THE LESOTHO GEODETIC CONTROL NETWORK

MOTLOTLO P. MATELA

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I hereby declare that this thesis is my own work and that it has not been submitted for a degree at any other university.

M. P. Matela

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ABSTRACT

The Geodetic network of Lesotho as established by the Directorate of Overseas Surveys in the 1950s, has been known to have distortions of several meters in some areas. This network is still very much in use today. Several attempts were made to strengthen the DOS network, but these attempts were not used for a complete readjustment.

The South African Control net, which completely surrounds Lesotho, has recently been readjusted so as to bring it into sympathy with the WGS reference system used by GPS. It has become urgent to similarly update the Lesotho control system, to enable economical use of GPS surveying methods. This thesis addresses the problems of updating the Lesotho control system and also of bringing existing data onto the updated system.

This thesis first reviews the historical background of Lesotho and that of its geodetic network.

Different sets of data were collected and common points in the compared sets selected for the analysis. The South African readjustment was chosen as the standard, because it is the most recent, derived with the support of the new zero-order South African control net.

The data sets were fitted to the reference system using conformal transformations from first up to fourth order. These comparisons were used to detect outliers. They revealed systematic distortions in the older data, which could be largely eliminated in the fourth-order transformation. The opportunity to update control point co-ordinates also gave an opportunity to revisit the existing choice of using two map panels of the Gauss Conform projection. The distortions involved in using a single Gauss Conform panel and also the UTM projection were investigated.

A comparison of all the methods and the recommendations concludes the section. Software was developed for transforming existing survey data onto the recommended updated reference system.

The height system used in Lesotho is also reviewed because it forms part of the control net. The focus is on heights in relation to gravity, because that bears on the relation of published orthometric heights, with GPS-derived ellipsoidal heights. This section is mostly a literature review, starting with the theory of heights and gravity, proceeding onto the applied corrections and then showing what relations have been found.

Acknowledgments

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Chapter 1 Situation, history and mapping of Lesotho.

1.1 Introduction

This chapter gives the geography of Lesotho. It looks briefly at the changing borders of Lesotho since the Boers trekked across into Southern Africa. Then it goes on to describe the evolution of the mapping of Lesotho up to the current borders. The present status of geodetic network is presented. The purpose is to create a picture of the location and size of, and features found in Lesotho.

1.2 The Geography of Lesotho

Lesotho is an independent country in Southern Africa. It is completely surrounded by South Africa with no coastline. It lies between latitude 28° 35'S and 30° 40'S and longitude 27° and 29° 30' E (LSPP, 1984). The terrain is mostly highland with mountains, hills and plateaus, as it lies in the Drakensburg range. The highest point is at Thabana-Ntlenyana, 3 482m (Merry, 1990), in the southeast of Mokhotlong district. The lowest point is on the southeastern side of the country at the junction of the Senqu (Orange) and the Makhaleng rivers at 1400m. The total land area is 30 350 square kilometres with a land boundary of 909 km. Only one third of the country area is considered lowlands; this is the Western part with an average width of 40 km.

The population estimate was 2.13m in 1999, with a growth rate of about 2 per cent per annum (Hutcheson, 1999). There is a higher concentration of people in the lowlands than in the highlands, perhaps because the western side, (the lowlands) is more developed than the mountainous side.

The origin of Lesotho as an identifiable country arose during the colonial rule of the British Empire in the 18th century when the first recognised king of Basotho, Moshoeshoe I, realised that his nation and land were about to be conquered by the Boer settlers. The Boers stripped him most of his arable land and annexed it to the Orange Free State, and Moshoeshoe was left with the land then called Basutoland and now Lesotho. According to Eldredge (1993), the following maps outline some of the captured, bought or agreed upon land pieces and the remaining portion:

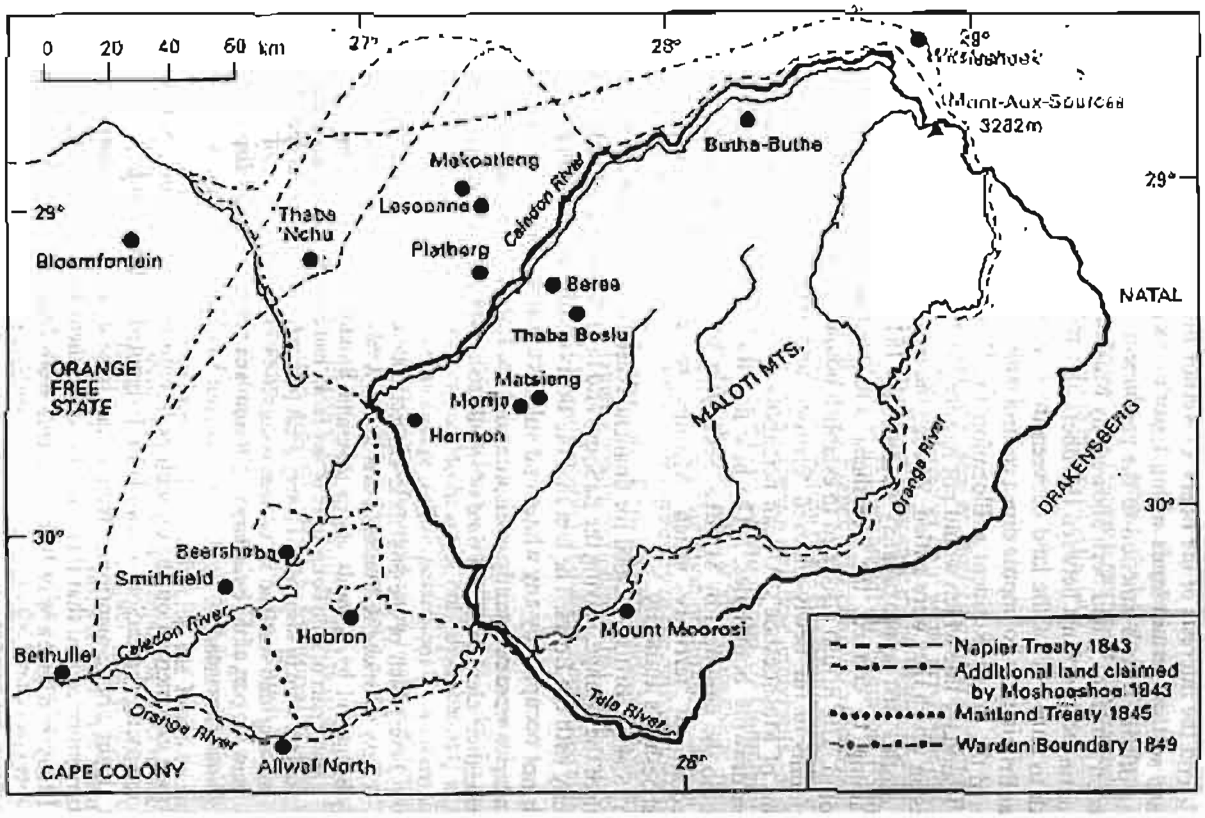


Figure 1.1 The changing borders of Lesotho, 1843 – 1849
(Eldredge, 1993)

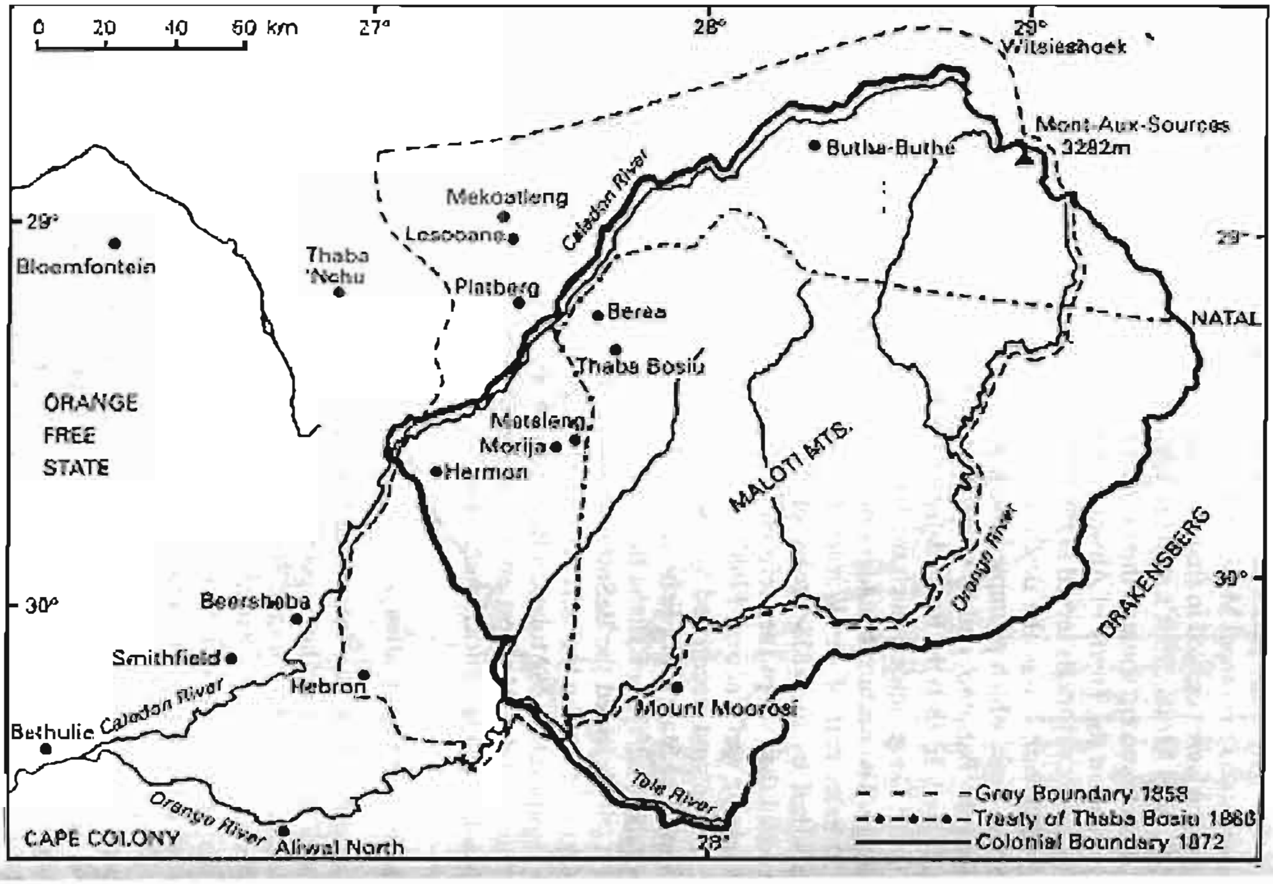


Figure 1.2 **The changing borders of Lesotho, 1858 – 1872**
(Eldredge, 1993)
(The scale on the above maps is about 1:2 857 143)

Ellenberger (1969) records the travels of the Basuto, their settlements in Bechuanaland and their migration and split towards the South until they settled at Ntsuanatsatsi, where most of them thought they originated from. This Ntsuanatsatsi, according to Ellenberger is "situated midway between the towns of Frankfort and Vrede in the Orange Free State", Ellenberger (1969, p18). Most of these places have Sesotho names, which are becoming more and more obsolete.

All the same, the present border of Lesotho is that of the colonial boundary of 1872, after swaying back and forth as dictated by the settlers then and the subsequent colonial rulers.

Note: The names of most places in most maps do not conform to the local use. For instance, Botha-Bothe is Butha-Buthe, Thaba-Putsoa is Thaba-Putsua, etc, but the wrong names were adopted for ease of reference. A compatible usage of place names is that of Thompson (1975).

1.3 The first maps of Lesotho

The earliest attempts to map Lesotho were carried out by the missionaries, Casalis and Arbousset, as is stated by Tylden, "The two earliest maps of Basutoland are the one by Casalis in the 'Livre d'Or' and another by Arbousset, both on a scale of twenty-five French leagues to one degree", Tylden, (1950, p.247). Different scales of Lesotho maps evolved over the years from about 1870 to 1900s, mostly unsatisfactory, until "in 1906, when the map of 12 miles to 1 inch (1:760320) was published by the surveyor-general of the Orange River Colony" (Tylden, 1950, p.248). Smith (1995) claimed that Lesotho was first mapped after the introduction of the British rule after 1884. The following paragraph summarises some comments by Smith (1995).

From 1904 to 1909, Captain Montague Charles Dobson carried out a reconnaissance survey of Lesotho, which resulted in the production of 1:250 000 map sheet GSGS2567 in 1911. In 1939 to 1940, Ronald S. Webb's work during World War II took him to the borders of Lesotho. While there, he revised and updated the 1:250 000 sheets. Two copies were passed onto the director of Trigonometric surveys in Mowbray in June 1945, and in September 1945, a copy was sent to the government secretary in Maseru, at the request of His Honour, the Resident Commissioner of Basutoland. In 1948, the Trigonometric Survey in Mowbray, Cape Town, published a 1:500 000 series covering the whole of Basutoland. Most names were not edited. Tylden (1950), confirms that Dobson's map of 1911 was still the standard of the country and was revised by R. S. Webb.

1.4 Creation of permanent control points

Several organisations came to Lesotho to establish some form of permanent geodetic control in the country. These are in chronological order: South African Trig. Surveys, DOS, DOPPLER Specialist Team of Royal Engineers (512 STRE), Lesotho Highlands Water Project, and Merminod, the last two operating almost at the same time. Finally, the S.A. Directorate of Control Surveys refixed some GPS points in their zero–order net and issued revised positions of many control points. The South African trigonometric and later the S.A. GPS networks around Lesotho were held fixed for the DOS and Merminod's adjustments.

1.4.1 First control points

The first permanent control monuments in Lesotho were erected in the course of the extension of the geodetic control of South Africa in the late 1800s to early 1900s, most notably under the supervision of Sir David Gill, into the Orange Free State and Transvaal. One geodetic chain went into the Western part of Lesotho, comprising these points: Dikhoele (BS29), Thaba Tsueu (BP25), Thaba Putsua (BP22), Machache (BP15) and Nthodimonate (BP36) as shown on figure 1.3. The closest base for these chains is in Wepener, on the western side. It is this South African control, in and around Lesotho, which DOS used to extend and form their first triangulation covering Lesotho.

1.4.2 DOS control points

From 1950 to 1955, the British Directorate of Overseas Surveys (DOS) fixed ground control for aerial photo cover. This control was based on the South African coordinates near the northwestern border in the anticipation that there would be more developments in the lowlands than in the southeastern, highland area. Windsor (1980) wrote that the accepted version of the Lesotho primary adjustment held fixed the South African points on the West and South of Lesotho, where they would be most useful for the future use. Windsor's letter is in appendix (1).

There was a choice of creating a new origin, but DOS chose not to, probably because it was both easier to connect to the already existing control, and this use also ensured the two systems would be compatible. The fact that there is not a proper fit of the geodetic network of Lesotho onto the South African network is one of the factors that inspired this research.

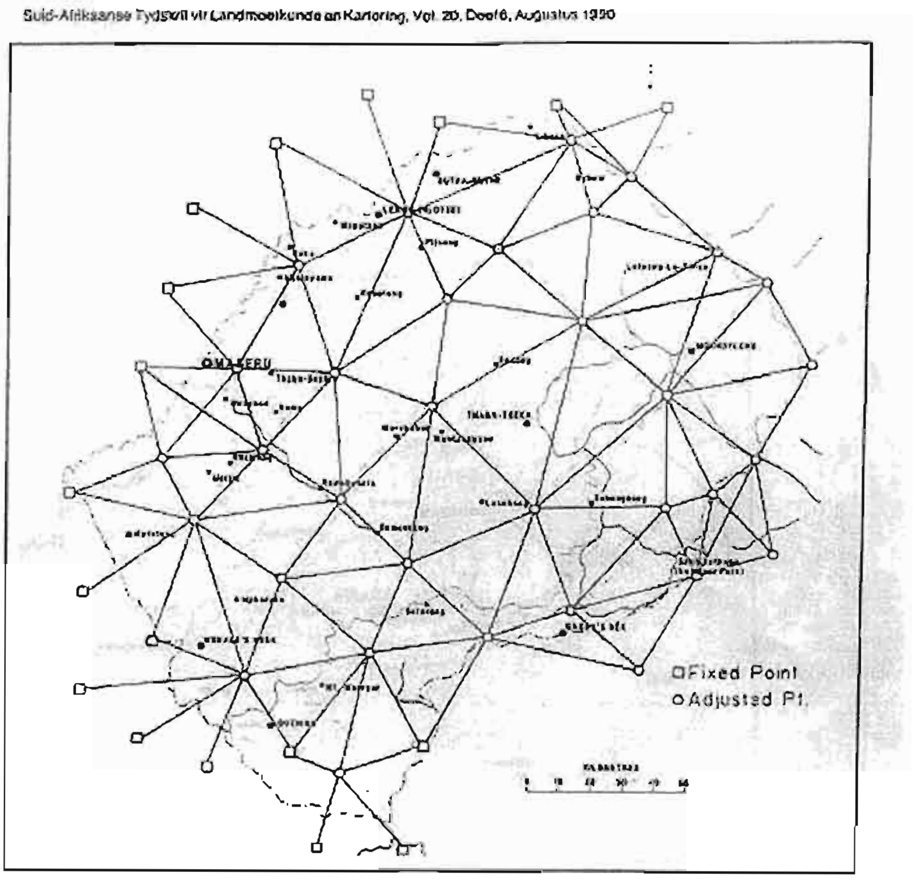


Figure 1.1 DOS Trigonometrical Network and the fixed S.A. Frame
(Merminod 1990)

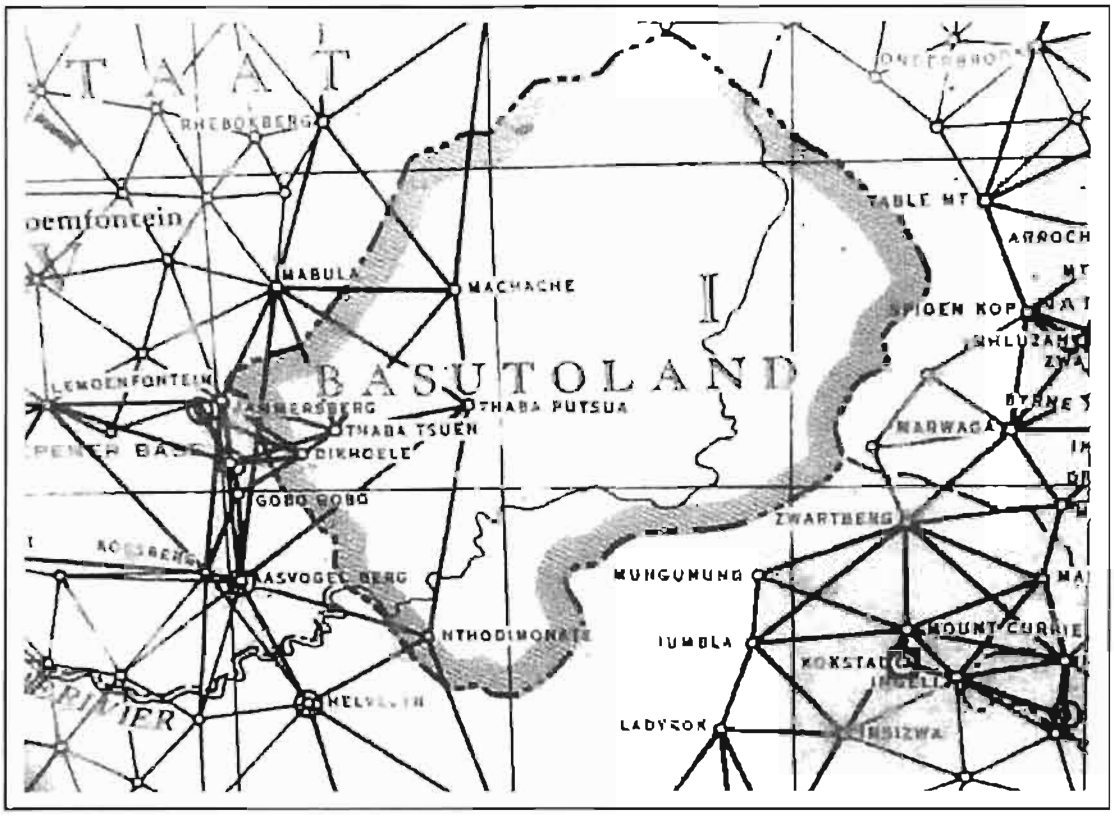


Figure 1.4 South African Control in and around Lesotho
(Trigsurvey, 1950)

1.4.3 Doppler points

In 1979, 5 existing DOS points were used by the DOPPLER 512 STRE (Specialist Team Royal Engineers) to coordinate them to other Doppler stations around the African continent under the African Doppler Survey Project (ADOS), which ended in 1986. This exercise was not very successful as these points were said to have contributed very little towards strengthening of the ADOS project. Ezeigbo says of the ADOS project, "It is pertinent to note that long after the project was completed, neither the datum nor a unified geodetic network for the continent has been realised. Instead, isolated efforts have been devoted to the determination of either the geoid or datums in various countries in Africa based on ADOS data", (Ezeigbo, 1994, p.385). In Lesotho, ADOS data has been lying idle all these years, and unfortunately, GPS which is considered more accurate than the Doppler survey method has overtaken it. Evaluation of the formation of African Reference Frame (AFREF) is underway, however, and this will probably achieve better realisation of the continental reference frame than the ADOS. Interest in and motivation for establishing a continental reference frame for Africa (AFREF) has been shown by Neilan and Boucher (2001), for instance (full text in appendix (1)). The African Organisation of Cartography and Remote Sensing (AOCRS) has also supported the development of a common continental geodetic infrastructure for Africa as Muftah (2001) points out (see Appendix (1)).

1.4.4 Merminod GPS points

In 1986, a geodetic section was introduced into the map production unit of the Land, Surveys and Physical Planning (LSPP). This integration was motivated principally by two reasons, namely:

- To examine fully and possibly correct the noticeable distortions of several metres in the entire Lesotho Geodetic Network;
- Further to replace and establish accurate and suitable higher order ground control points for photogrammetry, mapping, as well as for other engineering and geodetic works.

The governments of Lesotho and Switzerland entered into an agreement to help Lesotho with the technical aspects of geodetic issues. As a result, a Swiss geodesist, Mr. Bertrand Merminod, came to Lesotho to assess the situation and provide the best solution possible. In 1990-91, 34 new GPS points were erected and observed. Some 27 points of the existing DOS pillars were incorporated into the adjustment. The new points were built next to roads or airstrips for easy accessibility. The following diagram shows the location of these points.

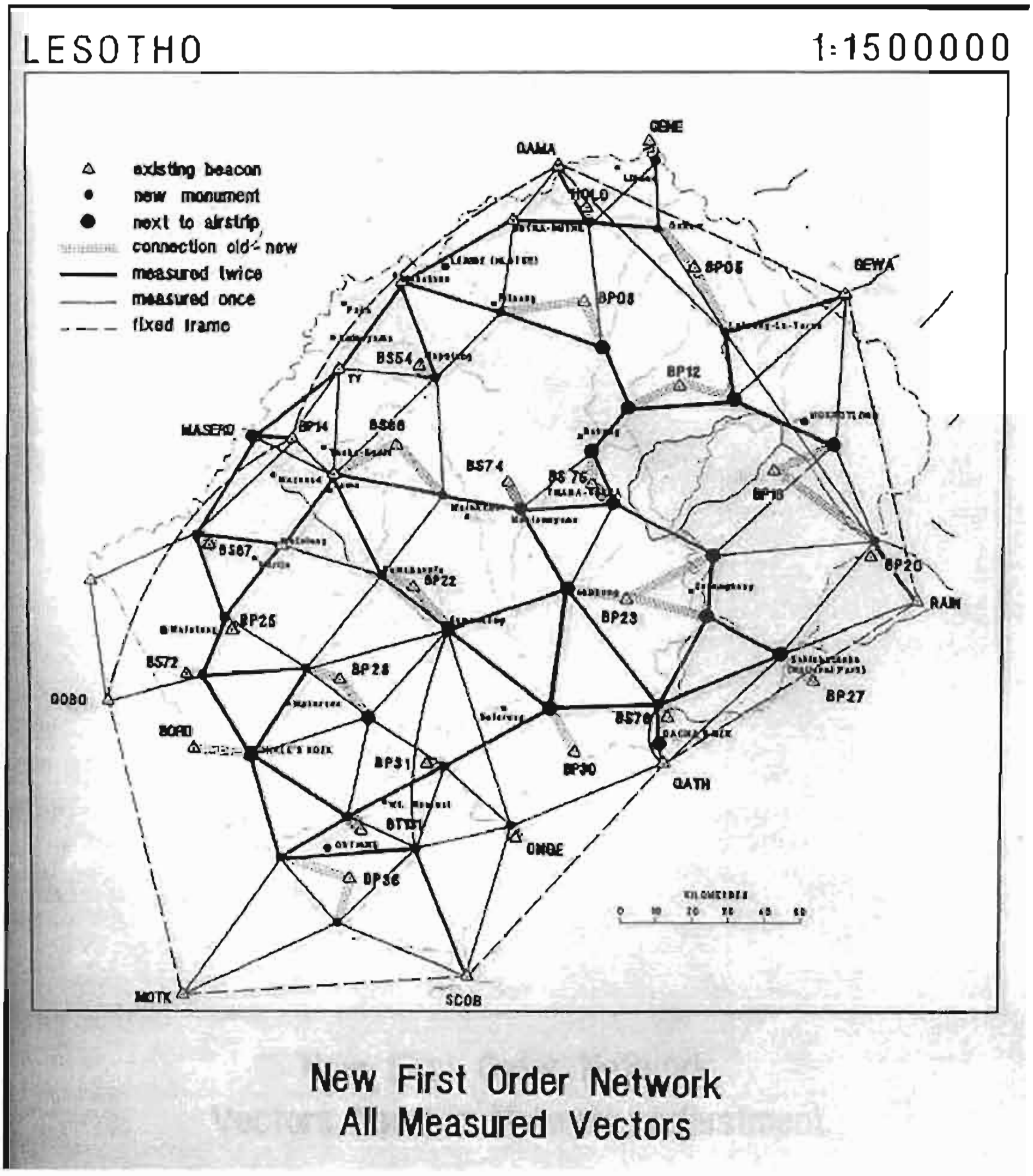


Figure 1.5 Merminod's GPS points in Lesotho

(Merminod, 1993)

1.4.5 Lesotho South African Set (South African coordinates)

South Africa horizontal control net was readjusted, starting in May 1991. According to Newling, GPS receivers were used "to strengthen the isolated areas where horizontal network is known to be poor and to provide an overall 'zero order' GPS net into which to adjust conventional triangulation observations" (Newling, 1993, p153). This network was to have station separation of about 100 kilometres, provided sufficient accuracy could be achieved over that range. A total of 205 points were fixed and these included a few points in Lesotho, Swaziland and Namibia. As Merminod (1993) pointed out, two points in Lesotho were to be included in the South African observation schedule to ensure that Lesotho would not form a gap and also prevent problems when coordinates were merged. Thus we have DOS list, Merminod's list and now the South African list as a third list, the last list having been compiled from the same terrestrial reference frame as the DOS one.

1.4.6 LHWP Coordinates list

As pointed out earlier, the control was better and denser at the north and western side of Lesotho where it was thought there would be more developments than elsewhere. From the 1950s, however, it had been foreseen that there was the possibility of a major engineering construction works in the mountains for the Lesotho Highlands Water Project (LHWP).

Because of discrepancies in the existing control network, the authorities found it necessary to hire consultants to establish a suitable control for the project sites. In addition to DOS monuments, 23 new monuments, (and 8 more added later) following the South African beacon building specifications, were constructed. Another list of coordinates was produced.

1.5 Conclusions

In this chapter, the geographical position of Lesotho has been introduced. The history of how Lesotho was mapped and the problems relating to mapping in Lesotho have been discussed. Then the current situation regarding the geodetic network has briefly been discussed. In the next chapter, geodetic control network will be discussed in more detail.

Chapter 2 Control Network

2.1 Introduction

The National Geodetic Survey defines a Control Network as "a geodetic control (a set of control stations established by geodetic methods) together with the measured or adjusted values of the distances, angles, directions or heights used in determining the coordinates of the control". The National Geodetic Survey (1986 p.42). In other words, a control network is a way of realising a reference system, using a set of rules and measurements to establish a set of coordinates associated with monumented points, i.e. a coordinate system.

The importance of control network is seen in many fields. Although traditionally, the aim of control networks was mapping, they have been very useful in engineering projects, space research, hydrography, environmental management, boundary demarcation, etc. In mapping, a good control network allows surveys to be related to each other, so that the coordinates of existing points can be used to recover the lost points in the same coordinate system. Then new surveys can be considered as extensions of older surveys which use the same system. Important theoretical concepts associated with control systems are: Reference ellipsoid, Geoid, Coordinate system. These will be discussed separately below.

2.2 Geodetic Datum

The National Geodetic Survey (1986), page 53 defines a datum as "any quantity or set of such quantities that may serve as a referent or basis for calculation of other quantities". According to Dana (1999), geodetic datums define the size and shape of the earth and the origin and orientation of the coordinate systems used to map the earth.

Although classic geodetic measurements between control points are taken on the highly irregular earth's surface, the calculations used to represent their position are carried out on a surface suitable for mathematical calculations. Thomson says

> Due to the natural shape of the physical earth, as represented by the geoid, the geodetic datum used for horizontal coordinates is a biaxial ellipsoid whose size and shape are given by the lengths of its semi-major and semi-minor axes, a and b respectively, or its semi-major axis and flattening f. The size, shape, and position of the reference ellipsoid is usually chosen such that it is 'best fitting' over the area of interest (Thomson 1980, p.8).

Modern satellite based calculation methods, however, compute networks directly in 3-D space. These positions may be referred to an ellipsoid reference surface which best approximates the geoid at the area of interest (ellipsoid of the best fit to the geoid in the area). It is defined by two constants, (figure 2.1) namely the semi-major axis a, the semi-minor axis b, or one of the constants and the flattening f and/or the eccentricity e where:

$$f = \frac{a-b}{a}$$

$$e^2 = \frac{a^2 - b^2}{a^2}$$

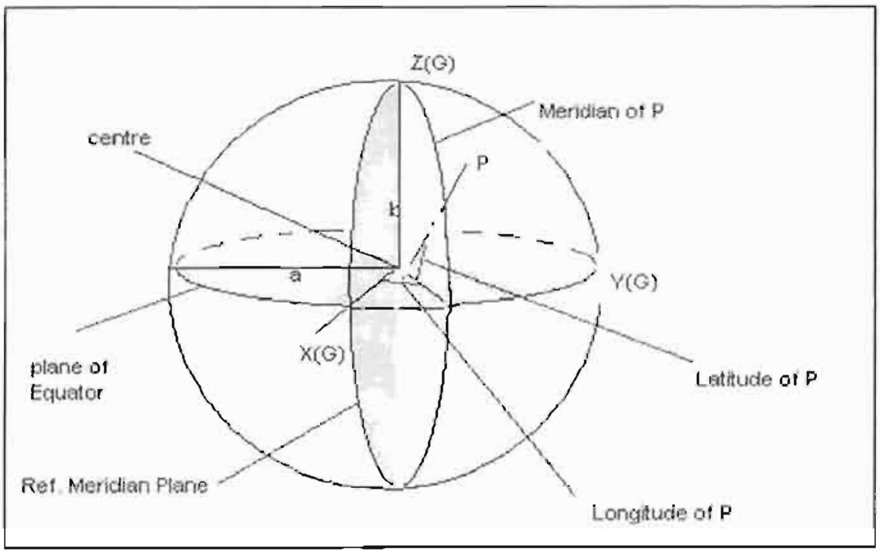


Figure 2.1 **Reference Ellipsoid**

It is because of this fit that different shapes and sizes of ellipsoids are in use in different countries. Broadly, however, ellipsoids can be global or regional. According to Thomson,

> A horizontal geodetic datum (reference ellipsoid, geodetic coordinate system) is positioned and oriented with respect to some physical properties of the earth by relating it to the Average Terrestrial coordinate system. The conditions to be fulfilled are, when dealing with a reference ellipsoid, that:
>
> (i) the equatorial plane be parallel to the earth's mean equatorial plane,
>
> (ii) the rotation (minor) axis be parallel to the earth's mean rotation axis, and
>
> (iii) the longitude reference plane be parallel to that of the Greenwich Mean Meridian plane (Thomson, 1980, p.8).

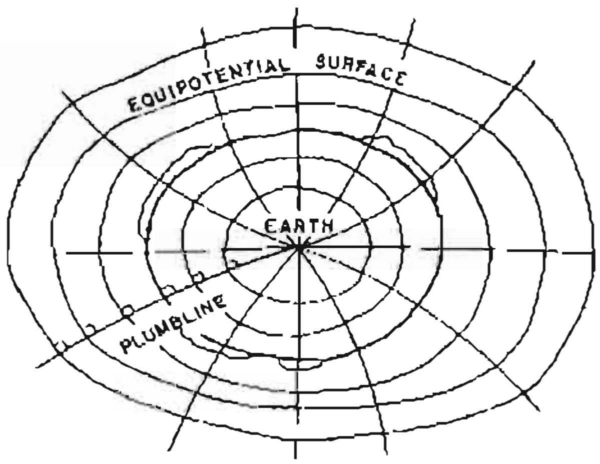
The centre of this ellipsoid can be positioned such that either it lies at the centre of mass of the earth i.e. geocentric or not at the centre, i.e. non-geocentric. In the latter case a translation vector between the two centres can be used to relate the ellipsoids.

There may also be some rotations and scale differences to be corrected for. An example of global ellipsoid is the World Geodetic System 1984 (WGS84).

Regional ellipsoids are many, each probably according to the latest model of the time or one favoured by surveyor of the time, as then, there were no methods of determining the geocentre accurately. For instance Great Britain at one time used Airy 1830 (Ordnance Survey, 1999) while South Africa and Lesotho use modified Clarke 1880 ellipsoid. In South Africa, as shown in Gill's Geodetic Report (1895) Maclear's[1] geodetic results were computed with both Airy's and Clarke's Elements of the Earth. Two more arcs (Kimberly and Natal Meridian Arcs) were also computed using the two ellipsoids. The comparison made suggested that the best representation of Maclear's arc was arrived at using Clarke's elements when 3 stations were omitted from the calculations. The Kimberly and Natal arcs were slightly in favour of Airy's elements. In the end, however, Clarke's arc was favoured and chosen for the South African ellipsoid. It seems that before the WGS84 era, once a country chose its ellipsoid, it would find it difficult to change to a new datum, which might have been better than the one used by that country whereas nowadays most countries are adopting the global WGS84 as a new datum. Whether global or regional, every control point and feature to be mapped is projected onto the ellipsoid and then onto the desired map.

2.3 Geoid

In classical surveying, the reference system for positioning is split into a 2 dimensional (latitude, longitude) horizontal and a one dimensional (height) vertical component. The horizontal datum is the ellipsoid. The elevation model is based on the geoid, which, according to Bomford (1971 page 109), is defined as "a surface conciding with mean sea level in the oceans, and lying under the land at the level to which the sea would reach if admitted by small frictionless channels. More precisely, it is that equipotential surface on the earth's attraction and rotation which, on average, concides with mean sea-level in the open ocean". The geoid is perpendicular to the



Equipotential surfaces and Plumblines

plumb line everywhere and is one of an infinite family of equipotential surfaces in space (figure 2.2).

Figure 2.2 Equipotential surfaces and plumblines
(Thomson, 1980)

2.4 The classical method for creating a geodetic control net

Before the advent of global 3-dimensional networks, regional nets were created by starting at one origin point whose position was measured astronomically, and then the

[1] Sir Thomas Maclear (1794 –1879) came to South Africa and carried out the verification and extension

azimuth was taken to another nearby point in the network for orienting the system. The ellipsoidal computing surface was normally assumed to coincide with geoid at the origin and the geoid position was sampled using tide gauges. For South Africa, the formal statement of origin is given by (Gill, 1895, p.148).

> the origin of the system of geodetic and astronomical latitudes, longitudes and azimuths is the station Buffelsfontein. The bearing of Zuurberg from Buffelsfontein was adopted from a preliminary comparison of the geodetic and astronomical results, rejecting the stations at which the astronomical results were most probably affected by considerable deviation of the plumb-line." He explains that the latitude of Buffelsfontein was adopted in a similar manner whereas the geodetic longitude was based on the telegraphic determination of the longitude of the Cape Transit Circle. The adopted values are:
>
> Bearing of Zuurberg from Buffelsfontein 183 58 15.000
>
> Log. Length of line Buffelsfontein-Zuurberg (in feet) [5.4332521]
>
> Latitude of Buffelsfontein 33 59 32.000
>
> Longitude of Buffelsfontein 25 30 44.622
>
> Elements of the earth from Clarke's Geodesy, p.319, a = 20926202 feet
>
> e = 20854895 feet"

2.5 The coordinate system

Co-ordinates of control points are published usually in one or more of the following forms:

1. 3-dimensional Cartesian co-ordinates (x,y,z), where points have been fixed by satellite methods;
2. 3-dimensional geographical coordinates (latitude, longitude, ellipsoidal height) if fixed by satellite methods or (latitude, longitude, orthometric height) if fixed by classical methods. This system, according to Bomford (1980), is the one most used in geodetic computations;
3. Map coordinates, e.g. Gauss Conform, which are preferred mostly in small size surveys such as in engineering, cadastral and in military surveys.

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of Abbe de Lacailles's meridian arc of 1840 – 1848.

2.6 Global co-ordinate reference systems – WGS84

Recently, the coordinate system mostly used is that of the world geodetic system 1984 (WGS84). This is defined so that its Cartesian axes and ellipsoid are geocentric. The orientation of the axes is such that the x-axis coincides with the intersection of the prime meridian and the equator, the z-axis coincides with the mean polar axis of rotation of the earth and the y-axis lies in the plane of the equator so as to form a right-handed system. WGS84 is realised through the ephemerides broadcast by GPS satellites. The GPS system uses 24 earth-orbiting artificial satellites as distant reference points from which positions on the surface of the earth can be calculated. Because of its easy availability and the high accuracy measurements it can attain, GPS is very useful as a geodetic surveying tool. (Thomson (1976)). More clearly, Ordnance Survey (1999) says that "the WGS84 definition includes the following items:

- The WGS84 cartesian axes and ellipsoid are geocentric; that is their origin is the centre of mass of the whole earth including oceans and atmosphere.
- The scale of the axes is that of the local earth frame, in the sense of relativistic theory of gravitation.
- The orientation (that is the direction of the axes, and hence the orientation of the ellipsoid equator and prime meridian of zero longitude) coincides with the equator and prime meridian of Bureau Internationale de l'Heure at the moment in time 1984.0 (that is at midnigh;t on New Year's Eve 1983).
- Since 1984.0, the orientation of the axes and ellipsoid has changed such that the average motion of the crustal plates relative to the ellipsoid is ellipsoid is zero. This ensures that the Z-axis of the WGS84 datum coincides with the International Reference Pole, and that the prime meridian of the ellipsoid (that is the plane containing the Z and X cartesian axes) coincide with the International Reference Meridian.
- The shape and size of the WGS84 biaxial ellipsoid is defined by the semi-major axis length a=6378137.0 metres and the reciprocal of flattening 1/f=298.257223563. This ellipsoid is the same shape and size as the GRS80 (Geodetic Reference System 1980).
- Conventional values are also adopted for the standard angular velocity and for the Earth gravitational constant. The first is for the time

measurement and the second to define the scale of the system in a relativistic sense".

Since South Africa participates in the VLBI (Very Long Baseline Interferometry) programmes through the HartRAO (Hartebeesthoek Radio Astronomy Observatory), the South African GPS network was tied on to this HartRAO point and therefore related to other VLBI points worldwide. Newling (1993) points out that the position of the Hartebeesthoek station has been fixed with respect to the ITRF91 (International Terrestrial Reference Frame 1991) system. WGS84 and ITRF91 are not the same, but the difference is less than one metre and therefore it was recommended that untransformed ITRF91 co-ordinates of HartRAO should be used to constrain the South African GPS network. This means that the South African network points coordinates are in ITRF91. (Newling (1993))

2.7 The future of Control Networks

The past events have suggested that control networks have been changing in one way or another as influenced by development in technology: from intervisible stations of highly densified visible monuments, to a situation of distant, but convenient GPS points which need no intervisibility between stations. The field survey parties have declined from at least four to a party of two people.

Recently , the concept of control networks has been extended to incorporate systems in which fixed active GPS receivers kept at known stations continually collect GPS satellite data and broadcast corrections, thereby behaving as the base stations for any roving receiver within a specified radius, say a 100 km.. According to Merry and

D'arcy-Evans. "These data collected can be used to compute precise satellite ephemerides, to predict range corrections for differential pseudo-ranging, to verify the integrity of the GPS system and to provide precise control without the requirement of the user to occupy the control stations", (Merry and D'arcy-Evans, 1991, p.136). Unlike using the passive control points where a surveyor would have to buy at least two or more receivers, one for the base station and the other the rover, the active GPS system allows a surveyor to work effectively using only one roving receiver. The service has to be paid for by the user (much like a prepaid television channel service). The implication is that the cost of surveying will be cheaper because a one-man team can carry out a successful survey with a single GPS receiver. There is no need to occupy one or more distant control points at the start of a survey.

In South Africa, a country-wide array of active GPS base stations is being established. This, it is hoped, will eventually replace the current passive network of highly visible survey monuments and marks. This network known as TrigNet will consist of 37 active base stations spaced at about 200 kilometres, and will provide both post-processing data and real time correction services.

Five of these base stations, namely 19 (Aliwal North), 22 (Bloemfontein), 24 (Bethlehem), 36 (Ladysmith) and 34 (Pietermaritzburg) will be of some importatnce to Lesotho if Lesotho is to consider this type of service (figure 2.3). The problem is whether Lesotho is ready for this type of change or not. As a matter of fact, its use cannot be delayed or avoided. Figure 2.4. give an idea of what an active control station would require to operate.

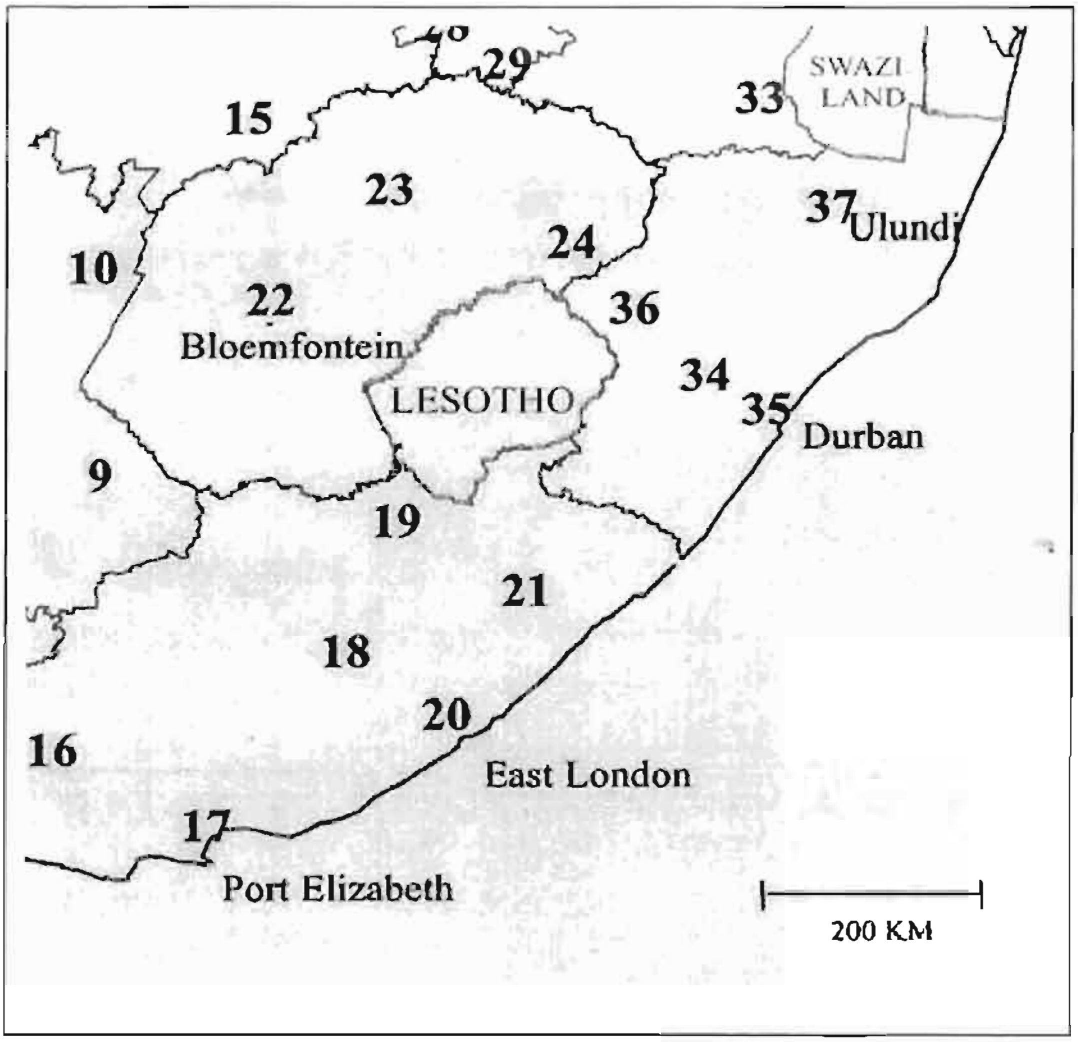

Figure 2.3 TrigNet Stations distribution around Lesotho
(Chief Directorate: Surveying and Mapping, Mowbray, Cape Town)

In South Africa, the question is whether the old control monuments are still needed, when the whole country has been covered by TRIGNET. Personally, I believe that this type of change will be gradual, as it would take time for most surveyors to acquire GPS equipment, or to use it for all surveys. Newling (1993) remarks that the Directorate of Control Surveys still has a responsibility towards surveyors who use traditional methods and do not intend to abandon the indigenous system of surveying for a few GPS control points. Although this remark related to the passive GPS control network, it would apply as well to the Active GPS control. The same situation applies to Lesotho. The conventional terrestrial surveys will still be carried out for a long time to come, so the beacons will still be used. What needs to be done is to update the existing control network, or establish the relation between the existing control and WGS system, so as not to lose the value of existing maps and cadastral data. A way has to be devised of bringing the old data onto the new system, such as calculating the transformation parametres for each, say, 15' by 15' grid square and applying these corrections to the old values to make them comply with the new values.

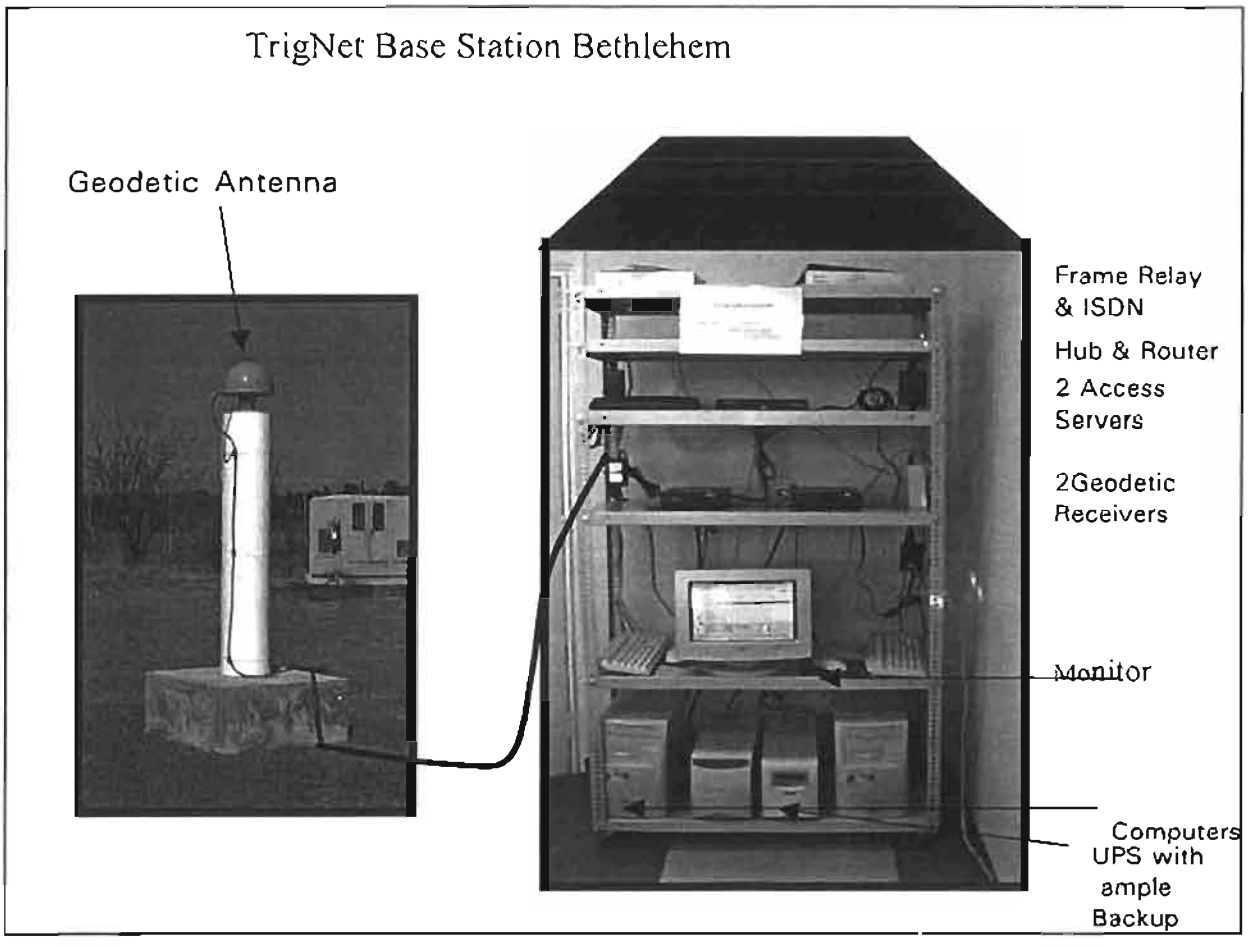

Figure 2.4 Required equipment for an Active Control Station
(.Hedling G., Parker A., Wonnacott R., 2000)

As a way of contributing towards establishment of global geodetic network, South Africa participates in the IGS (International GPS Service) through its HartRAO station operation (a regional data centre) together with three other stations known as operational data centres, situated at Sutherland, Richardsbay and Simonstown. (IGS Overview, (1998), Combrinck (1999)). One of the objectives of IGS is to densify the international nets using various methods including VLBI, SLR,, etc in which many organisations around the globe contribute by collecting data at their operational data centres, pass it on to their regional data centres, and then link with the global data centres.

2.8 Conclusions

In this chapter, an attempt has been made of defining a traditional and modern control network. Whereas classical way of surveying was very laborious and time consuming, modern systems are mostly automated, and comparatively quick in both collecting and analysing data. With technology changing so rapidly, prices of GPS equipment keep going down, and it is expected that some survey firms may want to have their own active base stations within their area of work.

Chapter 3 Problems in the Lesotho geodetic Network

3.1 Introduction

It has been shown in chapter 1 that there exist different sets of coordinates for the same points in Lesotho. This chapter elaborates on this by discussing problems associated with each set of coordinates.

3.2 Datum and Projection of Map coordinates used

The datum used in Lesotho is the Cape Datum, which is that used in South Africa, with its reference point (point of origin) at Buffelsfontein, near Port Elizabeth. The National Geodetic Survey (1986), describes this Cape Datum as defined by the following location and azimuth on the Clarke spheroid of 1880; the origin is at Buffelsfontein:

Longitude 25° 30′ 44″. 622E
Latitude 33° 59′ 32″. 000S
Azimuth from origin to Zuurberg:
(Clockwise from south) 184° 15′ 26″. 311.
(The bearing, as shown in Chapter 1, from Gill's report, is 183° 58′ 15″.000).

The Cape datum Clarke 1880 ellipsoid parameters are defined differently by DOS and the South African authorities:

	a	1/f	e	b
DOS	6378249.145		0.00680348102	6356514.870
S. A.	6378249.145326	293.466307656	0.006803481018843	6356514.966721

Note: DOS – Parameters used in Lesotho by the Directorate of Overseas Surveys (DOS) in the Report on DOS Computing Section work undertaken from 1964 – 80.

S. A. – Parameters mentioned in other texts (e.g. Merry & Rens 1989, Schrieber (1955). Wolfrum (1976), gives the constants of the Clarke 1880 spheroid to be:

a = 6 378 249.145 3621

b = 6 356 514.966 721

e^2 = 0.00680 34810 18843

f = 1/293.466 307 656

Even these values differ slightly from those found in the Projection Tables, South Africa Belts, e^2 being 0.00680 35112 82850 - (Author Unknown, LSPP archives).

Lesotho uses the Transverse Mercator projection (TM) using odd numbered central longitudes, as is done in South Africa. The country is covered by two map panels using Longitude 27 and Longitude 29 as central meridians. This topic will be dealt with in more detail under the investigation into the projections in chapter 8.

3.3 The Problem: coordinates in circulation in Lesotho

There are different sets of data for the same points in circulation in Lesotho:

- DOS list (published as geographical and map coordinates)
- Lesotho GPS list (Merminod's) (published as geographicals)
- South African Adjusted Lesotho list (South African) (Map coordinates)

- Lesotho Highlands Water Project list (LHWP) (Map coordinates)

The above also form the source of all data used in this research.

