

EXPLORING PRE-SERVICE TEACHERS' KNOWLEDGE OF PROOF IN GEOMETRY

BY

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DEDICATION

This work is dedicated

To:

My daughters **Zenande** and **Abenathi**

My wife **Nokukhanya**

My grandma, **Estinah Kufakuzile Ndlovu (UMaNgema)**,

And

My late dad, **Samuel David Ndlovu**,

who were the original sources of my inspiration.

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The Pre-Service Teachers who contributed in this work, without their participation this research study would not be finalised.

DECLARATION

I, **Bongani Reginald Ndlovu**, declare that this is my own work, submitted in partial fulfilment of the degree of Master of Education at the University of KwaZulu-Natal. I further declare that this dissertation has never been submitted at any other university or institution for any purpose, academic or otherwise.

Bongani Reginald Ndlovu

Date

As the candidate's supervisor, I agree to the submission of this thesis/dissertation for submission.

Dr Jayaluxmi Naidoo

Date

PREFACE

The work described in this thesis was carried out from March 2011 to November 2012 under the supervision of Dr J. Naidoo from the Mathematics and Computer Science Education Cluster, University of KwaZulu-Natal.

This study represents original work by the author and has not otherwise been submitted in any form for any degree or diploma to any tertiary institution. Where use has been made of the work of others, it is duly acknowledged in the text.

Abbreviation and Acronyms

Abbreviation	Descriptions
AMESA	Association for Mathematics Education of South Africa
ANA	Annual National Assessment
PST	Pre-Service Teacher
PSTs	Pre-Service Teachers
FGI	Focus Group Interview
FGI 1	Focus Group Interview Number 1
FGI 2	Focus Group Interview Number 2
II	Individual Interview
UOH	University of Hope
UKZN	University of KwaZulu-Natal
CAPS	Curriculum and Assessment Policy Statement
DGS	Dynamic Geometry Software
DBE	Department of Basic Education
DoE	Department of Education
C2005	Curriculum 2005
RNCS	Revised National Curriculum Statement
FET	Further Education and Training
GET	General Education and Training
TIMSS	Third International Mathematics and Sciences Study

Age brackets

Key	Age bracket
A	Below 15
B	Between 15 and 20
C	Between 21 and 25
D	Between 26 and 30
E	Above 30

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	Recorded as	Code	Year Level	Gender	Race
1	Learner 1	PST 01	3 rd	Female	I
2	Learner 2	PST 02	3 rd	Female	W
3	Learner 3	PST 03	3 rd	Female	I
4	Learner 4	PST 04	3 rd	Female	I
5	Learner 5	PST 05	4 th	Female	I
6	Learner 6	PST 06	3 rd	Male	I
7	Learner 7	PST 07	3 rd	Male	A
8	Learner 8	PST 08	4 th	Male	A
9	Learner 9	PST 09	4 th	Male	A
10	Learner 10	PST 10	3 rd	Male	I
11	Learner 11	PST 11	3 rd	Female	A
12	Learner 12	PST 12	4 th	Female	A
13	Learner 13	PST 13	4 th	Male	A
14	Learner 14	PST 14	4 th	Female	A

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ABSTRACT

Over the past years geometry has posed a challenge to most learners in South African schools. The Government, in particular the Department of Basic Education (DBE), have tried and are still trying to implement new innovations and strategies for teaching mathematics more effectively. South Africa has experienced many changes in mathematics curriculum with an aim of placing the country on an equal footing with countries globally. This study was conducted while there was the implementation of the new Curriculum and Assessment Policy Statement (CAPS), which reinstated the geometry section within the curriculum. Geometry was relegated to an optional paper in mathematics in 2006, 2007 and 2008 in Grades 10, 11 and 12 respectively.

This study is framed within the theoretical framework lens of social constructivism and situated learning, and is located within the qualitative research paradigm. It takes the form of survey research in one of the universities in KwaZulu-Natal, South Africa. This university is referred to as the University of Hope (UOH) in this study to protect its identity. The main aim of this study was to explore the pre-service teachers' (PSTs) knowledge of proof in geometry. The study used qualitative analysis of data generated through a survey questionnaire, task-based worksheets and semi-structured interviews for both the focus group and individual interviews.

In total 180 PSTs completed task-based worksheets. Within this group of 180 students, 47 were 4th year students, 93 were 3rd year and 40 were 2nd year students. After the analysis of a task-based worksheet, a total of 20 participants from the 3rd and 4th year were invited to participate in focus group interviews.

The findings of the study exhibit that the PSTs have very little knowledge of proof in geometry. The study revealed that this lack of the knowledge stems from the knowledge proof in geometry the PSTs are exposed to at school level.

CHAPTER 1

INTRODUCTION

1.1 The Overview

The quality of learning and teaching in South Africa is a serious issue for teachers and the citizens for a long time (DBE, 2011a). The South African administration has decided to deal with this matter head-on through curriculum changes. This study has been conducted during the implementation of the new Curriculum and Assessment Policy Statement (CAPS). The implementation of CAPS for Grades R – 12 was scheduled as follows:

- 2012 implementation in Grades R – 3 and Grade 10;
- 2013 implementation in Grades 4 – 9 and Grade 11; and
- 2014 implementation in Grade 12 (DBE, 2011a, p. 5).

The new CAPS reinstated Euclidean geometry as a compulsory section of mathematics after it had previously been relegated to an optional paper. In the new CAPS for Grades 10 – 12, which is being implemented in 2012, mathematics is divided into two papers; Paper 1 and Paper 2. Paper 1 encompasses algebra, pattern and sequences, finance and growth, functions and graphs, differential calculus and probability, and Paper 2 incorporates statistics, analytical geometry, trigonometry, and Euclidean geometry (DoE, 2010).

1.2 Motivation of the study

The study focuses on PSTs' knowledge of proving geometry. Effective teaching in mathematics geometry in particular requires an understanding of what learners and teachers know and what learners need to know. Learners need to know how to overcome challenges and how to obtain support from the teachers. Teachers ought to be able to establish what learners understand and what knowledge learners need to understand with respect to mathematics content knowledge. Naidoo (2011) emphasises that teaching does not mean standing in front of the class and imparting knowledge; rather, teaching is the ability to find the perfect strategies to ensure that effective teaching and learning take place.

Based on the CAPS in Grade 10 geometry, the focus is on knowledge, investigation and the construction of conjectures about the properties of quadrilaterals. Hence the proving

of these conjectures is crucial. At Grade 11 level, the focus shifts to the investigation and proofs of the circles and in Grade 12 the focus is on proportionality and similarity (DBE, 2011b). This reiterates the importance of the knowledge of proof for teachers before they embark on teaching as their career.

1.3 Focus and purpose of study

The primary focus of the study was to explore the PSTs' knowledge of proof in geometry. Within the South African context, teachers have been using rote learning in the teaching of mathematics. However, research has demonstrated that meaningful learning takes place when learners construct knowledge for themselves. The features attributed to meaningful learning are the ability to find out knowledge for yourself, being able to perceive relations of the knowledge, the ability to be able to apply knowledge in solving real life problems, being able to communicate knowledge to other people and being able to want to know more, as argued by Nightingale and O'Neil (1994). The above characteristics may resonate with Kilpatrick, Swafford, and Findell's (2001) 'strands of mathematical proficiency' which highlight the importance and integration of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition, as further discussed by Hobden (2009).

This study attempts to explore some of the relevant viewpoints and to offer background information on the subject of the nature of investigation within mathematics education. Naidoo (2011, p. 3) cites Schoenfeld (2000) in distinguishing two purposes of research in mathematics education. These purposes are pure and applied purposes. The pure purpose of mathematics education is to know the environment of mathematical knowledge, and the accepted wisdom, teaching and learning, whereas the applied purpose is to use this understanding to improve mathematical instruction. Schoenfeld (2000) also maintains that the primary ways in which research in mathematics assists practitioners include: hypothetical viewpoints for understanding thinking, learning, and teaching; descriptions of aspects of cognition e.g. thinking mathematically; existence of proofs and descriptions of consequences of various forms of instruction. This study has a pure purpose as it aims to explore PSTs' knowledge when working with geometry. In addition, this study also demonstrates the applied purpose, when PSTs focus on their ability to use the knowledge learnt, during teaching practice.

1.4 Problem statement

Many researchers have expressed concerns about the poor performance in mathematics, especially in geometry (Mthembu, 2007; Singh, 2006). Mathematics has always had a large failure rate (Naidoo, 2011). From the experience of teaching mathematics, interactions with other mathematics teachers and informal discussions with learners and parents, it is commonly agreed that many learners find mathematics, especially geometry, difficult and irrelevant to their lives (Lee & Ginsburg, 2009). It is then no secret that teaching mathematics in schools has been a challenge to most teachers, as evidenced by the low pass rate for the past few years.

In 2010 the South African General Education and Training (GET) learners in Grades 3 and 6 wrote an Annual National Assessment (ANA) of tasks in numeracy (mathematics) and literacy. The results for ANA in Grades 3 and 6 in GET also confirm this poor performance in mathematics. These basic results were earmarked as the baseline which the department would work from. The current baseline in Grade 3 mathematics is at 43% and in Grade 6 mathematics, at 19%. The Department of Basic Education has set a new pass rate target of 60% in 2014 for both literacy and mathematics in Grades 1 – 9 (DBE, 2011a, p. 22).

These ANA results exhibit evidence of a wide gap between the pupil's knowledge and the teacher's purpose. Among the multiple factors which may promote this situation, is that teachers' focal point tends to be on the role of shapes in the teaching and learning of geometry and not on the proof. It is therefore, a general statement that many learners find geometry difficult to understand as compared to algebra. From interactions with mathematics teachers in mathematics workshops or in meetings, the researcher gathered that they think that learners have developed negative attitudes to Euclidean geometry; teachers also blame themselves for not being innovative enough in classrooms, and for not having adequate pedagogical content knowledge in proofs in geometry. Teachers believe that some of them still subscribe to traditional strategies of teaching. On discussions with the teachers, they consider chalk and talk as an outdated strategy for the teaching mathematics, more especially the teaching and learning of geometry. Another issue raised was the shortage of qualified mathematics teachers in South Africa, as a result of which most principals are compelled to appoint unqualified teachers to teach mathematics.

From the discussion with the teachers, the researcher gathered that the relegation of geometry to an optional paper 3 in 2006 created mixed feelings. Some teachers accepted the relegation with joy, and others felt that the relegation might create an irreparable damage to the learners and future teachers. However, the introduction of the new CAPS in 2012 which reinstated Euclidean geometry as a required section of mathematics in Grade 10 has received a warm welcome by most of the teachers.

1.5 The critical research questions

The study sought to find out about the knowledge of PSTs and how this knowledge is used when teaching proof in geometry. This study sought to provide answers to the critical research questions that follow:

1. What is pre-service teachers' knowledge of proof in geometry?
2. How do pre-service teachers use their knowledge of proof in geometry?
3. Why do pre-service teachers use their knowledge of proof in geometry in the way that they do?

1.6 Significance of the study

This study highlights the problem teachers and learners are facing in schools everyday. The study aims at assisting the PSTs, teachers in the field and interested educationists, to find more innovative ways of gaining knowledge and imparting this knowledge to the learners. It also assists people in understanding how geometry is being taught in schools. The study will assist educationists who have an interest in seeking alternative approaches to teaching proofs in geometry. The study draws awareness to the understanding of the nature of mathematical knowledge, thinking, teaching, learning and the use of this understanding to improve mathematical instruction.

1.7 Outline of the study

This study is organised into, six chapters, the references and appendices. The content of these chapters are briefly highlighted as follows.

Chapter one

Chapter one of this study provides an overview of the study and provides a brief description of what will be discussed in the subsequent chapters. Additionally, chapter one presents the focus and the purpose behind the study and briefly discusses the significance of teacher's mathematical knowledge in proof.

Chapter two

Chapter two provides a review of the related literature in the field of geometry in mathematics education. This chapter also describes the relevant issues regarding curriculum change in South Africa in relation to mathematics, and geometry in particular.

Chapter three

Chapter three highlights the research design, the methodology used and the procedures undertaken to complete this study. The chapter outlines the critical research questions of the study. The data collection process and instruments used are also discussed.

Chapter four

Chapter four describes the theoretical framework which underpins this study. This chapter also reveals what other researchers have written in relation to the area of investigation. The theoretical frameworks discussed are social constructivism and situated learning.

Chapter five

Chapter five presents the data, data analysis and an interpretation of the findings. The chapter explores the results, the themes discovered and the main findings of the study.

Chapter six

Chapter six summarises the main findings of the study and discusses how the researcher has attempted to achieve the aims of the study. The chapter explores the results and attempts to respond to the critical research questions. The chapter further highlights the possible limitations of the study.

1.8 . Conclusion

This chapter has highlighted the general idea of the study, the focus, the purpose, the problem statement, the critical research questions, significance of the study and the outline of the study. The next chapter presents details of the literature reviewed. Chapter two provides the key definitions and core concepts within geometry. The chapter further describes the mathematics and curriculum changes in South Africa. In addition the Van Hiele theory and the function of proof in geometry are also discussed.

CHAPTER 2

REVIEW OF RELATED LITERATURE

2.1 Introduction

The trend of the matriculation pass rate in South Africa for the past few years has not been very promising (Naidoo, 2006). The Third International Mathematics and Sciences Study (TIMSS) provides an indication of how South African learners perform in mathematics (Mullis, Martin, Foy & Arora, 2012). To improve the pass rate interventions and innovations are needed in the learning and teaching of mathematics (Naidoo, 2011). However, when curriculum innovations are initiated, various problems arise, for example, political and economical issues. In 1994 South Africa emerged from the apartheid era with a huge resource backlog. After the democratic election of 1994, there was a need for national reconstruction and development in the country. The increasing demands of South African society led to rapid changes in education.

Teaching mathematics has been a challenge to most teachers (Naidoo, 2011); this is indicated by the lower pass rate over the past years. As discussed in chapter one, in the South African context teachers have been using rote learning in the teaching of mathematics whereas much research has proved that meaningful learning takes place when learners construct knowledge for themselves. Mthembu (2007) states that the teaching method most prominent in traditional mathematics classrooms; promotes memorisation and not understanding. Understanding is essential and crucial for success in mathematics, especially geometry. Teachers of mathematics ought to ensure that their teaching is meaningful, as this will result in learners who are not passive but rather active knowledge gatherers. There is thus a need to develop teachers professionally, as well as the need to make changes to the South African curriculum.

2.2 Curriculum changes in South Africa

Au (2012) defines the curriculum as the structure to run a formal course of study that the students complete within a specific time frame. Generally, educational and academic specialists develop curricula that will be suitable for the country. This curriculum changes from time to time to keep in line with new developments of a country. Curriculum changes are also prompted by pressures that emanate from people of the country. Changes after 1994 in South Africa came with tremendous transformations within the education system. The acceptance of the Constitution of the Republic of

South Africa presented a basis for curriculum transformation and development in South African education. The National Department of Education in South Africa initiated movement from the apartheid curriculum with the introduction of a all-inclusive curriculum change. Curriculum 2005 was constructed around the theoretical principles of Outcomes-Based Education (OBE) (DoE, 2010). The new curriculum emphasised the significance of outcomes as an alternative to input; it emphasised learner-centeredness as a replacement for teacher-centeredness, and active learning as a substitute for passive learning. This signalled a revolutionary new way of teaching and learning in South African classrooms. This curriculum was called Curriculum 2005, because it was predicted that it would be put completely into practice and it would be applied in all compulsory school grades by the year 2005 (Weber, 2008).

For 15 years, the South African education system has been based on what was considered as a radically different approach to education, OBE. In retrospect, the implementation of OBE, did not work as envisioned partially, due to insufficient resources. OBE was a system that flourished in a small group context which focused on teacher support in the form of teacher aids and administrative staff. South Africa in 1994 was far behind other developing countries in terms of education, literacy and numeracy levels (Booi, 2000). The Third International Mathematics and Sciences Study (TIMSS) confirms how South African learners perform in mathematics (Mullis, et. al, 2012). This was due to policy in education that was meant to delimit participation of the underprivileged. The underprivileged would form part of the labour force instead of the skilled labour division (Akhurst & Sader, 2009). This education system also disempowered teachers, despite the recognised high quality of the Revised National Curriculum Statement (RNCS).

2.3 Mathematics in South Africa

The main objectives of South African Curriculum are that the National Curriculum Statement for Grades R – 12 favours knowledge, skills and values. The National Curriculum Statement for Grades R - 12 ensures that learners obtain and use knowledge and proficiency which are meaningful to their own lives, it aims at encouraging learner centeredness and context based mathematics that is relevant to the learners and their experiences (DoE, 2003). In addition, the South African curriculum endorses the idea of preparatory knowledge in local contexts, and being sensitive to global essentials. The rationale for this is to provide learners, irrespective of their social setting, economic

background, race, gender, physical ability and intellectual ability, with the knowledge, skills and values required for self-fulfilment, and socially meaningful participation in South Africa; to give access to higher education; to make it possible for students to move smoothly from education institutions to the workplace; and to provide employers with an adequate profile of a learner's abilities (DoE, 2010).

All previous South African mathematics curricula included geometry in most grades, but in 2006 - 2008, geometry was made optional for Grades 10 - 12. The implementation of the new CAPS in 2012 reinstated Euclidean geometry as a compulsory section of mathematics after it had previously been relegated to an optional Paper Three. The knowledge and teaching of mathematics, which geometry is a part of, depends on the understanding of basic aims and principles of the Curriculum and Assessment Policy Statement.

Grade 10 Euclidean Geometry requires learners to find out, discover and form conjectures about the characteristics of some triangles, quadrilaterals and other polygons. Learners are required to validate or prove conjectures using any logical method as well as to disprove false conjectures by producing counter-examples. In Grade 11 learners start to prove that theorems about the line drawn parallel to one side of a triangle divides the other two sides proportionally; they will also prove that equiangular triangles are similar and that triangles with sides in proportion are similar. In addition they will prove the Pythagorean Theorem by similar triangles, in order to understand and accept what they have learnt in earlier grades. In Grade 12 learners are required to investigate and prove the theorems of the geometry of circles as a mini-axiomatic system and they will also solve circle geometry problems, providing reasons for statements when required (DBE, 2011).

The demand for knowledge in mathematics and mathematics careers has grown phenomenally in South Africa. It is now seen in a new perspective, new sections are introduced, and technology is now being integrated in problem solving. Mathematical knowledge has found a wide range of new applications as disciplines which relates this subject with other subjects. Mathematics knowledge can integrate with subjects like mathematical literacy, physical sciences, geography, economics, accounting and many more. This importance of mathematics teaching has caused the government, universities and non-governmental organisation to initiate programmes and courses that will help teachers and learners in mathematics, including geometry.

2.4 Geometry

Geometry is a section of mathematics that has different applications in many careers that involve advanced applications such as art, architecture, interior design and science; however it also has its operations in different technical careers such as carpentry, plumbing and drawing, as well as every day life (Knight, 2006). Usiskin (2002) proposes that there are two reasons why geometry is important to teach: firstly, geometry connects mathematics with the real outside world and secondly, geometry enables ideas from other areas of mathematics to be pictured. French (2004) enlisted three reasons why geometry is included in learning and teaching: to extend spatial awareness, to develop the skills of reasoning, and to inform challenges and stimulation. In every day life and occupational careers many ideas and practices are learned from the school geometry and applied in the relevant field.

Usiskin (2002) pointed out that geometry is important not only to the outside field work around us, but also to other parts of mathematics. For instance, learning the distributive property can be related to a learner using area models. Geometry can be used to concretely illustrate this and other abstract concepts to learners. Since geometry is important for accepting the real world and other aspects in mathematics; this research sought to determine how teachers are equipped for teaching geometry in their classrooms.

Soanes and Stevenson (2009) define geometry as the branch of mathematics that deals with solids, surfaces, lines, points, angles, properties, measurements and relationships appropriate to them and their positions in space. This includes higher dimensional analogues. In mathematics learners learn arithmetic and algebra by doing it or by being taught to follow the procedures and rules. Geometry, however, is taught logically and needs the construction of knowledge. Ideas in geometry are created when the proof are being done. The learner begins with learning definitions, postulates, and primitive terms; then proves the way through the course. The reason for this goes back centuries, to the times of Euclid. Euclid's book called the *Elements* contains only proofs and the proof of one proposition after another (Naidoo, 2006). Serra (1997) states that a proof in geometry is done by considering a sequence of formal statements, and each of these statements should be supported by a valid reason, that begins with the set of given properties and concludes with a valid conclusion. The underlying principles for

including formal proof in geometry in the school is twofold: it is perceived as a medium for teaching and learning deductive reasoning, and also as a first encounter with a formal axiomatic system (Mudaly & De Villiers, 2004). Thus the long-established view of deductive discourse as merely a strategy for creating authenticity is being replaced by a more valid viewpoint that envisages different roles to deductive argument.

In a modern sense geometry includes any mathematical system that is developed from a set of statements that are called axioms or postulates. A proof is a formal written argument of the complete thinking procedures that are used to reach a valid conclusion. The steps of these procedures are supported by a theorem, postulate or definition verifying the validity of each step and explaining why these steps are achievable. A logical series of statements that establish the truth of a proposition is called a proof. The believable expression that some mathematical statement is necessarily correct and valid in mathematics is a proof. Proofs are obtained from deductive ways of thinking. De Villiers (2004) maintains that the problems that learners have with seeing a necessity for proof is well understood by all high school teachers - it is recognised without exceptions in all educational research as a problem in the teaching of proof. In geometry, teachers ought to teach proof with an aim of verifying the correctness of mathematical statements, and proof ought to be used mainly to remove the doubts about what they know. When teaching geometry the teacher ought to consider various factors, for examples, how learners understand, perceive and think about geometry. Geometry is a type of mathematics which deals with position, shapes and visualisation, it is therefore important for the teachers to consider the development of the level of thinking proposed by Van Hiele model.

2.4.1 The Van Hiele theory

In Mathematics education, the Van Hiele model is a theory that describes how learners learn geometry. In their research on a learner's understanding of geometry Dutch teachers at Utrecht University, in the Netherlands, Pierre M. Van Hiele and Diana Van Hiele-Geldof (1957) (a husband and the wife team) noticed the difficulties that their learners had in learning geometry. Through their observations they created a theory that is related to Vygotsky's Theory. It argues that learners have five levels of understanding in mathematics that are achieved by consecutive steps, starting with a lower level and culminating at a higher level (Knight, 2006). It is also noted that sometimes these levels

can be numbered from 0 to 4 as opposed to 1 to 5 (Luchin, 2006). The Van Hiele Theory describes how a majority of learners face problems in learning traditional school geometry in mathematics. The Van Hiele's suggested that the cause for learners failing school geometry was that the curriculum was taught at a level higher than that of the learners' level of understanding. This implies that there is no mutual understanding by the learners and the teacher, the learners are not able to understand the teacher and the teacher is unable to understand why the learners could not understand. Dina and Pierre Van Hiele summarised the general characteristics of the Van Hiele Theory as follows:

Table 2.1: Characteristics of the Van Hiele theory (Adapted from Crowley, 1987)

Characteristic	Description
The model is sequential.	Each level builds on the thinking strategies developed in the previous level. Learners ought to finish all preceding levels before arriving at next level.
Advancement through the levels.	This level is dependent upon achieving the thinking strategies of the previous level. No level should be skipped when planning learning activities for development of spatial thinking.
Not age-dependent.	All levels at this stage are not dependent on age, in the way in which Piaget described development.
Implicit ideas become explicit ideas.	At this level as thinking advances, geometric ideas and concepts that are only implied at a level become the objects of study at another level and so become explicit ideas.
Each level has its own language.	This implies that learners reasoning at different levels cannot understand each other's explanations even though they might be describing the same idea or shape, neither can they follow the reasoning of each other.
Instruction should match thinking.	If the learner is on one level of thinking and the teacher's language, curriculum content, materials and language, etc. are on a different level, learners will not understand the language that is being used and as such, their progress may be obstructed. Learning and progress from different levels is further more reliant upon instruction and opportunities for learning than upon age.

The Van Hiele model has gained wide recognition and acceptance and is usually referred to as part of the model and is dominant in mathematics education. The teachers' presentation of material ought to be within the certain levels so that the learner will understand what is being taught and progress to the next level (De Villiers, 2004).

2.4.2 The significance of Van Hiele Theory levels in mathematics

The performance of learners in geometry has never been good. Teachers report that learners favour algebra over geometry (Naidoo, 2006). However, one may not be sure whether algebra is indeed the learners' favourite, or due to the teachers' bias whereby the teachers present algebra well to learners. Thus it is difficult to locate whether the problem is with the learners or with the teachers. It may also happen that teachers present algebra more efficiently than geometry. Pierre Van Hiele's (1995) dissertation attempted to explain why geometry education is a challenge to learners (De Villiers, 1996). The reason for learners not to do well in the traditional geometry curriculum, according to Van Hiele (1995) is that the geometry curriculum is not presented at the same level as that of the learners. The five Van Hiele levels of thought in geometry, i.e. recognition (visualisation), analysis, ordering (informal deduction), deduction and rigour are explained below.

Level 1: Recognition (Visualisation)

Learners recognise figures by an overall visual appearance without identifying properties. At this level of understanding the learner is expected to identify, name, and reproduce a given shape. The appearance or the visual characteristics of the shape are recognised, not the properties. They are unable to distinguish the properties of shapes at this stage. That is why Clement and Battista (1992) called this level pre-recognition and labeled it *level 0*. Learners are able to identify the given shape because they are able to associate the shape with what they know. For example they will be able to identify a circle because it resembles a ball and a coin, or a shape is a rectangle because it resembles an rectangular shape like window, book, and so on. A square does not resemble the shape of the rectangle, and a rhombus does not resemble other parallelograms, and these shapes are then recognised differently in the learners' minds. Learners see these figures as a whole without analysing their properties (Burger & Shaughnessy, 1986).

Visualisation plays a vital role in the development of thinking and understanding in teaching and learning in mathematics and in the change from concrete thinking to abstract thinking with regard to problem solving – this promotes deductive reasoning in mathematics.

Level 2: Analysis.

Learners start to classify properties of some different shapes and analyse the properties of the given figures; however at this stage learners are not making any connections between different shapes and their properties (Mason, 2010). At this level learners should be able to recognise, investigate, understand, deduce and make generalisations from the properties of the figures. A learner at this level is able to recognise that a square is a figure which has 4 equal sides and 4 equal angles. The diagonals of a square are equal and perpendicular bisectors. Learners at this level think inductively from examples; however they cannot at this stage think deductively because they still lack ideas of how the properties of shapes are related to each other.

Level 3: Ordering (informal deduction).

In this level of understanding the learners start putting the properties of the figure in correct order. At this stage learners are in a position to follow all the logical arguments. When learners follow these arguments the properties of the figures are used. Learners develop deductive thinking at this stage.

Level 4: Deduction.

At this stage learners start to develop series of statements and start to understand the importance of deduction and the vital role of axioms, theorems and proof. Learners at this level begin to understand the meaning of deduction. Learners are now learning to do formal proofs. Learners can develop geometric proofs at a high school level and understand the meaning of proofs. They understand the role that is played by terminology, definitions, axioms and theorems in Euclidean geometry. However, learners at this level think that axioms and definitions are not arbitrary but are fixed, thus reducing geometric thinking to objects in the Euclidean plane only.

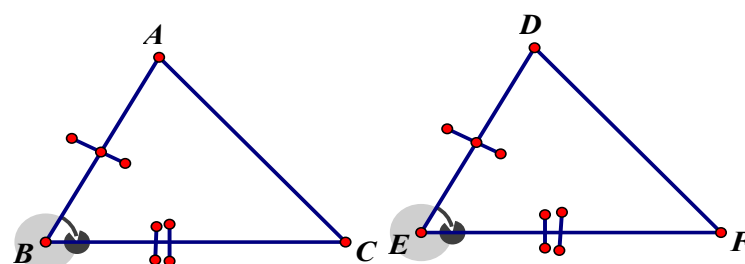


Figure 2.1: Two congruent triangles

Learners at this level can construct proofs in an axiomatic system. For example, from the above two triangles a learner can prove that if AB and BC and the included angle, $\hat{B}$ of one triangle are equal with the corresponding sides DE and EF and an angle $\hat{E}$ of another triangle, then the 2 triangles are congruent. However, at this level of thinking a learner cannot:

- Understand the importance of deduction or the role of axioms.
- Understand how to construct or alter a logical proof.
- Create a proof starting from unfamiliar or different premises

Level 5: Rigour.

At this level learners are able to reason formally about mathematical systems and they are in a position to understand geometric figures which are abstract. Level 5 is the highest level of thinking in the Van Hiele hierarchy. Learners at level 5 can engage in different geometric shapes and axiomatic systems. Learners who portray an understanding of this level are most likely to be enrolled in a university level course in geometry.

The Van Hiele levels of thought in geometry are sequential, and learners should progress from all preceding levels to arrive at any specific next level. These levels are not dependent on age, unlike the age-related way in which Piaget described stages of cognitive development. Luchin (2006) warns that teachers and learners should be aware that for the purposes of developing the knowledge and geometric thinking of the primary school learner, only the first two levels are considered. Research has shown that it takes approximately the whole of primary school to develop the first two thought levels completely. The development of learners' geometric thinking also includes the acquisition of technical terminology for describing properties of figures. The specific content to be focussed on in each of the grades of schooling is outlined in the assessment standards in the Revised National Curriculum Statements.

De Villiers (1996) outlined four characteristics of the Van Hiele theory as summarised by Usiskin (1982):

- **Fixed order:** This is the order in which learners' progress through the thinking levels. Learners need to go through all preceding levels to arrive at any specific level, as the levels are hierarchical.
- **Adjacency:** The properties which are intrinsic at one level of thought become extrinsic at the next level.
- **Distinction:** Each level of thought has its own linguistic symbols and its network of relationships which links those symbols. What is supposed to be correct at one level of thought is not necessarily correct at another level of thought. At Level 1 a square is something that looks like a box while at Level 3 a square is a special type of rectangle.
- **Separation:** When a learner is reasoning at the different level with the teacher, they cannot understand each other. The learner ends up not understanding the teacher, and also the teacher has the very same problem of understanding how the learner is reasoning. That may lead to the rejection of the answers from the learner by the teacher.

The Van Hiele also recognised characteristics of their representation, involving the fact that a learner must progress through the levels in a sequential instructions, that the progression from lower level to a higher level depends on subject matter and mode of instruction, rather than on age, and that each level has its own terminology and its unique method of operation of relations. That is why the Van Hiele proposed sequential phases of teaching to assist learners to move from one level to the other. Van Hiele (1986, p.53-54) and Luchin (2006) further categorised these sequential phases in which the child can understand geometry as follows:

- **Information:** In this phase learners are up to date with the field of exploration by using material given to them. The teacher identifies what learners know about the topic, and when they brainstorm what they already know about the topic, they are then introduced to the new topic.
- **Guided orientation:** Learners are guided by the tasks that are different. With these tasks they explore objects of instruction. These tasks are carefully structured tasks such as measurement and construction. The teacher has to ensure that the learners explore all the concepts involved.

- **Explication:** The teacher in this stage is able to introduce the new relevant mathematical terms. This allows the learners to build from their previous life experiences to express and exchange their up-and-coming views about the structure that they have observed. Learners become conscious of the relations, they describe what they have learnt in their own words. They learn new mathematical language that will be accompanied by subject content.
- **Free orientation:** At this stage learners are able to apply the relationships they have learned to solve problems and investigate the properties of the figures.
- **Integration:** They build a summary and integrate of all what they have learned from the subject content. They develop new network of the objects and relations.

