LINKS BETWEEN CONTENT KNOWLEDGE AND PRACTICE IN A MATHEMATICS TEACHER EDUCATION COURSE

By

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ABSTRACT

The link between content knowledge and how this influences classroom practice has been prominent in educational research in recent years. Shulman was the forerunner of research on this topic and his research dates back to 1986. Shulman's views on content knowledge were contrary to the views of his time. In South Africa, however, the Presidential Education Initiative Report which was published in 1999 initiated research on content knowledge and brought this topic into the forefront of educational research.

This study examined the link between content knowledge and practice from the perceptions of two university lecturers. The study was contextualized at a tertiary institution in South Africa where the two university lecturers were lecturing to a second year undergraduate teacher trainee class. The topic under discussion was calculus-rates of change.

The research was located in the interpretivist paradigm since it focuses on the individual and tries to understand the phenomenon that is being investigated from the individual's perspective. The research was also conceptualised in terms of Vygotsky's educational theory and the process of scaffolding.

A qualitative case study research methodology was employed. The data was gathered through semi-structured interviews with the two university lecturers and through the observation of video-recorded lessons that the lecturers conducted.

The study revealed that the two university lecturers saw a link between a teachers' content knowledge and his classroom practice. This study is by no means exhaustive, and is a case study of two university lecturers and their perception of the link between content knowledge

and practice. This topic can be explored further, and suggestions for further research have been made.

TABLE OF CONTENTS

CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

As a teacher of mathematics I have always been interested in the importance and role of content knowledge in the teaching of mathematics. This interest took on a new dimension at the beginning of 2006 when the Department of Education introduced a new curriculum for schooling in South Africa. The new curriculum is based on a National Curriculum Statement (NCS) which "aims to develop a high level of knowledge and skills in learners" (Department of Education, 2003, p.3). It has high expectations of South African mathematics learners. Implicit in the development of knowledge in learners would be a high level of knowledge in educators. There is however no explicit mention of this and it was therefore something that I wanted to pursue.

The NCS document also spells out the type of teacher that is envisaged. The teacher is expected to be qualified, competent and a specialist in his field. Here again one sees a subtle reference to the teachers' knowledge. In order for the teacher to be competent and skilful in his learning area he needs certain knowledge which I believe included content knowledge.

The new curriculum included topics that I (and many other teachers) had not studied in our undergraduate and post graduate degrees. For the first time after many years I had to now teach content that was new to me. I therefore found that I had to now question the importance and significance of having an in-depth content knowledge when teaching. This led to my having informal discussions with my colleagues, and many of them remarked on the significance of content knowledge for teaching. This was something that not only my colleagues and I saw the significance of, as in the very year of the implementation of the new curriculum the Department of Education conducted several workshops to discuss the new topics with teachers. The Department of Education, had therefore realized the need for teachers to be thoroughly familiar and knowledgeable about content that they were required to teach.

Since the publication of the Presidential Education Initiative (PEI) Report (Taylor & Vinjevold as cited in Brodie, 2004, p.65), “the issue of mathematics teacher knowledge has been on the research and teacher education agenda in South Africa” (Brodie, 2004, p.65). According to the PEI report it was found that teachers needed a better conceptual knowledge of mathematics in order to assist learners to gain access to the mathematical content. Also in America, according to Wu (2005, p.1), there has been a trend in mathematics education, which she found quite alarming, to avoid the importance of content knowledge. Wu believed that “sound pedagogical decisions are based on sound content knowledge” (Wu, 2005, p.1). I therefore embarked on this study in order to shed some light on the relationship between content knowledge and practice.

Also since I am a part of a National Research Foundation (NRF) project (NRF number FA2006041200004) that involves the implementation of activities designed by teacher education units at three South African universities, namely the University of the Witwatersrand, the University of Kwa-Zulu Natal and the Nelson Mandela Metropolitan

University, I decided to streamline my research and focus on the link between content knowledge and practice in specific relation to the teaching of calculus. In my research I used some data from the 'calculus project', in the form of videotaped lessons of two university lecturers teaching calculus to an undergraduate teacher trainee class. The other data such as the interviews that I conducted with the university lecturers was data that I retrieved on my own.

The 'calculus project' which was funded by Anglo American is characterised by the three universities developing material that is research based, learner centred and conceptually focused especially to teach calculus to teachers. The project focused on the "mathematical practices in calculus teacher education." The two university lecturers that I interviewed are lecturers that are also involved in the 'calculus project' and are based at a South African tertiary institution.

1.2 AIM AND CRITICAL QUESTIONS

The two main areas of investigation in mathematics education is the teaching of mathematics

> "which focuses on matters pertaining to organised attempts to transmit or bring about mathematical knowledge, skills, insights, competencies, and so forth, to well defined categories of recipients," (Niss, 1999, p.6)

and also the learning of mathematics "where the focus is on what happens around, in and with students who engage in acquiring such knowledge, skills… (Niss, 1999, p.1). In my research I focused on the teaching of mathematics which I believe is quite a complex issue. More especially I looked at the content knowledge of two university lecturers and how this informed their practice.

The aim of my research was to generate thick interpretations of data in the form of videotaped lessons, and interviews, in order to explain the lecturers' perceptions of the link between content knowledge and the activities that they engaged with in the classroom.

The key critical questions that were explored in this study were:

(a). What were the lecturers' perceptions of content knowledge?

(b). Did the lecturers engage their students in dialogue around the activities that occurred in the class and how did the nature of the dialogue influence the lecturer's classroom practice?

(c). How did the lecturers view the link between content knowledge and classroom practice?

CHAPTER TWO

REVIEW OF LITERATURE

2.1 INTRODUCTION

In this chapter I will present the theoretical framework that informed my study and also provide a review of the current literature that surrounds my topic, both nationally and internationally.

2.2 THEORETICAL AND CONCEPTUAL FRAMEWORKS

My research was based in the interpretivist paradigm. According to Cohen, Manion and Morrison (2007, p.21) the interpretivist paradigm focuses on the individual, in order to understand the phenomena that is being investigated from within the individual. In this instance the phenomenon that I investigated from within the individuals were their perceptions on the link between content knowledge and classroom practice. "An interpretivist researcher wants to learn what is meaningful or relevant to the people being studied, or how individuals experience daily life" (Neuman, 1997, p.69). These researchers look at social action that is meaningful, not just the observable or external behaviour of people. This characterized my research where I studied two university lecturers and how they engaged in meaningful activities with their students in order to teach a particular topic in mathematics.

According to Nieuwenhuis (2007, p.59) interpretivism is based on the following assumptions:

(a) Human life can only be understood from within - The focus is on the subjective experience of people, how they construct their social world by engaging in dialogue and sharing meaning with others. This is evident in my research which focused on the subjective experience of the two university lecturers. My research also focused on the dialogue that the lecturers engaged in with their students in order to construct meaning around the topic of calculus.

(b) Social life is a distinctly human product - By placing people in their social contexts the opportunity to understand their perceptions of their own activities is far greater. The data that was generated for my research was from the social contexts of the lecturers. The lecturers were observed in their contextual environment of teaching. Also the interviews that I conducted with the lecturers required that they reflect on their social practices in the teaching environment.

(c) The human mind is the origin of meaning –

> "By exploring the richness, depth and complexity of phenomena we begin to develop a sense of understanding of the meanings imparted by people to phenomena and their social context" (Nieuwenhuis, 2007, p.59).

This is characterized in my research by the interviews that I conducted with the participants in order to derive their meaning of the phenomenon under investigation.

(d) Human behaviour is affected by knowledge of the social world - Interpretivism proposes that there is more than one reality of a phenomenon and also that these realities change with time. In my research I presented one reality of the phenomenon under investigation and that was the perceptions of the two university lecturers of the link between content knowledge and practice.

(e) The social world does not exist independently of human knowledge - As researchers our knowledge and understanding is influenced by what we have been exposed to. Also as we proceed through our research we are influenced by our prior knowledge, intuition, beliefs and values. However I think that we have to be guarded against this so that is does not influence our objectivity of the research situation.

The aim of interpretivist research is to offer the perspective of the phenomenon under investigation from the viewpoint of the subject. This is criticized as it provides a subjective view and cannot be generalised. The nature of the interpretivist paradigm is such that although the findings cannot be generalised they can be transferred to similar situations, depending on the reliability of the data.

My research was conceptualized in terms of Vygotsky's educational theory and also the process of scaffolding. According to Vygotsky's theory "learning leads to the development of higher order thinking" (Dahms et al, 2007, p.1). Vygotsky believes that learning occurs through language and social interaction. The zone of proximal development (ZPD) is central to Vygotsky's view on how learning occurs. He describes this zone as the distance between the actual developmental level of a learner which is determined by independent problem solving and the potential developmental level of a learner which is determined by problem solving under the supervision of a teacher or a more capable peer. The ZPD can also be described as the area that separates what the learner is able to achieve on his own and what he can achieve under supervision and with assistance. The performance of a learner is lifted when she is working under supervision.

"Vygotsky defined those who teach as the 'More Knowledgeable Other' (MKO)" (Dahms et al, 2007, p.2). The MKO refers to anyone who has a higher level of understanding than the learner. The key characteristic of the MKO is that their knowledge of the topic that is being taught must be greater than that of the learner. In this way they can raise the competence of their learners. The MKO shares his knowledge with his students in order to bridge the gap between what they know and what they do not know. Once the student expands his knowledge the ZPD shifts. The ZPD is always changing as the students gain more knowledge. In my study the MKO would be the two university lecturers and it would be interesting to note their perception of how content knowledge was linked to the competencies of their students in order to allow a shift in the ZPD of their students.

According to Vygotsky (as cited in Dahms et al, 2007, p.2) the ideal role of the teacher is to provide scaffolding to assist the learner with their tasks. “The term ‘scaffolding’ was developed as a metaphor to describe the type of assistance offered by a teacher or a peer to support learning” (Lipscombe et al, 2008, p.3). Scaffolding is a process whereby the teacher helped a student to grasp a concept or master a task that he was initially unable to do. Assistance was only provided to those skills that were beyond the student’s capability. The student may make errors, but with feedback and prompting, the student is able to reach the required response. Once the student can perform the task on his own the teacher then removes the scaffolding which allows the student to work independently.

> “Scaffolding is actually a bridge used to build upon what students already know to arrive at something they do not know. If scaffolding is properly administrated, it will act as an enabler, not as a disabler” (Benson as cited in Lipscombe et al, 2008, p.3)

Many different tools can be used in order to scaffold student learning. This includes breaking the task down into smaller and more manageable subtasks; assisting students to verbalise their thinking processes; teamwork and co-operative learning among students, activating background knowledge, prompting and questioning. In order that scaffolding is successful the teacher needs to know what prior knowledge the student has. In this way the teacher connects the student’s prior knowledge to the new knowledge and makes it relevant for the student, so that he was motivated to learn. In instructional scaffolding the MKO leads the student from what he knows to an in depth understanding of new material and the MKO also provides support to the student at every step in the learning process.

According to Zhao and Orey (as cited in Lipscombe et al, 2008, p.5) scaffolded instruction has six general elements namely: “sharing a specific goal, whole task approach, immediate availability of help, intention assisting, optimal level of help and conveying an expert model” (Lipscombe et al, 2008, p.5)

(a) Sharing a specific goal – It is the responsibility of the MKO to establish a goal and share it with his students. Allowing for input from the students on the shared goal enhances their intrinsic motivation. It also assists the learner to master the goal that has been set, by developing new skills. In this way, learning is also enhanced and the student sees that he can attain success by putting in the required effort.

(b) Whole task approach - The focus of this approach is on the ultimate goal that is to be achieved. In this approach each task is seen as how it relates to the ultimate goal. This approach is only effective if the student does not experience any difficulty in any of the minor tasks in order to achieve the ultimate end result.

(c) Immediate availability of help - In scaffolded instruction success is important in order to control the frustration levels of the students. If the MKO provides assistance and support timeously, then the student experiences success and this motivates the student to be more productive.

(d) Intention assisting - In the scaffolding process it is important to provide assistance to the learner’s present focus in order to assist him with his current problems. In this way a more productive learning environment is developed as the learner pursues his current task.

Sometimes, however, it is necessary to redirect the learners thought processes, if they do not coincide with the current task at hand. The MKO needs to be aware of the different methods of completing a task and if the learner's method allows him to complete the task in a different way then the MKO must accept this method and allow the learner to proceed with the least amount of assistance. If however the learner needs constant assistance then the MKO could turn to coaching to assist the learner.

(e) Optimal level of help - The level of assistance that is provided to the learner must match what he is able to do. The learner must get just enough assistance in order

> "to overcome the current obstacle, but the level of assistance should not hinder the learner from contributing and participating in the learning process of that particular task" (Lipscomb et al, 2008, p.7).

(f) Conveying an expert model - An expert model provides the learner with an example of how to accomplish a particular task. The techniques that are to be used for completing the task are clearly expressed.

According to Lange as cited in Lipscomb et al (2008, p.7) there are five methods in instructional scaffolding, namely:

(a) Modelling - This is usually the first step in instructional scaffolding. In this step the MKO teaches the student behaviour that shows him how to think and act in a particular situation. Modelling can be demonstrated by thinking aloud, talking aloud, or by performance.

(b) Offering explanations - The MKO provides the learner with explanations about what is being learned, why it is being learned and how it is used. Initially explanations are detailed and are repeated often. As the student progresses the explanations could consist of key words and prompts.

(c) Inviting students to participate – This is usually done in the early stages of scaffolding. This allows for the ownership by students in the learning process and also enhances student engagement with the content being taught.

(d) Verifying and clarifying student understandings – As students become familiar with new content, the teacher must evaluate the student's knowledge and also provide feedback that is both positive and corrective, where necessary.

(e) Inviting students to contribute clues - The MKO can elicit the assistance of other students, who may have grasped the subject matter to provide clues to assist students who are still grappling with the new content.

Larkin as cited in Lipscombe et al (2008, p.7) suggests the following effective techniques of scaffolding that teachers can follow:

Teachers should begin by boosting the confidence of their students by introducing them to tasks that they can do quite well with limited or no assistance. Teachers should also provide the right amount of assistance to allow their students to achieve success quickly- this assists with the lowering of frustration levels and ensures that students are motivated to proceed to the next level. Teachers must also help their students to fit in with their peers, as they work harder if they feel that they belong with their peer group. The teacher must be guarded to prevent students from boredom. Once a skill has been learned, the scaffolding should be removed gradually until the skill is mastered.

The implications of Vygotsky's theory and the process of scaffolding for the teacher, is thus to guide the student's activity so that meaningful learning occurs.

2.3 CONTENT KNOWLEDGE AND CLASSROOM PRACTICE

2.3.1 INTRODUCTION

The conceptions of teachers' subject matter knowledge or content knowledge has changed from the beginning of the twentieth century. At the beginning of the century it was described in qualitative terms which made it very difficult to measure or evaluate the content knowledge of teachers. Towards the end of the nineteenth century this perception shifted with the emphasis now being placed on the number of courses taken by teachers and also their performance in standardised tests. Shulman's Presidential Address in 1986 (as cited in Even, 1990, p.322) saw a return to assessing teachers' subject matter knowledge in qualitative terms. According to Even

> "analyzing what it means to know mathematics, has some promise to contribute to the improvement of the quality of subject matter preparation of teachers and therefore the quality of teaching and learning" (1990, p.322).

2.3.2 DEFINITION OF KEY CONCEPTS

This study applies the following terminology within the constraints of the definitions provided:

(a) content / subject matter knowledge

These two concepts are used interchangeably in this research and mean the same thing. Content knowledge refers to the knowing about a subject, the disciplinary knowledge of a subject. "Mathematical content knowledge includes information such as mathematics concepts, rules and associated procedures for problem solving" (Chinnapen, 2003, p.1).

(b) pedagogical knowledge

Pedagogical knowledge refers to the broad knowledge that a teacher requires in order to be effective in the classroom. This includes content knowledge, knowledge about how to teach, knowledge about pupils and how they learn, knowledge about the curriculum and knowledge about discipline and classroom management.

(c) conceptual knowledge

This term is used by Adler, Slominsky and Reed (2002) and refers to the special way that a teacher uses the mathematical content in order to teach mathematics. Adler et al draws a distinction between the way a mathematician would view the mathematical content and the way a mathematics teacher views mathematical content. The teacher has to impart content knowledge to his students.

(d) pedagogical content knowledge

This term was first used by Shulman. It refers to a blend of content knowledge and pedagogical knowledge, this includes understanding why some children experience difficulties learning a concept while others find it easy to understand, the best method to approach or discuss a particular topic or concept, the quality of explanations that teachers give during a lesson.

This study does not draw a distinction between a teacher, an educator and a lecturer, in other words the MKO. These terms are used to refer to those that impart knowledge to their students, learners and pupils. The terms learners and pupils are used to refer to learners in a school and students refer to those studying at a university.

2.3.3 SHULMAN'S PRESENTIAL ADDRESS OF 1986

Shulman's inquiry into teacher education entailed him looking at literature that dated back to 1875, where he looked at tests that teachers were given. These tests demonstrated how teacher knowledge was viewed and defined. There were certain assumptions that underlined those tests and those were:

> "the person who presumes to teach subject matter to children must demonstrate knowledge of that subject matter as a prerequisite to teaching. Although knowledge of the theories and methods of teaching is important, it plays a decidedly secondary role in the qualifications of a teacher" (Shulman, 1986, p.5).

