

The Role of Practical Work in Learning the Division of Fractions by Grade 7 Learners in Two Primary Schools in Mpumalanga Ward of Hammarsdale Circuit in KwaZulu-Natal

by

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DECLARATION

I declare that this dissertation is my own work and that all references have been duly acknowledged. I was given support by my joint supervisors Dr. D. Brijlall and Mr. A. Maharaj.

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DEDICATION

This work is dedicated to my late mother u-Gladys Thandekile (u-MaMalevu) for her patience, guidance and motherly love that moulded me and saw me through my growing years.

ABSTRACT

The researcher's personal conviction that major problems in the teaching of mathematics are inherited from elementary levels inspired the investigation of the contribution of practical work in the teaching of fraction division in grade seven. The all encompassing approach of the study dictated the involvement of teachers and learners as participants. Teachers' perceptions of practical work and their classroom practices were investigated to confirm or refute existing assumptions and literature claims. Questionnaires in which teachers expressed their views on practical work and fraction teaching were administered to teachers. Lessons on the division of fractions were observed to determine teachers' practices in relation to the researcher's assumptions and claims by literature. Data yielded by these research instruments confirmed or refuted assumptions and literature claims.

Learners underwent an experiment and their views were sought to establish the value of practical work in the teaching of fractions and fraction division. Instruments used for the experiment were the pre-test, post-test and worksheets. Data from these instruments gave an indication of the value of practical work in enhancing learners' understanding of fraction division. Learners' responses to interview questions further elucidated and confirmed the valuable role played by practical work in learners' understanding of fraction division. Learners' responses also provided deeper insight into facets of learners' cognitive development as they engaged with different aspects of practical work in the division of fractions.

Besides confirmation and refutation of some established assumptions and literature claims, previously unknown realities about aspects of practical work and fraction division also emerged from findings. This wealth of the data carried crucial implications for teacher training, the teaching of fractions and fraction division, and further research. A look at these implications hopes to contribute to the enhancement and improvement of the teaching of fractions and fraction division. Teacher training institutions, designers of INSET programmes, policy makers and teachers should all benefit from findings of this study.

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CHAPTER 1

OVERVIEW OF THE STUDY

1.1 MOTIVATION

Learners usually learn operations on fractions through intensive training and drill in the use of appropriate algorithms applicable to specific operations. It's been asserted that "Procedural knowledge, such as algorithms for operations, is often taught without context or concepts, implying that algorithms are an ungrounded code only mastered through memorization" (Sharp, Garofalo & Adams, 2002, p.18). This also applies to the division of fractions. Rote learning leaves learners with a shallow understanding of underlying conceptual meanings and processes. This assumption was based on the writer's observation of poor performance by learners when they were involved in solving problems that required knowledge of fractions and operations on them.

The researcher's experience indicated that the lack of profound understanding of the associated conceptual meanings and processes involved, often proves to be a hindrance to learners' acquisition of further mathematical concepts. This affects their general performance, as they progress with the learning of mathematics. Difficulties that learners encounter in the acquisition and successful manipulation of fraction calculations occur in the contexts of the concepts percentages, ratio and algebraic simplifications. The difficulties usually manifest themselves in poor results that learners obtain where calculations with fractions are involved. A focus on the division of fractions through practical means is but one step in an effort to find a remedy to this sorry situation of poor performance.

Poor performance by learners when solving problems that involve fractions, and operations on them, leads to other assumptions about the potential causes of this unhealthy situation. Such assumptions, together with OBE (Outcomes Based Education) challenges to restructure instruction along learner-centred lines, form the motivation for this study. The assumptions on which this study was based are:

- a) Minimal use of practical work by educators is a source of impoverished development of concepts on fractions and operations on them, including division of fractions.
- b) Limited visual representation of the fraction concept with pictures of part-regions.
- c) Overemphasis of the algorithm as a goal of instruction. This leads to a shallow understanding of underlying conceptual meanings and processes involved in the division of fractions.
- d) OBE requires a learner-centred approach to teaching and learning. This implies that there should be a practical approach to instruction of fraction division that engages the learner.

1.1.1 Minimal use of practical work by teachers

Informal observation of practices by mathematics teachers, coupled with informal interactions at experience-sharing forums, suggested to the writer that teachers seldom include practical work in their teaching of fractions. The common approach by teachers hardly ever goes beyond pictorial representations of fractions, symbolical (often numerical) representation and manipulation of fractions. In the latter context the algorithm is applied to the solution of problems involving fractions. This often happens with hardly an effort to ensure that learners have the necessary

understanding of what the fraction concept is all about. The reason cited by teachers for the exclusion of practical work from their lessons is that it (practical work) takes a huge amount of time, both during the preparation and teaching stages. They claimed that this alleged shortcoming (of practical work) usually resulted in their efforts to complete the prescribed syllabus being seriously threatened and compromised. This position clearly shows a lack of appreciation for the positive role of practical work as a necessary and effective prerequisite in building a solid conceptual background that should, out of pedagogic necessity, precede the meaningful comprehension, acquisition and successful application of any subsequent algorithm. Such an unfavourable disposition towards practical work could have its origins in inadequate teacher-training in the valuable use of practical work, including the development and use of related materials.

Textbook publications of the pre-OBE era devoted very little attention to exercises that were responsive to the provisions and requirements of the inclusion of practical work in instruction sessions. The structuring and presentation of content in these textbooks hardly ever transcended pictorial (part-region or subset) and symbolic representations, and manipulation of a limited version of the fraction concept. Successful application of the relevant algorithm to find solutions to fraction problems often appeared to be the ultimate object of instruction. The structuring and presentation of content in textbooks influences the teaching practices of educators. This can be the case especially in the absence of alternative sources of content (other textbooks, syllabuses, departmental subject policies and curriculum guidelines). Under such circumstances, teachers often tend to rely heavily on the available textbook as the only source of guidance in their approach to the teaching of a

particular section of the syllabus. They end up following that textbook slavishly, sometimes at the expense of more effective alternatives worthy of exploration and trial. It is the writer's position that this lack of practical engagement of learners impacts negatively on their conceptual development in fraction learning. This made it necessary for this study to determine the views and perceptions of teachers on practical work and the teaching of fractions.

1.1.2 Limited visual representation of the fraction concept

When teaching fractions and operations on them, teachers have the tendency to use only examples that portray the part-region interpretation of the fraction concept, using only pictorial representations of the fraction. In discussing the importance of the exposure of learners to multiple representations of the fraction concept, observations have been made that "Pictures of subdivided regions to be shaded to indicate some fractional part accompany discussion of the real-life example of sharing a pizza" (Witherspoon, 1993, p.482). Witherspoon (ibid.) warned that if these are the only contexts in which learners encounter fractions, then they will learn only a small part of the underlying concepts. Consequently learners end up with a very limited ability at problem solving where fractions are involved. Their limited understanding of the fraction concept together with the associated limited understanding of concepts on fraction operations, have a negative impact on learners' ability to acquire further mathematical concepts. The same applies to their (learners') general performance as they progress with their learning of mathematics. Conceptual concepts in which learners end up experiencing difficulties have been noted under motivation. The importance of the representation and interpretation of the fraction concept beyond pictures was stressed by the suggestion that

“...instruction should focus on the interpretation of situations involving a product of two fractions, the modelling of those situations physically or pictorially, and the explanation of why the product of, for example, $\frac{1}{2}$ and $\frac{2}{3}$ is $\frac{1}{3}$ ” (Cramer & Bezuk, 1991, p.34). Although this comment was specific to fraction multiplication, the analogy with fraction division cannot be missed. The significance of the physical representation of $\frac{1}{2}$ and $\frac{1}{4}$ to explain why, for example, $\frac{1}{2} \div \frac{1}{4} = 2$, applies equally for learners to grasp the underlying conceptual meanings of fraction division. For the writer, a study therefore became necessary to explore the potential of practical work to enrich learners’ conceptual understanding of the concepts of the fraction and fraction division beyond part-regions and pictorial representations.

1.1.3 Overemphasis of the algorithm as a goal of instruction

Personal experience from observation of teachers’ practices showed that it’s a common tendency to overemphasise the algorithm as the primary object of instruction. This also applies to the division of fractions. A limited and impoverished understanding by learners of the underlying concepts involved, appear to be the final end-product of this tendency. The negative effect of teaching an algorithm, without understanding, is rote learning by learners. Many authorities in fraction learning have noted that this kind of learning leaves learners with a very limited understanding of conceptual meanings involved (Flores, 2002; Ott, Snook & Gibson, 1991). It has been pointed out that “Traditionally ... division of fractions has been taught often by emphasising the algorithm procedure ‘invert the second fraction and multiply’ with little effort to provide students with an understanding why it works” (Flores, 2002, p.237). A meaningful conceptual understanding of fractions and

operations on them, as clearly distinct from the ability to successfully manipulate algorithms, is a necessary prerequisite if learners are expected to draw any meanings from their learning about fractions. The ability to successfully manipulate the division algorithm may produce the desired result, but this does not necessarily guarantee an understanding of conceptual processes involved. Such cosmetic success can only be attributed to excessive training and drill in the appropriate use of the algorithm. There is no meaningful mathematics learning that can be said to be taking place under such circumstances. Teachers have been warned that “We should be careful not to assume that students ‘understand’ fractions merely because they are able to carry out an algorithm or recite a definition” (Witherspoon, 1993, p.484). She further argued that the successful application of rules in fraction problems is an exercise in futility if the learner cannot interpret the results of his or her labours.

The meaningful understanding of conceptual processes involved in fractions and operations (including division) on them, provides the prerequisite background necessary for learners to develop, refine and apply appropriate algorithms to the solution of problems involving fractions. Such understanding also helps to provide the background required for the meaningful acquisition of further mathematical concepts, as well as successful performance in problems where fractions are involved. Teachers are advised that “Once children possess meaningful images for fraction-division, they are then able to discover and find meaning for the IM rule” (Siebert, 2002, p.225). The IM rule refers to the ‘Invert-Multiply’ algorithm popularly taught in the division of fractions. Physical manipulation of concrete representations of the fraction concept could play a significant role in laying the necessary foundation for the meaningful understanding of conceptual processes

involved in fraction division. This could ensure the discovery and successful application of the division algorithm in the solution of fraction division problems. The meaningful understanding of concepts involved should serve as a countermeasure against unreasonably absurd and inaccurate results where operations on fractions are involved. For example, if a learner clearly understands from physical manipulations of concrete models what it means that there are two $\frac{1}{3}$'s in $\frac{2}{3}$, he or she is unlikely to come up with an unreasonably inaccurate answer to $\frac{2}{3} \div \frac{1}{3}$. A clear understanding of the concepts of $\frac{2}{3}$, $\frac{1}{3}$ and division serves as guidance towards a reasonably accurate answer. Conceptual understanding, which is necessary to help learners' understanding of fraction division, should emanate from practical manipulation of concrete representations of fractions. This led to the formulation of the third research question.

1.1.4 OBE requirements for a learner-centred approach

Among the key principles that guide the development and implementation of Curriculum 2005, the curriculum anchor of OBE, the education department's Policy Document (DoE, 1997) listed: (a) Participation and ownership, and (b) Learner-orientated approach. Also, Specific Outcome number 9 for MLMMS (Mathematical Literacy, Mathematics and Mathematical Sciences) aims at learners' achievement of '...the use of mathematical language to communicate mathematical ideas, concepts, generalizations and thought processes.' Further, the related Range Statement refers to 'presentation of real-life or simulated situations in mathematical format.' The wording of the principles, outcome and range statement strongly suggest serious

engagement of the learner in the learning process. Also, the approach to teaching and learning envisaged by OBE offers an appropriate platform for the multi-modal presentation of concepts and their perception by learners. Cramer & Bezuk (1991) suggested that this could be achieved by exposing learners to experiences of the concept with real-world situations, manipulatives, pictures, and written and spoken symbols. Engaging learners with practical activities in learning fraction division provides more than ample opportunity to put into practical implementation the ideals of Outcomes Based Education. If the principle of a learner-centred approach is to be upheld in real terms, then it becomes necessary to solicit the views of learners on practical work in the division of fractions, hence the fourth research question. After all, it's the learner who is supposed to occupy the centre stage during the learning process.

1.2 RELEVANCE OF THE STUDY

The study is of relevance to practising mathematics teachers, designers of in-service teacher development programmes, and teacher training institutions. Teachers usually struggle to teach concepts on fractions to learners. Their efforts, which are usually well intentioned and carefully considered, often bear no fruit because learners are left with no meaningful understanding of the concepts of fractions and fraction division. Instruction fails to connect the meaning of fractions and fraction division to the concrete reality of learners. This inability of learners to understand fractions proves to be a serious impediment to their ability to acquire further mathematical concepts and has dire consequences on learners' performance in the solution of problems that involve fractions. The perceptions of teachers on practical work and fractions could provide the necessary and useful insight into the needs and

challenges of teachers in relation to the teaching of fractions. This should contribute significantly to the identification of problem areas which require special attention, with the aim of improving the design and implementation of in-service programmes on the teaching and learning of fractions. Most pre-service teacher-training programmes on offer put enough emphasis on practical work as an important tool in the teaching and learning of mathematics. Yet in spite of this, one of the assumptions of this study points to the minimal use of practical work by teachers. Findings of this study should be able to add even more value to these programmes to ensure that teachers embrace the idea of including practical work in their lessons. Such developments should contribute to mathematics lessons becoming more learner-centred.

1.3 RESEARCH QUESTIONS

In the motivation assumptions have been made about teachers' non-commitment to the inclusion of practical work in their lessons on the division of fractions. These assumptions cannot be left as they are but need to be confirmed as true or otherwise be refuted. For this reason it was important for this study to ask questions about teachers' perceptions on practical work, with specific reference to their practices when they teach fractions and fraction division. These concerns gave rise to research questions 1 and 2: *1) What are the perceptions of teachers on practical work and the teaching of fractions in relation to their practices?*

2) What are the factors behind these perceptions?

The effectiveness of concrete experiences in the learning of fraction division needed to be examined within the context of the schools where we teach (the writer conducted the study in two South African township schools). The study needed to

find out about the benefits and disadvantages, if any, that learners experience as a result of engaging in practical activities when dividing fractions. This study aimed to find the answers within the specific contexts of part-region and subset representations of the fraction concept. This led to research question number 3:

3) Does the division of fractions by practical means result in better understanding of concepts involved?

The learner-orientated approach to learning advocated by Outcomes Based Education implies that the views of learners on the effectiveness of practical work in learning fraction division cannot be overlooked. They are the main players that every learning process should be concerned about. Does dividing fractions using practical means help them to understand the division of fractions better? This gave rise to research question number 4: *4) What are the views of learners about the use of practical work in the division of fractions?*

1.4 AIMS OF THE STUDY

The research questions suggested the following aims:

- a) Finding out whether the representation and interpretation of the fraction concept by means other than the usual part-region model has positive effects in enhancing learners' understanding of the concepts of the fraction and fraction division.
- b) Determining, by experimentation, the effect of using concrete models in learning the division of fractions.
- c) Determining the views of teachers and learners on practical work in the teaching and learning of fractions, and fraction division.

CHAPTER 2

REVIEW OF LITERATURE

2.1 INTRODUCTION

The literature review focused on answers to the following questions:

- a) What leads to enriched and diverse understanding of the fraction concept?
- b) What is the role of whole number division in the understanding of the meanings of fraction division?
- c) How do various interpretations of fraction division situations lead to the development and understanding of the fraction division algorithm?
- d) What is the role of understanding various interpretations in the development and understanding of the fraction division algorithm?
- e) What is the role of practical work in understanding fraction division?

2.2 ENRICHED AND DIVERSE UNDERSTANDING OF THE FRACTION CONCEPT

Limited exposure of learners to a single representation of the fraction concept has been identified to seriously impair learners' full development and understanding of the concepts of the fraction and operations on fractions (Witherspoon, 1993). This includes division of fractions. Subdivided regions for shading to indicate some fractional part of a real-life pizza, or a chocolate bar, are among some of the widely used examples for the fraction concept (Moskal & Magone, 2002; Witherspoon, 1993). This singular part-region representation of the fraction concept (Witherspoon, *ibid.*) prevails, although there are many representations and interpretations which could improve the understanding of the fraction concept.

2.2.1 Multiple perspectives of the fraction concept

It has been argued that “Central to the complexity of rational numbers is the fraction symbol” (Sinicrope & Mick, 1992, p.116). They listed various conceptions represented by the fraction symbol identified by the following researchers:

- a) Kieren in 1980 noted part-wholes, measures, divisions, operators and ratios.
- b) Usiskin & Bell in 1984 noted locations, ratios, counting units, variants of scientific notation, notations in algebra, scalars, multiplication across, division rates, division ratios and powering growth.

However, perceptions of the fraction concept are by no means a closed domain, prescribed only by the views of a few select authorities on fractions. Witherspoon (1993) citing Kennedy and Tipps viewed fractions as part-wholes, subsets, ratios, quotients and rational numbers. Teachers who understand a topic make connections with other mathematical concepts and procedures (Flores, 2002). Flores (ibid.) further suggested that some of the connections needed in the division of fractions are fractions and quotients, fractions and ratios, division as multiplicative comparison, reciprocals (inverse elements) and operators. Therefore teachers need to understand how the concepts of the fraction $\frac{3}{4}$, a quotient $3 \div 4$, and the ratio $3:4$ are different and related to each other.

Knowledge of the many perspectives of the fraction concept, although a valuable asset for learners, is not sufficient enough for meaningful and holistic understanding of the fraction concept itself, and fraction division. To gain a complete understanding of the fraction concept, learners need to be exposed to a variety of concept representations. Witherspoon (1993) suggested the following five representations

identified by Lesh *et al* in 1987: (a) symbols, (b) concrete models, (c) real-life situations, (d) pictures and (e) spoken language.

2.2.2 Part-region perspective of the fraction concept

In spite of the multitude of available perspectives of the fraction concept, and the desired necessity for the widest possible exposure of learners to these varieties, it is regrettable that instruction by most teachers still overemphasises the part-region perspective of the fraction concept. Sinicrope & Mick (1992) shared the views of Tobias who lamented as unfortunate that many students and math-anxious adults still view fractions strictly as part-wholes. Witherspoon (1993) concurred that the fraction interpretation that students probably encounter most frequently in elementary school is that of part of a region. As part of the problems associated with the overemphasis on the part-whole perspective of the fraction concept, it's been noted "Two specific areas are problematic for upper elementary school students in their ability to deal with fractions: the geometry of unmarked region models and the application of knowledge of regions to other interpretations" (Witherspoon, 1993, p.482). Her observations follow from students' inability to correctly partition and shade an unmarked model in accordance with a given fraction, and the inability to indicate correctly (by shading), a subset of marbles according to a given fraction. As a solution to the unmarked region problem, Witherspoon (*ibid.*) suggested that students must experience subdividing regions in various ways so that they become familiar with the geometry of various shapes. To overcome the problem of the application of knowledge of regions to other interpretations (e.g. the set interpretation), Witherspoon (*ibid.*) advised that learners must be able to understand that the 'one', in $\frac{1}{2}$ for example, is a set, not a single object. Further, learners should

understand a variety of fraction-number representations (e.g. set, ratio, division) and their interpretations.

Emphasis on the part-region perspective of the fraction concept is not as bad as suggested in the discussion so far. If anything, situations actually exist where such emphasis is quite desirable and useful. However, this should be within a broader context of exposure to other interpretations and representations of the fraction concept. Flores (2002) asserted that children go through several stages to develop the idea of the fraction in the context of subdividing areas. Flores (*ibid.*) further advised that teachers need to make sure learners have developed a fairly complete understanding of fractions before discussing division of fractions. In their study of children's informal knowledge of fractions Murray, Olivier & Human (1996) emphasised and exploited knowledge about fractions which involve the part-region concept of the fraction. Two sub-constructs of this concept are: (a) the part-whole relationship between the fractional part and the unit, and (b) the idea that the fractional part is that quantity which can be iterated a certain number of times to produce a unit. In a study that involved first and third-graders, Murray, Olivier & Human (*ibid.*) concluded, among others, that responses by first-graders showed that equal sharing situations elicited ideas about partitioning units into equal parts and about combining parts to form a unit. Murray, Olivier & Human (*ibid.*) went on to emphasise that both ideas (the part-whole and the iterative-part-to-form-a-whole) are crucial sub-constructs of the fraction concept. Therefore examples of situations which focus on the part-whole perspective of the fraction concept are desirable. They give rise to and lead to an understanding of an important perception of the fraction concept.

2.3 UNDERSTANDING WHOLE NUMBER DIVISION

Learners' knowledge of working with whole numbers is a valuable reservoir to the learning of multiplication and division of fractions. In their findings on young children's informal knowledge of fractions, Murray, Olivier & Human (1996) suggested that it's possible to elicit and build on (a) young children's conceptualization of computational problems and (b) the strategies they construct based on these conceptualizations. Murray, Olivier & Human (ibid.) further suggested encouraging and building on this base of children's informal knowledge. They argued that such informal knowledge about whole numbers (and problem situations involving whole numbers) is strong and almost completely free of misconceptions. The value of learners' knowledge of whole numbers had been echoed when it was advised that "...to help students extend the concept of whole-number multiplication to multiplication of fractions, we begin with such examples of whole-number multiplication as three packs of five sticks of chewing gum" (Sinicrope & Mick, 1992, p.117). Sinicrope & Mick's (1992) whole number multiplication is included in Murray, Olivier & Human's (1996) problem situations involving whole numbers where understanding by learners is said to be strong and completely free of misconceptions. Therefore learners' knowledge of whole numbers is a valuable asset in their ability to understand operations on fractions. It seems such knowledge is a basic necessity if they (learners) are to be successful with operations on fractions. Flores (2002) noted that a thorough understanding of the operations division and multiplication with whole numbers is a pre-requisite for understanding division of fractions. The continued link between multiplication and division takes us back to Flores' (ibid.) suggested connections, amongst which is division as multiplicative comparison. Even in his examples to illustrate the connection between

multiplication and division in the ‘Invert and Multiply’ algorithm, yet to be discussed, Flores (ibid.) used the multiplication and division of whole numbers as his starting point. Siebert (2002) suggested that for teachers to help children develop meaningful conceptions of division of fractions, they must first clearly understand whole number division. Siebert (ibid.) also advised that children can develop meaningful images for the division of fractions by reasoning about real-world contexts involving fraction division, and making connections between their solutions and their understanding of whole number division. The strong emphasis that literature puts on the knowledge of whole numbers as a prerequisite for meaningful understanding of operations on fractions, including fraction division, makes it necessary for *this study to investigate the teachers’ practices to secure this vital understanding. This is especially in relation to their teaching practices when they introduce and teach fraction division.* Do teachers’ practices show an appreciation for this crucial background to fraction division, and do these practices exploit to the fullest the potential of practical work to establish this vital link?

2.4 UNDERSTANDING VARIOUS INTERPRETATIONS OF FRACTION DIVISION

Various studies (e.g. Hart, 1981) have shown that in computations involving fractions, learners experienced the most difficulties with problems based on multiplication and division. At the heart of problems specific to division, the following have been identified:

- a) A general challenge for learners with regards to problems based on multiplication and division of fractions.

- b) Lack of enriched and diversified understanding of fraction division situations.
- c) Emphasis on meaningless application of the division algorithm.

2.4.1 Multiplication and Division of fractions: a general challenge for learners

In her study of children's understanding of mathematics, Hart (1981) found that the hardest group of problems for 14 and 15 year olds involved multiplication and division. It's been observed "The division algorithm is very difficult to apply (30 percent of the sample could deal with $\frac{3}{4} \div \frac{1}{8}$) and probably any computation which seemed to require its use was likely to upset the children" (Hart, 1981, p.75). One of the possible reasons Hart (ibid.) identified for difficulties with computations involving fractions was that learners often confused rules. It has been argued that "...many children do not feel confident in the use of fractions and try whenever possible to apply the rules of whole numbers to fractions" (Hart, 1981, p.76). Learners therefore have difficulties with fractional computations, especially those involving multiplication and division. The most tempting solution to this problem is intensive training and drill of learners in the correct use of the appropriate algorithm. With specific reference to fraction multiplication, Cramer & Bezuk (1991) warned that teachers must not conclude that a student's ability to answer correctly a fraction multiplication problem indicates that: (a) the student understands multiplication of fractions, or (b) the student can recognize problem situations requiring the multiplication of fractions. The researcher believes that the same applies to division of fractions. Contrary to claims by Hart (1981) that students generally experience difficulties with the multiplication and division of fractions, these and similar warnings by Witherspoon (1993) on the misinterpretation of the successful

application of the algorithm, suggest that it's possible for learners' performance to show success in the application of the algorithm (division or multiplication) to fraction problems. The main point though, is that such success should not be misconstrued to mean an understanding of concepts on fractions and fraction operations.

2.4.2 Enriched and Diverse Understanding of Fraction Division Situations

The Lesh translation model has been suggested as suitable for multiple representation and interpretation of the fraction concept, and other related concepts e.g. fraction multiplication and division (Cramer & Bezuk, 1991; Witherspoon, 1993). This model makes provision for five basic categories of concept representation: (a) real life situations, (b) symbols, (c) concrete models, (d) pictures, and (e) the spoken language. Teachers are advised that "Conceptual understanding is dependent on students having experiences representing multiplication of fractions in each of these modes" (Cramer & Bezuk, 1991, p.35). They argued that according to Lesh, there should be emphasis on the relationships between these modes of representation and within single modes. Among suggestions for translation from one mode of concept representation to another, Cramer & Bezuk (1991) mentioned real-life situations to manipulatives. This would mean representation of a real-life situation with concrete models in an effort to find a solution. Although the entire discussion by the authors was centred on fraction multiplication, application of the importance of the multi-modal representation of the fraction concept to fraction division remains apparent.

