figure out how to get there. The researcher discovered the use of artefacts in teaching mathematics and the merits it offered in enhancing the learning of mathematics.

Under-achievement in mathematics is of great concern in South Africa. Research conducted by the Human Sciences Research Council (HSRC) revealed that South African Grade 8 learners who participated in the Third International Mathematics and Science Study occupied the lowest position among 38 countries (Howie, 1999).

Lemke & Patrick (2006) cited in Strom (2009, p. 2) state that the world is changing and our learners are lagging behind in mathematics. Our education system is in dire need of improvement. The way in which we teach mathematics has to be changed so as to bring learners to a level of competence in keeping with the rest of the world, and to ensure that all learners have access to learning mathematics.

There are several methods of mathematics instruction. The effective use of artefacts as tools in mathematics instruction can assist all learners to understand mathematical concepts taught at all levels of schooling.

This research was therefore aimed at understanding how artefacts could assist in the learning process and what methods were suitable to guarantee that learners learn the concepts and that enable them to advance their understanding to an abstract level.

1.4.2 Learners' difficulties in understanding trigonometric ratios

The researcher has observed in her class that learners display a lack of knowledge when using the various trigonometric ratios. They are unable to visualise the adjacent side, the opposite side and the hypotenuse. Learners lack understanding and seem to memorise procedures. Some learners fail to realise that these trigonometric ratios can only be applied in a right-angled triangle.

1.4.3 Learners' inclination towards procedural methods rather than conceptual understanding in trigonometry

Trigonometry is an inseparable part of mathematics in the secondary school curriculum and can be described as a product of geometrical realities, algebraic techniques and trigonometric relationships.

Orhun (2004) claims that mathematics education is founded on problem solving, application of knowledge and manipulations. When learners encounter word problems it appears that their non-systematic and incomplete knowledge results in errors and conceptual mistakes. When the development and analysis of problems and the explanation of results and confirmation of processes are not fully comprehended, it

results in learners surrendering creativeness and leads them to learn by heart.

It is therefore important that development of teacher education programmes includes aspects of the pedagogical content knowledge (PCK) required for mathematics teachers. Brijlall and Ndlovu (2013) examined the link between mathematics content knowledge and classroom teaching activity of two university lecturers. They found that for effective teaching a strong link between these two aspects was necessary.

Cochran, Derutter and King (1993) claim that field knowledge and pedagogical knowledge of teachers play an important role in teaching and learning.

Tall (1997) states that when learners are faced with conceptual difficulties, they develop coping strategies such as computational and manipulative skills. Many South African schools have large classes and under-qualified teachers, and this contributes to the weak preparation and lack of interest in the subject. As a result learners engage in methods and techniques that ensure that they just pass the subject and meet the basic promotional requirements. Learners engage in drill and manipulative approaches which assist them to pass the examination by merely regurgitating what they have memorised without really applying their knowledge to problems that require insight and understanding.

Smith and Moore (1991) confirm that much of what learners have actually learnt is a set of coping skills for getting past the next assignment, the next quiz and the next examination. They therefore have no real opportunity of understanding mathematics. Teachers require innovative mechanisms to address this challenge.

1.5 Research questions and focus of enquiry

This study made use of artefacts (three-dimensional trigonometric models) in exploring learners' conceptual understanding of the trigonometric ratios in relation to real-life problems.

The aim of the study was:

- a) to determine whether the use of mathematical artefacts enhanced learning in trigonometry among Grade 10 learners; and
- b) to determine if the use of mathematical artefacts improved learning and understanding.

1.6 Terminology

The definition of artefact, concrete model and manipulatives as used in this study are defined below.

1.6.1 Artefacts

Fernandes, Carron and Ducasse (2008) state that when a teacher prepares a learning activity, he or she introduces special tools serving as auxiliaries between the learner, the concept and the activity. These auxiliaries assist the learner to construct a mental representation of the concept. These tools are called artefacts. Such artefacts can be physical objects such as tokens of varying colours, dice, or three-dimensional models, or they can be in a paper-and-pen format to draw a grid or to draw a series of objects. The artefact in most cases is an established object used in an exceptional way so as to support the learning process. The artefact acts as an intermediate link between the learnt concept and the internal representation of the learner. Learners internalise the concept in a step-by-step process.

1.6.2 Concrete model

According to Sowell (1974) there exist three kinds of materials, namely concrete, pictorial and abstract. The concrete materials can be moved around or manipulated by the learner. Materials that are essentially visual and include diagrams, charts and pictures are described as being pictorial. Numerals and words are referred to as abstract materials.

1.6.3 Manipulatives

McNiel and Jarvine (2007) describe 'manipulatives' as concrete objects used to help learners understand abstract concepts in the field of mathematics.

Physical manipulatives, usually referred to as 'manipulatives', are objects that can be physically touched and moved around by the learner. The objects and the manipulation of the objects represent abstract mathematical concepts (Kennedy, 1991; Williams, 1986; Moyer, 2001).

For the purpose of this study the term 'manipulative' will be applied to concrete objects that will be used to represent the context of several trigonometric tasks.

1.6.4 Embodied approach

Embodied cognition focuses on the bodily/biological mechanisms underlying cognition and this research lies within this broad scheme of ideas. According to Lakoff and Johnson (1999) all mathematics is embodied, meaning that it is dependent on constructions in human minds and shared meanings in mathematical cultures.

1.7 Importance of using artefacts in mathematics instruction

The use of artefacts (manipulatives) provides several advantages.

Extensive research shows that the use of manipulatives is a worthwhile method of instruction (Kennedy, 1991).

A well-selected manipulative mirrors the concept being taught and gives the learner an object on which to act. The learners become active learners of concepts that may otherwise just be symbols (Heddens, 1986; Boulton-Lewis, 1998; Kilpatrick, Swafford, & Findel, 2001). For this action to be a learning experience, learners must reflect on what their actions did and what it means for the mathematics concept being studied.

Thompson (1994) and Clements and McMillen (1996) state that manipulatives become tools for thinking that allow learners to correct their errors. Moyer (2004) explains that the use of manipulatives becomes extremely useful in situations where students may be self-conscious and unwilling to draw attention to themselves by asking questions. In addition, contact with the manipulatives gives students a visual aid to help with their memory and recall of the concept (Boulton-Lewis, 1998; Suh & Moyer, 2007).

Puchner, Taylor, O'Donnell and Fick (2008) explain that a major impact of the application of manipulatives is on the improvement of learners' thinking. Manipulatives assist learners in creating an internal representation of the external concepts being taught. Resnick (1983) maintains that all learning is based on prior learning and experiences. Learning involves connecting new concepts to prior knowledge. Manipulatives serve as tools for educators to link learners' experiences with the objects (prior knowledge) to the abstract mathematical concepts (new knowledge) that the artefacts represent.

Balka (1993) assert that the use of artefacts also helps advance learners to higher cognitive levels, including analysis, synthesis and evaluation levels of Bloom's taxonomy. Ball (1992) claims that manipulatives allow educators and learners to communicate their thinking by providing something at a concrete level. During this time educators are able to ascertain learners' thinking and steer them to the concept being taught.

1.8 How do artefacts help students learn mathematics?

Strom (2009) states that mathematical concepts are abstract. The process of learning mathematics involves internalising concepts (Cobb, Yackel & Wood, 1992). Beattie (1986) adds that as learners learn, they require representations of these concepts before they are able to internalise them and work with the concepts abstractly. Cain-Caston (1996) affirms this by stating that learners need to be able to relate the concepts to parts of their experiences in their own world. Strom (2009) agrees that manipulatives are a source of these world experiences.

1.9 Shortcomings of the use of artefacts

It is vital not to rush into using artefacts without proper guidelines.

Artefacts in mathematical instruction are excellent tools that need to be used correctly or they may result in confusion and just become a toy to the learner. Strom (2009) states that there are several mathematics instructors who, not being aware of the usefulness of manipulatives, tend to shy away from using them or use them incorrectly. Thompson (1994) advises that preliminary work needs to be done before using manipulatives.

1.10 Artefacts in pedagogy

Fernandes, Carron and Ducasse (2008) state that in order to understand what an artefact is, we have to first focus on mathematics. When the multiplication concept is first taught in primary school, teachers use tokens to represent the product of two numbers. Hence the product of 2×3 is represented as a rectangular series of tokens in 2 rows and 3 columns. Alternatively, pen and paper can be used to draw a grid with 2 rows and 3 columns. What we refer to as an artefact is therefore a tool used to give the learner a physical, touchable, kinetic and viewable representation of an intellectual concept.

According to the Vygotsky theory of activity, the learner functions in an activity with an intermediate auxiliary, namely a tool. The auxiliary could cause a representation more or less internalised by the learner. This representation relies on the learner and the domain (Baker, de Vries, Lund & Quignard, 2001).

In this study artefacts were used to solve problems using trigonometric methods.

1.11 Definition of trigonometric ratios

There are six trigonometric ratios, which relate the sides of a right-angled triangle to its angles. More precisely, these are ratios of two sides of a right-angled triangle and a related angle. Trigonometric functions are used to calculate unknown lengths or angles in a right-angled triangle. All six functions are related and are defined in terms of each other. These trigonometric ratios are:

$$\diamond \text{ Sine} \quad \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\text{cosec} \theta}$$

$$\begin{aligned}
&\text{❖ Cosine} && \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sec \theta} \\
&\text{❖ Tangent} && \tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{\cot \theta} \\
&\text{❖ Cosecant} && \operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin \theta} \\
&\text{❖ Secant} && \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{1}{\cos \theta} \\
&\text{❖ Cotangent} && \cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{\tan \theta}
\end{aligned}$$

The ratios of the three main functions, namely sine, cosine and tangent, may be recalled by using the acronym 'SOHCAHTOA'. The use of artefacts allows physical support for mental images.

In this study the researcher utilised an artefact to enable the learner to form a mental representation and to achieve an intended pedagogical outcome.

1.12 Overview of this study

The following structure has been used in this dissertation. The dissertation comprises five chapters, the bibliography and appendices. The chapters are as follows:

Chapter One introduces the background and purpose of this study. The motivation for doing this research is also discussed. The research questions, the use of artefacts in pedagogy, and the terminology related to this study are introduced. The importance of using artefacts

in mathematics and the shortcomings that present themselves are examined.

Chapter Two reviews the literature relevant to the use of artefacts in mathematics.

Chapter Three presents the theoretical framework for the research. In this section the researcher explores the concept of constructivism as a learning theory, as well as cognitive and social constructivism.

Chapter Four focuses on the research methodology, research design and the measures carried out to conduct the study. The outline, summary of the research design and the research instruments applied are presented. The research paradigm and how it applies to the study subjects, data collection methods, validity and reliability, triangulation, trustworthiness and ethical issues, and limitations of the study are discussed

Chapter Five discusses the analysis based on learners' written responses and responses to semi-structured interviews.

Chapter Six presents the findings, recommendations and limitations of the study.

The appendices section of this dissertation includes the activity sheet, a semi-structured interview schedule, consent form, gatekeeper's letter, research office ethical clearance certificate, editorial certificate, and a summary of the turn-it-in report.

1.13 Synopsis

In this chapter the researcher discussed the rationale that stimulated her interest in conducting this research. She has shown what led her to

select the use of artefacts to support an embodied approach to learning trigonometry: a case study of Grade 10 learners. In the next chapter the relevant literature is reviewed.

Chapter Two

Literature review

2.1 Introduction

In teaching mathematics the researcher's primary concern is with concept formation as compared to memorisation of facts. Reys (1971) states that mental processes used in concept formation are much more complex than those connected with the memorisation of a large number of isolated details. The work conducted by psychologists such as Jerome Bruner, Jean Piaget, Zolton Dienes and Richard Skemp is now starting to impact on mathematical pedagogy.

The South African teaching and learning setting has been transformed by policy-makers by revising curricula in accordance with the Curriculum and Assessment Policy Statement (CAPS) (DoBE, 2012). The revision intends improving teaching and learning in mathematics (Van Laren, 2012). This study will provide an alternate approach to teaching trigonometry by developing and using mathematical models to extend teachers' PCK for Grade 10 trigonometry.

The study will address the Minister of Education's challenge that teachers' knowledge and the supply of quality learning support material is inadequate in the area of trigonometry, among other teaching areas (Motshekga, 2012). The result is poor performance by learners in mathematics.

Trigonometry always was and still is a substantial component of the Grade 12 examination. Providing an active learning environment that enables learners to participate in the learning activity thus enhances learning. The research undertaken was to show real-life situations and trigonometric models and the application of these concepts. The representations were

explored to establish whether they improved concept development and problem-solving abilities in mathematics. Typical real-life scenarios included bridge construction, navigating at sea, and sending signals from communication towers.

Brijlall et al. (2006) maintained that learning without participation is in contradiction with the recommendations of constructivists such as Von Glasersfeld (1984), who strongly believed that reflective ability is a major source of knowledge at all levels of mathematics. This implies that it is important for learners to talk about their thoughts to each other and to the teacher. Constructivists focus on quality of learners' interpretative activity.

Brijlall et al. (2006) also pointed out that geometry, which is closely aligned to trigonometry, is involved in the specific context of designing and constructing artefacts, such as model houses, which makes learners interested in the subject. The benefits of using models (artefacts) in teaching mathematics include, but are not limited to, contributing to learners' visualisation of space and shape, as well as to their knowledge of mathematical terminology and concepts.

Similarly, Merrill et al. (2010) stated that improving and enhancing content knowledge requires mathematics teachers to implement three-dimensional (3D) solid modelling in mathematics classrooms to improve learners' understanding of mathematical concepts and principles. There is significant correlation between learners' knowledge and spatial visualisation. Their study points out that the more 'hands-on' and visualisation tools used by teachers, the greater the students' understanding. When students visualise, they see the relevance and this promotes rigour in their thinking. Herbst and Chazan (2011) corroborate this view by maintaining that artefacts can be used in activity systems for learners to interact with the object, other learners and the teacher, thereby enriching the lesson.

Weng (2011) reiterates that visible and visual 3D dynamic design has the potential to enhance interest in learning mathematics by improving learners' interdisciplinary and multimedia design abilities and fostering technological skills.

Stuart (1998), in her article 'Math curse or math anxiety?', offers important data about the usefulness of manipulatives as a tool for instruction:

'As manipulatives and cooperative groups become more widely used in mathematics classes, I wanted to know whether students perceived these aides and situations as being useful learning tools. Three-fourths of the students thought that using manipulatives when learning a new mathematical concept was helpful. Most of the comments indicated that using manipulatives first helped students see the origin of the numbers in formulas. Fewer than one-fourth of the students said that manipulatives were not helpful learning tools, stating that they were confusing.'

Near the end of the article Stuart quotes Williams (1998, p. 101), paraphrasing the Chinese proverb: 'Tell me mathematics and I forget; show me mathematics and I remember; involve me ... and I will be less likely to have math anxiety', to reinforce her belief in the value of manipulatives.

Resnick (1998) cites the following advantages of using manipulatives:

- ❖ Manipulatives are extraordinary tools used to assist weaker learners and a useful method to improve education in any mathematical class.
- ❖ They provide an environment to teach mathematics as well as pedagogy to teachers. Teachers are often ineffective because of their limited understanding and material available.
- ❖ Manipulatives are far more effective when used as a setting for problem solving, discussion, communication and reflection.

- ❖ Manipulatives should be a complement to and not a substitute for other representations.
- ❖ Specific attention must be given to assist students to transfer what they know in the context of the manipulative to other representations such as symbolic, numerical and graphical, as transfer does not occur freely.

Smith (2009, p. 20) states “ a good manipulative bridges the gap between informal math and formal math. To accomplish this objective, the manipulative must fit the development level of the child.”

In this research study the manipulatives were effective in problem solving. Learners were able to interact with the manipulative, discuss and communicate with each other. Learners were able to transfer their knowledge of trigonometric ratios to the context of the problem and the designed manipulative.

Elswick (1995) states that concrete experiences help to instil in students a sense of confidence in their ability to think mathematically and to communicate mathematically.

2.2 The process of utilising manipulatives in a mathematics classroom

It is important not to plunge into using artefacts (manipulatives) without suitable guidelines. The researcher discusses the teacher’s job prior to the use of artefacts (manipulatives), the process of teaching with the aid of artefacts (manipulatives), the role the teacher plays during instruction, and the time required for successful learning to take place.

Kelly (2006, p. 188) stated that “ teachers need to know when, why, and how to use manipulatives effectively in the classroom as well as opportunities to observe, first-hand, the impact of allowing learning through exploration with concrete objects.”

Smith(2009) states that there are most likely several wrong methods to teach with manipulatives as there are to teach without them. Manipulatives should be appropriate for the learners selected. "The complexity of the materials provided will increase as children's thinking and understanding of mathematical concepts increase" (Seefeldt & Wasik, 2006, p. 93).

Before using artefacts (manipulatives) it is vitally important to do preliminary work. Thompson (1994) states that when teachers are considering the use of manipulatives, they must focus on what they want the learners to understand and learn, and not on what they want the learners to do.

Reys (1971) strongly advocates that pedagogically there are several criteria that need to be considered when selecting manipulative materials. These include:

- ❖ The materials should provide a true embodiment of the mathematical concept or idea being explored. The materials must provide concrete representations of the mathematical principles. It is therefore imperative that the material be mathematically appropriate.
- ❖ The materials should clearly represent the mathematical concept. Concepts are so deeply set in some materials that learners experience difficulty in extracting relevant ideas from their personal experience with the materials. This problem is further compounded by distractors such as bright colours that act as impediments to concept formation.
- ❖ The materials should be motivating. Materials should be attractive yet simple. Materials that have good physical characteristics tend to stimulate a learner's interest and imagination.
- ❖ The materials should be multi-purpose if possible. They should be able to be used in the different grades and at different levels of concept formation. In an ideal situation the materials should be useful in developing more than one concept.

- ❖ The materials should provide a basis for abstraction. This emphasises the requirement that materials correctly express the concept. In addition the concept being developed needs to correspond to the level of abstraction needed to form a mental image.
- ❖ The materials should provide for individual manipulation. Each learner should be able to physically touch the materials. This can be done as a group or individually. These manipulations make use of several senses, such as visual, aural, tactile and kinaesthetic. Essentially the material should make use of as many senses as possible.

The manipulatives designed by the researcher were attractive yet simple and captured the interest of the learners. The manipulative provided a true embodiment of the trigonometric concept being explored. In addition the manipulative made use of several senses. It allowed for the learner to interact with the manipulative.

2.3 Comprehensive literature review and findings

Strom (2009) states that her use of manipulatives encouraged several learners who would otherwise just shrug their shoulders, to answer questions in the class. In addition she claims that the use of manipulaives allows for conversation and understanding between the teacher and the learners. She further states that the use of manipulatives enhanced her success with learning among disabled learners.