This emphasis on content knowledge contrasted with the emerging policies of the 1980's where the emphasis was on pedagogic knowledge, the knowledge to be able to teach. This did not meet with Shulman's approval and he questioned "where did the subject matter go? What happened to the content?" (Shulman, 1986, p.5). Shulman was disappointed that policymakers emphasised teaching procedures which were based on emergent research on teaching and the effectiveness of teaching. Shulman believed that this research simplified the complexities of classroom teaching and also ignored one important aspect of the classroom situation and that was content knowledge.

Shulman and his colleagues referred to this "absence of focus on subject matter among the various paradigms for the study of teaching as the 'missing paradigm' problem" (Shulman, 1986, p.6). Shulman believed that the consequences of not focusing on content would be serious for both education and research.

Shulman did not agree with the distinction between pedagogy and content. He found this recent distinction between pedagogy and content disturbing. Shulman was in agreement with the medieval philosophy on teaching where “content and pedagogy were part of one indistinguishable body of understanding” (Shulman, 1986, p.6). Shulman believed that the recent literature of research done on teaching avoided central issues. The literature emphasised classroom management, organisation of activities and time management. What was lacking was questions about the content that was being taught, the questions that pupils asked and the explanations that were being offered by their teachers. Shulman’s focus was on teacher development. He believed that there had been a lot of attention on learners and that the focus should now shift to teachers. Some of the issues that Shulman and his colleagues wanted to address were:

> “What are the sources of teacher knowledge? What does a teacher know and when did he or she come to know it? How is new knowledge acquired, old knowledge retrieved, and both combined to form a new knowledge base?” (Shulman, 1986, p.8).

Shulman’s main area of concern was how did the student teacher transform his expertise of his subject matter into a form that could now be presented to school pupils. With content expertise the teacher could correct flaws in textbooks and also offer explanations to pupils. Shulman believed that both pedagogical expertise and content knowledge are vitally important for effective teaching.

> “Mere content knowledge is likely to be as useless pedagogically as content-free skill. But to blend properly the two aspects of a teacher’s capacities requires that we pay as much attention to the content aspects of teaching as we have recently devoted to the elements of the teaching process” (Shulman, 1986, p.8).

Shulman began to probe teacher understanding and the transmission of content knowledge from the teacher to the pupil. In his research several questions came to the fore which he tried to find answers to and one of the important domains that Shulman focused on was content knowledge.

Shulman distinguished between three categories of content knowledge, namely:

(a) subject matter content knowledge- This refers to the knowledge that the teacher possesses about his subject. This goes beyond the knowledge of facts and concepts pertaining to a particular subject and encompasses knowledge about why concepts and topics are important to a subject, which topics are central and which are peripheral.

(b) pedagogical content knowledge- This goes beyond the knowledge of a particular subject and incorporates knowledge on how to teach the particular subject. This includes knowledge of topics most often taught in a particular subject, the most important and appropriate examples that are used and the most meaningful forms of representations used. All these come from experience and practice. Pedagogical content knowledge also includes knowledge about what makes a particular topic easy or difficult for pupils to understand, the common misconceptions that pupils may have on a particular topic and the strategies on how to deal with these issues.

(c) curricular knowledge- This refers to the teacher's knowledge firstly about his own subject matter, that is knowledge about the content taught in preceding years and the content to be taught in years that follow. In this way the teacher can adequately teach pupils present content as he is aware of what they already know and he is also aware of how the current topic will be extended in future years. Secondly curricular knowledge also refers to knowledge about other subjects and how this links to knowledge in your particular subject and field. In this way the teacher can relate what the pupil is learning in one subject to what is being taught in another subject and this makes learning more meaningful and relevant.

Shulman's ideas on content knowledge were contrary to the thinking of the 1980's. He went against the reforms of the time where the focus was on "how to improve teaching as both an activity and a profession" (Shulman, 1987, p.3). The PEI Report in South Africa however catapulted content knowledge into the forefront of research on education.

2.3.4 THE PRESIDENTIAL EDUCATION INITIATIVE (PEI) REPORT

Education systems presently not only act as vehicles for transformation and change but must also produce citizens that "will enable their countries to become globally competitive" (Taylor & Vinjevold, 1999, p.2). This challenge was faced by South Africa's first democratic government. In order to meet this challenge and also to make provision for an education system that is equitable, of high quality and also one that is relevant to all its citizens, the PEI was born.

The PEI research report identified five main issues that affect teaching and learning in South Africa currently. These issues are:

(a) institutional conditions

(b) the attitudes of teachers

(c) teacher knowledge

(d) classroom practices

(e) student learning.

These issues are interdependent and they influence each other, so it is difficult to separate them. However for the purposes of my research I looked at the link between two of these issues and that is how the teachers' content knowledge informs classroom practice.

> "One of the most consistent findings of a number of PEI projects pointed to teachers' low levels of conceptual knowledge, their poor grasp of their subjects and the range of errors made in the content and the concepts presented in their lessons" (Ibid, 1999, p.139).

According to a study by Pile and Symthe (as cited in Taylor & Vinjevold, 1999, p.139) their findings was that teachers do not have a holistic view and understanding of what they are teaching and it is therefore difficult for the teachers to see the links between the various parts of the curriculum. This inhibits the teachers' presentation of their lessons and also causes them to make factual errors (Ibid, 1999, p.139). This point is illustrated by Webb et al (as cited in Taylor & Vinjevold, 1999, p.141) who conducted tests on fractions with grade four pupils and also gave the same test to 19 first year teachers and 18 second year teachers and their findings were that the teachers' knowledge of key mathematics concepts "is a little better than that of their pupils, and that teacher knowledge is distressingly low in some topics" (Ibid, 1999, p.141).

The research studies conducted by the PEI strongly suggested that the teachers' poor and insufficient content knowledge of mathematics inhibited the teaching and learning of mathematics generally in South African schools. Poor content knowledge resulted in a superficial understanding of what entailed good teaching and learning. This resulted in a classroom situation that was teacher-centred and the pupils were engaged in a very superficial level with the content of the subject that was being taught. The PEI report was responsible to a large degree, in South Africa, of initiating and catapulting research on content knowledge. Research in this regard was conducted by, amongst others, Adler, Slonimsky, Reed, Ensor, Long and Brodie.

2.3.5 CONTENT KNOWLEDGE

The issue of content knowledge for teaching is not something new, it has a long international history and more recently a growing national history. Research into content knowledge distinguished between two major focuses namely quantitative studies and qualitative research. In quantitative studies the attention was on teacher characteristics for example "the number of mathematical courses taken by teachers; their qualifications; and their performances in tests" (Brodie, 2004, p.65). Qualitative studies on the other hand probed the nature of teacher knowledge in various topics in mathematics and these have shown deficits in teachers' content knowledge. However, what has not been examined was the link between teachers' knowledge and classroom practice, hence this research to shed some light on this issue.

What constitutes subject knowledge for teaching and how it is best to acquire this is a challenge that faces teacher education everywhere (Adler, Slonimsky & Reed, 2002, p.135). Subject knowledge for teaching is a very important aspect of the preparation that a teacher requires in order to deliver in the classroom. "A consideration of teachers' subject knowledge cannot be isolated from its use" (Ibid, 2002, p.135). This statement resonates with the link that I tried to establish between content knowledge and classroom practice.

In order to revitalize teacher education and classroom practice it is important to upgrade the teachers' subject matter knowledge. Only if you can engage with your subject matter in a meaningful way can you hope to impart this in a meaningful way to your students. Presently South Africa is faced with the challenge to improve the content knowledge of teachers. There are a number of reasons for this and the crucial one being

> "that knowledge of subject matter for teaching is of primary importance, for without this, teachers would not be able to engage their learners in high-level conceptual thinking" (Ibid, 2002, p.136).

This corresponds very closely with the findings of the PEI report. Although there is a great importance placed on teachers' content knowledge, what is not explained and explored is the nature of the subject knowledge that teachers require.

Ball and Bass on the other hand argued that "subject matter knowledge for teaching has too often been defined by the subject matter knowledge that students are to learn" (2002, p.95). They argued that it is generally assumed that teachers must know what they are required to teach and also have a broad idea of what their students already know and what they need to know in the years to come. Although they believed that there was nothing inherently wrong with this, they believed that

> "the unexamined conviction that possessing such knowledge is all that teachers need to know has blocked the inquiry to bring together subject matter and practice in ways that would enable teacher education to be more effective" (Ball & Bass, 2002, p.95).

They believed that there is a strong link between content knowledge and practice.

According to Long (2003, p.194), "the emphasis on improving teachers' subject knowledge is tied to the belief that this will improve mathematics achievement." Even also focused on this aspect and referred to reforms in education in the UK, for example "the Carnegie Task force, 1986: Holmes Group, 1986; NCTM, 1986b" (Even, 1990, p.251) that were designed to improve the content and professional knowledge of teachers. While current reforms were concentrating their efforts on strengthening the subject matter knowledge of teachers there was also increased interest in analyzing and defining what content knowledge for teaching means. I think that this is relevant for the South African context currently as we find ourselves undergoing curriculum changes and also reforms in the education system. Kilpatrick et al also affirmed this idea and they believed that "knowledge of content to be taught is the cornerstone of teaching for proficiency" (2001, p.372). They believed that improving teachers' knowledge is vital in developing students' mathematical ability and proficiency.

Long touched on the debate as to whether teachers need to know only the content that they were teaching or did their content knowledge need to extend beyond the curriculum that they were teaching. I tend to agree with Hilton (as cited in Long, 2003, p.196) who believed that pupils can ask questions that extend beyond the curriculum and teachers need to be equipped with the necessary expertise in order to answer these questions. Brodie in her interpretation of the PEI report suggested that teachers need to have more content knowledge about the subject that they were teaching. However she felt that there was a lack of suggestion as to what must be done in order to increase the content knowledge of teachers.

Even (1990) also focused on teachers' content knowledge for a particular mathematical topic. I found this important and relevant for my research since I also looked at a particular topic in mathematics and that was calculus. It was interesting to note that Even believed that the teachers' content knowledge about a particular topic in mathematics was influenced by their knowledge across a number of different areas, which included:

(a) the role of that particular topic in the mathematics curriculum and its importance in the curriculum

(b) theory and research on learning

(c) the understanding and knowledge of mathematical concepts

(d) understanding the role of teachers' content knowledge in teaching.

Even (1990) also identified seven aspects that formed the main facets of teachers' content knowledge about a specific topic. These aspects were:

(a) concept image- This refers to the essence of a concept. It also refers to the mental picture a person has of a concept in his mind together with the properties that he associates with the concept. Different people could have different concept images. In order for a teacher to have a well structured knowledge of mathematics, his concept image of the concepts that he teaches must correspond with the 'correct' mathematical concept.

(b) different representations- Concepts in mathematics often appear in different ways and also behave in different ways depending on the situation. Teachers need to understand the various representations of a concept and also be able to link and translate between these various representations. "Different representations give different insights which allow a better, deeper, more powerful and more complete understanding of a concept" (Even, 1990, p.524).

(c) alternative ways of approaching- A concept can be used differently in different topics or sections of mathematics. Teachers need to be familiar with the different approaches in order to make appropriate choices when necessary.

(d) the strength of a concept- The success of a concept depends on the new opportunities it creates. Concepts become powerful and important when they expand mathematical knowledge and thinking.

(e) basic repertoire- In every mathematical topic it is important to have access to many examples to demonstrate a particular concept or principle. The basic repertoire serves as a point of reference when the teacher is faced with complex problems.

(f) knowledge and understanding of a concept- Here Even differentiates between conceptual and procedural knowledge. Conceptual knowledge refers to meaningful learning and understanding whilst procedural knowledge refers to the memorisation of facts and procedures and also to the carrying out of procedures. Even argued that mathematical knowledge should be a balance between conceptual and procedural knowledge.

(g) knowledge about mathematics- This refers to a more general knowledge of mathematics as a discipline. “It includes ways, means and processes by which truths are established as well as the relative centrality of different ideas” (Even, 1990, p.527). It also refers to the dynamic nature of mathematics. Teachers need to thus be aware of the new developments in order to have an updated knowledge of mathematics.

Hill, Rowan and Ball, researchers in the United States since the 1960s, have explored the relationship between student achievement and the behaviour and the characteristics of teachers (2005, p.373). The different characteristics of teachers and how they impact on learner achievement, have been measured by different research programmes. Pertinent to my research is the “teachers in the teacher knowledge literature programme” (Hill et. al, 2005, p.376). This programme focused on the teachers’ knowledge about their subject and specifically on what content knowledge the teacher needed in order to teach his subject efficiently and effectively. Shulman (as cited in Hill et. al, 2005, p.376) was one of the front runners in this line of enquiry and his work focussed on what teachers needed to know in order to be accomplished in the classroom. Shulman’s work suggested that it was not only content knowledge that influences effective teaching, but also knowledge of how to teach the content contributed to this.

Margolinas, Coulange and Bessot (2005) developed research mainly in France in connection with the important topic of teachers' knowledge for mathematics education. Their approach to this topic was "in the context of research…around the basic notions of the theory of didactic situations" (Margolinas et. al, 2005, p.205). They concentrated on what they termed 'didactic knowledge'. They defined didactic knowledge as referring to the mathematical knowledge that needed to be taught and in terms of my research this would equate to content knowledge.
According to Wu (2005, p.1) "in the mathematics education mainstream of the past ten or fifteen years, there is an alarming trend which may be called avoidance syndrome." Wu argued that the importance of content had been avoided in the recent proposals for the improvements of education. The California Department of Education (CED), around 1990, which reflected the national trend at that time, decided that in order to improve mathematics education pedagogic techniques had to change. They also introduced group learning and the discovery method as the focus of mathematics instruction in classrooms. The fact that teachers in California [and elsewhere] were in dire need of better content knowledge was not mentioned. Also some educationists were urging teachers to concentrate on the cognitive capabilities of children. Here again what was over looked was the fact that in order for teachers to interpret the mathematical thinking of their pupils they needed strong content knowledge. In the last few years, however, new developments have emerged and one of these is the emphasis on teachers acquiring pedagogical content knowledge. Here again Wu felt that "pedagogic content knowledge is a refinement of content knowledge, and is built on that knowledge" (2002, p.2). There was, however, some indication that certain sectors of the

mathematics community were realising the urgency for teachers to know the required content knowledge for their particular subject.

To illustrate why content must dictate pedagogy Wu used a topic in elementary mathematics, that of invert and multiply. Wu argued that this topic could be explained “on the basis of a thorough understanding of mathematics” (2005, p.4). Wu therefore argued that “sound pedagogical strategies are those based on a firm grasp of the content” (2005, p.4).

As a result of research done in Turkey with pre-service primary mathematics teachers, it was found that “having a deep understanding of mathematical knowledge was necessary but not sufficient to teach mathematics” (Turnuklu & Yesildere, 2007, p.1). Although a number of factors influence the teaching of mathematics, the teachers’ role is a crucial one. It is a common belief in society that if a mathematics teacher knows his subject well then he is the best person to teach mathematics. Fennema and Frank (as cited in Turnuklu & Yesildere, 2007, p.1) however distinguished four components of a mathematics teachers’ knowledge. Pertinent to my research is the component that refers to the teachers’ knowledge of mathematics which encompasses content knowledge, the nature of mathematics and how the teacher organises his knowledge. According to Fennema and Frank (as cited in Turnuklu & Yesildere, 2007, p.1) if a teacher has a sound knowledge of mathematics then this influences the instruction in the classroom in a positive way.

2.3.6 PEDAGOGICAL KNOWLEDGE

Pedagogical knowledge refers to a professional knowledge base that is required for teachers (van Manen, 1999, p.13). Central to the idea of pedagogy is the notion of being able to distinguish between what is appropriate and what is less appropriate for children, and to be able to determine appropriate ways of teaching and assisting pupils.

Shulman also talked about the teachers' knowledge base and he argued that "teaching skills are bound up with teachers' thinking which draws upon their knowledge as a basis for judgement and action" (as cited in Kyriacou, 1999, p.5). Shulman argued that the teacher's knowledge base should include the following components:

(a) content knowledge

(b) curriculum knowledge

(c) knowledge about pupils

(d) knowledge about classroom management and discipline

(e) knowledge about the aims and values of education

(f) knowledge about the context of education and

(g) knowledge about the broad principles of education (as cited in Kyriacou, 1999, p.4).

Pedagogical knowledge is therefore an all encompassing term which refers to the broad base of information that a teacher requires in order to teach effectively.

2.3.7 CONCEPTUAL KNOWLEDGE

Adler, Slonimsky and Reed (2002) draw a differentiation between the knowledge that an expert may have on his subject, for example the mathematician's knowledge of mathematics, and that of a mathematics teacher. Their view is that a teacher needs a special kind of knowledge of his subject which they refer to as 'teachers' conceptual knowledge-in –practice. Adler (2005, p.4) supports this view when she advocates the growing support for the idea that there is a special way that teachers understand and use mathematics in their teaching. This she maintains is different from how a mathematician would use mathematics. Since teachers work with mathematics as something that needs to be learnt, they need to unpack the content and ideas of mathematics to make it accessible for their learners.

