CHAPTER ONE

DESCRIPTION OF CHAPTERS TWO TO SEVEN

1.1 Introduction

This chapter presents a descriptive overview of the following seven chapters. The chapters are numbered from chapter two to chapter seven. A description of each chapter is presented. Further, the purpose, and each chapter's contribution to the research project is considered.

1.2 Chapter Two: Introduction and overview

Chapter two presents a summary of the study topic and the participants' background information. The geographical location of the study is also detailed.

This is followed by the motivation for the research project. The researcher detailed issues concerning the teaching of mathematics in South Africa. These issues include: the recurrence of poor performance on the part of the National Senior Certificate (NSC) learners in their Mathematics examination, of which aspects of Euclidean Geometry were a contributing factor (especially with respect to the congruency of triangles). The researcher referred mostly to South African literature to address other issues and studies that were previously conducted on the learning of mathematics. The introduction of Transformation Geometry in grade ten was then mentioned and the researcher declared an interest in conducting a study to determine if, in order to better understand Euclidean Geometry, learners could study congruent triangles successfully in this section. The researcher raised concern about the high failure rate in Mathematics at NSC level in the country particularly with regard to its impact on governmental initiatives, which tend to rely upon the successful application of learned mathematics skills. Comparative studies, like Trend International Mathematics and Science Study (TIMSS) studies were mentioned, and the researcher noted that South Africa performed the poorest Howie (1999). Transformation Geometry was used as an aid to improve learners' understanding of Euclidean Geometry concepts.

The researcher drew up personal interest and experience with regard to mathematics teaching and learning. With reference to the researcher's personal teaching experience and information from cluster district colleagues, it was noted that learners did not perform well in Euclidean Geometry. Including the researcher's respective assessment strategies, comparisons were made between old teaching strategies and current teaching strategies, citing the advantages of each set. By making reference to the current education system (the National Curriculum Statement (NCS) under Outcomes-Based Education (OBE)), the researcher also considered what the Department of Education prescribes as the 'fashionable' form of teaching mathematics. Assessment strategies were also concurrently compared in the study.

Lastly, the research question is outlined, followed by the sub key questions. This is followed by a discussion of the data collection instruments required to assist in obtaining reliable findings. The researcher outlines the significant implications of this study. That is if study reveals that learners understand congruent triangles better in Transformation Geometry than in Euclidean Geometry the, it would be recommended that teachers integrate the two sections in their teaching. Limits of the research are also discussed along with their implications.

1.3 Chapter Three: Related Literature and Theoretical framework for the research

In this chapter, the researcher explains the selection of constructivism as a theoretical framework upon which the study would be based. Constructivism as a learning theory related to the current education system (OBE) is elucidated. The research also addresses the learning of congruent triangles.

The researcher outlines the role of the learner, the instructor, as well as the nature of the learning process as three main role players in the theory of constructivism for the purposes of this study. Relevant local and international literature was quoted to support the statements made. The researcher also quotes select pieces of literature to support criticisms of constructivism (both local and international).

The researcher then focuses on learning within the South African context in general. Relating this learning to the theory of constructivism, the researcher clarified whether what is occurring may actually be classified as being constructivism. The study referred to rote learning, a practice no longer perceived to occur in the country's OBE system. The researcher also referred to literature that demonstrates how some schools still mix constructivism with rote learning.

Both local and international researches in the teaching of Euclidean Geometry were discussed. Whilst acknowledging successful studies in the teaching of Euclidean Geometry, the researcher raised concerns that despite a paucity of research on Transformation Geometry in South Africa, the section had nevertheless been introduced in that country. The study also acknowledged the contribution of international authors' writing on Transformation Geometry.

The researcher discusses proofs and proving Euclidean Geometry riders in high schools. Citing relevant literature (particularly that which is locally published), mention was made of the poor performance of learners in tasks that involve proofs or proving. In addition, the study acknowledged the research findings on proofs in Euclidean Geometry. In many cases, constructivism still does not appear to be practised during the teaching of proofs of theorems and riders. Different views on why learners fail to construct proofs in Euclidean Geometry were discussed, as well as how authors defined proofs.

The researcher then presents views on how Euclidean Geometry should be learnt in order to ensure success. The study also made mention of instrumental and relational understanding, as defined by Skemp (1976), and related these understandings to the current education system.

Skemp (1976) also mentioned how congruent triangles could be taught in relation to the current education system. The role of teachers in the learning of Euclidean Geometry was also outlined.

Finally, the researcher looks at the mathematics classroom in South Africa and referred to the relationship between teachers and learners in traditional and contemporary classrooms.

1.4 Chapter Four: Research methodology

This chapter outlines the methodology employed during the research process. Firstly, the study named the qualitative paradigm as the preferred research method, thus using questionnaires and semi-structured interviews as the qualitative method of data collection. The details of these methods were then discussed. The researcher outlined the motivation for selecting these research methods in relation to the study of congruent triangles in Transformation Geometry in grade ten.

The researcher goes on to discuss the selected sampling techniques of the participants, clearly explaining the reasoning behind each choice. Relevant literature was quoted for this section of the chapter and the grouping of participants was outlined.

The researcher then mentions that learners are always participants of informal research, as teachers collect different kinds of data from the learners as part of the teachers' routine duties at school. This includes learning behaviour. The researcher describes the process followed before conducting the study and showed the detailed research questionnaire and the semi-structured interview questions. The researcher also outlines the conceptual framework used in setting the questionnaire design of questionnaires one, two and the semi-structured interview questions. Before the questions were shown the motivating conceptual framework was discussed.

1.5 Chapter Five: Results and analysis of questionnaires

The researcher presents the results and the analysis of written responses to the questionnaire questions, and the contextual information in terms of mathematics classes of the participated schools is shared.

The coding procedure of the responses in each set of questionnaires is discussed. The detailed responses for each questionnaire as well as the mean performance of the schools in each task are represented in the form of tables. After the appearance of each table, a discussion of the results it contained was presented. Scans of the responses of particular learners in each school are represented so as to clarify what is reported by the researcher. Interviews with particular learners were also included to verify the information on each table where necessary. Each school's responses were analysed thoroughly and compared to other school's responses.

1.6 Chapter Six: Results and analysis of semi-structured interview responses

In this chapter the researcher presents the results of the semi-structured interviews. As in the previous chapter, tables were used to display the quantity of learners who responded in a particular way. The aim of this data collection was to triangulate the data obtained from written responses. Each school learners' responses were analysed thoroughly in relation to their responses in written tasks.

1.7 Chapter Seven: conclusion and recommendations

In this chapter the researcher discusses concluding remarks and provides relevant recommendations on instructional matters surrounding congruent triangles, Transformation and Euclidean Geometry. The key findings of the research were outlined and noted together with possible implications. Finally, the researcher outlines the responses to the key research question of whether learners could benefit from learning congruent triangles in Transformation Geometry when studying Euclidean Geometry.

1.8 Conclusion

A brief description of the chapters of this study was presented in this chapter. The details and contents of each chapter were outlined.

CHAPTER TWO

INTRODUCTION AND OVERVIEW

2.1 Introduction

This comparative study sought to explore learners' conceptual understanding of congruent triangles, as learnt in Euclidean Geometry, versus learners' understanding of congruent triangles learnt in Transformation Geometry. The main aim of the study was to determine if Grade Ten learners understand the concept of congruency in Transformation Geometry. The concept of the congruency of triangles was introduced in Euclidean Geometry in grade nine. The participants in this study were Grade Ten mathematics learners from the Ugu district in KwaZulu-Natal. This chapter provides an outline of the study through a discussion of the motivations for this inquiry. In addition, this chapter also explains the reasoning for the use of quantitative and qualitative techniques as the research methodologies.

2.2 Motivation to undertake the study

From the personal teaching experiences of the researcher, it would appear that most learners performed poorly in Euclidean Geometry. Many learners seemed to have difficulty understanding proofs involving congruency and the concepts of congruent triangles required in the proofs of many geometry riders. Indeed, educators often blamed learners for failing to understand these proofs and concepts whilst learners blamed "boring" teaching strategies. Informal discussions between the researcher and some Mathematics educators revealed they were pleased with the shift of all Euclidean Geometry to the optional third paper in the National Curriculum Statement (NCS). However, as the third paper may become compulsory in the coming years, more research is needed in order to create effective instructional strategies to show the overlap between Euclidean Geometry and Transformation Geometry.

Euclidean Geometry studies have been conducted on issues of learning and instructional strategies (Brijlall, Maharaj & Jojo, 2006; de Villiers, 2008; Lauf, 2004; Mthembu, 2007; Mudaly, 2007). However, studies that relate Transformation Geometry and Euclidean Geometry are rare in South Africa, probably because the former was only recently introduced by the NCS (in 2006). Due to the persistent poor performance of senior certificate learners in Euclidean Geometry (KZN DoE, 2004; Mudaly, 2007), this study found it necessary to look for better instructional activities to foster successful learning. In order to find more effective instructional techniques, this study specifically explored how learners learnt the congruency of triangles in Transformation Geometry.

This investigation integrated congruent triangles into Transformation Geometry in order to assist learners in developing a better understanding when attempting tasks related to the congruency of triangles in Euclidean Geometry. This was one means of facilitating a concerted effort to improve the teaching and learning of Mathematics. Having been a mathematics teacher in various schools, the researcher has witnessed Mathematics educators themselves struggling with the solving of riders requiring the knowledge of congruent triangles with the result that they teach theorems as they appear in the textbook. This practice was criticised as it was felt that doing so did not improve understanding in the learners – but rather that it encouraged rote learning (Luthuli, 1996; Mudaly, 2007). In his study Mudaly (2007) affirmed that most learners were poorly prepared for the senior certificate examinations, and that they possessed inadequate knowledge of Euclidean Geometry.

Underachievement in mathematics acquisition contributed to a setback in government programmes that require mathematical skills acquisition, for example the Joint Initiative Programme of Skills Acquisition (DoE, 2003). The government (through the Department of Education) is continuously budgeting for the improvement of the teaching and learning of mathematics and science, DoE (2003)**.** Research revealed that, despite the newly recommended teaching improvement strategies, *e.g.* teaching using real-life examples, most South African learners did not do well in Mathematics (Bansilal, 2008). South Africa was placed at the bottom of the achievement list of thirty-eight countries in Africa in the TIMSS, (Howie, 1999).

In secondary schools, learners were found to be unable to identify geometry shapes like the rhombus, trapezium, kite and parallelogram in their formal mathematics lessons (Triadafillids, 1995). However, Brijlall *et.al.* (2006) found that learners have intuitive or in-born knowledge of different geometric shapes when they are engaged in their Technology lessons. Brijlall *et.al.* (2006) recommended that practical work, whereby learners identify or design basic geometric shapes, should be the starting point in the teaching of Euclidean Geometry. This recommendation prompted this study in the use of informal deductions to steer rigour in proof. In addition, it would appear that some Mathematics teachers were hasty to complete the syllabus and therefore did not spend sufficient time on the usage of informal deductions. This situation prompted the researcher to ascertain the feasibility of facilitating learners' conceptual knowledge of congruent triangles whilst the learners were engaged in Transformation Geometry.

As the head of department of mathematics at school, the researcher has observed that some teachers teach the solving of geometry problems using proof techniques found in the Mathematics textbook. These techniques primarily look at proving geometry riders in a stereotyped manner. The researcher observed (in school and in marking centres) that most learners have difficulty in solving proof-type problems of geometry. This was possibly due to the aforementioned teachers' habit of using the restricted problem solving strategies found in the Mathematics textbook. Mathematics teaching journals reveal many methods of teaching Euclidean Geometry (Mudaly, 2007). In Transformation Geometry, proof is about explanation, systematisation or discovery using visual examples that are done practically (De Villiers, 1990). In this regard, the researcher designed tasks that enabled learners to solve geometry problems using formal and informal reasoning.

History reveals that Mathematics instructional strategies started with arguments on the use of concrete (instead of abstract) examples on the use of patterns to discover laws, and to date the emphasis has been on the use of real-life or authentic examples (Bansilal, 2008). This motivated the researcher to demonstrate first informal proofs in Transformation Geometry, and then the more formal proofs, as required by Euclidean Geometry, to learners.

The NCS placed emphasis on Mathematics teaching that was "learner-centred" instead of traditionally "teacher-centred" approaches. As the new learner-centred approaches were time-consuming, many Mathematics teachers (aware of the time limit for syllabus coverage) were found to be still adopting teacher-centred approaches (Mthembu, 2007). Although understanding in mathematics is essential and crucial for success in Euclidean Geometry these traditional approaches adopted by teachers promoted memorisation and rote learning (Mudaly, 2007).

In an attempt to work towards promoting learning geometry with understanding, the researcher decided to investigate whether learners could understand congruent triangles using Transformation Geometry. It was envisaged that once the application of translation, reflection and rotation were found to be effective in learning congruency, both Mathematics educators and Mathematics textbook writers would be able to design instructional strategies of Euclidean Geometry using ideas found in Transformation Geometry. Gradually, concepts like similarity and properties of geometrical shapes (rhombus; kite; parallelogram and square) would be incorporated holistically in Transformation Geometry.

Informal discussions with Mathematics teachers during cluster workshops revealed that the traditional teacher-centred approach still dominates in Mathematics classrooms as some educators are not fully competent with the leaner-centred approaches. In addition, some schools are under-resourced according to my observation in my district.

2.3 Personal experience and interest

Due to an interest in Mathematics education and the issues which surrounded it, the researcher has taught Mathematics both at the General Education and Training (GET) and at Further Education and Training (FET) phases. The researcher has observed that many Mathematics learners are not eager to learn Euclidean Geometry. Some learners believed Mathematics to be about the manipulation of numbers at senior primary phase, and generalised to algebra at the secondary phase. Some Mathematics teachers gave a greater amount of homework in algebra as compared to Euclidean Geometry.

Mthembu (2007) feels that many teachers are less content with the fact that Mathematics involves spatial representation and language in the development and communication of Mathematical ideas essential in the teaching of geometry. Mthembu (2007) argued that the successful solving of geometry riders depends on learners’ ability to visualise the problem situation rather than their knowledge of the proofs of theorems. With transformations like translation, reflection and rotation on a Cartesian plane the researcher attempted to see if the idea of visualising the movement of triangles could be understood. If this idea works effectively, formal proofs in Euclidean Geometry may well be easier to understand.

Transformation Geometry involves mainly the practical location of new images after the original figure has been moved through translation, reflection or rotation. The introduction of this section in the NCS syllabus reminded the researcher of the "image figures" the researcher used to draw in Technical Drawing lectures at College. For example, a hexagonal shape was provided from which a new figure (as seen from the rear or front view) was required to be drawn. This is similar to the reflection of the hexagon in the x- or y- axis in Transformation Geometry. The researcher used to draw the figure first and then pull it to the desired plane using light lines to produce the identical figure. Non-congruent triangles could also be easily identified without mathematical proof where the figure appeared to be larger than, or even different from, the original. This could result from enlargement of shapes in Transformation Geometry and is a possibility for future research.

Learning Euclidean Geometry shapes through construction or observation in a non-mathematics lesson may help improve the understanding of such shapes in mathematics lessons (Brijlall *et.al.*, 2006; Mthembu, 2007). Research revealed many Mathematics teachers teach Euclidean Geometry proofs of theorems using direct textbook methods (De Villiers, 2008; Mudaly, 2007). A probable reason for this may have been that these Mathematics teachers were not comfortable with teaching Euclidean Geometry; further, the researcher doubted whether these teachers possess adequate content knowledge of the subject.

The pace at which some learners understand mathematical concepts was not the same as their colleagues; a few gifted learners were able to understand geometry directly from classroom lessons. When these gifted learners sometimes seemed to stop attempting to learn geometry, I found that teachers consequently referred to them as being "unable to understand", when this was not actually the case.

The education system has changed from one in which assessment was not based on formative assessment to the one in which assessment is based on regular feedback to the learners. The past education system left teachers with no option but to continue the syllabus at the pace of the faster learners in order to complete the syllabus for examination purposes. Thus, difficult geometry problems were not adequately taught to learners and teachers used ineffective instructional strategies. It appeared that as long as some learners understood the lesson, a common behaviour among many Mathematics teachers was to feel some sense of success and not concern themselves with attempts to increase the level of understanding amongst additional learners. Therefore, the challenge facing researchers studying Mathematics teaching is to find the most effective instructional strategies of Euclidean Geometry that will benefit the most learners in the Mathematics classroom.

2.4 Current teaching strategies

The current education system in South Africa emphasises that there should be a move towards learner-centred approaches with many examples of real-life situations being included in instructional strategies, (DoE, 2003). The use of formative kinds of assessment is also prescribed and the triangular relationship among the parent, learner and the educator was strengthened. However, some confusion seemed to have developed with this new education system from a teaching and assessment perspective. For example, Ntenza (2004) found that assessment on mathematical writing tasks *i.e.* essays, projects and assignments were not done properly in some schools. Mthembu (2007) found that some Mathematics teachers still follow the traditional methods of teaching based on summative assessment. The explanation teachers put forward for these findings is that there appears to be little difference between the traditional and new methods (in terms of teaching and assessment) as presented in the Mathematics textbooks. The researcher has personally observed that the types of questions found in the new external senior certificate examination do not differ significantly from the types of questions found in the old examination. As both time-bound examinations and timed-bound scopes were evident in current assessment practices, it may indeed be justifiable that some teachers saw no need to change to current instructional strategies.

As the Head of Department of Mathematics at a school, the researcher regularly finds the need to intercede when teachers apply the “drill” method of Mathematics teaching. As Bazzini & Inchley (2002) noted, Mathematics teachers complained that the need to meet the external assessment programme meant they were unable to change to time-consuming learner-centred strategies. For example, cutting and matching congruent triangular pieces of paper was not preferred to the traditional “chalk and talk” method in the teaching of the four cases of congruency.

Researchers studying Mathematics education were often faced with the challenge of determining whether or not new learner-centred approaches improve Mathematics attainment. In particular, Boaler (2002) warned of the misconception that the use of real-life examples in mathematics teaching created more understanding. For example, Mathematics teachers had achieved high pass rates long before these methods were introduced; these teachers would probably see no reason to change the style of teaching to the new approaches.

One feature of the Outcomes-Based Education (OBE) was that it was activity-based where learners actively participated in the presentation of a lesson; not only by doing class tasks but also by working out first-time proofs of theorems, DoE (2003). However, Mathematics teaching primarily consisted of the procedures and rules necessary for attaining to the correct answer. It was thus especially important that the teacher guided learners whilst proving a theorem. This increased need for attention and interaction between teacher and learner made it difficult for a Mathematics teacher to leave learners to work out the answer on their own with minimum facilitation.

Mthembu (2007) noted that some Mathematics teachers believed learners' participation required learners to talk amongst themselves when solving a Mathematics problem; the teacher only became involved when the students were unable to find the solution. It is worth noting that, in this method, no new information came from the learners, and instead they talk to one another about what they have already been taught. This was not what OBE was intended for. OBE emphasises that learners must be taught to be critical and creative thinkers (DoE, 2003). The researcher therefore believed there to be some degree of uncertainty about the implementation of OBE teaching strategies.

2.5 Focus of the study and research questions

The focus of the study was on the learners' conceptual understanding of congruent triangles as they appeared in Transformation Geometry using translations, reflections and rotations.

The research question guiding the research project was:

How well do grade ten learners understand congruency of triangles when working with their transformations as compared to formal reasoning?

To address this fundamental research query, the following questions were asked:

(a) How well do grade ten learners understand the four cases of congruency?

(b) To what extent do learners succeed in their conceptual understanding of congruency in Euclidean Geometry? (formal reasoning)

(c) To what extent do learners succeed in their conceptual understanding of congruency in Transformation Geometry? (informal reasoning)

(d) How does informal reasoning make a difference in the conceptual understanding of congruency in geometry?

In order to unpack these questions, the design of the data capture instruments considered:

For (a) the semi-structured interview questions
For (b) the questionnaire task 1 and verification interview
For (c) the questionnaire task 2 and verification interview
For (d) the semi-structured interview questions

2.6 The significance of the research

The research conducted in this study was important in the South African context of educational transformation. Euclidean Geometry learning was still difficult for many learners. The study provided knowledge of geometry teaching at the basic grade in the FET band at grade ten. If successful, it may provide a significant instructional link between Transformation and Euclidean Geometry in grade eleven and twelve. This study would also add to the literature of geometry teaching. As Transformation Geometry was previously not a part of the Mathematics syllabus, this study adds to the scarce literature

This study provided resource material for other researchers in the field of Mathematics education and supplemented those seeking effective approaches in geometry teaching through the integration of common ideas from Euclidean Geometry into Transformation Geometry.

This study thus intends enabling mathematics teachers to realise the advantages of the introduction of Transformation Geometry, and thereby aided in Mathematics curriculum transformation.

2.7 Constraints and issues of this research

Although this study was based on a very small part of Euclidean Geometry, it nevertheless attempted to facilitate the improvement in teaching of the entire section of Euclidean Geometry. However, the researcher hoped that some useful information will be obtained regarding the teaching of basic FET geometry; especially shapes which include triangles. In addition, the study will encourage other aspects in Euclidean Geometry learning to be achieved through the implementation of ideas contained in Transformation Geometry.
This study created a platform for further research. Some possibilities include:

(a) Similar triangles learnt through Transformation Geometry.
(b) Quadrilaterals in Transformation Geometry.
(c) Cyclic quadrilaterals in Transformation Geometry.

For the research project to obtain as much data as possible, it was necessary for the researcher to:

- Create a positive environment between me and my participants.
- Act as mathematics educator who is there to improve their attainment in geometry.
- Neutralise tension experienced by learners in the normal mathematics class.

The questionnaires given to participants took the form of a class exercise. This may have created the impression that the participants were being tested in the classroom setting, thus potentially creating a positive environment and neutralising tension.

In addition, at all times the researcher attempted to ensure that the participants were aware of the right to withdraw at any stage or not to answer the worksheet was maintained. Thus the researcher did not personally allocate marks for the questions. Instead, the researcher assured the participants that the subjects discussed would remain confidential.

Lastly, the researcher explained the details of consent letter.

2.8 Conclusion

The motivation of undertaking the study has been discussed. All the relevant aspects were outlined in this chapter. The next chapter looks at the related literature and the theoretical framework for the study.

CHAPTER THREE

RELATED LITERATURE AND THEORETICAL FRAMEWORK FOR THE RESEARCH

3.1 Introduction

This chapter considers arguments ranging from the learning of Mathematics to the learning of geometry at the Further Education and Training (FET) level. National and international views of geometry teaching were discussed, credited or discredited and reasons given for their failure or success. It was important for the researcher to consider literature on Euclidean Geometry teaching and learning issues because congruent triangles form the basis of most riders.

3.2 Constructivism

Constructivism is a learning theory explaining how learning happens, particularly within a classroom context. According to this theory, learners construct knowledge out of their experiences (Piaget, 1950). Constructivists like Piaget (1950) and Cobb (1994) associated constructivism with instructional approaches that promote active learning. This attribute also features in Outcomes Based Education (OBE) that emerged along with the National Curriculum Statement (NCS) (DoE, 2003). It is interesting to note that these active learning approaches, long argued for in Western countries, were implemented at the beginning of democracy in South Africa. According to the theory of constructivism, a learner may engage and argue with the teacher about content being taught, whereas in the previous traditional methods of teaching and learning, learners were expected to be passive recipients of knowledge presented to them by the teacher.

3.2.1 The nature of the learner

Werstsch (1997) argues that, according to constructivism, learners are unique and complex individuals, whilst at the same time forming an integral part of the learning process. According to OBE, each individual learner's solution of a mathematics problem is to be considered with the purpose of determining the arguments put forward. Learning emerged from classmates as well as from the teacher. In addition, Wertsch (1997) stresses the importance of the social background and culture within which learners develop. Social background and culture shape learners' knowledge; in other words, as they interact with more knowledgeable people, their thinking abilities continue to improve.

Learners accommodate new knowledge, either by replacing existing knowledge, or by incorporating new experiences into pre-existing understandings (Piaget, 1950). All these processes take place whilst learners try to construct their own understandings of what they are exposed to (von Glasersfeld, 1984). In many Mathematics classrooms, constructivism tends to occur partially or is compromised. The researcher has personally observed teachers standing in front of a class, demonstrating the solution to a problem using 'question and answer' techniques. The teachers seemed to believe that learners were participating actively as they gave correct answers in a chorus. The learners themselves seemed to enjoy this method, especially upon having been given correct answers all the way through to the solution of the problem. It has been argued that the successful completion of a challenging task results in confidence and motivation to learn more (Vygotsky, 1978).

3.2.2 The role of the instructor

The success of constructivist learning tends to depend on the approaches that are taken by respective education systems. The reason given by teachers, who used to attend information sharing cluster meetings in Ugu district, for the failure of OBE implementation in some schools is the lack of efficient and effective training in preparation for the OBE approach. Hence, teachers generally resort to a mixture of traditional and OBE approaches to fulfil their teaching duties. According to the theory of constructivism, the teacher 'reveals' learning to learners, lends support and provides guidelines, as well as creates an environment for learners that allows them to arrive at their own conclusions (Rhodes & Bellamy, 1999). Rhodes and Bellamy (1999) further argue that, in order to steer learning experiences toward what learners want, as well as to create value, teachers should be able to adapt learning experiences using their own sense of initiative.

3.2.3 The nature of the learning process

In constructivism, knowledge acquisition is first constructed in a social context and is then appropriated by individuals (Brunig, Schraw & Ronning, 1999; Cole, 1991; Eggan & Kauchak, 2004; Vygotsky, 1978). Group work learning, according to Van Meter & Stevens (2000), is the foundation of constructivism as it is through group work learning that learners construct understanding collaboratively, which Greeno, Collins and Resnick (1996) believe would be impossible if each learner was working on his/her own. In many Mathematics classrooms where the researcher observed group work being done, learners were given problems to solve. However, these groups were not adequately facilitated and were dominated by 'fast' learners. There is no means of negotiation towards a solution. Generally, Mathematics teachers seem to be impressed when the group presents the correct solution, irrespective of how it is found. It is clear to the researcher that the concept of group work learning may very well be viewed in different ways. Many South African studies (*e.g.* Brijlall & Maharaj, 2009) explored collaborative learning in higher education. These studies found that mathematics learning within a constructivist paradigm was effective. Constructivists like Duffy & Jonassen (1992) and Vygotsky (1978) believe that group work learning necessarily involves learners. In this context, they motivate towards the different possibilities of determining answers, where all views are equally considered. The OBE approach also shares a similar idea, but Mathematics teachers are sometimes tempted to overlook these other principles, allowing 'fast' learners to dominate lessons.

3.2.4 Criticism of educational constructivism

In theory, constructivism may initially appear to be sound, yet it might not work in practical terms. Similar approaches, like OBE in South Africa, were largely touted to be minimally successful and at best, misinterpreted. Seemingly, there are complications embedded in constructivist theory. Researchers like Kirschner, Sweller and Clark (2006) criticised the OBE approach even before it was fully implemented in South African schools. They perceive the constructivist approach to be unguided and, as a result, lacking in direction for learning. Apparently, by virtue of the OBE approach, Mathematics examination results have been poorer in the National Senior Certificate examinations in comparison to those of the past traditional examinations.

International critics tend to question the effectiveness of constructivism towards instructional design, pointing out the lack of substantive empirical evidence of its success in teaching and learning (Kirschner *et.al.* 2006; Mayer, 2004). The problem with constructivist-based approaches like OBE is that whilst they appear to be ideal in theory, they are ill-implemented.

3.3 Learning in the South African context

In the past, learning was mainly perceived as being the transfer of knowledge from the teachers' mind to that of the learners', in the form of "teacher-tell" methods, Adler (1991). This form of teaching corresponds with the apartheid legacy under which the country was then ruled. It was under the apartheid regime that authoritarian systems of leadership and management in classrooms appeared. Authoritarian systems of teaching were dominant in Mathematics classrooms, especially in the teaching of proofs of Euclidean Geometry. This situation led to an increase in the level of difficulty for the learners to understand as they could not challenge the teacher. Adler (1991) noted that many Mathematics educators were only one step ahead of their learners, particularly in indigent schools with predominantly black learners. In addition to this, the researcher has personally observed that some Mathematics educators had poor Mathematics results on their own senior certificates. This means that they were not rich in Mathematical content knowledge. For this reason, Adler (1992) believes the "transfer" manner of teaching and learning to be the most appropriate as learners were not encouraged to ask or challenge the educator. The idea of not questioning the decisions of the leader was one of the foundations of the apartheid regime.