3.3.1 DOS coordinates list

As noted in Chapter 2, the then Directorate of Colonial Surveys, now the Directorate of Overseas Surveys (DOS) which is a branch of the Ordnance Survey, with its head office in Southampton, U.K., established the first trigonometric stations in Lesotho in the early 1950s. These formed the primary network which was adjusted to the South African control. From the 1984 correspondence between D. P. M. Rousseau of the Trigonometrical Surveys and Ordnance Surveys and Lands, Surveys and Physical Planning (LSPP), the adjustment was dealt with during 1960 – 64. In this correspondence, it is said "it appears that the 10 – 15 ft 'discordancy' (Furmston, Nov 1964) has largely been eliminated. ...It was also found that the DOS team had incorrectly observed to a tertiary station Mungunung Aux instead of the geodetic station (32) Mungunung. " Rogers notes that,

> The first primary adjustment held only 3 South African points fixed, with a standard deviation of angle residuals of less than 1". Primary II held all 28 surrounding South African points fixed and gave a standard deviation of angle residuals of 2". 5 which is unacceptable for further breakdown. This adjustment suggested major systematic errors in one or both nets. A compromise adjustment between the internal consistency and proper fit into the South African Network using 18 South African points was done (figure 3.1). These points were on the west and south of Lesotho, where they were likely to be most useful for future use. It is clear that the discrepancies found on the east and north side had been forced to settle there. (Rogers, 1978 full text in appendix (1))

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[1] This is the way Wolfrum gives it but I suspect the last two digits have mistakenly been interchanged

Suid-Afrikaanse Tydskrif vir Landmeetkunde en Kartering, Vol. 20, Deel 6, Augustus 1990

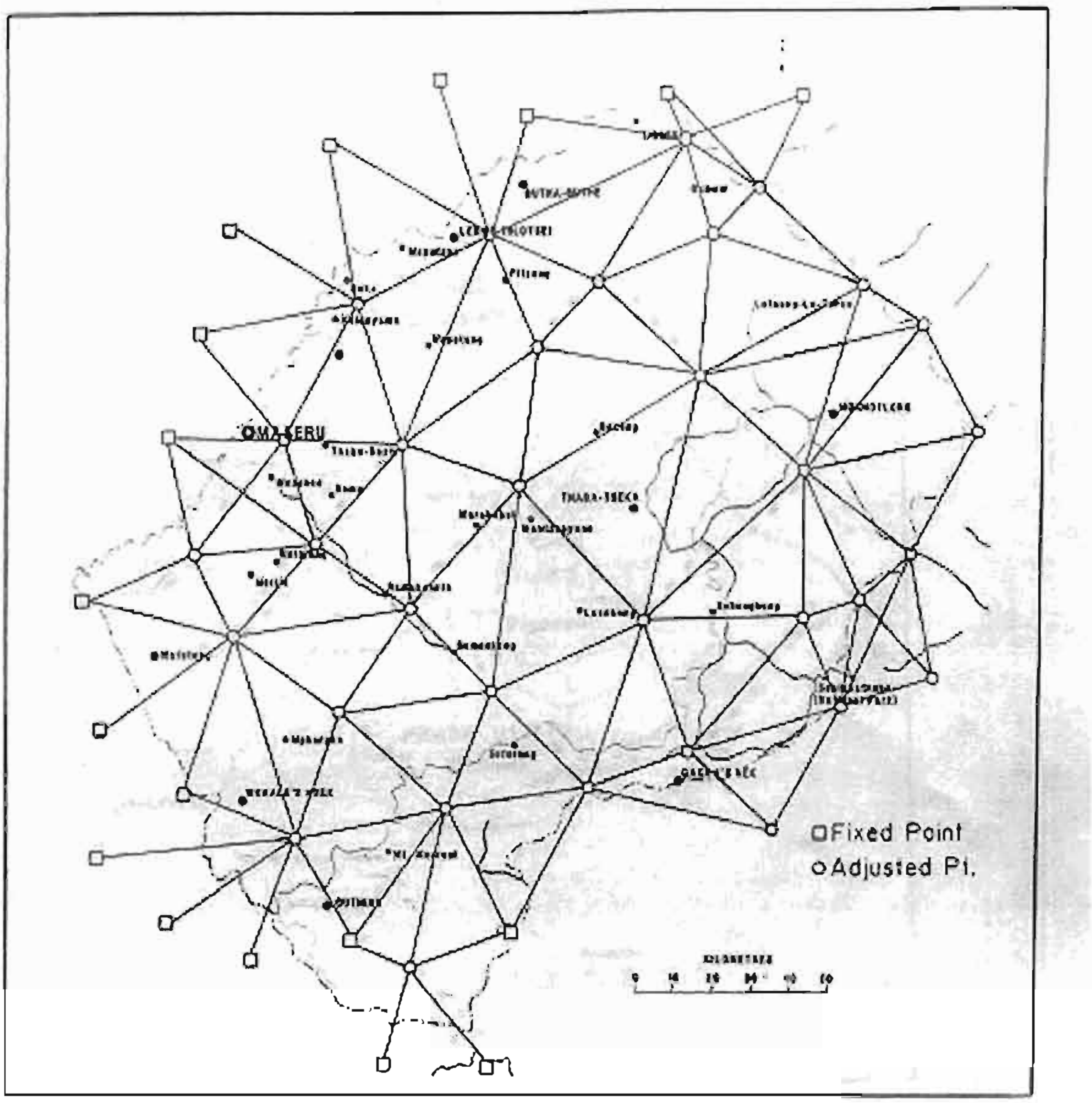


Figure 3.1 DOS Primary Triangulation holding 18 South African points fixed
(Merminod, 1990)

In 1973, the secondary triangulation (from Basotho Secondary (BS) 1 – 59) was adjusted to fit the primary network (DOS computing section, 1981). The associated computations were completed in 1980. The coordinates produced have been treated as the official coordinate set. It has been suspected by Lesotho surveyors that there still existed some distortions in the network, especially in the south-eastern part of the country. The physical condition of the monuments has also deteriorated owing to various factors. These pillars, most of which are on mountains tops, form a terrestrial reference frame comprising of 32 Basotho Primary (BP), 65 Basotho Secondary (BS) and 81 Basotho tertiary (BT) beacons and serve a major purpose as a basis for surveying and mapping and other engineering construction jobs. Further points of lower order are referred to as reference marks (RM), or town survey marks.

DOS Station Description

The monument consists of a 2.4m total height beacon, having a cylindrical concrete pillar of about 1.2m with a pipe 1.0m surmounted by 4 flaps. This pillar is built on a $1m^2$ concrete pedestal showing about 0.2m above ground level. The name of the pillar, the date of construction and the organisation (DOS) in most cases is marked on top of the pillar (figure 3.2). (Diagrams not to scale)

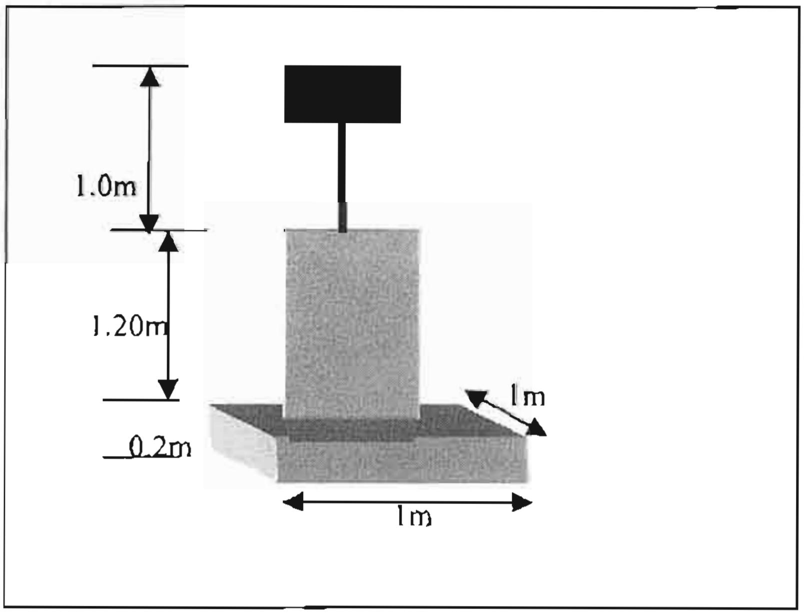


Figure 3.2(a) **A Trig. Pillar**

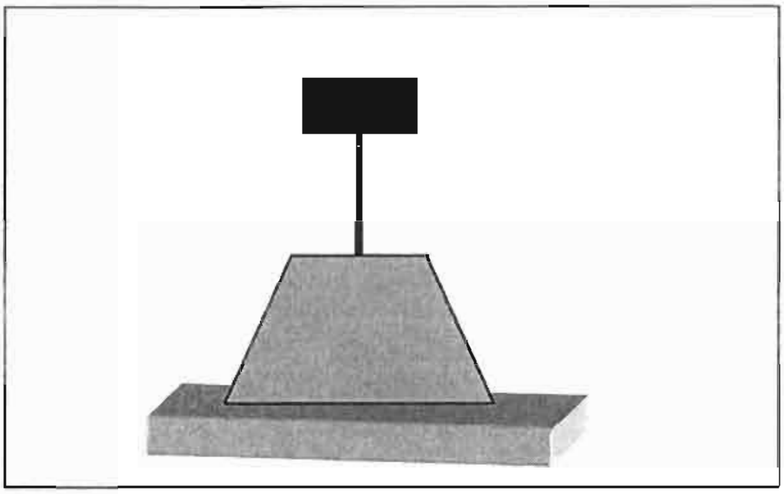


Figure 3.2(b) A Reference Mark

Normally, both types of beacons are painted white, even on a white background, while the vanes are black.

The DOS coordinates have been regarded as the official set and have been used as the control for almost all survey activities by both the government and private surveyors in Lesotho.

3.3.2 Lesotho GPS (Merminod's coordinates)

The establishment of GPS control in Lesotho produced another set of coordinates. Provisional coordinates given in the first stage of GPS survey were derived from single connections to old DOS beacons in the vicinity of new beacons. Merminod (1992), shows the 28 DOS beacons used to connect the new beacons to the old. The integration into the South African Network involved observing simultaneously at some agreed points with the South African surveying party since the SA GPS survey and the Lesotho GPS survey occurred at the same time. This involved 2 points (at Semonkong and Seshote) near the centre of Lesotho, which were part of the fixed frame (zero order) for Lesotho. There are 8 points in the fixed frame, 5 of which are South African trigonometric stations. These exclude the two points named above. Final coordinates were fixed holding the zero-order network as fixed. The values of the points held fixed are South African, on wgs84 with ellipsoidal heights.

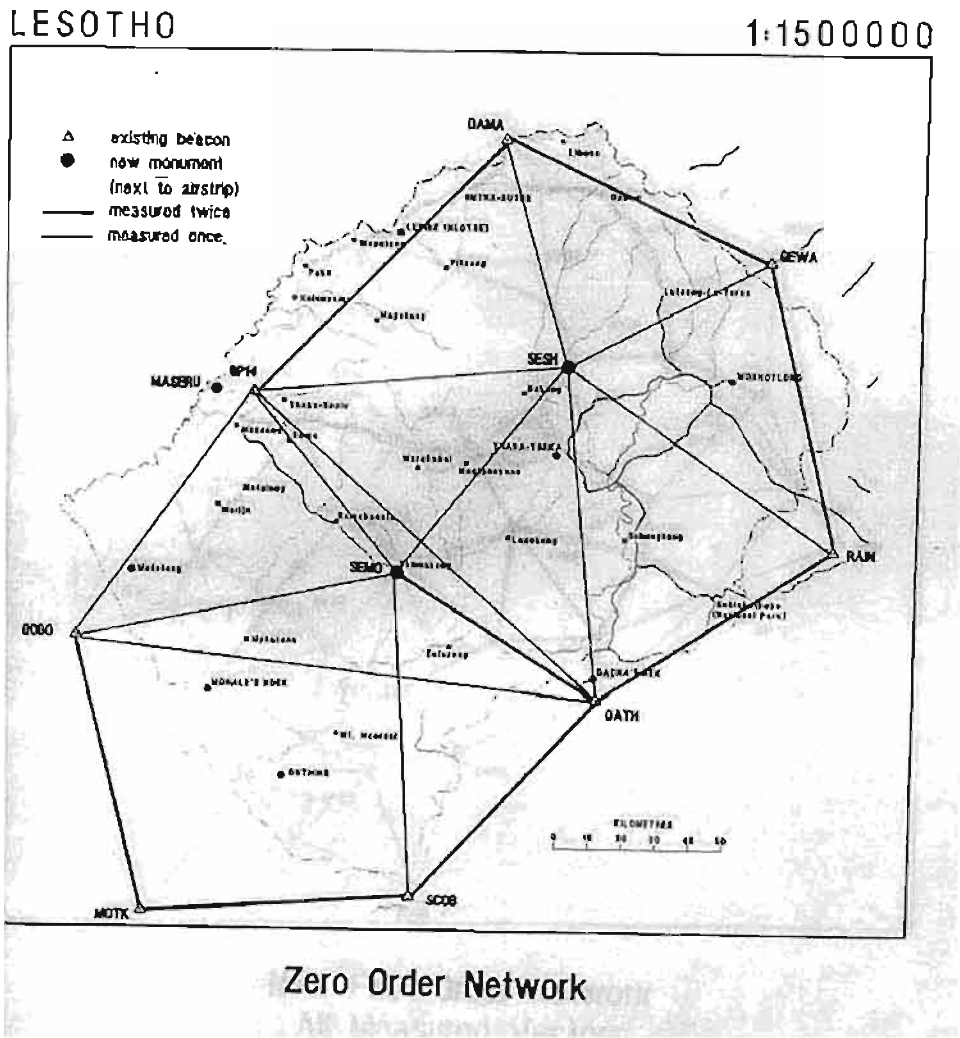


Figure 3. 3 Points held fixed for the new primary Lesotho Geodetic Network
(Merminod 1993)

name	Latitude	longitude	height	location
BP14	-29 20 40.08208	27 34 36.48556	1932.786	Berea Plateau, LS
DAMA	-28 36 34.61224	28 23 46.27000	1980.527	Damascus
GEWA	-28 58 52.08912	29 15 34.16221	1995.320	Gewaagd
GOBO	-29 59 26.78613	27 04 52.42176	1821.894	Gobo Gobo
MOTK	-30 51 24.53923	27 17 43.20094	2144.195	Motkop
QATH	-30 08 15.01783	28 40 09.55721	2204.908	Qathas Nek
RAIN	-29 44 19.27366	29 26 10.62012	1754.459	Rainbow
SCOB	-30 47 29.22953	28 07 14.82074	2768.852	Scobelleskop

. Points held fixed, on WGS84, with ellipsoidal height. (Merminod, 1993).

The GPS network in Lesotho has had very low recognition. Most GPS monuments are not visible from far, unlike the DOS trigonometric beacons, except for those points that used DOS trig. Beacons. The new GPS values of a few points in the old reference frame, even if converted to the Cape datum, have in the past not given good residuals when used with other points around them. Personal experience using BT71 has proven that the transformed GPS values are not compatible with the DOS values. A Survey Assistant program (LSPP) was used to transform WGS84 values to Cape datum values. It will be seen in the chapter on analysis that the results using GPS values of the point BS62 at 'Mamathe are not good.

GPS Point Description

These are at ground level, thus being less exposed to climatic variations and also are less attractive to vandals. These GPS points are not generally intervisible as they do not need to be. These points are spread evenly throughout the country so that one will always be within 30 kilometres of two GPS points. Their description is as follows:

A circular brass plate inscribed with the words ' BOPOTIELE LESOTHO – NATIONAL SURVEY ' on top and at the centre of a $1m^3$ of concrete slab flush with the ground (figure3.4)

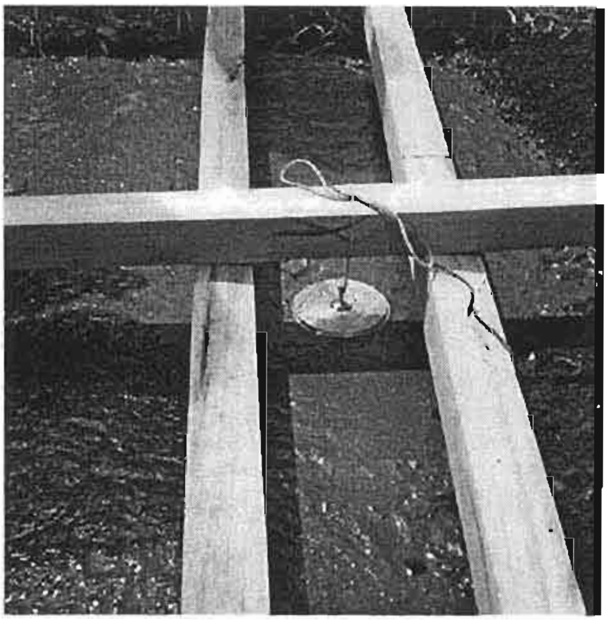


Figure 3.4 Lesotho GPS monument (LSPP Maseru)
(Merminod 1990 photo)

At present, these GPS points and their coordinates are of very limited use. Most of the privately practising surveyors have had no comprehensive training to operate GPS equipment and analyse the collected data. Also, the cost of GPS equipment is still very high considering the charges for survey services in Lesotho. Over and above that, almost all the surveyors in Lesotho are engaged in cadastral surveys most of the time and hence there is no immediate need to use the GPS system. It is only now that the government of Lesotho is trying to assess the amount of land available for different purposes (i.e. built up areas, land for agriculture, land for grazing, etc) that local surveyors will really need to use the existing GPS network of the 1990s. If other GPS methods, such as Omnistar and Trignet are introduced, then the existing GPS network will most likely have declining use. Otherwise, most of the presently practising surveyors have only heard about the GPS monuments and have not seen them except for the already existing DOS trigonometric beacons.

3.3.3 Lesotho South African Set on WGS84 system (South African coordinates)

The Lesotho South African coordinate list is as described in chapter 1. No new observations were taken of points in Lesotho. The old DOS observations were incorporated with the 1991 South African GPS adjustment. This explains why there are beacons in the South African list that no longer exist physically or are not being used anymore and do not appear in the official list of coordinates in Lesotho. The problem with this list is that errors in the DOS observations, if any, are carried into the adjustment. This set of data is however, believed to be better than all other sets of data because the South African network, being anchored to the Hartebeesthoek datum, is related to the global network. TrigNet is also tied to this network.

3.3.4 LHWP Coordinates list

Before implementation of the Lesotho Highlands Water project, the control was better and denser at the north and western side of the country where, as has been said, it was thought that there would be more development than elsewhere. After the military government took over in Lesotho in 1986, the Lesotho Highlands Water Project was started and its geodetic network had to be put in place. Study consultants from both the Lesotho Government and the Republic of South Africa began their feasibility study of the area. The area covered by the project in all its phases is more than one third of the whole country as it includes new designed roads, electric power lines, tunnels etc. all leading to the dam sites. The four major project sites are sites are Katse, 'Muela, Matsoku and Mohale.

According to the Lesotho Highlands Water Project (1986), the Republic of South Africa Study Consultants were responsible for all mapping to be derived from aerial surveys, both in Lesotho and South Africa. The Government of Lesotho Study consultants (GOL SC) prepared only the large scale mapping for the Phase 1A construction sites in Lesotho. At the start of the feasibility study, the available mapping was only the published small-scale maps of Lesotho and the 1: 10 000 mapping of some reservoir basins. The extensive new survey work, involving levelling, mapping and triangulation was carried out as part of the feasibility study.

For the plan control, the points BT 100 to BT125 established by DOS were used for surveys carried out during the feasibility study. LHWP (1980) pointed out that Lesotho Primary Network was recomputed by the Republic of South Africa Directorate of Survey and Mapping in 1983/84. It further mentions that at the time of the surveys the GOL SC (Government of Lesotho Study Consultants, represented by LSPP), were not aware of the changes in the national trig-network done in 1983/84. Therefore to LSPP, the standing large primary adjustment was that of the 1970s. Some 23 new beacons were constructed by the Department of Water Affairs, Pretoria (DWA) (LHWP 1986). 8 additional beacons were constructed later. The observations were made in April – May 1987 (LHWP, 1988). The adjustment followed in July 1987. A Helmert transformation using the above points and some others north of

Pelaneng was done. The readjusted primary points formed the base for the strengthening of triangulation south of Pelaneng carried out during the feasibility study by the South African Directorate of Water Affairs (DWA) acting as part of the RSA study consultants. Even though the project area is covered with a sufficient number of trigonometric points for any further survey work required for the project, the two areas, North of Pelaneng and South of Pelaneng, having evolved from two separate operations, do not bear a homogeneous network. The June 1988 adjustment provides final accepted coordinates for the project (LHWP 1988).

3.3.5 Doppler points

In January and February of 1979, a 512 Specialist Team of the Royal Engineers carried out a Doppler survey on 5 points as a contribution towards the ADOS survey. Because the Doppler points are few (5) and only two of them could be matched with those in the standard coordinates set these points could not be used in the horizontal control investigations.

3.3.6 The Height Control

DOS, from June 1977 to December 1979, established the original levelling line from Ficksburg (near Maputsoe in Leribe) to Makhaleng Bridge, in Mohale's hoek, as reported by Snowsill (1980). The terminals of the line are the fundamental bench marks (FBMs) at the South African site of the border at the two places mentioned above. The DOS line runs along the main road linking the main towns, thus it starts at Maputsoe, then to Teyateyaneng (TY), to Maseru, Morija, Mafeteng and ends at

Mohales hoek. The numbering was such that line 1 is between Mohales hoek and Mafeteng, line 2 Mafeteng and Morija, line 3 Morija to Maseru, line 4 Maseru to TY, and line 5 TY to Ficksburg. Depending on the terrain of the place, Bench Marks (BMs) were placed every 1 – 2kms. Where possible, these were cemented onto the live rock, otherwise marked on a raft of concrete. The FBMs are marked by a brass bolt from which suitable identifiers were marked and directions and distances measured. Approximately every 5th Bench Mark was marked by a buried as well as a surface mark.

Snowsill (1980) reports that the work was supervised by different DOS Staff during two different periods, the first three legs between June 1977 and June 1978 by Lynch and the remaining two legs by Snowsill between May and December 1979. This level line needed to be extended to cover most of the country. With LHWP, this was attempted, but they encountered problems. The 1972 line from Monontsa to Pelaneng was shown to be of lower order than the later lines. Although it is said that the values are dynamic, it seems that the values are simply derived from levelling uncorrected by any gravity considerations. LHWP explains that "In water engineering projects, dynamic heights must be used and the heights of all the surveys for this project are based on dynamic heights" LHWP, (1980 p.D-3.4). In the observation reductions, however, it is not shown how the heights of the starting point(s) were determined. During the reduction, an orthometric correction was applied to obtain the orthometric heights. The formula used is that of normal orthometric correction:

$$c = 0.005302.\sin 2\phi.\delta\phi.\sin 1''$$

As a student project at the Universty of Natal, Jefferys did some work on the LHWP level lines, in which gravity corrections were applied (Jefferys (1987)). The significance of his results cannot be overlooked and will be discussed again in the chapter on height.

3.4 Conclusions

Both the horizontal and vertical networks in Lesotho have problems. For the horizontal network, the isolated efforts to adjust parts of the network have resulted in different values of coordinates for the same points. For the Lesotho Highlands Water Project, the opportunity should have been taken to unify the lowlands and the highlands networks, as the project area spans about one third of Lesotho. The problem with the vertical network is similar to that of the horizontal network. LHWP has its own network, formed independently of the DOS network. These two networks have been loosely attached (at one point in Mazenod). The level lines have not been checked and maintained on a regular basis. The LHDA lines show both the orthometric and dynamic values, but there is nowhere in the calculations where actual gravity is included.

Chapter 4 Method of comparison of control point lists

4.1 Introduction

In the earlier chapters we noted that there were several data sets for horizontal control in Lesotho, with some common points. An aim of this research was to compile one consolidated list, suitable as control for modern surveys. Theoretically this list could be created by resorting to the raw observations used for each data set and to readjust the composite network, but because the observations were carried out by different agencies, the data is not all available to one computing agent and in any case such a recomputation would be beyond the resources available to this study.

4.2 Available methods of transformation

Whittal and Merry (1997), write

> The transformation between two datums is conveniently carried out using a three-dimensional conformal transformation of first-order (3D similarity transformation) ... For the new South African datum there will be no change in the height datum, so it makes sense to consider the coordinate transformation problem as a two-dimensional one. The two dimensional analogue of the three dimensional conformal transformation ... is the 2D conformal transformation also called similarity, or Helmert, transformation with four parameters.

Lauf (1961 page 3) says

> Using a complex polynomial of the fourth degree based on nine common stations, the coordinates of points on a local system (called "the Goldfields system") were transformed conformally to the Gauss Conform System (substantially the same as Transverse Mercator System) using the method of Least Squares.

Whittal and Merry continue to show that where a region is as small as a one degree square, successful results can be obtained using a Helmert transformation.

On this point Lauf (1961) shows that a 250 square miles area an average error of 0.20 English Foot was obtained for the Goldfields systems. Lesotho is in South Africa, uses same survey methods and techniques therefore the method of fourth order conformal polynomial transformation is feasible.

Various "rubber-sheeting" methods can also be used, in which the common points do not acquire ambivalent positions after transformations. For example a Triangular irregular network (TIN) may be created from the common points and within each triangle so formed an Affine transformation is applied. This 6-parameter transformation uses two scale factors along x and y axes and has no redundancies for 3 common points. Alternatively, collocation can be used to model systematic trends and for interpolation, also distance weighting which uses the weighted mean of the shifts at nearby points to interpolate values at the control points, nearest neighbour or minimum curvature can be used. (Whittal and Merry (1997)), The advantage of a single polynomial transformation over these rubber-sheeting methods, for purpose of this study, is that it models the distortion of the older net in simple terms and gives one a clear picture of the degree to which simple systematic distortions can account for discrepancies between the data sets. That is why a conformal polynomial approach was taken in this study, with the precedent of G.B. Lauf's use of it in the goldfields transformations.

After studying the problems in each coordinate list, it was necessary to choose one list as a standard of comparison. This choice will be justified. This is followed by a description of the preliminary editing of data. Finally, the method of comparison of data is introduced.

The following table gives the summary of data used in the analysis. The number of common points shows the number of points common to the file named in the contents and the South African data set. The reference ellipsoid is the ellipsoid the coordinates were on before the analysis.

Contents	Number of points in data set before data cleaning.	Number of points Common to S.A.	Reference ellipsoid
DOS points	227	117	Clarke1880
Merminod's points	113	29	WGS84
LHWP points	87	70	Clarke1880
ADOS points	5	2	WGS72
S. African points	183		WGS84

Table 4.1 Summary of files used in the analysis

4.2 Choosing a standard list for Comparison of coordinates

The DOS list of coordinates has been very useful and will continue to be for some time. Every other list includes some DOS points. This list includes all the primary points, secondary points, tertiary points and other miscellaneous points for lower order surveys.

The DOS list was not chosen as the standard of comparison since it has been known to contain some discrepancies which are due to lack of connection to South African surrounding control on the South East side of Lesotho. Besides, the same South African control, which served as a reference for Lesotho control, has been upgraded several times after the original connection of the DOS network.

The purpose of Merminod's list was to construct a new framework, which could be used as a network of higher order than the existing DOS network. But, these two are not strongly attached to each other.

Merminod adopted a few DOS points, but his DOS values for some of these points do not agree with the official values. The reason for this is not known. Merminod referring to provisional coordinates of new reference stations in Lesotho notes that some points were "not checked mutually like in the network" Merminod (1992, p.9). Even in some of the new monuments, there are many single vector measurements. Because of this unreliability and the fact that this list contains very few common coordinates to other lists, Merminod's coordinates list was not used as a standard.

The ADOS list was surveyed by Doppler methods which are regarded as of lower accuracy than GPS, and therefore could not be considered for the standard. The LHWP network has some discrepancies within itself and is a subset of the South African readjustment of 1984 on the Cape Datum, and could not be used as the standard either. The South African latest readjusted set was chosen as the standard for comparison. This set has many common points with DOS list, is in sympathy with GPS system and represents the system used in areas around Lesotho.

4.3 Data editing

Before comparing points, it was necessary to compile a file for those common coordinates between each set and the standard. Microsoft Access was used to select names of common coordinates to be forwarded in a readable form to Matlab on which the analysis was to be made. The original co-ordinates were given on the Lo27 and Lo29 map panels. For the comparison it was necessary to use a single map surface and this was chosen as Transverse Mercator with the central meridian 28 degrees East, which passes roughly through the middle of Lesotho. The co-ordinates were also referred to different reference ellipsoids and as a preliminary step, were transformed to the WGS84 ellipsoid. Transformation software XFORM, written at the University of Cape Town by Professor C. Merry, was used for the transformation from Clarke 1880 ellipsoid to the WGS84 system. The X-form trnaslation parameters for this transformation were: X-shift –135.4m, Y-shift –106.7m and Z-shift –291.7m. The rotation parameters in the 3 components are all 0 seconds. These parameters are the same as in Alice. However, because of user-friendliness of X_form, Alice was used only when and where X-form could not be employed, for instance, XFORM could not transform coordinates onto LO28, ALICE software of Mr. Jackson of the University of Natal, was employed for this stage.

In the first comparison of common-point co-ordinates, their co-ordinate shifts or differences were computed. This provided the first test for outliers.

Some manual editing was also carried out, as the naming system in the South African set is not the same as the one in Lesotho. Coordinates were compared, and points matched. In some cases, especially where there were two very close values, the right point could only be selected after a trial adjustment.

4.4 Calculation Method

The method used was two-dimensional conformal transformation, extended to the 4[th] order polynomial.

$$X = a_1 + a_2 * 0 + a_3 x - a_4 y \quad a_5(x^2 - y^2) - a_6(2xy) \quad a_7(x^3 - 3xy^2) + a_8(-3x^2 y + y^3)$$
$$a_9(x^4 + y^4 - 6x^2 y^2) - a_{10}(4(x^3 y - y^3 x))$$

$$Y = a_1 * 0 + a_2 + a_3 y - a_4 x \quad a_5(2xy) + a_6(x^2 - y^2) \quad a_7(-3x^2 y + y^3) + a_8(x^3 - 3xy^2)$$
$$a_9(4(x^3 y - y^3 x)) + a_{10}(x^4 + y^4 - 6x^2 y^2)$$

Equation 4.1

In the comparisons of common points because there were more equations than unknowns a_1 a_2 etc, it was necessary to apply least squares solution for the coefficients according to the following formulation:

$$Ax = b$$
$$\Rightarrow x = inv\ (A'*A) * A'*b$$

Equation 4.2

where A is the design matrix, derived from co-ordinates in System I.

x is the vector of unknown coefficients a_1 a_2 etc,

and b is the vector of coordinates in the second system. Ideally,

$$Ax - b = 0$$

It is not so since the control point positions have small errors because of various factors in the system of measurements. The residuals v are then calculated from the equation

$$v = Ax - b$$

Equation 4.3

The residuals are then used to calculate the precision of an observation of unit weight through the equation

$$\sigma_0 = \sqrt{\frac{v^2}{2m - 4Q}}$$ **Equation 4.4**

where m is the number of common points in the two systems and Q the transformation order.

The polynomial terms are as follows:

1st order or Helmert	a_1 to a_4
2nd order	a_5 and a_6
3rd order	a_7 and a_8
4th order	a_9 and a_{10}

The results are discussed in the chapter on results analysis.

In the case of the 1st order polynomial comparison, the coefficients had useful physical meanings, which were:

X_0 – translation in x given by a1 terms

Y_0 – translation in y obtained from a2 terms

a3 and a4 terms are used to calculate the rotation angle *Phi* and scale factor *sf*

where

$$phi = \tan^{-1}\left(\frac{a4}{a3}\right)$$ **Equation 4.5**

$$sf = \sqrt{(a3)^2 + (a4)^2}$$ **Equation 4.6**

These parametres lose their meaning as higher order transformations are applied.

When higher order transformations were used, at first there was a problem of badly scaled (ill-conditioned) matrices giving unreliable results. This was because the size of the coefficients became very large as higher order transformations were applied. For instance, the 2nd order transformation involves coefficients with the squares of both x and y. In the 3rd order transformation there are coefficients with the 3rd power of x and y values etc. This was counteracted by dividing coefficients by constants which were estimated by inspecting the design matrix.

The analysis program used the following sequential simple steps:

- Coordinates input (from a text file): A file consisting of beacon name (strictly 4 letters), y1, x1, y2, x2 delimited by tabs or spaces. The user was presented with a selection of available source files.
- Choice of transformation order: a list of available choices is displayed, user chooses by a mouse click on the button desired.
- Calculation of residuals and transformation parameters: This was done internally, but user could view the results, which could then be copied onto the clipboard and pasted in another application, such as Microsoft Word.
- Updating of coordinates: which again could be copied and pasted.
- Plotting of the residuals: Residuals were plotted at the point's location using an arrow symbol. The scale of the residual arrows could be controlled by a factor (usually set numerically to 5000).

4.5 Conclusions

In chapter 4. explanation of how the calculations will be done was given. The choice of standard data set depended on the best framework with regard to both angular and scale distortions and the number of data points common with other data sets. Data editing was done first to enable data points to be matched, that is, renaming of points was important at this stage, in preparation for the next chapters.

Chapter 5 Comparison of DOS co-ordinates with South African Hartebeesthoek Datum.

5.1 Introduction

The largest source of official co-ordinates currently available for Lesotho was derived by the Directorate of Overseas Surveys as mentioned in the last chapter. This chapter reports on the comparison of these points with those derived from the South African readjustment on the Hartebeesthoek datum, using conformal polynomial transformations. Before comparison, the two data sets were transformed to the Lo28 map panel, WGS ellipsoid. The following aspects of comparison are reported on in this chapter: the identification of outliers, finding the reason for outliers, comparison of residuals for different orders of polynomial.

5.2 Detecting Outliers

The research software was made to flag any residuals greater than 3 times the standard error, as a rejection criterion. The y and x residuals were presented. The rejection was applied at the first order transformation only, and not at the higher order transformations. If particular points were judged to be outliers, they were removed from the list. The program was then run again to determine further outliers that might have been masked by the ones already deleted. The following notes record how the tests proceeded.

During the first Helmert transformation run, only one residual was flagged as an outlier :

Name	Residual		Max Residual/σ_0	Place
	y	x		
S060	-5.577	-53.026	14.421	Qeme

Table 5.1 The First Rejected Point

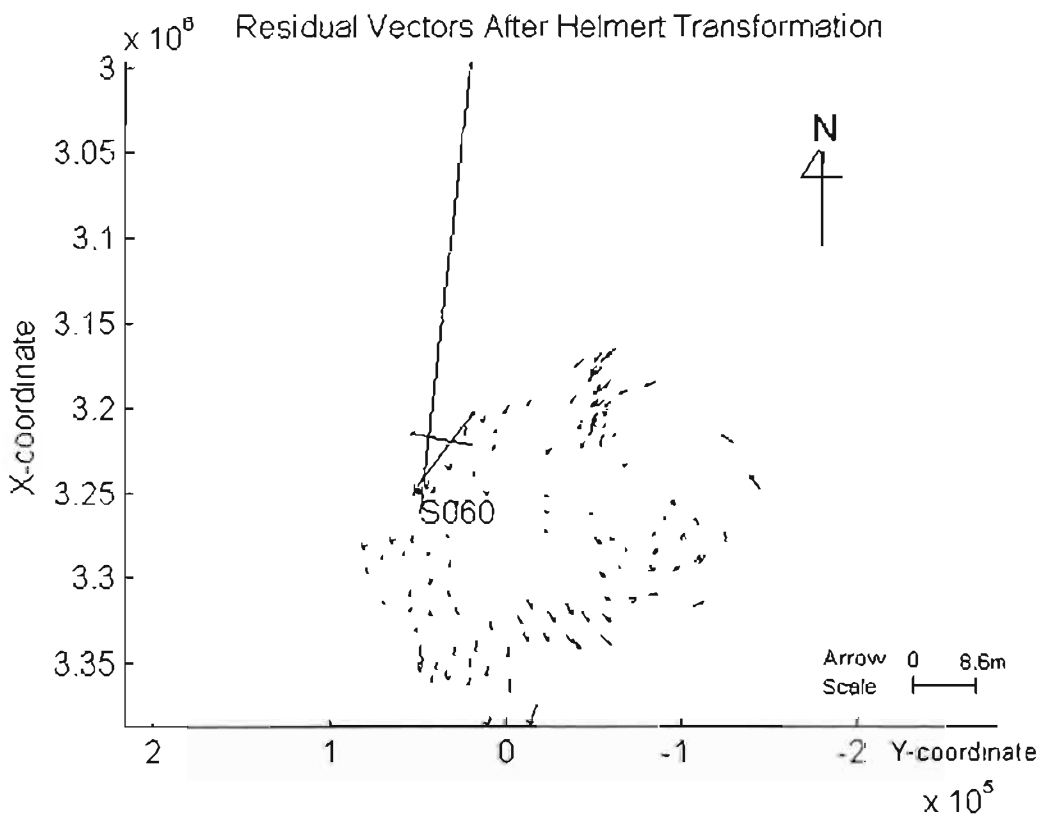


Figure 5.1 S060 as an outlier. The arrows indicate residual vectors.

In Figure 5.1, both the magnitude and the direction of the vector at S060 in the Helmert transformation are not in agreement with the rest of the vectors around it. This Point was then rejected.

The second run of the Helmert transformation gave the following points as outliers:

Name	Residual		Max Residual/σ_0	Place
	y	x		
S062	6.728	-2.184	6.803	'Mamathe
T074	-6.426	-9.269	9.372	Maseru

Table 5.2 Outliers after removing S060

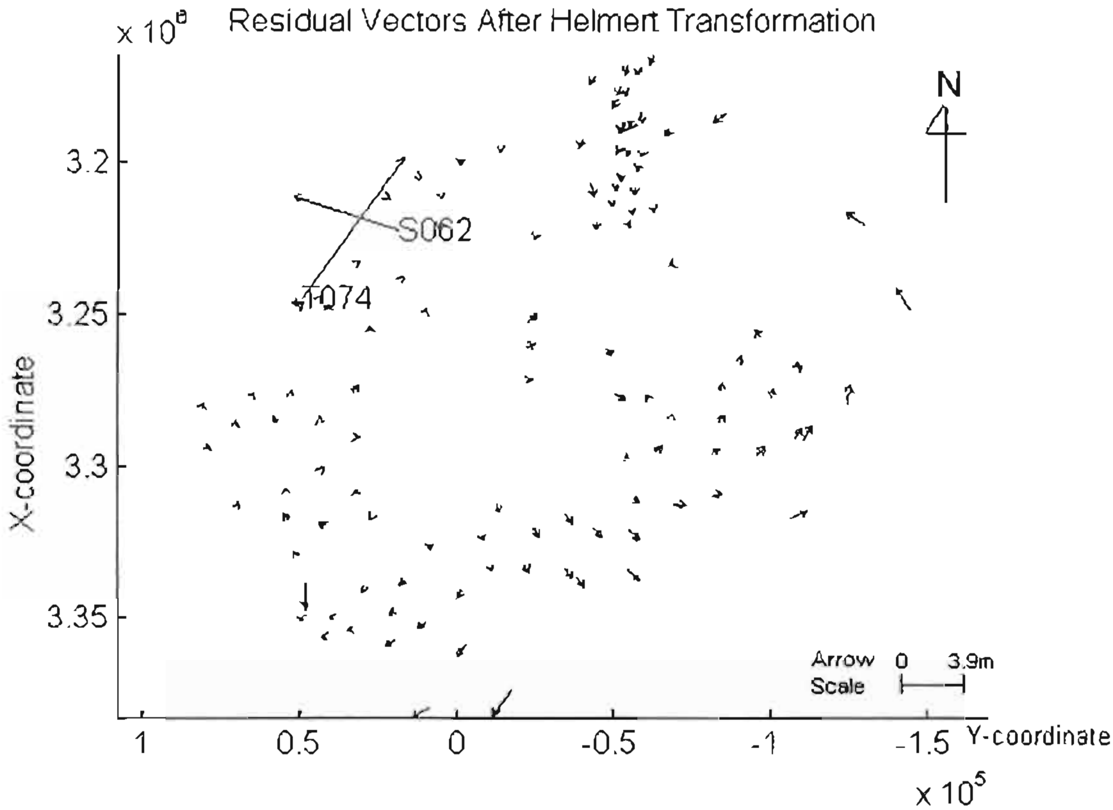


Figure 5.2 S062 and T074 vectors as outliers during 2nd Helmert transformation.

After removing both S062 and T074, 3 more outliers appeared when running the Helmert transformation and these are shown in table 5. 3.

Name	Residual		Max Residual/σ_0	Place
	y	x		
S071	0.114	1.408	3.18	Mohales hoek
P035	1.257	1.534	3.46	Quthing
P013	0.862	-1.428	3.22	Mokhotlong

Table 5.3 Third Run Outliers

The associated diagram is as in figure 5.3 below.

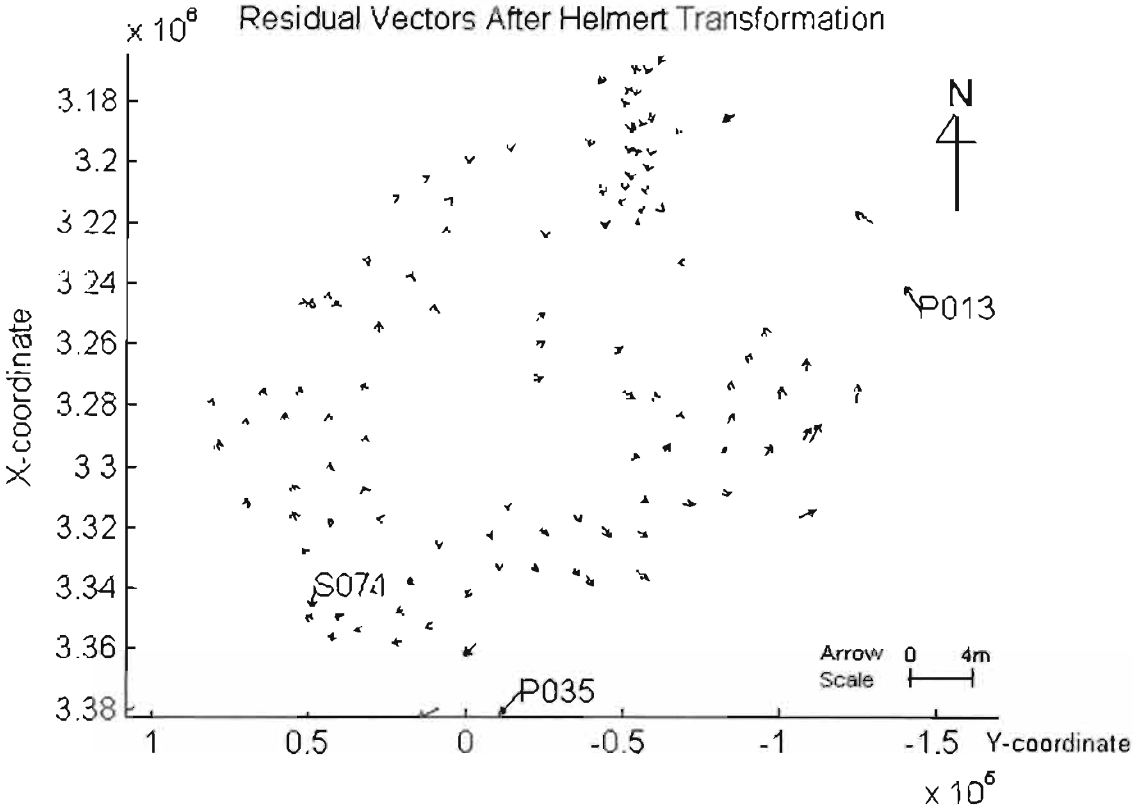


Figure 5.3 Third Helmert Transformation Run Outliers

On scrutinising figure 5.3, one can say that all flagged points, S071, P035 and P013 are only marginally outliers. The 2 last vectors generally conform to the rotation of the surrounding vectors but were flagged by the program because of residuals being greater than 3 standard error as observed in table 5.3. For these reasons and since the two points are at the edge of the investigation area, they were not removed from the list. S071 is the only point that is out of sympathy both in magnitude and direction with the other points, and therefore was rejected.

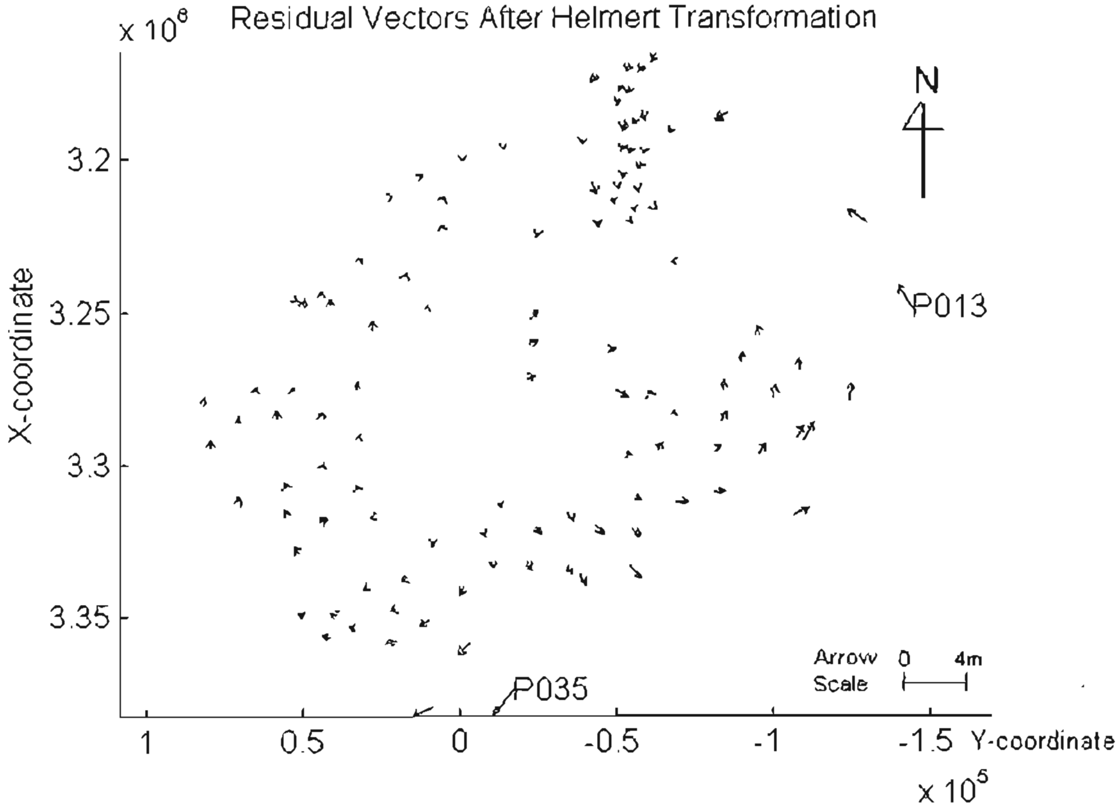


Figure 5.4 **Residuals after all outliers removed, Helmert Transformation.**

The final Helmert transformation after removing all outliers, is shown in figure 5.4. On applying the 2nd, 3rd and 4th order transformations, P035 was flagged, each time with decreasing residuals (See figure 5.5). This point was retained.

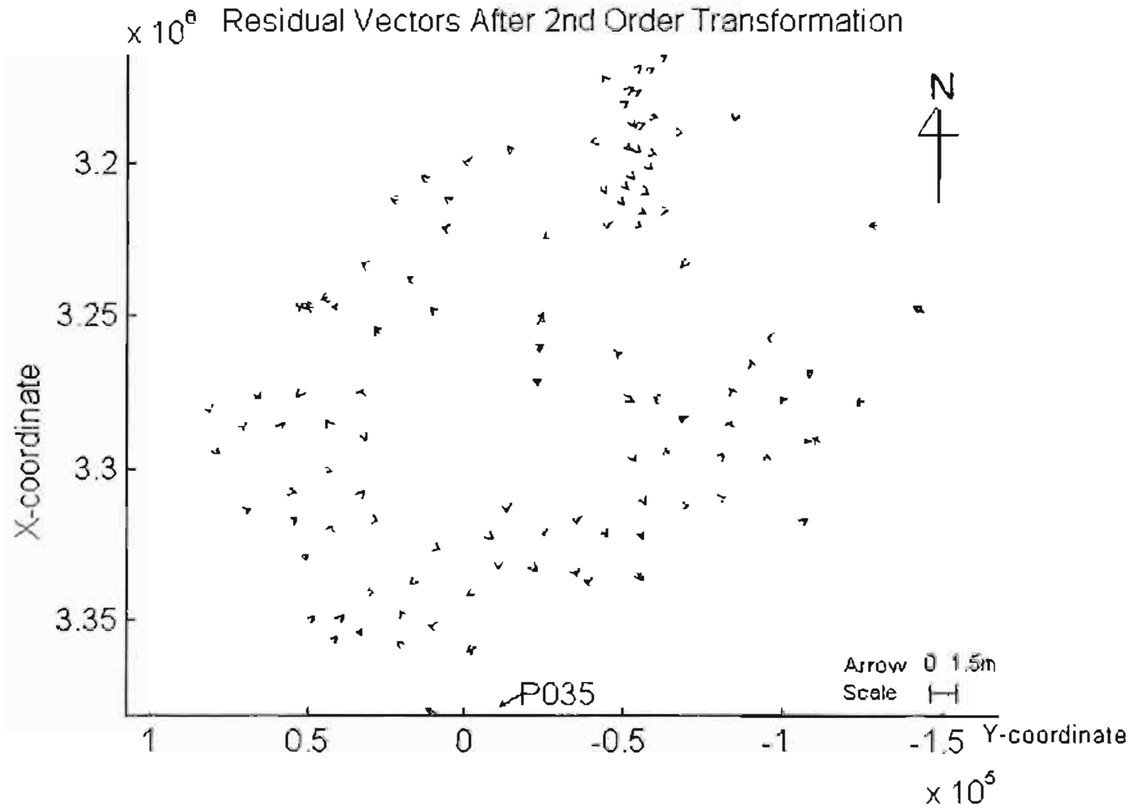


Figure 5.5 2nd Order transformation flagging BP35, point not rejected

Overall, 4 out of 117 points were thus eliminated. These points are S060, S062, T74 and S071. These form roughly 3% of the common points between Lesotho DOS and SA.

5.2.1 Visited points for a check survey

Two of four points identified as outliers were visited to check their consistency with close points for a resection. These points are T074, a tertiary beacon in the Maseru city and S062, a secondary pillar at 'Mamathe in the Berea district. A Wild T2 1-

second theodolite was used, and as both are strong pillars with standard removable vanes, forced centring using a suitable adapting plate was employed. The observations were taken around midday under hazy conditions.

At station T074, there were 5 fixed stations to observe to.

The coordinates of these points on Lo27 as listed by DOS are:

STN NO	Y	X	OBS DIRN ° ′ ″
9700071	-45244.595	3245950.184	000 00 00
2927115	-42639.080	3239146.800	074 07 40
9700053	-55870.407	3239595.776	175 53 50
9700075	-53289.316	3243859.036	200 06 09
9700051	-51556.669	3248233.840	252 03 49
9700074 (T074)	**-47827.160**	**3244591.809**	

Table 5.4(a) Coordinates of points and their observed direction from T074

Adopting the official DOS co-ordinates, the following residuals were obtained:

STN NO	ORIENT DIRN	DIST (KMS)	RESIDUAL	
	° ′ ″		SECS	METRES
9700071	62 15 23	2.918	-0.13	0.002
2927115	136 23 03	7.521	0.14	0.005
9700053	238 09 13	9.469	0.14	0.006
9700075	262 21 32	5.511	-0.39	0.010
9700051	314 19 13	5.213	0.24	0.006

Table 5.4(b) Bearing, distance and residuals at T074

At station S062, the following residuals were obtained relative to official DOS co-ordinates:

STN NO	Y	X	OBS DIRN ° ' "
9800051	-79755.793	3237999.094	000 00 00
9700077	-73931.614	3227058.589	49 10 00
5010011	-72184.757	3223196.142	86 44 41
2927035	-61258.520	3220230.200	101 05 15
9900009	-75490.186	3211875.234	167 22 44
9800062 (S062)	**-78638.735**	**3222364.575**	

Table 5.4(c) Coordinates and observed direction at point S062

STN NO	ORIENT DIRN ° ' "	DIST (KMS)	RESIDUAL	
			SECS	METRES
9800051	355 54 48	15.674	-0.04	0.003
9700077	45 04 48	6.648	0.47	0.015
5010011	82 39 30	6.507	-0.00	0.000
2927035	97 00 03	17.511	-1.21	0.103
9900009	163 17 32	10.952	0.70	0.037

Table 5 4(d) Bearing, Distance and Residuals at point S062

The points 2927115 and 2927035 are South African trigs.

With regard to the naming of points in the tables above, the following nomenclature was used:

Numbers starting with 97 are Tertiary points. e.g. 9700077 is T077 in this thesis and BT77 in Lesotho.

Numbers starting with 98 are Secondary points, e.g. 9800062 is S062 in this thesis, BS62 in Lesotho.

Numbers starting with 99 are primary points, e.g. 9900009 is P009 in this thesis, and BP9 in Lesotho.

These two points were refixed as resections from the DOS co-ordinates, to obtain the following differences from their official positions in table 5.5 below:

	DOS Official coordinates		Obtained coordinates		Comparison Differences		
At pt	y (m)	x (m)	y (m)	x (m)	y (m)	x (m)	Vector(m)
T074	-47827.085	3244591.823	-47827.160	3244591.809	0.075	0.014	0.076
S062	-78638.826	3222364.594	-78638.735	3222364.575	0.091	0.019	0.093

Table 5.5 DOS official versus calculated coordinates on T074 and S062

The results reveal a larger y-residual than the x-residual in both cases. This may be attributed to some systematic error during observation. These results do show, however, that T074 and S062 positions are in sympathy with the DOS coordinates set.

An attempt was made to calculate the same points using the SA data set, to see if the values would be in sympathy with those in the SA data set. Unfortunately, both points had only three external orienting stations and so the reliability of the fixes on the SA data set could not be checked. Three rounds of observations were taken.

