

THE CONSTRUCTION AND USE OF AN EVALUATION INSTRUMENT
TO MEASURE ATTAINMENT OF
OBJECTIVES IN MATHEMATICS LEARNING
AT SENIOR SECONDARY SCHOOL LEVEL

BY

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To my
ex school principals,
the late Messrs V.S. Pillay and V. Naidu,
for their dedication
to teaching and education.

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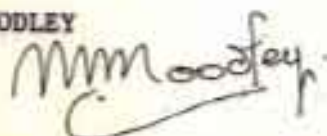
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It is hereby declared that the opinions expressed or conclusions reached are those of the author and are not to be regarded as a reflection of the views of the above-mentioned persons or organizations.

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S U M M A R Y

This research aimed at measuring the extent to which a group of senior secondary pupils were attaining desirable cognitive objectives in mathematics. The summary of the design and procedures adopted in this study and the major findings which emerged is presented here.

A scheme of objectives for mathematics learning at the senior secondary level was suggested in accordance with Bloom's Taxonomy of Educational Objectives and recent research relating to the Taxonomy and other classifications used in mathematics education.

Multiple choice-type test items were constructed with reference to the above scheme of objectives and to content areas selected from the standard grade senior secondary mathematics syllabus. A pilot test was administered and analysed. The selection of items for the final form of the test was based on a consideration of item analysis data, distractors, reliability, validity, rating of items according to objectives and length of test.

The final forms of the test and questionnaire were administered to a selected sample of 769 standard nine pupils from 14 Indian high schools in the Durban and District Area. The test was manually scored and the scores were subjected to statistical analyses by computerization.

The findings suggest that:

- (i) it is possible to devise a reasonably reliable and valid test instrument to test at least two different levels of objectives in mathematics learning at senior secondary school level;
- (ii) the lower level objectives in mathematics are significantly easier to attain than the higher level objectives, which tends to support - in at least two levels - the assumption of hierarchical structure of a taxonomic classification of objectives;
- (iii) the performance in mathematics of the higher grade pupils tends to be adversely affected by being taught mathematics in mixed higher and standard grade classes.

CHAPTER ONE

1. AIMS AND OBJECTIVES IN EDUCATION AS RELATED TO THE PROBLEM TO BE STUDIED

1.1 AIMS AND OBJECTIVES IN EDUCATION

One of the most complex problems with which educators throughout the ages have been faced is that of stating aims and objectives in education. In order to discuss these problems it is necessary to define and distinguish between certain terms. Since the defining of objectives is basic to this research, which leads to the construction of an evaluation instrument in mathematics, it is also essential to point out how objectives are derived and stated, and what role they play in curriculum development and syllabus construction .

1.1.1 Terms Used in this Study

Much of the confusion in regard to the discussion of aims and objectives may be attributed to the lack of clarity in communicating the ideas due to the inconsistent use of certain terms. It would, therefore, be useful to consider a list of definitive terms which are accepted and used, in general, in *this study*. The following is the list of terms presented in alphabetical order, with those terms which appear in the definitions and are themselves defined elsewhere placed in italics:

aims : general declarations of intent which give shape and direction to *education*.

behaviour : any human activity involving thinking or feeling or doing.



core syllabus : (also referred to as common basic syllabus)
a minimum listing of *subject matter* as prescribed
by the Joint Matriculation Board of South Africa
(J.M.B.), which is an examination board controlling
the syllabuses and standards of matriculation
examinations.

curriculum : the planned experiences offered to the learner
under the guidance of the school (curriculum is also
referred to as "educational programme").

education : a deliberate process for changing the *behaviour*
patterns of human beings.

evaluation : the process of determining how well pupils have
attained specified *objectives* (evaluation may take
several forms of which tests and examinations are the
most widely used in South Africa).

examination : *evaluation* which may be internal or external,
for purposes of promotion and certification. (The
internal examination is controlled by the school. The
external examination, e.g. matriculation examination
is controlled by the responsible Department of Education.
An examination is usually taken at the end of the school
year.)

general aims : aims which refer to the end-product of *education*
or to "the broad generalities which describe what the
school is trying to do".

instructional objectives : objectives stated in terms of both
content and *behaviour*, e.g. ability to recall the

definition of an isosceles triangle.

objectives : intended *behaviours* or statements of what pupils should be able to do at the end of any course of study, i.e. statements of expected outcomes.

school curriculum : (also referred to as the whole educational programme) a combination of several *subjects* offered to the learner, e.g. the *senior secondary curriculum*.

senior secondary level : fourth phase of schooling (in South Africa) extending over three years (tenth, eleventh and twelfth years).

specific aims : aims stated in terms of a specific field of study and at a level of education, e.g. aims relating to the teaching of mathematics at the senior secondary level.

specific or behavioural objectives : objectives stated purely in terms of *behaviour* and independent of *subject matter*, e.g. ability to recall knowledge.

subject : a field of study or discipline e.g. English, Afrikaans and Mathematics are subjects.

subject curriculum : an aspect of the *school curriculum* which is concerned with a particular subject, e.g. the mathematics curriculum.

subject matter : (also referred to as "content") the material to be studied in a particular *subject* or field of study.

syllabus : a listing of *subject matter* in broad outline, e.g. Syllabus for Mathematics (Standard Grade) in South Africa,

or GCE "A" Level Syllabus for Mathematics in Great Britain.

test : form of *evaluation* where questions are set and the responses are marked by class teacher at various stages in the school year.

The manner in which the terms have been defined for use in this study reflects the particular view point of education adopted here as indicated by the definition of the term "education". The discussion in the sections which follow will serve to make clear distinctions between some of the above closely related terms.

1.1.2 Stating Aims in Education

Although the problem of stating aims has a philosophical basis, it is not the purpose here to consider the philosophy of stating aims, but rather to clarify the notion of aims relevant to this study with a view to providing a basis for the discussion of objectives.

Dewey⁽¹⁾ discusses the nature of an aim insofar as it is internal to an activity and not something which directs it from outside. He contends that "to have an aim is to act with meaning and to perceive the meaning of things in the light of that intent". He also regards aims in education as arising from the persons involved in the process of education, and states certain criteria for good educational aims:

- "(i) An educational aim must be founded upon the intrinsic activities and needs (including original instincts and acquired habits) of the given individual to be educated.
- (ii) An educational aim must be capable of translation into a method of co-operating with the activities of those undergoing instruction."

Wheeler⁽²⁾ suggests a more comprehensive list of criteria by stating that aims "should be consistent with human rights, democratically oriented, socially relevant, tending to satisfaction of personal needs, and balanced". He proceeds to examine the aims in education promulgated over the last half a century by individuals or committees in the United States and Britain and points out the difficulty in distinguishing between "the ends of the total process of socialization and the particular aims which lie within the province of education".

Both these authors see aims as not being external to the process of education, but rather as arising out of the personal and social needs of the human beings involved in education. Aims are thus regarded as some end-products in the individual and the society.

It is understandable why there is difficulty in distinguishing between the ends of the total process of socialization and the aims or ends of education. Since education is inextricably bound up with the individuals in the society in which it is offered, the ends towards which the process of socialization strives are no different from the aims or ends towards which the education is geared.

The second condition as set down by Dewey is, in the opinion of the author, of paramount importance in the consideration of aims. There is little value in stating an aim which cannot be attained at some point in time. There is, therefore, a need to check, at various stages in the education of an individual, whether there is reasonable progress towards the attainment of stated aims. This will only be possible if the aims are capable of translation in terms of the activities (which may be observed and evaluated) at the various stages in the education of the individuals undergoing instruction.

The following are some illustrative examples of typical aims in education, and statements of policy which imply aims in education:

- (i) Whitehead⁽³⁾ suggests that we should aim at producing "men who possess both culture and expert knowledge in some special direction". He goes on to indicate in a more specific way that education has to impart "an intimate sense for the power of ideas, for the beauty of ideas and for the structure of ideas, together with a particular body of knowledge which has peculiar reference to the life of the being possessing it". Clearly, Whitehead refers to aims at two distinct levels; firstly, at a general level emphasizing the cultural and intellectual aspects, and secondly, at a more specific level in terms of knowledge and ideas.
- (ii) Wheeler suggests that the American Educational Policies Commission (1938)⁽⁴⁾ aimed at "self-realization, adequate human relationship, economic efficiency and civic responsibility". Wheeler tends to speak of these notions in terms of "main objectives" and "general aims" interchangeably. In order to avoid this kind of confusion in terminology the above "aims" will be interpreted as aims stated at a general level, i.e. general aims.
- (iii) The Harvard Committee's Report (1945)⁽⁵⁾ suggested that "education aims at the good man, the good citizen, and the useful man". These aims may be interpreted as being general aims, which emphasise the moral (provided 'good' is so interpreted) and utilitarian aspects of education.
- (iv) The Australian Teachers' Federation's Report (1964)⁽⁶⁾ indicated that the aim of education was "to help the

child to progress towards full attainment of his potentialities as an individual and a member of society". Here again the aim is stated at a general level emphasising the individualization and socialization processes of education.

- (v) The South African National Education Policy Act No. 39 of 1967⁽⁷⁾ lays down, *inter alia*, that education
 - (a) "shall have a Christian character",
 - (b) "shall have a broad national character".

