

# The effect of monetary policy regime switches on the application of different term structure models to estimate South African real spot rate curve

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## Declaration

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## Abstract

The global economy was recently brought to a standstill due to the Covid-19 pandemic. This resulted in a '1-in-100 years' stress in the global economy which saw the application of drastic monetary and fiscal policy adjustments to cushion the economy against this stress. South African bond market was also negatively affected, thus negatively affecting the ability to finance the increasing primary deficit due to increased funding costs and lower liquidity.

South africa uses inflation-indexed bonds as part of government funding; however, they are less tradable in the market, translating to inadequate bond pricing/valuation data. As such, this study aims to explore dynamism of different mathematical term-structure models during the heightened Covid-19 stress in estimating the South African inflation-indexed/real spot-rate curve. This study follows previous studies on the South African inflation-indexed bonds by Reid (2009) and Mashoene et al. (2021) where Nelson-Siegel and Svensson models posed a limitation in estimating spot-rates during a period of high volatility. As such, this study explores dynamic term-structure models, which follow the Nelson-Siegel framework, and static term-structure models with the option of recalibration of model parameters to account for a change in macro-economic dynamics brought by the effect of the Covid-19 pandemic.

A recalibration methodology has proven beneficial for Nelson-Siegel and Svensson term-structure models for model fitting and model forecasting process during the high volatility/period of total economic shutdown due to the Covid-19 pandemic. However, no improvements were observed in the Linear-parametric and Cubic-splines term-structure models. The effect of a dynamic decay rate ( $\lambda$ ) on the Dynamic Nelson-Siegel also did not improve the performance of the Dynamic Nelson-Siegel.

As such, it is recommended that the South African national government's debt managers and other bond fund managers explore this methodology for improved/enhanced estimations of debt management risk indicators. Compared to the current econometric methodology used by the National Treasury which is only able to produce two points on the entire term-structure; this methodology will enable the estimation of the entire curve and the bond pricing in longer maturities where most of government funding is based.

**Keywords:** Spot rate curve, South African inflation-indexed bonds, Model recalibration, Auto-Regressive model, Nelson-Siegel model, Forecasting.

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# **Chapter One**

## **Overview of the study**

### **1.1. Introduction**

This chapter contains the background of the issues that might influence this study's direction. It outlines the problem statement, the study's objectives, its significance to the overall corpus of knowledge, and the overview of the methodology applied in the study. It also gives a list of the notations used in this study.

### **1.2. South African bond market and the background of the South African monetary policy**

The first inflation-indexed bond was issued in March 2000 by the South African government (Harman and Mageza, 2009). The primary purpose of introducing inflation-indexed instruments in South Africa was to attract investors into an investment product that hedges their money's value against inflation and assure investors of the seriousness of the inflation-targeting strategy introduced by the South African Reserve Bank. However, due to a lack of expertise in the South African market for the pricing of these instruments; Mashoene, Doorasamy, and Rajaram (Mashoene et al., 2021: pp. 22) showed that "the introduction of the inflation-indexed debt became more expensive for the South African government when a 13-year R189 bond was launched at a real coupon rate of 6.25 per cent in the year 2000". This was above the current levels trading at around 3 per cent observed in the market (Bloomberg, 2023).

Over the years, the inflation-indexed debt market grew significantly to 22.4 per cent of the government's domestic debt as of 31st March 2021 (National Treasury, 2021). This increased from levels below 19 per cent observed before the 2009/10 fiscal year due to liquidity issues (National Treasury, 2010). Significant issuances into this debt

instrument post the 2009/10 fiscal year were fuelled by the popularity of the bond instrument in the market and significant growth in the South African budget deficit financed through the debt market. This is because most developing governments rely on bonds/loans to fund their budget deficits, given higher differences between realised revenue collections and given government expenditure.

Every week, the South African government go to the market to issue government bonds as part of a government funding/borrowing programme. Given the dynamic nature of the bond market, the cash collected in each auction is subject to some market risk factors (i.e. interest rate risk on the pricing of the bond, inflation risk on the bond indexation, settlement risk, etc.). The ability to perfectly price the inflation-indexed bond instrument with minimal deviations will help the government be well prepared to determine the nominal amount to be put on auction to receive the required weekly cash targets. This will also help to avoid overestimating the value of the South African government debt and the debt service cost during the budgeting process.

According to International Monetary Fund (IMF, 2020), during the 2020/21 fiscal year, when the global economy was affected by the Covid-19 pandemic, recommendations were made to sovereign debt managers to curb the effect of the stress on the fiscus by the Covid-19 pandemic. The pandemic resulted in many states shutting down their economic activities, resulting in poor revenue collections, poor funding performance in the secondary market, and drastically increasing government expenditures in addressing the effect of the pandemic. It was indicated that the effect of the global stress on short-term funding liquidity is critical as most governments might be expected to experience increased financing requirements due to policies/strategies to respond to the crisis. This was also evident in the case of South Africa, where significant adjustments were made by the South African Reserve Bank and the National Treasury to boost the unsustainable fiscal position observed over the past couple of years and then worsened by a severe decline in the economic and revenue outlook (National Treasury, 2020b).

### 1.3. The importance of inflation in the South African inflation-indexed bond market

Since the formation of the South African Reserve Bank (SARB) Act of 1989 and the SARB accepting its formal mission statement for the first time in 1990, South African Reserve Bank (2022), Ncube and Ndou (2013), and Coco and Viegi (2019) indicated that the mission of the SARB has evolved from protecting the internal and external value of the rand to protecting the value of the currency in 1996. This was highlighted in the 1996 Constitution of the Republic of South Africa in the interest of sustainable and balanced economic development and growth.

To achieve the South African Reserve Bank (SARB) adopted the inflation targeting framework in 2000, a measure of the price stability in the country. The inflation target was set in a range of between 3 and 6 per cent. Ncube and Ndou (2013) indicated that the SARB's framework under inflation-targeting is focused more on operational independence than goal independence. This implies that the SARB chooses any monetary policy instruments to pursue inflation targets the South African government sets. In line with the inflation targeting framework, the South African government introduced the inflation-indexed bond to assure investors stressing the importance of this framework to the government (Harman and Mageza, 2009).

### 1.4. Bond pricing and risks faced by the government

In line with suggestions made by IMF (2020) to address government funding liquidity issues in the wake of the effect of Covid-19 on the economy, the South African government made fiscal adjustments as indicated by the National Treasury (2021), which comprised of "drawing down its cash deposits held with the Reserve Bank, increased short-term borrowing (Treasury bills and bridging finance from the Corporation for Public Deposits) and obtained loans from international financial institutions". This was done in line with government risk management strategies of reducing currency and refinancing risk while accommodating the effectiveness of domestic bond market operations.

As part of government's risk management strategies in minimising refinancing risk for the government in the short term, the switch programme continued to play a bigger role. According to National Treasury (2000), the bond switch programme can be defined as a transaction of exchanging existing source bonds held by investors into selected destination bonds, mostly with longer maturities. However, external demand for emerging markets' debt declined significantly in March 2020, with \$31 billion in recorded outflows (IMF, 2020). This was also evident in the South African debt market, where all three rating agencies rated the local government on non-investment grade and thus resulting in the South African bonds being excluded from the World Government Bond Index (Anorld and Winning, 2020); as such, the overall non-resident bond holding declined by 6.7 percentage points to 30.6 per cent in June 2020 from 37.3 per cent in January 2020 (National Treasury, 2020b).

Variable	Period	Inflation-indexed bonds	Fixed-rate bonds
<b>Clearing yields (%)</b>	Average: 2019/20	3,60	9,31
	Average: 2020/21	4,25	9,69
	27 Mar 2020	5,38	13,17
	03 Apr 2020	3,19	9,61
<b>Bid-to-cover ratio</b>	Average: 2019/20	2,87	2,93
	Average: 2020/21	1,94	2,94
	27 Mar 2020	1,70	2,25
	03 Apr 2020	4,31	4,24

*Table 1.4.1: South African bond auction performance*

*Source: National Treasury (2021)*

In the case of South Africa, as indicated by National Treasury (2021, pp. 79), the South African Reserve Bank (SARB) "conducted a bond-buying programme in the government bond secondary market; and this step, announced in March 2020, contributed to continued market liquidity and stabilised government bond yields". This is observed in Table 1.4.1, where clearing yields for auctions held after the SARB announcement on both inflation-indexed bonds (ILBs) and fixed-rate bonds (FRBs) strengthened significantly by over 200 basis points. This is a recovery from over 100 basis points decline seen on the auctions just before the announcement. The same applied to the bid-to-cover ratio, which saw over 80 per cent recovery to levels above four after the announcement.

<b>Year</b>	<b>Moody's Investors Service</b>	<b>Fitch Ratings</b>	<b>S&amp;P Global Ratings</b>
2020	Ba1(Negative)	BB(Negative)	BB-(Stable)
2019	Baa3(Negative)	BB+(Negative)	BB(Stable)
:			
2011	A3(Negative)	BBB+(Stable)	BBB+(Stable)
2009	A3(Stable)	BBB+(Negative)	BBB+(Negative)
2008			BBB+(Negative)

*Table 1.4.2: South African historical credit ratings*

*Source: National Treasury (2021)*

In the 2008/09 fiscal year, in the wake of the 2008 global crisis, the real yield weakened significantly [i.e. the 5-year R189 bond yields climbed to 3.1 per cent in December 2008 from 1.7 per cent in June 2008 (National Treasury, 2010)]. This is the trend observed with the Covid-19 stress on the 5-year inflation-indexed (I2025) bond yields climbing from 3.3 per cent in April 2019 to 4.4 per cent in March 2020. However, the two economic environments have different inflation trajectories, as inflation peaked at over 11 per cent in 2008. In contrast, it declined to a record low of around 2 per cent during the 2020/21 fiscal year.

Based on Table 1.4.2, South African market sentiments were much better during the 2008 global crisis, with all three major rating agencies (i.e. Moody's, Fitch and Standard and Poor's credit rating agencies) rating the South African government stock way above the investment-grade (i.e. rated on A3, BBB+ and BBB+ respectively) (National Treasury, 2010). However, the picture has reversed drastically over the years due to local political issues and a dire state of the government's state-owned agencies, which rely on the fiscus for bailouts, thus significantly ballooning the government debt portfolio. This has resulted in the South African government stock being rated non-investment rating by all three major global rating agencies, thus worsened by the adverse effects of Covid-19 on the global economy (National Treasury, 2021). This has thus translated into a higher cost of borrowing (i.e. higher yields) for the government. The effect of the stress on the inflation-indexed bonds was less severe compared to the 2008 global financial crisis, given that the instrument is currently more popular in the market, coupled with interventions made by the South African Reserve Bank in buying South African government bonds (National Treasury, 2021).



As a risk management measure during the Covid-19 stressed period, the South African government, as indicated in National Treasury (2020a), has opted for a momentary measure by reviewing its borrowing strategy to concentrate on supplying shorter-dated bonds with a weighted average time-to-maturity of 7 to 10 years, compared to 15 years seen in the previous year. The strategy assists in balancing available market demand while managing the cost of issuing debt. To assess the nominal issuances into the inflation-indexed bonds between the 2019/20 and 2020/21 fiscal years, Table 1.4.3 shows the percentage issuances into each bond instrument.

The issuance strategy introduced by the National Treasury to issue short-dated maturities in line with market demand and manage the cost of raising debt was absent in inflation-indexed bonds. Based on Table 1.4.3, issuances in the shorter maturities (i.e. R212 and I2025 bonds) have marginally reduced. In comparison, significant reductions were observed in the longer maturities (i.e. I2033 and I2050 bonds). In contrast, significant increases were observed on the I2029 and the I2038 bonds. This is likely attributable to demand issues and the availability of issuance space into these maturities, given that the bonds are new in the market.

<b>Inflation-indexed bonds</b>	<b>Maturity</b>	<b>2019/20 nominal issuance</b>	<b>2020/21 nominal issuance</b>	<b>Absolute difference</b>
<b>R211</b>	31-Jan-22	1,7 %	0,8 %	-0,9 %
<b>I2025</b>	31-Jan-25	19,9 %	19,3 %	-0,6 %
<b>I2029</b>	31-Mar-29	8,1 %	10,9 %	2,8 %
<b>I2033</b>	28-Feb-33	14,8 %	11,4 %	-3,3 %
<b>I2038</b>	31-Jan-38	16,2 %	20,7 %	4,5 %
<b>I2046</b>	31-Mar-46	19,5 %	20,7 %	1,2 %
<b>I2050</b>	31-Dec-50	19,8 %	16,1 %	-3,7 %
<b>Total</b>		<b>100,0 %</b>	<b>100,0 %</b>	

*Table 1.4.3: Nominal issuances into inflation-indexed bonds*

*Source: National Treasury (2021)*

The performance of the inflation-indexed yield curve has a direct impact on the cash received on weekly auctions. This can be observed when issuances into illiquid bonds are made and might result in a lower take-up or costly transaction (i.e. issuance at higher interest rates), thus resulting in lower cash collection from the auction. Analysing the performance and the shape of the inflation-indexed yield-to-maturity

curve between the beginning and the end of the 2019/20 fiscal year is presented in Figure 1.4.1.

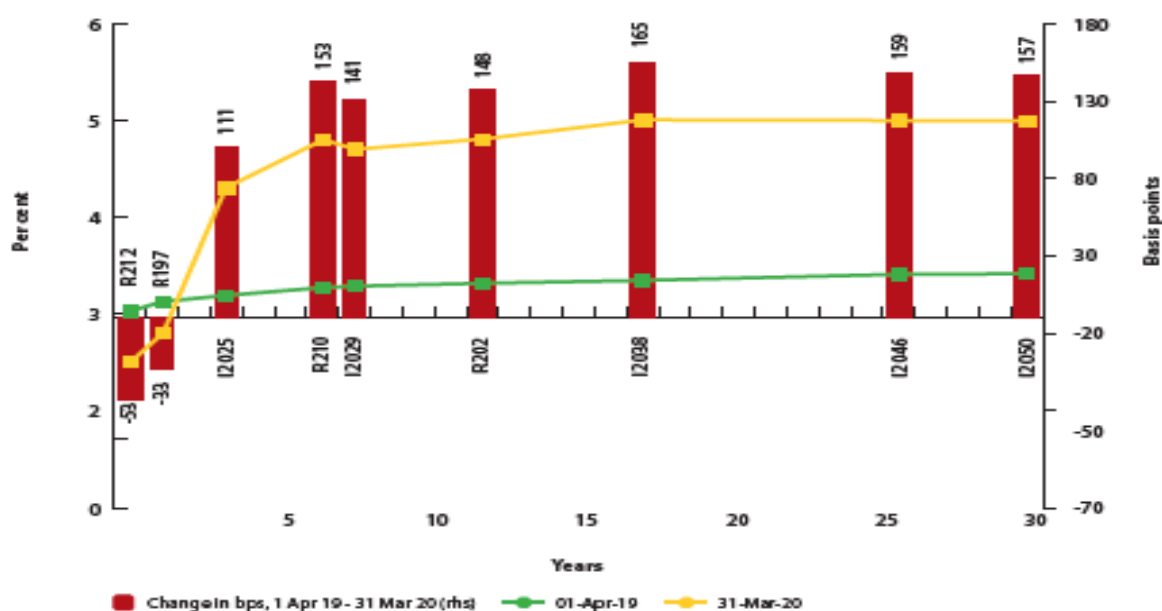


Figure 1.4.1: Yield curve movement in inflation-indexed bonds, 2019/20

Source: National Treasury (2020b: pp. 21)

It is evident from Figure 1.4.1 that the inflation-indexed yield-to-maturity curve has steepened at the end of the 2019/20 fiscal year compared to the beginning of the 2019/20 fiscal year. This could be attributable to regime switches in monetary policy regimes where significant cuts were observed on the repo rate to caution against the economic stress imposed by the Covid-19 pandemic (National Treasury, 2021). The steeper yield curve could be costly for the government, given that most government funding in the inflation-indexed bonds is on the longer maturities. This is in line with South African risk management strategies to lengthen the term-to-maturity of the government debt portfolio (National Treasury, 2020a).

It was detailed in work done by Mashoene et al. (2021) that the National Treasury's current budgeting processes and debt management risk measures are based on the econometric methodology to forecast the short (3-month) and long (10-year) term points. Other points on the yield curve are interpolated based on the output of these two benchmark points. The National Treasury uses the outcomes of the Johannesburg

Stock Exchange (2012) model to price a new bond. It was further indicated that inflation-indexed debt is part of South African government funding instruments, and proper pricing of these instruments is critical for budgeting processes and fair pricing of newly introduced bonds to minimise the risk of flawed budget figures due to potential gaps in the methodology used to price this instrument.

Given the current global market volatility and monetary policy regime changes brought forth by the outcome of the Covid-19 pandemic, finding a term structure model that can capture these volatilities is mandatory for the South African bond market. It was recently proven by Mashoene et al. (2021) that the Nelson-Siegel and Svensson term structure models could capture movements in the real spot rate curve in South Africa; however, these models lose fitting capabilities over periods of high market volatilities. This study will look at these models and the term structure models that incorporate regime switches to assess their performance on the real spot rate curve in South Africa.

## 1.5. Problem statement

For the South African domestic debt portfolio and funding instruments, over 22 per cent comes from the inflation-indexed debt portfolio (National Treasury, 2021). Mashoene et al. (2021: pp. 22) showed that "the current econometrics methodology used by the National Treasury to price this bond instrument is based on the Taylor rule, and it can give only two points on the real spot rate curve (i.e. the 3-month and 10-year benchmark points)". Most government funding happens in maturities over ten years (i.e. in the 25 to 30-year maturities). As such, reliance on this methodology might result in flawed prices for inflation-indexed bonds used for borrowing. To estimate yields on longer maturities, either an interpolation methodology is applied, or the 10-year bond yields are kept constant for ultra-long maturities (Naidoo, Nkuna, and Steenkamp [Naidoo et al.], 2020).

It is observed in Table 1.4.1 that almost 70 per cent of government funding in the inflation-linked bonds is based on the longer maturities (i.e. bonds with maturities of at least 10 years). To estimate the weekly nominal amount that has to be issued to clear

the required cash, future interest rates are required for pricing purposes. To illustrate the effect of interest rate movements on the nominal amount borrowed, refer to Table 1.5.1 below.

Nominal amount (R million)	2022/23	2021/22	2020/21	2019/20	2018/19
Budget	330 400	380 000	337 700	216 000	191 000
Actual	310 900	285 784	518 500	299 189	181 000
Deviation	19 500	94 216	(180 800)	(83 189)	10 000

*Table 1.5.1: South African domestic long-term borrowing*

*Source: National Treasury (2023)*

The primary deficit and revenue collections play a bigger role in determining the amount to be borrowed in the primary market. However, interest rate movements also play a role in determining funding costs which might result in a likelihood of not clearing the funding requirement given the nominal amount issued. This is reflected in Table 1.5.1 where deviations are observed on annual basis either above or below the budgeted figures. National Treasury (2019) indicated that weekly nominal issuance amounts increase as a result of weaker yields which makes it expensive for government to borrow. The weaker yields relative to the forecasted ones can be attributed to a lack of a proper methodology to estimate future yields that can be used to estimate debt levels and debt service costs in the medium term. As such, developing a reliable methodology to evaluate this instrument remains critical, especially under stressed market conditions where normality often breaks. This issue is brought forth by the deflationary economic environment experienced in 2020, which resulted in a significant decline in monthly headline CPI inflation to 2.1 per cent in May 2020, which was last realised in early 2000 when headline CPI inflation printed below 2 per cent (South African Reserve Bank [SARB], 2021).

It was proven in work done by Mashoene et al. (2021) that Nelson and Siegel (1987) and Svensson (1994) models gave a better fit on the real spot rate curve in South Africa and also gave a reasonable forecast. However, it was also noted that their forecasting capabilities are marginalised during stressed economic conditions. This could be attributed to the static parameters embedded in these models, which might not be able to capture more dynamic and complex yield curves. As such, relying on static term structure models throughout stressed market conditions might weaken the

ability of these models to perfectly capture movements of the yield curve and result in budgeted values that result in higher deviations from reality.

This research looks at the application of recalibration process to ensure that the model parameters are not kept constant over different monetary policy regimes. It was indicated in Figure 1.4.1 that the inflation-indexed coupon-bearing bond yield curve became steeper in March 2020 due to a cut in the repo rate as a result of the Covid-19 stress, National Treasury (2021). Yields in the short-end became negative and in the mid-to-long end of the curve increased significantly. The application of the same model parameters over the estimation period which ranges from 2010 to 2021 might result in the flawed output. The study will further incorporate the use of macroeconomic variables to estimate future model parameters, refer to Kobayashi (2017). This is to align future movements of the spot-rates with the performance in the economy.

## 1.6. Objectives

The key objective of this research is to explore the dynamism of several parametric term structure models (i.e. linear parametric models, cubic splines, extensions made on the Nelson Siegel framework) in estimating real spot rates in South Africa during the period of Covid-19 stress and monetary policy switches. However, it is noted that the South African inflation-indexed bond market is less liquid and has fewer points (Mashoene et al., 2021). According to Lartey and Li (2018) and Mashoene et al (2021), selected term-structure models have the capability to model a term-structure with fewer and illiquid spot-rates.

The main objective can be broken into the following specifics:

1. To fit the real spot rates in South Africa on parametric term structure models (i.e. linear parametric models, cubic splines, extensions made on the Nelson Siegel framework) that will capture different dynamics and shapes of the term structure over different monetary policy regimes.

2. To apply regression model on parameters of the selected parametric term-structure models to forecast/estimate future real spot rates in South Africa over different monetary policy regimes.
3. The application of equal mean methodology to backtest the performance of selected parametric term-structure models used to forecast the real spot rates in South Africa during different monetary policy regimes.

## 1.7. Research questions

In addressing the main objective, the study answers the following questions:

1. Which term-structure model is most suitable for capturing the dynamics and shapes of the real spot rates in South Africa under stressed market conditions?
2. What term-structure models can forecast the real spot rates in South Africa under a deflationary monetary policy regime?
3. How is the performance of the chosen term-structure model in forecasting the real spot rates in South Africa during different monetary policy regimes?

## 1.8. Significance of the study

This study forecasts the South African real spot rate curve, especially during a deflationary monetary policy regime. Mashoene et al. (2021) indicate that "the South African inflation-indexed debt market is illiquid in nature, given that most investors buy to hold the bonds". These are mainly pension fund managers who use this bond instrument to protect their long-term liabilities (National Treasury, 2020a). Also, given its newness in the market, a dynamic term structure model that can capture different shapes of the yield curve over different economic environments is very critical for emerging markets whereby, as indicated in Oji (2015), bond markets are relied upon for funding government deficit and other capital investments.

The study supplements the work done by Johannesburg Stock Exchange (2012) and the current econometric model currently used by National Treasury to price these instruments. Johannesburg Stock Exchange (2012) highlighted no real overnight rate trading in the market. The Johannesburg Stock Exchange (2012) methodology depends on the nominal overnight rate to calculate the real overnight rate. The estimated real spot rate will then price the entire spot rate curve. However, it was indicated by Johannesburg Stock Exchange (2012: pp.12) that "even though this methodology is consistent with the pricing of the inflation-indexed bonds, the indexation factor is seasonal and could result in some instability on shorter maturities of the real curve".

The econometric methodology highlighted in Mashoene et al. (2021) can only estimate the 3-month and 10-year points on the entire yield curve. This might limit the bond pricing given that most government funding is based on the ultra-long end of the curve (i.e. between 12-year and 30-year points), refer to Table 1.4.3. Using a term-structure methodology to estimate all points on the curve is critical for government budgeting processes and fairly pricing new inflation-indexed bonds when introduced in the market.

## 1.9. Methodology overview

The Bloomberg platform is user-friendly and can handle large datasets all at once; as such, it was used to retrieve inflation-indexed all-in prices for South Africa. South African real spot rates were retrieved from the Johannesburg Stock Exchange (JSE) platform. Even though the data is not actual real spot rates traded in the secondary market, but the estimated rates based on the Johannesburg Stock Exchange (2012) methodology, it is used for backtesting the output from selected term-structure models.

Relevant textbooks and most recently published journals/papers were used to find references on applying mathematical term-structure models during different monetary policy regimes. The inferential research approach was applied where a selected sample of coupon-bearing bond prices for South African inflation-indexed bonds was tested to conclude the entire population. This choice is due to the illiquid nature of the bond instrument, where coupon-bearing yields might experience none/minimal movements every month, especially when there are few or no changes in monetary

rates or issuance in the primary market for a particular point in the curve. Minimal movements are attributable to the fact that inflation-indexed bond yields react more to monetary policy changes than trends in the financial market (Christensen and Rudebusch, 2017). Given its free access and capabilities to deal with a large dataset, R-software was used to implement computational techniques for modelling spot-rates based on the selected term-structure models.

## 1.10. Dissertation structure

This study is dissected into six sections, as follows:

- Chapter 1 (Overview of the study) provides the study's background information. It also explains the problem statement and outlines the research significance and questions to be addressed.
- Chapter 2 (Theoretical literature review): explains the literature review on applying coupon-bearing inflation-indexed bond prices to extract the spot rates. To cater for the illiquid nature of the inflation-indexed bond market in South Africa, linear parameterised term structure models will be explored. Further consideration on the dynamism of term structure models that follow Nelson and Siegel (1987) outline will be explored with the possibility of including regime switches to cater for observed global changes in monetary policy regimes.
- Chapter 3 (Conceptual literature review): highlights the overview of the bond market and the effects of monetary policy regime switches on the risks affecting the bond market in South Africa. A further review of lessons learnt from previous studies, peer and big economies in navigating the bond pricing methodology during periods of monetary policy regime switches and deflationary economic environment.
- Chapter 4 (Methodology): details the overall methodology framework to be used in this study for analysing the data and fitting and forecasting the selected term structure models on the actual data.



- Chapter 5 (Data presentation, analysis and discussion): will give results of the model selection process, how well the selected models fit the underlying input data and how well the selected models perform in forecasting the entire term structure throughout the deflationary monetary policy.
- Chapter 6 (Summary, conclusion and recommendations): will give a conclusion and recommendations that could be derived from the findings made in this study.

## Chapter Two

### Theoretical literature review

#### 2.1. Introduction

The previous chapter provides, among other things, the background to this study, objectives and questions to be addressed by this study. As such, to address the aim of this research, this chapter looks at a literature study of the commonly used mathematical models in the market and further assesses the capabilities of these models in addressing the stated questions for this study. Furthermore, the derivation is analysed in detail to understand the background of each term-structure model.

#### 2.2. Bond pricing methodology

Based on the Johannesburg Stock Exchange (2012), the pricing of the inflation-indexed bond is defined as:

$$B(t, \tau) = \sum_{t=1}^{\tau} \left[ \frac{\{CF(t, \tau) * (1 + \pi_t)^{\tau}\}}{(1 + y(t, \tau))^{\tau}} \right] \quad (2.1)$$

where

- $t$  is today in years.
- $T$  is maturity in years.
- $\tau = T - t$  is the time left to maturity in years at time  $t$ .
- $CF(t, \tau)$  are cashflows which are indexed with inflation  $(1 + \pi_t)$  prevailing at each point in time.

- $y(t, \tau)$  is the per annum inflation-indexed (real) yield-to-maturity (YTM)/coupon-bearing yield at time  $t$  with term-to-maturity  $\tau$  paying coupon twice a year.
- $B(t, \tau)$  is the all-in-price of the inflation-indexed coupon-bearing bond at a given time  $t$  with term-to-maturity  $\tau$ .

Since inflation-accrual is uncertain going forward, inflation rate  $(1 + \pi_t)$  at each point is then added to real YTM to get the equivalent nominal YTM to calculate nominal bond pricing. The discount factor is also indexed with inflation accrual to be equivalent to the discount factor of a nominal bond:

$$B(t, \tau) = \sum_{t=1}^{\tau} \left[ \frac{\{CF(t, \tau) * (1 + \pi_t)^t\}}{\{1 + y(t, \tau) * (1 + \pi_t)^t\}^{\tau}} \right]$$

$$= \sum_{t=1}^{\tau} \left[ \frac{CF(t, \tau)}{(1 + y(t, \tau))^{\tau}} \right]$$

This is equivalent to the non-indexed price of a coupon-bearing bond. The only difference between the pricing of inflation-indexed bonds and nominal bonds is the use of real spot rates and real coupon rates versus nominal rates and nominal coupons.

For this analysis, a continuously compounded real spot curve (spot rates) is considered a theoretical proxy for real spot rates, which reduces the effect of coupon rates and results in a lower maturity-specific risk; the real spot rates are commonly favoured for term-structure analysis. However, these rates are mostly not existing in many countries (e.g. zero-coupon bonds are currently not issued in the South African financial market), or fewer points are obtainable on the whole maturity spectrum. It was further noted by Mashoene et al. (2021: pp. 34) that "the spot rate curve derived under arbitrage-free traditional one-factor models, which use the short rate to derive the whole spot rate curve, might not perfectly derive a spot rate curve for the South African inflation-indexed bonds". This is driven by the illiquid nature of this bond instrument and concerns about one-factor term-structure models, which might not be perfectly suitable for complex curve shapes over different economic environments. As

such, raising a critical need for using a coupon-bearing real YTM curve to construct a real spot rate curve.

Before the bootstrapping process can be introduced, let the price of a real zero-coupon bond with maturity  $T$  at time  $t$  be defined as (Hull, 2009):

$$\hat{P}(t, \tau) = P(\tau, \tau) \left(1 + \frac{R(t, \tau)}{m}\right)^{-m \cdot \tau} \quad (2.2)$$

where  $m$  is the compounding rate per year,  $R(t, \tau)$  is the real spot rate, and  $t$  is time in years. To derive a continuously compounding real spot rate curve, let the compounding rate  $m \rightarrow \infty$ , thus implying that the real spot rate compounds more frequently with a lower rate of return. Taking the limit of the zero-coupon bond price as  $m \rightarrow \infty$ .

$$\lim_{m \rightarrow \infty} \hat{P}(t, \tau) = \lim_{m \rightarrow \infty} \left(1 + \frac{R(t, \tau)}{m}\right)^{-m \cdot \tau} \quad \text{assume } \hat{P}(\tau, \tau) = 1$$

As such, given the continuously compounded inflation-indexed spot rate  $R(t, \tau)$ , the inflation-indexed zero-coupon bond price at a given point in time  $t$  with term-to-maturity  $\tau$  is defined as (Lyu, 2004):

$$\hat{P}(t, \tau) = [1 + R(t, \tau)]^{-\tau} \quad \text{in discrete terms, and}$$

$$\hat{P}(t, \tau) = e^{-R(t, \tau) \cdot \tau} \quad \text{in continuous terms.}$$

Under an arbitrage-free assumption, discount factors and the all-in bond price for coupon-bearing inflation-indexed bonds should satisfy the following expression to estimate the spot rates under the continuously compounding assumption (Voloshyn, 2015):

$$\hat{B}(t, \tau) = \sum_{t=0.5}^{\tau} [CF(t, \tau) \cdot d(\tau)] \quad (2.3)$$

where

- $d(\tau) = e^{-R(t, \tau) \cdot \tau}$  is a discount factor for inflation-indexed bonds using real spot rates  $R(t, \tau)$  under continuously compounding assumption.
- $R(t, \tau)$  is the continuously compounded inflation-indexed spot rate prevailing at time  $t$  with term-to-maturity  $\tau$ . This implies that the spot rate in the document refers to the real spot rate unless indicated otherwise.

Arbitrage-free inflation-indexed coupon-bearing prices can then be set up in the following system:

$$\begin{aligned} B(t, \tau_1) &= (CF_{\tau_1} + 1) \cdot d(\tau_1) \\ B(t, \tau_2) &= CF_{\tau_2} \cdot d(\tau_1) + (CF_{\tau_2} + 1) \cdot d(\tau_2) \\ &\vdots \\ B(t, \tau_n) &= CF_{\tau_n} \cdot d(\tau_1) + CF_{\tau_n} \cdot d(\tau_2) + \dots + (CF_{\tau_n} + 1) \cdot d(\tau_k) \end{aligned}$$

for  $k > n$  cashflows,  $CF_{\tau} \approx CF(t, \tau)$

For bonds paying coupons semi-annually, which can then be written in a matrix form as:

$$\begin{pmatrix} B(t, \tau_1) \\ \vdots \\ B(t, \tau_n) \end{pmatrix} = \begin{pmatrix} CF_{\tau_1} + 1 & 0 & 0 \\ \vdots & \ddots & \vdots \\ CF_{\tau_n} & \dots & CF_{\tau_n} + 1 \end{pmatrix} \begin{pmatrix} d(\tau_1) \\ \vdots \\ d(\tau_k) \end{pmatrix}$$

which can be simplified as a linear function:

$$B = CF \cdot d + e_t \quad (2.4)$$

where  $e_t$  is the error term.

Therefore, the optimisation function can be defined as:

$$\min e_t = B - CF \cdot d \quad (2.5)$$

Then the discount function is defined as:

$$d = CF^{-1}B \quad (2.6)$$

The discount function in (2.6) can only be applied, only if  $CF$  is invertible. However, in the case of South Africa, where only a few inflation-indexed bonds were issued with a large number of cashflows, the cashflow matrix  $CF$  might not be invertible.

This downside leads us to introduce the ordinary least squares (OLS) methodology, which might perform better. Based on the linear function defined by (2.4), the optimisation function for minimising squared errors by applying the OLS methodology is defined as:

$$\min e_t^T e_t = (B - CF \cdot d)^T (B - CF \cdot d) \quad (2.7)$$

Therefore, the discount function becomes:

$$d = (CF^T \cdot CF)^{-1} CF^T \cdot B \quad (2.8)$$

The discount function in (2.8) can also be applicable if and only if  $[CF^T, CF]$  is invertible. However, this might not always be the case since  $[CF^T, CF]$  might be a singular matrix with a 0 determinant.

## 2.3. Mathematical term-structure models

In exploring the effect of regime switches on calculating the spot rate curve, we refer to the work done on peer economies and other big economies. Further, the dynamism of term-structure models tested in Reid (2009) and Mashoene et al. (2021) is also explored (i.e. recalibration process on Nelson-Siegel and Svensson model) However, the term structure models with dynamic parameters were not ruled out even though their fitting capabilities were marginally lower. The Arbitrage-Free Generalised Nelson-Siegel model with the yield adjustment factor did not give any significant improvements compared to the dynamic generalised Nelson-Siegel model. As such, only the following models will be explored in this study.

### 2.3.1. Linear parametric model

To address a concern on a singular cashflow matrix in Section 2.2, some of the most common models used are linear parameterised yield curve models with model parameters based on the term-to-maturity. The discount curve defined as  $d(\tau) = d(\tau; a, b, \dots)$  is derived from a term-to-maturity (i.e.  $T - t = \tau$ ) as a function and other parameters (i.e.  $a, b \dots$ ). They can be expressed as a linear set of some basis elements. The discount function can then be expressed as (Suli, 2014):

$$d(\tau) = \sum_{i=1}^m \gamma_i \varphi_i(\tau) \quad (2.9)$$

where  $\varphi_i(\tau)$  is a given set of basis functions for  $i = 1, 2, \dots, m$  and  $\gamma_i$  is a vector of parameters estimates to derive the discount curve. The discount curve can then be defined in a matrix form as:

$$\begin{pmatrix} d(\tau_1) \\ \vdots \\ d(\tau_k) \end{pmatrix} = \begin{pmatrix} \varphi_1(\tau_1) & \cdots & \varphi_m(\tau_1) \\ \vdots & \ddots & \vdots \\ \varphi_1(\tau_k) & \cdots & \varphi_m(\tau_k) \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_m \end{pmatrix}$$

and can be expressed as a linear function:

$$d = \varphi\gamma \tag{2.10}$$

Given (2.10), then (2.4) can be expressed as:

$$B = C.\gamma + e_t \quad \text{where } C = CF.\varphi \tag{2.11}$$

Using OLS to minimise the  $e^T e$ :

$$\hat{\gamma} = (C^T C)^{-1} C^T B \tag{2.12}$$

The estimated vector of  $\hat{\gamma}$  can derive the discount curve in (2.10). To obtain the continuously compounding real spot rate  $R(t, \tau)$  curve, recall (2.3) where:

$$d(\tau) = e^{-R(t, \tau) \cdot (\tau)}$$



Then the continuously compounding real spot rate curve is then expressed as:

$$\hat{R}(t, \tau) = -\ln[d(\tau)]/\tau \quad \text{where } \tau = T - t \quad (2.13)$$

The parameterised yield curve model might be more suitable for deriving the real spot rate curve in South Africa; firstly, illiquid or undeveloped financial markets might result in the inability to buy/sell financial instruments anytime there is a need. Liquidity issues might also result in no/few bond prices that can be used to estimate the zero-coupon term structure. The benefit of parametric yield curve models is their ability to overcome this issue. Secondly, Fewer parameters are estimated to derive the entire curve, which results in a complete yield curve, not only a set of discount factors.

### 2.3.2.Cubic splines model

The cubic spline methodology is one of the standard linear function methods used in the market, and it is less complex. However, Suli (2014: pp. 30) indicated that "even though it uses a linear function, more parameters can be estimated to construct the entire discount factor curve". The use of more parameters is because different polynomial functions are joined on separate knots using knot points over the whole yield curve; as such,  $m(n - 1)$  parameters are needed to derive a discount factor curve using a cubic spline of a polynomial of order  $m$  with  $n - 1$  knot points. The discount factor curve is then presented as follows:

$$d(\tau) = \sum_{i=1}^{n-1} \gamma_{i,j} [\varphi_{i,j}(\tau)] \quad (2.14)$$

where  $\varphi_{i,j}(\tau)$  is a given set of basis functions for  $i = 1, 2, \dots, n - 1$  segments and  $j = 1, 2, \dots, m$  number of knots points; and  $\gamma_{i,j}$  are parameters that will be estimated to derive the discount curve.

The discount curve can then be defined in a matrix form:

$$\begin{pmatrix} d(\tau_1) \\ \vdots \\ d(\tau_k) \end{pmatrix} = \begin{pmatrix} [\varphi_{i,1}(\tau_1)] & \cdots & [\varphi_{i,m}(\tau_1)] \\ \vdots & \ddots & \vdots \\ [\varphi_{i,1}(\tau_k)] & \cdots & [\varphi_{i,m}(\tau_k)] \end{pmatrix} \begin{pmatrix} \gamma_{i,1} \\ \vdots \\ \gamma_{i,m} \end{pmatrix}$$

Given (2.10), then (2.4) can be expressed as:

$$B = C \cdot \gamma + e_t \quad \text{where } C = CF \cdot \varphi \quad (2.15)$$

Using OLS to minimise the  $e^T e$ :

$$\hat{\gamma} = (C^T C)^{-1} C^T B \quad (2.16)$$

Using different knots to dissect the curve into differentiable polynomial functions gives an advantage to the splines to fit complex shapes of the yield curve. The short-term maturities have more volatility as they mimic money market instruments mostly used for cash management. As such, applying this model might help capture this volatility. However, the downside of this model is the tedious exercise involved in the derivation of the model.

### 2.3.3. Nelson and Siegel (1987) model

Based on the intuitive appeal and implementational easiness of the Nelson and Siegel (NS, 1987) non-linear parameterised yield curve model; Coroneo, Nyholm and Vidova-Koleva (Coroneo et al., 2008) indicated that the model became more popular among central banks and most researchers relative to arbitrage-free yield curve models. The forward rate  $f(t, \tau)$  was then defined as:

$$f(t, \tau) = \sum_{i=0}^k \beta_i e^{-\lambda_i \cdot \tau} \quad (2.17)$$

This then implies that the forward rate with real and unequal roots<sup>1</sup> will be defined by:

$$f(t, \tau) = \beta_0 + \beta_1 e^{-\lambda_1 \cdot \tau} + \beta_2 e^{-\lambda_2 \cdot \tau} \quad (2.18)$$

However, (2.18) is over-parameterised, which might lead to non-convergence in non-linear model estimation. Nelson and Siegel (1987) then suggested a more parsimonious model that can generate the same range of shapes and was given by the solution for the case of equal roots defined as a Laguerre function<sup>2</sup> plus a constant:

$$f(t, \tau) = \beta_0 + \beta_1 e^{-\lambda \cdot \tau} + \beta_2 \lambda \tau e^{-\lambda \cdot \tau} \quad (2.19)$$

The simple parameterisation method proposed by Nelson and Siegel (1987) for the term structure of interest rates became more popular in the market. The method is the most well-known among families of smooth forward rate curves, and it is based on the parameterisation of the forward rate where  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$  and  $\lambda$  are constants. The model is based on the four parameters to model the forward rate. However, the main advantage is that this methodology guarantees an even and flexible curve. With term-

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<sup>1</sup> The roots of an equation are defined as real and unequal if the discriminant is positive and are real and equal if the discriminant is zero.

<sup>2</sup> Laguerre function is a method used for generalization to higher-order models and is defined by a polynomial multiplied by an exponential decay term.

to-maturity  $\tau$  at time  $t$ , the zero-coupon rate is then defined in Proof 1 (Annexure A) as:

$$\hat{R}(t, \tau) = a + b \left[ \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right] - ce^{\lambda\tau} + \varepsilon_t \quad (2.20)$$

where  $a = \beta_0$ ;  $b = \beta_1 + \beta_2$ ;  $c = \beta_2$  and  $\varepsilon_t \sim N(0, \sigma^2)$  are independent error terms for  $t = 1, \dots, T$ .

It is assumed that when time-to-maturity increases, the curvature  $\beta_2$  and the slope  $\beta_1$  and components disappear, and the long-term spot rate level converges to the constant component  $\beta_0$  of the model. In a case of a very small term-to-maturity, the spot rate converges to  $\beta_0 + \beta_1$ . The curvature  $\beta_2$  component of the model measures the intensity of the hump/trough, and the decay factor  $\lambda$  establishes the position of the hump/trough of the curvature and the steepness of the slope factor.