Garrity (1998) carried out action research to establish if hands-on learning with manipulatives would improve test scores of secondary education learners. The study detailed the difficulty and challenges of secondary learners in understanding geometry problem and sought to improve understanding by adopting a constructivist approach, which contained

manipulatives, co-operative learning and real-life problem solving. The experimental group consisted of 47 secondary school learners while the other was a control group. The findings that emerged were that the scores of the experimental group, which used manipulatives, were higher than those of the control group. This study arrived at the conclusion that traditional teaching methods are less effective when compared to using artefacts and manipulatives.

Chester, Davis and Reglin (1991) used manipulatives with an experimental group to teach a geometry unit and discovered that after teaching the same unit to a control group using traditional lecture-style instruction, the results in a subsequent test were much better in the experimental group than in the control group.

Steele (1993) supports the findings of the above researchers by claiming that students were more engaged and motivated when they became actively involved in the learning process and engaged in the use of manipulatives (artefacts), and also when working in co-operative groups.

Munger(2007) reported the results of a study designed to describe the benefits of manipulatives. The study revealed that the experimental group using mathematical manipulatives scored significantly higher in mathematical achievement on the posttest scores than the control group.

2.4 A critique of the use of manipulatives

The use of manipulatives in combination with various other methods can improve and increase understanding. However, educators should not rely entirely on the use of manipulatives as they can become ineffective. Learners could lose the chance for deeper conceptual learning if manipulatives are used without formal discussion, abstraction and mathematical conceptualisation.

2.4.1 Potential drawbacks and mistakes

The use of manipulatives can be a useful teaching tool that can be quite attractive to mathematics teachers, but they need to be aware that this is not a 'fool-proof' tool. Several mistakes and misconceptions can occur when using them for mathematics instruction. To prevent this from occurring it is therefore important to know what mistakes can be made.

A very common mistake that is recorded in reviewed research is that manipulatives are not transparent, which means that the mathematical concepts being taught using them are not automatically understood or grasped by the learners (Ball, 1992; Cobb, Yackel & Wood, 1992; Thompson 1994). It is argued that as manipulatives are designed by individuals who already know the mathematical concepts which the manipulative is supposed to teach, and as teachers are easily able to see the concept that the manipulative was designed for, they automatically assume that the learners will also easily be able to see it. Teachers fail to realise that learners may see other concepts in the very same manipulatives (Moyer, 2001; Ball, 1992; Puchner et al., 2008).

Thompson (1994) confirms this by adding that because learners may 'see' other concepts in manipulatives, there is a great need for teachers to become completely familiar with the manipulative being used. They need to be aware of the various representations and be able to recognise when their learners are using those rather than the one intended. Quite often teachers think that learners are using the intended representation, and as a result there is a breakdown in communication.

Kilpatrick, Swafford and Findel (2001) add that some learners are unable to make the connection between the mathematical concept being taught and the physical manipulative. It is not that the learners are unable to 'visualize' a different representation but that they are unable to see the connection. This results in just one more thing to learn, instead of being an aid in understanding mathematical concepts.

Suh and Moyer (2007) argue that the loss of a connection may be attributed to a cognitive overload when interacting with manipulatives and symbols simultaneously. As a result learners are unable to keep a record of everything at one time. Another contributory factor could be that teachers are not using manipulatives properly. Heddens (1986) confirms this, saying that teachers may not be guiding students to the concepts, and Ball (1992) states that teachers themselves may not understand the use of the manipulative.

Communication is another area where teachers tend to encounter problems when using manipulatives. Learners may develop misconceptions, which may go unnoticed. Heddens (1986) and Moyer (2001) confirm that communication is an area where teachers make mistakes when using manipulatives. They agree that learners need to be given an opportunity to communicate and reflect on how they are interacting with the manipulative. This process allows for the learners to formalise their understanding of the concepts learnt and to signal to the teacher any misconceptions. Resnick (1983) states that learners will try to make sense of what they are learning even if not provided with all the information. This results in incomplete, misguided and incorrect theories. These incorrect theories may never be addressed and corrected if there is a lack of communication.

Puchner et al. (2008) state that another misuse of manipulatives in the classroom is that they are chosen as a method of calculation instead of as tools to assist understanding. Thompson (1994) argues that when teachers select manipulatives for the mathematics classroom, they tend to select the manipulatives that help learners 'do' something instead of having them assist learners understand a mathematical concept. In addition, teachers do not think carefully about how manipulatives will assist their learners to learn a concept (Balka 1993; Puchner et al., 2008)

According to Thompson (1994) and Puchner et al. (2008), studies reveal that when teachers use manipulatives in a prescribed way, the teacher may be removing the purpose of using manipulatives during instruction. Instead of understanding a concept, learners are merely learning another process, which tarnishes the learning of the underlying concept. In addition, this may lead to ignoring learners' alternative methods or use of the manipulative that may be justifiable but is not the prescribed method. This results in a missed opportunity for beneficial communication and a deeper understanding in the learner.

Another common mistake in the use of manipulatives is the lack or shortage of time allocated to learners to work with manipulatives (artefacts) (Heddens, 1986). Moyer (2001) states that learners must be thoroughly familiar with the manipulative to ensure that learning takes place and to avoid cognitive overload. This can only be successful if adequate time is allowed.

How and why teachers decide to use or not use manipulatives for teaching may also lead to problems. Boulton-Lewis (1998) found that at the secondary school level it is not a common practice for teachers to use manipulatives. Regrettably, learners may encounter difficulty transitioning from learning with manipulatives to learning at an abstract level. Moyer (2001) states that teachers use manipulatives as a reward or take away manipulatives as punishment. This practice may result in learners seeing the use of manipulatives as being fun rather than as a tool for learning.

Finally, Clements and McMillen (1996) assert that not all learners require interaction with manipulatives; in fact, some learners may perform better with pen and pencil or may not need to interact with the manipulatives for as long as other learners.

2.4.2 Benefits and advantages

Strom (2009) states that manipulatives allow learners to become active participants and provide visual aids to their understanding, memory and

recall. Manipulatives allow passive learners to physically work with them, discuss actions with group members, attach notation to action, and share findings with the rest of the class. Using manipulatives encourages conversation and provides both learners and teachers with an avenue for verbal interchange and understanding. The use of manipulatives improves success with learning-disabled students. Chang (2008) studied the work of research scientist Jennifer Kaminski and discovered that children better understand mathematics when they use concrete examples.

According to Heddens (2005), manipulatives help learners learn:

- ❖ to relate real-world situations to mathematics symbolism
- ❖ to discuss mathematical ideas and concepts
- ❖ to work co-operatively to solve problems
- ❖ to make presentations in front of a large group
- ❖ to verbalise mathematical thinking
- ❖ that there are several different ways to solve problems
- ❖ that they are permitted to solve problems on their own using their own methods without just following the teacher's directions
- ❖ that mathematics problems can be symbolised in several ways.

The researcher designed manipulatives that relate to real-world situations that contain mathematical concepts. The use of manipulatives allowed for the participants to work co-operatively and to discuss mathematical trigonometric ratios. The use of manipulatives allowed learners to use their own methods to solve the required trigonometry.

2.5 Piagetian theory

Piaget (1970) studied the various stages of cognitive development of children from birth to maturity. According to Piaget, understanding occurs from actions carried out by an individual in response to the individual's environment. These actions change as time passes, from the physical actions to partially internalised actions that can be carried out with

symbols. According to Piaget's theory, this can be described as a continuous process of accommodation to and assimilation of the individual environment. The cognitive development begins with the use of physical actions to form schemas, which are then followed by the use of symbols.

Piaget stresses that learning involves both physical actions and symbols that represent previously performed actions. Learning environments therefore should include both concrete and symbolic models of the ideas that are to be learnt.

Piaget's study (1972) was founded on careful and thorough detailed observation of children in natural settings and utilised repeated naturalistic observations. This careful examination of the functioning of intelligence in children led Piaget to discover that at certain ages children have difficulty in understanding 'easy ideas'. He investigated the thinking patterns of children from birth through to adulthood and discovered that consistent systems existed within certain broad age ranges. Piaget illustrates four stages of the cognitive development, namely:

- Sensorimotor stage (birth to age 2)
- Pre-operational stage (ages 2 to 7)
- Concrete operations stage (ages 7 to 11)
- Formal operations stage (age 11 onwards).

Each major stage is a system of thinking that is qualitatively different from the preceding stage. A child must go through each stage in order and each stage must be mastered before proceeding to the next stage.

Sensorimotor stage: This stage is seen as being pre-symbolic and pre-verbal. Children acquire experience through their senses and the most important intellectual activity at this stage is the interaction between the environment and child's senses. Activities can be described as being practical. Children are able to feel and see what is occurring around them, but they are unable to sort their experience. At this stage children develop the concept of the permanence of objects and begin to develop basic

relations between similar objects. The rich sensory environment allows for movement to the next stage.

Pre-operational stage: In this stage objects and events begin to take on symbolic meaning. There is rapid language development. The natural speech of children is dominated by monologues. This stage is mostly intuitive. Children at this stage enjoy imitating sounds and trying out different words and are not too concerned with precision. Children display a heightened ability to learn more complex concepts from experience if provided with familiar situations that have common properties that were discovered at the previous stage. There is an increase in the child's capacity to retain images. In this stage thought processes are based on perceptual cues and children are unconscious of contradictory statements.

Concrete operations stage: In this stage the child begins to arrange data into logical relationships and starts to manipulate data in problem-solving scenarios. This learning situation will only transpire if concrete objects are provided. The child at this stage has the ability to make judgements in terms of reciprocal and reversible relations.

Formal operations stage: At this stage the child develops entire formal patterns of thinking and is capable of developing logical, rational and abstract strategies. Symbolic meanings and similes can be understood by the child. Cognitive growth improves when the symbolic process becomes more active. The child (adult/learner) is now able to formulate hypotheses and deduce possible results from them, form theories and arrive at conclusions in the absence of a direct experience in the subject. Learning now is reliant upon the individual's intellectual potential and environment experiences.

The studies of Bruner support Piaget's findings. Bruner (1966) describes three ways of knowing, namely enactive, iconic and symbolic. He states that a developing human being acts towards its environment through direct actions, imagery and language. A child begins to play with objects by touching, smelling and tasting them, which results in experiencing

characteristics the objects possess. Later, mental images are developed by the child and names are attached to objects. Bruner (1996) states that after children learn to distinguish objects by shape colour and size, they start mastering the concept of numbers. Later on, in school, children learn new mathematical concepts and need to proceed in the same sequence from concrete objects to pictorial and then to abstract symbols.

The use of manipulatives in this study allowed for learners to proceed from concrete objects to pictorial and then to abstract symbols and application of trigonometric ratios.

2.6 Manipulatives in present-day classrooms

The researcher observed in her school that several educators have not adhered to the use of manipulatives in their mathematics classrooms. A larger number of learners continue to see the study of concepts in trigonometry as a body of knowledge to be memorised rather than understood.

Marilyn Burns has been a teacher for more than 30 years. She has written several mathematics books for learners and teachers to help improve the teaching and understanding of mathematics. She strongly recommends the use of manipulatives as she believes that they help learners see mathematics as a subject to be understood and not memorised.

Burns (1996) outlines several advantages of using manipulatives, which aid in making abstract ideas concrete. A picture may be worth a thousand words, but while children learn to identify animals from picture books, they probably still don't have a sense of the animals' sizes, skin textures, or the sounds they make. Even videos fall short. There is no alternative for first-hand experience. Along the same lines, manipulatives provide students with ways to construct physical models of abstract mathematical ideas.

Manipulatives lift mathematics off textbook pages. While we want learners to become comfortable and proficient in the language of mathematics, everything from the plus sign to algebraic notations, words and symbols only represent ideas. Ideas exist in children's minds, and manipulatives assist them to construct an understanding of ideas that can then connect to mathematical vocabulary and symbols.

Manipulatives develop learners' confidence by giving them a way to test and confirm their reasoning. If learners have physical evidence of how their thinking works, their understanding is more robust.

Manipulatives are useful tools for solving problems. In searching for solutions, architects build models of buildings; engineers build prototypes of equipment; and doctors use computers to predict the impact of medical procedures. Similarly, physical manipulatives serve as concrete models for learners to use to solve problems in trigonometry.

Manipulatives make learning mathematics exciting, interesting and enjoyable. Give learners the choice of working on a page of problems or solving a problem with colourful and interesting-shaped blocks, and there is no contest. Manipulatives intrigue and motivate while helping students learn.

2.7 Closing the gap between abstract and concrete thinking

Several authorities offer suggestions about bridging the gap between concrete and abstract. Heddens (1986) explains that several learners experience difficulty in mathematics because they are unable to make the connection between the physical world and the world of thoughts, that is between the concrete and the abstract.

Underhill (1977) describes the learning stage between the concrete and the abstract levels as the semi-concrete stage. Heddens (1984) includes one more level, the semi-abstract level, to this scheme. The semi-concrete level is a representation of a real situation, such as pictures of objects instead of the real items. The semi-abstract level is the symbolic representation of concrete items. Pictures or symbols represent objects but do not always look like them. In mathematics tallies are used to represent objects. Heddens (1986) states that the gap between concrete and abstract functioning should be thought of as a continuum.

Likewise, Sowell (1989) divides the intermediate phase into concrete abstract and pictorial-abstract. Learners start to recognise relationships at the concrete-abstract level. Pictures and diagrams in conjunction with written symbols are used at the pictorial-abstract level. The learning experience becomes totally abstract at the end of the continuum and learners are taught to formulate relationships and use these to solve related problems.

Piaget (1972) stated that learners are unable to understand an abstract representation of new knowledge until they have internalised this knowledge. He defines two processes of interaction between reality and the mind as accommodation and assimilation. Some learners are able to assimilate new knowledge quickly while others need significantly more time to accommodate, or recognise their mental structures to incorporate new information (Hartshorn & Boren,1990).

Sowell (1989) added that it is important that learners have adequate concrete experiences before they are required to work with abstract matters. Learners normally learn to operate at the abstract level over a period of time, after gaining different experiences at other levels.

Heddens (1986) asserts that the use of concrete materials improves learners' thought processing skills, such as their logical thinking, and allows the transition from concrete to abstract. Concrete experiences allow learners to internalise mathematical concepts and develop them at the

abstract or symbolic level. Otherwise learners begin to view mathematics as rules to be memorised instead of as a unique and helpful way to see the world. Should learners be required to explain their procedures, they should be expected to confidently use their own words and display understanding of their work. If learners merely memorise procedures without understanding, they are quickly bound to get confused or forget.

According to Stanic and McKillip (1989), real understanding will occur in the learner's own words and will be retained for longer. Heddens (1986) believes that verbalisation is also important in developing thought-processing skills of learners. They must be given the chance to verbalise their thoughts, so as to ensure clarification of their mental processes.

Berman and Friederwitzer (1983) observed that mathematical concepts are best taught during activities that include the transition from concrete to abstract. During the first stage learners should participate in activities using concrete materials. Later on these concrete experiences offer the basis for understanding and performing abstract activities with paper and pencil.

2.8 Illustrations and physical models

It is known that learners make use of their several intelligences differently. Some learners are more abstraction-orientated while others are more comfortable with tangible, physical and visual things. For the latter it would appear that that physical models and illustrations assist towards better conceptual learning. McNeil and Jarvin (2007) state that manipulatives are an additional resource for learning mathematics, assisting learners to connect with the real world and increasing their memory and understanding. Wongapiwatkul, Laosinchai, Ruenwongsa and Panijpan (2011) argue that there are other concerns about the use of manipulatives, for instance that the 'gimmicky' aspect of and the fun associated with manipulatives may overshadow deep learning. Baba (2007) suggests that lesson study sessions by a group of teachers should reduce the highly

familiar or perceptually interesting manipulatives and concentrate more on the actual benefits of classroom practice using manipulatives (artefacts). According to Furner, Yahya and Duffy (2005), manipulatives instead of worksheets can be used creatively to enhance conceptual learning, assist learners to link concrete objects with abstract concepts, and thus match the learning styles of some students. The study conducted by Kikas (2006) revealed that apart from visio-spatial ability, verbal ability was also important to learners. Verbal abilities become more prominent in higher grades when abstract topics are taught. Conceptualisation is mediated by signs, which could be words, models, schemas or pictures. Several people have trouble with understanding as they have to assimilate visually and verbally perceived fragments of the world.

2.9 Translation model

Lesh's model (1979) shows the importance of using multiple models for teaching and learning in mathematics. Figure 1 is a diagrammatic representation of the model and displays the possibilities for translation between real-life situations and geometric and trigonometric models. The arrows show the variety of translations that are possible among the nodes that he identified as the real-life situations, pictures (geometric drawings, flow charts, diagrams, tables and graphs), verbal symbols (spoken language), written symbols (written explanations or mathematical symbols) and manipulatives (mathematical instruments for construction and measurement). In addition, the horizontal arrows within each node of the model show that flexibility and translation within each node are also vital for developing mathematical understanding.

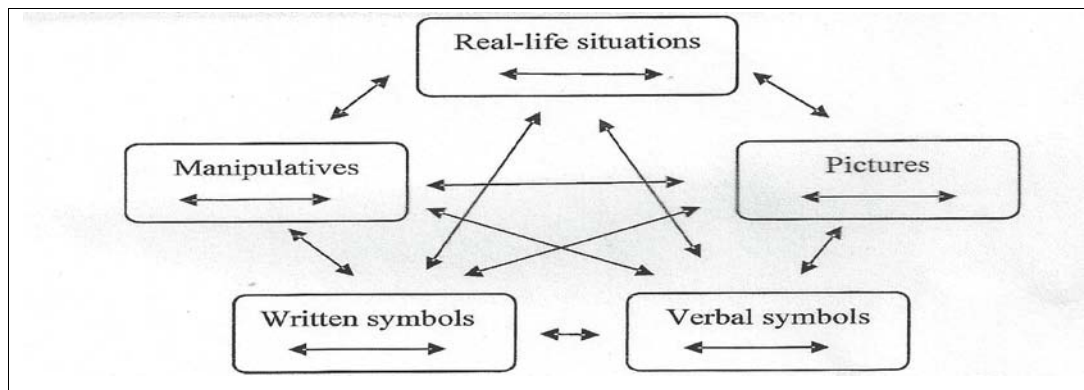


Figure 2.1: A diagrammatical representation of the Lesh (1979) model.

Adapted from Lesh, Post and Behr (1979, p.34).

Lesh (1979) regards the use of the variety of representations to be valuable for concept development and problem-solving abilities in mathematics. It is important to use manipulatives that can be translated into real-life situations—pictures such as geometric diagrams, verbal symbols or written symbols such as trigonometric ratios—to develop concepts in trigonometry.