Although these are not explicit statements of aims, they are policy statements which imply the direction in which the education shall proceed and hence imply the general aims of education.

- (vi) The following appears under the heading "General Aims" in the Syllabus for Mathematics (Higher Grade)⁽⁸⁾:

"To acquaint pupils with and train them in mathematical methods of thought and work". The "aim" here is clearly related to a specific field of study and hence must be regarded differently from those stated on a general level. In the present context this "aim" will be considered as a specific aim.

Two major problems emerge from the consideration of the above examples:

- (a) the different emphases in stating aims;
- (b) general confusion regarding the terms which refer to aims.

Firstly, the different emphases in stating aims in education give rise to several conflicts such as the intellectual versus the social

functions of education. In this regard it may be useful to consider Taba's⁽⁹⁾ suggestion that "a more systematic examination of all sources from which educational aims are derived would yield a more comprehensive and balanced statement of aims and eliminate the current conflicts". This would mean returning to an educational philosophy and the educational values stated in it.

Whatever the aims, or whatever the level at which they are stated, they stem from a philosophy of education which reflects the value judgments of a particular philosophical viewpoint. Since there are different philosophical viewpoints there will be different aims.

Secondly, in order to avoid the confusion regarding the terms which refer to aims, only two categories of aims are recognized in this study: general and specific aims as defined earlier.

In general, it must be pointed out that aims (general or specific) merely provide the necessary orientation to the emphasis in educational programmes by establishing a philosophy of education, and are thus "only a step toward translating the needs and values of a society and of individuals into an educational programme".⁽¹⁰⁾ They are, however, insufficient and of little use for making the more specific decisions about selection, organization and evaluation of specific learning experiences in the classroom.

Richmond,⁽¹¹⁾ for example, argues that educational thought is in no mood to settle for aims and since it is unsure of what is meant by the "good man" or "good citizen" it would rather set its sights on more obviously tangible accomplishments.

Lawton (1973)⁽¹²⁾ sees the lack of clarity about what the curriculum is meant to achieve as the main criticism of the typical secondary school

curriculum. There would, therefore, seem to be a need for more specific directions than the mere statement of aims.

It is clear that aims alone are insufficient and it is therefore necessary, in order to make aims more practically feasible, to describe in some detail and to specify the expected outcomes, or intended behaviours, in any particular field of study. When aims are refined in this way to an even more specific level in terms of intended behaviours they are referred to as objectives. (13)(14)

1.1.3 Distinction between Aims and Objectives

Some educators tend to confuse aims with objectives while others speak of "aims" as "general objectives". It is clear from the discussion in the previous section that "aims" and "objectives" have been used in two different senses. In order to make a clear distinction between these two terms their definitions will be restated and explained.

Aims, in general, can be thought of as general declarations of intent which give shape and direction to education. Two categories of aims can be distinguished: general aims and specific aims. Aims which refer to the end-products of education or to "the broad generalities which describe what the school is trying to do"⁽¹⁵⁾ are called general aims (also referred to as ultimate aims in several texts). Such aims do not relate to any particular field of study or level in the education of the individual, e.g. to produce a "good man" and a "good citizen".

Those aims which are stated in terms of a specific field of study and at a specific level in the education of the individual are called specific aims. The aims of teaching mathematics at the senior

secondary level, for example, will be regarded as specific aims. A specific aim might be to reveal the beauty of mathematics, or to create a love for mathematics.

It has already been pointed out that aims are insufficient for classroom practice and that it is necessary to specify expected outcomes. In this regard Brubacher⁽¹⁶⁾ has suggested that "in spite of their prime importance, the ultimate aims of education mark out the teacher's task in only the most general outlines". He further pointed out that in order to be effective in the classroom these aims must be broken down into "more immediate, specific or proximate objectives for the pupil and teacher to pursue".

Objectives in the present context are regarded as intended behaviours or statements of what pupils should be able to do at the end of any course of study, i.e. statements of expected outcomes. Two types of objectives are distinguished: specific (or behavioural) objectives and instructional objectives.

Objectives which are independent of subject matter and are stated purely in terms of behaviour are called specific (or behavioural) objectives, e.g. ability to recall knowledge, ability to apply knowledge. Since these objectives refer to processes (such as recalling or applying) they are also called process objectives.

Those objectives which are stated, in even more detail, in terms of both content and behaviour (process) are called instructional objectives, e.g. ability to apply the "mid-point theorem".

The definition of objective adopted here rests on a particular view of education. In it education is seen as a "process for changing the

behaviour patterns of human beings".⁽¹⁷⁾ Given this view it is expected that each school curriculum, subject curriculum and unit of instruction will bring about some significant changes in the learners. If education has to be effective then it must be decided what changes are desirable and what are possible. The teacher's job must then be to help the learner to modify his behaviours appropriately so that, when confronted with a problem situation, he can exhibit behaviours which will lead to a solution.

Unlike general aims which are remote, and specific aims which are intermediate (but still too general), objectives are immediate and clearly relevant to the classroom situation. Thus, while aims are general and strategic, objectives are specific and tactical in nature. It is possible, at the various stages in education, to determine, through evaluation, the extent to which pupils are attaining the stated objectives. However, it may not be possible to determine whether the aims have been attained at the end of schooling or even in a life-time.

In the South African context, the distinction between aims and objectives in education has not been defined. This is probably due to the fact that much of South African education is based on that of Britain and historically, it has been suggested that "the British have been fonder of stating aims than objectives, while American educationalists, who have been as prone as ourselves to sonorous declarations of intent, have shown themselves more willing to convert aims into objectives".⁽¹⁸⁾

In this section a major distinction between aims and objectives has been made. For further clarity in use of terms, distinctions have been made between two types of aims (general and specific) and between two

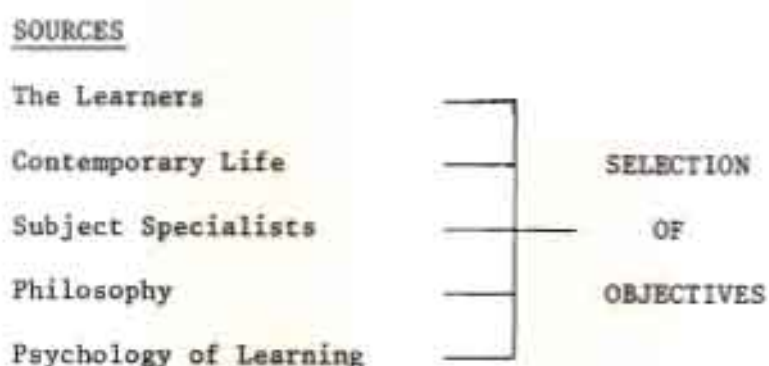


types of objectives (specific and instructional).

1.1.4 The Derivation of Objectives

The translation of general aims to objectives presents a problem to educators: how are the objectives to be derived? In the opinion of the author, two related issues need to be borne in mind when dealing with this problem: they are the questions of what objectives are desirable and what objectives are possible. The answers to these questions appear to lie in the sources of objectives.

Tyler⁽¹⁹⁾ examines the sources of objectives and contends that, in the final analysis, objectives become matters of the value judgments of those responsible for education. The following scheme emerges from his examination of five sources:



Tyler's approach points to the essential link between aims and objectives since he suggests that objectives become matters of value judgments. Thus value judgments, based on the examination of the above five sources, will point to a balance between desirable and possible objectives.

Kerr⁽²⁰⁾ refers to pupils, society and disciplines as sources for deriving objectives and specifies them as:

- (i) the level of development of the pupils, their needs and interests,
- (ii) the social conditions and problems which the children are likely to encounter and
- (iii) the nature of the subject matter and types of learning which can arise from it.

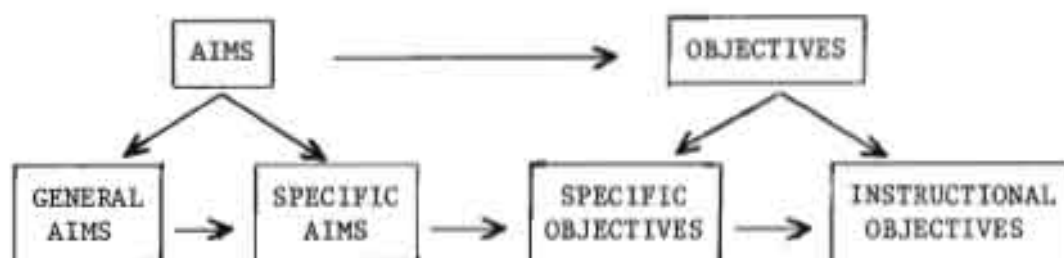
While this is a useful and practical approach in determining the objectives it seems to emphasize only the possible objectives and does not seem to take into account the value judgments which are indicative of the desirable objectives.

In an attempt to translate general aims to objectives Wheeler⁽²¹⁾ suggested a three step process: "Ultimate goals must be stated, mediate goals derived, and finally proximate goals set up, so that specific objectives can be planned at the classroom level." The term "goals" is used here to mean "expected behaviour patterns". "Ultimate goals" and "mediate goals" may be interpreted as corresponding to general aims and specific aims respectively, while "proximate goals" and "specific objectives" correspond to specific objectives and instructional objectives.

