THE PROPAGATION OF THE SOUND OF THE HORN

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Dissertation submitted in partial fulfilment of the requirements for the degree of Master of Music in the Department of Music, University of Natal.

Durban 1987.
The purpose of this study was to formulate a theory of the propagation of sound waves within the horn and to discover the sound spectrum and directional characteristics of the instrument. The dissertation also includes a brief discussion of the history and development of the horn.
DECLARATION

I declare that this dissertation is my own, unaided work. It is being submitted for the degree of Master of Music in the Department of Music, University of Natal. It has not been submitted before for any degree or examination in any other university.

[Signature]

Natalie Lemmer

10 December 1987
The instrument which, at one time will sway the feelings of our sweetheart through means of its melancholy tone, and at another time rouse the rough and ready huntsman to his wild and pitiless pursuits in field and forest; the instrument which, in the hands of a master, attracts the attention and commands the admiration of all music lovers to such a great extent, and at another time inspires the soldier to bloody battle - what other instrument could this be than the "Waldhorn", which is daily heard in field and forest, in the church and in the concert hall?

1. (E.L. Gerber - 1790)

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This research investigates the propagation of sound waves within the horn and examines the factors influencing the tone quality and directional characteristics of the instrument.

A knowledge of the above mentioned aspects has become of growing importance to musicians, conductors and acoustitians, in determining the seating arrangement of an orchestra and the acoustical design of concert halls, opera pits and recording studios.

Chapter 1 involves a brief discussion on the history and development of the horn. Chapter 2 examines the physical concepts involved in the sound generation along a confined tube. The final chapter experimentally investigates the sound spectrum and directivity of the horn.
CHAPTER 1

THE HISTORY AND DEVELOPMENT OF THE HORN

1.1 Early Ancestors

Mention of horns functioning as signalling devices can be traced in the histories of most of the ancient races. They were primarily used for announcing sacrificial offerings and assembling people for military and state occasions. These instruments were short in length, with a wide flaring bore, restricting their compass to one or two notes. They were usually made of animals’ horns, seashells, hollow logs, bronze or ivory (Figure 1.1). In Europe and Asia such horns were (Marcuse 1975, p. 745) endblown, while in parts of Africa and South America they were mostly sideblown.

The Scandinavian lur (Figure 1.2) is the earliest extant man-made horn and is reported to have been used in the period from the twelfth to the sixth centuries B.C. (Lur is (Marcuse 1975, p. 745) a modern name, dating from the 1797 excavations in Denmark). Usually made of bronze, the lur was two to three metres long and shaped in an elongated S curve. It had a fixed cup-shaped mouthpiece and a conical bore, which terminated in a flat disk-like bell, often studded with precious stones. These instruments were made in pairs, twisting in opposite directions and were played with the bells projecting high above the players’ heads. They were both at the same pitch and could produce about six
Figure 1.1 Animal’s horn which was used as a signalling device. (Franz 1906, p. 3)

notes. Archaeologists suggest (Apel 1970, p. 489) that the lur was, despite its loud and raucous tone, used in connection with a religious cult, rather than for military purposes.

The Jewish *shofar* was a ritual and martial signal-horn. Commonly made (Apel 1970, p. 774) of rams horn, it produced two harsh notes vaguely resembling the second and third harmonics. It is the only instrument to have survived into the twentieth century in the same form it had in antiquity. Today it is used in the Jewish rite for the celebration of New Year and on the Day of Atonement.

Other horns of note used (Marcuse 1975, p.749) during this period, are the Roman *cornu* and the *bucina*. Both were originally made of ox horns and later of metal. The *cornu* was chiefly of military use. It had a narrow conical bore with a slender bell. It was three metres long and shaped in the letter G. A cross-bar was used to steady the horn and the bell projected forward above the player’s head. The *bucina*, with much the same design as the *cornu*, was
a shepherd’s instrument and its martial uses were limited to waking soldiers and announcing the hours of the day.

All these various horns possessed one common factor, in that they were all
conical; that is to say, the bore increases with the length. In further development this was the decisive factor in the timbral difference between the horn and the trumpet.

1.2 The Hunting Horn

From the twelfth to the sixteenth century the shape of the horn was modified and increased in size to form a full circle. Natural animal materials were abandoned in favour of more malleable substances. The resultant instrument was the hunting horn (Figure 1.3). It fitted comfortably around the player’s shoulder and was easily played while on horseback.