This study exposed learners to four of the five categories of concept representation as suggested by the Lesh model. The drawing ruler and bottle-tops represented concrete embodiments of the concepts of the fraction and fraction division. Written tests and worksheets that learners worked on, offered opportunities for them to experience symbolic, pictorial and spoken-language representations of the concepts of the fraction and fraction division. In working through worksheets, learners worked in groups, and this called for a discussion of their efforts to find solutions to given problems. This way they experienced the different variations of fraction representations.

In addition to meaningful understanding of the fraction concept through exposure to multiple representations and interpretations as proposed by Lesh, learners need to have an enriched understanding of the meanings of fraction division situations. Such an understanding can derive from understanding division situations for whole numbers. The significance of understanding whole numbers to understand fractions has been discussed in this review. Literature showed that understanding situations for whole number division is also important for the understanding of fraction division. In their discussion of division situations which they termed interpretations, Sinicrope, Mick & Kolb (2002) noted:

For whole-number division, problem situations need to be categorised as measurement division (determining the number of groups), partitive division (determining the size of each group), or the inverse of the Cartesian product (determining the dimension of a rectangular array). Fraction division can be explained by extensions of all three of these whole-number interpretations (p.153).

However, Sinicrope, Mick & Kolb (ibid.) warned that these three extensions are not enough. A further two, which they termed: (a) division as the determination of the unit rate and (b) division as the inverse of multiplication, are also important fraction

division situations. Knowledge of these division situations is important for teachers to be able to impart a meaningful understanding of the division of fractions to their learners. Sinicrope, Mick & Kolb (2002) advised:

If our students are to construct a rich, rational understanding of fraction-division, we as teachers need a framework for fraction-division situations that will help us select problem types and to design tasks... we need to know what kind of situations are fraction-division situations, what reasoning occurs within these situations, and what mathematical generalizations can be made... (p.153).

Multiple representations of the concepts of the fraction (Witherspoon, 1993) and fraction multiplication (Cramer & Bezuk, 1991) should be used. However, other views exist that do not necessarily agree, especially when it comes to fraction division. After an example on a fraction division problem, Ott, Snook & Gibson (1991) argued:

Inability to interpret the results of division problems... is usually not the result of an inability to decode the meaning of the fraction symbols ($\frac{3}{4}$, $\frac{1}{3}$, $2\frac{1}{4}$) but, instead, is the result of a lack of understanding of what division of a fraction means (p.7).

On the importance of whole number division for the understanding of fraction division, it is asserted that “The meanings of fraction-division exercises are the same as those for the division of whole numbers” (Ott, Snook & Gibson, 1991, p.7). Sinicrope, Mick & Kolb (2002) supported this view. For the former, the measurement and partitive meanings of division are the most significant interpretations of division. The same interpretations can be found among the five listed by Sinicrope, Mick & Kolb (2002), although for them there are three other interpretations that are also important. The importance of understanding the meanings of fraction division is supported further by the suggestion that “Understanding division of fractions is helped by appreciating different meanings such as measurement division, sharing, finding a whole given a part, and missing

factors” (Flores, 2002, p.238). The sharing meaning of division is defined to mean the same thing as partitive division. It is interesting to note that among the various offerings of fraction division interpretations, the measurement and partitive meanings continue to form a common thread, even though different authors continue to have their own extra variations and additions. *This study chose to focus on the measurement interpretation of fraction division as its anchor for the investigation of the effectiveness of practical work in the teaching and learning of the division of fractions.*

2.4.3 Emphasis on meaningless application of the algorithm

An approach to the teaching of fraction division which upholds the algorithm as the primary object of instruction has been discussed in the motivation for this study, together with its negative consequences for learners. This obsession with the algorithm is often displayed with virtually no regard for the various division situations, whose importance to the understanding of fraction division situations has also been discussed. Lamenting this situation, Siebert (2002) noted:

Children often lack a ready understanding for operations involving fractions because these operations are often equated with seemingly nonsensical algorithms, such as the algorithm for division of fractions. For children, the traditional algorithm for division of fractions, the invert and multiply (IM) rule, does not seem connected to division in any way (p.247).

Flores (2002) echoed similar sentiments on the unexplained use of the algorithm in teaching the division of fractions. Emphasis on the algorithm, which isolates it from an understanding of fraction division situations, carries the danger of misleading teachers into thinking that there is an understanding of the division of fractions when learners are able to correctly apply the algorithm. There have been warnings against assuming an understanding of fractions merely on the basis of successful applications of the algorithm (Cramer & Bezuk, 1991; Witherspoon, 1993). Linking

understanding of fraction division situations to the development of the division algorithm and how it works should be the focus of instruction. Siebert (2002) stressed the importance of linking fraction division situations to the algorithm when he noted:

...by starting their study of the division of fractions with their informal thinking about two basic types of division situations, children can discover ways to draw pictures for fraction division in which they can actually see what it means to invert and multiply (p.248).

The two basic types of fraction division situations referred to are: (a) measurement division and (b) partitive division. The significance of establishing these links was emphasized by Sinicrope, Mick & Kolb (2002) who wrote:

For the teacher of mathematics, an exploration of different interpretations of fraction division forms a framework for designing instruction through posing problems. As students solve the teacher-posed problems, they can eventually generate algorithms for solving even 'larger sets' of problems (p.161).

The development of the fraction division algorithm within the context of understanding fraction division situations could lead to a meaningful understanding of the algorithm, and hence assist its successful application to the solution of problems on fraction division. Siebert (2002) advised that once children connect their images of sharing and measurement to the IM rule, this rule can become a meaningful tool to solve a wide range of interesting and important problems.

2.5 PRACTICAL WORK: A USEFUL VEHICLE FOR UNDERSTANDING VARIOUS INTERPRETATIONS OF FRACTION DIVISION

This study was specifically about the effectiveness of practical work in the teaching and learning of the division of fractions. The discussion of literature has so far outlined the importance of understanding fraction division situations first if one is to acquire a meaningful understanding of conceptual processes involved. Literature also attests to the important role that practical work can play in helping learners to

attain this meaningful understanding of fraction division situations, a critical requirement to the overall understanding of the division of fractions.

Concrete experiences are fundamental constituents of practical activities. In stressing the importance of using concrete experiences as the basis for abstraction, it is noted that "...familiar concrete experience - actual or recalled - should be a first step in the development of new abstract concepts and their symbolization" (Ott, Snook & Gibson, 1991, p.7). They also observed that although widely used with whole numbers, learners hardly ever use the principle of moving from concrete to abstract in the division of fractions. This led to fruitless consequences of the rote use of the algorithm. Concrete experience is directly useful when used as the basis for understanding fraction division situations. After identification of measurement and partitive interpretations of division as the most important in understanding fraction division, it is suggested that "...since these meanings are not obvious, students need experience dividing numbers in a more concrete and meaningful manner before moving on to more abstract means of dividing" (Ott, Snook & Gibson, 1991, p.8). They acknowledged that concrete experiences related to division of fractions are much more difficult for teachers to devise and for learners to follow, and that measurement interpretation is the easiest to represent using concrete models. On advice in examples for concrete experience, suggestions have been made that "instructional models like pattern blocks also use this measurement interpretation" (Sinicrope, Mick & Kolb, 2002, p.155). Sinicrope *et al* (2002) gave an example of how pattern blocks - yellow hexagons, red trapezoids, blue parallelograms, and green triangles - can be used to find a solution to $\frac{11}{12} \div \frac{1}{4}$. After detailing out the solution, both practically and symbolically, they concluded the algorithm representing the

procedural reasoning in that type of division is the common-denominator algorithm for the division of fractions. Algebraically this is represented by $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bd} \div \frac{bc}{bd} = \frac{ad}{bc}$. It was suggested “It is possible to relate the procedural reasoning used in the solution of measurement divisions to the invert - and - multiply algorithm” (Sinicrope, Mick & Kolb, 2002, p.154). Flores (2002) stressed the centrality of concrete experiences to the development of the division algorithm by suggesting that teachers need a complete picture that connects concrete approaches of division with the algorithm of multiplying by the reciprocal. On measurement interpretation of division, it has been suggested “With the help of concrete models of fractions, students can see that $\frac{1}{4}$ fits two times into $\frac{1}{2}$, therefore $\frac{1}{2} \div \frac{1}{4} = 2$ ” (Flores, 2002, p.238). In what is termed justification, with a view to the development of algorithms, it is advised that “Teachers can make concrete representations, empirical evidence and patterns, and properties of numbers and operations to explain the various approaches to division of fractions” (Flores, 2002, p.240).

As their concluding advice, Ott, Snook & Gibson (1991) felt that learners need early concrete experiences that clearly demonstrate the meaning of division of fractions. They claimed their belief was supported by the NCTM’s Curriculum and Evaluation Standards for School Mathematics of 1989, which stated that concepts are the substance of mathematics knowledge and that students can comprehend mathematics only if they understand its concepts and their meanings and interpretations. Dienes’ (1964) described the three levels of conceptual development as understanding: (a) pure concepts, (b) notational concepts and (c) applied concepts. He described pure

concepts as the understanding of intrinsic properties of numbers and operations on them e.g. what do 3 and 4 represent in $\frac{3}{4}$? The representation of pure concepts in written form represented Dienes' idea of notational concepts, while application of pure concepts to real life situations gave rise to his applied concepts. The understanding of properties of fractions implies Dienes' notion of pure concepts, while their notation in written form is indicative of his notational concepts. Application of knowledge of fractions in fraction division problems is an example of Diene's applied concepts. *It is appropriate, once again, to note that this study was about the effectiveness of practical work in the division of fractions. This focus of the study was particularly related to the understanding of concepts (of fractions and fraction division) and processes involved.*

2.6 SUMMARY

Literature suggests that it is important to expose learners to diverse interpretations of the fraction concept as a foundation to meaningful acquisition of the concept of fraction division. This is possible through presentations of the fraction concept through multiple perspectives, not the part-region only. Whole number division is advocated as a starting point towards understanding fraction division. To develop a meaningful understanding of how the fraction division algorithm works, understanding whole number division should serve as a basis for understanding various interpretations of fraction division situations. It is the understanding of these situations that will facilitate understanding of the fraction division algorithm. Practical work is placed at the centre of meaningful understanding of the division of fractions. The principle of moving from concrete to abstract, remains pivotal to the acquisition of this understanding.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 INTRODUCTION

Factors that motivated this study as well as associated research questions to find answers for, were discussed. A meaningful discussion of research methodology will be possible if these are recapped. This will paint a clearer picture of the research methodology. Factors behind the motive for this study were:

- a) Minimal use of practical work by teachers as a source of impoverished development of concepts of fractions and operations on them, including division.
- b) Limited representations of fractions with pictures of part-regions.
- c) Over-emphasis of the algorithm as a goal of instruction leading to a shallow understanding of underlying conceptual meanings and processes involved in the division of fractions.
- d) OBE requirements for a learner-centred approach to teaching and learning.

As a result of the concerns listed above, the research questions were:

- 1) *What are the perceptions of teachers on practical work and the teaching of fractions in relation to their practices?*
- 2) *What are the factors behind these perceptions?*
- 3) *Does the division of fractions by practical means result in better understanding of the concepts involved?*
- 4) *What are the views of learners about the use of practical work in the division of fractions?*

A clear perspective of these factors and questions will be of significant value in the discussion of research methodology.

3.2 QUALITATIVE NATURE OF THE STUDY

Investigating the effectiveness of practical work required an in-depth inquiry into the perceptions of teachers and learners about its use in learning about fractions and fraction division. It became necessary to test certain assumptions about teachers' practices. A naturalistic experiment on the effects of engaging learners with practical activities to find out if this had any positive benefits for the learning of fraction division was required.

All major areas that the study intended to look at qualified it to be categorised as qualitative. The research instruments used to bring out the required data were specifically associated with qualitative studies. Instruments involved (a) observation of lessons, (b) interviews with learners, (c) experimentation on learner practical work by using worksheets, (d) tests for learners and (e) questionnaires for teachers. Patton (2002) explicitly listed observations and interviews as instruments used in qualitative inquiry. Natural experiments are distinct from controlled experiments in that the observer is present during a real-world change to document a phenomenon before and after change (Patton, *ibid.*). This is the kind of experiment that the study undertook on learners' use of practical means to divide fractions. Though questionnaires are predominantly associated with quantitative studies (Cohen, Manion & Morrison, 2000), if they make provisions for open-ended responses, such questionnaires are capable of generating in-depth data on respondents' feelings, opinions, views, attitudes and perceptions about the phenomenon (division of

fractions by practical means). A questionnaire with all these attributes qualifies as a research instrument for a qualitative study. The questionnaire used in this study had similar qualities (see Appendix A). It made provision for open-ended responses where respondents could express their feelings and opinions on the use of practical work in the teaching of fractions.

3.3 THE PARTICIPANTS

The study on the effectiveness of practical work in learning the division of fractions sought to establish data in line with the assumptions stated and questions asked. These considerations determined the intended participants in this study. The participants were grade 7 learners and mathematics teachers associated with grade 7 mathematics education.

3.3.1 Teachers

Assumptions were made about the practices of teachers when teaching fractions and fraction division, and some of the underlying beliefs that inform these practices. These assumptions needed to be tested. To test assumptions on teachers' practices, lessons on fraction division had to be observed to ascertain the approach used by teachers. To find out more about the factors behind teachers' views on practical work in the teaching of fraction division, a questionnaire (see Appendix A) was designed for administration among teachers. Targeting only one teacher for both observation and questionnaire completion would have been insufficient for purposes of generating sufficiently credible data. The questionnaire was administered to all or several mathematics teachers in a school. However, due to time constraints, it was impossible to observe lessons on fraction division by more than one teacher in a school. Schools that granted access gave a maximum of three to four weeks within

which to conduct the study. Therefore, this called for a compromise arrangement to generate reasonably credible data on teachers' perceptions of practical work and the teaching of fraction division in relation to their practices. It was decided to administer the questionnaire to all mathematics teachers in the two schools, but to observe only the lessons of one grade 7 group per school. Given the time constraints, administering the questionnaire to several teachers and observing only one teacher per school seemed the only and most practicable way that could promise to yield data of any credible value.

3.3.2 Learners

This study focused on the effectiveness of practical work in learning the division of fractions. Since learning was at the heart of this study, this automatically placed the learner at the centre of the study. To establish the effects of engaging in practical activities, the study used worksheets (see Appendix B) which learners attempted. Besides this, learners wrote tests to determine the impact of practical work on their learning. The experimental nature of the study called for learners to be divided into two groups, the control and treatment groups.

3.4 SAMPLING

Teachers and learners were the main participants in this study. While all mathematics teachers in the two schools were requested to complete the questionnaire, only grade 7 mathematics teachers had their lessons observed. Small, purposefully selected groups of learners were the sample for this study. The purpose was to uncover in-depth information about what happens when learners learn fraction division using practical means. It is argued that "Qualitative inquiry typically focuses on relatively small samples, even single cases ($N=1$)....selected

purposefully to permit inquiry into and understanding of a phenomenon in *depth*” (Patton, 2002, p.46). This sampling strategy was used to form the two groups required by the experimental nature of the study. With average classes of more or less 60 learners, the experimental group from the first school consisted of 30 learners, while 33 learners constituted the control group. In the second school the experimental group consisted of 38 learners while 36 learners made up the control group. Learners in each of the groups were evenly spread in relation to levels of performance (i.e. above average, average and below average). Performance levels were decided on the basis of learners’ marks from previous summative assessment (tests, written work, assignments and projects - continuous assessment). Subject teachers made these available.

3.5 RESEARCH INSTRUMENTS

In the quest for answers to the first two research questions, the study used the research instruments:

- a) Questionnaires (for teachers), and
- b) Observation of lessons.

The remaining research questions, 3 and 4, were directly related to the learning of fraction division and the learner. To find answers to them, the study required the use of research methods that directly involved learners. Those methods were:

- c) Experimentation, and
- d) Group interviews.

3.5.1 Questionnaires

Questionnaires (Appendix A) were administered to teachers to find out their perceptions on practical work and fraction division. This called for the inclusion of

questionnaire items directly linked to perceptions of practical work by teachers. Although the questionnaire mostly consisted of closed items, eight items allowed for open-ended responses for teachers to express their opinions. This immediately rendered the questionnaire less structured. However, its use was justified by the suggestion that “If a site-specific case study is required, then qualitative, less structured, word-based and open-ended questionnaires may be more appropriate as they can capture the specificity of a particular situation” (Cohen *et al.*, 2002, p.247). This particular questionnaire, however, tried to find a balance between a highly structured questionnaire (with closed items only) and an unstructured questionnaire (open-ended items) to serve the purpose of the study, i.e. finding in-depth information about the effectiveness of practical work in the learning of fraction division. The questionnaire was designed, piloted and refined before the actual fieldwork. Inclusion of open-ended items was the product of these efforts. Teachers were given a week to complete the questionnaire. In line with ethical requirements, terms and conditions for their participation were fully explained to them in the company of the procedural letter of consent, which they signed and returned with completed questionnaires.

3.5.2 Observation

Teachers were observed teaching division of fractions to test assumptions on their practices. To capture unfolding events in depth, a semi-structured type of observation was deemed as suitable. According to Cohen *et al* (2000), a semi-structured observation has an agenda of issues of interest but gathers data in a far less pre-determined and systematic manner. This semi-structured character of the observation suited the qualitative nature of this study. The role of the researcher was made clear to the teacher and his learners before the observation of lessons. At the

initial stage of the research (before the experimental stage) the most appropriate role of an observer was observer-as-participant. A definition of this role states that “The ‘observer-as-participant’, like the participant-as-observer, is known as a researcher to the group, and maybe has less extensive contact with the group” (Cohen, Manion & Morris, 2000, p.310). Such a role allowed for the capture of events as they unfolded, with a special focus on what teachers did in relation to their practices assumed in the motivation. This implied that while the observer had specific issues of interest, the observation process itself was open to events as they unfolded.

3.5.3 The Experiment

Finding reliable data on the effectiveness of practical work in fraction division required engagement of learners with practical activities to determine their effect on learners’ learning. It entailed determining the difference in the understanding of fraction division between those learners who had been exposed to the use of practical work and those who had not. This required the study to take on an experimental shape at that particular stage. The most appropriate kind of experiment for this study was the quasi-experimental design, regarded as the closest compromise of the true experimental design. It is acknowledged that “...often in educational research, it is simply not possible for investigators to undertake true experiments” (Cohen, Manion & Morris, 2000, p.214). However, the salient and fundamental characteristics of an experimental study are still prominent in the quasi-experimental study. They include: (a) the experimental and control groups, (b) the treatment that the experimental group is exposed to (practical activities), (c) the pre-test and post-test that both groups undertake before and after the treatment to determine the difference it has made in the experimental group. The formation of the two groups was discussed under sampling. The treatment was the use of practical work in the division of

fractions. This came in the form of worksheets with exercises on the practical division of fractions, which the treatment group had to work on. Exercises included the use of a drawing ruler and sets of bottle-tops. Both groups wrote a pre-test and post-test (see Appendices C and D) before and after the treatment to measure levels of performance. These tests were set in accordance with the requirements of testing for research purposes. Since they were designed for a specific group, these tests of a non-parametric nature (Cohen *et al.*, 2000) were designed by the researcher. The tests focused on three basic categories:

- a) Fraction identification (multiple choice items)
- b) Fraction representation (shading appropriate fraction parts), and
- c) Division of fractions.

With the subject domain clearly prescribed, the tests were domain-referenced. A domain-referenced test was defined by Cohen *et al* (2000) as one where: (a) considerable significance is accorded to meticulous specification of content to be assessed, and (b) the domain is the particular field or area of the subject that is being tested. By virtue of seeking to establish whether learners could carry out the tasks listed above, the tests required learners to meet certain criteria and were therefore criterion-referenced. Bentley & Malvern (1983) defined criterion-referencing as testing where performance is measured against a description or model and judged as worthy or otherwise by how it matches that description. With learners expected to match specific response standards and models, both tests fitted into this category of testing. The normal duration for each test was 30 minutes to any average learner. But for purposes of removing time constraints as possible impediments to a true reflection of learners' abilities, learners were given 1 hour to write each test.

3.5.4 Interviews

After exposing learners from the experimental group to division of fractions by practical means, and evaluating each group's performance, the investigation focused on the views of learners about their experiences with practical work. Interviewees were drawn from the experimental group that had experience with the practical division of fractions. The interview was a group interview. The size of the group was six members drawn across the spectrum of performance levels. Included in the group were below average, average and above average learners. Post-test performance was used as the basis for the selection of interviewees. Areas that the interview focused on included the benefits and challenges, if any, that learners experienced with the practical division of fractions. The interview sought to find out learners' preferences between the two modes of fraction division that they had been exposed to i.e. the ruler (part whole) and bottle-tops (subsets). General comments by learners were also catered for.

3.6 TIME FRAMEWORK

After negotiations with the principals of the two schools in which the study was conducted, access was granted for periods before and after 2004 winter school holidays in schools A and B respectively. Each school granted at least 3 to 4 weeks to conduct the study. School A cited pending half-yearly exams for the request and school B the need to finish the syllabus as basis for a similar request. Table 3.1 gives the time frame it took to conduct and complete the study in each of the two schools. Due to the study's developmental needs that emerged with the progress of the study in school A, two additional worksheets were given to learners in school B. This meant that learners from school B ended up writing 12 worksheets.

Table 3.1: Study's Time Framework for Schools A and B

Activities	SCHOOL A	SCHOOL B
1) Observation 1	01-06-04	04-08-04
2) Observation 2	06-06-04	06-08-04
3) Questionnaires	01-06-04 to 08-06-04	04-08-04 to 13-08-04
4) Pre-test	08-06-04	11-08-04
5) Worksheets 1 & 2	10-06-04	13-08-04
6) Worksheets 3 & 4	14-06-04	18-08-04
7) Worksheet 5	15-06-04	19-08-04
8) Worksheets 6 & 7	17-06-04	23-08-04
9) Worksheets 8 & 9	21-06-04	24-08-04
10) Worksheet 10	22-06-04	25-08-04
11) Worksheet 11 & 12		27-08-04
12) Post-test	24-06-04	30-08-04

3.7 ANALYSIS

The questionnaires, observations, experiment worksheets, tests and interviews generated a wealth of data on a variety of aspects of practical work in the division of fractions. Although the study was primarily concerned with those issues that were explicitly stated in the motivation and research questions, its naturalistic nature and open-endedness of many of the research instruments used generated a substantial amount of data that was initially unexpected. It is argued that "Qualitative inquiry is particularly oriented toward exploration, discovery, and inductive logic" (Patton, 2002, p.55). Anticipation of this variety of data called for preparedness to organise it into categories of uniform patterns from which conclusions could be drawn. Patton (2002) defined such an approach as inductive analysis. It is argued that "The strategy of inductive designs is to balance the important analysis dimensions to emerge from patterns found in cases under study without presupposing in advance what the important dimensions will be" (Patton, 2002, p.56). The study intended to adopt an inductive approach to the analysis of data to make sense out of it.

CHAPTER 4

RESULTS AND ANALYSIS

4.1 INTRODUCTION

Chapter 4 looks at data that was generated by research instruments discussed in chapter 3. Data has been analysed in a manner that leads to general conclusions in relation to research questions asked in chapter 1. These general conclusions lead to assertions by the researcher, which serve as general answers to relevant research questions. The results and analysis are documented under the following headings: (1) Theory versus practice, (2) Factors behind teachers' views, (3) Strength of practical work in fraction division, (4) Learners' views, (5) Limitations of the study, and (6) Summary.

4.2 THEORY VERSUS PRACTICE

This section reports on teachers' perceptions of practical work in the teaching of fractions. To measure opinion against practice, data from questionnaire responses and observed lessons was compared. This data helped answer research question 1, namely: *What are the perceptions of teachers on practical work and the teaching of fractions in relation to their practices?* Teachers' perceptions are reported under the following sub-headings: (1) Teacher perceptions from questionnaire, (2) Teacher practices from observed lessons, and (3) Teachers' plea for help.

4.2.1 Teacher perceptions from questionnaire

These are documented under the following sub-headings: (1) Is there a place for practical work on fractions? (2) Teacher claims on their practices, (3) Are teachers

adequately skilled for practical work?

4.2.1.1 Is there a place for practical work on fractions?

Four respondents answered the questionnaire (see Appendix A). Data from questionnaires indicated that teachers attached a strong value to the role of practical work in teaching fractions and fraction division. All four respondents agreed that fractions offer enough opportunities for the learning of mathematics through practical means. The most preferred materials in teaching the division of fractions were: (a) groups of objects – sets, (b) pictures/diagrams, and (c) worksheets. Two respondents preferred each of these materials. Paper-folding and the graded ruler were each preferred by only one respondent. All four respondents strongly agreed that practical work has a place in the teaching of fractions.

4.2.1.2 Teacher claims on their practices

While one respondent claimed to always include practical work in his lessons (including fractions), one said he does it often and the remaining two said they only did it sometimes. All respondents indicated they would definitely recommend the use of practical work in the teaching of fractions. Respondents gave different reasons for their preferences. The graded ruler, groups of similar objects (sets) and paper-folding were preferred because of their easy accessibility by learners. Sets and pictures/diagrams were chosen for their ease of use by learners. These teachers considered worksheets to be easy for learners to understand and answer. Other favourites were the number line (one respondent) and physical objects that learners can handle (three respondents). Respondents gave different reasons why practical work seldom features in most teachers' lessons. Two respondents claimed it (practical work) is time consuming – both during preparation and actual teaching. Another respondent cited lack of passion for the subject as a factor. Lack of

resources and proper training was suggested by one respondent. One respondent blamed overcrowded classrooms as another factor behind omission of practical activities from lessons.

