THE MEAN VARIANCE EFFICIENCY OF THE JSE ALL SHARE INDEX (ALSI) AND IT'S IMPLICATIONS FOR PORTFOLIO MANAGEMENT

by

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EXECUTIVE SUMMARY

The use of proxies in the CAPM model to determine assets expected return has implications for portfolio management. An inefficient proxy can result in the lowering of beta estimates due to a weak regression relationship resulting in the misallocation of capital.

For the CAPM equation to be satisfied would require that the proxy should at least have an alpha that centred on zero over a period of time. This would allow the linearity of the model to hold and we would advocate a passive portfolio strategy. If the proxy were mean variance inefficient would indicate that alpha values are present in asset returns that can allow us to rebalance portfolios using optimisation techniques. We test the hypothesis that alpha averages around zero using the market model by regressing Industrial and Gold index excess returns on the market premium.

$$E(r_i) - r_f = \alpha_i + \beta_i\left[E(r_m) - r_f\right]$$

INDIX	Coefficients	Standard Error	t Stat	P-value	R Square
Intercept - α	0.015328	0.008109	1.890269	0.060329	
ALSIX	0.880827	0.032454	27.14081	1.58E-65	0.803627
GOLDIX					
Intercept - α	-0.07904	0.021226	-3.72368	0.000262	
ALSIX	1.181673	0.084952	13.90987	2.42E-30	0.518052

When tested from the SA investor perspective we find that the alpha of the ALSI regression is not zero for the Gold Index but centred on zero for the Industrial Index. The implications are that SA investors would get a fair return holding the ALSI index instead of trade in industrial shares. The result warrants a passive strategy. However, portfolio optimisation demonstrates that a higher return can be achieved by rebalancing the portfolio

The regression using the Gold index produced a negative alpha implying that investors should actively sell Gold shares from their portfolios. The ALSI was not an adequate proxy of risk to the SA investor for gold shares. Overall the ALSI is inefficient since it has a non-linear relationship to one sector of the JSE. Portfolio analysis and rebalancing is required to attain an optimal return. The Markowitz model recommends that all SA investment capital should be fully weighted in the Industrial index only.

Introducing an international investment proxied by the Dow Jones significantly improved the returns to SA investors. This is evident in the improved Sharpe ratio achieved by the rand adjusted Dow Jones available to the SA investor. In the absence of exchange restrictions the model recommends that 87% of local investors assets should be moved abroad under the present investment conditions.

ALSI	GOLDI	INDI	DOWLOCAL
0.06551	-0.15279	0.126315	0.664777

When tested from the US investor's perspective using dollar returns the data estimates achieved from the regression analysis were:

INDIXS	Coefficients	Std Error	t Stat	P-value	R Square
Intercept	0.016886	0.007724	2.186188	0.030092	
ALSIX$	0.96069	0.026158	36.72657	1.52E-85	0.882264
GOLDIXS					
Intercept -	-0.01665	0.0191	-0.87186	0.384446	
ALSIX$	1.070805	0.064683	16.55458	5.21E-38	0.603572

The alpha value of the Industrial index is non-zero and the Gold index alpha centres on zero. The results are a reversal of the Rand tests of the SA investor. Gold shares priced fairly in dollar terms as opposed to Industrial shares. Currency effects of Rand depreciation priced into the dollar return of Industrial shares led to their non-participation in the US investors' portfolio. Due to trade of gold in dollars, the gold shares were priced to provide a fair return to the dollar-adjusted ALSI as opposed to the rand denominated test. Overall, the ALSI was inefficient due to the Industrial sector pricing in dollars resulting in abnormal alpha values over time. Currency depreciation resulted in the distortion of the CAPM relationship between the INDI and ALSI.

The US investor's domestic index, the Dow Jones was found to lie on the efficiency frontier for tests using the ALSI and the INDI. There was no reason to invest in SA, but if the US investor did chose to invest in SA shares then gold had the lowest beta and the lowest correlation to the Dow Jones.

The beta values of the SA indices were all significant and the alpha values were negative when regressed against the Dow Jones. The implication of this would be to invest as much as possible in the international index portfolio as possible.

Regression Statistics	ALSIXS	INDIXS	GOLDIXS
P-values			
Intercept α	1.18E-05	0.001992	1.51E-05
DOWJONES	9.87E-15	1.32E-11	5.27E-05
Coefficients			
Intercept α	-0.09833	-0.07281	-0.15206
DOWJONES	1.082276	0.985812	0.831916

The Dow Jones introduces a significant diversification benefit to the SA investor's portfolio by increasing returns significantly per unit of risk. The Markowitz model recommends that 87% of SA investor's portfolio should be in the Dow Jones and 13% in the Industrial index.

Due to independent pricing of the gold and industrial sectors, the former by international markets in dollars and the latter in rands in SA, a dichotomy is created in the local market. From an SA investor's point of view the CAPM would not capture the correct return of gold shares. It would overstate the expected return since beta of the SA market premium will not include dollar returns. The ALSI is an incorrect proxy for the SA investor analysing gold shares. The Gold sector is only correctly priced from the US investor's perspective once the ALSI is dollar adjusted.

The industrial index can use CAPM analysis reliably with the ALSI as market proxy but higher returns are achievable through portfolio rebalancing. Active portfolio management is recommended. Nevertheless, this will not produce results significantly different to the CAPM once standard errors of the mean are accounted for.

The results found currency depreciation of the Rand as a major factor contributing to the exodus of SA capital. The dollar had an expected mean return of 12,6% p.a. This substantially increased the rand adjusted Sharpe ratio of the international portfolio compared to its dollar return. The increased Sharpe ratio of the rand denominated international portfolio resulted in a substantial shift of the optimal portfolios weighting away from the domestic portfolio and towards the Dow Jones. International investors optimal portfolios were similarly impeded due to the depreciating currency.

Chapter 1

1. INTRODUCTION

The South African investment community had been limited in its asset allocation decisions to local assets for a considerable period. Protectionist measures for the economy and currency in the form of foreign exchange regulations prohibited investors from diversifying their portfolio's internationally. These restrictions on the export of capital have the capability of distorting the domestic share market by the creation of artificial barriers on the movement of capital.

However with South Africa's readmission to the world political arena came economic liberalisation as well. This has had a profound effect on investment decision-making in the country both in terms of risk analysis and asset allocation. The variables interacting with investment models and the choices to consider in investment decision-making is now part of a broader and larger universal equity market. As part of a gradual relaxation of controls, financial institutions and unit trust's can allocate up to 15% of their assets abroad and individuals have freedom to invest up to R750 000 in foreign assets.

The resulting opening up of international investment opportunities has placed the returns offered by the ALSI into a larger basket of opportunities for South Africans, allowing investors for the first time to question the efficiency of returns in the South African market. Investing offshore through locally sold funds has allowed investors to gain exposure to overseas markets with simplicity. Without resorting to tedious administrative and legal procedures, investors are able to invest in funds locally consisting of foreign assets. The unit trust industry of SA has seen a deluge of international equity funds being introduced. Investor demand has resulted in these funds being capped and new accounts refused, to comply with foreign exchange regulations. The allowance to financial institutions provides for a maximum of 15% total assets to be placed abroad.

The new variable that has been introduced into the South African investment environment i.e. foreign investment, has obviously impacted on peoples local portfolios. Weightings have been rebalanced to benefit from international diversification. In view of this dynamic new environment, the author seeks to identify key relationships with respect to local and international markets and the implications for both local and foreign investors. The efficiency of the South African market and its implications for investment decision-making is crucial to this analysis.

To the extent that portfolio rebalancing and investor behaviour has been based on theoretical models of investor expectations we will use portfolio theory and pricing models to determine the efficiency of the South African market and its implications for decision making.

1.1. Problem Statement and Objectives

The most central model in finance today used to determine expected returns is the Capital Asset Pricing Model (CAPM). In 1964 William F. Sharpe wrote a paper in the journal of Finance entitled "Capital Asset Prices: A theory of market equilibrium under conditions of risk". This led to the introduction of the basic CAPM in investment decision-making. The model expresses a simple measurable linear relationship between risk and return. It states that the expected return on a share is a function of the risk of that share in relation to the market and the return earned by the market. For a fair return on a security, every investor expects to earn a market premium over the risk free rate of return that is commensurate with the risk of that security.

One of the simplifying assumptions contained in the CAPM is that investors are limited to a universe of publicly traded financial assets. The true market return (*Rm*) would theoretically include the return on all assets traded in an economy. Returns available on government assets, private companies and property should theoretically constitute the market return. However due to the lack of a relevant benchmark index of all assets in the economy typically this involves choosing a broad based stock index proxy to represent the market portfolio and hence market return. In the South African case, our broadest observable index of publicly traded assets is the All Share Index (ALSI). It records the movements of a large sample of shares listed on the Johannesburg Stock Exchange from a variety of primarily industrial and mining shares

What if the market return (*Rm*), using a market proxy like the ALSI, was inefficient? What implications would this have for the CAPM and would it still be a useful measure upon which to base investment decisions? Of concern to us then is whether the ALSI is providing investors with an acceptable return given the risks associated with it i.e. is it mean variance efficient? If it is, a passive portfolio is prescribed to maximise returns and minimise trading costs. Stated simply, if the expected return of an asset is related to its systematic risk and hence providing investors with a fair rate of return dictated by the CAPM then a passive portfolio of the index is optimal.

The CAPM states, where β_i is a measure of risk, R_f is the risk free rate and Rm is the market return, the expected return is on asset *i* is

$$E(r_i) = R_f + \beta_i(R_m - R_f) \quad (1.1)$$

The model portends that an assets expected return is dependant on the market excess return adjusted by a sensitivity coefficient of the assets risk relative to the market, plus the risk free rate.

The model has enabled portfolio managers to determine a benchmark for evaluating possible investments, make educated guesses about pricing initial public offerings and in merger and acquisition analysis. Corporations use the model in capital budgeting as a decision tool for project appraisal. The extent and importance of its use cannot be overemphasised.

The theoretical and inherently unobservable market portfolio is assumed in the model as lying on the Capital Market Line (CML) that is tangential to the efficiency frontier of the Markowitz model. The assumption is therefore that holding the market portfolio is also the optimal portfolio of the efficiency frontier. Our problem is that if the market proxy ALSI is not a mean variance efficient portfolio the linear relationship of the CAPM will no longer hold and the model breaks down as a useful measure of risk and return. For the model to hold it would have to satisfy the assumption of the market portfolio being an optimal portfolio. If we begin share selection analysis with an inadequate market proxy, we are lowering the market premium because an efficient proxy would have a higher return for the same level of risk. The CAPM predicted return of assets would therefore underestimate the fair return, leading to the selection of an inefficient portfolio as well.

Richard Roll in his article entitled "The Benchmark Error Problem with Global Capital Markets" explained the problem of using inadequate benchmarks, the benchmark typically being a security market index adjusted for risk by beta as used in the CAPM. He stated that an inappropriate proxy could have two effects. The systematic risk or beta computed for individual assets would be wrong because an inappropriate proxy is regressed in the CAPM and secondly the security market line (SML) would be misperceived as it would pass through an improperly specified proxy.

The purpose of the study is to determine the efficiency of returns of the ALSI index using the CAPM model of linearity and to compare these to returns offered by Markowitz portfolio selection model of efficiency frontiers using component indices like the Industrial and Gold Index locally and the Dow Jones internationally. We proposes to test the mean variance return of the All Share Index (ALSI) from the point of view of a local investor and determine its relevance if any, to the CAPM model. The contribution of this research will allow local and international investors in the South African market to understand the extent to which the market return, using the ALSI as a market proxy, does explain expected returns of individual assets in the Capital Asset Pricing Model and if so what effect international diversification has on the creation of an efficient portfolio.

The results will enable us to recommend the use of an adequate proxy and the implications for security selection and asset allocation. We will identify the risks associated with the various indices by estimating their betas and constructing a Capital Market Line (CML) and testing the relationship of expected return on assets to the ALSI through regression testing of the Security Characteristic Line (SCL). Based on these results we will draw conclusions on market efficiency in South Africa and its investment implications.

1.2. Limitations of Study

The portfolio theory model depends on the use of ex-post facto data. Empirical analysis attempts to predict future returns based on historic price movements. There is an implicit assumption that key statistical relationships will not alter over the life of the investment being analysed. By using a large sample of 182 observed returns we attempt to reduce any standard error of estimates and further set a 95% confidence level of on our estimates to obtain a high significance level of the findings.

Returns can be calculated for portfolios of different holding periods that can produce different means and standard deviations. Returns used for analysis in this research are calculated based on index values observed at month end. The indices therefore fail to capture hidden periodicities that might occur during the month. In order to get a more precise mean return in the Markowitz model involves obtaining data from longer periods of historical returns. However the older our data input, the higher the precision is improved at the risk of introducing potential bias from the older data in the calculations. Using mean returns are also subject to standard errors that grow larger for more volatile price movements and therefore cannot be utilised with a great degree of precision. The higher the confidence level that is set the higher the standard error that the expected return will have a larger range from the mean.

Beta estimates can change over time and future risk patterns associated with returns may change if there is a significant shift in an asset's characteristics. The article by Favish and Graves on estimating the market risk premium on the JSE found that risk premia are not consistent over time. Using the historic average market risk premium was not a useful predictor of the market premium. The trend line estimate was superior to the averaging method at predicting an ex-ante risk premium. Our study uses the averaging method as a point of reference to determine at the 95% confidence level whether the results of the CAPM are dependable in terms of a domestic and a global portfolio.

Chapter 2

2. THEORETICAL BACKGROUND

The CAPM assumes that all investors are mean variance optimisers having homogeneous expectations. Therefore, in a CAPM investment world all investors will hold the market portfolio. The assumption has its roots in the Markowitz Portfolio Selection Model. In 1952, Harry Markowitz wrote an article for an academic publication, the Journal of Finance. Mr. Markowitz was able to calculate a return for a portfolio and a measure of expected portfolio risk. He showed that the variance (or volatility) of the return was a good measure of portfolio risk. The portfolio selection model we use is a derivation of the technique used in his study.

The central theme of the Markowitz model was mean variance optimisation. The model contained three distinct stages. First, it identifies the risk return combinations available from the set of risky assets that result in the portfolio with the highest possible return for a given level of risk. Alternately, for a given return the model derived the portfolio that resulted in the lowest risk. These portfolio points were summarised by the minimum-variance or efficiency frontier. Next, it identifies the optimal portfolio of risky assets by finding the portfolio weights that result in the steepest Capital Market Line (CML) on the frontier. The slope of the CML is defined by the risk premium

The CML that was tangential to the efficiency frontier satisfied this criterion. Finally, mixing the risk free asset with the optimal risky portfolio completes the portfolio depending on the investors risk aversion. The stages of this analysis are covered in more detail in the methodology.

Foremost in the development of risk and return assumptions is the idea of an efficient frontier. This idea considers that diversification is important and it proposes that we can learn how to effectively diversify a portfolio through an optimisation process. The optimisation process is like any mathematical problem where we are trying to find the optimum solution based on certain inputs and constraints. In essence, the efficient frontier is the optimum solution. It is the mix of the various asset classes to derive the highest level of return at a given level of risk. The efficient frontier is the best mix of asset classes to maximise return at a certain level of risk.