![Figure 1.3 Hunting horn. (Franz 1906, p.4)](image)

New techniques of more refined horn-playing developed during the seventeenth century. This called for specific improvements to the instrument. A new funnel-shaped mouthpiece was fitted to a cylindrical leadpipe. Different coiled shapes
for the main body were experimented with and the flare of the bell was increased. This is (Tuckwell 1983, p.11) exemplified in a treatise by Marin Marsene (1588 - 1648), called Harmonie Universelle (1627), in which four types of horn are described: *Le grand cor* - a horn with a large circle; *Cor a plusieurs tours* - a horn that spirals in on itself; *Le cor qui n'a qu'un seul tour* - a horn that has only one spiral, shaped like an arc; and *Le huchet* - a smaller version of *le grand cor.* The improved tone quality and increased scale range encouraged composers to use the horns in fanfares during operatic hunting scenes. The earliest surviving fanfare is (Fitzpatrick 1970, p. 5) the *Chiamata alla Caccia* for four horns from the opera *Le Nozze di Teti e di Peleo* by Francesco Cavalli, premiered in Venice, in 1639.

1.3 Important New Developments

The development in horn manufacturing corresponded accordingly to the new role of the horn in the orchestra. Two of the most noteworthy instrument builders, Michael Leichnambschneider and his brother, Johannes, concerned themselves with instruments intended for chamber music and orchestral uses. The need for horns to play in different keys brought about the invention of crooks. There is much dispute about the date and place of origin of this invention. The earliest evidence of crooks being fitted, is (Fitzpatrick 1970, p.32) attributed to Michael Leichnambschneider. A bill, dated 1703, to the Abbot of Kremsmünster, includes charges for crooks. Coiled crooks (Figure 1.4), varying in length, were inserted immediately below the mouthpiece to lengthen the horn and thus alter the fundamental pitch. This was obviously a time-consuming process and therefore not very practical.
The horn player Anton Joseph Hampel of Dresden, is (Morley-Pegge 1973, p. 20) credited with the invention, in 1754, of a set of curved sliding crooks. They were centrally inserted and of varying length. In collaboration with the instrument maker Johann Werner, they built what is called, the Inventionshorn. This system was later improved on by J.G. Haltenhof of Hanau. He added a tuning slide to the crooked instrument (Figure 1.5), which facilitated fine tuning.

Following the invention of crooks, Hampel developed (Gregory 1969, p.29) the
technique of hand horn playing. He discovered that by closing the bell systematically with his right hand, he could artificially produce the missing notes between the harmonics. Although certain notes had either a muffled or muted sound, composers were quick to realize the potential of the hand horn. It became a popular solo instrument. The period 1750 to 1820 is (Tuckwell 1983, p.31) often referred to as the Golden Age of horn soloists.

Around 1760 the Bohemian horn player Kölbel, resident at the Imperial court at St. Petersburg, constructed (Fitzpatrick 1970, p.108) a horn which he supplied with a set of keys resembling that of a woodwind instrument. The keys were situated along the neck and throat of the bell. He called his invention an Amorschall. This, however, was unsuccessful. It did not provide a solution to the intonation problems and specific lower notes were still missing.

In 1788 the first attempts of building a double horn appeared. John Claggett, an Irish instrument manufacturer, combined (Morley-Pegge 1973, p. 26) an E and D horn. One mouthpiece was used and an attached valve directed the air through the desired horn. Unfortunately his adventure did not meet with success, as the two bells were impractical. In 1815 the Parisian Dupont, constructed (Morley-
Pegge (1973, p. 57) what he called a *Cor omnitonique* (Figure 1.6), in an attempt to obviate the need for changing crooks. Eight separate tube lengths lead from the mouthpiece onto the body of the horn, where all the crooks were built into one slide. By moving the mouthpiece, the horn could be set into different keys.

![Figure 1.6 Omnitonic horn. (Tuckwell 1983, p. 36)](image)

1.4 The Invention of Valves

The *Cor omnitonique* was superseeded by instruments which incorporated valves. This revolutionary system permitted instant lengthening of the instrument and change of key. It also meant that all the notes had the same tone quality. In
1814, the horn player Heinrich Stölzel and Friedrich Blühmel, supplied (Franz 1906, p.8) the horn with two piston valves (Figure 1.7). The first valve lowered the pitch by a semitone and the second, by a whole tone. In 1819 C.A. Muller and C.F. Sattler added a third valve, which lowered the horn by a minor third. This addition was not very popular as it added weight to the instrument. The tonal deficits were made up with the hand horn technique. The valve system did not meet with immediate success as the valves were not completely airtight. This caused uncertainty of attack and poor intonation. The added cylindrical sections and angular turns in the tubing detracted from the purity of the tone.