The change in government in the past fifteen years led to the implementation of teaching strategies which discourage rote learning. Indeed, the traditional methods of teaching which resulted in rote learning had already been criticised several years before the end of apartheid (Luthuli, 1996; Volmink, 1990). Volmink (1990) asserts that Mathematics learners had been made (by the previous education system) to believe that their own experiences, concerns and curiosity had no significant relation to Mathematics learning. The researcher concurs because, as personal experience of having been a Mathematics learner indicates, Mathematics lessons appeared to have no link with mathematical activities that were performed outside of school by learners. The new post-apartheid government began the process of transformation with respect to all governmental departments, including that of education. Transformation was focussed along democratic lines and adhered to “Batho Pele” principles. “Batho Pele” principles mean “peoples first” and they suggest that for every decision-making the leader needs to allow for inputs of those that are led. Management and leadership within schools, as well as the process of teaching and learning in classrooms, were also transformed accordingly.

OBE came to the fore as the new education system in South Africa, making use of the constructivist theory of learning. Among others, its key features include allowing for the participation of learners in “meaning-making” during lessons. It also encourages formative assessment approaches and the use of “real-life” or authentic examples in teaching. It has been argued that successful transformation of the country depends on an education system that promotes democracy (DoE, 2003; Taylor, 1991). According to the constructivist theory of learning, learners develop their own understanding that may even be different to what the educator intends (Confrey, 1990; Hiebert & Carpenter, 1992). Through negotiation among learners themselves, and between learners and the teacher, consensus on the correct meaning of terms or activities could be reached.

Learners would be able to prove themselves whether correct or not. This is one attempt in the creation of a learner who is creative, able to think critically, as well as able to make critical decisions (DoE, 2003). Clearly then, the task of the Mathematics teacher in OBE should not necessarily be to recite ‘facts’ and ‘rules’, but instead, to create situations that may enable learning (Mthembu, 2007). A pertinent issue facing Mathematics teachers is the extent of time they should allocate to learners for discussing solutions to Mathematics problems, even though it may be clear that learners might be progressing toward incorrect answers. As discussed before, this is one of the reasons why many Mathematics teachers continue to teach using the old “chalk and talk” method.

3.4 Research in Euclidean Geometry teaching

Research studies that were reviewed recommend changes in the teaching of geometry in order to improve understanding **(**Brijlall, Maharaj, & Jojo, 2006; De Villiers, 2008; Mudaly, 2007). Brijlall *et.al.* (2006) looked at how learners' experiences in a Technology class could be used to inform the effective teaching of geometry. They recommended that Euclidean Geometry teachers build upon learners' intuitive or in-born knowledge of geometrical shapes (like triangles) in their teaching. This would facilitate the understanding of van Hiele's levels one and two: the analysis and naming of shapes respectively (van Hiele, 1999). The Department of Education (DoE) (2003) concurs with this argument, recommending that teaching should be based on authentic or "real-life" examples. The researchers' personal experience in Mathematics teaching seems to show that learners enjoy drawing different patterns that they observed elsewhere (*i.e.* external to the classroom). Geometry instruction can be effective if these ideas are practised in group-work activities, with the teacher guiding the way to the more formal geometry riders. Vygosky (1974) defines the zone of proximal development as the gap between what is already known by the learner and what should be achieved through adult educator guidance.

It should be pointed out that Mathematics teachers need to master Euclidean Geometry knowledge before they can engage learners' learning through a "non-textbook" method. Adler (1991) argues that mathematics teachers need to have a conceptual understanding of the subject. In this way they can approach teaching through different Mathematical perspectives, depending on the context (Hall, 1999).

In his study of the instructional strategies followed by grade eleven teachers, Mthembu (2007) found that many Mathematics teachers teach geometry using the non-constructivist model of the "teacher-talk" method. In countries like Belgium, teacher-centred approaches are still followed (Fagnant, 2005). It is evident that, although many Mathematics teachers understand the constructivist approach to learning they tend not to implement it.

3.5 Research in Transformation Geometry

The researcher found a few studies that looked at the teaching of Transformation Geometry. This shortfall may have occurred due to this specific section not having been a part of the Mathematics syllabus in the past, and therefore not appearing to be a significant area for researchers to focus upon. For example, international literature argues that congruent and similar triangles can be understood better in Transformation Geometry (Beevers, 2001). Proofs of congruency in Transformation Geometry can be done by explanation instead of verification. Proving congruency by explanation is easier than the complications of Euclidean Geometry as any shape will be congruent to its image under translation, reflection and rotation (Fernandez, 2005; Rival, 1987). Proof by explanation may reinforce understanding as learners are encouraged to speak their minds about their observation of patterns in the contemporary education system. Hence it may be done before engaging with formal rigorous proofs. History reveals that Transformation Geometry was once popular in the discovery of congruent patterns of geometrical shapes, *e.g.* tiling of pentagonal shapes (Rival, 1987). Fernandez **(**2005) argues that Transformation Geometry is rich in possibilities of Mathematical investigation and discussions of “real-life” situations. The current NCS Mathematics syllabus includes Transformation Geometry. Some mathematics NCS textbooks show designs of patterns generated through congruent figures. These patterns are used to design, *inter alia*, clothes and tiles, and are also incorporated into art exhibitions.

3.6 Proofs and proving in secondary schools

Secondary school teachers are faced with the challenging task of teaching proofs. The researcher’s personal experience as a Mathematics learner and teacher revealed that many learners did not understand proofs of theorems. This problem became more pronounced in the proving of Euclidean Geometry theorems and riders. Mudaly (2007) compiled research findings showing that learners had been performing poorly in Euclidean Geometry in the past two decades. The KwaZulu-Natal Department of Education (KZN DoE) (2004) revealed similar findings in recent years. Sometimes this poor performance results in an unpleasant atmosphere in the Mathematics classroom. Indeed, as Davis and Hersh (1983) note, many teachers become frustrated when learners do not understand proofs and thus blame learners for being “stupid”. This tendency is exacerbated by the contrast between learners who experience difficulty in understanding the origins of a theorem's proof and a few 'gifted' learners who appear to achieve understanding through memorisation and rote learning for examination purposes (De Villiers, 2008; Mudaly, 2007). The reasons for learners' poor performance in constructing proofs include presenting proofs directly from the textbook and teaching proofs to learners who are not at the appropriate van Hiele level (Mudaly, 2007). The promotional requirements in South Africa in the FET phase allow learners to progress to the next grade whilst having only attained a minimum pass mark in Mathematics. This means that learners are likely to move to the next grade without having mastered significant van Hiele levels.

The common feeling amongst authors is that the failure of learners to construct proofs in Euclidean Geometry lies in the means by which proofs are taught (De Villiers, 2008; Mudaly, 2007). A quick perusal of the different Mathematics textbooks reveal that proving is seen as a verification of the truth. For example, proving that the opposite sides of a parallelogram are equal is seen to be similar to that of verifying that they are equal. In his study, De Villiers (1990) found that most Higher Education Diploma students believed that the function of a proof is to verify the truth of a statement. This method of teaching proofs has been criticised by many authors as it encourages rote learning to learners – a feature of the old, traditional method of teaching (Mudaly, 2007).

Mudaly (2007) cites several authors (like Coe & Ruthven (1984); Schoenfeld (1985); Gale (1990) and Hanna (1996)) who criticise proving by verification and who instead argue that proving should be the explanation of known facts. However, Hersh (1993) is of the view that as much as the above may be correct, there needs to be an explanation of why it is correct. Explanation may be done through proof heuristics using analogy and constructive definition (De Villiers, 2008). As an example, understanding a parallelogram can lead to the teaching of a new figure called parellelo-hexagon, which is a six-sided figure with opposite sides that are parallel Therefore, it could be argued that presenting a proof as verification does not necessarily engage learners into thinking about the theorem. Allowing learners to explain why the theorem is correct through appropriate guidance can stimulate their thinking (Mudaly, 2007).

Constructivism is a learning theory that allows learners to create knowledge using experiences that they are exposed to. Mudaly (2007), in his two studies, found that learners are eager to explain conjectures if they are engaged in interesting guided activities that aim to prove a conjecture. The results from these two studies indicate that, when teaching the proof of congruent triangles (for example), teachers need to engage learners in activities allowing learners to conclude that triangles are congruent and further, be able to explain why they believe this to be so. The formal textbook proof of the theorem does not fulfil the learners' natural desire to know why a conjecture holds true. In this study of congruent triangles, the questionnaire activity focusing on Transformation Geometry, is aimed at allowing learners to observe the movement of triangles on a plane, such that they could decide whether there is a change of size or not. Additional approaches are also suggested in the teaching of proofs in Mathematics journals.

De Villiers (2008) argued that teaching proofs using explanation can also be done using the "genetic" approach. This approach involves the utilisation of proof heuristics in contrast to that of direct textbook methods. In addition, De Villiers (2008) argues that reasoning by analogy can help stimulate the desire for proof understanding in learners. Analogy occurs when two figures share similar characteristics, for example, where the square is analogous to the cube as both have all sides equal (although the former is two-dimensional and the latter three-dimensional. In this case, a cube is defined by constructive definition as a set of squares put together to form faces of a three-dimensional shape. However, De Villiers (2008) admitted that this method of proving may be difficult to adopt as learners at school level are not likely to have been taught analogy. Perhaps the time will come when the contents of Mathematics textbooks might be transformed into ideas argued by authors who value proof by explanation.

Euclidean Geometry learning should be based on an understanding of the basic skills of the proofs of theorems. Examination questions are based on unseen problems in which learners cannot memorise facts but are instead required to apply their understanding of geometry riders. Mthembu (2007) argued that success in Euclidean Geometry depends on the knowledge of basic concepts as it is upon this knowledge that proofs and solutions of riders rely. Based on the researcher's experience, learners find it difficult to solve non-numerical riders.

According to Skemp (1976), ‘understanding’ may be divided into instrumental and relational understanding. Instrumental understanding refers to knowing the procedures and laws of solving a problem. Relational understanding refers to both knowing the procedures and the reasons for choosing them in solving a particular problem. In the main, OBE supports relational understanding. Relational understanding in Mathematics seems to be difficult to achieve as some Mathematics teachers teach for the purposes of attaining instrumental understanding on the part of learners (Skemp, 1976). It is also noted that the external examination for the NCS consists mainly of questions that encourage instrumental understanding. Learner-centred approaches to teaching could be integrated into assessment in order that teachers might be encouraged to assimilate them into their teaching.

3.7 Learning congruent triangles

Congruent triangles are introduced in grade nine. The knowledge of congruent triangles is important when learners are preparing for FET geometry riders. Mathematics teachers, many of whom may have mastered the current learner-centred approach to teaching, teach congruency in group settings using the “cut-and-match” method of teaching identical triangles. However, formal proofs of the four cases of congruency are mostly taught as ready-made theorems in the textbook. Luthuli (1996) argued that geometry teaching should be more learner-centred and closely based on learners’ experiences. Constructivists like Cobb (1994) and von Glasersfeld (1984), also argued for learner-participation in Mathematics lessons, where learners reflect on their learning and are able to communicate their thoughts to one another and to the teacher. Seemingly, Euclidean Geometry teachers find it difficult to apply a more learner-centred approach in teaching proofs.

Having looked at NCS mathematics textbooks, the researcher noticed that proofs of theorems and other riders are done in a similar way to those appearing in the old Mathematics textbooks. Mudaly (2007) argues that teaching proofs using the textbook method contributes to the learners’ poor performance in Euclidean Geometry. Furthermore, he recommends that proofs be taught using ‘proof heuristics’. De Villiers (1990) refers to proof heuristics as proof not only by verification, but also through the use of explanation and means of discovery, systematisation and communication. Studies show that learners can easily identify identical shapes in geometrical patterns and can further explain why particular shapes are identical, (Brijlall *et. al.*, 2006; Gerdes, 1997). It is for these reasons that the researcher concurs with

Mogari's (2000) suggestion that Euclidean Geometry be taught in practical and familiar contexts related to the interests and culture of learners. In addition, the researcher observed that learners in Technology lessons visibly enjoy drawing patterns, like face brick walls and floor tiles.

3.8 The role of teachers in Euclidean Geometry learning

An important role of geometry teachers is to design 'open' activities allowing learners to enjoy learning geometrical shapes. It has been contended that a teacher should be dynamic and able to design learning programmes and materials that could stimulate learners' interest (Ausabel, Novak & Hanesian, 1978; DoE, 2003). This has cross-disciplinary motives, calling for the creation of activities that integrate Mathematics and other learning areas, *viz*. Technology. The Mathematics teacher should be able to identify how learners seek to learn concepts, thereby selecting the appropriate instructional strategy to adopt.

Traditional teaching and learning took the form of miming the educator's actions in order that learners could be able to reproduce them in a neat, orderly manner (Faulkner, Littleton & Woodhead, 1998). This was compulsory and may have contributed to the high level of dropouts at school level. The implementation of the NCS was aimed at transforming education, including instructional strategies. It allowed learners to critically discuss their preferred method of learning. Formative types of assessments, like projects, assignments and tutorials were implemented (DoE, 2003). Rubrics, group and peer assessment take preference at the expense of marking memorandums. Teachers were expected to familiarise themselves with this transformation in assessment in order that they may have been able to discuss it with learners.

Congruent triangles formed part of the proofs of some Euclidean Geometry theorems, as well as the proofs of some riders. Teachers were thus expected to facilitate adequately for learners' understanding of congruency by means of effective strategies that could allow learners to participate in their own learning process.

3.9 Mathematics classrooms

Mathematics (especially Euclidean Geometry) is always done in a classroom setting, unlike the sciences. Traditionally, the classroom environment was stereotypically always tense and fearful to learners (Wehlage *et.al.*, 1989). This was caused by a combination of learners' difficulty in understanding Mathematical concepts, teachers' approaches that were not welcoming and rules preventing learners from challenging their teachers' style of teaching. In OBE teachers are tasked to change the classroom into a positive environment where learners jointly solve Mathematics problems.

Wehlage, Rutter, Smith, Lesko and Fernandez (1989) suggest that teachers should ensure a positive psychological investment in the minds of learners. Teachers therefore need to design interesting lessons and engage learners through challenging activities which both learners and teacher attempt to solve jointly. For instance, classroom walls could be decorated with attractive geometrical shapes in different settings. Another example is that there could be displays of charts for congruent triangles on a Cartesian plane and Euclidean Geometry. Furthermore, teachers could challenge learners to design these congruent patterns for display. By allowing learners to draw a house, for example, a pattern of congruent bricks and tiles could be observed.

3.11 Conclusion

There are huge challenges in Mathematics education based upon the constructivist model of learning. The NCS (DoE, 2005) delineates that the educator should master different instructional strategies in order to achieve the four Learning Outcomes of Mathematics. The research methodology for this study follows in chapter four.

CHAPTER FOUR

RESEARCH METHOLOGY

4.1 Introduction

This chapter considers the research methods that were employed in the research project. The research methods, and the reasoning for their adoption, will be discussed. In addition, solutions will be proposed and limitations discussed.

4.2 Methodological framework

This study was conducted according to the qualitative research method. Thus, the data used in this study was collected through questionnaires and a semi-structured interview according to the qualitative research paradigm. Before the beginning of the actual process of the research project the researcher met all participants (learners) from each school to discuss their engagement in the research into Euclidean Geometry and Transformation Geometry.

4.3 Motivation for employing the qualitative research method

The introduction of Transformation Geometry in grade ten presented a different method for the proving of congruent triangles. Literature reviewed shows that Euclidean Geometry learners experience difficulty in understanding proofs of congruent triangles using formal methods of Euclidean Geometry (De Villiers, 2008; KZN DoE, 2004; Mudaly, 2007). The researcher was thus interested in investigating learners' conceptual understanding of congruent triangles in Transformation Geometry and in Euclidean Geometry. The researcher believes that the qualitative approach best suits the outcomes of the study in determining the personal views of the learners themselves, as this approach potentially allows for greater insight into the nature of learners' learning proofs (Henning, 2004).

This study focused on the ability of grade ten learners to understand congruency in Transformation Geometry. In addition, this study sought to assist Geometry educators by potentially allowing them to transform their instructional strategies where necessary. The qualitative research approach was thus appropriate for this type of research as it is the most suitable tool for discovering and interpreting existing problems (Cohen, Manion & Morrison, 2007; Henning, 2004; Leedy & Ormrod, 2001). It is hoped that the nature of the answers learners provided to the clinical research questionnaire (and confirmed through semi-structured interview) reflect the learners 'in-depth feeling' about their preferred method of learning congruent triangles. Furthermore, as OBE is based on consultative forms of teaching and learning, the researcher hopes that interviewing learners may result in the collection of practically useful data on how the learners themselves feel they should be taught congruent triangles in order to aid their understanding. Although focussing on gathering data for the main research question, the semi-structured interview may also yield useful additional information (Henning, 2004). As the literature reviewed argues learners may provide more truthful answers to the questionnaires than they would in a personal interview (Henning, 2004; Leedy & Ormrod, 2001). Thus questionnaires were used first in this study.

4.4 Sampling and participants in the study

The two schools that were the main focus of this study are located in a rural area, with most learners' parents in the area currently employed. The learners were African predominantly IsiZulu speaking and were taught by black South African teachers. The majority of learners in the area live with single parents, grandparents or step-parents and most teachers in the area live in the nearby suburbs. The area is affected by social issues like HIV/AIDS, divorce and unemployment, with teenage pregnancy and alcohol abuse commonly affecting the school. Although most learners use public or private transport, some learners walk short distances to school.

The participants were grade ten learners from two schools in the Ugu district in the Paddock ward. As the schools that were chosen for the study are from the same ward, the findings of this study can potentially be assumed to be representative of this ward only. This sampling tied up with qualitative research. This study employed non-probability sampling techniques. Cohen *et.al.* (2007) argued that non-probability sampling ties sampling methods of convenience and purposeful sampling together, thus these sampling methods were used in the selection of participants.

Cohen *et. al.* (2007) explain that convenience sampling refers to the selection of participants that are convenient for the study while purposive sampling refers to choosing participants who possess a certain characteristic i.e. the participants were doing grade ten where Transformation Geometry is extended to congruent figures that result after performing transformations on a Cartesian plane. Each percentage achievement level was represented by two learners. That is, two learners from the percentage achievement levels: 0 to 19; 20 to 39; 40 to 59; 60 to 79 and 80 to 100. The performance of the learners was determined based upon the results they attained in the examinations they completed in the previous year.

Each of the five scale sections were put into separate containers and each learner was assigned a number. Pieces of paper, each with a number, were assigned to learners. The researcher drew two numbers from each container, resulting in a total of ten learners for each school. The two schools were coded School A and School B respectively. Learners that were interviewed in School A were coded as learner A_1; A_2; *etc.* The learners that were interviewed in School B were coded as learner B_1; B_2; *etc.*

4.5 Learners are always participants in informal research

Mthembu (2007) argued that teachers are everyday researchers at schools as they are continually studying the behaviour of learners from all aspects (*e.g.* educational achievement, emotional, psychological, *etc.*) in an informal manner. This implies that learners are potentially participating in casual research activities every day they are at school. Unconsciously, learners provide data that is then used by teachers to prepare lessons for the learners. The data gained from this form of research are unfortunately not submitted to formal bodies for analytic purposes (Mthembu, 2007). Mthembu (2007) indicated that, although the data collected is not intended for research purposes and is used instead for record keeping and submissions to educational authorities (like senior education managers), this data can nevertheless be seen as a form of data submission.

Smith and Lythe (1993) argue that teacher-learner research represents a radical challenge to assumptions about the nature of educational reform. The introduction of OBE was an indication that this teacher-learner research is of significance. OBE was introduced as a result of the observation that the old, traditional system was teacher-centred and based on rote learning. In research techniques like observation, participants release data not knowing that they are doing so (Henning, 2004). The longer the observation continues, the more in-depth the information the researcher may potentially gather.

4.6 The process followed in completing the study

After obtaining permission to interview learners (see Appendices C and D) from the two schools, and the approval letter (see Appendices A & B) from the Department of Education the researcher negotiated appointment dates with the participants. The schedule of events was created.

The questionnaires were handed out, and the interviews were consequently audio-taped on the scheduled dates. It was agreed that neither the real names of the participants, nor those of the participants' schools were to be mentioned.

4.7 Ethical issues

"A major ethical dilemma is that which requires researchers to strike a balance between the demands placed on them as professional scientists in pursuit of truth, and their subjects' rights and values potentially threatened by the research" (Cohen *et.al.*, 2007, p. 51).

With regards to potential ethical considerations, the researcher followed the procedures stipulated by the university's Faculty of Education. Specifically, learners attending the classes where the research was to be conducted were given letters of informed consent to complete (see Appendix C). Participation was totally voluntary and the confidentiality, privacy and anonymity of the participants were assured. The consent letters included details of the study and data collection procedures. The participants were also assured that, if they chose to be part of the study, they could withdraw at any time without suffering any form of negative consequence. Also, all written responses and recordings would be held in safe keeping until the study was over. Once the study was concluded this data would be locked in storage for a period of five years after which it would be destroyed. Before any results were published, the results were to be first shared with the participants. Lastly, the participants were given the choice to remain anonymous and guaranteed that any specific references and instructions which could potentially threaten their anonymity would be structured so as to ensure the protection of their privacy.

The university research office has acknowledged that this investigation conformed to all the necessary ethical conditions. This acknowledgement appears as the ethical clearance approval certificate number HSS/0105/09M (see Appendix E).

4.8 Reliability and validity

In qualitative research "validity might be addressed through the honesty, depth, richness and scope of the data achieved, the participants, the extent of the triangulation and the disinterestedness or objectivity of the researcher" (Winter as cited in Cohen *et.al.*, 2007, p. 133). Validity can be improved through careful sampling, using the appropriate instruments and data analysis techniques. Whilst validity is not something that can be achieved absolutely, it can nevertheless be maximised. In order to maximise validity, the research samples were chosen carefully. The learners selected were purposively chosen for their learning of Transformation Geometry and Euclidean Geometry at grade ten where they would study more examples of proving congruency in preparation for their National Senior Certificate examinations in grade twelve. The participants appeared to provide the researcher with honest, well thought out and thorough responses, as Agar (as cited in Cohen *et.al,* 2007, p. 134) argues, rich data and involvement of the participants secures a sufficient level of validity and reliability.

Also, the instruments chosen for data analysis and generation were carefully selected on the basis of their appropriateness. The semi-structured approach allowed the researcher to gain in-depth answers from the learners. According to Cohen *et.al.* (2007, p. 149), reliability can be seen as the correlation between the researcher's recorded data and what actually occurred in the natural setting where the research was conducted. The researcher achieved reliability in this manner by triangulating the data. After the researcher had captured the data from the written responses, the researcher was able to verify the correlation between the written responses and two sets of interviews. This potentially increased the reliability of the data.

4.9 The research questionnaires and the semi-structured interview questions

Written responses and interviews were the data-capture instruments used in this research. The participants were first handed out questionnaires containing two worksheets. The first worksheet was for Euclidean Geometry and the second was for Transformation Geometry. After the researcher had rated and analysed the responses to both questionnaires, the participants were each interviewed about their performance in the two tasks.

4.9.1 Conceptual framework adopted in questionnaire design

The activity sheet design was conceptualized in terms of constructivism and also adopted Vygotsky's educational theory and the process of scaffolding. According to Vygotsky's theory, "learning leads to the development of higher order thinking" (Dahms, Geonnotti, Passalacqua, Shilk, Wetzel & Zulkowsky, 2007, p. 1), Vygotsky believes that learning occurs through language and social interaction. The zone of proximal development (ZPD) is central to Vygotsky's view on how learning occurs. He describes this zone as the distance between the actual developmental level of a learner (determined by independent problem solving) and the potential developmental level of a learner (determined by problem solving under the supervision of a teacher or a more capable peer). The ZPD can also be described as the area that separates what the learner is able to achieve on their own and what they can achieve under supervision and with assistance. The performance of a learner is increased when they are working under supervision.

According to Vygotsky (as cited in Dahms *et.al.*, 2007, p. 2), the ideal role of the teacher is to provide scaffolding to assist the learner with their tasks. "The term 'scaffolding' was developed as a metaphor to describe the type of assistance offered by a teacher or a peer to support learning", (Lipscombe, Swanson & West., 2008, p. 3). Scaffolding is the process that occurs when a teacher helps a student to grasp a concept or master a task that the learner was initially unable to do. Assistance is only provided to those skills beyond the student's capability. The student may make errors, but with feedback and prompting, the student is able to reach the required response. Once the student can perform the task on their own, the teacher removes the scaffolding, allowing the student to work independently.

Scaffolding is actually a bridge used to build upon what students already know to arrive at something they do not know. If scaffolding is properly administrated, it will act as an enabler, not as a disabler. (Benson, cited in Lipscombe *et.al.*, 2008, p. 3)

Larkin, as cited in Lipscombe *et.al.* (2008, p. 7), suggests the following effective techniques of scaffolding teachers can follow:

"Teachers should begin by boosting the confidence of their students by introducing them to tasks that they can do quite well with limited or no assistance. The implications of Vygotsky's theory and the process of scaffolding for the teacher, is thus to guide the student's activity so that meaningful learning occurs."

4.9.2 QUESTIONNAIRE 1: Euclidean Geometry

The following questionnaire is a Problem-Centred Learning (PCL) activity involving of congruent triangles. The researcher decided to design the PCL with its featuring socio-constructivist theory as Oliver, Murray and Human (1996) argue that PCL engages learners actively in the process of acquiring knowledge. The learners also draw on past experiences and existing knowledge during learning. In this questionnaire, the learners will use their past knowledge of basic geometry (like parallel lines, isosceles triangles and alternating angles on parallel lines). The questionnaire also involves a formal proof of congruent triangles. As Mudaly (2007) admits that it is difficult to relate formal proof to learners' past experiences, and the researcher decided to use a form of guided activity whereby learners filled in the correct answers in spaces left intentionally blank. In South African schools, learners are not guided when doing these proofs and the researcher suspects that this may result in greater difficulties in learning such proofs. The researcher wanted to monitor how much relative success learners enjoy in determining formal proofs of congruent triangles. Hence, this activity is based on proof by verification. This kind of proving by verification has been criticised for its lack of improving learners' interest and understanding. The researcher then wanted to observe how learners would understand and stimulated to work out these kinds of proofs (De Villiers, 2008; Mudaly, 2007). De Villiers (2008) further suggests that it is about time teachers attempt to teach proofs by explanation using analogies and heuristics. However, he admits that learners might not be ready for such learning.

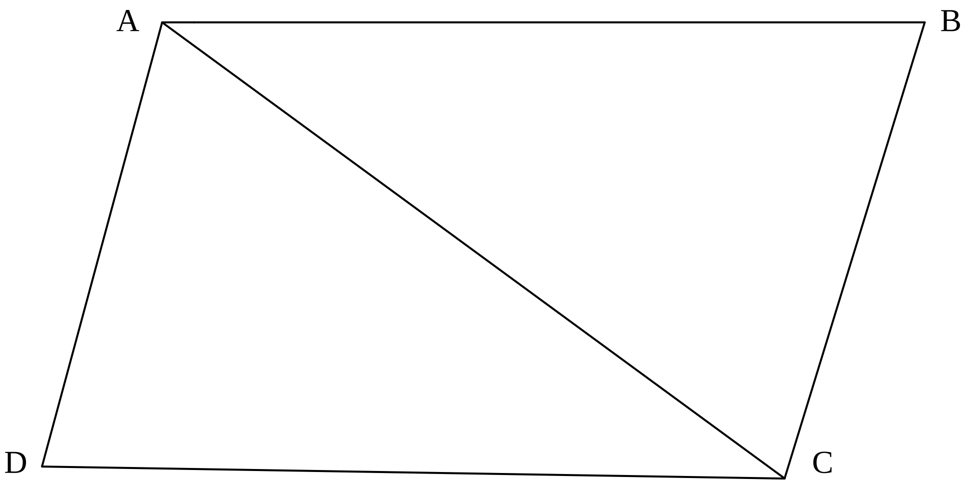
Each subsection of this questionnaire seeks the understanding of a specific case of congruency. In 1.1 the case of SIDE, SIDE, SIDE (S, S, S) is involved. To prove this, learners needed to know that both pairs of opposite sides of a parallelogram are equal. For 1.2

the learners required understanding of an isosceles triangle and the case of SIDE, ANGLE, SIDE (S, A, S). For 1.3 the case of 90^0, HYPOTENUS, SIDE (90^0, H, S) is deduced. Lastly, in 1.4 the case of ANGLE, ANGLE, SIDE (A, A, S) is tested and the learners needed to understand parallel lines and alternate angles.