2.4.3 The Van Hiele theory and the study

The Van Hiele model is relevant to this study because the study focuses on exploring PSTs' knowledge of geometric proof. The Van Hiele model was developed and has been able to explain why learners experience problems with high school geometry and also experience a problem in higher cognitive thinking (Knight, 2006). From this model it is evident that the difficulty in learning and teaching school geometry emanate from difficulties that Van Hiele suggested. As opposed to learners not having intelligence or being given poor explanations by the teacher. This is evidence that PSTs do not have to focus on content knowledge only, they ought to learn to understand why learners think the way they think. The PSTs are to face the challenge of bringing themselves to a learner's level of thinking, since if a learner is reasoning at a level different to that of the teacher; the chances of understanding each other are very minimal (Van Hiele, 1986). The Van Hiele levels have demonstrated success in helping learners grow in their geometric reasoning ability, as well as their ability to work with proof in mathematics.

2.5 Proof

Traditional teaching emphasises that a geometric statement is valid when that statement can be proved to be correct, and this guides learners to differentiate between the proof and the exploration of activities. De Villiers (1996) believes that mathematicians have to start by convincing themselves before convincing the next person that a mathematical

statement is valid, and must then find a way of doing a formal proof to that statement, and find out whether the statement is true. Proof plays a vital role in doing mathematics, which is reflected in most of the mathematics curricula.

2.5.1. Functions of proof

The main function of proof is to assist learners in developing reasoning and proving abilities, forming conjectures, evaluating arguments and using various methods of proof (Christou, Mousoulides & Pitta-Pantazi, 2004). Teachers have established a number of functions of proof writing, including explanation, systematisation, communication, discovery, justification, intuition development, and autonomy (Webber, 2003). Similarly Hanna (2000) described functions of proof as proving, verification, explanation, systematisation, discovery, communication, construction, exploration, and incorporation. Verification and explanation are considered as the basic essential functions of proofs, because they include the product of the long historical development of mathematical thought. Verification here indicates a correct statement, while explanation refers to a reason why the statement is correct and true. Teachers should be aware that proof will only be meaningful when it answers the learner's doubts and when it proves what is not obvious. The necessity or functionality of proof can only surface in situations in which the learner meets uncertainty about the truth of mathematical proposition.

De Villiers (2004) interrogated a model for the functions of proof as verification. Verification is the truth of the mathematical statement. Teachers believe that proof provides an absolute certainty and it is therefore the absolute authority in the establishment of validity of conjecture. Explanation; provides insight as to why a statement is true. However learners do not think that explanation is one of the functions of proofs. Varghese (2009) laments that very few learners in his study pointed out explanatory functions to secondary level PSTs. Through explanation alone it is easy to obtain confidence in the validity of a conjecture. However, this does not provide satisfactory explanation about why it may be true. Systematisation is the organisation of different outcomes into a deductive system of axioms, major concepts and theorems in geometry; in the discovery of new outcomes, proof plays a vital role in the finding of new geometry aspect (Knuth, 2002). Proof has a greater role to play as innovations in classrooms where technology is used and communication; which is the transmission of mathematical knowledge using deductive or inductive reasoning.

2.5.2. Deductive and inductive reasoning in proof.

Deductive reasoning is the process by which a learner makes some valid conclusions based on previously known facts. A specific example is made from a general statement. From the valid reasons and examples the deductive reasoning can be used about the statement. Deductive reasoning, also called deductive logic or logical reasoning is unlike inductive reasoning. It is a valid form of proof. It is, in fact, the way in which geometric proofs are written. An example of a deductive argument:

- All figures with four equal sides and four equal angles are squares.
- The window has four equal sides and four equal angles.
- Therefore, the window is a square.

The first premise gives a definition of a square with its property and the second premise identifies the characteristics of a window as one of the figures identified in the first premise. That will then lead to a conclusion that a window is a square because it has the characteristics of a square. This means that deductive reasoning is the process of demonstrating that if certain statements are accepted as correct or true, then more statements may be made from the initial true statements.

Geometric proof does not depend only on deductive reasoning. In contrast to deduction, inductive reasoning depends on working with cases, and developing a conjecture by examining instances and testing an idea about these cases. Inductive reasoning may also play a very important role in doing proofs from the observations. Inductive reasoning is the process of observing the given information or patterns and thereafter you make some generalisations from your observations. These generalisations are called conjectures.

In comparisons Serra (1997) highlights that inductive reasoning is based on the observation on geometric figures and have a conjecture about them. He gave an example of discovering that the sum of the angles of a triangle is 180° . The triangles are not measured but a study is done on a number of triangles to become convinced that the conjecture is true. However, Mason; Burton and Stacey (2010) highlights that, conjecture is a statement which appears reasonable, but whose truth has not been established. In other words it has not been convincingly justified and yet it is not known to be contradicted by examples, nor is it known to have any consequences which are false. A conjecture often begins as a vague feeling at the back of the mind. Gradually it

is dragged forward by attempting to state it as possible, so it can be exposed to the strong light of investigation. If it is found to be false, it is either modified or abandoned. If it can be convincingly justified, it takes its place in the series of conjecture and justification that will eventually make up the resolution. As such conjecture may be justified on the basis of direct, conditional or indirect proof.

2.5.3 Direct proof, conditional proof and indirect proof

Proofs may be categorised into three basic approaches in order to prove logical arguments. There are direct proofs, conditional proofs and indirect proofs. The direct proof refers to proofs where the given premises are stated, and then valid patterns of reasoning are used to arrive directly at the conclusion. For example if $M = P$, and $N = P$, therefore $M = N$, since both are equal to P . In indirect proof, the conclusions are made by logically including the axioms, definitions, and earlier theorems. Conditional proof refers to proofs where the assumption of the first part of the conditional statement is made and is true, and then the logical reasoning is used to demonstrate that the second part of the conditional statement is true. The third type of proof is an indirect proof, which is almost like a sneaky way of proving something. With indirect proof all possibilities that can be true are recognised, and thereafter, all but one possibility are eliminated when they are shown to contradict some given fact or accepted idea. It must then be accepted that the one remaining possibility is true (Serra, 1997). Serra used the example similar to this one to explain the example of indirect proof, trying to answer the following question:

In which year was your teacher born? The options are:

- A. 1902 B. 1968 C. 1998 D. 1910

You have to eliminate some options so as to find one last correct option. If the question was asked in 2012, 1902 and 1910 can be eliminated on the basis that the teacher could be more than hundred years old, which is impossible. Furthermore 1998 can also be eliminated because that will mean the teacher is less than 15 years old which is also impossible. From the eliminations above one could be sure that the correct option would be 1968. However the truth of the theorems must be proved by deductions and conclusions using previous axioms and theorems.

2.5.4 Axioms and theorems

The axioms or postulates are self-evident truths; for example, if two lines meet, the sum of the adjacent angles is 180° . Furthermore, from these axioms certain theorems can be deduced and from the theorems more theorems can be deduced and they must be proved using the deductions and conclusions of the previous axioms and theorems. Therefore a corollary is a direct outcome of a theorem but of sufficient importance for it to be given status of a theorem. Many learners do not understand the concept of a theorem very well. The reason lies with the way in which proof is taught in schools; sometimes the theorem is not explained well before the formal proof is done in class. Sometimes the theorems are just read to learners and proofs are given and learners are expected to proceed with difficult riders. Theorems must be fully explained and understood before doing riders. This understanding can be acquired by means of clear explanation, experimental work and even narrative work through the use of the content knowledge and the pedagogical knowledge of the teacher. Learners should be allowed to explore the theorem before the proof is done. They should draw measure and compare sizes of that particular shape as is outlined in the theorem. For the teachers to facilitate this to their learners, major professional development will be required.

2.6 Teachers' professional development

The knowledge the teachers ought to have depends on the professional development they have. The attempts to advance the knowledge and quality of South Africa's teachers and their teaching have been the focus of the Department of Education (DoE) for several years. Initiatives in the form of a new curriculum, the upgrading of qualifications, the development of subject competence and support for continuing professional development have had some success in providing teachers with knowledge, but much more remains to be done. It is suggested that years, perhaps decades, of concerted and sustained effort will be required to confront the numerous challenges facing teacher education and development. According to Deacon (2010), there is a need for better coordination, more effective funding, improved recruitment and retention and improved training and support. Importantly, though, South Africa is not unique in being faced with these challenges, and much of what needs to be done has already been done.

The approach to teacher professional development has been at the forefront of government plans; teachers have been given a chance to capacitate themselves in their respective fields. Many short courses and workshops by experts have been offered by

different universities; this was done to equip teachers with strategies to face their challenges. However, the past and present strategies of the government to continue with professional development for teachers have not produced the desired outcome, which means the country still needs new curriculum innovation. The intention of professional development is to effect individual change at classroom level, but also more broadly changes in particular content areas.

Akhurst and Sader (2009) have defined two categories of teachers, restricted and extended teachers. The restricted teachers refer to teachers whose understanding is only based on the classroom. The teaching of the restricted teacher is embedded only in his/her own experience rather than theory and strictly focussed on the well designed academic fixed programme, for example a restricted teacher may do a job of teaching learners during the scheduled lessons but will not be expected to be involved in further studies or community related activities. This type of teacher is inventive, skilful and sensitive to the cognitive and intellectual development of individual learners. The restricted teacher is a good mediator of learning, has a sound content of the learning area, has good management skills as classroom teacher and views learners as future adults.

On other hand, the extended teacher refers to a teacher who has located their work in a broader educational context and systematically evaluates their work in order to improve through research and development. The extended teacher starts by being a good mediator of learning and then extends work beyond the classroom and into the school community. This teacher reflects on their teaching and discusses the learners' work with colleagues, trying to understand it. The extended teacher is expected to play more than one role as defined by the South African Department of education norms and standard (DoE, 2010). Teachers ought to reflect on and improve their own practice in the own classrooms. The complexity of their daily work rarely allows them to communicate with colleagues about what they have discovered about teaching and learning (Cerin & Kopp, 2006).

To enhance teacher professional development, South Africa needs to focus deeply on the pedagogical content knowledge. Brijlall (2011) defines content knowledge as the disciplinary knowledge of a subject. By using Chinna (2003), Brijlall (2011) explains that mathematical content knowledge includes information such as mathematics terminology, rules, and associated procedures for problem solving.

Pedagogical knowledge refers to the broad knowledge that a teacher requires so that the learning and teaching are effective in schools. This includes content knowledge, knowledge about how to teach, knowledge about pupils and how they learn, knowledge about the curriculum and knowledge about discipline and classroom management.

Many teachers do not know what is expected of them, and find themselves playing the role of both teachers and caregivers. While struggling to teach, they have to face new challenges in the education system. Additionally while preparing for their work, they are required to join professional development programmes (Morrow, 2007). It is acknowledged that professional development programmes are more likely to achieve significant changes in the classroom practice if they are seen by teachers as being responsive to their needs. The sudden move to bring ordinary teachers into contact with a curriculum discourse is completely foreign to their understanding and practices. This becomes a major issue to the teachers since they are required to cope with the huge changes in the curriculum, even when they were not involved in its conceptualisation. Outcomes Based Education (OBE), the innovation which was brought in the 1990s, needs to be implemented by well trained or prepared teachers. OBE is based on the education system by which teaching and learning is aimed at a specific outcomes or end goals and it is characterised by the learner-centred approach.

A number of attempts have been made to ascertain teacher's preferences regarding the content, form, and style of professional development. One would agree with Morrow (2007), when he writes that the work of the teacher is to teach, and to teach effectively. However teachers' complain about the duration of the workshops held by the Department to capacitate them. Teachers are usually asked to attend a one-day workshop, while they are expected to teach the whole year. In research conducted by Zigarmi, Betz, and Jensen (1977) it was found that the most common forms of in-service training, which are after school or one-day regional workshops, were judged to be least useful. The problem with these educational programmes is that they do not reflect differences between the ideal and the actual conditions of teaching in mathematics.

2.7 Teachers' knowledge

Having the required understanding of subject matter allows the teacher to assist learners to create useful cognitive plans, relay ideas together, and address misconceptions from content knowledge in doing proof in geometry. Shulman (1987) suggests that all

teachers are required to have an understanding of the relationship between ideas within different fields of learning and real life experiences. This kind of understanding provides a strong foundation for pedagogical content knowledge that enables teachers to build on, and to make ideas accessible to learners. Shulman defines pedagogical content knowledge, as an understanding of “the most useful forms of representation of topics, the most powerful analogies, illustrations, examples, explanations, and demonstrations in a word, the ways of representing and formulating the subject that makes it comprehensible to others” (1987, p. 9).

Shulman (1986) proposed the concept of pedagogical content knowledge as a model for understanding teaching and learning (Veal & MaKinster, 1999; Ball, Thames, & Phelps, 2008). This was to help teachers acquire new understandings of their content, which is the knowledge of doing proof in geometry, and to apply these new understandings to strengthen their knowledge in teaching proof in geometry. Shulman created a Model of Pedagogical Reasoning, which included important aspects of learning that a teacher would need to complete for good teaching (Garritz, 2010). Some of the aspects he included were comprehension, transformation, instruction, evaluation, reflection and new comprehension. Good teaching should incorporate assessment, which includes checking the understanding and misunderstanding during teaching and learning. It is therefore very important for the teacher to reflect on his/her lesson presentations. Reflection should be done before and after the presentation. The reflection before presentation should concentrate on the anticipated misconceptions of the lesson content and reflection after should deal with what happens during the lesson.

Shulman (1986) also suggested three forms of knowledge namely; propositional knowledge, case knowledge and strategic knowledge. Propositional knowledge is knowledge acquired when researchers examine the research on teaching and learning and explore its implications for practice. Propositional knowledge has three major sources: disciplined empirical or principles, practical experience and moral reasoning. Case knowledge is the knowledge of specific, well-documented, and richly described events. Case knowledge has its own three facets; prototypes, which consists of theoretical principles; precedents, which capture and communicate the principles of practice, and lastly parables, which convey norms or values. Strategic knowledge is developed when the lessons of single principles contradict one another or the precedents of particular cases are incompatible (Shulman, 1986).

When teachers engage learners into a process of teaching and learning, they actually transmit knowledge to the learners. The starting point for transmission of explicit knowledge from specialist to learner is the expectation that a learner will receive the organisation of knowledge about a section in mathematics from another's understandings, into his own understanding. However, the explicit transmission of knowledge from a specialist hardly leads to the maximum understanding of the learner (Bond-Robinson, 2005). The constructivist model of learning requires that a teacher ought to encourage learners' efforts to understand the content so that they will remember it. The typical model of knowledge transmission does not compel that teachers openly push learners to reason, so that this teaching practice can inadvertently reinforce memorising rules, facts, algorithms and procedures. A mixture of transmission of knowledge, supplemented with attempts by the teacher to guide reasoning using the new information, is more likely to be effective for getting learners to understand difficult subject matter.

2.8 Pre-service teachers' knowledge of proof in geometry

A pre-service teacher is any individual that is in a programme designed to provide teacher certification upon completion. This study is about the PSTs' knowledge of proof in geometry, which it intends to explore. This study will also assist other mathematics teachers to find out more about how PSTs use this knowledge of proof. In addition, also the study intends to interrogate why PSTs use their knowledge of proof in geometry in the way that they do. If one studies the trend of the performance of mathematics from the past years one could easily observe that there is a problem in geometry. There is very little existing research on PSTs' knowledge of proof in geometry. However, Fonseca and Cunha (2011) report on the experience of mathematics educators in teacher training. They point out that teachers should reflect on how to adequately work methodologies in different mathematical subjects to answer the needs of the PSTs. Often they have little mathematical knowledge and reveal negative attitudes towards mathematics learning. Within geometry the teacher may use a dynamic geometry application (DGA) and pattern tasks to work geometrical concepts, in order to motivate PSTs to learn geometry and to encourage them to try to change their attitudes towards geometry. The aim of Fonseca and Cunha (2011) was to develop mathematical abilities and to explore how PSTs work with geometry in such a way that it is possible:

- To develop their mathematical knowledge;

- To develop their reasoning;
- To develop a positive attitude toward mathematics; and
- To gain more confidence in their capacities to do maths.

2.9 Conclusion

Researchers (Fonseca & Cunha, 2011) believe that learning mathematics will be meaningful, if and only if it is within the learner's experiences from the real world. Relevance in the teaching and learning of mathematics will only be created when learners can associate the mathematical concepts with their own experiences or the real world.

The development of an innovative mathematics teacher who possesses pedagogical content knowledge and whose practices reflect current knowledge on the teaching and learning of mathematics rests in part on how s/he engages learners with that knowledge. The teacher who presents content knowledge to learners with an adequate level of understanding would allow learners to socially construct their own knowledge and share mathematical ideas with peers. The way in which teachers and learners interact with mathematical ideas in the social context of the classroom, whether passively or actively, structures students' thinking about mathematics (Blanton, 2000).

The next chapter presents the research design and methodology of the study. It highlights the critical research questions and how the research tools were used to collect the data. It also describes the population and sample used on the study.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This researcher's study started with concern over unsatisfactory Grade 12 achievement in geometric mathematics. This intrigued the researcher to investigate and explore the pre-service teachers (PSTs) knowledge of proof. In this chapter the methodology is discussed. The chapter explains the location of the study in terms of the paradigm and approach. This chapter sets out to show the manner in which the research information for this study was collected. The chapter defines the targeted population and sample, the choice of research instruments, the process of obtaining access, the planning and implementation of the pilot study.

3.2 The critical research questions

The main objective of this study was to explore the PSTs knowledge of proof in geometry. The study sought to answer the following critical research questions:

1. What is pre-service teachers' knowledge of proof in geometry?
2. How do pre-service teachers use their knowledge of proof in geometry?
3. Why do pre-service teachers use their knowledge of proof in geometry in the way that they do?

3.3 Methodological approach

The study is located within the qualitative research paradigm, and takes the form of survey research in one of the universities in KwaZulu-Natal. The study focuses on the PSTs' knowledge of proof in geometry. According to Borrego, Douglas & Amelink (2009), qualitative research is characterised by the collection and interpretative analysis of written data obtained from surveys, interviews, questionnaires and focus groups.

In qualitative research, inductive and exploratory methods are used. These methods are used primarily for the purposes of description and exploration of data as well as to gain an understanding of how people think and experience their lives. The data is examined through interpretative analysis for patterns and themes (University of South Alabama, 2005). The rationale under which the qualitative approach is chosen is that qualitative

research problems are phrased as a research purpose or questions but not as a hypothesis. Usually problems are phrased more broadly to answer questions like What, How, and Why. This corresponds with the critical research questions of this study. Qualitative researchers investigate in-depth of small, distinct groups. These groups could for example include a Grade 10 class within a specific school.

Qualitative research methods include interviewing, observation and document analysis. This study was located within the interpretivist paradigm. The interpretivist researcher uses the qualitative approach to understand humans' behaviour within their own context in order to make sense of their world. When geometry in mathematics is studied, learners construct knowledge from the previous levels of understanding and from their own experiences. Similarly, according to Voce (2004), the interpretivist paradigm views knowledge as being constructed, additionally knowledge is about the way in which people make meaning in their lives. This study explored PSTs' knowledge of proof in geometry. This was done in order to find the nature of their knowledge and how they use their knowledge of proof in geometry. This study also questioned why PSTs used the knowledge of proof in the way that they did. The purpose of using the qualitative research paradigm was to describe and interpret PSTs' knowledge of proof in geometry.

3.4 Research design

The data was collected using four instruments: a survey questionnaire, a task-based worksheet, a semi-structured focus group interview schedule and a semi-structured individual interview schedule. Data was collected using a task-based worksheet because it provided additional information to assist in determining how pre-service teachers think when working with proofs in geometry. Data was also collected by a survey questionnaire with a aim of selecting a sample to be interviewed. This assisted in allowing the selection of the most appropriate group of PSTs to work with. The semi-structured focus group interview schedule was used. Each focus group interview was tape recorded. Finally, after the focus group interview was transcribed and analysed, the semi-structured individual interview schedule was used. The interviews provided important information with respect to PSTs' knowledge of proof in geometry. Semi-structured interviews were used in order to allow for probing during the process of the interview. This was done when further clarification of answers were required. The researcher ensured that enough time was provided for each respondent to respond.

The research design continued in an organised and detailed manner as represented to the flowchart in Figure 3.1. The flow chart on the table shows each item which depends upon the successful achievement of all the preceding items, therefore, it is important not to skip a single step. There are also feedback loops in the flow charts to allow revisions to the methodology and instruments. For example if the pilot study revealed some irrelevant information, the flow chart indicates how the researcher would respond.

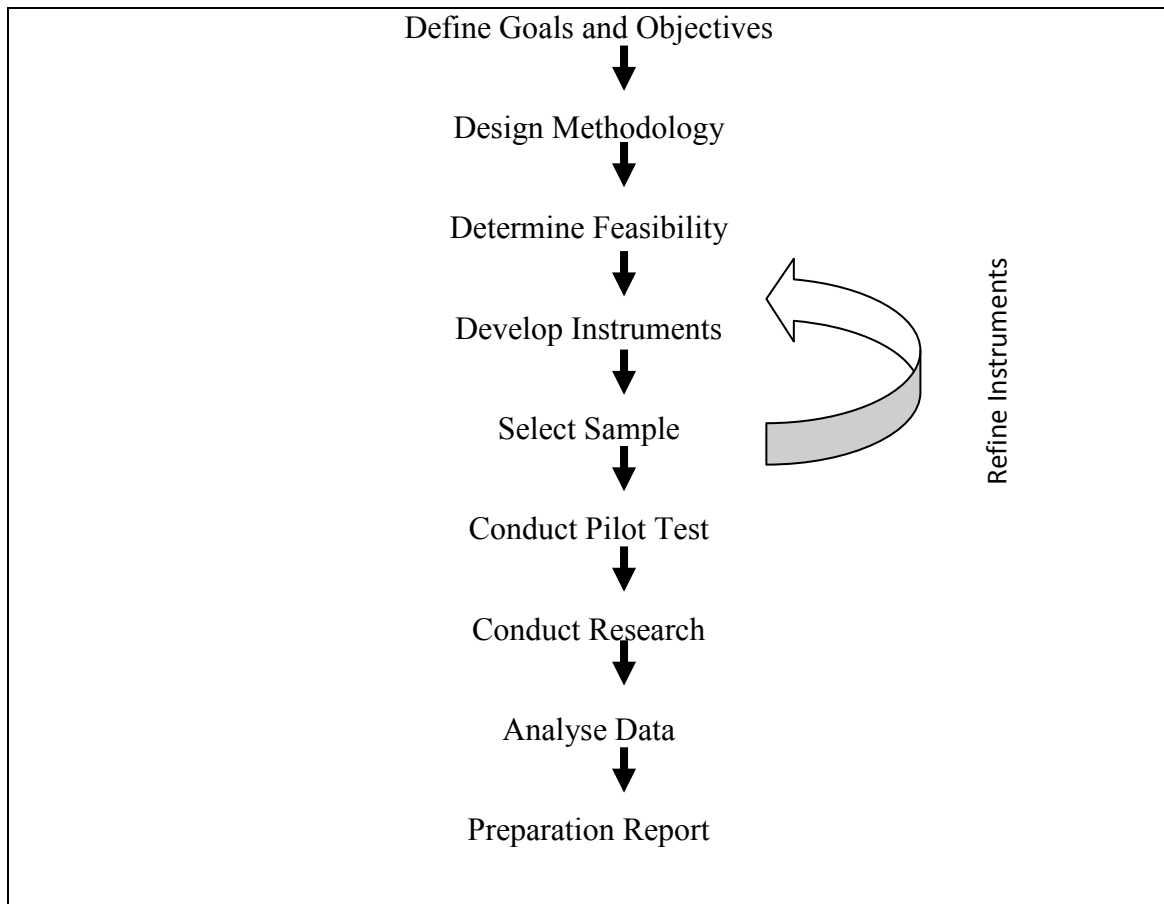


Figure 3.1: Research design flowchart-adapted from Naidoo (2006, p. 107)

The rest of the chapter describes the pilot study, gaining access to the participants, population and sampling, survey questionnaire, task-based worksheet, a semi-structured focus group interview schedule, and a semi-structured individual interview schedule.

3.5 Pilot study

A pilot study is usually conducted to test the reliability and to refine the measuring instruments. The pilot study is administered to a small group of participants similar to those to which the actual test will be administered. McMillan and Schumacher (2006, p. 202) point out that it is highly recommended that researchers conduct a pilot study of their questionnaires before using them in the main study. To do so, it is best to locate a

sample of subjects with characteristics similar to those that will be a part of the main study. From the pilot study the researcher can determine whether or not the instrument is appropriate. Another reason for conducting a pilot study is that the researcher can confirm whether or not the time frame is adequate, or whether the directions and items on the questionnaire are clear. The pilot study for this research was conducted at two different universities. The rationales under which these two universities were selected are based on the fact that the two universities were more convenient to the researcher and during the time of the pilot study these universities would have done geometry. In this pilot study, eight mathematics pre-service teachers who came to the researchers' school for practice teaching, were requested to complete the survey questionnaire and task-based worksheets.

3.6 Access

When the researcher was preparing to administer this study, it was evident that some ethical issues would arise. Ethical issues involving the institution and the PSTs participating in the study were taken into consideration. The researcher contacted the research office of the university in order to gain access to the PSTs to conduct both the pilot and the main study.

In addition the dean of the faculty and the mathematics lecturers were contacted, in order to gain access to the PST. In addition, an invitation to participate in the study was prepared and presented to each PST. As soon as permission was granted by all the relevant parties, the informed consent form was designed. This consent form informed the participants about their confidentiality and anonymity. The informed consent described the procedures which were to be followed and their purpose. Moreover, the informed consent also stipulated time frames that were expected for the completion of each instrument. In addition, each PST was informed that they were free to withdraw from the study at any stage of the research study. This could be done at any time without prejudice to the participant (Cohen & Manion, 1994).

Ethical clearance was approved for this study. The ethical clearance approval number is HSS/0850/011M.

3.7 Population and sample

The targeted population of the study were PSTs at the UOH. The data in this study was collected from second, third and fourth year PSTs at the UOH. The reason for selecting

second, third and fourth year PSTs is that, during the time of conducting research the proof would not have been taught to first year students, but the second, third and fourth year PSTs would have had some experience of university mathematics and teaching pedagogy. All second, third and fourth year students who were doing mathematics as one of their major subjects, were asked to complete the task-based worksheet.

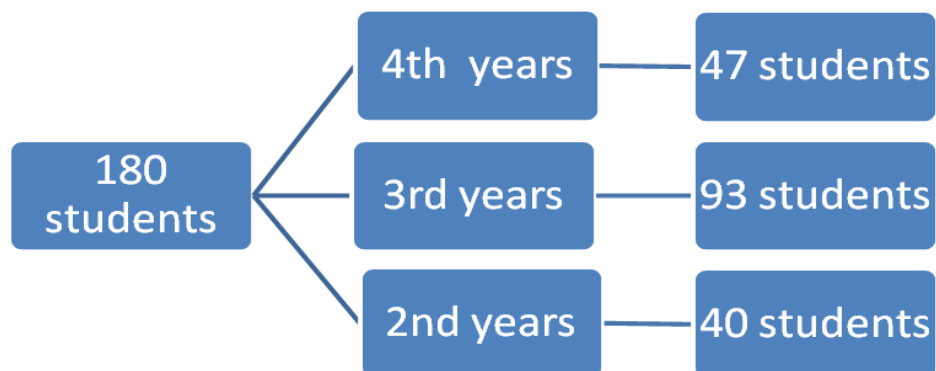


Figure 3.2: Distribution of 180 students by year level

In total, 180 PSTs completed the task-based worksheets. From this 180, 47 students were in 4th year, 93 students were in 3rd year and 40 students were in 2nd year. Even though it was not part of the research, it was very alarming to discover that female mathematics students were not well represented at the institution where the research was conducted. Out of 180 students who completed the task-based worksheet, there were 65 (36%) female students, while 115 (64%) male students. The researcher decided later not to include 2nd years in both focus group and individual interviews, due to inadequate information from their task-based worksheets. A total of 20 participants (3rd and 4th year students) were invited for a focus group interview and 10 participants (3rd and 4th year students) were invited for the individual interview. However, only 14 participants attended the focus group interview and nine attended individual interviews. The focus group participants were divided into two groups; Focus Group Interview One (FGI 1) and Focus Group Interview Two (FGI 2).

Based on the manner in which students answered the task-based worksheet and the analysis of the survey questionnaire, the researcher selected some students who demonstrated a better knowledge of geometry proof and of geometry problem solving, some students who displayed an average knowledge of geometry proof and of geometry problem solving and lastly, some students who demonstrated very little or no

knowledge of geometry proof. Even though the selection of the above participants was based on the performance reflected on the task-based worksheet, the purposive sampling technique was also used. Within the purposive sampling technique, the researcher handpicks participants to be included in the sample on the basis of the judgement of their typicality, like gender, demographics and also in their study year levels. Another reason for purposive sampling technique was the number of participants who declined the invitation to an interview. In this way, purposive sampling technique builds up a sample that is satisfactory to the specific needs (Cohen & Manion, 1994; Strydom, Fouché, & Delport, 2004).

This design was the most appropriate plan to be followed because the task-based worksheet provided information to assist in determining what PSTs know when working with proofs in Euclidean Geometry. The survey questionnaire assisted in enabling the selection of the most appropriate focus group of PSTs to work with. The interviews provided important additional information with respect to PSTs' knowledge of proof.

The selected sample was interviewed using a semi-structured interview schedule. The interviews stemmed from how the PSTs answered the task-based worksheet and the survey questionnaire. The task-based worksheet and survey questionnaire assisted in clarifying the PSTs' knowledge in geometry. All respondents were given enough time to respond to the task-based worksheet and survey questionnaire (McMillan & Schumacher, 2006). Interviews, focus group and individual interview were tape recorded.

During the interviews, codes were used instead of the real names of the PSTs to protect their identity. On the tape recorder they were referred to as Learner 1, Learner 2, Learner 3, et c. However, when the transcriptions were made Learner 1 was then changed to Pre-Service Teacher 01 coded as PST 01 and Learner 2 was also changed to Pre-Service Teacher 02 coded as PST 02, and Learner 3 became PST 03 and so on and on. The table 3.1 that follows indicates how the participants who participated on the interview focus groups were referred to the recorder during the interview and the table also illustrates the codes which are used for the participants. Table 3.1 also shows the year level, gender and race of each participant.

Table 3.1: List of pre-service teachers who participated in the interview focus group students

	Recorded as	Code ¹	Year Level	Gender	Race²
1	Learner 1	PST 01 ³	3 rd	Female	I
2	Learner 2	PST 02	3 rd	Female	W
3	Learner 3	PST 03	3 rd	Female	I
4	Learner 4	PST 04	3 rd	Female	I
5	Learner 5	PST 05	4 th	Female	I
6	Learner 6	PST 06	3 rd	Male	I
7	Learner 7	PST 07	3 rd	Male	A
8	Learner 8	PST 08	4 th	Male	A
9	Learner 9	PST 09	4 th	Male	A
10	Learner 10	PST 10	3 rd	Male	I
11	Learner 11	PST 11	3 rd	Female	A
12	Learner 12	PST 12	4 th	Female	A
13	Learner 13	PST 13	4 th	Male	A
14	Learner 14	PST 14	4 th	Female	A

The table 3.2 that follows also indicates how the participants who participated in the individual interview were referred to during the interview. This table also illustrates the codes used, year level, gender, race, age bracket, qualification and the year the PST completed Grade 12.

