AN APPLIED TRANCENDENTAL LOGARITHMIC COST FUNCTION: ECONOMIES OF SCALE AND ELASTICITIES OF SUBSTITUTION IN SELECTED SOUTH AFRICAN MANUFACTURING SECTORS (1972-1990)

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ABSTRACT

Moll (1991) has criticised the proposal that demand restructuring should act as the impetus for economic growth in a post-apartheid South Africa on the grounds of, a lack of empirical support. The demand restructuring thesis is premised on two empirically testable assertions: firstly that realisable economies of scale are greater in labour-intensive wage goods sectors than in luxury goods and secondly that in manufacturing as a whole labour can easily substitute for capital. While a number of studies employing either the Cobb-Douglas (Cobb & Douglas, 1948) or Constant Elasticity of Substitution (CES) (Arrow, Chenery, Minhas & Solow, 1961) functions have attempted to quantify these features of technology, their conclusions are potentially invalid.

Both functions impose the maintained hypotheses of homotheticity, homogeneity and seperability *a priori.* As primary hypothesis tests regarding the magnitude of parameters depend on the validity of both the hypothesis being tested and the underlying maintained hypotheses, the plausibility of maintained hypotheses is an important consideration when choosing a functional form for econometric analysis. Homotheticity and homogeneity constrain the theoretical determinants of economies of scale and seperability. The theoretical determinants of substitution thus limit the contexts in which functions which embody these hypotheses are likely to be appropriate.

The mathematical concept of duality has permitted the development of flexible, general functions, such as the Transcendental Logarithmic Cost Function (Christensen, Jorgensen and Lau, 1971, 1973), which rather than imposing, permits the testing of the most commonly imposed maintained hypotheses. By applying this function to three sub-sectors of South African manufacturing both the validity of the commonly imposed maintained hypotheses and the empirical premises of the demand restructuring position are assessed in this dissertation. This application indicates that not only are the hypotheses of homotheticity, homogeneity and seperability invalid but that the inappropriate imposition of homotheticity, homogeneity and seperability invalid but that the inappropriate imposition of homotheticity biases estimates of scale downwards. Evidence also emerges to challenge Moll's (1991) assertions regarding the empirical validity of demand restructuring.

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CHAPTER 1

INTRODUCTION

The first broad economic policy proposal advanced by the African National Congress (ANC) after its unbanning, and implicitly endorsed by the Congress of South African Trade Unions (Cosatu), was `... a programme of Growth through Redistribution in which redistribution acts as a spur to growth.' (McMenamin, 1992, p249 and Cosatu 1992, p14 ). Moll (1991, p314) has identified and criticised two variants of this growth model: the `spare capacity' approach and the demand restructuring approach. Neither approach, Moll (1991, p325) contends, has sufficient empirical support. This dissertation is explicitly concerned with addressing one, and partially another of the three empirical issues Moll (1991, p325) argues need to be addressed in order for a policy of demand restructuring to be successful; namely, the comparative extent of economies of scale in wage (basic) goods industries as opposed to luxury goods industries (Moll, 1991, p323) and the magnitude of elasticities of substitution in basic goods industries (Moll, 1991, p325). The conclusions are not, however, limited to the debate regarding demand restructuring, and could be useful in the wider context of the debates and empirical analyses surrounding industrial restructuring and policy.

The demand restructuring approach to growth through redistribution is premised on the argument that the historically unequal distribution of income in South Africa distorted industrial demand (Moll, 1991, p320). The distribution of income, it is argued, has led to a small high income market for durables and luxuries existing parallel to a limited market for labour intensive basic goods which are subject to high scale economies (Black, 1991, p165). Reducing the inequality in income distribution would, it is argued, stimulate demand in the latter market and consequently stimulate both employment and economic growth.

Moll (1991, p322) has argued that the demand restructuring approach could hold in South Africa if four conditions, three of which are empirically testable, held. Firstly, that the consumption basket of the poor is, indeed, more labour intensive than that of the rich. Secondly, that basic goods industries do experience greater economies of scale than do luxury goods industries. Thirdly, that manufacturing as a whole can easily substitute between capital and labour, so that the costs of shifting to a new more labour intensive productive structure are low. The fourth condition is that a suitable method of redistribution, which ensures that demand changes in the desired manner, exists. This paper is explicitly concerned with the second issue and partly the third (the focus here is limited to selected manufacturing sub-sectors, rather than manufacturing as a whole).

Empirical estimation of the characteristics of the technology underlying either the whole, or parts, of the South African manufacturing industry is not new. With two exceptions, all published work has involved estimation of either the Cobb-Douglas (Cobb & Douglas, 1928) or Constant Elasticity of Substitution (CES) (Arrow, Chenery, Minhas, & Solow, 1961) function. The choice of function being determined primarily by the objective of the application.

Both the Cobb-Douglas and CES functions are, however, limited. Both contain restrictive maintained hypotheses which may be undesirable representations of reality. *A priori* they impose constraints of homogeneity, homotheticity and separability on the underlying technology. A manifestation of these hypotheses is that the derived estimates of economies of scale and elasticities of substitution are constrained to being constant, irrespective of the level of output. This dissertation argues that not only are these hypotheses theoretically undesirable but empirically untenable. Moreover, the data suggest that imposing the hypothesis of homotheticty biases estimates of the magnitude of economies scale downwards.

Constrained Cobb-Douglas functions, where economies of scale are held constant, have been fitted by Browne (1943) and Enke (1962). Unconstrained applications appear in van der Dussen (1970); Spandau (1973); Matsebula (1979) and Standish and Galloway (1991). Notwithstanding the restrictions contained in the maintained hypotheses (in particular the assumptions of homogeneity and homotheticity in output) of the Cobb-Douglas form, applications of the unconstrained form which estimate economies of scale for the disaggregated components of South African manufacturing (van der Dussen (1970); Standish and Galloway (1991)) are perhaps useful for answering the first question to be addressed in this paper. Van der Dussen , however, concluded that the Cobb-Douglas form was not suitable for the manufacturing sectors with which he was concerned (Cluver, 1981, p61). A similar conclusion emerges from this study.

Cobb-Douglas estimates do not, however, contribute to an answer to the second question. The Cobb-Douglas form limits the elasticity of substitution between inputs to unity. This constraint on the elasticity of substitution between inputs was a motivating factor for the development of the CES form which permits an arbitrary constant elasticity of substitution between two inputs. In the South African context the CES has been applied by van der Dussen (1970) (Cluver, 1981, p62).

The application of duality theory to economic problems has provided an alternative approach to that afforded by production functions. Duality theory allows one to obtain estimates of the features of technology by empirically estimating a cost function which is dual to the production function. One such cost function is Christensen, Jorgensen and Lau's (1971, 1973) *Transcendental Logarithmic Cost Function* (Translog) consisting of a cost function and derived factor cost share equations. The Translog system constitutes a general, flexible functional form which enables the testing of the validity of the maintained hypotheses imposed by the conventional forms. Further, the derived estimates of economies of scale and elasticities of substitution vary with the level of output. The variation of the scale coefficient is particularly valuable as it provides an indication of the most efficient output level.

The Translog cost system has been used with South African data on one occasion. Cluver (1981)[1] applied a four input (capital, labour, energy and other inputs) Translog function, to eight sectors of South African manufacturing for the period 1961-1972 (Cluver & Contogiannis, 1984, p18). A Translog production function, which is in the same class of function as the Translog cost system, has been used by Van der Walt and Swanepoel (1987) who applied a two input (capital and labour) Translog function to aggregate manufacturing data for the period 1946-1983 (van der Walt & Swanepoel, 1987, p39). Cluver's (1981) application is flawed.

The Translog system, unlike either the Cobb-Douglas or CES functions, is not globally well-behaved. As a result, applications require testing for local `good behavior'. Badly behaved results suggest that for the data employed, the assumptions used to derive the dual Translog cost function are violated. Badly behaved results therefore imply that the assumption of the dual relationship is unwarranted. Cluver's (1981) results suggest a possible violation of the requirement of well-behaved results, although he fails to

[1]Cluver's (1981) results have subsequently been presented in Cluver and Contogiannis (1984).

comment on this. Furthermore the precise econometric technique employed[2] is not presented, and the method which appears to have been adopted is not appropriate.

This dissertation presents an application of the Translog system to three Standard Industrial Classification (SIC) South African manufacturing industries at the 2, 3 and 4 digit levels for the period 1972-1990. Two of the industries (Electrical Appliances and Household Goods (ISIC 3833) and Furniture (ISIC 3320)) are deemed to be wage goods industries and the third (Motor Vehicles Parts and Accesssories (ISIC (3840)) a luxury industry which is used to draw comparisons with the basic/wage industries.

The second chapter of this dissertation explains why the choice of a flexible non-homothetic form, which does not impose separability *a priori*, is justified. The modeling of two features of technology, namely scale and substitution effects, and the impact and implications of different maintained hypotheses on the modeling of these effects is addressed.

Chapter three describes the Translog model indicating the different constraints which can be imposed on the model, and the constraints which are required for successful estimation. In addition, the question of why the Translog is preferred to other flexible, general forms is addressed.

Chapter four presents a discussion of appropriate econometric techniques the data set which is employed and how different hypotheses will be tested.

Chapter five presents the econometric results while Chapter six is a concluding chapter which presents the implications for future research emerging from the estimates.

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[2]Estimation of the Translog system involves the dropping of one of the share equations. Appropriate econometric techniques are those which are invariant to which equation is dropped. Some techniques generate different results depending on which share equation is dropped.

CHAPTER 2

CHOICE OF AN APPROPRIATE FUNCTIONAL FORM

2.1. INTRODUCTION

The empirical analysis of technology is undertaken for many reasons in different contexts. No single `first-best' functional form exists for all purposes, `... to the contrary, many ... functional forms are well-suited for specific applications but poorly-suited for use as general purpose characterisations of technology' (Fuss, McFadden & Mundlak, 1978, p220). Indeed, the evolution of different functional forms for the analysis of technology has been influenced primarily by differing objectives of production studies. Fuss et al (1978, pp220-221) identify five main objectives of empirical analyses of technology:

- Distribution (i.e. the share of different factors of production in income) which provided the motivation for the development of possibly the most ubiquitous function of all - the Cobb-Douglas function (Douglas, 1948)

- scale (i.e. the existence of constant, increasing or decreasing returns)

- the degree of substitutability between factors of production[1]

- separability (i.e. whether or not the production process can be decomposed [i.e. separated] into additive components)

- technical change where the thrust of analysis tends to focus on three broad areas: whether technical change is embodied or disembodied[2]; whether technical change is factor, scale, or substitution augmenting[3]; or whether it is endogenous[4].

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[1]Arrow, Chenery, Minhas and Solow's (1961) development of the CES function, for example, was motivated by a desire to overcome the *a priori* restriction on elasticity of substitution imposed by the Cobb-Douglas form (Jorgensen, 1986, p1843).

[2]Embodied technical change is technical change which is embodied in a factor of production (usually capital but possibly other factors, most likely skilled labour). Disembodied technical change occurs when innovations require no specific capital (or presumably any other factor of production).

Apart from these five main thrusts of production studies, a number of auxiliary topics have also been the focus of econometric analyses: technological flexibility (i.e. the robustness of technology to adjust to changing environments); efficiency (i.e. operation on or within the technological boundary); and homotheticity (i.e. where factor shares are unchanged with changes in scale) (Fuss et al, 1978, p222).

While the objective of any empirical analysis will undoubtedly be a central concern in the choice of functional form, a number of other considerations ought to inform the practitioner's choice (Chambers, 1988, p159). In particular, cognisance ought to be given to the fact that the use of a specific functional form in econometric analyses (irrespective of the context) requires the acceptance of a number of '... maintained hypotheses which are not themselves tested as part of the analysis, but are assumed true' (Fuss et al, 1978, p222). Maintained hypotheses could possibly be classified according to the degree to which they may be regarded as universal truths. In production analyses, the most fundamental maintained hypotheses are the basic axioms of the nature of the technology which are widely held to be universal truths; namely that the production possibilities set is non-empty and closed (Fuss et al, 1978, p226 and Nadiri, 1982, p432). At the next level are both technological and behavioral hypotheses [such as monotonicity (Nadiri, 1982, p422) and convexity (Fuss et al, 1978, p222 & p226), or that behaviour is cost minimising] which while not accepted as universally true are regarded as plausible for the problem at hand. A third level of hypotheses are those which are made to facilitate the analysis (such as a stochastic structure of independent normal errors) and are deemed to be harmless approximations to reality. The final level hypothesis, which is the most restrictive, is the assumption that a specific parametric functional form is valid. Such hypotheses are made for convenience and are justified due to a perceived absence of negative consequences rather than the plausibility of the assumption.

The constraint of an implausible maintained hypothesis concerning the validity of a specific functional form, is manifested in the testing of specific primary hypotheses regarding the magnitude of estimated parameters and the overall fit of a model. The outcome of the testing of a primary hypothesis will depend on both the validity of the hypothesis in question and/or the validity of the underlying maintained hypotheses. 'This suggests a general principle that *one should not attempt to test a hypothesis in*

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[3]Factor augmenting technical change improves the effective quality of inputs; scale augmenting change expands the scale level where decreasing returns set in, while substitution augmenting change improves the substitutability of inputs.
[4]Technical change is endogenous if it occurs by learning-by-doing, for example.

the presence of maintained hypotheses that have less commonly accepted validity' [emphasis in the original] (Fuss et al, 1978, p223). Chambers (1988, p159) offers the following example: should one wish to test the hypothesis that a certain elasticity of substitution were 2, an obvious choice of model would be the CES form. A rejection of the null hypothesis that the substitution elasticity under consideration were 2 would not imply that that elasticity is never 2 because by employing the CES the econometrician has *a priori* restricted him/herself to considering only constant elasticities of substitution. A direct implication of the above principle is that (at least) for tests of the fundamental hypotheses of production theory in general flexible functional forms, embodying few maintained hypotheses, ought to be used[5].

Clearly a central, and perhaps primary, concern in the choice among different functional forms which are able to model the economic effect of interest ought to be the restrictiveness of maintained hypotheses. However, where a number of alternative forms are suited to the objective of the analysis at hand, and are compatible with the same distinct set of maintained hypotheses, additional criteria ought to be used to choose between competing forms. Fuss et al (1978, p224 & p225), suggest, inter alia, parsimony in parameters to avoid multicollinearity problems and to preserve degrees of freedom, ease of interpretation of the economic effects of interest, computational ease, and interpolative robustness (i.e. within the range of the observed data the chosen form should be well-behaved, displaying consistency with maintained hypotheses, such as convexity or concavity). Nevertheless, irrespective of the criteria adopted to choose a functional form, cognisance ought to be given to the impact that imposed maintained hypotheses have on the interpretation of estimates of the features of technology and the testing of statistical hypotheses.

By imposing specific restrictions upon the different economic effects (such as scale and substitution) modeled by production functions, different functional forms are obtained (Nadiri, 1982, p439). The maintained hypotheses implicit in any particular functional form will obviously be manifest in how these effects are modeled by different functions. An obvious corollary is that the presence and implication of different maintained hypotheses is often best described by their impact on the different effects modeled by production functions. How different effects are modeled by different functional forms, and the relationships between different effects when modeled by

[5]A further important implication, not explored here, is that given the qualitative, non-parametric nature of the fundamental axioms of production theory which make extensive use of implicit rather than explicit functions the more relevant tests will be non-parametric rather than based on parametric (even general) functional forms (Fuss et al, 1978, p223).

different forms, ought to provide important insights into the implicit maintained hypotheses.

Although the presence of homotheticity and separability are empirical questions in there own right, both the Cobb-Douglas and CES functions impose these characteristics as maintained hypotheses *a priori*. The Cobb-Douglas and CES functions are the two forms which have been used to model those features of South African manufacturing technology under investigation in this dissertation scale and substitution effects[6]. This chapter is concerned with assessing the impact of the hypotheses of homotheticity and separability on the modeling of scale and substitution effects respectively, and thus highlighting perceived deficiencies of past attempts to quantify these effects in South African manufacturing. How scale effects are described and presented in neoclassical production theory and the impact of homotheticity[7] on the modeling of scale is the concern of section 2.2. Modeling of substitution and the impact of separability on substitution is discussed in section 2.3.

2.2. ECONOMIES OF SCALE AND HOMOTHETIC FUNCTIONS

The existence of economies of scale[8] refers to all circumstances where the unit cost decreases with increasing output (Zamagni, 1987, p281). At the broadest level two types of economies can be identified according to their influence on the long-run average cost (LRAC) curve of the firm: economies which are internal to the firm and economies which are external to it. While internal economies determine the shape of the LRAC curve, it's position is influenced by external economies - technological and factor price changes exogenous to the firm's behaviour (Koutsoyiannis, 1979, p126). Internal economies are usually classified according to the phenomena which lead to their emergence, and two types are distinguished: *real* and *pecuniary* economies. Pecuniary economies which emerge as a result of a firm paying lower prices for factor inputs and distribution of output are derived from the degree of monopsony power enjoyed by the firm in labour markets (Koutsoyiannis, 1979, p128) and the firm's contractual strength, due to its size, in other markets (such as capital, materials and

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[6]Meaningful estimates of these two effects required the introduction of technological change into the model. A discussion of technical change in the context of the particular model used in this study is provided in Chapter 3.

[7]Homotheticity is closely related to homogeneity, another common maintained hypothesis. The discussion of homotheticity includes a discussion of homogeneity and the effects of homogeneity on the modeling of scale effects.

[8]While economies of scale refer to all sources of unit cost decreases from producing larger levels of output the term returns to scale refers to cost decreases due only to technical considerations (Koutsoyiannis, 1979, p77). Using this distinction, returns to scale would be associated with real economies as described below.

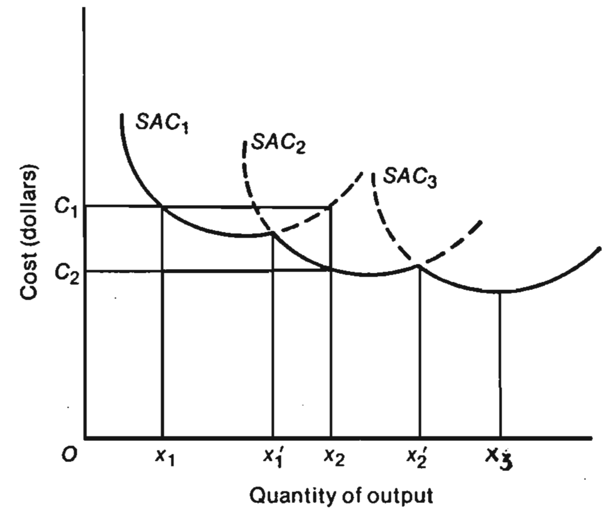
distribution markets) (Koutsoyiannis, 1979, p126 and Zamagani, 1987, p281). Real economies, on the other hand, are those associated with a reduction in the quantity of inputs employed by the firm per unit of output as output levels increase. Koutsoyiannis (1979, pp128-136) identifies four broad groupings of real economies of scale: production (which includes labour, technical and inventory economies); selling or marketing (which is associated with the distribution of the firm's product) managerial, and transport and storage. Internal economies can be analysed using either graphs or mathematical techniques.

2.2.1 ECONOMIES OF SCALE: A GRAPHICAL ANALYSIS

The LRAC curve is the locus of points of the lowest cost of producing different levels of output when all factors of production are variable (Lipsey, 1963, p227). While the long-run is conventionally defined as the period of time which is long enough for all factors to be variable, it is an analytical construct rather than an operating period as such. Indeed, all economic activity takes place in the short-run (Gould & Ferguson, 1980, p179). An entrepreneur can be regarded as operating in the long-run when he/she is about to make an investment and is able to choose among different short-run situations in which he/she will operate in the future. Hence, the long-run should rather be viewed as a planning horizon, encapsulating the fact that economic agents can plan and choose different aspects of future short-runs in which they will operate. This conceptualisation of the long-run is central to the derivation of the LRAC curve from short-run average cost (SAC) curves.

The shape and derivation of the LRAC curve is best described by initially employing the simplifying assumption that at a particular point in time available technology is such that only three methods of production, corresponding to three different plant sizes, are available. The smallest plant operates with costs given by SAC_1 in Figure 2.1, the medium size plant with costs given by SAC_2 and the largest firm with costs given by SAC_3.

Figure 2.1 Short-Run Average Cost Curves for plants of different sizes.



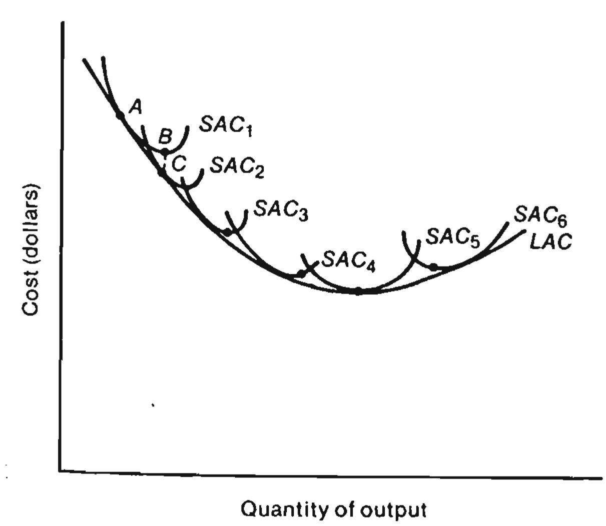
Source: Gould and Ferguson, 1980, p190.

In the long-run the entrepreneur is able to choose between three investment alternatives represented by the three different SAC curves. Choice of one of the three alternatives will be determined by expected demand for output. If, for example, the firm plans (on the basis of expected demand) to produce output of $0X_1$, it will invest in the smallest plant size. If it plans to produce $0X_2$, it will choose the medium plant size, and if it plans to produce $0X_3$, the largest plant size represented by SAC_3 will be chosen. Choice between alternative plant sizes is obviously informed by the lowest unit cost of producing different levels of output. At levels of output such as $0X_1$' and $0X_2$' two different plant sizes have the same average cost for producing a particular level of output and the firm can either continue to produce using the present plant size or it can move to a larger plant size. The decision here would depend on expectations about future demand. If demand were expected to expand in the future, the larger plant would be chosen by the cost minimising producer.

If when plant size was chosen planned output was $0X_1$, the plant represented by SAC_1 would be built. Similarly, if planned output was $0X_2$, the plant associated with SAC_2 would be built. The solid line in Figure 2.1 thus indicates which plant size would be chosen to produce different levels of output, with the choice of plant size being determined by cost minimisation considerations. This line, which is the locus of lowest cost points when all factors, and hence plant size, can be varied, is thus the planning curve of the cost-minimising firm.

Relaxing the assumption of only three available technologies to now assume that an infinite number of technologies and hence plant sizes exist; each suitable for the production of a certain level of output, a continuous smooth LRAC curve emerges (see Figure 2.2). Each point on this curve shows the least cost of producing different levels of output, when all factors are variable. Traditional classical theory of the firm assumes that the LRAC curve is U shaped[9]. This is based on the presumption that returns to scale will initially be increasing, become constant at some level of output, and thereafter decrease. Economies of scale are thus assumed to exist only up to a certain plant size, the *optimum plant size* . Larger plants invoke diseconomies of scale due to managerial inefficiencies.

Figure 2.2 Long -Run Average Cost Curve

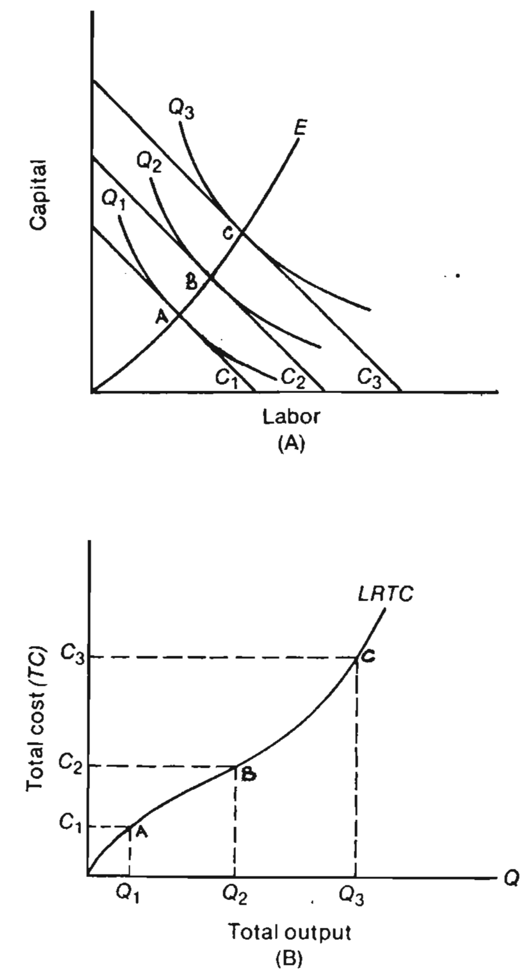


Source: Gould and Ferguson, 1980, p191.

[9]The traditional (classical) assumption of a U shaped LRAC which is based on the assumption of laws of returns to scale has been questioned both on theoretical and empirical grounds (Koutsoyiannis, 1979, p114).

How costs change as output changes, and therefore, whether returns to scale are increasing, constant or decreasing, can be analysed using an alternative construct: the firm's *expansion path* (EP)[10]. This is the locus of tangency points between isocosts and isoquants indicating the `rational'[11] input-combination for producing different levels of output. That long-run total costs are directly related to the expansion path can be seen from Figure 2.3, where panel A presents an EP and panel B a long-run total cost (LRTC) schedule.

Figure 2.3 The Expansion Path and Long-Run Total Costs schedule.



Source: Gould and Ferguson, 1980, p176.

Point A on the expansion path indicates that output level Q_1 can be produced by the firm for a minimum total cost of C_1. Similarly, points B and C indicate that the

[10]The concept of an expansion path, in particular its derivation, is discussed in more detail below.
[11]That is, consistent with the postulate that the entrepreneur is a cost-minimiser.

minimum total costs at which output levels Q_2 and Q_3 can be produced are C_2 and C_3, respectively. Greater levels of output are associated with higher costs. Repeating this procedure for all levels of output would yield a locus of output-cost combinations - the LRTC schedule. Such an exercise has been undertaken to derive the LRTC schedule in panel B, where the points corresponding to the EP of panel A are indicated. The LRTC curve is merely cost-output equivalent of the EP (Gould & Ferguson, 1980, p176). Where the slope of the LRTC curve is increasing, the implication is that expansion of output is becoming increasingly costly, implying that LRACs are increasing and that returns to scale are decreasing. Hence, the change in returns to scale along the expansion path (i.e. output responses to changes in costs) determines the shape of the LRAC curve.

2.2.2 ECONOMIES OF SCALE: A MATHEMATICAL EXPOSITION

Mathematically, two different concepts are used to describe the phenomenon of returns to scale, and derive measures of scale effects from different functions (production and cost) - the elasticity of scale and the elasticity of size (Chambers, 1988, p72 and Hanoch, 1975, p492). The two concepts are often mistakenly used interchangeably in the literature. While the two measures coincide at cost-minimising points (such as A, B, and C in panel A of Figure 2.3), they are different phenomena and unless the production function is homothetic the two measures will differ in response to a change in output. In this regard homothetic functions are unique. The elasticity of size is, however, the more relevant of the two measures for the firm.

2.2.2.1 ELASTICITY OF SCALE

For a regular[12] production function $f(x)$, where x is a vector of factor inputs $(x_1, x_2, \ldots, x_n)$, the elasticity of scale[13], which is the more commonly used definition of returns to scale, is defined as:

$$\varepsilon(x) = \left. \frac{\partial \ln f(kx)}{\partial \ln k} \right|_{k=1} \qquad 2.1$$

where: k is some scalar and kx is a formula for a ray through the origin in the input space. $\varepsilon(x)$ is, thus, an elasticity coefficient measuring the relative increase in output

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[12]A production function is deemed regular if that function is: positive; finite; continuously twice differentiable; strictly monotonic; and strongly quasi-concave.

[13]First identified by Johansen (1913), and known variously as the "elasticity of production"; "passus coefficient" and the "function coefficient".

as all input quantities are increased proportionally, i.e. along a ray from the origin in input space (Chambers, 1988, p72; Hanoch, 1975, p492 and Jehle, 1991, p227). The elasticity of scale delineates three types of returns to scale: decreasing, constant or increasing as $\varepsilon(x)$ is less than, equal to or greater than unity respectively. The delineation of returns to scale is often erroneously applied. The elasticity of scale, because it involves the use of a derivative, is a local measure, not a global one. For production to be characterised by either decreasing, constant or increasing returns to scale over the entire input space, output must always responds to proportional changes in inputs in the same quantitative manner, irrespective of the initial level of output (Jehle, 1991, p227). Most technologies exhibit increasing, constant or decreasing returns to scale over different ranges of output (Jehle, 1991, p227 and Chambers, 1988, p24). Assuming $\varepsilon(x)$ is constant, as in homogenous production functions, is obviously highly restrictive.

Marginal productivity may be measured by a unit-free measure, the elasticity of output, which for a production function $y = f(x)$ is defined as:

$$\varepsilon_i = \frac{\partial f(x)}{\partial x_i} \cdot \frac{x_i}{y} \qquad 2.2$$

The elasticity of output, is thus merely the ratio of the marginal product of the ith input to the average product[14], and provides a measure of the percentage change in output in response to a one percent change in the ith input (Chambers, 1988, p18 and Nadiri, 1982, p439). In other words, the elasticity of output provides a "normalized" measure of the relative importance of a particular input (Griliches & Ringstad, 1971, p6). The elasticity of scale can be shown to be mathematically equivalent to the sum of factor-output elasticities[15]:

[14]The first term in the formula is marginal product while the second is the reciprocal of average product. Multiplying marginal product by the reciprocal of average product is equivalent to dividing marginal product by average product.

[15]Given $\varepsilon(x) = \left.\frac{\partial \ln f(kx)}{\partial \ln k}\right|_{k=1}$ and letting $u = kx$ (implying that u is a vector) and $y = f(u)$

$$\Rightarrow \varepsilon(x) = \frac{\partial \ln f(u)}{\partial \ln k}$$

$$\Rightarrow \varepsilon(x) = \frac{k}{y} \cdot \frac{\partial f(u)}{\partial k} \qquad \text{[Given that } y = f(u)\text{]}$$

$$\Rightarrow \varepsilon(x) = \frac{k}{y} \sum_i \frac{\partial f(u)}{\partial u_i} \cdot \frac{\partial u_i}{\partial k} \qquad \text{[Using the Chain Rule given that } u \text{ is a vector]}$$

$$\Rightarrow \varepsilon(x) = \frac{k}{y} \sum_i f_i \cdot x_i$$

$$\varepsilon(x) = \sum_i \varepsilon_i \qquad 2.3$$

An implication of this relationship is that changes in the relative importance of different inputs impacts on the magnitude of the elasticity of scale.

2.2.2.2 ELASTICITY OF SIZE

The second measure of returns to scale is the elasticity of size, which measures the increase in output relative to costs for variations along the expansion path (i.e. the locus of cost minimising points in input space). Mathematically the elasticity of size is expressed as:

$$\psi(x) = \left.\frac{\partial \ln y}{\partial \ln c}\right|_{\overline{w}} \qquad 2.4$$

The elasticity of scale and the elasticity of size are equal at cost minimising points. Establishing the equality of the two measures at cost minimising points rests on two results, both of which stem from a manipulation of the first order conditions of a constrained cost minimisation problem (Henderson and Quandt, 1980, p77 and Hanoch, 1975, p493).

The Lagrangian for a constrained optimisation problem where the entrepreneur attempts to minimise the costs of producing a given output $y^0 = f(x)$, where x is a vector of factor inputs $(x_1, x_2, \ldots, x_n)$ and the total cost function has the specific form $c = \sum_{i=1}^{n} w_i x_i$, where w_i is the price of factor i, would be:

$$Z = \sum_{i=1}^{n} w_i x_i + \mu[y^0 - f(x)] \qquad 2.5$$

At constant prices the first order conditions for a minimum would be given by the following set of simultaneous equations:

$\Rightarrow \varepsilon(x) = k \sum f_i \frac{x_i}{y}$

$\Rightarrow \varepsilon(x) = k \sum \varepsilon_i$ [By definition (see equation 2.2)]

$\Rightarrow \varepsilon(x) = \sum \varepsilon_i$ when $k = 1$

$$\frac{\partial Z}{\partial x_i} = w_i - \mu f_i = 0, \qquad i = 1,2,\ldots,n$$
$$\frac{\partial Z}{\partial \mu} = y^0 - f(x) = 0 \tag{2.6}$$

Manipulating of this set of simultaneous equations yields the result that the Lagrangian multiplier μ equals marginal cost: Keeping w_i (for $i = 1,2,\ldots,n$) constant the differential of the cost function employed here is:

$$dc = \sum_{i=1}^{n} w_i dx_i \tag{2.7}$$

The first equation of 2.6 yields $w_i = \mu f_i$. Substituting this into 2.7 gives;

$$dc = \mu \sum_{i=1}^{n} f_i dx_i \tag{2.8}$$

Given that the differential of the production function is

$$dy = \sum_{i=1}^{n} f_i dx_i \tag{2.9}$$

dividing 2.8 by 2.9 will yield marginal cost:

$$\frac{\partial c}{\partial y} = \frac{\mu \sum_{i=1}^{n} f_i dx_i}{\sum_{i=1}^{n} f_i dx_i} = \mu \tag{2.10}$$

The equality of the Lagrangian multiplier with marginal cost is the first result needed to prove the equality between elasticities of scale and size at cost minimising points. The second result needed to prove the equality emerges from a manipulation of the first equation in 2.6. Rearranging that equation, and using the result of 2.10 yields:

$$\frac{\partial c}{\partial y} \cdot \frac{\partial f(x)}{\partial x_i} = w_i \tag{2.11}$$

which provides the result that the firm will minimise costs if it hires inputs, and produces the level of output where the product of marginal cost and marginal product of each input equals the price of each input. Substituting the definition of output elasticity (2.2) into 2.3 yields:

$$\varepsilon(x) = \frac{\sum f_i x_i}{f(x)} \qquad 2.12$$

Rearranging equation 2.11 by making marginal product the subject of the formula and substituting this into 2.12 allows one to express the elasticity of scale as:

$$\varepsilon(x) = \frac{\sum w_i x_i}{\mu f(x)} \qquad 2.13$$

The numerator of equation 2.13 is total costs, hence 2.13 can be expressed as:

$$\varepsilon(x) = \frac{c}{\mu f(x)} = \frac{AC}{\mu} \qquad 2.14$$

where AC is average costs (formally $AC = c/y$). Now, because (by virtue of 2.10) $\mu = \partial c/\partial y$,

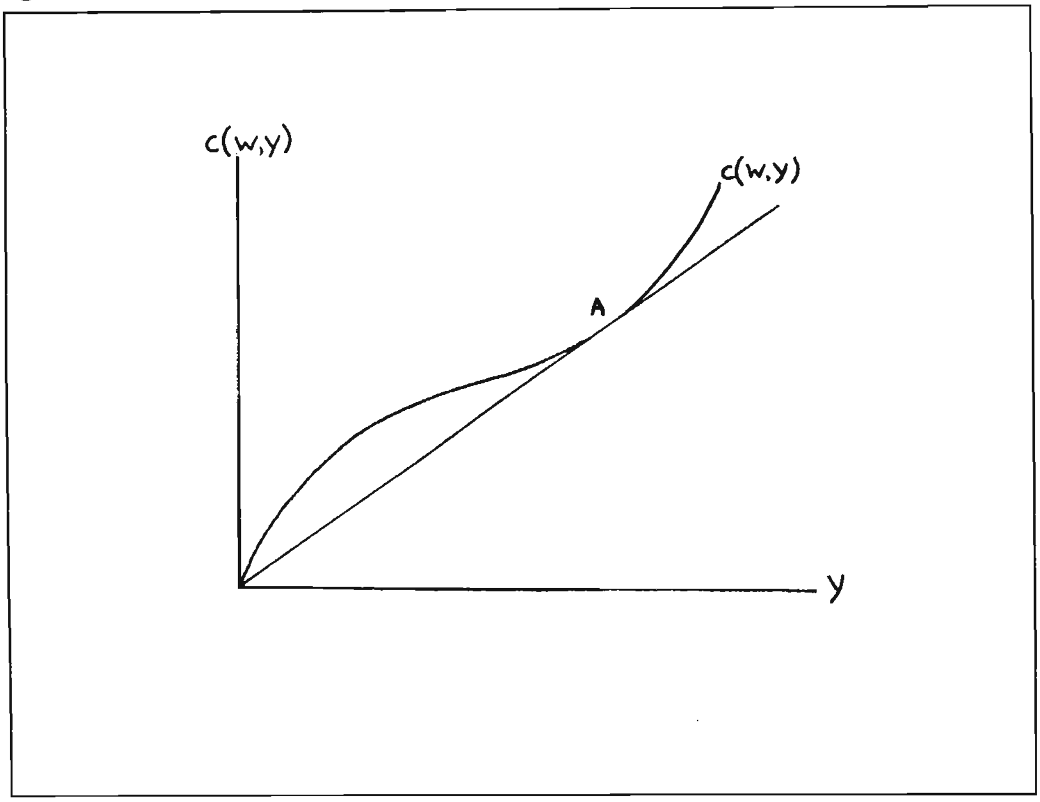
$$\varepsilon(x) = \frac{c}{y} \Big/ \frac{\partial c}{\partial y}$$

and, therefore;

$$\varepsilon(x) = \frac{\partial c}{\partial y} \cdot \frac{c}{y} = \frac{\partial \ln y}{\partial \ln c} = \psi(x) \qquad 2.15$$

An important issue is which of the two measures is the more relevant for the firm. The reciprocal of the elasticity of size is the ratio of marginal to average cost or the cost flexibility ratio. The cost flexibility ratio is a central determinant of the shape of the total cost function. Graphically, marginal cost is the slope of the total cost function while average cost for any level of output is the slope of ray from origin to the point on the total cost curve corresponding to that level of output.

Figure 2.4 Total cost, average cost, marginal cost and cost flexibility.



Source: Chambers, 1988, p78.

In Figure 2.4, when costs are given by point A on $c(w, y)$ marginal cost and average costs are equal and the cost flexibility ratio will be 1. For levels of output less than that corresponding to point A on the cost curve, a ray from the origin will cut a tangent to the curve from below implying that the slope of the ray from the origin is greater than the slope of the tangent to the curve at that point, indicating that average cost is greater than marginal cost and that the cost flexibility ratio is less than 1. Thus when cost flexibility is less than 1 economies of size are greater than 1 (economies of size being the reciprocal of cost flexibility). The converse will occur for levels of output greater than the level corresponding with point A on the cost function. Clearly, the magnitude of economies of size, which is inversely related to the cost flexibility ratio, is central to the determination of the shape of the average cost curve and leads Hanoch (1975, p492) to argue that the elasticity of size is the more relevant measure for the analysis of firm and industry behaviour. Just as the elasticity of scale delineates three types of returns to scale (usually referred to as economies of scale), the elasticity of size delineates three types of returns to size, depending on whether $\psi(x)$ is greater than, equal to or less than unity.

Although the two measures of returns to scale described above generate equivalent measures at cost minimising points, how each changes with changes in output is different. Only if a production function is homothetic will the two measures change in the same manner as output changes. While no proof of this result is provided here[16] an intuitive explanation of this result emerges from the fact that homothetic functions are the only class of functions where the expansion path is a straight line (ray) through the origin. It is to a discussion of the features of homothetic functions that I now turn.

2.2.3 HOMOTHETICITY

The concept of homotheticity is closely related to, albeit more general than, the concept of homogeneity. While every homogenous function is homothetic, homothetic functions are not necessarily homogenous (Chiang, 1984, p423 and Madden, 1986, p240). A function $f(x_1,\ldots,x_n)$ is said to be homogenous of degree ρ if multiplication of each of its independent variables by some constant k alters the value of the function by the proportion k^ρ, in other words if the following relationship holds:

$$f(kx_1,\ldots,kx_n) = k^\rho f(x_1,\ldots,x_n) \qquad 2.16$$

In production theory wide use is made of homogenous functions in general and functions which are homogenous of the first degree (i.e. linearly homogenous, where $\rho = 1$) in particular. The assumption of linear homogeneity imposes an assumption of global constant returns to scale on the underlying technology as raising all inputs (independent variables) k-fold will raise output (the value of the function) exactly k-fold. Similarly, for a function which is homogenous of a degree greater than (less than) unity, returns to scale will be globally increasing (decreasing).

That returns to scale are modeled as a global phenomenon for homogenous functions is readily apparent from an application of the definition of the elasticity of scale (equation 2.1). Homogeneity (defined by equation 2.16) can be expressed in logarithmic form, as:

$$\ln f(kx_1,\ldots,kx_n) = \rho \ln k + \ln f(x_1,\ldots,x_n) \qquad 2.17$$

Substituting 2.17 into the definition of elasticity of scale (2.1) yields:

[16]See, for example Hanoch (1975) or Chambers (1988) for proofs of this result.

$$\varepsilon(x) = \left. \frac{\partial[\rho \ln k + \ln f(x_1, \ldots, x_n)]}{\partial \ln k} \right|_{k=1} = \rho \qquad 2.18$$

implying that for homogenous functions returns to scale are always given by a constant (ρ) and therefore globally defined. This feature of homogenous functions is an important limitation. While economies of scale are in general a function of the input bundle and the level of output, for homogenous functions they are invariant to any of these features being modeled as global, rather than as local phenomena. The implications of this have already been discussed.

A function is classified as homothetic if it can be regarded as a monotonically increasing transformation of a homogenous function. Mathematically a function $f(x)$ is homothetic if it can be represented as: $f(x) = F[f^*(x)]$ (Clemhout, 1968, p91), where F is a monotonically increasing function of $f^*(x)$, which is regular and homogenous[17]. Because a homothetic function is a transform of a homogenous function, an intuitive explanation of a homothetic function requires an intuitive explanation of a transform.

A transform of a production function, as suggested by the definition of a homothetic function, is a function of a production function. Because output is simply an amalgam of inputs and technology, one can consider a production function as merely an `aggregate input' and consequently a transform can be viewed as a single input production function (Shepherd, 1970, cited in Chambers, 1988, p37).

An intuitive explanation of a homothetic function, requires a consideration of homogeneity and points to how scale effects are modeled by these functions. If a production function is homogenous an equi-proportional change in all inputs will lead to a proportional change in the value of the function. Hence homothetic functions (which are transforms of homogenous functions) are that class of transforms where proportionate changes in all inputs are accurately expressed by a proportionate change in the aggregate input (Chambers, 1988, p38). Thus for homothetic functions increasing the scale of operation for each of the actual inputs is equivalent to an

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[17]Different authors disagree as to the degree of homogeneity of $f^*(x)$. Chambers (1988, p37), Lancaster (1968, p334), Clemhout (1968, p91) and Madden (1986, p240) for example, indicate that $f^*(x)$ is linearly homogenous; while Hanoch (1971, p697 and 1975, p492) argues that $f^*(x)$ may be homogenous of any degree and the definition provided by Lancaster (1968) is erroneous. The confusion may however stem from the fact that Shepherd (1953 and 1970), who has been instrumental in introducing the concept into economic analyses, used a definition of homotheticity where $f^*(x)$ was linearly homogenous (Denny & May, 1978, p65).

increase in the scale of operation for the aggregate input, and where scale-type decisions must be made, no generality is lost in dealing only with the aggregate input (Chambers, 1988, p38). This intuition is obvious when one considers that an important feature of homothetic (and homogenous) functions is that they display straight line expansion paths. A straight line expansion path implies that when output is increased the ratio in which inputs are used remains fixed. An implication of this is that economies of scale for homothetic functions are not influenced by changes in relative input utilisation (which is fixed) but are a function of the level of output alone. Formal proofs of both the linearity of the expansion path and the result that scale effects are a function of output alone are presented below.

2.2.3.1 EXPANSION PATHS OF HOMOTHETIC FUNCTIONS

Proof of the linearity of the expansion path of homothetic functions is an extension of the proof that the expansion path of a homogenous functions is a straight line. Assuming only two factor inputs, the first equation of the first order condition of the cost minimising problem will provide the following equality[18]:

$$\frac{w_1}{f_1} = \frac{w_2}{f_2} = \mu \qquad 2.19$$

that is, at the point of optimal input combination, the input price-marginal product ratio for each input must be equal for both inputs (and equal to marginal cost). Equation 2.19 can be rearranged and expressed in the form:

$$\frac{w_1}{w_2} = \frac{f_1}{f_2} \qquad 2.20$$

[18]For the two input case, the cost function would be: $c = w_1x_1 + w_2x_2$ and the production function; $y^0 = f(x_1, x_2)$, yielding a Lagrangian for the constrained cost minimsation problem having the form $Z = w_1x_1 + w_2x_2 + \mu[y^0 - f(x_1, x_2)]$. First order conditions for the optimsation problem are:

$$\frac{\partial Z}{\partial x_1} = w_1 - \mu f_1 = 0$$

$$\frac{\partial Z}{\partial x_2} = w_2 - \mu f_2 = 0$$

$$\frac{\partial Z}{\partial \mu} = y - f(x) = 0$$

Manipulation of the first two equations of these first-order conditions will yield equation 2.19.

The f_1/f_2 ratio is the negative of the slope of the isoquant[19]; hence it is a measure of the marginal rate of technical substitution of input 1 for input 2. The price ratio in equation 2.20 represents the negative of the slope of an isocost line - the locus of input combinations yielding the same total cost. Total costs for the two input case are given by:

$$c = x_1 w_1 + x_2 w_2 \qquad 2.21$$

Making x_2 the subject of 2.21 provides the result that the price ratio w_1/w_2 (the term on the left-hand side of equation 2.20) is indeed the negative of the slope of the isocost line:

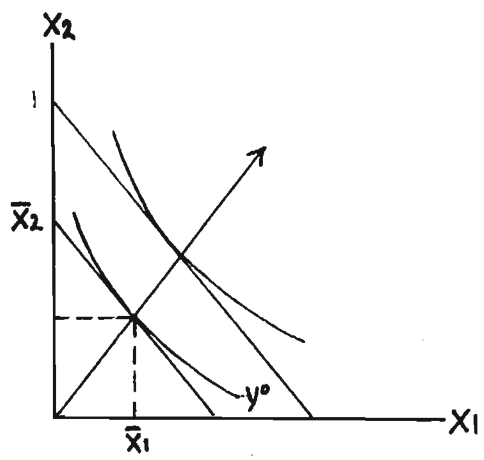
$$x_2 = \frac{c}{w_2} - \frac{w_1}{w_2} x_1 \qquad 2.22$$

When plotted in the $x_1 x_2$ plane, equation 2.22 will yield a family of straight lines whose slopes, given by the negative of price ratio, are equal while prices are fixed. The vertical intercept, given by the ratio c/w_2, is proportional to the level of costs (c). The equality implied by the first order condition of the cost minimisation problem and expressed in equation 2.20 is equivalent to the graphical condition of equality of slope between isoquant and isocost lines. Given that the isoquants are strictly convex to the origin[20] the requirement is one of tangency between the isocost and isoquant curves. Graphically:

[19]Along an isoquant output is constant, hence in the 2-input case $dy = f_1 dx_1 + f_2 dx_2 = 0$. Rearranging yields: $dx_2/dx_1 = -f_1/f_2$. Given that an isoquant is drawn in the input space, this equation provides an expression for the slope of an isoquant.

[20]The second order conditions of the cost minimization problem require the isoquants be strictly convex at the chosen input combination (Chiang, 1984, p362).

Figure 2.5 Expansion path of a homothetic function



Source: Adapted from Chambers, 1988, p 72.

The least cost combination for producing a level of output y^0 is the input combination $(\overline{x_1}, \overline{x_2})$. For successively higher levels of output, the least cost input combination will correspond to the tangency of both a higher isoquant and a higher isocost. As discussed earlier, the locus of points of tangency (i.e. the locus of cost-minimising points in input space describing the least cost combinations required to produce varying levels of output) is known as the expansion path of the firm.

The linearity of the expansion path of a homogenous function rests on the following argument: if a function $(y = f(x_1, x_2))$ is homogenous of degree ρ, then its marginal products f_1 and f_2 will both be homogenous of degree $\rho - 1$ in the inputs x_1 and x_2[21]. Hence multiplying both inputs by a constant k will raise the value of both marginal products by $k^{\rho-1}$, and leave their ratio unchanged. Thus if the first order condition 2.20 is satisfied for an input combination (x_1, x_2) then for a given fixed price ratio it will also be satisfied for a combination (kx_1, kx_2). Whenever the initial cost minimising quantities of inputs are multiplied by the same scalar the cost-minimising condition will remain fulfilled. Graphically this implies that the cost minimisation problem will be

[21]This follows from the general result that the n^{th} order partial derivatives of a function homogenous of degree ρ are themselves homogenous of degree $\rho - n$ (Lancaster, 1968, p335).

satisfied for any multiple of the initial input combination, implying that the locus of cost minimising points (the EP) will be linear for homogenous functions.

Turning to the case of a homothetic function: $y = F[f^*(x_1, x_2)]$, where $f(x_1, x_2)$ is homogenous, the slope of its isoquant will be:

$$-\frac{\partial y/\partial x_1}{\partial y/\partial x_2} = -\frac{F'(f^*).f_1^*}{F'(f^*).f_2^*} = -\frac{f_1^*}{f_2^*} \qquad 2.23$$

where the first equality emerges form the application of the chain rule. 2.23 indicates that the cost minimising condition will be given by:

$$\frac{\partial y/\partial x_1}{\partial y/\partial x_2} = \frac{w_1}{w_2} = \frac{f_1^*}{f_2^*}$$

Because $f^*(x_1, x_2)$ is homogenous multiplying both inputs by the same constant will leave the ratio f_1^* / f_2^* unchanged. The implication is that for a given price vector the locus of cost minimising points will be a straight line through the origin.

2.2.3.2 SCALE EFFECTS UNDER HOMOTHETICITY

The coincidence of the EP with a ray has important implications for the modeling of the elasticity of scale. In general the elasticity of scale, which is concerned with measuring how output varies along a ray from the origin, is a function of the input vector and the level of output. This can be seen from substituting equation 2.2 into equation 2.3:

$$\begin{aligned} \varepsilon(x) &= \sum_i \frac{x_i}{y} \cdot \frac{\partial f(x)}{\partial x_i} \\ &= g(x, y) \end{aligned} \qquad 2.24$$

This result does not, however, hold for either homogenous or homothetic functions, and is a feature which distinguishes those classes of functions from others.

For homogenous functions the elasticity of scale is a global constant and therefore independent of the input vector and the level of output (see equation 2.18 above). For a function which is homothetic, the elasticity of scale is merely a function of the level of output. To prove this result, an alternative, but equivalent, definition of a homothetic function needs to be employed (Chambers, 1988, p38). The original definition of homotheticity used above was that a function $f(x)$ is homothetic if it can be represented as: $f(x) = F[f^*(x)]$, where $f^*(x)$ is regular, and therefore strictly monotonic, and homogeneous. An alternative expression would be:

$$y = F[f^*(x)] \qquad 2.25$$

where y is output. Because $F(\cdot)$ is, by definition monotonic, the inverse of $F(\cdot)$ will be defined and exist (Chiang, 1984, p172). An implication of the existence of the inverse of $F(\cdot)$ is that a homothetic function can alternatively be expressed by:

$$f^*(x) = F^{-1}(y) \text{ or } F^{-1}(y) = f^*(x)$$

Specifying $h(y) = F^{-1}(y)$ a homothetic function may be described as:

$$h(y) = f^*(x) \qquad 2.26$$

where $f^*(x)$ is homogenous. Using 2.3, which shows that the elasticity of scale is the sum of output elasticities of different inputs, the elasticity of scale can be expressed as;

$$\varepsilon = \sum_i \frac{\partial f(x)}{\partial x_i} \cdot \frac{x_i}{y} \qquad 2.27$$

Using the chain rule of differentiation and applying 2.27 to 2.25 yields :

$$\varepsilon = \sum_i \frac{dy}{df^*(x)} \cdot \frac{\partial f^*(x)}{\partial x_i} \cdot \frac{x_i}{y} = \frac{\sum_i \frac{dy}{df^*(x)} \cdot \frac{\partial f^*(x)}{\partial x_i} \cdot x_i}{y} \qquad 2.28$$

Given that $f^*(x)$ is homogeneous 2.28 can be simplified by applying Euler's theorem[22]

$$\varepsilon = \frac{\frac{dy}{df^*(x)} \cdot \rho f^*(x)}{y} \qquad 2.29$$

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[22]Eulers theorem states that if some function $y = f(x)$ is linearly homogenous, then $\sum_i \frac{\partial y}{\partial x_i} \cdot x_i = y$ (Chiang, 1984, p413). If the function is homogenous of degree ρ then the Euler's theorem implies that $\sum_i \frac{\partial y}{\partial x_i} \cdot x_i = \rho y$, where ρ is the degree of homogeneity of the function.

where ρ is a constant indicating the degree of homogeneity of $f^{*}(x)$. By virtue of 2.26, equation 2.29 can be written as:

$$\varepsilon = \frac{\dfrac{dy}{dh(y)} \cdot \rho h(y)}{y} \qquad 2.30$$

Applying the inverse function rule[23], 2.30 yields:

$$\varepsilon = \frac{\rho h(y)}{h'(y)y} \qquad 2.31$$

where $h'(y)$ is the derivative of $h(y)$ with respect to y.

Equation 2.31 shows that for a homothetic production function, the elasticity of scale is a function of the level of output y only. Moreover, given the coincidence of the expansion path with a ray through the origin under homotheticity (and the consequent equality of the elasticity of scale and the elasticity of size) the elasticity of size is also a function of the level of output alone, responding in the same manner as the elasticity of scale to output changes. This equality of the two measures of scale also holds for homogenous functions which by definition are also homothetic.

The result that the elasticity of scale is a function of the level of output alone for homothetic functions is intuitively appealing. A straight line expansion path implies that the isoquants are parallel and their shape is independent of the scale of production (Clemhout, 1968, p94; Griliches & Ringstad, 1971, p7 and McFadden ,1978, p77), and therefore when output is increased the ratio in which inputs are used remains fixed.

The characteristic of a straight line expansion path under homotheticity (and the implied equality of the elasticities of scale and size) will also have an impact on the shape of the cost functions associated with homothetic production functions. The shape of the total cost function, as shown above, is determined by the cost flexibility and hence the elasticity of size. For homothetic functions the cost flexibility cannot be

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[23]If a function $f(x)$ is monotonic it has an inverse and the inverse function rule of differentiation $\dfrac{dx}{df(x)} = \dfrac{1}{df(x)/dx}$ holds (Chiang, 1984, p171-173). Given that $h(y)$ is by definition monotonic $\dfrac{dy}{dh(y)} = \dfrac{1}{dh(y)/dy} = \dfrac{1}{h'(y)}$.

influenced by changes in input use and hence the shape of the cost functions will be a function of the level of output only for a given input-price ratio. Changing relative input utilization as output expands (which is manifested in an expansion path deviating from a ray through the origin) will obviously impact directly on average costs and total costs. Explicitly preventing such chamges is obviously a restrictive feature of homothetic functions.

2.2.4 HOMOTHETICITY AND COBB-DOUGLAS AND CES FUNCTIONS

Both functional forms used, most often, to analyse features of the technology employed in South African manufacturing, the Cobb-Douglas and CES functions, are homogenous and thus homothetic: Consider an $n-$input Cobb-Douglas function:

$$y = A\prod_{i=1}^{n} x_i^{\alpha_i} \qquad 2.32$$

where A is an efficiency parameter and the x_i are the inputs. Input elasticities are given by;

$$\varepsilon_i = \frac{\partial f(x)}{\partial x_i} \cdot \frac{x_i}{y} = \alpha_i \qquad 2.33$$

The elasticity of scale is therefore:

$$\varepsilon(x) = \sum_{i=1}^{n} \alpha_i \qquad 2.34$$

This result indicates that for constant α_is, $\varepsilon(x)$ will be constant and will not vary with different levels of output, implying that the function is homogenous. Homogeneity of the Cobb-Douglas in turn implies homotheticity of the function, and the properties of homothetic functions described above apply to the Cobb-Douglas function.

The CES function can be expressed as:

$$y = \gamma[\delta x_1^{-\rho} + (1-\delta)x_2^{-\rho}]^{-v/\rho} \qquad 2.35$$

where γ is an efficiency parameter; δ $(0\langle\delta\langle 1)$ is the input intensity, ρ $(\infty \geq \rho \geq -1)$ is the substitution parameter and v represents the degree of homogeneity of the function and is the returns to scale parameter (Brown, 1968, p45). Clearly, the CES models economies of scale as a global phenomenon, independent of the level of output. Homogeneity of the CES implies homotheticity of the function, and the properties of homothetic functions described above apply to the CES function.

As argued in the introduction to this chapter the presence of restrictive or implausible maintained hypotheses will be manifested in the testing of specific primary hypotheses. Using either the Cobb-Douglas or CES forms to test the hypothesis that economies of scale are, for example, of a particular magnitude could rest on the validity of the underlying hypotheses of homotheticity and homogeneity which impose constraints on how scale is modeled. If one is to use either the Cobb-Douglas or CES forms with confidence the validity of the hypotheses of homotheticity and homogeneity ought to be established. The introduction of the mathematical concept of duality to the analysis of economic phenomena has led to the development of functional forms which are sufficiently flexible to allow for the testing of homotheticity and homogeneity, rather than imposing these hypotheses *a priori*. The *Transcendental Logarithmic cost function* (Christensen, Jorgensen and Lau, 1971 and 1973) (Translog) is an example of a flexible function emerging from the use of duality theory which allows the explicit testing of the validity of the hypotheses of homotheticity and homogeneity. Using this function to analyse scale is clearly superior to adopting either a Cobb-Douglas or CES form. Both the concept of duality and the specific form of the Translog are developed in Chapter 3.

2.3 ELASTICITIES OF SUBSTITUTION AND FUNCTIONAL SEPARABILITY

This dissertation is explicitly concerned with attempting to quantify two features of the technology employed in specific South African manufacturing subsectors: returns to scale and elasticities of substitution. The impact of the maintained hypotheses embodied in the most popular functional forms on the modeling of scale effects has been addressed above. How elasticities of substitution are modeled in theory and the impact that different maintained hypotheses have on elasticities of substitution is the concern of this part of this chapter.

Berndt and Christensen (1973a, p403-409) have shown that for production functions employing more than two factor inputs, the internal structure of a function, and in particular whether a function of several arguments can be separated into sub functions, is closely related to equality constraints on one particular measure of the partial elasticity of substitution between inputs, namely the *Allen partial elasticity of* *substitution* (AES) (Allen, 1938, p504). The corollary to their result is that, where separability is a maintained hypothesis, the modeling of AES will be restrictive. Although a number of different measures of the elasticity of substitution exist when more than two inputs are considered the AES is of particular interest. AESs can be obtained from simple algebraic manipulations of the parameters of the function employed here. Moreover AESs play an important role in the estimation of the function used in this analysis.

2.3.1. ELASTICITIES OF SUBSTITUTION

The possibility of producing a constant output level with a variety of input combinations and the degree to which inputs can be substituted for one another is an important question for economic decision making (Chambers, 1988, p28). The most common measure of substitutability is the *marginal rate of technical substitution* (MRTS) which measures the rate at which one factor can be substituted for another factor while the level of output remains constant. Graphically it is a measure of the slope of an isoquant. Mathematically, for a two-factor production function $y = f(x_1, x_2)$, it can be represented by:

$$MRTS = \frac{\partial x_2}{\partial x_1} = -\frac{f_1}{f_2} \qquad 2.36$$

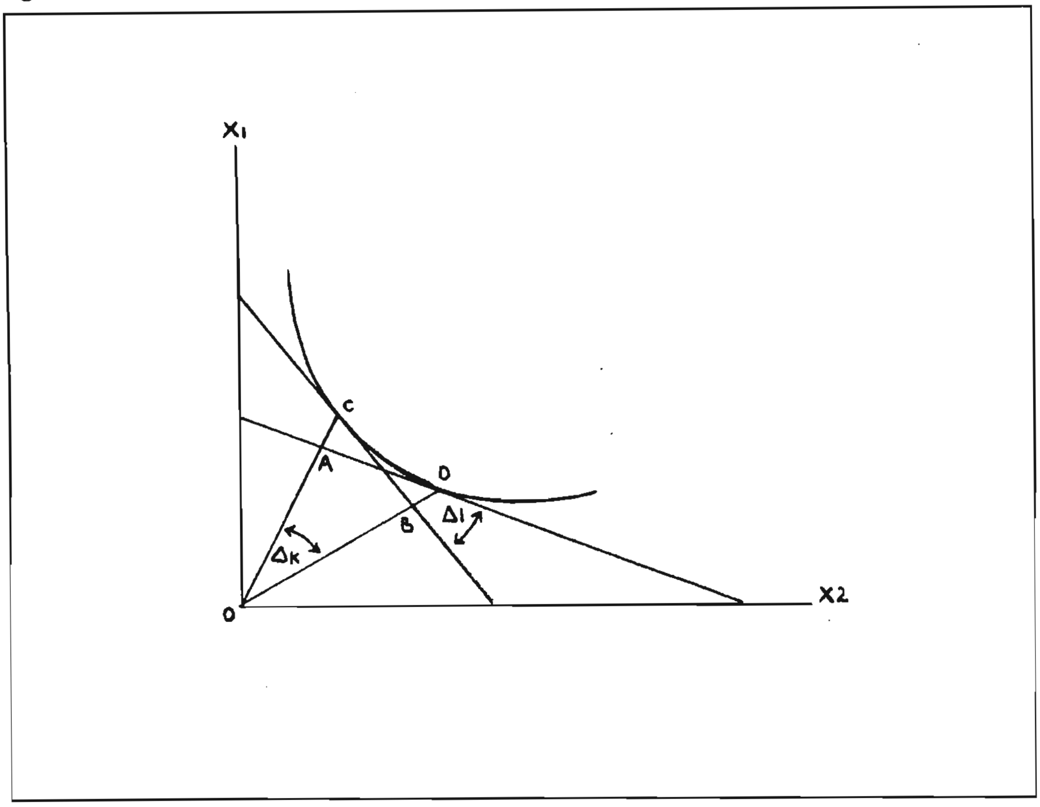
The assumption that the production function is monotonically increasing in each of the inputs (if $x_1' \geq x_1$, then $f(x_1') \geq f(x_1)$ and if $x_2' \geq x_2$ then $f(x_2') \geq f(x_2)$) implies that the MRTS must be everywhere negative, because output can only remain constant if when the use of one factor is increased, the use of another is decreased. The assumption that isoquants are convex to the origin means that the MRTS is everywhere diminishing in value (Jehle, 1991, p221). The existence of diminishing MRTS, which implies some friction in substitutability, is intuitively appealing as it is likely that it will become increasingly difficult to substitute one factor for another and still maintain output at a fixed level.

Elasticities of substitution are essentially unit-free measures of the MRTS. For the two-input case, the elasticity of substitution has been defined by Hicks (1963, cited in Chambers, 1988, p29 and Jorgensen, 1986, p1844) as:

$$\sigma_{12} = \frac{d\ln(x_2/x_1)}{d\ln(f_1/f_2)} = \frac{d(x_2/x_1)}{d(f_1/f_2)} \cdot \frac{f_1/f_2}{x_2/x_1} \qquad 2.37$$

Hence, σ is the elasticity of the input ratio with respect to the MRTS. In other words, it is a measure of the proportional change in the factor ratio (x_2/x_1) resulting from a proportional change in the MRTS of input 1 for input 2. When factor prices are fixed as quantities of factors demanded change (and the firm is a cost minimiser) σ may be interpreted as the percentage change in the ratio of inputs given a small percentage change in the ratio of marginal products or of the price ratio[24] (Nadiri, 1982, p442). An alternative explanation of the concept can be obtained from the following graph:

Figure 2.6 Elasticity of Substitution



Source: Chambers, 1988, p 31

If the initial input ratio was given by the ray OAC, the corresponding (initial) MRTS would be given by the slope of the tangent to the isoquant at point C. Should the input ratio change to one given by the ray OBD, the MRTS would now be given by the tangent to the isoquant at point D. σ can hence be regarded as the ratio of the two angles Δ_k/Δ_t, and can be regarded as a measure of the curvature of the isoquant. This can be seen from an interpretation of possible values for σ. The value of σ ranges form zero to infinity. The closer σ is to zero, the more difficult substitution between factors becomes. Because the ratio Δ_k/Δ_t becomes smaller as σ approaches zero, the

[24]The first order conditions of the cost minimising problem can be manipulated to obtain the equality: $w_1/w_2 = f_1/f_2$ (see equation 2.20).

implication is that the isoquants will be more convex the smaller σ is. In the limit, when $\sigma = 0$ and there is no substitutability between factors, isoquants will be right-angles to the origin. Conversely the larger is σ, the flatter the isoquants and the easier substitution between factors.

Two important features of the elasticity of substitution need to be noted: first that it is always positive in the two-input case and secondly that it is symmetric i.e. $\sigma_{12} = \sigma_{21}$. The first of these results can be explained using the above graph. A movement from point C to point D would imply an increase in both the x_2/x_1 ratio, implying that $d(x_2/x_1)$ would be positive, and in the f_1/f_2 ratio[25], implying that $d(f_1/f_2)$ would be positive. Given the definition of σ (equation 2.37) it is clear that moving from C to D would yield a positive σ. Movement the other way generates a negative numerator and denominator in the formula and hence a positive σ. In the two input case, then, factors can only be characterised as substitutes (Chambers, 1988, p32).

The second feature of the elasticity of substitution which warrants some explanation is the symmetry of the measure. Symmetry is intuitively appealing but can also be explained by considering an alternative formulation of the definition. σ can equivalently be expressed as (Chambers, 1988, p32):

$$\sigma_{12} = \frac{x_1 f_1 + x_2 f_2}{x_1 x_2} \cdot \frac{F_{12}}{F}$$

F is the determinant of the bordered hession of the production function:

$$F = \begin{vmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{21} & f_{22} \end{vmatrix}$$

and F_{12} is the cofactor of f_{12}. Finding σ_{21} would merely require the replacement of F_{12} with F_{21}, which will be equal given that if $f(x)$ is assumed to be regular, it will be twice continuously differentiable and Young's theorem ($f_{12} = f_{21}$) will hold.

The discussion up to this point has been limited to the case of only two inputs in the production process. In the more general n-factor case only the concept of partial

[25]Due to the assumption of diminishing marginal productivity using less of factor x_1 implies that its marginal product f_1 will rise and vice versa, hence as x_1 is substituted for x_2 the ratio f_1/f_2 will increase.

elasticity can be defined. Elasticity of substitution for the two input case is defined subject to the constraint that the level of output remains constant. Generalisation of the elasticity concept to the n-input case requires in addition to the constraint of constant output, the imposition of further constraints on the variables. Depending on the constraints imposed a number of different elasticities exist (Griliches & Ringstad, 1971, p6). Of the available alternatives, two will be developed here: the *direct elasticity of substitution* (DES), and the *Allen partial elasticity of substitution* (AES)[26]. Only the AES is of direct relevance to the function used here. It is not only directly computable from parameter estimates but also imperative for generating meaningful results[27]. The DES is developed here because it would appear to be the most natural definition in the n-input case, and provides a useful indicator of how the AES differs from the concept of elasticity of substitution as defined for the two-input case.

The DES is defined as:

$$\sigma_{ij}^{D} = \frac{\partial \ln x_j/x_i}{\partial \ln f_i/f_j} = \frac{\partial (x_j/x_i)}{\partial (f_i/f_j)} \cdot \frac{f_i/f_j}{x_j/x_i} \qquad 2.38$$

where all x_k $(k \neq i, j)$ are held constant. Because this measure assumes that inputs other than those with which it is directly concerned are held constant, it can be regarded as a short-run measure. The interpretation of the DES is essentially the same as that for the two-input elasticity: it is a measure of the proportional change in the factor ratio (x_j/x_i) resulting from a proportional change in the MRTS of input x_i for input x_j holding all other factors and the level of output constant. Moreover, as in the two input context, σ_{ij}^{D} is symmetrical. Goods are regarded as complements when the DES is negative and substitutes when it is positive.

Possibly the most used definition of elasticity of substitution for the n-input case is the AES (Allen, 1938, p504). This is a measure of the change in a firm's derived demand for factor j given a change in the price of factor i , all other factor prices and output

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[26]Further examples of elasticitys in the n-input case are the *Shadow elasticity of substitution* (McFadden, 1978, p80) and the *Morishima elasticity of substitution* (MES) (Chambers, 1988, p35). Different definitions of the elasticity of substitution in the n-input case display vastly differing characteristics. While both the DES and AES are symmetrical, the MES is not symmetrical. Furthermore, while Allen substitutes are always Morishima substitutes, Allen complements are not necessarily Morishima complements (Chambers, 1988, p35).

[27]The reasoning is developed in Chapters 3 and 4.

remaining constant (Berndt & Christensen, 1973a, p405). Formally, the AES is defined as[28]:

$$\sigma_{ij}^{A} = \frac{\sum_{k=1}^{n} x_k f_k}{x_i x_j} \cdot \frac{F_{ij}}{F} \qquad 2.39$$

where:

$$F = \begin{vmatrix} 0 & f_1 & & f_n \\ f_1 & f_{11} & & f_{1n} \\ & & & \\ f_n & f_{1n} & \cdots & f_{nn} \end{vmatrix}$$

and F_{ij} is the cofactor associated with the element f_{ij}. Stability conditions[29] are fulfilled if $F_{nn}/F \langle 0$, (i.e. if F_{nn} and F are opposite in sign) (Allen, 1938 pp502-504). Although the values of the AES can be positive or negative (a negative AES indicating complementarity and a positive AES, substitutability), the stability conditions require that, when weighted, the positive elasticities must counter balance the negative elasticities (Allen, 1938, p505 and Nadiri, 1982, p443). 'In particular, the $(n-1)$ partial elasticities of substitution between any one factor and the others cannot all be negative' (Allen, 1938, p505). In other words, in terms of the AES a factor of production cannot be a complement to all other factors of production. Chambers (1988, p35) argues that this restriction is intuitively appealing, given that in the two-factor case all inputs are substitutes. As in the case of the DES the AESs are symmetrical.

While the interpretation of the DES is relatively straightforward the interpretation of the AES is more complex. The AES provides information on the cross-demand elasticities of inputs (McFadden, 1978, p80). When production is efficient and when the supply of inputs is perfectly elastic (firms are perfect competitors in input markets) the AES and cross-demand elasticities of inputs are related in the following manner (Allen, 1938, p508):

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[28]If $i = j$ then the AES is termed an own AES. If $i \neq j$ then the AES is regarded as a proper AES.

[29] Stability conditions refer to the stability of demand for factors of production. In the 2-input case, for example, demands for factors of production are stable if the isoquants derived from the production function are convex to the origin at all relevant points (Allen, 1938, p502).

$$\sigma_{ij}^{A} = \frac{\eta_{ij}}{w_i x_i / C} \qquad 2.40$$

where, η_{ij} is the partial (cross) elasticity of demand for factor j with respect to the price of another factor i ($\eta_{ij} = \partial \ln x_j / \partial \ln w_i$), $w_i x_i$ is the expenditure on good x_i and C is total costs. The denominator of the term on the right-hand side will always be a positive fraction ($0 \langle p_i x_i / C \langle 1$), hence the sign of σ_{ij}^{A} will determined by the sign of η_{ij}. Should η_{ij} be positive, in terms of cross elasticity of demand factor j would be regarded as a substitute for factor i, σ_{ij}^{A} would also be positive and factor j would be deemed to be competitive with factor i for the grouping of factors considered. On the other hand, if η_{ij} were negative, σ_{ij}^{A} would also be negative and factors j and i would be considered complementary for the grouping of factors considered (Allen, 1938, p509).

2.3.2 SEPARABILITY

Both analytical and econometric considerations have influenced the tendency for production analysis in general, and applied production work in particular, to assume a relatively small number of input types. Analytical considerations arise from the difficulties associated with geometric expressions of dimensions higher than two, while econometric analyses have been constrained by the need for data with sufficient independent observations. Both considerations have resulted in economists generally adopting the classical categorization of inputs as land, capital and labour (Chambers, 1988, p41). The classification of inputs into different types is usually accompanied by the assumption that the degree of substitutability between inputs of one type differ from the degree of substitutability between inputs of other types (Jehle, 1991, p221). Production functions whose form embodies this type of assumption are deemed *separable* (Jehle, 1991, p221)

A more general explanation of separability is provided by Chambers (1988, pp42-45). Technology is separable if technology can be regarded as occurring in two distinct stages. In the first stage different inputs are combined (via what could be considered micro-production functions) to produce aggregate inputs[30] which are then combined (in the macro production function) to produce output. The existence of two stages is necessary, but not, sufficient to characterise a technology as being separable. Two types of separability can be identified by the conditions which are sufficient for their

[30]The aggregation through the creation of an index of different types of labor or capital is analogous to the existence of micro-production functions.

existence. A technology can be regarded as *weakly separable* if, in addition to production occurring in two logical stages, the micro-production functions are independent of one another. A *strongly separable* technology is one which, in addition to fulfilling the conditions of weak separability, has aggregate inputs which are perfect substitutes in the production of output. In general a function of several arguments is regarded as separable if it can be separated into sub-functions (Berndt and Christensen, 1973a, p403).

The imposition of a maintained hypothesis of separability on technology may be regarded as untenable in many contexts. The existence of separability is, however, crucial for justifying the aggregation of diverse heterogeneous inputs (Berndt & Christensen 1973b, p82 and Nadiri, 1982, p447). Disaggregation of indexes of factor inputs (such as labour into skilled and unskilled) can not be pursued in generalisations of the Cobb-Douglas or CES functions, for example. Those functions assume strong separability which is '... equivalent to assuming that the conditions for consistent aggregate capital and labour indexes are satisfied' (Berndt & Christensen, 1973b, p82).

