DOWNLINK CALL ADMISSION CONTROL IN MIXED SERVICE CDMA CELLULAR NETWORKS

Niven Bhimraj Ramlakhan
BSc Eng (Electronic)
Student Member IEEE

University of KwaZulu-Natal
Howard College Campus

2007

Submitted in fulfilment of the academic requirements for the degree of Master of Science in Engineering in the School of Electrical, Electronic and Computer Engineering, University of KwaZulu-Natal, Howard College Campus, Durban South Africa, 2007

ABSTRACT

Since the ascendance of CDMA (Code Division Multiple Access) as the generally accepted multiple access technology of choice for next generation networks, the rate of development of the wireless cellular communication industry has been phenomenal. Next generation wireless cellular networks offer a myriad of voice, video, data and text based information services for the future multimedia and information society. This mixed service scenario implies that the same finite resource i.e. the air interface must be shared amongst different classes of user, each with a specific quality of service. With multiple services competing for the same resource and with the inherent soft capacity nature of CDMA, call admission control becomes a formidable task. The problem is further compounded by the introduction of priorities between classes. Call admission control is an essential component of these next generation networks and the open nature of the current standards, such as UMTS (Universal Mobile Telecommunication System) allow for vendor implementation of different call admission control policies.

The main area of focus in this dissertation is on a proposed downlink, load-based, mixed service call admission policy. In a CDMA environment with symmetrical service and equal bandwidths in each direction, the uplink is commonly considered to be the bottleneck. Based on the asymmetric nature of the expected traffic in next generation networks the downlink is envisaged as the future bottleneck. Some of the more common choices for downlink call admission control include number based as well as power based call admission policies. A load-based call admission policy has been chosen as the maximum load threshold that can be supported varies with the state of the system and thus effectively models the behaviour of a soft capacity CDMA network. This dissertation presents a teletraffic performance analysis model of a load-based call admission control policy for downlink mixed service CDMA cellular networks.

The performance analysis yields customer oriented grade-of-service parameters such as call blocking probability which is essential for network planning. In our analysis we incorporate a Birth-Death Markov queuing model. This mathematical model is verified though computer simulation.

PREFACE

The research work presented in this dissertation was performed by Mr. Niven Ramlakhan under the supervision of Professor Fambirai Takawira in the School of Electrical, Electronic and Computer Engineering at the University of KwaZulu-Natal, Howard College Campus. This work was partially sponsored by Telkom South Africa and Alcatel South Africa as part of the Centre of Excellence Programme. The financial assistance of the Department of Labour (DoL) towards this research is hereby acknowledged. Opinions expressed and conclusions arrived at, are those of the author and are not necessarily to be attributed to the DoL, Telkom South Africa or Alcatel South Africa.

I hereby declare that all the material incorporated in this dissertation is the student's own original work, unless otherwise indicated by acknowledgement in name or reference. The work contained herein has not been submitted in whole or part for a degree at any other university.

Signed: ______________________________

Name: ______________________________

Date: ____________

As the candidate's supervisor I have approved this thesis for submission.

Signed: ______________________________

Name: ______________________________

Date: ____________

ACKNOWLEDGEMENTS

I would like to sincerely thank Professor Fambirai Takawira for his encouragement, advice, support and guidance during this arduous journey. His constructive criticism and patience is deeply appreciated.

I also wish to express a heartfelt appreciation to both my parents and family who have supported me endlessly in this as well as all my endeavours. Their unconditional patience and support is the reason I dedicate this dissertation to them.

I would also like to thank my wife for her constant encouragement and understanding during this compilation.

Finally I would like to acknowledge my fellow postgraduates, staff and colleagues for making this entire experience worthwhile and providing a much deserved break from the grindstone.

TABLE OF CONTENTS

TABLE OF FIGURES

TABLE OF TABLES

TABLE OF ACRONYMS

CDMA	Code Division Multiple Access
UMTS	Universal Mobile Telecommunication System
DoL	Department of Labour
AMPS	Analogue Mobile Phone Systems
TACS	Total Access Communication Systems
NMT	Nordic Mobile Telephone
D-AMPS	Digital AMPS
TDMA	Time Division Multiple Access
VPN	Virtual Private Networks
PCS	Personal Communication Services
FDMA	Frequency Division Multiple Access
PDC	Personal Digital Communications
GPRS	General Packet Radio Service
EDGE	Enhanced Data rate for Global Evolution
HSCSD	High Speed Circuit Switched Data
MMS	Mobile Multimedia Service
1G	First Generation Cellular Networks
2G	Second Generation Cellular Networks
2.5G	2.5 Generation Cellular Networks
3G	Third Generation Cellular Networks
ITU	International Telecommunications Union
RTT	Radio Transmission Technology
FPLMTS	Future Public Land Mobile Telecommunications Systems
DECT	Digital Enhanced Cordless Telephony
WCDMA	Wideband CDMA
CDG	CDMA Development Group

FDD	Frequency Division Duplex
UTRA	UMTS Terrestrial Radio Access
TDD	Time Division Duplex
UWC	Universal Wireless Communications
DECT	Digital Enhanced Cordless Telephony
3GPP	Third Generation Partnership Project
QoS	Quality of Service
SIR	Signal to Interference
BS	Base Station
MS	Mobile Subscriber
BBS	Best BS selection

TABLE OF SYMBOLS AND NOTATION

Chapter 2

i	Subscript variable used to refer to any parameter related to user i
N_u	Number of users currently being served
N_{max}	Maximum number of users
RES_i	Resource consumed by user i
RES_{new}	Resource request by new user
RES_{th}	Resource threshold

Chapter 3

$h_{k,G}$	Path gain between user k and BS G
$r_{k,G}$	Distance of user k to BS G
R	Radius of the hexagon
σ	Standard deviation for $\zeta_{k,G}$
$\zeta_{k,G}$	Decibel attenuation of user k to BS G

Chapter 4

$E(\ldots)$	Expectation of …
$Var(\ldots)$	Variance of …
$\ldots\|(r,\theta)$	… conditioned upon position (r,θ)
$\ldots\|BBS$	… conditioned upon event BBS
$f_{\ldots}(\ldots)$	*pdf* of …
$\underline{\ldots}$	Vector of …
$\Pr(\ldots)$	Probability of … occurring
$m_{\ldots}$	First moment of …
$\psi_{\ldots}$	Second moment of …

l	Reference variable used to refer to any parameter related to user l
$(E_b/N_0) = \gamma_l$	Bit energy to noise density ratio for user l
W	Bandwidth
G	Serving or reference BS
R_l	Bitrate of user l
$h_{l,G}$	Path gain between user l and BS G
ρ_l	Orthogonality factor for user l
N_0	Thermal noise power density
α_l	Activity factor for user l
K	Number of interfering BSs other than BS G
P_l	Required transmit power for user l
$P_{tot,G}$	Total transmitted power from the BS G
N_G	Total number of users in the reference BS G
$f_{l,G}$	Ratio of intercell interference to approximate intracell interference of user l to BS G
η_{DL}	Downlink current system load
$\eta_{DL,l}$	Load per user l
$\eta_{DL,max}$	Maximum system load
$P_{\max}$	Maximum BS power
C	Constant for maximum system load
$f_{pl,l}$	Other cell interference factor plus path loss for user l
$\hat{\xi}_l$	MS shadowing component for user l
ξ_G	BS shadowing component for BS G
(r,θ)	Polar co-ordinates of user l with respect to BS G

Chapter 5

$E^{v}_{Nv,Nd}$	Probability of accepting a new voice call
$E^{d}_{Nv,Nd}$	Probability of accepting a new data call
$S = (j,k)$	System state as a function of (j,k)
λ	Arrival rate
μ	Service rate
$\underline{\pi}$	Steady state distribution

CHAPTER 1

INTRODUCTION

1.1 Cellular Networks

Over time cellular networks have evolved from simple voice only fixed capacity networks to limitless applications of streaming data with the currently available CDMA (Code Division Multiple Access) networks. It is useful to examine some of the past generations of cellular technology to fully understand how far we have actually progressed with the currently available cellular networks.

When we speak of generation's we simply refers to a group of technologies that have certain fundamental characteristics in common. Below we examine each generation in a little detail and give some examples of each commercial technology.

1.1.1 First Generation Cellular Networks (1G)

Let us consider the 1st generation technologies. These were the first systems ever and were characterised by being analogue, circuit switched systems and carried predominately voice and little or no data services. Examples of these systems are:

Analogue Mobile Phone Systems (AMPS)

AMPS is the original standard specification for analogue mobile telephony systems. It was widely used in North America, Latin America, Eastern Europe, Australia and parts of Russia and Asia.

To optimise the use of transmission frequencies, AMPS divides geographic areas into cells. Each connection uses its own dedicated frequency - of which there are about 1,000 per cell. Two cells can use the same frequency for different connections so long as the cells are not adjacent to each other.

Total Access Communication Systems (TACS)

TACS is a mobile telephone standard originally used in Britain for the 900 MHz frequency band. The standard operates on the 900 MHz frequency band, allowing up to 1320 channels using 25 kHz channel spacing. It is designed with maximum consideration for very high subscriber densities in large urban areas as well as for sparsely populated rural area.

Nordic Mobile Telephone (NMT)

NMT is the common Nordic standard for analogue mobile telephony as established by the telecommunications administrations in Sweden, Norway, Finland and Denmark in the early 1980s. NMT systems have also been installed in some European countries, including parts of Russia, and in the Middle East and Asia.

1.1.2 Second Generation Cellular Networks (2G)

Second generations systems use a digital air interface technology yet are still circuit switched and offer voice as well as limited data facilities with data rates up to 9.6 kbps. Examples of these systems are:

Digital AMPS (D-AMPS)

This is the original American standard for digital mobile telephony used primarily in North America, Latin America, Australia and parts of Russia and Asia. D-AMPS is the digital evolution of the AMPS analogue mobile telephony system and is now known as TDMA (Time Division Multiple Access). In TDMA, the frequency band is split into a number of channels, which are stacked into short time units, so that several calls can share a single channel without interfering with one another. TDMA is used by the GSM (Global System for Mobile Communications) digital mobile standard.

TDMA is based on the IS-136 standard. It is one of the world's most widely deployed digital wireless systems. It provides a natural evolutionary path for analogue AMPS networks, offers efficient coverage and is well suited to emerging applications, such as wireless virtual private networks (VPN), and is the ideal platform for PCS (Personal Communication Services).

Global System for Mobile Communications (GSM)

GSM, which was first introduced in 1991, is one of the leading digital cellular systems. It uses a combination of FDMA (Frequency Division Multiple Access) and TDMA. Eight simultaneous calls can occupy the same radio frequency. GSM provides integrated voice mail, high-speed data, fax, paging and short message services capabilities, as well as secure communications. It offers the best voice quality of any current digital wireless standard and data rates up to 9.6kbps.

Originally a European standard for digital mobile telephony, GSM has become the world's most widely used mobile system in use in over 100 countries. GSM networks operate on the 900 MHz and 1800 MHz waveband in Europe, Asia and Australia, parts of Africa and on the 1900 MHz waveband in North America and in parts of Latin America and Africa.

CDMA Digital Cellular (IS-95, cdmaOne)

CDMA is an IS-95 based digital technology for delivering mobile telephone services. CDMA systems have been in commercial operation since 1995, and these systems now support over 95 million subscribers worldwide. CDMA networks operate in the 800 and 1900 MHz frequency bands with primary markets in the Americas and Asia. IS-95 CDMA technology provides for voice and data services up to speeds of 64 kbps.

IS-95 CDMA systems are marketed using the name cdmaOne. The next evolutionary step for 3G services is CDMA2000, or IS-2000.

Personal Digital Communications (PDC)

PDC is a Japanese standard for digital mobile telephony in the 800 MHz and 1500 MHz bands.

1.1.3 2.5 Generation Cellular Networks (2.5G)

The move to packet switched systems brought about the coining of the term 2.5 generation since these systems were not entirely new systems but rather enhancements of existing systems. Examples of these systems are GPRS (General Packet Radio Service), EDGE (Enhanced Data rate for Global Evolution) and HSCSD (High Speed Circuit Switched Data) which have been successfully deployed in South Africa. Much higher data rates are available with 2.5 generation systems, up to 115 kbps allowing for enhanced services such as MMS (Mobile Multimedia Service). Examples of these systems are

General Packet Radio Service (GPRS)

GPRS is a packet switched technology that enables high-speed (115 kbps) wireless Internet and other data communications. GPRS will offer a tenfold increase in data throughput rates, from 9.6kbit/s to 115kbit/s. GPRS uses a packet data service, which subscribers are always connected and always on line so services will be easy and quick to access. GPRS is an evolution path for GSM and other TDMA systems.

GPRS is implemented by adding new packet data nodes and upgrading existing nodes to provide a routing path for packet data between the mobile terminal and a gateway node. The gateway node will provide interworking with external packet data networks for access to the Internet and intranets.

Enhanced Data rate for Global Evolution (EDGE)

EDGE is a technology that gives GSM the capacity to handle services for the third generation of mobile telephony. EDGE introduces a new modulation technique along with improvements in the radio protocol that allow operators to use existing GSM frequency spectrums 800, 900, 1800 and 1900 MHz, more effectively. EDGE uses the same TDMA frame structure, logic channel and 200 kHz carrier bandwidth as today's GSM networks, which allows existing cell plans to remain intact. EDGE was developed to enable the transmission of large amounts of data at a high speed. EDGE supports data, multimedia services and applications at up to 384 Kbps. As EDGE progresses to coexistence with 3G WCDMA, data rates of up to 2 Mbps could be available.

High Speed Circuit Switched Data (HSCSD)

High Speed Circuit Switched Data (HSCSD) is a high speed implementation of GSM data techniques. It will enable users to access the Internet and other data services via the GSM network at considerably higher data rates than at present. HSCSD allows wireless data to be transmitted at 38.4 kilobits per second or even faster over GSM networks by allocating up to eight time slots to a single user.

Current data services over GSM generally allow transferring files or data and sending faxes at 9.6 kbps. With HSCSD the user will find wireless connection to the Internet much faster at 38.4 kbps, which is up to four times faster than standard GSM usage. HSCSD is especially well suited for time sensitive, real-time services. Examples could be transferring of large files with specified Quality of Service or video surveillance. Theoretical data rate of 57.6 kbps may be achieved by introducing new 14.4 kbps data coding.

1.1.4 Third Generation Cellular Networks (3G)

These types of networks offer wide area coverage up to 384kbps and local area coverage up to 2Mbps. The full range of data, voice and data services including audio, video and internet browsing is available.

The idea behind 3G is to unify the different standards that today's second generation wireless networks use. Instead of different network types being adopted in America, Europe and Japan, the plan is for a single network standard to be agreed and implemented.

In 1998, the International Telecommunications Union (ITU) called for Radio Transmission Technology (RTT) proposals for IMT-2000 (originally called Future Public Land Mobile Telecommunications Systems (FPLMTS)), the formal name for the Third Generation standard. Many different proposals were submitted; the DECT (Digital Enhanced Cordless Telephony) and TDMA/Universal Wireless Communications organisations submitted plans for the RTT to be TDMA-based, whilst all other proposals for non-satellite based solutions were based on CDMA. The main submissions were called Wideband CDMA (WCDMA) and cdma2000. The GSM players including infrastructure vendors such as Nokia and Ericsson backed WCDMA.

The North American CDMA community led by the CDMA Development Group (CDG) including infrastructure vendors such as Qualcomm and Lucent Technologies backed cdma2000.

The final recommendations by the ITU for IMT2000 stipulated the following five terrestrial Radio Interface Standards:

1. CDMA Direct Spread, known as Wideband CDMA-FDD (Frequency Division Duplex) WCDMA-FDD as well as UTRA (UMTS Terrestrial Radio Access) FDD

2. CDMA Multi-carrier known as CDMA 2000 or IS 2000

3. CDMA TDD Time Division Duplex known as WDCDMA-TDD , TD-SCDMA, UTRA (UMTS Terrestrial Radio Access) TDD

4. TDMA Single carrier known as EDGE, Enhanced Data Rates for GSM Evolution, as well as UWC, Universal Wireless Communications 136

5. TDMA Multi carrier known as DECT, Digital Enhanced Cordless Telephony

The following are two of the more prominent radio transmission standards approved for 3G systems under the IMT-2000 initiative and are commercially available across the world.

Wideband CDMA (W- CDMA)

W-CDMA, also known as CDMA Direct Sequence, is a 3G radio transmission technology favoured by Europe. It can be built upon existing GSM networks and represents the obvious next step for current system operators. As such, it is expected to gain widespread acceptance in most of the world, where GSM systems are prevalent.

cdma2000

Also called CDMA Multi-Carrier, cdma2000 is a 3G standard developed by the CDMA Development Group (CDG) and favoured by the U.S. It is derived from the narrowband cdmaOne digital standard and provides a clear evolutionary path for existing cdmaOne operators.

3G Data rates

The ITU has laid down some indicative minimum requirements for the data speeds that the IMT-2000 standards must support. These requirements are defined according to the degree of mobility involved when the 3G call is being made. As such, the data rate that will be available over 3G will depend upon the environment the call is being made in:

High mobility (144 kbps for rural outdoor mobile use) - This data rate is available for environments in which the 3G user is travelling more than 120 kph in outdoor environments.

Full mobility (384 kbps) - This data rate is for users travelling less than 120 kph in urban outdoor environments.

Limited mobility (At least 2 Mbps) - This data rate is with low mobility (less than 10 kph) in stationary indoor and short range outdoor environments. These kinds of maximum data rates that are often talked about when illustrating the potential for 3G technology will only therefore be available in stationary indoor environments.

UMTS (Universal Mobile Telecommunication System)

UMTS is the term used in Europe for 3G networks and is intended to make the transition from the 2nd generation networks smoother and eventually to replace them. This means that UMTS will, in the long term, support all applications current served by the 2nd generation cellular systems such as GSM and PDC [Ericsson 2006].

1.2 Admission Control in Cellular Networks

We focus our attention on admission control in the air interface as this is typically where the bottleneck occurs. The air interface is a finite resource and if the loading is allowed to increase excessively, the coverage area is reduced below the planned values and the quality of service of the existing connections cannot be guaranteed. The purpose of admission control is to ensure that the admission of a new user will not sacrifice the planned coverage area as well as the quality of the existing connections. The output of admission control is basically to accept or reject a new request to establish a radio link on the radio interface. The admission control algorithm needs to estimate the increased load of a new request on the radio interface and this must be done separately for the uplink and downlink. In fixed or hard capacity networks the admission control problem is rudimentary however with CDMA type networks it becomes somewhat challenging. CDMA networks are characterised by soft capacity in the sense that the capacity fluctuates with the number of users, each users bitrate, distance and other factors. In addition we have multiple classes or service each requiring specific bitrates as well as priorities between them which further compounds the problem. In Chapter 2 we elaborate on admission control.

1.3 Teletraffic Performance Analysis

Teletraffic theory is defined as the application of probability theory to the solutions of problems concerning planning, performance evaluation, operation and maintenance of telecommunication systems. More generally, teletraffic theory can be viewed as a discipline of planning where the tools of stochastic processes, queuing theory and numerical simulation are taken from the disciplines of operations research. Most of the detail in this section is taken from [Iverson V 2002] and [Leijon H].

The objective of teletraffic theory can be formulated as follows: to make the traffic measurable in well defined units through mathematical models and to derive the relationship between the quality of service and system capacity in such a way that the theory becomes a tool by which investments can be planned.

The task of teletraffic theory is to design systems based on the future demand and the capacity of the system elements within certain constraints. These design constraints are typically to be as cost effective as possible whilst maintaining a predefined quality of service. It is also the task to specify methods for controlling that the actual grade of service is fulfilling its requirements and to also specify emergency actions when systems are overloaded or technical faults occur. This requires methods for forecasting the demand, methods for calculating the capacity of the systems and specifications of quantitative measures for the grade of service.

1.3.1 Mathematical Modelling

Teletraffic theory consists of the mathematical modelling of a telecommunications system or some part of it, and its behaviour when demands are made on it or by it. The task of teletraffic theory is to configure optimal systems from knowledge of user requirements and habits. All such theory is a modelling exercise as we cannot set up and examine a truly identical system, and so instead we construct a hypothetical, simplified version, with well-defined inputs, and analyse that. The validity and usefulness of the theory we develop therefore rests entirely upon the answer to the question of how satisfactory is the model? If we have little confidence in that, then no matter how sophisticated our mathematics we can have little confidence in the final results of the theory.

In order to set up our model, we must take careful consideration of a number of points:

1) What part, exactly, of a system are we interested in? Can we separate out the relevant section, and look at this in isolation; or must we treat it all at once?

2) What is the precise technical behaviour of the section we have decided on, in terms of operating times, limitations on access, dead times, detailed response to a demand, etc.?

3) How does the input stream of demands on the system behave?

4) What information do we want from our model, and how accurate must it be?

The points are all mutually dependent. It is perhaps easier to start with the incoming demand stream, and enquire how this is structured, which will itself involve some assumptions.

Typically, this stream of demands is governed very largely by chance, so that the methods of stochastic processes will be appropriate.

It appears to split the description of the traffic properties into stochastic process for arrival of call attempts and processes describing service (holding) times. These two processes are assumed to be mutually independent, meaning that the duration of a call is independent of the time the call arrived. We then need to know:

a) What (in probabilistic language) are the arrival processes of demands? and

b) What is the distribution of the work (holding times) that they bring?

The description of the arrival process may require more or less detail. In a low congestion system fresh traffic is offered by a multitude of independent subscribers, we may assume with high accuracy that, at any rate over not too long periods of time, the arrivals are pure chance i.e. that they form a Poisson process. If we are examining a system offered overflow traffic, a more complex description will be needed; and more complex still if we expect a significant proportion of repeat attempts.

Now consider the second point viz. the distribution of the work brought by a single demand. Indeed, this may have considerable qualitative variation so that if we are concerned with circuit occupancy, the "work" consists of a single continuous holding-time; whereas if we are modelling the common control of a complex processor system, it may be a sequence of disjoint tasks, of widely different types and duration. Two particularly common and important cases are however where the holding-time has a negative-exponential distribution, and where it is effectively deterministic i.e. a constant.

The following figure describes the terminology normally used. The term arrival and call are used synonymously. The inter-arrival time is the time between arrivals.

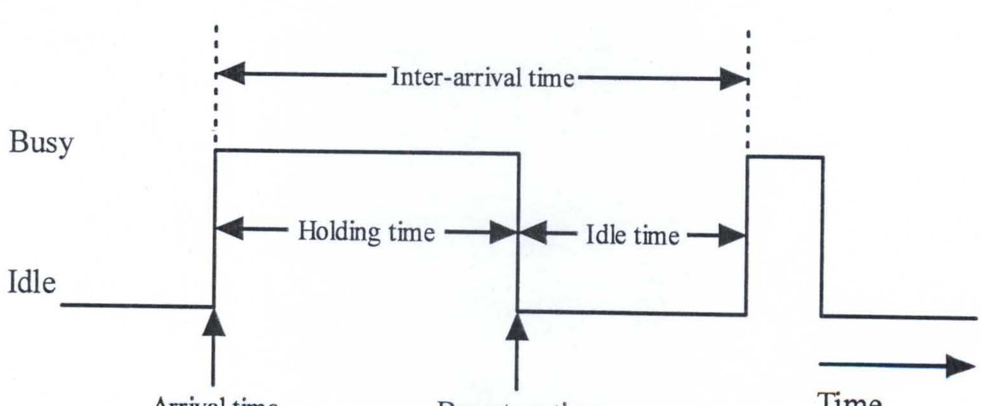


Figure 1-1 Terminology normally used for traffic processes [Iverson V 2002]

We can now turn our attention to point (4) above i.e. what information do we want from the model. This naturally requires at any rate some knowledge of the system behaviour (are blocked calls lost, or do they queue, for instance), and an understanding of the nature of the input process. Since that input is stochastic, the output from our model will be probabilistic, and may consist of probabilities of loss, mean delays, percentiles of processor occupancy or similar quantities. It might even be that this is only the first stage in the construction of a more complex model, in which case we may need to know complete details of the distributions of delays, of overflowing calls, or of some other quantity which affects the rest of the system. At this stage point (1) is considered.

We are now ready to attend to the details of the system engineering, point (2) above and it is at this stage that our mathematical model takes shape, and it becomes clear whether we have any hope of an analytic treatment. Finally, the whole process is repeated, until we have confidence that it is consistent and that the behaviour of the system is indeed compatible with the assumptions on the incoming demand stream and vice versa. We assume then that a model can be set up. It will necessarily be approximate in some way or other, and we must estimate how much effect this approximation will have on the results of model. If the answer is too much effect, we must reshape the model. The ultimate requirement is always for a number or set of numbers or a numerical calculation so useful tradeoffs can often be made between approximate models and approximate calculations. It may in fact not be possible to specify the input data of subscriber behaviour in as much detail as it needed, because the relevant quantities are unknown or even immeasurable. In such circumstances, the approaches are reasonable and to decide what ***The Reasonable Man*** should do and make a decision that, that is what the system shall be dimensioned to, or to analyse several models, differing only in the subscriber behaviour, and present a range of results for final decision making on other grounds. Fortunately, most quantities of interest as output from the mathematical models are remarkably robust with respect to variations in the input processes (provided, of course, that certain critical parameters like the overall traffic offered are kept constant), and so even quite simple models give remarkably accurate and useful results. It should never be forgotten however, that no matter how elaborate the mathematical solution, a bad model means unreliable results

1.4 Dissertation Outline

This dissertation focuses on downlink call admission control aspects in mixed service CDMA cellular networks. Our objective is to present an analytical model that can successfully be used in teletraffic performance analysis. This chapter introduced the different types of cellular networks and what characterises each generation. We also touched on admission control in cellular networks and finally we examined the basic principles of teletraffic modelling and performance analysis. We then discussed the outline of this dissertation followed by a section on original contributions in this dissertation.

Chapter 2 delves into a literature review of call admission control in CDMA cellular networks starting with the basic principles of admission control. We also cover the different classes of admission policies and lastly justify our choice of a downlink load-based admission control algorithm.

The simulation model is examined in detail in Chapter 3. We begin with basic principles of simulation modelling and then describe the basic design of our software simulator for a generalised call admission policy. We incorporate a wrap around technique for the basic cellular structure such that cells are modelled so as to prevent them from being lost to the network. We then describe the statistical distributions that we have assumed for the propagation gain followed by some of our assumptions for the simulation. Finally we describe a high level flow diagram of the entire simulation where the main routine is split into several subroutines where each subroutine performs a task specific to a particular event. A basic flow diagram and pseudo code for each major subroutine is included.

In Chapter 4 we introduce our load-based call admission control policy for downlink mixed service CDMA. For simplicity and ease of understanding we consider a single class of service to define our admission policy and then in the following chapter we extend these results to two classes of service, voice and data. We define our admission constraint or condition based on the individual user load, system load and maximum system load and derive analytical expressions for these. For our analytical model we need to statistically characterise the other cell interference plus path loss in order to probabilistically express the admission condition. We proceed to determine the first and second moment of the other cell interference plus path loss and then resort to Wilkinson's approximation to obtain a closed form expression. We incorporate into our analysis the best BS selection criteria where the user connects to the BS with the strongest path gain.

Chapter 5 extends our load-based call admission policy introduced in the previous chapter to two service classes, viz. voice and data and develops a mathematical model for this system. rate and holding time We introduce the concept of prioritising voice calls over data calls by using a resource reservation factor. Using the analytical expressions for the other cell interference plus path loss, with Wilkinson's approximation and other assumptions, we derive expressions for the acceptance probabilities for both voice and data calls. We then model the system with a two-dimensional Birth-Death Markov model. The effects of the time varying capacity are incorporated into the arrival process through the acceptance probabilities such that our model reduces to a $G/M/\infty$ system. From our model we determine the steady state distribution for this system and from this determine the blocking probabilities and mean number of calls in the system, for both voice and data calls, as a function of arrival rates. We lastly compare our analytical and simulation results.

In Chapter 6 we summarise this dissertation and briefly discuss each chapter. We also discuss some future directions for this work.

1.5 Original Contributions

In Chapter 5 we perform a teletraffic performance analysis for a proposed multi service downlink load-based call admission control policy. We use two classes of service viz. voice and data and our analysis yields the blocking probability as well as the mean number of calls. In addition we introduce prioritisation between classes by the introduction of a resource reservation factor as shown in [Narrainen 1999].