The optimisation process for finding optimal parameters that might fit the real spot rate curve in South Africa, the Nelson and Siegel (1987) model, can be built as a normal regression. The normal regression can only be possible if the decay rate  $\lambda$  is pre-defined.

Given (2.3), the theoretical zero-coupon bond price is defined as:

$$\begin{aligned} B(t, \tau) &= \sum_{\tau=0.5}^T [CF(t, \tau) \cdot d(\tau)] \\ &= \sum_{\tau=0.5}^T CF(t, \tau) \cdot e^{-\left[ a\tau + b \left[ \frac{1 - e^{-\lambda(\tau)}}{\lambda} \right] - c\tau e^{-\lambda(\tau)} \right]} \end{aligned} \quad (2.21)$$

where the discount factor  $d(\tau) = e^{-\tau \cdot R(t, \tau)} \approx e^{-\left[ a\tau + b \left[ \frac{1 - e^{-\lambda(T-\tau)}}{\lambda} \right] - c\tau e^{-\lambda(T-\tau)} \right]}$ .

To derive the spot rate curve, theoretical prices of coupon-bearing bonds can be represented in a matrix form:

$$B = CF \cdot d \quad \text{where } d = e^{-\tau.R(t,\tau)}$$

$$CF^{-1}B = e^{-\tau.R(t,\tau)} \quad \text{let } P \approx CF^{-1}B \text{ and}$$

Since  $CF^{-1}B$  [which is equivalent to the discount factor or the actual price of a zero-coupon bond  $P \approx P(t, \tau)$ ] is known and  $\lambda$  is pre-fixed, the theoretical price of the zero-coupon bond is estimated by using OLS to minimise the squared deviation term between the actual and the model estimated bond price:

$$\min_{\gamma} \varepsilon_t = [P(t, \tau) - \hat{P}(t, \tau)]^2 \quad (2.22)$$

where  $\hat{P}(t, \tau) = e^{-a.\tau - b\left[\frac{1-e^{-\lambda\tau}}{\lambda}\right] + c\tau e^{-\lambda\tau}}$  is the theoretical zero-coupon bond price and the estimation coefficients  $\gamma = a, b, c$ .

The log function can be applied on both sides to achieve a linear function. The function can then be presented linearly as:

$$\dot{P}(t, \tau) = -a.\tau - b\left[\frac{1-e^{-\lambda\tau}}{\lambda}\right] + c\tau e^{-\lambda\tau} + \varepsilon_t \quad \text{let } \dot{P}(t, \tau) = \ln \hat{P}(t, \tau) \quad (2.23)$$

This can be presented in a matrix form as:

$$P = \theta.\gamma \quad (2.24)$$

$$\text{where } \theta = \begin{bmatrix} -\tau_1 & -\frac{1-e^{-\lambda\tau_1}}{\lambda} & \tau_1 \cdot e^{-\lambda\tau_1} \\ \vdots & \vdots & \vdots \\ -\tau_n & -\frac{1-e^{-\lambda\tau_n}}{\lambda} & \tau_n \cdot e^{-\lambda\tau_n} \end{bmatrix} \text{ and } \gamma = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

Nelson and Siegel (1987) term-structure model is a non-linear model which might be challenging to find a perfect fit for the complex shape of a term-structure. By fixing the decay rate, the Nelson Siegel model becomes a simple linear problem. However, the following drawbacks could be derived from using the Nelson-Siegel model to estimate the spot rate curve. It was argued by Bjork and Christensen (1999: pp. 4) that Nelson and Siegel model has a possibility of arbitrage opportunity, thus implying that the 'consistency between the dynamic evolution of spot rates and the actual shape of the spot rate curve is not ensured at certain times'.

Annaert, Claes, Ceuster and Zhang (Annaert et al., 2012) indicated that multicollinearity between the slope and curvature factor might cause instability in the regression coefficients, thus, resulting in higher standard errors. The spot rate estimates might also be inconsistent, violating the economic intuition in the Nelson-Siegel model. Annaert et al. (2012) further argued that the spot rate curve defined by the Nelson-Siegel model permits a one hump/dip in the curve. This implies that the model might not perfectly fit the shape of a very complex spot rate curve. Van Elen, Mahieu and Schumacher (Van Elen et al., 2012) also argued that the Nelson-Siegel model gives a perfect fit for in-sample time series; however, it performs very poorly for out-of-sample forecasts due to constant parameters that are supposed to vary with time.

The following sections explore more sophisticated developments of the Nelson-Siegel framework (i.e. dynamic coefficients, an increase in the number of coefficients and rotated coefficients). This will determine the practicality of diverting from the Nelson-Siegel model to more sophisticated models following the Nelson-Siegel framework for using coupon-bearing bond prices for inflation-indexed instruments to extract the South African real spot rates.

#### 2.3.4. Dynamic Nelson-Siegel (2006) model

Nelson and Siegel (1987) model won its popularity due to its ability to perfectly capture the movements of a cross-section of spot rates at any given point in time. However, considering that financial markets operate in a dynamic environment, to comprehend the development of the bond market over time, a dynamic representation of Nelson and Siegel (1987) model was developed by Diebold and Li (2006) with the following assumptions by Koopman, Mallee and Van der Wel (Koopman et al., 2010). For each maturity, factor loadings remain constant over time to allow time-varying factors to be estimated in a linear form. The volatility of each factor also remains constant for the entire sample period. It was further argued that a series of time-varying factors seem to be highly correlated, thus implying that the coefficients are forecastable. As such, when compared with other models (i.e. random walk method, univariate autoregressive models and trivariate vector autoregressive models), the Diebold, Nelson and Siegel (2006) model, known as Dynamic Nelson-Siegel (2006) model, does much better than directly forecasting series of different spot rates, more so in the long-end of the curve.

Koopman et al. (2010) indicated that in fitting the Dynamic Nelson-Siegel (2006) model, the pre-fixed level of  $\lambda$  over time is not sensitive to time-varying factor loadings. Since  $\lambda$  governs spot rates for the maximum level of the curvature factor and the decay rate for the slope factor, the fixed level of  $\lambda$  also keeps these values fixed over time. The representation of the Dynamic Nelson-Siegel (2006) model is defined by a Nelson and Siegel (1987) model with time-dependent parameters:

$$\hat{R}(t, \tau) = a_t + b_t \left[ \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right] - c_t e^{\lambda\tau} \quad (2.25)$$

where  $a_t = \beta_{0,t}$ ,  $b_t = \beta_{1,t} + \beta_{2,t}$ ,  $c_t = \beta_{2,t}$  are time-varying factors that affect a dynamic structure of the spot rate curve and  $\varepsilon_t \sim N(0, \sigma^2)$ .

The theoretical zero-coupon bond price is then defined as:

$$\hat{P}(t, \tau) = e^{-a_t \cdot \tau - b_t \left[ \frac{1 - e^{-\lambda \tau}}{\lambda} \right] - c_t \cdot \tau e^{\lambda \tau}} \quad (2.26)$$

The same optimisation procedure, as in Section 2.3.3, is followed with  $\gamma = \begin{bmatrix} a_t \\ b_t \\ c_t \end{bmatrix}$ .

The same disadvantage of the spot rate curve having one hump/dip can also be observed in the Dynamic Nelson-Siegel (2006) model, thus implying that the model might not perfectly fit more complex models.

### 2.3.5. Svensson (1994) model

For the given level of the decay factor, in-sample shortcomings are observed in the Nelson-Siegel model (i.e. a rapid decay is observed on the slope and curvature factors as a function of term-to-maturity), which led to the development of the Svensson (1994) model. This implies that the level factor is the only variable available to fit spot rates with term-to-maturities of at least ten years, thus, presenting a poor fit for long-term spot rates (Christensen, Diebold and Rudebusch [Christensen et al.], 2008). Svensson (1994) model incorporated the second curvature with an equivalent decay factor as an extension of the Nelson-Siegel model, thus defining the forward rate as:

$$f(t, \tau) = \beta_0 + \beta_1 e^{-\lambda_1(T-t)} + \beta_2 \lambda_1 (T-t) e^{-\lambda_1(T-t)} + \beta_3 \lambda_2 (T-t) e^{-\lambda_2(T-t)} \quad (2.27)$$



Then the spot rate curve is then expressed as follows:

$$\hat{R}(t, \tau) = a + b \left[ \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right] - c e^{\lambda_1 \tau} + d \left[ \frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right] + \varepsilon_t \quad (2.28)$$

where  $a = \beta_0$ ,  $b = \beta_1 + \beta_2$ ,  $c = \beta_2$  and  $d = \beta_3$  are constant factors for the spot rate curve and  $\varepsilon_t \sim N(0, \sigma^2)$ .

It is assumed that when time-to-maturity increases, the slope  $\beta_1$  and the curvature  $\beta_2$  &  $\beta_3$  factors disappear, and the long-term spot rate level converges to the constant component  $\beta_0$  of the model. In a case of a very small term-to-maturity, the spot rate converges to  $\beta_0 + \beta_1$ . Curvature components (i.e.  $\beta_2$  and  $\beta_3$ ) of the model measure the intensity of the hump/trough in the short and long term, respectively, and the decay factors (i.e.  $\lambda_1$  and  $\lambda_2$ ) establish the position of the hump/trough of the curvature and the steepness of the slope factor in the short and long term, respectively.

A theoretical zero-coupon bond price is then defined as:

$$\hat{P}(t, \tau) = e^{-a \cdot \tau - b \left[ \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right] - c \cdot \tau e^{-\lambda_1 \tau} - d \cdot \tau \left[ \frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - c_t e^{-\lambda_2 \tau} \right]} \quad (2.29)$$

The same optimisation procedure as in Section 2.3.3 is followed with  $\gamma = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ .

### 2.3.6. The Dynamic Generalized Nelson-Siegel model

Compared to the Nelson and Siegel (1987) model, including more parameters in the Svensson (1994) model has aided the model in estimating different shapes of yield curves. Even so, Carvalho and Garcia (2019) indicated that the critical weakness of

the Svensson (1994) model is the capability to estimate spot rates below zero per cent and the presence of static parameters. The static parameters might fail to perfectly show the changing aspects over time in the financial markets.

Further, the arbitrage-free property might not be possible in the Svensson (1994) model to generate the factor loading structure of two curvatures with only one slope factor as follows:

$$R(t, \tau) = \beta_{0,t} + \beta_{1,t} \left[ \frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right] + \beta_{2,t} \left[ \frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right] + \beta_{3,t} \left[ \frac{1-e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right] \quad (2.30)$$

The use of two curvatures led to the introduction of a Dynamic Generalized Nelson-Siegel model (DGNS model) by including the fifth factor in the form of a second slope, thus defining the forward rate as:

$$f(t, \tau) = \beta_{0,t} + \beta_{1,t} e^{-\lambda_1 \tau} + \beta_{2,t} e^{-\lambda_2 \tau} + \beta_{3,t} \tau e^{-\lambda_1 \tau} + \beta_{4,t} \lambda_2 \tau e^{-\lambda_2 \tau} \quad (2.31)$$

The spot rate is then expressed as:

$$\begin{aligned} R(t, \tau) = & \beta_{0,t} + \beta_{1,t} \left[ \frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right] + \beta_{2,t} \left[ \frac{1-e^{-\lambda_2 \tau}}{\lambda_2 \tau} \right] + \beta_{3,t} \left[ \frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right] \\ & + \beta_{4,t} \left[ \frac{1-e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right] \end{aligned} \quad (2.32)$$

It is assumed that when time-to-maturity increases, the slope (i.e.  $\beta_1$  &  $\beta_2$ ) and the curvature (i.e.  $\beta_3$  &  $\beta_4$ ) factors disappear, and the long-term spot rate level converges to the constant component  $\beta_0$  of the model. In a case of a very small term-to-maturity, the spot rate converges to  $\beta_0 + \beta_1 + \beta_2$ . Curvature components (i.e.  $\beta_3$  and  $\beta_4$ ) of the model measure the intensity of the hump/trough in the short and long term,

respectively. The decay factors (i.e.  $\lambda_1$  and  $\lambda_2$ ) determine the steepness of the slope factor and the location of the hump/trough of the curvature in the short and long term, respectively.

However, given the stochastic dynamics of the DGNS factors, elimination of the arbitrage prospects remains challenging even with the application of the Nelson and Siegel (1987) model to estimate bond prices.

### 2.3.7. The Dynamic Nelson-Siegel model with regime switches

In analysing the work done on big economies, which includes the effect of regime switches; firstly, looking at the work done by Levant and Ma (2017), the regime switches were incorporated into the loading factor of the Dynamic Nelson-Siegel (2006) model and also by "introducing regime-switching factor disturbances by applying a hidden Markov switching component to the factor disturbances in the transition equation", Levant and Ma (2017, pp. 9).

The spot rates using the Dynamic Nelson-Siegel (2006) term structure model are expressed in the form of a matrix as follows:

$$R(t, \tau) = \theta(\lambda) \cdot \beta_t + \varepsilon(t, \tau) \quad \& \quad \beta_t = C + Ax_{t-1} + v_t \quad (2.33)$$

where the disturbance factor  $\varepsilon(t, \tau) \sim N(0, \sigma^2)$ , factor loading matrix

$$\theta(\lambda) = \begin{bmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \left[ \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - c_t e^{-\lambda\tau_1} \right] \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} & \left[ \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} - c_t e^{-\lambda\tau_n} \right] \end{bmatrix}, \text{ latent factor vector } \beta_t = \begin{bmatrix} \beta_{0,t} \\ \beta_{1,t} \\ \beta_{2,t} \end{bmatrix};$$

is a matrix of the autoregressive process coefficient;  $C$  is constant; and volatility  $v \sim N(0, \Sigma)$ .

The Dynamic Nelson-Siegel (2006) model with regime-switching loading parameter (DNS-MSL) was defined in a state-space model as:

$$R(t, \tau) = \theta(\lambda_{S_t}).\beta_t + \varepsilon(t, \tau) \quad (\text{the measurement equation}) \quad (2.34)$$

$$\beta_t = C + A\beta_{t-1} + v_t \quad (\text{the state equation}) \quad (2.35)$$

$$\lambda_{S_t} = \lambda_0(1 - S_t) + \lambda_1 S_t \quad (\text{the state equation}) \quad (2.36)$$

with  $S_t = 0$  or  $1$ , which defines the regime at time  $t$ .

The Dynamic Nelson-Siegel (2006) model with regime-switching volatility for factors (DNS-MSV) using the state-space model was defined as:

$$R(t, \tau) = \theta(\lambda).\beta_t + \varepsilon(t, \tau) \quad (\text{the measurement equation}) \quad (2.37)$$

$$\beta_t = C + A\beta_{t-1} + v_{S_t} \quad (\text{the state equation}) \quad (2.38)$$

$$v_{S_t} = v_0(1 - S_t) + v_1 S_t \quad (\text{the state equation}) \quad (2.39)$$

with  $S_t = 0$  or  $1$  which defines the regime at time  $t$  and volatility  $v_{S_t} \sim N(0, \Sigma_{v_{S_t}})$

The model was fitted on the US Treasury yields from January 1970 to December 2000 using the end-of-month bid-ask averages of the zero-coupon rates. Levant and Ma (2017, pp.12) indicated that the dataset used in the study was "produced through consistent and careful cleaning procedures that remove microstructure noise". It was further indicated that the dataset for the most recent periods was excluded from the study because it included an environment of significantly low nominal interest rates, which might not be able to be captured by the Gaussian type of term structure explored in the study.

<b>Maturity(months)</b>	<b>DNS RMSE</b>	<b>DNS-MSL RMSE</b>	<b>DNS-MSV RMSE</b>
<b>1</b>	0,7296	0,6506	0,7274
<b>3</b>	0,4919	0,4274	0,4840
<b>6</b>	0,2524	0,2875	0,2302
<b>9</b>	0,3087	0,3255	0,3101
<b>12</b>	0,3253	0,3219	0,3272
<b>15</b>	0,3147	0,2901	0,3123
<b>18</b>	0,2928	0,2702	0,2870
<b>21</b>	0,2777	0,2727	0,2719
<b>24</b>	0,2669	0,2774	0,2704
<b>30</b>	0,2702	0,2723	0,2741
<b>36</b>	0,2831	0,2712	0,2890
<b>48</b>	0,3201	0,3047	0,3233
<b>60</b>	0,3024	0,2847	0,3059
<b>72</b>	0,3243	0,3271	0,3223
<b>84</b>	0,3282	0,3350	0,3278
<b>96</b>	0,3264	0,3294	0,3249
<b>108</b>	0,3879	0,3756	0,3898
<b>120</b>	0,4122	0,4140	0,4145
<b>Average</b>	0,3453	0,3354	0,3440

*Table 2.3.1: Treasury yields in-sample evaluation – Root Mean Squared Errors*

*Source: Levant and Ma (2017: pp.26)*

Based on the result analysis presented in Table 2.3.1, it could be noted that when compared to the Dynamic Nelson-Siegel (2006) model estimates, the DNS-MSV model loses fitting capabilities mainly on the longer maturities due to an increased RMSE estimate of over 0.17 per cent on longer maturities of the yield curve. However, more significant improvements in the RMSE estimates for the DNS-MSL model were observed, thus suggesting that switching the loading factor  $\lambda$  resulted in a noticeable improvement in-sample fit for longer maturities compared to the original Dynamic Nelson-Siegel (2006) model.

The inflation-indexed debt portfolio in South Africa has the most extended maturity of 27 years, with most government funding based on the maturity spectrum of at least ten years (National Treasury, 2020b). The application of this methodology might not give any improvements when compared to the Dynamic Nelson-Siegel (2006) term-structure model given the critics above. The most extended maturity fitted in Table 2.3.1 is ten years, and it could be observed that there is a minimal difference in RMSE estimates between the proposed methodologies to the Dynamic Nelson-Siegel (2006) model. Applying this methodology to the South African case might not yield any improvements in calculating the real spot rates.

### 2.3.8. The Dynamic Nelson-Siegel model with time-dependent transition probabilities

Following the work done by Levant and Ma (2017), Kobayashi (2017) expanded the work on the Japanese corporate bond spreads by incorporating the effect of regime switches with time-varying transition probabilities on the Dynamic Nelson-Siegel (2006) model. The work also explored the inclusion of regime switches with both fixed [DNSRS (fp)] and time-dependent [DNSRS (tvp)] transition probabilities on the ordinary Dynamic Nelson-Siegel (2006) model. Based on the latent factor vector  $\beta_t$  in (2.35), for the Dynamic Nelson-Siegel (2006) model with regime switches as defined by the following state-space model:

$$\beta_t - \mu^{S_t} = A(\beta_t - \mu^{S_t}) + v_t \quad v_t \sim N(0, \Sigma) \quad (2.40)$$

where:

$S_t \in \{0,1\}$ : variables representing regime at time  $t$

$\mu^k = (\mu_{\beta_1}^k, \mu_{\beta_2}^k, \mu_{\beta_3}^k)' \in \mathbf{R}^3 (k = 0,1)$ : mean reversion vector with regard to  $k$ .

The constant transition probability is defined as:

$$p_{ik} = Pr[S_t = K | S_t = i] \quad (2.41)$$

with the fixed transition probability matrix defined as follows where  $p$  is the transition probability when regime  $S_t = 0$  is realised and  $q$  is when regime  $S_t = 1$  is realised:

$$\mathbf{P}^Z = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix} \in \mathbf{R}^{2 \times 2} \quad (2.42)$$

The time-dependent transition probability driven by a selected macroeconomic variable  $Z_{t-1}$  at time  $t - 1$  is defined as:

$$p_{ik,t} = Pr[S_t = K | S_{t-1} = i, Z_{t-1}] \quad (2.43)$$

with the transition probability matrix defined as follows:

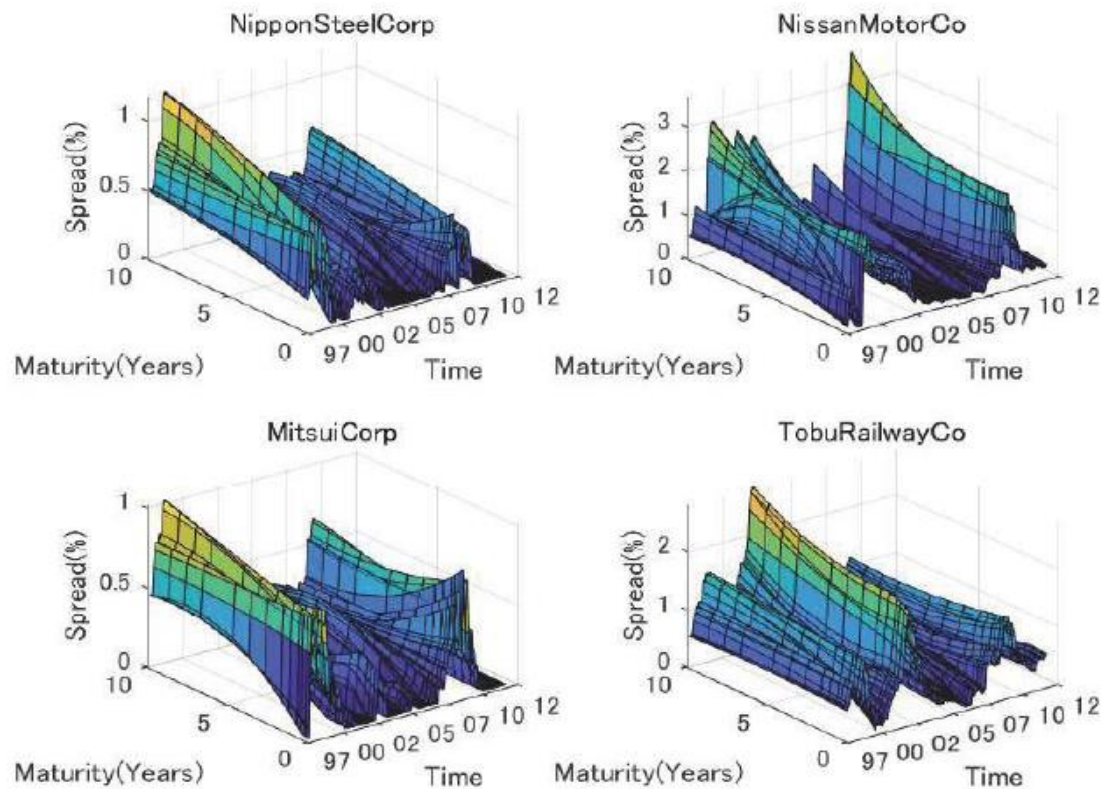
$$\mathbf{P}^{Z_t} = \begin{pmatrix} p(Z_{t-1}) & 1-q(Z_{t-1}) \\ 1-p(Z_{t-1}) & q(Z_{t-1}) \end{pmatrix} \quad (2.44)$$

A logit function was used to define the following transition probability functions:

$$p(Z_{t-1}) = Pr[S_t = 0 | S_{t-1} = 0, Z_{t-1}] = \frac{e^{(p_0 + p_1 Z_{t-1})}}{1 + e^{(p_0 + p_1 Z_{t-1})}} \quad (2.45)$$

$$q(Z_{t-1}) = Pr[S_t = 1 | S_{t-1} = 1, Z_{t-1}] = \frac{e^{(q_0 + q_1 Z_{t-1})}}{1 + e^{(q_0 + q_1 Z_{t-1})}} \quad (2.46)$$

The study was based on the selected 56 firms with maturity structures matching the government yield curve. The corporate bond interest rates were deducted from the corresponding government interest rates to obtain monthly credit spreads. The samples cover a period from September 1997 to December 2011. Figure 2.3.1 shows a time series of credit spreads of the selected sample of four firms.



*Figure 2.3.1: Credit spreads across companies over time*

*Source: Kobayashi (2017: pp. 12)*

In selecting macroeconomic variables that drive the transition probability of the regime switches, Kobayashi (2017) indicated that economic conditions, credit fundamentals and market sentiments were considered the driving factors for credit spreads. A logit regression model was used to estimate the smoothed probability given the selected macroeconomic variables as independent variables. As such, Nikkei, Ted Spread and equity volatility have proven to be the most significant drivers of the credit spread.

DNSRS(fp) / DNS	DNSRS (tvp) / DNS			DNSRS (tvp) / DNSRS (fp)		
	NIKKEI225	Ted Spread	Equity Vol	NIKKEI225	Ted Spread	Equity Vol
100 %	90 %	87 %	91 %	61 %	56 %	65 %

*Table 2.3.2: Ratio of AIC on the number of firms*

*Source: Kobayashi (2017: pp. 25)*



Based on the results in Table 2.3.2, it was proven that the Dynamic Nelson-Siegel model with time-dependent regime switches, DNSRS (tvp), might accurately represent the credit spread's evolution during periods of ambiguous economic conditions. The model displayed more robust in-sample model performance with a relatively higher Akaike Information Criterion (AIC) ratio, which is an indication of a better goodness-of-fit when compared to the original Dynamic Nelson-Siegel (2006) model and Dynamic Nelson-Siegel model with fixed regime switches DNSRS (fp). It was also indicated that the transition matrix was mainly driven by movements in the stock market index and liquidity indicators.

## 2.4. Forecasting methodologies for the real spot rate curve in South Africa

The inflation-indexed bond market in South Africa is very narrow, and few gaps are observable (i.e. the newness of the bonds in the market, lower liquidity and fewer maturities). These gaps might be the factors for the absence of practical studies for constructing the real spot rates. The following published methodologies (i.e. the JSE methodology and the forward rate methodology) will be analysed to make inferences in the study.

### 2.4.1. JSE real yield curve estimation methodology

The Johannesburg Stock Exchange (2012) methodology is currently applied in the market to calculate the real spot rates in South Africa. Given the coupon-bearing bond prices for inflation-indexed instruments, the model-implied spot rates are applied to the estimation methodology. Johannesburg Stock Exchange (2012: pp. 11) indicated that "the first step in estimating the real spot rate curve is finding the real overnight rate; however, the real overnight rate is non-existent, given that there is no overnight inflation-indexed securities trading on the JSE". As such, the nominal overnight rate  $\tilde{r}_t$  and future inflation expectations are used to calculate the real overnight rate:

a) The methodology starts by defining the amount paid at the redemption of the overnight inflation-indexed zero-coupon bond as:

$$FV_{real} = \frac{CPI_{t_0+1}}{CPI_{t_0}} \quad (2.47)$$

where  $CPI_{t_0}$  and  $CPI_{t_0+1}$  represent the 4-month, and 3-month lagged Consumer Price Index (CPI), respectively.

b) The definition of the present value of the zero-coupon is expressed as:

$$\begin{aligned} PV_{real} &= FV_{real} \cdot \text{Discount factor} \\ &= \left( \frac{CPI_{t_0+1}}{CPI_{t_0}} \right) \cdot \left( \frac{1}{1 + \hat{r}_t \left( \frac{1}{365} \right)} \right) \end{aligned} \quad (2.48)$$

where  $\hat{r}_t$  is the nominal short rate

c) The real overnight rate is then obtained from the following expression:

$$PV_{real} = \frac{1}{1 + r_t \left( \frac{1}{365} \right)} \quad (2.49)$$

d) As such, the real overnight rate is defined as:

$$\hat{r}_t = 365 \left[ \frac{CPI_{t_0} \left( 1 + \hat{r}_t / 365 \right)}{CPI_{t_0+1}} - 1 \right] \quad (2.50)$$

The calculated all-in price for an inflation-indexed price is defined as:

$$\hat{B}(t, \tau) = \Gamma[e^{R_{settle} \tau_{settle}} \sum_{i=1}^n CF_i e^{-R_i \tau_i}] \quad (2.51)$$

where:

- $\Gamma$  represents a bond's inflation indexation factor on  $\tau_{settle}$  calculated as the reference CPI corresponding to  $\tau_{settle}$  divided by the bond's-based index (Raffaelli, 2006).
- $CF_i$  is the cashflow with term-to-maturity  $\tau_i$  without inflation indexation for  $i \in \{1, 2, \dots, n\}$
- $R_i$  is the real spot rate with a term-to-maturity  $\tau_i$  for  $i \in \{1, 2, \dots, n\}$

The definition for the real spot rate is as follows:

$$\hat{R}_{\tau_n} = \frac{1}{\tau_n} \left[ \ln CF_n - \ln \left( \frac{B(t, \tau)}{\Gamma} \cdot e^{-R_{settle} \tau_{settle}} - \sum_{i=1}^{n-1} CF_i e^{-R_i \tau_i} \right) \right] \quad (2.52)$$

The calibration process is applied on (2.52) to calculate the spot rate with maturity  $\tau_i$  for  $i \in \{1, 2, \dots, n\}$ . The shortest maturity is calibrated, and the process is repeated until the calibration process is applied to the whole real spot rate curve. Johannesburg Stock Exchange (2012: pp.12) indicated that "even though this methodology is consistent with the pricing of inflation-indexed bonds, it could be noted that since the quantity  $\frac{CPI_{t_0}}{CPI_{t_0+1}}$  is seasonal, this could result with some instability in the short-end of the real curve".

#### 2.4.2. Forward rate methodology

The work by Reid (2009) aimed to look at different mathematical models to calculate the real forward rates for South Africa. Forward interest rates were used to gauge the South African financial market's inflation expectations. To model the inflation expectation using the *Fisher equation*:

$$\pi_t^e = \dot{R}(t, \tau) - R(t, \tau) \quad (2.53)$$

where  $\pi_t^e$  is future inflation expectation,  $R(t, \tau)$  is the real spot rate and  $\tilde{R}(t, \tau)$  is the nominal spot rate.

The modelling process on the real and nominal forward rates was done separately, and after that, estimated future inflation expectations. For calculating forward rates (real and nominal), the Nelson-Siegel model in Section 2.3.3 and the Svensson model in Section 2.3.5 were applied. Based on the zero-coupon bond prices, implied forward rates can be calculated on the assumption that:

$$P(t, \tau) = e^{-(\int_0^\tau f(t, \tau) dt)} \quad (2.54)$$

where  $P(t, \tau)$  is the zero-coupon bond price with term-to-maturity  $\tau$  and  $f(t, \tau)$  is the instantaneous forward rate. However, given that real zero-coupon prices are non-existent in the market, zero-coupon bond prices were extracted from coupon-bearing bond prices as follows

$$\hat{B}(t, \tau) = \sum_t^T CF(t, \tau) \cdot P(t, \tau) \quad (2.55)$$

The parametric models were adopted; where Nelson and Siegel (1987) model is defined as:

$$\hat{f}(t, \tau) = \beta_0 + \beta_1 e^{-\tau\lambda} + \beta_2 \tau \lambda e^{-\tau\lambda} \quad (2.56)$$

and the Svensson (1994) model as:

$$\hat{f}(t, \tau) = \beta_0 + \beta_1 e^{-\tau\lambda_1} + \beta_2 \tau \lambda_1 e^{-\tau\lambda_1} + \beta_3 \tau \lambda_2 e^{-\tau\lambda_2} \quad (2.57)$$

where  $\lambda$  is the exponential decay rate corresponding to the curvature,  $\tau$  is the term-to-maturity, and  $\beta$  is a vector of parameters to be estimated. The Svensson model was opted by imposing a second curvature on longer maturities to "address the issue of constant forward rates in the long end, which is embedded in Nelson-Siegel's methodology" (Svensson, 1994: pp. 16). To calculate ideal parameters  $\beta$  for fitting forward rates for the South African, the maximum likelihood methodology was applied

to minimise the differences between model estimated prices and actual prices. The results of the nominal interest rates are shown in Figure 2.4.1.

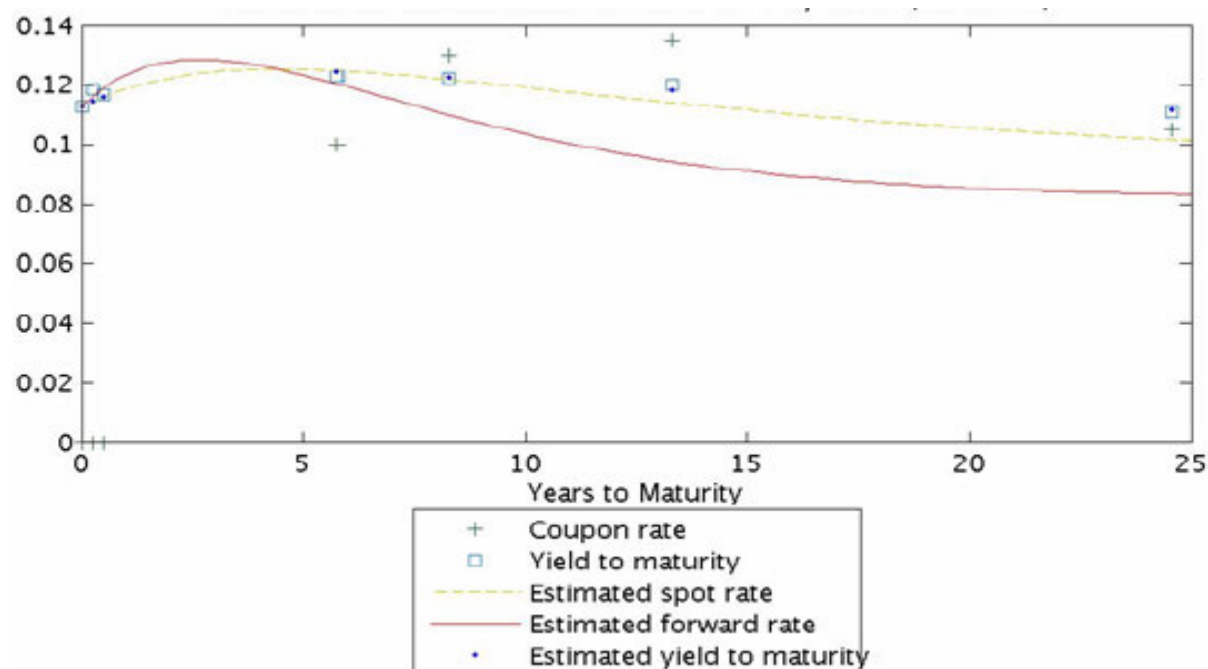
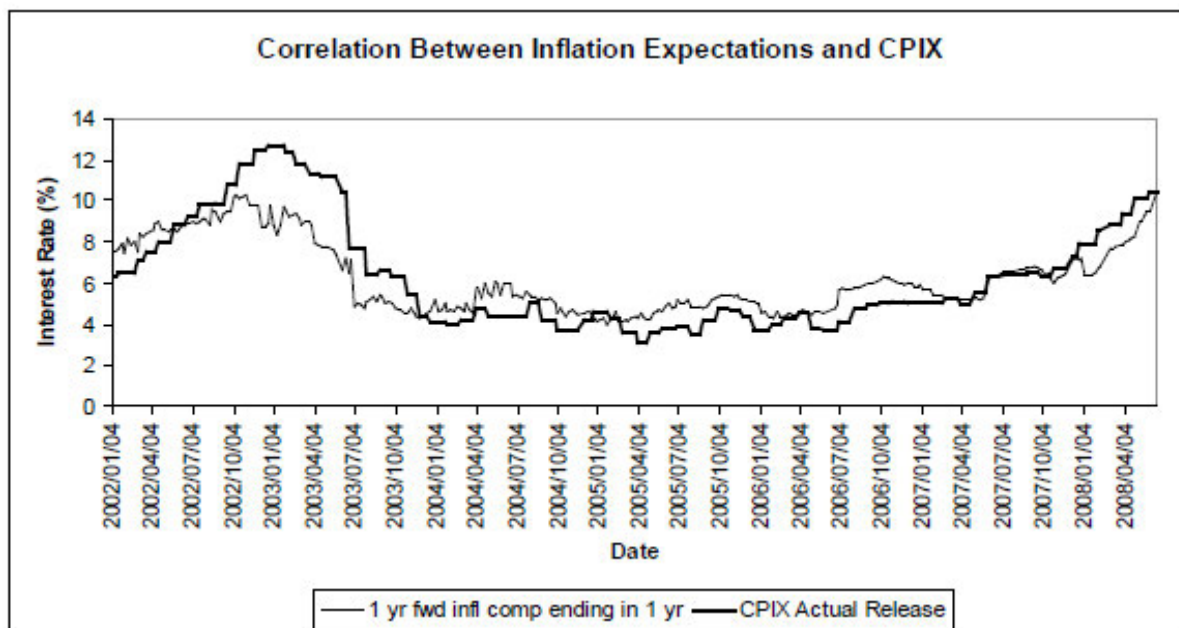


Figure 2.4.1: South African Nominal interest rates

Source: Reid (2009: pp. 13)

Given that only four government nominal bonds were available concurrently over the sample period, Treasury bills (i.e. 3-month, 6-month, 9-month and 12-month bills) were added to define interest rates in the short term. A perfect fit was observed between the actual and estimated yield-to-maturity nominal rates, refer to Figure 2.4.1. The same methodology was applied to the three government inflation-indexed bonds to calculate the real forward rate curve. Due to the absence of the real short rate in the market, the three Treasury bills and SARB overnight rate were first adjusted for inflation expectations using *Reuters Econometer* inflation estimates before being used to determine the real short rates. The daily variation between the real and nominal forward rates was calculated to further analyse the forward inflation compensation. Figure 2.4.2 shows the correlation between inflation expectations and the actual consumer price index.



*Figure 2.4.2: Correlation between inflation expectations and CPIX*

*Source: Reid (2009: pp. 14)*

The correlation analysis between the forward inflation compensation and the consumer price index (CPIX) with a horizon of one year indicates that it increased sharply above the actual CPIX at the end of 2000 and reduced to levels within the inflation target band of 3 to 6 per cent until mid 2007 just before the 2008 global financial crisis.

### 2.4.3. Spot rate methodology under normal market conditions

The most recent work by Mashoene et al. (2021) looked at different mathematical methodologies to calculate the real spot rates for South Africa. However, the analysis was based on periods of normal market conditions. A couple of mathematical methodologies used in the market were selected to calculate the real spot rates in South Africa.

To extract real spot rates from coupon-bearing bond prices for inflation-indexed debt, the ordinary least-squares (OLS) methodology was applied as follows:

$$\min_{\beta} e_t^T e_t = (B - CF \cdot d)^T (B - CF \cdot d) \quad (2.58)$$

where  $\beta$  represents model parameters to be optimised,  $d = e^{-R(t,\tau) \cdot \tau}$  is a discount factor for inflation-indexed bonds using continuously compounded real spot rates,  $B$  is the actual coupon-bearing inflation-indexed bond price, and  $CF$  is the cashflow matrix. Arbitrage-free property is assumed for the extraction of continuously compounded spot rate, thus implying that discount factors together with the coupon-bearing inflation-indexed all-in bond price should satisfy the following:

$$\hat{B}(t, \tau) = \sum_t^{\tau} CF(t, \tau) \cdot d(\tau) \quad (2.59)$$

The initial term-structure model to be applied was the one-factor Vasicek (1977) model with the mean-reversion characteristic. This suggests that for interest rates to not persist in being weak/strong over a more extended period, the short rate converges to the long-term mean. Vasicek (1977) model permits spot rates below zero, which were assumed appropriate for the South African case given negative inflation-indexed coupon-bearing yields, often observed in the short term.

Under the arbitrage-free assumption, the spot rate is expressed as follows:

$$\hat{R}(t, \tau) = - \left[ \ln[\hat{P}(t, \tau)] / \tau \right] \quad (2.60)$$

where

$$\hat{P}(t, \tau) = B_t(T) e^{-A_t(T) r_t} \quad (2.61)$$

$$r_t = r_0 e^{-kt} + \theta(1 - e^{-kt}) + \sigma \int_0^t e^{-k(t-u)} dW_u \quad (2.62)$$

$$A_t(T) = \frac{1}{k} (1 - e^{-k \cdot \tau}) \quad (2.63)$$

$$B_t(T) = e^{\left\{ \frac{\left( \frac{\sigma^2}{2} - k^2 \theta + \sigma k \lambda(t, r_t) \right)}{k^2} [A_t(T) - \tau] - \frac{\sigma^2}{4k} A_t(T)^2 \right\}} \quad (2.64)$$

Moreover,  $\theta$  is calculated as the long-term short-rate average of the inflation-indexed short-rate;  $k$  is estimated as the log function of the autocovariance/autocorrelation as a proportion of the number of days; and  $\sigma$  is estimated as the instantaneous volatility of the inflation-indexed short-rate.

The real short-rate  $r_t$  is critical for estimating the real spot-rate curve under the one-factor Vasicek (1977) model. However, based on the JSE methodology in Section 2.4.1, it was indicated that the real short rate is unavailable in the South African market. More reliance is put on the nominal short rate, which has to be adjusted for inflation to determine the real short rate.