At Station	Outside Orientation Stations		
S062	S051	P009	2927035
T074	T071	2927115	T075

Table 5.6 Outside fixed stations used for orientation observations

The external fixed stations coordinates for these points from the South African set of coordinates are:

Station Name	Y (m)	X (m)
S051	-79718.668	3238293.320
P009	-75453.418	3212169.498
2927035	-61221.50	3220524.368
T071	-45207.549	3246243.805
2927115	-42602.03	3239440.341
T075	-53252.372	3244152.803
S053	-78608.692	3222661.12
T074 (BG 12)	-47783.681	3244894.695

Table 5.7 South African coordinates of points used in the resection

The calculated resection fixes for the two points using the SA values above are:

Station	Y (m)	X (m)
S062 Resected position	-78601.82	3222658.85
T074 Resected position	-47790.08	3244885.54

Table 5.8 Values obtained by resection computed in the S.A. coordinate system

Since the naming of points is different in both the DOS and South African lists, it was necessary to match points by closeness of coordinates. It was found that S062 replaced S053. The coordinates given in the South African list are those of S053. The differences and join distance between these two points are as follows:

Name	Y (m)	X (m)	Vector (m)
S062 Resected	-78601.82	3222658.85	
S053 Closest point in SA list	-78608.692	3222661.12	
difference	-6.872	2.27	7.24

Table 5.9 Difference between S062 and S053

the calculations do show that S062 bears different coordinates from those of S053. A confirmation of this is found in a note written to some private surveyors by the then survey examination officer. Bullen (1986) (see appendix (1)), warning them of the pillars replaced by new ones with slightly different coordinates. This discrepancy is confirmed by DOS computing Section, (1980), which states that BS62 replaced BS53. The join distance between the S062 and S053 in the DOS coordinate system is 7.19m. This value compares with the join distance resected BS62 to BS53 of 7.24m. Therefore, this point is also in sympathy with the South African values. BS62 should therefore not be considered as an outlier. It has no image point in the South African set.

T074 has not been disturbed since it was erected. When the area around it was checked, a monument (pyramidal and without a vane) was found a few metres south of it. This may be the BG12 beacon that bears the coordinates in the South African list. Therefore T074 is not common to both DOS and SA sets of coordinates. Personal experience using T074 has proven that the point is in sympathy with other points around it, both in Lesotho and in South Africa along the border.

BS60 is on the Qeme plateau and is visible from most villages around Maseru. It has been used by several surveyors for cadastral jobs and has been found to be in sympathy with other Lesotho stations. BS71 is in Mohales hoek, BP38 in Thabantso, Quthing and BP13 in Giants Castle, Mokhotlong, all too far to visit at the time. No report of damage on any of these three trigonometric stations has reached the chief surveyor's office. The reasons for these points being out of position on either data set could not be established.

5.3 Analysis after removal of outliers

5.3.1 applying shifts only

First, a diagram was produced where the vectors obtained by applying only the shifts that had been reduced to the mean between the SA and the DOS coordinate sets were plotted. This was in effect a zero order transformation. The vectors show a remarkable pattern of rotation in the counterclockwise direction, as seen in figure 5.6. A large rotation angle of more than 3 seconds is illustrated in this diagram.

It can be seen from the same diagram that in general, the east and south residual vectors are larger than the west and north vectors. Also the vectors of points at the edge of Lesotho are larger than the inner ones. The largest vectors are the two easterly most vectors, P010 and P013. It is clear that the cumulative effects of systematic errors are largest in the eastern side of Lesotho.

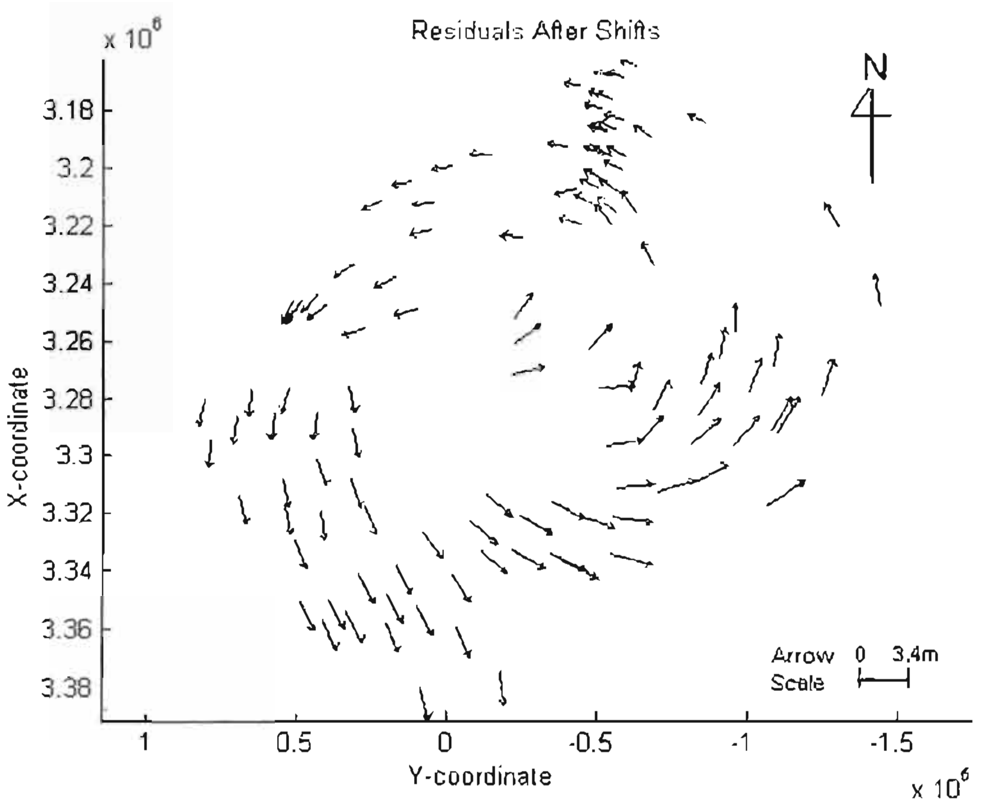


Figure 5.6 **Residual vectors between DOS and SA points, after removal of mean shifts.**

5.3.2 Application of Helmert Transformation on all points

Origin shifts of the Helmert transformation are negligible (of order $*10^{-9}$), as expected because as a preliminary step the two data sets were reduced to their common barycentres.

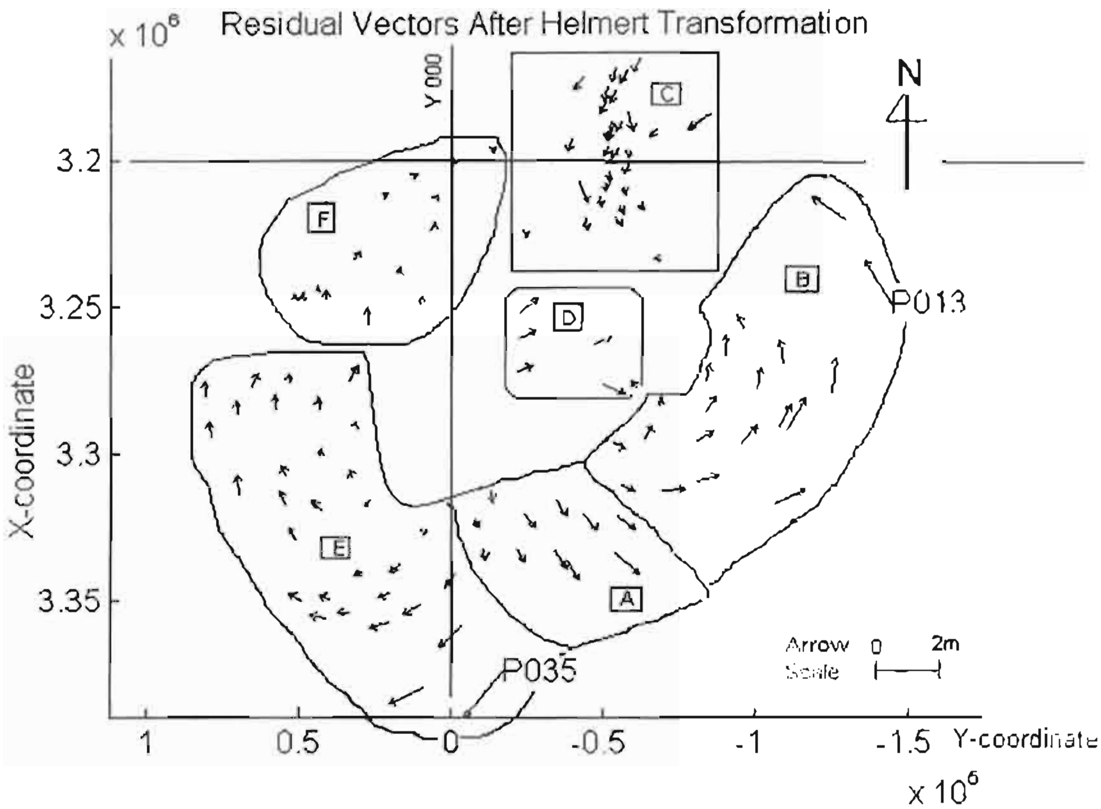


Figure 5.7 **Regional Patterns of Vectors**

From figure 5.4 to figure 5.7 the patterns are well formed and continuous. In these figures, P035 remains the largest vector. In Figure 5.7, apparent groups of points with shared patterns are labelled. Starting at region A, Quthing-Qacha's-nek, distortions split into the Southeast (E region) and Southwest (B region), which is Mokhotlong. From B region, vectors curve gently towards region C, Butha-Buthe, and there is a suggestion that if more points existed between these regions, there could be a continuous anticlockwise rotation, A-B-C. At C, (LHWP area), these vectors point south. Starting again from A, following the clockwise rotation, there appears to be a region E where distortions curve gently from southeast to north. At the northern part of region E, they start rotating to the northeast, and join those in the F area, which point northeast. Region D. at the centre. does not follow these continuous patterns. There are very few vectors in this area, most pointing northeast.

Vectors in regions C, F and upper E are relatively small as compared to those in other regions. It is evident that the southeastern vectors in general are large, suggesting that this is the weakest area in the network.

Since the rotation seems to originate around Longitude 28 degrees East, which is the boundary between Lo panels 27 and 29, one might suspect that the transformation from Lo27 and Lo29 to the common Lo28 is responsible for some of these patterns. Tests using data in their native Lo27 and Lo29 panels, however, revealed similar patterns.

Some of the systematic patterns shown in residuals from the Helmert transformation seem to be taken care of in figure 5.5 where second order conformal polynomial transformation was applied. The direction of vectors is more random than in previous diagrams. It is in this second order transformation that the sizes of the residuals are significantly reduced. Higher order transformations reveal little further improvement. Vectors in figure 5.8 are enlarged to clarify patterns in figure 5.5

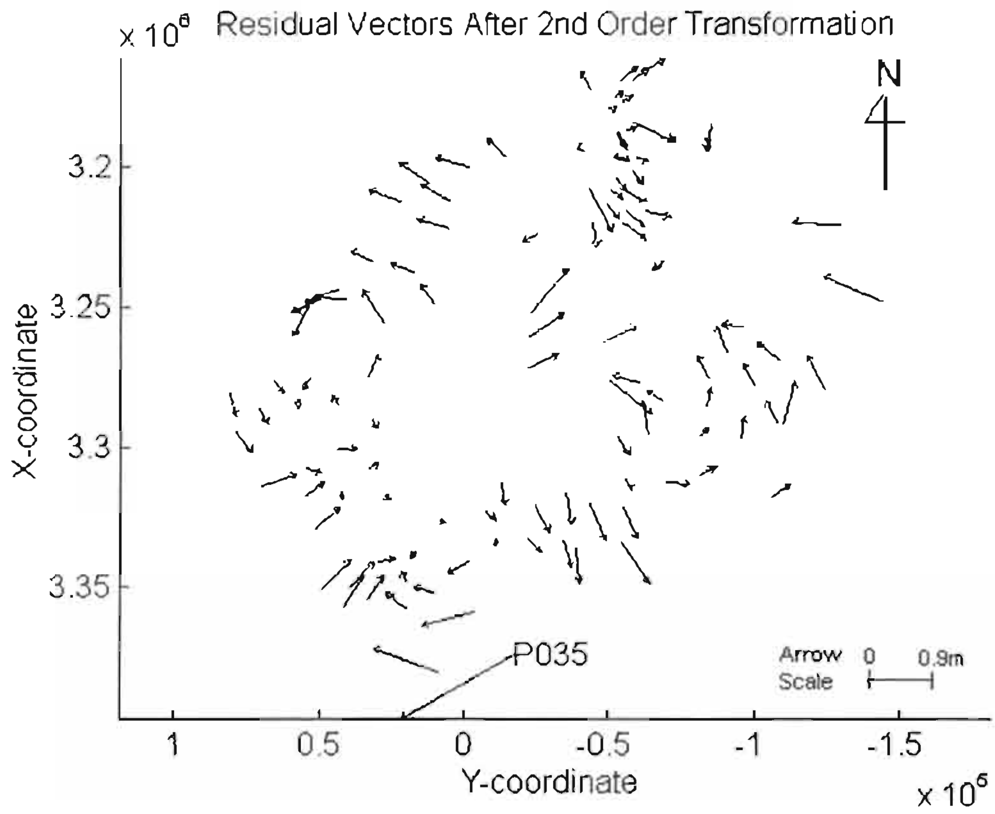


Figure 5.8 **2^{nd} Order transformation: points enlarged to show patterns of vectors**

5.3.3 Results when points are split into Primary and other points

A suspicion that distortions lie more in the primary adjustment arose from the observation of the diagrams of residuals, which showed that most primary points had large vectors. To investigate this, points were divided into primary only and other (all points and secondary and tertiary) points.

Figure 5.9 shows that the goodness of fit of the transformation as measured by the standard deviation of an observation of unit weight, improved dramatically from a first to a second-order transformation and improved very little after that.

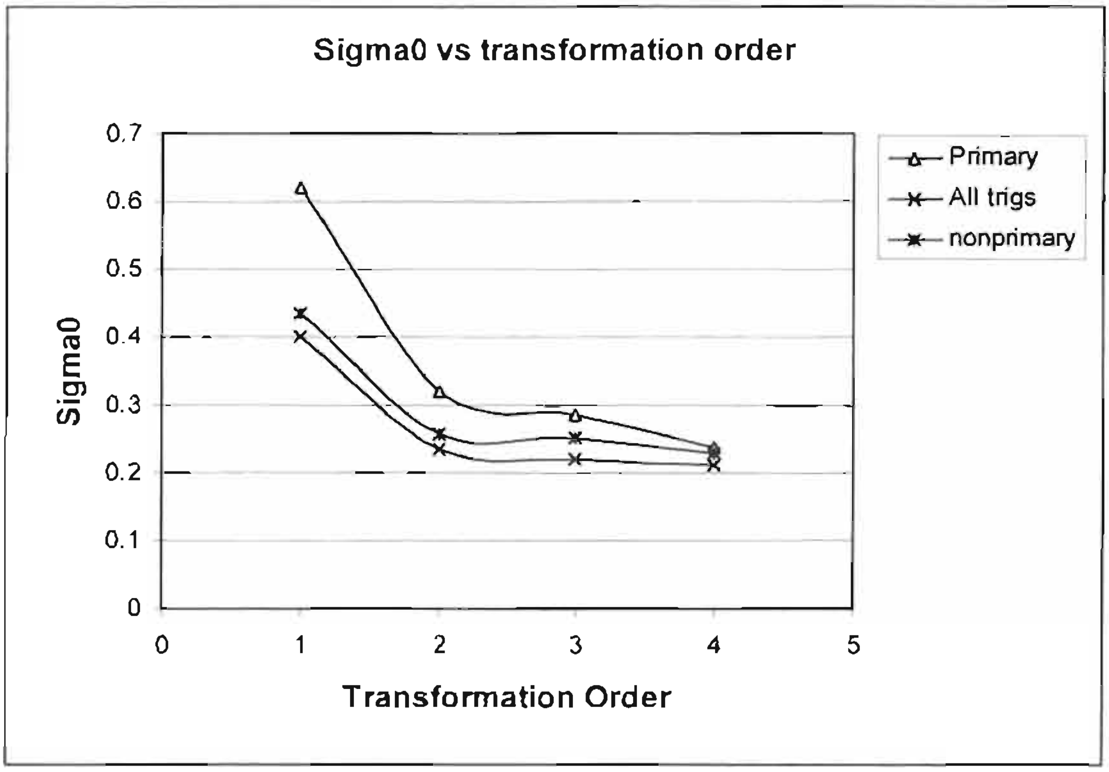


Figure 5.9 σ_0 Comparison at different transformation orders

The standard error in the Primary trigs only is much higher than the other two cases. A comparison of the three curves shows a more rapid recovery in the primary than the other curves, and it is not clear why this is the case.

The graph of the Maximum Displacement Vector against transformation order (Figure 5.10) shows how the displacement vectors vary with the transformation order.

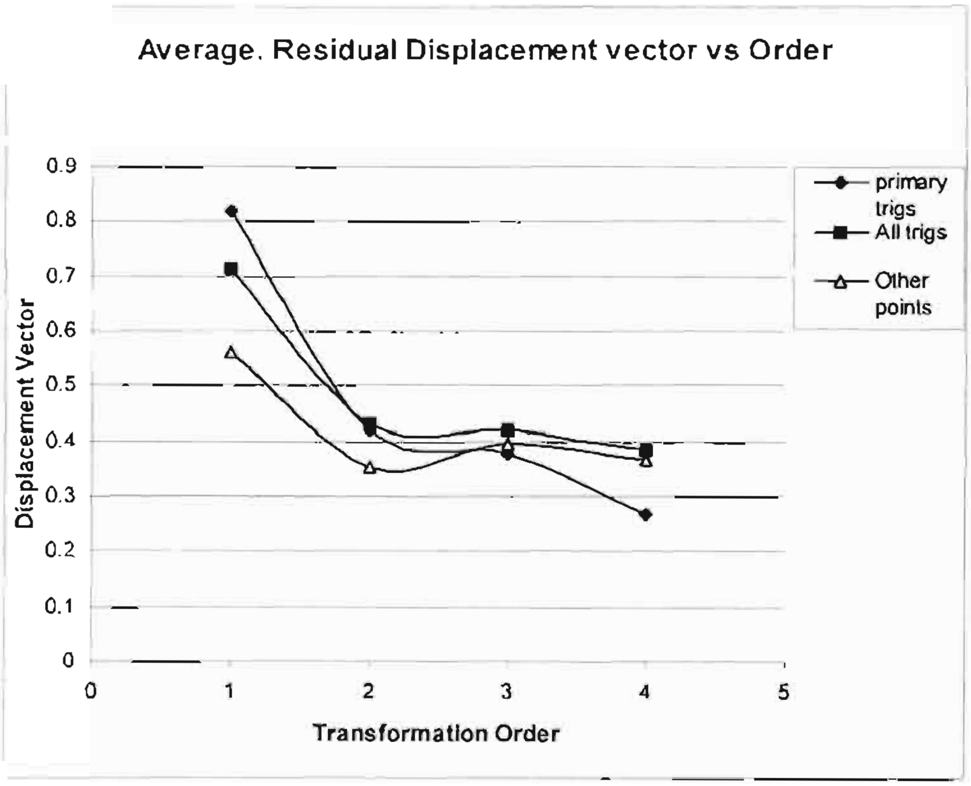


Figure 5.10 Average displacement vector variation with transformation order

Rogers (1978), Windsor (1980), DOS computing section, (June 1981), and Rousseau (1984) of the South African Directorate of Control Surveys all showed that the Lesotho primary adjustment held fixed the South African points on the West and South of Lesotho, but was unable to fit to South African points in the southeast. (See in appendix (I))

Rousseau (1984) noted that along the southern boundary of Lesotho, the DOS team had mistakenly observed to a tertiary station in Mungunung Aux instead of a geodetic station 32. He further comments that P030 (Komokha) had the largest angular corrections. P035, P038 and P030 are in the southeast area where it has been said there were no forwarding observations from the South African stations. The analysis so far confirms the distortion in the South East, and shows P010 and P013 on the eastern side to have been affected by these systematic effects.

This disagreement stresses the seriousness of the effects of systematic errors in the network. For most of the Lesotho points used in the SA readjustment, DOS observations were used, that is, practically no new observations taken in Lesotho were included in the adjustment. After all the South African network readjustments, it was assumed that if there were weaknesses in the South African control in the area southeast of Lesotho, probably they would have shown up. The picture might well be different if few new observations, such as GPS baselines, between points in the weak area in the geodetic network of Lesotho and newly readjusted points in SA were taken for adjustment.

When only primary points are adjusted, the maximum y-residual is 1.245m, and that of x is 1.565m, both in the Helmert transformation at point P035. The point with next largest residuals was P013. These two points are not rejected as was discussed earlier. The magnitude of most vectors is reduced as the transformation order increases. Figure 5.10 shows how the maximum vector varies with transformation order. Figures 5.11 and 5.12 show the vectors of primary only point residuals after 1^{st} and 2^{nd} order transformation respectively.

The imerging pattern for the first order transformation is the same as in the previous analysis where all the coordinates were used. The main improvements are reflected in the second order transformation (Figure 5.11). This agrees with the graph of precision, where the main improvement was between the first and second order, and very little improvement occurred after that. In the 4[th] order transformation, the magnitude and direction of some vectors has changed, and as an example, one can point at BP10, BP38, BP14, BP20, etc (Figures 5.12 and 5.13).

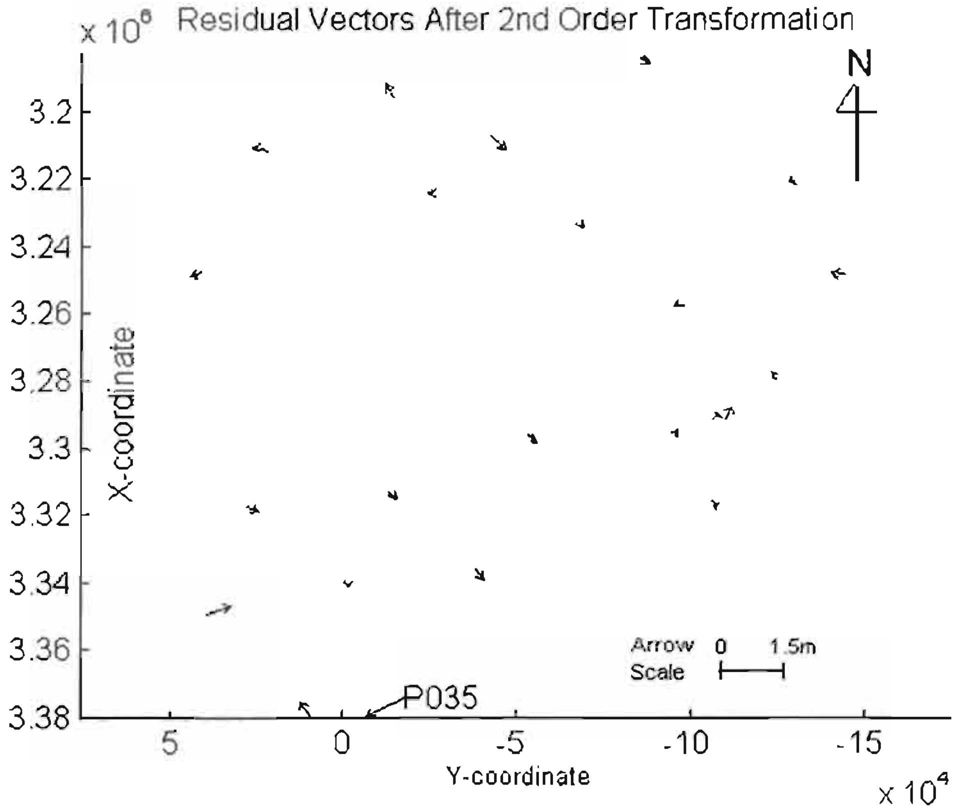


Figure 5.11 **2nd Order Transformation on Primary Trig Stations**

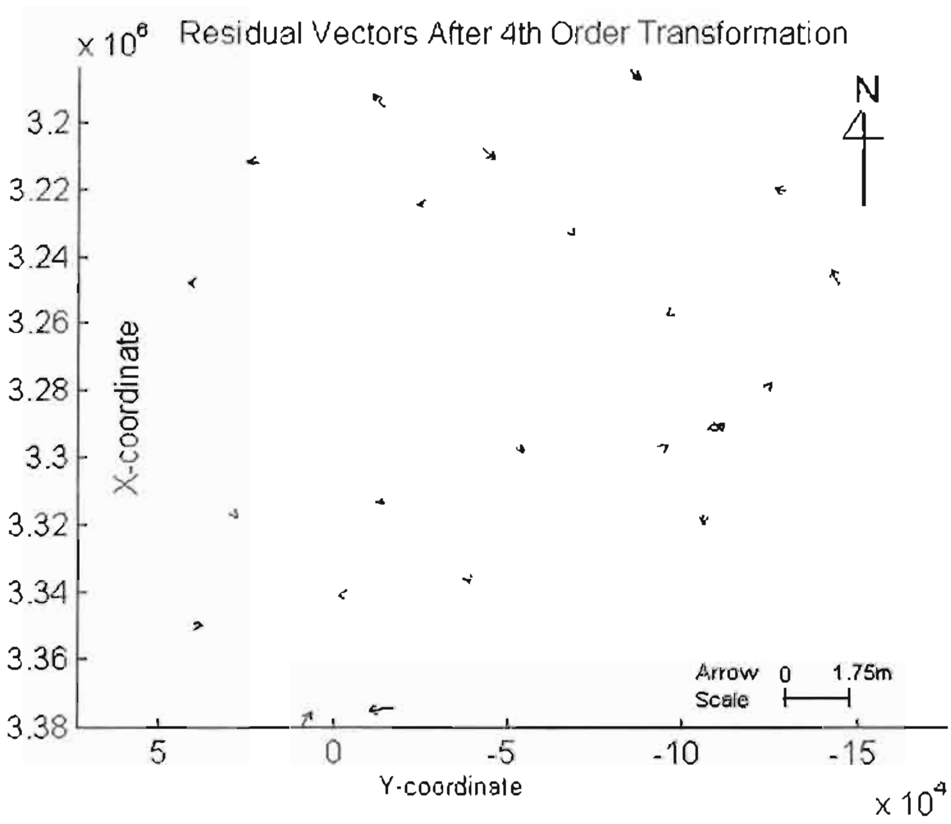


Figure 5.12 **Fourth Order Transformation on Primary Trigs.**

In figure 5.4, the sizes of lower order points were shown together with those of Primary points after the removal of outliers. Figure 5.13 indicates that the pattern formed by secondary and tertiary beacons together is similar to that of all points combined. The residual vectors for points in the north and west are smaller than those of points in the south and east. Second order transformation minimises most systematic error effects, and there is little improvement in higher order transformations.

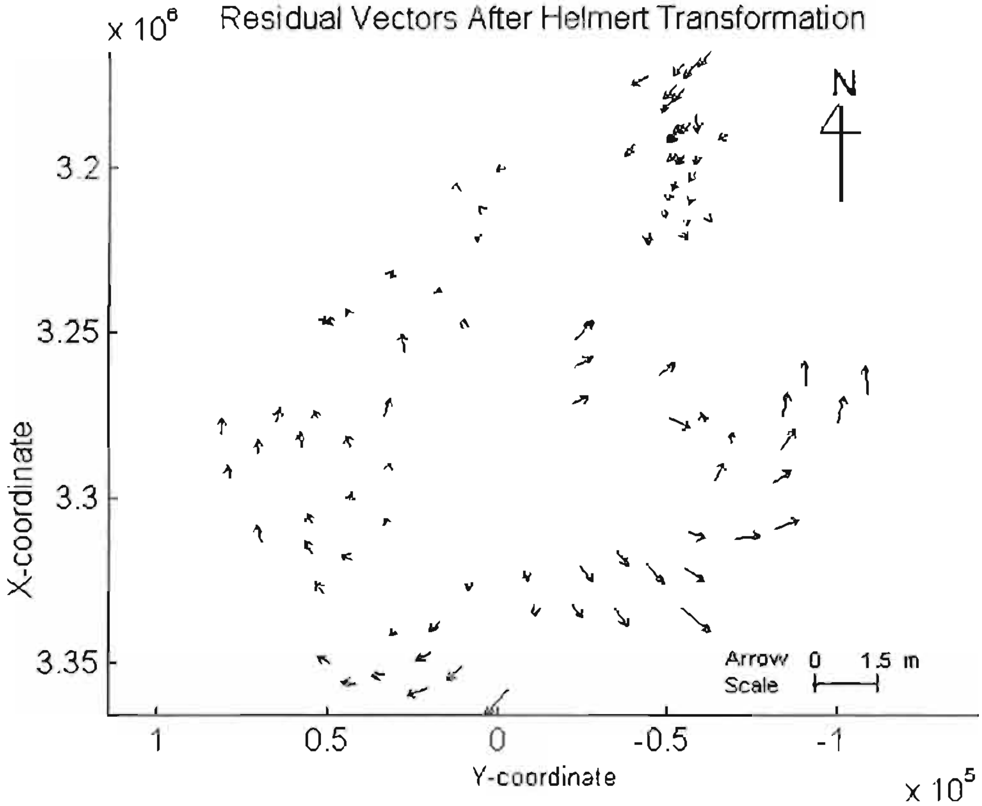


Figure 5.13 Non-primary trigs Helmert transformation adjustment vectors plot

5.4 Conclusions

Points that initially were considered as outliers were mostly mismatched. Some points were replaced, and updated in the Lesotho list but not in the South African list, while for other points erroneous matches were made.

The results reveal that primary points alone harbour more distortions than all points and non-primary points as measured by standard error of unit weight. Shift vectors showed a strong anticlockwise rotation affecting the whole country. Helmert transformation reduced these distortions, which were split by regions. The southeast region showed larger vectors than the other regions. Although most distortions were minimised in the 2nd order transformation, the 4th order polynomial transformation gives the minimum residuals.

Rousseau commented that P030 had the largest angular corrections. In this study, P013 and P035 have the large residuals as the vectors in the diagrams show, and therefore need further investigation. The tabulation below shows the ratios of the average discrepancies to the average ruling distance between points:

DOS non-primary points, average ruling distance = 8400m.

Transformation order	Average discrepancy	Discrepancy: ruling distance 1:	Ration in ppm
1	0.4407	19061	52ppm
2	0.2672	31437	32ppm
3	0.2398	35029	28ppm
4	0.2252	37300	30ppm

Table 5.10 Ratio of average vector discrepancies to ruling distance for DOS non-primary points

DOS primary points, average ruling distance = 30304m.

Transformation order	Average discrepancy	Discrepancy: ruling distance 1:	Ration in ppm
1	0.7408	40906	24ppm
2	0.3598	84224	12ppm
3	0.3212	94345	11ppm
4	0.2619	115707	9ppm

Table 5.11 Ratio of average vector discrepancies to ruling distance for DOS primary points

In both situations, the recovery is drastic from the 1st to the 2nd order i.e. by 50%, and a little after that, and so there was little point in using higher order polynomials.

Chapter 6 Lesotho GPS (Merminod's) and LHWP compared with South African Coordinates

6.1 General Introduction

In the previous chapter, the main data sets, DOS and SA, were compared. Two more data sets which make an important contribution to this research are Merminod's GPS points, since they are supposedly of higher accuracy than the others and the LHWP which has covered a large portion of Lesotho where previously the control was not dense enough for most jobs. Chronologically, LHWP control was established in the 1980s, with the last adjustment in 1984, while Merminod's control was erected in the early 1990s. The situation is that Merminod's points, like DOS's, cover the whole country and therefore are found in LHWP area. These sets will be treated separately as each has its unique coordinate values.

6.2 Merminod's points

6.2.1 Introduction

For conventional survey purposes Lesotho GPS monuments are too far apart, but are at easily accessible places all over the country. This makes them very convenient for GPS users, and they could also be used by other surveyors if the coordinates of such points were in sympathy with the older traditional control points around them. The compatibility between these GPS and DOS is tested indirectly through the SA data set, as explained in Chapter 4.

Methodology used in this chapter is similar to that used in chapter 5. Of 133 points in Merminod's list, there are only 29 matching points between the Lesotho GPS and the South African list. These two sets are related to the same WGS84 ellipsoid, and therefore the only transformation required before the comparison was begun was to move the coordinates from the two panel to a one panel, Lo28.

6.2.2 Detection of Outliers

The Helmert transformation flagged 2 points with large residuals as shown in the table below:

Name	Y(m)	X(m)	Max Residual/σ_0
P028	-5.246	2.151	5.42
P020	0.200	-3.743	3.87

Table 6.1 GPS-Helmert transformation first detected outliers

The general clockwise rotation is the result of the outlier forcing a false rotation onto the other points as shown in Figure 6.1.

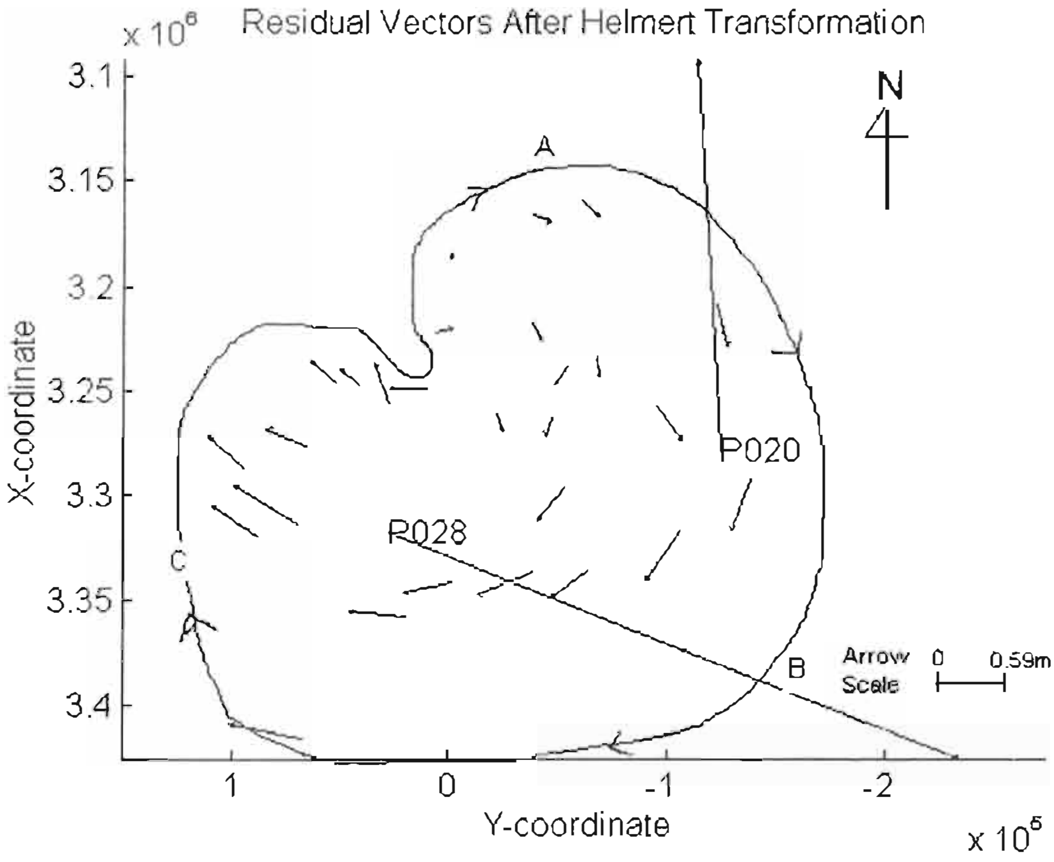


Figure 6.1 **Helmert transformation of Merminod's points onto SA coordinates before removal of any outliers**

In a second transformation after P028 and P020 were removed, P016 was flagged as an outlier, as shown in Figure 6.2 below with y-residual of -0.2881m. Compared with Figure 6.1, the residuals have become much smaller and the systematic rotation has almost disappeared.

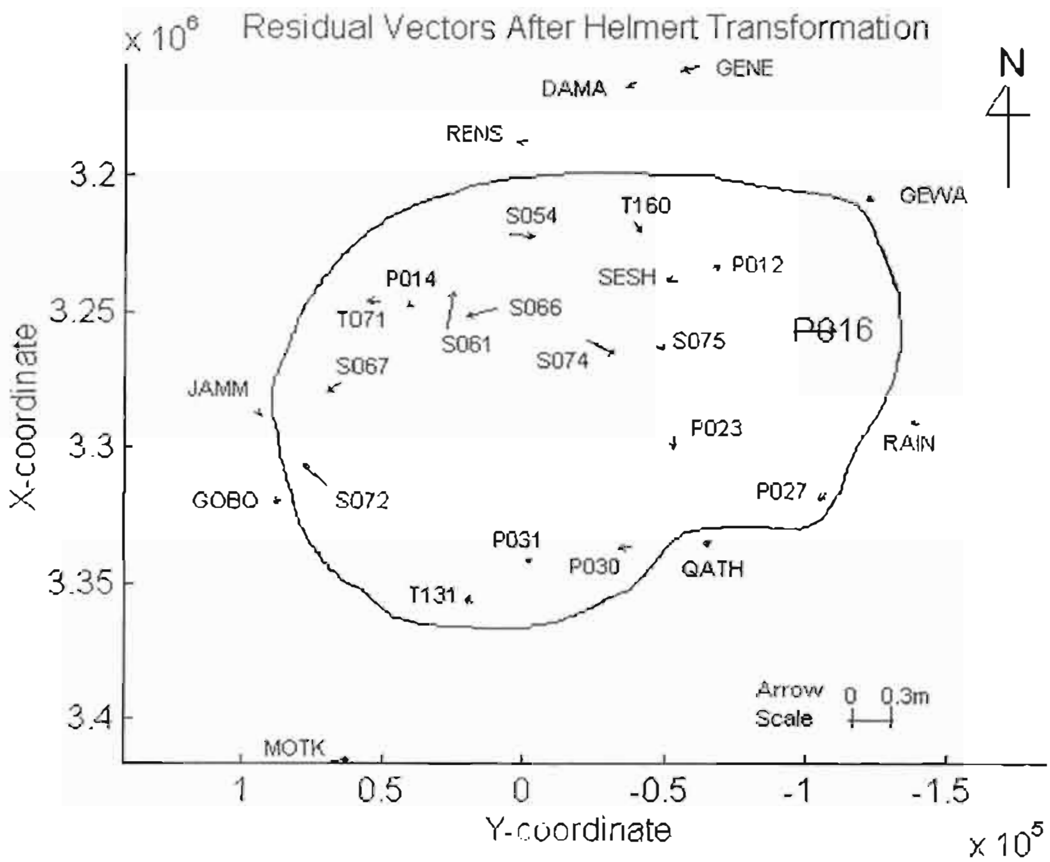


Figure 6.2 2nd Helmert transformation of Merminod's points onto SA after removal of P028, P020.

From Figure 6.2 above the encircled region excludes the South African trigs that were considered fixed for Merminod's adjustment. There is a suggestion of a slight clockwise rotation within this region. It can be seen in the diagram that these South African trigs have a general anticlockwise rotation. The only trigonometric point in Lesotho considered fixed during the same Merminod's adjustment was P014.

The higher transformation runs of the program of Merminod's points onto SA after the removal of P028 and P020 yielded results very similar to those of the 1st order transformation adjustment, except that only P016 was flagged as an outlier. The resulting diagrams are therefore not shown.

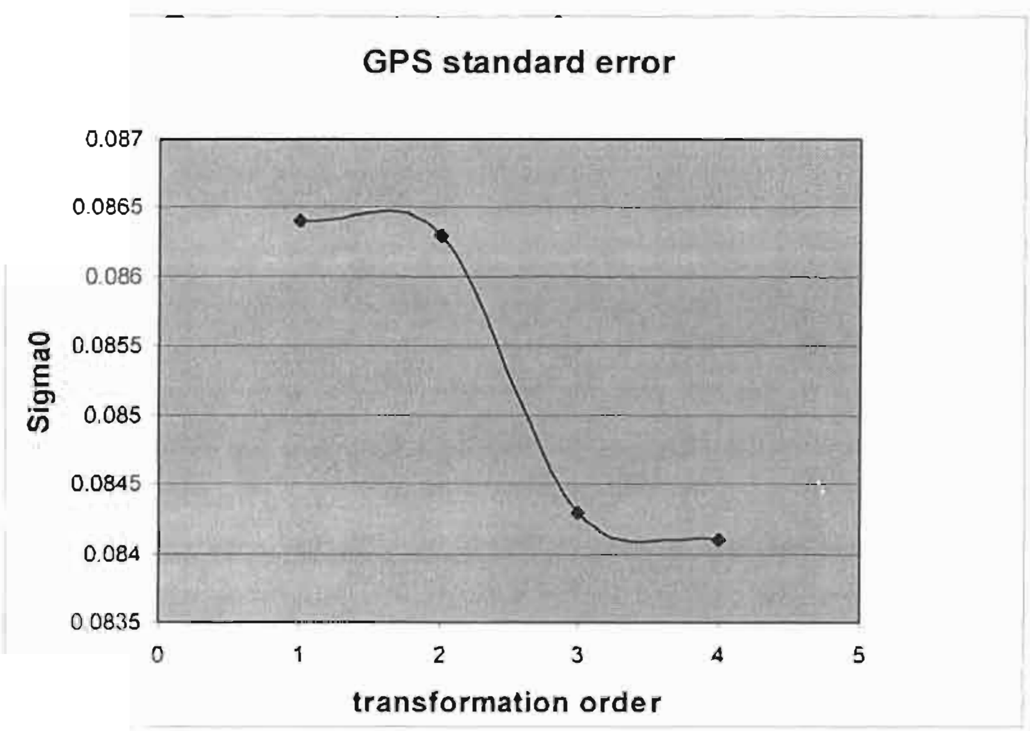


Figure 6.3 Graph of standard error against transformation order between Merminod's and SA coordinates

Figure 6.3 shows that there was a small improvement in the adjustment of the points between 1st and 2nd order, and 3rd and 4th order, as measured by the standard error of the residuals. Most improvement was obtained in the 3rd order transformation. (See also Figure 6.4). The relative improvement achieved by a higher order transformation was however, only about 3%, unlike in the previous DOS situation where it was more than 10%. The rotation angle is small, 0".01979, scale factor 1.000000082

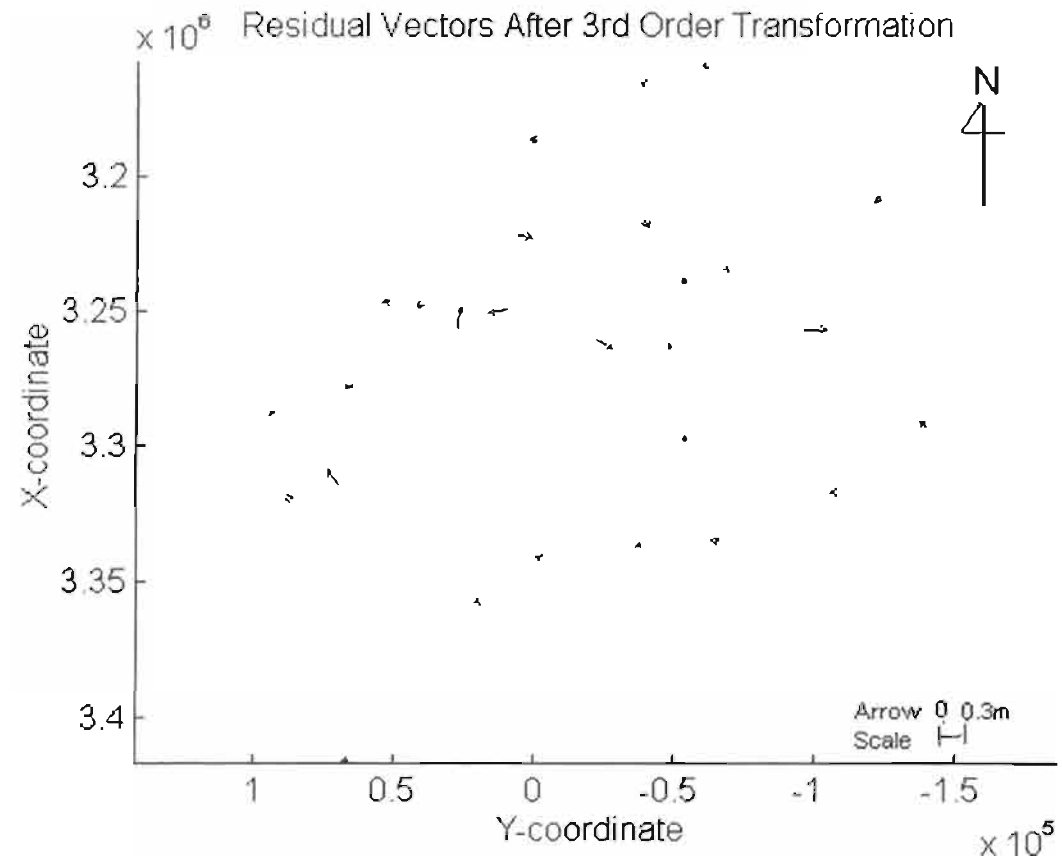


Figure 6.4 Third Order Transformation vectors

6.2.3 Causes of Discrepancies

It was found during Merminod's GPS campaign that the P028 pillar had been destroyed and the distance between the witness marks perceived as WM1 and WM2 was wrong by 21cm. Since the centre hole was not located, it was believed that WM1 had been used for the measurements (Merminod 1992).

P020 had also been destroyed. The centre hole was located in 1991 but was unusable since it was in a loose rock. One of the 3 holes believed to be WM2 found nearby was used (Merminod 1992).

6.2.4 Comparison of patterns between DOS/SA and Merminod's/SA

The rotation patterns of the vectors in both the DOS/SA and Merminod/SA situations are different. The shift patterns of the DOS/SA adjustment shows a strong, well-defined rotation. Merminod/SA shift pattern on the other hand is not so well structured. In the diagrams where outliers have not been removed (Figures 5.1 and 6.1), the rotations are clearly defined in both cases but this can be partly explained by the outliers forcing a compensating rotation onto the other common points. In the DOS/SA situation, the removal of outliers reduced the sizes of vectors slightly, and affected the direction of rotation so that all the vectors on the western side of the country, which were first in the clockwise direction, had an anticlockwise rotation (Figure 5.4). The direction and magnitude of vectors on the eastern side were affected very slightly. In Merminod/SA comparison, the removal of outliers reduces the rotation pattern (Figure 6.2) resulting in very nearly randomly directed vectors. Both these Figures relate to the Helmert adjustment.

In Chapter 5 we noted rotation patterns in DOS/SA comparison, suggesting systematic effects in the DOS horizontal network of Lesotho, which is apparently concentrated in the South-East. This is the side of the country where errors were forced to accumulate. On the other hand, one can not point out a specific area in Merminod/SA, with the present data and diagrams, to say where the sources of errors are.

Both the Lesotho and the South African GPS observations and adjustment were done almost at the same time and in the same system, WGS84 system. Merminod, (1993), records that he used 7 South African points out of 8 as his fixed control in the new primary points adjustment. The residuals are very small and the standard error is 0.086.

6.2.5 The Future use of GPS in Lesotho

Present use of the GPS control network

It has been found that the GPS monuments in Lesotho are at present of very little use to the surveying community since firstly, most do not have GPS equipment, and consequently, there is very little interest in the related activities. Secondly, the GPS network in Lesotho is not in sympathy with the DOS network. Thirdly, even when these GPS monuments are in the vicinity of the surveyor's area of work, they are not noticed because they are at ground level and as a result, a surveyor will establish his own control using the traditional manner.

The government survey office has made very little use of these monuments although it has had the GPS equipment for more than 4 years now. The typical project using GPS equipment is in real time kinematic (RTK) mode, which involves placing of cadastral beacons rather than enhancing the geodetic control.

It seems likely that, with time, this passive GPS network might be regarded as useless if not made homogeneous with the surrounding DOS network. Since the networks already exist, it would be a good idea for the government to resolve the differences between the two, by using a stronger network to restrain the weaker network. This will have more or less the same effect as adopting the coordinates of the latest South African adjustment.

6.2.6 Conclusions

Lesotho GPS/SA as compared to the DOS/SA analysis shows that indeed there are some systematic errors in the DOS observations. Comparing the two Helmert transformations onto the SA set, the rotation angle of the GPS set of data is small (0".01979) compared with the DOS (more than 3") and has a smaller standard error (0.086) with less proportional improvement between the 1st and 4th transformation order, whereas the DOS standard error is about 0.4 and 10% improvement occurs by using a higher order transformation. This might be because the DOS observation reductions were not complete, for instance, not applying deviation of the vertical correction. Also, traditional survey methods were laborious and prone to mistakes. Hence Merminod's GPS data is of better fit onto the SA network than is the DOS data set.

The following table shows the discrepancies ratio to the ruling distance of 22103m

Transformation order	Average discrepancy	Discrepancy: ruling distance 1:	Ration in ppm
1	0.1029	214801	5ppm
2	0.1024	215850	5ppm
3	0.1024	215850	5ppm
4	0.1022	216272	5ppm

Table 6.2 Lesotho GPS vector discrepancies

6.3 Lesotho Highlands Water Project (LHWP)

6.3.1 Introduction

The Lesotho Highlands Water Project (LHWP) is a large engineering water project involving both South Africa and Lesotho. It has taken several decades from the conception to the implementation stage for various reasons. The extent of the project in Lesotho is shown in the Figure 1.5 of chapter 1.

At the completion stage, the project will comprise the following, as shown by Spie Batignolles (undated), one of the project contractors:

- Katse dam, from which is linked a 45km transfer tunnel to 'Muela dam and Power station,
- 'Muela dam where there is a delivery tunnel into South Africa,
- Mohale Dam which feeds the Katse dam through an interconnecting tunnel of about 30km,
- Mashai dam with another transfer delivery tunnel to the Ash river outlet,
- Tsoelike dam with a water pumping station.
- Ntoahae dam also with a pumping station.

Together with access roads and power lines, this makes a huge project, which in all respects needs a good geodetic control. There are DOS trigs in the area, though not dense enough to cover the construction sites satisfactorily, there are LHWP beacons, some replacing destroyed DOS beacons and others newly established for the purpose of densifying control. There are also new GPS monuments. The list used for the analysis is the LHWP list. These coordinates were recomputed by Department of Public Works and Land Affairs (PWLA) in Pretoria, resulting in slightly different coordinate values, which are not considered in this thesis.

6.3.2 Provision of original control

The original plan geodetic control in the area is that established by DOS. Because of the inaccessibility of some points or lack of proper control in the vicinity of the construction sites, the contractors had to build 23 new trigonometric points, where appropriate, to facilitate mapping and construction of infrastructure.

The level control in Lesotho at the beginning of the project was the geodetic control from Makhaleng Bridge to Maputsoe, and the one from Witsieshoek in South Africa to Pelaneng in Lesotho. These had to be extended to the other sites of the project by the contractors of LHWP. Figure 6.5 shows the network at the beginning of survey.

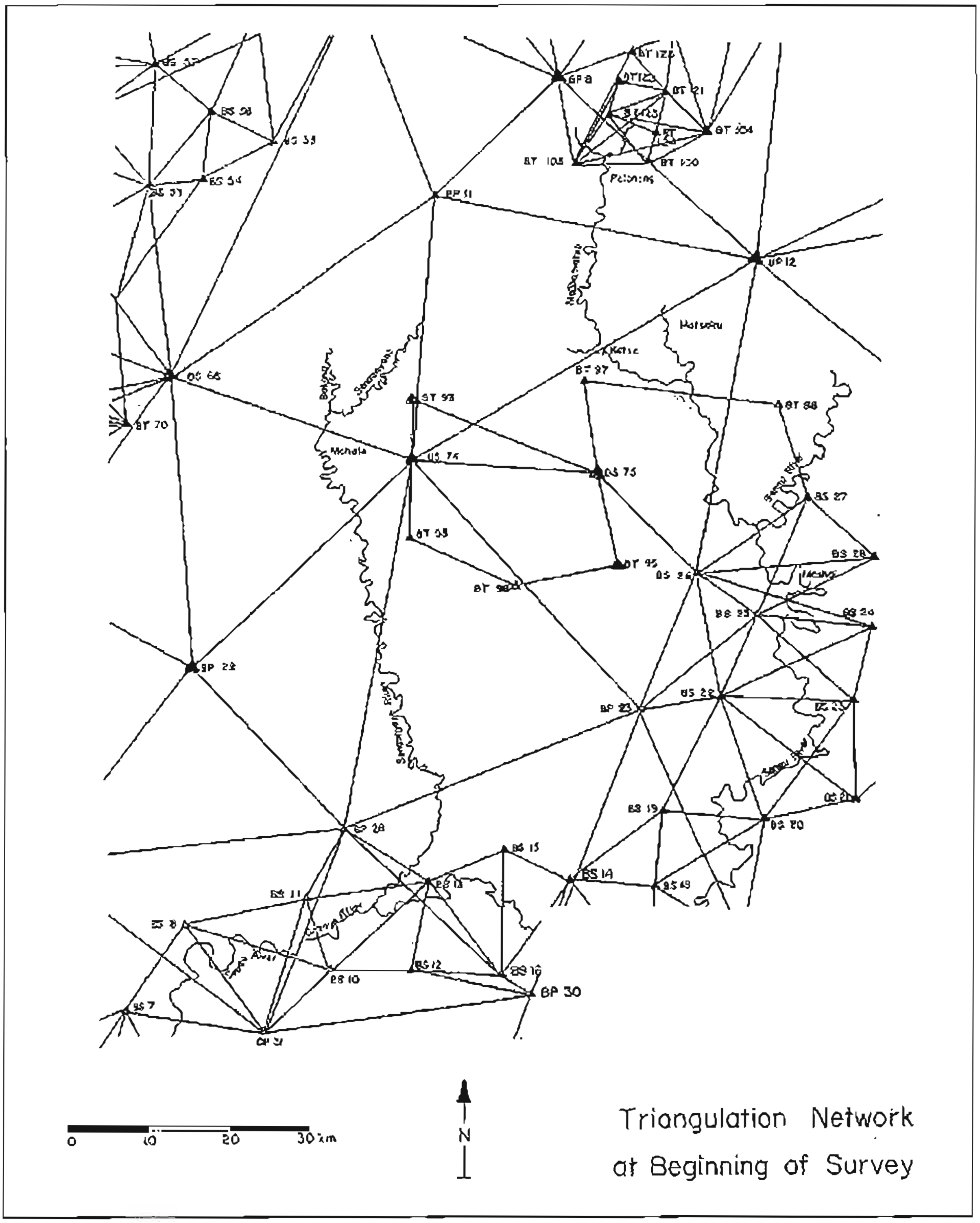


Figure 6.5 Triangulation Network at the beginning of the Survey

(Lesotho Highlands Water Project (1986))

6.3.3 Data analysis

There are 70 points common to the LHWP and the SA data sets. The first Helmert transformation program run yielded no outliers. Residual and shift vectors are large. The vectors produced revealed the patterns indicated in Figure 6.6.