The questionnaire is structured so as to lead learners into deducing the cases of congruency. The questions in this questionnaire rely on the knowledge of the properties of triangles, parallelograms, parallel lines and quadrilaterals to be solved. The questionnaire seeks the understanding of proofs using formal reasoning. This questionnaire appears as Appendix F at the appendices.

QUESTIONNAIRE 1

1.1 Below is parallelogram ABCD.



Prove that ΔADC ≡ Δ CBA by completing the following statements:

Statement Reason

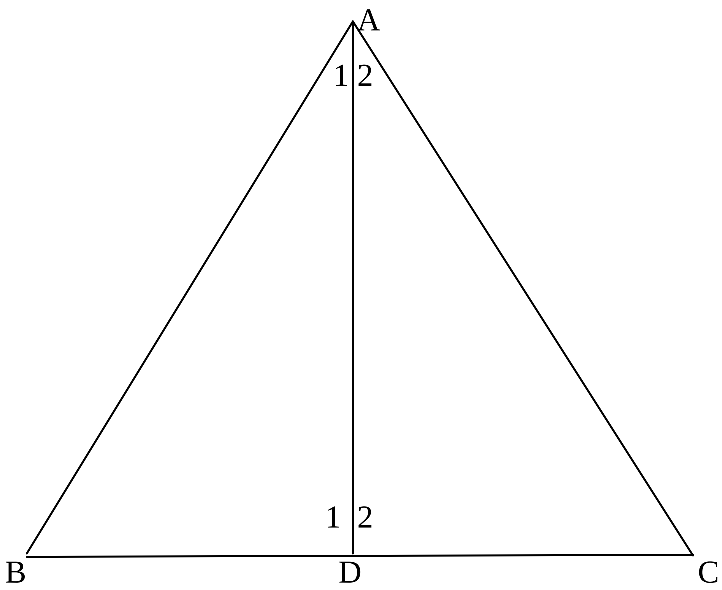
AB = ... (..)

AD = ...……. (..)

AC = ...……. (..)

so Δ ADC ≡ Δ CBA (..)

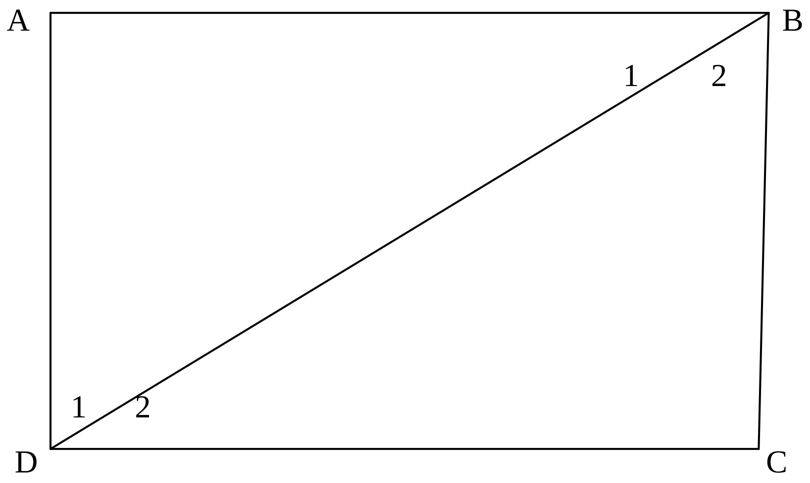
1.2 Below is isosceles Δ ABC with AB = AC. DA bisects Â.



Prove that Δ ABD ≡ ΔACD by completing the following statements:

Statement		Reason
…………. .	= ……….	(given)
$\hat{A}_1$	= ………...	(.......................………...……………..……………………)
AD	= AD	(........................………………………........………………......)
So Δ ABD	≡ Δ ACD	(.......................................…..………...…..………………)

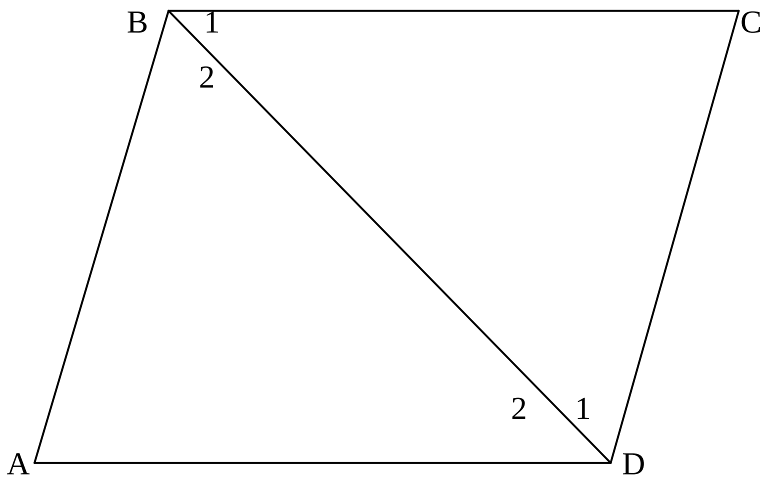
1.3 Below is quadrilateral ABCD with $\hat{A} = \hat{C} = 90^0$ and AB = DC.



Prove that ΔABD ≡ ΔCDB by completing the following statements:

Statement			Reason
Â	=	……….	(......................………...……………...…………………….)
BD	=	BD	(......................………...……………...…………………….)
AB	=	....	(......................………...……………...…………………….)
so ΔABD ≡		Δ CDB	(......................………...……………...…………………….)

1.4 Below is quadrilateral ABCD with $\hat{A} = \hat{C}$ and AB // DC.



Prove that ΔADB ≡ ΔCBD by completing the following statements:

Statement		Reason
$\hat{A}$	= ……	(.......................………...…………….....…………………….)
………..	= ……	(Alternate angles, AB // DC)
BD	= BD	(.......................………...…………….....…………………….)
so Δ ADB	≡ Δ CBD	(..….………………………..)

4.9.3 QUESTIONNAIRE 2: Transformation Geometry

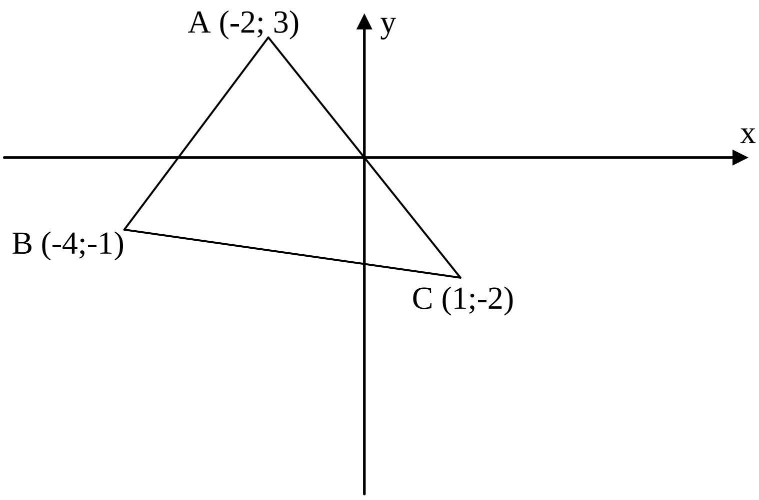
The following questionnaire is based on Transformation Geometry and the tasks are based on the informal proofs of Euclidean Geometry riders. The learners were guided through a practical activity where they would measure sides and angles to prove congruency.

Firstly, the learners were asked to draw the figure and its image on the same Cartesian plane under some kind of transformation, *viz.* translation, rotation or reflection. If the drawn image did not result in some kind of constructive defining, which De Villiers (2008) describes as a new concept created from the original figure, then the two triangles would be congruent. It is important for this activity to successfully stimulate learners' interest as Ausabel, Novak and Hanessian (1978) argue that meaningful learning occurs when learners are curious to determine the result in a discovery task. This activity is a guided activity as research has shown guided learning facilitates an increase in learners' discovery (Mudaly, 2007).

The purpose of these tasks is to check how much relative success learners achieve in doing informal proofs of riders. It is based on the knowledge of the four cases of congruency of triangles. In (a) the case of S, S, S is tested. To successfully complete this task, the learners need to know the rules of translation of points on a Cartesian plane as well as practical measurements of lines using a ruler. In (b) the learners are tested on the case of S, A, S. The knowledge of the rules of reflection on the Cartesian plane, as well as the ability to measurement of sides and angles is required in order to answer this question. For (c) the case of A, A, S is tested. The learners need the knowledge of the rules of rotation on the Cartesian plane as well as measurement of sides and angles. Lastly, for (d) the case of 90^0, H, S is tested along with the learners' knowledge of the rules of reflection and measurements. This questionnaire appears as Appendix G in the Appendices.

QUESTIONNAIRE TWO

Study the diagram below and then complete the tasks that follow:



ΔABC is translated by the rule: (x, y) → (x + 1, y + 2). Draw both ΔABC and its image ΔA′B′C′ on the same set of axis on the graph paper provided.

Complete the following statements by accurately measuring:

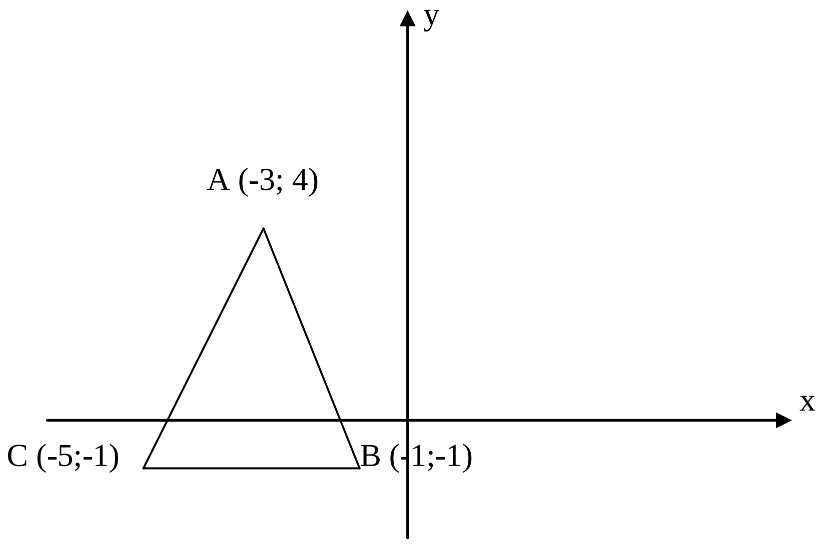
AB = ….............................. A′B′ = ...……

BC = ….............................. B′C′ = ...……

AC = ...……................... A′C′ = ...……

So Δ ABC ≡ ΔA′B′C′ (………...............................………..……………)

(b) Study the diagram below and then complete the tasks that follow:



In the above diagram, ΔABC is reflected about the y-axis. Draw both of ΔABC and its image ΔA´B´C´ on the same set of axis on the graph paper provided.

Complete the following statements:

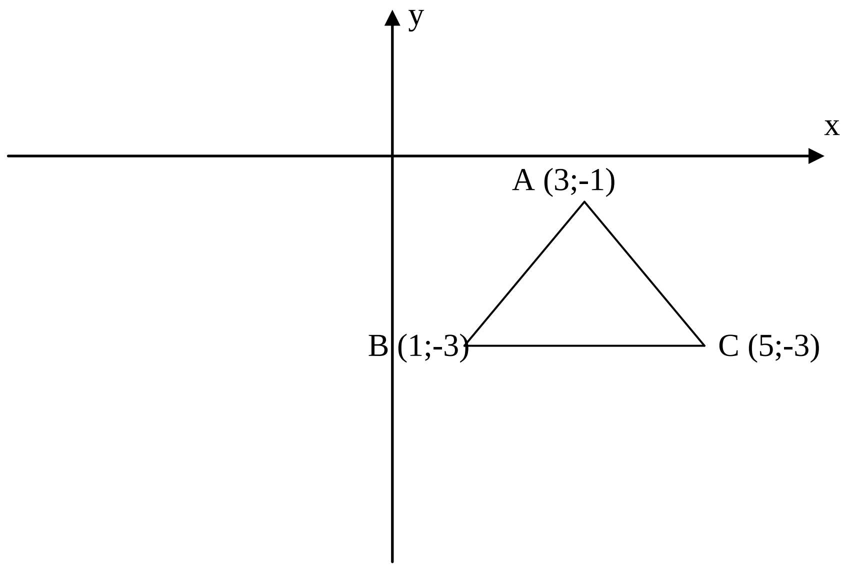
AB = ………………………………   A′B′ = ………………………………

Â = ………………………………   Â′ = ………………………………

BC = ………………………………   B′C′ = ………………………………

So ΔABC ≡ ΔA′B′C′   (………………………………………………)

(c) Study the diagram below and then complete the tasks that follow:



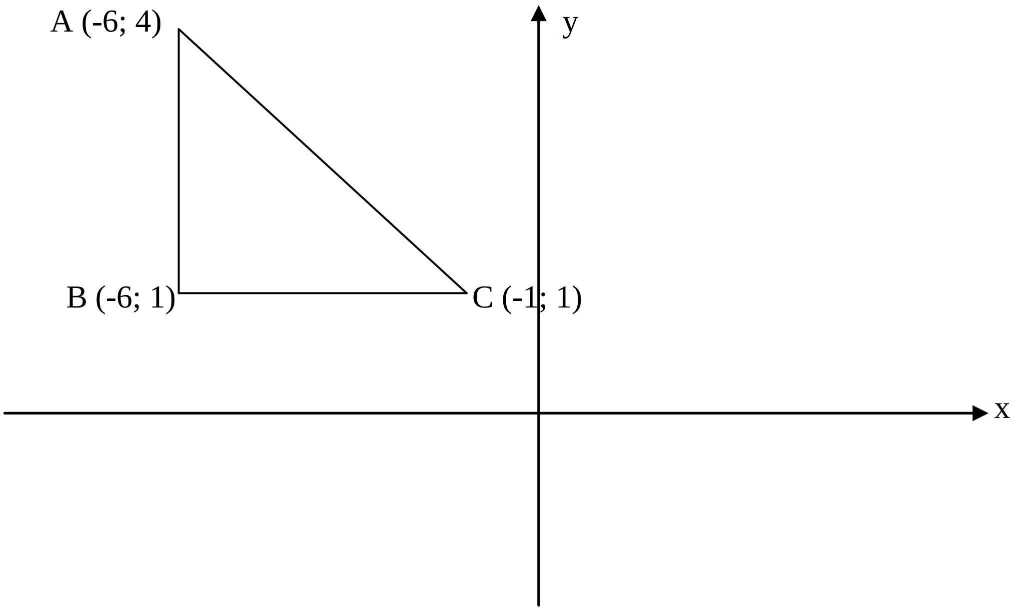
In the above diagram, ΔABC has been rotated through 180^0 clockwise. Draw both ΔABC and its image Δ A′B′C′ on the same set of axis on the graph paper provided.

Complete the following statements by accurately measuring:

Â	=	……………………….…..	Â′	=	…………………….………
Ĉ	=	……………………..…......	Ĉ′	=	…….…………….…….
BC	=	…………………………..	B′C′	=	……..……………….…

So ΔABC ≡ Δ A′B′C′ (……………………………….……………..)

(d) Study the diagram and then complete the tasks that follow:



In the figure above, ΔABC has been reflected along the x-axis. Draw both ΔABC and its image ΔA´B´C´ on the same set of axis.

Complete the following statements by accurately measuring:

AC = …….......................... A′C′ = ...………..

$\hat{B}$ = ….........................….. $\hat{B}'$ = ...….……

BC = ...….... B′C′ = ...….…….

So ΔABC ≡ ΔA′B′C′ (...........…..............……………………….………)

4.9.4 Semi-structured interview

The researcher wanted to confirm the extent of the validity of the written responses through interviews. The researcher also felt that through interviews, some questions would be clarified enabling the written responses to be changed accordingly. In this interview, the learners would clarify why they attained more success in learning congruent triangles between Euclidean Geometry and Transformation Geometry. Thus, the two learners who got the highest scores, two from the middle scores and two from the lowest were selected and interviewed. The participants selected by the procedure stated above were asked to answer the following questions after their written responses were analysed:

QUESTIONS

(a) Do you enjoy learning Euclidean Geometry? Why?

(b) Do you feel that the teachers can use other ways to teach Euclidean Geometry? Explain.

(c) Do you think that most learners enjoy working on problems involving congruency? Why do you think this is so?

(d) Tell me what we mean when we say “the two triangles are congruent”?

(e) Can you explain the cases of congruency? How many cases are there?

(f) Give me an example of two triangles that are not congruent.

(g) Can you tell me in Questionnaire number 2, what types of Transformation are displayed in each case?

(h) In which tasks did you enjoy working with? Why?

(i) Would you prefer using Transformation Geometry when studying ideas in Euclidean Geometry?

(j) In which tasks did you find it easier to identify the cases of congruency?

In (a), the researcher wanted to ascertain whether the learners' attitude towards Euclidean Geometry corresponded with the responses to the performance in the questionnaire. In (b) the researcher checked if the learners felt teachers' teaching strategies were responsible for their poor performance in Euclidean Geometry. Question (c) determined whether the learners had ever discussed the topic of congruency during their spare time and if they had, the question further attempted to ascertain how they 'felt' about it. In (d) the researcher wished to test the knowledge of congruent triangle possessed by the learner themselves. Questions (e) and (f) tested the extent to which the learner understood congruency. In (g) knowledge related to the identification of each type of transformation (translation, reflection and rotation) was tested. This was necessary as the researcher believes it to be important for learners to possess adequate knowledge of transformations before they can understand congruent triangles. In (h) the researcher attempted to discern the specific question the learner enjoyed the most in order to compare the verbal answer to the learners' performance on the questionnaire. Question (i) tested the learners' ability to recognise the significance of Euclidean Geometry diagrams in Transformation Geometry as introduced in grade ten. Question (j) specifically tested which method the learner enjoyed in solving each task between the two questionnaires of Euclidean Geometry and Transformation Geometry.

4.10 Conclusion

The qualitative research method used in the data collection was useful in the research project. An understanding of the learners' views on the learning in Euclidean and Transformation Geometry attributed to the design of the interview questionnaires. The findings of the research project will be discussed in the next chapter.

CHAPTER FIVE

RESULTS AND ANALYSIS: QUESTIONNAIRES

5.1 Introduction

This chapter considers the findings of the responses to the written questions presented in this research project. The analysis of the results will be discussed, thereby determining the extent to which learners understand congruent triangles in Euclidean and Transformation Geometry. The researcher wanted to determine how well grade ten learners understood congruency of triangles when working with their transformations, as compared to formal reasoning. To this end, an analysis of the responses, written by the learners, will be conducted in this chapter.

5.2 Contextual information

The context within which the research was conducted is significant as it may provide a better understanding of the nature of the results of the research. The study was conducted in two rural schools in the Ugu district in the Sayidi circuit of the Paddock ward. The number of learners in the mathematics classes in both schools ranged from thirty-five to fifty-five. In both schools learners were aware of the importance of mathematics attainment in the senior certificate examination and were often visited by different government officials and bursary organizations informing them of the importance of positive mathematics attainment. The learners' apparent interest for mathematics may thus be a result of their awareness of the need for mathematics as a prerequisite in their potential future careers. Mathematical literacy classes ranged from twenty to thirty-five learners in both schools. The above factors created the impression that the emphasis in both schools was on mathematics.

One particular learner indicated to the researcher that the learner was not sure whether to take mathematics or mathematical literacy, but decided to join his/her friends who were taking mathematics. Learners in both schools were largely unaware of the different sections of mathematics, such as Algebra, Euclidean Geometry, Trigonometry and Transformation Geometry. The researcher suspects that the learners' teachers might not have clarified these sections in their grade ten course outline. The researcher thus believed it to be important to engage participating learners in a general mathematics discussion, in which all the terms and classifications were clarified. The following day the questionnaires were handed out to the participants.

5.3 The coding procedure for questionnaire one

A convenient coding system was employed in rating the quality of responses in both schools. Tasks 1 to 4 were marked out of 8 marks each. In the formal reasoning tasks, a rating scale of 0 to 8 was categorized as indicated by Table 1 below.

Table 1: Coding mechanism

Mark Range	Category	Descriptor
0 - 2	A	Poor
3 – 5	B	Average
6 - 8	C	Good

The results for the two schools in this questionnaire appear in Table 2.

Table 2: Performance of learners for tasks on formal reasoning

Marks	0 - 2	3 - 5	6 - 8
School A – Task 1		10	
School B – Task 1		3	7
School A – Task 2	7	2	1
School B – Task 2		5	5
School A – Task 3	1	7	2
School B – Task 3		3	7
School A – Task 4	3	6	1
School B – Task 4		3	7

For school A, all learners scored in Category B in task 1. On viewing the responses, it was observed that learners could provide the correct sides involved. They could all indicate AB = DC, AD = BC and AC = AC. However, the reasons provided were incorrect as shown by the response for leaner A_3, in Figure 1.

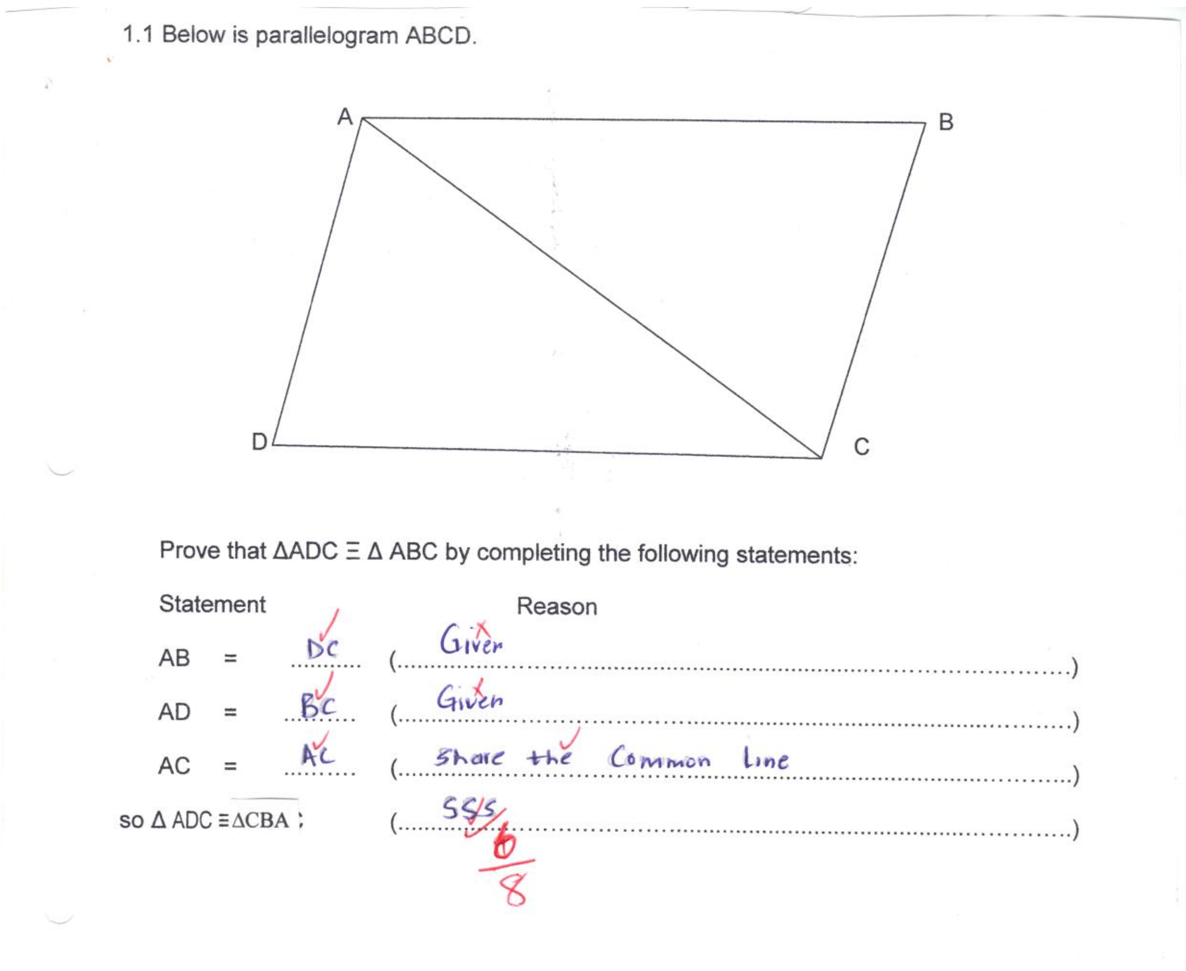


FIGURE 1: Written response for learner A_3

Learner A_3 seemed to confuse parallel sides with equal sides, as shown above where the symbols "=" and "//" were used in the same line. This misconception could possibly be due to the trend that most applications of Euclidean Geometry provide quadrilateral figures (like rectangle, square rhombus and parallelogram), which involve opposite sides equal and parallel. This may lead learners to believe that when one condition is satisfied then the other is automatic. Further, as is unusual for learners to deal with application tasks involving trapeziums, these tasks may well have exposed the potential lack of comprehension. Indeed, this specific quadrilateral is ideal to eradicate the above misconception, since it has exactly one pair of opposite sides parallel and not equal. In addition, this example could highlight the view that parallel sides need not be equal. To eradicate the converse we could promote the case of an isosceles triangle XYZ with XY = YZ by observing that XY cannot be parallel to YZ.

A positive note was that (for most of the learners), learners could identify the correct case of congruency. This could be that they were "scaffolded" into the correct conclusion by the structure of the task.

In order to triangulate the data, an interview was carried out with learner A_3.

Researcher: "*What did you mean when giving this reason?*" (Showing the reason.)

Learner A_3: *"AB is parallel to DC."*

Researcher: *"So that means that AB = DC?"*

Learner A_3: *"Ja, like this."* (Showing the given diagram.)

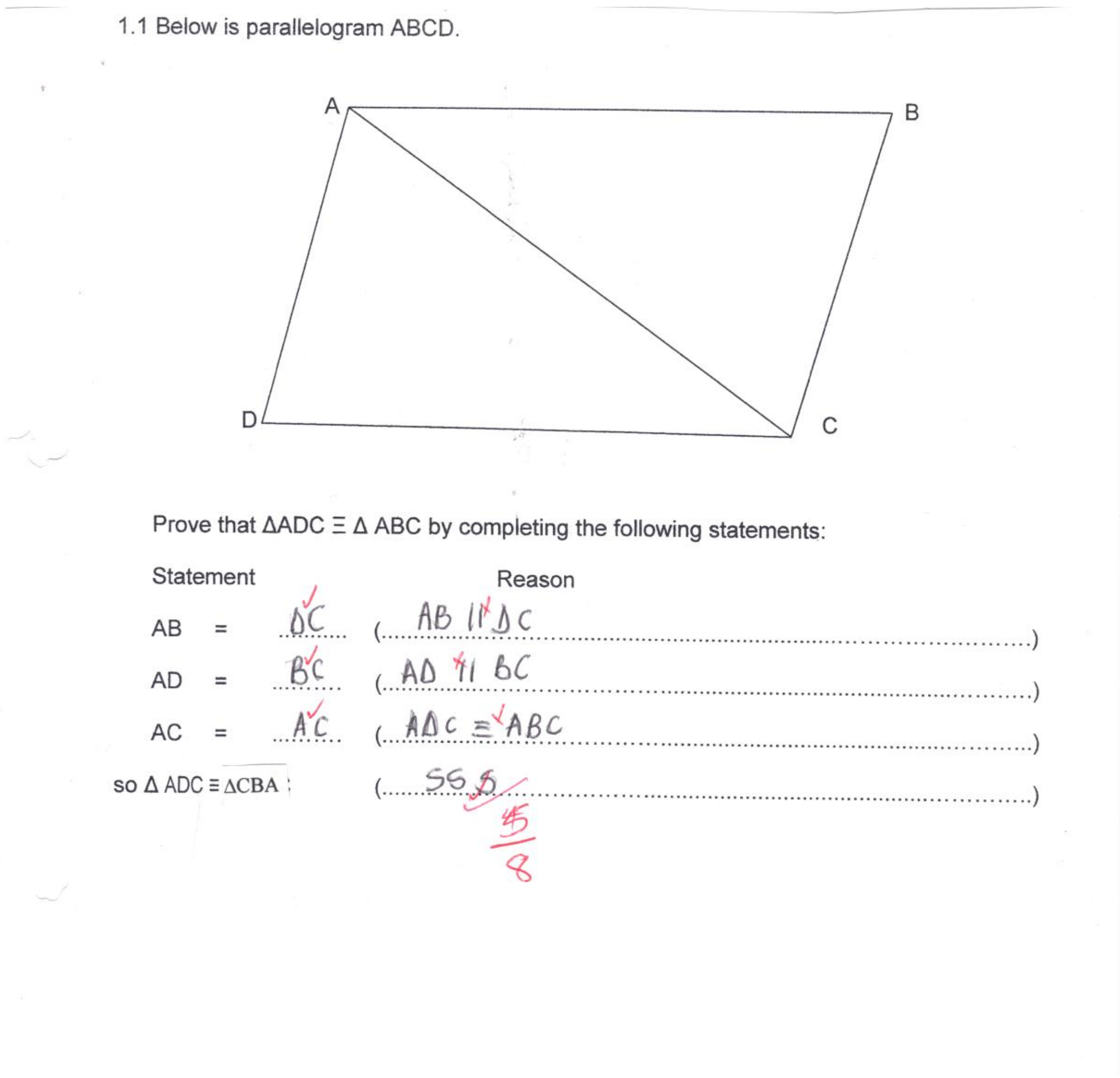
Researcher: *"So if the lines are parallel then they are also equal?"*

Learner A_3: *"Ja like this. I don't know. Am I wrong? They look to be equal."*

(Showing the diagram again.)