Non-linear parametric term-structure models were also applied to address any drawbacks of arbitrage-free/risk-neutral traditional models. Nelson and Siegel (1987: pp. 474) model is "proposed to model the forward rate using three latent factors by employing a relationship from expectation theory, and the main advantage of this method is that it ensures a smooth and fairly flexible curve".

$$R(t, \tau) = \beta_0 + \beta_1 \left[ \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right] + \beta_3 \left[ \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right] \quad (2.65)$$

The extension of Nelson and Siegel (1987) was introduced by Svensson (1994) model by incorporating the second curvature with the equivalent decay rate. This methodology addressed any possible drawbacks of the Nelson and Siegel (1987) model related to the inability to capture movements in longer maturities (Christensen et al., 2008).

$$R(t, \tau) = \beta_0 + \beta_1 \left[ \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right] + \beta_2 \left[ \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right] + \beta_3 \left[ \frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right] \quad (2.66)$$



Relative to the Nelson and Siegel (1987) model, increased model parameters on the Svensson (1994) model improved the model's performance in fitting more complex shapes; however, Carvalho and Garcia (2019) indicated that the model has constant parameters over time and has no capabilities to produce negative spot rates. As such, Diebold and Li (2006: pp.341) "introduced a time-dependent Nelson and Siegel (1987) model as follows":

$$R(t, \tau) = \beta_{0,t} + \beta_{1,t} \left[ \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right] + \beta_{3,t} \left[ \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right] \quad (2.67)$$

Dynamic Generalised Nelson-Siegel (DGNS) model with a five-factor loading framework was also applied by incorporating a second slope factor. The second slope factor was included to effect an arbitrage-free characteristic, which is impossible for the Svensson (1994) model with the one-slope factor loading framework.

$$\begin{aligned} R(t, \tau) = & \beta_{0,t} + \beta_{1,t} \left[ \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right] + \beta_{2,t} \left[ \frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} \right] + \beta_{3,t} \left[ \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right] \\ & + \beta_{4,t} \left[ \frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right] \end{aligned} \quad (2.68)$$

Bond prices implied by the Nelson and Siegel (1987) term-structure model have embedded arbitrage assumptions; as such, only the introduction of the Arbitrage-Free Generalized Nelson-Siegel/ AFGNS (2009) model could address this weakness:

$$\begin{aligned} R(t, \tau) = & X_t^1 + X_t^2 \left[ \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right] + X_t^3 \left[ \frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} \right] + X_t^4 \left[ \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right] \\ & + X_t^5 \left[ \frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right] - \frac{C_t(T)}{\tau} \end{aligned} \quad (2.69)$$

where  $-\frac{c_t(T)}{\tau} = -\frac{1}{2\tau} \int_t^T \sum_{j=1}^5 [\Sigma' B_u(T) B_u(T)' \Sigma]_{j,j} du$

This led to introduction of the Dynamic Nelson-Siegel (2006) model with the rotated factors of the yield curve so that one of the factors estimated is the short rate. The Rotated Dynamic Nelson-Siegel/RDNS (2018) model was introduced with external factors which might affect the short-rate dynamics being included in the methodology.

Spot rates are expressed in a matrix form following the Dynamic Nelson-Siegel (2006) model structure:

$$R(t, \tau) = \theta \cdot \beta + \varepsilon \text{ and } \beta_t = C + Ax_{t-1} + v_t \quad (2.70)$$

where  $\varepsilon \sim N(0, \sigma^2)$ ,

$$\theta = \begin{bmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \left[ \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - c_t e^{-\lambda\tau_1} \right] \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} & \left[ \frac{1-e^{-\lambda\tau_n}}{\lambda\tau_n} - c_t e^{-\lambda\tau_n} \right] \end{bmatrix}, \beta = \begin{bmatrix} \beta_{0,t} \\ \beta_{1,t} \\ \beta_{2,t} \end{bmatrix}$$

Then under the Rotated Nelson-Siegel (2018) model, the spot rate curve is defined as follows:

$$R(t, \tau) = G \cdot \gamma + \varepsilon \quad (2.71)$$

with the rotation matrix D, then

$$G = \theta \cdot D^{-1}$$

$$\gamma = D.\beta$$

$$m = D.C$$

$$F = D.A.D^{-1}$$

$$z_t = A.v_t$$

Quarterly data of South African inflation-indexed bond prices from April 2010 to April 2018 was used for model development and a two-step forecast covering May 2018 to October 2018. It was observed for model development that there were few variations between the model estimates and actual prices in the short term. This was attributed to the money-market characteristics embedded in this area of the curve in line with findings made by Reid (2009). Vasicek (1977) one-factor model priced the South African real zero-curve in deep discount, which is inconsistent with the bond pricing of the real spot rates.

<b>Root Mean Squared Error</b>	<b>Time step 1</b>	<b>Time step 2</b>
Linear model	0.016	0.015
Cubic splines model	0.017	0.014
Nelson-Siegel model	0.018	0.023
Dynamic Nelson-Siegel model	0.056	0.078
Svensson model	0.018	0.029
Dynamic Generalised Nelson-Siegel model	0.101	0.334
Arbitrage-Free Generalised Nelson-Siegel model	0.101	0.139

*Table 2.4.1: RMSE estimates on forecasted values*

*Source: Mashoene et al. (2021: pp.14)*

Based on the RMSE estimates, the rotation process on the Rotated Dynamic Nelson-Siegel (2018) model did not improve the original Dynamic Nelson-Siegel (2006) model. From Lartey and Li (2018) on the study of Ghanaian bond rates, it was determined that the Svensson (1994) model could perfectly fit the bond market with liquidity issues and longer maturities. It was further noted in Table 2.4.1 that the static term-structure

models performed relatively better than dynamic term-structure models in predicting future real spot rates in South Africa.

## 2.5. Conclusions

It was observed in the studies conducted on the real spot rates for South Africa that the Svensson (1994) term structure model gave better fitting capabilities under normal market conditions; however, Mashoene et al. (2021) indicated that the fitting capability diminishes over the periods of high stress. This suggested a need for a term-structure methodology that is suitable to calculate the real spot rates for South Africa over different market/ economic conditions and be able to give better forecasts in the foreseeable future for medium-term budget forecasts. According to Reid (2009) and Mashoene et al. (2021), it was also noted that the selected term-structure models might not be able to perfectly capture movements of short-term rates, given the money market characteristics embedded in this area of the curve.

This study tries to find a term-structure model that incorporates dynamism in the economy. The initial step involves the use of recalibration methodology to ensure that static model parameters reference the prevailing economic conditions. This is applied on linear parametric, cubic splines, Nelson-Siegel and Svensson models to address a poor performance during stressed condition indicated by Mashoene et al. (2021). Further, the study explores the implication of dynamic/time-dependent decay rate on the fitting of term-structure models using the Dynamic Nelson-Siegel (2006) model. Previous studies by Reid (2008) and Mashoene et al. (2021) were based on the constant decay rate for estimating South African real spot-rates. Based on Castello and Resta (2022), the dynamic/time-dependent decay rate on applying the Dynamic Generalised Nelson-Siegel model gave a relatively better performance for some leading emerging markets during the Covid-19 stress. This extension might improve the dynamism of the Dynamic Nelson-Siegel (2006) especially during the heightened Covid-19 stress.

Finally, the study extends the application of the autoregressive (AR) model by exploring the incorporation of macroeconomic variables in estimating future model parameters

to improve model performance for forecasting purposes. Caution was taken given that correlations do break during high stress; as such, it might not be possible to ultimately get macroeconomic variables that can explain movements of the model parameters during the Covid-19 stress period. These extensions are explored to improve the fitting and forecasting capabilities of the selected term-structure models in estimating the real spot rates for South Africa.

## **Chapter Three**

### **Conceptual literature review**

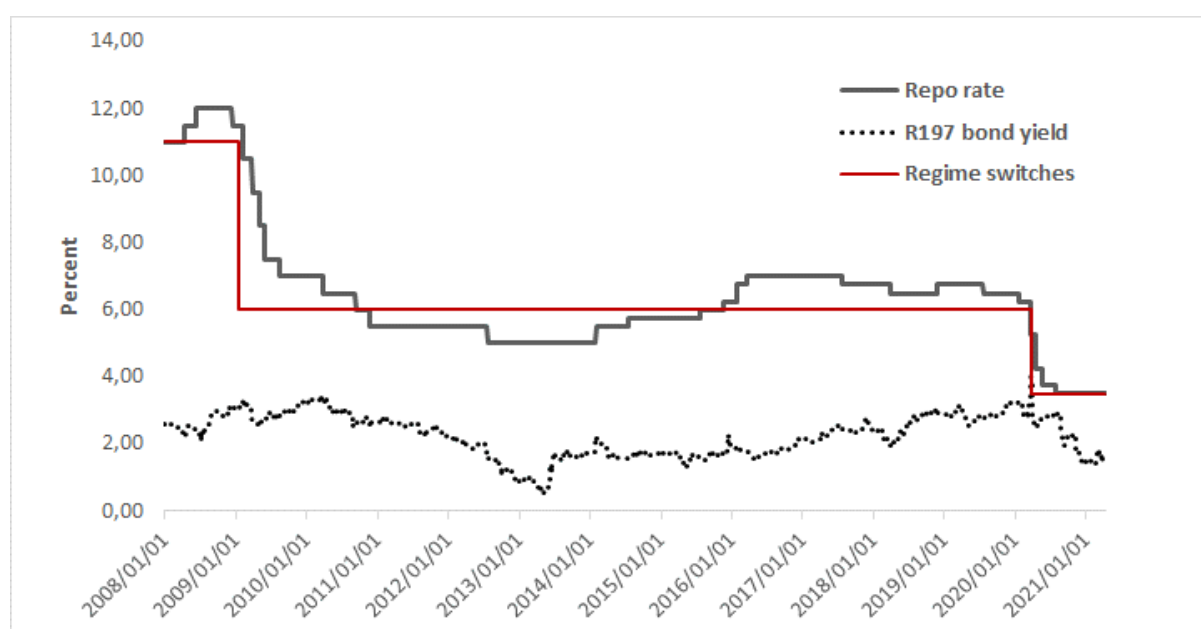
#### **3.1. Introduction**

The previous chapter noted that applying time-dependent term-structure models could capture movements and shapes of yield curves under different economic environments. However, given the "1-in-100 years" economic stress affected by the Covid-19 pandemic, correlations between macroeconomic factors do break, which can yield poor performance in using macroeconomic factors to estimate future model parameters. This chapter introduces the concept of monetary policy regime switches on bond markets and its application in the South African framework. It also looks at the effect of Covid-19 on the risk factors affecting the South African bond market. Furthermore, a review was also done on the latest term-structure models used by big economies and other emerging markets in modelling illiquid spot rate curves during periods of high stress and different monetary policy regimes.

#### **3.2. Monetary policy regime switches in South African funding requirements**

The emergence of the Covid-19 pandemic has been classified as a "1-in-100 years" event which negatively impacted the world's social, political and economic facets. A total closure in the global economy through lockdowns/restricted movements resulted in a dire situation globally. Naidoo et al. (2020) have indicated that the South African financial sector demonstrated its resilience and agility against the effect of the Covid-19 pandemic, which tested each country's ability to respond to the crisis. The capacity and availability of instruments to swiftly address this issue from a social, financial and economic point of view were critical to ensuring business continuity for both public and private sectors.

The main objective of the SARB, as indicated by South African Reserve Bank (2022), Ncube et al. (2013) and Coco et al. (2019), is to protect the value of the currency and ensure price stability through an inflation-targeting framework. Interest rates (i.e. repurchase/repo rate) are the current instrument used by the SARB to maintain price stability in the economy. The repo rate is set at a level that will bring the inflation forecasts close to the specified inflation target in the prescribed future (Croce and Khan, 2000). Different regime changes from 2008 to 2021, and the consequence of monetary policy regime changes on the movements of the inflation-indexed coupon-bearing yield can be observed in Figure 3.2.1.



*Figure 3.2.1: Yield curve movements*

*Source: Bloomberg and South African Reserve Bank*

Post the 2008 global financial crisis, a downward regime was observed where interest rates were significantly cut from 11 per cent to 6 per cent. The stable regime was observed from 2009 until 2020, when the downward regime was re-introduced to relieve liquidity stress in the market brought on by the effect of the Covid-19 pandemic, South African Reserve Bank (2022). The R197 (5.5%; 2023) bond was used as a benchmark to highlight this effect, specifically in the short-to-mid end of the curve. It

can be observed that regime changes in the monetary policy significantly impact the movements in the inflation-indexed yield curve. The inflation-indexed/real yield curve measures market expectations concerning movements in the country's economy in real terms. A significant rise in the cost of borrowing was observed in March 2020 following the economy's shutdown, given lockdown restrictions impacted by the Covid-19 pandemic in South Africa. However, immediately after one hundred basis points cut in the repo rate by the South African Reserve Bank, coupled with announcements that the reserve bank will be buying government bonds to boost liquidity issues (National Treasury, 2021), a 27 per cent decline in the R197 bond yield was observed at the beginning of April 2020. These dynamics in the yield curve might require a term structure model with time-varying parameters or regime switches to perfectly capture these movements.

On the funding side, some measures had to be taken to help the pressurised fiscal policy. Since more cash was needed to address the matter, debt managers had to compromise the government risk framework and use short-dated or over-issued bond instruments to raise cash. This was coupled with interventions from the South African Reserve Bank to buy government bond stock to minimise liquidity issues affected by the Covid-19 stress in emerging markets. Table 3.2.1 shows the budgeted and revised South African national government borrowing requirements in the 2020/21 fiscal year due to the volatility brought forth by the Covid-19 stress.



R million	2019/20 Outcome	2020/21 Budget Revised		2021/22	2022/23	2023/24
				Medium-term estimates		
<b>Main budget balance</b>	<b>-345 053</b>	<b>-367 999</b>	<b>-603 388</b>	<b>-482 580</b>	<b>-417 164</b>	<b>-389 011</b>
<b>Redemptions</b>	<b>-70 657</b>	<b>-64 699</b>	<b>-66 881</b>	<b>-65 280</b>	<b>-144 797</b>	<b>-152 657</b>
Domestic long-term loans	-19 428	-52 465	-52 465	-60 815	-129 357	-112 428
Foreign loans	-51 229	-12 234	-14 416	-4 465	-15 440	-40 229
<b>Total</b>	<b>-415 710</b>	<b>-432 698</b>	<b>-670 269</b>	<b>-547 860</b>	<b>-561 961</b>	<b>-541 668</b>
<b>Financing</b>						
<b>Domestic short-term loans</b>	<b>36 078</b>	<b>48 000</b>	<b>97 184</b>	<b>9 000</b>	<b>52 000</b>	<b>56 000</b>
Treasury bills (net)	26 001	48 000	124 539	9 000	52 000	56 000
Corporation for Public Deposits	10 077	—	-27 355	—	—	—
<b>Domestic long-term loans</b>	<b>305 449</b>	<b>337 700</b>	<b>518 457</b>	<b>380 000</b>	<b>440 800</b>	<b>396 900</b>
Market loans	305 738	337 700	518 500	380 000	440 800	396 900
Loans issued for switches	-289	—	-43	—	—	—
<b>Foreign loans</b>	<b>76 052</b>	<b>29 260</b>	<b>107 070</b>	<b>46 260</b>	<b>46 320</b>	<b>62 600</b>
Market loans	76 052	29 260	107 070	46 260	46 320	62 600
Loans issued for switches	—	—	—	—	—	—
<b>Change in cash and other balances<sup>2</sup></b>	<b>-1 869</b>	<b>17 738</b>	<b>-52 442</b>	<b>112 600</b>	<b>22 841</b>	<b>26 168</b>
Cash balances	2 474	12 596	-58 957	107 876	17 544	19 219
Other balances <sup>3</sup>	-4 343	5 142	6 515	4 724	5 297	6 949
<b>Total</b>	<b>415 710</b>	<b>432 698</b>	<b>670 269</b>	<b>547 860</b>	<b>561 961</b>	<b>541 668</b>
<i>Percentage of GDP</i>	<i>8.1%</i>	<i>8.0%</i>	<i>13.6%</i>	<i>10.2%</i>	<i>9.9%</i>	<i>9.0%</i>

1. A longer time series is presented in Table 1 of the statistical annexure at the back of the Budget Review

2. A positive value indicates that cash is used to finance part of the borrowing requirement

3. Differences between funds requested and actual cash flows of national departments

Source: National Treasury

*Table 3.2.1: South African national government borrowing requirement*

*Source: National Treasury (2021: pp. 82)*

It can be observed in Table 3.2.1 that the main budget increased by R 235 billion in the 2020/21 fiscal year just to address the effect of Covid-19 stress in the market. The most significant contribution to this increase is long-term bonds, Treasury bills and issuances in the foreign market. Over 76 per cent of the increased funding came from long-term bonds (i.e. fixed-rate bonds and inflation-indexed bonds), 32,8 per cent came from foreign issuances and 32,3 per cent from Treasury bills; however, this resulted in a R27 billion decrease in the Corporate for Public Deposit [1]balance and R70 billion increase in the cash balance.

Such significant changes in the borrowing requirements might lead to drastic changes in the bond maturity profile since issuance in most liquid assets might be required to clear the required cash weekly. This was realised on the I2025 (2%, 2025) bond, where an increase of R8,77 billion was effected relative to the previous fiscal year's issuance. In the 2020/21 fiscal year, the bond had less than five years to maturity and an outstanding nominal amount of R70 billion outstanding as of 31st March 2020. It is

expected to stop issuances into bonds with term-to-maturity of less than five years as it prepares for maturity. This reduces the volatility around this area of the curve as the bond conforms to money-market characteristics and might negatively affect the bond's performance should it be put for auction. However, such situations could be overruled by the pertinent issue of raising funds for cash management during periods of crisis.

### 3.3. Monetary policy regime switches on the risk factors affecting the bond market in South Africa

The total value of the South African government bonds/debt listed in the JSE was over R2 trillion in 2018, accounting for around 90 per cent of the reported liquidity (Johannesburg Stock Exchange, 2018). As such, the South African government debt is exposed to the following market risks:

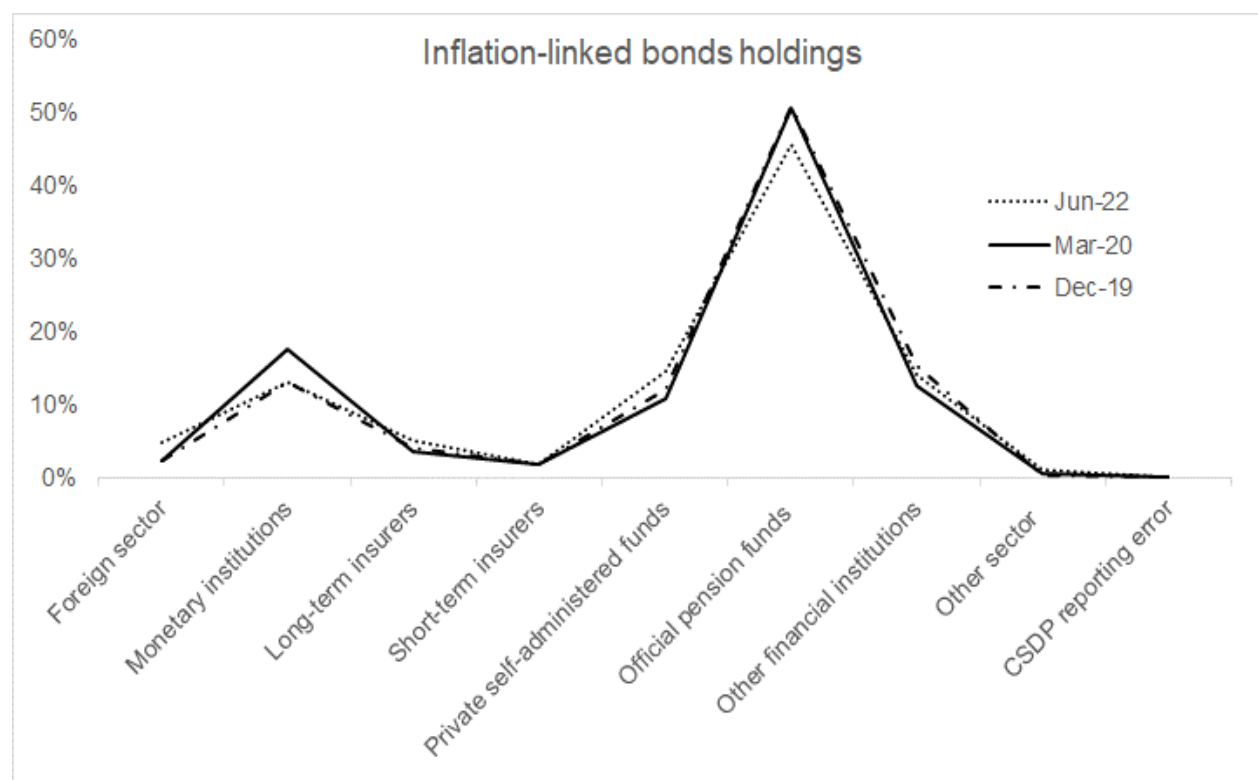
#### 3.3.1. Settlement risk

Settlement risk is defined by Johannesburg Stock Exchange (2018) as 'the risk of failing to deliver security or its equivalent cash value by a counterparty as per agreement when the security was initially traded after the other counterparty or counterparties have already delivered security or cash value as per the trade agreement'. Given that the auction system follows three days settlement period, the ability of investors to settle the amount allocated is critical (Bank of International Settlements, 2000), and this might negatively affect the government's ability to pay its financial obligations on time; this is mainly due to the high dependence on borrowings by government institutions to fund budget shortfalls.

#### 3.3.2. Liquidity risk

As Jonasson and Papapioannou (2018: pp. 7) indicate, liquidity risk is 'the risk of investors facing a sudden diminishing trading volume of a bond or a series of bonds

in the secondary market'. A lower trading volume/tradability in a bond might result in a higher cost of borrowing or low demand. A government institution issuing a bond should usually assess the market's demand around the prospective maturity before issuing the bond. To analyse government inflation-indexed bonds' liquidity in South Africa, the bond holding schedule is presented in Figure 3.3.1.



*Figure 3.3.1: Historical inflation-indexed bond holding in South Africa*

*Source: National Treasury (2022)*

It can be observed in Figure 3.3.1 that during the period of Covid-19 high stress in March 2020, monetary institutions came through for government inflation-indexed bonds. They increase their overall holdings in this instrument by five percentage points to 18 per cent; however, overall holdings for other institutions remained relatively the same compared to normal market conditions in December 2019. This implies that increased issuances into this bond instrument to cover the effect of the Covid-19 pandemic on the social economy are mainly carried by monetary institutions in line

with the South African Reserve Bank mandate to boost funding liquidity by buying government stock during this period.

A significant decrease of five percentage points in the overall holding was observed in June 2022 on both monetary institutions and official pension funds, while an increase of 3 percentage points and four percentage points was observed in foreign investors and private self-administered funds. This could be attributable to a cut in SARB's mandate to buy government stock and a redemption of the R212 (4,71%, 2022) bond in January 2022. The bond's holding was mainly dominated by monetary institutions and official pensions funds, which held 26 and 53 per cent of the nominal amount outstanding, respectively. However, given the illiquid nature of inflation-indexed bonds, official pension funds remain the instrument's biggest buyer, with the mandate to hedge their long-term liabilities against future inflation risk (National Treasury, 2021). Figure 3.3.2 shows historical auction bids and clearing yields in the primary market to further analyse the inflation-indexed bond market liquidity.

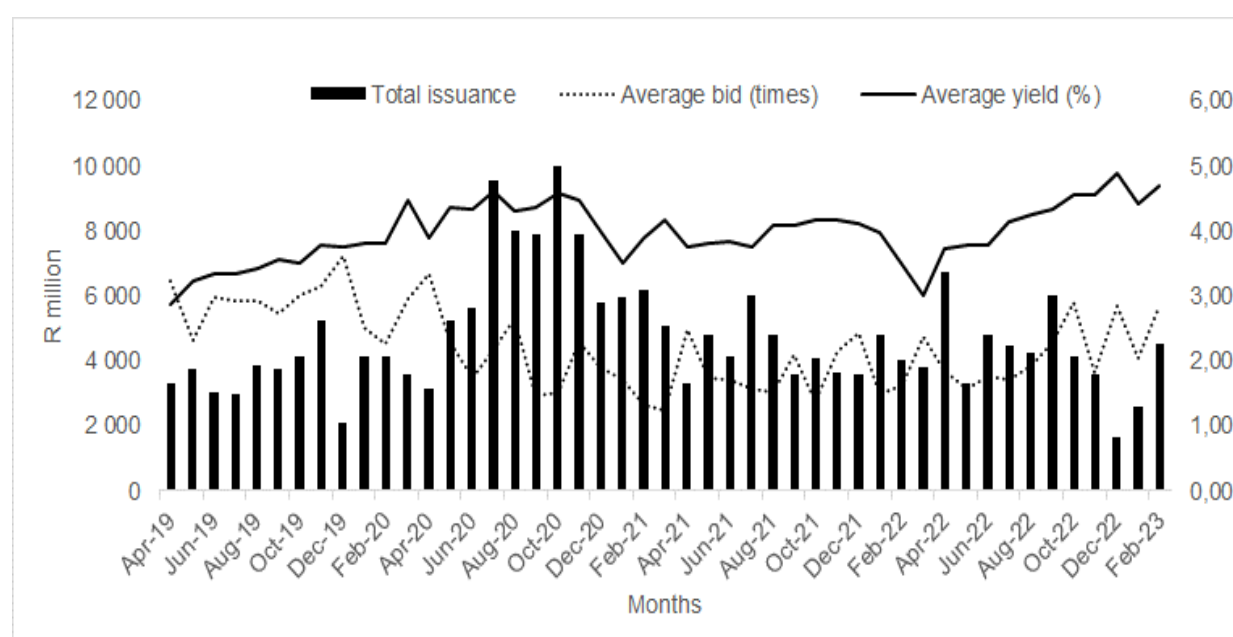


Figure 3.3.2: Primary market bond auction performance

Source: National Treasury

It is evident from Figure 3.3.2 that pre-Covid-19 pandemic, government-issued inflation-indexed bonds in the South African market had a slightly higher demand compared to during and post-Covid-19 crisis. Pre Covid-19, the average bid ratio was, on average, around 2.88 times the paper offered in the market. There is a practical level of stability in the average clearing yields over the 2019/20 financial year. A considerable dive was observed from May 2020, where issuances increased drastically to over twice pre-Covid levels. This happened at the same time when a significant decline in the demand for this bond instrument was observed. During the Covid-19 period, South African Reserve Bank's mandate to buy government stock helped to mop up most of the increase in government bond issuances which peaked at around R10 billion a month compared to an average of R3.6 billion a month prior in inflation-indexed bonds. The same liquidity continued in the 2021/22 financial year, where monthly bid-to-cover ratios averaged below two times the amount on offer; however, monthly average bond issuances in inflation-indexed bonds had declined to around R4.2 billion. In light of heightened global volatility and continued domestic political and economic instabilities in the 2022/23 financial year, average clearing yields weakened steadily by over 170 basis points between March 2022 and February 2023.

### 3.3.3.Sovereign risk

As defined by Jonasson and Papapioannou (2018: pp. 7), Sovereign credit riskiness is 'associated with the credit risk of a sovereign and the ability of a counterparty to fulfil its debt commitments'. Economic factors and the political environment are considered when determining this risk factor. Most foreign investors will require a government institution to achieve a particular credit rating standard by one or two big global rating agencies. To analyse changes in South African credibility over time, Figure 3.3.3 shows historical credit rating decisions by the global top three credit rating agencies (i.e. Standard and Poor, Fitch and Moody's).

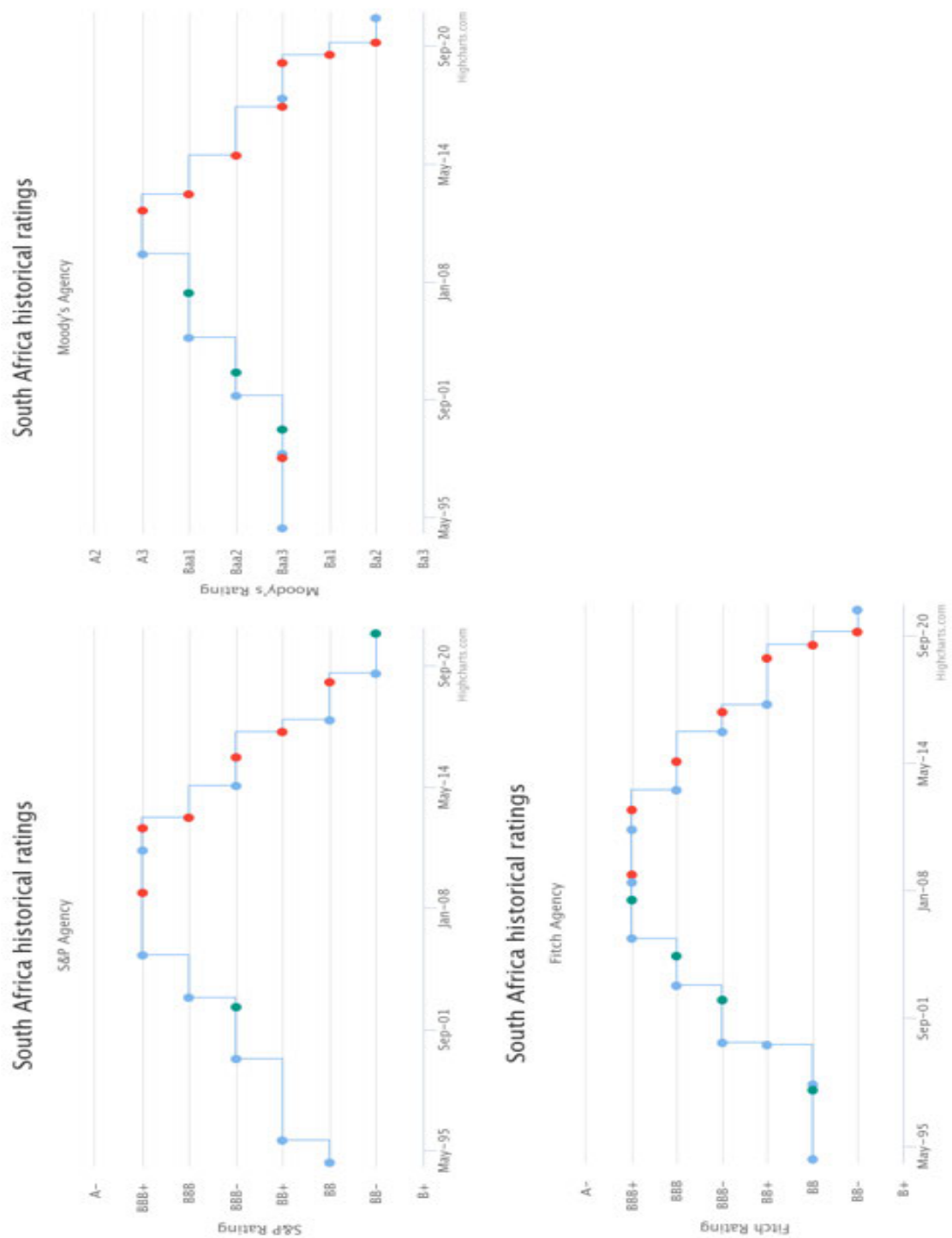
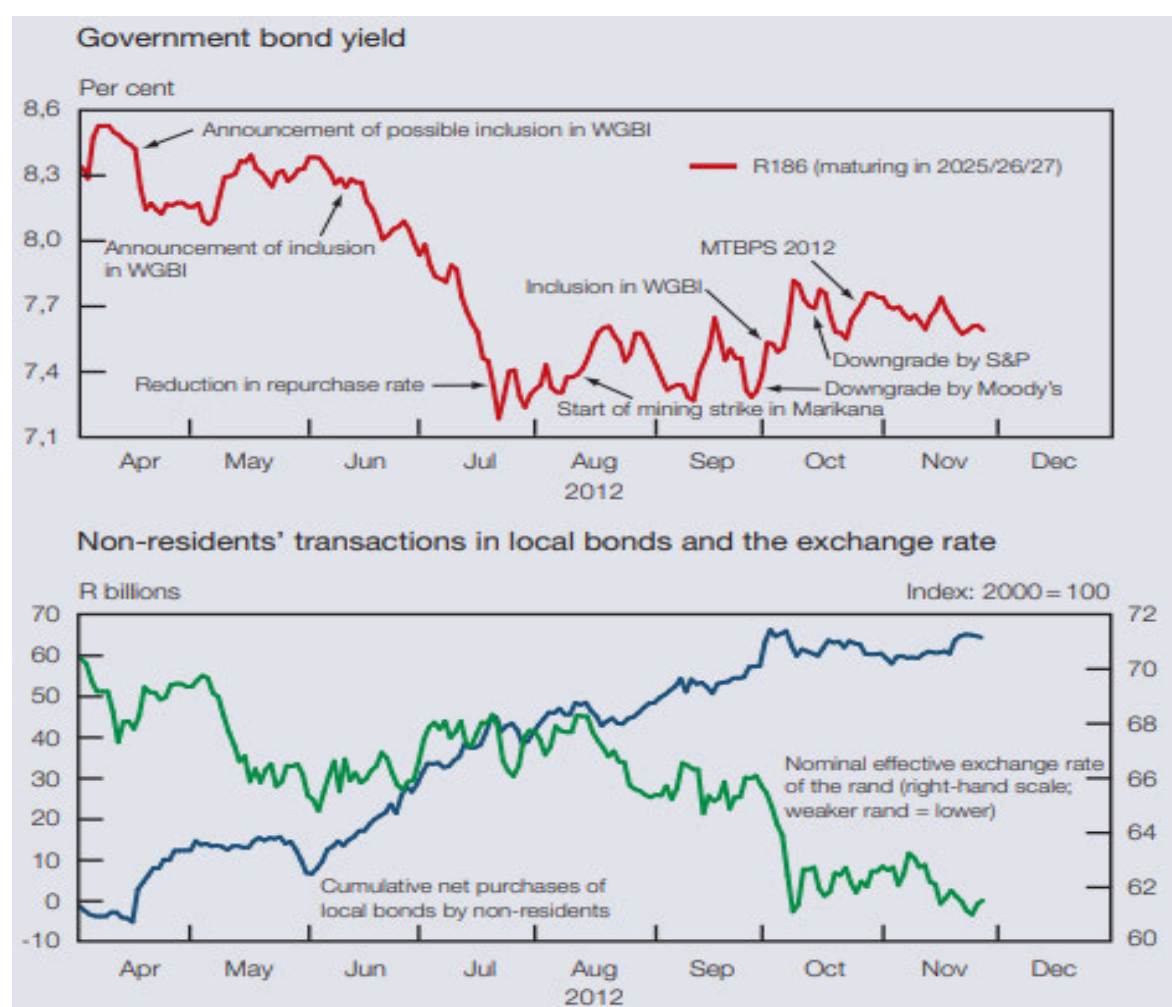


Figure 3.3.3: South African historical credit rating on local bonds

Source: World Government Bonds (2021)

NB: Green means a positive outlook assigned by the rating agency, blue means a stable outlook and red means a negative outlook.

Since the South African government gained independence in 1994, it is evident in Figure 3.3.3 that, over time, South African local debt has been regarded as of value by the top three global credit rating agencies. The local debt credit rating improved over time and peaked at BBB+ for S&P and Fitch and A3 for Moody's. It is also observed that during the 2008 global financial crisis, credit ratings remained resilient at the highest credit rating rank. This could be attributable to relatively better economic and political conditions during that period. To further analyse South African credit rating history, Figure 3.3.4 analyses the effect of South African bonds' inclusion in the Financial Times Stock Exchange World Government Bond Index (FTSE WGBI).



*Figure 3.3.4: South African bond inclusion in the WGBI*

*Source: South African Reserve Bank (2012: pp.1)*



Due to relatively better market sentiments in 2012 which included an adequately more robust domestic long-term credit rating and a market capitalisation exceeding US\$50 billion, it has made it possible for the South African domestic bonds to be included in the WGBI. As a result, South African bond yields strengthened significantly towards mid-2012, coupled with lower-than-expected inflation data and a cut in the repurchase rate, and further higher levels of global liquidity as foreign investors turned to emerging markets looking for higher returns. This is evidenced by a significant increase in "the foreign/non-resident investors' holdings of South African government domestic bonds from 12.8 per cent in 2008 to 29.1 per cent in 2011" (National Treasury, 2012: pp. 1). The first African country to participate in the WGBI was South Africa, accounting for 0.45 per cent of the index's market value, with 12 South African government bonds included in the WGBI in October 2012 (South African Reserve Bank, 2012).

However, it is noted in Figure 3.3.4 that these benefits were short-lived due to domestic volatility in the second half of 2012. The Marikana massacre, which Bruce (2015) indicated that it resulted in the fatalities of 34 mineworkers and seventy-eight left seriously injured following the open fire assault by the members of the South African Police Service in an attempt to contain a wildcat strike at Lonmin platinum mine in North West province. This resulted from a week-long protest where miners demanded remuneration increments. Secondly, sovereign credit rating downgrades initially by Moody's from A3 to Baa1 and later by Standard and Poor from BBB+ to BBB with a negative outlook from both credit rating agencies. South African Reserve Bank (2012) indicated the main drivers for this change were weakening government's institutional strength, reduced fiscal capacity, adverse investment climate because of infrastructure shortfalls, relatively high labour costs notwithstanding lower employment rate, and bigger concerns about future stability in the political space. Lastly, a more significant budget deficit was estimated to be 4.8 per cent of Gross Domestic Products (GDP) for the 2012/23 financial year in the 2012 Medium Term Budget Policy Statement from 4.2 per cent of GDP for the 2011/12 financial year (National Treasury, 2012). However, the cost of borrowing on South African bonds remained almost 100 basis points lower than before the announcement of possible inclusion in the WGBI. This could imply that these listed domestic issues could have been well expected and included in the bond yield estimation.



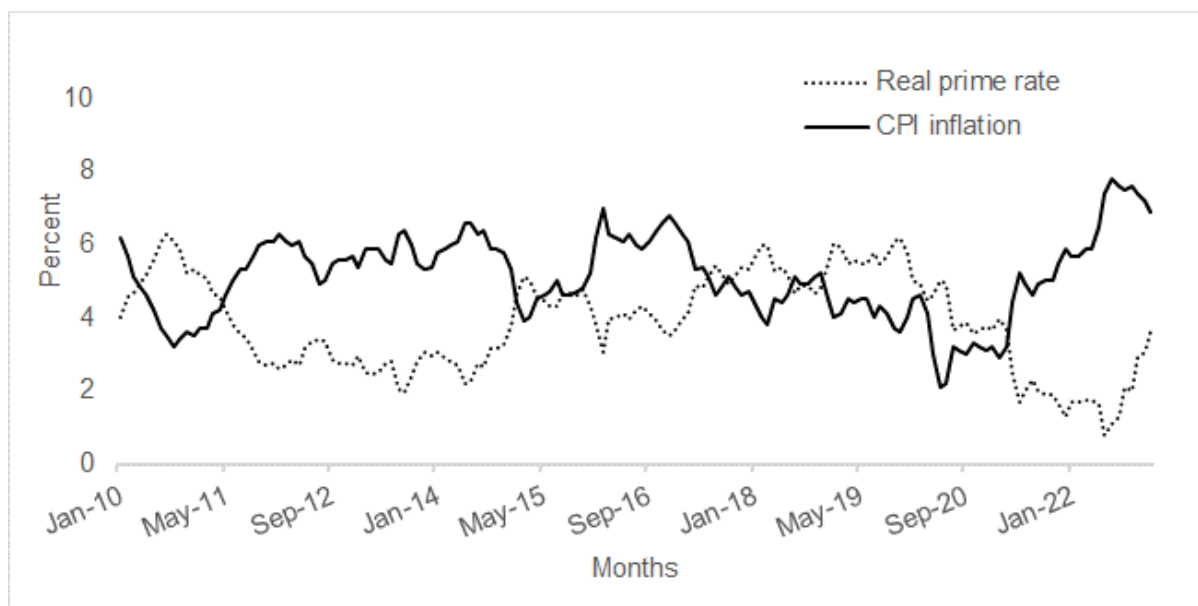
However, over the past decade to date, the South African political and economic state has deteriorated significantly. The government budget deficit worsened to 5.8 per cent of GDP (National Treasury, 2023); however, it was indicated by National Treasury (2012) that the budget deficit above 4.5 per cent of GDP is unsustainable. This has resulted in sub-investment sovereign credit ratings for South African debt, with "a clear path towards government debt stabilisation" being the main reason given by all three major credit rating agencies (Cliffe Dekker Hofmeyr, 2020: pp.1). Global financial volatilities (i.e. Covid-19 pandemic, Ukraine/Russia war, etc.) also aggravated the poor economic performance in emerging markets including South Africa. According to The World Bank (2022), the effect of the war in Ukraine will compound the damage in the global macroeconomic environment caused by the Covid-19 pandemic, which might see the situation in developing economies being worse than pre-pandemic levels. This is compared to the recovery from the stagflation of the 1970s; where steep increases in interest rates in major advanced economies were required, thus triggering a string of financial crises in emerging markets and developing economies.

Due to the global crisis and domestic volatilities (i.e. political instability, continued rolling electricity load-shedding and heavy reliance of poor-performing state-owned entities on the state for bailouts), the South African economic state continues to tumble. In trying to contain the situation, National Treasury (2023) indicated that the state continues to hand over bailouts to these state-owned to avoid total failures given the direct role they play in the well-functioning of the economy. Among other bailouts, the biggest one was to help the ailing state energy generator. National Treasury (2023) indicated that the state is proposing a R254 billion debt relief to Eskom over the medium term, which comprises R168 billion capital and R86 billion debt service cost. This continues to add to the already high level of debt and the debt service cost faced by the South African government, thus resulting in a poor credit rating and the credit outlook for government debt stock. Based on Figure 3.4.4, it could be observed that the South African domestic debt is rated BB- by S&P and Fitch; and Ba2 by Moody's, which is further down the investment grade of BBB-/Baa3 and also significantly lower than the credit rating of BB ranked by S&P and Fitch in 1995 when the South African government stock was first rated.

The situation in 1995 was far worse, given that the South African government experienced political and financial crises due to several sanctions imposed by several international bodies (Levy, 1999). The sanctions involved a ban on any form of trade, investments in the country and lending activities. It was estimated that the South African external debt was around \$24 billion by the mid-1980s, of which two-thirds was short-term (i.e. less than five years). After most lenders decided not to renew their short-term loans, South Africa ended in a liquidity crisis where the state depended on foreign lenders' willingness to refinance. The intensity of the crisis was so deep. It resulted in a significantly weaker Rand, and the state decided to close both the stock exchange and foreign exchange markets, with interest payments on the debt being suspended. This could be considered very bad compared to the current market conditions where South Africa still has access to funding. Anorld and Winning (2020) indicated that South African domestic bonds' attractiveness relative to other emerging markets peers and the depth of the South African domestic market has helped minimise the effect of the WGBI exit.