Fraser and Gillam⁽²²⁾ mention four levels in specifying objectives: general, mediate, specific and instructional. The first two levels refer respectively to the general aims in education and the aims of a particular course of study (i.e. specific aims). The "specific" level refers to objectives which are expressed purely in terms of observable pupil behaviour and are independent of subject matter (i.e. specific objectives). Instructional objectives are expressed in terms of both the behaviour of the pupil and the subject matter to be studied. The

authors suggest that all levels are necessary in the educational process and that each level depends, for its development, on the preceding level.

From the above approaches it is clear that the use of terms, regarding aims and objectives, lends itself to the following approach to moving from aims to objectives:



In general, aims originate from philosophical, sociological and psychological considerations. The objectives are then seen in terms of the possible behaviours which will lead to the realisation of these aims. These approaches to deriving objectives take into consideration the basic question of their relevance to education and society.

In the final analysis there would seem to be a two-way approach in terms of the derivation of objectives and attainment of aims:

- (i) the derivation of objectives points to a movement from aims to objectives;
- (ii) the attainment of aims points to a movement from attainment of objectives to the attainment of aims.

Schematically represented:-



In view of the above, the standpoint adopted here is that objectives should be considered in terms of behaviour modifications in relation to specific subject areas so that the attainment of these objectives will result, ultimately, in the attainment of the stated aims.

1.1.5 Stating Objectives

Having pointed out the need for objectives in the classroom approach to teaching-learning and evaluation, and the manner in which these objectives are derived, it is necessary to consider how they should be stated.

The stating of objectives is closely bound up with evaluation which is "the process of determining how well pupils have attained specified instructional objectives".⁽²³⁾ Evaluation, therefore, presupposes a clear statement of objectives. In order to evaluate pupil achievement, the specific ways in which the pupils should be able to exhibit that achievement must be known. This means that the objectives must refer to what the learner should be able to do at the end of a course of study. This in turn points to stating objectives in terms of pupil behaviour. Much of the literature in this field tends to support this approach.

Furst⁽²⁴⁾ contends that it is essential to define objectives in terms of pupil behaviour in order to know precisely what to seek in devising an evaluation procedure. He also suggests the following rules for specifying objectives:-⁽²⁵⁾

- (i) State objectives clearly in terms of behaviour.
- (ii) State the objectives at the right level of generality.
- (iii) Be sure that the objectives do not overlap.

Furst further stipulates the following requirements when stating objectives:-⁽²⁶⁾

- (i) They should include all the more important aspects of behaviour related to the problem of evaluation.
- (ii) They should specify the kinds of responses that may be accepted as evidence of these aspects of behaviour.
- (iii) They should specify the limiting conditions under which these responses are likely to take place.

Lindvall⁽²⁷⁾ sets down the following criteria for objectives to be of maximum utility in evaluation:

- (i) Objectives should be worded in terms of the pupil.
- (ii) Objectives should be worded in terms of observable behaviour.
- (iii) Objectives should refer to the specific content to which the behaviour is to apply.

Tyler⁽²⁸⁾ argues that the formulation of objectives in terms of behaviour alone is not satisfactory and states that "the most useful form of stating objectives is to express them in terms which identify both the kind of behaviour to be developed in the student and the content or area of life in which this behaviour is to operate".

Gagné⁽²⁹⁾ points out the value of behavioural objectives as a means of communication in selecting materials which can accomplish the desired outcomes and in planning instructional and assessment procedures. He stipulates that an objective should be designed to serve

all of its communicative purposes and it should indicate (a) what the student will have learned from instruction, and (b) what class of performances he will then be able to exhibit.

While the above approaches among others (Bloom,⁽³⁰⁾ Mager,⁽³¹⁾ Popham,⁽³²⁾ Taba,⁽³³⁾ Allendoerfer⁽³⁴⁾) lend strong support to specifying objectives in behavioural terms, some writers⁽³⁵⁾⁽³⁶⁾⁽³⁷⁾⁽³⁸⁾⁽³⁹⁾ have argued against it. In summarizing the most common arguments raised against behavioural objectives it may be stated that:

- (i) they stifle classroom creativity;
- (ii) the statements are prescriptive and lead to over-emphasis on conformity;
- (iii) educational outcomes cannot be accurately predicted;
- (iv) not every subject is amenable to this treatment;
- (v) the objectives are too atomistic and can become too numerous and unwieldy.

In summing up the debate on behavioural objectives, Stones⁽⁴⁰⁾ suggests that "the attraction of logic and rationality are so great that the literature arguing in favour of specifying objectives (behaviourally) is much more extensive than that arguing against it". The systematization of behavioural objectives by Bloom *et al*,⁽⁴¹⁾ for example, has proved to be a useful tool in educational research, test construction and classroom instruction. In this regard empirical evidence has been produced, which will be reviewed in the next chapter.

Teachers, however, have remained somewhat unconvinced about the advantages of specifying objectives. This attitude may be attributed to lack of knowledge of the rationale of the process and to the threat

posed by the approach which focuses clearly on teacher assessment and accountability.

While Wood⁽⁴²⁾ concedes that there are problems in this approach, he argues that it is up to the other systems of course-planning to show they have more to offer in making the teacher aware of what he is doing and why he is doing it.

The first step in constructing a test or examination is concerned with "what to evaluate?". Since tests and examinations serve the main purpose of providing evidence of pupil performance or of what the learner should be able to do at the end of a course of study, it is logical that objectives be stated in terms of both pupil behaviour and content. Only then will it be possible to know precisely what evidence to seek in a test or examination.

1.1.6 Aims and Objectives as Part of a Curriculum Plan

The concept of a curriculum has changed in recent years. It is now seen more in terms of the whole learning situation than in terms of the content of a teaching programme. Wheeler⁽⁴³⁾ defines curriculum as "the planned experiences offered to the learner under the guidance of the school". Other writers⁽⁴⁴⁾⁽⁴⁵⁾ give a similar definition.

A school curriculum refers to the whole educational programme offered to the learner. Such a programme, e.g. the senior secondary curriculum, is made up of a combination of several subjects.

A subject curriculum refers to one aspect of the school curriculum and is concerned with a particular subject, e.g. the mathematics curriculum. It is in this context that the word "curriculum" will be used in the present work. In the South African context, however, the term

"syllabus" has been used, which will be discussed later.

One of the earliest and clearest expositions of a curriculum was suggested by Tyler⁽⁴⁶⁾ who identified four fundamental questions which need to be answered in developing any curriculum:

- (i) What educational purposes should the school seek to attain?
- (ii) What educational experiences can be provided that are likely to attain these purposes?
- (iii) How can the educational experiences be effectively organized?
- (iv) How can we determine whether the purposes are being attained?

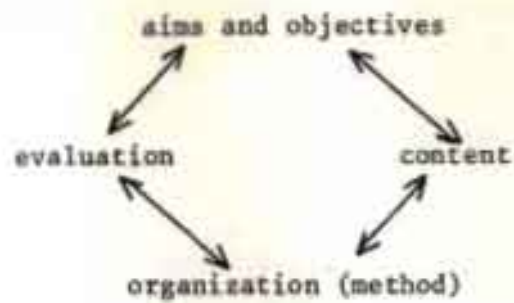
These are questions about aims and objectives, content, organization (instructional method), and evaluation.

Some writers have interpreted Tyler's approach to the development of a curriculum as a linear model:⁽⁴⁷⁾

aims and objectives → content → organization → evaluation.

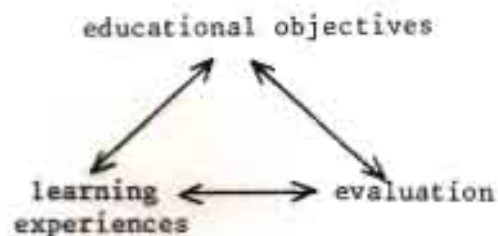
Tyler makes no mention of either a linear model or of interacting components, but emphasizes that "objectives become the criteria by which materials are selected, content is outlined, instructional procedures are developed, and tests and examinations are prepared".⁽⁴⁸⁾

However, it would be less restrictive and more useful to interpret the above approach in terms of interacting components showing the continuous reciprocal relationships:



This enables a dynamically constituted curriculum plan which shows a possible feedback of the effectiveness of the different components, each of which potentially bears on the others.

The above interpretation finds support in Furst's⁽⁴⁹⁾ model of the curriculum plan which clearly brings out the reciprocal relations:



Furst also emphasizes that objectives serve as the bases for developing both learning experiences and evaluation procedures, and that these in turn help to clarify the objectives.

Wheeler⁽⁵⁰⁾ suggests a five-phase cyclic model of a curriculum process with "selection of aims, goals and objectives" as the first phase:



All three models of a curriculum plan make the basic assumption that the general aim of education is to change behaviour and they emphasize the fundamental importance of aims and objectives in curriculum development.

1.1.7 Syllabus and Objectives with Special Reference to Mathematics

The syllabus has commonly come to be known as a listing of subject matter or content. When compared with the idea of a curriculum, a syllabus will have a restricted meaning in the sense that it is seen largely in terms of the content aspects of the educational programme. This is precisely the way in which "syllabus" is used in South Africa.