The last response raises another misconception that if the opposite sides of a quadrilateral (like a parallelogram, a rectangle, a square or a rhombus) look like they are equal, then it is true that they are equal without a formal reason. This behaviour seems common in the researcher's personal experience of teaching; especially in junior classes at high school level. Learners appear to prefer using visualisation instead of mental construction. This usage may be a factor which contributes to difficulty in proving Euclidean Geometry theorems and riders as concluded by Mudaly (2007).

For school B, thirty percent of the learners scored in Category B and seventy percent scored in Category C. The responses indicated that the learners could provide correct pairs of equal sides; the learners could provide the correct reason for AC = AC (common side). The reason for AB = DC and AD = BC was "given" in both cases. The researcher marked them wrong as the properties of a parallelogram were expected to be provided as justification in the proof. This can be seen in the response for learner B_1 in figure 2.


1.1 Below is parallelogram ABCD.

Prove that ΔADC Ξ Δ ABC by completing the following statements:

Statement			Reason
AB	=	DC	(AB ∥ DC)
AD	=	BC	(AD ∥ BC)
AC	=	AC	(ADC ≡ ABC)
so Δ ADC ≡ ΔCBA			(SSS)

5/8

FIGURE 2: Written response for learner B_1

In verifying the written response a discussion with learner B_1 was arranged. The following dialogue was recorded.

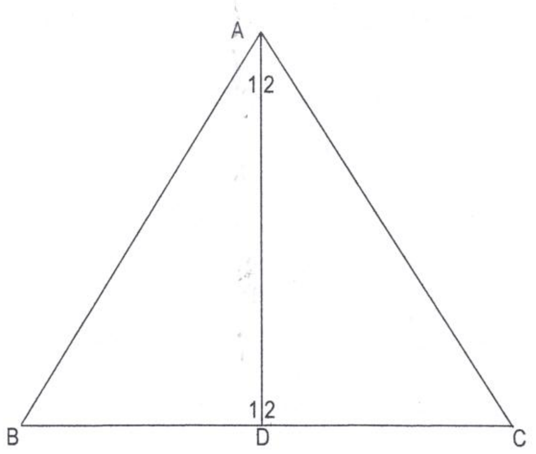
Researcher: *"What does "given" mean in these two reasons?"* (Pointing at the reasons.)
Learner B_1 :*"Sinekeziwe nje sir."* (Meaning we are given.)
Researcher: *"What is it that is given?"*
Learner B_1: *"That AB = DC and AD = BC."*
Researcher: "*But I can't see that in the problem, where is it?"*
Learner B_1: *"It says ABCD is a parallelogram this side is equal to this and this one is equal*

to this." (Pointing at AB = DC and AD = BC.)

From the above dialogue the researcher could deduce that the learners knew the opposite sides of a parallelogram are equal. However, the learners seemed to have been unable to use proper reasoning in their answers. This important implication is that mathematics assessment tasks sometimes do not measure the "exact" knowledge of the learner. In this case the learners were assessed on congruency, yet the knowledge of the properties of quadrilaterals (in this case a parallelogram) was part of the way to the correct answer. Fortunately in this case, the learners knew the properties of a parallelogram; otherwise they may have performed poorly on congruency due to their lack of the knowledge of the properties of a parallelogram.

In task 2 of questionnaire one, seventy percent of the learners in school A scored in Category A; twenty percent in Category B and only ten percent in Category C. After observing the responses, the researcher suspected that the learners were not aware of the importance of carefully reading through instructions and questions presented to them before attempting to answer. There was an indication that if the triangles look the same then they are congruent. Again, visualisation seems to be the justification, rather than formal reasoning. This was observed when the learners provided the reason that triangle ADB was congruent to triangle ADC for AD = AD instead of "common side". This argument could also imply circular reasoning. The child is using the situation of congruency before proving it. One can conclude that AD = AD since triangle ABD is congruent to triangle ACD after it is proved. It is also clear that the concept of a "common side" was not clearly understood as shown in the response for learner A_2 in figure 3.

1.2 Below is isosceles Δ ABC with AB = AC. DA bisects $\hat{A}$.



Prove that Δ ABD $\equiv$ ΔADC by completing the following statements:

Statement	Reason
AB = AC ✓✓	(Given)
$\hat{A}_1$ = $\hat{A}_2$ ✓	(AD ‖ AC ✗)
AD = AD	(ΔABC $\equiv$ ADC ✗)
So Δ ABD $\equiv$ ΔACD	(S LS ✓)

5/8

FIGURE 3: Written response for learner A_2

The dialogue that transpired between the researcher and learner A_2 appears below.

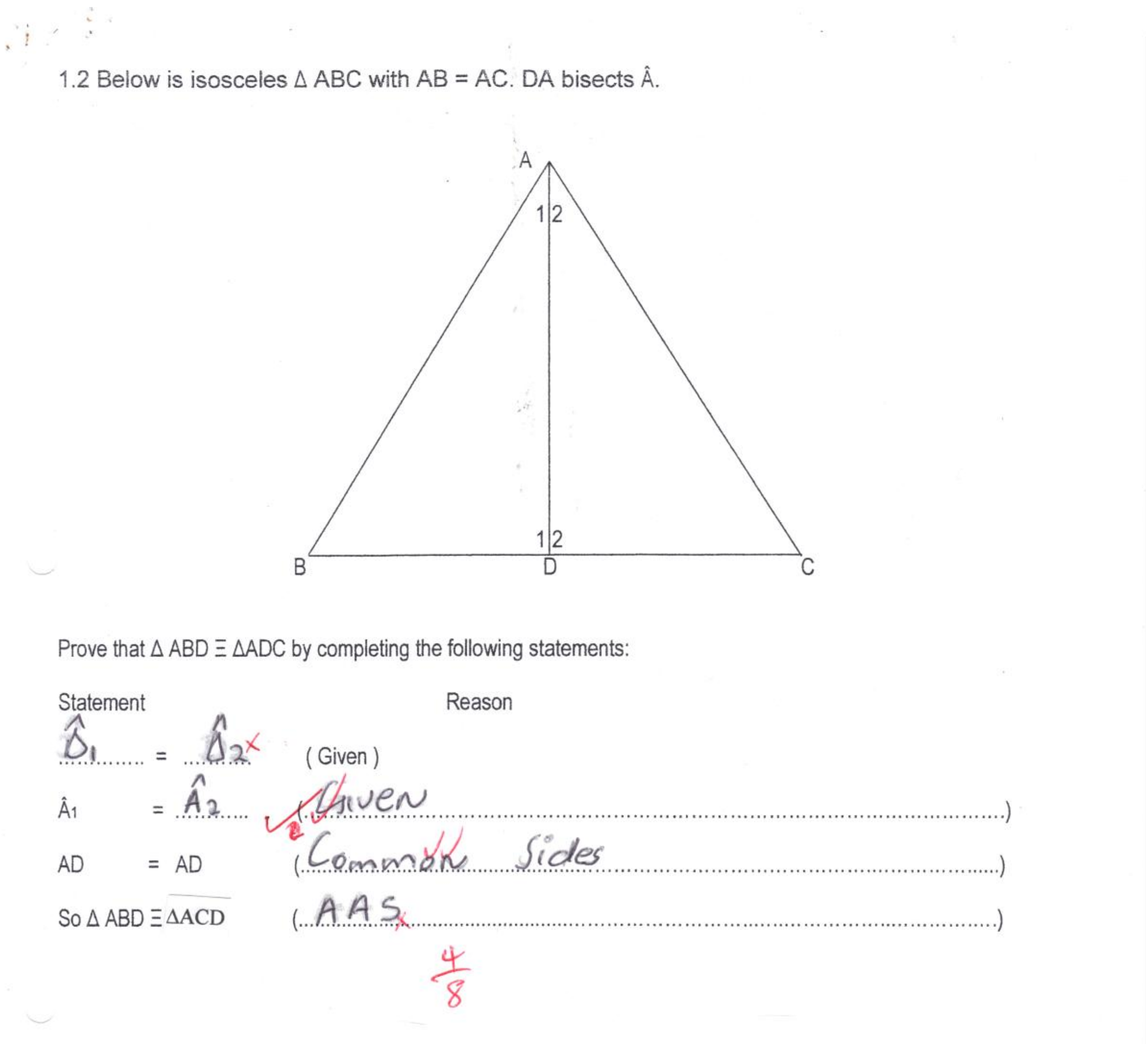
Researcher: *"Tell me how did go about answering this question."*
Learner A_2: *"I filled in the missing answering."*
Researcher: *"What was first step that you followed?"*
Learner A_2: *"I looked at the diagram and I filled in."*
Researcher: *"Do you read through the instruction?"*
Learner A_2: *"Yes, this one"* (Pointing at the instruction.)

Learner A_2 pointed at "Prove that triangle ABD is congruent to triangle ADC" in the worksheet.

However he did not refer to the information: “Below is isosceles triangle ABC with AB = AC. DA bisects A.” In problem solving reading and understanding the problem are vital, especially in Euclidean Geometry. The learners in this case seemed to have skipped the first step of problem solving as outlined by Polya (1988), that is “Read and understand the problem”.

From the above dialogue the researcher could deduce that learners in school A seemed to be not understanding of the procedure to be followed when answering a Euclidean Geometry question. This problem may be contributing the poor performance of shown by mathematics learners in geometry in the FET phase.

In school B, fifty percent of the learners scored in Category C and fifty percent scored in Category B. This question seemed to have presented greater difficulty to learners in school A than to learners in school B. The analysis of the responses of school B showed that the learners were first going through the “given” part, before answering the question. Most of the learners could see that AB = AC as it was given. For $\hat{A}_1 = \hat{A}_2$, some provided reasoning of “*bisect*” while some responded by writing “*given”*. A positive note, was that the learners' apparent ability to identify the common side when proving congruency? A written response for learner B_3 appears in figure 4.


1.2 Below is isosceles Δ ABC with AB = AC. DA bisects Â.

Prove that Δ ABD Ξ ΔADC by completing the following statements:

Statement	Reason
$\hat{D}_1$ = $\hat{D}_2$	(Given)
$\hat{A}_1$ = $\hat{A}_2$	(Given)
AD = AD	(Common Sides)
So Δ ABD Ξ ΔACD	(AAS)

4/8

FIGURE 4: Written response for learner B$_3$

The researcher spoke to learner B$_3$ to find out about his responses, in particular the reasoning as to why they could not provide the correct reason for $\hat{A}_1 = \hat{A}_2$.

Researcher: *"Show me where it is given that AB = AC."*

Learner B$_3$ *"Here."* (Pointing at the given part.)

Researcher: *"Why did you not provide the correct reason for $\hat{A}_1 = \hat{A}_2$?"*

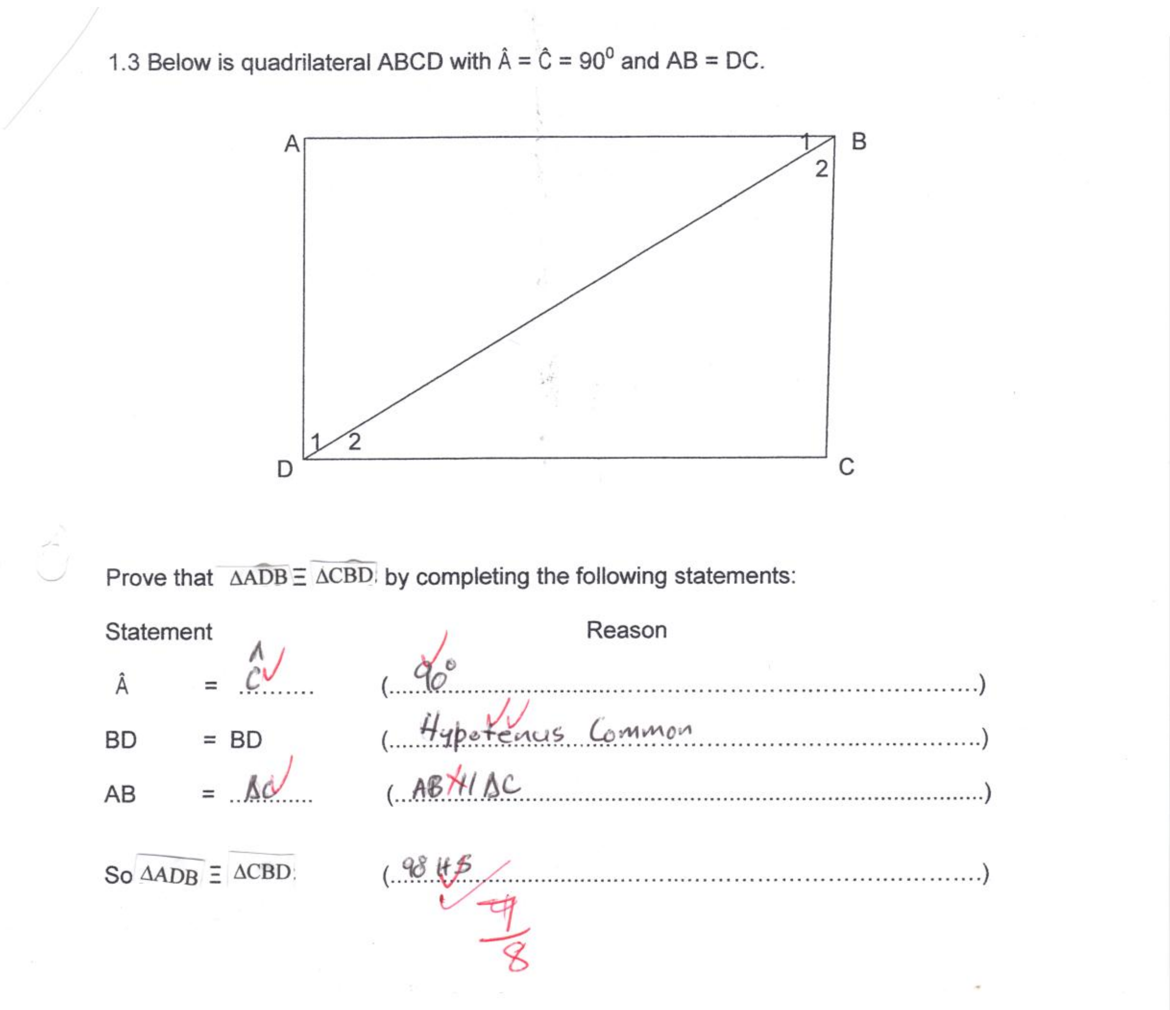
Learner B$_3$: *"I know $\hat{A}_1 = \hat{A}_2$."*

Researcher: *"What do you understand about the word "bisect"?"*

Learner B$_3$: *"Eh...... not sure."*

From the above, the researcher believed that this particular learner was unsure of the meaning of the word “bisect”. This may possibly be attributed to a language translation issue as the learners are taught by IsiZulu speaking teachers who normally use code-switching. Indeed, this finding reveals that code-switching is done in a way that disadvantages secondary English learners in mathematics teaching. At times, mathematics teachers may use code switching without actually giving the exact meaning of some of the technical terms they use. The use of precise meaning in defining some key words like “bisect” could also facilitate the learners’ understanding. For example, the teacher may need to join equal angles together using measurement and experimentation when describing the word “bisect”.

For task three in school A, eighty percent of the learners scored in Category B while twenty percent of them scored in Category C. A positive outcome of the responses of school A learners in this task was that they were able to provide the correct reason for angle A equal to angle C. Those who scored in Category C were the only two who gave correct reasoning for congruency; that is 90^0 H S as well as the reasoning for BD = BD (common side). In most responses similar misconceptions were observed, such as assuming that triangle ABD is congruent to triangle BDC. The case of 90^0 H S is rare in most geometry requiring proof of congruency. The response for learner A_2 who scored in Category C appears in figure 5.



1.3 Below is quadrilateral ABCD with $\hat{A} = \hat{C} = 90^0$ and AB = DC.

Prove that $\Delta\widehat{ADB} \equiv \Delta\widehat{CBD}$ by completing the following statements:

Statement			Reason
$\hat{A}$	=	$\hat{C}$	(90^0)
BD	=	BD	(Hypotenus Common)
AB	=	DC	(AB // DC)
So $\Delta ADB \equiv \Delta CBD$			(90 HS)

$\frac{4}{8}$

FIGURE 5: Written response for learner A_2

The researcher was eager to find out how the learner could identify the common side in this task whilst being unable to do so in the two previous tasks. Part of the dialogue appears below.

Researcher: *"In the two previous tasks you could not identify the common side and in this one you could. Why?"*

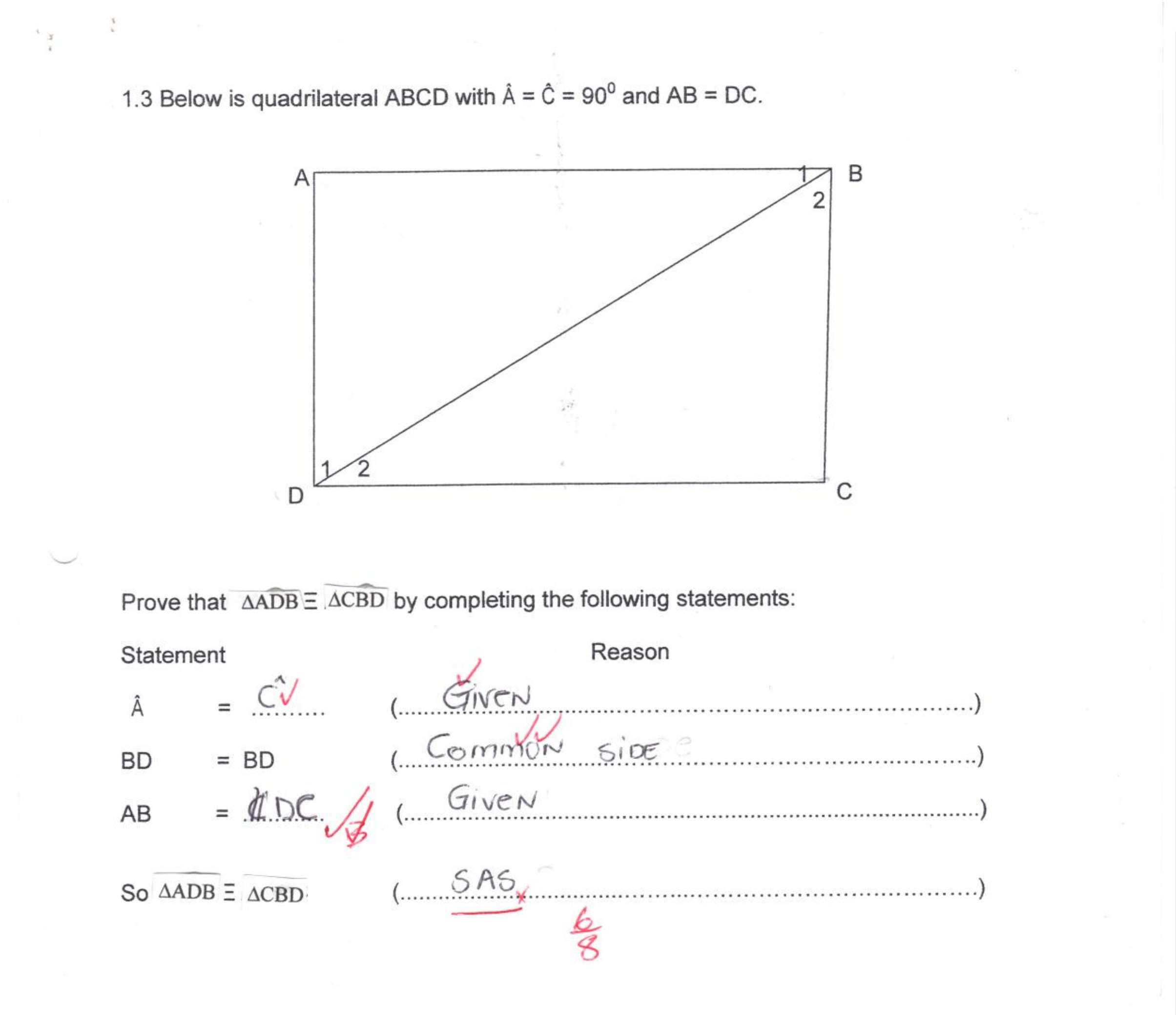
Learner A_2: *"I don't know, maybe I was confused. Geometry is difficult."*

Researcher: *"Why did you provide "90^0" as the reason for first statement?"*

Learner A_2: *"This is a corner and it is straight like house."*

The researcher suspected that this learner may potentially have been affected by the growing belief that geometry is difficult, whereby, as learners tackle geometry questions, their confidence seems to decrease. Again, the misconception arising from visualisation was observed. The learner seemed to believe that the corner of a quadrilateral which looks like a rectangle is a right angle. The researcher suspects that teachers rarely teach practical examples of angles that are close to 90^0, like 88^0, 92^0, etc. In these practical examples learners can observe that angles closer to 90^0 may seem like a "house corner" when looking at a glance, without actually measuring or using the given information, thereby answering incorrectly.

For task three in school B, seventy percent of the learners scored in Category C and thirty percent scored in Category B. In school B, it was positive to note that the answers provided by the learners were almost all correct. However, the learners were unable to provide proper reasoning for congruency of 90^0 H S; instead they gave A S S. A response for learner B_2 shown in figure 6.



1.3 Below is quadrilateral ABCD with $\hat{A} = \hat{C} = 90^0$ and AB = DC.

Prove that $\Delta A\widehat{D}B \equiv \Delta C\widehat{B}D$ by completing the following statements:

Statement		Reason
$\hat{A}$	= $\hat{C}$ √	(Given √)
BD	= BD	(Common √√ side)
AB	= DC √	(Given)
So $\Delta A\widehat{D}B \equiv \Delta C\widehat{B}D$		(SAS ×)

$\frac{6}{8}$

FIGURE 6: Written response for learner B_2

The dialogue between the researcher and learner B_2 about the written response occurred as follows:

Researcher: *"Can you name the four cases of congruency."*
Learner B_2: *"SIDE SIDE SIDE; SIDE ANGLE ANGLE."*
Researcher: *"You can't remember, can you?"*
Learner B_2: *"Sometimes I forget, but I know congruent triangles in maths."*
Researcher: *"What does this mean?"* (Pointing to A.S.S., reasoning for congruency.)
Learner B_2: *"Angles equal Sides equal Sides equal."*

The researcher suspected that these learners had done few (if any) examples on congruency involving 90^0 H S. The responses from the interview show that rote learning in mathematics classrooms still prevails; this form of teaching sometimes occurs when the teacher can see that the learners do not understand, or when it is difficult to explain the concepts. Learning congruency needs a lot of understanding, more especially in order to explain why 90^0 H S is not like S A S. In these circumstances, practical examples may better assist in learning congruency. The response for the last question appears to indicate that the learning and teaching of congruency rarely involved 90^0 H S in school B. It is possible that the learner observed that the triangle had two pairs of sides equal and one equal angle each. Hence, the learner responded with S A S. It is important in teaching to emphasise that the angle must be an included one.

For task four, thirty percent of school A learners scored in Category A; sixty percent scored in Category B and ten percent scored in Category C. It was worth noting that some learners could provide the correct reason for angle A equal angle C, that is “given”. However, some learners (like learnerA_2) could not identify equal alternate angles for parallel lines. The confusion of congruency was again evident in a manner similar to other responses recorded. A further example of this can be seen in the response for learner A_2 in figure 7.

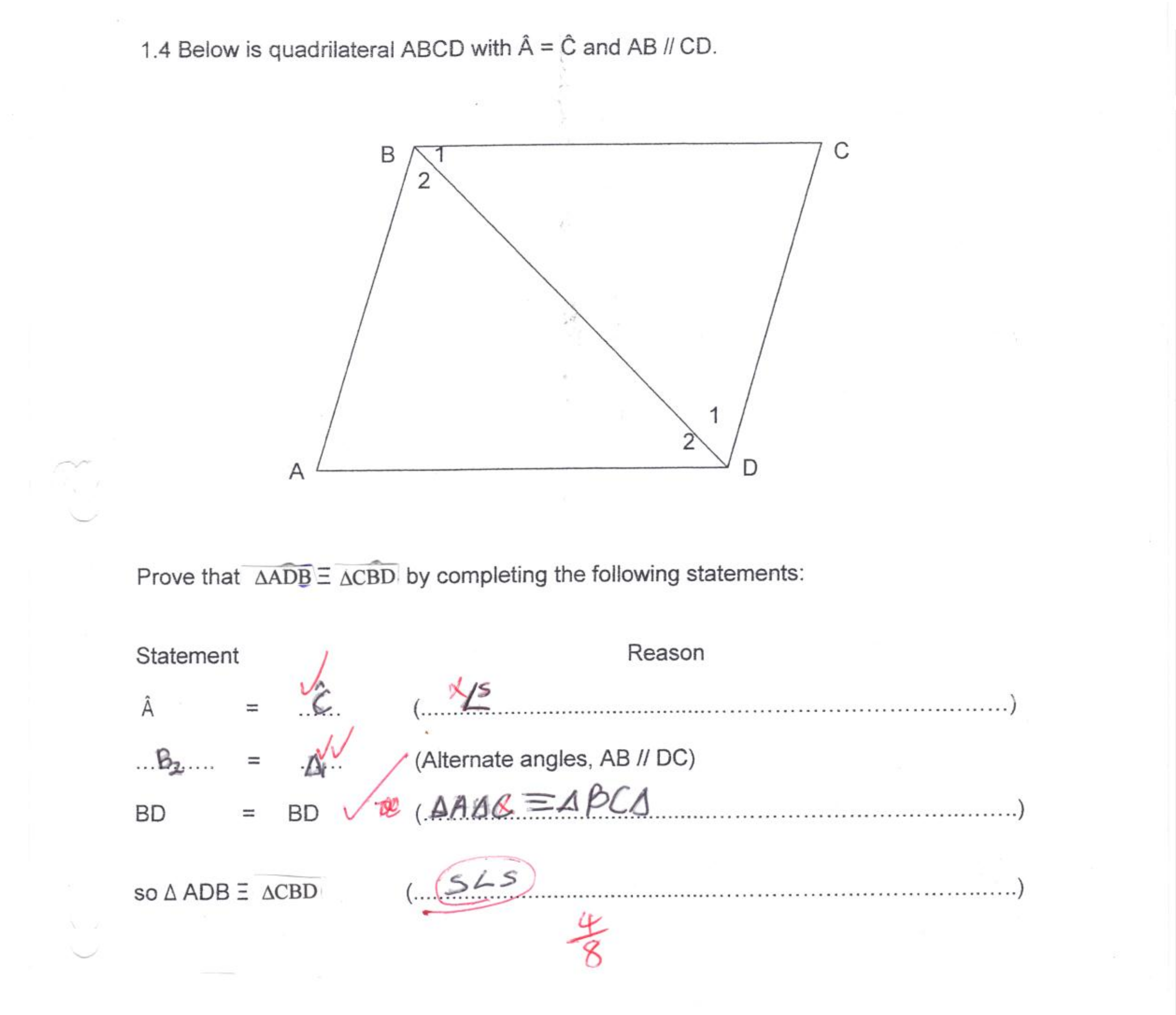


FIGURE 7: Written response for learner A_2

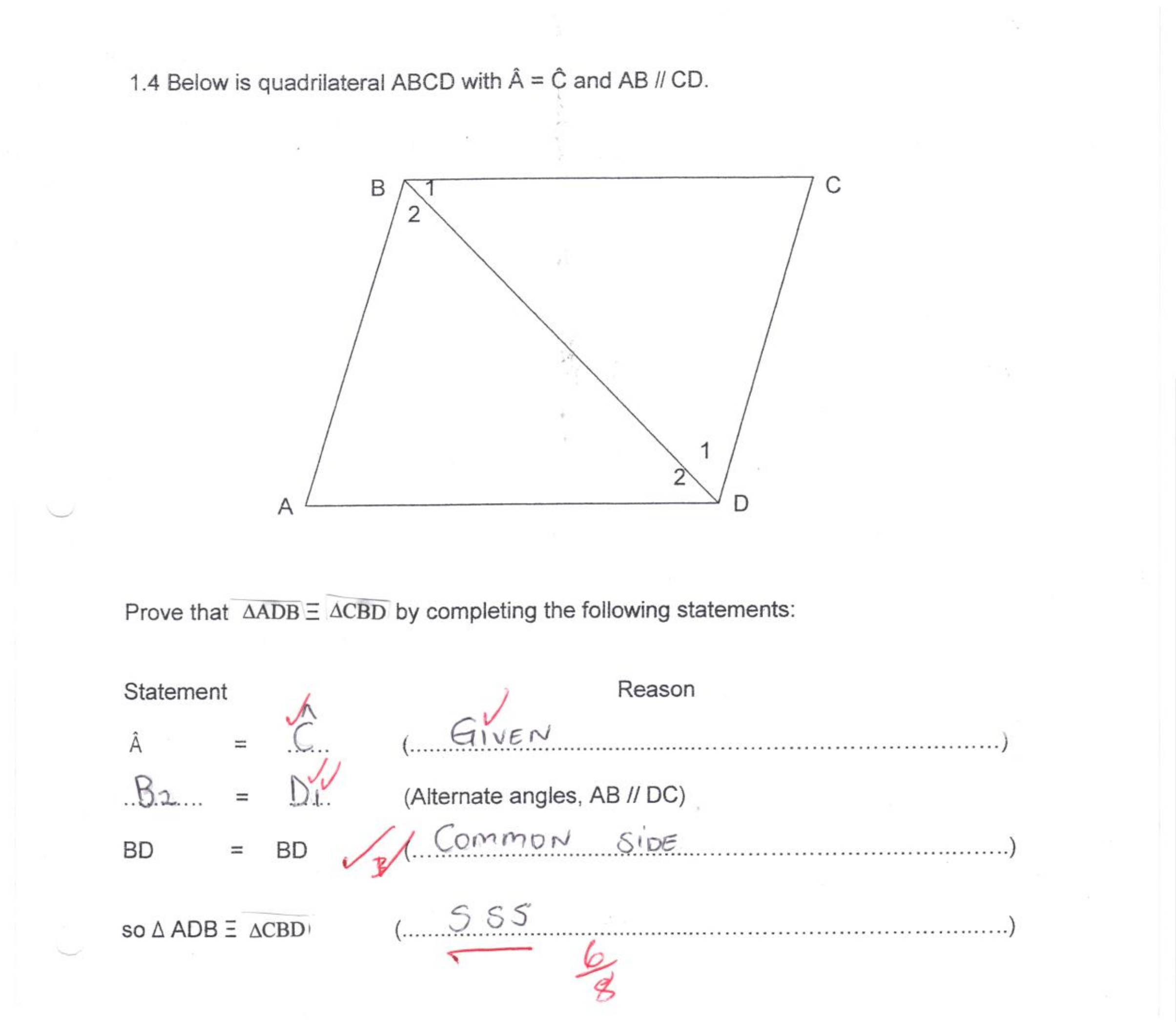
This learner seemed not to have a clear understanding of the difference between an angle and a side, as in concluding congruency, the learner provided S A S instead of A A S. The introductory stage of Euclidean Geometry teaching is important as it is at that point where such concepts should be defined using practical and concrete examples. A dialogue with learner A_2 revealed that some learners could correctly identify alternate angles in grade ten.