#### 3.3.4. Inflation risk

Inflation risk plays a critical role in bond pricing given that nominal bond yields are a function of, among others, inflation premium and real interest rate (Hordahl, 2008). The consumer inflation rate in emerging markets is relatively higher (Ha, Kose and Ohnsorge [Ha et al.], 2018), thus translating into a higher cost of borrowing in emerging economies. The historical association between the CPI inflation rate and the real prime rate is presented in Figure 3.3.5.



*Figure 3.3.5: Relationship between CPI inflation and real prime rate in South Africa*

*Source: South African Reserve Bank (2023)*

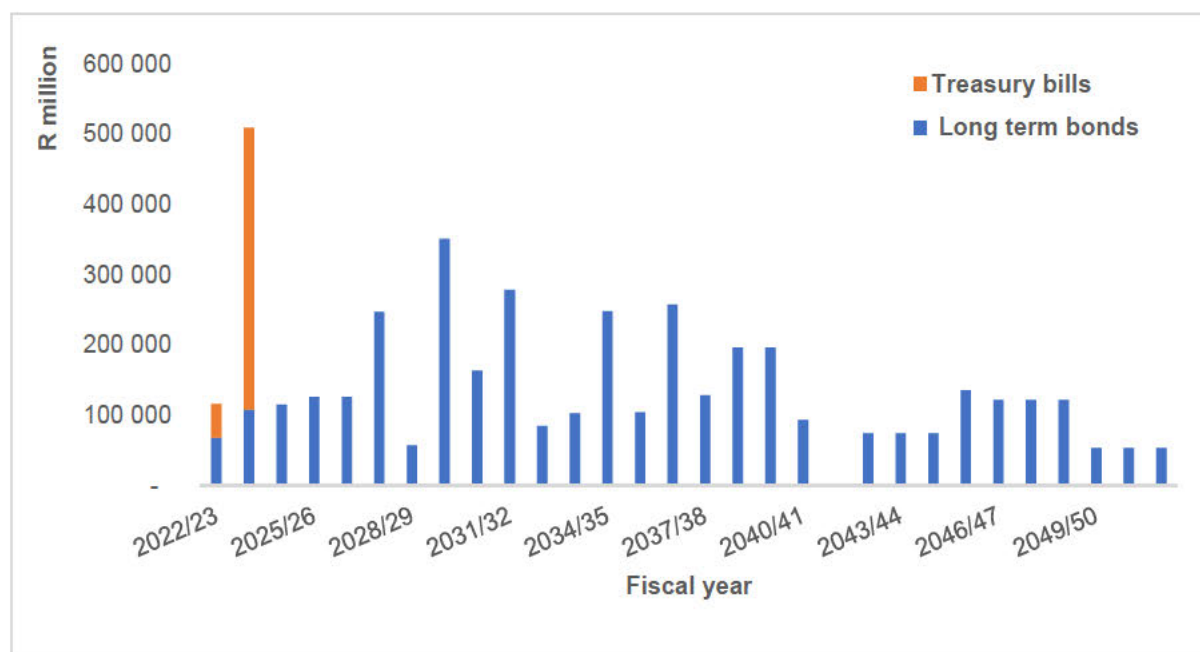
It could be observed in Figure 3.3.5 that over the past decade, the South African CPI inflation rate averaged above the 4.5 per cent midpoint; however only 25.48 per cent of the time, the CPI inflation rate was above/below the SARB's 6 or 3 per cent inflation target. A lower CPI inflation rate of 2.1 per cent was realised in May 2020, which was last seen over 15 years ago in September 2004, when CPI inflation rate of 1.3 per cent was realised. A lower CPI inflation rate might negatively affect demand for inflation-indexed bond instruments, given that interest rates already do not include the future inflation component; as such, the future value of the investment might be eroded by lower inflation accruals, making the bond instrument less attractive. Primary market auction bid-to-cover ratios support this; refer to Figure 3.4.3, which declined drastically over the Covid period and was influenced by a lower CPI inflation rate, among other factors.

It could also be observed that the cost of borrowing in real terms has a somewhat adverse relationship with annual changes in the headline CPI inflation rate. It was observed that over 60 per cent of the time, changes in the real prime rate and CPI inflation rate moved in opposite directions for the past decade. The CPI inflation rate is seen increasing towards the end of the 2021/22 financial year, and this peaked at

7.8 per cent in July 2022. This rate last seen 13 years ago in May 2009 when the CPI inflation rate was 8 per cent.

### 3.3.5. Refinancing risk

Government institutions rarely aim to pay off the capital amount owed when the bond gets to its maturity. This process is mainly due to most governments running substantial budget shortfalls. Jonasson and Papapioannou (2018) defined refinancing risk as 'the ability to refinance a debt exposure at maturity as a result of a loss of market access or low investor appetite'; this is very crucial, which might lead to a costly refinancing process for government institutions who are at the mercy of investors. To illustrate the South African government debt maturity profile on domestic debt, Figure 3.3.6 shows the maturity profile as of Dec 2022.



*Figure 3.3.6: South African government domestic debt profile*

*Source: National Treasury (2022)*

The government maturity profile for domestic debt in Figure 3.3.6 indicates a much-clustered profile in the medium term, with an average outstanding amount of at least R100 billion. A considerable amount is also outstanding on the Treasury bills, which must be rolled over weekly. The issue of rolling over the debt implies that even though it is not expected to redeem the amount outstanding on Treasury bills, new issuance is made every week to redeem the outstanding portion and raise funds for cash management purposes. The ability to raise cash every week to meet these responsibilities adds to the already pressurised government's ability to raise cash every week to fund the increasing budget deficit and also build cash reserves for a bond maturing in the short term.

It was observed in Table 3.2.1 that the South African government is projecting average bond redemptions of above R100 billion in the medium term for every single fiscal year. This is relatively higher than the average historical fiscal year bond redemptions of R60 billion each fiscal year (National Treasury, 2021). This brings the issue of bond switches to minimise the risk of being unable to meet financial obligations in a particular fiscal year. Bond switches phenomena/programme is defined as a transaction of exchanging a series of existing source bonds held by investors with a series of selected destination bonds where both source and destination bonds have to be determined by bond issuers. According to National Treasury (2000), this phenomenon was announced in South Africa in early 2000 when the South African government faced declining borrowing requirements and reduced the number of new bonds issued. The switch programme was introduced to manage liquidity issues by repurchasing in advance, less-liquid maturities while financing these bond purchases through more significant new issuances into the benchmark bonds.

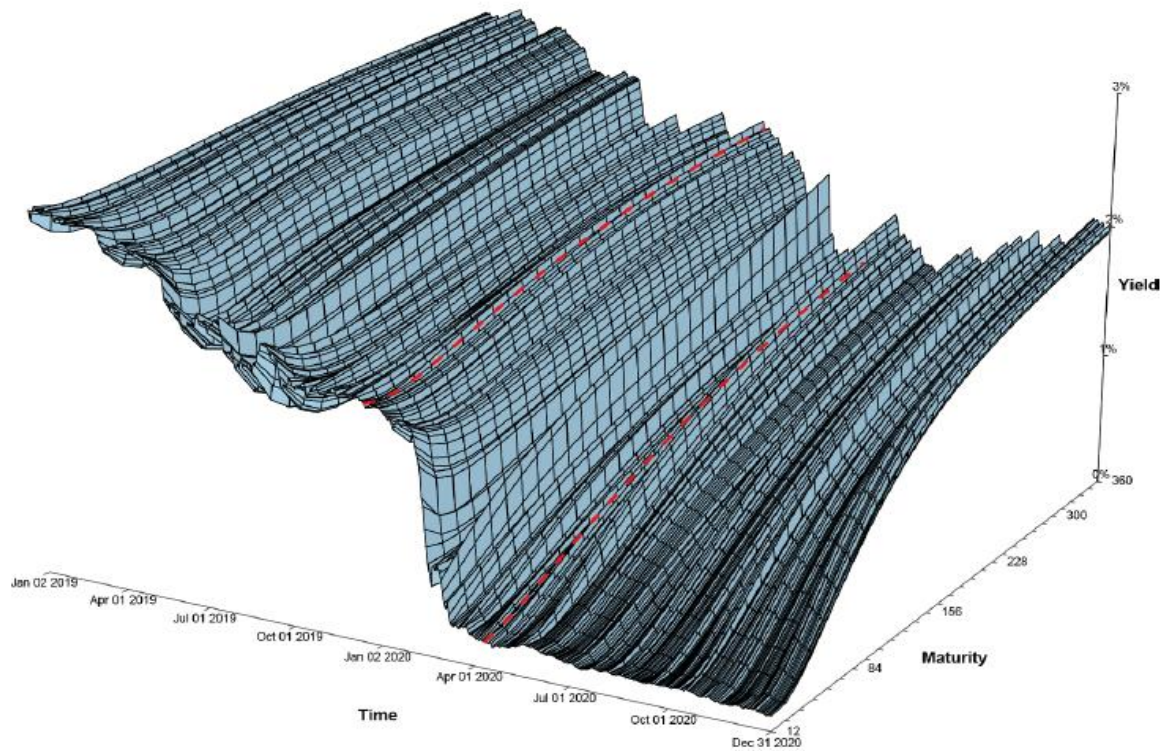
The switch programme gives the bond issuer the advantage of rapidly restructuring the maturity profile of outstanding debt. It becomes an issue when switches are done to minimise the government's redemption obligations in the short term by transferring the liability to longer maturities. Given the highly clustered South African government debt maturity profile in Figure 3.3.6 and the growing primary deficit, the issue of a switch programme will always be needed to minimise the eminent pressure on the government's ability to meet other fiscal policy needs in the short term. However, the switch programme might not always be feasible as it depends on the willingness of the

bondholders to switch maturities, which might not always favour their investment plans. This implies that the switch programme will remain costly for the government.

### 3.4. Survey of evidence from developed markets

#### 3.4.1. United States of America (USA)

The study by Papailias (2022) focused on examining the variation in persistence in the USA yield curve's short-term, medium-term and long-term factors. The study started by investigating the behaviour of the USA's yield curves by applying the Diebold and Li (2006) framework to assess the effect of the Covid-19 pandemic on the fixed-income space. The local linear framework defined in Cai (2007) was further applied to calculate the time-dependent variation in the persistence of the factors of a yield curve. It was indicated in Figure 3.4.1 that a decrease in persistence was observed around the end of 2019, resulting in more predictable factors of the yield curve and, subsequently, the yield curves. In contrast, evidence of explosive behaviour was realised in the USA's yield curve factors in the long-term and short-term during the pandemic. The study was based on the daily zero-coupon bond rates with maturities of 12 to 360 months for this analysis. A sample of 520 zero-coupon rates was obtained from the Federal Reserve Board (FED) from 2nd January 2019 to 29th January 2021.



*Figure 3.4.1: US daily yield curves*

*Source: Papailias (2022: pp. 2)*

For this analysis, the study was based on the daily interest rate for zero-coupon bonds for maturities of 12 to 360 months. For modelling purposes, sub-samples were defined as follows:

- The incubation phase runs from 2nd January 2020 to 17th January 2020, which covers the first Covid-19 declarations in Wuhan; subsequently, the shutting down of the Seafood Wholesale Market.
- The epidemic phase runs from 20th January 2020 to 21st February 2020, including the time the Covid-19 virus transmission was confirmed by the Chinese authorities and when the first report about the virus was published by the World Health Organisation (WHO).
- The fever phase runs from 24th February 2020 to 30th March 2020, including Italy's first economic shutdown.

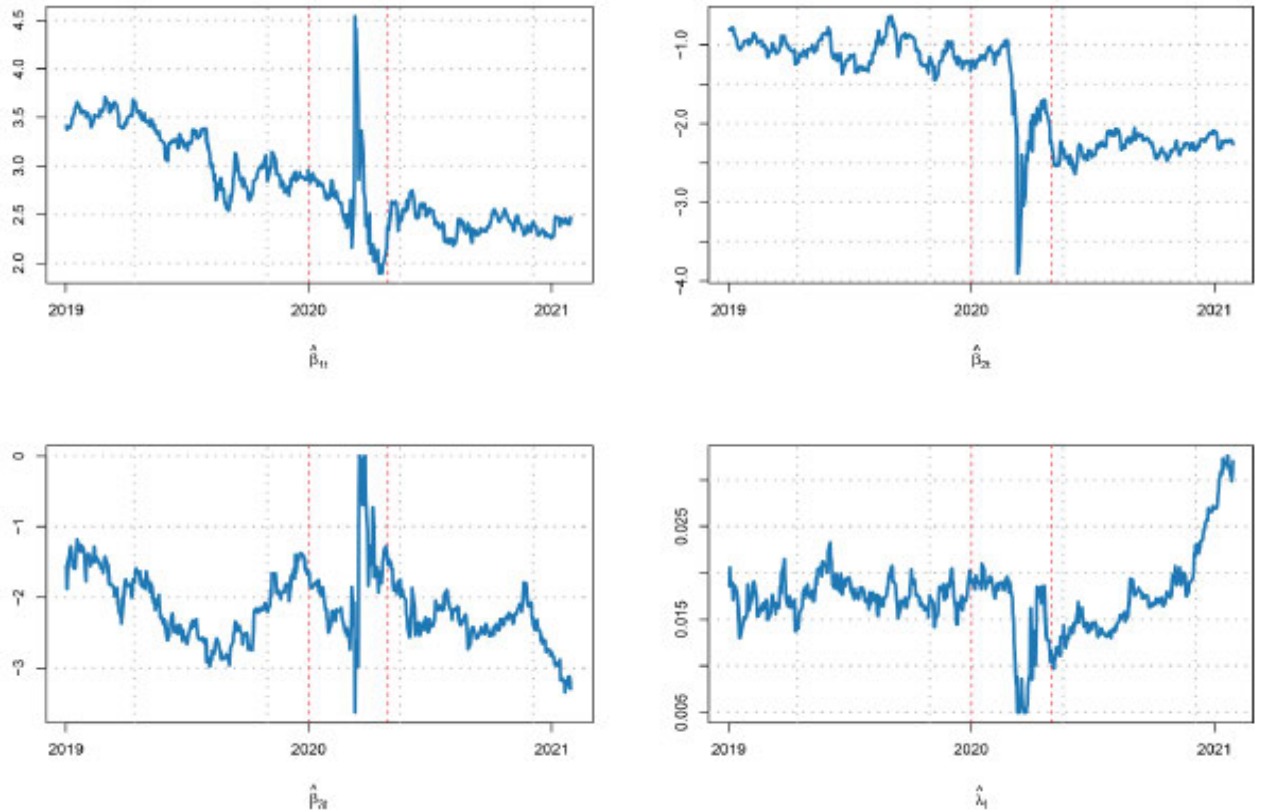
- From 23rd March 2020 to 30th April 2020, the rebound phase included the Federal Reserve (FED) 's interventions and the stock market rebound.

To assess the response of interest rates to the COVID-19 outcomes, yield curve factors in the short-, medium- and long-term were extracted following Diebold and Li (2006) term-structure model:

$$\hat{R}(t, \tau) = a_t + b_t \left[ \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right] - c_t e^{\lambda_t \tau} \quad (3.1)$$

where  $a_t = \beta_{1,t}$ ,  $b_t = \beta_{2,t} + \beta_{3,t}$ ,  $c_t = \beta_{3,t}$  are time-varying factors that effect a dynamic structure of the spot rate curve and  $\varepsilon_t \sim N(0, \sigma^2)$ .  $\hat{R}(t, \tau)$  is the bond yield at time  $t$  for  $t = 1, 2, \dots, T$  with maturity  $\tau$ . Model parameters  $\beta_{1,t}$ ,  $\beta_{2,t}$  and  $\beta_{3,t}$  which are factors of the yield curve and the decay factor  $\lambda_t$ . Bigger values of the decay factor result in a good fit on shorter maturities, whereas small values result in a good fit on the long end of the curve. The three factors are expressed as the short-term, medium-term and long-term factors and are taken as proxies for the yield curve's slope, curvature and level (Diebold and Li, 2006). A non-linear least squares methodology was applied to estimate optimal model parameters.





*Figure 3.4.2: Nelson Siegel factors on USA yield curve*

*Source: Papailias (2022: pp. 5)*

Based on the movements of the model parameters over the Covid period in Figure 3.4.2, it is evident that significant volatility was observed during the first wave of Covid-19 as indicated by the red dots. A significant increase in the level factor indicates increased volatility in the long-term rates. A decline in the slope factor was observed during the same period, indicating a yield curve inversion. This is reflected by changes in signs (i.e. to negative, thus reflecting a negative sloping yield curve over this period). A sudden decline followed by an increase in the curvature factor was also observed during this period.

For the estimation of the persistence in each model factor and subsequent calculation of the persistence of the interest rates, estimated autoregressive coefficients from the first order autoregressive model AR (1) were applied in line with Diebold and Li (2006). Papailias (2022) extended the application of the AR (1) model by including a time-dependent framework which allows for the investigation of changes in the

autoregressive coefficient over time, mainly around the pandemic. The model is defined as follows:

$$x_t = \varphi_{0,t} + \varphi_{1,t}x_{t-1} + \varepsilon_t \quad (3.2)$$

with  $t = 1, 2, \dots, T$  and the local linear approach was applied in the calculation.

It was then observed that during 2019, there was some relative stability in the long-term factor's persistence for the USA. Towards the end of 2019, a moderate decline in persistence is observed as the autoregressive coefficient moves from 1 to 0.5. Fluctuations around this level are observed from the end of 2019 to the beginning of 2020, thus indicating that the drop in persistence implies a variation in the fundamental dynamics, thus resulting in a more predictable long-term factor. For the slope factor, a drop in persistence is observed towards the end of 2019, a moderate rise during a period of explosive behaviour in the first two phases of the pandemic and a convergence to the pre-pandemic levels in the last two phases of the pandemic and the rest of 2020. The persistence of the curvature factor declines around the last quarter of 2019 and becomes more volatile. After the recovery, the autoregressive coefficient fluctuates between 0.5 and 1.

A moderate persistence decrease was observed before the pandemic for all three model factors. However, the short-term and long-term factors show volatile behaviour in the first two phases of the pandemic. For investors, this offers a prospect to go to instruments of lower persistence and, thus, a more probable estimation of a yield curve.

### 3.4.2. Canada

Severino, Cremona and Dadie (Severino et al., 2022) indicated that the study focused on unpacking the evolution of Nelson and Siegel (1987) model parameters over different periods that cover the effects of the Covid-19 pandemic on bond markets. Massive corporate bond sales were observed due to a rapid loss of trust in the financial system. However, disruptions in the debt market were subdued by interventions from the Federal Reserve. Therefore, the study focused more on the variations in the critical factors of the yield curve for the Canadian government bond market (i.e. level, slope and curvature). Daily data of yields from 1st May 2018 to 30th October 2020 was used with the following sub-samples for modelling purposes:

- The first period runs from 1st May 2018 to 21st March 2019, with 233 daily observations. This period is considered normal with a positive term spread, and the Bank of Canada increased the target rate twice. Even though global growth was solid, volatilities were associated with trade tensions between the United States of America and China.
- The second period runs from 22nd March 2019 to 27th February 2020 with 245 daily observations. The period was considered anomalous with a negative term spread. There were no observed interest rate interventions from the Bank of Canada and there were sombre economic perspectives anticipated by market actors. It was further noted that trade battles still existed between China and the United States of America. There was a further observed global slowdown in economic growth in China and the Eurozone due to Brexit concerns.
- The third period runs from 28th March 2020 to 30th October 2020, with 124 daily observations. The period is considered normal with a positive term spread. There were no interest rates interventions from the Bank of Canada; however, revamps economic support due to sharp contractions in the global economy as a result of the first wave of the Covid-19 pandemic (i.e. associated with an immense decrease in economic activities to curb the spread of the virus).

Dynamic Nelson-Siegel (2006) model with time-dependent parameters was applied:

$$\hat{R}(t, \tau) = a_t + b_t \left[ \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right] - c_t e^{\lambda_t \tau} \quad (3.3)$$

where  $a_t = \beta_{1,t}$ ,  $b_t = \beta_{2,t} + \beta_{3,t}$ ,  $c_t = \beta_{3,t}$  are time-varying factors that effect a dynamic structure of the spot rate curve and  $\varepsilon_t \sim N(0, \sigma^2)$ . The decay factor was selected to maximise the medium-term regressor when  $\tau = 30$  months and was estimated at  $\lambda_t = 0.0609$ ; furthermore, model parameters  $\beta$  were estimated using an ordinary least squares (OLS) methodology for each day  $t$ . The dynamics of the model parameters are presented in Figure 3.4.3 to trace their movements over different macroeconomic environments, which included a period of Covid-19 stress.

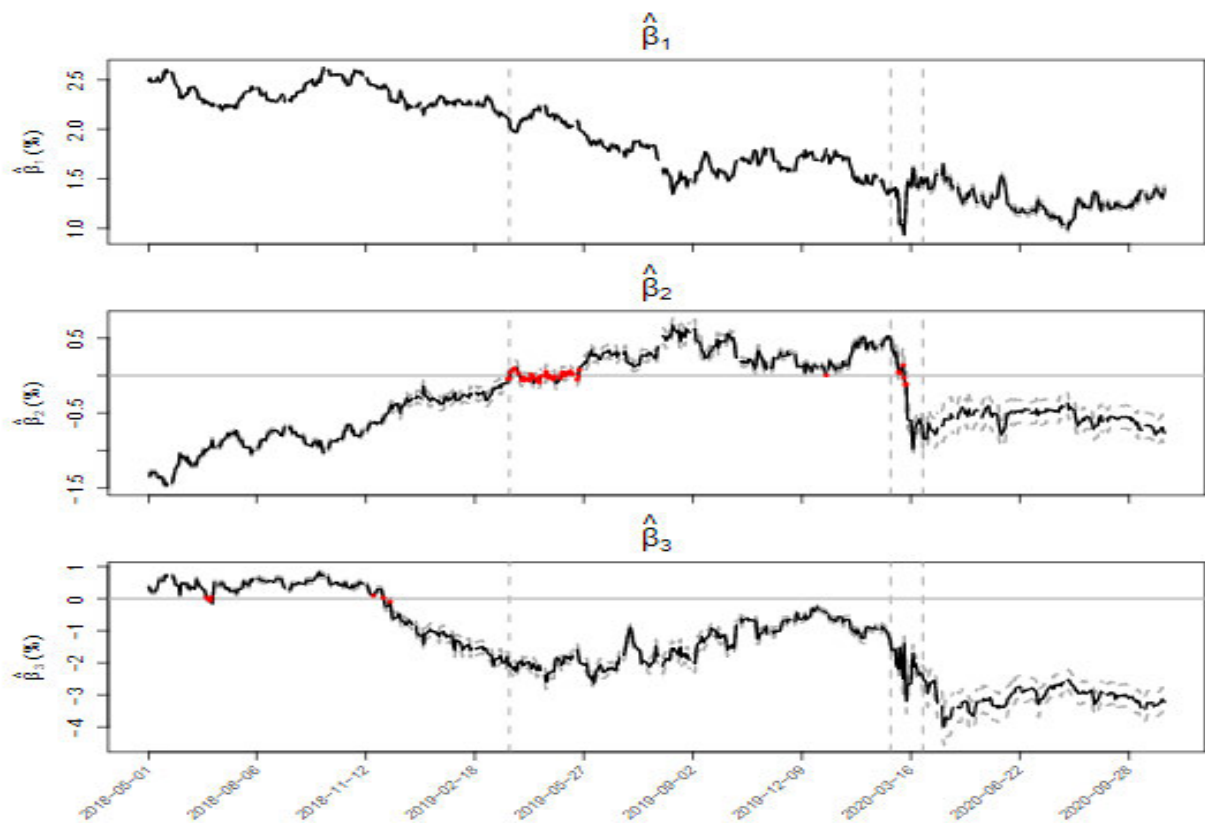


Figure 3.4.3: Nelson Siegel factors on the Canadian yield curve

Source: Severino et al. (2022: pp. 487)

On average,  $\beta_{1,t}$  estimates indicated a decline over the estimation period in line with observed declines for the three periods in the average long-term yields, refer to Figure 3.4.3. Severino et al. (2022) indicated that in March 2020, the Bank of Canada cut the target rate, which prompted a decline in the  $\beta_{1,t}$  between the second and the third periods. This reflects a relative instability on longer maturities; however, the analysis of relative volatility measured by the coefficient of variation has indicated lower relative volatility when compared to other model parameters.

The proxy for short-term rates  $\beta_{2,t}$  reflected a yield curve inversion in the second period. This is replicated by a sign changes (i.e. positive in the second period and back to negative in the third period). As a result of Covid-19, increasing yields were observed; however, in the second period, the central bank's interventions contributed to the correction of the curve inversion. Similarly, the relative instability on  $\beta_{2,t}$  was relatively higher in the second period echoing the curve inversion observed. The behaviour of  $\beta_{3,t}$ , which is a proxy for the curvature factor, showed a steady decline over the three periods. According to Diebold and Li (2006), the curvature factor is linked to twice the two-year yield minus the sum of the ten-year and three-month yields. A decline in the second period was associated with an inverted hump in the average yield curve. In the third period, a lower value was observed while the yield curve increase was associated with moderately weaker long-term rates on the two-year point.

### 3.5. Survey of evidence from emerging markets

For the country survey on peer countries, the analysis was based on the leading emerging economies for more progressive reforms to address stressed economic environments on managing the government debt and government funding strategies. South Africa is part of the world's leading emerging economies group BRICS (Brazil, Russia, India, China and South Africa), which was founded in 2009. According to the South African Government (2013, pp.1), the main aim of this group was "to promote peace, security, development and cooperation; and further contribute significantly to the development of humanity and establishing a more equitable and fairer world". The recognition of the country's contribution to shaping the socio-economic regeneration

of Africa and its involvement in peace, security and reconstruction efforts on the continent; led to South Africa's offer to join BRICS. Further, its developed financial system and fiscal and monetary policy frameworks added to South Africa's advantage. In analysing the world's leading emerging markets' historical riskiness based on the credit ratings by the three major rating agencies over time, results are presented in Table 3.5.1.

Country	Credit rating during 2008 crisis	Credit rating before Covid (2019)	Credit rating after Covid (2022)
Brazil	Standard and Poor: BBB negative Fitch: BBB- negative Moody's: Ba1 positive	Standard and Poor: BB-positive Fitch: BB- negative Moody's: Ba2 negative	Standard and Poor: BB-stable Fitch: BB- stable Moody's: Ba2 stable
Russia	Standard and Poor: BBB negative Fitch: BBB+ negative Moody's: Baa1 negative	Standard and Poor: BBB-negative Fitch: BBB negative Moody's: Baa3 negative	Standard and Poor: NR Fitch: NR Moody's: NR
India	Standard and Poor: BBB-negative Fitch: BBB- negative Moody's: Baa3 negative	Standard and Poor: BBB-negative Fitch: BBB- negative Moody's: Baa2 negative	Standard and Poor: BBB stable Fitch: BBB stable Moody's: Baa3 stable
China	Standard and Poor: A+ negative Fitch: A+ negative Moody's: A1 positive	Standard and Poor: A+ negative Fitch: A+ negative Moody's: A1 negative	Standard and Poor: A+ stable Fitch: A+ stable Moody's: A1 stable
South Africa	Standard and Poor: BBB+ negative Fitch: BBB+ negative Moody's: A3 negative	Standard and Poor: BB negative Fitch: BB+ negative Moody's: Baa3 negative	Standard and Poor: BB-stable Fitch: BB- stable Moody's: Ba2 stable

*Table 3.5.1: Credit ratings for emerging markets*

*Source: World Bank and Trading Economics*

It is evident that political instabilities and policy reforms have been significant drivers for deteriorating credit ratings in South Africa, Brazil and Russia, refer to Table 3.5.1. Brazil started to feel pressure from three major rating agencies in 2014 after being downgraded to one notch below the sub-investment grade by Standard and Poor. According to Korby (2014, pp. 1), Standard and Poor indicated a combination of 'fiscal slippage, the prospect that fiscal execution will remain weak amid subdued growth in the coming years, the constrained ability of government to adjust policy ahead of presidential elections, and some weakening in the country's external accounts'. It was

further indicated by Bisseker (2014) that these reasons, which resulted in a rating downgrade for Brazil, do apply equally to South Africa, which was also subjected to political instabilities, poor economic growth and increasing debt levels.

Brazil tasted the first sub-investment/junk credit rating in 2015 due to mounting political problems that have muddled economic policy, Brandimarte (2015). Fitch and Moody's followed in placing Brazil on sub-investment credit rating, citing a further deterioration in debt ratios amid economic contractions, Watts (2016). South Africa followed through in 2017, where it was rated sub-investment by Standard and Poor and Fitch citing economic contractions and political uproars that resulted in the removal of the finance minister in a 'late-night cabinet reshuffle' by then-President Jacob Zuma. Moody's finally followed through in 2020, which resulted in the exclusion of South Africa from the WGBI. This happened after being downgraded to sub-investment grade by all three major rating agencies, which is the minimum requirement to stay in the index. According to Hamill (2022) and FTSE Russel (2021), China was the only country in BRICS, which is part of the WGBI, thus holding 3.07 per cent of the WGBI on a market value-weighted basis at its total exposure. India remained on a watchlist by FTSE Russel for possible country reclassification and inclusion in the WGBI and Emerging Markets Government Bond Index (EMGBI). Both India and China have investment credit ratings with a stable outlook. However, South Africa, Brazil and China only remained in the EMGBI, which has the minimum requirements of a C rating from Standard and Poor and a Ca rating from Moody's.

Russia enjoyed being above investment grade with all three major rating agencies, even during the 2008 global crisis and the Covid-19 stress. However, a significant decline was realised in 2022 following the financial fallout over Russia's invasion of Ukraine. Russia saw a six-notch downgrade to B3, six notches below investment grade. It was further indicated that all three major rating agencies had withdrawn their rating mandates following European Union's decision to impose sanctions on Russia to ramp up economic pressure on the country (Chappell, 2022). In analysing the economic position of the selected emerging markets, critical economic results are presented in Table 3.5.2.

<b>Economic variable</b>	<b>Brazil</b>	<b>Russia</b>	<b>India</b>	<b>China</b>	<b>South Africa</b>
Economic growth (2021)	5,00%	5,60%	8,70%	8,40%	4,90%
Income per capita (2021)	\$15 600	\$32 070	\$7 130	\$19 160	\$14 340
CPI inflation (2022)	9,59%	13,80%	6,70%	2,00%	6,90%
Debt to GDP (2022)	72,90%	13,40%	55,10%	21,40%	71,00%
10-year gov yield (Dec 2022)	12,69%	10,34%	7,33%	2,88%	10,19%
Repo rate (Dec 2022)	13,75%	7,50%	6,50%	2,75%	7,00%

*Table 3.5.2: Deciding factors for emerging markets*

*Source: World Bank and Trading Economics*

South Africa and Brazil's economic positions are relatively the same, which could be attributed to the same political and economic instabilities realised. It is observed in Table 3.5.2 that the two countries are both rated on sub-investment credit rating where almost the same issues/drivers were cited by rating agencies. Subdued economic growth has been evident for the past decade, with an average growth rate of 0.98 per cent and 0.36 per cent for South Africa and Brazil, respectively. While South Africa's CPI inflation is currently above the target band of 3 and 6 per cent, it was observed that over the past decade, it remained well within the band with only two exceptions. A downward trend was observed a few years before the Covid-19 pandemic (Statistics South Africa, 2023). The uptick in 2022 above the upper band was mainly driven by heightened geopolitical uncertainty from the Ukraine/Russia war, which resulted in persistent increases in food and energy prices in both developed and emerging markets (National Treasury, 2023). The South African 10-year government bond yield remained relatively stable over the past decade, just before the Covid-19 pandemic, at an average of 8.5 per cent. A 300-basis point weakening was observed in March 2020 immediately after the WGBI exit, coupled with a total shutdown impacted by the Covid-19 shock. Even though some level of stability normalised back to pre-Covid shock, the global volatility impacted by Ukraine/Russian war had fuelled some instabilities and increases in the year 2022 (National Treasury, 2023). This has resulted in weaker yields above 10 per cent for most of the year 2022.

In the case of Brazil, CPI inflation has averaged around 5,79 per cent over the past decade, which was within the desired target range. However, there were some jumps in 2015 and 2016 way above the then target range of 2.5 and 6.5 per cent which resulted in band tightening in 2017 from +/-2 per cent allowance from the mid target of



4.5 per cent to +/-1.5 per cent (Oxford Analytica, 2015). After a spike in prices during 2015 and 2016, the move was to boost their credibility to regain the market's trust and ensure government commitment to lowering inflation. De Bolle (2015) indicated that the main driver for the inflationary spike was the electricity and fuel price correction policies implemented when the current government took office in 2015. The hikes constituted of 50.4 cent spike for residential energy, a 22 per cent spike for cooking gas and an 18.6 per cent spike for gasoline. This led to a 12-month inflationary spike of 14 per cent in administered prices, which accounted for 25 per cent of Brazilian CPI inflation. However, a couple of years pre-Covid stress, inflation in Brazil was well contained below the midpoint inflation target of between 4.25 and 4.5 per cent.

Given that Brazil is a commodity country, international increases in commodity prices and continued political instabilities fuelled the inflation spike of 8.3 per cent in 2021 (Carrara, 2022). The same volatilities were observed on the 10-year government bond yield over the past decade. At the height of political instabilities in 2015, the 10-year government yield weakened to above 15 per cent. However, a downward trend was also observed a couple of years before Covid-19. In line with global volatility tracking the pressures effected by the Ukraine/Russia war, weaker rates were observed in 2022.

Over the past decade, Russia's economic growth has been performing relatively better pre-Covid crisis, except for a 2 per cent contraction observed in 2015. According to Dabrowski and Collin (2019), the contraction was driven by a combination of a sharp decline in the international price of oil, which is Russia's main export item, and the conflict with Ukraine, which resulted in United States and European Union sanctions against Russia. This has also negatively affected the inflation rate, which increased to 15.53 per cent due to the same geopolitical issues. A recovery trajectory was observed pre/post-Covid stress; however, this was hammered by Russia's recent unfounded Ukraine attack in February 2022 (Welt, 2022). This has resulted in the European Council adopting a couple of sanctions against Russia and Belarus, which aimed to weaken Russia's ability to finance the war; and specifically target the political, military and economic abilities responsible for the invasion. As such, inflation increased by over 100 per cent to 13.8 per cent in 2022 compared to the prior year. A 10-year government bond yield also weakened by around 200 basis points over the same

period. While the economy still showed positive growth in 2022; the World Bank, the IMF, and the Organisation for Economic Cooperation and Development (OECD) expect the Russian economy to continue to shrink in the short term.

To further analyse capabilities of tackling the effects of heightened stress in the bond market and fiscal/monetary policy regimes for the world's leading emerging markets, the development of the bond market is critical to analyse the primary driver of increasing debt levels and costs of servicing the debt. Further analysis of the effect of Covid-19 stresses on the public debt and measures taken to address liquidity stresses in emerging markets to ensure continued availability of funds for social services and public investments. Subsequently, assess government yield curve/bond pricing over the peak of Covid-19 stresses and further monetary policy measures.

The study by Castello and Resta (2022) explores the impact of the Covid-19 pandemic on the application of different time-dependent term-structure models in fitting yield curves of the BRICS countries:

- The Five Factor De Rezende–Ferreira Model (5F-DRF)

This methodology depends on a two-dimension array  $\lambda$  of decay factors and a 5-parameter vector  $\beta$  to model a set of humps which determines the dynamics of a spot rate curve. This methodology is equivalent to the Dynamic Generalised Nelson-Siegel model in Section 2.3.6.

$$R(t, \tau) = \beta_{0,t} + \beta_{1,t} \left[ \frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right] + \beta_{2,t} \left[ \frac{1-e^{-\lambda_2 \tau}}{\lambda_2 \tau} \right] + \beta_{3,t} \left[ \frac{1-e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right] \\ + \beta_{4,t} \left[ \frac{1-e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right] + \epsilon_\tau \quad (3.4)$$

It was indicated that for the bigger values of the decay factor, term-structure results in a slower decay and ensures a better fit in the long end of the curve but not in the short end. Conversely, a smaller decay factor results in a quicker decay, hence a perfect fit on the short end of the curve but not on the long end. A slight modification is done on

the Dynamic Generalised Nelson-Siegel model by considering time-dependent/varying model decay rates. The model fitting is done at each point using a two-step calculation procedure. The optimal pair of decay rates is estimated as associated with the minimised Root Mean Square Error; subsequently, the optimal pair of decay factors is applied to estimate an optimal set of model parameters  $\beta$ .

- Feed Forward Neural Networks

As defined by Hornik, Stinchcombe and White (Hornik et al., 1989: pp. 359-361), "Feed Forward Neural Networks (FFNNs) are non-linear regression tools which do not need any prior assumption about the functional form or statistical properties of the data set under examination and can be used to identify and model nonlinearity in the data".

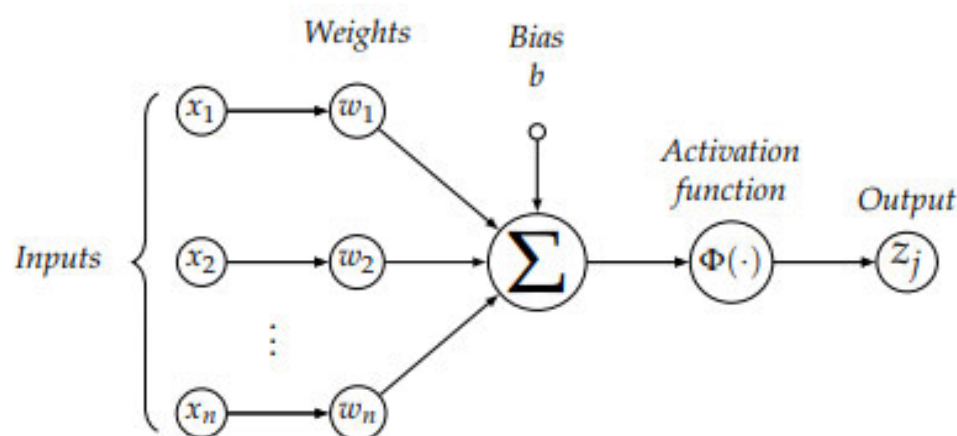
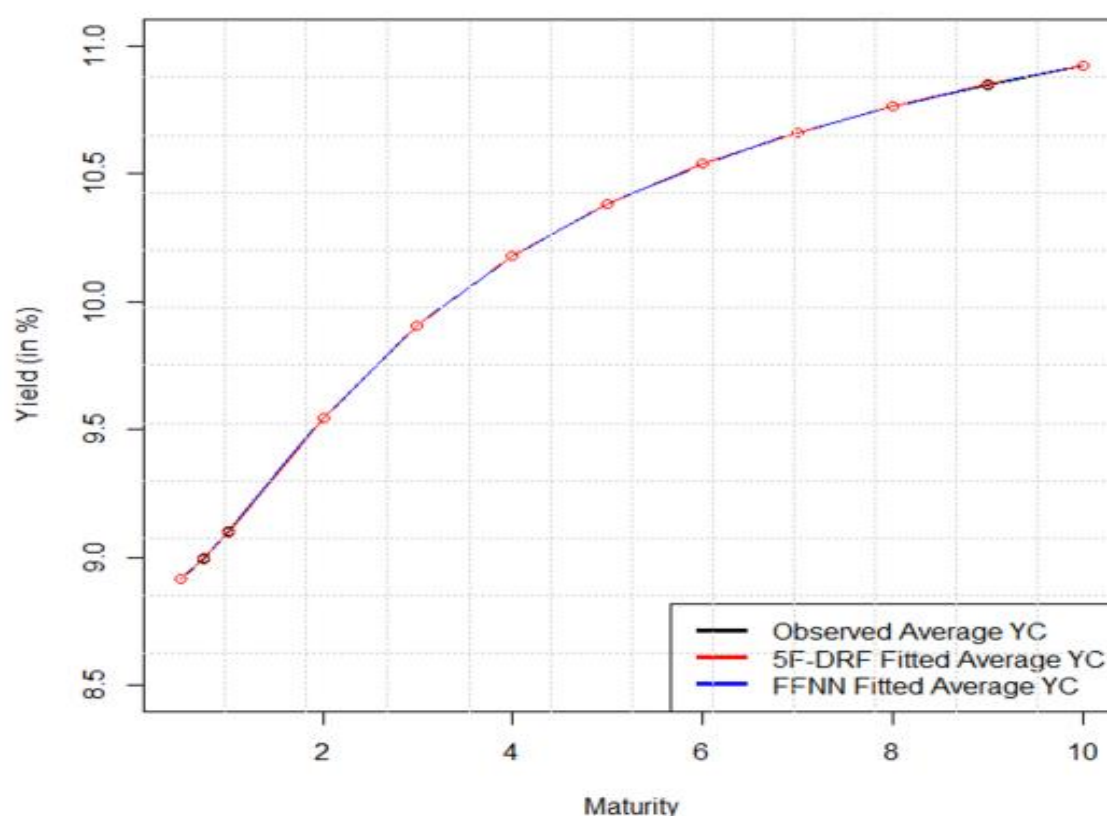


Figure 3.5.1: Node representation with  $n$  input features

Source: Castello and Resta (2022: pp. 5)

A graphical representation of an artificial neuron in Figure 3.5.1 indicates that at each input vector  $x$ , it is handled by a linear combiner  $\Sigma$  based on applying a weight vector  $w$ ; after that, it is transformed using a proper activation function. According to the gradient descent criterion, the process minimises a loss function (e.g. the MSE - Mean Squared Error) until the optimal mixture of weights is determined so that the target output is equivalent to the calculated output.