4.2.1.3 Are teachers adequately skilled for practical work?

All four respondents claimed to have received formal pre-service training in practical work and the teaching of mathematics in general. Except one respondent, all others agreed to materials development having been part of their pre-service training in practical work. The same respondent denied having ever received any form of training in the use of practical work for teaching fractions in particular. Two of the four respondents acknowledged having previously attended in-service courses on practical work in the teaching of fractions. The other two denied having had any such opportunities.

4.2.2 Teacher practices from observed lessons

These are documented under the following sub-headings: (1) A comedy of errors or just plain rote, (2) Real practices against teachers' claims.

4.2.2.1 A comedy of errors or just plain rote

In school A, the teacher's approach to the teaching of fraction division embraced the use of practical work, although he did most of the work himself and did not allow learners enough opportunities to explore practical work to find solutions to given problems. Lubienski (1999) called this approach, where solutions are demonstrated for learners, teaching mathematics for problem solving where learners learn key ideas and skills that they can later apply in problem situations. Also, the teacher's final solutions contained errors, or the example used did not relate to division of fractions, which was the intended outcome of the lesson. After giving two definitions

of division i.e. sharing and grouping, the teacher wrote a fraction division problem $2 \div \frac{1}{3}$ on the board and demonstrated the solution. The out-of-context problem was not related to any real life situation. Only later did the teacher attempt to contextualize the problem, equating 2 to two cakes divided by $\frac{1}{3}$, although there was no explanation of what $\frac{1}{3}$ might represent. The following is an illustration of the teacher's solution:

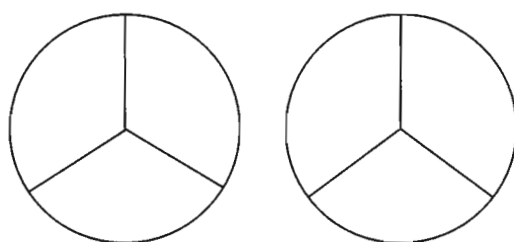


Figure 4.1: Teacher's Circle Solution of $2 \div \frac{1}{3}$

After depicting his solution, the teacher then asked learners how many pieces of $\frac{1}{3}$ were found in the 2 circles representing his two cakes. Learners correctly responded with 6. Erroneously, the teacher concluded and then wrote $\frac{6}{3} = 2$. This is equivalent to $6 \times \frac{1}{3} = 2$. The correct solution to the given problem would have been $2 \div \frac{1}{3} = 6$. The following is an illustration of how the teacher used the number line as an alternative approach to the solution:

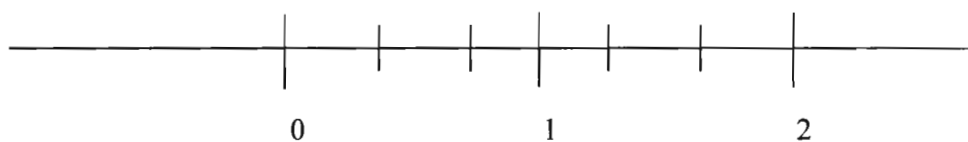


Figure 4.2: Teacher's Number Line Solution of $2 \div \frac{1}{3}$

Again the teacher erroneously concluded that the final solution was $\frac{6}{3}=2$. In his two attempts at the solution, the teacher never explained how his final solution was related to the original problem. As his last example, the teacher demonstrated the solution to the problem ‘find $\frac{1}{5}$ of $\frac{1}{2}$.’ The following is an illustration of the teacher’s solution:

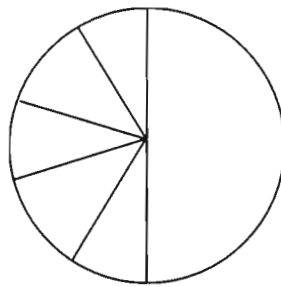


Figure 4.3: Teacher’s Circle solution to $\frac{1}{5}$ of $\frac{1}{2}$

After asking learners a number of leading questions, conclusion was finally reached that there are 10 fractions of $\frac{1}{5}$ in the two $\frac{1}{2}$ ’s, each of which is $\frac{1}{10}$ of the entire circle. Hence the conclusion that $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$. This is not an example of a fraction division problem and was thus irrelevant to the intended outcome of the lesson. The only visible involvement of learners during the lesson was their responses to teacher’s questions which probed desired cues towards final solution. As class work (to be done in groups), the teacher asked learners to find a solution to $2 \div \frac{2}{3}$. The following is an illustration of a presentation by one of the two groups (among six) that found time to offer their solution:

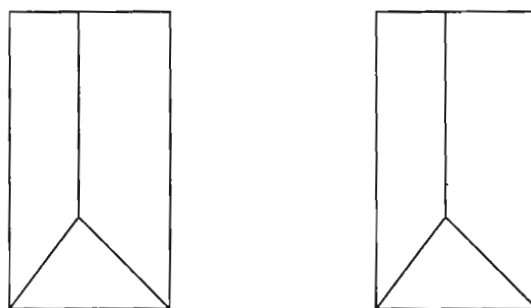


Figure 4.4: A group's solution of $2 \div \frac{2}{3}$

This was a manifestation of Witherspoon's (1993) problematic area of the geometry of unmarked region models. Also the influence of the teacher's erroneous conclusion of fraction division problems was reflected in the group's final conclusion that the sum of six $\frac{1}{3}$'s i.e. $\frac{6}{3}$, is the final answer. The other group also offered the same answer as their solution, although they never got the opportunity to demonstrate their solution. In school B, the lesson on fraction division focussed on revision of terminology and application of the algorithm, the origins of which learners were never assisted to understand, nor did they play any part in developing. To demonstrate application of the division algorithm, the teacher wrote the following out-of-context problems on the board: (1) $6 \div \frac{1}{2}$, (2) $4 \div \frac{1}{2}$, (3) $\frac{2}{3} \div \frac{1}{6}$, (4) $2\frac{1}{2} \div 5$, (5) $1\frac{1}{2} \div \frac{1}{4}$. Using $\frac{1}{2}$ as a referent, the teacher revised the definitions of: (a) numerator and (b) denominator. To revise reciprocals, the teacher asked learners to give reciprocals of $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{5}{6}$, for which he wrote $\frac{1}{2} = \frac{2}{1}$, $\frac{3}{4} = \frac{4}{3}$ and $\frac{5}{6} = \frac{6}{5}$ on the board. Although this expression of learners' oral responses may be understandable and perhaps acceptable within the context of giving reciprocals, the language of the symbols used suggests a different, incorrect and misleading story. In demonstrating

the solution to problem (1), the teacher suggested awareness on his part of learners' prior knowledge of the fraction division algorithm, for he opened with the statement:

'We all know that when we divide with a fraction we change the divisor into its reciprocal and multiply the dividend with the reciprocal instead of dividing with the original fraction.'

Through leading questions, the teacher demonstrated the application of the division algorithm to the solution of the problem. The following are some of his questions (and accompanying chorus responses by learners):

- a) What is the reciprocal of $\frac{1}{2}$? Response: $\frac{2}{1}$.
- b) What is 6×2 ? (After writing $6 \times \frac{2}{1}$) Response: 12.
- c) Therefore what is $6 \div \frac{1}{2}$? Response: 12.

When learners demonstrated solutions to subsequent problems, emphasis was also on reciprocals and accuracy in multiplication. The next lesson mainly dealt with the division of mixed numbers. Again, the approach used leading questions to solicit desired responses from learners to progress to the final solution. When the teacher requested learners to volunteer demonstrations of solutions to subsequent problems on the board, learners also focused in accurate reciprocals, conversion from mixed numbers and correct products. All these distinctive features of rote learning evident in this teacher's lessons are reminiscent of Siebert's (2002) parallels between operations involving fractions and seemingly nonsensical algorithms.

4.2.2.2 Real practices against teachers' claims

Evidence from real practices observed in both schools partly supported some of the claims made in responses to some questionnaire items, but also revealed serious and

interesting contradictions. The same data also opened the eye to crucial realities to which critical attention needed to be paid. While the teacher from school A displayed a degree of commitment to use of practical work in fraction division problems, the value of his efforts was seriously compromised by the erroneous conclusions he always arrived at. The researcher calls it a degree of commitment because use of practical work was only confined to teacher's demonstration of solutions. Learners were not assisted in discovering and mastering, on their own, practical skills useful in conceptual understanding of fraction division. However, data from this observation confirmed a number of claims made in the questionnaire. The teacher had claimed to often include practical work in his lessons and he used it in his demonstrations. The number line and pictures/diagrams, which he used in his demonstrations, were included among his preferred aids in the teaching of fraction division. Others were sets, the ruler, worksheets and physical objects that learners could handle. The restriction of the aids used to the number line and diagrams, when his range of preferences had been so wide, could perhaps be associated with his response to item 8 in part 3 of the questionnaire (see Appendix A): *Lack of resources and training*. The erroneous conclusions reached by the same teacher in his solution of fraction division problems were also cause for concern. Although he had agreed to having received training in using practical work in mathematics (including materials development), he denied ever attending an in-service course on practical work in the teaching of fractions.

Practices observed from the teacher in school B contradicted all claims made in the questionnaire. The teacher claimed having received pre-service and in-service training on practical work in the teaching of fractions. The ruler, sets,

pictures/diagrams and physical objects learners could handle were among his preferred materials. Easy accessibility was why he preferred most materials. Yet in spite of all these positive responses in favour of practical work, only evidence of rote learning of the algorithm by learners emerged from his lessons on fraction division. Perhaps an explanation for all these contradictions is summed up in his response to item 8 in part 3 of the questionnaire (see Appendix A): *They are time consuming. They refer to practical activities.*

4.2.3 Teachers' plea for help

Three respondents did not think that preparing and obtaining materials for practical work was either a long and tiring process or a difficult exercise. Neither did this trio think that obtaining materials was an expensive engagement. The other respondent differed. All respondents were unanimous that engaging learners in practical activities fitted well with OBE requirements for a learner-centred approach and therefore felt OBE workshops in mathematics education should put more emphasis on practical work. All respondents wished to see more practical work workshops on the teaching of fractions. Responses to part 5 item 7 of Appendix A were:

- a) *Development of materials because educators think it's expensive to find materials for practical work and it wastes a lot of time.*
 - Teacher observed in school A.
- b) *The workshop on practical work and teaching of fractions must include development of materials, easily accessible materials, learner activities, teacher's role during the lesson, assessment of practical work and lesson preparation to equip us (educators) with new developments.*
 - Teacher observed in school B.

This perhaps sums up the whole spectrum of developmental needs for a teacher whose lesson on fraction division begins and ends with memorization of the algorithm.

c) *Teachers need to be developed all the time since there are new things each day. Teachers should be developed on how to be innovative, competitive and also be life-long learners because they acquire new skills.*

- Another respondent from school A.

The emphasis on developing teachers to be innovative and to be life-long learners supports some of the values that the new OBE dispensation intends to inculcate in the new breed of teachers that it envisages. It also encapsulates the motive for the common desire in all respondents for OBE workshops in mathematics to put special emphasis on practical work. Perhaps, if these workshops were to evoke in teachers qualities of innovation and being life-long learners, teachers would cease to think that *it's expensive to find material for practical work* (response a) above). Such workshops would perhaps also go a long way in *equipping us (educators) with new developments* (response b) above).

ASSERTION I

Although teachers embrace the use of practical work in the teaching of fractions, they still need professional assistance to turn their positive disposition to practical work into practice.

4.3 FACTORS BEHIND TEACHERS' VIEWS

Favourable disposition towards practical work expressed in sub-section 4.2.1.1 was informed by specific beliefs respondents held about practical work in the teaching of mathematics in general, and fractions in particular. These beliefs were solicited by

items in part 4 of the questionnaire (see Appendix A). This section looks at the factors behind the views of teachers on practical work in the teaching of fractions as discussed in section 4.2. These factors are documented under the following sub-headings: (1) Understanding mathematical concepts, and (2) External factors. Data from this section helped answer research question 2, namely: *What are the factors behind these perceptions? These perceptions* refer to teachers' perceptions on practical work and the teaching of fractions with respect to their practices (see research question 1).

4.3.1 Understanding mathematical concepts

All respondents strongly agreed that the main objective of any teaching session should be the understanding of mathematical concepts by learners rather than completion of the syllabus. In a related reinforcement item (see Appendix A, part 4 item 5), one respondent felt that completion of the syllabus was equally important. This was the same teacher whose lessons did not include any practical activity. All respondents: (a) disagreed that learning activities that require learners to engage in practical activities are a waste of valuable teaching time, (b) agreed that practical work fitted well with OBE requirements for a learner-centred approach, (c) acknowledged the contribution of practical work to better understanding of fractions by learners, and (d) agreed that learners can learn fractions better by handling physical objects (three of them strongly agreed). However, their observed practices proved contradictory. These practices were discussed in sub-section 4.2.2. Although the teacher observed in school A showed a measure of commitment to practical work, his approach afforded learners little opportunity to explore practical work for their own benefit in the acquisition of concepts involved in fraction division.

Complete devotion to rote learning by the teacher from school B was also discussed. All these practices showed little or no evidence of espousing OBE's principles of: (a) participation and ownership, and (b) learner-oriented approach.

4.3.2 External factors

External factors included: (1) large numbers in classes, (2) pressure to finish the syllabus and (3) training in practical work.

4.3.2.1 Large numbers in classes

One of the responses (from school A) to item 8 in part 3 of the questionnaire (see Appendix A) was: *Huge numbers in the classroom to work with*. In school A the study was conducted with a class of 63 learners. This was before the group was split into the control group (33 learners) and experimental group (30 learners). The original size of the class confirmed the above claim by the respondent.

4.3.2.2 Pressure to finish the syllabus

A response to item 5 part 4 (see Appendix A) indicated that finishing the syllabus and understanding of mathematical concepts by learners, should both be objectives when teaching fractions. The respondent was the teacher from school B whose observed lessons did not feature any practical work. This was the same teacher who thought that *they (practical activities) were time consuming*.

4.3.2.3 Training in practical work

Except for one respondent, it can be safely concluded that all respondents had some training in the use of practical work in the teaching of fractions. This should be a strong factor behind the participants' favourable disposition to practical work (in theory). This is despite evidence to the contrary in teachers' observed practices.

ASSERTION 2

Factors behind teachers' perceptions of practical work in the teaching of fractions can be infrastructural, content and process related or associated with teachers' professional training. These should be exploited or addressed to help teachers put their positive ideas into practice.

4.4 STRENGTH OF PRACTICAL WORK IN FRACTION DIVISION

The potential of practical work as an aid in learners' (a) acquisition of concepts on fraction division, and (b) competence in fraction division, was at the centre of this study. An experiment with learners on the efficacy of practical work was discussed in sub-section 3.5.3. Data from these experimental activities constitute this section and is documented under: (1) Practical work and the fraction concept, (2) Practical work in fraction division and (3) Ruler or bottle-tops? A question of expediency. Results of learners' pre-test and post-test performance in both worksheets and tests are tabulated. In tables on worksheet performances, percentages for acceptable responses of the experimental groups of both schools are given for each item. In tables on learners' test performance, percentages of correct achievement levels for control and experimental groups in each school are given for each item. Data generated with these research instruments helped answer research question number 3, namely: *Does the division of fractions by practical means result in better understanding of concepts involved?*

4.4.1 Practical work and the fraction concept

Examination of learners' performance from selected worksheets and from sections of the pre-test and post-test was used to determine the contribution of practical work

towards development of a solid conceptual understanding of the fraction concept. Results are documented under the sub-headings: (1) worksheets based on the ruler, (2) worksheets based on bottle-tops, (3) pre-test and post-test, and (4) conclusions from worksheets and tests.

4.4.1.1 Worksheets based on the Ruler

Table 4.1: Performance of learners based on worksheet 1

	School A	School B
	Acceptable Responses (% age)	Acceptable Responses (% age)
1) 1 whole ruler	100	100
2) $\frac{1}{2}$ of 1 ruler	87	100
3) $\frac{1}{4}$ of 1 ruler	47	37
4) $\frac{3}{4}$ of 1 ruler	10	17
5) $\frac{1}{3}$ of 1 ruler	60	63
6) $\frac{2}{3}$ of 1 ruler	50	14

Table 4.1 shows the performance of learners from both schools for worksheet 1. Learners were required to use a drawing ruler (300mm) to determine given fractions in accordance with the length of the ruler. Table 4.1 indicates learners from school A obtained 60% and 50% acceptable responses for items 5 and 6 respectively. For school B, acceptable responses for the same fractions were 63% and 14% respectively. In school A, acceptable responses for items 3 and 4 were 47% and 10% respectively. In school B figures for these items were 37% and 17% respectively. Those two fractions proved to be the most difficult for learners in the entire study, both during the worksheet and test stages. However, after learners' continued exposure to using a ruler for concrete representations of fractions, their performance

for these fractions showed remarkable improvement in worksheets 4 and 6. Table 4.2 shows learners' performance for worksheet 4.

Table 4.2: Performance of learners based on worksheet 4

	School A Acceptable Responses (% age)	School B Acceptable Responses (% age)
Task 1		
1) $\frac{1}{2}$ the ruler	100	100
2) $\frac{1}{4}$ of 1 ruler	97	100
3) No. of $\frac{1}{4}$'s in $\frac{1}{2}$	86	100
4) $\frac{1}{2} \div \frac{1}{4}$	97	100
Task 2		
1) $\frac{1}{3}$ of 1 ruler	100	100
2) $\frac{2}{3}$ of 1 ruler	100	95
3) No. of $\frac{1}{3}$'s in $\frac{2}{3}$	100	53
4) $\frac{2}{3} \div \frac{1}{3}$	100	53

Together with learners' performance in worksheet 4 (see Table 4.2), the performance of learners for worksheet 6 was compared to learners' performance for worksheet 1 (see Table 4.1) to determine if there was any improvement in learners' ability to give fractions using a ruler. Learners' performance for worksheet 6 is shown in Table 4.3.

Table 4.3 Learners' performance based on worksheet 6

	School A Acceptable Responses (% age)	School B Acceptable Responses (% age)
Task 1		
1) $\frac{1}{2}$ the ruler	100	100
2) $\frac{2}{3}$ of 1 ruler	100	100
3) No. of $\frac{1}{2}$'s in $\frac{2}{3}$ of ruler	83	54
4) Remainder as a fraction of $\frac{1}{2}$	80	66
5) $\frac{2}{3} \div \frac{1}{2}$	27	51
Task 2		
1) $\frac{1}{3}$ of 1 ruler	100	97
2) $\frac{3}{4}$ of 1 ruler	77	66
3) No. of $\frac{1}{3}$'s in $\frac{3}{4}$ of 1 ruler	43	26
4) Remainder as a fraction of $\frac{1}{3}$	03	14
5) $\frac{3}{4} \div \frac{1}{3}$	00	17

In worksheet 4 (see Table 4.2), the performance of learners from school A improved to 100% for $\frac{1}{3}$ and $\frac{2}{3}$ (items 1 & 2 task 2), and that of learners from school B improved to 100% and 95% respectively for the same fractions. Again in worksheet 4 (see Table 4.2), the performance of learners from schools A and B improved to

97% and 100% respectively for $\frac{1}{4}$ (item 2 task 1). In worksheet 6 (see Table 4.3) the performance of learners from schools A and B for the fraction $\frac{3}{4}$ (item 2 task 2) improved to 77% and 66% respectively.

4.4.1.2 Worksheets based on Bottle-tops

Table 4.4 depicts the performance of learners from both schools for worksheet 2. Learners were required to use a total of 12 bottle-tops to give the correct number of bottle-tops that represent given fractions. In this and other subsequent tables on bottle-tops, bts is the abbreviation used for bottle-tops. In worksheet 2 (see Table 4.4), acceptable responses for item 5 were 63% and 71% in schools A and B respectively. For both items 4 and 6, acceptable responses were 17% in school A. In school B acceptable responses for the same items were 17% and 34% respectively. Once again, $\frac{3}{4}$ was the most difficult fraction for learners from both schools, as was the case with the ruler.

Table 4.4: Learners' performance based on worksheet 2

	School A Acceptable Responses (% age)	School B Acceptable Responses (% age)
1) 1 group of 12 bts	100	100
2) $\frac{1}{4}$ of 12	73	100
3) $\frac{1}{2}$ of 12	93	100
4) $\frac{3}{4}$ of 12	17	17
5) $\frac{1}{3}$ of 12	63	71
6) $\frac{2}{3}$ of 12	17	34

Learners' performance for $\frac{1}{3}$ and $\frac{3}{4}$ (see Table 4.4) was compared to their performance for the same fractions in worksheet 12. Table 4.5 shows learners' performance for worksheet 12.

Table 4.5: Learners' performance based on worksheet 12

	School A Acceptable Responses (% age)	School B Acceptable Responses (% age)
Task 1		
1) 1 group of 12 bts	100	97
2) Identifying $\frac{3}{4}$ group of 12 bts	73	100
Task 2		
1) Identifying $\frac{1}{3}$ of 12 bts	100	100
2) No. of $\frac{1}{3}$'s in $\frac{3}{4}$ of 12 bts	83	100
3) Identifying remainder	83	100
4) Remainder as a fraction	83	100
5) $\frac{3}{4} \div \frac{1}{3}$	83	97

In both schools acceptable responses improved to 100% for $\frac{1}{3}$ (item 1 task 2), while those for $\frac{3}{4}$ (item 2 task 1) improved to 73% and 100% for schools A and B respectively. Learners' performance for $\frac{2}{3}$ (see Table 4.4) was compared to their performance for the same fraction in worksheet 11. Table 4.6 shows learners' performance for tasks 2 and 3 of worksheet 11. Acceptable responses for $\frac{2}{3}$ (item 1 task 2) also improved to 73% and 100%, in schools A and B respectively.

Table 4.6: Learners' performance based on worksheet 11: Tasks 2 & 3

	School A Acceptable Responses (% age)	School B Acceptable Responses (% age)
Task 2		
1) Identifying $\frac{2}{3}$ of 12 bts	73	100
2) Identifying $\frac{1}{2}$ of 12 bts	100	100
Task 3		
1) No. of $\frac{1}{2}$'s in $\frac{2}{3}$ of 12 bts	20	100
2) Identifying Remainder	80	94
3) Remainder as a fraction	53	97
4) $\frac{2}{3} \div \frac{1}{2}$	30	94

4.4.1.3 Pre-test and Post-test

Learners' performance in the identification and representation of fractions in the pre-test and post-test was also compared, to determine the contribution of practical work in improving learners' understanding of the fraction concept. Table 4.7 shows the performance of learners from both groups (control and experimental) in the two schools for the identification and representation of fractions in the pre-test. The letters E and NE represent Equivalents and Non-equivalents respectively. The performance of learners from both groups in the two schools was reasonably good in fraction identification, except for item 1.2 in school A and item 1.1 in school B. The control group from school A also struggled with item 1.4. All learners experienced serious difficulties with representation of equivalent subsets (items 2.1 and 2.2).

Learners performed very well in the representation of non-equivalent part-regions (items 2.3 - 2.5).

Table 4.7: Learners' Pre-test performance: fraction identification & representation

	School A		School B	
	Correct Control	Responses (% age) Experimental	Correct Control	Responses (% age) Experimental
1. IDENTIFICATION: Part-regions (NE)				
1.1) $\frac{1}{3}$	76	90	31	53
1.2) $\frac{2}{3}$	30	40	81	63
Subsets (NE)				
1.3) $\frac{1}{4}$	67	77	72	71
1.4) $\frac{1}{2}$	36	60	58	55
1.5) $\frac{3}{4}$	61	87	72	76
2. REPRESENTATION: Subsets (E)				
2.1) $\frac{1}{3}$	15	13	06	03
2.2) $\frac{2}{3}$	12	27	08	03
Part-regions (NE)				
2.3) $\frac{1}{4}$	88	90	92	100
2.4) $\frac{1}{2}$	55	77	86	89
2.5) $\frac{3}{4}$	91	93	94	97

Table 4.8 shows the post-test performance of learners from both groups in the two schools in items similar to those in the pre-test. In school A, although the performance of both groups in fraction identification was generally unsatisfactory, the experimental group scored better in all items – equivalent subsets and equivalent part-regions. In school B, the control group did better in identification of equivalent subsets (items 1.1 and 1.2). With the exception of item 1.4, both groups in school B scored very low in the identification of equivalent part-regions, although again the

scores of the experimental group were higher than those of the control group. Learners from both schools did very well in the representation of non-equivalent part-regions. In school A, while the performance of the control group in the representation of equivalent subsets was remarkably low, that of the experimental group showed significant improvement. In school B, while the performance of both groups was generally poor in the representation of equivalent subsets, performance by the experimental group was better than that by the control group, except for item 2.3.

Table 4.8: Learners' Post-test performance: fraction identification & representation

	School A		School B	
	Correct Responses (% age)		Correct Responses (% age)	
	Control	Experimental	Control	Experimental
1. IDENTIFICATION: Subsets (E)				
1.1) $\frac{1}{3}$	15	34	56	22
1.2) $\frac{2}{3}$	27	55	58	27
Part-regions (E)				
1.3) $\frac{1}{4}$	06	34	25	32
1.4) $\frac{1}{2}$	39	72	89	84
1.5) $\frac{3}{4}$	12	24	11	24
2. REPRESENTATION: Part-regions (NE)				
2.1) $\frac{1}{3}$	91	86	97	95
2.2) $\frac{2}{3}$	97	97	97	97
Subsets (E)				
2.3) $\frac{1}{4}$	15	52	44	38
2.4) $\frac{1}{2}$	24	76	28	35
2.5) $\frac{3}{4}$	06	24	03	32

4.4.1.4 Special cases from pre-test and post-test

A number of special cases in the representation of fractions in the pre and post tests highlighted learners' difficulties in the perception of the subset perspective of the fraction. Firstly, most learners shaded only 3 circles out of a total of 12 in their efforts to show $\frac{3}{4}$. The correct response would have been shading 9 circles instead.

