The Effects of using Visual Literacy and Visualization in the Teaching and Learning of Mathematics Problem Solving on Grade 6 and Grade 7 learners

By

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DEDICATION

TO

The Divine Mother Saraswathi, the giver of knowledge, without whom this entire universe would be a living corpse for all time, I offer obeisance. By your will, I begin this process of learning. Let my efforts be crowned with success.

My mum Vindamuthee Budram and Late dad Budram Durgapersad for the sacrifices made in difficult times and instilling in me the value of education.

I, **RAJESH BUDRAM**, declare that the research involved in my dissertation submitted in the partial fulfillment of the Masters Degree in Mathematics Education, entitled **The Effects of using Visual Literacy and Visualization in the Teaching and Learning of Mathematics Problem Solving on Grade 6 and Grade 7 learners**, represents my own and original work.

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RAJESH BUDRAM

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DATE

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DR. V. MUDALY

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CONTENT

LIST OF FIGURES AND TABLES

ABSTRACT

In this study I examine the effects of visualization in the teaching of problem solving in grades 6 and 7 in a school south of Durban in KwaZulu Natal. One of the goals of mathematics instruction according to the Department of Education is to prepare learners to become proficient in solving problems (DoE, 2003). Whilst many studies have been conducted in the field of problem solving, using visualization as a strategy to solve problems has been a neglected area in mathematics teaching in some schools.

A literature survey shows that the link between solving problems and visualization strategies is making finding solutions easier for learners. The literature suggests that visualization assists learners to develop their problem solving skills as it allows them an opportunity to show their interpretation of the problem and the understanding of mathematical concepts. Through the use of problem centred mathematics, problem centred learning, growth of mathematical understanding and realistic mathematics education, learners see the connection and employ appropriate strategies to solve problems.

This study examines the strategies employed by educators in the teaching and learning of problem solving and the strategies used by learners when solving problems. Data was collected from educators using a questionnaire, observation of grade 6 and 7 learners in the classroom and semi structured interviews. The conclusions from the data analysis have shown that problem solving is been neglected and that visualization does assist learners in solving problems.

CHAPTER ONE

1.1 Introduction to the study

Expectations of educators for the 21st century are very different from what they were a generation ago. The mathematics curriculum is changing (Barb & Quinn, 1997) and the need for change paramount. The proposed changes include a revamping of essentially what mathematics educators teach, the method in which they teach, and the tools that they utilize while maintaining a rigorous mathematics curriculum. Willoughby (2000, p. 2) state that much of what was taught in school mathematics in the 19th and significant. Lott and Terry Souhrada cited in Burke and Curcio (2000, p. x) state that the *"mathematics of the twentieth century is not serving the needs of learners entering the twenty-first century"*.

With these changes has come the need to develop visually literate learners. We are living and teaching in a globally challenging and technologically evolving period in which visual literacy and visualization skills are critically needed for a clearer perception and interaction. Many 21st century educators are realising that visual literacy is crucial to life and are assisting learners develop visual literacies in order to exist and communicate in a vastly complex world (Tufte, 2008, p. 1). Diezmann (1995) state that visual literacy has become an increasingly significant component of communication and problem solving in everyday life and that visual literacy is now essential for extracting information, constructing knowledge and building successful educational outcomes (Bamford, 2003, p. 2).

Since the beginning of civilization, visual awareness has been a key component to communication and understanding. The drawings and symbols created by early man held a deep significance as written words holds significance for humanity in the 20th century (Pomona College, 1998).

Visual literacy is a relatively new research area in the field of mathematics and has not been able to attract enough interest from educators and curriculum planners. Whilst problem solving has been investigated from varying perspectives (Ballew and Cunningham, 1982; Malloy and Jones, 1998; Holton et al, 1999; Pantziara et al, 2004; Pape, 2004; Kotze & Strauss, 2007) there is a relative silence on using visual literacy as a skill and visualization as an alternate method in the teaching and learning of problem solving in mathematics. Balchin and Coleman cited in Diezmann (1995, p. 2) state that visual literacy is not considered significant and appropriate thus it *"remains a neglected field"*. Lida Cochran (1976) notes that in our present increasingly visual age where images are easily created more easily, learners need to be made aware of the power of visual literacy. Anything less denies them the preparation they need to be effective, knowledgeable and responsible learners.

The purpose of this study is to investigate whether visual literacy and visualization improves the teaching and learning of problem solving, which is a critical component of the mathematics curriculum (DoE, 2003, p. 27).

The National Education Policy documents call for a curriculum that is learner-centred, holistic, integrated and relevant to develop critical citizens who can participate actively and responsibly in a democratic, multi-cultural society. The reasoning behind such a move is to *"provide common goals for learners"* ... *"raise expectations"* and *"set directions for change"* (Burrill, 1998, p. 585). Through learner-centred teaching, learners will have to take responsibility for their own learning; they have to be independent in their learning and thinking processes and effective in their own learning (Department of Education, 1998).

Whilst learner-centred education creates hope for the masses, the South African mathematics curriculum requires deeper scrutiny. Lott and Souhrada cited in Burke and Curcio (2000, p. 103) maintain that the mathematics curriculum is not meeting the needs of our learners. Educators in countries like the United States of America and Canada now have the mathematics curriculum with specific content areas and levels to be attained by learners identified for them, something the South African education authorities are now advocating by making changes to the National Curriculum statement. This was only done after research was commissioned by the President Education Initiative (PEI) in 1998 when it was found that the implementation of the curriculum was proving problematic to educators and learners.

Changes in our society and the demographics of education as well as concerns about learners' achievements have prompted these changes. There is now a need to change the content material that we teach in mathematics as well as how we teach that content *"or we will not prepare learners for the world in which they will live"* (Burrill, 1998, p. 587). Due to the growing reliance of society on visual literacy, schools need to educate learners in visual literacy skills to allow them to participate equitably within society as *"schools have the responsibility to provide learners with opportunities to develop their visual faculties and in addition to provide an appropriate education for those learners who have a visual propensity"* (Diezmann, 1995). It is envisaged that visual literacy and visualization will be accorded its rightful place in the mathematics curriculum.

Due to the poor performance of the South African learners in international tests (Reddy, 2003) and the matriculation results over the years, the South African Education Department has implemented measures to ensure greater accountability on the part of educators in terms of their service delivery of the curriculum. This is particularly significant in the context of the national concern for improvement in the level of learning achievements. Schools academic performance are now been monitored by the Department of Education to ensure that educators and learners meet the norms as determined by the education department.

The question arises as to how mathematics can be taught more effectively. Educators have suggested that learners are unable to use school learned methods and rules due to not understanding them and the lack of understanding is contributed to the method that mathematics is taught (Boaler, 1998, p. 42). Fleisch (2007) state that although teaching methods have not changed dramatically the new methods implemented recently may be contributing to school failure.

Educators are now seeing mathematics in a fresh light with the emphasis now on making mathematics accessible and enjoyable for learners. There is a shift from the development of procedural knowledge and rote memorization to using open forms of mathematics to help solve real life problems and selecting appropriate methods to deduce solutions for these problems. Supporters of the open forms of mathematics argue *"that if learners are given open-ended, practical, and investigative work that requires them to make their own decisions, plan their own routes through tasks, choose methods, and apply their mathematical knowledge, the learners will benefit in a number of ways"* (Boaler, 1998, p. 42).

1.2 Motivation for this Study

As the study of learning and problem solving in mathematics advances and the complexity of the problem solving processes is recognised, there is still a need for further investigation in order to aid teaching practices. The researcher has been a mathematics educator for the last 24 years and has witnessed radical changes to the mathematics curriculum from the apartheid era to 2008. The mathematics curriculum in South Africa is becoming standardized and challenging, with advanced topics been introduced earlier. The standardization of the curriculum was to make mathematics more accessible to the masses. Based on the poor results as evident in the TIMMS 2003 (Reddy, 2003) and the systemic evaluation report (Department of Education, 2004), the education authorities did not consider the impact the changes will have on the learners. In their attempt to cure the ills in mathematics education the educational authorities, however, continue placing checks and balances to rectify the poor mathematics results. Contrasting styles of teaching and dissimilar research results (Holton et al, 1999; Pantziara et al, 2004; Lavy, 2006; Kotze and Strauss, 2007) have had significant implications in the way South African learners engage with problem solving.

The researcher has observed through the supervision of lesson preparation and attending workshops that educators in the schools are reluctant to incorporate the teaching of problem solving techniques into their teaching although it is a prescribed component as stated in the Mathematics Teachers Guide (Department of Education, 2003). This negativity has rubbed onto learners. As an educator and a sub-examiner for the AMESA Mathematics Olympiad, the researcher has noticed that learners are reluctant to participate in problem solving activities and programmes citing difficulty as the main reason. Although the researcher's learners have tasted success in the Mathematics Olympiads, the researcher was still discontent with the achievements of his learners.

The researcher started to seek alternate ways to teaching problem solving particularly for those learners who did not like mathematics and at the same time, he wanted mathematics teaching to place added importance on realistic problem solving situations. The National Curriculum Council in Britain (Jaworski, 1994, p. 8) state: *"using and applying mathematics... should stretch across and permeate all other work in mathematics providing both the means to, and the rationale for, the progressive development of knowledge, skills and understanding in mathematics"*.

The researcher intend to investigate whether visual literacy and visualization improves learners ability to problem solve.

1.3 Key research questions

This study was guided by the following questions to investigate

1. What visual strategies do educators engage in when encouraging learners to solve problems?
2. Do learners use visual strategies to solve mathematical problems?
3. How do visual strategies improve learners' ability to solve problems?

1.4 Structure of the Study

This study comprises six chapters, bibliography and appendices. The chapters in this study are as follows:

Chapter 1 introduces the background to the study. The key research questions are also stated together with the background of why this study was undertaken.

Chapter 2 presents the relevant literature on the areas under investigation namely visual literacy, visualization and problem solving.

Chapter 3 presents the theoretical framework for this study.

Chapter 4 presents the research design, the research methodology and procedures undertaken to complete this study. It also discusses the research instruments used to conduct this study.

Chapter 5 deals with the findings and analysis of the data obtained from the questionnaire, classroom observation, evaluation worksheet and semi structured interview.

Chapter 6 is the final chapter, which presents the conclusions to this study, and recommendations are made to be considered by the educators, those in the education fraternity and Department of Education.

CHAPTER TWO

LITERATURE REVIEW

2.1 Visual Literacy: Introduction

Children are accomplished visual learners and are able to read pictures well before they can read words. According to Ralph Haber cited in Sandell (2008, p. 2) *"the perceiver may see the world before he knows it"*. As infants, they scan the immediate surroundings to form an outline of details, which has significance to them. Since *"seeing comes before words"* the child looks and recognizes prior to it can speak (Berger cited in Stokes, 2002, p. 3). Children are able to assimilate visual images with ease. Education abandons using imagery after the formative years of schooling and *"even classifies the study of visual imagery as not only secondary to the acquisition of language, but as optional"* (Young, 2008, p.1). Visualization and visual literacy is grounded in children, the contentious issue is why we neglect this aspect, as children progress to become learners since it is documented that the use of visual literacy ideas and strategies enhances learning (Stokes, 2002).

Lida Cochran (1976) was one of the pioneers who championed the importance of visual literacy. Her work reminds us that visual literacy is not something that we add on in the classroom but is an integral part of meaning making. As educators, we must not only stress the addition of visuals in our learners work but we must provide them with information about how and when to use visuals.

Writer John Debes first used the term visual literacy in 1968 (Bamford, 2003). Many definitions and explanations of visual literacy have been considered over the years. According to Bamford (2003, p. 1) visual literacy *"is what is seen with the eye and what is seen in the mind"*. Robert Heinich et al cited in Rakes, Rakes and Smith (2008) state that *"visual literacy is the learned ability to interpret visual messages accurately and to create such messages. Thus the interpretation and creation in visual literacy can be said to parallel reading and writing in print legacy"*. Visual literacy is the ability to understand, create, and use visual images to communicate ideas and mathematical concepts in written form.

Visual Literacy, according to Lamb (2001); Stokes (2002) and Bamford (2003), is the ability to create and comprehend visual images in the mind. A visual image is a mental construct which helps the visualizer to understand and communicate to others what has been seen. Learners are constantly exposed to vast visual stimuli in the modern world, which the mind has to assimilate. In order to develop useful visual images the learner needs to acquire a range of visual literacy skills in order to read and comprehend images in a meaningful way.

Lamb (2001) state that visual literacy is a critical life skill and it has great implications in the teaching of mathematics. Bamford (2003) and Riesland (2005) state that visual literacy education must prepare learners from a young age to deal with what they will be exposed to during their lifetime because visual literacy is considered a critical life skill. This is further supported by Dake cited in Diezmann (1995, p, 2) who states that visual literacy is essential and foundational to learners' advancement in education.

The importance of visual literacy is evident in history. Eminent people like Kekule and Einstein acknowledged the importance of visual literacy as a result they had re-educated themselves to think more visually (Diezmann, 1995, p. 2).

Flattley cited in Stokes (2002, p. 3) state that *"the use of visual literacy ideas and strategies to enhance verbal learning is important"*. It is my belief that the non-acquisition of visual literacy skills is one of the main reasons for learners doing badly in mathematics.

2.1.1 Visual Literacy and Problem Solving

Stokes (2002, p. 1) defines the concept of visual literacy *"as the ability to interpret images as well as to generate images for communicating ideas and concepts"* whilst Hortin (cited in Diezmann, 1995) state that it is *"the ability to understand (read) and use (write) and to think and learn in terms of visual images"*. Problem solving is any complex situation that needs to be resolved, but which does not have a clear and unique method of solution.

Visual literacy and problem solving strategies are important tools to engage learners in visual processing, visual reasoning and critical thinking. Problem solving allows for innovation and encourages critical thinking in a learner. Visual Literacy gives the learners a chance to be creative and imaginative whilst the introduction to problem solving is a powerful way of creating significant learning for all. Both visual literacy and problem solving allows learners to elevate their thinking to a higher level. Hortin cited in Diezmann (1995, p. 2) argues that *"if verbal language or verbal literacy, i.e., to be literate, helps us to store information, provides us a means for transacting messages and gives us a method for problem solving and thinking, then visual language or literacy helps us to do the same"*, albeit differently.

Problem solving ought to be much easier with improved visual literacy skills.

2.1.2 Visual Thinking

Aristotle cited in Stokes (2002, p. 1) state, *"without images, thinking is impossible"*. Visual thinking is the ability to comprehend and communicate with the world around us. Sandell (2008, p. 2) state, *"images are key to comprehending and communicating with the world around us"*. It is an act of sense making, and rests on the process of specialising and generalising, conjecturing and justifying (Allen and Johnston-Wilder, 2004, p. 104). This component is closely connected to the process of questioning. Through questioning learners communicate their ideas verbally or through representation. They are able to transform thoughts, ideas into pictures or other images that will help them communicate the information. Thus through visual literacy learners can view their thought process as they represent their ideas visually.

Hadamard cited in Thornton (2008, p. 251) state that visual thinking has always been an important component in the philosophy of mathematicians but was not always an integral part of the mathematics classroom. Today mathematic educators are becoming more aware of the importance of mathematical and visual thinking (Nickson, 2000) and their responsibility to provide this link (Allen and Johnston-Wilder, 2004, p. 107). Mathematical thinking allows learners to explore related prior knowledge and to test its relevance to the problem. The learner is encouraged to use visual thinking, as *"the use of visualization in thinking appears to be increasing"* (Stokes, 2002, p. 2). Visual thinking assists learners to form mental images of their world as they provide reasons for their solution. This allows learners to make public their thought process.

Thornton (2008, p. 251) state that in view of the changing educational climate the role of visual thinking in mathematics must be re-evaluated. He put forth the following reasons:

1. There is a current trend to associate mathematics with patterns and modern technology. Using technology to discover a pattern is seen as devaluing algebraic thinking. My views are somewhat different. By using visual methods (which technology offers) to observe patterns it allows the observer to reflect on what has been seen, to generate and test hypothesis and then provide an algebraic proof. In effect, this does not devalue but rather increases the value of visual methods.

2. Visualization can present powerful approaches to developing mathematical results and solving problems and can also be used to form relations with other areas of mathematics.
3. Recognizing learner's diverse styles of learning and supporting them to develop a variety of techniques for examining mathematical situations.

According to Young (2008), learners must be given certain tools to think with. Visual thinking tools enhance chances of successful problem solving. Lavy (2007, p. 1075) state *"visualization enables various ways of thinking, different from traditional approaches where formalism and symbolism dominate teaching"*.

According to Hartley cited in Young (2008, p. 3) visual thinking tools **(Figure 1)** are symbols graphically linked by intellectual associations to produce a pattern of information and form knowledge about an idea. This helps to increase their understanding of the relationships between the elements of which the problem is composed and the construction of knowledge.

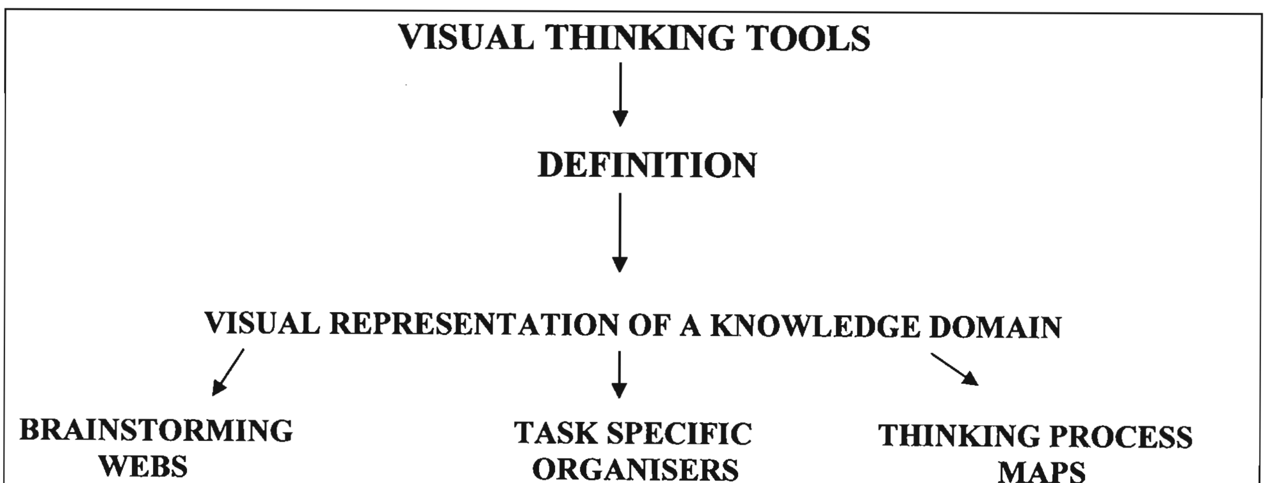


Figure 1: Graphic Representation of Visual Thinking Tool adapted from Young (2008, p. 3)

Young (2008, pp. 4-5) described the above as follows:

Brainstorming Webs are integrated, unstructured diagrams that start with a central idea and through free association, a relationship is formed with other ideas. Understanding grows as webs of interrelated ideas become more complex.

Task specific organisers are tools used to manage and display information and to learn particular skills and to understand processes in a specific area of knowledge.

Thinking-Process Maps are visual tools defined by fundamental and … global thinking processes, from constructing simple categories to developing new theories.

Visual Literacy allows for alternate thinking. Visual images are created in the mind and a representation of the problem is developed. Visual representation then frees the mind to create further representations to expand on the solution. Visual literacy as part of the teaching process is not only beneficial, but also necessary because visual literacy according to Dake cited in Diezmann (1995, p. 2) is considered as *"basic and foundational to the educational process"*.

2.1.3 Visual Communication

The Department of Education (2003) states that communication is one of the critical skills that needs to be developed. Learners need to be given various opportunities in mathematics to practice communicating through talking, reading, writing and listening and they must be able to *"use mathematical codes such as symbols and language that is peculiar to mathematics"* (DoE, 2003, p. 30).

In a visual communication era, the power of images is a rapidly evolving paradigm in the way humans communicate. *"Many types of messages are communicated visually and are inextricably intertwined ..."* (Fourie, 1996, p. 113). The goal is to present a clear, concise and easy to understand idea in a visualized form. Visual communication processes rely primarily on rich visual content as a means of conveying information. It involves expressing ideas and constructing meaning when seeing the visual image (Visual Literacy, 2007, p. 2). There are many forms of visual communication including gestures, objects, signs and symbols (Bamford, 2003, p. 1).

Visual literacy encourages an appreciation and understanding of visual communication. By understanding the fundamental principle of visual literacy, people can create images that communicate in more efficient ways (Bamford, 2003). Wheatley cited in English (1997, p. 295) state that by encouraging imagery, learners obtain greater success in mathematics because imaging as a mental activity *"plays a central role in learning and doing mathematics"*.

2.1.4 Visual Learning

Visual learning is the process of learning and constructing knowledge from seeing visual illustrations (www.educ.kent.edu, 2007). Visual Literacy is promoted through developed skills in visual problem solving and critical thinking. Critical reasoning and critical thinking are important components of learning visual literacy as a skill because critical thinking assists learners to decode ideas they perceive and interpret (Visual Literacy, 2007). The sharing of learners work through critical responses helps build visual literacy skills. According to Mudaly (2008, p.3) *"visual literacy is visualization combined with logical thought"*. Logical and critical thinking is needed in separating and examining facts and making decisions in problem solving. Learners are expected to use their intellect in thinking critically so that they can become competent to solve problems and make decisions (Smith, n/d, p. 1). Learners gather information, evaluate visual representations, and re-represent them in a logical manner to acquire a better understanding. They draw conclusions from the visual illustrations in order to arrive at possible solutions to the given problem. Learners continue observing and adjusting the solution and then choose and implement the best alternative (Smith, n/d, p. 2).

2.1.5 Visual Literacy and the Classroom

By promoting visual strategies in the classroom, mathematics teachers can empower learners to creatively express and critically respond to ideas. Whilst reading and writing is still considered as the main component of literacy teaching (Riesland, 2005, p.1) there is a call for a rethink on what it means to be literate. *"Proficiency with words and numbers is insufficient and must be supplemented with additional basic skills..."* (Stokes, 2002, p. 2). Kellner cited in Stokes (2002, p. 2) proposes that multiple literacies (visual literacy, print literacy, aural literacy) are essential to meet the challenges in today's classrooms. A group of educators and researchers known as the New London Group (NLG) in their examination of the current meaning of literacy found it to be outdated. The NLG state that the current meaning must include understanding and experienced control of representational formats that is necessary in the communications environment (Riesland, 2005, p. 2). The fundamental purpose of the current literacies is to ensure that all learners benefit from learning in ways that allow them to participate fully in public, community, and economic life (Riesland, 2005, p. 1).