The most important basic assumption about investors is that all investors are risk averse. This forms the core of the mean-variance analysis upon which Markowitz derives his efficient portfolio. Investors seek to maximise utility. For a given level of risk or standard deviation an investor will always chose the portfolio with the highest return for that level of risk. Investors will only accept higher risk if the expected return on the portfolio increases to compensate for the extra degree of risk. Bodie, Kane and Marcus stated the mean- variance criterion for portfolios A and B as: A dominates B if

$$E(r_A) \geq E(r_B)$$
$$and$$
$$\sigma_A \leq \sigma_B$$

One inequality has to be strict to rule out the equality.

Higher return/risk assets are likely to come into an investor's portfolio when the investor has a high-risk tolerance. Similarly, low return/risk assets will make up a large part of an investors portfolio when the risk tolerance is low.

An important factor here is the correlation between the asset classes. Asset classes do not always move up and down at the same time. Correlation is important because this is where diversification adds its benefit. This is why an asset on its own that may have a high return and high risk (e.g., gold shares) may be part of an investor's portfolio with a decidedly lower risk profile. If the gold sector does not perform similarly to the rest of his portfolio, it may provide a higher return and increase the investor's diversification without necessarily increasing the investors overall portfolio risk. In the article "Market Segmentation on the JSE" by Dr. P. van Rensburg and Kevin Slaney, research was conducted on the segmentation of processes that drove the returns of industrial and mining shares. The article found that "return generating process operational on the JSE is dichotomous in nature." The study "found strong evidence supporting the contention that the return generating process on the JSE is segmented in a manner that has economic significance. This would appear to support the view that gold would have a diversification benefit to a portfolio.

Bradfield in his research entitled "A Review of Systematic Risk Estimation on the JSE" stated that pricing of securities on the JSE between the Industrial and Mining indices revealed market segmentation of these two sectors. He argued that this dichotomy together with thin trading volumes impacted negatively on reliable beta estimates. Mining shares and gold shares in particular in his explanation were regarded as a different type of risk and hence forming a different market. Pricing on the JSE would therefore compensate investors separately for each index. He advocated the use of the relevant Mining or Industrial index in beta estimation rather than the use of the broader based ALSI and demonstrated that these sector proxies were more reliable at beta estimation than the JSE Overall Index. Bradfield stated, "An inappropriate proxy leads to a spurious reduction in the correlation between the securities and the returns measured. The reduction of correlation manifests itself in underestimates of systematic risk i.e. downward biased beta estimates."

The Markowitz model is explained in detail here due to the relevance is has for the CAPM model. In the CAPM, the efficient portfolio of the CML is used to determine the return on the market portfolio *(Rm)*. Whereas the use of the Markowitz model becomes tedious for large number of assets due to the high number of variance and covariance terms that need to be calculated, the CAPM reduces riskiness of the portfolio to beta. Beta is a coefficient of the market risk premium that expounds the degree of riskiness of an asset relative to the market in a linear relationship.

Markowitz assumed that expected return was the mean return over a sample period and that risk was measured by the standard deviation of returns from the mean. The standard deviation was a measure of total risk including unsystematic risk. The CAPM assumed that investors were adequately diversified to be holding portfolios similar to the market portfolio and hence unsystematic risk was irrelevant. Since investors were fully diversified and held the optimal portfolio, which was the market portfolio, the only concern in portfolio building was how the systematic risk of this market portfolio related to individual assets.

Richard Roll in his investigation of asset pricing theory noted the single testable hypothesis associated with the CAPM is that the market portfolio is mean variance efficient. He showed that the linear relationship of the model had its basis in the market portfolios efficiency. The vital implication of his article is that if there is no significant linear relationship between an assets returns and the market returns then the market return turns out to be mean variance inefficient.

Roll's (1977) criticised the CAPM because he found the use of market proxies such as the FTSE poor substitutes for the true market portfolio. If inferior proxies are used, then the expected return beta relationship is not based on the theoretical CAPM and there is a breakdown in the link of asset returns to the chosen market proxy return.

Rolls critique concluded that unless all assets are included in the sample to reflect the components of the true market portfolio then the CAPM is not testable. He mentioned that a difficulty of using proxies is that mean variance efficiency of a market proxy does not imply that the market portfolio itself is efficient and vice versa. Roll and Ross[4] in a further analysis of the CAPM wrote that if the positive relationship between average return and beta were rejected through hypothesis testing then the implication would be that the market proxy was inefficient and not that there was no evidence of linearity to the true market portfolio. The CAPM as a model could not be refuted but rather the appropriateness of the proxy used.

Utilising the conclusions of this article, we can test the beta, expected return relationship for the Gold Index (GOLDI) and the Industrial Index (INDI) to determine the efficiency of the ALSI. However, in order to complete the theoretical basis of this research we need to first expand our discussion to the implications of the Index Model of asset pricing. The model is termed the index model due to its use of a market index as a proxy for systematic or market risk.

The index model states the pricing relationship of assets to the market in terms of excess returns. It states that

$$R_i = \alpha_i + \beta_i R_M + e_i \qquad (2.1.)$$

Where R_i is the excess asset return over the risk free rate, R_M is the excess market return, e_i is a firm specific risk component and α_i is the return when the market is neutral or has zero excess return. It is obvious that the firm contains both market risk and unsystematic risk related to the asset.

For purposes of our analysis, we rearrange the CAPM equation as

$$E(r_i) - R_f = \beta_i(R_m - R_f) \qquad (2.2)$$

The equation states that the excess return on a particular risky investment is a function of the excess return on the market adjusted by the risk of the investment. Substituting λ for the excess returns the relationship can be postulated as follows.

In our test of this hypothesis, we regress the excess returns on the ALSI and solve for beta. The equation would hold if the relationship between the asset and the market proxy were efficient. There should therefore be no abnormal returns. The intercept of the equation must average to zero over a period of time.

$$\lambda_i = \beta_i \lambda_m \qquad (2.3)$$

Bodie, Kane and Marcus showed that the beta coefficient of the Index Model turns out to be the same beta as that of the CAPM except that the theoretical market portfolio of the CAPM is replaced by an observable market index. In our case, the ALSI is the index. They further stated that moving the single index model from an ex post to an ex ante model of expected returns would result in the following equation:

$$E(r_i) - r_f = \alpha_i + \beta_i[E(r_M) - r_f] \qquad (2.4.)$$

The firm specific component falls away from the ex ante model because it is unsystematic or unpredicted risk. We note that if the α_i turns out to be zero then the resulting equation is equal to the CAPM linear equation.

The difference between the fair and actually expected return is known as alpha (α). The CAPM therefore predicts that the α_i term should be zero if the asset is to be efficiently priced. If the alpha term is positive, security selection dictates that the weightings of assets with positive alphas are increased and negative alphas are decreased.

The index model holds that for a sample of historically observed returns, we should expect that the alpha term must centre on zero over time. If it does not Rolls critique tells us that we can infer that the market proxy is inefficient since a permanent alpha value implies non linearity of returns to the chosen proxy. No inference can be made about the true linear relationship of the market portfolios beta and the asset since the true market portfolio is not being tested. The beta measure can only tell us the degree of risk of the asset with relation to the chosen proxy. The result can therefore only make an inference as to the proxy's efficiency as a measure of systematic risk to the asset.

Using this result, we can test mean variance efficiency by testing for the presence of alpha terms in the asset returns regression over the market returns. Should this average be significantly different from zero over a period, would make the market proxy mean variance inefficient and would imply it was not an adequate proxy for determining fair asset returns on the CAPM. If an alpha value can be consistently achieved over a large sample average, would imply that there is no linearity of the ALSI market risk to that of the asset being tested, and that the CAPM would not hold.

We will test this relationship using the ALSI as the market proxy against the Gold Index and the Industrial Index, and thereafter solve for beta using simple linear regression. The results can be compared to those achieved through portfolio optimisation to determine the validity of the CAPM. We assume that prices follow a random walk, that all publicly available information is reflected in the stock price and that over time they return to their equilibrium values.

The slope of the SML for an inadequate proxy, by virtue of it not having a direct relationship to the proxy as predicted by the CAPM, would result in a lowering of the beta coefficient compared to the true market portfolio. The lower beta when used for ex ante forecasting of an assets return would lower the expected fair return of an asset resulting in misallocation of capital.

An alternative to the beta estimation method is Markowitz portfolio theory that ignores beta and rather measures total risk. The portfolio optimisation method is a data based method whereas the CAPM method is based on a theoretical pricing model. They represent extreme views on asset selection and portfolio optimisation. The CAPM assumes that all that needs to be done is hold the market portfolio. In the case of South Africa, the ALSI is assumed as the market portfolio to be held. It assumes this portfolio to be the most efficient and prices risk relative to the excess returns on the market portfolio. The Markowitz portfolio model ignores CAPM totally and bases its portfolio selection on maximising return to risk ratios. A comparison of the results using both techniques will determine the implications of the use of the ALSI as a market proxy.

The ALSI though is a domestic index and in a world where markets are increasingly becoming global, may be an incorrect proxy for asset returns in the portfolio that includes foreign assets.

South Africa's place in international investment markets introduces a new risk factor into expected returns of global equities, namely exchange rate risk. Due to the partial relaxation of exchange controls, it would be inappropriate to exclude international assets from the portfolio optimisation process. Mean variance efficiency in terms of international markets are relevant to decision making because globalisation of markets and the relaxation of investment restrictions imply that portfolio optimisation can be expanded to consider a global market portfolio rather than a narrowly focussed domestic index.

In calculating returns on international portfolio movements, both price movements and exchange rate returns need to be incorporated in the calculation of the total return to the investor. The incorporation of foreign exchange movements into the calculation can be shown as:

$$r_p = r_d + r_c + r_d r_c \qquad (2.5.)$$

Where r_p is the total return of the asset, r_d is the return of the asset in its domestic currency, and r_c is the return of the domestic currencies exchange rate.

Bhana in his research on the use of ex-post inter country correlation coefficients to predict gains from international diversification found that optimal portfolios were relatively unstable for short term horizons. He further found that the degree of stability in international correlation increases over longer-term horizons. Benefits from international portfolio diversification would realise gains if future relationships between two or more different national exchanges could be predicted. The condition would be fulfilled if the relationship of the national exchanges were stable over time. The results justified the practical use of the mean variance portfolio to allocate funds to international portfolios using ex-post data. The results were particularly useful for investors with medium to long-term investment horizons and inter country correlations did not vary significantly over time. Markowitz model can therefore be used reliably to determine an efficient international portfolio.

The assumptions of the CAPM regarding taxes and homogeneous expectations are unrealistic and ignore real world complexities. However, the simplification of the model provides us with an opportunity to capture a greater understanding of equilibrium in markets and investors expectations using simplifying logic to control for extraneous variables. It may therefore not be a precise reflection of the real world but is a useful measure of the rationality of market behaviour to risk and therefore is still useful to risk analysis. The concern though is whether the returns predicted by the model are a reasonable approximation of the fair return expected of the asset.

Chapter 3

3. METHODOLOGY

3.1. Sampling Design

A major issue for the sampling design is the selection of input data, and one possibility for generating the optimisation inputs is to use historical data. The simplest way to convert N years of historical data into inputs is to use the assumptions that Markowitz used. He assumed that the upcoming period would resemble one of the N previous periods, with a probability 1/N assigned to each. This assumption would imply that the expected return of the next holding period would equate to the samples mean estimate.

An existing convenience sample of secondary financial data already exists to complete the dissertation. As such, no measurement technique is employed to gather further data. The data sample was extracted from the INET database using closing market values of various economic data on the same date to ensure that the data was synchronous. The sample consists of 193 index values observed at month end of the All Share Index (ALSI), Gold Index (GOLDI), Industrial Index (INDI), and the rand adjusted Dow Jones (DOWLOCAL) from the period February 1983 to March 1999. For purposes of our analysis, we treat the ALSI as a proxy for the domestic market, the INDI for industrial shares, the GOLDI for gold shares and the DOWLOCAL as the international share market available to SA investors.

Using the sample, we are able to calculate 182 historical annual returns of each index. The Risk Free Rates include the observed month end interest rates of the BA rate and the 3 months US Treasury Rate for each period.

The research warrants the use of non-probability sampling of the financial values from the population, as we are interested in analysing and testing empirical data. In order to enable reliable long-term return and risk assessments we have used a sample period of 182 months. A convenience sample is utilised from February 1983 to January 1999 due to the ready availability of data for this period.

The data set is adequate to complete the research required and the writer does not contemplate expanding this data set. No further expense will be incurred to gather further data.

3.2. Research Design

The research design is an ex post facto exploratory study. No prior research on this topic could be found. The "ex post" approach to estimating returns assumes that investors expect future returns, on average, to equal past results. This methodology exposes the main limitation of the study in that we are using historical data to predict the future. The two critical theoretical models that guide the research design is the Markowitz theory of portfolio selection and the Capital Asset Pricing Model of William Sharpe.

The tests done assume a prior belief in the validity of the CAPM and tests are therefore based on the fit of the proxy to the linearity of the expected return beta relationship. The analysis begins with tests on the suitability of the market proxy in the CAPM model by using our index model equation.

$$E(r_i) - r_f = \alpha_i + \beta_i\left[E(r_m) - r_f\right] \qquad (3.1)$$

The equation turns out to be a simple regression equation. This relationship provides us with a testable hypothesis based on the index model. We note that the risk free rate in the equation is common to the left and right hand side of the equation and is therefore a constant variable, which will not influence the relationship. The assets premium over the risk free rate is treated as the dependant variable. The excess market return of the equation is the independent variable. The co-efficient of the independent variable turns out to be the beta or slope of the CAPM. The expected return on the asset can therefore be explained by its sensitivity coefficient (β) to the market index. If there were a weak relationship between the ALSI and the true market portfolio then the alpha term would turn out to be significantly different to zero.

The hypotheses to be tested are:

H_0: $\alpha = 0$ The ALSI is mean variance efficient

H_1: $\alpha \neq 0$ The ALSI is not mean variance efficient

And

H_0: $\beta = 0$ The asset returns are unrelated

H_1: $\beta \neq 0$ The asset returns are related

Using this methodology will allow us to solve for the ex post Beta and determine if the empirical beta does express a relationship with the market proxy. In the first instance using the ALSI as a proxy, we calculate the beta for the GOLDI and the INDI. The slope of the regression line is the ex post beta of the asset. Thereafter, using the Dow Jones as a market proxy we test the indices further. The Dow Jones test incorporates currency returns into total returns for the SA investor. The result will demonstrate the effect of international diversification and currency movements has in investment decision-making.

The results of the betas will be substituted into equation 3.2. below. From the results obtained, we will recommend the use of the index as a market proxy according to its ability to predict returns within the 95% confidence level of the mean return.

$$\lambda_i = \beta_i \lambda_m \tag{3.2.}$$

To determine how the results will compare from a US investor's point of view the analysis is repeated from a U.S point of view using dollar returns.

Once we have tested the ALSI, we will discuss the implications for an asset universe of four assets in terms of portfolio optimisation tools and compare the results of the methodologies. We assume that initially the SA investor is allowed to invest in SA only and is limited to a three-asset universe consisting of a market proxy and two major sub sectors, the gold sector and the industrial sector. This allows us to determine the benefits if any of holding a passive portfolio of all SA shares or diversifying into more efficient individual sector assets. At a later stage, a fourth asset representing international stocks will be introduced. A constraint of 15% is placed initially on the international stocks weighting and is later lifted. The ALSI will represent the local market portfolio. The INDI and the GOLDI will represent proxies for the industrial and mining assets of the country and the Dow Jones represents the portfolio of foreign shares.