Formally, separability can be explained in the following manner. If we let $N = \{1,2,\dots,n\}$ denote the set of n inputs, and if we assume that this set of inputs can be partitioned into r mutually exclusive and exhaustive subsets $[N_1,\dots,N_r]$, the production function is said to be *weakly separable* if the MRTS between any two inputs x_i and x_j from any subset N_s $(s = 1,\dots,r)$ is independent of all inputs which are not elements of that subset, i.e.:

$$\frac{\partial}{\partial x_k}\frac{\partial f(x)/\partial x_i}{\partial f(x)/\partial x_j} = 0, \text{ for all } i, j \in N_s \text{ and } k \notin N_s \qquad 2.41$$

The production function is regarded as *strongly separable* if the MRTS between any two inputs from subsets N_s and N_t is independent of the use of all inputs which are not elements of either subset, i.e.:

$$\frac{\partial}{\partial x_k}\frac{\partial f(x)/\partial x_i}{\partial f(x)/\partial x_j} = 0, \text{ for all } i \in N_s,\ j \in N_t,\ k \notin N_s \cup N_t \qquad 2.42$$

Berndt and Christensen (1973a, pp406-408)) show that separability restrictions on production functions are equivalent to certain equality restrictions on the AESs. Berndt and Christensen prove, inter alia, the following theorems:

- Weak separability of a production function at any point in input space is necessary and sufficient for all proper AESs σ_{ik}^{A}, σ_{jk}^{A} $(i, j \in N_s, k \notin N_s)$ to be equal at that point
- Strong separability of a production function at any point in input space is necessary and sufficient for all proper AESs σ_{ik}^{A}, σ_{jk}^{A} $(i \in N_s, j \in N_t, k \notin N_s \cup N_t)$ to be equal at that point.
- Complete strong separability of a production functions at every point in input space is equivalent to equality and constancy of all proper AESs.

Although the assumption of strong separability can reduce the number of parameters needed to be estimated in applied analysis, it does impose an important cost in that it constrains proper AESs to being constant over the entire input space (Chambers, 1988, p48). Chambers (1988, p46) shows that a completely strongly separable production function is also homothetic in aggregate inputs. Given that both the Cobb-Douglas and CES functions are homothetic in the aggregate inputs, both are also completely strongly separable and hence their modeling of substitution is restrictive. Not only is the elasticity of substitution constrained to be constant irrespective of the level of output, a constraint which is difficult to justify technologically (Fuss et al, 1978, p240), but when extended to include more than two inputs all proper AESs are equal (Diewert, 1971; Fuss et al, 1978 and Jorgensen, 1986). The potential problems associated with using a function which is *a priori* completely strongly separable have already been discussed in the introduction to this chapter. Flexible functions emerging from the application of duality theory are not *a priori* separable and hence allow for a more superior investigation of the possibilities of substitution than that offered by either the Cobb-Douglas or CES forms.

2.4 CONCLUSION

Econometric applications are often confronted with a trade-off between the generality of competing models and the analytical tractability of those models. Production analysis is no different. Prior to the extension of duality principles to production problems most applications employed either the Cobb-Douglas or CES functions or some generalisation or extension of these. Both these functions sacrifice generality for analytic tractability. The sacrifice of generality and the implicit assumptions, in the form of maintained hypotheses, which accompany it may have hidden costs. Indeed, the quantitative results emerging from the inappropriate adoption of the hypotheses of homotheticity, homogeneity and separability are potentially misleading. The main attraction of the dual approach is that it greatly mitigates the trade-off between generality and tractability, although by no means eliminating it (Chambers, 1988, p37). Using functions which exploit the duality between cost and production allows for testing of the hypotheses of homotheticity, homogeneity and separability and hence potentially more valid estimates of the characteristics of technology. The use of duality in production analysis and the specific form of the function employed here is the topic of Chapter 3.

CHAPTER 3

THE TRANSCENDENTAL LOGARITHMIC COST FUNCTION

3.1 INTRODUCTION

Recognition of the constraints implicit in the traditional production function approach to econometric modeling of technology (see Chapter 2) motivated the development of a number of more general, flexible functional forms which make use of the existence of duality between cost and production. Such forms are capable of representing non-homothetic production technologies where, inter alia, patterns of elasticities of substitution are not constrained *a priori*. Diewert (1971) developed the *Generalised Leontief production function* (GL). The GL is a quadratic function in the square roots of an arbitrary number of input prices, reducing to the Leontief (fixed input ratios) form as a special case (Diewert, 1971, p481). Christensen, Jorgensen and Lau (1971, 1973) developed the *Transcendental Logarithmic cost function* (Translog). This function is both linear and quadratic in the logarithms of the prices of an arbitrary number of inputs, and the level of output (if returns to scale are not restricted to being constant) and may be augmented to include an index of the level of technology (usually time in different forms) (Christensen et al, 1973, p28 and Jorgensen, 1986, p1848). The Translog reduces to the multi-input Cobb-Douglas function as a special case. Both the GL and the Translog can be regarded as flexible in the sense that they embody few maintained hypotheses[1]. Hypotheses of homogeneity, homotheticity and separability (for example) are not imposed on the underlying technology but rather are testable and can be adopted if compatible with the data being employed[2]. Furthermore, both functions are general in that they are easily adaptable to include not only multiple inputs but also multiple outputs.

The development of flexible functional forms is a direct result of the application of duality theory to economic problems in general and the theory of production in particular (Christensen and Greene, 1976, p658). The basic idea of the dual approach

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[1]Duality theory has also been employed to develop inflexible dual cost functions. Nerlove (1963), for example, developed the dual Cobb-Douglas cost function which, because it embodies the same maintained hypotheses as the Cobb-Douglas production function, cannot be regarded as either flexible or general (Berndt, 1991, p457).

[2]Not only are the most common maintained hypotheses statistically testable, when flexible functional forms are used, but the validity of the underlying theory of production is also testable (Jorgensen, 1986, p1847). How one would test maintained hypotheses is discussed in detail in Chapter 4.

to analysing technology is that because technology conditions the responses of producers to market phenomena, examining these conditioned responses of producers should provide insights into the structure of technology (Chambers, 1988, p49). Shepherd (1953) developed the duality between cost and production functions by showing that given certain regularity conditions[3] a cost function may be used to define a production function (i.e. there exist cost and production functions dual to each other) (Baumol, 1972 and Diewert, 1971 & 1982). An important conclusion of duality theory is that the structure of the technology underlying production may be analysed empirically using either a cost or a production function. The fundamental advantage of the dual approach is that the implications of optimizing behaviour as presented in the classic treatise of production theory (such as Hicks' *Value and Capital* (1946)), which used general functions, can be obtained without imposing *a priori* arbitrary constraints on the underlying technology, such as homogeneity or homotheticty (Jorgensen, 1986, p1843-1844).

Before turning to a presentation of the Translog model two issues need to be addressed, both of which are concerned with the validity of the choice of the Translog function in the present context. The theoretical advantages of a functional form which imposes few *a priori* maintained hypotheses on the underlying technology are obvious. Whether employing a dual cost function in a particular context is appropriate is not, however, determined merely by the perceived theoretical advantages of these forms. The existence of the dual function is premised on the presence of specific behavioural traits of economic agents, in this case the firm. Application of a dual cost function, should occur only where those traits exist. The first issue to be addressed below is the question: What behavioural traits is the firm assumed to display in order for the dual relationship between cost and production to exist? An ancillary consideration is the implications of the violation of that behaviour in a particular context. The first issue is then broadly concerned with the appropriateness[4] of cost functions, as opposed to production functions, for characterising the features of technology of subsectors of South African manufacturing. Because a number of dual cost functions exist[5], the second issue dealt with below is which of the available cost functions is most appropriate for this study.

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[3]Regularity conditions required to establish the duality between cost and production functions are that the cost function be: positive for positive input prices and a positive level of output; linearly homogenous in input prices; strictly monotonically increasing in outputs; monotonically increasing and concave in input prices and differentiable (and therefore continuous) with respect to both input prices and output quantities (Baumol, 1977, p366-367; Diewert, 1982, p554-555 and Jorgensen, 1986, p1185).

[4]Notwithstanding the obvious theoretical advantages of the flexibility of dual cost functions.

[5]Indeed the literature has produced a competition in the development of exotic functional forms (Greene, 1993, p504).

3.1.1 BEHAVIOURAL ASSUMPTIONS OF DUALITY AND THE ASSUMED NATURE OF REGRESSORS

Prima facie it would appear that using a cost function approach, such as the Translog, in applied work would be more desirable, given it's greater flexibility and generality[6]. Theoretically the choice of approach should, however, be determined by the nature of the data and is essentially a question to be decided on statistical grounds (Christensen & Greene, 1976, p658 and Fuss et al, 1978, p266). Indeed, while the dual cost function imposes few maintained hypotheses on the underlying technology, the construction of these functions does require maintained hypotheses on both market structure and firm behaviour, and as a consequence on the statistical nature of the data (Fuss et al, 1978, p266). In particular, the duality between cost and production functions is premised on the assumption of cost minimising behaviour and thus from the perspective of the producing unit, output is regarded as fixed (and therefore exogenous) and competitive markets deemed to exist for all inputs, implying that input prices are exogenous (Baumol, 1977, p364; Henderson and Quandt, 1980, p77 and Jorgensen, 1986, p1884).

Fuss et al (1978, p266) argue that violation of '... one of these maintained hypotheses *may* result in a model which does not have the postulated structural relationship to the underlying technological parameters' (own emphasis). None of the literature[7] (applied or theoretical) is, however, explicit regarding precisely when the dual relationship will be violated. It does, however, appear from the literature that where either output or prices (or both) are endogenous a cost function may still be estimated as long as an appropriate systems estimator is used.

Berndt (1991, p 474) and Fuss et al (1978, p276) explicitly argue that violation of the assumption of exogenous input prices is not a serious problem for the estimation of cost functions dual to the production function, and may be overcome by using an appropriate instrumental variable estimator[8]. Their argument may be vindicated by the many applications of the Translog in situations where the level of aggregation is high,

[6]A further obvious feature of using a dual cost function is the fact that the regressors required for empirical estimation are all economic observables (input prices and costs). Estimating a production function directly would require data on quantities of inputs, which are likely to be unobservable in many instances.
[7]See for example Baumol (1977), Berndt (1991), Berndt & Christensen (1973b), Berndt & Wood (1975), Chambers (1988), Christensen & Greene (1976), Denny & May (1978), Diewert (1982), Jorgensen (1986), and Nadiri (1982).
[8]Why an instrumental variable estimator would be appropriate and which instrumental variables ought to be used when estimating the Translog is discussed in detail in Chapter 4.

such as US. and Canadian manufacturing sectors, and prices are clearly not exogenous (see for example Berndt & Christensen (1973b); Berndt & Wood (1975); Denny & May (1978); and Fuss (1977)). Unfortunately, none of these applications of the Translog deal with the question of the nature of output. In all of them constant returns to scale are imposed *a priori*, and as a consequence output does not appear as a regressor in the system[9]. The applications are nevertheless premised on the existence of a duality between cost and production functions. It is reasonable to assume that despite the level of aggregation and the endogeneity of output the conditions for the existence of that duality were not violated in any of those applications. An implication of these applications, then, is that despite output being endogenous a dual cost function may be estimated where an appropriate estimator is available[10].

The argument above that the endogeneity of either (or both) price or (and) output does not violate the duality which exists between cost and production gains further currency from a review of econometric texts which survey appropriate estimators of seemingly unrelated (SUR) systems such as the Translog and attendant share equations[11]. Berndt (1991), Greene (1993) and Kmenta (1986) all discuss suitable instrumental variable techniques for estimating SUR systems, which are necessary when independent variables (in this case input prices and output) cannot be assumed to be exogenous.

While the duality between cost and production functions is premised on cost minimising behaviour, it would appear from the literature that violation of the two assumptions commonly used to depict cost minimising behaviour, exogeneity of prices and output, does not render the assumed duality invalid. For the present analysis, the assumption of exogeneity of output is violated. The present analysis is concerned with estimating economies of scale in specific 2, 3 and 4 digit standard industrial classification (SIC) industries. Following Berndt (1991, p460), it is argued that the

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[9]Why this is the case is explained in section 3.4.1 and 3.4.2.

[10]Of the applications of the Translog consulted, only one is explicitly concerned with returns to scale and hence the nature of output: Christensen and Greene (1976). While they (Christensen and Greene, 1976, p658) argue that estimating a cost function is more attractive than estimating a production function when output is exogenous, they do not provide reasons for their arguement, nor do they discuss the implications of attempting to estimate a cost function when output is assumed exogenous but is in fact endogenous. They applied the Translog to the U.S. electric power industry, an industry where output prices are regulated and hence output (which is a function of the output price) is exogenous to the firm (p658-659).

[11]While the discussion up to this point has referred to cost functions alone, empirical estimation typically involves estimating a system consisting of a cost function and associated share equations. How share equations are derived is discussed in detail below.

level of analysis is sufficiently disaggregated to assume that input prices are likely to be exogenous to the industry. Output cannot, however, be assumed exogenous[12].

3.1.2 CHOOSING BETWEEN DIFFERENT FLEXIBLE FORMS

A final issue which needs to be addressed before presenting the Translog model is the issue of why the Translog form has been chosen over another popular flexible form, the GL. Both forms can be regarded as second-order Taylor series approximations to an arbitrary cost function (Berndt, 1991, p469 and Chambers, 1988, p181). Hence, on theoretical grounds, one is not able to distinguish between them (Berndt, Darrough & Diewert, 1977, p661). Moreover, given that both forms have the same dependent variables, maximize similar likelihood functions and that maximization of the likelihood functions is invariant to scaling of normalised prices, *a priori* one is unable to chose between the two forms on econometric grounds (Berndt et al, 1977, p662). Hence, some other criterion needs to be used. Berndt et al (1977, p668) evaluated these forms on the basis of how well they fitted a set of observed data and found the Translog form to be preferred on Bayesian grounds *a posteriori*.

Guilkey, Lovell and Sickles (1983, p591) argue that the type of approach adopted by Berndt et al (1977) is not sufficiently general, and is only useful if interest is focused on the data set used for the Bayesian testing. They argue that a better approach would be to use a Monte Carlo technique, where one begins with a known technology and then examines the ability of different functional forms to track that technology. Adopting this approach Guilkey et al (1983) found that while all the forms they considered estimate economies of scale well, their results indicated a clear preference for the Translog form. The superiority of the Translog form is most apparent where inputs are complements (Guilkey et al, 1983, pp599-560). Based on Guilkey et al's (1983) results and the emphasis of this study on the magnitude of economies of scale, the Translog was chosen over the GL (or other flexible forms) for the present analysis.

Before turning to the Translog, an important caveat regarding flexible forms in general needs to be addressed. While these forms have the flexibility to model sophisticated technologies, that flexibility is only achieved at the expense of not displaying globally good behaviour. In neo-classical production theory production functions are assumed, inter alia, to be monotonic and either concave or quasi-concave in inputs (Lau, 1978, p409). Functions which automatically satisfy these requirements are well-behaved.

[12]Given the potential endogeneity of output an instrumental variable estimator was employed here. The choice of an appropriate estimator and instrumental variables is discussed in Chapter 4.

While the functional form of both the Cobb-Douglas and CES forms ensure that these requirements are globally satisfied, for an arbitrary set of parameters, the Translog cost function will not necessarily ensure the satisfaction of either requirement for the dual production function, either locally or globally (Lau, 1978, p411). In order for a cost function to completely describe a well-behaved production function, it will have to be, inter alia[13], monotonically increasing and concave in input prices. Neither condition is fufilled for the Translog cost function *a priori*, being determined by the nature of the data being analysed. Hence, there is a need to test these hypotheses over the range of data employed.

Violation of the concavity requirement may be particularly problematic. Duality between cost and production functions is, as argued above, premised on a maintained hypothesis of the existence of cost minimizing behaviour. Violation of this maintained hypothesis may undermine the postulated structural relationship between the cost function and the underlying technology. Concavity in input prices of the cost function is not only crucial for ensuring that the production function is well-behaved but for the fullfillment of the hypothesis of cost minimising behaviour. Parameter estimates derived in situations where concavity is violated should, therefore, be treated with circumspection.

3.2 THE TRANSCENDENTAL LOGARITHMIC COST FUNCTION

The discussion of the Translog so far has tended to refer only to a cost function. Any mention of a system of equations has been in passing. One of the broad fundamental advantages of employing a dual function (such as a cost function) is its computationally simple relation to the derived demand functions which impose few arbitrary maintained hypotheses on the underlying technology[14]. Indeed, by employing *Shepherd's lemma* (1953) and merely taking partial derivatives of any specification of the Translog, an appropriate set of share equations, which are closely related to derived demand equations, can be obtained. Those share equations together with the cost function constitute a system (of seemingly unrelated (SUR)[15]) equations. The form of the Translog cost function, the mechanics of *Shepherd's lemma*, the form of

[13]Further conditions of duality are: that the cost function is linearly homogenous for produceable outputs and strictly positive input prices; is strictly monotonically increasing in outputs; and is differentiable (and therefore continuous) with respect to both input prices and output quantities (Baumol, 1977, pp366-367 and Diewert, 1982, pp554-555).

[14]Obtaining demand functions is usually desirable given that many features of technology may be characterised by the derivatives of those functions.

[15]The differences between a SUR system and a conventional economic model, and appropriate methods for estimating SUR systems are discussed in greater detail in Chapter four.

the derived cost shares, the relation of those cost shares to derived demand equations and why the cost function together with cost shares constitutes a system of equations, is explored below. A further issue addressed below is the different constraints which must be imposed, due to either duality or production theory, on the system and those which may be imposed on the system by the econometrician wishing to test different hypotheses.

3.2.1 COMPONENTS OF THE SYSTEM

The Translog cost function can be interpreted as a second-order Taylor series approximation to an arbitrary twice differentiable cost function which is linearly homogenous in factor prices (Berndt, 1991, p469; Chambers, 1988, p180 and Greene, 1993, p504). The non-homothetic form of the function augmented for analysis of returns to scale can be expressed as:

$$\ln C = \ln\alpha_0 + \alpha_y \ln y + \frac{1}{2}\gamma_{yy}(\ln y)^2 + \sum_i \alpha_i \ln p_i + \frac{1}{2}\sum_i\sum_j \gamma_{ij} \ln p_i \ln p_j + \sum_i \gamma_{iy} \ln y \ln p_i \qquad 3.1$$

where C is total costs, y is output and p_i is the price of input i $(i = 1,\ldots,n)$. Two common variations of the Translog cost function involve the imposition of constant returns to scale *a priori*; and the incorporation of regressors reflecting technical change. Should returns to scale be assumed to be constant *a priori* , the third term and the last term in equation 3.1 would be dropped and the parameter on the second term (α_y) would be constrained to unity[16]. The function can be augmented to simultaneously model scale effects and fairly general types of technical change (Christensen (1977) (cited in Greene, 1983, p128) and Nadiri (1982)[17]):

$$\ln C = \ln\alpha_0 + \alpha_y \ln y + \frac{1}{2}\gamma_{yy}(\ln y)^2 + \sum_i \alpha_i \ln p_i + \frac{1}{2}\sum_i\sum_j \gamma_{ij} \ln p_i \ln p_j + \sum_i \gamma_{iy} \ln p_i \ln y + \alpha_t t + \frac{1}{2}\gamma_{tt} t^2 + \gamma_{yt} t \ln y + \sum_i \gamma_{it} t \ln p_i \qquad 3.2$$

[16]As shown in, Chapter two, economies of scale are measured by the reciprocal of the cost flexibility ratio: $\varepsilon = 1 \Big/ \frac{\partial \ln C}{\partial \ln y}$. Differentiating equation 3.1 partially with respect to the log of output would yield $\alpha_y + \gamma_{yy} \ln y + \sum_i \gamma_{iy} \ln p_i$. If the conditions for constant returns to scale were imposed ($\gamma_{iy} = 0 \forall i = 1,\ldots,n$; $\gamma_{yy} = 0$ and $\alpha_y = 1$) then cost flexibility ratio would equal one and hence economies of scale would be constant.

[17]Nadiri's specification differs slightly from Christensen. Instead of including time in its simple form, Nadiri uses a log specification.

This specification:[18] has been criticised by Greene (1983, p 128) on the grounds that it implicitly assumes that the efficient scale grows at a constant rate over time[19]. The restriction is accepted here on the grounds that the modeling of technical change (and changes in the efficient scale) is ancilliary to the task at hand: estimating the magnitude of economies of scale and elasticities of substitution. Technical change was modelled only after persistent violation of the concavity requirements for the specification given by equation 3.1, which implicitly assumes a constant state of technology, suggested the possibility that assuming constant technology may amount to a model misspecification.

'Patterns of producer behaviour can be described most usefully in terms of the behaviour of the derivatives of demand and supply functions' (Jorgensen, 1986, p1844). Following Hicks (1963) substitution effects, for example, can be specified in terms of the response of patterns of demand to changes in input prices (Jorgensen, 1986, p1844). Possibly the most important advantage of the dual approach is that demand and supply functions may be generated without imposing arbitrary constraints, which characterise the traditional approach, on production patterns. A further fundamental advantage of the use of a cost function in empirical analysis '... lies in its computationally simple relation to the cost minimising input demand functions' (McFadden, 1978, p3). This advantage is a product of *Shepherd's lemma* (1953) (Diewert, 1982, p574 and Fuss et al, 1978, p229). According to this lemma if a cost function satisfies the regularity conditions outlined earlier and in addition is differentiable with respect to input prices, then the derived demand for each input can be obtained by partially differentiating the cost function with respect to the factor price of that input (Nadiri, 1982, p467). Two important consequences flow from this lemma. Firstly, systematic investigations of cost minimising firms can be undertaken without having to establish the corresponding production function (Chambers, 1988, p66). Secondly, it is no longer necessary to derive input demand functions from the production function using Lagrangian techniques (Diewert, 1982, p547).

Following Diewert (1982, p576) and expressing *Shepherd's lemma* in logarithmic form ($\partial \ln C / \partial \ln p_i$) and applying it to either of the above specifications of the Translog (3.1 or 3.2) generates cost shares of the different factors, which are linear in the parameters (see below). For the specification of the cost function given by equation 3.1, the cost share functions will assume the following form:

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[18]This specification is modified slightly from the specification appearing in Greene (1988). In Greene (1983, p127) the term $\gamma_{yt} t \ln y$ appears as $\gamma_{yt} \ln yt$. Analysis of the rest of that paper suggests that the term $\gamma_{yt} \ln yt$ should in fact have been written as either $\gamma_{yt} \ln y.t$ or $\gamma_{yt} t \ln y$ to avoid it being confused for $\gamma_{yt} \ln(yt)$.

[19]This result is developed in section 3.4.3.

$$\frac{\partial \ln C}{\partial \ln p_i} = \alpha_i + \gamma_{iy} \ln y + \sum_j \gamma_{ij} \ln p_j \qquad 3.3$$

For the second specification incorporating technical change, the cost share functions for the different factors of production are given by[20]:

$$\frac{\partial \ln C}{\partial \ln p_i} = \alpha_i + \gamma_{iy} \ln y + \sum_j \gamma_{ij} \ln p_j + \gamma_{it} t \qquad 3.4$$

The reason why the logarithmic version of *Shepherd's lemma* yields cost share functions, as opposed to derived demand functions, and the relationship between derived demands and cost shares is premised on the fact that the logarithmic version of *Shepherd's lemma*, being a logarithmic derivative, can be alternatively expressed as

$$\frac{\partial \ln C}{\partial \ln p_i} = \frac{p_i}{C} \cdot \frac{\partial C}{\partial p_i} \qquad 3.5$$

The cost of input i is given by $p_i x_i$, and therefore the cost share is given by:

$$S_i = \frac{p_i x_i}{C} \qquad 3.6$$

where $C = \sum_{i=1}^{n} p_i x_i$. Now, x_i is also the derived demand for input i, and therefore, by *Shepherd's lemma*

$$x_i = \partial C / \partial p_i \qquad 3.7$$

Substituting 3.7 into 3.6 yields

$$S_i = \frac{p_i x_i}{C} = \frac{p_i}{C} \cdot \frac{\partial C}{\partial p_i} = \frac{\partial \ln C}{\partial \ln p_i} \qquad 3.8$$

As is the case with a number of other issues regarding flexible dual cost functions in general and the Translog cost function in particular, the literature is annoyingly ambiguous on the reasons why either the cost share equations or the cost share

[20]Should Nadiri's (1982) specification, referred to in footnote 17 above, which uses the logarithim of time instead of time, be used, then the last term of this equation would involve $\ln t$ instead of t.

equations together with the cost function constitute a system of equations[21] which need to be estimated simultaneously. All applications consulted either implictly assume, or explicitly state without adequate explanation, that the cost share equations, either with or without the cost equation appended, constitute a system of equations which need to be estimated simultaneously. This assumption is employed despite the fact that for all applications of the Translog, the cost equation contains all the parameters that appear in the share equations and the computational burden of estimating the cost function (a single equation) is considerably less than that of estimating either of the systems.

The reason why the cost share equations taken together constitute a system of equations would, nonetheless, appear to be intuitively obvious. The features of technology can be described using a number of different analytical devices, and production functions, dual cost, profit or revenue functions or derived demand functions are examples (Fuss et al. 1978, p266). Were one to use a factor-demand approach, a comprehensive analysis of the nature of production could only be achieved by using full sets of factor-demand equations. Single factor-demand equations would provide information only on the features of technology relating to each factor independently. Considering all share equations simultaneously would therefore provide more complete information. Not only do different share equations contain different information regarding each input individually[22] but regarding all factor demand equations as a system[23] would provide information relevant to the use of several inputs simultaneously and would thus provide additional information on the characteristics of technology.

The reason why in certain contexts the cost equation together with the derived demand equations constitute a system of equations is also intuitively reasonable. The magnitude of economies of scale is explicitly determined by the relationship between output and costs. That information is contained in the cost function. While the set of derived demand equations constitute a complete set of information in the case where scale is constrained to being constant they will not constitute a complete set when economies

[21]Which system is employed in a specific context is determined by *a priori* expectations regarding economies of scale. Which system would be appropriate in different contexts concerning scale is discussed in section 3.4.1 and 3.4.2.

[22]As witnessed by the fact that in the case of the Translog different share equations contain unique parameters: each share equation contains the parameters α_i and γ_{iY}, where i refers to the ith input.

[23]Not a truely simultaneous system but a seemingly unrelated system (SUR) where the component equations of the system are related by virtue of the fact that an exogenous shock to one of the equations of the system will also affect other equations in the system.

of scale are variable. In that case the cost function appended to the set of derived demand equations will constitute a complete system.

While it is intuitively clear why, when economies of scale are constant, a set of derived demand equations; and when economies of scale are variable, the set of derived demand equations together with the cost equation, constitute a SUR model, why estimation of such a set as opposed to merely the cost equation (which contains the same set of parameters) is advocated in the literature, is not obvious. Indeed, given that all the parameters which appear in the cost share equations appear also in the cost function, direct estimation of the cost function would appear to be desirable, particularly from the persepective of computational cost. Berndt (1991, p470) and Christensen and Greene (1976, p622) argue that such an approach would, however, not be efficient as it neglects the additional information contained in the cost share equations. What the source of the additional information is is not, however, clear from either Berndt (1991) or Christensen and Greene (1976) nor is it *prima facie* obvious. Intuition would suggest that the additional information emerging from the system stems from the fact that a number of parameters appear in more than one of the equations of the system. By obtaining estimates from the cost equation alone, other influences on parameters would be ignored.

3.2.2 CONSTRAINTS ON THE SYSTEM.

The Translog system is a product of duality theory which imposes a number of *a priori* constraints on the parameters of the Translog system. While these constraints amount to untestable maintained hypotheses they do not compromise the flexibility which makes the Translog an attractive form to estimate. Indeed, these maintained hypotheses are either universally acceptable axioms or are regarded as plausible for the problem at hand. A further set of constraints may be imposed at the discretion of the econometrician, and as a consequence are testable. This second set consists of less tenable hypotheses concerning the nature of technology, such as homotheticity and homogeneity, which are often embodied in less flexible representations of technology. That such hypotheses can be imposed and tested when the Translog is employed constitutes a fundamental advantage of this functional form. Not only does it allow the econometrician to test the consistency of these hypotheses with the data being employed but it also allows an analysis of the implications of these restrictions on the various features of technology, such as economies of scale (Christensen & Greene, 1976, p661). Both sets of constraints are described here.

3.2.2.1 *A PRIORI* IMPOSED CONSTRAINTS

The duality between cost and production functions is premised on a cost function fulfiling certain regularity conditions[24]. Some of these regularity conditions can be imposed on the Translog model via parameter constraints, while the remainder are testable using parameter estimates (Jorgensen, 1986, p1889). Three parameter constraints need to be imposed *a priori*: cost exhaustion (i.e. that the cost shares of different factors sum to unity); linear homogeneity of the cost function in input prices; and parameter symmetry. Monotonicity and concavity of the cost function are testable using parameter estimates. Testing for these two conditions is discussed below when AESs are discussed.

An important consequence of employing *Shepherd's lemma* is that cost shares of the different factors will always sum to unity ($\sum_i S_i = 1$), i.e. costs are exhausted (Jorgensen, 1986, p1890). This `adding up condition', which is intuitively appealing, has important implications for the econometric estimation of the model. In the case of a three-input specification, for example, only two of the factor share equations can be regarded as statistically independent (Diewert, 1982, p576). Estimation of the system (that is, either of the cost functions 3.1 or 3.2 and the corresponding set of share equations 3.3 or 3.4) requires the dropping of one of the share equations. The requirement that one share equation needs to be dropped is important for choice of econometric technique: the method of estimation needs to be invariant to which share equation is dropped[25].

The second constraint which needs to be imposed *a priori* is linear homogeneity in input prices. Thus if all input prices increase in the same proportion while output remains constant, total costs should increase in the same proportion. Given that the cost share equations are first-order partial derivatives of the cost function the requirement of linear homogeneity of the cost function is equivalent to the requirement that the cost share equations be homogenous of degree zero in input prices[26]. Linear homogeneity of the cost function is fulfilled when the following relationships among parameters is imposed on the system (Berndt, 1991, p469 and Christensen & Greene, 1976, p660):

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[24]See footnote 3.

[25]Estimators which are not invariant to which share equation is deleted are open to abuse by the econometrician seeking to support a particular hypothesis. The issue of invariance of estimators is discussed in greater detail in Chapter 4.

[26]This follows from the general result that the n^{th} order partial derivatives of a function homogenous of degree ρ are themselves homogenous of degree $\rho - n$ (Lancaster, 1968, p335).

$$\sum_i \alpha_i = 1 \qquad 3.9$$
$$\sum_i \gamma_{ij} = 0 \qquad 3.10$$
$$\sum_i \gamma_{iy} = 0 \qquad 3.11$$

When the Translog system is estimated, these constraints are fulfilled by normalising the prices of the factors represented by the share equations which remain in the model to be estimated in terms of the price of the factor whose share equation is dropped from the system (Greene, 1993, p505).

The third set of constraints which need to be imposed *a priori* are the following symmetry constraints:

$$\sum_i \gamma_{ij} = \sum_j \gamma_{ji} = 0 \qquad 3.12$$

The imposition of these cross-equation parameter constraints is crucial for ensuring that the model corresponds to the underlying theory (Jorgensen, 1986, p1890). While these theoretical restrictions are testable, the meaning of the model may be ambiguous should these constraints be violated (Greene, 1999, p499). The imposition of cross-equation symmetry constraints does however considerably reduce the number of parameters, hence conserving degrees of freedom and possibly eliminating problems of multicollinearity (Fuss et al, 1978, p229).

The need to impose certain constraints *a priori* has two important implications for estimating the Translog system and hence choice of estimator. Linear homogeneity in input prices and cost exhaustion together mean that one should estimate the Translog system by dropping one of the cost share equations and then normalising both total costs and the prices of the factor inputs in the price of the variable represented by the dropped share equation. The symmetry requirements will be fulfilled if in addition to the above the parameter constraints implied by 3.12 are imposed.

3.2.2.2 CONSTRAINTS FOR TESTABLE HYPOTHESES

A number of additional, empirically testable, constraints may be imposed on the cost function. These constraints, which are manifestations of a series of restrictive hypotheses concerning the underlying technology, highlight a fundamental advantage of employing a flexible functional form. Because different hypotheses can be imposed by parameter constraints their validity for a particular set of data can be tested using nested hypotheses. Rather than imposing *a priori* restrictions (as would be the case if either a Cobb-Douglas or CES function were employed) the econometrician is able to test the validity of different hypotheses and impose those which are appropriate. In addition, the imposition of constraints also allows the econometrician to analyse the implications of different maintained hypotheses for various aspects of technology, such as economies of scale (Christensen & Greene, 1976, p661). Hypotheses which are testable in the case of the Translog are: homotheticity, homogeneity in output, linear homogeneity in output, and Cobb-Douglas technology [that is, constant unitary elasticity of substitution and returns to scale which are either constant (the function is linearly homogenous in output) or variable (and the function is homogenous of a degree greater than one)[27]].

'A cost function corresponds to a homothetic production structure if and only if the cost function can be written as a separable function in output and factor prices' (Greene & Christensen, 1976, p661). Mathematically this implies that the Translog cost function will be homothetic if, and only if (Berndt, 1991, p470):

$$\gamma_{iy} = 0 \qquad 3.13$$

A homothetic production structure is further restricted to being homogenous in output if, and only if, the elasticity of cost with respect to output is constant. Mathematically, this implies that the underlying production function will be homogenous in output if in addition to the homotheticity constraint 3.13, the following holds[28]:

$$\gamma_{yy} = 0 \qquad 3.14$$

The reasoning behind this constraint is clear. The elasticity of the Translog cost function with respect to output is:

$$\partial \ln C / \partial \ln y = \alpha_y + \gamma_{yy} \ln y + \sum_i \gamma_{iy} \ln p_i . \qquad 3.15$$

This expression will be a constant if, and only if $\gamma_{iy} = 0$ and $\gamma_{yy} = 0$. $\gamma_{iy} = 0$ is the condition for homotheticity.Hence, linear homogeneity will be imposed if in addition to the homotheticity condition $\gamma_{yy} = 0$. Testing for homogeneity will be nested within a

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[27]Being homogenous the Cobb-Douglas function is also automatically homothetic. Indeed every homogenous function is also homothetic (Chiang, 1984, p423 and Madden, 1986, p240).
[28]The arguements developed below supporting the homogeneity constraints are based on the Translog cost specification given by 3.1. What the constraints would be, and why this is the case for the Translog specification given by 3.2 are presented in footnote 29.

test for homotheticity (Nadiri, 1982, p467) which follows from the general result that any homogenous function is also homothetic.

A more restrictive form of homogeneity, linear homogeneity (or constant returns to scale), may be imposed on the Translog system by imposing, in addition to the homotheticity and general homogeneity constraints, the constraint

$$\alpha_y = 1 \qquad 3.16$$

The reason why this constraint is required for the imposition of constant returns to scale is clear. A function is linearly homogenous if the elasticity of cost with respect to output equals one[29].

A variable returns to scale Cobb-Douglas technology (that is where the cost function is homogenous in output and elasticities of substitution are unitary and constant) may be imposed on the system if in addition to constraints 3.13 and 3.14 $\gamma_{ij} = 0$ is imposed (Berndt, 1991, p470). A constant returns to scale Cobb-Douglas technology may be imposed if in addition $\alpha_y = 1$ (Berndt, 1991, p470).

While the the flexibility of the Translog is possibly its most desirable feature, the fact that a number of different restrictions can be imposed with relative ease enhances the usefulness of this form considerably allowing the statistical testing of the most common maintained hypotheses and analysis of the impact of different hypotheses on the features of technology. How different features of technology are modeled by the Translog, are discussed below.

3.3 THE TRANSLOG AND SCALE, SUBSTITUTION AND TECHNICAL CHANGE

As argued above, possibly the most attractive feature of employing cost functions such as the Translog to analyse production, is the more sophisticated estimation of production characteristics afforded by these models. Indeed both partial elasticities of

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[29]Were one to employ a Translog function augmented for technical change (such as the one given by 3.2) elasticity of cost with respect to output would be:

$$\partial \ln C / \partial \ln y = \alpha_y + \gamma_{yy} \ln y + \sum_i \gamma_{iy} \ln p_i + \gamma_{yt} t$$

Homogeneity in output would be imposed on the system if, in addition to $\gamma_{iy} = 0$ and $\gamma_{yy} = 0$ being imposed, $\gamma_{yt} = 0$. Linear homogeneity in this case would be achieved if in addition to $\gamma_{iy} = 0$; $\gamma_{yy} = 0$ and $\alpha_y = 1$, $\gamma_{yt} = 0$.

substitution and economies of scale are not constrained to the same magnitude throughout the sample. Rather they are permitted to vary with the level of output.

3.3.1 ECONOMIES OF SCALE

Hanoch (1975, pp492-493) has shown that at cost minimising points:

$$\varepsilon(x) = \frac{\sum_i f_i x_i}{f(x)} = 1 \Big/ \frac{\partial \ln c(y,p)}{\partial \ln y} = 1 \Big/ \frac{\partial \ln C}{\partial \ln y} \qquad 3.17$$

which if applied to the first specification of the Translog (3.1) yields the following formula for economies of scale:

$$\varepsilon(x) = 1/[\alpha_y + \gamma_{yy} \ln y + \sum_i \gamma_{iy} \ln p_i] \qquad 3.18$$

If the Translog is augmented to be able to model technological change, then economies of scale will become[30]

$$\varepsilon(x) = 1/[\alpha_y + \gamma_{yy} \ln y + \sum_i \gamma_{iy} \ln p_i + \gamma_{yt} t] \qquad 3.19$$

Using either of these formulae clearly yield measures of economies of scale which vary as the level of output varies. Should homogeneity (and therefore homotheticity) be imposed, the scale would not vary with output. In this case, γ_{iy} and $\gamma_{yy} = 0$ and scale $\varepsilon(x)$ would be constant at $1/\alpha_y$. When the Translog is augmented to model technical change and homogeneity is imposed, economies of scale while invariant to output will nevertheless change over time. In that case scale will be the reciprocal of the sum of two terms: α_y and $\gamma_{yt}t$.

Calculation of scale in the general non-homogenous case which ignores technical change[31] requires values for three parameters: α_y; γ_{yy} and γ_{iy}. Of these, only γ_{iy} appears in the cost share equations (3.3). α_y and γ_{yy} appear in the cost equation (3.1). Estimation of the magnitude of scale would, therefore, require estimation of the cost function. While in instances where economies of scale are assumed constant (which is usually the case where elasticity of substitution is the objective of the analysis[32]) the appropriate system to estimate is the system of cost share equations, where the

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[30]Using the log of time as an index of technology, as suggested by Nadiri (1982), would obviously alter the last term of this formula to $\ln t$.
[31]The argument developed here is easily extended to the case where technical change is included.
[32]See for example Berndt & Wood (1975) and Berndt & Christensen (1973b).

objective of the analysis is the magnitude of scale the appropriate system should include the cost function as well (Berndt, 1991, p476).

3.3.2 ELASTICITIES OF SUBSTITUTION AND CONCAVITY TESTING

In the n-factor case only the concept of partial elasticities can be defined, for which no single definition exists. Elasticity of substitution for the two input case is defined subject to the constraint that the level of output remains constant. Generalistion of the elasticity concept to the n-input case requires in addition the imposition of further constraints. Depending on the constraints imposed a number of different elasticities exist (Griliches and Ringstad, 1971, p6). Computationally the obvious choice here is the AES. Not only is it relatively simple to obtain estimates directly from the cost share estimates and fitted values, but the AES is also involved directly in testing of local `good behaviour' and hence their calculation is imperative. Uzawa (1962) has shown that own and proper AESs can be obtained using the following formulae:

$$\sigma_{ii}^{A} = \frac{\gamma_{ii} + S_i^2 - S_i}{S_i^2} \text{ for } i = 1,\ldots,n \qquad 3.20$$

$$\sigma_{ij}^{A} = \frac{\gamma_{ij} + S_i S_j}{S_i S_j} \text{ for } i, j = 1,\ldots,n \text{ and } i \neq j \qquad 3.21$$

where S_i is the cost share of the i^{th} factor. These formula highlight an important advantage of estimating a flexible functional form such as the Translog: elasticities of substitution (like economies of scale) are not constant but rather vary with output. This is because they depend directly on the fitted values of cost shares, which in turn are a function of the level of output.

Where estimates of AESs are the only feature of technology of interest and economies of scale are assumed to be constant[33], all the information required for the compuation of AES (γ_{ii}; γ_{ij}; S_i and S_j) can be obtained form the share equations (the γ_{ij} parameter appears in both the cost function and the share equations). Hence, if one were interested only in elasticities of substitution, a system of share equations is all that one need to estimate (see for example Berndt & Wood (1975) and Cluver (1981)).

[33]This is likely to be the case in two instances: when elasticity of substitution is of direct concern (see for example Berndt & Wood (1975)) or when the issue of separability of inputs and the question of consistent aggregation of inputs is of interest (see for example Berndt & Christensen (1973b). Berndt and Christensen (1973a, pp403-409) show that separability (and hence the question of consistent aggregation) is closely related to equality constraints on AESs (Allen, 1938, p504).

Apart from providing measures of the degree of substitutability of inputs AESs have two other important uses. Firstly, a close relationship between AES and price elasticities exist which means that a slight modification of the own and proper AES formulae will yield formulae for own and proper price elasticities. The second important use of AESs is in testing whether the regularity conditions of monotonicity and concavity of the cost function are fulfilled for the data set being used.

AESs are related to price elasticities and cost shares in the following manner (Allen, 1938, p508):

$$\sigma_{ij}^{A} = \frac{\eta_{ij}}{p_i x_i / C} \qquad 3.22$$

where η_{ij} is the partial elasticity of factor j with respect to the price of factor i and the denominator is the share in costs of the ith factor S_i. Rearranging to make η_{ij} the subject of the formula, yiedls:

$$\eta_{ij} = \sigma_{ij}^{A} S_i \qquad 3.23$$

Using this result (3.23) and equations 3.20 and 3.21, one is able to calculate partial price elasticities directly from estimators generated from the Translog function and associated cost shares:

$$\eta_{ij} = \frac{\gamma_{ij} + S_i S_j}{S_j}, \text{ for } i,j = 1,\ldots,n, \text{ but } i \neq j \qquad 3.24$$

$$\eta_{ii} = \frac{\gamma_{ii} + S_i^2 - S_i}{S_i}, \text{ for } i = 1,\ldots,n \qquad 3.25$$

The second ancillary use of AESs is the most important. The derivation of the Translog cost function (and other general flexible cost functions) is based on a mathematical theorem concerning the dual[34] of a concave program (Madden, 1986, p265). As argued above (Section 3.3.1.1) Diewert (1982, pp554-555) and Baumol (1977, pp366-367) show that, according to this theorem, in order for a cost function

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[34]The term duality is often erroneously interchanged with *Shepherd's lemma* (Shepherd 1953), which is used to derive input demand equations from the cost function. The lemma is actually a corrollary to the theorem upon which dual cost functions are based (Diewert, 1982, p555 and Madden, 1986, p265).

to be a *sufficient statistic* for a production function[35], the cost function will have to be, inter alia[36], monotonically increasing and concave[37] in input prices. Violation of the concavity requirement is particularly problematic. Not only does it amount to a violation of one of the basic behavioural postulates of neo-classical production theory: that the entrepreneur is a cost minimiser, but, more importantly, the cost function may not have the postulated structural relationship to the underlying technological parameters (Fuss et al, 1978, p266 and Lau, 1978, p411). The parameter estimates derived in situations where concavity is violated should, therefore, be treated with considerable circumspection.

Unfortunately, for an arbitrary set of parameters the Translog cost function will not necessarily satisfy the monotonicity or concavity requirements, either locally or globally (Lau, 1978, p411). There are two simple tests for establishing the monotonicity and concavity requirements: the Translog cost function is monotonically increasing in input prices if the fitted factor shares are all positive and it is strictly quasi-concave if the $n \times n$ matrix of AESs is negative semi-definite[38] at each observation (Berndt, 1991, p477).

The only previous application of the Translog cost function found in the South African literature, Cluver (1981) (and Cluver and Contogiannis (1984) which is based on Cluver (1981)), would appear to violate the requirements that the cost function be monotonically increasing and concave (or quasi-concave) in input prices. No explicit testing of these two requirements appears. While Cluver (1981, p104) does present AESs for each of the sectors he analyses, he presents only a single AES[39] for each sector. Whether the AESs presented are for a particular point in time or are a mean of those for different years is unclear. Given these AESs one is unable to conclude whether the concavity requirements have been fulfilled or not. As a consequence, one

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[35]That is to completely describe a production function which is consistent with the `minimum' assumptions required for neo-classical production theory, i.e. that the function be real-valued, continuous, increasing and quasi-concave (McFadden, 1966, cited in Diewert, 1982, p553-554).
[36]Further conditions of duality are: that the cost function is linearly homogenous for produceable outputs and strictly positive input prices; is strictly monotonically increasing in outputs; and is differentiable (and therefore continuous) with respect to both input prices and output quantities (Baumol, 1977, pp366-367 and Diewert, 1982, pp554-555).
[37]Berndt (1991, p477) argues that theory requires that the cost function merely be quasi concave in input prices.
[38]A matrix $[B]$ is negative semi-definite if $|B_1| \leq 0, |B_2| \geq 0, \ldots |B_n| \leq 0$ if n is odd and $|B_n| \geq 0$ if n is even.The matrix will be negative definite if the weak inequalitites are replaced by storng inequalities (Chiang, 1984, p394). The precise form of the test of negative semi-definiteness of the matrix of AESs for the three input case is given in Appendix 3.
[39]AESs, as argued above, vary with the level of output. For time series applications a different AES for each observation in the sample should be generated.

is unable to determine whether the conditions required for the application of the duality theorem, upon which the validity of the Translog as a sufficient statistic for modelling the dual production function is based, are fulfilled. Given that the violation of concavity requirements may lead to the collapse of the theoretical relationship between the dual cost function and the underlying technological parameters, Cluver's (1981) results should be treated with due care.

Although the results presented by Cluver (1981) allow for calculation of fitted shares and thus all own and proper AESs, which would enable testing the monotonicity and concavity requirements, this has not been undertaken here. That the issue of concavity is not addressed in Cluver's (1981) application is nevertheless a serious flaw in the thesis. A further potential flaw in that thesis, the possible lack of invariance of the estimator employed, is addressed in Chapter 4.

3.3.3 MODELLING TECHNICAL CHANGE

The specification of the Translog given by equation 3.1 implicitly assumes that the state of technology is constant over the period under analysis. Persistent violation of the concavity requirements for estimators of that specification suggested the possibility that the assumption of constant technology may amount to a model misspecification. Rather than being non-concave, it is possible that the violation of the concavity requirement is due to the cost function shifting over time. To test this hypothesis an alternative form of the Translog, which does not constrain technical change to being constant, was also estimated. This alternative specification (3.2) which was developed by Christensen (1977) includes time (as a proxy for technology) in various forms. Christensen's (1977) specification of the Translog is quite general in the types of technical change it can accomodate. Indeed, simple parameter restrictions can be employed to impose a number of special cases of technical change on the underlying production structure (Greene, 1983, p127).

Because the modelling of technical change is ancillary to the main purpose of this dissertation, serving to assist the modelling of scale and substitution effects rather than being an area of interest in its own right, alternative approaches to modelling technical change and the impact of different maintained hypotheses on the modelling of technical change were not discussed in Chapter 2. `Technical change', although referring to the general effects that technological advances have on the production process, is measured in a plethora of different ways. Before interpreting the additional terms appended to the original Translog in the specification given by equation 3.2, a brief synopsis of some of the relevant alternative measures of technical change is provided below.

3.3.3.1 MEASURING TECHNICAL CHANGE

The theoretical analysis of technical change tends to focus on two broad areas: its nature and source (Fuss et al, 1978, p221). Attempts to measure technical change employing econometric techniques tend to focus primarily on the nature of technical change. Analysis of the source of technical change would appear to be largely limited to the theoretical literature. A common feature of the analysis of technical change is the extensive use of taxonomies to describe different aspects of the two focus areas: source and nature. Regarding the source of technical change, analysis is concerned with two broad areas: whether technical advances emerge from within or outside of the firm; and whether technical advance involves changes in the nature of inputs used in the process of production and hence whether the source of technical advance is new inputs. Analysis of the nature of technical change is concerned with measuring whether it may be deemed progressive or regressive, which aspect of technology may have been altered due to technical advances, and whether or not there is bias.

3.3.3.1.1 SOURCES OF TECHNICAL CHANGE

Regarding the sources of technical change, a broad distinction may be drawn between endogenous and exogenous technical change. Technical change is endogenous if it originates within the firm, for example as a result of learning-by-doing or innovation (Nadiri, 1982, p445). Of ancillary interest, in this regard, is what motivates endogenous technical change. Hicks (1963), for example, has proposed an *induced-invention hypothesis*: that technical change is a response to market phenomena such as relative price changes (Chambers, 1988, p204).

Closely related to the endogeneity-exogeneity dichotomy is the embodiment hypothesis which in a sense straddles the two focus areas of the theoretical literature: source and nature. Technical change may be embodied in a factor of production (usually capital[40] but possibly other factors, most likely skilled labour). When technical change is not embodied in any particular input or group of inputs, it is referred to as disembodied. Where technical change is embodied the basic form of the production function will change over time.

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[40]A classical example of embodied technical change is Eli Witney's cotton gin. The technical innovation was embodied in the gin in the sense that the gin had to be acquired to have access to the new technology (Chambers, 1988, p205).

3.3.3.1.2.THE NATURE OF TECHNICAL CHANGE

Technical change is deemed progressive if it expands the input requirement set, that is, if it allows input bundles formerly incapable of producing a certain level of output to produce that level of output. Graphically, progressive technical change would be depicted by an inward shifting of isoquants. Technical change is regressive if it reduces the input requirement set by eliminating bundles capable of producing a given level of output. Regressive technical change would be depicted graphically by an outward shift of isoquants. Such change may not be intuitively appealing. A more restricted version, locally regressive technical change, is however more appealing. Technical change is locally regressive if the new technology involves very intensive committal of certain inputs but not others. Graphically this would involve a rotation of isoquants toward the axis measuring the input being used more extensively as a consequence of technical advance, rather than a shift of isoquants (Chambers, 1988, p206-207).

Interest in which aspect of technology has been altered by technical change usually results in considering whether technical change is factor -, scale - or substitution - augmenting (Fuss et al, 1978, p221). Technical change is deemed factor-augmenting or input-augmenting if it improves input efficiency and therefore the effective quality of inputs (Chambers, 1988, p210 and Fuss et al, 1978, p221). Scale augmenting change expands the level where decreasing returns set in, while substitution augmenting change improves the substitutability of inputs.

The final taxonomy which exists regarding the nature of technical change is that associated with its bias. That technical advance may lead to the displacement of resources is widely conceded (Chambers, 1988, p203). When new production techniques are employed they will either have a neutral effect on the production process or alter the input-output relationship and thus be regarded as biased (Nadiri, 1982, p444). Identifying the nature of technical change is often regarded as being synonymous with characterising its bias. Indeed, technical progress is often defined according to its input bias. In this regard a number of different definitions of technical change exist, the most familiar being the Hicks, Harrod and Solow characterisations of technical progress (Nadiri, 1982, p444). All three are concerned with how the marginal rate of technical substitution among inputs changes over time. The differences emerge from which aspect of technology is held constant while changes in the marginal rate of technical substitution over time are analysed. In the Hicksian definition, the capital-labour ratio is held constant and change in the marginal rate of technical susbstitution over time is used as a measure of technical bias (Chambers, 1988, p207). Formally, for the two factor case Hicksian technical change is defined as:

$$\left.\frac{\partial(f_K/f_L)}{\partial t}\right|_{\substack{K/L \\ constant}} \begin{matrix} > \\ = \\ < \end{matrix} 0, \qquad Hicks\begin{cases} labour-saving \\ neutral \\ capital-saving \end{cases}$$

The Harrod definition of technical change holds the capital-output ratio constant and can be formally expressed as (Nadiri, 1982, p444) :

$$\left.\frac{\partial(f_K/f_L)}{\partial t}\right|_{\substack{K/Y \\ constant}} \begin{matrix} > \\ = \\ < \end{matrix} 0, \qquad Harrod\begin{cases} labour-saving \\ neutral \\ capital-saving \end{cases}$$

The Solow definition holds the labour-output ratio constant and can be formally expressed as (Nadiri, 1982, p444) :

$$\left.\frac{\partial(f_K/f_L)}{\partial t}\right|_{\substack{L/Y \\ constant}} \begin{matrix} > \\ = \\ < \end{matrix} 0, \qquad Solow\begin{cases} labour-saving \\ neutral \\ capital-saving \end{cases}$$

Given the above lexicon of characterisations of technical change it is clear that the empirical analysis of technical change will in all likelihood be constrained in the number of effects which can be modelled by the need to adopt a specific functional form. The most common approach to extending an analysis to include the effects of technical change is to append a time term to the function being used to model the characteristics of technology[41]. While the simple inclusion of a time term would appear to be relatively innocuous it entails the adoption of a number of limiting assumptions. Not only does it ignore the source or motivation for technical change but it also involves the tacit assumption that technical change is disembodied, that is that technical change is not embodied in new inputs. The approach which is adopted here is more sophisticated than merely appending a simply time term to the function. The precise implications of the terms appended to the Translog cost function are, however, not clear from the literature.

[41] The obvious motivation being that technical advances usually require the passage of time (Chambers, 1988, p204).

3.3.3.2 THE TECHNOLOGY AUGMENTED TRANSLOG

Following Christensen (1977) technical change has been modelled here by appending to the cost function (3.1.) a simple time variable, a time-squared variable, the product of time and the log of output and the sum of the product of the log of the prices of the different factors and time: $\left[\alpha_t t;\ \frac{1}{2}\gamma_{yt}t^2;\ \gamma_{yt} t \ln y \text{ and } \sum_i \gamma_{ti} t \ln p_i\right]$ Because the cost-share equations are obtained by employing *Shepherd's lemma* and differentiating the Translog cost function with respect to the log of the price of the different factor inputs, the cost share equations will also be modified somewhat. In particular, each of the cost share equations will have an additional term, time: $\gamma_{it} t$. By modifying the Translog system in this manner, a number of different types of technical change can be accommodated. Moreover a number of special cases of technical change can be imposed on the system via simple parameter restrictions (Greene, 1983, p127). Unfortunately, which types of technical change can be modelled by the modified Translog is not clear from the literature. The model was developed in an unpublished paper (Christensen, 1977) and while it is presented in other sources (Berndt, 1991; Berndt and Wood, 1982 and Greene, 1983) the precise types of technical change which can be modelled, when scale is not constrained to being constant, are not clear[42]. The literature does, nevertheless highlight three aspects of technical change which can be analysed using the model: whether or not technical change is regressive or progressive; the factor bias of technical change and the impact of technical change on efficient scale. Furthermore, it is apparent from the literature that when using the Translog cost function, one is unable to test for the possible endogeneity of technical change such as that suggested by Hicks's induced-innovation hypothesis.

For ease of exposition these different features the specification of the Translog which incorporates technical change and the associated cost share equations are repeated here as equations 3.24 and 3.25, respectively:

$$\ln C = \ln \alpha_0 + \alpha_y \ln y + \frac{1}{2}\gamma_{yy}(\ln y)^2 + \sum_i \alpha_i \ln p_i + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \sum_i \gamma_{iy} \ln p_i \ln y + \alpha_t t + \frac{1}{2}\gamma_{tt} t^2 + \gamma_{yt} t \ln y + \sum_i \gamma_{ti} t \ln p_i \qquad 3.26$$

[42]Berndt & Wood (1982, p206) do include a fairly detailed analysis on the types of technical chnage which can be modelled if economies of scale are constrained to being constant. That analysis is not particularly useful here given that an express objective of this paper is estimation of economies of scale. Nevertheless wher scale is constrained to being constant the specification of the Translog provided by 3.2 is able to model, inter alia, Hicks, Harrod, Solow and Leontief neutral technical change.

$$\frac{\partial \ln C}{\partial \ln p_i} = \alpha_i + \gamma_{iy} \ln y + \sum_j \gamma_{ij} \ln p_j + \gamma_{it} t \qquad 3.27$$

Including time as a variable in any cost function[43] allows the easy categorisation of technical change as either progressive or regressive. If technical change is progressive $c(p_i, y, t)$ is nonincreasing in t; if technical change is regressive, $c(p_i, y, t)$ is nondecreasing in t. This result rests on the fact that progressive technical change expands the input requirement set. The mimimum cost of producing a given level of output using the expanded input requirement set can be no larger than the minimum cost using the original input requirement set since the original cost-minimizing bundle remains feasible. A broad classification of technical change as progressive or regressive in any particular context can be achievedby analysing the *rate of cost dimunition* (Chambers, 1988, p 214):

$$\lambda = \frac{\partial \ln c(p_i, y, t)}{\partial t} \qquad 3.28$$

If $\lambda \leq 0$ technical change is progressive. Technical change is regressive if $\lambda \geq 0$. Applying 3.28 and differentiating 3.26 with respect to time yields the rate of cost dimunition for the Translog cost function as:

$$\frac{\partial \ln C}{\partial t} = \alpha_t + \gamma_{tt} t + \gamma_{yt} \ln y + \sum_i \gamma_{it} \ln p_i \qquad 3.29$$

Greene (1983, p126) defines factor bias b_i as:

$$b_i \equiv \frac{\partial S_i}{\partial t} \begin{matrix} < \\ = \\ > \end{matrix} 0 \qquad \begin{cases} \text{i} - \text{saving} \\ \text{i} - \text{neutral} \\ \text{i} - \text{using} \end{cases} \qquad 3.30$$

From 3.27 it is clear that $\partial S_i / \partial t = \gamma_{it}$. Hence the qualitative bias of technical change can be determined directly from the sign of the coefficient on the time variable in each share equation. Technical change is therefore input i-using when $\gamma_{it} > 0$, input i-saving when $\gamma_{it} < 0$ and input neutral when $\gamma_{it} = 0$. Berndt and Wood (1982, p203 -204) argue that because the bias coefficients γ_{it} are constants, they do not vary over the sample and remained fixed as relative input prices vary. As a consequence, they cannot

[43]Including time in a cost function will modify the general expression for a cost function from $c(y, p_i)$ to $c(y, p_i, t)$.

be used to assess Hick's induced innovation hypothesis: that technical change is a response to market phenomena such as relative price changes.

Berndt and Wood (1982, p204) raise a second caveat regarding bias coefficients. Bias parameters, they argue, represent relative rather than absolute changes in factor demands in response to technical change. This has important implications for the interpretation of the bias coefficients. $\gamma_{it} > 0$, which implies that technical change is factor i-using, does not necessarily imply that technical change has increased the amount of factor i which is demanded in production. Factor i-using technical change could mean that while the demand for all factors was reduced through technical progress the demand for factor i was not reduced by as much as the demand for other factors.

The incorporation of technical change into the Translog in the manner described by equation 3.27 impacts on the how scale is modelled in the system. As was indicated by equations 3.18 and 3.19 above, when technical change is modelled the formula for economies of scale employed in the case when technical change is assumed constant will be altered and have a the term $\gamma_{yt}t$ added to the denominator. That is, economies of scale will be measured by:

$$\varepsilon(x) = 1/[\alpha_y + \gamma_{yy} \ln y + \textstyle\sum_i \gamma_{iy} \ln p_i + \gamma_{yt}t] \qquad 3.31$$

Following Greene (1983, p127) and solving this expression for the efficient scale, that is the level of output where average costs reaches its minimum and $\varepsilon(x) = 1$, yields:

$$\ln y^* = \frac{1 - \alpha_y - \gamma_{yy} \ln y - \sum_i \gamma_{iy} \ln p_i - \gamma_{yt}t}{\gamma_{yy}} \qquad 3.32$$

where y^* denotes the level of output where efficient scale is achieved. Partially differentiating this expression with respect to time:

$$\frac{\partial \ln y^*}{\partial t} = -\frac{\gamma_{yt}}{\gamma_{yy}} \qquad 3.33$$

An implication of this results is that the level of efficient scale changes over time by the constant[44] $-\gamma_{yt}/\gamma_{yy}$. Greene (1983, p128) argues that it seems overly restrictive to assume that efficient scale grows at a constant rate over time.

[44] $-\gamma_{yt}/\gamma_{yy}$ is a constant as it is a ratio of two parameters which are fixed throughout the sample.

CHAPTER 4

DATA, ESTIMATION AND TESTING PROCEDURES

4.1 INTRODUCTION

The Translog cost function when combined with the derived cost-share equations, as argued in Chapter 3, constitutes what can be regarded as a seemingly unrelated regression (SUR) model (Greene, 1990, p528). A model (or system) is deemed seemingly unrelated if the component equations comprising the system are connected, not because of direct interaction between equations, but rather as a result of their disturbances being correlated (Kennedy, 1985, p137; Pindyck & Rubinfeld, 1991, p308). While *prima facie* the component equations of the Translog system appear unrelated in that the variables on the left hand side do not appear anywhere in the system as regressors the reasoning why the equations comprise a `simultaneous system' is far more subtle, yet intuitively clear. An exogenous shock to the cost of capital, for example, may impact on the demand for other factors and hence affect their cost shares and as a result also total costs. SUR models are in this sense unique - while not `truly' simultaneous - the component equations are nevertheless related via their disturbances. Because of the unique structure of a SUR model, the conventional methods for estimating both a system and single equations will generally not be appropriate. A primary concern of this chapter is establishing the appropriate method for estimating SUR systems under a number of different conditions: when regressors are endogenous rather than exogenous and when the disturbances of the component equations are correlated. Two other broad areas, both of which pertain directly to estimating the parameters of a Translog system, are addressed here: econometric testing procedures and data treatment. Testing for the fulfillment of the duality conditions has already been addressed in Chapter 3[1]. Consideration is here to the appropriate methods of testing in three other areas: determining which of a number of alternative estimators is appropriate in a given context; whether the various restrictions (such as homogeneity and homotheticity) which the researcher is able to impose on the underlying technology are valid, and how good the overall fit of the model is. The final concern of this chapter is the data which will be employed in the estimation of the model.

[1]Duality is valid only if the cost function can be regarded as concave in input prices. Testing for concavity is discussed in Section 3.4.2 and Appendix 3.2.

4.2 METHOD OF ESTIMATION

A model is deemed a SUR model if the equations of the model are related through non zero covariances of the errors across different equations at a given point in time (Pindyck & Rubinfeld, 1991, p326). Should the assumptions of the classical linear regression model be fulfilled for each of the component equations of the SUR model, application of OLS, on an equation by equation basis, while yielding unbiased and consistent parameter estimates will not, however, provide efficient estimates (Greene, 1993, p488; Kennedy, 1985, p137 and Kmenta, 1986, p637). Estimating each equation seperately and independently would disregard the information contained in the cross-equation correlation of disturbances. An improvement in efficiency will be gained by explicitly taking into account the fact that cross-equation error correlations may not be zero (Kennedy, 1985, p137, Kmenta, 1986, p637 and Zellner, 1962, p353).

The use of generalised least squares (GLS), which explicitly takes into account the fact that cross-equation error correlations may not be zero, to estimate a SUR model will bring gains in efficiency unless the equations comprising the system have identical explanatory variables in which case GLS and OLS will be identical[2] (Greene, 1993, p488 and Pindyck & Rubinfeld, 1991, p310). Gains in efficiency from the use of the GLS estimator will be positively related to the degree of correlation of the disturbances among equations, and negatively to the correlation of explanatory variables (Kmenta, 1986, p642). The application of GLS is based on knowledge of the elements of the variance-covariance matrix of disturbances. A lack of knowledge of this variance-covariance matrix can be overcome by a consistent estimate of it. The application of GLS using a consistent estimator of the matrix (estimated or feasible GLS (FGLS)) will yield parameter estimators which display the same asymptotic properties as the GLS estimator (Kmenta, 1986, p643). Four alternative FGLS estimators are available for estimation of the Translog model. Zellner (1962, p348-368) has developed a two-stage estimated generalised least squares (FGLS) method for estimating a SUR, often referred to in the literature as the Zellner Efficient Estimator (ZEF). The second estimator is Zellner and Theil's (1962, p54-78) three-stages least squares (3SLS), a systems estimator in the general sense, which is the appropriate estimator of the Translog model when at least one of the regressors is endogenous. Two further estimators can be obtained by adjusting the ZEF and 3SLS estimators to take autocorrelation into account - the ZEF(AR) and 3SLS(AR) estimators. Which of the

[2] A second, obvious situation where OLS and GLS will be identical is where cross-equation covariance of disturbances is zero.

four is appropriate can, theoretically, be determined by testing for the presence of both endogeneity and autocorrelation - issues dealt with in sections 4.2.4 and 4.2.5.

4.2.1 THE ZELLNER EFFICIENT ESTIMATOR (ZEF)

Being an FGLS estimator, the ZEF estimator emerges from a two-stage procedure. The first stage of the ZEF procedure involves estimating each of the equations of the system by OLS. An explicit assumption of the first stage is that the disturbances of each equation are homoscedastic and uncorrelated with each other[3] (Zellner, 1962, p350). The residuals from this process provide a consistent estimate of the variance-covariance matrix. The second stage of the ZEF involves applying Aitken's GLS procedure to the system of equations expressed as one large equation[4] and thus generating an FGLS estimator which is BLUE and asymptotically equivalent to the GLS estimator (Zellner, 1962, pp350 - 351). For small samples the ZEF is unbiased and efficient relative to the OLS estimator (Kmenta, 1986, p644).

Prior to the application of any estimator to a model or system the stochastic structure of the model needs to be specified. Following Christensen and Greene (1976, p662) and Jorgensen (1986, p1893), additive disturbances for each of the cost share equations and the cost function[5], which are homoscedastic and independently and normally distributed, are assumed[6]. This specification has important implications for the Translog system. Because the cost share equations always sum to unity, where there are n share equations, only $n-1$ will be linearly independent. An implication of this is that for each observation, the sum of disturbances across the cost share equations will always equal zero (Berndt, 1991, p472). This means that the OLS estimated disturbance variance-covariance matrix which is generated in the first stage

[3]Violation of this assumption will lead to inefficient estimates (Greene, 1993, p498). The issues of testing for autocorrelation and how the problem of autocorrelation may be overcome are discussed below in section 4.2.5.

[4]If the system consisted of N equations of the form: $Y_i = X_i\alpha_i + \varepsilon_i$ where the subscript i refers to the ith equation, writing the system as one large equation involves expressing the system as:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_n \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

[5]As the cost share equations are obtained by differentiating the cost function, the disturbance term of the cost function is not included in the share equations.

[6]In addition, following Jorgensen (1986, p1892) it is assumed that the disturbance vector for the system has a joint normal distribution, with mean vector zero and a constant covariance matrix

of the ZEF procedure will be singular, rendering this procedure inoperational (Christensen & Greene, 1976, p663). A possible solution to this problem would be to drop one of the share equations and to apply the ZEF procedure to the remaining cost function and share equations. Parameter estimates for the deleted share equation could be found by employing the constraints imposed by the requirement that the cost function be homogenous of degree one in input prices[7]. Application of this procedure is, however, only valid if the estimator used is invariant to which share equation is dropped[8] (Berndt, 1991, p473). The ZEF will be invariant to which share equation is dropped only if no cross equation symmetry constraints are imposed (Berndt, 1991, p474). Because the imposition of these cross-equation parameter constraints is crucial for ensuring that the model corresponds to the underlying theory[9] (Jorgensen, 1986, p1890), the ZEF is obviously not appropriate.

Barten (1969) has shown that maximum likelihood estimates of a system of share equations which has one equation deleted are invariant to which share equation is dropped (Christensen & Berndt, 1973b, p89 and Christensen & Greene, 1976, p663). Christensen and Greene (1976, p663) argue that Barten's (1969) conclusion can be extended to a system which includes the cost function. Maximum likelihood estimates can be obtained by iterating the Zellner method until convergence (Greene, 1993, p493). Iterative ZEF estimates (IZEF) will thus be invariant to which share equation will be dropped, and is consequently an appropriate estimator for applications of the Translog.

The application of the IZEF is premised on the assumption that all the component equations of the system satisfy the assumptions of the classical linear regression model. The presence of endogenous regressors in any of the component equations would violate the assumption that regressors and the errors are uncorrelated[10]. (Gujarati, 1988, p556). Violation of the assumption causes the least squares estimator to be biased[11], even asymptotically (Kennedy, 1985, p126). Hence, applying OLS to the

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[7]Specifically that the following equalities hold: $\sum_i \alpha_i = 1$; $\sum_i \gamma_{ij} = 0$ and $\sum_i \gamma_{iy} = 0$.

[8]Methods which are not invariant to the share equation which is dropped are obviously open to manipulation by the econometrician who may by dropping different equations be able to support *a priori* assertions.

[9]While cross-equation parameter constraints are testable, the meaning of the model may be ambiguous should these constraints be violated (Greene, 1999, p499)

[10]An explicit assumption of the OLS method is that the explanatory variables are either non stochastic, or if stochastic, are distributed independently of the stochastic disturbance term (Gujarati, 1988, p556).

[11]The source of the bias is intuitively obvious. An exogenous shock, manifested in a sudden change in the error term will influence the dependent variable both directly and indirectly via its influence on

Translog model where the regressors are endogenous would yield estimates which were not only inefficient but also inconsistent. While the use of the IZEF in this context would improve efficiency it would not improve consistency. An estimator which explicitly deals with both endogeneity of regressors and correlation between the errors of the different equations of the model would need to be employed. 3SLS is an example of such an estimator.

4.2.2 THREE STAGES LEAST SQUARES

The intuitively obvious approach to the resolution of the bias associated with using endogenous regressors would be to remove the source of that bias: the correlation between regressors and disturbances (Gujarati, 1988, p524). Replacing endogenous regressors with instrumental variables (IVs), which are highly correlated with the regressor being replaced but uncorrelated with the errors would achieve this end. Obvious candidates as IVs are exogenous variables appearing in the model. The problem of determining which of various exogenous variables would be the best IV in a particular context prompted the development of the two stages least squares (2SLS) approach, a single equation estimator which employs IVs which are combinations of all exogenous variables (Kennedy, 1985, p134). In the first stage of the 2SLS procedure, each endogenous regressor is regressed on all the exogenous variables in the system and the fitted values retained. In the second stage, the fitted values of the endogenous regressors are employed in place of the original endogenous regressors and OLS estimators are obtained in the usual manner (Gujarati, 1988, p608 and Kennedy, 1985, p134). Being an IV estimator which purges the correlation between regressors and errors, the 2SLS estimator will be consistent (Kennedy, 1985, p134).

While in the context of the Translog model with endogenous regressors the 2SLS estimator yields unbiased and consistent parameter estimates it will not provide efficient estimates. The lack of efficiency of the 2SLS estimator stems from the fact that using 2SLS amounts to estimating each equation seperately and independently and thus information about the correlation of disturbances between equations would be disregarded An improvement in efficiency will be gained by explicitly taking into account the fact that cross-equation error correlations may not be zero, and estimating all equations in the system simultaneously - in other words by employing a GLS estimator (in combination with an instrumental variable estimator) (Kmenta, 1986,

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the endogenous regressor with which it is correlated. The OLS technique attributes both influences on the dependent variable to the regressor, rather than only the indirect effect. Hence the OLS estimator will biased, even asymptotically (Kennedy, 1985, p127).

p695). The properties of the 2SLS estimator, and how these properties can be improved (by employing GLS) in the context of endogenous regressors, mirrors the properties of the OLS estimator, and how it's properties could be improved (by employing a GLS estimator - ZEF) when the regressors of the Translog model are exogenous.

Although the theoretical application of GLS requires knowledge of the elements of the variance-covariance matrix of errors, absence of this information can be overcome by a consistent estimate of the variance-covariance matrix. The application of GLS using a consistent estimate of that matrix (FGLS) will yield parameter estimators which display the same asymptotic properties as the GLS estimator (Kmenta, 1986, p643). Three stages least squares (3SLS) is an example of a FGLS estimator which simultaneously accounts for endogeneity of regressors and inter-equation disturbance correlation, and where regressors of the Translog are endogenous would yield estimates which are consistent and asymptotically efficient (Kennedy, 1985, p136).

The 3SLS estimator involves first applying the 2SLS procedure to all the equations in the system from which the residuals are retrieved and used to derive a consistent estimate of the covariance-variance matrix of errors of the system. The errors of each of the structural equations estimated using 2SLS are assumed to be both homoscedastic and independently distributed[12] (Zellner and Theil, 1962, p55). The consistent estimator of the variance-covariance matrix is then employed in the second stage where the Aitken GLS procedure is applied to a single large equation (the same equation as that employed by the ZEF estimator) which includes all the component equations of the system. Kmenta (1986, p697) argues that the 3SLS approach takes the correlation between disturbances of the different equations into account by treating the system of equations as a SUR system. The appropriateness of this estimator for Translog systems where regressors are endogenous is obvious, improving both consistency and efficiency. Improved efficiency derives from the explicit consideration of cross equation error correlations, improved consistency from the fact that the 3SLS estimator is an instrumental variable estimator (Greene, 1993, p612).

The 3SLS method can not, unfortunately, be applied to the Translog system which includes an additive disturbance term with each of the equations. Because the cost share equations always sum to unity should there be n share equations in the system, only $n-1$ will be linearly independent. An implication of this is that the 2SLS

[12]Violation of this assumption would lead to inefficient estimators (Kmenta, 1986, p706)

estimated disturbance variance-covariance matrix will be singular, rendering 3SLS inoperational. A possible solution to this problem, which also emerges if one attempts to apply the ZEF procedure to the entire system, would be to drop one of the share equations and to apply the 3SLS procedure to the remaining cost function and share equations. Parameter estimates for the deleted share equation could be found by employing the constraints imposed by the requirement that the cost function be homogenous of degree one in input prices[13].

As has been argued above, deleting a share equation is, however, only valid if the estimator used is invariant to which share equation is dropped. Iterating the 3SLS (I3SLS) procedure[14], does not, unlike iteration of the ZEF estimator, provide the maximum likelihood estimator, nor does it improve efficiency (Greene, 1993, p612 and Kennedy, 1985, p140). Despite not being the maximum likelihood estimator I3SLS estimates are nevertheless invariant to which of the share equations are deleted (Berndt & Wood, 1975, p261 and Berndt, 1991, p474). The I3SLS estimator is therefore an appropriate estimator of the Translog model.