Researcher: *"If parallel lines are cut by a transversal, what equal angles result there?"*
Learner A_2: *"Eehh, correspondence, alternate and straight angle."*
Researcher: *"Can you identify alternate angles that are equal in this diagram, now?"*
Learner A_2: *"$B_2 = D_2$ and $B_1 = D_2$."*

The researcher observed that, although the learners did learn about parallel lines in grade nine, their understanding needed some improvement. The learner gave $\hat{B}_1 = \hat{D}_2$ as they are alternate angles. A misconception is that all alternate angles are equal. This problem could be solved using examples of non-parallel lines cut by a transversal, where alternate angles would not be equal. Most learners come across the word “alternate angles” only when learning parallel lines in which the lines will be equal, with the result that learners misunderstand and assume that all alternate angles are equal. As in question one, the researcher assessed the knowledge of congruency, with the learners having to deal with parallel lines and alternate angles. The researcher could not be certain whether or not learners understood the A A S case of congruency. The researcher’s personal experience in high school geometry teaching appeared to show the trend that at early high school stages most learners find it difficult to learn Euclidean Geometry to the expected relevant van Hiele level at a particular grade.

In addition, it was observed that the written response of learner A_2 seemed to indicate that the learner did not understand the logic used in a proof. Learner A_2 used congruency as substantiation for BD = BD although the learner had not reached making a conclusion that the triangles were indeed congruent.

Thirty percent of learners in school B scored in Category B and seventy percent scored in Category C in task four. This was the pattern in almost all the four tasks for this school. The researcher believed there were differences in instructional approaches to Euclidean Geometry between the two mathematics teachers of the two schools. The performance of learner B_3 seems to support this belief as it can be seen in figure 8.



1.4 Below is quadrilateral ABCD with $\hat{A} = \hat{C}$ and AB // CD.

Prove that $\Delta ADB \equiv \Delta CBD$ by completing the following statements:

Statement			Reason
$\hat{A}$	=	C	(GIVEN)
B_2	=	D_1	(Alternate angles, AB // DC)
BD	=	BD	(COMMON SIDE)
so Δ ADB $\equiv$ ΔCBD			(S S S)

6/8

FIGURE 8: Written response for learner B_3

Learner B_3 seemed to lack some understanding of how to give a reason congruency. It is possible that learner B_3 simply guessed the reason for congruency in this task. As the four cases of congruency can be learnt thorough rote learning, they can thus be guessed. Learner B_3 may have concluded that, as the two triangles are congruent, one of the cases would be the reason. A discussion between the learner and the researcher is provided below.

Researcher: *"What does this answer mean to you?"* (Pointing at the last response of S S S.)
Learner B_3: *"It means sides are equal, angles are equal, sides are equal."*
Researcher: *"Can you show me these equal sides and angles in your answers."*
Learner B_3: *"this, and this and this."*(Pointing from the first down.)
Researcher: *"Which ones are sides and which ones are angles?"*

Learner B_3: *"this is angles, angles, sides."* (Pointing correctly.)

The researcher believes that the learner guessed the reason for concluding congruency. However, it was positive to note that learner B_3 could differentiate between a side and an angle. Congruent triangles are easy to learn through memorization, but this does not aid learner comprehension as the learners may end up guessing cases in difficult times when they do not know the answer.

The performance of learners in formal reasoning has been analysed above. A pattern of poor performance in school A and a pattern of good performance in school B were observed. This reveals that whilst some learners in certain schools benefit from successful teaching, learners in other schools suffer from ineffective teaching. This conclusion is derived on the basis of the general trend in written responses collected from each school. The similar responses were (the researcher believes) as a result of learning taking place in class taught by the same teacher (in each school). The issue of ineffective mathematics teaching in mathematics has been noted in many studies, where even the teachers themselves appear not to adequately understand the material they teach (Adler, 1991).

5.4 The coding procedure for questionnaire two

A convenient coding system was used to rate the quality of responses in both schools for the questionnaire addressing informal reasoning. Tasks 1 to 4 were marked out of 12 marks each. The performance of learners was rated from 12 to 0 as shown in Table 3.

Table 3: Coding mechanism

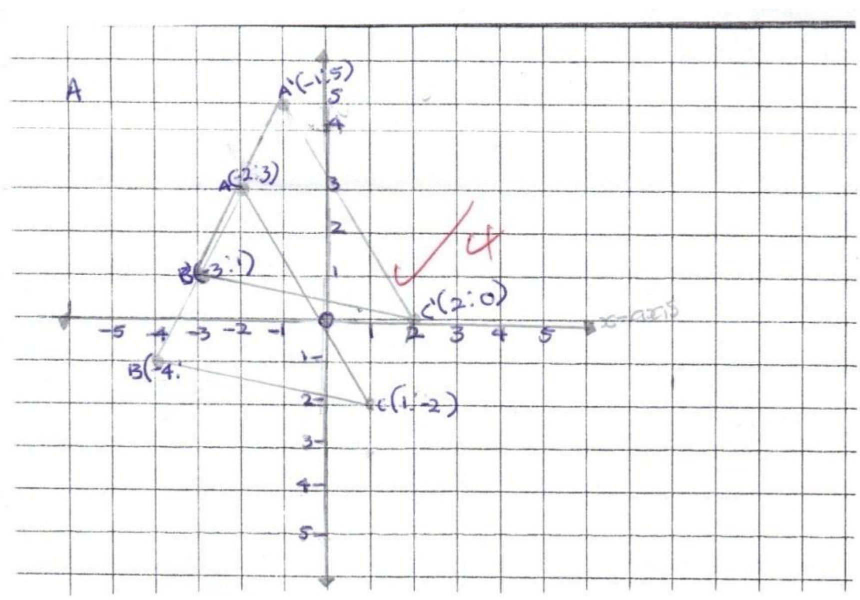
Mark Range	Catergory	Descriptor
0 – 3	A	Poor
4 – 5	B	Average
6 – 8	C	Good
9 – 12	D	Excellent

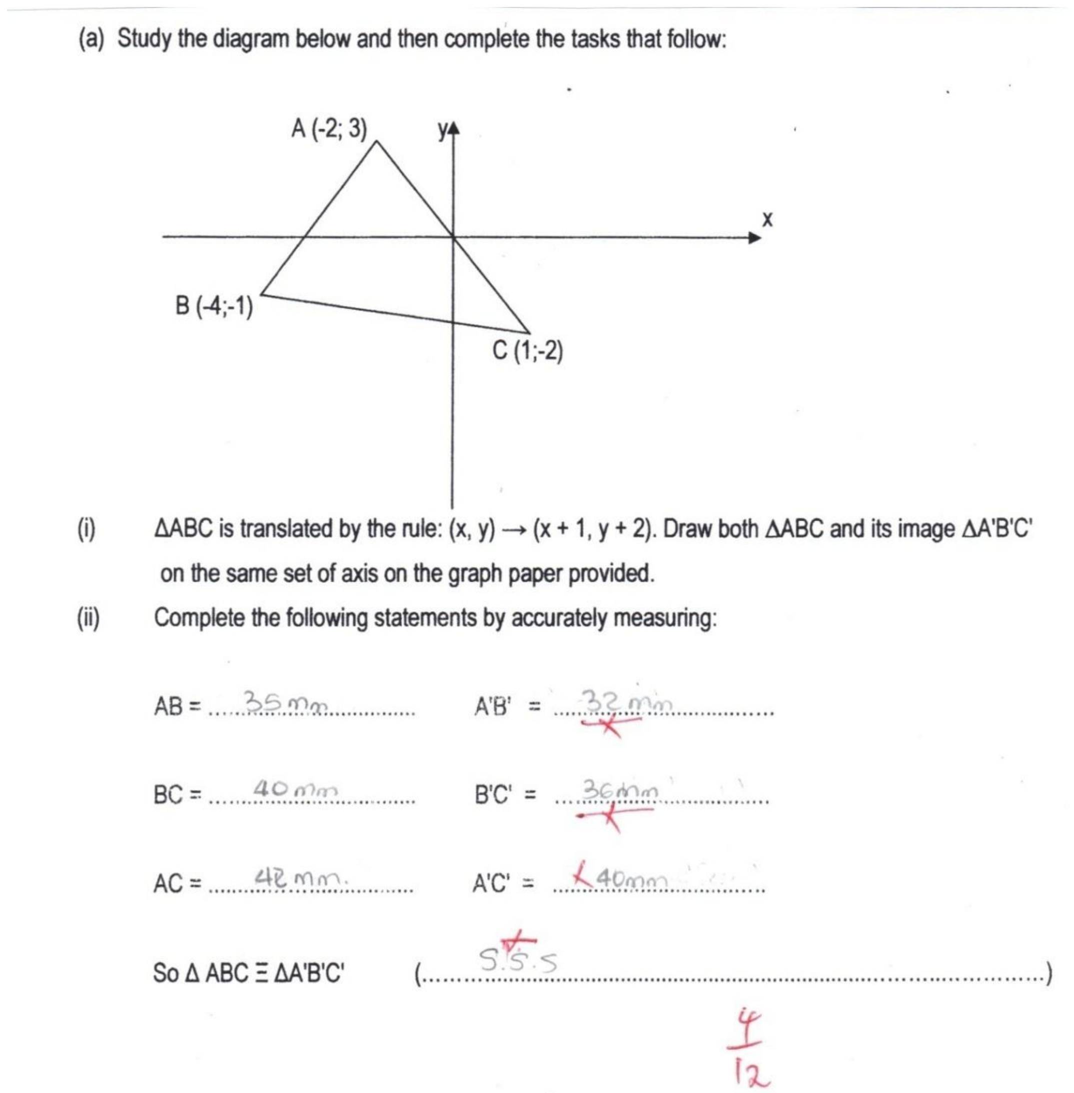
The results for the two schools in this questionnaire follow in Table 4.

Table 4: Performance of learners for tasks on informal reasoning

Marks	0 – 3	4 - 6	7 - 9	10 -12
School A Task 1		4	1	5
School B Task 1	4	4	2	
School A Task 2		2	4	4
School B Task 2	4	4	2	
School A Task 3	2	5	1	2
School B Task 3	7		3	
School A Task 4	2	3	5	
School B Task 4	7		3	

In task one, fifty percent of the learners from school A scored in Category D, while forty percent scored in Category B. Most of the learners were able to do draw the diagram and its image on the Cartesian plane. A negative aspect of the responses was that some learners presented incorrect measurements of the sides and were thus unable to see that the corresponding sides of the two triangles were unequal, contrary to the intention of the questionnaire. The response for learner A_2, who was more successful in formal reasoning than in the other tasks, is shown in figure 9.




(a) Study the diagram below and then complete the tasks that follow:

(i) ΔABC is translated by the rule: (x, y) → (x + 1, y + 2). Draw both ΔABC and its image ΔA'B'C' on the same set of axis on the graph paper provided.

(ii) Complete the following statements by accurately measuring:

AB = 35 mm A'B' = 32 mm

BC = 40 mm B'C' = 36 mm

AC = 42 mm A'C' = 40 mm

So Δ ABC ≡ ΔA'B'C' (S.S.S)

4/12

FIGURE 9: Written response for learner A_2

Learner A_2 provided a correct reasoning for concluding that triangle ABC was congruent to triangle A'B'C', despite incorrect measurements. A possible explanation for this may have been that the learner could see that only the sides are dealt with in the scaffolded task and that the reason for congruency was thus S, S, S.

A discussion between the researcher and learner A_2 appears below:

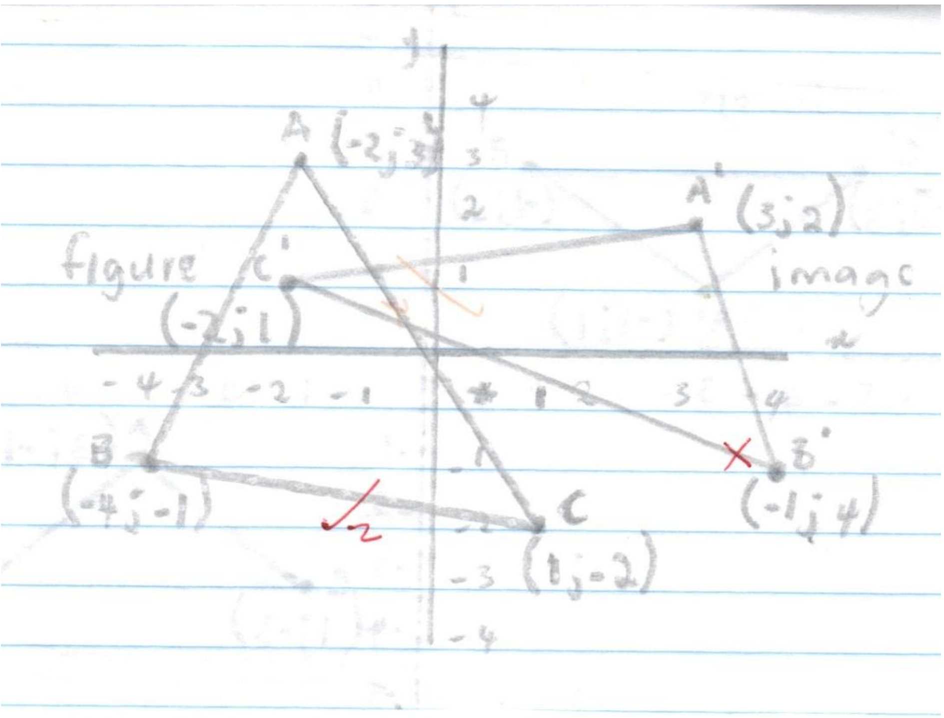
Researcher: *"How do you think you missed to find correct measurement with you ruler?"*
Learner A_2: *"Mistake."*
Researcher: *"Do you know how to measure a line, anyway?"*
Learner A_2: *"Yes."*

Researcher: *"Why did put S S S in this reason".* (Pointing at the last reason.)
Learner A_2: *"All three sides are equal."*

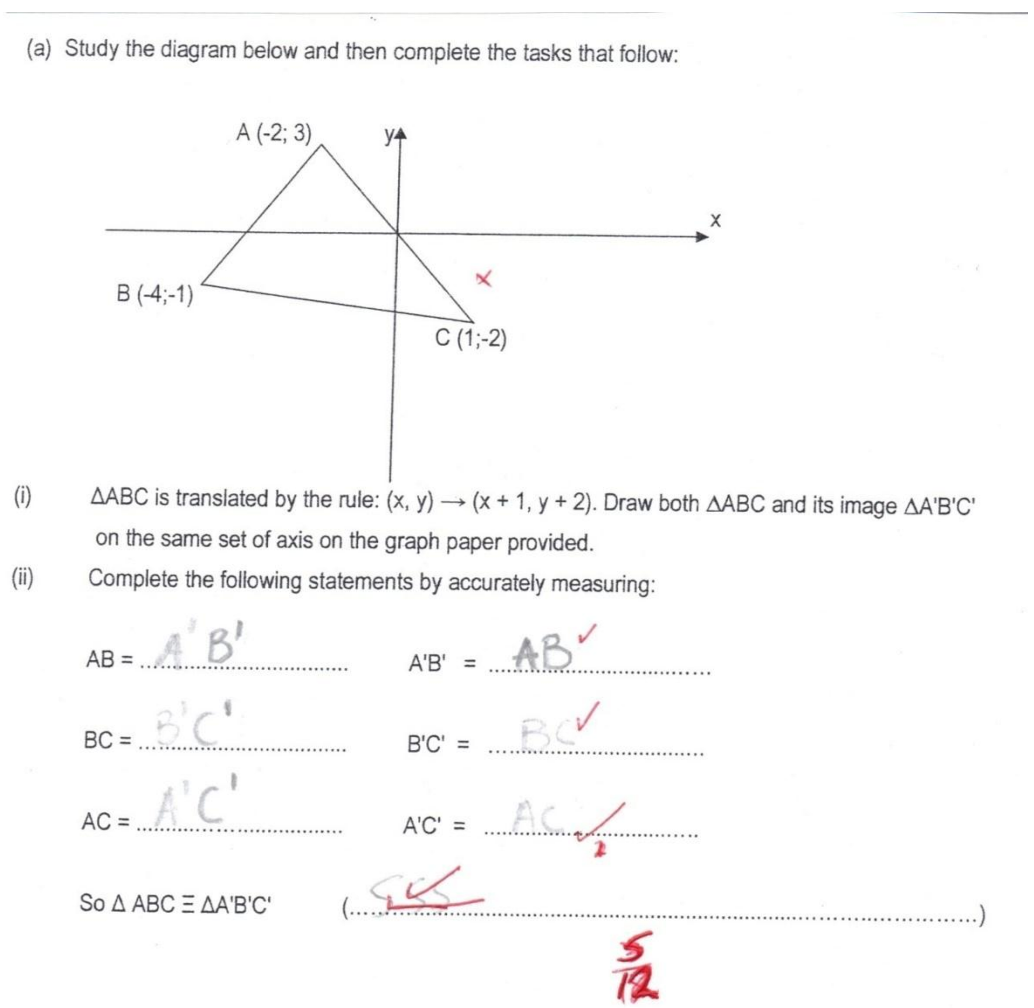
The researcher could see that the learner likely meant that the corresponding sides were equal as the learner did not mention angles. Learner A_2 seemed to have understood translation on a Cartesian plane which is taught from grade nine. However, learner A_2 seemed unable to do practical measurements of the sides of the two diagrams which are generally taught at the lower grades at primary school. This apparent dislocation may show potential knowledge gaps that learners suffer before their arrival at high school. Practical activities at primary school level are important as it is at this level that learners learn to perform basic proofs of equal quantities like sides and angles. Again, learner A_2 seemed to have used visualisation in concluding that the sides were equal. Learner A_2 might have inherently realised that the translation of triangle ABC was a rigid one.

For task one on informal reasoning, forty percent of school B learners scored in Category A, forty percent scored in Category B and twenty percent scored in Category C. However, it is important to point out that during the discussions undertaken before the study began concerning the questionnaires, the learners indicated that they had only recently started Transformation Geometry with their teacher and that they had not yet been introduced to drawing figures on squared graph paper. These learners said that they had finished studying geometric figures in Analytical Geometry. As a result, an agreement was made whereby the learners would redraw the image on the given Cartesian plane and find the required lengths of sides using the distance formula. Once this was completed, the learners would then confirm the results through measurements.

Most of the learners seemed to have trouble drawing the correct image as most of them drew the incorrect figure. However, a positive result was that most of the learners appeared capable of drawing the original figure, which seems to indicate that they had a good understanding of the coordinates system. An explanation for their inability to draw the correct image may be that they misunderstand translation, a key concept of transformation which successful completion of the question required. Leaner B_3 seemed to have confused reflection of C in the line y = x, reflection of A and B about y = -x and translation because as the coordinates were swapped as shown in figure 10.



A (-2; 3)
A' (3; 2)
figure
C'
(-2; 1)
image
B
(-4; -1)
C
(1; -2)
B'
(-1; 4)


(a) Study the diagram below and then complete the tasks that follow:

(i) ΔABC is translated by the rule: (x, y) ⟶ (x + 1, y + 2). Draw both ΔABC and its image ΔA'B'C' on the same set of axis on the graph paper provided.

(ii) Complete the following statements by accurately measuring:

AB = A'B' A'B' = AB

BC = B'C' B'C' = BC

AC = A'C' A'C' = AC

So Δ ABC Ξ ΔA'B'C' (SSS)

5/12

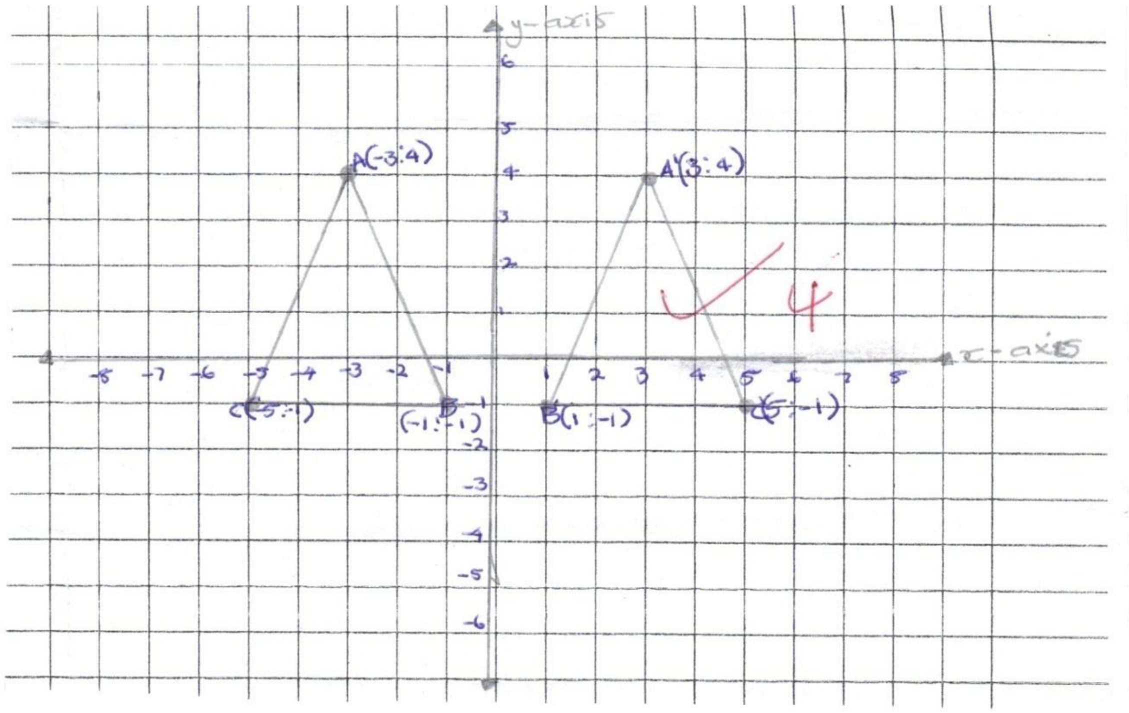
FIGURE 10: Written response for learner B_3

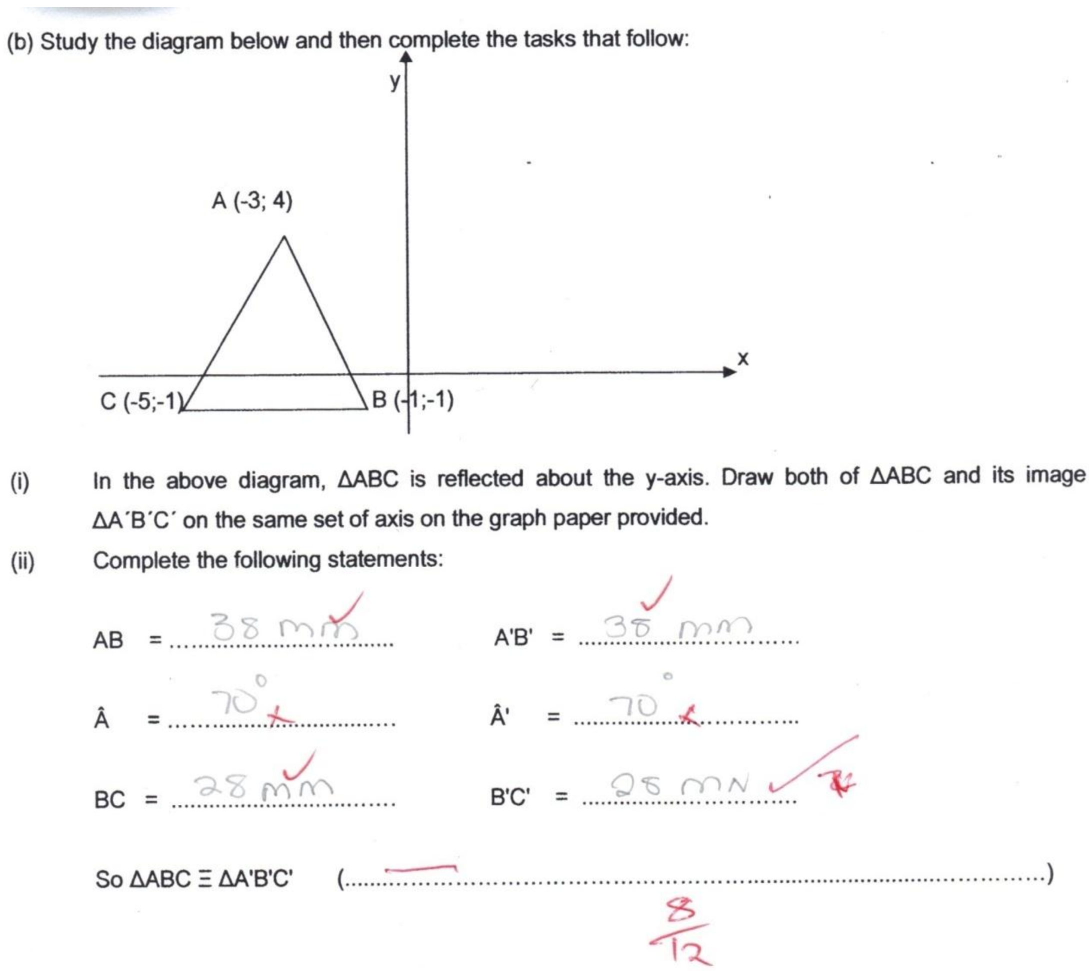
The answers presented by the learner prompted the researcher to believe that the learners had used Analytical Geometry. Instead of putting numerical values of the sides learner B_3 put equal sides. The researcher spoke to learner B_3 about these responses.