¹ Codes are used to protect the identity of PSTs.

² In the race column the abbreviations used are: A: African, I: Indian and W: White.

³ PST 01: Pre-Service Teacher number 1

Table 3.2: List of pre-service teachers who participated on the individual interview students

	Recorded as	Code	Year Level	Gender	Race	Age⁴ bracket	Other Qualification	Year completed Grade 12
1	Learner 1	PST 01	3 rd	Female	I	C	None	2004
2	Learner 2	PST 02	3 rd	Female	W	B	None	2009
3	Learner 3	PST 03	3 rd	Female	I	C	None	2007
4	Learner 4	PST 04	3 rd	Female	I	C	None	2007
5	Learner 5	PST 05	4 th	Female	I	C	Computer	2004
6	Learner 6	PST 06	3 rd	Male	I	B	None	2007
7	Learner 7	PST 07	3 rd	Male	A	C	None	2004
8	Learner 8	PST 08	4 th	Male	A	C	None	2006
9	Learner 9	PST 09	4 th	Male	A	B	None	2008

3.8 Research instruments

3.8.1 Survey questionnaire

At the beginning the researcher needs to find ways of understanding the participants as to who are they. It was therefore necessary for the researcher to find out some details of the participants by allowing them to complete the questionnaire. According to McMillan and Schumacher (2006) questionnaires are the most widely used technique for obtaining information from any subjects. A questionnaire is comparatively inexpensive, has the same questions for all subjects and may be anonymous. In any questionnaires, the researcher should make sure that the respondent is not taking too much time to complete. The steps to develop a questionnaire resemble the steps discussed in Figure 3.1. However, in developing questionnaires the researcher should begin by justifying the options of using this kind of technique before the goal and objectives are defined.

⁴ Age bracket: A - Below 15; B - Between 15 and 20; C - Between 21 and 25; D - Between 26 and 30; E - Above 30 (The table is provided on preliminary pages).

The questions asked in the study were both open ended and closed questions. The questionnaire also includes the scaled type of questions, which is the series of levels, or values describing various degrees of the responses (McMillan & Schumacher, 2006). The rationale of using the questionnaire was to gather information about the participants' knowledge of geometry. The questionnaire was also designed to obtain background information about each participant in the study. The questionnaire was specific enough to meet the objectives of the study. The participants were provided with the opportunity to complete the questionnaire in their own time. The survey questionnaire assisted in allowing the researcher to select the most appropriate group of PST to work with.

3.8.2 Task-based worksheet

The task-based worksheet provided information to assist in determining how PSTs worked with proofs in geometry. To clarify, the questionnaire was used to ascertain the depth of knowledge of proof in geometry. This task-based worksheet was based on solving problems based in proof of geometry in the following aspects:

Table 3.3: Task-based Worksheet questions

	Question	Types of proof per question	Grade level of task ⁵
1	Question 1	Proof of angle of straight lines.	Grade 8
2	Question 2	Proof of parallel lines.	Grade 9
3	Question 3	Proofs involving quadrilaterals.	Grade 10
4	Question 4	Proofs focusing on angles on triangles.	Grade 10
5	Question 5	Proofs involving chords and tangents.	Grade 11
6	Question 6	Proofs involving cyclic quadrilaterals.	Grade 11

⁵ These tasks are normally given to learners at the grade level stated in the table.

The example of question 3.

3. $PQRS$ is a parallelogram that is $PS \parallel QR$ and $PQ \parallel SR$. BP and AR bisect angles P and R respectively. Prove that $PBRA$ is a parallelogram.

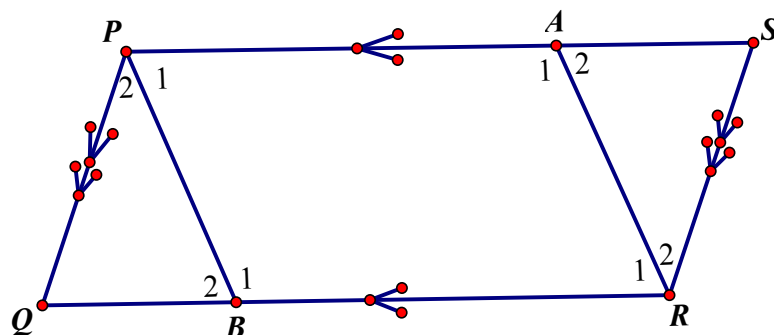


Figure 3.3: Diagram for Question 3

The example of question 5

5. Study the diagram below and answer the question.

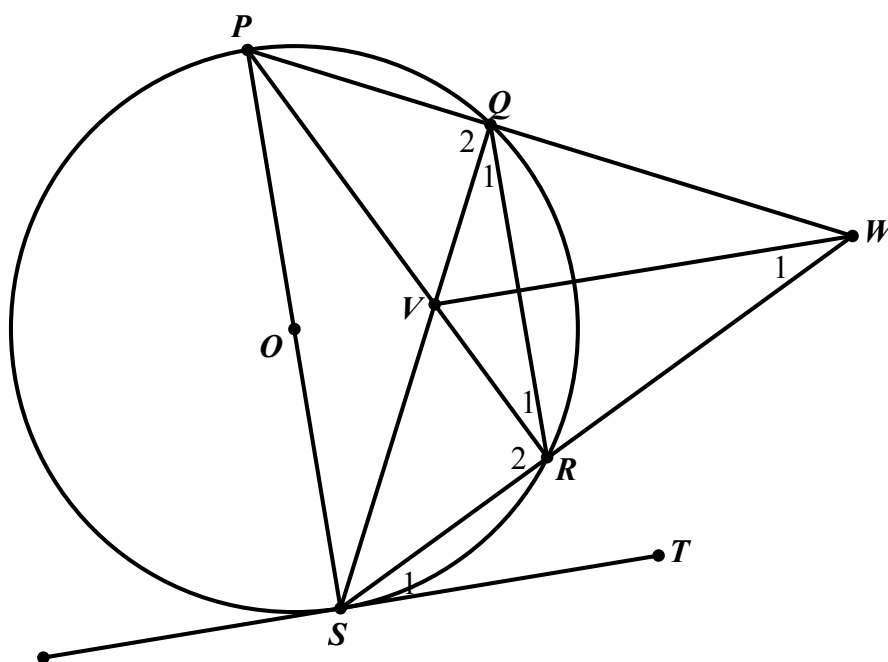


Figure 3.4: Diagram for Question 3

Given a circle O with a diameter POS and a tangent ST .

Prove that $WV \parallel TS$

The task-based worksheet was also used to assist in selecting participants for the interview. Second, third and fourth year students who enrolled for mathematics as one

of their major subjects, were asked to complete the task-based worksheet. A total of 180 PSTs completed the task-based worksheet. Based on the manner in which PSTs have responded to the task-based worksheet and the analysis of the survey questionnaire, students who demonstrated a good knowledge of geometry proof and of geometry problem solving, students who displayed an average knowledge of geometry proof and of geometry problem solving and some students who demonstrated very little or no knowledge of geometry proof and of geometry problem solving were chosen for the administration of the questionnaire. After the analysis of the collected data, the 20 PSTs were selected and invited for an interview.

3.8.3 The semi-structured interview schedules

The semi structured interviews in this study include both the focus group interview schedule as well as the individual interview schedule. A semi-structured interview is a method of research used in social sciences to collect data. Structured interviews has its own method which is more formal, it has a limited set of rigid questions, and a semi-structured interview is a flexible method of collecting data, allowing new questions to be asked during the interview process as a result of what the interviewee says; it also allowed for the researcher to probe. The interviews enabled the researcher of this study to probe difficult issues in more detail to provide clarification and to prompt the participants (Dowling & Brown, 2010). The interview provided important information with respect to PSTs' knowledge of proof in geometry. According to Trochim, (2006), the most efficient strategy to encourage someone to explain is to do nothing at all, just pause and wait. This is referred to as the silent probe. It works because the respondent is uncomfortable with pauses or silence. It suggests to the respondent that you are waiting, listening for what they will say next. This was done when further clarifications of answers were required.

The aim of interviewing the participants was to allow an opportunity to express perceptions and understanding of proof in geometry. Since the interview was semi-structured it allowed the researcher to probe the participants' responses. The advantages of using interviews to a large extent mirror the limitations of questionnaires. As indicated above, in this study, the number of participants invited for an interview was 20, who were selected from the 180 participants. All 20 participants were to be interviewed as a focus group and 10 participants were selected from the focus group for

an individual interview. The interview was based on their responses to the task-based worksheet and survey questionnaire.

In this study, the researcher ensured that enough time was provided for each respondent to respond. The focus group interview took about 30-40 minutes, while the individual interview lasted about 20-30 minutes for each participant. Interviews were conducted with 14 out of the 20 PSTs who made themselves available. The interviews were conducted in two variations as stated above; one was a focus group interview (FGI) and the other an individual interview (II).

A focus group is used to obtain a better understanding of a problem or an assessment through the interviewing of a purposefully chosen sample, rather than of a single person. The reason for conducting a focus group interview was to create the social environment in which group members are inspired by one another's responses. In this variation, the researcher is able to strengthen the level of quality and richness of the data through a more well-organised approach than one-on-one interviewing (McMillan & Schumacher, 2006). The individual interview (II) is a one-on-one interview, where a participant is with the researcher or interviewer only. An individual interview provides the freedom for the participant to talk freely without of fear of their colleagues or associates even when the response is sensitive.

3.9 Conclusion

This chapter presented an argument for the methodology used in this study. It outlined the main aspects of the research design and methodology, and explained how the data is collected. The responses collected from the questionnaire, task-based worksheet, focus group interview and individual interview provided data. Chapter 5, the data analysis, provides more detail and analysis of the responses collected.

The next chapter presents the theoretical frameworks within which the study is framed.

CHAPTER 4

THEORETICAL FRAMEWORKS

4.1 Introduction

The study explores pre-service teachers' (PSTs) knowledge of proof in geometry. The study is framed within the theories of social constructivism and situated learning. The rationale under which social constructivism is used in this study is that learners are learning mathematics by socially constructing meaning in response to what is given to them. Learners construct knowledge by linking their experiences in the past to present knowledge. The learning and teaching of mathematics is an active process whereby meaning is socially constructed. This is evidence that the learners themselves are in control of their learning processes (Singh, 2006). There are two types of constructivism (Atherton, 2011). The first is called cognitive constructivism, which is about the understanding of an individual learner in his or her developmental stages and learning styles, and the second is social constructivism. This type of constructivism is centred on how meanings and understandings of a learner may increase and improve as a result of social encounters.

Apart from socially constructing meaning, learners learn by socialisation. This type of learning resonates with the theory of situated learning. One needs to reiterate that learners are not empty vessels but rather, they need to be engaged in their learning and teaching. Teaching and learning are no longer teacher-centered but are learner-centered. Learners need to explore their knowledge and experiences, and be part of their learning process; teachers are to facilitate this process. Ernest (1986) strongly believed that the success of all mathematics teaching depends entirely to a large extent on the active involvement and the participation of the learner. Learners have to be educated, but they have also to be allowed to educate themselves. Singh (2006) quoted Williams (1988, p 101) when paraphrasing the Chinese proverb:

- *You tell me and I will forget*
- *You show me and I may remember*
- *You involve me and I will understand*

This implies that learners learn more effectively when they are practicing mathematics and by making the concepts and skills of mathematics their own. The participation of

learners in learning is a central component of development of learning and thinking in mathematics, and they learn to construct knowledge in a social context for themselves.

4.2 Social constructivism

4.2.1. What is social constructivism?

The implication of social constructivism for mathematics is that it is a theory which proposes that learners construct a meaningful understanding of mathematical knowledge from what they know to what they don't know. Knowledge should not be merely transmitted to the learner by the teacher but it must be constructed and reconstructed by the learner themselves. Learners' knowledge is constructed as the learner strives to organise the learning experiences in terms of pre-existing knowledge. The learners ought to be active, since they are not empty vessels to be filled with facts. The ability to learn any cognitive content is always related to the learners' stage of intellectual development. Learners should also be involved with the community and community activities so that they will engage and interact with the society, and that will help to adapt to the life style the community is living. It is out of this engagement and interaction that learners can readily construct meaningful knowledge.

Ernest (1995) described social constructivism as emphasising the social nature of learning. Learners should actively construct or create their own subjective representations of objective reality. For learning to take place, the learner should construct and transform external, social activities into internal activities. This implies that a learner needs to understand as well as reshape what the learner sees in their community.

4.2.2 Piaget's learning types

Social constructivism resonates with cognitive learning theory which deals mainly with the construction and attribution of meaning in learning. For a learner to know and construct knowledge in the world the learner must act on an object and it is this action which will create knowledge about those objects. According to Njisane (1992), Piaget distinguished three types of knowledge, referred to as; social, physical and logico-mathematical knowledge. Social knowledge is dependent on the particular culture, physical knowledge is gained when one abstracts information, logico-mathematical knowledge is made up of relationships between objects, as proposed by Njisane (1992).

Mayer (2009) defines constructivism as a theory of learning in which learners build knowledge in their working memory by engaging in appropriate cognitive processing during learning. He outlined the features of constructivism as: *who* (learner), *what* (build knowledge), *where* (in working memory), *how* (by engaging in appropriate cognitive processing) and *when* (during learning). Kintsch (2009) agrees with Mayer by emphasising that learning is an active process in the construction of knowledge.

Social construction in mathematics is important from a Piagetian perspective since it encourages the construction of logico-mathematical knowledge. Social constructivist theorists believe that knowledge is not easily obtained from the learner's experiences. The ability to develop this knowledge is based solely on the quality of learning that the learner has been exposed to. The learner adds knowledge to the existing knowledge by making sense of what s/he has been learning. Using different approaches to learning enhance the learning and teaching.

4.2.3 Vygotsky's approach to learning.

According to Vygotsky (1978), learning is a social interaction which plays an elemental role in the process of cognitive development, consciousness and cognition. Social interaction is the end product of socialisation and social behaviour. In contrast to Jean Piaget's understanding of child development, Vygotsky believed that social learning precedes development. The learners need to be given a chance to explore what they know from their own experience. Additionally they need be given a chance to interact with the ideas and knowledge with the peers. Learners should also be given a chance to reflect on their correct and incorrect solutions. Naidoo (2006) emphasises that teachers should not dismiss wrong or incorrect solutions, but teachers should rather allow the learners to explain and reflect on how they arrived at their solutions.

Vygotsky (1986) argues that scientific concepts do not come to the learner in a ready made form. Vygotsky viewed development as dependent on social interaction and that social learning leads to cognitive development. That is what he termed as the Zone of Proximal Development (ZPD). Vygotsky also believes in recognising the distance between the learners' ability to perform a task under adult supervision and the ability of the student to solve problems independently. Woolfolk (2007) views Vygotsky's learning as a collaborative construction of socially defined knowledge and values, which occur through socially constructed opportunities. Scott (2008) views the cultural version of ZPD as comprising the merging of different elements of learning. This

involves a distinction between what Vygotsky describes as scientific knowledge and everyday knowledge. In this version Scott (2008) refers to the distance between the everyday experience of a child and that body of cultural knowledge which is usually thought of as the contents of the formal curriculum. In contrast, Piaget's learning is viewed as an active construction, whereby prior knowledge is restructured. Learners have to be educated, but they also need to want to be educated. They need to be internally motivated to want to learn, acquire knowledge and to socially construct knowledge in mathematics.

4.2.4 Social constructivism and learning mathematics

The teaching and learning of mathematics has been under great scrutiny by researchers (Naidoo, 2011). Elmore (2002) compares the level of what is known theoretically with what is done practically. The understanding of mathematics developmental theory, the construction of knowledge, and a need for understanding of learning theories appropriate to the teaching and learning of mathematics, ought to be a priority. Learning to construct knowledge is missing in many mathematics education environments. Learners need to be given a chance to explore and construct knowledge for themselves. There have been a number of questions about how one can teach mathematics effectively or how learners construct knowledge in mathematics. Many initiatives have been implemented, however the knowledge of content, the knowledge of curriculum and the knowledge of teaching are still very limited.

4.2.5 Social constructivism and the study

The study focuses on exploring PSTs' knowledge of proof in geometry. The study of geometry and proofs therein are best taught by locating examples within real world contexts. As stated in Chapter 2, the reason for this emanates from the definition of geometry as one strand of mathematics that has applications in careers requiring advanced instruction and construction of knowledge, such as in art, architecture, interior design and science, but it also has its applications in technical careers such as carpentry, plumbing and drawing as well as daily life in the community. Usiskin (2002) states that geometry is important to teach in our daily life because:

- Geometry connects mathematics with the real outside world and
- Geometry enables ideas from other areas of mathematics to be pictured.

In daily life and vocational careers, many concepts and techniques are learnt and transferred from school geometry to the outside world. This is where the learners are able to relate what is learnt at school with their applications in the real world. Teachers are to produce the kind of learners who will fit in their community, by integrating the school environment with the outside world. PSTs are required to facilitate this knowledge and make sure that learners have the ability of constructing knowledge in their minds so that they will be able to discover new methods of problem solving within their societies.

4.3 Situated learning.

4.3.1 What is situated learning?

Situated learning was first used by Lave and Wenger (1991) as a model of teaching and learning within a community of practice. Their model of situated learning proposed that learning involved a process of engagement within a community of practice (Smith, 2009). This study is located within the framework of situated learning because it explores how PSTs' knowledge develops during activities. This type of learning allows an individual learner to learn by socialisation, visualisation, and imitation. This implies that a learner needs to interact with the surroundings and observe the behaviour of the environment and then act accordingly. It includes visualisation because the ability to reason visually is increasingly important in the learning of the child. Thus; the role that visualisation plays in learners' mathematical thinking and problem-solving experiences has become more significant (Ho, 2009). This emphasises that learning is specific to the situation in which the learner is exposed. For learning to become more socially effective, learners should be part of communities where they will have shared interests with, and benefit from the knowledge of, those who are more knowledgeable. This more knowledgeable person could be an adult or more knowledgeable peer.

4.3.2 Communities of practice

Wenger (2007) defines communities of practice as formed by people who engage in a process of collective learning within a shared domain of human endeavour; for example a group of learners may come together with an aim of sharing ideas on how to prove a certain theorem, a group of engineers working on similar problems, a clique of learners defining their identity in the school or a network of surgeons exploring novel techniques. Basically a community encompasses groups of people who share a common concern or a passion for something they do or they want to do and learn how to do it

better as they interact regularly. There are crucial elements in distinguishing a community of practice from other groups and communities as reflected in the table that follows:

Table 4.1: Characteristics of communities of practice. Adapted from Wenger (2007, p. 3)

Element	Description
The domain	A community of practice is more than a group of friends or a network of connections between people. It has an identity defined by a shared domain of interest. Membership therefore implies a commitment to the domain, and therefore a shared competence that distinguishes members from other people. For example in schools learners may form a peer group, while one may be a peer tutor.
The community	The main aim of gathering is to pursuing the interest in their domain. Members engage in joint activities and discussions, help each other, and share information. For example in this case the shared domain of interest may be discussion of what is learnt in mathematics
The practice	Members of a community of practice are practitioners. They develop a shared repertoire of resources: experiences, stories, tools, and ways of addressing recurring problems, in short shared practice. This takes time and sustained interaction. For example learners may gather with an aim of doing geometric problems discussed in class.

4.3.3 Legitimate peripheral participation

Legitimate peripheral participation is one of the very important concepts within situated learning. This is a process where the learner can learn by being involved in his or her social environment of practice (Wiesen, 2003). According to Lave and Wenger (1991) legitimate peripheral participation (LPP) provides a framework to look at how the individuals, can become involved with his/her community of learners. Legitimate peripheral participation is a vital principle of different kinds of learning theory, while situated learning is mainly social rather than psychological. This process of legitimate

peripheral participation introduces socio-cultural and historical realisations of power and access to the kinds of thinking and knowing. Atherton (2011) based the explanations on case-studies of how the new learners learn in different groups which are not described by any formal learning, and suggest that legitimate peripheral participation is the solution in that learning. He further analysed legitimate peripheral participation as follows:

- It is **legitimate** because all learners in a group accept the position of unqualified learners as potential members of the peer group or community.
- **Peripheral** because they hang around on the edge of the important knowledge, do the peripheral activities, and gradually get entrusted with more important ones.
- **Participation** because it is through socially constructing knowledge that they acquire it. Knowledge is situated within the practices of the group of learners or community of practice, rather than something which exists in books (Atherton, 2011, p. 1).

The study draws on situated learning in its exploration of knowledge of proofs in geometry. Geometry requires an exploration of real day to day experiences. PSTs are on their journey of preparing themselves to become teachers. PSTs are required to ensure that their learners learn from their own experiences. They are required to make learners develop thinking about properties of shape in order to classify them accordingly. To develop this thinking, learners ought to be furnished with the necessary skills that will provide reasoning ability, like exposing them in a situation or environment that will demand them to think.

4.3.4 Situated learning and this study

The theory of situated learning is relevant to this study, since situated learning theory is emerging as a learning theory that is relevant to the teaching and learning of geometry. Within this study PST knowledge of geometry was explored. This knowledge was a culmination of experiences and activities within different contexts and cultures. Situated learning is the study of how human knowledge develops in the course of activities and how learners create and interpret what they are doing. The study is about

the exploration of PSTs' knowledge, including content knowledge, knowledge about how to teach, knowledge about learners and how they learn, knowledge about the curriculum and knowledge about discipline and classroom management. Being community-minded, involving, consulting and being engaged within the local community is what situated learning theory suggests to teachers. Learning is a function of the activities, context and culture in which learning is happening.

4.4 Conclusion

Social constructivism is a sociological theory that encourages groups of people to construct their own knowledge from one another. This group of people have the potential to work collaboratively in creating a small culture of shared objectives with shared meanings and understanding. When one is immersed within a culture of this kind, one is learning all the time about how to be a part of that culture on many levels (Vygotsky, 1978). Situated learning is learning that takes place in the same context in which it is applied (Lave & Wenger, 1991). This chapter outlined the underpinning social constructivism and further explains Piaget and Vygotsky's approach in teaching mathematics. The chapter also compares social constructivism with learning mathematics. It also elaborates what situated learning is, how communities of practice are formed and how communities of practice may be linked with learning.

In the next chapter, the analysis of the data collected in this study will be discussed. The chapter also discusses the themes emerging from the data collected.

CHAPTER 5

DATA ANALYSIS

5.1 Introduction

The aim of the study was to explore pre-service teachers' (PSTs) knowledge of proof in geometry. The study also explored how PSTs use their knowledge of proof in geometry. Through the use of qualitative analysis, by means of task-based worksheets, survey questionnaire, focus group interview and individual interview, the researcher was able to collect the data for this study.

The study investigated and sought to answer the following critical research questions:

1. What is pre-service teachers' knowledge of proof in geometry?
2. How do pre-service teachers use their knowledge of proof in geometry?
3. Why do pre-service teachers use their knowledge of proof in geometry in the way that they do?

As discussed in Chapter 3 the participants in the study were 2nd, 3rd and 4th year students. A total of 180 participants completed the task-based worksheet. However interviews were conducted with 3rd and 4th year students only.

5.2 Data presentation

This section presents the results of the data analysis for the three critical research questions. Descriptive statistics of the participants, their knowledge of proof in geometry and the participants' performance on a task-based worksheet are also analysed in various tables that follow. To assist in answering questions 1 and 2, data was collected by means of a survey questionnaire, task-based worksheet and a semi-structured interview schedule. The third critical research question was addressed by the semi-structured interview schedule. Table 5.1 illustrates how data was collected for each critical research question:

Table 5.1: Data collection plan

Critical Research Questions	Participant	Data Collection Method
1. What is pre-service teachers' knowledge of proof in geometry?	Second, third and fourth year pre-service teachers	• Survey questionnaire. • Task-based work sheet. • Semi-structured focus group interview. (For only 3rd and 4th years) • Semi-structured individual interview. (For only 3rd and 4th years)
2. How do pre-service teachers use their knowledge of proof in geometry?	Second, third and fourth year pre-service teachers	• Survey questionnaire. • Task-based work sheet. • Semi-structured focus group interview. (For only 3rd and 4th years) • Semi-structured individual interview. (For only 3rd and 4th years)
3. Why do pre-service teachers use their knowledge of proof in geometry in the way that they do?	Third and fourth year pre-service teachers	• Semi-structured focus group interview. • Semi-structured individual interview.

Since the study focused on an exploration of PSTs knowledge of proof in geometry, the performance on various proof-based questions are discussed. All questions on the task-based worksheet encompassed the critical research questions. The performance of the PSTs were assessed by the researcher using marking memorandum as an assessment tool. The performance on the task-based worksheet demonstrated that most of the PSTs in the sample had little or no knowledge of proof. Based on the performance on each question from the task-based worksheet, it was evident that PSTs performed best in question one. The majority (51.67%) of PSTs, including 2nd years, showed that they have knowledge of proof relating to angles of straight lines, but very little of proof relating to parallel lines. This demonstrates that the PSTs have only the basic or fundamental knowledge of straight lines, angles on the straight line as well as angles of the triangle. This is surprising since the task-based questions were based on work extracted from the Grade 8 to 11 syllabuses. This is indicative of the fact that learners enter the university without adequate background knowledge of geometry at Grade 10, 11 and 12 level. This lack of background knowledge creates an enormous problem for

tertiary institutions, such as universities. Universities have their own scope of work to cover and expect learners to have foundational knowledge. They do not have time or capacity to bridge this gap. As a result the PSTs have a poor foundation in geometry. Table 5.2 that follows illustrates the performance of all participants who answered the task-based work sheet.

Table 5.2: Percentage of students who achieved correct responses on the task-based worksheet.

Task-based Questions	Types of proof per question	Questions taken from Grade:	Percentage of correct responses
Question 1	Proof on angle of straight lines.	8	51.67%
Question 2	Proof of parallel lines.	9	14.44%
Question 3	Proof on quadrilaterals.	10	5.00%
Question 4	Proof on angles in a triangle	10	5.56%
Question 5	Proof of chord and tangent	11	1.67%
Question 6	Proof on cyclic quadrilateral	11	1.67%

It was also noted in the study, that the level of knowledge of proof in geometry between 3rd and 4th years PSTs could not be distinguished. The fourth year and the third years seem to participate at the same level of knowledge of proof in geometry. Table 5.3 that follows presents the performance of each year level per question.

Table 5.3: Number of correct responses per question per year level of the 180 students

Year Level	Q 1⁶	Q 2	Q 3	Q 4	Q 5	Q 6
4 th year	27	09	03	06	01	00
3 rd year	52	17	06	04	02	03
2 nd year	14	00	00	00	00	00
Total	93	26	09	10	3	3
% correctly answered	51.67%	14.44%	5.00%	5.56%	1.67%	1.67%

⁶ Q1: Question one; Q2: Question 2 and so on and on from the task based worksheet.

As discussed earlier in Chapter 3, interviews were conducted with 3rd and 4th year students only. Twenty participants were invited to the focus group interview (FGI) and 10 of 20 were also invited to the individual interview (II). Fourteen participants attended the focus group interview, eight of the participants were 3rd years and six of the participants were 4th year students. Nine participants agreed to be part of the participants in the individual interview, six were 3rd year students and three were 4th year students. Based on the responses the participants have provided for the task-based worksheet and the analysis done on the responses to the survey questionnaire, the researcher selected PSTs who:

- demonstrated a better knowledge of geometry proof and of geometry problem solving,
- displayed an average knowledge of geometry proof and of geometry problem solving and
- demonstrated very little or no knowledge of geometry proof.

The details are indicated on Table 5.4 that follows. The last column on table indicates the questions each participant answered correctly from the six questions on the task-based worksheet.

Semi-structured interviews were conducted with each participant and the task-based worksheet was used to probe responses. The participants were interviewed using a semi-structured interview schedule. The design of the interviews was based on how the PSTs answered the task-based worksheet and the survey questionnaire. These worksheets and survey questionnaires assisted in clarifying the PSTs' knowledge in geometry. All participants were given ample time to respond to the task-based worksheet and survey questionnaire. Both the focus group and individual interview were tape recorded with the participants' permission. The aim of tape recording was to ensure that there were no misinterpretation and misunderstanding.

As indicated earlier in Chapter 3, in Table 3.2, pseudonyms were used to protect the identity of the PSTs. Table 5.4 that follows illustrates all questions that each PST has answered.

Table 5.4: List of pre-service teachers with the questions they have answered.

	Recorded as	Code ⁷	Year Level	Gender	Race	Question(s) correctly answered
1	Learner 1	PST 01 ⁸	3 rd	Female	I	1, 2,3& 6⁹
2	Learner 2	PST 02	3 rd	Female	W	1,2&4
3	Learner 3	PST 03	3 rd	Female	I	1,2,3&4
4	Learner 4	PST 04	3 rd	Female	I	1, 2,3& 5
5	Learner 5	PST 05	4 th	Female	I	2,3 &4
6	Learner 6	PST 06	3 rd	Male	I	1,3,4,5&6
7	Learner 7	PST 07	3 rd	Male	A	1
8	Learner 8	PST 08	4 th	Male	A	1&5
9	Learner 9	PST 09	4 th	Male	A	1&2
10	Learner 10	PST 10	3 rd	Male	I	1,2&3
11	Learner 11	PST 11	3 rd	Female	A	1&2
12	Learner 12	PST 12	4 th	Female	A	1,2 &4
13	Learner 13	PST 13	4 th	Male	A	1&3
14	Learner 14	PST 14	4 th	Female	A	1&4

5.3 Themes

In qualitative research the researcher triangulates the data collected from various resources. In this study the sources were the survey questionnaire, the task-based worksheet, the semi-structured focus group interview and semi-structured individual interview. Triangulation allows the researcher to gain multiple perspectives of a

⁷ Codes are used to protect identity of PSTs.

⁸ PST01: Pre-Service Teacher number 1

⁹ 1, 2, 3 & 6 means PST 01 got questions 1, 2, 3 and 6 correct and questions 4 and 5 incorrect.

phenomenon. In this study, the researcher collected data about PSTs' knowledge as well as the ideas about how proof in geometry is used through several processes. The survey questionnaire and semi-structured interview raised questions regarding the sections the participants liked the most and the least. The reason this question was asked is because the researcher wanted to find out about the PSTs' experiences from practice teaching about the learners preferences. Most PSTs in this study claimed that learners preferred algebra to geometry. The excerpts that follow provide evidence of this inclination. These excerpts are taken from focus group interview 1 and focus group interview 2.

Researcher: In your own opinion, which section (algebra, geometry and trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 02 : *They like algebra.*

PST 04 : *I think it will be algebra.*

PST 08 : *Algebra.*

PST 09 : *Well, I think they enjoy algebra*

PST 14 : *I think is mostly algebra.*

Responses of PSTs who participated in the individual interview confirmed this. Out of 14 participants, nine PSTs thought algebra was the most enjoyed section by learners in mathematics. Eight of the participants thought that geometry was enjoyed the least by learners in schools. Table 5.5 that follows reflects the results obtained for this question from the focus group and individual interviews.

Table 5.5: PSTs' opinion regarding comments made by learners regarding mathematics sections.