Kennedy (1997) also focuses on the teacher's conceptual understanding of subject matter since "the main aim of reformers is to instil a deeper understanding in students of the central idea and issues in various subjects and to enable students to see how these ideas connect to, and can be applied in, real world situations, it therefore makes sense to require that teachers themselves also understand the central concepts of their subjects, see these relationships, and so forth" (Kennedy, 1997, p.6). Kennedy distinguished between five distinct ideas which she felt described conceptual understanding:

(a) first notion- This refers to the sense of size and proportion. She argued that people should be able to grasp large numbers, be able to understand risk in terms of percentage and be able to understand weather reports.

(b) second notion- This refers to focusing on the central ideas of a subject rather than dealing with the minutiae.

(c) third notion- This refers to the relationship between ideas in a particular subject. Teachers should be able to determine which ideas are central to a topic, which ideas are needed to justify others and which ideas encompass other ideas. This is important since the teachers are to focus the students' attention on the central ideas and they can only do that, if they themselves are aware of the central ideas in a topic. Also, since teachers need to encourage discussion among students, they need to be able to judge whether or not a student's ideas need to be pursued and also how to guide and steer the student's thought processes to bring them to the desired result.

> "Without knowing how the various ideas in a discipline relate to one another, support one another, parallel one another, or subsume one another, teachers would have difficulty knowing whether student's questions and hypotheses will lead to greater understanding or instead to confusion or dead ends" (Kennedy, 1997, p.7).

(d) fourth notion- This notion that is sometimes attached to conceptual understanding and refers to a highly elaborate knowledge. It refers to an individual who has an in depth knowledge of a particular topic and knows lots of details and lots of examples on that topic. "Understanding, reasoning, and problem solving are all dependant on detailed specific knowledge" (Kennedy, 1997, p.7).

(e) fifth notion- This refers to the ability to "reason about phenomena, develop arguments, solve real problems, and justify one's solutions" (Kennedy, 1997, p.8). What Kennedy finds interesting is that although the literature of K-12 learners focuses on reasoning and problem solving, there is very little literature on teacher's knowledge that focuses on these issues. Implicit in this statement is that teachers have to have knowledge of problem solving in order to impart these skills to their students.

2.3.8 PEDAGOGICAL CONTENT KNOWLEDGE

This term was first introduced by Shulman to refer to the teacher's ability to represent and impart important ideas about a particular topic to students, so that they could understand these ideas. Pedagogical content knowledge was what enabled teachers to transform complex ideas into concepts that students could assimilate and grasp. Shulman advocated the use of metaphors in order to illuminate and illustrate complex ideas. Pedagogical content knowledge therefore relies on conceptual knowledge because in order for a teacher to use an apt and appropriate metaphor he needs to have a good conceptual knowledge of the topic that he is teaching.

Shulman also argued that pedagogical content knowledge allowed the teacher to choose metaphors that were not only appropriate to convey the required concept or idea but were also "within the realm of understanding of the students at hand" (Kennedy, 1997, p.9). In order for the teacher to know what metaphors and analogies to use requires a strong conceptual understanding of the topic, an understanding of what students currently think about the topic, any misunderstandings that they may have and what knowledge they may lack.

Au, Kulm and Wu as cited in Turnuklu and Yesildere (2007, p.2) view pedagogical content knowledge as comprising of three components, namely:

(a) knowledge of content

(b) knowledge of curriculum

(c) knowledge of teaching.

They argued that knowledge of teaching is the core component, but also that these three components are interrelated.

An important aspect of pedagogical content knowledge is that it requires the teacher to reason and think. Teachers have to first make sense of what they are teaching and then plan and implement the learning of the particular topic, so that it makes sense to and is accessible to their learners.

In South Africa Brodie (2001) expanded on this notion of pedagogical content knowledge by looking at the research of Grossman (as cited in Brodie, 2001) and Marks (as cited in Brodie, 2001) and she thus applied the term ‘teacher knowledge.’ Grossman identified four important components of pedagogical content knowledge namely:

(a) conceptions of purposes for teaching

(b) knowledge of student understanding

(c) curricular knowledge

(d) knowledge of instructional strategies.

Marks (as cited in Brodie, 2001) on the other hand identified the following components:

(a) knowledge of student understanding

(b) subject matter

(c) instructional processes

(d) instructional media.

From these categories Brodie (2001) came to the conclusion that pedagogic content knowledge represented an extended version of teaching methodology and content knowledge.

Brodie (2001) finds Ball and Cohen's (as cited in Brodie, 2001, p.19) outline of teacher knowledge very comprehensive. They categorize teacher knowledge as follows:

(a) conceptual knowledge of the subject- This refers to knowing the subject matter in a way that allows for effective teaching. This includes an in depth conceptual knowledge of different topics and their interrelatedness.

(b) knowledge of students- This refers to a general of knowledge of students and how they engage in learning and also a specific knowledge of students in order to assist them with their peculiar difficulties in the subject.

(c) knowledge of pedagogy- This refers to the teacher's access to different methodologies in order to engage the students in high level learning. It also includes reasons for teaching and learning, activities that the teacher engages the students in and ways of creating a classroom environment that is conducive to learning.

Brodie thus concluded that "teacher knowledge is a resource of the teacher which becomes a resource for teaching and learning for both teacher and students in the classroom" (2001, p.19).

Kilpatrick et al (2001) also expounded on this notion of teachers' knowledge and they refer to it as a knowledge base for teaching mathematics. They identify three components that are essential for teaching mathematics, namely:

(a) knowledge of mathematics

(b) knowledge of students

(c) knowledge of instructional practices.

These categories are very similar to the categories that are elaborated upon by Ball and Cohen (as cited in Brodie, 2001, p.19), there are however, some additions and differences in terms of how Kilpatrick et al (2001) describe these components.

Their description of these components is as follows:

(a) knowledge of mathematics- This

> "includes knowledge of mathematical facts, concepts, procedures, and the relationship among them; knowledge of the ways that mathematical ideas can be represented; and knowledge of mathematics as a discipline" (Kilpatrick et al, 2001, p.371).

Teachers must also know how mathematical knowledge is constructed, how to communicate the mathematics to their students in an effective manner and also the rules that governs proofs and logical arguments in mathematics.

(b) knowledge of students- This also includes how students learn mathematics. Teachers must be aware of how different mathematical concepts and ideas evolve in students' minds and how they develop over a period of time.

> "It includes familiarity with the common difficulties that students have with certain mathematical concepts and procedures, and it encompasses knowledge about learning and about the sorts of experiences, designs and approaches that influence students' thinking and learning" (Kilpatrick et al, 2001, p.372).

(c) knowledge of instructional practice- This includes knowledge of the mathematics curriculum, knowledge of how to impart crucial mathematical concepts and ideas to students, knowledge of how to organize the classroom space and knowledge of how to promote dialogue in the classroom that will allow for optimal learning.

2.3.9 CLASSROOM PRACTICE

Current research needs to focus on providing an analysis of teaching practices and the mathematical knowledge that is required to improve and also to sustain these practices (Ball et al as cited in Brodie, 2004, p.66). Ball et al also argued that there should be a strong link between the mathematics that teachers have to learn and the activities that define their practice in the classroom. In contrast to this, the PEI report argued that too much emphasis had been placed on teaching methodology in South African curriculum initiatives, at the expense of content knowledge that needed to be taught and learned. The report argued that in-depth content knowledge would promote better teaching and learning regardless of the teaching methodology that was used. I believe that of primary importance is content knowledge for without this no meaningful teaching and learning can occur. However, methodology is also important in order for the teacher to impart his content of the subject in an effective manner to his pupils.

Adler (2005) focused on the complex issues of the teaching and learning of mathematics. She felt it was important that we understood “how to make mathematics learnable by all children” (Adler, 2005, p.2). Her own area of interest is to know more about the support and mathematical preparation that teachers receive in order to make them more efficient and skilful in the classroom context. The question that she investigated in one of her studies was “what mathematics teachers (at different levels) need to know how to do, in order to teach well” (Adler, 2005, p.3). By looking at lessons in the classroom context Adler came up with a solution to the problem of how to deal with different learner responses. In order to deal with learner responses she argued that teachers needed ‘mathematical proficiency’ which included content knowledge, reasoning and problem solving skills and fluency in mathematical procedures. Here again content knowledge was included as part of the criteria necessary for skilful teaching.

Even (1990) also subscribed to this notion and argued that the way mathematics is taught is important for mathematics educators currently. The emphasis in recent times is to teach in such a manner that learners understand and so that meaningful learning takes place. The role of the teacher is to assist the learner to understand the subject matter. "But in order to do so the teachers themselves need to have a solid knowledge of the subject matter" (Even, 1990, p.521). A teacher that has a solid knowledge of the content that he is teaching will be able to impart this in a more meaningful manner. Subject matter knowledge or content knowledge is only one aspect of the knowledge that a well prepared teacher needs – nevertheless it is an important aspect.

Ensor looked at the possible explanation for the disjuncture between "what individual teachers said about how they thought children learn, and the classroom practices of those same teachers" (1999, p.2). Her explanation emerged from a two year longitudinal study where she researched the relationship between teacher education and classroom practice. Her findings were inconclusive, so the question:

'How does teachers' content knowledge dictate classroom practice?' was still regarded as needing further probing.

Margolinas et al (2005) use case studies in order to deepen their understanding of what teachers learn from the classroom experience. They focused on the teachers' didactic knowledge in relation to the observation in the classroom. This phenomenon is out of the ambit of my research, but what was relevant in their research was their use of the 'usefulness principle' which referred to the 'usefulness' of the teachers knowledge in building knowledge that was more permanent, that which he gained from his observations. They referred to another kind of knowledge that a teacher required and that was observational didactic knowledge. Also an important finding of their research was that teachers lacked didactical knowledge and they felt that there should be further inquiry into this lack of didactic knowledge.

2.4 PERCEPTION

Psychology refers to perception as the act of interpreting a stimulus by one or more of our senses (Sperling, 1967, p.36). The stimulus that I presented to the lecturers were the interview questions. The mechanics for receiving stimuli are similar for different individuals, however, different individuals may interpret the stimuli in different ways. This study intended to investigate how two different individuals interpreted the same stimuli. Our perceptions or interpretations are dependent on the background of the experience, our previous experiences, our feelings at the moment, our prejudices, attitudes, desires and goals. Perception depends on one's previous knowledge and experience. The study therefore probed the inter-relatedness of the lecturers' present experiences, feelings, prejudices and attitudes with their previous experiences.

"Perception is the act of interpreting a stimulus registered in the brain by one or more sense mechanism" (Sperling, 1967, p.36). Ordinarily people are not aware of the processes that determine how they perceive something. People rarely analyse the incoming sensations and the resultant interpretations that occur. It is a human characteristic to respond to a situation in a meaningful way. People generally organize things in their minds to make up a meaningful whole. Perception is thus a unified experience.

Perception, in the context of this study, therefore refers to the lecturers' views and interpretations and how this is shaped by their own experiences and context. These views are arrived at by "certain natural tendencies which might be related to an organizing and grouping function in the brain" (Sperling, 1967, p.38).

2.5 CONCLUSION

This chapter revealed the different types of knowledge that a teacher needs in order to be effective in the classroom. It also showed the relationship between a teachers' knowledge and his classroom practice. It was evident from the various researchers that were presented in this chapter that a teacher requires a broad knowledge base in order to make his classroom practice efficient and effective. The emphasis, however, in this chapter and the study as a whole, is the teachers' content knowledge and how this impacts on his classroom practice.

From the various researchers that have defined the concept of content knowledge it can be summarized that this concept refers to the knowledge pertaining to the subject matter of a particular discipline. It is also evident that an in-depth understanding of the subject matter enhances the delivery in the classroom.

The next chapter will address the methodology employed in the study.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

> "Methodology refers to the coherent group of methods that complement one another and that have the 'goodness of fit' to deliver data and findings that will reflect the research question and suit the research purpose" (Henning, 2004, p.36).

According to Cohen, Manion and Morrison research methods are a

> "range of approaches used in educational research to gather data which are to be used as a basis for inference and interpretation, for explanation and prediction" (2007, p.47).

With these definitions in mind the approach that I used for my research is a qualitative case study as I felt that this approach would best deliver my data, suit my research purpose and reflect my research question.

In this chapter I will also describe the participants involved in the study, the instruments that I used in order to generate data and the data analysis techniques employed to analyse the data and arrive at a conclusion.

3.2 THE CASE STUDY

In any type of dialogue it is effective when one uses a particular instance to illustrate something that is more general. It is easier to engage with your audience when you talk about real people and events instead of discussing theories and ideas that are abstract. People generally understand an idea better if an example is used to illustrate the idea. We are all familiar with specific details, “and the single instance helps us to see how the abstract principles fit together” (Bell et al, 1984, p.72). Using a case to illustrate a particular viewpoint makes it seem more real. Many researchers use individual cases in order to back up statistical and analytical studies and also to assist the reader to understand the conclusions that they arrive at. I therefore decided to use a case study in order to conduct my research as I found this in-depth investigation of a real life situation very rewarding and challenging.

Bromley (as cited in Maree, 2007, p.75) defines case study research as a “systematic inquiry into an event or set of related events which aims to describe and explain the phenomenon of interest.” Yin (as cited in Maree, 2007, p.75) on the other hand describes the case study research as an inquiry that investigates a current phenomenon in a real-life context. Both these definitions encapsulate the essence of my research. My inquiry was systematic and I studied related events. I interviewed the lecturers and then viewed the video-recordings of the lessons that were taught. The video-recordings, since they were actual lesson presentations, depicted the real-life context. The interviews were based on the lecturers’ teaching strategies and also their preparations for the lessons. The lessons themselves were the actual enactment of their strategies and preparation. These are thus related events. The phenomenon of interest in this case is the link between content knowledge and practice. The interviews assisted me to gather data in order to understand the lecturers’ perceptions on content knowledge whilst the video-recordings allowed for observation of their practice. The events of interviewing and observation were thus used to explain and describe the phenomenon that was being investigated.

Yin's description is also pertinent as content knowledge is a phenomenon of current interest. This is evident in chapter two where I described the current literature on this topic. By interviewing the lecturers and observing their lessons the topic of interest was brought into a real-life context.

"A case study is a specific instance that is frequently designed to illustrate a more general principle" (Nisbet & Watt as cited in Cohen et al, 2007, p.253). My research is specific in that I interview two university lecturers and observe specific lessons that they deliver. I also gain their specific perception of the link between content knowledge and practice.

Case studies also look at a context that is dynamic and unique. Therefore the case study investigates and reports on the interaction of events that are complex and dynamic for a particular and unique circumstance. In my research the unique circumstance are the two university lecturers and their interaction with their students and also their perceptions of the phenomenon that is being investigated.

Hitchcock and Hughes (as cited in Cohen et al, 2007, p.253) also point to certain characteristics of a case study which are evident in my research, for example: the case study provides rich and vivid descriptions (this is provided in the interview and the video-recorded lessons); it provides a description of events together with a detailed analysis and it focuses on individuals and gives their perceptions.

3.3 QUALITATIVE RESEARCH

My research is based in the qualitative paradigm. Qualitative research attempts to collect data that is rich and descriptive in order to understand the phenomenon that is being studied or observed. It thus focused on how groups or individuals view the world and how they derived meaning from their personal experiences (Niewenhuis, 2007, p.50).

Qualitative research is typically concerned with exploring the ‘what’ and ‘how’ questions of research. It is concerned with studying people in their natural environment and focusing on their perceptions and interpretations. This exemplifies my research as I studied the lecturers in their natural environment, their teaching of lessons. I also asked for their perceptions of the phenomenon that was being investigated.

In qualitative research “the emphasis is on the quality and depth of information…” (Niewenhuis, 2007, p.50). This therefore justifies the interpretivist paradigm where the individuals that I interrogated and their interaction with the phenomenon that was being investigated was regarded as being paramount.

3.4 PARTICIPANTS IN THE STUDY

The participants in my research were two university lecturers. The lecturers were chosen for two reasons: firstly both the lecturers are involved in the ‘calculus project’ and were therefore convenient and accessible and secondly they were chosen purposively. Access is a key issue in research and it is a factor that must be considered early in the research procedure (Cohen et al, 2007, p.109). The two university lecturers were accessible and that made my research practical to conduct. They work in the same university faculty that I studied in.

In terms of purposively selecting the two lecturers, they were handpicked for the knowledge that they possess in terms of the phenomenon that was under investigation.

My sample was thus built for a specific need. These lecturers are involved in the ‘calculus project’ and therefore have an interest in the phenomenon under investigation.

> “In many cases purposive sampling is used in order to access “knowledgeable people’ i.e. those who have in-depth knowledge about particular issues, maybe by virtue of their professional role, power, access to networks, expertise or experience” (Ball as cited in Cohen et al, 2007, p.115).

This type of sampling does not attempt to be representative and the findings are not generalizable, but the focus is to acquire in-depth information from knowledgeable people. Since the two lecturers are also involved in the ‘calculus project’ they are in a position to supply in-depth information about content knowledge in relation to calculus. The findings can however be transferable and extended to other settings. This will depend on the validity of the findings.