With the growing interest in curriculum development in mathematics, particularly in countries with decentralised educational systems, (e.g. U.S.A., England) several curriculum reform projects have emerged. In general, these have aimed at "improving the teaching of mathematics by developing new syllabuses, writing new texts and devising novel classroom materials".⁽⁵¹⁾ Such an approach, where teachers play a vital role in curriculum planning, is not feasible in a centralised educational system in which it is only possible for curriculum decisions to be made centrally and simply carried out (as orders) by teachers. In this regard, Lawton⁽⁵²⁾ claims that "in the United Kingdom, with its decentralised system, every teacher is, to some extent, his own curriculum planner". The J.M.B.⁽⁵³⁾ in South Africa, on the other hand, draws up the core syllabuses and also controls the standards of the examinations based on them. These syllabuses, it is felt, leave little room for manoeuvre and the teachers have no option but to follow them closely with a view to preparing the pupils for the examinations.

Curriculum researchers have criticised mathematics syllabuses for being vague and primarily concerned with covering topics. They have stated that "topics are usually given in broad terms with respect to specific content areas, with little indication of the depth of treatment desired".⁽⁵⁴⁾ Of the typical GCE or CSE syllabus it has been said that "it encourages a shallow and ephemeral style of learning largely because it offers little or no guidance to the teacher on the learning possibilities inherent in each topic".⁽⁵⁵⁾ It has also been suggested that these syllabuses are set out purely in terms of subject matter with such statements as "properties of triangles", "length of arc", "the sphere", "proportion and scale" and "pie diagrams".⁽⁵⁶⁾⁽⁵⁷⁾

In South Africa the senior secondary mathematics syllabus sets down, in addition to a listing of subject matter, certain "general aims", for example, "To acquaint pupils with and train them in mathematical methods of thought and work".⁽⁵⁸⁾ The syllabus includes such statements as "the set concept", "intersection and union of sets", "function of a function". Some aspects of the syllabus, which offer more detail include such statements as "Venn diagrams and their applications as an aid to illustrate solutions to problems" and "substitution in formulae". These latter statements are of greater use to the teacher in the sense that they provide some information on expected outcomes in terms of particular content areas.

In general, in South Africa, there seems to be a lack of clarity among teachers, examiners and pupils as to the depth of treatment of the subject matter in mathematics. This problem has been aggravated with the introduction of the New System of Differentiated Education⁽⁵⁹⁾ which requires that a differentiation be made between the Higher and Standard Grade Syllabuses. Since there is a lack of specific

direction in terms of the depth of treatment of the subject matter, which is only "prescribed in broad outline", the differentiation may well become meaningless.

1.2 NATURE AND BACKGROUND OF PROBLEM TO BE STUDIED

1.2.1 The Changing Mathematics Curriculum

Mathematics educators have made out a clear case for the modernization and reconstruction of school mathematics courses in recent years.⁽⁶⁰⁾⁽⁶¹⁾⁽⁶²⁾ This modernization, which has become inevitable mainly as a result of technological, social, economic and political changes, has left mathematics curricula the world over in "a state of flux".⁽⁶³⁾ Topics taught in higher grades in the past have now been brought down to the lower ones and newer topics taught at university have crept into the secondary school curriculum. In South Africa, for example, the latest Common Basic Mathematics Syllabus⁽⁶⁴⁾ for the senior secondary school, which was introduced under the New System of Differentiated Education⁽⁶⁵⁾ in 1973, includes such new topics as "vectors", "groups" and "mathematical induction".

Such changes raise several questions for mathematicians and mathematics educators: What are the purposes of these changes? How can they be met? How do we know whether they are being met? These are questions about educational objectives, and about curriculum construction, teaching and evaluation which must be based on these objectives.⁽⁶⁶⁾

1.2.2 Stating the Problem

Several curriculum reform projects have been developed in the United States, United Kingdom and Europe with the view to answering some of the above questions.⁽⁶⁷⁾⁽⁶⁸⁾ In general these have emphasized the

learning of mathematics with "insight and understanding" and a break-away from only simple recall and rote learning. In South Africa the J.M.B., much in line with this thinking and in conjunction with the new approach in the Common Basic Syllabus, rightly decided "that the mathematics examination should test not only formal knowledge but also insight and understanding".⁽⁶⁹⁾ Since the mathematics examination has become an indispensable part of the educational process, and since teachers and pupils inevitably prepare for the examination, their efforts to achieve success will be "inclined to over-ride educational considerations".⁽⁷⁰⁾ It is implied, therefore, that a "good" test or examination in mathematics will promote correct teaching methods and hence the attainment of desirable objectives. The construction of such a test or examination requires the basic consideration of "what to evaluate". This leads to the problem of suggesting a scheme of objectives on which evaluation in mathematics can be based.

While "insight and understanding" and "development of rational powers" are generally accepted as worthwhile aims of schooling today, it is questionable whether they are as clear to classroom teachers as they probably are to some specialists in education and psychology.⁽⁷¹⁾ It has already been pointed out that there is, in general, a lack of clarity about what the mathematics curriculum is meant to achieve. This points to a need for clearly defining objectives as opposed to merely stating aims.

The fact that the senior secondary mathematics syllabuses⁽⁷²⁾⁽⁷³⁾ contain vaguely stated "general aims" and lack any statement of clearly defined objectives, raises the problem of whether the teachers are teaching and examining the new syllabus in the traditional way: emphasis being on memory and proficiency rather than on "insight and

understanding". It must, therefore, be ascertained to what extent the senior secondary pupils (exposed to aspects of the new syllabus) are achieving certain objectives including those which go beyond the mere recall of formal knowledge. This will entail the construction of test items (in the absence of suitable tests) based on a range of objectives, and the administration of such a test.

Although there are several other problems in the teaching and learning of mathematics, the problems raised here are of fundamental importance in that they concern objectives and evaluation which are the closely-bound basic components⁽⁷⁴⁾ of an educational programme. Since there appears to be no published research in this area in this country, it is envisaged that this investigation will produce several possibilities.

In the light of the above discussion, the problems with which this study concerns itself are those of:

- (i) defining objectives for use in the teaching of mathematics at the senior secondary level,
- (ii) developing mathematics test items to bring out the stated objectives, and
- (iii) administering the test to selected senior secondary pupils to measure the extent to which they are achieving the objectives.

1.3 PURPOSE OF THIS STUDY

- (i) To suggest a suitable classification of cognitive objectives for use in the teaching of senior secondary school mathematics.

- (ii) To develop mathematics test items designed to elicit the cognitive outcomes outlined in such a classification.
- (iii) To administer the test to a selected group of pupils in the senior secondary phase in order to measure the extent to which they are achieving such objectives.

1.4 ASSUMPTIONS AND LIMITATIONS

The approach to the investigation of the problem outlined is based on certain assumptions and limitations.

- (i) The operational definition of an "objective" is given in terms of pupil behaviour as an account of "what a pupil should be able to do at the end of a course in terms of remembering, thinking and understanding".⁽⁷⁵⁾ This definition assumes the view that education is a process for changing behaviour.
- (ii) Objectives are stated only in the cognitive domain or area of thinking abilities. Consequently the evaluation is limited to measurement of cognitive abilities in mathematics.
- (iii) The content areas for which the evaluation instrument has been constructed were selected from the Standard Eight Standard Grade Mathematics Syllabus⁽⁷⁶⁾ of the New System of Differentiated Education.
- (iv) The sample selected for testing was restricted to Std IX pupils from 14 Indian High Schools in the Durban and District Area.

- (v) It is assumed that the evidence of test performance indicates a measure of attainment of the defined objectives. No assumptions, however, are made about the nature or process of learning.⁽⁷⁷⁾
- (vi) Positive correlations of the test scores with criterion scores such as standard eight examination results and teachers' ratings will be taken as evidence of the validity of the test.

1.5 OUTLINE OF DESIGN AND PROCEDURES OF THE STUDY

- (i) With reference to Bloom's Taxonomy of Educational Objectives,⁽⁷⁸⁾ and recent research relating to the Taxonomy and to the uses of other classifications of objectives in mathematics education, a scheme of objectives for mathematics learning at the senior secondary level is suggested.
- (ii) Multiple choice-type test items are constructed with reference to the above scheme of objectives. These items are based on content areas selected from the senior secondary mathematics syllabus. A pilot test is administered and analysed and a final form of the test is produced.
- (iii) The final form of the test is administered to a sample of 769 Std. IX pupils from 14 high schools serving the Indian Communities in the Durban and District Area.
- (iv) The test is scored manually and the scores are subjected to statistical analyses by computerization to yield means,

standard deviations, significance values, item analysis data and correlation coefficients for reliability and validity.