	Most liked section	Least liked section
Algebra	9	0
Trigonometry	2	5
Geometry	1	8
Other	1	0
No response	1	1
Total	14	14

In this study, task-based worksheets and the interviews assisted the researcher in identifying common themes about proof in geometry from the participants. Five themes emerging from data were identified as follows;

- fundamental knowledge
- traditional teaching
- non-traditional teaching
- visualisation
- concrete manipulatives

These themes were identified following the research questions that guided the exploration of PSTs' knowledge of proof in geometry (Gay, Mills, & Airasian, 2006). Question one of the critical questions assisted the researcher to identify the basic or fundamental knowledge the PSTs have. The second and third critical questions assisted to identify the methods and strategies used when teaching and learning mathematics. These themes also indicate the knowledge of proof in geometry the PSTs have. In many instances the reasons for the lack of knowledge are also provided. Five themes are discussed in detail in the following section.

5.3.1 Fundamental knowledge

The first steps in the study of geometry are concerns related to the naming, describing and classification of shapes. In addition, making links to measurements, position and movements of these shapes are also important. While children are growing, they learn from an early age through playing with different shapes, noting their obvious properties. They learn to identify different figures, for example squares, triangles and circles, and they also include the understanding of lines (French, 2004).

Ryan (2008) also explains that the study of geometry begins with the definitions of the five simplest geometric objects. He proposes that point, line and ray are the foundation when studying proof in geometry. He further differentiated between the two dimensional plane and three dimensional space. Moreover he suggested that the standard two-column geometry proof ought to contain the following elements (p. 50):

- The diagram
- The givens (the given information on the diagram)

- The prove statement
- The statement column
- The reason column

The new Curriculum and Assessment Policy Statement (CAPS) for Grades 10 – 12 in South Africa (DBE, 2010), which is currently being implemented in 2012 in Grade 10, defines mathematics as the study of quantity, structure, space and change. The Grade 10 mathematics work schedule in 2012 focuses on the revision of the fundamental results accomplished in lower grades as far as lines, angles and triangles go, and especially as far as the similarity and congruence of triangles are concerned. The document goes further to the investigation of the properties of line segments joining the midpoints of two sides of a triangle. After investigation it defines the following special quadrilaterals: the kite, parallelogram, rectangle, rhombus, square and trapezium. The fundamental knowledge required of learners includes the investigation and making of the conjectures about the properties of the sides, angles, diagonals and areas of these quadrilaterals. Proving of these conjectures for the parallelogram, solving of problems and proving conditions using the properties of parallel lines, triangles and quadrilaterals are very crucial at this level (DBE, 2011b, p. 25). For better understanding these investigations, making of conjectures and proving ought to be sequential to the learners. This work schedule forms a sound fundamental knowledge for the Grades 10, 11 and 12 if it is followed correctly and is done at the right stages. Table 5.6 that follows highlights the overview of topics that ought to be dealt with in Euclidean Geometry in Grades 10, 11 and 12.

Table 5.6: 2012 Topic overview adapted from CAPS document (DBE, 2011b, p. 14).

Grade 10 Being implemented in 2012	Grade 11 To be implemented in 2013	Grade 12 To be implemented in 2014
a) Revise basic results established in lower grade. b) Investigate line segments joining the mid point of two sides of a triangle. c) Properties of special quadrilaterals	a) Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with one other result concerning tangents and radii of circles. b) Solve circle geometry problems, proving reasons for statements when required. c) Prove riders.	a) Revise earlier work on the necessary and sufficient conditions for polygons to be similar. b) Prove (accepting results established in earlier grades): • that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and mid-point Theorem as a special case of this theorem) • that equiangular triangles are similar; • that triangles with sides in proportion are similar; • the Pythagorean Theorem by similar triangles; and riders.

By using the topic overview of the new CAPS, teachers will assist learners to accumulate adequate geometric knowledge in the last three years of their schooling (Grades 10, 11 and 12). This knowledge will then form a very crucial fundamental knowledge in geometry when they have to pursue their studies at universities. This study indicated that the PSTs entered university with a very limited basic knowledge in geometry. Most of the PSTs who participated in the study have indicated that, they did not have the fundamental knowledge of proofs in geometry. PSTs claim that they were

not taught proofs in geometry in high school at Grades 10, 11 and 12 levels. This is confirmed by excerpts taken from the individual interview as illustrated below.

Researcher: Can you explain how proof in geometry was taught at your high school?

PST 05 : *It was never taught, to prove something, eh not so much never taught, like for example you would prove that to a quadrilateral, say a cyclic quad, if you are to prove that ehh, if you are given a diagram and in the diagram, there was a quadrilateral, if we had to prove that we had to use a theorem with the angles and then try to see if it is 180° supplementary angles are 180° and then it would be a cyclic quad, but I didn't know that cyclic quad would touch four vertices of the circle. The smallest thing we didn't... wasn't really enforced on us.*

PST 09 : *No, we didn't prove anything in geometry. What I only remember is the sum of angles of a triangle is 180° and we didn't prove that, he just taught us it like that. so, and even for high school textbook, like I said I am teaching myself all this geometry and all the stuff in geometry. The textbook even that I used in high school geometry, they don't do proofs in most cases what they do, they just give theorems and say theorem number 1, theorem number 2, very few of them are doing the proof of the theorem, which means even the writers don't pass this logic in geometry and the meaning of geometry, and this application of geometry to real world to form their knowledge to learn us.*

The study exhibited that the only geometry knowledge the PSTs have was knowledge about angles on straight line, angles on parallel lines and angles in a triangle. A few PSTs mentioned circles, tangents and quadrilaterals. PSTs' performance on the task-based worksheet (as shown in Table 5.2), demonstrated that a higher percentage (51.67%) of participants achieved the question on angles on straight line and at least 14.44% on angles on parallel lines and 5.56% on angles of the triangles (see table 5.2 discussed for performances on the other questions). This performance correlates with what the PSTs have said in their interviews with the researcher. When asked about the geometry concepts they were aware of before they entered the university the PSTs responded as follows in the focus group interview:

Researcher : Which geometry concepts were you aware of when you enrolled at the university?

PST 01 : *... I really came in with is angles with the FUN and the circle geometry ... I think we did, do quite of geometry but it was, I just use what understanding I have.*

- PST 03 :** *The concepts that we were aware of, was basically angles everything in angles, eh... and then triangles all the different types of triangles and coming in few circle geometry that we came in with and quads that is the basic geometry that we came into university with.*
- PST 10 :** *Basically angles, yes angles is the one that stands out you are doing geometry because you work with every single day.*
- PST 13 :** *The part that, I remember the most, the one of the alternate angles and co-interior angles and co-exterior angles and all the stuff that one was familiar of and the like the straight line, we have got to add up to 180° and all the stuff.*
- PST 14 :** *Do you mean that something that I come here, I already know? Angles of triangle all add up to 180° .*

In an effort to find out more about how PSTs complete proofs in Euclidean geometry, the researcher studied the method PSTs used to respond to question 1. Question 1 focused on angles on a straight line. This question was generally well answered. It was interesting to note that most of the PSTs made deductions using the substitution and transitive properties in proving question 1 (see Fig 5.1 that follows). Substitution property refers to the case when two angles are congruent to each other and one of them is supplementary to the third angle. This implies that the other angle is also supplementary to the third angle. The transitive property states that if two angles are congruent to a third angle, then all the angles are congruent to each other (Ryan, 2008, pp 75 - 77). Stephan and Clements (2003, p. 5) outlined the transitivity as the understanding of the following points:

- If angle A is equal to angle B and angle B is equal to angle C, then angle A is equal to angle C;
- If angle A is greater than angle B and angle B is greater than angle C, then angle A is greater than angle C; and
- If angle A is less than angle B and angle B is less than angle C, then angle A is less than angle C (Stephan & Clement, 2003, p. 5).

It is however, noted that, even though the PSTs have responded well in question 1, the level of the tasks given were not the level of mathematical knowledge that PSTs were expected to be knowing at this stage, the tasks were taken from GET and FET phase.

This question was based on the fundamental knowledge of proof PSTs ought to have entered the university with. At the university the PSTs begin to study the methods and strategies of teaching and learning geometry in mathematics.

Figure 5.1 represents question 1 from the task-based worksheet. The discussion that follows is focused on the excerpts taken from the Individual Interview (II). Here PST 02 and PST 04 explain to the researcher their response:

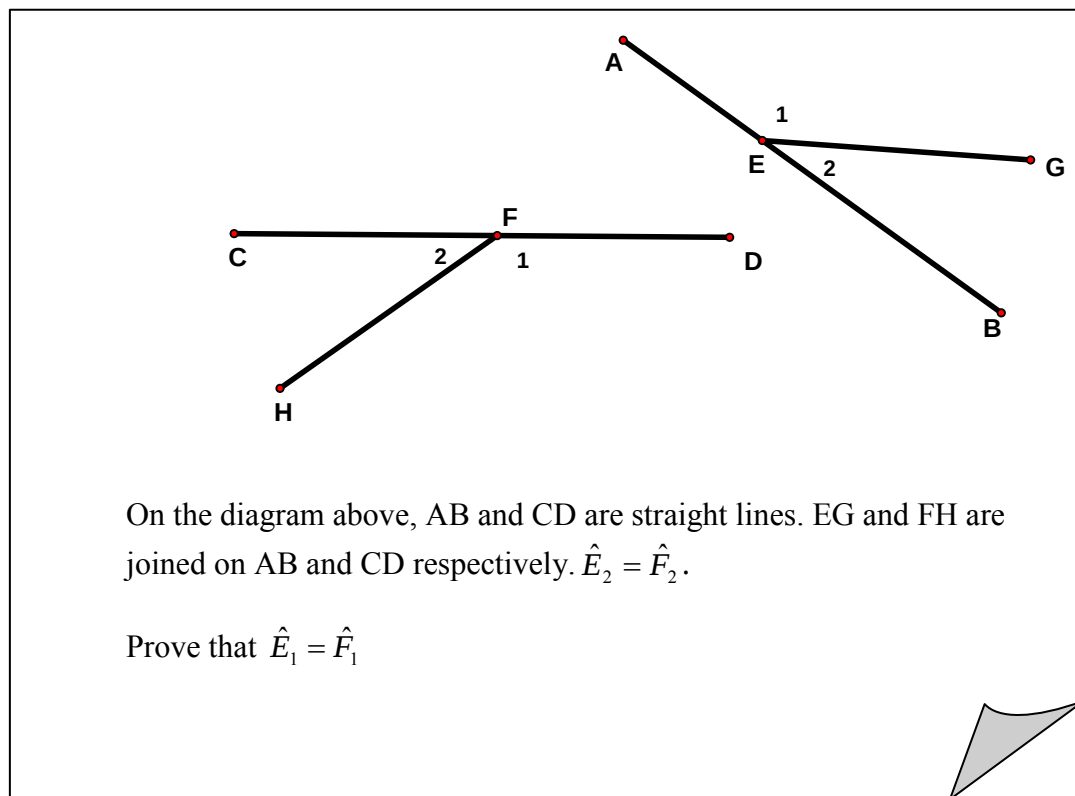


Figure 5.1 Question 1: Proof based on angles on straight lines. (Adapted from task-based worksheet)

Researcher : We are given that $\hat{E}_2 = \hat{F}_2$ and you said let $\hat{E}_2$ and $\hat{F}_2$ be equal to x and may I know why?

PST 02 : *Sometimes it is easier work with x.*

Researcher : There must be a reason why you have made both of them ($\hat{E}_2$ and $\hat{F}_2$) equal to x. Why have you made them equal?

PST 02 : *Because if two, just by putting $\hat{F}_2$ makes look different that if you put x you can see there is an angle. I've always done that same letter, same angles.*

Researcher : Now let us look at this one (pointing at the task based worksheet),

you said $\hat{E}_1 = 180^\circ - x$ and $\hat{F}_1 = 180^\circ - x$ and then you concluded that $\hat{E}_1 = \hat{F}_1$ why did you give this conclusion?

PST 02 : *First of all, they are straight line, so we have been given that angles equal if $\hat{E}_2 = \hat{F}_2$. So both are at the straight line, because both of a straight line, we can actually find that angles there, so I've said that an angle is $\hat{F}_1 = 180^\circ - x$ and $\hat{E}_1 = 180^\circ - x$ so because they are both $180^\circ - x$. So they are equal.*

The following excerpts are taken from the individual interview between PST 04 and the researcher on question 1:

Researcher : You were actually requested to prove that $\hat{E}_1 = \hat{F}_1$ and you said; since $\hat{E}_2 = \hat{F}_2 = a$ then $\hat{F}_1 = 180^\circ - a$ why do you say that?

PST 04 : *From the straight line theorem, we were taught that angles on a straight line is equal to 180° , so if I label one to be 'a' I know that the next angle will be $180^\circ - a$.*

Researcher : And you have proceeded to $\hat{E}_1 = 180^\circ - a$ and then you had your conclusion which is $\hat{E}_1 = \hat{F}_1$, why?

PST 04 : *I made connection that if one angle is equal to other ehh.... then more certainly then if one will be then equal to other the two are equal.*

From the excerpts above, it was evident that PST 02 used the substitution property. Since $\hat{E}_2 = \hat{F}_2$, these angles were then made to equal to x . Though it is not shown, the deductions are from understanding that on a straight line $\hat{E}_1 + \hat{E}_2 = 180^\circ$ and $\hat{E}_1 + \hat{F}_2 = 180^\circ$. This implies that $\hat{E}_1 = 180^\circ - \hat{E}_2$ and $\hat{F}_1 = 180^\circ - \hat{F}_2$ hence both $\hat{E}_2$ and $\hat{F}_2$ were made to equal to x . This means that $\hat{E}_1 = 180^\circ - x$ and $\hat{F}_1 = 180^\circ - x$. Transitive property was also used, based on the conclusion that $\hat{E}_1 = \hat{F}_1$ was based on the fact that each is equal to $180^\circ - x$. The same explanation also resonates with PST 04's attempt, with the exception of the use of variable a instead of x .

5.3.2 Traditional teaching

To enhance the quality of teaching and learning in the classroom non-traditional methods such as cooperative, collaborative and problem-based learning may be utilised. De Villiers (1987) noted that most teachers presented proof in geometry classrooms as

the only means of obtaining certainty. This approach creates doubts in learners' minds about the validity of empirical observations. This strategy creates an incorrect understanding of the function of proof as the only verification of the correctness of mathematical statements. De Villiers pointed out that about 60% of prospective mathematics teachers enrolled at South African universities sees the function of proof only in terms of verification, justification and conviction. Hence the prospective mathematics teachers were not able to identify other functions of proof like explanation, discovery and systematisation (these functions are explained in chapter 2 sub-section 2.5.1). The teaching of proof in geometry ought to reflect the nature of mathematics and what is meaningful. It ought to be mathematics that would allow learners to experience, to understand and know the usefulness of the activities they are involved in (De Villiers, 1990). In schools proof is taught as a mechanical procedure which carries truth in an unbreakable chain from assumptions to conclusions; they are not taught as a means of explanation or elaborations which make conjectures more plausible and more convincing (Lakatos, 1976).

Based on the data collected it was evident that the majority of the participants entered the university with the hope of getting new methods or approaches of teaching mathematics in particular geometry. This study has revealed that most of the PSTs do not have a sound knowledge of working with proof in geometry while at school. PSTs claim that the dominating methods at the schools they themselves had attended were chalk and talk or chalk and board. Their teachers would read the theorem without allowing learners to self discover. When the researcher asked PSTs about the manner in which they were taught geometry while they were at school, 10 out of 14 PSTs complained about traditional methods of teaching, some even claimed that they were not taught to do proofs at all. The following are excerpts taken from the two focus group interviews.

- Researcher :** Can you explain how geometry was taught at your high school?
- PST 02 :** *She basically, she just told us this how to prove it... she proved it with us she tell us ok how do you think it and a lot was...*
- PST 03 :** *... he will show us the proof that was, we just have to like memorise it off.*
- PST 04 :** *Chalk and board, ehh... chalk and talk, basically the proof was given to us to byheart it, never constructed on our own.*
- PST 05 :** *It was never taught, to prove something, ehh not so much never*

taught,

Moreover, the participants described the instructions they received while they were at school as directed by the teacher. They described their history of participating minimally in mathematics classrooms. Traditional teaching generally refers to situations where the teacher considers a learner as an empty vessel, something to be filled with knowledge. The examples of traditional teaching are textbook method, lecture method, chalk and talk method. This kind of teaching also views the teachers as the only person who is instrumental in the learning process (Novak, 1998).

Rote memorisation is the process of committing ideas to memory through repetition. Teachers teach ideas and then repeat these ideas or ask students questions to see if they remember ideas taught in the class. Teachers also have students read from a textbook and handouts that contain information. Learners memorise information and then repeat the information both in class activities and in class tests. However, this practice helps learners develop memory skills while also helping them learn facts that society considers important. The analysis presented by Johnson and Dasgupta (2005) indicates that there are still some PSTs who favour lecture style teaching. Overall, 37% of the PSTs in their sample preferred a traditional lecture-based approach to instruction. Traditional teaching strategies are often those that have lasted because they are effective. These techniques remain simply because they are easier to teach. For example, teaching learners scientific facts out of a textbook takes less work than coaching them through their own scientific experiments.

Mthembu (2007) has observed that some teachers still believe in the traditional way of teaching mathematics, especially geometry. The reason they give for teaching in the way they do, is that it saves them time and that they are able to cover a lot of work in a short time, thus are left with more time for revision. The traditional way of teaching of geometry is based on the transmission of axioms and theorems formulated by other mathematicians. These are recorded in texts for learners to study. Learners are not given the opportunity to question and understand them. This creates an impression that geometry comprises sequence of facts and formal proofs that ought to be followed as they are.

5.3.3 Non-traditional teaching

In this study non-traditional teaching involves innovative strategies of teaching which include the use of technology. The two themes to be discussed, that is visualisation and manipulatives, are also contributing to the non traditional teaching approaches, as they create the atmosphere of active learning amongst the learners. The non-traditional teaching should involve strategies like active learning, collaborative learning, cooperative learning, problem-based learning and small group teaching.

This study has revealed that most of the PSTs have a limited knowledge of working with proof in geometry. They claim that the way they were taught while they were at school is a major reason for their lack of knowledge. However, the PSTs suggested that innovative strategies that can be used in teaching proof in geometry would be more beneficial. They suggested that using non-traditional strategies and using new technology in teaching and learning of geometry would be useful. Most of the PSTs believed that learners should be exposed to constructing knowledge when doing proof. The following are extracts taken from the individual interview between PST 04 and the researcher:

Researcher : Which strategies do you think you would use when teaching proof in geometry to your learners?

PST 04 : *... I would definitely love to use constructive approach, when student are measuring and using tools to find out of the classroom situation allows me to do so. But I think with geometry it's easier for student to construct rather than just to learn of.*

Researcher : Why do you think that these strategies would work?

PST 04 : *I think it would work, because when they construct their knowledge, it's easier to remember and know what is true for them. As opposed to given to them and they are not understanding why is it true and why is that so that way.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 04 : *Most certainly, I think geometry becomes more believable that way as opposed to... As opposed to just given, then and they are learning but they never be able to apply, when they are able to construct they are able to apply.*

During the interview PST 04 made a distinction between constructing knowledge and providing learners with proofs to be done.

5.3.4 Visualisation

According to Level 1 of the Van Hiele (1995) levels of geometry understanding, as discussed in chapter 4, learners recognise figures by appearance alone, often by comparing them to known examples. Arcavi (2003) defines visualisation as the ability of interpretation, and reflection upon pictures, images and diagrams in minds, with the purpose of depicting information. It is an aid to an understanding or means towards an end and so one can therefore speak about visualising as a concept or a problem but not necessarily as a diagram (Presmeg, 1985). Visualising refers to mental images of a problem, and to visualise a problem means to understand a problem in terms of a diagram or visual image. Hence, according to Presmeg, the visualisation process is one which involves visual imagery with or without a diagram, as an essential part of the solution. Zaskis, Dubinsky and Dautermann (1996) describe visualisation as an act in which an individual establishes a strong connection between an internal construction and something to which access is gained through the senses.

The study revealed that most of the PSTs believed that proof in geometry can easily be understood when visualisation is considered as important. During interviews, PSTs indicated that they believed knowledge of proof originates from what one sees on the diagram. However, most of the PSTs stated that the role of the teacher and textbooks is to guide learners towards finding the knowledge within them on what they see. For example, when PSTs were asked about the use of colours, symbols and constructions on their task-based worksheet, most of them pointed out that they use colours or symbols in order to see what is given and what is required. The following excerpts taken from focus group interviews demonstrate these sentiments:

Researcher : What can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)

PST 03 : *... we were very used in using different colours, different symbols like, for example, if it was two triangles in a circle one will be one colour and other triangle will be other colour, so we can see which triangle are we talking about what ever...*

PST 04 : *I use symbols to allocate angles, using connection between from the first symbol, I would use alphabet 'a' to show that the angles are equal... I would use colours if the diagram is complex, I would use a highlighter if I see a parallelogram for*

instance; bring out the F shapes showing the corresponding angles. I would use colours to prove that. I would also use a construction.

PST 05 : *...we would put on the diagram so that we can see it, physically see it there, then a construction like, for example, I tried to prove ...*

PST 08 : *... if they are no symbols, then it is very hard to work out what is what, so you need to see something...*

From the above excerpts, it is evident that most PSTs value the use of visualisation in teaching and learning of proof in geometry. Visualisation in terms of using pictures and diagrams in proofs has played an immense role in geometric mathematics. In a study conducted by Gibson (1998), the data confirms that learners understand diagrams when learning using visualisation to construct proofs. Along similar lines, Mudaly (2011) confirmed this observation with 69 third year mathematics pre-service teachers. Mudaly (2011) used a question of a farmer who wanted to fence a square piece of land while insisting on using eight poles on each side of the square. The question of how many poles will the farmer need, may only be answered by visualising the square with poles which will eventually lead to the correct answer by drawing poles on each side. Teaching mathematics using diagrams or visual images helps to develop understanding of conceptual knowledge. Ho (2009) sees visualisation in the teaching of mathematics as providing assistance to learners in order to understand the problem.

Apart from diagrams and visual images, colours and pictures play a vital role in visualisation. Sometimes the problem can be made easier by showing some pictures or adding some colours to it, so as to be more visible to learners. This means the use of different colours may play a vital role in the teaching and learning of proof in geometry using visualisation. Carter (2009) emphasises that the understanding of mathematics using pictures is useful in a way that a simple example illustrating the act of understanding an expression by directing one's thoughts towards a picture consisting of an understanding of the facts. The pictures may provide more information than what is contained in the expression.

This sentiment is confirmed by PST 02, when asked about using colours on the task-based worksheet. The following excerpt is taken from the individual interview with the PST 02:

Researcher : Even though you think you have put these colours because you

have some special problems, but do you think this would help other students?

PST 02 : *Definitely, our teacher encourages us to use colours, when you are doing geometry, so we can see what we are doing parallel lines with that, and then you see angles that are equal and what you have to do.*

Visualisation helps to connect the related problems. It helps to provide the individual learning styles, as each learner has his/her own preferences when dealing with visuals in mathematics. Visualisation also acts as a substitute for computation; the answers may be obtained directly from the visual representation, without doing calculation, for example, with the farmer and the 8 poles discussed. With visualisation one can verify one's answers and one is able to transform the problem into a mathematical form as stated on the example. Visualisation is therefore an important aspect of mathematical understanding, insight and reasoning.

5.3.5 Concrete manipulatives

Mathematics manipulatives are defined as concrete objects used to help learners to understand abstract mathematical concepts (McNeil & Jarvin, 2007). One of the models for consideration when teaching and learning proof in geometry is the use of mathematics manipulatives. Manipulatives assist making mathematics instructions fun and meaningful. Mathematics manipulatives, for example pattern blocks, counting cubes, and pictures, stimulate physical activity and can provide a means to explore mathematics in a task (Clement & Battista, 1992).

In mathematics, different types of manipulative tools have been used to improve learners' understanding of proof in geometry with the aim of developing a learner's positive attitude toward geometry. Sowell (1974) has classified and defined these tools as three fold: concrete, pictorial, and abstract. Bayram (2004) suggested that concrete materials may be used or manipulated by learners in class during teaching and learning. Teachers may also use the new technology as a manipulative; they may allow their learners to explore various activities using anagrams, geoboards, sketchpad, interactive white board, smart board and any geometrical computer software to assist in proving the theorem. Many textbooks incorporating proof in geometry, focus mainly on calculations using formulae rather than asking learners to experiment. For example, learners should be encouraged to do manipulation of concepts, make conjectures about the properties, test conjectures and analyse various types of geometric shapes.

The study has shown that most of the PSTs favoured the use of concrete material when teaching and learning proof in geometry. Throughout the interview the PST recalled how they used visualisation and concrete mathematics manipulatives during their mathematics methodology lectures at university. They were often asked by the lectures to demonstrate the use of different concrete manipulatives, for example, geometry measuring tools. The participants suggested that learners ought to discover the theorem themselves through experimentation. The PSTs were of the belief that in order for the learners to be exposed to the theorem, they ought to be encouraged to do hands-on activities like paper cuttings, measurement and manipulating shapes. By using these concrete objects, it will be easier for learners to understand abstract concepts in the teaching and learning of proof in mathematics (McNeil & Jarvin, 2007). Learners ought to be allowed to experience and construct their own knowledge by themselves; teachers need only be facilitators. Based on the interviews, it was apparent that most of the PSTs believe in social constructivism. The excerpts below taken from the two focus groups demonstrate these sentiments:

Researcher : Which strategies do you think you would use when teaching proof in geometry to your learners?

PST 04 : *As I go out, I would definitely love to use constructive approach, when student are measuring and using tools to find out of the classroom situation allows me to do so. But I think with geometry it's easier for student to construct rather than just to learn of.*

PST 07 : *...using whatever is available, using practical ways as to make student follow an idea on what they see, and then following with more conceptual understanding a theory and proof.*

PST 09 : *...Some of the proof can be done by hands. You can let the learners do actual geometry by hands like paper folding, cutting and all those things or even doing actual measurement. So it depends on what you want to teach at that time, you can use experiments...*

During the interview, the PSTs recommended the use of new technologies such as sketchpad, interactive white board or smart boards in the teaching and learning of geometry. In the focus group interview, PSTs pointed out that the strategies that they would use to teach proof in geometry would involve more technology. They proposed that the use of technology would be one way to facilitate the constructing and testing of conjectures (Pandiscio, 2002). With the increase in the use of technology today, Dynamic Geometry Software (DGS), for example the sketch pad, have become

beneficial to all to teach geometry effectively since they support visualisation. Moreover, with the help of DGS, users can interact with geometrical objects and relations by manipulating these geometric shapes (De Villiers, 2004; Healy & Hoyles, 2001). The PSTs felt that if learners were left alone to explore this new technology they will be able to construct their own knowledge. This was evident in the following excerpts taken from the focus group interviews:

Researcher : Why do you think that these strategies would work?

PST 04 : *It think it would work, because when they construct their knowledge, it's easier to remember and know what is true for them. As opposed to giving them and they are not understanding why it is true and why is that, so that way.*

PST 05 : *.... with the regard to constructing your knowledge and learners will be able to prove things and we as teachers can also offer them or show them the contradictions to those proof, so it may complete in their mind and they will have to choose what is the correct proof and what is true.*

PST 06 : *... first of all you need to picture and the way that you know, the way that you know is normal concrete way. Sometimes we don't know exactly the shape. The concrete way of thinking helps to see these shapes that you try to find, or that going to help you as a link to solve equation. A abstract way of thinking is always important because if you don't have built in abstractly, I think the guy who has no that ability he has to quit mathematics.*

By engaging learners in examining, measuring, comparing, and contrasting a wide variety of shapes, this assists in developing essential teaching and learning skills (NCTM, 1989). This enables learners to learn to construct their own knowledge problem free.

5.4 Conclusion

This chapter presented an analysis of the data collected. The chapter highlighted some of the problems in the teaching and learning of proofs in geometry which led to PSTs' limited knowledge of proof. This data assisted the researcher in developing themes, which were discussed in this chapter. To clarify the themes of fundamental knowledge, traditional teaching, non-traditional teaching, visualisation and concrete manipulatives were discussed.

Evidence obtained from the task-based worksheet indicated that learners enter the university without adequate background knowledge in geometry in mathematics. This causes them to struggle a lot in trying to form a link between what they learnt at school with what they are learning at the university. The task-based worksheet also revealed that PSTs remember the geometry they have learnt in Grade 9. Some of the PSTs do not remember doing proof in Grade 10, 11 and 12. The performance on the task-based worksheet provided evidence that PSTs do not have enough basic knowledge in proof in geometry. It shows that PSTs have done very little in high school geometry while they were at school. The PSTs believe that the cause of the limited knowledge of geometry stemmed from high school, together with the use of traditional methods of teaching in the learning geometry. It was also interesting to discover out that some of the PSTs who wrote the mathematics paper 3 examination in their matric year, indicated that they did not start this paper in Grade 10, they did all the geometry in their matric year. Nevertheless, the PSTs are determined to change the way they were taught geometry which they believed did not benefit them much. They believe in non-traditional methods of teaching and learning which are based on theories of social constructivist learning. PSTs want their learners to socially construct their knowledge.

The next chapter will conclude the study by discussing the researcher's thoughts, and the findings and limitations of the study.

CHAPTER 6

SUMMARY AND CONCLUSION

6.1 Introduction

The study was based on the pre-service teachers' (PSTs) knowledge of proof in geometry. The discussion in this chapter summarises the main findings of the study and discusses how the researcher has achieved the aims of the study with respect to answering each of the critical research questions. It further highlights the implications and the possible limitations of the study and how this study is significant in solving some current problems in mathematics education.

6.2. Limitations

This study was conducted with a small number of participants. The results in the findings may be different for other studies conducted with larger numbers. It must be noted that the results of this research study are only relevant for PSTs at the university where the study was conducted.

This study was conducted at only one university in KwaZulu-Natal, South Africa. For future research studies more universities could be targeted. And possibly in future studies, the pilot study could be undertaken at the same university where the main study is conducted.

6.3 The main findings

The empirical evidence illustrated that a large percentage of the PSTs did not have an appropriate working knowledge of proof in geometry. For example 46.67 % of the PSTs had answered all questions incorrectly from the task-based worksheet, which is quite alarming because all questions were GET and FET phase questions.

Table 6.1: Number of participants with all incorrect responses on the task-based worksheet.

	Year Level	No
	4 th year	19
	3 rd year	39
	2 nd year	26
Total		84

Table 6.2 that follows represents the performance of all participants who wrote the task-based work sheet.

Table 6.2: Percentage of PSTs who obtained correct responses.

Task based questions	Percentage of the correct responses
Question 1	51.67%
Question 2	14.44%
Question 3	5.00%
Question 4	5.56%
Question 5	1.67%
Question 6	1.67%

A number of the PSTs tend to blame their teachers for poor knowledge of proof in geometry. Only a small percentage of the participants remember doing proofs as part of the school curriculum.

The study sought to explore the knowledge of PSTs and how this knowledge was used when teaching proof in geometry. The main findings of the study sought to answer the critical research questions that follow

1. What is pre-service teachers' knowledge of proof in geometry?
2. How do pre-service teachers use their knowledge of proof in geometry?
3. Why do pre-service teachers use their knowledge of proof in geometry in the way that they do?