3.4.1 ETHICAL ISSUES

> “A major ethical dilemma is that which requires researchers to strike a balance between the demands placed on them as professional scientists in pursuit of truth, and their subjects’ rights and values potentially threatened by the research” (Cohen et al, 2007, p.51).

In terms of ethical considerations I followed the following procedures as stipulated by the Faculty of Education: the university lecturers and the students that attended their lectures were given letters of informed consent to complete (see appendix A and appendix B). Participation was totally voluntary and their confidentiality, privacy and anonymity were assured. The consent letters included details of the study and data collection procedures. The participants were also assured that if they chose to be part of the study they could withdraw at any time without being prejudiced in any way. Also all video-recordings of the lectures and tape recordings of the interviews will be kept in safe keeping until the study is over and then it will be locked in storage for a period of five years after which it will be destroyed. Before any results were published they were shared with the participants. The participants then had the choice to remain anonymous, so that any references to the institutions could be anonymized so as not to reveal their identities.

The university research office has acknowledged that this investigation has conformed to all the necessary ethical conditions. This acknowledgement appears as the ethical clearance approval certificate number HSS/O758/08M (see appendix C).

3.5 VALIDITY AND RELIABILITY

In qualitative research "validity might be addressed through the honesty, depth, richness and scope of the data achieved, the participants, the extent of the triangulation and the disinterestedness or objectivity of the researcher" (Winter as cited in Cohen et al, 2007, p.133). Validity can be improved through careful sampling, using the appropriate instruments and data analysis techniques. Validity is not something that can be achieved absolutely but it can be maximized.

In order to maximize the validity of my research my samples were carefully chosen. The two university lecturers that I chose were purposively selected for their in-depth knowledge of the topic that was being investigated. They also provided me with honest, well thought out and thorough responses. According to Agar (as cited in Cohen et al, 2007, p.134) the rich data and involvement of the participants secure a sufficient level of validity and reliability.

Also the instruments that I chose for data analysis and generation were carefully chosen and appropriate. The semi-structured interviews allowed me to gain in-depth answers from the lecturers and the observation gave me an opportunity to verify the lecturers' responses.

According to Cohen et al (2007, p.149) reliability can be seen as the correlation between the researcher's recorded data and what actually happens in the natural setting of the research. This I achieved by triangulating the data. After I captured the data from the interviews I was able to verify the correlation between the interview responses and what actually happened in the classroom from the video recordings of the lessons. This ensured the reliability of the data.

3.6 DATA GENERATION INSTRUMENTS

Primary data was generated by conducting semi-structured interviews with the two university lecturers and secondary data was obtained by observing the video-taped lessons. The observations allowed me to gain clarity and also to verify what the lecturers told me in the interviews, in order to arrive at reliable conclusions.

3.6.1 SEMI-STRUCTURED INTERVIEW

I used the interview as one of my data generating instruments. With this instrument you can see the world through the eyes of someone else. It is especially useful in a qualitative study as you can gain rich and descriptive data (Nieuwenhuis, 2007, p.87). In particular the semi-structured interview required the respondent to answer predetermined questions but did also allow for clarification of responses and probing.

With the semi- structured interview rich and detailed data was captured. Also the very nature of my topic led itself to using the interview for data generation. Since I am looking at the link between the lecturers' content knowledge and the activities that are created in their classes, the interview will be a suitable instrument to gauge the lecturers' interpretations and also it will give them an opportunity to express the research situation from their viewpoint. By answering the interview questions the lecturers' shared their perceptions on the link between content knowledge and practice.

"The interview is a flexible tool for data collection," (Cohen, Manion and Morrison, 2007, p. 349). While interviewing, the interviewer can also observe non-verbal signs like body language, reluctance to answer questions and enthusiasm. The interviewer can also probe for complete answers and can also elicit responses to deeper and complex issues. Content knowledge for teaching is a deep issue and therefore probing may be required.

Also I will look at how the lecturers' perceptions of their content knowledge determines the activities that emerge in the classroom context and who better to explain this than the lecturers themselves.

I decided to use open ended questions in my interview which according to Cohen, Manion and Morrison (2007, p.357) helps to establish rapport and also encourages cooperation. This is essential in my research as I am depending on the lecturers to provide the data that I need. Open ended questions also allow for greater flexibility, they allow the interviewer to probe answers and to clear any misunderstandings (Cohen, Manion and Morrison, p.357).

I asked the respondents' permission to audio record the interview. According to Koul (1988, p.175) the recording of an interview allows the interviewer to devote her full attention to the respondent. It is also a time saving device as the interviewer does not have to record the interviewee's responses. It is also a very accurate form of recording responses as it eliminates distortions and omissions and it negates the unconscious or conscious selection of data by the interviewer. According to Koul

> "a good rapport helps the interviewee to feel at ease and express himself willingly. In order to establish a good rapport, the interviewer should greet the interviewee in a friendly manner so as to get settled in the new situation in a relaxed manner" (1988, p. 175).

I followed Koul's advice and then starting asking the questions that I had prepared.

I asked the questions one at a time. If a question needed repeating I did so. While the respondent was answering I listened attentively to the answers and maintained a neutral attitude. Koul also suggests that "the subject's facial expressions, gestures and tone of voice" (1988, p. 175) must be observed. This I also noted. I allowed the respondent sufficient time to answer each question, although being guarded that the interview was not dragging on. Since I was conducting a semi-structured interview, when the occasion presented itself I asked follow up questions and also probed certain responses for clarification and greater detail.

At the conclusion of the interview I thanked the respondents for their time and assured them that after I carefully studied the responses I would then double check with them to see if my interpretation of their responses coincided with what their actual responses were.

Like any other instrument the interview had strengths and weaknesses. The advantages according to Koul (1988, p.176) are as follows:

(a) It gives the interviewer the opportunity to question thoroughly certain areas.

(b) It allows for a greater depth of response which is not possible using any other means of enquiry.

(c) The interviewer can get responses concerning attitudes, values, feelings and emotions in relation to certain topics.

I found my interviews very rewarding. The respondents were very sincere and cooperative. I could clarify issues when the opportunity presented itself and also probe for more meaningful and in depth responses.

There are also weaknesses that are associated with interviews. Some of these as described by Koul (1988, p.176) are:

(a) The interview is quite time consuming.

(b) The effectiveness of the interview depends on the skill of the interviewer.

(c) There is the constant danger of the interviewer being subjective.

Even in the presence of a skilled interviewer some interviewees will not respond frankly, freely and accurately. Since the interview was a part of my studies I was prepared to sacrifice the time on interviewing. In terms of being skilful, I am still a novice researcher and therefore prepared for the interview by reading relevant literature. I was fortunate that my respondents were willing volunteers and as such I hope were frank and accurate. I tend to agree with Kitwood as cited in Cohen, Manion and Morrison “that there is a relatively permanent, consistent, ‘core’ to the personality, about which a person will give information” (2007, p.349). Also since my respondents are part of a larger project, namely the ‘calculus project’ they were willing to participate for the benefit of improvement in educational matters.

According to Cohen, Manion and Morrison (2007, p.350) the interviewer must posses three important attributes namely trust, curiosity and naturalness. Trust is described as a relationship between

> “the interviewer and interviewee that transcended the research, that promoted a bond of friendship, a feeling of togetherness and joint pursuit of a common mission rising above personal egos” (Cohen, Manion and Morrison, 2007, p,350).

Curiosity is described as a burning desire on the part of the interviewer to know and learn other people’s views and perceptions and also to hear their stories. This desire assists the researcher to overcome any difficulties that she may encounter. Naturalness refers to the unobtrusiveness of the interviewer. To blend in with your surroundings in order to hear what others are saying without tainting it with your own views and opinions. I found these three attributes very helpful and aspired to keep them in mind and practice them as far as possible.

My analysis of the interviews commenced with reading all the data and then dividing it into smaller and more meaningful units. These units I then organised into a system that was derived from the data that I generated. The analysis was therefore inductive. I allowed the data to dictate the categories and themes that I would pursue. The themes that emerged from the interviews were:

(a) an academic profile of the two university lecturers

(b) the participants' perceptions of content knowledge

(c) the participants view of their classroom practice

(d) the link between content knowledge and classroom practice

These themes are discussed in detail in chapter four.

In qualitative research the categories are flexible and may be modified during the analysis process. An important aspect of qualitative analysis is that it should reflect the perceptions of the respondents. I kept that in mind during the analysis process.

Using the following interview schedule I conducted the first round of interviews:

Interview schedule 1: (see appendix D)

1. What is it about teaching that you enjoy and why?
2. How would you describe your relationship with your colleagues and what impact does this have on your teaching?
3. Please list your qualifications and teaching experience?
4. What educational courses or training have you taken or received to teach Calculus?
5. Do you feel that your content knowledge of Calculus is adequate for teaching the particular class that you are teaching and why?

6. What other type of knowledge, besides content knowledge, do you need for teaching this topic? Could you please explain.

7. Briefly explain what you do to keep your knowledge of Calculus up to date?

8. In your opinion what are some of the most useful strategies you have learned from any conference, workshop or training on teaching Calculus?

9. How do you decide on what activities to engage your students in?

10. Do the activities that you engage your students in allow for the discovery of skills that they require in calculus?

11. How do you prepare your students to get ready for a lesson you are going to present?

12. Have you received any training either formal or informal to assist you in developing successful relationships with students? If yes, please describe.

13. Was there ever a time in your teaching where you felt that your content knowledge was lacking and how did this affect your teaching?

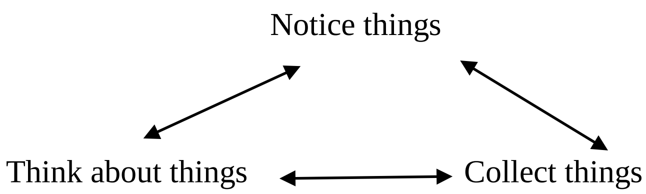
After conducting the first round of interviews and during the process of transcribing the interviews certain themes emerged from the raw data which sparked some ideas about how I should analyse the raw data which did not occur to me prior to the collection of the data. Three broad categories emerged from the data which I decided to pursue. These categories were:

(a) An academic profile of each of the participants.

(b) The participants' perceptions of content knowledge.

(c) The participants' perceptions of the link between content knowledge and practice.

There were, however, some gaps in these categories and I had to conduct a second round of interviews in order to fill in these gaps. “In qualitative studies researchers often find it advisable and necessary to go back…….to the participants to collect additional data…….(Niewenhuis, 2007, p.100). Data collection in qualitative research is thus an iterative process. Siedel as cited in Niewenhuis (2007, p.100) developed a model to explain this iterative approach of qualitative data analysis quite succinctly.



Siedel’s model consists of three essential categories namely: noticing, collecting and reflecting. These categories are intertwined and interlinked and necessary in the qualitative data analysis process. Sometimes while you are analysing and reflecting on the data that you have collected, you notice gaps in the data. You have to therefore go back to collect additional data in order to fill in the gaps. I therefore conducted a second round of interviews based on the following interview schedule: (see appendix E)

Interview schedule 2:

1. How would you define or describe content knowledge?

2. How would you describe the link between your content knowledge of calculus and how this influences your teaching?

3. In general what do you think is the relationship between content knowledge and classroom practice?

I then re-organised and merged both the interview schedules in order to reflect the broad categories that emerged from the transcription process and also to facilitate the analysis process. The re-organised interview schedule is presented as follows:

1. What is it about teaching that you enjoy and why?

2. How would you describe your relationship with your colleagues and what impact does this have on your teaching?

3. What educational courses or training have you taken or received to teach Calculus?

4. Have you received any training either formal or informal to assist you in developing successful relationships with students? If yes, please explain.

5. How would you define or describe content knowledge?

6. What educational courses or training have you taken or received to teach Calculus?

7. Do you feel that your content knowledge of Calculus is adequate for teaching the particular class that you are teaching and why?

8. What other type of knowledge, besides content knowledge, do you need for teaching this topic? Could you please explain.

9. Briefly explain what you do to keep your knowledge of Calculus up to date?

10. In your opinion what are some of the most useful strategies you have learned from any conference, workshop or training on teaching Calculus?

11. Was there ever a time in your teaching where you felt that your content knowledge was lacking and how did this affect your teaching?

12. How do you decide on what activities to engage your students in?

13. Do the activities that you engage your students in allow for the discovery of skills that they require in calculus?

14. How do you prepare your students to get ready for a lesson you are going to present?

15. How would you describe the link between your content knowledge of calculus and how this influences your teaching of Calculus?

16. In general what do you think is the link between content knowledge and practice?

From the participants responses to questions 1-4 (refer to appendices G and H) I drew up an academic profile for each of the participants. Using their responses to questions 5-11 (refer to appendices G and H) I discussed their perception of content knowledge and using their responses to questions 12-14 (refer to appendices G and H) I got an impression of their teaching style and classroom practice. Using their responses to questions 15 and 16 (refer to appendices G and H) I got an impression or their perception of the link between content knowledge and classroom practice.

3.6.2 ACTIVITY SHEETS

The activity sheets were given to the students prior to the lessons. The students worked on the questions in class and thereafter the questions were discussed by the lecturers.

The activity sheets were based on what research in calculus findings suggested texts should satisfy. It uses scaffolding and is student-centred and is based on self-discovery.

3.6.3 VIDEO RECORDINGS

I also observed video-taped lessons of the two university lecturers. The audio-visual department of the UKZN Edgewood campus staff provided the equipment and two final year students video-taped the lessons. Three lessons were taped.

The students were not involved in the class discussion but were only recording the lessons. It must be pointed out that these students had no training in video recording whatsoever. They directed the video recorder to the lecturer that was communicating with the students at that particular time. This was a spontaneous activity on their behalf.

Observation "offers an investigator the opportunity to gather 'live' data from naturally occurring social situations" (Cohen et al, 2007, p.396). The researcher can thus look directly at what is occurring and this allows for more accurate and valid data. This is the unique strength of observation. Two types of video recordings were used in order to record the lessons that were taught. There was one video recorder that was placed in a fixed position in the lecture room and that recorded the whole class interaction. There was another video recorder that was portable and that followed the lecturers as they engaged with students and also as they conducted the lessons.

Observation helps the researcher gain a deeper understanding and greater insight into the phenomenon that is being observed. In my observation of the videotaped lessons I was guided by my research questions. I observed what concepts of calculus the lecturers emphasized in their teaching. I also looked out for the mathematical practices that were constructed and watched closely the dialogue that emerged between the lecturers and the students around the content that was being taught.

I was a non-participant observer. This is the least obtrusive form of participation but the limitation was that I was not immersed in the research situation and may have therefore missed some important information. In order to try an overcome this I viewed the video recordings several times in order to become familiar with the context of the lessons and the dynamics that were occurring in the lecture room.

“In recording your observations you should capture two dimensions: your description of what you observed and your reflection about what happened” (Nieuwenhuis, 2007, p.85). I took note of this when I observed my subjects in their natural context. I observed the activities that the lecturers created in the class and also the dialogue between lecturers and students while they engaged in these activities.

Once I had defined the categories that emerged from the interviews the observation of the videotaped lessons was a matter of looking for details that further illustrated these categories. In my observation I actively sought out evidence to enhance and add to these categories. “The distinctive feature of observation as a research process is that it offers an investigator the opportunity to gather ‘live’ data from naturally occurring social situations” (Cohen et al, 2007, p.396). This allowed me to look directly at what was occurring rather than relying on second-hand information. This provided an opportunity to yield more authentic and valid data. This is the unique strength of observation.

Observation also provides a reality check, since what people say may differ from what they do. Observation also allows the researcher to look at everyday behaviour that may go unnoticed. Observation can focus on facts, events, behaviour and qualities. Since my research is a qualitative case study I decided to focus on all of the above in order to provide a thick description of the lessons that I observed.

“Observational data are sensitive to contexts and demonstrate strong ecological validity” (Cohen et al, 2007, p.396). In this way researchers can see things that might otherwise have been overlooked, can discover things that respondents do not talk freely about and can also access personal details. All of this information was necessary and pertinent to my research. Also since observation is less predictable there is a certain freshness to this type of data.

Observation can be structured where one knows in advance what one is looking for and follows an observation schedule. Semi-structured observation has an agenda of issues but gathers data to illuminate these issues in a far less systematic manner. Unstructured observation on the other hand is not very clear about the agenda and decides after gathering the data, its significance for the research. I opted for semi-structured observation since I had already decided on the categories that I wanted to pursue. However the data was not gathered in a very rigid format as the aim was to provide a rich description of the research situation which could lead to hypotheses generation.

Although my observation was not based on a rigid framework I did decide to use an observation checklist to guide me in my observation. “The intention here is to introduce some systematization into observations in order to increase their reliability” (Cohen et al, 2007, p.407).

I used the following observation checklist as a reminder to be alert and look out for certain pertinent details:

(a) number of students that were present

(b) were students working individually or in groups?

(c) the level of student participation

(d) layout of the lecture room

(e) delivery of lesson by lecturer

(f) resources that were being used during the lecturer

(g) dialogue between students and lecturer

(h) was there feedback from the lecturer?