2.2 Visualization: Introduction

According to Zimmermann & Cunningham and Hershkowitz cited in Arcavi (2003, p. 217),

"Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing

previously unknown ideas and advancing understandings".

Many learners do not develop the ability to use visualization automatically during mathematics problem solving. According to Montague (2005), most learners develop their visualization skills between the ages of 8 and 11. As they progress through the different grades, they continue to refine the skills and strategies that are necessary to solve real life mathematics problems.

According to Lynell Burmack cited in Sandell (2008, p. 2) *"a teacher can be 10 times more effective by incorporating visual information into a class discussion".* Effective visual representations whether on paper or in one's mind can make the lesson more visible. The learner is thus able to assimilate the information to create a better understanding. Instruction delivered simultaneously with visual support allows the learner to participate actively as they practice to become skilled in problem solving (Montague, 2005). This approach brings about keen interaction as visual representations supports their learning.

There is debate around the role of visualization in the teaching and learning of mathematics problem solving. Hadamard and Poincare cited in Stylianou (2002, p. 304) states that the use of visual representations *"was an essential element in problem solving and encouraged students to use visual representations in their own problem solving".* Sfard in Gierdien (2007, p. 61) cites certain mathematicians like Einstein and Glaser who states that visualization is not mathematics. Arcavi and Presmeg cited in Gierdien (2007) state that the possibility of the devaluation of visualization is likely to filter through right to the classroom, curriculum material and teaching methods.

Arcavi (2003, p. 226) states that although visualization is seen as important it *"remains a second-class citizen in both theory and practice of mathematics".* Presmeg cited in Arcavi (2003) state that learners have a greater need for visual methods of solving problems as visualization is gaining increased visibility in mathematics and mathematics education. Studies by Lesh (1981); Skemp (1982) and Presmeg, (1986) cited in Goldin (1998, p. 139) indicate that visualization in particular and imagistic representation in general came to be recognized as critical understanding to how mathematical and non-mathematical concepts are meaningfully understood. Davis, Maher & Noddings (1990, p. 94-97) state that there is a better way to assist learners discover mathematics, and goes further to mention that visualization allows them to think about the mathematics. This is called assimilation paradigm and since it involves teaching it is called assimilation-teaching paradigm. They also mention that if learners are not given something of this nature, as the assimilation paradigm then they will have no way of thinking about mathematics.

Symbolic representation may not be possible without explicit instruction from the educator that will assist learners move from concrete to a more symbolic, schematic level (Montague, 2005). Learners who have difficulty solving mathematic word problems usually draw a picture of the problem without considering the relationships among the different problem component. This leads to misunderstanding as learners cannot relate to the problem thus according to Montague (2005) learners need explicit instruction in how to use visualization to represent problems.

2.2.1 Visualization and Visual Reasoning

"Visualization is understood as linking images and diagrams ..." (Kadunz & Straber, 2004, p. 247). Visualization is not only related to illustrations but *"is also recognised as a key component of reasoning"* i.e. engaging with the conceptual and not the perceptual (Arcavi, 2000, p. 235). The cognitive demand is enormous when visualization acts upon conceptually rich imagery. Dreyfus cited in Wheatley (1997, p. 281) state that more recognition needs to be given to visual reasoning, the use of which can be traced back to the early days of mathematics in Mesopotamia and in Greece, by mathematics educators as *"visual reasoning plays a far more important role in the work of today's mathematicians than is generally acknowledged"*.

We have to understand the relation of images and diagrams and how to convert images into a diagram. According to Wheatley (1997, p. 282), an image is a mental construction which is transferred into diagrams which then creates a visual medium for the learner to understand a problem better. He argues that the ability to create, represent and convert images is linked to mathematics problem solving (Wheatley, 1997, p. 283), as a strong relationship exists between the use of imagery and the success in problem solving. It allows learners to look back at their represented solution, reflect and revise their visual representation. This process can help develop ideas towards a solution and demonstrate learners understanding of concepts that are being communicated. Wheatley found in his research that learners who used imagery in their reasoning were more successful in solving non routine mathematics problems than those who approached the tasks procedurally (Wheatley, 1997, p. 281).

Sfard cited in Wheatley (1997, p. 285) found that mathematicians relied on visualization but its importance in the teaching of mathematics has been widely ignored. The reason put forth by her is that mathematics as a subject is seen as *"learning and using rules which make little sense to the learners"*.

According to Wheatley (1997, p. 285) *"Diagrams do not communicate, they evoke"*. Diagrams evoke a variety of responses in different people depending on their level of comprehension of the representation and logical reasoning. Whilst certain problems can be solved by following logical reasoning others require visualization and imaging as *"a solution based on images may be more comprehensible and elegant"* (Wheatley, 1997, p. 284). Visual reasoning during the visual process allows learners to look back at what they have represented, reflect, revise and redraft their representations. This process can help develop the ideas that are being communicated.

2.2.2 Visualization and Problem Solving

Eminent people in early history recognised visualization as an indispensable problem-solving tool. Einstein cited in Diezmann (1995, p. 2) when reflecting on visualization as a problem solving tool stated that *"the psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be voluntarily reproduced or combined"*. Rival as cited in Stylianou (2002, p. 304) state that visual tools such as diagrams, graphs and sketches were seen as a vital component in the work of mathematicians. Eisenberg and Dreyfus cited in Diezmann (1995, p. 2) state that professional mathematicians have altered their representational bias to incorporate visual literacy as they had re-educated themselves now to think more visually than when they were in college.

Visualization is a process of creation of ideas through pen and paper, graphical presentation of concepts or by means of one's imagination (Lavy, 2007). Wickelgren (1974, p, 186) state that although occasionally one can unravel a problem *"in the head"* but the vast majority of all problems can be solved more swiftly by representing information on paper at an early stage. He lists the following reasons (Wickelgren, 1974, pp.186-187):

1. By writing down the components of the problem the learner now focuses on the important concepts of the problem;
2. Attention is automatically drawn to represent the information in symbols or diagrams;
3. Writing aids your memory. After working on a problem for sometime, it is easy to forget some of the vital information. Having represented the information on paper allows you to use visual scanning to jog your memory for information that will aid in solving the

problem;

4. Tables, graphs or matrices of information are difficult to retain as visual imagery. Visually represented information allows the brain to investigate the problem further and more efficiently.

Using visualization in problem solving may play an integral part in inspiring an explanation to the problem (Wickelgren, 1974; Arcavi, 2003). In the problem solving process or *"higher order skills"* (Peterson cited in Jaworski, 1994, p. 8), state that learners are confronted with a myriad of information in the problem. More often than not learners tend to want to use all the data in the problem or are distracted by it. They need to deduce what is relevant. Through the process of visual representation (Arcavi, 2003) certain data is eliminated as the solution to the problem is processed. Wickelgren (1974, p. 19) state we must prune as we progress through the problem so that *"there are not so many possible action sequences to investigate"*. Visual representation then assists in finding the solution to the problem speedily.

Visualization together with speaking allows learners to externalise their thinking. It assists them to clarify their own ideas when they are struggling to find words to communicate them to others. Therefore one can readily conclude that visual representations allow learners to overcome the language barrier as pictures can speak volumes.

Diezmann (1995, p. 5) state that the use of visualization as a valuable problem solving strategy is influenced by at least three factors and their relationships i.e. *"the structure of the problem; the predisposition of the learners towards visual methods of solution and the learners instruction in diagram use"*.

Kelly (1999) in her project with in-service educators introduced them to the power of visualization. Educators were given 'visualization friendly' problems and problems for which an algorithm was not obvious. After each problem, the educators carried out an analysis of their drawing and strategies. They found that although they arrived at the same answer their visual representations they created were different. The educators in this project agreed that *"visualization could be a powerful tool to enhance their learners problem-solving ability"* (Kelly, 1999, p. 49). They were asked to implement visualization with their learners. The educators were sceptical of using visualization to enhance their learner's problem-solving ability as they doubted their ability to *"visualize mathematics problems at all"* (Kelly, 1999, p. 49). Initially the educators were frustrated as their learners showed an inability to organise their thoughts in order to visualize a problem and create an illustration. The learners were gradually introduced to 'picture problems' and through representing the problem in a visual manner it was found that they could solve these problems (Kelly, 1999; Lavy, 2007). The educators found that the pictures were a foundation for *"future visualization endeavours"* (Kelly, 1999, p, 49). Hadamard and Poincare cited in Stylianou (2002, p. 304) argued that the use and value of diagrams and other visual tools was seen to be an indispensable component in problem solving and encouraged learners to use visual representations in their own problem solving. As mathematics educators, we can foster visualization skills by actively engaging learners in visual experiences, in which learners can gain first hand knowledge.

2.2.3 Spatial Visualization

According to McGee cited in Fennema and Tartre (1985, p. 184) spatial visualization involves *"the ability to mentally manipulate, rotate, twist or invert a pictorially presented stimulus object".* Silverman (2002, pp. 1-2) state that visual-spatial learners (VSL) are learners who make a replica of their ideas in pictures rather than words. They have a different brain organisation and always learn better visually. Haas (2003, p. 3) state that visual-spatial learners enhance their learning by seeing than by listening. They must be able see the big picture holistically in order for learning to occur as they have the aptitude to grasp concepts in numerous dimensions. Silverman (1983, p. 1) state that learners who are spatially oriented must image things first in their minds before they can reproduce them. It is a mental plan of things and involves representing objects in illustrative forms and visualizing objects from their drawings. It is their ability to form and retain visual images and mentally use these images in problem solving (Olkun, 2003, p. 8).

Freed et al (2006, p. 5) state that problems are usually an obstacle for these kind of learners *"so it forces them to use visual means to solve the problems".* They discovered in their research that all information needs to be stored as visual images during the learning process as *"visual people need to make visual pictures in order to learn".* Haas (2003, p. 4) found that *"visual-spatial learners are divergent thinkers, who prefer unusual solution paths and multiple strategies for problem-solving".* It is likely that they may arrive at an answer using more than one solution strategy. Problems might arise when an explanation is required as these learners are often not able to communicate to others how they arrived at an answer (Haas, 2003, p. 4). During the classroom observations, the researcher found learners had written down answers without showing any tangible signs of algebraic calculations but when asked by the educator to explain their answer they were able to give a verbal description that the educator translated into a visual representation of how they arrived at the answer.

Many children who enter school from deprived backgrounds are unable to express ideas because of language inadequacies and this restricts the learning of mathematics as they progress through the educational system. Haas (2003, p. 2) found that if such learners hoped to achieve in the classroom *"they must find ways to translate the material presented into a format that can be assimilated"*. Wheatley and Wheatley cited in Dickson et al (1984, p. 7) state that children who are low attainers find a spatial approach both more accessible. They found in their research that learners who worked with spatially designed activities benefited greatly. Booth and Thomas (2000, p. 179) found in their study that visual spatial learners performed significantly better in whatever form the question was presented and they also benefited when given a diagram to view. Woolner (2004) on the other hand found in her study that there was no tangible difference when using the visual and verbal approach to teaching mathematics.

2.3 Problem Solving: Introduction

The study of mathematical problem solving has always been an important area of investigation by researchers (Polya, 1945; Wickelgren, 1974; Schoenfeld, 1983; Barb & Quinn, 1997; Goldin, 1998; Lubienski, 2000; Montague, 2005). Problem solving requires learners to generate and develop their own mathematical solutions from particular situations using their prior knowledge.

Mathematics in schools, in my opinion, is based mainly on problem solving. Despite its wide spread use, mathematical problem solving is a complex cognitive activity involving a number of processes and strategies (Montague, 2005, p. 2). It involves a puzzling situation that needs to be resolved although there is no clear and unique solution. Problem solving is amongst the more challenging activities to master but not beyond the accomplishment of learners. Teaching learners how to process information can influence all facets of their lives. Problem solving is an important aspect of independent learning. The more independent learners become the more proficient they become in making sound decisions about what and how they learn something. This is strongly related to problem centred learning **(discussed in 3.3 in Chapter 3)**. The tasks set in problem centred learning promote competence and acquisition of specific mathematics skill and allows each learner to be an active participant. If learners can link the problems and acquired skills to their real life situation then problem solving becomes a powerful way of developing meaningful learning for them to make independent decisions.

Within this study three approaches to problem solving are distinguishable namely teaching about problem solving; teaching for problem solving and teaching through problem solving. The international mathematics community accepts these three approaches. It is necessary to distinguish between these three approaches:

1. Teaching for problem solving (Schroeder and Lester, 1989) follows Polya's four step approaches. Learners are taught mathematical ideas and strategies and then apply these strategies to solve problems (Murray, Olivier and Human, 1998; Rich and Meier, 2007) ;
2. In teaching for problem solving, learners apply acquired knowledge in order to solve problems. Educators prepare learners to transfer acquired knowledge to other areas of mathematical learning (Murray et al, 1998; Rich and Meier, 2007);
3. Learning through problem solving (Schroeder and Lester, 1989) starts with the mathematics problem (Murray et al, 1998). The problems are carefully constructed (Schroeder and Lester, 1989) and the learners develop their own problem solving and representation strategies to solve the problems and reflect on their own solutions. This approach is interrelated with Problem Centred Learning **(discussed in 3.2 in chapter 3).**

Researchers (Schroeder and Lester, 1989; Rich and Meier, 2007) state that learning through problem solving will make learners better problem solvers, as they will discover for themselves. The researcher agrees with this statement and he is of the opinion that learning for problem solving and teaching for problem solving is just as important especially in the primary schools. The foundation to becoming better problem solvers is laid in the intermediate and senior phases. Learners, many of whom are second language learners, need to develop their conceptual understanding, which is necessary for further progress.

In the reform movement of the 1990's, much emphasis was placed on the significance of problem solving in mathematics. *"solving problems is not only a goal of learning mathematics, but also a major means of doing so..."* (Pape, 2004, p. 187). The National Council of Teachers of Mathematics (NCTM) has called upon mathematics educators to teach mathematics through problem solving (Pape, 2004, p. 187). Problem solving has been given much attention internationally by The National Council of Teachers of Mathematics (USA); the 1982 Cockcroft Report (England and Wales) which stated that the ability to solve problems is at the heart of mathematics and was also highly prioritised in New Zealand in 1992.

In South Africa, problem solving has now become a critical area of focus (DoE, 2008) in mathematics teaching and learning and is considered to be the heart of mathematics (DoE, 2003, p. 27). Getting learners to solve problems is important as it reveals information about their mathematical knowledge and ability. Since most of the problems we solve in the classroom are really exercises, learners are content with the first solution and this leaves the impression that is the only way to solve the problem (Schoenfeld, 1993, p. 20-21). The study of problem solving should therefore provide learners with skills and processes that will enable them to cope in their lives. English cited in Brown (2008, p. 96) states problem solving must enable learners to develop mathematization skills that can become generative assets in life beyond the classroom

Mathematics is seen as a basic human characteristic and *"as a subject, it grows and changes as a result of problem-solving, trial and error and the interpersonal exchange of ideas"* (Nickson, 2000, p. 2). Most conceptions of the human act of problem solving reside in George Polya's identification of human thinking and action involved in problem solving.

"For a rich problem solving process to occur time must be allowed for investigating the nature of the problem, generating and proposing multiple solutions. . . and reflecting on what one has learned and what else could be learned" (Sawada 1999, p. 57).

One of the earliest attempts to specify stages in problem solving was that proposed by Wallas in 1926 in The Art of Thought (Reynolds and Flagg, 1983, p. 232). He stated that the problem solving process progressed through four steps namely, preparation; incubation; illumination and verification. Today much is still written about the significant contribution made by George Polya to problem solving. Polya (1945) proposed the following four steps: understanding the problem; devising a plan; carrying out the plan and looking back (Figure 2).

To make a general comparison Polya's steps are interwoven with that of Wallas'. The step of understanding corresponds with Wallas' preparation stage; checking the results is similar to that of Wallas' verification; Polya's devising a plan stage includes Wallas' preparation steps as well as incubation and illumination steps.

Problem solving is difficult for the majority of learners (Backhouse et al, 1972). Problem solving is not as straight forward as indicated by Wallas and Polya. One may conclude that a problem can be solved from proceeding neatly from the understanding stage to checking the results. Today various strategies and skills are used in solving a problem, as problem-solving strategies are not specific to an individual problem. Problems also tend to have sub problems, which learners need to identify and clarify. Learners need to pinpoint the problem area in order to progress towards a solution (Smith, n/d). Elsewhere in this chapter, I discuss how the process of problem solving has evolved due to using visual literacy and visualization skills.

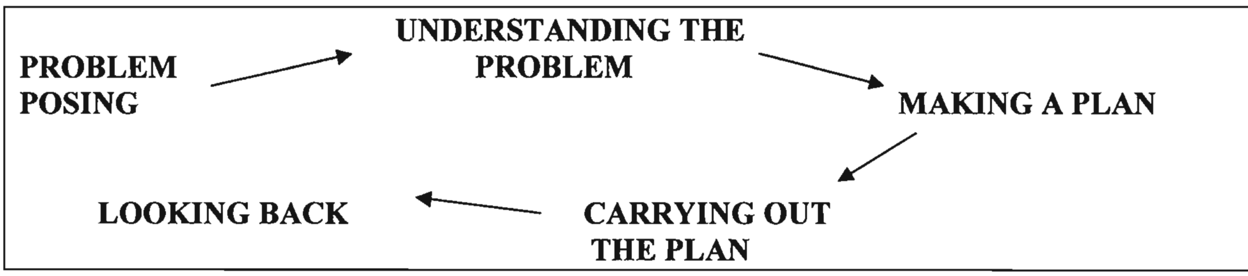


Figure 2: George Polya's Dynamic nature of problem solving adapted from Polya (1945)

The utilisation of problem solving is an essential aspect of effective teaching and learning.

Pirie cited in Jaworski (1994, p. 11) state mathematical problem solving process can be likened to the exploration of the unknown land where the journey and not the destination is the goal. It entails active and higher order learning. The Cockcroft Committee described problem solving as the ability to apply mathematics to a variety of circumstances in the reality (Backhouse et al, 1992).

Treffers cited in Van den Heuvel-Panhuizen (1998b, p. 2) formulated the idea of horizontal mathematization through which *"the learners come up with mathematical tools which can help them to organise and solve a problem located in a real-life situation"* and vertical mathematization, which allows learners in *"finding shortcuts and discovering connections between concepts and strategies and then applying these discoveries"*. If used effectively, these two processes of mathematization in problem solving can become a medium for developing pupils' mathematics and simultaneously help develop independent decision-making and independence of thought and action.

Problem solving has two stages namely, problem representation and problem execution (Meyer cited in Pape, 2004; Montague, 2005). Meyer cited in Pape (2004) state that to be successful in problem solving, learners must function between these two steps.

Montague (2005, p. 2) state that *"one of the most powerful problem representation strategies is visualization"* and that it is critical to **problem representation**. Successful problem solving is not possible without the appropriate representation of the problem. Appropriate problem representation indicates that the learner has understood the problem and guides the learners toward the solution (Montague, 2005, p. 2).

Diezmann (1995, p. 4) strongly believes that external and internal representations are necessary in knowledge acquisition. This type of representation modes are discussed by Lesh, Landau and Hamilton cited in Goldin (1998) when analyzing problem solving. External representations include diagrams, charts, graphs, manipulative models, symbols and real world situations whereas internal representations facilitated by mental processes are clear in imagery and mental models. Karmiloff-Smith as cited in Diezmann (1995, p. 4) state external representations play an important role in knowledge acquisition in which one representation is transformed into a more fruitful representation and internal representations are non verbal representations which are needed for the insightful interpretation of the problem (Goldin, 1998, p. 159). The role of representations in mathematics provides an additional dimension for visual literacy to be included in mathematics. It not only allows access to visual representations and reasoning, but also representations themselves are crucial in the learning of mathematics (Diezmann, 1995) as they provide important insights into problem solving. The objective is to construct sufficient understanding of concepts which leads to the construction of mathematical knowledge.

Visual representation (drawing diagrams, pictures, sketches, graphs) is one of the recommended visual problem solving strategies and is particularly important because of its role in understanding the problem. Visual representation of the relationships among the elements of the problem is beneficial in that it increases learner's conceptual learning (Serpil et al, 2005). By using visual representation many mathematical concepts become concrete and clear for the learners to understand. Van Essen and Hamaker; Larkin and Simon and Mayer and Gallini cited in Diezmann (1995, p. 5) state that there are cognitive advantages of using diagrams. They state that *"diagrams relieve working memory"; "diagrams facilitate the reorganization of information"; "diagrams provide a visual alternative to words"; "implicit information within a problem may become explicit to the solver on a diagram"* and *"diagrams display a solvers understanding of the structure of a problem"*. Success in problem solving is dependent on the incorporation of the data on the diagram and the recognition of relationships among the data.

Problem execution allows learners to construct an image of the problem physically on paper and mentally. By using a variety of visual representations, such as pictures, graphs, manipulatives and other forms of displays, learners must be taught how to translate the results of their thinking to symbolic representations using paper and pencil. As learners become more proficient, they will progress to using mental images. Meyer cited in Pape (2004) state that for learners to be successful in problem solving they must function between problem representation and problem execution.

According to Montague (2005) in order to solve problems successfully, learners will have to follow seven steps:

Firstly, the learners would have to read for understanding. Learners could read the problem more than once and may reread parts of the problems as they progress and think through the problem. They ask themselves if they have understood the problem. Critical comprehension strategies need to be used to *"translate the linguistic and numerical information in the problem into mathematical notations"* (Montague, 2005, p. 3).

Secondly, learners need to paraphrase the problem into their own words. The important information is identified and learners *"ask themselves what the question is and what they are looking for"* (Montague, 2005, p. 3).

The third step is very important. The learners should try to visualize the problem. The learners ought to develop a schematic illustration of the problem so that the picture or image reveals the associations between all the significant parts of the problem (Montague, 2005, p. 3). By using both visual representation and verbal communication, problem solvers are guided towards solving the problem. Learners have represented the problem and *"are now ready to develop a solution path"* (Montague, 2005, p. 3).

Through what has been visualised and observed the learners can hypothesize in the fourth step. By hypothezing, learners consider the logical solutions, types of operations and the number of steps that are needed to find the solution to the problem (Montague, 2005, p. 3). Learners make symbolic representations and choose the appropriate algorithms as they decide to execute their plan.

The last three steps involve estimation, computation and the process of checking. By utilising mental calculations or pen and pencil, learners try to estimate the answer. Calculations are made and checked against the estimation. The solution is checked to determine if it has answered the question and whether it makes sense. According to Montague (2005, p. 3) the learners have to ensure that they have used the correct steps and that their answer is correct.