Van Laren (2012) states that geometric concepts are required as learners require the use of similar triangles to understand the basic trigonometric functions. In addition, a variety of models (construction, geometric and trigonometric) is encouraged when introducing trigonometry. This would include the use of measurement and calculations of associated values using geometric shapes and is closely related to the translation model designed by Lesh (1979).

2.10 Conclusion

In this chapter the researcher has provided an overview of literature on manipulatives, its function in the classroom and the impact it has on learners understanding of mathematical concepts. Potential drawbacks and mistakes as well as benefits and advantages were discussed. Chapter three focuses on the theoretical framework of this study.

Chapter Three

Theoretical framework

3.1 Introduction

This research has been underpinned by cognitive and social constructivism. Constructivism can be described as the process whereby learners construct meaning where their understanding is highly dependent on pre-knowledge. The constructivist viewpoint on learning is that concepts are not taken directly from an experience, but instead from an individual's ability to learn. What the individual learns from an experience depends greatly on the quality of the ideas that the individual is able to bring to that specific experience. Constructivists focus on the quality of learners' interpretive activity. In this chapter the learning theory of constructivism in relation to cognitive and social constructivism is discussed and explained, engaging Piaget's theory of constructivism and Vygotsky's theory of social constructivism.

3.2 Conceptual framework

Bell (2005) describes the conceptual framework as being the basic structure that offers the necessary grounds on which a particular research study may be constructed. Maxwell cited in Miles and Huberman (1994) explains the conceptual framework of a study as being 'the system of concepts, assumptions, expectations, beliefs, and theories that supports and informs your research is a key part of your design'. A conceptual framework should make sense and facilitate contextual understanding of the findings of a research study for other researchers. In addition, Polit and Hungler (1995, p. 101), as cited in Al-Eissa (2009, p. 86), firmly believed that 'Frameworks are efficient for drawing together and summarising accumulated facts ...The linkage of findings into a coherent

structure makes the body of accumulated knowledge more accessible, and thus more useful for both practitioners who seek to implement findings and researchers who seek to extend the knowledge base.'

3.3 Constructivism: a learning theory

In this section the researcher explores the concept of constructivism as a learning theory as well as cognitive and social constructivism, which guide the overall direction of this study, as discussed in sub-section 3.3.1.

Pickard & Dixon (2004) state that a major advantage of constructivist inquiry is that it can offer understanding of the meanings behind the actions of individuals.

3.3.1 Core tenets of constructivism

The following statement is in keeping with constructivism, which is a cognitive learning theory with a clear focus on the mental processes that construct meaning:

'Learning is much more than memory. For students to really understand and be able to apply knowledge, they must work to solve problems, to discover things for themselves, to wrestle with ideas. The task of education is not to pour information into students' heads, but to engage students' minds with powerful and useful concepts' (Slavin, 1997, p. 269).

As Eggen and Kauchak (2007, p. 234) state: 'construct, rather than record knowledge'.

Van de Walle (2007) and Olivier (1989) assert that the principles of constructivism are founded largely on Piaget's processes of assimilation and accommodation, where assimilation refers to the use of existing schemas that give meaning to experiences, and accommodation is the process of altering ways of viewing things or ideas that contradict or do not fit into existing schemas.

Ornstein and Hunkins (2004) describe constructivism as being a realm concerned with how an individual learns, and places the individual as the active person in the process of thinking, learning and coming to know.

Piaget's study (1970) of the cognitive development of children led him to the conclusion that knowledge is actively constructed by each individual. He proposed that what is crucial to intellectual development is a shift in focus from the properties inherent in real-world objects as actions are applied to them, to a consideration of the actions themselves and the effect they have on objects. Through this shift in focus, knowledge is gained from the actions which the individual performs, leading to a constructed abstraction of the action process.

In his book, *Genetic Epistemology*, Piaget (1970, p. 16) wrote 'The abstraction is drawn not from the object that is acted upon, but from the action itself. It seems to me that this is the basis of logical and mathematical abstraction'.

According to Clayton (1994), Piaget can be considered as the father of constructivism and those who followed him have provided several useful insights which have led to a great change in the way mathematics is taught today compared with in the past.

Constructivist Von Glasersfeld (1984) argues that reflective ability is an important source of knowledge at all levels of mathematics and that it is therefore vitally important that learners are guided and directed to talk about their thoughts to each other as well as to the teacher. He emphasises that talking about what one is doing confirms that one is examining it. In this study the effect of such examination resulted in the learners discussing their view of the problem and their own tentative approaches to solving it. Learners gained self-confidence and developed

more viable conceptual strategies to solve real-life problems using trigonometric ratios.

Olivier (1989) asserts that knowledge does not arise merely from experience but rather from the interaction between experience and pre-existing knowledge structures.

The learner is no longer viewed as being a passive recipient of knowledge from the environment, but becomes an active participant in the construction of his or her knowledge. Olivier (1989) terms a unit of interrelated ideas in a child's mind a 'schema', where new ideas are interpreted and understood by the learner according to the learner's own current knowledge and previous experiences. These schemas can be considered to be useful tools, kept in the memory, which can be retrieved for use at a later stage. Learning now becomes an interaction between a child's schemas and new concepts, ideas or experiences.

When new ideas cannot be linked to any existing schema, then a learner will create a new 'box' and attempt to memorise the idea. Olivier (1989) terms this 'rote learning', as this new idea cannot be connected to any previous knowledge and as a result is not understood. This knowledge is isolated and becomes difficult to remember.

Brooks (1994) states that 'the learners have their own ideas, that these persist despite teaching and they develop in a way characteristic of the person and the way they experience things leads inevitably to the idea that, in learning people construct their own meaning'.

According to Nakin (2003), when a learner hears the word 'triangle', depending on the learner's experience, the learner may think of a tricycle, a tripod or a triangle road sign. The learner begins to understand that a triangle has 'three' of something.

Teachers are often guilty of making the incorrect assumption that learners are empty vessels which must be filled with knowledge. The constructivist view is totally different and is founded on the theory that learning is an active process and that learners construct their own meanings.

Brookes (1994, p. 12) states that in the constructivist theory it is accepted that 'learners have their own ideas, that these persist despite teaching and that they develop in a way characteristic of the person and the way they experience things, leads inevitably to the idea that, in learning, people construct their own meaning'.

When learners develop new knowledge, they often rely on pre-existing knowledge. Scott (1987) makes the following important points when considering the construction of meaning by learners:

- I. *That which is already in the learner's mind, matters.* This point reinforces the fact that learners' pre-existing knowledge is vital.
- II. *Individuals construct their own meanings.* Each learner can be at a totally different learning stage. A learner's experience can be different from others because of other contributory factors such as the environment, societal or cultural differences and even mental capabilities.
- III. *The construction of meaning is an active and continuous process.* Learners will often create ideas, test and evaluate them and then review them to validate these ideas and hypotheses.

Learners pre-existing knowledge and previous experiences has an impact on their interaction with the manipulatives and construction of new knowledge.

From a constructivist perspective teachers must take into consideration the learners' prior knowledge when designing and developing the use of

manipulatives in the classroom activities. A starting point must be made available for learners to restructure their ideas and knowledge.

3.4 Characteristics of a constructivist teacher

Brooks and Brooks (1993) offer the following characteristics of a constructivist teacher. The teacher should –

- ✓ Not become the primary source of information but rather one of several resources that learners may learn from.
- ✓ Involve students in experiences that offer challenges to previous conceptions or their existing knowledge.
- ✓ Promote thoughtful discussion among learners.
- ✓ Promote questioning by asking thought-provoking and open-ended questions.
- ✓ Permit learners' responses to steer lessons and seek elaboration of learners' initial responses.
- ✓ Give learners time to think once questions have been asked.
- ✓ Make use of cognitive terminology such as 'clarify', 'analyse', 'justify' when designing tasks.
- ✓ Not separate knowing from the process of finding out.
- ✓ Be prepared to use raw data and primary sources which may include artefacts and interactive physical materials.

The manipulatives designed by the researcher were motivating. The manipulatives were attractive yet simple, and their physical characteristics stimulated the participants' interest and imagination. The manipulative provided a basis for abstraction. Concepts being developed corresponded to the level of abstraction required to form mental images. The manipulatives allowed for discussion and encouraged conversation within groups to share findings.

3.5 Creating a constructivist learning environment

Jonassen (1991) offers the following principles as guidelines to educators to assist in creating a constructive learning environment:

- Create real-world environments that use the context in which learning is relevant.
- Focus on realistic approaches to solving real-world problems.
- Act as a coach and analyser of the strategies used to solve the problems.
- Instructional goals and objectives should be negotiated and not imposed.
- Stress conceptual interrelatedness, providing multiple representations or perspectives on content.
- Learning should be internally controlled and reconciled by the learner.
- Provide tools and environments that help learners interpret the several perspectives present in the world.
- Evaluation should serve as a self-analysis tool.

The manipulatives designed by the researcher related to real-world problems. The manipulatives allowed learners to see that trigonometry could be applied to their environment, such as calculating the height of a mountain, the distance of a boat away from the base of a mountain, the distance on the runway required for an aircraft to land, and calculating angles of elevation.

3.6 Social constructivism

Expanding on Piaget's theory that a learner constructs his/her own knowledge through encountering specific experiences within a stipulated

environment, social constructivism places more emphasis on the building of knowledge via social interaction (Eggen & Kauchak, 2007).

Vygotsky (1978) states that social constructivism is knowledge construction that is a shared experience rather than just an individual experience, and through the process of sharing individual perspectives learners construct understanding.

According to Noddings (1990), constructivists maintain that learning is an active process, a process that needs active participation of the learner.

Brijlall et al. (2006) add that learning without participation is in contradiction with the recommendations of constructivists such as Von Glasersfeld, who stresses that reflective ability is a major source of knowledge at all levels of mathematics and that it is therefore important that learners talk about their thoughts to each other and to the teacher.

3.7 Constructivism

Information processing constructivism was applied in this study, where students interacted with artefacts that they were familiar with in their daily lives. The flagpole elevation, the height of mountain, the distance of a runway the aircraft must cover, and a boat out at sea a certain distance from the cliff. The group activity transforms the mathematical concept into reality, resulting in the construction of more meaningful understanding of the concept. Students' prior knowledge coupled with their social interaction in their group provides a deeper understanding of the trigonometric concept. Knowledge is viewed as an active process embedded in their interaction with and understanding of each artefact in the study. It involves social interaction that supports thinking, brings prior knowledge to the surface, and allows skills to be applied in the context of the trigonometric content being taught (Hausfather, 2001).

3.8 Vygotsky and socio-cultural theory

To Vygotsky social relationships form a vital part for learning. He asserts: 'all higher mental functions are internalised social relationships' (Vygotsky, 1981, cited in Wertsch and Stone, 1986, p. 166). Vygotsky claims that the direction of learning stems from the social to the individual. Learning is first social, which is dominant, and the individual comes later. Vygotsky's socio-cultural theory is relevant to the present study as there was social interaction between learners.

Wertsch and Stone (1986) state that for learning to occur, the learner must reconstruct and convert external, social activity into internal, individual activity through a process of internalisation. The creation of such consciousness depends on social interaction and on 'mastering semiotically mediated processes and categories'. (Wertsch & Stone, 1986).

3.9 An embodied approach

Embodied cognition focuses on the bodily/biological mechanisms underlying cognition and this research lies within this broad scheme of ideas. According to Lakoff and Johnson (1999) all mathematics is embodied, meaning that it is dependent on constructions in human minds and shared meanings in mathematical cultures.

Tall (2002) agrees with Lakoff and Johnson, but believes that power is reduced in the word 'embodied' as it refers to all mathematical thinking. Tall (2002, p. 4) states that the term 'embodied' refers to thought built fundamentally on sensory perception as opposed to symbolic operation and logical deduction. This gives the term 'embodied' a more focused meaning in mathematical thinking.

3.10 Conclusion

This research has been underpinned by cognitive and social constructivism. Core tents of constructivism was discussed . Embodied cognition focuses on bodily/ biological mechanisms underlying cognition and this research lies in this wide scheme of ideas. The following chapter provides an outline of the methodology used in the study, the paradigm, data collection procedures, challenges conducting the research and limitations of the study.

Chapter Four

Research methodology

4.1 Introduction

Chapter three offered an overview of the theoretical framework applied in this study. This chapter begins with the reintroduction of the critical research questions and presents the subjects and instruments used in this study. Further on the researcher describes the methodological framework applied and the research paradigm within which the study was located. The researcher explains how the interpretative paradigm fits this study. In addition data sources and data collection processes are presented.

4.2 Critical research questions

The study used artefacts in exploring the conceptual understanding shown by Grade 10 mathematics learners in learning the application of trigonometric ratios to 3D problems. The research questions addressed by this study were:

- ❖ *How did the use of mathematical artefacts enhance learning in trigonometry among Grade 10 learners?*
- ❖ *How did the use of mathematical artefacts improve learning and understanding?*

4.3 Qualitative research methodology

Qualitative research methodology by its nature permits the use of different research strategies to gather data. It allows the voice of participants to be heard. Romberg (1992) asserts that when no numbers are used in categorising, organising and interpreting relevant information that has to be gathered, then this method can be described as being qualitative. Fouché and Delport (2001, p.79) delineate this type of research as follows:

- ❖ Qualitative research elicits participants' accounts of meaning, experience or perceptions about a concept.
- ❖ It produces descriptive data.
- ❖ Qualitative approaches allow for more diversity in responses as well as the capacity to adapt to new developments or issues.
- ❖ In qualitative methods, the data collected can include interviews and group discussions, observation and reflection, field notes, various texts, pictures, and other materials.

Schutt (2012, p. 325) states that “ the analysis of qualitative research notes begins in the field, at the time of observation, interviewing, or both, as the researcher identifies problems and concepts that appear likely to help in understanding the situation.”

The study followed a qualitative approach, which was considered a suitable method as it provided a deeper understanding and explanation of the use of artefacts in teaching trigonometry. The researcher constructed the artefacts and presented them to the learners.

4.3.1 Theoretical perspective

The research was underpinned by cognitive and social constructivism. The activities the learners engaged in allowed for social interaction to take place among them. It was observed by the researcher that learners drew on their existing schemas as they assimilated new information. The learners became active participants in the construction of their own knowledge. In this study learners talked to each other about their thoughts and the manipulatives.

4.3.2 Actual methods used for data collection

The study adopted a qualitative approach. Various methods of data collection were used: (1) observation, (2) written responses in worksheet activity, and (3) semi-structured interviews. The learners interacted with the manipulatives

and data were collected via the activity worksheet, which was administered to the five subjects. These subjects were carefully observed during their interaction with the manipulatives and completion of their activity worksheet. The researcher recorded detailed notes of the observation. The subjects were then subjected to a semi-structured interview to gain clarity on their written responses.

Since the core aim of this study was to analyse learners' mathematical thinking in trigonometry when using artefacts, an interpretative paradigm was utilised. In this study the participants are referred to as Learner 1 (L1), Learner 2 (L2), Learner 3 (L3), Learner 4 (L4) and Learner 5 (L5).

4.3.3 The interpretative paradigm

This research study followed an interpretivist paradigm and was based on cognitive and social constructivism of knowledge. Angen (2000) explains that interpretivism assumes that the researcher's values are inherent in all phases of the interview and that truth is negotiated throughout the interview process. Angen (2000) further gives the following characteristics of the interpretative paradigm:

- a) Interpretative approaches rely heavily on naturalistic methods such as interviews, observations and analysis of existing texts.
- b) These methods ensure an adequate dialogue between the researchers and those with whom they interact so as to collaboratively build meaningful reality.
- c) Generally, meanings emerge from the research process.
- d) Qualitative methods are applied.

According to Cohen, Manion and Morrison (2011), the interpretative enquiry discovers and interprets the perspectives of the learners in this study and the answers to the enquiry are practically dependent on the context. More specifically, the researcher examined the learners' attempts to answer the given questions with regard to their understanding of trigonometric ratios.

4.3.4 How the paradigm fits the present study

The interpretative paradigm fitted the study as the data collection methods included observation, semi-structured interviews and analysis of learners' written responses to activities. Cohen et al. (2011) reiterate that the interpretative approach relies heavily on interviewing, observation and analysis of written texts.

4.4 Data sources

Five Grade 10 learners were observed and interviewed at a secondary school in South Africa.

Data for analysis was obtained from learners' responses in the given activity sheet regarding their understanding of trigonometric ratios. Semi-structured interviews were conducted to analyse the learners' written response in the activity worksheet. In addition, the researcher observed how the learners worked through the tasks and the answers provided in their activity sheet based on trigonometric ratios.

4.5 Data collection procedures

The learners were observed during three 45-minute lessons and after the fourth lesson they were interviewed. Observation, semi-structured interviews and learners' written responses to activities were the data collection procedures used in this study.

In the following sub-sections the researcher discusses each data collection method used in this study.

4.5.1 Observations

During classroom activities data were collected through observations.

Marshall and Rossman (1995) describe observation as not merely looking, but as looking systematically and noting systematically, people, events, behaviour, settings, artefacts and routines. Bailey (1994) argues that observation studies are superior to experiments and surveys when data are being collected on non-verbal behaviour. In observation studies the researcher is able to discern ongoing behaviour as it happens and is able to make appropriate notes about its important features.

4.5.2 Instruments for data collection

Data were collected using an activity worksheet, which contained four trigonometric questions based on 3D problems and the application of the trigonometric ratios. Semi-structured interviews were conducted with all participants, based on their responses to the questions in the activity worksheet, and were the main source of understanding the subjects' responses. Some of the questions used in the semi-structured interview were:

- ❖ Have you ever used artefacts/models before to learn trigonometry? Explain.
- ❖ What trigonometric concepts are needed in order to solve the question in the activity?
- ❖ Why did you decide to use the trigonometric ratio you wrote down?
- ❖ Could you explain how the sides in your trigonometric ratio could be found in the artefact?
- ❖ What trigonometric concepts are needed in order to solve the question in the activity?

4.5.3 Semi-structured Interviews

Semi-structured interviews were conducted to establish what effect the use of artefacts had on the solving of 3D trigonometry problems among Grade 10 learners.

According to Maree (2007) the aim of qualitative interviews is to view the world through the eyes of the participants. Maree (2007) further explains that semi-structured interviews require the participants to answer a predetermined set of questions, which in addition allows for further probing and clarification of answers. Semi-structured interviews define the line of inquiry.

The interview schedule was designed with key questions. Interviewing was chosen for the current study for the following reasons:

- It provided the opportunity to generate rich data.
- Language use by participants was considered essential in gaining insight into their perceptions and values.
- Contextual and relational aspects were observed as significant to understanding others' perceptions.

Written responses from learners on completion of the trigonometric activity sheet were collected and learners' responses were analysed.