The following figure is a depiction of a learner's response.

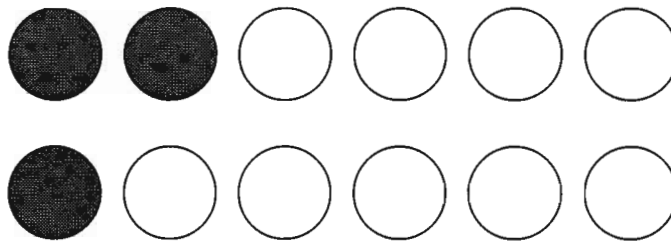


Figure 4.5: Learner's representation of $\frac{3}{4}$ of 12 in post-test

Secondly, another learner produced the following figure to represent $\frac{1}{4}$ when the correct response would have been shading 2 complete circles out of a total of 8.

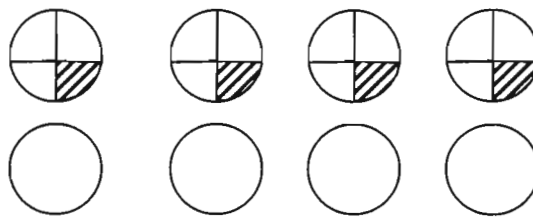


Figure 4.6: Learner's representation of $\frac{1}{4}$ of 8 in post-test

Figure 4.7 shows the same error in the representation of $\frac{3}{4}$. A similar item i.e. representation of the subset perspective of $\frac{3}{4}$, was part of worksheet 12 (see Table

4.5). Perhaps this explains the shading of 3 out of 4 equal parts of each of the 9 circles subdivided in similar fashion. The correct response was to shade 9 complete circles.

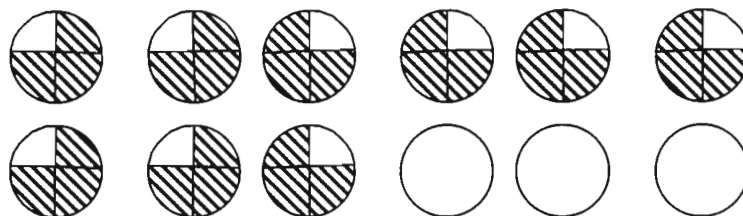


Figure 4.7: Learner's representation of $\frac{3}{4}$ of 12 in post-test

Thirdly and interestingly, a learner shaded 2 out of 3 equal parts in each circle to indicate $\frac{2}{3}$ of 6. This is equivalent to shading 4 complete circles out of a total of 6.

While the shading is correct, there is no doubt that this is a tedious and confusing way of showing $\frac{2}{3}$ of 6. Shading 4 complete circles would be more economic and

easily understandable. It would not be possible to tell how many marbles make $\frac{2}{3}$ of 6 marbles using this approach to practical fraction division. Figure 4.8 shows the learner's efforts.

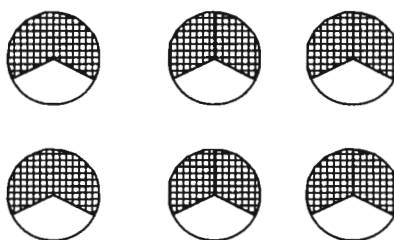


Figure 4.8: Learner's representation of $\frac{2}{3}$ of 6 in pre-test

4.4.1.5 Conclusions from Worksheets and Tests

- 1) Learners' competence in representing fractions improved after exposure to practical work based on ruler and bottle-tops. This includes fractions that initially proved to be very difficult for learners.
- 2) Test performances showed that learners found it difficult to work with equivalents, whether part-regions or subsets. Learners often did very well when working with non-equivalents.
- 3) Learners from the experimental groups in both schools, showed remarkable improvement in post-test items on representation of equivalent subsets after exposure to practical work.
- 4) In most post-test items, even those in which both groups (experimental and control) performed poorly, experimental groups from both schools often did better than their counterparts in the control groups.
- 5) Based on learners' performance in the identification and representation of fractions (worksheets and tests), it can be concluded that practical work contributed to improved understanding of the fraction concept by learners.

4.4.2 PRACTICAL WORK IN DIVISION OF FRACTIONS

Learners' performance in fraction division lessons observed was discussed in sub-section 4.2.2.1. In school A, learners were exposed to a very limited use of practical work with disastrous consequences. In school B, learners were equipped with skills at rote application of the division algorithm and desired results were achieved with remarkable success. This section gives an account of learner performance in fraction division during the experiment (worksheets and tests). Results are documented under the following sub-headings: (1) the introductory exercise, (2) worksheets based on

the ruler – without remainder, (3) worksheets based on the ruler – with remainder, (4) worksheets based on bottle-tops – without remainder, (5) worksheets based on bottle-tops – with remainder, (6) conclusions from worksheets, (7) pre-test and post-test, and (8) conclusions from pre-test and post-test.

4.4.2.1 The introductory exercise

Before learners were given worksheet tasks on the division of fractions, they were given an introductory exercise on the division of whole numbers. The aim was to help them understand the essence of and difference between two meanings of division i.e. measurement and sharing/partitive. Learners were given the following problems to discuss and find solutions to:

- 1) Themba was given 10 tablets as treatment for his flu. The doctor ordered him to always take 2 tablets per day, one in the morning and another at night. How many days did the tablets last Themba?
- 2) Thoko has 10 biscuits that she wants to give to her friends during school break. She has 2 friends, Lucy and Sarah. How many biscuits will each of Thoko's friends get?

All learners gave 5 as their response to both problems. Asked to explain how they got their answers, learners answered that they divided 10 by 2 and got 5 i.e. $10 \div 2 = 5$. A discussion then followed to explain to learners that in problem 1) 5 represented the number of groups of 2 in 10 tablets, and that in problem 2) 5 represented the number of items per group in each of the 2 groups from 10 biscuits. Then the terms measurement and partitive meanings of division were assigned to each of the two situations respectively.

4.4.2.2 Worksheets based on the Ruler – Without remainder

Worksheet 3 (see Appendix B) required learners to use a drawing ruler (300mm) to practically find solutions to non-remainder division problems. Table 4.9 summarises learner performance for worksheet 3. The following observations were made:

- 1) The performance of learners from school A for items 4 in tasks 1 and 2, improved dramatically, from 63% to 97% respectively. In school B, the success rate in the same items remained impressively high at 95% and 97% respectively.
- 2) Performance of learners from school A also showed remarkable improvement compared to their dismal performance in observed lessons.

Table 4.9: Learners' performance based on worksheet 3

	School A	School B
	Acceptable Responses (% age)	Acceptable Responses (% age)
Task 1		
1) 1 whole ruler	100	100
2) $\frac{1}{2}$ the ruler	100	100
3) No. of $\frac{1}{2}$'s in 1 ruler	100	97
4) Solution to $1 \div \frac{1}{2}$	63	95
Task 2		
1) 1 whole ruler	100	76
2) $\frac{1}{4}$ of 1 ruler	100	100
3) No. of $\frac{1}{4}$'s in 1 ruler	73	95
4) $1 \div \frac{1}{4}$	97	97

Performance for worksheet 3 (see Table 4.9) was compared with learners' performance for worksheet 4 (see Table 4.2), which required learners to use a ruler to find solutions to non-remainder division problems. The following observations were made on learners' performance in fraction division for worksheet 4:

- 1) Learners from school A maintained their improved performance in both tasks, scoring 97% and 100% for items 4 of tasks 1 and 2 respectively. In school B, while performance was impressively high in task 1, it dropped dramatically in task 2, i.e. 53% for items 3 and 4. This was due to learners' late arrival for the session because of poor communication of temporal changes to the time-table. This resulted in most learners not completing the second task.
- 2) The reduced gap in achievement levels between items 3 and 4 in both tasks 1 and 2 of worksheet 4 (see Table 4.2) suggested a successful connection by learners of practical manipulation of fractions and finding solutions to division problems.

4.4.2.3 Worksheets based on the Ruler – With remainder

Evaluation of the potential of practical work to assist learners in understanding division of fractions was extended to problems that involved the remainder. Table 4.3 showed learners' performance in worksheet 6 which required learners to use a ruler to find solutions to such problems. The following observations were made on learners' performance:

- 1) For school A the performance of learners in items 3 and 4 of task 1 was very good. However, in item 5 of the same task learners obtained a low of 27%. This suggested failure by learners to combine the remainder and the number

of times a fraction appeared in another to produce the final answer in which the solution included the remainder.

- 2) Working with $\frac{3}{4}$ proved to be exceptionally difficult for learners from both schools. While no learners from school A found the correct solution to $\frac{3}{4} \div \frac{1}{3}$, only 17% gave acceptable responses in school B (see Table 4.3, item 5 task 2).
- 3) Learners' performance in item 5 of both tasks showed that division problems whose solutions involved the remainder were generally difficult for most learners, including learners who were drilled in the application of the algorithm.

4.4.2.4 Worksheets based on Bottle-tops - Without remainder

Table 4.10: Learners' performance based on worksheet 7

	School A Acceptable Responses (% age)	School B Acceptable Responses (% age)
Task 1		
1) Identifying group of 12 bts	100	100
Task 2		
1) Making $\frac{1}{2}$ of 12 bts	100	100
2) Identifying $\frac{1}{2}$ of 12 bts	100	100
3) No. of bts in $\frac{1}{2}$ of 12 bts	100	95
4) No. of $\frac{1}{2}$ of 12 bts	100	95
5) $1 \div \frac{1}{2}$	86	100

Table 4.10 shows learners' performance for worksheet 7 (see Appendix B). Results from this worksheet were compared with those from worksheet 8 (see Table 4.11) to determine learners' competence in using bottle-tops to find solutions to non-remainder fraction division problems. Table 4.11 reflects learners' performance in task 2 of worksheet 8.

Table 4.11: Learners' performance based on task 2 of worksheet 8

	School A	School B
	Acceptable Responses (% age)	Acceptable Responses (% age)
Task 2		
1) Making $\frac{1}{4}$ from $\frac{1}{2}$ of 12 bts	83	92
2) No. of bts per group of $\frac{1}{4}$ of 12 bts	83	92
3) Groups of $\frac{1}{4}$ in $\frac{1}{2}$ of 12 bts	66	95
4) $\frac{1}{2} \div \frac{1}{4}$	62	100

The following observations were made:

- 1) Although both groups (from schools A and B) did impressively well in all items in both worksheets, learners from school B did better, except for items 3 and 4 of task 2 in worksheet 7 (see Table 4.10) .
- 2) The narrow gap in achievement levels in items 4 and 5 in task 2 of worksheet 7 (see Table 4.10), and items 3 and 4 in task 2 of worksheet 8 (see Table 4.11) suggested that learners were able to translate their successful physical manipulation of concrete representations of fractions to acceptable solutions of fraction division problems.

4.4.2.5 Worksheets on Bottle-tops – With remainder

Evaluation of the potential of bottle-tops to enrich learners' understanding of fraction division was extended to problems that involved the remainder. Table 4.12 reflects learners' performance for worksheet 10, their attempt with bottle-tops, at problems which involved the remainder.

Table 4.12: Learners' performance based on tasks 2 & 3 of worksheet 10

	School A	School B
	Acceptable Responses (% age)	Acceptable Responses (% age)
Task 2		
1) Making 3 equal groups from 12 bts	80	53
2) No. of bts Per group	80	53
3) Identifying $\frac{2}{3}$ of 12 bts	63	94
4) Groups of $\frac{2}{3}$ in 12 bts	00	65
Task 3		
1) Isolation of $\frac{2}{3}$ of 12 bts	53	97
2) Identifying remainder	53	97
3) Remainder as fraction of $\frac{2}{3}$	33	21
4) $1 \div \frac{2}{3}$	00	24

The following observations were made:

- 1) Although learners from school B did better than those from school A in most items, they struggled in the first two items of task 2, i.e. making 3 groups from 12 bottle-tops and giving the number of bottle-tops per group. The majority of

learners came late to school and had to serve punishment first before they were allowed to join classes. Consequently they were late for the session and they missed the early stages when instructions were explained. This is the possible reason for learners' poor performance in the first two items.

- 2) Learners from both schools experienced serious difficulties in items 3 and 4 of task 3. These items required learners to express the remainder as a fraction of the divisor and hence give the final solution to $1\div\frac{2}{3}$.
- 3) Although both groups' produced poor performance in item 4 (final solution) of task 3, learners from school B (algorithm group) did better than learners from school A (non-algorithm group).

Performance in worksheet 10 (see Table 4.12) was compared with performance for worksheet 11 (see Table 4.6), to determine if there was any improvement in learners' performance in division problems with the remainder. The following observations were made:

- 1) Again learners from school B did better than learners from school A in all items in worksheet 11.
- 2) For school A there was improvement in the identification of $\frac{2}{3}$, i.e. 63% in worksheet 10 (item 3 task 2) to 73% in worksheet 11 (item 1 task 2).
- 3) While learners from school A were still struggling with items 3 and 4 in task 3 of worksheet 11 (see Table 4.6), their performance for these items showed remarkable improvement from their performance for similar items in the previous worksheet 10 (see Table 4.12). Learners from school B showed big improvement in their performance for the same items compared to their performance in worksheet 10.

- 4) Improvement in the performance of learners from school B (the algorithm group) in items 3 and 4 of task 3 of worksheet 11 was far more significant than that of learners from school A who were not exposed to the algorithm. It was standard practice to discuss learners' performance in the last worksheet before learners worked on the next worksheet. This provided an opportunity for addressing learners' difficulties in the previous worksheet and this probably influenced learners' improved performance in similar tasks in subsequent worksheets. The performance improvement edge of school B learners was most probably due to a combination of their improved understanding of practical fraction division and their heavy reliance on the use of the algorithm.
- 5) In school A, there was a huge discrepancy between items 1 and 2 for task 3 of worksheet 11 (see Table 4.6). These items required learners to give the number of times a fraction appeared in another fraction, and to identify the remainder. While most learners struggled to give the number of times a fraction appeared in another fraction, they were nevertheless successful in identifying the remainder. While the achievement gap in these items was significantly small for learners in school B in worksheet 11, it was remarkably big for learners from school A. With problems experienced by school A learners in these items, more assistance was given to school B learners in the form of individual attention and guidance in similar items. This is the most probable reason for improved performance by school B learners in items in question (see section 4.6).

To determine further the contribution of practical work in assisting (a) learners' understanding of and, (b) competency in fraction division which involve the

remainder, performance in worksheet 6 (see Table 4.3) was compared with learners' performance in worksheet 12 (see Table 4.5). In worksheet 12, learners from school A showed dramatic improvement in their performance in all items compared to their performance in similar items for worksheet 6. Their performance almost matched that of learners from school B which had been better throughout. Performance of school B learners for the same items was also improved significantly. It is also noted that in worksheet 12 learners used bottle-tops while in worksheet 6 they used the ruler. The difference in performance could perhaps be explained by learners' responses to interview questions (see subsections 4.5.1 and 4.5.2).

4.4.2.6 Conclusions from Worksheets

- 1) The dominance in performance of learners from school B (the algorithm group) over learners from school A (the non-algorithm group) was reflected throughout the worksheets, even in items where both groups struggled.
- 2) The gradual improvement of the performance of learners from both schools as they progressed through worksheets indicated the contribution of practical work towards enhancement of learners' understanding of the concepts of the fraction and fraction division. Further, this was evidence of improving learner competence in the division of fractions by practical methods.
- 3) While learners from school A performed dismally in division problems in the lessons observed, the same learners became increasingly competent in finding solutions to division problems through engaging in practical activities as they progressed with worksheets. Again, this improvement of the performance of learners

from school A confirmed further the role of practical work in enhancing learners' competence in the division of fractions.

- 4) Division problems whose solutions involved the remainder generally presented learners with serious difficulties. This was the case with learners from both schools (A and B).

4.4.2.7 Pre-test and Post-test

Learners wrote the pre-test and post-test to determine the effectiveness of practical work in assisting learners in the understanding of and competence in fraction division. To this end, only division sections of the tests are reported on in this subsection. Table 4.13 shows the performance of learners from both schools in items on fraction division in the pre-test.

Table 4.13: Learners' performance based on fraction division in Pre-test

	School A		School B	
	Correct Responses (% age)	Control	Correct Responses (% age)	Control
3) DIVISION: Without remainder				
3.1) $1 \div \frac{1}{3}$	12	13	78	68
3.2) $\frac{2}{3} \div \frac{1}{3}$	00	00	75	63
3.3) $\frac{3}{4} \div \frac{1}{4}$	00	13	72	61
With remainder				
3.4) $1 \div \frac{2}{3}$	00	00	44	37
3.5) $\frac{3}{4} \div \frac{1}{2}$	03	00	56	45

The following observations were made from learners' performance in the pre-test:

- 1) Learners' performance in school A was very poor in all division items.

These learners were not exposed to the algorithm. Their teacher gave them a

glimpse of how to use diagrams to find solutions to fraction division problems. These inadequate efforts and their disastrous consequences for learners were discussed in subsection 4.2.2.1.

- 2) Learners from school B did well in fraction division, especially in problems where the solution did not involve the remainder. This group was drilled by their teacher in the use of the algorithm in the solution of fraction division problems.
- 3) Problems that involved the remainder proved to be more difficult than those that did not, even for better performing learners from school B.

Table 4.14 shows the performance of learners from both schools in the post-test. The following observations were made from the post-test:

- 1) The performance of both groups from school B continued to be significantly better than that of learners from school A. For both groups from school B there was also no substantial difference in performance between pre-test and post-test. Exposure of the experimental group to practical work did not seem to have made any significant impact on their competence in fraction division.
- 2) The performance of the experimental group from school A improved significantly while that of the control group remained at 0% for all items. Learners appeared to have benefited from exposure to practical work. A sizeable number of learners could now find solutions to certain division problems.
- 3) Division problems whose solutions involved the remainder continued to pose difficulties for learners from both schools. While learners from school B continued to do better than learners from school A, their performance in remainder problems was very low compared to performance in non-

remainder problems. In spite of dramatic performance improvement in non-remainder problems, the performance of the experimental group from school A showed almost no improvement in remainder type problems.

Table 4.14: Learners' performance based on fraction division in Post-test

	School A		School B	
	Correct Responses (% age)		Correct Responses (% age)	
	Control	Experimental	Control	Experimental
3. DIVISION:				
Without remainder				
3.1) $1 \div \frac{1}{2}$	00	48	75	62
3.2) $1 \div \frac{1}{4}$	00	45	75	54
3.3) $\frac{1}{2} \div \frac{1}{4}$	00	41	64	46
3.4) $\frac{2}{3} \div \frac{1}{3}$	00	17	83	57
With remainder				
3.5) $\frac{2}{3} \div \frac{1}{2}$	00	07	39	38
3.6) $\frac{3}{4} \div \frac{1}{3}$	00	00	33	46

4.4.2.8 Conclusions from Pre-test and Post-test

The following conclusions were made from learners' pre-test and post-test performances:

- 1) Engagement in practical activities made a positive contribution in better understanding of and competence in the division of fractions by learners. This is supported by improvements in the performance of learners from the experimental group (in school A) to fraction division problems.
- 2) Extensive drill in the use of the algorithm helped learners to arrive at correct solutions in most fraction division problems. However, these successes cannot be regarded as evidence of understanding conceptual processes involved in

fractions and fraction division. This assertion is supported by the poor performance of learners from school B in some items in the previous sections of both tests i.e. identification and representation of fractions. Performance in these sections was discussed in subsection 4.4.1.

- 3) Practical activity that comes after the algorithm does not help to enhance learners' understanding of concepts of the fraction and fraction division. Such activities should precede introduction to the algorithm for them to add any value to learners' better understanding of the division of fractions.

4.4.3 RULER OR BOTTLE –TOPS? A QUESTION OF EXPEDIENCY

The use of the ruler and bottle-tops as concrete embodiments of fractions in fraction division represented two perspectives of the fraction concept: (a) the ruler part-region perspective, and (b) bottle-tops subset perspective. Comparison of learners' performance using either of the practical aids became necessary to determine which of the two instruments was more efficacious and expedient in fraction division. Table 4.15 compares results with the ruler against results using bottle-tops. It shows the performance of learners in both schools in selected division problems in some worksheets. W is the abbreviation for worksheet, and is accompanied by a number to indicate the worksheet the problem is taken from e.g. W3 for worksheet 3.

The table indicates that the highest scores were obtained when learners used bottle-tops, except in 2) where learners from school A scored higher with a ruler. Therefore, it can be concluded that generally learners were more successful with bottle-tops than they were with the ruler when dividing fractions. Learners further expressed their preference for bottle-tops over the ruler in interviews. During

interviews, learners furnished reasons why they preferred bottle-tops over the ruler.

These are discussed in subsection 4.5.2.

Table 4.15: Comparison of learners' performance: ruler versus bottle-tops

Division problem	Correct Responses (% age): ruler			Correct Responses (% age): bottle-tops		
	Worksheet	School A	School B	Worksheet	School A	School B
1) $1 \div \frac{1}{2}$	W3	63	95	W7	86	100
2) $\frac{1}{2} \div \frac{1}{4}$	W4	97	100	W8	62	100
3) $\frac{2}{3} \div \frac{1}{2}$	W6	27	51	W11	30	94
4) $\frac{3}{4} \div \frac{1}{3}$	W6	00	17	W12	83	97

ASSERTION 3

Engaging learners in practical work helps them in the meaningful acquisition and understanding of the concepts of fractions and fraction division. Learners are better able to divide fractions with the help of practical work.

4.5 LEARNERS' VIEWS

Learners from the experimental groups of the two schools were interviewed to establish their views on specific aspects of their experiences with practical work in fraction division. Their views are documented under the following sub-headings: (1) attitudes towards practical work (2) use of ruler and (3) challenges in practical fraction division. Learners' responses to interview questions (see Appendix E) helped answer research question number 4, namely: *What are the views of learners about the use of practical work in the division of fractions?*

4.5.1 ATTITUDES TOWARDS PRACTICAL WORK

Question 6 (see Appendix E) sought to determine if learners had any particular method of fraction division that they preferred. Learners were offered a choice

between practical methods (the ruler, bottle-tops or diagrams) and the algorithm (referred to as the rule). The undeclared intention was to determine the learners' attitude towards practical work. The responses of learners from school A provided evidence that learners preferred to use practical methods to divide fractions. It is also noted that learners from school A had not been exposed to the algorithm (at least in the lessons that were observed). Therefore they had no recent experience of fraction division other than practical activities that their teacher had given them a glimpse of, and those that they had undergone during the experiment. It is also worth noting that learners would normally have encountered the algorithm for the division of fractions prior to grade 7. Evidence of this was found in test responses to some fraction division problems when some learners invoked the use of the algorithm, albeit with little success as there had been no recent revision of the algorithm prior to writing of tests. Learners also gave reasons for their preferences. Asked why she preferred to use diagrams, Phumla who had said that she preferred drawings, gave the following response:

Phumla : I can see my problem when I'm ... When I'm using drawings,
it's easy for me to see my problem.

For this learner diagrams carried the benefit of visual effect. She could see the fractions that she was required to divide, which made the problem easy for her. Her reasons were echoed by another learner who had also expressed a preference for drawings. The following conversation transpired with Phungula:

Phungula: Sir, yingoba uyakwazi ukuthi usho ukuthi ilo... ama-fractions
onikezwe wona siwafake kwi-drawing. Uyakwazi ukuwadivayda

uwabona (it's because drawings can help you to represent the fractions. This helps you to see the fractions as you divide them).

Interviewer: It's easy for you to see your problem when you've represented it with a drawing, eh?

Phungula & Phumla: Yes sir!

For Phungula and Phumla the advantage of using diagrams was clearly articulated. Diagrams enabled them to see the fractions that they wanted to divide, and hence made it easy for them to divide the fractions.

Reasons for using bottle-tops in the division of fractions were well articulated in the response by S'nenhlanhla. Her views were shared by other learners who had also opted for bottle-tops. Asked why she preferred to use bottle-tops, S'nenhlanhla gave the following response:

S'nenhlanhla: Because sir, bottle-tops you can divide them into groups.

Interviewer: *(Confirming his understanding of what S'nenhlanhla means by divide)* You can separate them into groups and you can move them around, eh?

All learners: Yes sir!

Interviewer: And you, Mbali?

Mbali: I also agree.

Interviewer: Mandisa?

Mandisa: Please sir, ngisho okushiwo uS'nenhlanhla (I agree with S'nenhlanhla).

For these learners bottle-tops were easy to use because of the ease with which they could be moved around. The resultant ability to group them into required fractions further endeared bottle-tops to these learners.

The attitude of learners towards practical work was categorically declared in responses by most learners from school B. These learners were exposed to both the algorithm (in lessons observed) and practical work (during the experiment) approaches. It is assumed that their views were informed by their experiences with both these approaches. The following is an extract from a conversation with a learner from one of the groups interviewed in school B, with other learners providing chorus support for his views (three groups representing below average, average and above average learners were interviewed in school B):

Interviewer: Which of these instruments make it easier for you to understand... and do *(interrupted by learners as they give their response before the question is completed)*

All learners: *(Interrupting the interviewer in unison)* bottle-tops.

Interviewer: Reason? *(Silence)* Why? *(Further silence)* So you say it's easier to divide fractions by using bottle-tops, eh?

All learners: Yes sir!

Interviewer: Can you give me a reason? *(Silence)* or maybe, is it the same reason as the one that you gave before?*

All learners: Yes sir!

Interviewer: Why?

Sabelo: Because sir, the answer you get easy when you are using bottle-tops.

Interviewer: Right! You get the answer easily with bottle-tops. What makes it easy to get the answer when you use bottle-tops?