2.3.1 Problem Solving and Memory

Problem solving is a complex activity and requires a plan, previous experience and engaging the mind to evaluate the solution. Since the learners have to rely on previous experience, problem-solving skills depend greatly on the way information is stored in memory and the circumstances under which it is retrieved. Memory plays an important role in problem solving activities.

Greeno cited in Reynolds and Flagg (1983, p. 250) was one of the foremost to explain the connection between memory structures and processes and problem solving. He explains that, even though problem solving uses information and techniques from our experiences that is stored in our memory, problem solving still, requires a unique form of processing. According to Greeno the solution to the problem is often found through the retrieval of relational properties. Greeno's memory model (Figure 3) cited in Reynolds and Flagg (1983, pp. 250-251) contains:

Short-term memory from which information is easily retrieved;

Working memory contains a moderate quantity of information that is used actively at any given time. Information can only be stored as factual knowledge after it has been processed in the working memory. The arrows refer to the interrelated networks of the information processing process which shows the flow of information from perception (recognition) to the working memory and consolidation as factual knowledge. Older information is then replaced by the new information and kept available until needed again in a similar situation.

Semantic and factual knowledge refers to the long-term memory span and contains semantic and factual knowledge.

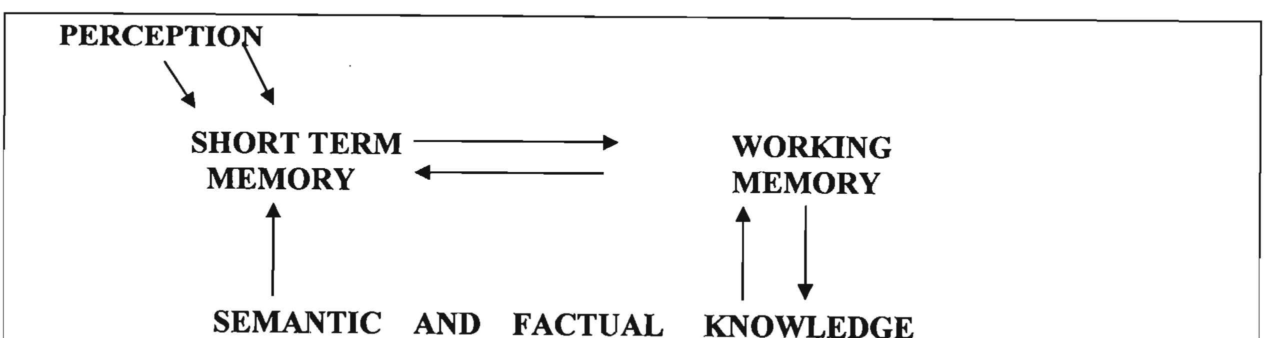


Figure 3: Greeno's Memory Model adapted from Reynolds & Flagg (1983, p. 250)

There are two stages of problem solving in Greeno's problem Solving Model (Figure 4) (Reynolds and Flagg, 1983, p. 250-252):

Firstly, the formation of a network for the problem and the formation of a set of relationships between the problem representation and the network of relations are established. This is referred to as the goal state. The construction of the representation of the problem takes place in working memory **(Figure 3).**

Secondly, associations need to be made between these facts and the desired goal. Information from the semantic memory is used to modify the structure in working memory. Information is retrieved and used to transform the structure of working memory and solve the problem.

The external information becomes understood once it is incorporated into the existing knowledge, linked to already learned internal knowledge. The strength of the links determines the level and degree of understanding. From the above discussion on Greeno's Memory Model **(Figure 3)** and Greeno's Problem Solving Model **(Figure 4)** it is evident that learner's prior knowledge has an influence on learning.

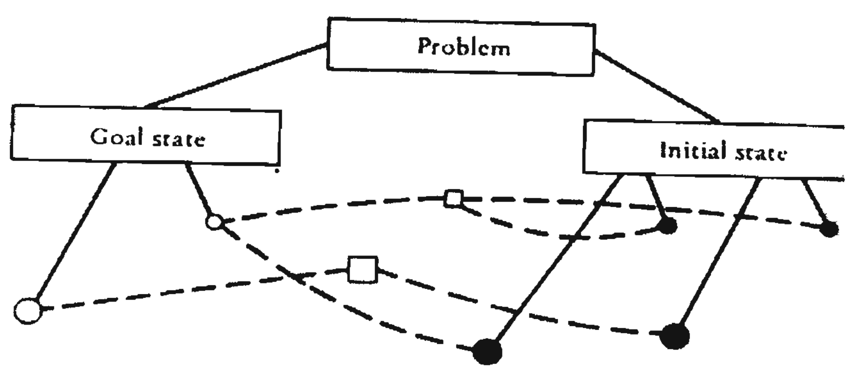


Figure 4: Greeno's Problem Solving Model (Reynolds & Flagg, 1983, p. 250-252)

The following symbols •; □ and ○ used in **Figure 4** refer to the nodal structure in which the nodes are interrelated in complex ways within and across the networks. Information is organised into conceptual networks (concept learning) and the lines connecting the nodes represents the association between the concepts. It is through this process that the learner is able to process the information identified in the problem and work towards a solution.

The researcher strongly believes that Greeno's model **(Figure 3 and Figure 4)** is relevant in the teaching and learning of problem solving using visual literacy and visualization. When the learner recognizes key concepts in a problem the working memory **(Figure 4)** enters into information processing stage trying to relate to concepts and building of ideas in the mind. The exchange of information between the short term memory and the working memory can be translated into a visual representation and by using a strategy a solution (goal state) can be reached.

2.3.2 The Educator and Problem Solving

The educators in today's classrooms serve many roles and their importance cannot be ignored. The greatest service an educator can provide to learners is to assist them to discover for themselves. One of their many roles is to shape learners thinking and nurture their learning in the right direction and at the same time provides experiences that are both instructive and rewarding.

Many educators still consider mathematics teaching as a transmission of knowledge in a recipe like manner by placing emphasis on rules and procedures. *"it is important to recognize that, as teachers, we convey messages about the nature of mathematics by the way we teach it"* (Nickson, 2000, p. 43). A paradigm shift is needed by educators. Educators normally see themselves as the originators of knowledge and their responsibility is to transmit this knowledge to learners who are considered empty vessels. A common belief is that learners learn the content by listening well and then demonstrate the procedures learnt. If a learner displays understanding this serves as an indicator to the educator that learning has taken place. The educator in modern day classroom is the facilitator of the learning process and not the provider of the solutions. In Problem Centred Learning, educators must provide the necessary information to the learners to understand the problem and how to use mathematical instruments e.g. calculator (Murray et al, 1998, p. 175).

The objective of mathematics educators should be to facilitate learners understand mathematics and to encourage them to believe it is both natural and enjoyable to continue using and learning mathematics. Backhouse et al (1972, p. 13) state that educators would like learners to have a positive attitude towards mathematics. It is essential that educators teach in such a way that learners will see mathematics as a sensible, natural and enjoyable part of school life. We live in a highly visual world and therefore educators should be using realistic mathematics education (discussed in chapter 3) to teach problem solving. They should be seeking ways to teach learners ways of solving problems visually.

Winter cited in Nickson (2000, p. 2) argues that, *"pupils have their particular view of what mathematics is all about"*. Learners from an early age see mathematics as a subject of facts and rules that is to be learnt by rote but the Department of Education (2003, p. 24) wants learners to move away from this mindset as *"performing operations by rote or following a recipe simply will not help the development of understanding and mathematical knowledge, skills or values"*. To change this mindset learners have to be told that facts and procedures are essential for understanding mathematics. They will appreciate mathematics if they come to know and understand that there is a meaningful reason for doing so (Nickson, 2000, p. 43).

Teaching in today's climate is a challenge for many educators and research has often been conducted with the aim of identifying factors, which will enable educators to be more effective in the classroom. Schoenfeld (1983); Kelly (1999) and Montague (2004) discovered that educators found teaching mathematics problem solving an immense challenge. Schoenfeld (1983) state that problem solving is difficult. In her project with in-service teachers Kelly (1999, p. 48) observed that teachers were most comfortable with the algorithmic approach *"because that is how they were taught mathematics when they were in school"*. Educators admitted that using only the algorithmic approach *"posed obstacles for their learners"* (Kelly 1999, p. 48). The algorithmic approach is still problematic to learners (DoE, 2004). The researcher concurs with the systemic report as he has observed that learners in the primary schools still have difficulty computing basic algorithms.

In the mathematics curriculum, there are many principles or theories that learners need to know and that the educator has the responsibility to teach. Therefore, educator support is of vital importance to the learner. The educator's role is to assist learners *"make connections between the learning that takes place in the classroom and their experience in the real world"* (Nickson, 2000, p. 6). Problems must relate to the application of mathematics to everyday situations within the learners' experience. DoE (2003, p. 27) teachers need to ensure that all learner are given opportunities to solve problems appropriate to their skills. The educator must know his learners. Nambiar (2000) state that information and knowledge about learners is a powerful tool as educators are able to devise strategies to involve them in the learning process.

The Professional Standards for Teaching Mathematics and Principles and Standards for School Mathematics (Artzt and Armour-Thomas, 2002, p. 5) state that educators must create opportunities that stimulate, guide, and support learners to make associations among mathematics concepts, construct mathematical ideas, unravel problems through reasoning, and take responsibility for their individual learning. A shift in teaching methods is necessary in today's evolving curriculum. *"Learners need to learn visually and teachers need to learn to teach* *visually"* (Stokes, 2002, p. 4). In order to teach problem solving effectively using visual skills educators need to be trained in this field. *"Teachers need specific and intensive hands on preparation and empathetic support over a sufficiently long period of time to be able to utilise problem solving to the benefit of all the learners in the class"* (Nieuwoudt et al, 2000, p. 336). This changing role will require additional educator training. Educators responsible for teaching mathematics must undergo professional development in mathematics pedagogy. By obtaining ongoing support in professional development, educators will be able to meet the mathematics learning of all learners (Walden University, n/d, p. 2).

2.3.3 Textbooks and Problem Solving

Adler (2001) and Montague (2004) state that the textbook remains a key resource to educators as they rely exclusively on mathematics textbooks to guide classroom instruction.

Many textbooks do not offer concrete problem solving examples on which learners can build on. Problems in textbooks are not really problems. They are mostly of the computational type couched in words where one or two direct translational steps are required. The learners learn, practise and apply these steps procedurally. Problem solving in textbooks is sometimes limited to artificial story problems, where the goal is to translate from words using a standard algorithm (Goldin, 1998, p. 159) and have no relevance to the learner's world. The problems used in most *"textbooks do no invite pupils' involvement essentially because they do not arise from any of the pupils' activity and they cannot relate them to anything remotely tangible in their lives; thus they have no ownership over them"* (Nickson, 2000, pp. 153-154). In order for learners to benefit from problem solving the problem must be realistic so that they can relate to. Hamilton cited in Brown (2008) notes with concern that there is little evidence to suggest that textbook problem solving will show the way to improved mathematical performance beyond the classroom.

"The mathematics presented in school mathematics textbooks and curriculum is neat and polished" (Betts, 2007, p. 27). Textbooks provide illustrations not often related to the problem provided. These illustrations are often representations that tend to distract learners from the actual problem or sets mathematics in a *"neat and polished form"* (Betts, 2007, p. 27). Although the newer textbooks refer to problem solving strategies these problems solving activities do not occur adequately. Textbook authors must provide problems, with visual stimuli that will challenge learners and bring about critical thinking.

Educators today teach learners to do but not to think in mathematics. Mathematics is taught procedurally and when problem-solving strategies are taught, they are often taught in isolation and as a result learners do not see the relevance of these strategies to problem solving activities. Educators are too involved delivering what the education system is asking i.e. prepare learners to achieve levels as laid down by the DoE that they forget that mathematics has other purposes in learner's lives. Schoenfeld (1983) state that many educators are concerned with the amount of material they must cover that they forget about the actual purpose of mathematics.

2.3.4 Questioning and Problem Solving

The Professional Standards for Teaching Mathematics (Artzt and Armour-Thomas, 2002, p. 18) acknowledges the role of questioning in mathematics problem solving. It has been found that mathematics has shifted from telling to listening, questioning and probing for understanding. Questions encourage discussion. In this approach learners grow more comfortable and challenge ideas and accept others suggestions.

Questioning in teacher-learner interaction is important because through questioning the educator encourages learners to make public their thinking and determines how they make connections with mathematical ideas. The questioning of learners allows their responses in the classroom domain to be assessed and built by other learners. If we want learners to build on their existing knowledge, questions should be provoking and effective so that it allows them to speak and explain freely. Questioning will challenge learners to examine their thoughts and create arguments that will support their answers. For this reason, questioning is an important part of mathematics.

Educators must determine the amount of time that will be allowed for responses to be forthcoming. The ability levels of learners need to be considered because the way learners respond will determine the tempo of teaching and how concepts are taught. Persistence with proper questioning techniques will enhance learners understanding and involvement in the lesson.

2.3.5 Problem Solving Strategies

Learners' strategies and solutions in solving problems have become a major area of focus in teaching mathematics world over (Schoenfeld, 1983; Barb & Quinn, 1997; Malloy & Jones, 1998; Sawada, 1999; DeYoung, 2001; DoE, 2008). These researches provide an interesting insight on how learners engage in problem solving using various problem solving strategies and visual methods. According to DeYoung (2001) and Wheatley (2004) problem solving involves a variety of strategies **(Appendix H)**, approaches and visualization, which is a key aspect.

George Polya (1945) stated that the creation of a plan is the main work in problem solving. Problem solving is an activity that requires learners to engage in decision making, analytical, creative and critical thinking. (Ollerton, 2007) state that problems must incorporate a component of challenge and puzzlement to enable learners to relate prior knowledge, skills, and understanding to establish an answer to a question. There is no ready solution available and strategies are used to find solutions. The learners construct their own knowledge using their experience and educators need to guide learners through the problem solving process.

Pantziara et al (2004) in their study found that when learners were provided with problems with and without visual representations, there was no significant difference in their performance. The diagrams used were seen as a form of distraction as learners tried to concentrate on the diagram than the actual problem. Studies by Fischbein, 1987; Greer, 1992 and Alseth, 1998 (cited in Nickson, 2000) show that learners used mostly diagrams in problem-solving situations, but were not able to do so unless they had an understanding of the concept embedded in the problem. Educators must teach mathematical concepts in order for learners to obtain a better understanding and the best way to reinforce a concept is through visual representation. Nickson (2000, p. 25) state, *"to be effective, diagrams should be used alongside the teaching of a concept to help pupils to reach an understanding of the concept"*.

In primary school learners continue to learn a variety of mathematical concepts by doing mathematics problems that often only require basic computation. Peterson cited in Jaworski (1994, p. 8) refers to this as lower level skills as it does not tackle skills beyond their routine use. They engage in traditional word problems, which involves the translation of numbers in the order they appear in the problem. The learners can understand the presented problem but are unable to solve it due to not being adept at using problem solving skills. They often have difficulty solving the word problems because they are unsure what to do to solve the problem. They need practice to use mathematical problem solving skills and strategies to solve problems in their daily lives. These learners need explicit instruction in how to use visualization to represent problem as effective visual representations show the relationships among the problem parts.

Good problem solvers use a variety of processes and strategies as they read and represent the problem before they prepare to solve it. Problem solving approaches must enable pupils to process and develop their mathematical knowledge and it is for this reason that non-routine problems will be used in this study. Some of the strategies that can be used in the teaching and solving of non-routine problems are listed in **Appendix H.**

2.4 Conclusion

The different ways of understanding the teaching and learning of mathematics itself and how it has to take place are factors that shape how learners act and interact within the mathematics classroom. Many techniques are employed by educators in the hope of assisting learners in school. Literature suggests that visual literacy and visualization enrich mathematical understanding especially in helping learners to solve problems. Visual Literacy and Visualization can capture learner's imagination and when the two are properly combined they can improve learners' attitudes towards mathematics. Research by Malloy and Jones (1998) found that learners who used visualization were more successful than those who used traditional methods. The use of visualization provides learners a look at mathematics from a different perspective. It can be a method and powerful resource for learners doing mathematics which can change their ways of thinking about mathematics. Problem solving enables learners to critically analyse and provide mathematical arguments to solutions in problem solving. Visual literacy and visualization offer educators a great opportunity in overcoming the challenges of the new demanding curriculum where language is a barrier. It helps learners understand mathematical concepts in the face of poor reading and comprehension abilities. The learners and educators are at liberty to utilise any one or combination of the strategies or manufacture their own strategy in the process of finding a solution when presented with a problem. English cited in Brown (2008, p. 96) state that because the children's final products embody the factors that they consider important, powerful insights can be gained into learner's mathematical and scientific thinking as they work the problem sequentially. This can be supported with visual representations to reinforce their understanding.

CHAPTER THREE

THEORETICAL AND CONCEPTUAL FRAMEWORK

3.1 Introduction

In this chapter, the researcher explores the theories relating to the research study. The focus of this study is how educators teach problem solving, how learners engage in problem solving, and whether visual literacy and visualization improves their performance.

Taking into consideration the issues under investigation the researcher has drawn on more than one theory namely, Problem-centred Mathematics; Problem-centred Learning; Theory of the Growth of Mathematical Understanding and Realistic Mathematics Education. The researcher finds these theories have important implications for both teaching and learning of mathematics.

Problem solving is an area which has been investigated from various angles both locally and internationally. The main area of concern is how to help learners become proficient problem solvers. Two types of teaching methods were recognizable during this study, namely, the learners were taught using the traditional method and the learner centred approach. In the latter approach learners had to construct their own knowledge on activities that were based on realistic mathematics.

The learning theories discussed in this chapter have strong links to constructivism and socio-constructivism and are important theoretical foundations for developing mathematical thinking in the classroom, as the learner is actively involved with the educator in creating knowledge. Its relation to these theories will be discussed in this chapter.

3.2 Problem Centred Mathematics (PCM)

Wheatley cited in Walbert (2001) demonstrated the Problem Centred Approach (PCA0) for the Advancement of Teaching in 2001. This idea is derived from research which has shown that learners perform better when they develop their own strategies and procedures (Walbert, 2001; Zambo and Zambo, 2008). It follows the theory that learning occurs when learners construct their own knowledge and emphasises the active role of the learner in the construction of knowledge.

PCM is linked to constructivism. According to constructivism, learning is viewed as a human act and an act of meaning seeking by an individual when faced with new information in a problem-solving situation. Most conceptions of the human act of problem solving resides in George Polya's identification of the four phases of problem solving (Polya, 1945), human thinking and actions involved in problem solving. If a learner is able to perform in a problem-solving situation, then meaningful learning should occur *"because he has constructed as interpretation of how things work using pre-existing structures"* (Thanasoulas, 2008, p. 2). Constructivists state that learners do not merely absorb information but rather construct knowledge from the things they experience. Knowledge is built on existing structures as learners assimilate new information into existing schemas (Zambo & Zambo, 2008) thus constructivism allows us to think critically and creatively about the teaching-learning process. This was important to this study as the researcher wanted to discover how learners engaged in problem solving and the strategies they used in arriving at the solutions.

According to constructivists, we learn through the process of trial and error. This is one of the first problem solving strategies taught to learners. Through the process of elimination, learners arrive at the answer. Constructivists maintain that learners need to be given opportunities to participate in problem solving activity. A radical shift is required in South African classes. Educators must allow learners to make mistakes, discover those mistakes by trial and error and discuss and develop their own strategies to solve problems. Barb & Quinn (1997) mention that Japanese learners have to participate in the lesson where they are given many opportunities to solve problems. There is a two-directional flow of information (dialogue) involving the educator and learner. This supports the learner's construction of knowledge as the learner is guided through the process of problem solving.

The PCM goal is to guide learners, who are the focus of attention in the teaching and learning situation. The learners must create their own understanding after interacting with other learners solutions. Walbert (2001, p. 1) state that learners must *"construct their own understanding of mathematics through solving problems, presenting their solutions, and learning from one another's methods"*. When learners work together collaboratively various solutions to a problem is possible. They have to determine which one of the solutions fully describes the problem. Learners often make an association between something learnt and the context in which it was first acquired. When solutions to problems are produced visually, collaborative learning takes place amongst learners. Problem solving thus becomes a creative activity as learners use alternate interpretations to find different solutions to the question. Through diagrammatic representations, learners can see the solution unfolding in a more practical way and can therefore make suggestions as on how to improve the solution. By focusing on the schematic, a plan is devised in a form of pictorial representation solutions, which has a potential to widen the scope of solution strategies for the learners (Maharaj, 2007). By seeing the solution on paper, learning is facilitated when learners link new information to their prior knowledge thereby generating new understandings (Davis et al, 1990) to develop new knowledge. In developing new knowledge, learners use their pre-existing and existing knowledge against new ideas so that they can make meaningful connections. Pre-existing knowledge is stored in the sub-conscious and is brought into focus and expanded as they continue to learn. This is related to Greeno's Model **(Figure 3 and Figure 4 discussed in Chapter 2).** The learner constructs his own meaning and finds his own solutions to problems.

The theory and use of PCM has created some differences of opinion in California, United State of America. The traditionalists and reformers have made strong representations regarding the implementation of PCM. The traditionalists regard PCM as "fuzzy maths" stating that learners must first develop basic mathematics skills before they can understand concepts and the most effective way to learn skills is through memorization and practice (Walbert, 2001, p. 1). The supporters of the traditionalist approach, which includes educators and parents, maintain that if drill and practice worked for them then it is fine enough for today's children (Walbert, 2001, p. 2). The reformers argue that they want to produce citizens who can use mathematics to understand the world around them through problem solving (Walbert, 2001, p. 1). They substantiate this claim using research findings that shows that "children harness the power of mathematics when they devise their own procedures for solving problems" (Walbert, 2001, p. 2). The researcher agrees with the reformers view as learners develop mathematical understanding, effectively learn new skills, and are better able to apply those acquired skills to new problems.

3.3 Problem centred Learning (PCL)

The researchers Hanlie Murray, Alwyn Olivier and Piet Human at the University of Stellenbosch developed the PCL approach (Murray, Olivier and Human, 1998). One of the goals of mathematics is to prepare learners to become problem solvers, and problem solving is regarded as a medium for learning (Murray, Olivier and Human, 1998). The world they exist in will require them to collect, organize and interpret information to solve complex problems. The situation presented to learners must allow them to engage in mathematical tasks that is personally and realistically meaningful (Meyer, 2001, p. 238). This is based on the principle of Realistic

Mathematics Education **(discussed in 3.5)**. It is only logical that learners are provided with a classroom situation (a social environment), where they are exposed to similar problems (Kestner, 2001, p. 1). In each of the observed lessons, the researcher noticed that tasks were set to promote competence and acquisition of a specific mathematical skill and allow each learner to be an active participant. PCL can be compared to student-discovered application (Schoenfeld, 1983, p. 23) through which a lot of knowledge is discovered. Learners learn in different ways and have different learning styles and through talking, writing and reflection they construct their own meaning.