4.6 Challenges of conducting classroom-based research

These are discussed under sections on validity and reliability, triangulation and participation below.

4.6.1 Validity and reliability

Validity determines whether a research instrument investigates what it was intended to investigate, while reliability refers to how consistent or reliable the

results are. In this study the issue of validity spans triangulation and participation.

4.6.2 Triangulation

Cohen et al. (2011) define triangulation as the use of two or more methods of data collection in the study of a specific aspect of human behaviour. Denzin (1978) described methodological triangulation as involving more than one method of gathering data, such as interviews, observations, questionnaires and documents.

In this study, triangulation to ensure the validity of data was achieved by using observation, semi-structured interviews and analysis of learners' written responses to activities. Data were collected over a period of time. Activity sheets contained learners' written responses were used to collect data. It was important to obtain the thinking behind the learners' written answers despite responses being correct or incorrect. Learners were interviewed to validate the answers given in their activity sheets.

4.6.3 Participation

The study was conducted with five Grade 10 learners from a public school in South Africa. Codes were allocated to the participants: L1, L2, L3, L4 and L5, to represent Learner 1, Learner 2, Learner 3, Learner 4 and Learner 5 respectively. Qualitative research was conducted with learners to record cognitive learning and understanding of artefacts and their application to trigonometric ratios with regard to three- dimensional problems.

The researcher used purposive sampling in this study. Maree (2007) describes purposive sampling as a method of sampling that is utilised in special situations where the sampling is done with a specific purpose in mind. Henning (2004) adds that purposive sampling looks towards the people who satisfy the criteria of desirable participants. Cohen et al. (2011) justifies that in

a qualitative study of 30 participants with a similar socio-economic background, a sample of five or six may suffice the researcher who is to acquire additional corroborative data to ensure validity. The five participants were Grade 10 mathematics learners and satisfied the criteria for the study.

4.7 Trustworthiness and ethical issues

Cohen et al. (2011, p. 542) state “that as qualitative analysis frequently concerns individual cases and unique instances, and may involve personal and sensitive matters, it raises the question of identifiability, confidentiality and privacy of individuals. The researcher has an ethical obligation to reflect on the principles of non-maleficence, loyalty and beneficence and to ensure that the principle of *primum non nocere* is addressed—first, do no harm (to participants)”.

In this study the researcher addressed the following issues:

- ❖ Permission was obtained from parents and guardians as well as from the learners to voluntarily participate in the study without any rewards.
- ❖ Participation was completely voluntary and has no impact or bearing on evaluation or assessment of the learner in any studies or courses while at school.
- ❖ Participants were asked to take part in the interviews after the worksheets had been completed.
- ❖ All participants were noted on transcripts and data collections by a pseudonym (i.e. fictitious name). The identities of the interviewees were kept strictly confidential.
- ❖ All data were stored with a secure password and was not used for any other purpose except for the research.
- ❖ Participants were allowed to leave the study at any time by notifying the researcher.

❖ Gatekeepers' permission was sought (Department of Basic Education).

Participants were permitted to review and comment on any parts of the researcher's written reports.

According to Cohen et al. (2011, p. 91), the essence of anonymity is that information provided by participants should in no way reveal their identity. A participant or subject is therefore considered anonymous when neither the researcher nor another person can identify the participant or subject from the information provided.

In this study the learner activity worksheets contained a group number and not the names of the participants. The participants' privacy was guaranteed. Cohen et al. (2011) explain that a questionnaire that has no identifying marks such as names, addresses, occupational details or coding symbols ensures complete and total anonymity.

Cohen et al. (2011, p. 92) state "that another way of protecting a participant's right to privacy is through the promise of confidentiality: not disclosing information about a participant in any way that might identify that individual or that might enable the individual to be traced. It also means not discussing an individual with anybody else. So although researchers know who has provided the information, they will in no way make the connection known publicly; the boundaries surrounding the shared information will be protected. The essence of the matter is the extent to which investigators keep faith with those who have helped them."

In this study the researcher explained to the participants what the meaning and limits of confidentiality were. The letter of consent explained in detail the steps taken to ensure confidentiality. All participants were noted on transcripts and data collections by a pseudonym. The identities of the interviewees were

kept strictly confidential. All data were stored securely and was not used for any other purpose except for the research.

4.8. Limitations of the study

Since the study followed an interpretative paradigm, the sample size was small and generalisations of the findings could not be made.

4.9. Conclusion

In this chapter methodological issues relating to this study were considered. The critical research question and research instruments were discussed. The qualitative paradigm and how it was linked with the theoretical framework adopted for this study was also discussed. The data capture methods are in keeping with a qualitative approach. To ensure reliability and validity, compliance, triangulation and participation were discussed. In Chapter Five the researcher discusses the results of the study on the basis of the data obtained.

Chapter Five

Results and analysis of written responses and interviews

5.1 Introduction

In the previous chapter the research design, methodology and procedures applied in this study were discussed. Comprehensive descriptions of each tool of enquiry and data sources were provided. Furthermore, the data analysis process was disclosed. In this chapter findings and discussions of the study are presented.

Qualitative methods were used in this study and data were collected through an activity sheet, observation and semi-structured interviews. The semi-structured interview was designed to obtain an insight into the learners' knowledge of and skill in trigonometric functions. The data were analysed to investigate how purposely designed manipulatives enhanced the learning of trigonometry among Grade 10 learners and how the use of manipulatives improved learning and enhanced teaching of mathematics. The five participants interacted with the manipulatives designed by the researcher and attempted to complete the activity sheet. While the learners were engaged in the activity the researcher observed them. Finally, semi-structured interviews were conducted to elicit the learners' understanding of trigonometric ratios based on the previous task.

5.2 Analysis of the four tasks on the activity sheet (refer to Appendix 1)

Each question on the activity sheet is referred to as a task (e.g. Task 1 corresponds to question 1 on the activity sheet in Appendix 1).

5.2.1 Analysis of task 1 on the activity sheet

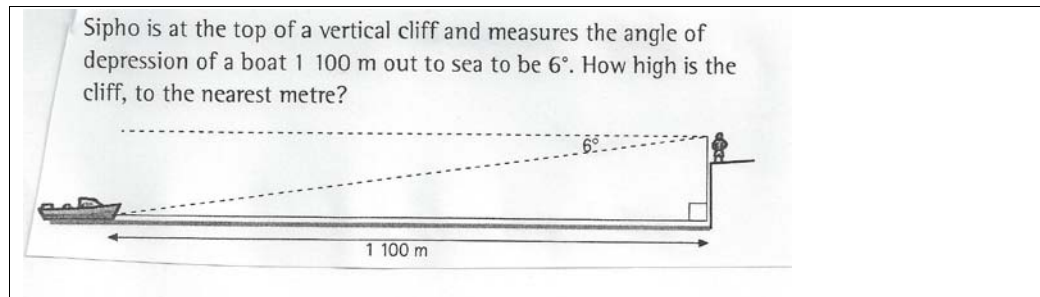


Figure 5.1 Question 1 of the activity sheet

The first question of the activity sheet required the learners to use their knowledge of the six trigonometric ratios and to select the correct one to solve the required unknown based on the information given. In this question the magnitude of an angle was given and the measurement of the adjacent side. The length of the opposite side had to be calculated, so the most suitable trigonometric ratio was $\tan \theta = \frac{\text{opp}}{\text{adj}}$. It should be noted that Sipho's height was to be ignored on the basis that his height was insignificant in terms of the height of the cliff.



Figure 5.2 Manipulative designed by researcher

Figure 5.2 shows the manipulative designed by the researcher. This was presented to the participants to use in solving task 1 on the activity sheet. The manipulative displays the scenario in question 1 where Sipho is at the top of a vertical cliff and a boat is in the ocean 1100 m away from the cliff.



Figure 5.3 Learners interacting with the manipulative

The researcher changed learners' places as the boys and the girls were congregating in separate groups. This was done to encourage interaction between the genders. The learners were given time to interact with the manipulative before looking at the activity worksheet. Figure 5.3 shows the interaction between the learners and the manipulative. Learners pointed at, touched and engaged with the manipulative and with each other. They spoke aloud and described and discussed what they saw.

The researcher asked the participants the following question:

Researcher: 'What do you see?'

Learner 1: 'I see a mountain with a person standing on it.'

Learner 3: [Shouts out] 'I see a boat away from the mountain.'

Learner 5: 'I see the beautiful blue ocean.'

The other learners were in agreement and nodded their heads to confirm what could be seen.

Learners interacted with each other, wrote information on their diagrams and interacted with the manipulative. All participants were successful in solving the question, and all obtained the correct answer.

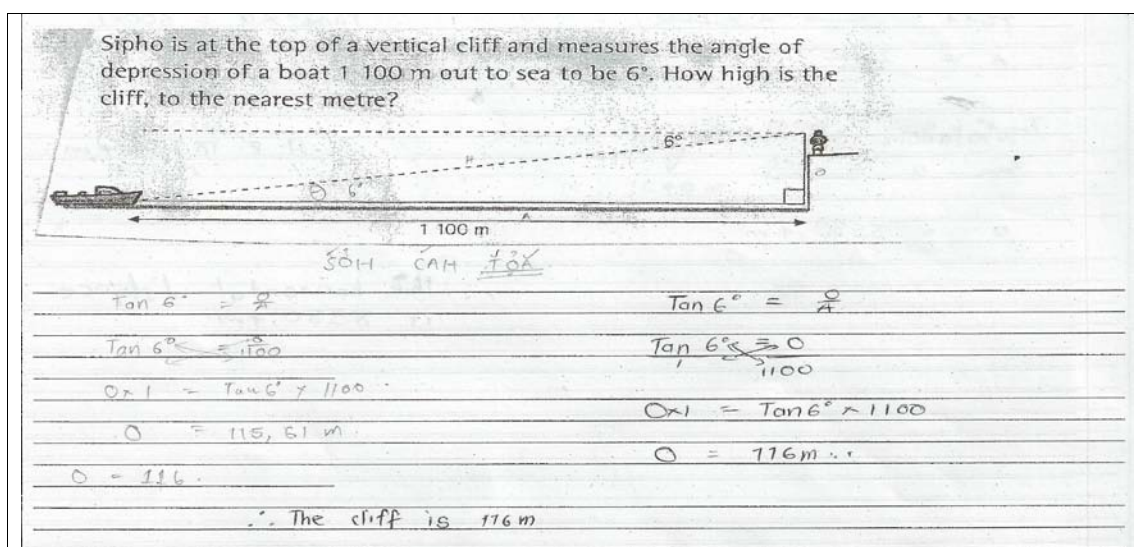


Figure 5.4 Written response of L3 to task 1

Figure 5.4 displays the written response of L3. I observed that L3 first inserted 6° in a right-angled triangle. L3 also wrote the mnemonic SOH CAH TOA and used ticks to assist the process of elimination. He managed to select the correct trigonometric ratio $\tan 6^\circ$ and was able to arrive at the correct answer (see line 9, Figure 5.4). From his written response we notice that he displayed effective algebraic skills of multiplication. He also manipulated the calculation correctly to determine $\tan 6^\circ$ (see line 9, Figure 5.4).

5.2.2 Analysis of task 2 on the activity sheet

Task 2

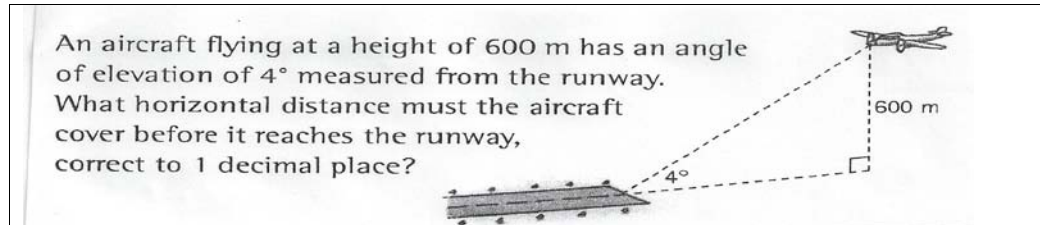


Figure 5.5 Question 2 on the activity sheet

The second question of the activity sheet required the learners to use their knowledge of the trigonometric ratios and to select the correct one to solve the required unknown based on the information given. In this question the measurement of an angle was given and the measurement of the opposite side. The measurement of the adjacent side had to be calculated, so the most suitable trigonometric ratio was $\tan \theta = \frac{\text{opp}}{\text{adj}}$.



Figure 5.6 Manipulative designed by researcher

The manipulative (Figure 5.6) consisted of an aircraft in flight and a runway/landing strip. This was a three-dimensional model of the scenario in task 2, where the measurement of the adjacent side and the magnitude of 4 are given, and was presented to the participants to use in solving question 2 from the activity sheet. ⁰

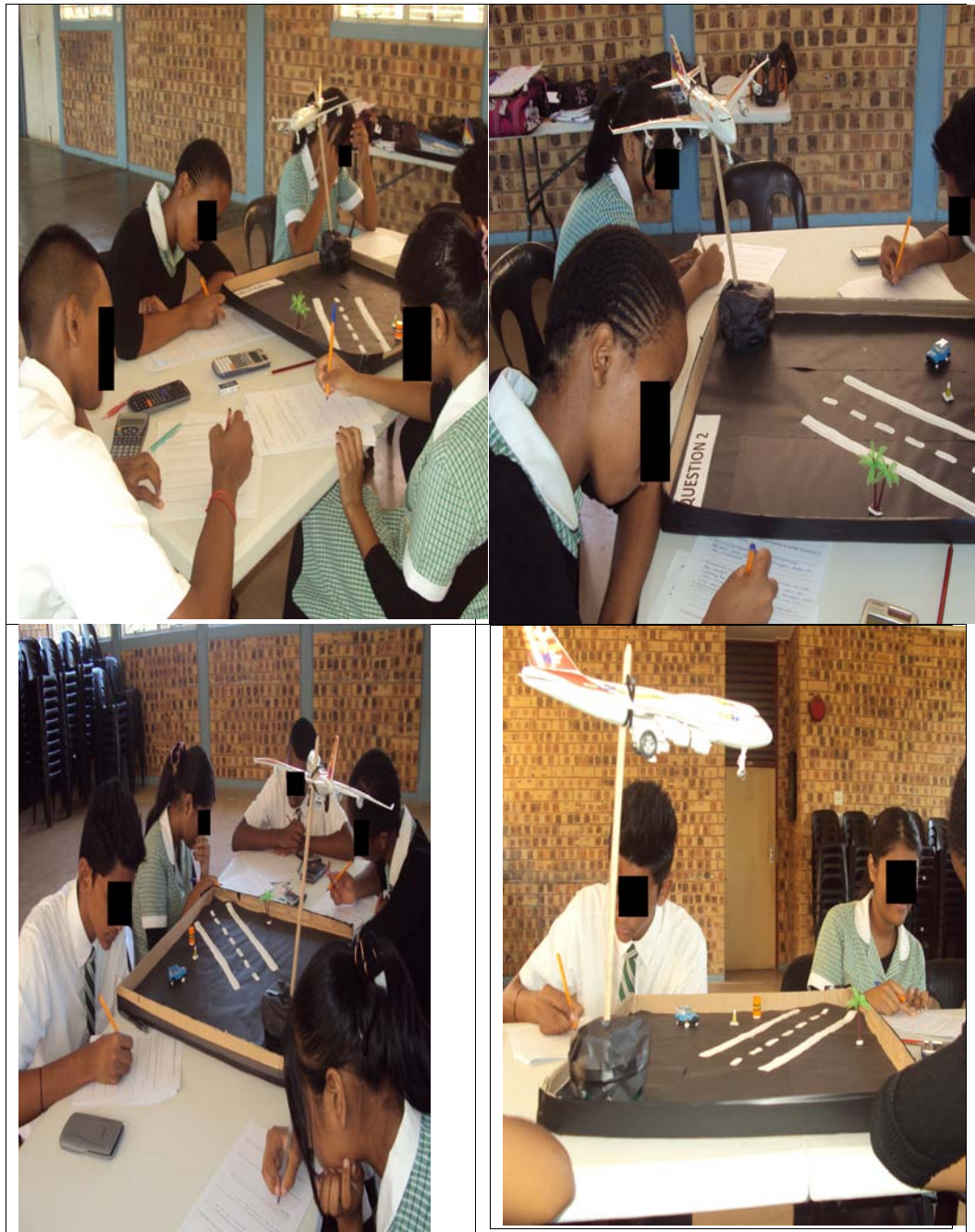


Figure 5.7 Learners interacting with the manipulative

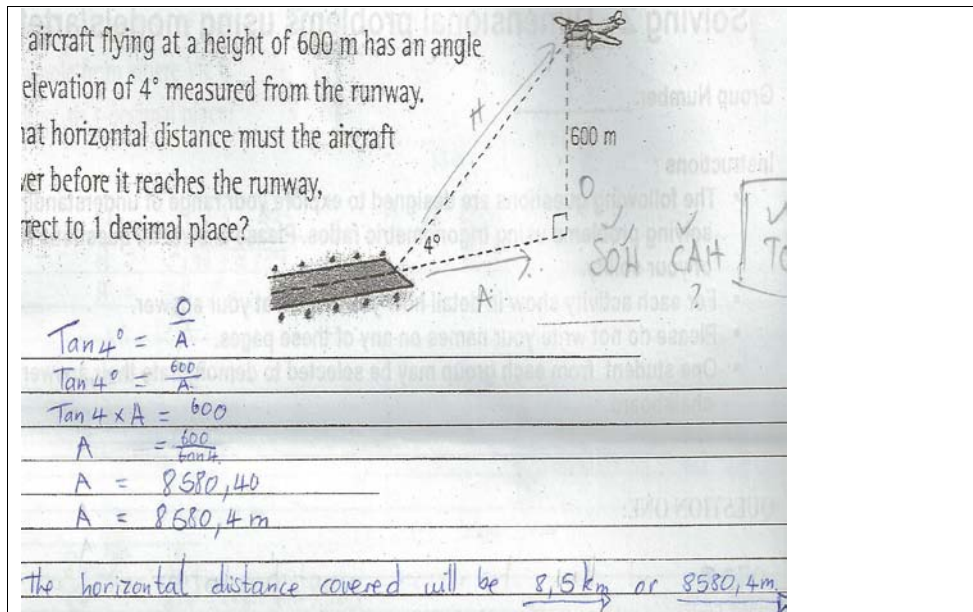


Figure 5.8 Learner 1: Written response to task 2 on activity sheet

Again we observed that L1 used the mnemonic SOH CAH TOA and used ticks to assist in the process of elimination, thus selecting the $\tan \theta$ trigonometric ratio. Line 3, Figure 5.8 shows that L1 was able to correctly substitute. In addition, L1 displayed algebraic skills in cross-multiplication and the use of the scientific calculator to successfully arrive at the correct answer (see line 5, Figure 5.8). Furthermore, L1 looked back at the problem and added a concluding statement to address the demands of the problem.