Sabelo: *(Probably using divide in the same context as before)** *
because sir, when you need a half you can divide in the bottle-tops.

Interviewer: *(Confirming his understanding of learner's use of the term divide)* you can actually separate the fractions from each other, eh?

All learners: Yes sir!

Interviewer: So you all agree that, again, it was easy to divide fractions by using bottle-tops because bottle-tops can be moved around?

All learners: Yes sir!

*Questions 1 and 3 had asked learners which between the ruler and bottle-tops made it easy for them to represent and divide fractions. Learners had expressed a preference for bottle-tops and the main reason for their choice had been the ease with which bottle-tops can be manoeuvred and grouped into desired fractions. ** Several learners, including Sabelo, had previously used the term *divide* to mean *separate*. After these responses by learners there was no doubt as to learners' preferences. Learners were favourably disposed to the use of practical work in the division of fractions. Learners in school A had mentioned bottle-tops and diagrams as their preferred aids in dividing fractions. In school B learners chose bottle-tops. However, one learner from one of the groups interviewed in school B responded differently. This learner was also very eloquent in articulating reasons for his position. The following extract comes from a conversation with this learner:

Interviewer: *(Continues to list methods of division that learners have been exposed to)* Using the drawing ruler and bottle-tops. We did those with you, angithi (not so)?

Sihle: Mina sir, the rule! (For me sir, it was the rule!)

Interviewer: The rule?

Sihle: Yes sir!

Interviewer: Now if you say the rule, I'm going to ask you...did you understand *(with emphasis)* what was happening when you...when you used the rule?

Sihle: Yes sir! Because we used...we used to use the rule when we divide *(interrupted)*

Interviewer: *(Interrupting)* Sihle! Am I correct to say...you think you understood because you got the answers correct?

Sihle: Yes sir!

Interviewer: So you think you understood because you got the answers correct. But did you understand what you were doing? Did you understand why you were multiplying instead of dividing?

Sihle: *(After short silence)* Yes sir!

Interviewer: You understood?

Sihle: Yes sir!

Interviewer: Sure! *(Silence)* So, so...so, so in short, the rule made it easier for you to divide fractions, eh?

Sihle: Yes sir!

For Sihle the major yardstick for deciding on the effectiveness, and hence the preference of the method of fraction division, was getting the correct answer. Other learners also articulated their reasons for their choices. Sihle thought that the algorithm helped him to understand the division of fractions because it helped him to get the correct answers. However, his hesitation before he agreed that he understood why the algorithm required him to multiply, suggested uncertainty on the justification for multiplying when dividing fractions. Nevertheless, Sihle believed he understood fraction division better when he used the algorithm because it helped him to get the correct answers.

The various responses by different learners to question 6 brought the researcher to the following conclusions:

- 1) Learners are generally well disposed to the use of practical work in the division of fractions. Asked which approach made it easier for them to understand and carry out the division of fractions, most learners mentioned practical activities (bottle-tops and diagrams).
- 2) Obtaining the correct answer can be wrongly perceived by learners as understanding conceptual processes involved. Views by Sihle are confirmation of this perspective. Such misconceptions can result from an approach whose sole and primary objective is the correct manipulation of the algorithm to arrive at the correct answer.
- 3) Diagrams can be a powerful tool of practical work in the division of fractions. Their strength lies in their potential to provide learners with visual perceptions of the fraction concept, thus removing misconception of the fraction concept

as a hindrance towards acquisition of the concept of fraction division. They enable learners to divide fractions easily.

- 4) Physical objects that learners can move around with relative ease are well suited to children's inherent desire to experiment with and move things around. Most learners preferred bottle-tops because they enabled them to form required fractions by separating them from each other.

4.5.2 USE OF RULER

Interview questions 1 and 3 (see Appendix E) sought to find out learners' preferences between the ruler and bottle-tops in the representation and division of fractions respectively. In responding to question 1, the following conversation transpired between learners from school A and the researcher:

Mandisa: Bottle-tops.

Interviewer: So you found it easier to show fractions by using bottle-tops?

Mandisa: Yes sir!

Interviewer: *(To Mandisa)* Why did you find it easier to use bottle-tops?

Mandisa: Please sir ngoba ngiyakwazi ukuwadivayda (because I can easily divide the fractions, sir).

Interviewer: *(Suspecting a different meaning could be associated with the term divide)* What do you mean by ngiyakwazi ukuwadivayda (I can divide them)? You can separate them *(interrupted)*

Mandisa: *(Interrupting the interviewer)* Yes sir!

Interviewer: From each other?

Mandisa: Yes sir!

Interviewer: Is that what you mean?

Mandisa: Yes sir!

Interviewer: So you can move them around and separate them from each other?

Mandisa: *(Joined by other learners)* Yes sir!

The ease with which bottle-tops could be moved around made them a favourite with learners. Mandisa had expressed the same sentiments when she supported S'nenhlanhla on why bottle-tops made it easier to divide fractions. The views of Mandisa were shared by Phungula. He gave the following justification for his preference of bottle-tops:

Phungula: Sir, mina ngithi ngisebenzisa ama bottle-tops (I am saying that I use bottle-tops). Sir... *(interrupted)*

Interviewer: *(Interrupting)* So you also find it easier to show fractions using bottle-tops?

Phungula: Yes sir!

Interviewer: Can you give me a reason for that?

Phungula: Sir, because they are... *(the learner moves his hands in a circular motion in apparent gesture to indicate the ease with which bottle-tops can be manoeuvred)*

Interviewer: *(Helping the learner to find the right words)* They are movable?

Phungula: Yes sir!

Interviewer: You can move them around?

Phungula: Yes sir!

For Mandisa and Phungula their manoeuvrability made bottle-tops a favourite instrument for the representation of fractions. The same question solicited a

different and interesting response from a learner from one of the groups interviewed in school B:

Sabelo: Bottle-tops.

Other learners: *(In support of Sabelo)* Bottle-tops.

Interviewer: Bottle-tops?

All learners: Yes sir!

Interviewer: So you all agree that it was easier to show fractions using bottle-tops?

All learners: Yes sir!

Interviewer: Can you give me a reason why it was easy... to show fractions using bottle-tops? Why? *(Silence)* What made it easier? Sabelo!

Sabelo: Ngoba uyithola kalula i-answer (it's easier to get the answer).

Interviewer: *(To Sabelo)* what makes it easier for you to get the answer? *(Silence)* what makes it easier for you to find the answer when you use bottle-tops?

Sabelo: When you use a ruler you count so many times. *(Other learners laugh)*

Interviewer: So you count so many times?

Sabelo: Yes sir!

Interviewer: So you don't count a lot with bottle-tops?

All learners: Yes sir!

For Sabelo, the need for accuracy as he tried to find the required fractions with a ruler was clearly an obstacle towards easy expression of fractions. Having to count

accurately to get the correct fractions distracted this learner from the primary objective of acquiring the fraction concept when he uses this instrument.

Another learner from the same group shared similar sentiments as Sabelo. This is how he responded to the same question:

Mkhize: Because bottle-tops make it easy to find the answer.

Interviewer: Alright! What makes it easy to find answers with bottle-tops?

That's my original question. What makes it easy to find answers when you use bottle-tops? Why is it easy compared to a ruler?

Mkhize: Because bottle-tops are 12, but the ruler is 300 (*in apparent reference to the total number of bottle-tops that each learner was using and the length of a complete ruler*).

Interviewer: (*Trying to make sense of Mkhize's response*) so, again, you go back to what Sabelo said. You count a lot when you use a ruler... (*Interrupted by learners*)

Other learners: (*Interrupting the interviewer*) yes sir!

Interviewer: But with bottle-tops, because there are only few of them, it's easy to use them.

All Learners: Yes sir!

Again the need to count as required by using the ruler was found to have a distractive influence as learners tried to give the required fractions. When learners were asked which between the two instruments used in the study, the ruler or bottle-

tops, was easier to use in the division of fractions (question 3, Appendix E), the following views emerged from a conversation with learners from school A:

Interviewer: So you all found it easier to work with bottle-tops than work with a ruler?

All learners: Yes sir!

Interviewer: Okay! The reason why you found it easier to work with bottle-tops? Why did you find it easier to use bottle-tops? *(Silence- and then Mthandeni indicates that he wants to respond)* Yeah Mthandeni!

Mthandeni: Ngoba ama bottle-tops uyakwazi ukuwahlukanisa kabili (because you can separate bottle-tops into two groups).

Interviewer: So you can... separate them into different *(interrupted by learners)*.

Learners: Groups...

Interviewer: *(Completing the statement)* to show different fractions.

All learners: Yes sir!

Interviewer: Just like you said in question 1! You said it was easier for you... to show fractions using bottle-tops.

While Mthandeni reiterated the popular reason why most people preferred bottle-tops i.e. manoeuvrability, the following response to the same question gave a fresh and deeper insight into reasons why bottle-tops were a favourite with Sihle:

Interviewer: Can anyone give me a reason why? *(Silence)* What made it easier for you to divide fractions... using bottle-tops?
(Silence) I mean , I mean there must be a reason . Yeah

Sihle! You seem to be having something to say.

Sihle: Because you can show by moving...You can show fractions that you take away with bottle-tops.

Interviewer: So, with bottle-tops it's actually easier because, again, you can move bottle-tops?

Sihle: Yes sir!

Interviewer: Now when you, when, when you count the number of fractions you actually can, can move them away, take them away and see how many times a fraction appears in another fraction?

All learners: Yes sir!

In stating '*...You can show the fractions that you take away with bottle-tops*', Sihle unconsciously expressed the interpretation of the division concept as repeated subtraction/removal of an equal quantity from the original group, and then counting the number of subsets formed. This is the essence of the measurement interpretation of the division concept, which was incidentally the study's choice of division interpretation. In general, according to Flores (2002), measurement situations involve finding how many groups can be made when the total amount and amount per group are known. The same perspective of division by Sihle could also be used to help learners in the acquisition of the partitive/sharing interpretation of division. Flores (2002) defined this interpretation as finding out how much is in each group, when the total amount and number of groups are known. Chorus concurrence of other learners with Sihle's view confirmed that they also shared a similar perception of what division of fractions, as portrayed with bottle-tops, meant to them. This was

a categorical vindication of practical work as a powerful tool in assisting learners in the understanding of the concept of fraction division.

After the initial three interviews with learners from school B, another interview was conducted with a fourth group of learners from the same school. The fourth group was made of learners from all previous three groups who had actively participated in interviews of their respective groups. The undeclared intention was to find out if learners would stick to their original positions on issues they had been previously interviewed in. Most learners did, but Sihle interestingly added a new dimension to his previous response to question 3. His new position was in direct support of a position put forward earlier by Sabelo and Mkhize. The following conversation transpired with Sihle:

Interviewer: Can you give me a reason why you think bottle-tops were easier to use to divide fractions?

Sihle: Sir, because you can count better with bottle-tops *(He makes gestures with his hands indicating movement from one position to another, and other learners laugh).*

Interviewer: *(Confirming his understanding of the meaning behind Sihle's gestures)* You can count better with bottle-tops?

Sihle: *(Together with Sabelo)* Yes sir! *(Pause)* Sir! When you count with a ruler, you can get disturbed and you forget where you were.

Interviewer: *(Nodding in understanding)* Okay, okay! Because with a ruler there's a lot of counting, it's easy for you to get disturbed...

Sihle: *(Interrupting)* Yes sir!

Interviewer: *(Continues)* along the way?

Sihle: Yes sir!

Interviewer: And you can't remember where you stopped?

Sihle: And you start afresh.

Interviewer: *(Clearly impressed)* You start afresh. Now, that becomes a problem?

Sihle: Yes sir!

Interviewer: Whereas that's not a problem with bottle-tops, eh?

Sihle: Yes sir!

Sabelo and Mkhize only mentioned the need to count repeatedly when using the ruler as an obstacle towards acquisition of the fraction concept and use thereof to divide fractions. Sihle went further to explain how this inconvenience actually affected him. According to Sihle, *'When you count with a ruler, you can get disturbed and you forget where you were,'* and as a result, *'...you start afresh.'* From this conversation with Sihle, the researcher gained a clearer and deeper insight into problems that learners associated with the use of the ruler. These problems of a distractive nature took learners' focus away from understanding the fraction concept. The focus becomes the detailed accurate measurements on the ruler.

Various responses by learners to questions 1 and 3 brought the researcher to the following conclusions:

- 1) The choice of all learners was clear and unanimous. It was bottle-tops.
- 2) The reason for learners' choice of bottle-tops was clearly articulated. It was because of their manoeuvrability.

- 3) Learners had valid reasons for the unfavourable light in which they viewed the ruler. The fact that it is graded meant that the need for accuracy (during the counting) often interfered with the prime objective of finding or dividing required fractions. Use of the ruler therefore defeated the ultimate objective of assisting learners in the acquisition of concepts of fractions and fraction division.
- 4) The potential and effectiveness of bottle-tops as a practical aid in learners' meaningful acquisition of the concept of fraction division was confirmed.

4.5.3 CHALLENGES IN PRACTICAL FRACTION DIVISION

Two areas stood out as posing serious difficulties when learners engaged in practical division of fractions. The first was concrete representation of the fractions $\frac{3}{4}$ and $\frac{2}{3}$.

The second problem was finding solutions to division problems that involved the remainder. Question 2 (see Appendix E) required learners to give reasons why it was difficult for them to show the fractions $\frac{3}{4}$ and $\frac{2}{3}$ using a ruler or bottle-tops. The

following conversation with learners from school B provided the reasons:

Sihle: Sir! Because we were not familiar with $\frac{3}{4}$ and $\frac{2}{3}$.

Interviewer: You were not familiar with those fractions?

Sihle: Yes sir! (*Silence*)

Interviewer: Xaba!

Xaba: (*Referring to Sihle*) Ngihambisana naye sir (I go along with him, sir). (*Other learners laugh*)

Interviewer: You agree with him?

Xaba: Yes sir!

Interviewer: *(To all learners)* Right! Can you, can, can you give me a reason? What, what makes these fractions unfamiliar and what makes other fractions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{3}$ familiar?

Xaba: Besingawafundi, Sir! (They were not taught to us, Sir!)

Interviewer: Beningawafundi? (You never learned them?)

Xaba: Yes sir!

Interviewer: So, whenever your teachers teach you fractions, they don't usually use these fractions $\frac{3}{4}$ and $\frac{2}{3}$? Is that correct?
(Thobile nods approvingly and the interviewer offers her a chance to respond) Thobile!

Thobile: Yes sir!

Sihle clearly articulated learners' difficulties in showing $\frac{3}{4}$ and $\frac{2}{3}$ with a ruler or bottle-tops when he stated, '*we were not familiar with $\frac{3}{4}$ and $\frac{2}{3}$.*' The reason for learners' unfamiliarity with these fractions was explained by Xaba's response to the question '*What makes these fractions unfamiliar...?*' The response was '*Besingawafundi, Sir! (They were not taught to us, Sir!)*' Learners' familiarity with certain fractions because of overexposure to those fractions was also suggested by Sabelo, a learner from another group interviewed in school B. Because of the groups' original non-response when the question was asked for the first time, the researcher had to rephrase the question to encourage learners to respond. The following conversation transpired with Sabelo as he responded to a modified version of question 2:

Interviewer: *(Silence)* Or maybe, maybe let us put the question in another way. Why was it easy for you to show the fractions

$\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{3}$?

Sabelo: Because sir $\frac{1}{2}$ is easy to get.

Interviewer: $\frac{1}{2}$ is easy to get?

Sabelo: Yes sir!

Interviewer: What makes it easy to get?

Sabelo: Because sir you show half of the ruler.

Interviewer: *(Making sure that he understands the learner correctly)* so in other words, am I correct if I think that $\frac{1}{2}$ is easy because it's a fraction you work with most of the time?

Sabelo: Yes sir!

Interviewer: Okay! Does everyone agree? $\frac{1}{2}$ is easy because each and every time you work with fractions you work with $\frac{1}{2}$? $\frac{1}{2}$ is always there.

All learners: Yes sir!

Interviewer: And $\frac{1}{4}$ too, eh?

All learners: Yes sir!

Interviewer: And $\frac{1}{3}$, eh?

All learners: Yes sir!

Interviewer: But $\frac{3}{4}$ is not a fraction that you usually find when you do fractions, eh?

All learners: Yes sir!

With the guidance from the interviewer, Sabelo agreed that $\frac{1}{2}$ was easy to show using a ruler because in the interviewer's words '*...it's a fraction that you work with most of the time,*' or '*...every time you work with fractions you work with $\frac{1}{2}$...and $\frac{1}{4}$...and... $\frac{1}{3}$?*' Positive chorus responses by all learners to interviewer's questions confirmed that these were their teachers' favourite fractions, whenever they learned fractions. All learners agreed that $\frac{3}{4}$ was not a familiar fraction.

Question 5 (see Appendix E) asked learners why they experienced difficulties with division problems that involved the remainder. The following conversation transpired with a learner from school A:

Phungula: Sir! Mina i-problem ila kumikswa khona sir. Mina ngikwazi la sir kungamikswa khona, mhlampeni kuthiwa $1 \div \frac{1}{2}$. (Sir! My problem is when we have to include the remainder. I can only manage where no remainder is involved e.g. in $1 \div \frac{1}{2}$.)

Interviewer: (To all learners) Right! Now, there's something that I noticed while I was working with you. For instance, if you talk about $\frac{1}{2}$, or... or let me say $\frac{2}{3}$, which is 200 on the ruler, divided by $\frac{1}{2}$, which is 150, most of you were able to tell me ukuthi

(that)... in 150 or in... in 200 or $\frac{2}{3}$ we find only one half,

angithi (not so)?

All learners: Yes sir!

Interviewer: But most of you were not able to give me the fraction for the remaining part. *(Silence)* Is that correct?

All learners: Yes sir!

Interviewer: So you were able to say, you were able to see that there is only one half... in $\frac{2}{3}$, but you were not able to tell me the fraction that represents the remaining part, eh?

All learners: Yes sir!

Interviewer: So that was your problem?

All learners: Yes sir!

Phungula stated that he experienced difficulties with writing the remainder. He said '*he can only manage where no remainder is involved...*' Other learners agreed that they also experienced difficulties when they had to write the remainder as a fraction. The problem of writing the remainder as a fraction was reiterated in another interview with a group of learners from school B. The following extract gives a response to question 5 by a learner:

Sabelo: *(Interrupting the interviewer)* because sir, when you said

$\frac{2}{3} \div \frac{1}{2}$, there is a remainder.

Interviewer: *(Impressed)* very good! *(Pause- and to the rest of the class)* He *(referring to Sabelo)* has said it very clearly that the problem was that there is a remainder.

Sabelo: Yes sir!

Interviewer: What was difficult? *(Pause)* Why did the remainder make it difficult for you to give the answer? *(Silence)* Yes? Sabelo has said because there was a remainder, then that was the problem. What was the problem with the remainder? *(Silence- and then he directs the question to a specific learner)* Mvubu! *(Silence again)* because you had to write the remainder... as a fraction?

Sabelo: *(Emphatically)* Yes! Yes sir! We can't write the remainder as a fraction.

In the same interview another learner gave the following account of her difficulties:

Ndawo: *(Referring to the initial total of 12 bottle-tops)* Sir! Makuwu-
 $\frac{3}{4}$, angithi sir ama tin-tips enza $\frac{3}{4}$ abawu-9? (For $\frac{3}{4}$ we require
9 bottle-tops?)

Interviewer: Right!

Ndawo: Bese kusala lawa amanye awu-3. (Then 3 are left as the remainder.) Besekumele sibhale ukuthi u- $\frac{3}{4}$ mungaki kuma tin-tips awu -12. (Then we have to write how many times $\frac{3}{4}$ appears in 12 bottle-tops.)

Interviewer: Right, right!

Ndawo: I-remainder asiyibhali. (We do not write the remainder.)

Ndawo's account reiterated the views of Sabelo in so far as the problem of writing the remainder goes. However, Ndawo's account gave a detailed insight into what

she understood by $1 \div \frac{3}{4}$ when she used 12 bottle-tops. While Ndawo clearly understood that '*Makuwu- $\frac{3}{4}$, angithi sir ama tin-tips enza u- $\frac{3}{4}$ abawu-9? (For $\frac{3}{4}$ we require 9 bottle-tops?)*' she also understood that the remainder is 3 bottle-tops: '*Bese kusala lawa amanye awu-3. (Then 3 are left as the remainder.)*' But her problems began '*Besekumele sibhale ukuthi u- $\frac{3}{4}$ mungaki kuma tin-tips awu-12. (Then we have to write how many times $\frac{3}{4}$ appears in 12 bottle-tops.)*' Ndawo's problem was '*I-remainder asiyibhali. (We do not write the remainder.)*' All learners agreed with Ndawo that the main problem was writing the remainder.

The writer came to the following conclusions on learner challenges in practical division of fractions:

- 1) The concentration of teachers on examples with 1 in the numerator, to the exclusion of others, limits learners' understanding of the fraction concept to teachers' favourite examples. As a result learners encounter difficulties when they have to transfer their understanding of the fraction concept to other fractions that they are not familiar with.
- 2) Learners' limited understanding of the fraction concept makes it difficult for learners to relate fractions to each other. This is the main source of learners' difficulties in expressing, for example, the remainder as a fraction of $\frac{1}{3}$ in $\frac{3}{4} \div \frac{1}{3}$. Learners find it difficult to relate the remaining part of the ruler or bottle-tops to the divisor, namely $\frac{1}{3}$. They cannot say what fraction of $\frac{1}{3}$ the

remainder is. The results are that learners fail to find the correct solution to problems that involve the remainder. They cannot write the remainder as a fraction.

- 3) Learners' difficulties in finding solutions to fraction division problems that involve the remainder are prevalent even with learners who use the algorithm. Interviews with learners from school B, the algorithm group, confirmed this reality.

ASSERTION 4

Teachers should take advantage of learners' positive disposition and responsiveness towards practical work and employ practical activities in teaching fractions and fraction division.

4.6 LIMITATIONS OF THE STUDY

Utmost care was taken by the researcher to capture as much useful data as was possible to find reliable answers to the research questions. However, certain limitations made it imperative to confine findings from emergent data strictly to the two schools that were involved. The following were the major limitations of this study:

- 1) Research activities for this study were piloted before the actual fieldwork. Questionnaires were piloted with non-participating teachers from nearby schools. Also, worksheet activities were piloted on a select small group of grade 8 learners. Yet in spite of these piloting initiatives, the researcher did not get the opportunity to experience interaction with real life grade 7 classes (learners and their teachers) before real fieldwork. The researcher had also never experienced working with large groups of grade 7 learners. This could have impacted negatively in initial interpersonal and working

relationships with learners, especially those from the first school, the first research site. This is suggested from the background that the researcher is a secondary school teacher who normally teaches mathematics to older children. Through continued interaction with grade 7 learners and the associated accumulated experience, it is possible that the aforementioned relations could have improved by the time the study was conducted in school B. A possibility exists that such developments could have tipped the scales in favour of school B in so far as learners' performance in the two schools is concerned, thus making any comparative analysis of performance in the two schools less conclusive.

- 2) Four questionnaires were administered to four teachers out of a possible total of five from the two schools where the study was conducted. The small number of teachers who answered the questionnaires made it extremely difficult to generalize that conclusions reached would apply equally in other similar situations. The principles of transferability and generalization could not be guaranteed for conclusions reached from data generated by such a small sample.

4.7 SUMMARY

In theory teachers are favourably disposed towards practical work in fraction teaching. This position is mainly inspired by pre-service and/or in-service training in practical work teaching that they have received in their career pathways. However, conditions teachers have encountered in real practice, infrastructural or process related, make implementation of their ideas on practical work in fraction teaching difficult to put into practice. Also, the depth of training teachers might

have received could be inadequate to equip them for the challenges that accompany the advent and implementation of new curriculum changes. These factors which can be content or process related, or of an infrastructural nature, are some of the main impediments preventing teachers from adopting a practical approach to fraction teaching. On the other hand, practical work proved effective as an aid in enhancing learners' understanding of and competence in the concepts of fractions, and fraction division. Through practical manipulation of concrete representations of fractions, learners were better able to find solutions to fraction division problems. Learners' responses to interview questions confirmed their positive inclination towards practical work in fraction division. They particularly embraced bottle-tops which they claimed made it easy for them to work with fractions as they are easy to move around. However, all the positive conclusions to emerge cannot be automatically generalized as applying to all schools. This is because of unique relationships that the researcher had with learners in each of the schools where the study was conducted. Also, the limited number of participants who took part in the study made application of principles of transferability and generalization impossible.

CHAPTER 5

DISCUSSION OF DATA FINDINGS

5.1 INTRODUCTION

This chapter discusses findings by looking at: (a) confirmed assumptions and literature claims, and (b) refutations of some literature claims. Discussion also extends to some previously unanticipated data on learners' experiences with practical fraction division. Findings are documented under the following headings derived from the study's research questions: (1) teachers' perceptions on practical fraction division, (2) factors behind teachers' perceptions, (3) practical work and conceptual development, and (4) learners' views on practical work in fraction division.

5.2 TEACHERS' PERCEPTIONS ON PRACTICAL FRACTION DIVISION

Teachers' responses to questionnaires and practices in observed lessons confirmed all assumptions in the study's motivation. A number of literature claims were also confirmed by data generated by these research instruments. Confirmation of assumptions on teachers' perceptions on practical fraction division and claims by literature is documented under the following sub-headings: (1) confirmation from questionnaires, and (2) confirmation from observations.

5.2.1 Confirmation from questionnaires

Data confirmed from questionnaires is discussed under the following sub-headings: (1) teachers' difficulties in constructing practical fraction division activities, and (2) relevance of practical fraction division to OBE.