Figure 1.7 Horn with two piston valves. (Franz 1906, p.4)

Refinement of the valves increased the popularity of the horn. Three main valve systems have been used. In the piston valve, the piston is positioned in a cylindrical casing. It has four openings, connecting three potential airways. With the piston at rest, the air is directed straight from the leadpipe to the
main tube (Figure 1.8.1). When the piston is depressed (Figure 1.8.2), the air is diverted through the valve tube before joining the main section.

Figure 1.8.1 Piston at rest. Figure 1.8.2 Depressed piston. (Adapted from Bate 1980, p.512)

Piston valves are not built into horns anymore, unless they are specifically requested. Another type of piston valve, called the Vienna valve, is still in use in the horns played in the Vienna Philharmonic Orchestra. This valve has two separate but parallel pistons (Figure 1.9.1 and 1.9.2), which are operated by a connecting rod. The advantage of the Vienna valve is the absence of sharp curves, constricting the airway.
The most commonly used valve for the horn today, is the rotary valve. A rotor, operated by a metal spring, which is connected to a finger-plate, is situated in
a cylindrical casing with four ports. The rotor has two curved airways. At rest, the rotor provides a direct link between the leadpipe section and the main body (Figure 1.10.1). When the finger-plate is depressed, the rotor revolves ninety degrees to connect the extended valve tubing (Figure 1.10.2).

Figure 1.10.1 Rotary valve at rest. Figure 1.10.2 Depressed rotary valve. (Adapted from Bate 1980, p.512)

1.5 Design of the Modern Double Horn

The modern-day standard $B^b/F$ double horn is (Morley-Pegge 1973, p.51) based on a design done by Fritz Kruspe of Erfurt, in collaboration with a nephew of the famous horn player Friedrich Gumbert, in 1898. The valves have double airways and are connected to two sets of slides of varying length. A thumb valve directs the air through either the F or the $B^b$ horn.
The length of the horn is 3.81 metres, with the lowest note (or pedal tone) being $F_1$ - concert pitch (horn notation would indicate the note $C_2$, as the horn is pitched a fifth higher than what it sounds - refer to appendix A). However, when all three valves are depressed, the length of the tubing is extended to 4.715 metres, thus lowering the horn by another six semitones. These pedal tones are difficult to excite and are very low in intensity.

The wall thickness remains unchanged throughout the horn at $0.375\,\text{mm}$, except for specific areas which need protection, such as the initial section of the leadpipe and the handgrip. The overall bore expansion of the horn, including the cylindrical valve section, is plotted to scale in Figure 1.11.
Figure 1.11 Bore expansion of the horn, plotted to scale.
CHAPTER 2

THEORY OF SOUND WAVES IN PIPES

2.1 The Nature of Sound

The sensation of sound is caused by small variations in air pressure about atmospheric pressure which are recorded by the eardrum. While a full exposition of the physical laws that govern sound propagation would go beyond the main purpose of this thesis, some of this material is necessary in order that the material of Chapter 3 can be presented in a logical and compact way. More specifically, attention is focussed on the propagation of sound along confining tubes (of which brass instruments like the trumpet, the trombone and the horn, are rather complicated examples) and the examination in a qualitative way, as to initially what physical concepts enter into the description of a sound wave.

As stated above, the sensation of sound is related to pressure variations that are picked up by the eardrum. This pressure variation (or simply pressure, for short) is denoted by \( p \) and the absence of any sound wave, i.e. any pressure variation, is characterised by \( p = 0 \), and not \( p = \text{atmospheric pressure} \). The pressure \( p \) can vary both spatially (different pressures at different positions in a room at the same instant of time) and temporally (different pressures at different times at the same position in the room). The standard mathematical notation for this is \( p = p(x; t) \), where \( x \) indicates the position where the pressure is measured and
Sound waves in unrestricted space are basically a three dimensional phenomena so that the position at which \( p \) is detected, requires three spatial coordinates for its specification. However, the metal tubing that goes into the manufacturing of a brass instrument has a diameter much smaller than the length of tubing used. Consequently, variations in pressure along the tube are much more important than variations over the tube cross-section. This means that only a single space coordinate \( x \) will be required to measure the distance along the central line in such a tube.