Researcher: *"Do you understand translation since you started learning it?"*
Learner B_3: *"A little but not sure."*
Researcher: *"Explain how arrived at the answers in (ii)."*
Learner B_3: *"I used this."* (Showing the Casio calculator.)
Researcher: *"What was the length of AB and A'B'?"*
Learner B_3: *"2√5"*

From the dialogue above, the researcher noted that translation appeared to be inadequately understood in school B. This may be due to the fact that the learners had only recently commenced learning the section of Transformation Geometry. What appeared positive from the learners responses was that the learners seemed capable of using other informal methods to prove that the sides were equal; in this case using Analytical methods. The use of Analytical methods in proving congruency was another form of reasoning. As a result, the researcher wonders if, perhaps, congruency should be incorporated in all geometry.

In task two of informal reasoning, forty percent of school A learners scored in Category D; forty percent in Category C and twenty percent in Category B. This performance is better than what they did in formal reasoning tasks. A response for learner A_2 who scored in Category B (average performance) in formal reasoning tasks is shown below in figure 11.





(b) Study the diagram below and then complete the tasks that follow:

(i) In the above diagram, ΔABC is reflected about the y-axis. Draw both of ΔABC and its image ΔA´B´C´ on the same set of axis on the graph paper provided.

(ii) Complete the following statements:

AB = 38 mm ✓ A'B' = 38 mm ✓

Â = 70° ✗ Â' = 70° ✗

BC = 28 mm ✓ B'C' = 28 mN ✓

So ΔABC ≡ ΔA'B'C' (—)

8/12

FIGURE 11: Written response for learner A_2

Most learners correctly drew both diagrams on a Cartesian plane with the result that they may have been successful in learning reflections in their study of transformations. Learner A_2 seemed to be unable to use a protractor when measuring angles. This is another potential example of the importance of doing practical measurements of angles at lower grades, indeed, Learner A_2 could not provide a reason for concluding congruency. The misconception may be that congruency in Euclidean Geometry did not mean the same as congruency in Transformation Geometry. It is therefore important that all sections in mathematics teaching be linked (especially when a concept like congruency could be taught using more than one section).

The discussion between the researcher and learner A_2 is presented below.

Researcher: *"Do you enjoy learning Euclidean Geometry? Why?"*

Learner A_2: *"Yes, because we measure angles and side."*

Researcher: *"Do you feel that teachers can use other ways to teach Euclidean Geometry? Explain."*

Learner A_2: *"Yes, by making more examples so that we understand."*

Researcher: *"In questionnaire number two, what types of transformations are displayed in each case?"*

Learner A_2: *"Translation; reflection; rotation and reflection."*

Researcher: *"Which type of transformation is the easiest to work with? Why?"*

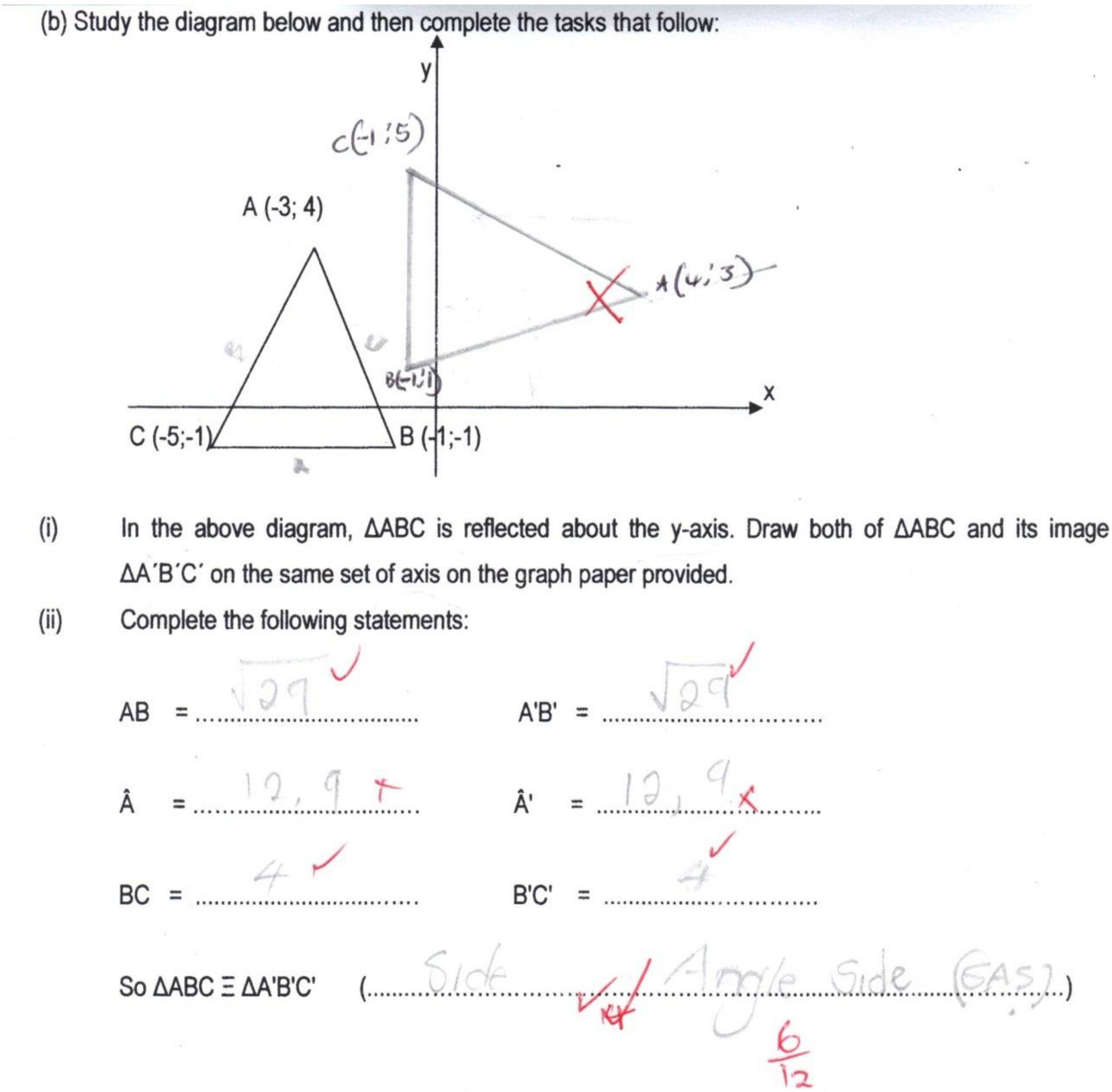
Learner A_2: *"Reflection not complicated."*

Researcher: *"Would you prefer using Transformation Geometry when ideas in Euclidean Geometry?"*

Learner A_2: *"yes."*

From the above dialogue, learner A_2 seemed to be enjoying Transformation Geometry and congruent triangles. This learner did not correctly measure the size of the angles but did indicate that they enjoyed performing such activities in Euclidean Geometry. Again, learner A_2 may have been treating Euclidean and Transformation Geometry as two separate concepts.

The learners in school B appeared to have a problem with the fundamental starting point in understanding congruency in Transformation Geometry; that is in their understanding of the different kinds of transformation. In task two, forty percent of the learners scored in Category A, forty percent in Category B and only twenty percent in Category C. No one scored in Category D. An example of a response for learner B_4 (who scored in Category B) provided in figure 12 seems to reveal that most of the learners in school B required some reinforcement of the concepts of transformations.


(b) Study the diagram below and then complete the tasks that follow:

(i) In the above diagram, ΔABC is reflected about the y-axis. Draw both of ΔABC and its image ΔA′B′C′ on the same set of axis on the graph paper provided.

(ii) Complete the following statements:

AB = √29   A'B' = √29

Â = 12,9   Â' = 12,9

BC = 4   B'C' = 4

So ΔABC ≡ ΔA'B'C' (Side Angle Side (SAS))

6/12

FIGURE 12: Written response for learner B_4

In this specific question, the learner was required to consolidate angles and sides. However, instead of measuring the angle, the learner seemed to have calculated side BC. A possible explanation for this was that the learner believed they needed to calculate the third side. This potentially shows that the learner may have believed that angles are for Euclidean Geometry only and not for coordinate system. It was useful to note that most school B learners could provide correct reasoning for the case of congruency of triangles.

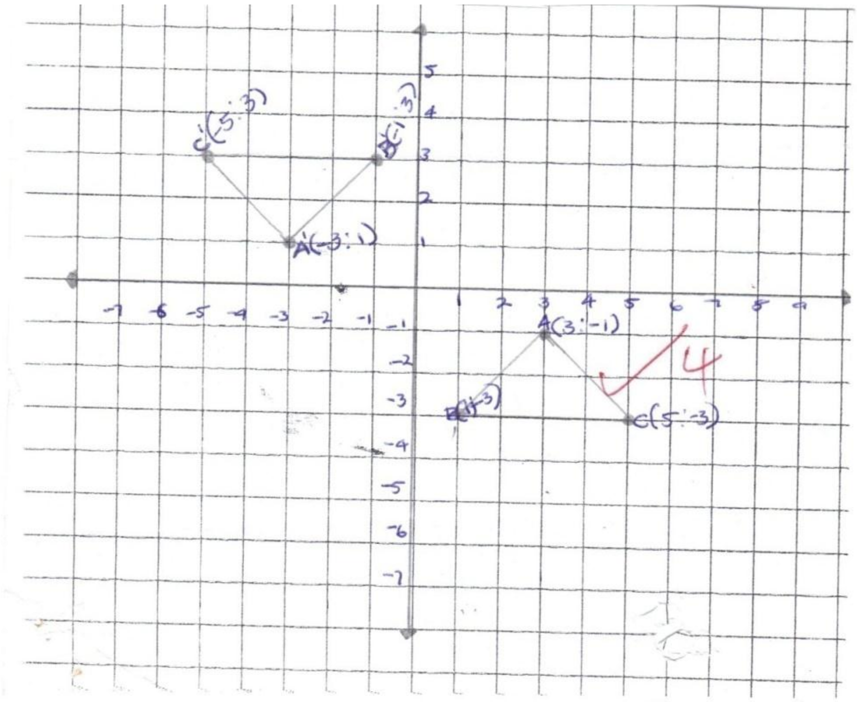
When asked about the knowledge of transformations, learner B_4 responded the following way.

Researcher: *"Name the kinds of Transformation displayed in each case in questionnaire two?"*
Learner B_4: *"Translation, reflection, rotation and reflection."*
Researcher: *"How did you arrive at this image?"* (In task two.)
Learner B_4: *"I was rotating."*
Researcher: *"Is this rotation?"*
Learner B_4: *"not sure."*

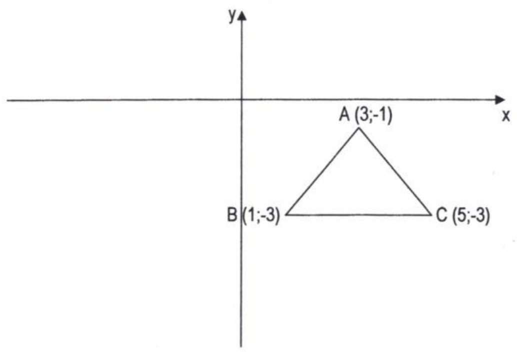
From these responses, the researcher believes that the learner seemed to have only a very basic understanding. A positive feature of the responses of learners from school B seemed to be that, as time went on; their understanding of understood Transformation Geometry appeared to increase. Indeed, once they grasped the concepts correctly they seemed to be able to work successfully with triangles.

In task three, fifty percent of school A learners scored in Category B, twenty percent in Category A and only twenty percent in Category D. This task thus appeared to be difficult to the learners and caused the researcher to feel that transformation involving rotation is potentially problematic for grade ten learners. However, a positive outcome of this task is that most of the learners seemed to be capable of correctly locating the image on the Cartesian plane.

Learner A_3 had a problem in measuring the size of an angle. As learner A_3 found angle C to be equal to 180^0, learner A_3's response appeared to indicated that this learner were not aware that the sum of the angles of a triangle is equal to 180^0. This can be seen in figure 13.



(c) Study the diagram below and then complete the tasks that follow:



(i) In the above diagram, ΔABC has been rotated through 180⁰ clockwise. Draw both ΔABC and its Image Δ A'B'C' on the same set of axis on the graph paper provided.

(ii) Complete the following statements by accurately measuring:

Â = ... Â' = ...

Ĉ = ... Ĉ' = ...

BC = ... B'C' = ...

So ΔABC ≡ Δ A'B'C' (..)

4/12

FIGURE 13: written response for learner A_3

The researcher spoke to learner A_3 about this task.

Researcher: *"What aspect of this task were you confident in answering? Why?"*

Learner A_3: *"number (i) because I know rotation."*

Researcher: *"What is the sum of the angles of a triangle?"*

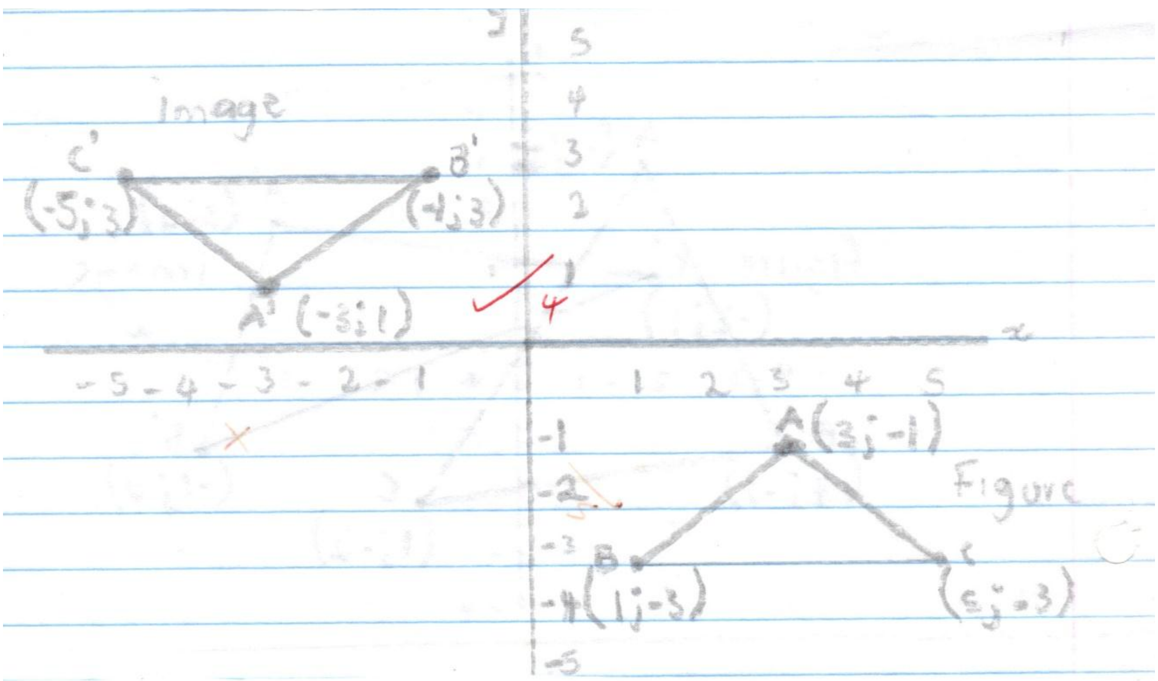
Learner A_3: (No answer).

Researcher: *"Show me how you measured angles A and C."*

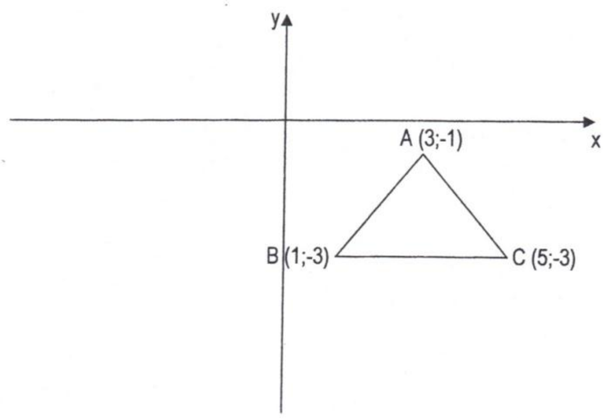
Learner A_3: (wrongly done).

The researcher noticed that the learner did not attempt to employ practical measurement of angles. It was also observed that many learners worked successfully with rotation. During interviews some of the learners appeared to enjoy learning transformations on a Cartesian plane, yet seemed unable to perform the activities related to the transformed shape.

School B learners scored poorly in task three as seventy percent of them scored in Category A. Only thirty percent of them could score in Category C. The researcher became interested in the responses for learner B_3 who scored in Category C because he correctly located the figure and its image on the Cartesian plane. The learner could not locate the image for task one on translation. Learner B_2's response in task three appears in figure 14.



(c) Study the diagram below and then complete the tasks that follow:



(i) In the above diagram, ΔABC has been rotated through 180^0 clockwise. Draw both ΔABC and its Image Δ A'B'C' on the same set of axis on the graph paper provided.

(ii) Complete the following statements by accurately measuring:

$\hat{A}$ = $\hat{A}'$ ✓ $\hat{A}'$ = $\hat{A}$ ✓

$\hat{C}$ = $\hat{C}'$ ✓ $\hat{C}'$ = $\hat{C}$ ✓

BC = B'C' ✓ B'C' = BC ✓

So ΔABC ≡ Δ A'B'C' (AAS ✓)

12/12

FIGURE 14: Written response for learner B_2

This learner might have used visualisation in proving congruency as both diagrams looked exactly the same. However, the learner had to observe that rotation through 180^0 did not alter the shape and size of the figure. To confirm this assumption the researcher spoke to learner B_2:

Researcher: *"How did you know that angle A equals angle A'?"*

Learner B_2: *"I measured."*

Researcher: *"Show how."*

Learner B_2: (done correctly.)

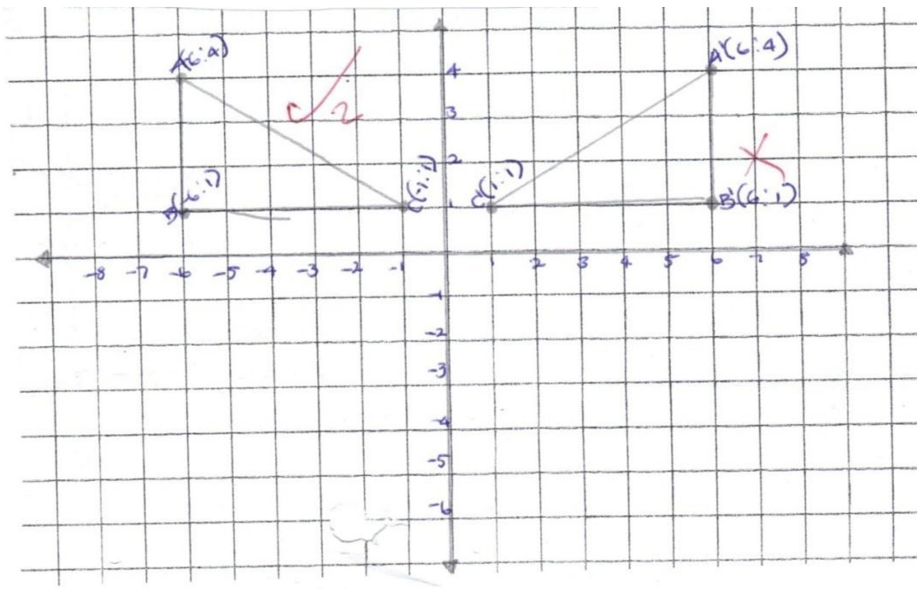
Researcher: *"(why did you not do well in (i) in task one involving translation?"*

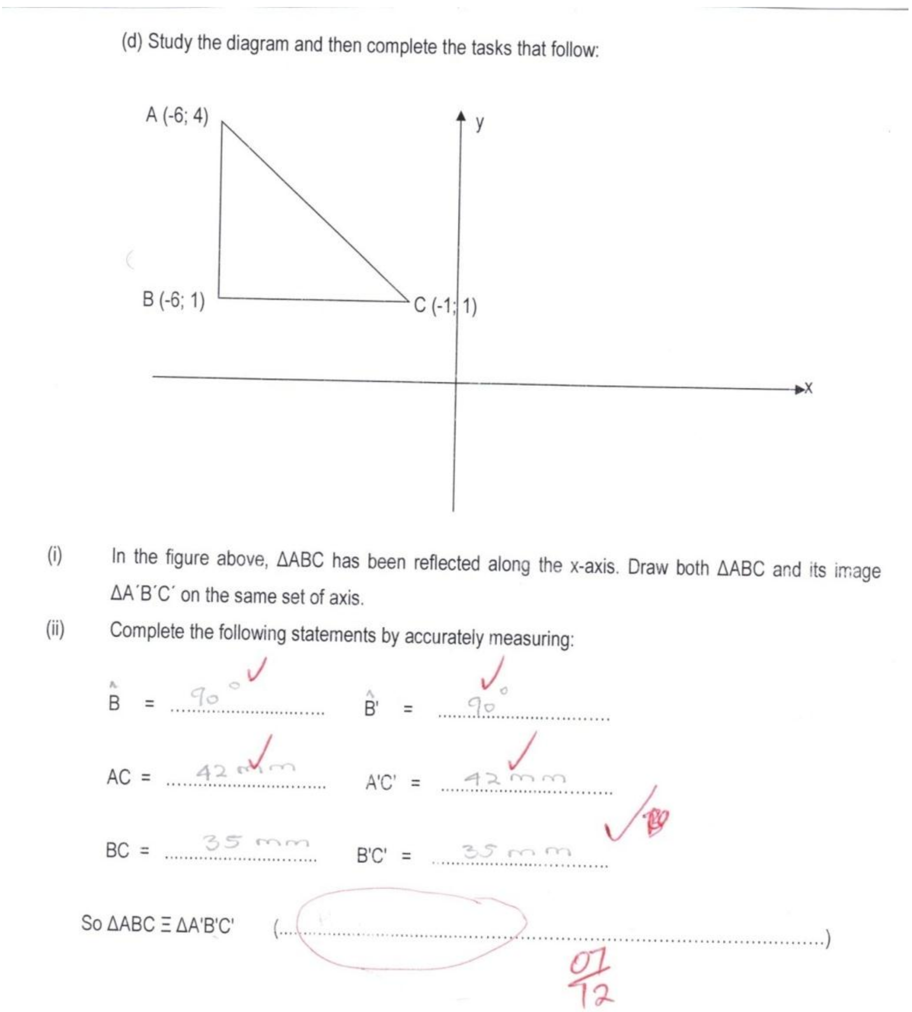
Learner B_2: *"complicated."*

Researcher: *"Are you comfortable with learning congruent triangles this way?"*
Learner B_2: *"Ja it helps."*

The researcher observed that the learner had employed effective strategies of measuring, that is extending the lines to enable correct reading on the protractor. The learner seemed to have a better understanding of rotation than translation. The researcher had a feeling that learners might have a better understanding of translation than that of rotation because in translation the both shapes remain upright whereas in rotation the image is sometimes tilted. Before the interview the researcher suspected that the learner used visualisation to arrive at conclusions. However, during the interview it appeared apparent that the learner had used practical methods in responding to the questions.

In task four (involving reflection in the x-axis), twenty percent of school A learners scored in Category A, thirty percent in Category B and fifty percent in Category C. None scored in Category D whilst most of the learners seemed able to locate the figure on the Cartesian plane; they located the image on the first quadrant as if it was the reflection in y-axis. It is possible that the learners were confusing the two transformations involving the concept of the reflection in the x- or y-axis. The responses of learner A_2 (who did well locating all the diagrams on the Cartesian plane, except for the image of task four) are shown below in figure 15.




(d) Study the diagram and then complete the tasks that follow:

(i) In the figure above, ΔABC has been reflected along the x-axis. Draw both ΔABC and its image ΔA´B´C´ on the same set of axis.

(ii) Complete the following statements by accurately measuring:

$\hat{B}$ = 90° $\hat{B}'$ = 90°

AC = 42 mm A'C' = 42 mm

BC = 35 mm B'C' = 35 mm

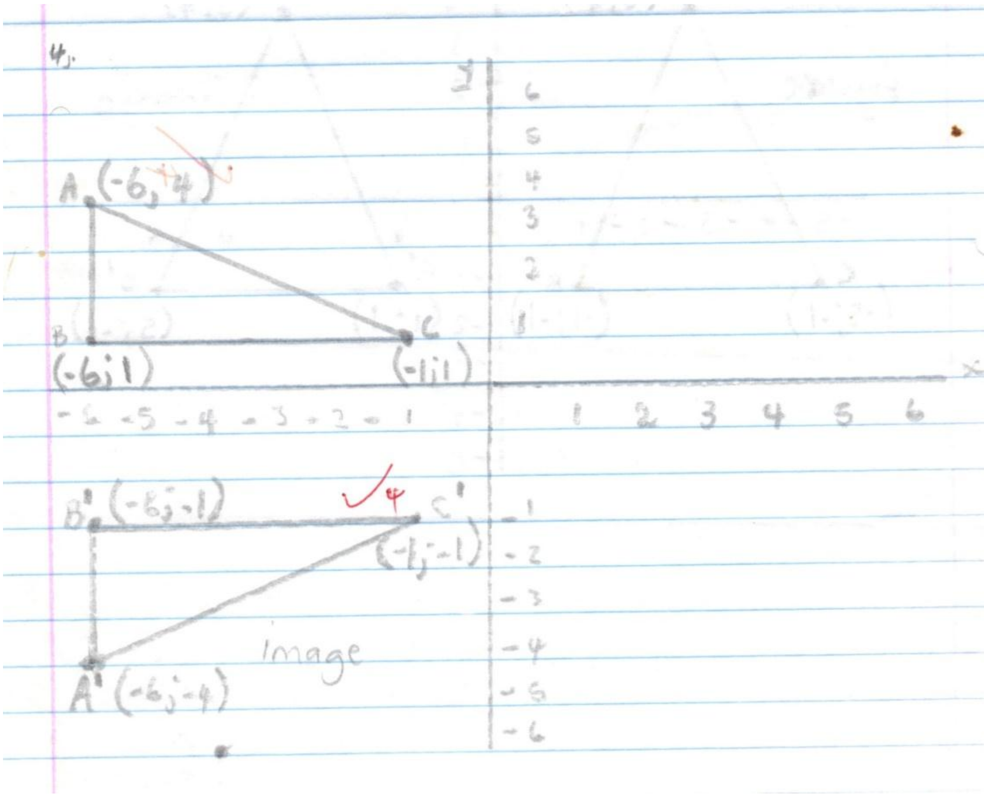
So ΔABC ≡ ΔA'B'C' (.......)

07/12

FIGURE 15: written response for learner A_2

Discussion between the researcher and learner A_2 revealed that most learners appeared to confuse transformations involving the reflection in the x-axis with the one involving the reflection in the y-axis. This may have resulted from the learners being still at the introductory stage of learning Transformation Geometry. This could mean they do not understand the meaning of reflection or are confusing the axes.

For task four in school B, seventy percent of the learners scored in Category A and thirty percent scored in Category C. From these results, it would appear that these learners were at the basic introductory stage in the learning of Transformation Geometry, and that the teacher seemingly had not introduced the section using squared paper. Instead, the teacher may have introduced the section using ideas of Analytical Geometry. The researcher was interested in the response of learner B_3 who was the only participant to correctly locate the image of the transformation for task four. Learner B_3's answers appear below in figure16.