5.2.1.1 Teachers' difficulties in constructing practical fraction division activities

The following are some of the reasons advanced for teachers' reluctance to include practical activities in their lessons (see item 8 part 3, Appendix A):

- a) *They are time consuming.*
- b) *Maybe educators do not have love for mathematics. If they do have love they will be able to move from the abstract world of mathematics to the concrete world of mathematics.*
- c) *Lack of resources and training.*
- d) *Requires a lot of planning and preparation.*

The common message is that preparation of practical activities is a laborious exercise. With specific reference to the measurement and partitive/sharing interpretations of division, Ott, Snook & Gibson (1991) argued:

Such concrete experiences are easy to devise and are relatively easy for students to follow as long as the numbers are whole numbers. However, meaningful concrete experiences related to division of fractions are much more difficult for teachers to devise and for learners to follow (p.8).

Although teachers' responses were related to general inclusion of practical work in their mathematics lessons, within the context of fraction division their justification of their reluctance to include practical activities in their lessons supports the argument of Ott *et al* (1991). While failure of teaching to relate abstract concepts to learners' concrete experience is interpreted in response b) as lack of passion for mathematics, it is insinuated in responses a), c) and d) that practical fraction teaching is a difficult task. These insinuations support the argument of Ott *et al* (1991).

5.2.1.2 The relevance of practical fraction division to OBE

One of this study's motives was the relevance of practical work to OBE requirements for a learner-centred approach to teaching and learning. All respondents agreed that engaging learners in practical fraction division fitted well

with OBE requirements for a learner-centred approach. Subsequently all respondents agreed that OBE workshops in mathematics should put more emphasis on practical work. Different aspects of practical lessons in fraction teaching that OBE workshops should address were discussed in subsection 4.2.3, together with respondents' reasons thereof. In view of serious difficulties encountered by the implementation of OBE in schools, it is imperative for these workshops to pay attention to details that are informed by the genuine needs of teachers. In discussion on implications for further research in teachers' understanding and use of assessment in the OBE context, it has been observed that "Workshops in OBE have not shed any light on educators because OBE facilitators have been unable to address educators' concerns" (Langa, 2003, p.65). It is such concerns that attention to detail by practical work workshops in fraction teaching should seek to address.

5.2.2 Confirmation from observations

Lesson observations confirmed the following assumptions: (1) minimal use of practical work by teachers, (2) limited visual representation of the fraction concept, and (3) overemphasis of the algorithm as a goal of instruction.

5.2.2.1 Minimal use of practical work by teachers

Although Ott *et al* (1991) suggested that familiar concrete experience should be the first step in the development of new abstract concepts and their symbolisation, they also acknowledged that this was hardly the case in the division of fractions. Their claims were confirmed by the observation of teachers' practices. In school A, while the teacher gave his learners severely limited experience with practical work, his efforts did not carry much weight as learners were not afforded any meaningful opportunities at own experiences in practical fraction division. This, coupled with

erroneous conclusions the teacher always arrived at in his demonstrated examples resulted in learners not benefiting much from their experiences. In school B, all lessons in fraction division were characterised by a complete absence of any practical activity in favour of absolute devotion to rote application of the fraction division algorithm. Cosmetic successes that such an approach had with learners were discussed in subsection 4.2.2.

5.2.2.2 Limited visual representation of the fraction concept

Another motive for this study was limited visual representation of the fraction concept with pictures of part-regions. The standard sub-divided regions for shading to indicate some required fractional part of a real life pizza have been cited and used by Witherspoon (1993) and Moskal & Magone (2002) respectively. The teacher from school A replaced the pizza with circles representing cakes (see Figure 4.1). His alternative, the number line, was still another representation of the part-region perspective of the fraction. These examples of the fraction perspective supported assumptions and claims of the restriction of the fraction concept to the part-region perspective. Dangers of the narrow view of the fraction as a part-region were highlighted by Witherspoon (1993) as: (a) the geometry of unmarked region models, and (b) application of knowledge of regions to other fraction interpretations. The erroneous partition of rectangular shapes into uneven parts by learners in school A (see Figure 4.4), is manifestation of problem (a). The incorrect shading to show the subset perspective of the fraction (see Figures 4.5, 4.6 & 4.7) is an example of problem (b). The negative effects of limited visual representation of the fraction concept on learners were evident in school A, even though learners had been exposed to demonstrations using drawings. One of the factors behind this overemphasis on the part-region perspective of the fraction concept is the over-

concentrated focus of textbooks on this fraction perspective. It has been observed that “When it comes to fractions, it is not unusual for textbooks to emphasize the part-whole representations and fraction symbols, to the exclusion of other forms of expression” (Empson, 2002, p.35). This view directly supports claims on pre-OBE textbooks made in the motivation (see sub-section 1.1.1).

5.2.2.3 Overemphasis of the algorithm as a goal of instruction

Religious devotion to the algorithm by the teacher in school B was consistent with laments by Flores (2002) on overemphasising the algorithm procedure ‘invert the second fraction and multiply’, with little effort to provide learners with an understanding why it works. This also supported Siebert’s (2002) assertion that children often lack a ready understanding for operations involving fractions because these operations are often equated with seemingly nonsensical algorithms, such as the fraction division algorithm. Practices in school B also supported observations by Sharp *et al* (2002) that procedural knowledge such as algorithms for operations is often taught without context or concept, implying that algorithms are an ungrounded code only mastered through memorization. The impressive ease with which learners from school B obtained solutions to fraction division problems by use of the algorithm in pre and post test (see Tables 4.13 & 4.14) amid evidence of absence of understanding of underlying concepts (see subsections 4.5.1 & 4.5.3) validated warnings by Cramer & Bezuk (1991) and Witherspoon (1993) against assuming an understanding of fractions merely on the basis of successful application of the algorithm. It is inconceivable that learners who had an incorrect perception and understanding, of $\frac{3}{4}$ (*because such fractions had never been taught to them*), could

suddenly understand what $\frac{3}{4} \div \frac{1}{3}$ meant. This is regardless of whether those learners gave the correct solution or not.

5.3 FACTORS BEHIND TEACHERS' PERCEPTIONS

Factors that informed teachers' perceptions on practical work and the teaching of fractions and fraction division are discussed under the following sub-headings: (1) teachers' beliefs, (2) convenience, efficacy and expedience, and (3) teachers' level of training.

5.3.1 Teachers' beliefs

The underlying belief by all respondents to the questionnaire that learners' understanding of mathematical concepts should be primary objective of instruction (see items 1 & 5 part 4 in Appendix A) informed further beliefs that: (a) practical work fitted well with OBE requirements for a learner-centred approach to teaching, (b) learning activities that require learners to engage in practical work are not a waste of time, (c) practical work contributes to learners' better understanding of fractions, and (d) learners can learn fractions better by handling physical objects. Belief (a) was discussed in sub-section 5.2.1.2. Beliefs (b) to (d) support the assertions on the value of practical work in aiding learners' better understanding of fraction division (Flores, 2002; Siebert, 2002; Sinicrope *et al*; 2002). Sinicrope *et al* (2002) offered advice on examples for concrete experiences for learners by suggesting instrumental models, like pattern blocks, can be used for the measurement interpretation of fraction division. Siebert (2002) gave examples of how diagrams can be used to find solutions to fraction division problems.

5.3.2 Convenience, efficacy and expediency

The convenience of practical activities to peculiar conditions they may be faced with, was another determining factor behind teachers' perceptions on practical work in fraction teaching. Large numbers in classes and pressure to complete the prescribed syllabus were cited among some of the conditions facing teachers, which determine the convenience and suitability of practical work in fraction division. The efficacy and expedience of various instruments of practical work were other factors behind teachers' positive disposition towards practical work (see sub-sections 4.2.1.1 and 4.2.1.2). However, sub-section 4.2.2.2 and sub-section 4.2.3 revealed serious difficulties that teachers encounter when they consider implementation of practical work. These difficulties were manifestations of claims by Ott *et al* (1991) on difficulties teachers encounter in their attempts to construct practical activities for learners. These were discussed in sub-section 5.2.1.1. Whitworth & Edwards (1969) offered a range of suggestions on instruments and activities for practical work in fraction teaching that teachers could find useful to address their difficulties.

5.3.3 Teachers' level of training

Their level of training was another driving factor behind teachers' favourable disposition towards practical work. Yet in spite of their claims of adequate training in practical work in fraction teaching, teachers' observed fraction teaching practices (see sub-section 4.2.2) revealed half-measures and errors, or complete omission of practical work from their lessons on fraction division. Ott *et al's* (1991) difficulties that teachers experience in designing practical activities contributed to teacher's shortcomings. These shortcomings in use of practical methods in fraction teaching, together with glaring errors made by the teacher in school A, call for the design of training programmes to assist teachers with their difficulties.

5.4 PRACTICAL WORK AND CONCEPTUAL DEVELOPMENT

The success of practical work in enhancing learners' understanding of conceptual processes involved in fraction division is documented under the following sub-headings: (1) whole numbers in fraction division, (2) concrete experience in fraction division, (3) the problem of writing the remainder, (4) overemphasis of the part-region fraction perspective, and (5) learners' successful application of the algorithm.

5.4.1 Whole numbers in fraction division

Time constraints did not allow the researcher opportunities to determine if teacher practices tried to secure learners' understanding of fraction division situations from the background of whole number division. However, the preparatory exercise which served as an introduction to worksheets on fraction division helped to give an indication of the strength of whole numbers in aiding learners' understanding of fraction division (see subsection 4.4.2.1). Limited as it was to measurement and partitive interpretations of division the exercise contributed immensely to learners' better understanding of fraction division. It also helped to prepare them for competent performance in related worksheet tasks. Learners' successes in this exercise and the subsequent ability to translate their successes to fraction division, supported Siebert's (2002) claim that learners must first clearly understand whole number division for them to develop meaningful conceptions of fraction division. Subsequent successes in division of fractions by practical means provided further support to Siebert's (ibid.) suggestions that children can: (a) develop meaningful images for the division of fractions by reasoning about real-world contexts involving fraction division, and (b) make connections between their solutions and their understanding of whole numbers. The value of whole numbers in meaningful understanding of and competence in division of fractions was also resonated in

Murray *et al* (1996) who advocated soliciting, encouraging and building on learners' base of informal knowledge of whole numbers and problem situations involving them. Recommendations by Murray *et al* (ibid.) had been previously made by Sinicrope & Mick (1992) who suggested progression from whole number multiplication to multiplication of fractions. Further, learners' successes in whole number division supported assertions by Flores (2002) that thorough understanding of operations division and multiplication of whole numbers is a prerequisite for understanding division of fractions. Understanding whole number division and its importance to meaningful understanding of fraction division is fundamentally linked to enriched and diverse understanding of fraction division situations. Although the study focused in the measurement interpretation of division, many researchers have offered many varieties of such interpretations. After listing measurement, partitive/sharing and Cartesian product interpretations as important categories of whole number division, Sinicrope *et al* (2002) advised that fraction division can be explained by extensions of all three of these whole number interpretations. During this study, learners' success in whole number division and the subsequent ability to translate their successes to fraction division supported this argument, especially as learners were able to extend the measurement interpretation of division from whole numbers to fractions. Learners' success also vindicated Ott *et al*'s (1991) observation that the meanings of fraction division exercises are the same as those for the division of whole numbers. Lastly, the successful transfer of learners' competence in whole number division to understanding of fraction division also justified the argument by Ott *et al* (ibid.) that inability to interpret the results of division problems, has more to do with lack of understanding what division of a fraction means than inability to decode the meaning of fraction symbols.

5.4.2 Concrete experience in fraction division

Concrete experience is advocated by different researchers as a valuable basis for meaningful acquisition of the fraction division concept (e.g. Flores, 2002; Ott *et al*, 1991; Sinicrope *et al*, 2002). Learners' successful manipulation of concrete representations of fractions to find solutions to fraction division problems was discussed in section 4.4. The value that learners attached to these exercises was discussed in section 4.5. Learners' success with concrete experience as basis for meaningful understanding of fraction division supported claims by Ott *et al* (1991) that familiar concrete experience should be the springboard for the development of new abstract concepts and their symbolization. Ott *et al* (ibid.) claimed concrete experiences related to division of fractions to be difficult for teachers to devise and for learners to follow. Confirmation of teachers' difficulties was discussed in subsection 5.2.1.1. However, learners' successes in worksheets (see subsections 4.4.1 & 4.4.2) refuted claims that learners found it difficult to follow practical activities on fraction division. Sinicrope *et al*'s (2002) suggestion of using instructional models like pattern blocks for teaching measurement interpretation of fraction division was supported by learners' successful manipulation of concrete representations of fractions, especially since fraction division problems in worksheets also focused on the same interpretation of division. Flores (2002) recommended that teachers need a complete picture that connects concrete approaches of fraction division with the algorithm. Learners' successes with concrete approaches to fraction division make it the more imperative to explore ways by which these successes can be translated to meaningful development and understanding of the fraction division algorithm.

5.4.3 The problem of writing the remainder

Evidence that writing the remainder as a fraction was a problem for learners in division problems that involved the remainder first emerged in learners' performance in worksheets 6, 10, 11 and 12 (see Tables 4.3, 4.12, 4.6 & 4.5). Learners confirmed their difficulties with writing the remainder in their responses to interview questions (see subsection 4.5.3). Learners' difficulties with writing the remainder confirmed the observation that "Another sign of lack of ease with fractions is the insistence on giving the answer in *remainder* form rather than *fractional* form" (Hart, 1981, p.68). After a real-life division problem in which learners were offered a choice between answers where the remainder was given as a concrete expression of the remaining part e.g. 1cm, and where the remainder was given as a fraction (of the divisor), it was observed that "They much prefer the remainder type answer than one which states a fraction" (Hart, 1981, p.76). Learners' failure to write the remainder was directly linked to their inability to interpret the remainder as a part of another fraction – the divisor (see subsection 4.5.3). This particular source of learners' predicament resonated of Hart's (1981) observed tendency among learners to ignore the question '*a fraction of what?*'

5.4.4 Overemphasis of the part-region fraction perspective

Overemphasis of the part-region perspective of the fraction concept was seen to cause problems for learners on two occasions. Firstly, Figure 4.4 shows learners' erroneous partition of two rectangular shapes to show $\frac{1}{3}$ in an effort to find a solution to $2 \div \frac{2}{3}$. This was a manifestation of the first part of Witherspoon's (1993) two-part problem of the limited part-region perspective of the fraction concept i.e. the geometry of unmarked region models. Learners did not take care that all the three

parts of the partitioned figure representing $\frac{1}{3}$ are equal. Witherspoon (ibid.) suggested intensive experience by learners in subdividing regions in various ways to become familiar with the geometry of various shapes. Secondly, Figures 4.5, 4.6 & 4.7 show wrong representations of the subset perspective of $\frac{3}{4}$ and $\frac{1}{4}$ by some learners in the post-test (see subsection 4.4.1). This confirmed Witherspoon's (ibid.) other problem of overemphasis of the part-region fraction perspective, i.e. incorrect application of knowledge of regions to other interpretations. In all likelihood, these problems stemmed from restricting learners' perception of the fraction concept to the part-region perspective. Cramer & Bezuk (1991) and Witherspoon (1993) cited Lesh who suggested five different ways that can be used to represent concepts (see subsection 2.2.1). Witherspoon (ibid.) also cited Kennedy & Tipps on alternatives to the part-region fraction perspective. Sinicrope & Mick (1992) cited Kieren and Usiskin & Bell on further perspectives of the fraction symbol. Witherspoon's (ibid.) advice on the solution to the second problem was to make learners understand that the 'one', in $\frac{1}{2}$ for example, is a part of a set, and not a single object.

5.4.5 Learners' successful application of the division algorithm

Subsection 4.4.2.6 showed how successful in fraction-division problems learners from school B were compared to learners from school A. Learners from school B had been trained and drilled in the use of the algorithm while those from school A had not. Pre and post test performances by learners from school B showed how inclined these learners were to always use the algorithm. This applied even to the experimental group who had experience with practical work. Such successes with the division algorithm contradicted observations that "The division algorithm is very

difficult to apply... and probably any computation which seemed to require its use was likely to upset the children” (Hart, 1981, p.75). Further, Sihle’s preference of the algorithm and the reasons he advanced for his position (see subsection 4.5.1) were further proof that learners can be comfortable with application of the algorithm, provided they have been adequately trained and drilled. This was further refutation of Hart’s (ibid.) claims on the difficulty of applying the algorithm. But Sihle’s hesitation before he declared his understanding of why the algorithm required him to multiply instead of dividing cast doubts over the validity of his claim. His admission of *‘assuming understanding because he got the answers correct,’* was reminiscent of warnings by Cramer & Bezuk (1991) and Witherspoon (1993) (see subsection 5.2.2.3). Learners’ poor performance in pre and post test on some fraction identification and representation items compared to much better performance in fraction division items justified warnings that “We should be careful not to assume that students ‘understand’ fractions merely because they are able to carry out an algorithm or recite a definition” (Witherspoon, 1993, p.84).

5.5 LEARNERS’ VIEWS OF PRACTICAL FRACTION DIVISION

The views of learners on practical fraction division are documented under the following sub-headings: (1) preference of concrete experience, (2) the value of the subset perspective, (3) learners’ challenges, and (4) ungrounded teaching of the algorithm.

5.5.1 Preference of concrete experience

The usefulness of concrete experience in assisting learners’ understanding of and competence in fraction division was evident in learners’ performance in worksheets

and post-test (see section 4.4). In response to question 6 in the interview, learners confirmed their preference of the concrete approach to fraction division over all others they had been exposed to (see subsection 4.5.1). Learners' successes in fraction division in worksheets and post-test, coupled with learners' pronouncements in favour of practical work confirmed claims that "With the help of concrete models of fractions learners can see that $\frac{1}{4}$ fits two times into $\frac{1}{2}$, therefore $\frac{1}{2} \div \frac{1}{4} = 2$ " (Flores, 2002, p.238). That learners embraced practical work supported Flores' (2002) advice that teachers can make concrete representations, empirical evidence, patterns and properties of numbers and operations to explain various approaches of fraction division. Although the majority of learners favoured bottle-tops as preferred tools of practical fraction division, two learners favoured diagrams. The use of diagrams to find solutions to fraction division problems was widely employed and recommended by Siebert (2002). Asked for reasons for her preference of diagrams, Phumla responded: *'...when I'm using diagrams it's easy for me to see my problem.'* Phumla's view confirmed claims that "Many students who make drawings understand mathematical operations better than those who use only symbols or observe the drawings made by someone else" (Dirkes, 1991, p.28). Phumla had successfully demonstrated her faith in diagrams by using them in solutions to a number of fraction division problems in the pre-test and post-test. Phungula, a co-admirer of diagrams justified his affinity towards diagrams: *'it's because drawings can help you to represent fractions. This helps you to see the fractions as you divide them.'* The views of these two learners on the benefits of using diagrams supported observations that "Diagrams often helped towards the solution of the problem...It was sometimes apparent on interview that the child needed a diagram to help him see what a word problem required" (Hart, 1981, p.70). The resolution of fraction

division problems with diagrams was also witnessed in Stephanie, a learner in a study of children's development of meaningful fraction algorithms who "...used explicit pictorial and symbolic strategies to divide whole numbers with fractions" (Sharp, Garofalo & Adams, 2002, p.19). This attraction towards diagrams was therefore a continuation of a tendency already identified by several other researchers before this study.

5.5.2 The value of the subset perspective

Several researchers have expressed their concerns over repeated emphasis of fraction teaching on the part-whole perspective of the fraction concept (e.g. Empson, 2002; Sinicrope & Mick, 1992; Witherspoon, 1993). This is despite a wide range of available perspectives. Sinicrope & Mick (1992) cited Kieren and Usiskin & Bell in listing numerous other perspectives of the fraction concept while Witherspoon (1993) cited Kennedy & Tipps who offered further perspectives of the fraction (see subsection 2.2.1). In reference to pre-marked region models, teachers were cautioned "...if these are the only contexts in which students encounter fractions, they learn only a small part of underlying concepts. Their repertoire for problem solving involving fractions becomes quite limited" (Witherspoon, 1993, p.482), hence the study's choice to explore the potential of the subset interpretation of the fraction concept to enhance learners' understanding of the concepts of fractions and fraction division (see subsection 1.1.2). In response to interview questions 1 and 3, learners openly embraced bottle-tops as preferred instruments of concrete fraction representation and division (see subsection 4.5.2). Learners' reasons for opting for bottle-tops were their easy manoeuvrability. The ease with which learners could use bottle-tops to divide fractions had been previously witnessed in their performance on

worksheets (see subsection 4.4.3). Therefore, the subset interpretation proved to be a powerful alternative to the part-region perspective of the fraction concept. It proved itself to be a useful option that teachers should consider seriously to enhance and broaden learners' understanding of the concepts of fractions and fraction division.

5.5.3 Learners' challenges

Challenges encountered by learners in practical division of fractions fell under the following categories: (1) the problem of familiar and unfamiliar fractions, (2) writing the remainder and (3) the ruler's problem of repeated counting.

5.5.3.1 The problem of familiar and unfamiliar fractions

Learners' constant reference to $\frac{1}{2}$ drove the researcher to assume that $\frac{1}{2}$ was a familiar fraction to learners, hence the researcher's inquiry if fractions $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ were familiar fractions to learners and to which the response was positive. The idea of $\frac{1}{2}$ as one of the familiar fractions was suggested in the assertion that "A half seems to be very much easier to deal with than any other fraction..." (Hart, 1981, p.69). Sabelo's response to why the aforementioned fractions are easy to show, i.e. *'because sir half is easy to get,'* confirmed Hart's claim. Learners' concurrence with the researcher's suggestion that $\frac{1}{2}$ was easy *'because it's a fraction that you work with most of the time'* confirmed the familiarity of half and other fractions listed thereafter. Working with the same familiar fractions all the time proved to be a handicap to learners when they worked with less familiar fractions. Asked why it was difficult to show the fractions $\frac{3}{4}$ and $\frac{2}{3}$ (see question 2, Appendix E), Sihle's

response was *'Because we were not familiar with $\frac{3}{4}$ and $\frac{2}{3}$ '* and Xaba concurred.

Asked why these fractions were not familiar, Xaba's response was *'Besingawafundi, Sir! (They were not taught to us, Sir!).'* Thobile agreed with the researcher's interpretation of Xaba's response, i.e. *'whenever your teachers teach you fractions they don't usually use the fractions $\frac{3}{4}$ and $\frac{2}{3}$ '*.

5.5.3.2 Writing the remainder

Learners' problems with writing the remainder were discussed at length in subsection 5.4.3. During interviews several learners confirmed and gave reasons for their difficulties. Sabelo admitted *'Yes! Yes sir! We can't write the remainder as a fraction.'* In a separate interview, Ndawo supported Sabelo's view when she responded with *'I-remainder asiyibhali. (We do not write the remainder.)'* Besides being evident in worksheet performances, learners' actually confessed their difficulties with remainder-type fraction division problems in interviews.

5.5.3.3 The ruler's problem of repeated counting

Learners clearly articulated the difficulties they experienced with using the ruler for fraction division. Sabelo's problem was that *'when you use a ruler you count so many times.'* For Sihle the effects of the ruler's demand for repeated and accurate counting were *'When you count with a ruler, you can get disturbed and you forget where you were.'* For him the results of this were *'and you start afresh.'* Therefore using the ruler compounded the problem of dividing fractions, than ease it, by adding the extra dimension of the need for accuracy. Besides having to represent fractions and divide them correctly, now learners had to deal with the issue of accuracy to determine fractions correctly. This proved to be an obstacle in learners' progress

towards easy acquisition of and competence in the concepts fractions and fraction division.

5.5.4 Ungrounded teaching of the algorithm

Sihle was the only learner who favoured using the algorithm. His views and reasons behind them were discussed in subsection 5.4.5. Further, it was established that he preferred the algorithm because he was assured of getting correct solutions when he used it. Associated dangers of this position to understanding conceptual processes were discussed. Such ungrounded disposition towards the algorithm, unconnected to understanding division situations, runs against Siebert's (2002) assertion that the IM rule can become a meaningful tool to solve important problems if connected to images of sharing and measurement. The importance of a solid conceptual understanding of the fraction as a foundation to meaningful understanding and development of the algorithm is advocated in most literature on fraction learning (e.g. Sharp *et al.*, 2002; Sinicrope & Mick, 1992). It has been generally established that once children have developed a conceptual knowledge base for fraction sense and operation sense, they can meaningfully learn, or even create for themselves, appropriate fraction algorithms (Sharp *et al.*, 2002).

5.6 SUMMARY

Data on teachers' perceptions and their practices confirmed several assumptions in the study's motivation, and supported a number of literature claims. Practical activities were confirmed to be difficult for teachers to construct. This was implied in various reasons given for the omission of practical activities from teachers' lessons. Teachers' difficulties in constructing practical activities led to the following assumed

practices which were themselves later confirmed in observations: (a) minimal use of practical work and (b) limited visual representation of fractions with pictures of part-regions. Convenience of practical activities and teachers' level of training were other factors behind teachers' perceptions towards practical fraction teaching. All respondents to questionnaires acknowledged the relevance of practical fraction division to OBE requirements for a learner-centred approach. Learners' performance in the experiment confirmed and refuted a number of literature claims. The following literature claims were confirmed: (a) the importance of whole numbers in understanding fraction division, (b) the value of concrete experience in understanding fraction division, (c) learners' difficulties with writing the remainder as a fraction and (d) the danger of overemphasizing the part-region perspective. Although literature claimed practical activities to be difficult for learners to follow, learners' successes in worksheets on practical fraction division refuted those claims. Successful application of the fraction division algorithm by learners from school B also refuted literature claims that the algorithm is difficult for learners to apply. Interviewing learners on their experiences with practical fraction division revealed that learners actually embrace and enjoy practical work in learning about fractions and fraction division. The subset perspective of the fraction concept, bottle-tops, proved to be a particular favourite with learners. The potential of the subset perspective to enhance learners' understanding of fractions and fraction division emerged from learners' successes and their self-declared preference for this fraction perspective. Interviews revealed the following main problems encountered by learners: (a) restriction of learners' understanding of fractions caused by focus on the same familiar fractions, (b) the problem of writing the remainder and (c) the distracting and obstructive effects of using the ruler.

