Geogebra, a Tool for Mediating Knowledge in the Teaching and Learning of Transformation of Functions in Mathematics

by

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ABSTRACT

As a teacher of mathematics, I always taught the topic functions (graphs such as linear, quadratic, hyperbola, exponential, trigonometric functions) in the same way all of my twenty-three years in the profession. I often assumed that the learner understood a concept that had been presented only to find, in subsequent lessons, that the learner could not recall it or talk about it. I referred to the constant value c in the function $f(x) = ax^2 + c$ or $f(x) = ax^2 + bx + c$ as the y-intercept informing my learners that it is a point on the y-axis of the Cartesian plane. I also taught transformation of functions as the vertical and horizontal shift without much visual demonstration beyond pen and paper. Whilst using dynamic mathematics geometry software, last year namely, *Geogebra*, I realized that this section could be taught more effectively through interaction with this software. *Geogebra*, is a freely available interactive dynamic software for the teaching and learning of mathematics that combines geometry and algebra into a single user-friendly package.

Within this research I set out to explore firstly, the function of *Geogebra*, as a pedagogical tool and mediating artifact in the teaching and learning of transformation of functions in secondary school mathematics; and secondly whether interaction with these virtual manipulatives enhance the understanding of mathematics concepts.

The study is rooted in a social constructivist view of learning and mediated learning and the approach used is a case study. The research was carried out in an independent school that involved 8 learners. My data consisted of feedback from two sets of student worksheets, the first being from prior to using the *Geogebra* applets and the other from post engagement with the applets, classroom observations during the practical use of *Geogebra* and finally with learner interviews. On analysis of the data it seems that the introduction of *Geogebra* did indeed influence the educational practice in three

dimensions, namely: the development of mathematical ideas and concepts through computer-based teaching and the role *Geogebra* plays in the understanding of and visualization of certain mathematical concepts in high school algebra topics.

DECLARATION

"I, Razack Sheriff Uddin, hereby declare that the work on which this dissertation is based, is original (except where acknowledgements indicate otherwise) and that neither the whole work nor any part of it has been, or being, or shall be submitted for another degree at this or any other university, institution for tertiary education or examining body".

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CHAPTER ONE

INTRODUCTION

1.1   Background

The South African government has made a pledge to develop the Information and Communication Technology (ICT) skills of its people to address the inevitable technological demands to compete in the global village. The attempt to roll out the laptop programme to teachers to take technology to the classrooms is evidence of its commitment to address the challenges of 21st century schooling. We are now immersed in a society that is becoming increasingly dependant on technology for its survival and as a nececessity to compete globally. Teachers are working with learners whose lives have becomesubsumed into this 21st century media culture. Today's learners are digital learners – they literally take in the world through the myriad of computing devices such asdigital cameras, music players, cellular phones, handheld gaming devices, smart phones, ipods laptops and ipads, in addition to computers, TVs, and gaming consoles at home and global positioning devices in their travels. Therefore, education ought to be structured to meet the needs of these learners. There is a dramatic departurefrom the top-down, authoritarian type of education of the past. It is a relinquishing, finally, of a textbook-driven, teacher-dominated, paper-and-pencil schooling. It means a new way of understanding the concept of "knowledge", a new definition of the "educated person". The 21st century will require knowledge generation or creation, not just information delivery, and schools will need to create a "culture of inquiry". A new way of designing and delivering the curriculum is therefore necessary. Thus, research in finding avenues to address the needs of a 21st Century child in class is of paramount importance to prepare him/her for jobs that may not exist now but are sure to surface soon. Adherence to current traditional teaching methods without a partnering them with new technologies will certainly deprive the child of his/her future.

With tests and exams as the final determining factors for passing, learners are not oftenencouraged to study the developmentof processes necessary to arrive at a formula. Instead, formulaeare given to learners to memorize with the aim of applying them directly, to solve typical exercises. Much of the learning is procedural yet the aim should be conceptual understanding. According to Kilpatrick, Swafford, and Findell (2001, pp. 380-382),

conceptual understanding is consideredsignificant to understandingmathematical concepts and ideas.

Conceptual understanding is a central element in mathematical adeptness. Interconnection between mathematical concepts and representations is as a result of conceptual understanding. Conceptual understanding offersa base from which learners cultivateinsight into mathematical concepts and ideas and skillfullyapply them in solving non-routine mathematical problems.

1.2   Purpose of the study

As a teacher of mathematics, I have always taught functions in mathematics (graphs such as linear, quadratic, hyperbola, exponential and trigonometric functions) in the same way through all of my twenty-three years in the profession. I often assumed that the learner understood a concept that had been presented only to find, in subsequent lessons, that the learner could not recall it or talk about it. I referred to the constant value c in the function defined by $f(x) = ax^2 + c$ or $f(x) = ax^2 + bx + c$ as the y-intercept informing my learners that it is a point on the y-axis of the Cartesian plane. I also taught transformation of functions as the vertical and horizontal shift without much visual demonstration beyond pen and paper. Whilst using dynamic mathematics geometry software last year, namely *Geogebra*, I realized that this section could be taught more effectively through interaction with this software. Ever since, I have wanted to explore the influence of this dynamic software on the learners' understanding of these mathematical concepts in the teaching and learning of the topic, ie. functions at secondary school level. Within this study I intended to explore whether computer software could influence the understanding of mathematical concepts, particularly through the use of dynamic geometry software (DGS), such as. *Geogebra.*

The study aimed to :

> explore the role of *Geogebra*, as a pedagogical tool and mediating artefact in the teaching and learning of transformation of functions in secondary school mathematics;

> explore whether interaction with these virtual manipulatives will enhance the understanding of mathematics concepts.

One intention of this research was to contribute to studies in understanding how learners acquire and develop their conceptual understanding of mathematics using computer technology in technology supported environments.

Questions to be answered in the research:

- How does *Geogebra* serve as a didactic tool and a mediating artefact in the teaching and learning of transformations of functions?
- What influence does the use of *Geogebra* have on the understanding of mathematical concepts embedded in the section on transformation of functions?

1.3   Structure of the Research Study

Chapter 1 is an introduction to the thesis. It provides background information to the research being undertaken. It also discusses the rationale, significance of the study and an overview of the study.

Chapter 2 presents a review of research literature illuminating amongst other aspects; the understanding of mathematical concepts; the role of computer technology in mathematics teaching and learning; how technology,particularly virtual manipulatives impacts on concept development in mathematics learning, and finally on *Geogebra* as a dynamic learning mathematics software tool to mediate learning of mathematical concepts.

Chapter 3 presents the theoretical framework that guides the research, determining the structure that holds or supports the theories on which this research is based. This chapter explores how learning theories, especially the activity theory and constructivism, support the use of computer technology in a mathematics classroom.

Chapter 4 looks at the research approach that was used in this research. This study used the case study approach and explains the reasons why case study research was chosen as the research approach for this study. Next the areas of reliability and validity of research are explored. Ethical issues are also considered.

Chapter 5 presents the findings emerging from the case study. The findings will be drawn from three stages; namely, the pre-activity stage, the activity stage and the post-activity stage. The pre-activity stage involves a pen-paper exercise based on features of the functions, which leads to the section on transformation of functions in the grade 10 syllabus. The activity stage involves hands-on manipulation of *Geogebra* applets, which aims to enhance the conceptual understanding of transformation of functions. Finally, the third stage of the study involves a pen-paper test evaluating the level of understanding as a result of learners' engagement with virtual manipulatives during the activity stage.

Chapter 6 presents the analysis of the data collected from the three stages mentioned above to conclude whether *Geogebra*, was indeed a pedagogical tool and mediating artefact in the teaching and learning of transformation of functions in secondary school mathematics; and whether interaction with these virtual manipulatives did indeed enhance the understanding of mathematics concepts involving transformation of functions. This chapter also presents some of the limitations experienced during the study as well as a discussion of the study.

Chapter 7 aims to conclude the study andalso presents the recommendations and suggestions for further research in this field.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

According to the National Curriculum and Assessment Policy Statement (2011),

> *Mathematics is a distinctly human activity practiced by all cultures, for thousands of years. Mathematical problem solving enables us to understand the world (physical, social and economical) around us, and, most of all, to teach us to think creatively.*

One of the general principles the Curriculum and Assessment Policy Statement (CAPS) document advocates in teaching is the idea of investigations which ought to provide the opportunity to develop in learners the ability to be methodical, to generalize, make conjectures and try to justify or prove them. This implies that learners need to reflect on processes and not be concerned only with getting correct answers. These investigations are paramount in order to establish deeper meaningful understanding of mathematical concepts and mediating tools such as virtual manipulatives provides a bases for such investigations.

Establishing deep conceptual understanding in learners is an important role of mathematics teachers. The development of mathematical knowledge involves a process of creating new knowledge by linking and associating different piecesof knowledge that should alreadyhave been understood by the learners. The literature review will focus on, amongst other aspects, the understanding of mathematical concepts, the role of computer technology in mathematics teaching and learning, how technology, particularly virtual manipulative, impacts on concept development in mathematics learning and finally on *Geogebra* as a dynamic learning mathematics software tool to mediate learning of mathematical concepts.

2.2 Understanding of mathematical concepts

For mathematics learning and understanding to take place, at any level, learners need to engage with the concept being taught. Mathematics is not an observer sport. Learners must be actively engaged in the learning process through practical applications of mathematics or that

which endeavours to make mathematics less abstract. A constant challenge faced by learners of mathematics is its abstract nature.

According to Martínez, Bárcena and Rodríguez, (2005, p. 1),

> *"true understanding of mathematics takes place as learners progress through phases of action (physical and mental), abstraction (the process by which actions become mentally entrenched so that learners can reflect and act on them), and reflection (deliberate analysis of one's thinking). Moving through these phases time after time enables learners to construct increasingly sophisticated mental models of the abstraction".*

To facilitate knowledge construction through mathematical reasoning and communication would involve learners having to make conjectures, testing these conjectures, proving, attaining high levels of conviction and being able to convince others that their conjectures are true, to critique or disprove conjectures and, if the conjectures are found to be false, to develop new conjectures. In order for learners to develop their innate logical sense, and a working knowledge of the mathematics concepts, they must have a great variety of interactions with their environment, exploring, manipulating, comparing, arranging and rearranging real objects and sets of objects. In my opinion, many of these types of interactions and experiences may occur incidentally whilst others need carefully planned interventions.

This study aims to explore whether computer generated virtual manipulatives provide learners with the necessary tools to enable them to understand mathematical concepts through their engagement with these manipulatives. This research looks at the understanding of concepts relating to functions and transformation of functions in high school mathematics. Harel and Dubinsky (1992, p. ix) feel that the concept of a functionis the "single most important" concept in mathematics education at all grade levels. Learners have trouble with the language of functions(e.g., image, domain, range, pre-image, one-to-one, reflection) which subsequently impacts negatively on their abilities to work with graphical representations of functions (Markovits, Eylon & Bruckheimer, 1988, pp. 43-60). Dreyfus (1990, pp. 53-59) identified three problem areas in his summary of the research on learners' working towardsunderstanding the concepts of functions. These are:

- *The mental concept that guides a learner when working with a function in a problem tends to differ from both the learner's personal definition of a function and the mathematical definition of a function.*

- *Learners have trouble graphically visualizing attributes of a function and interpreting information represented by a graph.*
- *Most learners aree unable to overcome viewing a function as a procedural rule, with few able to reach the level of working with it as a mathematical object*

These problems are significant and need to be addressed by educators during the course of their teaching. Not much time is spent in schools with the so-called "mundane" aspects of definitions. Definitions present useful building blocks for meaning making. This may contribute to the learners' lack of visual understanding of some concepts. Very rarely do educators encourage learners to visualize concepts. Finally, the idea that learners view functions procedurally is a common one and, hopefully, this study can show that deep understanding can be established through the use of virtual manipulatives.

Goldenberg (1988, pp. 135–173) identified pedagogical challenges in the educational use of graphical representation of functions. He stated that graphing technologies sometimes produced negative effects :

> *In an algebra or pre-calculus context, visual illusions can arise that actually are learner misinterpretations of what they see in a function's graphical representation.For example, learners view vertical shifts as horizontal shifts when comparing linear graphs (such as the graphs of y=2x+3 and y=2x+5). Also, learners falsely conclude that all parabolas are not similar due to the misleading effects of scaling. Learners often conclude that a function's domain is bounded due to misinterpretations of the graphing window.*

Goldenberg also highlights that, it is still the task of every educator to ensure that these misconceptions and misrepresentations are eradicated. One possible way of overcoming this misrepresentation is through virtual manupulatives, where the learner can view for himself/herself the different shifts and what produces them.

Exploration of problems from a number of representations adds depth to learners' understanding of concepts and new mathematical ideas are best remembered and understood if the learner can link them to their previous knowledge. One of the most important issues that arise in mathematics education scenarios is the fact that strategies need to be found to promote understanding in mathematics (Hiebert and Carpenter, 1992, p. 67). Kaput (1989, pp. 167 -194), as well as, Keller and Hirsch (1998, pp. 1-17) found that the use of multiple

representations provide diverse concretisations of a concept, carefully emphasize and suppress aspects of complex concepts, and promote the cognitive linking of representations. This is supported by Dreyfus (1991, p. 32) when he stated that:

> *"to be successful in mathematics, it is desirable to have rich mental representations of concepts. A representation is rich if it contains many linked aspects of that concept.... One does not get the support that is needed to successfully manage the information used in solving a problem unless the various representations are correctly and strongly linked. One needs the possibility to switch from one representation to another one, whenever the other one is more efficient for the next step one wants to take....Teaching and learning this process of switching is not easy".*

Eisenberg & Dreyfus conducted extensive explorations on learners' understanding of function transformations, focusing mainly on visualization of transformations. They acknowledged the difficulty in visualizing a horizontal translation in comparison to a vertical one, suggesting that "there is much more involved in visually processing the transformation of *f(x)* to *f(x + k)* than in visually processing the transformation of *f(x)* to *f(x) + k*",(1994, p. 58). Using *Geogebra,* learners can easily and effortlessly input a function, $f(x) = 3x^2 - x - 2$ and thereafter $g(x) = f(x) + 3$ or $h(x) = f(x - 1)$ and immediately see the results of their enquiry. Learners need to see what happens next is possible with dynamic geometry tools.

Learning is the importance of building new knowledge on the foundation of learners' existing knowledge and understanding. Because learners have several encounters with functional relationships in their everyday lives, they bring a great deal of relevant knowledge to the classroom. That knowledge can help learners' reason carefully through algebra problems. Learners also need a strong conceptual understanding of function as well as procedural fluency. The very essential concept with functions is that of a dependent relationship: the value of one thing depends on, is determined by, or is a function of another. This is where the learner can observe how one variable affects other properties of a function, that is, how the vale of (*c*) moves/translates/shifts the function vertically. This move in relation to the value of (*c*) on a dynamic geometry software implants in the child a visual effect of the function which forms a deeper understanding with conviction in the mind of the child, hence conceptualization of the value of (*c*) on the graph.

The learner now focuses on the rule or expression that tells him how one thing (c) is related to another (vertical shift). The "translation" is formally defined in mathematics as "a vertical shift".

Each of these representations describes how the value of one variable is determined by the value of another. Good understanding is not just about developing learners' facility with performing various procedures, such as finding the value of y given x or creating a graph given an equation. Instruction should also help students develop a conceptual understanding of function, the ability to represent a function in a variety of ways, and fluency in moving among multiple representations of functions. For instance, the slope of the line as represented in an equation, for example, should have a "meaning" in the verbal description of the relationship between two variables, as well as a visual representation on a graph.

2.3   The role of computers in education

As a result of technology dominating our lives and society becoming immersed in a digital world, a new paradigm has evolved in education, changing how teaching is delivered and how learning is processed. Learning is no longer confined to the physical school building or the classroom but can take place anytime and anywhere, such as in computer labs and via the radio, television, podcasts or internet. It is often assumed that when one speaks of technology one is referring to calculators and computers. Yet the concept of technology includes non-electronic media and tools(paper cuttings, bending of wires to form parallelograms, etc.) as well. Technology refers to all the tools or gear that human beings use to search for meaning, to resolve problems, to convey their findings and, to measure and to explain phenomena around them. Technology therefore includes all the tools we use to search, sort, create and report information in our own unique socio-cultural context. According to the National Council of Teachers of Mathematics:

> *electronic technologies ...furnish visual images of mathematical ideas, they facilitate organising and analysing data, and they compute efficiently and accurately. They can support investigations by learners in every area of mathematics. When technology tools are available, learners can focus on decision making, reflection, reasoning, and problem solving" (NCTM, 2000, p. 24).*

This study aimed to explore the efficacy of these technologically based tools: how theyassist teachers and learners alike to create and develop knowledge, to develop and respect the many unique ways of quantifying, comparing, classifying, measuring and explaining mathematical concepts in a more palatable way.

There is a growing range of mathematics teaching software available for use in schools. Weist (2001, p. 47) suggests that this may be classified into two types of programs, and she distinguishes between what she calls instructional software and tool software.

> *Instructional software is designed to teach learners skills and concepts….[while]*
> *Tool software is used as an aid towards another goal. It does not teach but rather*
> *performs a function that facilitates attainment of some objective.* (Weist (2001, p. 47)

With the emphasis on mathematical processes technology in the mathematics classroom becomes increasingly necessary. With the aid of technology, tedious computations are easily performed, multiple variations of examples are effortlessly produced. Together with vivid, dynamic visuals, technology can provide a strategy that encourages mathematical thinking. This may, therefore, allow learners and teachers more time to concentrate on the mathematical processes in the classroom. The illustrative properties of the software allow learners to visualize and refer to the charts, images and diagrams thus facilitating both the conceptualization of the mathematical ideas and concepts as well as in conjecturing and reasoning during their engagement with the problem.

This research attempted to investigate the relevance of computer software as a tool to assist teaching and learning. This may inevitably encourage the migration from concrete to abstract, externalization to internalization. In fact it may support the idea of iteration between the processes as postulated by Mudaly and Rampersad (2010, pp. 36-48).

Technology's influence on mathematical learning is either amplified or limited through the kinds of mathematical tasks and activities teachers provide. Hollebrand's (2007, pp. 695-705) views on learning in a technological environment is that, "learners can benefit in different ways from technology integration into everyday teaching and learning". A technological environment has the potential to offer learners opportunities to engage with different mathematical skills and levels of understanding through variedmathematical tasks and activities. Van Voorst (1999, p. 2) emphasisedthe idea that technology was "useful in helping learners view mathematics less passively, as a set of procedures, and more actively as reasoning, exploring, solving problems, generating new information, and asking new questions." In addition, he maintainedthat technology helps learners to "visualize certain math concepts better" and that it adds "a new dimension to the teaching of mathematics"

2.4   Teacher-researcher role

According to Hoyles and Sutherland dynamic and symbolic nature of computer environments is capable of arousinglearners to make links between their intuitive notions and more formal aspects of mathematical knowledge (Hoyles & Sutherland, 1989; Sutherland, 1998). The illustrations of geometric objects in a dynamic geometry software, as for example in *Geogebra*, are a way of bridging together formal and intuitive elements.It is also established that mathematical understandings do not develop spontaneously and that there is a need for a teacher to support learners to travel between informal mathematical knowing and the virtual world of mathematics (Balacheff & Sutherland, 1994, pp. 137-150).   Therefore, the role of the teacher in developing the applets together with a guiding worksheet together helps the learner make the necessary connection between the abstract to the obvious on the screen. These guided worksheet in conjunction with the appropriate applets form a means to scaffold learning.   Knowing and doing of mathematics is underpinned by reasoning. Conjecturing and exhibiting the logical validity of conjectures are the essence of the creative act of doing mathematics. To give more learners access to mathematics in a powerful way of making sense of the abstractness of certain concepts in mathematics, it is crucial that a prominence on reasoning pervade all mathematical activity. Students need a great deal of time and many experiences to develop their ability to construct valid arguments in problem settings and evaluate the arguments of others. "Dependent" and "independent" variables form the basis of functions and how these variables interact with each other becomes apparent through these applets that were custom made by the teacher to illustrate exactly that which he wishes his learners to conceptualise. The variables in the quadratic function defined by $y = a(x - p)^2 + q$ and $y = k/(x-p) + q$ are almost inconceivable to a 'weak' learner when drawing graphs. The applets as is shown in the study were a means for certain learners to explain in words how the values representing the 'p' and 'q' affected the translation of each of the graphs. The intention of the teacher in developing such applets was to highlight these 'poorly' understood concepts, thus allowing for deeper understanding of these components in a of the above forms.

2.5   Virtual Manipulatives

Encounter with concrete objects in mathematics, as compared to abstract concepts, allows learners to construct their own meaning of mathematical concepts. These concrete materials are referred to as manipulatives. According to Moyer, Bolyard and Spikell's study in 2001 in search of a definition for virtual manipulatives and the unique properties they possess, they posited that virtual manipulatives are dynamic and allow the user to control the input and the

various objects. "Just as a student can slide, flip, and turn a concrete manipulative by hand, he or she can use a computer mouse to actually slide, flip, and turn the dynamic visual representation" (Moyer, Bolyard and Spikell's, 2001, p. 373). This study uses virtual manipulatives as the basis upon which learners develop a conceptual understanding of transformations of functions. There are currently two forms of virtual manipulative: static visual representations and dynamic visual representations. Static visual representations are graphical representations of concrete manipulatives and are not true virtual manipulatives as onecannot interact or physically engage with them (drag, reflect, enlarge, flip etc.) as onewould with dynamic manipulatives. Dynamic visual representations, however, are visual representations that can be manipulated just like their concrete counterparts.

Virtual manipulatives is situated within the realm of dynamic learning systems In addition to the different representations of virtual manipulative, there are also microworlds, intelligent tutoring systems and applets that are available to learners. This study shows how the use of

applets as the mediating tool can scaffold the learning of certain concepts in mathematics. Dynamic learning systems allow for the development of learners' cognitive skills and promote discovery learning. Moyer et al (2001, p.373) defined virtual manipulative as an "interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge". Virtual manipulatives are moved or dragged and manipulated using the computer mouse. Dragging the points or sliders in dynamic interactive softwareallows for visual discovery of properties of objects, hence, transforming these figures in ways beyond the constraintsof traditional paper-and-pencil geometry (Laborde, 2001; Ruthven, 2005). Virtual manipulatives have many unique properties. One of them is the ability to visually track objects as they move, called "tracing". Encouraging to educators is that virtual manipulatives may be obtained free of charge by schools. As they are available online, virtual manipulatives may also be accessed by learners from home. Virtual manipulatives can be physically altered by the user.For example, when using virtual tangrams, colours of shapes may be changed. This is not possible when using textbooks or the traditional pencil and paper method.

According to Dorward and Heal (1999, pp. 1510-1512) "visual representations of concepts and relations help learners gain insight in mathematics. According to (Clements, 1999, pp. 45-60) virtual manipulatives can also encourage problem solving and conjecture. As learners test their ideas using virtual manipulatives they can easily move from 'empirical to logical

thinking'. Virtual manipulatives can also reduce the cognitive load of the learner using cognitive tools.

The problem of understanding mathematics is directly linked to how mathematical knowledge and its nature is conceived. Mathematical terms and expressions denote abstract entities that are supported by Yetkin (2003, pp. 3-6) where he "points out that the written symbols of mathematics create confusion for many learners". Therefore, he suggests using manipulatives, in an "effort to more concretely visualize the abstract symbol". The need for internalisation is further supported by Sfard (1991, p. 28) and Tall (2000, pp. 5-24). They claimthat one way to capture mathematical understanding is to describe it as a process, where a mathematical object transforms from being a process to become a mental object. Thus, a deep understanding of a mathematical object is not mainly about manipulating complicated expressions.Rather, it rests on the ability to create an internal picture of an abstract mathematical concept. Virtual manipulatives have a vital role to play in the internal conceptualisation of abstract mathematical ideas.

Hiebert and Carpenter (1992, p. 67) defined mathematical understanding as involving the building up of the conceptual 'context' or 'structure'.

> *The mathematics is understood if its mental representation is part of a network of representations. The number and strength of its connections determine the degree of understanding. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections.*

In relation to this study, the manipulation of the applets, for example, a vertical shift is seen as a move on the Cartesian plane resulting in a change or transformation of all coordinates of the graph thus making clear to the learner a visual displacement of the graph. The introduction of the "c" in the equation $y = ax^2 + bx + c$as part of this visual displacement. This movement of the graph, together with the changing coordinates,tendsto foster a "mental" understanding of this vertical shift and its resulting effect on the function as a whole. This move is very noticeable in the parabolic, hyperbolic, exponential and trigonometric functions. Theinstantaneous visual insight that this virtual manipulative provides is invaluable for internalization, as compared to the traditional pencil-and-paper method.

Another important questionthat emerges is related to whether understanding is relatedto an action or whether it is the result of an action through the use of mediating tools. Sierpinska

(1994, p. 2) clarified this by listing three different ways of looking at understanding.