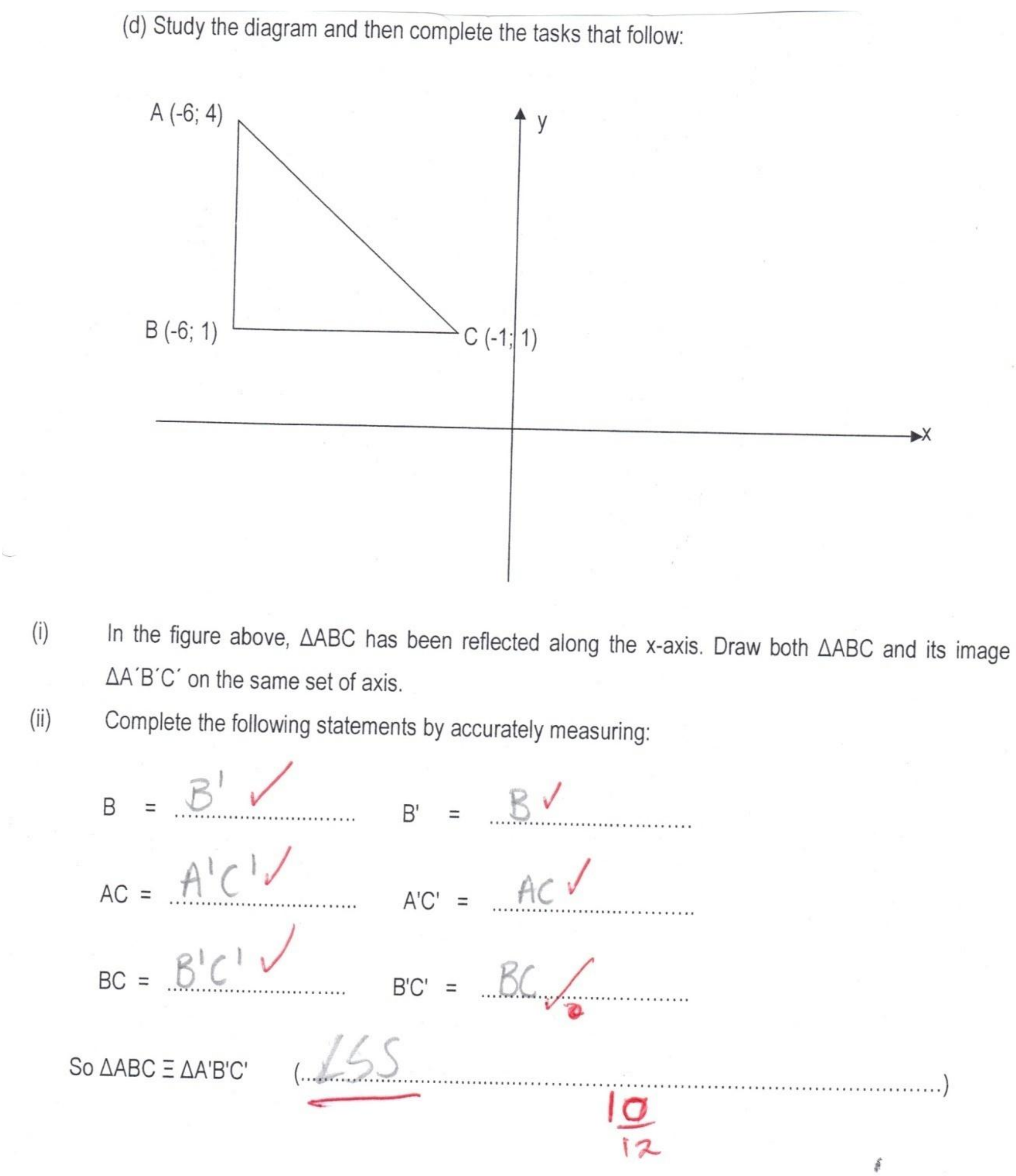


FIGURE 16: written response for learner B_3

Although learner B_3 was awarded full marks for the above task, he graduated the negative part of the y-axis incorrectly. Visually triangle A'B'C' "looks" as reflection. It is indeed a reflection of triangle ABC about the line y = -1. Learner B_3 could not detect his mistake since he possibly did not realise that the original shapes and its reflection should be the same distance away from the line of reflection.

Learner B_3's reasoning for congruency showed that the case of 90^0 H S is unfamiliar as the responses for the same case in formal reasoning in this case was incorrectly answered. It should also be noted that Learner B_3 is the same learner who could not locate the image of the transformation in task one, which involved translation.

The researcher spoke with learner B_3 about the learner's responses in task four:

Researcher: *"Do you think it is easy to learn congruent triangles while learning about Transformations?"*
Learner B_3: *"Ja."*
Researcher: *"Why do you think other learners don't understand?"*
Learner B_3: *"They will understand later."*

The above dialogue would seem to indicate that this learner was confident with learning transformations and congruent triangles; the learner even claimed that fellow classmates would be similarly successful in their understanding. Learner B_3 alludes to the point that more time and effort is required to obtain success in the learning of geometrical concepts. These responses suggest that some learners may benefit in the introduction of Transformation Geometry at high school in respect of the learning of congruent triangles. It could be argued that informal reasoning should be a stepping stone to the introduction of formal reasoning. When learners do not understand the complication of formal proofs the teacher could then consolidate with informal means.

5.5 Mean performance of the participants

The analysis of the responses of the learners in the questionnaires appears to show that, depending on how well learners understand Transformation Geometry, learners may be capable of successfully learning congruent triangles. The researcher was interested in finding the comparative mean performance of learners in both schools for each task as this would possibly provide an overview of how successful learners had been in understanding Transformation Geometry through their learning of the congruency of triangles. The overall performances in Euclidean Geometry, as well as in congruent triangles, could also be observed. Table 5 on the next page shows comparatively the mean performance of the two schools in each task of both formal and informal reasoning.

Table 5: Mean performance of schools for each respective task on both questionnaires

Mean	Formal reasoning	Informal reasoning
School A - Task 1	4	8,3
School B - Task 1	7	4
School A - Task 2	2,3	6
School B - Task 2	6	4
School A - Task 3	4,3	9
School B - Task 3	7	5,2
School A - Task 4	8,4	9
School B - Task 4	6,2	4

For each task, School B did well in questionnaire one (involving Euclidean Geometry) and did poorly in questionnaire two (involving Transformation Geometry). The converse appeared true for School A learners. The researcher believes that there were some instructional differences of mathematics in the two schools in terms Euclidean Transformation Geometry teaching as shown in Table 6 below where the overall mean performances of the two schools are comparatively analysed for the two questionnaires.

Table 6: Overall mean performance of the two schools for both questionnaires

School	Mean Performance: Formal reasoning	Mean Performance: Informal reasoning
School A	11.9	28.7
School B	23.8	13.3

The mean performance for school A in formal reasoning is almost two times lower than their mean performance in informal reasoning in all the tasks whilst the mean performance for school B in formal reasoning is almost two times higher that their mean performance in informal reasoning.

5.6 Conclusion

The results and analysis of the responses of the two questionnaires have been presented and it would appear that both schools perform differently in formal and informal reasoning tasks. In some cases, Transformation Geometry has been found to be useful in the understanding of congruency in Euclidean Geometry. However, the researcher cannot argue for a positive correlation between the two types of reasoning. One cannot conclude that overall good performance in one leads to "automatically" to the other or vice versa. In fact, the data shows that these two types of reasoning are somewhat independent of each other. What is evident in the learning of congruency of triangles, in both Transformation Geometry and Euclidean Geometry, is that if pre-existing understanding is absent then new experiences are hampered. This confirms the notion of equilibration (Piaget, 1950). The researcher found that present tasks were hampered by learners:

- Lack of knowledge of properties of regular shapes (like parallelogram, rectangle, isosceles triangle, etc.).
- Inability to accurately measure distance and angles.
- Misconceptions about alternate angles.
- Confusion of the use of visualisation rather than mathematical reasoning and the use of given information.
- Misconception that parallel lines are equal.
- Incorrect graduation of axes.
- Inability to relate correct case of congruency with already proved properties.
- Habitual neglect of given information in the mathematics task at hand-not making an effort to understand the problem.
- Lack of clarity in meaning of words (like bisect, reflect, rotate, common, etc
- .).
- Unclear interpretation of the cases of congruency (S A S for 90^0 H S and not realising the angle in S A S is an included one.
- Lack of confidence in mathematics (geometry) problem solving (perceive geometry as difficult).
- Lack of exposure to a greater number of problems involving the case 90^0 H S.
- Rote learning without conceptual understanding of each case of congruency.

- Seeming to perceive Transformation Geometry and Euclidean Geometry as completely divorced of each other.
- Inability to apply general rule for a particular transformation to specific points with coordinates given.

  The results and analysis of the semi-structured interview will be discussed in the next chapter.

CHAPTER SIX

RESULTS AND ANALYSIS: SEMI-STRUCTURED INTERVIEWS

6.1 Introduction

This chapter presents the results and provides an analysis of the interviews with the intention of triangulating data collected from the written responses in chapter five. Each individual question was analysed comparatively between the two schools. Four learners were interviewed in each school and certain responses were analysed statistically by the number of "yes" or "no" answers. Tables 1 to 10 were used for each respective question and indicate the quality of responses. Coding of responses was done for each individual question.

6.2 Question one: Do you enjoy learning Euclidean Geometry? Why?

The responses were coded in the yes/no format and Table 7 shows the results.

Table 7: responses for both schools to question 1

Learners	Yes	No
School A	4	0
School B	4	0

All the learners who were interviewed in both schools responded that they enjoyed learning Euclidean Geometry. This was a contradiction to the learners who performed poorly in the written tasks. To clarify this contradiction, a second round of interviews was carried out. The researcher spoke to learner A_2 about the responses. The following dialogue transpired:

Learner A_2: *"Yes."*
Researcher: *"Why?"*
Learner A_2: *"It is easy."*
Researcher: *"Is it easy from grade 9 or from all grades?"*
Learner A_2: *"All grades."*
Researcher: *"What is easy there?"*

Learner A_2: *“Ehh, the angles.”*

Researcher*: “What about triangles, easy or not?”*

Learner A_2: *“Easy.”*

Researcher*: “…and congruent triangles?”*

Learner A_2: *“Yes.”*

Researcher: *“At what grade did you do congruent triangles?”*

Learner A_2: *“Grade 9.”*

Learner A_2 performed at an average level in formal task. Thus it would appear that the actual performance of the learner did not coincide with the perceptions that were portrayed. Table 8, showing the responses for question two, follows.

6.3 Question Two: Do you feel that the teachers can use other ways to teach Euclidean Geometry? Explain.

The responses were coded using the yes/no analysis and Table 2 shows the results:

Table 8: Responses for question 2

Learners	Yes	No
School A	3	1
School B	2	2

Three learners from school A responded “yes” and two learners from school B responded “yes”. When asked to explain, learner B_1 responded: *“I think the current method is easy”* while learner B_3 said *“it is fine for now”*. The learners from school B did not perform well in the task on informal reasoning which involved Transformation Geometry. Learner B_1 was among the learners who performed well in the task on formal reasoning which involved Euclidean Geometry. Table 9 on the next page shows the responses for question three.

6.4 Question three: Do you think that most learners enjoy working on problems involving congruency?

The responses were coded in the yes/no format and the results are shown in Table 9.

Table 9: Responses for question three

Learners	Yes	No
School A	1	3
School B	4	0

Three learners from school A indicated that some of their classmates do not enjoy learning congruent triangles, whereas in school B the opposite seemed to occur. The researcher was relatively unsurprised by these responses as the mean performance for the two schools coincides with this pattern. Table 10 below analyses the responses for question 4:

6.5 Question four: Tell me what we mean when we say the two triangles are congruent

The responses were coded using the "correct" and "not correct" format and Table 4 shows the results:

Table 10: Responses for question four

Learners	Yes	No
School A	3	1
School B	4	0

Being English second language speakers, the participants were marked as correct when they responded with *"they are similar in size"* or *"when they are equal"*. The researcher suspected that their respective teachers might have explained congruency in their vernacular language or by code switching. It seemed from the responses that the learners had a satisfactory level of knowledge of congruent triangles and the learners gave the correct symbol for congruency. The researcher felt that their responses in the questionnaires could be trusted as it appeared that the learners knew the concept of congruency. Table 11 shows the responses for question five.

6.6 Question five: Can you explain the cases of congruency?

The responses were coded using the scale ranging from "good" for all four cases, "Satisfactory" for at least two cases and "Weak" for at most one case. Table 11 shows the results.

Table 11: Responses for question 5

Learners	Good	Satisfactory	Weak
School A	2	0	2
School B	1	2	1

Two learners in school A performed well and two did not do well. In school B three learners did not perform badly, one of which gave all four cases. Again, in school B most learners performed well in the tasks for formal reasoning on congruency. The learners were also able to list the cases of congruency in this question. In school A the performance of learners in the formal tasks also tallied with their responses in interview as most of the learners seemed unable to provide the correct case of congruency.

6.7 Question six: Give me an example of two triangles that are not congruent.

The responses for this question were coded using "correct" and "incorrect" format. Table 12 shows the analysis of the responses.

Table 12: Responses for question 6

Learners	Correct	Incorrect
School A	4	0
School B	4	0

All the learners that were interviewed were successful in drawing two triangles that were not congruent. This was seemingly an easy question because the answer required the drawing of any two non-identical triangles. The researcher thus felt that the learners had a conceptual understanding of congruency. Personal experience in teaching Euclidean Geometry seemed to indicate that most learners understand concepts like congruency, similarity, concurrency, *etc*. However, formal proof of riders involving these concepts would appear to be difficult.

6.8 Question seven: Can you tell me in Questionnaire number 2, what types of transformation are displayed in each case?

The responses for the learners in this question were analysed in terms of the number of learners who mentioned the correct transformation in each task focused on informal reasoning. The results are shown in Table 13.

Table 13: Responses for question 7

Learners	Question (a)	Question (b)	Question (c)	Question (d)
School A	3	3	2	2
School B	2	2	3	1

It is encouraging to note that in school A most learners could correctly name the performed transformations (especially in the first two tasks). Similarly in school B, except that most of the learners performed poorly in question (d) which involved reflection in the x-axis. These results also coincide with the performance of each school in the questionnaire involving informal tasks.

6.9 Question eight: Which tasks did you enjoy working with? Why?

The responses to this question were analysed in terms of the number of learners who appeared to enjoy working in one of the two questionnaires. The researcher explained to certain learners that this question referred to a set of questions in Questionnaire one and Questionnaire two. This question was aimed at finding out which set of tasks were enjoyed more; either formal or informal. The results are shown in Table 14.

Table 14: Responses for question 8

Learners	Questionnaire 1	Questionnaire 2
School A	3	1
School B	3	1

As most of the learners in school B chose formal tasks, this appeared to coincide with their performance in the questionnaires. However, learners from school A responded in a way which would seem to contradict their performance in questionnaires. The possible implication in the school A responses is that some learners enjoy making an effort to understand Euclidean Geometry. This may be caused by the belief that if one is successful in Euclidean Geometry then one is good a mathematics learner. The tendency of learners to get bored by difficult tasks in Euclidean Geometry is seemingly not common to all learners.

6.10 Question 9: Would you prefer using Transformation Geometry when studying ideas in Euclidean Geometry?

The responses were analysed in terms of yes/no format and the results are shown in Table 15.

Table 15: Responses for question 9

Learners	Yes	No
School A	3	1
School B	2	2

In school A, most learners indicated that they would like to learn to relate Transformation Geometry and Euclidean Geometry in studying common concepts like congruency.

The same responses were found in half of school B learners. The implication in this case is likely to be that learners do indeed recognise familiar concepts when they are introduced in other sections of mathematics. Therefore it may help improve understanding amongst the learners if their teachers could link sections when teaching common concepts.

6.11 Question 10: In which tasks did you find it easier to identify the case of congruency?

The researcher explained to the learners that they could choose any tasks from each questionnaire. The responses were analysed in terms of each learner's responses; as shown in Table 16.

Table 16: Responses for question 10

Learners	Responses
School A	A 1: 90^0 H S A 2: All Questionnaire 1 A 3: All Questionnaire 1 A 4: Number (a) in Questionnaire 2
School B	B 1: Number 1.1 in Questionnaire 1 B 2: Number 1.1 in Questionnaire 1 Number (a) & (b) in Questionnaire 2 B 3: All Questionnaire 1 B 4: Number 1.1 in Questionnaire 1

Learner A_1 (who was one of the few to answer this specific question correctly) chose the 90^0 H S. The case was missed by most learners in both schools. Learners can surprise teachers sometimes by showing competency in pieces of knowledge that is rarely taught in a section. Learner A_4 chose question number (a) in Questionnaire two that involved translation. This learner seemingly could relate the case of congruency and the movement of a triangle on a Cartesian plane. Learner A_3 and A_4 indicated that they enjoyed doing formal tasks. It is possible that they were unfamiliar with studying congruent triangles in Transformation Geometry, and thus felt uncomfortable in this situation.

Learner B_3 also indicated enjoyment in working with all formal tasks; probably as this learner performed relatively well in this instance. Learner B_1 and B_4 appeared to enjoy working with number 1.1 for Questionnaire one. It is probable that their teacher had employed similar methods in teaching them congruent triangles in grade nine. Learner B_2 chose number 1.1 in Questionnaire one, number (a) and (b) in Questionnaire two. Learner B_2 was the only learner who chose tasks from both formal and informal tasks. The responses in school B may provide a certain level of optimism that learners enjoy learning Transformation Geometry.

6.12 Conclusion

The responses recorded in the interview and collected as data have been analysed. Most learners responded in a way that coincided with their responses to the questionnaires. Therefore the triangulation procedure of the data was somewhat efficient. We now make concluding remarks in chapter seven.

CHAPTER SEVEN

CONCLUSION AND RECOMMENDATIONS

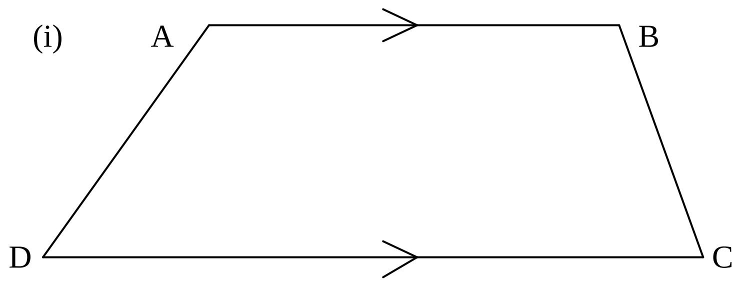
7.1 Introduction

This chapter makes concluding remarks concerning the research as well as presenting some recommendations for the teaching of congruent triangles. Reference will be made to the relationship between Euclidean and Transformation Geometry in relation to the learning of congruent triangles. The discussion will be based on the analysis of data that was presented in chapter five and chapter six.

7.2 Findings for the formal tasks on Euclidean Geometry

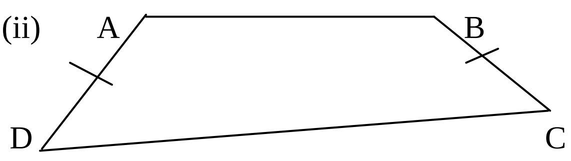
In the course of this study, it was found that learners from school A confuse parallel and equal lines; the learners indicated that parallel lines are equal in quadrilaterals, which is not always the case. According to discussions with teachers in the district cluster workshops, this appears to be common practice in mathematics classrooms. At the introductory phase of the teaching of quadrilaterals, it may be more effective to provide a larger number of examples with one condition, rather than the other. The examples below could be used.

(i)



In the above case learners could be asked: Is AB = DC? Explain.

(ii)



In the above case where AD = BC learners could be asked to discuss whether AD is parallel to BC.

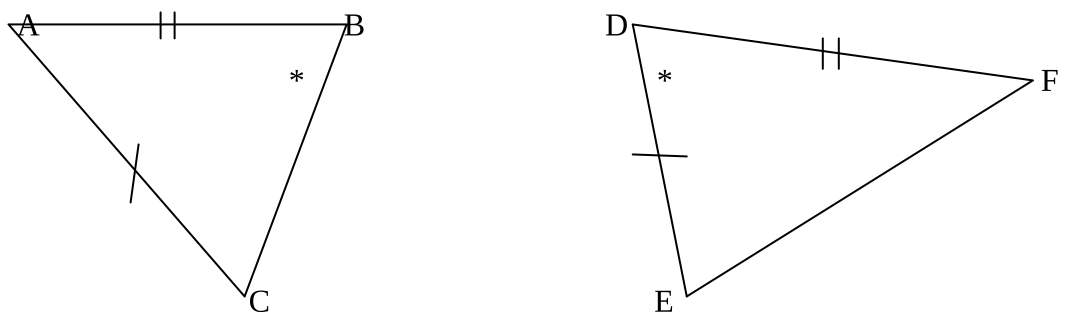
In school B, learners appeared to use improper geometrical reasoning. When they were given a quadrilateral as a parallelogram, the learners argued that opposite sides are equal because it was given, rather than using the reason: "opposite sides of parallelogram are equal". The learners' method of reasoning thus left the researcher doubting whether they properly understood the properties of a parallelogram.

When engaging learners with tasks that involve the properties of a quadrilateral, teachers would need to first emphasise the procedure for solving riders. Learners appeared to be unable to use the reasoning of "given" properly. More examples of riders involving the reasoning of "given" and the properties of quadrilaterals may help clarifying this mistake.

It was found that learners prefer using visualisation to arrive at conclusions rather than mental construction. This may be a factor contributing to the difficulty in proving Euclidean Geometry theorems and riders as deduced by Mudaly (2007). This was observed particularly when a learner indicated that a corner of a house is equal to 90^0. Were teachers to teach more examples of angles that are very close to a 90^0 (like 92^0 or 89^0); it may well help resolve this misconception. Other potential examples could include quadrilaterals that look like a square, a rhombus, parallelogram or a rectangle. Learners could be asked to measure the lengths of the opposite sides of these figures so that they could see that these figures are not as they appear to be.

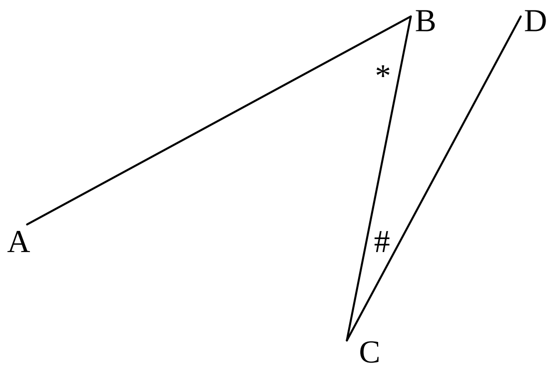
In number three of the questionnaire, very few learners who participated in the research could correctly answer and identify the case of 90^0, H, S in proving congruency. This would seem to indicate that learners are not as familiar with this case of congruency as they should be at grade ten. A quick perusal of mathematics textbooks reveals that even mathematics authors do not appear to include as many examples of the 90^0, H, S as other cases like S, S, S; S, A, S and A, A, S. Experience has also shown that even in the classroom this case of congruency is rarely done; probably because most teachers rely on textbooks when preparing for lessons. The researcher recommends that the editors of mathematics textbooks rectify this situation by including as many of these types of cases as possible. As Adler (2001) found, many mathematics teachers (especially inexperienced teachers) are likely to rely inordinately upon the use of textbooks in their teaching. With this recommendation, the researcher hopes other misconceptions around this case could be solved. For example, other learners involved in this study confused the 90^0, H, S case with the S, A, S case. Teachers need to emphasise that the angle in this case is the included one. Thorough teaching of congruency using relevant examples could overcome this problem. For example the learners can be asked to attempt the following task:

For the triangle below discuss whether the two triangles are always congruent.



Some learners in task four seemed to believe that all alternate angles are equal in a quadrilateral. This was observed when learners indicated a pair of alternate angles equal when this was not the case. The researcher is of the view that this problem is caused by the tendency to introduce alternate angles when teaching parallel lines, only when they are equal. Experience has shown that very few teachers expose learners to alternate angles that are not equal. This problem could be a factor contributing to the poor performance of learners in National Senior Certificate assessment, and one is led to suspect that the same may well happen for corresponding and co-interior angles. Teachers may need to provide more counter-examples before introducing the concept of parallel lines. An example of this appears below.

Name a pair of alternate angles in the figure below. Measure each of these alternate angles and discuss their relationship.



It was also found that assessing learners' knowledge of congruent triangles in the context of quadrilaterals (like a parallelogram, a rhombus or a rectangle) does not give a clear indication of the learners' knowledge of congruent triangles. It was observed in some cases that learners did not possess adequate knowledge of the properties of quadrilaterals and hence could not prove congruency. This would seem to be an indication of the knowledge gaps learners develop when leaving for the next grade. This appears likely to result in high levels of rote learning; especially in concepts like the four cases of congruency. More time needs to be spent on teaching of the basic properties of figures like regular quadrilaterals and triangles at the appropriate level of learning.

The overall finding in this research is that school B performed almost two times better than school A in the formal tasks. This was deduced from the mean performance in each task, as well as the overall mean performance on the whole questionnaire on Table 3 and Table 4 of chapter four respectively. An inappropriate van Hiele level could be the cause of poor performance of learners in this questionnaire. This was apparent as learners could not answer questions related to alternate angles while proving congruency. To facilitate understanding, informal reasoning may be introduced to the learners while they are studying Euclidean Geometry especially the properties of geometrical shapes and patterns. While this is done the teachers have to ensure that learners are at the appropriate van Hiele level for that particular grade.

7.3 Findings for informal tasks on Transformation Geometry

Most school A learners were able to correctly locate triangles on the Cartesian plane which likely means that the learners had an understanding of the points in the coordinate system. The learners could also locate the image of the triangle after a performed transformation, meaning that the learners had an understanding of translation, rotation and reflection. Teachers would need to strengthen learners' knowledge on Transformation Geometry as this could contribute to an improvement of their performance in the NSC. Some school A learners were unable to prove congruency in a convincing manner as they failed to correctly measure the sides and angles of the located triangles. Practical measurements of sides and angles are introduced at Senior Phase levels. Hence failure for learners to find correct measurements is an indication of the knowledge gaps present in learners upon their entry to grade ten. Improved facilitation and supervision of the performance of the Senior Phase teachers in mathematics teaching could therefore minimise this problem.

Most school B learners could correctly locate triangles after having performed transformation on the Cartesian plane. This appears to indicate that the learners were familiar with the coordinate system. However, most learners could not locate the image after having performed transformation on a Cartesian plane. This means that the comprehension of translation, rotation and reflection of the learners needs to be improved. This understanding may be improved by encouraging mathematics teachers to conduct their reflection after lessons in order for them to identify the strengths and weaknesses of learners. This could potentially aid in making an informed decision about the strategies of improvement for their Integrated Quality Management System (IQMS).

School B learners who were successful in proving congruency used their knowledge of Analytical Geometry. It was encouraging to find that the learners were able to calculate distances as this would help them in their NSC tasks. However, knowledge gaps were evident when learners were unable to find the measurements of angles. These knowledge gaps may have contributed to lower overall mean learner performance as shown in Table 6 of chapter six. This creates opportunity for future research on exploring links in learning mathematics concepts in Analytical Geometry, Euclidean Geometry and Transformation Geometry.

In both schools confusion of the kinds of transformation in the tasks was observed, this was especially noticeable in the reflection along the x- axis and the reflection along the y-axis. It is hoped that this confusion could be resolved at the early stages of mathematical development. Mathematics teachers should adhere to teaching methods involving similar tasks as well as providing appropriate feedback after assessing these tasks. Category D learners in both schools performed well in both questionnaires. It is hoped that the introduction of Transformation Geometry in grade ten could then contribute to the improvement of the understanding of congruent triangles and vice versa.

7.4 Findings for the semi-structured interview in relation to both tasks

Interviewing learners about their performance in the questionnaires was done with intention of triangulating data obtained from written tasks. Hence the researcher was interested in determining whether there was any discrepancy between the data that was collected from questionnaires and the data from interviews. In most instances the interviews confirmed the deductions the researcher had previously made from the written responses.

7.4.1 Congruent triangles in Euclidean Geometry

Learners from both schools indicated that they enjoy learning Euclidean Geometry. The researcher felt that these learners are keen to be successful Euclidean Geometry learners, probably because the section is very challenging. It was also encouraging to find that learners understood the meaning of congruency and that they were able to tell if the triangles were not congruent. This finding would seem to contradict the assumption that, as a result of poor performance in Euclidean Geometry, learners completely lack the memory of certain relevant geometry content. This implies that mathematics teachers need to understand that although learners do not necessarily appear competent in the usage of proofs using logical constructions of Euclidean Geometry, learners may nevertheless still be keen to learn and succeed since they enjoy learning of concepts in Euclidean Geometry. Once this is understood, teachers could then design Euclidean Geometry activities that would improve understanding.

Most of the learners' responses to interview questions were in line with their responses in questionnaires. The researcher noted that the learners interviewed remained capable of recalling their thought processes that had occurred during their answering of the questionnaires.