As I observed the lessons I wrote down a description of what I observed. My description included the physical setting of the lecture room, the events that occurred in the lecture room and the activities that the students and the lecturers engaged in. The descriptions were then categorized into the themes that I had already decided upon from the interviews.

3.7 DATA ANALYSIS

Qualitative data analysis involves "making sense of the data in terms of the participants' definition of the situation, noting patterns, themes, categories and regularities" (Cohen, Manion & Morrison), 2007, p.461). There is no one fixed method to analyse the data but it must fit the purpose of your research. Qualitative data relies heavily on interpretation and there are frequently multiple interpretations that arise from qualitative data.

The approach that I used was to summarize and interpret the findings from the two lecturers. I drew comparisons and generated common themes and patterns. This type of analysis I did for both the interviews and the observation. I also included some direct quotations from the lecturers to illustrate their viewpoints. This is more effective than my translating what they said.

I also reduced the data that I gathered to bring out the relevant themes that are needed for my research topic. I described the rich and detailed data that I generated, I then drew explanations and conclusions from the data and finally I generated hypotheses which answered my research questions.

3.8 CONCLUSION

In this chapter I have presented the different methods that I have used for data collection. For this research which is located in the qualitative paradigm, the main method of data collection was the semi-structured interview. Observation has also supported and added to the data collected. In the next chapter I raise discussion in terms of the critical questions using the data that I obtained.

CHAPTER FOUR

DISCUSSION AND FINDINGS

4.1 INTRODUCTION

Qualitative data

> “tries to establish how participants make meaning of a specific phenomenon by analysing their perceptions, attitudes, understanding, knowledge, values, feelings and experiences in an attempt to approximate their construction of the phenomenon” (Niewenhuis, 2007, p.99).

In qualitative research this is best achieved through inductive analysis of the data where the main purpose is to allow the significant themes to emerge from the raw data itself rather than imposing a more rigid and theoretical framework.

It was therefore decided to use grounded theory and also the immersion style in order to analyse the data that was retrieved, from the interviews that were conducted with the university lecturers and from the observation of their lessons. Grounded theory and the immersion style complement the qualitative paradigm. This chapter addresses the issue of the university lecturers’ perceptions of the link between content knowledge and practice. Using grounded theory their responses in the interviews were coded and categorized. The videotaped lessons that were conducted were also analysed to bring out the relevant themes that were needed for the research topic.

4.2 ANALYSIS OF ACTIVITY SHEETS

Using Zhao and Orey's (as cited in Lipscombe et al, 2008, p.5) six general elements on scaffolded instruction the activity sheets were analysed. Of the six elements the elements of 'sharing a specific goal' and 'whole task approach' were relevant to written work.

(a) Sharing a specific goal- The two university lecturers, who in this case would be the MKO, established the goal and shared this with their students via the activity sheets (refer to appendix F) that were given to them. The activity sheets made it explicit to the students what was expected of them, for example, in the section entitled 'Getting started' the students had to solve the problems and also reflect on their solutions. In the section entitled 'Key Task 1' under the 'Tasks' section in question 1, the students were asked to use sketch graphs in their discussion of the solution to the question posed. In this way the students were fully aware of what was expected of them. The lecturers therefore fulfilled their responsibility to establish the goal of the topic under discussion and shared this with their students.

(b) Whole task approach- The activity sheets made it clear to the students what the ultimate goal was. The activity sheets were designed in a systematic manner. It starts with a section entitled 'Getting started' (refer to appendix F), it then proceeds to a section entitled 'Rates of change' where the concepts of 'average gradient' and 'tangent lines', which are pertinent to the topic of calculus were discussed. The activity sheets then discuss 'Notation' relevant to calculus. Thereafter there are 'Tasks' that the students have to engage in and this is followed by 'Consolidation' and 'Assessment'. The students therefore had a holistic view of the topic that they were engaging with and they could also see how the various components related to the ultimate end result.

4.3 ANALYSIS OF OBSERVATION

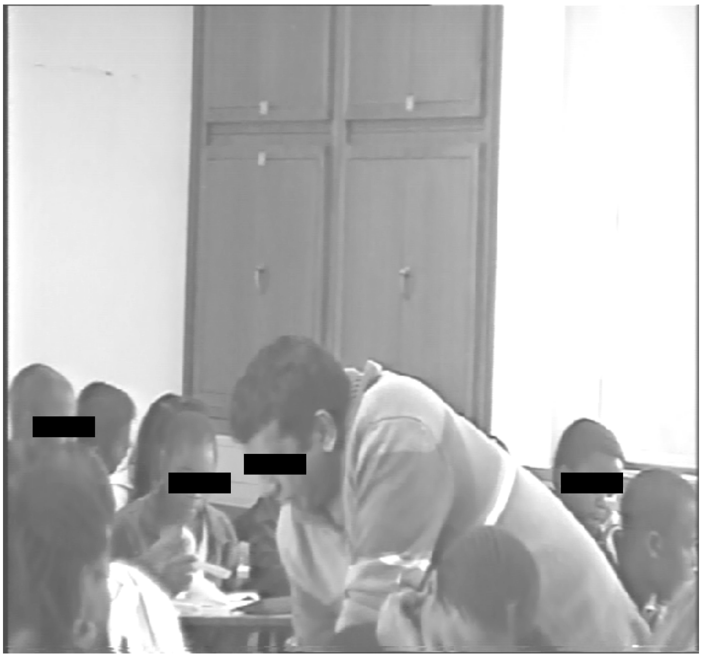
The lecturers were observed lecturing to a class of second year undergraduate students. There were seventy eight students in the class of which fifty three were male and twenty five were female. The course module was: Mathematics for Education 310 which dealt with Differential Calculus. The topic under discussion was ‘rates of change’. The tasks that the students had to engage in appear in appendix F.

Two video-taped lessons conducted by lecturer 1 and lecturer 2 were observed. The first lesson started with the students working on the solutions to the questions that were given to them in the activity sheet. The students were given time in class to work on the solutions.
The students were seated in a lecture room. The desks were all single desks that were arranged in rows facing the front of the lecture room. There were three rows of desks, the row on the left had three desks grouped together, the middle row had two desks grouped together and the row on the right had two desks grouped together. The students were either working on their own or in groups. The groups the students formed on their own by either turning around and working with students behind them or by working with students that were sitting next to them. This is illustrated in the still photograph below that was taken from the video recording of lesson one.

Frame one: the groups that students formed.

As the students worked on their responses both lecturer 1 and lecturer 2 walked around the lecture room to either assist the students or check on what they were doing. Several times both lecturer 1 and lecturer 2 stopped at the desks of students and assisted them with their queries. This can be observed from the still photograph below which was taken from the video recording of lesson one.

Frame two: lecturer assisting students.



In this way students were provided with immediate assistance, this in keeping with Zhao and Orey’s (as cited in Lipscombe et al, 2008, p.5) scaffolded instruction. Students also raised their hands to get the attention of either lecturer 1 or lecturer 2 to answer their queries or questions.

After about twenty minutes lecturer 1 asked for a volunteer to come to the front of the class and work out question one on the board. The students were initially reluctant but after some persuasion and coaxing a student eventually came to the front of the class and attempted question 1 on the board. Lecturer 1 guided him as he was working out the solution. Thereafter another student worked out question 2 on the board.

Lesson two also began in a similar manner, but this time the students were working on the solution to question three. Only lecturer 2 was present at this lecture. Here again students volunteered to work out the subsections of question 3 on the board. Lecturer 2 encouraged the students to come to the front of the class and also assisted and guided them when they were working out the solutions on the board. This is evident in the still photograph below which was taken from the video recording of lesson two.

Frame three: student working on board, assisted by lecturer.

Both the lecturers socially interacted with the students to promote learning. Using language and social interaction the lecturers engaged with their students in order to promote learning. This is in keeping with Vygotsky's learning theory. Vygotsky's learning theory advocates that learning is enhanced through the social interaction between the student and a teacher. Vygotsky views the teacher as the MKO who is able to lift the student's achievement level. The lecturers also lifted the performance of the students that they were supervising by providing immediate assistance, this in keeping with the scaffolding approach. The ‘immediate availability of help’ is one of the six general elements of the scaffolding approach which is discussed in detail in the literature review.

4.4 ACADEMIC PROFILE OF PARTICIPANTS

In keeping with the interpretivist paradigm, which focuses on the individual, providing an academic profile of each of the participants would be relevant and interesting. Also since this is a case study this type of information would give more credibility to the case. It also personalised the case.

Also since the focus is on the lecturers’ perceptions, “which is a result of the interplays between past experiences, one’s culture and the interpretation of the perceived” (Wikepedia, 2009, p.1) a profile of the lecturers would provide useful information about their experiences and viewpoints.

4.4.1 PROFILE OF LECTURER 1

Lecturer 1 is a mathematics lecturer at a tertiary institution in South Africa. He has been teaching for twenty four years. His teaching experience includes teaching at a high school, a teacher training institution and a university. His academic qualifications include a Bachelor of Science degree, a Bachelor of Science Honours degree, a Bachelor of Education Honours degree, a Masters of Science degree and a Doctorate in Pure Mathematics. Lecturer 1 is thus an experienced teacher who is highly qualified.

Lecturer 1 enjoys teaching, he enjoys teaching the mathematical content and he also enjoys *putting ideas across to others and seeing their face light up when they actually understand what I'm getting across to them*

This statement reflected a passion for teaching and also a commitment to imparting knowledge to his students.

Lecturer 1 also engaged in dialogue with his colleagues so as to improve and adjust his teaching to suit the calibre of students that he is exposed to and also to gain insight and knowledge about his students.

But even on the informal level when we chat in corridors and at meetings we actually speak about students and what transpires in our lecturers and that does have an impact on our teaching, in the sense that we get to know our students more. If the student has been dealt with by lecturers in the past, so we get some past history of the students. At the same time it also helps us to actually check our teaching in the sense that we know the calibre of the students that we are dealing with so we know how to adjust our teaching in that respect.

Although lecturer 1 had not received any formal training on developing successful relationships with his students, he had actually developed this in an informal manner from his interactions with students over the years.

The actual relationships that I encountered with students transpired during my years of teaching, that was developed incidentally

From my observation I also noticed that the students reacted in a positive manner to lecturer 1. If they have any queries or problems they were comfortable enough to ask him for assistance. The students also seemed quite relaxed in the classroom and were allowed to work at their own pace without much pressure. This also displayed the rapport that lecturer 1 shared with his students.

4.4.2 PROFILE OF LECTURER 2

Lecturer 2 is also a mathematics lecturer at a South African institution. She is an experienced lecturer who has been teaching for fifteen years. Her qualifications include a Masters degree in Mathematics and a Doctorate degree in Mathematics Education.

She enjoyed interacting with students and also believed that her teaching allowed her to learn more.

because as you teach you learn more

This enjoyment of pupil interaction was also quite evident in my observation of lecturer 2 in the classroom context. She constantly interacted with her students by either asking questions or assisting them in their work.

She also believed that her interaction with her colleagues allowed her to broaden her academic horizons.

I enjoy working with them because of the opportunities to learn, because different people have different perspectives and different strengths. And sometimes you can even look at one problem differently and you learn from the other persons approach, you learn a different method.

Lecturer 2 is obviously very enthusiastic about learning and developing her skills. Also what was apparent from the above statement is her openness to new and different ideas. She is prepared to learn from other people. Also what came across very strongly in the interview was her enthusiasm, passion and commitment to teaching. Apart from her actual responses to the questions, her body language and tone of voice portrayed her passion for teaching.

Although lecturer 2 had not received any formal training on developing successful relationships with her students she believed that successful relationships depended on your commitment to the teaching profession.

In the academic field as well developing successful relationships depends on your own need, your sense of fulfilment, whether you get a sense of fulfilment from developing relationships with students or whether you are more comfortable doing research, or whether you are more comfortable working somewhere else. It's about your commitment to your job.

I also got the impression that lecturer 1 seemed to be disappointed with some of her colleagues lack of commitment to their profession.

…you see it all the time, you can have ten people in a department and you can see how different people interact at different levels, different people are prepared to go to different lengths to help students and others are not prepared to go even if you send them for training they'll do the bare minimum.

This statement also demonstrated the type of personality that lecturer 1 has, she is quite emotional and involved in her profession. She is not apathetic but is rather concerned about the education profession as it is currently. This was also evident in the following comment that she made:

…it's the same thing, like with teaching, it's a passion. Are you passionate about teaching? It's the problem with our education system as you know because we have too many teachers who are not passionate about teaching.

4.5 PARTICIPANTS' PERCEPTIONS OF CONTENT KNOWLEDGE

Lecturer 1 defined content knowledge as:

To me content knowledge is knowledge that is pertinent to a particular topic you are teaching, in other words it does not entail didactic aspects of knowledge. I do not integrate it with pedagogics. It is dealing with a particular topic and the mathematics around it.

Lecturer 2 also expressed similar sentiments:

Content knowledge…knowing the mathematics. The content is about the content, how well you know the content, how it fits in with other topics…about the concepts, having an understanding of how it works, when it works, knowing interrelationships within the content.

I think what came across very strongly is that content knowledge refers to the actual mathematics that you are required to teach. These sentiments are echoed by Kilpatrick et al who believe that content "includes knowledge of mathematical facts, concepts, procedures, and the relationships among them; knowledge of the ways that mathematical ideas can be represented; and knowledge of mathematics as a discipline" (2001, p.371). These lecturers responses also agree to Chinnapen's (2003, p.1) view to content knowledge when he stated that content knowledge refers to the knowing about a subject, the disciplinary knowledge of a subject.

When content knowledge is looked at in quantitative terms then both the lecturers are adequately qualified to teach the course. When asked: "What educational courses or training have you taken or received to teach Calculus?" Lecturer 1 explained:

For this topic is part of the first year, university undergraduate at the B.Sc. level. You do calculus, a whole course in calculus, and then later on you actually study advanced topics in calculus at a higher level like in Measure Theory and Real Analysis.

Lecturer 2 on the other hand refers to the methodology of teaching and comments:

I'm sure we must have done methods in teaching maths at university.

Lecturer 1 also believed that his content knowledge was adequate to teach the particular course since his content knowledge of calculus far exceeded that which he had to teach since Measure Theory is a study of various types of integrals other than those dealing with Riemann Sums. This idea is supported by Long (2003) and Hilton (as cited in Long, 2003) who argued that it was beneficial and advantageous for the teacher to know content that extended beyond the curriculum in order to answer pupils queries. Lecturer 1 had studied calculus up to the honours level and this was reflected by the following comment:

I would say yes because, as I mentioned, the depth of the calculus course. Calculus is not just plain first year differential and integral calculus one sees in textbooks at the classical course given to first year students. In fact calculus has been studied much deeper. As I mentioned if you look at it from an analytical point of view in where you do real analysis in second year and third year courses, where you really have to look at calculus at an advanced level.

This response alludes to the depth of subject matter knowledge that lecturer 1 has on this particular topic. This depth of knowledge contributes to the high level of conceptual thinking experienced by students. The need for in-depth content knowledge "for teaching is of primary importance, for without this, teachers would not be able to engage their learners in high-level conceptual thinking" (Adler et al, p.136).

Lecturer 2 on the other hand did not specialize in calculus, but nevertheless she also believed that her content knowledge was adequate to teach the course.
That class that I was teaching calculus I just went in for a few weeks as part of the project. I didn't have any hesitation in managing because I knew I would know the calculus, except that

I haven't taught it for a while. No I don't have any problems in maths with content knowledge.

Also from the observation of their lessons it was quite evident that both lecturer 1 and lecturer 2 were quite comfortable to teach calculus to their classes. They could answer student's queries and questions adequately. They were also able to guide students' thinking to bring them to the correct answer. To demonstrate this reference is made to the activity sheet 'Key Task 1: Tasks: number 1 (see appendix F). The exchange between lecturer 1 and a student exemplifies this point:

Lecturer 1: *Oh, right, so what is your answer then to this question?* (referring to question 1). *Would you say that the statement is always true, never true or sometimes true? So you say it's always true. So what do you mean by always true? What is always true?*

Student: *We said it's always true according to this function.*

Lecturer 1: *Very good. He says that according to this function it's always true.*

Student: *Ya.*

Lecturer 1: *So, can there be another graph? There can be another graph.*

Student: *Yes.*

Lecturer 1: *For which this will be true?*

Student: *No it's not true.*

Lecturer 1: *So it might not be true. So what will be our choice among our three options?*

In this way the student arrived at the correct answer. This exchange is also a good example of the scaffolding process. There was an immediate availability of help to the student from lecturer 1. Lecturer 1 also redirected the student's thought processes to bring him to the correct solution.

In order to keep his knowledge of calculus current and updated Lecturer 1 attends conferences, presents research papers, analyses students' work and also reads current literature on the topic.

Just to highlight, here's one copy of a paper I just printed by another academic (points to a research paper on his desk)*, and I will read this paper obviously and see how this topic is being taught internationally and what successes they have gained so I can implement similar strategies in my teaching.*

Lecturer 2 on the other hand had to prepare for the calculus classes as she had not taught this topic recently.

When I was asked to teach that course I looked at three textbooks, I went over them properly. I looked at Dr. X's notes, I worked out every possible question before I taught.