The use of index returns for such diverse sectors and the broad ALSI index provides a useful opportunity to determine the effects of the sectors on an optimal portfolio and whether passive strategies are applicable in the SA investment environment. The rand adjusted Dow Jones applicable to South African shareholders assumes that the investor buys dollars and the Dow Jones as a portfolio and then sells the portfolio and dollars for Rands at the end of the period.

Our analysis will conclude with the construction of the efficient frontier for differing scenarios. These would include local investment only, 15% foreign investment limit and no foreign investment limit. We will determine graphically and mathematically whether the ALSI plots on the efficient frontier. Mathematically the market portfolio is an inefficient portfolio in this asset universe if there exists a higher Sharpe ratio than the market's ratio. A higher Sharpe ratio than the market portfolio implies a positive CAPM-measured alpha i.e. a return in excess of the fair return predicted by the CAPM. Individual assets are therefore able to outperform their expected return.

In order to accomplish this task the author develops a spreadsheet model in Microsoft Excel based on the guidelines of Bodie, Kane and Marcus on how to create an efficiency frontier. This dual framework of both data analysis and CAPM testing will help us determine how well the CAPM results stand up to optimisation techniques in portfolio selection.

The monthly arithmetic mean return is calculated for the ALSI, Industrial Index (INDI), Gold index (GOLDI) and Dow Jones Industrial (DJI). The Dow Jones was used to reflect international diversification because it is based on companies on the New York Stock Exchange, which is one of the largest stock market in the world in terms of market capitalisation. Because the Dow Jones returns are dollar based, we controlled for the movement of exchange rates by multiplying the index values by the exchange rate on the same date before calculating mean returns. This provided us with Rand returns on the DJI. The BA rate and U.S. 3 month Treasury rate were chosen as risk free rates. We derive an input list for the South African market premium by finding the difference between the ALSI and the BA rate. Similar asset premiums are derived for the INDI and GOLDI.

The mean and standard deviation of each index are found from a statistics table derived by a statistical analysis program. Using the program, we will further create a correlation and covariance matrix for our indices. The matrix method of calculating portfolio expected return and variance is a powerful method that avoids long equations and makes easy spreadsheet analysis. A 4x4 matrix including the ALSI, INDI, GOLDI, and DOWLOCAL will allow us to calculate 16 estimates of variances. The four diagonal amounts will show the assets variance σ_i^2. The rest of the coordinates results are covariance terms.

A bordered covariance matrix is created for a base case scenario of an equally weighted portfolio. The bordered matrix is calculated by multiplying the matrix coordinate weights by the corresponding covariance matrix coordinate. Portfolio variance will result from the sum of each column of the bordered covariance. Using the bordered covariance matrix for the equally weighted portfolio, we derive the portfolio mean and variance. The portfolio's expected return is the weighted sum of each assets return.

These results are denoted mathematically as:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i r_j)$$

In order to solve for the efficiency frontier the solver in Excel was used. To solve for points on the efficiency frontier we solved for the minimum portfolio variance for a range of portfolio mean inputs by only varying the portfolio weights. One of the constraints was that the weights of the portfolio had to sum to one. To reflect different investment scenarios the weight in the

DOWLOCAL was altered from zero, to less than 15% and finally unlimited to determine the frontier and optimal recommended portfolio in each case.

The range of points that resulted plots on the frontier since they represent the portfolio weights that will result in the least risk for the expected return. An XY scatter diagram was used to plot the points and a curved regression line of best fit was plotted through the points to graphically represent the efficiency frontier. The individual returns of the indices were plotted on the chart to show their positions in terms of the efficient range.

Adding an additional constraint of 15% maximum international investment creates a new efficient frontier and lifting the 15% constraint yet another larger frontier. The reasoning behind the constraint was to reflect the situation currently prevailing for institutional investors of their 15% limit for international investment and to illustrate the effects constraints on returns. By lifting the constraint and superimposing each efficient frontier, we can demonstrate the benefits of international diversification to SA investors. The results will be compared to tests using the CAPM on the rand adjusted Dow Jones to determine if the optimisation recommendations are upheld.

The next step in our analysis would be to introduce the risk free rate so that the Capital Asset line can be derived. The efficient portfolio is the determined by the portfolio weights of the point at which the CAL is tangent to the efficiency frontier. This point represents the reward to variability ratio with the highest value. To solve for this point we need to solve the equation that maximises the reward to variability ratio by allowing the weights of the portfolio to be altered. If the weightings for the optimal portfolio are not invested in the market proxy provides us with a first indication of inefficiency. The sum of the weights as a constraint should add up to 1.

If the tangency point is denoted as Sp then we solve for it as

$$\underset{w_i}{Max} S_p = \frac{E(r_p) - r_f}{\sigma_p} \quad (3.4.)$$

The return of the optimal points will then be compared to our ALSI expected return or mean to determine if the points coincide. Using the solver program developed above we will further test the ALSI point by trying to find an expected return other than the ALSI that has lower risk. If this is possible the ALSI is an inefficient portfolio since the Sharpe ratio which tests reward to volatility will not be the highest for the ALSI, indicating that the ALSI cannot be a tangency portfolio required by the CAPM. The CAPM will therefore have positive alpha values indicating a need to rebalance the portfolio.

By definition, no asset can plot above the efficiency frontier since the frontier represents the set of results that maximise portfolio returns. If a point is inefficient, it has to plot inside the frontier borders. The estimates of our portfolio selection analysis will be compared to the tests on the CAPM to determine how results of the CAPM differ from results achieved in our portfolio optimisation analysis. Our empirical analysis will compare the extent to which the optimal holding differs from the ALSI as the market portfolio. The analysis ignores the effects of transaction costs and taxes.

In terms of international mean variance efficiency, we need to test whether the ALSI is a mean variance efficient portfolio with respect to a world market portfolio using the dollar as the currency of the international portfolio. All constraints on investment weightings are lifted except that short sales are still not permitted. We can then create a new portfolio from the perspective of the international investor to determine whether SA equities are mean variance efficient in a global market. Tests of efficiency using the CAPM are then utilised to test the mean variance efficiency of the indices with regard to a US investor's return in SA utilising the Dow Jones Industrial Index as a proxy.

3.3. Data Analysis

A summary of the statistical analysis of all the sample indices is provided below.

Table 3.1 Descriptive Statistics

	ALSI	GOLDI	INDI	DOWLOCAL	DJINDI	BA	US3M
Mean $\overline{X}$	15.9228%	8.2404%	17.3104%	28.7049%	14.7664%	14.7043%	5.9077%
Standard Error	1.7672%	2.9695%	1.7308%	1.5994%	1.0523%	0.2538%	0.1323%
Median	12.1578%	7.9534%	14.2081%	26.9344%	14.2648%	14.8250%	5.6700%
Std. Deviation σ	23.8413%	40.0603%	23.3498%	21.5771%	14.1966%	3.4242%	1.7845%
Sample Variance	0.0568	0.1605	0.0545	0.0466	0.0202	0.0012	0.0003
Kurtosis	-0.6550	0.9391	-0.3160	-0.3171	-0.3175	-0.6051	-0.3635
Skewness	0.1264	0.8260	0.3794	0.2547	-0.0269	0.1251	0.3763
Range	1.0929	2.2313	1.0949	0.9769	0.7174	0.1391	0.0769
Minimum	-0.3624	-0.5229	-0.3481	-0.1587	-0.2302	0.0840	0.0275
Maximum	0.7305	1.7084	0.7468	0.8182	0.4872	0.2231	0.1044
Sum	28.9796	14.9975	31.5050	52.2430	26.8749	26.7619	10.7521
Count	182	182	182	182	182	182	182
Conf. Level 95%)	3.4870%	5.8592%	3.4152%	3.1559%	2.0764%	0.5008%	0.2610%

We note that the standard errors of our mean results are below 2% in all cases except for the GOLDI at 2.97%. It is noted from the standard deviation of the GOLDI that it is extremely volatile compared to the other indices. The GOLDI also has a high positive kurtosis compared to the other indices implying it is positively skewed compared to the other indices. The other indices are also slightly positive skewed but have a relatively normal distribution.

The return distribution of the ALSI appears normal and we can therefore predict that our mean estimates are reasonably accurate predictors of the actual population mean. Normal distribution satisfies the condition of parametric testing that the sample distribution should be approximately normally distributed. At the 95% confidence level gold has the widest range of the mean implying that is experiences the most volatility and is therefore also the riskiest.

Despite the gold sector, having the highest risk it has the lowest return and consequently the lowest market premium in on the JSE. The Dow Jones has the lowest mean range at the 95% confidence level showing that it is less volatile and is a safer investment than the SA alternatives. The Dow Jones is still the safest investment even after adjusting for currency risk and denominating it in Rands.

The Industrial and ALSI index have very close mean and standard deviation results and taking standard error into account the mean of the ALSI is expected to be between 12.44% and 19.41%. The INDI mean is expected to be between 13.90% and 20.73%. The area between 13.9% and 19.41% is common to both indices. Results of the CAPM would be inconclusive as to which portfolio is mean variance efficient in this region as the standard error is common in this area. The standard deviations are similar implying that their Capital Allocation Lines will have similar slopes.

Currency effects on portfolios introduce new volatility into risk analysis. The table that follows demonstrates how returns expressed in dollars differ from the portfolio retur above. Introducing the currency effect results in a downward adjustment for all SA equity returns because the mean return of the dollar is expected to be a positive 12,6% per annum.

Table 3.2 Dollar Adjusted Statistics

	ALSI$	GOLDI$	INDI$	DOWJONES	SRETURN
Mean	5.6619%	-1.9285%	7.3602%	14.7664%	12.6033%
Standard Error	2.1849%	3.0251%	2.2366%	1.0523%	1.2468%
Median	0.7676%	-11.7166%	6.0700%	14.2648%	9.8399%
Mode	#N/A	#N/A	#N/A	#N/A	#N/A
Standard Deviation	29.4764%	40.8102%	30.1738%	14.1966%	16.8205%
Sample Variance	0.0869	0.1665	0.0910	0.0202	0.0283
Kurtosis	0.9340	1.0775	1.2210	-0.3175	1.5991
Skewness	0.8221	1.1891	0.6914	-0.0269	1.0738
Range	1.5231	1.9997	1.6821	0.7174	0.9433
Minimum	-0.5073	-0.5673	-0.5233	-0.2302	-20.2692%
Maximum	1.0158	1.4324	1.1587	0.4872	74.0563%
Sum	10.3047	-3.5098	13.3956	26.8749	22.938069
Count	182	182	182	182	182
Confidence Level(95.0%)	4.3112%	5.9689%	4.4132%	2.0764%	2.4602%

The Rand is therefore expected to consistently devaluate against the dollar implying that for a US investor returns would have to consider the effect of this consistent depreciation of their dollar returns on SA equity in their investment decisions. An interesting point is that the range of the ALSI$ and INDI$ has increased compared to the rand results but the range of gold decreases. This demonstrates an increased risk in industrial shares but a decreased risk in gold. The increased risk of the ALSI and INDI is shown in the larger standard deviations of the dollar denominated table. GOLDI in dollars does not alter significantly in dollars. The mean return of gold over the sample period in dollars is negative due to currency depreciation. We will examine these implications for portfolios in detail in the results section.

Chapter 4

4. RESULTS

The Johannesburg Stock Exchange and in fact the South African economy consists of two major components. These are the industrial sector including services, and mining sector. In the mining sector the gold mining shares; are the most prominently quoted due to their link to international gold markets? We will therefore use these two broad asset classes to represent local assets and test the ALSI on each in turn. Where we find that the ALSI is not an adequate proxy we will use the Dow Jones to determine whether the sector return is related more to international market movements extraneous to the South African market.

(a) Null Hypothesis

H_0: $\alpha = 0$ The ALSI is mean variance efficient

H_1: $\alpha \neq 0$ The ALSI is not mean variance efficient

(b) Statistical Test

We will use two parametric tests, the *t* test and the *F* test to analyse the results. From the analysis of data, the sample is normally distributed thus satisfying the assumption of the *t* test. The data are ratio measurements of return. The *F* test is also incorporated to reinforce our results. The *F* test is suitable as our sample includes interval data for asset returns on the indices.

(c) Significance Level

Let significance level be .05 with n = 182 for the *t* test

Let significance level be .05 with d.f. = (1, 180) for the *F* test.

(d) Critical Test Values

For 180 degrees of freedom and significance level of .05, the critical value is 1.96 for the *t* test.

With degrees of freedom (1, 180) and significance level .05 the critical value is 3.84 for the *F* test. These critical values will be common to all the regression tests that follow.

4.1. Tests of the ALSI Using the Gold Index

Using the BA rate as the risk free rate in each case, we regress the excess returns of the gold index (GOLDX) over the excess returns on the market proxy (ALSIX). The results are summarised below

Table 4.1. Regression Analysis of Gold index on the ALSI

SUMMARY OUTPUT

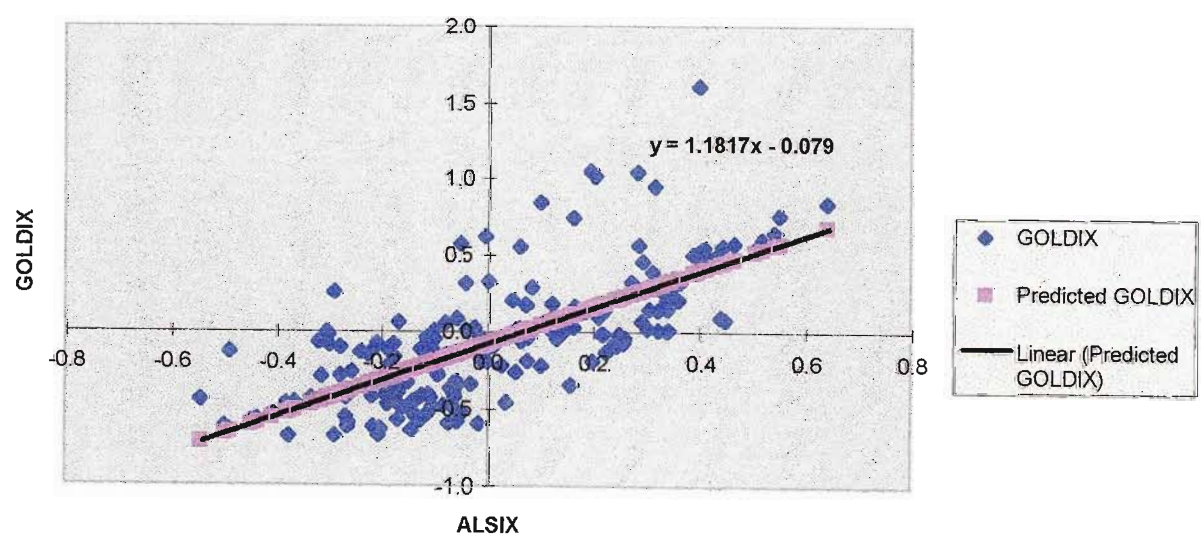
Regression Statistics	
Multiple R	0.719759
R Square	0.518052
Adjusted R Square	0.515375
Standard Error	0.286012
Observations	182

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	15.82762	15.82762	193.4846	2.42275E-30
Residual	180	14.72454	0.081803		
Total	181	30.55216			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept – α	-0.07904	0.021226	-3.72368	0.000262
ALSIX	1.181673	0.084952	13.90987	2.42E-30

Fig.4.1 ALSIX Line Fit Plot



The regression equation that results is $y = -7.9\% + 1.1817x$. Stated in terms of the index model equation we treat the excess return on GOLDI as the dependant variable and the excess ALSI return as the independent variable.

$$E(r_{goldi}) - r_f = \alpha_{goldi} + \beta_{goldi}\left[E(r_{alsi}) - r_f\right]$$

The null hypothesis can be rejected if the constant alpha term (-7.9%) of the regression can be proven statistically different to the zero mean alpha demanded by the CAPM. The calculated t value 3.72 of the regression intercept is greater than the critical value 1.96. Further, the intercept's P value of 0.000262 is smaller than the chosen significance level of .05.