The above contribution utilises the statistical characterisation of the downlink other cell interference plus path loss that we introduced in Chapter 4 based on [Cho J and D. Hong, 2002]. Based on Wilkinson's approximation we incorporate the best BS selection criteria into the above analysis and although numerical computation is used the end result is an expression in closed form.

The following publications have resulted from this work or as an extension thereof:

[Ramlakhan, N. B. & Takawira, F. 2002] "Prioritised Call Admission Control for 3rd Generation Cellular Networks" *Proceedings of SATNAC (South African Telecommunications and Network Applications Conference) 2002*

[Ramlakhan, N. B. & Takawira, F. 2003] "Outage Analysis of a Downlink Load-Based Admission Policy" *Proceedings of SATNAC (South African Telecommunications and Network Applications Conference) 2003*

CHAPTER 2

CALL ADMISSION CONTROL IN CDMA CELLULAR NETWORKS

2.1 Introduction

Call admission control is essentially deciding whether to admit a new user to the system given certain constraints on the system. The basic idea is to make some estimate of the existing limited resource and then consider whether sufficient resources are available to accommodate a new request. Unlike FDMA/TDMA systems with hard capacity, CDMA systems, have a soft interference limited capacity. This means that the capacity in a CDMA system is variable and fluctuates according to many different parameters. Each new call that is added to the system increases the interference level of all other ongoing calls thus affecting the quality of their connection. When we have multiple service classes as well as a priority between them the problem of admission control is further compounded, as now we have to incorporate their priority into the admission procedure. Without admission control the loading of the air interface would increase excessively resulting in the planned coverage area being reduced and the quality of the existing connections being compromised. In this manner there is a trade-off between capacity and quality [Ishikawa and Umeda 1996, 1997]. Before admitting a new user to the system, admission control needs to check that if the user is added to the system, the planned coverage area and quality of the existing connections will not be sacrificed. Admission control will either accept or reject a request from a user. The admission control algorithm estimates the load increase that the new user will cause in the radio network for both the uplink and downlink directions separately. The uplink and downlink can be decoupled, since admission control in radio resource management is made practically independent in the uplink and downlink through using different sets of codes. The uplink and downlink are run on a different basis, as the access in the former is many to one as opposed to one to many in the latter. Admission control is used to effectively achieve high traffic capacity as well as stability of the radio access network [Knutsson et.al. 1997].

UMTS has well defined structures and procedures for implementation of call admission; however the actual admission policy is open to vendor implementation. The following two principles regarding call admission are defined in [3GPP (Third Generation Partnership Project) TR 25.922 2002]. **Principle 1**: Admission Control is performed according to the type of required QoS (Quality of Service). "Type of service" is to be understood as an implementation specific category derived from standardised QoS parameters. **Principle 2**: Admission control is performed according to the current system load and the required service. The call should be blocked if none of the suitable cells can efficiently provide the service required by the user at call set up (i.e. if considering the current load of the suitable cells, the required service is likely to increase the interference level to an unacceptable value). This would ensure that the user avoids wasting power and affecting the quality of other communications. In this case, the network can initiate a re-negotiation of resources of the on-going calls in order to reduce the traffic load. Based on the above two principles we can infer that the CAC must be interference based as well take into account priority of traffic classes.

In a CDMA environment with symmetrical service and equal bandwidths in each direction, the uplink is commonly considered to be the bottleneck. Based on the asymmetric nature of the expected traffic in next generation networks the downlink is envisaged as the future bottleneck.

Many different types of admission control have been suggested and in Section 2.2 we review and discuss some of the more popular choices starting with the basic principles of admission control and in Section 2.3 we briefly justify our choice of a downlink load-based admission policy. In Section 2.4 we summarise this chapter.

2.2 Admission Control Schemes

Admission control in WCDMA (UMTS) is inherently different from systems whose resources are finite and specified. The number of channels per sector is fixed in FDMA and TDMA systems such as GSM. The capacity limit is thus a hard limit and the call admission control has only to take care of the allocation of available channels, time slots, in the case of GSM. CDMA has no hard limits on the maximum capacity, which makes admission control a complex soft capacity management problem [Pollonen 2001].

At the highest level we can define a generic call admission policy using the concept of some finite resource where RES_{th} = resource threshold, RES_i = resource consumed by user i, RES_{new} = resource request by new user and N_u = number of users currently being served. We further assume that a one to one mapping between a user and resource. The generic call admission policy would then be:

$$\text{If } \left(\sum_{i=1}^{N_u} RES_i + RES_{new} \right) < RES_{th} \text{ ACCEPT else BLOCK} \tag{2.1}$$

If the current resource consumption plus the new request is less than the threshold then accept the new user request else if greater than the threshold block the new user request. RES_{th} must take into account reserving some resource for erroneous transmissions as well as fluctuations in usage patterns. The above is a simple policy with no prioritisation or reservation, known as complete sharing. It can easily be extended to differentiate between different classes of users.

2.2.1 Fixed Capacity Based Admission Control

One of the simpler design choices is to consider a "worst case" scenario of a fixed and equal number of calls in each cell or the maximum total bitrate of the cell. In terms of our generic call admission policy above this is equivalent to assuming a fixed threshold of $RES_{th} = N_{max}$ equal to the maximum number of concurrent users and equal resource consumption per user [Capone 2001]. Equation (2.1) then reduces to:

$$\text{If } (N_u + 1) < N_{max} \text{ ACCEPT else BLOCK} \tag{2.2}$$

Fixed capacity admission control uses an average value of the other cell interference i.e. the ratio of the total intercell interference power to the total intracell interference power, to compute a fixed average CDMA capacity. Intracell interference refers to the interference within the cell and intercell interference refers to the interference between cells [Viterbi 1993], [Gilhousen et al. 1991], [Ishikawa and Umeda 1997], [Holma 1998]. A fairly conservative estimate is in order to ensure a good quality of service. Fixed capacity networks have always been associated with FDMA/TDMA networks and teletraffic modelling of such systems is a mature field. The basic performance analysis method is using an M/M/C queue [Cooper 1981], [Syski 1986]. This shorthand notation for describing a queuing process was mainly developed by [Kendall 1953] and has become the standard method utilised in queuing analysis. The first two letters refer to the arrival and service statistics respectively and the third the cell capacity. In the above representation M refers to an exponential distribution and C is deterministic. This approach however ignores the inherent ability of CDMA systems to cater for a flexible number of users [Narrainen 1999].

2.2.2 Infinite Capacity Based Admission Control

In these types of admission policies, we assume no hard limit on the cell capacity and accept all calls. In terms of our generic call admission policy above this is equivalent to assuming an infinite threshold. Equation (2.1) then reduces to an always accept policy:

$$\text{If } \left(\sum_{i=1}^{N_u} RES_i + RES_{new} \right) < \infty \text{ ACCEPT else BLOCK} \tag{2.3}$$

As the number of calls increases, there is a presumed graceful degradation in communication quality. Infinite capacity admission control was initially analysed by [Fapojuwo 1993] and other attempts that appear in the literature are based on modelling each cell as as independent M/M/∞ queue [Evans and Everitt 1999(a)], [Viterbi and Viterbi 1993], [Evans and Everitt 1999(b)]. This model corresponds to a system where no calls are blocked and no calls are prematurely terminated. Infinite capacity is not really an admission policy and the network performance is poor when the number of users in the network is high [Narrainen 1999].

2.2.3 Soft Capacity Based Admission Control

Soft capacity based admission control take into account the time varying capacity of CDMA networks using the time varying interference present. In terms of our generic call admission policy above this is equivalent to assuming a variable threshold for RES_{th}. Equation (2.1) is then unchanged, bearing in mind RES_{th} will fluctuate.

$$\text{If } \left(\sum_{i=1}^{N_u} RES_i + RES_{new} \right) < RES_{th} \text{ ACCEPT else BLOCK} \tag{2.4}$$

Soft capacity admission control can be further broken down into Interference Based and SIR (Signal to Interference) Based admission control. The fundamental difference is that in the former the total interference in the network is used in the admission decision while in the latter the user's SIR is used.

SIR Based

A SIR based policy was proposed by Liu and Zarki [1994] and here the admission decision is made on an individual basis comparing each user's SIR to a SIR threshold value [Jeon and Jeong 2002], [Hjelm 2000], [Narrainen 1999].

Interference Based

The total interference strategy was first proposed by Huang and Yates [1996]. With this type of admission policy a call is denied admission when the measured total power for a cell exceeds the predetermined threshold. This allows the admission policy to essentially retain the soft capacity feature of CDMA. The interference based approach support many more users than a non-interference based approach as a result of the soft capacity utilisation [Holma 1998]. This effect can be anticipated considering the inherent interference limitation of CDMA. The advantage of the total interference based solution is that all interference is treated equally without any explicit assumptions about the strength of the interference source. The total wideband interference is measured, and the admission control algorithm estimates the load increase that the establishment of a new call would cause. If the new resulting total interference would be unacceptably high, according to a predefined threshold value, the admission request would be denied [Pollonen 2001].

Within the interference based admission control schemes we also have Effective Bandwidth Based admission control [Evans and Everitt 1999(a)]. In this type of scheme each type of call is assigned an effective bandwidth lying somewhere between the mean rate and peak rate. An effective bandwidth equal to the peak rate corresponds to a very conservative admission policy which obtains no benefit from the statistical multiplexing of calls, while an effective bandwidth equal to the mean rate corresponds to the expectation that so many calls will be multiplexed together that the variations, expected in bursty data traffic, will be averaged out to produce a non-bursty aggregate traffic stream. The actual values of effective bandwidth assigned will depend on among other factors the statistical properties of the individual traffic streams. Admission policies based on effective bandwidth are becoming more popular due to the nature of data traffic seen in third generation cellular networks [Wang & Zhuang 2006].

Another class of interference based admission control is power based admission control. The idea here is that the CDMA capacity is strictly related to power limits which prevent the power control mechanism from reaching a new equilibrium when the load is too high. In fact, the power levels are increased by the power control mechanism when interference increases to keep the SIR at the target value and, as a result, the level of power emitted with respect to the limit can be adopted as load indicator in the admission decision [Huang and Yates 1996], [Knutsson et.al. 1998], [Capone 2001]. With the uplink the primary admission control decision criterion is often determined using the total received power at the BS (Base Station) relative to the noise level. This ratio between the total received wideband power and the noise level is often referred to as the noise rise. Similarly, with the downlink the total BS downlink transmission power is used [Holma, H. and Toskala 2002], [Knutsson et.al. 1998]. Partitioning of the received signal power can also be used as an alternative to sharing the signal power across all classes. [Choi et.al 2007].

2.3 Load-based Admission Control

With load based admission control the resource as described in equation (2.1) is the load which can be calculated in different ways. The load factor can be throughput based, where it takes into account among other factors the bitrate or the load factor can be power based as described in the previous paragraph [Holma, H. and Toskala 2002]. The air interface load needs to be measured in an appropriate way if we are to base admission control on the interference levels in the air interface.

In [Sipilä et.al. 2000], [Burley 2001(a)] and [Burley 2001(b)] downlink pole equations are introduced and verified with respect to downlink capacity planning in WCDMA networks. These pole equations are the foundation of load based admission control policies. In WCDMA networks different services demand different capacity as well as different quality. Hence service dependant admission control thresholds should be employed. These service dependant thresholds should preferably depend on load estimates [Dahlman et.al. 1998]. In [Knutsson et.al. 1998] downlink admission decisions based on total received power at the BS was shown to have the best performance compared to other policies.

In this dissertation a power based load-based call admission policy has been chosen for the reasons above and since the maximum load threshold that can be supported varies with the state of the system, thus effectively modelling the behaviour of a CDMA network. Based on the asymmetric nature of the expected traffic in next generation networks the downlink is envisaged as the future bottleneck and this is where the focus of this dissertation lies.

2.4 Summary

In this chapter we explored the basic principle of call admission control in CDMA cellular networks and elaborated on the basic classes of call admission control policies. We introduced the class of load-based call admission control policies and the reasons for focusing on a downlink load-based call admission policy, which we elaborate on in Chapter 4.

CHAPTER 3

SIMULATION MODEL

3.1 Introduction

The network performance of call admission policies can be determined either through analytical or simulation models. A model of a system can be defined as a description that eliminates non-essential detail and captures the essence of the system being described and simulation may be described as a methodology whereby a system is studied by observing the response of its model to artificially generated input [Cooper, 1981 pp. 281]. Our simulation model is based heavily on the model presented by Narrainen [1999] and uses the simulation techniques proposed therein.

A simulation model is a description of the structure of the system under study and attempts to simulate a real world model. Even though we are attempting to model real world behaviour, it is inevitable that certain assumptions must be made. For example, we need to assume some distributions for the arrival statistics, service statistics, and channel fading statistics and so on. The validity of these distributions is questionable and therefore reasonable assumptions must be made. Since a simulation is an experiment in which the response to input data is observed, the output itself will be statistical in nature and will therefore require some statistical analysis for its interpretation. The proper analysis of output data is critical in order to draw general conclusions and gain real insight from the data [Cooper, 1981 pp. 284], [Gross and Harris, 1985 pp 459].

In this chapter we describe the basic design of our software simulator for a generalised call admission policy. We give a basic overview of the different components of our simulator without going into any complicated details. We begin by looking at the basic cellular structure in Section 3.2 where we incorporate a wrap around technique such that cells are modelled so as to prevent them from being lost to the network. In Section 3.3 we then describe the statistical distributions that we have assumed for the propagation gain followed by some of our assumptions for the simulation in Section 3.4. We finally describe a high level flow diagram of the entire simulation in Section 3.5 where the main routine is split into several subroutines. Each subroutine performs a task specific to a particular event. There are various processes or events that can occur during a simulation and these have been defined as a specific task that needs to be performed during the simulation. An example of these events would be a new call arrival, a call departure and so on. We also include the pseudo code for each subroutine. Lastly we summarise this chapter in Section 3.6.

3.2 Cellular Structure

Ideally our network structure would consist of a very large number of cells; however such a system can easily become cumbersome. An alternative approach is to use a smaller number of cells that adequately approximate the larger network. The immediate disadvantage of using fewer cells is the introduction of edge effects. Edge effects occur when a user is at the network boundary where effectively the user does not have any immediate neighbouring cells. To avoid this effect we model the network as a spherical grid of hexagonal cells, much like the shape of a soccer ball. Figure 3-1 illustrates how the network wraps around itself. The network consists of 30 cells with a single BS centrally located within each cell. Each cell is modelled as a hexagonal structure and the cells are numbered from 0 to 29, shaded in grey in Figure 3-1.

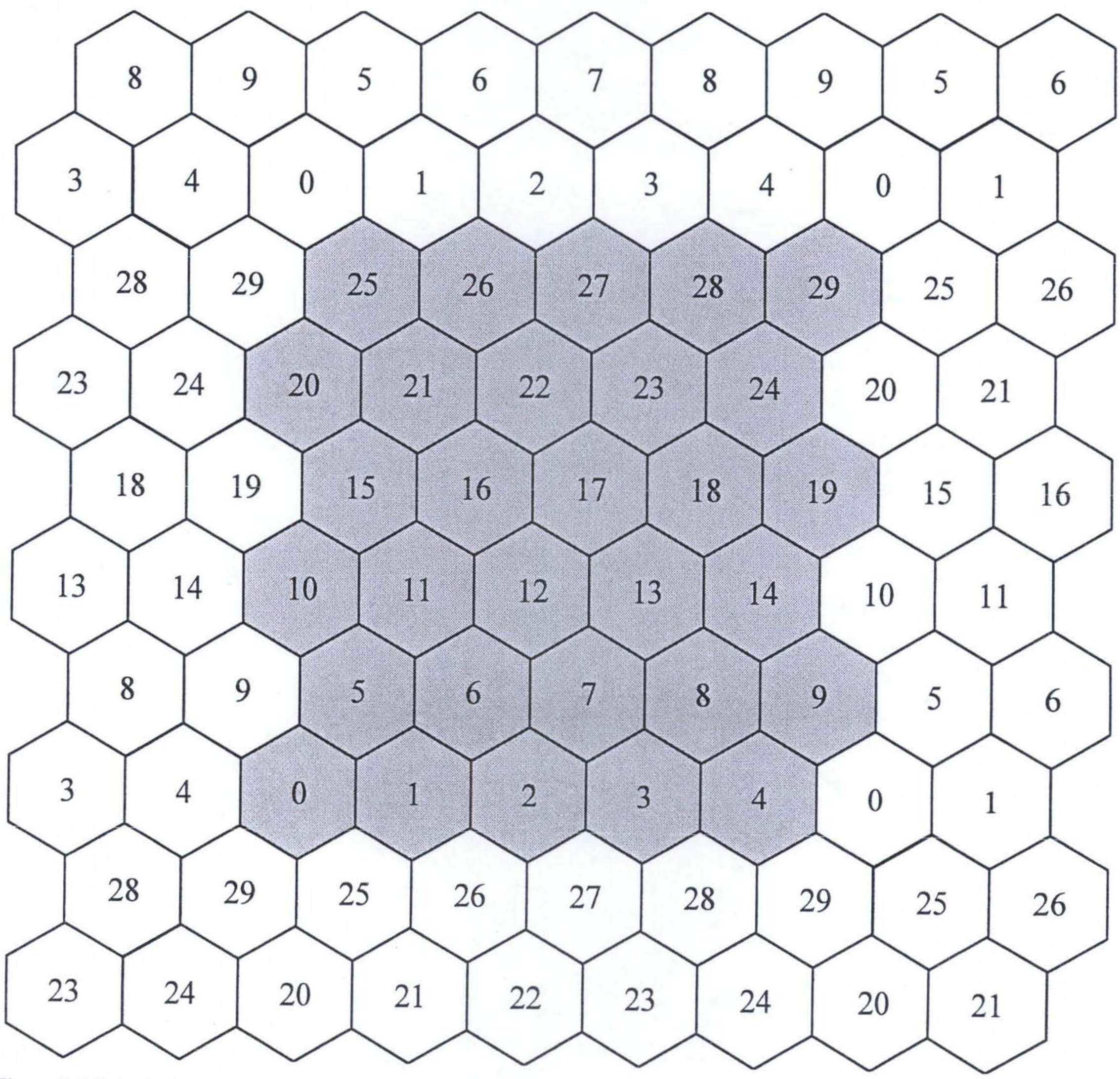


Figure 3-1 Spherical network structure laid out flat

We convert the above network structure to a Cartesian co-ordinate (x,y) system by using the following relationship in Figure 3-2 between the radius of a regular hexagon and its width and height. The regular hexagon is made up of six equilateral triangles.

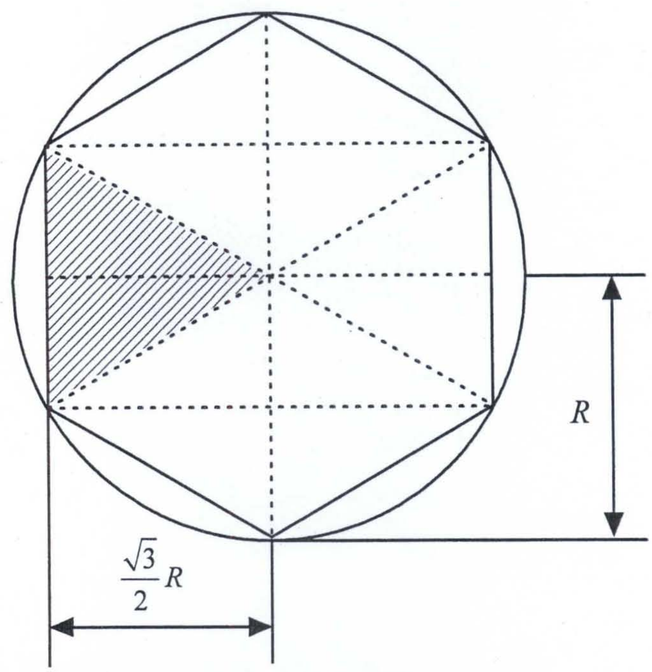


Figure 3-2 Relationship between radius, width and height of a regular hexagon

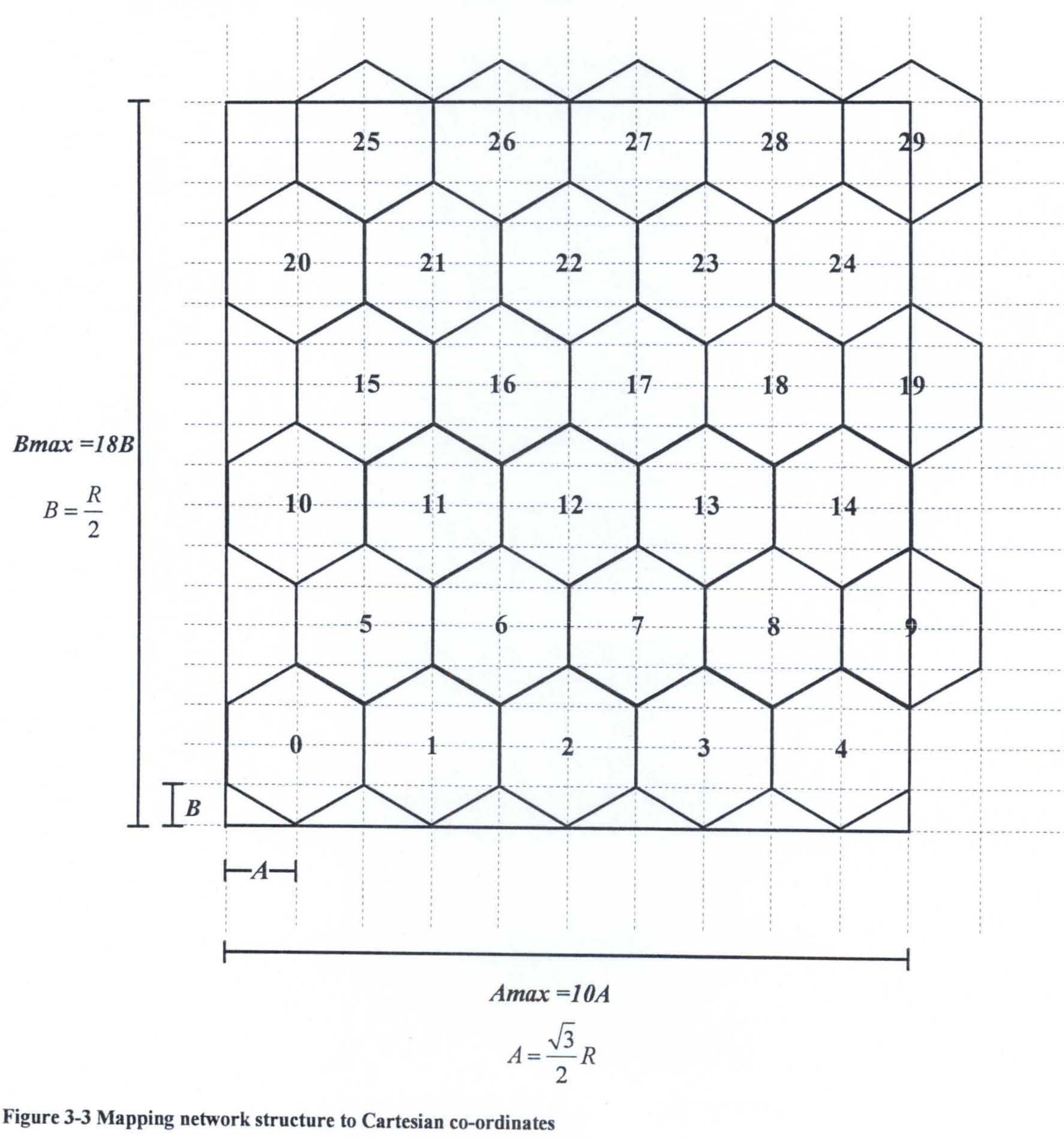


Figure 3-3 Mapping network structure to Cartesian co-ordinates

Using the standard trigonometric ratios we can define the width and height of the hexagon in relation to the radius of the hexagon *R*, which is equal to the length of the side of each equilateral triangle. If we define A and B as shown in Figure 3-3, we can then map the entire 30 cells network structure to the rectangle, and then reference each cell using Cartesian co-ordinates. With the above Cartesian co-ordinate rectangle in Figure 3-3 we can then randomly generate uniformly distributed users across the network and calculate the relative distances to each and every BS.

3.3 Path Gain

Due to shadowing, the mean value of the received signal fluctuates with a log-normal distribution. Shadowing is caused by properties of the terrain such as buildings and hills. These large objects cause diffraction and scattering losses. The effect is a very slow change in the local mean value. Shadowing is typically modelled by a log-normal random variable and consist of two components i.e. path loss due to distance and log-normal shadowing. In a typical scenario, the rate at which path loss due to distance changes is much smaller than that of log-normal shadow fading. Shadow fading has two components: one in the near field of the user, i.e. the MS (Mobile Subscriber)'s shadowing and a component which pertains solely to a link between MS and a particular BS. The BS shadowing component is independent from one BS to another. The MS shadowing component is independent between users.

The received signal power in a given position in a cell falls off with distance according to a power law. The path loss between the user and the BS is proportional to r^m where r is the distance between the user and BS and m is a constant. m ranges from 2 in free space to 5.5 in a very dense urban environment, with a typical value of 4 [Glisic and Vucetic, 1997].

$h_{k,G}$ is the path gain and is generally modelled as the product of the m^{th} power of distance, r, and a lognormal component representing shadowing losses.

$$h_{k,G} = r_{k,G}{}^{-m} 10^{\zeta_{k,G}/10} \tag{3.1}$$

In the above equation $r_{k,G}$ is the distance of user k to BS G, $\zeta_{k,G}$ is the decibel attenuation due to both MS and BS shadowing, and is a Gaussian random variable with zero mean and standard deviation, σ. Usually m is chosen to be 4 and σ^2 is equal to 8dB. The path gain expression is then:

$$h_{k,G} = r_{k,G}{}^{-4} 10^{\zeta_{k,G}/10} \tag{3.2}$$

3.3.1 Greatest Path Gain

A mobile will connect to the BS with which it has the strongest path gain. This strongest path gain may not necessarily come from the closest BS. When determining the strongest path gain, we consider only two tiers of surrounding interfering BSs and assume that outside this range the interference is negligible. In Figure 3-4 below the cell that the user is physically in is indicated by the number 0 and shaded in grey. The first tier of interferers consists of six BSs numbered 1 to 6, indicated by the hatched hexagon and the second tier of interferers consists of twelve BSs numbered 7 to 18, shown in white. This gives us a total of 18 BSs which are taken into account during call admission. Once we have determined the greatest path gain, we have effectively determined which BS we will attempt to admit a new user to.

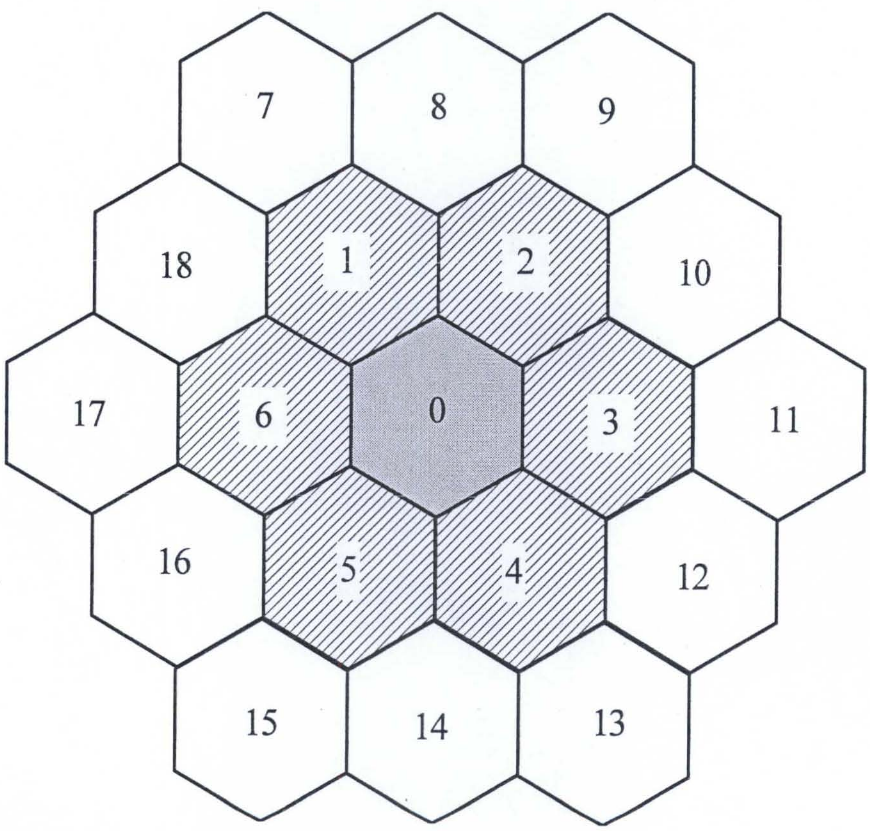


Figure 3-4 Two tiers of interferers

3.4 Assumptions

Path Gain

We assume that no fast fading occurs. We update both MS and BS components of shadow fading in our shadow fading update routine.