6.4. The critical research questions

6.4.1 What is pre-service teachers' knowledge of proof in geometry?

What was evident from the data collected was that the participants demonstrated very little knowledge of geometric proof in mathematics. Based on the data collected, it is evident that the knowledge that the PSTs have is in two parts. Firstly the knowledge of geometry achieved at school and secondly the knowledge achieved at university. This knowledge focuses on lines, parallel lines with transversal line, angles, alternate angles corresponding angles, alternate angles circles and tangents. Some PSTs also mentioned triangles and rules of congruency. The evidence collected in this study indicates that there is very little or no knowledge of proof from what the participants were exposed to at school level. The lack of this knowledge creates the perception to PSTs that proof is not important in high school geometry. That perception might be disseminated to their learners as well. This lack of knowledge also created a gap in the PSTs' understanding

of writing a formal proof in geometry. The responses from the PSTs showed that there were difficulties in teaching theorems and their proofs at the school level.

The second knowledge the PSTs have, is knowledge gained while studying at university. The evidence in this study revealed that the PSTs have gained knowledge of Dynamic Geometry Software (DGS), for example sketch pad, intuitive smart boards and other advanced technology. The findings of the study confirm that participants perceive the use of dynamic geometry software in geometry as very effective since it provides a dynamic learning environment and enjoyable geometry tasks.

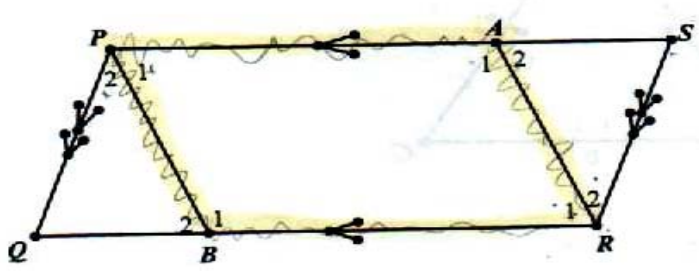
6.4.2 How do pre-service teachers use their knowledge of proof in geometry?

Based on responses collected from the task-based worksheets, it is clear that when working with proof, the PSTs use deductive reasoning using the substitution and transitive property. Deductive reasoning is discussed in chapter 2 in section 2.5.2. Substitution and transitive property are discussed in Chapter 5, in section 5.3.1. Deductive reasoning is the process of demonstrating that if certain statements are accepted as true, then other statements can be shown to be true, following from them. It is the process by which one makes conclusions based on previously known facts (Serra, 1997). Deductive reasoning is logically valid and it is the fundamental method in which mathematical facts are shown to be true. This is evident when looking at PST 03's response from the task-based worksheet in question three. The question required participants to prove that PBRA is a parallelogram. Based on PST 03's response that $\hat{P}_1 = \hat{B}_2$ and $\hat{A}_2 = \hat{R}_1$ because they were alternate angles and from there, the PST 03 went further to work with co-interior angles $(\hat{P}_1 + \hat{P}_2) + \hat{S} = 180^\circ$ and $(\hat{R}_1 + \hat{R}_2) + \hat{S} = 180^\circ$. Using the transitive property this lead to the conclusion that $(\hat{P}_1 + \hat{P}_2) + \hat{S} = (\hat{R}_1 + \hat{R}_2) + \hat{S}$ which means $\hat{P}_1 + \hat{P}_2 = \hat{R}_1 + \hat{R}_2$ but $\hat{P}_1 = \hat{B}_2$ and $\hat{R}_1 = \hat{R}_2$, given that PB and AR bisect $\hat{P}$ and $\hat{R}$ respectively. If adding $\hat{P}_1$ and $\hat{P}_2$, using the substitution property for both $\hat{P}_1$ and $\hat{P}_2$ to be $\hat{P}_1$ since they are both equal, they produce $2\hat{P}_1$. Doing the same with $\hat{R}_1$ and $\hat{R}_2$, using the substitution property for both $\hat{R}_1$ and $\hat{R}_2$ to be $\hat{R}_1$, since they are also equal, they give $2\hat{R}_1$. This means $2\hat{P}_1 = 2\hat{R}_1$ which is then translated to $\hat{P}_1 = \hat{R}_1$. However, it is stated above that $\hat{R}_1 = \hat{A}_2$, therefore $\hat{P}_1 = \hat{A}_2$ which are also corresponding angles and that would make PB and AR to be

parallel to each other. Hence it is given that $PS \parallel QR$ then $PBRA$ is a parallelogram. Figure 6.1 is an example of a correct solution to question 3.

3. PQRS is a parallelogram that is $PS \parallel QR$ and $PQ \parallel SR$. BP and AR bisect angles P and R respectively.

Prove that PBRA is a parallelogram.



$$\begin{aligned} \hat{P}_1 &= \hat{B}_2 \\ \hat{A}_2 &= \hat{R}_1 \\ \hat{P}_1 + \hat{P}_2 + \hat{S} &= 180^\circ \\ \hat{R}_1 + \hat{R}_2 + \hat{S} &= 180^\circ \\ \therefore \hat{P}_1 + \hat{P}_2 + \hat{S} &= \hat{R}_1 + \hat{R}_2 + \hat{S} \\ \hat{P}_1 + \hat{P}_2 &= \hat{R}_1 + \hat{R}_2 \\ \text{but } \hat{P}_1 &= \hat{B}_2 \\ \text{and } \hat{P}_2 &= \hat{R}_2 \\ \therefore 2\hat{P}_1 &= 2\hat{R}_1 \\ \hat{P}_1 &= \hat{R}_1 \\ \text{But } \hat{R}_1 &= \hat{A}_2 \\ \therefore \hat{P}_1 &= \hat{A}_2 \\ \therefore PB &\parallel AR \end{aligned}$$

$\therefore PBRA$ is a parallelogram

alt $\angle$'s
alt $\angle$'s
co-int $\angle$'s
co-int $\angle$'s
both = 180°
'S' is common
Given PB bisects $\hat{P}$
Given AR bisects $\hat{R}$

alt $\angle$'s
corrsp $\angle$'s
PA $\parallel$ BR $\rightarrow$ given
PB $\parallel$ AR $\rightarrow$ proven

Figure 6.1: PST 03's response for question 3

As exhibited above it is evident that some PSTs are able to use the knowledge they have. The knowledge demonstrated by PSTs above includes deductive reasoning using substitution and transitive property. Reasoning is a required basis for the knowing and doing of mathematics. This type of deductive reasoning was observed with responses from the other PSTs who completed the task-based worksheet, for example the response PST 06 presented when answering question 5. In this question PSTs were given a diagram and requested to prove that two lines WV and TS were parallel ($WV \parallel TS$) to each other. PST 06 deduced that $QVRW$ is a cyclic quadrilateral because of the sum of

the opposite interior angles. This will then mean $\hat{Q}_1$ and $\hat{W}_1$ are equal because they are subtended by the same chord VR . But on the existing circle $\hat{Q}_1 = \hat{SPR}$ subtended by the same chord SR . Based on the transitivity property, it will mean $\hat{W}_1$ is also equal to $\hat{SPR}$ since both are equal to $\hat{Q}_1$. From the tangent-chord theorem $\hat{S}_1 = \hat{SPR}$, and it has been proved that $\hat{W}_1 = \hat{SPR}$ therefore $\hat{S}_1 = \hat{W}_1$, but $\hat{S}_1$ and $\hat{W}_1$ are alternate angles, therefore $WV \parallel TS$. Figure 6.2, which follows, shows the correct response to question 5 for PST 06.

5. Study the diagram below and answer the question.

Given a circle O with a diameter POS and a tangent ST.

Prove that $WV \parallel TS$

Proof: From theorem of tangency we know that
 $S_1 = m(\widehat{SPR})$ (1)

we also know that $m(\widehat{SPR}) = Q_1$ (2)

Since PS is diameter $R_2 = Q_2 = 90$.

$\Rightarrow m(\widehat{SQW}) + m(\widehat{VRW}) = 180 \Rightarrow QVRW$ is cyclic.

$\Rightarrow Q_1 = W_1$ (3)

(2) $\Rightarrow m(\widehat{SPR}) = W_1$ as well.

(1) $\Rightarrow m(\widehat{SPR}) = S_1 = W_1$. Proof completed since W_1 and S_1 are alternate angles.

Figure 6.2: PST 06's response for question 5

6.4.3 Why do pre-service teachers use their knowledge of proof in geometry in the way that they do?

Based on focus group interview 1, it was evident that the PSTs use their knowledge of proof in order to assist learners in the teaching and learning of geometry. They work with proofs with the aim of developing reasoning and proving abilities, forming conjectures, evaluating of arguments and the use of various methods of solving problems in geometry (Christou et al, 2004). Hanna (2000) and Webber (2003) described the aim of doing proof in the functions of proof as proving: verification, explanation, systematisation, discovery, communication, construction, exploration, and incorporation. From the task-based worksheets, PSTs were able to use proof in order to prove that $\hat{E}_1 = \hat{F}_1$ in question one. In this case they have used proof to verify whether or not $\hat{E}_1$ is equal to $\hat{F}_1$. A formal proof here is a way of explaining the understanding and provides an insight as to why the conjecture is true, which is something more than just knowing. Verification and explanation are considered as the basis of proofs, because they comprise the product of the long historical development of mathematical thought. And verification here refers to the truth of a statement while the explanation provides a reason why this statement is true. The excerpt that follows is taken from the FGI 2 transcript describes why PSTs use their knowledge of proofs in geometry.

PST 13 : the proofs are necessary, to see that if it always work regardless of the size the shape to see it, it is always true that may be in a circle, in a small circle...

PSTs have also used their knowledge in order to systematise the deductive reasoning system of axioms, major concepts and theorems and they use proof so that they will be able to discover new outcomes (De Villiers, 1990). PSTs use proof to communicate or transmit the ideas and mathematical knowledge learnt; proofs help them to understand the world.

PST 14 : We have to proof in order to understand the world we live in. We can not understand this world we live in if we don't understand how certain things came about, to understand how certain things came about you have to come to do proof ...

Teachers should be aware that proof will only be meaningful when the proof answers the learner's doubts and when it proves what is not obvious. The necessity or

functionality of proof can only surface in situations in which the learners meet uncertainty about the truth of mathematical proposition. PSTs ought to establish multiple purposes for proof writing, including explanation, systematisation, communication, discovery, justification, intuition development, and autonomy (Webber, 2003). It is therefore important to consider the functions of the proof when teaching proofs. Understanding how proofs are done may help the teacher to teach them well. This provides the reasons why PSTs are using the knowledge of proof in geometry in the way that they do. Mathematics teachers ought to promote proof writing as a means of explaining and communicating in mathematics.

REFERENCES

- Akhurst, J., & Sader, S. (2009). *Education policy and professionalism*. Learning guide 2009. School of education & development. Pietermaritzburg: University of KwaZulu-Natal.
- An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics teacher education*, 7, 145 - 172.
- Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational studies in mathematics*, 52, 215 – 241.
- Atherton, J. S. (2011). *Learning and teaching; constructivism in learning*. Retrieved 8 February, 2012, from, <http://www.learningandteaching.info/learning/constructivism.htm>.
- Au, W. (2012). *Critical curriculum studies: Education, consciousness and the politics of knowing*. New York: Routledge.
- Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of teacher education*, 59, 389 - 407.
- Battista, M. T., & Clements, D. H. (1995). Geometry and proof. *Mathematics teacher*, 88(1), 48 - 54.
- Bayram, S. (2004). *The effect of instruction with concrete models on eighth grade students' geometry achievement and attitudes toward geometry*. A masters of science thesis submitted to the graduate school of natural and applied sciences. Middle East Technical University, Ankara. Turkey.
- Blanton, M. L. (2000). Using an undergraduate geometry course to challenge pre-service teachers' notions of discourse. *Journal of mathematics teacher education*, 5, 117 – 152.
- Bond-Robinson, J. (2005). Identifying pedagogical content knowledge (PCK) in the chemistry laboratory. *Chemistry education research and practice*, 6(2), 83 - 103.

- Booi, K. (2000). *The implications of the introduction of outcomes based education in the natural sciences curriculum at Cape college of education*. Submitted in partial fulfilment of the requirements for the degree of master of education (science education). Rhodes university.
- Borrego, M., Douglas, E. P., & Amelink, C. T. (2009). Quantitative, qualitative and mixed research methods in engineering education. *Journal of Engineering Education*, 98(1), 53 - 66.
- Brijlall, D. (2011, January 18 - 21). *A case study of learners views on content knowledge and practice*. Proceedings of the nineteenth annual meeting of the South African Association for Research in Mathematics, Science and Technology Education (SAARMSTE).
- Burger, W., & Shaughnessy, M. (1986). Characterizing the Van Hiele levels of Development in Geometry. *Journal for Research in Mathematics Education* 17(1), 31 – 48.
- Carter, J. (2009). *Visualization and understanding in mathematics*. University of Southern Denmark, Odense. Denmark.
- Cerbin, W., & Kopp, B. (2006). Lesson study as a model for building pedagogical knowledge and improving teaching. *International journal of teaching and learning in higher education*, 18(3), 250 - 257.
- Chinnapen, M. (2003). Mathematics learning forum: Role of ICT in the construction of pre-service teacher's content knowledge. *Australian Journal of Educational Technology*. 19(2), 176 - 191.
- Christou, C., Mousoulides, N, P., & Pitta-Pantazi, D. (2004). Proofs through exploration in dynamic geometry environments. *International Journal of Science and Mathematics Education* 2, 339 – 352.
- Clement, D., & Battista, M. (1992). Geometry and spatial reasoning. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420 - 464). New York. Macmillan Publishing Co.
- Cohen, L., & Manion, L. (1994). *Research methods in education* (4th ed.). London: Routledge.

- Crowley, M. L. (1987). The van Hiele Model of the Development of Geometric Thought. In M.M. Lindquist & A.P. Shulte (Eds.), *Learning and Teaching Geometry, K-12*. National Council of Teachers of Mathematics Yearbook. pp. 1-17.
- De Villiers, M. D. (1987, June 11- 13). *Research evidence on hierarchical thinking, teaching strategies, and the Van Hiele Theory: Some critical comments*. Paper presented at learning and teaching geometry: Issues for research and practice working conference, Syracuse University, Syracuse New York.
- De Villiers, M. D. (1996). *Some development in geometry education (1). The future of secondary school geometry*. Retrieved 11 June, 2011, from <http://www.didactique.imag.fr/preuve/Resumes/deVilliers/deVilliers98/deVilliers98.html>
- De Villiers, M. D. (2004). Using dynamic geometry to expand mathematics teachers' understanding of proof. *International Journal of Mathematical Education in Science and Technology*, 35(5), 703–724.
- De Villiers, M.D. (1990). *The role of the function of proof in mathematics*. Research unit on mathematics education. University of Stellenbosch: South Africa.
- Deacon, R. (2010). Educating the educators: Challenges facing teachers' education and development in South Africa. *The Journal of the Helen Suzman Foundation*, 59, 38-43.
- Department of Basic Education. (2010). *Curriculum and assessment policy statement (CAPS) grade 10 – 12: Mathematics*. Pretoria: National Education Department.
- Department of Basic Education. (2011). *Curriculum and assessment policy statement grade 10 – 12: Mathematics*. Pretoria: National Education Department.
- Department of Basic Education. (2011a). *Curriculum news*. Pretoria: National Education Department.
- Department of Basic Education. (2011b). *Curriculum and assessment policy statement grade 10 – 12: Mathematics*. Pretoria: National Education Department.

- Department of Education. (2003). *National curriculum statement grade 10 – 12 Policy: Mathematics*. Pretoria: National Education Department.
- Department of Education. (2005). *National curriculum statement grade 10 – 12 (General) Policy: Mathematics*. Pretoria: National Education Department.
- Dowling, P., & Brown, A. (2010). *Doing research/Reading research* (2nd ed.). Re-interrogating education. London: Routledge.
- Elmore, R. F. (2002). *The limits of change*. Harvard Education Letter Research. Retrieved 13 October, 2012, from, <http://sparkaction.org/node/25084>
- Ernest, P. (1986). Games: A rationale for their use in the teaching of mathematics in school. *Mathematics in School*, 15(I), 2 - 5.
- Ernest, P. (1995). *The one and the many*. Hillsdale: N.J. Lawrence Erlbaum Associates.
- Fonseca, L., & Cunha, E. (2011). *Pre-service teachers and the learning of geometry*. School of Education of the Polytechnic Institute of Viana do Castelo. Portugal.
- French, D. (2004). *Teaching and learning geometry*. London: Continuum.
- Garritz, A. (2010). Pedagogical content knowledge and the affective domain of scholarship of teaching and learning. *International Journal for the Scholarship of Teaching and Learning*. Retrieved 11 October 2012, from, http://academics.georgiasouthern.edu/ijstol/v4n2/personal_reflections/PDFs/_Garritz.pdf
- Gay, L. R., Mills, G. E., & Airasian, R. (2006). *Educational research: Competencies for analysis and applications* (8th ed.). Upper Saddle River, NJ: Pearson/Merrill/Prentice Hall.
- Gibson, D. (1998). Students' use of diagram to develop proofs in an introductory analysis course. In A.H. Schoenfield, J. Kaput, & E. Dubinsky (Eds.), *CBMS: Issues in mathematics Education III*, (pp. 284 - 307). Rhode Island: American Mathematical Society.
- Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational Studies in Mathematics*, 144 (1-2), 5 - 23.

- Healy, L., & Hoyles, C. (2001). Software tools for geometrical problem solving: Potentials and pitfalls, *International Journal of Computers for Mathematical Learning*, 6, 235 – 256.
- Ho, S. Y. (2009). *Visualisation in primary school mathematics: Its roles and processes in mathematical problem solving*: Unpublished doctoral dissertation, National Institute of Education, Nanyang Technology University, Singapore.
- Hobden, S. (2009). *Teaching and learning mathematics in the FET*. Unpublished lecture notes. University of KwaZulu-Natal, Durban.
- Hodge, A., & Frick, K. (2009). University preparation of pre-service secondary geometry teachers: A need for research. *Journal of mathematical sciences & mathematics education*, Vol. 4 No. 1
- Johnson, H. D., & Dasgupta, N. (2005). Traditional versus Non-traditional Teaching: Perspectives of Students in Introductory Statistics Classes. *Journal of Statistics Education*, 13(2). Retrieved 5 July, 2011, from, www.amstat.org/publications/jse/v13n2/johnson.html
- Kilpatrick, J., Swafford, J & Findell, B (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington D.C.: National Academy Press.
- Kim, B. (2001). Social constructivism. In M. Orey (Ed.), *Emerging perspectives on learning, teaching, and technology*. Retrieved 18 June, 2011, from, <http://projects.coe.uga.edu/epltt/>.
- Knight, K.C. (2006). *An investigation into the change in the van hiele level of understanding geometry of pre-service elementary and secondary mathematics teachers*. A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in teaching. The Graduate School The University of Maine.
- Kintsch, W. (2009). Learning and constructivism. In S. Tobias, & T.M. Duffy (Eds.), *Constructivist instruction: Success or Failure?* (pp. 223 - 241). New York: Routledge.
- Knuth, E. J. (2002). Secondary school mathematics teachers' conceptions of proof. *Journal for Research in Mathematics Education* 33(5), 379 -405.

- Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery*. Cambridge: Cambridge University Press.
- Lave J., & Wenger, E. (1991). Situated learning: Legitimate peripheral participation. (R. Pea & J. S. Brown, Eds.), *American anthropologist* (Vol. 95, p. 138). Cambridge University Press. Retrieved 21 March, 2011, from, <http://books.google.com/books?id=CAVIOrW3vYAC&pgis=1>.
- Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood – Volume 34 (4) pp. 37–45*
- Luchin, B.M. (2006). *Geometry patterns and process: Making the TAKS connection*. Numbers mathematics professional development. Retrieved 13 October, 2012, from, <http://staff.beaumont.k12.tx.us/babel/Documents/geometrypatternsprocess>
- Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathematically*. Wokingham, England: Prentice Hall.
- Mason, M. (2010). *The Van Hiele Levels of geometric understanding: Professional Handbook for Teachers. Geometry: Explorations and Applications*. Charlottesville, Virginia: McDougal Little Inc.
- Mayer, R. E. (2009). Constructivism as a theory of learning versus constructivism as a prescription for instruction. In S. Tobias, & T.M. Duffy (Eds.), *Constructivist Instruction: Success or Failure?* (pp. 184 - 200). New York: Routledge.
- McMillan, J.H., & Schumacher, S. (2006). *Research in education. Evidence-Based Inquiry* (6th ed.). New York: Pearson.
- McNeil, N. M., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory into Practice*, 46(4), 309 - 316.
- Morrow, W. (2007). What is teachers' work? *Journal of Education*, 41, 3 – 19.
- Mthembu, S. G. (2007). *Instructional approaches in the teaching of Euclidean geometry in grade 11*. Thesis submitted for the degree of Master of Education. University of KwaZulu-Natal. Durban.

- Mudaly, V. (2011). *Diagrams: To draw or not to draw?* Unpublished lecture notes. University of KwaZulu-Natal, Durban.
- Mudaly, V., & de Villiers, M. (2004, June 30 – July 4). *Mathematical modelling and proof*. Paper presented at 10th AMESA Congress, AMESA, University of the North West, Potchefstroom.
- Mullis, I.V.S., Martin, M.O., Foy, P., & Arora, A. (2012). *TIMSS 2011 International Results in Mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.
- Naidoo, J. (2006). *The effect of social class on visualisation in geometry in two KwaZulu-Natal schools, South Africa*. Thesis submitted for the degree of Master of Philosophy. University of Nottingham, Nottingham.
- Naidoo, J. (2011). *Exploring master teachers' use of visuals as tools in mathematics*. A full dissertation for the Degree of Doctor of Education. University of KwaZulu-Natal, Durban.
- NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA. Retrieved 13 October, 2012, from, <http://www.ams.org/notices/200008/comm-ferrini.pdf>
- Neuman, W. L. (2000). *Social Research Methods: Qualitative and quantitative approaches*. (4th ed.). London: Allyn and Bacon.
- Nightingale, P., & O'Neil, M. (1994). *Achieving quality learning in higher education*. London: Kogan Page Limited.
- Njisane, R. A. (1992). Constructivism. In M. Moodley, R.A. Njisane & N. Presmeg, (Eds.), *Mathematics Education for in-service and pre-service teachers* (pp. 26 – 37). Petermaritzburg: Shuter and Shooter.
- Novak, J. (1998). *Learning, creating and using knowledge. Concept Maps as facilitative tools in schools and in corporations*. London: Lawrence Erlbaum.

- Pandiscio, E. A. (2002). Exploring the link between pre service teachers' conception of proof and the use of dynamic geometry software. *School Science and Mathematics*, 102(5), 216–221. Retrieved 22 May, 2012, from, <http://onlinelibrary.wiley.com/doi/10.1111/j.1949-8594.2002.tb18144.x/full>.
- Presmeg, N. (1992). Prototypes, Metaphors, metonymies and imaginative rationality in high school mathematics. *Educational Studies in Mathematics*, 23, 595 - 610.
- Ryan, M. (2008). *Geometry for dummies*. (2nd ed.). Danvers. Wiley Publishing Inc.
- Schoenfeld, A. H. (2000). Purpose and methods of research in mathematics. *Notices of the American Mathematical Society*, 47(6), 641- 649.
- Scott, D. (2008). *Critical essays on major curriculum theorists*. London: Routledge.
- Serra, M. (1997). *Discovering geometry: An inductive approach*. (2nd ed.). California. Key Curriculum Press.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4 - 14.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1 - 22.
- Singh, R. I. (2006). *An investigation into learner understanding of the properties of selected quadrilaterals using manipulatives in a grade eight mathematics classes*. Thesis submitted for the degree of Master of Education. University of KwaZulu-Natal. Durban.
- Smith, M. K. (2009). *Communities of practice, the encyclopaedia of informal education*, Retrieved 12 May, 2011, from, http://www.infed.org/biblio/communities_of_practice.htm
- Soanes, C., & Stevenson, A. (2009). *Concise Oxford English dictionary*. (11th ed.). New York: Oxford University Press.
- Sowell, E. (1974). Another look at materials in elementary school mathematics. *School Science and Mathematics*, 74(3), 207–211.

- Stephen M., & Clements. D.H. (2003). Linear and Area Measurement in Prekindergarten to Grade 2. In D.H. Clements & G. Bright (Eds.), *Learning and Teaching Measurement 2003 Yearbook*, (pp.3-16). Reston VA: National Council of Teachers of Mathematics.
- Strydom, H., Fouché, C.B., & Delport, C.S.L. (2004). *Research at grass roots: For social sciences and human service professions*. Pretoria: Van Schaik Publishers.
- Thompson, P. W. (1994). Concrete materials and teaching for mathematical understanding. *Arithmetic Teacher*, 41(9), 556-558. Retrieved 26 August, 2012, from, <http://patthompson.net/PDFversions/1994Concrete.pdf>
- Trochim, W. M. K. (2006). *The research methods knowledge base*. Retrieved 19 August, 2011, from, www.socialresearchmethods.net/kb/
- University of South Alabama. (2005). *Quantitative, qualitative and mixed research*. Retrieved 21 May, 2011, from, www.southalabama.edu/coe/bset/johnson/lectures/lec2.pdf.
- Usiskin, Z. (2002). Teachers need a special type of content knowledge. *ENC Focus*, 9(3), pp. 14-15.
- Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* (Final report of the Cognitive Development and Achievement in Secondary School Geometry Project: ERIC Document Reproduction Service No. ED 220 288). Chicago: University of Chicago. Retrieved 11 September, 2012, from, http://ucsmp.uchicago.edu/resources/van_hiele_levels.pdf
- Van Hiele, P.M. (1957). The problem of insight, in connection with school-children's insight into the subject matter of geometry. English summary (by P.M. Van Hiele) of De Problematiek van het Inzicht Gedemonstreerd wan het Inzicht von Schoolkindren in Meetkundeleerst of Doctorial dissertation, University of Utrecht.
- Van Hiele, P. M. (1986). *Structure and insight: A theory of Mathematics Education*. Orlando, FL: Academic Press Inc.

- Van Hiele, P. M. (1995). *Development of learning processes. A study of some aspects of Piaget's psychology in relationship with the didactics of mathematics*. Groningen, Netherlands: J.B. Wolters
- Varghese, T. (2009). *Secondary-level student teachers' conceptions of mathematical proof*. Department of Mathematical and Statistical sciences CAB 632, University of Alberta, Edmonton.
- Veal, W. R., & MaKinster, J. G. (1999). Pedagogical content knowledge taxonomies. *Electronic Journal of Science Education*, 3(4): Retrieved 29 October, 2012, from, <http://ejse.southwestern.edu/article/view/7615/5382>
- Voce, A. (2004). Introduction to research paradigms, lecture notes: *Qualitative research*.
- Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Vygotsky, L. (1986). *Language through interaction*. Cambridge, MA: MIT Press.
- Webber, K. (2003). *Students' difficulties with proof* (Research Sampler No. 8). Retrieved on 23 June, 2003, from, the Mathematical Association of America website: http://www.maa.org/t_and_1/sampler/rs_8.html
- Weber, E. (2008). *Educational Changes in South Africa: Reflections on Local Realities, Practices, and Reform*. Rotterdam: Sense Publishers
- Wenger, E. (2006). *Communities of practice. A brief introduction*. Retrieved on 14 January, 2012, from, <http://www.ewenger.com/theory/>.
- Wiesen, G. (2003). *What is a situated learning?* Conjecture Corporation. Retrieved on 12 November, 2011, from, <http://www.wisegeek.com/what-is-situated-learning.htm>.
- Woolfolk, A. (2007). *Educational psychology* (10th ed.). New York: Pearson.
- Zaskis, R., Dubisnky, E., & Deutermann, J. (1996). Coordinating visual and analytic strategies: A study of students' understanding of the group D4. *Journal for Research in Mathematics Education*, 27, 435-457.

Zigarmi, P., Betz, L., & Jensen, D. (1977). Teachers' preferences in and perceptions of in-service education. *Educational leadership* 34 (1), pp. 545-551.

Appendices

Ethical clearance certificate



Research Office, Govan Mbeki Centre
Westville Campus
Private Bag x54001
DURBAN, 4000
Tel No: +27 31 260 3587
Fax No: +27 31 260 4609
mohunp@ukzn.ac.za

13 September 2011

Mr BR Ndlovu (202518677)
School of Science, Mathematics & Technology
Faculty of Education
Edgewood Campus

Dear Mr Ndlovu

PROTOCOL REFERENCE NUMBER: HSS/0850/011M
PROJECT TITLE: Exploring Pre-Service Teacher's Knowledge of proof in geometry

In response to your application dated 7 September 2011, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol has been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment /modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

Professor Steven Collings (Chair)
HUMANITIES & SOCIAL SCIENCES RESEARCH ETHICS COMMITTEE

cc. Supervisor: Dr J Naidoo
cc: Mr N Memela/Mrs S Naicker, Faculty Research Office, Edgewood Campus

Letter from the Registrar of the university



10 August 2011

Mr BR Ndlovu
School of Science, Mathematics & Technology
Faculty of Education
Edgewood campus
UKZN

Dear Mr Ndlovu,

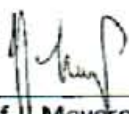
RE: PERMISSION TO CONDUCT RESEARCH

Gatekeeper's permission is hereby granted for you to conduct research at the University of KwaZulu-Natal towards your postgraduate qualification, provided Ethical clearance has been obtained via the Research Office. The topic of your project/area of research is noted as:

- 1) Exploring pre-service teachers' knowledge of proof in geometry

Please note that the data collected must be treated with confidentiality and anonymity.

Yours sincerely,



Prof. J. Meyerowitz
Registrar

Office of the Registrar
Postal Address: Private Bag X54001, Durban 4000, South Africa
Telephone: +27 (0) 31 260 8005/2206 Facsimile: +27 (0) 31 260 7824/2204 Email: registrar@ukzn.ac.za Website: www.ukzn.ac.za

 1910 - 2010 
100 YEARS OF ACADEMIC EXCELLENCE
Founding Campuses:  Edgewood  Howard College  Medical School  Pietermaritzburg  Westville

Informed consent for participants



Dear Student

My name is Bongani Ndlovu, student number: 202518677. I am a postgraduate student at the University of KwaZulu-Natal in the Science, Mathematics and Technology Education Department at Edgewood Campus. I am currently conducting research in mathematics education under the supervision of Dr. J. Naidoo. The purpose of this research is to assist me in exploring Pre-Service Teachers' knowledge of proof in geometry.

If you agree to participate in this research study, the following will occur:

1. Each student will be asked to complete a task based worksheet as well as participate in an individual interview; the interview will be between 30 min – 45 min. Selected students may be asked to participate in focus group interviews.
2. A semi-structured interview schedule will be used. All interviews will be audio taped.
3. If you agree to participate in this research study, an audio tape of the interview will be made for research purposes.

CONFIDENTIALITY:

The records from this study will be kept as confidential as possible. No individual identities will be used in any reports or publications resulting from the study. All audiotapes, transcripts and summaries will be given codes and stored separately from any names or other direct identification of participants. Research information will be kept in locked files at all times. After the study is completed and all data has been transcribed from the tapes, the audiotapes will be held for five years and then destroyed. You will receive a copy of the final transcript, so that you have the opportunity to suggest changes to the researcher, if necessary. Participation in this study is voluntary and you are free to withdraw your participation from this research study at any point. You will be given a copy of this consent form for perusal.