CHAPTER 6

IMPLICATIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

The section on limitations of the study acknowledged the limits of the findings in their applicability to other similar situations (see subsection 4.7). Nevertheless, the study's findings carry a number of important implications for the training of teachers, the teaching of fractions and fraction division, and further research. These are documented under: (1) implications for teacher training, (2) teaching implications and (3) implications for further research.

6.2 IMPLICATIONS FOR TEACHER TRAINING

Observed teacher practices and their perceptions on practical fraction teaching necessitate a look at the way in which teachers are trained in fraction teaching. Practices include excessive focus on the part-region perspective, rote application of the algorithm, and erroneous conclusion of fraction division problems. Teacher training should take into account teachers' perceptions which: (a) are likely influence their practices, and (b) seriously compromise learners' meaningful understanding of the concepts of fractions and fraction division. Implications for teacher training are discussed under the following subheadings: (1) pre-service training and (2) in-service training.

6.2.1 Pre-service training

On trainee teachers' implicit theories of mathematics teaching, it has been observed that "Pre-service mathematics teachers regard personal or formal theories of teaching and learning mathematics and classroom practice as separate areas of study"

(Hobden, 1991, p.76). In this study, the observed contradiction between teachers' classroom practices and their self-declared positive attitudes towards practical fraction teaching looks like a continuation of Hobden's observed pre-service tendencies of trainee teachers to regard theory and practice as two separate entities. Pre-service teacher training needs to take into account the teachers' reasons for excluding practical work and implementing teaching strategies that are not centred on practical work. Therefore, teacher training needs to provide programmes that directly address these concerns, especially issues of overcrowded classrooms and perceptions that practical activities take up a lot of time, both during preparation and implementation. It is a known fact that the issue of overcrowded classrooms is still a thorn in the side of our public education system. Yet the self-denial approach of our teacher training programmes continues to tailor the training of teachers along methods that are suitable for normal-sized classes. The notion that practical activities are time consuming suggests a lack of clear understanding of the nature, scope and functional potential of practical work by teachers, the origins of which are summed up by the suggestion that teachers *'lack proper training'* in practical work. Therefore, pre-service teacher training on practical fraction teaching needs to be revisited with an eye to addressing these and many other concerns which further research should help bring to the fore.

6.2.2 In-service training

Teachers' concerns, their observed practices and their acknowledgement that practical fraction division is relevant to OBE requirements for a learner-centred approach, call for a demand to look at how in-service training can assist to address teachers' needs. The restriction of instruction to rote application of the algorithm by teachers is a serious impediment to understanding. As practising teachers, in-service

training seems to be the most immediately accessible remedy to their deficiencies. Flores (2002) advised that teachers who understand a topic should be able to make connections with other mathematical concepts and procedures. Recommended and approved in-service training programmes should be informed by teachers' perceptions of their needs directly solicited from them through relevant and appropriate research strategies. Teachers' embracing attitude towards the relevance of practical fraction division to OBE is an encouraging point of departure. The ideas of the teacher from school B on aspects of practical fraction division that OBE workshops should address just about sums up all the teachers' needs in this regard (see subsection 4.2.3). Such workshops should also ground teachers in more profound aspects of the concepts of fractions and fraction division (e.g. other fraction perspectives and fraction division situations).

6.3 TEACHING IMPLICATIONS

The implications of findings of this study are discussed under the following subheadings: (1) whole number division, (2) concrete experience: a point of departure and (3) accommodating learners' problems.

6.3.1 Whole number division

Subsection 5.4.1 mentioned how the introductory exercise on whole number division helped to enhance and consolidate learners' understanding of the measurement interpretation of division. The exercise also successfully prepared learners for fraction division tasks in worksheets. The positive results of introducing fraction division with whole number division make it imperative to continue using whole numbers to explain the meaning of fraction division situations. To broaden learners' understanding of the meaning of the division concept, it is important not to confine

learners' understanding to the measurement interpretation of division. Instruction needs to ensure that learners' understanding is extended to other division interpretations. These were discussed in subsection 2.4.2. Whole numbers will continue to play an important role in assisting learners' meaningful understanding of other division situations. Therefore instruction on fraction division continues to rely on understanding whole number division and should continue to use whole numbers as a starting point.

6.3.2 Concrete experience: a point of departure

Instruction needs to capitalize on learners' attraction towards the subset perspective i.e. bottle-tops, to extend learners' understanding of fractions. This is especially against difficulties learners experienced in the identification and representation of the subset perspective of fractions, especially the equivalent type (see subsection 4.4.1). Use of the subset perspective should not be limited to understanding fractions. It should be extended to help learners understand fraction division situations. Just like it was possible for learners to meaningfully experience the measurement meaning of fraction division through use of the subset perspective of the fraction, learners should be assisted with understanding the sharing/partitive and other meanings of division using practical representations of fractions. That this is not an easy task is supported by the view that "...a review of literature indicates that the partitive meaning for division has almost been totally ignored... The partitive meaning of division of fractions has been very resistant to clear concrete explanations" (Ott, Snook & Gibson, 1991, p.8). This calls for a commitment from teachers to seek and design effective strategies to help learners with the understanding of partitive and other meanings of fraction division. For them to be successful, teachers' efforts in this

regard need to be the overall outcome of teacher training initiatives both at pre-service and in-service levels.

6.3.3 Accommodating learners' problems

The design of teaching programmes and sessions should anticipate problems that learners are likely to encounter in their division of fractions by practical means.

Learners' problems that should be accommodated and addressed by instruction are:

- (1) the remainder problem, (2) overemphasis of the part-region perspective, (3) familiar and unfamiliar fractions, (4) the problem with graded instruments, and (5) the algorithm problem.

6.4 IMPLICATIONS FOR FURTHER RESEARCH

The following themes for further research are suggested:

- (1) Practical solution of real life fraction division problems.
- (2) Resolving the accuracy problem of graded instruments.
- (3) Extension of practical fraction division to multiples of fractions, and
- (4) Practical work in division situations other than the measurement interpretation.

APPENDIX A

QUESTIONNAIRE ON PRACTICAL WORK IN MATHEMATICS EDUCATION

TARGET GROUP OF RESPONDENTS: Grade 6, 7 and 8 Mathematics educators

This questionnaire is part of the overall study that investigates the effectiveness of practical work in the teaching of the division of fractions to grade 7 learners at two senior primary schools in Mpumalanga circuit, Hammarsdale. With the aid of the questionnaire, the researcher seeks to establish in detail, the perceptions of the above-mentioned group of educators on the use of practical work in the teaching of fractions, to grade 7 learners in particular. Your cooperation by taking your time to answer questions in this questionnaire will be highly appreciated. To answer the questions, please put a cross in the appropriate box or give a written reply where applicable.

Respondents are assured of the following:

1. Information provided by respondents will remain confidential and will not be used for any purposes other than those intended for this study.
2. To protect their identity, respondents are not required to give their names, surnames nor addresses.
3. To ensure that information provided is not traceable back to respondents, data will be handled on an aggregate basis i.e. no data will be dealt with as an individual case.

PART 1

This section of the questionnaire is designed to assist the researcher to build a **personal profile of the respondent as a Mathematics educator**. To answer the questions, please put a cross in the appropriate box or give a written reply where appropriate.

1) Please give an indication of your gender.

1. Male ☐
2. Female ☐

2) Indicate the grades to which you teach Mathematics at present.

1. Grade 1 ☐
2. Grade 2 ☐
3. Grade 3 ☐
4. Grade 4 ☐
5. Grade 5 ☐
6. Grade 6 ☐
7. Grade 7 ☐
8. Grade 8 ☐
9. Grade 9 ☐
10. Grade 10 ☐
11. Grade 11 ☐
12. Grade 12 ☐

3) Write down any other grades that you have taught before.

1. _____
2. _____
3. _____
4. _____
5. _____

4) Indicate the number of years that you have taught each of the following grades for.

1. Grade 1

2. Grade 2

3. Grade 3

4. Grade 4

5. Grade 5

6. Grade 6

7. Grade 7

8. Grade 8

9. Grade 9

10. Grade 10

11. Grade 11

12. Grade 12

5) What age group do you belong to?

1. 00- 19 yrs. ☐
2. 20- 29 yrs. ☐
3. 30- 39 yrs. ☐
4. 40- 49 yrs. ☐
5. 50- 59 yrs. ☐
6. 60 yrs. and above ☐

6) What type of school do you teach in?

1. Urban ☐
2. Rural ☐
3. Township ☐
4. Farm ☐

- 7) What is your highest qualification in Mathematics education?
1. Certificate in education ☐
 2. Diploma in education ☐
 3. Further Diploma in education ☐
 4. Advanced Certificate in education ☐
 5. Degree in education ☐
 6. Post-graduate Degree in education ☐
 7. None ☐
 8. Other. Specify. _____
- 8) From which of the following institutions did you obtain your qualifications?
1. College of education ☐
 2. University ☐
 3. Technikon ☐
 4. Private College ☐
 5. Not qualified ☐
 6. Other. Specify. _____

PART 2

This section of the questionnaire aims to establish the level of your **training in the use of practical work** in the teaching of Mathematics in general, and fractions in particular. Training includes both pre-service and in-service training received up to now. To answer questions, please put a cross in the appropriate box or give a written reply where applicable.

- 1) Fractions offer enough opportunities for learning Mathematics through practical work.
1. Strongly disagree ☐
 2. Disagree ☐
 3. Neither agree nor disagree ☐
 4. Agree ☐
 5. Strongly agree ☐
- 2) Did your teacher-training course include the use of practical work in the teaching of Mathematics?
1. Yes ☐
 2. No ☐
- If the answer is No, please go to Q.4
- 3) Did the course on practical work include materials development?
1. Yes ☐
 2. No ☐
- 4) Have you ever received any form of training in the teaching of fractions through practical work?
1. Yes ☐
 2. No ☐

5) Have you ever attended an in-service course on the use of practical work in the teaching of fractions?

1. Yes ☐
2. No ☐

If the answer is No, please go to Q.7

6) Which of the following materials would you be happy to use in a practical lesson on fractions?

1. Paper-folding ☐
2. Graded instruments e.g. a drawing ruler ☐
3. Marked beakers filled with water ☐
4. Matter in the form of particles e.g. sand ☐
5. Groups of similar objects e.g. marbles ☐
6. Pictures or diagrams ☐
7. Worksheets ☐
8. Other. Explain _____

7) Practical work has a place in the teaching of fractions.

1. Strongly disagree ☐
2. Disagree ☐
3. Neither agree nor disagree ☐
4. Agree ☐
5. Strongly agree ☐

PART 3

This section is intended to inform the researcher about the **current practices of educators when they teach fractions**. To answer questions, please put a cross in the appropriate box or give a written reply where applicable.

1) How often do you include practical activities when teaching Mathematics?

1. Always ☐
2. Most of the time ☐
3. Sometimes ☐
4. Never ☐

2) Have you ever tried practical work in teaching fractions?

1. Yes ☐
2. No ☐

If the answer is No, please go to Q.4

3) How often do you include practical work when teaching fractions?

1. Always ☐
2. Most of the time ☐
3. Sometimes ☐
4. Never ☐

4) Would you recommend the use of practical work in teaching fractions?

1. Yes ☐
2. No ☐

If the answer is No, please go to the next section.

5) What materials would you prefer to use in a lesson on fractions?

1. Paper-folding ☐
2. Graded instruments e.g. a drawing ruler ☐
3. Marked beakers filled with water ☐
4. Sand ☐
5. Groups of similar objects e.g. marbles ☐
6. Pictures or diagrams ☐
7. Worksheets ☐
8. Other. Specify _____

6) State your reasons for your choices in Q.5

1. Paper-folding _____
2. Graded instruments _____
3. Beakers with water _____
4. Sand particles _____
5. Groups of similar objects e.g. marbles _____
6. Pictures or diagrams _____
7. Worksheets _____
8. Other. Specify and then state reason _____

7) Which of the following aids do you prefer to use when you teach operations on fractions?

1. The number line ☐
2. Diagrams of various shapes ☐
3. A rule or set of rules given in the book ☐
4. Physical objects that learners can handle ☐
5. Other. Specify _____

8) Although most mathematics educators agree to the value of practical work in mathematics education, most of them find it difficult to include this in their lessons. What do you think is the reason behind this?

PART 4

This section aims to help the researcher to establish the **beliefs that inform educators when they teach fractions**. To answer questions, please put a cross in the appropriate box or give a written reply where applicable.

1) The main objective of any teaching session should be the understanding of mathematical concepts by learners, rather than the completion of the syllabus.

1. Strongly disagree ☐
2. Disagree ☐
3. Neither agree nor disagree ☐
4. Agree ☐
5. Strongly agree ☐

2) Learning activities that require learners to do practical work are a waste of valuable teaching time.

1. Strongly disagree ☐
2. Disagree ☐
3. Neither agree nor disagree ☐
4. Agree ☐
5. Strongly agree ☐

3) Engaging learners in practical activities when teaching fractions fits well with OBE requirements for a learner-centred approach.

1. Strongly disagree ☐
2. Disagree ☐
3. Neither agree nor disagree ☐
4. Agree ☐
5. Strongly agree ☐

4) Practical work makes a huge contribution to a better understanding of fractions by learners.

1. Strongly disagree ☐
2. Disagree ☐
3. Neither agree nor disagree ☐
4. Agree ☐
5. Strongly agree ☐

5) When teaching fractions, the teacher's overall objective should be:

1. To finish the syllabus ☐
2. Understanding of mathematical concepts by learners ☐
3. Both 1 and 2 ☐
4. None of the above ☐
5. Other. Specify. _____

6) Learners can learn fractions better by handling physical objects that represent fractions.

1. Strongly disagree ☐
2. Disagree ☐
3. Neither agree nor disagree ☐
4. Agree ☐
5. Strongly agree ☐

PART 5

This section is intended to inform the researcher about **the challenges and needs of educators** in the use of practical work for the teaching of fractions. To answer questions, please put a cross in the appropriate box or give a written reply where applicable.

1) Preparing materials for practical work is a tiring long process.

1. Strongly disagree ☐
2. Disagree ☐
3. Neither agree nor disagree ☐
4. Agree ☐
5. Strongly agree ☐

2) Obtaining materials for practical work is difficult.

1. Strongly disagree ☐
2. Disagree ☐
3. Neither agree nor disagree ☐
4. Agree ☐
5. Strongly agree ☐

3) Obtaining materials for practical work is an expensive exercise.

1. Strongly disagree ☐
2. Disagree ☐
3. Neither agree nor disagree ☐
4. Agree ☐
5. Strongly agree ☐

4) Would you like to see more workshops for educators on practical work and the teaching of fractions?

1. Yes ☐
2. No ☐

If the answer is No, please go to Q.6.

5) Please put a cross in the box to indicate any issue or issues you would like a workshop on practical work and the teaching of fractions to include

- | | |
|-------------------------------------|--------------------------|
| 1. Development of materials | <input type="checkbox"/> |
| 2. Readily available materials | <input type="checkbox"/> |
| 3. Easily accessible materials | <input type="checkbox"/> |
| 4. Learner activities | <input type="checkbox"/> |
| 5. Model lessons | <input type="checkbox"/> |
| 6. Teacher's role during the lesson | <input type="checkbox"/> |
| 7. Assessment of practical work | <input type="checkbox"/> |
| 8. Lesson preparation | <input type="checkbox"/> |
| 9. Other. Explain. | |

6) OBE workshops in Mathematics education should put more emphasis on practical work.

- | | |
|-------------------------------|--------------------------|
| 1. Strongly disagree | <input type="checkbox"/> |
| 2. Disagree | <input type="checkbox"/> |
| 3. Neither agree nor disagree | <input type="checkbox"/> |
| 4. Agree | <input type="checkbox"/> |
| 5. Strongly agree | <input type="checkbox"/> |

7) What would you like a workshop on practical work and the teaching of fractions to address, and why?

Note: The questionnaire has come to an end. Please go through the questionnaire again and check if no question has been unintentionally left unanswered. Thank you for your participation. Your valuable contribution is highly appreciated. Should you be interested in the major findings after analysis of data, do not hesitate to contact me. The contact number is provided in the covering letter

APPENDIX B

WORKSHEET 1

Follow the instructions below carefully.

- a) Use a ruler of 300mm to do the tasks which follow.
- b) Write the correct length of the ruler that makes each of the following fractions given below.
- c) Write your answer in the space next to the given fraction.

1) 1 whole ruler = ----- mm

2) $\frac{1}{2}$ of the ruler = ----- mm

3) $\frac{1}{4}$ of the ruler = ----- mm

4) $\frac{3}{4}$ of the ruler = ----- mm

5) $\frac{1}{3}$ of the ruler = ----- mm

6) $\frac{2}{3}$ of the ruler = ----- mm

WORKSHEET 2

Follow the instructions below carefully.

- a) Use a group of 12 bottle-tops to do the tasks which follow.
- b) Use the 12 bottle-tops that you were given to form smaller groups that represent each of the following fractions.
- c) Write the total number of bottle-tops for each fraction in the space next to that fraction.

1) 1 whole group of 12 bottle-tops = ----- bottle-tops.

2) $\frac{1}{4}$ of 12 bottle-tops = ----- bottle-tops

3) $\frac{1}{2}$ of 12 bottle-tops = ----- bottle-tops

4) $\frac{3}{4}$ of 12 bottle-tops = -----bottle-tops

5) $\frac{1}{3}$ of 12 bottle-tops = -----bottle-tops

6) $\frac{2}{3}$ of 12 bottle-tops = -----bottle-tops

WORKSHEET 3

Follow the instructions below carefully.

- a) In worksheet 1 you used a ruler to find given fractions. Now use a ruler of 300mm to find answers to the following divisions by fractions.
- b) Write your answer to each question in the space given.
- c) You can go back to correct answers for worksheet 1 and use them to do the following tasks.

TASK 1: Finding $\frac{1}{2}$ of your ruler.

1) 1 whole ruler = ----- mm

2) $\frac{1}{2}$ of the ruler = ----- mm

3) How many times does $\frac{1}{2}$ of the ruler appear in 1 complete ruler at one given time?

Answer: -----

4) Therefore $1 \div \frac{1}{2} = \text{-----}$

TASK 2: Finding $\frac{1}{4}$ of your ruler.

1) 1 whole ruler = ----- mm

2) $\frac{1}{4}$ of the ruler = ----- mm

3) How many times does $\frac{1}{4}$ of the ruler appear in 1 complete ruler at one given time?

Answer: -----

4) Therefore $1 \div \frac{1}{4} = \text{-----}$

WORKSHEET 4

Follow the instructions below carefully.

- a) In worksheet 1 you used a ruler to find given fractions. Now use a ruler of 300mm to find answers in the following tasks on division of fractions.
- b) Write your answer to each question in the space given.
- c) You can go back to correct answers for worksheet 1 and use them to do the tasks which follow.

TASK 1: Finding $\frac{1}{4}$ in $\frac{1}{2}$ of your ruler.

1) $\frac{1}{2}$ of the ruler = ----- mm.

2) $\frac{1}{4}$ of the ruler = ----- mm.

3) How many times does $\frac{1}{4}$ appear in $\frac{1}{2}$ of the ruler at one given time?

Answer: -----

4) Therefore $\frac{1}{2} \div \frac{1}{4} = \text{-----}$

TASK 2: Finding $\frac{1}{3}$ in $\frac{2}{3}$ of your ruler.

1) $\frac{1}{3}$ of the ruler = ----- mm

2) $\frac{2}{3}$ of the ruler = ----- mm

3) How many times does $\frac{1}{3}$ appear in $\frac{2}{3}$ of the ruler at one given time?

Answer: -----

4) Therefore $\frac{2}{3} \div \frac{1}{3} = \text{-----}$

WORKSHEET 5

Follow the instructions below carefully.

- In worksheet 1 you used a ruler to find given fractions. Now use a ruler of 300 mm to do the following tasks on the division of fractions.
- Write your answer to each question in the space given.
- You can go back to correct answers for worksheet 1 and use them to do these tasks.

TASK 1: Finding $\frac{1}{2}$ in $\frac{1}{4}$ of your ruler.

- $\frac{1}{4}$ of the ruler = ----- mm.
- $\frac{1}{2}$ of the ruler = ----- mm.
- Because $\frac{1}{2}$ is bigger than $\frac{1}{4}$, a complete $\frac{1}{2}$ of the ruler cannot be found in $\frac{1}{4}$ of the ruler. Only a fraction of $\frac{1}{2}$ the ruler can be found in $\frac{1}{4}$ of the ruler. What fraction of $\frac{1}{2}$ the ruler can be found in $\frac{1}{4}$ of the ruler?
Answer: -----

4) Therefore $\frac{1}{4} \div \frac{1}{2} = \text{-----}$

TASK 2: Finding $\frac{1}{2}$ in $\frac{1}{3}$ of your ruler.

- $\frac{1}{3}$ of the ruler = ----- mm.
- $\frac{1}{2}$ of the ruler = ----- mm.
- Because $\frac{1}{2}$ is bigger than $\frac{1}{3}$, a complete $\frac{1}{2}$ of the ruler cannot be found in $\frac{1}{3}$ of the ruler. Only a fraction of $\frac{1}{2}$ can be found in $\frac{1}{3}$ of the ruler. What fraction of $\frac{1}{2}$ the ruler can be found in $\frac{1}{3}$ of the ruler?
Answer: -----

4) Therefore $\frac{1}{3} \div \frac{1}{2} = \text{-----}$

WORKSHEET 6

Follow the instructions below carefully.

- 1) In worksheet 1 you used a ruler to find given fractions. Now use a ruler of 300mm to do the following tasks on division of fractions.
- 2) Write your answer to each question in the space given.
- 3) You can go back to correct answers for worksheet 1 and use them to do these tasks.

TASK 1: Finding $\frac{1}{2}$ in $\frac{2}{3}$ of your ruler.

1) $\frac{1}{2}$ of the ruler = ----- mm

2) $\frac{2}{3}$ of the ruler = ----- mm

3) How many times does $\frac{1}{2}$ appear in $\frac{2}{3}$ of the ruler at one given time?

Answer: -----

4) The remaining part of $\frac{2}{3}$ does not make another complete $\frac{1}{2}$. What fraction of $\frac{1}{2}$ does the remaining part of $\frac{2}{3}$ of the ruler make?

Answer: -----

5) Therefore $\frac{2}{3} \div \frac{1}{2} =$ -----

TASK 2: Finding $\frac{1}{3}$ in $\frac{3}{4}$ of your ruler

1) $\frac{1}{3}$ of the ruler = ----- mm

2) $\frac{3}{4}$ of the ruler = ----- mm

3) How many times does $\frac{1}{3}$ appear in $\frac{3}{4}$ of the ruler at one given time?

Answer: -----

4) The remaining part of $\frac{3}{4}$ does not make another complete $\frac{1}{3}$. What fraction of $\frac{1}{3}$ does the remaining part of $\frac{3}{4}$ of the ruler make?

Answer: -----

5) Therefore $\frac{3}{4} \div \frac{1}{3} =$ -----

WORKSHEET 7

Follow the instructions below carefully.

- 1) Use the group of 12 bottle-tops given to you to do the following tasks.
- 2) You can go back to correct answers for worksheet 2 and use them to do these tasks.
- 3) To answer questions, circle the letter of the correct answer or write the correct answer in the given space.

TASK 1: Finding a group of 12 bottle-tops.

- 1) Which of the following groups of buttons represents 1 whole of 12 bottle-tops?

a. • •
• •
• •
• •
• •
• •

b. • •
• •
• •
• •

c. • •
• •
• •

d. • •
• •

TASK 2: Finding groups of $\frac{1}{2}$ of 12 bottle-tops.

- 1) Make groups of $\frac{1}{2}$ of 12 bottle-tops.
- 2) Which of the following groups is $\frac{1}{2}$ of 12 bottle-tops?

a. • •
• •
• •
• •

b. • •
• •
• •

c. • •
• •

d. • •
•

- 3) How many buttons make $\frac{1}{2}$ of 12 bottle-tops?

a. 3

b. 4

c. 6

d. 8

- 4) How many groups of $\frac{1}{2}$ of 12 bottle-tops do you get from 12 bottle-tops?

a. 2

b. 3

c. 4

d. 6

- 5) Use your answer to question 4 to complete the following:

$$1 \div \frac{1}{2} = \text{-----}$$

WORKSHEET 8

Follow the instructions below carefully.

- 1) Use the set of 12 bottle-tops given to you to do the following tasks.
- 2) You can go back to correct answers for worksheet 2 and use them to do these tasks.
- 3) To answer questions, circle the letter of the correct answer or write the correct answer in the space given.

TASK 1: Finding groups of $\frac{1}{2}$ of 12 bottle-tops.

- 1) Make groups of $\frac{1}{2}$ of 12 bottle-tops.
- 2) How many groups do you get?
a. 2 b. 3 c. 4 d. 6
- 3) How many bottle-tops does each group have?
a. 3 b. 4 c. 2 d. 6

TASK 2: Finding groups of $\frac{1}{4}$ in $\frac{1}{2}$ of 12 bottle-tops.