The PCL approach is based on the social constructivist theory where learning is regarded as a social process and learners are engaged in social activities. It also considers the nature of knowledge. It follows the theory that learning occurs when learners construct their own knowledge through their interactions with one another in order to improve the nature and quality of those constructions (Murray, Olivier & Human, 1998). Learners are encouraged to discuss, evaluate and reflect on the different strategies they used and also make sense of other learner's explanations in this manner learning from each other.

The devising of plans for solving a problem ought to be taught to learners, as it is a form of representation, which is an important part of problem solving (Maharaj, 2007, p. 41). Engaging learners in a visual process as they progress will encourage them to talk about what they are doing and why because *"we cannot separate visual representations from the talk that surrounds them"* (Whitin and Whitin, 2001, p. 230). By using visualization and problem solving strategies, solutions are discovered creatively and this creates openness to new ideas. The learners make connections and invent solutions as they see the picture.

Learning is part of the social process and through social interaction between learners and educators knowledge is acquired. Social interaction creates opportunities for learners to talk about their thinking. Open dialogue between the educator and learners and learners themselves in a consensual domain helps to develop their cognitive processes. The role of an effective classroom educator is that of an active problem facilitator and they should involve themselves as little as possible in the learning process (Murray, Olivier and Human, 1998). They must make instructional decisions in such a way that not everything is told to learners beforehand and the learners are able to learn something. The educator must provide the necessary information for learners to understand the problem, communicate with each other and capture their thoughts on paper (Murray, Olivier and Human, 1998, p. 175). They should merely state the problem and allow ample time for the solution to be discovered. If the problem is deemed difficult then the educator needs to restructure the problem by providing a diagram to support the visualization of the given information (Maharaj, 2007, p. 39). Samson (2007, p. 40) state that since pictorial representations are easily understood, questions can be set in a variety of practical contexts in order to allow learners for greater scope in terms of solution strategies and can be made more learner centred (Maharaj, 2007, p. 38). *"Pictorial representation solutions"* have the potential to expand the range of solution strategies and allows for better learner participation in the learning process (Maharaj, 2007, p. 39). According to Samson (2007, p. 8) the use of representation allows for a variety of visual strategies as the *"diversity of visualization strategies has direct pedagogical application within the classroom context"*.

The educator should not interfere but rather spend a great deal of time listening to learners with the purpose of understanding and monitoring their social procedures and thinking processes. This enables the educator to present appropriate learning experiences that will facilitate learner's growth in mathematics especially becoming able problem solvers. Stylianou (2002, p. 303) stresses that in order to become able problem solvers learners need to acquire both the knowledge base and the strategies that will enable them to solve problems effectively. Strategies here do not only refer to problem solving strategies but also using various visual techniques. Educators must be able to design activities that will assist learners attain designated objectives (Kestner, 2001, p.1). Murray, Olivier and Human (1998, p. 172) state that no matter how well designed a problem is the amount of quality learning that takes place will depend on the learners and teachers expectations.

PCL creates a model where the learner becomes the thinker and is engaged in a self-directed search for solutions. If applied appropriately, the PCL approach to teaching and learning particularly in co-operative social settings can support learners from different social and cultural backgrounds in managing, overcoming, or eliminating traditional limitations (Nieuwoudt, 2000, p. 337).

3.4 Theory of the Growth of Mathematical Understanding

Very often, we hear pupils say, "I don't understand" to "I think I understand". Meaningful learning results in mathematical understanding.

In 1989 Susan Pirie and Thomas Kieren developed the theory of the Growth of Mathematical Understanding **(Figure 5)** cited in Cobb (1994). This theory discusses in detail the constructivist's explanation of understanding as a continuing process of organising one's knowledge structures. According to Meyer (2001, p. 239) *"Mathematical understanding is structured and interconnected"* and the learning of mathematics occurs by *"the progress through the levels of understanding"*. This theory has eight levels namely, primitive knowing, image making, image having, property noticing, formalising, observing, structuring and inventising.

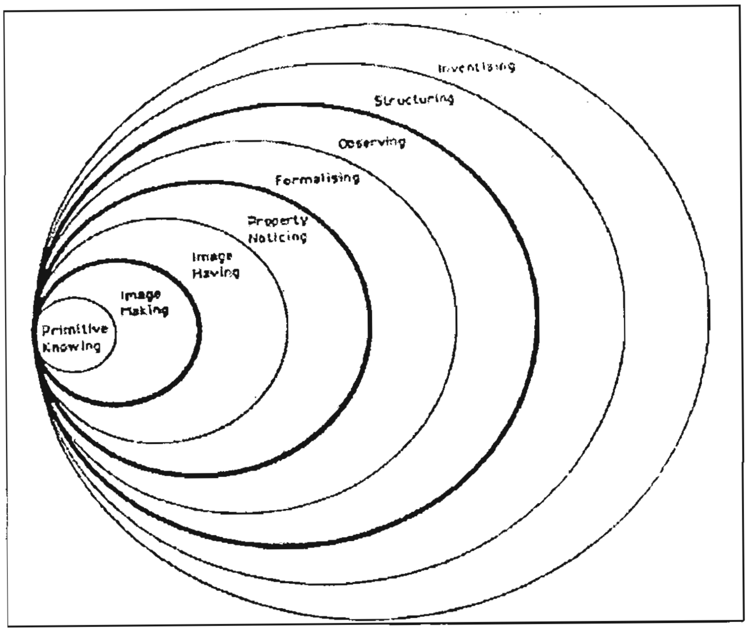


Figure 5: Susan Pirie and Thomas Kieren Model of Mathematical Understanding cited in (Cobb, 1994, p. 167)

Level one is primitive knowing. It is the starting point of mathematical understanding. It is where a person draws from previous experience.

Level two is image making. A mental picture is created of the concept or of the given problem. If the learner cannot create an image of what is seen then the other levels cannot be understood.

Level three is image having which is seeing. The learner constructs mental representations of what is asked. He attaches meaning to demonstrate an understanding.

Level four is property noticing. At this level understanding occurs. The learner draws from previous experience and combines with existing images. Teacher intervention occurs at this level to check learner understanding.

Level five is formalising. The learner creates a mathematical definition. It is a process of seeing and communicating the thought.

Level six is observing. Learner starts to make comparisons and looks for patterns to cement his understanding.

Level seven is structuring which involves the formal application of the theory.

Level eight is inventising. Learner asks questions and looks for alternate solutions to get a better understanding.

Brownell cited in Willoughby (2000, p. 2) conducted experiments that showed that if learners learn with understanding then they are able to use their understanding to transfer the acquired knowledge to new situations. Hiebert cited in (Willoughby, 2000, p. 2) states there is much evidence that *"mathematics learned first with understanding and then practiced for retention is more useful and transferable than mathematics learned by rote"*.

According to Meyer (2001, pp. 238-239) mathematics represents itself in *"real phenomena"*. Meyer elaborates that in order for learners to gain an understanding of mathematics the starting point must be real to learners **(reference made to realistic mathematics education discussed in 3.5)** and must be based on learners understanding of concepts which is learnt over an extensive period. As learners progress through the various levels, they make connections through their creation and use of models, drawings, diagrams, tables, or symbolic notations (Meyer, 2001, p. 238). The researcher is of the opinion that visualization is one of the most powerful representation strategies as it plays an important role in mathematics as it allows conceptual knowledge to be constructed in the mind and this forms the foundation for understanding and retention.

3.5 Realistic Mathematics Education (RME)

The term Realistic Mathematics Education (RME) is somewhat confusing. One will readily refer to it as realistic which shows its connection with the real world but it is actually related to problem situations which learners can relate to and imagine. This shows its strong connection to visual literacy which requires learners to create visual images in their minds. Learners are encouraged to construct ideas for problem situations using mental representations. It is the importance on making something real in your mind that gave RME its name (Van den Heuvel-Panhuizen, 1998a).

Realistic Mathematics Education (RME) started almost thirty years ago (Meyer, 2001, p. 238) and has evolved since then. RME is linked to the Realistic Theory developed by Hans Freudenthal in Netherlands (Van den Heuvel-Panhuizen, 1998a). It represents a significant departure from traditional ideas about learning and teaching of mathematics.

RME was a reaction to the Dutch approach to mathematics which was seen as *"mechanistic mathematics education"*. Van den Heuvel-Panhuizen (1998a, p.1) in making reference to the reformation of mathematics in the Netherlands state *"the reason why the Dutch reform of mathematics education was called realistic is not just the connection with the real-world, but it is related to the emphasis that Realistic Mathematics Education puts on offering the learners problem situations which they can imagine"*. Van den Heuvel-Panhuizen (1998b) saw mathematics as a human activity and it must be connected to real world mathematics education. She stressed that education should give learners an opportunity to re-invent mathematics by continually doing it and relating it to their real world. In South Africa much emphasis is placed on teaching mathematics by using real life situations to assist learners transfer school acquired knowledge to the real world situation in order to gain a better understanding of the real world. To learn mathematics, learners must be put in situations that are real to them and with which they can identify. Problems given to learners must relate to their world and experience and must make sense to them (Department of Education, 2003), as there is a link between the learner's experience and what is learnt. With the introduction of RME in South Africa, it gave rise to the Realistic Mathematics Education in South Africa (REMESA).

Aubrey cited in Nickson (2000, p. 5) state in order to make sense of the mathematics the learners meet in school and to make it their own, they have to link it with the reality of their world and what they already know. The Department of Education recommends that educators in South Africa expose learners to problem-solving activities using life related problems. Nickson (2000) state that if we desire children to learn in a meaningful manner, the purpose must relate to their development and adaptation to the world around them. Learners can be provided with opportunities to widen their experience but there is a danger of setting problems, which are not meaningful to them. Ball cited in Lubienski (2000) found using life related problems could be problematic due to different interpretation and cultural approaches. She found that when her learners were exploring real world problems they were distracted and confused. Adler cited in Pournara (2000) state that when a problem is taken from its original context and drawn into the school environment, it becomes a school task, not a real world task. The learner cannot see the relevance of the problem to the real world. Gravemeijer cited in (Nickson, 2000, p. 3) is of the view that *"mathematics has to be realistic to be accessible to them"*. Gravemeijer's view is supported by Skovsmose and Reid cited in Nickson (2000) who state that the world is the community in which learners live and this forms much of their reality and mathematics will be meaningful if learners see its relevance to their world. De Lange cited in Pournara (2000) state that if learners see the relevance of school mathematics to everyday life, they will be more motivated to learn maths and will transfer their mathematical skills beyond the classroom. These differing views open the door for further research.

According to Van den Heuvel-Panhuizen (1998b, p. 3) RME rejects the traditional procedural approach to mathematics and the splitting up of the curriculum in meaningless parts. It moves away from teaching learners small parts of the curriculum in isolation in which they have to work individually. RME encompasses meaningful conceptualization of learning where learners are seen as active participants in the teaching and learning process and they are given opportunities to share their knowledge with others.

Ball cited in Lubienski (2000) state that although real world mathematics may be unsuitable for developing mathematical knowledge, the researcher states from experience as a mathematics educator that there is a strong need to include real world mathematics in the curriculum as it will provide the means to address, social, cultural and economic imbalances that we have inherited from the apartheid era. Nickson (2000) states that increased attention must be paid to these aspects as it now affects the way mathematics is taught in the classroom. The need to address the social, political and economic influences of mathematics must be taken seriously and for this we need to force ourselves out of our comfort zone of rule-based mathematics and procedural thinking.

Streefland cited in Nickson (2000, p. 7) identified the following educational principles associated with realistic mathematics education and the growth of understanding **(discussed in 3.4 in Chapter 3):**

1. The source of concept formation is reality. The learning of concepts and skills is achieved over a period of time as the learner moves through the different levels of abstraction. The learners bridge these gaps through their creation and use of models, drawings, diagrams, tables and symbolic notations.
2. Pupils are given the opportunity to be constructors and actively contribute to the learning process by this involvement.

3. The learning process must be interactive so that in the course of constructing knowledge from real-life situations, pupils discuss and collaborate when necessary.
4. Different lines of learning are entwined so that both vertical and horizontal mathematization can take place. Mathematical understanding is interconnected therefore learning strands should be intertwined.
5. The various tools used in the process of coming to understand mathematics symbols, diagrams and visual models, should result from the need to describe and use what they have found out for themselves. Learners must have the opportunity to reflect on what they have learned as they progress through the levels of understanding (discussed in 3.4 in Chapter 3).

Gravemeijer cited in Nickson (2000, p. 7) state that realistic mathematics is *"concerned with generalizing (analogues, classifying and structuring) and formalizing (modelling, symbolizing, schematizing and defining)"*.

The researcher finds Nickson's (2000, pp. 4-5) view on the implications of the constructivist approach in the teaching and learning of mathematics very significant as it sheds light on the relevance to realistic mathematics. The learners are no longer seen as receivers of knowledge but makers of it. According to Piaget knowledge is constructed as the learner strives to organise his experiences in terms of pre-existing mental structure. According to constructivism, learners need to develop their own knowledge from their understanding and experience by selecting, absorbing and adjusting what they experience in the world around them. Individuals create meaning through their interactions with each other and the environment they live in (Kim, 2008). The representation of reality differs from one individual to another therefore individuals need to construct their own mental representations of situations, events, and conceptual structures (Nickson 2000, p. 4).

The social constructivist theory acknowledges that both the social process and the individual play a central and essential role in the learning of mathematics as the source of knowledge is through interactive work with other people. The key feature of social constructivism is that knowledge is constructed on the basis of experience and previous knowledge. This provides the basis for understanding and serves the purpose of guiding further actions.

Van den Heuvel-Panhuizen (1998b, p. 1) states that RME focuses on the development of learners' knowledge and understanding of mathematics and continually works toward the progress of learners. Considering the TIMSS results, it seems that RME can bring forth progress in achievements.

3.6 Conclusion

The researcher has reviewed the above theories as it has significant implications in the teaching and learning of problem solving. The curriculum in South African schools calls for learners to participate actively in mathematics lessons therefore educators need to ensure that activities are more learner centred. Learner centeredness will develop independent learning and using their cognitive abilities learners will be able to enhance their ability to understand the real world.

Problem solving ties in with to these theories in that learners will be able to construct their own knowledge. The type of problems given to learners must be realistic in nature to enable them to find solutions that will give them a better understanding of the world.

Mathematical understanding is crucial to learning mathematics. Through the process of visual literacy and visualization, the learner can represent and make public his understanding so that the educator is able to correct any misconceptions before it becomes embedded in the learner's subconscious. A better understanding of what learners actually do to solve mathematics word problems may lead to the consistent use of visualization and fewer mathematical errors (Pape, 2004). If the learners are encouraged to comprehend and expressively represent mathematical word problems rather than convert them into mathematical operations then they are more likely to solve these problems on their own (Pape, 2004, p. 214).

As stated in the National Curriculum Statement, educators are referred to as facilitators in the classroom. They now need to facilitate the process of learning in the classroom. Mathematics through problem centred mathematics and problem centred learning can now be given prominence so that learners understanding can be developed using realistic mathematics. The educator uses learners' ideas and weaves it into the learning process so that knowledge constructed in a social environment becomes concrete in the real world. The question as to whether learners can apply school mathematics to real life situations needs to be further investigated.

CHAPTER FOUR

RESEARCH METHODOLOGY

4.1 Introduction

In this chapter the researcher discuses the how the research study was conducted with reference to the research instruments used, pilot study, data collection and ethical considerations.

The methods of research are tools by which we investigate the problem. Methodology consists of the systematic procedures by which we travel from the initial identification of the problem to the conclusion. The role of the research methodology is to complete the research in a valid manner.

Research methodology involves identifying of the research problem, formulating hypotheses, review of literature, designing of methodology, identification and designing of research instruments, sampling procedures, data collection, data analyses, drawing conclusions and making recommendations and the preparation of the final report.

4.2 Methodological Approach

The aim of a research methodology is to assist the researcher to develop a research strategy, understand the process of data collection and to be able to analyse the data. According to Cohen, Manion and Morrison (2000) the underlying principle of research assists in deciding the methodology and design of the research study and the methods, which are a range of approaches, used to gather data in research.

The researcher engaged a mixed method methodology in this study. According to Creswell cited in Maree (2007, p. 260) the mixed method is defined as a procedure for collecting , analysing and 'mixing' both quantitative and qualitative data at some stage of the research process to understand a research problem more completely. The motivation to use such a method was to gain a more in-depth understanding of the teaching and learning of problem solving and to produce a validated conclusion. Maree (2007, p. 290) state that using the mixed methods might lead to definite and validated findings.

In this research, the mixed method approach was used to collect both quantitative and qualitative data as the researcher intended to investigate whether visual literacy and visualization assist learners in becoming better problem solvers. The researcher wanted to interpret how learners created visual images, used visual illustrations in the field of problem solving, and focused on their mathematical reasoning and understanding. Therefore, this study was placed within the interpretive research paradigm.

According to Cohen et al (2000), the interpretative paradigm focuses on the individual and his understanding of the world around him. The researcher began with the individual and tried to understand his interpretation of the world. This research focused on the learner's problem-solving skills using visual literacy and visualization techniques and their understanding, reasoning skills and strategies employed during the problem solving lessons and the evaluation worksheet. After observing and talking to both educators and learners, the researcher attempted to interpret what was observed and said.

The process of using multiple methods to acquire knowledge of the same phenomenon using different research measures is known as triangulation (Maree, 2007). Triangulation involves the amalgamation of quantitative and qualitative methods when an area under enquiry is investigated from different angles in order to get a enhanced understanding. The sequence of data collection in this study was a questionnaire, class observation, evaluation worksheet and an interview **(See Appendices)**. These are discussed in this chapter.

4.3 Research Design

This study will follow a mixed methodological approach by conducting both a quantitative and qualitative study. The researcher used a questionnaire for educators, observations of lessons in the classroom and learners activities and semi structured interviews with learners. These instruments were used to elicit in-depth knowledge. These instruments are discussed in **4.4; 4.5; 4.6; 4.7.**

4.4 Questionnaire

Baro cited in Koul (1988, p. 142) describes a questionnaire *"as a systematic compilation of questions that are administered to a sample of population from which information is desired"*. Questionnaires are popular means of collecting all kinds of data since it is through this medium that the data is generated.

The most obvious consideration involved in the selection of the population for answering a questionnaire is to contact people who will be able to supply the crucial information you need for the research. Questionnaires were administered to two hundred mathematic educators within the intermediate and senior phases. The researcher used a questionnaire in this study **(Appendix A)** as he found it advantageous to gather data regarding a wide range of issues. The rationale of using a questionnaire was to obtain an in-depth understanding on the problem solving strategies, visual literacy and visualization skills, and methods used by the teachers to engage learners in problem solving.

The questionnaires, for convenience, were distributed to the schools within the South Durban ward where the researcher teaches. The learners attending these former House of Delegates, House of Representatives and Model C controlled schools are from different social, racial and cultural backgrounds. Some of the school's quintile ranking is within the 2 to 7 range making them well resourced to under resourced. Thus, the data collected via the questionnaire for this study was obtained from different performing schools. The questionnaire was distributed personally to these schools. The reason thereof was two fold: one was to establish a rapport with the Principal and educators and the other was to arrange with the Principal that he collect the questionnaires on my behalf by the due date. Harold cited in Best (1977, p. 167) found in his study of questionnaire returns that when a request is made to the administrative head of an institution to assist in the collection there was higher return. Of the two hundred questionnaires that were distributed 120 were returned to me.

Leedy (1974); Best (1977); Lydeard (1991); Maree (2007); Mitchell and Jolley (2007) all state that questionnaires should be designed to fulfil a specific research objective. The significance of the questionnaire was clearly and carefully stated since it seeks only that information which is relevant to the study. The questionnaire used was to obtain the views of mathematics educators on the teaching of problem solving, visual literacy and visualization. The questionnaire was brief, simple to read and easy to respond to allowing the respondents to organise their own thinking. It was hoped that it would solicit only that data essential for the study in question.

There were both closed (dichotomous and Likert scale questions) and open-ended questions asked in the questionnaire. The closed questions permitted only certain responses therefore provision was made for unanticipated responses by allowing a category for "other" or "please specify". Maree (2007, p. 161) state that there are advantages of using open-ended questions. Using open-ended questions allowed for greater depth of responses as the educators were given the freedom to reveal their opinions, experiences and to clarify their responses. Open-ended questions allow the respondents *"to explain and qualify their responses"* (Cohen et al, 2000).

This gave the researcher a deeper understanding of the area under investigation.

When designing the questionnaire as one of my research instrument all the relevant factors were taken into consideration. The questionnaire was piloted, as it allowed the researcher to obtain a reliable overview since the terminology used in the crux of the study was relatively new. The purpose of the pilot study was *"to test whether there are items that they may have difficulty in understanding or in knowing precisely what the writer of the questionnaire is seeking to find out"* (Leedy, 1974, p. 81) and the time needed to complete the questionnaire. Responses from the pilot resulted in improvements been made to the questionnaire. Koul (1988); Lydeard (1991); Maree (2007) and Mitchell and Jolley (2007) state that the pilot study should be carried out from a population similar to those from the main study, which in this instance were primary school mathematic educators and learners. The questionnaire was also piloted to ensure that the data collected was relevant to the field of study and to ensure reliability and validity.

Cohen et al (2000), Maree (2007) state that the following aspects, informed consent; confidentiality and right to withdraw without prejudice, have to be considered when working with questionnaire. A covering letter outlining the rationale of the study and the above-mentioned factors were explained on the questionnaire distributed to educators **(Appendix B).**

The questionnaire was divided into three sections **(Appendix A)**. The first section focused on educator's knowledge of problem solving and the content on problem solving in the National Curriculum Statement (NCS). The second section was the crux section of the questionnaire. It was designed to obtain information from educators on their understanding of visual literacy, visualization and problem solving strategies and their ability to use these skills in the teaching of problem solving. The third section focused on learners and their mathematical ability to use visual skills and problem solving strategies to solve problems. The questionnaire covered the relevant areas that the researcher proposed to investigate.

4.5 Observation

Observation, according to Cohen et al (2000) refers to the gathering of fresh data as they occur and access is gained to personal knowledge and happenings at the site it occurs. Observation of real-life situations took place as they afforded the researcher the opportunity to gather 'live' data from 'live' situations" and pertinent recordings are made thus enabling the researcher to enter and understand the situation that is being described (Cohen et al, 2000).