Mobility Model

We assume no mobility and thus no handover between cells. We do however model varying conditions using the shadow fading update routine.

Power control

We assume perfect power control per individual MS to BS connection that combats the effect of fast fading. The total BS power is shared amongst all users in the downlink and the maximum BS power limit is considered as part of the admission decision. There is no individual limit on the power per user, but rather on the total BS power.

User distribution

We assume a uniform user distribution across the entire network.

General

- There is a large number of spreading codes available at the base station such that blocking is not limited by the unavailability of spreading codes. We make this assumption since if a code shortage were to occur the system is now hard limited and the ordinary Erlang models apply
- All new arrivals follow a Poisson arrival process
- All call holding times follow an exponential distribution
- The other cell interference and intracell interference can be accurately and instantaneously measured at each MS
- A large number of mobile users is assumed in each cell such that the mean arrival rate is independent of the number of calls in progress
- Blocked calls are cleared and are assumed not to retry access immediately

3.5 Flow Diagram

The main routine is split into several subroutines where each subroutine performs a task specific to a particular event. There are various processes or events that can occur during a simulation and these have been defined as a specific task that needs to be performed during the simulation. These events can occur at specific times, random times, sequentially and may or not be interdependent. The events generally occur relative to a time frame structure where the simulation starts at a reference time, Time_Start and ends at a defined Time_End. There are two possible ways to increment the reference time in a simulator. The first method is to continuously increment the reference time by a fixed infinitesimal amount. Events are then processed whenever they match the reference time. This is known as a *continuous time* simulator. The second method increments the reference time only when the next event occurs. Basically the simulator automatically jumps to the time of the next event. This type of simulator is commonly known as an *event driven* simulator. The one advantage of an event driven simulator is that the simulations are faster for large and complex models and time isn't wasted waiting for the reference time to increment.

Our simulation model is thus event driven and operates on a time frame structure. Our reference time is incremented only when the next event occurs. The choice of Time_End is important as it is necessary for the system to be in steady state when we are capturing statistics. Steady state is defined as a state of statistical equilibrium and in theory can never be reached, only approached asymptotically [Cooper, 1981 pp. 289]. The practical solution is to simply ignore the initial transient events in our simulation and capture statistics after some time referred herein as Time_Steady. This time, Time_Steady must be sufficient that the initial transients have been sufficiently smoothed. To determine the Time_Steady in our simulation we tried arbitrary increasing values of Time_Steady until the statistics of the output data appear to be constant between runs. When the output statistics did not change after the Time_Steady was increased, then that particular Time_Steady was sufficient.

For our random number generation we employed the use of the IMSL(R) C Numerical Libraries Version 2.0 [Visual Numerics, Inc. 1995].

3.5.1 Main Routine

There are four main events that drive this simulation and we examine each event in a bit more detail below. The events that we have are as follows:

- Call arrival process
- Call departure process
- Update shadow fading process
- Steady state process

In addition to the above there are other routines and events that relate to initialising processes, resetting counters and capturing information for statistical analysis. These are basic events and processes that will not be elaborated on herein. At the start of the simulation, we load the network with calls to reach steady state faster. In Figure 3-5 below we describe the main routine. The simulator effectively jumps to the next event and increments they time accordingly as described in the previous section.

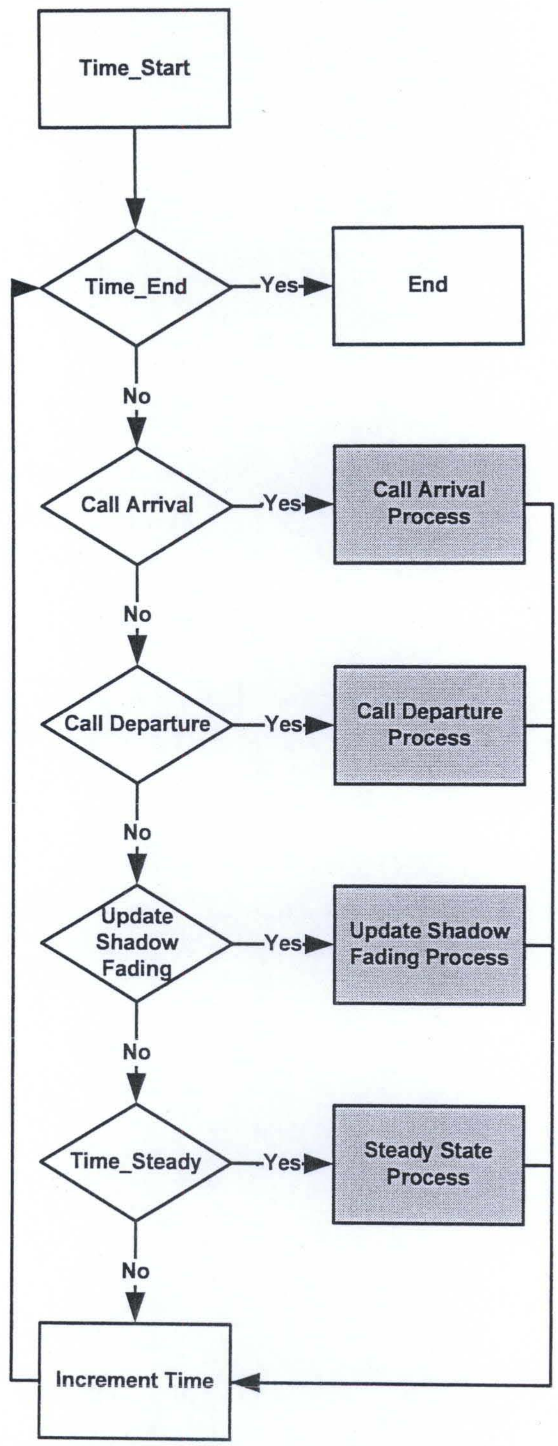


Figure 3-5 High Level Flow Diagram for Main Routine

3.5.2 Call Arrival Process

The basic call arrival process deals with all factors relating to the arrival of a new call as explained in the following flow diagram and pseudo code.

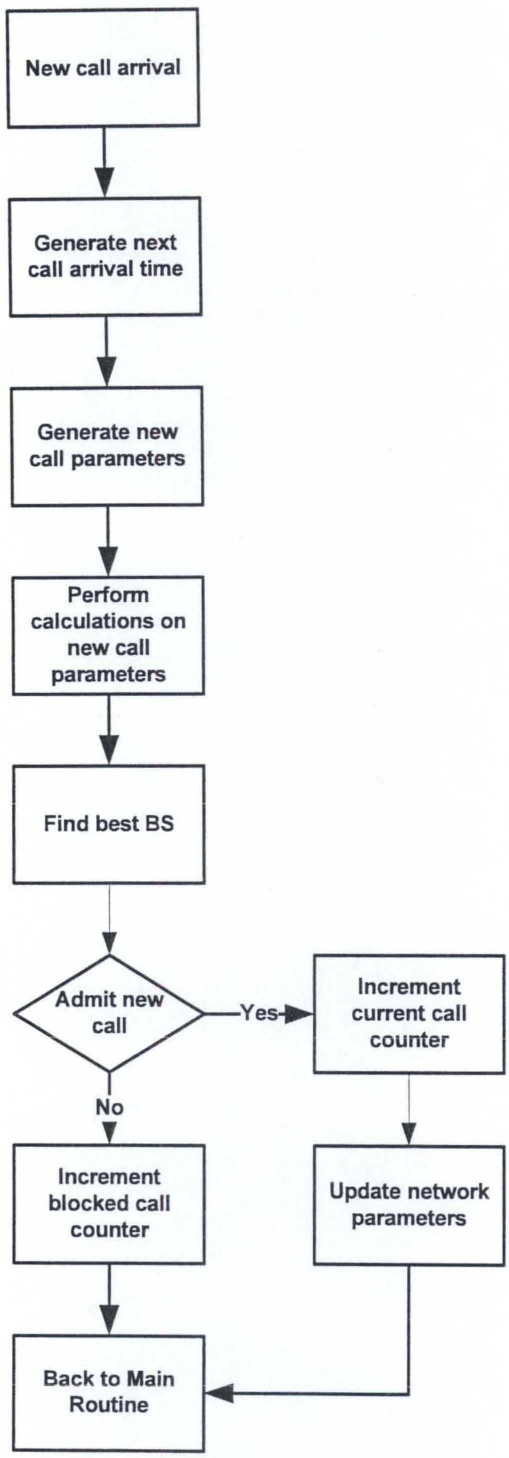


Figure 3-6 High Level Flow Diagram for Call Arrival Process

Pseudo Code

- <u>Generate next new call arrival time</u>

  → Use random number generator and arrival rate

- <u>Generate new call parameters</u>

  → Holding time

→ Position in Cartesian plane *(x,y)*

- Perform calculation on new call parameters

  → Determine which BS new call is in

  → Calculate distance loss component of path gain

  → Generate new call shadow fading parameters

- Find best BS

  → Calculate path gain to two tiers of neighbouring BS

  → Determine best path gain and corresponding best BS to connect to

- Admit new call

  → If admission decision is false

  - Increment blocked call counters

  → If admission decision is true

  - Increment current call counters
  - Update network parameters such as BS power, load, etc

At the end of the call arrival process we return to the main routine where we await the next event.

3.5.3 Call Departure Process

The basic call departure process deals with all factors relating to the departure of a current call as explained in the following flow diagram and pseudo code.

Pseudo Code

- Current call departure

  → Decrement current call counters

  → Update network parameters such as BS power, load, etc

At the end of the call departure process we return to the main routine where we await the next event.

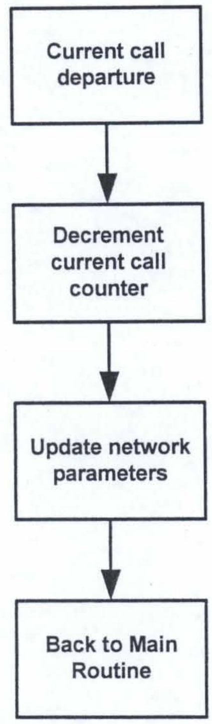


Figure 3-7 High Level Flow Diagram for Call Departure Process

3.5.4 Update Shadow Fading Process

The update shadow fading process cycles through all calls and BS updating the shadow fading parameter as explained in the following flow diagram and pseudo code.

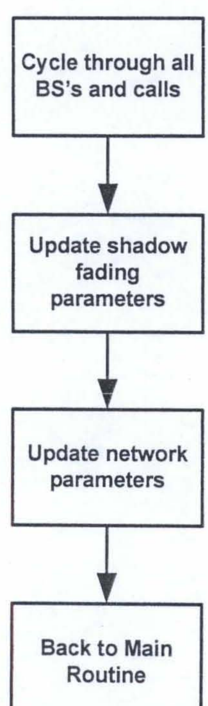


Figure 3-8 High Level Flow Diagram for Update Shadow Fading Process

Pseudo Code

- Update shadow fading parameters

    → Use log-normal random number generator

    → Update network parameters such as BS power, load, etc

At the end of the update shadow fading process we return to the main routine where we await the next event.

3.5.5 Steady State Process

The steady state process simply resets the relevant counters such that only calls occurring after steady state has been reached will be used for reporting.

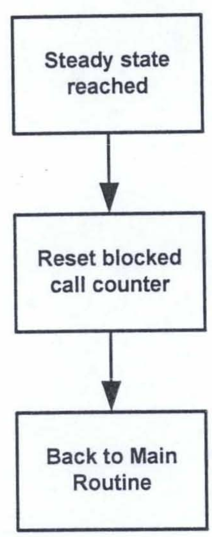


Figure 3-9 High Level Flow Diagram for Steady State Process

At the end of the steady state process we return to the main routine where we await the next event.

3.6 Summary

In this chapter we discussed the basic principles of simulation modelling. We described the basic design of our software simulator for a generalised call admission policy. We began by looking at the basic cellular structure where we incorporate a wrap around technique such that cells are modelled so as to prevent them from being lost to the network. We then described the statistical distributions that we have assumed for the propagation gain followed by some of our assumptions for the simulation. We finally described a high level flow diagram of the entire simulation where the main routine is split into several subroutines. Each subroutine performs a task specific to a particular event. We elaborated with a basic flow diagram and pseudo code for each major subroutine.

CHAPTER 4

LOAD-BASED CALL ADMISSION CONTROL POLICY

4.1 Introduction

In this chapter we introduce our load-based call admission control policy for downlink mixed service CDMA. For simplicity and ease of understanding we consider a single class of service to define our admission policy and then in the following chapter we extend these results to two classes of service, voice and data. A user can only explicitly be a voice call or data call as defined by the voice and data service class respectively. A service class is defined by its bitrate, (E_b/N_0), activity factor, arrival rate and holding time. We define our admission constraint or condition based on the individual user load, system load and maximum system load and derive analytical expressions for these. For our analytical model we need to statistically characterise the other cell interference plus path loss in order to probabilistically express the admission condition. We proceed to determine the first and second moment of the other cell interference plus path loss and then resort to Wilkinson's approximation to obtain a closed form expression. We incorporate into our analysis the best BS selection criteria where the user connects to the BS with the strongest path gain.

This chapter is organised as follows. We first present our load-based admission policy for a single class of service in Section 4.2. In Section 4.3 we statistically characterise the other cell interference plus path loss. We present our simulation and analytical results for the other cell interference plus path loss in Section 4.4 and provide a summary in Section 4.5.

4.2 Load-based Admission Policy

Assuming perfect power control, the average received $(E_b/N_0) = \gamma_l$ for user l is given by equation (4.1) [Sipilä et.al. 2000], where W is the bandwidth, R_l is the bitrate of user l, $h_{l,G}$ represents the path gain from the G^{th} BS to user l, ρ_l is the orthogonality factor and is equal to zero for perfect orthogonality between channels and equal to one for no orthogonality, N_0 is the thermal noise power density, α_l is the activity factor and K is the number of interfering BSs other than the serving BS G. P_l is the required transmit power for user l and $P_{tot,G}$ is the total transmitted power from the serving or reference BS G and similarly $P_{tot,N}$ is the power from neighbouring BSs. Furthermore the total BS power $P_{tot,G}$ is expressed as the sum of all the users required transmit powers within the cell, $P_{tot,G} = \sum_{l=1}^{N_G} P_l$ with N_G = total number of users in the reference cell BS G.

$$\left(E_b/N_0\right)=\gamma_l=\frac{\left(\frac{W}{\left(\alpha_l R_l\right)}\right)P_l h_{l,G}}{\rho_l\left(P_{tot,G}-P_l\right)h_{l,G}+N_0W+\sum_{M=1}^{K}P_{tot,M}h_{l,M}} \tag{4.1}$$

As described in equation (4.1) above γ_l is the bit energy to noise density ratio and can be expressed as follows:

$$\left(E_b/N_0\right)=\gamma_l=\frac{\text{Bit Energy}}{\text{Noise density}}=\frac{\text{Energy per bit}}{\text{Noise power per Hertz}} \tag{4.2}$$

The numerator in (4.1) is the energy per bit and is expressed as the received despread power divided by the average bitrate, taking into account the activity factor. The first term in the denominator is the intracell interference, the second is the thermal noise power and the third term is the intercell interference as described in equation (4.3) below

$$\left(E_b/N_0\right)=\gamma_l=\frac{\overbrace{\left(\frac{W}{\left(\alpha_l R_l\right)}\right)P_l h_{l,G}}^{\text{Energy per bit}}}{\underbrace{\underbrace{\rho_l\left(P_{tot,G}-P_l\right)h_{l,G}}_{\text{Intracell interference}}+\underbrace{N_0W}_{\text{Thermal noise power}}+\underbrace{\sum_{M=1}^{K}P_{tot,M}h_{l,M}}_{\text{Intercell interference}}}_{\text{Noise power per Hertz}}}. \tag{4.3}$$

From (4.3) by cross multiplying and rearranging terms we may express the required transmit power for user l as:

$$P_l=\left(\frac{W}{\left(\gamma_l\alpha_l R_l\right)}+\rho_l\right)^{-1}\left(P_{tot,G}\rho_l+\frac{N_0W}{h_{l,G}}+\frac{\sum_{M=1}^{K}P_{tot,M}h_{l,M}}{h_{l,G}}\right) \tag{4.4}$$

We can then define

$$A_l=\left(\frac{W}{\left(\gamma_l\alpha_l R_l\right)}+\rho_l\right)^{-1}. \tag{4.5}$$

to express the power per user l as:

$$P_l=A_l\left(P_{tot,G}\rho_l+\frac{N_0W}{h_{l,G}}+\frac{\sum_{M=1}^{K}P_{tot,N}h_{l,M}}{h_{l,G}}\right) \tag{4.6}$$

Using this above expression for the power per user we may then express the total BS power equal to the sum of the power of all N users that the BS is transmitting to as:

$$P_{tot,G}=\sum_{l=1}^{N_G}P_l=\sum_{l=1}^{N_G}A_l\left(P_{tot,G}\rho_l+\frac{N_0W}{h_{l,G}}+\frac{\sum_{M=1}^{K}P_{tot,M}h_{l,M}}{h_{l,G}}\right). \tag{4.7}$$

After effectively multiplying the 3rd term of the summation in (4.7) above by 1 i.e. $\left(P_{tot,G}/P_{tot,G}\right)$, we get:

$$P_{tot,G} = \sum_{l=1}^{N_G} A_l \left(P_{tot,G}\rho_l + \frac{N_0 W}{h_{l,G}} + \left(\frac{P_{tot,G}}{P_{tot,G}}\right) \frac{\sum_{M=1}^{K} P_{tot,M} h_{l,M}}{h_{l,G}} \right). \tag{4.8}$$

We define the other cell interference factor, $f_{l,G}$ as the ratio of the intercell interference to the approximate intracell interference as follows:

$$f_{l,G} = \frac{\sum_{M=1}^{K} P_{tot,M} h_{l,M}}{P_{tot,G} h_{l,G}} \tag{4.9}$$

The intercell interference i.e. the numerator in the above expression is the total received power for user l. The denominator is the approximate intracell interference since it is the total BS power and includes the power transmission for user l despite the fact that this power is not interference. We assume that this power is negligible compared to the total BS power.

Using our newly defined $f_{l,G}$ (4.9) along with A_l (4.5), we rewrite the total BS power expression, (4.8) as:

$$P_{tot,G} = P_{tot,G} \sum_{l=1}^{N_G} A_l \left(\rho_l + f_{l,G}\right) + \sum_{l=1}^{N_G} A_l \left(\frac{N_0 W}{h_{l,G}}\right). \tag{4.10}$$

We finally express the total BS power as:

$$P_{tot,G} = \frac{\sum_{l=1}^{N_G} A_l \left(\frac{N_0 W}{h_{l,G}}\right)}{\left(1 - \sum_{l=1}^{N_G} A_l \left(\rho_l + f_{l,G}\right)\right)}. \tag{4.11}$$

From the denominator of (4.11) we can define the downlink current system load as:

$$\eta_{DL} \equiv \sum_{l=1}^{N_G} A_l \left(\rho_l + f_{l,G}\right). \tag{4.12}$$

and the load per individual l^{th} user is given by:

$$\eta_{DL,l} = A_l \left(\rho_l + f_{l,G}\right). \tag{4.13}$$

Using (4.12), our unique load expression we can now express the total BS power (4.11) in the familiar pole equation form below:

$$P_{tot,G} = \frac{\sum_{l=1}^{N_G} A_l \left(\frac{N_0 W}{h_{l,G}}\right)}{\left(1 - \eta_{DL}\right)}. \tag{4.14}$$

The pole capacity is achieved when η_{DL} approaches 1 and the required transmit power approaches infinity. In practice however, η_{DL} is kept well below 1 with the downlink transmit power limited to some $P_{\max}$ to ensure stability of the network [Burley 2001(a)], [Burley 2001(b)] [Sipilä et.al. 2000], [Shin et.al 2002].

Using the fact that the total BS power must be less than some maximum BS power, $P_{tot,G} \le P_{\max}$ we can derive a maximum value for the total system load as follows. Using (4.11) and (4.12) we can express the maximum BS power inequality as:

$$\frac{\sum_{l=1}^{N_G} A_l \left(\frac{N_0 W}{h_{l,G}} \right)}{\left(1-\eta_{DL}\right)} \le P_{\max} \tag{4.15}$$

and thus express the maximum system load using $C = \left(N_0 W\right)/P_{\max}$:

$$\eta_{DL,\max} = 1 - \sum_{l=1}^{N_G} A_l \left(\frac{C}{h_{l,G}} \right). \tag{4.16}$$

4.2.1 Admission Condition

At this stage we have an expression for the current system load (4.12) and we have an expression for the maximum system load (4.16). We can now define the following admission constraint that must be satisfied in order to accept the new $\left(N_G+1\right)^{th}$ user. After adding the contribution of a new user to both the total system load and the maximum system load, the inequality below must be satisfied in order to accept this new user,

$$\underbrace{\eta^{N_G+1}{}_{DL}}_{\text{System load after adding new user}} \le \underbrace{\eta^{N_G+1}{}_{DL,\max}}_{\text{Maximum system load after adding new user}} \tag{4.17}$$

where

$$\eta^{N_G+1}{}_{DL} = \underbrace{A_{N_G+1}\left(\rho_{N_G+1} + f_{N_G+1,G}\right)}_{\text{Load of a new user}} + \underbrace{\sum_{l=1}^{N_G} A_l \left(\rho_l + f_{l,G}\right)}_{\text{Current system load}} \tag{4.18}$$

and

$$\eta^{N_G+1}{}_{DL,\max} = \underbrace{1 - \sum_{l=1}^{N_G} A_l \left(\frac{C}{h_{l,G}} \right)}_{\text{Current maximum system load}} - \underbrace{A_{N_G+1} \left(\frac{C}{h_{N_G+1,G}} \right)}_{\text{Contribution of new user}}. \tag{4.19}$$

We can then substitute (4.18) and (4.19) into the inequality in (4.17) to get

$$A_{N_G+1}\left(\rho_{N_G+1} + f_{N_G+1,G}\right) + \sum_{l=1}^{N_G} A_l \left(\rho_l + f_{l,G}\right) \le 1 - \sum_{l=1}^{N_G} A_l \left(\frac{C}{h_{l,G}} \right) - A_{N_G+1} \left(\frac{C}{h_{N_G+1,G}} \right) \tag{4.20}$$

which after some manipulation will yield:

$$\sum_{l=1}^{N_G+1} A_l\left(\rho_l + f_{l,G} + \left(\frac{C}{h_{l,G}}\right)\right) \le 1. \tag{4.21}$$

We then define the other cell interference factor plus path loss for the l^{th} user as

$$f_{pl,l} = f_{l,G} + \left(C/h_{l,G}\right) \tag{4.22}$$

to arrive at the following inequality:

$$\sum_{l=1}^{N_G+1}\left(A_l\rho_l + A_l f_{pl,l}\right) \le 1 \tag{4.23}$$

We have now reduced our admission decision to the above constraint and can determine what we need to statistically characterise.

4.3 Statistical Characterisation of the Admission Condition

Upon examination of equation (4.23) we need to statistically characterise $f_{pl,l}$, the other cell interference factor plus path loss for the l^{th} user. We require the first moment (expectation) and the second moment in its lowest possible form. We can then assume a statistical distribution and proceed to determine the probability of accepting a new call based on the admission constraint. From equation (4.22) we have

$$f_{pl,l} = f_{l,G} + \left(C/h_{l,G}\right) \tag{4.24}$$

In [Cho J and D. Hong, 2002], [Cho J, 2001] the other cell interference, $f_{l,G}$ is statistically characterised taking into account the best BS selection criteria. We follow a similar development.