To analyse the impact of Covid-19 stress on using the selected time-dependent term-structure models in Section 3.6, daily returns from 2128 observations of government zero-coupon bonds (ZCB) were used. The data used ranged from 30th September 2011 to 30th December 2020. Fitting results are presented on Figures 3.5.2, 3.5.3, 3.5.4 and 3.5.5 below:



*Figure 3.5.2: Brazil - Realised vs fitted average yield curves* Source: Castello and Resta (2022: pp. 10)

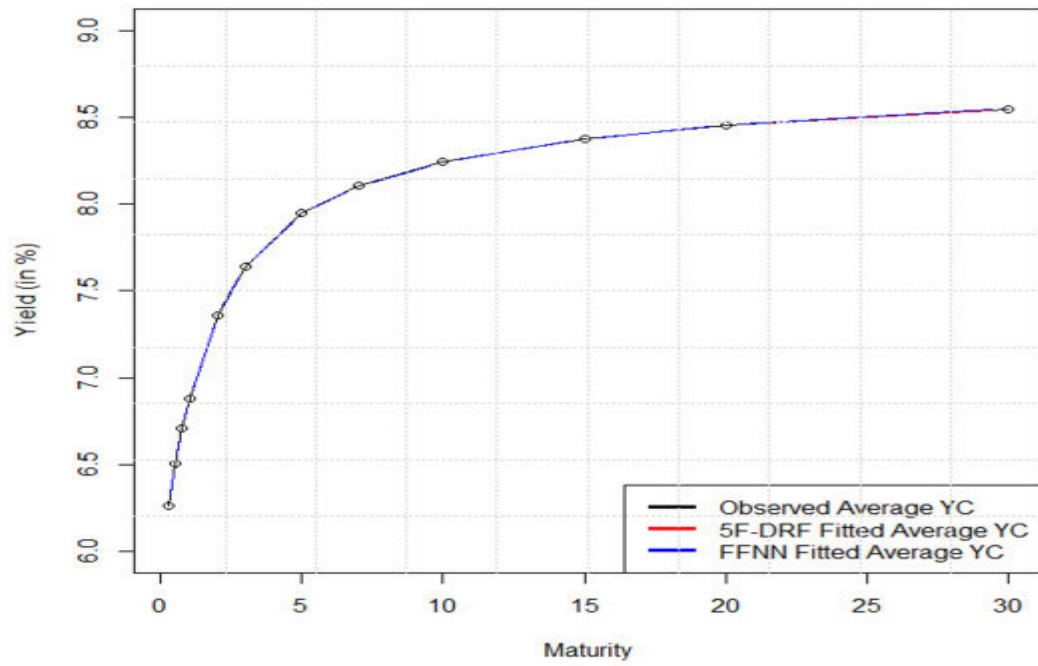


Figure 3.5.3: Russia - Realised vs fitted average yield curves

Source: Castello and Resta (2022: pp. 10)

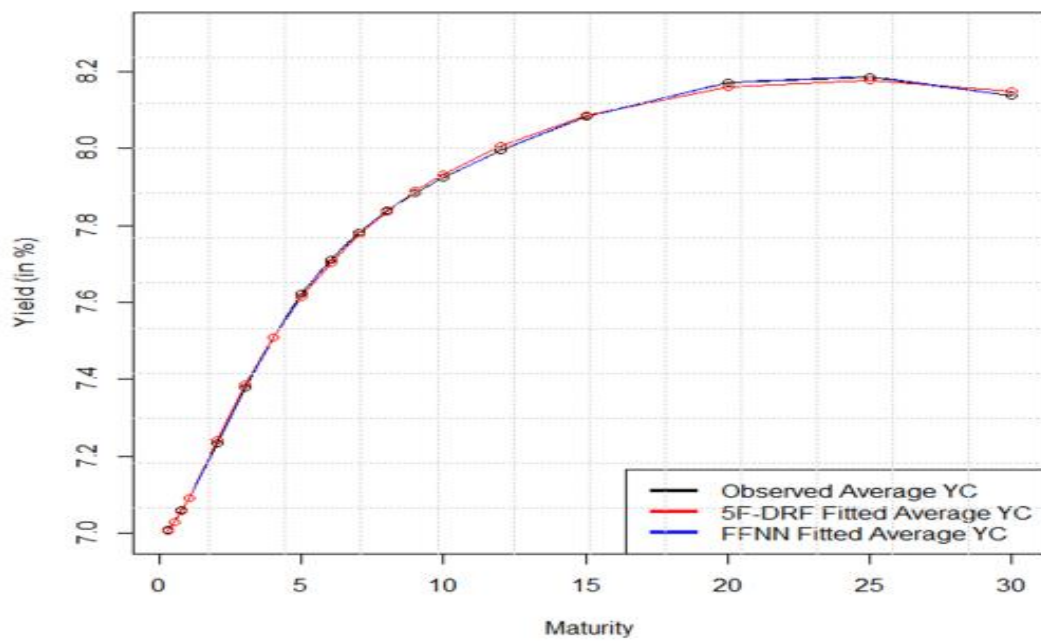
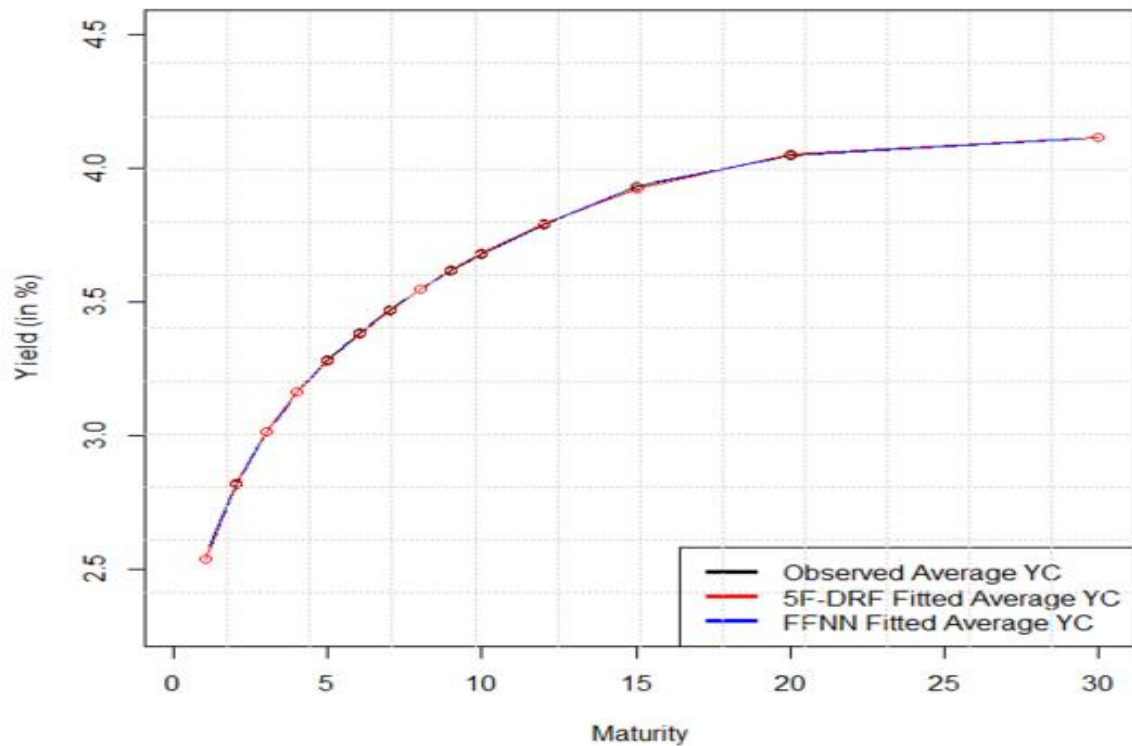


Figure 3.5.4: India - Realised vs fitted average yield curves

Source: Castello and Resta (2022: pp. 10)



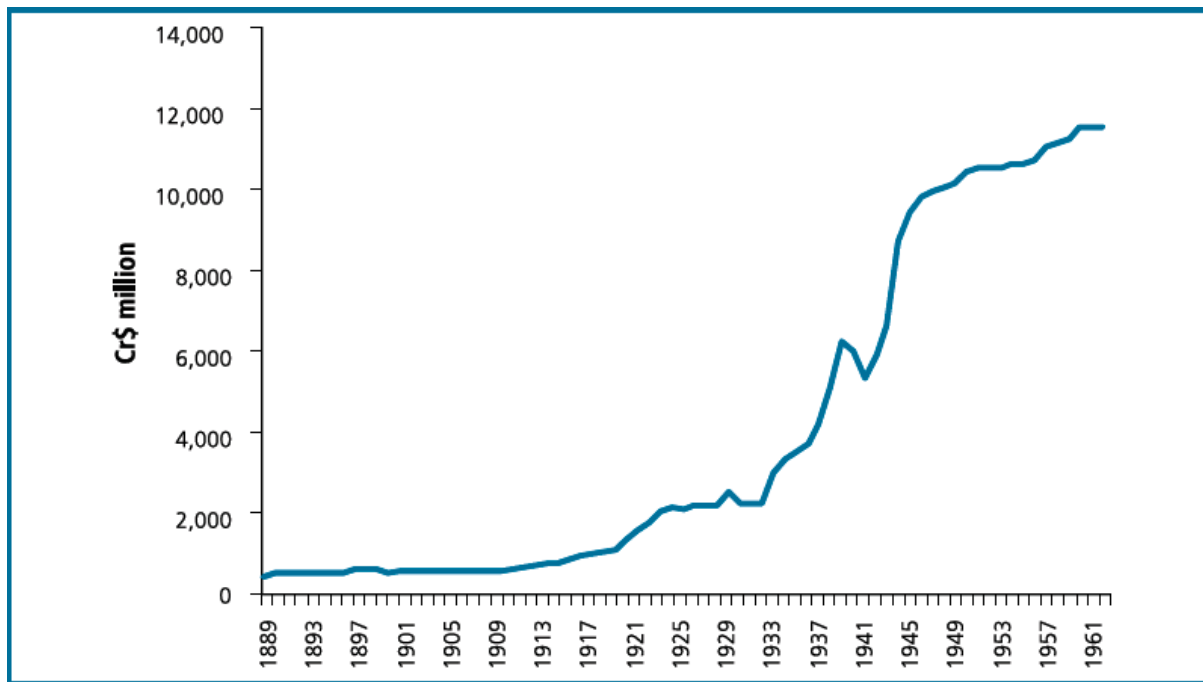
*Figure 3.5.5: China - Realised vs fitted average yield curves*

*Source: Castello and Resta (2022: pp. 10)*

Significantly lower deviations were observed between the average yield curves and those estimated by the 5F-DRF model and the FFNNs methodology. This implies that, on average, the two methodologies can fit an extensive diversity of shapes/types displayed by the yield curve over different macroeconomic conditions, thus explaining their superior fitting capabilities even during a period characterised by high volatilities and uncertainties in the market. The effect of time-dependent decay factor on the 5F-DRF model helped the dynamism of the model during the heightened Covid-19 stress.

### 3.5.1. Brazil

Sovereign debt in Brazil could be traced back to the 1500 century; however, it was indicated by Caputo Silva, Oliveira de Carvalho and Ladeira de Medeiros (Caputo Silva et al., 2010) that in the colonial period, everything including the magnitude of the debt, the motive for the loan and circumstances around the borrowing was concealed. This is because the loans were privately taken out by the rulers/governors. The legalisation of the bond market in Brazil happened during the Imperial Period in the 18th century, which led to the creation of the first debt management office, institutionalised domestic debt, enhanced financing mechanisms and instruments, and introduction of debt restructuring operations. This was during the country's first years of independence; however, historical financial difficulties that happened before the independence made the process more challenging. As such, 1822 - 1850 was considered a challenging period of conflicts and consolidation, and the construction period was considered to be mainly during 1850 – 1889. This has been highly admired by American Latin nations for their success in issuing debt, especially domestic debt, which included large amounts of long-term debt and further being able to meet their external financial commitments/responsibility. At the same time, other countries failed to do so and even defaulted in settling their domestic financial commitments. Towards the end of the Imperial Period, Brazil faced a couple of challenges in domestic debt negotiations and bond liquidity, mainly driven by the highly fragmented structure of the public debt. To further analyse public debt issuance in Brazil for both Imperial and Republican periods, a schedule of outstanding debt per period is presented in Figure 3.5.6.



*Figure 3.5.6: Balance of policies and outstanding debt in Brazil, 1889-1963*

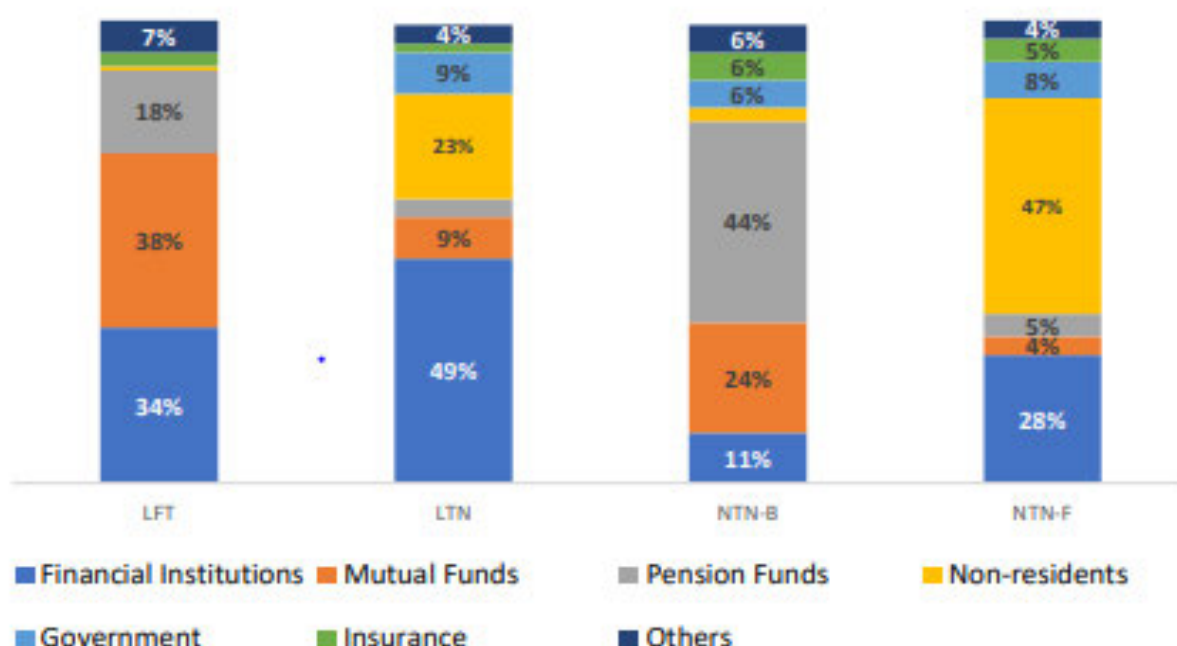
*Source: Caputo Silva et al. (2010: pp. 43)*

It is observed in Figure 3.5.6 that bond issuances started to gain momentum in 1920 when 145 authorisations for bond issues were granted with annual interest rates varying from 3 to 7 per cent. The loans were issued to cover different mandates (i.e. budget shortfalls, sterilisation of additional liquidity, funding civil works, obtaining fixed assets or companies, and paying-off compulsory loans). Drastic issuances were observed from 1942 where war bonds were issued for the Republic's funding policy during World War II (Caputo Silva et al., 2010).

It was further indicated that Brazil's public credit position was compromised by years of not paying interest or redeeming outstanding securities. As such, from the mid-1950s, stagnation in its voluntary issuance of public securities made it difficult to finance the growing budget deficit. The state resorted to issuing its currency to finance almost its entire budget deficit, which largely contributed to increases in inflation, thus resulting in negative real yields and further reducing liquidity or demand for public securities in Brazil.



Over the years, substantial progress was made in developing the government bond market in Brazil, which included the lengthening of the yield curve, reduction of foreign exposure and diversification of the investor base (Park, 2012). Improvements in macroeconomic conditions supported this, thus making the government bond market more resilient to various market risk factors. However, the extension of the yield curve or term-to-maturity of the government debt portfolio remained challenging due to private investors' preference for shorter-term floating-rate debt and indexed bonds. Demand for fixed-rate bonds was mainly driven by non-resident investors who significantly provided liquidity for a fixed-rate bond market in Brazil. However, given the global riskiness associated with non-resident investors, a high likelihood of volatility is expected during periods of global stress, which is associated with sudden exit from emerging markets in search of a safer haven. To further analyse Brazil's investor base for government bond instruments, refer to Figure 3.5.7.



*Figure 3.5.7: Government bond holdings for Brazil*

*Source: National Treasury of Brazil (2021: pp. 21)*

According to the National Treasury of Brazil (2021), diversification of the investor base is considered a risk management strategy used as a guideline for borrowing strategy and contributes to debt management efficiency. Diversifying the investor base with different risk profiles and investment horizons or mandates minimises debt risks by reducing their volatility and increasing bonds' liquidity.

It is observed in Figure 3.5.7 that non-resident holdings on the long-term fixed-rate (NTN-F) improved to 47 per cent in 2021 from 45.1 per cent in 2020. This was mainly driven by a recovery in the global economy with increased long-term interest rates, which stimulated liquidity or demand from non-residents and pension funds investors for long-term bonds. However, of the overall Brazilian Domestic Federal Public Debt (DFPD), financial institutions contributed the highest composition of the investor base at 29.5 per cent in 2021. Non-residents contributed only 10.6 per cent, a marginal increase of 1.4 per cent from 2020. Even though incentives were made available for non-residents investors to invest in domestic debt (i.e. exemption of income tax on gains from investments in domestic debt bonds as with most emerging markets), the participation of non-resident investors in the DFPD remained relatively lower (Caputo Silva et al., 2010). Compared to the preceding years, a significant increase of BRL124.5 billion in non-resident holdings of DFPD was observed in 2021 which is the first significant increase realised in Brazil in a while. To further analyse the composition of DFPD, which comprises monthly average issuances and average issuance term-to-maturity over the Covid-19 stress period, refer to Figure 3.5.8.

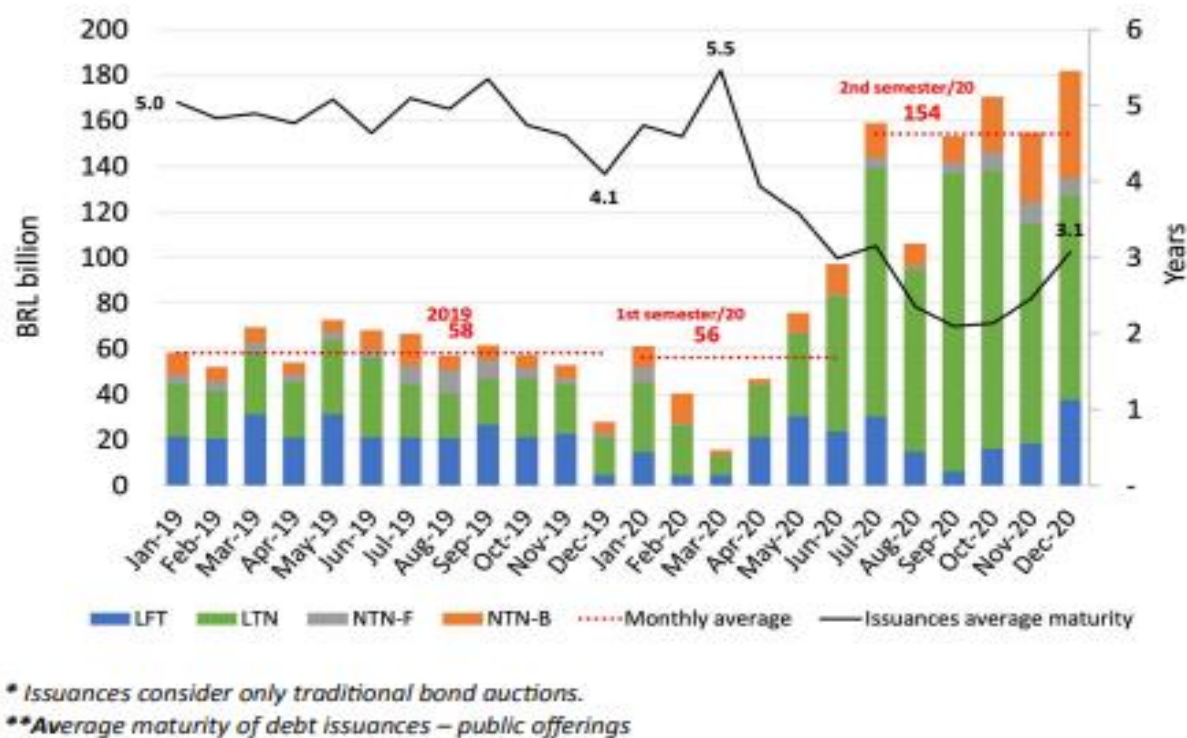


Figure 3.5.8: Bond issuances and the average maturity of DFPD issuances

Source: National Treasury of Brazil (2020: pp. 11)

Based on Figure 3.5.8, it can be observed that the effect of Covid-19, as compared to the South African case, had negatively impacted government borrowing plans. Average monthly issuance levels tripled to BRL 154 billion in the second half of 2020 compared to BRL 58 billion in the first half of 2020. This significantly reduced the average issuance maturity from 5 years in 2019 to less than three years in the second half of 2020, which is significantly lower than the lower limit of 3.9 years. Issuances in short-dated maturities were driven mainly by a scenario of uncertainties around the Covid-19 pandemic, risk aversion by non-resident investors and a steeper yield curve. It was observed that more issuance was based on long-term notes (LTN). It was further observed that given the negative impact the Covid-19 pandemic had on liquidity levels in emerging economies, more issuances were based on the short end of the curve to ensure enough capital for cash management strategies. By December 2020, the share of debt with a maturity of at most 12 months was 27.6 per cent relative to 18.7 per cent in December 2019, thus bridging the upper limit of 23 per cent. While the strategy was viable for cash management purposes, it was riskier from a risk management

perspective. The strategy puts more short-term pressure on the state/debt managers, given an increased budget deficit to finance the effects of Covid-19 coupled with lower bond liquidity levels in emerging economies. Issuances on shorter maturities of the curve were, to some extent, driven by the steeper yield curve as a result of lower interest rate effected by monetary policy stimuli to address the effect of Covid-19 in the local economy. However, movements of interest rates on the short end of the curve were not seen in the mid-to-long end, which remained sticky and resulted in a relatively steep curve on the short end of the curve.

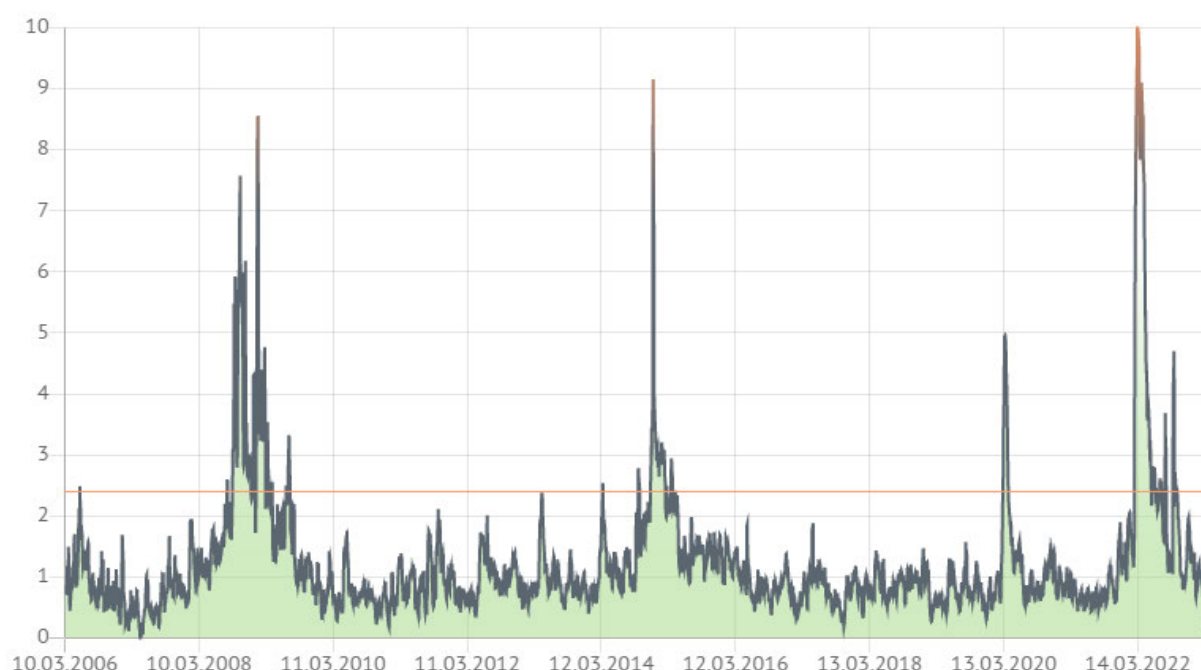
According to the National Treasury of Brazil (2021), it was evident that Brazil's economic sentiments have seen a significant improvement post the Covid-19 as the global economy normalises to pre-Covid levels. Better performance in revenue collections has positively affected the budget deficit in 2021, thus reducing government borrowing requirements. This has resulted in a 6.6 per cent reduction in the share of debt maturing in 12 months to 21 per cent, thus slightly increasing the average term-to-maturity of the overall debt portfolio to 3.8 years. However, the recovery in the local economy was short-lived due to high global inflation rates and a rise in interest rates imposed by international scenarios, which were exacerbated by the outbreak of the Ukraine/Russian war (National Treasury of Brazil, 2022).

### 3.5.2. Russia

The bond market in the Russian economy is relatively new, with the first corporate bond issued in 1990. External and local challenges caused instability in the financial and economic environment which resulted in a limitation in developing the Russian bond market (Baranova and Kulikov, 2019). However, the stable growth in the Russian bond market was mainly pushed by lower inflation, the de-dollarisation of payments and savings, and the amalgamation with international settlement organisations, which reduced transaction costs for non-resident investors who wish to hold internally issued debt. According to Lu and Yakovlev (2017), it was indicated that Russia had been blessed with a commodity-driven fiscal surplus; as such, a need for issuing local currency government bonds (OFZ bonds) was mainly to preserve a sovereign yield curve. However, negative real OFZ bond yields were unattractive to institutional and

local long-term investors. As such, based on its investment policy, the State Pension Fund of Russia (SPFR) became one of the prominent investors of OFZ bonds.

Based on several external and local challenges faced by Russia over the past decade, Analytical Credit Rating Agency Financial Stress Index ((ACRA FSI) in Figure 3.5.9 is used to analyse the proximity of the Russian financial system to the crisis and its reaction to different events.

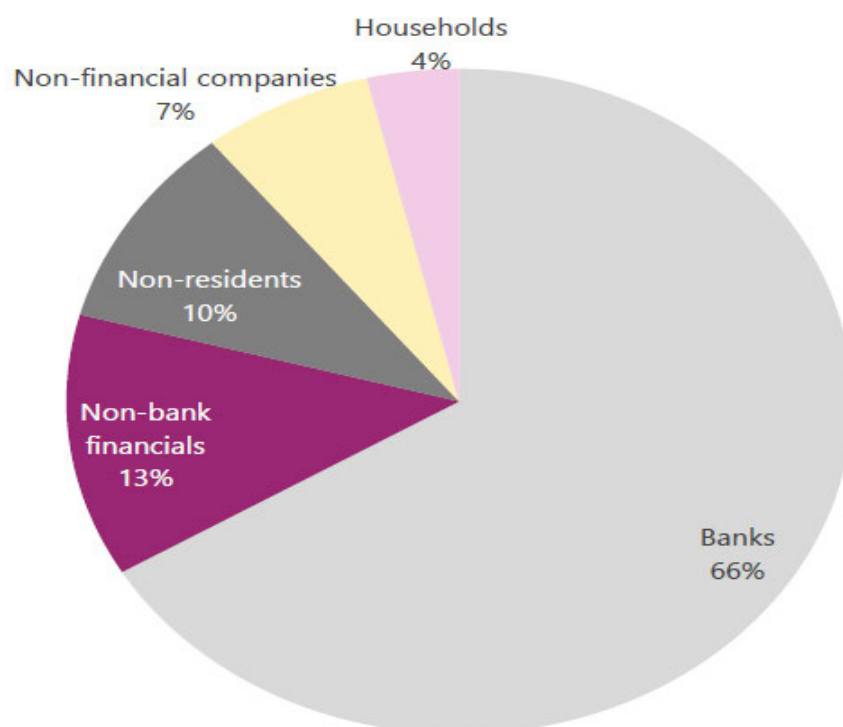


*Figure3.5.9: Russia's dynamics in the ACRA FSI for 2006-2022*

*Source: Bloomberg (2022)*

It is observed in Figure 3.5.9 that the ACRA FSI index has been relatively stable over the past decade. This was well below the 2.5 points boundary for the system's transition to a state of crisis except for a couple of periods of high stress (i.e. the 2008 global stress, the 2014 Ukraine/Russian war, the Covid-19 pandemic in 2020 and the 2022 Ukraine/Russian war). Over these periods, the ACRA FSI index climbed to over 8 points except for the Covid-19 period, where the index slightly jumped to a maximum of 5 points. The 2022 Ukraine/Russian war resulted in an index of closer to 10 points

which was never realised during the previous global and local crises. According to Simola (2022), most countries, including the United States of America and the European Union, imposed extensive economic sanctions on Russia to restrict Russia's financial and technological capabilities for warfare. As such, Russia has not been able to obtain significant new sources of foreign finance, given that most of its reliance on foreign financing is based on the economies that have imposed sanctions or prohibitions for buying Russia's stock. This came during the recovery period from the Covid-19 impact after a year of global economic shutdowns, drastically affecting emerging economies' funding liquidity and increased budget deficits. As such, this saw a drastic decline of over US dollar 23 billion in the net flow of inward financial direct investments (FDI) in Russia. This is a significant blow for Russia compared to all previous external and local stresses, which only resulted in an average outflow of US dollar 5 billion.



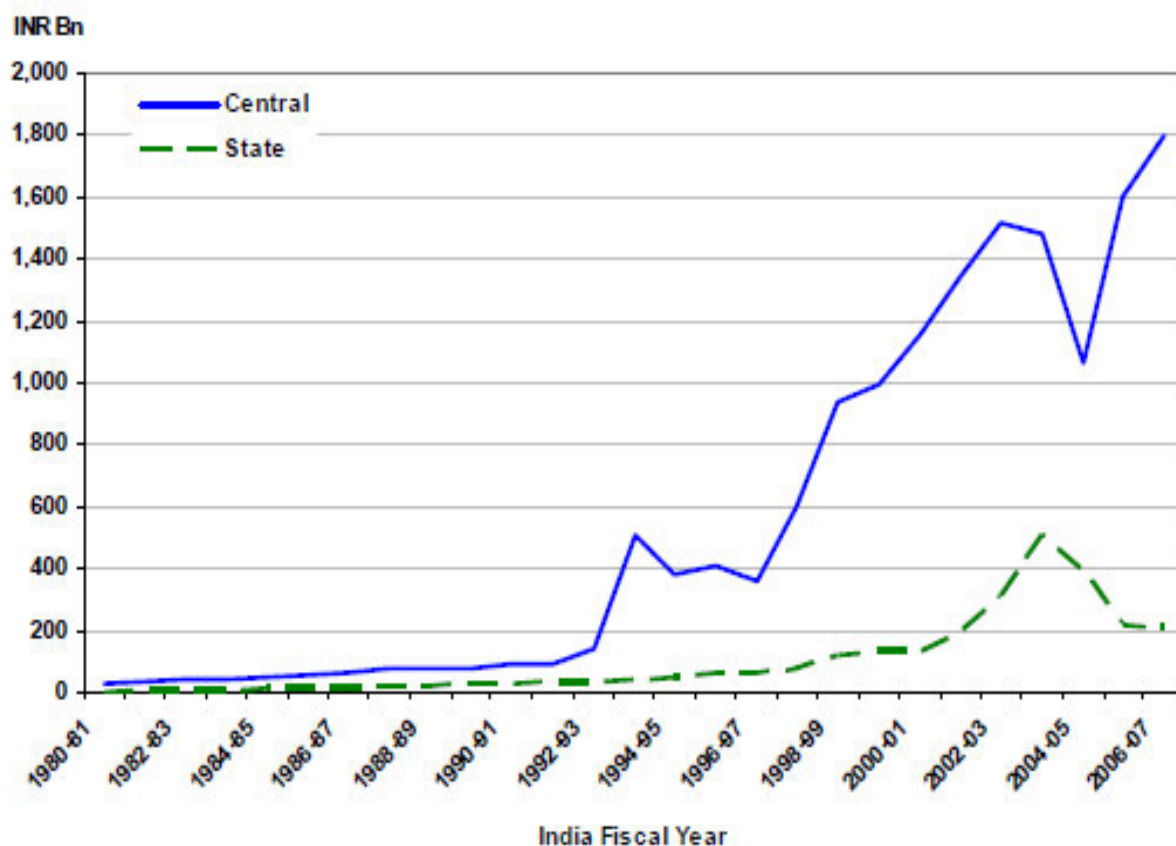
*Figure 3.5.10: Russian distribution of bondholders as of June 30, 2018*

*Source: Baranova and Kulikov (2019: pp. 9)*

Based on Figure 3.5.10, it is evident that by 2018, most of the government bonds in Russia were held by non-bank financial companies and residential commercial banks, with a combined total of 79 per cent of the total debt. Baranova and Kulikov (2019) indicated that in 2015 during periods of structural liquidity shortfall, the Central Bank used government bonds to drive liquidity by using repurchase agreements (REPO), considered relatively liquid versus ordinary loans. This is due to the investment risk perception that has always been embedded in most households in Russia, which began with the perception that all types of financial savings are risky and connected to the potential loss of wealth. Due to these issues, financial institutions and financial instruments in Russia do not have a high level of confidence. This resulted in negative real interest rates, typical in Brazil before 2014, for a very long time.

### 3.5.3.India

Even though the public debt in India could be traced back to the 1950s, according to Reddy (2002), it was until the late 1980s and early 1990s that there was a transformation in the pattern of economic developments adopted by India. As such, between 75 and 80 per cent of financial intermediation, which significantly helped the development process of the public debt market, was predominantly from the public sector banks and financial institutions. In order to attract investors, interest rates on government/public debt were administered and kept below the market level. To further analyse the growth of India's government borrowing over time, refer to Figure 3.5.11.



*Figure 3.5.11: Indian government market borrowing*

*Source: Wells and Schou-Zibell (2008: pp. 20)*

It is observed in Figure 3.4.14 that from the early 1990s, the government bond market in India has developed drastically, mainly driven by market reforms, infrastructure enhancements and an increased supply of government bonds. Wells and Schou-Zibell (2008) indicated that while this is the case, a new fiscal discipline aimed at managing the government budget deficit might negatively affect levels of new bond issuances in India. However, as indicated by Institute for International Monetary Affairs (IIMA, 2020), the growth was also substantiated by obligations for domestic financial institutions to maintain a particular percentage of government securities to facilitate a smooth absorption of government bonds issued to finance the Indian fiscal deficit. To analyse the Indian public debt maturity profile over time, refer to Figure 3.5.12.



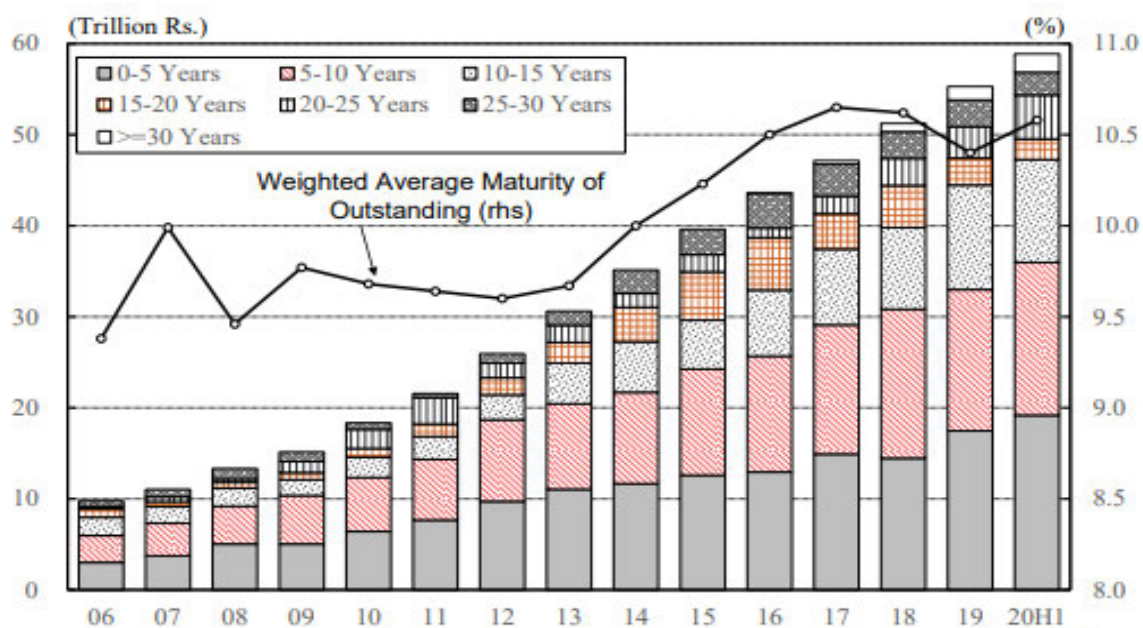


Figure 3.5.12: Maturity profile for India's government debt

Source: Institute for International Monetary Affairs (2020: pp. 17)

With steady growth in the overall public debt in India over the past decade, most of the debt dominated the short-to-mid end of the curve (i.e. at most ten years). It is observed in Figure 3.5.12 that by the end of the first half of 2020, a bigger portion of the overall public debt had maturities of at most ten years. According to Sitharaman (2022), the use of switches has played a critical role in transferring the short-term refinancing pressures to the mid and long-end of the curve. The same strategy has been followed by South Africa (National Treasury, 2021) in managing short-term refinancing pressures, given the clustered government maturity profile with higher outstanding amounts redeemable. As such, to analyse the debt indicators of India over the Covid-19 period, refer to Table 3.5.3.

<b>Debt indicator</b>	<b>2016-17</b>	<b>2017-18</b>	<b>2018-19</b>	<b>2019-20</b>	<b>2020-21</b>
General Govt. Liabilities (in ₹ crore) as % of GDP	68.4	68.5	68.7	73.7	87.8
Weighted average maturity (years)	10.65	10.62	10.40	10.72	11.31
Short-term debt (as % of GDP)	1.0	1.3	1.4	1.3	1.7
Ownership of dated Govt securities (%)					
Insurance companies	22.9	23.5	24.3	25.1	25.3
Provident funds	6.3	5.9	5.5	4.7	4.4
Commercial banks	40.5	42.7	40.3	40.4	37.8

*Table 3.5.3: Debt indicators for India*

*Source: Sitharaman (2022: pp. 10)*

It is observed in Table 3.5.3 that during the peak of Covid-19 stress in the 2020/21 fiscal year, the weighted average maturity of the Indian debt had lengthened by over six months mainly due to switches/conversions. These strategies were adopted for debt management in support of the extension of the term-structure and further assist with minimising the refinancing risk in the short-term (Sitharaman, 2022). This is commendable given emerging markets' liquidity pressures, resulting in issuances in the short-dated long-term bonds, as observed in the South African case (National Treasury, 2021). Furthermore, wider budget deficits observed in the 2020/21 fiscal year to cover fiscal pressure emanating from Covid-19 stress resulted in the short-term debt as a percentage of GDP increasing by 0.4 percentage points to 1.7 per cent. This also happened while the general government debt as a percentage of GDP increases by 14.1 percentage points to 87.8 per cent. Sitharaman (2022) further indicated that the public debt in India is mainly dominated by fixed-rate components, which provide some level of stability and protection against any interest rate volatilities to interest payments, even during periods of market stress. It is also observed that the investor base remained well diversified, with commercial banks being the dominant investor. However, a decline of 2.6 percentage points in commercial banks' holding was realised to 37.8 per cent in the 2020/21 fiscal year. Insurance companies and provident funds also contribute a combined 29.7 per cent of the investor base for Indian public debt, which provides some level of stability in demand for long-term securities.

### 3.5.4.China

Even though the Chinese bond market has grown to recently be one of the world's largest government bond markets, the market remained relatively marginal for many years. According to Bai, Fleming and Horan (Bai et al., 2013), from 1861 to 1950, China issued foreign bonds to finance military expeditions during the World War I and II era; however, issuances in the Chinese bond market were terminated in 1950. As such, domestic issuances resumed in 1981 post the liberalisation reforms of 1979, mainly to address a shortfall in funding for national construction projects. However, trading in the secondary market commenced in 1988 in selected cities and only happened countrywide when the Shanghai and Shenzhen Stock Exchange commenced in 1990, and the auction system opened in 1996. Figure 3.5.13 further summarises a growth in public debt issuances for the Chinese bond market over time.