4.6 CLASSROOM PRACTICE

"The type of classroom climate generally considered to best facilitate pupil learning is one that is described as being purposeful, task-orientated, relaxed, warm, supportive and has a sense of order" (Kyriacou, 1991, p.65). This was evident from my observation of the lessons and also the activity sheets that the students were provided with. There was an atmosphere of purpose that pervaded the lecture room. The activity sheets provided the questions that the students were purposefully engaged with. The students were actively working on the solutions to the problems that they were given. The activity sheets also orientated the students to the tasks at hand. The students were also relaxed as they worked in groups that they had formed on their own, yet order prevailed. Both lecturer 1 and lecturer 2 have warm personalities and interacted with the students in a congenial manner. Students were encouraged and assisted where necessary.

Lecturer 1 generally introduces a new topic with a problem or mathematical task.

At the commencement of the lecture I always present them maybe a small mathematical task that they have not done before but they have some idea about how to get along, but they probably would not solve it. But the whole intention would be to say at the end of the lecture is that they can now solve the problem. I don't know if I'm clear. Ya, a simple example I could give you, for example in the grade eleven class, the child can solve the trinomial using the factor method. So after you do the factor method you will probably throw one that does not factorize, so that at the end of the lesson he will learn how to use the quadratic formula, so at the end of the lesson he has now learned a new technique.

From the video recording, I observed lecturer 1 demonstrate this. One group of students, when working with question 1 from “getting started” (see appendix F) indicated that they had failed to solve the problem. Lecturer 1 then asked them to explain geometrically what the derivative meant. After a discussion with the students he then asked them to evaluate f(-3) and told them that they should now be able to apply themselves and solve the task.

This relates to Shulman’s ideas on classroom practice and he states that “teaching necessarily begins with a teacher’s understanding of what is to be learned and how it is to be taught” (Shulman, 1987, p.7). Shulman believes that teachers know something that is not understood by others and that they can transform their understanding

> “into pedagogical representations and actions. These are ways of talking, showing, enacting, or otherwise representing ideas so that the unknowing can come to know, those without understanding can comprehend and discern, and the unskilled can become adept” (Ibid, p.7).

Lecturer 2 would either use a problem to start a new topic or the students would be expected to read their notes, which are given to them before the lesson, in order to prepare for the lesson.

Well if possible I give them a problem to work out beforehand otherwise they always have notes beforehand, they have a breakdown of what’s going to happen. When the maths is quite complicated it really helps if they read beforehand so it helps if they have some sort of idea of some of the new terms.

This was evident in my observation where both the lessons began with the students working out problems that they were given in advance to prepare for the lessons.

The activities that lecturer 1 engaged his students in would depend on the topic that he was teaching. In this case since the topic under discussion was ‘calculus for teaching’ the activities that the students were engaged in are reflected in the activity sheets (refer to appendix F). These activities included questions on gradients, derivatives and tangents, all of which are relevant to the teaching of calculus.

It would all depend on the situation. On what I wanted to teach within the topic. For instance if I wanted to teach, for example, the derivative concept via first principles, the student has got no notion of the definition at that stage. So from the average gradient I would now lead on to speaking about the approach of one point to another in the classical way where you arrive at the gradient of the tangent at a particular instance on the graph. So what I’m saying is the particular, the concept that I put across determines the activities that I design so as to gain students’ understanding of the concept.

Lecturer 2 is approaches her teaching in the following manner:

I will, firstly if I am teaching a course I will look at what I’m supposed to teach. I design the course outlines if I’m teaching it for the first time. Then I go through everything. I work through every possible problem in that text book. Then I’ll go find other books and look at how they approach it.

Lecturer 1 allows students to discover skills and also lectures in the classical style depending on the context of the learning situation.

I would say yes, both skills and conceptual understanding…and skills yes. Many of these skills are not self discovered, they are taught to students in a lecture style.

This was observed in the video recording. This lecturer always provided clues that could (in his mind) assist students in succeeding in the task. He did not provide them with answers.

Lecturer 2 on the other hand

If I need them to understand something I try to find a motivating question. It could be a maths question, it doesn't have to be a concrete activity, that will make them think about the need for what I'm going to introduce. Or I present them this whole big *idea to them and show them how this little thing fits*

4.7 THE LINK BETWEEN CONTENT KNOWLEDGE AND CLASSROOM PRACTICE

In terms of the link between content knowledge of calculus and classroom practice lecturer 1 believed that there was definitely a strong link between content knowledge and classroom practice. He elaborated:

To me I feel there is a very strong link between my content knowledge and the way I teach. I am able to emphasize on particular aspects of the content

Kilpatrick et al (2001, p.372) also argued that the teachers' content knowledge is important for effective teaching. They argued that the teachers' content knowledge is important in the development of the students' proficiency and ability in mathematics.

Lecturer 2 also supported this notion:

A deeper knowledge of calculus… affects how you teach because the deeper your knowledge is, you have a bigger repertoire of examples to draw upon and you can readily come up with counter examples in order to help learners to see conditions when theorems hold and or conditions when rules hold. You are able to, without any problem, come up with relevant examples.

These sentiments expressed by lecturer 2 are supported by Ball and Bass who argued that " knowing content is also crucial to being inventive in creating worthwhile opportunities for learning that takes learners' experiences, interests, and needs into account" (2000, p.86). Even also endorsed these views when she stated that "acquiring the basic repertoire gives insights into and a deeper understanding of general and more complicated knowledge" (1990, p.525).

In general both lecturer 1 and lecturer 2 saw a link between content knowledge and classroom practice. Lecturer 1 believed:

Certainly there is…my classroom practice, the approach that I use is one that can forsee solutions based on the content knowledge, in other words, the classroom practice, the *approach that I use is dictated by content knowledge. I have a whole global picture of where I'm going.*

Lecturer 2 added a further dimension:

I don't think you got a one to one relationship, but definitely a deeper content knowledge results in better classroom practice…it is necessary. To have good classroom practice it is necessary to have good content knowledge but not sufficient.

In terms of her classroom practice lecturer 2 was able to respond to students' queries on several occasions. The one instance was when she explained the link between the gradient of f(x) with f (x) = g(x) in question 2 of getting started (see appendix F).

4.8 CONCLUSION

Content knowledge for teaching and the link between content knowledge and practice is a topic that has been researched for many years, both nationally and internationally. In the course of research done on teaching the emphasis has shifted from pedagogy to content knowledge.

What was clear from the findings was that both the lecturers viewed content knowledge as the knowledge that pertains to the understanding of a particular concept or topic in mathematics. This knowledge involves more than knowing the mathematics, it also involves knowing the relationship between various topics and where a particular topic fits into the bigger picture.

The two university lecturers definitely saw a positive correlation between content knowledge and classroom practice. A thorough content knowledge enhanced their classroom practice. It allowed them to assist students by providing clues rather than merely supplying them with answers.

CHAPTER FIVE

CONCLUSION

5.1 INTRODUCTION

Content knowledge is one component of the knowledge that a teacher requires in order to deliver in the classroom context. It is however a very important and crucial component. There has been a lot of research done on this topic, and there is still research being done on this topic currently. Content knowledge is therefore a dynamic field of research. This is echoed by Adler et al who believe that “a great deal of work lies ahead in tackling challenges related to the nature and place of subject knowledge in teacher education” (2002, p.136).

This small-scale study involved two university lecturers and investigated their perception of content knowledge in the context of teaching calculus to an undergraduate second year class. This chapter presents the summary of the findings in the context of the three critical questions raised in chapter one. It also identifies the strengths and limitations of this research and provides issues for further consideration and exploration by other researchers and policy makers.

5.2 STRENGTHS AND LIMITATIONS

Although I had read extensively on how to conduct interviews and went into the interview situation with the best intentions I felt that my inexperience made it difficult for me to seize opportunities for probing of responses when they presented themselves. It was only after I had read the transcript of the interviews that I saw the opportunities that I had missed. In order to compensate for this I did a follow up interview where I asked further questions to fill in the gaps.

It was also difficult to coordinate the times for the interviews. Since there were two lecturers involved I had to find a time that would be suitable and convenient for both the lecturers, and also a time that would be suitable for me since I also had work commitments. However what I found very rewarding was the dialogue that emerged between the respondents and me. The research topic was something that was very relevant and pertinent to me and to the lecturers as we are involved in education. Also the fact that the lecturers were also involved in the 'calculus project' and were willing volunteers they had a genuine interest in the research that was being done. This made my task much easier as they were always cooperative and helpful.

Since I was only conducting a small scale study I only observed two video-taped lessons. I would have also preferred to have been present at the lessons and witnessed them for myself. However due to circumstances, like my own work commitments, this was not possible. I did, however watch the video-recordings several times in order to try and immerse myself in the teaching and learning environment.

The fact that I was conducting a case study was beneficial to my investigation as it very effectively portrayed the lecturers' perceptions. I also had personal contact with the lecturers via the interview process and could thus gauge their body language and facial expressions which added to the thick data which was necessary for the interpretivist approach that I employed.

5.3 IMPLICATIONS OF THE RESEARCH

While researching and reading information on my research topic, there were other topics and subjects that I encountered, which did not pertain directly to my research topic, but which I felt could be pursued further. Time and the length of this research did not permit me to follow these avenues. Some of these topics were:

(a) teachers' attitudes towards mathematics- Clemens (as cited in Kennedy, 1997, p.11) argued that even if the mathematics teacher had an acceptable understanding of the nature of knowledge required in his subject, he still needed to have an acceptable attitude towards his subject. This attitude should be demonstrated by an inquiring mind, an openness to new ideas and concepts and a scepticism that characterises mathematics.

(b) the link between pedagogical knowledge, conceptual knowledge and pedagogical content knowledge and classroom practice- In my research I did look at these other types of knowledge that a teacher requires, but my focus was on content knowledge. There is a lot of information available on the other types of knowledge that a teacher requires for effective classroom practice, however time constraints did not allow me to pursue these other avenues.

The findings of this study has shown that the depth of content knowledge determine the strength of the activities designed for teaching. Kazima and Adler (2006) state that mathematical knowledge for teaching involves the restructuring of knowledge to make it accessible to learners. Much more research is therefore needed on a range of classroom contexts. Also researchers have not yet reached consensus on what exactly comprises mathematical knowledge for teaching and this is therefore something that could be pursued further.

5.4 SUMMARY

In conclusion I revert back to the research questions that were raised in chapter one:

What were the lecturers' perceptions of content knowledge?

Content knowledge was viewed by both lecturers as knowledge that pertained to Mathematics as a subject. Lecturer 1 elaborated:

It is dealing with a particular topic and the mathematics around it.

Lecturer 2 echoed these sentiments:

Content knowledge…knowing the mathematics.

In this instance content knowledge would refer to the content that the lecturers were required to teach in the calculus module.

Content knowledge also refers to the inter-relationships between concepts in a particular topic, and also across various topics. Shulman (1986, p.9) encapsulated this effectively when he stated that "to think properly about content knowledge requires going beyond knowledge of the facts or concepts of a domain. It requires understanding the structures of the subject matter."

Did the lecturers engage their students in dialogue around the activities that occurred in the class and how did the nature of the dialogue influence the lecturers' classroom practice? Both the lecturers engaged their students in meaningful dialogue. The dialogue centred on the classroom activities that the students had to participate in. There was also dialogue in the form of questions and answers. Both the lecturers and the students asked and answered questions.

Due to the meaningful dialogue that occurred in the classroom there was an interactive style of classroom delivery.

How did the lecturers' view the link between content knowledge and classroom practice?

The pedagogical decisions that teachers make, for example the "questions they ask, activities they suggest- are based on their subject matter knowledge" (Even, 1990, p.524). Both lecturer 1 and lecturer 2 agreed with Even's views on the link between content knowledge and classroom practice.

Lecturer 1 said:

There is a very strong link between my content knowledge and the way I teach. I am able to emphasize on particular aspects of the content.

…my classroom practice, the approach that I use is one that can forsee solutions based on the content knowledge

Lecturer 2 added:

A deeper knowledge of calculus… affects how you teach

Turnuklu and Yesildere (2007, p.12) extended this notion and they argued that "a deep understanding of mathematics is necessary but not sufficient to teach mathematics. Additionally, it is not possible to teach mathematics without having mathematical knowledge as well. Mathematics teachers must be educated both from 'mathematics knowledge' and 'pedagogical content knowledge' aspects in universities." Shulman also supported this idea and he stated that “content and pedagogy were part of one indistinguishable body of understanding” (1986, p.6).

Lecturer 2 summed up this idea eloquently:

I don’t think you got a one to one relationship, but definitely a deeper content knowledge results in better classroom practice…it is necessary. To have good classroom practice it is necessary to have good content knowledge but not sufficient.

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APPENDICES

APPENDIX A

UNIVERSITY OF
KWAZULU-NATAL

Associate Prof Iben M.Christiansen, PhD.
School of Education and Development
Faculty of Education
Private Bag X01

Scottsville 3209

Tel: 033-2606092

Fax: 033-2605080

Dear [name of lecturer/tutor]

Consent Letter for the project:
Recontextualisation of a material for teaching calculus to prospective teachers

You have been approached to take part in the above project. In this letter, I, the project leader, will describe to you the aims of the project, explain what is required from you if you agree to participate in the project, explain the benefits you may derive from participating in the study, and explain how we will protect you in the project.

The project is developed around investigating what happens when a material developed specifically for the purpose of teaching calculus to pre-service teachers is being used. We are interested in the interplay between lecturer/tutor and material in shaping the mathematical practices of the classroom, including the types of activity that are recognised as relevant and what counts as mathematics for teachers. The project uses classes at University of KwaZulu-Natal and Nelson Mandela Metropolitan University.

We intend to investigate this through observing the lectures and afterwards interviewing you about the decisions you make as a tutor/lecturer. We are interested in the reasons you have for making educational choices in your lecturing/tutoring. You were identified as a participant in this project because you will be lecturing the relevant course at one of the participating institutions.

The project is headed by Associate Professor Iben Maj Christiansen, University of KwaZulu-Natal, with the participation of Professor Hugh Glover, Nelson Mandela Metropolitan University and Dr Deonarain Brijlall, University of KwaZulu-Natal. If you have further questions about the project, you can direct it to any of these participants. Professor Christiansen's contact details are stated in the letterhead. A number of post-graduate students will also be associated with the project.

You will of course have to teach the module as you have intended to do, allow us to video record it, and collect copies of any handouts, assignments and tests. In addition, the project requires of you that you are willing to let us interview you at least 10 times throughout the semester. These interviews can be expected to last 30-50 minutes per time. We may also be conducting interviews with students and collect copies of their work.

We are sure you will benefit from participating in this project, as the interviews will encourage you to reflect on your own educational choices, and thus become more aware of your own teaching.

We will keep all video recordings of lectures/tutorials and tape recordings of interviews in safe keeping, until the project has been completed, upon which they will be transferred to locked storage at University of KwaZulu-Natal for a five year period.

Before any results are published from the project, they will be shared with you. At that time, you can indicate if you are willing to let us use your details or want us to make you anonymous. Should you choose the latter, we will make sure that any reference to the institution will also be anonymized, so that it is not possible to deduce your identity from the information.

Thus, we assure you full confidentiality and anonymity.

The participation in the study is voluntary. If you choose not to participate in the study, it will not result in any form of disadvantage. If you choose to participate in the study, you are free to withdraw at any stage and for any reason.

I ……………………………………………………………… (full name of participant) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project.

I understand that I am at liberty to withdraw from the project at any time, should I so desire.

………………………………………………………………………… ………………………...

(Signature of participant) (Date)

UNIVERSITY OF
KWAZULU-NATAL

Associate Prof Iben M.Christiansen, PhD.
School of Education and Development
Faculty of Education
Private Bag X01
Scottsville 3209
Tel: 033-2606092
Fax: 033-2605080

Dear student

Consent Letter for the project:
Mathematical thinking embedded in the construction of a bubble machine

You have been approached to take part in the above project. In this letter, I, the project leader, will describe to you the aims of the project, explain what is required from you if you agree to participate in the project, and explain how we will protect you in the project.

The project is developed around investigating what happens when a material developed specifically for the purpose of teaching calculus to pre-service teachers is being used. We are interested in the interplay between lecturer/tutor, students and material in shaping what happens in the classroom, including the types of activity that are recognised as relevant and what counts as mathematics for teachers. The project observes classes at University of KwaZulu-Natal and Nelson Mandela Metropolitan University.

We intend to investigate this through observing the lectures, interviewing the tutors/lecturers, collecting materials used by the lecturers/tutors and possibly collecting examples of students' test papers, assignments as well as interviewing students. As you will be attending the relevant course at one of the participating institutions, we need your consent to collect the data.

The project is headed by Associate Professor Iben Maj Christiansen, University of KwaZulu-Natal, with the participation of Professor Hugh Glover, Nelson Mandela

Metropolitan University and Dr Deonarain Brijlall, University of KwaZulu-Natal. If you have further questions about the project, you can direct it to any of these participants. Professor Christiansen's contact details are stated in the letterhead. A number of post-graduate students will also be associated with the project.