As a research tool, observation needs proper planning, execution and adequate recording.

The planning for observation must take into consideration the subjects to be observed, the activities to be observed, the length of each observation and the tools to be used to observe and record (Koul, 1988, p. 168). The researcher chose a grade 6 class comprising of 45 learners and two grade 7 classes comprising of 45 learners in each class to be observed. The learners attending this school are multiracial but predominantly Black. The ratio between Blacks to Indian learners is 3:1 and the classes are of mixed ability. The educators in these grades were given an opportunity to participate in the studies. At the site of study there are five units of grade sevens and two educators who share the teaching load. Both educators agreed to be part of the study and through a process of negotiation, they nominated the class in which they wished to be observed. There are three grade six classes and three teachers. Through a negotiated process between the grade six educators, the educator with the most number of years of mathematics teaching experience was chosen to participate in this study.

Observations were made of three educators teaching problem solving lessons and three classes of learners completing their activities in the classroom. The educators were observed using a structured observation criterion **(Appendix C)**. Koul (1988) state *"it is advisable to develop an observation form while making observations"*. Specific behaviour (teaching style) was observed and recordered on the schedule. Field notes were made of each lesson observed to record teaching style, learner's interaction in the classroom and their methods of solving problems, chalkboard work and the nature of any materials (worksheets and textbooks) used by the educator and learners during the problem solving lessons. The learners were observed whilst they were engaged with their written tasks in the classroom. This took place concurrently with teacher observation. The learners' books were examined to determine the type of skills learners used in other areas of mathematics.

The observations took place four times and these observations were spread out systematically during the course of the second and third term with one mathematics lesson per week per grade observed to record educator-learner interaction and learner performance in problem solving. *"The greater the number of observations, the greater the reliability of the data might be"* (Cohen et al, 2000, p. 314). The lessons observed were all one hour problem solving lessons. These visits were negotiated with the grade six and seven educators. They were happy to co-operate as they felt it would be beneficial to the teaching profession.

The researcher used an observational schedule **(Appendix C)** and monitored the educators and learners. The observational schedule consisted of definite categories for observation. The focus was on how educators taught problem solving, the skills and strategies used and their interaction with learners.

The observed data must be recorded immediately after the observation as it is still fresh in the observers mind. The observation report was written as soon as the lesson was completed thus giving the researcher an opportunity to describe exactly what had transpired in the classroom. When recording the observation the observer must minimise the influence of his biases on the observation report and must objectively record the relevant data (Koul, 1988, p. 170).

According to Koul (1988, p. 172) the observation method has the following advantages:

1. It provides a system for studying various aspects of human behaviour, which may be the only effective way to gather data in a particular situation.
2. It enables the observer to code and record activities at the time of its occurrence.

4.6 Evaluation Worksheet

The evaluation worksheet **(Appendix F)** was a problem solving activity that the grade six and grade seven learners engaged with in class. The purpose of the worksheet was to evaluate visual skills and visualization methods and strategies used by learners to solve problems. This worksheet consisted of six problem questions of which two included diagrams. The problems that were used in this study were specifically chosen such that their solution is facilitated by the use of visual representations. The questions were of the non-routine type and was chosen after consulting the Mathematics Teachers Guide (Department of Education, 2003), observing the lessons and consulting the textbooks and Association of Mathematics Educators of South Africa (AMESA) question booklets used by the educators and learners. This was to ensure that learners had not solved the same sort of question before. The aim of using an evaluation worksheet was to afford the learners a chance to utilise their own methods and skills to provide solutions to the problems. This was also an opportunity for me to explore the visual skills and problem solving strategies used by the learners.

4.7 Semi-Structured Interview

An interview is classified as a two directional conversation (Cohen et al, 2000; Maree, 2007) as the researcher was interested in what is inside the participants head. For the purposes of this research, a semi-structured interview was used. According to Maree (2007, p. 87) a semi-structured interview is used *"to corroborate data emerging from other data sources"*. Other data sources in this study are educator questionnaires, teacher-learner observation, and the evaluation worksheet.

The interview assisted in obtaining a better understanding on how learners construct ideas when engaging in problem solving. The interview process involved a face-to-face interview with learners as this kind of interviews provides greater flexibility. This was done after the classroom observations. According to Mitchell and Jolley (2007) these kinds of interviews allow additional questions to be asked to follow up on interesting or unexpected answers. The interviewer is at liberty to *"rephrase the questions, modify them, and add some new questions to his list"* (Koul, 1988, p.174). This interview consisted of predetermined core questions **(Appendix G)** to guide the process. The learners were given problems and were asked to explain their reasoning as they solved the problem. The researcher probed their responses by asking additional questions to get a better understanding of what the learners were trying to express in the evaluation worksheet.

A random selection was done using the school's administrative Master C programme. This programme arranged the learners according to their first term assessment aggregate scores. The learners were randomly selected from the high to the lower achievement group. Every fifth learner on the list was selected. Further random sampling occurred to ensure that learners had a good command of the language of instruction. Ten learners from grade 7 and five learners from grade 6 were chosen for the interview process. The questions were piloted with the learners from the same school where the study was undertaken. The educators and learners who participated in this pilot were not part of the main study. This was done to ensure that the language and terminology used was understandable, the questions were meaningful and they elicited the relevant data.

These interviews, limited to 30 minutes, were audio taped with the consent of the learners and their parents. When all the interviews were completed, the audiotapes were transcribed and copies of the learner's worksheet were merged with each transcript. This was done to maintain the authencity of the data and to ensure there was no misinterpretation and misquotes. Recording the interview is a vital aspect as it helps eliminates omissions, misrepresentation and other amendments and *"provides an objective basis for evaluating the adequacy of the interview data"* (Koul, 1988, p. 175). Notes were taken to support the recordings. Koul (1988, p. 176) state that *"the notes should include unusual and significant behaviour as well as the responses to questions of the interviewees"*. To ensure validity of the interview the responses of the interviewee must be corroborated with other sources of data. The corrobation of the evidence obtained from the lesson observations and the interviews enabled the researcher to develop a comprehensive understanding of learners' experience and see mathematics from the learner's perspective.

Koul (1988, p. 176) state that the interview process has advantages as it:

1. Allows for an opportunity to extensively probe certain areas of inquiry.
2. Permits greater depth of response, which is not possible through other means of inquiry.
3. Enables the interviewer is able to obtain information concerning attitudes to certain questions.

Piloting the interview is important.

4.8 Pilot Study

A pilot study was carried out at the school where the study was undertaken. Best (1977); Cohen et al (2000); Mitchell and Jolley (2007) state that a pilot study is a dry run. All data collecting instruments must be piloted as it rids the instruments of defects and ambiguity so that the respondents in the main study will not experience problems in completing the instrument.

According to Cohen et al (2000), a pilot study helps to increase the reliability, validity, and practability of using a questionnaire as a tool. Cohen et al (2000) state that all data collecting instruments must be piloted. Piloting assisted the researcher with the questionnaire layout and length, the questions used in the questionnaire and semi structured interview and its ability to provide data relevant to the research (Cohen et al, 2000; Maree, 2007). The piloting of the questionnaire, semi structured interview and evaluation worksheet also assisted in determining the amount of time that was required in the completion of the data collecting instruments.

4.9 Data Analysis

Mayan cited in Maree (2007, p. 295) state that data analysis is the process of observing patterns in the data, asking questions of those patterns, asking additional questions, seeking more data, furthering the analysis by sorting, questioning, thinking, constructing and testing conjecture. Data analysis in mixed methods approach involves analysing quantitative and qualitative data (Creswell cited in Maree, 2007). Data was collected using a questionnaire, observation, an evaluation worksheet and interviews. The quantitative and qualitative data (questionnaire, lesson observation and evaluation worksheet) was coded and classified into specific categories. The qualitative data were coded, classified and categorised. The data was collected, processed, condensed and then interpreted using triangulation in order to make this study trustworthy, reliable and valid (Maree, 2007).

4.10 Access

In this study, the school chosen out of convenience falls under jurisdiction of the Department of Education and the research involved learners and educators, hence this necessitated written consent from all stakeholders. The researcher needed permission from School Governing Body, Principal, Department of Education, parents of learners and learners. It is vital to seek authorization early from all stakeholders to carry out an investigation.

The school principal and School Governing Body were approached and a letter describing the research study was discussed **(Appendix E)**. A background to the research and rationale thereof was discussed. The principal of the chosen school granted permission.

Permission was sought from Department of Education via the institution of my study to conduct research in the school in Merebank, KwaZulu-Natal, an area inhabited by low to middle-income families.

4.11 Sampling

Sampling is a process by which a relatively small number of individuals or events is selected and analysed in order to find out something about the entire population from which it was selected Koull (1988). In studying a specific group a generalization can be made to the larger group. (Koul, 1988; Cohen et al, 2000) state that the following steps must be considered when choosing the sample:

1. Defining the population

The smaller group involved in the study is a sample of the larger group called the population. As a researcher I needed to identify who is taking part, the nature of the participants, their statuses and roles in the study. This study required mathematics educators and learners.

2. Sample size

A sample is a subgroup of the population that is being investigated. The entire population (educators and learners) cannot be measured. For the purpose of this study, one class of 45 grade six learners and two classes of 45 grade seven learners at the school was chosen.

3. Selecting a representative sample

It is not possible to get every member of the population involved in the study. A good sample must be representative of the entire population. When a *"suitable sampling design has been applied"* Maree (2007, p. 291) *"it will have the potential to generalise to large populations"*. The population for this study was selected on the basis that they would help answer the questions relevant to the study. In this study for the questionnaire was meant specifically for the mathematics educators and for the observation and evaluation worksheet I chose the grade 6 and grade 7 learners. The reason for choosing Grade 6 and Grade 7 learners was based on previous studies (Kotze and Strauss, 2007; Lavy, 2007) and the results of the TIMMS (Reddy, 2003) where it was found that these grades produced mixed results. The researcher wanted to discover for himself whether these were truly the problematic grades in the education system.

4. Obtaining an adequate sample

Depending on the type and style of the study, the sample is chosen. This study was on a relatively new investigative area in mathematics in grade six and seven (visualization) and the researcher required in-depth knowledge from a wide range of experienced mathematics educators. As a matter of convenience, the questionnaire was distributed in the South Durban ward where the researcher presently teaches.

For the purpose of this study, the researcher used probability sampling which includes random sampling. For the interviews, random sampling was used **(discussed in 4.7 above).** If the participants are randomly selected then those participants will be representative of the entire population. The educators in the schools in the Durban South area are multi racial and are representative of the population.

4.12 Ethical Issues

Ethical issues are a vital part of a research study (Setati, 2000; Terre Blanche and Durrheim, 2002; Maree, 2007). Obtaining consent does not only involve the signing of a consent form. It is a complex yet important. Bak (2004, p. 28) state that ethical guidelines must be considered by the researcher whose overall responsibility is to:

1. Design, conduct and report research in agreement with recognised standards of scientific aptitude and ethical research;
2. Comply with national and provincial law and policy and with professional standards governing the conduct of research;
3. Minimise the possibility that the result will be misleading ;
4. Acquire appropriate permission from the human subjects;
5. Protect the rights and interests of humans and the welfare of animals;
6. Protect the identities and interests of those involved;
7. Guarantee the confidentiality of the information given to the researcher;
8. Consult the university Ethics committee about any unclear ethical issues.

Written consent was obtained from the Principal, School Governing Body, and the Department of Education to use the site for my study. Ethical clearance was obtained from the institution through which the researcher is studying.

As the researcher, I had to take into consideration the three ethical principles namely autonomy, nonmaleficence and beneficence.

In the first principle, autonomy had to be considered (Terre Blanche and Durrheim, 2002). Informed written and signed consent was obtained from educators, parents/guardians of the participants **(Appendix D)** and the participants themselves. Diener and Crandall cited in Cohen et al (2000) defines informed consent as the procedures in which individuals choose whether to participate in an investigation after being informed of facts that would be likely to influence their decisions. A letter was given to participants. In this letter, the researcher introduced himself, the purpose of the study, indicating that the respondent was under no obligation to participate and if they did they had the right to withdraw without any consequences; ensuring their confidentiality; anonymity and non-traceability in the case of publications as no identification was needed (Cohen et al, 2000; Maree, 2007).

Regarding the second principle of nonmaleficence, as the researcher I had to ensure that no physical, emotional , social or other harm came to the participants (Terre Blanche and Durrheim, 2002).

The third principle of beneficence (Terre Blanche and Durrheim, 2002) had to ensure that the research benefited the participants and the society. Problem solving has become a critical area of focus (DoE, 2008) and the results obtained from using visual techniques in this study will address critical issues that will assist educators and the teaching fraternity at large in aiding their learners becoming proficient problem solvers.

Holding a management position, the researcher had to consider power relations and removed himself from occupying any positions on the Mathematics Learning Area committee and teaching mathematics for the year of study. All ethical issues were considered for this study. A covering letter accompanied the questionnaire to the educators.

4.13 Conclusion

This chapter serves an overview of how this study was conducted with respect to methods and procedures. The research methodology therefore serves as a guideline with respect to data collection and procedures followed.

The analysis of my data is discussed in **Chapter 5.**

CHAPTER 5

DATA ANALYSIS

5.1 Introduction

In this chapter, the researcher will analyse the questionnaire, classroom observations, problem solving evaluation worksheets and the interviews.

The focus of this study was to understand how learners used visual skills to solve problems and to gain an insight into the teaching of problems in the classroom.

5.2 Analysis of questionnaire

Below is the discussion of the data obtained from the educators in the questionnaire. From the 120 educators who completed the questionnaire the following statistics became available.

Is the teaching of problem solving neglected in our curriculum?

	Strongly agree	Agree	Disagree	Strongly Disagree
%	33	48	19	0

Table 1

Eighty one of the educators agreed that problem solving is a neglected component in the mathematics curriculum. This is an important finding within the sample because often educators are aware that it is neglected but yet have not ensured that they engage with it more often, as can be seen in the next table.

Do educators teach problem-solving strategies?

	Everyday	Once a week	Twice a week	Other
%	24	52	12	12

Table 2

Fifty two percent of the educators taught problem solving on a weekly basis using one hour out of the allocated five hours for mathematics in a week. Those educators that fell into the 'other' category taught problem solving only when the need arose. It should be noted that problem solving is part of all mathematics and the researcher therefore questions the idea that problem solving is taught only when the need arises. Perhaps the researcher should have probed the idea of need in problem solving. Twenty four percent of the educators taught at least one problem-solving sum in their everyday lesson as stipulated by the Foundations for Learning Document (DoE, 2008) which requires that problem solving be taught on a daily basis. As it has been already stated only 24 % of the educators felt a need to engage with problem solving activities daily. This deviates from the fact that learners are confronted with problem solving daily in real life.

How much of an educator's lesson time (1-hour period) in a day is spent on teaching problem solving?

Minutes	1 - 15	16 - 30	31 - 45	46 - 60
%	52	24	7	14

Table 3

Many educators indicated that they spend approximately quarter of an hour in the teaching of problem solving. The majority (81 %) responded that the time was not sufficient due to constraints related to syllabus coverage. Some of the responses, which have been categorised, are mentioned below.

Some of the educators' responses indicated that the learners are not trained to solve problems as:

- *More time is needed to teach them the basics in mathematics that they can use in their application to problem solving;*
- *Most learners require indepth explanation;*
- *We are still trying to get the basics right;*
- *Learners don't have the mental capacity to sit with problem solving for long periods;*
- *Children today do not do much thinking. Logic needs to be pointed out to them;*
- *They are being spoon-fed.*

Some of the educators indicated that trying to complete the syllabus was also problematic due to high demands made on them by the curriculum.

- *The syllabus is extensive – unable to spend more time on problem solving;*
- *Have to complete other aspects of the syllabus;*
- *Lesson is content driven. The curriculum is overburdened with specific skills and concepts which need to be developed.*

Educators admitted that teaching problem solving was time consuming as

- *Children struggle with problem solving and this takes up a lot of time;*

- *Time is limited when the problem centred approach is used;*
- *Learners require more time to apply their maths knowledge and skills;*
- *Learners need more time to understand and grasp the concept;*
- *Learners are of different capabilities and work at different speed.*

Educators also indicated that learners are confronted by various other factors such as reading and language.

- *Children are from various backgrounds and language issue to deal with;*
- *Learners are unable to read – find problem solving difficult;*
- *Language of teaching is a problem.*

Twenty one percent of the educators who spent more than 30 minutes teaching problem solving felt that there was a need:

- *To focus on teaching a strategy and providing application and if there is need, remediation is provided;*
- *To teach one strategy at a time and let learners master such a strategy;*
- *For more time to be allocated to problem solving;*
- *There should not be a time-constraint on problem solving;*
- *To teach problem solving as a life skill more time should be spent on it.*

Most of the responses tended to give the impression that problem solving activities were separate from other teaching activities. Yet it can be argued that much of the teaching in a class can be centred around problem solving methods and strategies.

Furthermore, the tendency of educators to see problem solving activities as a time consuming process is probably based on the idea that many problems need to be completed in the allocated lesson time. One could argue that if a few strategic problems are completed well then learners may develop good skills in solving similar or related problems.

Although the curriculum is content driven, it is still imperative that the teaching focuses on problem solving strategies. In any case, problem solving tends to lend itself to mathematical modelling. This is essential when one considers real world mathematics.

Table 4 shows that the educators interviewed believed that the learners would perform much better if they used visual techniques. The researcher can only assume that this conclusion was established due to their own experiences in teaching over a number of years.

Educators' rating of their learner's performance when not using visual strategies

	Poor	Good	Excellent	Unchanged	No Answer
%	81	7	0	5	7

Table 4

Table 4 showed that 81% of the educators interviewed actually believed that learners would perform poorly if no visual techniques were used. The data in Table 4 is significant for this research as it answers one of the key research questions. This then raises the question as to why, if they see the use of visual techniques as important, do only 24% **(Table 2)** of the educators use visual techniques daily? This is an important point and it should be researched further.

Do visualization skills assist learners in mathematics lessons?

When the educators were asked to rate their learners' performance when using visual techniques, 88% of them felt that their performance will be fairly good. It is also significant to note that 10% of the educators actually felt that when using visual techniques the learner's performance was very poor.

The empirical evidence strongly supports visualization as a necessity in the teaching of mathematics as compared to learners' performance without the assistance of visual techniques. Educators should encourage learners to draw more frequently if that solution strategy makes sense to them. They may not be able to use this method for all problems but they should be aware that diagrammatic representations are acceptable.

Do educators think that diagrams and pictures can overcome language barriers?

	Yes	No	No answer
%	82	14	4

Table 5

Educators stated using visualization can overcome language barriers which are a problem in many schools today. As mathematics educators we should acknowledge that language plays an integral part in mathematics considering the fact that majority of the learners in public schools have English as their second language.

Some of the statements made by educators on how visualization can overcome the language barrier and create a better understanding in learners are:

- *Pictorial representations help the barriers of language;*
- *Language barrier – mental picture overcomes the barriers;*
- *Visuals break down the language barrier;*
- *Seeing is believing. Gets first hand impression of the problem;*
- *A picture is easier to visualize – you don't then battle with the language;*
- *They can see and initiate use of pictures can enhance their understanding of the problem;*
- *Illustrations may assist understanding- learners come from a multilingual background;*
- *If they don't understand the language, pictures and diagrams can assist.*

These responses indicate that the educators actually believe that despite the language barriers that exist in the problem solving process, visual representations can and do help learners to understand the problem better. Understanding the problem is a good starting point for finding a reasonable solution. This emphasizes the point that not only is mathematics a written language but it is also amenable to visual methodologies as well.

It seems that the educators believe that the visuals inherent in the problem, removes the difficulties learners experienced with a lack of mathematical vocabulary. An example of such a sentiment was expressed when two educators stated that:

- *sometimes the children do not understand the vocabulary but when they see the objects and act out the situation they have a clearer understanding and communicate better;*
- *learners with reading and language disabilities can manage better with pictures.*

The comments made by the educators demonstrate their belief in the idea that seeing a problem has greater value than just reading it. They believed in the idea that seeing is often worth more than just trying to interpret. Their belief in visualization confirms their faith in using visualization as a means to overcome the language barrier in problem solving. This barrier can be overcome if words were translated into pictures or diagrams. Of greater concern to us should be the 14% who believed that pictures, diagrams or any other visual would not improve the learners understanding of the problem. Does this mean that these educators do not attempt to simplify the problems for their learners or is it that they engage in other means to make the problem understandable, thus ensuring an easier solution process?

Do learners enjoy problem solving?

	Yes	No	No Response
%	40	55	5

Table 6

Some of the educators' responses indicated that learners did not enjoy problem solving because they had difficulty with language, comprehension and literacy. The lack of these skills seriously hampered the learner's problem solving ability. Two educators stated that learners did not enjoy problem solving:

- *because of poor understanding of the language and concepts; and*
- *they find difficulty in reading and understanding problems.*

Whilst the above reasons impeded learners in their ability to solve problems there were other reasons for learners to dislike problem solving. Most of these reasons revolved around the learners.

Some of the reasons why learners disliked problem solving were:

- *cannot work on their own;*
- *lacked confidence and mechanical skills;*
- *have no previous strategies to assist them, so they merely give up;*
- *do not have an aptitude for it.*
- *did not know basics, example their tables;*
- *become demotivated and freeze;*
- *find it difficult to apply themselves.*

It is important to note that the 40% who answered yes came from the group who said that visualization will assist learners in problem solving.

The responses show that learners battle with language and as a result, lack the necessary confidence. It can be hypothesized that if more visualization is encouraged in a class, the learners may develop greater confidence.

The learners are taught in their mother tongue in the foundation phase and also in the school's medium of instruction. Are learners not learning to read, use language and communicate in an acceptable manner in the foundation phase and intermediate phase that they are battling to read and write at grade seven levels? Bohlmann and Pretorius (2008, p. 51) state that *"it is a matter of concern that even after seven years of primary schooling, many learners enter high schools with exceedingly low reading levels"*. A concerted effort needs to be made by all educators to ensure that learners obtain the proper training in word identification at a very early age to eliminate the reading problem. *"If schools wish to improve their mathematics teachings"* they must *"give serious attention to improving reading levels"* so that learners can increase their literacy levels to support their mathematical growth (Bohlmann and Pretorius, 2008, p. 51). So, if learners had all of these problems, as represented by the educators comments, would their teaching not improve learner attitude and performance if they used more visual spatial methods

Do educators manage learners who show a lack of interest in problem solving?