4.2.3 INVARIANCE OF ESTIMATORS: CLUVER (1981) REVISITED[15]

That an estimator which is invariant to which share equation is dropped needs to be chosen highlights a further potential flaw in Cluver's (1981) application of the Translog system[16]. That application used 3SLS (Zellner & Theil, (1962) (Cluver, 1981, p103)). As argued above, 3SLS would usually only be employed where regressors were endogenous: a situation which is more likely to occur when highly aggregated data is used[17]. Cluver's (1981) study is, however, concerned with relatively disaggregated data and IZEF may also have been appropriate. Indeed were the regressors exogenous IZEF would yield consistent and efficient estimates, while 3SLS, if properly applied and iterated, would yield estimates which were inefficient compared to the IZEF estimates (Berndt, 1991, p379). The potential lack of efficiency is not that serious a problem, but the fact that 3SLS appears not to have been iterated is potentially damaging.

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[13] Specifically that the following equalities hold: $\sum_i \alpha_i = 1$; $\sum_i \gamma_{ij} = 0$ and $\sum_i \gamma_{iy} = 0$.

[14] That is, using residuals from the estimated 3SLS equations to obtain new estimates of the variance-covariance matrix and then reapplying the second stage of the 3SLS process and repeating this process until there is no change in the parameter estimates (Kmenta, 1986, p700).

[15] A second potential flaw in Cluver's (1981) thesis, failure to adequately test for fulfillment of concavity conditions, has already been discussed in Chapter 3, section 3.4.2

[16] The flaw applies also to Cluver and Contogiannis (1984) which is based directly on Cluver (1981).

[17] Whether a 3SLS estimator is appropriate for the present analysis is dealt with in more detail below.

As noted earlier, iteration is necessary to yield parameter estimates which are invariant to which share equation is dropped. Whether I3SLS was employed by Cluver (1981) is not clear, since reference is made only to the use of 3SLS as developed by Zellner and Theil (1962) (Cluver, 1981, p103). Given that Zellner and Theil's (1962)development of 3SLS merely suggests the possibility of iterating the procedure without specifying the implications and method of iteration (Zellner and Theil, 1962, p78) it is probable that Cluver (1981) did not apply iteration and his results are not invariant to which share equation was dropped. This conclusion is supported by the fact that although symmetry constraints, which require the use of iterative estimators, are explicitly discussed, and presumably applied, there is no discussion of the issue of the invariance of different estimators.

Clearly, whether the regressors of the component equations of the Translog model are exogenous or not, by iterating either the ZEF or 3SLS estimators parameter estimates can be obtained which are both consistent and efficient, and invariant to which share equation is dropped. An important issue is determining which estimator would be most appropriate as incorrect choice would lead to inefficiency at best (using I3SLS instead of IZEF) or inconsistency at worst (using IZEF instead of I3SLS). The choice of which estimator (IZEF or I3SLS) is employed should be determined by the characteristics of the regressors. The choice between either the IZEF or I3SLS procedures in other applications (and `textbook' expositions) is motivated by intuition regarding the probable endogeneity of input prices[18] rather than statistical testing of that intuition (see for example Berndt (1991); Berndt & Wood (1975); Berndt & Christensen (1973), Cluver (1981) and Jorgensen (1986)). While intuition may be used to make assertions about the nature of input prices for this application of the Translog a more reliable method ought to be used to decide upon the nature of output.

The present analysis is concerned with estimating economies of scale and elasticities of substitution in specific 2, 3 and 4 digit standard industrial classification (SIC) industries. Following Berndt (1991, p460) it is argued that the level of analysis here is

[18]The Translog as originally specified has both input prices and output (in various forms) as regressors. Most applications are explicitly concerned with estimates of elasticities of substitution and hence impose constant returns to scale *a priori*. Imposing constant returns to scale reduces the appropriate system to the cost share equations alone (see section 3.3.1), where those cost share equations have only prices as regressors (output is deleted (see 3.3.1.2 for details)). Hence the nature of input prices, and not output, would need to be addressed when choosing between IZEF or I3SLS. In applications such as this, where the magnitude of scale is an explicit concern, output appears as a regressor (in both the share equations and the cost function which is included in the system) and the nature of output also needs to be addressed.

sufficiently disaggregated to assume that input prices are likely to be exogenous to the different industries who are unable to assert monopsony power in input markets. Output cannot, however, be assumed *a priori* to be exogenous. Although theoretical intuition would suggest that output is likely to be endogenous, the absence of precedents in the applied literature prompted the need to test rather than speculate upon the statistical nature of output. The theoretically appropriate method for testing the nature of output is, however, not suitable for the Translog in this application.

4.2.4 TESTING ENDOGENEITY: THE SPENCER AND BERK (1981) TEST.

Hausman (1978) has developed specification tests concerned with testing whether the OLS assumption of exogeneity of regressors (orthogonality) is violated, in a variety of different contexts (Hausman, 1978, p1251). Spencer and Berk (1981) provide a simpler version of the test which Hausman proposed for the simultaneous equation context (Spencer & Berk, 1981, p1079). Spencer and Berk's (1981) version of Hausmans test is concerned with testing the specification of the component equations of systems (Spencer & Berk, 1981, p1079). The proposed test comprises two stages. In the first stage, the suspected endogenous regressors in the equation under scrutiny are regressed on all the exogenous variables in the system, and the fitted values, which constitute instrumental variables, are retrieved. In the second stage the instrumental variables are added to the original equation and OLS is performed on this expanded equation (Berndt, 1991, p566; Greene, 1993, p618 and Kmenta, 1986, p718). The null hypothesis that the regressors under consideration are exogenous is equivalent to testing the null hypothesis that the coefficients on the instrumental variables are insignificant. While the Spencer and Berk Test will provide an indication of whether the ZEF procedure or the 3SLS procedure ought to be used, precisely which form of those estimators[19] should be employed would require testing for the presence of autocorrelation.

Application of Spencer and Berk's (1981) test to the Translog model using the samples employed here is, unfortunately, not possible. The second stage of the test could not be applied for one of two reasons: a lack of degrees of freedom or excessive multicollinearity. The degrees of freedom problem emerged when the nature of output and other regressors which are linear functions of output[20] were tested in the cost

[19] In the case of the Zellner procedure IZEF or IZEF adjusted for autocorrelation (IZEF(AR)) and in the case of 3SLS, I3SLS or I3SLS adjusted for autocorrelation (I3SLS(AR))

[20] Output appears in the system as output squared, the product of time and output and the product of different input prices and output.

function of the technical change augmented cost function[21]. The addition of the fitted values, obtained in the first stage of the test, to the appropriate equation increased the number of regressors in that equation beyond the number of observations in the sample thus eliminating all available degrees of freedom and rendering OLS impossible[22].

The multicollineairty problem emerged in all other equations where output appears as a regressor. Attempts to apply the second stage of Spencer and Berk's (1981) test to the cost functions of the homothetic, homogenous and Cobb-Douglas variable returns to scale versions of the Translog[23], and the cost share equations of the technical-change augmented and non-homothetic forms of the Translog[24] all failed due to the presence of excessive multicollinearity. Why the problem of extreme multicollinearity emerges is intuitively obvious: the IVs used in the second stage of Spencer and Berk's (1981) test are merely linear combinations of the other variables appearing as regressors in that second stage.

The failure of the Spencer and Berk (1981) test means that the most appropriate estimator cannot be determined statistically. Should output be exogenous IZEF estimators (either IZEF or IZEF(AR) depending on the nature of the errors of the component equations) would be appropriate yielding consistent and efficient estimates. Employing I3SLS estimators would yield consistent but inefficient estimators. Should output be endogenous I3SLS estimators would be appropriate yielding consistent and efficient estimators. IZEF estimates would in this case be both inefficient and inconsistent relative to the I3SLS estimates. Given that I3SLS estimates are consistent irrespective of the context in which they are applied and the only penalty for applying the I3SLS process in the wrong context is a loss of efficiency (which merely imposes the potentially beneficial result that statistical inference is more conservative) one could be tempted to merely apply I3SLS. One problem with adopting that approach here is that the sample used is very small and while I3SLS estimates will be consistent

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[21] See Appendix 5.1 for detailed functional forms of the Translog when different maintained hypotheses are imposed *a priori*.

[22] The sample being used here contains 19 observations. The cost function for the technical-change augmented specification of the Translog contains 15 regressors (including the constant). 7 of those regressors are some function of output. Were one to employ Spencer and Berk's (1981) test, fitted values for each of these 7 regressors would need to be appended to the cost function generating an equation with 22 regressors. No degrees of freedom would be available to perform OLS on that equation and the test cannot be performed.

[23] Output does not appear as a regressor in the cost function of the linear homogenous form of the Translog.

[24] Output does not appear as a regressor in the cost share equations of any of the other specifications of the Translog.

there is no guarantee of their unbiasedness. For this reason, and other arguments[25], both classes of estimator (IZEF and I3SLS) are used here. The precise form of the two classes of estimator which would be appropriate[26] was determined by testing for autocorrelation.

4.2.5 AUTOCORRELATION

The asymptotic efficiency of the IZEF and I3SLS estimators is derived from the assumption in both cases [Zellner, 1962, p350 (for IZEF) and Zellner and Theil, 1962, p55 (for I3SLS)] that disturbances of the equations estimated in the first stage are independently distributed[27] (Greene, 1993, p498 and Kmenta, 1986, p706). Berndt and Christensen (1973b, p95) citing both Durbin (1957) and Malinvaud (1970) suggest employing a conventional single equation Durbin-Watson statistic to test for the presence of autocorrelation in the component equations of the Translog model. The test, which is performed individually for the separate equations of the system, involves the computation of the test statistic using the residuals from the final stage estimates (Malinvaud, 1970, p509). The appropriate degrees of freedom for the test differ for the IZEF and I3SLS estimators. When applying the Durbin-Watson test to the IZEF estimates the number of degrees of freedom are the number of regressors in each equation and the number of observations in the sample. For the I3SLS estimates, the number of degrees of freedom are the number of exogenous variables employed in the first stage of the procedure and the number of observations in the sample (Berndt and Christensen, 1973b, p95 and Malinvaud, 1970, p509).

Should autocorrelation be detected, the estimator being employed ought to be modified to take this phenomenon into account. For the ZEF estimator, Greene (1993, p498) and Kmenta (1986, p646-647) show that by preceding the usual ZEF procedure with a stage which first estimates the coefficient of correlation (ρ) for each equation of the system and then transforms the data to remove any autocorrelation the resulting ZEF estimator would be efficient. For the I3SLS procedure, autocorrelation can be adjusted for in an analogous manner. Rather than employing 2SLS in the first stage of the 3SLS a weighted 2SLS estimator which explicitly accounts for the serial correlation of the errors of the component equations would be employed (Kmenta, 1986, p706).

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[25] See footnote 5 of Chapter 5.

[26] That is the autocorrelation-augmented version as opposed to the version which assumes that the component equations of the Translog system fulfill the assumptions of the CLR model.

[27] Because the data used here are time-series it is assumed that the errors of the component equations are homoscedastic and no testing of this hypothesis was undertaken.

According to Greene (1993) iteration of the ZEF process modified in this manner would not yield maximum likelihood estimates. Maximum likelihood estimates are important for ensuring that parameter estimates are invariant to which share equation is dropped. Given that the I3SLS estimates, although not maximum likelihood estimates are nevertheless invariant to which equation is deleted from the system, iteration of the ZEF process modified for autocorrelation ought to be invariant to which share equation is dropped. The literature is, however, conspicuously silent on this point. Despite giving considerable attention to estimating the Translog system when autocorrelation is suspected and to the issue of invariant estimators. Berndt (1991), for example, does not mention that the IZEF estimator modified for autocorrelation is not the maximum likelihood estimator nor whether or not it is invariant to which share equation is dropped. The same arguement can be applied to the I3SLS estimator adjusted to account for autocorrelation.

Applying the Durbin-Watson Test, as described above, often yields test statisitcs which fall within the region of indecision. While one can employ Theil and Nagar's (1961) result that where the regressors are changing slowly, the upper distribution of the D-W statistic is the appropriate distribution (Kennedy, 1985, p106) to reach a decision regarding the presence of autocorrelation, it was decided that further testing may be needed. Using an estimator which specifically accounts for autocorrelation, by assuming a different error structure (i.e. errors which are serially correlated), changes the specification of the model. The specification assumed when an autocorrelation-augmented estimator is used differs from the specification assumed when a conventional estimator is employed in that for the former the coefficient of correlation (ρ) is not zero. The validity of the specification assumed by an autocorrelation-augmented estimator can be established by testing the null hypothesis that $\rho = 0$. The Likelihood Ratio Test, discussed below, is an example of a test which may be employed in the systems context to test such a nested hypothesis. The mechanics of applying the test in this context are discussed further in section 4.3.1.

4.3 METHODS OF STATISTICAL INFERENCE

In this study statistical testing is required in three areas: determination of the appropriate estimator; determination of which of the various restrictions (such as homogeneity and homotheticity) which the researcher is able to impose on the underlying technology are valid, and determination of how good the overall fit of the model is. The choice of appropriate estimator rests on establishing the statistical nature (i.e. whether endogenous or exogenous) of the regressors and the relationship among the disturbances of the component equations of the system (i.e. whether component equation disturbances are autocorrelated). Both these issues have been dealt with above (see sections 4.2.4 and 4.2.5). Should autocorrelation be suspected on the basis of the Durbin-Watson test described in section 4.2.4 and an autocorrelation-augmented estimator be employed the validity of the derived estimates can be tested using a nested-hypothesis test: the Likelihood ratio test. The same general procedure can be employed for testing which of the various restrictions (such as homogeneity and homotheticity) which the researcher is able to impose on the underlying technology are valid. Nested hypothesis tests in general and the Likelihood ratio test in particular are discussed in section 4.3.1. Section 4.3.2 is concerned with the final area in which testing is required: the overall fit of the model.

4.3.1 NESTED HYPOTHESIS TESTS

Three alternative asymptotically equivalent tests are available for testing the validity of parameter restrictions (nested hypotheses) imposed on systems of equations: the *Likelihood ratio test* (LR test); the *Wald test* and the *Lagrange multiplier test* (Greene, 1993, p129 and Kennedy, 1985, p58).The three tests are asymptotically equivalent generating chi-square statistics with degrees of freedom equal to the number of restrictions imposed. As the small sample properties of the three tests are only known for a few special cases[28], choice among the three tests is most often made on the basis of ease of computation[29]. *Prima facie* either the Wald or Lagrange multiplier tests would appear the most attractive. While the LR test requires the calculation of both the constrained and unconstrained estimators, the Wald test requires only the unconstrained estimator and the Lagrange multiplier test only the constrained estimator. Computational consideration inform the choice here: not only is the LR test simpler computationally when constrained and unconstrained estimators are easily obtained[30] (Kennedy, 1985, p59) but the program used for the econometric work, Micro-TSP, generates the determinant of the variance-covariance matrix of residuals, which can be used to construct a LR test statistic directly when estimating systems of equations.

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[28] A weakness common to all three tests when the sample is small is the fact that critical values from the chi-square distribution are used despite the fact that in small samples they are not distributed as chi-square (Kennedy, 1985, p64).

[29] Choice of test may lead to conflicting results in the small sample case. Berndt and Savin (1977) show that in small samples the value of the three tests adopts the following inequality: Wald> Likelihood ratio>Lagrange multiplier (Berndt, 1991, p467 and Kennedy, 1985, p64).

[30] The Wald or Lagrange multiplier test would be preferred where constraints impose or remove non-linearities, respectively (Greene, 1993, p129-130)

Intuitively, the LR test is based on the idea that any restriction on parameters would be valid if there were no significant reduction in the value of the log-likelihood function once the restriction is imposed (Kmenta, 1986, p491). The test is based on comparing the difference between the log-likelihood function of the constrained and unconstrained estimates. Formally, if we denote the maximum of the likelihood function for the constrained estimator (which constitutes the null hypothesis) as $\hat{L}_C$, and the maximum of the likelihood function of the unconstrained estimator as $\hat{L}_U$, the LR is defined as:

$$\lambda = \frac{\hat{L}_C}{\hat{L}_U} \qquad 4.1$$

which will lie between 0 and 1 as both likelihood functions are positive and because a restricted optimum can never be greater than an unrestricted optimum (i.e.$\hat{L}_U > \hat{L}_C$). The test procedure is based on the result that the large sample distribution of $-2\ln\lambda$ is chi-squared with degrees of freedom equal to the number of restrictions imposed (Greene, 1993, p130-131 and Kennedy, 1985, p66).

Two alternative formulae can be used to calculate the LR test statistic. The first formula which is merely a mathematical manipulation of the definition of the LR as defined by 4.1[31] (Berndt, 1991, p466):

$$LR = -2(\ln \hat{L}_C - \ln \hat{L}_U) \sim \chi^2 \qquad 4.2$$

Where m refers to the number of restrictions imposed in the constrained model. The null hypothesis that a constraint is valid will be rejected if the value of the test statistic is greater than the appropriate critical chi-square value.

The alternative formula for the LR test statistic (for a proof see Cramer, 1986, p122) is:

$$LR = n(\ln|\Omega_c| - \ln|\Omega_{uc}|) \sim \chi_m^2 \qquad 4.3$$

Where n is the size of the sample, $|\Omega_{uc}|$ and $|\Omega_c|$ are the determinants of the residual variance-covariance matrices for the unconstrained and constrained models

[31]The equivalence between formulation 4.2 and that given by 4.1 is obvious when considering that the log of a ratio is equal to the difference between the log of the numerator and the log of the denominator.

respectively, and m refers to the number of restrictions imposed on the unconstrained model in order to generate the constrained model. Interpretation of the test statistic generated by the second formula is equivalent to the interpretation applied to the first formula.

The LR test is used to test the validity of imposing restrictions (usually nonlinear restrictions) on the parameters of a model, and as a consequence is appropriate for testing nested as opposed to non-nested hypotheses (Kennedy, 1985, p58 and Kmenta, 1986, p491). Fortunately, a large amount of the statistical testing undertaken in the context of estimating the Translog model involves the testing of nested hypotheses. Indeed determining both the correct specification of the Translog model and whether a chosen error specification is correct, both involve the testing of nested hypotheses.

Alternative maintained hypotheses, such as homotheticity, homogeneity, constant returns to scale (homogeneity of degree one), Cobb-Douglas technology, can all be specified, as described in section 3.3.1.2, by constraining the parameters of the non-homothetic form of the Translog. Thus, determining which of the alternative maintained hypotheses is valid for the different sectors, can be ascertained by using a LR test of the validity of the parameter constraints associated with different maintained hypotheses.

In addition to being able to test which specification of the Translog is appropriate, the LR test may also be used to establish whether an estimator which accounts for presumed autocorrelation of disturbances is valid (Berndt, 1991, p497 and Kmenta, 1986, 711). The test amounts to testing a null hypothesis that the disturbances are nonautocorrelated against an alternate hypothesis that the disturbances are, in fact, autocorrelated. The test involves estimation of the parameters of the system with and without the null hypothesis imposed, and then a comparison of the values of the maximised likelihood functions (or determinants of the variance-covariance matrices) in the manner described above, with degrees of freedom equaling the number of equations in the system[32] (Kmenta, 1986, p711).

[32] As the I3SLS, I3SLS (AR) and the IZEF (AR) estimates are not the maximum likelihood estimates they should not, theoretically be used for LR testing. However, given that the LR test is an asymptotic test and given that I3SLS, I3SLS (AR) and IZEF (AR) have the same asymptotic distribution as the maximum likelihood estimator, the asymptotic validity of the test should be unaffected (Kmenta, 1986, p711).

4.3.2 GOODNESS OF FIT

In the single equation context, the coefficient of determination (R^2) is usually employed as a measure of the goodness of fit of a model. This coefficient is calculated as one minus the ratio of unexplained variation of the dependent variable (i.e. the sum of the squared residuals) to the total variation of the dependent variable (i.e. the sum of the squared deviations of the estimated values of the dependent variable around their mean) (Kennedy, 1985, p11; Gujarati, 1988, p176) i.e.:

$$R^2 = 1 - \frac{\sum e_i^2}{\sum (\hat{y} - \bar{\hat{y}})^2} \qquad 4.4$$

where, e_i denotes the residuals at each observation $\hat{y}$ the fitted value of the dependent variable at each observation and $\bar{\hat{y}}$ the mean of the fitted values of the dependent variable. Because the OLS estimator minimises the sum of the squared residuals (i.e. $\sum e_i^2$), R^2 will be maximised when least squares is employed.

This single-equation measure of goodness-of-fit may not, however, be appropriate in the context of a system of equations. Least squares estimation ensures that the sum of residuals is zero, implying that the mean of the residuals will also be zero. As a result the numerator of the above expression for R^2 is equivalent to the sum of the square of deviation of the residuals around their mean. For system estimation in general, the sum of the residuals for each equation is not necessarily zero, implying that the mean of the residuals may not be zero and it is therefore possible that $\sum e_i^2$ could be greater than $\sum (\hat{y} - \bar{\hat{y}})^2$ which would yield a negative R^2 (Berndt, 1991, p468). A second problem regarding the use of the single equation R^2 emerges from the estimation technique used to estimate systems of equations. The estimators employed here minimise the determinant of the variance-covariance matrix (rather than the sum of squared residuals of each equation), and hence do not necessarily maximize the R^2 of each equation (Berndt, 1991, p468 and Kennedy, 1985, p144).

Given these two problems with the single equation coefficient of determination, an alternative measure is needed. McElroy (1977, p384) has developed a measure of goodness of fit for SUR systems:

$$R_z^2 = 1 - \frac{|E'E|}{|y'y|} \qquad 4.5$$

The numerator, $|E'E|$, is the determinant of the variance-covariance matrix of the errors of the system, and is therefore the sum of squared residuals (RSS) of the system (Berndt, 1991, p468 and Gujarati, 1988, p176). The denominator, $|y'y|$, is the total sum of squares of the system. The measure is confined to the range $0 < R_z^2 < 1$ and may be related to either an (asymptotic) F test statistic (McElroy, 1977, p384) or a LR test statistic (Berndt, 1991, p469):

$$LR = -T \cdot \ln(1 - R_z^2) \sim \chi^2 \qquad 4.6$$

Where T is the number of observations in the sample, and the number of degrees of freedom is given by the number of independent slope coefficients in the system of equations.

4.4 DATA CONSIDERATIONS

Estimating the Translog system here requires data on two types of variables: the variables appearing in the system directly and the instrumental variables used in I3SLS. Sections 4.4.1 to 4.4.3 are concerned with the variables appearing directly in the system while section 4.4.4 is concerned with the choice of appropriate instruments and their sources.

Four types of variables appear in the Translog system: the level of output y for the sector under analysis, the prices of the different factors of production P_i, the share of the different factors in total costs S_i and total costs C. Cost shares of different factors are calculated as the ratio of total expenditure on each input to total costs. Where total expenditure on an input is not directly available, expenditure can be derived from the product of the price of the factor P_i and the quantity employed Q_i. While data on the level of output produced in the different sub-sectors is readily available[33], input price and cost share data, particularly for capital, are less readily available, requiring the manipulation of other data. The data employed in this analysis have been obtained from two sources. The price of Capital (P_K) for the manufacturing sector[34] was calculated employing data appearing in Lombard and van den Heever (1990, p19) and data

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[33] The IDC publish series of total production for different subsectors of manufacturing. They derive total production figures using CSS input-output tables for the years when those tables are available. Total production for the interim years is calculated using trends in the indices of physical volume of manufacturing production per sector (IDC, 1992, p7).

[34] Separate prices of Capital could not be obtained for all the sectors under analysis. The problems involved in calculating the price of Capital for different sectors are discussed in detail below.

published by the Industrial Development Corporation (IDC, 1992). All other variables employed in this analysis were derived from the later source. The problems associated with the development of price and cost share data differ with the inputs. Before discussing the problems associated with the price and cost share series employed here for the different factors of production, some comment on the choice of inputs is required.

Applications of the Cobb-Douglas and CES generally employ only two inputs: capital and labour[35]. Applications of flexible forms such as the Translog generally employ more than two inputs. The convention of using aggregated capital and labour as inputs has, however, been adopted in most empirical applications of the Translog system[36]. Indeed, it would appear that it is only where the objective of analysis has been the estimation of elasticities of substitution between components of an aggregate input (in order to test whether the requirements for consistent aggregation have been fulfilled) that aggregates have not been used[37]. Choice of other inputs has largely been informed by the objective of the analysis at hand. For example, where the nature of the relation of energy to other inputs has been concerned, energy together with materials, capital and labour have been used[38].

Applications of the Translog which are concerned with the estimation of economies of scale use, in addition to capital and labour, those inputs which are important in the generation of the output of the industry with which they are concerned. For this reason both Christensen and Greene (1976, p663) and Greene (1983, p131), whose concern is the estimation of scale economies for U.S electric power generation, use fuel as a third input. While it would be desirable to include the most important factor, other than capital or labour, for each of the industries analysed here, the difficulty of establishing which `other' factor is most important and obtaining appropriate data precludes adopting Christensen and Greene (1976) and Greene's (1983) approach. Instead, a third input, `materials' - which is essentially a composite input of all factors of production other than capital or labour - is used.

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[35]Some South African applications, for example Browne (1943) and Spandau (1973) have, however, disaggregated labour into White and Black categories.

[36]See for example Berndt and Wood (1975); Christensen and Greene (1976); Contogiannis and Cluver (1984); Denny and May (1978); Denny and Pinto (1978); and Green (1983).

[37]See for example Berndt & Christensen (1973b).

[38]See Berndt & Wood (1975) and Cluver & Contogiannis (1984)

4.4.1 CAPITAL

The generation of prices and quantities of capital services is beset by problems which are very different from those associated with either labour or materials. While active markets exist for transacting the services of labour and materials, in general no market exists for capital services since "... the supplier of the capital service and its ultimate user are typically within the same economic unit" (Christensen and Jorgensen, 1969, p293). As a result data on the value of transactions in capital services do not exist and general procedures for constructing price and quantities cannot be adopted. Consequently, alternative procedures have been developed. These involve fairly lengthy chains of indirect inference, beginning with data on the value of transactions in investment goods and involving the imposition of fairly strong assumptions (Jorgensen & Griliches, 1967, p255). As already stated, the quantity of different factors is required for the generation of cost shares and is not, in itself, needed for the estimation of the Translog model. In the case of capital, total expenditure on capital services in each sub-sector can be directly calculated as the sum of depreciation and interest paid minus interest received.

The absence of a market in capital services and hence an explicit price of capital necessitates the construction of an `implicit rental price' for capital services. Because both supplier and consumer of capital are within the same economic entity, the implicit rental value is conceptually the same as the user cost of capital (Mark & Waldorf, 1983, p5). Formulae for the user cost of capital emerge from manipulating the neoclassical theory of optimal capital accumulation - in particular from a manipulation of the conditions required for the fulfillment of the assumption of perfect capital markets.

The precise form that the function describing perfect capital markets takes depends on the assumptions which are adopted regarding the survival function[39]. The choice of a specific form is constrained by Arrow (1964), Hall (1968) and Jorgensen's (1974), establishment of the duality between gross capital stock and the user cost of capital (Bi φrn, 1989, p51). An important implication of this duality is that, for consistency, the same assumption regarding age-efficiency (survival) ought to be adopted in the generation of both price and quantity series. Should the procedure used for creating capital stocks assume a linear survival function, so to should the calculation of the user

[39] That is the function expressing the relationship between the retirement of capital units over time and the loss of efficiency of remaining units.

cost of capital adopt a linear survival function. In South Africa, fixed capital stock figures are produced using a perpetual inventory method where provision for depreciation is on a straight-line basis for most economic sectors (Mohr, 1988, p66 and Moll, 1990, p200). The use of a straight-line method of depreciation embodies the assumption of a linear survival function.

Adopting the assumption of a linear survival function and manipulating the condition expressing the requirements for the existence of a perfect capital market yields the expression[40]

$$q(t) = \int_t^{\infty} e^{-r(s-t)} c(t) e^{\gamma(s-t)} (1 - \delta(s-t)) ds \qquad 4.7$$

where, q is the price of capital goods, r is the nominal rate of return on financial assets, γ is the investment price index and δ is the rate of decline of the efficiency of capital goods and s is the assumed life span of the investment good. The equation states that the current purchase cost of an investment good at a particular time is equal to the present value of its future service price, when allowance is made for retirement and a linear decline in efficiency with age (Biφrn, 1988, p53)

An expression for the user cost of capital (c) may be obtained by first differentiating equation 4.7 with respect to the time of purchase[41],

$$\frac{\partial q}{\partial t} = q(r+\delta) - c(t) \qquad 4.8$$

and then rearranging the result making $c(t)$ the subject of the formula yields:

$$c(t) = q(r+\delta) - \frac{\partial q}{\partial t} \qquad 4.9$$

Because the differential $\partial q/\partial t$ is concerned with instantaneous rates of change which are not empirically tractable applications using this formula substitute $\partial q/\partial t$ with $q_t - q_{t-1}/q_{t-1}$ which is essentially the differential for discrete periods of time. The formula employed by Lombard and van den Heever (1990, p19) to calculate the user

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[40] See Biφrn (1988) for a detailed proof.

[41] In general, given a function $q(t) = \int_t^{\infty} f(t,s) ds$, $\frac{\partial q}{\partial t} = \int_t^{\infty} \frac{\partial f}{\partial t}(t,s) ds - f(t,t)$

cost of capital for South African manufacturing is similar to that given by equation 4.9. They employ the following:

$$c(t) = q(r+\delta) - q[(1-\delta)(q_t - q_{t-1}/q_{t-1})] \qquad 4.10$$

where: r which represents the interest rate is the annual average rate on long-term company securities; δ represents the depreciation rate and employs national accounts assumptions; q represents the purchase price of capital which for the purposes of their application, was assumed to be the deflator for gross fixed investments. Although aware of the impact of taxes on the user cost of capital[42], as manifested in investment incentive schemes, Lombard and van den Heever (1990, p8) do not take these effects into account "for practical reasons." They do, however, concede that such schemes were likely to have decreased the user cost of capital as calculated using their formula.

What the source of the difference between the formula employed by Lombard and van den Heever (1990) and the one presented in 4.9 is, is unclear. Lombard and van den Heever (1990, p7) although not explicit, appear to cite Jorgensen[43] (1963) as the source of their formula. Jorgensen (1963) does not explicitly deal with the derivation of a user cost formula although a modified version (incorporating the effects of tax) employed by him (Jorgensen, 1963, p249) is consistent with the formulation given by 4.9. A possible explanation of the source of the difference between equation 4.9 and equation 4.10 is that Lombard and van den Heever (1990) employed different assumptions regarding the age-efficiency of assets and changes in the service price of assets over time to those employed above. In particular, it is possible that the survival function adopted by Lombard and van den Heever (1990) was non-linear[44].

Given the uncertainty regarding the assumptions implicit in equation 4.10, and as a result the source/s of the difference between equations 4.9 and 4.10, equation 4.9, which is consistent with the assumption employed in the perpetual inventory method of deriving capital stock[45] and hence fulfills the consistency required by the duality between the quantity and price of capital, has been used here. The application of equation 4.9, requires data on a nominal rate of interest, a rate of depreciation, and the

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[42]How the user-cost of capital is effected by different types taxation, both direct and indirect, has been explored by, inter alia, Christensen and Jorgensen (1969); Hall and Jorgensen (1967).

[43]Jorgensen (1963) is the only reference in Lombard and van den Heever (1993) which deals with the capital theory.

[44]Use of the simple exponential function would, however, yield equation 4.9 suggesting that a more complex non-linear survival function may have been used.

[45]That the efficiency of an asset declines in a linear fashion (Mohr, 1988, p66 and Moll, 1990, p200)

purchase price of capital. Ideally series of these variables which are specific to each sector under analysis ought to be employed. While it is plausible to assume that the rate of interest faced by different firms can be represented by the same composite rate of return[46], it is possibly less plausible to assume that the price of investment goods used by different firms and the rate of depreciation of that capital, is the same. Unfortunately, the data set employed prevents the generation of prices and depreciation rates specific to the different sectors[47]. As a result, the same user cost of capital series had to be used for all the sectors under analysis. The raw data used to generate the user cost of capital and the user cost of capital are presented in Table 4.1 below.

The development of the user cost of capital presented here is further flawed in one crucial respect: the effect of taxation on the user cost of capital is ignored. While theoretically incorporating the effects of taxation is relatively straightforward (see for example Christensen & Jorgensen (1969) or Jorgensen (1963)), the practical constraint of data unavailability has prevented the inclusion of the effects of taxation in the calculation of the user cost of capital for South African manufacturing. That the user cost of capital is likely to have been effected by taxation is without doubt.

4.4.2 MATERIALS

Although markets exist, and hence transactions occur, in material inputs, data on the value (either in nominal or real terms) of expenditure on materials appears only to be available via indirect inference using (or rather reversing) the conventions of national income accounting. Where value-added is available, reversing the method used to generate value-added enables one to construct the price of, and total expenditure on, materials. There is, fortunately, no need to construct material prices for the different sectors analysed here. The IDC publish data on material prices[48] for the various sub-

[46] The interest rate used in equation 4.9 represents the opportunity cost which firms incur as a consequence of them holding real capital assets as opposed to interest bearing financial assets. As firms are able to invest in a spectrum of different financial assets the opportunity cost incurred by the firm investing in real assets can best be represented by a composite rate of interest.

[47] While the data set contains series on the constant Rand value of capital, no data on the current Rand value of capital is available, and as a consequence a price of capital series cannot be created. Further, although a series of the current Rand value of depreciation does appear, in order to generate a rate of depreciation rate (i.e. the percentage of capital depreciated each time period) specific to each sector, either a constant Rand value of depreciation, or a current Rand value of capital would need to be generated.

[48] The IDC publish indexes of the prices of inputs, excluding labour or capital, specific to different manufacturing sub-sectors. Local prices and input prices are published seperately and combined into a single input price index which has 1990 as 100 (IDC 1992, p9). The price index used here is the composite index modified so that 1970 = 1.

sectors of manufacturing. Data on the total expenditure on materials is not, however, directly available and needs to be created. An indirect method for calculating total expenditure on materials, analogous to that used by Denny and May (1978), which is based on the conventions of national income accounting, has been employed here.

Table 4.1. User cost of Capital

Year	Purchase price of capital (q)	Interest Rate (r)	Depreciation Rate (δ)	Capital Gains ($\dot{q}$)	User Cost of Capital: Value $c(t)$	User Cost of Capital: Index (P_k)	User Cost of Capital: Log of Index ($\ln P_k$).
1972	1.575	0.095	0.124	0.094	0.251	1.000	0.000
1973	1.752	0.095	0.118	0.112	0.261	1.038	0.038
1974	2.001	0.116	0.115	0.142	0.320	1.274	0.243
1975	2.406	0.131	0.113	0.202	0.385	1.531	0.426
1976	2.809	0.135	0.115	0.167	0.535	2.129	0.756
1977	3.075	0.136	0.114	0.095	0.674	2.684	0.987
1978	3.425	0.120	0.111	0.114	0.677	2.697	0.992
1979	3.917	0.109	0.101	0.144	0.679	2.703	0.994
1980	4.466	0.115	0.093	0.140	0.789	3.140	1.144
1981	5.044	0.141	0.092	0.129	1.046	4.164	1.426
1982	5.866	0.156	0.093	0.163	1.298	5.166	1.642
1983	6.633	0.152	0.095	0.131	1.508	6.002	1.792
1984	7.204	0.177	0.097	0.086	1.888	7.516	2.017
1985	8.489	0.189	0.103	0.178	2.300	9.159	2.215
1986	10.373	0.177	0.106	0.222	2.714	10.804	2.380
1987	11.661	0.167	0.108	0.124	3.083	12.273	2.507
1988	13.384	0.171	0.109	0.148	3.600	14.332	2.662
1989	15.682	0.182	0.106	0.172	4.345	17.298	2.851
1990	17.301	0.181	0.106	0.103	4.862	19.358	2.963

Note: 1. The user cost of capital was calculated using the following formula $c(t) = q(r + \delta) - \dot{q}$, where capital gains ($\dot{q}$) is calculated as $\dot{q} = q_t - q_{t-1} / q_{t-1}$

2.For 1972, the lagged value of the price of capital needed for calculating $\dot{q}$ was 1.44 (Lombard and van den Heever, 1990, p19).

3.The purchase price of capital (q) used here is a price index of the deflator for gross fixed investment which has 1960 as it's base year.

Gross Domestic Product (GDP) can be calculated using three different methods which are equivalent *ex post*: the production, income and expenditure methods (Mohr et al, 1988, p38). Manipulation of the production and income methods has been used here to generate data on expenditure on materials. The production method provides an estimate of GDP based on the sum of the contribution of each industry to GDP. Double counting is avoided by measuring the contribution of each industry in terms of value added (net output) rather than gross output. Value-added is merely the difference between the value of any industry's output and its purchases of intermediate products (Mohr et al, 1988, p39). An alternative definition of value-added emerges from the income method of calculating GDP and rests on the equality between the alternative methods for calculating GDP. The income method of calculating GDP involves summing all income received by factors of production. The equivalence of the income and production methods of calculating GDP rests on the equality of the value-added of any industry, as defined, above and the incomes received by factors of production in that industry. Hence value-added for any industry can be calculated either as the sum of wages and salaries, profits, rent, depreciation allowances, interest and dividends or the difference between the value of the industryies output and its purchases of intermediate products (Mohr, et al, 1988, p39).

Denny and May (1978, p59) whose analysis required data on the use of materials for Canadian manufacturing as a whole reversed the procedure employed by *Statistics Canada* to derive real domestic product. Statistics Canada calculate real domestic product, using the production method for calculating GDP, as the difference between deflated gross output and deflated materials (Denny & May, 1978, p59). Denny and May (1978), reversing the production method of calculating GDP, calculated current and constant dollar materials as the difference between current and constant dollar gross output and domestic product. They then derived an implicit price of materials by dividing the current dollar value of materials by the constant dollar value (Denny & May, 1978, p59).

Reversing the production method of calculating GDP and finding the difference between the gross output of any industry and the contribution of that industry to domestic product (i.e. value-added or net-output) would yield the value of the intermediate products employed by that industry. Finding this difference, both in real and nominal terms, for the Canadian manufacturing sector as a whole is what Denny and May (1978) did to derive a price series for materials. As the IDC publish data on both the value of output of an industry and the value-added; data on expenditure on materials can be computed directly for the different subsectors of South African manufacturing. There is, unfortunately, a flaw in this approach which Denny and May (1978) appear not to have considered, which leads to an upward bias in the value of expenditure on materials derived in this manner. The IDC (1990, p6) point out that rather than merely being the difference between gross output and intermediate products used in an industry, value-added is the difference between the gross output and intermediate products plus indirect taxes less subsidies. Reversing the production method of finding value-added, and subtracting value-added from gross output will not yield data on expenditure on materials alone but will generate data on expenditure on materials and indirect taxes and subsidies. Unless separate data on indirect taxes and subsidies are available it would appear that it is not possible to obtain a `true' measure of expenditure on materials.

The IDC has calculated value-added for the sub-sectors of South African manufacturing for the period 1972-1990 using the income method described above. The series were adjusted so as to equal the contribution of each sub-sector to GDP (IDC, 1992, p7), and are thus equivalent to value-added calculated using the production method. Real total expenditure on materials (using 1990 as the base) was calculated here by subtracting real value-added from the real value of output (in 1990 prices). In order to be able to employ expenditure on materials in the calculation of cost shares, a nominal value had to be employed. This was achieved by multiplying the real cost of materials (i.e. in 1990 prices) by a price index which had 1990 = 1. That price index was obtained by dividing each observation of the composite price of materials index published by the IDC, which has 1990 = 100[49].

4.4.3 LABOUR

The IDC (1992) publishes time-series of both the total number of labourers employed in each sector of manufacturing and the nominal rand value of expenditure on labour. The nominal rand value of expenditure on labour can be used in the computation of total costs and thus the cost share of expenditure on labour. This precludes the need to derive a labour quantity variable for the different sectors. A price of labour series was derived by dividing nominal total expenditure on labour by the total number of labourers employed in each sector. This price of labour was then converted into a price index by dividing each observation by the first observation[50]. Deriving the cost share of labour and the price of labour in this manner, embodies a number of potentially untenable assumptions and is thus problematic in a number of respects.

Total expenditure on labour, while reflecting the total amount expended on wages and salaries, does not reflect total expenditure on labour services. The approach adopted here, by regarding the quantity of labour as merely the sum of the number of labourers implicitly assumes that labour services are proportional to the stock of labour. Such an assumption is obviously naive' (Jorgensen and Griliches, 1967, p266). The most obvious solution to this problem would be to measure labour in terms of man-hours.

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[49] Raw data and the final expenditure on materials series, together with the relevant price of materials data, used in this analysis are found in Appendix 4.

[50] The raw data required to generate both the total expenditure on labour and the price index of labour for the different sectors, together with the derived series, are presented in Appendix 4.

However, assuming that one could measure services of labour in terms of man-hours would be equally incorrect, for it fails to account for variations in the intensity of effort and the impact that qualitative differences between workers make on labour inputs.

`[T]he intensity of effort varies with the number of hours worked per week, so that the labour input can be measured accurately only if data on man-hours are corrected for variations in the number of hours per man on labour intensity' (Jorgensen and Griliches, 1967, p266). Although ratios have been developed elsewhere to adjust man-hours for intensity effects (Denison (1962) cited in Jorgensen & Grilliches (1967, p266)), none exist for the present study. While in principle, a fairly accurate measure of the flow of labour services can be constructed and have been employed in applications of the Translog system (see for example Denny & May (1978) and Denny & Pinto (1978)) lack of the required information precludes such accuracy in this study.

A further problem in this regard is that in the data set employed, data on labour remuneration reflects remuneration to all employed in a a particular sub-sector during a specific period of time (including casual and seasonal employees (IDC, 1992, p3)). Hence multiplying the number of workers by the average number of hours (even if adjusted for hours worked and intensity) would not provide an accurate indication of the flow of labour services. Indeed, the fact that in the data set used the number of employees includes casual and seasonal employees introduces a bias into the price of labour. The price of per unit of labour is derived by dividing total expenditure on labour by the total number of employees. Because the total number of employees includes casual labourers the price of labour will be biased downwards.

4.4.4. INSTRUMENTAL VARIABLES

The 2SLS procedure, which is the first stage of the 3SLS procedure, uses all the exogenous variables in a system as instruments (Kennedy, 1985, p134). The Translog cost function and cost share equations can be regarded as the `supply side' of a broader market model. As a consequence exogenous variables appearing in both the Translog system and the demand side of the model would be appropriate instruments for 3SLS. Variables in the Translog model which are definitely exogenous are those which do not include output in any form and are therefore appropriate instruments and are used here.

While no demand model has been specified to determine appropriate instruments the variables which are likely to be employed in such a model are intuitively obvious.

Conventionally, demand for a commodity is modeled as a function of, inter alia, the price of the commodity concerned, the prices of other related commodities, disposable income, the size of the population, and the level of expenditure. Following this convention, in addition to the exogenous variables in the Translog, the following variables have been used as instruments in the different sectors analysed here: an output price index, a consumer price index (as a proxy for the prices of related commodities), an index of the level of manufacturing employment (as a proxy for the population), an index of real personal disposable income, and indexes of the level of real private and government consumption expenditure. These instruments together with details of how they were generated are presented in Appendix 4.2.

CHAPTER 5

ECONOMETRIC RESULTS

5.1 INTRODUCTION

Given that the central concern of this dissertation is a comparison of the technology used in wage and luxury goods sectors examples of both types of sectors have been analysed. Motor Vehicles, Parts and Accessories (ISIC 3840) is analysed as a representative luxury goods sector. The Electrical Appliances and Household Goods (ISIC 3833) and Furniture (ISIC 3320) sectors are analysed as representative wage goods sectors. While the output of the Motor Vehicles[1] sector can quite reasonably be deemed a luxury the output of the Appliance[2] and Furniture[3] sectors is not necessarily exclusively limited to commodities which could be regarded as `basic' or `wage' goods. Indeed, given that electricity is a prerequisite for the purchase of an electric appliance and in addition that 23 million South Africans (approximately 60% of the population) (I.D.T, 1993, p10) do not have electricity, it is likely that consumers of appliances would tend to fall into middle or upper income brackets rather than lowest income brackets (however these may be defined). Similarly furniture would appear to be *prima facie* a luxury commodity. Nevertheless, both these sectors were chosen for analysis on the grounds that not only is any growth path which improves the distribution of income likely to increase demand for these commodities in the future but also that the proposed national housing and electrification campaigns (ANC, 1994, p22 and p33) are likely to effect demand for these goods in the short-term.

Using the Translog system to obtain estimates of economies of scale and elasticity's of substitution involves use of both statistical and theoretical criteria to choose between

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[1] The Motor Vehicles sector covers the "...specialised manufacture of motor vehicles, caravans, trailers, vehicle bodies, motor vehicle parts and accessories such as engines, brakes, radiators, transmissions, frames etc." (IDC, 1992, p22).

[2] The electrical appliances and household goods sector is involved in the manufacture of "...smaller electrical appliances and housewares, such as electric space heaters; blankets and heating pads; hot plates, boilers, roasters, toasters and food mixers; irons and mangles; fans, vacuum cleaners and floor waxers and polishers; hair driers, toothbrushes, hair clippers, shavers and water heaters." (SIC, 1988, p61-62).

[3] The Furniture sector involves "... the manufacture of household, office, public building, professional and restaurant furniture and fixtures which are made mainly of wood or other materials other than metal." (SIC, 1988, p49).

competing estimates. Theoretical criteria ought, however, to be afforded a superior status. Translog estimates are valid only if a dual relationship between production and cost holds for the sample under analysis. Two conditions of duality, that the cost function be monotonically increasing and concave in input prices, are not imposed *a priori* and need to be tested. Violation of concavity is particularly problematic as it may imply that the cost function does not have the structural relationship postulated by duality theory to the underlying technological parameters of production (Fuss et al, 1978, p266). Parameter estimates, and the associated estimates of scale and substitution, which satisfy rigid statistical criteria but which violate the concavity condition should, therefore, be treated with considerable circumspection. Clearly emphasis ought to be afforded theoretical, rather than statistical, criteria.

Four alternative econometric methods are available for estimating the parameters of the Translog: the iterative Zellner efficient estimator (IZEF) which is applicable for seemingly unrelated regression (SUR) models in general (Zellner, 1962); the IZEF modified to take serial correlation in the component equations of the system into consideration (IZEF(AR)) (Greene, 1993, p498); the iterative three-stages least squares (I3SLS) estimator which is appropriate if regressors of the SUR system are endogenous (Zellner and Theil, 1962) and I3SLS modified to take serial correlation in the component equations into account (I3SLS (AR))(Kmenta, 1986, p708). Which of the four alternative regressors will yield the statistically best estimates requires knowledge of the nature of regressors (whether they are exogenous or endogenous) and the relationship among errors of the component equations.

Determining the nature of the regressors is required for determining which broad class of estimator, ZEF or 3SLS, is most appropriate. Should any of the regressors[4] be endogenous the appropriate class of estimaptors, yielding consistent and asymptotically efficient estimates (Kennedy, 1985, p136) are 3SLS estimators. Employing a ZEF estimator in this context, while efficient, would not, however, be consistent (Kmenta, 1986, p718). Should all regressors be exogenous ZEF estimators would be appropriate, yielding consistent and efficient parameter estimates. Although consistent, 3SLS estimators would, in this case, be inefficient relative to ZEF estimators (Berndt, 1991, p379). While tests do exist for establishing the nature of regressors in systems of equations, and therefore, which broad class of regressors would be appropriate as argued in Chapter 4 these tests are not suitable in this context. As a consequence it is

[4] Regressors in the Translog model are combinations of input prices, output and time. Time is always exogenous, while the relatively disaggregated level of this analysis permits the reasonable assumption that input prices are exogenous to the industries under analysis. The nature of output, and therefore all regressors which are functions of output, is, however, debatable.

not possible to determine statistically which of the two broad classes of estimator is appropriate. Both classes were therefore used[5] and the results generated compared.

While it is not possible to determine which class of estimator is most appropriate the related question of the appropriate error structure[6] can be established. Conventional Durbin-Watson diagnostic testing for the presence of first-order autocorrelation among the errors of the component equations is available to determine whether the autocorrelation modified version (IZEF (AR) and I3SLS (AR)) of the two classes of estimator would be appropriate. Post estimation LR comparisons of estimates emerging from different assumptions regarding error structure are available to determine whether autocorrelation specifications (implied in the adoption of either the IZEF(AR) or the I3SLS(AR) estimators) are statistically significantly different from the nonautocorrelated error specifications.

LR testing is not only useful for determining which error structure is valid, it is also useful for deriving the most appropriate estimates of technology. The modelling flexibility of the Translog provides the opportunity to determine statistically whether the commonly imposed restrictive maintained hypotheses of homotheticity or homogeneity, and associated estimates of scale and substitution, are appropriate. Homotheticity and homogeneity can be imposed by constraining parameters and hence their validity determined by LR comparisons of the constrained and unconstrained specifications.

For each estimator used here five different specifications of the Translog system were estimated[7]. The first specification (Model 1) which was estimated is the specification augmented to simultaneously estimate scale, substitution and technical change effects, as proposed by Christensen (1977, cited in Greene, 1983, p127). The second specification (Model 2) is the specification employed by Christensen and Greene (1976) which assumes variable economies of scale in a production structure which is non-homothetic. The third specification (Model 3) allows economies of scale to vary

[5] 3SLS estimators are consistent wherever they are applied. It could be argued that they alone ought to be employed given that the only loss associated with their inappropriate use is a loss of efficiency. Indeed, efficiency losses give rise to more conservative statistical hypothesis testing which could be deemed a virtue rather than a cost. Despite this argument, and the additional computational burden, both classes of estimator were employed here. The principle motivation for using both classes of estimator emerges from two areas of interest associated with concavity: Firstly, whether concavity is invariant to different assumptions regarding the nature of regressors, and therefore the class of estimator used. Secondly, to establish empirically the theoretical assertion that using estimates which are non-concave are potentially misleading.

[6] As the data used here are time-series it is assumed that error variances are homoscedastic.

[7] The form (for three inputs) of the different specifications analysed here are presented in Appendix 5.1.

with output but assumes that the dual production function is homothetic. The fourth specification (Model 4) imposes homogeneity (and hence homotheticity) on the underlying technology thus preventing economies of scale from varying with output. The fifth specification, (Model 5) assumes a Cobb-Douglas technology with variable returns to scale.

The maintained hypotheses embodied in Model's 2 through 5 are introduced into the Translog system by imposing parameter restrictions on Model 1. The validity of these maintained hypotheses can therefore be tested using nested hypothesis tests, such as the LR test, of the validity of parameter constraints. This approach was adopted here. In addition to testing the appropriate specification of the Model, each of the specifications estimated was tested (using a LR test) to establish whether the emerging estimate of economies of scale was significantly different from unity[8].

The above testing procedures were employed for both classes of estimator using the same general procedure. Parameter estimates were first obtained assuming *a priori* that the component equations of the SUR system fulfilled the assumptions of the CLR model. Those estimates were then tested to determine which of the five alternative specifications of the model (see Appendix 5.1) was statistically valid, and whether the assumption that the errors of the component equations were uncorrelated across observations was valid. Where autocorrelation was detected, autocorrellation-augmented estimators were employed. Parameter estimates derived from the autocorrelation augmented estimators were analysed to determine the correct specification of the model: both in terms of comparisons among the different specifications which assume autocorrelated errors and between specifications which assume non-autocorrelated and autocorrelated errors. Testing of the hypothesis that economies of scale are constant were then performed on all estimated specifications. Finally testing was undertaken to determine whether or not the duality conditions of monotonicity and concavity in input prices were fulfilled for different parameter estimates for the sample under analysis.

A persistent feature of the parameter estimates generated here is the violation of the concavity requirement. Given that non-concavity threatens the validity of the duality between cost and production functions few theoretically sound estimates of technology emerged from this analysis. A number of useful conclusions do, nevertheless, emerge. The data clearly suggest that flexible, non-homothetic specifications such as the

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[8] Constant returns to scale can be imposed on each of the models by imposing constraints on a number of parameters.

Translog are more appropriate than functions which are homothetic, such as the Cobb-Douglas. Moreover, not only is homotheticity found to be statistically inappropriate but it clearly biases estimates of scale downwards. Finally the data provide some support for the argument that economies of scale are greater in wage goods industries than in luxury industries.

The data used in this analysis[9] are presented in Appendix 4. The `raw data[10]' and certain manipulations of the data were used here as part of an *a priori* attempt to form expectations regarding the magnitude and sign of elasticity's of substitution and economies of scale. A similar regime of *a priori* analysis was conducted for each of the sectors. Four 'devices' were used to form expectations about elasticity's of substitution: changes in the cost shares of inputs; both plots and descriptive statistics (means and standard deviations) of input-output coefficients[11] and relative prices of the three inputs[12]; and the correlation matrix of the different input-output coefficients and relative input prices. In order to form expectations about economies of scale the trend of real average costs[13] over the period was analysed.

5.2 ELECTRICAL APPLIANCES AND HOUSEHOLD GOODS (ISIC 3833)

5.2.1 INTRODUCTION and *A PRIORI* DATA ANALYSIS

Of the three sectors analysed here the estimates emerging for the appliances sector are most useful. Firstly, these estimates fulfil the concavity condition most often - allowing for the most complete set of results. The data indicate that all three inputs substitute each other in the production of Appliances, with the degree of substitutability greatest between capital and labour. Conclusions regarding the magnitude and trend of scale depend on assumptions made about the nature of output. The data do, however, suggest that not only is imposing homotheticity statistically invalid, but in addition

[9] Total Costs C; Cost shares of Capital S_K; Labour S_L and Materials S_M; Input prices of Capital P_K; Labour P_L and Materials P_M and output y. Estimation of the model requires manipulation of these series. In particular homogeneity in prices requires that total costs and the input prices retained in the system are divided by the price of the factor whose cost-share is dropped from the system; and the natural logarithm of the variables needs to be found in accordance with the specification of the model.

[10] That is, the non-logarithmic and non-normalised form.

[11] Input-output coefficients, which are the ratio of the quantity of different factors to the quantity of output, were calculated by defining the quantities of factors as the constant Rand expenditure on the different factors and the quantity of output as the constant Rand value of output.

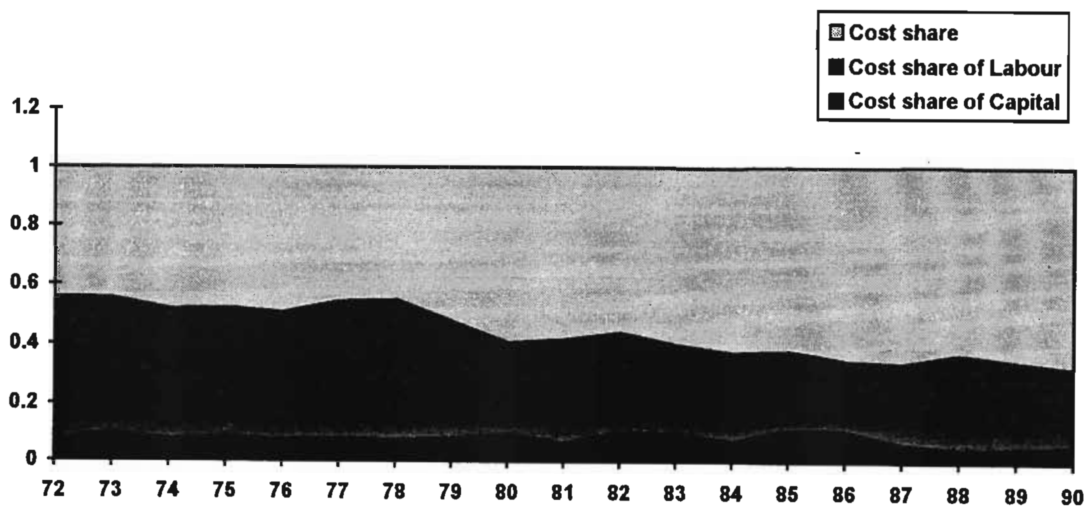
[12] Relative prices are defined as the ratio of the price index of each input to the price index of output as calculated by the I.D.C (1992).

[13] Real average costs is nominal average costs deflated using a price index of output, where nominal average costs are calculated as the ratio of the current value of total costs to the constant rand value of output.

imposing this hypothesis *a priori* biases estimates of the magnitude of scale downward. The second useful feature of these estimates also concerns concavity: most IZEF and I3SLS results are concave while most IZEF (AR) and I3SLS (AR) estimates non-concave. A consequence of this phenomenon is that for most specifications of the system a concave and a non-concave set of estimates are available for comparison. While differences are not systematic concave estimates of scale and substitution do differ from non-concave estimates. Before turning to a more detailed consideration of these conclusions the *a priori* analysis is presented.

The appliance sector is characterised by a cost structure where the relative importance of the three inputs (capital, labour and materials) in total costs has remained the same (materials contributing the most, and capital the least, to total cost) over the period 1972-1990. A slight fluctuation in relative shares between 1978 and 1990 when relative expenditure on materials increased at the expense of labour (and after 1986, capital) appears to indicate that materials may have substituted both capital and labour, which complement one another (see Figure 5.2.1).

Figure 5.2.1 Cost shares of Capital, Labour and Materials for ISIC 3833 (1972-1990)



This conclusion is supported by trends of input-output coefficients (see Figures A5.2.1 - A5.2.3, Appendix 5.2.1) and simple correlation's between input-output coefficients and relative input prices. While the input-output coefficient of materials fluctuates in a small band around an overall upward trend the input-output coefficients of capital and labour both display general downward trends. The similarity in the capital and labour trends[14] suggests that the two inputs complement one another in the sector. Probable complementarity and susbtitutability between inputs can also be inferred from the sign of the correlation between input-output coefficients of inputs and the sign of cross correlation's between input-output coefficients and relative prices[15]. The data presented in Table 5.2.1 suggest that materials is a substitute for both capital and labour and that capital and labour are complements in the production of appliances.

The positive correlation between the input-output coefficient of materials with the relative prices of both capital and labour is a possible indication that materials are a substitute for both labour and capital. This conclusion is supported by the negative correlation's between the input-output coefficient of materials and those of capital and labour respectively[16 17]. The substitutability relationship would, however, appear to be asymmetrical. The input-output coefficients of both labour and capital are negatively correlated with the relative price of materials.

Complementarity between capital and labour can be inferred from the negative correlation's between the input-output coefficients of capital and the relative prices of labour and capital respectively. The possibility of such complementarity is supported by the strong (0.94) positive correlation between the capital and labour input-output coefficients.

Table 5.2.1 Correlation matrix of Relative prices and Input-output coefficients (ISIC 3833)

	Relative price of Capital	Relative price of Labour	Relative price of Materials	Input-output coefficient of Capital	Input-output coefficient of Labour	Input-output coefficient of Materials
Relative price of Capital	1.0000					
Relative price of Labour	0.40547	1.0000				

[14] This assertion of similarity in the trends is supported by the ratio of standard deviation to mean of the two input-output coefficients. For capital the ratio is 0.385 and for labour 0.333 (see Table A5.2.2 Appendix 5.2).

[15] A positive correlation between the input-output coefficient of an input x with the relative price of another input y suggests, *ceteris paribus*, that as the relative price of y rose more of variable x was employed. Were that to occur x would be regarded as a substitute for y. A corollary to this conclusion is that complementarity between inputs would be reflected in a negative correlation between input-output coefficients and relative input prices.

[16] A negative correlation between two input-output coefficients would imply that when the relative use of one input increases the relative use of the other decreases.

[17] The suggestion that materials are a substitute for both capital and labour may be explained by evidence that materials is an essential input. The correlation between the relative price of materials and the input-output coefficient of materials is positive. This suggests that demand for materials is invariant to the price of materials and, therefore, that materials are an essential input. This conclusion would be strengthened if the correlation was near to zero, which it is not.

Relative price of Materials	0.84800	0.71051	1.0000			
Input-output coefficient of Capital	-0.83442	-0.43509	-0.73563	1.0000		
Input-output coefficient of Labour	-0.71333	-0.36033	-0.57128	0.94253	1.0000	
Input-output coefficient of Materials	0.55160	0.28571	0.38985	-0.42224	-0.50724	1.0000

The final objective of undertaking an *a priori* analysis is to establish the relationship between real average costs and output and hence obtain some indication of the magnitude of economies of scale. Figure 5.2.2, which plots real average cost against time, indicates that prior to 1978 the industry experienced a period of increasing real average costs (suggesting diseconomies of scale), while between 1978 and 1986 the industry experienced, in general, decreasing average costs (suggesting economies of scale).

Figure 5.2.2 Real Average Costs over the period 1972-1990 (ISIC 3833).

5.2.2 REGRESSION ANALYSIS

An important consideration for establishing the properties of estimates of the Translog is the relationship among errors. While both the IZEF and I3SLS[18] procedures explicitly take inter equation error correlation's into account, providing more efficient estimators than OLS or 2SLS respectively, both procedures are premised on the assumption that the errors of each individual equation are normally and independently

[18] Parameter estimates are presented in Tables A5.5.3.1 and A5.5.3.2 in Appendix 5.5

distributed. Should the errors of each equation not be independently distributed, these estimators would not be efficient. The presence of serial correlation can be detected by calculating the single equation Durbin-Watson (D-W) statistic for the errors corresponding to the component equations of the SUR system[19]. Computed and critical D-W statistics are presented in Tables A5.5.2.1 and Table A5.5.2.2 - Appendix 5.5.[20]. Following Theil and Nagar[21] (1961) for the case where the test statistic falls in the region of indecision, and the usual Durbin-Watson conclusion for cases where the test statistic falls below the lower limit of the critical value, first-order autocorrelation would appear to exist in all the component equations of the different specifications of the model, irrespective of whether IZEF or I3SLS is used.

The implication emerging from the Durbin-Watson testing is that IZEF (AR) and I3SLS (AR) [22] estimates are more appropriate than IZEF and I3SLS respectively. Estimates for Model 1 using I3SLS(AR) could not, however, be obtained as estimates did not converge on iteration[23]. Whether the results generated by adopting this alternative error specification are statistically different to those obtained when the errors of the component equations are specified to be non-correlated can be tested using LR tests. Table 5.2.2 and 5.2.3 present computed LR test statistics, for the five specifications of the model, of the null hypothesis that the error specification assumed for the IZEF/I3SLS estimators is valid. For Models 1 through 4 the number of degrees of freedom is 3, while for the 5th model the number of degrees of freedom is 1[24]. For both classes of estimator for all specifications apart from Model 5, the computed LR

[19] The different methods which need to be adopted when testing for autocorrelation of IZEF and I3SLS estimators is discussed in Chapter 4. An important difference between the two tests is the calculation of degrees of freedom.

[20] No tests are performed on the cost share equations of the Cobb-Douglas form of the Translog (Model 5) as that function has no regressors, only a constrained constant.

[21] Theil and Nagar (1961) have shown that where regressors are changing slowly the upper distribution of the D-W statistic is the appropriate distribution. Kennedy (1985, p106) argues that it is likely that in economic time series that the regressors would be changing slowly.

[22] Parameter estimates of the different specifications using IZEF (AR) and I3SLS (AR) are presented in Tables A5.5.3.3 and A5.5.3.4, respectively.

[23] By imposing an autoregressive error structure on this Model, the number of parameters being estimated rises from 14 to 17 (each equation of the system being estimated was presumed to posses a different coefficient of autocorrelation which meant that three different autocorrelation coefficients needed to be estimated). As one observation is lost when employing the autoregressive estimator, the number of degrees of freedom for estimating this specification is only one - hence the instability of the parameter estimates emerging. Although 3SLS(AR) estimates could be obtained, the lack of invariance of such estimates to which share equation was dropped from the system precluded there use here.

[24] The number of degrees of freedom is determined by the number of constraints imposed on the unrestricted version of the model to generate the restricted form of the model. Where all three equations of the Translog system were modified to take autocorrelation into account, the number of restrictions imposed on the unconstrained model (IZEF(AR)) to generate the constrained model (IZEF) would be three. For the 5th specification, only the cost equation is modified and hence only one constraint is imposed to generate the constrained model.

statistic is greater than the critical chi-squared statistic leading to a rejection of the null hypothesis that the non-autocorrelation augmented estimator is correct. Clearly, aside from specification five, more efficient estimates will emerge if an autocorrelated error structure is adopted, irrespective of the nature of the regressors.

Table 5.2.2 LR test statistics for comparison of IZEF and IZEF(AR) estimators (ISIC 3833)

Model	1	2	3	4	5
Likelihood Ratio Statistic	36.707	17.645	31.129	30.919	-11.711
Critical Chi-squared statistic	7.81	7.81	7.81	7.81	3.84

Table 5.2.3 LR test statistics for comparison of I3SLS and I3SLS(AR) estimators (ISIC 3833)

Model	2	3	4	5
Likelihood Ratio Statistic	16.815	30.783	30.623	-12.591
Critical Chi-squared statistic	7.81	7.81	7.81	3.841

An interesting conclusion emerging from the all four sets of parameter estimates is a statistical preference for the most general functional specifications and hence a rejection of the hypotheses of homotheticity and homogeneity. The validity of the alternative specifications relative to the most general model estimated by each estimator[25] was tested by performing LR tests - results of which are presented in Table 5.2.4. For the IZEF estimates testing the null hypothesis that the Translog model should not include time, in any form, as a regressor or as a multiple of another regressor (Model 2) against the alternative hypothesis that time ought to be included in the Translog as specified by Model 1 the LR test statistic is 17.12[26]. The appropriate chi-squared statistic for 5 degrees of freedom[27] at the 0.05 level of significance is 11.07 (Gujarati, 1988, p685). As the test statistic is greater than the appropriate chi-squared statistic the null hypothesis, that Model 2 is a valid specification, cannot be accepted with 95% confidence. Similarly the null-hypotheses implied by any of the alternative models cannot be accepted for any of the estimators suggesting that the most flexible, general representation of technology is appropriate for these data. This result potentially damages analyses of these data using functions, such as the Cobb-Douglas or CES, which impose the hypotheses of homotheticity and homogeneity *a priori*.

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[25] In the cases of IZEF, IZEF (AR) and I3SLS Model 1, while in the case of I3SLS (AR) Model 2.

[26] $LR = -N(\ln|S_{uc}| - \ln|S_c|)$ = -19 [ln(2.49E-11)-ln(6.13E-11)]=17.12. See Chapter 4 for more detail regarding LR testing.

[27] The number of degrees of freedom equals the number of restrictions being tested. Five parameters appearing in Model 1 are constrained to zero when estimating Model 2. A sixth appears to be 'missing' from the results table γ_{Mt}. That parameter is, due to the linear homogeneity in input prices constraints discussed in Chapter 3, a linear combination of the other parameters and is not estimated directly.

Table 5.2.4. LR test statistics and appropriate chi-square statistics for ZEF and 3SLS estimates of ISIC (3833)

Null Hypothesis	Critical χ^2 statistic and df for IZEF, IZEF (AR) and I3SLS estimates	Computed L. R. statistic IZEF estimates	Computed L.R. statistic IZEF (AR) estimates	Computed L.R. statistic I3SLS estimates	Critical χ^2 statistic and df for I3SLS (AR) estimates	Computed L.R. statistic I3SLS (AR) estimates
Non-homothetic	11.07 (5)	17.12	37.24	17.14		
Homotheticity	14.07 (7)	46.78	52.66	46.22	5.99 (2)	13.59
Homogeneity in Output	15.51 (8)	47.73	53.84	47.19	7.81 (3)	14.66
V.R.T.S. Cobb-Douglas	19.7 (11)	78.62	129.72	78.51	12.59 (6)	87.55

Note: 1. For IZEF, IZEF (AR) and I3SLS the alternate hypothesis is that the technology augmented form is valid. For I3SLS (AR) estimates the alternate hypothesis is that the non-homothetic form is valid.
2. Numbers in parenthesis appearing after χ^2 indicate Degrees of Freedom (df) (at the 5% level of significance) which are determined by the number of parameter constraints required to obtain the specification of the different null hypothesis from the alternative specification.

As argued in the introduction to this chapter a crucial consideration in deriving estimates of scale and substitution is the theoretical tractability of the estimates. The Translog is neither monotonically increasing nor concave in input prices *a priori*. Non-concave estimates are potentially meaningless as the assumptions upon which the dual relationship is premised are violated. The appropriate procedures for testing monotonicity and concavity have been discussed in Chapter 3[28], and the data relevant to the testing of both conditions are presented in Appendix 5.5 (Tables A5.5.4.1 - A5.5.4.30). Although the monotonicity requirements are fulfilled for all sets of parameter estimates an interesting anomaly regarding the fulfilment of the concavity requirement for different estimators emerges. The concavity condition is fulfilled for all specifications[29], over the whole sample, when the IZEF and I3SLS estimators are used[30], but only intermittently when the auto-correlation augmented estimators are used. Concavity is only fulfilled for a few observations in two of the specifications

[28] Monotonicity is fulfilled if the fitted cost shares are positive at every observation. Concavity is fulfilled if the matrix $[A]$ of AESs is negative semi-definite at every observation. Negative semi-definiteness emerges when $|A_1| \leq 0, |A_2| \geq 0, \ldots |A_n| \leq 0$ if n is odd and $|A_n| \geq 0$ if n is even. Violation of one of these conditions will mean that the matrix of AESs is not negative semi-definite and that concavity has been violated. The precise form of the concavity test in the three input context is presented in Appendix 3.1.

[29] There is no need to test the concavity condition for Model 5 as it fulfilled *a priori*.

[30] A requirement of concavity is that the determinant of the 3×3 matrix of AESs be negative or zero. If the value of that determinant is positive but very small it will still be acceptable as deviations from zero could be due to rounding errors

when the IZEF (AR) is used[31]. When I3SLS (AR) is used concavity is fulfilled for Model 2 over the whole sample but not at all for any of the other specifications.

These concavity violations present an important choice regarding the ZEF estimates. Statistical considerations reveal a preference for the IZEF (AR) estimates in general. The concavity violation suggests that these estimates may, however, be theoretically meaningless. A choice, therefore, exists between the efficiency of the IZEF (AR) estimates and the theoretical validity of the IZEF estimates. Given that the cost of inefficiency is merely more conservative hypothesis testing (which may even be deemed desirable), while non-concave estimates are potentially meaningless, the IZEF estimates are preferred. Should the ZEF class of estimator be valid, the appropriate estimates of substitution and scale would, therefore, be those emerging from the IZEF estimates of Model 1[32]. The same dilemma does not exist for the 3SLS set of estimates. Combining specification and autocorrelation tests the I3SLS (AR) estimates of Model 2 are preferred[33], and these estimates fulfil concavity over the whole sample.

Having determined which sets of parameter estimates are appropriate derived estimates of substitution, as embodied in proper AESs, and economies of scale can be obtained. The analysis of both features is concerned with both the magnitude of estimates and the impact of concavity violation on estimates. The analysis of scale has a third objective: assessing the impact of different maintained hypotheses on modelling scale.

5.2.2.1 ELASTICITY'S OF SUBSTITUTION - ISIC 3833

Whether the ZEF or 3SLS class of estimator is appropriate, the same conclusion regarding the relationship among inputs emerges[34]. Contrary to the *a priori* expectation that capital and labour are complementary inputs proper AESs indicate that all factors are substitutes in the production of appliances[35]. This relationship among inputs is, however, quantitatively different if different assumptions about the nature of output are adopted. If output is assumed to be endogenous the degree of substitutability between capital and labour is considerably greater than if output is assumed to be exogenous. Such comparisons may, however, be invalid given that

[31] For Model 2 concavity is fulfilled for 1972 and 1973 and 1977 and 1978. For Model 3 concavity is fulfilled for 1988.
[32] LR testing reveals a preference for this specification.
[33] LR tests indicate a preference for more general specifications and estimators which assume an autocorrelated error structure. Because no I3SLS (AR) estimates of Model 1 emerge the statistically most desirable estimates are the I3SLS (AR) estimates of Model 2.
[34] Derived estimates of proper AESs for all models are presented in Tables A5.5.4.1 to A5.5.4.30.
[35] There is, however, one exception to this conclusion. The I3SLS (AR) estimates of Model 2 indicate that in 1990 labour and materials are complements.

different model (and less importantly error) specifications are valid for the two assumptions regarding the nature of output[36].

Patterns of AESs emerging here provide empirical support for the theoretical assertion that non-concavity may effect estimates of technology. Indeed, a comparison of the proper AESs emerging from concave and non-concave sets of estimates of both classes of estimator suggests that reliance on generally non-concave sets of parameters for estimates of technology is potentially misleading. While for all concave parameter estimates all inputs are (for the most part) substitutes[37] the same does not hold for the generally non-concave parameter estimates[38]. No systematic relationship exists between the violation of concavity and the pattern of AESs emerging[39].

5.2.2.1 ECONOMIES OF SCALE - ISIC 3833

As argued in Chapter 4 the statistical significance of estimates of scale emerging from Translog estimates can be determined using LR tests of the validity of appropriate parameter constraints. Results, presented in Table 5.2.5, reveal that for theoretically valid estimates only Model 2 and Model 5 generate estimates of scale which suggest non-constant returns to scale.