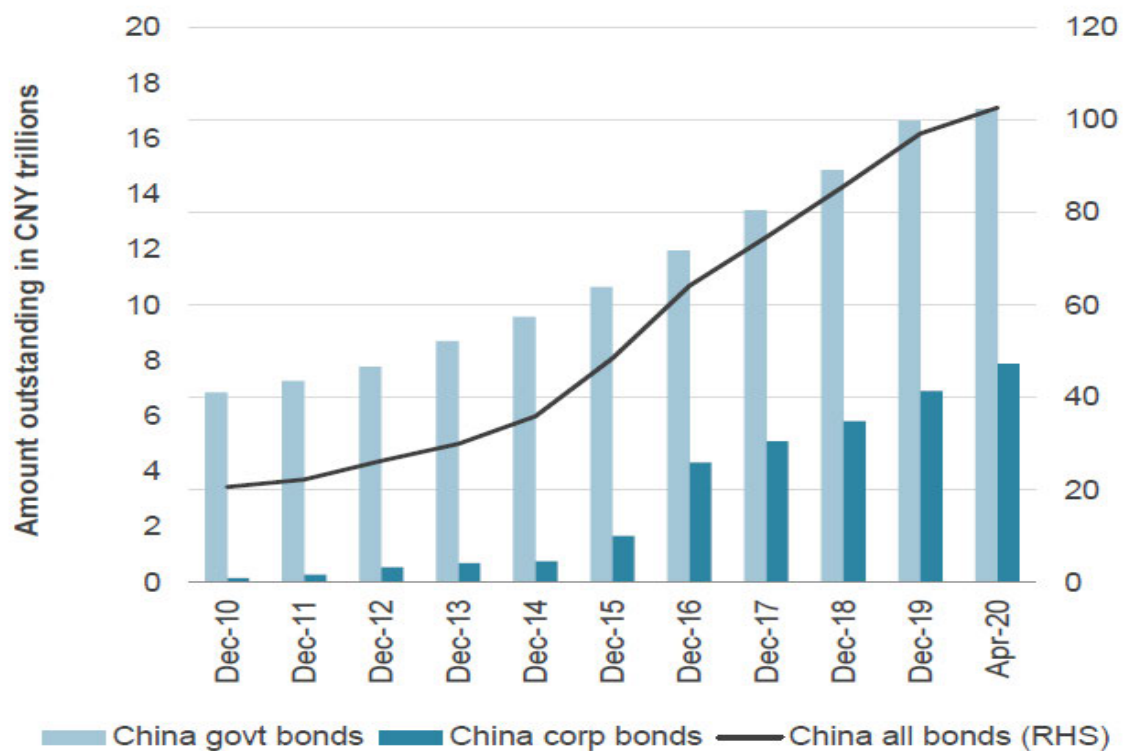


Figure 3.5.13: Growth in issuance in the Chinese bond market

Source: Bloomberg (2020)

It is evident in Figure 3.5.13 that over the past decade, the Chinese bond market has increased significantly. It is further indicated in Seafarer Capital Partners (2023) that the total bond issuance in China has grown from US\$ 286 billion in the year 2000 to US\$ 15 trillion as of December 2020, with almost 65 per cent coming from the government sector and its agencies. As part of risk management against currency risk, over 93 per cent of government and corporate bonds in China are issued in the local currency. However, the average term-to-maturity of the overall sovereign debt has declined from around ten years in 2000 to below six years in 2020. This is a result of China's facilitation of extending short-term maturities. This might be a downside for China should there be issues of capital availability which can affect the ability to refinance this magnitude of debt in the short term.

### 3.6. The gaps to be addressed by this study

The currently published methodologies by Reid (2009), Johannesburg Stock Exchange (2012) and Mashoene et al. (2021) have proven to capture the movements in the real spot rate curve in South Africa. These studies were carried out over stable market and economic periods (i.e. after the 2008 global financial crisis and before the Covid-19 pandemic). It was further indicated by Mashoene et al. (2021) that term-structure models with constant parameters might not have the capabilities to perfectly fit the yield curve over periods of high volatility. It is observed in the study by Papailias (2022) in the case of the USA and Severino et al. (2022) in the case of Canada that the application of the Dynamic Nelson-Siegel (2006) model was able to fit the dynamism of the term-structure over the Covid-19 period. On emerging markets, the study by Castello and Resta (2022) indicated that the Five Factor De Rezende–Ferreira Model (5F-DRF) which is equivalent to the Dynamic Generalised Nelson-Siegel model performed relatively well during the period of heightened Covid-19 stress.

To address the issue of model limitations over periods of high volatility in the South African case, this study explores further dynamism by incorporating the model recalibration process on static term-structure models (i.e. linear parametric, cubic spline, Nelson-Siegel and Svensson models) over different macroeconomic environments. Further, the application of dynamic/time-dependent decay rates on the

Dynamic Nelson-Siegel (2006) model over periods of non-normal or highly volatile market and economic conditions (i.e. including the data covering the period of the Covid-19 pandemic). The study further extends the use of the AR (1) process by exploring the incorporation of macroeconomic factors in estimating future model parameters for the forecasting process of future spot rate curves.

### 3.7. Conclusions

It is observed in the study that the South African bond market is facing a couple of challenges which might limit the likelihood of maintaining a sustainable debt level in the country. Covid-19 exacerbated the fiscal position, which is reflected in drastic increases in the overall public debt level, which is currently expected to breach the 70 per cent mark by the end of 2023 (National Treasury, 2023). Current global pressures, partially affected by the Ukraine/Russian war, have also worsened the situation in emerging markets, with the possibility of recession being inevitable. South Africa also has a clustered maturity profile across the entire maturity spectrum, with large outstanding amounts redeeming in the short term. Reliance on the switch programme has effectively reduced short-term financial pressures. The same issues are also observed in some of the world's leading emerging markets (i.e. Brazil and Russia) which are also rated further down the sub-investment rating category by all three major international rating agencies. In the analysis of the effects of Covid-19 on the estimation of a yield curve based on mathematical term-structure models following the Nelson and Siegel (1987) framework, it was observed from bigger economies (i.e. Canada and the United States of America) that time-dependent/ varying model parameters were able to fit the volatilities observed in 2020 which are considered '1-in-100' year's event.

## **Chapter Four**

### **Methodology review**

#### **4.1. Introduction**

It was mentioned in the previous chapter that Covid-19 had a negative impact on the majority of emerging markets, and some adjustments were made on fiscal and monetary policies to support the country's financial/economic and social welfare. These adjustments had a significant impact on the cost of borrowing. As such, the use of Nelson and Siegel (1987) framework with time-varying model parameters has proven to be adequate for estimating yield curves under economic stress for both selected big economies and emerging markets. Given that the focal point of this research is to discover a term-structure for the real spot rates in South Africa, this chapter looks at the research design and approach using different term-structure models with Nelson and Siegel (1987) background. Furthermore, data collection and analysis techniques are detailed for data quality control; and the sampling techniques are also detailed.

#### **4.2. Research design and approach**

The study is quantitative by nature and will look at different mathematical term structure models (i.e. linear parametric, cubic splines and Nelson-Siegel extensions) to capture the movements of the real spot rates for South Africa over different monetary policy regimes. The exercise aims to arrive at a significant conclusion on the term structure model that can fit and forecast the movements of the South African spot rate curve over normal and stressed market conditions. It was suggested by Mashoene et al. (2021) to extend the scope of work throughout stressed market conditions to assess the performance of different term structure models following the Nelson-Siegel (1987) framework.

The fitting capabilities of each fitted term structure model will be explored using a couple of goodness-of-fit measures that will be detailed in Section 4.3. These measures will include the use of Root Mean Square Error (RMSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). To further analyse the performance of the chosen term structure models, Kikuchi and Shintani (2012) indicated that the model must satisfy these aspects of a spot-rate curve. The spot rate estimates should not result in abnormal estimates (i.e. drastically high/low); further, a significant level of smoothness in the yield curve is critical. Given the volatility surrounding the newness of the instrument in the market, Barclays and the Bond Exchange South Africa (2006) indicated that 2006 inflation-indexed bond yields were distorted by the issues surrounding a significant growth in demand which was not reciprocated by supply. However, National Treasury (2010) indicated that post the 2008 global crisis, liquidity showed many improvements in the South African inflation-indexed bond market, given portfolio restructuring by investors and increased bond issuances into this bond instrument.

As such, bond data from June 2010 to June 2022 was considered for this study. This will ensure that the data used in this study covers periods of different economic environments which also include a period of heightened Covid-19 stress (i.e. from March 2020 to December 2021). However, in deciding whether to use quarterly or monthly data for this study, the systematic sampling methodology used by Mashoene et al. (2021) was considered. A systematic sample is obtained by selecting a "random start at the beginning of the population list and then taking every unit equally spaced after that" (Bellhouse, 2014: pp. 1). Given the kind of data used in this study (i.e. bond prices), this methodology can also be used on periodical data and help decide on the frequency of the data series.

The analysis is based on the daily coupon-bearing yield-to-maturity data on government inflation-indexed bonds from South Africa to analyse the systematic sampling methodology. This is due to the market's unavailability of the real zero-coupon curve. The coupon-bearing bond prices will be used to calculate the real spot rates and be compared to the output from the Johannesburg Stock Exchange (2012) methodology. The study only focused on the government inflation-indexed bonds issued by South Africa as the biggest issuer of inflation-indexed bonds at 85 per cent

of the total outstanding amount in the year 2020 (Botha and January, 2020), and it further analysed the bonds on the curve with the same credit rating.

The significance test was based on the two-sample t-test for means ( $\mu$ ) assuming equal variance. Gujarati and Porter (2009) indicated that the null hypothesis can be defined as:

$$H_0: \mu_1 - \mu_2 = 0 \quad (4.1)$$

where  $\mu_1$  is the mean for the first sample and  $\mu_2$  for the second sample; and the test statistic is:

$$T = \frac{\bar{X}_1 - \bar{X}_2}{SE_{\bar{X}_1 - \bar{X}_2}} \quad (4.2)$$

with degrees of freedom  $df = n_1 + n_2 - 2$  and standard error  $SE_{\bar{X}_1 - \bar{X}_2} = \sqrt{\left(\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$  at 5 per cent level of significance ( $\alpha$ ). The null hypothesis is not rejected if  $p - value \leq \alpha$ . The analysis of a test of differences in the monthly averages is presented on *Table 4.2.1*.

Timeframe	Test statistics	P-value	Pass (Y/N)
Mar-2018 vs Apr-2018	22 639,89	8,7987E-144	Y
Oct-2018 vs Nov-2018	-5,68	1,13273E-06	Y
Mar-2019 vs Apr-2019	6,36	1,47091E-07	Y
Oct-2018 vs Nov-2019	-14,33	9,05261E-18	Y
Mar-2020 vs Apr-2020	7,06	1,21067E-08	Y
Oct-2020 vs Nov-2020	8,12	5,43874E-10	Y

*Table 4.2.1: Test for difference in mean/average*

*Source: Author's own work (2023)*



To assess any deviations from the work done by Mashoene et al. (2021), especially throughout global market volatility, the analysis of the sensitivity of the bond yields covered the period between March 2018 and November 2020. From Table 4.2.1, it can be observed that there is a significant difference in the monthly averages over the selected period. This differs from the analysis made by Mashoene et al. (2021), where no significant differences were observed between the monthly average of the selected months. This could be attributed to changes in economic landscapes between the two analysis periods.

Gujarati and Porter (2009) indicated that the stationary stochastic process has a mean and variance that are constant over time, and the value of the covariance between the two time periods depends only on the distance or gap or lag between the two time periods and not the actual time at which the covariance is computed. Hyndman and Athanasopoulos (2018, pp. 1) also indicated that "a stationary time series will have no predictable patterns in the long term". However, it is often assumed that stock prices (including bond prices) follow a random walk; which is defined by Gujarati and Porter (2009) as non-stationary or the presence of unit root. This implies that the future value of the bond price is unpredictable to avoid any speculations/arbitrage opportunities in the debt market. The analysis of stationarity on the R197 bond rates is presented in Table 4.2.2. It was observed that the historical data of the R197 coupon-bearing bond rates failed both stationarity and unit-root tests. This indicates that the R197 bond rates have non-stationarity property, thus mimicking a random walk. This implies that no arbitrage opportunity available in the pricing of this instrument given that the changes in interest rates follow a random and the future values of interest rates are unpredictable.

<b>Description of Assumption</b>	<b>Description of Statistical Test</b>	<b>Test Statistics</b>	<b>P-value</b>	<b>Pass (Y/N)</b>
<b>Test for stationarity</b>	KPSS Stationarity Test – Trend	0,51614	0,01	N
<b>Test for unit root</b>	Aug. Dickey-Fuller Test – zero mean	-1,9648	0,591	N
	Phillips-Perron Unit Root Test	-6,512	0,7413	N

*Table 4.2.2: Statistical tests for stationarity and unit root*

*Source: Author's own work (2023)*

As such, monthly data covering the period between June 2010 and June 2022 was used in this study. To cater for the period of unfavourable market conditions, the development phase covered the period between June 2010 and December 2021 (to ensure that the period covering the Covid-19 pandemic is included in the development). Since macroeconomic factor forecasts are revised quarterly, the testing phase only covered six months between January 2022 and June 2022. The data used was a sample of over 160 points covering periods of normal and stressed market conditions with monetary policy switches. The sample size of 30 data points is considered satisfactory based on the Central Limit Theorem (CLT). As such, the data used in this study was sufficient to decide on the model's capability to estimate current and future real spot rates for South Africa over normal and non-normal market conditions.

The study also analysed the link between macroeconomic factors and regime switches on the term-structure models. The work done by Kobayashi (2017) indicated that economic conditions, credit fundamentals and market sentiments were selected as macro-economic variables that drive the transition probability of the regime switches. This study included the following macro-economic factors in line with the work done by Ding (2020) on Chinese bond yields using the Nelson and Siegel (1987) model:

- FTSE/JSE Inflation-indexed Government Index (IGOV) is an index that measures the performance of the most liquid South African inflation-indexed government bonds. It is a proxy for movements in the bond market conditions.
- Real gross domestic product growth rate (real GDP) to analyse the country's economic health in real terms.
- Consumer Price Inflation (CPI) to analyse changes in prices in a country. Higher CPI future expectations translates into future higher indexations and has an inverse relationship with the cost of borrowing on inflation-indexed bonds.
- Auction bid ratio to analyse the market liquidity for the inflation-indexed bond instrument. A higher bid ratio translates into higher liquidity/appetite for a particular bond instrument.

### 4.3. Data collection methods and data quality control

All data sources used in this study do not require any consent to source the information. These sources are reliable official institutions for publishing the selected macroeconomic and financial data used in this study. Many institutions consider the Bloomberg platform reliable since it relies mainly on the same institutions for the reliability of the published macroeconomic information.

The bond information, including the bond name, the issue date, maturity date and coupon rates, were sourced directly from the National Treasury. For consistency in the data, closing all-in bond prices for the inflation-indexed bonds (which were used to derive the real spot rate curve) were sourced from the Bloomberg platform. All-in-bond prices were used to retrieve real spot rates to account for all future cashflows. All-in-bond prices accounted for all cashflows due for the bond's life over different periods. Macro-economic data (i.e. macro-economic factors used for the analysis of regime switches) was directly sourced from the South African Reserve Bank.

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### 4.4. Data analysis

To minimise the model risk that could be embedded in using Excel to model different term structures, the work done in this study is automated in the R software. The modelling process minimises the error between the actual and model-generated coupon-bearing bond prices. Gujarati and Porter (2009) defined the Ordinary Least-Squares (OLS) methodology as a statistical method that assesses the association between two variables by minimising the squared error between the actual and model-generated values. Gujarati and Porter (2009) and Verbeek (2017) indicated that OLS estimators should suggest the best linear unbiased estimators (BLUE) characteristic. These characteristics require that the estimators should not have any bias between the OLS estimator's expected value and the population's true value; and should further be consistent even for large samples. As such, the OLS methodology is used to estimate model parameters by minimising the squared error between actual and model-generated coupon-bearing bond prices.

The same process followed by Mashoene et al. (2021) to estimate the continuously compounded real spot rates supposing a risk-neutral assumption, discount factors and the all-in bond price for coupon-bearing inflation-indexed bond should satisfy the following condition:

$$\hat{B}(t, \tau) = \sum_{t=0.5}^{\tau} [CF(t, \tau) \cdot d(\tau)] \quad (4.3)$$

where  $CF(\tau, \tau) = 100 + Coupon_i$  is the cashflow at maturity which is made up of the coupon-payment for bond  $i$  and the par value, and  $d(\tau) = e^{-R(t, \tau) \cdot \tau}$  as a theoretical discount factor applied on continuously compounded real spot rates for inflation-indexed bonds. This implies that, under the risk-neutral assumption, a coupon-bearing bond price for inflation-indexed instruments is expressed in a matrix form as:

$$B = CF \cdot d + e_t \quad (4.4)$$

Given the exponential nature of the discount factor, a log conversion is used to transform to linear regression. Minimising the squared errors by applying the OLS methodology, the optimisation process is defined as:

$$\min_{\beta} e_t^T e_t = (B - CF \cdot d)^T (B - CF \cdot d) \quad (4.5)$$

where a set of model parameters  $\beta$  is optimised.

The analysis of static term-structure models comprises two dimensions: using initial parameters (i.e., as of June 2010) over the estimation period and using the recalibrated parameters to account for different economic environments. The recalibration process is applied at the end of 2011 after the South African Reserve Bank made the last

interest rate cut before stability was maintained with a couple of small increases to tame the volatile local currency, and also in March 2020 to account for the effects of Covid-19 pandemic in the global economy.

A couple of goodness-of-fit measures will be applied to further analyse the goodness of fit in the model output for backtesting analysis. This ensures that a term structure model meets a couple of hypothesis tests to ensure it is significant in capturing movements in the yield curve.

- Root Mean Square Error (RMSE) methodology is the standard deviation of prediction errors to analyse the spread of prediction error:

$$RMSE = \sqrt{\sum_{i=1}^n \frac{(P(t, \tau_i) - \hat{P}(t, \tau_i))^2}{n}} \quad (4.6)$$

where  $n$  is the sample size,  $P(t, \tau_i)$  is the actual price of the real zero-coupon bond, and  $\hat{P}(t, \tau_i)$  is the model estimated price of the real zero-coupon bond using the selected term structure models. The lower RMSE estimate is preferred as it implies a better model fit to the actual prices.

Willmott, Matsuura and Robeson (Willmott et al., 2009) indicated that mean absolute error (MAE) is mostly preferred over the RMSE in model valuation statistics citing that RMSE is ambiguous. However, it was proven by Chai and Draxler (2014: 1) that 'RMSE is more appropriate to represent the model performance than the MAE when the error distribution is expected to be Gaussian'. The Gaussian property is in line with assumptions embedded in the OLS methodology. The Akaike and Bayesian information criteria were used to further evaluate the model's performance to supplement RMSE results.

- Akaike information criterion (AIC) methodology is another measure of goodness-of-fit used in this study. Gujarati and Porter (2009) indicated that the AIC methodology enforces a stricter penalty than the coefficient of variation ( $R^2$ ) by adding more

regressors, and it is more suitable for both in-sample and out-of-sample performance of the regression model. The AIC methodology is defined as follows:

$$AIC = e^{2k/n \frac{\sum (P(t, \tau_i) - \hat{P}(t, \tau_i))^2}{n}} \quad (4.7)$$

where  $k$  is the number of regressors (including the intercept), and  $n$  is the number of observations. The methodology prefers the model with the lowest AIC estimate.

- Bayesian information criterion (BIC) methodology is defined as:

$$BIC = n^{k/n} \frac{\sum (P(t, \tau_i) - \hat{P}(t, \tau_i))^2}{n} \quad (4.8)$$

where  $k$  is the number of regressors (including the intercept), and  $n$  is the number of observations. Like AIC, the methodology prefers a model with a lower BIC estimate and is suitable for both in-sample and out-of-sample model performance analysis. It provides a stricter penalty than the AIC methodology by penalising more regressors.

- Hypothesis testing for equal means:

Marcus and Elias (1998: pp. 1549) indicated that "although hypothesis tests can never show that the model is right (i.e. model verification), they can be used to validate the model (i.e. show that the application of the model is not wrong)". It is expected for the predicted values to be different from actual observations; however, the mean observation and the mean forecast from a sound model should not have significant differences from one another.

Statistical methods were applied to analyse the significance of deviations observed between actual values and forecasts (Marcus and Elias, 1998). These tests are

based on comparing averages/ means from the two datasets (i.e. actual observations and forecasts made). In comparing the two-sample means, the primary null hypothesis is that the means are equal:

$$H_0: \mu_{actual} = \mu_{forecast}$$

with the alternative hypothesis for the two-sided distribution:

$$H_0: \mu_{actual} \neq \mu_{forecast}$$

where  $\mu_{actual}$  is the average/mean of the actual values, and  $\mu_{forecast}$  is the average/mean of the predicted/forecasted values. The probability value ( $p - value$ ) defines the probability that the test statistic will take a value at least as extreme as the observed value. A confidence level of  $\alpha = 0.05$  is assumed. This implies that if the  $p - value$  is less than the prescribed confidence level  $\alpha$ , the null hypothesis is not accepted, thus implying that the test is significant.

The test is based on the Aspin-Welch test, which is robust under normal distribution regardless of the sample sizes and group variances (Ahad and Yahaya, 2014). The T-statistic used in this regard is estimated as follows:

$$T - statistics = \frac{\mu_{actual} - \mu_{forecast}}{SE_{\mu_{actual} - \mu_{forecast}}} \quad (4.9)$$

with the standard error (SE) for unequal variance is defined as:

$$SE_{\mu_{actual} - \mu_{forecast}} = \sqrt{\frac{S_{actual}^2}{n_{actual}} + \frac{S_{forecast}^2}{n_{forecast}}} \quad (4.10)$$

The degrees of freedom is defined as:

$$dof = \frac{\left( \frac{S_{actual}^2}{n_{actual}} + \frac{S_{forecast}^2}{n_{forecast}} \right)^2}{\left( \frac{(S_{actual}^2)^2}{n_{actual}^{-1}} + \frac{(S_{forecast}^2)^2}{n_{forecast}^{-1}} \right)} \quad (4.11)$$

where  $S^2$  is the sample variance and  $n$  is the sample size.

Forecasting process remains critical as part of this study to assess the model's capability on the South African real term structure. Given that most macroeconomic data is released on a periodic interval (i.e. monthly or quarterly), the forecasting process will be done monthly for six months ahead. The forecasting process will be used for the calculation of the spot rates in the future; as such, the estimation for each model was based on the following assumptions:

- The original model stated in Section 3 will be used for the forecasting process for linear parametric and cubic splines models.
- For static term structure models that follow the Nelson and Siegel (1987) basis, the future methodology will be used; and it is defined as:

$$\hat{R}(t + l, \tau) = \beta_0 + \beta_1 \left( \frac{1 - e^{-\lambda \cdot \tau}}{\lambda \cdot \tau} \right) + \beta_2 \left( \frac{1 - e^{-\lambda \cdot \tau}}{\lambda \cdot \tau} - e^{-\lambda \cdot \tau} \right) + \dots \quad (4.12)$$

where:

- The first scenario will be based on betas observed as of June 2010 and are kept constant over the estimation period,  $l = 0.25, 0.5, \dots \infty$  is future time-steps in years; and
- Another scenario will look at the recalibrated model betas given the swing in the monetary policy regime in March 2020 due to the Covid-19 pandemic stress. Recalibrated betas as of March 2020 were used for the forecasting process.



➤ For dynamic term structure models that follow the Dynamic Nelson and Siegel (2006) basis, parameters to estimate future discount factors, which ultimately derive future spot rates, were estimated using the following:

- The autoregressive  $AR(p)$  model of order one  $p = 1$ . The process follows a linear function that forecasts current parameters using the most recent parameters. This methodology was recommended by Van Elen et al. (2012) because the vector autoregressive  $VAR(1)$  model was unable to give optimal results:

$$\beta_{i,t+1} = \alpha_i + \theta_i \beta_{i,t} + \varepsilon_{i,t} \quad (4.13)$$

where  $i = 0, 1, 2$  and  $\varepsilon_{i,t}$  is the error term. Future discount factors/zero-coupon bond prices are then estimated as:

$$\hat{P}(t + l, \tau) = e^{-\tau \cdot \beta_{0,t+l} - \beta_{1,t+l} \left( \frac{1 - e^{-\tau \lambda}}{\lambda} \right) - \tau \cdot \beta_{2,t+l} \left( \frac{1 - e^{-\tau \lambda}}{\tau \lambda} - e^{-\tau \lambda} \right) + \dots} \quad (4.14)$$

where  $\tau = T - (t + l)$ . Spot rates are then estimated as follows:

$$\hat{R}(t + l, \tau) = -\ln[P(t + l, \tau)]/\tau \quad (4.15)$$

- A macroeconomic factor process was also considered to estimate future parameters. A macroeconomic factor may be defined as a state (i.e. economic, environmental, etc.) that affects the development of a given large-scale economy. Macroeconomic factors considered in this case, in line with Kobayashi (2017), were monetary policy switches, government inflation-indexed bond index (IGOV), Consumer Headline Inflation rate (CPI) and auction bid-ratio. Due to data availability issues on the auction bid prior to 2013,

this implies that the macro-economic factor process will start from April 2013. Linear regression will be fitted as:

$$\beta_{i,t+1} = \alpha_i + \theta_i \beta_{i,t} + \theta_j SWITCH_t + \theta_3 IGOV_t + \theta_4 CPI_t + \theta_5 BID_t + \varepsilon_{i,t} \quad (4.16)$$

where  $i = 0, 1, 2$  and  $\varepsilon_{i,t}$  is the error term.

- For the application of linear regression, single-factor analysis was done to determine statistically significant independent variables to estimate movements/changes in the model parameters. According to Mishra et al. (2019), normality in the data is the underlying assumption in parametric testing; as such, normality assessment is critical. Normality analysis should be performed before applying a single factor/analysis or any statistical analysis to assess the existence of any linear association between the dependent and independent variables. To arrive at an objective judgement on normality, the statistical normality test will be applied; however, the Shapiro-Wilk Normality test, which is more suitable for smaller sample tests ( $n < 50$ ), is applied to address the issue of statical test not always being sensitive enough at low sample size. Normality analysis is done using the most popular Shapiro-Wilk Normality test (Rochon, Gondan and Kieser [Rochon et al.], 2012) and the Anderson-Darling Normality test with the following hypothesis:

Null hypothesis  $H_0$ : *the data is not different from normal*

Alternative hypothesis  $H_a$ : *the data is different from normal*

According to Ramachandran and Tsokos (2020), Shapiro-Wilk Normality/goodness-of-fit test is used to determine if a random sample is drawn from a normal Gaussian probability distribution with a true mean/average  $\mu$  and a variance  $\sigma^2$  such that  $X \sim N(\mu, \sigma^2)$ . King and Eckersley (2019) defined the Shapiro-Wilk Normality/goodness-of-fit test as ordering and standardising the sample (i.e. converting the data to a distribution with a mean  $\mu = 0$  and standard

deviation  $\sigma = 1$ ). The Shapiro-Wilk test statistic is given by (Ramachandran and Tsokos, 2020):

$$W = (\sum_{i=1}^n a_i x_i)^2 / \sum_{i=1}^n (x_i - \bar{x})^2 \quad (4.17)$$

where  $x_i$  are ordered sample values,  $\bar{x}$  is the sample mean/average and  $a_i$  are constants that are generated by the following function:

$$(a_1, a_2, \dots, a_n) = \frac{m^T V^{-1}}{\sqrt{m^T V^{-1} m}} \quad (4.18)$$

with  $m = (m_1, m_2, \dots, m_n)^T$  being expected values of the ordered statistics that are independent and identically distributed random variables that follow the standard normal  $N(0,1)$ , and  $V$  is the covariance matrix of the ordered statistics. A decision to reject or accept the null hypothesis is based on the associated  $p - value$ , which should be bigger than the selected significance level  $\alpha = 0.05$ ; thus implying that at a 95 per cent degree of confidence, we reject or accept the null hypothesis that the data is not different from a normal distribution.

According to Anderson and Darling (1952), the principal innovation of this test was to incorporate the weighting function to allow more flexibility in the test. Even though the Anderson-Darling Normality test is considered as of the most powerful normality tests because it is less sensitive to outliers, Ghasemi and Zahediasl (2012) indicated that the test has a disadvantage of having a higher Type I error rate relative to other normality tests. As such, the test will not be used independently but it is used to complement the Shapiro-Wilk test.

AD test statistic is defined as:

$$AD = -n - \frac{1}{n} \sum_{i=1}^n (2i - 1) [\ln F(X_i) + \ln (1 - F(X_{n-i+1}))] \quad (4.19)$$

where  $n$  is a sample size,  $F(X)$  is a cumulative distribution function (CDF) for the specified distribution, and  $i$  is the  $i$ th sample/observation when the data is sorted in ascending order.

Further, a correlation analysis is applied to further analyse the presence of a linear relationship between the model parameters. Mukaka (2012) stated that a correlation is measured by using a statistic called a correlation coefficient, which represents the strength of the assumed linear association between variables in question and ranges between -1 and 1. There are two standard correlation methodologies (i.e. Pearson's product-moment correlation coefficient and Spearman's rank correlation coefficient), and the correct usage of correlation coefficient type solely depends on the studied variables.

Pearson's product-moment correlation coefficient is used when both variables being studied are normally distributed and are both quantitative; however, the performance of Pearson's correlation is affected by extreme values, which may exaggerate or dampen the strength of relationship, especially when one or both variables are not normally distributed.

A sample correlation coefficient  $r$  is defined as:

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{[\sum_{i=1}^n (x_i - \bar{x})^2][\sum_{i=1}^n (y_i - \bar{y})^2]}} \quad (4.20)$$

where  $x_i$  and  $y_i$  are values for variables  $x$  and  $y$ . Turney (2022) also indicated that Pearson's correlation coefficient could be treated as an inferential statistic. This implies that the correlation coefficient can be used to test the statistical hypothesis of whether a significant linear relationship between two variables does exist. Given the sample Pearson's correlation coefficient  $r$  and the sample

size  $n$ , it can be inferred whether the population Pearson's correlation coefficient  $\rho$  is significantly different from zero based on the following hypothesis:

*Null hypothesis ( $H_0$ ):  $\rho = 0$*

*Alternative hypothesis ( $H_a$ ):  $\rho \neq 0$*

A test statistic  $t$  is defined as:

$$t = r / \sqrt{\frac{1-r^2}{n-2}} \quad (4.21)$$

with the degrees of freedom  $df = n - 2$ . At  $\alpha = 0.05$  significance level, the decision to reject or accept the null hypothesis is defined when the associated  $p$  - *value* is less or equal to the significance level.

Spearman's rank correlation coefficient is more appropriate when one or both variables have a presence of extreme values and there is a presence of skewness in the distribution that determines the studied variables. A sample correlation coefficient  $r_s$  is defined as:

$$r_s = 1 - 6 \sum_{i=1}^n d_i^2 / n(n^2 - 1) \quad (4.22)$$

where  $d_i$  is the difference in the ranks for  $x$  and  $y$ .

Size of Correlation	Interpretation
0.9 to 1.0 (-0.9 to -1.0)	Very high positive (negative) correlation
0.7 to 0.9 (-0.7 to -0.9)	High positive (negative) correlation
0.5 to 0.7 (-0.5 to -0.7)	Moderate positive (negative) correlation
0.3 to 0.5 (-0.3 to -0.5)	Low positive (negative) correlation
0.0 to 0.3 (0.0 to -0.3)	Negligible correlation

*Table 4.4.1: Interpreting the size of a correlation coefficient using a rule of thumb*

*Source: Hinkle, Wiersma and Jurs (Hinkle et al.,2003)*

Based on the rule of thumb in Table 4.4.1, correlation coefficients, and a test for the statistical hypothesis, one can conclude on whether there is a positive or negative linear/general relationship between the variables studied. A positive relationship is an indication of a co-movement in the same direction, whereas a negative relationship is an indication of co-movements in opposite directions.

## 4.5. Ethical clearance

The information used in this study is available for public use on official government websites; as such, it could be concluded that ethical issues were upheld. Ethical clearance in this regard was acquired from the UKZN Research Ethics Office, and the proof is attached to the submission of this study.

## 4.6. Conclusions

It was noted in this section that the availability of data prior 2008 global financial crisis remains a challenge for inflation-indexed bonds in South Africa due to the newness of the bond instrument in the market. Furthermore, since investors buy to hold this bond instrument, liquidity is relatively lower. However, there is a sufficient sample size when considering monthly data from June 2010 to June 2022 for model development and backtesting analysis. The OLS methodology is used to extract real spot rates from traded inflation-indexed coupon-bearing bond prices. A couple of statistical tools are used to analyse the fitted results and perform the future estimation of the real spot rates in South Africa.

## **Chapter Five**

### **Data presentation, analysis and discussion**

#### **5.1. Introduction**

It was noted in the previous chapter that the data prior to 2010 on the inflation-indexed bonds had a couple of gaps due to the newness of the bond in the South African market, which was further affected by liquidity issues during the 2008 global financial crisis. However, the selected sample from June 2010 to June 2022 is adequate to draw a statistically significant conclusion. Given the nature of the data, statistical tests are used to extract real spot rates from the coupon-bearing bond prices and further analyse the model's capability. Given the objective of this study, this chapter explores the practicality of the selected term-structure models on the South African inflation-indexed bond data given selected statistical tests. The study incorporates the Covid-19 stress period to analyse the capabilities to model and estimate the future real spot rate curve in South Africa under non-normal market/economic conditions.

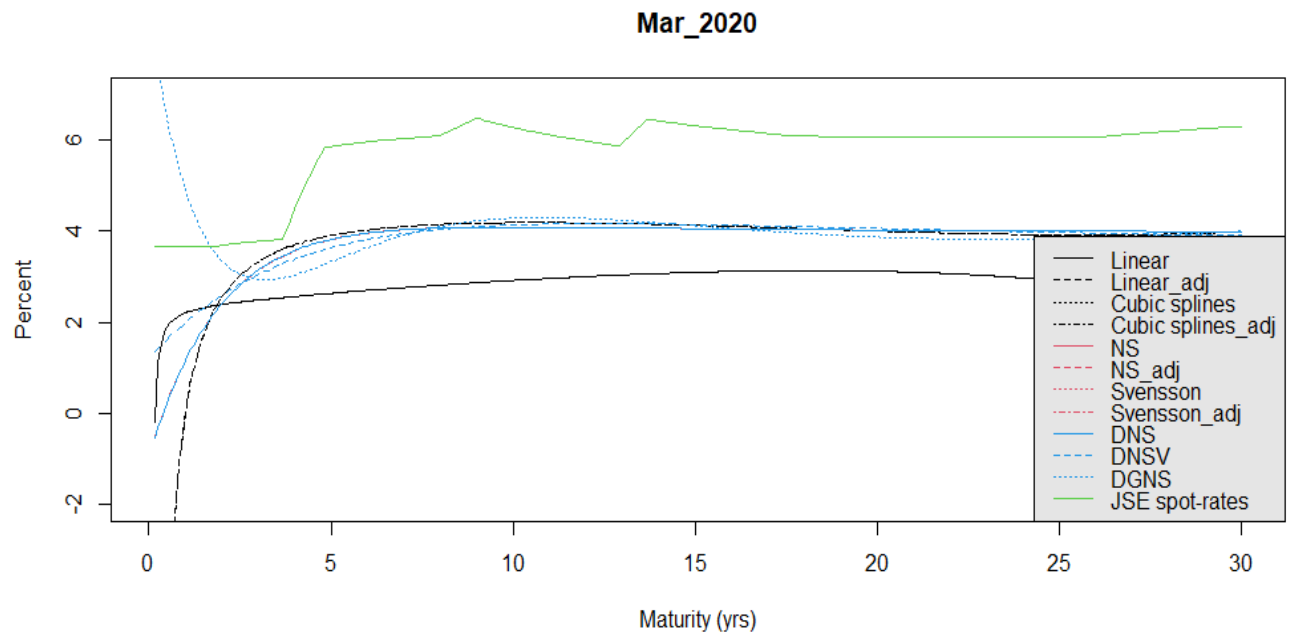
#### **5.2. Fitting**

To assess the fitting capability of the selected models, the spot rate curve should have the following characteristics as described by Kikuchi and Shintani (2012):

##### **5.2.1. Abnormal spot rates**

Before the 2020 Covid-19 global stress, the real spot rate curve in South Africa has been trading below 4 per cent across the curve. An up-pick in real spot rates was realised in the second week of March 2020 after the first case of Covid-19 was realised in the country (National Treasury, 2020a). The cost of borrowing in the inflation-indexed bonds (yield-to-maturity) weakened by 240 basis points (bps) on the I2050 (2.5%, 31-Dec-2050) bond between the 6th of March 2020 and the 27th of March 2020. This is due to uncertainties brought by the effects of the Covid-19 pandemic,

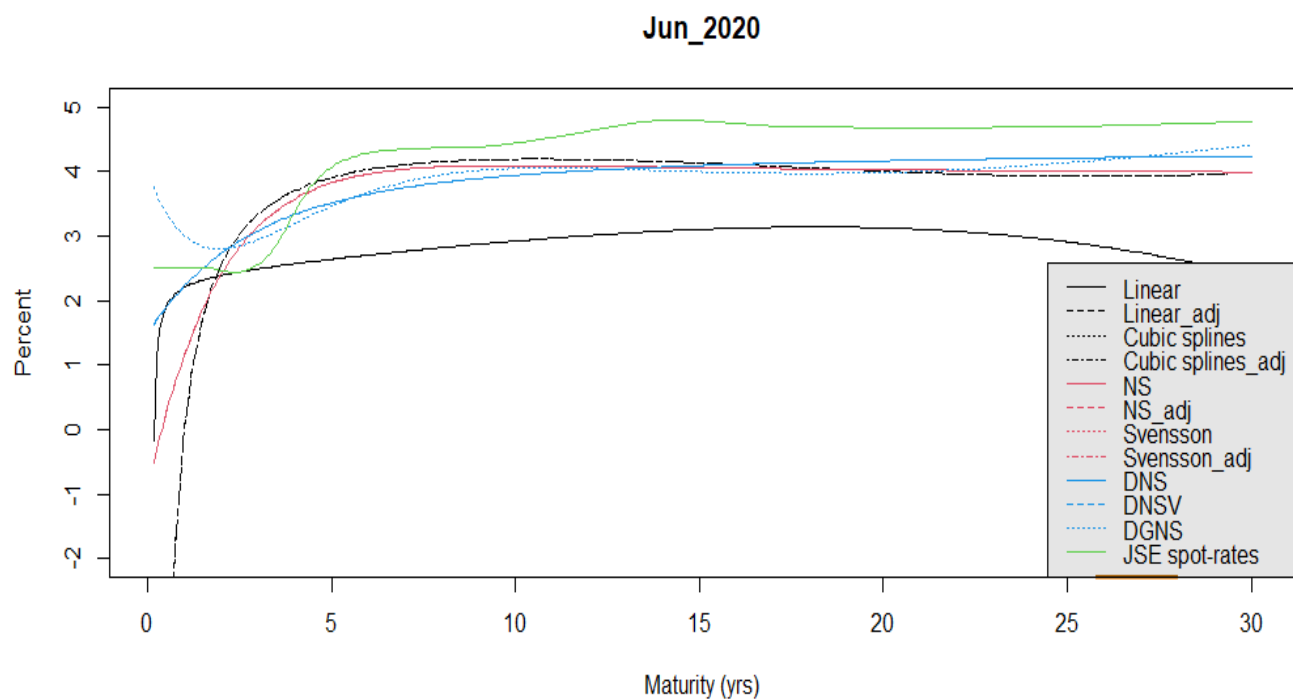
which led to the global economic shutdown. To analyse the effect of the Covid-19 pandemic on the estimation of the real spot rate curve in South Africa, Figures 5.2.1, 5.2.2 and 5.2.3 show the real spot rate curve during March 2020 – after the first economic shutdown was introduced in South Africa, June 2020 – post interventions by the South African Reserve Bank on the buying government bonds and December 2021 when the economy has stabilised.



*Figure 5.2.1: Estimated South African real spot rates – March 2020*

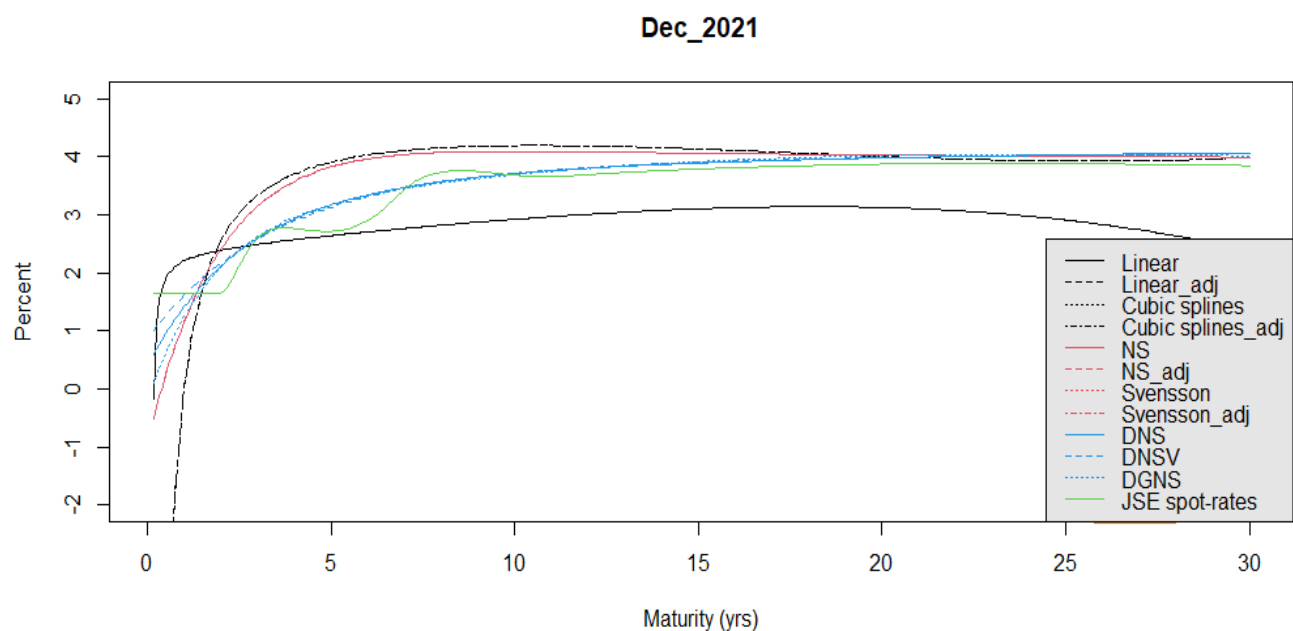
*Source: Author's own work (2023)*





*Figure 5.2.2: Estimated South African real spot rates – June 2020*

*Source: Author's own work (2023)*



*Figure 5.2.3: Estimated South African real spot rates- December 2021*

*Source: Author's own work (2023)*

It is observed in Figure 5.2.1 that the actual real spot rates from the JSE have painted a very volatile environment, with spot rates trading at levels around 4 per cent on the short end of the curve and around 6 per cent on the longer maturities. None of the selected term-structure models was able perfectly capture this volatility but resulted in the estimated rates of around 4 per cent on the longer maturities. However, the estimated real spot rate at the short end of the curve is estimated to be on levels below 2 per cent; and even negative rates on most of the static term-structure models.