We therefore reject the null hypothesis that alpha is equal to zero and accept the alternate hypothesis. Our conclusion is that the ALSI is mean variance inefficient. It is therefore not an adequate proxy of the market portfolio to use in the CAPM to determine fair return of the gold sector. The negative value of alpha would imply that the gold sector is consistently under performing the ALSI index. Gold shares should therefore not be held by the SA investor and should be actively sold.

The correlation coefficient (r) also known as Pearson's product moment is labelled Multiple R (0.719) on the statistics table and the R Square measure is 0.518. The correlation coefficient is an important measure of the direction and intensity of movements between assets and is a vital tool used in the portfolio model to achieve diversification. The coefficient scale moves between −1 and +1. Assets that move in an exactly opposite direction are perfectly negatively correlated and marked −1. Similarly, assets that move in exact tandem are +1 with coefficients in between reflecting less perfect co movements. It also signifies to us how straight the regression line is that we are testing. The R Square measure which is simply a higher moment (r^2) of the

Pearsons moment (r) is known as the coefficient of determination. It signifies how much of the variance of the dependant variable can be explained by variances in the independent variables.

The relationship between the GOLDI and the ALSI excess returns shows that even though there is a 71.9% correlation between the two indices, the movements of the ALSI only explain 51.8% of the relationship of the GOLDI according to the R square measure. This signifies the existence of other variables that are responsible for explaining at least half the gold share movements. We therefore test the hypothesis of the relationship between the GOLDI and the ALSI by examining their beta coefficients

Hypothesis

H_0:	$\beta = 0$	There is no market risk between the ALSI and GOLDI
H_1:	$\beta \neq 0$	Market risk exists between the ALSI and the GOLDI

The critical values at the 95 % confidence level are retained in this test. The calculated *t* value of the ALSI (13.91) is higher than the critical value. Using the *F* test, the calculated *F* value of 193.49 is greater than the 3.84 critical value. Both the *F* test and *t* test reject the null hypothesis. There is therefore a strong relationship between the GOLDI and the ALSI. Our results appear to be contradictory. Whereas on the one hand we find the ALSI an inadequate proxy, the relationship is still relevant. The beta of the regression equation (1.18) implies that the volatility of the gold sector is higher than the overall SA share market and that gold shares have higher systematic risk than the ALSI. Our R square measure however shows that the ALSI only explains about 52% of the GOLDI. The standard error of the R-square value is unacceptably high at 28.9% and the dispersion from the mean R-square indicates it is not a reliable estimate. We therefore need to identify other possible important variables that can explain its variance.

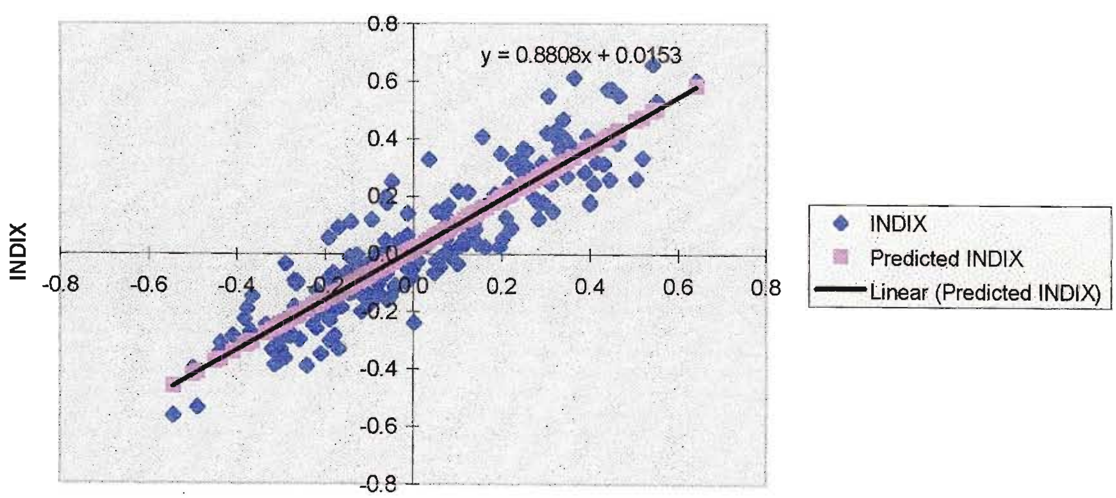
4.2. Tests on the ALSI using the Industrial Index

Table 4.2. Regression Analysis of the Industrial Index on the ALSI
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.896452
R Square	0.803627
Adjusted R Square	0.802536
Standard Error	0.109264
Observations	182

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept – α	0.015328	0.008109	1.890269	0.060329
ALSIX	0.880827	0.032454	27.14081	1.58E-65

Fig. 4.2 ALSIX Line Fit Plot



In the case of the Industrial index excess returns the linear equation resulting from our regression is $y=1.5\% + 0.88x$. Testing the hypothesis of the alpha value, we find that the calculated t value is less than the critical 1.96 value. The P-value of .06 is also greater than the chosen .05 significance level.

In the case of the Industrial index, we fail to reject the null hypothesis. We conclude that the ALSI index is mean variance efficient in the case of the industrial index thus proving to be an adequate proxy for use in CAPM analysis. Further, the industrial index is highly correlated with movements in the ALSI (0.896). The R-square measure of 80,36% demonstrates that there is a strong explanatory relationship between the ALSI and the INDI. The standard error of the R-square measure at 10.9% is much lower than that of the Gold index and considering the high R-square value is acceptable.

The implications for portfolio management is that it would be more efficient to hold the market portfolio represented by the ALSI than invest in the INDI. Passive portfolio management is optimal. We are however cautious of these results as the P-value of alpha is only .01 above the .05 significance level and is close to the rejection point. Further the results of the gold sector advocated active portfolio management due to the presence of negative abnormal alpha presence for the SA investor. The ALSI and INDI have an 80% correlation coefficient and the ALSI R square value implies it explains 80% of the INDI returns. Taking into account that we are using historical data and that a different sampling period might produce different results, we accept these results with a degree of caution.

4.3. Relationship between the Industrial and Gold Index

Table 4.3. Regression Analysis between the Industrial and Gold Indices

Statistics	GOLDIX as Explanatory Variable			
Multiple R	0.450868			
R Square	0.203282			
Adjusted R Square	0.198856			
Standard Error	0.220085			
Observations	182			
	Coefficients	Standard Error	T Stat	P-value
Intercept	0.043503	0.016516	2.634068	0.009172
GOLDIX	0.269838	0.039817	6.776944	1.69E-10
SUMMARY OUTPUT	INDIX As Explanatory Variable			
Statistics				
Multiple R	0.450868			
R Square	0.203282			
Adjusted R Square	0.198856			
Standard Error	0.367737			
Observations	182			
	Coefficients	Standard Error	t Stat	P-value
Intercept - α	-0.08427	0.027412	-3.0743	0.002439
INDIX	0.75335	0.111164	6.776944	1.69E-10

The above regression tests the relationship of the Industrial Index excess return (INDIX) onto the Gold Index excess return (GOLDIX) and vice versa. We wish to determine the risk characteristics of each index in relation to the other.

The P-value is stated in compact form for very low values by the statistics program. Therefore 1.69E-10 is defined, as a decimal number with nine zero decimal points before the one. It signifies a very low decimal value. Since the P-value is less than the .05 significance level, we can reject the hypothesis of beta being equal to zero. The gold and industrial sectors are interrelated.

In the first case where the GOLDIX is used as the explanatory variable the excess return equation is $y=0.0435+0.2698x$. In order for the excess return formula to be linear, we know that alpha should be approximately zero. The P value of alpha 0.009 is less than .05 implying that we can reject alpha being equal to zero. The P-value of beta is also less than .05. Beta is also greater than zero. The positive alpha value implies that the returns of the Industrial index are superior to the Gold index over a period. SA Portfolios should hold Industrial shares instead of Gold shares as these provide consistently higher returns than gold for lower risk.

Using the INDIX as the explanatory variable the equation is $y= - 0.0843+0.7534x$. The alpha's P-value of 0.0024 suggests that the alpha of GOLDIX in relation to the INDIX is a negative non zero alpha. This suggests that it under performs the INDIX consistently for an SA investor. The results confirm that SA investors should hold industrial and not gold shares. The examination of the R-square values indicates that even though statistically the indices have a 20,3% coefficient of determination this is subject to high standard errors. This implies that the dispersion of the range is too wide to have confidence in the R-square estimates. In the case of the GOLDIX as the explanatory variable the standard error of R-square is 22% and using the INDIX the standard error climbs to 36.7%. This can only point to the fact that the two indices are independent variables that operates in a segmented manner on the JSE. This confirms the research results of Bradfield and Van Rensburg on the dichotomy on the JSE. There is therefore segmented movement of returns on the JSE with neither the Industrial sector having any meaningful explanatory relationship towards the Gold sector or vice versa.

The segmented nature of the JSE would imply that the gold and industrial sectors would have different relationships to international markets like the Dow Jones. We therefore test the relationship of the indices to the rand adjusted Dow Jones. We test the returns in terms of the excess return relationship of the index model. The Dow Jones return in Rands represents the market proxy to determine its efficiency and the implications this would have for the SA investors' portfolio.

4.4. The Relationship of the ALSI to the Dow Jones

In order to determine the relationship of international indices from the view of the SA investor we will examine the actual price movements in Rand terms of the indices. The relationship of returns without currency fluctuations will be examined later. From a South African investors perspective, we need to adjust the Dow Jones by the relevant exchange rate prevalent on the same day to determine Rand returns to SA investors. A uniform return currency is required to test the effective returns using the CAPM. Exchange rate movements are a component of returns generated from overseas investment and need to be incorporated to test the expected return of international investment. A Rand adjusted Dow Jones excess return will determine the effects of international markets on SA shares. This will assist us in making decisions on the mean variance efficiency of the Dow Jones to SA investors and create implications for portfolio asset allocation.

The risk free rate for an international investment will be the risk free rate in the investor's home currency if his portfolio returns are adjusted to his home currency. This domestic risk free rate excludes currency risk. The investor therefore needs to be rewarded for risk of both the portfolio and the currency to accept the international investment. The risk free rate of an investor must satisfy the condition of being reasonably predictable and stable from the investor's point of view. The test of a risk free rate therefore should be to determine the safe haven status as seen from the investor's perspective. To determine the risk premium of an investment in the US for an SA investor we therefore rand adjust the returns and subtract the

BA rate to determine the true risk premium to the SA investor. It is argued that using the US 3 month Treasury Bill to determine a risk premium is flawed since from the South African investors perspective this rate of return would be subject to the volatility of the rand / dollar exchange rate. The return of an overseas risk free instrument is therefore risky for SA investors because it is unpredictable in rand terms and does not satisfy the criteria of a safe investment.

Fig 4.3 DOWXRAN Line Fit Plot

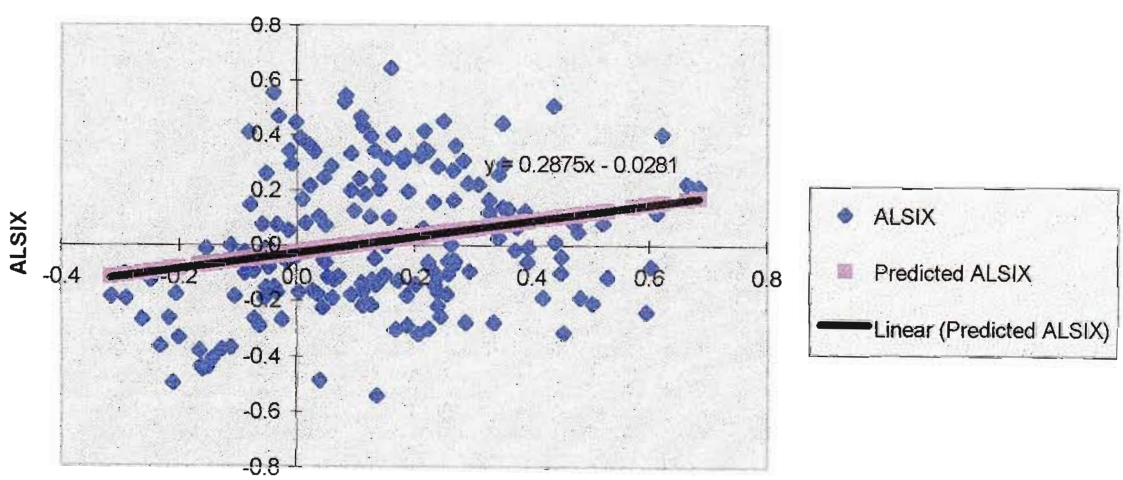


Table 4.4. Regression of ALSI on Rand Adjusted Dow Jones

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.237913
R Square	0.056602
Adjusted R Square	0.051361
Standard Error	0.243737
Observations	182

	Coefficients	*Standard Error*	*T Stat*	*P-value*
Intercept – α	-0.02807	0.021827	-1.28581	0.200161
DOWXRAN	0.287494	0.087483	3.286294	0.00122

The regression is done from a South African investors point of view of expected returns of the indices. The regression is modelled on the excess return of indices implying that the Dow Jones is Rand adjusted before being used in the regression. The variable is termed the DOWXRAN.

The regression equation states the relationship as $y = -0.02807 + 0.2875x$. The R-square measure indicates that variation in the Dow Jones only explains 5.7% of the ALSI returns showing that it is not a significant variable. The beta's P-value of 0.001 indicates that we can reject the null hypothesis of beta being equal to zero. There is therefore a relationship of the Dow Jones risk to the ALSI.

Furthermore using the ALSI as a test, we find that the P-value of the intercept (0.20) is greater than .05 implying that we fail to reject the hypothesis that alpha is equal to zero. The Dow is therefore a mean variance efficient proxy of the market portfolio for an SA investor. An important conclusion from this test is that using the Dow Jones as the market proxy, SA investors are better off holding a passive index portfolio of the Dow Jones than holding a broad domestic index like the ALSI. The international market proxy is more efficient than the local market proxy.