Table 5.2.5 Statistical Significance of Estimates of Scale (ISIC 3833)

	Critical Ch-Squared Statistic	Computed LR - IZEF estimates	Computed LR - IZEF (AR) estimates	Computed LR - I3SLS estimates	Computed LR - I3SLS (AR) estimates
Model 1 (5)	11.071	10.876	305.555	11.01	--
Model 2 (4)	9.488	67.475	23.232	68.53	46.08
Model 3 (2)	5.991	0.114	2.301	0.391	2.582
Model 4 (1)	3.841	1.236	0.112	0.563	0.117
Model 5 (1)	3.841	9.270	12.029	10.787	18.975

Note: 1. Numbers in parenthesis after the model number are the number of degrees of freedom
2. Source of critical chi-squared values: Gujarati (1988, p685)

[36] If output is exogenous IZEF estimates of Model 1 are valid, however is output is endogenous I3SLS (AR) estimates of Model 2 are appropriate.
[37] The exception are the I3SLS (AR) estimates of Model 2, which show that the relationship between labour and materials may have changed to one of complementarity in 1990.
[38] For the IZEF (AR) estimates of Model 1 capital and labour are everywhere complements, capital and materials are substitutes only between 1972 and 1978 and in 1984 and 1985. The signs of the proper AESs for the other non-concave estimates (IZEF (AR) estimates of Models 2, 3 and 4 and the I3SLS (AR) estimates of Models 3 and 4) follows the same pattern - capital and labour and labour and materials are everywhere substitutes while capital and materials are everywhere complements.
[39] Compare, for example, the IZEF (AR) and IZEF estimates of Model 2. The IZEF estimates are everywhere concave and produce a positive capital:materials AS over the whole sample. The IZEF (AR) estimates are concave in 1972, 1973, 1977 and 1978 yet produce proper capital:materials AESs which are everywhere negative.

A comparison of the mean[40] magnitude of economies of scale suggests that imposing the assumption of homotheticity as a maintained hypothesis produces lower estimates of scale. Table 5.2.6 reveals that models 1 and 2 which are non-homothetic, produce larger mean estimates of scale than the homothetic specifications suggesting the conclusion that imposing homotheicity biases estimates of scale downwards. This conclusion is, however, contradicted by the tests of the constant returns to scale hypothesis which reveal that for model 1 estimates of scale are statistically not different to unity.

Table 5.2.6 Mean economies of scale ISIC (3833)

	IZEF	IZEF (AR)	I3SLS	I3SLS (AR)
Model 1	1.331	1.620	1.332	--
Model 2	2.046	1.537	2.133	2.104
Model 3	1.005	1.011	0.988	1.017
Model 4	1.019	1.010	1.013	1.010
Model 5	1.083	1.36	1.073	1.504
Note: 1.For Models 4 and 5 economies of scale are calculated as $\varepsilon(x) = 1/\alpha_y$.				

An analysis of the trends of the estimates of scale which vary with output (Models 1, 2 and 3) reveals two interesting results. Firstly, the data suggest that non-concave estimates differ from concave estimates - albeit in a non-systematic manner. The second result is that depending on the assumption made about the nature of regressors different conclusions emerge regarding the trend of scale over the period.

A visual inspection of the trends of estimates of scale which vary with output[41] indicate differences in the volatility of concave and non-concave estimates of the same model. The observation regarding volatility is supported by comparisons of standard deviations. For models 1 and 3, concave estimates[42] produce estimates of scale which are more stable over the sample, while in the case of model 2, the concave estimates[43] are more volatile. An implication of this is that estimates of scale are not robust to the fulfilment of concavity. Indeed, given that both features of technology analysed display dissimilarities between the concave and non-concave sets of results care should be

[40] Estimates of scale emerging from the nonhomogenous specifications (Models 1, 2 and 3) vary with output. In order for comparisons to be drawn between these estimates and the estimates emerging from the homogenous specifications where scale is invariant to the level of output means of the nonhomogenous estimates were calculated. Trends are analysed below.

[41] See figures A5.5.6.5 to A5.5.6.7.

[42] For both models the IZEF and I3SLS estimates are concave over the whole sample.

[43] For this specification the IZEF (AR) estimates are non-concave.

taken in drawing inferences from estimates of features of technology emerging where concavity is violated.

Table 5.2.7 Standard deviations of scale estimates ISIC 3833

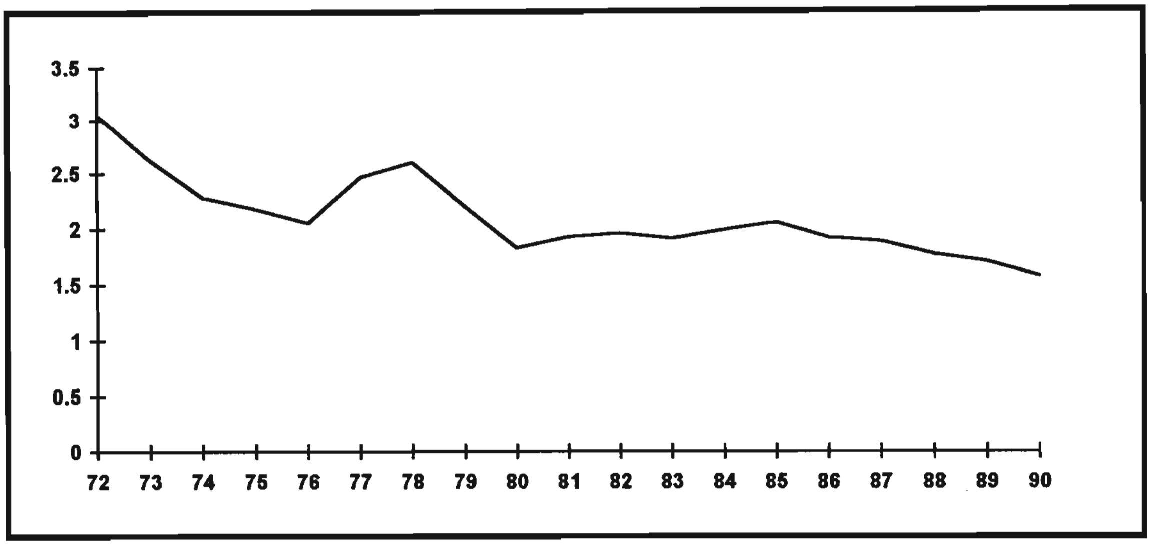
	IZEF	IZEF (AR)	I3SLS	I3SLS (AR)
Model 1	0.0855	0.4446	0.1247	--
Model 2	0.1064	0.0747	0.1387	0.3670
Model 3	0.0001	0.0498	0.0046	0.0532

Depending on the assumptions made about the nature of regressors, different conclusions emerge regarding trends in scale. Assuming that all regressors are exogenous, the ZEF class of estimates are valid and statistical and theoretical considerations suggest that the most appropriate estimates of scale are those emerging from Model 1. Not only are these estimates statistically not significantly different from unity but they fail to display any obvious trend[44].

Should output be endogenous and the 3SLS class valid, as argued above, the appropriate estimates of the characteristics of technology are those emerging from the I3SLS (AR) estimates of Model 2. These estimates indicate that while economies of scale do prevail, as witnessed by a mean greater than 2, they have decreased in magnitude over the sample (see figure 5.2.3 below), yielding the conclusion that should output be endogenous, the statistically and theoretically valid conclusion emerging is that economies of scale exist but are decreasing as output increases in the appliances sector. A disturbing feature of the results is that should one discount efficiency and employ the I3SLS estimates of either model 1 or 2 the conclusion that economies of scale are diminishing is rejected (see Figures A5.5.6.5 and A5.5.6.6).

Figure 5.2.3 Economies of Scale - ISIC 3833

[44] The IZEF results display an interesting feature: Model 2 generates estimates of scale which are greater than the estimates emerging from Model 1. This result is, however, intuitively reasonable as changes in costs due to technical change are likely to appear as scale effects where technical change is not explicitly modelled.



5.2.3. CONCLUSION - ISIC 3833

A number of important conclusions can be drawn from the preceding analysis. The data clearly suggest, contrary to *a priori* expectations, that all three inputs are substitutes for each other in the production of appliances, with the degree of substitutability greatest between capital and labour. Should output be exogenous, the sector appears not to experience scale advantages. If, however, output is endogenous the data suggest the presence of economies which appear to have decreased in size over the sample. A further result regarding scale regards the impact of different maintained hypotheses on the magnitude of estimates of scale: there is evidence to suggest that not only is imposing homotheticity statistically invalid, but in addition imposing this hypothesis *a priori* biases estimates of the magnitude of scale downward. The final conclusion emerging from an analysis of the Appliances sector regards the effect that concavity violations have on estimates of the features of technology. While no systematic differences appear between estimates which are concave and those which are non-concave - the features of technology emerging from non-concave estimates differ from those emerging from the concave results.

5.3 FURNITURE (ISIC 3320)

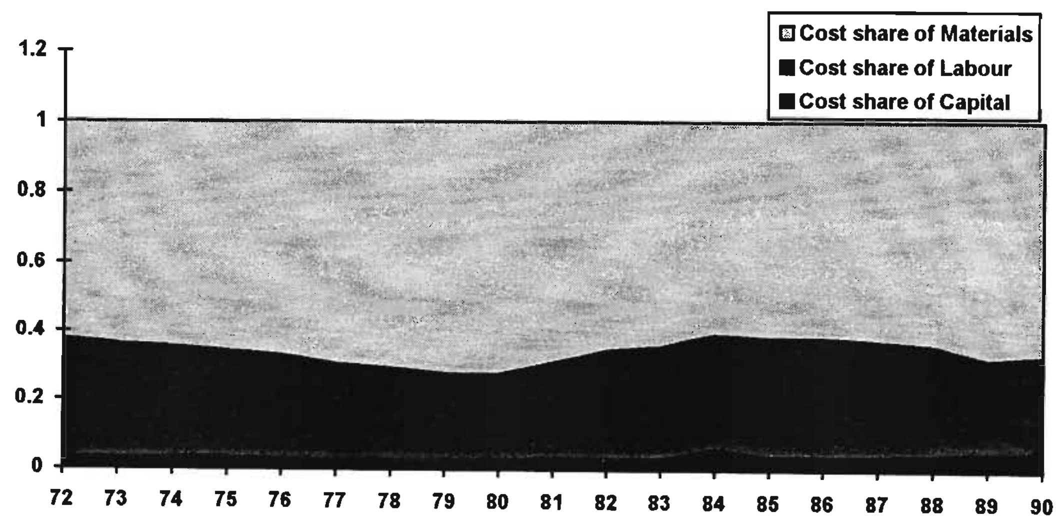
5.3.1 INTRODUCTION and *A PRIORI* DATA ANALYSIS- ISIC 3320

Of the three sectors analysed in this dissertation the furniture sector estimates are least useful. All four estimators generate parameter estimates which violate the concavity condition at every point in the sample. Non-concavity, as argued above, may imply that the cost function does not have the structural relationship postulated by duality theory to the underlying technological parameters of production thus rendering parameter estimates untenable (Fuss et al, 1978, p266). Indeed, the appliance sector results provide empirical support for this theoretical assertion that non-concave estimates are unreliable. Clearly, the results for the furniture sector ought to be treated with caution.

Prior to performing the regression analysis the furniture data was analysed in the same manner as the appliance sector data in an attempt to illicit expectations about scale and substitution. That analysis suggests that the sector experienced slightly diseconomies over the period under analysis. Furthermore, the data suggest complementarity between capital and both materials and labour and that labour may be an essential input.

The relative cost structure faced by the furniture sector is similar to that of the appliances sector: materials share in total costs is greatest while capital's share is the smallest (see Figure 5.3.1). While shifts in relative costs over the period under analysis, do not reveal any obvious patterns of complementarity/substitutability there is some evidence[45] to suggest complementarity between capital and labour. This suggestion is supported by trends in input-output coefficients.

Figure 5.3.1 Cost shares of Capital, Labour and Materials for ISIC 3320 (1972-1990)



The input-output coefficients of both capital and labour have followed the general trend of their respective cost shares over the period under analysis (Figures A.5.2.2.1 and A5.2.2.2 - Appendix 5.2), decreasing between 1974 and 1979 and increasing after 1980. More relevant for this analysis though, is the distinct similarity in the trend of

[45] Between 1974 and 1981 and again between 1982 and 1986 the cost shares of capital and labour changed in the same manner suggesting complementarity.

these coefficients which suggests the complementarity of capital and labour. The similarity in the trends is underlined by the fact that for both series the ratio of standard deviation to mean is 0.137 (see Table A5.2.2.2)

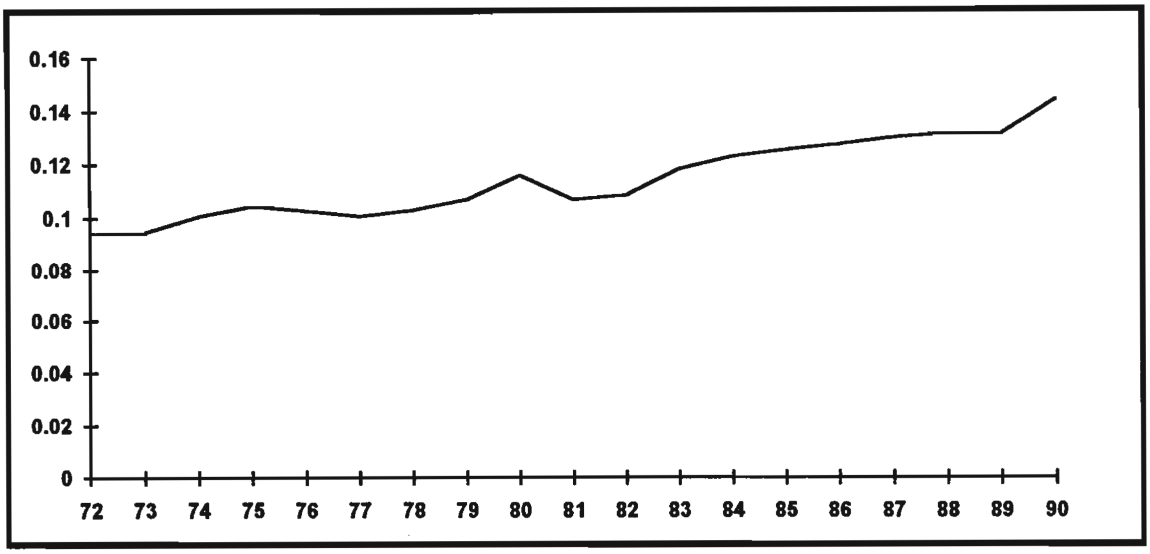
The above conclusion is supported by the correlation matrix of relative prices and input-output coefficients (Table 5.3.1) which shows a positive correlation (0.638) between the capital and labour input-output coefficients. While this is evidence of complementarity any such conclusion is potentially damaged by the positive relationship between the labour input-output coefficient and the relative price of capital. The matrix does, nevertheless, provide a clear conclusion regarding the relationship between capital and materials. The negative correlation between the input-output coefficient of capital (materials) and the relative price of materials (capital) together with the positive correlation between the capital and materials input-output coefficients indicate complementarity between capital and materials. The correlation matrix suggests a further conclusion: the positive correlation between the relative price of labour and its input-output coefficients suggests that labour is an essential input. This inference is strengthened by the fact that the correlation is near to zero (0.0269).

Table 5.3.1 Correlation matrix of Relative prices and Input-output coefficients (ISIC 3320)

	Relative price of Capital	Relative price of Labour	Relative price of Materials	Input-output coefficient of Capital	Input-output coefficient of Labour	Input-output coefficient of Materials
Relative price of Capital	1.0000					
Relative price of Labour	0.42187	1.0000				
Relative price of Materials	0.88345	0.52683	1.0000			
Input-output coefficient of Capital	-0.681E-01	-0.30511	-0.47790	1.0000		
Input-output coefficient of Labour	0.65548	0.269E-01	0.31411	0.63804	1.0000	
Input-output coefficient of Materials	-0.33967	-0.64542	-0.32032	0.26850	0.161E-01	1.0000

Turning to the possible presence of scale, a plot of real average costs over time (Figure 5.3.2) indicates a general increase in real average costs over the period under analysis, suggesting that diseconomies prevail in the sector.

Figure 5.3.2 Real Average Costs over the period 1972-1990 (ISIC 3320)



5.3.2 REGRESSION ANALYSIS

A disturbing feature of the regression results emerging from the furniture sector data is the persistent violation of concavity. Indeed for all models and the different estimators labour and materials both have positive own AES[46] (see Tables A5.4.4.1 to A5.4.4.28). A consequence of these results is that no confidence can be placed on estimates of the different features of technology emerging from these sets of results.

LR tests of the appropriate functional form for the different estimators generate the same results for the IZEF, IZEF (AR) and I3SLS estimators as those obtained for the appliance sector - that the more general specification is preferred (see Table A5.4.1.1 - Appendix 5.4). For both the IZEF and I3SLS estimates, the data indicate that the technology augmented specification (Model 1) is preferred. Despite manipulation of the convergence criterion, no estimates for Model 2 could be generated using the IZEF (AR) estimator. However, for that estimator LR tests reveal a preference for Model 1 over Models 3, 4 and 5. The I3SLS (AR) results contradict the usual preference for a more general specification. As with the Appliances sector no estimates could be generated for Model 1 using I3SLS (AR). A comparison of the estimates emerging from Models 2, 3, 4 and 5 revealed that Model 2 was preferred only to Model 5.

Although Durbin-Watson testing of the errors emerging from the IZEF and I3SLS estimates suggested the presence of autocorrelation in all specifications estimated (see Table A5.4.2.1 and Table A5.4.2.2), prompting the use of the IZEF (AR) and I3SLS (AR) estimators respectively, a comparison of the estimates generated when

[46] This is not the only violation of the concavity requirements, in most case at least two of the determinants of the 2×2 matrixes of AESs are negative while concavity requires that they be positive. For details on the requirements of concavity see Appendix 3.1.

autocorrelation was not accounted for to those estimates accounting for autocorrelation contradict this finding. LR testing of the null hypothesis that IZEF estimates were preferred to IZEF (AR) estimates (see Table A5.4.1.2) showed a rejection of the null hypothesis for Models 3 and 4. A similar test comparing I3SLS and I3SLS (AR) estimators (see Table A5.4.1.3) indicate that the I3SLS (AR) estimates are preferred only for Model 4. Combining the results of this set of LR tests with results of the model specification tests indicates that if output is exogenous (endogenous) the IZEF (I3SLS) estimates of Model 1 are preferred[47].

A comparison of proper AESs reveals a pattern which contradicts that emerging from the Appliances sector. Patterns of proper AESs differ according to model specification rather than estimator - which was the case in the Appliances sector. All four estimates of Models 3 and 4 reveal the same signs on the three proper AESs: capital:labour is everywhere positive; while both capital:labour and labour materials are everywhere negative. Moreover, all three estimates of Model 1 generate the same pattern of proper AESs: capital:labour and capital:materials are everywhere positive, while labour:materials is everywhere negative. The only specification where the pattern is inconsistent across all estimators is Model 2.

Using the earlier assertion that whether output is exogenous or endogenous the non-autocorrelation augmented estimates of Model 1 are most appropriate suggests that irrespective of the assumption made about output the same conclusion holds regarding the relationship between inputs: that capital can substitute for both labour and materials, and that materials and labour are complements in the production of furniture. This conclusion should, nevertheless, be qualified by the fact that it emerges from sets of non-concave estimates.

Tests of the hypothesis that economies of scale are constant for the different estimates of the various models reveal an interesting difference between the autocorrelation augmented estimates of the different specifications and those estimates which do not take autocorrelation into consideration. For both the IZEF and I3SLS estimates of the five models, the hypothesis of constant returns to scale are rejected (see Table A5.4.1.4 and Table A5.4.1.5). For the IZEF (AR) estimates, the hypothesis of constant

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[47] The choice of appropriate estimates when output is exogenous is clear. When output is endogenous LR tests are less revealing about a preferred model. LR tests comparing I3SLS estimates show Model 1 to be preferred. No I3SLS (AR) estimates of this model emerge preventing a test of the D-W indication that errors are autocorrelated. Given that for most specifications LR testing revealed a preference for estimates which assume errors are uncorrelated it is submitted that if output is endogenous that I3SLS estimates of Model 1 are most appropriate.

returns can be rejected only for Model 1, while for the I3SLS (AR) estimates constant returns to scale is valid for the Cobb-Douglas specification.

Comparing mean economies of scale of the different estimators of the five models (see Table 5.3.2) reveals some startling differences in the estimates of scale. While IZEF and IZEF (AR) estimates show a similar pattern to that emerging from the Appliance sector - models which are nonhomothetic (Models 1 and 2) producing larger estimates of scale than non-homothetic specifications (Models 3, 4 and 5), the same does not hold for I3SLS and I3SLS (AR) estimates. The data suggest that should output be exogenous and the ZEF class of estimators is preferred then non-homothetic specifications generate larger estimates of scale than homothetic specifications. If, output is endogenous and the 3SLS class of estimator is appropriate then homothetic specifications yield larger estimates of scale. Despite no clear pattern of influence the results indicate that homotheticity does effect the magnitude of estimates of scale.

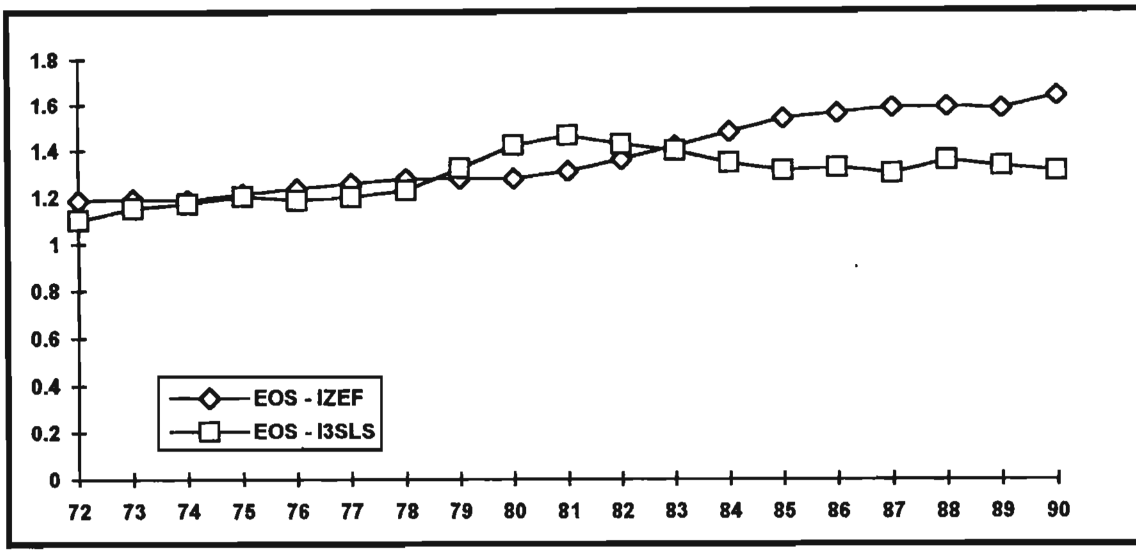
Table 5.3.2 Mean estimates of Economies of Scale - ISIC 3320

	IZEF	IZEF (AR)	I3SLS	I3SLS (AR)
Model 1	1.374*	1.358	1.290*	--
Model 2	1.361*	--	0.674*	-0.278
Model 3	1.191	1.287*	1.190*	0.183
Model 4	1.176	1.193*	1.188	1.253*
Model 5	1.263*	1.136	1.263*	1.116

Note: Asterixs indicate estimates which are statistically preferred on the basis of LR tests comparing autocorrelated augmented and non - autocorrelated augmented estimators.

As is the case with elasticity's of substitution economies of scale appear to be similar irrespective of the assumption made about output. Both the IZEF and I3SLS estimates of Model 1 yield similar mean estimates of scale (1.374 and 1.290 respectively). The trends of these scale estimates do, however, differ. A plot of the IZEF estimates of (see Figure 5.3.3) suggest that economies of scale have prevailed over the entire period under analysis and are increasing. The I3SLS estimates, on the other hand, indicate that since 1981 the magnitude of scale has decreased and any available economies are already exhausted.

Figure 5.3.3 Economies of Scale - IZEF and I3SLS estimates of Model 1 - (ISIC 3320)



5.3. 3 CONCLUSION - ISIC 3320

While the extensive violation of concavity in this sector prevents any confidence being placed on the estimates emerging here the data do provide some interesting results. The data suggest, contrary to *a priori* expectations, that capital and labour are substitutes in the production of furniture. Furthermore, patterns of proper AESs in this sector appear to be sensitive to model specification rather than to the estimator used, which was the case in the Appliances sector. The data also suggest that homotheticity does, indeed, influence estimates of economies of scale. The impact of homotheticity on scale estimates depends, however, on the nature of output. If output is exogenous then non-homothetic specifications generate larger estimates of scale than homothetic specifications. If, output is endogenous the converse holds. Furthermore the data suggest that if output is exogenous, further economies of scale may be realised but if output is endogenous economies of scale may already have been exhausted.

5.4 MOTOR VEHICLES, PARTS AND ACCESSORIES (ISIC 3840)

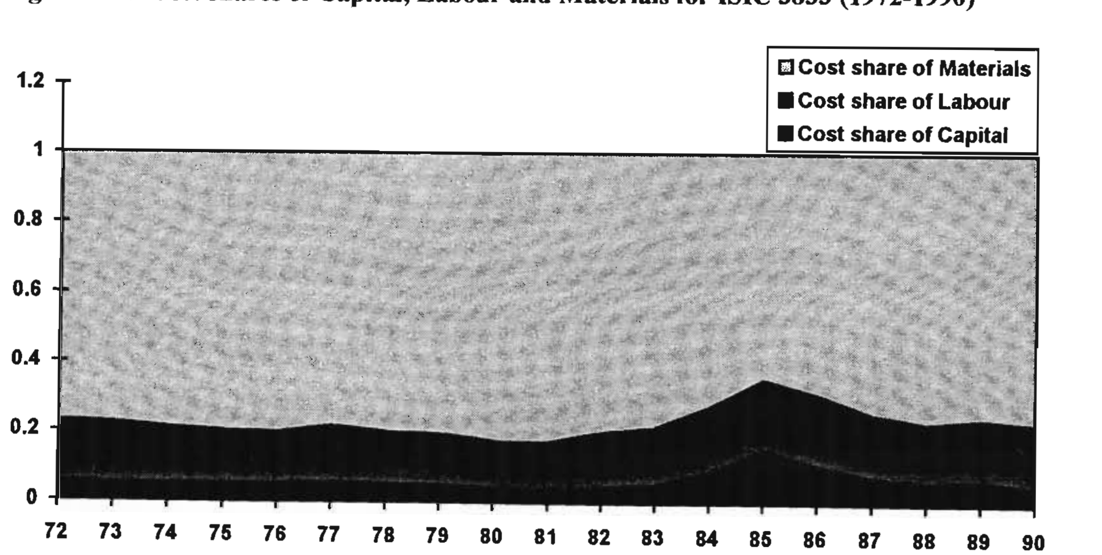
5.4.1 INTRODUCTION and *A PRIORI* ANALYSIS

As argued in the introduction to this dissertation the research questions posed by Moll (1991) can only be answered by comparing the features of technology of wage goods and luxury goods sectors. Motor Vehicles and Accessories (ISIC 3840) is used here as a suitable example of a luxury good industry which is believed to experience increasing returns to scale (see for example Standish and Galloway (1991)). An attempt to address Moll's assertions by comparing the technology of this industry with that of the two representative wage good industries (Appliances and Furniture) using the Translog system was, unfortunately, not entirely successful. As was the case with the Furniture sector the Motor Vehicles estimates are plagued by non-concavity. Indeed, none of the sets of parameter estimates fulfilled the requirements of quasi-concavity over the entire sample. A consequence of non-concavity is that very few theoretically valid estimates of the features of technology emerged. The data do, nevertheless, yield both quantitative and theoretical conclusions regarding Motor Vehicle technology and in addition they allow valid quantitative comparisons of the Motor vehicles and Appliances technologies. That comparison is deferred until section 5.5.

As was the case with the other two sectors manipulations of the data were analysed in an attempt to form *a priori* expectations about substitution possibilities and economies of scale. That analysis indicates that capital and labour may be complements and materials a substitute for both capital and materials in the production of Motor Vehicles. No obvious trends in real average costs prevent any expectations being formed about economies of scale.

The composition of total costs faced by the Motor Vehicles sector is similar to that faced by both the Appliances and Furniture sectors with materials contributing the most and capital the least to total costs. The contribution of materials to total costs in this sector, is however, considerably greater than in either of the other sectors. More relevant for this dissertation, in particular for identifying possible complementarity/supplementarity, are patterns in the relative cost shares of the three inputs. The cost shares of capital and labour display similar trends over the period both moving in a counter-cyclical manner to materials' cost share. These movements suggest complementarity of capital and labour and the substitutability between materials and both capital and labour.

Figure 5.4.1 Cost shares of Capital, Labour and Materials for ISIC 3833 (1972-1990)



The above inferences are supported by plots of the input-output coefficients of the three inputs (see figures A5.2.3.1 to A5.2.3.4 - Appendix 5.2). which reveal that between 1981 and 1985 the input-output coefficients of both capital and labour moved together increasing considerably (decreasing again after 1986) while, despite fluctuating, the material input-output coefficient generally declined over that period.

These conclusions are partially supported by the correlation matrix of input-output coefficients and relative prices (see Table 5.2.3) The positive correlation between capital and labours input-output ratios suggests that these two inputs are complements. A conclusion which is supported by the negative corrrelation between the input-output coefficient of capital and the relative price of labour. The conclusion is, however, contradicted by the positive correlation between the input-output coefficient of labour and the relative price of capital. The substitutability between materials and both labour and capital is suggested by the negative correlation's between the input-output coefficient of materials and those of labour and capital, respectively. It is, however, contradicted by the negative correlation between the relative price of materials and the input-output coefficients of capital and labour.

Table 5.2.3 Correlation matrix of Relative prices and Input-output coefficients (ISIC 3840)

	Relative price of Capital	Relative price of Labour	Relative price of Materials	Input-output coefficient of Capital	Input-output coefficient of Labour	Input-output coefficient of Materials
Relative price of Capital	1.0000					
Relative price of Labour	0.35293	1.0000				
Relative price of Materials	0.27280	0.59548	1.0000			
Input-output coefficient of Capital	0.55179	-0.249E-01	-0.45214	1.0000		
Input-output coefficient of Labour	0.41469	-0.30031	-0.40274	0.88435	1.0000	
Input-output coefficient of Materials	-0.120E-01	-0.42615	-0.880E-01	-0.794E-01	-0.4318E-01	1.0000

A plot of real average total costs over the period under consideration does not reveal nor discount the presence of scale effects. Real average total costs remained relatively constant and stable until 1982 whence they began fluctuating until 1986. After 1986 average costs decline only to increase again in 1988.

Figure 5.2.6 Real Average Costs over the period 1972-1990 (ISIC 3840)

5.4.2 REGRESSION ANALYSIS

The parameter estimates emerging for the Motor vehicles sector display a number of parallels to those of the Appliances sector. In both sectors the errors of the component equations are correlated[48] and more flexible forms are statistically preferred FOOTNOTE. Both these parallels are unremarkable. Autocorrelation is to be anticipated given that the data are time series. The statistical desirability of more flexible forms is reasonable given that the maintained hypotheses of the more restrictive specifications are untenable. A third parallel between both sets results is, however, remarkable. For both sectors, both non-autocorrelation augmented estimators generated results which display concavity more frequently than the results emerging from the autocorrelation augmented estimators. Moreover IZEF and I3SLS estimates display the same patterns of concavity[49] as do the IZEF (AR) and I3SLS (AR) estimates[50] [51] (see Tables A5.3.4.1 to A5.3.4.30.). The source of the more

[48] Durbin-Watson tests indicated the presence of autocorrelation in all specifications estimated using both IZEF and I3SLS (see Tables A5.3.2.1 and A5.3.2.2) prompting the use of IZEF (AR) and I3SLS (AR) respectively. LR tests of the appropriateness of these autocorrelation augmented estimators indicated their validity for all specifications (where comparisons were possible[48]) other than the Cobb-Douglas form (see Tables A5.3.1.2 and A5.3.1.3).

[49] For IZEF and I3SLS estimates, Model 1 was non-concave everywhere; Model 2 concave in 1972 and 1985 and 1986; Model 3 was concave in 1983, 1984, 1985, 1988 and 1989; and Model 4 was concave between 1983 and 1989.

[50] For IZEF (AR) and I3SLS (AR) estimates, Models 1, 2 and 3 were everywhere non-concave, while Model 4 was concave between 1983 and 1989.

[51] These similarities between the two non-autocorrelation augmented estimators and the two autcorrelation augmented estimators prevail to a large extent with respect to the sign and magnitude of the proper AESs (see Tables A5.3.4.1 to A5.3.4.30). Consider for example estimates of Model 2. The IZEF and I3SLS estimates produce the same pattern of AESs: both the capital:labour and capital:materials AESs are positive while the labour:materials AS is negative. The IZEF (AR) and

prevalent non-concavity among autocorrelation augmented estimates is not unfortunately clear.

An implication of the widespread violation of concavity is that the theoretically sound estimates of technology are not the most desirable statistically. LR and autocorrelation testing revealed a preference for autocorrelation augmented estimators of the most general specifications. All autocorrelation augmented estimators violated concavity at all points in the sample as did the non-autocorrelation estimates of Model 1. Given that priority ought to be afforded theoretical validity[52] the most appropriate[53] estimates for analysing scale and substitution are the IZEF and I3SLS estimates of Model 2 at three points in the sample 1972, 1985, 1986. This limited number of valid observations impairs an important advantage of using non-homothetic and non-homogenous models such as the translog: that estimates of scale vary with output.

5.4.2.1 ELASTICITY'S OF SUBSTITUTION - ISIC 3840

Proper AESs emerging from the IZEF and I3SLS estimates of Model 2 (Table 5.4.2.) yield two important results. Firstly the data indicate that whether output is exogenous (and IZEF estimates are appropriate) or endogenous (and I3SLS estimates are appropriate) capital and labour are strong substitutes, capital and materials are weak substitutes and labour and materials are weak complements. Secondly a comparison of the generally concave estimates with the generally non-concave estimates indicates that non-concavity does impact on estimates of substitution although no systematic impact is obvious.

Table 5.4.2 Selected elasticity's of substitution ISIC 3840

Capital-Labour Proper AES				
Year	IZEF estimates	I3SLS estimates	IZEF (AR) estimates	I3SLS (AR) estimates
1972	3.7682	3.7266	4.4940	4.4709
1985	3.5247	3.4964	3.4955	3.4779
1986	3.5344	3.4997	3.5510	3.5266
Capital-Materials Proper AES				
Year	IZEF estimates	I3SLS estimates	IZEF (AR) estimates	I3SLS (AR) estimates
1972	0.2459	0.2609	-0.5428	-0.5381

I3SLS (AR) estimates produce similar patterns of AESs which differ significantly from the non-autocorrelation estimates in that the capital:materials AES is everywhere negative.

[52] The impact of non-concavity on parameter estimates is unknown.

[53] These results are statistically flawed in that they are less efficient than estimates which take account of autocorrelation. Furthermore this specification is less flexible than Model 1 which was shown to be statistically the most desirable.

1985	0.2127	0.2263	-0.2865	-0.3382
1986	0.2367	0.2517	-0.3451	-0.3102
Labour-Materials Proper AES				
Year	IZEF estimates	I3SLS estimates	IZEF (AR) estimates	I3SLS (AR) estimates
1972	-0.0894	-0.0880	-0.1628	-0.1778
1985	-0.0030	-0.0016	-0.0414	-0.0970
1986	-0.0283	-0.0275	-0.1353	-0.1260

5.4.2.2 ECONOMIES OF SCALE - ISIC 3840

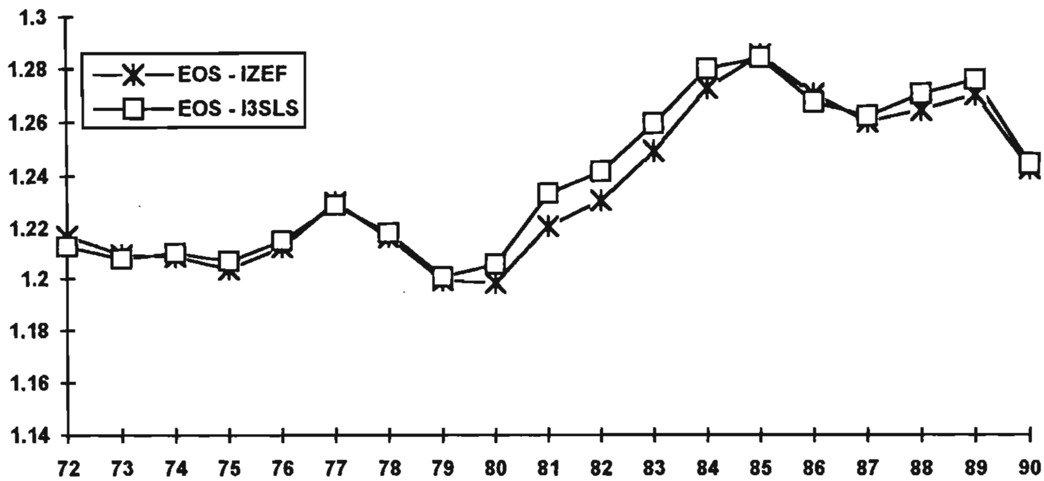
The conclusion emerging from the AESs that concavity does influence estimates of the different features of technology is supported by an analysis of estimates of economies of scale. Trends in estimates of scale emerging from the four estimators suggests the possibility of an inverse relationship between the stability of scale estimates and the concavity of the estimates. For this sector non-autocorrelation augmented estimators generated results which display concavity more frequently than the results emerging from the autocorrelation augmented estimators. For all three specifications where scale estimates vary with output (Models 1, 2 and 3) a comparison of trends indicates far greater deviations from the mean in the IZEF (AR) and I3SLS (AR) estimates than the IZEF and I3SLS estimates (see figures A5.3.6.5, A5.3.6.6 and A5.3.6.7.). This casual observation regarding the volatility of non-concave results is supported by a consideration of the standard deviations of scale estimates (see Table 5.4.3.). These data indicate that for all specifications the autocorrelation augmented estimators generate estimates of scale which vary more than the those estimates which ignore autocorrelation. Given that the autocorrelation augmented estimators violate concavity more generally it would appear that an inverse relationship exists between the volatility of estimates of scale and the fulfilment of the concavity requirements.

Table 5.4.3 Means and standard deviations of estimates of Economies of Scale - ISIC 3840

	IZEF		IZEF (AR)		I3SLS		I3SLS (AR)	
	MEAN	S.D	MEAN	S.D	MEAN	S.D.	MEAN	S.D.
Model 1	1.245	0.049	1.465	0.102	1.251	0.061	--	--
Model 2	1.234	0.027	0.774	0.235	1.238	0.028	1.317	0.049
Model 3	1.033	0.005	1.154	0.045	1.027	0.00001	1.121	0.054

As argued above the most appropriate estimates of scale are the IZEF and I3SLS estimates of Model 2 in 1972, 1985 and 1986. Those estimates indicate that irrespective of the assumption made about the nature of output the same conclusion emerges about trends of economies of scale: that economies of scale have improved over the period under analysis but may be on the wane (see Table 5.4.4). An interesting feature of the results is that while only a few points in the sample are valid plots of the whole sample (see figure 5.4.3) reveal very similar trends, suggesting that while economies of scale do prevail in the sector and increased considerably between the early 1970's and 1985, they appear to be following a downward trend towards the end of the sample.

Figure 5.4.3 Economies of Scale in Model 2 based on IZEF and I3SLS estimates - ISIC 3840



The conclusion emerging from the Appliances sector regarding the impact of homotheticity on estimates of scale is vindicated by the results emerging from this sector. The only non-homothetic estimates of scale which fulfil concavity are the IZEF and I3SLS estimates of Model 2 for 1972, 1985 and 1986. Conclusions regarding the impact of homotheticity emerge if one can compare these estimates of scale with concave estimates of homothetic models at the same points in the sample. Although the IZEF and I3SLS estimates of Model 3 are concave at five points in the sample only one point, 1985, is common with the estimates of Model 2. For that observation both the IZEF and I3SLS estimates of scale emerging from Model 2 are greater than those emerging from Model 3 (see Table 5.4.4. below). Furthermore, Model 5, the Cobb-Douglas specification which is homogenous and thus homothetic, fulfils concavity *a priori* and for both the IZEF (1.1056) and I3SLS (1.0868) parameter estimates produces estimates of scale which are smaller than the Model 2 estimates for 1976, 1985 and 1986.

Table 5.5.3 Economies of Scale, Models 2 and 3 - ISIC 3840

Year	Economies of Scale - Model 2 - IZEF estimates	Economies of Scale - Model 3 - IZEF estimates	Economies of Scale - Model 2 - I3SLS estimates	Economies of Scale - Model 3 - I3SLS estimates
1972	1.2168		1.2126	

1983		1.0254		1.0267
1984		1.0296		1.0267
1985	1.2842	1.0390	1.2833	1.0267
1986	1.2695		1.2665	
1988		1.0307		1.0267
1989		1.0313		1.0267

5.4.3. CONCLUSION - ISIC 3840

Despite the widespread violation of concavity among the estimates generated using the Motor Vehicles data a number of useful results emerge. Not only do these estimates support the theoretical conclusions emerging from the Appliance data but also provide estimates of the different features technology. These estimates support the conclusion emerging from the Appliances sector that imposing homotheticty is invalid and that it systematically biases estimates of scale downwards. Moreover the data indicate that non-concavity impacts on estimates of technology. Indeed there is evidence to suggest that an inverse relationship exists between non-concavity and stability of estimates of scale.

Turning to estimates of different features of technology the data indicate that during the period under analysis, and contrary to *a priori* expectations, that capital and labour were strong substitutes while capital and materials were weak substitutes. Furthermore as anticipated by the *a priori* analysis the estimates suggest that labour and materials are complements, albeit weak complements. While no trends regarding scale are obvious from an *a priori* analysis the regression analysis indicates that whether output is exogenous or endogenous that economies of scale prevail in the sector and have improved over the period under analysis but may be on the wane.

5.5 CONCLUSION

The application of the Translog system to the three sectors under analysis yields two broad groups of conclusions - a theoretical set and a quantitative set. The theoretical conclusions provide a useful contribution to attempts to quantify features of technology, and hence the use of empirical estimates to support a position in debate on industrial strategy. In this regard the data clearly indicate that the maintained hypothesis embodied in the most common functional forms are not only statistically inappropriate for the sectors analysed here but furthermore that imposing homotheticity inappropriately biases estimates of scale downwards. Whether the downward bias is always, in relative terms, of the same magnitude is an important empirical question. If it is not, using estimates of scale which emerge from homothetic functions such as the Cobb-Douglas to rank sectors according to the degree of scale experienced would be misleading.

A second theoretical conclusion emerging from the preceding analysis concerns the impact of non-concavity on estimates of the different features of technology. Duality theory suggests that non-concave estimates ought to be treated with care. A comparison of concave and non-concave estimates indicates that concavity violations effect estimates of both AESs and economies of scale. In general no systematic relationship exists between these features and concavity[54].

The persistent violation of concavity among the furniture and motor vehicles estimates considerably undermined the attempt of this dissertation to address the empirical questions posed by Moll's (1991) critique of inward industrialisation as a post-apartheid growth strategy for South Africa. That evidence which does emerge does, however, suggests that inward industrialisation is empirically appropriate. This conclusion, which emerges from a comparison of only two 'representative' sectors, should nevertheless be treated with respect.

Concavity violations prevented the use of any of the estimates for the furniture sector and prevented the use of the statistically most appropriate estimates for the other two sectors. For the appliance sector concavity violations merely meant that less efficient estimates had to be used. For the motor vehicles sector the concavity problem was more serious. Again no efficient estimates were available, but in addition the most suitable specifications were precluded and the 'second best' estimates were only appropriate at three points in the sample: 1972, 1985 and 1986. Consequently, in order for a valid comparison to be drawn between the appliances and motor vehicles sectors the 'second best' estimates of both sectors at the appropriate three points in the sample were analysed.

A comparison of estimates of economies of scale at these points in the sample (see Table 5.5.1) indicates that whether output was exogenous or endogenous both sectors experienced economies of scale which followed the same trends: increasing between 1972 and 1985 and then decreasing slightly between 1985 and 1986[55]. In terms of

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[54] While in the case of the Motor Vehicles sector non-concave estimates produce more volatile estimates of scale than concave estimates there does not appear to be any systematic relationship between concavity and the volatility of scale estimates in the Appliance sector.

[55] A disturbing feature of the conclusion emerging here is that the analysis of the Appliances sector suggested that if output was exogenous that returns to scale were constant and that if output were endogenous that scale was decreasing. Concavity problems prevented the use of the statistically most desirable estimates of the Appliance sector for the comparison with the Motor Vehicles sector. A

relative magnitude of economies of scale, these estimates indicate that the appliance sector enjoyed considerably larger economies of scale than the motor vehicle sector. Should one argue that the appliance sector is a truly representative wage good sector - there would appear to be some empirical evidence supporting the argument of inward industrialisation. That argument is premised on wage goods sectors having large economies of scale. A scale coefficient of 2 indicates that any percentage change in output is double the associated percentage change in cost - indicating that there are considerable benefits to be reaped from expanding production.

5.5.1 Comparative estimates of economies of scale, Appliances and Motor vehicles, Model 2

Year	IZEF estimates for Motor Vehicles (ISIC 3840)	I3SLS estimates for Motor Vehicles (ISIC 3840)	IZEF estimates for Appliances (ISIC 3833)	I3SLS estimates for Appliances (ISIC 3833)
1972	1.2168	1.2126	1.9069	1.9331
1985	1.2842	1.2833	2.0259	2.0955
1986	1.2695	1.2665	1.970	2.0388

The inward industrialisation argument is also premised on easy substitutability between capital and labour. These data indicate that both sectors use technologies where capital and labour are substitutes, albeit the degree of substitutability is greater in the motor vehicles sector.

5.5.2 Comparative estimates of proper AESs, Appliances and Motor vehicles, Model 2

Capital - Labour proper AS				
Year	IZEF estimates - ISIC 3840	I3SLS estimates - ISIC 3840	IZEF estimates - ISIC 3833	I3SLS estimates - ISIC 3833
1972	3.7682	3.7266	1.2263	1.3698
1985	3.5247	3.4964	1.3930	1.6402
1986	3.5344	3.4997	1.4245	1.6897
Capital - Materials AS				
Year	IZEF estimates - ISIC 3840	I3SLS estimates - ISIC 3840	IZEF estimates - ISIC 3833	I3SLS estimates - ISIC 3833
1972	0.2459	0.2609	1.3293	1.3228
1985	0.2127	0.2263	1.2956	1.2823
1986	0.2367	0.2517	1.2836	1.2693
Labour - Materials				
Year	IZEF estimates - ISIC 3840	I3SLS estimates - ISIC 3840	IZEF estimates - ISIC 3833	I3SLS estimates - ISIC 3833
1972	-0.0894	-0.0880	0.2161	0.2216
1985	-0.0030	-0.0016	0.2150	0.2210
1986	-0.0283	-0.0275	0.1918	0.1969

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consequence is that the conclusion of the comparison of these sectors rests on using 'second best' estimates of the Appliances sector.

CHAPTER 6

CONCLUSION

The characteristics of the technology used in the sub-sectors of South African manufacturing were central to the early debate concerning the appropriate industrial strategy for the post-apartheid era. The demand restructuring variant of the growth through redistribution approach, for example, was premised on the validity of at least two empirically testable conditions. Firstly, that realisable economies of scale are greater in labour-intensive wage goods production than in the production of luxury goods, and secondly that considerable possibilities for substituting labour for capital exist in manufacturing as a whole.

Moll (1991) has questioned the validity of these two conditions and hence the viability of the demand restructuring thesis. While a number of studies employing either the Cobb-Douglas (Cobb & Douglas, 1948) or CES (Arrow, Chenery, Minhas & Solow, 1961) functions have attempted to quantify these features of techonology their conclusions are potentially flawed. Both specifications impose the hypotheses of homogeneity, homotheticity and seperability *a priori*. The most obvious manifestation of these hypotheses is that they constrain estimates of the magnitude of scale and substitutability to a constant over the sample - constraints which are intuitively and theoretically unreasonable. Given that primary hypothesis tests regarding the magnitude of parameters depend on the validity of both the hypothesis being tested and the underlying maintained hypotheses, the presence of implausible maintained hypotheses is potentially damaging to any econometric analysis. Indeed, establishing the validity of imposed maintained hypothesis ought to be a central concern of any analysis.

This dissertation - by employing a flexible non-homothetic Translog function (Christensen, Jorgensen and Lau, 1971, 1973) - is explicity concerned with the issue of

This dissertation - by employing a flexible non-homothetic Translog function (Christensen, Jorgensen and Lau, 1971, 1973) - is explicity concerned with the issue of the validity of the most commonly imposed maintained hypotheses while simultaeously addressing Moll's (1991) critique of the demand restructuring thesis.

The introduction to the study of economic phenomena of the mathematical tool of duality has permitted the development of flexible functional forms which allow the simultaneous modelling of a number of diffferent aspects of technology while imposing few *a priori* maintained hypotheses. The Translog, an example of a function emerging from the duality between cost and production, does not impose the commonly maintained hypotheses of homtheticity, homogeneity or seperability *a priori*. Indeed these hypotheses can be imposed on the function through parameter constraints and thus are rendered testable.

The added flexibility afforded by duality is however, at a cost. Duality between cost and production is premised on the validity of certain behavioural assumptions which are manifested in the concavity of the cost function. Violation of these assumptions renders the estimates of technology theoretically void. While non-concavity hampered this application of the Translog the data do, nevertheless, yield some interesting and potentially useful results and point to a number of areas of interest for future researcch.

Broadly, three sets of conclusions emerge from this dissertation. The first set concerns the empirical questions posed by Moll (1991), the second the validity of homotheticity, homogeneity and seperablity and the impact of homotheticity on estimates of economies of scale while the third set concerns the impact of concavity on estimates of scale and substitution. All three sets of conclusions suggesst that useful insights could emerge form further applications of the Translog. Indeed all three conclusions would be stregthened by both an analysis of a wider cross section of sub-sectors together with an aattempt to improve the data.

In this regard an attempt could be made to create individual price of capital series for the different sectors analysed and longer series of the other variables used here.

While concavity problems prevented a complete comparison of scale and substitution in the sectors under analysis the data do sugggest a conclusion to both empirical questions posed by Moll (1991). The data indicate that economies of scale are greater in wage goods industries (as represented by the Electrical Appliances and Household goods sector) than in luxury production (as represented by the Motor Vehicles, Parts and Accessories sector). Furthermore the data indicate that labour is a good substitute in both sectors - albeit that the degree of substitutablity is greater in the Motor Vehicles sector. The fact that these conclusions emerge from a comparison of only two sectors and using less than the best estimates of the model, at very few points in the sample, points to the need for two areas of future research before the Translog will yield convincing contributions to the industrial strategy debate.

Firstly the present study could be extended to incorporate a broader cross- section of manufacturing sub-sectors. Using both the same data set and methods as employed here but extending the analysis to all wage and luxury goods manufacturing sub-sectors would serve two purposes. Not only would it provide more evidence to draw conclusions about the validity of demand restructuring but it may highlight causes of the concavity problems experienced here. Should an extended analysis yield estimates which are as badly conditioned as the etimates which emerge here a second area of research would present itself which may contribute to resolving the concavity issue. Concavity problems could stem from the IDC data set or the restrictive assumptions made here concerning the price of capital. Attempts should be made to develop more appropriate price of capital series and alternaative, longer series of the other variables.

The second important conclusion of the dissertation concerns the appropriate maintained hypotheses. The data clearly indicate that not only are the hypotheses of homogeneity, homotheticity and seperability invalid but that homotheticity biases estimates of economies of scale downwards. While the invalidity of the commonly imposed maintained hypotheses is sufficient to statistically discredit the results of applications using either the Cobb-Douglas or CES forms, the finding that imposing homotheticity inappropriately biases estimates of scale downwards presents an interesting avenue for research which could partially resurrect the usefulness of Cobb-interesting avenue for research which could partially resurrect the usefulness of Cobb-Douglas and CES based analyses. In this regard whether the downward bias emerging from homtheticity is always, in relative terms, of the same magnitude is an important consideration. If it is, estimates of scale which emerge from homothetic functions such as Cobb-Douglas could be used to give a crude ranking of sectors according to the degree of scale experienced. Pursuing the avenues for research and homogeneity on scale estimates and hence better use of traditional production function analyses for industrial strategy debates.

The final conclusion to emerge here concerns concavity. The data provide empirical evidence for the theoretical argument that non-concave estimates ought to be treated with care. Indeed, the data clearly indicate that concave estimates of scale and substitution differ from non-concave estimates - albeit in a non-systematic manner. Again the research programmes suggested above could prove useful for assessing more completely the effect of non-concavity on estimates of scale and substitution.

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APPENDIX 3

CONCAVITY TESTING

Quasi - concavity (concavity) is fulfilled if the matrix of AESs is negative semi-definite (negative definite). A matrix $[B]$ is negative semi-definite if $|B_1| \leq 0, |B_2| \geq 0, \ldots, |B_n| \leq 0$ if n is odd and $|B_n| \geq 0$ if n is even. The matrix will be negative definite if the weak inequalities are replaced by strong inequalities (Chiang, 1984, p394).

Where three inputs are employed: capital (k), labour (l) and materials (m), the matrix of AESs $|\sigma|$ will be negative semi-definite, and the Translog quasi-concave, if $|\sigma_1| \leq 0, |\sigma_2| \geq 0$ and $|\sigma_3| \leq 0$.

In order for the mechanics of the concavity test to be established the matrix of AESs needs to be obtained. The matrix of AESs, which is symmetrical, can take one of six forms:

$$\begin{bmatrix} \sigma_{kk} & \sigma_{lk} & \sigma_{mk} \\ \sigma_{kl} & \sigma_{ll} & \sigma_{ml} \\ \sigma_{km} & \sigma_{lm} & \sigma_{mm} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{kk} & \sigma_{mk} & \sigma_{lk} \\ \sigma_{km} & \sigma_{mm} & \sigma_{lm} \\ \sigma_{kl} & \sigma_{ml} & \sigma_{ll} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{ll} & \sigma_{kl} & \sigma_{ml} \\ \sigma_{lk} & \sigma_{kk} & \sigma_{mk} \\ \sigma_{lm} & \sigma_{km} & \sigma_{mm} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{ll} & \sigma_{ml} & \sigma_{kl} \\ \sigma_{lm} & \sigma_{mm} & \sigma_{km} \\ \sigma_{lk} & \sigma_{mk} & \sigma_{kk} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{mm} & \sigma_{km} & \sigma_{lm} \\ \sigma_{mk} & \sigma_{kk} & \sigma_{lk} \\ \sigma_{ml} & \sigma_{kl} & \sigma_{ll} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{mm} & \sigma_{lm} & \sigma_{km} \\ \sigma_{ml} & \sigma_{ll} & \sigma_{kl} \\ \sigma_{mk} & \sigma_{lk} & \sigma_{kk} \end{bmatrix}$$

Despite six alternative matrices of AESs existing only three elements appear as the first element on the principle diagonal: σ_{kk}, σ_{ll} and σ_{mm}. Thus in order for the first condition of negative semi-definiteness ($|\sigma_1| \leq 0$) to be fulfilled three conditions need to be satisfied: $\sigma_{kk} \leq 0, \sigma_{ll} \leq 0$ and $\sigma_{mm} \leq 0$

The second condition for negative semi-definiteness is that $|\sigma_2| \geq 0$. Six different $[\sigma_2]$ matrixes exist:

$$\begin{bmatrix} \sigma_{kk} & \sigma_{lk} \\ \sigma_{kl} & \sigma_{ll} \end{bmatrix} \quad \text{A3.1}$$

$$\begin{bmatrix} \sigma_{kk} & \sigma_{mk} \\ \sigma_{km} & \sigma_{mm} \end{bmatrix} \quad \text{A3.2}$$

$$\begin{bmatrix} \sigma_{ll} & \sigma_{kl} \\ \sigma_{lk} & \sigma_{kk} \end{bmatrix} \quad \text{A3.3}$$

$$\begin{bmatrix} \sigma_{ll} & \sigma_{ml} \\ \sigma_{lm} & \sigma_{mm} \end{bmatrix} \quad \text{A3.4}$$

$$\begin{bmatrix} \sigma_{mm} & \sigma_{km} \\ \sigma_{mk} & \sigma_{kk} \end{bmatrix} \quad \text{A3.5}$$

$$\begin{bmatrix} \sigma_{mm} & \sigma_{lm} \\ \sigma_{ml} & \sigma_{ll} \end{bmatrix} \quad \text{A3.6}$$

The determinants of matrixes A3.1 and A3.3; A3.2 and A 3.5; and A3.4 and A3.6 are identical. Testing whether $|\sigma_2| \geq 0$ will, therefore require finding the determinant of three matrixes (either A3.1 or A3.3; and either A3.2 or A3.5; and either A3.4 or A3.6).

The third, and final, condition for negative semi-definiteness is $|\sigma_3| \leq 0$. Although six different $[\sigma_3]$ exist, because they are symmetrical the value of the determinant will be identical in all cases. Hence the third condition merely requires a test of whether the determinant of any of the versions of the matrix of AESs is less than or equal to one.

APPENDIX 4.1

DATA SERIES

A4.1.1 ELECTRICAL APPLIANCES AND HOUSEHOLD GOODS (ISIC 3833)

Table A4.1.1.1 Cost of Capital: Electrical Appliances and Household Goods (ISIC 3833)

Year	Depreciation (δ)	Interest Paid (*ip*)	Interest Received (*ir*)	Cost of Capital (C_K)
1972	1000000	0	0	1000000
1973	1000000	1000000	0	2000000
1974	1000000	1000000	0	2000000
1975	2000000	1000000	0	3000000
1976	2000000	1000000	0	3000000
1977	2000000	1000000	0	3000000
1978	2000000	1000000	0	3000000
1979	2000000	2000000	0	4000000
1980	2000000	4000000	0	6000000
1981	3000000	2000000	0	5000000
1982	3000000	7000000	1000000	9000000
1983	4000000	6000000	1000000	9000000
1984	4000000	4000000	2000000	6000000
1985	3000000	10000000	2000000	11000000
1986	5000000	10000000	2000000	13000000
1987	6000000	6000000	2000000	10000000
1988	6000000	5000000	2000000	9000000
1989	6000000	8000000	3000000	11000000
1990	9000000	11000000	4000000	16000000

Note: 1. Cost of Capital (C_K) is calculated as $C_K = \delta + ip - ir$
2. Source of δ; *ip* and *ir*: IDC (1992)

Table A4.1.1.2 Cost of Labour and Price of Labour: Electrical Appliances and Household Goods (ISIC 3833)

Year	Cost of Labour (C_L)	Number of Labourers (L)	Average Price of Labour (AvP_L)	Price Index of Labour (P_L)
1972	7000000	3600	1944.4399	1.0000
1973	9000000	3980	2261.3066	1.1630
1974	12000000	4410	2721.0884	1.3994
1975	15000000	4760	3151.2605	1.6207
1976	17000000	5140	3307.3931	1.7009
1977	17000000	4710	3609.3418	1.8562
1978	19000000	4420	4298.6426	2.2107
1979	19000000	3910	4859.3350	2.4991
1980	20000000	3820	5235.6021	2.6926
1981	26000000	3870	6718.3462	3.4552
1982	29000000	3870	7493.5400	3.8538
1983	27000000	3470	7780.9800	4.0017
1984	25000000	3160	7911.3926	4.0687
1985	26000000	2710	9594.0957	4.9341
1986	29000000	2630	11026.6162	5.6708
1987	43000000	3040	14144.7373	7.2745
1988	67000000	4220	15876.7773	8.1652
1989	68000000	4120	16504.8535	8.4882
1990	72000000	4290	16783.2168	8.6314

Note: 1. $AvP_L = C_L / L$
2. $P_L = AvP_L / 1944.4399$
3. Source of C_L and L: IDC (1992)

Table A4.1.1.3. Output, Cost of Materials and Price of Materials: Electrical Appliances and Household Goods (ISIC 3833)

Year	Output (y)	Value-Added (va)	Real Cost of Materials (rC_M)	Material Price Index ('90=100) (P_{M1})	Material Price Index (1990=1) (P_{M2})	Cost of Materials (C_M)	Price Index of Materials (P_M)
1972	115000000	41000000	74000000	8.5000	0.0850	6290000	1.0000
1973	141000000	48000000	93000000	9.5000	0.0950	8835000	1.1176
1974	175000000	59000000	116000000	11.2000	0.1120	12992000	1.3176
1975	192000000	62000000	130000000	12.7000	0.1270	16510000	1.4941
1976	195000000	66000000	129000000	15.1000	0.1510	19479000	1.7765
1977	138000000	45000000	93000000	18.1000	0.1810	16833000	2.1294
1978	136000000	46000000	90000000	19.8000	0.1980	17820000	2.3294
1979	172000000	59000000	113000000	22.2000	0.2220	25086000	2.6118
1980	225000000	76000000	149000000	25.4000	0.2540	37846000	2.9882
1981	230000000	77000000	153000000	28.5000	0.2850	43605000	3.3529
1982	218000000	73000000	145000000	33.2000	0.3320	48140000	3.9059
1983	216000000	70000000	146000000	37.1000	0.3710	54165996	4.3647
1984	195000000	66000000	129000000	40.3000	0.4030	51987000	4.7412
1985	194000000	63000000	131000000	46.2000	0.4620	60522000	5.4353
1986	205000000	67000000	138000000	57.2000	0.5720	78936000	6.7294
1987	230000000	72000000	158000000	65.9000	0.6590	104122000	7.7529
1988	262000000	85000000	177000000	73.6000	0.7360	130272000	8.6588
1989	251000000	83000000	168000000	89.1000	0.8910	149688000	10.4824
1990	276000000	91000000	185000000	100.0000	1.0000	185000000	11.7647

Note 1: $rC_M = y - va$

2. $P_{M2} = P_{M1} / 100$
3. $C_M = rC_M \times P_{M2}$
4. $P_M = P_{M1} / 8.5$
5. Source of y, va, and P_{M1}: IDC (1992)

Table A4.1.1.4 Costs and Cost Shares of Capital, Labour and Materials: Electrical Appliances and Household Goods (ISIC 3833)

Year	Cost of Capital (C_K)	Cost of Labour (C_L)	Cost of Materials (C_M)	Total Costs (TC)	Cost Share of Capital (S_K)	Cost Share of Labour (S_L)	Cost Share of Materials (S_M)
1972	1000000	7000000	6290000	14290000	0.0700	0.4899	0.4402
1973	2000000	9000000	8835000	19835000	0.1008	0.4537	0.4454
1974	2000000	12000000	12992000	26992000	0.0741	0.4446	0.4813
1975	3000000	15000000	16510000	34510000	0.0869	0.4347	0.4784
1976	3000000	17000000	19479000	39479000	0.0760	0.4306	0.4934
1977	3000000	17000000	16833000	36833000	0.0814	0.4615	0.4570
1978	3000000	19000000	17820000	39820000	0.0753	0.4771	0.4475
1979	4000000	19000000	25086000	48086000	0.0832	0.3951	0.5217
1980	6000000	20000000	37846000	63846000	0.0940	0.3133	0.5928
1981	5000000	26000000	43605000	74605000	0.0670	0.3485	0.5845
1982	9000000	29000000	48140000	86140000	0.1045	0.3367	0.5589
1983	9000000	27000000	54165996	90166000	0.0998	0.2994	0.6007
1984	6000000	25000000	51987000	82987000	0.0723	0.3013	0.6264
1985	11000000	26000000	60522000	97522000	0.1128	0.2666	0.6206
1986	13000000	29000000	78936000	120936000	0.1075	0.2398	0.6527
1987	10000000	43000000	104122000	157122000	0.0636	0.2737	0.6627
1988	9000000	67000000	130272000	206272000	0.0436	0.3248	0.6316
1989	11000000	68000000	149688000	228688000	0.0481	0.2973	0.6546
1990	16000000	72000000	185000000	273000000	0.0586	0.2637	0.6777

Note: 1. $TC = C_K + C_L + C_M$
2. $S_K = C_K / TC$
3. $S_L = C_L / TC$
4. $S_M = C_M / TC$

A4.1.2 MOTOR VEHICLES, PARTS AND ACCESSORIES (ISIC 3840)

Table A4.2.1.1 Cost of Capital: Motor Vehicles, Parts and Accessories (ISIC 3840)

Year	Depreciation (δ)	Interest Paid (*ip*)	Interest Received (*ir*)	Cost of Capital (C_K)
1972	35000000	15000000	5000000	45000000
1973	40000000	18000000	7000000	51000000
1974	47000000	30000000	10000000	67000000
1975	58000000	41000000	16000000	83000000
1976	60000000	50000000	21000000	89000000
1977	52000000	59000000	17000000	94000000
1978	87000000	46000000	25000000	108000000
1979	78000000	56000000	15000000	119000000
1980	104000000	53000000	18000000	139000000
1981	136000000	66000000	28000000	174000000
1982	154000000	111000000	37000000	228000000
1983	208000000	118000000	36000000	290000000
1984	336000000	230000000	72000000	494000000
1985	602000000	324000000	112000000	814000000
1986	441000000	264000000	90000000	615000000
1987	500000000	197000000	81000000	616000000
1988	539000000	217000000	142000000	614000000
1989	668000000	351000000	214000000	805000000
1990	626000000	329000000	288000000	667000000

Note: 1. Cost of Capital (C_K) is calculated as $C_K = \delta + ip - ir$
2. Source of δ; *ip* and *ir*: IDC (1992)

Table A4.1.2.2 Cost of Labour and Price of Labour: Motor Vehicles, Parts and Accessories (ISIC 3840)

Year	Cost of Labour (C_L)	Number of Labourers (L)	Average Price of Labour (AvP_L)	Price Index of Labour (P_L)
1972	138000000	57510	2399.5828	1.0000
1973	165000000	61230	2694.7576	1.1230
1974	200000000	66590	3003.4539	1.2517
1975	238000000	70840	3359.6838	1.4001
1976	249000000	74450	3344.5266	1.3938
1977	263000000	70230	3744.8384	1.5606
1978	297000000	70700	4200.8486	1.7507
1979	342000000	73300	4665.7573	1.9444
1980	446000000	81310	5485.1802	2.2859
1981	617000000	90250	6836.5649	2.8491
1982	764000000	96260	7936.8379	3.3076
1983	849000000	89540	9481.7959	3.9514
1984	1009000000	90090	11199.9111	4.6674
1985	1055000000	84840	12435.1719	5.1822
1986	1162000000	81200	14310.3447	5.9637
1987	1399000000	80110	17463.4883	7.2777
1988	1675000000	79980	20942.7363	8.7277
1989	2061000000	80100	25730.3359	10.7228
1990	2280000000	82100	27771.0117	11.5733

Note: 1. $AvP_L = C_L / L$

2. $P_L = AvP_L / 2399.5828$

3. Source of C_L and L: IDC. (1992)

Table A4.1.2.3 Output, Cost of Materials and Price of Materials: Motor Vehicles, Parts and Accessories (ISIC 3840)

Year	Output (y)	Value-Added (va)	Real Cost of Materials (rC_M)	Material Price Index (1990=100) (P_{M1})	Material Price Index (1990=1) (P_{M2})	Cost of Materials (C_M)	Price Index of Materials (P_M)
1972	11025000000	2807000000	8218000000	7.5000	0.0750	616350000	1.0000
1973	12421000000	3287000000	9134000000	8.2000	0.0820	748988000	1.0933
1974	14093000000	3629000000	10464000000	9.5000	0.0950	994080000	1.2667
1975	15094000000	3803000000	11291000000	11.2000	0.1120	1264592000	1.4933
1976	13959000000	3478000000	10481000000	13.0000	0.1300	1362530000	1.7333
1977	11732000000	2918000000	8814000000	14.7000	0.1470	1295658000	1.9600
1978	13507000000	3485000000	10022000000	16.2000	0.1620	1623564000	2.1600
1979	13615000000	3341000000	10274000000	18.4000	0.1840	1890416000	2.4533
1980	17487000000	4340000000	13147000000	20.8000	0.2080	2734576000	2.7733
1981	21252000000	5341000000	15911000000	23.5000	0.2350	3739085000	3.1333
1982	19417000000	5008000000	14409000000	27.4000	0.2740	3948066000	3.6533
1983	18456000000	4633000000	13823000000	29.9000	0.2990	4133077000	3.9867
1984	15963000000	4298000000	11665000000	33.7000	0.3370	3931105000	4.4933
1985	11613000000	2968000000	8645000000	39.2000	0.3920	3388840000	5.2267
1986	10707000000	2771000000	7936000000	49.2000	0.4920	3904512000	6.5600
1987	13244000000	3206000000	10038000000	58.5000	0.5850	5872230000	7.8000
1988	15357000000	3801000000	11556000000	66.4000	0.6640	7673184000	8.8533
1989	15054000000	3728000000	11326000000	79.1000	0.7910	8958866000	10.5467
1990	13277000000	3278000000	9999000000	100.0000	1.0000	9999000000	13.3333

Note 1: $rC_M = y - va$
2. $P_{M2} = P_{M1} / 100$
3. $C_M = rC_M \times P_{M2}$
4. $P_M = P_{M1} / 7.5$
5. Source of y, va, and P_{M1}: IDC (1992)

Table A4.1.2.4 Costs and Cost Shares of Capital, Labour and Materials: Motor Vehicles, Parts and Accessories (ISIC 3840)

Year	Cost of Capital (C_K)	Cost of Labour (C_L)	Cost of Materials (C_M)	Total Costs (*TC*)	Cost Share of Capital (S_K)	Cost Share of Labour (S_L)	Cost Share of Materials (S_M)
1972	45000000	138000000	616350000	799350000	0.0563	0.1726	0.7711
1973	51000000	165000000	748988000	964988000	0.0529	0.1710	0.7762
1974	67000000	200000000	994080000	1261080000	0.0531	0.1586	0.7883
1975	83000000	238000000	1264592000	1585592000	0.0523	0.1501	0.7976
1976	89000000	249000000	1362530000	1700530000	0.0523	0.1464	0.8012
1977	94000000	263000000	1295658000	1652658000	0.0569	0.1591	0.7840
1978	108000000	297000000	1623564000	2028564000	0.0532	0.1464	0.8004
1979	119000000	342000000	1890416000	2351416000	0.0506	0.1454	0.8039
1980	139000000	446000000	2734576000	3319576000	0.0419	0.1344	0.8238
1981	174000000	617000000	3739085000	4530085000	0.0384	0.1362	0.8254
1982	228000000	764000000	3948066000	4940066000	0.0462	0.1547	0.7992
1983	290000000	849000000	4133077000	5272077000	0.0550	0.1610	0.7840
1984	494000000	1009000000	3931105000	5434105000	0.0909	0.1857	0.7234
1985	814000000	1055000000	3388840000	5257840000	0.1548	0.2007	0.6445
1986	615000000	1162000000	3904512000	5681512000	0.1082	0.2045	0.6872
1987	616000000	1399000000	5872230000	7887230000	0.0781	0.1774	0.7445
1988	614000000	1675000000	7673184000	9962184000	0.0616	0.1681	0.7702
1989	805000000	2061000000	8958866000	11824866000	0.0681	0.1743	0.7576
1990	667000000	2280000000	9999000000	12946000000	0.0515	0.1761	0.7724

Note: 1. $TC = C_K + C_L + C_M$

2. $S_K = C_K / TC$

3. $S_L = C_L / TC$

4. $S_M = C_M / TC$

A4.1.3 FURNITURE (ISIC 3320)

Table A4.1.3.1 Cost of Capital: Furniture (ISIC 3320)

Year	Depreciation (δ)	Interest Paid (*ip*)	Interest Received (*ir*)	Cost of Capital (C_K)
1972	3000000	3000000	1000000	5000000
1973	3000000	3000000	1000000	5000000
1974	4000000	5000000	1000000	8000000
1975	5000000	6000000	2000000	9000000
1976	5000000	5000000	2000000	8000000
1977	4000000	3000000	1000000	6000000
1978	5000000	4000000	1000000	8000000
1979	5000000	4000000	1000000	8000000
1980	8000000	5000000	1000000	12000000
1981	16000000	9000000	2000000	23000000
1982	15000000	9000000	2000000	22000000
1983	16000000	13000000	4000000	25000000
1984	24000000	29000000	4000000	49000000
1985	15000000	22000000	5000000	32000000
1986	23000000	20000000	5000000	38000000
1987	30000000	20000000	4000000	46000000
1988	34000000	33000000	7000000	60000000
1989	45000000	46000000	9000000	82000000
1990	51000000	57000000	12000000	96000000

Note: 1. Cost of Capital (C_K) is calculated as $C_K = \delta + ip - ir$
2. Source of δ; *ip* and *ir*: IDC (1992)

Table A4.1.3.2 Cost of Labour and Price of Labour: Furniture (ISIC 332)

Year	Cost of Labour (C_L)	Number of Labourers (L)	Average Price of Labour (AvP_L)	Price Index of Labour (P_L)
1972	47000000	25950	1811.1801	1.0000
1973	55000000	26820	2050.7085	1.1323
1974	68000000	29680	2291.1052	1.2650
1975	76000000	28220	2693.1255	1.4870
1976	79000000	28570	2765.1382	1.5268
1977	80000000	26700	2996.2546	1.6544
1978	83000000	25200	3293.6509	1.8186
1979	100000000	26320	3799.3921	2.0979
1980	135000000	31000	4354.8389	2.4046
1981	175000000	34900	5014.3267	2.7687
1982	212000000	36770	5765.5698	3.1835
1983	244000000	36400	6703.2969	3.7013
1984	259000000	36100	7174.5151	3.9615
1985	278000000	34180	8133.4111	4.4909
1986	324000000	35000	9257.1426	5.1114
1987	356000000	35000	10171.4287	5.6162
1988	414000000	37300	11099.1953	6.1285
1989	409000000	36300	11267.2178	6.2213
1990	495000000	36300	13636.3633	7.5294

Note: 1. $AvP_L = C_L / L$

2. $P_L = AvP_L / 1811.1801$

3. Source of C_L and L: IDC (1992)

Table A4.1.3.3 Output, Cost of Materials and Price of Materials: Furniture (ISIC 332)

Year	Output (y)	Value-Added (va)	Real Cost of Materials (rC_M)	Material Price Index (1990=100) (P_{M1})	Material Price Index (1990=1) (P_{M2})	Cost of Materials (C_M)	Price Index of Materials (P_M)
1972	1459000000	492000000	967000000	8.8000	0.0880	85096000	1.0000
1973	1613000000	551000000	1062000000	10.0000	0.1000	106200000	1.1364
1974	1708000000	591000000	1117000000	12.4000	0.1240	138508000	1.4091
1975	1762000000	600000000	1162000000	14.1000	0.1410	163842000	1.6023
1976	1700000000	582000000	1118000000	15.8000	0.1580	176644000	1.7955
1977	1717000000	607000000	1110000000	17.7000	0.1770	196470000	2.0114
1978	1782000000	641000000	1141000000	19.2000	0.1920	219072000	2.1818
1979	2059000000	746000000	1313000000	21.6000	0.2160	283608000	2.4545
1980	2346000000	839000000	1507000000	25.4000	0.2540	382778000	2.8864
1981	2411000000	900000000	1511000000	29.3000	0.2930	442723000	3.3295
1982	2231000000	890000000	1341000000	33.1000	0.3310	443871000	3.7614
1983	2081000000	761000000	1320000000	37.0000	0.3700	488400000	4.2045
1984	1888000000	694000000	1194000000	39.9000	0.3990	476406000	4.5341
1985	1759000000	647000000	1112000000	45.2000	0.4520	502624000	5.1364
1986	1788000000	680000000	1108000000	53.1000	0.5310	588348000	6.0341
1987	1734000000	631000000	1103000000	62.0000	0.6200	683860000	7.0455
1988	1894000000	707000000	1187000000	71.5000	0.7150	848705000	8.1250
1989	1894000000	648000000	1246000000	84.5000	0.8450	1052870000	9.6023
1990	1797000000	603000000	1194000000	100.0000	1.0000	1194000000	11.3636

Note 1: $rC_M = y - va$

2. $P_{M2} = P_{M1} / 100$
3. $C_M = rC_M \times P_{M2}$
4. $P_M = P_{M1} / 8.8$
5. Source of y, va, and P_{M1}: IDC (1992)

Table A4.1.3.4 Costs and Cost Shares of Capital, Labour and Materials: Furniture (ISIC 332)

Year	Cost of Capital (C_K)	Cost of Labour (C_L)	Cost of Materials (C_M)	Total Costs (*TC*)	Cost Share of Capital (S_K)	Cost Share of Labour (S_L)	Cost Share of Materials (S_M)
1972	5000000	47000000	85096000	137096000	0.0365	0.3428	0.6207
1973	5000000	55000000	106200000	166200000	0.0301	0.3309	0.6390
1974	8000000	68000000	138508000	214508000	0.0373	0.3170	0.6457
1975	9000000	76000000	163842000	248842000	0.0362	0.3054	0.6584
1976	8000000	79000000	176644000	263644000	0.0303	0.2996	0.6700
1977	6000000	80000000	196470000	282470000	0.0212	0.2832	0.6955
1978	8000000	83000000	219072000	310072000	0.0258	0.2677	0.7065
1979	8000000	100000000	283608000	391608000	0.0204	0.2554	0.7242
1980	12000000	135000000	382778000	529778000	0.0227	0.2548	0.7225
1981	23000000	175000000	442723000	640723000	0.0359	0.2731	0.6910
1982	22000000	212000000	443871000	677871000	0.0325	0.3127	0.6548
1983	25000000	244000000	488400000	757400000	0.0330	0.3222	0.6448
1984	49000000	259000000	476406000	784406000	0.0625	0.3302	0.6073
1985	32000000	278000000	502624000	812624000	0.0394	0.3421	0.6185
1986	38000000	324000000	588348000	950348000	0.0400	0.3409	0.6191
1987	46000000	356000000	683860000	1085860000	0.0424	0.3279	0.6298
1988	60000000	414000000	848705000	1322705000	0.0454	0.3130	0.6416
1989	82000000	409000000	1052870000	1543870000	0.0531	0.2649	0.6820
1990	96000000	495000000	1194000000	1785000000	0.0538	0.2773	0.6689

Note: 1. $TC = C_K + C_L + C_M$

2. $S_K = C_K / TC$

3. $S_L = C_L / TC$

4. $S_M = C_M / TC$

APPENDIX 4.2

INSTRUMENTAL VARIABLES

Table A4.2.1 Instrumental Variables: Prices and Employment data.