Any analysis of other cell interference involves comparison of propagation losses among two or more BS's [Viterbi, 1995 pp. 185-186], [Viterbi 1994]. The model must then take into consideration the dependence of the propagation losses to two different BS's from a mobile user. The classical model for the path gain shown in equation (3.2) does not consider the dependence of propagation gains from one point to two or more others. Since the propagation gains in dB are Gaussian, we assume a joint Gaussian probability density for losses to two or more BS's. Equivalently we may express the random component of the path gain as the sum of a component in the near field of the user, $\hat{\xi}_l$ i.e. the mobile's shadowing which is common to all BSs, and a component which pertains solely to a particular BS, ξ_G and is independent from one BS to another. Thus we may express the random component of the dB loss of the l^{th} user to the G^{th} BS as

$$\zeta_{l,G} = a\hat{\xi}_l + b\xi_G \qquad \text{where} \qquad a^2 + b^2 = 1 \tag{4.25}$$

with

$$\begin{aligned} &\mathrm{E}(\zeta_{l,G}) = \mathrm{E}(\hat{\xi}_l) = \mathrm{E}(\xi_G) = 0 \\ &Var(\zeta_{l,G}) = Var(\hat{\xi}_l) = Var(\xi_G) = \sigma^2 \quad \forall G \\ &\mathrm{E}(\hat{\xi}_l \, \xi_G) = 0 \quad \forall G \\ &\mathrm{E}(\xi_G \, \xi_H) = 0 \quad \forall G \neq H \end{aligned} \tag{4.26}$$

It is generally assumed that the near field and the BS specific path gain component have equal standard deviations giving us $a^2 = b^2 = \frac{1}{2}$. The path gain expression is then

$$h_{l,G} = r_{l,G}{}^{-4} 10^{\left(a\hat{\xi}_l + b\xi_G\right)/10} . \tag{4.27}$$

Let us now consider a cellular system where a number of BSs are distributed according to a hexagonal pattern. The user of interest, l, is connected to BS, G at (r,θ) which are the polar co-ordinates of the user with respect to the BS G. We have previously defined the other cell interference factor for the downlink, (4.9) as:

$$f_{l,G} = \frac{\sum_{N=1}^{K} P_{tot,N} h_{l,N}}{P_{tot,G} h_{l,G}} \tag{4.28}$$

By assuming that on average all BS's have equal transmitted powers [Viterbi, 1995], the other cell interference becomes

$$f_{l,G} = \frac{\sum_{N=1}^{K} h_{l,N}}{h_{l,G}} . \tag{4.29}$$

The other cell interference factor plus path loss for the l^{th} user (4.24), can then be expressed as:

$$f_{pl,l} = \frac{C + \sum_{N=1}^{K} h_{l,N}}{h_{l,G}} \tag{4.30}$$

Using our definitions for path gain from equation (4.27) we can write $f_{pl,l}$ as

$$f_{pl,l} = \frac{C + \sum_{N=1}^{K} r_{l,N}{}^{-4} 10^{\left(a\hat{\xi}_l + b\xi_N\right)/10}}{r_{l,G}{}^{-4} 10^{\left(a\hat{\xi}_l + b\xi_G\right)/10}} . \tag{4.31}$$

After simplifying we get:

$$f_{pl,I} = Cr_{I,G}{}^{4}10^{-(a\hat{\xi}_I + b\xi_G)/10} + \sum_{N=1}^{K} \left(\frac{r_{I,G}}{r_{I,N}}\right)^4 10^{b(\xi_N - \xi_G)/10} \tag{4.32}$$

If we define:

$$M_N \equiv T_N 10^{b(\xi_N - \xi_G)/10} \tag{4.33}$$

with

$$T_N \equiv \left(\frac{r_{I,G}}{r_{I,N}}\right)^4 \tag{4.34}$$

we can then express the other cell interference plus path loss as

$$f_{pl,I} = Cr_{I,G}{}^{4}10^{-(a\hat{\xi}_I + b\xi_G)/10} + \sum_{N=1}^{K} M_N \tag{4.35}$$

We can now begin to determine the expectation and second moment of the above expression. The expectation of the sum of random variables is the sum of the expectations irrespective of whether the random variables are independent or not [Yates and Goodman, 1999]. Using this result we can distribute the expectation operator to obtain $m_{fpl|(r,\theta)}$, the first moment of $f_{pl,I}$ at the given position (r,θ) as:

$$\begin{aligned} m_{fpl|(r,\theta)} &\equiv \mathrm{E}\left(f_{pl,I} \middle| (r,\theta)\right) \\ &= \mathrm{E}\left(\left(C r_{I,G}{}^{4}10^{-(a\hat{\xi}_I + b\xi_G)/10} + \sum_{N=1}^{K} M_N\right) \middle| (r,\theta)\right) \\ &= C\,\mathrm{E}\left(\left(r_{I,G}{}^{4}10^{-(a\hat{\xi}_I + b\xi_G)/10}\right) \middle| (r,\theta)\right) + \sum_{N=1}^{K} \mathrm{E}\left(M_N \middle| (r,\theta)\right) \end{aligned} \tag{4.36}$$

and $\psi_{fpl|(r,\theta)}$, the second moment of $f_{pl,I}$ at the given position (r,θ) as:

$$\begin{aligned} \psi_{fpl|(r,\theta)} &\equiv \mathrm{E}\left(\left(f_{pl,I}\right)^2 \middle| (r,\theta)\right) \\ &= \mathrm{E}\left(\left(C r_{I,G}{}^{4}10^{-(a\hat{\xi}_I + b\xi_G)/10} + \sum_{N=1}^{K} M_N\right)^2 \middle| (r,\theta)\right) \end{aligned} \tag{4.37}$$

which after expanding out all the terms we get:

$$\begin{aligned} \psi_{fpl|(r,\theta)} &= \sum_{N=1}^{K} \mathrm{E}\left(M_N{}^2 \middle| (r,\theta)\right) + \sum_{V=1}^{K} \sum_{N=1, N\neq V}^{K} \mathrm{E}\left(M_N M_V \middle| (r,\theta)\right) \\ &+ C^2\,\mathrm{E}\left(\left(r_{I,G}{}^{8}10^{-2(a\hat{\xi}_I + b\xi_G)/10}\right) \middle| (r,\theta)\right) + C\sum_{N=1}^{K} \mathrm{E}\left(\left(M_N r_{I,G}{}^{4}10^{-(a\hat{\xi}_I + b\xi_G)/10}\right) \middle| (r,\theta)\right) \end{aligned} . \tag{4.38}$$

4.3.1 Best BS Selection

We now begin to examine each term in (4.36) and (4.38) in more detail however we must first consider the best BS selection criterion. The received signal quality on the downlink is mainly influenced by the ratio of interference from the connected BS and that from adjacent BS's. Since we have made the assumption of equal BS transmitted powers as described in the previous section, we have removed the dependency on BS power. Thus the only other interference component that remains is the path gain. The best BS selection criterion is where we chose the BS with the greatest path gain and not simply the closest BS. This choice has a considerable effect on the signal quality and thus must be considered in our analysis. The best BS selection criteria means that the inequality $h_{l,G} > h_{l,N}$ must be met for all $N, N \neq G$ BS in (4.36) and (4.38), we describe this as event *BBS*. Recall that BS G is our serving BS. Without considering the inequality, the random variables that represent the BS specific shadowing component, ξ_G are independent with one another and thus their *pdfs* can easily be multiplied to yield a joint *pdf*. The joint *pdf* assuming a Gaussian distribution as per the previous section is given by:

$$\begin{aligned} f_{\underline{\xi}}(\underline{\xi}) &= f_{\xi_1\xi_2\cdots\xi_G\cdots\xi_K}(\xi_1\xi_2\cdots\xi_G\cdots\xi_K) \\ &= \prod_{N=1}^{K} f_{\xi_N}(\xi_N) \\ &= \prod_{N=1}^{K} \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{\xi^2{}_N}{2\sigma^2}\right)} \end{aligned} \tag{4.39}$$

where $\underline{\xi} \equiv \{\xi_1, \xi_2, \ldots \xi_G, \ldots \xi_K\}$. The MS specific shadowing term must also be included in the analysis. Since the near field shadowing component, $\hat{\xi}$ is independent from the BS shadowing component, ξ we can simply multiply the *pdfs* $f_{\hat{\xi}_l}$ and $f_{\underline{\xi}}$ to obtain a joint *pdf*. Using this joint *pdf* and with the inequality $h_{l,G} > h_{l,N} \forall\ N, N \neq G$ (Event *BBS*) for the best BS selection we can begin to compute the expected values in equations (4.36) and (4.38).

4.3.2 First Moment of $f_{pl,l}$ at the Given Position (r,θ)

We begin by determining the expected value of $M_N \mid (r,\theta)$ using the following method. To evaluate the expected value in (4.36) we need to use the following definition from [pp 151, Papoulis 1991]. Consider two random variables x, y and $g(x,y)$ a function of x and y, with $f_{x,y}(x,y)$ being the joint *pdf* of x and y. The expected value of $g(x,y)$ is given by:

$$\mathrm{E}\left(g(x,y)\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{x,y}(x,y)\, dx\, dy\,. \tag{4.40}$$

Using the above methodology and we begin with the last term of (4.36) and expand using (4.33) as well as condition upon event *BBS* to get:

$$\begin{aligned}\mathrm{E}\left(M_N|(r,\theta)\right) &= \int T_N 10^{b(x_N-x_G)/10} f_{\underline{\xi}}\left(\underline{x}|BBS\right)d\underline{x} \\ &= \int T_N 10^{b(x_N-x_G)/10} f_{\underline{\xi}}\left(\underline{x}\middle|\left(h_{I,G}>h_{I,1}\dots h_{I,G}>h_{I,J}\dots h_{I,G}>h_{I,K}\right)_{\forall J,J\neq G}\right)d\underline{x}\end{aligned} \quad (4.41).$$

Using the expression for path gain, (4.27) we may express each of the inequalities of the conditional portion of the *pdf* in the following way:

$$\begin{gathered} h_{I,G} > h_{I,N} \\ r_{I,G}^{-4}10^{\left(\frac{a\hat{\xi}_I+b\xi_G}{10}\right)} > r_{I,N}^{-4}10^{\left(\frac{a\hat{\xi}_I+b\xi_N}{10}\right)} \\ 10^{\left(\frac{a\hat{\xi}_I}{10}\right)}10^{\left(\frac{b\xi_G}{10}\right)} > \left(\frac{r_{I,G}}{r_{I,N}}\right)^4 10^{\left(\frac{a\hat{\xi}_I}{10}\right)}10^{\left(\frac{b\xi_N}{10}\right)} \\ 10^{\left(\frac{b\xi_G}{10}\right)} > \left(\frac{r_{I,G}}{r_{I,N}}\right)^4 10^{\left(\frac{b\xi_N}{10}\right)} \end{gathered} \quad (4.42)$$

Using (4.34) we can then simplify the inequality above in (4.42) to:

$$\begin{gathered} 10^{\left(\frac{b\xi_G}{10}\right)} > T_N 10^{\left(\frac{b\xi_N}{10}\right)} \\ \log 10^{\left(\frac{b\xi_G}{10}\right)} > \log T_N + \log 10^{\left(\frac{b\xi_N}{10}\right)} \\ \left(\frac{b\xi_G}{10}\right) > \log T_N + \left(\frac{b\xi_N}{10}\right) \\ \xi_G > \xi_N + \left(\frac{10}{b}\right)\log T_N \\ \xi_N < \xi_G - \left(\frac{10}{b}\right)\log T_N \end{gathered} \quad (4.43)$$

The conditional *pdf* with best BS selection of BS G can then be represented as:

$$\begin{aligned} f_{\underline{\xi}}\left(\underline{\xi}|BBS\right) &= f_{\underline{\xi}}\left(\underline{\xi}\middle|\left(h_{I,G}>h_{I,1}\dots h_{I,G}>h_{I,J}\dots h_{I,G}>h_{I,K}\right)_{\forall J,J\neq G}\right) \\ &= f_{\underline{\xi}}\left(\underline{\xi}\middle|\left(\xi_1<\xi_G-\left(10/b\right)\log T_1\dots\xi_J<\xi_G-\left(10/b\right)\log T_J\dots\xi_K<\xi_G-\left(10/b\right)\log T_K\right)_{\forall J,J\neq G}\right)\end{aligned} \quad (4.44)$$

To evaluate the conditional expected value in (4.41) we need to use the following definition from [pp 171, Papoulis 1991]. Consider two random variables x, y some event W in sample space of x, y with $\Pr(W)$ being the probability of W occurring and $g(x,y)$ a function of x and y and $f_{x,y}(x,y|W)$ being the joint *pdf* of x and y conditioned upon event W. The expected value of $g(x,y)$ conditioned upon event W is then given by:

$$\mathrm{E}\left(g(x,y)\,|\,W\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{x,y}(x,y\,|\,W)\,dx\,dy\,. \quad (4.45)$$

From [pp. 156,Yates 1999] we have:

$$f_{x,y}(x,y \mid W) = \begin{cases} \dfrac{f_{x,y}(x,y)}{\Pr[W]} & (x,y) \in W \\ 0 & \text{otherwise} \end{cases} \tag{4.46}$$

so the expectation becomes then:

$$\mathrm{E}\left(g(x,y) \mid W\right) = \iint_{(x,y)\in W} g(x,y) \frac{f_{x,y}(x,y)}{\Pr(W)} \, dx \, dy \,. \tag{4.47}$$

Back to our equation (4.41), we have a *pdf* that is conditioned about an event and thus when we evaluate the expectation we must divide by the probability of the event occurring and integrate the product of the joint *pdf* and the function over the values that constitute the sample space of the event. We begin with the probability of choosing the best BS i.e. event BBS.

The probability of event *BBS* occurring is the probability of $h_{j,G} > h_{j,N} \forall\ N, N \neq G$. From (4.43) we can express each inequality above in terms of the BS specific shadowing as shown in (4.48) below.

$$BBS = \left(\xi_1 < \xi_G - \left(10/b\right)\log T_1 \ldots \xi_J < \xi_G - \left(10/b\right)\log T_J \ldots \xi_K < \xi_G - \left(10/b\right)\log T_K\right)_{\forall J, J \neq G} \tag{4.48}$$

In order to determine the probability of this event occurring we must then integrate over all possible values of ξ for all neighbouring BSs as well as the serving BS *G*. We use the general formula for calculating the probability of a value range occurring given its *pdf* as follows:

$$\Pr\left(BBS \middle| (r,\theta)\right) = \int_{-\infty}^{\infty} f_{\xi_G}(x_G) \prod_{J=1, J\neq G}^{K} \left(\int_{-\infty}^{x_G - \left(10/b\right)\log T_J} f_{\xi_J}(x_J)\, dx_J \right) dx_G \,. \tag{4.49}$$

Using the general expression for solving a Gaussian *pdf* as per our earlier assumption for ξ_G:

$$\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}} dx = \left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{x}{\sqrt{2}\sigma}\right)\right) \tag{4.50}$$

as well the Gaussian *pdf* itself we can express (4.49) as:

$$\Pr\left(BBS \middle| (r,\theta)\right) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}} \prod_{J=1}^{K} \left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{bx - 10\log T_J}{\sqrt{2}b\sigma}\right)\right) dx \,. \tag{4.51}$$

We can now rewrite equation (4.41) using (4.47) as:

$$\mathrm{E}\left(M_N \mid (r,\theta)\right) = \int \frac{T_N 10^{b(x_N - x_G)/10} f_{\underline{\xi}}(\underline{x})}{\Pr\left(BBS \middle| (r,\theta)\right)} d\underline{x} \,. \tag{4.52}$$

Expanding out the numerator as in (4.49) we arrive at:

$$E\left(M_N \mid (r,\theta)\right) = T_N \frac{\displaystyle\int_{-\infty}^{\infty}\left(10^{\frac{-bx_G}{10}} f_{\xi_G}(x_G) \int_{-\infty}^{x_G-\left(10/b\right)\log T_N} 10^{\frac{bx_N}{10}} f_{\xi_N}(x_N)\,dx_N \prod_{J=1,J\neq N}^{K}\left(\int_{-\infty}^{x_G-\left(10/b\right)\log T_J} f_{\xi_J}(x_J)\,dx_J\right)\right)dx_G}{\Pr\left(BBS\left|(r,\theta)\right.\right)} \tag{4.53}$$

To illustrate how we solve the above consider the following calculation. Define

$$Y \equiv \int_{-\infty}^{x_G-\left(10/b\right)\log T_N} 10^{\frac{bx_N}{10}} f_{\xi_N}(x_N)\,dx_N \tag{4.54}$$

To compute Y we use a change of variables as well as the error function. The error function is given by [pp.77, Taub & Schilling 1986]:

$$\operatorname{erf}(u) \equiv \frac{2}{\sqrt{\pi}} \int_0^u e^{-y^2}\,dy \tag{4.55}$$

The error function has the value $\operatorname{erf}(0)=0$, $\operatorname{erf}(\infty)=1$ and $\operatorname{erf}(-\infty)=-1$. The complementary error function written as erfc is defined by:

$$\operatorname{erfc}(u) \equiv 1-\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} e^{-y^2}\,dy \tag{4.56}$$

The integral in equation (4.54) is not readily evaluated however tabulated values of erf and erfc are readily available hence we perform a change of variables to bring equation (4.54) to the form of (4.56). We must first do a change of base as follows:

$$\begin{aligned} 10^{\frac{bx_N}{10}} &= e^{\ln\left(10^{\frac{bx_N}{10}}\right)} \\ &= e^{\frac{bx_N}{10}\ln 10} \end{aligned} \tag{4.57}$$

Along with the Gaussian *pdf* definition, equation (4.54) then becomes:

$$Y = \int_{-\infty}^{x_G-\left(10/b\right)\log T_N} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{bx_N}{10}\ln 10} e^{\frac{-x_N^2}{2\sigma^2}}\,dx_N\,. \tag{4.58}$$

We can rewrite the above as:

$$\begin{aligned} \int_{-\infty}^{x_G-\left(10/b\right)\log T_N} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{bx_N}{10}\ln 10} e^{\frac{-x_N^2}{2\sigma^2}}\,dx_N &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{bx_N}{10}\ln 10} e^{\frac{-x_N^2}{2\sigma^2}}\,dx_N - \int_{x_G-\left(10/b\right)\log T_N}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{bx_N}{10}\ln 10} e^{\frac{-x_N^2}{2\sigma^2}}\,dx_N \\ Y &= H - F \end{aligned} \tag{4.59}$$

with

$$H \equiv \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{bx_N}{10}\ln 10} e^{\frac{-x^2_N}{2\sigma^2}} dx_N$$

$$F \equiv \int_{x_G-\left(10/b\right)\log T_N}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{bx_N}{10}\ln 10} e^{\frac{-x^2_N}{2\sigma^2}} dx_N \tag{4.60}$$

Now we know that H the first term on the right hand side (RHS) in (4.59) is simply the infinite integral over the Gaussian *pdf* and hence has the value 1; however since we perform a change of variables later on, we leave the expression in its current form. The second term F needs to be changed to the form of the complimentary error function, erfc. Using $2\sigma^2$ as a lowest common denominator (LCD), the second term may be written as:

$$F = \int_{x_G-\left(10/b\right)\log T_N}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{2\sigma^2\left(\frac{bx_N}{10}\right)\ln 10 - x^2_N}{2\sigma^2}} dx_N . \tag{4.61}$$

We now rewrite the quadratic as the square of a binomial to yield:

$$F = \int_{x_G-\left(10/b\right)\log T_N}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-\left(x_N-\sigma^2\left(\frac{b}{10}\right)\ln 10\right)^2}{2\sigma^2}} e^{\frac{\sigma^4\left(\frac{b}{10}\right)^2(\ln(10))^2}{2\sigma^2}} dx_N . \tag{4.62}$$

Simplifying the above by pulling the constant term out of the integral we then arrive at:

$$F = e^{\frac{\sigma^2 b^2(\ln(10))^2}{200}} \int_{x_G-\left(10/b\right)\log T_N}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-\left(x_N-\sigma^2\left(\frac{b}{10}\right)\ln(10)\right)^2}{2\sigma^2}} dx_N . \tag{4.63}$$

If we define

$$y = \frac{x_N - \sigma^2\left(\frac{b}{10}\right)\ln(10)}{\sqrt{2}\sigma} \tag{4.64}$$

we can write the differential as:

$$dx_N = \sqrt{2}\sigma dy . \tag{4.65}$$

After performing a change of variables on equation(4.63) [pp 342, Thomas and Finney], using the above transformation we get:

$$F = e^{\frac{\sigma^2 b^2(\ln(10))^2}{200}} \int_{\frac{x_G-\left(10/b\right)\log T_N-\sigma^2\left(\frac{b}{10}\right)\ln(10)}{\sqrt{2}\sigma}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-y^2} dy \tag{4.66}$$

Performing the same change of variables on integral H from equation (4.59) yields

$$H = e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-y^2} dy \tag{4.67}$$

With the aid of (4.56) we may then express these 2 integrals in terms of the error function as:

$$F = e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \frac{1}{2} \operatorname{erfc}\left(\frac{x_G - \left(10/b\right)\log T_N - \sigma^2 \left(\frac{b}{10}\right)\ln(10)}{\sqrt{2}\sigma} \right) \tag{4.68}$$

and

$$\begin{aligned} H &= e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \frac{1}{2} \operatorname{erfc}(-\infty) \\ &= e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \frac{1}{2} \left(1 - \operatorname{erf}(-\infty)\right) \\ &= e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \frac{1}{2} \left(1 - (-1)\right) \\ &= e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \end{aligned} . \tag{4.69}$$

Using (4.68),(4.69) and (4.59) Y is then given by:

$$Y = H - F \tag{4.70}$$

$$Y = e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} - e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \frac{1}{2} \operatorname{erfc}\left(\frac{x_G - \left(10/b\right)\log T_N - \sigma^2 \left(\frac{b}{10}\right)\ln(10)}{\sqrt{2}\sigma} \right) \tag{4.71}$$

Substituting the complimentary error function with the error function from (4.56) we get

$$Y = e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} - e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \frac{1}{2} \left(1 - \operatorname{erf}\left(\frac{x_G - \left(10/b\right)\log T_N - \sigma^2 \left(\frac{b}{10}\right)\ln(10)}{\sqrt{2}\sigma} \right) \right) \tag{4.72}$$

which with some simplification leads to

$$Y = e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{b x_G - 10\log T_N}{\sqrt{2} b \sigma} - \frac{\sigma b \ln 10}{10\sqrt{2}} \right) \right). \tag{4.73}$$

Using the above methodology we can then express equation (4.53) as:

$$\mathrm{E}\left(M_N \middle| (r,\theta)\right) = T_N \frac{\int_{-\infty}^{\infty} \left(\begin{array}{c} \frac{10^{\frac{-bx}{10}}}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}} e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}} \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{bx - 10\log T_N}{\sqrt{2} b \sigma} - \frac{\sigma b \ln 10}{10\sqrt{2}} \right) \right) \\ \prod_{J=1, J\neq N}^{K} \left(\frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{bx - 10\log T_J}{\sqrt{2} b\sigma} \right) \right) \end{array} \right) dx}{\Pr\left(BBS \middle| (r,\theta)\right)} \tag{4.74}$$

We can more succinctly write the above expression if we define the following function:

$$S_N(x,y) \equiv \frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{bx - 10\log T_N}{\sqrt{2}b\sigma} - y\frac{\sigma b \ln 10}{10\sqrt{2}}\right) \tag{4.75}$$

The expectation now becomes:

$$\mathrm{E}\left(M_N \middle| (r,\theta)\right) = \frac{e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}}}{\sqrt{2\pi}\sigma} T_N \frac{\int_{-\infty}^{\infty}\left(10^{\frac{-bx}{10}} e^{\frac{-x^2}{2\sigma^2}} S_N(x,1) \prod_{J=1,J\neq N}^{K} S_J(x,0)\right)dx}{\Pr\left(BBS \middle| (r,\theta)\right)} \tag{4.76}$$

where using (4.51) and (4.75) we have:

$$\Pr\left(BBS \middle| (r,\theta)\right) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}} \prod_{J=1}^{K} S_J(x,0)\,dx\,. \tag{4.77}$$

We now evaluate the expectation of the path loss itself which is the other term in (4.36) and (4.38). Now that the near field or MS shadowing term is no longer cancelled out as in (4.41) we must include it in the conditional *pdf*. Since the near field shadowing component is independent from the BS shadowing we can simply multiply $f_{\underline{\xi}}$ and $f_{\hat{\xi}_I}$ to obtain the joint *pdf*.

$$\begin{aligned}&\mathrm{E}\left(\left(r_{I,G}{}^4 10^{-\left(a\hat{\xi}_I + b\xi_G\right)/10}\right)\middle|(r,\theta)\right) = \\ &r_{I,G}{}^4 \int 10^{-\left(az + bx_G\right)/10} f_{\hat{\xi}_I} f_{\underline{\xi}}\left(z, \underline{x} \middle| \left(h_{I,G} > h_{I,1} \ldots h_{I,G} > h_{I,J} \ldots h_{I,G} > h_{I,K}\right)_{\forall J, J\neq G}\right) dz d\underline{x}\end{aligned} \tag{4.78}$$

Expanding out the integration as in (4.53) we get:

$$\begin{aligned}&\mathrm{E}\left(\left(r_{I,G}{}^4 10^{-\left(a\hat{\xi}_I + b\xi_G\right)/10}\right)\middle|(r,\theta)\right) = \\ &r_{I,G}{}^4 \frac{\int_{-\infty}^{\infty} 10^{\frac{-az}{10}} f_{\hat{\xi}_I}(z)\,dz \int_{-\infty}^{\infty}\left(10^{\frac{-bx_G}{10}} f_{\xi_G}(x_G) \prod_{J=1}^{K}\left(\int_{-\infty}^{x_G - (10/b)\log T_J} f_{\xi_J}(x_J)\,dx_J\right)\right)dx_G}{\Pr\left(BBS \middle| (r,\theta)\right)}\end{aligned} \tag{4.79}$$

After substituting using the error function as in (4.74) we get:

$$\begin{aligned}&\mathrm{E}\left(\left(r_{I,G}{}^4 10^{-\left(a\hat{\xi}_I + b\xi_G\right)/10}\right)\middle|(r,\theta)\right) = \\ &r_{I,G}{}^4 \frac{\int_{-\infty}^{\infty} 10^{\frac{-az}{10}} f_{\hat{\xi}_I}(z)\,dz \int_{-\infty}^{\infty}\left(\frac{10^{\frac{-bx}{10}}}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}} \prod_{J=1}^{K}\left(\frac{1}{2} + \frac{1}{2}\operatorname{erf}\left(\frac{bx - 10\log T_J}{\sqrt{2}b\sigma}\right)\right)\right)dx}{\Pr\left(BBS \middle| (r,\theta)\right)}\end{aligned} \tag{4.80}$$

and using equation (4.75) we get:

$$\mathrm{E}\left(\left(r_{I,G}{}^4 10^{-\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\Big|(r,\theta)\right)=\frac{2}{\sqrt{2\pi}\sigma}r_{I,G}{}^4\frac{\int\limits_{-\infty}^{\infty}10^{\frac{-az}{10}}e^{\frac{-z^2}{2\sigma^2}}dz\int\limits_{-\infty}^{\infty}\left(10^{\frac{-bx}{10}}e^{\frac{-x^2}{2\sigma^2}}\prod_{J=1}^{K}S_J(x,0)\right)dx}{\Pr\left(BBS|(r,\theta)\right)}. \tag{4.81}$$

Using the above equation (4.81) and (4.76), $m_{fpl|(r,\theta)}$ the first moment of $f_{pl,I}$ at the given position (r,θ), (4.36) can now be expressed as:

$$\begin{aligned}
m_{fpl|(r,\theta)}&=C\,\mathrm{E}\left(\left(r_{I,G}{}^4 10^{-\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\Big|(r,\theta)\right)+\sum_{N=1}^{K}\mathrm{E}\left(M_N|(r,\theta)\right)\\
&=C\frac{2}{\sqrt{2\pi}\sigma}r_{I,G}{}^4\frac{\int\limits_{-\infty}^{\infty}10^{\frac{-az}{10}}e^{\frac{-z^2}{2\sigma^2}}dz\int\limits_{-\infty}^{\infty}\left(10^{\frac{-bx}{10}}e^{\frac{-x^2}{2\sigma^2}}\prod_{J=1}^{K}S_J(x,0)\right)dx}{\Pr\left(BBS|(r,\theta)\right)}\\
&+\sum_{N=1}^{K}\frac{e^{\frac{\sigma^2b^2(\ln(10))^2}{200}}}{\sqrt{2\pi}\sigma}T_N\frac{\int\limits_{-\infty}^{\infty}\left(10^{\frac{-bx}{10}}e^{\frac{-x^2}{2\sigma^2}}S_N(x,1)\prod_{J=1,J\neq N}^{K}S_J(x,0)\right)dx}{\Pr\left(BBS|(r,\theta)\right)}
\end{aligned} \tag{4.82}$$

4.3.3 Second Moment of $f_{pl,I}$ at the Given Position (r,θ)

We now consider $\psi_{fpl|(r,\theta)}$, the second moment of $f_{pl,I}$ from (4.38) and follow a similar development as in the previous section. We start with the first term being summed, expand using (4.33) as well as condition upon event *BBS* to get:

$$\begin{aligned}
\mathrm{E}\left(M_N{}^2|(r,\theta)\right)&=\int T^2{}_N 10^{2b(x_N-x_G)/10}f_{\underline{\xi}}\left(\underline{x}|BBS\right)d\underline{x}\\
&=\int T^2{}_N 10^{2b(x_N-x_G)/10}f_{\underline{\xi}}\left(\underline{x}\Big|\left(h_{I,G}>h_{I,1}\dots h_{I,G}>h_{I,J}\dots h_{I,G}>h_{I,K}\right)_{\forall J,J\neq G}\right)d\underline{x}
\end{aligned} \tag{4.83}$$

Evaluating the above by expanding out and using (4.47) yields

$$\mathrm{E}\left(M_N{}^2|(r,\theta)\right)=T^2{}_N\frac{\int\limits_{-\infty}^{\infty}\left(10^{\frac{-bx_G}{5}}f_{\xi_G}(x_G)\int\limits_{-\infty}^{x_G-(10/b)\log T_N}10^{\frac{bx_N}{5}}f_{\xi_N}(x_N)dx_N\prod_{J=1,J\neq N}^{K}\left(\int\limits_{-\infty}^{x_G-(10/b)\log T_J}f_{\xi_J}(x_J)dx_J\right)\right)dx_G}{\Pr\left(BBS|(r,\theta)\right)} \tag{4.84}$$

After substituting using the error function we get

$$\mathrm{E}\left(M_N{}^2|(r,\theta)\right)=T_N{}^2\frac{\int\limits_{-\infty}^{\infty}\left(\begin{array}{l}\frac{10^{\frac{-bx}{5}}}{\sqrt{2\pi}\sigma}e^{\frac{-x^2}{2\sigma^2}}e^{\frac{\sigma^2b^2(\ln 10)^2}{50}}\left(\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{bx-10\log T_N}{\sqrt{2}b\sigma}-\frac{\sigma b\ln 10}{5\sqrt{2}}\right)\right)\\ \prod_{J=1,J\neq N}^{K}\left(\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{bx-10\log T_J}{\sqrt{2}b\sigma}\right)\right)\end{array}\right)dx}{\Pr\left(BBS|(r,\theta)\right)} \tag{4.85}$$

and using equation (4.75) we get

$$\mathrm{E}\left(M_N^{\,2}\left|(r,\theta)\right.\right)=\frac{e^{\frac{\sigma^2 b^2(\ln(10))^2}{50}}}{\sqrt{2\pi}\sigma}T_N^{\,2}\frac{\int_{-\infty}^{\infty}\left(10^{\frac{-bx}{5}}e^{\frac{-x^2}{2\sigma^2}}S_N(x,2)\prod_{J=1,J\neq N}^{K}S_J(x,0)\right)dx}{\Pr\left(BBS\left|(r,\theta)\right.\right)}. \tag{4.86}$$

Note that the constant exponential term has been multiplied by the square of 2 to yield a denominator of 50. Next we compute the expectation of the cross product terms from (4.38), expand using (4.33) as well as condition upon event *BBS* to get:

$$\begin{aligned}\mathrm{E}\left(M_N M_V\left|(r,\theta)\right.\right)&=\int T_N T_V 10^{b(x_N+x_V-2x_G)/10} f_{\underline{\xi}}\left(\underline{x}\left|BBS\right.\right)d\underline{x}\\&=\int T_N T_V 10^{b(x_N+x_V-2x_G)/10} f_{\underline{\xi}}\left(\underline{x}\left|\left(h_{1,G}>h_{1,1}\ldots h_{1,G}>h_{1,J}\ldots h_{1,G}>h_{1,K}\right)_{\forall J,J\neq G}\right.\right)\end{aligned} \tag{4.87}$$