The educators provided varied responses. The following two responses indicate negativity or helplessness on the part of the educator:

- *I have no time to deal with individual learners as such;*
- *I don't really know what to do.*

If learners are not assisted with their difficulties then educators will be contributing to the poor results in mathematics and making learners become poor problem solvers. It is probably this apathy that instills in learners the idea that mathematics is difficult. Creative or interested educators could find alternate time slots in order to assist these learners and these slots could include lunch breaks and after school has ended.

Even the sense of helplessness should not be condoned because of the availability of a vast database of knowledge in books, journals and the world wide web.

Unlike the educators mentioned in the preceding paragraphs others felt that there was a need to assist learners who were facing difficulty with problem solving. They indicated that they tried to:

- *get peer assistance;*
- *organize group activities and this assists learners as they learn by solving the problems co-operatively;*
- *help by reading the problem;*
- *use visualization techniques;*
- *use concrete examples;*

- *provide examples that would interest the learners for example, sport, music;*
- *ensure that data is related to current events;*
- *give situations specific to a learner.*

Using peer and group activity to assist with co-operative learning is emphasised in problem centred learning. The theory encourages social interaction as it contributes to learners understanding concepts better.

By using concrete examples the educators were trying to make the problems as realistic as possible to match the learner's experience. By making them active participants, learners become active learners and they remember what they have done. The use of visualization techniques was to ensure that learners understood the problem through representation. In essence, the learners were beginning to construct their own meaning and this helped make sense to them.

Some responses were not ideal and may have contributed to more problems. The response that, "*I give them more homework*", was a case in point. The researcher disagrees that by giving homework learners will demonstrate a liking for problem solving. By giving learners additional homework without any support will increase their dislike for problem solving. From my personal experience I have observed that learners do not do the homework due to the frustration of not understanding the work. The researcher believes in more homework if it is given as consolidation of work done in the classroom. Learners must show an ability to solve problems in the classroom. Homework must be given not to frustrate the learner but to test their level understanding.

There was only one educator who believed that if one used visualization techniques, learner interest in problem solving would improve. Often, a lack of interest in an aspect of mathematics results in poor performance in that aspect. If educators can get learners to perform better, it might create the necessary interest. As has been stated previously, visual techniques could improve learner results, yet only one educator saw this as a possible way of improving learner results.

It might also point to the fact that educators have very little time to reflect on these issues and they often feel overwhelmed by the required syllabus. Perhaps, reflection (Artzt and Armour Thomas, 2002) should be encouraged by the Department of Education and it should be included in teacher assessment programmes.

5.3 Analysis of the classroom lesson observation

The purpose of the observations was to examine educator's instructional practices and learner's responses in the classroom. This was done to obtain an insight into how educators engaged learners in problem solving exercises.

The observation lesson focused on three aspects:

- The introduction and whole class teaching;
- Learners carrying out their written tasks; and
- The discussion at the end of the lesson

The researcher observed in the initial observation lessons that educators were using the traditional chalk and talk teaching strategy. After the first observation, I informally discussed visual strategies with the educators and they were given related articles by Kelly (1999); Sawada (1999); Arcavi (2003) and Lavy (2007). In subsequent lessons, the educators began asking learners to think creatively and work co-operatively as suggested in the given articles. The learners however needed a period of adjustment in order to work with problem solving in a different way. The educator's response could possibly be related to what they read. It must also be stated that there is a strong possibility that they changed their strategy because they knew what I was looking for.

Observation of the Grade 7 classes

In the initial lessons (1 and 2), the educators presented a clear lecture type lesson providing all the mathematical definitions and rules. Using the exposition and explanation method or as sometimes referred to as direct instruction (Jaworski, 1994, p. 8) the educators presented the mathematical content. The educators engaged in extensive one way verbal communication. Brown (2008) refers to this type of teaching by the educator as an attempt at depositing mathematical knowledge into the learners' heads. There were relatively no real verbal responses from the learners and communication between the learners themselves. A noticeable feature was the amount of time educators spent on teaching a problem skill. An estimated 10 minutes was spent on teaching. Sawada (1999, p. 57) state that for the problem solving process to occur, time must be allowed for investigating the nature of the problem and if a few minutes was spent on problems then he has *"grave doubts that a problem solving approach was being used at all"*.

The learners were given a few exercise problems, which they had to do as written work. An example of one of the problem is:

A farmer sold three prize sheep and bought two cows, but was short of R20, which he had to pay in from his own pocket. The next day, he sold two prize sheep and bought one cow and this time he had just the right amount of money. All the prize sheep had the same sale prize and all the cows the same cost price. What was the selling price of a sheep?

The learners busied themselves without any real direction. It seemed that the learners showed disinterest in this problem as it had no particular relevance in their lives. The context was not relevant. The educator brought the class together in the latter part of the lesson to provide the answers to these questions. An estimated 10 minutes was spent giving answers during this very important aspect of the lesson. None of the learners managed to get the correct solution to the above problem. The researcher is of the opinion that this amount of time is inadequate especially if an educator has to deal with learners' misconceptions and arguments. The conclusion of lessons must also cater for the consolidation of mathematical concepts that will lay the foundation for future learning.

The educators were asked in the questionnaire to describe the discussion part of the lesson and the majority of them indicated that they recapped the lesson and provided answers to the written exercise. Yet in the lesson observation very little time was spent discussing learners' answers. A great limitation of this type of a general conclusion was that the educators were not engaging with the learners in order to determine their understanding, and at the same time inhibiting learners from demonstrating their potential as problem solvers. What was also evident was that educators were not reviewing the errors made by the learners and the answer was either right or wrong. This should not be the case as learners' misconceptions are not rectified and the learners may become dejected and lose motivation if they find that they are continually getting their solutions incorrect without understanding why. Sawada (1999) states that the lesson must end with a discussion, as it is an integral part of the lesson. All learners must be able to display an understanding of the problem by displaying the correct solution for all to learn. Errors in mathematics must not be rejected but valued. The misconceptions and errors must be discussed as it can be used to enhance learning (Sawada, 1999).

The majority of the learners were Black and second language learners and it was evident that some of them were battling to comprehend the words used in the problem. This may have explained why they showed little interest in the written work. Based on the results of their investigation Bohlmann and Pretorius (2008, p. 43) state that Grade 7 learners should have sufficient competence in mathematics, language and reading to manage with the demands of the national curriculum. The educator adopted the attitude of reading and re-reading out the problems to learners who were having difficulty.

By the beginning of the third term, there was evidence that the learners were beginning to use appropriate forms of reasoning and visual skills and they slowly started to become involved in the problem solving process. The change may be attributed to the researcher drawing attention to the relevant section in the mathematics policy document on problem solving and the discussion of problem solving strategies and the use of visual techniques as requested by the educators during the mathematics learning area committee meetings. These meetings are held as part of on going staff development to discuss issues relating to mathematics and its teaching.

The researcher observed during the classroom visits that the learners were encouraged by the educators to reflect and use their prior knowledge of situations to show procedures for solving problems. The learners were also encouraged to discuss their solutions by working with learners in their groups. This signalled that the educator valued social interaction with peers as a means of generating solutions. This type of interaction is encouraged in problem centred learning. After allowing appropriate time for the completion of the problem, the educator proceeded to the whole class discussion. The educator progressed logically and wrote down each step as she obtained a response from learners. Simultaneously an illustration was started and built on, as the learners responded. Foster (2007, p. 200) agrees *"that creating a visual to go along with the word problem"* gives learners a *"better understanding of what the problem was asking"*. Foster (2007) emphasizes that when we read we create images in our minds to help us understand better what is being conveyed with words.

The educators became the support structure (facilitators) and encouragement was forthcoming from the educators as learners' responses were acknowledged. Learners were now showing a keener interest in the lesson. This change was noticed after a general discussion of educator's classroom lesson observation reports as part of the IQMS process during the mathematics learning area committee meetings. After allowing sufficient time for the completion of the task, the educator proceeded to the whole class discussion. The discussion component was very interesting. It proved very lively as learners expressed their agreements and disagreements to illustrate and justify that their answer was correct. This kind of interaction allowed learners to take responsibility for their own thinking and learning. The social constructivists state that this kind of interaction allows learners to take responsibility for their thinking, especially in a social environment (Davis, Maher and Noddings, 1990). Good communication was established as learners answered without restraint. They were randomly selected to demonstrate on the chalkboard and offer an explanation as to how they arrived at an answer. Schoenfeld (1987) state that comparing and discussing methods of problem solving are tremendously useful to learners. Explanations were called for and the educator provided the necessary support to ensure that the learners understood the concepts before the discussion was halted.

This study also focused on the visual tools used by educators in their teaching of problem solving. In subsequent lessons the educators made maximum use of visual mediums of instruction. The use of illustrations on the chalkboard and displaying concept related charts created a dynamic working environment for learners. Backhouse et al (1972, p. 157) state that a vivid presentation can make a difference in how learners retain mathematics. The educator demonstrated the use of tables and diagrams in order to solve problems. In some instances, learners were given the problems and asked to fend for themselves. This resulted in learners imitating the teacher in trying to produce solutions using visual means **(Figure 6)**.

The educators in this study used demonstrations to create a better understanding of the problem. The following problem was practically used in the classroom:

Five friends attended a birthday party. Each one of them shook hands once with each other.
How many handshakes were made altogether?

This problem could be physically demonstrated and the educator called forward five learners. They demonstrated the solution to their fellow learners through shaking hands with each other. This approach allowed learners to solve a realistic problem practically. The learners used stick figures to represent learners when given this question to solve **(Figure 6)**.

The educators often used diagrams, charts, tables and the chalkboard but never used the television. The school used in this study signs up annually to receive information from the South African Broadcasting Corporation (SABC) on how to use and incorporate the Learning Channel into the daily teaching programme. The researcher perused through the guide and discovered that there were many programmes in mathematics that lent itself to visual teaching and learning. However, no attempt was made by educators to utilize the television as a teaching resource despite its availability.

The type of visual techniques used by grade 6 and 7 educators in the teaching of problem solving skills

Type of Visual Technique	Never	Seldom	Often	Always
Diagrams			√	
Charts			√	
Tables			√	
Television	√			
Chalkboard				√
Demonstrations			√	

Table 7

Observation of the Grade 6 class

The learners in the grade 6 class displayed difficulty with the problems they were presented with and were reluctant to participate actively in the classroom. An example of one such problem:

There will be 8 people eating dinner at Pam's house. She is using 14 potatoes to make her famous mashed potatoes. She expects to have no leftover potatoes. How many potatoes will each person eat?

This resulted in learners been inattentive and fidgety. Backhouse et al, (1972, pp. 138-139) state that if problems are not within the experience of learners it can lead to frustration. In this type of pseudo-realistic problem, it is necessary to ensure that the learners understand that a 'realistic' solution is dependent on the potatoes being the same size. Learners know that this is often not the case. One can, if left on their own, expect the learners to find solutions dependent on their experiences.

The educator realised that learners were having difficulty and chose problems that were at a suitable level of difficulty and open ended. Learners were encouraged to use illustrations to support their problem solving skills. Backhouse et al, (1972, p. 138) believe that if problems are at an appropriate level of difficulty the activity of problem solving can be highly motivating. The educators posed several realistic mathematical problems and learners were encouraged to solve them using their own strategies including algebraic techniques. Examples of problems given to learners:

1. *You are the timekeeper at a boxing match. The fight is made up of 10 rounds. Each round lasts for 3 minutes and there is a 30 second break between each round. If the match starts at 17:00 and lasted the whole ten rounds, at what time did the fight end?*

2. *To make one square 4 matchsticks are used. To make two squares 7 matchsticks are used. To make three squares 10 matchsticks are used. How many matchsticks are be used to make 10 squares?*

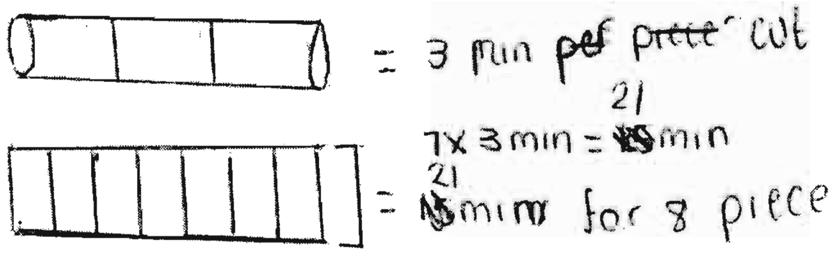
Whitin and Whitin (2001, p. 228) state that encouraging learners to generate their own visual representations *"is an act of sense making"*. This has many benefits because when learners are given the opportunity to demonstrate mathematical ideas in their own way, they display significant insight and creativeness and it challenges them to classify their thoughts and ideas in their own distinctive way. The researcher was interested in attempting to determine if the learners were more inclined to solve the problems using their visualization techniques or not.

The educator requested her learners to work in pairs and discuss the problem and identify key features in a problem. The learners talked about their answers and representations explaining to each other in their groups how they arrived at the solution. Whitin and Whitin (2001, p. 229) state that learner's oral responses reflect their observations of this visual display, their understanding and knowledge of the given problem. This is in keeping with the underlying principle of problem centred math where learners construct their own knowledge. It is very important for learners to know what kind of information to look for in a problem because it is a critical step in problem solving and we must teach them to *"prune the tree"* so that there are not so many possible action sequences to investigate (Wickelgren, 1974, pp. 19-22). By pruning, the goal of solving the problem thus becomes more focused. The observed educators demonstrated to learners an example of how to read and interpret the mathematics problem in order to identify key information. Foster (2007, p. 197) stated that she, *"had to show the learners how to read the mathematics problem with the purpose of gaining full understanding of the situation"*. This was done to assist learners who were second language learners and were experiencing difficulties with the interpretation of the first language (English). Learners were encouraged to create visual images or visual representations to make sense of the words in the problem. Foster (2007, p. 197) refers to this as *"mental modelling"*. Foster went further and asked her learners to *"describe to me what goes on inside your head when you are working on a math problem"*. The learners had to orally explain to the class what was expected in order to arrive at a solution. In some instances learners provided a visual presentation to the class as they worked sequentially through the problem.

The readjustment also resulted in a shift to the learner centred approach (LCA) which is linked to the problem centred learning (PCL) theory. In both the LCA and PCL the emphasis is on the learners and how they take responsibility for their own learning. The educator reduced the amount of coaching allowing learners to work on their own. The learners were encouraged to ask questions and discuss with their peers. Some of the learners made requests for assistance (*"mam I am stuck"* ; *"can I get some help with this step")* when faced with difficulty. The tasks set provided rich opportunities for independent problem solving. The educator encouraged her learners to provide explanations of their thoughts as they progressed by asking reasons for certain visual representations or when not used asking learners to try and use one as she moved around the class whilst learners were engaged in their task. Constructive and continuous feedback in the form of open dialogue between the educator and learners is of paramount importance as learners engage in problem solving. Learners need to be guided to ensure that no misconception is taking place as they attempt to solve the problem. The learners as they became confident began using their own techniques as they worked their way through the problems **(Figure 6)**. The researcher observed that learners were now enjoying themselves with the activities as they were being allowed to be creative.

An observed feature was that learners appropriated educator like strategies in solving their problems. They used stick figures to represent people as was demonstrated in the classroom.

4. A machine can cut a metal rod into 3 pieces in six minutes. How many minutes will it take the machine to cut the bar into 8 parts working at the same speed?

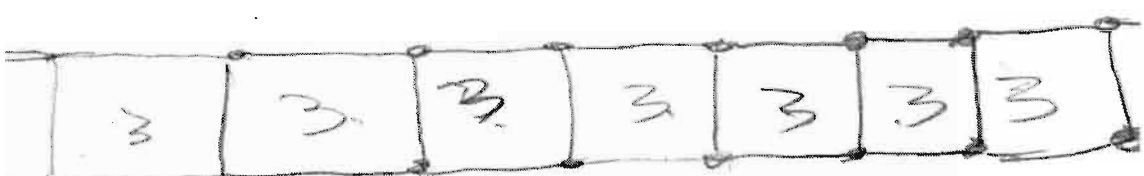


5. Four cars red, blue, green and yellow are entered in a race. The cars have no numbers but each is painted a different colour. If all the cars finish the race and there are no ties, how many different orders of finish are possible?

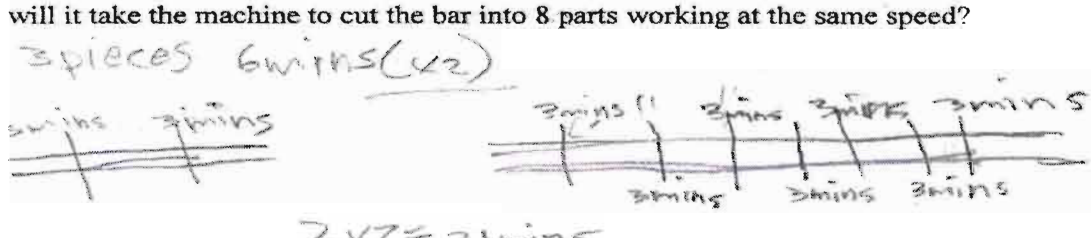
1	R	R	R	R	R	R	B	B	B	B	B	B	Y	Y	Y	Y	Y	Y	G	G	G	G	G	G
2	B	B	Y	Y	G	G	R	R	Y	Y	G	G	R	R	B	B	G	G	R	R	B	B	Y	Y
3	Y	G	B	G	Y	B	Y	G	G	R	Y	R	G	B	G	R	Y	R	Y	B	R	Y	R	B
4	G	Y	G	B	B	Y	G	Y	R	G	R	Y	B	G	R	G	R	Y	B	Y	Y	R	B	R

=24 orders

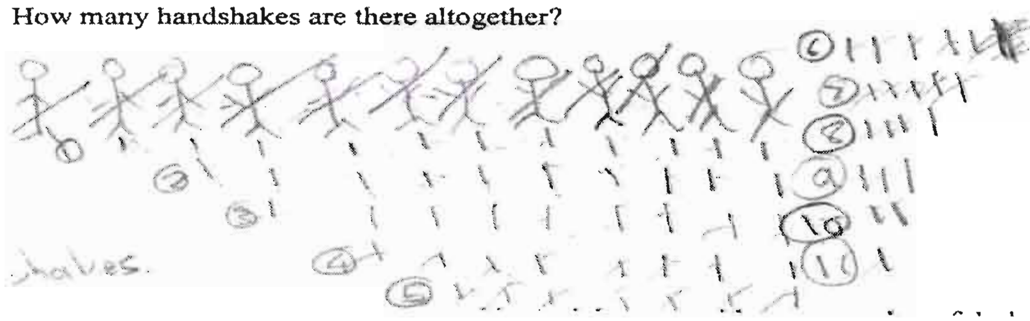
To make 1 square 4 matchsticks were used. To make 2 squares 7 matchsticks were used. How many matchsticks would you need to make 10 squares?



A machine can cut out a metal rod into 3 pieces in 6 minutes. How many minutes will it take the machine to cut the bar into 8 parts working at the same speed?



Each of the 12 people in a room greet every one else once by shaking hands. How many handshakes are there altogether?



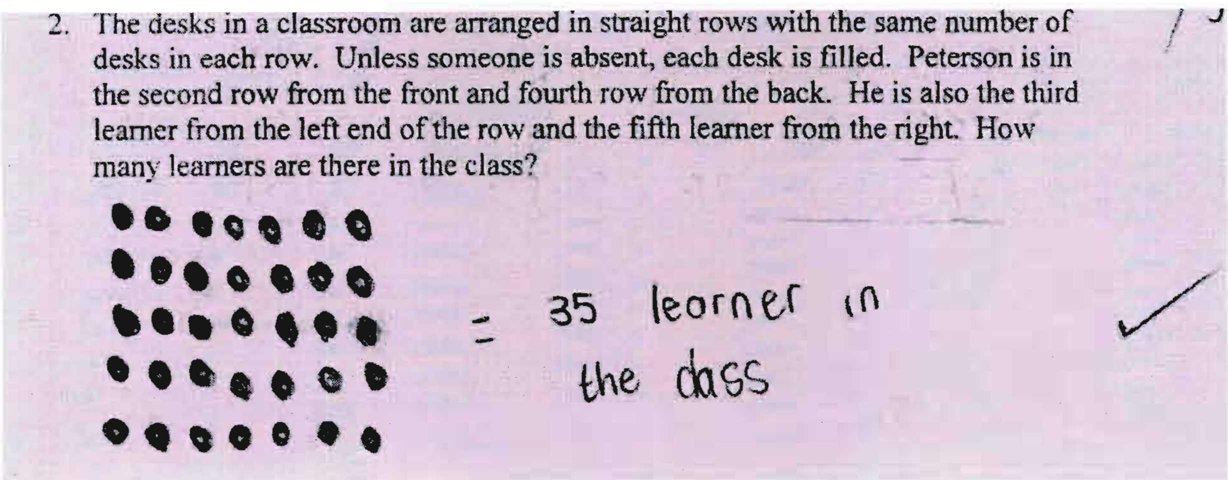

2. The desks in a classroom are arranged in straight rows with the same number of desks in each row. Unless someone is absent, each desk is filled. Peterson is in the second row from the front and fourth row from the back. He is also the third learner from the left end of the row and the fifth learner from the right. How many learners are there in the class?

Figure 6 Learners' Solutions from the Evaluation Worksheet

It was noticeable during this part of the observation that learners' responses were not disregarded. An important observed feature of learner participation in the discussion stage provided an insight to the level of their understanding. The following problem was posed to the learners during an observed lesson:

Mr Khumalo wanted to wire fence his garden. He wanted to use 12 poles on each side to support the fencing and they must be ten metres apart.

The educator asked the following questions:

1. How many poles must Mr Khumalo buy?
2. What is the total distance from first pole to the last pole on one side?

   Using the information from 1 and 2 above, help Mr Khumalo calculate the amount of wire fencing required to complete his job of fencing his garden.

Educator: (walking around the class) *What is the shape of the garden?*

Class : *"square"* (in unison)

Educator: (checks on learners work and calls upon a learner to draw a square on the board)

Learner: goes to the board and draws a triangle **(Figure 7).**

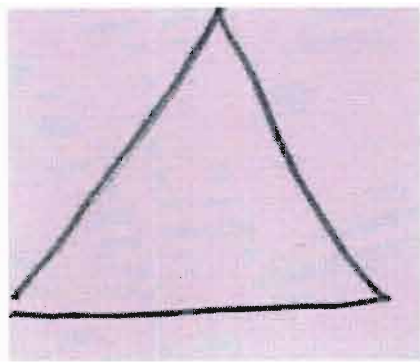


Figure 7

By drawing a triangle, the learner displayed that there was a fundamental conceptual misunderstanding regarding his idea of a square. The educator got all learners to **draw** a square in their books. This was done to examine learner's understanding of a square. Some of the learners drew a rectangle. This example also demonstrates that learners must have an understanding of fundamental concepts before solving problems. Realising that her learners had a problem she reinforced the concept of a square by displaying a chart which had diagrams of squares of different sizes. Learners were called randomly to measure the squares on the chart, which was pinned on the chalkboard. The educator asked the learners to identify similar type of shapes in the classroom. The conclusion learners arrived at was that all sides in a square are equal in length. In this instance, learners needed to know what a square was as it was vital to solving this problem. As mathematics educators, we must not take it for granted that all learners know their basic geometric shapes and these must be reinforced regularly in the primary phase.