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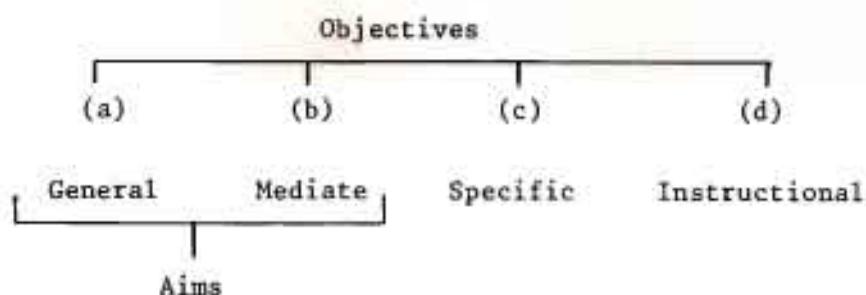
CHAPTER TWO

2. OBJECTIVES FOR MATHEMATICS LEARNING

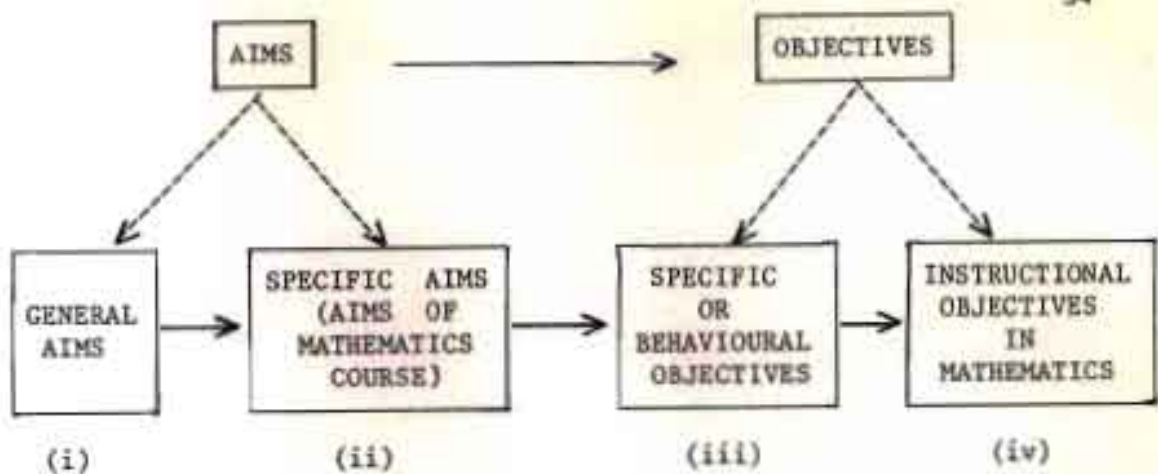
2.1 DERIVATION OF EDUCATIONAL OBJECTIVES IN MATHEMATICS

In the previous chapter aims and objectives in education were considered in general. An aim was differentiated from an objective and the need for the latter was stressed. It would be relevant to pursue this distinction further with respect to mathematics.

A useful distinction between four types or levels of "objectives" has been made by Fraser and Gillam:⁽¹⁾



In this representation (a), (b), (c) and (d) are regarded as different types of "objectives" while the first two are labelled as "aims". The global label "objectives" tends to obscure the kind of distinction between aims and objectives that has been made in this study. The following representation is offered as a suitable adaptation with special reference to mathematics:



This scheme provides the necessary distinction between aims and objectives and also demonstrates the manner in which aims may be refined to yield objectives. The dotted-line arrows indicate the types of aims and objectives while the solid-line arrows show the movement from aims to objectives. The objectives are derived from the aims in four levels, where each level is dependent on the preceding one. All these levels are essential in guiding the planning of the educational process. (2)

2.1.1 General Aims

Several examples of general aims in education have already been cited and it has been pointed out that they have their origins in "the philosophical approach and cultural environment of the people in charge of planning the educational process". (3) They are, therefore, the same for all subjects of a school curriculum offered in a particular educational system. Since mathematics becomes an integral part of the educational system in which it is offered, it has to play its role in attaining the general aims of that system. If, for example, the general aims are mainly utilitarian, then the utilitarian or practical value of mathematics in society will be emphasised.

2.1.2 Aims of the Mathematics Course (or Specific Aims)

The general aims of education of a particular society determine, in turn, the aims of its mathematics course. The aims of the mathematics course are all-embracing statements of intent which give direction and shape to the teaching and learning of mathematics. They emphasise *why* the subject is being studied but do not spell out the desired outcomes of a course of instruction in mathematics. The following are examples of specific aims relevant to secondary mathematics courses:

- (a) usage in everyday life;⁽⁴⁾
- (b) to interest pupils in mathematics and to train them to use the language;⁽⁵⁾
- (c) mathematics as a means of communicating quantifiable ideas and information;⁽⁶⁾
- (d) to begin to understand the powers and limitations of mathematics;⁽⁷⁾
- (e) the inculcation of a feeling, almost a love, for mathematics;⁽⁸⁾
- (f) to contribute to the general education of the pupils with special emphasis on the development of logical thought and of habits of systematic, accurate and neat methods of working.⁽⁹⁾

In general, the aims of a mathematics course tend to be stated in terms of the utilitarian, disciplinary or cultural value of the subject. Aims stated at such a general level are, however, of little value for classroom practice. What is needed is a set of objectives in terms of the skills and abilities to be developed by the pupils in order to attain the aims of the course.

2.1.3 Specific or Behavioural Objectives

These are also referred to as process objectives which translate the aims of the course into detailed statements of observable pupil behaviour which indicate the achievement of the objectives. These objectives are independent of subject matter and refer to behaviours in terms of thinking (cognitive), feeling (affective) and acting (psychomotor).

For example, the Taxonomy of Educational Objectives⁽¹⁰⁾ provides a detailed classification of behavioural objectives in the cognitive domain. Since most behavioural outcomes in mathematics seem to have cognitive origins⁽¹¹⁾⁽¹²⁾⁽¹³⁾ several researchers have found an adaptation of this classification useful in the teaching and learning of mathematics, particularly in the construction of mathematics test items. Stating objectives in terms of behaviour alone is not sufficient. Having decided on what behaviours are desirable, it is necessary to consider these in conjunction with the subject matter.

2.1.4 Instructional Objectives

An instructional objective is expressed in terms of the behaviour the pupil is expected to exhibit and the subject matter used to produce this behaviour, e.g. (a) ability to recall the group properties of real numbers with respect to addition; (b) ability to apply the Theorem of Pythagoras.

Instructional objectives are thus best represented along two dimensions:⁽¹⁴⁾⁽¹⁵⁾ one of subject matter (content) and the other of behaviours (processes). Such an analysis yields a two-dimensional grid:

P R O C E S S

C				
O				
N				
I				
E				
N				
T				

This grid not only provides a clear picture of the subject curriculum but also serves as a design for an achievement test. Each cell in a grid for mathematics will clearly mark out a particular component of mathematical ability.

2.2 TAXONOMY OF EDUCATIONAL OBJECTIVES : COGNITIVE DOMAIN

2.2.1 Need for a Model of Classification

Having decided on a list of behavioural objectives which are regarded as desirable for a course or examination in mathematics, it is necessary to organize them in some way in order to avoid overlap. It would be useful to classify the objectives into categories or families under headings such as knowledge, application and evaluation, each designed to evoke the associated behaviours. Since mathematics is basically a logical, deductive system in which one proceeds from simple definitions and axioms to more complex proofs of theorems and solutions to problems, the organization of the categories from the simplest to the more complex will further increase the potential usefulness of objectives for the teaching and learning of mathematics.

Avital and Shettleworth⁽¹⁶⁾ warn against the danger of emphasising

low-level or easily attainable objectives, such as mastering notation and terminology, when, in fact, mathematical methods need to be applied to new situations. These authors suggest the need for a comprehensive model of levels of performance from the lowest to the highest so that the teacher will be able to construct the objectives to include the full range of mathematical performance.

Such a model is available in the Taxonomy of Educational Objectives.⁽¹⁷⁾

It is claimed that, in the absence of such a model as a basis to formulate objectives, "the tendency has been, and still is to a great extent, to emphasize only knowledge and comprehension and to avoid the higher mental processes involving understanding and critical thinking".⁽¹⁸⁾

This problem has been aggravated by the inclusion of a wealth of formal definitions and new concepts in mathematics curricula. Thus, teachers may easily be led to emphasize simple recall and comprehension and neglect the development of the applicational ability in mathematics. Clearly, the latter will require the acquisition of higher level cognitive abilities. This necessitates the organization of all cognitive abilities in such a way that it is possible to distinguish between the higher level and the lower level abilities.

2.2.2 The Structure and Function of the Taxonomy

The Taxonomy of Educational Objectives Handbook I: Cognitive Domain,⁽¹⁹⁾ which will be referred to as the Taxonomy in this treatise, has been acclaimed as one of the most useful frameworks for the classification of educational objectives. This classification represents one of the three interacting areas of behaviour roughly corresponding to thinking, feeling and acting. These areas of behaviour are denoted by three domains: Cognitive, Affective and Psychomotor. Handbook II concerns



itself with the Affective Domain⁽²⁰⁾ which contains objectives dealing with attitudes, values, interests, appreciation and social-emotional adjustment. The Psychomotor Domain has been planned to deal with objectives relating to manual and motor skills.

It has already been pointed out that most behavioural outcomes in mathematics seem to have cognitive origins. Therefore, the Taxonomy which provides a detailed classification of behaviours in the cognitive domain has special significance for mathematics and hence for this study. Thus, this and the following two sections are devoted to a description and discussion of the Taxonomy.