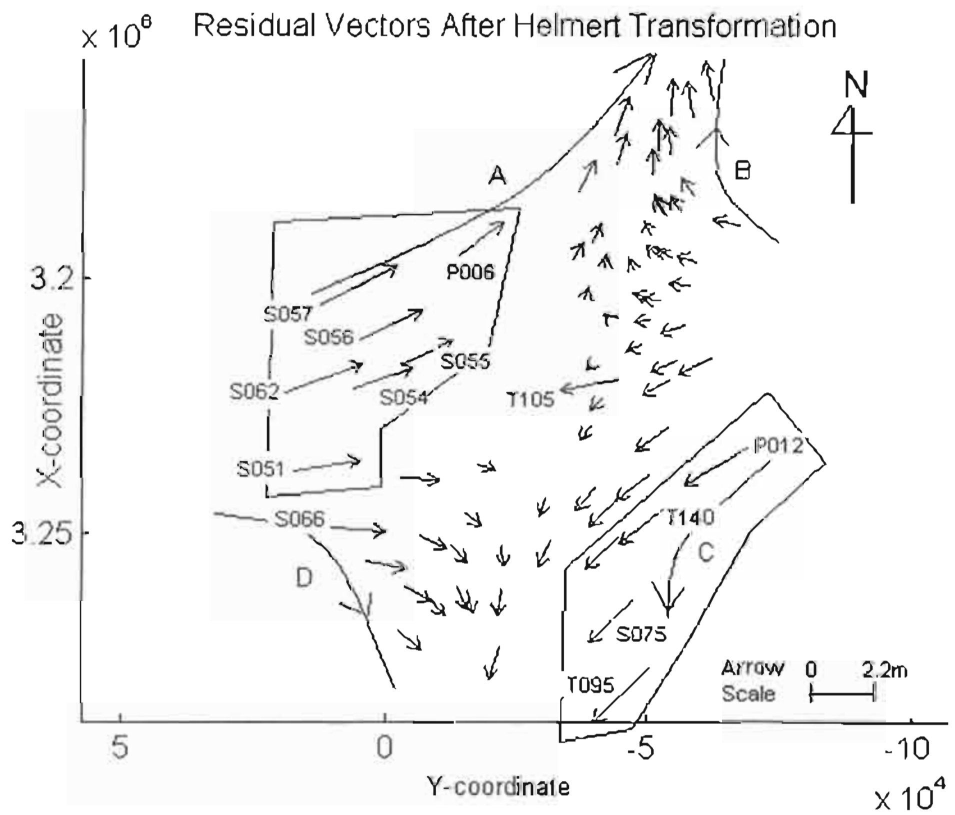


Figure 6.6 Helmert transformation residual vector plot of LHWP network

The rotation patterns are as in Figure 6.6. Easterly vectors follow curves A and D, splitting to the North and South and joining those of B and C from the West at the centre. The magnitude of vectors along curve A and C is larger than at other areas.

These vectors are enclosed in polygons. A labelled diagram (Figure 6.7) gives the names of these vectors.

The following values were obtained in the same Helmert transformation:

Standard error = 1.050m

Maximum absolute residuals: y = 2.940m x = 2.110m

Minimum absolute residuals: y = 0.021m x = 0.041m

Rotation angle 5". 6936

Scale factor 0.999743

The above values suggest something is wrong with the Lesotho Highlands Water Project network. Merminod (1992 p.7), says this about the LHWP topographic surveys "... and the coordination of their topographic work is far from being an example". It is surprising that using the South African 1983/84 adjustment values, which were the latest at the time, their values came out the way they did.

The 3rd order transformation flagged points S057 and P012. The 4th order transformation flagged P012 and P006. Even the 4th order polynomial transformation could not change the rotation patterns noticeably. As can be evidenced from the diagrams, the further one goes in the west, the larger the vectors become. The patterns suggest that whatever types of systematic errors were incurred, they were propagated further and were allowed to accumulate. In contrast, on the eastern side the vectors were being slightly reduced.

LHWP (1986 p. D1-2) reports that

There is however a problem that these beacons have evolved from two independent operations interconnected only at BP8, BT100 and BT105. The Southern and Northern part of the net will have to be connected by additional observations and consideration will have to be given to the establishment of the first order triangulation network which will cover the complete project area.

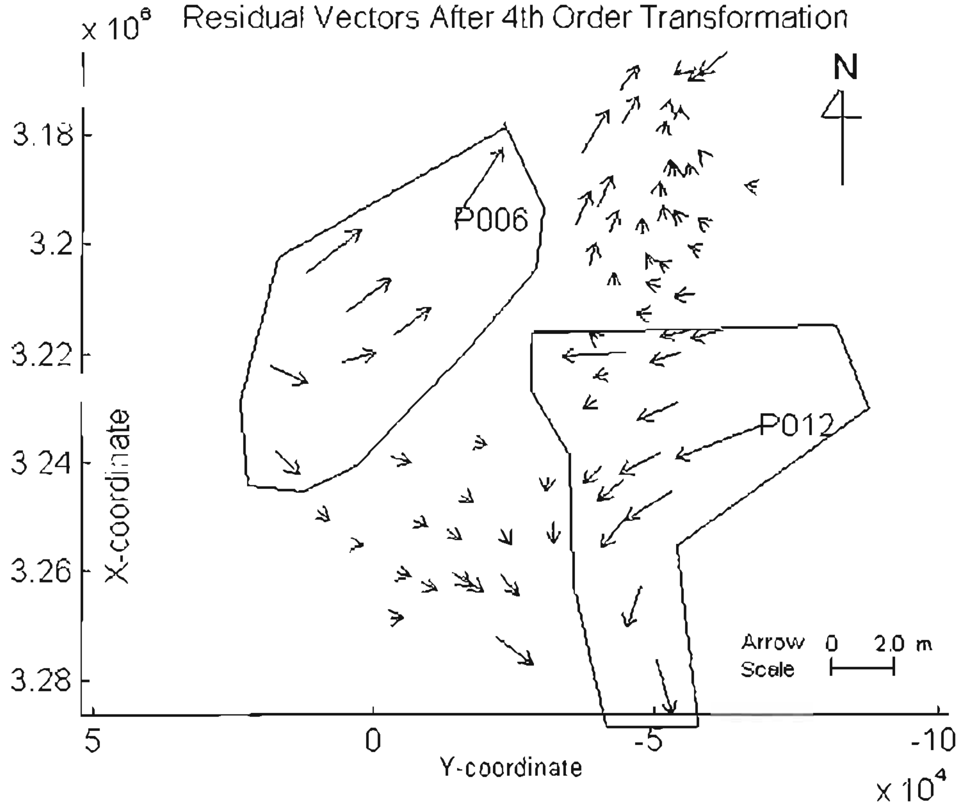


Figure 6.7 Fourth Order transformation for LHWP network

This network forms a large part of the Lesotho geodetic network, but it has discrepancies within itself. It is mentioned that because of the high cost and clear visibility in the mountains, lamps and helios were not used (Feasibility Study 1986). It is also reported that some beacons were not always visible due to cloud cover, haziness or other weather interruptions. Perhaps helios should have been used during their observations, just to be on the safe side. The large residuals obtained were concluded to come partly from observational errors and partly from the points selected to constrain the adjustment. These errors show lack of proper supervision and motivation and will prove costly in the future, since the work might have to be done again.

6.3.4 Conclusions

The systematic errors in the LHWP network are large as revealed in the diagrams. Merminod, (1992) pointed out that the LHWP was interested only in areas where it was affected, as far as mapping is concerned. He also commented that all their jobs were contracted, with the knowledgeable people in Pretoria, and no one locally from the LHWP knowing what was going on. Although the government survey office is involved mostly in cadastral surveys (because it generates revenue), the evidence from the LHWP suggests that the Lesotho government should play a role in activities of geodetic nature and other topographic surveys everywhere in the country, to check that appropriate specifications are achieved and by the right people.

The following table shows the discrepancies as found in the LHWP control: The average distance between trigonometric stations was 7200m.

Transformation order	Average discrepancy	Discrepancy: ruling distance 1:	Ration in ppm
1	1.2882	5603	178ppm
2	1.2013	5962	168ppm
3	0.9830	7286	137ppm
4	0.9532	7513	133ppm

Table 6.3 LHWP vector discrepancies

The LHWP network forms part of the geodetic network of Lesotho, but has so far contributed little towards strengthening it. The project dams have created a centre of attraction. which will soon need to be surrounded with good infrastructure to accommodate tourist's needs. Monitoring of dams and other earth movements in the vicinity of dam sites caused by dam loading is important, and requires the network to be checked on a regular basis, to monitor movement of the whole subsiding area. Weak surrounding network may lead to more confusion, especially because Lesotho relies on control points in its cadastral fixed boundary surveys.

It is unfortunate that the opportunity was not used to strengthen the whole network in the area of the LHWP. The Lesotho government survey office has now to work out a way to deal with the situation in the area. and it is hoped that informed members of the LHWP will be involved this time. The distortions in the area are large. It would therefore make little sense to apply the calculated parameters to non-common coordinates in the LHWP set to try to bring them into the consolidated coordinate set. Merminod's points can be used as a fixed frame for the GPS observation and adjustment of the LHWP points. This would help homogenise the networks.

The grouping of distortions shown in figures 6.6 and 6.7 suggest that a rubber sheeting approach to transformations might be more appropriate for transforming non-common points than the polynomial approach used. However, the polynomial approach enables these systematic effects to be clearly revealed.

Chapter 7 Choice of Projections for updated control system

7.1 Introduction

Up to now Lesotho has published control point co-ordinates on the two-panel Gauss Conform or Transverse Mercator (TM) projections. centred on the Lo27 and Lo29 meridians. This follows what is done in South Africa. A large part of the country lies in the overlap area (one quarter degree each side of Lo28 degrees). In this overlap region it is not clear which map panel to use, corrections are large and it is often necessary to transform point co-ordinates from one map panel to another. When an updated list is prepared, it will be appropriate to make another choice of map projection. This chapter investigates possible choices of projections to use in Lesotho, including the use of one panel. and the use of the UTM projection of which there is a likelihood that some donor countries might support its use in Lesotho. This last projection has also found a wide recognition around the world, and is not very different from the presently used projection, therefore it is likely that Lesotho would choose it if it decides to go for a change. This chapter goes on to demonstrate how the errors affect measurements in different places across the country. The shortcomings of the projections in use are highlighted. Then a system is suggested which could be used in place of the existing projection.

Snyder (1987) refers to other conformal projections used in different parts of the world according to its position. extent or shape. including the.conic e.g. Bipolar Oblique Conic conformal. Lambert Conformal or cylindrical e.g. Oblique Mercator. Mercator or azimuthal e.g. Stereographic. These were not considered because only the UTM and Gauss are used in neighbouring countries.

7.2 Properties of Projections used

7.2.1 Transverse Mercator

According to Davis, Foote, Anderson and Mikhail (1981, p. 572-573), the properties of the transverse Mercator projection are the following:

a) It uses a cylinder in a transverse position such that it touches the earth along the central meridian,
b) The scale is true along the central meridian,
c) The central meridian is used as the origin for the y-coordinates of the map in the Southern hemisphere,
d) Origin of the x-coordinate is the equator,
e) Both the central meridian and the equator are straight lines,
f) Other meridians are complex curves that are concave toward the central meridian,
g) Parallels are concave toward the poles,
h) Projection used mostly for state plane coordinate system of states (or zones thereof) of greater North-South than East-West extent.

7.2.2 Universal Transverse Mercator

Davis et al, (1981, p. 574) state that the UTM is based on the transverse Mercator projection. The characteristics of the UTM as of 1958 are:

1. Transverse Mercator projection is in zones that are 6° wide,
2. The reference ellipsoid Clarke 1866 is used in North America,
3. The origin of longitude is at the central meridian,
4. The origin of latitude is at the equator,
5. The unit of measure is the metre,
6. For the Southern Hemisphere, a false northing of 10,000,000m is used,
7. A false easting of 500,000m is used for the central meridian of each zone,
8. The scale factor at the central meridian is 0.9996,

9. The zones are numbered beginning with 1 for zone between 180°W and 174°W meridians and increasing to 60 for zone between meridians 174°E and 180°E.
10. The latitude for the system varies from 80°N and 80°S.

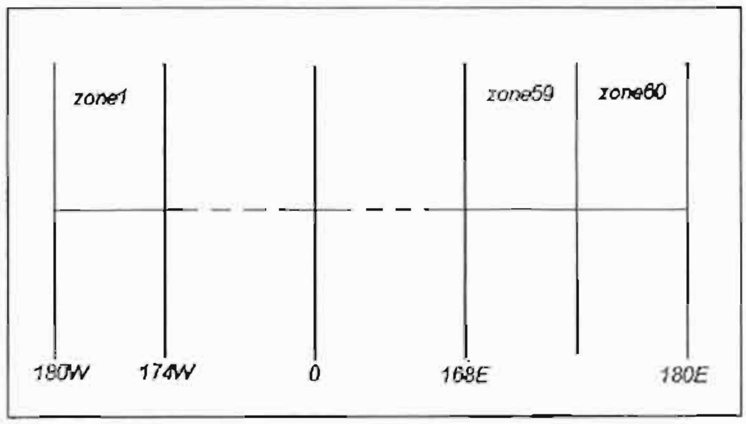


Figure 7.1 **UTM zones**

Figure 7.1 and 7.2 show the UTM zones. Lesotho is in zone 35, using Longitude 27° East as central meridian.

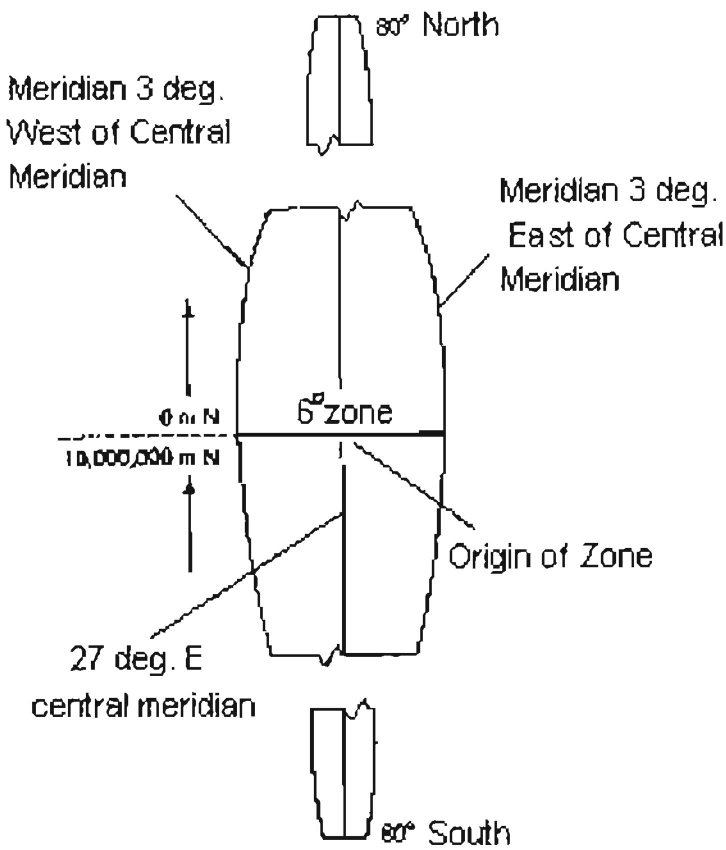


Figure 7.2 **One UTM zone**

According to the Ministry of Defence, (1965, pp. 97-98), the following is stated about the UTM:

> In order that the longitude covered by the projection may be extended as widely as possible without introducing intolerable scale errors, it is normal to introduce a scale factor. This has the effect of reducing the scale on the central meridian. The scale factor is chosen in such a way that the scale at the east and west limits of the projection is made too great, by the same amount as the scale on the central meridian is made too small. This will have the effect of making the scale correct on lines situated about two thirds of the distance from the central meridian to the limits of the projection.... The Universal Transverse Mercator projection..... is a development of this idea.

7.3 Distortions when projecting measurements from the ground to the map.

Survey measurements are distorted when projected onto the projection, principally by the sea level, scale enlargement and (t-T) corrections. Clearly, surveyors would prefer a map projection where for most surveying purposes, these corrections could be ignored. To test the size of corrections under different scenarios, a section was drawn across Lesotho on a 1:250 000 map, along latitude 29° 30′ and heights measured at test points along it. At each test point the following corrections were calculated and graphed:

1. (t-T) correction in seconds for a ray of 1km length North-South.
2. Sea-Level correction on a 1000m line,
3. Scale Enlargement on a 1000m line,
4. Combined Sea-Level and Scale Enlargement corrections on a 1000m line .

The following equations simplified from Schreiber (1955) were applied to both the Gauss Conform and Universal transverse Mercator projections:

1) For the scale enlargement

$$Scale = S(f + f\frac{y^2}{2R^2} - 1)$$ **Equation 7.1**

2) The sea level correction (which is always negative) is given by

$$clevel = S\frac{h}{R}$$ **Equation 7.2**

3) t-T correction:

$$t - T = \frac{\rho.\Delta x.y}{2R^2}$$ **Equation 7.3**

where:

S is the distance measured (=1000m)

f is the scale factor at the central meridian

For Gauss Conform scale (f) =1; for UTM scale (f) = 0.9996

y is the distance from the central meridian

R (is the radius of the earth) = 6356000m

ρ =206265

Δx =1000m.

These formulae were used in a spreadsheet where the following graphs were drawn. The Tables of these computations are in appendix (4).

7.4 Lesotho on Gauss Conform Projection

For projects extending beyond the edge of one panel, a 15′ overlap on either side of the next central meridian, Lo28 is used as seen from Figure 7.3. This means that a substantial part of Lesotho lies in the overlap between the boundaries shown by dotted lines (within 25 000m from Lo28).

For the sea-level correction it can be noted that generally the Western side of the country is the lowest at about 1500m while the eastern side is in the high Drakensberg range and is above 2 000m. Most development and therefore survey work is carried out in the Western and central areas.

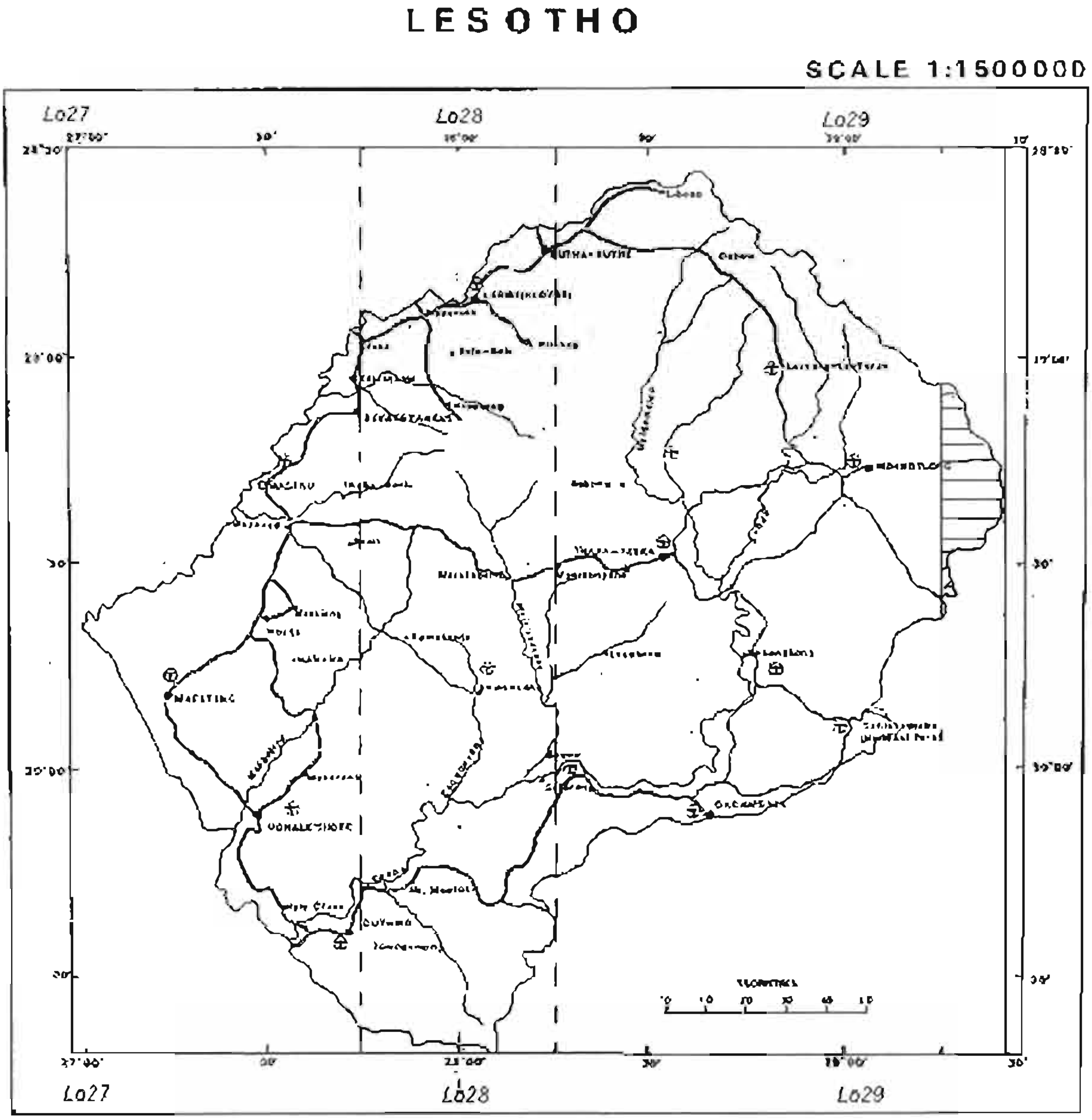


Figure 7.3 Different Los in Lesotho
(Department of Land, Surveys and Physical Planning – Lesotho)

7.5 Results of scenario testing.

7.5.1 Using two panels of Gauss Conform Projection: On Lo27 and Lo29.

(Refer to Table 1 in appendix (4) and Figure 7.4)

On both panels the minimum correction for the scale enlargement and (t-T) corrections is at the central meridian, where both are zero. The t-T correction increases linearly with distance from the central meridian, reaching a maximum of 0.61 seconds for a 1000m dray at the outer edge of the overlap. For almost all engineering and cadastral survey purposes using traversing methods, this correction can therefore be neglected.

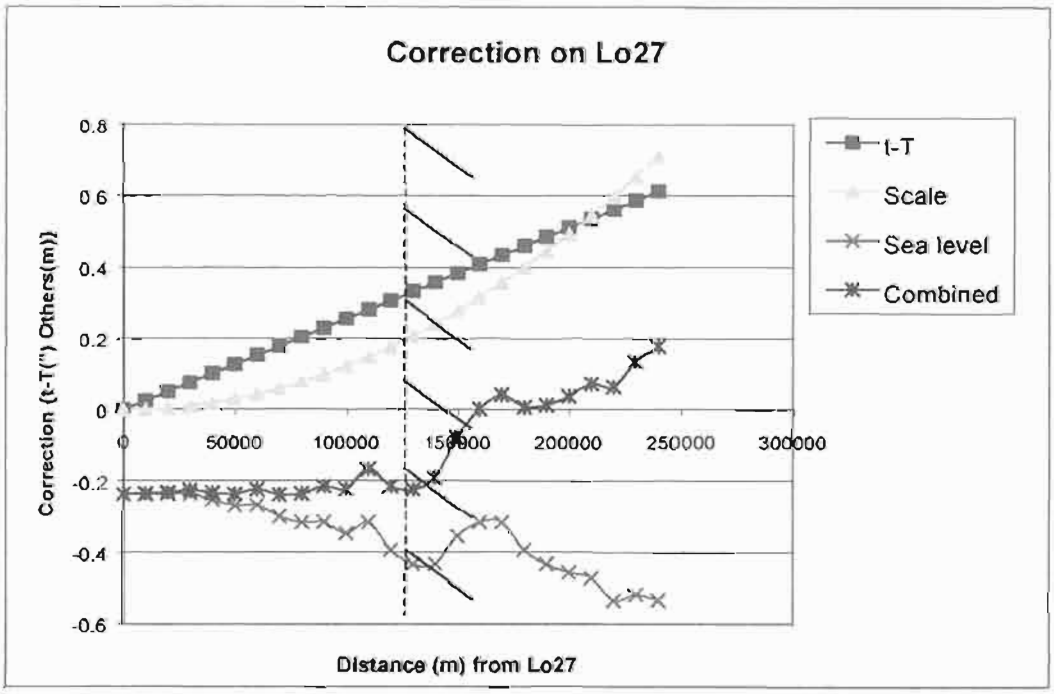


Figure 7.4 Corrections on TM projection on lo27 along 29° 30′ parallel

(The area indicated by the diagonal lines to the right of the dotted line signifies the area beyond the normal overlap region where the 2-degree Gauss Conform panel is normally not used because of rapidly increasing corrections)

The scale correction graph is a parabolic curve. The rate of change of this curve varies exponentially with the distance from the Lo. At a distance of 125km from the central meridian, the e correction for a 1km line reaches about 0.2m

The Sea level correction starts at –0.24m for a 1km line and becomes negatively larger as the test point moves to higher ground in the East. This correction is inversely proportional to height and therefore where the graph shows a trough, we find high point and where there is a bump, we find a valley on the ground..

The combined sea-level and scale enlargement correction therefore starts at the central Lo with the sea level correction, bears the same shape as the sea level correction but departs from it gradually owing to the influence of the scale correction on it. The turning point of this curve is around 130km from Lo27 where it becomes positive. For the region where Lo27 panel has been used up till now, the combined correction graph is fairly horizontal at about –0.24m for a 1km line.

On Lo29, (Refer to Table 3 in appendix (4) and Figure 7.5) the scale enlargement curve is parabolic with a minimum value less than 0.0002m occurring at a distance between 200km from Lo27. Lo29 is only about 40km from the eastern border of Lesotho, and the scale correction is 0.026m and small on this portion of land. On the western side, the scale correction increases to 10cm at 100km and 0.190m at 125800m from Lo29.

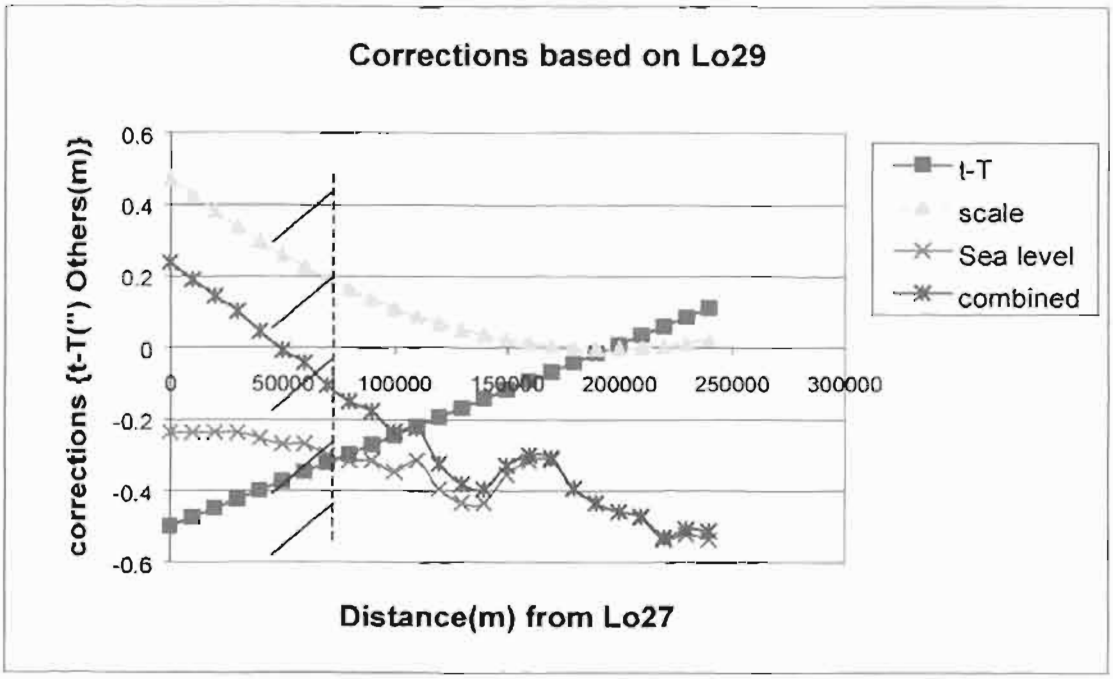


Figure 7.5 corrections on TM projection on lo29 along 29° 30′ parallel

(The area indicated by the diagonal lines to the left of the dotted line signifies the area beyond the normal overlap region where the 2-degree Gauss Conform panel is normally not used because of rapidly increasing corrections)

The sea level correction curve is exactly like the one on Lo27, starting at –0.24m on Lo27, skewing slightly to the right with two peaks at around 110km and 150-160km. The combined graph still resembles the sea level graph in that the same heights and distances give it the same bumps as those in the sea level curve. As the scale correction curve approaches zero at Lo29, the combined correction gets closer and closer to the sea level correction. The area around Lo29 is mountainous. Since the scale correction is zero at the central meridian, it is to be expected that the combined correction would be close or equal to the sea level correction near the central meridian.

The t-T correction for a 1km North-South ray is again a linear curve that reaches only 0,5 seconds at the Western edge of Lesotho.

The combined representation of corrections using the two panels is shown in Figure 7.6.

It can be seen that the t-T correction can be neglected as it reaches the value of only about 0.3 seconds at the distance of 1° 15′ from the Lo. The scale enlargement corrections are about 0.2m at 125 000m from both central meridians, and this cannot be neglected over a distance of 1km. The sea level correction has the dominating influence in the combined correction. The corrections are largest in the overlap region which is 15′ on each side of longitude 28, indicated by the shaded area in Figure 7.6.

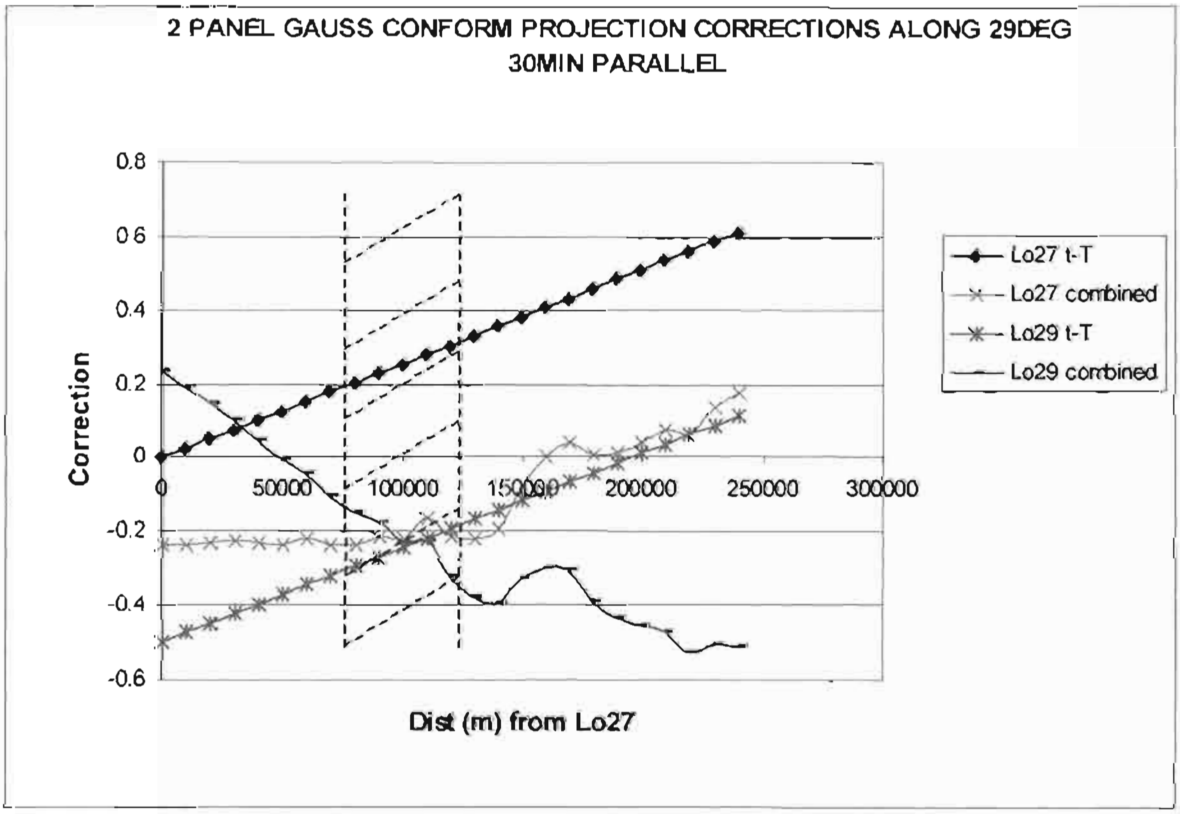


Figure 7.6 **Combined corrections for Lo27 and Lo27**

7.5.2 Using a single Gauss Conform panel with central meridian 28 degrees East.

(Refer to Table 2 in appendix (4) and Figure 7.7)

Lo28 is situated near the centre of the country. The eastern border of the country is nearly 140km away from Lo28 while the western border is only about 96km away. The most eastern part of the country is outside the boundary of Lo28 corrections, but this portion of land is about 15km wide. The maximum combined sea-level and scale enlargement correction applicable from Lo28 in this part would amount to 0.25m over 1km.

From the results, the combined correction is closest to zero at the lowlands and at its maximum on the west at around 50km from Lo28.

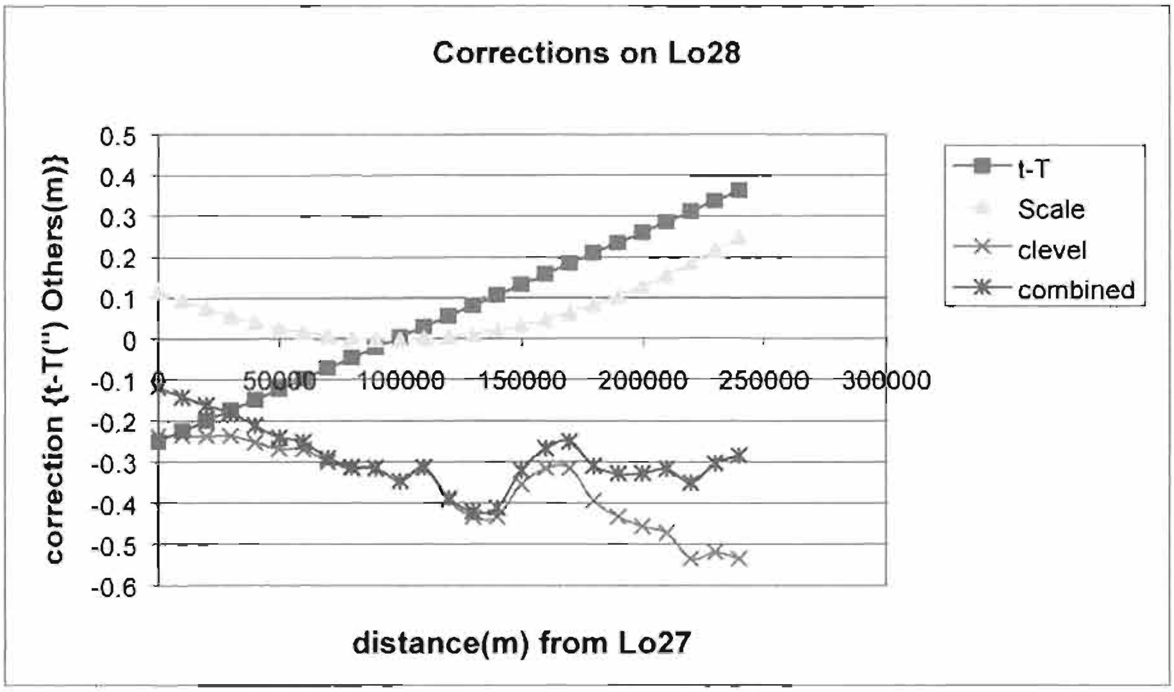


Figure 7.7 **Corrections on TM projection on lo28 along 29° 30' parallel**

7.6 Conclusions on Transverse Mercator projection

The maximum combined sea-level and scale enlargement correction with this option of a single Lo panel would be 0.43m on 1000m. This is not good for long legged traverses and road survey works, though considerably better than the 2 panel Lo with 0.53m per km on Lo29.

In Lesotho, the difficult terrain causes the combined correction to be reliant on height as well as the Lo used. Lo28 would seem the most suitable for most of the country as it gives a fair representation of corrections in all graphs. The coverage of the country is good, whether one panel or two panel are followed.

The t-T graph shows that this correction is not significant for most engineering and cadastral purposes.

7.7 Mapping Lesotho on the UTM zone 35

(Refer to Table 4 in appendix (4) and Figure 7.8)

With UTM, each zone spans 6°, with 3° on either side of the central meridian. Lesotho lies in zone 35, with the central meridian on Lo27, the lower bound at 24°E and the upper bound at Lo30°E. Lesotho lies between Lo27 and Lo33, as the most western border is about 2° 24′ from Lo27.

From Figure 7.8, the false scale factor of 0.9996 produces a scale enlargement correction at the central meridian of –0.4m. This correction decreases (i.e. gets closer to zero) as the distance from the central meridian increases, and is zero at around

180km, which is one quarter of the total distance of the upper limit from the central meridian. The maximum scale correction goes above 0m is 0.31m. The sea level correction graph is the same as in the Gauss Conform graphs.

The combined correction is –0.6m at the central meridian and remains near this value up to a distance of 100km-110km, where we find the Senqunyane and Senqu valleys. Thereafter, the slope of the curve becomes generally positive.

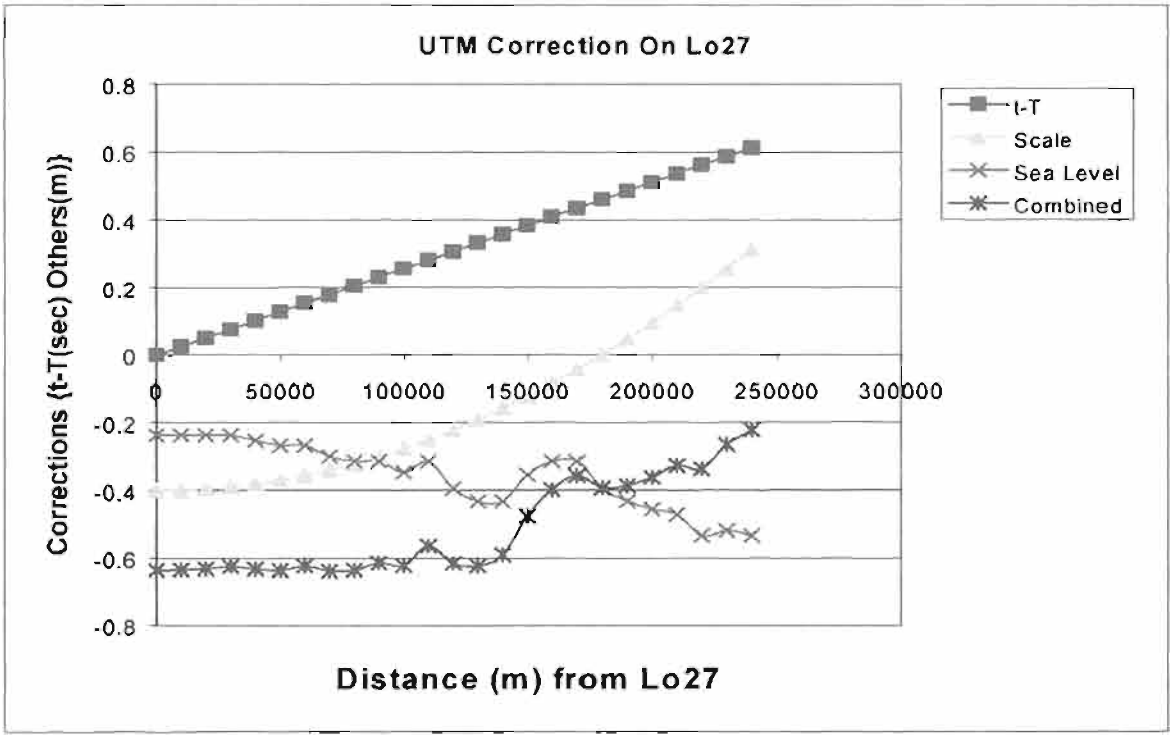


Figure 7.8 **Corrections on UTM projection on lo27 along 29° 30′ parallel**

The minimum combined correction of -0.22m is at the most eastern and highest point.

7.7.1 Conclusions on Universal Transverse Mercator projection

With the UTM, the maximum combined corrections of magnitude 0.63m are in the lowlands (see Figure 7.8). This value stays almost constant up to around 100 000m from Lo27, where the terrain starts changing. Although the country can be covered under one zone or panel, the 0.6m per km correction is the largest in the projections under investigation, and can be disturbing since it is largest in that part of the country where most developments take place and where it would be more convenient if the corrections could be small. It can be seen that as in the case of Lo27 in the 2-panel situation, the UTM combined correction then rises steeply, getting closer to zero as one goes higher in altitude and further away from the central meridian.

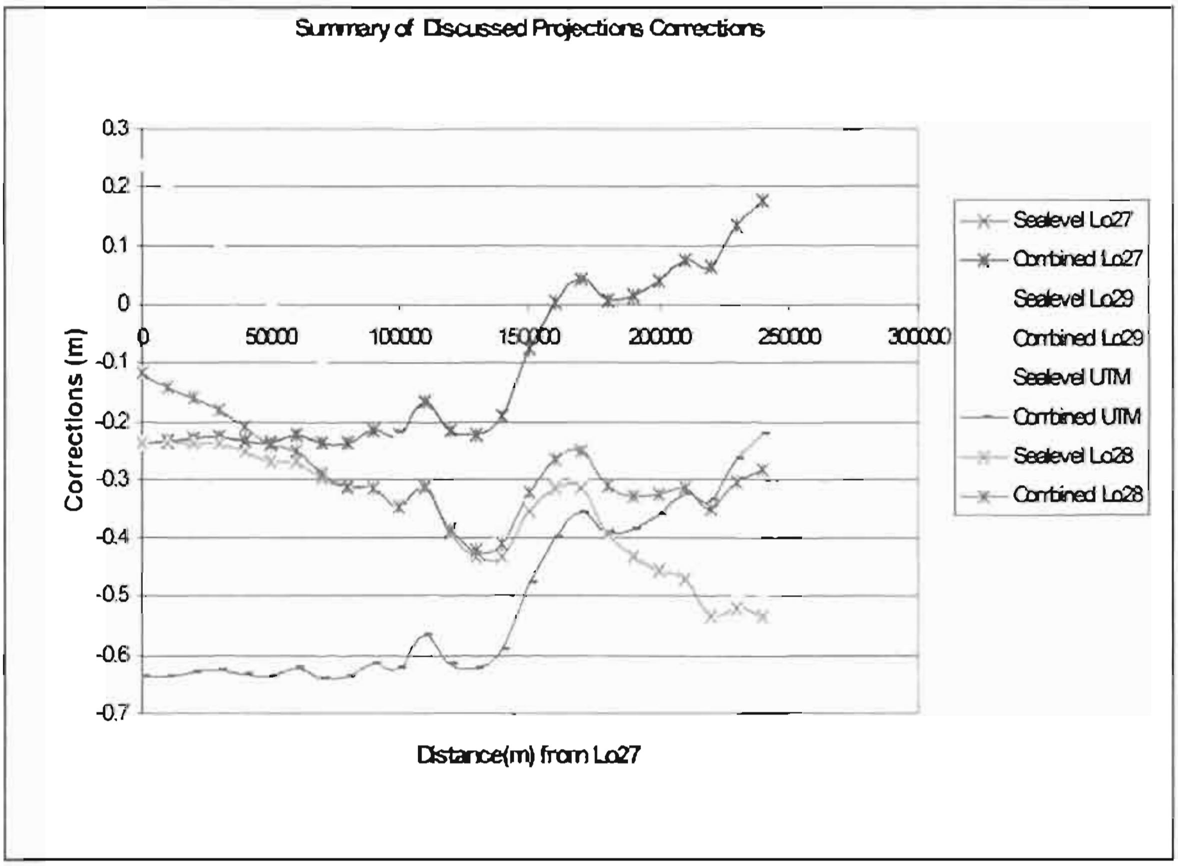


Figure 7.9 **A summary of the corrections for the projections under investigation**

As shown in Figure 7.8, the corrections for Lo27 on a Gauss conform projection are parallel to those on the UTM projection but differ in magnitude.

7.8 General conclusions and Recommendations

7.8.1 Implications of remaining with 2-Lo-panels:

It has been shown that on the Gauss Conform, the two Los used in Lesotho cover the country well in extent. Confusion arises, however, with the use of points in the overlap zone. Abandoning this system may involve more work, money and time, but if the general feeling is that the two panel system is satisfactory, especially with the privately practising surveyors, then there will not be any need, at least for the near future, to change it. The office of the chief surveyor should conduct a survey to find what the users' views regarding this matter are, and only then should action be taken.

7.8.2 Option of a single Lo-Panel centered at Lo28 degrees.

It would seem that Lo28 is the solution to the above problem because of its location and the size of corrections that need to be applied if it is used. There is, however, a very small portion of land, of about 15km, in Mokhotlong which falls outside the upper limit of Lo28.

7.8.3 Option of basing control on UTM zone 35.

Lesotho falls on the eastern side of zone 35, which uses Lo27 as a central meridian. As each zone covers 3° on either side of the central meridian, Lesotho is well covered as it is nowhere further than 2°30′ from the central meridian.

The adoption of UTM would require all the present data to be changed. Then the government would have the duty to alert all concerned parties, or even provide support in the form of courses for those surveyors who might feel uncomfortable with the new system. On the other hand, this system might be familiar to other foreign investors, such as the German companies which have already started work helping the Lesotho Government to survey the whole country, in order to establish the land database.

7.9 Feasibility of using WGS84 as a new reference ellipsoid

Once a country has decided to use one specific datum, that country finds it difficult to change to a later and even if a better, one. This reluctance to change may be caused by one or more of the following factors:

a) Convenience – A decision should be made first as to the convenience of changing to the new datum, based on the needs of the surveying community in general, and of the long term benefits of such a change. With its treeless and mountainous terrain highly suitable for GPS, a change to WGS84 would seem to be sensible, but this decision would depend on the privately practising surveyors having easy access to GPS equipment.

b) GPS equipment. Presently, only the government survey office is known to own such equipment. Currently, provision of control based on WGS84 has to be transformed onto the local datum for the benefit of private surveyors.

c) Coordinates conversion from the Cape Datum to WGS84 system – This needs to be done for all data at hand, and it might take a few years to complete. To do the conversion. suitable transformation hardware. software and personnel should be available.

d) Norms and Regulations – Changes need to be incorporated into the regulations governing the practices on land surveying (Chief Surveyor's Directions). This would affect the specifications on the placement and surveying of beacons in both the geodetic/topographic and cadastral surveying.

e) Training – Although it takes only a few days to learn to operate GPS equipment, it can take a much longer time to understand fully how GPS works. In order to be able to plan, carry out a survey and analyse the GPS results, training is required. Training would also help in updating of surveying regulations, as it is people with insight into GPS who would know which GPS error causes a particular displacement error. Currently, few Basotho nationals are competent in GPS data analysis.

Fortunately, a project focusing on land inventory in Lesotho is under way. This project helped by GTZ, a German company, with funding from the German federal government, aims to develop a GIS database in Lesotho, known as the National Inventory of Land Potential (NILP). GTZ sponsors the project, supplying all the equipment for data capture and processing and providing experts to train the technicians and land surveyors in technical matters. The surveyors and technicians (called *land technicians*) involved will undergo some basic GPS training, even though the data capture will be done using the Digital Orthophoto Mosaics (DOM), Digital Plane Tables (DPT) and measuring tapes. According to Zülsdorf (2000, p. 9), the equipment to be used will include "1 DPT, GPS dual frequency with one base station and 2 rovers, all linked by radio". One of the aims is that at the end of the project, surveyors will have the option of buying the equipment that they were using. If the exercise is successfully done, most of the points above would have been covered. A change to WGS84 would help the usefulness of this project and data conversion may as well start so that when surveyors own the GPS equipment, the Chief surveyor's office will be readily prepared for their needs.

A question arises: What will happen to the existing data once a change occurs?

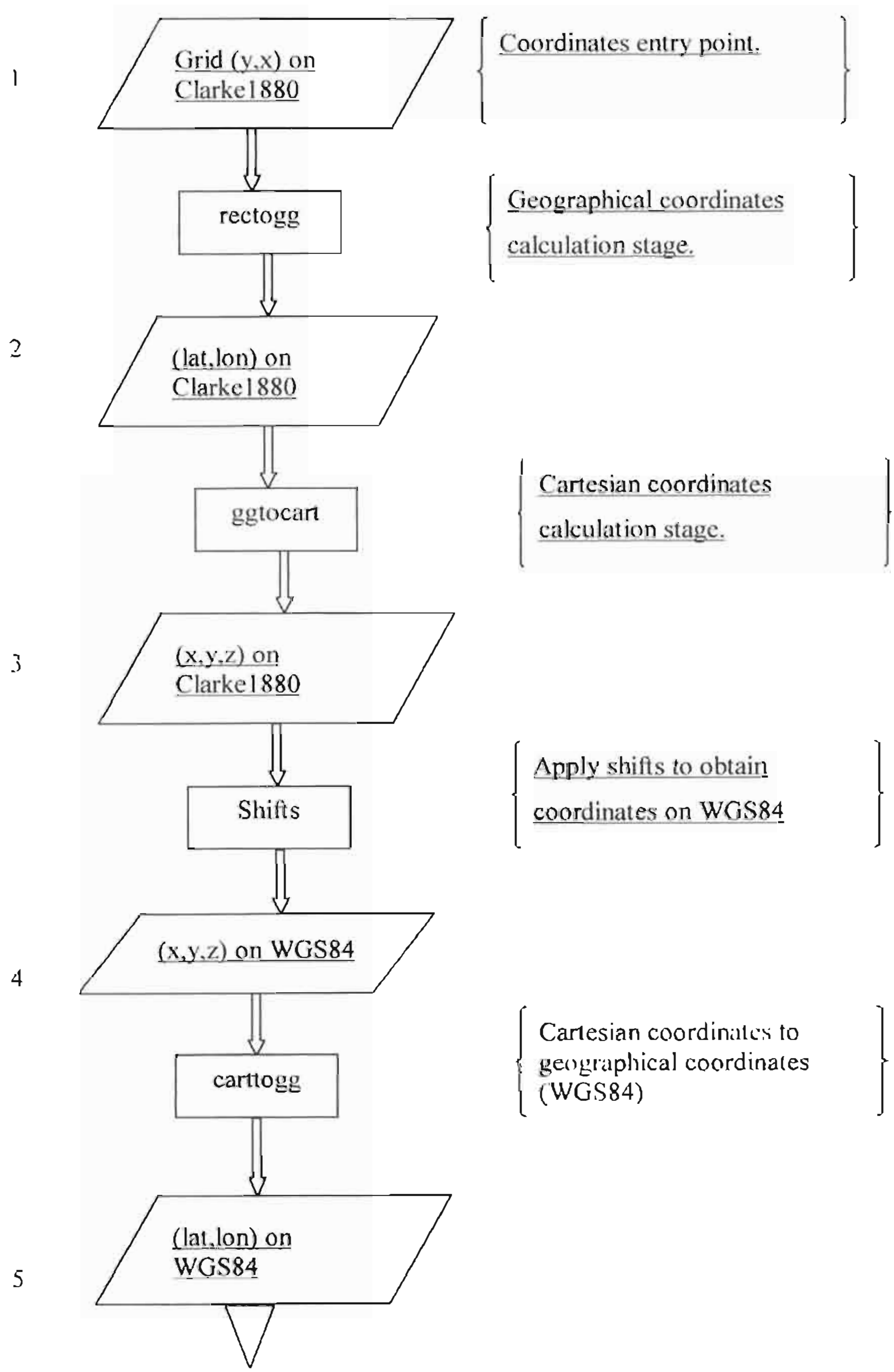
It is expected that not many survey firms in Lesotho will have the capacity to implement the change within a short time, say five years. The government still has its duty to provide services to such firms and therefore it is the responsibility of the government to provide safety of such data. Besides, a lot more materials such as maps and charts will still need to be referenced to the present data. Right now, the existing data is still very much in use and will remain in use for some time to come.