The basic physical theory underlying the propagation of sound waves was already formulated (Benade 1973, p.35) in the eighteenth century. As with all undulatory waves (e.g. light waves), the pressure variation \( p(x; t) \) is governed by a wave equation. The form that the solutions of this equation take, i.e. how \( p \) depends in detail on \( x \) and \( t \), come in a great variety. Such general solutions are too complicated for this study. Since the pressure variations that correspond to a musical sound usually have a definite frequency \( (f) \) and wavelength \( (\lambda) \) associated with them, only very simple solutions of the wave equation, called standing wave solutions, will be required.

### 2.2 Standing Waves

The air within the tube of a brass instrument provides the medium through which sound energy is transmitted in the form of a sound wave. Since the passage of sound through a gas is accompanied by an oscillatory motion of the gas molecules in the direction of propagation of the wave (Morse 1948, p.217), sound waves are longitudinal. Such oscillations of the air molecules

---

1. *wavelength*: the distance between two wavecrests.
change the air density (and therefore the pressure) along the tube of a brass instrument, which are the source of the sound energy it produces. The physical boundary conditions imposed on such oscillations at the open and closed ends (the bell and mouthpiece, respectively) create standing sound waves within the instrument. Such standing waves arise from the interference between the two longitudinal waves that are excited at the mouthpiece and reflected at the open end of the instrument, respectively. Figure 2.1.1 illustrates the interference of two longitudinal waves within a tube, moving with the same speed in opposite directions. Their interference sets up nodes \(N_1, N_2, \text{etc.}\) which are fixed positions of no gas motion, and antinodes \(A_1, A_2, \text{etc.}\) which are positions of maximum vibration of the gas in the tube. This is more clearly illustrated in the adjacent diagram of Figure 2.1.2, by plotting the gas displacement at each position in the tube.

The cross-sectional shape of the tube and how it is terminated (closed or open at both or either end) determines the possible resonant modes that can be excited along the tube. This comes about as follows. Consider first a standing wave in a cylindrical tube that is open at both ends (an open-open resonator), as shown in Figure 2.2.

Since the gas density at each open end must match that outside the tube, antinodes in the gas displacement form at each of the open ends. Equivalently this means that the total pressure at these points is equal to atmospheric pressure, i.e. \(p = 0\). Consequently, these points are pressure nodes. A node (or pressure antinode) forms in the middle. The gas displacement in this mode of oscillation is plotted as a function of distance, in Figure 2.3.

Figure 2.3 shows at once that the longest wavelength that the tube can accommodate is twice its length, i.e. \(\lambda_1 = 2L\). This is called the wavelength of the
fundamental mode. Since $\lambda f = c$, where $f$ is the frequency of oscillation and $c$ the speed of sound in air, one has

$$f_1 = \frac{c}{\lambda_1} = \frac{c}{2L}$$

for the resonant frequency of the fundamental mode of an open-open tube. Clearly other modes of shorter wavelengths (and correspondingly higher frequencies) are also possible. Two higher modes of oscillation of an open-open tube are depicted in Figure 2.4 in addition to the fundamental mode. The sequence of wavelengths are seen to be $\lambda_1 = 2L$, $\lambda_2 = L$, and $\lambda_3 = \frac{2L}{3}$. The
Figure 2.2 Standing wave of the fundamental tone (the longest possible wavelength) in an open-open cylindrical tube, with a node \( N \) and antinodes \( A_1 \) and \( A_2 \).

Figure 2.3 The gas displacement in the standing wave in Figure 2.2, plotted as a function of distance along the tube.

The general rule is obviously

\[
\lambda_n = \frac{2L}{n}; \quad n = 1, 2, 3 \ldots
\] 

(2.2)
with associated resonant frequencies

\[ f_n = \frac{c}{\lambda_n} = \frac{nc}{2L} = nf_1 \quad ; \quad n = 1, 2, 3 \ldots \]  (2.3)

where \( f_1 \) is the fundamental frequency given by Equation (2.1).

In the musical literature the \( n = 1 \) mode (which has the longest wavelength, or lowest frequency) is usually referred to as the fundamental (or pedal) tone; the higher modes with \( n > 1 \) are then variously referred to as harmonics of each order \( n \), or overtones. Table 2.1 sets out this nomenclature in tabular form. This nomenclature is used in Figure 2.4. The situation for an open-closed tube is different. Here the action of the player’s lips at the mouthpiece causes a node to form at that end of the tube. Consequently, the allowed oscillations that can occur are different from those in Figure 2.4. They are instead given by the patterns of Figure 2.5.