4.5. The Relationship of the SA Gold Index to the Dow Jones

Fig. 4.4 DOWXRAN Line Fit Plot

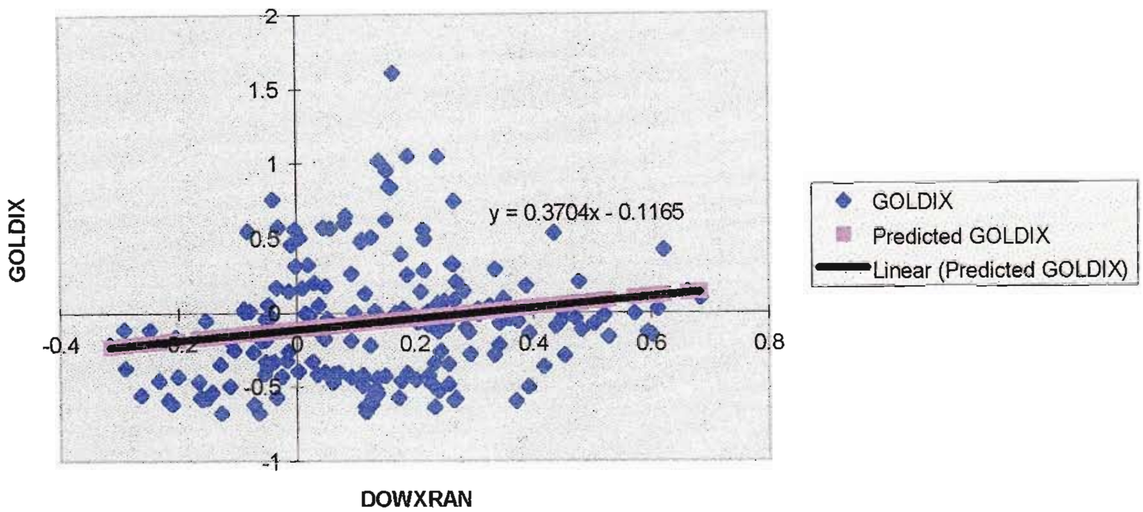


Table 4.5 Regression of Gold Index on Rand Adjusted Dow Jones
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.186693
R Square	0.034854
Adjusted R Square	0.029492
Standard Error	0.404745
Observations	182

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept – α	-0.1165	0.036246	-3.21402	0.001551
DOWXRAN	0.370382	0.145272	2.549578	0.011619

To determine the above relationship the excess Dow Jones in Rands return over the BA as a risk free rate (DOWXRAN) is regressed over the excess gold index returns (GOLDIX).

The regression relationship from the empirical data is stated as $y = - -0.1165 + 0.370x$.

The intercept of the GOLDI has a P value of 0.001. Since this is less than the chosen confidence level of .05 we reject the null hypothesis that the alpha of the regression is equal to zero. Consequently, the Dow Jones is also not an adequate proxy to use for gold shares. A similar result was achieved for the ALSI implying that both market proxies are inadequate in explaining gold returns. We test this relationship intuitively to determine if perhaps international markets have a better explanatory relationship of the gold sectors market risk than the local market. Our results thus far show that the ALSI and the DOWLOCAL are not suited to CAPM analysis of the GOLDI in SA.

The conclusion reached is that gold shares are not dependant on market risk of either the SA ALSI index or the international market risk proxied by the rand adjusted Dow Jones. Both display a non-linear relation to gold shares. We therefore need to determine from a multiple regression analysis similar to the Arbitrage Pricing Theory (APT) multifactor model, the determinants of gold share performance.

The R square measure of 0.0349 shows that variance in the DOWXRAN adjusted in Rand terms has no explanatory significance to the movement of the South African Gold Index. The P value of 0.012 for the Dow's beta signifies that we can reject the null hypothesis that $\beta = 0$ as it is less than .05. There is a relationship of the GOLDI to the adjusted Dow Jones albeit a significantly reduced one compared to the GOLDI and the ALSI. The value of 0.37 is significantly less than the 1.18 achieved against the ALSI. The test against the DOWLOCAL shows a reduced relation of market risk evident in the reduced beta. Correlation is further reduced from 0.72 with the ALSI proxy to 0.19 with the DOWLOCAL proxy.

From the definition of beta in the CAPM, we know that beta is a sensitivity coefficient that measures the sensitivity of an asset to the market proxy. This would provide us with information as to the relationship of gold to international market risk. The beta of the rand adjusted Dow Jones is approximately zero implying that it does not share any systematic risk with gold shares. The sign of the alpha has been shown to be a non-zero negative according to the equation implying that the Dow Jones is the preferred investment as the returns to an SA investor of gold shares is expected to on average under perform the Dow.

4.6. The Relationship of the Dow Jones to the Industrial Index

Fig. 4.5 DOWXRAN Line Fit Plot

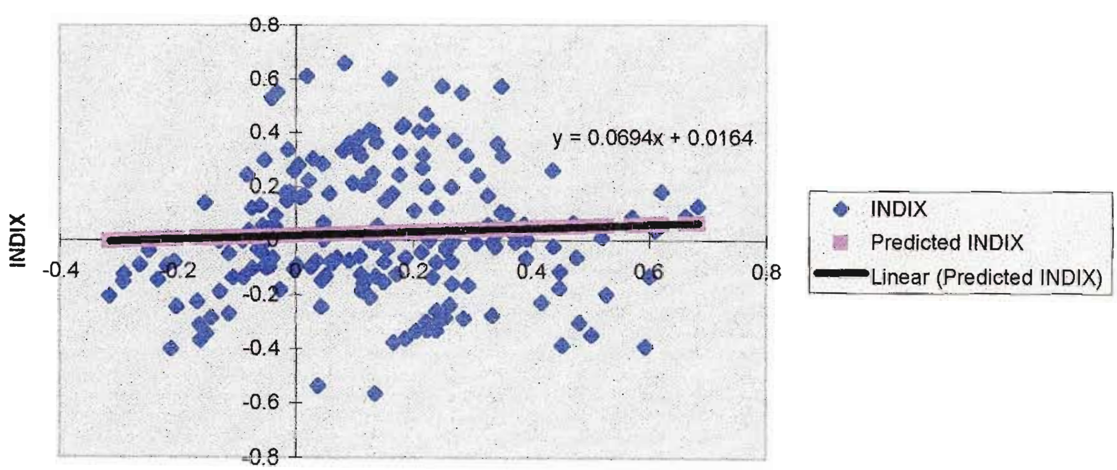


Table 4.6. Regression of Industrial Index on Rand Adjusted Dow Jones
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.058414
R Square	0.003412
Adjusted R Square	-0.00212
Standard Error	0.246147
Observations	182

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept – α	0.01635	0.022043	0.741746	0.459208
DOWXRAN	0.069357	0.088348	0.785045	0.433459

The R-square value is extremely small and has a high standard error range at the 95% confidence level, signifying that there is no explanatory effect of the Dow Jones variance in Rands to the variances in the industrial index. The P value of beta (0.4335) is greater than .05. The null hypothesis that beta is zero cannot be rejected. This is evidence of the two indices independence. Their risks are therefore not related providing an opportunity for diversification.

The P value of the intercept (0.4592) is also greater than .05 implying that we cannot reject the null hypothesis of alpha equal to zero. The Rand adjusted Dow Jones in relation to the Industrial index displays efficiency. Both the ALSI and the Industrial Index show that the Dow Jones is mean variance efficient. The rand adjusted Dow Jones displays linearity that satisfies the CAPM.

We have determined thus far that gold has a negative alpha against the Dow. The Dow is also the mean variance efficient portfolio using both the ALSI and the INDI. In a portfolio consisting of a universe of the three SA indices and the Dow Jones, the Dow Jones provides the best return to investors.

4.7. The Multifactor Gold Model

Table 4.7 Gold Index Regression on Gold Price
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.529975
R Square	0.280874
Adjusted R Square	0.276879
Standard Error	0.349371
Observations	182

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	8.5813	8.5813	70.30377	1.43E-14
Residual	180	21.97086	0.12206		
Total	181	30.55216			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept – α	-0.21835	0.031729	-6.88169	9.47E-11
GOLDPR	1.580503	0.188498	8.384734	1.43E-14

We noted previously that the price movements of the Gold index could not be adequately explained by the ALSI index or a rand adjusted Dow Jones. We therefore test the relationship of the Gold index as a function of movements in price of the Rand price of gold as traded in international precious metal markets.

The sample determination coefficient of the regression, R Square, is estimated at 28.09%. The P-value of beta is significantly smaller than .05 indicating that we can reject the hypothesis that beta is equal to zero. The regression equation is therefore valid.

The test tells us that there are two major variables playing a role in the determination of the variance in the price of gold shares. The one variable is the ALSI itself and the other variable that is external to the South African market is the dollar price of gold. The price of gold is not determined by South African market conditions but rather through international trade in precious metal markets. South Africa is a price taker in this case. Its supply is price inelastic. This would imply that the prices of South African gold mining shares are dependent on two different variables. Both the ALSI and the Dow Jones are therefore inefficient proxies for the CAPM in the case of gold because they fail to capture the effect of gold price fluctuations.

In order to determine the exact regressive relationship we test the two factors of market risk from the ALSI and the price of gold on the performance of gold shares. The price of gold is adjusted to Rands and then expressed as an excess return over the risk free rate in a multi factor APT regression model. Partial t tests will be utilised to test the significance of the relationship.

This is stated mathematically as:

$$E(r_{GOLDI}) = \alpha + \beta_{ALSI}[E(r_{ALSI}) - r_f] + \beta_{GOLDP}[E(r_{GOLDP}) - r_f]$$

The result of the multiple regression is:

Table 4.8. Gold Multifactor Model
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.833179
R Square	0.694187
Adjusted R Square	0.69077
Standard Error	0.228467
Observations	182

ANOVA

	Df	*SS*	*MS*	*F*	*Significance F*
Regression	2	21.2089	10.60445	203.1622	8.88E-47
Residual	179	9.343256	0.052197		
Total	181	30.55216			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept - α	-0.01441	0.018111	-0.79578	0.427216
ALSIX	0.99549	0.070293	14.16191	4.97E-31
GOLDPX	1.2524	0.123345	10.15361	2.04E-19

The equation is $y = -0,01 + 0.996x_1 + 1.252x_2$

The partial regression coefficients of the ALSIX and the GOLDPX are smaller than .05 so we can reject the null hypothesis that the betas coefficients are equal to zero. The F statistic is also less than .05 signifying that all the corresponding independent variables have coefficients that are different from zero. The relationship is therefore statistically significant

Previously in the ALSI CAPM test we had a negative alpha of -7,9% that resulted in us rejecting the use of the ALSI as a market proxy for the GOLDI. In the multiple regression the P-value of alpha is 0.42 implying that we cannot reject the hypothesis of alpha being zero. The multifactor model with the price of gold shares and market risk displays no abnormal returns. Alpha averages to zero over time. The multiple regression therefore has a much tighter fit and is more appropriate for gold shares than the CAPM. The R-square measure increases to close to 70% using the two-factor model. This is a marked improvement on the R-square value of 51.8% in the CAPM model using the systematic risk of the market index alone.

It should be noted that the multiple regression estimates a beta of almost one for the excess return on the ALSI index. The model implies that the ALSI contributes all of its market risk to the expected return on gold shares. This suggests that the returns of gold shares are determined by a base return of the entire market index or ALSI, plus a further return for additional risk of gold price fluctuations. It is expected that we would achieve similar results for all commodities or precious metals whose earnings where dependent on a market price set in a highly inelastic price environment where such an environment bears no relationship to the market risk of the stock exchange. For instance, the dollar price of gold will not be affected by increases or decreases in South African gold supply significantly. The gold price is set on international gold markets that bear no relationship to the JSE itself. They however do play a significant role in the earnings of shares that sell their gold at the price level set on the international commodity market.

We conclude that a single factor model like the CAPM is inappropriate for use in the case of an asset like gold where the returns are dependant on more than a single market. Gold will have systematic risk related to the ALSI and to the Rand price of gold.

4.8. The Efficient Frontier of the South African Investor

Two investment philosophies are dominant in investment management. They are active or passive portfolio management. Passive portfolio management presupposes as in the CAPM that markets are efficient. Assets are therefore priced to provide a fair return. There is therefore no possibility of earning superior returns by incurring costs of research and trading to discover positive alphas or abnormal returns. The most efficient and cost effective method of portfolio building is to hold a portfolio that mimics a broad based stock index like the ALSI. There is minimal trading except to rebalance the portfolio so that returns are not diminished by costly research, administration and transaction costs that eat into the returns. This has sparked a relatively large number of investment funds that emulate particular indices termed index funds

The alternate philosophy is to assume that markets are not always efficient. They may overreact both up and down. The strategy of such a portfolio manager would be to invest in research staff etc. to discover assets that are under priced i.e. they are expected to have positive alphas and actively trade to buy shares that display this attribute. The belief is that the abnormal gain from these positive alpha shares will exceed the cost of active trade and administration costs. The efficiency of the ALSI index would have implications for investors' if holding the index portfolio were the optimal portfolio. If not, it would imply that weightings can be rebalanced to achieve superior returns to that of the passive index portfolio.

Now that we have determined the efficiency of the ALSI in terms of various indices, we need to more importantly examine the implications for investors in portfolio construction. To accomplish this we need to apply the Markowitz portfolio selection model

To create the efficient frontier the starting point is to use the means and standard deviations of the sample data contained in table 3.1 as estimates of expected return and risk of the relevant assets. The returns are expressed in Rand terms to firstly determine the SA investor's perspective of the SA market.

Table 4.9. Mean Standard Deviation Table of Indices

	ALSI	GOLDI	INDI	DOWLOCAL
Mean	15.9228%	8.2404%	17.3104%	28.7049%
Standard Deviation	23.8413%	40.0603%	23.3498%	21.5771%

The DOWLOCAL is the average return that would have been achieved by a South African investor taking into account the gains achieved through Rand depreciation. The standard deviation of an asset being a measure of the volatility or risk of an asset shows us some immediate implications for investment risk. Gold for instance has the highest volatility and the lowest expected return. The ALSI and the INDI exhibit similar risk return attributes. This isn't unexpected since the INDI is really a major sub index of the ALSI for the industrial sector. Interestingly though the INDI does have a slightly higher expected return for a lower risk.

Portfolio analysis however, teaches us that evaluation of single assets is inefficient. What is more important is how the assets correlate with other assets over time. This is where the diversification benefit of portfolios arises. Even if assets are positively correlated, they can still provide a diversification benefit as long as they are not perfectly positively correlated or have a correlation coefficient equal to 1.

Table 4.10. Correlation Matrix for SA Investor

Correlation Matrix	ALSI	GOLDI	INDI	DOWLOCAL
ALSI	1.0000			
GOLDI	0.7010	1.0000		
INDI	0.8856	0.4107	1.0000	
DOWLOCAL	0.2191	0.1569	0.0365	1.0000

From the analysis of historic returns, the following correlation matrix provides an indication of the movements of indices in relation to each other. The GOLDI and the INDI have high positive correlations with the ALSI as expected since they are sub indices of the ALSI on the JSE.

The correlation of the ALSI with the DOWLOCAL is only 0.22 which presents opportunities for risk reducing diversification. However, it must be remembered that there is a limit to how much can be invested overseas in the present SA environment so we may not be able to capture the full benefit of international diversification. The INDI and the DOWLOCAL have the lowest correlation coefficient of all the assets included in the portfolio. The low correlation and high returns of the DOWLOCAL and the INDI point towards them being favoured assets in the final portfolio weightings.

To determine the efficient frontier lends itself to multiple regression analysis where the expected portfolio return is the weighted sum of the component asset returns. However, the weights we require need to make up the set of points that plot on the efficient frontier. Therefore, the points on the regression equation need to solve for a given portfolio return, whose weights result in the portfolio of least risk.

Using the previous two tables, we are able to create a covariance matrix. Each coordinate result is the product of the coordinate assets standard deviation and correlation coefficient e.g. the covariance of the ALSI with GOLDI can be stated as

$$Cov(r_{ALSI}, r_{GOLDI}) = \sigma_{ALSI}\sigma_{GOLDI}\rho_{ALSI,GOLDI}$$

Table 4.11 Covariance Matrix for SA Investor

Covariance Matrix	ALSI	GOLDI	INDI	DOWLOCAL
ALSI	0.0565			
GOLDI	0.0666	0.1596		
INDI	0.0490	0.0382	0.0542	
DOWLOCAL	0.0112	0.0135	0.0018	0.0463

For the covariance terms above a base case scenario is set for a bordered covariance matrix containing an equally weighted portfolio. The first investment scenario will focus on SA investors being limited to the local market. This will give us a chance to analyse the characteristics faced by local investors in the SA market while exchange controls were in place. The matrix therefore excludes the Dow Jones index from the analysis for the time being. The portfolio variance is calculated by summing the total at the bottom of each column.