Evaluating the above expanding out and using (4.47) we get:

$$\mathrm{E}\left(M_N M_V\left|(r,\theta)\right.\right)=T_N T_V\frac{\int_{-\infty}^{\infty}\left(\begin{array}{l}10^{\frac{-bx_G}{5}}f_{\xi_G}(x_G)\int_{-\infty}^{x_G-(10/b)\log T_N}10^{\frac{bx_N}{10}}f_{\xi_N}(x_N)dx_N\int_{-\infty}^{x_G-(10/b)\log T_V}10^{\frac{bx_V}{10}}f_{\xi_V}(x_V)dx_V\\ \prod_{J=1,J\neq N\cup V}^{K}\left(\int_{-\infty}^{x_G-(10/b)\log T_J}f_{\xi_J}(x_J)dx_J\right)\end{array}\right)dx_G}{\Pr\left(BBS\left|(r,\theta)\right.\right)} \tag{4.88}$$

Using the error function we get:

$$\mathrm{E}\left(M_N M_V\left|(r,\theta)\right.\right)=T_N T_V\frac{\int_{-\infty}^{\infty}\left(\begin{array}{l}\frac{10^{\frac{-bx}{5}}}{\sqrt{2\pi}\sigma}e^{\frac{-x^2}{2\sigma^2}}e^{\frac{\sigma^2 b^2(\ln 10)^2}{100}}\left(\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(\frac{bx-10\log T_N}{\sqrt{2}b\sigma}-\frac{\sigma b\ln 10}{10\sqrt{2}}\right)\right)\\ \left(\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(\frac{bx-10\log T_V}{\sqrt{2}b\sigma}-\frac{\sigma b\ln 10}{10\sqrt{2}}\right)\right)\prod_{J=1,J\neq N\cup V}^{K}\left(\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(\frac{bx-10\log T_J}{\sqrt{2}b\sigma}\right)\right)\end{array}\right)dx}{\Pr\left(BBS\left|(r,\theta)\right.\right)} \tag{4.89}$$

Note that the constant exponential term has two contributions to yield a denominator of 100, one from the N^{th} BS and one from the V^{th} BS. Using equation (4.75) we get:

$$\mathrm{E}\left(M_N M_V\left|(r,\theta)\right.\right)=\frac{e^{\frac{\sigma^2 b^2(\ln 10)^2}{100}}}{\sqrt{2\pi}\sigma}T_N T_V\frac{\int_{-\infty}^{\infty}\left(10^{\frac{-bx}{5}}e^{\frac{-x^2}{2\sigma^2}}S_N(x,1)S_V(x,1)\prod_{J=1,J\neq N\cup V}^{K}S_J(x,0)\right)dx}{\Pr\left(BBS\left|(r,\theta)\right.\right)}. \tag{4.90}$$

The expected value of the square of the path loss from (4.38) and conditioned upon event BBS is given by:

$$
\begin{aligned}
&\mathrm{E}\left(\left(r_{I,G}{}^{8}10^{-2\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|(r,\theta)\right) \\
&= r_{I,G}{}^{8}\int 10^{-2(az+bx_G)/10} f_{\hat{\xi}_I} f_{\underline{\xi}}\left(z,\underline{x}\middle|BBS\right) dz d\underline{x} \\
&= r_{I,G}{}^{8}\int 10^{-2(az+bx_G)/10} f_{\hat{\xi}_I} f_{\underline{\xi}}\left(z,\underline{x}\middle|\left(h_{I,G}>h_{I,1}\ldots h_{I,G}>h_{I,J}\ldots h_{I,G}>h_{I,K}\right)_{\forall J,J\neq G}\right) dz d\underline{x}
\end{aligned}
\tag{4.91}
$$

Expanding out the integration using (4.47) we get:

$$
\begin{aligned}
&\mathrm{E}\left(\left(r_{I,G}{}^{8}10^{-2\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|(r,\theta)\right) \\
&= r_{I,G}{}^{8}\frac{\int_{-\infty}^{\infty} 10^{\frac{-az}{5}} f_{\hat{\xi}_I}(z)\,dz \int_{-\infty}^{\infty}\left(10^{\frac{-bx_G}{5}} f_{\xi_G}(x_G)\prod_{J=1}^{K}\left(\int_{-\infty}^{x_G-\left(10/b\right)\log T_J} f_{\xi_J}(x_J)\,dx_J\right)\right)dx_G}{\Pr\left(BBS\middle|(r,\theta)\right)}.
\end{aligned}
\tag{4.92}
$$

Using the error function we get

$$
\begin{aligned}
&\mathrm{E}\left(\left(r_{I,G}{}^{8}10^{-2\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|(r,\theta)\right) \\
&= r_{I,G}{}^{8}\frac{\int_{-\infty}^{\infty} \frac{10^{\frac{-az}{5}}}{\sqrt{2\pi}\sigma} e^{\frac{-z^2}{2\sigma^2}}\,dz \int_{-\infty}^{\infty}\left(\frac{10^{\frac{-bx}{5}}}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}}\prod_{J=1}^{K}\left(\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left(\frac{bx-10\log T_J}{\sqrt{2}b\sigma}\right)\right)\right)dx}{\Pr\left(BBS\middle|(r,\theta)\right)}
\end{aligned}
\tag{4.93}
$$

and using equation (4.75) we get

$$
\begin{aligned}
&\mathrm{E}\left(\left(r_{I,G}{}^{8}10^{-2\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|(r,\theta)\right) \\
&= \frac{2}{\sqrt{2\pi}\sigma} r_{I,G}{}^{8}\frac{\int_{-\infty}^{\infty} 10^{\frac{-az}{5}} e^{\frac{-z^2}{2\sigma^2}}\,dz \int_{-\infty}^{\infty}\left(10^{\frac{-bx}{5}} e^{\frac{-x^2}{2\sigma^2}}\prod_{J=1}^{K} S_J(x,0)\right)dx}{\Pr\left(BBS\middle|(r,\theta)\right)}.
\end{aligned}
\tag{4.94}
$$

Lastly we evaluate the fourth and final term in (4.38), expand using (4.33) as well as condition upon event *BBS* to get

$$
\begin{aligned}
&\mathrm{E}\left(\left(M_N r_{I,G}{}^{4}10^{-\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|(r,\theta)\right) \\
&= \int T_N r_{I,G}{}^{4}10^{\left(bx_N-az-2bx_G\right)/10} f_{\hat{\xi}_I} f_{\underline{\xi}}\left(z,\underline{x}\middle|BBS\right) dz d\underline{x} \\
&= \int T_N r_{I,G}{}^{4}10^{\left(bx_N-az-2bx_G\right)/10} f_{\hat{\xi}_I} f_{\underline{\xi}}\left(z,\underline{x}\middle|\left(h_{I,G}>h_{I,1}\ldots h_{I,G}>h_{I,J}\ldots h_{I,G}>h_{I,K}\right)_{\forall J,J\neq G}\right) dz d\underline{x}
\end{aligned}
\tag{4.95}
$$

Expanding out the integration using (4.47) we get

$$\mathrm{E}\left(\left(M_N r_{I,G}{}^4 10^{-\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|(r,\theta)\right)$$

$$= T_N r_{I,G}{}^4 \frac{\displaystyle\int_{-\infty}^{\infty} 10^{\frac{-az}{10}} f_{\hat{\xi}_I}(z)\,dz \int_{-\infty}^{\infty}\left(\begin{array}{c} 10^{\frac{-bx_G}{5}} f_{\xi_G}(x_G) \displaystyle\int_{-\infty}^{x_G-\left(10/b\right)\log T_N} 10^{\frac{bx_N}{10}} f_{\xi_N}(x_N)\,dx_N \\ \displaystyle\prod_{J=1,J\neq N\cup V}^{K}\left(\int_{-\infty}^{x_G-\left(10/b\right)\log T_J} f_{\xi_J}(x_J)\,dx_J\right)\end{array}\right)dx_G}{\Pr\left(BBS\middle|(r,\theta)\right)} \tag{4.96}$$

Using the error function we get

$$\mathrm{E}\left(\left(M_N r_{I,G}{}^4 10^{-\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|(r,\theta)\right)=$$

$$T_N r_{I,G}{}^4 \frac{\displaystyle\int_{-\infty}^{\infty}\frac{10^{\frac{-az}{10}}}{\sqrt{2\pi}\sigma}e^{\frac{-z^2}{2\sigma^2}}dz\int_{-\infty}^{\infty}\left(\begin{array}{c}\dfrac{10^{\frac{-bx}{5}}}{\sqrt{2\pi}\sigma}e^{\frac{-x^2}{2\sigma^2}}e^{\frac{\sigma^2b^2(\ln 10)^2}{200}}\left(\dfrac{1}{2}+\dfrac{1}{2}\mathrm{erf}\left(\dfrac{bx-10\log T_N}{\sqrt{2}b\sigma}-\dfrac{\sigma b\ln 10}{10\sqrt{2}}\right)\right)\\ \displaystyle\prod_{J=1,J\neq N}^{K}\left(\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{bx-10\log T_J}{\sqrt{2}b\sigma}\right)\right)\end{array}\right)dx}{\Pr\left(BBS\middle|(r,\theta)\right)} \tag{4.97}$$

and using equation (4.75) we get

$$\mathrm{E}\left(\left(M_N r_{I,G}{}^4 10^{-\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|(r,\theta)\right)=$$

$$\frac{2e^{\frac{\sigma^2b^2(\ln(10))^2}{200}}}{\sqrt{2\pi}\sigma}T_N r_{I,G}{}^4\frac{\displaystyle\int_{-\infty}^{\infty}10^{\frac{-az}{10}}e^{\frac{-z^2}{2\sigma^2}}dz\int_{-\infty}^{\infty}\left(10^{\frac{-bx}{5}}e^{\frac{-x^2}{2\sigma^2}}S_N(x,1)\prod_{J=1,J\neq N}^{K}S_J(x,0)\right)dx}{\Pr\left(BBS\middle|(r,\theta)\right)}. \tag{4.98}$$

Using (4.86), (4.90), (4.94) and (4.98) $\psi_{fpl|(r,\theta)}$ the second moment of $f_{pl,I}$ at the given position (r,θ) (4.38) can now be expressed as

$$
\begin{aligned}
\psi_{fpl|(r,\theta)} &= \sum_{N=1}^{K} \mathrm{E}\left(M_N^{\,2}\,|(r,\theta)\right) + \sum_{V=1}^{K}\sum_{N=1,N\neq V}^{K} \mathrm{E}\left(M_N M_V\,|(r,\theta)\right) \\
&+C^2\,\mathrm{E}\left(\left(r_{l,G}^{\,8}10^{-2\left(a\hat{\xi}_l+b\xi_G\right)/10}\right)|(r,\theta)\right)+C\sum_{N=1}^{K}\mathrm{E}\left(\left(M_N r_{l,G}^{\,4}10^{-\left(a\hat{\xi}_l+b\xi_G\right)/10}\right)|(r,\theta)\right) \\
&= \sum_{N=1}^{K} \frac{e^{\frac{\sigma^2 b^2(\ln(10))^2}{50}}}{\sqrt{2\pi}\sigma} T_N^{\,2} \frac{\int_{-\infty}^{\infty}\left(10^{\frac{-bx}{5}} e^{\frac{-x^2}{2\sigma^2}} S_N(x,2) \prod_{J=1,J\neq N}^{K} S_J(x,0)\right)dx}{\Pr\left(BBS|(r,\theta)\right)} \\
&+\sum_{V=1}^{K}\sum_{N=1,N\neq V}^{K} \frac{e^{\frac{\sigma^2 b^2(\ln 10)^2}{100}}}{\sqrt{2\pi}\sigma} T_N T_V \frac{\int_{-\infty}^{\infty}\left(10^{\frac{-bx}{5}} e^{\frac{-x^2}{2\sigma^2}} S_N(x,1) S_V(x,1) \prod_{J=1,J\neq N\cup V}^{K} S_J(x,0)\right)dx}{\Pr\left(BBS|(r,\theta)\right)} \\
&+C^2 \frac{2}{\sqrt{2\pi}\sigma} r_{l,G}^{\,8} \frac{\int_{-\infty}^{\infty} 10^{\frac{-az}{5}} e^{\frac{-z^2}{2\sigma^2}} dz \int_{-\infty}^{\infty}\left(10^{\frac{-bx}{5}} e^{\frac{-x^2}{2\sigma^2}} \prod_{J=1}^{K} S_J(x,0)\right)dx}{\Pr\left(BBS|(r,\theta)\right)} \\
&+C\sum_{N=1}^{K} \frac{2e^{\frac{\sigma^2 b^2(\ln(10))^2}{200}}}{\sqrt{2\pi}\sigma} T_N r_{l,G}^{\,4} \frac{\int_{-\infty}^{\infty} 10^{\frac{-az}{10}} e^{\frac{-z^2}{2\sigma^2}} dz \int_{-\infty}^{\infty}\left(10^{\frac{-bx}{5}} e^{\frac{-x^2}{2\sigma^2}} S_N(x,1) \prod_{J=1,J\neq N}^{K} S_J(x,0)\right)dx}{\Pr\left(BBS|(r,\theta)\right)}
\end{aligned}
\tag{4.99}
$$

At this juncture we have expressions for the first and second moment of $f_{pl,l}$ that take into account the best BS selection criteria (Event BBS) and are conditioned upon a specific position (r,θ). The next step is to average over all feasible values of (r,θ) to yield expressions that can more readily be calculated albeit numerically.

4.3.4 Averaging Over All Positions

As discussed in Section 4.3.1, the effect of the best BS selection should be included in the distribution of the position of user l for an exact analysis. In our case BS G represents the best BS and thus user l is connected to it. $f_{r,\theta}(r,\theta|BBS)$ is the conditional distribution of the position of user l given that it is connected to best BS G. With considering the best BS selection and using [pp. 84, Papoulis 1991] this conditional distribution may be expressed as:

$$
\begin{aligned}
f_{r,\theta}(r,\theta|BBS) &= \frac{\Pr\left(BBS|r,\theta\right) f_{r,\theta}(r,\theta)}{\iint_{A_0}\Pr\left(BBS|r,\theta\right) f_{r,\theta}(r,\theta)\,dr\,d\theta} \\
&= \frac{\Pr\left(BBS|r,\theta\right) f_{r,\theta}(r,\theta)}{\Pr(BBS)}
\end{aligned}
\tag{4.100}
$$

where $f(r,\theta)$ is the joint *pdf* of (r,θ) over the possible positions of user l region regardless of the connected BS and A_0 is the entire region that is feasible for the selection of BS G. To simplify the analytical procedure the region A_0 that the user could possibly be in and connected to BS G is restricted to the sector of radius D_A (1½ times the distance between BSs) and of angle $\pi/6$ (1/12$^{\text{th}}$ of 2π) as show in the figure below. By symmetry, the relative position of users and BSs is the same throughout as for the sector of the figure. In our analysis we consider 11

BSs near BS G which are influential in the received other cell interference of users located on region A_0 and connected to BS G.

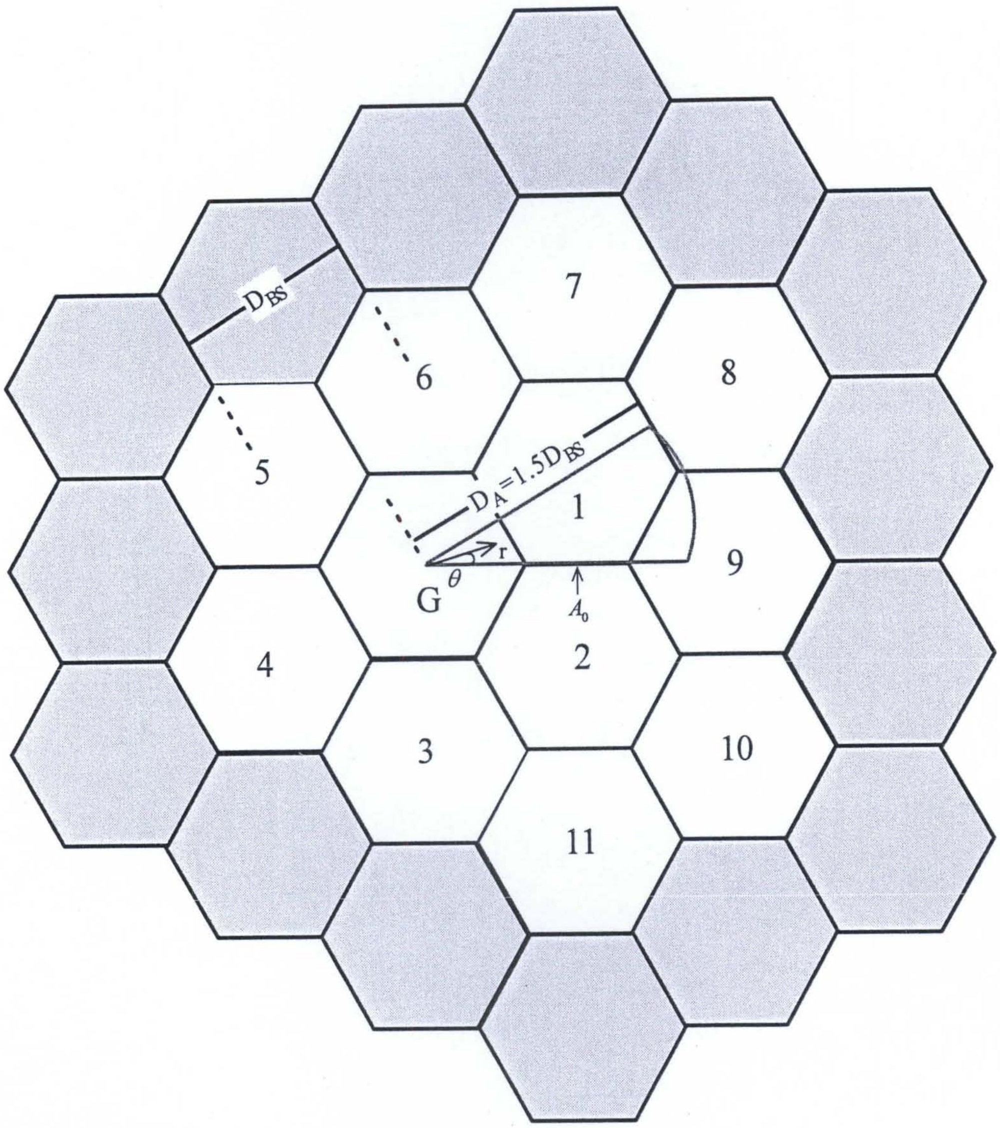


Figure 4-1 Cell Site deployment model

With a uniform distribution of users we have:

$$f(r,\theta)=\begin{cases} \dfrac{12r}{\pi D^2_A} & 0<r\le D_A \text{ and } 0\le\theta<\pi/6 \\ 0 & \text{otherwise} \end{cases} \tag{4.101}$$

The uniform distribution is simply the inverse of the region of integration i.e. area of a circle. Since we integrate over only 1/12$^{\text{th}}$ of the circle we must multiply by 12 and r is the Jacobian to take care of the change of variables from Cartesian co-ordinates to polar co-ordinates. As defined previously in (4.77) the probability of selecting BS G at the given position is given by:

$$\Pr\left(BBS\middle|(r,\theta)\right)=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-x^2}{2\sigma^2}}\prod_{J=1}^{K}S_J(x,0)\,dx \tag{4.102}$$

and using the *pdf* defined in (4.101) and integrating over the area A_0 we can then express the probability of choosing the best BS (Event BBS) as:

$$\Pr(BBS)=\int_0^{\pi/6}\int_0^{D_A}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-x^2}{2\sigma^2}}\prod_{J=1}^{K}S_J(x,0)\left(\frac{12r}{\pi D_A^2}\right)dxdrd\theta \tag{4.103}$$

We can then use the above equations (4.101), (4.102) and (4.103) and substitute back into the conditional distribution (4.100) to yield

$$\begin{aligned}f(r,\theta|BBS)&=\frac{\Pr(BBS|r,\theta)f(r,\theta)}{\Pr(BBS)}\\&=\frac{\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-x^2}{2\sigma^2}}\prod_{J=1}^{K}S_J(x,0)\,dx\frac{12r}{\pi D_A^2}}{\int_0^{\pi/6}\int_0^{D_A}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-x^2}{2\sigma^2}}\prod_{J=1}^{K}S_J(x,0)\left(\frac{12r}{\pi D_A^2}\right)dxdrd\theta}\qquad 0<r\le D_A \text{ and } 0\le\theta<\pi/6\\&=0\qquad\text{otherwise}\end{aligned} \tag{4.104}$$

From equation (4.36) $m_{fpl|(r,\theta)}$ is the first moment of $f_{pl,l}$ given a particular position. We require the value of $m_{fpl|(r,\theta)}$ averaged over all possible positions of user l when connected to G. The mean value m_{fpl} averaged over all positions and taking into account best BS selection is given by

$$\begin{aligned}m_{fpl}&=\mathrm{E}\left(f_{pl,l}|BBS\right)\\&=C\,\mathrm{E}\left(\left(r_{l,G}{}^{4}10^{-\left(a\hat{\xi}_l+b\xi_G\right)/10}\right)|BBS\right)+\sum_{N=1}^{K}\mathrm{E}\left(M_N|BBS\right)\end{aligned} \tag{4.105}$$

Starting with the last term in (4.105) above and integrating over all possible values of (r,θ) we get

$$\mathrm{E}\left(M_N|BBS\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{E}\left(M_N|(r,\theta)\right)f\left((r,\theta)|BBS\right)dr\,d\theta. \tag{4.106}$$

After substituting for the conditional *pdf* from (4.104) and using (4.76), with some manipulation we arrive at

$$\begin{aligned}&E\left(M_N|BBS\right)\\&=\frac{e^{\frac{\sigma^2b^2(\ln(10))^2}{200}}}{\Pr(BBS)}\int_0^{\pi/6}\int_0^{D_A}\int_{-\infty}^{\infty}\left(T_N\times f(r,\theta)\times10^{\frac{-bx}{10}}\times\frac{e^{\frac{-x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}\times S_N(x,1)\times\prod_{J=1,J\ne G\cup N}^{K}S_J(x,0)\right)dxdrd\theta\end{aligned} \tag{4.107}$$

and following a similar derivation from (4.81) we can express the mean path loss as:

$$\mathrm{E}\left(\left(r_{I,G}{}^{4}10^{-\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|BBS\right)=$$

$$\frac{2}{\sqrt{2\pi}\sigma\Pr(BBS)}\int_{-\infty}^{\infty}10^{\frac{-az}{10}}e^{\frac{-z^2}{2\sigma^2}}dz\int_0^{\pi/6}\int_0^{D_A}\int_{-\infty}^{\infty}\left(r_{I,G}{}^{4}\times f(r,\theta)\times 10^{\frac{-bx}{10}}\times\frac{e^{\frac{-x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}\times\prod_{J=1,J\neq N}^{K}S_J(x,0)\right)dx\,dr\,d\theta$$

(4.108)

From (4.105) using (4.107) and (4.108), m_{fpl} can now be expressed as

$$m_{fpl}=C\,\mathrm{E}\left(\left(r_{I,G}{}^{4}10^{-\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|BBS\right)+\sum_{N=1}^{K}\mathrm{E}\left(M_N\,|BBS\right)$$

$$=\frac{C2}{\sqrt{2\pi}\sigma\Pr(BBS)}\int_{-\infty}^{\infty}10^{\frac{-az}{10}}e^{\frac{-z^2}{2\sigma^2}}dz\int_0^{\pi/6}\int_0^{D_A}\int_{-\infty}^{\infty}\left(r_{I,G}{}^{4}\times f(r,\theta)\times 10^{\frac{-bx}{10}}\times\frac{e^{\frac{-x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}\times\prod_{J=1,J\neq N}^{K}S_J(x,0)\right)dx\,dr\,d\theta$$

$$+\sum_{N=1}^{K}\frac{e^{\frac{\sigma^2b^2(\ln(10))^2}{200}}}{\Pr(BBS)}\int_0^{\pi/6}\int_0^{D_A}\int_{-\infty}^{\infty}\left(T_N\times f(r,\theta)\times 10^{\frac{-bx}{10}}\times\frac{e^{\frac{-x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}\times S_N(x,1)\times\prod_{J=1,J\neq G\cup N}^{K}S_J(x,0)\right)dxdrd\theta$$

(4.109)

Similarly from (4.38) $\psi_{fpl|(r,\theta)}$ is the second moment of $f_{pl,I}$ given a particular position and the second moment ψ_{fpl} averaged over all possible positions and taking into account best BS selection (Event BBS) is given by

$$\begin{aligned}\psi_{fpl}&=\mathrm{E}\left(\left(f_{pl,I}\right)^2|BBS\right)\\&=\sum_{N=1}^{K}\mathrm{E}\left(M_N{}^2\,|BBS\right)+\sum_{V=1}^{K}\sum_{N=1,N\neq V}^{K}\mathrm{E}\left(M_NM_V\,|BBS\right)\\&\quad+C^2\,\mathrm{E}\left(\left(r_{I,G}{}^{8}10^{-2\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|BBS\right)+C\sum_{N=1}^{K}\mathrm{E}\left(\left(M_Nr_{I,G}{}^{4}10^{-\left(a\hat{\xi}_I+b\xi_G\right)/10}\right)\middle|BBS\right)\end{aligned}\qquad(4.110)$$

where using (4.86) and the conditional *pdf* (4.104) we have

$$\mathrm{E}\left(M_N{}^2\,|BBS\right)$$

$$=\frac{e^{\frac{\sigma^2b^2(\ln(10))^2}{50}}}{\sqrt{2\pi}\sigma\Pr(BBS)}\int_0^{\pi/6}\int_0^{D_A}\int_{-\infty}^{\infty}\left(T_N{}^2\times f(r,\theta)\times 10^{\frac{-bx}{5}}\times e^{\frac{-x^2}{2\sigma^2}}\times S_N(x,2)\times\prod_{J=1,J\neq N}^{K}S_J(x,0)\right)dx\,dr\,d\theta\qquad(4.111)$$

and similarly using (4.90) we have

$$\mathrm{E}\left(M_NM_V\,|BBS\right)$$

$$=\frac{e^{\frac{\sigma^2b^2(\ln 10)^2}{100}}}{\sqrt{2\pi}\sigma\Pr(BBS)}\int_0^{\pi/6}\int_0^{D_A}\int_{-\infty}^{\infty}\left(T_NT_V\times f(r,\theta)\times 10^{\frac{-bx}{5}}\times e^{\frac{-x^2}{2\sigma^2}}\times S_N(x,1)S_V(x,1)\times\prod_{J=1,J\neq N\cup V}^{K}S_J(x,0)\right)dx\,dr\,d\theta$$

(4.112)

and similarly using (4.94) we have

$$\mathrm{E}\left(\left(r_{I,G}{}^{8} 10^{-2\left(a\hat{\xi}_I + b\xi_G\right)/10}\right) \middle| BBS\right)$$

$$= \frac{2}{\sqrt{2\pi}\sigma \Pr(BBS)} \int_{-\infty}^{\infty} 10^{\frac{-az}{5}} e^{\frac{-z^2}{2\sigma^2}} dz \int_{0}^{\pi/6} \int_{0}^{D_A} \int_{-\infty}^{\infty} \left\{ r_{I,G}{}^{8} \times f(r,\theta) \times 10^{\frac{-bx}{5}} \times e^{\frac{-x^2}{2\sigma^2}} \times \prod_{J=1}^{K} S_J(x,0) \right\} dx\, dr\, d\theta \quad (4.113)$$

and lastly using (4.98) we have

$$\mathrm{E}\left(\left(M_N r_{I,G}{}^{4} 10^{-\left(a\hat{\xi}_I + b\xi_G\right)/10}\right) \middle| (r,\theta)\right)$$

$$= \frac{2e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}}}{\sqrt{2\pi}\sigma \Pr(BBS)} \int_{-\infty}^{\infty} 10^{\frac{-az}{10}} e^{\frac{-z^2}{2\sigma^2}} dz \int_{0}^{\pi/6} \int_{0}^{D_A} \int_{-\infty}^{\infty} \left(T_N r_{I,G}{}^{4} \times f(r,\theta) \times 10^{\frac{-bx}{5}} \times e^{\frac{-x^2}{2\sigma^2}} \times S_N(x,1) \times \prod_{J=1, J\neq N}^{K} S_J(x,0) \right) dx\, dr\, d\theta$$