The Taxonomy identifies six major categories of cognitive behaviours (those behaviours primarily involved in thought processes) which are listed in the following order:⁽²¹⁾

- 1.00 Knowledge
- 2.00 Comprehension
- 3.00 Application
- 4.00 Analysis
- 5.00 Synthesis
- 6.00 Evaluation

The Taxonomy is seen in terms of two parts: (i) knowledge, and (ii) the development of those intellectual abilities and skills which are necessary to use the knowledge. The categories of the Taxonomy are further divided into several sub-categories.⁽²²⁾ For example, Comprehension has three sub-categories: Translation, Interpretation and Extrapolation.

It has been suggested that the Taxonomy basically grew out of attempts

"to resolve some of the confusion in communication which resulted from the translation of such general terms as 'to understand' into more specific behaviours".⁽²³⁾

Thus, the Taxonomy was conceived as "an educational-logical-psychological classification system".⁽²⁴⁾ The terms in this order express the emphasis placed on the different principles on which the Taxonomy was developed. Krathwohl⁽²⁵⁾ explains the structural organization of the Taxonomy as follows:

"It makes educational distinctions in the sense that the boundaries between categories reflect the decisions that teachers make among student behaviours in their development of curricula and in choosing learning situations. It is a logical system in the sense that its terms are defined precisely and are used consistently. In addition each category permits logical subdivisions which can be clearly defined and further subdivided as necessary and useful. Finally the Taxonomy seems to be consistent with our present understanding of psychological phenomena."

The generic term "taxonomy" indicates that the classification is not arbitrary but represents "something of the hierarchical order of the different classes of objectives".⁽²⁶⁾ The objectives are organised in such a way that each successive category is built upon, and is in turn dependent upon, those which precede it. This arrangement of the objectives from the simple to the complex is based on the assumption that a simple behaviour becomes integrated with other similar behaviours to form complex behaviours. Seen symbolically, if behaviour

A forms one class, behaviour type AB will form another, while type ABC will form yet another class:



The authors of the Taxonomy suggest that one of the implications of the hierarchical structure is that there should be a relationship between complexity of behaviour and facility of problem solving.⁽²⁷⁾ For example, problems requiring behaviour A alone should be correctly answered more often than problems requiring behaviour AB.

The presentation of the Taxonomy includes the definition of objectives in three ways, provided by:

- (i) a verbal description or definition of each class and subclass, where every effort has been made to describe the major aspects of each category as carefully as possible;
- (ii) a list of illustrative educational objectives which are included under each subclass of the classification;
- (iii) several illustrative examination questions and test items which clarify the behaviour appropriate to each category.

The last type of definition is considered by the authors as representing "the most detailed and precise definitions of the subclass since it includes the tasks the student is expected to perform and the specific behaviour he is expected to exhibit".⁽²⁸⁾

2.2.3 Uses of the Taxonomy

The Taxonomy has been regarded as a useful classification device in that it has provided a basis for precision in the communication of objectives by teachers, administrators, professional specialists and research workers who deal with curricular and evaluation problems. For almost two decades since its publication the Taxonomy has proved to be a valuable tool for educational research workers in aiding them to formulate hypotheses about the learning processes. Its "neutral" structure allows for a wide range of applicability to "educational programs which can be specified in terms of intended student behaviours".⁽²⁹⁾ Thus it becomes useful to teachers and testers, in different educational systems, for providing a basis for suggestions regarding methods of developing curricula, instructional procedures and testing techniques.

Several research studies have shown how the Taxonomy has been utilized and studied in a variety of ways. Some of those that bear relation to the present work are reviewed here.

McFall⁽³⁰⁾ used the Taxonomy as a method for classifying test items of an experimental achievement test. The test was constructed to identify and evaluate the ability to recall specific material and to deal with higher level cognitive tasks in general science, grades seven through eleven. Data were produced to support the hypothesis that a significantly low correlation exists between performance on test items requiring higher level cognitive behaviour and performance in current tests evaluating student achievement. McFall suggests that the use of tests involving largely recall items limits the measuring of learning outcomes and might restrict the type of learning which might occur.

Pfeiffer and Davis⁽³¹⁾ used the Taxonomy to analyse teacher-made semester examinations for ninth grade courses. It was found that the similarity between cognitive objectives was indicative of the great emphasis placed on the acquisition of knowledge and the mental process of memory in these examinations. The authors suggest that, while teachers may very well have been unaware of such emphases, these test items did not reflect the entire range of objectives which were implemented in the classroom.

Marksberry *et al*⁽³²⁾ used the six levels of the Taxonomy to analyse recommendations of some national committees and the cognitive objectives from a group of selected texts at the elementary level in language arts, mathematics, reading and social studies. It was found that all six categories were implied in the objectives inferred from the recommendations of the national committees, but not all six were in the objectives from texts analysed.

Klein⁽³³⁾ demonstrated the use of the Taxonomy in constructing tests for primary school pupils. It was shown that a paper-and-pencil instrument can be developed for the age range of seven, eight and nine, which will measure separately and distinctly the behaviours defined in the Taxonomy. Klein suggests that the Taxonomy is not just a theoretical definition of cognitive behaviours but a valuable tool to educators concerned with developing the rational powers of our future citizens.

Wood,⁽³⁴⁾ in reviewing several classifications of objectives in the teaching of mathematics and suggesting a scheme based on the Taxonomy for the Item Bank Project, observes that all these classifications "postdate the Taxonomy and each owes some debt to it". Lewis,⁽³⁵⁾

in a similar approach to objectives in the teaching of science, suggests three broad categories of objectives and points out the resemblance of each to one or more levels of the Taxonomy.

In general, these studies point to the increasing usefulness of the Taxonomy as a tool in education and educational research. Perhaps, the greatest value of the Taxonomy lies in the field of evaluation. Several other studies⁽³⁶⁾⁽³⁷⁾⁽³⁸⁾⁽³⁹⁾⁽⁴⁰⁾⁽⁴¹⁾⁽⁴²⁾ illustrate the use of the Taxonomy, or an adaptation of it, in test construction.

2.2.4 Validation of the Taxonomy

The Taxonomy is a theoretical construct which, it would seem, is based on the following major assumptions:

- (a) The processes which it stipulates are behavioural and all the behaviours are cognitive processes.
- (b) The arrangement of the categories is hierarchical and cumulative.

The Taxonomy is clearly the result of logical and psychological analysis which is based on the experiences of several examiners, educators and psychologists rather than on empirical evidence. While there is little doubt about its communicability and usefulness in education and educational research, few research studies concerning the existence of empirical foundations for the Taxonomy are available.

The authors of the Taxonomy have pointed out that communicability and usefulness are not sufficient conditions for validity and that "a taxonomy must be validated by demonstrating its consistency with the theoretical views in research findings of the field it attempts to order".⁽⁴³⁾

Kropp *et al*⁽⁴⁴⁾ stated that sound evidence concerning the validity of the Taxonomy was not available. They presented some of the major problems relating to response measures, test formats and other observational conditions, and selection of valid statistical methodology, which need to be considered in an attempt to provide evidence of validity. However, they suggested that the Taxonomy was becoming a tool of growing importance to developers of curricula, tests and teaching methods.

Smith⁽⁴⁵⁾ used hierarchical syndrome analysis⁽⁴⁶⁾⁽⁴⁷⁾ to investigate possible ways of combining the cognitive classifications suggested in the Taxonomy in order to validate the assumption that the cognitive processes are cumulative and hierarchical. In general, the analysis of data gathered from responses to four taxonomic tests supported the Taxonomy rationale of a cumulative and hierarchical continuum of cognitive processes. However, the Knowledge and Evaluation categories were found to be inconsistent with the theoretical formulation.

In one of a series of studies designed to investigate the empirical validity of the Taxonomy, Stoker and Kropp⁽⁴⁸⁾ found that there was inter-judge agreement with respect to cognitive processes being sought in test items. The data gave general support to the hierarchical structure with a possible misplacement of the Evaluation category. Factor analysis, however, failed to support the hypothesized structure of the Taxonomy.

In a Schools Council research study⁽⁴⁹⁾ on examinations, involving the use of different types of test items, the mathematics panel classified the test items according to four levels of cognitive behaviour (*viz.* Comprehension, Application, Analysis and Synthesis)

based on the Taxonomy. Candidates taking the main tests were also rated by their teachers according to the same schedule. It was found that the correlations of the tests with teacher ratings ranged in size according to the hierarchical order of the categories. The panel concluded that the data suggested "that it is possible to devise papers which test specified taxonomic levels of mathematical ability".⁽⁵⁰⁾

The authors of the Taxonomy have subjected the classification scheme to a series of checks, primarily of communicability and comprehensiveness.⁽⁵¹⁾ In terms of communicability the most complete test of the classification has been their attempt to classify a large number of test items. The authors report that the major problem revealed by the study was that in all cases it was necessary to know or assume the nature of the examinees' prior educational experiences. It was concluded that, in general, "test material can be satisfactorily classified by means of the taxonomy only when the context in which the test problems were used is known or assumed".⁽⁵²⁾ In terms of the comprehensiveness of the Taxonomy the authors claim that "as yet, in the cognitive domain, we have encountered few statements of student behaviours which could not be placed within the classification scheme".⁽⁵³⁾

In general, the above researches tend to support the basic assumptions on which the Taxonomy is founded.