QUESTIONS

If you have any further questions or queries about the study, please contact:
Mr. Bongani Ndlovu: 082 7651121 / bonganirndlovu@webmail.co.za (email)
Dr. J. Naidoo: (031) 260 1127 (W) / naidooj2@ukzn.ac.za (email)

CONSENT FORM FOR PARTICIPANTS: Exploring Pre-Service Teachers' knowledge of proof in geometry.

I agree to take part in the study on: Exploring Pre-Service Teachers' knowledge of proof in geometry. I am aware that the researcher is going to ask me to complete a task based questionnaire and conduct interviews. I am willing to take part in focus group interviews as well as individual interviews.

I am aware that each interview will be audio taped. I have read and understood the accompanying letter. I know what the study is about and the part I will be involved in. I know that I do not have to answer all of the questions and that I can decide not to continue with this research at any time.

Name _____

Signature _____ **Date** _____

Participant

Signature _____ **Date** _____

Researcher

Turnitin certificate

EXPLORING PRE-SERVICE TEACHERS' KNOWLEDGE OF PROOF IN GEOMETRY

ORIGINALITY REPORT

14 %

SIMILARITY INDEX

11 %

INTERNET SOURCES

3 %

PUBLICATIONS

7 %

STUDENT PAPERS

PRIMARY SOURCES

1	Submitted to University of KwaZulu-Natal <i>Student Paper</i>	1%
2	www.skoolwiskunde.co.za <i>Internet Source</i>	1%
3	www.edwardsmaths.com <i>Internet Source</i>	< 1%
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Editors' report

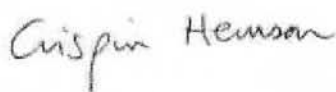
Crispin Hemson
15 Morris Place
Glenwood
Durban
South Africa 4001

hemson@ukzn.ac.za
C: 082 926 5333
H: 031 206 1738

9th October 2012

TO WHOM IT MAY CONCERN

This is to record that I have carried out a language editing of the dissertation by Bongani Reginald Ndlovu, entitled **Exploring pre-service teachers' knowledge of proof in geometry.**



Crispin Hemson

Survey Questionnaires for Pre-Service Teachers

The aim of this questionnaire is to obtain information from Pre-Service Teachers about their knowledge of proofs in geometry. I would appreciate any assistance that you could provide. Please complete the following questionnaire as honestly as possible. Please remember that there are no right or wrong answers, we are interested in your input.

Name: _____

NB: Please remember that all details will remain confidential

Section A: Background

Complete the following, mark with an X in the blocks where necessary.

1. Age:

Below 15	Between 15 and 20	Between 21 and 25	Between 26 and 30	Above 30

2. Male or Female:

M	F

3. Year Completed Grade 12 :

4. Subjects completed in Grade 12:

1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	

5. Qualifications: _____

6. Any working experiences: Yes

☐

No

☐

6.1 If yes state the nature of work: _____

6.2 Position: _____

6.3 Years of experience: _____

7. Any Professional

Development: _____

8. Which professional organisations do you belong to:

Thank you

Task Based Worksheet

Exploring Pre-Service Teachers' knowledge of proof in geometry.

Dear Student.

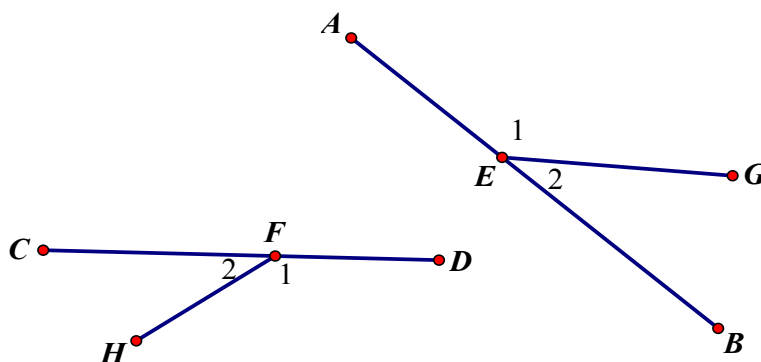
Thank you for agreeing to take part in this study. Please remember that your responses in this worksheet will be treated with the strictest confidence. Please answer the following questions in the spaces provided. The diagrams are not drawn to scale. Please fill in your details for research purposes.

Your Name:.....

Your Student Number:.....

Level (2nd/3rd/4th year):.....

QUESTIONS



b)

On the diagram above, AB and CD are straight lines EG and FH are joined on AB and CD respectively and $\hat{E}_2 = \hat{F}_2$.

Prove that $\hat{E}_1 = \hat{F}_1$

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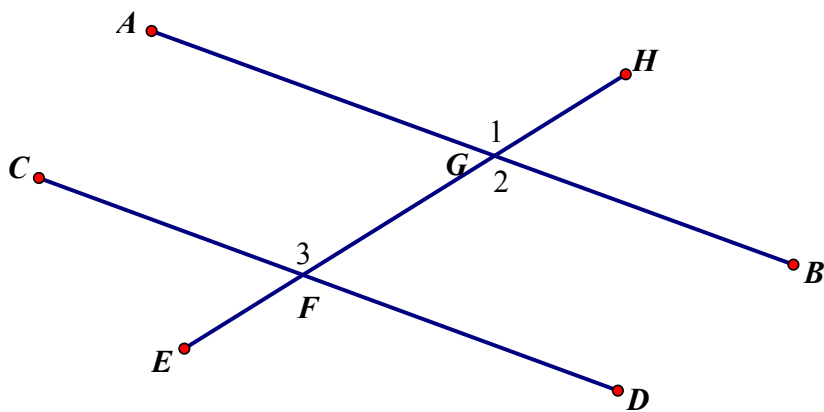
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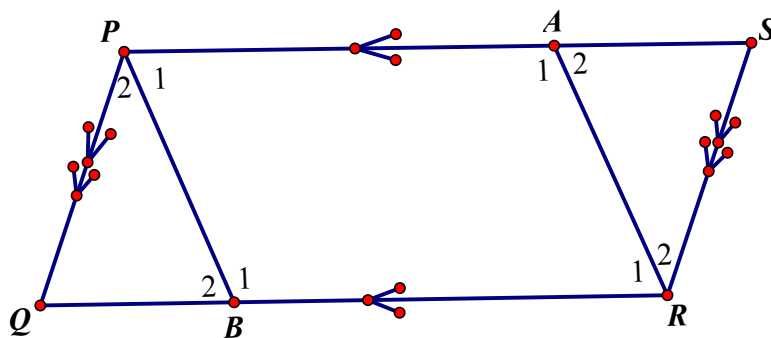
.....

- Given: EH cuts AB in G and CD in F, such that $\hat{G}_2 = \hat{F}_3$

Prove that $AB \parallel CD$.

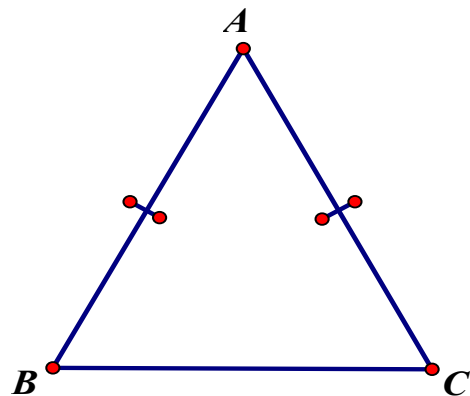
This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced, horizontal dotted lines across the entire width of the page. The background is plain white, providing a clear guide for letter height and placement. There are no margins, titles, or other markings present.

- Prove that PBRA is a parallelogram.

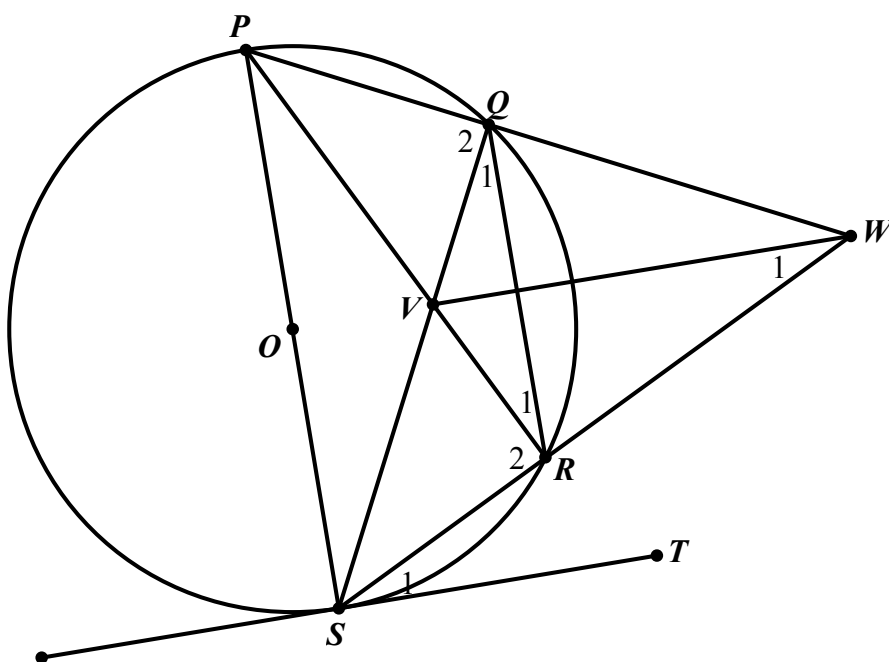
This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

e) In $\triangle ABC$ below, $AB = AC$

Prove that $\hat{B} = \hat{C}$

[illegible]

This image shows a full page of white paper designed for handwriting practice. It features 18 evenly spaced horizontal dotted lines running from left to right across the entire width of the page. There are no margins, text, or other markings present.



Prove that $WV \parallel TS$



Given: AOD and EOB are diameters

$$AF \perp EB$$

Prove: a) $EFHD$ is cyclic

b) $BAD = H_1$

c) $C_2 = A_1$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

EXPLORING PRE-SERVICE TEACHERS' KNOWLEDGE OF PROOF IN GEOMETRY.

Semi – structured interview schedule

Schedule for follow up interview with Pre-Service teachers who have been selected after analysing the task based questionnaire and survey questionnaire.

Focus Group

PRE-SERVICE TEACHERS' INTERVIEW SCHEDULE

1. In your own opinion, which section (Algebra, Geometry or Trigonometry) in mathematics do you think is enjoyed the most by learners?

.....

.....

.....

Why do you say so??

.....

.....

2. Which section (Algebra, Geometry or Trigonometry) in mathematics do you think is enjoyed the least by learners?

.....

.....

.....

Why do you say so??

.....

.....

3. What do you remember about the Geometry that you learnt in high school?

.....

.....

.....

4. Can you explain how Geometry was taught at your high school?

.....

.....

.....

5. Which Geometry concepts were you aware of when you enrolled at the University?

.....

.....

.....

6. Do you think you have gained any deeper or new knowledge now that you have been studying at the University for 3 or more years?

.....

.....

.....

7. Which strategies do you think you would use when teaching Geometry to your learners?

.....

.....

.....

8. Why do you think that these strategies would work?

.....

.....

.....

.....

9. Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

.....

.....

.....

10. Are these teaching strategies similar or dissimilar to the way that you were taught?

.....

.....

.....

Researcher reflections/ comments:

The rest of the interview will be based on individual responses to the task based questionnaire and survey questionnaire.

Thank you for your co-operation.

EXPLORING PRE-SERVICE TEACHERS' KNOWLEDGE OF PROOF IN GEOMETRY.

Semi – structured interview schedule

Schedule for follow up interview with Pre-Service teachers who have been selected after analysing the task based questionnaire and survey questionnaire.

Individual Interview

PRE-SERVICE TEACHERS' INTERVIEW SCHEDULE

Name: _____

NB: Please remember that all details will remain confidential

Section A: Background

Complete the following, mark with an X in the blocks where necessary.

1.

Below 15	Between 15 and 20	Between 21 and 25	Between 26 and 30	Above 30

 Age: _____

2. Year Completed Grade 12 :

3. Qualifications:

4. Any working experiences: Yes ☐ No ☐

4.1 If yes state the nature of work:

4.2 Position:

4.3 Years of experience:

5. Any Professional Development:

6. Which professional organisations do you belong to:

.....
.....
.....

Section B: Knowledge of proof in Geometry

1. In your own opinion, which section (Algebra, Geometry or Trigonometry) in mathematics do you think is enjoyed the most by learners?

.....
.....
.....

Why do you say so?

.....
.....
.....

2. Which section (Algebra, Geometry or Trigonometry) in mathematics do you think is enjoyed the least by learners?

.....
.....
.....

Why do you say so??

.....
.....
.....

3. What do you remember about the Geometry that you learnt in high school?

.....
.....
.....

4. Can you explain how proof in Geometry was taught at your high school?
-
-
-
5. Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)
-
-
-
6. Do you think you have gained any deeper or new knowledge now that you have been studying at the University for 3 or more years?
-
-
-
7. Which strategies do you think you would use when teaching proof in Geometry to your learners?
-
-
-
8. Why do you think that these strategies would work?
-
-
-
9. Do you think that these teaching strategies would make the teaching and learning of proof in Geometry better for your learners?
-
-
-

10. Are these teaching strategies similar or dissimilar to the way that you were taught?

.....

.....

.....

Researcher reflections/ comments:

The rest of the interview will be based on individual responses to the task based questionnaire and survey questionnaire.

Thank you for your co-operation.

Interview Transcription

Focus Group one

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 01 : *I suppose, it depends what they like to do, some learners like the methods so they like algebra or like trigonometry.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 04 : *I will probably say its geometry.*

Researcher : Why do you say so?

PST 04 : *In terms of marks, it's a section that students' score the less marks in, because they don't quite get the different properties under theorems that go with it. As opposed to algebra which is easier for learners.*

PST 02 : *Geometry.*

Researcher : Why do you say so?

PST 02 : *So because the kids do not have fundamental, only if they have a bad teacher you are not going to enjoy doing geometry because it is a lot of understanding so you got a bad teacher you can have, you are in trouble.*

PST 01 : *Absolutely it geometry.*

Researcher : Why do you say so?

PST 01 : *When it comes to mathematics (algebra) when you talking about learners not having definition for mathematics you still learn strategies involved but when it comes to geometry you have to have a talent in order to apply to geometry, so already in geometry you see who is mathematical inclined.*

Researcher : What do you remember about geometry you learnt in high school?

PST 01 : *Fun..! (Laughing) Fun, you know the angles, co-interior, alternate and corresponding angles. Ja that basically I remember.*

PST 02 : *I did maths paper 3, so everyday or every Monday after school, I was...I had to do maths paper 3, to do extra geometry. I learnt all circle theorems, something like that.....*

Researcher : Can you explain how geometry was taught at your high school?

PST 03 : *I think, we were not ideally given a chance to ehm...! Find out for ourselves, like for example if you say the sum of angles of triangle is 180° . We won't be given really a chance to... to discover it for ourselves. It was just thought was ok, this is it, and this is how it is done and this is it like it's my way and that is it. It likes, it was never, and I don't think that was ever ideally known as that...*

PST 05 : *Ehh....! I came from the old system, so we had all geometry that we had to learn, so it was given to us as theorems and we had to like by heart it. It wasn't make..., It never make any connections in our minds, so it was very route.*

Researcher : Which geometry concepts were you aware of when you enrolled at the university?

PST 03 : *The concepts that we were aware of, was basically angles everything in angles, ehh... and then triangles all the different types of triangles and coming in few circle geometry that we came in with and quads that is the basic geometry that we came into university with.*

PST 05 : *When I came here (university) I had very less understanding of geometry. I had ...when I was doing maths method 11, I had to go to library and borrowed ehh...all geometry books from grade 8 to 12 to.. in order to ...learn it on my own.*

PST 01 : *I'm the same, because I'm also from the old system before, so it was also because the time lapse, though basically the only thing I really came in with is angles with the FUN and the circle geometry because we didn't really, I think we did, do quite of geometry but it was, I just use what understanding I have.*

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 05 : *I understand what is non Euclidean geometry and we didn't learn that in school, and it makes a lot of sense. We can understand how are aeroplane travels and lines of latitude and longitudinal. So that was very interesting.*

PST 03 : *I will also think because of various reasons which were taught how to teach it. It like, it's brought our own understanding of what exactly geometry it is. What consist of and how to get where we need to be.*

PST 04 : *I t hink t hat, al though w e kn ew t he ge ometry c oming ou t s chool t o university, ahmm... because we were taught to memorise theorems and just know to know off just for exam purposes. We never really understood where came from and how to apply it. When c oming i nto u niversity an d doing mathematics method II, we learn where t he theorem comes from, h ow to prove and how to construct this kind of knowledge.*

PST 01 : *As well as, Jah I also believe that, specifically coming into teaching college is more about u nderstanding why is, you di d the work you did when you were a t s chool. B ecause I h ave attended o ther u niversities where t he ex - modules are j ust basically set ou t the way you have don e in high s chool. This is the knowledge you have to have by the end and you must write exam where in maths here in the university in teaching is understanding the bases how you got to limits in much as we are talking about geometry.*

Researcher : Which s trategies do you t hink you w ould us e w hen t eaching G eometry t o your learners?

PST 01 : *Strategy I would be...I would want to really use with my learners, is it like PST3 said, where you basically learners discover for themselves that what I m saying, what is written in the book is actually true if you do in physical terms but obviously coming to understanding, we always have time to do it. So I would like to do it but probably not for the entire section but it will be strategy I will use in the classroom.*

Researcher : Why do you think that these strategies would work?

PST 02 : *Student will remember more if they construct and actually apply themselves within t he kn owledge. O therwise it ju st r ead of th e th eorem, th e le arner could not remember a thing.*

PST 04 : *They gonna be like us going into university not understanding why, and not be able to apply the kn owledge if they have to go t o the field of maths or engineer or teaching for that matter.*

PST 05 : *Ehmm...also with the regard to constructing your knowledge and learners will be able to prove things and we as teachers can also offer them or show them the contradictions to those proof, so it may complete in their mind and they will have to choose what is the correct proof and what is true.*

PST 01 : *One more thing to add, problem with mathematics is that people say is not very interactive subject or interactive of course, that is why people don't enjoy it but if you allow learners to discover on their own, it brings that interactivity that people thought could never have in mathematics, so it brings a different views, changing view that mathematics is like all other subjects, you can apply individual strategies in classroom.*

PST 02 : *There is more programmes in our days so if you use sketch pad or interactive white board or smart board, you can actually show them different proofs and prove to them by using actual examples.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 06 : *Constructive; no I don't think it will help, because always they need a guide and it does not work actually. But as a person I was actually educated in a pragmatic approach, I mean, what I mean by this I was educated in the....in the one classroom. No one was there. It was something like just for me indication and that time it apply differently, it works. But otherwise where you dealing with forty students it hard to deal with them. So I don't think it will help.*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 04 : *It is different, ahmm... coming into university and learning different styles of teaching. It far different from the way we were taught, it was basically the way I've been taught was chalk and board the class of 45 to 46 learners and it was ehh... very rushed. It like I'm writing the theorem on the board, you learn it and if it comes out in the exam you are expected to know it, is not about understanding or why you should do it or why it's done that way because there was no time and no control over syllabus as such.*

PST 01 : *I when... I didn't have the problem when came to time not being the large class but they separated us into levels of understanding, so we definitely had lot of time but we were also taught chalk and board, so I feel that since we had the time they could have apply more of constructivist approach towards mathematics. But I mean I came out fine(laughing). I came out fine I did not complain, it just, it because it talk about it I'm talking about from my point of view and when I was a learner I don't have an issue that our learners now a days have, but then again I was a learner I found it very easy to understand if someone explain the topic.*

Researcher : So when you go out to teach you will be using the strategies that will be

similar or different?

PST 01 : *No, I wouldn't be using the similar strategies, I will be using the very different strategies, because a majority of is boarder line or moderate because of*

PST 05 : *Another thing is; in our method 1, 2, and 3 for mathematics educators, we do not really learn how to teach. So we as teacher (PSTs) have to take an initiative on our own and go and try to learn different things on our own. So that we can make it easier for the learners, so if we understand how they learn, then we can also help them, but it is like we still not get enough teaching methods strategies, in our methods class we do not learn the CAPS documents we don't learn even the R NCS documents so if that was incorporated and we had more knowledge of that may be it will also help us.*

PST 02 : *I will keep mine similar to the way I was taught because I was taught, the teacher was a facilitator and we used sketch pad and interactive white board most of the time, we did use lecture hall better the times but most of the times interactive white board, so that way we could get one on each.*

PST 03 : *I think is important to kind of like strike the balance between the two that you at certain times you need to, as a teacher you need to take control and at certain times you need to know when do you give learners control so it difficult but I think that will be idea.*

Researcher : Okay, meaning that yours will also be different?

PST 03 : Yes.

PST 06 : *I think I will teach only individuals, I won't teach the entire class hopefully, I will just deal with some sort of, you know, the student with high level. I think because and right now, right now I'm dealing with students who are actually competing internationally. So hopefully in the future I hope I won't deal with entire class.*

Researcher : But if you can be in a situation of the entire class, what strategies would you use?

PST 06 : *I will just quite my teaching. (laughter broke out)*

PST 01 : *PST 6 is actually at Star Collage teaching mathematics there, so he is privileged but you know the rest of us when we got to schools for P T*

(Practice Teaching) is a different story, you are dealing with average learners and below average learners and you don't get learners who would get 60.

PST 06 : *I mean, when you teach geometry I think you don't need to use concrete things because everything related geometry is abstract things and geometry has a lot with the philosophy, so in order to teach philosophy you mustn't use concrete things about abstract things.....so it doesn't apply for geometry again.*

PST 01 : *I totally disagree, because I use geometry in my everyday life and there are many people using geometry in their everyday life. You know when considering buildings or when considering you just furnish in your house that you do want personal may obviously it has to fit in a particular way and styled... according to a particular size, you can apply things in your real life.*

PST 06 : *No I don't likethe main thing that I think is, not teaching how to build building; the main thing is why you teach them and why they learn sort of things. Is that okay for them to learn this or that something else more than this we can teach. And I think philosophy is the best for that one.*

PST 01 : *If you are learning something, you are learning it to use in your life. What is the point of learning something if you are not going to use.*

Researcher : So in other words you want to develop context on what you are teaching.

PST 01 : *Yes, a context of understanding; I mean try geometry by given.....but I see those shapes in everyday life if I like construct models, what ever..... I have to construct models I need to measure and know the angles.*

Interview Transcription

Focus Group Two

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 14 : *I think is mostly algebra.*

Researcher : Why do you say so?

PST 14 : *Because , they are kindly like mostly to solve from unknown of which that is something we start at very early ages of our life, for example grade one, when you have a box plus two equals to 10 ($\square + 2 = 10$) and you have to find a box. So because of it is explained that could started early in the years, they are able to develop with until they get to matric and you find that they excel more in algebra, rather than trigonometry, they only get introduced to trig only in grade 10 and then it becomes a problem because now they have started thinking on another level.*

PST 13 : *Mina(I) would say that is trigonometry.*

Researcher : Why do you say so?

PST 13 : *Because in most cases you find that learners are able to transform, may be from Sine to Cosine or the stuff and or using the graph of a right angle with the hypotenuse side and y-axis and x-axis. So that one I think is enjoyed the most , ehh.. regarding my school while I was a learner so we enjoyed the part of trigonometry.*

PST 10 : *I feel, its more more like a half-half. I feel algebra, depend on the learner, algebra or geometry can be most enjoyed. For those who are more , they like the method. They can link, they can work with numbers much better, algebra is much better, for them because they can work with method, and they know that method they can played it every single time and get the answer. But for those who are more hands on interactive more for the practical they would prefer geometry because geometry is something you can see and work with and work with your hands so you can prove it yourself why it happens and how?*

Researcher : Let us say from your school where have been while you were at school, you were If you can categorise your school mates, they were actually, which part they were interested the most, were they equally or they are equal now?

PST 10 : *Ehh... I believe algebra they enjoyed the most.*

PST 09 : *I think is algebra.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 10 : *Trigonometry*

Researcher : Why do you say so?

PST 10 : *Trigonometry, although give kind equation to some former algebra regards to formulas and geometry regarding sheet. But ... However, trigonometry is a section where learners feel how am I going to use this in real life. Why am I learning this, I m not gonewhy am I learning about sine cosine and tan, how this gone help me and this theories how is $\sin^2 x + \cos^2 x = 1$, how is going to help me. And also the complexity of advance of like proving advance trigonometry formulas is a problem for learners who like critical thinking skills.*

Researcher :even identities hehh?

PST 10 : *Yes*

Researcher : I would like to find out from PST 13, which section do you think learners enjoyed the least?

PST 13 : *Ehh ...I will link with my previous school, where I was, so the section is geometry.*

Researcher : Why do you say so?

PST 13 : *Because I have seen most of learners they can not necessary identify something that is given in a graph, let us say mhlambe (Maybe) they are given rectangle with two parallel lines and two short ones at the end but they can not understand that definition of parallel lines, all the stuff, they can not even see the symbol that indicate, that this one is parallel to that one and this one is parallel to this one. And this is when I find myself having enjoying that part because for my point sake I was enjoying geometry that much because I can be able to see those differences between parallel lines and all the stuff.*

PST 12 : *Ok, in our school thina (we) we used to enjoy geometry and we enjoyed the least trigonometry. Our teacher was not good himself. The problem was the teacher.*

PST 14 : *I would say something also add to the difficulty in trigonometry. Is that trigonometry is.....(Interrupted by the researcher)*

Researcher : So you also confirm that is trigonometry?

PST 14 : *.....Ja, that is at least, like enjoy, people are not interested in that because we only start trigonometry at grade 10 and is not like we learn trigonometry for the whole year. It just a small section and we are suppose for some odd reasons to keep that in our minds and carry it to grade 11 of which is very complex we also have other section that are colliding with this information we already have, so coming to say suppose you did the trig in grade 10 first term you get into grade 11 first term, you also doing something else in trigonometry now its clustering, you can remember you did, so children are not able to.... Are not given enough time to grasp those concepts even not really dwelling into trig not even define what is sin rather they just drawing Cartesian plane and say y/r is sine but what does y really mean what does r really mean?*

Researcher : What do you remember about geometry that you learnt in high school?

PST 14 : *If I'm correct, If I'm correct that, the gradient of the tangent (the tangent) is always perpendicular to the radius*

Researcher : Gradient of the tangent, ok. You have learnt tangents?

PST 14 : *Something like that or the gradient is perpendicular to the radius to every point to the curve, I'm not quite sure.*

PST 10 : *Well, even someone who did mathematics paper 3 in matric, I can probably tell you a lot of circle theories, like examples, the tangent chord theorem, the cyclic quads all I had to remember when come to that paper, geometry is not just about proofs is about circle, also involves angles and shape and properties because we need, we have to use them because we have these question when get situation, we have to think about this.*

Researcher : Don't we need to prove those properties?

PST 10 : *Of course.*

PST 12 : *What I remember in geometry is that when ever the diameter meets the chord it bisects that chord.*

PST 11 : *Like the sum of angles of a triangle and like vertical opposite angles are equal and corresponding angles.*

PST 09 : *Ja, they have already said some of the stuff, like sum of angles of a triangle, vertical opposite angles, alternating angles are equal, angles separated with the transversal line such things. But the challenge I had in my high school geometry was that it was optional, so as a learner for rural place or rural settlement my teacher didn't boarder himself by doing anything in geometry he just did geometry that was all in grade 9.*

Researcher : PST 10, may I also come back to you. You said you did paper 3

PST 10 : *Yes.*

Researcher : When did you start to do paper 3? In grade 12 or in grade 10?

PST 10 : *In grade 12.*

Researcher : Grade 12, no paper 3 in grade 10, no paper 3 in grade 11, and then suddenly in grade 12 you get paper 3?

PST 10 : *Because I don't remember marking (doing) paper 3 in grade 10 and grade 11, so in grade 12 is when I started paper 3. And we had basically one class every week for paper 3.*

Researcher : Was that ok with you?

PST 10 : *Ehh... suppose I did manage an A, but it wasn't easy, I needed... , because one class per week and pay the fees, all these theorems is not, is not enough for grade 12, we also have to accommodate 7 other subjects also needed some tuitions of about to help me.*

Researcher : Can you explain how geometry was taught at your high school?

PST 10 : *Basically, in high school when we were taught geometry we..., it was basically textbook and diagrams on the board the teacher drawn those shape and such.. such and explain what goes on in the lesson, like exterior angle of triangle she best drew a diagram exterior angle and broke that theory some on the board and we also has textbook with examples to help us*

basically do more examples and grasp the concepts.

PST 14 : *Can I also add on with the teaching methods not just particularly geometry, but we found out with most of the things or sections in maths in high school? Most of the times when open classroom textbook there is an examples introducing the unit, example introducing the list of complex to the complex thing and the teacher will do exactly what is listed on the board . So they are giving examples that you are already looking rather just explain to you, not explaining reading out to you " $x + 2 = 2$ " which is written there and then they say go to exercise 5.1, do a, b and c.*

PST 13 : *Ehh...in my school, they were using mostly compass like to measure the sides and angles of ehh..is it isosceles? that all angles and a side equal, equilateral triangle, to prove equilateral triangle we use compass that we put end sharp and you put a pencil and arch then you also put an arch where they meet and then you join your diagram then meaning that the lines must be equal and also the angles will be 60° all sides and then you use that thing that prove it and put the protractor to measure it is exactly 60° and you find out it is 60° and then with the other one we use rulers that straight draw the line and then you draw a transversal and all this thing to see to introduce us to U shapes and F shapes and important angles and important styles.*

Researcher : If you can be given a chance to go back to your school now, would you teach in very same way you were taught, if you can teach differently, what can you change, or how can you teach differently?

PST 11 : *Actually that is the reason why I choose to do mathematics at the university, because my teacher really he didn't have a background of geometry, then I decide that I have to go and study mathematics so that I will come back and give my learners information like go an extra miles find more information not just use textbook and use exactly what is on the textbook.*

Researcher : So how would you present mathematics, I mean geometry?

PST 11 : *Like, I wouldn't like , in the textbook they have the examples, I would like to go extra miles find something new to introduce different types of theorems not just use the method that is on the textbook.*

PST 14 : *Ehh... the thing is, the trick is with anything that you about teach children. You have to first understand how they view life. There are other kids that would actually understand things but rather just find, given them notes and simple take it in . But majority of children will rather deal with concrete things that is the one problem with maths, the nature of maths it is not concrete, they are very little things in maths that you take and leave on the*

board and say now feed on this and you expect a certain answer. But with geometry you can, you can bring into class pieces like our maths lecturer has done with us, he brought pieces of circles into class ask us to do certain things and we all got all, the same conclusion or he ask to draw your own circle cut and do certain things by that you show that every body circle is different in the class but at the end we are getting the same results. Meaning this is a proof, it is a theorem and this is nothing gone change it.

PST 09 : *I would say, including experiments, actual measurement and proving some theorems or any geometric information can help the learners grasp the concepts more, like for example if you saying a square has all sides equal you let the learners construct a figure like the.... Ehh.. by doing that they can be able to visualise and never forget the information which I think can also help to the issue of pass rate which were introduced at the beginning of the session.*

Researcher : Which geometry concepts were you aware of when you enrolled at the university?