Table 4.12 Base Case of Equally Weighted Portfolio for SA Investment Only

		ALSI	GOLDI	INDI
	Weights	**0.3333**	**0.3333**	**0.3333**
ALSI	**0.3333**	0.0063	0.0074	0.0054
GOLDI	**0.3333**	0.0074	0.0177	0.0042
INDI	**0.3333**	0.0054	0.0042	0.0060
	1.0000	0.0191	0.0294	0.0157

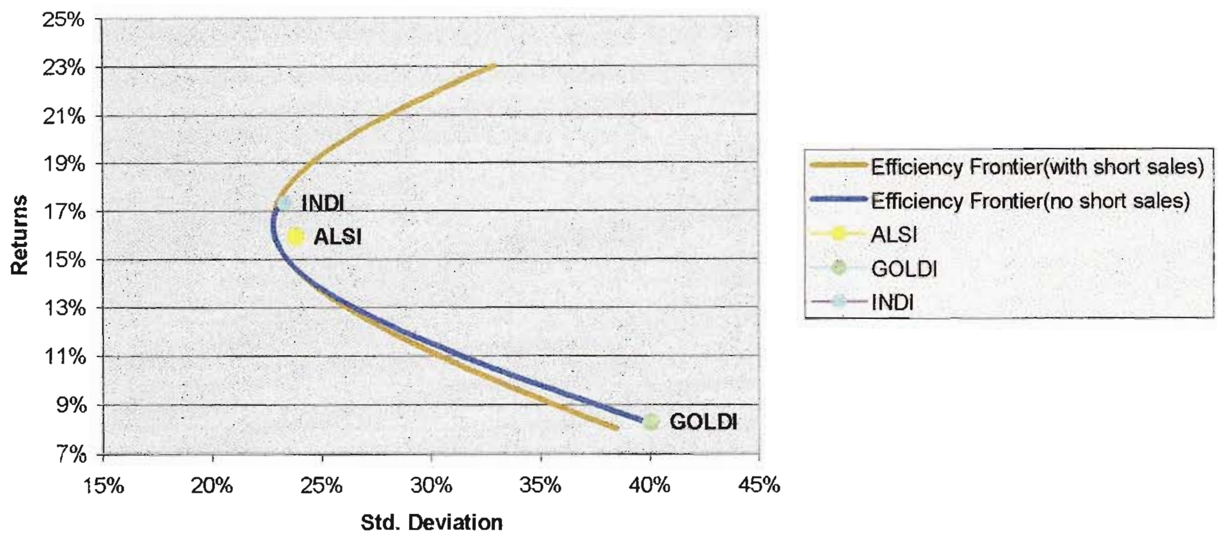
Portfolio Variance	0.0642
Portfolio Standard Deviation	25.3421%
Portfolio Mean	13.8246%

There are two types of frontiers. The unrestricted efficiency frontier allows the free movement of capital between assets with no constraints on what percentage can be invested. Short selling of assets is possible and utilised in the creation of the frontier. Arbitrage Pricing Theory states that short selling is possible when an asset is inefficiently priced. This creates an opportunity for a zero cost arbitrage by selling the inefficiently priced asset short to use the proceeds in an alternative superior return investment.

The second or restricted efficiency frontier adds constraints like no short selling to the equation thus resulting in a frontier positioned within the unrestricted frontier. This implies that a restricted efficient frontier incurs a penalty for its restrictions. The restricted frontier results in a sub optimal risk minimisation portfolio that decreases the Sharpe ratio for points on it compared to an unrestricted frontier. We will see the impact of lifting restrictions in the South African environment and how this impacts the frontier but for the time being we need to determine the optimal portfolio in an environment of no foreign investment, as has been the case in SA for some time now.

Using the inputs above and an optimisation program we solve for the weights that minimise the portfolio variance by altering the weights only, for a set of expected returns. The only restrictions placed are that the sum of the weights should add up to 1 and that no short selling is allowed. This reflects the present situation with brokers. Brokers will not allow buy orders to be placed by an investor unless it is verified that the investor has sufficient cash in the JSE trustee account to execute the order. Further, sell orders cannot be placed for stock not registered in the investors name and being held in the custody of the broker. (Source: Tradek.com). The resulting XY points join to form the blue frontier on the graph below: For comparative purposes an unrestricted frontier in orange is superimposed onto the restricted frontier to demonstrate the effects of short selling on the portfolio feasibility set.

Fig. 4.6 SA Efficiency Frontiers



What is evident from the two frontiers is that the South African restricted efficient frontier is not the typical curve we would expect from a normal frontier. The restricted frontier stops at the point where the efficient area of the frontier begins in a typical frontier of assets. What is also evident is that the INDI is a point on the restricted frontier border at the maximum point and not the ALSI. The GOLDI provides the lower boundary for the restricted frontier in the inefficient region of the frontier.

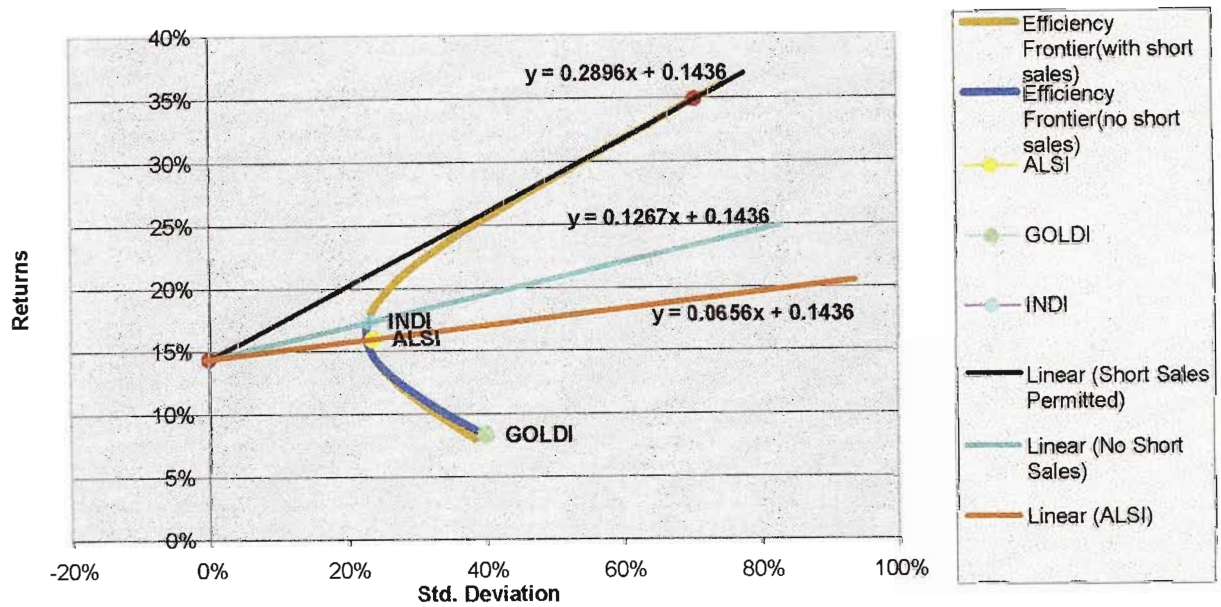
In order to determine the point of the tangency or optimal portfolio we need to solve for the CML that is tangential to frontier. Using the an optimisation program we find that the optimal point and portfolio weights are:

Table 4.13 Optimal Portfolio for SA Investment Only

Optimal Portfolio	Expected Return	Std Deviation	Sharpe Ratio	ALSI %	GOLDI%	INDI%
Short Sales incl.	34.8424%	70.7263%	0.2896	478.8788%	-266.5566%	-112.3221%
Risk Free Rate	14.3610%	0.0000%				
No Short Sales	17.3104%	23.2856%	0.1267	0.0000%	0.0000%	100.0000%
Risk Free Rate	14.3610%	0.0000%				

The negative position for the portfolio allowing short sale displays that to obtain the most efficient portfolio mixed with a risk free asset would require us to sell the GOLDI and INDI short and invest the proceeds in the ALSI. From Arbitrage Pricing Theory we know that if a zero cost investment like this is possible then the ALSI has to be inefficient. The CML lines are presented graphically below:

Fig. 4.7 CML Slopes of SA Portfolios



The restricted efficient frontier, as is noticeable from the diagram, does not have a significant efficient area for portfolio creation compared to a typical efficient frontier such as that exemplified by the unrestricted frontier in the graph. Since many institutions are prevented from taking short positions in assets, the restricted flatter sloped portfolio is the decisive frontier to the portfolio creation decision. The effect of the restriction demonstrates the concept of penalties being involved by restricting the portfolio choice to a much smaller area marked in blue compared to the larger efficient area of the unrestricted frontier. The more restrictions one places on the efficient set of portfolios the smaller the efficient frontier becomes. Restricted frontiers will always plot within the unrestricted frontier.

The slope of each portfolio represents the degree of return per unit of risk. Investors attempt to maximise the slop in order to have the highest return per unit of risk. The lowest slope line passing through the ALSI demonstrates the CML slope of holding the ALSI market portfolio. Clearly to a South African investor, there are higher slopes available to improve the reward to risk ratio. This is the basic law of investors in the Markowitz model. Investors in risky assets attempt to maximise the reward for risk taken. Since the ALSI portfolio is not the tangency portfolio South African investors would be holding a sub optimal portfolio if they chose to hold the market portfolio or ALSI.

The results of the restricted frontier show that given the risk free rate of 14.36% at that date, the portfolio should contain no GOLD shares and be 100% invested in industrial shares. This is an unusual decision given the fact that gold mining companies are such a major component of the SA economy.

It should be noted that the market proxy cannot be tested statistically against the true market portfolio. The true market portfolio is a theoretical concept of the CAPM that is not empirically observable. Bodie, Kane and Marcus define the market portfolio as the aggregate of all individual investors portfolio, causing lending and borrowing to cancel out. The value of the aggregate risky portfolio will equal the entire wealth of the economy, which is termed the market portfolio. This market portfolio includes all traded assets. The difficulty of measuring the true market portfolio becomes obvious. We can however determine whether the market proxy is mean variance efficient by solving for the tangency CML. If we are able to create, a portfolio with a higher CML than the ALSI implies the ALSI is inefficient. The CAPM requires the market proxy to be an optimal portfolio.

The results from the analysis produce the following CML slopes also known as Sharpe ratios. These lines are included in the graph above. The slope of the CML with no short sales is also the Sharpe ratio of the industrial index since our optimal restricted frontier is invested 100% in the industrial index. The Sharpe ratio of the gold index is –0.1528 thus implying that the gold sector holds no benefit to return maximisation and is irrelevant to decision making to the South African investor.

Table 4.14 Sharpe Ratios

	ALSI Slope	CML no Short Sale	CML Short Sales
Sharpe Ratio	0.0655	0.1267	0.2896

Evidently more efficient portfolios can be created with or without short selling to reduce portfolio risk further than investing in the ALSI only. We have therefore demonstrated mathematically and graphically that the ALSI is mean variance inefficient to the JSE as a whole. Our problem is, given its inefficiency, what are the implications for portfolio building. Is the ALSI still a useful measure for the CAPM? Before this argument is investigated, the constraints on foreign investment is lifted to 15% as is the present case and then unlimited foreign investment is permitted. This will give us insight into what the implications of inefficiency are for foreign investment.

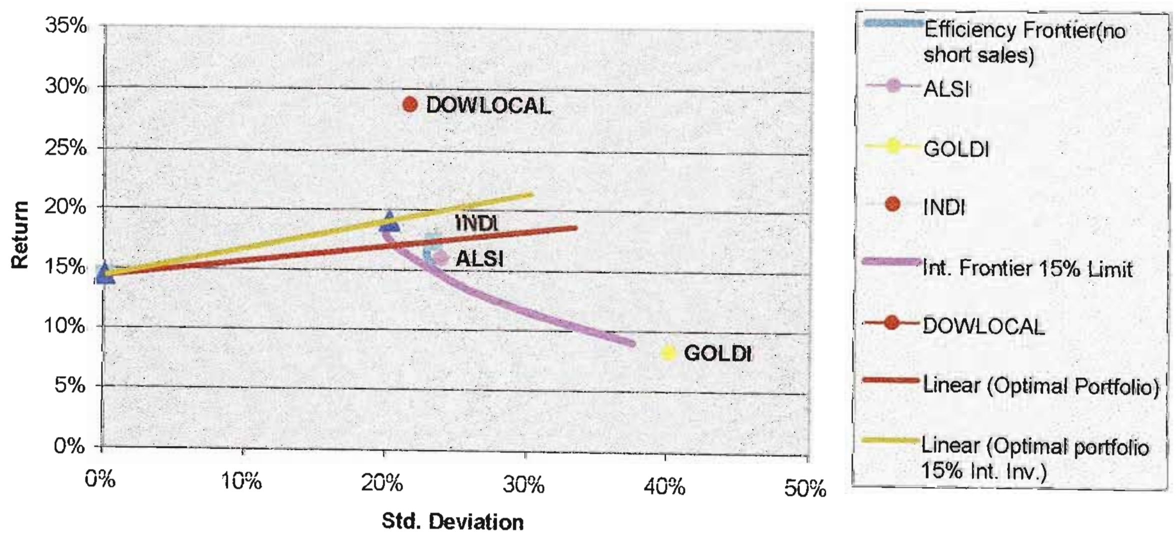
The allowance of a 15% offshore allowance to financial institutions changes the local investment scenario. The DOWLOCAL, being our rand adjusted return of the Dow Jones Index is brought into the investment universe. A constraint that is added to the optimisation process is that the weight of the DOWLOCAL must be less than or equal to 15%.

Table 4.15. Portfolio Variance for Equally Weighted Portfolio with Dow Jones

		ALSI	GOLDI	INDI	DOWLOCAL
	Weights	0.25	0.25	0.25	0.25
ALSI	0.25	0.003533	0.004162	0.003064	0.000701
GOLDI	0.25	0.004162	0.009975	0.002388	0.000843
INDI	0.25	0.003064	0.002388	0.003389	0.000114
DOWLOCAL	0.25	0.000701	0.000843	0.000114	0.002894
	1	0.011460	0.017368	0.008956	0.004552
Portfolio Variance		0.042335			
Portfolio Standard Deviation		20.5755%			
Portfolio Mean		17.5447%			

The base case for the equally weighted portfolio is the starting point again with a constraint of 15% on the DOWLOCAL. After optimising the variance to a minimum for a set of expected returns the resulting data points plot is graphed below. The graph is super imposed onto the efficiency frontier before international diversification to show how the portfolio benefits from introducing an international asset.

Fig. 4.8 SA Portfolio with 15% Foreign Investment Allowance



The increased reward for risk taken is evident in the push of the frontier envelope to the left in the small efficient area. Portfolio efficiency area has thus increased by introducing the international diversification constraint resulting in more advantageous portfolio weightings. The areas to the left of South Africa's domestic efficient frontier are beyond the boundaries previously capable of attainment. The push of the boundary further to the left has the effect of reducing risk for a given expected return. The optimal portfolio with the 15% investment allowance has a higher slope than the local investment efficient frontier, thus demonstrating that the immediate effect of international diversification is to improve the Sharpe ratio and confirms that investment returns relative to risk is improved by the investment allowance.