(4.114)

From (4.110) and using (4.111), (4.112), (4.113) and (4.114) ψ_{fpl} can now be expressed as

$$\psi_{fpl} = \sum_{N=1}^{K} \mathrm{E}\left(M_N{}^2 | BBS\right) + \sum_{V=1}^{K} \sum_{N=1, N\neq V}^{K} \mathrm{E}\left(M_N M_V | BBS\right)$$

$$+ C^2\, \mathrm{E}\left(\left(r_{I,G}{}^{8} 10^{-2\left(a\hat{\xi}_I + b\xi_G\right)/10}\right) \middle| BBS\right) + C \sum_{N=1}^{K} \mathrm{E}\left(\left(M_N r_{I,G}{}^{4} 10^{-\left(a\hat{\xi}_I + b\xi_G\right)/10}\right) \middle| BBS\right)$$

$$= \sum_{N=1}^{K} \frac{e^{\frac{\sigma^2 b^2 (\ln(10))^2}{50}}}{\sqrt{2\pi}\sigma \Pr(BBS)} \int_{0}^{\pi/6} \int_{0}^{D_A} \int_{-\infty}^{\infty} \left(T_N{}^{2} \times f(r,\theta) \times 10^{\frac{-bx}{5}} \times e^{\frac{-x^2}{2\sigma^2}} \times S_N(x,2) \times \prod_{J=1, J\neq N}^{K} S_J(x,0) \right) dx\, dr\, d\theta$$

$$+ \sum_{V=1}^{K} \sum_{N=1, N\neq V}^{K} \frac{e^{\frac{\sigma^2 b^2 (\ln 10)^2}{100}}}{\sqrt{2\pi}\sigma \Pr(BBS)}$$

$$\times \int_{0}^{\pi/6} \int_{0}^{D_A} \int_{-\infty}^{\infty} \left(T_N T_V \times f(r,\theta) \times 10^{\frac{-bx}{5}} \times e^{\frac{-x^2}{2\sigma^2}} \times S_N(x,1) S_V(x,1) \times \prod_{J=1, J\neq N\cup V}^{K} S_J(x,0) \right) dx\, dr\, d\theta$$

$$+ \frac{C^2 2}{\sqrt{2\pi}\sigma \Pr(BBS)} \int_{-\infty}^{\infty} 10^{\frac{-az}{5}} e^{\frac{-z^2}{2\sigma^2}} dz \int_{0}^{\pi/6} \int_{0}^{D_A} \int_{-\infty}^{\infty} \left\{ r_{I,G}{}^{8} \times f(r,\theta) \times 10^{\frac{-bx}{5}} \times e^{\frac{-x^2}{2\sigma^2}} \times \prod_{J=1}^{K} S_J(x,0) \right\} dx\, dr\, d\theta$$

$$+ C \sum_{N=1}^{K} \frac{2e^{\frac{\sigma^2 b^2 (\ln(10))^2}{200}}}{\sqrt{2\pi}\sigma \Pr(BBS)}$$

$$\times \int_{-\infty}^{\infty} 10^{\frac{-az}{10}} e^{\frac{-z^2}{2\sigma^2}} dz \int_{0}^{\pi/6} \int_{0}^{D_A} \int_{-\infty}^{\infty} \left(T_N r_{I,G}{}^{4} \times f(r,\theta) \times 10^{\frac{-bx}{5}} \times e^{\frac{-x^2}{2\sigma^2}} \times S_N(x,1) \times \prod_{J=1, J\neq N}^{K} S_J(x,0) \right) dx\, dr\, d\theta$$

(4.115)

At this stage we have expanded expressions for both the mean m_{fpl} (4.109) and second moment ψ_{fpl} (4.115) for the other cell interference plus path loss for the l^{th} user $f_{pl,l}$, where we have taken into account the best BS selection criteria and averaged over all possible positions of user l. These two expressions must be solved numerically, taking all BSs into account as is it not easy to obtain a closed form expression. The results of these two expressions will yield numerical values for m_{fpl} and ψ_{fpl} which will be used in the next chapter in our teletraffic analysis to determine the probability of accepting a new call. We use the conventional Wilkinson's approximation shown in [Cho J and D. Hong, 2002] and [Beaulieu, 1995], to approximate $f_{pl,l}$ as a lognormal random variable with mean and second moment given by (4.109) and (4.115) respectively.

4.4 Results

We briefly illustrate the effect of the best BS selection on the distribution of connected users to BS G to show that the closest BS is not necessarily the one that user l will connect to. In Figure 4-2 below we have plotted the conditional *pdf* from (4.104) where we show the distribution of user l connected to BS G with $\theta = 0$. Some of the parameters we have used are listed in Table 4-1 below.

Table 4-1 Table of parameters and values related to Figure 4-1

Variable	Value
Standard deviation σ	8
Cell diameter D_{cell}	1
Intersite distance $D_{BS} = \sqrt{3} \times D_{cell}$	$\sqrt{3}$
Radius of the sector of interest $D_A = 1.5 \times D_{BS}$	$1.5 \times \sqrt{3}$
Coefficient of BS shadowing b	$\sqrt{0.5}$

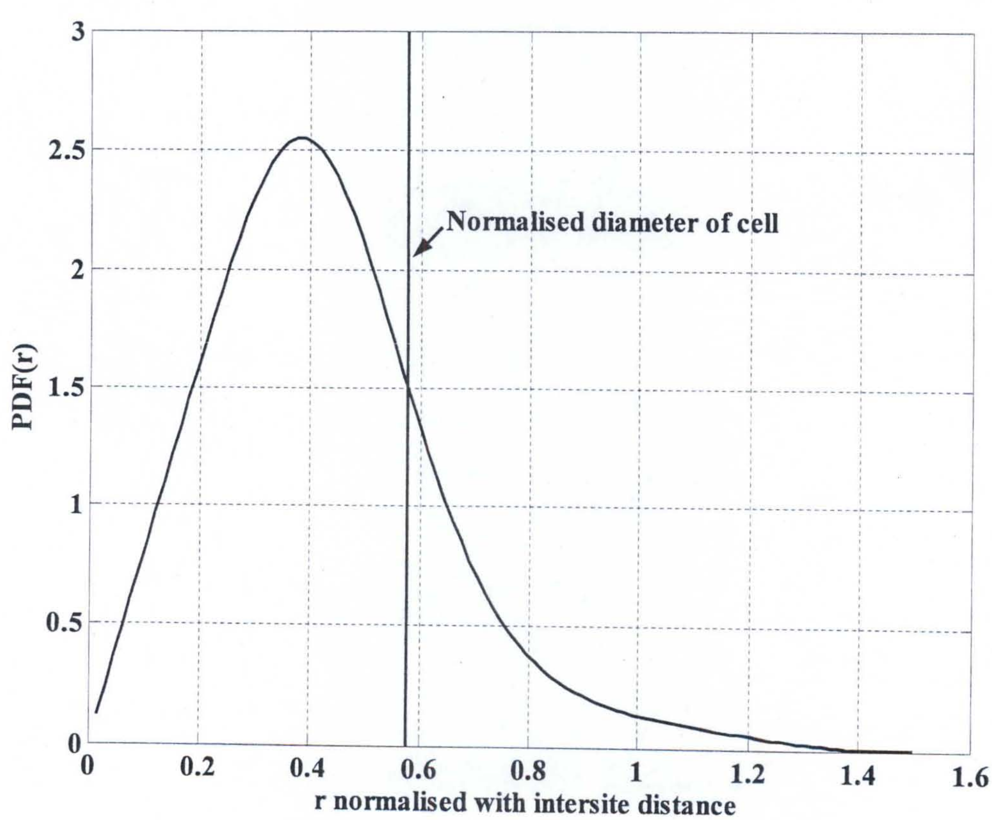


Figure 4-2 The conditional distribution of user l that is connected to BS G

Without the best BS selection criterion and assuming that the user connects to the closest BS, the probability of the user being outside the diameter of the cell and still connected to BS G is zero. We can clearly see in Figure 4-2 that when we consider the best BS selection criterion instead of simply the closest BS the user distribution outside the diameter of the cell is too large to be neglected. This result thus confirms that the effect of the best BS selection should be included in the user distribution for an exact analysis. The result obtained for the user distribution in Figure 4-2 agrees with that presented in [Cho, J. 2001].

We also compare the lognormal *pdf* of $f_{pl,l}$ generated using the mean and second moment by (4.109) and (4.115) from our developed analytical model with that from our simulation model from Chapter 3. Some of the parameters are listed in Table 4-2 below.

In Figure 4-3 below, we can see that the analytical result from the proposed model agrees with that from our simulation model for values of the *pdf* greater than approximately 10^{-7}. Values outside this range have such a low probability of occurring that we ignore them for our purposes. This analytical model can now be used with confidence in our teletraffic performance analysis in Chapter 5 and can reasonably replace a simulation model for $f_{pl,l}$, the other cell interference plus path loss.

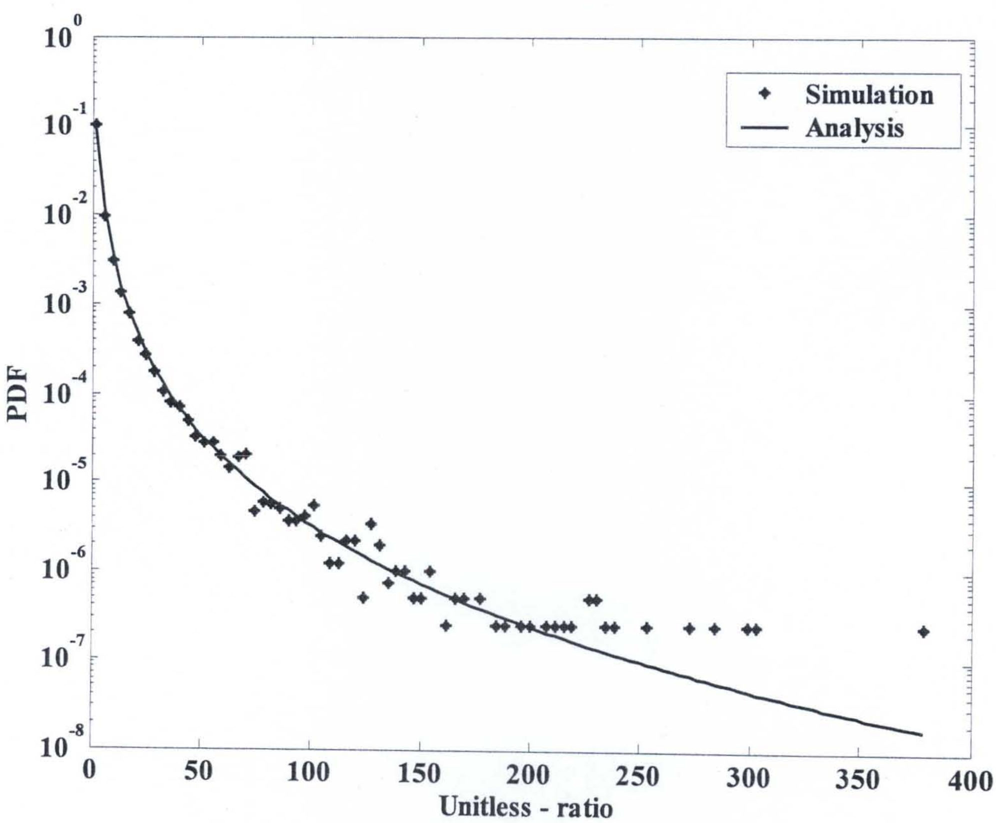


Figure 4-3 Histogram and *pdf* of other cell interference plus path loss

Table 4-2 Table of parameters and values related to Figure 4-3

Variable	Value
Coefficient of MS shadowing a	$\sqrt{0.5}$
Path gain exponent m	4
Thermal noise power density N_0	10^{-13}
Bandwidth W	3.84Mhz
Maximum BS power P_{max}	1

4.5 Summary

In this chapter we presented our load-based admission policy for a single class of service. Examining this admission policy in detail we needed to statistically characterise the other cell interference plus path loss in order to express our admission policy in a closed form to subsequently determine an admission probability. We proceeded to determine the first and second moment of the other cell interference plus path loss and then resorted to Wilkinson's approximation to obtain the closed form expression. We incorporated into our analysis the best BS selection criteria where the user connects to the BS with the strongest path gain. Then we presented a favourable comparison between our simulation and analytical results for the other cell interference plus path loss as well as the justification for the best BS selection criteria. Based on these reasonable results we can then proceed with a certain level of confidence into the next chapter.

CHAPTER 5

ANALYSIS OF A MULTI-SERVICE LOAD-BASED CALL ADMISSION CONTROL POLICY

5.1 Introduction

In this chapter we extend our load-based call admission policy introduced in the previous Chapter to two service classes, viz. voice and data and develop a mathematical model for this system. The two classes of service that we consider differ in bitrate, (E_b/N_0), activity factor, arrival rate and holding time We introduce the concept of prioritising voice calls over data calls by using a resource reservation factor. A user is defined as being explicitly either a voice call or a data call and thus the term user and call are used interchangeably. Voice calls are prioritised since they are adversely affected by latency in the network and thus we must attempt to ensure that sufficient throughput is achieved.

We use the principles of mathematical modelling discussed in Chapter 1 to perform our teletraffic analysis. Using the analytical expressions for the other cell interference plus path loss, with Wilkinson's approximation and other assumptions, we derive expressions for the acceptance probabilities for both voice and data calls. We then model the system with a two-dimensional Birth-Death Markov model. The effects of the time varying capacity are incorporated into the arrival process through the acceptance probabilities such that our model reduces to a $G/M/\infty$ system. From our model we determine the steady state distribution for this system and from this determine the blocking probabilities and mean number of calls in the system, for both voice and data calls, as a function of arrival rates.

This chapter is organised as follows. In Section 5.2 we present our multi-service load-based admission policy where we introduce two classes of service as well as resource reservation. We derive expressions for the admission condition as an extension from the previous chapter. In Section 5.3 we derive deterministic analytical expressions for the probability of accepting a new voice or data call using the new admission conditions as well as the statistical characterisation and assumptions from the previous chapter. Section 5.4 describes our system model where we perform a teletraffic analysis and in Section 5.5 we present our results, comparing our simulated model from Chapter 3 to the analytical results from this chapter. Lastly in Section 5.6 we summarise this chapter.

5.2 Multi-service Load-Based Admission Policy

We now consider a system with two types of services: voice and data, and that a single user is exclusively a voice or data call. What differentiates each type of service is bitrate, R_k, bit energy to noise density ratio, γ_k and activity factor α_k. We assume that all voice calls share the same R_{voice}, γ_{voice} and α_v and similarly all data calls share the same R_{data}, γ_{data} and α_d. For the purposes of simplification we assume that all calls, within both voice and data service classes, in the system experience the same orthogonality factor, $\rho_k = \rho \forall k$. We define N_v as the number of admitted voice calls currently in the system and N_d as the number of admitted data calls currently in the system. The total number of calls in the system is $N_d + N_v$. We use j to represent the j^{th} voice user and v to represent any parameter associated with voice users. Similarly we use k to represent the k^{th} data user as well as d to represent any parameter associated with data users. With these assumptions we can then define the following constant for all voice calls from (4.5) as

$$A_v = \left(\left(\frac{W}{\alpha_{voice} R_{voice} \gamma_{voice}} \right) + \rho \right)^{-1} \tag{5.1}$$

and similarly for all data calls

$$A_d = \left(\left(\frac{W}{\alpha_{data} R_{data} \gamma_{data}} \right) + \rho \right)^{-1} \tag{5.2}$$

We can then express the load per individual voice user as in (4.13) to be

$$\eta_{DL,j} = A_v \left(\rho + f_{j,G} \right) \tag{5.3}$$

and similarly the load per individual data user as in (4.13) to be

$$\eta_{DL,k} = A_d \left(\rho + f_{k,G} \right). \tag{5.4}$$

As per (4.12) the current system load is given by

$$\eta_{DL} = \underbrace{\sum_{j=1}^{N_v} A_v \left(\rho + f_{j,G} \right)}_{\text{Load contribution from voice users}} + \underbrace{\sum_{k=1}^{N_d} A_d \left(\rho + f_{k,G} \right)}_{\text{Load contribution from data users}} \tag{5.5}$$

and the maximum system load is now

$$\eta_{DL,\max} = 1 - \underbrace{\sum_{j=1}^{N_v} A_v \left(\frac{C}{h_{j,G}} \right)}_{\text{Load contribution from voice users}} - \underbrace{\sum_{k=1}^{N_d} A_d \left(\frac{C}{h_{k,G}} \right)}_{\text{Load contribution from data users}} \tag{5.6}$$

5.2.1 Admission Condition

Using our current system load (5.5), along with the maximum system load (5.6) and load per individual voice user (5.3), we can express the following admission constraint that must be satisfied in order to accept the new $(N_v+1)^{th}$ voice call, as in the simplified form shown in (4.23) to be

$$\underbrace{\sum_{j=1}^{(N_v+1)} A_v\left(f_{pl,j}\right)+A_v\rho\left(N_v+1\right)}_{\text{Load contribution from voice users}}+\underbrace{\sum_{k=1}^{(N_d)} A_d\left(f_{pl,k}\right)+A_d\rho\left(N_d\right)}_{\text{Load contribution from data users}}<1. \tag{5.7}$$

Similarly we can define the following admission constraint that must be satisfied to accept the new $(N_d+1)^{th}$ data call, as in the simplified form shown in (4.23) to be

$$\underbrace{\sum_{j=1}^{(N_v)}\left(A_v f_{pl,j}\right)+A_v\rho\left(N_v\right)}_{\text{Load contribution from voice users}}+\underbrace{\sum_{k=1}^{(N_d+1)} A_d\left(f_{pl,k}\right)+A_d\rho\left(N_d+1\right)}_{\text{Load contribution from data users}}<1 \tag{5.8}$$

5.2.2 Resource Reservation

The basic idea from [Narrainen 2000] is that the higher priority classes have privilege over the lower priority classes at call admission phase. This policy ensures that even under relatively heavy loads, the higher priority classes have more opportunities to be accepted, resulting in a low blocking probability for these higher classes at the expense of the lower classes.

In order to prioritise voice calls over data calls at admission we employ a resource reservation scheme. We reserve (1- β) capacity by modifying the admission conditions (5.7) and (5.8) to include β as described below. β is some fixed value < 1. If β=1 then voice and data have equal priority since we do not reserve any of the maximum system load. We can now express the admission condition (5.7), including the resource reservation, for the admission of a new new $(N_v+1)^{th}$ voice call as

$$Z^v{}_{Nv,Nd} \leq D^v{}_{Nv,Nd} \tag{5.9}$$

where

$$Z^v{}_{Nv,Nd} = A_v\sum_{j=1}^{(N_v+1)}\left(f_{pl,j}\right)+A_v\rho\left(N_v+1\right)+A_d\sum_{k=1}^{(N_d)}\left(f_{pl,k}\right)+A_d\rho\left(N_d\right) \tag{5.10}$$

and

$$D^v{}_{Nv,Nd} = 1. \tag{5.11}$$

The admission condition (5.8), including the resource reservation for the admission of a new new $(N_d+1)^{th}$ data call is given by

$$Z^d{}_{Nv,Nd} \leq D^d{}_{Nv,Nd} \tag{5.12}$$

where

$$Z^d{}_{Nv,Nd} = A_v\sum_{j=1}^{(N_v)}\left(f_{pl,j}\right)+A_v\rho\left(N_v\right)+A_d\sum_{k=1}^{(N_d+1)}\left(f_{pl,k}\right)+A_d\rho\left(N_d+1\right) \tag{5.13}$$

and

$$D^{d}{}_{Nv,Nd} = \beta . \tag{5.14}$$

The above equations in effect mean that voice and data calls share a fraction β, of the entire capacity = 1 and that voice calls only, have access to this reserved remainder of (1- β) of the total capacity. The following figure illustrates this concept.

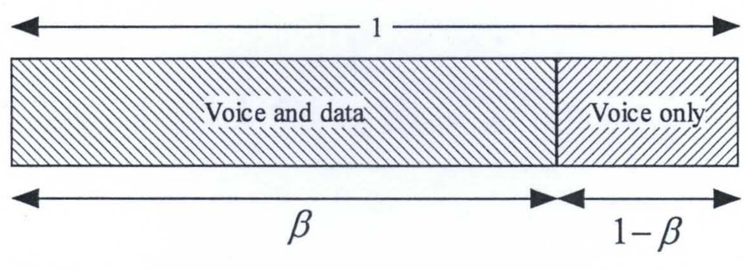


Figure 5-1 Illustration of resource reservation

5.3 Probability of Accepting a New Call

Given our admission conditions above which include resource reservation we can now define the probability of accepting a new voice call, given that there are N_v voice calls and N_d data calls currently admitted into the system and in progress to be

$$E^{v}{}_{Nv,Nd} \equiv \Pr\left(Z^{v}{}_{Nv,Nd} \le D^{v}{}_{Nv,Nd}\right). \tag{5.15}$$

We also define the probability of accepting a new data call given that there are N_v voice calls and N_d data calls currently admitted into the system and in progress to be

$$E^{d}{}_{Nv,Nd} \equiv \Pr\left(Z^{d}{}_{Nv,Nd} \le D^{d}{}_{Nv,Nd}\right). \tag{5.16}$$

In order to numerically determine the above probabilities to be used in our teletraffic analysis we model both $Z^{v}{}_{Nv,Nd}$ and $Z^{d}{}_{Nv,Nd}$ as lognormal random variables. We require numerical values for the mean and second moment of both $Z^{v}{}_{Nv,Nd}$ and $Z^{d}{}_{Nv,Nd}$ in order to use the lognormal distribution to determine an actual probability. Both $Z^{v}{}_{Nv,Nd}$ and $Z^{d}{}_{Nv,Nd}$ are functions of the random variable $f_{pl,l}$ which we have statistically characterised in the previous chapter and assumed a lognormal distribution for. The other components of $Z^{v}{}_{Nv,Nd}$ and $Z^{d}{}_{Nv,Nd}$ are constants which can be readily determined.

Since $Z^{v}{}_{Nv,Nd}$ and $Z^{d}{}_{Nv,Nd}$ are in fact summations of $f_{pl,l}$ and other constants and if we further assume independence between the components of $Z^{v}{}_{Nv,Nd}$ and $Z^{d}{}_{Nv,Nd}$ we can use Wilkinsons's approximation that the sum of independent lognormal random variables is another lognormal random variable [Cho, 2002], [Beaulieu, 1995]. The conventional Wilkinsons approximation states that the sum of lognormal random variables, $\sum L_i$ is another lognormal random variable L_s. The mean and variance of L_s is determined by matching the first and second moment of $\sum L_i$ with that of L_s. The mean and second moment of $Z^{v}{}_{Nv,Nd}$ and $Z^{d}{}_{Nv,Nd}$ is determined below using the mean and second moment of $f_{pl,l}$ derived in Chapter 4.

5.3.1 First Moment

Using Wilkinson's moment matching approach [Beaulieu, 1995], and using the fact that you can distribute the expectation operator and that $f_{pl,l}$ is identically distributed for every user, upon taking the expectation of (5.10) and using the substitution for $\mathrm{E}(f_{pl,l})$ from (4.105) we get

$$\begin{aligned} m_{Z^v(Nv,Nd)} &\equiv \mathrm{E}\left(Z^{v}{}_{Nv,Nd}\right) \\ &= \mathrm{E}\left(A_v \sum_{j=1}^{(N_v+1)}\left(f_{pl,j}\right) + A_v\rho(N_v+1) + A_d \sum_{k=1}^{(N_d)}\left(f_{pl,k}\right) + A_d\rho(N_d)\right) \\ &= (N_v+1)\left(A_v\,\mathrm{E}\left(f_{pl,j}\right)+A_v\rho\right)+(N_d)\left(A_d\,\mathrm{E}\left(f_{pl,k}\right)+A_d\rho\right) \\ &= (N_v+1)\left(A_v m_{fpl}+A_v\rho\right)+(N_d)\left(A_d m_{fpl}+A_d\rho\right) \end{aligned} \tag{5.17}$$

and taking the expectation of (5.13) and using the substitution for $\mathrm{E}(f_{pl,l})$ from (4.105) yields

$$\begin{aligned} m_{Z^d(Nv,Nd)} &\equiv \mathrm{E}\left(Z^{d}{}_{Nv,Nd}\right) \\ &= A_v \sum_{j=1}^{(N_v)}\left(f_{pl,j}\right) + A_v\rho(N_v) + A_d \sum_{k=1}^{(N_d+1)}\left(f_{pl,k}\right) + A_d\rho(N_d+1) \\ &= (N_v)\left(A_v\,\mathrm{E}\left(f_{pl,j}\right)+A_v\rho\right)+(N_d+1)\left(A_d\,\mathrm{E}\left(f_{pl,k}\right)+A_d\rho\right) \\ &= (N_v)\left(A_v m_{fpl}+A_v\rho\right)+(N_d+1)\left(A_d m_{fpl}+A_d\rho\right) \end{aligned} . \tag{5.18}$$

5.3.2 Second moment

The second moment of $Z^{v}{}_{Nv,Nd}$ using (5.10) is then given by

$$\psi_{Z^v(Nv,Nd)} \equiv \mathrm{E}\left(\left(Z^{v}{}_{Nv,Nd}\right)^2\right) = \mathrm{E}\left(\left(\sum_{j=1}^{(N_v+1)}\left(A_v f_{pl,j}+A_v\rho\right)+\sum_{k=1}^{N_d}\left(A_d f_{pl,k}+A_d\rho\right)\right)^2\right) \tag{5.19}$$

which we simply to

$$\psi_{Z^v(Nv,Nd)} = \mathrm{E}\left(\left(\underbrace{\sum_{j=1}^{(N_v+1)}\left(A_v f_{pl,j}\right)}_{a_1}+\underbrace{\sum_{k=1}^{N_d}\left(A_d f_{pl,k}\right)}_{b_1}+\underbrace{\left((N_v+1)A_v\rho+(N_d)A_d\rho\right)}_{c_1}\right)^2\right). \tag{5.20}$$

We begin with the square of a trinomial rule shown below and distribute the expectation operator and use the fact that $f_{pl,l}$ is identically distributed for every user. We then have

$$\mathrm{E}\left(a_1+b_1+c_1\right)^2 = \mathrm{E}\left(a_1^2\right)+\mathrm{E}\left(b_1^2\right)+\mathrm{E}\left(c_1^2\right)+\mathrm{E}\left(2a_1b_1\right)+\mathrm{E}\left(2a_1c_1\right)+\mathrm{E}\left(2b_1c_1\right) \tag{5.21}$$

with

$$\begin{aligned} a_1 &= \sum_{j=1}^{(N_v+1)} \left(A_v f_{pl,j}\right) \\ b_1 &= \sum_{k=1}^{(N_d)} \left(A_d f_{pl,k}\right) \\ c_1 &= \left((N_v+1) A_v \rho + (N_d) A_d \rho\right) \end{aligned} \quad . \tag{5.22}$$

Starting with the first term we get

$$\mathrm{E}\left(a_1^2\right) = \mathrm{E}\left(\sum_{j=1}^{(N_v+1)} \left(A_v f_{pl,j}\right)\right)^2 = A_v^2 \sum_{j=1}^{(N_v+1)} \mathrm{E}\left(f_{pl,j}\right)^2 + A_v^2 \sum_{j=1}^{(N_v+1)} \sum_{jj=1, jj \neq j}^{(N_v+1)} \mathrm{E}\left(f_{pl,j}\right) \mathrm{E}\left(f_{pl,jj}\right) \tag{5.23}$$

and rewriting using our expression for the mean and second moment of $f_{pl,l}$, (4.105) and (4.110) we get

$$\mathrm{E}\left(a_1^2\right) = A_v^2 \left(N_v+1\right) \psi_{fpl} + A_v^2 \left(N_v+1\right)\left(N_v\right) m_{fpl}^{\ 2} . \tag{5.24}$$