L1 indicated that trigonometric ratios could only be used in right-angled triangles. When asked how the manipulative assisted her in solving the problem, L1 responded by saying that the manipulative helped her visualise the situation and the information given. She further explained where the nose of the aircraft would land and pointed out on the manipulative the path the aircraft would take.

When asked how she related the 600 m and 4° to the manipulative, L1 responded by stating that she could imagine 600 m from the tip of the aircraft to the ground.

L1 inferred that on the manipulative the aircraft would not land in such a short distance or at such a sharp angle and that the aircraft needed to descend gradually. She mentioned the mathematical concept of gradient and stated that the aircraft needs to have a gentle gradient, as it would crash-land if the gradient were too large. L1 added that the wing of the aircraft would crash into the cars shown on the manipulative (see Figure 5.6).

All five participants made use of the mnemonic SOH CAH TOA and selected the correct trigonometric ratio. Four of the participants obtained a correct answer, while one participant struggled with solving the equation. Learner 3 cross-multiplied incorrectly and then, through discussion with the other learners, realised that an error had been made. L3 then struck off the incorrect solution and proceeded with the correct solution.

The learners engaged in discussions and assisted each other, and this bears out the finding by Heddens (2005) that manipulatives help learners learn to discuss mathematical ideas and concepts and to work co-operatively to solve problems.

5.2.3 Analysis of task 3 on the activity sheet

Task3

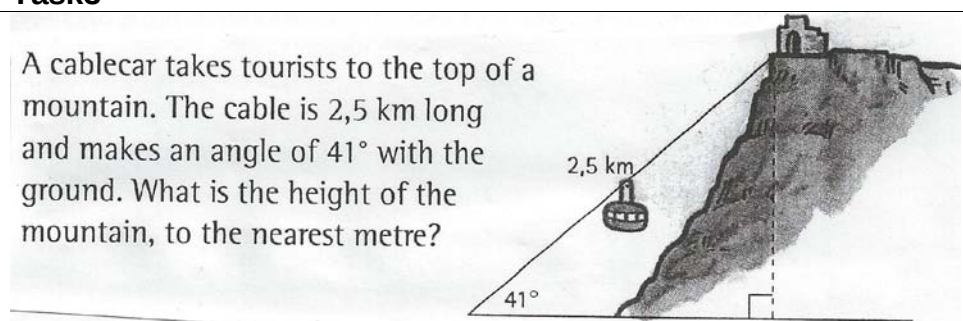


Figure 5.9 Question 3 on the activity sheet

The third question of the activity sheet required the learners to use their knowledge of the six trigonometric ratios and to select the correct one to solve the required unknown based on the information given. In this question the

measurement of an angle was given as well as the measurement of the hypotenuse. So the most suitable trigonometric ratio was $\sin \theta = \frac{opp}{adj}$.

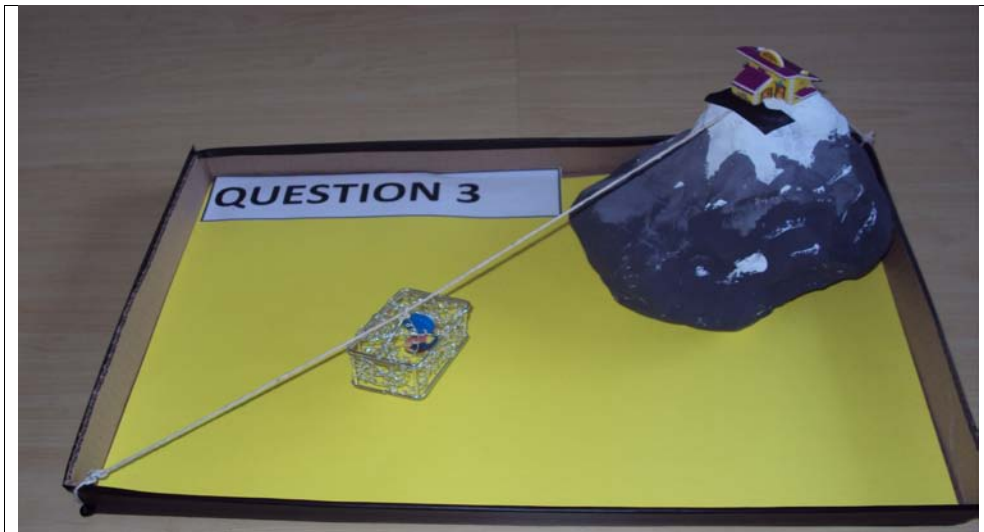


Figure 5.10 Manipulative designed by researcher

Figure 5.10 shows the manipulative designed by the researcher. This was presented to the participants to use in solving question 3 on the activity sheet. The manipulative displays a scenario where a cable 2.5 km long is at an angle of 41° to the ground.





Figure 5.11 Learners interacting with the manipulative

Figure 5.11 shows learners working with the manipulative and writing on their activity worksheets.

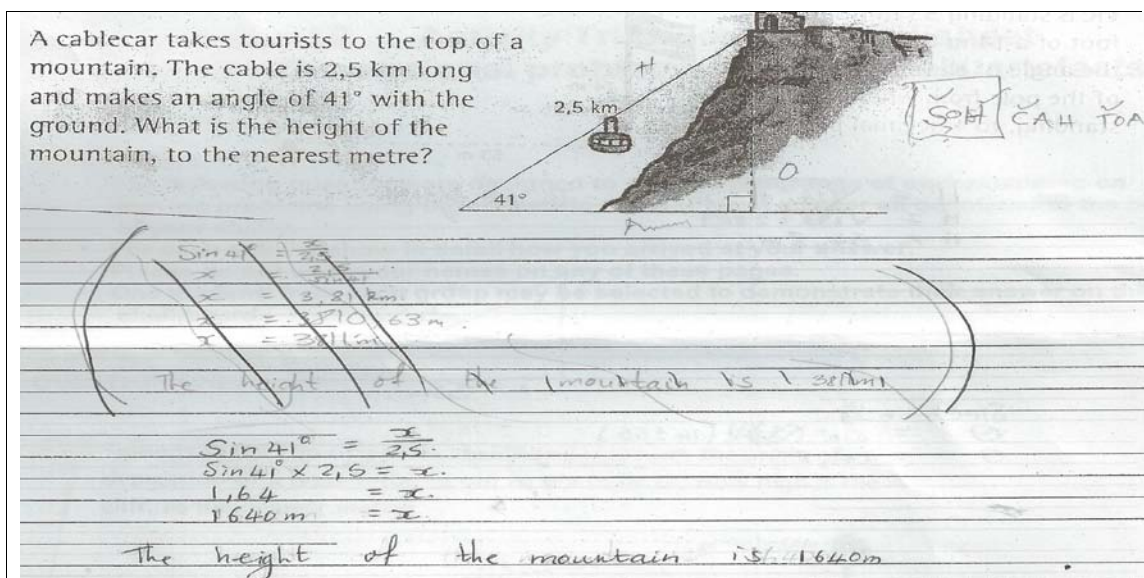


Figure 5.12 Written response of L1

Although L1 selected the correct trigonometric ratio by using the mnemonic SOH CAH TOA and ticks to assist the process of elimination, she displayed difficulty in cross-multiplying the equation (see line 2, Figure 5.12). L1 realised that her answer was incorrect when the other participants in the group discussed their answers. In her second attempt she cross-multiplied correctly (see line 11, Figure 5.12). In line 12, Figure 5.12, L1 displayed mathematical knowledge of conversion of kilometres to metres.

L1 stated that with the manipulative she could not see 90° but she was able to see the cable holding the cable-car and visualise 2.5 km. L1 further elaborated that the manipulative does not give actual measurements and that the cable-car was too big and not in proportion to the length of the cable. Asked if she could see 41° and the hypotenuse of a right-angled triangle on the manipulative, L1 responded by saying she could see an acute angle on the manipulative from the ground and could see that the cable was the hypotenuse, and she visualised the opposite side and 90° as it passed through the mountain. L1 added that it was hard to see the 90° triangle as the mountain caused an obstruction and you had to focus on one part at a time (L1 pointed to the base of the mountain).

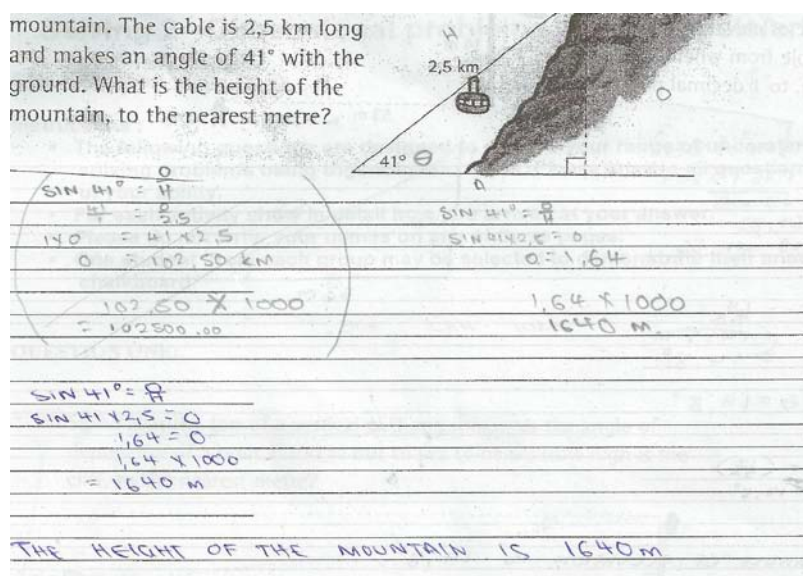


Figure 5.13 L4's written response to task 3 on activity sheet

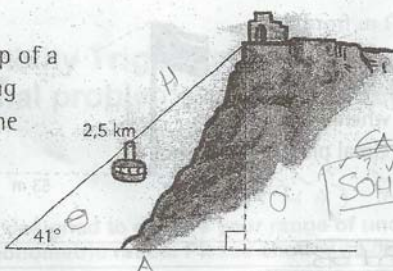
L4 in his first attempt did not carry $\sin \theta$ but θ alone. He merely omitted $\sin$ (line 2, Figure 5.13). His explanation was that he had just forgotten to add it. L4 did not acknowledge that $\sin \theta$ was one function and was inseparable. He cross-multiplied and arrived at an answer of 102 500 m (see line 7, Figure 5.13). He reasoned that the answer was not realistic as the mountain could not be 102 500 m high. Once he had discussed his answer with the other participants, he discovered that his answer was incorrect. His second attempt

yielded the correct answer. This time L4 carried $\sin 41^\circ$ to the second step (see line 3, Figure 5.13).

L4 indicated that the manipulative helped him understand the set-up. He was able to see that 2.5 km was the length of the cable and a right-angled triangle in the manipulative. However, he experienced difficulty seeing 41° in the manipulative. L4 further added that the artefact helped him a little and that it was easier working with the diagram on the activity sheet. It is evident that this learner did not need to interact with the manipulatives and found the diagram to be sufficient. This finding that not all learners require interaction with manipulatives is corroborated by Clements and McMillen (1996). In fact, some learners may perform better with pen and pencil or may not need to interact with the manipulatives for as long as other learners.

QUESTION THREE:

A cablecar takes tourists to the top of a mountain. The cable is 2,5 km long and makes an angle of 41° with the ground. What is the height of the mountain, to the nearest metre?



Handwritten solutions:

$\sin 41^\circ = \frac{H}{2,5}$
 $\sin 41^\circ = \frac{H}{2,5}$
 $x = 41^\circ \times 2,5$
 $= 102.90 \times 1000$
 $=$

$\sin 41^\circ = \frac{H}{2,5}$
 $\sin 41^\circ = \frac{H}{2,5}$
 $x = \frac{H}{\sin 41^\circ}$
 $= 3.81m$

$\sin 41^\circ = \frac{H}{2,5}$
 $\sin 41^\circ = \frac{H}{2,5}$
 $\sin 41^\circ \times 2,5 = x$
 $1.64 = x$
 Height of the mountain = 1,64m

Figure 5.14 L5's written response to Task 3 of activity sheet

L5 used the mnemonic SOH CAH TOA and used ticks to aid in the process of elimination. L5 incorrectly ticked A in the CAH and TOA of the mnemonic (see Figure 5.14) as this measurement was not in the given information. When asked why she ticked the adjacent side A in the mnemonic SOH CAH TOA, L5 could not justify why she had done so. It would appear that L5 merely

ticked above the mnemonic without understanding her action. L5 experienced difficulty with the solution (see lines 1 – 5, Figure 5.14).

At first L5 omitted $\sin$ from $\sin \theta$ (line 4, Figure 5.14), the same difficulty experienced by L4. L5's second attempt revealed that she was unable to cross-multiply (see line 2, Figure 5.14). This time L5 divided 2.5 by $\sin 41^\circ$ and then abandoned the solution. After interacting with the other participants, L5 finally obtained the answer of 1.64 (line 12, Figure 5.14). L5 used an incorrect unit of metres instead of kilometres. Once again, L5 failed to realise that her answer was not realistic and that the height of the mountain could not be 1.64 m. L5 was unable to convert kilometres to metres by multiplying 1.64 by 1000 to obtain the correct answer of 1640 m.

Although all five participants selected the correct trigonometric ratio, three of the participants had difficulty in arriving at the correct answer at the first attempt.

5.2.4 Analysis of task 4 on the activity sheet

Task4

Vic is standing 53 m from the foot of a 14 m flagpole. What is the angle of elevation of the top of the pole from where Vic is standing, to 1 decimal place?

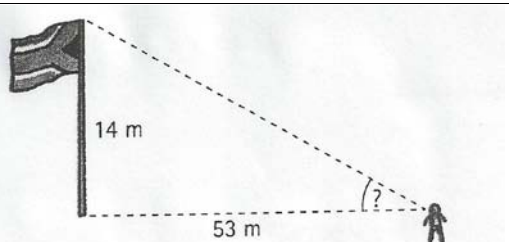


Figure 5.15 Question 4 on the activity sheet

The fourth question of the activity sheet required the learners to use their knowledge of the six trigonometric ratios, but this time they were required to solve the size of an angle. In this question the measurements of the opposite

and adjacent sides were given. An angle had to be calculated, so the most suitable trigonometric ratio was $\tan \theta = \frac{\text{opp}}{\text{adj}}$.



Figure 5.16 Manipulative designed by researcher



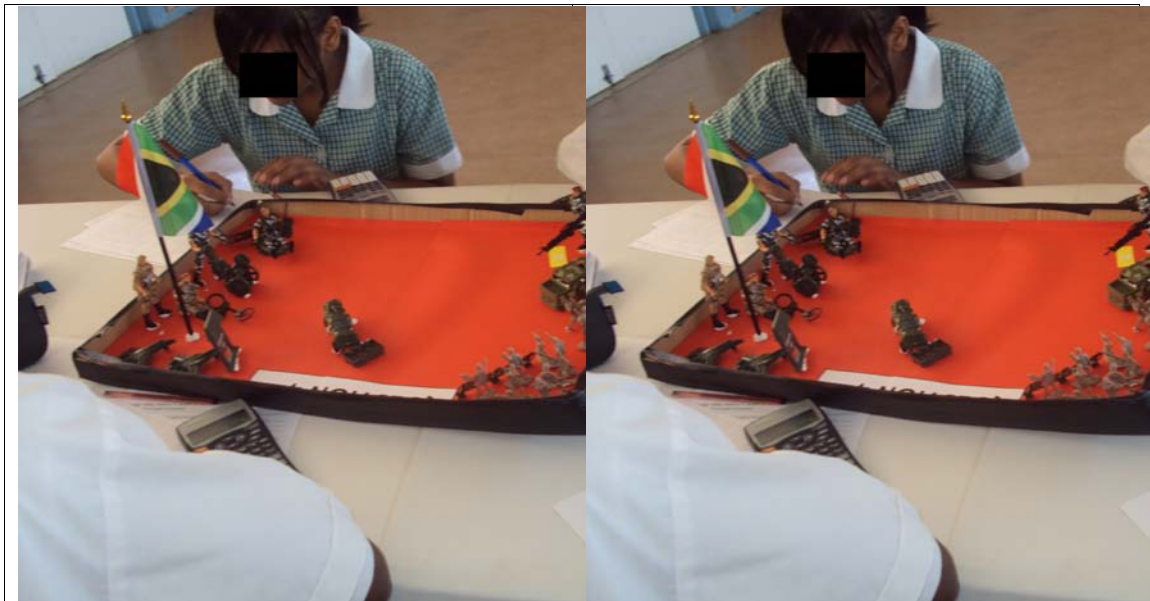


Figure 5.17 Learners interacting with manipulative

Figure 5.16 illustrates the manipulative depicting the problem for task 4. Figure 5.17 shows the interaction of the learners with the manipulative. In Figure 5.18 the written response of L1 is depicted.

$$\begin{aligned}
 H &= \sqrt{14^2 + 53^2} \quad \dots \text{Th of Pyth.} \\
 H &= \sqrt{196 + 2809} \\
 H &= 54,8 \text{ m.}
 \end{aligned}$$

$$\sin \theta = \frac{14}{55}$$

$$\theta = \sin^{-1} \left(\frac{14}{55} \right)$$

$$\theta = 14,8^\circ$$

Figure 5.18 L1's written response

L1 sketched a triangle in her solution. It was observed that she calculated the measurement of the hypotenuse and then proceeded to use the trigonometric ratio $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ to calculate θ . L1 was unable to see that there was no need for the measurement of the hypotenuse and that θ could be calculated with

the given information. L1 used the theorem of Pythagoras, substituted correctly and was able to calculate the measurement for the hypotenuse (see line 3, Figure 5.18).

When asked why she did not use the trigonometric ratio $\tan \theta = \frac{opp}{adj}$ L1

explained that she had later realised that there was no need to calculate the measurement for the hypotenuse, and that only after solving the problem realised that she had taken a long route to find the solution. Initially she did

not see that the trigonometric ratio $\tan \theta = \frac{opp}{adj}$ could be used, but later

realised that it could be. L1 explained that she felt comfortable working with

$\sin \theta = \frac{opp}{hyp}$ as she had previously worked with it and thereby calculated the

measurement of the hypotenuse. L1 added that the manipulative assisted her in that it was of a good size when compared with the other manipulatives and that the sizes of the men and the flag were in proportion (see Figures 5.16 and 5.17).