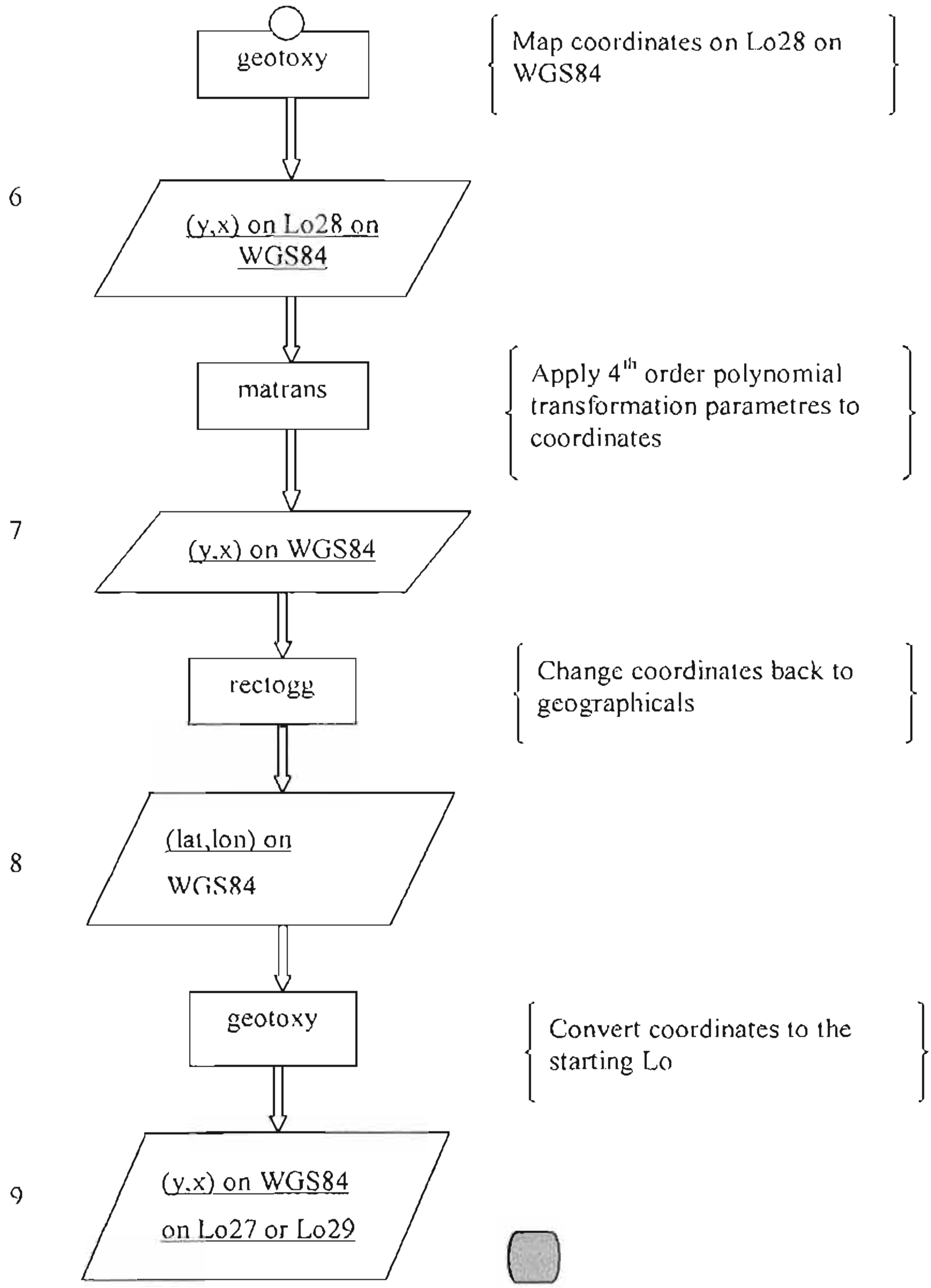
7.10 Software for conversion of DOS co-ordinates to WGS84.

Chapter 5 described how a fourth-order conformal polynomial transformation could be applied to transform DOS co-ordinates to the Hartebeeshoek datum. This transformation could be used to transform non-common DOS control points and also surveyed points measured from the DOS control. Using this transformation, the user is able to transform points without concern for seamlines or non-conformality, as happens when one applies a transformation using only closest control.

For this purpose, software was written to transform the existing data from the Clarke1880 map coordinates onto the WGS84 Lo27, 28, or 29. The variety of output form was to cope with different decisions about the best map to use. Because the polynomial transformation had been derived for points on WGS84, Lo28, this kernel had to be embedded in routines that would get the data into this form. This program is summarised using the flow diagram below. The program listing is in appendix (3), and the sample results in appendix (4).

Flowchart for the Lesotho Program





geotoxy
Map coordinates on Lo28 on WGS84
6
(y,x) on Lo28 on WGS84
matrans
Apply 4th order polynomial transformation parametres to coordinates
7
(y,x) on WGS84
rectogg
Change coordinates back to geographicals
8
(lat,lon) on WGS84
geotoxy
Convert coordinates to the starting Lo
9
(y,x) on WGS84 on Lo27 or Lo29

7.10 Shortfalls of the transformation software Lesotho.m

The program starts by input of local map co-ordinates. A choice has not been made allowing geographical co-ordinates to be used as input. This is because most point co-ordinates in Lesotho are computed and/or stored as map co-ordinates. There are instances, however, especially in geodetic control, where geographical co-ordinates are used. Therefore, as an improvement to the program, a subroutine to make a choice of input co-ordinates may be added.

Another problem is that of choice of transformation parametres. The way this program works is such that the parametres have to be changed in the program, which is not easy for users to trace. The transformation parametres need to be calculated first using Transf15.m, for insertion into Matrans.m. For further improvement, the choice should have one default set of parameters and a choice depending on the data set used, or inputting new parameters.

7.11 Chapter summary

The Transverse Mercator projection presently in use in Lesotho, and the Universal Transverse Mercator projection have been studied. Properties of each projections have been listed, and the problems for each projection, as applied to Lesotho were discussed. The implications of using one projection over the other were also discussed. The conclusions drawn show that the TM projection one panel system based on longitude 28°E is the best for Lesotho, as judged by the sizes of corrections over a distance of 1000m. The chapter concluded by looking at the viability of using WGS84 as a new datum. It is suggested that the preparations for a change of datum should start as soon as possible, and to facilitate this change, software has been developed for the conversion of present map coordinates onto WGS84 system.

Chapter 8 Heights

8.1 Introduction

The height system forms an important dimension in the geodetic survey. It provides not only the technique for measuring the distance above a specified reference surface, but also assists in geophysical prospecting and scientific study of the form of the geoid. The main value of a height system is to assist in planning and setting out civil works including road and hydro works, and for control for mapping. This chapter reviews the methods of heighting used in Lesotho and in South Africa as a basis for Lesotho height systems.

8.2 The South African Heights System

As with the plane control, the height system in Lesotho is based on the South African height system. The South African system of heights, according to Merry (1977),

> is based on a publication by G. A. Rune in 1949. It is supposedly an orthometric height system, in which the correction to an observed height difference is given in two parts:
>
> a) The 'spheroidal orthometric' correction C_S,

$$C_S = -\frac{d\gamma}{G} h_m$$ **Equation 8.1**

$$h_m = \frac{1}{2}(h_1 + h_2)$$

$$d\gamma = \gamma_2 - \gamma_1$$

$$G = 980000 \quad mgals$$

$$\gamma = \gamma_0(1 + \alpha_1 \sin^2\phi + \alpha_2 \sin^2 2\phi)$$

or

$$\gamma = \gamma_{45}(1 + \beta_1 \cos 2\phi + \beta_2 \cos^2 2\phi)$$

Where γ_0 and γ_{45} are the values of normal gravity at latitudes 0° and 45°. ϕ the latitude of the point, h_1 and h_2 are heights of two consecutive observed points of a level line section and α_1, α_2, β_1, β_2 are constants. As γ is a function of ϕ only, C_S can be expressed as

$$C_S = -2\beta_1 \sin 2\phi.\ d\phi.h_m$$

b) The "Geoidal orthometric" correction C_G,

$$C_G = -\frac{\Delta_m}{G} dh$$ **Equation 8.2**

$$\Delta_m = \frac{1}{2}(\Delta_1 + \Delta_2)$$

$$\Delta_1 = g_1 - \gamma_1 + kh_1$$

$$\Delta_2 = g_2 - \gamma_2 + kh_2$$

k is the free air gradient of gravity (+0.3086 mgals/metre)". (Merry, 1977, pp 45-47).

Trigonometrical Survey Geodetic Branch (1966) shows that $\beta_1 = 0.002644$ and $d\phi$ is in radians in the formula for C_S.

The spheroidal orthometric correction has been applied to all the Primary levelling in South Africa as it is the easier of the two to calculate, being a function of the latitude only, whereas the geoidal orthometric correction requires that free air gravity anomalies be known along the level line, and these were not available at the time of the adjustment of the levelling network (Merry, 1977, Trigonometrical Survey, 1966).

The corrections applied to South African height system are not rigorously orthometric as the closure of the levelling line will not be zero. The closure error as given by Trigonometrical Survey Geodetic Branch, 1966, p. 33 is as follows:

$$Closure\_error = [dh] - \frac{1}{980}[\Delta.10^{-3}.dh]$$ **Equation 8.3**

where Δ is the free air anomaly in milligals and dh is the difference in height obtained after application of spheroidal orthometric corrections.

The combination of the correction is comparable to Vignal's normal height correction, as explained by Merry, (1977).The following diagram shows the primary levelling of South Africa around Lesotho:

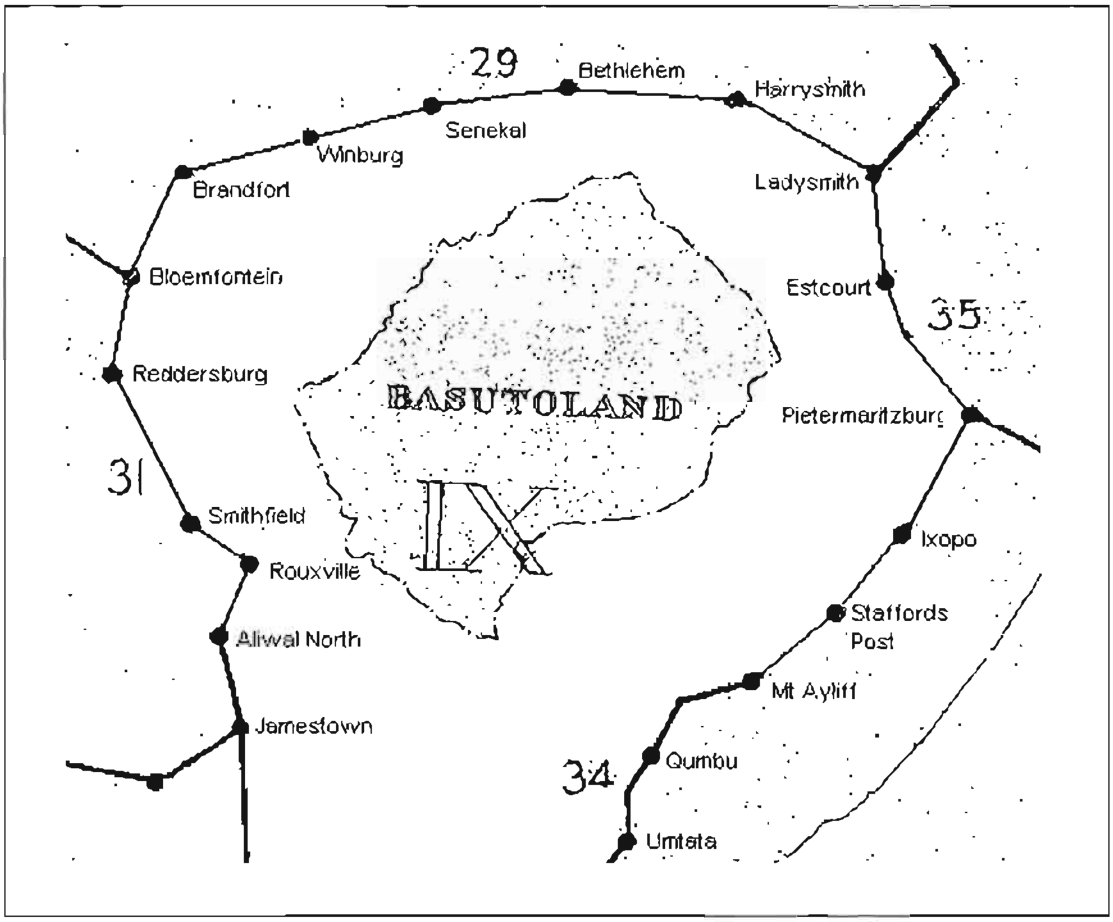


Figure 8.1 South African Primary Levelling around Lesotho
(Trigonometrical Survey Geodetic Branch 1966)

8.2.1 Height Connection to trig beacons

The Trigonometrical Survey (1966) shows that where topographical features were favourable and the beacons were not further than a mile away from the levelling route, trigonometric beacons were connected to the level lines by running a line of spur levels. If however the terrain was difficult, then trigonometric heighting using a theodolite was to be used.

8.3 Lesotho levelling

In the same way as the horizontal network's case, the height network in Lesotho had to be connected to South African levelling, because past technology differed from the present when orthometric datum can be established in a landlocked country from an ellipsoidal height plus information on the geoidal undulation. The connections were made on the spurs from Smithfield to Makhaleng Bridge and from Senekal to Ficksburg (Merminod, 1993).

8.3.1 Precise Levelling

The first levelling route in Lesotho runs from Makhaleng Bridge from South Africa, into Lesotho (in Mohales Hoek district) to Ficksburg Bridge. The two terminals are the Fundamental Bench Marks (FBMs) on the South African side of the border, constructed to provide control for the DOS levelling. DOS then constructed their FBMs at Mohale's Hoek, Mafeteng, Morija, Maseru, Teyateyaneng (TY) and Maputsoe, the route following the main road between these towns (See Figure 8.2).

Snowsill says "the placement interval of the BMs range between 1 and 2 km depending on the slope. Where possible, bench marks were cemented onto live rock, otherwise constructed as large rafts of concrete" (Snowsill, 1980, p. 1). He further points out that there is a surface and a buried mark at roughly every 5th BM, and in such a situation, the levelling closes on to the buried mark.

The line consists of a total of 163 level marks, 134 benchmarks and 29 buried marks. All BMs have appropriate descriptions and accompanying diagrams at LSPP. Where reference survey marks were within a distance of one set-up from a staff position, levellers had to offset to those survey marks, even though it was not done consistently. Not many trig beacons were connected to the level line as indicated by Snowsill, 1980. when he mentions that some connections to hilltop trig stations were observed for line 1.

A tolerance of $0.003\sqrt{km}$ metres was allowed for the misclosure between forward and back levelling. According to Snowsill, (1980) cumulative misclosures for the lines 1-3, where line 1 is from Mohales' hoek to Mafeteng, line 2 Mafeteng to Mazenod and line 3 Mazenod to Maseru, were over the tolerance limit of $0.003\sqrt{km}$. These misclosures were attributed to observations being done at different periods under different supervisors and observations not being made by the best observers. The overall misclosure for the whole line between Makhaleng and Ficksburg FBMs was within the expected tolerance of $0.003\sqrt{km}$.

Line	From	To	Ht difference	Distance
1	FBM Makhaleng	FBM2 Mafeteng	+249.3197	58.483
2	FBM2 Mafeteng	BM 2/20 Morija	-47.2515	35.912
3	BM 2/20 Morija	FBM4 Maseru	-79.1134	41.719
4	FBM4 Maseru	FBM TY	+99.1836	44.489
5	FBM TY	FBM Ficksburg	-100.3143	42.903
	Ht difference		+121.8241m	223.5km

Table 8.1 Height Differences
(Snowsill, 1980)

Elevations quoted (provisional) for the South African BMs are:

FBM Ficksburg	1551.612
FBM Makhaleng	1429.755
Ht difference	+121.859m

The misclosure is:

DOS	121.824
S.A.	121.859
Misclosure	0.035m

This misclosure difference is 0.035m in 223.5km, equivalent to 0.0023√km which is smaller than the allowable misclosure using the 0.003√km tolerance of 0.045m.

8.3.2 Maintenance of the Level Line

The maintenance aspect was to be incorporated in the activities involved in a joint effort between the government of Lesotho and the Swiss Development Corporation that started in 1986 and aimed at renovating the existing and upgrading or even extending, where possible, the level line. It appears, however, that all that was done was to visit the BMs and document the existing situation then, as deduced from Merminod 1992.

In December 1991 the situation was found to be as reflected in the Table 8.2 below. The numbers in curly brackets indicate the number of the benchmarks affected. In the table 8.2 MKGB: Makhaleng Bridge, MFT: Mafeteng, MRA: Morija, MSR: Maseru, TY: TY, FBG:Ficksburg, MPE: Maputsoe. The line numbers in Table 1 and 2 are for the same lines as marked in figure 8.2.

Line/route	No. of BMs	BM minor/no damages	BMs major damages	BMs destroyed/not found
1. MKGB-MFT [FBM1-BM1/31]	32	18	5 {1,15,16,18,26}	9
2. MFT-MRA [FBM2-BM2/21]	22	9	3 {3,5,10}	10
3. MRA-MSR [FBM3-BM3/22]	23	14	1 {2}	8
4. MSU-TY [FBM4-BM4/29]	30	25	2 {19,22}	3
5. TY-FBG/MPE FBM5- BM5/26. FBM6	27	22	4 {14,16,17,18}	1
TOTAL	134	88	15	31

Table 8.2 Bench Marks status of DOS precise levelling line as at 1992
(Merminod 1992)

8.3.3 Lesotho Highlands Water Project

As this is one of the biggest water projects in southern Africa, and in a very small and poor country, there were problems regarding its height control. The height system used is said to be a dynamic system, as mentioned on p. D.3.4 of the Supporting Report D, 1986. According to the reductions given, however, (e.g. Tlhaka-Oxbow line), the calculations were based on the height of the starting bench mark, and the results were termed 'dynamic heights' where they were derived from the levelled height differences, without any concern for gravity or shape of equipotential surfaces. When corrections based on spheroidal shape for level surfaces were applied, the results were called 'orthometric heights' (See appendix (5)). Jefferys says "All dynamic heights were based on the orthometric listed height of B. M. 29E 78A" (Jefferys 1987, p. 8), which is the Kestell Bench Mark. This suggests that the orthometric height of this bench mark was converted to, or considered as, dynamic height. Either way, this would not give the correct dynamic height, as actual gravity has to be applied on all the levelled points. Van Gysen, shows that "These 'spheroidal orthometric' heights are often referred to (erroneously) as 'dynamic heights'". (Van Gysen, 1993, p. 3).

The existing control in the area had been a level line observed in 1971 from Monontsa to Pelaneng. The final geodetic ties were carried out in 1983 to the BMs at Kestell on the South African levelling line from Ficksburg to Harrismith. All other height control available was established by reciprocal trigonometric heighting for aerial photo control. The accuracy is quoted as ±1m locally and ±5m overall and is therefore of limited use for detailed project mapping.

The additional height control, consisting of 11 new routes in Lesotho were observed by the department of Public Works and Land Affairs (PWLA) of South Africa, from November 1983 to December 1984.Table 8.4 gives a summary of lines closures in metres that did not attain the primary specifications.

Line	From – to	actual	Allowable	Difference
29E78A	Kestell - Mono	0.0123	0.0111	0.0012
IX AS 39-FBM TT	FBM Soai-TTseka	0.0060	0.0059	0.0001
IX AM 81-IX AM 94	TTseka – Likalaneng	0.0095	0.0092	0.0003
IX AN 40-IX AN 60	Likalaneng- Mazenod	0.0082	0.0081	0.0001
IX AM 121-IX AM 144	Likalaneng – Mohale	0.0083	0.0067	0.0016
IX AE 15 FBM Molumo	Khukhune – Molumo	0.0142	0.0125	0.0017
IX AF3-EX AH 13	Bohlane – Sentelina	0.0126	0.0103	0.0023

Table 8.3 Levelling Closures for sections not to primary specification
(LHWP, 1986, D-3.3).

The specifications used are as follows:

Primary 0.0030m *√ D

Secondary 0.0075m * √D

Tertiary 0.0150m * √D

where D is the distance is kilometres. (LHWP 1986)

A misclosure of 0.0622m out of an allowable of 0.0623 on the line from Kestell to Maseru FBM4 was obtained. The total distance of the line was about 430km.

For secondary levelling, steel pegs were put into the ground or fixed on solid rock outcrops. Permanent level benchmarks were built according to RSA standards and specifications with solid brass bolts in rock or concrete. Temporary benchmarks were marked with paint on rock-outcrops or with steel bolts in rock. Results were within 0.005±0.01D.

Two sections in the Oxbow levelling route did not achieve secondary specifications, a source being a suspected systematic error as there were 75% positive and only 25% negative closures. The ±0.075m height difference was however, considered acceptable for tunnelling purposes.

In an attempt to use dynamic heights at LHWP, the levelled heights to which some form of orthometric correction was applied were used. LHWP heights were corrected to orthometric heights to check for closing accuracy as the levelling was based on the South African orthometric-levelling network. All the checking and reductions were done by the Department of Water Affairs. The Kestell beacon, 29E 78A, was considered the datum and all the heights were based on it. Fouriesburg (IX-Y 33) – Clarens was corrected for dynamic height differences relating to the Kestell datum. The Fouriesburg-Tlhaka route was based on IX-Y 33 and the rest of the routes from Pelaneng to Maseru were based on the Maseru FBM4. (LHWP, 1986).

In his final year project at the University of Natal Durban, Sean B. Jefferys (1987). applied gravity data to the levelled heights in the Lesotho Highlands Water Project area to get the dynamic, orthometric and normal heights of all level lines in the project area. (See appendix (5)).

Jefferys' results show that the magnitude of correction varies inversely with the profile, and are at the maximum where there is a sudden change in the terrain profile. Of the three types of height, the smallest corrections were those of dynamic corrections followed by orthometric corrections. These large corrections occurred in the line Likalaneng – Mazenod at point IXAN28 with a height difference of –296.78m, normal correction of 0.0846976m, orthometric correction of 0.031876m and the dynamic correction of 0.024632m. A section BM28 –BM41 on the line Monontsa - Pelaneng follows with height difference –126.97m and dynamic correction of 0.022651m. Other graphs show varying degrees of corrections of lesser magnitude to the ones stated above, but still significant. The following map shows the level routes in the country.

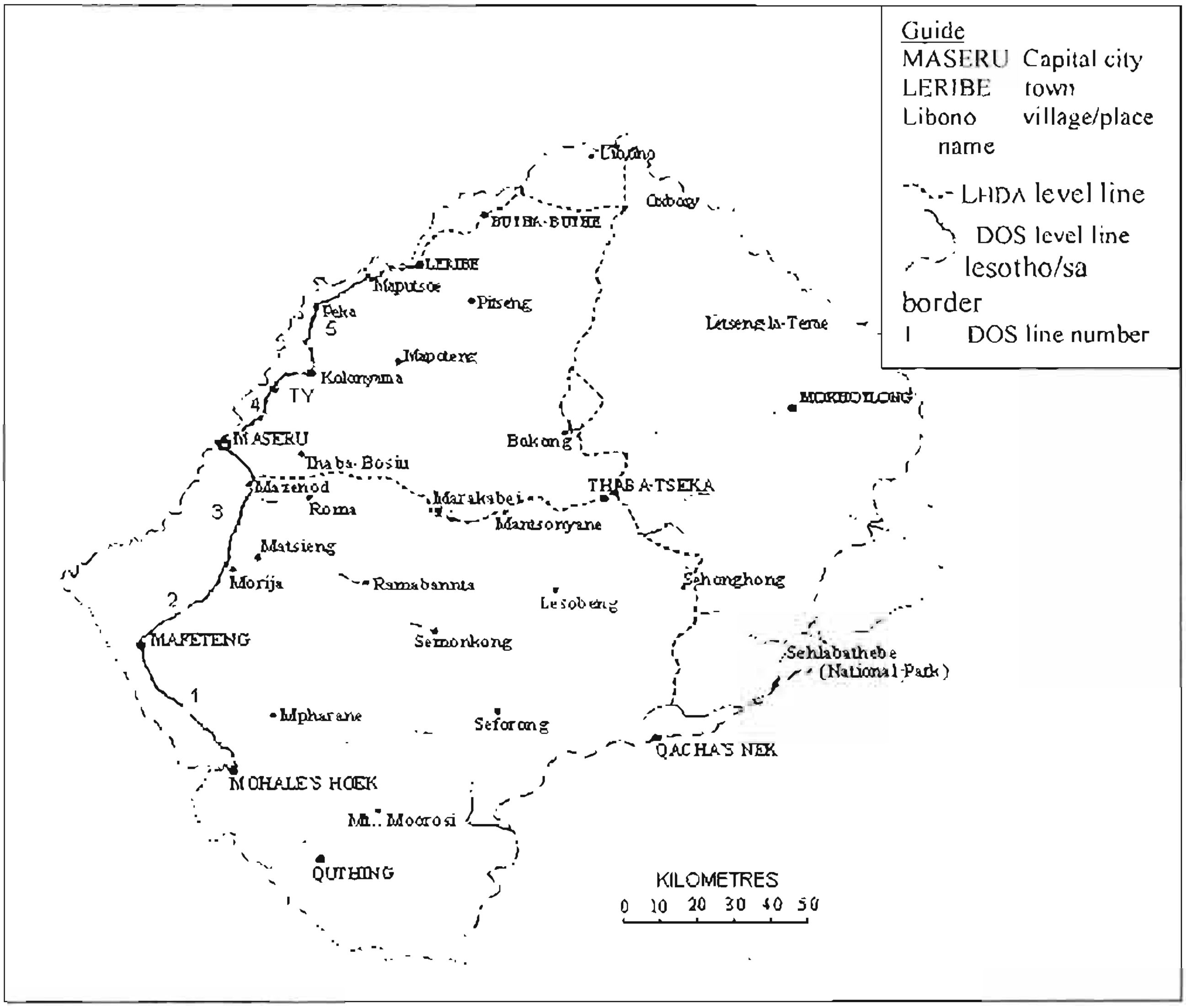


Figure 8.2 Levelling Lines in Lesotho
(Merminod, 1993)

8.3.4 Conclusions

It has been pointed out by different authors (Van Gysen, 1993, Merminod, 1993) that the system of heights in Lesotho is nominally of orthometric heights. The only correction applied assumes a spheroidal shape for the equipotential surfaces but the gravity corrections were neglected. In passing, the South African system, which forms the basis for the Lesotho height system is not strictly orthometric, but is equivalent to Vignal normal heights and is referenced to the quasi-geoid.

Although initially the water project inspired the first levelling events in Lesotho, these levelling lines were rather far from the area of the project, and the type of heights used were aimed at the general heighting purposes. Due to lack of expertise and proper equipment, the first precise level lines were not extended until the water project was under way, with the exception of the Monontsa to Pelaneng line, which was levelled in 1972. The LHWP level lines bridged the gap by connecting the levelling line on the northern site of the country from Kestell in South Africa, via the project areas in the central parts of Lesotho to the first precise line in the lowlands.

The heighting system in Lesotho is a combination of several lines:

a) DOS precise level line in the lowlands, connected to the South African primary level line at Makhaleng Bridge and Ficksburg.
b) DOS 1972 level line from Monontsa to Pelaneng, connected to South African line at Kestell. This line was aimed at the Highlands Water Project. Together these lines were short of primary specifications because sectors within them which were only to secondary standard.
c) For trig stations, trig heighting was used, as most of these stations are on mountain tops. If however, the trig was within two setups to the levelling line, that trig was

connected. This was partly done on line one only of the first DOS line. Thus there is a very weak link between trigs and the precise level lines.

d) Subsequently, in the 1980s, DWA and other contracted firms did the levelling for the LHWP, connecting the two existing lines via all the project areas. 11 lines were levelled. The Monontsa-Pelaneng line was based on Kestell BM, and all other lines on Maseru FBM4.

e) For newly established project trig stations in the highlands, trig heighting was used.

The link between the DOS and LHWP vertical systems is at Mazenod and Maputsoe. Generally, the work may be considered of lower quality than that expected for a project of the magnitude of LHWP. Two factors might have contributed to this:

1. Lack of local expertise. Everything was done by people who left the country at the end of their contracts, so naturally their focus was on doing the job at hand, with no future interest.
2. Urgency of the job. For DOS, developments were evident in the lowlands. In the 1980s, LHWP dam sites, access roads and tunnels were already undergoing construction, so heights were needed immediately.

It still has to be decided by the departments concerned whether or not a unified height system for both the highlands and lowlands needs further attention. If the vertical datum is redefined, it needs to be decided whether to use gravity corrections, or whether this would bring no material benefits to the country. Whatever decisions are taken, for the sake of densification of levelled points and strengthening of the weak sections, alternative routes could be levelled at a convenient time, to avoid a repeat of the poorly planned and integrated LHWP work.

Gravity data has been supplied for the whole country by the Department of Mines and Geology of the Institute of Natural Resources and this data can be used to correct for the heights. This correction would be an extensive task, requiring time and expertise. The initial stages have been done by Jefferys and what is important is to work from there on. Table 8.5 gives a sample of the differences between the LHWP and Jefferys' work on orthometric heights on the Kestell to Monontsa level line:

Bench Mark	Levelled height	Jefferys Orthometric height (m)	LHWP orthometric height (m)	Difference (m)
29E 78A	1716.521			
IXU1	1697.4514			
IXU3	1670.9158	1670.9177	1670.9143	-0.0034
IXU5	1688.7449	1688.7436	1688.7412	-0.0024
IXU7	1711.6316	1711.6299	1711.626	-0.0039
IXU9	1720.1165	1720.1159	1720.1093	-0.0066
IXU11	1680.6623	1680.6652	1680.6527	-0.0125
IXU12	1665.0259	1665.027	1665.0149	-0.0121
IXU14	1683.5974	1683.5961	1683.5839	-0.0122

Table 8.4 Differences between Jefferys' and LHWP orthometric heights

Differences of such magnitudes cannot be overlooked. These differences emphasise the point that LHWP heights were not corrected for gravity. An extract of LHWP heights and reductions is given in appendix (5).

The problem of lack of communication which exists between the government departments themselves and the parastatals and other organisations with regard to developments concerning spatial data should be attended to as early as possible to avoid duplication of effort on these issues. This problem could be corrected by forming a body composed of delegates from different relevant organisations whose responsibility would be to research on spatial information, form a database and be in a position to inform stakeholders of any developments relating to spatial information. South Africa is considering redefining its vertical control system, which would impact on Lesotho. It may be worthwhile waiting for South Africa to complete its redefinition so as to compare the old and new systems before embarking on an action plan.

8.4 The shape of the Geoid below Lesotho

To aid mapping and also facilitate the water project implementation in Lesotho, the geoid determination for Lesotho has been a necessity. The geoid is important because it provides a means for converting between orthometric heights measured by spirit Levelling, and ellipsoidal heights, measured by GPS. As these two methods of fixing heights are both used extensively, knowledge of the geoid undulation has become an important adjunct of the control net. Merry shows that global models are not suitable for

Lesotho: "It is apparent that the short wavelength geoid features found in mountainous terrain cannot be adequately represented by global models". Having said that Merry continues "the nature of this model is such that ...only features larger than 50 km in extent will be resolved" about OSU91A (Merry, 1993, p. 84). Other models cover the whole of the Southern African region, and can be considered highly generalised for areas the size of Lesotho, with its distinct topographical features. The models include one by Newling (1993), derived by using GPS and the UCT87A by Merry.

8.4.1 Geoid profile along 30° latitude

Another contribution to the study of the geoid in Lesotho was carried out by Dr. B. M. Jones (1970), but only along the 30°S parallel. The aim was to determine the geoid profile starting in Durban, at the Indian Ocean, proceeding west through to the Atlantic Ocean. The technique used was astronomical levelling. The direction of the outward normal was measured using an astronomical astrolabe, at trigonometrical stations approximately 22km apart for the first 28 stations, which covers the area of study. By comparing the geodetic and astronomical longitudes, the slope of the geoid in the East-West direction was measured along the 30th parallel.

The height is referenced to the spheroid, assuming the geoid and the spheroid coincide at station 1, Durban.

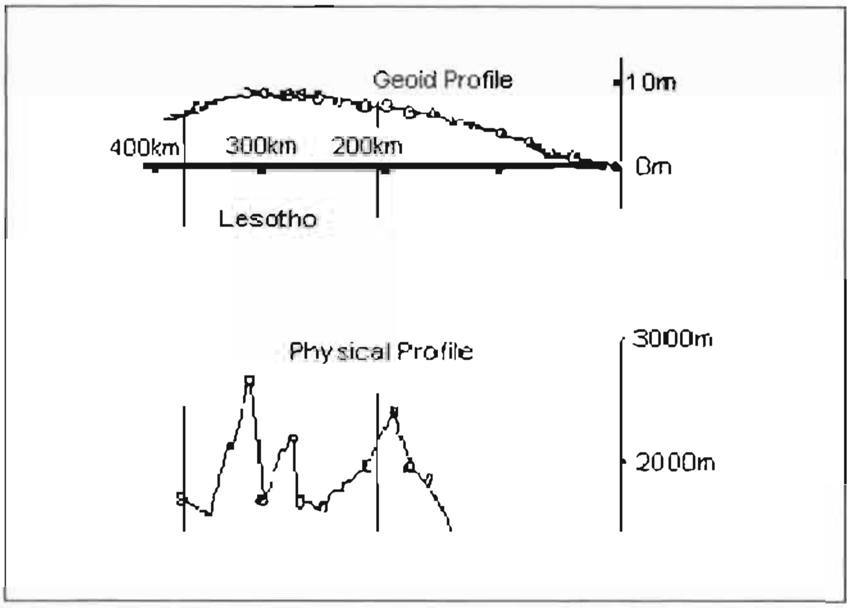


Figure 8.3 A section of Jones' geoid profile and terrain along 30° S parallel (Jones, 1970)

From the geoid profile diagram, the geoid height peaks in Lesotho, sloping from there in both East and West directions, more rapidly to the West than to the East, from point 17. It should be noted that the results were not corrected for the personal observation errors when star observations were taken, as is mentioned in Jones (1970).

8.4.3 1993 geoid Model

In 1992-3, GPS surveying techniques were employed to fix 34 new monuments in Lesotho. As GPS is a three-dimensional measuring tool, ellipsoidal heights of those points were derived simultaneously with the WGS84 geographical coordinates. After transforming the coordinates to the appropriate reference ellipsoid, orthometric heights could be obtained by subtracting the geoid-ellipsoid separation N from the ellipsoidal heights, according to the formula:

$$h = H + N$$

where h is the ellipsoidal height obtained through GPS,

H the orthometric height, and

N the geoidal height obtained by levelling.

Van Gysen says "the geoid used in Lesotho at present (referred to in the Step 3 report as the 1990 model) is an extract from the SAGEOID86 model for the whole of southern Africa" (Van Gysen, 1993, p. 3).

Van Gysen (1993) calibrated the existing (SAGEOID86) geoid used in Lesotho using the ellipsoidal height values obtained during the GPS survey of the 1990s in Lesotho. Point FBM4 in Maseru was an outlier, and was therefore removed from the calibration points. After proper corrections of the bias and tilt were applied to the SAGEOID86 geoid model, a calibrated model for Lesotho as shown in Figure 8.5 was produced. This models the quasi-geoid. The contour interval is 0.5m. In his comments, Van Gysen notes: "In the absence of such (gravimetric) corrections, I have taken the published orthometric heights in Lesotho to be normal heights, and thus have calculated height anomalies (or quasi-geoid heights) rather than geoid heights" (Van Gysen 1993, p. 3).

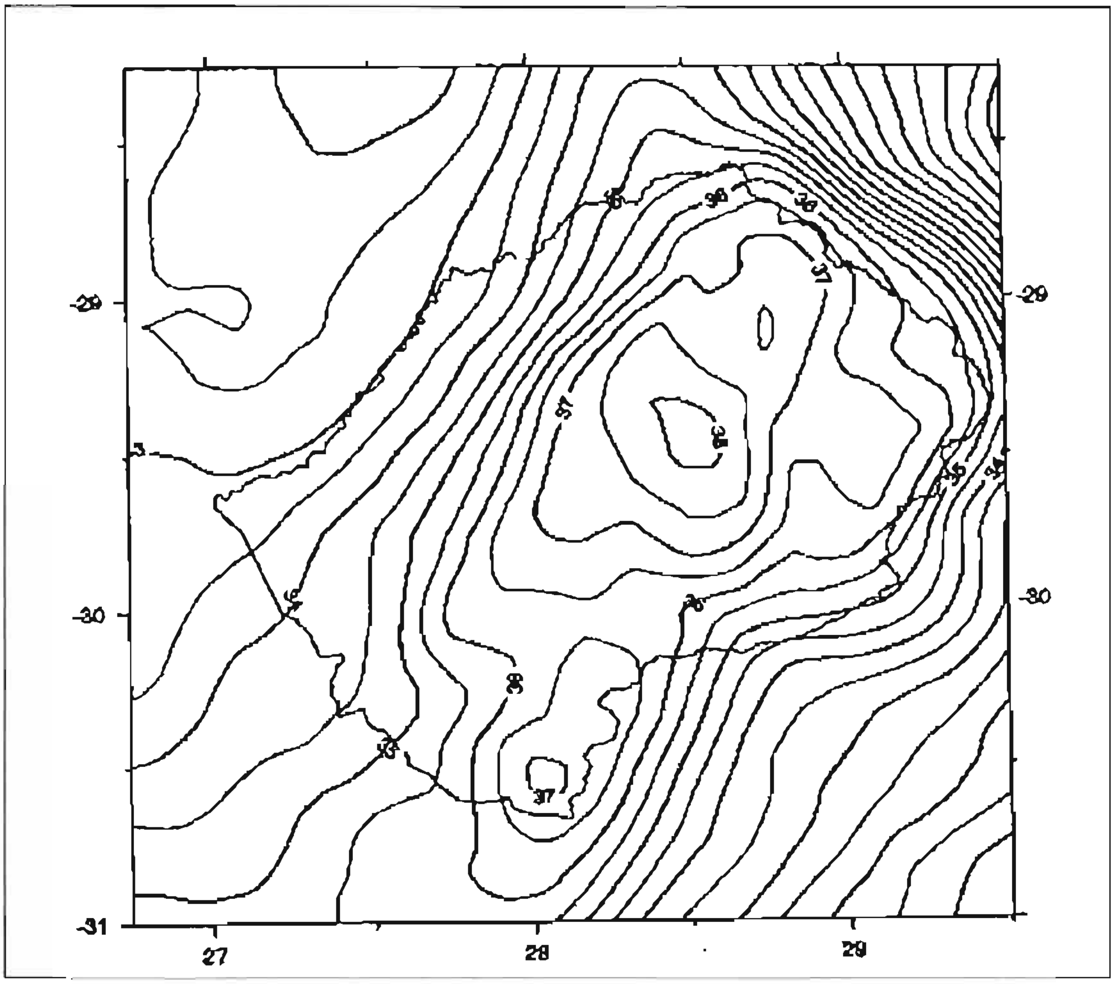


Figure 8.4 1993 calibrated geoid for Lesotho on WGS84
(Van Gysen 1993)

The geoid profile along the 30°S parallel as derived from 1993 Geoid model is shown in Figure 8.5.

8.4.2 Comparison of Jones geoid model with Van Gysen's along 30°S parallel

In order to compare Jones's profile with modern data, it was adapted by conversion of geodetic co-ordinates to the Hart94 system, so that it gave relative geoid undulation above the WGS84 ellipsoid. Geoid undulation values were interpolated from Van Gysen's (1993) geoid model at intervals of about 9.3 km along the 30^{th} parallel, starting from 27°E to 29°30'E. These gave a total of 26 points, on Table 2 in appendix (5). Jones relative geoid heights were brought to the approximate geoid height of Van Gysen by adding a constant. The two profiles are compared in Figure 8.6.

The general trend of the two curves is the same; a positive slope from about 145 km (continuous from Durban) up to about 276km, the peak, then a negative slope to the last point, about 380 km from Durban, the starting point. Van Gysen's geoid is the steeper of the two. This Van Gysen's profile depicts more details than Jones' and therefore is more informative.

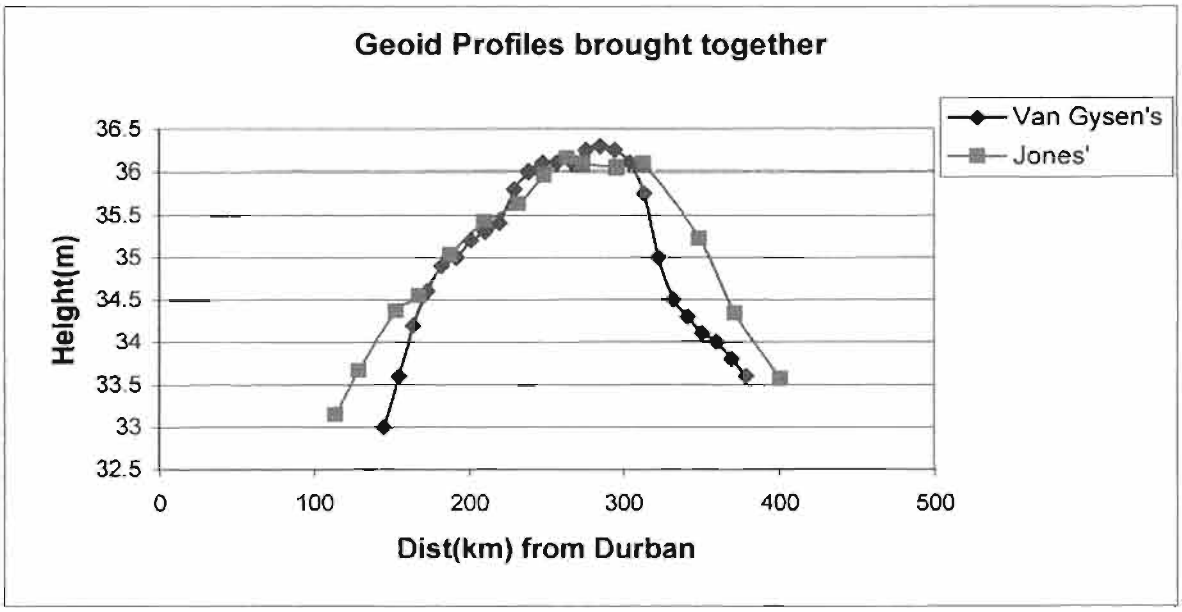


Figure 8.5 Geoid profiles along 30° S parallel in Lesotho

The geoid height range of Van Gysen's geoid profile is about 3.3m while that of Jones is 3m. Of importance in water projects is the slope of the geoid, because the greater it is the larger the impact. Merry shows that "...in the Transvaal and the Drakensberg the slope reaches 20", which translates into a metre over 10 km. Furthermore, the slope changes rapidly, ... and a uniform slope cannot be assumed for even a moderately sized area" (Merry, 1993, p. 82). The shown geoid profiles confirm these statements. On both sides of the maxima, the slope of the geoid is very steep and there is nowhere along the parallel where the difference between orthometric and ellipsoidal heights can be ignored. Although the general positive slope of Van Gysen's geoid profile is about 0.02m/km to the west (that is between the peak and the 1st point on the profile diagram) and the negative one about 0.03m/km to the east, other sections of this geoid have a slope reaching 0.08m/km. Other parts of Lesotho would probably show greater geoid slope along other parallels as can be deduced from Van Gysen's geoid model.

8.4.3 Conclusion

Up until 1993, Lesotho was using the same geoid as South Africa, SAGEOID86. With the conception of the GPS network and the LHWP, the opportunity was taken to refine the SAGEOID86, to produce the 1993 Lesotho geoid. This geoid model (calculated by Van Gysen) is the latest and probably the best for Lesotho at present. Van Gysen (1993) and Merminod (1993) p. 6 suggest that point FBM4 seems to be an outlier, using the 1993 geoid model. With proper checks, the error seems to be in the geoid model and not with the observations and/or computations. It is therefore still necessary to improve the geoid model further.

The profile derived from this 1993 geoid model agrees with the one found using Jones's data along the 30°S parallel. Since the 1993 model used better techniques and gives data at more frequent intervals, it is probably the better choice to be relied on for most purposes. The Jones's profile however, provides an interesting confirmation of van Gysen's geoid. The comparison of the ellipsoidal, geoidal and orthometric heights at the calibration points is given on Tables in appendix (5).

A serious consideration needs to be taken when considering the height system in Lesotho. This is in view that the present situation reveals that the said orthometric heights are not rigorously orthometric and those of the Lesotho Highlands Water Project are not dynamic as it appears in the LHWP reports. Jefferys heights are so far the closest to the orthometric heights and should be considered if a change is deemed necessary. The height system should then be extended to the trigonometric stations and related closely to the geoid. In this way a truly 3D network could be achieved Altenatively, bench marks would be required at regular intervals in all towns and along the main roads and major projects. This is an enormous task.

Chapter 9 General Conclusions

9.1 Introduction

The geodetic network of Lesotho has had problems of distortions since its first adjustment by DOS. Later measurements have not completely eliminated these distortions. All groups whose measurements have been reviewed i.e. DOS, LHWP and Merminod, adopted as fixed, the latest South African values. This resulted in different values for the same points. The South African network has been upgraded several times, these adjustments including some Lesotho points. Their latest readjustment is one of the best in the world, being based on a VLBI origin and a GPS zero-order network. It was thus necessary to find how this last readjustment compares with Lesotho geodetic control.

9.2 Comments on the Results and Analysis section

Comparing DOS and Hart94 datum positions, the area of greatest distortion is the Southeast. In particular, the position of point P035 needs further investigation. Most points believed to be outliers were found to result from mismatched points in the two data sets. After removing those points, 113 points were common to DOS and SA sets. The lower order points were found to have less distortion than the primary control. This was ascribed to the measurements between lower order points, opposing the distortion forced on the primary net by the South African control of that time. The fourth order conformal polynomial transformation worked well for the DOS/Hart94. This shows that the distortion of the DOS net was simple.

The comparison of Merminod's GPS net with the Hart94 datum points produced relatively small residual vectors with small systematic error effects. This confirms the earlier assumption that the DOS co-ordinates are more distorted than the Hart94 datum points. Even though Merminod/Hart94 is of better fit than DOS/Hart94, Merminod's data set is less useful to most surveyors in Lesotho as the points are very far apart. Up to now they have not been in sympathy with the official published (DOS) points around them. Since Merminod's points were connected to the SA control using the latest values, they are comparable to the SA control. It may be possible in the future to connect DOS points to Merminod's in the weaker areas. 29 points were used in this analysis. Thus Merminod's points can be used to constrain future networks. (This applies to the points which were used in this analysis and not to the new points erected in 1991 which were not in the South African set of coordinates).

The LHWP fit onto South African net is the poorest of those reviewed, with the Northwest and the Southeast showing very large vectors. This does not appeared to have prejudiced the civil works. Unlike the DOS/Hart94 comparison, higher order conformal polynomial comparison of LHWP with Hart94 points did not significantly reduce the residuals. This suggests that the distortions in LHWP control are relatively complex and that a readjustment of their observations may be needed.

Both Merminod's and DOS points are found in the LHWP area. These points could be used with their updated coordinates, to reduce the discordance and to homogenise the national geodetic network. Merminod's points may be used in the future as zero order points in a readjustment of LHWP points.

9.3 Comment of choice of projection for revised co-ordinates.

Considering various transverse Mercator projection possibilities. the corrections are most affected by the Lo used and the height of the point. Both the two panel (Lo27, Lo29) and one panel (Lo28) systems cover Lesotho well in extent but the two panel system has larger corrections than the one panel Lo, which uses the central meridian near the centre of Lesotho. Only a small portion of Lesotho falls outside the recommended 1 degree belt limit from Lo28, which can therefore be used for the whole country.

A consideration of the possibility of using UTM showed that length corrections will be maximal in the lowlands, reaching more than 0.6m/km, and constant for about half of the country from longitude 27 to longitude 28. Further East the length corrections become smaller as the compensating sea level correction becomes larger. However this correction would be significant and troublesome in civil works such as road construction. As with the option of adopting Lo28, the advantage of using UTM is that the whole country falls on one panel. Considering the size of Lesotho and the corrections for the Gauss Conform and the Universal Transverse Mercator, the preference is for Gauss Conform on Lo28. However the transformation software used allows for optional output.

9.4 Comments of Height

Lesotho height system is that of normal heights, after the South African height system, though in most cases, of lower accuracy than originally aimed at. The first precise level line covered only five districts, all in the lowlands, and had sections not satisfying primary standards. The next precise levelled line established in the 1970s between Monontsa and Pelaneng was also not to primary specifications. However, considering the topography of Lesotho, these lines were accepted for the subsequent level readjustments. Monitoring and use of these level points has been close to non-existent, probably because of lack of proper equipment. However, it would be better to keep a yearly record of the situation of these points than to visit them only when they are needed.

Confusion about the terminology used by the LHWP can lead to more misconceptions about the system of height used in Lesotho. Though the LHWP heights are claimed to be dynamic heights, the results, and perhaps the method of computation shown, does not indicate that actual gravity or normalised gravity value was used. The levelled height differences themselves without any adjustment, were used to derive these "dynamic" heights. Therefore, in this paper, the height system was considered to be normal and not dynamic. Jefferys' work which showed values of orthometric, dynamic and normal heights for the points at LHWP using actual (interpolated) gravity (appendix 4) may be good enough to be adopted in place of the official LHWP values.

Concerning the geoid, the tailored model produced by Van Gysen is the best. However, it needs some investigation around FBM14 in Maseru. ADOS data is not enough to give a detailed geoid, but shows the general slope of the geoid to be in the Northeast to Southwest. Along the 30° parallel, Jones profile verifies the equivalent profile drawn from van Gysen's geoid.

9.5 Conclusion

This study revealed that Lesotho geodetic network has problems, both in the horizontal and vertical networks. These problems have been addressed in this thesis, some partly answered and some just brought into the light. It was found out that DOS errors were not very serious as compared with the LHWP errors. Both networks can be adjusted to Merminod's network, which is comparable to the SA network.

There are few local trained people in the field of geodesy, and if nothing is done towards improving expertise, the problems will persist. Although the defence force is secretive about its activities, it is doubtful whether they find mapping very useful. Many other ministries require mapping services, but are reluctant to make effort towards improving geodetic control in the country. As has happened on several occasions, some ministries use control points for planning their projects and then destroy or obstruct the monuments after using them. People need to be made aware of the importance of a homogeneous accessible control, from some top government officers to the ordinary citizen, and this will take time and effort to accomplish.

In the highlands, especially LHWP, the loading of the reservoirs will have an effect on the stability of the area with respect to earth movements. In other areas, such as those affected when the water tributaries were redirected to supply water to the main reservoirs, people may have to draw underground water, which would cause subsidence. How this movement may be monitored has to be planned carefully because it needs a highly reliable and broad network in place.

Merminod (1990) pointed out that the uplift of the Maloti mountains is small, about 1mm in one year. He stressed the urgency of monitoring networks and referred to Egger, 1982 recording that the extent of traditional monitoring networks is too limited, because the most external points are also affected by local tectonic movements. But this situation can not be left in the hands of LHWP alone. The government, through its different ministries, should take action and state clearly what needs to be done to reduce the risks of being caught unaware regarding these problems.

A new software has been used successfully for converting old co-ordinates to the new system and a uniform list for Lesotho points based on Hart94 datum can be created.

9.6 The overall view of the new data set, the transformation results and future use

It's a long time since Sir David Gill disclosed his idea of a continental geodetic network, in an attempt to contribute to a global network. Even now, there are some countries which are still using their national networks. For Lesotho things are beginning to change, and the new coordinates could be viewed as a step towards the achievement of joining the global geodetic network.

The overall view of the analysis can be summarised in the following table which compare the 4^{th} order transformation vectors discrepancies:

	Distance (m)	Average discrepancy (m)	Discrepancy: ruling distance 1: (m)	Ration in ppm
Merminod	22103	0.1022	216272	5ppm
DOS primary	30304	0.2619	115707	9ppm
DOS non-primary	8400	0.2252	37300	30ppm
LHWP	7612	0.9532	7513	133ppm

Table 9.1 4^{th} order discrepancies of different agencies

As can be seen from the table. it can be said that Merminod's values can be accepted as zero order point values. A readjustment of DOS points is necessary. probably after correcting for the deviation of the vertical. Lesotho Highlands Water Project needs total reconsideration. 1:25000 is mostly considered the largest ratio for geodetic work.

The new DOS values calculated using the new transformation software could be used to compare new values when transformation is done between the local and WGS84 datums in Lesotho. It should be remembered that whatever errors were there in the DOS values, they were carried over in the transformation. and that reobservation is the best but very costly solution. But before reobservation is done for the DOS points, the present new values could be accepted and used as provisional for the control. For GPS users, if this is done, then there won't be any need for transformation. since the datum used would be that of WGS84. What is needed in future is that Lesotho needs to homogenise its different networks, which can be done using Merminod's points as fixed.

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Appendices

Sources of coordinate files

1. South Africa- Directorate of control Surveys – On WGS84 (ITRF91). Map
2. Mermind – LSPP – On WGS84 (ITRF91) geographical
3. DOS – On Clarke1880, Gauss
4. LHWP – On Clarke1880, Gauss

Appendix (1)

Notes, correspondence and reports

Supporting documents used in the thesis, some of which could not be referenced properly in the text, i.e. source or author unknown.

DOS computing section (1981) notes on Lesotho triangulation

Windsor's letter on Lesotho triangulation

Rosseau's letter on Lesotho triangulation

Rogers' notes on Lesotho triangulation

Example of Bench Mark description

Bullen's letter of replaced beacons

Projection Tables – South African Belts

Rogers' notes on Doppler check on ground survey in Lesotho

ADOS sample of summary of observation stations

Two letters on AFREF, the African reference frame.

LESOTHO

Report on DOS Computing Section work 1964 - 80

1.1 Primary Triangulation

Lesotho is surrounded by S African territory but the S A Primary trig is not homogeneous in this area. Two adjustments were run to determine the consistency of DOS observations and their fit to the S A points.

Primary I held only 3 S A points and gave a standard deviation of angle residuals of less than 1". This shows that the DOS observations were good.

Primary II held all 28 surrounding SA points fixed and gave a standard deviation of angle residuals of 2".5; an unacceptable distortion for further breakdown.

A compromise Primary adjustment was therefore accepted holding 18 S A Primaries on the west of Lesotho, the more developed side. This gave a standard deviation of 1".5 leaving considerable disagreement between Lesotho and SA coordinates for the S A points to the east of the country. BP pts nos 1 - 38 were fixed in this adjustment.

1.2 Datum

The compromise primary adjustment holds the following S A points fixed :

BABANGIBONI	GENERALSKOP
VENTERSBERG	GIBRALTAR
ZWARTKRANS	MABULA
RHEBOKBERG	GOBO GOBO
ORANGE SPRINGS	LANGKRANS
JAMMERSBERG	NTHODIMON[illegible]
AASVOGELBERG	STERKFONTEIN
NOJIKI	SCOBELLSKOP
TSATSANA	
HOLDERNESS	

Spheroid Clarke 1880 (Modified)

a 6 378 249.145m

e^2 0.00630348102

2. Secondary Triangulation

In 1973 all existing secondary triangulation was adjusted, including a base measurement, to fit the primary network. This fixed BS 1-59 and a few SA points on the eastern borders.