Similarly the second term in (5.21) can be expressed as

$$\mathrm{E}\left(b_1^2\right) = A_d^2 \left(N_d\right) \psi_{fpl} + A_d^2 \left(N_d\right)\left(N_d - 1\right) m_{fpl}^{\ 2} \tag{5.25}$$

and the third as

$$\mathrm{E}\left(c_1^2\right) = \left((N_v+1) A_v \rho + (N_d) A_d \rho\right)^2 . \tag{5.26}$$

The fourth term in (5.21) is

$$\mathrm{E}\left(2 a_1 b_1\right) = A_v A_d \left(N_v+1\right)\left(N_d\right) m_{fpl}^{\ 2} \tag{5.27}$$

the fifth term,

$$\mathrm{E}\left(2 a_1 c_1\right) = 2\left(c_1\right)\left(N_v+1\right) A_v m_{fpl} \tag{5.28}$$

and finally the sixth term is

$$\mathrm{E}\left(2 b_1 c_1\right) = 2 c_1 \left(N_d\right) A_d m_{fpl} . \tag{5.29}$$

Using the above expressions (5.21) through to (5.29) we can now substitute back into (5.20) to yield the second moment of $Z^{v}{}_{Nv,Nd}$ to be

$$\begin{aligned} \psi_{Z^v(Nv,Nd)} &= \mathrm{E}\left(a_1^2\right) + \mathrm{E}\left(b_1^2\right) + \mathrm{E}\left(c_1^2\right) + \mathrm{E}\left(2a_1 b_1\right) + \mathrm{E}\left(2 a_1 c_1\right) + \mathrm{E}\left(2 b_1 c_1\right) \\ &= A_v^2 \left(N_v+1\right) \psi_{fpl} + A_v^2 \left(N_v+1\right)\left(N_v\right) m_{fpl}^{\ 2} \\ &\quad + A_d^2 \left(N_d\right) \psi_{fpl} + A_d^2 \left(N_d\right)\left(N_d-1\right) m_{fpl}^{\ 2} \\ &\quad + \left((N_v+1) A_v \rho + (N_d) A_d \rho\right)^2 \\ &\quad + A_v A_d \left(N_v+1\right)\left(N_d\right) m_{fpl}^{\ 2} \\ &\quad + 2\left(c_1\right)\left(N_v+1\right) A_v m_{fpl} \\ &\quad + 2 c_1 \left(N_d\right) A_d m_{fpl} \end{aligned} \quad . \tag{5.30}$$

The second moment of $Z^{d}{}_{Nv,Nd}$ is given by

$$\psi_{Z^d(Nv,Nd)} \equiv \mathrm{E}\left(\left(Z^{d}{}_{Nv,Nd}\right)^2\right) = \mathrm{E}\left(\left(\sum_{j=1}^{N_v}\left(A_v f_{pl,j} + A_v \rho\right) + \sum_{k=1}^{(N_d+1)}\left(A_d f_{pl,k} + A_d \rho\right)\right)^2\right) \tag{5.31}$$

which we simply to

$$\psi_{Z^d(Nv,Nd)} = \mathrm{E}\left(\left(\underbrace{\sum_{j=1}^{N_v}\left(A_v f_{pl,j}\right)}_{a_2} + \underbrace{\sum_{k=1}^{(N_d+1)}\left(A_d f_{pl,k}\right)}_{b_2} + \underbrace{((N_v)A_v\rho + (N_d+1)A_d)\rho}_{c_2}\right)^2\right) \tag{5.32}$$

We again use the square of a trinomial rule and distribute the expectation operator and use the fact that $f_{pl,l}$ is identically distributed for every user as below. We then have

$$\mathrm{E}\left(a_2 + b_2 + c_2\right)^2 = \mathrm{E}\left(a_2^{\,2}\right) + \mathrm{E}\left(b_2^{\,2}\right) + \mathrm{E}\left(c_2^{\,2}\right) + \mathrm{E}\left(2a_2b_2\right) + \mathrm{E}\left(2a_2c_2\right) + \mathrm{E}\left(2b_2c_2\right) \tag{5.33}$$

with

$$\begin{aligned} a_2 &= \sum_{j=1}^{(N_v)}\left(A_v f_{pl,j}\right) \\ b_2 &= \sum_{k=1}^{(N_d+1)}\left(A_d f_{pl,k}\right) \\ c_2 &= \left((N_v)A_v\rho + (N_d+1)A_d\rho\right) \end{aligned} \quad . \tag{5.34}$$

Starting with the first term and using our expression for the mean and second moment of $f_{pl,l}$, (4.105) and (4.110) we get

$$\mathrm{E}\left(a_2^{\,2}\right) = A_v^{\,2}(N_v)\psi_{fpl} + A_v^{\,2}(N_v)(N_v - 1)m_{fpl}^{\;2}. \tag{5.35}$$

Similarly the second term in (5.33) can be expressed as

$$\mathrm{E}\left(b_2^{\,2}\right) = A_d^{\,2}(N_d+1)\psi_{fpl} + A_d^{\,2}(N_d+1)(N_d)m_{fpl}^{\;2} \tag{5.36}$$

and the third as

$$\mathrm{E}\left(c_2^{\,2}\right) = \left((N_v)A_v\rho + (N_d+1)A_d\rho\right)^2. \tag{5.37}$$

The fourth term in (5.33) is

$$\mathrm{E}\left(2a_2b_2\right) = A_vA_d(N_v)(N_d+1)m_{fpl}^{\;2} \tag{5.38}$$

the fifth term,

$$\mathrm{E}\left(2a_2c_2\right) = 2(c_2)(N_v)A_vm_{fpl} \tag{5.39}$$

and finally the sixth term is

$$\mathrm{E}(2b_2c_2) = 2c_2(N_d+1)A_d m_{fpl}. \tag{5.40}$$

Using the above expressions (5.33) through to (5.40) we can now substitute back into (5.32) to yield the second moment of $Z^d{}_{Nv,Nd}$ to be

$$\begin{aligned}
\psi_{Z^d(Nv,Nd)} &= \mathrm{E}(a_2^2)+\mathrm{E}(b_2^2)+\mathrm{E}(c_2^2)+\mathrm{E}(2a_2b_2)+\mathrm{E}(2a_2c_2)+\mathrm{E}(2b_2c_2) \\
&= A_v^2(N_v)\psi_{fpl} + A_v^2(N_v)(N_v-1)m_{fpl}^2 \\
&+A_d^2(N_d+1)\psi_{fpl} + A_d^2(N_d+1)(N_d)m_{fpl}^2 \\
&+((N_v)A_v\rho+(N_d+1)A_d\rho)^2 \\
&+A_vA_d(N_v)(N_d+1)m_{fpl}^2 \\
&+2(c_2)(N_v)A_v m_{fpl} \\
&+2c_2(N_d+1)A_d m_{fpl}
\end{aligned} \tag{5.41}$$

At this stage we have the first and second moments for both $Z^v{}_{Nv,Nd}$ and $Z^d{}_{Nv,Nd}$ given by (5.17), (5.18), (5.30) and (5.41) respectively, in a form that be easily evaluated using our numerical values for m_{fpl} and ψ_{fpl} that can be calculated from (4.109) and (4.115) respectively as described in Chapter 4.

Using a lognormal distribution and the above numerical values we can deterministically evaluate (5.15) and (5.16) to yield the probability of accepting a new voice or a new data call. The probabilities are a function of the random variable $f_{pl,l}$, the other cell interference plus path loss and the number of currently admitted voice and data calls. These admission probabilities are then incorporated into the rate of transitions between states as discussed in the section below.

5.4 System Model

We begin by modelling the system using state variables and a two dimensional state space. We determine all possible transition between states in this state space. We define the system state as S = (j,k) where $j = N_v$, the number of admitted voice calls currently in the system, $k = N_d$, the number of admitted data calls currently in the system. The behaviour of the system is best described by the following state transitions.

5.4.1 Driving Processes and State Transitions

We begin with the generic state S = (j,k) and then determine all feasible neighbouring states. Since at any given point in time only one call, either a voice or data call may be admitted or may depart the system we can significantly reduce the number of feasible neighbouring states. This means that from S = (j,k) we can transition only from one state to the immediate neighbour by either incrementing or decrementing j or k. The probabilities of transitioning to all other states are zero. The four possible state transitions that we have are:

- New voice call arrives and is accepted:

$$S = (j,k) \rightarrow S = (j+1,k)$$

- New data call arrives and is accepted:

$$S = (j,k) \rightarrow S = (j,k+1)$$

- Voice call in system departs:

$$S = (j,k) \rightarrow S = (j-1,k)$$

- Data call in system departs:

$$S = (j,k) \rightarrow S = (j,k-1)$$

Before we examine the probabilities associated with each of four these state transitions above we describe our traffic processes as discussed in Section 1.3. Recall that we need to describe each traffic class, voice and data in terms of its arrival rate and holding time. The arrival rate determines how quickly calls arrive and the interarrival time, which is the inverse of the arrival rate, determines the time between calls. The service rate determines how quickly calls depart and the holding time, which is the inverse of the service rate, determines how long each call lasts. We make the conventional assumption that the random variable for the arrival rate follows a Poisson process and consequently the interarrival time becomes a negative exponential process. In addition we assume that the random variable for the service rate follows a negative exponential process [Narrainen 1999]. We define the total system arrival rate, λ_{tot} as the sum of the voice and data arrival rates such that

$$\lambda_{tot} = a_v \lambda_v + (1 - a_v) \lambda_d \qquad 0 < a_v < 1 . \tag{5.42}$$

We also define the system service rate for voice calls as μ_v, the inverse of the call holding time for voice calls, and similarly we define the service rate for data calls as μ_d.

We now focus our attention on the rate of transitions for these states where we incorporate the admission probabilities from (5.15) and (5.16).

- New voice call arrives and is accepted:

$$S = (j,k) \rightarrow S = (j+1,k)$$

Without any admission control the rate of this transition would simply be the rate of arrival of new voice calls given by λ_v, and if we assume some fixed number based capacity for voice users C_v, the end state in the dimension j would then be S = (C_v ,k). The state model in this dimension would then reduce to a fixed capacity model (M/M/C) as discussed in section 2.2.1.

We incorporate the admission probability for voice users from (5.15) to modify this rate transition to be

$$\omega_{j,k} = \lambda_v E^{v}{}_{j,k}\,. \tag{5.43}$$

The end state in the dimension j now becomes S = (∞,k) and the state model is now an infinite capacity model where the arrival process is described by a general distribution. Using Kendall's notation the state model in dimension j is now a $G/M/\infty$ model [Kendall 1953]. The probability of accepting a new voice call will asymptotically tend to zero as j increases so in effect at some value of j the probability of accepting a new voice call will cause the transition rate to tend to zero.

- New data call arrives and is accepted:

S = (j,k)→ S = (j,$k+1$)

Similarly for data calls we modify the rate of this transition to include the probability of accepting a new data call. The rate of this transition is then given by

$$\varepsilon_{j,k} \equiv \lambda_d E^{d}{}_{j,k}\,. \tag{5.44}$$

- Voice call in system departs:

S = (j,k)→ S = ($j-1$,k)

The rate of this transition is determined by how quickly voice calls depart from the system and is thus governed by the service rate. We multiply the voice call service rate by the current number of voice calls to yield a rate of transition of

$$\sigma_j = j\mu_v \tag{5.45}$$

- Data call departs:

S = (j,k)→ S = (j,$k-1$)

Similarly for data calls we multiply the data call service rate by the current number of data calls to yield a rate of transition of

$$\alpha_k = k\mu_d \tag{5.46}$$

Figure 5-2 below summarises our feasible neighbouring states and rates of state transitions for state S = (j,k).

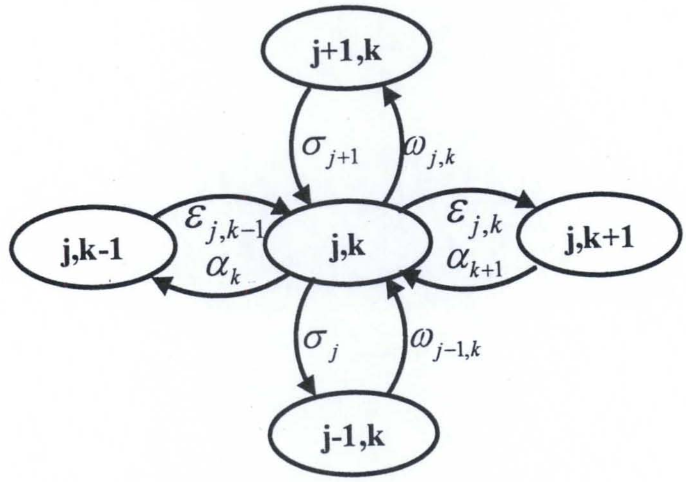
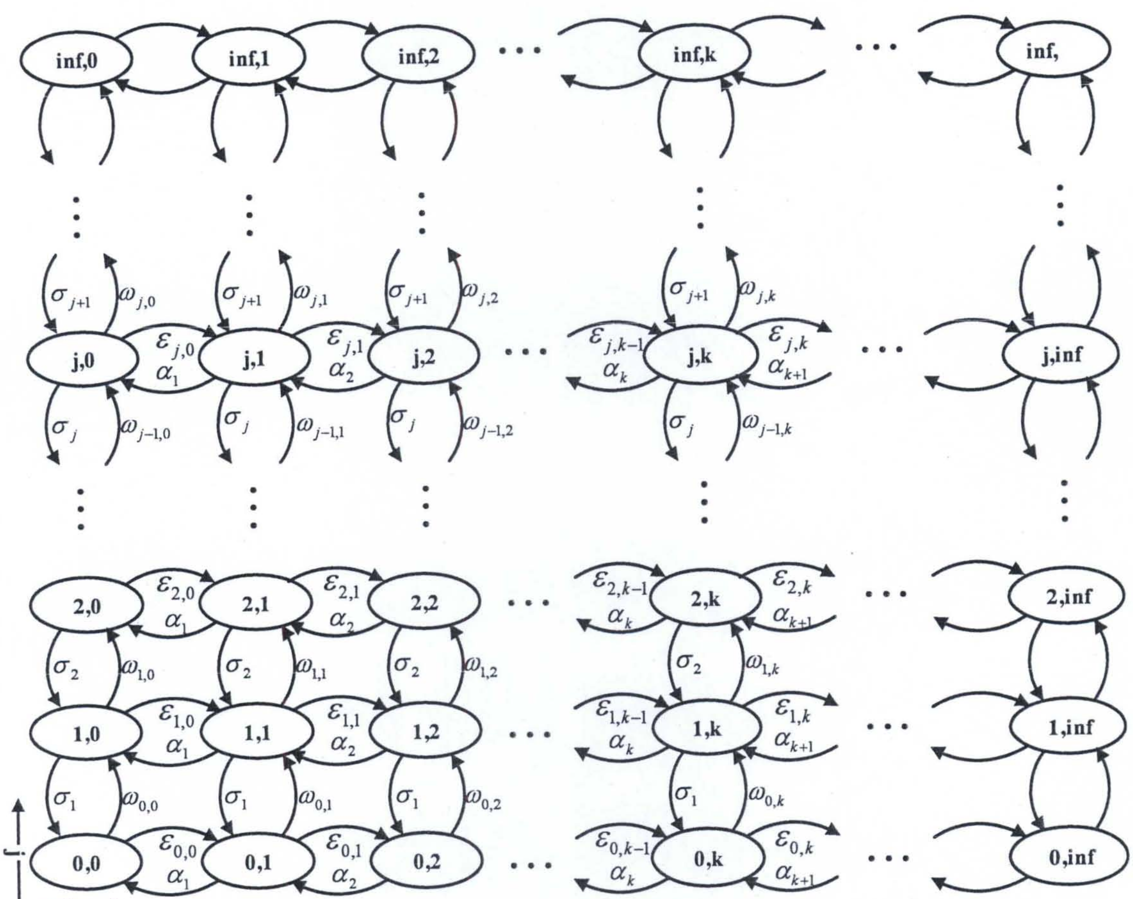


Figure 5-2 Feasible neighbouring states and rates of state transitions

We can then express a similar figure to show all possible states and state transition rates as shown in Figure 5-3 below.



$\alpha_k = k\mu_d$ $\varepsilon_{j,k} = \lambda_d E^d{}_{j,k}$ $\omega_{j,k} = \lambda_v E^v{}_{j,k}$ $\sigma_j = j\mu_v$

Figure 5-3 System state diagram for voice and data calls

5.4.2 Steady State

Every state in the system can be described in terms of the flow in and the flow out of that state into all other states. The flow is basically the rate of transitions in and out of that state. When we have reached steady state conditions the total flow into a state is equal to the total flow out of the state [Gross and Harris, 1985]. This means that the system is in equilibrium and the net rate of flow is zero for every state and we can thus determine the probabilities associated with every state. We can now define the steady state probability of the system being in state S = (*j*,*k*) as $\pi_{j,k}$. In addition we have the constraint below since the total sum of all probabilities must equal one,

$$\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\pi_{j,k}=1. \tag{5.47}$$

From Figure 5-2 above and assuming we are in steady state so that the total flow in is equal to the total flow out, we can write the following flow balance equation for state S = (*j*,*k*) including the probability of that state and its neighbouring states occurring as

$$\underbrace{\left(\varepsilon_{j,k}+\alpha_k+\omega_{j,k}+\sigma_j\right)\pi_{j,k}}_{\text{Total flow out}}=\underbrace{\alpha_{k+1}\pi_{j,k+1}+\varepsilon_{j,k-1}\pi_{j,k-1}+\sigma_{j+1}\pi_{j+1,k}+\omega_{j-1,k}\pi_{j-1,k}}_{\text{Total flow in}}. \tag{5.48}$$

We can then simplify the above to yield

$$\left(\varepsilon_{j,k}+\alpha_k+\omega_{j,k}+\sigma_j\right)\pi_{j,k}-\left(\alpha_{k+1}\right)\pi_{j,k+1}-\left(\varepsilon_{j,k-1}\right)\pi_{j,k-1}-\left(\sigma_{j+1}\right)\pi_{j+1,k}-\left(\omega_{j-1,k}\right)\pi_{j-1,k}=0.$$

$$\left[\left(\varepsilon_{j,k}+\alpha_k+\omega_{j,k}+\sigma_j\right)\quad-\left(\alpha_{k+1}\right)\quad-\left(\varepsilon_{j,k-1}\right)\quad-\left(\sigma_{j+1}\right)\quad-\left(\omega_{j-1,k}\right)\right]\begin{bmatrix}\pi_{j,k}\\ \pi_{j,k+1}\\ \pi_{j,k-1}\\ \pi_{j+1,k}\\ \pi_{j-1,k}\end{bmatrix}=[0]$$

(5.49)

The above flow balance equation is now in the form of $\underline{A}\underline{x}=\underline{b}$ where $\underline{A}$ is a vector of constants, $\underline{x}$ is the vector of variables that we are trying to solve for and $\underline{b}$ is a vector of constants which in this case is a zero vector.

At this stage we still have infinite values in our state space and thus in order to compute the steady state probabilities we use a very large value L_j and L_k relative to the number of feasible states in each dimension *j* and *k*, to emulate an infinite value in our state space [Rajaratnam and 2001].

For every state S = (*j*,*k*), we can write the above flow balance equation in the form of (5.49). We must take care when we determine the flow balance equations for the states at the limit of the state space, to ensure that the relevant state flows are set to zero. Once we have flow balance equations for each state, in the form of (5.49) we have a set of linear equations in the form of $A\underline{\pi}=\underline{0}$ where A is a square matrix as shown below including the emulated infinite values L_j and L_k. $\underline{\pi}$ is the vector of steady state probabilities that we are trying to solve for and $\underline{0}$ is zero vector. Note that the matrix has a dimension of $\left(L_j+1\right)\times\left(L_k+1\right)$ since it includes all flows from each state to every other state.

$$\begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & \cdots & \cdots & \cdots & \cdots & A_{1,((L_j+1)\times(L_k+1))} \\ A_{2,1} & A_{2,2} & & & & & & \vdots \\ \vdots & & \ddots & & & & & \vdots \\ \vdots & & & \ddots & & & & \vdots \\ \vdots & & & & \ddots & & & \vdots \\ \vdots & & & & & \ddots & & \vdots \\ \vdots & & & & & & \ddots & \vdots \\ A_{((L_j+1)\times(L_k+1)),1} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & A_{((L_j+1)\times(L_k+1)),((L_j+1)\times(L_k+1))} \end{bmatrix} \begin{bmatrix} \pi_{L_j,L_k} \\ \pi_{L_j,L_k-1} \\ \vdots \\ \pi_{L_j,0} \\ \pi_{L_j-1,L_k} \\ \pi_{L_j-1,L_k-1} \\ \vdots \\ \pi_{0,0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

(5.50)

The relationship between the elements of A above and the flow for a particular state is shown below. We reference elements within A using the variables (*row,col*) to identify rows and columns and map this to out state variables j and k as follows. Consider S = (j,k) and the four neighbouring states as shown in Figure 5-2.

For all the five state flow expressions below the initial values for (*row,col*) as a function of (j,k) are as follows

$$\begin{aligned} row &= \left(L_j+1\right)\times\left(L_j-j\right)+\left(L_k-k+1\right) \\ col &= row \end{aligned} \tag{5.51}$$

- Flow into S = (j,k)

The value in A (*row = row, col = col*) is given by the sum of the flow out of S = (j,k) into S = $(j+1, k)$, S = $(j-1, k)$, S = $(j, k+1)$ and S = $(j, k-1)$ given by

$$A\left(row=row,col=col\right)=\left(\varepsilon_{j,k}+\alpha_k+\omega_{j,k}+\sigma_j\right). \tag{5.52}$$

- Flow out of S = $(j+1, k)$ into S = (j,k)

The value in A (*row = row, col = col* - (L_k+1)) is given by the flow out of S = $(j+1, k)$ into S = (j,k) and is given by

$$A\left(row=row,col=col-\left(L_k+1\right)\right)=\alpha_{k+1}. \tag{5.53}$$

- Flow out of S = $(j-1, k)$ into S = (j,k)

The value in A ($row = row$, $col = col + (L_k+1)$) is given by the flow out of S = (j-1, k) into S = (j,k) and is given by

$$A\left(row = row, col = col + \left(L_k + 1\right)\right) = \omega_{j-1,k}. \tag{5.54}$$

- Flow out of S = (j, k+1) into S = (j,k)

The value in A ($row = row$, $col = col$ - 1) is given by the flow out of S = (j, k+1) into S = (j,k) and is given by

$$A\left(row = row, col = col - 1\right) = \alpha_{k+1}. \tag{5.55}$$

- Flow out of S = (j, k-1) into S = (j,k)

The value in A ($row = row$, col =col + 1) is given by the flow out of S = (j, k-1) into S = (j,k) and is given by

$$A\left(row = row, col = col + 1\right) = \varepsilon_{j,k-1} \tag{5.56}$$

This particular linear system in (5.50) is however underdetermined as one of the linear equations is a multiple of the other i.e. the rank of A is less than the number of rows in $\underline{\pi}$. We now have a system with more unknowns than equations which can lead to multiple solutions. In order to determine a unique solution we replace the last equation in (5.50) above with the constraint from (5.47). Using the form of (5.49) we can express the constraint from (5.47) as

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \pi_{L_j,L_k} \\ \pi_{L_j,L_k-1} \\ \vdots \\ \pi_{L_j,0} \\ \pi_{L_j-1,L_k} \\ \pi_{L_j-1,L_k-1} \\ \vdots \\ \pi_{0,0} \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} \tag{5.57}$$

and include this into (5.50) to yield

$$\begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & \cdots & \cdots & \cdots & \cdots & A_{1,((L_j+1)\times(L_k+1))} \\ A_{2,1} & A_{2,2} & & & & & & \vdots \\ \vdots & & \ddots & & & & & \vdots \\ \vdots & & & \ddots & & & & \vdots \\ \vdots & & & & \ddots & & & \vdots \\ \vdots & & & & & \ddots & & \vdots \\ \vdots & & & & & & \ddots & \vdots \\ 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \pi_{L_j,L_k} \\ \pi_{L_j,L_k-1} \\ \vdots \\ \pi_{L_j,0} \\ \pi_{L_j-1,L_k} \\ \pi_{L_j-1,L_k-1} \\ \vdots \\ \pi_{0,0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}. \tag{5.58}$$

We now have a square system that will yield an exact solution. We obtain the solution by using the traditional linear algebra technique of inverting matrix A and left multiplying by the vector on the RHS of (5.58) as shown below

$$\begin{bmatrix} \pi_{L_j,L_k} \\ \pi_{L_j,L_k-1} \\ \vdots \\ \pi_{L_j,0} \\ \pi_{L_j-1,L_k} \\ \pi_{L_j-1,L_k-1} \\ \vdots \\ \pi_{0,0} \end{bmatrix} = \begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & \cdots & \cdots & \cdots & \cdots & A_{1,((L_j+1)\times(L_k+1))} \\ A_{2,1} & A_{2,2} & & & & & & \vdots \\ \vdots & & \ddots & & & & & \vdots \\ \vdots & & & \ddots & & & & \vdots \\ \vdots & & & & \ddots & & & \vdots \\ \vdots & & & & & \ddots & & \vdots \\ \vdots & & & & & & \ddots & \vdots \\ 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \tag{5.59}$$

After plugging in the value for the elements of A using (5.51) through to (5.56), inverting the matrix and multiplying by the vector on the RHS, we will have obtained the steady state distribution $\underline{\pi}$ for all values of (j,k) in the state space constrained by the emulated infinite values L_j and L_k. We must then reconstruct this column vector of probabilities into a matrix of probabilities of dimension $(L_j + 1)$ by $(L_k + 1)$ to get the steady state probabilities in the following form to better illustrate $\pi_{j,k}$ as a function of number of current voice and data calls, (j,k)

$$\begin{bmatrix} \pi_{0,0} & \cdots & \cdots & \pi_{0,k} & \cdots & \cdots & \pi_{0,L_k} \\ \vdots & \ddots & & & & & \vdots \\ \vdots & & \ddots & & & & \vdots \\ \pi_{j,0} & & & \pi_{j,k} & & & \pi_{j,L_k} \\ \vdots & & & & \ddots & & \vdots \\ \vdots & & & & & \ddots & \vdots \\ \pi_{L_j,0} & \cdots & \cdots & \pi_{L_j,k} & \cdots & \cdots & \pi_{L_j,L_k} \end{bmatrix}. \tag{5.60}$$

5.4.3 System Model Output

At this stage we have a system that supports two types of calls, voice and data. Using the parameters that we defined in Table 4-1 and Table 4-2 we extend this to include the following parameters.

Table 5-1 Table of parameters and values for system model

Parameter	Value (Voice)	Value (Data)
Bitrate R	12.2Kbps	64Kbps
Activity factor α	0.5	0.8
(E_b/N_0)	7dB	3.5dB
Emulated infinite value	15	15

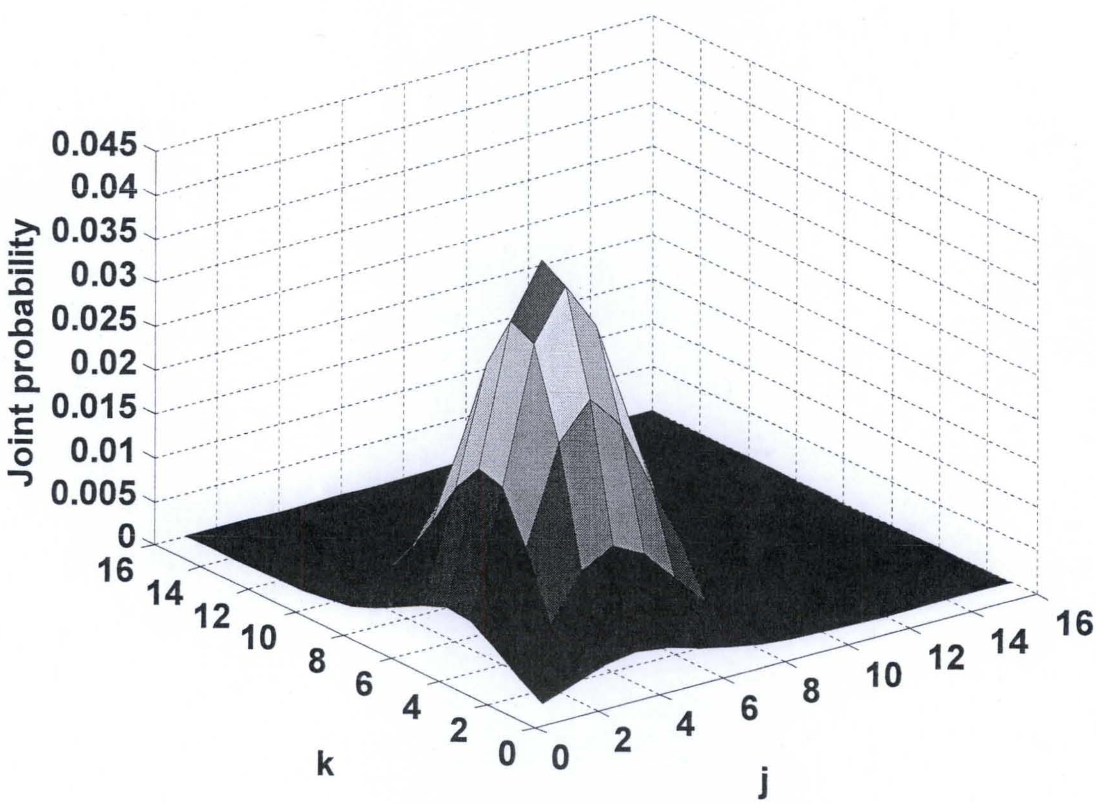


Figure 5-4 Surface plot of joint *pdf* $\pi_{j,k}$

We arrive at the emulated infinite value in Table 5-1 by empirically examining the steady state distribution as shown in Figure 5-4. From this figure we can see that value of the joint *pdf* tends to zero well before our emulated infinite values.