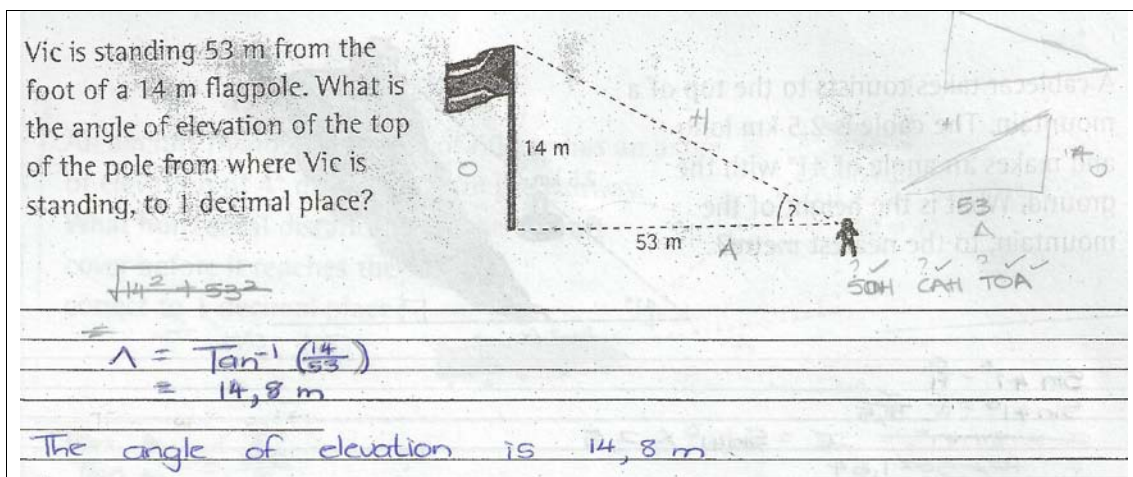


Figure 5.19 L2's written response

L2 sketched two triangles on the right-hand corner (see Figure 5.19). Her explanation of the two triangles was to depict the manipulative and that she rotated the first triangle to obtain the bottom triangle. L2 then proceeded to

insert the information given on the sketch and used the mnemonic SOH CAH TOA along with ticks to complete the process of elimination and select.

L2 explained that she had struck off the answer in line 1, Figure 5.19 as she had suddenly realised that there was no need to calculate the measurement of the hypotenuse as the measurements for the adjacent and opposite sides were given.

L2 stated that the manipulative provided a larger view of the situation. She could imagine the height of the flag (see Figures 5.16 and 5.17) on the manipulative being 14 m and the distance from Vic to the base of the flag being 53 m. L2 acknowledged that these measurements were smaller versions of the actual measurements. L2 was able to successfully manipulate her calculator and arrive at the correct answer (see line 4, Figure 5.19).

All five participants arrived at the correct answer. However, four did not use the given information and used $\tan \theta = \frac{opp}{adj}$. These participants calculated the

length of the hypotenuse and then used the trigonometric function $\sin \theta = \frac{opp}{hyp}$.

In his interview L3 stated that he used both $\sin \theta = \frac{opp}{hyp}$ and $\tan \theta = \frac{opp}{adj}$ to verify the answer.

5.3 The structure and analysis of the interviews

Semi-structured interviews of 40 minutes to an hour's duration each were conducted by the researcher with each of the five participants from the Grade 10 mathematics class. A semi-structured interview schedule was prepared to optimise the use of time. The purpose of the interview was explained to each participant before the interview. The codes L1, L2, L3, L4 and L5 were used

instead of the names of the participants to ensure confidentiality and anonymity. These codes correspond to those already used in the discussion of tasks on the activity sheet. Probing questions were asked to elicit information on trigonometric ratios and the use of manipulatives.

5.4 Analysis and discussion of Interviews

In addition to written responses to activities and observations, semi-structured interviews were conducted with the five participants. The objectives of the interviews were to:

- obtain clarity on written responses on the activity sheet;
- determine the impact the use of manipulatives had on their understanding in solving the tasks given; and
- check understanding of trigonometric ratios.

In these semi-structured interviews the participants were asked to respond to open-ended questions to:

- justify their responses to particular questions in the research instruments;
- indicate the trigonometric ratio applied to solve each task;
- describe the impact the manipulative had on them; and
- Explain the mathematics used to solve the tasks.

5.5.1 Interview question 1 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>Have you ever used artefacts/models before to learn trigonometry? Explain.</i>
--

Learner 1: <i>No, we used diagrams and textbooks before.</i>

Learner 2: <i>No, because our teacher did not use artefacts/models.</i>
Learner 3: <i>No, because we have a large class and it would be difficult.</i>
Learner 4: <i>No, I have not. Our teacher did not use artefacts/models to teach us.</i>
Learner 5: <i>No we've used right-angled triangles drawn on the chalkboard.</i>

It is evident that these learners did not have the opportunity to use manipulatives in their classrooms, and this confirms the finding of Boulton-Lewis (1998) that at the secondary school level it is not common practice for teachers to use manipulatives. L5 remarks that sketches with right-angled triangles were used instead of concrete objects replicating the task. It seems that another reason for not using artefacts, as L3 indicated, was that classes were too large.

5.5.2 Interview question 2 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>What trigonometric concepts are needed in order to solve question 1 in the activity'</i>
Learner 1: <i>We have to know about right-angled triangles. How to operate a scientific calculator. You need to know about 'sin', 'cos', 'tan'. Also hypotenuse, adjacent and opposite sides.</i>
Learner 2: <i>One of the six trig ratios - tan.</i>
Learner 3: <i>SOH;CAH; TOA. Then I chose TOA which is $\tan = \frac{OPP}{ADJ}$.</i>
Learner 4: <i>We used the six trigonometric ratios. Of the six, tan was used to solve the question.</i>
Learner 5: <i>We need to know which angles we are solving for. Know about right-angled triangles and rules on how to solve like SOH CAH TOA and also know trigonometric ratios.</i>

L1 acknowledged that trigonometric ratios could only be applied to right-angled triangles and indicated that the ability to use a scientific calculator to solve the problem was important.

This question was designed to test the learners' knowledge of the use of trigonometric ratios. It was pleasing to note that all participants were able to select the correct trigonometric ratio by using the process of elimination and the mnemonic. Learners were able to set up an equation, cross-multiply and use the calculator correctly.

5.5.3 Interview question 3 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>Why did you decide to use the trigonometric ratio you wrote down in question 1 in the activity?</i>
Learner 1: <i>We learnt to do it in class from our mathematics teacher. We got into a habit to use SOH CAH TOA.</i>
Learner 2: <i>Because the figures were best suited to TAN.</i>
Learner 3: <i>Because that was the only one we had learnt.</i>
Learner 4: <i>It was the best suitable to find out the height of the cliff.</i>
Learner 5: <i>We needed to solve for O. We learnt that to solve for O we needed to look for the trigonometric ratio we needed to use.</i>

Learners reasoned that the information given and what was required to solve best suited the $\tan \theta$ trigonometric function. L1 indicated that they had learnt about trigonometric ratios from their mathematics educator who had introduced them to the mnemonic SOH CAH TOA. L2 explained that the information given best suited the $\tan \theta$ trigonometric ratio. L5 inferred that the opposite side of the right-angled triangle had to be calculated. The manipulative assisted L4 to identify that the height of the cliff could be calculated by using the $\tan \theta$ ratio.

It is evident from the above excerpts that the participants were familiar with the mnemonic and the sides of a right-angled triangle. The participants displayed a good understanding of the trigonometric ratios and of the process to follow to select the correct trigonometric ratio when information was given.

5.5.4 Interview question 4 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>Could you explain how the sides in your trigonometric ratio could be found in the artefact'</i>
Learner 1: <i>The triangle is imaginary on the artefact, so we imagine them but the artefact has the angles found in them. They help us see the trigonometric ratios better.</i>
Learner 2: <i>By imagining the sides on the artefact.</i>
Learner 3: <i>We could see Sipho who was the army man standing on the cliff that was the opposite side, the hypotenuse was an imaginary line and the sea was the adjacent line.</i>
Learner 4: <i>By looking at the artefact and looking at the given diagram.</i>
Learner 5: <i>From Sipho to the sailing boat it was the hypotenuse and the cliff where Sipho was standing was the right-angle triangle. The imaginary line was from Sipho to the sailing boat.</i>

L3 explained that the manipulative helped him visualise better and he was able relate that in a right-angled triangle the cliff was the opposite side, the sea was the adjacent side and the hypotenuse was the imaginary line. From the learners' responses it can be seen that the use of manipulatives assisted them in visualising and understanding the problem better. This confirms Strom's view (2009) that manipulatives allow learners to become active participants and provide a visual aid to their understanding, memory and recall. In addition, this finding agrees with that of Heddens (2005) that manipulatives help learners to verbalise mathematical thinking.

5.5.5 Interview question 5 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>What trigonometric concepts are needed in order to solve question 2 in the activity?</i>
Learner 1: <i>Height; Horizontal; the 'H,O,A' sides of a triangle and the tan.</i>
Learner 2: <i>Tan.</i>
Learner 3: <i>SOH CAH TOA and I used TOA which is $\tan = \text{OPP/ADJ}$.</i>
Learner 4: <i>The six trig ratios could be used. Tan was used once again.</i>
Learner 5: <i>Horizontal line, the height and the imaginary line.</i>

All five participants once again indicated that the trigonometric ratio

$\tan \theta = \frac{\text{opp}}{\text{adj}}$ was required to solve the task. L1 stated that the hypotenuse,

opposite and adjacent sides of the triangle were required.

5.5.6 Interview question 6 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>Why did you decide to use the trigonometric ratio you wrote in question 2?</i>
Learner 1: <i>By checking what figures were given on the diagram and working it out using the artefact. I used 'SOH CAH TOA'.</i>
Learner 2: <i>The figure best fits in with tan.</i>
Learner 3: <i>Because that is the only one we learnt.</i>
Learner 4: <i>$\tan = \text{O/A}$ was used because it was best suitable for the answer.</i>
Learner 5: <i>We used tan because the trigonometric ratio for tan is A/O. We had the value of tan and the value of A and we needed to calculate O.</i>

L1 explained that she had used the information given in the diagram as well as the manipulative together with the mnemonic SOH CAH TOA to select the trigonometric ratio needed to solve Task 2. This bears out the finding of Elswick (1995) that concrete experiences help to instil in students a sense of confidence in their ability to think and communicate mathematically.

L5 displayed understanding of the trigonometric ratio $\tan \theta = \frac{opp}{adj}$ and that the measurement of θ was 4° and measurement of the opposite side 600 m (see Figure 5.5).

5.5.7 Interview question 7 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>Could you explain how the sides in your trigonometric ratio could be found in the artefact?</i>
Learner 1: <i>Imaginary triangle. I imagine the angles or sides. We used a logical explanation to know how a plane would land.</i>
Learner 2: <i>By imagining the sides on the artefact.</i>
Learner 3: <i>I drew lines in my head and the ground as the adjacent side.</i>
Learner 4: <i>Looking at the given diagram and comparing to the artefact could help find the sides.</i>
Learner 5: <i>The imaginary line, height and the landing of the plane.</i>

L1 and L2 stated that they could imagine a triangle in the artefact. L3 indicated that he drew a line in his head and could see the ground as the adjacent side. This corroborates the finding of Merrill *et al.* (2010) that improving and enhancing content knowledge requires mathematics teachers to implement three-dimensional solid modelling to improve learners'

understanding of mathematical concepts and principles. There is significant correlation between learners' knowledge and spatial visualisation.

5.5.8 Interview question 8 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>What trigonometric concepts are needed in order to solve question 3 in the activity?</i>
Learner 1: <i>SOH, CAH, TOA, hypotenuse, opposite, adjacent</i>
Learner 2: <i>Sin.</i>
Learner 3: <i>SOH, CAH, TOA and choose sin= O/H.</i>
Learner 4: <i>the six trig ratios. Of the six sin was used to solve the question.</i>
Learner 5: <i>The trig ratios SOH CAH TOA.</i>

All five participants indicated that aknowledge of trigonometric ratios was required. L3 was more specific and stated that the trigonometric ratio

$$\sin \theta = \frac{opp}{hyp} \text{ was required.}$$

5.5.9 Interview question 9 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>Why did you decide to use the trigonometric ratio you wrote down in question 3?</i>
Learner 1: <i>SIN because 'SOH' is given and hypotenuse so we could find the opposite.</i>
Learner 2: <i>Because the figures given are best suited with sin.</i>
Learner 3: <i>Because that's the only one we had learnt.</i>
Learner 4: <i>Sin=O/H. This trigonometric ratio was best to solve the answer.</i>
Learner 5: <i>We needed to calculate the opposite angle. The SOH</i>

was the concept. The hypotenuse and the alternate angle were given.

L5 makes mention of an alternate angle which was not given. Perhaps she meant an acute angle. She further explains that based on the information given, the ratio $\sin \theta = \frac{opp}{hyp}$ was most suitable to solve the task. It is evident that the participants were confident in selecting the correct trigonometric ratio to solve the task.

5.5.10 Interview question 10 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: *Could you explain how the sides in your trigonometric ratio could be found in the artefact depicting question 3?*

Learner 1: *Imaginary, the rope of the cable-car is the hypotenuse of the triangle.*

Learner 2: *Used the cable as a hypotenuse. Imagining the sides on the artefact.*

Learner 3: *Mountain was opposite side, adjacent was the ground and cable-car line was the hypotenuse.*

Learner 4: *By looking at the mountain and the cable and referring to the diagram that was given.*

Learner 5: *From the top of the rock to the cage was the hypotenuse, the height was the rock.*

L3 explained that in the artefact he was able to see the mountain as the opposite side of a right-angled triangle, the adjacent side was the ground on the artefact and the cable was the hypotenuse. The other participants offered a similar explanation.

5.5.11 Interview question 11 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>What trigonometric concepts are needed in order to solve Question 4 in the activity?</i>
Learner 1: <i>SOH CAH TOA, angle of elevation.</i>
Learner 2: <i>Tan.</i>
Learner 3: <i>We had two sides, so I used $\tan^{-1}$ and then to check my answer I found H and checked it was $\sin^{-1}$.</i>
Learner 4: <i>The six trig ratios. Of the six, tan was used to solve the question.</i>
Learner 5: <i>Trig ratios SOH CAH TOA.</i>

All five participants indicated that a knowledge of trigonometric ratios was required. L3 was more specific and stated that the trigonometric ratio $\tan \theta = \frac{\text{opp}}{\text{adj}}$ was required. He later explained that two sides were given and an angle had to be calculated. He added that he also calculated $\sin^{-1}$ to confirm his answer.

5.5.12 Interview question 12 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>Why did you decide to use the trigonometric ratio you wrote down in Question 4?</i>
Learner 1: <i>All sides were calculated and we could use any one.</i>
Learner 2: <i>Tan is the best trig ratio to use.</i>
Learner 3: <i>Because it was the most suitable one for the problem.</i>
Learner 4: <i>This trigonometric ratio works out the best to find the angle of elevation.</i>
Learner 5: <i>We were given the opposite and alternate angle and had to calculate the hypotenuse and use sin.</i>

L1 had calculated the measurement of the hypotenuse and therefore stated that any trigonometric ratio could be applied to solve the task. Once again L5 displays a misconception between an alternate angle and an acute angle. L5 also chose to calculate the measurement of the hypotenuse and then the trigonometric ratio $\sin \theta = \frac{opp}{hyp}$ to solve the angle. L2 was able to see that there was no need to calculate the measurement of the hypotenuse and that the trigonometric ratio $\tan \theta = \frac{opp}{adj}$ could be used to solve the task.

5.5.13 Interview question 13 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>Was it easy to solve the problem when using models/artefacts?</i>

Learner 1: Yes. Showed more details, imagine the triangle. Gave a better picture instead of equation to solve. Was enjoyable. Allowed for interaction because don't usually interact with learners, only the teachers. Enjoyed interacting with classmates.
Learner 2: Yes, more fun way to calculate the answers because you can already see the triangle with the model. Enjoyed working with group to discuss answers to see if answer is right or wrong. Liked the plane model the best as it showed the plane in the air and the landing path is the hypotenuse.
Learner 3: Yes. We could identify sides of the triangle and understand the problem better by using the model. Favourite model was the airplane. The stick was the opposite side. Looked very realistic. Looked like real life. Model was to scale. Models showed trigonometry existed in real life. You can identify trigonometry in real life.
Learner 4: Models made it a little more simple. With models we could visualise the scenario as to what was happening. Not much of a difference when looking at diagrams and models. A diagram is sufficient for me. No need for a model.
Learner 5: Yes, it made it much easier and simple. Enjoyed it. Got an opportunity to calculate other trigonometric questions using word problems and models. We worked in groups and discussed the problem.

L1 indicated that the use of artefacts added an element of reality instead of there just being an abstract equation. Heddens (2005) also confirmed that the use of manipulatives is to help learners to relate real-world situations to mathematics symbolism.

L2 and L1 both indicated that the use of artefacts allowed them to communicate with other classmates, which was a rare practice in their class. Brijlall et al. (2006) concur that learning without participation is in contradiction to the recommendations of constructivists such as Von Glasersfeld, who

strongly believe that reflective ability is a major source of knowledge at all levels of mathematics. This implies that it is important for learners to talk about their thoughts to each other and to the teacher. Constructivists focus on quality of learners' interpretive activity.

L3, L4 and L5 indicated that the use of artefacts helped them understand the problem better, visualise the situation and discuss their answers. This finding was also made by Strom (2009), namely that manipulatives allow learners to become active participants by providing a visual aid to their understanding, memory and recall. Manipulatives allow passive learners to physically interact with them, discuss actions with group members, attach notation to action and share findings with the rest of the class. Using manipulatives encourages conversation.

It is clearly evident from the above responses that the use of manipulatives assisted in solving the trigonometric problems. Learner 4 was an exception to the norm in that he stated that a diagram was sufficient for him and that there was no need for a manipulative. This indicates that this learner is functioning at a higher cognitive level.

5.6 How models/manipulatives assisted participants to learn during the lesson, and analysis of question 14 (refer to Appendix 2)

Excerpt from interviews with the participants:

Researcher: <i>Describe how the models/artefacts helped you learn during the lesson.</i>
--

Learner 1: <i>It gave a better view of what we needed to work out. Better picture on what we wanted to work. Gave different situations</i>

on how equations can be used. Different artefacts, airplane, mountain, army. Gave a wider view of where these rules can be applied in real life. Rules like SOH CAH TOA and theorem of Pythagoras. Would like teachers to use models. **Exciting** and better to picture [visualise] and we remember how to solve. We are not only writing, we are learning by visualising things. **It improved my learning.** Gave a wider view on how situations can be solved. Enjoyed the participation. Liked group work. Models were good. Accurate, how we get the triangles.

Learner 2: It **helped me understand** trigonometry better in a **more fun** and easy way. Builds my knowledge on trigonometry. Makes me **understand better**. Excited to participate. Nothing like what I expected, thought it was going to be bad, like we were going to be asked questions and have to just work out answers without discussion or models. Turned out to be fun and easy. Would like teacher to use models to teach because school will be fun. Learning and understanding will be better. People understand better by using models.