Many learners gave 48 poles as the solution to the first question. The following explanation was commonly given:

A square has 4 sides and if 12 poles are used on each side then 4 x 12 = 48 **(Figure 8)**

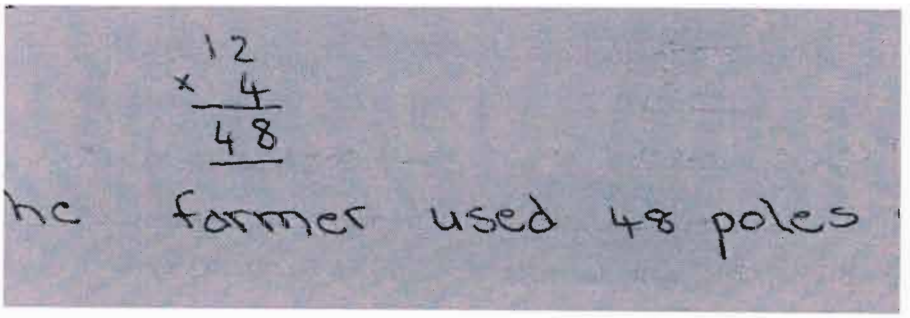


Figure 8

The reason for the above solution was that the learners were focusing on the number of poles and not the shape of the garden. A few learners had different explanations and the educator allowed them to explain to the class. One such explanation is replicated together with a diagram **(Figure 9)** that the learner drew on the chalkboard.

"The four corners of the square will be common" (he drew the four corners with a coloured chalk).

"I already have two poles on one side and I will use 10 more poles in-between which will give me 12 poles". (The learner used 10 strokes to indicate the poles in-between the two common poles).

"I will do the same on the other three sides. The answer is 44".

The learner's who managed to use this strategy were able to shift their focus to visual representation of a square.

By allowing such discussions of solutions in the classroom, the educator gave learners an opportunity to show their level of understanding and rectify certain misconceptions.

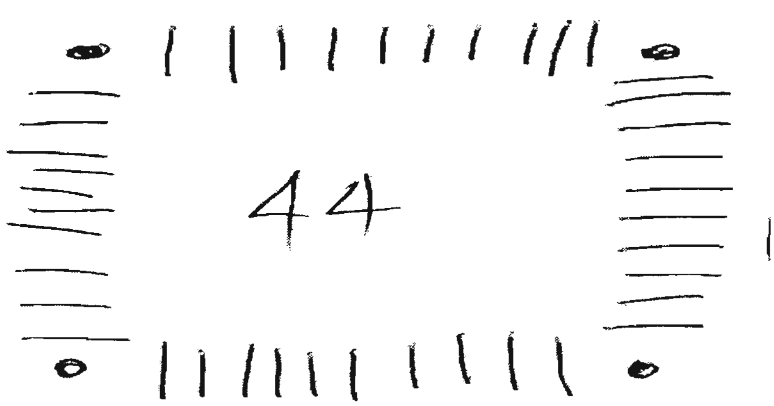


Figure 9

5.4 Analysis of the evaluation worksheet

A worksheet consisting of 6 questions **(Annexure F)**, of which 2 had diagrams were administered to the grade 6 learners and grade 7 learners as individual classroom based activity.

Here the researcher sought to investigate how learners solved mathematical problems. The broad goal was to determine the different steps they engaged in when solving problems. I was not interested in the correct solutions in this study but was interested in the visual techniques and the problem solving strategies that learners engaged in when attempting these questions.

Learner's responses were classified according to whether they used visual techniques **(Table 9 and Table 10).**

Grade 6 learners understood the problems. The learners mainly used a single strategy to solve problems. Some tried to translate the problem using the algorithmic method but the majority of them attempted visual techniques **(Figure 6)** to arrive at the solution. Some of the problem solving strategies used to solve the problems included drawing of a diagrams, guess and check, and finding a pattern. The visual techniques used to represent the problems included drawing a picture or diagram. The problem strategies and visual techniques used in conjunction with each other was an indication that the learners had interpreted the problem correctly and could articulate their ideas through visual means.

Grade 6

Question	Used Visual Techniques (%)	No Visual Techniques (%)	Not Attempted (%)
1	96	2	2
2	98	2	
3	32	58	10
4	71	29	
5	68	32	
6	49	51	

Table 9

Grade 7

Question	Visual Techniques Used (%)	No Visual Techniques (%)	Not Attempted (%)
1	66	22	12
2	88		12
3	78		22
4	84	2	14
5	66	12	22
6	63	27	10

Table 10

In questions 1 and 2 **(Table 9)** the majority of the learners demonstrated their understanding of the problem and the solution arrived at was correct and it was evident that learners experienced difficulty comprehending question 3, 5 and 6 **(Table 9)**. Although they used visual techniques to solve these problems, it was obvious that learners still needed more explicit instruction and practice in order to master the solving techniques. Those learners, who were successful, demonstrated a clear understanding of the visual techniques. They had applied some of the skills taught in the class. By continually applying the relevant methods to a similar class of problems, learners ought to gain considerable skills. The practice of the correct techniques will make perfect. When problems are more complicated, problem-solving skills can be a substantial aid (Wickelgren, 1974, p. 4).

Grade 6 learners achieved some success in using visual representations as a skill to solve problems. Learners that used visual representation in combination with the problem solving strategies demonstrated a clear understanding and arrived at the correct answer. Although 44% of the learners had chosen the incorrect problem solving strategy, it was evident from their diagrammatic representations that they understood the process involved in solving problems. They translated the given problem into diagrams, tables and pictures.

In their attempts to solve the problem, **Grade 7** learners used various visual techniques to solve the problems **(Table 10)**. Although the learners used visual strategies in their attempts, they could not discover the correct method. This was evident in their responses that showed unrelated calculations and in some instances, minimal understanding. Some learners were repeating the same mistakes as observed in the class. The educator plays a pivotal role in the classroom particularly in supporting learners in classroom activities but it seems that educators had not reviewed learners' misconceptions in their teaching. If they had done so then learners may not have made repeated the same mistakes. It may be quite difficult to understand problem-solving methods in the field of mathematics if taught incorrectly (Wickelgren, 1974).

It is necessary to state that there were a number of learners who made successful attempts to solve the problems, whilst other learners did not answer the question, possibly due to not being able to read and comprehend the problem. It may be that language barriers hindered their problem-solving ability. This is supported by research done by Bohlmann and Pretorius (2008).

5.5 Analysis of semi structured interview

An interview is necessary if we cannot observe behavioural patterns on how people interpret the world around them. The aim of the semi-structured interview was to determine what the learners were thinking about during the problem solving exercise administered during the interview. This interview process was important as it was providing the researcher with data from the learners themselves. The idea was to gather the information about what learners thought when working with these problems. Learners' responses to the questions mainly provided data about their understanding of the problem. The use of visual representation indicated their visual response when confronted with the problem.

All participants were asked the same set of prepared questions **(Appendix G)**. The interview questions were based around George Polya's problem solving strategies (1945) and Montague's Solve it (2005) framework. The interview sessions were audio taped whilst the learners were busy with the problem-solving task. They were prompted as they worked through the solution. The reason for prompting was to get them to talk aloud in order to record what they were thinking as they solved the problems.

The questions posed were of the non-routine type.

During the interview process the learners were asked to read the problem aloud as the researcher wanted to determine whether they could read the problem. After a few basic questions, it was evident that some of the participants had difficulty understanding the problem. Fasi and Woo-Hyung Whang cited in Kazima (2008, p. 57) explored how learners understand word problems in mathematics. They found that language difficulties interfered with learner's ability to solve mathematics word problems. Some of the learners did not fully comprehend the problem as they were not reading for understanding.

When asked whether they understood the problem, some responses given were:

"I didn't understand it at first and I had to read it a second time to get a better understanding", "confused" and "give me a little bit more time to think about it". It is always possible that some learners will not understand the problem the first time they read it. The researcher found that the participants had to read the problem more than once and engage in deep thought before they could obtain any understanding. It seems that when learners read the problem more than once it brought about greater clarity and further understanding. By identifying key information, they were able to in some cases convert the information to visual representations. This seems to have assisted them in obtaining a solution.

Orton (1992) mentions that mathematics as a language is grounded in word problems and it cannot be read quickly. Although language can be a barrier to learning (Setati, 2008; Kazima, 2008) the researcher noted in his classroom observations that visualization supported learners to overcome the language difficulty and it created a better understanding. Through teacher scaffolding and by creating diagrammatic representations, learners were translating what they were seeing in their minds and it seems that they understood what was required. Learners were able to identify key features which assisted in producing a solution to the problem.

In summary it must be stated that an observable feature was the learner's inability to read and understand. It was evident that the inability to read and understand created some anxiety in the learners. This evoked comments such as *"I cannot think"*; *"The question is hard"*. With prompting and encouragement, the educator was able to get the learners to understand what was expected of them. It is important for educators to assist learners to understand the mathematics problem so that they could at least attempt a solution.

When prompted to describe exactly what they thought about when they read this question, the learners responded in different ways.

The question that they were asked to solve was:

The middle toy soldier is the 12th one in a row of toy soldiers. How many toy soldiers are there in the row?

The learners were first asked to read the question and they were interviewed.

Researcher: *What are you thinking right now?*

Participant: *I am trying to picture the soldiers in a line.*

Researcher: *I want you to explain to me as you are working?*

Participant: *The middle toy soldier is the 12th one from one side of the row. Okay, so you have 11 before it* (drawing a diagram), *so there should be another 11 after it* (counts up the soldiers in the diagram). *There should be 23 soldiers in the line.*

Researcher: *How did you arrive at the answer of 23?*

Partcipant: *Because they say the middle toy soldier is 12th, so they can only be one middle (points to the number 12 in the diagram). So there will be 11 in front of him and 11 behind him.*

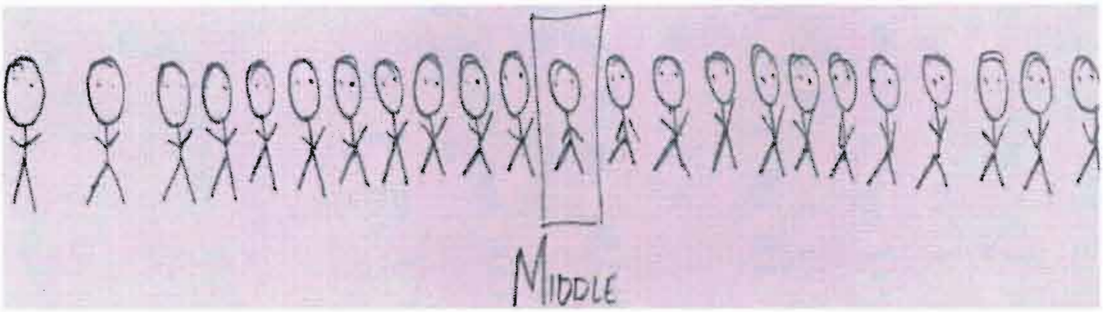


Figure 10

Majority of the learners were able to translate the verbal information into a representation, which demonstrated their understanding of the problem. *"I looked at this question and this diagram came to my head".* Representation played a major role in problem solving as it allowed the learners to show their ideas on paper. Whitin and Whitin (2001, p. 236) state that *"transforming an idea involves rethinking, re-creating, and reconstructing it in a new form".* They made changes to their representations and it was evident that the visual representation allowed the learners to make adjustments as they worked towards a solution.

When asked why they thought of using a diagram or picture, the following were some responses:

"I have something to work with", "it is easier to understand"; "because it was the easiest way I could think of to solve this question". The learners tried to be creative and found alternate diagrams to solving the problem.

Researcher: *Is there a difference between what you did before and what you did now in order to solve this problem?*
Participant: *Ja, I used a different diagram.*

Learners drew a diagram or some picture indicating a starting point and then proceeded towards arriving at a solution. Learners demonstrated a plan and using a problem solving approach proceeded systematically towards a solution.

When prompted to indicate a preference for a method of solution, most of the learners responded that they preferred using visual means of solving the problem.
Researcher: *Now if you had to choose, which method will you choose?*
Participant: *The first one.* (pointing to the diagram that was drawn)
Researcher: *Why?*
Participant: *It was easier.*

The learners where then asked why they used that strategy as they proceeded with their solution. Many learners responded that it made it easier for them to understand and solve the problem.

Researcher: *Why would you want to use a diagram to solve this problem?*

Participant: *The diagram is easier.*

It was possible for them to see their errors as they progressed and at the same time correct them. By seeing their errors, learners were able to revise their thinking.

The following question gives an example of such a happening:

You are facing east. You turn through seven right angles in a clockwise direction. In which direction will you now be facing?

Researcher: *Now what are you thinking about?*

Participant: *Directions and angles.*

Researcher: *What will you do first?*

Participant: *Try to draw one angle.*

Researcher: *What are you drawing?* **(Figure 11)**

Participant: *Right angles.*

Researcher: *How many right angles will you need?*

Participant: (reads the question again) *7.*

Researcher: *Why 7?*

Participant: (looks at the diagram). *East, South, West and North. There are four points. I will need 4 right angles not 7.*

Researcher: *Okay, you turned through four right angles. In which direction are you now facing?*

Participant: *East.*

Researcher: *How many more turns do you require?*

Participant: *3* (continues to draw). *The answer is north.*

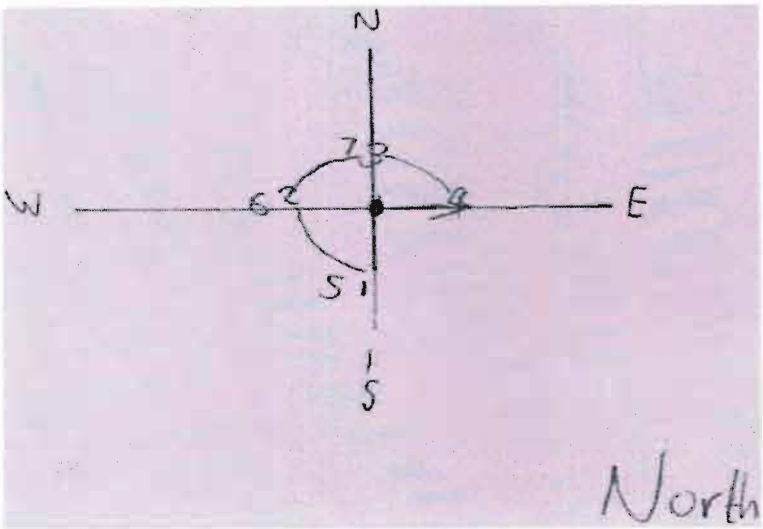


Figure 11

In conclusion the learners were asked whether they thought drawing a diagram made them understand the problem better and easier to solve.

The participants agreed that a diagram aided their attempts to problem solve. Whitin and Whitin (2001, p. 236) state that *"learner generated representations enrich mathematical understanding"* as *"learners are given a choice in how they represent their understanding"*. Learners responded that using a diagram, as a problem solving strategy would be an ideal way to solve problems, as they are able to see their thought process in the solution.

Researcher: *Now why did you want to use a diagram?*
Participant: *Because it made the question easier to answer.*
Researcher: *Now do you think you will be able to solve the problem much better by using a diagram?*
Participant: *Yes.*

Majority of the learners responded in the affirmative whilst a few learners expressed a difficulty translating the problem into a visual representation. Perhaps greater probing would have revealed more details. The learners may have felt compelled to answer 'yes' due to the teacher-learner relationship.

5.6 Conclusion

Throughout this chapter, the research has provided data on the impact of visual literacy and visualization and problem solving strategies on the teaching and learning of problem solving. As the researcher I observed learner's attempting to represent their problem solving efforts and mathematical thinking in the classroom. I also witnessed how learner's use of visual representation can be a forerunner to problem solving. The learners responded positively in using visualization and problem solving strategies when faced with a problem. They were able to use analytical reasoning and transfer information into mathematical operations. Using Polya (1945) and Montague's problem solving framework (2005) learners were able to use an appropriate and reasonable approach to find a solution.

Experiencing difficulty with the language of learning and teaching (LOLT), second language learners were able to communicate their understanding of mathematical concepts and the problem by using visual representations.

The wealth of insights gained from the data (questionnaire, observation, interview) explicitly validates that visual literacy and visualization does assist learners in becoming better problem solvers.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Introduction

The objective of this study was to investigate the effects of using visualization in the teaching and learning of mathematics problem solving in grade 6 and grade 7 classrooms. This research attempted to explore how learners utilized visual literacy and visualization techniques when solving problems.

6.2 Conclusion

The first question focused on the visual skills and problem solving strategies used by the educators in the classroom. The majority of the educators indicated that visual strategies were necessary when teaching problem solving but few actually used these strategies in class. They felt that these strategies should be used only when there was a need. After a limited intervention by the researcher, at the mathematics departmental meeting, the observed educators began to employ visual strategies in all their lessons relating to problem solving. The learners were immediately receptive and there was a distinct change in the way they responded.

An important finding here was that educators only taught problem solving strategies when they thought that it was necessary. They did not see the need to introduce problem solving as a regular activity within the classroom.

The second question focused on how learners engaged with visualization when solving problems. The level of visual understanding shown by learners indicates that visualization plays a vital role in problem solving. It was observed that learners have the ability to become creative in the classroom and therefore they must be encouraged to approach their solutions in different ways, at the same time examining the effectiveness of their strategies and decide whether a different strategy could prove more useful or not (Nambiar, 2000, p. 134). Learners were able to construct their own strategies when attempting to solve a problem **(Figure 7 in chapter 5)**. An observation made during the classroom visits revealed that if learners can relate the problem to their real life experiences and contexts then they are more likely to solve it. The starting point should be the familiar concrete world of experience of the child as Nieuwoudt et al (2000, p. 333) argues that *"realistically contextualised solving forms an integral part of all learning and* *teaching events"*.

The creative attempts of the learners were apparent when the researcher examined some of the solutions and analysed the diagrams they used.

Competence in problem solving is traditionally viewed as a desired result of learning which is not mastered by all. This study, through the collected data, has shown that using visual literacy and visualization skills can enhance learner's ability to problem solve.

6.3 Recommendations

With huge emphasis now been placed on problem solving, a problem solving curriculum needs to be considered. What is required is an integrated mathematics curriculum that will get learners to actively investigate, analyse and reflect on realistic mathematics problems. An effective problem solving mathematics curriculum will place emphasis on skills and knowledge of problem solving, communication and reasoning.

Mathematics is a subject with a large amount of what is perceived, as serious content. An attempt to integrate subject content with visual material will help cement learners understanding of mathematical concepts. The findings of this study suggest that mathematics educators should be concerned with the status quo of visualization in mathematics. Educators who participated in this study agree that visualization plays an essential role as it complements the teaching and learning of mathematics. This finding is consistent with literature (Whitin and Whitin, 2001, Arcavi, 2003; Bamford, 2003; Pantziara, 2004; Lavy, 2007; Murata, 2008) and is supported by research by Cunningham and Hubbold cited in (Diezmann, 1995, p. 10) amongst others who state that, *"the educational experience cannot be considered as complete unless visual literacy plays its fullest part"*. Using visual literacy and visualization in classroom tasks will lead to better understanding, as learners will organize their solutions as they see it in their minds and this will assist the educator to determine any misconceptions that may materialise as the visual feedback is immediate. Although there is a choice as to whether or not visual literacy is included in the curriculum, its growing importance suggests that to omit visual literacy could result in the marginalization of learners in a visually oriented society (Diezmann, 1995).

Visualization is seen as a natural process and educators can nurture it. This study has shown that educators believe that using visualization as a teaching aid in problem solving is important as learners can benefit immensely. This is supported by Kelly (1999, p. 51) as she found in her research that "visualization is a powerful tool for enhancing problem solving ability". Japan is one of the consistent performing countries internationally (Reddy, 2003) and it was found that their mathematics instruction made two to four times more use of visual representations than the instruction of other countries in their teaching (Murata, 2008). With support for visual literacy and visualization they should be extolled as valuable tools for the learning and doing of mathematics and should be engaged as springboards for the sharing and talking mathematics. Therefore curriculum planners should, at its earliest opportunity incorporate visualization into the South African mathematics curriculum as a definite strategy. In this way, educators will be able use this technique to teach learners who will obtain knowledgeable assistance allowing them their freedom to make use of strategies that will steer them in making independent decisions and point them in the correct direction. Murata (2008, p. 379) state that if visual representations are used consistently with instruction this will become part of a learner's mathematical thinking and lay the foundation for future understanding and this is supported by Whitin and Whitin (2001, p. 236) who state *"the power of learners generated representations challenges us to capitalize on this potential in the future"*. It will create a positive sentiment towards mathematics and eliminate anxiety associated with problem solving (Kelly, 1999, p. 51).

Reflection is something that educators should engage in which was not visible in my observation. Educators did not allow for reviewing of learners' errors and reinforcement of concepts taught. According to Artzt and Armour-Thomas (2002), reflection involves thinking about a lesson after it has been taught. In teaching a challenging subject like mathematics, educators must allow themselves an opportunity to revisit their lessons. Using reflection in problem solving lessons will allow educators to critically evaluate what they are doing prior to, throughout and after the lesson and approach problem solving from different angles. This can be compared to Polya's looking back stage (Polya, 1945) where one examines the steps involved in arriving at the solution. It will allow the educator an opportunity to think about the lesson after it has been taught. Whitin and Whitin (2001, p. 236) state *"reflective teachers are constantly reviewing an unending series of cherished moments, missed opportunities, and promising plans for the future"*. Through reflection, educators will be able to recognize problems which if rectified will support the learner's development. This will enable them to discover the reasons for problems and consider alternate methods that will adjust their teaching in the classroom. Artzt and Armour-Thomas (2002, p.7) state that the habitual use of the reflection process will eventually lead to transformation in teaching methods. Educators should have a section of their preparation schedule set aside for reflective aspects of their teachings.