First of all, there is the 'act of understanding' which is the mental experience associated with linking what is to be understood with the 'basis' for that understanding. Examples of basis could be mental representations, mental models, and memories of past experiences. Secondly, there is 'understanding' which is acquired as a result of the acts of understanding. Thirdly, there are the 'processes of understanding' which involve links being made between acts of understanding through reasoning processes, including developing explanations, learning by example, linking to previous knowledge, and carrying out practical and intellectual activities.

The illustrations above point to the factthat understanding is a network of internalised concepts and that understanding is both related to an action and a result of an action.This reinforces the view that mediating tools such as applets, which, when manipulated and engaged with, may result in better understanding of concepts that arise out of actions carried out during the engagement with these tools.

2.6 Virtual manipulative used as didactic tools

Virtual manipulatives can be thought of as cognitive technological tools (Zbiek, Heid, Blume, & Dick, 2007, pp. 1169-1207). Their features as cognitive tools are evident in theircapabilities that permits users to act on the virtual manipulatives as demonstrations of objects, with the consequences of the user's manipulations resulting in visual on-screen feedback from the virtual tool. Although virtual manipulatives have some similarities with their physical manipulative counterparts, as cognitive tools, virtual manipulatives have unique characteristics that go beyond the capabilities of physical manipulatives. Their potential is thus increased for mathematically meaningful actions by users and influences the user's learning. In this way *Geogebra* is a tool that promotes a way of thinking and reasoning about mathematical objects. It offers an environment where learners can observe and describe the relationships within and among objects, analyze what changes and what stays the same when functions are transformed, and make generalizations. When shapes or objects are transformed or moved, their properties such as location, length, angles, shapes and area changes. These properties are quantifiable and may vary with each other. Therefore, it was therefore possible to design the lesson with*Geogebra* which used the visual component of *Geogebra* and to animate algebraic concepts of variables and functions. Noticing varying quantities or positioning is a pre-requisite skill towards understanding transformation of function. Noticing varying quantities is as important as pattern recognition.

Fundamentally, the virtual manipulative brings together the visual or pictographic representation of a mathematics concept, along with symbolic notation for that concept, or even a demonstration of the procedure one follows for a particular algorithm. Learners do not always make connections,for example, the value of (*b*) in the function defined y = ax^2 +bx + c to that of the horizontal shift of the and its direct in relation to the sign of (*b*), that is, how a negative sign affects the translation from a positive sign. Therefore, combining multiple representations in a virtual environment allows learners to manipulate and change the representations to develop their relational thinking and to generalize mathematical ideas.Virtual manipulatives are also a powerful cognitive tool for learners because they constrain the user's actions on the mathematical object in the virtual environment, directing the user to focus on the mathematics in the environment; they react to user input with visual and verbal/symbolic feedback showing the user the results of their actions on the object; and, they enforce mathematical rules of behaviour (Zbiek et al., 2007).

When working with groups of third-grade students learning algebraic concepts, Suh and Moyer (2005, pp. 5-11) reported that unique features, both in the physical and virtual environment, encouraged relational thinking and promoted algebraic reasoning. For example, the activities using virtual algebra applets promoted the understanding of the fundamental algebraic idea of equality using the dynamic feature of the tilting balance scales.Greenes and Findell (1999, pp. 127-137) state that in order to develop mathematical reasoning in algebra, students need to be able to interpret algebraic equations in various representations like pictorially, graphically or symbolically. Meyer (2001, pp. 238-250) states that the bridge between concrete and abstract is through students' creation and use of models, drawings, diagrams, tables or symbolic notation.

Hitt (1998, pp. 123-134) stated that the understanding of functions does not emerge to be easy, given the multiplicity of representations related to this concept. Sierpinska (1992, pp. 25-58) denoted that learners experience difficulties making the connections between different representations of the notion (formulas, graphs, diagrams, and word descriptions), in interpreting graphs and manipulating symbols related to functions. Some learners' difficulties in the construction of concepts are linked to the restriction of representations when teaching. According to Eisenberg & Dreyfus, (1991, pp. 9-24) and Kaldrimidou & Iconomou, (1998, pp. 271-288) teachers, at the secondary level, traditionally have fixated their teaching on the use of algebraic representations of functions rather than the approach of them from the graphical point of view. Markovits, Eylon & Bruckheimer (1986, pp. 18-28) observed in their study that translation from graphical to algebraic form was more difficult than the reverse conversion and that the examples given by the learners were limited in the graphical and algebraic form.

The above arguments are suggestive of the role of visual objects in the learning of abstract mathematical concepts.

2.7   Geogebra, a dynamic interactive mathematics software

GeoGebra is a Dynamic Mathematics Software (DMS) for teaching and learning mathematics that includesmany aspects of the variousmathematical packages in the market. Created by Markus Hohenwarter, *Geogebra*is an open-source mathematics software that is freely downloadable which offers a flexible tool for visualizing mathematical notionsfrom primary schoolto university level, "ranging from simple to complex constructions" (Hohenwarter & Jones, 2007, pp. 126-131). It dynamically combinesgeometry, algebra and calculus a fully connected software environment (Hohenwarter & Lavicza, 2007, pp. 49-54). It offers a user-friendly computer interface providing basic features of Computer Algebra Systems (CAS). GeoGebra is an open-code math software (GNU General Public License) which can be freely downloaded from www.geogebra.org. *GeoGebra* works on various operating system platforms that have Java virtual machine installed.

This study demonstrates how java applets created through *Geogebra* serves as a virtual manipulative for learners to engage in the development of the concept of function. Having been created particularly for educational purposes, "*GeoGebra* can help learners grasp experimental, problem-oriented and research-oriented learning of mathematics, both in the classroom and at home" (Diković, 2009, p. 1). *GeoGebra's* interface provides two presentations of each mathematical object, one in its graphic (geometry) window and the other in its algebra window, hence the name *Geogebra*. A change of an object in one of these windows will immediately result in a change in the other window thus increasing the learner's ability to recognisesignificant cognitive relationships. Whilst a great deal can be listed about this software, this study does not require the learners to learn how to use the software. Rather, they merely interact with applets created using this software.

The software providesa geometry window or working area, a toolbar, an algebra window, an input field, a menu-bar and navigation bar (figure 2.1).
Although *GeoGebra*best provides a platformfor the teaching of geometry, itoffers equally good featuresfor the teaching of algebra particularly in functions and graphs. "Functions can be

defined algebraically and then changed dynamically afterwards" (Sangwin, 2007, pp. 36-38). For instance, by entering the equation $y=x^2$ the resultantgraph is immediately produced in the geometry area with the corresponding algebraic component of the graph appearing simultaneously in the algebra window. Dragging the line or curve of the graph instantaneously brings about a change in the equation inthe algebraic window. "This encourages the investigation of the connection between variables in the equations and graphs in a bidirectional experimental way" (Hohenwarter & Preiner, 2007).
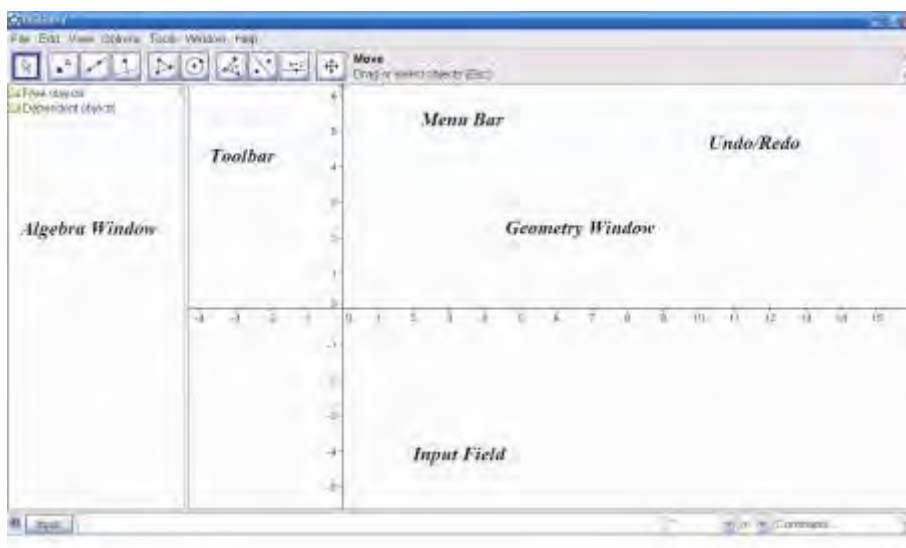
Figure 2.1: Screenshot from a Geogebra window.

Geogebra can be used in variousways in the teaching and learning of mathematics: for exploration and discovery since it can provide different representations; as a construction tool since it possessesthe facilityfor constructing shapes such as circles and polygons, lines and angles; as aknowledge creation tool providing proofs of investigations, helping to compare and contrast mathematical ideas, and as a communication and discussion tool. (Hohenwarter & Fuchs, 2004, pp. 21-35).

2.8   Geogebra Applets

An applet is a java program that can be embedded into web pages. A Java applet is a computer program ("applet" means "small application") that can be run as part of a web page. Java applets runs on the java-enabled web browsers such as Mozila, Safari and Internet Explorer. Applets are designed to run remotely on the client browser without the application software it was originally created in, therefore rendering it to be a stand-alone program for the specific

purpose for which it was created. They require an internet browser and a java interpreter to run, which are freely available as open source software. These minimum requirements may posea challenge. However, technology at the forefront have circumvented this limitation by placing, even in smartphones, the capabilities of running java applets. Hence, learners of the 21st century are now in a position to engage with applets on their palm-held devices. Applets are visual representations that are used as models for mathematical concepts within which learners can work on the basis of their own ideas and experiment freely. Web-based interactive graphics have the abilityto enhanceknowledge creationin ways that printedmaterialslack. Applets allow learners to work at their own level of thinking and thus better provide for individual differences between learners; the visual, interactive and dynamic features of applets makes the mathematics more natural to understand; the applets form a model learners can work on, revert to and practise. The feedback features are much more powerful than pencil-and-paper exercises.

According to Arcavi (1999, p. 56),

> *"visualisation is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding."*

Duval (1999, pp. 3-26) further asserts that only when individuals can go back and forth between various representations of mathematical concepts (for example, the visual and the analytic) does mathematical understanding occur.

The visualization that is possible with dynamic software enables the learner to explore and witness mathematical relations and concepts that were difficult to "illustrate" in the past, prior to the advent of such technology. Applets in this study (example figure 2.2) were created by the researcher particularly to explore the behaviour of functions when different variables are manipulated by simply dragging "free" objects around the plane of drawing, or by using sliders. Learners can generate changes using a technique of manipulating free objects, and they can then learn how the dependent objects will be affected. In this way, learners have the opportunity to solve problems by investigating mathematical relations dynamically. These applets are portable, independent of environment and browser, and interactive, thus providing ideal investigation tools that produceimmediate response to any number of trials.
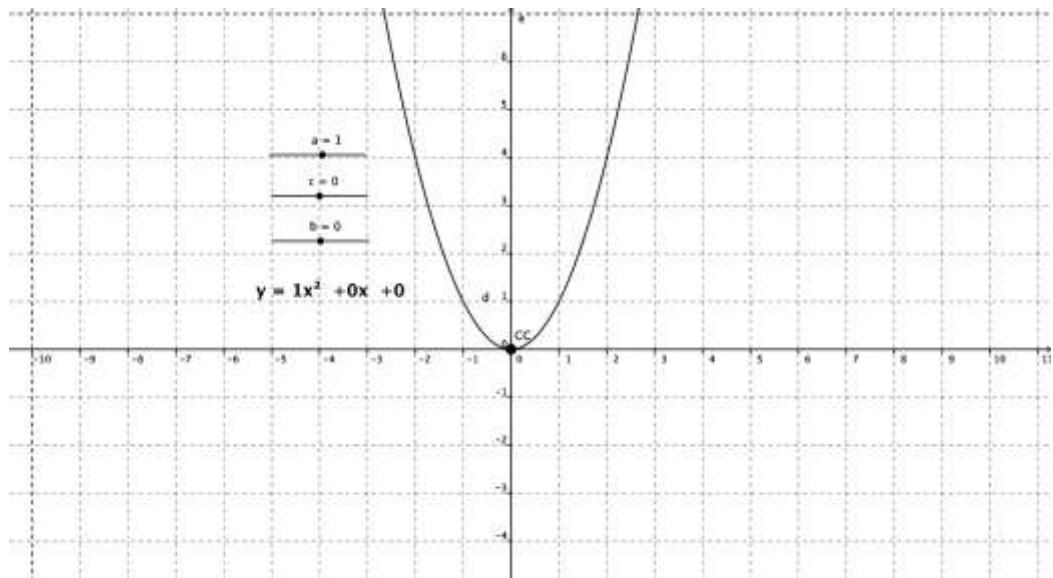
Figure 2.2: Screenshot of an applet used to explore transformation of a quadratic function

This applet in Figure 2.2 is an example of what would be provided for the learner to engage with, explore, develop and test conjectures through manipulation.Three sliders namely, *a, b* and *c* have been created to dynamically change the values of the respective variables. With all values being set or reset at zero, no graph or function would result. Each value needs to be explored one at a time to see the resultant outcome in relation to a quadratic function in the form $y = ax^2 + bx + c$ *and* $y = a(x – p)^2 + q$ where:

> *a* - represents the characteristic of the shape of the parabola
>
> *c* - represents the vertical shift and y-intercept
>
> *p* - represents the horizontal shift
>
> *q* - represents the maximum value or minimum value of the turning point.

By viewing the general forms of the function and the movement of the slider from side to side simultaneously, the learner would be able to see the resultant outcome.By moving the slider "*a*" to the right a minimum value function would result, and a maximum value graph would result should the slider be moved in the opposite direction. This also indicates to the learner the effect of positive "*a*" as compared to negative "*a*".The manipulation of "*c*" results in the vertical shift up or down.The manipulation of "*b*" results in the axis of symmetry being changed, confirming the formula x = -b/2a. This move simultaneously shows the effect the maximum or minimum value has on theturning point of the function.The resulting outcomes after several manipulations of the variables ought toconvince the learner of their effect in the general form.

Whilst many learners may use this software as a learning tool, others may experience challenges with the software.Some learners maynot see it as tool mediating their learning. Some of these deficiencies are listed below:

- Learners may not understand the functioning of the software.
- Learners could sometimesfind difficulty in using the input box to input mathematical or algebraic sentences, particularly when they need to know that "^" denotes exponential power or "*" denotes multiplication or no spaces are required between sin and (x) as in sin(x) and also that x must be written within round brackets otherwise an invalid error statement will appear.
- Independent exploring and investigation may not be appropriate for some learners.
- Learners who are not comfortable using computers will be reluctant to engage in computer-driven environments.

A collective case study on the use of manipulatives in teaching in primary school mathematics lessons carried out by (Puchner, Taylor, O'Donnell & Fick, 2008, pp. 313-325) found that in three out ofthe four lessons observedmanipulative use was seenasan end in and of itself rather than a tool, and that in the fourth lesson manipulative use hampered learning. It was found that there was no direct correlationbetween an external representation and an internal one. Hence, even when the teacher used an externalrepesentation to teach a concept, the desired internal representation was not noticeable in the students' learning. Goldin and Shteingold (2001, pp 1-23) also agreed that an external representation is only meaningful if people grasped it internally. According to Lamon (2001, pp. 145-165) teachers often expected learners to internalise some representation by engaging with manipulatives. However, mathematics learning is a constructive process, and good teachers would not encourage regurgitation of rules, rather an explanation of methods , procedures or concepts. Schram, Fieman-Nemser and Ball (1990, pp. 2-23) found many teachers assume manipulatives will automatically aid in understanding and that learners need to simply touch and look at manipulatives to gain understanding. Manipulatives can actually impede learning (Ambrose,2002, pp. 16-21). Although manipulatives vary in usefulness, many teachers do not believe that the particular type of manipulative or how they are incorporated makes much difference (Schram et al., 1990). Teachers often use manipulatives in a procedural manner, instructing students to apply a manipulative in a particular manner to obtain the correct answer. Such use obstructs rather than helps conceptual learning (Kamii & Warrington, 1999, pp. 82-92).

2.9   The complexities of *Geogebra* as a didactic tool

Geogebra, as a didactic tool may provide the learner with some complexities such as:

Learners or teachers without previous programming experience may find difficulty in inserting algebraic commands in the input box.   Some software require specific syntax when commands are to be input. For instance, entering a quadratic function requires the user to use a "^" sign and an asterix, "*", for multiplication or product when entering exponents. The syntax for the function defined y = ax2 is y = a*x^2 or else an "illegal" format error will appear. Although the basic commands are not difficult to learn, learners may feel embarrassed or quite at a loss of what to do. However, this study did not require the learners to have knowledge about *Geogebra* as software, rather manipulating of sliders in ready-made applets to observe the necessary effect produced by such a manipulation.

Independent exploring and experimenting without the researcher's intervention may not have been appropriate for many learners. If learners were not informed of the objectives of such an exercise, they may have been at a loss as to what to do with the sliders and how appropriate it would have been for their lesson.

Computer tools such as *Geogebra* can become intellectual partners to support learning. While this study supports the use of cognitive computer tools in a high school mathematics classroom, more research is required to examine whether the engaging of learners in high levels of cognitive demand during technology enhanced activities actually support long-term retention of mathematical understanding.

The introduction and integration of the software in the learners' mathematical activities made the teaching situation complex for some and more observable for others and a differentiation of the learners' responses were observed. For some learners the use of the software seemingly supportedtheir mathematical work, and at the same time for some learners the result was a bit to time consuming and unnecessary, as they preferred learning and applying the rules than manipulating the applets. The use of the software was seen as a disturbing factor in their mathematicalactivities.

When it comes to the teachers work with *Geogebra* different types of obstacles would prevent them from utilizingthe full didactical potential of the software in their teaching of

mathematics. This will be as a result of them not being able to create applets using *Geogebra* because of their inadequacies with learning the software themselves. A teacher may not be aware of the didactical potential of *Geogebra* and how to exploit it in in a way that support learners' learning of mathematical concepts;a teacher may not be aware of the complexity of technology based environments. However, to circumvent the latter situation, numerous applets are freely downloadable from the internet that could be used as an application to enhance the teaching and learning of mathematics in the classroom. These applets demand negligible or no knowledge of computers per se.

Finally, not all problems are best investigated with *Geogebra*. Problems should be carefully selected or designed according to themathematical maturity of participants, to maintain a certain level of cognitive complexity andpedagogical flexibility.

2.10 The "instructional affordances" of *Geogebra* such as multiple representations of functions.

Geogebra provides the functionality of dynamic software and the user can work with points, angles, segments, lines, curves etc. The software also allows some capabilities of computeralgebra systems in that equations and coordinates can be entered directly. When it comes to functions they can be defined algebraically and then changed dynamically. The later capabilities are characteristic of *Geogebra* and the default screen provides two windows Each object in the left window (algebra window) corresponds toan object in the right window (geometry window) and vice versa (Hohenwarter & Jones,2007, pp. 126-131).

*Geogebra*allows seamless movement between the algebra window and the geometrywindow, where it is possible for the user, on the one hand, to investigate the parameters ofthe equation of a curve by dragging the curve with the mouse and observing the equationchange, or, on the other hand, to change the equation of the curve directly and observe theway the objects in the geometry window change (Hohenwarter & Jones, 2007, pp. 126-131). Also, gettingan immediate feedback on their work in Geogebra. Preiner (2008, pp. 30-38) states that visualizing and exploring mathematical concepts in multimedia environments can foster students' understanding in a new way compared with non-dynamical environments.In addition the software allows creationof web-based interactive instructional materials, so called dynamic worksheets as applets andthese interactive materials can be used both on off-line computers or via the Internet.

Geogebra provides immediate graphing and calculation, thereby benefits mathematics teaching and learning. Wright (2005, pp. 11-14) also asserts that ICT, particularly mathematical software, helps to provide better visual and dynamic representations of abstract ideas and the links between symbols, variables and graphs. Pederson (2004, pp. 158-159) also claims that, "geometry is a skill of the eyes and the hands as well as the minds." There are more visual and dynamic areas in geometry than in algebra. Since mathematical software offers great visualisation capability and dynamic changeability for teaching, it is well placed to support this important element.

Furthermore, according to Healy and Hoyles, (2001, pp. 121-128), the efficient coupling of visual representation with other forms of representations and interactivity between students and mathematics can enhance learning and DGS is not only for teacher demonstrations but also for students' interactive learning."

Multiple representations are ways to symbolize, explain and represent the same mathematical entity. They are used to conceptualise and to communicate different mathematical qualities of the same object or operation, as well as associations between different properties. Multiple representations may include graphs and diagrams, tables and grids, formulas, symbols, words, animations, tracing, pictures, and sounds.The dynamic mathematics software *Geogebra* provides three different views of mathematical objects: a Graphics view, a, numeric Algebra view and a Spreadsheet view. They allow one to display mathematical objects in three different representations: graphically (e.g., points, function graphs), algebraically (e.g., coordinates of points, equations), and in spreadsheet cells. Thereby, all representations of the same object are linked dynamically and adapt automatically to changes made to any of the representations, no matter how they were initially created.

Using the Input bar one can directly enter algebraic expressions in Geogebra. After hitting the Enter-key the algebraic input appears in the Algebra view while its graphical representation is automatically displayed in the Graphics view. For example, the input f(x) = x^2 gives you the function in the Algebra view and its function graph in the Graphics view

In the Algebra view, mathematical objects are organized as free or dependent objects. If one wants to create a new object without using any other existing objects, it is classified as a free object. If your newly created object was created by using other existing objects, it is classified as a dependent object.

In *Geogebra's* Spreadsheet view every cell has a specific name that allows one to directly address each cell. For example, the cell in column A and row 1 is named A1.

These cell names can be used in expressions and commands in order to address the content of the corresponding cell. Into the spreadsheet cells one can enter not only numbers, but all types of mathematical objects that are supported by *Geogebra* (e.g., coordinates of points, functions, commands). If possible, *Geogebra* immediately displays the graphical representation of the object you enter into a spreadsheet cell in the Graphics view as well. Thereby, the name of the object matches the name of the spreadsheet cell used to initially create it (e.g., A5, C1).

CHAPTER 3
Theoretical Framework

3.1   Introduction

Computer-assisted teaching and learning dominates the forefront in classroomsaround the world, with positive learning outcomes being conveyedby variousresearchers (Sivin- Kachala, 1998; Holmes, Savage & Tangney, 2000). This is especially true in relation to the use of computer technology to teach mathematics (Papert, 1980, 1990; Campbell, 1991). Research into mathematics classrooms shows that computer technology can support problem solving skills (Fey, 1989, pp. 237-272), decrease the amount of time required to master skills, allowing for more time to be spent on developing conceptual understanding (Wagner & Parcker, 1993, pp 119-139) and facilitate the development of deeper understanding of algebraic ideas (Kaput, 1992, pp. 515-556).

According to Vygotsky (1962, p. 55), mediation is the use of tools such as computers to achieve a goal, and these mediate learners' activity. Learning is socially mediated (Leontev, 1978), for instance with interactivity through the use of the Internet, helping in construction of meaning. Vygotsky's basic idea was that each human mental function has a mediated structure, and can therefore be analysed as a triad: individual, goal and mediating artefact (Vygotsky, 1978; Cole, Engeström & Vasquez 1997). Mediation is the use of artificial means, such as a tool or symbolic artefacts to achieve a goal, (Vygotsky, 1962, p. 55). According to Leont'ev (1981, pp. 262-278), human beings mediate their activities using artefacts/tools. These activities are driven by certain needs where people wish to achieve certain objectives.

Central to this study is how learners use artefact, particularly virtual manipulatives in the form of interactive, dynamic *Geogebra* applets to enhance their understanding of certain mathematical concepts in the study of transformation of functions. In view of how computers potentially impact or mediate learning, the Activity Theory and the Social Constructivism theory underpin this study.

3.2 The Activity Theory

Activity theory is a cultural-historical theory of activity that was initiated by a group of Russian psychologists in the 1920s and 1930s. This approach was led by Lev Vygotsky (1896-1934) and his colleagues A. N. Leont'ev and A. R. Luria (Engestrom, 2003).