When there is no available capacity voice and data calls are blocked and lost to the system. Voice calls have priority over data calls by using a different admission criterion (resource reservation factor). Using our steady state distribution and the probability of accepting a new voice call (5.15) we can determine the blocking probability for voice calls as follows. Since a call is blocked if it not accepted the blocking probability of voice calls is given by the steady state distribution multiplied by one minus the probability of accepting a new voice call. The blocking probability, P_{b_v} is then given by

$$P_{b\_v} = \sum_{j=0}^{L_j} \sum_{k=0}^{L_k} \pi_{j,k} \left(1 - E^{v}{}_{j,k}\right) \tag{5.61}$$

Similarly for data calls the performance metric is the blocking probability, P_{b_d} and is given by

$$P_{b\_d} = \sum_{j=0}^{L_j} \sum_{k=0}^{L_k} \pi_{j,k} \left(1 - E^{d}{}_{j,k}\right) \tag{5.62}$$

We also calculate the mean number of voice calls in the system as

$$m_v = \sum_{j=0}^{L_j} \sum_{k=0}^{L_k} j\pi_{j,k} \tag{5.63}$$

and similarly for data calls we have

$$m_d = \sum_{j=0}^{L_j} \sum_{k=0}^{L_k} k\pi_{j,k} \tag{5.64}$$

5.5 Results

From the analytical model in this chapter, numerical results were generated and compared to simulation results obtained from the software simulation described in Chapter 3. We examine the blocking probabilities for voice and data calls from (5.61) and (5.62) respectively. The resource reservation factor β is set to 0.8 and the relationship between voice and data arrival rates is given by (5.42) with $a_v = 0.5$.

There is a substantial difference between the analytical and simulation results for voice calls as the arrival rate increases. Possible causes for this are that for higher arrival rates more calls are generated and thus a wider range of values in terms of the other cell interference plus path loss. If we examine our result for the other cell interference plus path loss in Figure 4-3 we see that the analytical model starts to deviate from the simulation at higher values of $f_{pl,l}$ albeit at very low probabilities. Nevertheless it is assumed that this has an effect on our analytical results. The validity of our assumptions particularly Wilkinsons approximation must also be questioned as possible causes for deviation. Bear in mind that the expression for the sum of independent lognormal random variables is intractable and thus we must resort to approximation.

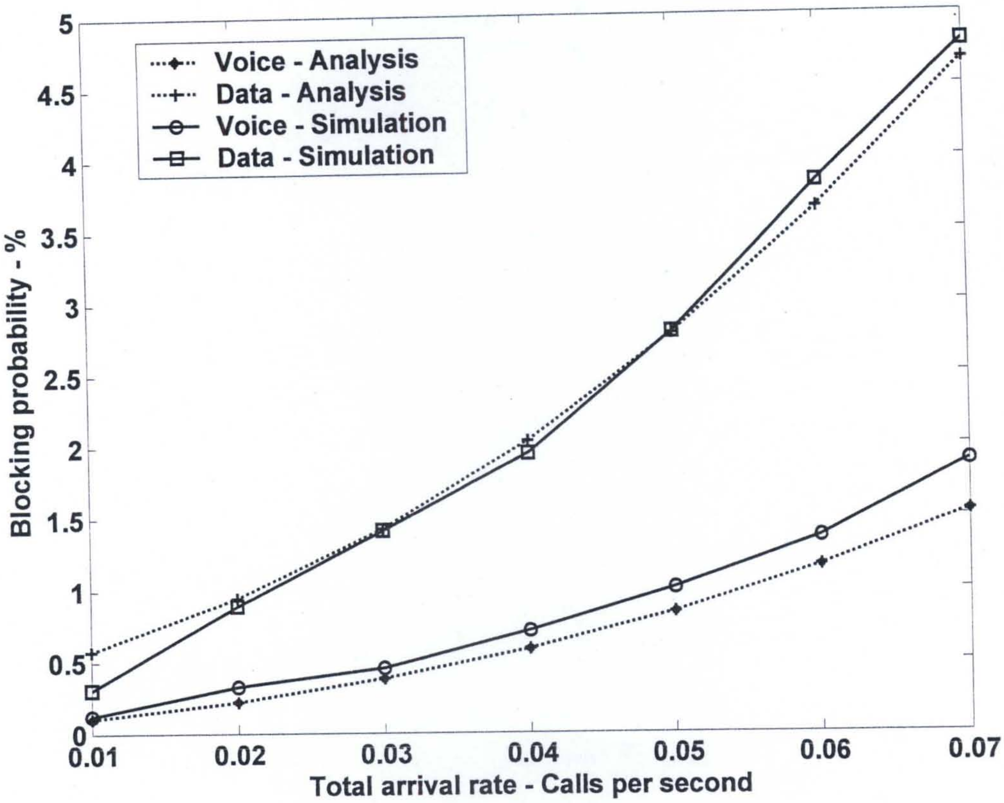


Figure 5-5 Blocking probability for voice and data call as a function of arrival rates $(\beta=0.8, a_v=0.5)$

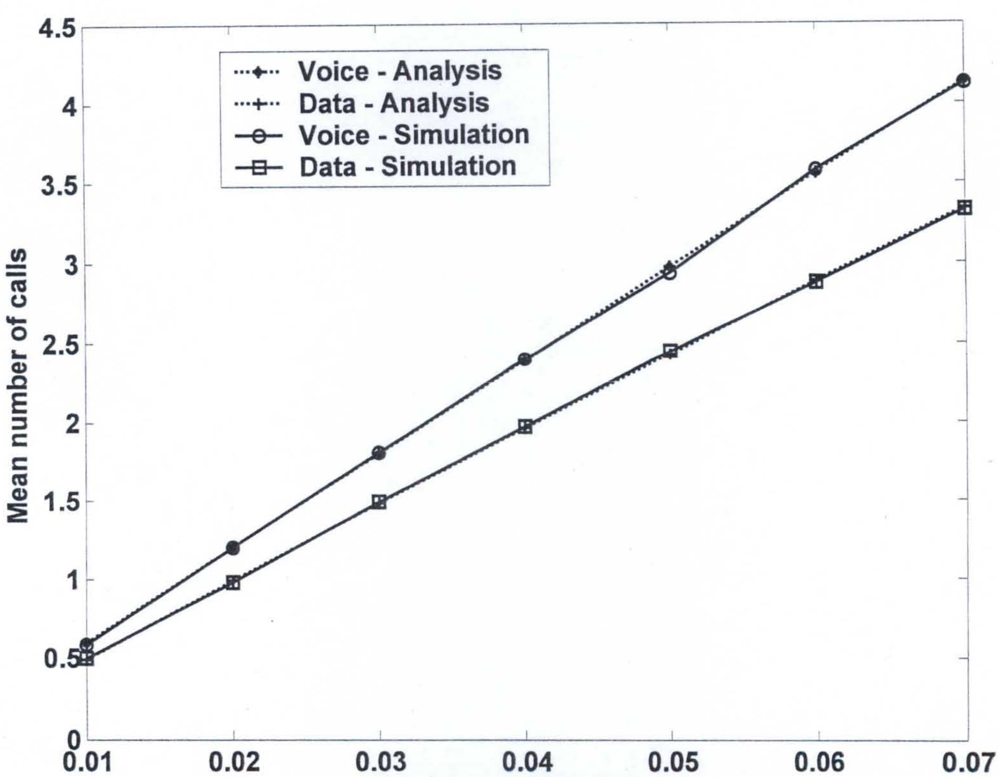


Figure 5-6 Mean number of voice and data calls as a function of arrival rates $(\beta=0.8, a_v=0.5)$

The mean number of voice and data calls from (5.63) and (5.64) respectively is plotted in the Figure 5-6 as a function of arrival rates. For this metric results compare favourably however this metric is not generally used in radio planning.

We also examine the effect of varying the resource reservation factor β on the blocking probabilities for both voice and data calls. We keep our total arrival rate fixed at 0.03 calls per second to produce Figure 5-7 below. The result below illustrates that as we decrease the resource reservation factor β, voice calls have a decreasing blocking probability and data calls have an increasing blocking probability. We are effectively prioritising voice calls at the expense of data calls. It is also evident that the blocking probabilities for data calls are severely impacted by varying β and the blocking probabilities for voice calls are much less impacted. This can be seen by examining the different scales of the left and right axes in Figure 5-7 below. This method of resource reservation is thus only effective for values of β close to 1. Smaller values of β produce marginal decreases in voice call blocking probabilities with substantial increase in data call blocking probabilities.

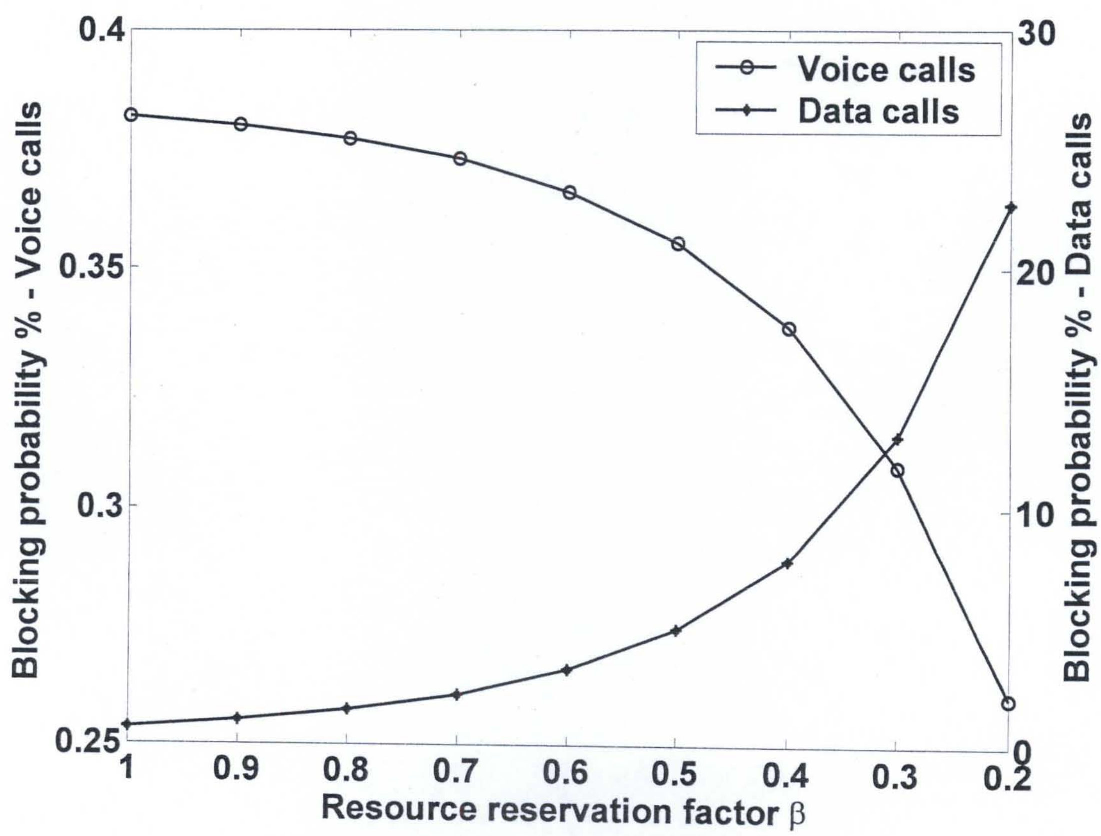


Figure 5-7 Blocking probabilities for voice and data calls as a function of β

5.6 Summary

In this chapter we extended our load-based call admission policy introduced in the previous chapter to two service classes, viz. voice and data and developed a mathematical model for this system. We introduced the concept of prioritising voice calls over data calls by using a resource reservation factor. Using the analytical expressions for the other cell interference plus path loss, with Wilkinson's approximation and other assumptions, we derived expressions for the acceptance probabilities for both voice and data calls. We then modelled the system with a two-dimensional Birth-Death Markov model. The effects of the time varying capacity were incorporated into the arrival process through the acceptance probabilities such that our model reduced to a $G/M/\infty$ system. From our model we determined the steady state distribution for this system and from this determined the blocking probabilities and mean number of calls in the system, for both voice and data calls, as a function of arrival rates. We presented our results illustrating some discrepancies between our analytical and simulation model and briefly attempted to explain this deviation. The possible causes for this deviation are the Wilkinsons approximation with respect to the assumptions regarding lognormal random variables. Bear in mind that the expression for the sum of independent lognormal random variables is intractable and thus we must resort to approximation. Furthermore the analytical model for the other cell interference plus path loss starts to deviate from the simulation model at higher values of $f_{pl,l}$ albeit at very low probabilities. It is assumed that this has an effect on our analytical results. We also investigated the effect of varying the resource reservation factor β on the blocking probabilities for both voice and data calls. With this method we are effectively prioritising voice calls at the expense of data calls. This method of resource reservation is only effective for values of β close to 1.

CHAPTER 6

CONCLUSION

6.1 Dissertation Summary

Next generation wireless cellular networks offer a myriad of voice, video, data and text based information services for the future multimedia and information society. With multiple services competing for the same resource and with the inherent soft capacity nature of CDMA, call admission control becomes a formidable task. The problem is further compounded by the introduction of priorities between classes. In this dissertation we focused on downlink call admission control in mixed service CDMA cellular networks. This work was prompted by the ever increasing adoption of CDMA as the wireless technology of choice and its potential to provide ubiquitous computing and communication capabilities.

Our objective, which we have achieved, was to present an analytical model that can successfully be used in teletraffic performance analysis. Our analytical model can be used to yield customer oriented grade-of-service parameters such as call blocking probability which is essential for network planning. In our analysis we incorporated a Birth-Death Markov queuing model. This mathematical model was verified though computer simulation and some discrepancies and well as the boundaries of accuracy between the two were briefly discussed. One of the shortcomings of our approach has been the assumptions regarding the independence of certain lognormal random variables as well as the intractable nature of the distribution of the sum of independent lognormal random variables.

We began this dissertation by introducing the different types of cellular networks and what characterises each generation. We then touched on admission control in cellular networks and finally we examined the basic principles of teletraffic modelling and performance analysis.

In our second chapter we reviewed the literature on call admission control in CDMA cellular networks starting with more details on the basic principles of admission control. We covered the different classes of admission policies and lastly justify our choice of a downlink load-based admission control algorithm. Some of the more common choices for downlink call admission control include number based as well as power based call admission policies. A load-based call admission policy was chosen as the maximum load threshold that can be supported varies with the state of the system and thus effectively models the behaviour of a soft capacity CDMA network.

The simulation model was examined in detail in Chapter 3. We began with the basic principles of simulation modelling and then described the basic design of our software simulator for a generalised call admission policy. We incorporated a wrap around technique for the basic cellular structure such that cells are modelled so as to prevent them from being lost to the network. We then described the statistical distributions that we have assumed for the propagation gain followed by some of our assumptions for the simulation. Finally we described a high level flow diagram of the entire simulation where the main routine is split into several subroutines where each subroutine performs a task specific to a particular event. A basic flow diagram and pseudo code for each major subroutine was included.

In Chapter 4 we presented our proposed load-based admission policy for a single class of service for downlink CDMA networks. Examining this admission policy in detail we needed to statistically characterise the other cell interference plus path loss in order to express our admission policy in a closed form to subsequently determine an admission probability. We proceeded to determine the first and second moment of the other cell interference plus path loss and then resorted to Wilkinson's approximation to obtain the closed form expression. We incorporated into our analysis the best BS selection criteria where the user connects to the BS with the strongest path gain. Then we presented a favourable comparison between our simulation and analytical results for the other cell interference plus path loss as well as the justification for the best BS selection criteria. Based on these reasonable results we proceeded with a certain level of confidence into the next chapter.

Chapter 5 extended our proposed load-based call admission policy introduced in the previous chapter to two service classes, viz. voice and data and we developed a mathematical model for this system. We introduced the concept of prioritising voice calls over data calls by using a resource reservation factor. Using the analytical expressions for the other cell interference plus path loss, with Wilkinson's approximation and other assumptions, we derived expressions for the acceptance probabilities for both voice and data calls. We then modelled the system with a two-dimensional Birth-Death Markov model. The effects of the time varying capacity were incorporated into the arrival process through the acceptance probabilities such that our model reduced to a $G/M/\infty$ system. From our model we determined the steady state distribution for this system and from this determined the blocking probabilities and mean number of calls in the system, for both voice and data calls, as a function of arrival rates. We presented our results illustrating some discrepancies between our analytical and simulation model and briefly attempted to explain this deviation.

Despite some discrepancies between our analytical and simulation model, we have nevertheless presented an original analytical model that has built a foundation for further work as discussed below.

6.2 Future Work

- This analytical model can be extended to a lossless system where blocked calls are queued. A similar model can be developed based on [Rajaratnam 2001] who applied [Cobham 1954]'s splitting formula for queued calls in a lossless system. The basic idea is to use splitting formula to extend a single queued class of calls to multiple classes

- The validity and effect of Wilkinsons approximation on this analytical model is another area of focus since the expression for the sum of independent lognormal random variables is intractable.

- The mobility model can also be extended such that intracell and intercell handover can be incorporated into the analytical model.

- Recent work has found that the data traffic in wireless data networks can be described statistically by self-similarity models. It is worth exploring the use of these models to represent data users.

REFERENCES

[3GPP TR 25.922 2002] 3GPP Radio resource management strategies, http://www.3gpp.org, TR 25.922 V5.0.0, Mar. 2002.

[Beaulieu, N.C., Abu-Dayya, A.A. & McLane, P.J. 1995] "Estimating the Distribution of a Sum of Independent Lognormal Random Variables", *IEEE Transactions on Communications*, vol. 43, No. 12, Dec. 1995.

[Burley, S. 2001(a)] "Downlink Capacity Estimation in a WCDMA Cellular Network" *12th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, vol. 1, pp. A26 - A30, Oct. 2001.

[Burley, S. 2001(b)] "Capacity of an Integrated LCD/UDD WCDMA Cellular Network" *3G Mobile Communication Technlogies*, Publication No. 477, pp. 58-62, Mar 2001.

[Capone, A., and Redana, S. 2001] "Call Admission Control Techniques for UMTS" *Vehicular Technology Conference*, vol. 2, pp. 925-929, Oct. 2001

[Cho, J. 2001] "Tradeoff Analysis of Throughput and Fairness on CDMA Packet Downlinks with Location-Dependent QoS" http://www.samsung.com/AboutSAMSUNG/SocialCommitment/HumantechThesis/downloads/8th/s10.*pdf*

[Cho, J. & Hong, D. 2002] "Statistical Model of Downlink Interference for the Performance Evaluation of CDMA Systems" *IEEE Communications Letters*, vol.6, No. 11, pp. 494-496, Nov 2002.

[Choi, J., Choi, Y. & Bahk, S. 2007] "Power-Based Admission Control for Multiclass Calls in QoS-Sensitive CDMA Networks" *IEEE Transactions on Wireless Communications*, vol. 6, No. 2, Feb 2007

[Cobham, A. 1954] "Priority Assignment in Waiting Line Problems", *Journal of the Operations Research Society of America*, vol. 2, pp. 70 – 76, 1954.

[Cooper, R. B. 1981] "*Introduction to Queueing Theory 2nd edition.*", Elsevier North Holland, United States of America, 1981.

[Dahlman E., Beming P., Knutsson J., Ovesjo F., Personn M. and Roobol C. 1998] "WCDMA-The Radio Inteface for Future Mobile Multimedia Communications", *IEEE Transactions On Vehicular Technology*, Vol. 47, No. 4, pp. 1105-1117, Nov 1998.

[Ericsson 2006] "*Ericsson WCDMA System Overview, Student Book LZT 123 6208 R5B*", 2006

[Evans J.S. and Everitt D. 1999(a)] "Effective Bandwidth-Based Admission Control for Multiservice CDMA Cellular Networks", *IEEE Transactions On Vehicular Technology*, Vol. 48, No. 1, pp. 36-46, Jan 1999.

[Evans J.S. and Everitt D. 1999(b)] "On the Teletraffic Capacity of CDMA Cellular Networks", *IEEE Transactions On Vehicular Technology*, Vol. 48, pp. 153-165, Jan 1999.

[Fapojuwo A.O. 1993] "Radio capacity of direct sequence code division multiple access mobile radio systems,"*IEE Proceedings*, Vol. 140, No. 5, pp. 402–408, Oct. 1993. – Cited in [Narrainen 1999]

[Gilhousen K.J., Jacobs I.M., Padovani R., Viterbi A.J., Weaver I.A. and Wheatly C.E 1991], "On the capacity of a cellular CDMA system", *IEEE Transactions on Vehicular Technology*, Vol. 40, pp. 303-312, May 1991.

[Glisic, S. & Vucetic, B. 1997] "*Spread Spectrum CDMA Systems for Wireless Communications*" Artec Publishers, Norwood, MA, 1997.

[Gross, D. and Harris, C.M. 1985] "*Fundamentals of Queuing Theory, 2nd edition*", John Wiley and Sons, United States of America, 1985.

[Hjelm B. 2000] "Admission Control in Future Multi-Service Wideband Direct-Sequence CDMA Systems", *IEEE Vehicular Technology Conference*, vol.3, pp. 1086-1093, Sep. 2000.

[Holma, H. and Toskala 2002] "*WCDMA for UMTS*", 2nd Edition, Wiley Technology Publishing, John Wiley & Sons, 2002.

[Holma, H. and Laakso, J. 1998] "Uplink admission control and soft capacity with MUD in CDMA", *IEEE Vehicular Technology Conference*, Vol.1, September 1998, pp. 431-435.

[Huang C. and Yates R.D. 1996] "Call admission in power controlled CDMA systems", *IEEE Vehicular Technology Conference*, Vol.3, pp. 1665-1669, 1996

[Ishikawa, Y. and Umeda, N. 1997] "Capcity Design and Performance of Call Admission Control in Cellular CDMA Networks", *IEEE Journal on Selected Areas in Communications*, Vol. 15, No. 8, 1997, pp. 1627 – 1635

[Ishikawa, Y. and Umeda, N. 1996] "Method of designing CDMA Capcity Considering Rate of Deterioration in Communication Quality", *Technical Report of IEICE RCS95-132*, Jan 1996

[Iverson V 2002] *"Handbook - Teletraffic Engineering"* ITU-D Study Group, Question 16/2, http://www.tele.dtu.dk/teletraffic March 2002, http://www.com.dtu.dk/teltraffic

[Jeon W. S. and Jeong D.G. 2002] "Call Admission Control for CDMA Mobile Communication Systems Supporting Multimedia Services" *IEEE Transactions on Wireless Communications*, Vol. 1, No. 4, pp. 649-659. Oct 2002.

[Kendall, D.G. 1953] "Stochastic Processes Occuring in the Theory of Queues and Their Analysis by the Method of Imbedded Markovb Chains", *Ann. Math Statistics*. 24, pp. 338-354, 1953 – Cited in [Narrainen 1999]

[Knutsson, J., Butovitsch, M., Persson M. and Yates R.D. 1997] "Evaluation of admission control algorithms for CDMA systems in a Manhattan environment", *Proceedings 2nd CDMA International Conference, CIC 1997*, pp 414-418. 1997. Cited in [Dahlman et.al. 1998]

[Knutsson, J., Butovitsch, M., Persson M. and Yates R.D. 1998] "Downlink Admission Control Strategies for CDMA is a Manhattan Environment", *IEEE Vehicular Technology Conference*, pp. 1453-1457, 1998.

[Leijon H] *"Introduction to Basic Teletraffic theory, ITU"* http://www.itu.int/itudoc/itu-d/dept/psp/ssb/planitu/plandoc/ttt.pdf

[Liu Z. and Zarki M 1994], "SIR based call admission control for DS-CDMA cellular systems", *IEEE Journal on Selected Areas in Communications*, Vol. 12, pp. 638-644. 1994

[Narrainen, R. P. 1999] "Call Admission Control in CDMA Cellular Networks" PhD Thesis, School of Electrical and Electronic Engineering, University of Natal, Durban, South Africa, 1999.

[Narrainen, R. P. & Takawira, F. 2000] "A Traffic Model for CDMA Cellular Systems with Soft Capacity Taken into Account" *IEEE Sixth International Symposium on Spread Spectrum Techniques and Applications*, vol. 1, Sept. 2000

[Pollonen, J. 2001] "Quality of Service Based Admission Control for WCDMA Mobile Systems", MSc Thesis, Department of Engineering Physics and Mathematics, Helsinki University of Technology, Finland, 2001.

[Papoulis, A. 1991] "*Probability, Random Variables, and Stochastic Processes, 3rd edition*" McGraw-Hill Book Company, Singapore, 1991.

[Rajaratnam M. & Takawira, F. 2001] "A Single Cell Model for the Performance Analysis of the Radio Layer in GSM Phase 2+ (GRPS) Networks under Voice and Data Traffic" *Proceedings of Personal Indoor Mobile and Radio Communications (PIMRC) Conference* , vol. 1, pp. C118-C123, 2001.

[Shin, S., Lee, K., and Kim, K. 2002] "Performance of the Packet Data Transmission Using the Other-Cell-Interference Factor in DS/CDMA Downlink", *Wireless Communications and Networking Conference, 2002,* vol.1 pp. 400-404, Mar 2002.

[Sipilä, K., Honkasalo, Z., Laiho-Steffens, J., and Wacker, A. 2000] "Estimation of Capacity and Required Transmission Power of WCDMA Downlink Based on a Downlink Pole Equation", *IEEE Vehicular Technology Conference*, vol. 2, pp. 1002-1005, 2000.

[Syski, R. 1986] "*Introduction to Congestion Theory in Telephone Systems, 2nd Edition*", Elsevier Science Publishers B.V., The Netherlands, 1986.

[Taub, H. & Schilling, D.L. 1986] "*Principles of Communications Systems, 2nd Edition*" McGraw-Hill Book Company, Singapore, 1986

[Visual Numerics, Inc. 1995] "IMSL(R) C Numerical Libraries Version 2.0, 1995.

[Viterbi, A.J., Viterbi, A.M., & Zehavi, E. 1994] "Other-cell interference in cellular power-controlled CDMA", *IEEE Transactions on Communications*, vol. 42, Issue: 234, pp:1501 – 1504, Feb/Mar/Apr 1994.

[Viterbi, A. M. & Viterbi, A. J. 1993] "Erlang Capacity of a Power Controlled CDMA System", *IEEE Journal on Selected Areas in Communications*, vol. 11, No. 6, pp. 892 – 900, Aug. 1993.

[Viterbi, A. J. 1995] "*CDMA - Principles of Spread Spectrum Communication*" Addison-Wesley Publishing Company, Massachusetts, United States of America, 1995.

[Wang, L & Zhuang, W. 2006] "A Call Admission Control Scheme for Packet Data in CDMA Cellular Communications", *IEEE Transactions on Wireless Communications*, vol. 5, No. 2, pp. 406 – 416, Feb 2006

[Yates, R. D. & Goodman, D. J. 1999] "*Probability and Stochastic Processes: A friendly introduction for Electrical and Computer Engineers*" John Wiley and Sons, United States of America, 1999.