Year	C.P.I	C.P.I. '72=1	App. Price Index	App. Price Index `72=1	Mot. Price Index	Mot. Price Index `72=1	Furn. Price Index	Furn. Price Index '72=1	Man. Emp.	Emp. Index `72=1
1972	10.40	1.000	13.10	1.000	7.70	1.000	14.6	1.000	1135160	1.000
1973	11.40	1.096	14.30	1.092	8.20	1.065	15.9	1.089	1196160	1.054
1974	12.70	1.221	15.70	1.198	8.80	1.143	18.2	1.247	1266170	1.115
1975	14.60	1.404	17.10	1.305	10.40	1.351	19.7	1.349	1313140	1.157
1976	16.00	1.538	19.20	1.466	12.20	1.584	22.0	1.507	1355210	1.194
1977	17.90	1.721	20.70	1.580	13.70	1.779	23.8	1.630	1317120	1.160
1978	19.80	1.904	21.60	1.649	15.10	1.961	24.6	1.685	1314650	1.158
1979	22.40	2.154	24.20	1.847	16.90	2.195	25.9	1.774	1336070	1.177
1980	25.50	2.452	27.30	2.084	18.60	2.416	28.2	1.932	1423180	1.254
1981	29.30	2.817	28.50	2.176	21.30	2.766	36.3	2.486	1510480	1.331
1982	33.70	3.240	35.30	2.695	25.20	3.273	40.8	2.795	1544280	1.360
1983	37.80	3.635	43.00	3.282	23.10	3.000	44.9	3.075	1466890	1.292
1984	42.20	4.058	44.30	3.382	31.50	4.091	49.3	3.377	1478520	1.302
1985	49.10	4.721	54.30	4.145	37.60	4.883	53.7	3.678	1428990	1.259
1986	58.20	5.596	65.90	5.031	51.00	6.623	60.7	4.158	1415560	1.247
1987	67.60	6.500	69.80	5.328	62.90	8.169	70.1	4.801	1429270	1.259
1988	76.30	7.337	75.60	5.771	73.20	9.506	77.0	5.274	1442580	1.271
1989	87.50	8.413	80.90	6.176	80.70	10.48	90.2	6.178	1449760	1.277
1990	100.0	9.615	100.0	7.634	100.0	12.99	100.0	6.849	1451610	1.279

Notes: 1. Source of C.P.I, Appliance Prices, Motor Vehicle Prices, Furniture Prices and Manufacturing employment IDC (1992) *Sectoral Data Series 1972 to 1990*3.

2. Emp Index (`72=1) are manufacturing employment numbers indexed with 1972 being the base year.

Table A4.2.2 Instrumental variables: Government and Private Consumption data

Year	Real Pvt. Con: `85 prices	Nom Pvt Con	Pvt Con `72=1	Real Govt Con: `85 prices	Nom Govt Con	Govt Con `72=1
1972	43452	9503	1.000	10991	1937	1.000
1973	46746	11134	1.089	11475	2219	1.097
1974	49198	13106	1.218	12363	2802	1.286
1975	50984	15167	1.360	13859	3687	1.510
1976	51788	17116	1.511	14621	4465	1.733
1977	53219	18914	1.625	15181	5034	1.882
1978	52180	21086	1.848	15283	5526	2.052
1979	53472	24427	2.089	16017	6329	2.242
1980	58065	30797	2.425	17477	8158	2.649
1981	62306	38086	2.795	17808	9877	3.147
1982	63613	44564	3.203	18934	12361	3.704
1983	65525	51596	3.600	19277	14115	4.155
1984	68536	59705	3.983	20589	17927	4.941
1985	66167	66167	4.572	21297	21297	5.674
1986	66272	77965	5.379	21785	25672	6.687
1987	68827	93353	6.202	22600	30599	7.683
1988	72453	111324	7.026	22975	35276	8.712
1989	74191	131309	8.093	23786	44308	10.57
1990	75319	152475	9.256	24025	50476	11.92

Note 1. Source of Private Consumption Expenditure and Government Consumption Expenditure is: SARB (1991) *South Africa's National Accounts 1946 to 1990.*

2. Pvt. Con (`72=1) emerges from applying the formula (Nom. Pvt. Con./Real Pvt. Con: `85 prices) and indexing the series by setting the 1972 value to 1.

3. Nom. Pvt. Con is Nominal Private Consumption Expenditure and Real Pvt. Con: `85 prices is Real Private Consumption Expenditure with 1985 prices as the base.

4 Govt Con Nom. is Nominal Government Consumption Expenditure and Real Govt. Con: `85 prices is Real Government Consumption Expenditure with 1985 prices as the base.

Table A4.2.3 Instrumental Variables: Gross Domestic Product and Disposable Income data

Year	Nom GDP	Real GDP: `85 prices	GDP DEF. `72=1	Nom PDY	PDY Index `72=1
1972	15535	87599	1.000	10836	1.000
1973	19218	91604	1.183	12062	0.941
1974	23690	97202	1.374	14217	0.955
1975	26646	98850	1.520	16810	1.021
1976	30020	101074	1.675	18131	0.999
1977	33263	100979	1.857	21342	1.060
1978	38247	104023	2.073	22884	1.019
1979	45772	107966	2.391	27595	1.065
1980	60328	115114	2.955	34710	1.084
1981	71080	121285	3.305	39067	1.091
1982	80531	120820	3.758	45393	1.115
1983	91457	118589	4.349	52696	1.118
1984	107221	124636	4.851	62556	1.190
1985	123126	123126	5.639	70760	1.158
1986	142135	123148	6.508	80106	1.136
1987	164524	125735	7.378	98091	1.227
1988	198110	130888	8.535	115248	1.246
1989	232532	133636	9.812	133674	1.257
1990	262650	132405	11.19	154888	1.278

Note: 1. Source of Gross Domestic Product and Personal Disposable Income is: SARB (1991) *South Africa's National Accounts 1946 to 1990.*

2 GDP DEF (`72=1) emerges from applying the formula (Nom. GDP/Real GDP: `85 prices) and indexing the series by setting the 1972 value to 1. Nom GDP is Nominal Gross Domestic Product and Real GDP: `85 prices is Real GDP with 1985 as the base year. GDP DEF (`72=1) is therefore the GDP deflator with 1972 as the base year.

3 PDY Index (`72=1) emerges from applying the formula (Nom PDY/GDP DEF (`72 = 1)) and indexing the series by setting the 1972 value to 1. Nom. PDY is Nominal Personal Disposable Income.

APPENDIX 5.1

FORMS OF ALTERNATIVE SPECIFICATIONS OF THE TRANSLOG SYSTEM[1]

5.1.1 MODEL 1: TECHNICAL CHANGE AUGMENTED NONHOMOTHETIC FORM

Cost Function:

$$\ln C = \ln \alpha_0 + \alpha_y \ln y + \tfrac{1}{2}\gamma_{yy}(\ln y)^2 + \alpha_K \ln p_K + \alpha_L \ln p_L + \alpha_M \ln p_M + \tfrac{1}{2}\gamma_{KK}(\ln p_K)^2$$

$$+ \tfrac{1}{2}\gamma_{KL} \ln p_K \ln p_L + \tfrac{1}{2}\gamma_{KM} \ln p_K \ln p_M + \tfrac{1}{2}\gamma_{LL}(\ln p_L)^2 + \tfrac{1}{2}\gamma_{LM} \ln p_L \ln p_M + \tfrac{1}{2}\gamma_{MM}(\ln p_M)^2$$

$$+ \gamma_{Ky} \ln p_K \ln y + \gamma_{Ly} \ln p_L \ln y + \gamma_{My} \ln p_M \ln y + \alpha_t t + \tfrac{1}{2}\gamma_{tt} t^2 + \gamma_{yt} t \ln y + \gamma_{tK} t \ln p_K + \gamma_{tL} t \ln p_L$$

$$+ \gamma_{tM} t \ln p_M$$

Cost Shares:

$$\frac{\partial \ln C}{\partial \ln p_K} = S_K = \alpha_K + \gamma_{Ky} \ln y + \gamma_{KK} \ln p_K + \gamma_{KL} \ln p_L + \gamma_{KM} \ln p_M + \gamma_{tK} t$$

$$\frac{\partial \ln C}{\partial \ln p_L} = S_L = \alpha_L + \gamma_{Ly} \ln y + \gamma_{LK} \ln p_K + \gamma_{LL} \ln p_L + \gamma_{LM} \ln p_M + \gamma_{tL} t$$

$$\frac{\partial \ln C}{\partial \ln p_M} = S_M = \alpha_M + \gamma_{My} \ln y + \gamma_{MK} \ln p_K + \gamma_{ML} \ln p_L + \gamma_{MM} \ln p_M + \gamma_{tM} t$$

[1] Specifications provided below are theroretical specifications. As discussed in Chapters 3 and 4, the econometric estimation of the Translog requires the dropping of one of the share equations and normalisation of the prices and total costs in the remaining system by the price of the variable whose share equation has been dropped. The econometric estimation here involved dropping the cost share of materials.

5.1.2 MODEL 2: NONHOMOTHETIC FORM (CONSTANT TECHNOLOGY)

Cost Function:

$$\ln C = \ln\alpha_0 + \alpha_y \ln y + \tfrac{1}{2}\gamma_{yy}(\ln y)^2 + \alpha_K \ln p_K + \alpha_L \ln p_L + \alpha_M \ln p_M + \tfrac{1}{2}\gamma_{KK}(\ln p_K)^2$$

$$+\tfrac{1}{2}\gamma_{KL}\ln p_K \ln p_L + \tfrac{1}{2}\gamma_{KM}\ln p_K \ln p_M + \tfrac{1}{2}\gamma_{LL}(\ln p_L)^2 + \tfrac{1}{2}\gamma_{LM}\ln p_L \ln p_M + \tfrac{1}{2}\gamma_{MM}(\ln p_M)^2$$

$$+\gamma_{Ky}\ln p_K \ln y + \gamma_{Ly}\ln p_L \ln y + \gamma_{My}\ln p_M \ln y$$

Cost Shares:

$$\frac{\partial \ln C}{\partial \ln p_K} = S_K = \alpha_K + \gamma_{Ky}\ln y + \gamma_{KK}\ln p_K + \gamma_{KL}\ln p_L + \gamma_{KM}\ln p_M$$

$$\frac{\partial \ln C}{\partial \ln p_L} = S_L = \alpha_L + \gamma_{Ly}\ln y + \gamma_{LK}\ln p_K + \gamma_{LL}\ln p_L + \gamma_{LM}\ln p_M$$

$$\frac{\partial \ln C}{\partial \ln p_M} = S_M = \alpha_M + \gamma_{My}\ln y + \gamma_{MK}\ln p_K + \gamma_{ML}\ln p_L + \gamma_{MM}\ln p_M$$

5.1.3 MODEL 3: HOMOTHETIC FORM (CONSTANT TECHNOLOGY)

Cost Function:

$$\ln C = \ln\alpha_0 + \alpha_y \ln y + \tfrac{1}{2}\gamma_{yy}(\ln y)^2 + \alpha_K \ln p_K + \alpha_L \ln p_L + \alpha_M \ln p_M + \tfrac{1}{2}\gamma_{KK}(\ln p_K)^2$$

$$+\tfrac{1}{2}\gamma_{KL}\ln p_K \ln p_L + \tfrac{1}{2}\gamma_{KM}\ln p_K \ln p_M + \tfrac{1}{2}\gamma_{LL}(\ln p_L)^2 + \tfrac{1}{2}\gamma_{LM}\ln p_L \ln p_M + \tfrac{1}{2}\gamma_{MM}(\ln p_M)^2$$

Cost Shares:

$$\frac{\partial \ln C}{\partial \ln p_K} = S_K = \alpha_K + \gamma_{KK}\ln p_K + \gamma_{KL}\ln p_L + \gamma_{KM}\ln p_M$$

$$\frac{\partial \ln C}{\partial \ln p_L} = S_L = \alpha_L + \gamma_{LK}\ln p_K + \gamma_{LL}\ln p_L + \gamma_{LM}\ln p_M$$

$$\frac{\partial \ln C}{\partial \ln p_M} = S_M = \alpha_M + \gamma_{MK}\ln p_K + \gamma_{ML}\ln p_L + \gamma_{MM}\ln p_M$$

5.1.4 MODEL 4: HOMOGENOUS FORM (CONSTANT TECHNOLOGY)

Cost Function:

$$\ln C = \ln \alpha_0 + \alpha_y \ln y + \alpha_K \ln p_K + \alpha_L \ln p_L + \alpha_M \ln p_M + \tfrac{1}{2}\gamma_{KK}(\ln p_K)^2$$

$$+ \tfrac{1}{2}\gamma_{KL} \ln p_K \ln p_L + \tfrac{1}{2}\gamma_{KM} \ln p_K \ln p_M + \tfrac{1}{2}\gamma_{LL}(\ln p_L)^2 + \tfrac{1}{2}\gamma_{LM} \ln p_L \ln p_M + \tfrac{1}{2}\gamma_{MM}(\ln p_M)^2$$

Cost Shares:

$$\frac{\partial \ln C}{\partial \ln p_K} = S_K = \alpha_K + \gamma_{KK} \ln p_K + \gamma_{KL} \ln p_L + \gamma_{KM} \ln p_M$$

$$\frac{\partial \ln C}{\partial \ln p_L} = S_L = \alpha_L + \gamma_{LK} \ln p_K + \gamma_{LL} \ln p_L + \gamma_{LM} \ln p_M$$

$$\frac{\partial \ln C}{\partial \ln p_M} = S_M = \alpha_M + \gamma_{MK} \ln p_K + \gamma_{ML} \ln p_L + \gamma_{MM} \ln p_M$$

5.1.5 MODEL 5: VARIABLE RETURNS TO SCALE COBB-DOUGLAS FORM

Cost Function:

$$\ln C = \ln \alpha_0 + \alpha_y \ln y + \alpha_K \ln p_K + \alpha_L \ln p_L + \alpha_M \ln p_M$$

Cost Shares:

$$\frac{\partial \ln C}{\partial \ln p_K} = S_K = \alpha_K$$

$$\frac{\partial \ln C}{\partial \ln p_L} = S_L = \alpha_L$$

$$\frac{\partial \ln C}{\partial \ln p_M} = S_M = \alpha_M$$

APPENDIX 5.2 *A PRIORI* DATA ANALYSIS

A 5.2.1 ELECTRIC APPLIANCES AND HOUSEHOLD GOODS (ISIC 3883)

Table A5.2.1.1 Input-output coefficients, relative input prices and real average costs (ISIC 3833)

Year	Input-output of Capital	Input-output of Labour	Input-output of Materials	Relative price of Capital	Relative price of Labour	Relative price of Materials	Real Average Costs
1972	0.4174	0.000031304	0.6435	1.0000	1.0000	1.0000	0.1243
1973	0.4184	0.000028227	0.6596	0.9509	1.0654	1.0239	0.1289
1974	0.4000	0.0000252	0.6629	1.0630	1.1677	1.0994	0.1287
1975	0.4167	0.000024792	0.6771	1.1729	1.2416	1.1446	0.1377
1976	0.3795	0.000026359	0.6615	1.4526	1.1605	1.2121	0.1381
1977	0.4783	0.00003413	0.6739	1.6986	1.1747	1.3476	0.1689
1978	0.4191	0.0000325	0.6618	1.6357	1.3408	1.4127	0.1776
1979	0.3081	0.000022733	0.6570	1.4632	1.3528	1.4138	0.1513
1980	0.2311	0.000016978	0.6622	1.5067	1.2921	1.4339	0.1362
1981	0.2174	0.000016826	0.6652	1.9140	1.5882	1.5412	0.1491
1982	0.2385	0.000017752	0.6651	1.9171	1.4302	1.4495	0.1466
1983	0.2176	0.000016065	0.6759	1.8285	1.2191	1.3297	0.1272
1984	0.2205	0.000016205	0.6615	2.2226	1.2032	1.4020	0.1258
1985	0.2268	0.000013969	0.6753	2.2096	1.1904	1.3113	0.1213
1986	0.2098	0.000012829	0.6732	2.1477	1.1273	1.3377	0.1173
1987	0.1783	0.000013217	0.6870	2.3034	1.3653	1.4551	0.1282
1988	0.1565	0.000016107	0.6756	2.4835	1.4149	1.5004	0.1364
1989	0.1633	0.000016414	0.6693	2.8010	1.3745	1.6974	0.1475
1990	0.1486	0.000015543	0.6703	2.5359	1.1307	1.5412	0.1296

Note: 1 Input-output coefficients were calculated by defining the quantities of factors of production as the constant Rand expenditure on the different factors and the quantity of output as the constant Rand value of output.

2. Relative prices are defined as the ratio of the price index of each input to the price index of output as calculated by the IDC (1992)

3. Real average costs is defined as the ratio of nominal average costs to the price index of output, where nominal average costs are calculated as the ratio of current value of total costs to the constant rand value of output.

Table A5.2.1.2 Descriptive statistics of relative factor prices and input-output coefficients (ISIC 3833)

Variable	Mean	Standard Deviation.
Relative price of Capital	1.8056	0.54839
Relative price of Labour	1.2547	0.14454
Relative price of Materials	1.3502	0.18409
Input-output coefficient of Capital	0.28662	0.11039
Input-output coefficient of Labour	0.000020903	0.0000069507
Input-output coefficient of Materials	0.66725	0.96169E-02

Figure A.5.2.1.1 Input-output coefficient of capital (ISIC 3833)

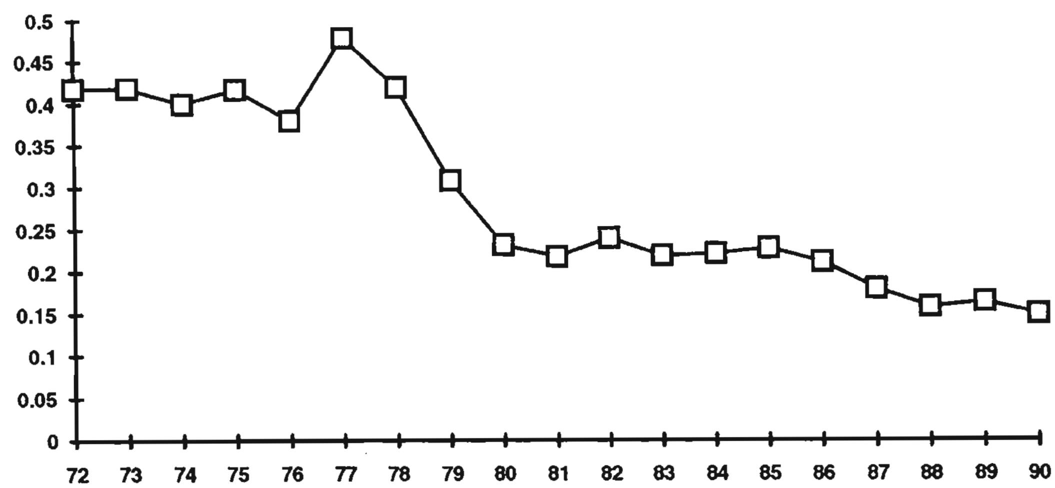


Figure A.5.2.1.2 Input-output coefficient of labour (ISIC 3833)

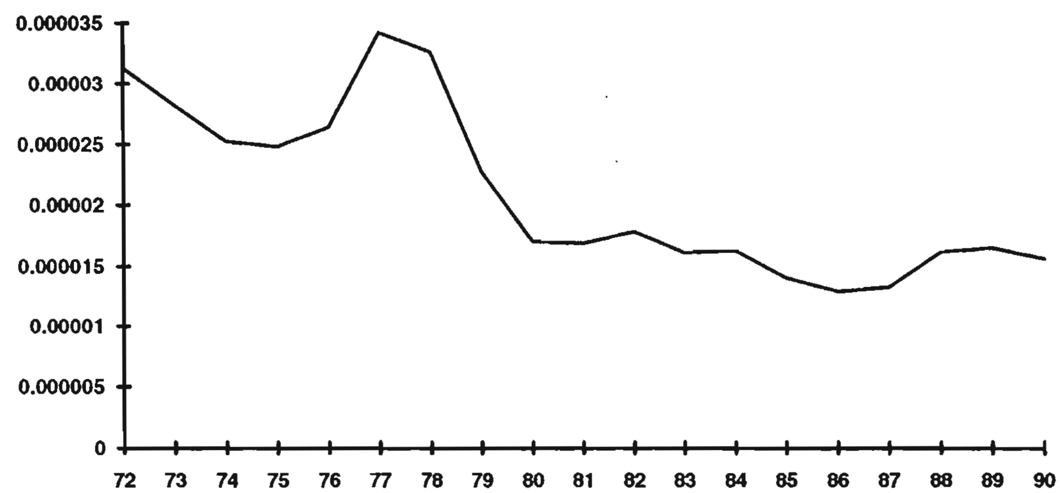
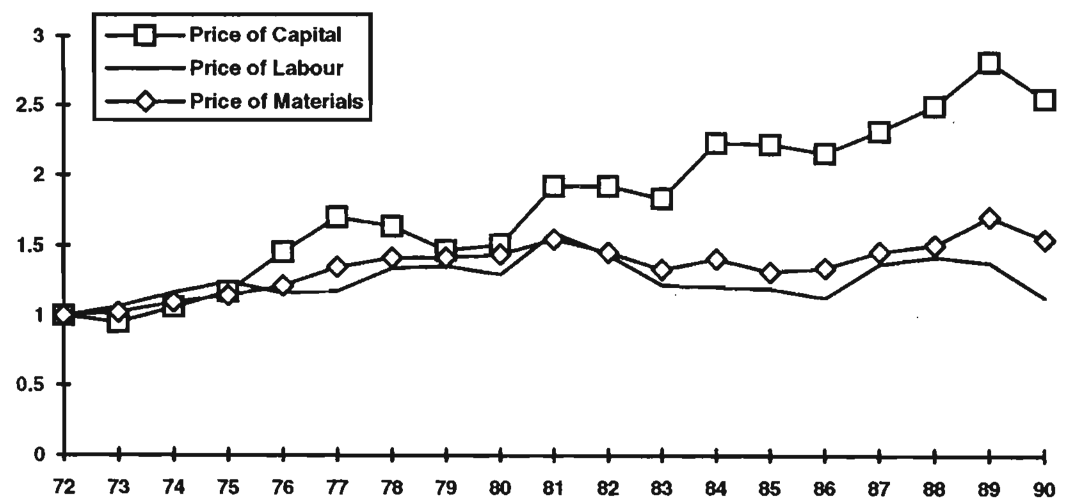


Figure A.5.2.1.3 Input-output coefficient of materials (ISIC 3833)

Figure A.5.2.1.4 Relative price of capital, labour and materials (ISIC 3833)



A5.2.2 FURNITURE (ISIC 3220)

Table A5.2.2.1 Input-output coefficients, relative input prices and real average costs (ISIC 3840)

Year	Input-output of Capital	Input-output of Labour	Input-output of Materials	Relative price of Capital	Relative price of Labour	Relative price of Materials	Real Average Costs
1972	0.1480	1.78E-05	0.6628	1.0000	1.0001	1.0000	0.0940
1973	0.1333	1.66E-05	0.6584	0.9526	1.0391	1.0429	0.0946
1974	0.1247	1.74E-05	0.6540	1.0206	1.0134	1.1288	0.1006
1975	0.1209	1.60E-05	0.6595	1.1326	1.1001	1.1854	0.1045
1976	0.1229	1.68E-05	0.6576	1.4096	1.0109	1.1888	0.1027
1977	0.1182	1.56E-05	0.6465	1.6421	1.0122	1.2306	0.1007
1978	0.1066	1.41E-05	0.6403	1.5962	1.0763	1.2913	0.1030
1979	0.0928	1.28E-05	0.6377	1.5191	1.1790	1.3795	0.1069
1980	0.0801	1.32E-05	0.6424	1.6088	1.2320	1.4789	0.1157
1981	0.0946	1.45E-05	0.6267	1.6725	1.1121	1.3374	0.1067
1982	0.1098	1.65E-05	0.6011	1.8450	1.1370	1.3433	0.1085
1983	0.1206	1.75E-05	0.6343	1.9470	1.2006	1.3639	0.1181
1984	0.1319	1.91E-05	0.6324	2.2196	1.1699	1.3390	0.1227
1985	0.1302	1.94E-05	0.6322	2.4823	1.2172	1.3921	0.1252
1986	0.1292	1.96E-05	0.6197	2.5851	1.2230	1.4438	0.1272
1987	0.1251	2.02E-05	0.6361	2.5423	1.1634	1.4594	0.1297
1988	0.1098	1.97E-05	0.6267	2.6884	1.1496	1.5241	0.1310
1989	0.1103	1.92E-05	0.6579	2.7776	0.9990	1.5419	0.1309
1990	0.1174	2.02E-05	0.6644	2.8069	1.0918	1.6477	0.1440

Note: 1 Input-output coefficients were calculated by defining the quantities of factors of production as the constant Rand expenditure on the different factors and the quantity of output as the constant Rand value of output.

2. Relative prices are defined as the ratio of the price index of each input to the price index of output as calculated by the IDC (1992)

3. Real average costs is defined as the ratio of nominal average costs to the price index of output, where nominal average costs are calculated as the ratio of current value of total costs to the constant rand value of output.

Table A5.2.2.2 Descriptive statistics of relative factor prices and input-output coefficients (ISIC 3840)

Variable	Mean	Standard Deviation.
Relative price of Capital	1.8657	0.63557
Relative price of Labour	1.1119	0.81819E-01
Relative price of Materials	1.3326	0.17112
Input-output coefficient of Capital	0.11719	0.16105E-01
Input-output coefficient of Labour	0.17164E-04	0.23575E-05
Input-output coefficient of Materials	0.64161	0.16851E-01

Figure A.5.2.2.1 Input-output coefficient of capital (ISIC 3320)

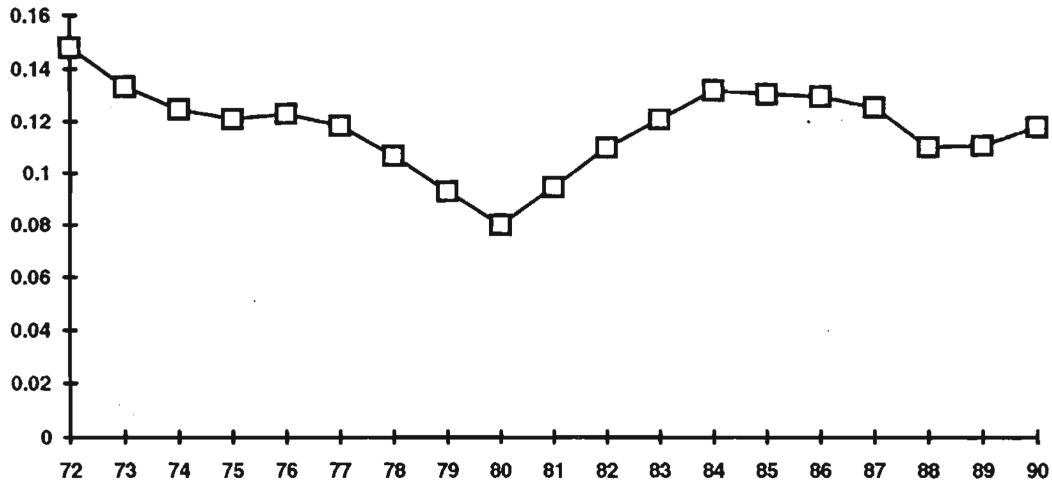
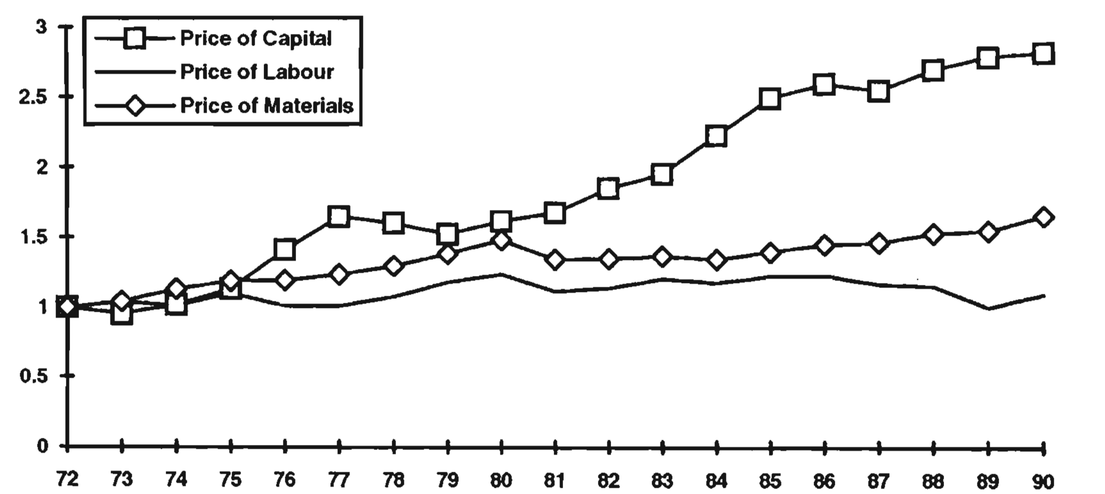


Figure A.5.2.2.2 Input-output coefficient of labour (ISIC 3320)

Figure A.5.2.2.3 Input-output coefficient of materials (ISIC 3320)

Figure A.5.2.2.4 Relative price of capital, labour and materials (ISIC 3320)



A5.2.3 MOTOR VEHICLES PARTS AND ACCESSORIES (ISIC 3840)

Table A5.2.3.1 Input-output coefficients, relative input prices and real average costs (ISIC 3840)

Year	Input-output of Capital	Input-output of Labour	Input-output of Materials	Relative price of Capital	Relative price of Labour	Relative price of Materials	Real Average Costs
1972	0.1638	5.22E-06	0.7454	1.0000	1.0000	1.0000	0.0725
1973	0.1533	4.93E-06	0.7354	0.9747	1.0545	1.0267	0.0730
1974	0.1416	4.73E-06	0.7425	1.1148	1.0952	1.1083	0.0783
1975	0.1322	4.69E-06	0.7480	1.1335	1.0366	1.1056	0.0778
1976	0.1365	5.33E-06	0.7508	1.3437	0.8797	1.0940	0.0769
1977	0.1532	5.99E-06	0.7513	1.5085	0.8771	1.1016	0.0792
1978	0.1276	5.23E-06	0.7420	1.3753	0.8927	1.1015	0.0766
1979	0.1296	5.38E-06	0.7546	1.2315	0.8859	1.1178	0.0787
1980	0.1148	4.65E-06	0.7518	1.2999	0.9463	1.1481	0.0786
1981	0.1167	4.25E-06	0.7487	1.5053	1.0299	1.1327	0.0771
1982	0.1536	4.96E-06	0.7421	1.5785	1.0107	1.1163	0.0777
1983	0.1777	4.85E-06	0.7490	2.0007	1.3171	1.3289	0.0952
1984	0.2254	5.64E-06	0.7308	1.8372	1.1409	1.0984	0.0832
1985	0.3281	7.31E-06	0.7444	1.8756	1.0613	1.0704	0.0927
1986	0.3385	7.58E-06	0.7412	1.6312	0.9004	0.9904	0.0801
1987	0.2591	6.05E-06	0.7579	1.5024	0.8909	0.9548	0.0729
1988	0.2140	5.21E-06	0.7525	1.5076	0.9181	0.9313	0.0682
1989	0.2128	5.32E-06	0.7524	1.6505	1.0231	1.0063	0.0749
1990	0.2439	6.18E-06	0.7531	1.4906	0.8911	1.0267	0.0751

Note: 1 Input-output coefficients were calculated by defining the quantities of factors of production as the constant Rand expenditure on the different factors and the quantity of output as the constant Rand value of output.

2. Relative prices are defined as the ratio of the price index of each input to the price index of output as calculated by the IDC (1992)

3. Real average costs is defined as the ratio of nominal average costs to the price index of output, where nominal average costs are calculated as the ratio of current value of total costs to the constant rand value of output.

Table A5.2.3.2 Descriptive statistics of relative factor prices and input-output coefficients (ISIC 3840)

Variable	Mean	Standard Deviation.
Relative price of Capital	1.4506	0.28540
Relative price of Labour	0.99220	0.11434
Relative price of Materials	1.0768	0.87638E-01
Input-output coefficient of Capital	0.18538	0.67783E-01
Input-output coefficient of Labour	0.54475E-05	0.86508E-06
Input-output coefficient of Materials	0.74704	0.68690E-02

Figure A.5.2.3.1 Input-output coefficient of capital (ISIC 3840)

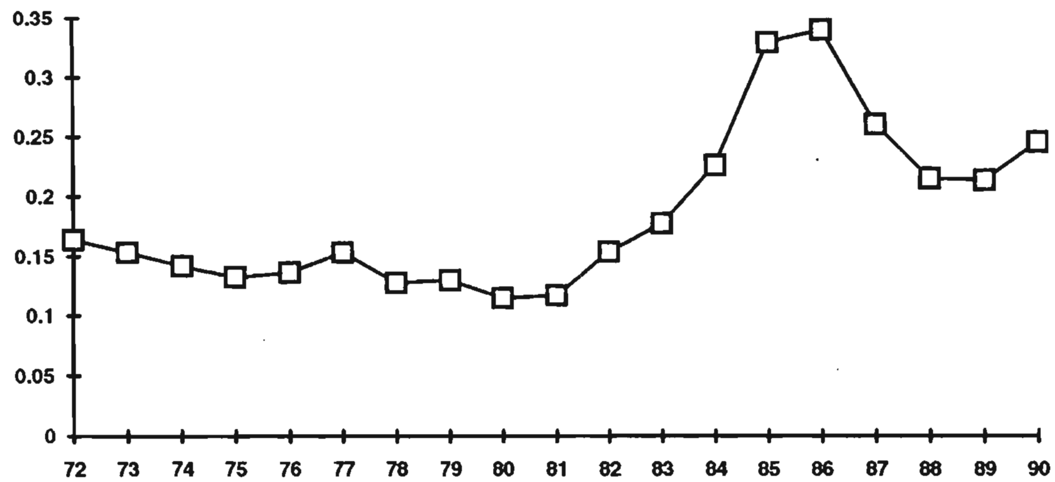


Figure A.5.2.3.2 Input-output coefficient of labour (ISIC 3840)

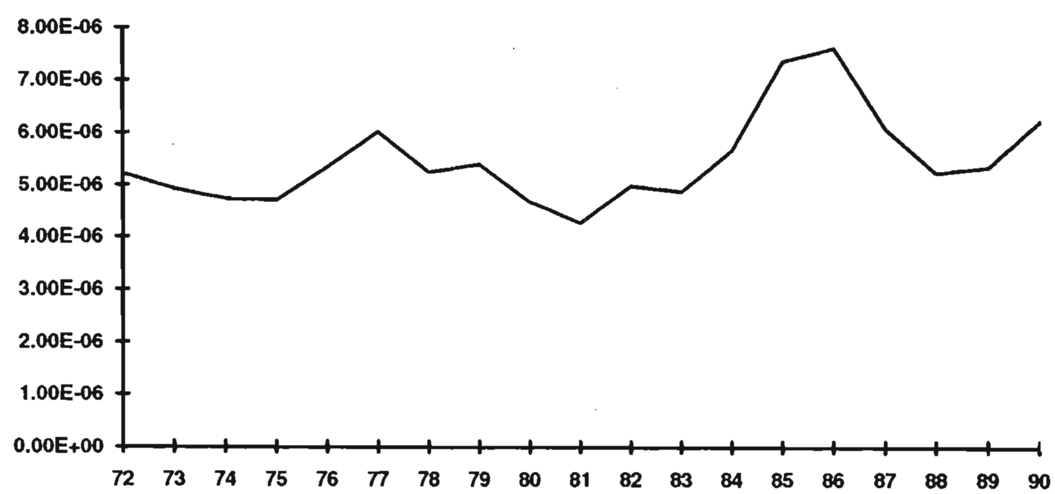
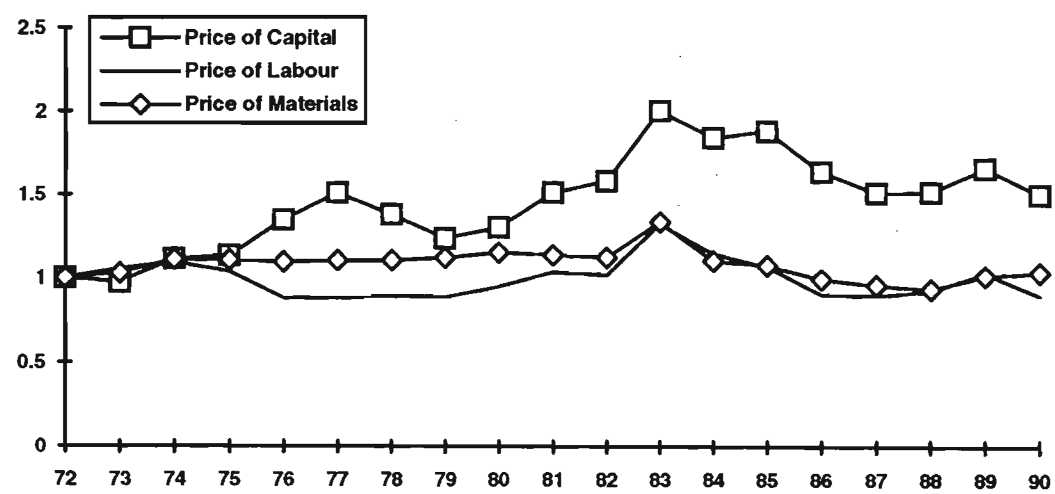


Figure A.5.2.3.3 Input-output coefficient of materials (ISIC 3840)

Figure A.5.2.3.4 Relative price of capital, labour and materials (ISIC 3840)



APPENDIX 5.3

MOTOR VEHICLES, PARTS AND ACCESSORIES (ISIC 3840)

5.3.1 Hypothesis Testing:

Table A5.3.1.1 LR test statistics and appropriate chi-square statistics for ZEF and 3SLS estimates of ISIC (3840)

Null Hypothesis	Critical χ^2 statistic and df for IZEF, IZEF (AR) and I3SLS estimates	Computed L. R. statistic IZEF estimates	Computed L.R. statistic IZEF (AR) estimates	Computed L.R. statistic I3SLS estimates	Critical χ^2 statistic and df for I3SLS (AR) estimates	Computed L.R. statistic I3SLS (AR) estimates
Non-homothetic	11.07 (5)	20.539	19.451	20.359		
Homotheticity	14.07 (7)	56.969	48.584	56.627	5.99 (2)	28.564
Homogeneity in Output	15.51 (8)	57.651	51.428	57.418	7.81 (3)	31.177
V.R.T.S. Cobb-Douglas	19.7 (11)	75.031	86.309	75.361	12.59 (6)	67.549

Note: 1. For IZEF, IZEF (AR) and I3SLS the alternate hypothesis is that the technology augmented form is valid. For I3SLS (AR) estimates the alternate hypothesis is that the non-homothetic form is valid.

2. Numbers in parenthesis appearing after χ^2 indicate Degrees of Freedom (df) (at the 5% level of significance) which are determined by the number of parameter constraints required to obtain the specification of the different null hypothesis from the alternative specification.

Table A5.3.1.2 LR test statistics for comparison of IZEF and IZEF(AR) estimators ISIC 3840

Model	1	2	3	4	5
Likelihood Ratio Statistic	9.608	9.615	14.995	12.797	-5.618
Critical Chi-squared statistic	7.81	7.81	7.81	7.81	3.84

Table A5.3.1.3 LR test statistics for comparison of I3SLS and I3SLS(AR) estimators ISIC 3840

Model	2	3	4	5
Likelihood Ratio Statistic	8.833	14.627	12.765	-6.610
Critical Chi-squared statistic	7.81	7.81	7.81	3.84

Table A5.3.1.4 LR test of Economies of Scale - IZEF estimates (ISIC 3840)

	Critical Chi-Squared Statistic	Computed Wald - IZEF estimates	Computed Wald - IZEF (AR) estimates
Model 1 (5)	11.071	205.345	441.078
Model 2 (4)	9.488	121.158	151.682
Model 3 (2)	5.991	9.899	79.499
Model 4 (1)	3.841	16.629	23.580
Model 5 (1)	3.841	51.028	22.210

Note: 1. Numbers in parenthesis after the Model Number are the number of degrees of freedom
2. Source of critical chi-squared values: Gujarati (1988, p685)

Table A5.3.1.5 LR test of Economies of Scale - I3SLS estimates (ISIC 3840)

	Critical Chi-Squared Statistic	Computed Wald - I3SLS estimates	Computed Wald - I3SLS (AR) estimates
Model 1 (5)	11.071	208.839	
Model 2 (4)	9.488	115.586	154.732
Model 3 (2)	5.991	6.954	55.053
Model 4 (1)	3.841	16.648	16.714
Model 5 (1)	3.841	30.906	11.317

Note: 1. Numbers in parenthesis after the Model Number are the number of degrees of freedom
2. Source of critical chi-squared values: Gujarati (1988, p685)

5.3.2 Durbin-Watson Test Results

Table A5.3.2.1 Computed and critical Durbin-Watson statistics for IZEF estimates of ISIC 3840.

Model	Equation	Number of Explanatory's	DL	DU	Computed D-W Statistic	H_0: $\rho=0$ vs. H_1: $\rho>0$
Model 1	Cost function	14	0.070	3.642	1.080	N.D.
	Cost share of Capital	4	0.859	1.848	0.989	N.D.
	Cost share of Labour	4	0.859	1.848	1.620	N.D.
Model 2	Cost function	9	0.369	2.783	0.618	N.D.
	Cost share of Capital	3	0.967	1.685	0.626	Reject
	Cost share of Labour	3	0.967	1.685	1.508	N.D.
Model 3	Cost function	7	0.549	2.396	0.838	N.D.
	Cost share of Capital	2	1.074	1.536	0.755	Reject
	Cost share of Labour	2	1.074	1.536	1.038	Reject
Model 4	Cost function	6	0.649	2.206	0.838	N.D.
	Cost share of Capital	2	1.074	1.536	0.747	Reject
	Cost share of Labour	2	1.074	1.536	1.017	Reject
Model 5	Cost function	3	0.967	1.685	0.804	Reject
	Cost share of Capital	--	--	--	--	--
	Cost share of Labour	--	--	--	--	--

Note: Significant Lower and Upper D-W statistics are given for the 0.05 level of significance (Source: Gujarati, 1988, p687).

Table A5.3.2.2 Computed and critical Durbin-Watson statistics for I3SLS estimates of ISIC 3840

Model	Equation	Number of Exogenous Variables	DL	DU	Computed D-W Statistic	H_0: $\rho=0$ vs. H_1: $\rho>0$
Model 1	Cost function	15	0.063	3.676	1.097	N.D
	Cost share of Capital	15	0.063	3.676	0.996	N.D.
	Cost share of Labour	15	0.063	3.676	1.631	N.D.
Model 2	Cost function	11	0.220	3.159	0.611	N.D.
	Cost share of Capital	11	0.220	3.159	0.619	N.D.
	Cost share of Labour	11	0.220	3.159	1.517	N.D.
Model 3	Cost function	11	0.220	3.159	0.841	N.D.
	Cost share of Capital	11	0.220	3.159	0.759	N.D.
	Cost share of Labour	11	0.220	3.159	1.042	N.D.
Model 4	Cost function	11	0.220	3.159	0.839	N.D.
	Cost share of Capital	11	0.220	3.159	0.747	N.D.
	Cost share of Labour	11	0.220	3.159	1.016	N.D.
Model 5	Cost function	8	0.456	2.589	0.813	N.D.
	Cost share of Capital	--	--	--	--	--
	Cost share of Labour	--	--	--	--	--

Note: Significant Lower and Upper D-W statistics are given for the 0.05 level of significance (Source: Gujarati, 1988, p687).

5.3.3 Parameter Estimates

Table A5.3.3.1 IZEF parameter estimates for Motor Vehicles (ISIC 3840).

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o	20.5118 (1526.62)	20.5454 (2095.70)	20.5084 (2244.63)	20.5103 (2417.36)	20.5255 (2810.09)
α_K	0.0700 (7.9168)	0.0896 (11.9729)	0.0656 (11.9376)	0.0664 (12.2346)	0.0659 (12.0302)
α_L	0.1744 (55.7134)	0.1778 (60.9623)	0.1647 (35.7946)	0.1638 (35.5762)	0.1648 (40.9885)
α_M	0.7556	0.7326	0.7697	0.7698	0.7693
α_Y	0.7755 (17.1890)	0.8218 (22.1241)	0.9610 (32.1764)	0.9593 (96.0428)	0.9045 (67.6657)
α_t	0.0047 (2.4907)				
γ_{KK}	0.0467 (3.3846)	0.0054 (0.7636)	0.0109 (1.3662)	0.0089 (1.1478)	
γ_{KL}	0.0430 (5.1074)	0.0441 (8.2463)	0.0320 (3.6116)	0.0336 (3.7887)	
γ_{KM}	-0.0897	-0.0495	-0.0429	-0.0425	
γ_{LL}	0.0957 (7.7777)	0.0978 (8.1845)	0.1079 (5.3868)	0.1023 (4.7437)	
γ_{LM}	-0.1387	-0.1419	-0.1399	-0.1359	
γ_{MM}	0.2284	0.1914	0.1828	0.1784	
γ_{KY}	-0.0761 (-3.7649)	-0.0798 (-3.2842)			
γ_{LY}	-0.0666 (-10.297)	-0.0664 (-9.9745)			
γ_{MY}	0.1457	0.1462			
γ_{YY}	0.1557 (2.2549)	0.0215 (0.2592)	0.0277 (0.2943)		
γ_{tt}	-0.0001 (-0.8793)				
γ_{Yt}	5.29E-05 (0.02116)				
γ_{Kt}	0.0005 (0.6687)				
γ_{Lt}	0.0004 (1.0394)				
γ_{Mt}	-0.0009				
Dtrm	1.91E-13	5.63E-13	3.83E-12	3.97E-12	9.91E-12

Note: 1. Numbers in parenthesis are t-statistics referring to parameter estimates above. t-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covariance matrix.

Table 5.3.3.2 IZEF (AR) parameter estimates for Motor Vehicles (ISIC 3840).

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o	20.6046 (587.168)	20.5633 (1236.01)	20.5690 (458.71)	20.5208 (1245.86)	20.5213 (2187.61)
α_K	0.0994 (8.4328)	0.0850 (8.1679)	0.0522 (2.0149)	0.0630 (6.5489)	0.0614 (11.8744)
α_L	0.1786 (11.6508)	0.1761 (42.4787)	0.1477 (17.3615)	0.1438 (15.5765)	0.1625 (41.0398)
α_M	0.722	0.7389	0.8001	0.7932	0.7761
α_Y	0.5803 (12.1830)	0.7693 (20.9784)	0.8189 (28.8541)	0.9215 (56.9829)	0.9037 (44.1960)
α_t	-0.0025 (-0.6131)				
γ_{KK}	0.1130 (9.0302)	0.0446 (2.6401)	0.0714 (2.9026)	0.0253 (1.7897)	
γ_{KL}	0.0399 (4.5619)	0.0523 (7.2021)	0.0562 (3.1976)	0.0609 (3.9282)	
γ_{KM}	-0.1529	-0.0969	-0.1276	-0.0862	
γ_{LL}	0.0510 (2.4153)	0.0990 (6.2152)	0.0089 (0.2914)	0.0044 (0.1396)	
γ_{LM}	-0.0909	-0.1513	-0.0651	-0.0649	
γ_{MM}	0.2438	0.2482	0.1927	0.1511	
γ_{KY}	-0.1418 (-10.531)	-0.1027 (-4.5118)			
γ_{LY}	-0.0786 (-7.4313)	-0.0680 (-8.6887)			
γ_{MY}	0.2204	0.7070			
γ_{YY}	0.1028 (1.4889)	0.0745 (0.7149)	0.1870 (2.0476)		
γ_{tt}	7.64E-05 (0.2898)				
γ_{Yt}	0.0112 (4.2270)				
γ_{Kt}	-0.0024 (-2.3324)				
γ_{Lt}	6.05E-05 (0.0575)				
γ_{Mt}	0.0023				
Dtrm	1.12E-13	3.30E-13	1.665E-12	1.95E-12	1.354E-11

Note: 1. Numbers in parenthesis are t-statistics referring to parameter estimates above. t-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covariance matrix.

Table A5.3.3.3 I3SLS parameter estimates for Motor Vehicles (ISIC 3840)

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o	20.5125 (1523.17)	20.5454 (2057.33)	20.5064 (2169.61)	20.5104 (2418.04)	20.5197 (2658.82)
α_K	0.0699 (7.9301)	0.0904 (11.8879)	0.0651 (11.7853)	0.0664 (12.2421)	0.0643 (11.7320)
α_L	0.1743 (55.5600)	0.1777 (60.6326)	0.1651 (36.0186)	0.1638 (35.5759)	0.1638 (40.6735)
α_M	0.7558	0.7319	0.7698	0.7698	0.7719
α_Y	0.7682 (16.5304)	0.8247 (20.2754)	0.9740 (27.7927)	0.9590 (95.3638)	0.9201 (64.0343)
α_t	0.0045 (2.3873)				
γ_{KK}	0.0479 (3.4770)	0.0051 (0.7193)	0.0120 (1.4781)	0.0089 (1.14327)	
γ_{KL}	0.0437 (5.1641)	0.0438 (8.1860)	0.0312 (3.5276)	0.0337 (3.7935)	
γ_{KM}	-0.0916	-0.0489	-0.0432	-0.0426	
γ_{LL}	0.0955 (7.7270)	0.0977 (8.2086)	0.1097 (5.6831)	0.1022 (4.7271)	
γ_{LM}	-0.1392	-0.1415	-0.1409	-0.1359	
γ_{MM}	0.2308	0.1904	0.1841	0.1785	
γ_{KY}	-0.0755 (-3.7405)	-0.0826 (-3.3236)			
γ_{LY}	-0.0664 (-10.2371)	-0.0654 (-9.7151)			
γ_{MY}	0.1419	0.1480			
γ_{YY}	0.1941 (2.3689)	0.0058 (0.0596)	-6.482E-05 (-0.0006)		
γ_{tt}	-0.0001 (-0.7344)				
γ_{Yt}	-0.0002 (-0.0755)				
γ_{Kt}	0.0005 (0.6333)				
γ_{Lt}	0.0003 (0.9805)				
γ_{Mt}	-0.0008				
Dtrm	1.934E-13	5.647E-13	3.809E-12	3.971E-12	1.021E-11

Note: 1. Numbers in parenthesis are *t*-statistics referring to parameter estimates above. *t*-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covariance matrix.

Table A5.3.3.4 I3SLS(AR) parameter estimates for Motor Vehicles (ISIC 3840)

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o		20.5644 (1216.58)	20.5570 (500.192)	20.5166 (1313.91)	20.5186 (1932.92)
α_K		0.0864 (8.1624)	0.0545 (2.1641)	0.0641 (7.2228)	0.0636 (10.5731)
α_L		0.1754 (43.7719)	0.1443 (16.1818)	0.1432 (15.5670)	0.1629 (37.4923)
α_M		0.7382	0.8012	0.7927	0.7735
α_Y		0.7611 (17.2166)	0.8323 (24.1256)	0.9336 (57.4715)	0.9234 (40.5366)
α_t					
γ_{KK}		0.0455 (2.6437)	0.0660 (2.8967)	0.0223 (1.7380)	
γ_{KL}		0.0526 (7.4699)	0.0620 (3.8317)	0.0633 (4.2116)	
γ_{KM}		-0.0981	-0.1280	-0.0856	
γ_{LL}		0.0999 (6.4407)	0.0012 (0.0377)	0.0080 (0.2513)	
γ_{LM}		-0.1525	-0.0632	-0.0173	
γ_{MM}		0.2502	0.1912	0.1569	
γ_{KY}		-0.1092 (-4.7814)			
γ_{LY}		-0.0658 (-8.5811)			
γ_{MY}		0.1750			
γ_{YY}		0.0938 (0.6466)	0.2343 (1.8995)		
γ_{tt}					
γ_{Yt}					
γ_{Kt}					
γ_{Lt}					
γ_{Mt}					
Dtrm		3.457E-13	1.690E-12	1.954E-12	

Note: 1. Numbers in parenthesis are *t*-statistics referring to parameter estimates above. *t*-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covariance matrix.

5.3.4: Monotonicity and Concavity Test Results

Table A5.3.4.1 Fitted cost shares and Own AESs for IZEF estimates, Model 1 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0705	0.1748	0.7547	-3.7885	-1.5888	0.0760
1973	0.0607	0.1676	0.7721	-2.7929	-1.5596	0.0880
1974	0.0526	0.1584	0.7898	-1.1250	-1.4986	0.1000
1975	0.0465	0.1500	0.8045	1.0989	-1.4131	0.1099
1976	0.0548	0.1487	0.7973	-1.6904	-1.3964	0.1050
1977	0.0732	0.1644	0.7627	-3.9432	-1.5417	0.0815
1978	0.0594	0.1531	0.7881	-2.5965	-1.4490	0.0989
1979	0.0525	0.1455	0.8027	-1.0962	-1.3518	0.1087
1980	0.0369	0.1341	0.8304	8.2166	-1.1358	0.1270
1981	0.0342	0.1378	0.8299	11.6168	-1.2175	0.1267
1982	0.0443	0.1465	0.8109	2.2042	-1.3668	0.1141
1983	0.0555	0.1616	0.7844	-1.8606	-1.5237	0.0964
1984	0.0740	0.1807	0.7464	-3.9851	-1.6032	0.0702
1985	0.0989	0.1998	0.7014	-4.3369	-1.6077	0.0386
1986	0.0989	0.1947	0.7063	-4.3366	-1.6116	0.0420
1987	0.0822	0.1814	0.7369	-4.2546	-1.6045	0.0636
1988	0.0752	0.1785	0.7474	-4.0381	-1.5986	0.0709
1989	0.0791	0.1837	0.7381	-4.1788	-1.6078	0.0644
1990	0.0767	0.1721	0.7518	-4.0988	-1.5795	0.0739

Table A5.3.4.2. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 1 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.4893	-0.6859	-0.0514	-14.1347	-0.7583	-0.1233	-1.30E-15
1973	5.2302	-0.9154	-0.0719	-22.9997	-1.0836	-0.1424	-0.0138
1974	6.1649	-1.1603	-0.1090	-36.3206	-1.4587	-0.1618	-0.0429
1975	7.1675	-1.3986	-0.1495	-52.9255	-1.8353	-0.1776	-0.0771
1976	6.2810	-1.0542	-0.1702	-37.0910	-1.2888	-0.1757	-0.0413
1977	4.5761	-0.6078	-0.1064	-14.8614	-0.6907	-0.1369	-0.0048
1978	5.7292	-0.9167	-0.1494	-29.0614	-1.0970	-0.1656	-0.0285
1979	6.6336	-1.1297	-0.1879	-42.5234	-1.3954	-0.1822	-0.0418
1980	9.6927	-1.9288	-0.2454	-103.2802	-2.6770	-0.2044	-0.2075
1981	10.1103	-2.1560	-0.2127	-116.3609	-3.1769	-0.1994	-0.3332
1982	7.6212	-1.4952	-0.1677	-61.0947	-1.9841	-0.1841	-0.1580
1983	5.7921	-1.0598	-0.0940	-30.7134	-1.3025	-0.1556	-0.0772
1984	4.2158	-0.6241	-0.0283	-11.3842	-0.6693	-0.1134	-0.0227
1985	3.1761	-0.2933	0.0105	-3.1156	-0.2533	-0.0621	-0.0010
1986	3.2326	-0.2837	-0.0087	-3.4611	-0.2627	-0.0678	0.0006
1987	3.8819	-0.4803	-0.0374	-8.2427	-0.5011	-0.1034	-0.0083
1988	4.2056	-0.5969	-0.0399	-11.2314	-0.6425	-0.1149	-0.0200
1989	3.9584	-0.5361	-0.0228	-8.9506	-0.5566	-0.1041	-0.0153
1990	4.2583	-0.5561	-0.0720	-11.6593	-0.6123	-0.1220	-0.0115

Table A5.3.4.3 Fitted cost shares and Own AESs for IZEF estimates, Model 2 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0896	0.1778	0.7326	-9.4881	-1.5306	-0.0084
1973	0.0810	0.1702	0.7488	-10.5244	-1.4994	0.0059
1974	0.0695	0.1606	0.7699	-12.2682	-1.4347	0.0240
1975	0.0618	0.1517	0.7864	-13.7621	-1.3426	0.0379
1976	0.0623	0.1499	0.7879	-13.6673	-1.3183	0.0391
1977	0.0763	0.1653	0.7585	-11.1802	-1.4700	0.0143
1978	0.0653	0.1536	0.7811	-13.0417	-1.3647	0.0335
1979	0.0630	0.1453	0.7916	-13.5058	-1.2503	0.0422
1980	0.0449	0.1337	0.8213	-18.5799	-1.0094	0.0662
1981	0.0346	0.1375	0.8280	-23.4084	-1.0990	0.0714
1982	0.0419	0.1458	0.8123	-19.7816	-1.2576	0.0590
1983	0.0503	0.1608	0.7889	-16.7455	-1.4362	0.0400
1984	0.0645	0.1796	0.7559	-13.2019	-1.5360	0.0120
1985	0.0881	0.1983	0.7136	-9.6543	-1.5558	-0.0254
1986	0.0904	0.1924	0.7171	-9.3983	-1.5555	-0.0223
1987	0.0744	0.1788	0.7468	-11.4719	-1.5338	0.0041
1988	0.0651	0.1756	0.7592	-13.0820	-1.5232	0.0149
1989	0.0681	0.1806	0.7513	-12.5115	-1.5385	0.0081
1990	0.0705	0.1681	0.7614	-12.0915	-1.4876	0.0168

Table A5.3.4.4. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 2 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	3.7682	0.2459	-0.0894	0.3232	0.0190	0.0048	1.75E-17
1973	4.1991	0.1837	-0.1133	-1.8528	-0.0957	-0.0217	-2.23E-17
1974	4.9506	0.0751	-0.1477	-6.9067	-0.3005	-0.0563	2.01E-16
1975	5.7010	-0.0181	-0.1891	-14.0242	-0.5221	-0.0867	-4.14E-16
1976	5.7255	-0.0090	-0.2017	-14.7633	-0.5343	-0.0922	-4.14E-16
1977	4.4981	0.1445	-0.1322	-3.7976	-0.1803	-0.0384	-2.94E-16
1978	5.3959	0.0300	-0.1830	-11.3183	-0.4374	-0.0792	-4.00E-16
1979	5.8143	0.0080	-0.2334	-16.9199	-0.5702	-0.1073	2.82E-16
1980	8.3381	-0.3412	-0.2918	-50.7694	-1.3462	-0.1520	-1.99E-15
1981	10.2802	-0.7294	-0.2468	-79.9565	-2.2039	-0.1394	1.92E-15
1982	8.2165	-0.4536	-0.1983	-42.6332	-1.3730	-0.1135	-3.29E-15
1983	6.4532	-0.2473	-0.1188	-17.5944	-0.7306	-0.0715	1.13E-16
1984	4.8051	-0.0150	-0.0451	-2.8105	-0.1587	-0.0205	-5.88E-17
1985	3.5247	0.2127	-0.0030	2.5964	0.2004	0.0396	1.95E-17
1986	3.5344	0.2367	-0.0283	2.1273	0.1532	0.0338	-1.37E-16
1987	4.3163	0.1086	-0.0625	-1.0356	-0.0594	-0.0103	3.59E-17
1988	4.8554	-0.0011	-0.0641	-3.6483	-0.1952	-0.0268	-1.56E-16
1989	4.5841	0.0332	-0.0461	-1.7650	-0.1019	-0.0145	6.15E-17
1990	4.7202	0.0784	-0.1090	-4.2932	-0.2091	-0.0368	3.55E-16

Table A5.3.4.5 Fitted cost shares and Own AESs for IZEF estimates, Model 3 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0656	0.1647	0.7697	-11.7110	-1.0939	0.0093
1973	0.0659	0.1659	0.7682	-11.6660	-1.1076	0.0080
1974	0.0653	0.1636	0.7711	-11.7606	-1.0811	0.0106
1975	0.0638	0.1585	0.7776	-11.9947	-1.0148	0.0164
1976	0.0609	0.1478	0.7914	-12.4875	-0.8256	0.0283
1977	0.0617	0.1502	0.7881	-12.3383	-0.8745	0.0254
1978	0.0613	0.1491	0.7896	-12.4131	-0.8540	0.0267
1979	0.0592	0.1427	0.7981	-12.7787	-0.7093	0.0340
1980	0.0608	0.1478	0.7914	-12.5043	-0.8269	0.0283
1981	0.0657	0.1635	0.7708	-11.7023	-1.0803	0.0103
1982	0.0662	0.1651	0.7687	-11.6193	-1.0980	0.0085
1983	0.0698	0.1768	0.7534	-11.0928	-1.2045	-0.0053
1984	0.0724	0.1853	0.7423	-10.7295	-1.2540	-0.0154
1985	0.0714	0.1817	0.7468	-10.8619	-1.2355	-0.0113
1986	0.0680	0.1704	0.7616	-11.3503	-1.1523	0.0022
1987	0.0683	0.1717	0.7600	-11.3013	-1.1643	0.0006
1988	0.0704	0.1786	0.7510	-11.0062	-1.2163	-0.0074
1989	0.0715	0.1823	0.7462	-10.8507	-1.2388	-0.0119
1990	0.0651	0.1614	0.7735	-11.7837	-1.0532	0.0127

Table A5.3.4.6. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 3 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	3.9618	0.1504	-0.1036	-2.8847	-0.1321	-0.0210	-2.73E-16
1973	3.9269	0.1524	-0.0976	-2.4985	-0.1166	-0.0184	-1.50E-16
1974	3.9963	0.1478	-0.1090	-3.2562	-0.1466	-0.0233	-2.47E-16
1975	4.1632	0.1354	-0.1347	-5.1604	-0.2145	-0.0347	-4.96E-16
1976	4.5583	0.1093	-0.1964	-10.4685	-0.3649	-0.0619	-3.93E-16
1977	4.4516	0.1182	-0.1821	-9.0274	-0.3278	-0.0554	-1.02E-15
1978	4.5006	0.1136	-0.1881	-9.6549	-0.3444	-0.0582	-6.81E-16
1979	4.7865	0.0922	-0.2283	-13.8458	-0.4428	-0.0762	-3.45E-16
1980	4.5625	0.1080	-0.1959	-10.4765	-0.3655	-0.0618	-5.65E-16
1981	3.9803	0.1523	-0.1098	-3.2000	-0.1440	-0.0232	-3.17E-16
1982	3.9288	0.1570	-0.1025	-2.6771	-0.1234	-0.0198	-3.78E-16
1983	3.5935	0.1839	-0.0501	0.4480	0.0247	0.0038	7.33E-18
1984	3.3849	0.2020	-0.0173	1.9972	0.1244	0.0190	1.32E-16
1985	3.4648	0.1959	-0.0308	1.4155	0.0838	0.0130	1.26E-16
1986	3.7624	0.1715	-0.0781	-1.0765	-0.0539	-0.0086	-1.55E-16
1987	3.7274	0.1738	-0.0720	-0.7346	-0.0375	-0.0059	-9.40E-17
1988	3.5457	0.1885	-0.0431	0.8144	0.0460	0.0072	1.58E-17
1989	3.4540	0.1961	-0.0284	1.5124	0.0903	0.0139	-3.28E-17
1990	4.0448	0.1485	-0.1209	-3.9502	-0.1719	-0.0280	-5.32E-16

Table A5.3.4.7 Fitted cost shares and Own AESs for IZEF estimates, Model 4 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0664	0.1638	0.7698	-12.0416	-1.2922	0.0020
1973	0.0668	0.1648	0.7684	-11.9693	-1.3012	0.0007
1974	0.0661	0.1628	0.7712	-12.0998	-1.2824	0.0033
1975	0.0645	0.1580	0.7775	-12.3723	-1.2317	0.0089
1976	0.0609	0.1484	0.7907	-13.0198	-1.0934	0.0206
1977	0.0615	0.1511	0.7874	-12.8992	-1.1367	0.0177
1978	0.0613	0.1498	0.7889	-12.9417	-1.1162	0.0191
1979	0.0595	0.1433	0.7973	-13.3025	-0.9960	0.0264
1980	0.0610	0.1482	0.7908	-12.9996	-1.0898	0.0207
1981	0.0657	0.1636	0.7706	-12.1529	-1.2905	0.0028
1982	0.0661	0.1653	0.7686	-12.0844	-1.3054	0.0009
1983	0.0697	0.1766	0.7536	-11.5086	-1.3826	-0.0128
1984	0.0723	0.1850	0.7428	-11.1350	-1.4163	-0.0230
1985	0.0711	0.1818	0.7471	-11.3033	-1.4053	-0.0189
1986	0.0676	0.1708	0.7615	-11.8391	-1.3482	-0.0055
1987	0.0681	0.1719	0.7600	-11.7643	-1.3556	-0.0070
1988	0.0702	0.1785	0.7513	-11.4380	-1.3916	-0.0150
1989	0.0714	0.1821	0.7465	-11.2657	-1.4066	-0.0194
1990	0.0650	0.1618	0.7732	-12.2847	-1.2732	0.0051

Table A3.5.4.8. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 4 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.0893	0.1685	-0.0778	-1.1624	-0.0526	-0.0086	-1.12E-16
1973	4.0505	0.1724	-0.0733	-0.8320	-0.0383	-0.0063	-6.36E-17
1974	4.1252	0.1656	-0.0826	-1.4997	-0.0668	-0.0110	-2.69E-16
1975	4.2984	0.1519	-0.1060	-3.2369	-0.1337	-0.0222	-2.67E-16
1976	4.7174	0.1175	-0.1581	-8.0180	-0.2825	-0.0476	-7.02E-16
1977	4.6144	0.1230	-0.1426	-6.6307	-0.2440	-0.0405	-6.68E-16
1978	4.6589	0.1214	-0.1502	-7.2601	-0.2616	-0.0439	-5.05E-16
1979	4.9447	0.1034	-0.1897	-11.2008	-0.3617	-0.0623	-4.68E-16
1980	4.7161	0.1191	-0.1596	-8.0746	-0.2836	-0.0481	-9.93E-16
1981	4.1238	0.1610	-0.0777	-1.3221	-0.0596	-0.0096	-2.19E-16
1982	4.0737	0.1640	-0.0699	-0.8199	-0.0379	-0.0061	-8.48E-17
1983	3.7274	0.1914	-0.0209	2.0177	0.1108	0.0173	2.32E-16
1984	3.5139	0.2081	0.0109	3.4226	0.2123	0.0324	1.26E-16
1985	3.5996	0.2000	-0.0007	2.9272	0.1733	0.0265	1.50E-16
1986	3.9082	0.1749	-0.0447	0.6875	0.0346	0.0054	6.70E-17
1987	3.8693	0.1789	-0.0401	0.9763	0.0500	0.0078	9.05E-17
1988	3.6808	0.1942	-0.0133	2.3693	0.1338	0.0207	1.48E-16
1989	3.5854	0.2022	0.0004	2.9907	0.1780	0.0273	1.45E-16
1990	4.1958	0.1539	-0.0860	-1.9635	-0.0860	-0.0139	-2.17E-16

Table A5.3.4.9 Fitted cost shares and Own AESs for IZEF (AR) estimates, Model 1 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0970	0.1787	0.7243	2.7005	-2.9994	0.0841
1973	0.0729	0.1686	0.7584	8.5478	-3.1365	0.1053
1974	0.0576	0.1591	0.7832	17.7306	-3.2705	0.1206
1975	0.0455	0.1519	0.8025	33.6065	-3.3736	0.1325
1976	0.0685	0.1574	0.7739	10.4956	-3.2941	0.1149
1977	0.1026	0.1750	0.7221	1.9859	-3.0490	0.0827
1978	0.0705	0.1612	0.7680	9.5444	-3.2407	0.1113
1979	0.0520	0.1545	0.7932	23.6159	-3.3358	0.1268
1980	0.0187	0.1380	0.8430	270.4179	-3.5686	0.1568
1981	0.0107	0.1341	0.8548	898.2686	-3.6208	0.1638
1982	0.0279	0.1435	0.8281	110.0835	-3.4915	0.1479
1983	0.0434	0.1547	0.8014	37.9075	-3.3331	0.1318
1984	0.0754	0.1728	0.7514	7.6252	-3.0796	0.1009
1985	0.1215	0.1973	0.6807	0.4254	-2.7582	0.0571
1986	0.1201	0.1969	0.6824	0.5062	-2.7638	0.0581
1987	0.0835	0.1797	0.7362	5.2428	-2.9854	0.0915
1988	0.0655	0.1721	0.7618	12.0897	-3.0891	0.1074
1989	0.0686	0.1758	0.7549	10.4330	-3.0382	0.1031
1990	0.0639	0.1728	0.7625	13.0086	-3.0791	0.1079