Post the SARB intervention in bond buying; actual real spot rates were trading at around 5 per cent which was still 50 bps above the estimates on the real spot rates in South Africa made by the selected term-structure models. In December 2021, after the economy gained some stability with resumption in the global economic activities, real spot rates strengthened to slightly below 4 per cent on longer maturities and around 2 per cent on the short end. During this period, most of the dynamic and recalibrated term-structure models were able, to some extent, perfectly capture movement in the real spot rates in South Africa.

### 5.2.2. Negative spot rates

Real spot rates at the short end of the curve mimic the performance/movements in the real economy. Over four years before the Covid-19 pandemic, the real economic growth rate was around zero per cent on average, refer to Figure 5.2.4. The global economic shutdown drove a significant dip in June 2020; however, a big jump in the subsequent quarter was mainly due to the repricing effect. Even though there are more positives post that, there are few negative real growth rates between December 2020 and June 2022. Table 5.2.1 presents the analysis of the effect of Covid-19 on the real spot rates in South Africa, especially on shorter maturities, in line with the real growth rate in Figure 5.2.4.



*Figure 5.2.4: South African real growth rate*

*Source: South African Reserve Bank (2022)*

Maturity	Linear	Cubic splines	NS	NS_adj	DNS	Svensson	Svensson_adj	DGNS
1-month	139	139	139	139	139	139	139	139
3-month	0	0	139	121	36	139	121	38
6-month	0	0	0	98	33	0	98	35
9-month	0	0	0	0	30	0	0	29
1-year	0	0	0	0	24	0	0	22
2-year	0	0	0	0	11	0	0	14
3-year	0	0	0	0	7	0	0	6
4-year	0	0	0	0	4	0	0	4
5-year	0	0	0	0	3	0	0	3
6-year	0	0	0	0	0	0	0	3
7-year	0	0	0	0	0	0	0	0
8-year	0	0	0	0	0	0	0	0
9-year	0	0	0	0	0	0	0	0
10-year	0	0	0	0	0	0	0	0
30-year	0	0	0	0	0	0	0	0

*Table 5.2.1: Realised frequency of zero rates*

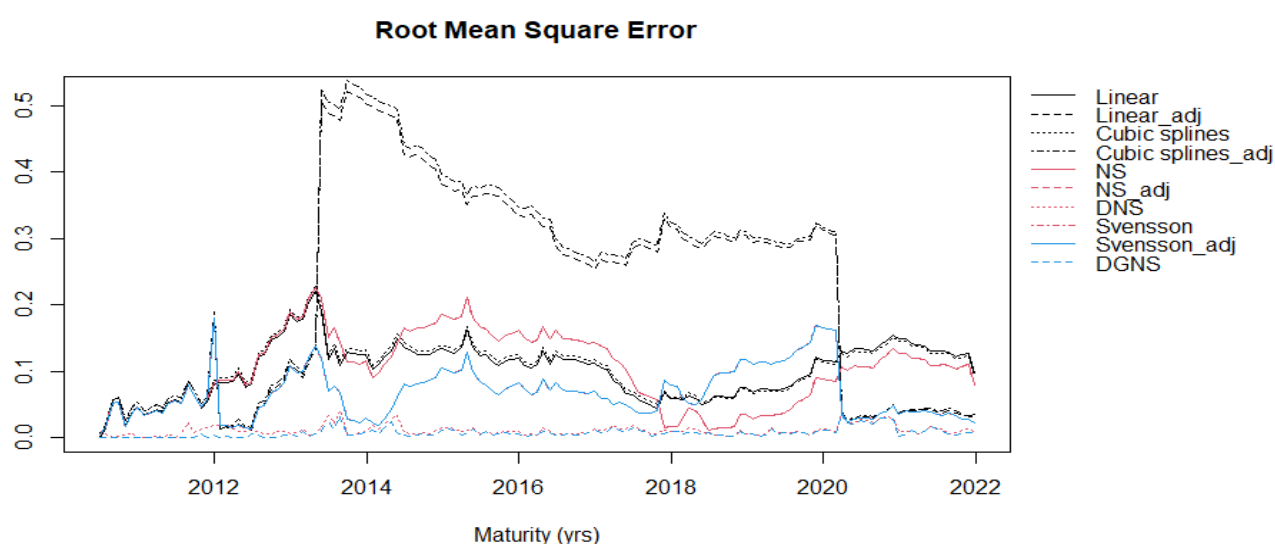
*Source: Author's own work (2023)*

It is observed in Table 5.2.1 that the time-dependent/dynamic models have resulted in a higher number of negative spot rates, especially on term-to-maturities of above six months. Most negative spot rates on these maturities were realised during the Covid-

19 stress period (i.e. March 2020 to January 2021). This is consistent with the past trends of South Africa's real economic growth of around zero per cent, thus implying that the short-term real spot rates in South Africa mimics the movements of the South African real economic growth.

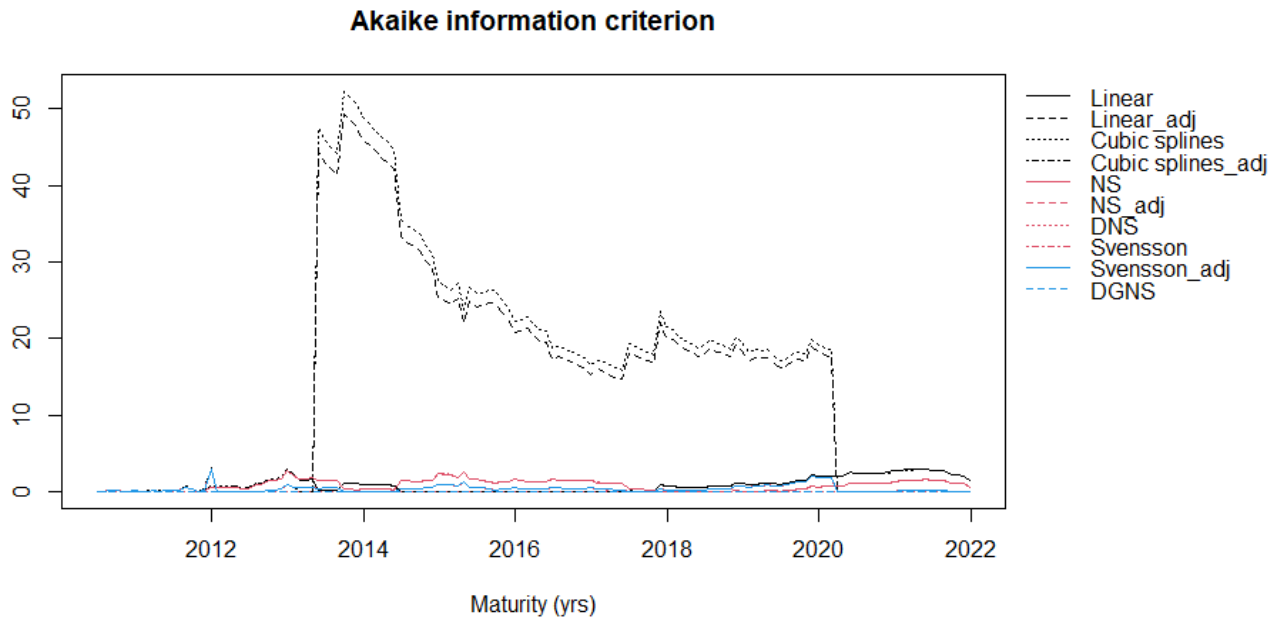
### 5.2.3. A perfect fit

To analyse the perfect fit of the selected term-structure models on the real spot rates in South Africa, a couple of statistical methods are used to ascertain the capability of the term-structure model in estimating the real spot rate curve. Firstly, root mean squared error (RMSE) is applied to analyse squared errors between the actual and estimated bond prices; secondly, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) to assess the model's performance by enforcing a stricter penalty by adding more regressors. The analysis of RMSE, AIC and BIC estimates of selected term-structure models are presented in Figure 5.2.5, Figure 5.2.6 and Figure 5.2.7, respectively.

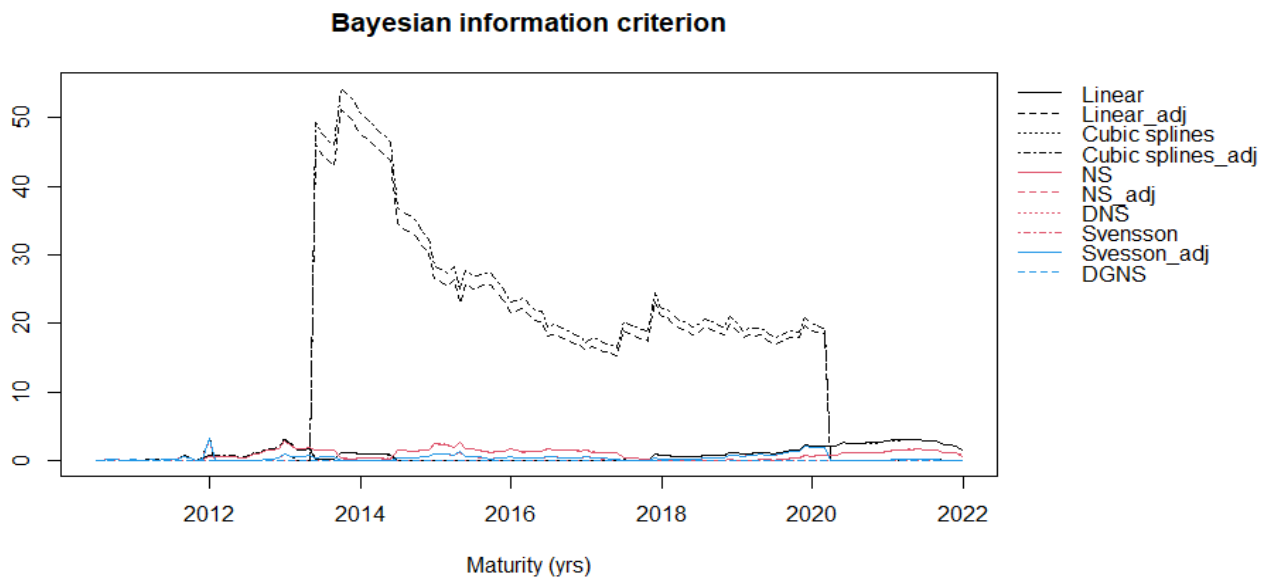


*Figure 5.2.5: RMSE for the estimated term-structure models*

*Source: Author's own work (2023)*



*Figure 5.2.6: AIC for the estimated term-structure models*  
Source: Author's own work (2023)

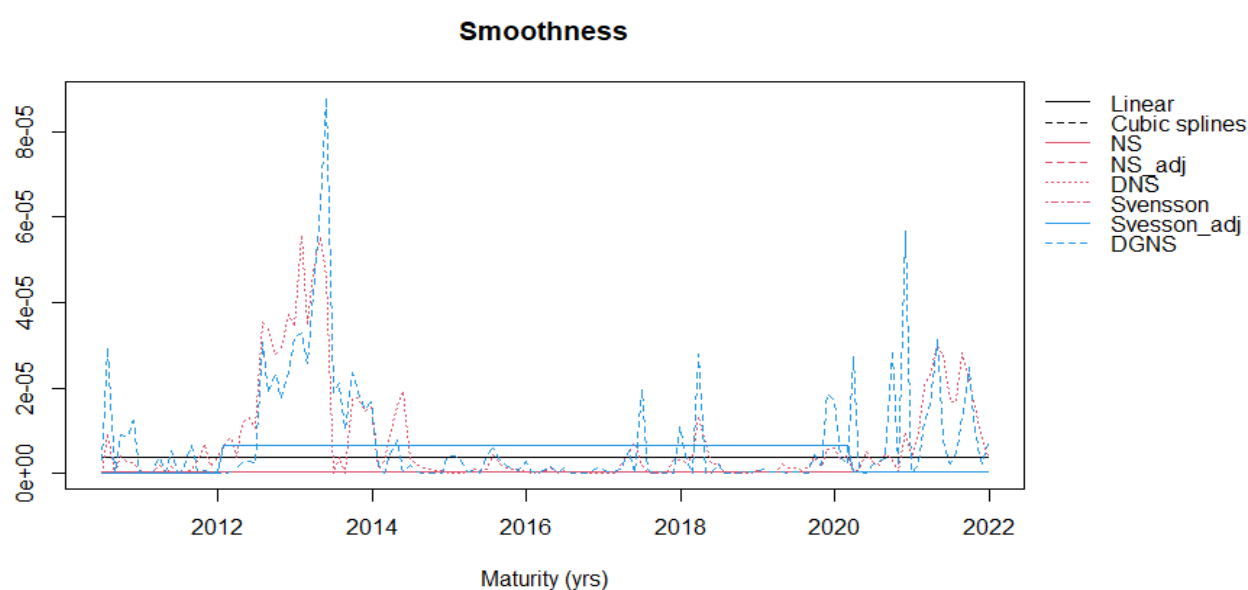


*Figure 5.2.7: BIC for the estimated term-structure models*  
Source: Author's own work (2023)

A lower estimate is considered for a good-performing model in analysing the goodness of fit using RMSE, AIC and BIC (Gujarati and Porter, 2009)). It is observed in Figures 5.2.5, 5.2.6 and 5.2.7 that both recalibrated linear parametric and cubic splines models gave a poor performance with significantly higher estimates of closer to 50 on AIC and BIC; and closer to 0.5 on the RMSE estimates. Other term-structure models resulted in RMSE estimates of below 0.2, and AIC and BIC estimates of below 5. This implies that the recalibration process on these term-structure models did not give an improved performance; as such, both recalibrated linear parametric and cubic splines models are not a good fit for estimating the real spot rate curve in South Africa during a period of high stress.

#### 5.2.4. Lower curvature or smoothness of the term-structure

According to Kikuchi and Shintani (2012), a more significant unevenness in the zero curve is undesirable since it might lead to erroneous results. This is driven by the estimation method used to derive the zero curve. The analysis of the smoothness/evenness of the real spot rate curve in South Africa based on selected mathematical term-structure models is presented in Figure 5.2.8.



*Figure 5.2.8: Smoothness measure*

*Source: Author's own work (2023)*

In analysing the smoothness/evenness of the yield curve, a lower estimate is preferred. It is observed in Figure 5.2.8 that all selected term-structure models have resulted in a smooth real spot rate curve in South Africa. The smoothness estimates across the estimation period were below 0.00008 for all term-structure models. This was observed even during periods of high Covid-19 stress period.

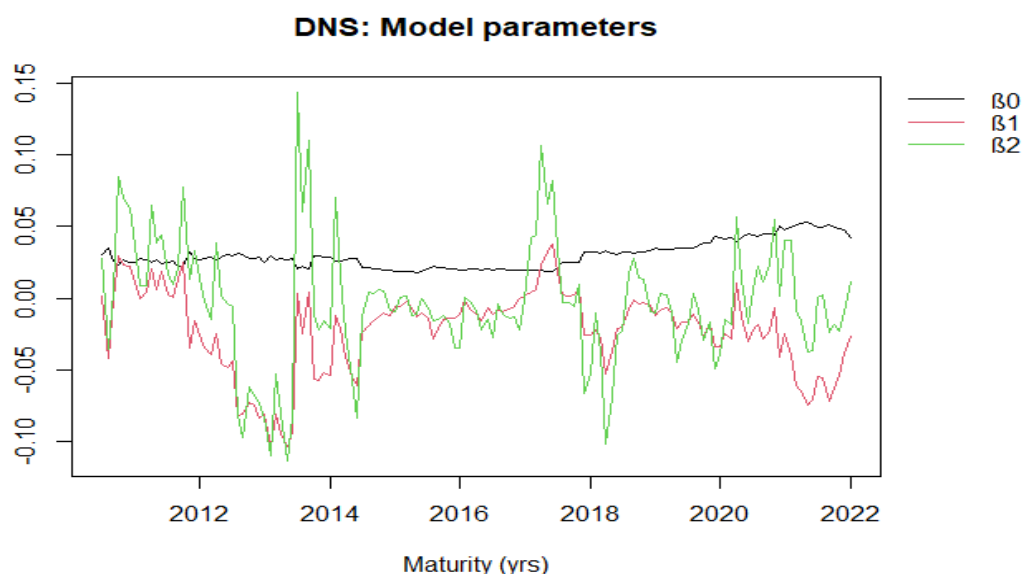
### 5.2.5. Movements in the model parameters: Dynamic Nelson-Siegel (2006) model

Given a different economic landscape during the Covid-19 stress period, it will be beneficial to analyse the performance of model parameters for the Dynamic Nelson-Siegel (2006) model over the estimation period. This is because a drastic cut in the repo rate is realised in most economies as part of central banks' stimulus to aid the ailing economy during periods of heightened economic/market volatility. These results are presented in Table 5.2.2 and Figure 5.2.9.

Statistical test	Level ( $\beta_{0,t}$ )	Slope ( $\beta_{1,t}$ )	Curvature ( $\beta_{2,t}$ )
Average	2,99%	-2,27%	-0,40%
Min	1,76%	-10,32%	-11,36%
Max	5,28%	3,82%	14,40%
Coefficient of variation	0,319	-1,291	-10,632
Skewness	0,844	-0,686	0,250
Kurtosis	2,744	3,210	4,275

*Table 5.2.2: Statistical analysis of Dynamic Nelson-Siegel model parameters*

*Source: Author's own work (2023)*



*Figure 5.2.9: Movements of Dynamic Nelson-Siegel model parameters*

*Source: Author's own work (2023)*

The statistical analysis of model parameters in Table 5.2.2 and results in Figure 5.2.9 indicates that when the decay rate is kept constant at ( $\lambda=0.5$ ), there is some stability in the level factor  $\beta_{0,t}$ , a proxy for a long-term average for the real spot rates. Over the estimation period which covers more than a decade of monthly real spot rates, the average long-term rate was 2.99 per cent with a lower coefficient of variation than other model parameters. When analysing the tail risk, it is observed that the kurtosis measure is less than three, which implies that, on average, the frequency of significant values in the tail was relatively lower than what is considered normal. However, some increases were observed during the Covid-19 period while significant cuts in target rates (i.e. a cumulative 275 basis points cut in the repurchase rate) to cushion the economy against the effect of Covid-19 shocks. This contradicts the results observed in Severino (2021), where it was indicated that a cut in the target rate by the Bank of Canada triggered a decline in the level factor. As such, indicating that even though cuts were made, long-term risks remain heightened in real terms.

The slope factor  $\beta_{1,t}$ , a proxy for the short-term rates, indicates that, on average, the real spot rate curve has been pricing below the zero per cent mark. A slight increase associated with the curve inversion observed at the beginning of 2020. This could be



associated with the volatilities that followed the first total economic shutdown in South Africa. However, this was short-lived, given interventions made by the central bank to cut target rates.

The curvature factor  $\beta_{2,t}$ , a proxy for the medium-term rates, remained highly volatile over the estimation period, with the highest coefficient of variation and kurtosis estimate of 4.275. A heightened volatility was also observed in 2020 during the peak of Covid-19 stress.

### 5.3. Forecasting

Forecasting exercise on the real spot rate for South Africa is one of the objectives of this study. National Treasury (2012) indicated that forecasts on economic factors are updated/revised every quarter to account for the most recent information. However, a monthly forecasting exercise was done for this study for six months. This aligns with the National Treasury budgeting process, which is reviewed twice a year.

The forecasting process is used relative to the forward rate estimation method. The forecasting exercise calculates the future spot rates; as such, the estimation for each model was based on the assumptions indicated in Section 4.4.

#### 5.3.1. Normality analysis

Before the analysis of the forecasting process, a normality test is made on the selected macroeconomic factors and model parameters data to ascertain the type of a regression analysis to be applied. Table 5.3.1 show normality test results based on Shapiro-Wilk and Anderson-Darling Normality tests.

	Shapiro-Wilk Normality Test			Anderson-Darling Normality Test		
	Test statistic	P-value	Significant	Test statistic	P-value	Significant
Switches	0,63	0,00	No	4,37	0,00	No
IGOV	0,94	0,16	Yes	0,61	0,10	Yes
CPI	0,91	0,06	Yes	0,94	0,01	No
Bid ratio	0,94	0,18	Yes	0,47	0,22	Yes
$DNS_{\beta_{1,t}}$	0,94	0,18	Yes	0,57	0,12	Yes
$DNS_{\beta_{2,t}}$	0,96	0,47	Yes	0,34	0,47	Yes
$DNS_{\beta_{3,t}}$	0,96	0,46	Yes	0,29	0,59	Yes
$DNS_{\beta_{1,t-1}}$	0,94	0,17	Yes	0,58	0,12	Yes
$DNS_{\beta_{2,t-1}}$	0,96	0,49	Yes	0,33	0,50	Yes
$DNS_{\beta_{3,t-1}}$	0,95	0,26	Yes	0,40	0,33	Yes
$DGNS_{\beta_{1,t}}$	0,91	0,05	No	0,57	0,12	Yes
$DGNS_{\beta_{2,t}}$	0,91	0,05	No	0,80	0,03	No
$DGNS_{\beta_{3,t}}$	0,95	0,28	Yes	0,38	0,37	Yes
$DGNS_{\beta_{4,t}}$	0,93	0,14	Yes	0,61	0,10	Yes
$DGNS_{\beta_{5,t}}$	0,93	0,13	Yes	0,52	0,16	Yes
$DGNS_{\beta_{1,t}}$	0,90	0,03	No	0,68	0,07	Yes
$DGNS_{\beta_{2,t}}$	0,90	0,03	No	0,87	0,02	No
$DGNS_{\beta_{3,t}}$	0,94	0,20	Yes	0,43	0,28	Yes
$DGNS_{\beta_{4,t}}$	0,93	0,10	Yes	0,66	0,07	Yes
$DGNS_{\beta_{5,t}}$	0,92	0,06	Yes	0,65	0,08	Yes

*Table 5.3.1: Normality results on the selected variables*

*Source: Author's own work (2023)*

Based on both Shapiro-Wilk and Anderson-Darling Normality tests presented in Table 5.3.1, it can be significantly proven that the dataset comes from a normal distribution only if the p-value is more than the selected level of significance ( $\alpha = 0,05$ ). This implies that 95 per cent of the time, there is a level of confidence that the data has passed the normality test. However, it could not be significantly proven that the data for the selected variables is generated from a normal distribution given that the *p – value* is less than the level of significance in some cases.

### 5.3.2. Correlation analysis

Furthermore, correlation analysis is done to establish if there is any linear/non-linear relationship between the selected independent and dependent variables. Correlation is a statistical measure that defines the size and direction of a relationship between two or more variables; however, this statistical measure does not automatically imply

that a movement in one variable causes movements in the values of the other variable (Gujarati and Porter, 2009). In analysing the correlation between the selected values for the forecasting process, it is then proven that the Pearson method will not be applicable since not all data for the selected variables comes from the normal distribution. Either Spearman or Kendall can be applied, given that it is not necessarily crucial for these methodologies to be applied on a normal distribution. The correlation analysis using a Kendal methodology on the selected variables is presented in Table 5.3.2 with  $p$  – values for the significance test in the parentheses.

Variable	$DNS_{\beta_{1,t}}$	$DNS_{\beta_{2,t}}$	$DNS_{\beta_{3,t}}$	$DGNS_{\beta_{1,t}}$	$DGNS_{\beta_{2,t}}$	$DGNS_{\beta_{3,t}}$	$DGNS_{\beta_{4,t}}$	$DGNS_{\beta_{5,t}}$
Switches	0,38 (0,03)	<b>-0,36</b> <b>(0,05)</b>	<b>-0,24</b> <b>(0,17)</b>	<b>-0,31</b> <b>(0,08)</b>	<b>0,10</b> <b>(0,56)</b>	0,38 (0,03)	<b>-0,02</b> <b>(0,91)</b>	<b>0,27</b> <b>(0,13)</b>
IGOV	0,40 (0,01)	-0,52 (0,00)	-0,41 (0,01)	<b>-0,18</b> <b>(0,26)</b>	<b>0,06</b> <b>(0,70)</b>	<b>0,20</b> <b>(0,20)</b>	<b>-0,06</b> <b>(0,70)</b>	<b>0,19</b> <b>(0,24)</b>
CPI	<b>0,10</b> <b>(0,51)</b>	<b>-0,23</b> <b>(0,15)</b>	<b>-0,18</b> <b>(0,25)</b>	<b>-0,10</b> <b>(0,51)</b>	<b>-0,08</b> <b>(0,59)</b>	<b>0,15</b> <b>(0,35)</b>	<b>-0,20</b> <b>(0,20)</b>	<b>0,04</b> <b>(0,80)</b>
Bid ratio	-0,39 (0,01)	<b>0,28</b> <b>(0,07)</b>	<b>0,09</b> <b>(0,58)</b>	<b>0,06</b> <b>(0,70)</b>	<b>-0,00</b> <b>(1,00)</b>	<b>-0,09</b> <b>(0,58)</b>	<b>0,02</b> <b>(0,91)</b>	<b>-0,06</b> <b>(0,74)</b>
$DNS_{\beta_{1,t-1}}$	0,55 (0,00)	-0,60 (0,00)	-0,43 (0,00)					
$DNS_{\beta_{2,t-1}}$	-0,42 (0,00)	0,51 (0,00)	0,45 (0,00)					
$DNS_{\beta_{3,t-1}}$	<b>-0,11</b> <b>(0,50)</b>	0,20 (0,20)	<b>0,29</b> <b>(0,06)</b>					
$DGNS_{\beta_{1,t-1}}$				-0,32 (0,04)	<b>0,08</b> <b>(0,62)</b>	0,38 (0,01)	<b>-0,01</b> <b>(0,96)</b>	<b>0,24</b> <b>(0,13)</b>
$DGNS_{\beta_{2,t-1}}$				<b>-0,12</b> <b>(0,47)</b>	<b>0,04</b> <b>(0,82)</b>	<b>0,16</b> <b>(0,31)</b>	<b>-0,02</b> <b>(0,91)</b>	<b>0,07</b> <b>(0,66)</b>
$DGNS_{\beta_{3,t-1}}$				<b>-0,30</b> <b>(0,05)</b>	<b>0,10</b> <b>(0,54)</b>	0,36 (0,02)	<b>0,00</b> <b>(1,00)</b>	<b>0,22</b> <b>(0,16)</b>
$DGNS_{\beta_{4,t-1}}$				<b>0,00</b> <b>(0,00)</b>	<b>0,00</b> <b>(0,00)</b>	<b>0,00</b> <b>(0,00)</b>	<b>0,00</b> <b>(0,00)</b>	<b>0,00</b> <b>(0,00)</b>
$DGNS_{\beta_{5,t-1}}$				<b>0,00</b> <b>(0,00)</b>	<b>0,00</b> <b>(0,00)</b>	<b>0,00</b> <b>(0,00)</b>	<b>0,00</b> <b>(0,00)</b>	<b>0,00</b> <b>(0,00)</b>

Table 5.3.2: Correlation analysis between selected variables

Source: Author's own work (2023)

The correlation analysis in Table 5.3.2 indicates some level of relationship between the movements of the selected dependent and independent variables. This is a general relationship/association, given that normality does not have to hold for this methodology. However, to significantly consider this relationship, hypothesis testing has to be analysed with the null that the relationship is statistically significant. In

parenthesis, it could be noted that there is no relationship between the movements of most independent variables and the selected dependent variables. This is indicated by a  $p - value$  greater than the significance level ( $\alpha = 0,05$ ). This implies that not all selected independent variables will be applied to the macro factor DNS model to forecast movements in the model parameters.

### 5.3.3. Linear regression analysis

Based on the correlation analysis in Table 5.3.2, it could be proven that not only a linear relationship does not exist in most of the selected pairs. Further, a general relationship based on the Kendall correlation analysis could not prove statistically significant. However, a linear regression was applied between the model parameters and the selected independent variables, which were deemed statistically significant. A coefficient of determination and parameter analysis for the selected linear regressions are tabled in Table 5.3.3 and Table 5.3.4.

R-squared	DNS-AR (1)	DNS-Macrofactor	DGNS-AR (1)	DGNS-Macrofactor
$\beta_1$	0,902	0,675	<b>0,197</b>	
$\beta_2$	0,629	0,606	<b>0,078</b>	
$\beta_3$	<b>0,219</b>	<b>0,153</b>	<b>0,236</b>	<b>0,300</b>
$\beta_4$			<b>0,073</b>	
$\beta_5$			<b>0,098</b>	

*Table 5.3.3: Coefficient of determination for selected linear regression for model parameters*

*Source: Author's own work (2023)*

Estimate (P-value)	DNS-AR (1)	DNS- Macrofactor	DGNS-AR (1)	DGNS- Macrofactor
$\beta_1$				
Intercept	0,004	0,017	0,028	
$\beta_{1,t-1}$	0,918 (<0,000)	0,794 (0,012)	0,446 (0,006)	
Switches		<b>0,000 (0,878)</b>		
IGOV index		<b>-0,000 (0,691)</b>		
Bid ratio		<b>-0,001 (0,509)</b>		
$\beta_2$				
Intercept	-0,007	0,106	<b>-0,077</b>	
$\beta_{2,t-1}$	0,777 (<0,000)	<b>0,428 (0,069)</b>	<b>0,281 (0,095)</b>	
IGOV index		<b>-0,000 (0,086)</b>		
$\beta_3$				
Intercept	-0,002	0,002	-0,007	-0,001
$\beta_{3,t-1}$	0,470 (0,004)	<b>0,292 (0,193)</b>	0,488 (0,002)	<b>0,398 (0,064)</b>
Switches		<b>-0,011 (0,403)</b>		<b>0,0511 (0,184)</b>
$\beta_4$				
Intercept			-0,036	
$\beta_{4,t-1}$			<b>0,273 (0,105)</b>	
$\beta_5$				
Intercept			-0,031	
$\beta_{5,t-1}$			<b>0,316 (0,058)</b>	

*Table 5.3.4: Significance analysis on selected independent variable*

*Source: Author's own work (2023)*

Linear regression is applied to independent variables with a highly significant correlation/relationship with dependent variables. According to Gujarati and Porter (2009) and Chicco et al. (2021), R-squared is a statistical measure that quantifies the dependence of a dependent variable determined by the independent variables in proportion to its variance. It could be observed in Table 5.3.4 that Dynamic Nelson-Siegel (2006) model resulted in a bigger coefficient of determination (R-squared) on the first parameters for both AR (1) process and the macro-economic factor process. However, the Dynamic Generalized Nelson-Siegel (DGNS) model resulted in R-squared below 30 per cent for all parameters on both processes. For the DGNS-Macrofactor process, the only fitting was on the  $\beta_3$  parameter since all other parameters did not pass correlation analysis on all selected independent variables. This implies that the selected set of models are not significantly perfect in explaining/determining the movements/changes in the DGNS model parameters.

It was noted in Table 5.3.4 that a 95 per cent level of significance ( $\alpha = 0.05$ ), one-month lagged parameters are significantly suitable for explaining the movements of the DNS models parameters following the AR (1) process. However, the same could

not be significantly proven for all other selected term-structure models. For the DGNS model following the AR (1) process, one-month lagged parameters are significantly suitable for explaining the movements of the DNS models parameters 90 per cent of the time ( $\alpha=0.1$ ). This analysis implies that applying macroeconomic factors in estimating future parameters for the DNS and DGNS models is deemed futile in this study. The selected macroeconomic factors used in this study align with those used in Kobayashi (2017) study and might have been significantly suitable for the nominal yield curve. T this further proves the existence of different dynamics in the inflation-indexed yield curve relative to the nominal yield curve.

#### 5.3.4. Forecasting output

To analyse the perfect forecast fit of selected mathematical term-structure models on the real spot rates in South Africa, a couple of statistical methods were also used to ascertain the performance of the term-structure model in estimating the real spot rate curve. The AIC, BIC and RMSE methodologies were used for out-of-sample model performance analysis. The analysis of AIC, BIC and RMSE estimates of selected term-structure models are presented in Tables 5.3.5, 5.3.6 and 5.3.7, respectively.



AIC	Linear	Cubic splines	NS	NS_adj	DNS-AR (1)	DNS-Macrofactor	Svensson	Svensson_adj	DGNS-AR (1)	DGNS-Macrofactor
Jan-2022	<b>1,574</b>	<b>1,574</b>	0,813	0,004	0,002	<b>1,459</b>	0,817	0,004	0,002	0,369
Feb-2022	<b>1,283</b>	<b>1,283</b>	0,619	0,003	0,007	<b>1,778</b>	0,623	0,003	0,010	0,131
Mar-2022	<b>1,641</b>	<b>1,641</b>	0,897	0,011	0,004	<b>1,400</b>	0,903	0,011	0,002	0,262
Apr-2022	<b>1,306</b>	<b>1,306</b>	0,666	0,001	0,006	<b>1,753</b>	0,670	0,001	0,007	0,547
May-2022	<b>1,169</b>	<b>1,169</b>	0,580	0,006	0,020	<b>1,924</b>	0,584	0,006	0,018	0,680
Jun-2022	<b>1,405</b>	<b>1,405</b>	0,771	0,001	0,001	<b>1,637</b>	0,775	0,001	0,000	0,518

*Table 5.3.5: Out-of-sample AIC estimates*

*Source: Author's own work (2023)*

BIC	Linear	Cubic splines	NS	NS_adj	DNS-AR (1)	DNS-Macrofactor	Svensson	Svensson_adj	DGNS-AR (1)	DGNS-Macrofactor
Jan-2022	<b>1,644</b>	<b>1,644</b>	0,840	0,004	0,002	<b>1,508</b>	0,854	0,004	0,003	0,390
Feb-2022	<b>1,339</b>	<b>1,339</b>	0,640	0,003	0,007	<b>1,838</b>	0,651	0,003	0,011	0,138
Mar-2022	<b>1,713</b>	<b>1,713</b>	0,928	0,012	0,005	<b>1,447</b>	0,944	0,012	0,003	0,277
Apr-2022	<b>1,364</b>	<b>1,364</b>	0,689	0,001	0,006	<b>1,813</b>	0,700	0,001	0,007	0,578
May-2022	<b>1,220</b>	<b>1,220</b>	0,600	0,006	0,021	<b>1,990</b>	0,610	0,006	0,019	0,719
Jun-2022	<b>1,467</b>	<b>1,467</b>	0,797	0,001	0,001	<b>1,693</b>	0,811	0,002	0,000	0,548

*Table 5.3.6: Out-of-sample BIC estimates*

*Source: Author's own work (2023)*

RMSE	Linear	Cubic splines	NS	NS_adj	DNS-AR (1)	DNS-Macrofactor	Svensson	Svensson_adj	DGNS-AR (1)	DGNS-Macrofactor
Jan-2022	<b>0,112</b>	<b>0,112</b>	0,094	0,012	0,011	<b>0,212</b>	0,094	0,012	0,009	<b>0,156</b>
Feb-2022	<b>0,100</b>	<b>0,100</b>	0,082	0,008	0,010	<b>0,225</b>	0,082	0,008	0,015	<b>0,283</b>
Mar-2022	<b>0,114</b>	<b>0,114</b>	0,097	0,018	0,017	<b>0,214</b>	0,097	0,018	0,015	<b>0,311</b>
Apr-2022	<b>0,100</b>	<b>0,100</b>	0,083	0,013	0,015	<b>0,225</b>	0,083	0,013	0,016	<b>0,328</b>
May-2022	<b>0,095</b>	<b>0,095</b>	0,079	0,019	0,022	<b>0,231</b>	0,079	0,019	0,022	<b>0,335</b>
Jun-2022	<b>0,110</b>	<b>0,110</b>	0,094	0,018	0,016	<b>0,214</b>	0,094	0,018	0,016	<b>0,320</b>

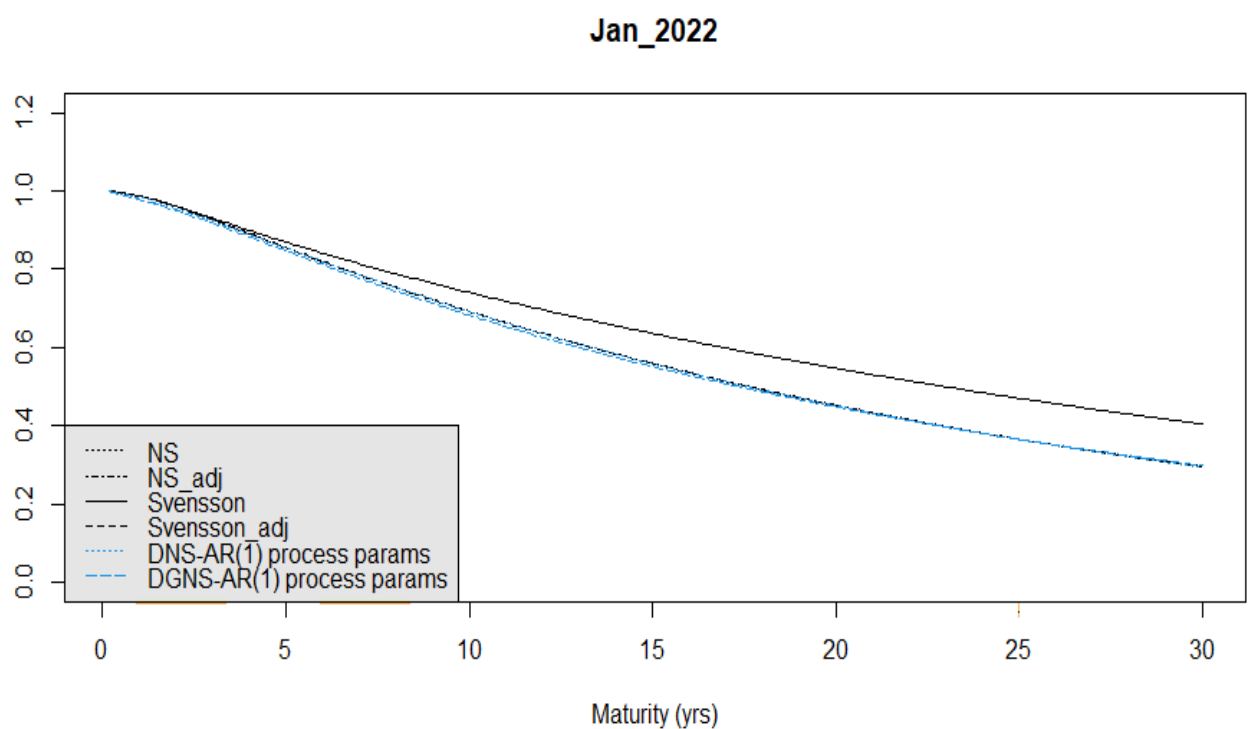
*Table 5.3.7: Out-of-sample RMSE estimates*

*Source: Author's own work (2023)*



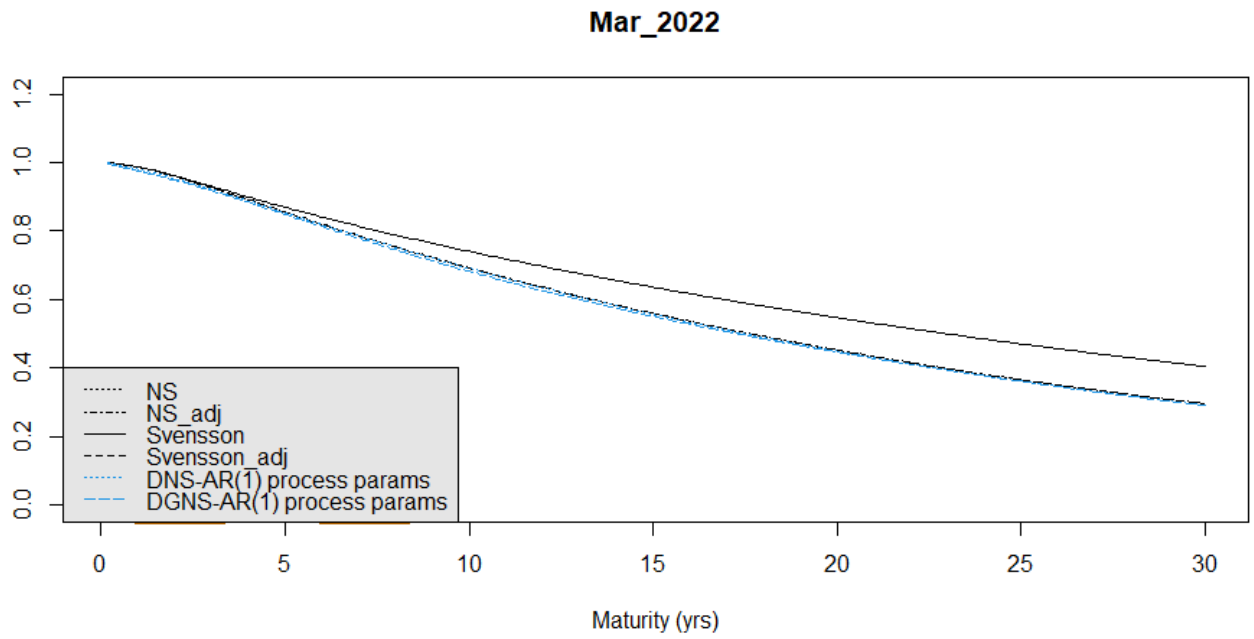
Based on Tables 5.3.5, 5.3.6 and 5.3.7, it is noted that dynamic term-structure models using AR (1) process to forecast the yield curve gave the lowest AIC, BIC and RMSE estimates for the entire forecasting period (i.e. between January 2022 to June 2022). The recalibrated Nelson-Siegel and Svensson term-structure models also gave better estimates due to the most recent information used for the forecasting exercise (i.e. recalibrated parameters as of the 31st of March 2020 were used).