The first three allowed wavelengths are now seen to be \( \lambda_1 = 4L \); \( \lambda_2 = \frac{4L}{3} \); and
Table 2.1 Nomenclature in common use for harmonics and/or overtones. The associated resonance frequencies and resulting wavelengths are also listed.

\[ \lambda_3 = \frac{4L}{5} \quad \text{or} \]

\[ \lambda_m = \frac{4L}{2m-1} \quad ; \quad m = 1, 2, 3 \ldots \]  

(2.4)

In general, \( m \) has purposely been used instead of \( n \) in the sequence, to emphasize the fact that here \( \lambda_m \) is formed by dividing the constant \( 4L \) by an odd integer \( (2m - 1) \), instead of all integers as in Equation (2.2). Any even numbered wavelengths would call for antinodes at both ends, which is excluded in this case. The associated resonance frequencies are thus odd multiples of the fundamental.
Figure 2.5 Fundamental mode of an open-closed tube, together with the third and fifth harmonics. \( f_1 = \frac{c}{4L} \)

frequency \( f_1 \) \( (f_1 = \frac{c}{4L} \) in this case) : thus

\[
 f_n = nf_1 \quad ; \quad n = 1, 3, 5 \ldots
\]  

(2.5)

reverting to the \( n \) of Equation (2.3) again, but now noting that all even \( n \)'s are suppressed. Therefore an open-closed tube can only support odd overtones in addition to the fundamental tone, in its sound spectrum.

2.3 Brief Mathematical Description of Standing Sound Waves

The main points relating to the allowed frequencies and wavelengths that cylindrical open-open or open-closed tubes can support, have been made pictorially in the previous section. The physics underlying these pictures can be simply described in terms of trigonometric functions of space and time. It can be shown
(Coulson 1952, p.91), for example, that the gas displacement \( y = y(x; t) \) at each point \( x \) along the axis of an open-open tube at time \( t \) must be of the form

\[
y_n(x; t) = A_n \cos\left(\frac{2\pi x}{\lambda_n}\right) \sin(2\pi f_n t + \delta_n) \tag{2.6}
\]

in the \( n^{th} \) mode of oscillation, where \( \lambda_n \) and \( f_n \) are given by Equations (2.2) and (2.3), while \( A_n \) and \( \delta_n \) are arbitrary constants. The pressure wave associated with Equation (2.6) has nodes at each end of the tube; \( x = 0 \) and \( x = L \). Consequently, the pressure variation away from atmospheric pressure along the tube, will be of the form

\[
p(x; t) = A'_n \sin\left(\frac{2\pi x}{\lambda_n}\right) \sin(2\pi f_n t + \delta_n) \tag{2.7}
\]

where \( A'_n \) is a new constant.

### 2.4 Excitation of Overtones

Equation (2.7) raises another important point. Suppose a brass instrument is tuned to have \( f_1 = 65 \) Hz for its fundamental mode, i.e. at the frequency for the note \( C_2 \).
However, when the player blows this note, some or all of its overtones will be excited at the same time, with amplitudes and phases $A'_n$ and $\delta_n$, which are determined by the actual details of excitation, i.e. the particular blowing technique of the player. A partial set of overtones that can accompany the fundamental vibration $f_1 = 65$ Hz associated with the note $C_2$, is given in Figure 2.6.

Mathematically, this circumstance is expressed by writing the actual pressure along the tube as a sum of the resonance pressure modes, given by Equation (2.7): 

$$p(x; t) = A'_1 \sin\left(\frac{2\pi x}{\lambda_1}\right) \sin(2\pi f_1 t + \delta_1) + A'_2 \sin\left(\frac{2\pi x}{\lambda_2}\right) \sin(2\pi f_2 t + \delta_2) + \ldots$$ (2.8)

Figure 2.6 The first seven overtones associated with the fundamental $C_2$. The mode number (in brackets) is attached to each note, with its corresponding frequency given directly below. (Adapted from Meyer 1978, p.22)
where \( p(x; t) \) is the resulting pressure variation along the tube, and \( A_1', A_2' \) and \( \delta_1, \delta_2 \) etc. are determined by the initial (and not boundary) conditions at \( t = 0 \) relating to how the tube was excited. The \( \lambda_1, \lambda_2 \) and \( f_1, f_2 \) etc. are given by Equations (2.2) and (2.3) as before.