A point to observe is that the efficient frontier that has a 15% offshore allowance precludes the efficient frontier from reaching its maximum point. This is the point marked DOWLOCAL on the graph. The 15% allowance therefore restricts the portfolio from attaining a more optimal position

The weights characteristics of the new optimal portfolio are as follows:

Table 4.16. Optimal Portfolio with Dow Jones Limited to 15%

ALSI%	GOLDI%	INDI%	DOWLOCAL%
0.0000%	0.0000%	85.0000%	15.0000%
Portfolio Return	19.0196%		
Std Deviation	20.1702%		
Sharpe Ratio	0.2309645		

The new optimal portfolio recommends that the full 15% investment allowance should be used. The balance of the 85% of the risky portfolio should be invested in the INDI. It is noted that the model still recommends that South Africans should make no investment in the gold sector or in the ALSI index. The return improves from 17,3 % to 19,01% and risk reduces from 23,2% to 20,17% as a result of the limited foreign investment allowance.

The last South African perspective to be created is an investment environment with unlimited foreign investment. Using the same bordered covariance matrix incorporating the DOWLOCAL, the constraint on a 15% foreign investment allowance is lifted. This scenario is vital for us to predict the possible implications for both portfolio investment and the economy should all restrictions on capital flows be lifted. The optimal points on the efficiency frontier are recalculated using the new constraints. Short selling is still not allowed. The rationale behind this is that due to distance, duration and timing of market trading it would not be possible to implement short selling between separate exchanges like the NYSE and the JSE.

Fig. 4.9 SA Frontier with Unlimited International Portfolio

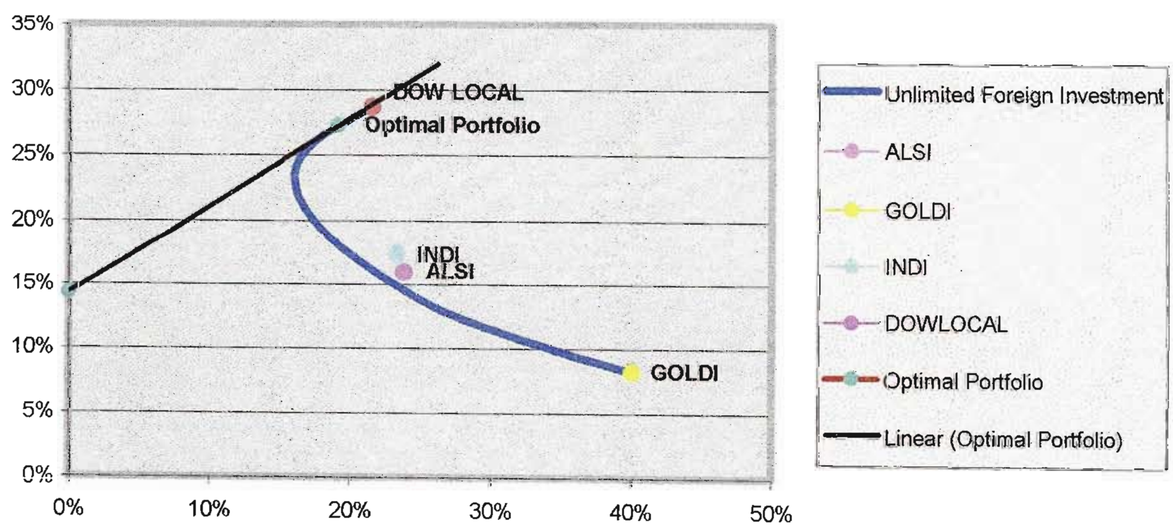


Table 4.17. Optimal Portfolio with Unrestricted International Investment

Optimal Portfolio	ALSI%	GOLDI%	INDI%	DOWLOCAL%
	0.0000%	0.0000%	12.4972%	87.5028%
	Expected Ret.	**Std. Deviation**		
Portfolio	27.2809%	19.1569%		
Risk Free Rate	14.3610%	0.0000%		
Sharpe Ratio	67.4427%			

The graph of the efficient frontier with no restrictions on offshore allowance is the one represented above. Short sales are still prohibited but the introduction of unlimited international investments creates a much larger frontier than previous estimates. The CML slope of this optimal portfolio also has the steepest slope of any previous CML line and by definition the highest return.

If South Africa had to lift its 15% offshore investment allowance would imply that investors should keep 87.5% of their portfolios invested in the Dow Jones and only 12.50% should be invested in South Africa's industrial sector of the JSE only. The gold sector is not a relevant sector for SA investors as it has too high a risk for low expected returns

Compared to the previous case of a 15% investment allowance the unrestricted international investment portfolio improves returns from 19,01% to 27,28% by investing the optimal 87,5% of investment capital offshore and reduces risk even further from 20,17% to 19,15%. There is a strong argument for SA investors to invest substantially abroad as opposed to locally not only due to returns overseas but also to take advantage of the consistent depreciation predicted of the rand at an approximate mean of 12% per annum.

5. INTERNATIONAL INVESTORS PERSPECTIVE OF THE JSE

In this chapter, we will examine the characteristics of SA from an international investors perspective. All returns are stated in dollar terms and the risk free rate will be the US 3 month Treasury bill.

5.1. Test of ALSI Efficiency in Dollars Utilising the Gold Index

Table 5.1. Dollar Tests of ALSI Efficiency using the Gold index
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.776899
R Square	0.603572
Adjusted R Square	0.601369
Standard Error	0.257664
Observations	182

	Coefficients	*Standard Error*	*T Stat*	*P-value*
Intercept – alpha	-0.01665	0.0191	-0.87186	0.384446
ALSIX$	1.070805	0.064683	16.55458	5.21E-38

The tests of efficiency using the dollar returns provide a different conclusion to the rand return tests of the gold sector on the ALSI. The P-value of beta implies that it is not zero and that of alpha (0.38>0.05) tells us that we cannot reject the alpha being zero. The ALSI in dollar returns is mean variance efficient its abnormal alpha return averages to zero in dollar terms. In rand value we obtained a consistently negative alpha.

The dollar R-square value is 60,36% which is higher than the Rand R-square value. Beta is lower in dollar terms reducing from 1,18 to 1,07.

5.2. Test of ALSI Efficiency in Dollars Utilising the Industrial Index

Table 5.2. Dollar Tests of ALSI Using the Industrial Index
SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.939289
R Square	0.882264
Adjusted R Square	0.88161
Standard Error	0.104199
Observations	182

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.016886	0.007724	2.186188	0.030092
ALSIX$	0.96069	0.026158	36.72657	1.52E-85

In dollar terms, the ALSI index becomes an inefficient proxy for testing dollar returns of the Industrial index since the P-value of the intercept (0.03) is less than .05. In rand terms, it was an efficient proxy. The beta value estimated of the INDI in dollar terms of 0.96 shows that risk associated with Industrial shares is close to all the market risk of the ALSI.

5.3. Tests of the Dow Jones

Table 5.3. Tests on the Dow Jones

Regression Statistics	ALSIX$	INDIX$	GOLDIX$
Multiple R	0.532731	0.474438	0.295029
R Square	0.283802	0.225092	0.087042
Adjusted R Square	0.279823	0.220787	0.08197
Standard Error	0.251271	0.267322	0.393762
Observations	182	182	182
P-values			
Intercept α	1.18E-05	0.001992	1.51E-05
DOWJONES	9.87E-15	1.32E-11	5.27E-05
Coefficients			
Intercept α	-0.09833	-0.07281	-0.15206
DOWJONES	1.082276	0.985812	0.831916

The P-values of all the alpha estimates using the ALSI, INDI and the GOLDI are less than .05. We can reject that alpha will be zero for the SA market as a whole including the industrial and gold sectors. Each intercepts P-value is less than .05. Each intercepts value is also a negative alpha implying that a US investor would be better off investing in the Dow Jones than in any SA shares including the ALSI index. The equity returns for SA shares in rand terms produced a zero alpha for the INDI and ALSI regression. Dollar adjusting them results in a negative alpha value. The effect of currency depreciation in SA makes the local equity environment unworthy to the US investors' portfolio because the currency has a steep estimated mean depreciation.

The P-values of beta are similarly all less than .05 so we can conclude that the Dow Jones beta is not equal to zero for all the SA indices. The beta of the ALSI at 1.08 implies that the ALSI is riskier than the Dow Jones.

From the US investors view the gold sector has the lowest correlation (0.295) to the Dow Jones and hence the lowest coefficient of determination (0.087) as well. The beta of gold shares has the lowest sensitivity coefficient (0.832) to the market risk of the US. Gold is a safer investment than the ALSI index from the US investors' perspective. This would imply that US investors who were interested in SA equities would benefit the most from gold shares. The investment that is the most risky for the SA investor is the safest sector for the US investor.

5.4. The Efficient Frontier of the US Investor

Fig. 5.1 US Investors Perspective of SA Portfolios

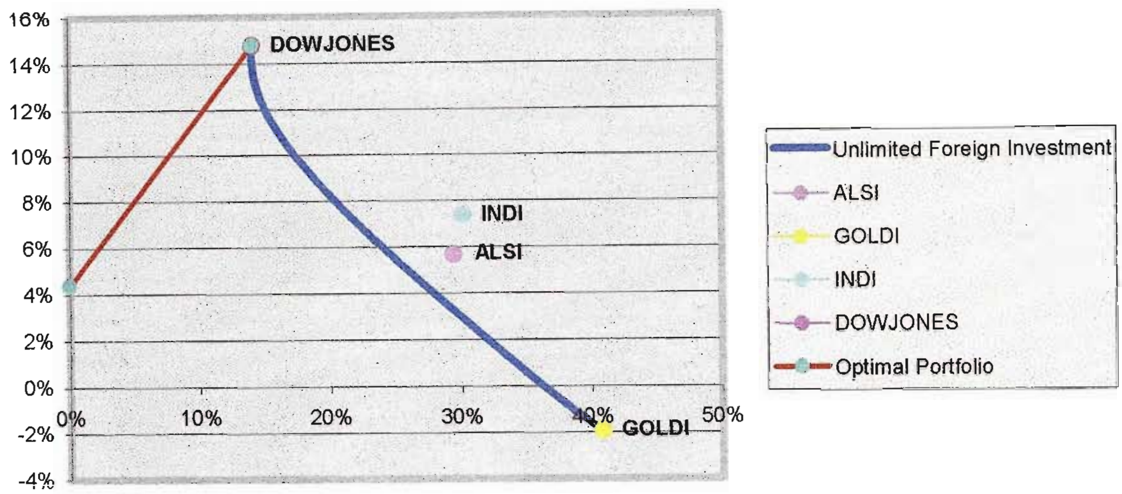


Table 5.4. Optimal Portfolio for US Investor

Return	Std Dev	ALSI%	GOLDI%	INDI%	DOWJONES%
14.7664%	14.1575%	0.0000%	0.0000%	0.0000%	100.0000%
Sharpe Ratio	73.4339%				

An immediate observation of the graph is that the Dow Jones is much further northwest on the graph than any of the South African indices. Portfolios that move in this direction increase return while simultaneously decreasing risk. The basic concept of an efficient frontier is to create portfolios that result in a set of points that plot as far northwest on the graph as possible.

The optimal portfolio for the US investor is to be totally invested in the domestic portfolio and not to invest in SA equity at all. This is due to the mean variance efficiency of the Dow Jones. If the investor however chose to invest part of his portfolio in SA shares the optimisation table below shows that up to about 15% of his portfolio will initially only be invested in gold shares to optimise the return on the portfolio. Gold provides the best opportunity from the investment choices in SA to diversify US systematic risk. None of the optimal portfolios recommends investing in the ALSI index and a miniscule weighting is given to the industrial sector.

Table5.5. Portfolios on the Efficiency Frontier

Return	Std. Dev		ALSI%	GOLDI%	INDI%	DOWJONES%
10.0001%	17.3650%		0.0000%	27.1353%	3.1881%	69.6766%
11.0001%	16.1175%		0.0000%	21.7647%	1.7922%	76.4431%
12.0000%	15.1280%		0.0000%	16.3946%	0.3966%	83.2088%
13.0000%	14.4510%		0.0000%	10.5807%	0.0000%	89.4193%
14.0000%	14.1367%		0.0000%	4.5908%	0.0000%	95.4092%
14.7664%	14.1575%		0.0000%	0.0000%	0.0000%	100.0000%

Chapter 6

6. DISCUSSION OF FINDINGS

6.1. The South African Investment Environment

The basic tenet of the CAPM is that holding the market portfolio is efficient. All investors would invest in the market portfolio in the same proportion as its constituent assets. The decision of the investor would only be how much to invest in the market portfolio versus the risk free rate. In a practical sense, markets are not at equilibrium. Some are overpriced and some are under priced compared to their fair return. But imperfections should not last forever and over time should abnormal returns measured by the alpha intercept should centre on zero. This would imply that over time assets would be roughly fairly priced as dictated by the CAPM. It theorizes, if the expected return of an asset is higher than the fair return predicted by the model then the investors portfolio should be rebalanced toward the asset that provides the superior return and alpha returns should disappear.

We find that the dichotomous nature of the JSE has made the CAPM applicable only in certain circumstances. The findings show that industrial index of the JSE has an approximately zero alpha with respect to the ALSI whereas the Gold Index demonstrates a non-zero negative alpha. These estimates are shown to be consistent over a large sample for a period of 15 years. The ALSI is therefore not an adequate proxy to use for all share returns. In particular, CAPM will be useful to SA investors only in the industrial sector, where we have demonstrated that a one-factor model is applicable efficiently.

Portfolio selection using the CAPM suggests that we realign assets towards assets with positive alphas and reduce the negative alphas. Recalling our empirical regression equation for the Gold Index $(R_{GOLDI} - R_f) = -7.9\% + 1.1817(R_{ALSI} - R_f)$ we note that our empirical alpha value is estimated to be –7.9%. The consistent negative alpha value would imply that we should not hold gold shares in the South African investors portfolio but further that the ALSI is also not an adequate proxy. Our optimisation model derives a similar result telling us that we should invest 100% of the portfolio in Industrial shares. Gold shares and the ALSI are not in the optimal portfolio. This implies that the tangential efficient frontier portfolio is the Industrial index and not the ALSI index. The ALSI is inefficient. The result advocates the use of active portfolio management.

A matrix of the coefficient of determination results for a South African investor are summarised below.

Table 6.1. R-Square Table for SA Investor

R - Square	ALSI	GOLDI	INDI	DOWLOCAL
ALSI	1.0000			
GOLDI	0.4914	1.0000		
INDI	0.7843	0.1687	1.0000	
DOWLOCAL	0.0480	0.0246	0.0013	1.0000

The R-square value of the ALSI to the GOLDI is only 49% whereas the INDI is at 78%. We can conclude that although the market risk does explain almost 50% of the gold sectors returns it is obvious that systematic risk is not the only factor affecting gold share returns.

The tests of the efficiency of the ALSI with respect to the INDI indicate that the difference in returns is not statistically significant to render the ALSI irrefutably mean variance inefficient. The mean market premium of the ALSI is expected to be 1,2% with a standard error of approximately +-3,7% around the mean at the 95% confidence level. The INDI premium is estimated at 2,6% with a standard error range of +-3,6% around the mean. The estimated means of both indices are close and overlap at the 95% confidence level. This explains why our regression test resulted in us accepting the ALSI as mean variance efficient with a P-value of .06, which was only .01 above the significance level whereas the optimisation method shows the ALSI as slightly inefficient. The conflicting result is due to the overlapping standard errors in the sample, due to close proximity of the means.