2.2.5 Criticisms of the Taxonomic Approach

Since the Taxonomy is itself a classification of intended behaviours, several of the criticisms are, in general, similar to those raised against the statement of objectives in terms of behaviour. The need to state objectives behaviourally has already been stated in the preceding chapter. It may further be argued that, since this research rests on

the basic assumption (as stated earlier) that education is a process for changing behaviour, any criticism against stating objectives behaviourally must be regarded as irrelevant to the use of the Taxonomy in this study.

Although the authors of the Taxonomy claim that it is neutral and does not rest on any one view of education,⁽⁵⁴⁾ the fact remains that it is a classification of intended behaviours (as they themselves admit⁽⁵⁵⁾); and as such, "only those educational programs which can be specified in terms of student behaviours can be classified".⁽⁵⁶⁾ This would seem to be an apparent contradiction in an otherwise logical structure.

It is also claimed that the Taxonomy includes all possible behaviours and, therefore, any objective which describes an intended behaviour should be classifiable in this system.⁽⁵⁷⁾ The essential behaviours related to manipulative and computational skills in mathematics do not, however, seem to fit into any particular category of the Taxonomy. This need for a broad category relating to skills in mathematics has been recognised by the several adaptations of the Taxonomy, which will be presented later.

Some critics⁽⁵⁸⁾ have argued against the assumptions regarding the hierarchical structure of the Taxonomy. In general, these arguments are theoretical and lack supporting empirical evidence. The authors of the Taxonomy carefully argue the basic problems of ordering phenomena in ways which will reveal their essential properties and interrelationships and suggest that, in the absence of "a larger synthetic theory of learning", the order used in the Taxonomy is consistent with research findings.⁽⁵⁹⁾ The research studies reviewed in the previous section also generally support the basic assumptions underlying the hierarchical structure of the Taxonomy.

Ausubel⁽⁶⁰⁾ claims that few curriculum specialists are trained to define objectives behaviourally and that behavioural terminology more often obscures than clarifies objectives. Firstly, the lack of trained specialists does not point to a problem with the defining of objectives behaviourally, but rather to a lack of foresight in the administration of education. Secondly, it must be pointed out that the terms used in the Taxonomy are defined and used (consistently) in a special way. Clearly, there will be some misunderstanding if they are seen to "have different meanings for psychologists and educators of different theoretical persuasion".⁽⁶¹⁾

It may be concluded that the Taxonomy has been successful in establishing a classification device which communicates objectives precisely. It has also been shown that it is a fairly comprehensive scheme which has been found to be useful in education and educational research in general and in the field of educational measurement in particular. Moreover, the available empirical evidence tends to support the hypothesis that the Taxonomy is hierarchical in structure.

2.3 CLASSIFICATIONS OF OBJECTIVES USED IN MATHEMATICS EDUCATION

The Taxonomy is intended to have universal application and hence it is phrased in general terms so that it does not relate to any particular subject matter. It has been stated earlier that "it is particularly relevant to mathematics where most significant behaviours appear to have cognitive origins".⁽⁶²⁾ Further, its applicability to mathematics is increased by its logical and hierarchical structure. The Taxonomy, therefore, readily lends itself to an adaptation to mathematical performance. Several adaptations suitable for use in mathematics

teaching have been produced. These have served to increase the potential usefulness of the Taxonomy to both the mathematics teacher and the research worker in mathematics education. Some of these classifications which have been developed for use in overseas projects are presented here.

2.3.1 The Indian National Council of Educational Research (63)

The major categories are as follows:

Objective I: The pupil acquires *KNOWLEDGE* of mathematical terms, symbols, concepts, assumptions, principles, formulae and processes.

Objective II: The pupil develops *SKILL* in

- (a) handling the mathematical instruments;
- (b) drawing geometric figures and graphs;
- (c) reading tables, graphs, etc.;
- (d) computation.

Objective III: The pupil acquires *UNDERSTANDING* of mathematical terms, symbols, concepts, principles, formulae and processes.

Objective IV: The pupil *APPLIES* the knowledge of mathematics to unfamiliar situations.

Each of the categories has several subcategories of detailed outcomes, for example:

- (i) the pupil recognises terms, instruments, processes, etc.;

- (ii) the pupil reads tables, charts and graphs quickly and accurately;
- (iii) the pupil verbalizes symbolic relationships and vice versa;
- (iv) the pupil selects most appropriate formula, method or process to solve a problem.

While there is a striking similarity between this scheme and the Taxonomy, Objective II is clearly a useful departure which takes care of the manipulative and computational skills which are essential to mathematics learning. Wood suggests that this scheme offers "a detailed inventory of terminal behaviour which may be of more immediate practical use than some of the other schemes".⁽⁶⁴⁾

2.3.2 The International Study of Achievement in Mathematics (IEA)⁽⁶⁵⁾

This was a cross-national study designed to investigate the mathematics achievement of secondary school children in twelve countries. As a starting point for the construction of the test instruments each of the countries was required to submit a list of behavioural objectives. The research committee then agreed upon the following "short list of behaviours (or objectives) which they believed would be accepted as desirable by most teachers of mathematics regardless of their nationality":⁽⁶⁶⁾

- A. *Knowledge and information:* recall of definitions, notation, concepts.
- B. *Techniques and skills:* solutions.

- C. *Translation* of data into symbols or schema and vice versa.
- D. *Comprehension*: capacity to analyse problems, to follow reasoning.
- E. *Inventiveness*: reasoning creatively in mathematics.

This scheme has much in common with both the Indian National Council Classification and the Taxonomy. However, there appears to be a difference in terminology. "Translation" is seen to be different from "Comprehension" which is concerned with "Analysis". "Application", though not stated as one of the above five broad categories, is included in a slightly more detailed scheme⁽⁶⁷⁾ where a behaviour, such as "ability to apply concepts to mathematical problems", is stated.

It should be noted that the behaviours outlined above are mainly confined to the cognitive domain. The IEA investigation omitted the non-cognitive behaviours and endeavoured to eliminate as far as possible student personality traits.

2.3.3 The Ontario Institute for Studies in Education: Objectives for Mathematics Learning⁽⁶⁸⁾

Avital and Shettleworth⁽⁶⁹⁾ suggested a model of levels of performance in mathematics, based on the Taxonomy, for the upper grades of elementary school and the secondary school. The authors noted three levels in mathematical thinking and distinguished five taxonomic categories of mathematics teaching objectives:

<u>Levels in Mathematical Thinking</u>	<u>Taxonomic Level</u>
recognition, recall	1. <i>Knowledge</i>
algorithmic thinking, generalization	{ 2. <i>Comprehension</i> 3. <i>Application</i>
open search	{ 4. <i>Analysis</i> 5. <i>Synthesis</i>

The authors have argued that although Evaluation is assumed to be the most complex level of performance, in mathematical performance "we cannot distinguish evaluation as psychologically distinct". Thus tasks involving the judgement of the correctness of a proof by internal analysis of the steps, which is an integral part of the process of proof itself, must belong to the category of Analysis or Synthesis.

The authors have also taken cognisance of the fact that the formulation of objectives is closely bound up with evaluation and have accordingly produced several test items to illustrate each level of thinking and the corresponding taxonomic category.

2.3.4 National Longitudinal Study of Mathematical Abilities

The Research and Test Development Section of the School Mathematics Study Group in U.S.A. suggested the following scheme of objectives for the development of mathematics achievement tests for the National Longitudinal Study of Mathematical Abilities: (70)

Levels of cognitive behaviour in mathematics:

Knowing : knowing terminology, facts, properties,
reasons, principles, structure.

Manipulating : carrying out algorithms.

- Translating* : changing from one language to another.
- Applying* : making comparisons.
- Analysing* : analysing data;
recognizing relevant and irrelevant
information;
seeing patterns, isomorphisms, and symmetries.
- Synthesizing* : specializing and generalizing;
formulating problems;
constructing a proof or a problem.
- Evaluating* : validating answers.

These seven levels of cognitive behaviours in mathematics were further grouped into two basic categories: low-cognitive (knowing, manipulating, translating and applying) and high-cognitive (analysing, synthesizing and evaluating). This arrangement clearly implies a hierarchical structure. In this study an attempt was made to establish content validity by mapping out components of mathematical ability resulting from a two-dimensional representation in terms of content areas and major levels of cognitive behaviours.

It is clear that this scheme closely resembles the Taxonomy. The essential differences are the inclusion of "manipulating" and the listing of "translating" as a separate behaviour on its own.