Table A5.3.4.10. Proper AESs and determinants of matrixes of AESs for IZEF (AR) estimates, Model 1 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	3.3024	-1.1763	0.2975	-19.0056	-1.1566	-0.3407	0.0014
1973	4.2457	-1.7658	0.2893	-44.8360	-2.2181	-0.4139	0.0062
1974	5.3565	-2.3914	0.2705	-86.6799	-3.5798	-0.4678	0.0167
1975	6.7747	-3.1876	0.2541	-159.2706	-5.7097	-0.5114	0.0391
1976	4.7010	-1.8853	0.2539	-56.6731	-2.3485	-0.4430	0.0187
1977	3.2218	-1.0633	0.2807	-16.4352	-0.9663	-0.3311	0.0075
1978	4.5100	-1.8234	0.2658	-51.2711	-2.2627	-0.4312	0.0240
1979	5.9705	-2.7102	0.2583	-114.4251	-4.3506	-0.4897	0.0574
1980	16.4569	-8.6957	0.2185	-1235.8405	-33.2140	-0.6073	0.6180
1981	28.8591	-15.7501	0.2071	-4085.3413	-100.9273	-0.6360	2.2074
1982	10.9542	-5.6116	0.2352	-504.3532	-15.2035	-0.5719	0.3194
1983	6.9401	-3.3941	0.2668	-174.5136	-6.5239	-0.5105	0.1287
1984	4.0644	-1.7001	0.2997	-40.0020	-2.1207	-0.4007	0.0364
1985	2.6646	-0.8492	0.3232	-8.2735	-0.6968	-0.2618	0.0099
1986	2.6873	-0.8651	0.3234	-8.6205	-0.7191	-0.2653	0.0110
1987	3.6606	-1.4887	0.3129	-29.0518	-1.7364	-0.3711	0.0339
1988	4.5419	-2.0658	0.3065	-57.9745	-2.9689	-0.4258	0.0670
1989	4.3084	-1.9524	0.3150	-50.2601	-2.7359	-0.4126	0.0627
1990	4.6121	-2.1367	0.3101	-61.3264	-3.1623	-0.4283	0.0791

Table A5.3.4.11 Fitted cost shares and Own AESs for IZEF (AR) estimates, Model 2 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0850	0.1761	0.7389	-4.5917	-1.4862	0.1012
1973	0.0718	0.1679	0.8242	-4.2781	-1.4443	0.1521
1974	0.0594	0.1585	0.9137	-3.1974	-1.3687	0.2029
1975	0.0505	0.1497	0.9683	-1.3070	-1.2618	0.2320
1976	0.0585	0.1492	0.9188	-3.0670	-1.2555	0.2056
1977	0.0807	0.1658	0.7869	-4.5435	-1.4297	0.1300
1978	0.0631	0.1531	0.8927	-3.6423	-1.3081	0.1913
1979	0.0555	0.1438	0.9139	-2.5372	-1.1666	0.2029
1980	0.0331	0.1321	1.0822	11.5676	-0.8965	0.2879
1981	0.0253	0.1369	1.1897	31.1176	-1.0230	0.3348
1982	0.0371	0.1459	1.1205	6.4225	-1.2031	0.3052
1983	0.0499	0.1616	1.0649	-1.1189	-1.3970	0.2798
1984	0.0719	0.1816	0.9450	-4.2819	-1.5047	0.2197
1985	0.1042	0.2011	0.7226	-4.4887	-1.5247	0.0914
1986	0.1053	0.1947	0.6843	-4.4747	-1.5245	0.0687
1987	0.0828	0.1805	0.8351	-4.5714	-1.5014	0.1584
1988	0.0717	0.1773	0.9287	-4.2715	-1.4910	0.2110
1989	0.0759	0.1824	0.9087	-4.4346	-1.5069	0.2001
1990	0.0751	0.1689	0.8556	-4.4090	-1.4506	0.1703

Table A5.3.4.12. Proper AESs and determinants of matrixes of AESs for IZEF (AR) estimates, Model 2 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.4940	-0.5428	-0.1628	-13.3719	-0.7595	-0.1770	-1.42E-15
1973	5.3352	-0.6366	-0.0932	-22.2858	-1.0558	-0.2283	-2.1333
1974	6.5522	-0.7848	-0.0445	-38.5553	-1.2645	-0.2796	-6.5142
1975	7.9229	-0.9824	-0.0440	-61.1230	-1.2684	-0.2947	-12.2746
1976	6.9875	-0.8018	-0.1035	-44.9749	-1.2735	-0.2689	-7.2480
1977	4.9088	-0.5256	-0.1600	-17.6009	-0.8669	-0.2115	-0.9508
1978	6.4169	-0.7212	-0.1070	-36.4125	-1.2169	-0.2616	-5.2523
1979	7.5538	-0.9108	-0.1513	-54.1007	-1.3444	-0.2596	-7.8717
1980	12.9786	-1.7088	-0.0584	-178.8139	0.4105	-0.2615	-46.3137
1981	16.0915	-2.2181	0.0713	-290.7681	5.4987	-0.3476	-97.5676
1982	10.6560	-1.3293	0.0745	-121.2772	0.1933	-0.3728	-37.0377
1983	7.4903	-0.8247	0.1207	-54.5416	-0.9932	-0.4054	-15.7850
1984	5.0042	-0.4258	0.1183	-18.5995	-1.1221	-0.3446	-4.2581
1985	3.4955	-0.2865	-0.0414	-5.3747	-0.4925	-0.1411	-0.2755
1986	3.5510	-0.3451	-0.1353	-5.7875	-0.4265	-0.1230	0.1979
1987	4.5016	-0.4021	-0.0039	-13.4010	-0.8860	-0.2379	-1.8665
1988	5.1131	-0.4552	0.0813	-19.7747	-1.1085	-0.3212	-4.2138
1989	4.7748	-0.4042	0.0873	-16.1160	-1.0507	-0.3091	-3.2816
1990	5.1201	-0.5073	-0.0467	-19.8199	-1.0081	-0.2492	-2.7493

Table A5.3.4.13 Fitted cost shares and Own AESs for IZEF (AR) estimates, Model 3 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0522	0.1477	0.8001	8.0463	-5.3625	0.0512
1973	0.0500	0.1450	0.8050	9.5622	-5.4724	0.0551
1974	0.0519	0.1479	0.8001	8.2119	-5.3537	0.0512
1975	0.0504	0.1485	0.8011	9.2989	-5.3294	0.0520
1976	0.0546	0.1573	0.7881	6.6203	-4.9971	0.0413
1977	0.0618	0.1633	0.7748	3.4999	-4.7886	0.0304
1978	0.0562	0.1583	0.7854	5.7908	-4.9617	0.0392
1979	0.0461	0.1511	0.8029	12.9505	-5.2292	0.0534
1980	0.0502	0.1530	0.7968	9.4105	-5.1573	0.0485
1981	0.0672	0.1628	0.7700	1.9401	-4.8055	0.0263
1982	0.0713	0.1663	0.7624	1.0099	-4.6919	0.0198
1983	0.0809	0.1706	0.7485	-0.4531	-4.5554	0.0079
1984	0.0911	0.1769	0.7320	-1.3715	-4.3671	-0.0065
1985	0.0918	0.1792	0.7291	-1.4189	-4.3046	-0.0091
1986	0.0825	0.1749	0.7426	-0.6273	-4.4269	0.0029
1987	0.0807	0.1726	0.7468	-0.4244	-4.4963	0.0065
1988	0.0858	0.1746	0.7396	-0.9552	-4.4341	0.0002
1989	0.0885	0.1757	0.7359	-1.1800	-4.4046	-0.0031
1990	0.0709	0.1674	0.7617	1.1066	-4.6563	0.0193

Table A5.3.4.14. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 3 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	8.2893	-2.0552	0.4491	-111.8606	-3.8120	-0.4761	-1.12E-14
1973	8.7511	-2.1704	0.4423	-128.9107	-4.1838	-0.4973	-2.08E-15
1974	8.3146	-2.0702	0.4500	-113.0974	-3.8651	-0.4766	-1.34E-14
1975	8.5142	-2.1630	0.4529	-122.0486	-4.1951	-0.4822	-1.67E-14
1976	7.5396	-1.9640	0.4749	-89.9281	-3.5836	-0.4321	-1.86E-14
1977	6.5639	-1.6631	0.4856	-59.8443	-2.6595	-0.3812	-4.41E-15
1978	7.3118	-1.8884	0.4764	-82.1942	-3.3390	-0.4215	-1.58E-14
1979	9.0774	-2.4510	0.4633	-150.1199	-5.3156	-0.4939	-2.54E-14
1980	8.3187	-2.1897	0.4659	-117.7341	-4.3382	-0.4673	-2.29E-14
1981	6.1390	-1.4675	0.4808	-47.0105	-2.1024	-0.3576	-6.50E-15
1982	5.7368	-1.3458	0.4865	-37.6493	-1.7912	-0.3298	-4.13E-15
1983	5.0710	-1.1069	0.4902	-23.6507	-1.2289	-0.2764	-1.90E-15
1984	4.4876	-0.9142	0.4974	-14.1488	-0.8268	-0.2190	-1.36E-15
1985	4.4183	-0.9071	0.5016	-13.4130	-0.8099	-0.2125	-1.34E-15
1986	4.8966	-1.0835	0.4988	-21.1995	-1.1757	-0.2614	-4.69E-15
1987	5.0373	-1.1181	0.4948	-23.4667	-1.2530	-0.2738	-6.33E-15
1988	4.7510	-1.0111	0.4960	-18.3364	-1.0225	-0.2467	-2.46E-15
1989	4.6169	-0.9602	0.4964	-16.1184	-0.9184	-0.2329	-2.78E-15
1990	5.7377	-1.3638	0.4895	-38.0741	-1.8386	-0.3295	-1.06E-14

Table A5.3.4.15 Fitted cost shares and Own AESs for IZEF (AR) estimates, Model 4 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0630	0.1438	0.7932	-8.4986	-5.7413	-0.0206
1973	0.0633	0.1408	0.7959	-8.4828	-5.8808	-0.0179
1974	0.0624	0.1442	0.7935	-8.5271	-5.7235	-0.0203
1975	0.0597	0.1452	0.7952	-8.6516	-5.6786	-0.0186
1976	0.0549	0.1556	0.7896	-8.8202	-5.2457	-0.0241
1977	0.0571	0.1622	0.7809	-8.7542	-4.9976	-0.0328
1978	0.0558	0.1567	0.7877	-8.7950	-5.2022	-0.0260
1979	0.0513	0.1490	0.7999	-8.8796	-5.5116	-0.0140
1980	0.0544	0.1509	0.7950	-8.8339	-5.4329	-0.0187
1981	0.0644	0.1612	0.7749	-8.4275	-5.0357	-0.0389
1982	0.0657	0.1650	0.7698	-8.3589	-4.8997	-0.0441
1983	0.0728	0.1692	0.7585	-7.9619	-4.7555	-0.0557
1984	0.0783	0.1759	0.7464	-7.6430	-4.5429	-0.0686
1985	0.0767	0.1786	0.7454	-7.7388	-4.4616	-0.0696
1986	0.0698	0.1745	0.7564	-8.1327	-4.5856	-0.0580
1987	0.0702	0.1719	0.7586	-8.1084	-4.6677	-0.0556
1988	0.0743	0.1739	0.7526	-7.8751	-4.6035	-0.0620
1989	0.0765	0.1749	0.7495	-7.7472	-4.5722	-0.0653
1990	0.0638	0.1669	0.7702	-8.4579	-4.8330	-0.0436

Table A5.3.4.16. Proper AESs and determinants of matrixes of AESs for IZEF (AR) estimates, Model 4 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	7.7223	-0.7250	0.4310	-10.8406	-0.3509	-0.0677	-0.0068
1973	7.8316	-0.7104	0.4208	-11.4484	-0.3531	-0.0720	-0.0080
1974	7.7662	-0.7404	0.4328	-11.5095	-0.3752	-0.0712	-0.0088
1975	8.0252	-0.8155	0.4379	-15.2748	-0.5045	-0.0864	-0.0128
1976	8.1267	-0.9875	0.4717	-19.7757	-0.7628	-0.0962	-0.0168
1977	7.5777	-0.9340	0.4876	-13.6725	-0.5852	-0.0739	-0.0128
1978	7.9620	-0.9604	0.4742	-17.6398	-0.6937	-0.0896	-0.0176
1979	8.9664	-1.1008	0.4556	-31.4547	-1.0878	-0.1306	-0.0326
1980	8.4219	-0.9942	0.4591	-22.9341	-0.8227	-0.1089	-0.0264
1981	6.8676	-0.7273	0.4803	-4.7257	-0.2012	-0.0348	-0.0063
1982	6.6176	-0.7041	0.4890	-2.8364	-0.1274	-0.0232	-0.0041
1983	5.9425	-0.5608	0.4944	2.5497	0.1293	0.0207	0.0044
1984	5.4200	-0.4744	0.5057	5.3446	0.2989	0.0558	0.0099
1985	5.4476	-0.5083	0.5125	4.8504	0.2804	0.0480	0.0094
1986	5.9981	-0.6323	0.5083	1.3162	0.0717	0.0075	0.0027
1987	6.0426	-0.6175	0.5024	1.3347	0.0697	0.0072	0.0030
1988	5.7111	-0.5412	0.5042	3.6366	0.1950	0.0309	0.0083
1989	5.5488	-0.5029	0.5050	4.6331	0.2527	0.0434	0.0113
1990	6.7176	-0.7538	0.4952	-4.2492	-0.1995	-0.0345	-0.0096

Table A5.3.4.17 Fitted cost shares and Own AESs for I3SLS estimates, Model 1 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0704	0.1746	0.7550	-3.5398	-1.5947	0.0804
1973	0.0606	0.1673	0.7721	-2.4553	-1.5651	0.0920
1974	0.0526	0.1580	0.7894	-0.7045	-1.5037	0.1036
1975	0.0466	0.1496	0.8039	1.6184	-1.4170	0.1132
1976	0.0549	0.1483	0.7968	-1.3241	-1.4007	0.1085
1977	0.0733	0.1640	0.7627	-3.7279	-1.5465	0.0857
1978	0.0595	0.1526	0.7879	-2.2807	-1.4515	0.1026
1979	0.0525	0.1447	0.8028	-0.6543	-1.3501	0.1125
1980	0.0371	0.1333	0.8296	8.8771	-1.1282	0.1299
1981	0.0348	0.1371	0.8281	11.7939	-1.2125	0.1290
1982	0.0449	0.1457	0.8094	2.4774	-1.3642	0.1168
1983	0.0562	0.1607	0.7831	-1.6302	-1.5249	0.0994
1984	0.0748	0.1797	0.7455	-3.8058	-1.6075	0.0739
1985	0.0995	0.1987	0.7018	-4.2121	-1.6138	0.0437
1986	0.0993	0.1934	0.7072	-4.2126	-1.6174	0.0475
1987	0.0827	0.1801	0.7371	-4.0891	-1.6082	0.0682
1988	0.0758	0.1771	0.7471	-3.8571	-1.6016	0.0750
1989	0.0798	0.1822	0.7380	-4.0096	-1.6117	0.0687
1990	0.0770	0.1704	0.7525	-3.9097	-1.5797	0.0787

Table A5.3.4.18. Proper AESs and determinants of matrixes of AESs for I3SLS estimates, Model 1 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.5552	-0.7234	-0.0560	-15.1049	-0.8078	-0.1313	-8.67E-16
1973	5.3124	-0.9582	-0.0777	-24.3787	-1.1441	-0.1501	-3.74E-15
1974	6.2559	-1.2053	-0.1160	-38.0767	-1.5257	-0.1692	-8.16E-15
1975	7.2750	-1.4474	-0.1577	-55.2194	-1.9118	-0.1852	-2.79E-15
1976	6.3669	-1.0938	-0.1780	-38.6833	-1.3400	-0.1837	-5.13E-15
1977	4.6360	-0.6382	-0.1131	-15.7268	-0.7266	-0.1453	-1.08E-15
1978	5.8125	-0.9531	-0.1581	-30.4742	-1.1424	-0.1739	-3.71E-15
1979	6.7569	-1.1753	-0.1981	-44.7729	-1.4549	-0.1911	-3.58E-15
1980	9.8403	-1.9783	-0.2584	-106.8474	-2.7602	-0.2134	-1.09E-14
1981	10.1573	-2.1771	-0.2263	-117.4711	-3.2182	-0.2076	-1.52E-14
1982	7.6788	-1.5194	-0.1806	-62.3435	-2.0191	-0.1920	-8.29E-15
1983	5.8372	-1.0810	-0.1060	-31.5868	-1.3306	-0.1628	-5.74E-15
1984	4.2522	-0.6435	-0.0389	-11.9633	-0.6954	-0.1203	-2.63E-15
1985	3.2104	-0.3122	0.0020	-3.5090	-0.2814	-0.0705	-1.07E-15
1986	3.2740	-0.3038	-0.0175	-3.9055	-0.2922	-0.0771	-9.27E-16
1987	3.9324	-0.5019	-0.0484	-8.8878	-0.5306	-0.1120	-1.13E-15
1988	4.2545	-0.6169	-0.0522	-11.9232	-0.6699	-0.1228	-2.23E-15
1989	4.0050	-0.5553	-0.0351	-9.5772	-0.5839	-0.1120	-2.52E-15
1990	4.3283	-0.5800	-0.0853	-12.5578	-0.6441	-0.1316	-2.37E-15

Table A5.3.4.19 Fitted cost shares and Own AESs for I3SLS estimates, Model 2 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0904	0.1777	0.7319	-9.4379	-1.5335	-0.0109
1973	0.0815	0.1702	0.7483	-10.5073	-1.5030	0.0037
1974	0.0696	0.1607	0.7696	-12.3101	-1.4398	0.0221
1975	0.0618	0.1519	0.7863	-13.8554	-1.3496	0.0362
1976	0.0624	0.1500	0.7876	-13.7139	-1.3241	0.0373
1977	0.0769	0.1651	0.7580	-11.1431	-1.4730	0.0121
1978	0.0656	0.1536	0.7808	-13.0670	-1.3696	0.0316
1979	0.0633	0.1454	0.7913	-13.5288	-1.2568	0.0403
1980	0.0445	0.1341	0.8215	-18.9104	-1.0238	0.0648
1981	0.0335	0.1379	0.8286	-24.3215	-1.1149	0.0704
1982	0.0411	0.1461	0.8128	-20.3286	-1.2682	0.0579
1983	0.0495	0.1611	0.7894	-17.1075	-1.4425	0.0388
1984	0.0641	0.1797	0.7561	-13.3557	-1.5394	0.0105
1985	0.0886	0.1980	0.7134	-9.6376	-1.5584	-0.0277
1986	0.0912	0.1922	0.7167	-9.3530	-1.5581	-0.0246
1987	0.0745	0.1788	0.7467	-11.4994	-1.5368	0.0022
1988	0.0649	0.1757	0.7594	-13.2062	-1.5268	0.0133
1989	0.0679	0.1806	0.7515	-12.6175	-1.5417	0.0064
1990	0.0707	0.1680	0.7612	-12.1157	-1.4910	0.0149

Table A5.3.4.20. Proper AESs and determinants of matrixes of AESs for I3SLS estimates, Model 2 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	3.7266	0.2609	-0.0880	0.5853	0.0345	0.0089	2.13E-17
1973	4.1583	0.1978	-0.1107	-1.4991	-0.0776	-0.0178	-1.57E-16
1974	4.9137	0.0875	-0.1438	-6.4204	-0.2800	-0.0525	9.09E-17
1975	5.6676	-0.0070	-0.1843	-13.4221	-0.5012	-0.0828	-1.49E-15
1976	5.6796	0.0052	-0.1979	-14.0995	-0.5112	-0.0885	1.67E-16
1977	4.4495	0.1609	-0.1304	-3.3845	-0.1607	-0.0348	-2.02E-16
1978	5.3492	0.0447	-0.1797	-10.7177	-0.4148	-0.0756	-1.47E-15
1979	5.7592	0.0235	-0.2296	-16.1665	-0.5461	-0.1034	-1.93E-15
1980	8.3465	-0.3388	-0.2847	-50.3041	-1.3403	-0.1474	8.21E-16
1981	10.4852	-0.7630	-0.2380	-82.8238	-2.2955	-0.1352	-3.04E-15
1982	8.2987	-0.4652	-0.1912	-43.0870	-1.3930	-0.1100	-4.49E-15
1983	6.4895	-0.2504	-0.1130	-17.4361	-0.7258	-0.0687	-1.28E-15
1984	4.8005	-0.0086	-0.0411	-2.4850	-0.1404	-0.0179	-2.61E-16
1985	3.4964	0.2263	-0.0016	2.7943	0.2153	0.0431	1.77E-16
1986	3.4997	0.2517	-0.0275	2.3255	0.1672	0.0377	5.72E-16
1987	4.2871	0.1213	-0.0599	-0.7069	-0.0405	-0.0070	-7.67E-17
1988	4.8431	0.0072	-0.0603	-3.2925	-0.1763	-0.0240	-4.94E-16
1989	4.5703	0.0419	-0.0425	-1.4351	-0.0829	-0.0117	-3.20E-16
1990	4.6842	0.0920	-0.1062	-3.8763	-0.1889	-0.0335	-4.51E-16

Table A5.3.4.21 Fitted cost shares and Own AESs for I3SLS estimates, Model 3 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0651	0.1651	0.7698	-11.5295	-1.0324	0.0116
1973	0.0653	0.1664	0.7683	-11.4979	-1.0480	0.0103
1974	0.0648	0.1640	0.7712	-11.5747	-1.0185	0.0129
1975	0.0634	0.1588	0.7778	-11.7893	-0.9472	0.0186
1976	0.0608	0.1476	0.7916	-12.2069	-0.7396	0.0306
1977	0.0618	0.1499	0.7883	-12.0451	-0.7893	0.0277
1978	0.0612	0.1490	0.7898	-12.1345	-0.7697	0.0290
1979	0.0590	0.1426	0.7984	-12.5003	-0.6184	0.0363
1980	0.0606	0.1478	0.7917	-12.2407	-0.7435	0.0306
1981	0.0655	0.1635	0.7709	-11.4635	-1.0131	0.0126
1982	0.0662	0.1650	0.7688	-11.3740	-1.0313	0.0108
1983	0.0697	0.1769	0.7534	-10.8727	-1.1473	-0.0030
1984	0.0725	0.1853	0.7422	-10.5153	-1.2019	-0.0131
1985	0.0716	0.1817	0.7468	-10.6303	-1.1806	-0.0090
1986	0.0681	0.1702	0.7617	-11.0948	-1.0886	0.0044
1987	0.0684	0.1716	0.7600	-11.0582	-1.1025	0.0029
1988	0.0704	0.1786	0.7510	-10.7787	-1.1597	-0.0051
1989	0.0716	0.1824	0.7461	-10.6317	-1.1849	-0.0096
1990	0.0652	0.1612	0.7736	-11.5210	-0.9819	0.0150

Table A5.3.4.22 Proper AESs and determinants of matrixes of AESs for I3SLS estimates, Model 3 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	3.9029	0.1380	-0.1086	-3.3290	-0.1531	-0.0238	-2.93E-16
1973	3.8705	0.1391	-0.1020	-2.9316	-0.1376	-0.0212	-3.85E-16
1974	3.9365	0.1355	-0.1142	-3.7065	-0.1675	-0.0262	-6.37E-16
1975	4.0994	0.1238	-0.1407	-5.6389	-0.2351	-0.0375	-8.50E-16
1976	4.4787	0.1019	-0.2059	-11.0301	-0.3834	-0.0650	-4.85E-16
1977	4.3697	0.1127	-0.1923	-9.5869	-0.3467	-0.0588	-1.07E-15
1978	4.4215	0.1064	-0.1975	-10.2097	-0.3633	-0.0613	-4.51E-16
1979	4.7073	0.0830	-0.2374	-14.4279	-0.4604	-0.0788	-3.51E-16
1980	4.4865	0.0989	-0.2044	-11.0282	-0.3842	-0.0645	-7.39E-16
1981	3.9107	0.1451	-0.1176	-3.6801	-0.1656	-0.0266	-8.26E-16
1982	3.8582	0.1507	-0.1107	-3.1563	-0.1454	-0.0234	-5.10E-16
1983	3.5294	0.1777	-0.0573	0.0182	0.0010	0.0002	1.41E-18
1984	3.3235	0.1967	-0.0244	1.5927	0.0993	0.0152	2.11E-16
1985	3.3998	0.1917	-0.0386	0.9914	0.0587	0.0091	1.36E-16
1986	3.6911	0.1673	-0.0868	-1.5461	-0.0772	-0.0124	-2.38E-16
1987	3.6584	0.1687	-0.0802	-1.1927	-0.0608	-0.0097	-1.06E-16
1988	3.4808	0.1833	-0.0507	0.3848	0.0218	0.0034	9.94E-17
1989	3.3911	0.1908	-0.0356	1.0976	0.0656	0.0101	2.53E-16
1990	3.9704	0.1430	-0.1298	-4.4518	-0.1933	-0.0316	-7.60E-16

Table A5.3.4.23 Fitted cost shares and Own AESs for I3SLS estimates, Model 4 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0664	0.1638	0.7698	-12.0416	-1.2959	0.0022
1973	0.0668	0.1648	0.7684	-11.9689	-1.3048	0.0009
1974	0.0660	0.1628	0.7712	-12.1000	-1.2862	0.0034
1975	0.0644	0.1581	0.7775	-12.3734	-1.2358	0.0091
1976	0.0609	0.1484	0.7907	-13.0240	-1.0987	0.0208
1977	0.0615	0.1511	0.7874	-12.9035	-1.1419	0.0179
1978	0.0613	0.1498	0.7889	-12.9456	-1.1213	0.0192
1979	0.0594	0.1433	0.7973	-13.3071	-1.0016	0.0265
1980	0.0610	0.1482	0.7908	-13.0033	-1.0949	0.0209
1981	0.0657	0.1637	0.7706	-12.1545	-1.2946	0.0029
1982	0.0661	0.1653	0.7686	-12.0861	-1.3094	0.0010
1983	0.0697	0.1767	0.7536	-11.5087	-1.3860	-0.0127
1984	0.0723	0.1850	0.7427	-11.1345	-1.4194	-0.0228
1985	0.0711	0.1818	0.7471	-11.3034	-1.4085	-0.0187
1986	0.0676	0.1709	0.7615	-11.8407	-1.3520	-0.0054
1987	0.0681	0.1720	0.7599	-11.7654	-1.3593	-0.0068
1988	0.0702	0.1786	0.7512	-11.4382	-1.3950	-0.0149
1989	0.0714	0.1822	0.7465	-11.2654	-1.4097	-0.0193
1990	0.0649	0.1619	0.7732	-12.2872	-1.2776	0.0052

Table A5.3.4.24 Proper AESs and determinants of matrixes of AESs for I3SLS estimates, Model 4 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.0985	0.1666	-0.0778	-1.1928	-0.0540	-0.0089	-1.14E-16
1973	4.0596	0.1705	-0.0733	-0.8632	-0.0397	-0.0065	2.02E-17
1974	4.1345	0.1637	-0.0826	-1.5306	-0.0682	-0.0112	-2.27E-16
1975	4.3083	0.1499	-0.1059	-3.2705	-0.1351	-0.0225	-1.79E-16
1976	4.7287	0.1150	-0.1578	-8.0519	-0.2838	-0.0477	-9.50E-16
1977	4.6252	0.1205	-0.1422	-6.6582	-0.2452	-0.0406	-1.27E-15
1978	4.6700	0.1190	-0.1499	-7.2923	-0.2630	-0.0440	-4.35E-16
1979	4.9571	0.1009	-0.1895	-11.2448	-0.3633	-0.0625	-1.60E-15
1980	4.7276	0.1167	-0.1594	-8.1124	-0.2850	-0.0483	-8.83E-16
1981	4.1329	0.1589	-0.0775	-1.3449	-0.0607	-0.0098	-1.37E-16
1982	4.0825	0.1619	-0.0696	-0.8405	-0.0389	-0.0062	-9.62E-17
1983	3.7349	0.1894	-0.0207	2.0012	0.1100	0.0171	4.55E-17
1984	3.5206	0.2062	0.0111	3.4088	0.2115	0.0323	3.82E-16
1985	3.6065	0.1980	-0.0004	2.9139	0.1726	0.0264	5.99E-16
1986	3.9162	0.1728	-0.0444	0.6719	0.0338	0.0053	1.16E-16
1987	3.8773	0.1768	-0.0398	0.9598	0.0492	0.0077	3.35E-17
1988	3.6881	0.1923	-0.0131	2.3544	0.1330	0.0206	2.19E-16
1989	3.5923	0.2003	0.0006	2.9764	0.1773	0.0272	2.36E-16
1990	4.2050	0.1516	-0.0857	-1.9844	-0.0870	-0.0140	-1.33E-16

Table A5.3.4.25 Fitted cost shares and Own AESs for I3SLS (AR) estimates, Model 2 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0864	0.1754	0.7382	-4.4789	-1.4541	0.1045
1973	0.0724	0.1675	0.7600	-4.1332	-1.4094	0.1174
1974	0.0592	0.1584	0.7823	-2.9130	-1.3311	0.1306
1975	0.0498	0.1496	0.8004	-0.7471	-1.2207	0.1412
1976	0.0585	0.1489	0.7924	-2.8020	-1.2102	0.1365
1977	0.0819	0.1651	0.7527	-4.4272	-1.3918	0.1131
1978	0.0633	0.1527	0.7837	-3.4400	-1.2648	0.1314
1979	0.0555	0.1434	0.8007	-2.2545	-1.1152	0.1414
1980	0.0315	0.1323	0.8358	15.0913	-0.8502	0.1617
1981	0.0227	0.1377	0.8392	45.4208	-0.9929	0.1637
1982	0.0351	0.1465	0.8179	9.4040	-1.1704	0.1514
1983	0.0483	0.1621	0.7890	-0.1951	-1.3674	0.1345
1984	0.0714	0.1819	0.7461	-4.0800	-1.4783	0.1092
1985	0.1058	0.2006	0.6929	-4.3870	-1.5025	0.0779
1986	0.1073	0.1940	0.6979	-4.3679	-1.5003	0.0808
1987	0.0834	0.1803	0.7356	-4.4483	-1.4731	0.1029
1988	0.0714	0.1775	0.7502	-4.0792	-1.4630	0.1116
1989	0.0758	0.1826	0.7407	-4.2725	-1.4803	0.1060
1990	0.0756	0.1686	0.7547	-4.2673	-1.4171	0.1142

Table A5.3.4.26 Proper AESs and determinants of matrixes of AESs for I3SLS (AR) estimates, Model 2 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.4709	-0.5381	-0.1778	-13.4763	-0.7575	-0.1835	0.0099
1973	5.3359	-0.7821	-0.1979	-22.6460	-1.0970	-0.2046	0.0171
1974	6.6083	-1.1173	-0.2310	-39.7927	-1.6286	-0.2271	0.0322
1975	8.0546	-1.4592	-0.2736	-63.9653	-2.2346	-0.2472	0.0560
1976	7.0361	-1.1156	-0.2925	-46.1153	-1.6270	-0.2507	0.0455
1977	4.8890	-0.5907	-0.2273	-17.7404	-0.8495	-0.2090	0.0210
1978	6.4427	-0.9782	-0.2741	-37.1574	-1.4088	-0.2413	0.0428
1979	7.6051	-1.2060	-0.3283	-55.3233	-1.7730	-0.2654	0.0655
1980	13.6208	-2.7249	-0.3794	-198.3571	-4.9848	-0.2815	0.2294
1981	17.8530	-4.1564	-0.3199	-363.8292	-9.8423	-0.2649	0.4425
1982	11.2242	-2.4143	-0.2731	-136.9895	-4.4053	-0.2518	0.1880
1983	7.7188	-1.5748	-0.1921	-59.3130	-2.5064	-0.2208	0.0908
1984	5.0504	-0.8417	-0.1236	-19.4754	-1.1539	-0.1767	0.0350
1985	3.4779	-0.3382	-0.0970	-5.5048	-0.4562	-0.1265	0.0122
1986	3.5266	-0.3102	-0.1260	-5.8838	-0.4493	-0.1372	0.0139
1987	4.5009	-0.6000	-0.1502	-13.7055	-0.8179	-0.1742	0.0309
1988	5.1517	-0.8319	-0.1451	-20.5724	-1.1474	-0.1844	0.0465
1989	4.8021	-0.7479	-0.1276	-16.7354	-1.0122	-0.1731	0.0409
1990	5.1246	-0.7189	-0.1982	-20.2151	-1.0043	-0.2012	0.0510

Table A5.3.4.27 Fitted cost shares and Own AESs for I3SLS (AR) estimates, Model 3 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0545	0.1443	0.8012	4.8717	-5.8724	0.0497
1973	0.0527	0.1411	0.8062	5.7711	-6.0263	0.0537
1974	0.0541	0.1446	0.8012	5.0452	-5.8562	0.0497
1975	0.0521	0.1458	0.8021	6.0939	-5.8038	0.0504
1976	0.0546	0.1568	0.7887	4.8464	-5.3293	0.0394
1977	0.0611	0.1635	0.7754	2.3061	-5.0707	0.0283
1978	0.0561	0.1578	0.7861	4.1341	-5.2884	0.0373
1979	0.0465	0.1500	0.8035	10.0338	-5.6120	0.0516
1980	0.0507	0.1518	0.7975	6.9452	-5.5370	0.0467
1981	0.0674	0.1618	0.7708	0.6976	-5.1340	0.0245
1982	0.0712	0.1657	0.7631	-0.0261	-4.9927	0.0179
1983	0.0810	0.1697	0.7494	-1.2817	-4.8526	0.0060
1984	0.0908	0.1762	0.7329	-2.0086	-4.6354	-0.0084
1985	0.0910	0.1791	0.7299	-2.0186	-4.5470	-0.0111
1986	0.0815	0.1751	0.7434	-1.3354	-4.6713	0.0008
1987	0.0801	0.1723	0.7476	-1.1996	-4.7627	0.0044
1988	0.0854	0.1741	0.7404	-1.6605	-4.7027	-0.0018
1989	0.0882	0.1750	0.7368	-1.8526	-4.6752	-0.0050
1990	0.0703	0.1672	0.7624	0.1246	-4.9363	0.0173

Table A5.3.4.28 Proper AESs and determinants of matrixes of AESs for I3SLS (AR) estimates, Model 3 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	8.8837	-1.9314	0.4533	-107.5284	-3.4880	-0.4975	-7.40E-15
1973	9.3319	-2.0110	0.4444	-121.8632	-3.7339	-0.5214	-5.10E-15
1974	8.9170	-1.9507	0.4547	-109.0579	-3.5543	-0.4980	-1.00E-14
1975	9.1564	-2.0602	0.4594	-119.2068	-3.9371	-0.5039	-6.54E-15
1976	8.2488	-1.9751	0.4889	-93.8702	-3.7099	-0.4491	-2.82E-16
1977	7.2035	-1.7010	0.5015	-63.5847	-2.8279	-0.3951	1.13E-15
1978	7.9996	-1.9012	0.4905	-85.8567	-3.4606	-0.4377	-6.36E-15
1979	9.8906	-2.4273	0.4757	-154.1336	-5.3739	-0.5158	3.93E-15
1980	9.0558	-2.1649	0.4778	-120.4635	-4.3624	-0.4871	8.56E-15
1981	6.6870	-1.4648	0.4933	-48.2978	-2.1285	-0.3690	-1.91E-15
1982	6.2563	-1.3557	0.5001	-39.0103	-1.8383	-0.3396	-4.49E-15
1983	5.5143	-1.1099	0.5029	-24.1880	-1.2397	-0.2823	-1.90E-15
1984	4.8739	-0.9231	0.5107	-14.4446	-0.8352	-0.2217	-1.98E-15
1985	4.8050	-0.9271	0.5165	-13.9094	-0.8371	-0.2162	-4.59E-16
1986	5.3430	-1.1122	0.5145	-22.3096	-1.2381	-0.2683	-1.04E-15
1987	5.4907	-1.1371	0.5094	-24.4349	-1.2983	-0.2807	-4.96E-16
1988	5.1685	-1.0241	0.5099	-18.9052	-1.0458	-0.2515	-1.04E-15
1989	5.0177	-0.9700	0.5099	-16.5159	-0.9316	-0.2366	-3.62E-16
1990	6.2710	-1.3871	0.5044	-39.9406	-1.9219	-0.3399	1.32E-15

Table A5.3.4.29 Fitted cost shares and Own AESs for I3SLS (AR) estimates, Model 4 (ISIC 3840)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0641	0.1432	0.7927	-9.1733	-5.5931	-0.0118
1973	0.0646	0.1401	0.7952	-9.1334	-5.7290	-0.0094
1974	0.0635	0.1435	0.7931	-9.2195	-5.5814	-0.0115
1975	0.0606	0.1443	0.7952	-9.4310	-5.5475	-0.0095
1976	0.0549	0.1545	0.7906	-9.8171	-5.1384	-0.0138
1977	0.0567	0.1613	0.7820	-9.7011	-4.8930	-0.0222
1978	0.0558	0.1556	0.7887	-9.7623	-5.0973	-0.0157
1979	0.0515	0.1475	0.8010	-10.0073	-5.4130	-0.0039
1980	0.0546	0.1495	0.7959	-9.8326	-5.3305	-0.0088
1981	0.0644	0.1604	0.7751	-9.1494	-4.9221	-0.0290
1982	0.0655	0.1643	0.7701	-9.0669	-4.7889	-0.0339
1983	0.0727	0.1690	0.7583	-8.5387	-4.6362	-0.0459
1984	0.0780	0.1761	0.7460	-8.1567	-4.4216	-0.0586
1985	0.0761	0.1786	0.7453	-8.2922	-4.3471	-0.0593
1986	0.0692	0.1740	0.7568	-8.7945	-4.4823	-0.0474
1987	0.0698	0.1713	0.7588	-8.7479	-4.5639	-0.0453
1988	0.0739	0.1736	0.7525	-8.4458	-4.4956	-0.0518
1989	0.0762	0.1747	0.7492	-8.2841	-4.4634	-0.0553
1990	0.0635	0.1657	0.7709	-9.2211	-4.7447	-0.0332

Table A5.3.4.30 Proper AESs and determinants of matrixes of AESs for I3SLS (AR) estimates, Model 4 (ISIC 3840)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	7.8961	-0.6846	0.3719	-11.0411	-0.3603	-0.0722	7.97E-16
1973	7.9888	-0.6653	0.3602	-11.4950	-0.3569	-0.0759	3.89E-17
1974	7.9510	-0.7005	0.3734	-11.7596	-0.3849	-0.0753	5.80E-16
1975	8.2437	-0.7771	0.3784	-15.6397	-0.5148	-0.0908	-8.27E-16
1976	8.4663	-0.9726	0.4162	-21.2338	-0.8105	-0.1023	-9.74E-16
1977	7.9239	-0.9309	0.4347	-15.3212	-0.6516	-0.0805	1.54E-15
1978	8.2982	-0.9468	0.4189	-19.0988	-0.7432	-0.0954	1.83E-16
1979	9.3273	-1.0733	0.3964	-32.8291	-1.1129	-0.1360	1.75E-15
1980	8.7493	-0.9687	0.4008	-24.1377	-0.8519	-0.1137	2.30E-15
1981	7.1243	-0.7142	0.4267	-5.7222	-0.2451	-0.0395	2.37E-16
1982	6.8778	-0.6961	0.4366	-3.8835	-0.1768	-0.0281	-7.13E-17
1983	6.1539	-0.5535	0.4437	1.7159	0.0853	0.0158	-8.63E-17
1984	5.6105	-0.4716	0.4571	4.5875	0.2556	0.0501	-7.02E-16
1985	5.6582	-0.5099	0.4645	4.0323	0.2317	0.0420	-4.81E-16
1986	6.2570	-0.6347	0.4586	0.2692	0.0142	0.0023	1.83E-17
1987	6.2913	-0.6156	0.4516	0.3442	0.0175	0.0029	3.36E-18
1988	5.9323	-0.5386	0.4541	2.7767	0.1477	0.0268	9.40E-17
1989	5.7575	-0.4998	0.4551	3.8267	0.2080	0.0396	-1.33E-17
1990	7.0216	-0.7500	0.4417	-5.5518	-0.2564	-0.0376	-4.43E-16

A5.3.5 Economies of Scale

Table A5.3.5.1 IZEF estimates of Economies of Scale - ISIC 3840

Year	Economies of Scale - Model 1 - IZEF estimates	Economies of Scale - Model 2 - IZEF estimates	Economies of Scale - Model 3 - IZEF estimates	Economies of Scale Model 1 - IZEF (AR) estimates	Economies of Scale - Model 2 - IZEF (AR) estimates	Economies of Scale - Model 3 - IZEF (AR) estimates
1972	1.2894	1.2168	1.0406	1.6906	1.2999	1.2212
1973	1.2553	1.2096	1.0370	1.6123	1.2055	1.1888
1974	1.2271	1.2086	1.0333	1.5643	1.0934	1.1563
1975	1.2074	1.2036	1.0312	1.5176	0.9905	1.1394
1976	1.2300	1.2122	1.0336	1.5419	0.9297	1.1587
1977	1.2841	1.2297	1.0387	1.5943	0.8944	1.2041
1978	1.2393	1.2160	1.0345	1.5047	0.8411	1.1670
1979	1.2201	1.1990	1.0343	1.4365	0.7861	1.1650
1980	1.1706	1.1981	1.0269	1.3772	0.7401	1.1048
1981	1.1543	1.2202	1.0213	1.3757	0.7102	1.0620
1982	1.1788	1.2301	1.0239	1.3882	0.6768	1.0814
1983	1.2049	1.2487	1.0254	1.4078	0.6632	1.0926
1984	1.2553	1.2720	1.0296	1.4535	0.6463	1.1260
1985	1.3384	1.2842	1.0390	1.5070	0.6238	1.2068
1986	1.3409	1.2695	1.0415	1.4662	0.5777	1.2294
1987	1.2801	1.2592	1.0351	1.3898	0.5427	1.1721
1988	1.2518	1.2635	1.0307	1.3558	0.5222	1.1352
1989	1.2606	1.2692	1.0313	1.3469	0.4990	1.1401
1990	1.2590	1.2416	1.0350	1.2979	0.4665	1.1714
MEAN	1.2446	1.2343	1.0328	1.4646	0.7742	1.1538

Table A5.3.5.2 I3SLS estimates of Economies of Scale - ISIC 3840

Year	Economies of Scale - Model 1 - I3SLS estimates	Economies of Scale - Model 2 - I3SLS estimates	Economies of Scale - Model 3 - I3SLS estimates	Economies of Scale - Model 1 - I3SLS (AR) estimates	Economies of Scale - Model 2 - I3SLS (AR) estimates	Economies of Scale - Model 3 - I3SLS (AR) estimates
1972	1.3021	1.2126	1.0267	.	1.3139	1.2015
1973	1.2609	1.2078	1.0267		1.2883	1.1625
1974	1.2261	1.2100	1.0267		1.2750	1.1238
1975	1.2037	1.2067	1.0267		1.2625	1.1039
1976	1.2316	1.2146	1.0267		1.2901	1.1267
1977	1.2979	1.2285	1.0267		1.3376	1.1808
1978	1.2447	1.2176	1.0267		1.2992	1.1365
1979	1.2260	1.2003	1.0267		1.2729	1.1341
1980	1.1636	1.2052	1.0267		1.2445	1.0634
1981	1.1384	1.2327	1.0267		1.2533	1.0141
1982	1.1678	1.2409	1.0267		1.2773	1.0364
1983	1.1969	1.2587	1.0267		1.3066	1.0493
1984	1.2562	1.2790	1.0267		1.3566	1.0881
1985	1.3630	1.2833	1.0267		1.4179	1.1842
1986	1.3733	1.2665	1.0267		1.4082	1.2115
1987	1.2971	1.2613	1.0267		1.3636	1.1425
1988	1.2599	1.2694	1.0267		1.3504	1.0990
1989	1.2713	1.2746	1.0267		1.3602	1.1046
1990	1.2792	1.2434	1.0267		1.3384	1.1417
MEAN	1.2505	1.2375	1.0267		1.3167	1.1213

A5.3.6 Graphs of Economies of Scale

Figure A5.3.6.1 Economies of Scale based on IZEF estimates - ISIC 3840

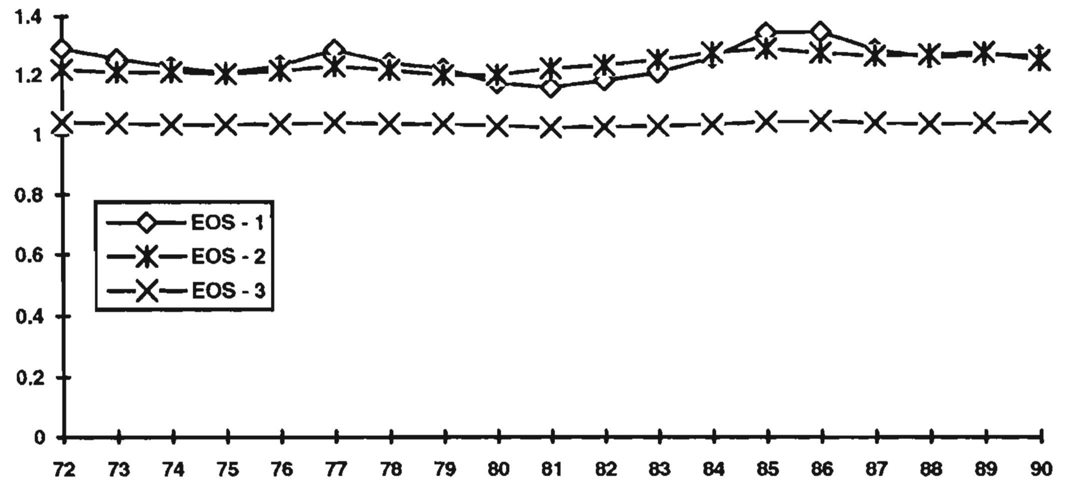


Figure A5.3.6.2 Economies of Scale based on IZEF (AR) estimates - ISIC 3840

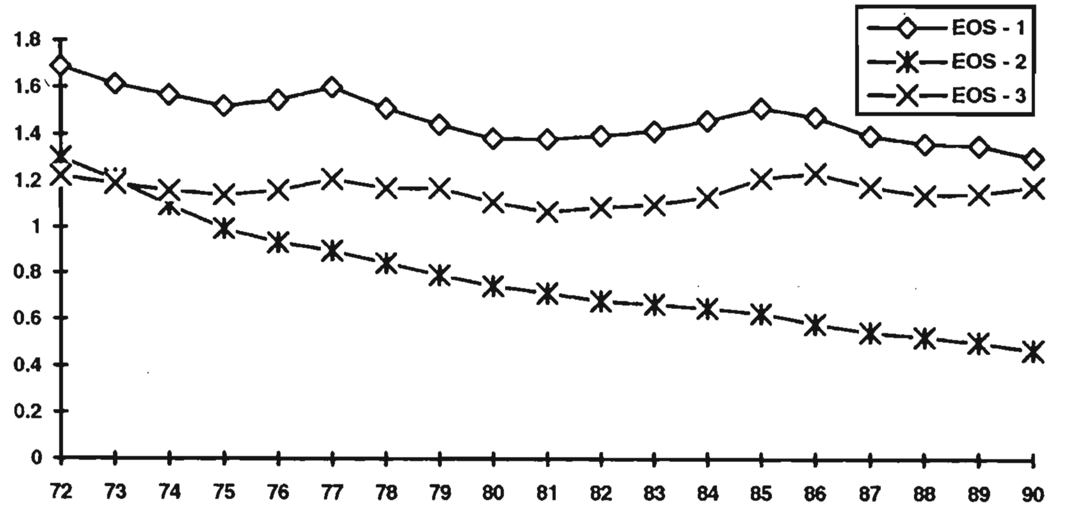


Figure A5.3.6.3 Economies of Scale based on I3SLS estimates - ISIC 3840

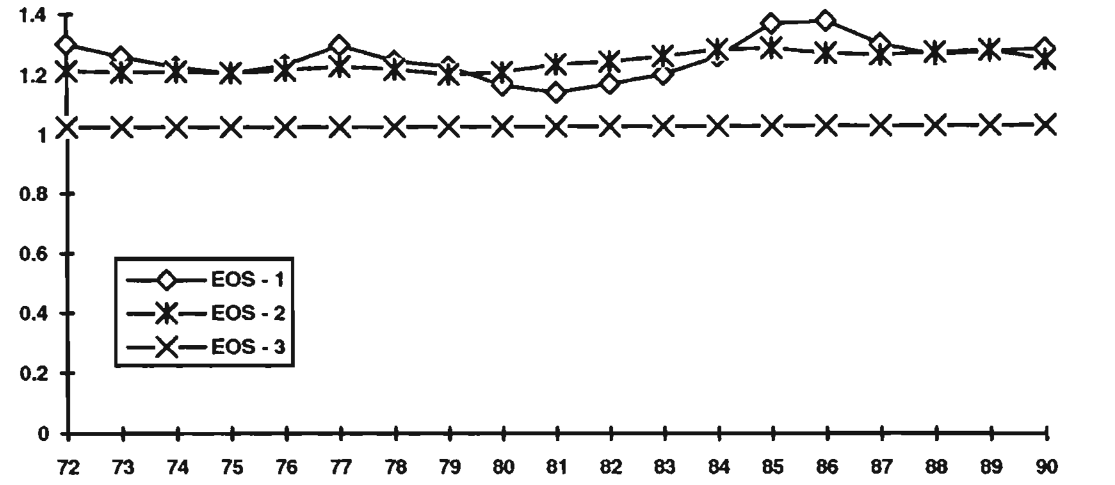


Figure A5.3.6.4 Economies of Scale based on I3SLS (AR) estimates - ISIC 3840

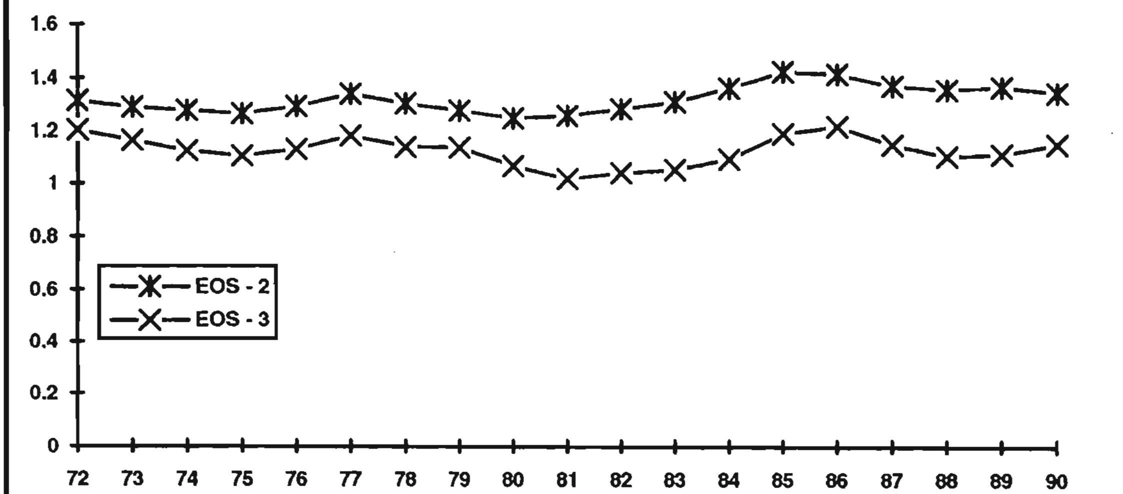


Figure A5.3.6.5 Economies of Scale - Model 1 - IZEF, IZEF (AR) and I3SLS (AR)

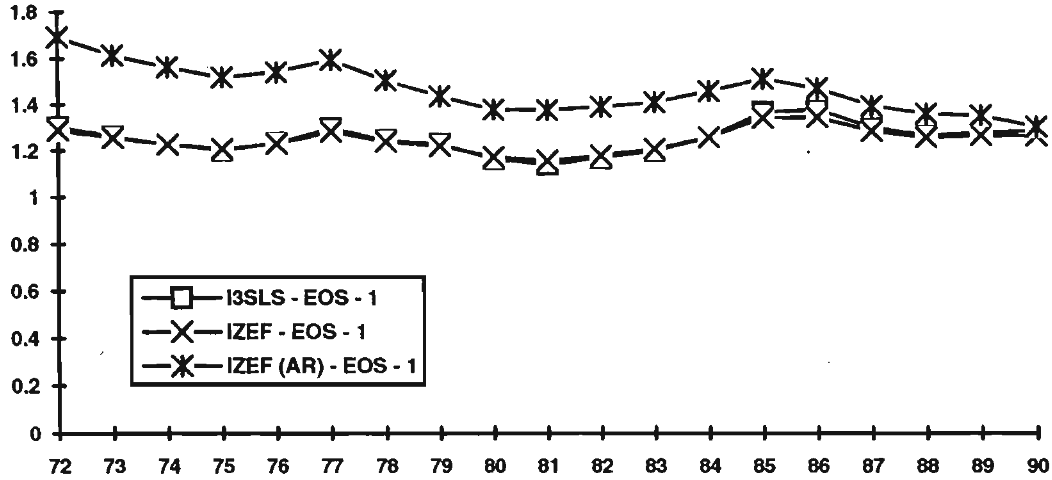


Figure A5.3.6.6 Economies of Scale - Model 2 - IZEF, IZEF (AR), I3SLS and I3SLS (AR)

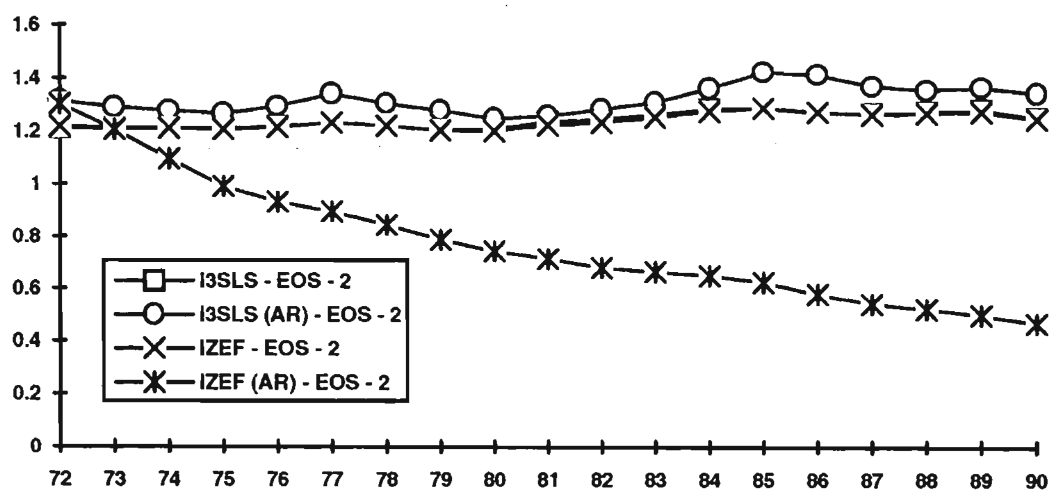
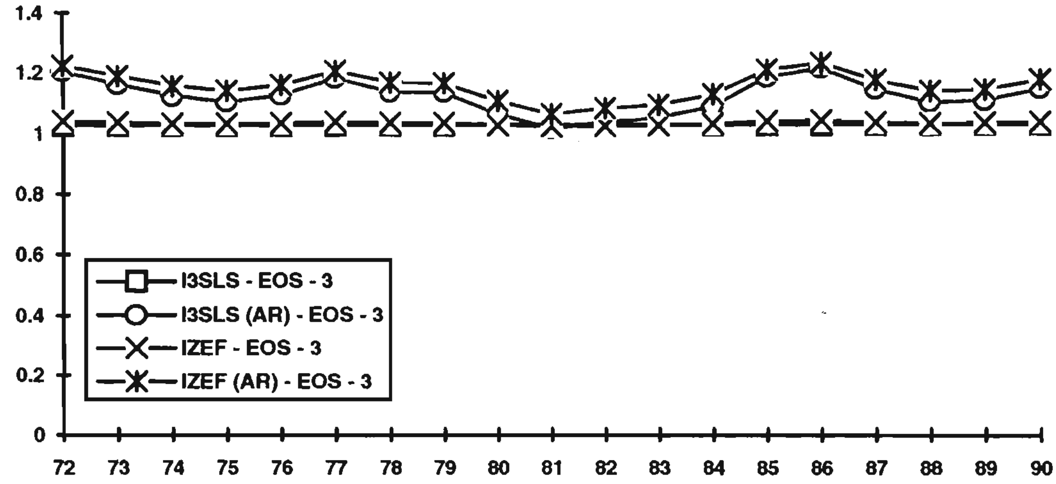


Figure A5.3.6.7 Economies of Scale - Model 3 - IZEF, IZEF (AR), I3SLS and I3SLS (AR)



APPENDIX 5.4

FURNITURE (ISIC 3320)

5.4.1 Hypothesis Test Results

Table A5.4.1.1 LR test statistics and appropriate chi-square statistics for ZEF and 3SLS estimates of ISIC (3320)

Null Hypothesis	Critical χ^2 statistic and df for IZEF, IZEF (AR) and I3SLS estimates	Computed L. R. statistic IZEF estimates	Computed L.R. statistic IZEF (AR) estimates	Computed L.R. statistic I3SLS estimates	Critical χ^2 statistic and df for I3SLS (AR) estimates	Computed L.R. statistic I3SLS (AR) estimates
Non-homothetic	11.07 (5)	22.683	--	21.685	--	--
Homotheticity	14.07 (7)	27.637	17.159	26.692	5.99 (2)	-1.111
Homogeneity in Output	15.51 (8)	27.990	18.161	26.692	7.81 (3)	-23.142
V.R.T.S. Cobb-Douglas	19.7 (11)	62.748	58.878	61.406	12.59 (6)	17.320

Note: 1. For IZEF, IZEF (AR) and I3SLS the alternate hypothesis is that the technology augmented form is valid. For I3SLS (AR) estimates the alternate hypothesis is that the non-homothetic form is valid.

2. Numbers in parenthesis appearing after χ^2 indicate Degrees of Freedom (df) (at the 5% level of significance) which are determined by the number of parameter constraints required to obtain the specification of the different null hypothesis from the alternative specification.

Table A5.4.1.2 LR test statistics for comparison of IZEF and IZEF(AR) estimators ISIC 3320

Model	1	2	3	4	5
Likelihood Ratio Statistic	N/A	-0.233	8.792	8.124	0.335
Critical Chi-squared statistic	7.81	7.81	7.81	7.81	3.84

Table A5.4.1.3 LR test statistics for comparison of I3SLS and I3SLS(AR) estimators ISIC 3320

Model	2	3	4	5
Likelihood Ratio Statistic	-20.042	-14.188	7.843	0.268
Critical Chi-squared statistic	7.81	7.81	7.81	3.84

Table A5.4.1.4 LR test of Economies of Scale - IZEF estimates (ISIC 3320)

	Critical Chi-Squared Statistic	Computed Wald - IZEF estimates	Computed Wald - IZEF (AR) estimates
Model 1 (5)	11.071	34.600	14.640
Model 2 (4)	9.488	54.171	--
Model 3 (2)	5.991	49.848	4.932
Model 4 (1)	3.841	47.417	4.385
Model 5 (1)	3.841	44.803	2.096

Note: 1. Numbers in parenthesis after the Model Number are the number of degrees of freedom
2. Source of critical chi-squared values: Gujarati (1988, p685)

Table A5.4.1.5 LR test of Economies of Scale - I3SLS estimates (ISIC 3320)

	Critical Chi-Squared Statistic	Computed Wald - I3SLS estimates	Computed Wald - I3SLS (AR) estimates
Model 1 (5)	11.071	32.675	--
Model 2 (4)	9.488	54.150	19.599
Model 3 (2)	5.991	51.838	12.058
Model 4 (1)	3.841	50.678	7.334
Model 5 (1)	3.841	42.824	1.276

Note: 1. Numbers in parenthesis after the Model Number are the number of degrees of freedom
2. Source of critical chi-squared values: Gujarati (1988, p685)

5.4.2 Durbin-Watson Test Results

Table A5.4.2.1 Computed and critical Durbin-Watson statistics for IZEF estimates of ISIC 3320.

Model	Equation	Number of Explanatories	DL	DU	Computed D-W Statistic	H_0: ρ=0 vs. H_1: ρ>0
Model 1	Cost function	14	0.070	3.642	0.888	N.D.
	Cost share of Capital	4	0.859	1.848	1.733	N.D.
	Cost share of Labour	4	0.859	1.848	1.103	N.D.
Model 2	Cost function	9	0.369	2.783	0.435	N.D.
	Cost share of Capital	3	0.967	1.685	1.041	N.D.
	Cost share of Labour	3	0.967	1.685	0.408	Reject
Model 3	Cost function	7	0.549	2.396	0.377	Reject
	Cost share of Capital	2	1.074	1.536	1.104	N.D.
	Cost share of Labour	2	1.074	1.536	0.335	Reject
Model 4	Cost function	6	0.649	2.206	0.391	Reject
	Cost share of Capital	2	1.074	1.536	1.119	N.D.
	Cost share of Labour	2	1.074	1.536	0.339	Reject
Model 5	Cost function	3	0.967	1.685	0.391	Reject
	Cost share of Capital	--	--	--	--	--
	Cost share of Labour	--	--	--	--	--

Note: Significant Lower and Upper D-W statistics are given for the 0.05 level of significance (Source: Gujarati, 1988, p687).

Table A5.4.2.2 Computed and critical Durbin-Watson statistics for I3SLS estimates of ISIC 3320

Model	Equation	Number of Exogenous Variables	DL	DU	Computed D-W Statistic	H_0: ρ=0 vs. H_1: ρ>0
Model 1	Cost function	15	0.063	3.676	0.961	N.D.
	Cost share of Capital	15	0.063	3.676	1.741	N.D.
	Cost share of Labour	15	0.063	3.676	1.103	N.D.
Model 2	Cost function	11	0.220	3.159	0.454	N.D.
	Cost share of Capital	11	0.220	3.159	1.071	N.D.
	Cost share of Labour	11	0.220	3.159	0.405	N.D.
Model 3	Cost function	11	0.220	3.159	0.395	N.D.
	Cost share of Capital	11	0.220	3.159	1.114	N.D.
	Cost share of Labour	11	0.220	3.159	0.335	N.D.
Model 4	Cost function	11	0.220	3.159	0.395	N.D.
	Cost share of Capital	11	0.220	3.159	1.112	N.D.
	Cost share of Labour	11	0.220	3.159	0.337	N.D.
Model 5	Cost function	8	0.456	2.589	0.391	Reject
	Cost share of Capital	--	--	--	--	--
	Cost share of Labour	--	--	--	--	--

Note: Significant Lower and Upper D-W statistics are given for the 0.05 level of significance (Source: Gujarati, 1988, p687).

5.4.3 Parameter Estimates

Table A5.4.3.1 IZEF parameter estimates for Furniture (ISIC 3320).

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o	18.7265 (1044.19)	18.7551 (878.141)	18.7293 (1281.77)	18.7231 (1730.12)	18.7357 (1584.90)
α_K	0.0298 (6.4772)	0.0404 (7.4758)	0.0360 (12.2287)	0.0358 (12.1968)	0.0365 (14.6303)
α_L	0.3343 (46.6709)	0.3504 (37.9877)	0.3348 (49.1518)	0.3339 (50.1495)	0.3020 (44.8775)
α_M	0.6359	0.6092	0.6292	0.6303	0.6615
α_Y	0.8584 (4.5440)	0.6983 (6.3029)	0.7952 (8.8000)	0.8492 (38.7837)	0.7916 (25.4297)
α_t	-0.0047 (-0.7747)				
γ_{KK}	-0.0044 (-0.2417)	0.0245 (4.7877)	0.0251 (5.2046)	0.0265 (5.7032)	
γ_{KL}	9.40E-05 (0.0059)	0.0457 (5.4972)	0.0364 (4.2228)	0.0374 (4.3089)	
γ_{KM}	0.0043	-0.0702	-0.0615	-0.0639	
γ_{LL}	0.3028 (8.6365)	0.2293 (11.0997)	0.2382 (10.6702)	0.2349 (10.9819)	
γ_{LM}	-0.3029	-0.2750	-0.2746	-0.2723	
γ_{MM}	0.2986	0.3452	0.3361	0.3362	
γ_{KY}	-0.0354 (-2.1396)	-0.0101 (-0.5188)			
γ_{LY}	-0.1279 (-5.2163)	-0.0815 (-2.5057)			
γ_{MY}	0.1633	0.0916			
γ_{YY}	0.0946 (0.2065)	0.1054 (0.3322)	0.1869 (0.6008)		
γ_{tt}	0.0013 (1.9738)				
γ_{Yt}	-0.0157 (-1.2642)				
γ_{Kt}	0.0017 (1.8831)				
γ_{Lt}	0.0053 (4.7521)				
γ_{Mt}	-0.0070				
Dtrm	9.97E-13	3.29E-12	4.27E-12	4.35E-12	2.71E-11

Note: 1. Numbers in parenthesis are *t*-statistics referring to parameter estimates above. *t*-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covariance matrix.

Table A5.4.3.2 IZEF AR(1) parameter estimates for Furniture (ISIC 3320).

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o	18.7344 (446.019)		18.7637 (471.858)	18.7287 (782.140)	18.7160 (771.828)
α_K	0.0290 (5.4532)		0.0358 (6.8730)	0.0363 (8.4681)	0.0366 (15.5720)
α_L	0.3294 (24.3411)		0.3284 (19.7286)	0.3336 (19.8748)	0.3026 (45.0868)
α_M	0.6416		0.6358	0.6301	0.6608
α_Y	0.7350 (2.0129)		0.6034 (2.7361)	0.8381 (10.8367)	0.8799 (10.6029)
α_t	-0.0046 (-0.6969)				
γ_{KK}	5.1E-05 (0.0026)		0.0317 (3.0595)	0.0260 (3.1229)	
γ_{KL}	0.0024 (0.1529)		0.0430 (2.5357)	0.0374 (2.3841)	
γ_{KM}	-0.0025		-0.0747	-0.0634	
γ_{LL}	0.2902 (6.5941)		0.2169 (4.5046)	0.2312 (4.7835)	
γ_{LM}	-0.2926		-0.2599	-0.2686	
γ_{MM}	0.2951		0.3346	0.3320	
γ_{KY}	-0.0335 (-1.9550)				
γ_{LY}	-0.1112 (-2.7786)				
γ_{MY}	0.1447				
γ_{YY}	0.2462 (0.3478)		0.7602 (1.2015)		
γ_{tt}	0.0011 (1.5342)				
γ_{Yt}	-0.0066 (-0.3615)				
γ_{Kt}	0.0016 (1.7208)				
γ_{Lt}	0.0050 (4.1016)				
γ_{Mt}	-0.0066				
Dtrm	1.01E-12		2.62E-12	2.77E-12	2.66E-11

Note: 1. Numbers in parenthesis are *t*-statistics referring to parameter estimates above. *t*-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covariance matrix.

Table A5.4.3.3 I3SLS parameter estimates for Furniture (ISIC 3320).

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o	18.7291 (1052.33)	18.7492 (809.314)	18.7255 (1098.64)	18.7250 (1734.44)	18.7357 (1571.29)
α_K	0.0298 (6.4595)	0.0398 (7.4180)	0.0358 (12.1265)	0.0359 (12.2893)	0.0365 (14.6304)
α_L	0.3342 (46.4143)	0.3493 (36.8824)	0.3345 (47.9128)	0.3343 (49.7975)	0.3020 (0.0067)
α_M	0.6360	0.6109	0.6297	0.6298	0.6615
α_Y	0.9120 (4.7551)	0.7514 (5.4979)	0.8384 (6.9707)	0.8416 (37.8196)	0.7919 (24.9053)
α_t	-0.0076 (-1.2056)				
γ_{KK}	-0.0065 (-0.3489)	0.0256 (4.8512)	0.0255 (5.0606)	0.0259 (5.5829)	
γ_{KL}	-0.0009 (-0.0576)	0.0447 (5.3090)	0.0357 (4.0950)	0.0369 (4.2284)	
γ_{KM}	0.0074	-0.0703	-0.0612	-0.0628	
γ_{LL}	0.3023 (8.4905)	0.2258 (10.3918)	0.2353 (9.9341)	0.2360 (10.8070)	
γ_{LM}	-0.3014	-0.2705	-0.2710	-0.2729	
γ_{MM}	0.2940	0.3408	0.3322	0.3357	
γ_{KY}	-0.0371 (-2.2135)	-0.0099 (-0.5147)			
γ_{LY}	-0.1285 (-5.1708)	-0.0784 (-2.3425)			
γ_{MY}	0.1656	0.8830			
γ_{YY}	-0.3824 (-0.6911)	-0.0835 (-0.1959)	0.0079 (0.0188)		
γ_{tt}	0.0013 (2.0379)				
γ_{Yt}	-0.0051 (-0.3492)				
γ_{Kt}	0.0018 (1.9458)				
γ_{Lt}	0.0054 (4.7979)				
γ_{Mt}	-0.0072				
Dtrm	1.07E-12	3.35E-12	4.36E-12	4.36E-12	2.71E-11

Note: 1. Numbers in parenthesis are t-statistics referring to parameter estimates above. t-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covariance matrix.

Table A5.4.3.4 I3SLS AR(1) parameter estimates for Furniture (ISIC 3320).

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o		18.5046 (111.630)	18.5418 (134.981)	18.7388 (819.578)	18.7108 (690.890)
α_K		0.0339 (4.9303)	0.0322 (6.9514)	0.0366 (8.3166)	0.0365 (14.1616)
α_L		0.2396 (1.8319)	0.3393 (13.0888)	0.3329 (19.1357)	0.3003 (42.9850)
α_M		0.7265	0.6285	0.6305	0.6632
α_Y		2.5387 (1.9785)	2.3240 (2.2786)	0.7981 (10.7084)	0.8959 (9.7263)
α_t					
γ_{KK}		0.0305 (2.9084)	0.0360 (3.2999)	0.0267 (3.0043)	
γ_{KL}		0.0044 (0.1945)	0.0329 (1.7508)	0.0397 (2.5003)	
γ_{KM}		-0.0349	-0.0689	-0.0644	
γ_{LL}		0.2291 (3.9789)	0.2418 (4.4774)	0.2319 (4.7675)	
γ_{LM}		-0.2335	-0.2747	-0.2716	
γ_{MM}		0.2684	0.3436	0.3380	
γ_{KY}		-0.0214 (-1.0205)			
γ_{LY}		-0.0691 (-1.6471)			
γ_{MY}		0.0905			
γ_{YY}		-6.1347 (-1.5235)	-5.0693 (-1.5672)		
γ_{tt}					
γ_{Yt}					
γ_{Kt}					
γ_{Lt}					
γ_{Mt}					
Dtrm		1.02E-11	9.59E-12	2.82E-12	2.67E-11

Note: 1. Numbers in parenthesis are t-statistics referring to parameter estimates above. t-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covariance matrix.