Vygotsky's triangular model (*Figure 3.1*) of "a complex, mediated act" (Vygotsky, 1978, p. 40) was central in showing the relationship between human agent and objects of environment as mediated by cultural means, tools and signs.
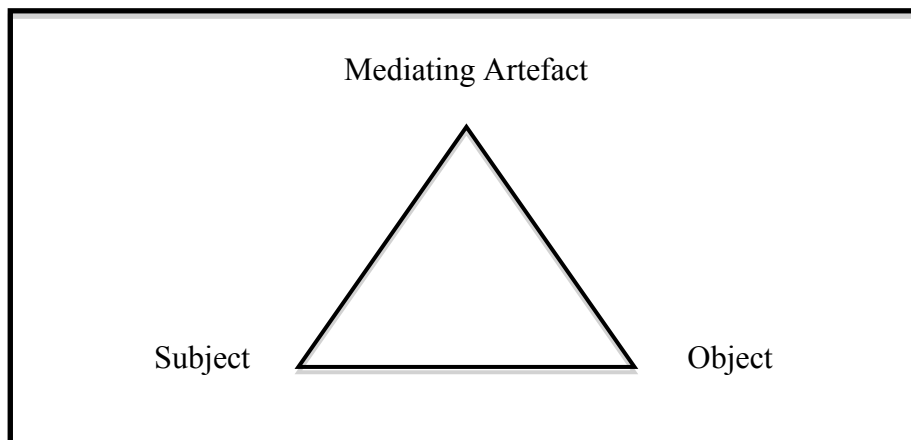
Figure 3.1 : Vygotsky's triangular model of "a complex, mediated act"

In this structure, an activity comprises a subject and an object, mediated by a tool, where the subject is learner or learners involvedin an activity, while an object is held by the subject and motivates the activity, giving it a specific direction. The mediation can take placethrough the use of different types of tools, material as well as mental, including culture, ways of thinking and language (Mappin, D., Kelly, M., Skaalid, B., and Bratt S. , 1999, p. 33).

According to Leont'ev (1979, pp. 37-71), analysis, using activity theory, takes into account three levels: analyzing the activity and its motive, analyzing the action and its goal, and analyzing the operation and its conditions. This is translated into the activity the learners will engage in towards an objective. The activity is the manipulation of *Geogebra* applets by dragging the sliders with the particular motive of visualising the resultant effect of a particular variable slide. This action of dragging the sliders leads to the goal of conceptualising transformation of the function through changes in the function variable(s), hence a visual interaction with once abstract concepts.

The conditions refer to the computer driven environment within which this activity takes place and how learners use these applets to discover, explore, confirm, contest, prove and conclude.

Activity theory (AT) is a general framework for studying different forms of human activity as development processes (Kuutti, 1996, pp. 17-44). Within this general context, Engeström (1987, p. 189) proposed a model (*Figure 3.2*) that conceptualises all purposeful human activity as the interaction of the elements: subject, object, tools, community, rules and division of labour.

In this model of an activity system, the *subject* refers to the individual or group whose point of view is taken in the analysis of the activity. The *object* (or objective) is the target of the activity within the system. *Tools* refer to internal or external mediating artefacts that help to achieve the *outcomes* of the activity. The *community* is comprised of one or more people who share the objective with the subject. *Rules* refer to the explicit and implicit regulations, norms and conventions that constrain actions and interactions within the activity system. The *division of labour* discusses how tasks are divided horizontally between community members as well as referring to any vertical division of power and status.
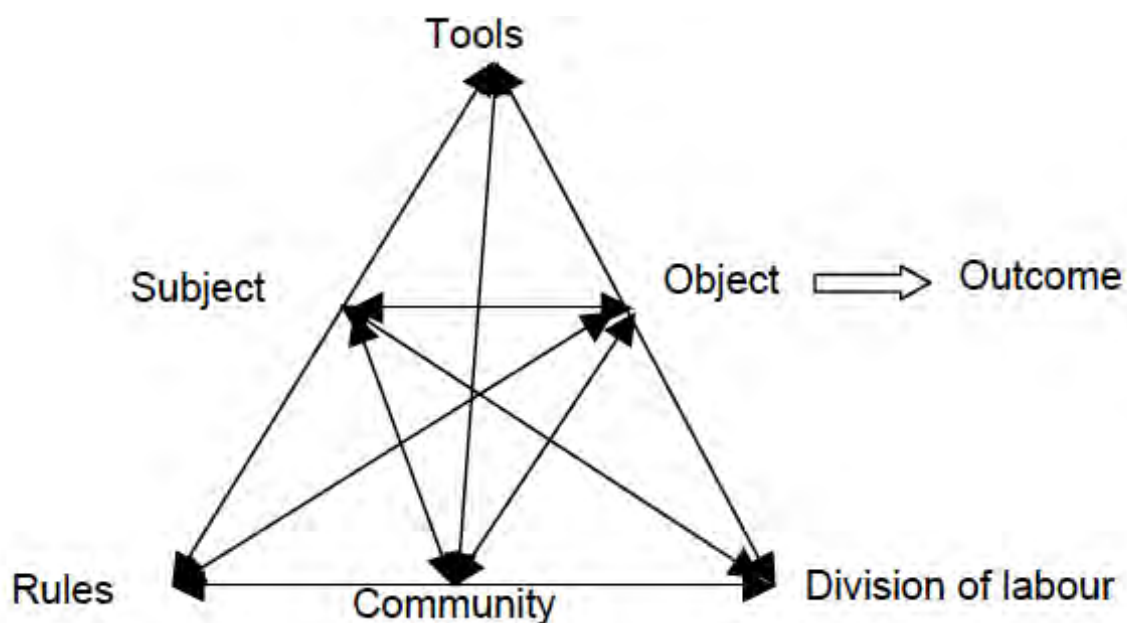
Figure 3.2 : The structure of an activity system

Using this model to analyse the activity involved in the learners' engagement with *Geogebra* applets, the following elements can be mapped:

Subject	Learner	Learner, use of the computer, prior pen & paper knowledge of functions (linear, parabolic & hyperbolic) Minimal knowledge of using computer is required
Object	Visualising the effects of variable change to each of the functions in the form: y = mx + c y = ax^2 + bx + c y = a(x – p)^2 + q y = k/x y = k/x+ q y = k/(x-p)+ q	The goals of using virtual manipulatives in teaching-learning process (knowledge and skills acquisition, and problem solving). Observing the effects on change in : value of *m* and *c* values of *a* and *c* values of *a, p* and *q* values of *k* values of *q* and *k* values of *p* and *q* and *k*
Tools	Manipulating and engagement with *Geogebra* applets	Computers and *Geogebra* applets (virtual manipulatives
Rules		The evaluation criteria, expectations of the teacher, rules of the school, secondary school mathematics curriculum
Community		Learners, teachers, department of education
Division of labour		The roles and responsibilities of learners and teachers, cooperation among teachers, the support of ICT Coordinator.
Outcome	Conceptualisation of transformation of functions	The reflection of the use of virtual manipulatives in teaching-learning process to the learning of students and instruction

This model is useful for conveying information about the factors that impact on the activity. The human **activity** is typically mediated by the meditational **tools** used and **artefacts** that

are considered in relation to the activities, eg. the virtual manipulative and the computer. The process of dragging or moving the slider allows for the actual activity which leads to the outcome. Pea (1987, pp. 89-122) suggests that there are five general categories of process functions that are identified with cognitive technologies. Cognitive computer tools in mathematics should:

- *provide support for developing conceptual fluency,*
- *aid in mathematical exploration,*
- *allow for integration of different mathematical representations,*
- *promote learning how to learn, and*
- *encourage learning of problem-solving methods.*

The activity is also mediated by the **community (teacher and learners as well as subject head)** in which the activity is being carried out. Assistance by the teacher or fellow learners may also be required to move from a point of not knowing to a point of knowledge or clarification.The subject may be subjected to **rules** whilst engaging with the activity which are imposed by the community.The learners were required to work with worksheet 1 in conjunction the the appropriate applets and record their findings on the worksheet. The curriculum sets boundaries on topics and sections to cover for each grade. The created applets also impose boundaries unless changed, for example the sliders are set to -10 to +10 for convenience, yet it can be expanded to cover a greater domain.The subject may collaborate with the community for the achievement of the object. This results in some form of **division of labour.**

Engestrom's model includes both historical and situated aspects of human activity. The model also represents the motive behind situation-bound actions that individuals within the activity system are a part of (Engestrom, 1987, p. 189).

3.3 Social Constructivism

The early roots of Constructivism are from the educational theories of John Dewey and Jean Piaget (Brown & Green, 2006). Dewey set the foundation for constructivism by finding inquiry to be a fundamental part of learning. Piaget's theories also helped to shape constructivism with the key concepts of assimilation, accommodation and schema. Combined, these theories

constituted the beginning of the constructivist learning process by focusing on how learning is processed and structured (Neo, 2007, pp. 149-158).

Vygotsky (1978, p 85) expanded the constructivist epistemology by arguing that social interaction plays a key role in the development of cognitive function and higher order thinking results from relationships between individuals. According to Social Constructivism, learning is a collaborative process which is differentiated between two developmental levels. Distinguishing between these levels of actual development and potential development resulted in Vygotsky's identification of the "zone of proximal development" (Vygotsky, 1978, p. 85). The zone of proximal development (ZPD) is the potential level of cognitive development learners have if they are provided with the appropriate support. Scaffolding is the process that supports individual efforts through the structuring of interactions and the breakdown of instruction into steps that are manageable by the student in response to their level of performance (Brown & Green, 2006). The ZPD is the distance between what a learner can achieve alone and what can be achieved with the assistance of a more advanced partner or mentor. The ZPD in this study is that area where the learner needs to move from the abstract knowledge, for example, the horizontal and vertical asymptote of the hyperbola to an area of understanding after engaging with the applets. The applet in this case is the scaffold that assists the learner to another level of understanding or knowing. Learning is an active process that involves the learners' personal interpretations created through experience. Instructors would need to take an interactive role providing applets or assistance that allows for scaffolding.

On the belief that learning occurs as learners are actively involved in the process of knowledge construction and meaning, instead of passively receiving information, the theory of constructivism complements the activity theory.

Van de Walle (2004: p. 22) states that:

> *The most widely accepted theory known as constructivism, suggests that children must be active participants in the development of their own understanding. Constructivism provides us with insights concerning how children learn mathematics and guides us to use instructional strategies that begin with children rather with ourselves.*

Van de Walle also affirmed that a learning environmentpromoting constructivism, providing an effective method of teaching, will help expandstudent understanding of mathematical concepts. The computer-based environment in this study provides the necessary learning environment for learning to take place. The computer environment embodies asociocultural dimension, that is, communicationbetween teachers and learners, amongst fellow learners, and between people and technology, in order to examine how different participation relationshipsoffer opportunities for learners to engage constructively and critically with mathematical ideas. While technology may be regarded as a mathematical tool or as a transforming tool, it may also be regarded as a cultural tool which allows for changes in relationships between people, and between people and tasks.

Jaramillo (1996, pp. 113-140) also believes that learning through self-discovery, and social interactions is necessary toactivate learners' higher cognitive levels (Zuckerman, 2004, pp. 9-18). Borthick, Jones and Wakai (2003, pp. 107-134) have found that in Vygotsky's theory of socio-cultural theory, learners were be able to construct their own knowledge and progressivelydeveloped independence in learning new concepts through social interaction. Schmittau (2004, pp. 19-43) also acknowledgesVygotsky's constructivist theory with emphasison conceptual understanding rather than procedural understanding of the concepts.

Kamii (1985, pp. 123-135) indicates that in constructivist learning, students buildtheir own knowledge. Mvududu (2005, pp. 49-54) and Geary (1995. pp. 24-37) found that, in a constructivist environment, students are constructorsof their own knowledge. Richardson (1997, pp. 3-14) believes that in a constructivist environment teachers should help learners develop meaning from the known to the unknown, from simple to involved.

Kim, Fisher, and Fraser (1999, pp. 239-249) found favourable students' attitudes were promoted when students experienced more personal relevance, shared control, and negotiated their learning. Kamii, Manning, and Manning (1991, pp. 17-29) found that learners do not acquire knowledge and understand it from the outside; they must construct it from the inside, which can be achieved through interaction with other people. The engagement with the applets in this study allows for learners to construct their knowledge about transformations of functions. Technology is viewed as one of several types of cultural tools-sign systems or material artefacts that not only amplify, but also re-organise, cognitive processes through their integration into the social and discursive practices of a knowledge

community (Resnick,Pontecorvo & Säljö, 1997, pp. 1-20).

Wertsch is of the assumption, "that action is mediated and that it cannot be separated from the milieu in which it is carried out" (p. 18). The central principleof sociocultural theory is that human action is mediated by cultural tools, and is primarilyalteredin the process (Wertsch, 1985, pp. 35-44). The swiftdevelopment of computer and virtual manipulatives makes availablenumerous examples of how such tools altermathematical tasks and the reasoning behind them.

According to Wertsch mentions three basic assumptions.

1. *Human action is mediated by cultural tools, and is fundamentally transformed in the process.*
2. *The tools include technical and physical artefacts, but also concepts, reasoning, structures, symbol systems, modes of argumentation and representation.*
3. *Learning is achieved by appropriating and using effectively cultural tools that are themselves recognised and validated by the relevant community of practice.*

If the teacher is to ensure that learners learn by using specific artefacts, he or she will have to study the activities or processes within which these learning aids are used. This implies that the teacher has to be computer literate, and study learning processes in progress as well as engage with the applets himself or herself or sometimes be able to create the applets himself or herself.

From experience, learning of algebra, more frequently than not, offered hindrances to learners understanding algebra. Understanding of variables often posed problems to learners taking algebra in grade 8 and grade 9 levels. According to Skemp (1986, p. 213), " a variable is in fact a key concept in algebra" stressing that this concept formed the basis of algebra. For learners to succeed in the understanding of algebra, the concept of variables must be thoroughly understood. One of the reasons thatmake the concept of variable difficult is the recurrent use of "letters" and "verbalsymbols". "Letters" and "verbal symbols" used in algebra are complex and multiplerepresentations of this concept (Schoenfeld & Arcavi, 1988, pp. 420-427) and, at least dueto such representations, it is a difficult task to express it.

Functions are grounded by the concept of variables. Variables are unknowns that represent specific the values taken for each variable as in the case of: $y = \dfrac{k}{x-p} + q$, where values for (p) and (q) transform the function defined by $y = \dfrac{k}{x}$ accordingly. This visual transformation is expected to produce a deeper meaning of variables in the function. Attempting to show this visual transformation to learners other than by dynamic means may not achieve the desired outcome as is purported by Nathan, Kintsch & Young (1992, pp. 329 - 389) who are of the view that "this connection between therepresentation of a situation and the representation of a mathematical function is difficult to achieve in astatic environment". Indeed, Kaput (1992, pp. 515 - 556) argues that a primaryaffordance of representational software is that actions taken in one representational system are automatedin another, which can help students to make important connections between them.

3.3   Conclusion

This chapter explored how learning theories, especially constructivism and the activity theory, support the use of computer technology in a mathematics classroom. The importance of manipulatives and virtual manipulatives in the teaching of mathematics was also outlined. Finally, it describes the challenges learners face with regards to understanding the concepts of variables in algebra. The next chapter will explore the research approach and the data collection methods which were chosen for this study.

CHAPTER 4

Methodology

4.1   Introduction

Thepurposeofthischapteris to explain the research setting and target group, the research approach, research plan and how data was collected and managed. To answer the research questions posed in chapter one, a suitable research design had to be identified. The study explores Grade 10 learners' conceptual understanding of transformation of functions through the engagement with a dynamic geometry software in an independent school in Durban, a city in the province of Kwa-Zulu Natal.

4.2   Statement of Problem/Research Question

The purpose of this study was to explore the specific functionof *Geogebra*, as a pedagogical tool and mediating artefact in the teaching and learning of transformation of functions in secondary school mathematics, and to explore whether interaction with these virtual manipulatives enhances the understanding of mathematics concepts.

This study aimed to investigate the following:

- How do virtual manipulatives serve as mediating artefacts in the teaching and learning of certain mathematical concepts?
- How effective are these applets in the learning of transformation of functions in secondary school mathematics?

4.3   Research Setting

4.3.1   Physical setting

The research was conducted at an independent combined school situated in Durban, a city, in the province of Kwa-Zulu Natal, South Africa. This co-educational school has a roll of 896

learners from Grades 0 to 12 and is considered a well-resourced school with two computer laboratories and five interactive whiteboards on the camp.Children come from predominantly affluent homes. Permission fromthe school'sBoard of Directors wasrequested for the study to be carried out at the school as well as for the use of the school computer laboratory. The physical setting for this study was the researcher's classroom. The classroom, a computer laboratory, is a modern room, rectangular in shape, off-white in colour and is bright and air-conditioned. The room has eighteen computers with broadband Internet connection. The facilities in this room met the needs of the intervention. Accordingly, the researcher, a mathematics teacher working at the school,randomly selected8 learners/participants from two out of four Grade 10 mathematics classes. Two of the grade 10 classes were the researcher's classes who were taught the sections being researched via computer technology.The researcher did not choose learners from the classes he taught. The participants chosen in the research study were from the other two classes, who were not exposed to computer technology in their mathematics lessons and they were taught the topic under research via the traditional method, namely the chalk and talk method, devoid of avirtual manipulative by their teacher. However, it must be stated that most learners of the school own computers at home and use computers for Internet access and school assignments. The study was carried outin the classroom over a period of five days during 7 lessons each of which was 35 minutes long.

4.3.2   The target population

The studytook place in September 2010. The target group for this research study included eight Grade 11 learners (aged between 16-17 years) who were selected withthe cooperation of the Principal, the Board of Directors, parents and participants themselves. The selection of these participants wasrandomly donefromthe two remaining grade 10 classes, yet purposive as the research sought to determine whether virtual manipulatives had a role to play in the conceptual understanding of transformation of functions with learners that were taught the topic via the traditional method of delivery. Eight learners, namely, six females and two males voluntarily agreedto participate in the study.

This particular target group was chosen as the researcher is teaching in the school and could thus easily access the group, which made for convenient sampling.

4.3.3   Pilot Study

Robson (2002, pp. 115-131) refers to the pilot study as a stage version of the real research. It is a trial of the anticipated research to establishits viability. Prior to starting the main study, I conducted a pilot study in which I invited 2 of my learners toparticipate. The learners worked on worksheet 1 that I preparedusing the pencil-and-paper method. I subsequently marked these worksheets and asked them to, thereafter, undertake the on-computer task with the four applets that were prepared. I used freely downloadable software called *Camstudio*, which recorded their interaction with the applets to determine the level of engagement with these applets. It also allowed me to determine if they experienced any errors or technical issues during their interaction. Yin (2003, pp. 78-80) views pilot tests as assistingresearchers improve their data collection plans regarding the content of the data and the method that needsto be adopted. I was also able to judge the effectiveness of the applets and the boundaries or limitation each of the applets presented and whether the results were correct. Three questions were re-worded to eliminate some ambiguity that existed. Also, one of the diagrams was modified to accommodate the general form of the type $y = a(x – p)^2 + q$.

4.4   Research Approach

Derived from the works of Thomas Kuhn, a historian of science, Maxwell (2005, p.36) refers to the term 'paradigm' as:

> *"A set of very general philosophical assumptions about the nature of the world (ontology) and how we can understand it (epistemology)….*
> *Paradigms also typically include specific methodological strategies linked to these assumptions and identify particular studies that are seen as exemplifying these assumptions and methods".*

Patton (2002, p. 69) likewise claimedthat:

> *"A paradigm is a world view, a general perspective, a way of breaking down the complexity of the real world. As such, paradigms are deeply embedded in the socialization of adherents and practitioners."*

Aparadigmcanbedefinedaswhatshouldbestudied,what

questionsshouldbeaskedandwhatrulesshouldbefollowedininterpretingtheanswersobtained(Grobler,1995, p.27).A paradigm is important as it describes the frame of reference and a research problem. There are two paradigmsin literature, the qualitative and the quantitative. The qualitative paradigm is termed the 'constructivist' or aturalist' approach (Lincoln & Guba, 1985, pp. 135-182). The quantitative paradigm is termed the 'traditional', the 'positivist', the 'experimental',orthe'empiricist'paradigm.

The aim of this study was to explore the role of *Geogebra*, as a pedagogical tool and mediating artefact in the teaching and learning of transformation of functions in secondary school mathematics and to explore whether interaction with these virtual manipulatives enhanced the understanding of mathematics concepts. In order to achieve this aim, it was necessary to provide a detailed account of how learners, through the engagement with these mediating tools, were able to conceptualise certain mathematical concepts. I found the interpretativist paradigm fittingfor my study. In a qualitative study I sought to unearth and understand the phenomenon, the process, the particular attitude and worldviews of the people involved in this study, namely learners at a school. The interpretive paradigm is grounded on the premise that each person's way of making sense of the world is as convincingand as worthy of respect as any other (Patton, 2002, p. 97). Merriam (2002, p. 6) claimedthat in a qualitative study the concern of the researcher lies in understanding how participants makemeaning of a situation or phenomenon. This study provides a qualitative, indepth examination of how learners conceptualise certain mathematical concepts through exploiting computer technology and are able to progressto the next level of cognitive understanding with the help of these virtual manipulatives. As individuals we infer what others mean from observing what they say and do.

Thisstudy is an exploratory case study, in that, the researcher pursues his questto explore or understand how a particular phenomenon, that is, how the manipulation of *Geogebra* applets affect understanding of mathematical concepts in a technology-based environment. Yin (2003, pp. 5-6) claimedthat case study research method can be dividedinto exploratory, descriptive and explanatory approachesin an effortto address the "who," "what," "where," "how," and "why" research questions. This method fits the purpose for my study as I seek to elicit responses to these questions from learners through their interaction with this software.
The study does not attemptto formgeneralizations as this is not the purpose of a case study.
Given the above mentioned characteristics the case study approach was suited to the exploration of this phenomenon, the use of mediating tools in the quest to understanding of

certain mathematical concepts. The deep observational data of the case study approach will provide the most insight as to the value of these virtual manipulatives in the understanding of these concepts. This approach was chosen as it allows an individual researcher the opportunity to study one situation in detail over a limited period of time (Bell, 1993, pp. 235-255).

Case study research can be a valuable research approach; however, there are limitations to case study research. Often case studies focus on a single situation and, therefore, generalisation of findings is not possible (Bell, 1993, pp. 10-17). This creates problems with regard to the reliability, credibility and validity of the study. For the purpose of this study a target population which was familiar to the researcher was chosen.

This study will include the procedure of acquiring data from multiple sources, a semi-structured interview, a participatory observation and study of artefacts. This study adopted a qualitative paradigm that facilitated exploration of a phenomenon within a mathematical context using *Geogebra* applets.Interpretivisttheoryofknowledgeisconstructednotonlyby observable p h e n o m e n a ,but also by descriptions of people's intentions, beliefs and self-understanding (Henning, van Rensberg & Smit, 2004, p. 20). In order to understand a phenomenon, worksheets were answered by learners, and learners were observed while they were engaging with the applets.Furthermore, interviews were conducted to explore Grade 10 learners' views as towhether computer utilisation in learning was beneficial to them.

4.4.1 *Qualitative research paradigm*

Qualitative studies usually aim for depth rather than "quantity of understanding" (Henning et al; 2004, p. 3). In this research, the learner's perspectives and understanding were interrogated and the study investigated the process of math learning and the resulting outcomes. These methods of data gathering w e r e u s e d because they allowed for open-ended responses. Furthermore, it encouraged participants to articulate for themselves their experiences, perceptions, their knowledge building, confirmation of assumptions, etc., whilst engaging with the applets andassessing how thesehelped them understand transformations of functions. They were also in a position to test their understanding when they answered a post-engagement worksheet related to their self-directed learning.

4.5   Design of instructional materials

4.5.1   Electronic Applets

GeoGebra was used to createapplets representing **transformations** of the linear function, parabolichyperbolic and the exponential functions taught in this research study. Four applets were devised to represent the intentional transformations (translation and reflection concepts).