3. Secondary/Tertiary Traverse/Triangulation

From 1974 onwards several small adjustments fixed extra framework control either replacing or enhancing the existing primary and secondary nets which were held fixed. These included a few secondary (BS) points, tertiary and other points were inclusion strengthened the network.

4. Reference Mark Traverses

From 1975–80, DOS observed Reference Mark traverses in several townships throughout Lesotho. These have been individually adjusted to the existing framework.

5. Photo Control Points

These have been fixed and computed by various means and are all based on higher order work.

6. Miscellaneous Points

Some local points were coordinated from DOS work and some old resected pts recomputed from adjusted values.

7. Heights

All heights are based on S African trigs and are provisional. Unless otherwise stated, heights refer to the tops of pillars, ground mark etc. The majority of difference heights are computed from simultaneous reciprocal vertical angles.

8. Distribution

Geographical and Local TM Grid Coordinate lists have been sent to Survey Data Library and to the Lesotho Survey Department. These coordinates should supersede any existing coordinates for these points.

9. Doppler

512 STRE have observed five precise ephemeris Doppler stations in Lesotho. These are too close to be held fixed in any new adjustment but show fairly random difference from the terrestrial values. There is some suspicion of a systematic azimuth difference. MCE are making a detailed comparison.

Computing Section
JUNE 1981

OVERSEAS [illegible]
D[illegible]

The Chief Surveyor
Department of Lands and Surveys & Physical Planning
Ministry of Interior
Maseru
Lesotho
(Attention - Mr J W McCulloch)

230/97/31

12 November 1980

Dear Mr McCulloch

I am enclosing copies [illegible] the comprehensive [illegible] list [illegible]. It includes [illegible] points [illegible] ground.

The list has been compiled in a number of sections [illegible] FP points, BS points [illegible] nip survey [illegible] easy to use. [illegible] of 25°E [illegible] Lo. 27 and Lo. 2[illegible].

As you know, the accepted version of the Lesotho primary adjustment held fixed the South African points on the West and South of Lesotho, where they would be [illegible] for future [illegible] to [illegible] South African points in the North and East. The points 12.Y.[illegible] to 12.Y.[illegible] which were fixed for the original mapping control of Lesotho, fall over the border outside the area where the South African points were held. The list containing all these points is based on the South African values and is not in sympathy with the Lesotho coordinates. [illegible] to 12.Y.[illegible] South Africa [illegible] the only practical [illegible]. 12.Y.[illegible] to Lesotho and [illegible] a separate list giving its coordinates in sympathy with the Lesotho values.

I am afraid the pr[illegible] this list has taken rather longer than I had hoped but we had computer [illegible] earlier in the year and a number of the older points have had to be recomputed to bring them into sympathy with the accepted adjustment.

Yours sincerely

[signature]

[illegible] Windsor

Encl.

The Chief Surveyor
Dept of Lands, Surveys and Physical Planning
P O Box 876
Maseru 100
Lesotho

Copy for your information. A copy of the co-ordinate list is attached.

D P M Rousseau
for/CHIEF DIRECTOR OF SURVEYS AND MAPPING

Overseas Surveys Directorate
Ordnance Survey
Romsey Road
Maybush
Southampton S09 4DH
ENGLAND

X105

D P M Rousseau

CD 8/4

PRIMARY TRIANGULATION : LESOTHO

1. The adjustment of the Lesotho primary triangulation received a good deal of attention from both yourselves and our organisation during the period 1960 - 64.

2. Recent fieldwork connected with our re-triangulation programme both north and west of the Lesotho border as also the possibility of investigations concerning mutual water resource surveys have rekindled interest in this adjustment which never really was very satisfactory. Copies of recent recomputations are enclosed and it appears that the 10 - 15 ft "discordancy" (Furmston, Nov 1964) has largely been eliminated.

3. With modern software at our disposal it seemed logical to attempt a simultaneous adjustment for the entire Lesotho in stead of the earlier fragmented sections. As was done originally, the co-ordinate values of the early South African Geodetic Triangulation were retained. Some of the stations of the western chain situated in Lesotho, could not be located by the DOS survey team and naturally could not be regarded as fixed.

4. The enclosed triangulation plan indicates EDM traversing recently completed north and west of Lesotho. The scale of the existing triangulation varied from 1/75000 to 1/350000 and was regarded as acceptable for this exercise.

5. To assess the quality of the South African stations along the northern border of Lesotho, a trial adjustment the southern limit of which is shown in a broken blue line, was done using only South African primary triangulation observations. Only the station (50) Ventersberg showed any marked deviation from the available co-ordinates. The other South African primary stations were accepted as fixed for the Lesotho adjustment.

2/.....

6. Both double points 213/214 Mechachaneng (BP1 and BP2) and 216/217 Mt Aux Sources (BP3 and BP4) were incorporated into the adjustment with the taped distances by DOS regarded as fixed.

7. DOS and South African primary observations at common border points were suitably combined and used in the adjustment.

8. South African stations (25) Indows and (29) Kozani along the eastern escarpment were not included in the main adjustment due to poor connections with adjoining Lesotho points.

9. Along the southern boundary of Lesotho, no forward observations are as yet available from the South African geodetic stations (67) Zwartberg and (16) Jumbla. It was also found that the DOS team had incorrectly observe to a tertiary station Mungunung Aux in stead of the geodetic station (32) Mungunung.

10. Reliable secondary triangulation co-ordinates recently became available for the common border point (54) Tsatsana BP35 and the available co-ordinates for the tertiary station Mungunung Aux are quite good.

11. The adjustment was completed with reasonable control all round Lesotho with the expected weakest area being the south-east due to the lack of forward observations from three South African stations.

12. All "surrounding" stations held fixed for the purpose of the adjustment have been circled in blue on the enclosed triangulation plan. As was expected the largest angular corrections reflected on the print-out were obtained for Lesotho station (161) Komokha BP30 and closer inspection of these corrections would indicate a relative uncertainty of not more than 0,5 m in both y and x values. The main network is fairly homogeneous and slots in well with the South African control along the western, northern and eastern boundaries.

13. In common with the existing South African triangulation in the surrounding areas, no attempt has been made to introduce scale into the adjustment. Fieldwork is still in progress in these areas and a complete re-adjustment with adequate scale control will only be attempted after the completion of our grid iron first order traverse network. I may add that trial adjustments of the lower order Lesotho and adjoining South African networks have gone down well into the re-adjusted primary.

14. I shall be pleased to receive your comments on the enclosures - you may have further suggestions which may assist in improving the state of the cover in the area.

15. A copy of this letter has been sent to :-

The Chief Surveyor
Dept of Lands, Surveys and Physical Planning
P O Box 876
Maseru 100
Lesotho

Yours faithfully

D: f

for/CHIEF DIRECTOR OF SURVEYS AND MAPPING

(15)

1978

NOTES ON LESOTHO TRIANGULATION

References: A. [illegible] STR 2512/6/7/158 dated 11 Aug [illegible]

B. [illegible] (52300) dated 9 Aug 78

C. [illegible] Primary Trig" by [illegible] dated Nov 64

1. It appears from reference B that the nature of the trigonometrical control in Lesotho has NOT been fully understood. It is also [illegible] that there may be some confusion over the [illegible] taken the [illegible] to discuss this triangulation with Mr [illegible], now Deputy Director [illegible] at DOS, who adjusted it and with Miss Windsor, the DOS Chief Computer, whose section is computing a considerable amount of [illegible] connected to it.

2. The author of reference B has assumed that the definitions of "geodetic" "primary" [illegible] in South Africa. This is NOT the [illegible]. The main Lesotho triangulation was observed on 16 zeros with glass arc [illegible] theodolites by DOS and is thus comparable with the "geodetic" specification [illegible] in South Africa.

3. [illegible] accuracy of the South African "geodetic" triangulation which is wildly optimistic. It must be remembered that at the time this [illegible] to supplement the [illegible] the world has shown how [illegible] adjustment was done before the days of large electronic computers and so could not have been adjusted as a whole. This would have caused further deterioration even though the obser-[illegible]

4. [illegible] is completely surrounded by [illegible] independent scale and azimuth control is unlikely [illegible] the main purpose of the 1964 DOS adjustment was to achieve [illegible] compromise between fixed [illegible] still valid. For the record, [illegible] Chief Surveyor, [illegible] modern EDM traverses using both long and short range equipment close satisfactorily on the 1964 coordinates [illegible] control as a whole should [illegible] then be of great value. Until [illegible]

5. [illegible] of the five points [illegible] with them. There is [illegible] coordinates because the triangulation was originally computed on Clarke 1880 (modified) and was subsequently changed to Clarke 1880 (unmodified) to agree with the [illegible] African control.

6. [illegible] old triangulation stations in Lesotho. The original values were derived [illegible] the heights of adjoining South African stations. Since then many South African values have been improved and levelling has appeared [illegible] of the Lesotho values [illegible] changed too. It must be realised that final values [illegible] heights should be relied on to better than ± 3 metres. The values [illegible] shown to the nearest tenth of a metre only because they were converted from imperial units.

H R

H M ROGERS

for Head of Establishment

BENCH MARK DESCRIPTION No. FBM 4

COUNTRY LESOTHO D.O.S. Sheet No.

Sited by A. J. SAVAGE	Date 1976	Distances to next B.M.	
Built by "	Date 1976	B.M. 3/22	~~Ft~~/M/~~Miles~~ 564
Described by D. M. SNOWSILL	Date 10.1.80	B.M. 4/1	" " 848

GRID COORDINATES	PHOTO-IDENTIFICATION
E ~~459~~ -47 100	Air photo nos. 147 LE4 035 037
N ~~4892~~ -3244 200	Measurements in box are in mm. to Reference Mark on photo - no. 035

53 | 21

Title edge of photo

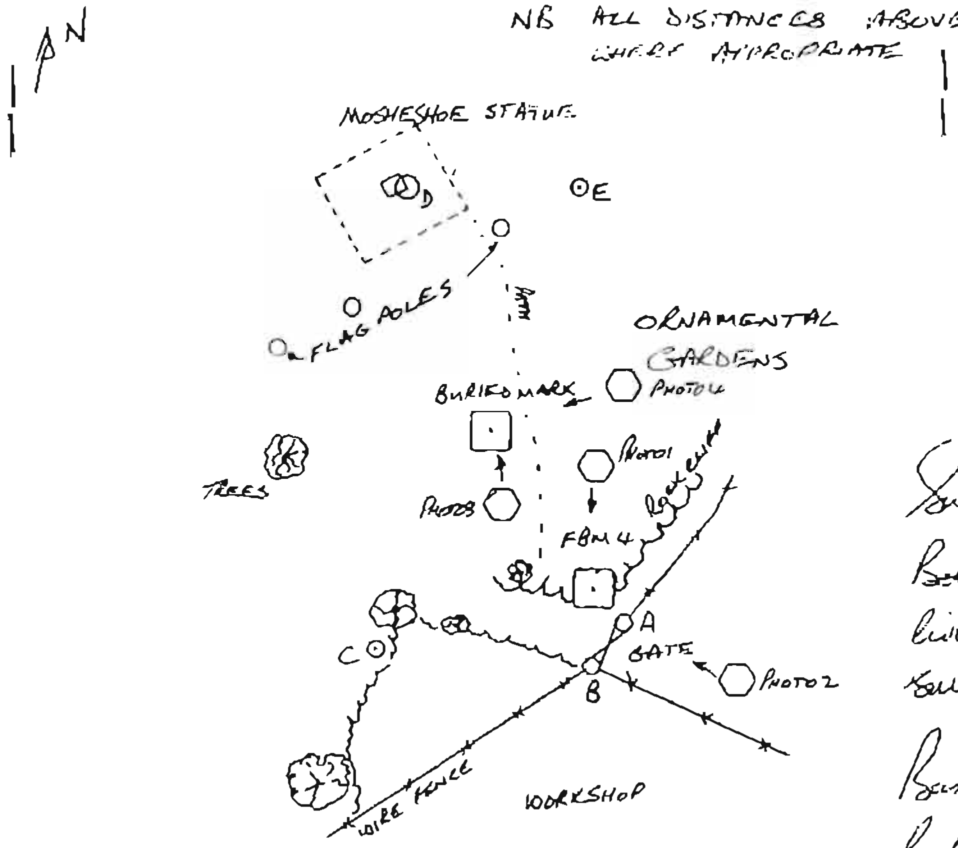
REFERENCE MARKS — Field Book

To	At Surface Mark Brg	At Surface Mark Dist	At Surface Mark Ht Diff	At Buried Mark Brg	At Buried Mark Dist	At Buried Mark Ht Diff	Description
B.M.	328°	12.5m	—				
S.M.				—	—	—	
a	139°	3.1m	—	146°	15.5m	—	Iron gate post
b	181°	5.4m	—	157°	17.2m	—	Iron gate post
c	252°	14.2m	—	268°	16.0m	—	Electricity pole
d	336°	28.7m	~~16.3m~~	340°	16.3m	—	Corner Moshesho statue (under right buttock)
e	358°	26.4m	—	17°	16.4m	—	Flag pole ~~nea~~ nearest Kingsway

Bearings are magnetic (variation). Distances in Ft/M

DESCRIPTION and LOCATION SKETCH of BURIED and SURFACE MARKS

NB ALL DISTANCES ABOVE ARE SLOPE DISTANCES WHERE APPROPRIATE



Surface Mark - Brass bolt cemented into live rock at top of small cliff.

Buried mark - Brass bolt in square hollow in reinf. slab-conc. buried approx 0.2m. Hollow covered by lid.

D.O.S. 2766

F. Leseryeho Licensed Surveyor.
V. Kumane. Licenced Surveyor.

Please note that the following triangulation points have been replaced by pillars with coordinates that are slightly different. You should in future use the new pillars with their coordinates. Please update your records so that confusion does not arise.

- BP15 is replaced by BS66
- BP17 is replaced by BS74
- BP18 is replaced by BS67
- BP19 is replaced by BS65
- BP29 is replaced by BS76
- BS3 is replaced by BT131
- BS33 is replaced by BS71
- BS37 is replaced by BS72
- BS42 is replaced by BS63
- BS45 is replaced by BS64
- BS49 is replaced by BS60
- BS50 is replaced by BS61
- BS53 is replaced by BS62
- BS55 is replaced by BS69
- BS58 is replaced by BS70
- BS59 is replaced by BS73
- BS27 is replaced by BT96
- [illegible]Y77 is replaced by BT94.

R. N. Butler Examiner 23.05.'[illegible]

PROJECTION TABLES

SOUTH AFRICA BELTS

Projection: Transverse Mercator

Spheroid: Clarke 1880, the dimensions of which, as given in the Royal Geographical Society Technical Series No. 4, are as follows:

$$a = 6,378,249.145 \text{ meters}$$

$$f = \frac{1}{293.465}$$

$$n = 0.00170\ 66885\ 12280$$

whence

$$e^2 = 0.00680\ 35112\ 82850$$

$$e'^2 = 0.00685\ 01161\ 25$$

$$^* A' = 111,131.861691$$

$$^* B' = 16,300.70065$$

$$^* C' = 17.38763$$

$$^* D' = 0.02308$$

* See page vi

Unit of Measurement: Yard (1 yard = 0.9143917962 meter)

Origin:- Equator and odd numbered meridians

Belt	
F	11°E
G	13
H	15
J	17
K	19
L	21
M	23
N	25
P	27
R	29
S	31
T	33
U	35
X	37
Y	39
Z	41

False Coordinates of Origin:- 250,000 yards East
5,000,000 yards North

Scale Factor:- Unity

MAPPING & CHARTING ESTABLISHMENT RE

COMPUTER & GEODESY SUPPORT GROUP

WORKING PAPER NO 14/81

DOPPLER CHECKS ON GROUND SURVEYS IN LESOTHO

by

H H M ROGERS

SUMMARY

This working paper describes computations carried out in CGSG to compare the coordinates obtained by Doppler fixes with the ground survey coordinates at five stations in LESOTHO. It discusses the geodetic evidence these provide.

Ref: MCE/Daq

Date: September 1981

ADOS STATION COORDINATES

COUNTRY	COMPUTED COORDINATES OF STATION MARK (SOFTWARE DOPPLR 79) (ELLIPSOID USED: a=6378145m, f=1/298.25)			HEIGHT OF ANTENNA E.C. ABOVE STATION MARK (m)	NO. OF PASSES FOR SOLUTION INPUT/USED
	X (m)	Y (m)	Z (m)		
	LATITUDE (φ)	LONGITUDE (λ)	ELLIPSOID HT. (m)		
6. GUINEA					
AGV001	6 096 077.25 N 12° 04' 44".063	-1 329 209.90 E 347° 41' 58".382	1 326 252.83 1 490.98	0.50	84/64
AGV002	6 118 912.52 N 11° 29' 27".485	-1 282 799.67 E 348° 09' 34".923	1 262 436.53 816.62	0.49	90/71
AGV003	6 054 320.24 N 12° 25' 30".732	-1 468 462.45 E 346° 21' 58".736	1 363 408.35 145.20	0.53	37/36
7. IVORY COAST					
AIV001	6 306 984.70 N 4° 25' 13".988	-814 426.96 E 352° 38' 31".410	488 328.96 53.77	0.940	63/52
AIV002	6 335 420.69 N 5° 19' 36".255	-443 674.80 E 355° 59' 38".648	588 183.92 153.72	2.167	145/135
AIV003	6 297 288.29 N 7° 42' 04".830	-551 192.20 E 354° 59' 51".846	849 110.41 373.64	0.498	52/47
AIV004	6 263 835.48 N 7° 15' 10".584	-897 837.23 E 351° 50' 34".709	799 943.76 410.41	1.218	56/51
AIV005	6 236 934.81 N 9° 30' 04".447	-828 173.74 E 352° 26' 10".342	1 045 956.57 461.86	1.058	54/48
AIV006	6 274 623.62 N 7° 57' 31".735	-734 916.58 E 353° 19' 10".765	877 326.09 404.38	1.426	41/38
AIV007	6 308 696.63 N 8° 02' 41".806	-307 959.77 E 357° 12' 19".146	886 762.38 423.75	2.20	132/111
AIV008	6 257 251.26 N 9° 58' 35".160	-564 892.48 E 354° 50' 29".157	1 097 746.87 370.99	2.10	149/139
AIV009	6 335 421.27 N 5° 19' 36".134	-443 676.01 E 355° 59' 38".611	588 180.24 154.04	2.50	242/212
8. LESOTHO					
ALT001	4 879 098.94 S 30° 25' 36".582	2 551 595.48 E 27° 36' 28".690	-3 212 185.89 1 816.21	0.548	44/43
ALT002	4 937 801.25 S 29° 38' 20".012	2 533 432.83 E 27° 09' 39".434	-3 136 570.82 1 864.83	1.993	43/42
ALT003	4 848 942.39 S 29° 46' 54".762	2 686 278.77 E 28° 59' 10".081	-3 151 132.30 3 457.98	0.578	45/44
ALT004	4 898 844.37 S 29° 27' 51".504	2 630 172.14 E 28° 13' 52".295	-3 120 252.48 2 924.20	0.548	46/45

COUNTRY	COMPUTED COORDINATES OF STATION MARK (SOFTWARE DOPPLR 79) (ELLIPSOID USED: a=6378145m, f=1/298.25) X (m) / LATITUDE (φ)	Y (m) / LONGITUDE (λ)	Z (m) / ELLIPSOID HT. (m)	HEIGHT OF ANTENNA E.C. ABOVE STATION MARK (m)	NO. OF PASSES FOR SOLUTION INPUT/USED
8. LESOTHO					
ALT005	4 902 086.87 S 28° 46' 20".370	2 703 058.84 E 28° 52' 21".985	-3 053 407.67 3 319.11	1.721	45/43
9. LIBERIA					
ALI001	6 280 755.08 N 4° 51' 14".691	-971 500.55 E 351° 12' 26".034	536 110.17 24.72	1.798	41/39
ALI002	6 255 275.88 N 7° 31' 12".191	-936 891.18 E 351° 28' 54".400	829 372.21 1 410.98	0.921	40/36
ALI003	6 299 343.42 N 4° 52' 01".760	-842 666.72 E 352° 22' 50".679	537 562.01 156.90	1.638	40/39
ALI004	6 219 828.47 N 6° 52' 55".476	-1 189 649.89 E 349° 10' 19".083	759 199.46 83.71	1.841	40/35
10. MALAWI					
AMI001	4 987 267.99 S 16° 48' 36".631	3 525 374.78 E 35° 15' 19".919	-1 832 783.48 165.37	0.980	47/45
AMI002	5 009 040.60 S 15° 52' 19".422	3 546 173.54 E 35° 17' 48".345	-1 733 325.94 765.54	0.963	52/52
AMI003	5 054 475.32 S 14° 19' 38".828	3 558 763.58 E 35° 08' 55".231	-1 568 263.20 612.15	0.967	48/48
AMI004	5 160 794.83 S 13° 39' 14".656	3 436 782.83 E 33° 39' 40".528	-1 496 154.89 1 419.42	0.980	49/48
AMI005	5 162 762.97 S 11° 39' 25".111	3 519 591.20 E 34° 16' 59".816	-1 280 446.18 901.52	0.988	88/80
AMI006	5 242 825.47 S 9° 39' 10".660	3 475 688.90 E 33° 32' 31".442	-1 062 742.63 1 875.67	1.048	53/52
11. MALI					
AML001	6 052 551.64 N 14° 28' 43".594	-1 233 332.77 E 348° 28' 56".937	1 584 344.55 64.48	1.50	376/205
12. MAURITANIA					
AMR001	5 964 479.73 N 16° 03' 42".148	-1 418 426.43 E 346° 37' 22".248	1 753 315.74 82.52	1.50	55/37
13. SENEGAL					
ASG001	5 896 776.50 N 14° 41' 41".276	-1 818 959.25 E 342° 51' 24".290	1 607 485.16 100.02	1.50	219/62

INTRODUCTION

1. During the first three months of 1979, 512 STRE fixed five Doppler stations in LESOTHO. All of these stations were existing primary or secondary stations but at two of them it was necessary to set up over witness marks as the original pillar had been destroyed.

2. This working paper examines the relationship between the Doppler positions and the ground survey coordinates.

EXISTING GROUND SURVEYS

3. LESOTHO is approximately 250km long by 150km wide and is completely surrounded by SOUTH AFRICA. The LESOTHO primary triangulation was observed by the Directorate of Overseas Surveys who adjusted it in 1964. It has since been broken down by second order EDM traverses.

4. The same specification was used for the LESOTHO primary triangulation as was used in SOUTH AFRICA. However the surrounding South African framework had already been adjusted and DOS found that it was not possible to fit their own triangulation into the remaining gap without seriously distorting the angles. A compromise was adopted whereby the South African coordinates in the flat cultivated areas near the Northwestern border were accepted and the DOS framework adjusted to them. South African coordinates on the other side of the country, which is mountainous and unlikely to undergo intensive development, were not used. The LESOTHO triangulation is discussed in greater detail in Reference 1.

Note that height values for trigs in LESOTHO can NOT be relied on to better than ± 3 metres.

5. The stations occupied by the Doppler team were as follows:

31016	Thaba Phechela	(WM4, BS43)
31017	Alwyns Kop	(3027/04)
31018	Ditsuming	(BS74)
31019	Mekoaneng	(BP24)
31020	Mount aux Sources	(WM2, BP3)

6. Stations 31016 and 31020 had to be set up over witness marks. At 31016 it did not prove possible to find the original witness mark observations so the Doppler point was resected using eight other stations well spread round the horizon. The standard error of this fix was 0.16m vector. At 31010 the original buried mark from beneath the pillar was located. Witness mark observations had been abstracted onto the DOS Station Description. As a result, there should be no error in the relationship between centre and the Doppler station which is significant in the present context. In passing, it must be recorded that the missing DOS pillar was itself a replacement for the original mark.

THE DOPPLER SURVEY

7. Reference 2 presents a full account of the field work.

8. All five Doppler summaries claim an accuracy of "1.5 meters in each axis (90% linear error)". It need only be said that:

a. Uncertainties at 31016 and 31020 in the relationship between the Doppler stations and the ground survey framework will be much smaller than the errors in the Doppler fixes.

Heights in Metres of the Geoid above the Clarke 1880 Spheroid, Cape Datum

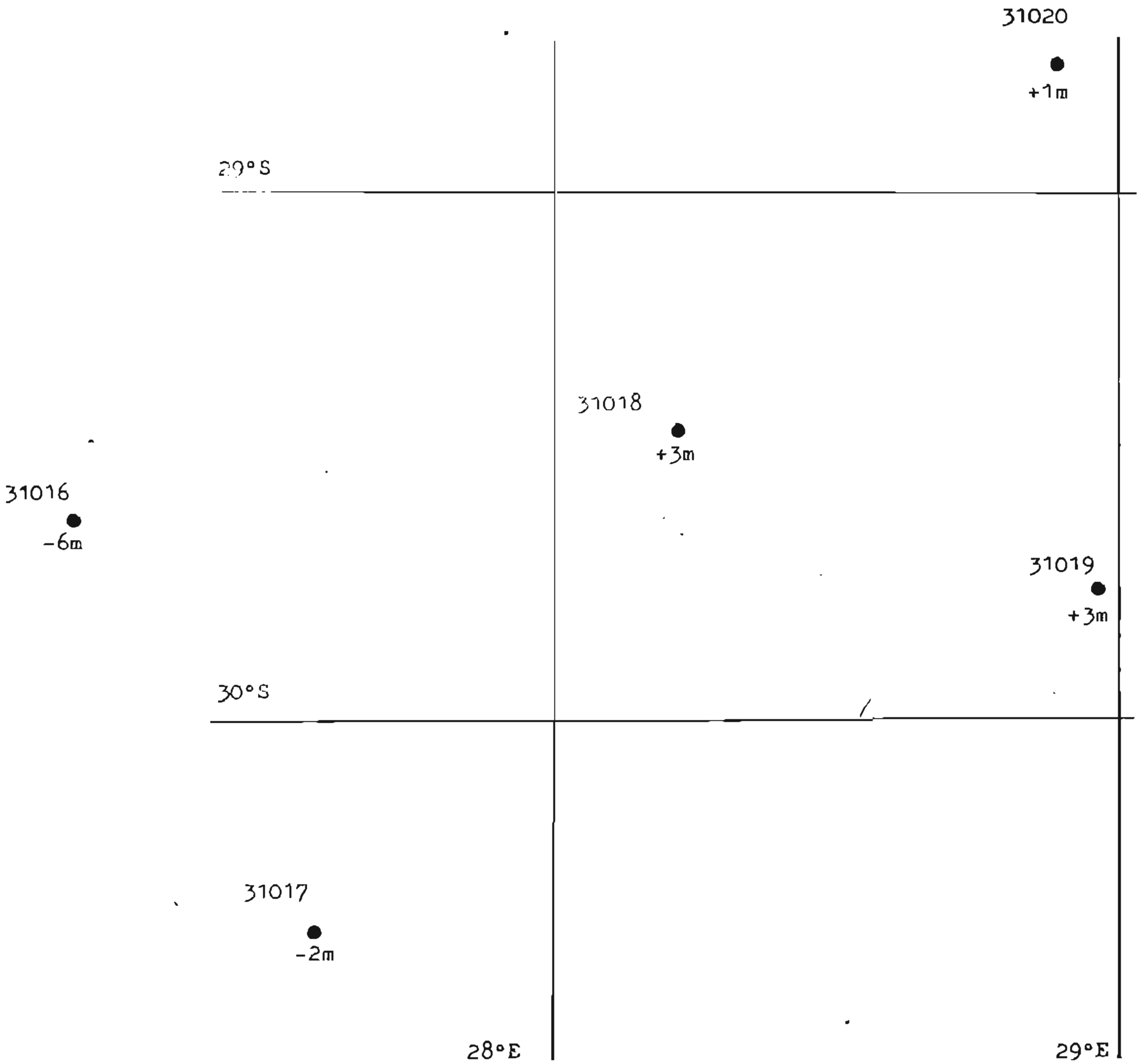
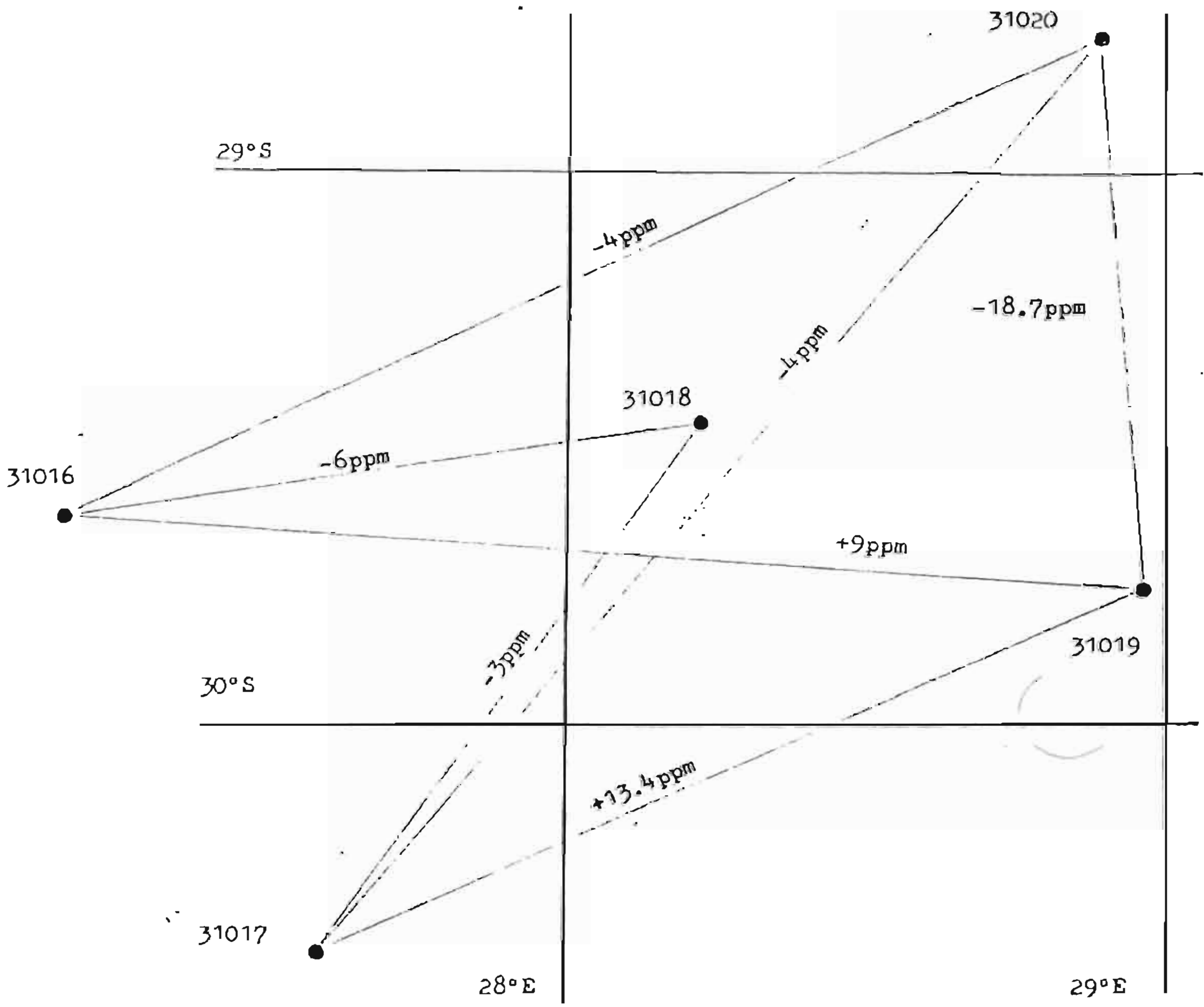


Figure 1

Scale Differences

Differences in chord length expressed as parts per million in the sense Ground Survey minus Doppler. Only chords exceeding 100km have been considered.



Mean scale error -1 ± 11ppm

Figure 2

8. b. In a country as small as LESOTHO where the stations can NOT be very far apart, only limited conclusions may be drawn from Doppler/ Ground comparisons about the scale of the ground survey framework.

COORDINATES

9. The Doppler survey coordinates are as follows:-

Datum and Spheroid: WGS 72

Station	Ø	λ	Spheroidal Height
31016	29° 38' 20.029" S	27° 09' 39.675" E	1870.34
31017	30 25 36.584	27 36 28.934	1822.28
31018	29 27 51.492	28 13 52.574	2931.00
31019	29 46 54.750	28 59 10.322	3464.20
31020	28 46 20.325	28 52 22.216	3325.18

10. The ground survey coordinates are as follows:-

Datum: Cape

Spheroid: Clarke 1880 a 6 378 249.145
e^2 0.006 803 48102

Station	Ø	λ	Height AMSL
31016	29° 38' 18.705" S	27° 09' 41.618" E	1832.0
31017	30 25 35.436	27 36 30.876	1779.0
31018	29 27 50.127	28 13 54.386	2884.4
31019	29 46 53.375	28 59 12.151	3417.5
31020	28 46 18.812	28 52 23.951	3281.1

COMPARISONS

11. The best mean fit between coordinates in the two systems, in the sense Ground minus Doppler (Clarke 1880, Cape Datum minus WGS 72) proved to be:-

$\Delta X = 113.8 \pm 0.3$ metres, spread −0.5 to +0.2 metres
$\Delta Y = 117.5 \pm 1.1$ −0.9 to +1.7
$\Delta Z = 304.1 \pm 0.8$ −0.6 to +1.5

12. The geoidal separations in the sense geoid above Clarke 1880 spheroid, Cape Datum, were:-

Stations	N
31016	-6.3m
31017	-1.5
31018	+2.9
31019	+3.4
31020	+1.5

and these are plotted in figure 1.

13. Surprisingly, there were 7 chords between Doppler stations whose length exceeded 100km. The scale differences in the sense Ground minus Doppler between the computed lengths of these chords were:-

Stations	Scale Difference	Length of Line
31016 - 31018	-5.5 ppm	106 km
31019	9.5	177
31020	-4.1	192
31017 - 31018	-3.7	123
31019	13.4	112
31020	-3.7	221
31019 - 31020	-18.9	112

and these are plotted in figure 2.

14. It must be said at once that the values obtained for N are of the same order as the errors to be expected in the trigonometrical heights and that not only is the mean scale difference only -1ppm but that it is derived from differences of chord length of the same order as the random errors to be expected in Doppler fixes.

15. It is now many years since the DOS primary adjustment. In the meantime, they have never found any difficulty closing breakdown traverses and this applies equally to short-range and long-range EDM work. Taking Figures 1 and 2 at their face value, we can see good reason for this. Although there does appear to be an East/West slope on the geoid, the overall magnitude is negligibly small. It takes a change of approximately 6m in the mean height of the terminals of a line to cause a scale change of 1 ppm. There is certainly no evidence of a systematic scale error underlying the values plotted in Figure 2.

CONCLUSIONS

16. The validity of the ground coordinates for 31016 and 31020 is confirmed.

17. There is no evidence to be found here suggesting that the scale of the LESOTHO framework is in any way substandard or that a readjustment is necessary. Even if one were to be undertaken these Doppler fixes could contribute little to the strength of the framework.

18. To establish definitive datum shifts, and to obtain better separation and scale checks, much more accurate trigonometrical heights would be required than are currently available. Should these become available it would be worthwhile to repeat the computations carried out for this paper which are annexed to the top copy only. However, for many purposes the datum shifts quoted will suffice.

H H M Rogers

H H M ROGERS

20th September 1981

Sample of ADOS Summary of satellite-observed station

SUMMARY OF SATELLITE-OBSERVED STATION

STATION NAME/LOCAL NUMBER	LOCATION	DOPPLER NO.
THABA PHECHELA (BS43,WM4)	LESOTHO	31016

STAMPING ON MARK

AGENCY (CAST IN MARK)	TYPE OF STATION MARK
	SURVEY PILLAR

DOPPLER OBSERVATIONS

EQUIPMENT/SERIAL NO.	HEIGHT OF TRACKING EQUIPMENT REFERENCE POINT ABOVE STATION MARK	TRACKING EQUIPMENT REFERENCE POINT
DHQ 008	1.993m	RED BAND ON ANTENNA
OBSERVED BY (AGENCY)	SATELLITE(S) OBSERVED	PERIOD OF OCCUPATION
DMAHTC/512 STRE	30200 30130, 30140, 30190, 30120,	23-26 JANUARY 1979

SATELLITE-DERIVED COORDINATES

PASSES ACCEPTED	DEGREES OF FREEDOM	RESIDUAL RMS	STATION SET	GRAVITY MODEL	ELLIPSOID	MINIMUM ELEV. ANGLE
43	712	0.21m	NSWC 9Z-2	NSWC 10E-1	NWL 8E	10°

(Satellite-derived coordinates referred to station mark)

φ	λ	h	ACCURACY
S 29° 38' 20".048	E 27° 09' 39".415	1 865.78m	1.5 METERS IN EACH AXIS (90% LINEAR ERROR)
X 4 937 801.74m	Y 2 533 432.48m	Z -3 136 572.28m	

(Satellite-derived coordinates of station mark transformed to local datum)

φ	λ	h	DATUM
S 29° 38' 20".029	E 27° 09' 39".675	1 870.34m	WGS 72
X 4 937 794.46m	Y 2 533 435.61m	Z -3 136 569.68m	ELLIPSOID WGS 72
ΔX	ΔY	ΔZ	DATE OF TRANSFORMATION

GROUND SURVEY COORDINATES OF STATION MARK

φ	λ	DATUM (HORIZONTAL)	ELLIPSOID
S 29° 38' 18".985	E 27° 09' 41".825	ARC 1950	CLARKE 1880

DATE OF ADJUSTMENT	ORDER	SURVEY BY (AGENCY)	DATE	LOCATION OF SURVEY DATA

ELEVATION (H)	DATUM (VERTICAL)	GEOID HEIGHT (N)	ELLIPSOID HEIGHT (h)
1 832.0m	SOUTH AFRICAN TRIG		

ORDER (ELEV.)	ESTABLISHED BY (AGENCY)	DATE	SOURCE OF (N)

CONNECTION TO LOCAL CONTROL

FROM	TO	AZ FROM NORTH	DISTANCE

REMARKS

OTHER RELATED DATA FOR THIS STATION

DATA	AVAIL.	LOCATION/REMARKS
STATION OCCUPATION REPORT	X	DMAHTC
GEODETIC INFORMATION REPORT		
STATION DESCRIPTION	X	DMAHTC
SURVEY DIAGRAM		
STATION SITE SKETCH	X	DMAHTC
PHOTOIDENTIFICATION	X	DMAHTC
ASTRONOMIC COORDINATES		
STATION PHOTOS		

PREPARED BY/DATE	CHECKED BY/DATE	REVISED BY/DATE	CHECKED BY/DATE
DMAHTC/RJ /JUN 79	DMAHTC/RBK/JUL 79		

DMA FORM 8290-1-R

AFREF - Reference System for Africa

AFREF-3

[] 'AFREF' - Continental Reference System for Africa

To: afref@igscb.jpl.nasa.gov, igsmail@igscb.jpl.nasa.gov
Subject: [AFREF-3] 'AFREF' - Continental Reference System for Africa
From: "Ruth E. Neilan" <ruth.neilan@jpl.nasa.gov>
Date: Wed, 14 Feb 2001 15:45:18 -0800
Sender: owner-afref

****
AFREF Mail 14 Feb 15:46:58 PST 2001 Message Number 3
****

Author: Ruth Neilan & Claude Boucher

Dear colleagues,

Enclosed below is the text of a letter describing the interests and motivations for establishing a continental reference system for Africa - 'AFREF'. This letter is being widely distributed by email and by hardcopy to as many additional interested persons within Africa and elsewhere as we have been able to obtain. Please feel free to forward this message. We plan to compile a mailing list for this initiative over the coming months, instructions on how to be included in this directory are at the end of this message.

The PDF version of this message contains a map, and the directory update form.please see:
http://igscb.jpl.nasa.gov/mail/afref/letter.010214.pdf

As this message is sent to multiple mailing lists, we apologize if you receive more than one copy.

with best regards,

Ruth Neilan and Claude Boucher

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February 14, 2001

Subject: Continental Reference System for Africa, African Reference System 'AFREF'

Dear colleague,

A number of discussions over the past year strongly suggest that it is time to initiate an effort towards realizing a modern continental geodetic reference system for Africa. The International Association of Geodesy (IAG) Commission X on Global and Regional Geodetic Networks and the International GPS Service (IGS), through contacts with various organizations and people within Africa, support the establishment of such a project within Africa. This letter solicits your interest in such an effort.

The most effective way to achieve a robust and globally consistent continental eference system is through the technology of the Global Positioning System (GPS). The explosive growth of GPS applications and the economics of GPS make it the technique of choice for sustainable geodetic operations within Africa. The IGS, as an existing IAG service, provides high quality GPS data, products and information resources that can further the realization of an African continental reference system. The IGS strongly supports the charter of the International Terrestrial Reference System (ITRS), part of the International Earth Rotation Service (IERS), which relies to a great extent on IGS and GPS for densification of the International Terrestrial Reference Frame (ITRF). We recall that ITRS is the global terrestrial reference system officially adopted by the IAG, and that the WGS84 (World Geodetic System 1984) reference system of the GPS, which is widely used by several communities, is now identical to ITRS at the centimeter level.

Due to the permanent global infrastructure of the IGS, a flexible approach can be designed to accomplish a continental reference system for Africa. First, permanent GPS stations within Africa are or can be linked to the highly consistent daily processing of the IGS. A sparse number of these currently exist and provide a backbone of precise control - it would be clearly beneficial to increase the number of permanent IGS stations within the African area (see IGS map at http://igscb.jpl.nasa.gov/network/netindex.html). Secondly, additional stations are required for the regional densification such as establishing GPS national networks through either permanent or semi-permanent GPS networks; or in specific areas through campaign style or single point measurements with mobile GPS receivers.

It is important to note that contrary to previous GPS network observations where it was important to have an entire network observe simultaneously, it is no longer necessary to do so. Through the continuity and permanency of the global infrastructure provided by the IGS, GPS observations taken at one time can be linked to observations taken at a subsequent locations and times with little degradation of accuracy. In fact, given the vast extent of Africa and logistical difficulties of coordinating between more than 50 nations, a more regional approach tied to a fiducial continental network seems more feasible and prudent.All subsequent analysis and results can be based on the precise products and orbits produced by the IGS to position stations in the ITRF, as well as providing the basis for transformations between it and any national geodetic datum. By following IGS/ITRF recommendations and conventions, centimeter level 3-dimensional positioning can be obtained within this framework. The applications of realizing this have vast potential from geodesy, mapping, surveying, geoinformation, geomanagement, natural hazards mitigation, Earth sciences, etc. Additionally, the project will provide a major springboard for the transfer and enhancement of skills in surveying and geodesy and especially GPS technology. Surveyors and geodesists from participating African countries will be strongly encouraged to actively participate in all phases of the project.

Over recent years there are models of other continents and countries realizing a continental reference system. For example, EUREF, the European Reference System for Europe, and SIRGAS, Sistema de Referencia Geocentrico para America del Sur, a continental reference system throughout South America. Currently a new initiative to join reference networks within North America is being pursued by the national agencies and organizations within the U.S., Canada, Mexico, the Caribbean nations, etc.

It is clear that the leadership for such a project must come from within the African professional community. Preliminary meetings within Africa to discuss project interest and approach will be initiated. IGS and IAG experts and advisors will be available to advocate the formation of such a project within Africa. The IGS and IAG would be willing to attempt coordinating international resources and assistance to facilitate such an activity. It would be important over the longer term to ensure that this project will produce established, dedicated analysis and data information centers, links between African agencies and links with international organizations. Venues for such organizational meetings are currently being considered and it is envisioned that a series of discussions may take place as opportunities arise at scheduled conferences and meetings.

On behalf of IAG and IGS, we would like to invite your participation in this effort. Please respond to this letter by sending a statement of your interest along with your contact information to the address below or email to igscb.jpl.nasa.gov with subject 'AFREF Interest'. A primary contact list will be compiled from the responses; these people will be included in any correspondence and notified of future meetings to discuss plans and organization of such a project. An AFREF mail list has been established by the IGS to facilitate discussion. You may view the correspondences and other information at: http://igscb.jpl.nasa.gov/mail/afref/afref.html, as well as instructions to subscribe.

Looking forward to your positive reply, and with best regards.

Claude Boucher
Chair, IAG Commission X, Global and Regional Networks
Head, International Terrestrial Reference Frame, IERS
IERS Representative to IGS Governing Board

Ruth E. Neilan
Director, International GPS Service Central Bureau
Member, Federation of Astronomical and Geophysical Data Services

AFREF-5

[] Re: [AFREF-4] Author: Richard Wonnacott

To: "Richard Wonnacott" <RWONNACOTT@sli.wcape.gov.za>, <afref@igscb.jpl.nasa.gov>, "Ludwig Combrinck" <ludwig@hartrao.ac.za>
Subject: [AFREF-5] Re: [AFREF-4] Author: Richard Wonnacott
From: "Muftah Unis" <oact@wissal.dz>
Date: Mon, 12 Mar 2001 08:21:25 +0100
Cc: "Khier Bouchibi" <imagis@wissal.dz>
References: <saa8bc4d.027@sli.wcape.gov.za>
Sender: owner-afref

****
AFREF Mail 12 Mar 08:48:04 PST 2001 Message Number 5
****

Author: M. Muftah UNIS

Dear Mr Ludwig Combrinck

Dear Mr RT Wonnacott

I have just received Mr RT Wonnacott' s email on AFREF project, primarily among Southern African countries to be held in Cape Town on the 13th of March 20011 q: very happy that the Southern African Region has taken some action in this matter, but i am very sorry not being able to prepare my self for participation.

I was not able to react to your previous mails because I was very busy preparing and organising The 10 th session of our Administration Council that just took place in Tunis last Week

You know that AOCRS, the African Organization of Cartography and Remote Sensing representing the African Countries, is actively sustaining efforts aiming at the development of a common continental geodetic infrastructure for Africa, the organization actively brought its contribution to the realization of ADOS project. She also sustained efforts of the AIG on the Integrated Geodetic Network for Africa (1989-1996). With the same constancy AOCRS encourages and also sustains actions aiming the unification of geodetic networks on a regional level, these networks constitutes for us the foundations of the future unified geodetic network for the continent. Currently, AOCRS is coordinating the preparation of the second Workshop on the Unified geodetic Network for North Africa that will be held in Algiers May 2001. The first Workshop was organized in Tunis May 2000. The third one will be held in Morocco late 2001 or early 2002. Morocco, Algeria, Tunisia, Libya, Mauritania and Egypt as well as representatives of European or international scientific institutions, EUREF BGI and IGN participated in the first Workshop.

As previous Important commitments prevents me from participating in this very Important meeting to be held in CAPE TOWN (South Africa) . i would like to express our real interest and our commitment to AFREF Project and our consideration for efforts provided by participants to make this importing project advancing. We kindly ask you Mr Ludwig Combrinck to represent AOCRS during this meeting and hope to receive all the presented papers and communications of this meeting as well as the report in order to inform our members on the orientations and decisions reached and be able to participate in your future meeting to bring our modest contribution.

I hope to you all success in your meeting .

Best Regards

M. Muftah UNIS

Secretary General

Appendix (2) Data files

Coordinate files used in research

Transf15.m data files

Transf15.m needs a file with 5 columns, beacon name, y1 and x1 of the 'from file' and y2 and x2 of the standard file (without headings). There should be 4 or more points. The following files were used for Transf15.m

79dossa.txt

GPS28_4.txt

Lhwp4.txt

79dossa.txt (DOS and SA)

name	d sy	dosx	say	sax
P014	4 095.51	3247524.51	41098.73	3247523.97
P028	2 797.83	3317327.13	27802.03	3317326.17
P033	3 761.73	3349589.92	39766.16	3349588.76
P038	8 54.210	3379950.65	8958.430	3379949.15
P0 9	2 361.39	3211943.35	21964.29	3211943.01
S001	4 609.09	3356515.30	41613.47	3356514.09
S010	- 0795. 1	3332414.87	-10790.85	3332414.05
S012	- 1760. 4	3332370.39	-21754.96	3332369.66
S0 2	3 544.4	3354032.71	33548.95	3354031.50
S032	4 294.0	3350241.49	49298.40	3350240.32
S034	5 063.27	3329269.80	51067.43	3329268.86
S035	4 566.42	3319055.25	42570.39	3319054.33
S036	5 449.44	3317222.23	54453.45	3317221.33
S038	5 730.12	3307600.85	54734.08	3307599.85
S004	2 652.64	3340616.13	29657.09	3340614.93
S040	7 960.09	3294194.91	78963.81	3294193.78
S041	7 274.4	3286389.87	70278.14	3286388.87
S043	8 226.5	3280371.89	81230.19	3280370.82
S044	5 034.6	3275995.56	53038.24	3275994.73
S046	3 753.6	3290393.81	31757.58	3290393.02
S047	4 380.5	3300718.46	43384.62	3300717.62
S048	3 518.2	3307937.15	32522.33	3307936.39
S005	2 006.19	3347455.42	20010.59	3347454.26
S051	1 474.22	3238028.10	17477.46	3238027.83
S052	3 919.5	3233325.88	31922.76	3233325.48
S054	5 00.52	3221574.53	5903.50	3221574.35
S056	4 34.16	3212233.32	4987.04	3212233.23
S057	234.5	3205160.11	12237.41	3205159.93
S006	287.0	3351690.07	10291.51	3351688.88
S061	2 634.7	3255943.89	27638.14	3255943.79
S063	5 834.15	3284939.43	57837.85	3284938.69
S064	4 487.58	3285018.16	43491.28	3285017.55
S065	3 129.6	3275350.33	33133.56	3275350.02
S066	1 148.9	3249007.90	10152.41	3249007.82
S067	6 430.7	3276832.61	65434.50	3276831.72
S007	1 890.4	3337573.84	16894.78	3337572.68
S070	- 096.1	3199474.63	-1093.45	3199474.43
S072	6 610.9	3313937.70	69614.96	3313936.79
S008	8 06.18	3326103.41	8510.68	3326102.52
T131	2 255.1	3357614.78	20259.47	3357613.53
T071	5 914.98	3246273.59	51918.28	3246272.49
T072	4 800.16	3247532.23	49803.42	3247531.48
T073	4 145.5	3246384.71	49148.77	3246383.91
T075	4 888.47	3244113.52	43891.63	3244112.80
T092	- 1910. 4	3271852.80	-21906.17	3271852.98
T093	- 2574. 4	3252339.61	-22570.17	3252340.24
P039	- 10067.41	3291732.68	-110062.25	3291734.97
P0 1	- 668.4	3340787.22	-1663.93	3340786.13
P030	- 3553. 7	3336125.04	-38548.50	3336124.28
P027	- 16554.70	3317014.88	-106548.79	3317016.43
P026	- 3181. 8	3312788.85	-13176.52	3312788.15
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SO2[illegible]	-[illegible]34.[illegible]4	3295624.38	-81329.15	3295625.47
SO[illegible]	-[illegible]62.[illegible]9	3294997.49	-63857.44	3294998.48
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T1[illegible]	-[illegible]12.[illegible]8	3175946.34	-54310.30	3175946.79
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T1[illegible]	-[illegible]37.[illegible]4	3168374.96	-54235.65	3168375.43
T10[illegible]	-[illegible]287.[illegible]7	3168668.20	-58284.84	3168668.73
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T10[illegible]	-[illegible]86.[illegible]2	3192985.28	-39584.15	3192985.51
T10[illegible]	-[illegible]87.[illegible]7	3201120.35	-57984.39	3201120.84
T10[illegible]	-[illegible]55.[illegible]7	3219880.34	-44551.96	3219880.51
T1[illegible]	-[illegible]54.[illegible]3	3215634.57	-62351.41	3215635.29
T1[illegible]	-[illegible]85.[illegible]5	3189582.82	-67683.04	3189583.60
T10[illegible]	-[illegible]33.[illegible]5	3186875.88	-56031.37	3186876.44
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T10[illegible]	-[illegible]407.[illegible]5	3219862.46	-54404.52	3219862.91
TO[illegible]	-[illegible]493.[illegible]7	3276200.28	-50488.34	3276200.35
PO[illegible]	-[illegible]410.[illegible]9	3374203.02	-17406.85	3374200.92
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Gps28_[illegible].txt (merminod's and SA)

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S0[illegible]	[illegible]	3255943.530	27638.147	3255943.799
S0[illegible]	[illegible]	3249007.875	10152.24	3249007.820
S0[illegible]	[illegible]	3276831.785	65434.504	3276831.722
S0[illegible]	[illegible]	3313936.629	69614.968	3313936.790
T1[illegible]	[illegible]	3357613.483	20259.474	3357613.539
T0[illegible]	[illegible]	3246272.492	51918.282	3246272.494
GOB[illegible]	[illegible]	3319446.011	88659.050	3319446.014
JAM[illegible]	[illegible]	3287681.440	94336.313	3287681.426
MO[illegible]	[illegible]	3415312.401	67399.930	3415312.410
P0[illegible]	[illegible]	3233408.88	-68540.812	3233408.883
P0[illegible]	[illegible]	3296428.479	-53311.307	3296428.384
P0[illegible]	[illegible]	3317016.495	-106548.791	3317016.439
P0[illegible]	[illegible]	3336124.323	-38548.502	3336124.286
P0[illegible]	[illegible]	3340786.173	-1663.934	3340786.130
S0[illegible]	[illegible]	3260751.397	-22447.920	3260751.291
S0[illegible]	[illegible]	3262886.597	-47345.534	3262886.576
T1[illegible]	[illegible]	3216942.516	-39192.913	3216942.433
DAM[illegible]	[illegible]	3166066.053	-38748.278	3166066.028
GEN[illegible]	[illegible]	3159728.517	-61614.662	3159728.486
GEW[illegible]	[illegible]	3207832.275	-122750.831	3207832.234
QA[illegible]	[illegible]	3335545.452	-64491.805	3335545.469
RA[illegible]	[illegible]	3292010.829	-138950.719	3292010.838
REN[illegible]	[illegible]	3187027.687	-412.084	3187027.672
SE[illegible]	[illegible]	3238648.774	-54183.13	3238648.77

Lhwp4.txt (LHWP and SA)

Nam	LHWPy	LHWPx	SAy	SAx
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S	18686.29	3222366.29	18720.37	3222406.79
S	17443.24	3237988.37	17477.46	3238027.83
S	4953.40	3212192.01	4987.04	3212233.23
S	5869.76	3221533.77	5903.50	3221574.35
S	12203.67	3205118.43	12237.41	3205159.93
T	3461.43	3255125.84	3495.40	3255164.42
T	-54732.39	3195975.63	-54700.69	3196018.07
P	-68572.19	3233368.84	-68540.81	3233408.88
S	-22481.04	3260712.84	-22447.92	3260751.29
S	-3417.49	3216450.86	-3384.01	3216491.86
T	-54436.40	3219822.07	-54404.52	3219862.91
T	-62383.10	3215594.14	-62351.41	3215635.29
T	-49174.49	3212771.46	-49142.50	3212812.72
T	-55671.47	3215764.18	-55639.64	3215805.28
T	-43880.98	3177667.74	-43849.07	3177711.21
T	-36729.35	3183082.44	-36697.22	3183125.54
T	-50596.55	3191307.82	-50564.72	3191350.46
T	-41834.49	3197845.08	-41802.40	3197887.27
T	-47558.67	3197998.38	-47526.72	3198040.59
T	-38138.34	3203606.07	-38106.15	3203647.87
T	-35677.26	3196346.64	-35645.07	3196388.89
T	-49242.71	3203434.32	-49210.74	3203476.17
P	-14125.85	3195436.95	-14092.93	3195479.21
T	-51949.33	3175080.41	-51917.64	3175124.15
T	-43642.14	3171871.73	-43610.27	3171915.59
T	-56063.06	3186833.49	-56031.37	3186876.44
T	-67714.41	3189540.76	-67683.04	3189583.60
T	-39616.26	3192943.00	-39584.15	3192985.51
T	-58016.12	3201078.78	-57984.39	3201120.84
T	-51871.80	3195360.51	-51840.02	3195402.96
T	-59207.35	3196550.76	-59175.67	3196593.18
T	-54267.24	3168331.19	-54235.65	3168375.43
T	-58316.27	3168624.44	-58284.84	3168668.73
T	-62390.13	3164967.00	-62358.83	3165011.55
T	-54341.97	3175903.10	-54310.30	3175946.79
T	-51044.55	3179558.68	-51012.82	3179602.03
T	-52674.61	3186996.65	-52642.89	3187039.61
T	-53350.20	3187429.16	-53318.50	3187472.10
T	-58348.59	3183725.13	-58316.98	3183768.27
T	-52229.22	3204100.52	-52197.38	3204142.34
T	-50683.18	3207764.13	-50651.29	3207805.69
T	-56853.21	3209165.57	-56821.42	3209207.08
P	-42553.04	3207323.35	-42520.88	3207364.91
T	-21939.36	3271815.08	-21906.17	3271852.98
T	-22603.26	3252301.34	-22570.17	3252340.24
T	-50520.61	3276162.58	-50488.34	3276200.35
T	-44516.24	3250624.45	-44483.90	3250663.56
T	-54040.98	3229005.89	-54009.05	3229046.18
T	-44305.56	3243073.99	-44273.25	3243113.49
T	-3268.61	3238974.71	-3234.97	3239014.17
T	-31796.71	3251155.05	-31763.95	3251194.04
T	-52739.48	3245412.54	-52707.43	3245451.97
T	-15558.94	3245852.66	-15525.65	3245891.84
T	-17991.26	3236201.68	-17958.10	3236241.40
T	-14048.20	3260308.97	-14014.78	3260347.39

N	LHWP y	LHWPx	SAy	SAx
T	-8330.07	3262193.45	-8296.46	3262231.75
T	-7078.91	3250525.89	-7045.31	3250564.77
T	-30970.92	3242674.14	-30938.19	3242713.59
T	-50619.69	3238084.76	-50587.61	3238124.56
T	-40363.76	3240694.21	-40331.34	3240733.81
T	-15937.01	3261092.00	-15903.65	3261130.40
T	-2931.82	3268601.14	-2897.97	3268639.09
T	-41355.73	3223556.06	-41323.45	3223596.60
T	-13050.92	3252397.08	-13017.51	3252435.90
T	-39339.50	3228587.56	-39307.13	3228627.81
T	-39225.20	3216901.54	-39192.91	3216942.43
T	-4264.43	3260196.30	-4230.68	3260234.67
S	-47377.85	3262848.13	-47345.53	3262886.57
T	-44584.13	3219839.73	-44553.69	3219880.50

Appendix (3) Programs listing

Programs code (Scripts and Functions).