PST 14 : *Do you mean that something that I come here, I already know? Angles of triangle all add up to 180°*

PST 10 : *Basically angles, yes angles is the one that stands out you are doing geometry because you work with every single day.*

PST 12 : *We did geometry thoroughly, I matriculated long time ago, so geometry was the part of the paper, we did geometry, everything.*

PST 13 : *The part that, I remember the most, the one of the alternate angles and co-interior angles and co-exterior angles and all the stuff that one was familiar of and the like the straight line. we have got add up to 180° and all the stuff.*

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 14 : *Yes of course; especially when we are in Prof de Villiers modules. I have gained a lot because in that class we actually learnt to construct figures using ehh...sketchpad and it was so difficult at times and you... when you heard him saying construct equilateral triangle. What is all about of course, we all do that but actually trying to doing the sketchpad was very difficult, because now it not just on having the sides being equal, its about those angles also being equal.*

PST 13 : *I would say, I been most, in terms of proving some theorem because I came here knowing that in a angles subtended by same chord must be equal, so those theorem is just a theory in my mind that angles should be equal but I never knew of how to prove them and all the stuff, and then also to use worksheet using a sketchpad I never knew how can I use a worksheet that people make learners understand what I m talking about is exactly true. If you move may be, you draw ehh...chord and then ehh...you do a line from a centre to the chord it will bisect the chord if the line is perpendicular then if you move the chord in every side of the circle, it will still bisect the chord. The two sides will be equal.*

PST 09 : *As my colleague, have just said, the geometric sketchpad has put more emphasis in proves like you can construct a very dynamic ehh...figure. And the...the...the measurement that you did hold for what ever size it can be. Its unlike if you looking at it on the board if you see 60^0 there and may be 5cm, you can change the size of the figure, so showing you or convincing you more that the result hold no matter what, you know its unlike just telling you...teacher telling learners that the result hold it better if you see it.*

Researcher : Which strategies do you think you would use when teaching Geometry to your learners?

PST 13 : *Ja...I would say, I will give the learners the task to do themselves and I'm going to facilitate if they are going to the wrong direction, I m going to take them back and make them see what we are talking about is this side. For instance if we are talking about shapes I give them plain paper and ask them to take a pair of scissors and then cut square and then a definition will be given to them , a square is a four sided figure.....and at least one angle is 90^0*

PST 10 : *Basically, the strategy I would perform is equate question to real life, make sure mathematics doesn't just exist in mathematic world, because if it quite in your life you will understand better and you will keep and retain, that is the main aspect of learning experience.*

PST 12 : *I would add more practical, like in geometry we were told that sum of angles of a triangle are add up to 180^0 , so now but we didn't know how to prove it but now I can let them know how to prove it on practical.*

Researcher : Why do you think that these strategies would work?

PST 14 : *Most of the strategies that PST 12 talked about are more concrete and getting involve with rather than... I really compare with high school, what happens in school we are given things. We are given the answers and we being told to verify that these answers are correct, rather than these people*

are saying they are going out there and say prove. No just do so and so and prove what you get at the end. So children would not have the answer so it will be not effective because children will be more hands on and they will be discovery, is more discovering learning there will be a discovering answer rather than just proving the answer is.

PST 09 : *Ehh... I'm just taking the point of integration to real life; it is easy to learn something that is within your scope of experience then doing something for the first time. So I think it will work for them because they have the theory already in mind or even if they don't know that the theory we need in geometry but if we just identify as a theory that we need they can grasp and remember them quickly.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 14 : *I think the teaching... I know the teaching will be better but it not something can happen over four years, it does not something can happen over five years either. The result, there are very few teachers and making difference and the is already the children's mind are already destroyed because they already have perspective of maths, they already think at maths as difficult being this, geometry is this, an animal thing so will go back and have to first destroy this mentality before we ever try making them interested, you can send, they say you can take the cow to the river but you can never make the cow to drink water, so we have to first start to make them want to drink water. If we want the change and it only then teaching and learning will be effective.*

PST 10 : *I feel these strategies, yes they will work.....*

PST 11 : *I think it will work, because like taking the examples using maths, examples of real life, like if you use real life examples learners will now know that maths actually is related to real life, because you find out that learners don't even know why we are doing maths they are not motivated by their teachers. So by using real life examples they will be motivated and do more.*

PST 13 : *If I may make an example that ehhm.....so the more practicals, it will work.*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 09 : *I think they are completely different, because when I was used to in high*

school, a maths teacher always carry all textbook and T-square, the chalk that all, then with these strategies that we just mentioned now, ehh... a teacher can now come up with papers something is relevant and something that is, that will keep learners busy rather than just keeping them seating and listening to what you are saying and it like you are preaching to them and even introduce the technology that we have like computers, you know kids are very attached to computers they like them a lot and then if you just put geometry in what they like obviously they will like geometry, they will like proof, and they be changes

PST 10 : *I feel these strategies yes are very different, for me at high school when we were taught mathematics, we were taught everything if not majority of what was taught mathematical concepts are theory, that the impression were given that all this is a theory, and that ehh... you know, theory neither accept them or not accept them and that is why you make it known that these theories they work, that these theories work through our real life through out the world and that is why mathematics being a universal language it is ehh.. requires this bring out mathematics to real world to everyone understanding.*

PTS 13 : *I would say, they are different because during our time we didn't have computers at school, so I like examples of creation a worksheet using sketchpad because that thing is, will make the learner do it themselves, even if they are, they have got free time free periods during school hours, so they go to the LAN(Computer lab) and then they going to do this thing, because they enjoy when they play with thing, then you keep on moving all the points and you will see yourself that this thing it will automatically be true for each and every side.*

PST 14 : *Something else that makes, I think the teaching strategies, that we are going to introduce now are totally different from the methods that were used back in the days, is because a teacher every maths teacher almost all of them that I can quite remember that walk into door, they walk into the door and automatically there was a bridge between the learner and the teacher. Because everything that the teacher talked about some how come from his own world, he was like ehh... when you go to Eskom and most of us agree, is Eskom, we got electricity ja but where is Eskom, they move on to another complex world, they took something we learn from other modules here that, what ever like if we will take everything Dr De Villiers and Mudaly are teaching us today and say it example to the learners, the examples that they are using are examples of this urban area. So its ok for them to teach us like that but for me to go back at Eshowe, Ntumeni at the farm and go start talking those examples that exist here, it is totally unacceptable because they cannot visualise things at urban settings. I have to step to their level, I have to go down to the kids level first and go up again.*

Researcher : In school level, what steps do we have to consider when we do proof?

PST 14 : *If I were to construct a textbook today I would eliminate.....(out of point)*

PST 09 : *I would say if you doing proof, you must be able to see what you know from the proof and I identify exactly what is the ... then use what you have to prove ehh ... the result that is being asked from you to prove, because you cannot assume what you have to prove. So you just have to see or look what is given and what can you use within proof or within the problem that can give you the solution, so use a problem as your solution.*

PST 13 : *I would say, the first step is that you should do is to use or take all that information which is given and write them. Another thing then from then you take something that you think is equated it will look like similar and then you can try to form up your own way of calculating what you want to get and therefore prove that is it necessary give you, if go back is it going to give the same thing that you get from the beginning.*

Researcher : Why do we do proof at a school level?

PST 10 : *My proofs are basically an understanding why this phenomenon occurs like I get situation, why are all triangles cyclic, why is when you draw the line from the centre of the circle to the chord bisect the chord, there has to be a proof to understand why does this thing occurs because of a reason and proof is a reason.*

PST 12 : *We use proof knowledge when we are solving problems like geometry problems when we give examples ahh! Not examples, reasons, you will say may be you want to prove something line is perpendicular to another line, so you have to prove there is 90° that is formed so you use knowledge of proofs.*

PST 09 : *Within the scope of formulating or constructing knowledge, it is said that at the end of everything or at the end of experiment you must have generalisation or a general statement that you will come up with. So I think a proof is kind of showing us a result how did you go about getting the results, and then it for convincing us to take mathematics as something that is true, rather than just take a theory that exist in the head of those intelligent people.*

PST 13 : *Ja, the proofs are necessary, to see that if it always work irregardless of the size the shape to see it, it is always true that may in a circle, in a small circle, we say a chord subtend to two ends then it mean the two ends are equal so if now the circle is now big and we are going to get the same answer or not.*

PST 14 : *We have to proof in order to understand the world we live in. We can not understand this world we live in if we don't understand how certain things came about, to understand how certain things came about you have to come to do proof(interrupted by researcher)*

Researcher : So specifically in geometry?

PST 14 : *..... to understand why we construct houses like this, how we have a balcony, we can't just place a balcony anywhere. To understand the medians, centroids, how those things happen, if you understand those things you understand the world we live in, and also what PST 9 has said, to form a generalisation, forming a generalisation is something we can carry with us every where. So if we form this proof in grade 7 or grade 10, if you found is a generalisation, there is no need for you to keep going back for every big circle going back and doing generalisation because you already got this generalisation that you can carry through mathematics till have instrumental or correctional understanding.*

Interview transcription

Individual interview

Section B: Knowledge of proof in Geometry

PST 01

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 01 : *I would say algebra and trigonometry, between the two; there was a question for most of us.*

Researcher : Why do you say so?

PST 01 : *For most learners' mathematics eh... is a concept they can grasp very easily. So they like patterned things, they like things with... they learn and they just apply and when it comes to trigonometry and algebra you do have a lot of application where you don't have to visualise on your own which geometry is.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 01 : *I would it is probably geometry that students enjoy the least.*

Researcher : Why do you say so?

PST 01 : *Because you have to be able to see, and no one can teach you how to see, you see it yourself and so learners who haven't the ability find it very frustrating and as soon as the learner is frustrated is 'I can't do it' and they just don't do it.*

Researcher : What do you remember about geometry you learnt in high school?

PST 01 : *I remember the angles basically the FUN, co-interior, alternate and corresponding. Eh... triangles the most I think the shape we dealt with, sure we dealt with other shape quadrilaterals but basically just triangles and angles.*

Researcher : Can you explain how proof in Geometry was taught at your high school?

PST 01 : *The proof, it was, you were given an example and you were shown how it worked and then you were... you wrote it down so in a very rigid form and that was your proof and it was you write your proof this particular was and you go this way not your own way.*

Researcher : Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)

PST 01 : *Well basically, when it comes to signs and symbol, they are able to identify when lines are parallel or perpendicular and when angles are equal. I use the , I use basically the signs I was taught in high school with angles you have got the arch it and if it, you have more then one type of angles that is not equal you double the arch and stuff like that, and when it comes to parallel lines it just the arrows on the lines and when lines are equal is the sort of parallel the two slanted lines and how basically I label my work. But I try not to when it comes to geometry because the problem when comes to geometry what you seeing like our lecturer says, what you seeing you seeing things you know, you not, you like that so I try very hard not to make any assumptions of, something is that because it looks that way to me.*

Researcher : I have seen here on your task you said if $\hat{E}_2 = \hat{F}_2$, and you said if $\hat{E}_1 + \hat{E}_2 = 180^\circ$ and also $\hat{F}_1 + \hat{F}_2 = 180^\circ$ then you have made your conclusion, why do you think like this?

PST 01 *Well, I can see. I can see the answer, but the thing is when doing geometry, when you are writing your answers, you basically making a plan of what your thinking is when you come to answer, so because I automatically see it but I have to break down what is the thought of all process that I actually have, I actually come to this answer and that I write down what I saw that's why I wrote it down.*

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 01 : *I believe I have deeper knowledge, but it is not too much knowledge in context, it more about why we do the context we do and how the context came about. So it is behind the curtains magic when you are in school, you are front of the curtain and the teacher knows something else and then now starting to be an educator, you learn how it easier they know what it easier they know how they com.. how they make examples for you to do and they*

know that the example is going to work.

Researcher : Which strategies do you think you would use when teaching proof in Geometry to your learners?

PST 01 : *Well, obviously I have to look at the type of learners you have, I would like to use constructivist approach, where you build your own information but then going back on what I said, that learners find easier when it comes to algebra and trigonometry because they can learn things when it comes to mathematics, when it comes to writing proof, learners use to get full marks, if they had it done very beautifully. So try to balance between the two, where, yes I would love them to understand what is it, they are doing. But considering the whole point of high school is to get into university and they need to pass. I would also then just really keep them on the track, this is what you need to do, remember learners informative_(sic) and they write an exam.*

Researcher : Why do you think that these strategies would work?

PST 01 : *Because; what is the outcome at the end of the day, you want your learners to understand, yes wonderful, that's such a broad concept and it's so very..... but at the end of the day you are a teacher if your learners are not passing, they look on you and so your aim is to get them to pass all, it's no longer about getting them to understand their work, it is about getting them through the work. So what strategies do you need to get them through the work. It's no longer a question about getting their understanding it will be a borderline of, yes you learn these, what you need to learn, but again when it comes to geometry, they have to be able to visualise. I can teach them all the theories under the sun, if they can't see that a problem.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 01 : *Yes, you know what. When I go for the TP (Teaching Practice), I try to adapt and use those strategies, but I'm only there for a month and it's basically fun time for the learners, and when it comes to their understanding because we do do assessment with them. Yes it's fun strategies and learners really have a lot of fun when it comes to constructivist approach but when it comes to understanding, I think they have a little bit too much fun and not understanding, so it will have to be a gain it just a balance.*

Researcher : So it will be better if it is more fun?

PST 01 : *Yes its better if it is more fun but learners with too more fun they don't pay attention, you know you only remember that you had a funny day, but you don't remember what you did on the day*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 01 : *They are dissimilar, because we were basically chalk and talk but then again when I was in seven when I was in school it was pre OBE, so it was still using old, old system but had teachers who were capable, because those teachers that come out and was still an old system because I had general old teachers who know more about what was going on. So though it was chalk and talk I think they address the misconceptions that we had but the strategies I would have to employ will be slightly different because I would want to have learners build their own understanding but it again depend to where you are , what kind of learners you end up with.*

Researcher : If you remember very well, my first question was to which section is much more eh... enjoyed by learners, which one is not enjoyed by learners. If I can ask you now, which section do you enjoy?

PST 01 : *I always, always enjoy geometry, but not because it's my best section, it actually my worst section, but I always enjoy a challenge. I would like to do well the challenge but I prefer a challenge and I particularly like where comes to how your mind works and not how they want you to generic the answer because someone in the 16 century developed this method you know because geometry is not always something that is like algebra where does come from past history and we condensed information that happen over quite few hundred years we are doing in class for this week, but when it comes to geometry it really how your mind is working and how is ticking and so the answers that you are getting are your own and it makes you feel proud and you can see something through your understanding and it is correct.*

Researcher : So I don't want to assume that you can enjoy teaching geometry?

PST 01 : *I do enjoy teaching geometry it a lot of fun but again geometry is a topic where you need learners not to be destructed by everything else because is about visualisation so if you are destructed you are not looking where you are suppose to be looking and then there should lies a problem.*

Interview transcription

Individual interview

Section B: Knowledge of proof in Geometry

PST 02

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 02 : *I think it depends on the learner, so you can't just say they are all enjoy this, it depends on the learner, some people enjoy algebra, some people enjoy trigonometry and some people enjoy geometry.....its not really a lot on syllabus at the moment. So it's just depends on the person and how you teach it.*

Researcher : Why do you say so?

PST 02 : *Algebra you can actually put it down, ok you got an equation and you can take the equation find x and you can find x, remove this remove that divide this divide that and you can find x easily.*

Researcher : But your overall viewing, you can sense that this class is actually like this section.

PST 02 : *They like algebra.*

Researcher : And you? Which section did you like?

PST 02 : *Algebra and trigonometry.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 02 : *Definitely Geometry*

Researcher : Why do you say so?

PST 02 : *Because ehm... they and again so the teacher teaches early in the years, so when you actually get to an other grade you don't actually understand the previous years work. Say you can actually build up, and just nothing really*

tell you this is what happen, this is what it is but prove it but doesn't, it doesn't there is something abstract.(added later). If you got a special problem, sometimes you can't see object on top of one another, I had a special problem as a child I couldn't really do geometry, because I couldn't see the circle and triangles within the circle and something in side..... People have problem of that, lot of kids, parents don't know that their children has a special problem but they do actually have a problem with it.

Researcher : What do you remember about geometry you learnt in high school?

PST 02 : *Besides creating..... I am not quite good in geometry, because I know grade 8 and grade 9 and then I did maths paper III in high school work, I know circle theorems and beat of or everything in geometry.*

Researcher : Can you explain how proof in Geometry was taught at your high school?

PST 02 : *She basically, she just told us this how to prove it...(laughing and after being probed)...she proved it with us she tell us ok how do you think it and a lot was... she just facilitate the knowledge the lot of the time but otherwise she could really, we could do it on our own. So she would have to say curtsies of what we got to do, this the steps and the proves, so the lot of time she did try to facilitate the knowledge and she use a sketchpad quite a beat so that we would show us on sketchpad the different thing and try and prove things on sketchpad.*

Researcher : Was that helping you?

PST 02 : *Yes*

Researcher : Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (Colour/symbols/constructions).

PST 02 : *It basically talking me through it, so then if I'm putting construction I can then see, it better when I see things, I write something down I might not pickup something but if you see it, I can then pickup something that is similar or different or I can go from that point.*

Researcher : No to be specific in this, why did you put some colours?

PST 02 : *Ehm ... I saw what I had to prove, so many lines like I said I had a special problem as a child, so then I couldn't, I won't be able to see but then I went to a psychologist or whatever so I work through the problem but still*

difficult for me now to see object on top of each other, but I can see it , I can see things but I might not be able to see what I have to prove. If I put the colour I can then point what I have to prove.

Researcher : Even though you think you have put these colours because you have some special problems, but do you think this would help other students?

PST 02 : *Definitely, our teacher encourages us to use colours, when you are doing geometry, so we can see what we are doing parallel lines with that, and then you see angles that are equal and what you have to do.*

Researcher : So when dealing with proof you must not forget adding colours and all these indications that you did here, constructions and ehh... oh! Specifically, why did you put this construction, this one, AF?

PST 02 : *(Laughing)...What about construction?... because I want to make two triangles and prove that they are congruent.*

Researcher : Correct. I'm interested in what you are saying, the question is not about congruencies is about proving whether the angles $\hat{B}$ and $\hat{C}$ are equal but you have to think beyond that. Why?

PST 02 : *Jah. There is no, you have to prove that, if you prove that two triangles are congruent , means that all sides are equal and angles are equal so I thought to prove the $\hat{B}$ or $\hat{C}$ are two angles equal in the two triangles. I can prove that the triangles are congruent meaning that implies that the two angles are equal.*

Researcher : Which means you have to move from what you know to what you don't know?

PST 02 : *Yes*

Researcher : We are given that $\hat{E}_2 = \hat{F}_2$ and you said let $\hat{E}_2$ and $\hat{F}_2$ be equal to x, may I know why?

PST 02 : *Sometimes it is easier work with x.*

Researcher : Does it gives any impact in doing the problem or may be you like to work

with x ?

PST 02 : *We are familiar with x , we use x in everything in maths. How many people change things from x , so I make x it more familiar with me. But I am trying to see why I did this now.*

Researcher : There must be a reason why you have made both of them ($\hat{E}_2$ and $\hat{F}_2$) equal to x ?

PST 02 : *Because if two, just by putting $\hat{F}_2$ makes look different that if you put x you can see there is an angle. I've always done that same letter same angles.*

Researcher : Now let us look at this one, you said $\hat{E}_1 = 180 - x$ and $\hat{F}_1 = 180 - x$ and then you concluded that $\hat{E}_1 = \hat{F}_1$ why did you give this conclusion?

PST 02 : *First of all, they are straight line, so we have been given that angles equal if $\hat{E}_2 = \hat{F}_2$. So both are at the straight line, because both of a straight line, we can actually find that angles there, so I've said that angle is $\hat{F}_1 = 180 - x$ and $\hat{E}_1 = 180 - x$ so because they are both $180 - x$. So they are equal.*

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 02 : *Yes, I definitely have.*

Researcher : Which strategies do you think you would use when teaching proof Geometry to your learners?

PST 02 : *Ehh... try and get them do proof for themselves, learning themselves then me facilitating, sometimes it is not easy.*

Researcher : What is the different between facilitating and doing proof by themselves?

PST 02 : *Ehm... sometimes, if a child can't get something, so if they are doing the proof for themselves and they come to a stumbling block and just going to leave it there, I had this too difficult I am not going to do it all but if I'm here as a facilitator, I can say ok look at this clue, ok you start and move on. They got a clue they can then move on but if they just come to a stumbling*

block they can stop there and then 'I don't want to do it any more I hate geometry, worse thing I ever I saw, they come to other conclusions from that.

Researcher : Why do you think that these strategies would work?

PST 02 : *Because you remember more by doing work by yourself that the only way.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 02 : *I think, will make it better but it is depending on the teacher again, teachers, another ways are not facilitator they wanna get through the work so they are not gonna be facilitators they want... and I like, I said earlier on that we can use sketchpad and new technology to actually teach the children. I was teaching my sister the other day one of the proofs, let just take an example the tangent proof, tangent-chord theorem. I was proving to her, I drew the circle, I changed the size of the circle and then you just click on the line that measures the angle that measures the angles on an alternate segment and I prove to her and she is like why my teacher didn't do that in the first place. It makes the whole of sense to them specially when they see things for themselves so using that showing the proof and helping them through the proof, it makes life that easier.*

Researcher : Which one is better to use sketchpad or to use concrete things to bring material to class?

PST 02 : *Would like, will depend on you on what resources you have, I would...I taught in another school from teaching practice last year they didn't have electricity, so what I had to do was, like at times I had to ok, take a normal piece of paper and teach them proof through a piece of paper. But if I'm going to a school which got a projector in a classroom that might be easier for children to see things on the laptop but you could also if that is not working for them they can move to concrete things, try and error between the class.*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 02 : *Similar, I had a very good teacher.*

Researcher : If now you can go back to your learners, would you teach proof in geometry

the same way you were taught or you would teach differently?

PST 02 : *I would do it in a similar. I am happy with the way she taught me the work, I was happy the way she taught me the work and it worked for me.*

Researcher : But as you are here at the university you said you do have knowledge that you have gain.

PST 02 : *I can incorporate that means knowledge is more of back on knowledge we didn't have like if you take an example of this question here, question two ohh! No! prove that $\hat{B} = \hat{C}$ in isosceles triangle no one ever teach us that proof no ever, I stare into the lecturer and said can I make congruent triangle. At the end of it I said that is how I thought of it and said I never ever seen that proof, never in my life. So, I'm going to gain more back on knowledge here and how I can back on knowledge but whether other people us it.*

Researcher : In a nutshell it means you will teach differently because you will be incorporating what you have learnt.

PST 02 : *Yes, similar but differently.*

Interview transcription

Individual interview

Section B: Knowledge of proof in Geometry

PST 03

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 03 : *Ehh well personally I think is algebra*

Researcher : Why do you say so?

PST 03 : *Because, it gives a general structure of you know what like for example if sum to find oh sum to find x can follow same sort of methods. So it is like easier for them to put it in to terms you know to understand it all let it to apply it to get mark.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 03 : *I think is geometry.*

Researcher : Why do you say so?

PST 03 : *I think is because ehhm like although we given background knowledge. It very difficult to the learner to... how can I say... to change it ehh to delete what al ready i nto l ike a pr oble m gi ven. S o I think t hat main, the most important thing the most difficult part is to regulate what you know and use it in what is required. To find out.*

Researcher : What do you remember about geometry you learnt in high school?

PST 03 : *I remember triangles, because that section was fine and the teacher was fine explaining it. So I think is section I remember it*

Researcher : Can you explain how proof in Geometry was taught at your high school?

PST 03 : *Ok, well, it was... I was from the old school, where OBE was not in as yet, so it was the teacher, were putting the.. let say circle geometry put circle on the board enjoy t he pr oof of h ow we g et, l et s ay l i n e f r o m t h e c e n t r e t o t h e c h o r d, i t p e r p e n d i c u l a r t o t h e c h o r d, h e w i l l s h o w u s t h e p r o o f t h a t w a s, w e*

just have to like memorise it of.

Researcher : Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)

PST 03 : *Ok, I think like when I was in school, we were very use in using different colours, different symbols like for example if was two triangles in a circle one will be one colour and other triangle will be other colour, so we can see which triangle are we talking about what ever. I'm very used to that system. What ever information is given to me I always generally transfer it to the sketch, so the reason why I have all these marks and symbols.*

Researcher : So, does it help?

PST 03 : *It does, it really does especially the colour coding, it gives you like if you are looking let say a circle with about four or five triangles and you are asked to put of two congruent, it really difficult to look at it as one but you disintegrate it and put it different in pieces and you colour code it, it much easier you can see things that you wouldn't just see like that.*

Researcher : You were requested to prove that $\hat{E}_1 = \hat{F}_1$ then you stated that $\hat{E}_1 + \hat{E}_2 = 180^\circ$ and $\hat{F}_1 + \hat{F}_2 = 180^\circ$ then you generalised that the two are equal, now then after that you concluded that $\hat{E}_1 = \hat{F}_1$ why do you have to do that?

PST 03 : *I think is to show the relation like in maths method I we learnt a bout instrumental and relational knowledge and how, I makes like so much easier if you know the simple basic rules to move to a more difficult task in knowing this, it... you don't have to put the whole lot of working out to get to the final answer. You will be able to identify much easier, ok.....vertical opposite angles or alternate angles or what ever angle it may be ; it just makes the final work much easier.*

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 03 : *I really think so, ehh at school level it was just what ever was given to you that what you gonna take and apply it on the exam and that was it. And at the university things are not handed to you is for you to think for yourself, this is how to get there, this is what you can do and you think of new innovative ways, be cause of knowledge that you have to impart onto students is not simple given it to them and that is because learners are not are the same level, so obviously you have to account for all of that. So I think I does make a be at of a lot of difference actually and lecturers are*

also, level of speaking to us is not at children level, is at student level so that also helps a lot.

Researcher : In this way it means you are now different from the situation when you were just entering the university as compare as now.

PST 03 : *Yes. Definitely.*

Researcher : Which strategies do you think you would use when teaching proof Geometry to your learners?

PST 03 : *Although, not totally, for the old system, it does work. So I feel it's a balance between the two, it knowing when to give them everything that they need and when to allow them the space to discover for themselves that the angles of a triangle are 180^0 or what ever. So is to strike the balance between the two so I definitely use both.*

Researcher : Why do you think that these strategies would work?

PST 03 : *I think it will work because it would allow them to discover themselves makethem interact_{sic} and make them think of different, it like opens the minds to different types of possibilities what can help them what can work for them to do like what will work what wont work and I can use the old method also you can't rule it out because it does work, so that it, I think were system to get the knowledge that they need.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 03 : *Definitely will, it definitely will it's because not only about learners it not only about teacher it about both of them working together to achieve a common goal of that knowledge.*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 03 : *It is different, because ehh when I was in school just only the conventional and traditional methods. Now in me implementing the method that I want, it ehhm... the balance between the two so is not totally the traditional methods and not to totally an OBE based method.*

Interview transcription

Individual interview

Section B: Knowledge of proof in Geometry

PST 04

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 04 : *I think it will be algebra*

Researcher : Why do you say so?

PST 04 : *I would say learners tend to find it easier, as opposed to geometry when they see geometry they tend to be scared about it and has a lot to do with the fact that it is not taught properly but basically when they see geometry they see a whole lot of diagram, whole lot of sketches they tend to be a tread as oppose to writing an equations and solve for x.*

Researcher : So do you know why it is easier?

PST 04 : *I think with algebra there' are steps to follow, there is lot of steps to follow as oppose to geometry, if you see it you see it if you don't see it then you can't, you can't prove it.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 04 : *I would say there is a tie between trigonometry and geometry because they are similar in terms of when you try to prove trigonometric identities you have to know everything in order for you to do it. With algebra you can get method marks or you can, you know you can solve for x, you know you have to take this above quality signs. I would say both geometry and trigonometry.*

Researcher : Which one exactly that you can think?

PST 04 : *Ok if I would say geometry again. It will be, because of the fact that there is a lot to know, there is a lot to know about geometry and it all given probable in a question will be given in one diagram, so an application of all those might not been easy to see at the time as oppose to seeing an equation and*

say solving for x.

Researcher : What do you remember about geometry you learnt in high school?

PST 04 : *Well, basically, I remember everything, we did. Ok first pure geometry, triangles, congruency, I remember some proofs concepts like cyclic quadrilaterals, tangents.*

Researcher : Can you explain how proof in Geometry was taught at your high school?

PST 04 : *Chalk and board, eh...chalk and talk, basically the proof was given to us to byheart it, never constructed on our own, which I never like because when I came to the university I now know to apply it as opposed to just byheart it.*

Researcher : Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)

PST 04 : *I use symbols to allocate angles, using connection between from the first symbol I would use alphabet 'a' to show that the angles are equal, that how I learnt in school to do that and then I would use colours if the diagram is complex, I would use a highlighter if I see a parallelogram for instance, bring out the F shape showing the corresponding angles. I would use colours to prove that. I would also use a construction.*

Researcher : You were actually requested to prove that $\hat{E}_1 = \hat{F}_1$ and you said; since $\hat{E}_2 = \hat{F}_2 = a$ then $\hat{F}_1 = 180 - a$ why do you say that?

PST 04 : *From the straight line theorem, we were taught that angles on a straight line is equal to 180° , so if I label one to be 'a' I know that the next angle will be $180 - a$.*

Researcher : And you have proceeded to $\hat{E}_1 = 180 - a$ and then you had your conclusion which is $\hat{E}_1 = \hat{F}_1$, why?

PST 04 : *I made connection that if one angle is equal to other eh... then more certainly then if one will be then equal to other the two are equal.*

Researcher : Do you think you have gained any deeper or new knowledge now that you

have studying at the University for 3 or more years?

PST 04 : *More certainly have, most definitely I have gained a lot of new knowledge apart from teaching maths, and I have acquired lot of knowledge of maths. That a lot of further knowledge on derivative, integration also thing I suppose to be doing in school which I didn't do is method of contradiction, I don't ever remember doing but coming into university, that we are doing if now I understand this when I suppose to do.*

Researcher : Which strategies do you think you would use when teaching proof Geometry to your learners?

PST 04 : *As I go out, I would definitely love to use constructive approach, when student are measuring and using tools to find out of the classroom situation allows me to do so. But I think with geometry it's easier for student to construct rather than just to learn of.*

Researcher : Why do you think that these strategies would work?

PST 04 : *I think it would work, because when they construct their knowledge, it's easier to remember and know what is true for themselves. As opposed to given to them and they are not understanding why it's true and why is that so that way.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 04 : *Most certainly, I think geometry becomes more believable that way as opposed to. As opposed to just given, then and they are learning but they never be able to apply, when they are able to construct they are able to apply.*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 04 : *It is very different, different, the way I was taught as I said earlier, was chalk and talk, I was given the proof and likely for me I was able to make sense out of it from myself. But I know, with other class mates we came extreme difficult. But ehh... I think with constructing it's, it make everybody on the same level and able to understand geometry better as opposed to by hearing the proof.*

Researcher : The first question if you can remember it, was asking you about which section is enjoyed by learners the least and the most, you, which section were you

enjoying, when you were at school?

PST 04 : *Actually to be honest it was geometry the task for me was when everyone was felt it was difficult, I was the one that use to succeed in it, so it become an enjoyable task for me because I use to be better in it I guess.*

Researcher : So, it is obvious but I don't want to speculate. Now as you are at the university and you are about to live, which section do you enjoy the most in teaching?