Figure 4.1 Geogebra applet representing the linear function

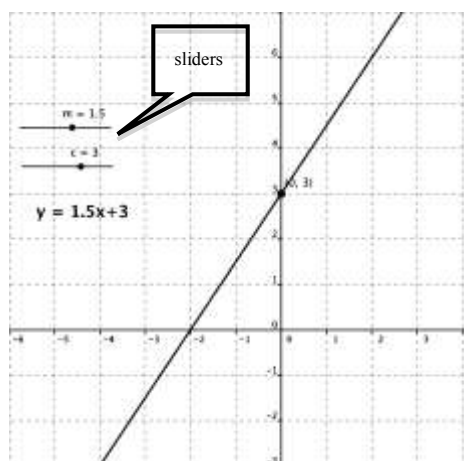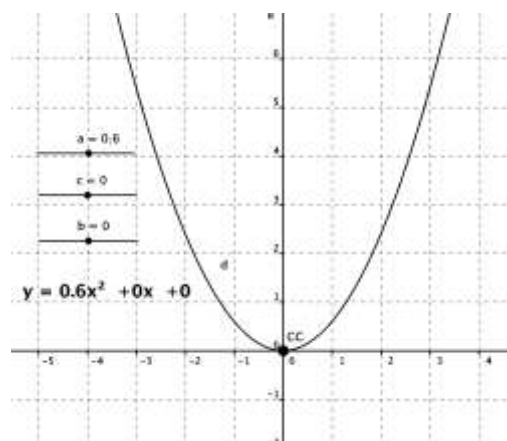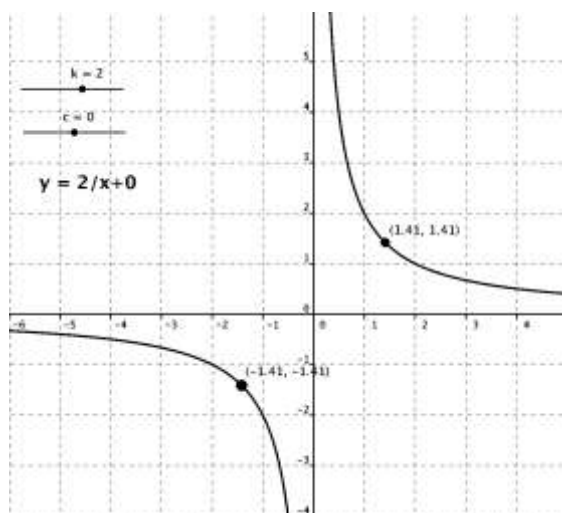*Figure 4.2 Geogebra applet representing the parabola*

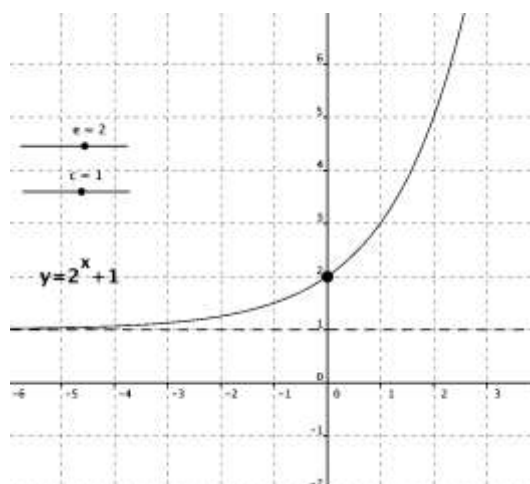*Figure 4.3 Geogebra applet representing the hyperbola*

Figure 4.4 Geogebra applet representing the exponential function

The first applet (Figure 4.1) represented a straight line or linear function in the form $y = mx + c$. The second (Figure 4.2) represented the parabola in the form $y = ax^2 + bx + c$. The third

(Figure 4.3) represented the hyperbola in the form $y = \dfrac{k}{x-p} + q$

Finally the fourth applet (Figure 4.4) represented the exponential function in the form $y = a^x + p$. The applets were designed to illustrate the effects of the variables namely *m, a, b, c, k and p* via their respective sliders. Hence, by moving the sliders, the resultant effect was the transformation of each of the functions. All of the applets provided the learners with thepossibility to transform each of the functions. Each variable was linked to a particular transformation and this could easily be observed by moving the slider related to that variable. By moving the sliders the learners produced their own outcomesand they were expectedto observe and record the resulting transformationas the variables assumed different values in the function. Based on the changing variables and its resultant effect, related to the appropriate applet learners had to answer the questions in the worksheet.

4.5.2   *Worksheets*

In order to support the learners' interactionwith the applets, two different worksheets were developed (see Appendix V Worksheets). Worksheet 1 had several questions based on each of the four functions. Each question presentedfour possible answers.The learner was required to read the question and choose an appropriate answer based on his/her understanding of the properties of each of the functions as well as on transformation of these functions.

Worksheet 2 contained questions particularly on transformation of functions without the provision of possible answers. The intention here was to see whether the learner would be able to answer these questions based on his/her successful understanding of the concepts learned after engaging with these applets which acted as mediating tools. Learning ought tohave taken place if the learner was able to successfully answer this worksheet.

4.6   Research Procedure

The diagram below sets out my research procedure:

```
┌─────────────────────┐      ┌─────────────────────┐      ┌─────────────────────┐
│ Learner attempting  │      │ Teacher marking the │      │ Teacher model the   │
│ Worksheet 1 —       │ ───▶ │ Worksheet 1         │ ───▶ │ use of Applets on   │
│ traditional         │      │                     │      │ Smartboard or Data  │
│ pen/paper method    │      │                     │      │ Projection          │
│ Day 1 ( Period 3/4) │      │                     │      │ Day 2 (Period 6/7)  │
│ 45 mins             │      │                     │      │ 10 mins             │
└─────────────────────┘      └─────────────────────┘      └─────────────────────┘
                                                                      │
                                                                      ▼
┌─────────────────────┐      ┌─────────────────────┐      ┌─────────────────────┐
│ Observation and     │      │ Observation and     │      │ Worksheet 1         │
│ video recording of  │      │ video recording of  │      │ answered again      │
│ learners engaging   │ ◀─── │ learners engaging   │ ◀─── │ whilst engaging     │
│ with mediating      │      │ with mediating      │      │ with the Geogebra   │
│ artefacts (applets) │      │ artefacts (applets) │      │ applets on Computer │
│ During periods (6/7)│      │ During periods (6/7)│      │ Day 2 (Period 6/7)  │
│ Day 2               │      │ Day 2               │      │ 60 mins             │
└─────────────────────┘      └─────────────────────┘      └─────────────────────┘
          │
          ▼
┌─────────────────────┐      ┌─────────────────────┐      ┌─────────────────────┐
│ Worksheet 2         │      │ Teacher marking the │      │ Teacher Interview   │
│ answered WITHOUT    │ ───▶ │ Worksheet 2         │ ───▶ │ with learners about │
│ the use of the      │      │                     │      │ their experiences   │
│ computer            │      │                     │      │ and the learning    │
│ Day 3 (Period 4)    │      │                     │      │ Day 4 (Period 8/9)  │
│ 35 mins             │      │                     │      │ 70 mins             │
└─────────────────────┘      └─────────────────────┘      └─────────────────────┘
```
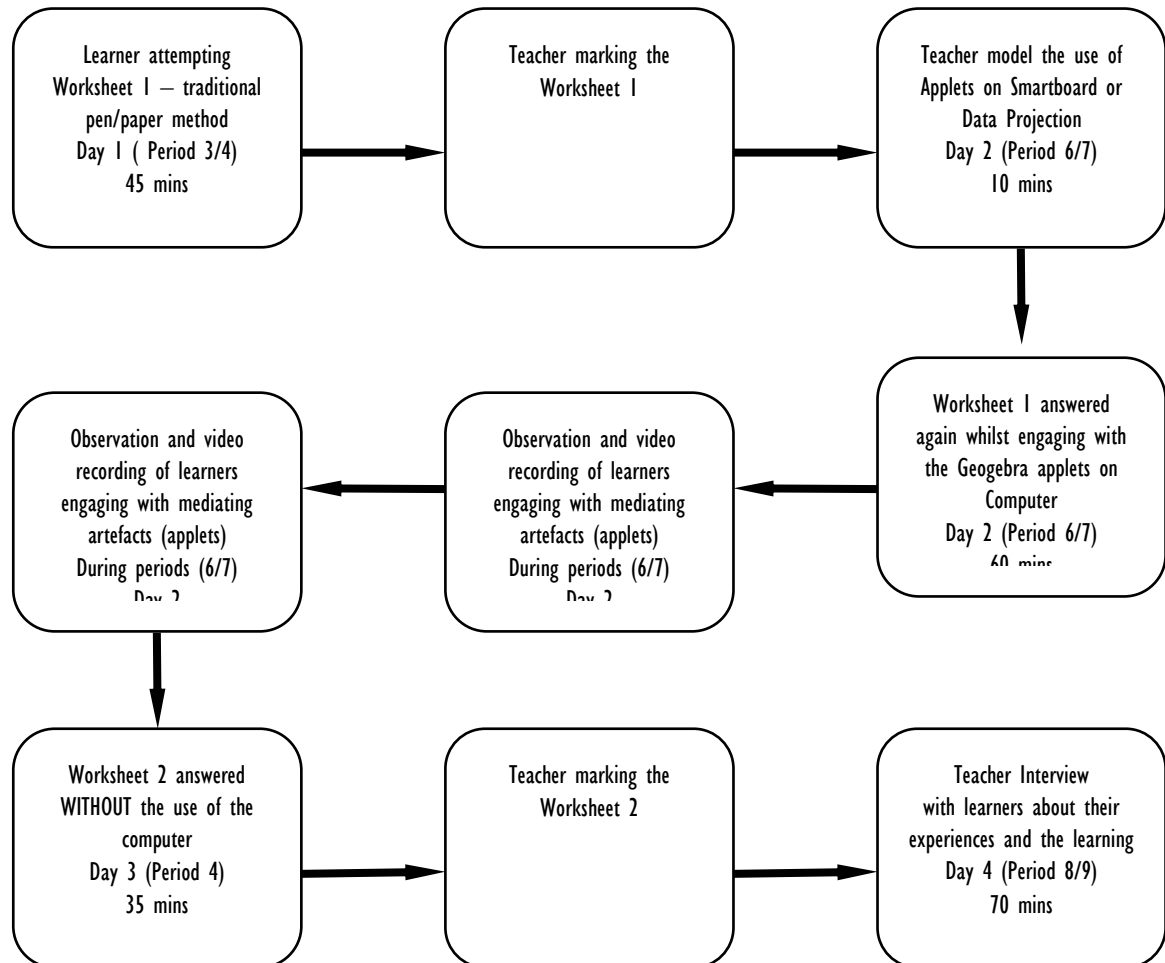
Figure 4.5 : Illustrative view of the Research procedure

This research was conducted in four stages in the school computer laboratory over a period of four days.

Stage 1 – Day 1 (Pre-engagement exercise)

> Worksheet 1 was issued to the participants to be answered individually. This worksheet contained 17 questions each with four possible answers. The learner was required to read the questions based on each of the four functions, namely, linear, parabola, hyperbola and exponential graphs with emphasis on highlighting the understanding behind the concepts of horizontal and vertical shifts in each of these graphs.

These graphs have been taught to the learners using the traditional "chalk and talk" method three weeks earlier. Once the learners had completed the worksheets, theywere collected and marked by the researcher.

Stage 2 – Day 2 (Engagement with *Geogebra* Applets)

The next day, the researcher, by means of the smart board, (Figure 4.6) explained to the learners what applets were and how they were used. He modeled this, using an example to demonstrate the dragging action of the mouse and its resultant effect. Learners were not expected to use *Geogebra* as software to draw graphs etc. as this was not the intended purpose of the study. Rather, learning to manipulate applets created using *Geogebra* pre-designed by the researcher was the objective of this quick tutorial. They were subsequently;shown how to access the applets related to this research study and once again demonstrated how the sliders worked. Learners were then asked to engage with each of the applets, dragging each of the variable sliders to visualise the resulting effect. This explorative exercise was intended for self- learning, self-discovery, visualizing, conjecturing, evaluating, and testing and for internalization to take place. Whilst learners were engaged, the researcher made observations of their experiences, and their attitudes.The researcher also engaged in discussions with the learners to ascertaindeeper meaning of their understanding and meaning making. He also video recorded two learners interacting with the applets. Fifty minutes into this double lesson, a fresh worksheet 1 was handed to each learner.The worksheets were collected at the end of the lesson.
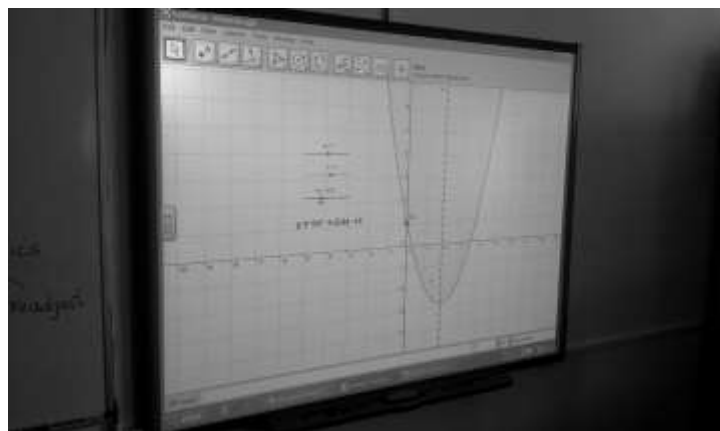

Figure 4.6 : Smart board (Interactive whiteboard)

Stage 3 – Day 3 (Post-engagement exercise)

On the third day, worksheet 2 was handed to the learners to complete. This worksheet required responses to questions relating to transformation of functions without any possible answers, unlike the previous worksheet that did provide possible answers. The worksheet was intended to test the recall of concepts learned through the learners' engagement with the applets the previous day as well as an evaluation of their knowledge about transformation of functions.

Stage 4 – Day 4 (Focus group interview with participants)

On the final day, the researcher held an interview with all learners together to learn about their experiences, their perceptions, and their subsequent understanding of the concepts regarding the topic under study. The interview was directed so as to inform the researcher about their understanding of the concepts through the use of this mediating tool, their attitude towards learning mathematics concepts and their views about the efficacy of the introduction of computers as learning tools.

4.7   Data Collection

In addition to the designed applets some research instruments were developed to assess the findings. These were classroom observations with and without digital camera recordings, interviews/discussions with the learners, a questionnaire and pre- and post-activity with *Geogebra* applets.

4.7.1   Observation of classroom activities

Observational research gains data on the physical setting, human setting (i.e. the way people are organised), interactional setting (type of interaction that the subjects engage in verbal/non verbal) and the programme setting (pedagogy used, curriculum etc.) (Cohen, Manion & Morrison, 2000, p. 112). With the permission of the school administration, classroom activities were video recorded. During observation, the researcher offered assistance, or prompts or technical assistance where necessary, while at the same time observing the learners. Furthermore, notes on the crucial points observed during the

classroom activities were made. These notes were then used to analyse the degree to which learners were inspired to learn graphing concepts whilstengaging with adynamic geometry software (DGS). The information from thissource also helped assess learners' self-directedness and mediated learning and the learners' collaborationwith each other and with the teacher. Moreover, this data assistedin analysing the function of DGS in a learner-centred learning environment. Thus, this type of data assistedin gauging whether the purpose of the study was achieved or not.

4.7.2   Interview with learners

Constructivismadvocatesthat knowledge is constructed between humans, and interviews can aidas a substantial tool in this process of construction. An interview isa two-way exchange of knowledge, ideas, beliefs, agreements or disagreements. Cannell and Kahn (1968) have describedaninterview as a conversation between two individuals, with one being the interviewer,whose purpose is to amass data relevant to the researchfocusing on content directing the research's goals;the other being the interviewee who provides the desired or undesiredresponse. However, this conversation cannot resemble an ordinary conversation , butmust be specific as it is aims to jot down responses that are as explicit as possible (Cohen *et al.*, 2007, pp. 181-186).   After the intervention all learners were interviewed together in a group (see Appendix III Interview responses). Focus group interviews are structured small group discussions. The purpose of this group interview was to gather information about the learner's attitudes, emotions, feelings and whether they were motivated to learn math by interacting with the applets or not. Participants interacted with the leader and either supported or disagreed with oneanother. Group interviews yield information different from that of people interviewed individually.   Also, focus groups allow the researcher to develop a broad and deep understanding rather than a quantitative summary.This data helped the researcher analyse learners' reflections and views on the role of DGS as a supporting tool in the teaching and learning of transformation of functions. Their answers were then paralleled with the other data sources to determine consistency.

The interview was guided by questions as per Appendix G.

The open-ended nature of the questions allowed for useful discussion, encouraged cooperation among the learners and allowed the researcher to have a more informed account oflearners' deep understanding of concepts.

4.7.3   Pre and post exercises

A pre- and a post-engagement worksheet was administered. The difference inthe results betweenthe pre- and post-engagement allowed for an analysis of possible changes made in learners' understanding of the concepts before they were exposed to the applets and after they engaged with the applets. Upon analysing the results derived from two worksheets the researcher was in a position to establish whetherrole of dynamic geometry softwaredid in fact supportthe development of learners' conceptual understanding.

4.8   Triangulation

Triangulation is defined as '*the use of two or more methods of data collection in the study of some aspect of human behaviour*' (Cohen, Manion & Morrison, 2000, p. 112). Triangulation, according to Lichtman (2006, p.194), is a method of making '*qualitative research more objective and less subjective-in other words more scientific*'. When a researcher has used two of theabove methods of data collection, and results from both methods correspond, the researcher can have confidence in his/her results (Cohen et al, 2000). Credibility of research can therefore be enhanced through triangulation (Hoepfl, 1997, pp. 47-63).

Triangulation however is not without its disadvantages and does not guarantee reliability (Patton, 1990). There are many types of triangulation. Methodological triangulation involves the use of different data gathering methods (Willis, 2007, pp. 219-223). There are two forms of methodological triangulation; 'within methods' triangulation and 'between methods' triangulation. 'Within methods' triangulation happens when a study is duplicated and reliability is being confirmed (Cohen et al 2000, p. 143). 'Between methods' triangulation occurs when more than one method is used in order to gain a particular outcome (Cohen et al 2000, p. 143).   Despite the benefits of triangulation, sometimes a study conducted extensively using one research tool could prove to be more reliable and valid than a study using multiple research tools (Willis, 2007 pp. 219-223). This case study adopted the 'Between Methods' triangulation in its approach. This approach is evident in the quantitative and qualitative research tools that are used. Observation was used throughout the case study to observe the learners. In addition to observation, video-recording and the pre- and post-engagement exercises were used to triangulate the data collected. These informed the researcher about

the learners' attitudes to and experiences with virtual manipulatives and theiruse in the teaching and learning of particularmathematics topics in secondary schools. The results from these three methods allowed for triangulation and thus enhanced the validity and reliability of this case study.

4.9   Ethics

'Ethics has to do with the application of moral principles to prevent harming or wronging others, to promote the good, to be fair' (Sieber, 1993, p. 14).   The ethics of research is concerned with the level of honesty of the researcher when analysing and reporting results (Greenfield, 2002, pp. 33-39).

In this study the researcher attempted to fulfil all acceptable requirements with regards to safeguarding and protecting the rights of all concerned. The school principal gave his consent (on behalf of the Board of Directors) for the research study. Letters were sent to parents of volunteering learners to seek their permission to include their child in the study as participants.The children were also fully aware of their involvement in the study and were informed oftheir rights as participants as well as the possibility of withdrawal if they so wished. The learners' identity was not revealed in the studyreport.

4.10   Conclusion

This chapter has outlined the research setting and target group, the research approach, research plan, how data was collected and managed and the methodologies that this study engaged in. The study will take the form of a case study and will utilize both qualitative and quantitative data collection methods. The next chapter will present the results which arose from the data collection methods of results that emerged from the three worksheets, observation and learner interviews.

CHAPTER FIVE

FINDINGS

5.1   Introduction

The strategy in answering research questions in this study was based on an examination of the quantitative and qualitative data collected. In trying to simplify the process of qualitative analysis the results emerging from the data sources were tabulated. Included as well, were learners' interviews and snapshots of learners at work via a screen capture software called *Camstudio.* The findings from the different sources were analysed, then crossed-referenced and combined.

5.2   Stages of Data Handling

Figure 5.1 encapsulates graphically how the researcher analysed the data collected
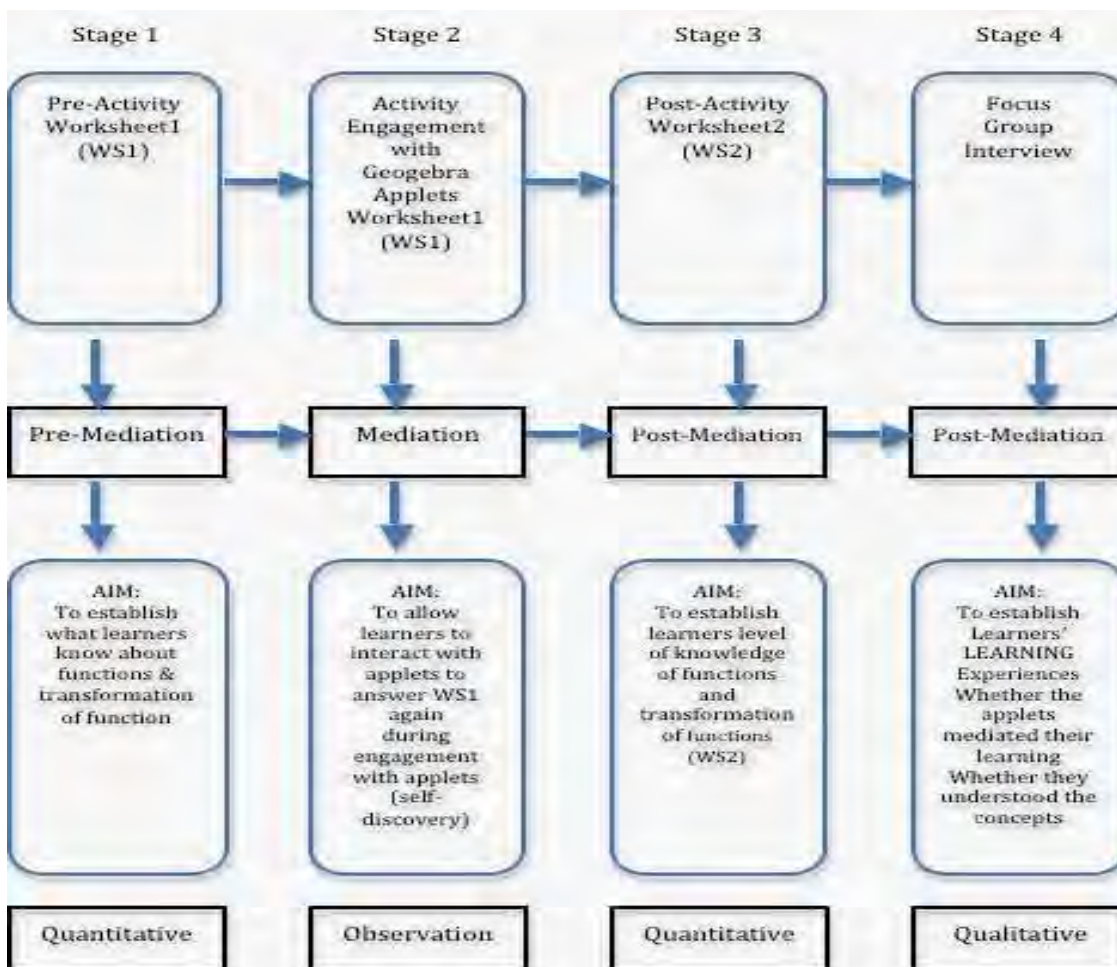

Fig. 5.1 : Data Collection and Data Analysis Flow Diagram

The aim of Stage 1 was to establish what learners knew about functions and transformation of functions at Grade 10 level. The researcher sought to determine the 'a priori' knowledge of the learners owing to the fact that this particular aspect was already taught a few weeks earlier using the ordinary "chalk and talk" method. It was an important precursor for the activities that followed over the next two weeks. The tool used to ascertain this knowledge of the learners was worksheet WS1, a pencil and paper exercise. The results obtained from this stage emerged from counting the correct and incorrect responses to each question. For instance, it was established that six out of eight learners could not explain what real roots meant in spite of the section being taught a few weeks before. Five out of eight learners were not able to explain the concept, the transformation or identify whether the graph of $y = x^2$ was transformed in any way if it was represented as $y = x^2 + 2$.

The second stage provided the learners with *Geogebra* applets that served to mediate the participants' learning. Through manipulation of these applets, through trial and error, discovery, enquiry and self-learning, learners were required to answer the same worksheet, namely, Worksheet 1 (WS1). During this stage the researcher assisted learners with queries and technical problems. Learners were also questioned for example, as to why a particular answerwas written or how they discovered their answers. Learners were expected to engage with the applets to obtain answers, give reasons or to even convince themselves. Participants' responses helped the researcher establish whether learning was taking place. Learners were not prohibited from discussing with their classmates when "discoveries" were made. Some learners also helped others to "see" the result of dragging certain variable sliders. Although learners sought assistance sometimes from others, they mainly engaged with applets on their own. The results from this stage informed the researcher as to whether the applets mediated learning via observation and occasional questioning.

Stage three formed the crux of the analysis in this study. It provided the platform where an activity was observed and, at the same time, responses were elicited from the participants in order to establish whetherlearning took place. This stage measured the extent to which the engagement or interaction with virtual manipulatives enhanced the understanding of the concepts of transformation of functions. This stage also allowed the researcher to reflect upon the learners' responses in the pre-activity worksheet and how the learners reacted to the same worksheet following the assistance of the applets or the mediating artefact.

The last stage sought to determine participants' learning experiences and how the interaction assisted them in understanding of the concepts. The data collected at this stage provided information on learners' feelings, experiences and attitudes towards learning in the computer based environment.

The three stages, according to Leont'ev's (1979, pp. 37-71) analysis, using activity theory, analyses the activity and its motive, analyses the action and its goal, and analyses the operation and its conditions. The activity is the manipulating of *Geogebra* applets by dragging the sliders with a particular motive of visualising the resultant effect of a particular variable slide. This action of dragging the sliders led to the conceptualising of transformation of the function through changes in the function variable(s); hence a visual interaction with abstract concepts.