Transf15.m
Zoom.m
Utm.m
Lesotho.m

Functions

Rectogg.m
Ggtocart.m
Carttogg.m
Geotoxy.m
Matrans.m

Sub-functions

Radtodms.m
Dtodms.m
Degdec.m

Transf15.m

```
echo on
 % A program to calculate the Helmert up to the 4th order polynomial
 % conformal Transformation parameters between the Lesotho coordinates
 % and the S.A. computed coordinates.
echo off

%*****************************************************************
%********** READ COORDS FROM THE DATA FILE ***************

disp('          Hit any key to continue after pause');
disp('          Press any key to continue');
pause
clc
clear all
format long;

[filename,path]=uigetfile('*.txt','Select a file')
fid =fopen(filename,'rt')

S=fscanf(fid,'%s %f %f %f %f\n\t',[8 inf]);
[S1]=S';
a1=setstr(S1(:,1));b1=setstr(S1(:,2));c1=setstr(S1(:,3));d1=setstr(S1(:,4));

name=[a1 b1 c1 d1];

do84sa=S1;
ncoords=length(do84sa);
%ncoords=4;

name=name;
dosy=do84sa(:,5);    %DOS y-coords
dosx=do84sa(:,6);    %DOS x-coords
say=do84sa(:,7);     %S.A. y-coords
sax=do84sa(:,8);     %S.A. x-coords
%ht=do84sa(:,9);     %Orth. ht (dos)

meandosy=mean(dosy);
meandosx=mean(dosx);
meansay=mean(say);
meansax=mean(sax);

diffy=say-dosy;
diffx=sax-dosx;
meany1=mean(diffy);
meanx1=mean(diffx);
vecraw=sqrt(diffy.^2+diffx.^2);
```

```
dosy1=dosy-meandosy;
dosx1=dosx-meandosx;
say1=say-meansay;
sax1=sax-meansax;
y1=say1-dosy1;
x1=sax1-dosx1;


%*******************************************************************
%*************ENTER THE COORDS INTO THE COEF. MATRIX ********

k=1;
i=0;

scale1=10000;
scale2=1e5;
scale3=1e10;
scale4=1e11;

disp(' 1 for Helmert transformation');
disp(' 2 for 2nd order polynomial');
disp(' 3 for 3rd order polynomial');
disp(' 4 for 4th order polynomial');
order=menu('what polynomial/order do you want','1','2','3','4')
if isempty(order)~=0
  order
end

for k=1:ncoords
  i=i+1;

      %*************** For the x-terms ****************
%------------------ 1st order polynomial terms--------------------
if order==1
  A(i,1)=scale1;
  A(i,2)=0;
  A(i,3)=dosx1(k);
  A(i,4)=-dosy1(k);

%------------------ 2nd order polynomial terms--------------------
elseif order==2
  A(i,1)=scale1;
  A(i,2)=0;
  A(i,3)=dosx1(k);
  A(i,4)=-dosy1(k);

  A(i,5)=((dosx1(k))^2-(dosy1(k))^2)/scale2;
  A(i,6)=(-2*dosx1(k)*dosy1(k))/scale2;

%------------------ 3rd order polynomial terms--------------------
```

```
elseif order==3
  A(i,1)=scale1;
  A(i,2)=0;
  A(i,3)=dosx1(k);
  A(i,4)=-dosy1(k);

  A(i,5)=((dosx1(k))^2-(dosy1(k))^2)/scale2;
  A(i,6)=(-2*dosx1(k)*dosy1(k))/scale2;

  A(i,7)=((dosx1(k))^3-3*dosx1(k)*(dosy1(k))^2)/scale3;
  A(i,8)=(3*(dosx1(k))^2*dosy1(k)-(dosy1(k))^3)/scale3;

%-------------------- 4th order polynomial terms----------------------
elseif order==4
  A(i,1)=scale1;
  A(i,2)=0;
  A(i,3)=dosx1(k);
  A(i,4)=-dosy1(k);

  A(i,5)=((dosx1(k))^2-(dosy1(k))^2)/scale2;
  A(i,6)=(-2*dosx1(k)*dosy1(k))/scale2;

  A(i,7)=((dosx1(k))^3-3*dosx1(k)*(dosy1(k))^2)/scale3;
  A(i,8)=(3*(dosx1(k))^2*dosy1(k)-(dosy1(k))^3)/scale3;

  A(i,9)=((dosx1(k))^4+(dosy1(k))^4-6*(dosx1(k))^2*(dosy1(k))^2)/scale4;
  A(i,10)=(-4*((dosx1(k))^3*dosy1(k)-(dosy1(k))^3*dosx1(k)))/scale4;
end

  b(i)=sax1(k);

%**************** For the y-terms *****************

  i=i+1;
% ----------------1st order(Helmert) polynomial terms------------------
if order==1
  A(i,1)=0;
  A(i,2)=scale1;
  A(i,3)=dosy1(k);
  A(i,4)=dosx1(k);

%------------------ 2nd order polynomial terms-------------------
elseif order==2
  A(i,1)=0;
  A(i,2)=scale1;
  A(i,3)=dosy1(k);
  A(i,4)=dosx1(k);

  A(i,5)=(2*dosx1(k)*dosy1(k))/scale2;
  A(i,6)=((dosx1(k))^2-(dosy1(k))^2)/scale2;
```

```
%-------------------- 3rd order polynomial terms------------------
elseif order==3

   A(i,1)=0;
   A(i,2)=scale1;
   A(i,3)=dosy1(k);
   A(i,4)=dosx1(k);

   A(i,5)=(2*dosx1(k)*dosy1(k))/scale2;
   A(i,6)=((dosx1(k))^2-(dosy1(k))^2)/scale2;

   A(i,7)=(-1*(-3*(dosx1(k))^2*dosy1(k)+(dosy1(k))^3))/scale3;
   A(i,8)=((dosx1(k))^3-3*dosx1(k)*(dosy1(k))^2)/scale3;

%---------------------4th order polynomial terms--------------------
elseif order==4
   A(i,1)=0;
   A(i,2)=scale1;
   A(i,3)=dosy1(k);
   A(i,4)=dosx1(k);

   A(i,5)=(2*dosx1(k)*dosy1(k))/scale2;
   A(i,6)=((dosx1(k))^2-(dosy1(k))^2)/scale2;

   A(i,7)=(-1*(-3*(dosx1(k))^2*dosy1(k)+(dosy1(k))^3))/scale3;
   A(i,8)=(((dosx1(k))^3)-3*dosx1(k)*(dosy1(k))^2)/scale3;

   A(i,9)=(4*(((dosx1(k))^3)*dosy1(k)-((dosy1(k))^3)*dosx1(k)))/scale4;
   A(i,10)=((dosx1(k))^4+(dosy1(k))^4-6*(dosx1(k))^2*(dosy1(k))^2)/scale4;
end

  b(i)=say1(k);

end

%*****************************************************************************
%***************** solve normal equations ***********************


b=b';                    %
%sigpriori=input('enter sigma priori estimate:');    %sigma priori estimate
sigpriori=1;
W=eye(2*ncoords,2*ncoords);                    %size of wt matrix
P=(1/sigpriori^2)*W;                    %Wt matrix
B=A'*P*A;                    %
r=A'*P*b;                    %
%pause

C=inv(B);                    %
```

```
pause(2)
x=C*r                       %soltn for the unknown x
disp('  Enter to continue');
pause

v=A*x-b;                    %residuals Ax-b=0
sigma0=sqrt(v'*v/(2*ncoords-4)) %std error
nresid=length(v);
n=0; m=0;
 for n=1:(nresid/2)
    m=m+1;
    resmat(m,1)=v(2*n-1);
    resmat(m,2)=v(2*n);
    xx=resmat(:,1);
    yy=resmat(:,2);
 end

vbig=find(abs(v)>3*sigma0);
yyvbig=find(abs(yy)>3*sigma0);
xxvbig=find(abs(xx)>3*sigma0);


%end
%pause(3)

%   ********* give large resid. zero weight **********

w=0;
if isempty(yyvbig)& isempty(xxvbig)
   yyvbig
   xxvbig
   %if yyvbig~=[] & xxvbig~=[]
   %w=input('enter the weight of each coord to reweigh w:')
   w==0;
   ny=length(yyvbig);
   nx=length(xxvbig);
   for n=1:ny
   P(yyvbig(n),yyvbig(n))=w;
   end
   for m=1:nx
   P(xxvbig(m),xxvbig(m))=w;
   end
   B1=A'*P*A;               %
   pause(3)

end


disp('                 residuals:');
disp('                 ==========');
```

```
disp('                              ');
resmat=[yy xx]
pause


dosyy=dosy+yy;
dosxx=dosx+xx;

format bank;


disp('             updated coordinates:');
disp('             ======= ============');
disp(' ')
disp('        update1=original coords + residuals');
update1=[dosyy dosxx]
pause

format short;

%**************************************************************

disp('          Transformation Parameters');
disp('          ============== ==========');

sigma0=sqrt(v'*v/(2*ncoords-4)) %std error

disp('           Translation in metres');
disp('           =========== == ======');

Xo=x(1)*scale1          %translation in x
Yo=x(2)*scale1          %translation in y
p=x(3)              %direction cosines
q=x(4)

pause
%clc

Q=sigma0^2*C
var=diag(Q)            %variance
se=sqrt(diag(Q))
CC=corrcoef(Q);           %correlation coefficient
pause

% ************ Angle and Scale Factor Calculations ****************

disp('            Rotation');
disp('            ========');

theta=atan2(q,p)                 % angle in radians
```

```
phi=(theta)*(180/pi);                % angle in seconds
const=60;

phi
r1=fix(phi);r2=(phi-r1);r3=const*r2;
r4=fix(r3);r5=r3-r4;r6=r5*const;
rotangle=[r1 r4 r6];
fprintf(1,'Rotation Angle is %3.0f deg %3.0f min %10.8f sec\n',r1,r4,r6);

format long

SF=sqrt(p^2+q^2)          %Scale Factor

pause

% **************** Shift vector calculations ********************
%            raw difference between coords

format bank
  y1=say1-dosy1;
  x1=sax1-dosx1;
  vecshift=1.*sqrt(x1.^2+y1.^2);
  vecres=sqrt(yy.^2+xx.^2);
  f=5000;                         %Magnification factor

th=atan2(x1,y1)+pi;              % angle in radians
ph=(th)*(180/pi);                % angle in seconds
const=60;

format long
ph;
r1=fix(ph);r2=(ph-r1);r3=const*r2;
r4=fix(r3);r5=r3-r4;r6=r5*const;

% ************** Plot of points positions *********************
clc
  disp('         For the following diagrams:');

disp('****************************************************************
*** ');
  disp('* The left mouse button for zooming into point under mouse')
  disp('*  pointer or by pressing and dragging into selected area');
  disp(' ');
  disp('*  ');
  disp('* The right mouse button for zooming out');
  disp(' ');
  disp('* ');
  disp('* Doube click on the right mouse-button to reset figure ');
  disp(' ');
  disp('* ')
```

```
  disp('* Zoom factor is 2');
  disp(' ');
  disp('* ');
  disp('* type f and enter to veiw magnification factor of residuals');
  disp('*');

disp('*****************************************************************
*** ');
  pause

  cla;


  %*********** draw displacement vectors after Helmert transformation

  th=atan2(xx,yy);                % angle in radians
  ph=(th)*(180/pi);               % angle in seconds
 %  vecres=1.*sqrt(xx.^2+yy.^2);

disp('maximum and minimum residuals:')
max_residuals=max(abs([yy xx]))
min_residuals=min(abs([yy xx]))
disp('maximum and minimum residual vectors:')
max_vector=max(vecres)
min_vector=min(vecres)
disp('maximum and minimum shift vectors:')
max_shiftvector=max(vecshift)
min_shiftvector=min(vecshift)

if order==1, title('Residual Vectors After Helmert Transformation'),
elseif order==2, title('Residual Vectors After 2nd Order Transformation'),
elseif order==3, title('Residual Vectors After 3rd Order Transformation'),
elseif order==4, title('Residual Vectors After 4th Order Transformation'),
end

axis ('equal')
  if ~isempty(yyvbig)|~isempty(xxvbig)
  if yyvbig~=xxvbig
  n=length(yyvbig)+length(xxvbig);
  else n=find(max(length(xxvbig|yyvbig)));
  end
  nx=xxvbig;
  ny=yyvbig;

  if isempty(yyvbig)& isempty(xxvbig)
  nx=xxvbig;
  ny=yyvbig;
  n=length(xxvbig):
  nx=xxvbig;
  ny=yyvbig;
```

```
    end
    end

    if ~isempty(yyvbig) | ~isempty(xxvbig)
    for yyvbig=1:n
    name1=([a1(ny) b1(ny) c1(ny) d1(ny)]);
    residual=[yy(ny) xx(ny)];
    %fprintf('%s %1.7f %1.7f\n',[name1]',[yy(ny)],[xx(ny)]);
    text(say(ny),sax(ny),(name(ny,1:4)))
    end
    if ~isempty(yyvbig)
     name1
     residual
    end
    for xxvbig=1:n
    name1=([a1(nx) b1(nx) c1(nx) d1(nx)]);
    residual=[yy(nx) xx(nx)];
    text(say(nx),sax(nx),name(nx,1:4))
    end
    if ~isempty(xxvbig)
    name1
    residual
    end
end


    if isempty(yyvbig)& isempty(xxvbig)
    text(say,sax,name)
    end
    hold on
    h=uicontrol('style','edit','position',[75 50 100 20],...
            'string','ArroScale 250','backgroundcolor','white');

    %f=5000;
    tiplen=3000;
    for k=1:ncoords

    %************ draw dispacement vector

    tipx=say(k)+f*vecres(k)*cos(th(k));
    tipy=sax(k)+f*vecres(k)*sin(th(k));

     line([say(k)  tipx] , [sax(k) tipy ]);
     dirn=th(k)+(30-180)*pi/180;
     tipx2=tipx+tiplen*cos(dirn);
     tipy2=tipy+tiplen*sin(dirn);
     line([tipx tipx2],[tipy tipy2]);
     dirn=th(k)+(-30-180)*pi/180;
     tipx2=tipx+tiplen*cos(dirn);
     tipy2=tipy+tiplen*sin(dirn);
```

```
   line([tipx tipx2],[tipy tipy2]);
  end
   set(gca,'ydir','reverse')
   set(gca,'xdir','reverse');
zoom on
  hold on
y=1; n=0;
disp('     ..............................');
disp('     ..............................');

ansr=menu('label all points','y','n');

if ansr==y
   for k=1:ncoords
  text(say,sax,name);
end

elseif ansr==n
   break
end
hold off
 pause
 th=atan2(x1,y1)+pi;                    % angle in radians
ph=(th)*(180/pi);                     % angle in seconds
const=60;

 cla;
  title('Residuals After Shifts'),...
  xlabel('Y-coordinate Residual'),ylabel('X-coordinate residual')

h=uicontrol('style','edit','position',[75 50 100 20],...
          'string','ArroScale 250','backgroundcolor','white');
  zoom on

  axis('equal');
  f=2500;
  tiplen=3000;
  for k=1:ncoords

  %************ draw dispacement vector

  tipx=say(k)+f*vecraw(k)*cos(th(k));
  tipy=sax(k)+f*vecraw(k)*sin(th(k));

   line([say(k)  tipx] , [sax(k) tipy ]);
   dirn=th(k)+(30-180)*pi/180;
   tipx2=tipx+tiplen*cos(dirn);
   tipy2=tipy+tiplen*sin(dirn);
   line([tipx tipx2],[tipy tipy2]);
   dirn=th(k)+(-30-180)*pi/180;
```

```
    tipx2=tipx+tiplen*cos(dirn);
    tipy2=tipy+tiplen*sin(dirn);
    line([tipx tipx2],[tipy tipy2]);
   end
    set(gca,'ydir','reverse')
    set(gca,'xdir','reverse');
zoom on
hold off
    end
```

zoom.m

The following zoom subfunction has been borrowed from the Matlab zoom function to work with the transformation program above:

```
function out = zoom(m)
%ZOOM       Zoom in and out on a 2-D plot.
%       ZOOM ON turns zoom on for the current figure.  Click
%       the left mouse button to zoom in on the point under the
%       mouse.  Click the right mouse button to zoom out
%       (shift-click on the Macintosh).  Each time you click,
%       the axes limits will be changed by a factor of 2 (in or out).
%       You can also click and drag to zoom into an area.
%
%       ZOOM OFF turns zoom off. ZOOM with no arguments
%       toggles the zoom status.  ZOOM OUT returns the plot
%       to its initial (full) zoom.

%       Clay M. Thompson 1-25-93
%       Revised 11 Jan 94 by Steven L. Eddins
%       Copyright (c) 1984-94 by The MathWorks, Inc.
%       $Revision: 1.11 $  $Date: 1994/04/26 22:13:30 $

%       Note: zoom uses the userdata of the zlabel of the axis and
%       the figure buttondown and buttonmotion functions
%
%       ZOOM XON zooms x-axis only
%       ZOOM YON zooms y-axis only

if nargin==1, % Catch off first
  if isstr(m),
    if strcmp(m,'off'),
      set(gcf,'windowbuttondownfcn','','windowbuttonupfcn','',...
            'windowbuttonmotionfcn','','buttondownfcn','');
      return
    end
  end
end

if any(get(gca,'view')~=[0 90]), error('Only works for 2-D plots'); end

rbbox_mode = 0;
zoomx = 1; zoomy = 1; % Assume no constraints

if nargin==0, % Toggle buttondown function
   if strcmp(get(gcf,'windowbuttondownfcn'),'zoom(''down'')'),
      set(gcf,'windowbuttondownfcn','','windowbuttonupfcn','', ...
            'windowbuttonmotionfcn','','buttondownfcn','');
   else
```

```
    set(gcf,'windowbuttondownfcn','zoom(''down'')', ...
         'windowbuttonupfcn','1;', ...
         'windowbuttonmotionfcn','','buttondownfcn','', ...
         'interruptible','yes');
    set(gca,'interruptible','yes')
    figure(gcf)
  end
  return

elseif nargin==1, % Process call backs
 if isstr(m),
  m = lower(m);

  % Catch constrained zoom
  if strcmp(m,'xdown'),
   zoomy = 0; m = 'down'; % Constrain y
  elseif strcmp(m,'ydown')
   zoomx = 0; m = 'down'; % Constrain x
  end

  if strcmp(m,'down'),
   % Activate axis that is clicked in
   ax = get(gcf,'Children');
   ZOOM_found = 0;
   for i=1:length(ax),
    if strcmp(get(ax(i),'Type'),'axes'),
     ZOOM_Pt1 = get(ax(i),'CurrentPoint');
     xlim = get(ax(i),'XLim');
     ylim = get(ax(i),'YLim');
     if (xlim(1) <= ZOOM_Pt1(1,1) & ZOOM_Pt1(1,1) <= xlim(2) & ...
       ylim(1) <= ZOOM_Pt1(1,2) & ZOOM_Pt1(1,2) <= ylim(2))
      ZOOM_found = 1;
      axes(ax(i))
      break
     end
    end
   end
   if ZOOM_found==0, return, end

   % Check for selection type
   selection_type = get(gcf,'SelectionType');
   if (strcmp(selection_type, 'normal'))
    % Zoom in
    m = 1;
   elseif (strcmp(selection_type, 'open'))
    % Zoom all the way out
    zoom('out');
    return;
   else
    % Zoom partially out
```

```
    m = -1;
  end

  ZOOM_Pt1 = get(gca,'currentpoint');
  ZOOM_Pt2 = ZOOM_Pt1;
  center = ZOOM_Pt1(1,1:2);

  if (m == 1)
    % Zoom in
    rbbox([get(gcf,'currentpoint') 0 0],get(gcf,'currentpoint'))
    ZOOM_Pt2 = get(gca,'currentpoint');

    % Note the currentpoint is set by having a non-trivial up function.
    if min(abs(ZOOM_Pt1(1,1:2)-ZOOM_Pt2(1,1:2))) >= ...
          min(.01*[diff(get(gca,'xlim')) diff(get(gca,'ylim'))]),
      % determine axis from rbbox
      a = [ZOOM_Pt1(1,1:2);ZOOM_Pt2(1,1:2)]; a = [min(a);max(a)];
      rbbox_mode = 1;
    end
  end
  limits = zoom('getlimits');

elseif strcmp(m,'on'),
  set(gcf,'windowbuttondownfcn','zoom(''down'')', ...
        'windowbuttonupfcn','1;', ...
        'windowbuttonmotionfcn','','buttondownfcn','', ...
        'interruptible','yes');
  set(gca,'interruptible','yes')
  figure(gcf)
  return

elseif strcmp(m,'xon'),
  set(gcf,'windowbuttondownfcn','zoom(''xdown'')', ...
        'windowbuttonupfcn','1;', ...
        'windowbuttonmotionfcn','','buttondownfcn','',...
        'interruptible','yes');
  set(gca,'interruptible','yes')
  figure(gcf)
  return

elseif strcmp(m,'yon'),
  set(gcf,'windowbuttondownfcn','zoom(''ydown'')', ...
        'windowbuttonupfcn','1;', ...
        'windowbuttonmotionfcn','','buttondownfcn','',...
        'interruptible','yes');
  set(gca,'interruptible','yes')
  figure(gcf)
  return

elseif strcmp(m,'out'),
```

```
  limits = zoom('getlimits');
  center = [sum(get(gca,'Xlim'))/2 sum(get(gca,'Ylim'))/2];
  m = -inf; % Zoom totally out

elseif strcmp(m,'getlimits'), % Get axis limits
  limits = get(get(gca,'ZLabel'),'UserData');
  if size(limits,2)==4 & size(limits,1)<=2, % Do simple checking of userdata
    if all(limits(1,[1 3])<limits(1,[2 4])),
      getlimits = 0; out = limits(1,:); return   % Quick return
    else
      getlimits = -1; % Don't munge data
    end
  else
    if isempty(limits), getlimits = 1; else getlimits = -1; end
  end

  % If I've made it to here, we need to compute appropriate axis
        % limits.

  if isempty(get(get(gca,'ZLabel'),'userdata')),
    % Use quick method if possible
    xlim = get(gca,'xlim'); xmin = xlim(1); xmax = xlim(2);
    ylim = get(gca,'ylim'); ymin = ylim(1); ymax = ylim(2);

  elseif strcmp(get(gca,'xLimMode'),'auto') & ...
       strcmp(get(gca,'yLimMode'),'auto'),
    % Use automatic limits if possible
    xlim = get(gca,'xlim'); xmin = xlim(1); xmax = xlim(2);
    ylim = get(gca,'ylim'); ymin = ylim(1); ymax = ylim(2);

  else
    % Determine which IMAGE coordinate system is being used.
    s = [version ' ']; k = find(s<46 & s>58);
    if ~isempty(k), s = s(1:min(k)); end
    [ver,count,msg,next] = sscanf(s,'%f',1);
    if ver > 4.1
      useNew = 1;
    elseif ver < 4.1,
      useNew = 0;
    else
      if s(next)=='a', useNew = 1; else useNew = 0; end
    end

    % Use slow method only if someone else is using the userdata
    h = get(gca,'Children');
    xmin = inf; xmax = -inf; ymin = inf; ymax = -inf;
    for i=1:length(h),
      t = get(h(i),'Type');
      if ~strcmp(t,'text'),
        if strcmp(t,'image') & useNew, % Determine axis limits for image
```

```
        x = get(h(i),'Xdata'); y = get(h(i),'Ydata');
        x = [min(min(x)) max(max(x))];
        y = [min(min(y)) max(max(y))];
        [ma,na] = size(get(h(i),'Cdata'));
        if na>1, dx = diff(x)/(na-1); else dx = 1; end
        if ma>1, dy = diff(y)/(ma-1); else dy = 1; end
        x = x + [-dx dx]/2; y = y + [-dy dy]/2;
      else
        x = get(h(i),'Xdata'); y = get(h(i),'Ydata');
      end
      xmin = min(xmin,min(min(x)));
      xmax = max(xmax,max(max(x)));
      ymin = min(ymin,min(min(y)));
      ymax = max(ymax,max(max(y)));
    end
  end

  % Use automatic limits if in use (override previous calculation)
  if strcmp(get(gca,'xLimMode'),'auto'),
    xlim = get(gca,'xlim'); xmin = xlim(1); xmax = xlim(2);
  end
  if strcmp(get(gca,'yLimMode'),'auto'),
    ylim = get(gca,'ylim'); ymin = ylim(1); ymax = ylim(2);
  end
 end
 limits = [xmin xmax ymin ymax];
 if getlimits~=-1, % Don't munge existing userdata.
   % Store limits in ZLabel userdata
   set(get(gca,'ZLabel'),'UserData',limits);
 end

 out = limits;
 return

elseif strcmp(m,'getconnect'), % Get connected axes
 limits = get(get(gca,'ZLabel'),'UserData');
 if all(size(limits)==[2 4]), % Do simple checking
   out = limits(2,[1 2]);
 else
   out = [gca gca];
 end
 return

else
  error(['Unknown option: ',m,'.']);
 end

else
 error('Only takes the strings "on","off", or "out".')
end
```

```
end

%
% Actual zoom operation
%
if ~rbbox_mode,
  xmin = limits(1); xmax = limits(2); ymin = limits(3); ymax = limits(4);
  if m==(-inf),
    dx = xmax-xmin;
    dy = ymax-ymin;
  else
    dx = diff(get(gca,'Xlim'))*(2.^(-m-1)); dx = min(dx,xmax-xmin);
    dy = diff(get(gca,'Ylim'))*(2.^(-m-1)); dy = min(dy,ymax-ymin);
  end

  % Limit zoom.
  center = max(center,[xmin ymin] + [dx dy]);
  center = min(center,[xmax ymax] - [dx dy]);
  a = [max(xmin,center(1)-dx) min(xmax,center(1)+dx) ...
       max(ymin,center(2)-dy) min(ymax,center(2)+dy)];
end

% Update circular list of connected axes
list = zoom('getconnect'); % Circular list of connected axes.
if zoomx,
  set(gca,'xlim',a(1:2))
  h = list(1);
  while h ~= gca,
    set(h,'xlim',a(1:2))
    % Get next axes in the list
    next = get(get(h,'ZLabel'),'UserData');
    if all(size(next)==[2 4]), h = next(2,1); else h = gca; end
  end
end
if zoomy,
  set(gca,'ylim',a(3:4))
  h = list(2);
  while h ~= gca,
    set(h,'ylim',a(3:4))
    % Get next axes in the list
    next = get(get(h,'ZLabel'),'UserData');
    if all(size(next)==[2 4]), h = next(2,2); else h = gca; end
  end
end
```

Utm.m

Calculates the corrections for the TM and UTM projections. Formulae used here are similar to those used in the spreadsheet, where graphs were drawn.

```
%function [tlessT,scalecorr]=utm(y,delx,dist)
%To calculate the arc to chord and scale corrections given the heights,
%the distance measured and the difference in x (delx)

r=6.356514967394748e+006;
dist=1000;
%scale1=1;
%scale2=0.9996;
load hts1.txt;
h=hts1(:,1);
rho=1/sin(1/3600*pi/180);  %206264.806248;
y=0:10000:240000;

pr=menu('which projection', 'TM', 'UTM');
if pr==1
scale=1;
disp('        On Traverse Mercator Projection');
Lo=menu('Choose Lo','27','28','29');

elseif pr==2
scale=0.9996
disp('        On Universal Traverse Mercator');
end

disp('  y    t-T    Sf.corr   clevel   projcorr');
disp(' (m)  (sec)    (m)       (m)       (m)  ');
disp(' =    ===    ======   ======   ========');
 i=0;

for y=0:10000:240000
i=i+1;

delx=1000;
clevelcorr=-dist*(h(i)/r);

if pr==1
scale=1;
if Lo==1
tlessT=206265*delx*y/(2*r^2);
scalecorr=+dist*(scale+scale*y.^2./(2*r.^2))-dist;
projcorr=scalecorr+clevelcorr;
elseif Lo==2
tlessT=206265*delx*(y-97800)/(2*r^2);
scalecorr=+dist*(scale+scale*(y-97800).^2./(2*r.^2))-dist;
projcorr=scalecorr+clevelcorr;
elseif Lo==3
```

```
tlessT=206265*delx*(y-195600)/(2*r^2);
scalecorr=+dist*(scale+scale*(y-195600).^2./(2*r.^2))-dist;
projcorr=scalecorr+clevelcorr;
end
elseif pr==2
scale=0.9996;
tlessT=206265*delx*y/(2*r^2);
scalecorr=+dist*(scale+scale*y.^2./(2*r.^2))-dist;
projcorr=scalecorr+clevelcorr;
end
table=[y tlessT scalecorr clevelcorr projcorr];

fprintf('%6.0f %6.4f %7.6f %7.6f %7.6f\n',table);
end
```

lesotho.m

This program transforms coordinates from local to WGS84 system on both starting Lo and Lo28. Sample of results printout is shown in appendix 3.2.

```
function lesotho
%test data on Lo29:beacon P039:Dos (Cape datum) values
global a b ee ee2 name Lo1
%******************************************************************
*****%
   Lo1=input('Lo: ')

i=0;  k=0;
inpmode=menu('Choose mode of input','Typein','File');
if inpmode==1
   fid=fopen('coordsfile.txt','at+');
   quest=input('how many points:' );
   quest
   %coordsfile=struct('name','','y',[],'x',[])

   ncoords=quest;
   while k~=ncoords
      i=i+1;
      fprintf(fid,'%s %10.3f 10.3f\n',name,y,x);
  end %for
S=fscanf(fid,'%s %f %f\n\t',[6 inf]);
[S1]=S';
aa=setstr(S1(:,1));bb=setstr(S1(:,2));cc=setstr(S1(:,3));dd=setstr(S1(:,4));
name=[aa bb cc dd];
filen=S1;
y1=filen(:,5);
x1=filen(:,6);

elseif inpmode==2
      [filename,path]=uigetfile('*.txt','Select a file')
      fid=fopen(filename,'rt')
      ncoords=length(filename);
      S=fscanf(fid,'%s %f %f\n\t',[6 inf]);
      [S1]=S';
      a1=setstr(S1(:,1));b1=setstr(S1(:,2));c1=setstr(S1(:,3));d1=setstr(S1(:,4));

name=[a1 b1 c1 d1];
filen=S1;
y1=filen(:,5);
x1=filen(:,6);
  end

   for k=1:ncoords
      i=i+1;
```

```
  end
%name(i,1:4)=char(S1(k,1:4));  %'P039'%input('beacon name in single quotes: ') ;
%y(i,5)=y1;           %-13353.746%input('y_value: ')
%x(i,6)=x1;           %3290903.638%input('x_value: ')
disp('________________________________________________________________');
%Transformation 1 :Cape map to Geographicals
disp('  **************** TRANSFORMATION 1 ******************');
disp('               ----------------              ');
disp('  Rectangular (x,y) to geogrphical (lat,lon) coordinates');
disp('                  on the Cape datum                  ');
disp('             function: rectogg               ');
disp('');
disp('-----------------------------------------------')
disp('')
        Lo=Lo1;
        ellipsee(1);
        [lat,lon]=rectogg(y1,x1,Lo1);
disp('');
disp('________________________________________________________________');
pause

%Transformation 2 :Geographicals to Cartesian
disp('  **************** TRANSFORMATION 2 ******************');
disp('               ----------------              ');
disp('  Geographical (lat,lon) to Cartesian coordinates (x,y,z)');
disp('                  on Cape datum                          ');
disp('             function: ggtocart              ');
disp('');
disp('-----------------------------------------------')
disp('');
        [x,y,z]=ggtocart(lat,lon);

disp('');
disp('________________________________________________________________');
pause

%Transformation 3 : Shift of origion using Merry's shifts parameters
disp('  **************** TRANSFORMATION 3 ******************');
disp('               ----------------              ');
disp('                  Done in 2 above               ');
disp('             function: shifts              ');
disp('');
disp('-----------------------------------------------')
format bank;
wgsx=x;
wgsy=y;
wgsz=z;
disp('  wgsx            wgsy              wgsz');
[wgsx wgsy wgsz]
disp('________________________________________________________________');
```

```
pause
format long
%Transformation 4: Cartesian to Geographicals on WGS84
disp('    ****************** TRANSFORMATION 4 ********************');
disp('                    ----------------                    ');
disp('          Cartesian to Geographicals on WGS84          ');
disp('               function: carttogg                 ');
disp('');
disp('-----------------------------------------------------------')
disp('');
        ellipsec(3);
        [lat,lon,h]=carttogg(wgsx,wgsy,wgsz);
disp('');
disp('_____________________________________________________________');
pause
format bank;
%Transformation 5: Geographical to Map
disp('    ****************** TRANSFORMATION 5 ********************');
disp('                    ----------------                    ');
disp('     Geographicals to Grid (map) coordinates on Lo28    ');
disp('               function: geotoxy                 ');
disp('');
disp('-----------------------------------------------------------')
disp('                  ');
        ellipsec(3)
        [y,x]=geotoxy(lat,lon,28);
disp('');
disp('_____________________________________________________________');
pause

%Transformation 6: Polynomial transformation on map
disp('    ****************** TRANSFORMATION 6 ********************');
disp('                    ----------------                    ');
disp('     Applying 4th Order polynomial transformation      ');
disp('               function: matrans                  ');
disp('');
disp('');
disp('-----------------------------------------------------------')
[y x];
disp('');
        [y,x]=matrans(y,x);
disp('');
[y' x'];
disp('Check your coordinates in file outmatr.txt');
disp('_____________________________________________________________');
pause

%Transformation 7: Map on Lo28 to Geographicals
disp('    ****************** TRANSFORMATION 7 ********************');
disp('                    ----------------                    ');
```

```
disp('     o28 rectangular back to Geographical on WGS84      ');
disp('              function: rectogg                   ');
disp('');
disp('-----------------------------------------------------------');
disp('');
       o2=28;
[y' x'];
       lat.lon]=rectogg(y',x',Lo2);
[lat lon
disp('');
disp('____________________________________________________________');
pause

%Tran  ormation 8: Geographicals to Lo27 or Lo29
disp('   *************** TRANSFORMATION 8 *******************');
disp('                  ---------------           ');
disp('          Geographicals to Lo27 or Lo29              ');
disp('              function: geotoxy                 ');
disp('')
disp('-----------------------------------------------------------');
disp('')
   ellip  ec(1)
       y.x]=geotoxy(lat,lon,Lo1);

disp('').
disp('')
disp('____________________________________________________________');


disp('_      _____END___________END___________END___________');
disp('')
disp('').
disp('')
type h:  natela/outmatr.txt
disp('')
pause(.

disp('')
disp('')
type h:  natela/wgsco.txt

end
```

Functions

The fo owing subfunctions work with lesotho.m program

Rectc jg.m

Rectog .m transforms coordinates from rectangular to geographicals.

```
%to co vert from rectangular to geographical coordinates
functi [lat,lon]=rectogg(Y,X,Lo)

global  b ee ee2 ncoords i k name

[Y X]:

n=(a-b (a+b);
rho=l  n(1/3600*pi/180);  %206264.806248;
Cr=a^ b;
format ong;
kx=a* l-n)*(1-n^2);
AI=(3  *n-27/32*n^3+67/256*n^5);
AII=(  /16*n^2-55/32*n^4);
AIII=(  1 96*n^3-417/125*n^5);
AIV=(  097/512*n^4);
AV=(  11/2561*n^5);

BI=kx  1+9/4*n^2+225/64*n^4);
BII=k  3/2*(n+15/8*n^3+175/64*n^5);
BIII=k  *15/16*(n^2+7/4*n^4);
BIIV=  *35/48*(n^3+27/16*n^5);
BV=k  315/512*(n^4);
BVI=k *693/1280*(n^5);
fpcoe  1 BI;

Q=X*  coef;
fplat1  Q+AI*sin(2*Q)+AII*sin(4*Q)+AIII*sin(6*Q)+AIV*sin(8*Q)+AV*sin(10*Q)
;

tau1=  n(fplat1);
eta=sq  (ee*(cos(fplat1)).^2);
V=sqr  1+ee2*(cos(fplat1)).^2);
M=Cr  V.^3;
N=Cr  V;
MN=M  *N;
CC=t  1.*(N./M);
signy  ign(Y);
CV=a  (Y./N);
R=sqr  MN);
%PHI  fplat1+CV^2*CC/2;
```

```
%tail=  3*tau1^2+eta^2-9*eta^2*tau1^2+9*eta^2*tau1;
%PHI2  PHI1+CV^4*CC*tail/24;
%tailat  Y^6*tau1/(720*M*N^5)*(61+46*eta^2+90*tau1^2+45*tau1^4-
252*et  2*tau1^2-3*eta^4);
%PHI=  HI2-CV^6*CC*tailat/720

lat=fpl  -(((tau1).*(Y.^2))./(2*MN)...
      -  Y.^4.*tau1)./(24*M.*N.^3)).*(5+3*tau1.^2+eta.^2-9*eta.^2.*tau1.^2-
4*eta.  )...
      -  Y.^6.*tau1)./(720*M.*N.^5)).*(61+46*eta.^2+90*tau1.^2+45*tau1.^4-
252*et  2.*tau1.^2-3*eta.^4));
lat=-lat
%Lo=i  ut('enter the central meridian Lo:' )

%dlon  CV/cos(PHI)+CV^3/(6*cos(PHI))*(1+2*tau1^2+eta^2)
%tailo  5+6*eta^2+28*tau1^2+8*eta^2*tau1^2-24*tau1^4-3*eta^4)
%dlon  dlon1-CV^5*tailon/(120*cos(PHI))

%lamb  Lo/(rho/3600)+signy*dlon2

lon=Lo  ho/3600)-(Y./(N.*cos(fplat1))...
    +Y  /(6*N.^3.*cos(fplat1)).*(1+2*tau1.^2+eta.^2)...
    -Y.  (120*N.^5.*cos(fplat1)).*(5+6*eta.^2+28*tau1.^2+8*eta.^2.*tau1.^2-
24*tau  -3*eta.^4));
disp('  at                 lon');
[lat lon
```

ggtocart.m

```
ggtoca  ansforms geographical to cartesian
functio  x,y,z]=ggtocart(phi,lambda)

global  h ee ee2
h=0;
phi;
W=sqr  -ee*(sin(phi)).^2);
N=a./W
M=a*(  ee)./W.^3;
%R=b  2            %radius of curvature at a pt with lat phi
%Rphi  cos(alfa))^2/M+(sin(alfa))^2/N %radius of curvature at az. alfa

p=(N+  .*cos(phi);
x=p.*c  (lambda);
y=p.*s  lambda);
z=(N.*  -ee)+h).*sin(phi);

%to co  rt cartesian xyz from cape to wgs84
disp('  ******** cartesian coordinates on WGS84 *********');
```

```
[x y z]
disp('')
delx=- 5.4; dely=-106.7; delz=-291.7;
wgsx= delx;
wgsy= dely;
wgsz= delz;
x=wgs
y=wgsy
z=wgsz
```

carttogg.m

carttogg.m converts the cartesian coordinates to geographical coordinates:

```
function phi,lambda,h]=carttogg(x,y,z)
global  b ee ee2 Lo1

h=0;
phi=1.c
phi=ph  pi/180;
p=sqrt(  ^2+y.^2);
lambda= atan2(y,x);    %lambda is in radians
%lamb a=radtodms(lambda)  %to convert to dms

for counter=1:20
  counte  ;
  W=sq t(1-ee*(sin(phi)).^2);
  N=a./W;
  %phi  atan(z/(p*((1-ee)*N+h)/(N+h)))
  phi=a n(z./(N.*(1-ee)+h));
  h1=p. os(phi)-N;
  delht= 1-h;
  h=h1;
     if a  (delht)<0.0005
        break
     end
end
[phi lam oda h]
```

geotoxy.m

geotoxy.m transforms the geographicals to map coordinates:

```
%to co  ert geographical to grid coordinates
function y,x]=geotoxy(lat,lon,Lo)

global a b ee ee2 R y x name ncoords i k Lo1
```

```
n=(a b  a+b);
rho= 1/  n(1/3600*pi/180);  %206264.806248;
rhod  r  /3600;
C=a  2
nam

form at  ng;

kx=a*(  n)*(1-n^2);
AI=( 3/  n-27/32*n^3+67/256*n^5);
AII= 2  16*n^2-55/32*n^4);
AIII (  1/96*n^3-417/125*n^5);
AIV (  97/512*n^4);
AV= (8  11/2561*n^5);

BI=kx*  1+9/4*n^2+225/64*n^4);
BII=kx  3/2*(n+15/8*n^3+175/64*n^5);
BIII k  15/16*(n^2+7/4*n^4);
BIV  k  35/48*(n^3+27/16*n^5);
BV= kx  315/512*(n^4);
BVI  k  693/1280*(n^5);
fpco f  (kx*(1+9/4*n^2+225/64*n^4));

fid=fop  n('wgsco.txt','wt');
%disp(  Check the coordinates in file wgsco.txt');
if Lo =  27
fprintf( d,'******  Coordinates on WGS84 Lo27  *******\n\n');
elsei l  1 =29
fprintf( d,'******  Coordinates on WGS84 Lo29  *******\n\n');
end
fprintf( d,'name          y            x \n ');

Q=la*  coef;
fplat  -  +AI*sin(2*Q)+AII*sin(4*Q)+AII*sin(6*Q)+AIV*sin(8*Q)+AV*sin(10*Q);

tau1  t  (lat);
eta= sq  ee*(cos(lat)).^2);
V=sqrt  +ee2*(cos(lat)).^2);
M=(  ^3;
N=C  V
MN= M  *N;
CC= ta  .*N;
CV= (  rhod-lon).*cos(lat);
sign  s  ign(CV);
CV= ab  CV);
R=sqrt  N);
Y=C V  N+CV.^3.*N.*(1-tau1.^2+eta.^2)/6;
taily 5  8*tau1.^2+tau1.^4+14*eta.^2-58*tau1.^2.*eta.^2+13*eta.^4;
form t  ank;
y=sign  .*(Y+CV.^5.*taily.*N/120);
```

```
forn     ng;
BB=     n-BII*sin(2*lat)+BIII*sin(4*lat)-BIV*sin(6*lat)+BV*sin(8*lat);
X=B     V.^2.*CC/2+CV.^4.*CC/24.*(5-tau1.^2+9*eta.^2+4*eta.^4);
tailx     -58*tau1.^2+tau1.^4+270*eta.^2-330*eta.^2.*tau1.^2+445*eta.^4);
x=X     .*tailx.*CV.^6/720;format bank;
x=al
coor     s=[y x]
for i     oords
  fp     fid,'%s %15.3f %15.3f\n',name(i,1:4)',y(i),x(i));
end
fclo     );
```

mat   s.m

mat   n applies the transformation parameters calculated in transf15.m to the newly calc   coordinates:

```
func     yout,xout]=matrans(y,x)
% tc     y 4th order tranformation polynomial parameters to
% p     e a new set of coordinates given old coordinates
glob     me k i ncoords ys xs
pron     enu('which parameters do you want','DOS_SA','GPS_SA');
if pr     =2|promt=='G'|promt=='g'
  X     716e-9;
  Y     969e-12;
  th     957e-8;
  %     degdec(th);
  sf     00000082698;
else     t==1|promt=='D'|promt=='d'
  X     2418e-9;
  Y     0626e-11;
  th     35e-5;
  %     degdec(th);
  sf     999557248407;
end
  cl
nam     me;
  %
  y
  x
nco     length(name);
fid=     'outmatr.txt','wt+');
  f     intf(fid,'*******Coordinates on WGS84 Lo28********\n\n');
  fprintf(fid,'     name        y         x \n ');
for k=1:ncoords
  i=i+1;
  y=yn(k);
  x=xn(k);
```

```
    Tr(1,1)=Yo;
    Tr(2,1)=Xo;

    pp=cos(th); qq=sin(th);
    Rot(1,1)=pp; Rot(1,2)=qq;
    Rot(2,1)=-qq; Rot(2,2)=pp;

    Trfn=[Tr]+sf*[Rot]*[yn(k);xn(k)];
    fprintf(fid,'%s %15.3f %12.3f\n\t',name(k,1:4)',Trfn');
    %fclose(fid)
    Trfn'
    yout(k)=Trfn(1)';
    xout(k)=Trfn(2)';

end
```

Sub-functions

In most of the functions above, some other sub-functions have been included to convert angles between DMS, RADS and Degree decimals. These are:

Radtodms.m

Radtodms.m to convert radian to degrees minutes and seconds:

```
%function converts angle in radians to ddd.mmss
function[degphi,minphi,secphi,dms]=radtodms(phi)
phi=phi*180/pi;        %from rads to decdeg.
degphi=fix(phi);
rem1=phi-degphi;
rem2=rem1*60;
minphi=fix(rem2);
rem3=rem2-minphi;
secphi=rem3*60;
dms=fprintf('%3.0d %2.0d %7.4f\n',[degphi' minphi' secphi']);
```

Dtodms.m

Dtodms to convert from degree decimals to DMS

```
function[geog]=ddotms(decdeg)
%to convert angle in decimal degrees to format ddd.mmss
r1=fix(decdeg);
r2=decdeg-r1;
```

```
r3=r2*60;
r4=fix(r3);
r5=r3-r4;
r6=r5*60;
geog=r1+r4/100+r6/10000;
```

degdec.m

degdec.m converts DMS to D.ddd. degree decimals (and also to radians):

```
%function converts angle in format ddd.mmss into dec. deg. & radians
function [rads]=degdec(dms)
indeg=fix(dms);
remdeg=dms-indeg;
remdeg1=remdeg*100;
inmin=fix(remdeg1);
remin=remdeg1-inmin;
insec=remin*100;
decdeg=((insec/3600+inmin/60)+indeg)
rads=decdeg*pi/180
```

Appendix (4) Program sample results

Sample results of each program

A printout of a program results showing the calculations involved between DOS and SA coordinates. For distances, the units are metres. Theta is in radians, phi in decimal degrees and rotation angle in degrees minutes and seconds.