5.4.4 Monotonicity and Concavity Test Results

Table A5.4.4.1 Fitted cost shares and Own AESs for IZEF estimates, Model 1 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0315	0.3396	0.6289	-35.1804	0.6809	0.1649
1973	0.0300	0.3310	0.6390	-37.1553	0.7428	0.1663
1974	0.0298	0.2974	0.6729	-37.5816	1.0613	0.1733
1975	0.0301	0.3088	0.6611	-37.0642	0.9375	0.1706
1976	0.0321	0.2922	0.6757	-34.3995	1.1245	0.1741
1977	0.0329	0.2861	0.6809	-33.4076	1.2035	0.1754
1978	0.0337	0.2907	0.6756	-32.5863	1.1432	0.1740
1979	0.0308	0.2851	0.6841	-36.1596	1.2178	0.1763
1980	0.0279	0.2659	0.7061	-40.5103	1.5210	0.1827
1981	0.0280	0.2672	0.7048	-40.3098	1.4984	0.1823
1982	0.0320	0.2878	0.6801	-34.4899	1.1812	0.1752
1983	0.0360	0.3139	0.6500	-30.1318	0.8873	0.1683
1984	0.0405	0.3294	0.6301	-26.3529	0.7550	0.1650
1985	0.0444	0.3440	0.6116	-23.7467	0.6521	0.1632
1986	0.0455	0.3376	0.6169	-23.0954	0.6947	0.1636
1987	0.0484	0.3284	0.6232	-21.5337	0.7626	0.1642
1988	0.0469	0.3057	0.6474	-22.3073	0.9691	0.1678
1989	0.0485	0.2649	0.6865	-21.4791	1.5392	0.1769
1990	0.0523	0.2837	0.6639	-19.7182	1.2366	0.1712

Table A5.4.4.2. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 1 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.0088	1.2171	-0.4182	-24.9724	-7.2820	-0.0627	0.0008
1973	1.0095	1.2240	-0.4322	-28.6164	-7.6784	-0.0633	0.0009
1974	1.0106	1.2148	-0.5138	-40.9066	-7.9904	-0.0800	0.0013
1975	1.0101	1.2160	-0.4839	-35.7689	-7.8014	-0.0742	0.0014
1976	1.0100	1.1981	-0.5343	-39.7009	-7.4231	-0.0897	0.0017
1977	1.0100	1.1917	-0.5546	-41.2245	-7.2802	-0.0965	0.0019
1978	1.0096	1.1891	-0.5422	-38.2715	-7.0852	-0.0951	0.0018
1979	1.0107	1.2043	-0.5530	-45.0577	-7.8246	-0.0911	0.0022
1980	1.0127	1.2183	-0.6129	-62.6433	-8.8851	-0.0978	0.0030
1981	1.0126	1.2178	-0.6084	-61.4238	-8.8300	-0.0971	0.0033
1982	1.0102	1.1973	-0.5474	-41.7585	-7.4765	-0.0927	0.0026
1983	1.0083	1.1835	-0.4845	-27.7531	-6.4717	-0.0854	0.0020
1984	1.0070	1.1684	-0.4595	-20.9098	-5.7144	-0.0866	0.0017
1985	1.0062	1.1583	-0.4399	-16.4967	-5.2177	-0.0870	0.0015
1986	1.0061	1.1532	-0.4545	-17.0573	-5.1084	-0.0929	0.0016
1987	1.0059	1.1425	-0.4801	-17.4340	-4.8415	-0.1053	0.0017
1988	1.0066	1.1415	-0.5307	-22.6321	-5.0460	-0.1190	0.0021
1989	1.0073	1.1291	-0.6653	-34.0751	-5.0751	-0.1703	0.0029
1990	1.0063	1.1238	-0.6079	-25.3968	-4.6388	-0.1579	0.0024

Table A5.4.4.3 Fitted cost shares and Own AESs for IZEF estimates, Model 2 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0404	0.3504	0.6092	-8.7417	0.0137	0.2886
1973	0.0370	0.3373	0.6257	-8.1321	0.0508	0.2835
1974	0.0314	0.3082	0.6604	-6.0045	0.1692	0.2773
1975	0.0340	0.3158	0.6502	-7.2058	0.1325	0.2785
1976	0.0356	0.3086	0.6558	-7.7651	0.1675	0.2778
1977	0.0369	0.3055	0.6576	-8.1057	0.1835	0.2776
1978	0.0353	0.3020	0.6627	-7.6522	0.2027	0.2770
1979	0.0321	0.2907	0.6772	-6.3792	0.2733	0.2761
1980	0.0293	0.2737	0.6970	-4.6066	0.4077	0.2758
1981	0.0324	0.2774	0.6902	-6.5139	0.3750	0.2758
1982	0.0363	0.2920	0.6717	-7.9449	0.2644	0.2763
1983	0.0397	0.3085	0.6518	-8.6452	0.1679	0.2783
1984	0.0440	0.3215	0.6345	-9.0728	0.1079	0.2814
1985	0.0465	0.3308	0.6227	-9.1757	0.0725	0.2844
1986	0.0450	0.3224	0.6326	-9.1249	0.1043	0.2818
1987	0.0419	0.3097	0.6484	-8.9104	0.1617	0.2788
1988	0.0388	0.2904	0.6708	-8.4958	0.2754	0.2764
1989	0.0324	0.2565	0.7111	-6.5012	0.5864	0.2764
1990	0.0325	0.2634	0.7041	-6.5911	0.5087	0.2761

Table A5.4.4.4. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 2 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.2283	-1.8523	-0.2883	-17.9979	-5.9543	-0.0792	3.81E-15
1973	4.6616	-2.0317	-0.3031	-22.1441	-6.4334	-0.0775	5.59E-15
1974	5.7203	-2.3843	-0.3511	-33.7378	-7.3499	-0.0763	8.01E-15
1975	5.2599	-2.1784	-0.3392	-28.6214	-6.7528	-0.0781	1.24E-14
1976	5.1577	-2.0049	-0.3590	-27.9025	-6.1766	-0.0823	7.62E-15
1977	5.0542	-1.8934	-0.3688	-27.0324	-5.8351	-0.0851	4.53E-15
1978	5.2921	-2.0049	-0.3739	-29.5575	-6.1395	-0.0836	9.41E-15
1979	5.8960	-2.2288	-0.3969	-36.5055	-6.7286	-0.0821	1.00E-14
1980	6.6957	-2.4350	-0.4417	-46.7104	-7.2001	-0.0826	5.40E-15
1981	6.0887	-2.1413	-0.4363	-39.5151	-6.3817	-0.0869	1.13E-14
1982	5.3154	-1.8821	-0.4019	-30.3538	-5.7379	-0.0885	8.47E-15
1983	4.7308	-1.7124	-0.3676	-23.8317	-5.3384	-0.0884	4.56E-15
1984	4.2297	-1.5142	-0.3481	-18.8689	-4.8459	-0.0908	3.82E-15
1985	3.9681	-1.4222	-0.3351	-16.4107	-4.6319	-0.0917	8.49E-15
1986	4.1477	-1.4643	-0.3484	-18.1549	-4.7159	-0.0920	6.60E-15
1987	4.5224	-1.5844	-0.3694	-21.8932	-4.9946	-0.0914	1.07E-14
1988	5.0576	-1.6984	-0.4116	-27.9187	-5.2327	-0.0933	7.57E-15
1989	6.5071	-2.0514	-0.5075	-46.1554	-6.0053	-0.0955	1.29E-14
1990	6.3330	-2.0644	-0.4829	-43.4603	-6.0814	-0.0928	1.93E-14

Table A5.4.4.5 Fitted cost shares and Own AESs for IZEF estimates, Model 3 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0360	0.3348	0.6292	-7.4105	0.1382	0.2596
1973	0.0336	0.3307	0.6357	-6.5279	0.1544	0.2586
1974	0.0295	0.3054	0.6650	-4.0923	0.2792	0.2563
1975	0.0321	0.3154	0.6525	-5.8157	0.2241	0.2569
1976	0.0344	0.3024	0.6632	-6.8499	0.2980	0.2563
1977	0.0361	0.2988	0.6651	-7.4496	0.3215	0.2563
1978	0.0347	0.2991	0.6662	-6.9702	0.3190	0.2562
1979	0.0327	0.3009	0.6664	-6.1098	0.3075	0.2562
1980	0.0315	0.2944	0.6742	-5.4297	0.3518	0.2562
1981	0.0349	0.2990	0.6661	-7.0456	0.3199	0.2562
1982	0.0379	0.3066	0.6555	-7.9095	0.2723	0.2567
1983	0.0403	0.3174	0.6423	-8.3580	0.2139	0.2578
1984	0.0438	0.3210	0.6352	-8.7453	0.1963	0.2587
1985	0.0456	0.3239	0.6305	-8.8602	0.1833	0.2594
1986	0.0446	0.3165	0.6389	-8.8019	0.2185	0.2582
1987	0.0417	0.3010	0.6573	-8.5438	0.3069	0.2566
1988	0.0400	0.2883	0.6717	-8.3094	0.3973	0.2562
1989	0.0350	0.2528	0.7122	-7.0728	0.7710	0.2585
1990	0.0344	0.2561	0.7095	-6.8544	0.7265	0.2582

Table A5.4.4.6. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 3 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.0200	-1.7151	-0.3035	-17.1849	-4.8656	-0.0563	3.14E-15
1973	4.2766	-1.8793	-0.3063	-19.2970	-5.2199	-0.0539	2.82E-15
1974	5.0335	-2.1301	-0.3519	-26.4787	-5.5861	-0.0523	5.59E-16
1975	4.5911	-1.9325	-0.3345	-22.3820	-5.2284	-0.0543	1.96E-15
1976	4.5016	-1.6974	-0.3692	-22.3056	-4.6369	-0.0599	2.97E-15
1977	4.3722	-1.5593	-0.3819	-21.5109	-4.3404	-0.0635	-1.43E-16
1978	4.5074	-1.6611	-0.3780	-22.5397	-4.5451	-0.0611	4.32E-15
1979	4.6988	-1.8219	-0.3694	-23.9573	-4.8848	-0.0577	3.64E-15
1980	4.9298	-1.8991	-0.3837	-26.2137	-4.9975	-0.0571	3.35E-15
1981	4.4883	-1.6456	-0.3788	-22.3987	-4.5133	-0.0615	3.89E-15
1982	4.1329	-1.4760	-0.3663	-19.2341	-4.2086	-0.0643	2.61E-15
1983	3.8463	-1.3763	-0.3470	-16.5818	-4.0486	-0.0653	3.36E-15
1984	3.5903	-1.2120	-0.3466	-14.6069	-3.7313	-0.0694	1.13E-15
1985	3.4631	-1.1377	-0.3448	-13.6172	-3.5929	-0.0713	1.81E-15
1986	3.5800	-1.1591	-0.3580	-14.7398	-3.6160	-0.0718	3.56E-16
1987	3.9015	-1.2448	-0.3879	-17.8439	-3.7416	-0.0717	4.96E-15
1988	4.1581	-1.2900	-0.4180	-20.5913	-3.7927	-0.0729	1.35E-15
1989	5.1162	-1.4690	-0.5250	-31.6286	-3.9864	-0.0763	4.27E-15
1990	5.1324	-1.5208	-0.5111	-31.3213	-4.0828	-0.0736	4.29E-15

Table A5.4.4.7 Fitted cost shares and Own AESs for IZEF estimates, Model 4 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0358	0.3339	0.6303	-6.2563	0.1120	0.2597
1973	0.0333	0.3297	0.6371	-5.1147	0.1280	0.2587
1974	0.0291	0.3048	0.6661	-2.0667	0.2476	0.2565
1975	0.0318	0.3147	0.6535	-4.2427	0.1944	0.2570
1976	0.0343	0.3022	0.6635	-5.6082	0.2631	0.2565
1977	0.0361	0.2988	0.6651	-6.3801	0.2843	0.2565
1978	0.0346	0.2991	0.6663	-5.7694	0.2826	0.2565
1979	0.0325	0.3006	0.6669	-4.6700	0.2728	0.2564
1980	0.0312	0.2941	0.6746	-3.8293	0.3152	0.2564
1981	0.0348	0.2989	0.6662	-5.8655	0.2834	0.2565
1982	0.0380	0.3066	0.6554	-6.9559	0.2373	0.2569
1983	0.0405	0.3173	0.6423	-7.5285	0.1817	0.2580
1984	0.0441	0.3211	0.6348	-8.0543	0.1640	0.2590
1985	0.0461	0.3240	0.6299	-8.2230	0.1513	0.2598
1986	0.0450	0.3167	0.6383	-8.1384	0.1844	0.2585
1987	0.0420	0.3014	0.6566	-7.7911	0.2680	0.2568
1988	0.0403	0.2889	0.6708	-7.4958	0.3531	0.2564
1989	0.0352	0.2540	0.7109	-6.0073	0.7045	0.2586
1990	0.0345	0.2571	0.7083	-5.7313	0.6637	0.2583

Table A5.4.4.8. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 4 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.1288	-1.8319	-0.2938	-17.7475	-4.9805	-0.0573	1.29E-15
1973	4.4101	-2.0151	-0.2965	-20.1039	-5.3838	-0.0548	1.13E-15
1974	5.2171	-2.2970	-0.3412	-27.7303	-5.8064	-0.0529	1.62E-15
1975	4.7373	-2.0745	-0.3241	-23.2672	-5.3939	-0.0551	2.51E-15
1976	4.6131	-1.8114	-0.3579	-22.7560	-4.7199	-0.0606	7.19E-16
1977	4.4636	-1.6587	-0.3703	-21.7378	-4.3877	-0.0642	4.87E-16
1978	4.6137	-1.7710	-0.3665	-22.9166	-4.6160	-0.0618	-2.96E-16
1979	4.8300	-1.9498	-0.3582	-24.6031	-4.9993	-0.0584	1.70E-15
1980	5.0750	-2.0356	-0.3722	-26.9628	-5.1256	-0.0577	-9.26E-16
1981	4.5923	-1.7539	-0.3672	-22.7515	-4.5804	-0.0622	-3.58E-15
1982	4.2127	-1.5675	-0.3551	-19.3979	-4.2441	-0.0651	-2.19E-15
1983	3.9133	-1.4587	-0.3363	-16.6819	-4.0705	-0.0662	1.82E-17
1984	3.6386	-1.2804	-0.3360	-14.5606	-3.7256	-0.0704	-1.67E-15
1985	3.5038	-1.2003	-0.3343	-13.5205	-3.5768	-0.0724	-2.43E-15
1986	3.6225	-1.2233	-0.3471	-14.6238	-3.6005	-0.0728	-3.70E-15
1987	3.9525	-1.3157	-0.3760	-17.7096	-3.7319	-0.0726	-7.58E-16
1988	4.2130	-1.3641	-0.4051	-20.3964	-3.7826	-0.0736	-3.77E-15
1989	5.1879	-1.5562	-0.5083	-31.1459	-3.9751	-0.0762	1.67E-16
1990	5.2131	-1.6131	-0.4950	-30.9807	-4.0826	-0.0736	-1.69E-15

Table A5.4.4.9 Fitted cost shares and Own AESs for IZEF (AR) estimates, Model 1 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0306	0.3344	0.6350	-31.6253	0.6047	0.1570
1973	0.0288	0.3270	0.6444	-33.6361	0.6559	0.1588
1974	0.0282	0.2953	0.6776	-34.3472	0.9411	0.1669
1975	0.0289	0.3066	0.6666	-33.5721	0.8251	0.1640
1976	0.0315	0.2908	0.6815	-30.7260	0.9933	0.1680
1977	0.0327	0.2853	0.6869	-29.5728	1.0603	0.1696
1978	0.0330	0.2898	0.6820	-29.2237	1.0044	0.1682
1979	0.0298	0.2858	0.6893	-32.4511	1.0544	0.1703
1980	0.0270	0.2688	0.7099	-35.9650	1.2965	0.1769
1981	0.0277	0.2706	0.7088	-35.0571	1.2684	0.1766
1982	0.0319	0.2895	0.6867	-30.2748	1.0080	0.1696
1983	0.0359	0.3138	0.6592	-26.7801	0.7606	0.1621
1984	0.0408	0.3278	0.6414	-23.4843	0.6503	0.1582
1985	0.0448	0.3410	0.6252	-21.3147	0.5630	0.1555
1986	0.0457	0.3350	0.6310	-20.8418	0.6006	0.1564
1987	0.0482	0.3257	0.6385	-19.7236	0.6651	0.1577
1988	0.0467	0.3049	0.6616	-20.3856	0.8417	0.1627
1989	0.0479	0.2658	0.7003	-19.8384	1.3446	0.1738
1990	0.0513	0.2831	0.6803	-18.4591	1.0891	0.1677

Table A5.4.4.10. Proper AESs and determinants of matrixes of AESs for IZEF (AR) estimates, Model 1 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.2345	0.8713	-0.3780	-20.6490	-5.7259	-0.0479	0.0025
1973	1.2547	0.8654	-0.3887	-23.6362	-6.0909	-0.0469	-0.0077
1974	1.2878	0.8694	-0.4621	-33.9817	-6.4893	-0.0564	-0.0851
1975	1.2711	0.8701	-0.4315	-29.3161	-6.2614	-0.0509	-0.1358
1976	1.2623	0.8834	-0.4766	-32.1130	-5.9435	-0.0602	-0.2553
1977	1.2576	0.8886	-0.4930	-32.9377	-5.8060	-0.0632	-0.3379
1978	1.2507	0.8890	-0.4802	-30.9174	-5.7056	-0.0616	-0.3232
1979	1.2814	0.8785	-0.4854	-35.8598	-6.2997	-0.0560	-0.3679
1980	1.3307	0.8696	-0.5335	-48.3982	-7.1187	-0.0553	-0.5401
1981	1.3204	0.8726	-0.5258	-46.2114	-6.9510	-0.0525	-0.6460
1982	1.2597	0.8860	-0.4717	-32.1038	-5.9181	-0.0516	-0.5503
1983	1.2128	0.8945	-0.4147	-21.8397	-5.1412	-0.0487	-0.4424
1984	1.1795	0.9045	-0.3918	-16.6638	-4.5340	-0.0506	-0.4004
1985	1.1572	0.9107	-0.3724	-13.3390	-4.1433	-0.0512	-0.3696
1986	1.1566	0.9134	-0.3840	-13.8553	-4.0934	-0.0536	-0.4053
1987	1.1529	0.9188	-0.4069	-14.4474	-3.9540	-0.0607	-0.4356
1988	1.1685	0.9191	-0.4505	-18.5246	-4.1612	-0.0660	-0.5546
1989	1.1883	0.9255	-0.5716	-28.0866	-4.3041	-0.0931	-0.8083
1990	1.1652	0.9284	-0.5196	-21.4622	-3.9572	-0.0873	-0.6789

Table A5.4.4.11 Fitted cost shares and Own AESs for IZEF (AR) estimates, Model 3 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0358	0.3284	0.6358	-2.1990	-0.0339	0.2549
1973	0.0328	0.3237	0.6435	-0.0019	-0.0194	0.2540
1974	0.0280	0.3007	0.6714	5.7706	0.0733	0.2528
1975	0.0311	0.3103	0.6586	1.5691	0.0302	0.2530
1976	0.0342	0.3006	0.6652	-1.1604	0.0739	0.2529
1977	0.0365	0.2984	0.6650	-2.6273	0.0846	0.2529
1978	0.0347	0.2980	0.6673	-1.4847	0.0866	0.2528
1979	0.0321	0.2985	0.6694	0.6075	0.0843	0.2528
1980	0.0306	0.2924	0.6770	2.1556	0.1169	0.2529
1981	0.0350	0.2980	0.6670	-1.6657	0.0867	0.2528
1982	0.0387	0.3059	0.6554	-3.6681	0.0490	0.2532
1983	0.0416	0.3161	0.6423	-4.7209	0.0074	0.2541
1984	0.0460	0.3208	0.6331	-5.7609	-0.0098	0.2553
1985	0.0484	0.3241	0.6275	-6.1237	-0.0207	0.2561
1986	0.0471	0.3175	0.6354	-5.9464	0.0022	0.2550
1987	0.0436	0.3031	0.6533	-5.2707	0.0618	0.2533
1988	0.0417	0.2916	0.6667	-4.7404	0.1213	0.2528
1989	0.0358	0.2596	0.7046	-2.1962	0.3667	0.2547
1990	0.0350	0.2620	0.7030	-1.6855	0.3427	0.2546

Table A5.4.4.12. Proper AESs and determinants of matrixes of AESs for IZEF (AR) estimates, Model 3 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.6575	-2.2818	-0.2448	-21.6177	-5.7673	-0.0685	-7.73E-16
1973	5.0525	-2.5417	-0.2476	-25.5277	-6.4610	-0.0662	-3.29E-15
1974	6.1133	-2.9783	-0.2875	-36.9493	-7.4115	-0.0641	-2.35E-15
1975	5.4496	-2.6414	-0.2719	-29.6511	-6.5801	-0.0663	3.04E-16
1976	5.1792	-2.2805	-0.2999	-26.9102	-5.4942	-0.0713	-7.88E-16
1977	4.9428	-2.0737	-0.3096	-24.6538	-4.9646	-0.0744	-2.67E-15
1978	5.1593	-2.2270	-0.3069	-26.7466	-5.3350	-0.0723	1.85E-16
1979	5.4872	-2.4759	-0.3007	-30.0583	-5.9763	-0.0691	-2.36E-15
1980	5.8031	-2.6040	-0.3129	-33.4238	-6.2358	-0.0684	4.79E-15
1981	5.1276	-2.2035	-0.3075	-26.4369	-5.2768	-0.0726	7.20E-16
1982	4.6339	-1.9459	-0.2964	-21.6533	-4.7153	-0.0754	2.01E-15
1983	4.2705	-1.7954	-0.2802	-18.2716	-4.4234	-0.0766	1.62E-15
1984	3.9125	-1.5640	-0.2794	-15.2511	-3.9166	-0.0806	3.22E-15
1985	3.7431	-1.4616	-0.2778	-13.8839	-3.7048	-0.0825	3.59E-15
1986	3.8741	-1.4944	-0.2885	-15.0219	-3.7494	-0.0826	1.83E-15
1987	4.2506	-1.6200	-0.3126	-18.3931	-3.9593	-0.0821	3.98E-15
1988	4.5387	-1.6892	-0.3367	-21.1748	-4.0519	-0.0827	3.24E-15
1989	5.6280	-1.9616	-0.4210	-32.4793	-4.4074	-0.0838	4.17E-15
1990	5.6904	-2.0371	-0.4109	-32.9581	-4.5790	-0.0816	6.41E-15

Table A5.4.4.13 Fitted cost shares and Own AESs for IZEF (AR) estimates, Model 4 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0363	0.3336	0.6301	-6.8167	0.0799	0.2492
1973	0.0338	0.3294	0.6368	-5.8333	0.0950	0.2484
1974	0.0296	0.3049	0.6655	-3.1490	0.2072	0.2470
1975	0.0323	0.3146	0.6530	-5.0533	0.1572	0.2472
1976	0.0347	0.3025	0.6628	-6.2121	0.2208	0.2470
1977	0.0365	0.2992	0.6643	-6.8795	0.2403	0.2470
1978	0.0350	0.2994	0.6656	-6.3475	0.2390	0.2470
1979	0.0329	0.3009	0.6662	-5.3930	0.2302	0.2470
1980	0.0317	0.2945	0.6738	-4.6463	0.2700	0.2471
1981	0.0352	0.2993	0.6655	-6.4312	0.2397	0.2470
1982	0.0383	0.3069	0.6548	-7.3880	0.1963	0.2471
1983	0.0408	0.3174	0.6418	-7.8885	0.1442	0.2479
1984	0.0444	0.3213	0.6343	-8.3329	0.1273	0.2486
1985	0.0463	0.3242	0.6295	-8.4706	0.1152	0.2492
1986	0.0452	0.3170	0.6377	-8.4006	0.1461	0.2483
1987	0.0423	0.3019	0.6558	-8.1035	0.2241	0.2471
1988	0.0405	0.2896	0.6699	-7.8418	0.3035	0.2470
1989	0.0354	0.2553	0.7094	-6.4901	0.6307	0.2501
1990	0.0348	0.2584	0.7069	-6.2485	0.5931	0.2498

Table A5.4.4.14. Proper AESs and determinants of matrixes of AESs for IZEF estimates, Model 4 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.0884	-1.7719	-0.2778	-17.2598	-4.8380	-0.0573	-5.73E-16
1973	4.3581	-1.9445	-0.2805	-19.5469	-5.2298	-0.0551	2.15E-15
1974	5.1375	-2.2136	-0.3238	-27.0466	-5.6779	-0.0537	1.36E-15
1975	4.6772	-2.0034	-0.3072	-22.6702	-5.2627	-0.0555	-2.43E-15
1976	4.5663	-1.7590	-0.3396	-22.2232	-4.6285	-0.0608	1.72E-15
1977	4.4250	-1.6153	-0.3513	-21.2337	-4.3082	-0.0641	-3.32E-15
1978	4.5685	-1.7215	-0.3478	-22.3885	-4.5314	-0.0619	-2.54E-15
1979	4.7740	-1.8898	-0.3400	-24.0321	-4.9032	-0.0587	-3.90E-16
1980	5.0109	-1.9720	-0.3535	-26.3640	-5.0370	-0.0582	-4.47E-16
1981	4.5482	-1.7054	-0.3485	-22.2271	-4.4967	-0.0622	-7.64E-16
1982	4.1808	-1.5273	-0.3366	-18.9291	-4.1585	-0.0648	-1.46E-15
1983	3.8887	-1.4221	-0.3185	-16.2596	-3.9779	-0.0657	-1.69E-15
1984	3.6223	-1.2516	-0.3180	-14.1816	-3.6383	-0.0695	8.60E-16
1985	3.4909	-1.1745	-0.3162	-13.1623	-3.4908	-0.0713	1.45E-15
1986	3.6079	-1.1975	-0.3285	-14.2440	-3.5197	-0.0717	1.22E-15
1987	3.9317	-1.2881	-0.3565	-17.2738	-3.6616	-0.0717	-4.93E-16
1988	4.1877	-1.3364	-0.3845	-19.9162	-3.7232	-0.0728	-1.75E-15
1989	5.1422	-1.5268	-0.4834	-30.5354	-3.9542	-0.0759	-3.63E-15
1990	5.1650	-1.5805	-0.4707	-30.3834	-4.0587	-0.0735	-2.79E-15

Table A5.4.4.15 Fitted cost shares and Own AESs for I3SLS estimates, Model 1 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0316	0.3396	0.6288	-37.1549	0.6766	0.1532
1973	0.0303	0.3311	0.6386	-39.1291	0.7372	0.1550
1974	0.0301	0.2976	0.6723	-39.3887	1.0526	0.1630
1975	0.0304	0.3090	0.6606	-38.9867	0.9295	0.1599
1976	0.0322	0.2924	0.6754	-36.3714	1.1158	0.1639
1977	0.0329	0.2864	0.6808	-35.4509	1.1944	0.1655
1978	0.0338	0.2911	0.6752	-34.3166	1.1326	0.1638
1979	0.0309	0.2856	0.6835	-38.1201	1.2051	0.1663
1980	0.0280	0.2665	0.7055	-43.0156	1.5045	0.1733
1981	0.0279	0.2677	0.7044	-43.2360	1.4830	0.1729
1982	0.0319	0.2883	0.6798	-36.6924	1.1682	0.1652
1983	0.0360	0.3145	0.6495	-31.7649	0.8766	0.1573
1984	0.0405	0.3300	0.6295	-27.6768	0.7456	0.1534
1985	0.0444	0.3447	0.6109	-24.8043	0.6434	0.1509
1986	0.0456	0.3384	0.6160	-24.0430	0.6849	0.1514
1987	0.0488	0.3294	0.6218	-22.2270	0.7505	0.1522
1988	0.0473	0.3067	0.6460	-23.0553	0.9531	0.1565
1989	0.0491	0.2661	0.6848	-22.0705	1.5105	0.1666
1990	0.0532	0.2851	0.6617	-20.1040	1.2114	0.1602

Table A5.4.4.16. Proper AESs and determinants of matrixes of AESs for I3SLS estimates, Model 1 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	0.9161	1.3724	-0.4114	-25.9774	-7.5772	-0.0656	-1.66E-15
1973	0.9102	1.3828	-0.4254	-29.6743	-7.9772	-0.0667	-3.71E-15
1974	0.8996	1.3656	-0.5063	-42.2694	-8.2860	-0.0848	-3.99E-16
1975	0.9041	1.3689	-0.4764	-37.0573	-8.1092	-0.0783	3.68E-15
1976	0.9043	1.3406	-0.5261	-41.4012	-7.7588	-0.0939	-2.98E-15
1977	0.9044	1.3308	-0.5461	-43.1603	-7.6365	-0.1006	1.51E-15
1978	0.9084	1.3246	-0.5337	-39.6928	-7.3768	-0.0993	3.32E-15
1979	0.8981	1.3500	-0.5441	-46.7436	-8.1602	-0.0957	4.03E-15
1980	0.8794	1.3747	-0.6031	-65.4895	-9.3424	-0.1031	2.20E-15
1981	0.8794	1.3768	-0.5983	-64.8911	-9.3707	-0.1016	-2.57E-15
1982	0.9022	1.3409	-0.5379	-43.6788	-7.8579	-0.0964	3.84E-15
1983	0.9206	1.3163	-0.4756	-28.6916	-6.7284	-0.0883	3.47E-15
1984	0.9326	1.2904	-0.4508	-21.5059	-5.9097	-0.0889	-3.88E-15
1985	0.9412	1.2727	-0.4314	-16.8460	-5.3616	-0.0891	-1.23E-15
1986	0.9417	1.2633	-0.4460	-17.3530	-5.2365	-0.0952	2.05E-15
1987	0.9440	1.2439	-0.4716	-17.5718	-4.9298	-0.1082	-2.03E-15
1988	0.9379	1.2423	-0.5212	-22.8528	-5.1517	-0.1224	-3.45E-15
1989	0.9311	1.2202	-0.6538	-34.2052	-5.1667	-0.1758	-6.22E-16
1990	0.9406	1.2103	-0.5976	-25.2391	-4.6857	-0.1630	-3.31E-15

Table A5.4.4.17 Fitted cost shares and Own AESs for I3SLS estimates, Model 2 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0398	0.3493	0.6109	-7.9644	-0.0122	0.2763
1973	0.0363	0.3366	0.7018	-7.1294	0.0221	0.2670
1974	0.0308	0.3081	0.7784	-4.5091	0.1331	0.2778
1975	0.0334	0.3156	0.7915	-6.0065	0.0983	0.2806
1976	0.0354	0.3083	0.7701	-6.8213	0.1319	0.2761
1977	0.0368	0.3053	0.7791	-7.2820	0.1470	0.2779
1978	0.0351	0.3020	0.8119	-6.7138	0.1646	0.2853
1979	0.0318	0.2911	0.9336	-5.1545	0.2291	0.3199
1980	0.0291	0.2746	1.0501	-3.1242	0.3529	0.3568
1981	0.0323	0.2783	1.0635	-5.4260	0.3224	0.3610
1982	0.0363	0.2925	0.9875	-7.1086	0.2202	0.3368
1983	0.0397	0.3086	0.9162	-7.9457	0.1306	0.3145
1984	0.0442	0.3212	0.8266	-8.5167	0.0753	0.2890
1985	0.0468	0.3302	0.7623	-8.6773	0.0426	0.2747
1986	0.0453	0.3219	0.7842	-8.5988	0.0725	0.2790
1987	0.0422	0.3094	0.7771	-8.3171	0.1268	0.2775
1988	0.0391	0.2905	0.8646	-7.8387	0.2331	0.2993
1989	0.0329	0.2572	0.9043	-5.7360	0.5259	0.3109
1990	0.0330	0.2638	0.8584	-5.7826	0.4536	0.2975

Table A5.4.4.18. Proper AESs and determinants of matrixes of AESs for I3SLS estimates, Model 2 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.2153	-1.8914	-0.2676	-17.6718	-5.7775	-0.0750	2.48E-15
1973	4.6556	-1.7574	-0.1452	-21.8321	-4.9922	-0.0152	-3.3724
1974	5.7046	-1.9284	-0.1279	-33.1423	-4.9711	0.0206	-6.8137
1975	5.2365	-1.6569	-0.0828	-28.0114	-4.4306	0.0207	-6.6503
1976	5.0950	-1.5784	-0.1392	-26.8586	-4.3749	0.0170	-5.3745
1977	4.9740	-1.4493	-0.1372	-25.8112	-4.1243	0.0220	-5.3670
1978	5.2163	-1.4665	-0.1033	-28.3146	-4.0661	0.0363	-6.7798
1979	5.8221	-1.3651	0.0048	-35.0783	-3.5123	0.0733	-11.7235
1980	6.5961	-1.3014	0.0618	-44.6112	-2.8083	0.1221	-17.5628
1981	5.9722	-1.0461	0.0859	-37.4170	-3.0532	0.1090	-14.8950
1982	5.2139	-0.9632	0.0636	-28.7505	-3.3221	0.0701	-10.4978
1983	4.6490	-0.9329	0.0432	-22.6512	-3.3695	0.0392	-7.5978
1984	4.1521	-0.9262	-0.0188	-17.8813	-3.3193	0.0214	-5.0851
1985	3.8957	-0.9724	-0.0747	-15.5463	-3.3289	0.0061	-3.6959
1986	4.0664	-0.9798	-0.0715	-17.1591	-3.3589	0.0151	-4.2431
1987	4.4267	-1.1456	-0.1252	-20.6505	-3.6205	0.0195	-4.4971
1988	4.9307	-1.0773	-0.0768	-26.1385	-3.5067	0.0639	-7.2314
1989	6.2861	-1.3642	-0.1633	-42.5309	-3.6443	0.1368	-11.2485
1990	6.1380	-1.4836	-0.1944	-40.2980	-3.9217	0.0972	-9.2296

Table A5.4.4.19 Fitted cost shares and Own AESs for I3SLS estimates, Model 3 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0358	0.3345	0.6297	-7.0366	0.1134	0.2497
1973	0.0334	0.3304	0.6362	-6.0645	0.1287	0.2489
1974	0.0294	0.3055	0.6651	-3.4953	0.2477	0.2474
1975	0.0320	0.3153	0.6527	-5.3331	0.1952	0.2477
1976	0.0344	0.3024	0.6632	-6.5039	0.2660	0.2474
1977	0.0362	0.2988	0.6650	-7.1595	0.2886	0.2474
1978	0.0347	0.2992	0.6661	-6.6426	0.2861	0.2474
1979	0.0327	0.3010	0.6664	-5.7087	0.2749	0.2474
1980	0.0314	0.2945	0.6740	-5.0015	0.3172	0.2476
1981	0.0349	0.2991	0.6660	-6.7242	0.2870	0.2474
1982	0.0379	0.3066	0.6555	-7.6415	0.2416	0.2476
1983	0.0403	0.3172	0.6425	-8.1168	0.1860	0.2483
1984	0.0439	0.3208	0.6354	-8.5448	0.1693	0.2490
1985	0.0458	0.3236	0.6307	-8.6751	0.1570	0.2496
1986	0.0447	0.3162	0.6390	-8.6112	0.1906	0.2486
1987	0.0419	0.3010	0.6572	-8.3364	0.2751	0.2475
1988	0.0402	0.2884	0.6714	-8.0972	0.3615	0.2475
1989	0.0353	0.2534	0.7113	-6.8697	0.7183	0.2507
1990	0.0347	0.2567	0.7086	-6.6365	0.6757	0.2504

Table A5.4.4.20. Proper AESs and determinants of matrixes of AESs for I3SLS estimates, Model 3 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	3.9812	-1.7148	-0.2866	-16.6479	-4.6977	-0.0538	-6.20E-16
1973	4.2383	-1.8832	-0.2891	-18.7439	-5.0561	-0.0515	-3.02E-15
1974	4.9770	-2.1319	-0.3336	-25.6365	-5.4100	-0.0500	-1.20E-15
1975	4.5409	-1.9324	-0.3168	-21.6613	-5.0550	-0.0520	-4.77E-15
1976	4.4355	-1.6858	-0.3511	-21.4036	-4.4513	-0.0574	-3.29E-15
1977	4.3018	-1.5436	-0.3637	-20.5720	-4.1542	-0.0609	-1.51E-15
1978	4.4378	-1.6475	-0.3597	-21.5942	-4.3580	-0.0586	-1.18E-15
1979	4.6323	-1.8127	-0.3512	-23.0278	-4.6985	-0.0553	-1.95E-15
1980	4.8567	-1.8890	-0.3651	-25.1744	-4.8068	-0.0547	-1.18E-15
1981	4.4185	-1.6317	-0.3605	-21.4527	-4.3262	-0.0590	-1.97E-15
1982	4.0695	-1.4611	-0.3485	-18.4070	-4.0267	-0.0617	1.39E-15
1983	3.7910	-1.3623	-0.3298	-15.8808	-3.8713	-0.0626	-6.00E-16
1984	3.5370	-1.1958	-0.3297	-13.9571	-3.5576	-0.0665	-1.32E-15
1985	3.4115	-1.1208	-0.3280	-13.0003	-3.4214	-0.0684	-1.37E-15
1986	3.5238	-1.1411	-0.3410	-14.0587	-3.4432	-0.0689	5.17E-16
1987	3.8338	-1.2248	-0.3702	-16.9906	-3.5635	-0.0689	-1.62E-15
1988	4.0788	-1.2672	-0.3996	-19.5638	-3.6101	-0.0702	-2.98E-16
1989	4.9896	-1.4364	-0.5036	-29.8310	-3.7856	-0.0735	-1.60E-15
1990	5.0096	-1.4896	-0.4899	-29.5802	-3.8805	-0.0709	-5.12E-17

Table A5.4.4.21 Fitted cost shares and Own AESs for I3SLS estimates, Model 4 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0359	0.3343	0.6298	-6.7591	0.1204	0.2585
1973	0.0334	0.3301	0.6365	-5.7345	0.1364	0.2575
1974	0.0293	0.3051	0.6656	-2.9700	0.2575	0.2553
1975	0.0320	0.3150	0.6530	-4.9371	0.2038	0.2559
1976	0.0343	0.3023	0.6633	-6.1537	0.2743	0.2554
1977	0.0362	0.2988	0.6650	-6.8477	0.2964	0.2554
1978	0.0347	0.2991	0.6662	-6.2967	0.2943	0.2553
1979	0.0326	0.3008	0.6666	-5.3061	0.2838	0.2553
1980	0.0313	0.2943	0.6744	-4.5401	0.3268	0.2553
1981	0.0349	0.2990	0.6661	-6.3834	0.2952	0.2553
1982	0.0380	0.3066	0.6554	-7.3703	0.2487	0.2557
1983	0.0404	0.3173	0.6422	-7.8864	0.1923	0.2568
1984	0.0440	0.3211	0.6349	-8.3499	0.1747	0.2577
1985	0.0459	0.3240	0.6301	-8.4946	0.1619	0.2585
1986	0.0449	0.3166	0.6385	-8.4217	0.1958	0.2573
1987	0.0419	0.3013	0.6568	-8.1148	0.2809	0.2557
1988	0.0402	0.2887	0.6711	-7.8477	0.3678	0.2553
1989	0.0351	0.2536	0.7113	-6.4786	0.7265	0.2576
1990	0.0345	0.2568	0.7087	-6.2289	0.6844	0.2573

Table A5.4.4.22 Proper AESs and determinants of matrixes of AESs for I3SLS estimates, Model 4 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.0746	-1.7776	-0.2962	-17.4166	-4.9072	-0.0566	-2.85E-15
1973	4.3444	-1.9522	-0.2989	-19.6554	-5.2877	-0.0542	-5.27E-15
1974	5.1258	-2.2192	-0.3438	-27.0390	-5.6832	-0.0524	-8.12E-15
1975	4.6644	-2.0083	-0.3266	-22.7630	-5.2967	-0.0545	-4.79E-15
1976	4.5550	-1.7576	-0.3608	-22.4359	-4.6607	-0.0601	-4.34E-15
1977	4.4145	-1.6114	-0.3732	-21.5174	-4.3453	-0.0636	-4.48E-15
1978	4.5577	-1.7189	-0.3694	-22.6258	-4.5625	-0.0613	-3.53E-15
1979	4.7626	-1.8896	-0.3610	-24.1884	-4.9254	-0.0579	-7.07E-15
1980	5.0003	-1.9713	-0.3750	-26.4872	-5.0450	-0.0572	-6.48E-15
1981	4.5374	-1.7026	-0.3701	-22.4719	-4.5286	-0.0616	-3.52E-15
1982	4.1697	-1.5240	-0.3579	-19.2199	-4.2074	-0.0645	-7.86E-15
1983	3.8771	-1.4195	-0.3390	-16.5484	-4.0404	-0.0655	-3.40E-15
1984	3.6114	-1.2476	-0.3387	-14.5008	-3.7087	-0.0697	-2.69E-15
1985	3.4803	-1.1701	-0.3369	-13.4877	-3.5649	-0.0716	-5.22E-15
1986	3.5977	-1.1923	-0.3499	-14.5920	-3.5883	-0.0720	-7.09E-15
1987	3.9225	-1.2814	-0.3791	-17.6654	-3.7167	-0.0719	-3.72E-15
1988	4.1800	-1.3281	-0.4086	-20.3589	-3.7673	-0.0730	-5.68E-15
1989	5.1422	-1.5133	-0.5130	-31.1488	-3.9593	-0.0760	-6.13E-15
1990	5.1637	-1.5680	-0.4994	-30.9262	-4.0615	-0.0733	-1.39E-14

Table A5.4.4.23 Fitted cost shares and Own AESs for I3SLS (AR) estimates, Model 2 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0339	0.2396	0.7265	-1.9586	0.8171	0.1321
1973	0.0290	0.2315	0.7396	2.8145	0.9561	0.1386
1974	0.0270	0.2036	0.7695	5.8374	1.6163	0.1537
1975	0.0281	0.2093	0.7626	3.9721	1.4531	0.1502
1976	0.0351	0.1926	0.7722	-2.7408	1.9821	0.1551
1977	0.0384	0.1849	0.7768	-4.3397	2.2946	0.1575
1978	0.0353	0.1850	0.7797	-2.8428	2.2887	0.1590
1979	0.0288	0.1802	0.7910	3.0798	2.5038	0.1647
1980	0.0255	0.1653	0.8092	8.6887	3.3345	0.1741
1981	0.0292	0.1636	0.8072	2.5757	3.4462	0.1731
1982	0.0338	0.1734	0.7928	-1.8572	2.8506	0.1657
1983	0.0366	0.1874	0.7760	-3.5516	2.1865	0.1570
1984	0.0432	0.1931	0.7637	-5.8060	1.9663	0.1508
1985	0.0469	0.1985	0.7546	-6.4624	1.7780	0.1461
1986	0.0466	0.1901	0.7633	-6.4119	2.0794	0.1506
1987	0.0461	0.1782	0.7757	-6.3457	2.6046	0.1569
1988	0.0444	0.1595	0.7962	-6.0484	3.7385	0.1674
1989	0.0444	0.1247	0.8309	-6.0432	7.7093	0.1853
1990	0.0439	0.1332	0.8229	-5.9482	6.3989	0.1811

Table A5.4.4.24 Proper AESs and determinants of matrixes of AESs for I3SLS (AR) estimates, Model 2 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.5417	-0.4171	-0.3414	-3.9772	-0.4326	-0.0087	-3.44E-16
1973	1.6561	-0.6285	-0.3641	-0.0516	-0.0051	-0.0001	-3.50E-18
1974	1.8012	-0.6812	-0.4908	6.1908	0.4333	0.0076	6.79E-16
1975	1.7471	-0.6260	-0.4632	2.7196	0.2048	0.0037	-9.50E-17
1976	1.6505	-0.2871	-0.5695	-8.1567	-0.5076	-0.0169	-4.48E-16
1977	1.6206	-0.1714	-0.6261	-12.5841	-0.7127	-0.0307	-9.06E-16
1978	1.6741	-0.2685	-0.6188	-9.3088	-0.5240	-0.0191	1.53E-16
1979	1.8483	-0.5332	-0.6378	4.2950	0.2230	0.0057	5.45E-16
1980	2.0438	-0.6913	-0.7456	24.7954	1.0348	0.0246	7.12E-16
1981	1.9222	-0.4827	-0.7679	5.1813	0.2129	0.0068	4.25E-16
1982	1.7516	-0.3041	-0.6982	-8.3622	-0.4002	-0.0152	-6.25E-16
1983	1.6415	-0.2290	-0.6055	-10.4603	-0.6102	-0.0233	-1.28E-15
1984	1.5275	-0.0577	-0.5835	-13.7493	-0.8788	-0.0440	-7.14E-16
1985	1.4722	0.0149	-0.5592	-13.6578	-0.9447	-0.0529	-1.64E-15
1986	1.4969	0.0185	-0.6092	-15.5734	-0.9659	-0.0580	-2.10E-15
1987	1.5353	0.0248	-0.6895	-18.8849	-0.9963	-0.0668	-1.28E-15
1988	1.6216	0.0124	-0.8392	-25.2415	-1.0126	-0.0785	-3.98E-15
1989	1.7953	0.0531	-1.2531	-49.8115	-1.1224	-0.1420	-1.68E-15
1990	1.7526	0.0334	-1.1296	-41.1340	-1.0785	-0.1169	-4.59E-15

Table A5.4.4.25 Fitted cost shares and Own AESs for I3SLS (AR) estimates, Model 3 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0322	0.3393	0.6285	4.6650	0.1531	0.2788
1973	0.0288	0.3355	0.6357	9.6389	0.1677	0.2772
1974	0.0250	0.3099	0.6651	18.5277	0.2908	0.2732
1975	0.0281	0.3198	0.6521	10.9928	0.2376	0.2745
1976	0.0330	0.3057	0.6613	3.7533	0.3162	0.2735
1977	0.0362	0.3016	0.6623	0.8791	0.3429	0.2734
1978	0.0338	0.3022	0.6639	2.8855	0.3383	0.2733
1979	0.0305	0.3045	0.6650	6.9050	0.3237	0.2732
1980	0.0292	0.2979	0.6729	8.9349	0.3678	0.2727
1981	0.0342	0.3021	0.6638	2.5557	0.3396	0.2733
1982	0.0381	0.3094	0.6525	-0.4685	0.2938	0.2745
1983	0.0408	0.3202	0.6390	-1.8922	0.2354	0.2765
1984	0.0460	0.3233	0.6308	-3.7133	0.2204	0.2782
1985	0.0486	0.3259	0.6255	-4.3354	0.2084	0.2795
1986	0.0477	0.3183	0.6340	-4.1444	0.2447	0.2775
1987	0.0447	0.3027	0.6525	-3.3607	0.3351	0.2745
1988	0.0434	0.2898	0.6669	-2.9126	0.4286	0.2731
1989	0.0391	0.2537	0.7072	-1.0331	0.8149	0.2730
1990	0.0378	0.2573	0.7049	-0.2819	0.7659	0.2729

Table A5.4.4.26 Proper AESs and determinants of matrixes of AESs for I3SLS (AR) estimates, Model 3 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.0113	-2.4045	-0.2882	-15.3765	-4.4814	-0.0404	2.56E-15
1973	4.4027	-2.7602	-0.2881	-17.7667	-4.9471	-0.0365	1.38E-16
1974	5.2423	-3.1400	-0.3328	-22.0931	-4.7973	-0.0313	2.61E-15
1975	4.6608	-2.7590	-0.3174	-19.1118	-4.5947	-0.0355	5.65E-16
1976	4.2610	-2.1572	-0.3588	-16.9695	-3.6267	-0.0423	-3.57E-16
1977	4.0174	-1.8772	-0.3755	-15.8379	-3.2834	-0.0472	3.55E-15
1978	4.2166	-2.0667	-0.3689	-16.8034	-3.4825	-0.0437	2.61E-15
1979	4.5419	-2.3965	-0.3566	-18.3930	-3.8566	-0.0387	3.00E-15
1980	4.7790	-2.5040	-0.3704	-19.5529	-3.8329	-0.0369	6.19E-15
1981	4.1865	-2.0367	-0.3701	-16.6586	-3.4497	-0.0442	3.20E-15
1982	3.7883	-1.7691	-0.3607	-14.4886	-3.2582	-0.0495	2.07E-15
1983	3.5173	-1.6415	-0.3426	-12.8169	-3.2180	-0.0523	4.29E-15
1984	3.2146	-1.3771	-0.3471	-11.1521	-2.9294	-0.0592	1.99E-15
1985	3.0773	-1.2662	-0.3476	-10.3730	-2.8148	-0.0626	3.00E-15
1986	3.1662	-1.2780	-0.3612	-11.0392	-2.7835	-0.0625	3.06E-15
1987	3.4300	-1.3610	-0.3905	-12.8913	-2.7747	-0.0606	4.31E-15
1988	3.6187	-1.3832	-0.4215	-14.3434	-2.7085	-0.0606	1.80E-15
1989	4.3156	-1.4912	-0.5310	-19.4661	-2.5057	-0.0595	5.47E-15
1990	4.3796	-1.5836	-0.5147	-19.3966	-2.5846	-0.0559	2.03E-15

Table A5.4.4.27 Fitted cost shares and Own AESs for I3SLS (AR) estimates, Model 4 (ISIC 3320)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0366	0.3329	0.6305	-6.3905	0.0886	0.2642
1973	0.0340	0.3285	0.6375	-5.3349	0.1049	0.2631
1974	0.0296	0.3039	0.6665	-2.3356	0.2204	0.2605
1975	0.0324	0.3138	0.6538	-4.4432	0.1684	0.2612
1976	0.0347	0.3021	0.6632	-5.6505	0.2309	0.2606
1977	0.0365	0.2990	0.6644	-6.3721	0.2492	0.2606
1978	0.0350	0.2991	0.6659	-5.7888	0.2489	0.2605
1979	0.0329	0.3003	0.6667	-4.7513	0.2414	0.2605
1980	0.0316	0.2939	0.6745	-3.9059	0.2823	0.2604
1981	0.0352	0.2990	0.6657	-5.8802	0.2494	0.2605
1982	0.0385	0.3068	0.6547	-6.9479	0.2042	0.2611
1983	0.0410	0.3175	0.6415	-7.5144	0.1510	0.2625
1984	0.0447	0.3217	0.6336	-8.0119	0.1325	0.2637
1985	0.0467	0.3247	0.6286	-8.1713	0.1197	0.2646
1986	0.0456	0.3175	0.6369	-8.0864	0.1507	0.2631
1987	0.0424	0.3024	0.6552	-7.7357	0.2293	0.2611
1988	0.0406	0.2900	0.6694	-7.4246	0.3089	0.2604
1989	0.0351	0.2556	0.7093	-5.8116	0.6370	0.2620
1990	0.0345	0.2586	0.7069	-5.5450	0.6008	0.2618

Table A5.4.4.28 Proper AESs and determinants of matrixes of AESs for I3SLS (AR) estimates, Model 4 (ISIC 3320)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	4.2583	-1.8774	-0.2940	-18.6998	-5.2131	-0.0630	4.22E-15
1973	4.5505	-2.0599	-0.2971	-21.2665	-5.6464	-0.0606	5.30E-15
1974	5.4093	-2.3626	-0.3410	-29.7751	-6.1904	-0.0588	3.41E-15
1975	4.9023	-2.1325	-0.3239	-24.7809	-5.7081	-0.0609	4.41E-16
1976	4.7859	-1.8841	-0.3557	-24.2094	-5.0224	-0.0663	3.34E-15
1977	4.6325	-1.7346	-0.3670	-23.0478	-4.6691	-0.0697	5.89E-15
1978	4.7891	-1.8466	-0.3637	-24.3762	-4.9179	-0.0675	2.94E-15
1979	5.0131	-2.0233	-0.3564	-26.2786	-5.3313	-0.0641	5.29E-15
1980	5.2750	-2.1154	-0.3701	-28.9286	-5.4919	-0.0635	2.71E-15
1981	4.7669	-1.8296	-0.3644	-24.1897	-4.8793	-0.0678	3.32E-15
1982	4.3652	-1.6376	-0.3520	-20.4736	-4.4960	-0.0706	3.79E-15
1983	4.0470	-1.5220	-0.3336	-17.5126	-4.2890	-0.0717	3.60E-15
1984	3.7590	-1.3426	-0.3327	-15.1918	-3.9151	-0.0757	2.69E-15
1985	3.6173	-1.2615	-0.3307	-14.0631	-3.7532	-0.0777	2.62E-15
1986	3.7439	-1.2881	-0.3430	-15.2351	-3.7871	-0.0780	4.10E-15
1987	4.0954	-1.3891	-0.3709	-18.5462	-3.9493	-0.0777	4.85E-15
1988	4.3749	-1.4457	-0.3989	-21.4330	-4.0236	-0.0787	6.89E-15
1989	5.4268	-1.6682	-0.4980	-33.1518	-4.3055	-0.0811	4.88E-15
1990	5.4522	-1.7240	-0.4857	-33.0575	-4.4236	-0.0787	7.08E-15

A5.4.5 Economies of Scale

Table A5.4.5.1 IZEF estimates of Economies of Scale (ISIC 3320)

Year	Economies of Scale - Model 1 - IZEF estimates	Economies of Scale - Model 2 - IZEF estimates	Economies of Scale - Model 3 - IZEF estimates	Economies of Scale - Model 1 - IZEF (AR) estimates	Economies of Scale - Model 2 - IZEF (AR) estimates	Economies of Scale - Model 3 - IZEF (AR) estimates
1972	1.1867	1.4320	1.2575	1.3729		1.6573
1973	1.1903	1.4083	1.2286	1.3335		1.4713
1974	1.1854	1.3798	1.2126	1.2998		1.3827
1975	1.2127	1.3798	1.2041	1.3074		1.3390
1976	1.2360	1.3776	1.2139	1.3299		1.3896
1977	1.2588	1.3727	1.2112	1.3377		1.3752
1978	1.2768	1.3660	1.2011	1.3312		1.3237
1979	1.2787	1.3397	1.1633	1.2800		1.1557
1980	1.2781	1.3116	1.1313	1.2345		1.0368
1981	1.3079	1.3089	1.1248	1.2412		1.0150
1982	1.3587	1.3272	1.1434	1.2905		1.0796
1983	1.4138	1.3468	1.1607	1.3414		1.1450
1984	1.4748	1.3673	1.1857	1.4070		1.2510
1985	1.5314	1.3829	1.2046	1.4616		1.3413
1986	1.5558	1.3748	1.2002	1.4599		1.3194
1987	1.5801	1.3710	1.2085	1.4738		1.3611
1988	1.5819	1.3458	1.1849	1.4298		1.2473
1989	1.5744	1.3241	1.1849	1.4105		1.2473
1990	1.6297	1.3361	1.1988	1.4517		1.3126
MEAN	1.3743	1.3607	1.1905	1.3576		1.2869

Table A5.4.5.2 I3SLS estimates of Economies of Scale (ISIC 3320)

Year	Economies of Scale - Model 1 - I3SLS estimates	Economies of Scale - Model 2 - I3SLS estimates	Economies of Scale - Model 3 - I3SLS estimates	Economies of Scale - Model 1 - I3SLS (AR) estimates	Economies of Scale - Model 2 - I3SLS (AR) estimates	Economies of Scale - Model 3 - I3SLS (AR) estimates
1972	1.1027	1.3308	1.1927		0.3939	0.4303
1973	1.1531	1.1823	1.1916		0.5193	0.5508
1974	1.1709	0.9802	1.1910		0.6323	0.6557
1975	1.2038	0.8956	1.1906		0.7209	0.7313
1976	1.1869	0.8232	1.1910		0.6218	0.6456
1977	1.1997	0.7659	1.1909		0.6463	0.6672
1978	1.2265	0.7316	1.1905		0.7577	0.7633
1979	1.3226	0.6908	1.1889		2.3037	1.7312
1980	1.4188	0.6379	1.1874		-2.7437	-11.9165
1981	1.4613	0.5961	1.1871		-1.8699	-4.4982
1982	1.4222	0.5622	1.1880		-16.1364	5.8456
1983	1.3924	0.5349	1.1888		2.7666	1.9088
1984	1.3410	0.5163	1.1899		1.0464	0.9832
1985	1.3074	0.4900	1.1907		0.7202	0.7267
1986	1.3202	0.4608	1.1905		0.7749	0.7731
1987	1.2938	0.4346	1.1908		0.6743	0.6904
1988	1.3501	0.4148	1.1898		1.0576	0.9986
1989	1.3256	0.3913	1.1898		1.0465	0.9986
1990	1.3012	0.3714	1.1904		0.7829	0.7889
MEAN	1.2895	0.6743	1.1900		-0.2781	0.1829

A5.4.6 Graphs of Economies of Scale

Figure A5.4.6.1 Economies of Scale - IZEF estimates - ISIC 3320

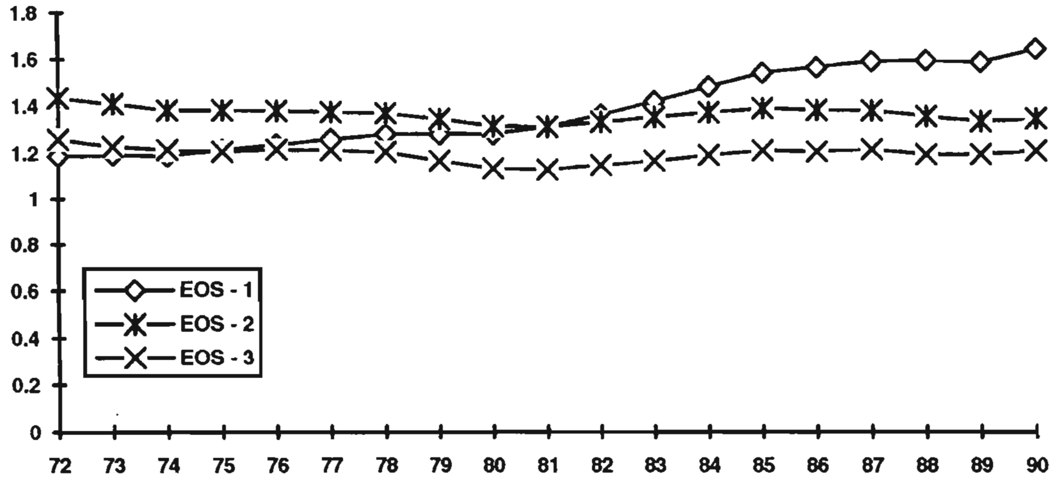


Figure A5.4.6.2 Economies of Scale - IZEF (AR) estimates - ISIC 3320

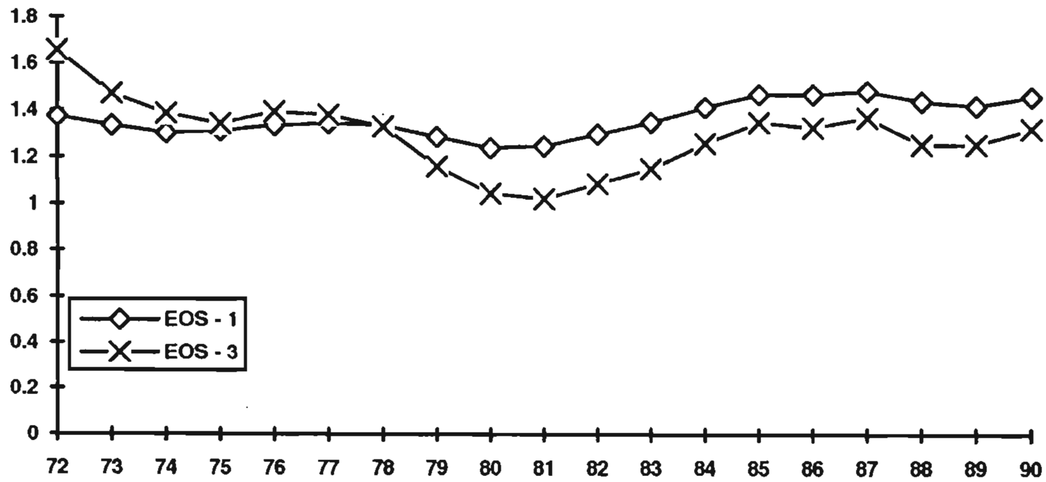


Figure A5.4.6.3 Economies of Scale - I3SLS estimates - ISIC 3320

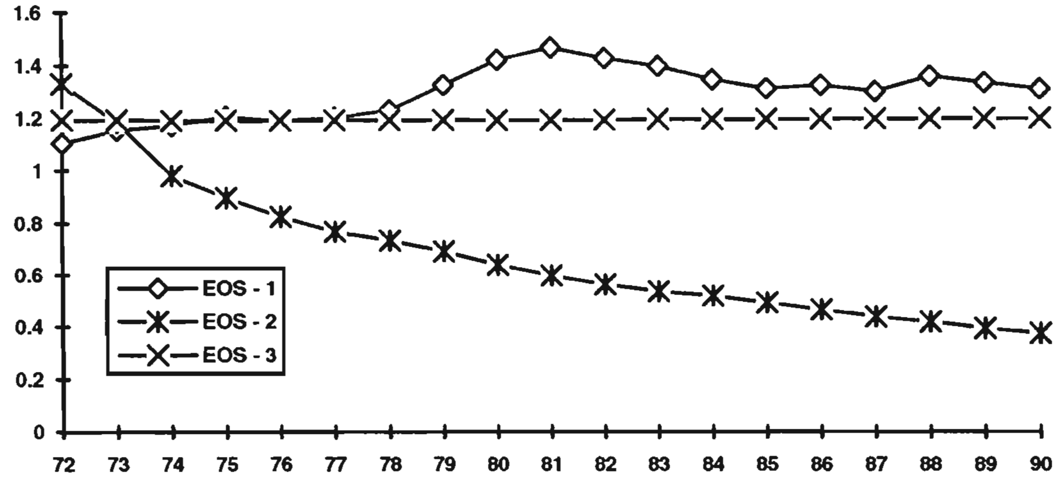


Figure A5.4.6.4 Economies of Scale - I3SLS (AR) estimates - ISIC 3320.

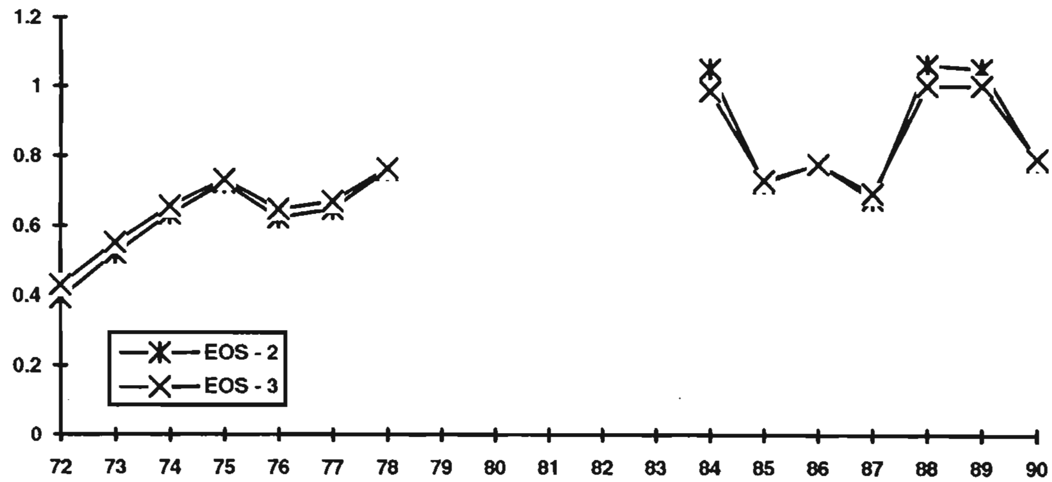


Figure A5.4.6.5. Economies of Scale - Model 1 - ISIC 3320

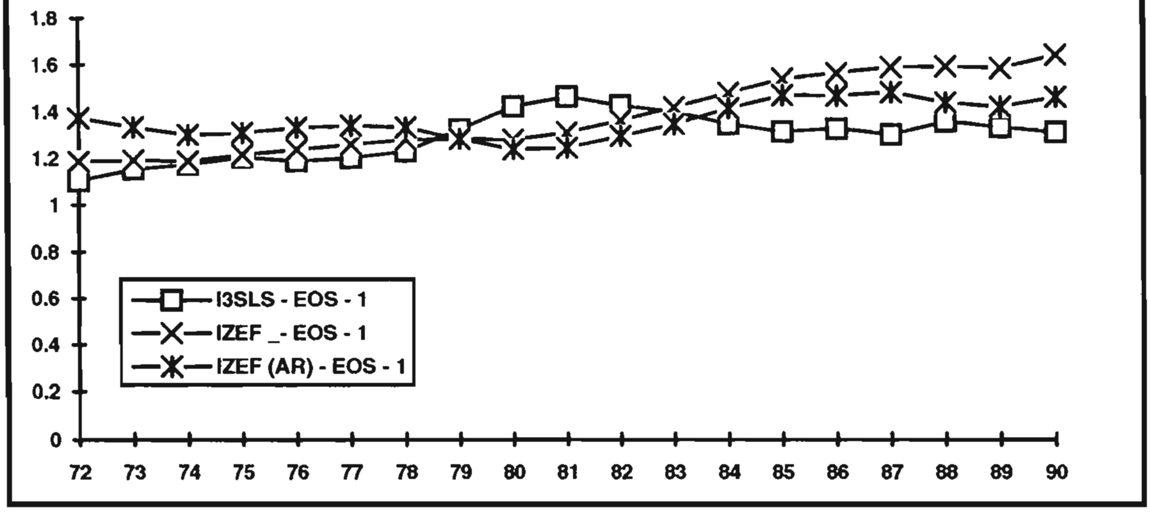


Figure A5.4.6.6. Economies of Scale - Model 2 - ISIC 3320

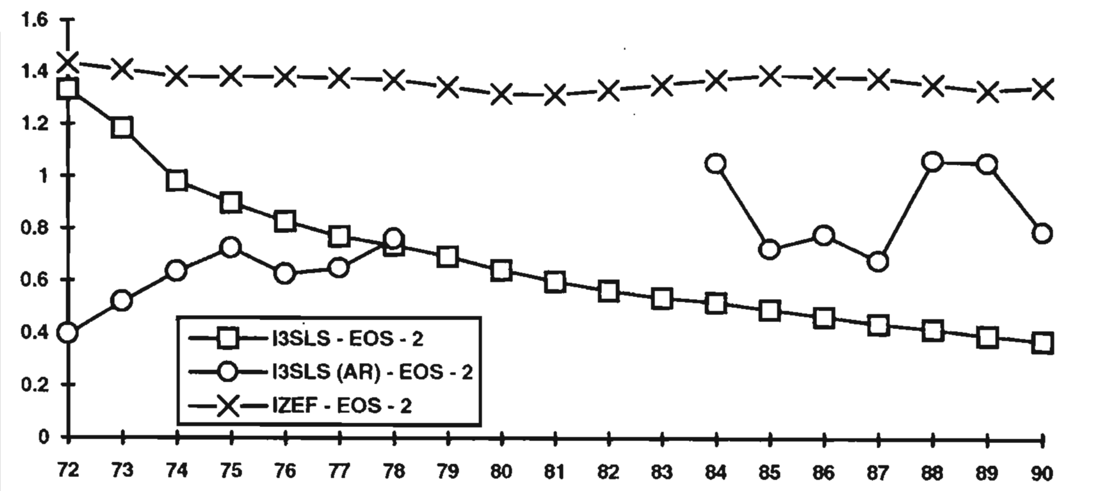
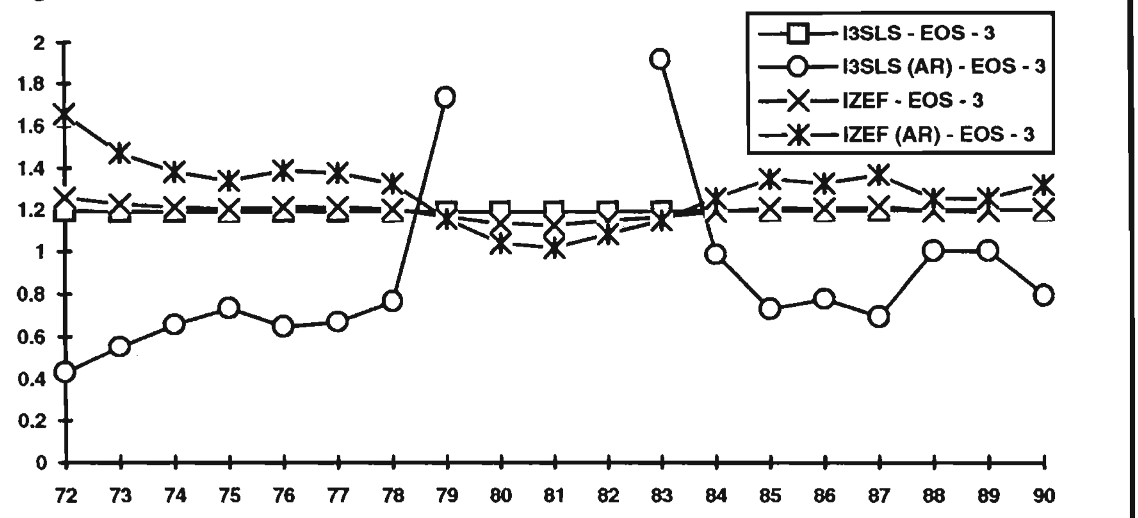


Figure A5.4.6.7. Economies of Scale - Model 3 - ISIC 3320



APPENDIX 5.5

ELECTRICAL APPLIANCES AND HOUSEHOLD GOODS (ISIC 3833)

5.5.2 Durbin-Watson Test Results

Table A5.5.2.1 Computed and critical Durbin-Watson statistics for IZEF estimates of ISIC 3833.

Model	Equation	Number of Explanatory's	DL	DU	Computed D-W Statistic	$H_0: \rho=0$ vs $H_1: \rho>0$
Model 1	Cost function	14	0.070	3.642	0.7848	N.D.
	Cost share of Capital	4	0.859	1.848	1.4960	N.D.
	Cost share of Labour	4	0.859	1.848	0.6343	Reject
Model 2	Cost function	9	0.369	2.783	0.5368	N.D.
	Cost share of Capital	3	0.967	1.685	1.3731	N.D.
	Cost share of Labour	3	0.967	1.685	0.4943	Reject
Model 3	Cost function	7	0.549	2.396	0.1923	Reject
	Cost share of Capital	2	1.074	1.536	1.4026	N.D.
	Cost share of Labour	2	1.074	1.536	0.2132	Reject
Model 4	Cost function	6	0.649	2.206	0.1906	Reject
	Cost share of Capital	2	1.074	1.536	1.4193	N.D.
	Cost share of Labour	2	1.074	1.536	0.2062	Reject
Model 5	Cost function	3	0.967	1.685	0.1782	Reject
	Cost share of Capital	--	--	--	--	--
	Cost share of Labour	--	--	--	--	--

Note: Significant Lower and Upper D-W statistics are given for the 0.05 level of significance (Source: Gujarati, 1988, p687).

Table A5.5.2.2 Computed and critical Durbin-Watson statistics for I3SLS estimates of ISIC 383333

Model	Equation	Number of Exogenous Variables	DL	DU	Computed D-W Statistic	H_0: ρ=0 vs H_1: ρ>0
Model 1	Cost function	15	0.063	3.676	0.7985	N.D
	Cost share of Capital	15	0.063	3.676	1.5004	N.D.
	Cost share of Labour	15	0.063	3.676	0.6377	N.D.
Model 2	Cost function	11	0.220	3.159	0.5693	N.D.
	Cost share of Capital	11	0.220	3.159	1.3545	N.D.
	Cost share of Labour	11	0.220	3.159	0.5303	N.D.
Model 3	Cost function	11	0.220	3.159	0.1933	Reject
	Cost share of Capital	11	0.220	3.159	1.3895	N.D.
	Cost share of Labour	11	0.220	3.159	0.2184	Reject
Model 4	Cost function	11	0.220	3.159	0.1913	Reject
	Cost share of Capital	11	0.220	3.159	1.4172	N.D.
	Cost share of Labour	11	0.220	3.159	0.2071	Reject
Model 5	Cost function	8	0.456	2.589	0.1780	Reject
	Cost share of Capital	--	--	--	--	--
	Cost share of Labour	--	--	--	--	--

Note: Significant Lower and Upper D-W statistics are given for the 0.05 level of significance (Source: Gujarati, 1988, p687).

5.5.3 Parameter Estimates

Table A5.5.3.1 IZEF parameter estimates for Electrical Appliances and Household Goods (ISIC 3833).

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o	16.5504 (598.017)	16.5493 (437.412)	16.2943 (482.088)	16.3011 (499.395)	16.3063 (544.891)
α_K	0.0961 (8.9697)	0.0908 (8.9555)	0.0827 (12.8733)	0.0830 (12.9137)	0.0803 (18.4029)
α_L	0.4973 (25.3261)	0.5012 (21.4250)	0.3713 (19.9111)	0.3700 (19.5238)	0.3465 (19.1818)
α_M	0.4066	0.4080	0.5460	0.5470	0.5732
α_Y	0.7529 (6.9942)	0.5244 (6.7923)	0.9943 (20.7089)	0.9817 (59.5296)	0.9233 (36.6742)
α_t	-0.0147 -3.2152				
γ_{KK}	0.0231 (0.5269)	-0.0225 (-1.2139)	-0.0089 (-0.5762)	-0.0087 (-0.5593)	
γ_{KL}	-0.0085 (-0.1772)	0.0103 (0.5016)	0.0155 (0.8660)	0.0208 (1.1463)	
γ_{KM}	-0.0146	0.0122	-0.0066	-0.0121	
γ_{LL}	0.1418 (2.3773)	0.1500 (5.2564)	0.1679 (6.2324)	0.1648 (6.2168)	
γ_{LM}	-0.1333	-0.1603	-0.1834	-0.1856	
γ_{MM}	0.1479	0.1481	0.1900	0.1977	
γ_{KY}	-0.0099 -0.2775	-0.0101 (-0.4564)			
γ_{LY}	-0.1016 -1.6519	-0.2524 (-5.8322)			
γ_{MY}	0.1115	0.2625			
γ_{YY}	0.3083 (0.7497)	-0.0955 (-0.8449)	0.0006 (0.0058)		
γ_{tt}	0.0009 (1.1526)				
γ_{Yt}	-0.0163 (-1.0072)				
γ_{Kt}	-0.0017 (-0.5695)				
γ_{Lt}	-0.0072 (-1.8300)				
γ_{Mt}	0.0089				
Dtrm	2.49E-11	6.13E-11	2.92E-10	3.07E-10	1.56E-09

Note: 1. Numbers in parenthesis are t-statistics refering to parameter estimates above. t-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covaraince matrix.

Table A5.5.3.2 IZEF AR(1) parameter estimates for Electrical Appliances and Household Goods (ISIC 3883).

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o	17.2338 (108.687)	16.4796 (229.222)	16.2533 (244.493)	16.2264 (237.248)	16.4235 (326.193)
α_K	0.1632 (7.2401)	0.1061 (5.5632)	0.0704 (4.1855)	0.0752 (4.5064)	0.0801 (18.243)
α_L	0.2889 (4.2649)	0.3988 (7.5443)	0.3064 (6.0229)	0.2954 (5.4589)	0.3669 (20.746)
α_M	0.5479	0.4951	0.6232	0.6294	0.5530
α_Y	0.3484 (5.8932)	0.5869 (4.9258)	0.8890 (11.978)	0.9902 (33.756)	0.7489 (10.344)
α_t	-0.1018 (-5.3029)				
γ_{KK}	0.1807 (6.3684)	0.0139 (0.5281)	0.0343 (1.0530)	0.0242 (0.7416)	
γ_{KL}	-0.1340 (-3.3832)	0.0582 (2.2454)	0.0417 (1.4040)	0.0524 (1.8168)	
γ_{KM}	-0.0467	-0.0721	-0.0760	-0.0766	
γ_{LL}	0.2935 (4.0092)	0.1107 (2.5425)	0.1082 (2.3244)	0.0993 (2.1941)	
γ_{LM}	-0.1595	-0.1689	-0.1499	-0.1517	
γ_{MM}	0.2062	0.2410	0.2259	0.2283	
γ_{KY}	-0.0152 (-0.5487)	-0.0475 (-1.6851)			
γ_{LY}	-0.0956 (-2.2950)	-0.1178 (-2.5098)			
γ_{MY}	0.1108	0.1653			
γ_{YY}	0.6159 (4.4423)	0.1327 (0.8148)	0.1974 (1.2870)		
γ_{tt}	0.0071 (5.7044)				
γ_{Yt}	-0.0018 (-0.2995)				
γ_{Kt}	-0.0124 (-4.7011)				
γ_{Lt}	0.0132 (3.1161)				
γ_{Mt}	-0.0008				
Dtrm	3.24E-12	2.30E-11	5.18E-11	5.51E-11	2.99E-09

Note: 1. Numbers in parenthesis are *t*-statistics refering to parameter estimates above. *t*-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covaraince matrix.