5.2.1 Pre-Activity Exercise

Inordertoestablish how *Geogebra* applets served as a mediating tool, quantitative data was analysed. This was attained using *Excel* spreadsheets where responses from worksheet 1 (see *AppendixB*) were recorded. Questions 1 to 16 of WS1 were based on the properties or behaviour of functions. Considering that the topic on functions was taught to these learners using traditional teaching methods, it was necessary to assess whether learners could recall salient features of functions. Questions 17 to 24 helped to assess whether learners could link variables in a general form of a function to the graphical representation of the function. This was done in order to establish whether learners understood how each component of a function influenced the function itself. For instance, in $y=-2x^2–3$, the negative sign(-) represented a maximum value graph, the absence of bx meant the graph would turn on the y-axis and so on.

Table 5.2 displays the quantitative responses of the learners from the pre-activity worksheet. The "1" represents the correct or desired response; the "x" represents the incorrect or undesired response and the "0" represents a blank which translated into the learner not knowing the answer, was not sure of the answer, chose not to write an answer, or that the learner did not comprehend the question. In most cases learners indicated that they left a solution blank because they did not know how to establish the answer.

Table 5.2 indicates that the learners were able to answer questions that depended on procedural or rote-learning knowledge. In most cases the learners were able to provide answers to questions that also depended on straight recall. However, in questions were learners were expected to demonstrate a deeper understanding, they faltered. Given the equation $y = 3x^2 + 4$, learners could not describe the graph by simply stating whether it had a minimum or a maximum value. It may be true that if the learners were allowed to procedurally draw the graph then they may have found the solution but they could not determine from the equation itself whether the graph had a minimum or maximum value. This may imply that the learners had little or no understanding of the values of the variables in the equation $y = ax^2 + bx + c$. Only two of the eight learners answered most of the questions correctly. At this stage it was not possible to establishwhether these learners had a good conceptual understanding of the concepts that were tested. Perhaps this should have been probed further. The other six learners showed little or no conceptual understanding of the more involved items in the exercise. The concept of equal roots of an equation, for example, was answered correctly by two learners only, yet this might be a fundamental requirement for an in-depth understanding of functions.

5.2.2. The Activity Stage

In this stage the learners engaged with applets created by the researcher using *Geogebra* which aimed to develop certain mathematical concepts involved in the teaching and learning of transformation of functions. Three applets were designed to represent the graphs of linear, parabola and hyperbola. All of the applets provided the learners with possibilities to transform the functions. In the applets toggling witha given point or variable on a slider produced certain consequences for the profilesand properties of each of the functions. The geometry software provided dynamic diagrams that learners could manipulate to understand concepts, which cannot be done with traditional paper diagrams, except for re-drawing them several times without an in-depth visual internalisation of the resultant transformation of the functions. When a component of such a diagram is dragged with the mouse, the diagram is modified while all the functions are preserved. Unlike constructing diagrams with pencil and paper, this dynamic geometry software helped learners discover new information using previous knowledge. It provided a means of exploration and investigation without learners constructing each of the graphs.

In order to support learners' engagementwith the applets and establish whether these applets did in fact influence the learners' understanding of the concepts, a fresh copy of WS1 was supplied to the learners to complete, this time with the help of the applets.

An introduction to the applets was given and the learners were told how to manipulate the sliders by means of a demonstration on the interactive smartboard connected to thelaptop in the computer lab . The applets were loaded on each of the computers' shared folder so that each learner used a stand-alone copy of the applets. The learners did not have to have any computer-operating knowledge except for mouse movement and slider manipulation; this eliminated software knowledge as a pre-requisite, technical issues and non-user friendliness. During this session, the researcher walked around the classso as to observe and provideany assistance whenever asked for. Learners appeared to be working enthusiastically with the applets during this stage of the study. The interacted with one another and asked or "showed-off " their findings to their fellow classmates.

Theirengagement with the applets, for example, of the graph of the parabola introduced learners to *GeoGebra* sketches and sliders that permittedthem to manually manipulate the values of variables via the sliders and simultaneously observe the resulting impact on the sketches. After opening an applet of the quadratic equation *y = 2x²* on a coordinate system, learners were invited to change the values of "*a*" and "*q*" in the *y = ax² + q* form by dragging the sliders for these variables. Learners were then asked todescribe the effect after several adjustments to *a*'s slider resulting in limited values of "*a"* that were positive, negative and zero. Learners were expected to observe the effect on the shape of parabola as a result of these manipulations. Learners then answered a corresponding set of questions about adjustments to "*b*" 's slider and the effect this had on the graph. Learners then answered the question in WS1. Since all other parabolas "inherit" the basic characteristics of the $y=x^2$ parabola, the learners can learn most of what they need to know about parabolas by studying the parent parabola in detail.

Table 5.2 displays the responses recorded from the learners' WS1 before engagement with the applets.

Question No.	Question	Expected Answer	Ras (PupilA)	Naz (PupilB)	Ayesha (PupilC)	Firdos (PupilD)	Asma (PupilE)	Moh (PupilF)	Zai (PupilG)	Jo (PupilH)	Incorrect Responses per question	Percentage incorrect responses
1	Graph identification	Visual identification of each of the four function considered in the study	1	1	1	1	1	1	1	1	0	0
2	Write down general form of each function		1	1	1	1	1	1	1	1	0	0
3	What do you understand by the term y-intercept ?	point where the graph cuts the y-axis	1	1	1	1	1	1	1	1	0	0
4	What do you understand by the term x-intercept ?	points where the graph cuts or touches the axis	1	1	1	1	1	1	0	1	1	13
5	Draw a rough sketch of each of the graphs below:		x	1	1	x	1	1	1	1	2	25
6	Describe the shape of the function $y = 3x^2 + 4$, will it have a maximum or minimum value?	minimum value	x	1	0	0	x	1	0	1	5	63
7	Give an explanation of the term turning point of a parabola ?	when the graph turns	x	1	1	1	1	0	1	1	2	25
8	Explain the term roots	x-intercepts	1	1	1	1	1	1	0	1	1	13
9	How would you explain real roots ?	when the graph intersects the x-axis	x	1	x	1	1	0	0	1	4	50
10	How would you explain equal roots ?	when the graph touches ONE point on the x-axis	x	1	0	0	0	0	0	1	6	75
11	How would you explain unreal/non-real roots ?	when the graph does NOT touch or intersect x-axis	x	1	0	0	0	0	0	1	6	75
12	How would you explain the term asymptote to a friend ?	a line the graph approaches but does not touch	x	1	x	0	x	1	0	1	5	63
13	Explain the term symmetry.	line that brings about symmetry/mirror image	1	1	1	1	0	1	1	1	1	13
14	Explain axis of symmetry	line the bisects a parabola	1	1	1	0	0	1	0	1	3	38
15	Explain the concept, transformation.	change in the graph	x	1	1	0	0	1	0	x	5	63
16a	Explain the concept, translation.	Horizontal/vertical shift or glide of the graph	1	1	1	0	0	1	0	1	3	38
16b	Explain the term reflection.	mirror image	1	1	1	0	0	1	0	0	4	50
	Correct answers out of 17 per participant		9	17	12	8	8	13	6	15	3	
	Correct answers in percentages per participant		52,9	100	70,6	47,1	47,1	76,5	35,3	88,2	18	

Table 5.2 : Responses from pre-activity worksheet 1

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Table 5.2, illustrates important information.All learners were able to identify each of the graphs and match it with its general form. Most learners were able to explain the y-intercept and x-intercepts as points cutting the y and x-axis respectively. Learners merely recalled that which they were taught by rote which displayed little conceptual understanding. All except two learners were also able to draw rough sketches of each of the functions correctly showing shape and the correct quadrants they occupied on the Cartesian plane.   Three out of eight learners did not know what "minimum value" represented in a parabola; two of the eight answered it incorrectly whilst the other three learners answered this question correctly. This resulted in five of the eight participants not being able to provide the correct answer to this question. The question on the explanation of the turning point was answered correctly by six of the eight learners. All participants except one, successfully explained the term "roots". However, participants struggled to explain the meaning of "nature of roots"except Naz and Jo. Although nature of roots did not form part of the curriculum, it was felt that these concepts be discussed in class although not required in an exam. Finding solutions to a quadratic equation or determining domain of a function or establishing roots when graphing a function requires knowledge of the nature of roots. I believe, that teaching learners this concept will in no way impede their learning of functions but more so will allow for a deeper understanding of roots. In orderto explain to a learner why a quadratic function hangs 'suspended' when roots are not possible it was necessary to discuss the nature of roots. The learner indeed needs to be exposedto the holistic view of functions rather than being limited by a document that suggest that parts of a section should be excluded in a particular grade or grades.

Three out of eight learners were able to explain the term "asymptote" whilst five learners had little or no idea of what it was. Five learners were able to explain the term "symmetry"whilst three were unable to explain "line/axis of symmetry." Almost two thirds of the learners could not explain the term "transformation"as it occurs in graphs or functions. Five learners successfully explained the concept of "translation" as a "glide, shift or move". Lastly four learners were able to explain the term "reflection" correctly. Overall, for the first 17 questions which assessed the learners' basic knowledge of functions, four out of eight learners scored below average in this section of WS1. One learner scored 100% whilst 3 learners scored between 50 and 100%.

5.2.2   Post-Activity Exercise

Table 5.3 represents the responses of the learners to the second worksheet (WS2) which was presented to the learners a day after they had engaged with the applets in the computer lab. Learners were expected to fill in the answers to worksheet 2 on paper without the use of the applets. The objective of this worksheet was to establish whether learners were able to recall and utilise the knowledge gained during the activity stage and whether they were successful in acquiring a deep conceptual understanding of the concepts given in the task.

ANALYSIS OF RESPONSES FROM WORKSHEET 2
Transformation of Functions

	Names of Participants ——>			Ras	Naz	Ayesha	Firdos	Asma	Moh	Zai	Jo	Incorrect Responses	Percentage incorrect
	Given the general form of the functions : $f(x) = ax + t$; $g(x) = a(x-p)^2 + q$ and $h(x) = k/(x+p) + q$ answer the questions:			PupilA	PupilB	PupilC	PupilD	PupilE	PupilF	PupilG	PupilH		
1	What effect does *a* have on the shape of f(x) ?			1	1	1	1	1	1	1	1	0	0
2	What effect does *a* have on the shape of g(x) ?			1	1	1	1	1	1	1	1	0	0
3	What effect does the value of *k* have on the shape of h(x) ?			1	1	1	1	1	1	1	1	0	0
4	How is each of the functions transformed with the introduction of the values *t* and *q* on their respective graphs ?			0	1	1	1	1	1	1	1	1	13
	Original function	Transformation	Equation of function after transformation										
5	$f(x) = 3x$	vertical shift up 3 units	$y = 3x + 3$	1	1	1	1	1	1	1	1	0	0
6	$f(x) = -4x$	vertical shift down 2 units	$y = -4x - 2$	1	1	1	1	1	1	1	1	0	0
7	$f(x) = 2x + 1$	vertical shift down 3 units	$y = 2x - 2$	1	1	1	1	1	1	1	1	0	0
8	$f(x) = 4x - 3$	vertical shift down 1 unit	$y = 4x - 4$	1	1	1	1	1	1	1	1	0	0
9	$g(x) = 3x^2$	reflection along x-axis	$y = -3x^2$	1	1	1	1	1	1	1	1	0	0
10	$g(x) = 2x^2$	vertical shift up 3 units	$y = 2x^2 + 3$	1	1	1	1	1	1	1	1	0	0
11	$f(x) = -4x^2$	vertical shift down 2 units	$y = -4x - 2$	1	1	1	1	1	1	1	1	0	0
12	$f(x) = 2x^2 + 1$	vertical shift down 3 units	$y = 2x^2 - 2$	1	1	1	1	1	1	1	1	0	0
13	$f(x) = 2x^2 + 1$	reflection	$y = -2x^2 - 1$	1	1	1	1	1	1	1	1	0	0
14	$h(x) = 4/x$	vertical shift up 3 units	$y = 4/x + 3$	1	1	1	1	1	1	1	1	0	0
15	$h(x) = 4/x + 1$	vertical shift down 2 units	$y = 4/x - 1$	1	1	1	1	1	1	1	1	0	0
16	$h(x) = 4/x - 2$	Vertical shift down 3 units	$y = 4/x - 5$	1	1	1	1	1	1	1	1	0	0
17	$h(x) = 4/x + 4$	vertical shift down 1 unit	$y = 4/x + 3$	1	1	1	1	1	1	1	1	0	0
18	$g(x) = 3x^2$	translation 2 units to the right	$y = 3(x - 2)^2$	1	1	1	1	1	1	1	1	0	0
19	$g(x) = 2x^2$	translation 2 units to the left	$y = 2(x + 2)^2$	1	1	1	1	1	1	1	1	0	0
20	$g(x) = -4(x +1)^2 - 2$	translation 3 units to the right	$y = -4(x + 4)^2 - 2$	0	1	0	1	0	1	x	1	4	50
21	$g(x) = 2(x-4)^2 + 1$	translation 1 units to the left	$y = 2(x - 3)^2 + 1$	0	1	0	1	0	1	0	1	4	50
22	$g(x) = 3(x-2)^2 - 3$	vertical translation of 2 & horizontal translation of 1 to the right	$y = 3(x - 3)^2 - 1$	0	1	x	x	0	1	x	1	5	63
23	$h(x) = 4/x$	horizontal translation of 1 to the right	$y = 4/(x-1)$	1	1	1	1	1	1	1	1	0	0
24	$h(x) = 4/x + 1$	vertical shift down 2 units & horizontal translation of 2 to the left	$y = 4/(x-2) - 1$	x	1		1	1	1	1	1	2	25
25	$h(x) = 4/x - 2$	vertical shift down 3 units & horizontal translation of 1 to the right	$y = 4/(x - 1) - 5$	1	1	1	1	1	1	1	1	0	0
26	$t(x) = x^2 - 3$	reflect at the turning point	$t(x) = -x^2 - 3$	0	1	1	1	1	1	0	1	2	25
27	$m(x) = x^2 - 3$	reflect on the x-axis	$m(x) = -x^2 + 3$	0	1	1	0	1	1	x	1	3	38
28	$p(x) = 3/x$	reflect along the y = -x line	$p(x) = -3/x$	0	1	1	1	1	1	x	1	2	25
29	$r(x) = 3x^2 + 1$	$s(x) = r(x) + 2$	$s(x) = 3x^2 + 3$	x	1	1	x	x	1	x	1	4	50
				20	29	25	26	25	29	22	29		
				69	100	86	90	86	100	76	100		

Table 5.3

The worksheet used in the activity and the post-activity were not the same. In view of the number of correct answers obtained in WS2, it was evident that learners displayed a better understanding of the concepts. They were able to produce responses that involved understanding of concepts dealt with in WS1. The responses in WS2 required a certain degree of practical application of questions 16 to 25 of WS1.All eight learners were able to successfully associate the relationship between the variables a and k to the shape of each of the functions defined by , f(x) = ax + t; g(x) = a(x – p)^2 + q and h(x) = $\dfrac{k}{x+p}+q$.

In a typical classroom situation, learners would generally make conjectures about what is obvious to them. Whereas, in a dynamic geometry situation learners are able to make conceivable conjectures as a result of their engagement with the applets. The click and drag of the slider, representing the "a" in the general of the parabola, results in widening or narrowing of the curve displaying how "a" affects the shape of the curve.

Except for Raz all other learners were able to link the values of "t" and "q" to the vertical transformation of the graphs. All learners successfully answered the questions that required single transformations, that is either a translation up or down, left or right or a reflection. There were no incorrect responses to questions 5 to 19. This may indicate a better understanding of vertical translation. The usefulness of the applet was observed when the learners were asked to show that their conjectures are in fact true. This allowed the learners to reason as to whether their original conjectures could be accepted or rejected.

Between 4 to 5 learners found some difficulty in questions 20, 21 and 22 which required translation in the form y = a(x – p)^2 + q.Two out of the eight learners were unsuccessful in answering the question on reflection. Finally, four learners did not provide answers for question 29, which resulted in the translation of a function by 2 units, in the form
r(x) = 3x^2 + 1 to s(x) = r(x) + 2.

Overall, the percentage of correct answers was far better than the results obtained in Worksheet 1. In view of the number of correct answers obtained in WS2 than in WS1, it was evident that learners displayed a better understanding of the concepts.

5.2.3   Focus group interview schedule

When asked about their concerns when responding to WS1, learners stated that they had great difficulty in completingthe worksheet except for the identification and general form of each of the functions. Some even forgot the formula used to determine the nature of the roots. Two learners were unable to recall the term 'nature of roots'. Two learners, namely Naz and Jo found most of the questions easy because they knew the formula. The reasons for their not answering the questions in the desired response was as a result of their merely regurgitating the rule as per their learning of it.

On being probed about the use of applets, there was a general consensus that the virtual manipulatives assisted learners understand the concept of transformation. Examples of this theme were comments such as: "*It's like a computer game that helps me learn*" (Mo); "*It shows the movement of the graph, which helps*" (Firdous); and, "*I learned how the hyperbola behaves with the asymptote. It [virtual manipulatives] helps me to understand more about the values in thehyperbola graph*" (Zai). Jo, on the other showed little excitement in the engagement with the applets. She obtained a score of 15/17 in her pre-activity exercise. She felt that too much time was wasted "*playing around*" with technology. She believed that "technology was over-rated and normal chalkboard teaching was more effective".   She believed that if there wererules or procedures, oneshould follow them and be certain of getting the answer correct. It seems, in her case, she enjoyed procedural understanding of concepts which allowed her to apply rules rather than learning with understanding.

Another consistent theme extracted fromlearners' comments was that the **applets made it easier to learn** about the hyperbola and parabola than when paper-and-pencil methods were used. These ideas focussedon the beliefthat the learners wereable to try several combinations of the variables to see the resultant outcome.   Learners reported the following: "*It [the applets] helped me more than when Sir [referring to the chalk-talk method] did it on the board because it's simpler with the computerthan the teacher drawing many graphs on the board. It confuses us.* (Raz).   Three participants indicated that computers allowedthem visual representations that were often impossible to "see" with pen and paper or on a chalkboard. In addition, computers' visual representations helpedthem better understand different concepts. They also commented on the unfolding of other "things" whilst working on a certain aspect. When asked to elaborate on one such episode, Raz explained, "*I was working with the hyperbola applet and whilst I dragged the slider'p' to see the effect of the'sign' of the vertical*

asymptote I noticed the turning point of the hyperbola also changed. I was always under the impression that the turning point was the square root of 'k' ". This realisation had provided the learner with further discovery which had then convinced her of her new findings, hence, experiential learning.

But it must be noted that the applets were used as explanatory tools, and not tools for solving problems. In the exercises given to learners an attempt was made to increase understanding rather than teach procedural ways of solving similar problems.

When asked whether they would like their future mathematics lessons to include virtual manipulates,all learners agreed, except Jo. Her comment was that since computers will not be used to answer a math exam, there was no need in doing it that way. *"We are allowed to use a calculator in our exams, so a calculator is fine, not applets"*. Although Jo engaged with the applets, she was still not convinced about their value as a learning tool. She, unfortunately, prefers rules and procedures, since it helps her get good test marks. The other learnersstated that the computer assisted them in that they now had a better understanding of the concepts.They also felt that textbooks were "flat" and did not offer a lot of help with regards to transformation; they [the texbooks] were limited in their visuals. The meaning did not "really come out" as expressed by Asma.

In summary, the findings from the interview revealed that most of the learners loved the algebra lessons with the DGS and thus, they were stimulated to participate in the lesson. They wished all their lessons could be done in the computer lab. The learners enjoyed interacting with each other during which time they shared and discussed their findings and ideas regarding concepts in transformation of functions.

CHAPTER SIX

DATA ANALYSIS

6.1   Introduction

The analysis of the data in this thesis is framed within Activity Theory. Thestudy is based on the assumptions that the "activity" is the basic unit of analysis and that the relationship between the subject (who is involved in the activity) and object (why the activity is taking place) is mediated by artefacts or tools, which in this case was the *Geogebra* applets. Thestudy roots its analysis in the first generation activity theory and makes brief references to the second generation model. According to Johassen andRohrer-Murphy (1999, pp. 66-67), a primaryassumption of the activity theory is that tools facilitateor changethe nature of human activity and, when understood, influence humans' mental development.

Cole and Engeström (1993, pp. 1-46) define an activity as a form of action through engagement with an object, which is deliberateandaimedtowards the formationof a physical or mental object. This in turn leads to an outcome.
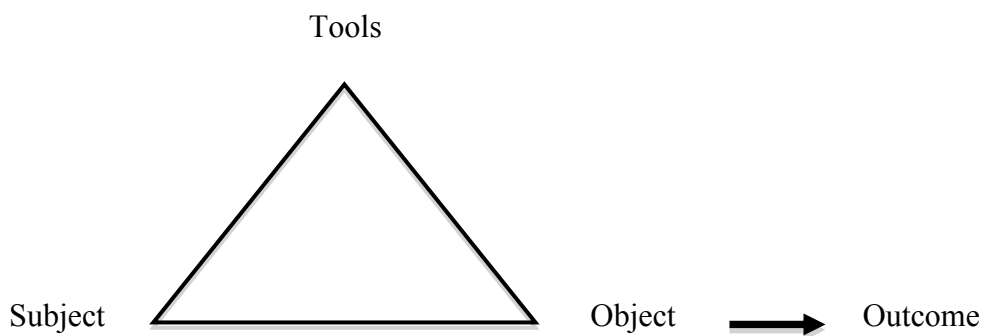
Fig. 6.1 : First Generation Activity Theory Model

The analysis of the data collected is divided in to two episodes: the first as being the pre-computer intervention stage and the other asthe computer-intervention stage, known as the activity stage. Both of these episodes will be analysed using the Activity Theory (AT) model with the unit of analysis being the activity. The"teacher-talk" will be the activity in the first episode and "engaging with *Geogebra* applets"will bethe activity in next episode. The outcome in each of the stages is a level of understanding of the concepts relating to the transformation of functions.

6.2   EPISODE 1

Episode 1 involved the mathematics teacher completing the Grade 10 lessons on transformation of functions using pencil and paper methods.   The relationship between the learners (being the subject), the teacher (being the tool) and the object (being the procedural knowledge) was mapped on the AT model. The findings from the pre-activity exercise carried out withthe participants was used to analyse whether learners understood the concepts taught during thelessons taught the traditional way.

In order to ascertain whether the introduction of *Geogebra* applets (a response generating tool) brought about a pedagogical shift, it was necessary to first elaborate on the structure of face-to-face lecture type lessons where no computers were used.

Tool
Teacher : Chalk & Talk

Subject
Passive Learners

Object
ProceduralKnowledge
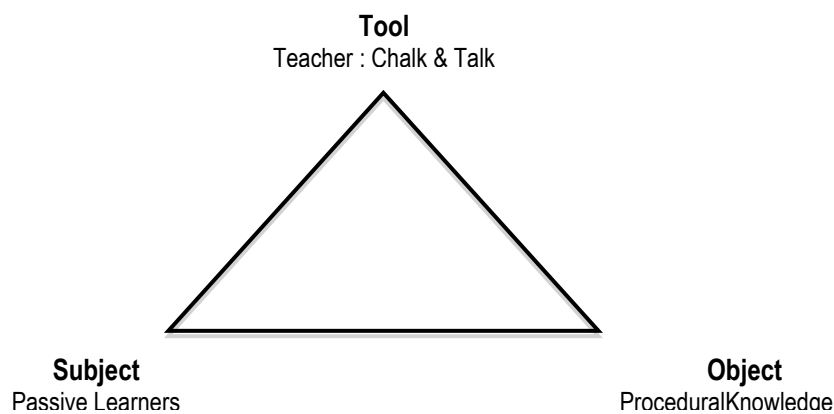
Fig. 6.2 : Adapted First Generation Activity Theory Model

The purposeof an activity system is the problem space that the subject engages withand transforms. The object or purpose during the teacher's lesson was to cover the content of the Grade 10 syllabus and to develop learners' understanding of the concepts taught.   The mediational tool in this instance was the teacher using the traditional method of instruction. At the time the research was carried out, the topic had already beentaught by the teacher. He was convinced that learners understood the concepts.The teacher then gave the learners an exercise to complete which was subsequently corrected. He was obviouslyconvinced at that time that the learners understood the concepts. By examining the responses from the pre-activity exercise it can be conjectured that the learners' understanding of the concepts was limited. The pre-activity exercise was a set of questions based on basic knowledge of the parabola, for instance, questions on understanding of the terms y-intercept,

x-intercepts, roots, description of the nature of roots, turning point of the parabola, axis of symmetry and so on. The deep understanding of these terms forms the basis for transformation of functions. If four out of eight learners could not explain what real roots were and six out of eight learners could not explain the concept of equal roots,this implied that learning may not have taken place. This exercise was aimed at establishing what knowledge learners possessed about functions and transformation of functions at Grade 10 level. The results obtained from this exercise informed the researcher of the participants' level of knowledge at that stage considering that this particular topic had been taught three weeks previously.