The analysis was done on the zero-spot rate/discount rate curve over six months to further assess the performance of selected term-structure models based on the bond prices. Results are presented in Figures 5.3.1, 5.3.2 and 5.3.3.



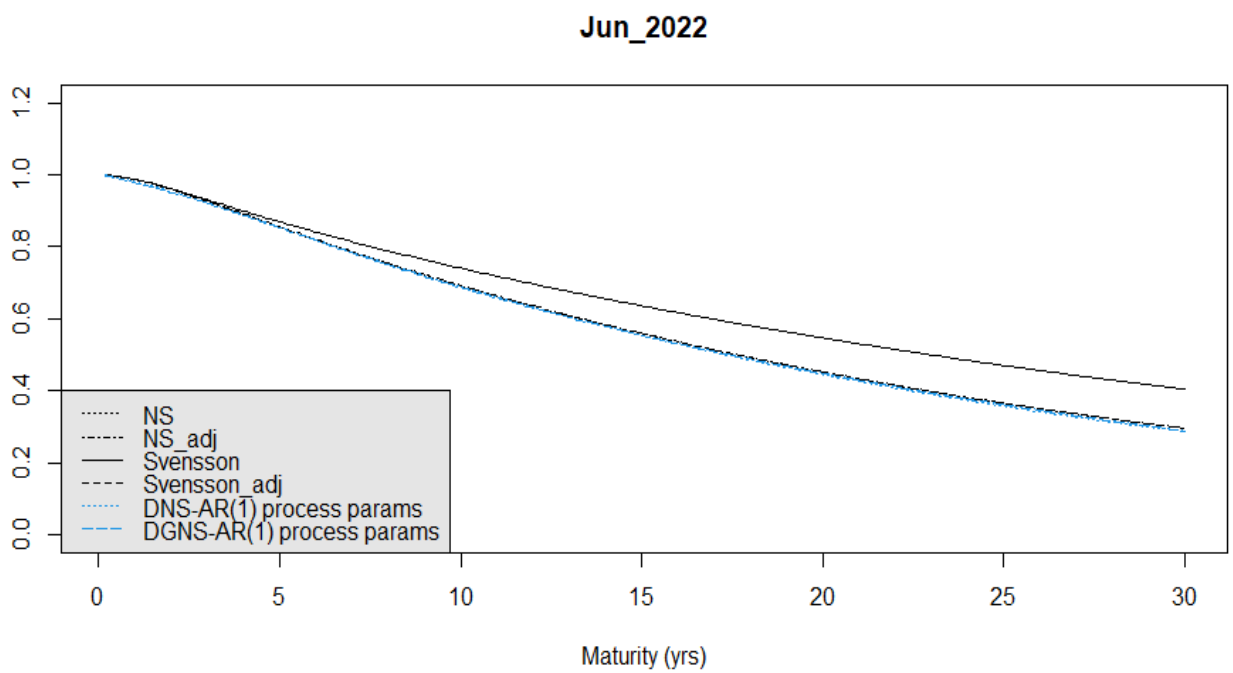
*Figure 5.3.1: One month forecasted discount rate curve*

*Source: Author's own work (2023)*



*Figure 5.3.2: Three months forecasted discount rate curve*

*Source: Author's own work (2023)*



*Figure 5.3.3: Six months forecasted discount rate curve*

*Source: Author's own work (2023)*

Based on the forecasted discount rate curves presented in Figures 5.3.1, 5.3.2 and 5.3.3, it is observed that static models (i.e. Nelson-Siegel and Svensson models) resulted in higher prices on longer maturities when compared to recalibrated static models and dynamic/time-dependent models. There are some similarities across the selected models at the short end of the curve (term-to-maturity <5 years). This aligns with bond pricing basics, indicating that bond prices converge to par pricing closer to bond maturity.

## 5.4. Backtesting

The other objective of this study was to do an out-of-sample backtesting analysis on the chosen term-structure models to assess the model's capability in estimating/forecasting future South African real spot rates.

### 5.4.1. Traditional backtesting analysis

Traditional backtesting analysis is done where a comparison of the real spot rates in South Africa is made between the estimated/forecasted spot rates and actual spot rates from JSE. Table 5.4.1 shows the backtesting results for one-month forecasts using selected term-structure models relative to the JSE actual real spot rates.

Maturity	NS	NS_adj	DNS-AR (1)	Svensson	Svensson_adj	DGNS-AR (1)	JSE spot rates
3-month	<b>0,626</b>	<b>0,674</b>	<b>0,626</b>	<b>0,674</b>	1,325	2,106	1,762
6-month	<b>0,914</b>	<b>0,946</b>	<b>0,914</b>	<b>0,946</b>	1,516	2,186	1,762
9-month	<b>1,168</b>	<b>1,192</b>	<b>1,168</b>	<b>1,192</b>	1,690	<b>2,267</b>	1,762
1-year	<b>1,392</b>	<b>1,415</b>	<b>1,392</b>	<b>1,415</b>	1,850	<b>2,347</b>	1,762
2-year	2,051	2,115	2,051	2,115	<b>2,366</b>	<b>2,657</b>	1,807
3-year	2,448	2,595	2,448	2,595	2,737	2,932	2,671
4-year	2,687	2,929	2,687	2,929	3,009	3,164	2,785
5-year	2,829	3,167	2,829	3,167	3,211	<b>3,353</b>	2,771
6-year	2,914	3,341	2,914	3,341	3,364	<b>3,504</b>	2,995
7-year	<b>2,965</b>	3,47	<b>2,965</b>	3,47	3,483	3,624	3,517
8-year	<b>2,994</b>	3,569	<b>2,994</b>	3,569	3,576	3,718	3,805
9-year	<b>3,011</b>	3,646	<b>3,011</b>	3,646	3,651	3,790	3,891
10-year	<b>3,02</b>	3,708	<b>3,020</b>	3,708	3,712	3,847	3,913
25-year	<b>3,023</b>	4,031	<b>3,023</b>	4,031	4,048	4,032	4,010
30-year	<b>3,021</b>	4,066	<b>3,021</b>	4,066	4,085	4,039	4,064

*Table 5.4.1: One month spot rate forecasts versus JSE actual spot rates – January 2022*

*Source: Author's own work (2023)*

The analysis of a one-month forecast on the real spot rates in South Africa relative to the actual JSE real spot rates is presented in Table 5.4.1, indicates some uncertainty in the short end of the curve. This is due to the forecasted spot rates pricing 50 bps below the actuals for term-to-maturity of at most one year. This is in line with findings made by Reid (2009), Lartey and Li (2018), and Carvalho and Garcia (2019), which indicated that term-to-maturities around this area of the curve have the same dynamics as the money market instruments; as such, term-structure models might not be able to perfectly define movements of these spot rates. To further analyse the ability of the term-structure models to estimate the future real spot rates in South Africa over a more extended period (i.e. over three months and six months), Figures 5.4.1, 5.4.2 and 5.4.3 represents the differences/deviations from the JSE actual real spot rates, and Figures 5.4.3, 5.4.4 and 5.4.5 represents the forecasted and actual spot rates.

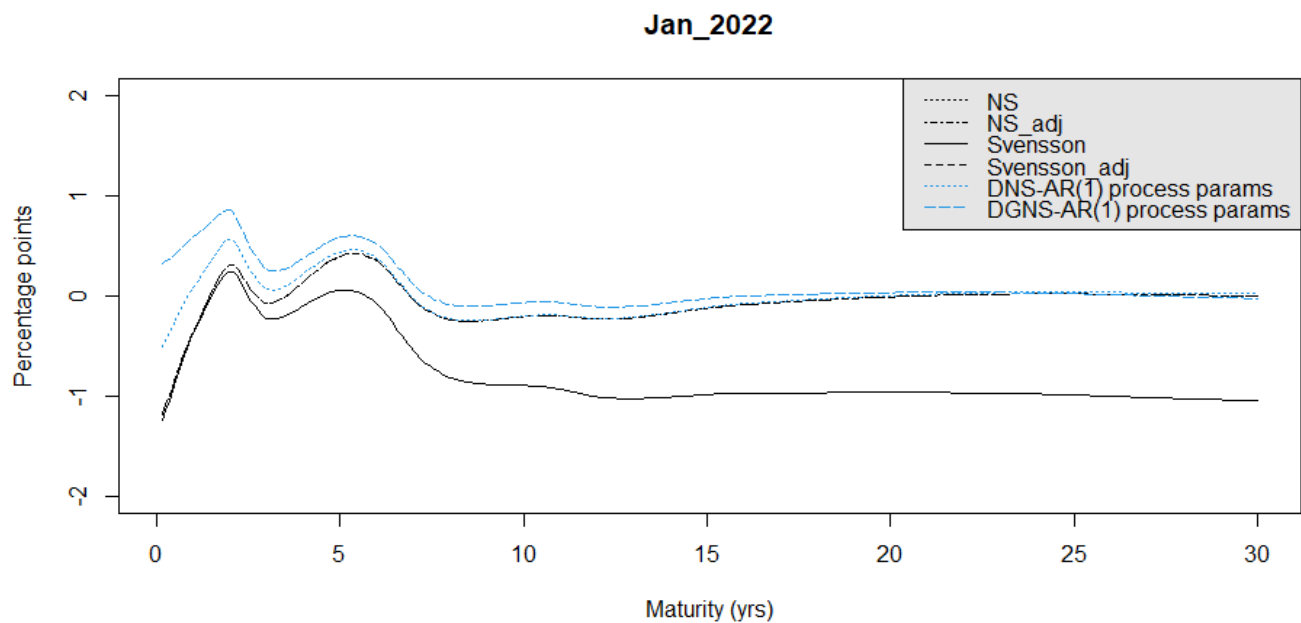


Figure 5.4.1: Differences between model forecasts and JSE actual spot rates for January 2022

Source: Author's own work (2023)

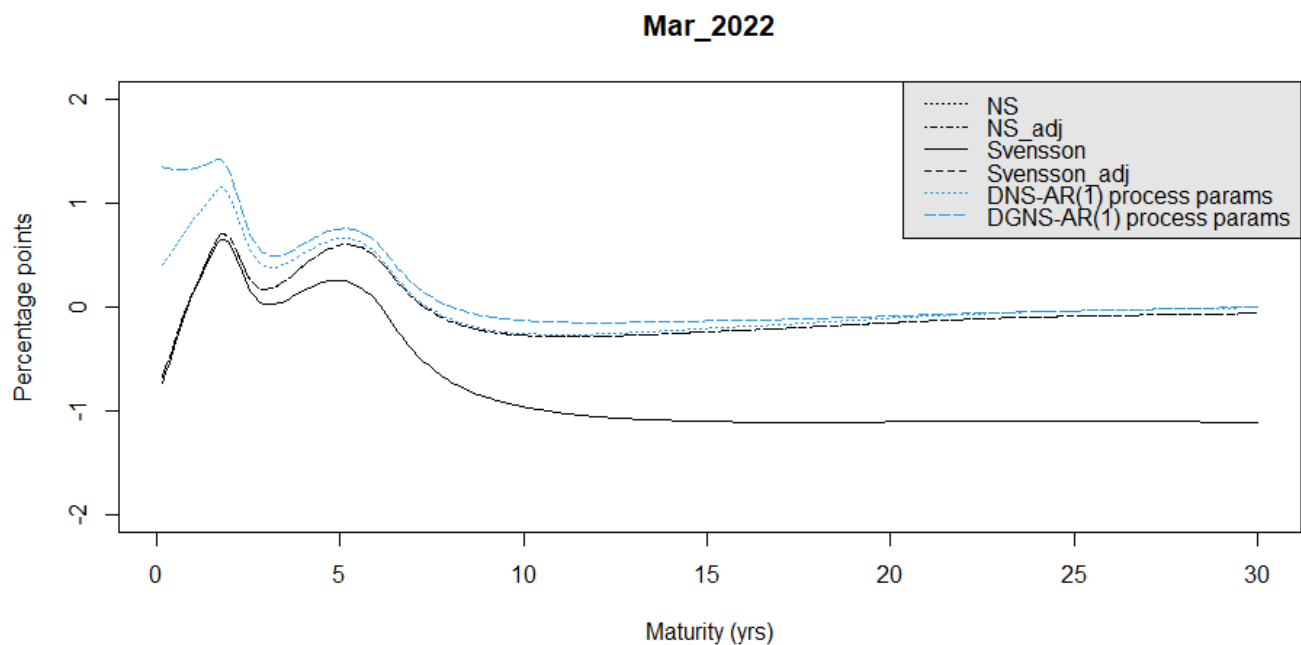
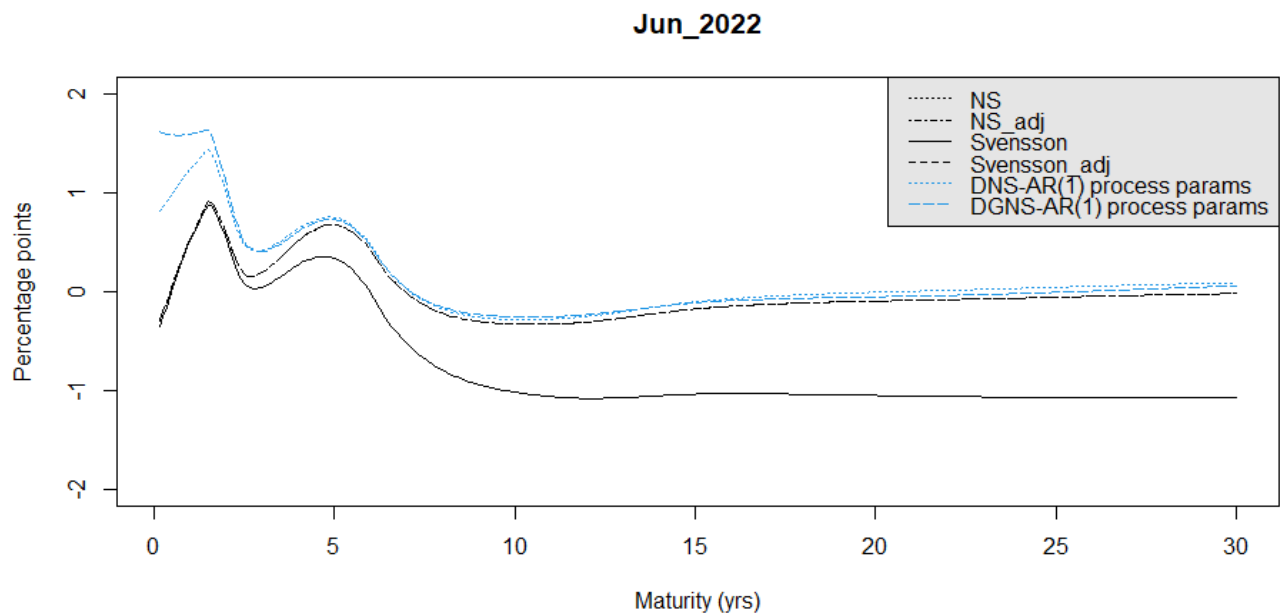


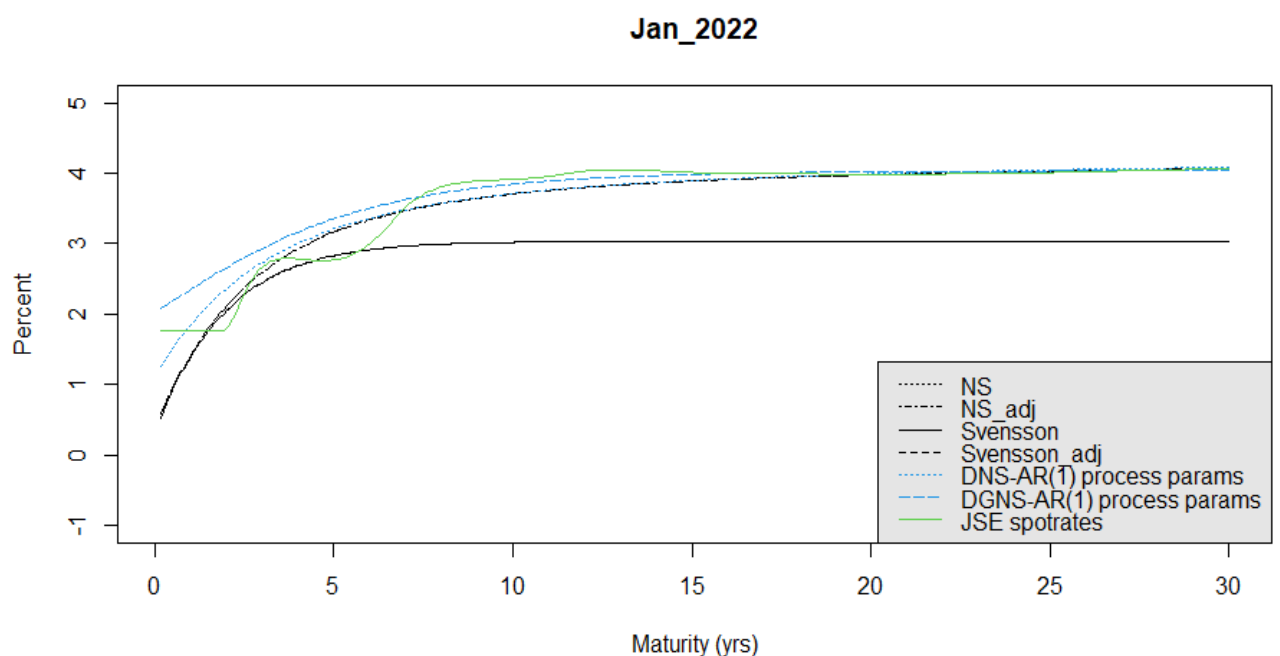
Figure 5.4.2: Differences between model forecasts and JSE actual spot rates for March 2022

Source: Author's own work (2023)



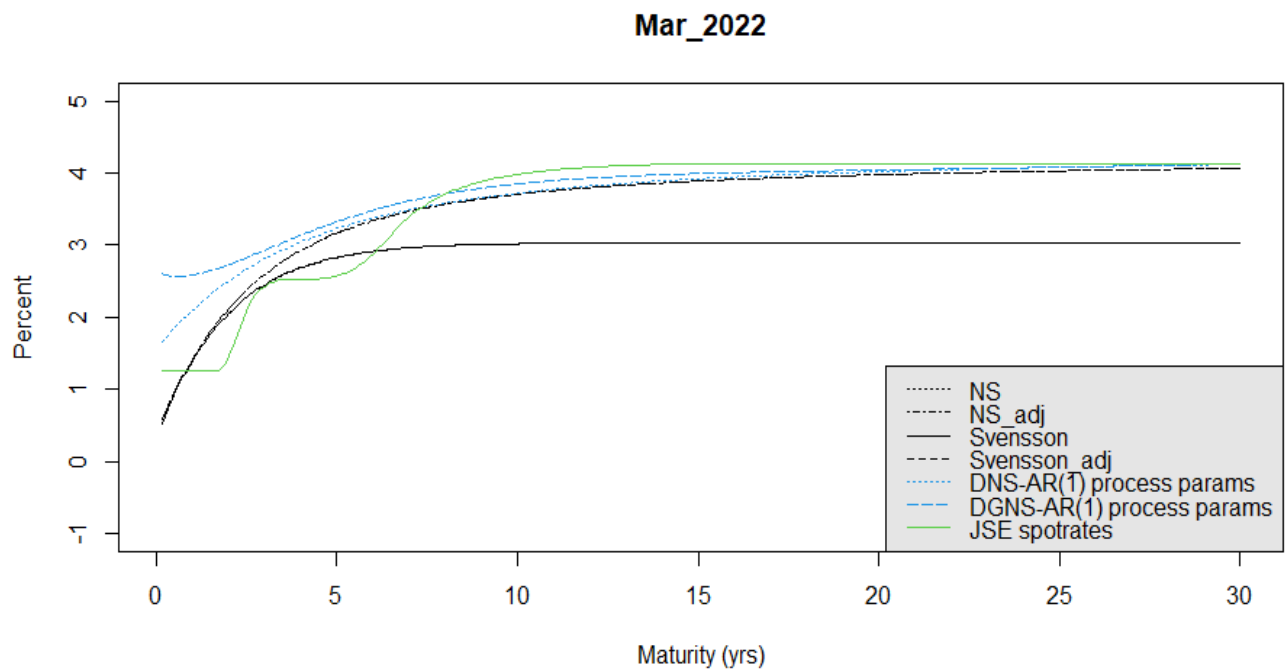
*Figure 5.4.3: Differences between model forecasts and JSE actual spot rates for June 2022*

*Source: Author's own work (2023)*



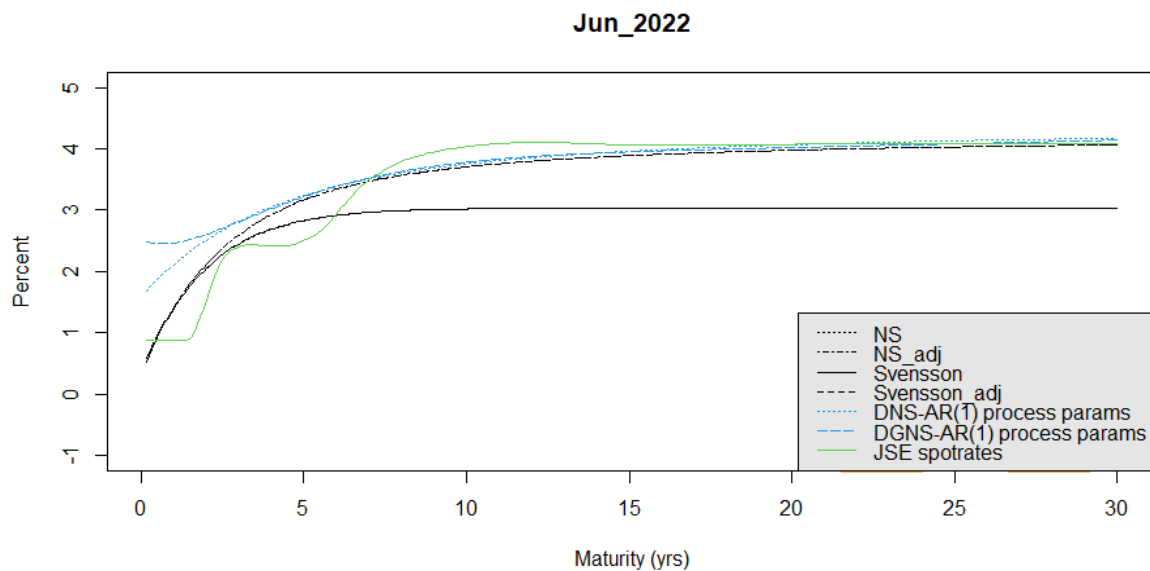
*Figure 5.4.4: Model forecasted spot rates versus JSE actual spot rates for January 2022*

*Source: Author's own work (2023)*



*Figure 5.4.5: Model forecasted spot rates versus JSE actual spot rates for March 2022*

*Source: Author's own work (2023)*



*Figure 5.4.6: Model forecasted spot rates versus JSE actual spot rates for June 2022*

*Source: Author's own work (2023)*

The results of backtesting analysis on Figures 5.4.1, 5.4.2 and 5.4.3 indicated that dynamic term-structure models and recalibrated term-structure models gave a better forecast on the mid-to-ultra-long end of the yield curve for the selected forecast period. This is where most government funding is based (National Treasury, 2020a). There is so much volatility in the short-to-mid end of the curve (i.e. term-to-maturity  $\leq 5$  years). It was noted by Reid (2009) and Lartey and Li (2018) that the short end of the curve mimics the money-market instrument, implying that uncertainty dynamics determined by the monetary policy are consistent with movements in the short end of the yield curve.

Carvalho and Garcia (2019) also indicated that due to the volatility of short-term yields and the unstable monetary policy, forecasted spot rates might not be so accurate in the short term. This is observed in the JSE actual real spot rate in Figures 5.4.4, 5.4.5 and 5.4.6 when compared to forecasted spot rates based on the selected term-structure models, which showed significant smoothness around this area of the yield curve.

## 5.4.2. Test for equal means

The statical test of equal means is analysed to further complement the traditional backtesting analysis. Given uncertainties between actual and estimated values, Marcus and Elias (1998) indicated that to validate the performance of the model, it is expected for the predicted values to be different from actual observations; however, the mean observation and the mean forecast from a sound model should not have significant differences from one another. Hypothesis tests can be used to validate the model (i.e. show that the application of the model is not wrong). However, they can never be used for model verification. A detailed analysis of the equal means test is presented in Table 5.4.2.



Parameters	NS	NS_adj	DNS-AR (1)	Svensson	Svensson_a dj	DGNS- AR (1)
<b>1-month forecasts</b>						
Test-statistic	-18,36	-0,95	-101,81	-18,36	-0,95	-82,62
P-value	<b>0,00</b>	0,34	<b>0,00</b>	<b>0,00</b>	0,34	<b>0,00</b>
Forward rate mean	2,86	3,59	-0,02	2,86	3,59	0,38
JSE spot rate mean	3,64	3,64	3,64	3,64	3,64	3,64
<b>3-months forecasts</b>						
Test-statistic	-15,56	-0,89	-80,68	-15,56	-0,89	-32,21
P-value	<b>0,00</b>	0,37	<b>0,00</b>	<b>0,00</b>	0,37	<b>0,00</b>
Forward rate mean	2,86	3,59	0,02	2,86	3,59	1,32
JSE spot rate mean	3,65	3,65	3,65	3,65	3,65	3,65
<b>6-months forecasts</b>						
Test-statistic	-14,19	-0,25	-75,31	-14,19	-0,25	-28,36
P-value	<b>0,00</b>	0,81	<b>0,00</b>	<b>0,00</b>	0,81	<b>0,00</b>
Forward rate mean	2,86	3,59	0,03	2,86	3,59	1,42
JSE spot rate mean	3,61	3,61	3,61	3,61	3,61	3,61

*Table 5.4.2: Analysis of equal means on model forecasted estimates*

*Source: Author's own work (2023)*

The analysis of equal means in Table 5.4.2 indicates that the recalibrated Nelson and Siegel (1987) and Svensson (1994) term-structure models could perfectly forecast the real spot rate curve in South Africa over six months. *P – values* of  $>0.05$  support this. This implies 95 per cent confidence that the real spot rates in South Africa forecasted by these two term-structure models could fit the actual spot rates realised after one, three and six months. In Table 5.4.1, most of the selected term-structure models estimated/forecasted spot rates that were either 50bps above or below JSE actual spot rates, notably observed in Nelson and Siegel (1987) and Dynamic Nelson-Siegel (2006) models. However, to validate the application of the selected models, the test for equal means indicated that almost all of the selected models, except for the recalibrated Nelson and Siegel (1987) and Svensson (1994) term-structure models, are not perfect for forecasting the real spot rate curve in South Africa.

## 5.5. Findings

The following findings were observed on the fitted models:

- a) Given that the inflation-indexed yield curve has a direct link with movements in macroeconomic factors, it was noted in the study that the application of macroeconomic factors in estimating/forecasting term-structure model parameters did not provide any better performance in both Dynamic Nelson-Siegel (2006) and Dynamic Generalized Nelson-Siegel models. This might, to some extent, be dependent on the choice of variables, especially during a period of high stress (i.e. considering the data that covers the Covid-19 era where normality did break).
- b) It was observed in Table 5.4.1 that for a one-month forecast, most of the selected term-structure models forecasted the real spot rate curve in South Africa at most 50bps below/above the actual JSE spot rates. This is supported by results in Figure 5.4.4, where differences between the forecasted and actual spot rates were lower. However, the test for equal means by Marcus and Elias (1998) to prove that the application of the model is not wrong indicated that the recalibrated Nelson and Siegel (1987) and Svensson (1994) term-structure models have the perfect mathematical models for forecasting the real spot rates in South Africa.
- c) The choice of a perfect term structure for forecasting exercise depends entirely on the institution's mandates/scope of work. Stability in the term-structure model should also be considered to avoid discrepancies/uncertainties in the budgeting process.

## 5.6. Conclusions

In determining a perfect term-structure methodology to estimate the real spot rates in South Africa, it was observed that dynamic models had overfitting characteristics by consistently producing negative real spot rates, especially in the 1 to 5 years maturity spectrum. This is expected in the short-term (i.e. at-most 12 months term-to-maturity)

given the money-market characteristics often observed on this area of the curve. As with the analysis by Castello and Resta (2022) on BRICS countries, the effect of the dynamic decay factor on the Dynamic Nelson-Siegel (2006) model yielded no improvements. This could be attributable to dynamics of the inflation-indexed bond market in the South African case, which is less liquid/less volatile relative to nominal bonds used in the study Castello and Resta (2022).

In forecasting the real-spot rates in South Africa, it was noted that incorporating macroeconomic variables in estimating future model parameters was also unsuccessful in improving the original AR (1) methodology applied in Mashoene et al. (2021). This is a perfect example of correlations that break over periods of high economic stress. Based on the traditional backtesting analysis and a couple of statistical methodologies to attach a level of significance to the performance of the selected term-structure models over different market conditions, it could be concluded that the application of model parameter recalibration on Nelson-Siegel (1987) and Svensson (1994) models were successful in improving the performance of these methodologies/models over different market/economic conditions.

## **Chapter Six**

### **Summary, conclusions and recommendations**

#### **6.1. Introduction**

As we recall, the main objective of this study is to explore the dynamism of different term-structure model in calculating the current and future real spot rates in South Africa. This chapter summarises significant findings and looks at the conclusion that could be derived from the model results and forecasting analysis; as such, giving recommendations and an overview of possible future studies.

#### **6.2. Summary**

Mashoene et al. (2021: pp. 14) indicated that "two decades after the introduction of the first South African government inflation-indexed bond, the market remained illiquid". Further, investors choose to keep the bonds to maturity to optimise their returns due to inflation accumulation over the bond's life. As such, these issues led to the absence of data or resulted in real rates being static over time.

Different economic and monetary policy regimes characterise the period under review. The effect of the Covid-19 pandemic on the country's economic and social aspects necessitated increased borrowing requirements and some measures from monetary policy to help the government borrowing plans, which were faced with increased liquidity issues. In March 2020, South Africa was downgraded to sub-investment grade by Moody's, the third most prominent credit rating agency, to place South African sovereign bonds on junk/sub-investment grade following Standard and Poor and Fitch rating agencies in 2017. Current domestic political volatilities and worsening debt levels fuelled by the growing budget deficit and poor economic conditions remained the most significant drivers for this decision, where a lack of "a clear path towards

government debt stabilisation" was labelled as the core of the downgrade decision by all three major credit rating agencies (Cliffe Dekker Hofmeyr, 2020: pp. 1).

This resulted in a dire situation where South African bonds were excluded from the WGBI, and foreign investors sell-off of government bonds amounting to R3.2 billion a month after the exclusion (Anorl and Winning, 2020). However, this was a long-awaited decision and has long been priced in the bond prices and further clouded by the effect of the Covid-19 crisis. Due to heightened uncertainties around the Covid-19 pandemic, the global economy was shut down from trading, and emerging markets realised further sell-off from most foreign investors as they looked for safer markets. Further, to address the social impact of the continued shutdowns in the country, the South African government made fiscal adjustments to minimise the impact on livelihood. This has further aggravated the poor position of the South African government's funding/borrowing levels while faced with liquidity issues driven by the WGBI exit and the Covid-19 crisis. South African government resorted to tapping on available resources for cash management purposes in line with IMF (2020: pp. 2) guidelines which included "drawing down cash deposits held with the Reserve Bank, increased short-term funding and receiving loans from international financial institutions". The South African Reserve Bank helped manage liquidity issues by buying government stock. This has helped in reducing the cost of borrowing by almost 200 and 350 basis points on inflation-indexed bonds and fixed-rate/nominal bonds, respectively, during the peak of Covid-19 stress around March/April 2020; and further boosted primary market bond auction's bids from 1.7 to 4.31 and 2.25 to 4.24 for both inflation-indexed bonds and fixed-rate/nominal bonds, respectively.

As such, finding a term-structure model to perfectly capture the movements of the real spot rates in South Africa is essential to help as a benchmark for budgeting processes in government. We have noted in this study that during non-normal market conditions, normality does break, and traditional methodologies might not be applicable to capture the heightened volatility.

To derive the real spot rates, different term-structure models were explored on the coupon-bearing bond prices for South African inflation-indexed debt. The recalibration process was also applied to term-structure models with constant parameters to account for a change in the economic environment. These methodologies were used to derive the real spot rate curve in South Africa and further do a forecasting exercise to explore the capabilities of the selected models under a stressed macroeconomic environment. It was observed that during the Covid-19 peak in March 2020, none of the selected models could capture the heightened volatility across all maturities. However, three months later, as volatility reduces in line with strategies to minimise the impact of Covid-19 stress, recalibrated and dynamic/time-dependent term-structure models aligned with spot rates estimated using the JSE methodology.

For the forecasting process, the use of macroeconomic factor methodology was applied to complement the use of AR (1) methodology. FTSE/JSE Inflation-indexed Government Index (IGOV) was used as a proxy for movements in the bond market conditions. Real GDP was also used to analyse the country's economic health in real terms, and CPI to analyse changes in prices in a country. Auction bid ratios were also used to analyse the market liquidity for the inflation-indexed bond instrument. A higher bid ratio translates into higher liquidity/appetite for a particular bond instrument. However, it was observed that the application of the selected macroeconomic factors in analysing the movements in the betas/model parameters was not significantly perfect. As such, AR (1) methodology was used for estimating future model parameters for forecasting spot rates using dynamic term structure models.

Statistical methodologies applied indicated that the recalibration methodology on static models and the use of AR (1) methodology on the Dynamic Nelson-Siegel and Dynamic Generalized Nelson-Siegel models were able to perfectly forecast the South African real spot rates for six months with deviations of less than 50 basis points on longer maturities. Deviations were observed in the short-end of the curve (i.e. maturities of at most 12 months) as a result of money-market characteristics embedded in this area of the curve in line with findings made in Reid (2009), Lartey and Li (2018), and Carvalho and Garcia (2019). As such, mathematical term-structure models might not be able to perfectly define the movements of these spot rates.

## 6.3. Conclusions

It was observed in the study that during the period of the Covid-19 pandemic, drastic monetary policy measures were taken by the South African Reserve Bank to ensure that National Treasury's increased borrowing requirement was not negatively affected by liquidity issues that affected most emerging markets as a massive sell-off of domestic bonds was realised from foreign investors as they look for a safer market. It should also be noted that inflation-indexed bonds have lower liquidity due to investors using them to hedge their long-term obligations against inflation. As such, it is critical to have a term-structure methodology to capture movements/changes observed in the cost of borrowing during normal and non-normal market conditions.

Both the estimation and forecasting processes have indicated that term-structure models with static parameters could not capture the volatility that emanated from the Covid-19 stress. However, incorporating the recalibration process into these models benefited Nelson and Siegel (1987) and Svensson (1994) term-structure models. The same performance was observed on the term-structure models with time-dependent parameters (i.e. Dynamic Nelson-Siegel (2006) and Dynamic Generalized Nelson-Siegel models). In the forecasting exercise, incorporating macroeconomic factors in estimating future parameters was not beneficial compared to the AR (1) methodology. Including the hypothesis test of equal means to supplement traditional backtesting analysis proved that the recalibration process added some advantage to Nelson and Siegel (1987) and Svensson (1994) term-structure models. Based on the study by Mashoene et al. (2021), it was indicated that these term-structure models were perfect for the real spot rates in South Africa; however, it was observed that they do lose capabilities in fitting real spot rates during periods of high stress. As such, it can be concluded that both Nelson and Siegel (1987) and Svensson (1994) term-structure models with the option to recalibrate model parameters over different economic environments remain perfect for estimating current and future real spot rates in South Africa.

## 6.4. Recommendations

Based on this study, it was found that the South African government relied more on the econometric methodology to estimate the real spots, which can produce only two points (i.e. 3-month and 10-year points) on the entire yield curve of the maturity spectrum of over 30 years. This might result in over- or under-valuation of the government debt or projected nominal paper to be issued in the primary market to clear government funding requirements. It was also noted that the JSE methodology relies on a nominal short rate to estimate the real short rate and, subsequently, the entire real spot rate curve. This is due to the non-availability of real overnight rate trading. As such, having an internal methodology that can capture movements of the real spot rates over the entire maturity spectrum, even during periods of high market stress, remains critical for government budgeting and funding processes.

Furthermore, the study revealed that the recalibration process on the Nelson and Siegel (1987) and Svensson (1994) term-structure and dynamic term-structure models provided a better fit for estimating current and future real spot rates in South Africa. It is recommended that the output of this research be explored in South African budgeting and funding processes for more realistic medium-term projections. This could complement the currently used econometrics methodology, which has limitations for longer maturities where most government borrowings are based.



## 6.5. Suggestions for future studies

The study was based on the inflation-indexed bond market, which is deemed complex given that there is no real short-rate trading in the market (Johannesburg Stock Exchange, 2012) and the illiquid nature given its newness in the market for the South African case. It is suggested that this study be extended to the pricing of the nominal spot rates in South Africa to supplement the econometric methodology, which can only forecast two benchmark points (i.e. 3-month and 10-year maturities) of the whole spot rate curve. This process might enhance government budgeting and risk management, especially during market stress.

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## Annexure A

### 8.1. Proofs

#### 1. Spot rate on Nelson Siegel

Given that the forward rate under Nelson-Siegel is estimated as:

$$f(t, \tau) = \beta_0 + \beta_1 e^{-\lambda \tau} + \beta_2 \lambda \tau e^{-\lambda \tau}$$

The spot rate is then given by the following differential equation:

$$\begin{aligned} R(t, \tau) &= \frac{1}{\tau} \int_t^\tau f(u, \tau) du \\ &= \frac{1}{\tau} \int_t^\tau [\beta_0 + \beta_1 e^{-\lambda(T-u)} + \beta_2 \lambda(T-u) e^{-\lambda(T-u)}] du \\ &= \frac{1}{\tau} \left[ \tau \beta_0 - \beta_1 \left( \frac{1-e^{-\lambda \tau}}{\lambda} \right) + \beta_2 \left( -\tau e^{-\lambda \tau} + \left\{ \frac{1-e^{-\lambda \tau}}{\lambda} \right\} \right) \right] \\ &= \left[ \beta_0 + \beta_1 \left( \frac{1-e^{-\lambda \tau}}{\lambda \tau} \right) + \beta_2 \left( \left\{ \frac{1-e^{-\lambda \tau}}{\lambda \tau} \right\} - e^{-\lambda \tau} \right) \right] \end{aligned}$$

Integration by parts:

$$\begin{aligned} \int w v dx &= w \int v dx - \int w' \int v dx dx \\ \text{for } \int_t^T \lambda u e^{-\lambda u} du &= (T-u) \int_t^T \lambda e^{-\lambda(T-u)} du + \int_t^T e^{-\lambda(T-u)} du \\ &= -\tau e^{-\lambda \tau} + \left\{ \frac{1-e^{-\lambda \tau}}{\lambda} \right\} \end{aligned}$$

where

$$\begin{aligned} w &= u & w' &= 1 \\ v &= \lambda e^{-\lambda u} & \int v du &= -e^{-\lambda u} \end{aligned}$$



## 8.2. Figures

Grade	Description	S&P	Moody's	Fitch	DBRS
Investment Grade	Prime	AAA	Aaa	AAA	AAA
	High Medium Grade	AA+	Aa1	AA+	AA(high)
		AA	Aa2	AA	AA
		AA-	Aa3	AA-	AA(low)
	Upper Medium Grade	A+	A1	A+	A(high)
		A	A2	A	A
		A-	A3	A-	A(low)
	Lower Medium Grade	BBB+	Baa1	BBB+	BBB(high)
		BBB	Baa2	BBB	BBB
		BBB-	Baa3	BBB-	BBB(low)
Speculative Grade	Speculative	BB+	Ba1	BB+	BB(high)
		BB	Ba2	BB	BB
		BB-	Ba3	BB-	BB(low)
	Highly Speculative	B+	B1	B+	B(high)
		B	B2	B	B
		B-	B3	B-	B(low)
	Substantial Risk	CCC+	Caa1	CCC+	CCC(high)
		CCC	Caa2	CCC	CCC
		CCC-	Caa3	CCC-	CCC(low)
	Extremely Speculative	CC	Ca	CC	CC
		C	C	C	C
	In Default	RD	/	RD	RD
		SD	/	SD	SD
		D•NR	D•NR	D•NR	D•NR

Figure 8.2.1: Credit rating scales

Source: World Government Bonds (2023)

## 8.3. Ethical Clearance Letter



15 June 2022

Mrs Mmakganya Colleen Mashoene (217077847)  
School Of Acc Economics&Fin  
Westville

Dear Mrs Mmakganya Colleen Mashoene,

**Original application number:** 00017404

**Project title:** The effect of monetary policy regime switches on the application of different term structure models to estimate South African real spot-rate curve

### Exemption from Ethics Review

In response to your application received on 8 June 2022, your school has indicated that the protocol has been granted **EXEMPTION FROM ETHICS REVIEW**.

Any alteration/s to the exempted research protocol, e.g., Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through an amendment/modification prior to its implementation. The original exemption number must be cited.

For any changes that could result in potential risk, an ethics application including the proposed amendments must be submitted to the relevant UKZN Research Ethics Committee. The original exemption number must be cited.

In case you have further queries, please quote the above reference number.

#### PLEASE NOTE:

Research data should be securely stored in the discipline/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours sincerely,



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Academic Leader Research  
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