Learner 3: It gave us an in-depth look at the problem to **help us understand**. By in-depth I mean I could understand the problem better to find a solution and an answer. We should definitely use it in future lessons. Makes mathematics **more fun** than normal mathematics, which is boring. Only use numbers and only use textbooks. Here had fun using models. We could talk to each other. Worked together, shared and supported each other. Helped each other when others got it wrong. We made him understand. Enjoyed participating. We did not have to be quiet, we could talk to each other and interact by talking and pointing out the right sides of the triangle.

Learner 4: It **helped us to visualise** the scenario. Working with models you get **more interested** and you want to solve the problem. Favourite model was the army, what was happening and

how it was set up interested me. In real life we can use mathematics to solve a problem. In real life we can use mathematics, but not all the time. Some things we don't need to use mathematics in life.' [Researcher: Can you give an example?] ... I can't think of an example. I would like to participate because it was a nice experience. Working with models made it fun and interesting. Worked as a team together to solve problems.

Learner 5: It **helped me visualise** and see which angle I had to calculate. The models helped me identify the angles and which angle to calculate. The mountain model, I experienced the angle at which the person stood and the distance at which the boat was. Models **helped me to see better**. [Researcher: Would you like to use models?] Yes, they help me understand better. My favourite model was Siphon and the cliff. I was able to calculate the hypotenuse, then the other angles. I prefer calculating lengths rather than angles.

We discuss our findings from these interview responses in the next stage in the form of themes that arose.

5.7 Themes that emerged from the (above excerpts) analysis of question 13

5.7.1 Fun, enjoyable, interesting, exciting

L2 explained that the use of manipulatives assisted her in understanding trigonometry in a fun way. This provided her with a better understanding. L3 stated that the use of manipulatives made mathematics more fun than normal 'boring' mathematics. Burns agrees (1996) that manipulatives make learning mathematics exciting, interesting and enjoyable. Manipulatives intrigue and motivate while helping students learn.

5.7.2 Visualisation/imagine

The manipulatives assisted the participants in visualising the problem and understanding the problem better. Strom (2009) also found that manipulatives allow learners to become active participants and that they provide a visual aid to their understanding.

5.7.3 Improved understanding

Manipulatives provided an in-depth look at the problem and helped learners understand. This in-depth view of the problem provided a better understanding of how to find a solution. This confirms Heddens' finding (1986) that the use of concrete materials improves learners' thought processing skills, such as their logical thinking, and allows the transition from concrete to abstract. Concrete experiences allow learners to internalise mathematical concepts and develop them at the abstract or symbolic level.

5.7.4 Encourages teamwork

The use of manipulative encouraged conversation, co-operation and teamwork. When a learner experienced difficulty, the other participants acted as 'scaffolding' and supported this learner. This finding is confirmed by Steele (1993), who claimed that students are more engaged and motivated when they become actively involved in the learning process and when they engage in the use of manipulatives (artefacts), and also when working in co-operative groups. In addition

Strom (2009) confirms this finding by stating that manipulatives allow passive learners to physically work with manipulatives, discuss actions with group members, attach notation to action, and share findings with the rest of the

class. Using manipulatives encourages verbal interchange. It provides both learners and teachers with an avenue for conversation and understanding.

5.8 Conclusion

The learners' responses discussed above indicate that the use of manipulatives aids the understanding of mathematical concepts when taught using activities, which includes the transition from concrete to abstract. This chapter gave explanations of how Grade 10 mathematics learners solved real-life problems by using trigonometric ratios and manipulatives. Learners' written response extracts served to explore the conceptual understanding of trigonometric concepts and the influence manipulatives have on their understanding.

The next chapter concludes the study by discussing the findings, recommendations, the researcher's thoughts, and the limitations of the study.

Chapter Six

Conclusions and recommendations

6.1 Introduction

This chapter presents the findings, conclusions, recommendations, limitations of this study, and themes for further research.

6.2 Findings and conclusions

The four manipulatives designed in this study were carefully designed to develop learners' conceptual understanding of the application of trigonometric ratios to real-life situations. The activities were designed to engage the learners in a meaningful, hands-on activity through the use of manipulatives.

The outcome of this study proved to be very informative. The use of the manipulatives in teaching and learning trigonometric ratios in a Grade 10 mathematics class proved to be successful in the following ways. It:

- ❖ provided a tangible context to what would have been an otherwise abstract concept to many learners.
- ❖ motivated and inspired learners through hands-on interaction.
- ❖ mediated learning by fostering meaningful and contextualised learning through learner - learner conversation.
- ❖ encouraged teacher - learner conversation.
- ❖ provided learners with the opportunity to explore and experience the application of trigonometric ratios in the world around them.

The findings of my case study support Bayram's (2004) and Strom's (2009) research findings which confirm that manipulatives are important mediating

tools in the development of conceptual and procedural understanding of mathematical concepts.

6.2.1 Learners' understanding of trigonometric ratios

The learners' responses about the use of concrete manipulatives proved to be positive. The learners stated that they learn better when they can manipulate and see an object instead of just a two-dimensional drawing on a chalkboard. During their interaction with the manipulatives they indicated that they were involved in active learning. They stated that they could visualise the scenarios and could visualize the right-angled triangles in their minds. They were able to connect trigonometric ratios with the manipulatives. They further added that they found the tasks enjoyable, creative and helpful in their learning process.

Learners indicated that the use of the manipulatives allowed them to interact with their classmates. They were able to discuss their solutions and ask their classmates who had a better understanding of the solution, unlike in their traditional mathematics lessons where the teacher discussed the solutions on the chalkboard. They also mentioned that they learned through the discussions that transpired within the group.

They added that with the manipulatives it was easier to concentrate on learning. It was noted that in traditional classroom activities and environments it was easier for learners not to learn and not to influence the learning of others.

6.2.2 Difficulties experienced by learners

There were several factors that contributed to making trigonometry difficult for learners. These include a poor understanding of trigonometric notation, poor calculator skills, lack of knowledge of the theorem of Pythagoras and ability to apply it appropriately, inability to solve simple linear equations, a poor concept of ratio, and basic difficulties with algebraic manipulation such as cross-

multiplication in basic equations to calculate an unknown value. In this research some of the learners experienced difficulty in algebraic skills of multiplication. L1 displayed difficulty in cross-multiplying an equation (see line 2, Figure 5.12). L1 realised that her answer was incorrect when the other participants in the group discussed their answers.

L4 in his first attempt did not carry $\sin \theta$ but θ alone. He merely omitted $\sin$ (line 2, Figure 5.13). His explanation for his action was that he merely had forgotten to add it on. L4 did not acknowledge that $\sin \theta$ was one function and was inseparable. He cross-multiplied and arrived at the large answer of 102 500 m (see line 7, Figure 5.13). He reasoned that the answer was not realistic as the mountain could not be 102 500 m high. Once he had discussed his answer with the other participants, he discovered that his answer was incorrect. This further supported the positive attributes of peer-peer discussion/interaction.

6.2.3 Impact manipulatives had on learners' understanding of trigonometry

The use of manipulatives had the following effect on learners' understanding of trigonometry:

- ❖ The use of manipulatives allowed learners to become active participants and provided a visual aid to their understanding, memory and recall.
- ❖ The use of manipulatives allowed for learners to relate real-world situations to mathematics symbolism.
- ❖ The use of manipulatives permitted learners to discuss mathematical ideas and concepts.

- ❖ The use of manipulatives provided an opportunity for learners to work together co-operatively.
- ❖ The use of manipulatives allowed for learners to use their own methods to solve the required trigonometry problems.
- ❖ Manipulatives make learning mathematics exciting, interesting and enjoyable.
- ❖ Manipulatives intrigue and motivate while helping students learn.
- ❖ Manipulatives are useful tools for solving problems.
- ❖ Manipulatives develop learners' confidence by providing them with a way to test and confirm their reasoning.
- ❖ Manipulatives aid in making abstract ideas concrete.
- ❖ Manipulatives lift mathematics off the textbook pages.

6.3 Recommendations

The following recommendations can be made from this study:

- ❖ Teachers require training, exposure to and knowledge about the use of manipulatives in discovery learning and co-operative learning methods in mathematics classrooms in South Africa. Therefore teacher training is crucial for the effective use of these methods.
- ❖ Future research should be conducted –a longitudinal study to evaluate the long-term effect of the use of manipulatives.

- ❖ Considering that the sample consisted of only five participants, generalisations cannot be made. Future research should be conducted with a larger sample of participants to increase the validity of the findings and results.
- ❖ Several secondary school teachers steer away from the use of manipulatives and believe that they should be used in primary schools. They believe that they are time-consuming and unnecessary. Some are unsure how to use manipulatives in their classrooms. It is therefore recommended that workshops and training be provided for teachers on how and when to use manipulatives.
- ❖ Workshops need to be conducted to introduce teachers to new innovative methods of teaching by using manipulatives, and to extract them from their tradition of slavishly following the textbook. Manipulatives should be supplied as teachers' aids to enhance student learning.

6.4 Sampling

A sample of five Grade 10 mathematics learners were the participants in the study. This implies that the data used in the analysis have been based entirely on those learners' written responses from the activity sheet and the semi-structured interviews. This study may be extended to any Grade 10 mathematics learner of any secondary school.

6.5 Limitations

This was a study that comprises five Grade 10 learners at a specific school. The results could therefore not be generalised.

6.6 Themes for further research

- ❖ A similar study of the use of manipulatives at higher learning institutions where mathematics is taught, for example an engineering faculty.

- ❖ The use of virtual manipulatives in the classroom. Virtual manipulatives help learners to make the link between concrete and abstract. Virtual manipulatives may be more appealing than the physical objects for older learners.

6.7 Conclusion

Research on the application of manipulatives in mathematics classrooms is extensive. The utilisation of manipulatives in mathematics classrooms has several advantages and most certainly improves learners' understanding of abstract concepts by bridging the gap between concrete and abstract concepts in mathematics. Manipulatives permit learners to work at a concrete level and to advance to an abstract level. It affords learners the opportunity to experience and connect their previous knowledge to new knowledge.

Manipulatives subject learners to both verbal and visual mental representations that pave the way for better understanding. This is a technique that has been employed by some mathematics educators and has yielded positive results.

Proper professional development should be made available for educators. The researcher believes that workshops and training be offered for teachers on how and when to use manipulatives.

Although mathematic results in South Africa continue to be poor, mathematics teachers must continue to persevere to explore innovative methods to improve learner understanding, which would result in obtaining better mathematics results.

References

1. AL-Eissa, A. I. (2009). *Non-Audit Services and Auditor Independence: The Case of Saudi Arabia*. Victoria University: Melbourne.
2. Angen, M. J. (2000). Evaluating interpretive inquiry: Reviewing the validity debate and opening the dialogue. *Qualitative Health Research*, 10(3), 378-395.
3. Baba, T. (2007). *Japanese education and lesson study: An overview* In M. Isoda, M Stephens, Y. Ohara & T. Miyakawa (Eds) *Japanese lesson study in mathematics: Its impact, diversity and potential for educational improvement*. Singapore: World Scientific Publishing.
4. Bailey, K.D. (1994). *Methods of social research*. New York: The Free Press.
5. Baker, M.J., de Vries, E., Lund, K. & Quignard, M. (2001). Computer - mediated epistemic interactions for co-constructing scientific notations: Lessons learned from a five-year research programme. In P. Dillenbourg, A. Eurelings & K. Hakkarainen (Eds) *Proceedings of EuroCSCL 2001: European Perspectives on Computer-Supported Collaborative Learning*, Maastricht McLuhan Institute (ISBN 90-5681-097-9), pp. 89-96. Maastricht, March 22-24, 2001.
6. Balka, D. S. (1993). Making the connections in mathematics via manipulatives. *Contemporary Education*, 65(1), 19-23.
7. Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of mat education. *American Educator*, 16, 14-18, 46-47.
8. Bayram, S. (2004). *The effect of instruction with concrete models on eighth grade students' geometry achievement and attitudes towards*

geometry. (Master of science in secondary science and mathematics education), Middle East Technical University.

9. Beattie, I. D. (1986). Modeling operations and algorithms. *The Arithmetic Teacher*, 33(6), 23-28.
10. Bell, J. (2005). *Doing your research project: A guide for first time researchers in education, health and social science*. (4th ed.). England: Open University Press.
11. Berman, B. & Friederwitzer, F. J. (1983). Teaching fractions without numbers. *School Science and Mathematics*, 83(1), 77-82.
12. Boulton-Lewis, G. (1998). Childrens strategy use and interpretations of mathematical representations. *Journal for Mathematics Behaviour*, 17(2), 219-237.
13. Brijlall, D., Maharaj, A. & Jojo, Z. M. M. (2006). The development of geometric concepts through design activities during a Tecnology education class. *An African Journal of Research in SMT Education*, 10(1), 37-45.
14. Brijlall, D. & Ndlovu, Z. (2013). Exploring high school learners' metal construction during the solving of optimization. *South African Journal of Education*, 33(2).
15. Brooks, D. W. (1994). *The nature of conceptual learning and thinking*. Unpublished master's thesis. University of Durban Westville, South Africa.
16. Brooks, J. G. & Brooks, M. G. (1993). *In search of understanding: the case for constructivist classrooms*. Alexandria: VA: Association for Supervision and Curriculum Developement.

17. Bruner, J. S. (1966). *Toward a theory of instruction*. Cambridge, Mass.: Harvard University Press. In R. C. Sprinthall, & N. A. Sprinthall (1977). *Educational Psychology: USA. A Developmental Approach*. Addison-Wesley Pub. Comp., USA.
18. Burns, M. (1996). Manipulatives in Today's Classroom. Retrieved 3 July, 2013, from <http://scholastic.com/Instructor>
19. Cain-Caston, M. (1996). Manipulative queen. *Journal of Instructional Psychology.*, 23, 270-274.
20. Chang, K. (2008, April 25). Study suggests math teachers scrap balls and slices. *New York Times*. Retrieved 22 February, 2013, from <http://www.nytimes.com/2008/04/25/science/25math.html>
21. Chester, J., Davis, J. & Reglin, G. (1991). *Math manipulatives use and math achievement of third grade students*. Unpublished master's thesis. University of North Carolina, USA.
22. Clayton, C. (1994). How do learning theories affect what happens in my classroom. *Cemp VI*, 1(Jan/Feb).
23. Clements, D. H., & McMillen, S. (1996). Rethinking "concrete" manipulatives. *Teaching Children Mathematics.*, 2(5), 270-279.
24. Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education*, 23(1), 2-33.
25. Cochran, K. F., Derutter, J. A. & King, R. A. (1993). Pedagogical content knowing: an integrative model for teacher preparation. *Journal of Teacher Education*, 44(4), 3.
26. Cohen, L. , Manion, L. , & Morrison, K. (2011). *Research methods in Education* (7th ed.). London: Routledge.

27. Denzin, N. K. . (1978). *The research act: A theoretical introduction to sociological methods*. New York: McGraw-Hill.

28. DoBE, Department of Basic Education. (2012). *National curriculum statement Grades R- !2*. Pretoria: Department of Basic Education.

29. DoEB, Department of Basic Education. (2003b). *National curriculum statement, Grades 10-12: Mathematics*. Pretoria: Department of Education.

30. Eggen, P. & Kauchak, D. (2007). *Educational Psychology: Windows on classrooms* (7th ed.). Upper Saddle River, NJ: Merrill Prentice Hall.

31. Elswick, V. A. (1995). Using manipulatives to develop understanding. In *Integrated mathematics 1, Teachers Edition*. Boston: Houghton Mifflin Company.

32. Fernandes, H., Carron, T., Ducasse, S. (2008, 14-16 January). Paper presented at the Sixth International Conference on Creating, Connecting and Collaborating through Computing, Universite' of Savoie.

33. Fouche', C. B. & Delport, M. (2001). *Introduction to the research process*. In A. S. de Vos (Ed), *Research at Grassroots*. Pretoria: Van Schaik Publishers.

34. Furner, J. M., Yahya, N. & Duffy, M. L. (2005). Teach mathematics: Strategies to reach all students. *Intervention in School and Clinic*,, 41(1), 16-23.

35. Garrity, C. (1998). *Does the use of hands - on learning with manipulatives improve the test scores of secondary education geometry students?* Unpublished master's thesis. University of Chicago, Illinois.

36. Hartshorn, R. & Boren, S. (1990). Experimental learning of mathematics: Using manipulatives. *ERIC Digest (ED 321967)*.
37. Hausfather, S. (2001). Where's the Content? The Role of Content in Constructivist Teacher Education. *Educational Horizons, Fall 2001*, 15-19.
38. Heddens, J. W. (1986). Bridging the gap between the concrete and the abstract. *The Arithmetic Teacher*, 33(6), 14-17.
39. Heddens, J. W. (2005). Improving mathematics teaching by using manipulatives. Retrieved 25 July, 2013, from <http://www.fed.cuhk.edu.hk/fllee/mathfor/edumath/9706/13hedden.html>
40. Henning, E. (2004). *Finding your way in qualitative research*. Pretoria: Van Schaik.
41. Herbst, P. & Chazan, D. (2011). On creating and using representations of mathematics teaching in research and teacher development. *ZDM Mathematics Education*, 43, 1-5.
42. Howie, S. J. (1999). Third International Mathematics and Science Study Repeat. Executive summary. Pretoria: Human Sciences Research Council.
43. Jonassen, D. (1991). Evaluating constructivist learning. *Educational Technology*, 36(9), 28-33.
44. Kelly, C. A. (2006). Using manipulatives in mathematical problem solving: A performance Based analysis. *The Montana Mathematics Enthusiast*, 3(2), 184-193
45. Kennedy, S. (1991). *Historical Topics in Mathematics Education*. New York: Macmillan.

46. Kikas, E. (2006). The effect of verbal and visuo-spatial abilities on the development of knowledge of the Earth. *Research in Science Education*, 36(3), 269-283.
47. Kilpatrick, J., Swafford, J., & Findel, G. (Eds). (2001). Teaching for mathematical proficiency. In *Adding it up: Helping children learn mathematics*. Retrieved from National Academies Press database.
48. Lakoff, G. & Johnson, M. (1999). *Philosophy in the Flesh*. New York: Basic Books.
49. Lesh, R. (1979). *Mathematical learning disabilities: Considerations for identification, diagnosis and remediation*. In R. Lesh, D. Mierkiewicz & M. G. Kantowski Eds., *Applied Mathematical problem solving*. Columbus, Ohio: ERIC/SMEAC.
50. Lesh, R., Post, T. & Behr, M. (1979). *Representations and translations among representations in mathematics learning and problem solving*. In C. Janvier Ed., *Problems of representation in the teaching and learning of mathematics*. Hillsdale: NJ: Lawrence Erlbaum.
51. Maor, E. (1998). *Trigonometric Delights*. Princetown: Princetown University Press.
52. Maree, K. (2007). *First steps in research*. Pretoria: Van Schaik.
53. Marshall, C. & Rossman, G. B. (1995). *Designing qualitative research*. New Park: Sage.
54. McNeil, N. M. & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory into Practice*, 46(4), 309-316.
55. Merrill, C. , Devine, K. L., Brown, J. W. & Brown, R. A. (2010). Improving Geometric and Trigonometric Knowledge and Skill for High School Mathematics Teachers: A Professional Development Partnership. *The Journal of Technology Studies*, 36(2), 16-41.