You will simply have to attend the module as normal, allow us to video record you, and collect copies of your work. In addition, we may ask to interview you 3-4 times throughout the semester. These interviews can be expected to last 30 minutes per time.

We will keep all video recordings of lectures/tutorials, tape recordings of interviews and copies of materials and student work in safe keeping until the project has been completed, upon which they will be transferred to locked storage at University of KwaZulu-Natal for a five year period.

Before any results from the project are published, we will anonymise all references to students. Thus, we assure you full confidentiality and anonymity.

The participation in the study is voluntary. If you choose not to participate in the study, it will not result in any form of disadvantage. If you choose to participate in the study, or parts thereof, you are free to withdraw at any stage and for any reason.

I ……………………………………………………………… (full name of participant) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to allow the researchers to include me in their video recordings.

I understand that I am at liberty to withdraw from the project at any time, should I so desire.

? I agree
? I do not agree to having the researchers obtain copies of my work upon request.

? I agree
? I do not agree to being interviewed 3-4 times throughout the semester at times convenient to me.

……………………………………………………………………… ………………………...
(Signature of participant) (Date)

UNIVERSITY OF
KWAZULU-NATAL

RESEARCH OFFICE (GOVAN MBEKI CENTRE)
WESTVILLE CAMPUS
TELEPHONE NO.: 031 – 2603587
EMAIL : ximbap@ukzn.ac.za

28 NOVEMBER 2008

MS. V ISAAC (8421455)
MATHEMATICS, SCIENCE, COMPUTER AND TECHONOLOGY EDUC

Dear Ms. Isaac

ETHICAL CLEARANCE APPROVAL NUMBER: HSS/0758/08M

I wish to confirm that ethical clearance has been approved for the following project:

"Case study of two university lecturers and their perception of the link between content knowledge and practice"

PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years

Yours faithfully

PXimba
....
MS. PHUMELELE XIMBA

cc. Supervisor (Dr. D Brijlall)
cc. Mr. D Buchler

RECEIVED
2008 -12- 09
FAC RESEARCH OFFICE

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

APPENDIX D

Interview schedule 1

1. What is it about teaching that you enjoy and why?

2. How would you describe your relationship with your colleagues and what impact does this have on your teaching?

3. Please list your qualifications and teaching experience?

4. What educational courses or training have you taken or received to teach Calculus?

5. Do you feel that your content knowledge of Calculus is adequate for teaching the particular class that you are teaching and why?

6. What other type of knowledge, besides content knowledge, do you need for teaching this topic? Could you please explain.

7. Briefly explain what you do to keep your knowledge of Calculus up to date?

8. In your opinion what are some of the most useful strategies you have learned from any conference, workshop or training on teaching Calculus?

9. How do you decide on what activities to engage your students in?

10. Do the activities that you engage your students in allow for the discovery of skills that they require in calculus?

11. How do you prepare your students to get ready for a lesson you are going to present?

12. Have you received any training either formal or informal to assist you in developing successful relationships with students? If yes, please describe.

13. Was there ever a time in your teaching where you felt that your content knowledge was lacking and how did this affect your teaching?

APPENDIX E

Interview schedule 2

1. How would you define or describe content knowledge?

2. How would you describe the link between your content knowledge of calculus and how this influences your teaching?

3. In general what do you think is the relationship between content knowledge and classroom practice?

Name of student:………………………………….

Mathematics in Education C: Calculus for Teaching
Getting started

You may work in pairs or independently on the following examples. For each problem reflect on your attempt at solving the problem after you present your solution to the problem.

Question 1

Given $f(x) = 2x^2 + x - 1$, calculate the equation of the tangent to the curve of f where the gradient equals -3.

Solution:

Reflection

Question 2

The line $g(x) = 5x + 1$ is a tangent to the curve of a function f at the point where x = 2.

Calculate the value of $f(2) + f'(2)$

Solution:

Reflection:

Question 3

You know the following from school:

If $f(x) = x^2$, then $f'(x) = 2x$.

A Grade 12 learner asks you:

If $f(x) = x^3$, then is $f'(x) = 3x$?

What will you say to this learner? How will you convince her of your ideas?

Question 4

Determine the following: $\lim_{x \to -2} \log(x+2)+1$

Solution:

Reflection:

Mathematics in Education C: Calculus for Teaching
Getting started II

Question 5

You have looked at the following question:

> The line $g(x) = 5x + 1$ is a tangent to the curve of a function f at the point where x = 2. Calculate the value of $f(2) + f'(2)$

What if the question said: "Calculate the value of $f(3) + f'(3)$"?

Question 6

What does the statement $f'(2) = -3$ tell us about the graph of $f(x)$?

What does the statement $f'(x) = -3$ tell us about the graph of $f(x)$?

Question 7

You are teaching a Grade 12 class and you write the following on the board:

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} (x - 1) = -3$$

A learner says he doesn't understand what you have written. How will you help him to understand the meaning of what you have written?

Question 8

Do you agree with the following statements? Justify your answer.

1. "The derivative is the same as the tangent line"
2. "The derivative is the same as average gradient"

After the discussion, you will hand in your answers for assessment. Over the next few weeks you will have a chance to improve your answers as your understanding develops.

1. You know from school that if $f(x) = x^2$, $f'(x) = 2x$. Explain fully why this is so (without just stating the differentiation rule).
2. Do you think that $\frac{d}{dx}2^x = x2^{x-1}$? How could you prove if this is right or wrong?
3. What does the statement $f'(2) = -3$ tell us about the graph of $f(x)$?
4. The line g(x) = 4x – 3 is a tangent to the curve of f(x) at x = 2.
   a) Find f(2).
   b) Must f(x) be a parabola? What can you say with certainty about f(x)?
   c) What is $f'(2)$? Explain your thinking.
   d) Can you find $f'(4)$? If not, explain why not and state what other information you would need.
5. Write down a formula for finding the derivative of a function. Explain each part of the formula.
6. What is the relationship between the derivative and a tangent line?
7. What is the relationship between the derivative and average gradient?

Do $f'(a)$ and $f'(x)$ mean the same thing? Explain.

Purpose

- revise the difference and ratio aspects of gradient
- revise $f'(a)$ notation as the gradient of a curve at the point (a, f(a))

This is to prepare for Task 2, which will define the derivative as the limit of average gradients.

Assumed knowledge and notation

- the average gradient between two points is given by $\dfrac{\Delta y}{\Delta x}$
- gradient is an average rate of change
- a secant line joins two points on a curve.
- a tangent touches a curve locally at one point

Targeted knowledge

- The gradient of a tangent to a curve is the same as the gradient of the curve at that point.
- $f'(a)$ is the gradient of the curve $y = f(x)$ at the point $(a, f(a))$.

Background information for discussion

For the tutor

Revise the information below about tangent and secant lines. (You might want to mention that the term secant comes from the Latin "secare: to cut" and does not relate to the trigonometric function.) Ask students if it is possible to find the gradient of a given tangent line from one point only (without calculus). They should agree that we can only approximate the value. Also revise the meaning of the notation $f'(a)$ as the gradient of the curve and the tangent to the curve at a point. Encourage students to read $f'(a)$ as "the gradient of the graph at $x = a$".

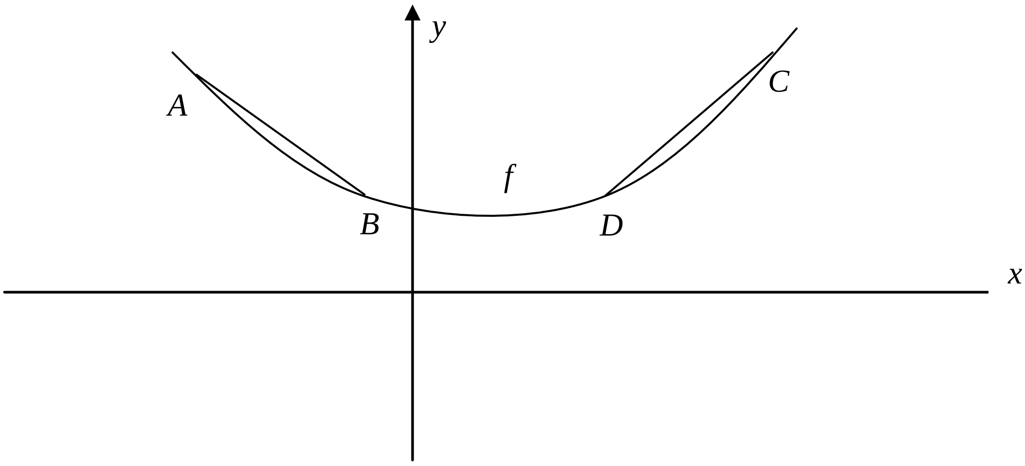
Average gradient

The gradient of a secant line joining two points on a graph compares vertical with horizontal change: $\frac{\Delta y}{\Delta x}$. This is called the *difference quotient*. Can you explain why? The gradient value tells us how much the dependent variable changes as the independent variable changes. This is called an average *rate of change* of the function. What does the ratio $\frac{\Delta y}{\Delta x} = \frac{-3}{2}$ tell us about how y changes as x changes?
[Plenty of work on this in Layer A?]

In the graph below, the average rate of change of the function from A to B is negative, shown by the negative slope of the secant line AB $\left(\frac{\Delta y}{\Delta x} = \frac{-}{+} \; or \; \frac{+}{-}\right)$. The graph is *decreasing* from A to B (going down to the right).

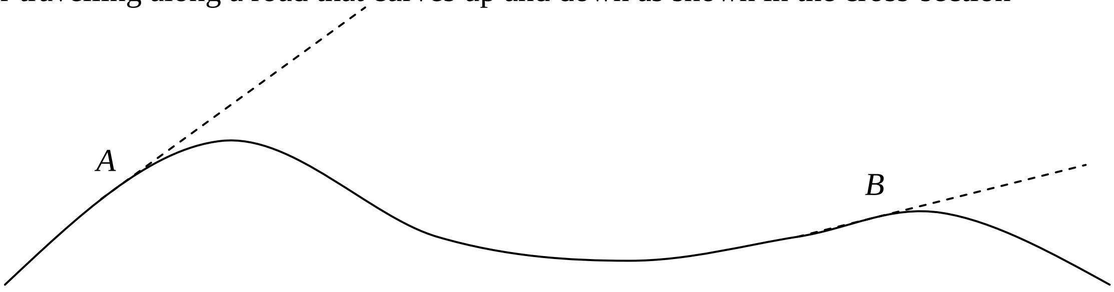
The average rate of change (gradient) from D to C is positive $\left(\frac{\Delta y}{\Delta x} = \frac{+}{+} \; or \; \frac{-}{-}\right)$.

The graph is *increasing* from D to C (going up to the right).



Tangent lines

Imagine a car travelling along a road that curves up and down as shown in the cross-section below.



At points A and B, the dotted lines show the directions of the headlights. The headlights point higher into the sky when the road has a steeper gradient. These dotted lines form *tangents* to the curve of the road. At a given point, if the car continued in a constant direction without following the road, it would travel along the tangent line.

Each dotted line shows how steep the road is at that point. Similarly, a tangent to a graph at a point has the same gradient as the graph at that point. The gradient of the tangent indicates how the dependent variable would change *if the rate of change stayed constant*.

effective rate of change?

Definition

A tangent to a graph:

- touches the graph at that point, and
- has the same gradient as the graph at that point.

(The tangent line could cut the graph again at another point, depending on how the graph curves.)

Notation

- The gradient of the curve $y = f(x)$ at the point (a, $f(a)$) is denoted by $f'(a)$ (called the *derivative* of f at a), which is also the gradient of the tangent drawn at that point.

[sketch]

Tasks

Here are the graphs of a linear function f(x) and a curve g(x).

(sections of the graphs, the origin not shown)

1. a) If point C(6, 8) lies on the graph of f(x), is the gradient of the line $\frac{4}{3}$?

   b) Can you use point A only to find the gradient of the line y = f(x)? Explain.

   c) Use points A and B to find the gradient of the linear function, and show your working.

   d) Move point B to the right of A and then to the left of A, so that the gradient of the line AB is still the same as it is now.

   e) If we keep point A as it is, explain how you could move point B so that

      i) the line AB is steeper than it is now

      ii) the line AB is less steep than it is now

      iii) the line AB has a negative gradient

      iv) the y-value of the linear function increases three times as fast as the x-value

      v) the y-value of the linear function decreases twice as fast as the x-value

   f) Use the gradient of the line to find p if the point (12, p) lies on the graph of f(x).

2. Answer these questions for the graphs of *f* and *g* given above.

   a) On the graphs, show these calculations:

      i) $f(5)-f(2)$ ii) $\frac{f(5)-f(2)}{5-2}$ iii) $\frac{g(3)-g(1)}{2}$.

   b) Which is greater: $f(5)-f(2)$ or $g(5)-g(2)$?

   c) State whether each of the following is true or false, and explain your answers.

      i) $\frac{g(3)-g(7)}{3-7}>0$ ii) $\frac{f(b)-g(a)}{b-a}=2$ iii) $\frac{g(5)-g(7)}{-2}=\frac{\Delta y}{\Delta x}$

   d) Find two points (a, g(a)) and (b, g(b)) so that

      i) $\frac{g(b)-g(a)}{b-a}=3$ ii) $\frac{g(b)-g(a)}{b-a}=\frac{f(b)-f(a)}{b-a}$

      iii) $\frac{g(b)-g(a)}{b-a}<0$.

   e) Find value(s) for *a* (where possible) so that

      i) $\frac{g(2)-g(a)}{2-a}>0$ ii) $\frac{g(2)-g(a)}{2-a}=0$ iii) $\frac{f(3)-f(a)}{3-a}=1$

   f) On the given axes above, draw a linear function y = h(x) so that $\frac{h(b)-h(a)}{b-a}=-\frac{1}{2}$.

3. Use the given graphs of f(x) and g(x) to answer these questions.

   a) Complete (and explain your answer): $f'(x)=$.....

   b) Draw a tangent line to show $g'(2)$.

   c) Find *a* so that $g'(a)=1$.

   d) Write in order from smallest to largest value: $g'(2),\ g'(4),\ g'(5)$.

   e) For what value(s) of x is $g'(x)>f'(x)$?

   f) For what value(s) of x is $g'(x)=0$?

   g) Fill in < or >:

i) $g'(2) .... g(2)$ ii) $g'(2) .... f'(2)$ iii) $g(4) .... f'(3)$

h) Find (where possible) values for *a* and *b* so that

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v) $g'(a) = f'(a)$ vi) $g'(a) > f'(a)$

vii) $f'(b) \geq f(b)$

4. Below is the graph of $h(x)$.

[graph]

a) Fill in the table of values for the function $g(x) = h(x) + 3$ and plot the graph of *g* on the same axes above.

[table]

b) In what ways are the graphs of h(x) and g(x) the same or different?

c) Estimate the values of $h'(1)$ and $g'(1)$. Do you think that the result is a coincidence? Explain.

d) For which value(s) of x is $h'(x) = g'(x)$?

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f) For which value/s of *x* will *h* and *g* both have a negative gradient?

g) Write a general conclusion about the gradient of a function f(x) and the function f(x) + C.

Consolidation

Write a short summary about the average gradient between two points on a graph, and the gradient of a graph at one point. Use a graph as part of your explanation, and use correct notation.

Assessment

1. The diagram shows two parallel tangents to the graph of *f*. The equation of the tangent line at the point (3, $f(3)$) is $y = 3x - 4$.

[graph]

a) Find $f(3)$, $f'(3)$ and $f'(-1)$.

b) Find the equation of the parallel tangent through the point (-1, 2).

c) Find (where possible) values for *a* so that:

i) $\frac{g(4)-g(a)}{4-a} = f'(3)$ ii) $\frac{g(4)-g(a)}{4-a} = 0$ iii) $f'(a) = -1$

2. a) Sketch the graph of the function $f(x) = 5$, and hence find $f'(-1)$.

b) If $f(x) = 5x$, use a graph to explain why $f'(-1) = f'(3) = 5$. What is $f'(x)$?

3. Sketch a possible graph of f for x between 0 and 4, if $f(3) = f(1) = 2$, $f'(3) > 0$ and $f'(1) < 0$.

State if each of the following is possible for your graph, giving reasons.

a) $f'(c) = 0$ for $1 < c < 3$

b) $f(2) < 0$

c) $f(2) > 0$

d) $f'(3) = -f'(1)$

4. The point A(3, -1) lies on the graph of g, and $g'(3) > 0$.

a) Use a graph to show the calculation $\dfrac{g(b)+1}{b-3}$.

b) Find the coordinates of a point B(b, f(b)) so that $\dfrac{g(b)+1}{b-3} > 0$.