Thus the same note of a brass instrument blown differently, will have a different mix of overtones in its harmonic make-up in Equation (2.8), i.e. different values of \( A'_n \) and \( \delta_n \). Incidentally, this feature of exciting overtones, should not be confused, in the case of a piano string, with the phenomena of sympathetic vibration, where a vibrating piano string sets other select piano strings into vibration as well. The phenomena of sympathetic vibration merely makes audible (Bernstein 1976, p. 21) the overtones that are in any event present in the motion of the single string.

2.5 The Modern Horn Design

As discussed in section 2.3, only odd-numbered harmonics are excited in an open-closed cylindrical tube. The horn, however, needs the full harmonic range in order to produce the desired bright timbre. It is found (White 1980, p. 241) that if a bore progressively varies in diameter, it will generally produce an overtone series with inharmonic partials (i.e. the overtones are not exact multiples of the fundamental frequency), as in the case of brass instruments.

An extreme example is (Morse 1968, p.332) that of a conical horn, given by the dunces hat shape in Figure 2.7.

The horn has, amongst others, the simple spherical wave solution

\[
p(r; t) = \frac{\sin\left(\frac{2\pi r}{\lambda}\right)}{r} \sin(2\pi ft) \quad (2.9)
\]
for the pressure variation inside the cone. Here $r$ is the distance from the apex $A$ to any point on the wavefront. The condition that the pressure in Equation (2.9) should vanish everywhere on the surface of a sphere of radius $L$ (refer to Figure 2.7), leads back to exactly the same condition as Equation (2.2) for the allowed wavelengths in an open-open tube, and hence, to an harmonic sequence that contains the even, as well as, the odd harmonics. Thus, the tapering of the half-closed tube has restored the missing even harmonics.

The design of the modern horn is much more involved as it has to accommodate a lengthy cylindrical valve section (refer to Figure 1.11). The taper of the mouthpipe and flare of the bell are the main features effecting the harmonic content of the sound spectrum generated by the instrument. These shapes are (Benade 1974, p.80) mathematically difficult to handle. For application and comparison between theory and experiment in Chapter 3, results that have been established in the literature will be used, without discussing their derivation in too much detail.
ANALYSIS OF THE SOUND SPECTRUM OF THE HORN AND ITS DIRECTIVITY

3.1 The Sound Spectrum of the Horn

An acoustical analysis of the tone quality of a musical note can be graphically illustrated by its sound spectrum. This is a graphical representation of the frequency of each harmonic that has been excited, versus its relative intensity, for the note in question.

The experiments to be described, were conducted in an anechoic chamber under exactly the same conditions, so that no results were influenced by a changed environment. Two classes of measurements were made. In class A, the harmonic content of a series of notes on the horn of varying pitch, was determined by using a filter technique. The bell of the horn was also removed to determine its influence on the sound spectra of notes chosen from the above mentioned series. In class B, the influence of varying dynamic levels on the sound spectrum of a note, was determined.

3.1.1 Sound Spectra of Various Tones - Class A Experiments

The experimental set-up used for the experiments of classes A and B, is depicted
in Figure 3.1.

A Brühl & Kjaer 25 mm microphone (M) was placed at a distance of three metres from the player (P), in line with the horizontal axis of the bell of the horn. The opening of the bell was facing the microphone. The microphone was linked to a Brühl & Kjaer audio frequency spectrometer, type 2113. The frequency response control was switched to LIN (linear - i.e. all frequencies were measured). The signal was further connected to a UHER tape recorder, type
4000 REPORT IC, which recorded the individual notes played on the horn. The signal from the tape recorder was then reversed through the audio frequency spectrometer and connected to a Brühl & Kjaer level recorder, type 2305, as shown in Figure 3.2.

![Diagram of audio equipment](image)

Figure 3.2 Recorded notes are analysed and graphically printed by the level recorder.

The recorded tones were played back on the tape recorder, analysed by the audio frequency spectrometer, which was set on measuring in one third octave.
frequency bands, and the final readings were taken from the spectra graphed by
the level recorder. It must be pointed out that these spectra will differ slightly
with different horn players, depending on each player's technical ability and
sound concept (refer to Chapter 2, p.26).

Figure 3.3 illustrates the sound spectra of eight different notes, each played at
a comfortable mezzo-forte dynamic level.

![Sound Spectra Graph](image)
Figure 3.3 Sound spectra of eight different notes ranging from the lower to the upper register. The point 0 represents the highest intensity level.