It was also encouraging to note that preferred Euclidean Geometry activities are the activities which learners apparently found relatively easy to work with. This could help to remove the conception that only 'fast' learners like Euclidean Geometry.
Mathematics teachers need to improve the design of their lessons by including a variety of concepts. Concepts need to be defined and explained as thoroughly as possible through the use of various relevant examples. The above two points are recommended as a result of the finding that the familiarity of a concept depends on how regular it was taught.

7.4.2 Congruent triangles and Transformation Geometry

Data collection from the interview revealed that translation, rotation and reflection were understood by most learners who participated in the research. This was apparent from the learners' responses when asked to name the performed transformation in each case of questionnaire two.

Most learners who were interviewed also indicated that they would prefer to study congruent triangles using ideas of Transformation Geometry. This was seen when learners were asked if they would prefer to use Transformation Geometry in studying congruent triangles. This finding potentially opens another dimension of the improvement of teaching of Euclidean Geometry (especially of congruent triangles). The researcher's purpose in conducting this study was to determining whether or not learners were able to successfully learn congruent triangles using ideas of Transformation Geometry. This implies that teachers could employ the strategy of introducing or reinforcing the ideas of congruent triangles using translation, rotation and reflection.

Some learners were at the introductory stage of learning Transformation Geometry in school B. It was interesting to note that learners performed satisfactorily in the tasks related to transformations. It would appear that Transformation Geometry is a section that may be relatively easy to learn. Although some learners seemed to confuse the types of transformations but it is hoped that they would eventually overcome this difficulty once they progress past the introductory stage of these transformations.

7.5 Conclusion

In this chapter the researcher looked at the concluding points of the research and considered recommendations for the teaching of congruent triangles. The researcher found that learners can learn congruent triangles successfully using the ideas of Transformation Geometry. The researcher also found that learners still appear to enjoy learning congruent triangles in Euclidean Geometry. The researcher thus recommended that teachers use both approaches in teaching congruency so as reinforce these concepts. Further research focused on the learning approaches of other concepts (like similar triangles) in Transformation Geometry may be necessary. Data has shown that there is no clear answer to learners understanding of congruency when working with transformations as compared to formal reasoning; the one category did not imply the other. Learners who were successful in formal reasoning did not necessarily perform well the informal situation, and vice versa.

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APPENDIX A

PROVINCE OF KWAZULU-NATAL
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Mr Mbili LA
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PERMISSION TO INTERVIEW LEARNERS AND EDUCATORS

The above matter refers.

Permission is hereby granted to interview Departmental Officials, learners and educators in selected schools of the Province of KwaZulu-Natal subject to the following conditions:

1. **You make all the arrangements concerning your interviews.**
2. **Educators' and work programmes are not interrupted.**
3. **Interviews are not conducted during the time of writing examinations in schools.**
4. **Learners, educators and schools and other Departmental Officials are not identifiable in any way from the results of the interviews.**
5. **Your interviews are limited only to targeted schools.**
6. **A brief summary of the interview content, findings and recommendations is provided to my office.**
7. **A copy of this letter is submitted to District Managers and principals of schools or heads of section where the intended interviews are to be conducted.**

The KZN Department of education fully supports your commitment to research: **Grade ten learners' conceptual understanding of congruency in Transformation trigonometry.**

It is hoped that you will find the above in order.

Best Wishes

R Cassius Lubisi, (PhD)
Superintendent-General

APPENDIX B

PROVINCE OF KWAZULU-NATAL
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228 Pietermaritz Street
PIETERMARITZBURG

INHLOKOHOVISI PIETERMARITZBURG HEAD OFFICE

Imibuzo:	Reference:	Date:
Enquiries: Sibusiso Alwar	Inkomba: 0012/09	Usuku: 23 February 2009

Mr Mbili LA
P.O. Box 142
Anerley
4230

RP: GRADE TEN LEARNERS' CONCEPTUAL UNDERSTANDING OF CONGRUENCY IN TRANSFORMATION GEOMETRY

Your application to conduct the above-mentioned research in schools in the attached list has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.

2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.

3. You make all the arrangements concerning your investigation.

4. Educator programmes are not to be interrupted.

5. The investigation is to be conducted from 02 March 2009 to 02 March 2010.

6. Should you wish to extend the period of your survey at the school(s) please contact Mr Sibusiso Alwar at the contact numbers above.

7. A photocopy of this letter is submitted to the principal of the school where the intended research is to be conducted.

8. Your research will be limited to the schools submitted.

9. A brief summary of the content, findings and recommendations is provided to the Director: Resource Planning.

APPENDIX B CONTNUED

10. The Department receives a copy of the completed report/dissertation/thesis addressed to

The Director: Resource Planning
Private Bag X9137
Pietermaritzburg
3200

We wish you success in your research.

Kind regards

R. Cassius Lubisi (PhD)
Superintendent-General

APPENDIX C

RE: Letter of Consent

To: Participant(s) and Parent/Guardian

Research Project: Grade ten learners' conceptual understanding of congruency in Transformation Geometry

Year: 2009

Lingelo Aaron Mbili (Final year Masters Student) is doing a study through the **School of Education, Mathematics Education at the University of KwaZulu-Natal** with **Dr Deonarain Brijlall.** His contact numbers are 031-260 3491(work). We want to research congruent triangles in Transformation Geometry at a secondary school in KwaZulu-Natal: South Africa.

Grade ten learners are asked to help by taking part in this research project as it would be of benefit to teachers and interested educationists and/or mathematics teachers. However, participation is completely voluntary and has no impact or bearing on evaluation or assessment of the learner in any studies or course while at school. Participants may be asked to take part in the interviews after the worksheets have been completed. These interviews will be tape-recorded in the future. All participants will be noted on transcripts and data collections by a *pseudonym* (i.e. fictitious name). The identities of the interviewees will be kept strictly confidential. All data will be stored in a secured password and not been used for any other purpose except for the research.

Participants may leave the study at any time by telling the researcher. Participants may review and comment on any parts of the researchers' written reports.

________________________ ______________

(Researcher's Signature) *(Date)*

DECLARATION

I,______________________________ *(Participant's NAME)*__________________*(Signature)*

________________________*(Parent's/ Guardian's NAME)*__________________*(Signature)*

__________________ *(Date)*

☐ Agree.

☐ Disagree.

N.B. Tick ONE

To participate/allow participation in the research being conducted by Lungelo aaron Mbili; concerning *An investigation of the learners' conceptual understanding of congruent triangles in Transformation Geometry.*

APPENDIX D

Application to do a Research in two Public Schools

To: Communications/Research officer: KwaZulu-Natal Department of Education
Circuit Manager: NGESI S.O.
Principal of SHEZI V.S

Year: 2008

Research Project:

Lungelo Aaron Mbili (Final year Masters student) is conducting a study through the **School of Education, Mathematics Education at the University of KwaZulu-Natal** under the supervision of **Dr Deonarain Brijlall**. His contact number at work is (031) 2603491. Proposed research looks towards the grade ten learners' conceptual understanding of congruent triangles in Transformation Geometry. In particular, this inquiry looks the better learning strategy of congruent triangles between Euclidean Geometry and Transformation Geometry.

Learners are requested to assist through participating in this research project as it would be of benefit to education practitioners and interested educationalists/researchers and/or mathematics teachers. However, participation is *completely voluntary* and has no impact or bearing on evaluation or assessment of the learner/teacher in any studies or course while at school. Participants may be asked to take part in the semi structured interviews after the worksheets have been completed. These interviews will be recorded (*only for analysis purposes*) as the time progresses, but initially no recording will take place. All participants will be noted on transcripts and data collections by a *pseudonym* (i.e. fictitious name). The identities of the interviewees will be kept strictly confidential. All data will be stored in a secure password protected server where authentication will be required to access such data.

Participants may revoke from the study at any time by advising the researcher of this intention. Participants may review and comment on any parts of the dissertation that represents this research before publication.

(*Researcher's Signature*) 02/02/09 (*Date*)

PROVINCE OF KWAZULU-NATAL
UMNYANGO WEMFUNDO NAMASIKO
DEPARTMENT OF EDUCATION & CULTURE
DEPARTEMENT VAN ONDERWYS
2009 -02- 02
HE 4249
INGWEMABALA H EMIS 163762

DECLARATION

I, Shezi V. S ______ (**NAME and SIGNATURE**)

principal/~~circuit manager~~ on this day of 02 month 02 2009, hereby grant permission to the researcher to go ahead with the research in the above-mentioned schools following the terms of reference noted in this request letter.

APPENDIX D CONTINUED

Application to do a Research in two Public Schools

To: Communications/Research officer: KwaZulu-Natal Department of Education
Circuit Manager: NGESI S.O. (Mv)
Principal of MAKHANDA HIGH

Year: 2008

Research Project:

Lungelo Aaron Mbili (Final year Masters student) is conducting a study through the **School of Education, Mathematics Education at the University of KwaZulu-Natal** under the supervision of **Dr Deonarain Brijlall**. His contact number at work is (031) 2603491. Proposed research looks towards the grade ten learners' conceptual understanding of congruent triangles in Transformation Geometry. In particular, this inquiry looks the better learning strategy of congruent triangles between Euclidean Geometry and Transformation Geometry.

Learners are requested to assist through participating in this research project as it would be of benefit to education practitioners and interested educationalists/researchers and/or mathematics teachers. However, participation is *completely voluntary* and has no impact or bearing on evaluation or assessment of the learner/teacher in any studies or course while at school. Participants may be asked to take part in the semi structured interviews after the worksheets have been completed. These interviews will be recorded (*only for analysis purposes*) as the time progresses, but initially no recording will take place. All participants will be noted on transcripts and data collections by a *pseudonym* (i.e. fictitious name). The identities of the interviewees will be kept strictly confidential. All data will be stored in a secure password protected server where authentication will be required to access such data.

Participants may revoke from the study at any time by advising the researcher of this intention. Participants may review and comment on any parts of the dissertation that represents this research before publication.

____________________ 14/05/09
(*Researcher's Signature*) (*Date*)

DECLARATION

MAKHANDA SEC. SCHOOL

I, MNCWELA MPS 2009 -05- 15 (**NAME** and **SIGNATURE**)

principal/~~circuit manager~~ on this day of 15 MAY month______ 2009, hereby grant permission to the researcher to go ahead with the research in the above-mentioned schools following the terms of reference noted in this request letter.

PORT SHEPSTONE
4240

APPENDIX E

UNIVERSITY OF
KWAZULU-NATAL

10 February 2009

Faculty Research Committee
Faculty of Education
Edgewood Campus
University of KwaZulu-Natal

Dear Dr Brijlall,

Consideration of Ethical Clearance for student:

Mbili, - 964109142

Your student's ethical clearance application has met with approval in terms of the **internal review process** of the Faculty of Education.

Approval has been obtained from the Faculty Research Committee, and the application will be forwarded for ratification (MEd) or recommended in the case of PhD and Staff applications, to the Ethics Sub-Committee of the University of KwaZulu-Natal. All Masters applications approved by Faculty Research Committe may commence with research.

Both you and the student will be advised as to whether ethical clearance has been granted for the research thesis (PhD), once the Ethics Sub-Committee has reviewed the application. An ethical clearance certificate will be issued which you should retain with your records. The student should include the ethical clearance certificate in the final dissertation (appendixes).

Should you have any queries please contact the Faculty Research Officer on (031) 260 3524 or on the email buchler@ukzn.ac.za

Yours faithfully

UNIVERSITY OF KWAZULU-NATAL
EDUCATION
2009 -02- 10
FACULTY RESEARCH OFFICE

Professor D. Bhana
Acting Deputy Dean Postgraduate Studies and Research

APPENDIX E CONTINUED

UNIVERSITY OF
KWAZULU-NATAL

RESEARCH OFFICE (GOVAN MBEKI CENTRE)
WESTVILLE CAMPUS
TELEPHONE NO.: 031 – 2603587
EMAIL : ximbap@ukzn.ac.za

17 APRIL 2009

MR. MBILI (964109142)
EDUCATION

Dear Mr. Mbili

ETHICAL CLEARANCE APPROVAL NUMBER: HSS/0105/09M

I wish to confirm that ethical clearance has been granted for the following project:

"Grade ten learners' conceptual understanding of congruency in Transformation Geometry"

PLEASE NOTE: Research data should be securely stored in the school/department for a period of 5 years

Yours faithfully

PXimba

MS. PHUMELELE XIMBA

cc. Supervisor (Dr. D Brijlall)
cc. Mr. D Buchler

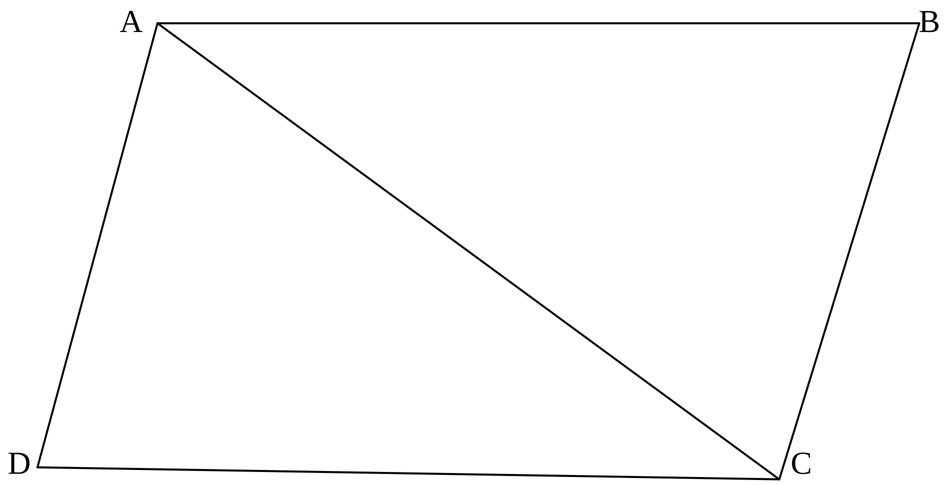
RECEIVED
2009 -04- 28
FAC RESEARCH OFFICE

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

APPENDIX F

QUESTIONNAIRE 1

1.1 Below is parallelogram ABCD.



Prove that ΔADC ≡ Δ CBA by completing the following statements:

Statement Reason

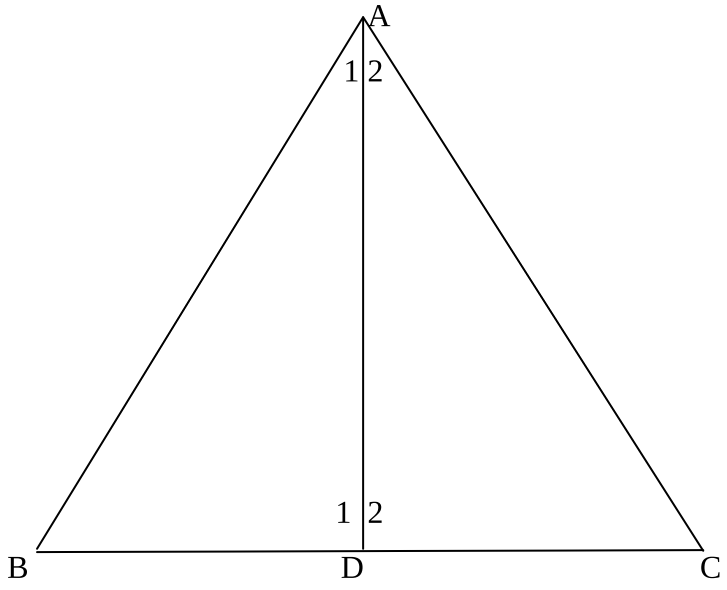
AB = ... (...)

AD = ... (...)

AC = ... (...)

so Δ ADC ≡ Δ CBA (...)

1.2 Below is isosceles Δ ABC with AB = AC. DA bisects Â.



Prove that Δ ABD ≡ ΔACD by completing the following statements:

Statement Reason

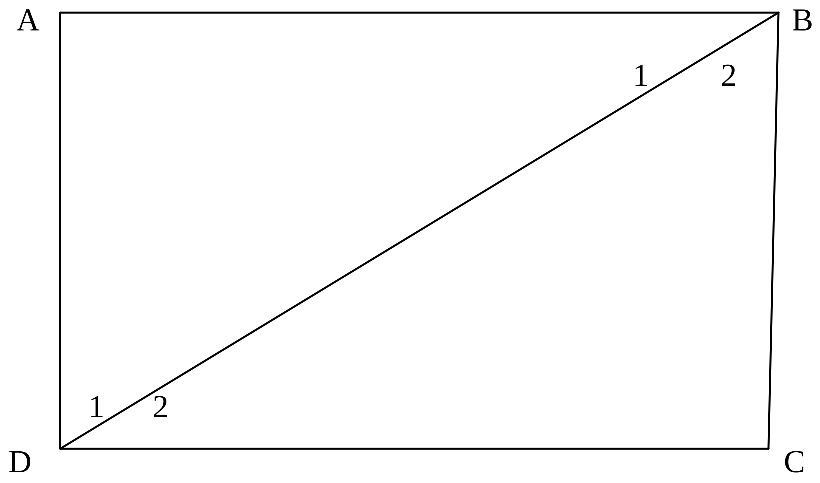
…………. . = ………. (given)

$\hat{A}_1$ = ………... (.......................……….…………….……………………….)

AD = AD (.......................…………………….........………………......)

So Δ ABD ≡ Δ ADC (..….……….….…………….)

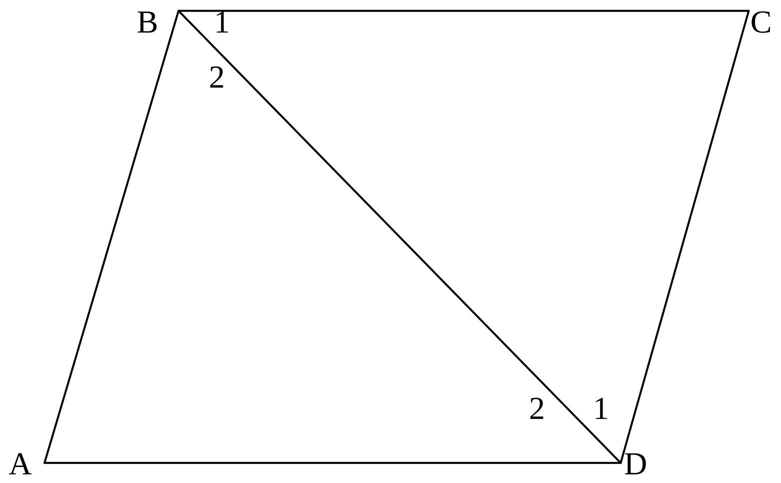
1.3 Below is quadrilateral ABCD with $\hat{A} = \hat{C} = 90^0$ and AB = DC.



Prove that ΔABD ≡ ΔCDB by completing the following statements:

Statement			Reason
$\hat{A}$	=	……….	(.......................………...……………..……………………….)
BD	=	BD	(.......................………...……………..……………………….)
AB	=	....	(.......................………...……………..……………………….)
so ΔABD ≡		Δ CDB	(.......................………...……………..……………………….)

1.4 Below is quadrilateral ABCD with $\hat{A} = \hat{C}$ and AB // DC.



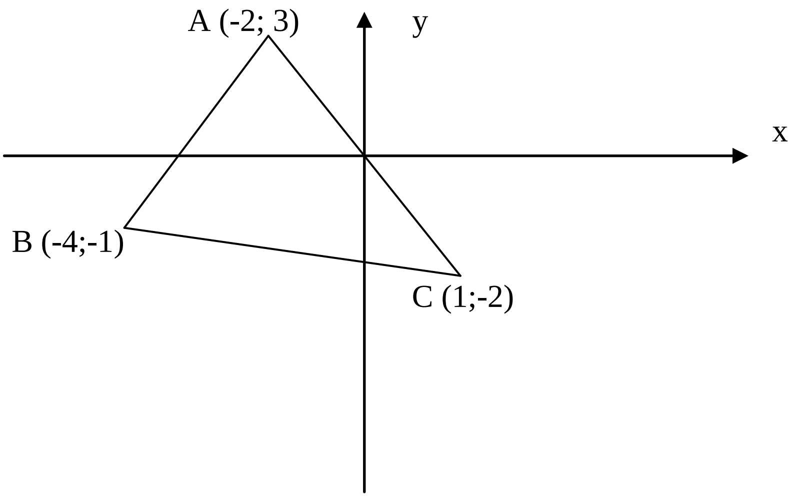
Prove that $\Delta ADB \equiv \Delta CBD$ by completing the following statements:

Statement		Reason
$\hat{A}$	= ……	(...)
………..	= ……	(Alternate angles, AB // DC)
BD	= BD	(...)
so Δ ADB	$\equiv$ Δ CBD	(..)

APPENDIX G

QUESTIONNAIRE TWO

Study the diagram below and then complete the tasks that follow:



ΔABC is translated by the rule: (x, y) → (x + 1, y + 2). Draw both ΔABC and its image ΔA′B′C′ on the same set of axis on the graph paper provided.

Complete the following statements by accurately measuring:

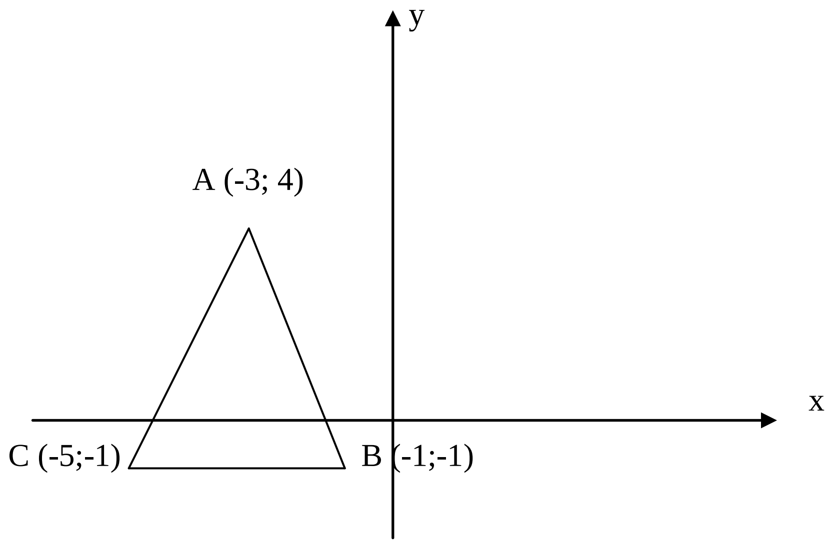
AB = ……………………………… A′B′ = ………………………………

BC = ……………………………… B′C′ = ………………………………

AC = ……………………………… A′C′ = ………………………………

So Δ ABC ≡ ΔA′B′C′ (………………………………………………………)

(b) Study the diagram below and then complete the tasks that follow:



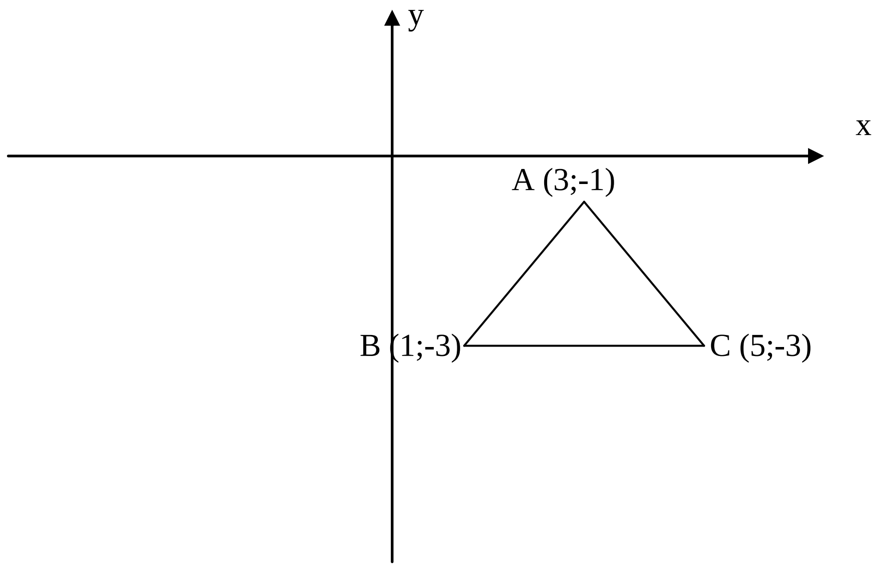
In the above diagram, ΔABC is reflected about the y-axis. Draw both of ΔABC and its image ΔA´B´C´ on the same set of axis on the graph paper provided.

Complete the following statements:

AB	= ………………………………	A′B′	=	………………………………
Â	= ………………………………	Â′	=	………………………………
BC	= ………………………………	B′C′	=	………………………………

So ΔABC ≡ ΔA′B′C′ (…………………………………………)

(c) Study the diagram below and then complete the tasks that follow:



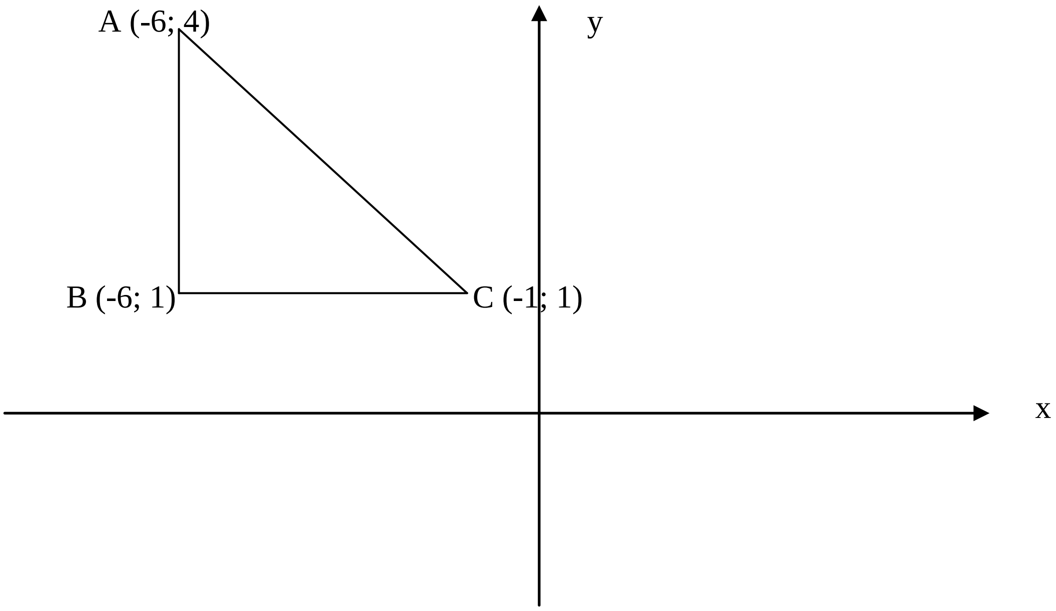
In the above diagram, ΔABC has been rotated through 180^0 clockwise. Draw both ΔABC and its image Δ A′B′C′ on the same set of axis on the graph paper provided.

Complete the following statements by accurately measuring:

$\hat{A}$	=	…..........................…..	$\hat{A}'$	=	......................………
$\hat{C}$	=	…...................…......	$\hat{C}'$	=	.......….............……
BC	=	…….........................	B′C′	=	........…..................…

So ΔABC ≡ Δ A′B′C′ (...……………..)

(d) Study the diagram and then complete the tasks that follow:



In the figure above, ΔABC has been reflected along the x-axis. Draw both ΔABC and its image ΔA´B´C´ on the same set of axis.

Complete the following statements by accurately measuring:

AC = …….......................... A′C′ = ...………..

$\hat{B}$ = …........................….. $\hat{B}'$ = ...…..……

BC = ...…..... B′C′ = ...……….…..

So ΔABC ≡ ΔA′B′C′ (............…...............………………….……………)

APPENDIX H

For att: Dr Brijlall

Faculty of Education, University of KwaZulu-Natal

Re: **Confirmation letter with respect to the proofreading and editing of Mr Lungelo Aaron Mbili's Masters thesis**

Dear Dr Brijlall,

This letter serves as proof that the Masters thesis presently being completed by Mr Lungelo Aaron Mbili, via the University of KwaZulu-Natal's Faculty of Education, has been proofread and edited by Mr Robert Tyrrell. Further, the proofreading work has been overseen by Ms Jacintha John, a prescribed proofreader for UKZN, and who served in the capacity of mentor proofreader.

Best regards,

Mr Robert Tyrrell

(Cell: 074 104 4151 / Email: robertjtyrrell@gmail.com)

Ms Jacintha John

(Cell: 074 116 3360 / Email: jacintha_john@yahoo.com)