Answers

Purpose

- revise the difference and ratio aspects of gradient
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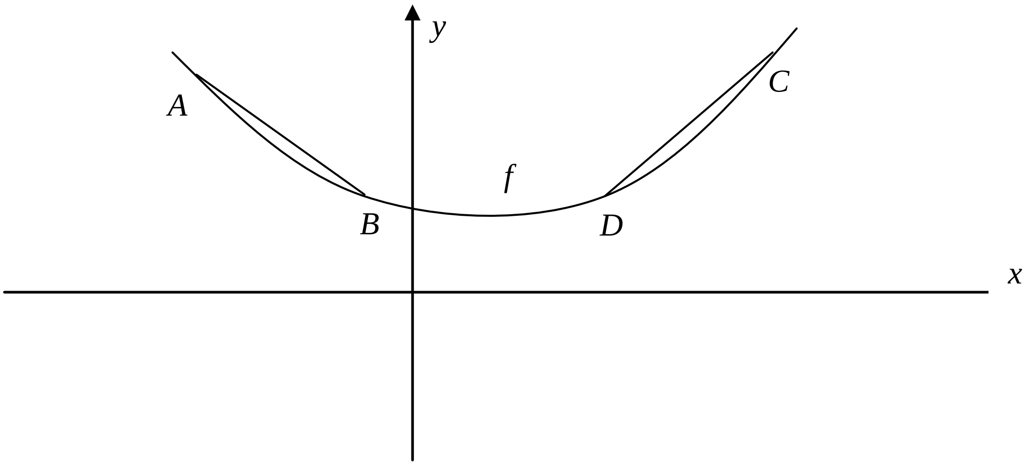

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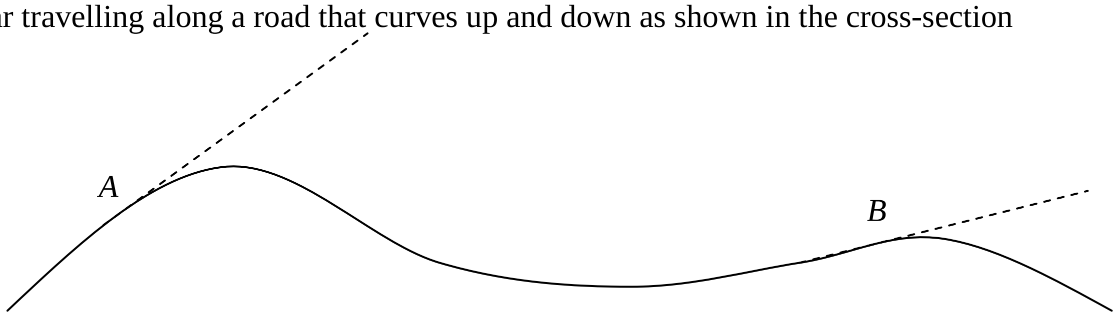

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c) Use a graph to show the calculation $\dfrac{g(b)+1}{b-3}$.

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Answers

Interview 1 with lecturer 1

1. What is it about teaching that you enjoy and why?

Lecturer 1: I think basically the issues of teaching is, first of all the mathematics content that I enjoy, the mathematics. And secondly is putting ideas across to others and seeing their face light up when they actually understand what I'm getting across to them. In other words you know the ZPD- the gap is closed.

2. How would you describe your relationship with your colleagues and what impact does this have on your teaching?

Lecturer 1: Well I'm sure you really mean the academic relationship?

Researcher: Yes.

Lecturer 1: But even in the informal level when we chat in corridors and at meetings we actually speak about students and what transpires in our lectures and that does have an impact on our teaching in the sense that we get to know the students more. If the student has been dealt with by lecturers in the past so we get some past history of the students. At the same time it helps us to actually check our teaching in the sense that we know the calibre of the students that we have, that we are dealing with so we know how to adjust our teaching in that respect.

3. Please list your qualifications and experience.

Lecturer 1: Teaching experience… I think I'm involved in teaching for 10 and 4, 14… 22 years now at various levels. I've been a school teacher for 4 years at one school, 2 years at another school. I've taught at a school for 6 years altogether. 10 years at a teacher training institution, I year with Unisa and thereafter I joined the University of Natal and then the UKZN now.

In terms of qualifications I have a B.Sc, B. Sc honours, B.Ed honours degree, Masters of Science and a Doctorate in Pure Mathematics.

4. What educational courses or training have you taken or received to teach Calculus?

Lecturer 1: For this particular topic is part of the first year undergrad at the B.Sc. undergrad level. You do calculus. A whole course in calculus. Then later you actually study advanced levels of calculus at a high level like in Measure Theory and Real Analysis.

5. Do you feel that your content knowledge of Calculus is adequate for teaching the particular class that you are teaching and why?

Lecturer 1: I would say yes because, as I mentioned, the depth of the calculus course. Calculus is not just plain first year Differential and Integral Calculus one sees in textbooks at the classical course given to first year students.

In fact calculus has been studied much deeper. As I mentioned if you look at it from an analytical point of view where you do the real analysis in second year and third year courses, where you really have to look at calculus at an advanced level. You can even look at it from a level of for instance measure theory, where for instance integration in the first year you look at the Riemann Sum. You can actually look at other types of integrals like what is called the Le Bake integral at the advanced level like the honours level.

6. What other type of knowledge, besides content knowledge, do you need for teaching this topic? Explain.

Lecturer 1: Ya obviously to teach calculus, you need to know the pre-calculus part of it. You need to know what topics the child has to be fluent in order to be successful in the calculus topics an example of that is normal school mathematics. The student for instance should know how to sketch functions, the graphs of certain functions like the parabola and so on….

and that is being extended. So that’s one aspect other than the actual content knowledge of calculus that he needs …pre-calculus knowledge.

Also we need pedagogical knowledge…pedagogical methodology, in other words you need to know how to put things across to students, pending the quality, the standard of the students. Ya, you need to know how to teach…teaching strategies.
I use a lot of variety in teaching, in other words, I use for instance worksheets.
I will provide them with ideas where they will sit together and work collaboratively.

7. Briefly explain what you do to keep your knowledge of Calculus up to date?

Lecturer 1: Ya, well, attending conferences, presenting papers, research papers, looking at students tasks.

Just to highlight, here's one copy of a paper by another academic (shows me a paper that has been lying on his desk) and I will read this paper obviously and see and see how this topic is being taught internationally and what successes they have gained so I can implement similar strategies in my teaching.
Attending workshops, doing more research, analysing students gaps. trying to adjust methodology, so that those gaps will not appear in the future, in other students.

8. In your opinion what are some of the most useful strategies you have learned from any conference, workshop or training on teaching Calculus?

Lecturer 1: As mentioned earlier, the inductive method especially in terms of definition making. The students self discover. You can integrate collaborative learning within that, within self discovery, where students actually interact with each other and they discuss these concepts.

9. How do you decide on what activities to engage your students in?

Lecturer 1: It would all depend on the situation. On what I wanted to teach within the topic. For instance if I wanted to teach, for example, the derivative concept via first principles, the student has got no notion of the definition at that stage. So from the average gradient I would now lead on to speaking about the approach of one point to another in the classical way where you arrive at the gradient of the tangent at a particular instance on the graph. So what I'm saying, the particular topic, the concept that I put across determines the activities that I design so as to gain students’ understanding of the concept.

10. Do the activities that you engage your students in allow for the discovery of skills that they require in calculus?

Lecturer 1: I would say yes, both skills and conceptual understanding… and skills, yes. Many of these skills are not self discovered, they are taught to students in a lecture style.

11. How do you prepare your students to get ready for a lesson you are going to present?

Lecturer 1: I generally introduce them with a problem that they have not a seen before that lends its solution to the future lesson. So at the commencement of the lecture I always present them maybe a small mathematical task that they have some idea about how to get along, but they probably would not solve it. But the whole intention would be to say at the end of the lecture is that they can now solve the problem.

I don’t know if I’m clear. Ya , a simple example I could give you, for example in the grade eleven class, the child can solve the trinomial using factor method so after you do the factor method you will probably throw one that does not factorize, so that at the end of the lesson he will learn how to use the quadratic formula, so at the end of the lesson he has now learned a new technique.

12. Have you received any training either formal or informal to assist you in developing successful relationships with students? If yes, please describe.

Lecturer 1: I think whatever I’ve learnt in terms of students is at an informal level. It was very theoretical within the teaching qualification, within the PGC and the UHDE teaching diploma where the reading of textbooks led to the discussion of ideas of student relationships. No formal or practical training that I have underwent. The actual relationships that I encountered with students transpired during my years of teaching, that was developed incidentally.

13. Was there ever a time in your teaching where you felt that your content knowledge was lacking and how did this affect your teaching?

Lecturer 1: I would say not really at this level in calculus, but there has been other avenues. Just to give you an idea when I was teaching in a high school in Stanger I was asked to teach Technika Electronics which was a new subject that was introduced in the FET, in the high school at that time, and when I started teaching it I realized that I didn't know much electronics and in fact the amusing incident which I always relate…. we had to do practical work and none of the pracs worked in grade 10 and this affected my teaching in the sense that the theory I was teaching was not fitting in with the pracs, I couldn't get the results via practical experimentation. And yes that is how that affected that particular teaching over there.

The lack of content knowledge, thorough content knowledge affected teaching because there was no correlation between what I was telling them theoretically with the actual practical findings o the experiment.

Researcher: Would you say therefore that if your content knowledge was lacking it would hinder your progress in teaching?

Lecturer 1: It definitely hinders you in that sense. The important thing is that the onus lies on the teacher. He has to take the initiative to do something about it. I actually felt that I was deficient in that respect and so I had to study and I did a National Technical Diploma with NT 6 in Electronics and Electrotechnics so that I would be better prepared to teach Electronics.

Interview 1 with lecturer 2

1. What is it about teaching that you enjoy and why?

Lecturer 2: I love interacting with the students. I love learning more, because as you teach you learn more. But basically interacting with the students and learning more about that and improving your knowledge as you teach.

2. How would you describe your relationship with your colleagues and what impact does this have on your teaching?

Lecturer 2: Different with different colleagues. There are certain colleagues that I work with well and others not so well. Particularly my work with Dr. X and Y. I enjoy working with them because of the opportunities to learn, because different people have different perspectives and different strengths. And sometimes you can even look at one problem differently and you learn from the other persons approach, you learn a different method.

3. Please list your qualifications and experience.

Lecturer 2: I got a lot of teaching experience. I started at Springfield College and then I moved to University of Natal for four years. With Casme and then back to Sprinfield College for two years. Since 2002 I've been here.

My qualifications is a Masters in Maths and a doctorate in Maths Education.

4. What educational courses or training have you taken or received to teach Calculus?

Lecturer 2: I'm sure we must have done methods in teaching Maths at university.

5. Do you feel that your content knowledge of Calculus is adequate for teaching the particular class that you are teaching and why?

Lecturer 2: That class that I was teaching calculus I just went in for a few weeks as part of the project. I didn't have any hesitation in managing because I knew I would know the calculus except that I haven't taught it for a while.

No I don't have any problems in Maths with content knowledge.

6. What other type of knowledge, besides content knowledge, do you need for teaching this topic? Explain.

You would need pedagogic content knowledge. Pedagogic content knowledge is about knowing the content, knowing the best way to teach it to students. Teaching in a way that is accessible to them. Knowing the common misconceptions in that topic so when you teach it you are aware of the problems that the students could be having and pedagogic content knowledge also involves knowing different representations of the same concept and then choosing the best representation or various representations to best mediate it, that's pedagogic content knowledge.

You would also need pedagogic knowledge, I think. Knowing how best to teach people, issues like the dynamics of grouping, issues about how long you can speak for, issues about when to write summaries on the board, practical issues of classroom management, that's what I mean by pedagogic knowledge. Pedagogic content knowledge is knowing about the content and how to teach it.

7. Briefly explain what you do to keep your knowledge of Calculus up to date?

Lecturer 2: When I was asked to teach that course I looked at three textbooks. I went over them properly. I looked at Dr. X's notes. I worked out every possible question before I taught.

8. In your opinion what are some of the most useful strategies you have learned from any conference, workshop or training on teaching Calculus?

Lecturer 2: From my conversations with Y and with the materials that he has been working on. I think the classical approach getting them to understand the idea of a changing slope because after that it is application of calculus, simple calculus.

9. How do you decide on what activities to engage your students in?

Lecturer 2: I will, firstly if I am teaching a course, I will look at what I'm supposed to teach. I design the course outline if I'm teaching it for the first time.

Then I go through everything. I work through every problem in that textbook. Then I'll go find other books and look at how they approach it, then I'll come up and I,ll break it up into my lesson units, and then I'll decide how I'm going to approach it.
And then lesson by lesson I think about what I'm going to say, what I'm going to work out and what I'm going to apply. Everything I do I write down. Every simple problem I will work out and I will have it in my hand. Because sometimes you'll just make a little slip and it confuses the students. I'm very particular about that.

Especially sometimes we don't realize how when we use a different word, how it affects students. So I'm always…I think my students get used to me, I always have my paper in my hand.

I'm too scared to make a mistake and to confuse them because if I make a slip with a 4 or a 5 on the board I don't know the repercussions that will have for them before I go back and correct it.

Then I decide how I'm going to introduce a lesson and then I spend time thinking about it. I'm always nervous before a lesson. I spend about one and a half hours preparing for a lesson even if I've taught it before.

10. Do the activities that you engage your students in allow for the discovery of skills that they require in calculus?

Lecturer 2: Now I'm going to move a bit out of calculus. If I need them to understand something I try to find a motivating question. It could be a maths question. It doesn't have to be a concrete activity that will make them think about the need for what I'm going to introduce. Or I present this whole big idea to them and show them how this little thing fits.

I always start off with what we're going to do, and why we're going to do it and where we're going to go.

11. How do you prepare your students to get ready for a lesson you are going to present?

Lecturer 2: Well if possible I give them a problem to work out beforehand, otherwise they always have notes beforehand. They have a breakdown of what's going to happen. When the maths is quite complicated it really helps if they read beforehand, so it helps if they have some sort of idea of some of the new terms.

12. Have you received any training either formal or informal to assist you in developing successful relationships with students? If yes, please describe.

Lecturer 2: No I think this here depends on peoples' personality and their commitment to teaching. That no matter how much training you have to develop successful relationships, I cannot think how that can help because in the end it depends on your commitment, and you see it all the time. You can have ten people in a department and you can see how different people interact at different levels. Different people are prepared to go to different lengths to help students and others are not prepared to go…even if you send them for training they'll do the bare minimum.

It's a personal commitment, it's the same thing, like with teaching, it's a passion. Are you passionate about teaching? It's the problem the problem with our education system, as you know. Because we have too many teachers who are not passionate about teaching.

In the academic field as well, developing successful relationships depends on your own need, sense of fulfilment, whether you get fulfilment from developing relationships with students or whether you are more comfortable doing research, or whether you are more comfortable working somewhere else. It's about your commitment to your job.

13. Was there ever a time in your teaching where you felt that your content knowledge was lacking and how did this affect your teaching?

Lecturer 2: In theory I don't worry about content knowledge because I've done extremely well in all the Maths I've studied. But the reality is before every lesson I get nervous. Whether it's a grade eleven maths lesson or even if it's a grade nine Maths lesson. I'm always worried that I may not explain it properly. I always am prepared. I have my notes beforehand and if necessary I actually practice it. That's what I do.

Before I go into any class, any workshop, any conference, even if it's up to grade nine, I will practice… I'll know I'm going to start off with something, and I'm going to back it up with this and I'll think about it. I won't do anything the night before besides think about what I'm going to do.

APPENDIX H

Interview 2 with lecturer 1

1. Researcher: How would you define or describe content knowledge?

Lecturer 1: There are different angles to the definition of content knowledge. To me content knowledge is knowledge that is pertinent to a particular topic that you are teaching, in other words it does not entail the didactic aspects of knowledge. I do not integrate it with pedagogics. It is dealing with a particular topic and the mathematics around it.

2. Researcher: How would you describe the link between your content knowledge of calculus and how this influences your teaching?

Lecturer 1: To me I feel there is a very strong link between my content knowledge and the way I teach. I am able to emphasise on particular aspects of the content, when teaching whatever topic.

3. Researcher: In general what do you think is the relationship between content knowledge and classroom practice?

Lecturer 1: Certainly there is. It depends on the topic. If I'm teaching equations, my content knowledge of equations will enable me to approach the teaching of equations in a particular way. My classroom practice, the approach that I use is dictated by content knowledge. I have a whole global picture of where I'm going.

Interview 2 with lecturer 2

1. Researcher: How would you define or describe content knowledge?

Lecturer 2: Content knowledge….knowing the mathematics. The content is about the content, how well you know the content, how it fits in with other topics….about the concepts, having an understanding of how it works, when it works, knowing the interrelationships within the content.

2. Researcher: How would you describe the link between your content knowledge of calculus and how this influences your teaching?

Lecturer 2: A deeper knowledge of calculus, as in a deeper knowledge of anything, affects how you teach because the deeper your knowledge is, you have a bigger repertoire of example to draw upon and you can readily come up with counter examples in order to help learners see conditions when theorems hold, and or conditions when rules hold. You are able to without any problem come up with relevant examples.

3. Researcher: In general what do you think is the relationship between content knowledge and classroom practice?

Lecturer 2: I don’t think you got a one to one relationship, but definitely a deeper content knowledge will impact on classroom practice. You cannot say that a deeper content knowledge results in better classroom practice….it is necessary. To have good classroom practice it is necessary to have good content knowledge but not sufficient.

APPENDIX A

APPENDIX B

APPENDIX C

APPENDIX D

APPENDIX E

APPENDIX F

APPENDIX G

APPENDIX H

APPENDIX H