(Reference to pitches are at concert pitch and not at written horn pitch.)
The vertical axis represents the relative intensity measured in dB, while the horizontal axis gives the frequency plotted in one third octave frequency bands. It can be seen from these plots that harmonics above about 5 kHz (= 5000 Hz) do not have any appreciable effect on the timbre of the note (see also Meyer 1968, p.33).

The first important feature illustrated by these graphs, is that the intensity distribution changes as the fundamental frequency is increased. In particular, the fundamental frequency determining the pitch, is not necessarily the loudest harmonic. In fact, in the lower register (Figure 3.3 a - c), the fundamental is seen to be relatively weak, with the third, fourth, fifth and sixth partials featuring prominently. The second partial (i.e. the first octave) is distinctive in the middle register (Figure 3.3 d - f). The fundamental (Figure 3.3 g,h) only dominates in the upper register.

The prominence of the fundamental in the upper register is far more distinctive than those partials that dominate in the lower registers. In, for instance Figure 3.3 h, there is an intensity difference of 25 dB between the fundamental and the nearest harmonic. Such differences in the lower register only amount to about 5 or 10 dB. The radiational efficiency of the higher frequencies of the horn, is (Martin 1942, p. 312) due to the fact that the circumference of the bell is larger than the wavelengths of these fundamentals. As the flare of the bell is gradual compared to the shorter wavelengths, very little reflection occurs at the bell. To illustrate the influence of the bell on the impedance peaks, sound spectra for four notes measured without the bell, are shown in Figure 3.4.
$B_2$ (116 Hz)

$C_4$ (262 Hz)
Figure 3.4 Sound spectra of four notes measured without the bell of the horn.
It can clearly be seen from these graphs that the higher frequencies are less prominent, than with the bell present and that the spectra differ from those of the same notes measured with the bell on.

Another influential factor in the sound colouring of a note, is the *frequency - extent* of the spectrum. In the bottom register (Figure 3.3 a - c) the peaks seem to fade to a *noise level* around 2000 Hz. In the middle and upper registers the peaks extend to the higher frequency *edge* around 4500 - 5000 Hz. There also appears to be no preferential excitation of either the odd - or the even harmonics in the development of any of the sound spectra that have been measured.

The occurrence (as shown in Figure 3.3) of changing groups of harmonics gaining prominence as the fundamental frequency is increased, is characteristic of the sound of each individual instrument. Such a group indicates (White 1980, p.92) the frequency band of the note, where the sound energy is largely concentrated and is referred to as a *formant*. A formant is thus an indication of the timbre of the instrument as well as the sound colouring of the tone. This leads to the next important point of the influence of the dynamic level on the harmonic structure of a specific tone.

### 3.1.2 Sound Spectra of Varying Dynamic Levels - Class B Experiments

The experimental set-up used to determine the influence of varying dynamic levels on the sound spectrum of a note, has already been described under section 3.1.1. Figure 3.5 illustrates three spectra of the note $F_4$ (349 Hz) played at a *fortissimo*, a *mezzo-forte*, and a *pianissimo* dynamic level.
Figure 3.5 The note $F_4$ (349 Hz) played at a $ff$, a $mf$ and a $pp$ dynamic level.
The change in an increasing dynamic level, brings about an increase in the harmonic content, which in turn changes the sound colouring. It is found that the large number of upper partials present at a fortissimo dynamic level, add a certain brightness to the tone (also described as a brassy or metallic sound), whereas the sparse harmonic content at the pianissimo level, lends itself to a darker and more somber tone colouring. It will also be noted that at the fortissimo dynamic, the second harmonic has the highest intensity level, but is strongly supported by the fundamental and the upper partials. With decreasing dynamic levels, the number and intensity of the overtones drop sharply. Thus, the main formant of the note shifts slightly higher in the sound spectrum with increasing dynamic level, distinctly influencing the tonal colouring. Once again, it must be stressed that the detailed harmonic content of these spectra depends upon the technique of the individual player.

3.2 Directivity

The sound of any musical instrument is not radiated in all directions with the same intensity. In the case of the horn, its sound radiation is influenced by the flare of the bell, the oblique position of the instrument when held in playing position, as well as the interference of the player's body.