Upon engaging with the responses from the pre-activity exercise the analyis for this episode was based on the following three questions:

♦ in what context did the learners receive this knowledge ?
♦ how did they receive this knowledge ?
♦ how did they use the knowledge learned ?

6.2.1   In what context or milieu did the learners receive this knowledge?

The traditional classroom organization could be explain using the following diagram:
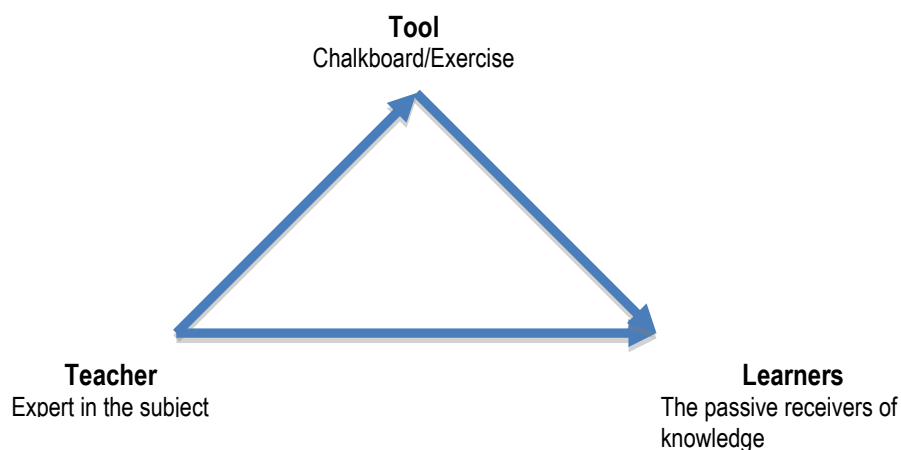
Fig. 6.3: Adapted First Generation Activity Theory Model

The teacher is the specialistand authority in the classroom and manages the classroom as such. The chalkboard (a non-responsive medium) is his main medium of delivery supported by exercises that he believes assesses learner understanding. The learners

understand the authority in front of the class, listen and, whilst listening, take down some procedureor rule by which they can learn their mathematics in preparation to answer questions in a test or exam.

This is evident on the diagram (from the direction of the arrow), that is, from teacher to learner, from chalkboard to learner. Learning in this environment is one-way, linear in its control and solely for the purpose of exam or tests. The classroom organisation and management essentially subscribes to a teacher-centred situation with the object of passing an assessment through recall of some method or formula.

6.2.2 How did they receive this knowledge ?

Procedural knowledge refers to the procedure or algorithm used to carry out an action. For instance, the method to remove a flat tyre can be considered procedural knowledge. Knowledge about "how" to remove a flat tyre, what tools one would need, the sequence of events, ensuring that the park-brake is enabled, the nuts to be loosened before using the jack etc. Procedural knowledge is rule-oriented. It focuses on application of a set of rules to obtain a result. This epitomizes the traditional method of delivery.

It was important to attempt to establishhow learners came to know what they know. Whilst the learners were interacting with the applets in the activity stage, I observed Naz, who was generally amongst the higher achievers in the grade and who obtained 100% in the pre-activity exercise. She was asked what she felt about the questions in the worksheet. She explained that the exercise was easy and that she knew all of the concepts. The researcher probed to find out how she answered questions 9, 10 and 11 in the pre-activity exercise relating to the nature of roots of a quadratic equation. The question was clearly and deliberately worded as "How would you explain - " 9. real roots?, 10. equal roots? and 11. unreal/non-real roots? The responses expected from learners would have beenthat the graphs cut/touches the x-axis, the graphs touched the x-axis at one point and the graph never touched the x-axis respectively. These explanations would have emerged if there wassome conceptual understanding, as it tries to explain the phenomena as it occurs or ought to occur.

However, the answer received from Naz was a well-formulated formula-based response. She explained that she used the formulae $b^2 - 4ac > 0$, $b^2 - 4ac = 0$ and $b^2 - 4ac < 0$ respectively to determine her answers.

In the exercise she explained her answer strictly via the procedural methods (refer to Figure 6.5) that she was taught. She directed the researcher to her mathematics notebook (refer to Figure 6.2) which provides the formulae.

Extract from Naz's mathematics notebook.
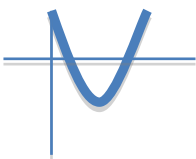
Value of the discriminant	Type and number of Solutions	Example of graph
Positive Discriminant $b^2 - 4ac > 0$	**Two Real Solutions** If the discriminant is a perfect square the roots are rational. Otherwise, they are irrational.	
Discriminant is Zero $b^2 - 4ac = 0$	**One Real Solution**	
Negative Discriminant $b^2 - 4ac < 0$	**No Real Solutions** Two **Imaginary** Solutions	

Figure 6.4 : Extract from a worksheet given by the teacher in Naz's mathematics notebook

9.	How would you explain real roots ? $b^2 - 4ac > 0$ an equation which consists of a real no. real root refers to the solution of eg. $\sqrt{5}$; $\sqrt{16}$
10.	How would you explain equal roots ? $b^2 - 4ac = 0$ If line of graph touches but does not cross x-axis , the 2 roots are equal .
11.	How would you explain unreal/non-real roots ? $b^2 - 4ac < 0$ non-real roots are roots no's of a negative square root . $\sqrt{-25}$; $\sqrt{-7}$

Figure 6.5 : Extract from Naz's responses to question 9 to 11 of Worksheet 1

Naz's notes (Figure 6.4) and her answer sheet (Figure 6.5) show a strong resemblance, and it may imply a mechanical method of her answering. In my opinion, a conceptual understanding of the above concepts would have resulted in a more descriptive response related to the actual graph of the function. The $b^2 - 4ac$ used would certainly produce the desired result, but strictly from a procedural approach rather than from a conceptual one.She felt that this exercise with the computer wasted her time because she knew the answer to the questions hence there was "*no need for this practical explanation.*"

Testing, traditionally can be described as recalling offacts, systematic application of rules or laws learnt during instruction. The recognition of a problem type (even in an unfamiliar context), the linking of the correctformula and its systematic application in obtaining the correct answer usually implies understanding of the concepts underlying that task. As in Naz's case, when asked to describe the nature of the roots of the parabola, she recognized the problem type, linked it to the formula $b^2 - 4ac$ to determine whether the result of such formula yields a negative value, a positive value or an unreal value. Therefore, best performances may sometimes be attributed to the reproduction of learned rules and algorithms. Perhaps she was not exposed to anything other than rules, procedures or algorithms.

The results indicated that in each of these questions more than half the learners either did not answer the question or answered them incorrectly. This implies that the learners either knew how to do a procedure (and therefore could execute it successfully) or did not know how to do the procedure. Hiebert and Lefevre (1986, p. 7) concur, stating that procedural knowledge is defined as "*rules or procedures for solving mathematical problems*". This research attempts to show how virtual manipulatives may transform or shape the understanding of certain mathematical concepts so that learners may be able to explain a concept from their experiences as a result of working with virtual manipulatives. Procedures have their functional place in solving a problem, but should follow conceptual understanding as apriori understanding. One of the benefits of emphasizing conceptual understanding is that a person is less likely to forget concepts than procedures. If a step is forgotten in an algorithm, it is likely that the result may be incorrect.

6.2.3   How did they use the knowledge ?

Evidently from Naz's case the knowledge learned was used clinically to solve a problem. The application of the rule seemed to fit the need. As important as it is to be able to follow a method to quickly and efficiently obtain a certain kind of answer to a certain kind of problem, mathematics problems require deep understanding as well so that similar problems in varying contexts will be answered in a similar way. An extract from Naz's answer sheet further confirms this assumption that Naz reproduced her notes or rules she was taught(refer to Figure 6.6).
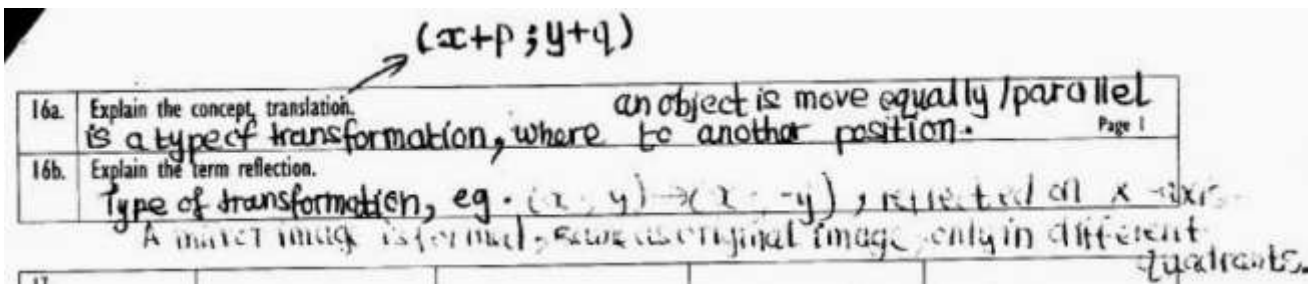
Fig. 6.6 : Extract from Naz's responses to question 16a/b of Worksheet1

The use of the formula (x + p; y + q) in the explanation of the term, "translation" is evidence of Naz being a "formula using machine" to explain her reasoning, which is certainly not aligned with conceptual understanding. A simple term "shift" would have encapsulated the concept of translation. The question asked in the worksheet did not relate the term "translation" to any specific question, but, rather, intentionally aimed at eliciting the conceptual meaning of the term.

The analysis above was used to set the stage for analysing the activity that involved virtual manipulatives, viz dynamic interactive applets in attempting to enhance the conceptual understanding of similar mathematical concepts discussed above.

6.3   EPISODE 2

Practically positioning myself inside the Activity Theory triangle permitted me to observe the subject accomplishing the object whilst engaging this time with a response-generating tool, namely, the *Geogebra* applet. This position also allowed me to take the context and analyze it from the point of view of the whole interaction of the learner (subject/agent), the object (goal/objective), and the behaviour that gives the learner a specific direction.

The two-way connection between the subject and the tool and the two-way connection between the subject and the object allowed the learners to try and retry theirfindings through explorations and discovery, to achieve the object of theirdiscoveries so that conviction couldoccur. This relationship between the subject (learners who are involved in the activityand object (why the activity is taking place: to conceptualise the transformation of functions) is mediated through tools (applets) in a triangular form. Here, the activity or engagement with the tools is seen as dynamic, contextually bound (within a certain topic in Grade 10 mathematics syllabus) and the basic unit of analysis.

65
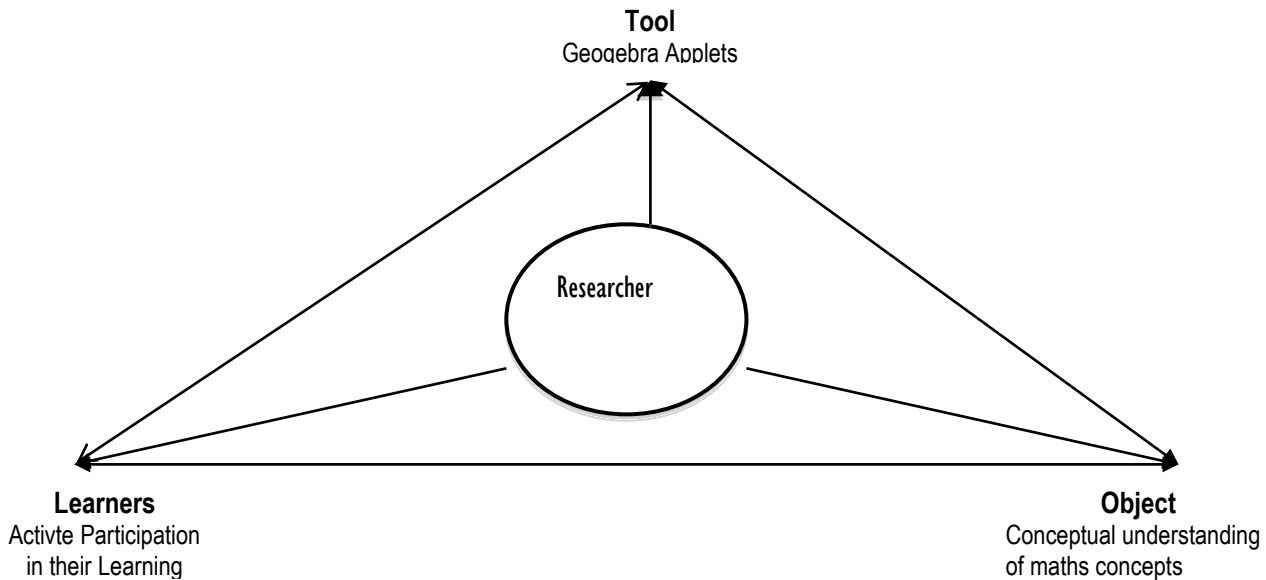
Figure 6.7 : Activity Theory model incorporating the researcher as an insider

Constructivism maintainsthat learners cannot be handedknowledge; theylearn best when they discover things, developtheir own ideas and actthem out rather than when they are merelytold or trainedwith a set of rules to apply. Vygotsky argues that :

> *Direct teaching of concepts is impossible and fruitless. A teacher who tries to do this accomplishes nothing but empty verbalism, a parrot-like repetition of words by the child, simulating a knowledge of the corresponding concepts but actually covering up a vacuum (Vygotsky, 1962, p. 83).*

Immersed in a social constructivist activity, opportunity arise for learners not only to learn mathematical skills and procedures, but also to expoundand validatetheir own intellectand alsoprovides an arena to discuss their observations. (Silver, 1996, pp. 127-159). O'Neill (1998, pp.144)states thatICT presents teachers with a significant pedagogical tool-kit from a social constructivist perspective. Hoyles (1991, p. 221) argued learning is achieved through social interaction for three reasons in a lesson involving computers: "the social nature of mathematics; the collaboration that computer-based activities invite and the basis for viewing the computer as one of the partners of the discourse. "
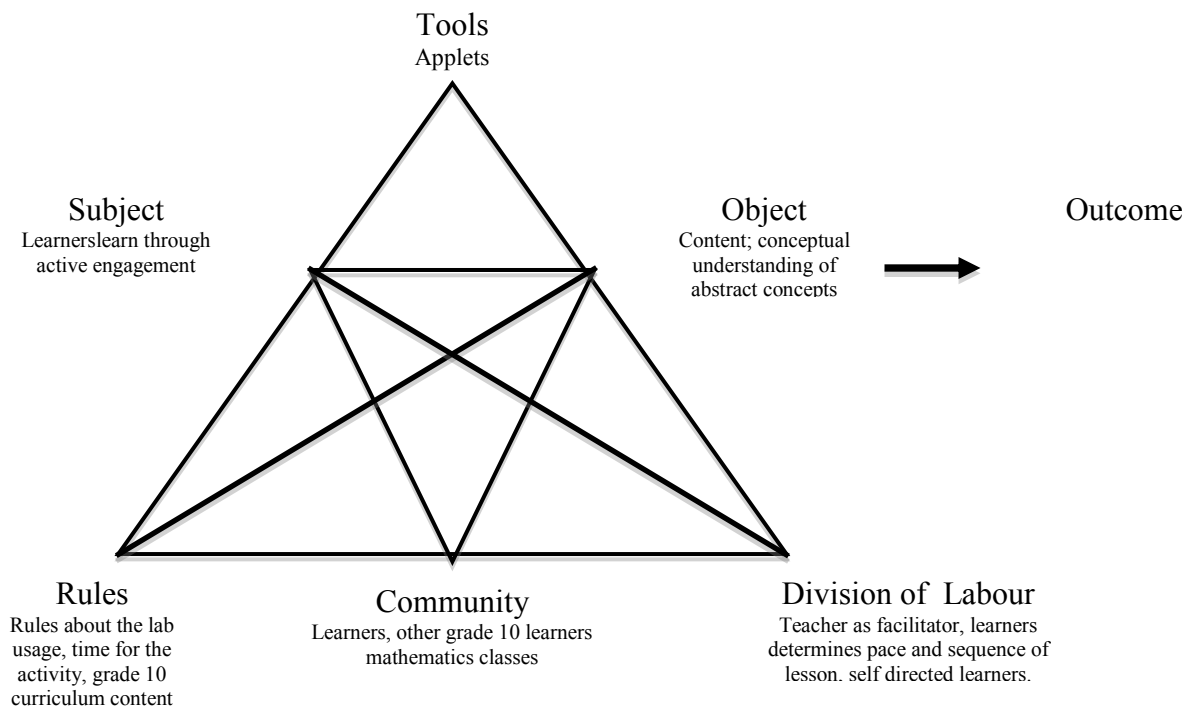
Figure 6.8 : First & Second Generation Activity Theory Model

Extending the AT to the second generation sees the teacher interacting with the learners in a computer environment which is informed by rules and norms specifically regardinghow learners conduct the activity together with the rules of the computer lab, engaging with the activity within the framework of the Grade 10 mathematics syllabus. The community involves the researcher and the learners, who belong to a wider community, to influence the object. The object of the activity attempted to develop the learners' understanding of the concept of transformation of functions i.e the movement from abstract concepts to concrete, visual understanding the ultimate outcome being the conceptualisation of the abstract concepts involved with transformations. The division of labour involved, primarily, the researcher and the learners with the researcherbeing the facilitator and the learners directing their own pace and sequencing where learning shifted from a teacher-centred to learner-centred approach. Whilst the researcherwas still responsible for the curriculum content, the division of labour shifted with the learners taking a more active role in their learning. Where the learners sometimes worked in collaboration with their peers, there was horizontal division of labour between them and,learners sharing knowledge and skills, vertical division of labourwas noted.

Figure 6.9 – Computer Lab (Learners engaging with the applets)

Whilst engaging with all the data collected, common elements were identified that permeated the study. They were classified into the following categories:

- escape from the routine
- exploratory, interactive learning
- extenuate learners' weakness
- change in the status of mistakes
- increasing engagement
- assisting learners' thinking
- meaning making amongst learners

Through this classification I congregated the categories into three distinctive themes, namely learning environment, cognitive amplification, role of the mediating tool and learning attitudes.

6.3.1   Learning environment

This theme concerns the change in the classroom environment.It involved the extension from a chalkboard and worksheet (these are non-responsive media) to a response-generating, teaching/learning aid (dynamic geometric software, *Geogebra*applets).

6.3.1.1 escape from the routine

Using computer technology in the mathematics class was somewhatof an escape from the routine for the learners. This escape contributed to the lesson being more interesting, fun and enjoyable. Learners were empowered by the tool they engaged with; they determined their own pace of learning and were in control of their learning. The study was based on the learner interaction with dynamic applets created for concept development.

6.3.1.2 "we can see our answers"

Learners absorb knowledgein different ways through different learning styles. In a self-discovery exercise personalized and uniqueresponses are promoted. This helps foster creativity, inventiveness, inspiration and originality in students. The applets facilitated the classroom activity in that the execution of the tasks were accurate, produced results quickly and allowed learners to "see our answers", as commented by Firdous. Teaching is often geared towards the 'average' student and everyone is obligedto advanceat the same rate,followed by assessment which unsurprisingly takes the ritualof traditional exams. This type of teaching promotes learning that make learners 'reproduce' that which they have accumulated during a given lesson. Often this is done at the expense of the learners being able to process this knowledge accumulation into 'knowledge' thatis 'usable' and 'transferable' in their lives). The concept of transformation that Zai referred to as "movement" became clear to Zai, who commented that "this concept was difficult to see on the chalkboard" referring to the above applets (6.10.1 to 6.10.3). "This is a cool program" meaning something he enjoyed or liked to work with.

6.3.1.3 "I really enjoyed it...."

In the focus group interview, Asma commented that she liked the lesson on the computer. "This was the first time I have done such a thing on computer and I really enjoyed it …"
Student-centred learning involves the student making sense or meaning what he/she is acting upon, hence as is the case in this study manipulating applets provides such a basis for the accomplishment of this type of learning.The applet acted as a mediator between learners and that which needed to be learned. This mediation affected learners' learning experience, particularly interaction with the applets. Thus, interacting with dynamic geometry software, learners received feedback on the basis of their new interactions. In this type of learning environment learners took chargeof their learning. They were active knowledge seekers. They constructed knowledge by relatingboth with themselves and the teacher and the data was gathered through manipulation of the applets, with the objectof solving a problem/task that they had been given.

6.4 Cognitive amplification

6.4.1 Learning as an active dynamic process

Learning is acknowledged as an active dynamic process in which connections (between different facts, ideas and processes) are constantly changing and their structure is continually reformatted. Such connections were fostered through interaction with the applets. By dragging the sliders in each of the applets, the learners altered the graphs and were required to observe the resulting effects on the functions to attempt to understand the transformation that resulted. Based on the changingvalues, numbers or shapes describingthe different states of the functions, they were expected to arrive atanswers to the questions given in worksheet 1.

The case of Asma
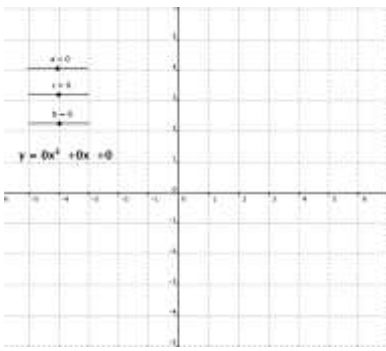
Figure 6.10.1 : where a = 0   Figure 6.10.2 : where a = 1.6   Figure 6.10.3 :where a = 7.2
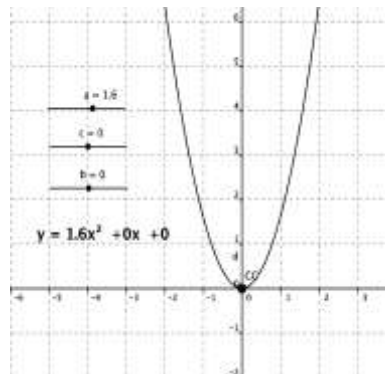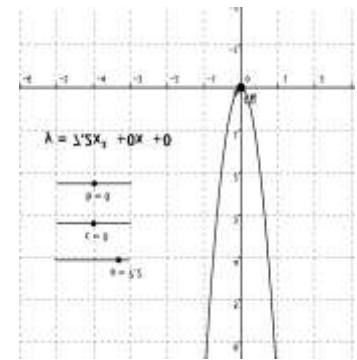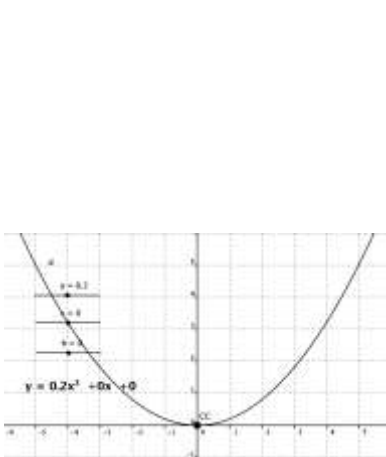
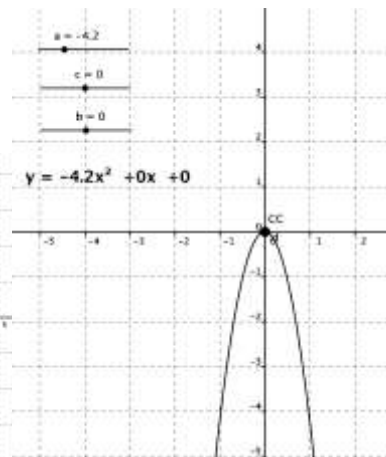Figure 6.10.4: where a = 0.2   Figure 6.10.5: where a = -0.2   Figure 6.10.6 : where a = -4.2
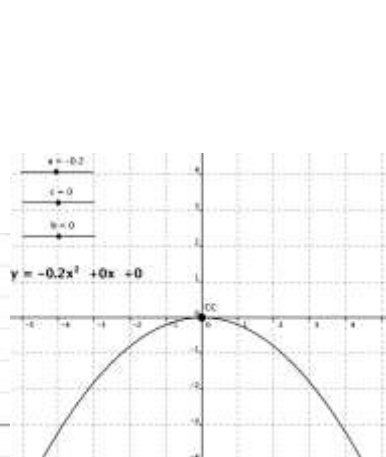
The above figures 6.10.1 to 6.10.6 are screen shots of Asma's interaction with the applets. The slider reflected in Fig. 6.10.1, serves as a variable, that shows how a certain graph or function behaves as one changes the values (or drag the slider). One can make as many sliders (or variables) depending on the need of the function or graph. To graph the parabola, the function was typed at the input bar. Each slider created represented the variable of the function. For instance, in $y = ax^2 + bx + c$, each slider represented each of the variable *a,b,* and *c* of the function respectively.