56. Miles, M. P. & Huberman, A. M. (1994). *An expanded Source Book- Qualitative Data Analysis*. United Kingdom: Sage Publications Inc.
57. Motsheka, A. (2012). *Keynote address at the Public Sector Managers Forum*. By Mrs Angie Motshekga. Pretoria, 30 January 2012.
58. Moyer, P. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, 47, 175-197.
59. Munger, D. (2007, October 9). Children learn and retain math better using manipulatives. Retrieved 21 February, 2013, from http://scienceblogs.com/cognitivedaily/2007/10/children_learn_and_retain_math.php
60. Nakin, J. B. N. (2003). *Creativity and divergent thinking in geometry education*. Unpublished doctoral thesis. UNISA, Pretoria.
61. Noddings, N. (1990). Constructivists views on the teaching and learning mathematics. *Journal for Research in Mathematics Education, Monograph No (4)*.
62. Orhun, N. (2004). Students' mistakes and misconceptions on teaching of trigonometry. International conference on new ideas in mathematics education. Palm Cove, Australia
63. Ornstein, A. C. & Hunkins, F. P. (2004). *Curriculum-Foundations, Principles and Issues*. United States of America: Pearson Education.
64. Piaget, J. (1970). *Genetic Epistemology*. New York: Columbia University Press.
65. Piaget, J. (1972). *The Principles of Genetic Epistemology*. London: Routledge & Keegan Paul.

66. Pickard, A., & Dixon, P. (2004). The applicability of constructivist user studies: How can constructivist inquiry inform service providers and systems designers ?. *Information Research*, 9(3), 1-9.
67. Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics*, 108(7), 313-325.
68. Resnick, L. B. (1983). Mathematics and science learning: A new conception. *Science*, 220, 477-478.
69. Resnick, M. (1998). *Digital manipulatives : new toys to think with*. In *proceedings of CHI' 98*. Retrieved 14 September, 2014, from <http://citeseer.ist.psu.edu/resnick98digital.html>
70. Reys, R. E. (1971). Considerations for teachers using manipulative materials. *The Arithmetic Teacher*, 18, 551-558.
71. Romberg, T. A. (1992). *Problematic features of the school mathematics curriculum*. In P.W. Jackson (Ed), *Handbook of research on curriculum*. New York: Macmillan.
72. Schutt, R. K. (2012). *Investigating the social world. The Process and Practice of Research*. London: Sage.
73. Scott, P. (1987). A constructivist view of learning and teaching in science. Children's Learning in Science Project. Centre for studies in science and mathematics education: University of Leeds, England.
74. Seefeldt, C., & Wasik, B. A. (2006). *Early education: three-,four-,and five-year-olds go to School* (2nd ed.). Upper Saddle River: Pearson Education.

75. Slavin, R. E. (1997). *Educational Psychology: Theory and Practice* (5th ed). USA: Allyn & Bacon.
76. Smith, S. S. (2009). *Early Childhood Mathematics* (4th ed.). Boston: Pearson Education
77. Smith, D. A. & Moore, L. C. (1991). Project CALC : An Integrated Laboratory Course. In Leinbach et al (Eds), *The Laboratory Approach to Teaching Calculus*, MAA Notes volume 20, Mathematical Association of America, 81-92.
78. Sowell, E. (1974). Another look at materials in Elementary school mathematics. *School Science and Mathematics*, 74(3), 207-211.
79. Sowell, E. J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education*, 20, 498-505.
80. Stanic, G. M. A & McKillip, W. D. (1989). Developmental Algorithms have place in elementary school mathematics instruction. *Arithmetic Teacher*, 36(5), 14-16.
81. Steele, D. (1993). *What mathematics students can teach us about educational engagement: Lessons from the middle school*. Paper presented at the meeting of the American Educational Research Association, Atlanta, Georgia.
82. Strom, J. (2009). *Manipulatives in Mathematics Instruction*. (Masters of Science in Mathematics Unpublished), Bemidji State University, Bemidji, Minnesota, USA.
83. Stuart, V. B. (1998). *Math curse or math anxiety?* Retrieved 6 August, 2013, from <http://mathforum.org/t2t/faq/faq.manipulatives.html>
84. Suh, J. & Moyer, P. S. (2007). Developing students' representational fluency using virtual and physical algebra balances. *The Journal of Computers in Mathematics and Science Teaching*, 26(2), 155-173.
85. Tall, D. O. (1997). Functions and Calculus. In A. J. Bishop et al (Eds.), *International Handbook of Mathematics Education*, 289-325, Dordrecht: Kluwer.

86. Tall, D. O. (2002). *Using Technology to Support an Embodied Approach to Learning Concepts in Mathematics*. Paper presented at the First Coloquio de Historia e Tecnologia no Ensino de Matematica, Rio De Janiero.
87. Thompson, P. W. (1994). Concrete materials and teaching for mathematical understanding. *Arithmetic Teacher*, 41(9), 556-558.
88. Underhill, R. G. (1977). *Teaching Elementary School Mathematics*. Columbus, Ohio: Charles E. Merrill Publishing Company.
89. Van Laren, L. (2012). Mathematics Teachers at the intersection of policy interventions: What practicing teachers offer as challenges in teaching trigonometry. *Education as Change*, 16(2), 203-215.
90. Von Glasersfeld, E. (1984). *An introduction to radical constructivism*. In P. Watzlawick (Ed), *The invented reality*. New York: Norton.
91. Vygotsky, L. S. (1978). *Mind in society*. Boston: Harvard University Press.
92. Weng, T. S. (2011). 3D Mathematics - Seeing is believing. *International Education, e-Business, e-Management and e-Learning*, 1(1), 52-55.
93. Wertsch, J. & Stone, C. (1986). *The concept of internalisation in Vygotsky's account of the genesis of higher mental functions*, In Wertsch, J. (Ed) *Culture, communication and cognition: Vygotskian perspectives*. New York: Cambridge University Press.
94. Wongapiwatkul, P., Laosinchai, P., Ruenwongsa, P. & Panijpan, B. (2011). Enhancing High-school Students' Conceptual Understanding of the Linkages between the Earth and Trigonometry through an Inquiry-based Learning Unit. *The International Journal of Learning*, 18(2), 1-21.

Appendix 1

INSTRUMENTS

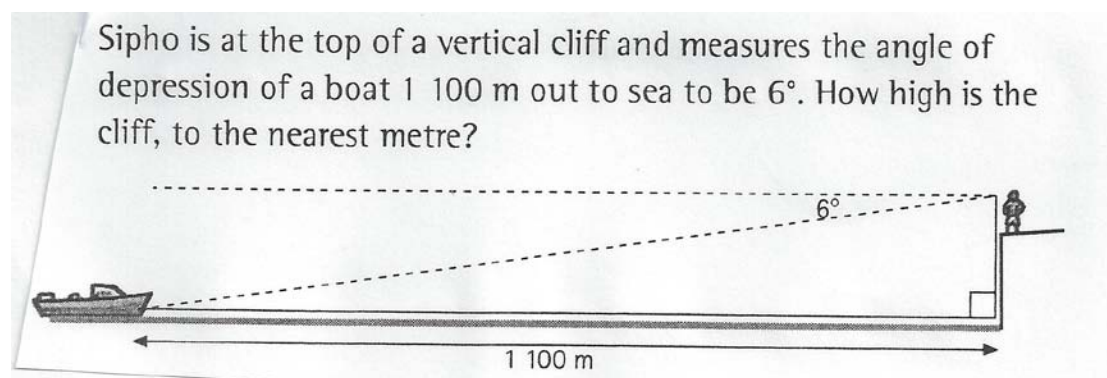
Grade 10 Activity Trigonometry worksheet Solving 2 - Dimensional problems using models/artefacts

Group Number: _____

Instructions :

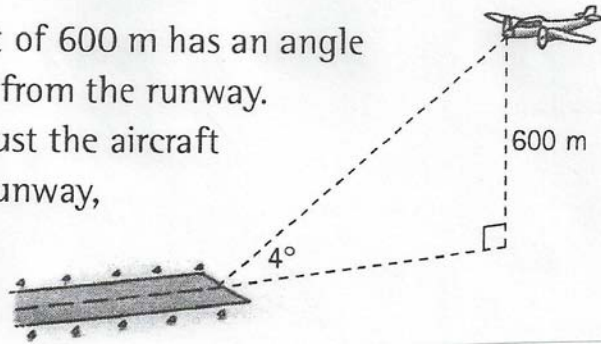
- The following questions are designed to explore your range of understanding on solving problems using trigonometric ratios. Please answer all questions to the best of your ability.
- For each activity show in detail how you arrived at your answer.
- Please do not write your names on any of these pages.
- One student from each group may be selected to demonstrate their answer on the chalkboard.

QUESTION ONE:



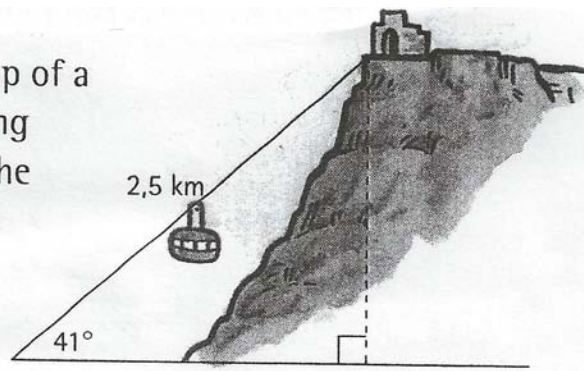
QUESTION TWO:

An aircraft flying at a height of 600 m has an angle of elevation of 4° measured from the runway. What horizontal distance must the aircraft cover before it reaches the runway, correct to 1 decimal place?



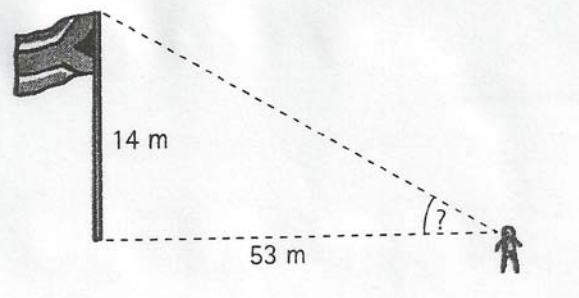
A cablecar takes tourists to the top of a mountain. The cable is 2,5 km long and makes an angle of 41° with the ground. What is the height of the mountain, to the nearest metre?

The diagram shows a right-angled triangle representing the mountain's profile. The hypotenuse, representing the cable, is labeled '2,5 km'. The angle between the hypotenuse and the horizontal ground is labeled '41°'. A cable car is shown on the cable. A vertical dashed line from the top of the mountain to the ground indicates the height, with a right-angle symbol at the base.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

QUESTION FOUR:

Vic is standing 53 m from the foot of a 14 m flagpole. What is the angle of elevation of the top of the pole from where Vic is standing, to 1 decimal place?

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Appendix 2

INTERVIEW SCHEDULE (Semi –Structured)

Date: _____

Grade: _____

Name of Learner: _____ (Pseudonym): _____

**Topic: Using Artefacts to Support an Embodied Approach to Learning
Trigonometry: A case study of Grade 10 Learners**

-
1. Have you ever used artefacts/models before to learn trigonometry?
Explain .

2. What trigonometric concepts are needed in order to solve the
QUESTION 1 in the activity?

3. Why did you decide to use the trigonometric ratio you wrote down in
QUESTION 1 ?

4. Could you explain how the sides in your trigonometric ratio could be found in the artifact?

5. What trigonometric concepts are needed in order to solve the QUESTION 2 in the activity?

6. Why did you decide to use the trigonometric ratio you wrote down in QUESTION 2 ?

7. Could you explain how the sides in your trigonometric ratio could be found in the artifact?

8. What trigonometric concepts are needed in order to solve the QUESTION 3 in the activity?

9. Why did you decide to use the trigonometric ratio you wrote down in QUESTION 3 ?

10. Could you explain how the sides in your trigonometric ratio could be found in the artifact?

11. What trigonometric concepts are needed in order to solve the QUESTION 4 in the activity?

12. Why did you decide to use the trigonometric ratio you wrote down in QUESTION 4 ?

13. Was it easy to solve the problem when using models/artefacts ?

14. Describe how the models/artefacts helped you learn during the lesson.

Some Standard Probes for the interviewer

<u>FOR CLARITY/SPECIFICITY</u> • Can you be more specific? • Can you tell me more about that?	<u>FOR COMPLETENESS:</u> • Anything else? • Tell me more.
<u>OTHER PROBING TECHNIQUES:</u> • Repeat the question • Echo their response • Pause a second • Baiting • What is your best estimate?	<u>OTHER PROBING TECHNIQUES</u> • Which would be closer? • Which answer comes closest to how you feel/ think? • If you had to pick one answer, what would you choose? • What do you think?

APPENDIX 3 Ethical Clearance Certificate



YAKWAZULU-NATALI

28 November 2013

Mrs Caresse Niranjani (212559567)
School of Education
Edgewood Campus

Protocol reference number: HSS/0829/013M

Project title: Using artefacts to support an embodied approach to learning Trigonometry: A case study of Grade 10 learners

Dear Mrs Niranjani,

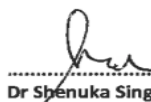
Expedited Approval

I wish to inform you that your application dated 27 March 2013 has now been granted Full Approval.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. Please note: Research data should be securely stored in the discipline/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully


.....
Dr Shenuka Singh (Chair)

/ms

cc Supervisor: Professor D Brijlall
cc Academic Leader Research: Dr MN Davids
cc School Administrator: Mr Thoba Mthembu

Humanities & Social Sciences Research Ethics Committee

Dr Shenuka Singh (Chair)

Westville Campus, Govan Mbeki Building

Postal Address: Private Bag X54001, Durban 4000

Telephone: +27 (0) 31 260 3587/8350/4557 Facsimile: +27 (0) 31 260 4609 Email: ximbap@ukzn.ac.za / snymanm@ukzn.ac.za / mohunp@ukzn.ac.za

Website: www.ukzn.ac.za



1910 - 2010
100 YEARS OF ACADEMIC EXCELLENCE

Founding Campuses:

Edgewood Howard College Medical School Pietermaritzburg Westville

Appendix 4 Informed consent for participants

Letter of Consent

To: Participant(s) and Parent/Guardian

Research Project: Using Artefacts to Support an Embodied Approach to Learning Trigonometry: A case study of Grade 10 Learners.

Year: 2013

Mrs C. Niranjan is doing a study through the **School of Education, Mathematics Education at the University of KwaZulu-Natal** with **Dr Deonarain Brijlall**. His contact numbers are 031-260 3491(work) and 083 555 2390. We want to research the use of artefacts in the learning of trigonometry at a secondary school in KwaZulu-Natal: South Africa.

Learners and educators are asked to help by taking part in this research project, as it would be of benefit to interested educationists and/or mathematics teachers. However, participation is completely voluntary and has no impact or bearing on evaluation or assessment of the learner in any studies or course while at school. Participants may be asked to take part in the surveys and interviews after the worksheets have been completed. These interviews will be tape-recorded. All participants will be noted on transcripts and data collections by a *pseudonym* (i.e. fictitious name). The identities of the interviewees will be kept strictly confidential. All data will be stored in a secured password and not been used for any other purpose except for the research.

Participants may leave the study at any time by notifying the researcher. Participants may review and comment on any parts of the researchers' written reports.

(Researcher's Signature)

(Date)

DECLARATION

I, _____ (Participant's NAME) _____
_____(Signature)

_____(Parent's/ Guardian's NAME) _____
_____(Signature)

_____(Date)

☐ Agree.

N.B. Tick ONE

☐ Disagree.

To participate/allow participation in the research being conducted by **Mrs C. Niranjan** concerning *Using Artefacts to Support an Embodied Approach to Learning Trigonometry: A case study of Grade 10 Learners*.

Appendix 5 Letter from Editor

L. Gething, M.Phil. (Science & Technology Journalism) (*cum laude*)

WHIZZ@WORDS

PO Box 1155, Milnerton 7435 Cape Town, South Africa; cell 072 212 5417
leverne@eject.co.za

1 December 2013

DECLARATION OF EDITING:

**Using artefacts to support an embodied approach to learning
trigonometry: A case study of Grade 10 learners**

By **Caresse Niranjan**

(212 599 567)

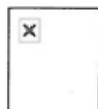
In partial fulfilment of the requirement for the **DEGREE OF MASTER OF
EDUCATION** In the school of Mathematics Education (Edgewood Campus),
Faculty of Humanities, University of KwaZulu-Natal. **Supervisor: Professor
Deonarain Brijlall.**

I hereby declare that I carried out language editing of the above thesis by Caresse Niranjan. I am a professional writer and editor with many years of experience (e.g. 5 years on *SA Medical Journal*, 10 years heading the corporate communication division at the SA Medical Research Council), who specialises in Science and Technology editing - but am adept at editing in many different subject areas. I am a full member of the South African Freelancers' Association as well as of the Professional Editors' Association.

Yours sincerely

LEVERNE GETHING
leverne@eject.co.za

Appendix 6 Turnitin Report

**Turnitin Originality Report**

Using artefacts to support an embodied
approach to learning trigonometry by
Caresse Niranjana

From M Ed dissertation (Masters)

Similarity Index	Similarity by Source
22%	Internet Sources: 15%
	Publications: 6%
	Student Papers: 9%

Processed on 03-Dec-2013 09:07 CAT

ID: 379475006

Word Count: 23527

I hereby acknowledge that I have accurately referenced the work used by others from internet sources, publications and student papers.

.....
Caresse Niranjana (212599567)



Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

PALMVIEW SECONDARY SCHOOL

26/28 CANTERPALM ROAD
PALMVIEW
PHOENIX
E MAIL: palmviewsst@hotmail.com



P.O. BOX 2058
VERULAM 4340
TEL: 031 5054462
FAX: 031 5055332

8 April 2013

Dear Mrs C. Niranjani

Permission is hereby granted to do scholarly research with grade 10 mathematics learners at Palmview Secondary school.

Yours faithfully

Mr B. Deepnarian (Principal)