PST 04 : *Eish! I haven't had much exposure to teach all sections but what I did teach when I was in school is algebra, so I haven't had exposure in teaching geometry, I haven't had practice teaching geometry.*

Interview transcription

Individual interview

Section B: Knowledge of proof in Geometry

PST 05

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 05 : *Algebra*

Researcher : Why do you say so?

PST 05 : *Algebra, well from grade, from smallest grade they learn about numbers and even when they go to the higher grades they just deal with mostly numbers, and geometry although its there in primary school, but they get hit to it high school. Jah I would say algebra, they like to do solving problem with the variable some of them don't understand the variables but they like doing because of simple multiplication, division, subtraction.*

Researcher : What course that, what course them to like even algebra while they are at lower classes?

PST 05 : *Only be they had specific method of doing it and followed that same method all along.*

Researcher : Which can not be translated to geometry?

PST 05 : *Jah, it not abstract, it just something taught to them and they do lot of examples they do many examples in the school, with regards with geometry is not like that you don't do it all the time.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 05 : *Geometry*

Researcher : Why do you say so?

PST 05 : *Well from my experiences with children, they just, they don't like it because is not, may be, the teacher doesn't know how to teach it. That's the main*

thing.

Researcher : You mean it emanates from the teacher.

PST 05 : *Yes, from the teacher, if you have a good teacher who knows geometry, and many teachers don't know how to teach it, that is why I am struggling, I, I'm the one that come out and say I don't know geometry. I will never say that I know it because I really don't.*

Researcher : What do you remember proof in geometry you learnt in high school?

PST 05 : *I remember that, you must use colours, use different colours, say if you have a theorem, in cyclic geometry, a tan-chord theorem, so you must eh... identify the angles they subtended by the chord to equal angle and alternate segment. So look for those type of angles, not so much proofs, congruencies, not even medians and all that, not even that..... So mostly parallel lines, the FUN word, corresponding angles and co-interior.*

Researcher : Can you explain how proof in Geometry was taught at your high school?

PST 05 : *It was never taught, to prove something, eh not so much never taught, like for example you would prove that to a quadrilateral say a cyclic quad if you are to prove that eh, if you are given a diagram and in the diagram, there was a quadrilateral, if we had to prove that we had to use a theorem with the angles and then try to see if it is 180° supplementary angles are 180° and then it would be a cyclic quad but I didn't know that cyclic quad would touch four vertices of the circle. The smallest thing we didn't... wasn't really enforced on us.*

Researcher : Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)

PST 05 : *Ok, if there was given anything equal to each other, then what was given we would put on the diagram so that we can see it physically see it there, then a construction like for example I tried to prove congruency I think.*

Researcher : And even though the question was not about congruency you can see that it will lead to the correct answers, in other words you are saying you have to use what you know to get what you don't know.

PST 05 : *Yes, you have to use, jah.*

Researcher : Now about these arks, why do you put some marks?

PST 05 : *Oh, I try to like, you know bow tie ehh it was a bow tie theorem, in school we learnt as bow tie or butterfly but is not actually that, if it..I thought I was equilateral triangle ehh isosceles triangle sorry if two sides are equal, well I put these constructions, to try to start from some where, and I try to get the same angle, I tried to work from the end. You know even know your answer you can try to use other angles to get to the answers.....*

Researcher : Now, let me take this one because it is correct. You were given that $\hat{G}_2 = \hat{F}_3$ and you marked that as the correct one and you said $\hat{G}_1 = \hat{G}_2$ vertical opposite angles, then there after you reminded us that $\hat{G}_2 = \hat{F}_3$ because it is given. And you made some conclusion from there. Right you said therefore $\hat{G}_1 = \hat{F}_3$ why? From your conclusion why, why do you say $\hat{G}_1 = \hat{F}_3$?

PST 05 : *Ehhm... Because it both equal to same number, if it will equal the same thing, so if it equal the same thing then it has to equal.*

Researcher : So in other words is vital to collect all the information that you know so that it will form a basic of what you want.

PST 05 : *Yes and those we learn a lot in maths methods of proof because if something is equal to something and one of those thing is equal to another variable that has to be equal. Jah but in school I didn't know why I was not doing that.*

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 05 : *Definitely, yes.*

Researcher : That means you can now teach your learners much more better than you were taught.

PST 05 : *No, No, I need more, I need more time to learn it and maybe to even, I had to go back to school on different perspective I would go to learn how the teacher teaches it and even if the teacher is right I would adopt that type of method that the teacher would use, if it's the good teacher then I would want to go back to learn to. So that I can go to my school and I can teach them.*

Researcher : Which strategies do you think you would use when teaching proof Geometry to your learners?

PST 05 : *I would start from basic. From basic like a point, they need to understand what is a point what a line is. And then from there how the triangle is formed and how to prove. It must be from basic and use ehh use item, use object to show learners and like for example volume how to calculate the area of a triangle and show them you know 3D and 2D problems, even in trigonometry, show them like if you stand at the bottom of the building and if you used chronometer and they need to measure, you know, do things practical. I would, although time I would fit it in so that they can do it for themselves and also the teacher need to bring like coke and teach them about radius, or and even give them like circle and take you, know like cotton wool and they must measure it so that they can find out what is pie and all of that.*

Researcher : Why do you think that these strategies would work?

PST 05 : *Because learners are familiar with those things, if you go and draw a big circle with lines on the board they can't touch it so it will be very abstract to them.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 05 : *It would because ... They will make connections, with what they know like furnish for example that we were talking about furnish like even now that I'm aware of geometry even if I have to look my bedroom door or the window or even..... a TV is a square you know, rectangle it all over us.*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 05 : *Ehhm, not to what I had, because I don't think I was at the advantage. I would want learners to know geometry and to understand it. So I wouldn't teach them the way I was taught. I would take more approaches to it more strategies, things that I learnt now, and things I will learn I will go and research it, if, I wouldn't want a child not know where they writing an exam or test they must just be able to understand how to do it.*

Researcher : What would be your comments, when one would say we have to teach mathematics by removing the misconceptions by eliminating misconceptions

from the learner?

PST 05 : *I need an examples*

Researcher : Examples of misconceptions?

PST 05 : *Yes an example of misconception*

Researcher : Sometimes you would take things for granted as if they are same and let me say, let me say you have 2^3 and the learner would say its 6, do you see that because of multiplication whereas it suppose to be 8 something like that. Is it possible to eliminate all these misconceptions in mathematics?

PST 05 : *Yes i t i s pos sible, be cause w hen l ike f or e xample y ou f ind ou t the misconception w hat the l earner di d, t hey... y ou t ry t o e radicate t he misconception by using a technique to show and prove to them that is not how you do it, this is how we do it and may be make them know. But we did study that , t hey are lot of misconceptions so the main thing is when you give a test or assignment and you received that that back so when you mark ehh y ou w ill kn ow w hat t he l earners i s doi ng wrong f or e xamples i n trigonometry they are using Soh Cah Toa but they use a reciprocal of Sin, so you gon na show on the triangle first e radicate it , n o i t oppos ite over hypotenuse . Jah so you to... when you get test back assess them see their wrong and then show it so that they can learn from those and eradicate it so that they don't make that mistake again so even if you writing in a test make a big circle or do something so that they remember if was wrong and may be correct it yourself show them in the test and they will never forget that.*

Researcher : Not embarrassed by a big circle?

PST 05 : *Yes no.*

Interview transcription

Individual interview

Section B: Knowledge of proof in Geometry

PST 06

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 06 : *It depends on the teacher; it depends on how teacher teaches, actually the topic, because all of this topic may be enjoyable. It was combinatory.*

Researcher : Why do you say so?

PST 06 : *It is one section in mathematics and dealing with logical equations and you know, you don't need too much theorems or proofs, you know you just need to think properly and come up with solution.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 06 : *Trigonometry, I think. But doesn't mean that I don't know trigonometry that I have bad background in trigonometry no what I mean is I'm talking about student. I think they find it hard to grasp trigonometry'*

Researcher : Why do you say so?

PST 06 : *Because original trigonometry is dealing with triangles, you know it derives it is origin root you know Cos, Sin Tan functions from the unit circle, it is hard to explain them, when these are in grade 8 or grade 9 you talk about unit circle they have no idea what you talking about. So sometimes it hard to make the sense of what you are teaching then about trigonometry.*

Researcher : What do you remember about geometry you learnt in high school?

PST 06 : *Can I tell you something, I finish school curriculum in grade 6, because what I mean is I was independent, I was only me in the class, and there teachers coming, I was prepared for international Olympiad and it was great.....*

Researcher : Can you explain how proof in Geometry was taught at your high school?

PST 06 : *It was about, you know, making sense of what you see, it not about what you try to see, you already see but you can make sense of what you see. So geometry is about making sense of about what you see.*

Researcher : Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)

PST 06 : *Constructions, because sometimes you know that ehh when we think about question that might be some sort of you know part of pool you know, that is sometimes like this. Some one wants someone was actually, I mean preparing that question, maybe he erase the past so I was trying to find out which..... is the misbelief some sort of thing, I just try to construct some lines or for x is an element_{sic}.*

Researcher : I want us to look at this one I'm not sure whether you can still remember; if two lines are cut by transversal and alternate angles are equal then the lines are parallel, you were given that EH cuts AB in G and CD in F, such that $\hat{G}_2 = \hat{F}_3$ and you were required to prove that AB//CD. You said we know that $\hat{F}_3$ and $\hat{G}_2$ are alternate angles which implies from the given information about that AB//CD. Why do you say that?

PST 06 : *Ja, actually, as I said you know, I didn't care with this thing. I didn't take it serious. But if you want a real explanation for that one, you, when we come back to definition of the parallel lines it means they don't intersect, they don't have a common point and infinite as well because some of the mathematicians say that, they intersect and infinite, but I don't believe that one because they don't intersect.*

Researcher : No, but it's not about your answers it's just about you to respond to them.

PST 06 : *Ja, but what I say is when you go back to the definition of parallel lines, they don't intersect, am I right? Now we give that there are alternate angles, am I right, so if they are equal, just let check the supplementary angles one of them and that will be 180° - when you will sum up it will be 180° in a contrary when you assume that they intersect, sum of the three angles must be 180° but the sum of two is 180° and the third angle is 0° .*

Researcher : So you are saying it yourself that is not enough?

PST 06 : *No*

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 06 : *No, n o, n o I di dn't learn any i nformation at t he u niversity but may be I learn something but I'm studying additionally I'm not continuing with the thing I am studying additional with sources, so.....*

Researcher : You are doing other studies with other institution?

PST 06 : *No not institution, I'm studying on myself, I'm teaching Olympiad guys. I'm training some of them.*

Researcher : In other words you won't be teaching after this?

PST 06 : *No, t hat doe s n ot mean I w on't be t eaching. I 'm not s ure y ou kn ow something may happen.*

Researcher : But I heard that you are teaching some of the schools.

PST 06 : *Ja. I 'm teaching in th e college but n ot lik e what you think, I teach like Olympiad guys that are genius.*

Researcher : Which s trategies do you t hink you w ould us e w hen t eaching G eometry t o your learners?

PST 06 : *Normally I would use two strategies, where my own students, the guys who are ge tting pr epared f or O lympiad I nternationally. O ne of t hem i s, y ou know , t he s ide w edging c ontrolling y our t he s ide w edging ,y ou k now dealing with some sort of pictures you know, that can assist to see you know some sort of different shapes. There are some teasing shapes you know, you think th e shape is th at but there are a nother things in side shape, so it is nice to deal with that shapes in geometry, because it helps too much in case you are solving the deeper equations*

Researcher : In other words you will expose your learner in more deeply in figures?

PST 06 : *Ja. In figures is one way, and a s econd way that I'm using is philosophy, because the first one is improving the concrete thinking, you know concrete way of thinking. The second one philosophy improves the abstract thinking, so when they combine together or when they come together add something*

like two wings of birds that can fly.

Researcher : If I may just go back to what you said earlier on in our interview about teaching geometry in a concrete way. What would you say?

PST 06 : *I said, it is not the only thing. It can be done but it is not... it can not be done in that way the lady told you, it must be ...I think, the way that I told you, that something that improve the vision , ability to see, the other thing inside the same thing.*

Researcher : Why do you think that these strategies would work?

PST 06 : *Because, your roots for geometric equations, first of all you need to picture and the way that you know, the way that you know is normal concrete way. Sometimes we don't know exactly the shape, we know they exist but we can ignore exactly in which..... and some thing like...question about the God, it is something like we don't know exactly how He looks like, but at least we can know that He exist. So...so... (Interrupted). The concrete way of thinking helps to see these shapes that you try to find, or that going to help you as a link to solve equation. Abstract way of thinking is always important because if you don't have built in abstractly, I think the guy who has no that ability he has to quit mathematics.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 06 : *I'm not sure because everyone has no potential to grasp the geometry equation especially the deeper equations the higher level equations because all of them all equation are different. I mean and level of thinking also different they are not just you know, they are not like chocolate from the same type, they are different you know all of them are different, you, me everyone else that you see in this world and everyone past and present that will gonna come in the future, all of them are different , unique , so what I say is , since they are unique the potentials are also different, they can not be the same, since their potential are different we can not expect all student to grasp what we are teaching, so what I'm talking about or what I am suggesting actually is the strategy that can be apply to the leadenness of Olympiad.*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 06 : *No, I won't use the same methods, I have some innovations, special that concrete thing, they taught us how to think abstractly, but they I think neglect the concrete part.*

Researcher : So they were teaching abstractly and they forget what concrete.

PST 06 : *I don't say they forget they didn't emphasis, put an exact, some pressure but is fine.*

Researcher : Unless you have more to say, do you have something to say?

Interview transcription

Individual interview

Section B: Knowledge of proof in Geometry

PST 07

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 07 : *Algebra*

Researcher : Why do you say so?

PST 07 : *Because eh h... s tudents o r l earners f ind al gebra e asier, i t m ore l ike following r ules an d l ogarithm. E hh like may be i n ge ometry an d o ther sections i s more l ike y ou got t o u se di agram y ou got t o an alyse c ertain things but algebra is merely about using the rules most of the time although not all cases.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 07 : *I t hink al l of t he pape r I I, bu t m ostly t rignonometry, l earners f ear trigonometry, learners say it's hard.*

Researcher : Why do you say so?

PST 07 : *Because I use to here comments from learners where ever I meet with the. My view is that ok now_{sic} as of my self trigonometry is beat tricky then any other s ection i n mathematics specially i n p aper II i ts tricky, I c an say i t hard.*

Researcher : What do you remember about geometry you learnt in high school?

PST 07 : *I t hink I r emember a number of t heorems. I w as s hown by t he t eacher demonstrating on the board may be on their proofs and then ultimately on the theories itself this and that is equal to that so the teacher will do for us on the board and then you know and practice proving in case it is asked it in an exam.*

Researcher : Can you explain how proof in Geometry was taught at your high school?

PST 07 : *Ok, it was through teacher, the teacher will lead everything on the board and then as learners will follow it was chalk and talk. At a time I was still a student I thought it was the way of doing things, but I mean as of now I am here at the university, we should have been somehow be involved a bit so that our own development was gonna be accounted for.*

Researcher : Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)

PST 07 : *I just do it, but there is a reason, for the sake of helping myself as I'm proving the problem sort of like understand other links, are there in the diagram, try to form the link, a nigger picture of what is going on.*

Researcher : If I may be specific, you said $\hat{F}_1 + \hat{F}_2 = 180^\circ$ and you said $\hat{F}_1 = 180^\circ - \hat{F}_2$ and also you $\hat{E}_1 + \hat{E}_2 = 180^\circ$ and also you have $\hat{E}_1 = 180^\circ - \hat{E}_2$ then you generalised, you said therefore $\therefore \hat{E}_1 = \hat{F}_2$. Why?

PST 07 : *Oh I just ehh... I did that because from above as if said I have generalised that $\hat{E}_1 = 180^\circ - \text{other angle}$ so the other words $\hat{F}_1$ and $\hat{E}_1$ are both equal to $180^\circ - \text{angle}$, although I don't know the angle, I think it with uncertainty though but then for the purpose of this, there were both equal to $180^\circ - \text{angle}$, so I just concluded but although I felt uncertainty because the angle could be different so making unequal.*

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 07 : *Of course ja I have gained a bit of knowledge.*

Researcher : Which strategies do you think you would use when teaching Geometry to your learners?

PST 07 : *As of now I'm... I'm ehh doing method II, most of the time we are told about using whatever is available using practical ways as to make student follow an idea on what they see and then following with more conceptual understanding a theory and proof.*

Researcher : But in a nutshell would you teach this in different way from the way you were taught?

PST 07 : *Yes*

Researcher : Why do you think that these strategies would work?

PST 07 : *Ok because , taking from my experience, we were taught mathematics in a way so as to pass ,but then find that I do not understand the whole thing that the teacher has talked about, I just go t here and study this thing and know it and know how to prove it and its on my head . I write a test and then I forget. But now I would want to make a poi nt that student our learners understand why A , B , C, D is don e as it is done s o th at th ey will k now mathematics through out life not only for the purpose of a test.*

Researcher : What is it that you are going to change from the way that you were taught?

PST 07 : *Eiya, I will try to involve them more practical, involving practical and then I will truly explain and put emphasis o n every step I will n ot fell false and memorise or to cram the theorem.*

Researcher : Do you t hink t hat t hese t eaching s trategies w ould m ake t he t eaching a nd learning of geometry better for your learners?

PST 07 : *Yes, I think it should make it better.*

Researcher : Are these teaching s trategies s imilar or d issimilar to the way that you were taught?

PST 07 : *Yes, they gonna be different, that will be easier for learners to understand then to cram or memorise. I'm going for understanding then cramming and memorising.*

Interview transcription

Individual interview

Section B: Knowledge of proof in Geometry

PST 08

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 08 : *Algebra*

Researcher : Why do you say so?

PST 08 : *I don't know, I just think it most common maths that is available, that is always there, I don't know. People tend to dislike geometry for some reason I don't know.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 08 : *Geometry*

Researcher : Why do you say so?

PST 08 : *They think it hard for some reasons. Ehh I think that geometry I think is pretty basic when you know your theorems, pretty doable. It not that hard if you know your theorems.*

Researcher : What do you remember about geometry you learnt in high school?

PST 08 : *Theorems like how to prove ehm I know there was... I learnt ratios.*

Researcher : Can you explain how proof in Geometry was taught at your high school?

PST 08 : *Ok, if I remember correctly, basically what my mistress is to do, she would come with an example and do it on the board and we have to follow what she was ever doing and I do not remember anything special that she use to do, she just, she would write an example there, she would make us follow her like step by step what she is doing, we would follow her until it is proven and she use to give us problems to work out which was helpful, sometimes when you follow somebody doing an example you do not really*

understand what they are doing.

Researcher : Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)

PST 08 : *Ehm... If Oh! If they are no symbols, then it is very hard to work out what is what, so you need to see something, there has to be something.*

Researcher : What something?

PST 08 : *For example I put 90^0 angle, here, like in my mind I know, I knew it was 90^0 but still I had to say it, so that, I could find out other things, ja.*

Researcher : So you mean if it is not there it won't be easier?

PST 08 : *Ja, it won't be easier to figure out the other stuff if its not there, you can figure it out but I think it will take longer. I don't know your mind; your mind wants to see something there.*

Researcher : Right. Now, let us be specific to this one because I'm not asking the correct answers. I just want to know why you have done this. Now you said here, $\hat{E}_2 = \hat{F}_2$ given, and you let $\hat{E}_2$ and $\hat{F}_2$ be equal to x and then you said thereafter, you said $\hat{E}_2 = 180^\circ - x$ and $\hat{F}_2 = 180^\circ - x$ and you concluded by saying $\hat{E}_1 = \hat{F}_1$. Why do you say that?

PST 08 : *Like everything?*

Researcher : Yes everything.

PST 08 : *Ok, I said $\hat{E}_2 = \hat{F}_2$, ok said and it given, then I said, I made everything equal to x , ok I made $\hat{E}_2$ above I have said that $\hat{E}_2 = \hat{F}_2$ obvious I decided to call then x both. Ehh the reason why I decided to give them variable is because ...why? Ehhm.... It was gonna easier for me and the next step if I had a variable. Ehhm for example, I said that $\hat{E}_1 = 180^\circ - x$ it was gonna be beat confusing if I have said $\hat{E}_1 = 180^\circ - \hat{F}_2$ or $\hat{E}_2$, I think It was gonna be beat confusing, so that is why I swap those angles for x . Yes.*

Researcher : Now did this help you, then? In terms of x ? When we look at this lead you to conclude with this ehh, Now how did this helped you to conclude with this?

PST 08 : *If be cause, bot h of t hese an gles ar e e qual t o $180^0 - x$. S o t hat i s h ow I concluded ehh but you have been asking how? I think is just something I was taught to do it.*

Researcher : You really don't know why you did it?

PST 08 : *I think I knew why I did it, it but it very hard for me to explain why I did it, because some how that make things more simpler.*

Researcher : Ok, you were actually making this simpler?

PST 08 : *Yes, buy introducing x*

Researcher : If it is simpler, then to lead you to the answer, remember we are not saying the answer is wrong or right. By the way this one is right, but I just want to know why you have done this?

PST 08 : *Mm..., because I worked out the solution in my mind, I could, I could.....*

Researcher : Jah, PST 08, the only thing I want from you is that, which is in your.

PST 08 : *What in my mind.*

Researcher : Yes, because you said you worked out the problem in your mind.

PST 08 : *I didn't work out the solution, work out solution but I could work out thing as I go, like you can visualise what, you know what you are looking for in the end. So is just why I manipulate the situation to go where you want it to go. But of course.....*

Researcher : As it lead you to this answer?

PST 08 : *Yes*

Researcher : But if one learner may ask why do you say this?

PST 08 : *That $\hat{E}_1 = \hat{F}_1$?*

Researcher : Yes

PST 08 : *Ehh but there is a reason back, because, I have work out that they are both equal to $180 - x$.*

Researcher : That is what I was looking for, that is what I was looking for.

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 08 : *I seriously do not remember anything new about geometry that I learnt here, because there is no much of geometry we did only did geometry in methods, I think method III I'm not so sure but I think that the only time or may be second time, but not much just I don't remember actually learning geometry here, I remember answering things about geometry.*

Researcher : Which strategies do you think you would use when teaching Geometry to your learners?

PST 08 : *I think, that it is the best to start by teaching the learners all the theorems so that they are used to them and ehm.., like for example because most of the time, with geometry you need to have theorems in order for you to be able to answer the question. Ehm.. for example here if I didn't know that the angle of a straight line is 180° I wouldn't have known that if the other angle is x then the other angle has to be $180^\circ - x$. I think it is important to teach the learners theorem first.*

Researcher : You think the strategy you can use is to teach the theorem?

PST 08 : *Yes and provide the visualisations like I was saying the other time, because we have to see and I know that they taught us here, some other doctors, taught us that we have to make examples related to real world. But for me, it was different when I learnt it because I first learnt the academic stuff first, the stuff on paper then I was later able to relate to real world.*

Researcher : So then your strategy will be doing what? Now if you do that in real world, what would you do? You would?

PST 08 : *In reverse, introduce real world problems and examples.*

Researcher : Why do you think that these strategies would work?

PST 08 : *Because people are familiar toward what they see, what is around them, they understand it much better. So I think I would do that.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 08 : *Well is not something tried and proven just that, what ever, but yes I think it would. Because people are more interested in what they know then what they don't know, what are familiar with as opposed to what they don't know.*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 08 : *A beat dissimilar, because I would be going the other way , I would be going in reverse because like I said , I first learn the theory part, the part in books then I think out how to apply it to the real world, so mine is reverse.*

Researcher : In the beginning I have asked you to tell, which section was liked the most by learners and the which one was liked the least by learners. Now let be specific to you. Which one did you like the most which one did you like the least when you were at school?

PST 08 : *I used to like algebra.*

Researcher : Algebra, and liked the least?

PST 08 : *It had to be ok let say geometry.*

Researcher : Now that you are about to go out and teach, which section do like the most?

PST 08 : *I think it still algebra, yes.*

Researcher : Would you teach geometry in different way now?

PST 08 : *Yes, I would certainly try, yes, ja I would teach it differently.*

Interview transcription

Individual interview

Section B: Knowledge of proof in Geometry

PST 09

Researcher : In your own opinion, which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the most by learners?

PST 09 : *Well, I think they enjoy algebra*

Researcher : Why do you say so?

PST 09 : *Because if you just ask a learner can you do something in maths or can you show me what did you do at school in mathematics, taking grade 11 learner who has done geometry, trigonometry and algebra, that learner can just take any ... in most cases I have tried this, they just do x plus something and then solve x , they just enjoy solving for x and if you are talking about maths they would just say solving for x , so ehh solving for x in a context of equation not may be x representing an angle or something, so due to that I can just say they enjoy algebra.*

Researcher : Which section (Algebra, geometry and Trigonometry) in mathematics do you think is enjoyed the least by learners?

PST 09 : *Jah, well it geometry*

Researcher : Why do you say so?

PST 09 : *Because, it involve lots of logical reasoning and proofs especially proofs, learners don't like proving, you know if you are solving just an algebraic equation, may be $x + 1 = 0$ saying $x = -1$ is an answer, they enjoy that but they can not just take that one and replace it or substitute it on the equation where x was and see if they can get the zero. They are not interested in proving their mathematics what they just enjoy is achieving an answer with no care whether the answer is correct or wrong.*

Researcher : What do you remember about geometry you learnt in high school?

PST 09 : *Well, my high school geometry is terrible, I must say that, ehh... the last time I did geometry was in grade 9, and unfortunately we didn't have maths teacher up to June in my grade and maths teacher only came after June and one thing he insisted in geometry was reasoning. He said geometry*

requires a reasoning, that what I took from him. And then the other grades we were not doing any geometry, the geometry I know is the geometry I taught myself. Because I'm also a physics training teacher, I'm training to teach maths and physics. So like if you are teaching vectors you have to know geometry, for example if you are using parallelogram method you use co-interior angles that sum of the co-interior of a parallelogram is 180° then you use such theorems. So then I did geometry on my own.

Researcher : Can you explain how proof in Geometry was taught at your high school?

PST 09 : *No, we didn't prove anything in geometry. What I only remember is the sum of angles of a triangle is 180° and we didn't prove that, he just taught us it like that. So, and even for high school textbook, like I said I am teaching myself all this geometry and all the stuff in geometry. The textbook even that I used in high school geometry they don't do proofs in most cases what they do, they just give theorems and say theorem number 1, theorem number 2, very few of them are doing the proof of the theorem, which means even the writers don't pass this logic in geometry and this meaning of geometry, and this application of geometry to real world to form their knowledge to learn us.*

Researcher : Based on Task base questions, what can you tell about your solutions? Why did you answer the way you did? (colour/symbols/constructions)

PST 09 : *Well, ehh... basically the main reason for doing that is when you try to prove something, it is important that you don't assume what you have to prove, so you have to use something to prove what you have, like for example in laboratory if you have to prove the existence of starch you can not just say..just look at this it here, but you just add a certain solution and then you look at the changes so geometry works the same one, if you have to prove ehh... something then try and see that, what is it that you know, from the diagram that can help you to prove the information that is given. I can not visualise what I don't know so I must see what I know first so that I can move forward.*

Researcher : Now let us just check one problem here, not that I'm interrogating you but I just want to find out why do you do that, now you are given here, this diagram you are given a diagram and its also stated that it is not drawn to scale, right and ehh. You are supposed to prove that $\hat{E}_1 = \hat{F}_1$ and you said $\hat{E}_1 + \hat{E}_2 = 180^\circ$ and $\hat{F}_1 + \hat{F}_2 = 180^\circ$ then thereafter you simplified as $\hat{E}_2 = 180^\circ - \hat{E}_1$ and $\hat{F}_2 = 180^\circ - \hat{F}_1$ and then thereafter you stated clearly that $\hat{E}_2 = \hat{F}_2$, so right and you made some conclusions at the end, based on what you have done that $\hat{E}_1 = \hat{F}_1$. Why?

PST 09 : *Well, by looking at what was given, ehh I was given that $\hat{E}_2 = \hat{F}_2$ and I know these angles are lying on the straight line, these just one line separating them. You have an angle and an angle the sum of two angles is forming a straight line which means is 180° , if you add the unknown angle with the known angle. Now this appears, to help me to see that the unknown angle can be represented by what is known and then doing the same thing with the other angle representing what unknown by what is known. But if I look at this I subtract the two equal angles from 180° and then to get the two angles that are unknowns and that means to me that if I subtract the same number here and same number here obviously the two that I don't know there must be equal. So the manipulation of these equations to write in this form helps me to visualise what I don't know, how can I represent it, in case I'm given something if I was given maybe the value for $\hat{E}_2$ or $\hat{F}_2$ then I was going to see ohh! This is how I can find the answer.*

Researcher : Do you think you have gained any deeper or new knowledge now that you have studying at the University for 3 or more years?

PST 09 : *Well, there is some knowledge I have gained especially eh teaching methods, because teaching is not just a profession that you can just have your content and go focus on it, you have to also focus on how the human minds work, how students behave and do you they work or they make their senses. Well ehh, even though there are some of the things that I personally I'm not so satisfied ehh particularly about the university curriculum. Ehh however, there is some experience that I've got just listening and getting some information from the professors and all those professional guys. Well ja, it does make sense you know they just change my understanding of mathematics because I think this is normal I'm teaching on Sundays well just for my personal experience and what I've noticed normally to learners, as you ask them what is mathematics, they can not give the proper meaning of mathematics and you can even hear them saying e yi.. I only like mathematics because maths is numbers I just add numbers subtract numbers and last all. I hate other subjects because they are just a collection of notes so you know there is this thing which I think is a misconception that mathematics is just a set of numbers. Mathematics is not numbers even though we use numbers in mathematics. So ja they have just brought meaning and understanding of what mathematics is and you know all this contradictions of culture and mathematics like saying mathematicians are against human religion and culture all those things, ja through videos and some extra classes, there is a lot I have learnt.*

Researcher : Which strategies do you think you would use when teaching Geometry to your learners?

PST 09 : *Well, it is not an easy question for me. But ehh... when I am thinking carefully I think it depends on the context or the content or the proof that you want to teach. Some of the proof can be done by hands you can let the*

learners do actual geometry by hands like paper folding, cutting and all those things or even doing actual measurement. So it depends on what you want to teach at that time, you can use experiments, you can use theory sometimes or just integrate what they know, just from nature or home.

Researcher : Why do you think that these strategies would work?

PST 09 : *I think they will work because, one I assume that there are... there is something that is in the learners mind about those strategies like if I say we use information or integrating from nature or from home experience, I mean learners already know what... how to solve something or how to do something like that and that at home then if I apply that in geometry they can see ok this is what we do at home and this how it is applied in mathematics then they can think they can make links very easily and using the constructions and experiment cutting and paper folding I think that can help them to be convinced and see that the proof is actually true before you even start proving it using ehh mathematical representation.*

Researcher : Do you think that these teaching strategies would make the teaching and learning of geometry better for your learners?

PST 09 : *Well, I think ja, because one thing I've noticed even if we are doing geometry like with other students, when comes to proving theorems and proving some, ja theorem I'm right, most of the learners don't like proving they just want to apply what is already there but if you try and say you know the paper, you know how to use the protractor, and measurement now do this and this and this. If they actually see the proof in front of them without using any mathematics I think it can help them visualise the actual concept behind ehh just cutting paper folding and all those strategies.*

Researcher : Are these teaching strategies similar or dissimilar to the way that you were taught?

PST 09 : *They are completely different*