The effect of using the appletsappeared to have shifted the pedagogical focus from studying the individual and isolated effects of coefficients of quadratic equations to a holistic and dynamic exploration of relationships between the coefficients and the appearance and shape of the parabola. When all three variables were zero, no graph existed.

The mere manipulation of the variable *a* in the equation $y = ax^2 + bx + c$ had given rise to a

parabola. Asma was amazed by her exploration. She exclaimed, "*Sir, does each of these variables do something to the graph?*" She was now able to attach some significance to the variable *a* in the general form. "*Yes, Asma, as you engage with the other variables you should notice other "surprises"; just continue.*" Without telling her much,the researcher prodded her to discover for herself more changes to the graph.

Kuutti (1996, pp. 17-44) emphasised the fact that individual actions are always situated in a meaningful context and are impossible to understand in isolation without the meaningful context as the basic unit of analysis. This affirms that seeing the value of '*a*' as an isolated variable in the general form, does not do much for the conceptualization of this variable in the general form. In seeing for herself, Asma "suddenly" discovered that each of the variables in the function defined by $y = ax^2 + bx + c$ had a particular role. This discovery occurred in spite of the fact that this section was taught previously. Her reaction to this discovery requires some reflection. It is clear thatthe previous non-responsive, "chalk-and-talk" method prevented the mediation of understanding. It is possible that had the teacher spent more time and effort, similar results could have been achieved but the level of conviction attained through the use of the applets was a key factor resulting inher quickly grasping the roles of each variable. Following her "discoveries", Asma made the connection between the **shape** of the graph and "***a***", the **vertical movement** of the graph and *c* and finally the **horizontal movement** and "***b***".

6.4.2   Tools used to alleviate learners weaknesses

It seemed that students who were presented with alternate ways to view concepts hada better chance of understanding complex concepts. Underwood (2005, pg. 56) stated that "*the design of technology tools has the potential to dramatically influence how students interact with tools, and these interactions in turn may influence students' content area understanding and problem solving.*" Cognitive amplification connectsthe use of applets to diminish learners' weaknesses. Firdos scored 8 out 17 in her pre-activity exercise and scored 26 out 29 in her post-activity test;this may attest to the fact that the tools did play a role in alleviating her weaknesses. She was unable to explain the terms "equal roots", "unreal roots", "asymptote", "translation", "reflection" and "transformation". She successfully applied the knowledge gained from her engagement with the applets to a situation that required her conceptual understanding of the phenomena of transformation. The answer in the post-activity exercise

did not require recall of a rule or procedure but an understanding of concepts of transformation. She probably was one of the learners who found difficulty understanding the concept theoretically or seeing them on the board. When asked why she had left these questions blank in her pre-activity exercise, she replied:"*The rules (she meant the algebraic rules) of reflections are very confusing; they seem alike*".

During the activity all learners were actively engaging with the applets. Generally, in a traditional classroom, the weaker learners remain passive and often unnoticed by the teacher in terms of whether they grasped the concepts. It is generally observed that some learners dominate the discussions in the lessons. In this situation, Firdous was able to work at her own pace and worked on what she was challenged by. It seems that this active self-discovery type of lesson changed some students from passive observers to energetic, self-motivated members of the classroom.

6.4.3   Immediate and dynamic feedback

Computer generated applets, also called virtual manipulatives, often provide interactive environments where learners could solve problems by forming connections between mathematical concepts and operations, and get immediate feedback about their actions. More importantly, though, it enables the learners to see exactly what is happening on the screen relating to changes requested by the learner using the applet. Learners commented during the focus group interview that they preferred that method rather than doing it in their books, "*because, if you make a mistake, the computer tells you. Otherwise you'll have to wait until the teacher marks it the next algebra period, but the computer can tell you right away*" (Asma). One of the main advantages of computer-based learning is the ability to provide immediate feedback on individual responses as compared to group feedback. In general terms, feedback is any message generated in response to a learner's action. Feedback helped Asma identify her errors,which enabled her tobecame aware of her misconceptions. Feedback is also an integral factor in motivating further learning. As described by Cohen (1985, pg. 33), "*this component (feedback) is one of the more instructionally powerful and least understood features in instructional design*"

The case of Zai

The concept of transformation that he referred to as "movement" became clear to Zai, who commented that "*this concept was difficult to see on the chalkboard*" referring to the applets in Figure 6.11. He added, "*This is a cool program*", meaning that it was something he enjoyed working with. The applet assisted the visualization of his concepts of roots and nature of roots by dragging the sliders appropriately and observing its effect on the graph. This ultimately led to high levels ofconviction when he stated,"*It is easier for us to come to an understanding when we drag one point to see how it affects the movement of another point. It makes life so much easier having this software*". With immediate feedback Zai was able to evaluate his actions. Zai was busy with the <u>slider b</u>, which yielded the horizontal shift. At this point the researcher decided to probe further.
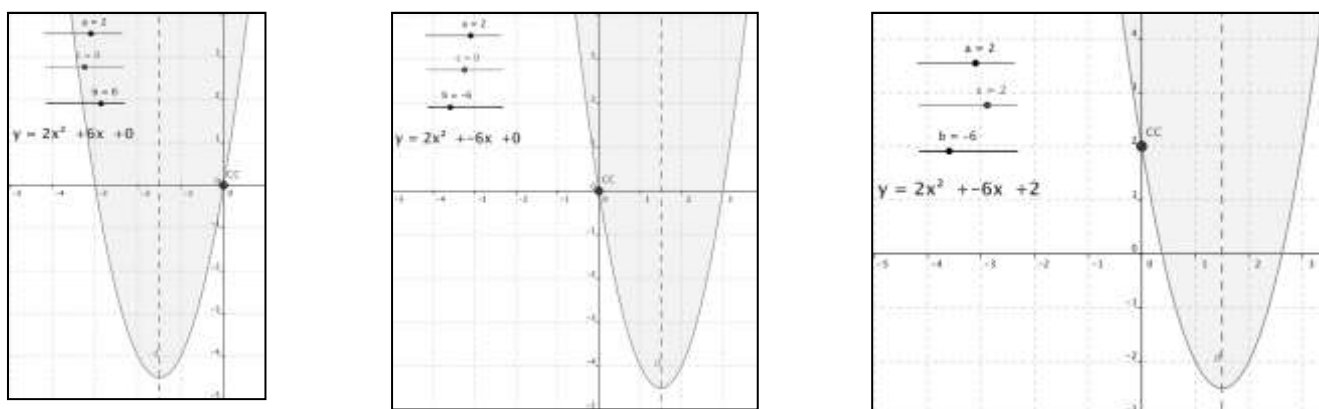
Figure 6.11 : Screenshot of Zai's engagement with variable (b)

Researcher :	Zai :
What are you busy with?	*I am busy with the variable b.*
What did you find?	*I found that by changing b, the graph moves sideways.*
What do you mean by 'sideway'?	*The c moved it upwards or downwards; the b moves it right or left.*
What is this movement referred to?	*A shift,*
How would you determine the translation if you were just given the function?	*The negative b means shift to the right and the positive b means shift to the left*
Excellent !!!	
So, if I give you an equationy = 2x² – 8x + 4, where will the graph appear on the Cartesian plane?	*On the right side.*
Can you demonstrate maximum and Minimum values on the applet?	*Yes,* *(he goes on to move <u>slider a</u> left and right)*

Zai further made the following pertinent comment:

"You know, Sir, I always got the sign of axis of symmetry wrong, when we used the

$y = a(x - p)^2 + q$ form. Now I see, if the graph is on the positive side the sign is negative and when

the graph is on the negative side the sign is positive."

Zai experienced instantaneous and dynamic feedback that provided him several occasions to try various possibilities whilst witnessing the consequences of his manipulations of the slider "b". It was evident that Zai's discoveries and conclusions were as a direct result of his engaging with the applet, considering that he was one of those participants who scored 6/17 in his first exercise without the use of the applets. He had beenunable to explain the concepts "vertical shift", "horizontal shift" and "transformation" which was crucial to this study and the research question on the effectiveness of applets in the understanding of mathematical concepts. He also had a conceptual understanding of minimum and maximum value by demonstrating on the applet the resulting effect of manipulating the value of "*a*". When asked, what he understood by reflection, Zai was able to link it to the sign of "*a*", thereby concluding that the equations $y = x^2$ and $y = -x^2$ were a reflection of each other. He was asked to demonstrate this using the applet which he easily andsuccessfully did.

6.4.4 Applet as a mediating artefact

To term 'mediate'refers*to "bringing about something"*. It is something that enhances reflection or that which enhances discussion, or something that brings about 'focus'. Mediating tools in this study may be used with the purpose ofdescribing problems; eliciting some requirements; generating a response; visualising a response; generating ideas and concepts; evaluating solutions; help explore or investigate, conclude or bring about some conviction or it make serve as a stimulus, triggering some reactions or reflections; creating experience or help reinforcing concepts.

Discussion with Naz

Researcher :	Naz :
In your engagement with the applets where you able to see the nature of the roots ?	*Yes*
Can you demonstrate to me what you understand by the roots being equal, real and unreal using the parabola applet ?	*She immediately switched onto the parabola applet, using the drag function she was able to demonstrate the nature of roots.*
Why then did you use the formula $b^2 - 4ac$ to answer the question in the exercise ?	*I did not understand it this way; I know the formula*
Would you be able to explain the nature of roots without using the formula to a friend ?	*Yes, now that I have seen it in the applet*

Evidently, Naz did not have a conceptual understanding of the nature of roots until her engagement with the applets. Hence the applet proved beneficial and effective in her case. The findings of the pre-activity and activity exercises show that the learner made considerable progress in acquiring a conceptual understanding as a result of the classroom intervention. This confirms Vygotsky's notion of zone of proximal development, where the learner's applet filled the gap between unassisted to the assisted region through mediation of a tool. The learner expounded a better understanding of the concepts through reinforcementby working with the applets. However, she felt that the rules or formula sufficed and getting it right was important. Her remarks indicated that she was driven by procedural methods rather than conceptual understanding since it benefited her in getting the answer right rather than understanding the deep workings of objects. Naz's outcome was achieved by getting the answer correct through some memorized method. However, the outcome required by the activity theory in this study was a conceptual understanding through the use of tools rather than the application of some learned formula. Wiggins (1998, p. 2), supports this claim that "*even good students don't always display a deep understanding of what's been taught even though conventional measures certify success*". He further comments that "*correct answers offer inadequate evidence for understanding or good test results can hide misunderstanding*" (p. 40).

The case of Asma (2)

Asma, who was busy with the hyperbola applet, exclaimed loudly "I get it, the vertical shift in the graph accompanies a broken line with it; this means there's a new x-axis. Sir, how is there a new x-axis"? The researcher asked her to demonstrate her findings. The following represent Asma's engagement with the hyperbola applet.
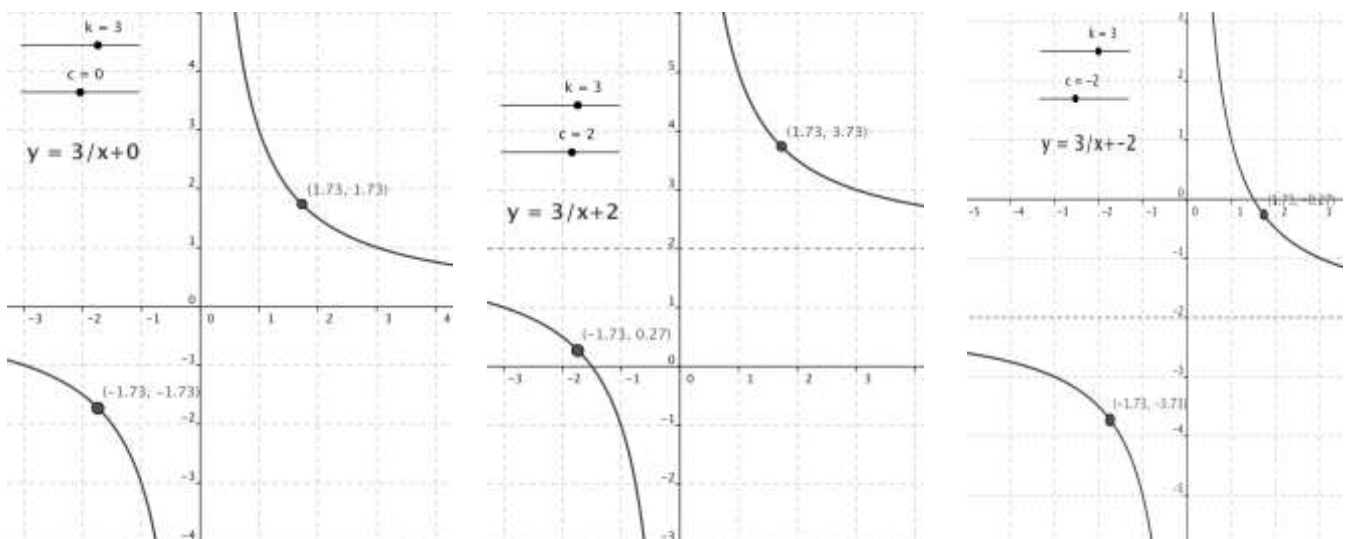
Figure 6.12.1: where c = 0   *Figure 6.12.2 : where c = 2*   *Figure 6.12.3 : where c = -2*

Responses given to questions posed by the researcher whilst Asma engaged with the applet.

Researcher:	Asma:
What do you notice about the value of 'c' and the value which corresponds with the broken line ?	*They are the same*
Describe the movement of the graph when 'c' is changed or dragged	*Vertical shift*
When 'c' is zero where is the broken line ?	*On the x-axis*
When does the function touch the x-axis ?	*When 'c' is positive or negative* *When the function shifts up or down*
What do you notice about the graph and the broken line ?	*It never touches the line.*
Is this line an imaginary line ?	*I suppose you could say that.*

Asma had understood that the asymptote was a line in place of the x-axis when the graph makes a vertical shift. Meaning making was evident in her case as well as scaffolding of knowledge was apparent. Engagement with the applet assisted her, taking her from a realm of uncertainty or little certainty to some clarity and understanding. This could be regarded as the applets supporting or acting as a meditational tool for deeper understanding.

Suydum and Higgins (1976, pp. 2-5) also highlighted the fact that the use of manipulatives heighten the probability of increasing achievement and are important in providing a solid foundation for development of mathematical concepts.

The following graphs emerged when the value of c was manipulated.
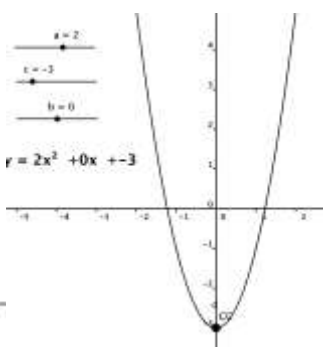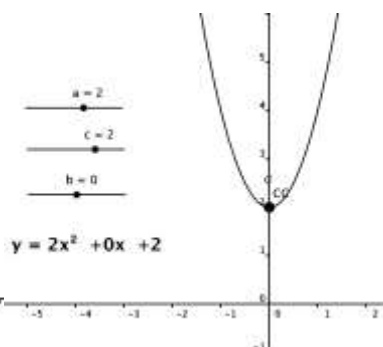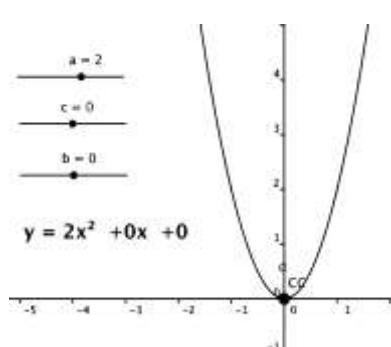
Figure 6.13.1 : where c = 0   *Figure 6.13.2: where c = 2*   *Figure 6.13.3: where c = -3*

Learners discovered that vertical shift took place when the value of c was manipulated. Learners also discussed the nature of roots. Figures 6.13.1 indicated equal roots; 6.13.2 indicated no roots or unreal roots and 6.13.3 yielded real roots but unequal roots.

The discussion with Raz also revealed important information.

Researcher:	Raz:
Why are you writing stuff in your worksheet ?	*For me to learn from*
Would you not be able to answer questions of this type in the future ?	*No, it's not that, it just helps me when I learn*
Do you like learning rules when it comes to maths ?	*Ya, everything in maths is about rules and formulas. I am writing these rules in my own words so that I can use them when I study for maths.*
How do you study for maths ?	*I learn the rules and then try to remember them when I'm answering a test or exam.*
And if you forget a rule ?	*Then I try whatever.*
Do you think you would still need to learn a rule to determine the nature of roots now that you have used this applet ?	*I suppose I don't have to now, because I can see the nature of roots when I move the graph, but in the exams I won't have this program.*

Raz'scomments are worthy of note because of her innocence and the way she looked at learning mathematics. Perhaps this may be the reason she had eight incorrect answers in her first exercise. She probably could not answer any better because she did not rememberthe rules.Evidently, she preferred learning rules. Also, cognisance must be taken of the fact that with the applets Raz could "see" the roots as compared to having just a theoretical understanding.

Juxtaposed against pre- and post-exercise scores, the responses of learners indeed showed a marked difference between what learners knew about features and behaviours of parabolas and hyperbolas before the activity andwhat they knew about translations and reflections of these functions after the activity. Results of the pre-activity exercise showed that learners had difficulty explaining features of the parabola and hyperbola. In the pre-test, almost all the students were unable to explain translation and reflection.

	Original function	Transformation	Equation of function after transformation										
5	$f(x) = 3x$	vertical shift up 3 units	$y = 3x + 3$	1	1	1	1	1	1	1	1	0	0
6	$f(x) = -4x$	vertical shift down 2 units	$y = -4x - 2$	1	1	1	1	1	1	1	1	0	0
7	$f(x) = 2x + 1$	vertical shift down 3 units	$y = 2x - 2$	1	1	1	1	1	1	1	1	0	0
8	$f(x) = 4x - 3$	vertical shift down 1 unit	$y = 4x - 4$	1	1	1	1	1	1	1	1	0	0
9	$g(x) = 3x^2$	reflection along x-axis	$y = -3x^2$	1	1	1	1	1	1	1	1	0	0
10	$g(x) = 2x^2$	vertical shift up 3 units	$y = 2x^2 + 3$	1	1	1	1	1	1	1	1	0	0
11	$f(x) = -4x^2$	vertical shift down 2 units	$y = -4x - 2$	1	1	1	1	1	1	1	1	0	0
12	$f(x) = 2x^2 + 1$	vertical shift down 3 units	$y = 2x^2 - 2$	1	1	1	1	1	1	1	1	0	0
13	$f(x) = 2x^2 + 1$	reflection	$y = -2x^2 - 1$	1	1	1	1	1	1	1	1	0	0
14	$h(x) = 4/x$	vertical shift up 3 units	$y = 4/x + 3$	1	1	1	1	1	1	1	1	0	0
15	$h(x) = 4/x + 1$	vertical shift down 2 units	$y = 4/x - 1$	1	1	1	1	1	1	1	1	0	0
16	$h(x) = 4/x - 2$	Vertical shift down 3 units	$y = 4/x - 5$	1	1	1	1	1	1	1	1	0	0
17	$h(x) = 4/x + 4$	vertical shift down 1 unit	$y = 4/x + 3$	1	1	1	1	1	1	1	1	0	0

Figure 6.14 : Extract from responses from analysis of the post-activity exercise

Examining the responses from the intervention, as reflected in Figure 6.14, I noticed some improvement when the learners applied themselves to the given tasks. All of them were able to answer questions on translation and reflection. Results of the post-test showed a remarkable improvement in the explanation of concepts involving functions, their special features and the influence of the variables on the graph.

In analysing the data collected in Figure 6.14, all learners were able to vertically translate the parabola and hyperbola. It is important to note that these questions were answered without the use of the applets. These responses were as a result of their engagement with the applets the day before. It seems that the applets contributed to knowledge construction. The applets provided seem to have been the means for fundamentally changing the way the instruction (learning of the vertical shift) was delivered to learners. When Raz was asked about her experience with the applets, she replied that "*the applets were probably the best way of simplifying all the concepts by way of live animations*". These applets brought "life" into every concept for her. As a result she was able to "see" the vertical shift. This web-based learning produced an effective learning experience for her. Traditional teaching emphasizes content-learning the "what", whereas, the interactive learning improves understanding through asking "how" something is done.

When Zai used the worksheet in conjunction with the applet during the activity, the interaction between the curriculum and the technology was different from previous mathematical activities. The textbook/teacher's notes no longer guided his learning. He was able to use his knowledge of the mathematics and the applet to present his answers. He produced 22 out of 29 correct answers as compared to his pre-activity. Scoring 12 out of 12 as per Figure 6.14 showed an inprovement in his understanding of vertical translation. He was able to apply his knowledge and would have made meaning in this aspect of transformation of functions.

Learners no longer used a rule for re-writing a vertically or horizontally transformed function, but through their visual experiences, connections and patterns were in a position to translate a graph accordingly and correctly.   Transformation in their learning convincingly took place during the intervention stage.

The post-test scores showed an improvement in all learners' results especially in the more complex questions on graphs. This meant that the applets aided the learners' understanding and allowed for internalisation of mathematical concepts relating to transformation. Certainly the intervention transformed the way learners saw graphs again.

6.5   Limitations

According to Cohen, Manion and Morrison (2007, pp. 101-106) the sample size should be large enough to generate ample data. The results of this study is limited since it is a reflection of only eight learners in a mathematics class. The study cannot generalize the effects computer-generated virtual manipulatives has on other Grade 10 mathematics learners. However, this study does make available material that can be made available to teachers and researchers when attempting to teach or research this particular Grade 10 mathematics topic. It is mybelief that appropriately designed mathematical tools can enrichinstruction. To determine the extent to which these response generating applets may affect learning of these mathematical concepts, many more studies will need to be conducted and on a larger scale. I can only conclude that, in this computer environment, interaction with the dynamicapplets had a positive effect on learners' conceptual understandingof concepts.

Limitations were also acknowledged in the data collection technique. The pre-activity and post-activity exercises used were teacher-made, and, therefore, not standardized. Also the

pre- and post- exercises were not identical; there may have been discrepancies in the levels of difficulty in each exercise. In addition, responsesregarding the attitudes of learners towards these computer generated applets may have been influencedby learners' reluctance to report truthfully because the researcher was also a teacher at their school. This may have caused students to choose responses they thought their teacher would like to hear. This may have also been true of responses received duringthe interviews on the days when their teacher was their interviewer.

The technology itself can be time consuming and frustrating if learners have difficulty with operating the computer. Also, if learners are not proficient in the use of the mouse, this may lead to frustration and distraction since the entire activity is underpinned by the dragging of the mouse. Also, computers can be a tempting way for learners to go off task.

6.6   Discussion

Notwithstanding the limitations in the previous chapter, I firmlybelieve that this study embodies valuableinformation worth communicatingto the mathematics community. The conceptual understanding of the abstract mathematical concepts involved in the learning of transformation of functions as indicated by the results obtained from the post-activity exercise and the discussions during the activity exhibitednoteworthygains in this relatively small sample of learners, with over two thirds of the participants in this classroom improving their scores after using these applets. One explanation is that the applets used were interactive, dynamic and response-generating visual images of the concept of transformation of functions. The pre- and post-exercises included questions requiring learners to explain concepts from an in-depth understanding.

They were **<u>better</u>** able to explain concepts after engaging with applets than they were when they were not exposed to these tools as was the case duringthe pre-exercise. Working with graphiccomputer applets could have enrichedlearners' abilities to describeand demonstratetheir thinking using visual tools; hence, the tools may have mediated their knowledge.   The virtual manipulatives also provided opportunities to interact with the applets that the learners themselves manipulated and it provided opportunities for them to generate dynamic responses desired or undesired immediately. Learners are deprived of the opportunity topractise with dynamic visual representations when they view graphs or

functions in a worksheet or textbook.

The activity provided an opportunity for learners to have acquired conceptual understanding as compared to their traditional-class lesson. After learning about transformation of functions in traditional class system, two learners' procedural knowledge showed a marked improvement. The learners in the focus group interview were asked whether they were able to understand the questions in worksheet 2 despite there being no diagrams. All answered in the affirmative. The scores they obtained in the post-activity exercise indicates that learners had a good grasp of the procedures for determining the nature of roots or translating a graph across the Cartesian plane. Although some learners werefamiliar with accurate rules or formulae or algorithms for solving problems, they may not have understood or may not be in a position to explain the reasoningbehind procedures or rules. Improvement in learners' scores regarding conceptual understanding after engaging withvirtual manipulatives may be indicative that working with dynamic visual interactive images may have supported or mediated their understanding.