3.2.1 Experiments to Determine the Directivity of the Horn

Experiments to investigate the directional characteristics of the horn, where done in the same anechoic chamber, with a similar experimental set-up as was used for the previous experiments. This is illustrated in Figure 3.6.
The player \((P)\) stood on a Brühler & Kjaer turntable, type 3922, which revolved at 0.75 rpm. The microphone \((M)\) was again placed at a distance of three metres from the player, in line with the horizontal axis of the bell of the horn. The microphone was linked to the audio frequency spectrometer, with the frequency response control switched to \(LIN\). The signal was then connected to the level recorder which was supplied with polar graph paper. Polar diagrams of the directivity of the notes played, were recorded on the graph paper which also
rotated at 0.75 rpm.

Each experiment was started with the bell of the horn facing at a 90° angle to the microphone. Thus, the 0° (microphone) point was situated slightly to the right of the player. The photos in Figure 3.7 illustrate the four 90° positions in relation to one of the polar diagrams.

Four different notes were chosen and played at a comfortable mezzo-forte level:

1) $F_2$ - (87 Hz)
2) $F_3$ - (175 Hz)
3) $C_4$ - (262 Hz)
4) $A_4$ - (440 Hz)

Polar diagrams for the selected frequencies are plotted in Figure 3.8 to illustrate the results of the measurements. In order to obtain the correct visual perception of the directivity of the horn when held in playing position, the graphs have been turned so that 240° (on the graph) corresponds with the front of the player.
Figure 3.7 Photos indicating the four 90° positions in relation to a polar diagram.

(The photos were taken by Mr. D.J. Claude).
a) \( F_2 \) (87 Hz)

b) \( F_3 \) (175 Hz)
Figure 3.8 a - d. Polar diagrams. The sharp indentations are the breath snatches of the player.
At each point in the graphs, the maximum sound projection of either the fundamental or the overtones, is indicated. Thus, the ordinal number of the harmonics is not considered in the directionality of the sound (refer to section 3.1.1, p.36).

As can be seen from each graph, most sound energy is directed towards the right and back of the player (i.e. between 330° and 30°). A secondary maximum is found towards the left-front side of the player, between 150° and 210°. This appears to provide a certain symmetry to the directionality. The least radiation is recorded at the back-left side, where the interference of the player's body is at a maximum.

In the lower frequencies the sound intensity distribution is fairly omnidirectional. Because of the longer wavelengths of the lower frequencies, most of the sound is reflected and a very small percentage is radiated beyond the bell (refer to section 3.1.1, p.36). With increasing frequency and thus, more efficient radiation, areas of distinct directionality, together with interposing dull patches, become more apparent.

The above diagrams show that the position of the listener has a direct bearing on the perceived timbre and sound spectrum of the horn.
CONCLUSION

The characteristic timbre of an instrument, is a result of the individual set of patterns of sound spectra formed by that instrument. Within the scope of these patterns, the final tone production depends on two important factors: firstly, the manner in which the note is excited (i.e. the attack transient) and secondly, the dynamic level of the tone. Both these aspects effect the frequency-extent of the spectrum, as well as the intensity of the harmonics, which influence the tone colouring.

The sound spectrum of a brass instrument is determined by the standing waves formed within the instrument and the radiation efficiency of the bell. Although the horn does not have a constant conical bore, the design of the taper of the mouthpipe and in particular, the flare of the bell, distorts the sound waves to ensure an harmonic series of odd - and even overtones, vital to the bright timbre of the instrument.

A knowledge of the changing distribution of the sound energy surrounding the horn player, is of importance in determining microphone and reflecting panel positions. Polar diagrams (such as those illustrated in Chapter 3, Figure 3.8) give clear indications of the areas where the highest sound intensity is being radiated and thus also where the brightest timbral colour of the instrument can be recorded.
The notable tone of the "Waldhorn", the peculiarity of its varied characteristics, together with its resounding tonal volume, fit it admirably for every style of composition. Its peculiar tonal character adapts itself equally well to the demands of joyous, boisterous music for the hunt, as well as for expressive, dreamy and melancholy music!

1. (J. Rühlmann - Neue Zeitschrift für Musik, 1871)

APPENDIX A

USA STANDARD
C₁ C₂ C₃ C₄ C₅ C₆ C₇ C₈

HELMHOLTZ
C₁ C C¹ Cⁿ C'' C''' C''''

HORN NOTATION
G₁ G₂ G₃ G₄ G₅

(Adapted from Backus 1970, p.135)
LIST OF REFERENCES


Coulson, C.A. 1952, *Waves: A Mathematical Account of the Common Types of*