The researcher is of the opinion that reading and mathematics are strange bedfellows; one cannot do without the other. This is supported by literature (Foster, 2007; Bohlmann & Pretorius, 2008). Today much emphasis is placed on the language aspect in mathematics and reading is found to be crucial to comprehending and understanding mathematics. Asking learners to read and explain the context will indicate whether understanding has occurred. Understanding opens doors to learning mathematics and if one cannot read, it can be problematic. Reading lays the foundation to the learning of mathematics therefore mathematics educators need to find suitable solutions to improving the literacy problem so that the illiteracy gap is not widen in South Africa.

The Department of Education has implemented its Foundations for Learning Programme (DoE, 2008b) and Reading Programme (DoE, 2008a). What is needed urgently is support for educators by all stakeholders to implement these policies. Monitoring at school level should be a matter of priority and ongoing to ensure that it is successfully implemented and the researcher supports the initiative of the Department of Education to assess the level of Mathematics and English in the form of systemic evaluation on an annual level.

Pimm (1987, p. 5) states that *"language without meaning is meaningless"*. Mathematical language can be problematic because the learners only come across these unfamiliar words in mathematics lessons. Nesher and Teubal cited in (Dickson et al, 1984, p. 360) carried out a research involving children's strategies in solving word problems and they found that mathematical concepts played an important role in understanding. Words that are used in mathematics and other learning areas may offer diverse meanings in different context (cultural and linguistic differences). *"part of what makes learning Mathematics in non-mother tongue instruction difficult is that there are poorly developed Mathematical lexicons for most of South Africa's indigenous languages"* (DoE, 2003, p. 32). To cope with these infrequently used words it is suggested that learners create a mathematical dictionary and that educators repeatedly teach mathematical concepts using mathematical language supported by illustrative visualization. This will help internalize their understanding and offer them an opportunity to display their communicative competence when they come across it again.

The South African mathematics curriculum needs to take cognizance of the Japanese curriculum where the focus is on a few core topics (which minimize the need to reteach). Topic presentation is carefully planned with common visual representation to connect core ideas and this minimizes the need for reteaching (Murata, 2008, p. 379). A lot of emphasis needs to be placed problem centred learning. Learners must be allowed to engage in problem solving and discover solutions and at the same time learn life skills to problems that may affect them later in their lives.

6.4 Limitations

The Principal did not want lesson contact time lost. The researcher had to make alternate arrangements to gain entry into the observed class. It was difficult staying within the confines of the learner's daily schedules to perform the interviews. Having learners removed from the classes during the contact time proved difficult. The interviews were conducted during the breaks as learners have pre-arranged transport times in the afternoons. The researcher would have liked to visit better-resourced schools and observe more educators and learners in the classroom but this was not possible due to financial constraints.

6.5 Further Research

The mathematization of the present world has brought about anxiety in learners in schools. Recommendations for future research include the investigation of the reasons for the cause of anxiety in mathematics focusing on why problem solving causes anxiety. Further research is required to measure learner's visual literacy skills of creating and interpreting visual language in mathematics.

6.6 Conclusion

The different stages of schooling should provide learners with skills (reading, visual literacy, visualization, problem-solving strategies) that will enable them to access doors of opportunities and succeed in education.

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ANNEXURE A

FORM NO: _____

I AM A VOLUNTARY PARTICIPANT IN THIS STUDY YES NO

QUESTIONNAIRE

WHERE OPTIONS ARE GIVEN TICK (√) ONE ONLY

GENERAL

1. Number of years of teaching experience. __________
2. Number of years of teaching mathematics. __________
3. Grade you are currently teaching. ____________

NATIONAL CURRICULUM STATEMENT

1. Have you read the Mathematics Teachers Guide **(2003)**, **National Curriculum Statement (NCS) Document,** on Problem Solving?

   YES NO

2. What is your understanding of Problem Solving?

   ____

   ____

   ____

   ____

If your response is *YES* to any of the following below, please offer an explanation

3. Do you understand the following concepts:

3.1 routine problems YES NO

____

____

____

3.2 non-routine problems YES NO

____

____

____

3.3 multiple approach problems YES NO

3.4 open ended questions YES NO

4. Which of the above do you use in your lessons?

5. Do learners understand these types of problems? YES NO

6. Teaching Problem solving is neglected in the mathematical curriculum.

STRONGLY DISAGREE DISAGREE

AGREE STRONGLY AGREE

FOCUS ON YOUR TEACHING/LESSON

1. How often do you teach problem solving strategies?

EVERYDAY ONCE A WEEK

TWICE A WEEK OTHER

If you have answered OTHER, PLEASE SPECIFY

2. How much of your lesson time (1-hour period) in a day is spent on **TEACHING** problem solving? **(time shown in minutes).**

1 – 15 16 – 30 31 – 45 46 – 60

3. Is this sufficient time? YES NO

Justify your answer

3.

4. How do you make problem-solving **lessons enjoyable**?

5. Do you teach **problem-solving strategies**? YES NO

If YES, name a few.

6. Where do you source your questions?

TEXTBOOKS AMESA COLLEAGUES OTHER

If other, please specify.

7. Can your learners relate to these problems? YES NO

8. What is your **understanding of visual literacy**?

9. What is your **understanding of visualization**?

10. Do you use **visualization techniques** in your lessons? YES NO

If YES give a few examples.
If NO please specify why.

11. Do you believe **learners understand the lesson better** when provided with **visual stimuli?**

YES NO

If YES/NO please offer an explanation.

12. What are some of the **visual skills** that one can use when **teaching problem solving**?

13. Considering your response above, do you think most problems can be taught using visual skills.

YES NO

15. How would you rate your learner's performance **without using visual techniques**?

POOR GOOD EXCELLENT UNCHANGED

16. How would you rate your learner's performance **using visual techniques**?

POOR GOOD EXCELLENT UNCHANGED

17. Visualization skills assist learners in mathematics lessons.

YES NO

18. Do your learners experience any language difficulty in problem solving?

YES NO

19. Do you think that visualization can overcome language barriers?

YES NO

If yes, please specify how.

____

____

____

____

LEARNERS

1. Do your learners **enjoy problem solving**?

YES NO

If NO, please specify WHY.

____

____

2. Do your learners provide visual solutions in their work books?

YES NO

3. If YES, give examples.

____

____

____

____

4. How do you handle learners who show a lack of interest in problem solving?

____

____

____

____

ANNEXURE B

Tel./Fax. 031-4624491
032-5338682
Cell: 0725384309

ALIPORE PRIMARY SCHOOL
P.O. Box 31313
MEREBANK
4065

10 MARCH 2008

TO: ***THE PRINCIPAL***
MATHEMATICS TEACHER

Sir/Madam

I, RAJESH BUDRAM, am a registered Masters Student in the Faculty of Mathematics, Student Number 207525539, at the University of Kwa-Zulu Natal, Edgewood Campus in the current year 2008.

I am in the process of undertaking research in the field of ***MATHEMATICS***, the topic been, **The Effects of using Visual Literacy and Visualization skills in the teaching and learning of problem solving in a Grade 6 and Grade 7 class.**

This research is concerned specifically with determining the status of problem solving in the mathematics curriculum and whether visual literacy and visualization aids the teaching and learning of problem solving or not.

I am particularly desirous of obtaining your response because of your experience in the field of mathematics and your indepth knowledge will contribute significantly to this study as it will provide rich data from a South African perspective. It is anticipated that the findings and recommendations of this study will be used effectively to improve learner performance in this crucial area of mathematics education.

I hereby request that you spend a few moments of your precious time in completing the enclosed questionnaire.

As the instrument does not require any form of identification, your confidentiality, anonymity and non-traceability is guaranteed. The number indicated is to track the number of questionnaires distributed and returned.

It will be appreciated if you will complete the enclosed questionnaire prior to **23 MAY 2008** and return it in the enclosed envelope to the administrative Head of the Institution.

My supervisor for this study is **Dr. V. Mudaly** in the **Faculty of Science, Mathematics & Technology** and he can be contacted **on 0829770577.**

I want to thank you for your co-operation and once again I cannot stress the importance of your contribution to this study.

Yours faithfully

____

RAJESH BUDRAM

PONENT	ALWAYS	OFTEN	OCCASSIO NALLY	SELDOM	NEVER
irners exhibit a desire to pate in the lesson					
rners asked questions					
rners asked for ıce					
rners asked by teacher : solutions to the class					
:her asked for other ıs					
ners used the ard to show solutions					
ners completed given eously					
her use of visual ıes to enhance lesson					
ıers use of multiple chniques					
ere a change from a al strategy to a visual					
earners show more ı the visual compared ebraic one?					
em solving used in the lesson					
er encouraged use visual ıs in p.s					

LESSON NO. ______

YPE OF VISUAL ECHNIQUE	NEVER	SELDOM	OFTEN	ALWAYS
IAGRAMS				
HARTS				
ABLES				
ELEVISION				
HALKBOARD				
EMONSTARIONS				
OLE PLAY				
SE OF MANIPULATIVES				
YPE OF PROBLEM OLVING STRATEGY				
UESS AND CHECK				
RAW A DIAGRAM				
RAW A TABLE				
ORK BACKWARDS				
IMINATION				
YPOTHESIS AND PROOF				
PE OF PROBLEM				
OUTINE				
ON-ROUTINE				
ULTIPLE APPROACHES				
PEN ENDED PROBLEMS				
% NEVER				
0 % SELDOM				
– 60 % OFTEN				
% and over ALWAYS				

ANNEXURE D

TEL./FAX: 031-4624491

ALIPORE PRIMARY SCHOOL
P.O. BOX 31313
MEREBANK
4065
19 March 2008

Dear Parent/Guardian

I am a registered Masters student in the Faculty of Mathematics at the University of Kwa-Zulu Natal, Edgewood Campus, Student Number 207525539 in the current academic year. The research programme is a two year degree which involves course work and a dissertation. The dissertation entails undertaking research in the field of Mathematics.

My research topic is: The Effects of using Visual Literacy and Visualization in Problem Solving on Grade 6 and Grade 7 learners in the Senior Primary Phase.

I request your permission to interview your child. The interview will be conducted during the school year 2008, during the second term. It will be done individually for a duration of 30 minutes. Participation is voluntary and your child may withdraw at any time without any penalty.

The interview questions will be based on work that is done in the classroom during the Mathematics lesson. The purpose is to gauge the level of understanding and knowledge in the field of problem solving. The identity of your child will be protected at all times and information passed on will remain confidential. All names will remain anonymous and will be substituted with a fictitious one.

It is anticipated that the recommendations made from this research study will be used to effectively improve mathematics teaching-learning.

My supervisor for this is **Dr. V. Mudaly** in the **Faculty of Science, Mathematics & Technology** and he can be contacted **on 0829770577** in the event of any enquiries.

Yours faithfully

____

R BUDRAM

0725384309

CONSENT FORM

I, ______________________________, parent/guardian of ________________ in Grade ____ fully understand the above and herby give permission to participate in this study.

____

SIGNATURE (Parent)

I, ____________________________ , in Grade _____ hereby agree to participate in the above study voluntarily and I understand the contents of this letter.

____________________________ **SIGNATURE (Learner)**

ANNEXURE E

TEL/FAX. 031-4624491
CELL: 0725384309

ALIPORE PRIMARY SCHOOL
P.O.Box 31313
MEREBANK
4065

19 March 2008

THE PRINCIPAL
ALIPORE PRIMARY SCHOOL

Sir

I am registered as a Masters Student in the Faculty of Mathematics, Student Number 207525539 at the University of Kwa-Zulu Natal, Edgewood Campus in the current academic year (2008). The programme is a two year degree which involves course work and a dissertation.

The dissertation would entail undertaking research in the area of MATHEMATICS. My research topic is: **The Effects of using Visual Literacy and Visualization in Problem Solving on Grade 6 and Grade 7 learners in the Senior Primary Phase.**

The importance of Problem Solving in Mathematics is emphasized in the Mathematics Curriculum Statement.

I hereby request your kind permission and support to enable me to undertake the study at ALIPORE PRIMARY SCHOOL. The research study will be conducted during the school year 2008 during the second term. It will be a quantitative and qualitative study and will involve observation of teacher's lessons and semi-structured interviews with learners (10 learners per grade).

These observations will be undertaken during my non teaching time or with your prior permission and that of the teacher concerned. I will at all times ensure that the teaching programme is not disrupted. The interviews will be conducted out of lesson contact time.

It is anticipated that the recommendations of the findings of the research project will be used effectively to improve the teaching-learning of mathematics.

My supervisor for this study is **Dr. V. Mudaly** in the **Faculty of Science, Mathematics and Technology**. He can be contacted on **0829770577** in the event of any enquiries.

Yours faithfully

____

R. BUDRAM
0725384309

CONSENT FORM

I, ________________________________, Principal of the school, hereby consent to Mr. R. Budram undertaking his research study at Alipore Primary school during the academic year 2008.

____

PRINCIPAL

SCHOOL STAMP

ANNEXURE F

MATHEMATICS PROBLEM SOLVING

NAME: ________________________________ GRADE: 6 ___

You are allowed to use any method to solve the following problems. ALL WORKING MUST BE DONE ON THE QUESTION PAPER.

1. You are facing East. You turn through seven right angles in a clockwise direction. In which direction will you now be facing?

2. One table can seat 4 people around it. When 2 tables are put together there are 6 people who could be seated. How many tables need to be put together to accommodate 24 people?

3. At an army camp, soldiers were given numbers and had to stand in 2 rows as in the diagram. At another army camp soldiers were also given numbers and had to stand opposite each other in 2 rows. If the number 5 stood opposite 20 and 9 stood opposite 24, how many soldiers were there in the 2 rows?

1	2	3	4	5
↕	↕	↕	↕	↕
6	7	8	9	10

4. Jill got into an elevator. She went down 5 floors, up 4 floors and down 7 floors. She was then on the second floor. On which floor did she get into the elevator?

5. There are 8 rows of orange trees on your farm. Each row had 10 trees. Due to the huge demand for oranges, you decided to increase the number of rows by 2 and in addition 5 more trees were planted in each row. How many trees were there now on the farm?

6. The middle toy soldier is the 12th from one end in a row of toy soldiers. How many toy soldiers are there in the row?

MATHEMATICS PROBLEM SOLVING

NAME: ______________________________ GRADE: 7____

You are allowed to use any method to solve the following problems. ALL WORKING MUST BE DONE ON THE QUESTION PAPER

1. The time of your Youth Club Meeting has changed. As the secretary you phone 4 members. These 4 members each call 3 other members, and those 3 each call 2 more members. Each of those 2 in turn call 1 member. How many members have been contacted to attend the meeting?

2. The desks in a classroom are arranged in straight rows with the same number of desks in each row. Unless someone is absent, each desk is filled. Peterson is in the second row from the front and fourth row from the back. He is also the third learner from the left end of the row and the fifth learner from the right. How many learners are there in the class?

3. An incomplete net shows 5 faces of a cube. One face is missing to complete the making of a gift box. In how many places could we attach the missing face to complete the gift box?

4. A machine can cut a metal rod into 3 pieces in six minutes. How many minutes will it take the machine to cut the bar into 8 parts working at the same speed?

5. Four cars red, blue, green and yellow are entered in a race. The cars have no numbers but each is painted a different colour. If all the cars finish the race and there are no ties, how many different orders of finish are possible?

6. A golf ball is dropped from a height of 32 metres onto a concrete floor. Each time it bounces back half the height from it fell. If the ball was caught when it reached a maximum height of one metre, how many times did the ball strike the ground?

- golf ball

FLOOR

ANNEXURE G

QUESTIONS

1. Did you understand the problem?
2. Have you seen this type of problems before? If yes, where and when? Where you able to solve it?
3. Can you describe exactly what you thought about when you read this question? Can you translate this thought into a diagram/picture on paper? Draw it.
4. Is the picture/diagram related to what you have read?
5. Why did you think of using a diagram/picture? (If a diagram is used ask why/ no diagram used ask Do you think that it would be better to use a diagram?
6. Is there any other way/method to solve this problem.
7. (Learner will be given a question with a diagram) Look at this diagram, can you make sense of the given question by studying the given diagram.
8. Do you find working with this diagram better to solve this problem?
9. Explain why/why not.
10. Is there a difference between what you did before and what you did just now?
11. If you had choose which method will you use?
12. (Learners will be given another problem) Try solving this problem.
13. Why did you use this strategy?
14. Do you think drawing a diagram made you understand the problem better and easier to understand and solve?

PROBLEM SOLVING STRATEGIES **APPENDIX H**

1. Guess and check

George Polya's famous film 'Let us teach guessing' in the United States (Jaworski, 1994, 5) promoted the use of guess and check. Learners are encouraged to get involved with the problem and subsequently improve their initial thinking. It is not possible to arrive at the answer through routine means. By constantly guessing and checking, the solution can be found.

An example of such a problem: A carpenter makes some coffee tables with 3 legs and some with 4 legs. If he uses 96 legs to make 30 coffee tables, how many 3 legged coffee tables does he make?

2. Drawing inferences

Drawing inferences is an important part of solving any problem (Wickelgren, 1974, p. 24). It is probably the first problem solving method one employs to solve a problem. This is when all the relevant information is retrieved from memory associated with the problem and then translated into diagrammatic representation.

An example of such a problem: One of the three ladies (Devi, Pam and Saras) is the grandmother of one of the other and the mother of the remaining one. Saras is older than Pam's eldest brother. One of the other two is Devi's daughter. Saras is the daughter of one of the other two. Who is the grandmother?

Devi is the grandmother

3. Draw/make a table

Drawing a table is a useful strategy especially if data needs to be tabulated to get a better understanding. The table must be labelled to get a better understanding.

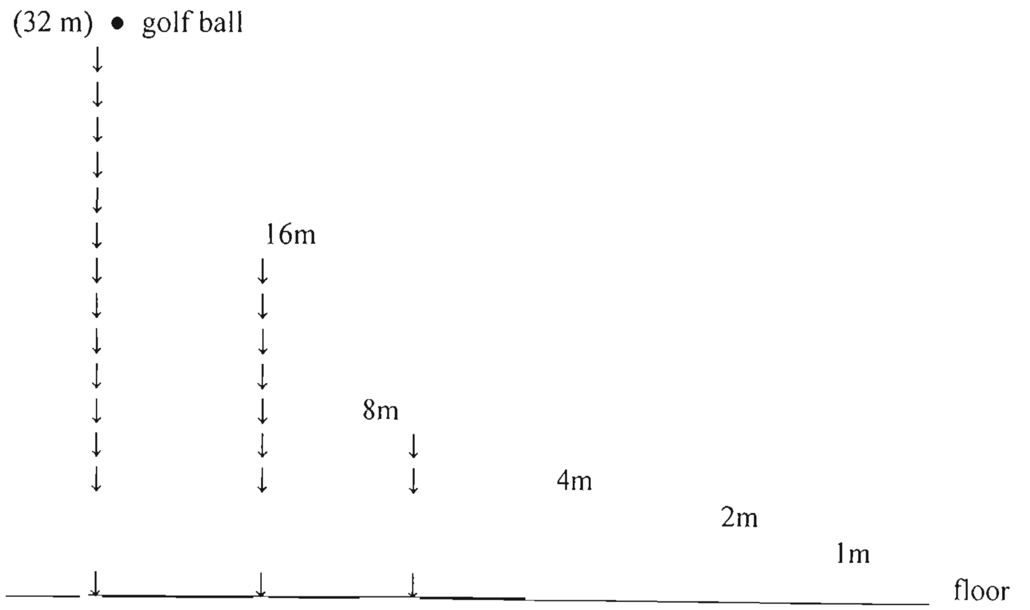
An example of such a problem: During a basketball game Shivesh placed the ball in the basket 3 times out every 5 attempts he took. How many times will he place the ball in the basket if he had 30 shots during the match?

Successful baskets	3	6	9	15	20	25
No of attempts	5	10	15	20	25	30

4. Draw a diagram/picture

Drawing a diagram/picture is a freehand strategy that is very helpful to learners to solve problems. Diagrams/pictures are used to help children conceptualize or create a better understanding of the problem leading to a solution. Draw a picture strategy often sets the stage for a more efficient solution to the problem by helping learners see that other strategies can also be used.

> **An example of such a problem:** A golf ball was dropped from a height of 32 metres onto a concrete floor. Each time it bounces back half the height from where it fell. If the ball was caught when it reached a maximum height of one metre, how many times did the ball strike the ground?



5. Logical reasoning

Logical reasoning is a more general strategy than such strategies as make a table or guess and check. The data is first organised in the problem.

An example of such a problem: Ann, Ben, Carla and Dina like different types of books: Humour, mystery, sports and adventure. One of Ann's classmates in the group likes mystery books the best. Carla and Dina do not like adventure books. Ben's favourite type of book is sports. Dina did like humour books but has changed her favourite. What is Dina's favourite book?

Book Type	ANN	BEN	CARLA	DINA
HUMOUR	No	No	Yes	No
MYSTERY	No	No		?
SPORTS	No	Yes		
ADVENTURE	Yes	No	No	No

Using the given information and logical reasoning, one can deduce that Dina is the only possible person to have mystery books as her favourite.

6. Role playing

Leaners take the role of people mentioned in the problem. They then play out the scenario as described in the problem.

An example of such a problem: At a birthday party, all seven of the friends who came had to shake hand with each other. How many handshakes were there?

Everybody gets an opportunity to shake hands, the answer is 21, and not 49 as learners will want to believe is the answer.

7. Use of manipulatives

Learners are able to act out a problem or use manipulatives to solve problems. This involves the use of concrete materials in teaching, example, and strips of paper, collection of materials, containers and people. The importance of using structured materials in mathematical teaching involves the "*physical manipulation of objects or the actual physical enacting in some way*" (Nickson, 2000, p. 19).

8. Find a pattern

Learners who look for patterns in problem solving situations will be able to predict what will come next.

An example of such a problem:	9 x 9 + 7	=	88
	98 x 9 + 6	=	888
	987 x 9 + 5	=	8888
	If □ x 9 + 2	=	8888888, then
	□	=	?

□ = 987654

9. Working backwards

We start at the end and gradually work our way backwards (inverse operations). Inverse operations are operations that move from the output statement back to the input statement.

> **An example of such a problem**: Ashlek thinks of a secret number. If he multiplies it by 5, then subtracts 5 and thereafter divides the result by 5, the answer is 14. What is his secret number?

15 x 3 → - 5 → ÷ 5 = SECRET NUMBER

10. Counting strategies and number sentences

Using counting strategies and number sentences is a way for learners to keep records of data collected leading to a solution.

Example: Study the following sum carefully

$$\begin{array}{l} 1 + 2 + 3 + 4 + 5 \\ \underline{5 + 4 + 3 + 2 + 1} \\ \underline{6 + 6 + 6 + 6 + 6} = 30 \end{array}$$

Now, find the sum of the first 99 natural numbers.