During the activity, learner progressionmay have been ascribedto the immediate and specific feedback students received whilstengaging with virtual manipulativesowing to its immediate response-generating capabilities. These specific instances of feedback in dynamic/interactive form may have served the function of alleviating weaknesses or illuminating learner' errors, making students more aware of their own misconceptions. This immediate feedback served as a connection between their previous knowledge to new knowledge formed. Some form of verification or conviction may have resulted from this feedback. Learners may have seen this concept in action, for example, that of the shift axis of symmetry in the direction opposite to that of the general form of the function. This terminology of horizontal translation while using the virtual manipulatives may have enhanced meaning or internalisation to the learner. According to Hiebert and Lefevre (1986, p.4) conceptual knowledge is achieved in two ways: by "*the construction of relationships between pieces of information*" or by the "*creation of relationships between existing knowledge and new information that is just entering the system*". Upon reflection of the responses from Zai, Asma and Naz, I am convinced that the fourth principle of activity theory - that of internalization – externalization (Vygotsky, 1978, p. 144), which describes the mechanisms underlying the originating of mental processes had come to bear. The internalization according to Vygotsky, refers to the range of actions that can be performed by a person in cooperation with others;this comprises the so-called "zone of proximal development". Activity theory emphasizes that internal activities cannot be

understood if they are analysed separately, in isolation from external activities, because there are mutual transformations between these two kinds of activities : internalization and externalization.

Different learning ability levels were catered for in this activity. Learners worked at their own pace; hence,learners completed many more tasks than theywould have completedhad they attempted them on paper.   This assisted in keeping advanced learners interested and engaged. These applets also supported visual learners by representations in the form of dynamic visual objects. Hence, this would have scaffolded learning for the less able learners in the group. It was noted that the learners that performed poorly in the pre-activity performed better after using these virtual manipulatives. Thesevisualtools may have helped those learners who needed assistancetocomplete the worksheets during the activity stage successfully. The results from the post-activity exercise certainly indicate that exploiting virtual manipulatives did not negatively affect learners' knowledge.

Based on the results of the pre-activity exercise, most learners did not possess a good procedural or conceptual knowledge of the concepts. Using technology saved time and gave learners access to a powerful new way to explore concepts in depth thanwaspossible in the traditional pen and paper method. The power of computers led to fundamental changes in mathematics instruction. For instance, the ability to explore the "what if" questions through variations has opened up new avenues for mathematics.

During the focus group interview after the participants' interaction with the applets and their answering of the post-activity exercise, learners commented that the applets helped them gain "a better understanding of translations and reflections".   They also expressed that their experiences with the engagement of these tools had given them confidence and believe that they understood these concepts as a result of their interaction with these applets. Many learners displayedpositive attitudes regarding their learning experiences, and thoroughly enjoyed working with these "*computer thingies*", meaning the applets. The themes manifested in the interviews and responses to the questionnaires reinforces this conclusion.

As a teacher-researcher I believe that these virtual manipulatives are indispensibleas an instructional tool or as a discovery tool. It helps mediate the learning process. Also, preparing these applets prior to the lesson for specific knowledge development, helps focus the lesson and learners in achieving the desired outcomes through the objects which learners act upon.

Fortunately, as a mathematics teacher with a strong background in computer programming, I am in a position to create specific tools for specific outcomes. Similar research on physical manipulatives hasconcurredthat teachers who are more experienced and proficient at using manipulatives will yield more positive results in their classroom (Raphael & Wahlstrom, 1989, pp. 173-190).

CHAPTER SEVEN

CONCLUSION

Typically teachers are too often satisfied with teaching mathematics as manipulating symbols and focus on learning rules and procedures and doing routine problems, without ever ensuring that their learners acquire deep, conceptual understanding. Learners usually learn and believe what they are taught and reproduce the same in a test or examination.

Challenging learners to think and reason about mathematics allows learners to construct their own meaning about mathematics and in turn develops deep, conceptual understanding. Creating opportunities for learners to investigate, discover, examine, apply, prove and communicate mathematics will not only give meaning to their learning, but also develop a deeper understanding of mathematics.

In providing an arena for learners to investigate, discover, examine, apply and prove, technology proves to be a powerful tool; yet in the South African education forecourt, it has not reached its potential as an instructional tool. Dynamic interactive software technology is a promising tool for improving learners' visual and conceptual abilities in mathematics. The dynamic nature of virtual manipulatives, along with colour, graphics, and interactivity can capture and hold the attention of learners so that they persist at mathematics tasks, hence being motivated to learn and make meaning of their learning. Deeper understanding occurs when learners engage with mathematical models.

The purpose of the study was firstly to explore the role of *Geogebra* as a pedagogical tool and mediating artefact in the teaching and learning of transformation of functions in secondary school mathematics and, secondly, to explore whether interaction with these virtual manipulatives enhanced the understanding of certain mathematical concepts. Furthermore, one intention of this research was to contribute to the studies in understanding how learners acquired and developed their conceptual understanding of mathematics using computer technology in a technology supported environment.

The conclusion of this study has been drawn from data analysed in chapter five. In answering the research questions,the findings identified in chapter six indicated that

Geogebra did indeed fulfill its role as a pedagogical tool in an attempt to mediate the learning of transformation of functions in a grade 10 mathematics class. Theconceptions were measured against expected outcomes,notably within the theoretical framework of Activity Theory.

Based on the learners'verbal responses during the activity stage and their written responses in the post-activity exercise, this study confirms that learners have moved from a stage of routine learning to that of conceptual understanding subsequent to the intervention. The intervention, being engagement with *Geogebra* applets, has elevated them to a stage where they are able to explain features of the various functions as well as rewrite functions after transformation has taken place resulting from the visual experiences they aquired during the activities they engaged with. A marked improvement in the explanations of concepts from the pre-activity exercise to the post-activity exerciseindicates the significant progress made through intervention.

The use of a variety of tools in mathematics is invaluable in helping learners understand abstract concepts. The uses of various representations can improvethe learners' abilitiy to think flexibly about mathematics topics. Worksheet 2 aimed at assessing the conceptual understanding of concepts relating to transformation of functions. The responses obtained from this worksheet, highlighted the importance of using virtual manipulatives in teaching and learning mathematics. The responses were indicative that the applets have provided support to learners' existing knowledge. The use of dynamic visual applets is worthy of further study to sustainthe impact it has on students' learning and understanding of mathematical concepts.

Interactive applets are an inventiveand constructive way to enrich self-discovery and meaning making in mathematics learning. Virtual manipulatives in this researchattestedto be effective for students.

Geogebra provides a "live"way to demonstrate manipulationof mathematical objects thereby bringing about a deepunderstanding of mathematical concepts."Dragging" as a powerfulaction makes these environments much more effectiveand "live" than traditional paper and pencil learning.

Integrating virtual manipulatives such as *Geogebra* applets, allowed for this break from routine. It facilitatedclassroom activity and enhanced its self-study and learner control situations. The teacher's role need not diminish in any way, but can increase, as a guide so that learning can take place by learners being directly involved with their own meaning-making and learning. The teachers have to identify the suitablelearning outcomes, choose or create appropriate software activities inguiding the learning process. The software activities should serve as vehicles to mediate learning or help foster fundamental concepts. To be able to use a formula, the learner ought to have a conceptual understanding of the roots of this formula, or the short method of solving the problem. Evidently, this study demonstrates the effectiveness of the use of computer software artefacts as meditational tools as well as vehicles for the internalisation of transformation of functions.

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APPENDIX A

Request permission letter to the School Board of Directors

Enq: Sheriff Uddin R   P.O. Box 19580
Cell: 082 768 1969   DORMERTON
         4015
         12 July 2010

The Chairman
cc The Principal
Al-Falaah College
99 Lotus Road
SPRINGFIELD
4091

Sir

Request for permission to do data collection at your school: Al-Falaah College

I am an educator at Al-Falaah College undertaking a research study as a master's student of the University of KwaZulu Natal. My study aims to explore the role of a computer software called *Geogebra* as a teaching and learning tool in secondary school mathematics. The study also aims to explore whether learners' interaction with this software enhances their understanding of certain mathematical concepts.

With reference to the above kindly consider my request to collect data at school whilst engaging learners in this study. The study will be carried out in school with eight grade 11 learners. Learners' participation will be completely voluntary and their parents will receive a similar letter seeking their permission.

The results of this study will be used for my dissertation. Pseudonyms or codes will be substituted for the names of children and the school. This would ensure participants' confidentiality.

Attached, please find copy of the letter that would be sent to parents for their consent.

Your cooperation is always appreciated.

Yours in the advancement of education.

_____
R. Sheriff Uddin

APPENDIX B

Request permission letter to Parent of Minors

Enq: Sheriff Uddin R   P.O. Box 19580
Cell: 082 768 1969   DORMERTON
              4015
              12 July 2010

Dear Parent,

Request for permission to allow your child to participate in a study

I am an educator at Al-Falaah College undertaking a research study as a master's student of the University of KwaZulu Natal. My study aims to explore the role of a computer software called *Geogebra* as a teaching and learning tool in high school mathematics. The study also aims to explore whether learners' interaction with this software enhances their understanding of certain mathematical concepts.

I do not anticipate any risk greater than normal life. Your child may enjoy this research while learning more about transformation of graphs. Your child's participation in this project is completely voluntary. In addition to your permission, your child will also be asked if he or she would like to take part in this project. Any child may stop taking part at any time. The choice to participate or not will not impact your child's grades or status at school.

The video-recording and all other documented information that will be obtained during this research project will be kept strictly secure and will not become a part of your child's school record. The video-recordings will be kept in a locked file cabinet and will be accessible only to project personnel. These recordings will be transcribed and coded to remove children's names and will be erased after the project is completed.

The results of this study will be used for my dissertation. Pseudonyms or codes will be substituted for the names of children and the school. This helps protect confidentiality.

Attached, please find a consent returnform indicating whether you do or do not want your child to participate in this project. Ask your child to bring one copy of this completed form to his or her teacher by 16 July 2010. The second copy is to keep for your records. If you have any questions about this research project, please feel free to contact me or my supervisor either by e-mail or telephone. Please keep a copy of this form for your records.

Thank you.

Sincerely,

Razack Sheriff Uddin, Researcher   Dr Vimolan Mudaly, University Supervisor
082 768 1969   031 260 3587
sheriff786@telkomsa.net   mudalyv@ukzn.ac.za

Appendix C

PARENT CONSENT – Research Study

I, Mr/Mrs _____

parent of _____ in gr_____

    do agree ☐ / do not agree ☐ to allow my child to participate in this project and also

    do agree ☐ / do not agree ☐ to allow my child to be video-recorded for transcription only.

Parent's signature: _____ _____ Date _____

If you have any questions about your rights as a research participant please contact UKZN Research and Ethics Office, Ms. Phume Ximba, Humanities and Social Science Research Ethics Office on 031 260 3587.

Appendix D

Researcher: R. Sheriff Uddin

Project title: ***Geogebra*, a Tool for Mediating Knowledge
in the Teaching & Learning of Transformation of Functions in Mathematics**

Purpose: to explore the role of *Geogebra*, as a teaching tool in the teaching and learning of transformation of functions in secondary school mathematics;

to explore whether interaction with these virtual manipulatives enhance the understanding of mathematics concepts.

The research will take place in four maths lessons in the computer room. This project will be carried a follows :

(i) you will be asked to answer questions based on transformation of graphs viz. parabola, hyperbola and the exponential graphs on a worksheet;
(ii) you will be required with the aid of the computer, to engage with ready-made *Geogebra* applets involving transformation of functions and to record your findings in another worksheet;
(iii) your engagement with this software may be video-recorded for the sole purpose of analysis and
(iv) a discussion will be held with you regarding your use of this software and also your understanding of mathematical concepts involved in this section in mathematics.

You are under no obligation to participate and may withdraw at any time. Only aggregated results will be reported.

If you participate in this study, the information will not be linked back to you as an individual. The information will be stored in a secure environment and access to the data will be made available only to the members of the research team. Your comments will be kept confidential and any information provided will only be used for the purposes of this research.

You are welcome to discuss your participation in this study with the researcher or his/her academic advisor or to impose conditions, or withdraw from the study at any time.

If you would like to speak to an officer of the University not involved in this study, you may contact the University's Ethics Officer on 336 53924.

I, _____ *(name of learner)*, read the above and have been explained by the researcher on the above.
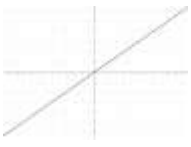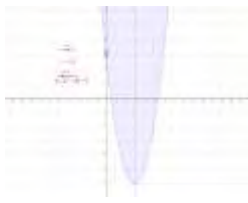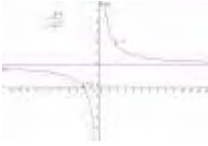
I will ☐ / will not ☐ participate in the above study and am fully aware that I can withdraw anytime from the study.
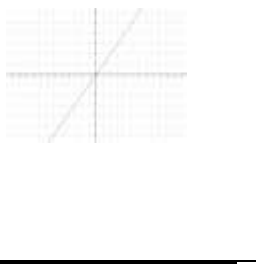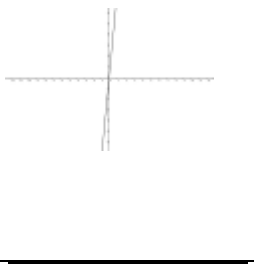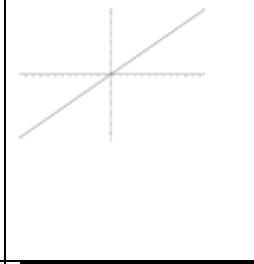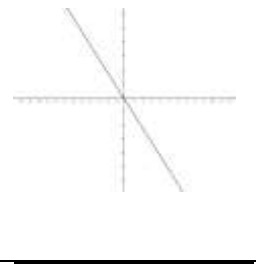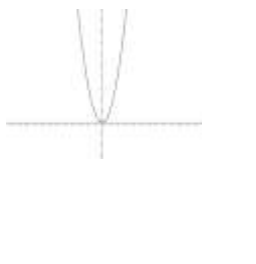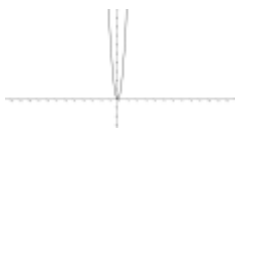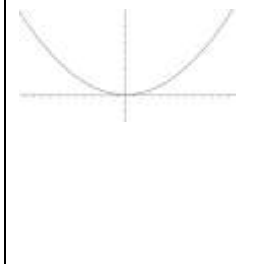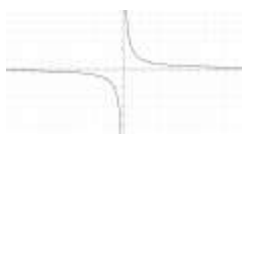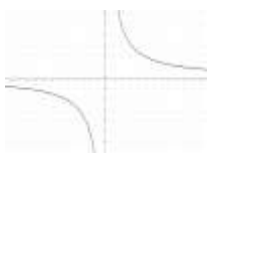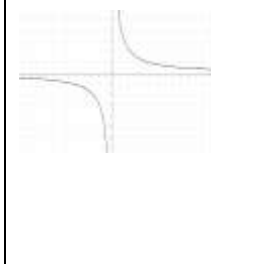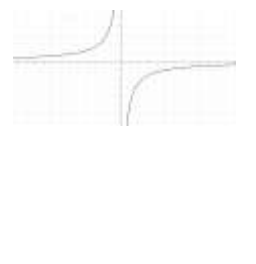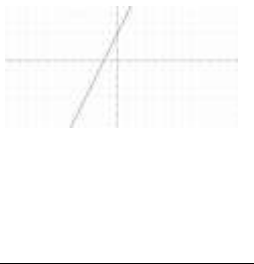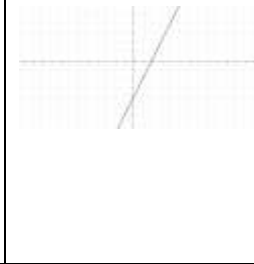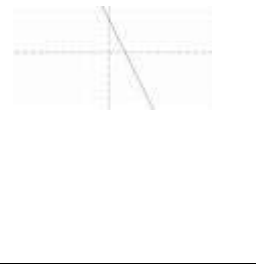
_____   _____
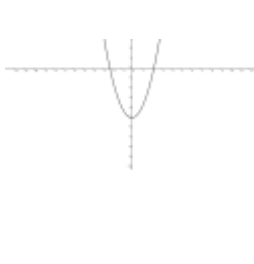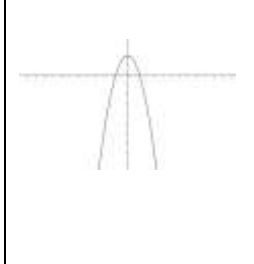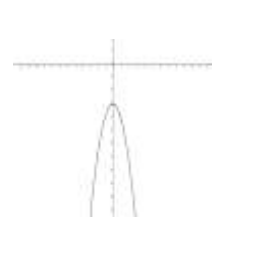Student signature   Date

APPENDIX E

Worksheet 1a– Functions (Grade 11)

1.	Identify each of the graphs below *(circle the correct answer)*:			
	a. Straight Line b. Exponential Function c. Parabola d. Hyperbola	a. Straight Line b. Exponential Function c. Parabola d. Hyperbola	a. Straight Line b. Exponential Function c. Parabola d. Hyperbola	a. Straight Line b. Exponential Function c. Parabola d. Hyperbola
2.	Write down the general form of the above graph y =	Write down the general form of the above graph y =	Write down the general form of the above graph y =	Write down the general form of the above graph y =
3.	What do you understand by the term y-intercept ?			
4.	What do you understand by the term x-intercept ?			
5.	Draw a rough sketch of each of the graphs below:			
	Linear function with a +ve gradient	Linear function with a –ve gradient	Parabola with a min. value turning point	Parabola with a max. value turning point
6.	Describe the shape of the function $y = 3x^2 + 4$, will it have a maximum or minimum value?			
7.	Give an explanation of the term turning point of a parabola ?			
8.	Explain the term roots			
9.	How would you explain real roots ?			
10.	How would you explain equal roots ?			
11.	How would you explain unreal/non-real roots ?			
12.	How would you explain the term asymptote to a friend ?			
13.	Explain the term symmetry.			

14.	Explain axis of symmetry
15.	Explain the concept, transformation.
16a	Explain the concept, translation.
16b	Explain the term reflection.

18. Which of these functions best describe the graphs drawn A. y = x B. y = 10x C. y = ½ x D. y = -x				
	A B C D	A B C D	A B C D	A B C D
19. Which of these functions best describe the graphs drawn A. $y = 10x^2$ B. $y = -x^2$ C. $y = ½ x^2$ D. $y = x^2$				
	A B C D	A B C D	A B C D	A B C D
20. Which of these functions best describe the graphs drawn A. $y = \dfrac{-5}{x}$ B. $y = \dfrac{10}{x}$ C. $y = \dfrac{5}{x}$ D. $y = \dfrac{2}{x}$				
	A B C D	A B C D	A B C D	A B C D
21. Which of these functions best describe the graphs drawn A. y = x + 3 B. y = -x + 3 C. y = 2x + 3 D. y = 3x - 4				
	A B C D	A B C D	A B C D	A B C D
22. Which of these functions best describe the graphs drawn A. $y = x^2 + 3$ B. $y = -2x^2 - 3$ C. $y = x^2 - 5$				

D. $y = -2x^2 + 2$																

	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D

23.
Which of these functions best describe the graphs drawn

A. $y = \dfrac{5}{x} + 1$

B. $y = \dfrac{10}{x} - 3$

C. $y = \dfrac{-5}{x} - 3$

D. $y = \dfrac{-10}{x} + 1$

	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D

24.
Which of these functions best describe the graphs drawn

A. $y = x^2 + 6x + 3$
B. $y = x^2 + 4x - 2$
C. $y = -x^2 - 2x - 2$
D. $y = -2x^2 - 4x + 2$

	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D

25.
Which of these functions best describe the graphs drawn

A. $y = \dfrac{5}{X+2} + 1$

B. $y = \dfrac{10}{X+2} - 3$

C. $y = \dfrac{-5}{X+2} - 3$

D. $y = \dfrac{10}{X+1} + 1$

	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D

APPENDIX F

Given the general form of the functions viz. :
Linear : $f(x) = ax + t$ Quadratic : $g(x) = ax^2 + bx + c$ Hyperbola : $h(x) = \dfrac{k}{x+p} + q$

answer the following questions:

1. What effect does the value of *a* have on the shape of the linear function ?

    _____

2. What effect does the value of *a* have on the shape of the quadratic function ?

    _____

3. What effect does the value of *k* have on the shape of the hyperbola ?

    _____

4. How is each of the functions transformed with the introduction of the values t, c and q on their respective graphs?

    _____

5. In what way is the graph transformed with the introduction of *b* and *p* in the parabola and hyperbola respectively ?

    _____

6. Given the following functions fill in the table:

WORKSHEET 2 - Transformation of Functions (Grade 11)

	Names of Participants ———>			
	Given the general form of the functions : $f(x) = ax + t$; $g(x) = a(x-p)^2 + q$ and $h(x) = k/(x+p) + q$ answer the questions:			
1	What effect does *a* have on the shape of f(x) ?			
2	What effect does *a* have on the shape of g(x) ?			
3	What effect does the value of *k* have on the shape of h(x) ?			
4	How is each of the functions transformed with the introduction of the values *t* and *q* on their respective graphs ?			
	Original function	Transformation	Equation of function after transformation	
5	$f(x) = 3x$	vertical shift up 3 units	$y = 3x + 3$	
6	$f(x) = -4x$	vertical shift down 2 units	$y = -4x - 2$	
7	$f(x) = 2x + 1$	vertical shift down 3 units	$y = 2x - 2$	
8	$f(x) = 4x - 3$	vertical shift down 1 unit	$y = 4x - 4$	
9	$g(x) = 3x^2$	reflection along x-axis	$y = -3x^2$	
10	$g(x) = 2x^2$	vertical shift up 3 units	$y = 2x^2 + 3$	
11	$f(x) = -4x^2$	vertical shift down 2 units	$y = -4x - 2$	
12	$f(x) = 2x^2 + 1$	vertical shift down 3 units	$y = 2x^2 - 2$	
13	$f(x) = 2x^2 + 1$	reflection	$y = -2x^2 - 1$	
14	$h(x) = 4/x$	vertical shift up 3 units	$y = 4/x + 3$	
15	$h(x) = 4/x + 1$	vertical shift down 2 units	$y = 4/x - 1$	
16	$h(x) = 4/x - 2$	Vertical shift down 3 units	$y = 4/x - 5$	
17	$h(x) = 4/x + 4$	vertical shift down 1 unit	$y = 4/x + 3$	
18	$g(x) = 3x^2$	translation 2 units to the right	$y = 3(x - 2)^2$	
19	$g(x) = 2x^2$	translation 2 units to the left	$y = 2(x + 2)^2$	
20	$g(x) = -4(x+1)^2 - 2$	translation 3 units to the right	$y = -4(x + 4)^2 - 2$	
21	$g(x) = 2(x-4)^2 + 1$	translation 1 units to the left	$y = 2(x - 3)^2 + 1$	
22	$g(x) = 3(x-2)^2 - 3$	vertical translation of 2 & horizontal translation of 1 to the right	$y = 3(x - 3)^2 - 1$	
23	$h(x) = 4/x$	horizontal translation of 1 to the right	$y = 4/(x-1)$	
24	$h(x) = 4/x + 1$	vertical shift down 2 units & horizontal translation of 2 to the left	$y = 4/(x-2) - 1$	
25	$h(x) = 4/x - 2$	vertical shift down 3 units &horizontal translation of 1 to the right	$y = 4/(x - 1) - 5$	
26	$t(x) = x^2 - 3$	reflect at the turning point	$t(x) = -x^2 - 3$	
27	$m(x) = x^2 - 3$	reflect on the x-axis	$m(x) = -x^2 + 3$	
28	$p(x) = 3/x$	reflect along the y = -x line	$p(x) = -3/x$	
29	$r(x) = 3x^2 + 1$	$s(x) = r(x) + 2$	$s(x) = 3x^2 + 3$	

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APPENDIX G

The interview included the following questions:

1. Were you able to answer the questions in Worksheet 1 on day one? If not explain why, if yes explain why?
1. How did you feel whilst engaging with the applets?
2. Did the applets assist your understanding of translation of each of the functions?
3. Do you think you benefited from the use of *GeoGebra* in the activity lesson, or not? How do you support your answer?
4. Did the use of *GeoGebra* facilitate your learning?
5. Were you able to answer the questions in Worksheet 2? How would you compare your feeling after completing worksheet 1 on day one from your completion of worksheet 2 on day 4?
6. Were you able to understand the questions in worksheet 2 despite there being no diagrams?
7. Was there a significant moment in activity lesson that you would like to mention?
8. Would you like your future mathematics lessons to include virtual manipulates of this nature and why?