Table A5.5.3.3 I3SLS parameter estimates for Electrical Appliances and Household Goods (ISIC 3883)

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o	16.5515 (598.487)	16.5573 (430.076)	16.2873 (470.832)	16.2981 (494.906)	16.3122 (549.177)
α_K	0.0958 (8.9262)	0.0876 (8.3544)	0.0825 (12.8434)	0.0829 (12.8951)	0.0803 (18.3882)
α_L	0.4976 (25.2327)	0.5093 (20.9961)	0.3721 (20.1549)	0.3701 (19.5746)	0.3470 (19.2032)
α_M	0.4066	0.4031	0.5454	0.5470	0.5727
α_Y	0.7193 (6.5405)	0.5173 (6.5282)	1.0016 (20.0166)	0.9873 (58.4458)	0.9138 (34.8265)
α_t	-0.0138 (-2.9929)				
γ_{KK}	0.0243 (0.5491)	-0.0279 (-1.4845)	-0.0089 (-0.5829)	-0.0083 (-0.5340)	
γ_{KL}	-0.0081 (-0.1655)	0.0165 (0.7950)	0.0116 (0.6461)	0.0201 (1.1079)	
γ_{KM}	-0.0162	0.0114	-0.0027	-0.0118	
γ_{LL}	0.1386 (2.2510)	0.1433 (4.9568)	0.1690 (6.3001)	0.1651 (6.2785)	
γ_{LM}	-0.1305	-0.1598	-0.1806	-0.1852	
γ_{MM}	0.1467	0.1484	0.1833	0.1970	
γ_{KY}	-0.0086 (-0.2321)	-0.0004 (-0.0179)			
γ_{LY}	-0.1015 (-1.5978)	-0.2720 (-6.0244)			
γ_{MY}	0.1101	0.2724			
γ_{YY}	0.4953 (1.1569)	-0.1266 (-1.0533)	0.0198 (0.1780)		
γ_{tt}	0.0012 (1.4198)				
γ_{Yt}	-0.0224 (-1.3477)				
γ_{Kt}	-0.0018 (-0.5792)				
γ_{Lt}	-0.0073 (-1.7918)				
γ_{Mt}	0.0091				
Dtrm	2.52E-11	6.21E-11	2.87E-10	3.02E-10	1.57E-09

Note: 1. Numbers in parenthesis are t-statistics refering to parameter estimates above. t-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covaraince matrix.

Table A5.5.3.4 I3SLS(AR) parameter estimates for Electrical Appliances and Household Goods (ISIC 3883)

Parameter	Model 1	Model 2	Model 3	Model 4	Model 5
α_o		16.6000 (298.552)	16.2592 (244.292)	16.2267 (237.981)	16.4578 (302.244)
α_K		0.1021 (7.3573)	0.0691 (3.9853)	0.0751 (4.5303)	0.0804 (17.8105)
α_L		0.4808 (13.3939)	0.3081 (6.0150)	0.2957 (5.4667)	0.3526 (18.7296)
α_M		0.4171	0.6228	0.6292	0.5670
α_Y		0.3279 (2.6837)	0.8780 (11.4151)	0.9898 (33.3289)	0.6645 (8.6257)
α_t					
γ_{KK}		-0.0108 (-0.5531)	0.0371 (1.1012)	0.0243 (0.7532)	
γ_{KL}		0.0380 (1.5366)	0.0395 (1.2900)	0.0521 (1.8078)	
γ_{KM}		-0.0272	-0.0766	-0.0764	
γ_{LL}		0.1408 (4.1079)	0.1101 (2.2649)	0.0998 (2.1776)	
γ_{LM}		-0.1788	-0.1496	-0.1519	
γ_{MM}		0.2060	0.2262	0.2283	
γ_{KY}		-0.0308 (-1.2218)			
γ_{LY}		-0.2253 (-4.4711)			
γ_{MY}		0.2561			
γ_{YY}		0.2929 (1.7089)	0.2079 (1.3825)		
γ_{tt}					
γ_{Yt}					
γ_{Kt}					
γ_{Lt}					
γ_{Mt}					
Dtrm		2.44E-11	5.19E-11	5.51E-11	

Note: 1. Numbers in parenthesis are t-statistics refering to parameter estimates above. t-statistics do not exist for indirectly calculated parameters.
2. Dtrm is the determinant of the residual covaraince matrix.

5.5.4 Monotonicity and Concavity Test Results

Table A5.5.4.1 Fitted cost shares and Own AES's for IZEF estimates Model 1 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0944	0.4901	0.4155	-7.0010	-0.4501	-0.5500
1973	0.0886	0.4685	0.4429	-7.3418	-0.4885	-0.5039
1974	0.0856	0.4419	0.4726	-7.5326	-0.5369	-0.4538
1975	0.0841	0.4277	0.4882	-7.6247	-0.5628	-0.4279
1976	0.0869	0.3999	0.5131	-7.4471	-0.6138	-0.3871
1977	0.0906	0.4141	0.4953	-7.2228	-0.5879	-0.4162
1978	0.0864	0.4212	0.4924	-7.4816	-0.5749	-0.4208
1979	0.0797	0.3923	0.5281	-7.9116	-0.6278	-0.3633
1980	0.0762	0.3491	0.5747	-8.1460	-0.7010	-0.2922
1981	0.0770	0.3573	0.5657	-8.0918	-0.6880	-0.3055
1982	0.0776	0.3488	0.5735	-8.0477	-0.7014	-0.2940
1983	0.0776	0.3318	0.5906	-8.0535	-0.7258	-0.2692
1984	0.0807	0.3244	0.5948	-7.8435	-0.7351	-0.2631
1985	0.0800	0.3252	0.5948	-7.8907	-0.7342	-0.2632
1986	0.0773	0.3023	0.6205	-8.0728	-0.7563	-0.2275
1987	0.0732	0.2987	0.6281	-8.3508	-0.7585	-0.2172
1988	0.0712	0.2786	0.6502	-8.4891	-0.7625	-0.1882
1989	0.0711	0.2542	0.6746	-8.4926	-0.7395	-0.1573
1990	0.0693	0.2234	0.7073	-8.6222	-0.6350	-0.1182

Table A5.5.4.2. Proper AESs and Determinants of matrixs of AESs IZEF Estimates, Model 1 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	0.8163	0.6278	0.3454	2.4845	3.4568	0.1282	1.18E-15
1973	0.7953	0.6281	0.3575	2.9541	3.3047	0.1183	1.36E-16
1974	0.7752	0.6389	0.3616	3.4431	3.0101	0.1128	7.55E-16
1975	0.7637	0.6444	0.3616	3.7082	2.8471	0.1101	5.77E-16
1976	0.7555	0.6727	0.3505	4.0006	2.4304	0.1148	9.74E-16
1977	0.7735	0.6746	0.3501	3.6478	2.5508	0.1221	1.52E-15
1978	0.7663	0.6567	0.3573	3.7138	2.7171	0.1142	1.01E-15
1979	0.7281	0.6530	0.3565	4.4366	2.4480	0.1010	1.00E-15
1980	0.6804	0.6665	0.3356	5.2470	1.9363	0.0922	1.31E-15
1981	0.6910	0.6648	0.3405	5.0901	2.0303	0.0943	-2.01E-16
1982	0.6862	0.6721	0.3337	5.1737	1.9141	0.0948	1.63E-16
1983	0.6697	0.6813	0.3198	5.3966	1.7036	0.0931	1.64E-15
1984	0.6754	0.6959	0.3093	5.3096	1.5796	0.0978	1.01E-15
1985	0.6733	0.6932	0.3109	5.3398	1.5965	0.0966	1.12E-15
1986	0.6361	0.6955	0.2892	5.7011	1.3531	0.0884	1.16E-15
1987	0.6112	0.6824	0.2896	5.9606	1.3485	0.0809	1.99E-15
1988	0.5715	0.6846	0.2642	6.1462	1.1288	0.0737	1.87E-15
1989	0.5300	0.6958	0.2228	5.9994	0.8519	0.0667	6.74E-16
1990	0.4507	0.7020	0.1564	5.2723	0.5259	0.0506	1.38E-16

Table A5.5.4.3 Fitted cost shares and Own AESs for IZEF estimates Model 2 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0908	0.5012	0.4080	-12.7423	-0.3981	-0.5613
1973	0.0908	0.4550	0.4542	-12.7397	-0.4733	-0.4837
1974	0.0879	0.4039	0.5081	-13.2813	-0.5564	-0.3944
1975	0.0859	0.3843	0.5298	-13.6883	-0.5865	-0.3599
1976	0.0809	0.3633	0.5558	-14.7878	-0.6161	-0.3198
1977	0.0823	0.4370	0.4807	-14.4642	-0.5029	-0.4394
1978	0.0853	0.4525	0.4622	-13.8218	-0.4773	-0.4703
1979	0.0855	0.3933	0.5212	-13.7722	-0.5728	-0.3735
1980	0.0818	0.3167	0.6015	-14.5798	-0.6620	-0.2532
1981	0.0792	0.3330	0.5878	-15.2047	-0.6503	-0.2726
1982	0.0779	0.3406	0.5814	-15.5419	-0.6429	-0.2818
1983	0.0764	0.3324	0.5913	-15.9514	-0.6509	-0.2676
1984	0.0735	0.3497	0.5768	-16.7631	-0.6330	-0.2886
1985	0.0728	0.3601	0.5671	-16.9875	-0.6203	-0.3028
1986	0.0725	0.3345	0.5930	-17.0594	-0.6489	-0.2652
1987	0.0728	0.3214	0.6058	-16.9792	-0.6593	-0.2472
1988	0.0705	0.2898	0.6397	-17.6979	-0.6646	-0.2013
1989	0.0695	0.2777	0.6528	-18.0558	-0.6559	-0.1843
1990	0.0676	0.2389	0.6935	-18.7301	-0.5577	-0.1340

Table A5.5.4.4. Proper AESs and Determinants of matrixs of AESs IZEF Estimates, Model 2 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.2263	1.3293	0.2161	3.5686	5.3851	0.1767	-6.24E-16
1973	1.2493	1.2958	0.2243	4.4693	4.4835	0.1786	-8.84E-16
1974	1.2900	1.2730	0.2190	5.7251	3.6172	0.1715	-5.02E-17
1975	1.3120	1.2680	0.2126	6.3070	3.3178	0.1658	1.27E-16
1976	1.3503	1.2712	0.2060	7.2878	3.1133	0.1546	-1.32E-15
1977	1.2863	1.3082	0.2368	5.6196	4.6439	0.1649	-4.05E-16
1978	1.2669	1.3096	0.2336	4.9922	4.7856	0.1699	1.19E-16
1979	1.3062	1.2738	0.2180	6.1828	3.5218	0.1664	1.53E-16
1980	1.3975	1.2479	0.1584	7.6996	2.1343	0.1425	-5.48E-16
1981	1.3904	1.2620	0.1810	7.9546	2.5529	0.1445	-1.15E-16
1982	1.3881	1.2693	0.1907	8.0657	2.7683	0.1448	-1.55E-15
1983	1.4058	1.2702	0.1843	8.4061	2.6560	0.1402	-9.07E-16
1984	1.4006	1.2877	0.2053	8.6491	3.1800	0.1406	-1.67E-15
1985	1.3930	1.2956	0.2150	8.5963	3.4651	0.1416	6.39E-16
1986	1.4245	1.2836	0.1918	9.0415	2.8773	0.1353	6.05E-16
1987	1.4401	1.2766	0.1767	9.1198	2.5676	0.1317	-5.69E-16
1988	1.5039	1.2704	0.1352	9.4999	1.9491	0.1155	4.04E-17
1989	1.5339	1.2690	0.1158	9.4905	1.7174	0.1075	-9.29E-16
1990	1.6381	1.2604	0.0325	7.7619	0.9211	0.0737	8.20E-17

Table A5.5.4.5 Fitted cost shares and Own AESs for IZEF estimates Model 3 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0827	0.3713	0.5460	-12.3932	-0.4754	-0.1942
1973	0.0840	0.3768	0.5392	-12.1706	-0.4713	-0.2011
1974	0.0839	0.3809	0.5352	-12.1776	-0.4681	-0.2052
1975	0.0837	0.3853	0.5309	-12.2104	-0.4644	-0.2095
1976	0.0804	0.3668	0.5528	-12.8117	-0.4783	-0.1872
1977	0.0785	0.3518	0.5697	-13.1808	-0.4859	-0.1699
1978	0.0806	0.3648	0.5546	-12.7797	-0.4796	-0.1854
1979	0.0817	0.3644	0.5539	-12.5713	-0.4798	-0.1861
1980	0.0806	0.3546	0.5648	-12.7686	-0.4848	-0.1749
1981	0.0812	0.3797	0.5391	-12.6582	-0.4691	-0.2012
1982	0.0800	0.3734	0.5466	-12.8900	-0.4739	-0.1935
1983	0.0785	0.3617	0.5598	-13.1794	-0.4814	-0.1800
1984	0.0762	0.3528	0.5710	-13.6501	-0.4855	-0.1686
1985	0.0766	0.3631	0.5603	-13.5808	-0.4805	-0.1795
1986	0.0758	0.3499	0.5743	-13.7344	-0.4866	-0.1652
1987	0.0776	0.3677	0.5547	-13.3596	-0.4778	-0.1853
1988	0.0773	0.3693	0.5534	-13.4250	-0.4768	-0.1866
1989	0.0750	0.3436	0.5814	-13.9219	-0.4882	-0.1579
1990	0.0735	0.3270	0.5995	-14.2604	-0.4879	-0.1394

Table A5.5.4.6. Proper AESs and Determinants of matrixs of AESs IZEF Estimates, Model 3 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.5048	0.8538	0.0953	3.6270	1.6773	0.0832	5.31E-16
1973	1.4898	0.8542	0.0974	3.5168	1.7176	0.0853	4.63E-16
1974	1.4848	0.8531	0.1003	3.4958	1.7707	0.0860	8.24E-17
1975	1.4803	0.8516	0.1035	3.4789	1.8324	0.0865	1.75E-16
1976	1.5255	0.8515	0.0955	3.8013	1.6739	0.0804	-1.79E-16
1977	1.5611	0.8524	0.0849	3.9673	1.5134	0.0754	1.08E-15
1978	1.5273	0.8523	0.0935	3.7963	1.6423	0.0801	9.76E-17
1979	1.5205	0.8542	0.0914	3.7196	1.6103	0.0810	1.76E-16
1980	1.5421	0.8551	0.0842	3.8124	1.5026	0.0777	-1.65E-16
1981	1.5025	0.8493	0.1040	3.6802	1.8259	0.0836	9.40E-16
1982	1.5189	0.8491	0.1014	3.8015	1.7738	0.0814	1.06E-16
1983	1.5458	0.8499	0.0942	3.9546	1.6504	0.0778	-3.18E-16
1984	1.5764	0.8484	0.0895	4.1426	1.5810	0.0738	2.08E-16
1985	1.5575	0.8461	0.0986	4.1002	1.7224	0.0765	9.32E-16
1986	1.5841	0.8484	0.0873	4.1731	1.5493	0.0728	5.84E-16
1987	1.5430	0.8467	0.1008	4.0018	1.7590	0.0784	-1.97E-16
1988	1.5430	0.8457	0.1026	4.0196	1.7894	0.0784	-7.93E-16
1989	1.6016	0.8486	0.0820	4.2315	1.4782	0.0704	1.01E-16
1990	1.6452	0.8502	0.0645	4.2512	1.2649	0.0638	4.32E-16

Table A5.5.4.7 Fitted cost shares and Own AES's for IZEF estimates Model 4 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0830	0.3700	0.5470	-12.3111	-0.4989	-0.1674
1973	0.0845	0.3750	0.5405	-12.0579	-0.4947	-0.1734
1974	0.0845	0.3792	0.5362	-12.0451	-0.4910	-0.1773
1975	0.0845	0.3839	0.5316	-12.0564	-0.4866	-0.1815
1976	0.0805	0.3666	0.5529	-12.7609	-0.5015	-0.1620
1977	0.0781	0.3522	0.5697	-13.2243	-0.5108	-0.1462
1978	0.0806	0.3644	0.5549	-12.7391	-0.5031	-0.1600
1979	0.0818	0.3634	0.5548	-12.5280	-0.5038	-0.1602
1980	0.0804	0.3539	0.5657	-12.7833	-0.5099	-0.1499
1981	0.0817	0.3795	0.5388	-12.5361	-0.4908	-0.1750
1982	0.0803	0.3736	0.5461	-12.8047	-0.4959	-0.1682
1983	0.0784	0.3623	0.5593	-13.1661	-0.5046	-0.1560
1984	0.0758	0.3544	0.5698	-13.7047	-0.5096	-0.1461
1985	0.0764	0.3649	0.5586	-13.5694	-0.5028	-0.1566
1986	0.0753	0.3516	0.5730	-13.8100	-0.5110	-0.1430
1987	0.0777	0.3691	0.5533	-13.3154	-0.4997	-0.1616
1988	0.0774	0.3708	0.5518	-13.3731	-0.4983	-0.1630
1989	0.0743	0.3456	0.5801	-14.0454	-0.5137	-0.1363
1990	0.0722	0.3293	0.5985	-14.5133	-0.5170	-0.1190

Table A5.5.4.8. Proper AESs and Determinants of matrixs of AESs IZEF Estimates, Model 4 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.6773	0.7335	0.0830	3.3287	1.5230	0.0766	7.39E-16
1973	1.6566	0.7350	0.0844	3.2212	1.5505	0.0787	6.06E-16
1974	1.6488	0.7331	0.0873	3.1959	1.5984	0.0794	1.09E-15
1975	1.6413	0.7306	0.0906	3.1731	1.6547	0.0801	7.60E-16
1976	1.7046	0.7282	0.0843	3.4943	1.5364	0.0741	1.15E-15
1977	1.7559	0.7281	0.0749	3.6711	1.4031	0.0691	1.25E-15
1978	1.7078	0.7296	0.0823	3.4929	1.5064	0.0738	9.62E-16
1979	1.6998	0.7333	0.0795	3.4228	1.4691	0.0744	1.05E-15
1980	1.7311	0.7340	0.0729	3.5211	1.3776	0.0711	1.25E-15
1981	1.6706	0.7253	0.0922	3.3618	1.6674	0.0774	1.10E-15
1982	1.6934	0.7240	0.0903	3.4828	1.6300	0.0753	3.90E-16
1983	1.7320	0.7241	0.0840	3.6439	1.5293	0.0716	9.31E-16
1984	1.7742	0.7199	0.0809	3.8356	1.4836	0.0679	3.34E-16
1985	1.7456	0.7167	0.0895	3.7754	1.6109	0.0707	6.15E-16
1986	1.7853	0.7197	0.0789	3.8699	1.4573	0.0669	1.12E-15
1987	1.7256	0.7185	0.0910	3.6755	1.6354	0.0725	1.63E-15
1988	1.7248	0.7167	0.0929	3.6884	1.6656	0.0726	1.06E-15
1989	1.8104	0.7191	0.0744	3.9377	1.3980	0.0645	6.23E-16
1990	1.8745	0.7201	0.0583	3.9895	1.2081	0.0581	6.68E-16

Table A5.5.4.9 Fitted cost shares and Own AESs for IZEF (AR) estimates Model 1 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.1508	0.3021	0.5471	2.3148	0.9058	-0.1389
1973	0.1166	0.3174	0.5660	5.7121	0.7629	-0.1231
1974	0.1055	0.3105	0.5840	7.7644	0.8232	-0.1077
1975	0.0993	0.3133	0.5874	9.2492	0.7984	-0.1048
1976	0.1317	0.2674	0.6009	3.8245	1.3649	-0.0931
1977	0.1463	0.2794	0.5744	2.6105	1.1812	-0.1160
1978	0.1073	0.3303	0.5624	7.3682	0.6628	-0.1262
1979	0.0700	0.3385	0.5915	23.5963	0.6075	-0.1012
1980	0.0643	0.3063	0.6294	29.1427	0.8633	-0.0683
1981	0.0638	0.3344	0.6018	29.7345	0.6341	-0.0923
1982	0.0694	0.3316	0.5990	24.1062	0.6538	-0.0947
1983	0.0740	0.3189	0.6071	20.4736	0.7505	-0.0877
1984	0.0977	0.3034	0.5989	9.6878	0.8926	-0.0949
1985	0.0889	0.3254	0.5857	12.6124	0.6988	-0.1063
1986	0.0769	0.3180	0.6051	18.5553	0.7580	-0.0894
1987	0.0458	0.3536	0.6006	65.3062	0.5194	-0.0934
1988	0.0388	0.3498	0.6114	95.2199	0.5397	-0.0840
1989	0.0469	0.3228	0.6302	61.7558	0.7186	-0.0676
1990	0.0458	0.2984	0.6558	65.3716	0.9452	-0.0454

Table A5.5.4.10. Proper AESs and Determinants of matrixs of AESs IZEF (AR) Estimates, Model 1 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	-1.9414	0.4340	0.0350	-1.6723	-0.5099	-0.1271	7.35E-17
1973	-2.6204	0.2925	0.1121	-2.5088	-0.7889	-0.1065	-1.62E-16
1974	-3.0914	0.2418	0.1205	-3.1651	-0.8950	-0.1032	3.70E-16
1975	-3.3064	0.1995	0.1333	-3.5478	-1.0093	-0.1014	6.23E-16
1976	-2.8047	0.4099	0.0074	-2.6466	-0.5242	-0.1272	-1.08E-16
1977	-2.2798	0.4441	0.0060	-2.1138	-0.5000	-0.1370	1.52E-16
1978	-2.7799	0.2263	0.1413	-2.8441	-0.9810	-0.1036	3.12E-16
1979	-4.6561	-0.1279	0.2034	-7.3457	-2.4050	-0.1028	5.81E-16
1980	-5.8022	-0.1538	0.1727	-8.5054	-2.0148	-0.0888	6.16E-16
1981	-5.2818	-0.2166	0.2075	-9.0422	-2.7923	-0.1016	1.89E-15
1982	-4.8234	-0.1233	0.1969	-7.5034	-2.2985	-0.1007	5.51E-16
1983	-4.6777	-0.0392	0.1761	-6.5148	-1.7971	-0.0968	9.08E-16
1984	-3.5196	0.2021	0.1221	-3.7400	-0.9598	-0.0996	-2.61E-17
1985	-3.6319	0.1032	0.1631	-4.3771	-1.3510	-0.1009	4.37E-16
1986	-4.4806	-0.0036	0.1711	-6.0099	-1.6592	-0.0970	1.49E-15
1987	-7.2743	-0.6976	0.2489	-18.9956	-6.5836	-0.1105	4.17E-15
1988	-8.8705	-0.9684	0.2542	-27.2938	-8.9367	-0.1100	3.13E-15
1989	-7.8458	-0.5791	0.2161	-17.1800	-4.5076	-0.0952	3.97E-15
1990	-8.8096	-0.5554	0.1849	-15.8182	-3.2741	-0.0771	2.60E-15

Table A5.5.4.11 Fitted cost shares and Own AESs for IZEF (AR) estimates Model 2 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.1061	0.3988	0.4951	-7.1903	-0.8115	-0.0366
1973	0.0977	0.3749	0.5274	-7.7789	-0.8798	-0.0297
1974	0.0892	0.3540	0.5568	-8.4644	-0.9414	-0.0186
1975	0.0868	0.3488	0.5643	-8.6737	-0.9570	-0.0153
1976	0.0810	0.3423	0.5767	-9.2266	-0.9766	-0.0094
1977	0.0927	0.3756	0.5317	-8.1727	-0.8777	-0.0283
1978	0.0971	0.3818	0.5211	-7.8224	-0.8598	-0.0315
1979	0.0849	0.3485	0.5666	-8.8512	-0.9580	-0.0142
1980	0.0688	0.3111	0.6201	-10.5927	-1.0706	0.0141
1981	0.0779	0.3331	0.5890	-9.5428	-1.0045	-0.0031
1982	0.0788	0.3382	0.5829	-9.4491	-0.9889	-0.0062
1983	0.0755	0.3335	0.5910	-9.8030	-1.0033	-0.0021
1984	0.0785	0.3465	0.5750	-9.4812	-0.9641	-0.0102
1985	0.0829	0.3569	0.5603	-9.0417	-0.9330	-0.0171
1986	0.0753	0.3393	0.5854	-9.8331	-0.9856	-0.0050
1987	0.0759	0.3368	0.5873	-9.7676	-0.9931	-0.0040
1988	0.0706	0.3246	0.6048	-10.3785	-1.0300	0.0054
1989	0.0637	0.3126	0.6236	-11.2720	-1.0660	0.0162
1990	0.0534	0.2904	0.6562	-12.8499	-1.1309	0.0358

Table A5.5.4.12. Proper AESs and Determinants of matrixs of AESs IZEF (AR) Estimates Model 2 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	2.3755	-0.3725	0.1446	0.1919	0.1245	0.0088	-2.17E-19
1973	2.5890	-0.3992	0.1458	0.1412	0.0713	0.0048	-4.73E-18
1974	2.8430	-0.4519	0.1432	-0.1148	-0.0464	-0.0029	1.64E-17
1975	2.9216	-0.4715	0.1420	-0.2354	-0.0899	-0.0056	1.84E-17
1976	3.0988	-0.5435	0.1444	-0.5924	-0.2088	-0.0117	-5.76E-18
1977	2.6722	-0.4632	0.1543	0.0329	0.0164	0.0010	3.28E-18
1978	2.5695	-0.4246	0.1510	0.1233	0.0662	0.0043	6.67E-18
1979	2.9673	-0.4990	0.1446	-0.3257	-0.1232	-0.0073	1.00E-17
1980	3.7175	-0.6890	0.1244	-2.4787	-0.6239	-0.0306	-3.16E-17
1981	3.2420	-0.5707	0.1391	-0.9255	-0.2960	-0.0162	-1.02E-16
1982	3.1828	-0.5691	0.1434	-0.7865	-0.2648	-0.0144	3.47E-17
1983	3.3106	-0.6152	0.1430	-1.1252	-0.3582	-0.0184	5.90E-17
1984	3.1393	-0.5969	0.1522	-0.7149	-0.2596	-0.0133	-1.49E-16
1985	2.9677	-0.5527	0.1552	-0.3716	-0.1508	-0.0081	-4.11E-18
1986	3.2791	-0.6364	0.1497	-1.0603	-0.3562	-0.0175	-1.73E-16
1987	3.2779	-0.6184	0.1462	-1.0444	-0.3435	-0.0174	-1.51E-16
1988	3.5402	-0.6892	0.1397	-1.8435	-0.5312	-0.0251	-2.00E-16
1989	3.9220	-0.8147	0.1338	-3.3663	-0.8460	-0.0351	3.39E-19
1990	4.7525	-1.0570	0.1136	-8.0541	-1.5770	-0.0534	3.43E-16

Table A5.5.4.13 Fitted cost shares and Own AESs for IZEF (AR) estimates Model 3 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0704	0.3064	0.6232	-6.2839	-1.1112	-0.0230
1973	0.0695	0.3076	0.6229	-6.2873	-1.1074	-0.0232
1974	0.0718	0.3115	0.6167	-6.2745	-1.0951	-0.0275
1975	0.0746	0.3162	0.6092	-6.2411	-1.0803	-0.0328
1976	0.0748	0.3092	0.6160	-6.2386	-1.1023	-0.0281
1977	0.0726	0.3012	0.6262	-6.2664	-1.1274	-0.0209
1978	0.0732	0.3069	0.6199	-6.2593	-1.1098	-0.0253
1979	0.0697	0.3031	0.6272	-6.2867	-1.1216	-0.0201
1980	0.0678	0.2972	0.6351	-6.2875	-1.1398	-0.0145
1981	0.0791	0.3187	0.6022	-6.1606	-1.0725	-0.0376
1982	0.0794	0.3166	0.6040	-6.1531	-1.0791	-0.0364
1983	0.0777	0.3103	0.6120	-6.1886	-1.0990	-0.0308
1984	0.0798	0.3091	0.6111	-6.1445	-1.1028	-0.0315
1985	0.0843	0.3177	0.5980	-6.0368	-1.0757	-0.0405
1986	0.0795	0.3076	0.6129	-6.1516	-1.1074	-0.0302
1987	0.0835	0.3187	0.5978	-6.0566	-1.0726	-0.0406
1988	0.0852	0.3211	0.5937	-6.0110	-1.0650	-0.0435
1989	0.0788	0.3045	0.6168	-6.1669	-1.1173	-0.0275
1990	0.0746	0.2937	0.6318	-6.2420	-1.1506	-0.0169

Table A5.5.4.14. Proper AESs and Determinants of matrixs of AESs IZEF Estimates Model 3 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	2.9332	-0.7323	0.2150	-1.6211	-0.3919	-0.0207	1.67E-16
1973	2.9499	-0.7551	0.2177	-1.7393	-0.4242	-0.0217	7.86E-17
1974	2.8656	-0.7174	0.2198	-1.3399	-0.3418	-0.0181	1.02E-16
1975	2.7671	-0.6718	0.2218	-0.9145	-0.2464	-0.0137	6.86E-17
1976	2.8028	-0.6496	0.2131	-0.9790	-0.2468	-0.0144	1.72E-17
1977	2.9066	-0.6714	0.2052	-1.3838	-0.3202	-0.0186	-5.85E-17
1978	2.8554	-0.6738	0.2120	-1.2067	-0.2957	-0.0168	3.74E-16
1979	2.9730	-0.7375	0.2114	-1.7877	-0.4174	-0.0221	3.34E-16
1980	3.0709	-0.7663	0.2058	-2.2639	-0.4958	-0.0258	5.77E-16
1981	2.6546	-0.5958	0.2190	-0.4396	-0.1231	-0.0076	1.14E-16
1982	2.6581	-0.5842	0.2161	-0.4261	-0.1171	-0.0074	5.96E-17
1983	2.7295	-0.5981	0.2106	-0.6491	-0.1668	-0.0105	-6.83E-18
1984	2.6902	-0.5579	0.2063	-0.4610	-0.1179	-0.0079	3.89E-17
1985	2.5577	-0.5081	0.2110	-0.0484	-0.0137	-0.0010	3.23E-18
1986	2.7051	-0.5598	0.2049	-0.5054	-0.1273	-0.0085	7.85E-17
1987	2.5672	-0.5225	0.2132	-0.0943	-0.0268	-0.0018	2.07E-17
1988	2.5238	-0.5018	0.2136	0.0323	0.0094	0.0007	-9.00E-18
1989	2.7386	-0.5641	0.2017	-0.6096	-0.1486	-0.0099	2.34E-17
1990	2.9044	-0.6133	0.1920	-1.2532	-0.2708	-0.0175	3.21E-16

Table A5.5.4.15 Fitted cost shares and Own AESs for IZEF (AR) estimates Model 4 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0752	0.2954	0.6294	-8.0185	-1.2473	-0.0125
1973	0.0755	0.2955	0.6290	-8.0000	-1.2470	-0.0128
1974	0.0775	0.2996	0.6228	-7.8716	-1.2315	-0.0170
1975	0.0800	0.3048	0.6152	-7.7157	-1.2122	-0.0223
1976	0.0773	0.3006	0.6221	-7.8863	-1.2279	-0.0175
1977	0.0736	0.2939	0.6325	-8.1191	-1.2529	-0.0104
1978	0.0760	0.2979	0.6261	-7.9678	-1.2379	-0.0148
1979	0.0737	0.2928	0.6335	-8.1119	-1.2570	-0.0097
1980	0.0709	0.2877	0.6414	-8.2877	-1.2763	-0.0041
1981	0.0820	0.3097	0.6082	-7.5951	-1.1935	-0.0270
1982	0.0813	0.3087	0.6100	-7.6411	-1.1973	-0.0258
1983	0.0784	0.3035	0.6182	-7.8205	-1.2170	-0.0202
1984	0.0783	0.3044	0.6173	-7.8220	-1.2137	-0.0208
1985	0.0828	0.3131	0.6041	-7.5499	-1.1808	-0.0298
1986	0.0777	0.3032	0.6191	-7.8623	-1.2179	-0.0196
1987	0.0830	0.3131	0.6039	-7.5367	-1.1808	-0.0299
1988	0.0843	0.3160	0.5997	-7.4559	-1.1702	-0.0327
1989	0.0763	0.3007	0.6230	-7.9515	-1.2274	-0.0169
1990	0.0710	0.2907	0.6382	-8.2824	-1.2648	-0.0064

Table A5.5.4.16. Proper AESs and Determinants of matrixs of AESs IZEF (AR) Estimates Model 4 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	3.3589	-0.6184	0.1841	-1.2807	-0.2821	-0.0183	1.34E-16
1973	3.3491	-0.6130	0.1838	-1.2405	-0.2737	-0.0179	7.54E-17
1974	3.2555	-0.5861	0.1871	-0.9049	-0.2094	-0.0140	1.66E-16
1975	3.1480	-0.5554	0.1909	-0.5570	-0.1367	-0.0094	6.78E-17
1976	3.2552	-0.5928	0.1887	-0.9129	-0.2131	-0.0141	7.59E-17
1977	3.4222	-0.6453	0.1839	-1.5392	-0.3323	-0.0208	1.09E-16
1978	3.3144	-0.6097	0.1866	-1.1215	-0.2539	-0.0165	1.78E-16
1979	3.4274	-0.6403	0.1822	-1.5508	-0.3314	-0.0210	8.48E-17
1980	3.5679	-0.6835	0.1778	-2.1519	-0.4328	-0.0263	3.68E-16
1981	3.0627	-0.5355	0.1948	-0.3156	-0.0818	-0.0057	1.38E-17
1982	3.0887	-0.5452	0.1945	-0.3913	-0.1002	-0.0069	2.95E-17
1983	3.2036	-0.5814	0.1913	-0.7456	-0.1797	-0.0120	8.32E-17
1984	3.1978	-0.5840	0.1926	-0.7329	-0.1782	-0.0118	-1.03E-17
1985	3.0220	-0.5322	0.1981	-0.2176	-0.0585	-0.0041	6.46E-17
1986	3.2245	-0.5926	0.1919	-0.8214	-0.1970	-0.0129	8.77E-17
1987	3.0166	-0.5287	0.1978	-0.2010	-0.0540	-0.0038	-7.10E-18
1988	2.9668	-0.5148	0.1994	-0.0766	-0.0213	-0.0015	3.63E-18
1989	3.2850	-0.6121	0.1903	-1.0315	-0.2403	-0.0155	2.28E-16
1990	3.5376	-0.6898	0.1825	-2.0393	-0.4232	-0.0253	-5.18E-17

Table A5.5.4.17 Fitted cost shares and Own AESs for I3SLS estimates Model 1 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0940	0.4903	0.4157	-6.8882	-0.4630	-0.5567
1973	0.0883	0.4684	0.4433	-7.2067	-0.5032	-0.5094
1974	0.0855	0.4417	0.4728	-7.3728	-0.5536	-0.4588
1975	0.0841	0.4274	0.4884	-7.4534	-0.5809	-0.4324
1976	0.0870	0.4000	0.5130	-7.2832	-0.6337	-0.3919
1977	0.0902	0.4144	0.4954	-7.1015	-0.6061	-0.4208
1978	0.0857	0.4210	0.4932	-7.3576	-0.5932	-0.4245
1979	0.0791	0.3919	0.5289	-7.7566	-0.6491	-0.3663
1980	0.0759	0.3489	0.5752	-7.9586	-0.7275	-0.2951
1981	0.0769	0.3567	0.5665	-7.8972	-0.7142	-0.3081
1982	0.0774	0.3483	0.5743	-7.8635	-0.7287	-0.2964
1983	0.0772	0.3314	0.5914	-7.8746	-0.7555	-0.2715
1984	0.0803	0.3242	0.5955	-7.6851	-0.7659	-0.2655
1985	0.0796	0.3247	0.5957	-7.7297	-0.7652	-0.2652
1986	0.0767	0.3019	0.6214	-7.9060	-0.7917	-0.2293
1987	0.0727	0.2979	0.6294	-8.1564	-0.7950	-0.2185
1988	0.0708	0.2777	0.6515	-8.2741	-0.8037	-0.1894
1989	0.0706	0.2537	0.6758	-8.2910	-0.7882	-0.1586
1990	0.0687	0.2231	0.7082	-8.4086	-0.6976	-0.1195

Table A5.5.4.18. Proper AESs and Determinants of matrixs of AESs I3SLS Estimates Model 1 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	0.8243	0.5854	0.3597	2.5099	3.4916	0.1283	2.52E-16
1973	0.8042	0.5862	0.3715	2.9794	3.3273	0.1183	-1.72E-16
1974	0.7855	0.5992	0.3751	3.4644	3.0234	0.1132	-7.01E-16
1975	0.7747	0.6057	0.3749	3.7294	2.8563	0.1106	1.95E-18
1976	0.7673	0.6370	0.3640	4.0269	2.4486	0.1159	-8.52E-17
1977	0.7832	0.6374	0.3644	3.6905	2.5818	0.1222	-8.52E-16
1978	0.7756	0.6169	0.3716	3.7631	2.7424	0.1137	-4.51E-16
1979	0.7388	0.6129	0.3705	4.4893	2.4653	0.1005	-9.46E-16
1980	0.6941	0.6288	0.3498	5.3083	1.9536	0.0924	-5.27E-16
1981	0.7045	0.6279	0.3541	5.1441	2.0390	0.0947	-6.12E-16
1982	0.6995	0.6356	0.3476	5.2404	1.9268	0.0952	-7.54E-16
1983	0.6835	0.6453	0.3341	5.4821	1.7216	0.0935	-5.25E-16
1984	0.6888	0.6612	0.3240	5.4115	1.6034	0.0984	-4.64E-16
1985	0.6865	0.6582	0.3253	5.4433	1.6169	0.0971	7.46E-17
1986	0.6502	0.6602	0.3043	5.8364	1.3773	0.0890	1.03E-16
1987	0.6261	0.6460	0.3040	6.0928	1.3649	0.0813	-5.59E-16
1988	0.5883	0.6490	0.2787	6.3042	1.1456	0.0745	-1.03E-16
1989	0.5475	0.6603	0.2387	6.2355	0.8787	0.0680	-4.51E-16
1990	0.4713	0.6670	0.1740	5.6437	0.5599	0.0531	1.81E-16

Table A5.5.4.19 Fitted cost shares and Own AESs for I3SLS estimates Model 2 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0876	0.5093	0.4031	-14.0513	-0.4110	-0.5675
1973	0.0902	0.4583	0.4514	-13.5083	-0.4997	-0.4870
1974	0.0894	0.4032	0.5075	-13.6836	-0.5987	-0.3943
1975	0.0881	0.3819	0.5300	-13.9546	-0.6359	-0.3585
1976	0.0816	0.3624	0.5560	-15.4396	-0.6682	-0.3186
1977	0.0788	0.4439	0.4773	-16.1824	-0.5256	-0.4436
1978	0.0826	0.4586	0.4588	-15.2002	-0.4992	-0.4746
1979	0.0858	0.3941	0.5202	-14.4554	-0.6149	-0.3739
1980	0.0842	0.3126	0.6031	-14.8047	-0.7325	-0.2501
1981	0.0818	0.3286	0.5896	-15.4011	-0.7160	-0.2692
1982	0.0793	0.3380	0.5826	-16.0412	-0.7042	-0.2792
1983	0.0770	0.3307	0.5923	-16.6847	-0.7136	-0.2653
1984	0.0720	0.3513	0.5766	-18.2674	-0.6853	-0.2879
1985	0.0712	0.3618	0.5670	-18.5359	-0.6692	-0.3021
1986	0.0713	0.3353	0.5933	-18.5007	-0.7077	-0.2639
1987	0.0735	0.3192	0.6073	-17.7842	-0.7264	-0.2442
1988	0.0722	0.2852	0.6425	-18.1880	-0.7446	-0.1969
1989	0.0698	0.2750	0.6551	-19.0417	-0.7415	-0.1806
1990	0.0682	0.2350	0.6967	-19.6434	-0.6605	-0.1296

Table A5.5.4.20. Proper AESs and Determinants of matrixs of AESs I3SLS Estimates Model 2 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.3698	1.3228	0.2216	3.8989	6.2240	0.1841	6.55E-16
1973	1.3989	1.2799	0.2277	4.7925	4.9403	0.1915	2.67E-16
1974	1.4580	1.2514	0.2189	6.0673	3.8297	0.1882	-1.52E-15
1975	1.4906	1.2443	0.2106	6.6518	3.4542	0.1836	-1.40E-15
1976	1.5578	1.2512	0.2069	7.8905	3.3533	0.1701	1.70E-15
1977	1.4717	1.3031	0.2458	6.3396	5.4813	0.1728	1.25E-15
1978	1.4357	1.3009	0.2405	5.5265	5.5214	0.1790	9.29E-16
1979	1.4883	1.2556	0.2204	6.6732	3.8292	0.1813	-3.34E-16
1980	1.6266	1.2244	0.1525	8.1986	2.2028	0.1599	-2.40E-16
1981	1.6140	1.2365	0.1753	8.4229	2.6171	0.1620	1.83E-15
1982	1.6154	1.2467	0.1886	8.6869	2.9239	0.1610	-1.34E-15
1983	1.6478	1.2499	0.1841	9.1912	2.8643	0.1554	-2.26E-16
1984	1.6522	1.2745	0.2113	9.7898	3.6345	0.1527	-5.92E-16
1985	1.6402	1.2823	0.2210	9.7141	3.9562	0.1534	1.88E-15
1986	1.6897	1.2693	0.1969	10.2383	3.2708	0.1480	5.65E-16
1987	1.7037	1.2555	0.1757	10.0156	2.7669	0.1465	-1.22E-15
1988	1.8007	1.2456	0.1281	10.2993	2.0297	0.1302	-8.71E-16
1989	1.8591	1.2492	0.1131	10.6631	1.8791	0.1211	-1.77E-16
1990	2.0288	1.2397	0.0241	8.8588	1.0079	0.0850	1.05E-15

Table A5.5.4.21 Fitted cost shares and Own AESs for I3SLS estimates Model 3 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0825	0.3721	0.5454	-12.4288	-0.4669	-0.2173
1973	0.0836	0.3780	0.5384	-12.2319	-0.4628	-0.2250
1974	0.0835	0.3819	0.5346	-12.2528	-0.4598	-0.2292
1975	0.0832	0.3861	0.5307	-12.3004	-0.4563	-0.2335
1976	0.0804	0.3669	0.5528	-12.8175	-0.4701	-0.2092
1977	0.0788	0.3516	0.5696	-13.1143	-0.4771	-0.1907
1978	0.0806	0.3650	0.5544	-12.7789	-0.4712	-0.2073
1979	0.0817	0.3650	0.5533	-12.5764	-0.4712	-0.2086
1980	0.0809	0.3551	0.5641	-12.7300	-0.4759	-0.1967
1981	0.0809	0.3797	0.5394	-12.7170	-0.4615	-0.2239
1982	0.0799	0.3731	0.5471	-12.9182	-0.4662	-0.2155
1983	0.0787	0.3611	0.5602	-13.1518	-0.4732	-0.2010
1984	0.0766	0.3516	0.5718	-13.5664	-0.4771	-0.1883
1985	0.0767	0.3618	0.5615	-13.5436	-0.4729	-0.1996
1986	0.0763	0.3487	0.5750	-13.6347	-0.4779	-0.1847
1987	0.0777	0.3667	0.5557	-13.3497	-0.4703	-0.2060
1988	0.0773	0.3680	0.5546	-13.4190	-0.4694	-0.2071
1989	0.0756	0.3422	0.5822	-13.7860	-0.4791	-0.1769
1990	0.0745	0.3255	0.6000	-14.0319	-0.4771	-0.1575

Table A5.5.4.22 Proper AESs and Determinants of matrixs of AESs I3SLS Estimates Model 3 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.3779	0.9400	0.1101	3.9041	1.8172	0.0893	-6.50E-16
1973	1.3670	0.9400	0.1125	3.7916	1.8684	0.0915	-9.06E-16
1974	1.3638	0.9395	0.1154	3.7734	1.9254	0.0920	-6.53E-17
1975	1.3610	0.9389	0.1186	3.7606	1.9911	0.0925	-6.27E-16
1976	1.3934	0.9392	0.1094	4.0845	1.7992	0.0864	-1.61E-16
1977	1.4185	0.9399	0.0981	4.2446	1.6173	0.0813	3.25E-16
1978	1.3944	0.9396	0.1075	4.0773	1.7666	0.0861	-5.17E-16
1979	1.3890	0.9403	0.1058	3.9962	1.7397	0.0871	-1.33E-15
1980	1.4041	0.9408	0.0983	4.0864	1.6191	0.0840	-6.52E-16
1981	1.3775	0.9381	0.1182	3.9707	1.9675	0.0894	-5.34E-16
1982	1.3894	0.9382	0.1151	4.0923	1.9032	0.0872	-4.68E-16
1983	1.4084	0.9387	0.1073	4.2403	1.7619	0.0836	-1.29E-15
1984	1.4306	0.9384	0.1016	4.4257	1.6734	0.0795	-2.96E-16
1985	1.4178	0.9373	0.1110	4.3944	1.8247	0.0821	-3.17E-16
1986	1.4360	0.9385	0.0992	4.4539	1.6375	0.0784	-1.23E-15
1987	1.4073	0.9374	0.1136	4.2972	1.8711	0.0840	-1.64E-15
1988	1.4076	0.9371	0.1152	4.3182	1.9012	0.0840	-3.80E-16
1989	1.4484	0.9386	0.0936	4.5066	1.5576	0.0760	-3.04E-16
1990	1.4785	0.9396	0.0754	4.5091	1.3274	0.0695	-1.23E-15

Table A5.5.4.23 Fitted cost shares and Own AESs for I3SLS estimates Model 4 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0829	0.3701	0.5470	-12.2705	-0.4966	-0.1698
1973	0.0843	0.3752	0.5405	-12.0282	-0.4925	-0.1758
1974	0.0844	0.3794	0.5362	-12.0152	-0.4888	-0.1797
1975	0.0843	0.3840	0.5317	-12.0250	-0.4845	-0.1840
1976	0.0805	0.3666	0.5529	-12.6987	-0.4993	-0.1642
1977	0.0782	0.3521	0.5697	-13.1412	-0.5084	-0.1483
1978	0.0806	0.3644	0.5550	-12.6785	-0.5009	-0.1623
1979	0.0817	0.3635	0.5548	-12.4782	-0.5015	-0.1625
1980	0.0804	0.3539	0.5657	-12.7228	-0.5074	-0.1521
1981	0.0817	0.3794	0.5389	-12.4824	-0.4888	-0.1773
1982	0.0803	0.3735	0.5462	-12.7387	-0.4939	-0.1705
1983	0.0785	0.3622	0.5593	-13.0837	-0.5024	-0.1582
1984	0.0760	0.3541	0.5699	-13.5946	-0.5073	-0.1481
1985	0.0766	0.3646	0.5588	-13.4643	-0.5007	-0.1587
1986	0.0755	0.3514	0.5731	-13.6946	-0.5087	-0.1451
1987	0.0778	0.3688	0.5534	-13.2233	-0.4976	-0.1638
1988	0.0775	0.3705	0.5519	-13.2775	-0.4963	-0.1651
1989	0.0745	0.3453	0.5802	-13.9180	-0.5113	-0.1384
1990	0.0725	0.3290	0.5985	-14.3624	-0.5142	-0.1209

Table A5.5.4.24 Proper AESs and Determinants of matrixs of AESs I3SLS Estimates Model 4 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.6551	0.7398	0.0852	3.3545	1.5356	0.0770	3.41E-16
1973	1.6354	0.7411	0.0867	3.2489	1.5653	0.0791	2.17E-16
1974	1.6278	0.7392	0.0896	3.2232	1.6131	0.0798	-2.16E-16
1975	1.6207	0.7368	0.0929	3.1995	1.6692	0.0805	1.17E-15
1976	1.6810	0.7350	0.0862	3.5152	1.5451	0.0746	1.66E-16
1977	1.7299	0.7352	0.0767	3.6884	1.4088	0.0695	4.87E-17
1978	1.6841	0.7363	0.0842	3.5145	1.5154	0.0742	-3.09E-16
1979	1.6766	0.7397	0.0816	3.4472	1.4801	0.0748	4.72E-16
1980	1.7065	0.7405	0.0749	3.5441	1.3870	0.0716	7.88E-17
1981	1.6484	0.7320	0.0942	3.3837	1.6774	0.0778	3.93E-16
1982	1.6701	0.7310	0.0922	3.5022	1.6378	0.0757	2.55E-16
1983	1.7069	0.7313	0.0857	3.6602	1.5346	0.0721	2.61E-16
1984	1.7469	0.7276	0.0823	3.8454	1.4847	0.0684	-2.51E-16
1985	1.7194	0.7244	0.0910	3.7857	1.6120	0.0712	-5.51E-16
1986	1.7574	0.7274	0.0803	3.8786	1.4578	0.0674	8.82E-16
1987	1.7004	0.7259	0.0926	3.6889	1.6386	0.0729	-1.91E-16
1988	1.6996	0.7243	0.0944	3.7008	1.6680	0.0730	3.33E-16
1989	1.7813	0.7270	0.0756	3.9437	1.3972	0.0650	6.57E-17
1990	1.8422	0.7282	0.0594	3.9915	1.2061	0.0586	-2.49E-16

Table A5.5.4.25 Fitted cost shares and Own AESs for I3SLS (AR) estimates Model 2 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.1021	0.4808	0.4171	-9.8303	-0.4708	-0.2134
1973	0.0981	0.4377	0.4642	-10.3120	-0.5498	-0.1982
1974	0.0918	0.3934	0.5148	-11.1718	-0.6322	-0.1652
1975	0.0891	0.3777	0.5332	-11.5777	-0.6606	-0.1509
1976	0.0822	0.3626	0.5552	-12.7583	-0.6870	-0.1329
1977	0.0888	0.4292	0.4820	-11.6360	-0.5656	-0.1880
1978	0.0934	0.4412	0.4654	-10.9497	-0.5432	-0.1976
1979	0.0877	0.3852	0.5271	-11.8141	-0.6471	-0.1557
1980	0.0769	0.3168	0.6063	-13.8228	-0.7536	-0.0890
1981	0.0796	0.3371	0.5834	-13.2769	-0.7274	-0.1089
1982	0.0789	0.3454	0.5757	-13.4149	-0.7149	-0.1155
1983	0.0759	0.3387	0.5854	-14.0399	-0.7252	-0.1071
1984	0.0750	0.3578	0.5672	-14.2425	-0.6950	-0.1228
1985	0.0767	0.3692	0.5541	-13.8775	-0.6756	-0.1338
1986	0.0727	0.3445	0.5829	-14.8038	-0.7165	-0.1093
1987	0.0734	0.3331	0.5935	-14.6359	-0.7331	-0.1001
1988	0.0691	0.3062	0.6248	-15.7430	-0.7641	-0.0728
1989	0.0646	0.2943	0.6411	-17.0577	-0.7723	-0.0586
1990	0.0580	0.2589	0.6831	-19.4565	-0.7619	-0.0224

Table A5.5.4.26 Proper AESs and Determinants of matrixs of AESs I3SLS (AR) Estimates Model 2 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	1.7741	0.3613	0.1084	1.4806	1.9674	0.0887	5.37E-16
1973	1.8848	0.4029	0.1199	2.1171	1.8820	0.0946	5.16E-16
1974	2.0520	0.4245	0.1171	2.8518	1.6655	0.0907	1.04E-15
1975	2.1287	0.4277	0.1121	3.1173	1.5643	0.0871	4.39E-16
1976	2.2745	0.4042	0.1118	3.5914	1.5319	0.0788	1.57E-15
1977	1.9974	0.3643	0.1358	2.5917	2.0545	0.0879	1.37E-15
1978	1.9225	0.3740	0.1293	2.2519	2.0238	0.0906	6.12E-16
1979	2.1254	0.4113	0.1195	3.1278	1.6701	0.0865	3.35E-16
1980	2.5591	0.4168	0.0691	3.8684	1.0563	0.0623	3.00E-16
1981	2.4170	0.4139	0.0907	3.8162	1.2743	0.0710	8.13E-16
1982	2.3947	0.4010	0.1009	3.8560	1.3884	0.0724	8.69E-16
1983	2.4775	0.3882	0.0981	4.0434	1.3532	0.0681	1.39E-15
1984	2.4152	0.3610	0.1189	4.0660	1.6183	0.0712	7.09E-16
1985	2.3423	0.3599	0.1260	3.8898	1.7267	0.0745	1.35E-15
1986	2.5179	0.3579	0.1094	4.2663	1.4899	0.0663	1.50E-15
1987	2.5548	0.3754	0.0956	4.2027	1.3239	0.0642	1.03E-15
1988	2.7970	0.3696	0.0653	4.2063	1.0102	0.0514	1.25E-15
1989	2.9980	0.3436	0.0522	4.1854	0.8818	0.0425	1.26E-15
1990	3.5313	0.3134	-0.0110	2.3533	0.3379	0.0170	7.36E-16

Table A5.5.4.27 Fitted cost shares and Own AESs for I3SLS (AR) estimates Model 3 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0691	0.3081	0.6228	-5.7018	-1.0858	-0.0225
1973	0.0679	0.3096	0.6225	-5.6811	-1.0815	-0.0227
1974	0.0702	0.3134	0.6164	-5.7170	-1.0699	-0.0270
1975	0.0732	0.3180	0.6088	-5.7373	-1.0559	-0.0323
1976	0.0741	0.3105	0.6154	-5.7385	-1.0787	-0.0277
1977	0.0723	0.3021	0.6256	-5.7337	-1.1037	-0.0205
1978	0.0725	0.3081	0.6194	-5.7347	-1.0858	-0.0249
1979	0.0686	0.3046	0.6268	-5.6942	-1.0963	-0.0197
1980	0.0668	0.2986	0.6346	-5.6564	-1.1142	-0.0141
1981	0.0783	0.3200	0.6017	-5.7199	-1.0499	-0.0372
1982	0.0789	0.3177	0.6034	-5.7142	-1.0569	-0.0360
1983	0.0775	0.3111	0.6114	-5.7264	-1.0767	-0.0305
1984	0.0802	0.3095	0.6104	-5.7014	-1.0818	-0.0312
1985	0.0846	0.3181	0.5973	-5.6360	-1.0557	-0.0402
1986	0.0799	0.3080	0.6121	-5.7042	-1.0863	-0.0300
1987	0.0836	0.3192	0.5971	-5.6530	-1.0522	-0.0403
1988	0.0855	0.3215	0.5930	-5.6213	-1.0451	-0.0431
1989	0.0793	0.3047	0.6160	-5.7102	-1.0962	-0.0273
1990	0.0753	0.2937	0.6310	-5.7370	-1.1285	-0.0167

Table A5.5.4.28 Proper AESs and Determinants of matrixs of AESs I3SLS (AR) Estimates Model 3 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	2.8554	-0.7799	0.2204	-1.9618	-0.4801	-0.0241	8.57E-18
1973	2.8785	-0.8115	0.2237	-2.1420	-0.5297	-0.0255	-4.47E-17
1974	2.7947	-0.7696	0.2256	-1.6939	-0.4379	-0.0220	-4.36E-17
1975	2.6965	-0.7186	0.2273	-1.2132	-0.3311	-0.0175	-1.81E-16
1976	2.7170	-0.6797	0.2170	-1.1917	-0.3033	-0.0173	1.82E-16
1977	2.8092	-0.6943	0.2085	-1.5633	-0.3646	-0.0209	-1.63E-16
1978	2.7689	-0.7065	0.2162	-1.4403	-0.3564	-0.0197	8.88E-17
1979	2.8895	-0.7807	0.2164	-2.1063	-0.4975	-0.0253	1.86E-16
1980	2.9797	-0.8064	0.2105	-2.5764	-0.5704	-0.0286	1.53E-16
1981	2.5762	-0.6254	0.2230	-0.6313	-0.1785	-0.0107	4.76E-17
1982	2.5751	-0.6081	0.2195	-0.5917	-0.1640	-0.0101	1.80E-16
1983	2.6385	-0.6169	0.2135	-0.7956	-0.2060	-0.0128	1.32E-16
1984	2.5925	-0.5657	0.2080	-0.5536	-0.1423	-0.0095	-5.11E-17
1985	2.4673	-0.5152	0.2125	-0.1376	-0.0390	-0.0028	1.02E-17
1986	2.6052	-0.5661	0.2064	-0.5909	-0.1496	-0.0101	5.95E-17
1987	2.4796	-0.5340	0.2152	-0.2009	-0.0574	-0.0039	4.37E-17
1988	2.4372	-0.5113	0.2154	-0.0650	-0.0191	-0.0014	7.64E-18
1989	2.6340	-0.5672	0.2028	-0.6787	-0.1660	-0.0113	2.16E-17
1990	2.7852	-0.6113	0.1927	-1.2830	-0.2779	-0.0183	1.64E-16

Table A5.5.4.29 Fitted cost shares and Own AESs for I3SLS (AR) estimates Model 4 (ISIC 3833)

Year	Fitted cost share of Capital	Fitted cost share of Labour	Fitted cost share of Materials	Own AES Capital	Own AES Labour	Own AES Materials
1972	0.0751	0.2957	0.6292	-8.0071	-1.2404	-0.0126
1973	0.0754	0.2958	0.6288	-7.9899	-1.2400	-0.0129
1974	0.0774	0.3000	0.6226	-7.8625	-1.2246	-0.0172
1975	0.0799	0.3051	0.6150	-7.7075	-1.2055	-0.0224
1976	0.0772	0.3008	0.6220	-7.8739	-1.2215	-0.0176
1977	0.0736	0.2941	0.6324	-8.1028	-1.2465	-0.0104
1978	0.0759	0.2981	0.6259	-7.9548	-1.2315	-0.0149
1979	0.0736	0.2931	0.6333	-8.0987	-1.2501	-0.0098
1980	0.0709	0.2879	0.6412	-8.2718	-1.2694	-0.0043
1981	0.0819	0.3100	0.6081	-7.5855	-1.1874	-0.0271
1982	0.0812	0.3089	0.6099	-7.6301	-1.1913	-0.0259
1983	0.0783	0.3036	0.6181	-7.8069	-1.2110	-0.0203
1984	0.0783	0.3044	0.6172	-7.8062	-1.2079	-0.0209
1985	0.0827	0.3132	0.6040	-7.5365	-1.1753	-0.0298
1986	0.0777	0.3033	0.6190	-7.8458	-1.2122	-0.0197
1987	0.0829	0.3133	0.6038	-7.5243	-1.1752	-0.0300
1988	0.0843	0.3161	0.5996	-7.4438	-1.1648	-0.0328
1989	0.0763	0.3007	0.6230	-7.9335	-1.2217	-0.0169
1990	0.0711	0.2907	0.6382	-8.2599	-1.2589	-0.0064

Table A5.5.4.30 Proper AESs and Determinants of matrixs of AESs I3SLS (AR) Estimates Model 4 (ISIC 3833)

Year	Proper AES Capital : Labour	Proper AES Capital : Materials	Proper AES Labour: Materials	Dtrm of Matrix 1	Dtrm of Matrix 2	Dtrm of Matrix 3	Dtrm of Matrix 4
1972	3.3461	-0.6168	0.1836	-1.2641	-0.2792	-0.0180	-1.18E-17
1973	3.3367	-0.6119	0.1834	-1.2257	-0.2713	-0.0176	-2.29E-16
1974	3.2436	-0.5850	0.1867	-0.8922	-0.2071	-0.0138	3.05E-17
1975	3.1366	-0.5543	0.1904	-0.5464	-0.1345	-0.0092	2.20E-18
1976	3.2426	-0.5904	0.1881	-0.8965	-0.2097	-0.0138	-5.30E-17
1977	3.4082	-0.6421	0.1831	-1.5155	-0.3277	-0.0205	1.33E-16
1978	3.3015	-0.6073	0.1860	-1.1037	-0.2503	-0.0162	-3.64E-17
1979	3.4140	-0.6383	0.1816	-1.5312	-0.3280	-0.0207	5.64E-17
1980	3.5534	-0.6810	0.1772	-2.1262	-0.4286	-0.0260	3.12E-16
1981	3.0515	-0.5335	0.1942	-0.3046	-0.0792	-0.0055	-2.97E-17
1982	3.0770	-0.5428	0.1938	-0.3787	-0.0972	-0.0067	-2.50E-17
1983	3.1910	-0.5784	0.1906	-0.7288	-0.1759	-0.0117	3.55E-17
1984	3.1849	-0.5803	0.1916	-0.7139	-0.1737	-0.0115	6.06E-17
1985	3.0103	-0.5287	0.1971	-0.2038	-0.0548	-0.0038	-1.61E-17
1986	3.2112	-0.5887	0.1909	-0.8011	-0.1923	-0.0126	7.84E-17
1987	3.0051	-0.5256	0.1969	-0.1882	-0.0507	-0.0036	1.95E-17
1988	2.9555	-0.5117	0.1986	-0.0648	-0.0180	-0.0013	-1.00E-18
1989	3.2712	-0.6077	0.1892	-1.0082	-0.2350	-0.0151	1.20E-16
1990	3.5216	-0.6845	0.1813	-2.0035	-0.4158	-0.0248	-6.20E-17

A5.5.5 Economies of Scale

A5.5.5.1 IZEF estimates of economies of scale - ISIC 3833

Year	EOS 1 - IZEF	EOS 2 - IZEF	EOS 3 - IZEF	EOS 1 - IZEF (AR)	EOS 2 - IZEF (AR)	EOS 3 - IZEF (AR)
1972	1.3576	1.9069	1.0057	2.8852	1.7039	1.1249
1973	1.2823	2.0176	1.0056	2.1383	1.6319	1.0762
1974	1.2082	2.1302	1.0055	1.6769	1.5696	1.0289
1975	1.1944	2.1993	1.0054	1.5414	1.5524	1.0099
1976	1.1950	2.0699	1.0054	1.5014	1.5302	1.0068
1977	1.3832	1.8542	1.0056	2.1763	1.6227	1.0811
1978	1.4402	1.9227	1.0056	2.2631	1.6438	1.0845
1979	1.3320	2.0131	1.0055	1.7076	1.5531	1.0326
1980	1.2147	2.0572	1.0053	1.3249	1.4580	0.9790
1981	1.2525	2.2300	1.0053	1.3313	1.5036	0.9748
1982	1.2998	2.1557	1.0053	1.3898	1.5150	0.9850
1983	1.3205	2.0708	1.0054	1.3918	1.5023	0.9867
1984	1.3990	1.9688	1.0054	1.5201	1.5311	1.0068
1985	1.4481	2.0259	1.0054	1.5466	1.5554	1.0078
1986	1.4300	1.9700	1.0054	1.4564	1.5122	0.9969
1987	1.4130	2.1293	1.0053	1.3412	1.5047	0.9748
1988	1.3687	2.1954	1.0052	1.2152	1.4724	0.9510
1989	1.3955	2.0078	1.0053	1.2357	1.4460	0.9587
1990	1.3520	1.9458	1.0052	1.1423	1.3968	0.9418

A5.5.5.2 I3SLS estimates of Economies of Scale - ISIC 3833

Year	EOS 1 - I3SLS	EOS 2 - I3SLS	EOS 3 - I3SLS	EOS 1 - I3SLS (AR)	EOS 2 - I3SLS (AR)	EOS 3 - I3SLS (AR)
1972	1.4349	1.9331	0.9984		3.0497	1.1390
1973	1.2952	2.0802	0.9944		2.6252	1.0865
1974	1.1706	2.2332	0.9902		2.2813	1.0360
1975	1.1427	2.3240	0.9884		2.1788	1.0157
1976	1.1472	2.1636	0.9881		2.0543	1.0124
1977	1.4553	1.8816	0.9948		2.4685	1.0918
1978	1.5394	1.9599	0.9951		2.6022	1.0954
1979	1.3447	2.0907	0.9905		2.1993	1.0398
1980	1.1624	2.1709	0.9853		1.8301	0.9828
1981	1.1995	2.3736	0.9849		1.9324	0.9784
1982	1.2680	2.2734	0.9859		1.9622	0.9892
1983	1.3008	2.1692	0.9861		1.9147	0.9910
1984	1.4261	2.0331	0.9881		1.9887	1.0124
1985	1.4925	2.0955	0.9882		2.0543	1.0135
1986	1.4643	2.0388	0.9871		1.9187	1.0018
1987	1.4150	2.2387	0.9849		1.8828	0.9784
1988	1.3370	2.3320	0.9824		1.7644	0.9531
1989	1.3893	2.1022	0.9832		1.6989	0.9613
1990	1.3254	2.0387	0.9814		1.5655	0.9434

A 5.5.6 Graphs of Economies of Scale

Figure 5.5.6.1 IZEF estimates of scale- ISIC 3833

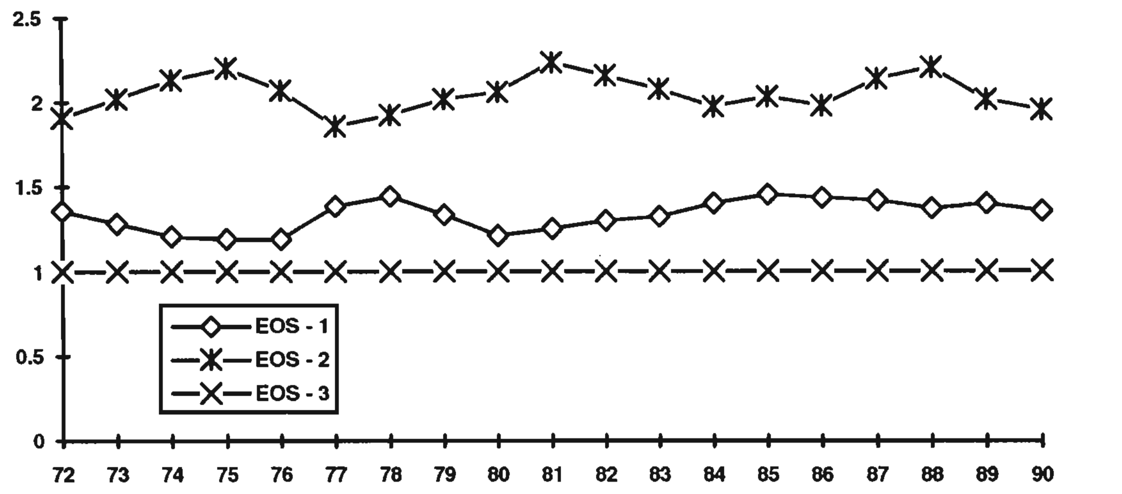


Figure 5.5.6.2 IZEF (AR) estimates of scale- ISIC 3833

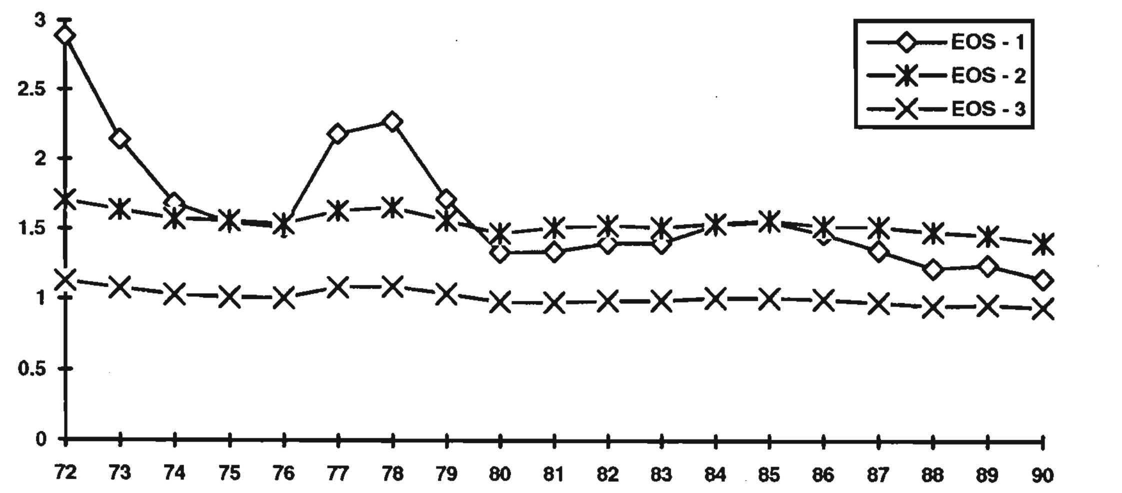


Figure 5.5.6.3 I3SLS estimates of scale- ISIC 3833

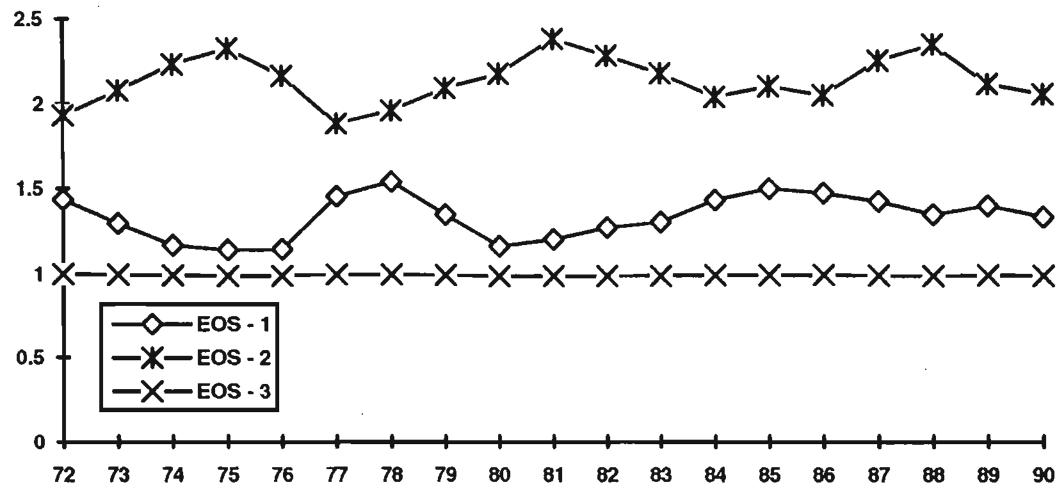


Figure 5.5.6.4 I3SLS (AR) estimates of scale- ISIC 3833

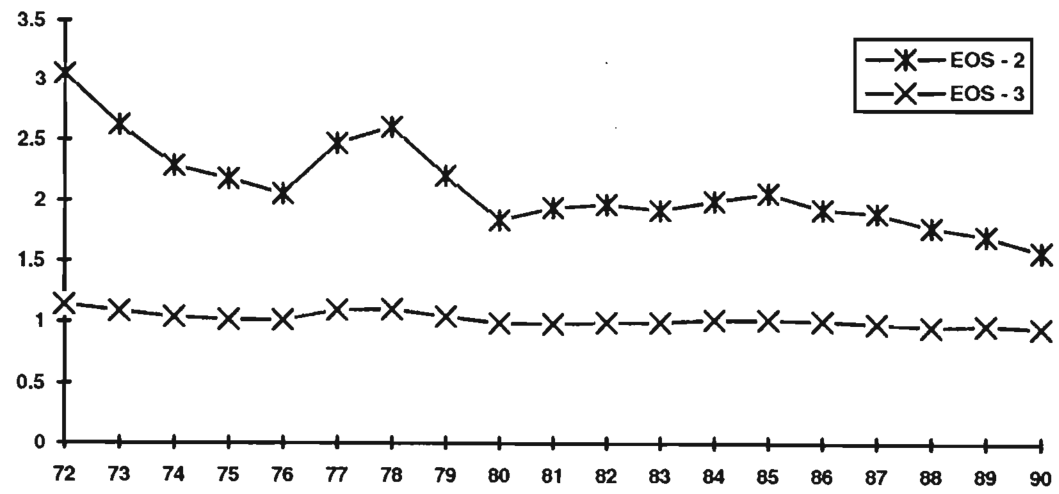


Figure 5.5.6.5 IZEF, IZEF (AR) and I3SLS estimates of Model 1 - ISIC 3833

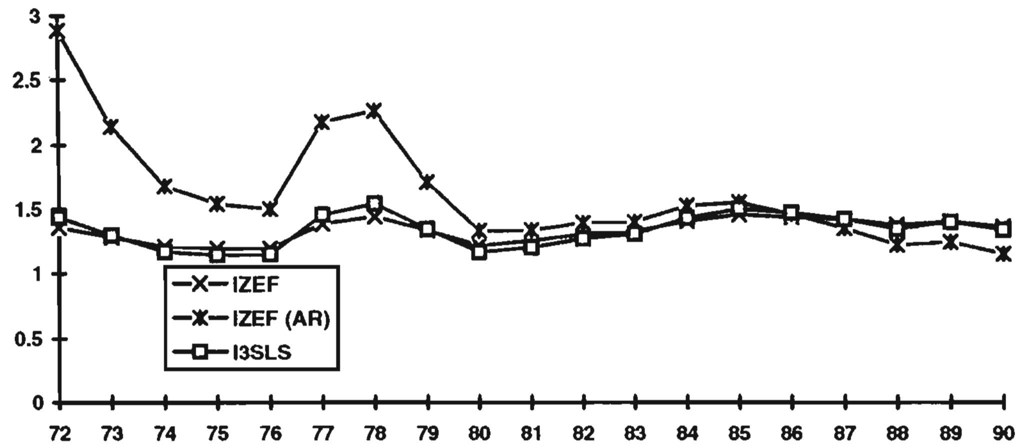


Figure 5.5.6.6 IZEF, IZEF (AR), I3SLS and I3SLS (AR) estimates of Model 2 - ISIC 3833

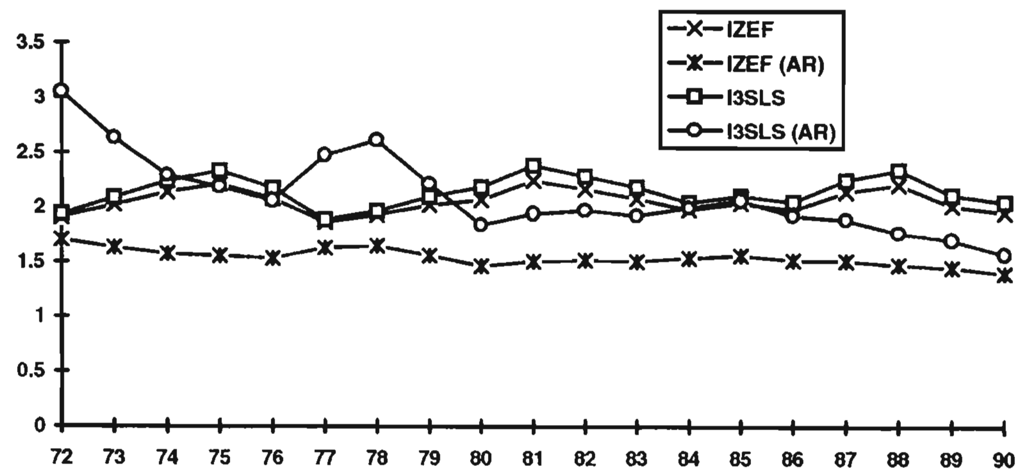


Figure 5.5.6.7 IZEF, IZEF (AR), I3SLS and I3SLS (AR) estimates of Model 3 - ISIC 3833