- 1) Go back to your groups of $\frac{1}{2}$ of 12 bottle-tops that you made in task 1. From one of these groups, make other groups that represent $\frac{1}{4}$ of 12 bottle-tops.
- 2) How many bottle-tops make $\frac{1}{4}$ of 12 bottle-tops?
a. 3 b. 4 c. 2 d. 6
- 3) How many groups of $\frac{1}{4}$ of 12 bottle-tops does one group of $\frac{1}{2}$ of 12 bottle-tops have?
a. 2 b. 6 c. 3 d. 4
- 4) Use your answer to question 3 to complete the following:
$$\frac{1}{2} \div \frac{1}{4} = \text{-----}$$

WORKSHEET 9

Follow the instructions below carefully.

- 1) In worksheet 2 you used a group of 12 bottle-tops to find given fractions. Now use your group of 12 bottle-tops to do the following tasks on the division of fractions.
- 2) Write your answer to each question in the space given.
- 3) You can go back to correct answers for worksheet 2 and use them to do these tasks.

TASK 1: Finding $\frac{1}{2}$ of 12 bottle-tops in $\frac{1}{4}$ of 12 bottle-tops.

- 1) Make a group that represents $\frac{1}{2}$ of 12 bottle-tops. Which of the following groups

are $\frac{1}{2}$ of 12 bottle-tops?

a. ● ● ●
● ● ●

b. ● ●
● ●

c. ● ●
●

d. ●

- 2) Make another group that represents $\frac{1}{4}$ of 12 bottle-tops. Which of the following

groups are $\frac{1}{4}$ of 12 bottle-tops?

a. ● ●
● ●

b. ● ●

c. ●

d. ● ●
●

- 3) Because $\frac{1}{2}$ is bigger than $\frac{1}{4}$, a complete $\frac{1}{2}$ of 12 bottle-tops cannot be found in $\frac{1}{4}$ of 12 bottle-tops. Only a fraction of $\frac{1}{2}$ of 12 bottle-tops is found in $\frac{1}{4}$ of 12 bottle-tops. What fraction of $\frac{1}{2}$ of 12 bottle-tops can be found in $\frac{1}{4}$ of 12 bottle-tops?

Answer:.....

- 4) Therefore $\frac{1}{4} \div \frac{1}{2} = \text{-----}$

TASK 2: Finding $\frac{1}{2}$ of 12 bottle-tops in $\frac{1}{3}$ of 12 bottle-tops.

- 1) Make a group that represents $\frac{1}{2}$ of 12 bottle-tops. Which of the following groups

is $\frac{1}{2}$ of 12 bottle-tops?

a. ● ● ● ●
● ● ● ●

b. ● ● ●
● ● ●

c. ● ●
● ●

d. ● ●

2) Make another group that represents $\frac{1}{3}$ of 12 bottle-tops. Which of the following groups is $\frac{1}{3}$ of 12 bottle-tops?

a. ●

b. ● ●

c. ● ●
●

d. ● ●
● ●

3) Because $\frac{1}{2}$ is bigger than $\frac{1}{3}$, a complete $\frac{1}{2}$ of 12 bottle-tops cannot be found in $\frac{1}{3}$ of bottle-tops. What fraction of $\frac{1}{2}$ of 12 bottle-tops can be found in $\frac{1}{3}$ of 12 bottle-tops?

Answer: -----

4) Therefore $\frac{1}{3} \div \frac{1}{2} = \text{-----}$

WORKSHEET 10

Follow the instructions below carefully.

- 1) Use the group of 12 bottle-tops given to you to do the following tasks.
- 2) You can go back to correct answers for worksheet 2 and use them to do these tasks.
- 3) To answer questions, circle the letter of the correct answer or write the correct answer in the given space.

TASK 1: Finding a group of 12 bottle-tops.

- 1) How many bottle-tops make 1 whole group of 12 bottle-tops?
- a. 9 b. 8 c. 6 d. 12

TASK 2: Finding $\frac{2}{3}$ of 12 bottle-tops.

- 1) Make 3 equal groups from the 12 bottle-tops given to you.

- 2) How many bottle-tops does each group have?

a. 3 b. 4 c. 8 d. 6

- 3) Which of the following groups is $\frac{2}{3}$ of 12 bottle-tops?

a. • • b. • • • c. • • • • d. • •
• • • • • • • • • •

- 4) How many groups of $\frac{2}{3}$ can you find from 12 bottle-tops?

Answer: -----

TASK 3: Dividing 1 by $\frac{2}{3}$

- 1) Go back to the three groups that you formed in 2 above. Put away all the bottle-tops that represent $\frac{2}{3}$ of 12 bottle-tops.

- 2) Which of the following groups represents bottle-tops that you are now left with?

a. • b. • • c. • • d. • • •
• • • • • • •

- 3) The bottle-tops that are left do not make a complete group of $\frac{2}{3}$ of 12 bottle-tops.

What fraction of $\frac{2}{3}$ of 12 bottle-tops do these remaining bottle-tops make?

a. $\frac{1}{2}$ b. $\frac{3}{4}$ c. $\frac{8}{8}$ d. $\frac{3}{5}$

- 4) Use your answers to questions 4 in task 2, and 3 above to complete the following:

$$1 \div \frac{2}{3} = \text{-----}$$

WORKSHEET 11

Follow the instructions below carefully.

- 1) Use the group of 12 bottle-tops given to you to do the following tasks.
- 2) You can go back to correct answers for worksheet 2 to do these tasks.
- 3) To answer questions, circle the letter of the correct answer or write the correct answer in the given space.

TASK 1: Finding a group of 12 bottle-tops.

1) How many bottle-tops make 1 whole group of 12 bottle-tops?

- a. 12 b. 6 c. 8 d. 4

TASK 2: Finding groups of $\frac{1}{2}$ and $\frac{2}{3}$ of 12 bottle-tops.

1) Make 3 equal groups from the 12 bottle-tops given to you and then form a group that shows $\frac{2}{3}$ of 12 bottle-tops. Which of the following groups is $\frac{2}{3}$ of 12 bottle-tops?

- a. $\begin{array}{c} \bullet \bullet \\ \bullet \end{array}$ b. $\begin{array}{c} \bullet \bullet \\ \bullet \bullet \end{array}$ c. $\begin{array}{c} \bullet \bullet \bullet \\ \bullet \bullet \bullet \end{array}$ d. $\begin{array}{c} \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \end{array}$

2) Which of the following groups is $\frac{1}{2}$ of 12 bottle-tops?

- a. $\begin{array}{c} \bullet \bullet \\ \bullet \bullet \end{array}$ b. $\begin{array}{c} \bullet \bullet \bullet \\ \bullet \bullet \bullet \end{array}$ c. $\begin{array}{c} \bullet \bullet \bullet \\ \bullet \bullet \bullet \\ \bullet \bullet \bullet \end{array}$ d. $\begin{array}{c} \bullet \bullet \\ \bullet \end{array}$

TASK3: Dividing $\frac{2}{3}$ of 12 bottle-tops by $\frac{1}{2}$.

1) Go back to your group of $\frac{2}{3}$ of 12 bottle-tops. How many groups of $\frac{1}{2}$ of 12 can you find in $\frac{2}{3}$ of 12 bottle-tops?

Answer: -----

2) Take away $\frac{1}{2}$ of 12 from your group of $\frac{2}{3}$ of 12 bottle-tops. Which of the following groups show bottle-tops that are left?

- a. $\begin{array}{c} \bullet \\ \bullet \end{array}$ b. $\begin{array}{c} \bullet \bullet \\ \bullet \end{array}$ c. $\begin{array}{c} \bullet \bullet \\ \bullet \bullet \end{array}$ d. \bullet

3) The bottle-tops that are left do not make a complete group of $\frac{1}{2}$ of 12 bottle-tops. What fraction of $\frac{1}{2}$ of 12 bottle-tops do bottle-tops that are left make?

- a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. $\frac{1}{3}$ d. $\frac{3}{4}$

4) Use your answers to 1 and 3 to complete the following: $\frac{2}{3} \div \frac{1}{2} = \text{-----}$

WORKSHEET 12

Follow the instructions below carefully.

- 1) Use the group of bottle-tops given to you to do the following tasks.
- 2) You can use correct answers for worksheet 2 to do these tasks.
- 3) To answer questions, circle the letter of the correct answer or write the correct answer in the given space?

TASK 1: Finding $\frac{3}{4}$ of 12 bottle-tops.

- 1) How many bottle-tops make 1 whole group of 12 bottle-tops?
a. 8 b. 4 c. 12 d. 6
- 2) Make 4 equal groups from the 12 bottle-tops given to you and then form a group that shows $\frac{3}{4}$ of 12 bottle-tops. Which of the following groups is $\frac{3}{4}$ of 12 bottle-tops?

a. •••••
•••••

b. ••
•

c. •••
•••

d. ••
••

TASK 2: Dividing $\frac{3}{4}$ of 12 bottle-tops by $\frac{1}{3}$.

- 1) Which of the following groups is $\frac{1}{3}$ of 12 bottle-tops?

a. •••
••

b. ••
•

c. ••
••

d. •••
•••

- 2) Go back to your group of $\frac{3}{4}$ of 12 bottle-tops. How many groups of $\frac{1}{3}$ of 12 bottle-tops can you find in $\frac{3}{4}$ of 12 bottle-tops?

Answer: -----

- 3) Take away groups of $\frac{1}{3}$ of 12 from your group of $\frac{3}{4}$ of 12 bottle-tops. Which of the following groups show bottle-tops that are left?

a. •
•

b. ••
••

c. ••
•

d. •

- 4) The bottle-tops that are left do not make a complete group of $\frac{1}{3}$ of 12 bottle-tops.

What fraction of $\frac{1}{3}$ of 12 do remaining bottle-tops make?

a. $\frac{1}{2}$

b. $\frac{1}{4}$

c. $\frac{2}{3}$

d. $\frac{1}{3}$

- 5) Use your answers to 2 and 4 to complete the following: $\frac{3}{4} \div \frac{1}{3} = \text{-----}$

APPENDIX C

TEST 1

GRADE 7

DIVISION OF FRACTIONS

MARKS : 25

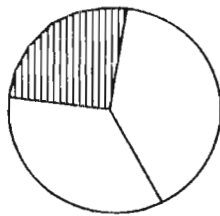
TIME : 1 Hour

INSTRUCTIONS

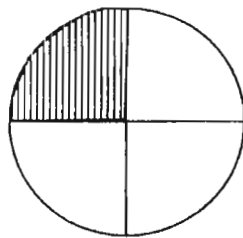
1. Answer the following questions by following instructions for each question carefully.
2. Answer all questions on the question paper.
3. Show all your work when you answer questions.
4. Use a pencil for circling letters for correct answers.
5. If you have circled the wrong letter, use an eraser to erase the wrong circling and then circle the correct letter.
6. Use a pencil to shade or draw diagrams.

1. Circle the letter of the figure in which the shaded part represents the given fraction.

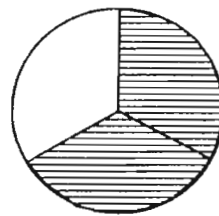
1.1) $\frac{1}{3}$



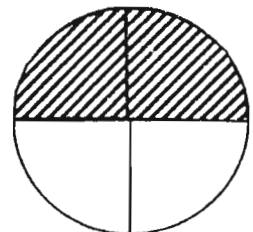
a.



b.

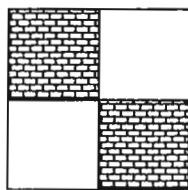


c.

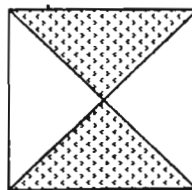


d.

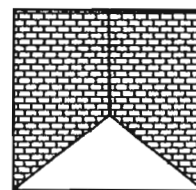
1.2) $\frac{2}{3}$



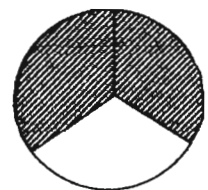
a.



b.



c.



d.

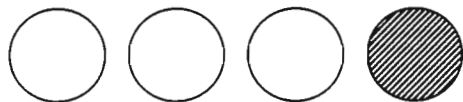
1.3) $\frac{1}{4}$



a.



b.



c.



d.

1.4) $\frac{1}{2}$



a.



b.



c.



d.

1.5) $\frac{3}{4}$



a.



b.



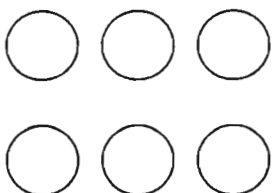
c.



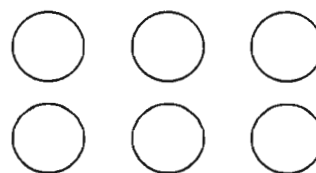
d.

2. Show the required fraction by shading the part or parts that represent that fraction.

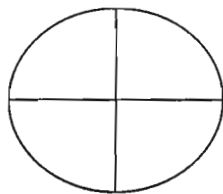
2.1) $\frac{1}{3}$



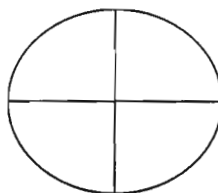
2.2) $\frac{2}{3}$



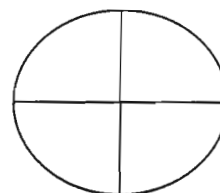
2.3) $\frac{1}{4}$



2.4) $\frac{1}{2}$



2.5) $\frac{3}{4}$



3. a) Find answers to the following problems on the division of fractions.
 b) You can draw diagrams to find answers to the problems.
 c) Write your solutions in the space given.
 d) Show all your work.

3.1) $1 \div \frac{1}{3}$

3.2) $\frac{2}{3} \div \frac{1}{3}$

3.3) $1 \div \frac{2}{3}$

3.4) $\frac{3}{4} \div \frac{1}{4}$

3.5) $\frac{3}{4} \div \frac{1}{2}$

APPENDIX D

TEST 2

GRADE 7

DIVISION OF FRACTIONS

MARKS : 20

TIME : 1 Hour

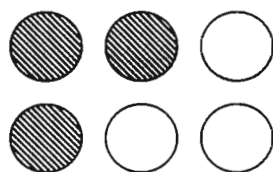
INSTRUCTIONS

1. Answer the following questions by following instructions for each question carefully.
2. Answer all questions on the question paper.
3. Show all your work when you answer questions.
4. Use a pencil for circling letters for correct answers.
5. If you have circled the wrong letter, use an eraser to erase the wrong circling and then circle the correct letter.
6. Use a pencil to shade or draw diagrams.

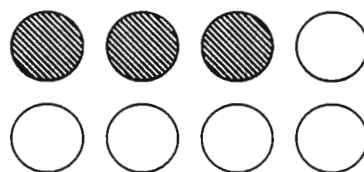
1. Circle the letter of the figure in which the shaded part represents the given fraction.

1.1)

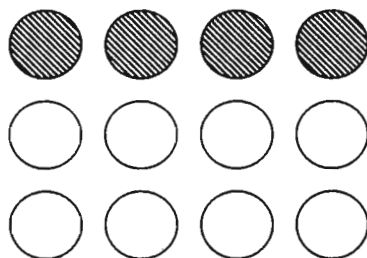
$$\frac{1}{3}$$



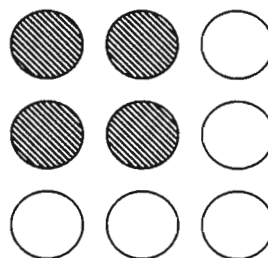
a.



b.



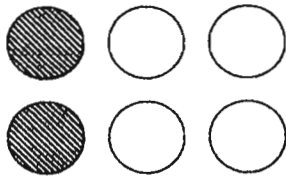
c.



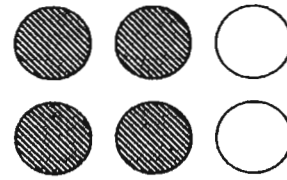
d.

1.2)

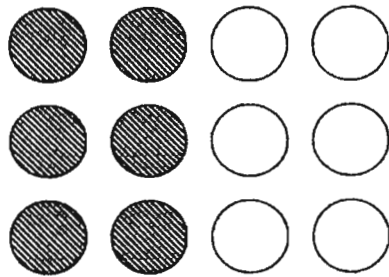
$$\frac{2}{3}$$



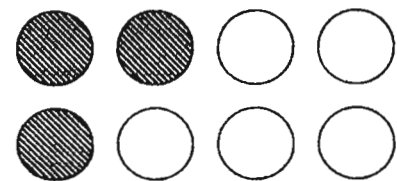
a.



b.



c.



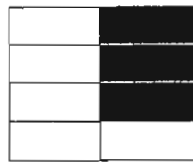
d.

1.3)

$$\frac{1}{4}$$



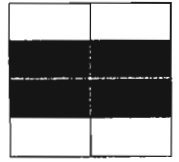
a.



b.



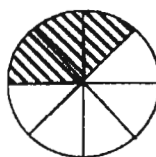
c.



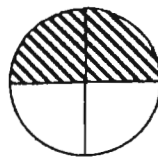
d.

1.4)

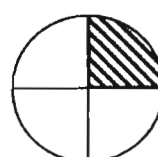
$$\frac{1}{2}$$



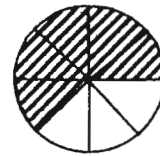
a.



b.



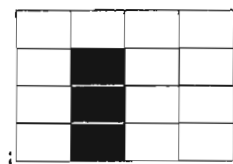
c.



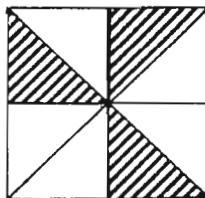
d.

1.5)

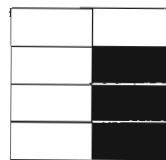
$$\frac{3}{4}$$



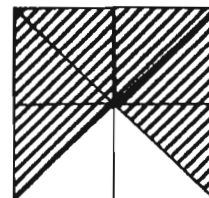
a.



b.



c.



d.

2. Show the required fraction by shading the part or parts that represent that fraction.

2.1)

$$\frac{1}{3}$$



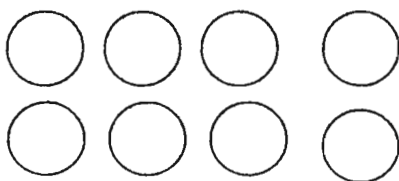
2.2)

$$\frac{2}{3}$$



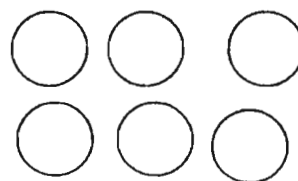
2.3)

$$\frac{1}{4}$$



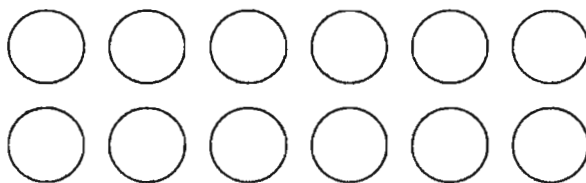
2.4)

$$\frac{1}{2}$$



2.5)

$$\frac{3}{4}$$



3. a) Find answers to the following problems on the division of fractions.
b) You can draw diagrams, or use a ruler or bottle-tops to find answers to these problems.
c) If you choose to use a ruler, use a ruler with 300mm.
d) If you choose to use bottle-tops, use a group of 12 bottle-tops.
e) Write your solutions in the space given.
f) Show all your work.

3.1) $1 \div \frac{1}{2}$

3.2) $1 \div \frac{1}{4}$

3.3) $\frac{1}{2} \div \frac{1}{4}$

$$3.4) \frac{2}{3} \div \frac{1}{3}$$

$$3.5) \frac{2}{3} \div \frac{1}{2}$$

$$3.6) \frac{3}{4} \div \frac{1}{3}$$

APPENDIX E

INTERVIEW QUESTIONS

The following were the interview questions:

- 1) Worksheets 1 and 2 asked you to show fractions using a ruler or bottle-tops. Which of the two instruments did you find easier to use, and why?
- 2) Most of you did not have problems when asked to show $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{3}$ with a ruler or bottle-tops. However most of you had difficulties with $\frac{3}{4}$ and $\frac{2}{3}$. Can you give reasons for this?
- 3) After showing fractions with a ruler or bottle-tops, you were asked to divide fractions with the same instruments. Which of the two instruments did you find easier to use, and why?
- 4) Most of you did not have difficulties to find solutions to division problems $1 \div \frac{1}{2}$, $1 \div \frac{1}{4}$, $\frac{1}{2} \div \frac{1}{4}$ and $\frac{2}{3} \div \frac{1}{3}$. What made it easy to find solutions to problems with these fractions?
- 5) Most of you had problems dividing fractions that gave mixed numbers as answers e.g. $\frac{2}{3} \div \frac{1}{2}$. Why?
- 6) We have seen different methods of dividing fractions. These include diagrams, the rule for dividing fractions, using the drawing ruler and bottle-tops. Which of these instruments makes it easier for you to understand and do division of fractions? Why?

APPENDIX F

Box 194
P.O. Hammarsdale
3700
20 Aug. 04

Dear Colleague

RE: Letter of Consent

I am an M.Ed student in Mathematics Education at the University of KwaZulu-Natal, and my course presently requires me to conduct research in an area of importance to the teaching and learning of mathematics. My area of interest is the use of practical work in the division of fractions by grade 7 learners, with a view to enhanced understanding of conceptual meanings involved. This requires me to work with experienced educators like you, people with the necessary expertise in the subject.

The research project requires me, amongst others, to:

- Observe a set of mathematics lessons on the division of fractions to establish the teaching practices of mathematics educators when they teach this section.
- Administer a questionnaire among mathematics educators to establish their views on practical work and the teaching of fractions.

Your cooperation in respect of the two areas mentioned above is invaluable and will be highly appreciated.

Your participation in this project is entirely voluntary, and should you at any stage wish to withdraw, you will be free to do so. You are assured of complete confidentiality of your identity as a participant in this project. No real names, either of persons or institutions, will be used in the write-up of the findings of this study. Also the findings of this study will be used for no purposes other than those of this study. Should you be interested in the findings of this study, these will be made available to you through the necessary arrangements.

In the event of you having any questions, you are free to contact one of my supervisors, Dr. D. Brijlall, at 031-2603491 (office hours).

Yours truly,
J.J.L. Molebale

Please read and sign

I, _____, fully understand the conditions of my participation in this project. I also understand that this participation is voluntary, and can be terminated as and when I think necessary. I also understand that no real names will be used in the write-up of this study.

Signature: -----

APPENDIX G₁

Box 194
P.O. Hammarsdale
3700
13 August 04

Mzali

Isicelo Sokusebenza nabantwana

Nginguthishela oqhuba izifundo zakhe ze-M.Ed (Mathematics Education) e-University of KwaZulu-Natal. Sengifike esigabeni sokuba ngenze ucwaningo ngayinoma iyiphi ingxenye ebalulekile ekufundweni kwe-Mathematics. Ngikhethe ukugxila ekufundweni kokuhlukaniswa kwamaqhezu (Division of fractions) ngabantwana baka-grade 7.

Kuloluphenyo ngidinga ukusebenza nabafundi baka-grade 7, omunye wabo okungumntwana wakho. Ngakhoke ngiyacela ungivumele ngisebenze nomntwana wakho, ehlangene nabanye afunda nabo. Ngiyakuqinisekisa ngokuthi asiyukubakhona isidingo sokudalulwa kwamagama abantwana uma sekukhishwa imiphumela, futhi imiphumela angeke isetshenziselwe ezinye izinhloso ngaphandle kwezaloluphenyo.

Ukubamba komntanakho iqhaza kuncike othandweni lwakho njengomzali, kanye nakuye umntwana uqobo lwakhe. Umntwana uyovumeleka ukuyeka ukuba yingxenye yalomsebenzi nayinoma yinini uma wena noma yena efisa kube njalo.

Uma kungenzeka ube nemibuzo mayelana naloludaba, uvumelekile ukushayela ucingo uthisha wami khona e-University of Kwazulu-Natal, u-Dr. D. Brijlall, kulenombolo:

031-2603491 (ngesikhathi somsebenzi).

Yimi Ozithobayo,
J.J.L. Molebale

Funda bese usayina

Mina _____, ongumzali ka _____, ngiyavuma ukuthi ngiyayiqonda kahle yonke into ebhalwe ngasenhla futhi ngiyahambisana nayo. Ngiyakuqonda futhi nokuthi umntanami akaphoqelekile ukuqhubeka nokuba yingxenye yalomsebenzi, nokuthi angahoxa noma nini uma yena noma mina sifisa kube njalo. Ngiyakuqonda futhi nokuthi amagama angempela ezingane kanye nesikole okube yingxenye yalomsebenzi, angeke adalululwe, kepha ayohlale evikelekile ngaso sonke isikhathi.

Signature: -----

APPENDIX G₂

ENGLISH VERSION OF APPENDIX G₁

Box 194
P.O. Hammarsdale
3700
13 August 2004

Dear Parent/Guardian

Request to work with your child

I am a practising teacher who is currently furthering his studies at M. Ed (Mathematics Education) level in the University of KwaZulu-Natal. I've reached a critical point in my studies where I'm required to conduct research on any important aspect of mathematics education of my choice. I've chosen to research the Division of Fractions by grade seven learners.

In my investigation I need to work with grade seven learners, one of whom is your child. I therefore request your permission to work with your child, in the company of other grade seven learners. You are assured that real names of participants will not be revealed upon the release of findings of the study. You are also assured that findings will not be used for any purposes other than those to do with the objectives of the study.

Your child's participation depends on your parental will and that of your child him or herself. His or her participation will be duly terminated if you and/or your child so wishes.

For further inquiries, you may contact Dr. D. Brijlall, my chief supervisor at the university at the following number: 031-2603491 (offices hours).

Yours truly,
J.J.L. Molebale

Read and sign

I _____, the parent/guardian of _____, hereby declare that I fully understand the contents of the above letter. I also understand that my child is under no compulsion to participate in the study and that his/her participation will be terminated at any moment if he/she or I so wishes. I also understand that real names will not be used in reporting the findings, but that these will always be protected.

Signature:-----

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