The optimisation model graph plots the INDI on the efficiency frontier whereas the ALSI does not. Using the mean return 15.9228% of the ALSI as the surrogate of market return and the present risk free rate 14.361% the CAPM model predicts a return of 15.74% on the Industrial index. The mean return of the INDI is 17.3104% and at the 95% confidence level is expected to be between 13.9% and 20.73%. The CAPM return prediction falls within the parameters predicted by the descriptive statistics.

Since the industrial index has a consistently higher Sharpe ratio of 0.1267 compared to the ALSI 0.0655, we conclude that the ALSI is slightly inefficient, but that this is not significant enough to render it an inappropriate market portfolio to analyse the industrial sectors shares The linearity of the CAPM model still holds using the ALSI as the market proxy.

Tests of the Gold index produce are at an extreme difference to those achieved using the Industrial Index. The Gold Index does not have a linear relationship with the ALSI. The use of CAPM analysis using the ALSI as a proxy for market return is therefore flawed and irrelevant. The R-square measure shows that the ALSI has a low explanatory effect for returns in gold shares.

The mean expected return of gold is 8.2404% with a standard error of 5.86% at the 95% confidence level. The upper limit of the expected mean is 14.1%. The CAPM relationship with the ALSI predicts the return of the Gold index to be 16,21%. The CAPM equation produces an overestimate of returns outside of the 95% confidence range of the mean. The ALSI is therefore not a useful predictor of gold share returns. It fails to account for a –7.9% predicted alpha value in our regression that the market risk of the ALSI cannot explain. The R-square measure was not sufficiently high in the Gold sector to allow us to conclude that market returns were a major coefficient of determination of the sectors asset returns.

Using the multi factor model we previously estimated, and using the mean return of the ALSI (15.9228%) and the gold price in Rands 9.7253%, the expected return of gold is 10.11% which falls within the 95% confidence level of the mean gold return.

The Multiple factor model demonstrates that the returns on gold had other relevant variables besides the ALSI with a predictor value to the Gold share returns. These included the dollar gold price and the rand/dollar exchange rate both of which are accounted for in the Rand price of gold. These other two factors are asset specific risks, and therefore relate to unsystematic, and not market risk. The multiple regression technique achieved a much higher R-square value (69.5% vs.51.8%) than the CAPM one factor model that tries to explain risk through the beta on a market proxy only. CAPM analysis therefore has no useful purpose for SA investors in the gold sector. Unsystematic risk is therefore a key factor that still needs to be considered for gold shares as it produces a large alpha value that distorts CAPM linearity. The Industrial Index on the other hand had an adequately high R-square value of 80.3% to the ALSI and no other significant variables were estimated.

The research results discussed above supports the findings of Bradfield and van Rensburg that the market is significantly segmented. Tests of the ALSI show that while it can be used as an adequate proxy of market risk for the industrial shares that is not the case for gold shares. The expected return relationship for the two major asset sectors demonstrate a non-linear function of ALSI market risk to all South African shares. The non-linearity is introduced by the gold sector, which have other significant influences. We have shown that there is no significant relationship between the ALSI and the GOLDI due to a low R square measure. Hence utilising a market proxy for CAPM analysis that does not measure the systematic risk of gold shares is irrelevant.

Since gold shares do not have an adequate proxy in the ALSI, utilisation of the CAPM in South Africa needs to view the JSE not as a unitary market, but rather as two distinct markets trading on one exchange. The implications of this are extreme. Market portfolios by definition should include all traded assets therefore the broader the index the closer we should be to correlation with the true market portfolio.

CAPM utilisation according to our research would be better served by separating the JSE into two autonomous segments. The implication would be that the Gold index would be a proxy for the mining sector and the Industrial index for the industrial universe. Only in this manner will CAPM analysis be rational and relevant to the analysis of an assets fair return. The Industrial Index is efficient in terms of industrial shares and the Gold Index is efficient in the universe of the gold shares. These have been demonstrated on the domestic efficient frontier. The Industrial index creates the maximum point on the frontier and the Gold index the minimum. The ALSI lies in between and within the frontier.

The dichotomous nature of the results can only imply that the gold sector of the JSE is the factor contributing to the inefficiency of the ALSI to the efficient frontier and that the market is not as perfect as assumed by the CAPM.

Perhaps a more useful methodology in estimating returns would be to use a Bayesian framework in which the data are adjusted to reflect prior knowledge or judgment. We have seen that expected returns for the Gold index for example can be calculated either from a sample mean, the CAPM, or more reliably from multiple regression considering other variables. All estimates of expected returns from these three methodologies can be utilised, and a weighted average return could be calculated. The weightings would reflect the belief we have in each model and how well the model compares to the empirical data analysis and theoretical foundation. Since we have rejected the mean variance efficiency of the ALSI index model to the CAPM, it would receive a relatively low weighting in the estimate. Our analysis has shown that the multiple regression technique had the highest explanatory effect. In a Bayesian framework, we would therefore place a higher weighting on this result than the other models. The exact weighting is a matter of judgement.

6.2. The Implications of International Diversification for SA Investors

It is noted that the DOW although not a broad based index like the S&P 500 for instance, is a highly recognised and frequently quoted index on international markets. Most of its component shares are also highly recognized global blue chip companies that trade on other share markets as well. The shares that form part of the Dow are likely to form part of most internationally diversified portfolios. In this sense, we consider it a useful measure of how international markets will affect South African investors perspective of the investment environment.

In the tests of the Dow with the SA indices, we found that ALSI and INDI showed that the alpha value of the regression averaged zero but the GOLDI rejected alpha as being zero to SA investors. The CAPM suggests that holding a passive international portfolio like the Dow Jones would provide a fair return to SA investors rather than the ALSI or the INDI. While we rejected beta as being zero for the ALSI and the GOLDI, the INDI beta value was approximately zero. The INDI had no risk associated with the US. The R-Square values of all the indices were below 6%. This meant that there was no significant explanatory effect of the Dow Jones variance on the movements of the SA share market. Since the SA investors' optimal domestic portfolio recommends investing in the Industrial index the introduction of the superior Dow return recommends investing the majority of the portfolio overseas. The negative alpha value of gold indicates that the SA investor should sell gold shares actively from a South African portfolio.

The optimisation model shows that the Rand rate of return on US shares is substantially higher than the dollar rate of return due to the Rands depreciation. Before currency risk the return was 14.77% but the rand return to SA investors is 28.71%. The rand virtually doubles the return of the portfolio. Its mean return over the 15 years is 12.61% per annum, which is a sizeable return to the investor considering that it does not yet include the international portfolios expected return.

Table 6.2. Sharpe Ratios of Indices

ALSI	GOLDI	INDI	DOWLOCAL
0.06551	-0.15279	0.126315	0.664777

The effect of international diversification can be demonstrated by the Sharpe ratio. Investors try to attain the highest ratio possible because it maximises return per unit of risk. The effect of international diversification on the reward to risk ratio of the SA investor is substantial. Clearly portfolio theory would dictate that in the absence of any limitations that the Dow would have a large weighting in an internationally diversified portfolio. Its Sharpe ratio is significantly larger than the ratios of any of the constituent portfolios available to the SA investor including the industrial index recommended in the domestic portfolio.

The beta of industrial sector is approximately zero implying that for local investors purposes international investment does not contain any SA risk related to industrial shares. We have determined that the SA investor would invest in industrial shares only. The zero risk of the Industrial index to the Dow makes the Dow an effective hedge against SA market risk. Logic would imply that if it was free of SA market risk and provides a larger return per unit of risk we should maximise the amount we can invest in the Dow.

In the portfolio model with the Dow, both the Industrial Index and the Gold Index turn out to be inefficient portfolios. The model shows a high weighting towards moving South African investment capital into US shares, since not only are they providing a higher rand rate of return but also have the lowest total risk. Tests to determine the efficiency of the rand denominated Dow Jones show that it is mean variance efficient to the ALSI index and the Industrial Index but not to the Gold index.

Using the mean return on the Dow as the market return and the estimated betas from the regression analysis we can test the CAPM results. The ALSI expected return is 18.49% and the INDI return is 15.35%. At the 95 % confidence level the mean of the ALSI is expected to be 15.9228% +-3.5% and the INDI is 17.3104% +-3.4%. The Dow Jones can therefore be adequately be used as a market proxy for returns on the ALSI and the INDI but due to the presence of the abnormal alpha in the GOLDI will not be an adequate proxy to the GOLDI.

6.3. The Implications of SA Investment for US Investors

Investors require lower risk premiums from assets that diversify asset specific unsystematic risk away (Chatterjee, 1999). The rationale behind this is that assets that reduce unsystematic risk of an unrelated asset will be viewed as being less risky to the portfolio than the assets risk implies. The risk-reducing asset introduces an element of additional safety in portfolio returns and acts as a hedging tool. Chatterjee states that assets that act as hedges partially buffer the portfolio returns from macroeconomic uncertainties in the investors domestic market. South Africa's gold shares are one such particular sector that can reduce macroeconomic risk of US investors. Gold shares reduce their exposure both to the industrial sector and to country risk.

Table 6.3. Correlation Matrix of US Investor

Correlation Matrix	ALSIS	GOLDIS	INDIS	DOWJONES
ALSI$	1			
GOLDI$	0.7719641	1		
INDI$	0.9388119	0.5896143	1	
DOWJONES	0.5270226	0.275379	0.46840686	1

The table above summarises correlation coefficient amongst the indices from a US investor's perspective. South Africa's industrial sector has a higher correlation to the US market than gold shares. The ALSI with a correlation coefficient of 0.53 has the highest correlation of all the indices with the Dow. The implication is that the gold sector offers a benefit for international diversification of a US portfolio

Table 6.4. R-Square Table of US Investor

R-Square	ALSIS	GOLDIS	INDIS	DOWJONES
ALSI$	1			
GOLDI$	0.595929	1		
INDI$	0.881368	0.347645	1	
DOWJONES	0.277753	0.075834	0.219405	1

An immediate result of the US investor's perspective of the South African environment tells us that gold shares hold the lowest correlation (0.275) to US shares and have the lowest R-square value (0.075). Beta of the gold shares was lower than the ALSI and the INDI. For a US investor gold shares therefore are the most independent asset class on the JSE that will assist in diversifying USA country risk.

An important result of our tests of mean variance efficiency is that to a US investor the ALSI is mean variance efficient using the GOLDI as a test. The above explanation of the safety of gold shares to US investors as opposed to its extreme risk to SA investors provides an intuitive explanation of the reason for its low excess return in Rands despite its high estimated beta. The proof lies in the pricing of returns. Our tests have proven that gold is priced efficiently in dollar returns to make the ALSI mean variance efficient to the US investor of gold shares. It is the dollar returns that are relevant to its pricing not the Rand return. Our estimates of beta (0.83) demonstrate gold sector has a risk that is lower than the risk of the ALSI (1.08) when stated in dollars. US investors will require a lower premium from gold shares than SA investors will.

Chapter 7

7. CONCLUSION

The ALSI index is inefficient in pricing gold shares to SA investors but displays a degree of efficiency when pricing industrial shares. We find that the cause of this disparity is that gold shares do not reflect an expected return relationship with the ALSI in Rands. Once the ALSI and the GOLDI are stated in dollars the linear relationship to the ALSI holds. The relationship exists because of trade of gold internationally in dollars only. The ALSI fails to capture the beta return relationship if it is utilised in Rands, hence its inefficiency as a proxy. It is submitted that this differential pricing mechanisms of the gold and industrial sectors is the principal contributor of the market segmentation proposed by Bradfield and Van Rensburg on the JSE.

Our portfolio analysis determined that gold shares were irrelevant to SA investors as the return relative to risk was too high. Beta of gold shares showed that it was riskier than the market index but had a lower risk premium. A passive market portfolio emulating the ALSI was equally inadequate because it was mean variance inefficient. SA investors would maximise domestic returns by holding the industrial index.

Movements of capital across regional and national boundaries, and across currencies, have exploded in volume, thanks to the dismantling of currency and exchange controls and other financial regulations and to revolutionary economies in technologies of communication and transactions. The integration has had a positive effect on the benefits of diversification to SA investors.

The ALSI is inefficient in the portfolio that includes international investment. Investors are able to substantially increase their returns on their portfolio by diversifying internationally. Restrictions on international investment constrain the SA investor to the creation of a sub-optimal portfolio compared to the unlimited international portfolio. The portfolio analysis using the Dow as an international portfolio found that SA investors should in the absence of regulations, invest 87% of their portfolios in the Dow and only 13% locally. The rand depreciation was a major contributor to the increased benefits of international investments.

The US investors' perspective of SA found the Dow Jones to be mean variance efficient. They would gain no benefit from investing in SA equities. Those US investors wanting to invest in SA will optimise their portfolios by focussing on the purchase of gold shares. The reason for this is that currency depreciation make the ALSI and the INDI riskier than gold. Gold shares were determined to be efficiently priced by the beta return relationship of the CAPM utilising the dollar-adjusted ALSI as a proxy. Hence gold shares excluded the US investors from exposure to currency risk prevalent in the rands depreciation that negated severely the return on the ALSI and the INDI.

Currency risk is demonstrated as a serious impediment to investment in SA as it penalizes foreign investment and encourages outflow of local capital. The Rands depreciation is a major deterrent to investment in South Africa's non-gold sector for US investors. Similarly the superior return offered by the Dow in rand terms is detraction for SA investors to invest locally. The rand denominated Sharpe ratios of the SA sectors compared to the dollar ratios exemplifies the penalty incurred by the overseas investor as a result of currency risk. The SA equities experience a decrease in the Sharpe ratios of all sectors when dollar adjusted whereas the Dow Jones increases its ratio. This demonstrates the impact that currency risk can have on a portfolios return. The increased return of dollars and superior return of the Dow are factors that create an SA investors portfolio with a majority international investment and deter US investors from investing locally in SA. None of the SA portfolios in rand or dollars are able to match the reward to risk ratio of the Dow Jones.

Sharpe Ratio	ALSI	INDI	GOLDI	DOW JONES
Rands	0.0655	0.1267	-0.1528	0.6648
Dollars	0.0438	0.0991	-.1543	0.7343

The use of CAPM analysis in SA needs to take cognisance of the sector being examined before determining expected returns. If we wish to determine returns on the Industrial index, the normal CAPM analysis is applicable. Gold returns require that we dollar adjust the expected return on the market proxy and use a dollar risk free rate such as the US Treasury Bill to determine returns. An alternative would be to use a multifactor model that incorporates the expected return on the Rand price of gold in addition to market returns. The portfolio analysis recommends that SA investors should fully utilise the foreign investment allowance of 15%.

The model indicates that should foreign exchange regulation be lifted SA investors would benefit by investing at least 87% of their portfolio abroad. The rebalancing would further pressurise the Rand and result in a continuing depreciation. The use of high domestic interest rates is therefore a natural tool to act as a deterrent to capital leaving and at the same time to lure foreign capital into the country. Simultaneously though, high interest rates tend to reduce growth of local companies and thereby their returns. In order to keep capital within the country macroeconomic policy needs to take note of the implications of the portfolio optimisation findings and create a domestic environment that encourages savings and simultaneously positions the SA economy for growth. Until this balancing act is perfected lifting exchange controls could be potentially disastrous. It would lead to a mass exodus of capital that would be detrimental to the countries long-term survival and impair the ability of the SA economy to grow.

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