Transf15.m results
Lesotho.m results
Spreadsheet/Utm.m results

Transf15.m results

```
transf15
% A program to calculate the Helmert Transformation parameters
% between the Lesotho coordinates and the S.A. computed coordinates
% on the WGS84 datum on LO28.
echo off
        Hit any key to continue after pause
        Press any key to continue

filename =

79DOSSA.TXT


path =

H:\MATELA\


fid =

    9

 1 for Helmert transformation
 2 for 2nd order polynomial
 3 for 3rd order polynomial
 4 for 4th order polynomial

what polynomial/order do you want [1,2,3,4]:  4

order =

    4

x =

  -0.00000172243081
   0.00001694879689
   0.99999571941976
   0.00001556919153
  -0.00000830334005
  -0.00000041521902
  -0.00000197878952
   0.00000034678057
   0.00000000000926
  -0.00000000020859

 Enter to continue
```

sigma0 =

0.22817181913177

residuals:

resmat =

0.42237576105981	-0.08352821037261
-0.09557167383900	0.04051636344229
-0.32505018054508	-0.13019880026695
0.36188420833059	-0.23768693933380
0.41306903666555	-0.26758850248734
-0.28311027440941	-0.14768937333429
-0.02552813342299	0.00283625857992
-0.16075153695124	0.03161436099617
-0.27914295949449	-0.11647701989568
-0.34038663559477	-0.14399343183322
-0.24713671662903	-0.20244527520117
-0.01102023801650	-0.09479407556501
-0.17862570278521	-0.15496489508951
-0.16774250481103	0.02310829598719
-0.24937712597603	0.02887799905147
-0.19015799293993	0.17217637414433
-0.06656221303274	0.12873423150813
-0.08762529624801	0.22641716238832
0.14390253189777	0.07743054642924
-0.04770688297140	0.06648656000471
-0.19428819579480	-0.02696565743827
-0.07870784313855	-0.10281334446336
-0.04277522349003	-0.06302511547983
0.30708524130023	-0.18856924506690
0.31840465751156	-0.18850364469836
0.34831238586776	-0.19457093221718
0.31965847615720	-0.31510962858738
0.32458733994281	-0.36410978522326
0.10410967616190	-0.06527918517531
0.29603658620908	-0.44633711869028
-0.00408201398386	-0.08278266094567
0.09332260032534	-0.14948261547033
-0.07711600387120	-0.33660371087899
0.20111399864982	-0.30086946859592
-0.07794208187261	0.09519319342144
0.00547342369100	0.07606138377741
0.30577938770148	-0.19379652017960
-0.35317373952421	-0.18548304233263
-0.10649753509279	0.00118844522513

0.06351418788108	-0.10908186668530
0.34447291994002	0.43448439355052
0.38594274377101	0.09065199116594
0.44300275806745	0.14293837175137
0.47141807273147	0.08253844593855
-0.33267915240003	-0.21329466167572
-0.43328069245064	-0.58460050883150
-0.22263906267472	-0.21553999091338
0.17931582309757	0.06394912660471
0.02057731898094	0.22374287758430
-0.00353374707629	0.21699467923463
-0.01910487451460	0.18753900779848
-0.01603499731573	-0.05261867628724
-0.08025669409108	0.17941232820158
0.02714906768233	0.00944074747895
-0.14154803093697	-0.08546107070106
0.13567519403296	-0.03038924690918
0.12343600409804	0.05286684011662
0.16865863431212	0.02964717756549
0.44008522611693	-0.57794582880888
-0.29847129823611	0.51538315785001
0.11932322269786	-0.24932303125388
0.36707059719629	0.29889286328398
0.36654589323007	0.22087696378003
-0.38965968167759	-0.22895377047962
-0.39213767425474	-0.32855154542722
0.05798649729695	-0.10833488359140
0.03003704210278	-0.12063606081392
0.03594855408301	-0.27785648117776
0.09979358737473	-0.20149545634831
0.33961430563795	-0.10738403214236
0.22736393842933	-0.10447571561963
-0.04757627229264	-0.14029982664943
-0.04719195995131	-0.00451035836886
0.13419793888170	-0.37156799834338
-0.04201820796152	0.00569726144749
-0.14540515535191	0.02400228794431
0.04685427199729	0.10521888243238
-0.02119850960662	0.29905751272599
-0.14975373031848	0.32798115552578
-0.03054895034620	0.14499677062850
-0.02671963581270	0.28886976996000
-0.07211577696216	0.32843021080771
-0.12677170552274	0.22195947837463
-0.10521946367044	0.03932318469015
0.44500582862383	0.06280483768205
-0.41106700206728	0.32403818999592
-0.19050008230988	0.14677537204261
-0.13202148867276	0.16952719770052
-0.11058816545847	0.11936511142994

-0.15906495360105	0.08862328961550
-0.09393628043472	0.14230064707954
-0.01789082292453	0.08688731437724
-0.02837730112151	0.08037751566735
-0.08827706769807	0.05393747698690
-0.06350944418227	0.27763688097184
-0.06245482218947	0.26354466755583
-0.05392901110099	0.09120041677670
-0.10502819514295	0.09785421912966
0.02032292899094	-0.04988464419148
-0.19181433102494	0.07590007549152
-0.11671955396014	0.10475717089139
-0.15028583465028	0.14866221688862
-0.01070401269681	0.04486445616931
-0.07801514755556	0.15773398822057
-0.03277114428056	0.25504064731649
-0.23921119656734	0.03414020177297
-0.03517036890116	0.03891270415625
-0.06632371768501	0.03104837649153
-0.10533578363902	0.08646814298118
-0.26147557902732	0.15932342668384
-0.46022385608012	0.38531041181341
1.06421986726855	0.44408986590861
-0.02400859465706	-0.53078762907899

updated coordinates:

update1=original coords + residuals

update1 =

41095.93	3247524.43
27797.73	3317327.17
39761.40	3349589.79
8954.57	3379950.41
21961.80	3211943.08
41608.81	3356515.15
-10795.64	3332414.87
-21760.20	3332370.42
33544.20	3354032.59
49293.72	3350241.35
51063.02	3329269.60
42566.41	3319055.16
54449.26	3317222.08
54729.95	3307600.87
29652.39	3340616.16
78959.90	3294195.08
70274.40	3286390.00
81226.50	3280372.12

53034.82	3275995.64
31753.58	3290393.88
43380.37	3300718.43
32518.17	3307937.05
20006.15	3347455.36
17474.53	3238027.91
31919.85	3233325.69
5900.87	3221574.34
4984.48	3212233.00
12234.90	3205159.75
10287.19	3351690.00
27635.04	3255943.44
57834.15	3284939.35
43487.67	3285018.01
33129.60	3275349.99
10149.16	3249007.60
65430.71	3276832.71
16890.44	3337573.92
-1095.80	3199474.44
69610.59	3313937.51
8506.07	3326103.41
20255.21	3357614.67
51915.32	3246274.02
49800.55	3247532.32
49146.01	3246384.85
43888.94	3244113.60
-21910.87	3271852.59
-22574.77	3252339.03
-110067.63	3291732.46
-1668.23	3340787.28
-38553.75	3336125.26
-106554.70	3317015.10
-13181.10	3312789.04
-95386.11	3296336.35
-53316.18	3296428.38
-107777.16	3291220.60
-124506.80	3279147.59
-95969.74	3257107.70
-68543.99	3233408.08
-25169.56	3224171.40
-129591.29	3220362.44
-42524.42	3207365.58
-14095.50	3195478.91
-85142.88	3184383.06
-85262.72	3184335.64
-47350.30	3262885.66
-22452.57	3260750.61
-108762.47	3269069.49
-100183.83	3277706.95
-90203.69	3266426.65

-84142.51	3275031.97
-60800.83	3277361.50
-68672.41	3283526.51
-83691.07	3285367.37
-81334.19	3295624.38
-63861.96	3294997.12
-81379.00	3309496.11
-69585.30	3312388.41
-56307.98	3310785.66
-55209.61	3321170.52
-54441.26	3333857.16
-34597.96	3333473.14
-35026.16	3315964.01
-43857.34	3320359.67
-24668.93	3320507.71
-7847.71	3322591.73
-3334.40	3358774.98
-58320.25	3183768.28
-55643.11	3215804.95
-49145.80	3212812.56
-50654.45	3207805.41
-56824.70	3209206.62
-52200.43	3204142.08
-54703.43	3196017.67
-51842.77	3195402.60
-59178.56	3196592.62
-53321.15	3187472.12
-52645.53	3187039.61
-51015.28	3179601.73
-54312.79	3175946.44
-43612.41	3171915.18
-54238.13	3168375.04
-58287.19	3168668.30
-62361.17	3165011.12
-39586.83	3192985.32
-57987.35	3201120.51
-44555.20	3219880.60
-62354.97	3215634.60
-67685.69	3189582.86
-56034.02	3186875.91
-51920.11	3175123.83
-54408.21	3219862.62
-50493.53	3276200.67
-17409.93	3374203.46
-144070.45	3248518.59

Transformation Parameters

sigma0 =

0.2282

Translation in metres

Xo =

-0.0172

Yo =

0.1695

p =

1.0000

q =

1.5569e-005

Q =

1.0e-011 *

Columns 1 through 7

0.5651	0.0066	-0.0012	-0.0034	0.0019	0.0324	0.0150
0.0066	0.5577	-0.0034	-0.0042	-0.0305	0.0044	0.0072
-0.0012	-0.0034	0.0084	0.0002	-0.0003	-0.0004	-0.0002
-0.0034	-0.0042	0.0002	0.0094	0.0012	-0.0008	-0.0049
0.0019	-0.0305	-0.0003	0.0012	0.0146	-0.0004	-0.0020
0.0324	0.0044	-0.0004	-0.0008	-0.0004	0.0146	0.0020
0.0150	0.0072	-0.0002	-0.0049	-0.0020	0.0020	0.0144
-0.0052	0.0058	-0.0028	0.0007	0.0015	-0.0016	-0.0035
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Columns 8 through 10

-0.0052	0.0000	0.0000
0.0058	0.0000	0.0000
-0.0028	0.0000	0.0000
0.0007	0.0000	0.0000
0.0015	0.0000	0.0000

-0.0016	0.0000	0.0000
-0.0035	0.0000	0.0000
0.0130	0.0000	0.0000
0.0000	0.0000	0.0000
0.0000	0.0000	0.0000

var =

1.0e-011 *

0.5651
0.5577
0.0084
0.0094
0.0146
0.0146
0.0144
0.0130
0.0000
0.0000

se =

1.0e-005 *

0.2377
0.2362
0.0289
0.0307
0.0382
0.0382
0.0380
0.0361
0.0000
0.0000

Rotation

theta =

1.5569e-005

phi =

8.9205e-004

Rotation Angle is 0 deg 0 min 3.21139002 sec

SF =

0.99999571954096

For the following diagrams:

****

* The left mouse button for zooming into point under mouse
* pointer or by pressing and dragging into selected area

*

* The right mouse button for zooming out

*

* Doube click on the right mouse-button to reset figure

*

* Zoom factor is 2

*

* type f and enter to veiw magnification factor of residuals
*

****

maximum and minimum residuals:

max_residuals =

1.06421986726855 0.58460050883150

min_residuals =

0.00353374707629 0.00118844522513

maximum and minimum residual vectors:

max_vector =

1.15316075847724

min_vector =

0.02568520894979

maximum and minimum shift vectors:

max_shiftvector =

3.38979972219446

min_shiftvector =

0.34171956699804

name1 =

P035

residual =

1.06421986726855   0.44408986590861

name1 =

''

residual =

[]

....
....

Lesotho.m results

Sample results of the program Lesotho, which performs multiple transformations from local system to WGS84 on Lo28 and back to the starting Lo.

```
» lesotho

Lo: 27

Lo1 =

      27.00


filename =

TESTLO27.TXT


path =

H:\MATELA\


fid =

      3.00

______________________________________________________________________
         ***************** TRANSFORMATION 1 ******************
             ---------------
         Rectangular (x,y) to geogrphical (lat,lon) coordinates
                        on the Cape datum
             function: rectogg
---------------------------------------------------------


fplat1 =

  0.51217326895785
  0.52318264486454
  0.52824596217491
  0.53306903312768
  0.50659576185064
  0.52895407757105
  0.52833534302199
  0.52503103852006
  0.52343446082393
  0.52312953455273
  0.52161428128323
  0.52684703856866
```

0.51947044874072
0.51825341588907
0.51729100812470
0.51664024158543

lat lon

ans =

-0.51215148201239	0.48131216830840
-0.52314902697809	0.48367061634273
-0.52822298913661	0.48148615157041
-0.53301391405921	0.48706869460134
-0.50655675834786	0.48476287187070
-0.52892576041765	0.48261121307798
-0.52831946792070	0.47975688206255
-0.52501632866876	0.47945367201804
-0.52341373435530	0.48099889078402
-0.52311692992847	0.47885145159142
-0.52160183773603	0.47880927884203
-0.52681513669745	0.48332388862091
-0.51946821688739	0.47444903722941
-0.51824846156308	0.47602512033716
-0.51728929110480	0.47405816243036
-0.51662675689244	0.47914263855773

***************** TRANSFORMATION 2 ******************

Geographical (lat,lon) to Cartesian coordinates (x,y,z)
on Cape datum
function: ggtocart

********* cartesian coordinates on WGS84 *********

ans =

1.0e+006 *

4.93223422229546	2.57600629311250	-3.10695898030586
4.89554886719432	2.57155976771044	-3.16765054571350
4.88682218751710	2.55337074958705	-3.19552500418022
4.85888336097745	2.57340428907251	-3.22177008351529
4.93864375605570	2.60108350285797	-3.07594101521004
4.88195251609161	2.55782244577608	-3.19937938551535
4.89095635308760	2.54477374504089	-3.19605423873488
4.90108392339220	2.54815508202773	-3.17791819788566
4.90165709369697	2.55808234784901	-3.16910674441409
4.90797512915353	2.54798446603842	-3.16747395969459
4.91234352819671	2.54998933191395	-3.15913482788130

4.88610708027079	2.56443892339326	-3.18779889145213
4.92941018399139	2.53162494744563	-3.14737916410002
4.92882830005336	2.54115126127724	-3.14065229612954
4.93649953246495	2.53282786125896	-3.13535929884819
4.92540302900769	2.55885322855753	-3.13170157009758

****************** TRANSFORMATION 3 *******************

Done in 2 above

function: shifts

----

wgsx	wgsy	wgsz

ans =

4932098.82	2575899.59	-3107250.68
4895413.47	2571453.07	-3167942.25
4886686.79	2553264.05	-3195816.70
4858747.96	2573297.59	-3222061.78
4938508.36	2600976.80	-3076232.72
4881817.12	2557715.75	-3199671.09
4890820.95	2544667.05	-3196345.94
4900948.52	2548048.38	-3178209.90
4901521.69	2557975.65	-3169398.44
4907839.73	2547877.77	-3167765.66
4912208.13	2549882.63	-3159426.53
4885971.68	2564332.22	-3188090.59
4929274.78	2531518.25	-3147670.86
4928692.90	2541044.56	-3140944.00
4936364.13	2532721.16	-3135651.00
4925267.63	2558746.53	-3131993.27

****************** TRANSFORMATION 4 *******************

Cartesian to Geographicals on WGS84

function: carttogg

----

ans =

-0.51215765919412	0.48130643614365	23.70201601460576
-0.52315459491212	0.48366492062355	24.04713960085064
-0.52822827339926	0.48148037200829	24.28187725227326
-0.53301895290159	0.48706307096090	24.28781029023230
-0.50656326305783	0.48475726226735	23.38152947928756
-0.52893100926403	0.48260546574376	24.27452262118458
-0.52832474253390	0.47975104904566	24.33292554412037
-0.52502178321558	0.47944784084034	24.22880356106907

-0.52341928101916	0.48099311231771	24.12989646755159
-0.52312248753944	0.47884560833692	24.17919615469873
-0.52160747902894	0.47880343936580	24.12649226095528
-0.52682050241634	0.48331817020658	24.18390101473778
-0.51947396556738	0.47444307167023	24.17297149822116
-0.51825428235331	0.47601920700614	24.08309513516724
-0.51729516050470	0.47405219232410	24.10381731018424
-0.51663267636314	0.47913682574404	23.93450531084090

***************** TRANSFORMATION 5 *******************

\----------------

Geographicals to Grid (map) coordinates on Lo28

function: geotoxy

\--

coordswgs =

41096.21	3247524.53
27799.19	3317327.16
39762.50	3349589.94
8957.06	3379950.67
21963.04	3211943.37
33545.53	3354032.73
49294.51	3350241.50
51063.68	3329269.82
42567.08	3319055.27
54449.76	3317222.25
54730.43	3307600.87
29653.90	3340616.15
78960.13	3294194.92
70274.56	3286389.88
81226.62	3280371.91
53035.03	3275995.58

***************** TRANSFORMATION 6 *******************

\---------------

Applying 4th Order polynomial transformation

function: matrans

\--

ans =

41095.90 3247524.56

ans =

```
    27798.87    3317327.19

ans =

    39762.18    3349589.97

ans =

     8956.73    3379950.70

ans =

    21962.73    3211943.40

ans =

    33545.21    3354032.76

ans =

    49294.19    3350241.54

ans =

    51063.36    3329269.85

ans =

    42566.76    3319055.30

ans =

    54449.45    3317222.28

ans =

    54730.12    3307600.90

ans =
```

```
    29653.58    3340616.18


ans =

    78959.82    3294194.96


ans =

    70274.25    3286389.92


ans =

    81226.31    3280371.94


ans =

    53034.71    3275995.61

  Check your coordinates in file outmatr.txt
```

***************** TRANSFORMATION 7 ******************

Lo28 rectangular back to Geographical on WGS84
function: rectogg

```
fplat1 =

  0.512169375079 86
  0.52316009640268
  0.52823965572459
  0.53301954110377
  0.50656656894771
  0.52893912486969
  0.52834223759252
  0.52504041483597
  0.52343218200487
  0.52314357924377
  0.52162871453097
  0.52682681472581
  0.51951794388524
  0.51828902041069
  0.51734146572661
  0.51665238905318
```

```
      lat                 lon

ans =

  -0.51215766422117   0.48130654069254
  -0.52315459977717   0.48366499407112
  -0.52822827854472   0.48148047810278
  -0.53301895747964   0.48706313049344
  -0.50656326766497   0.48475732465140
  -0.52893101429312   0.48260555292676
  -0.52832474786822   0.47975119852909
  -0.52502178855437   0.47944799831188
  -0.52341928617644   0.48099322713504
  -0.52312249292718   0.47884578553290
  -0.52160748440815   0.47880361725423
  -0.52682050735004   0.48331824795484
  -0.51947397140040   0.47444348688531
  -0.51825428800496   0.47601951474402
  -0.51729516636076   0.47405263458126
  -0.51663268166324   0.47913698925567
```

***************** TRANSFORMATION 8 *******************

Geographicals to Lo27 or Lo29
function: geotoxy

```
Lo1 =

   27


coordswgs =

    -56020.87   3247332.95
    -68715.21   3317246.51
    -56469.17   3349402.74
    -87007.68   3380034.57
    -75457.32   3211916.36
    -62647.06   3353899.97
    -46931.66   3349970.39
    -45346.55   3328985.16
    -53932.11   3318845.71
    -42065.85   3316909.22
    -41868.97   3307286.40
    -66656.71   3340518.27
    -17758.18   3293671.94
    -26510.11   3285943.25
    -15611.57   3279831.61
```

-43837.93 3275699.03

___________END_______________END_______________END___________

*******Coordinates on WGS84 Lo28********

name	y	x
BP14	41095.902	3247524.563
BP28	27798.868	3317327.186
BP33	39762.180	3349589.975
BP38	8956.734	3379950.696
BP09	21962.727	3211943.401
BS02	33545.210	3354032.761
BS32	49294.187	3350241.537
BS34	51063.356	3329269.850
BS35	42566.762	3319055.300
BS36	54449.447	3317222.280
BS38	54730.115	3307600.904
BS04	29653.578	3340616.180
BS40	78959.816	3294194.957
BS41	70274.246	3286389.919
BS43	81226.305	3280371.944
BS44	53034.713	3275995.609

****** Coordinates on WGS84 Lo27 *******

name	y	x
BP14	-56020.869	3247332.952
BP28	-68715.207	3317246.506
BP33	-56469.173	3349402.735
BP38	-87007.680	3380034.566
BP09	-75457.316	3211916.357
BS02	-62647.056	3353899.968
BS32	-46931.660	3349970.390
BS34	-45346.545	3328985.156
BS35	-53932.109	3318845.710
BS36	-42065.845	3316909.219
BS38	-41868.970	3307286.404
BS04	-66656.714	3340518.273
BS40	-17758.183	3293671.935
BS41	-26510.110	3285943.252
BS43	-15611.568	3279831.606
BS44	-43837.928	3275699.029

»

Lo29 transformation results
*******Coordinates on WGS84 Lo28********

name	y	x
P039	-110067.704	3291732.723
P038	8948.950	3379950.689
P036	23950.555	3374690.002
P035	-17413.381	3374203.070
P031	-1672.255	3340787.268
P030	-38554.884	3336125.087
P027	-106555.008	3317014.919
P026	-13183.755	3312788.902
P024	-95386.399	3296336.444
P023	-53316.733	3296428.243
P022	7659.287	3290216.944
P021	-107777.496	3291220.626
P016	-95970.188	3257107.770
P013	-144070.330	3248519.149
P012	-68544.504	3233408.078
P011	-25171.463	3224171.424
P010	-129591.902	3220363.060
P009	21954.636	3211943.386
P008	-42525.047	3207365.111
P007	-113758.082	3210020.912
P006	-14098.097	3195479.202
P004	-85143.558	3184382.809
P003	-85263.394	3184335.460
P002	-66426.414	3171556.248
P001	-66728.283	3171423.697
S076	-66776.089	3328491.457
S075	-47350.698	3262885.931
S074	-22454.114	3260750.988
S073	-27303.538	3188646.400
S070	-1099.788	3199474.669
S069	-3390.421	3216492.131
S068	26543.715	3217401.048
S066	10143.857	3249007.947
S062	18717.863	3222404.917
S057	12229.300	3205160.148
S056	4979.785	3212233.365
S054	5896.021	3221574.572
S052	31910.921	3233325.901
S051	17468.095	3238028.142
S031	-108762.833	3269069.642
S030	-100184.163	3277707.113
S029	-90204.040	3266426.973
S028	-84142.922	3275032.206
S026	-60801.662	3277361.654
S025	-68673.033	3283526.654
S024	-83691.342	3285367.553

S023	-81334.467	3295624.416
S022	-63862.539	3294997.774
S021	-81379.280	3309496.140
S020	-69585.541	3312388.434
S019	-56308.606	3310785.597
S018	-55210.190	3321170.261
S017	-54441.724	3333856.871
S016	-34599.217	3333473.047
S015	-35027.392	3315963.770
S014	-43858.175	3320359.385
S013	-24670.637	3320507.532
S012	-21762.077	3332370.436
S011	-7850.778	3322591.736
S010	-10798.527	3332414.912
S009	-3338.536	3358774.965
S008	8501.112	3326103.447
S007	16884.084	3337573.882
S006	10281.705	3351690.113
S005	19999.295	3347455.454
T132	26585.489	3363027.166
T131	20248.187	3357614.810
T130	-66234.207	3331844.538
T129	31657.612	3365578.044
T128	-94219.071	3239277.335
T127	-103494.689	3238449.526
T126	-104088.303	3245413.646
T125	-58320.359	3183768.008
T124	-55643.496	3215804.841
T123	-49146.396	3212812.435
T122	-50655.031	3207805.341
T121	-56825.095	3209206.579
T120	-52200.994	3204141.985
T119	-54703.999	3196017.620
T118	-51843.400	3195402.565
T117	-59178.981	3196592.618
T116	-53321.716	3187471.883
T115	-52646.106	3187039.390
T114	-47225.811	3183777.990
T113	-51015.904	3179601.690
T112	-54313.275	3175946.389
T111	-43613.313	3171915.275
T110	-54238.536	3168375.003
T109	-58287.595	3168668.244
T108	-62361.477	3165011.015
T107	-39587.851	3192985.329
T106	-57987.802	3201120.394
T105	-44556.034	3219880.388
T104	-62355.191	3215634.617
T103	-67686.043	3189582.868
T102	-56034.517	3186875.924

T101	-51920.648	3175123.883
T100	-54408.549	3219862.509
T098	-71635.583	3253858.801
T097	-45848.949	3250041.749
T096	-75301.565	3266857.354
T095	-50493.773	3276200.321
T094	-36816.060	3279107.800
T093	-22576.260	3252339.661
T092	-21912.523	3271852.851
T089	-25517.963	3183652.269
T088	-23703.311	3181104.161
T087	-19369.151	3183184.141
T084	-7879.382	3190924.352
T083	-6995.162	3193899.917
T082	4819.704	3197837.830
T081	6109.748	3202275.925
T080	17805.261	3204003.890
T079	22658.098	3204197.112
T078	23945.752	3206339.873
T077	23383.916	3227138.414
T076	28442.917	3225577.664

****** Coordinates on WGS84 Lo29 *******

name	y	x
P039	-13321.371	3290940.784
P038	104915.254	3380193.391
P036	119965.609	3375065.362
P035	78601.358	3374212.028
P031	94638.387	3340934.942
P030	57794.690	3335949.364
P027	-10029.255	3316247.622
P026	83370.660	3312835.705
P024	1316.188	3295670.680
P023	43379.282	3296127.165
P022	104411.817	3290444.313
P021	-11027.405	3290448.633
P016	1070.663	3256444.909
P013	-46939.210	3247446.569
P012	28694.306	3232983.645
P011	72144.695	3224116.802
P010	-32227.373	3219422.325
P009	119379.502	3212288.311
P008	54933.287	3207164.281
P007	-16311.505	3209217.096
P006	83461.557	3195518.865
P004	12512.259	3183825.661
P003	12392.842	3183777.314
P002	31334.435	3171158.025

P001	31033.705	3171022.965
S076	29641.810	3328069.416
S075	49634.524	3262639.492
S074	74549.727	3260718.476
S073	70312.892	3188575.010
S070	96427.377	3199624.006
S069	93992.542	3216622.467
S068	123923.067	3217785.345
S066	107251.421	3249254.739
S062	116053.487	3222723.182
S057	109710.109	3205422.308
S056	102399.790	3212434.518
S054	103236.916	3221783.875
S052	129155.794	3233757.390
S051	114670.521	3238336.869
S031	-11821.375	3268294.293
S030	-3319.397	3277003.728
S029	6755.575	3265811.791
S028	12741.490	3274467.629
S026	36059.708	3276997.955
S025	28135.881	3283094.225
S024	13103.810	3284805.072
S023	15371.527	3295080.582
S022	32846.640	3294605.451
S021	15206.283	3308949.584
S020	26973.183	3311944.080
S019	40262.871	3310457.025
S018	41270.707	3320850.150
S017	41928.204	3333542.145
S016	61773.608	3333332.146
S015	61498.417	3315820.134
S014	52629.341	3320138.425
S013	71815.808	3320454.027
S012	74620.796	3332342.004
S011	88618.576	3322685.058
S010	85584.674	3332482.486
S009	92813.679	3358908.318
S008	104941.552	3326339.754
S007	113225.225	3337884.234
S006	106497.909	3351943.255
S005	116254.116	3347793.845
T132	122704.112	3363424.848
T131	116413.586	3357956.131
T130	30154.299	3331426.794
T129	127754.536	3366020.777
T128	2973.864	3238633.034
T127	-6292.470	3237726.269
T126	-6945.370	3244683.776
T125	39337.449	3183436.259
T124	41743.829	3215492.178

T123	48265.986	3212555.117
T122	46799.783	3207535.717
T121	40618.177	3208884.619
T120	45284.851	3203859.645
T119	42850.526	3195714.977
T118	45716.172	3195124.093
T117	38370.972	3196252.188
T116	44304.682	3187181.749
T115	44983.897	3186754.980
T114	50431.406	3183539.432
T113	46676.495	3179331.691
T112	43409.921	3175649.102
T111	54143.307	3171707.974
T110	43548.039	3168079.119
T109	39496.761	3168338.466
T108	35453.742	3164647.614
T107	57991.823	3192810.309
T106	39523.877	3200789.501
T105	52796.306	3219661.387
T104	35034.046	3215265.083
T103	29923.670	3189171.669
T102	41597.034	3186563.043
T101	45809.329	3174846.728
T100	42944.293	3219559.895
T098	25428.886	3253405.077
T097	51246.361	3249809.276
T096	21651.824	3266370.226
T095	46376.986	3275925.628
T094	60029.377	3278950.853
T093	74499.704	3252306.330
T092	74995.884	3271824.700
T089	72140.522	3183596.068
T088	73976.656	3181063.278
T087	78293.591	3183179.602
T084	89719.166	3191016.396
T083	90578.408	3193999.438
T082	102361.369	3198037.062
T081	103614.114	3202486.228
T080	115296.654	3204313.109
T079	120148.615	3204547.361
T078	121418.363	3206701.188
T077	120680.033	3227496.716
T076	125753.146	3225978.856

»

Spreadsheet/UTM.m results

The Transverse Mercator and Universal Transverse Mercator results from Excel spreadsheet. The program listed in appendix 2.3 gives similar results.

R	6356000	Lo27 Table		Yconst:	0
Delx=	1000				
CentSc.	1				
Rho	206265				
Dist	1000				
y	Av. ht	t-T	Scalecorr	Clevelcorr	Combined
0	1500	0	0	-0.236	-0.236
10000	1500	0.025529	0.001238	-0.236	-0.23476
20000	1500	0.051057	0.004951	-0.236	-0.23105
30000	1500	0.076586	0.011139	-0.236	-0.22486
40000	1600	0.102115	0.019803	-0.25173	-0.23193
50000	1700	0.127643	0.030942	-0.26746	-0.23652
60000	1700	0.153172	0.044556	-0.26746	-0.22291
70000	1900	0.178701	0.060645	-0.29893	-0.23828
80000	2000	0.204229	0.07921	-0.31466	-0.23545
90000	2000	0.229758	0.100251	-0.31466	-0.21441
100000	2200	0.255286	0.123766	-0.34613	-0.22236
110000	2000	0.280815	0.149757	-0.31466	-0.16491
120000	2500	0.306344	0.178223	-0.39333	-0.21511
130000	2750	0.331872	0.209165	-0.43266	-0.2235
140000	2750	0.357401	0.242582	-0.43266	-0.19008
150000	2250	0.38293	0.278474	-0.354	-0.07552
160000	2000	0.408458	0.316842	-0.31466	0.002178
170000	2000	0.433987	0.357684	-0.31466	0.043021
180000	2500	0.459516	0.401003	-0.39333	0.007674
190000	2750	0.485044	0.446796	-0.43266	0.014134
200000	2900	0.510573	0.495065	-0.45626	0.038803
210000	3000	0.536102	0.545809	-0.47199	0.073814
220000	3400	0.56163	0.599029	-0.53493	0.064101
230000	3300	0.587159	0.654723	-0.51919	0.135529
240000	3400	0.612687	0.712894	-0.53493	0.177966

Table 4.1A... TM Projection corrections on Lo27

r=	6356000	Lo28		yconst:	97800
delx=	1000				
CentSc.	1				
rho	206265				
Dist	1000				
YfromLo27	Av. ht	t-T	Scalecorr	clevelcorr	Combined
0	1500	-0.24967	0.118380436	-0.235997483	-0.117617046
10000	1500	-0.22414	0.095409421	-0.235997483	-0.140588062
20000	1500	-0.19861	0.07491373	-0.235997483	-0.161083753
30000	1500	-0.17308	0.056893364	-0.235997483	-0.179104119
40000	1600	-0.14755	0.041348323	-0.251730648	-0.210382325
50000	1700	-0.12202	0.028278608	-0.267463814	-0.239185206
60000	1700	-0.09649	0.017684217	-0.267463814	-0.249779597
70000	1900	-0.07096	0.009565151	-0.298930145	-0.289364994
80000	2000	-0.04544	0.00392141	-0.31466331	-0.3107419
90000	2000	-0.01991	0.000752994	-0.31466331	-0.313910316
100000	2200	0.00561	5.99029E-05	-0.346129641	-0.346069738
110000	2000	0.03114	0.001842137	-0.31466331	-0.312821173
120000	2500	0.05667	0.006099696	-0.393329138	-0.387229442
130000	2750	0.08220	0.01283258	-0.432662052	-0.419829472
140000	2750	0.10773	0.022040789	-0.432662052	-0.410621263
150000	2250	0.13325	0.033724322	-0.353996224	-0.320271902
160000	2000	0.15878	0.047883181	-0.31466331	-0.266780129
170000	2000	0.18431	0.064517365	-0.31466331	-0.250145945
180000	2500	0.20984	0.083626874	-0.393329138	-0.309702264
190000	2750	0.23537	0.105211708	-0.432662052	-0.327450344
200000	2900	0.26090	0.129271866	-0.4562618	-0.326989934
210000	3000	0.28643	0.15580735	-0.471994965	-0.316187615
220000	3400	0.31196	0.184818159	-0.534927627	-0.350109469
230000	3300	0.33748	0.216304292	-0.519194462	-0.30289017
240000	3400	0.36301	0.250265751	-0.534927627	-0.284661877

Table 4.1B... TM Projection corrections on Lo28

r	6356000	Lo29		yconst:	195600
delx=	1000				
CentSc.	1				
rho	206265				
Dist	1000				
YfromLo27	Av. ht	t-T	Scalecorr	clevelcorr	Combined
0	1500	-0.49934	0.473522	-0.236	0.237524
10000	1500	-0.47381	0.426342	-0.236	0.190345
20000	1500	-0.44828	0.381638	-0.236	0.14564
30000	1500	-0.42275	0.339409	-0.236	0.103411
40000	1600	-0.39723	0.299655	-0.25173	0.047924
50000	1700	-0.3717	0.262377	-0.26746	-0.00509
60000	1700	-0.34617	0.227573	-0.26746	-0.03989
70000	1900	-0.32064	0.195246	-0.29893	-0.10368
80000	2000	-0.29511	0.165393	-0.31466	-0.14927
90000	2000	-0.26958	0.138016	-0.31466	-0.17665
100000	2200	-0.24405	0.113114	-0.34613	-0.23302
110000	2000	-0.21853	0.090688	-0.31466	-0.22398
120000	2500	-0.193	0.070737	-0.39333	-0.32259
130000	2750	-0.16747	0.053261	-0.43266	-0.3794
140000	2750	-0.14194	0.038261	-0.43266	-0.3944
150000	2250	-0.11641	0.025735	-0.354	-0.32826
160000	2000	-0.09088	0.015686	-0.31466	-0.29898
170000	2000	-0.06535	0.008111	-0.31466	-0.30655
180000	2500	-0.03982	0.003012	-0.39333	-0.39032
190000	2750	-0.0143	0.000388	-0.43266	-0.43227
200000	2900	0.011233	0.00024	-0.45626	-0.45602
210000	3000	0.036761	0.002566	-0.47199	-0.46943
220000	3400	0.06229	0.007369	-0.53493	-0.52756
230000	3300	0.087819	0.014646	-0.51919	-0.50455
240000	3400	0.113347	0.024399	-0.53493	-0.51053

Table 4.1C... TM Projection corrections on Lo29

r	6356000	Lo27 Table		Yconst:	0
delx=	1000				
CentSc.	0.9996				
rho	206265				
Dist	1000				
y	Av. ht	t-T	Scalecorr	Clevelcorr	Combined
0	1500	0	-0.4	-0.236	-0.636
10000	1500	0.025529	-0.39876	-0.236	-0.63476
20000	1500	0.051057	-0.39505	-0.236	-0.63105
30000	1500	0.076586	-0.38887	-0.236	-0.62486
40000	1600	0.102115	-0.38021	-0.25173	-0.63194
50000	1700	0.127643	-0.36907	-0.26746	-0.63653
60000	1700	0.153172	-0.35546	-0.26746	-0.62293
70000	1900	0.178701	-0.33938	-0.29893	-0.63831
80000	2000	0.204229	-0.32082	-0.31466	-0.63548
90000	2000	0.229758	-0.29979	-0.31466	-0.61445
100000	2200	0.255286	-0.27628	-0.34613	-0.62241
110000	2000	0.280815	-0.2503	-0.31466	-0.56497
120000	2500	0.306344	-0.22185	-0.39333	-0.61518
130000	2750	0.331872	-0.19092	-0.43266	-0.62358
140000	2750	0.357401	-0.15752	-0.43266	-0.59018
150000	2250	0.38293	-0.12164	-0.354	-0.47563
160000	2000	0.408458	-0.08329	-0.31466	-0.39795
170000	2000	0.433987	-0.04246	-0.31466	-0.35712
180000	2500	0.459516	0.000842	-0.39333	-0.39249
190000	2750	0.485044	0.046617	-0.43266	-0.38604
200000	2900	0.510573	0.094867	-0.45626	-0.36139
210000	3000	0.536102	0.145591	-0.47199	-0.3264
220000	3400	0.56163	0.198789	-0.53493	-0.33614
230000	3300	0.587159	0.254462	-0.51919	-0.26473
240000	3400	0.612687	0.312608	-0.53493	-0.22232

Table 4.1D... UTM Projection corrections on Lo27

Appendix (5) Height tables and calculations

Computations relating to height

The following articles are included in this section:

Table for the calibration points for the improved geoid

Table of Geoid Model 1993 values along 30°S parallel

Jefferys' heights and corrections

LHWP height values

LHWP sample of derivation of heights.

Tables of geoid profile derivations

a= Ellipsoidal height WGS84
b=Geoid separation according to calibrated geoid model
c=Approximate orthometric height
d=Discrepancy
e=Published orthometric height

Various points countrywide

stn	a	b	c	d	e	
DAMA	1980.527	35.221	1945.306	-0.122	1945.184	DAMAscus
BM03	2010.664	35.436	1975.228	0.182	1975.41	MONOntsa
XZ32	1669.16	34.045	1635.115	-0.067	1635.048	IX Z 32
BT82	1670.88	33.737	1637.143	-0.071	1637.193	
G151	1864.609	35.936	1828.673	-0.053	1828.602	MOTEng
AG41	2845.285	36.61	2808.675		2808.622	IX AG 41
BT76	1697.377	33.421	1663.956		1664.944	
FBM4	1583.783	33.049	1550.734	0.282	1552.735	MASERU
BS61	1980.055	34.344	1945.711	0.07	1945.993	
M114	2290.821	37.877	2252.944	0.211	2254.014	IX AM 114
BM64	2201.543	38.07	2163.473	0.035	2163.684	IX AR 64
MAT1	2170.075	37.718	2132.357	-0.136	2132.392	SESHote
M305	1633.144	33.845	1599.299	-0.158	1599.163	DOS 3/5
G451	2252.604	37.409	2215.195	0.436	2215.037	MANTsonya
G461	2254.745	37.919	2216.826	0.19	2217.261	thaba-TSEK
FBMM	1972.079	36.871	1935.208	0.051	1935.398	FBM Mashai
M209	1694.825	34.054	1660.771	0.101	1660.822	DOS 2/9
M121	1645.68	34.061	1611.619	0.077	1611.719	DOS1/21
G512	1681.852	34.08	1647.772	-0.15	1647.849	SILOoe
BM11	1738.904	36.142	1702.762	-0.283	1702.612	Matebeng
G611	1592.565	34.47	1558.095	-0.348	1557.812	MOHAlesho
BM25	2050.695	35.295	2015.4		2015.052	Tsoelike

Traverse along the LHWP tunnel route

stn	a	b	c	d	e
I001	2141.294	37.547	2103.747	0.041	2103.788
VS11	2093.325	37.531	2055.794	0.08	2055.874
VS22	2205.039	37.478	2167.561	0.122	2167.683
P001	2227.075	37.423	2189.652	0.133	2189.785
VS33	2299.714	37.315	2262.399	-0.086	2262.313
H003	1985.548	36.596	1948.952	-0.159	1948.793
KT06	2371.826	36.601	2335.225	-0.204	2335.021
M006	1918.154	36.038	1882.116	-0.017	1882.099
M001	1852.818	35.991	1816.827	-0.015	1816.812
HL01	1814.999	35.89	1779.109	-0.019	1779.09
N002	1840.001	35.618	1804.383	-0.1	1804.283
C004	1725.877	35.35	1690.527	-0.075	1690.452

Calibration points for the improved geoid (Merminod (1993))

Map dist (km)	Distance from lo27 (km)	Distance from Durban (km)	GEOID height estimate. (m)
0	0	379	33.6
0.005	9.3495	369.651	33.8
0.01	18.699	360.301	34
0.015	28.0485	350.952	34.1
0.02	37.398	341.602	34.3
0.025	46.7475	332.253	34.5
0.03	56.097	322.903	35
0.035	65.4465	313.554	35.75
0.04	74.796	304.204	36.1
0.045	84.1455	294.855	36.25
0.05	93.495	285.505	36.3
0.055	102.8445	276.156	36.25
0.06	112.194	266.806	36.1
0.065	121.5435	257.457	36.1
0.07	130.893	248.107	36.1
0.075	140.2425	238.758	36
0.08	149.592	229.408	35.8
0.085	158.9415	220.059	35.4
0.09	168.291	210.709	35.3
0.095	177.6405	201.36	35.2
0.1	186.99	192.01	35
0.105	196.3395	182.661	34.9
0.11	205.689	173.311	34.6
0.115	215.0385	163.962	34.2
0.12	224.388	154.612	33.6
0.125	233.7375	145.263	33

Geoid Model 1993 values along 30°S parallel

LEVELLING DATA & CORRECTED HEIGHTS

BENCH MARK	LATITUDE	LONGITUDE	LEVELLED HEIGHT(m)	GRAVITY (milligals)	DYNAMIC HEIGHT(m)	ORTHOM HEIGHT(m)	NORMAL HEIGHT(m)
			Section 1: Kestell - Monantsa				
IXU1	-28.304167	28.708055	1697.4514	978722.09			
IXU3	-28.311389	28.711667	1670.9158	978726.84	1670.9151	1670.9177	1670.9215
IXU5	-28.327500	28.709167	1688.7449	978724.85	1688.7454	1688.7436	1688.7411
IXU7	-28.344722	28.714722	1711.6316	978720.12	1711.6321	1711.6299	1711.6266
IXU9	-28.358055	28.706944	1720.1165	978721.00	1720.1167	1720.1159	1720.1147
IXU11	-28.373889	28.701944	1680.6623	978730.73	1680.6613	1680.6652	1680.6708
IXU12	-28.386389	28.700555	1665.0259	978734.73	1665.0254	1665.0270	1665.0292
IXU14	-28.404444	28.691944	1683.5974	978733.63	1683.5980	1683.5961	1683.5934
IXU17	-28.413889	28.691389	1715.1205	978727.91	1715.1215	1715.1182	1715.1138
IXU18	-28.423333	28.695833	1676.8572	978735.03	1676.8560	1676.8600	1676.8654
IXU20	-28.439167	28.698056	1680.3948	978734.64	1680.3949	1680.3945	1680.3940
IXU21	-28.447778	28.691944	1686.6622	978734.75	1686.6624	1686.6617	1686.6609
IXU23	-28.462500	28.681667	1697.6161	978734.64	1697.6165	1697.6153	1697.6137
IXU25	-28.478333	28.687778	1817.4953	978710.70	1817.4981	1817.4862	1817.4692
IXU26	-28.490556	28.696389	1848.5498	978703.91	1848.5500	1848.5473	1848.5429
IXU30	-28.500833	28.685833	1734.4246	978727.99	1734.4227	1734.4335	1734.4499
IXU33	-28.496944	28.710000	1715.6083	978728.61	1715.6078	1715.6097	1715.6124
IXU34	-28.490000	28.717222	1733.5836	978723.79	1733.5841	1733.5823	1733.5796
IXU36	-28.500833	28.728889	1776.8697	978714.32	1776.8705	1776.8664	1776.8600
IXU38	-28.512222	28.733889	1871.9786	978695.62	1871.9791	1871.9711	1871.9569
IXU40	-28.523889	28.737500	1892.4076	978691.80	1892.4075	1892.4059	1892.4029
IXU43	-28.531944	28.722778	1896.3496	978693.21	1896.3496	1896.3493	1896.3487
IXU46	-28.544167	28.715555	1871.1448	978699.52	1871.1449	1871.1469	1871.1506
FBM616	-28.560000	28.717500	1959.2555	978682.60	1959.2547	1959.2482	1959.2351
			Section 2: Monantsa - Pelaneng				
BM09	-28.580555	28.699167	2219.6799	978633.74	2219.6688	2219.6563	2219.6176
BM04	-28.591944	28.683055	2039.8412	978670.00	2039.8560	2039.8579	2039.8844
BM01	-28.578333	28.661111	1973.9260	978684.97	1973.9275	1973.9317	1973.9414
BM8	-28.587500	28.620555	1989.2579	978683.88	1989.2577	1989.2563	1989.2544
BM13	-28.617500	28.596111	2017.0546	978684.56	2017.0542	2017.0522	2017.0482
BM14	-28.618333	28.587778	1965.9643	978695.91	1965.9048	1965.9087	1965.9158
BM17	-28.632222	28.575833	1946.3765	978703.02	1946.3765	1946.3782	1946.3808
BM19	-28.647500	28.586944	2036.4129	978685.59	2036.4124	2036.4051	2036.3928
BM23	-28.659167	28.603611	2180.3672	978656.66	2180.3630	2180.3541	2180.3342
BM28	-28.658611	28.613333	2551.0119	978582.66	2550.9815	2550.9739	2550.9222
BM33	-28.670278	28.626111	2771.2856	978538.99	2771.2543	2771.2602	2771.2292
BM37	-28.669444	28.642778	2895.6782	978512.21	2895.6566	2895.6629	2895.6452
BM41	-28.683611	28.638611	2768.7093	978538.60	2768.7320	2768.7249	2768.7431
BM44	-28.695278	28.634166	2728.6140	978547.04	2728.6204	2728.6188	2728.6245
BM49	-28.722778	28.626111	2641.2153	978560.38	2641.2284	2641.2255	2641.2383
FBM2	-28.735278	28.617500	2616.7659	978562.81	2616.7694	2616.7687	2616.7724
BM55	-28.750833	28.619167	2560.8406	978569.01	2560.8485	2560.8471	2560.8558
BM60	-28.771111	28.620555	2485.8304	978577.46	2485.8401	2485.8386	2485.8508
BM62	-28.780833	28.620278	2507.5863	978570.51	2507.5835	2507.5839	2507.5803
BM64	-28.790000	28.607778	2432.9065	978584.18	2432.9159	2432.9145	2432.9273
BM65	-28.799166	28.610555	2420.9748	978584.24	2420.9762	2420.9761	2420.9781
BM68	-28.812778	28.612500	2408.9455	978584.17	2408.9469	2408.9468	2408.9489
BM71	-28.828055	28.617222	2421.5833	978579.72	2421.5818	2421.5820	2421.5797
BM74	-28.843055	28.592500	2332.7454	978598.08	2332.7555	2332.7545	2332.7707
FBM3	-28.859722	28.592500	2313.6325	978601.19	2313.6345	2313.6344	2313.6379
BM85	-28.879444	28.579167	2267.4106	978611.16	2267.4156	2267.4152	2267.4237
BM91	-28.897778	28.568889	2301.7233	978605.58	2301.7201	2301.7199	2301.7136

CORRESPONDING HEIGHT CORRECTIONS

HEIGHT DATUM: IXU1 = 1697.4514 m
GRAVITY CONSTANT: Gamma = 978700.0 mgals

BENCH MARK	DYNAMIC CORRN	ORTHOM CORRN	NORMAL CORRN	HEIGHT DIFF	ACCUMULATED DYNAMIC CORR	ACCUMULATED ORTHOM CORR	ACCUMULATED NORMAL CORR
			Section 1: Kestell - Monontsa				
IXU3	-.000663	.001936	.005767	-26.5356	-.000663	.001936	.005767
IXU5	.000471	-.001297	-.003838	17.8291	-.000193	.000639	.001869
IXU7	.000526	-.001686	-.004961	22.8867	.000333	-.001047	-.003091
IXU9	.000178	-.000631	-.001845	8.4845	.000512	-.001678	-.004936
IXU11	-.001043	.002906	.005565	-39.4540	-.000531	.001228	.003569
IXU12	-.000523	.001133	.003346	-15.6364	-.001054	.002361	.006915
IXU14	.000649	-.001347	-.003964	18.5715	-.000405	.001015	.002951
IXU17	.000991	-.002321	-.006748	31.5232	.000586	-.001306	-.003797
IXU18	-.001230	.002811	.008211	-38.2637	-.000645	.001505	.004414
IXU20	.000126	-.000257	-.000760	3.5376	-.000519	.001248	.003654
IXU21	.000222	-.000457	-.001349	6.2674	-.000297	.000791	.002306
IXU23	.000388	-.000803	-.002352	10.9539	.000091	-.000012	-.000047
IXU25	.002777	-.009127	-.026121	119.8792	.002868	-.009139	-.026167
IXU26	.000232	-.002466	-.006918	31.0545	.003100	-.011605	-.033085
IXU30	-.001860	.008857	.025263	-114.1257	.001240	-.002748	-.007822
IXU33	-.000544	.001466	.004130	-18.8163	.000696	-.001342	-.003693
IXU34	.000481	-.001343	-.003977	17.9757	.001177	-.002685	-.007670
IXU36	.000843	-.003291	-.009691	43.2867	.002020	-.005976	-.017361
IXU38	.000483	-.007517	-.021707	95.1089	.002503	-.013493	-.039068
IXU40	-.000131	-.001666	-.004730	20.4290	.002372	-.015159	-.043797
IXU43	-.000030	-.000323	-.000913	3.9476	.002342	-.015482	-.044710
IXU46	.000094	.002057	.005800	-25.2048	.002435	-.013425	-.038910
FBM016	-.000805	-.007310	-.020412	88.1107	.001630	-.020736	-.059323
			Section 2: Monontsa - Palameng				
BM09	-.011131	-.023573	-.062299	260.4244	-.009506	-.044308	-.121622
BM04	.008771	.016592	.043186	-179.8387	-.000730	-.027716	-.078435
BM01	.001489	.005731	.015374	-65.9152	.000760	-.021985	-.063061
BM8	-.000244	-.001316	-.003532	15.3349	.000516	-.023302	-.066593
BM13	-.000448	-.002412	-.006357	27.7967	.000068	-.025714	-.072950
BM14	.000510	.004413	.011539	-51.1503	.000578	-.021301	-.061411
BM17	.000011	.001655	.004341	-19.5277	.000589	-.019646	-.057070
BM19	-.000524	-.007767	-.020092	90.0368	.000065	-.027413	-.077162
BM23	-.004247	-.013149	-.033030	143.9540	-.004182	-.040561	-.110192
BM28	-.030426	-.037986	-.089668	370.6447	-.034608	-.078547	-.199860
BM3X	-.031324	-.025396	-.056374	220.2731	-.065932	-.103943	-.256233
BM37	-.022166	-.015271	-.032988	124.3921	-.088098	-.119214	-.289221
BM41	.022651	.015580	.033789	-126.9685	-.065447	-.103634	-.255433
BM44	.006439	.004775	.010524	-40.0953	-.059006	-.098859	-.244909
BM49	.013064	.010166	.022978	-87.3987	-.045944	-.088693	-.221931
FBM2	.003458	.002785	.006485	-24.4494	-.042487	-.085908	-.215446
BM56	.007662	.006272	.015001	-55.9253	-.034825	-.079636	-.200446
BM60	.009716	.008200	.020437	-75.0104	-.025109	-.071435	-.180009
BM62	-.002801	-.002353	-.006027	21.7535	-.027910	-.073789	-.186036
BM64	.009359	.007992	.020800	-74.6796	-.018551	-.065796	-.165236
BM65	.001412	.001255	.003329	-11.9317	-.017139	-.064542	-.161907
BM68	.001423	.001259	.003390	-12.0293	-.015716	-.063283	-.158517
BM71	-.001524	-.001322	-.003604	12.6376	-.017240	-.064606	-.162121
BM74	.010085	.009149	.025344	-88.8379	-.007156	-.055457	-.136777
FBM3	.001960	.001924	.005429	-19.1174	-.005196	-.053533	-.131348
BM85	.004431	.004587	.013124	-46.2219	-.000765	-.048946	-.118224
BM91	-.003212	-.003396	-.009749	34.3127	-.003977	-.052342	-.127973
BM95	-.000279	-.000290	-.000829	2.9073	-.004256	-.052632	-.128802

BENCH MARK HEIGHT LIST

THLAKA - OXBOW

B.M.No.	DYNAMIC	ORTHOMETRIC
IXAG 7	1794.907	1794.848
IXAG 8	1795.731	1795.673
IXAG 9	1868.896	1868.836
IXAG10	1924.800	1924.738
IXAG11	1937.168	1937.105
IXAG12	1932.184	1932.120
IXAG13	1951.642	1951.578
IXAG14	1922.816	1922.750
IXAG15	1884.382	1884.315
IXAG16	1845.216	1845.148
IXAG17	1843.247	1843.179
IXAG18	1843.936	1843.868
IXAG19	1914.863	1914.794
IXAG20	1924.270	1924.200
IXAG21	1909.704	1909.633
IXAG22	1904.757	1904.685
IXAG23	1920.282	1920.208
IXAG24	1924.520	1924.446
IXAG25	1948.862	1948.787
IXAG26	1994.322	1994.246
IXAG27	2046.264	2046.187
IXAG28	2082.142	2082.064
IXAG29	2085.387	2085.309
IXAG30	2086.240	2086.162
IXAG31	2109.007	2108.928
IXAG32	2132.831	2132.751
IXAG33	2197.370	2197.288
IXAG34	2286.308	2286.226
IXAG35	2362.913	2362.830
IXAG36	2414.911	2414.827
IXAG37	2498.458	2498.373
IXAG38	2572.774	2572.688
IXAG39	2640.508	2640.420
IXAG40	2708.631	2708.543
IXAG41	2808.711	2808.622
IXAG42	2809.124	2809.035
IXAG43	2808.227	2808.138
IXAG44	2802.629	2802.539
IXAG45	2775.854	2775.763
IXAG46	2715.068	2714.976
IXAG47	2712.400	2712.308
IXAG48	2639.650	2639.556
BM 52	2616.634	2616.538

PRECISE LEVELLING REDUCTIONS

LEVELLERS: J. H. STEYN & P. M. FRANCK.

LOCALITY: LESOTHO

INSTRUMENT: ZEISS NI 1. ; JENA 007

ROUTE NO: IX AG

THLAKA – OXBOW.

STAVES: SET 1: 162967 & 162968 ; SET 2: 162963 & 162964

CALIBRATION: 0,999984 1,000006

SUB SECTIONS B.M. NUMBER	Length Km	DIFFERENCES OF LEVEL + UPHILL & – DOWNHILL: FORWARD Staff Units (m)	FORWARD S.U.	BACKWARD Staff Units (m)	BACKWARD S.U.	Diff Staff Units (m)	MEAN Staff Units	MEAN m	PROVISIONAL HEIGHTS B.M. NUMBER	PROVISIONAL HEIGHTS m	Ortho Corrn m	FINAL HEIGHTS m above M.S.L.	STAVES, BOOK & PG No. FORWARD	STAVES, BOOK & PG No. BACKWARD
									IXAG7	1794,9066	-0,0585	1794,8481		
IX AG7 - IXAG5	1,345	+51,1697	38	-51,1718	38	-0,0021		+ 0,0008 +51,1699	IXAG5	1846,0773	-0,0600	1846,0173	Set 1 2729/6	Set 1 2729/2
IX AG5 - IXAG9	0,840	+22,8185	24	-22,8190	24	-0,0005		+0,0008 +22,8184	IXAG9	1868,8965	-0,0610	1868,8355	Set 1 2729/11	Set 1 2728/42
IX AG9 - IXAG10	0,870	+55,9010	26	-55,9033	26	-0,0023		+0,0008 +55,9026	IXAG10	1924,7999	-0,0620	1924,7379	Set 2 2730/1	Set 2 2730/5
IX AG10 - IXAG11	0,815	+12,3663	18	-12,3690	18	-0,0027		+0,0008 +12,3677	IXAG11	1937,1684	-0,0630	1937,1054	Set 2 2730/5	Set 2 2730/6
IX AG11 - IXAG12	0,845	-4,9856	20	+4,9839	20	-0,0017		+0,0008 -4,9848	IXAG12	1932,1844	-0,0640	1932,1204	Set 2 2730/7	Set 2 2730/9
IXAG12 - IXAG13	0,870	+19,4558	24	-19,4584	24	-0,0026		+0,0008 +19,4573	IXAG13	1951,6425	-0,0650	1951,5775	Set 2 2730/13	Set 2 2730/11
IXAG13 - IXAG14	0,765	-28,8263	26	+28,8291	26	+0,0028		+0,0008 -28,8272	IXAG14	1922,8161	-0,0660	1922,7501	Set 1 2729/13	Set 1 2729/15
IXAG14 - IXAG15	0,910	-38,4365	34	+38,4353	34	-0,0012		+0,0008 -38,4353	IXAG15	1884,3816	-0,0670	1884,3146	Set 1 2729/20	Set 1 2729/17
IXAG15 - IXAX17	1,035	-41,1373	34	+41,1322	34	-0,0051		+0,0008 -41,1351	IXAG17	1843,2473	-0,0680	1843,1793	Set 2 2730/15	Set 2 2730/18.
	8,295	+48,3256	244	-48,3410	244	-0,0154		+48,3335						