2.3.5 The Item Bank Project⁽⁷¹⁾ (Examinations and Tests Research Unit, NFER)

The Item Bank Project is a research project which came into being,

with the inception of the CSE in England, as a result of the need to produce a school-based examination. The item bank which is a pool (bank) of test questions (items) has been confined to mathematics. One of the major tasks of the project was the preparation of a blueprint which is a detailed specification for the writing of test items. The blueprint was drawn up by reference to "(a) a list of objectives which are regarded as being those that the examination is intended to measure and (b) the relevant content areas".⁽⁷²⁾

A five-point classification of objectives relevant to the teaching situation was adopted for the project.⁽⁷³⁾ The following are the categories of objectives together with some illustrative examples of behaviour assumed under each objective:

A. *Knowledge and information:*

recall of definitions, notations, concepts:

- (i) knowledge of terminology and conventions;
- (ii) knowledge of specific facts;
- (iii) knowledge of principles and generalizations.

All three types of knowledge behaviour are present in the following examples of a pupil's terminal behaviour:

- (a) accepts the idea of a vector as representing direction and force;
- (b) knows that the area of a triangle equals half the base times the height;
- (c) awareness of axioms in geometry and their special status.

B. *Techniques and Manipulative Skills:*

computations, manipulation of symbols.

A skill is defined as "anything that the individual has learnt to do with ease and precision". A pupil who can demonstrate that he can carry out the following tasks is displaying this type of behaviour:

- (a) manipulate formulae involving length, area, volume, capacity, time, speed and money;
- (b) use measuring instruments (micrometer, ruler, etc.) to stipulated accuracy.

C. *Comprehension:*

capacity to understand problems, to translate symbolic forms, to follow and extend reasoning.

- (i) Translation: transforming a communication into other terms, into another language or into another form of communication.

Examples of translation behaviour are:

- (a) translation of illustrations, models, tables, diagrams, and graphs to verbal form and vice versa;
 - (b) translation of geometric concepts expressed in verbal terms, into spatial form.
- (ii) Interpretation: rearrangement of material so as to secure a total view of the content of the message.
Example of interpretation behaviour is:

making inferences from data presented in tabular or graphic form.

- (iii) Extrapolation (and Interpolation): ability to extend any trends perceived in given data and to specify implications and corollaries.

Extrapolation behaviour is exemplified in the following:

- (a) predicting population characteristics from sample data and vice versa;
- (b) extending ideas from one topic or subject to another relevant one.

D. *Application:*

Application of appropriate concepts to unfamiliar mathematical situations. This is the ability to transfer learning from one situation to another. Example of application behaviour is:

"Assuming that all the pages of a telephone directory are of equal thickness, how would you find the approximate thickness of one page? Express your method of obtaining a result in the form of an algebraic formula, explaining clearly the meaning of the letters used".

E. *Inventiveness:*

reasoning creatively in mathematics.

This is the highest level of behaviour and it involves the assembling of elements so as to form a structure or pattern not clearly visible before, and which for a given student is original or unique.

Example of a situation which might induce inventiveness is:

"Say how you would measure the diameter of the moon.

Give actual numbers where possible."

This classification has been derived from both the Taxonomy and the classification of the International Study of Achievement in Mathematics. The higher objectives of the Taxonomy were omitted because they were considered to be beyond the reach of the majority of the population under study. For the same reason "inventiveness" was regarded as an experimental category "which needs a lot more attention before it can be confidently used".⁽⁷⁴⁾

2.4 CLASSIFICATION OF OBJECTIVES FOR MATHEMATICS LEARNING USED IN THIS STUDY

In this study, the Taxonomy is considered to be the essential framework for developing a classification of categories of objectives for mathematics learning at the senior secondary level. The review of the classifications of objectives used in secondary school mathematics, in the previous section, clearly suggests their affinities with the Taxonomy. Certain useful modifications of the Taxonomy, relevant to the teaching of mathematics, have emerged from these classifications.

Using these modifications of the Taxonomy and drawing from the author's own experiences with the content and methodology of mathematics instruction, the following scheme of objectives is suggested for use in the construction of an evaluation instrument in mathematics:

2.4.1 The Categories of Objectives for Mathematics Learning ⁽⁷⁵⁾

The major categories of objectives are labelled A, B, C etc. The subsections (labelled 1, 2, 3 etc.) are presented to clarify and define each major level of behaviour.

A. *KNOWLEDGE*

1. *Specific Facts*

ability to recall definitions of terms, notation (symbols), formulae.

2. *Universal facts or generalizations*

ability to recall axioms/postulates, theorems, conventions, methods, techniques, patterns, structure, conditions (criteria), classifications.

B. *SKILLS*

1. *Manipulative skills*

ability to handle instruments, draw graphs/figures, read tables.

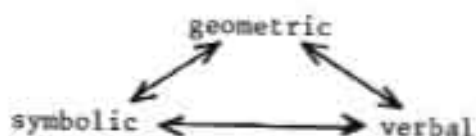
2. *Computational skills*

ability to perform operations, factorise, solve, substitute, change subject of formula.

C. *COMPREHENSION*

1. *Translation*

ability to translate from the verbal to the symbolic and vice versa, from the geometric to the verbal and vice versa, from the symbolic to the geometrical and vice versa:



2. *Interpretation*

ability to illustrate terms/concepts, to explain mathematical terms, notation, concepts and principles in own words.

3. *Extrapolation*

ability to perceive and extend a trend/pattern/idea.

D. *SELECTION-APPLICATION*

1. *Selection*

ability to select appropriately the principle, method, formula, axiom or theorem required for the solution of a problem. Ability to reduce an unfamiliar situation to a familiar situation.

2. *Application*

ability to apply correctly a principle, method, formula, axiom or theorem in a problem situation.

E. *ANALYSIS-SYNTHESIS*

1. *Analysis*

ability to analyse data (parts) with the view to forming relationships, to compare related mathematical concepts/terms, to discriminate between concepts/terms.

2. *Synthesis*

ability to generalize, to establish relationships, to construct problems/solutions/proofs.

3. *Evaluation*

ability to check the validity of a solution, proof or generalization.

The essential difference between this classification and the Taxonomy is that it (i) includes skills and (ii) groups Analysis, Synthesis and Evaluation under one category.

Manipulative and computational skills (as defined in the above classification) are essential in mathematics. While several of the classifications of objectives in mathematics teaching have recognized the importance of this category of behaviour, the Taxonomy makes no provision for it. Several sections of the senior secondary mathematics syllabus,⁽⁷⁶⁾ e.g. factorization, simplification of fractions, substitution in formulae, graphical representation and geometric constructions, lend themselves to this kind of behaviour. Thus, the suggested classification includes manipulative and computational skills under the major category, Skills.

In the suggested classification, Analysis, Synthesis and Evaluation behaviours have been placed under a single broad category : Analysis-Synthesis. In mathematical performance, Analysis and Synthesis appear to be complementary, since the one implies the other and vice versa. For example, the analysis takes place with the view to forming relationships or producing a solution, which clearly implies synthesis. Evaluation involves both analysis and synthesis as pointed out under 2.3.3. In view of these interrelationships among the three behaviours the need for listing them separately as major categories was not recognized.

A prerequisite to applying a principle, method or formula in mathematics is the ability to select (from the several that are available) the appropriate principle, method or formula. Selection is emphasized here as an important aspect of the major category, Selection-Application.

In general, the suggested classification includes all the more important behaviours relevant to mathematics, which have been presented in the Taxonomy and in the other classifications (stated in 2.3) used in secondary school mathematics.

2.4.2 The Suggested Classification of Objectives and Illustrative Test Items

Test items, perhaps, represent the most detailed and precise definitions of objectives. A test item pinpoints the task the pupil is expected to perform and the specific behaviour he is expected to exhibit. In this section each major category of behaviour will be clarified by an appropriate sample test item.

2.4.2.1 Knowledge

"Knowledge" is used in the sense of repetition of material in the form in which it was learned. Knowledge objectives emphasize, most of all, the psychological processes of remembering and recall. Knowledge is usually regarded as the lowest category. It is defined here in terms of the recall of specific facts and universal facts.

Both types of knowledge behaviours are present in the following examples:

- (a) the pupil should be able to state the formula for the area of a triangle;
- (b) the pupil should be able to state the conditions for congruency of triangles;
- (c) the pupil should be able to define "equivalent sets".

It is clear from the above examples that there is a close connection between the statement of an objective and the plan for evaluation.⁽⁷⁷⁾

If, for example, "the pupil should be able to" is removed from each of the objectives, then what remains may be used as an item to assess whether the pupil has attained the objective.

Although Knowledge represents a lower level of mathematical performance, it is indispensable to higher level categories which assume it to be a prerequisite. Clearly, therefore, a pupil cannot perform at a higher level without knowledge of the required facts. In general, the measurement of attainment of a higher level objective will mean that the necessary knowledge is also assessed.

The following examples of test items illustrate the knowledge objectives:

- (i) Objective : ability to recognise set notation.
 Test item : 'X is the subset of Y' is denoted by:

A. $X = Y$

B. $X \cap Y$

C. $Y \subset X$

D. $X \subset Y$

E. $X \cup Y$

- (ii) Objective : ability to recall the property of zero.
 Test item : If a, b are integers which one of the following is *not* true?

A. $a + 0 = a$

B. $b \times 0 = 0$

C. $0 \div b = 0$

D. $b \div 0 = 0$

E. $a - a = 0$

(iii) Objective : ability to state a theorem.

Test item : State the theorem of Pythagoras
for a plane right-angled triangle.

2.4.2.2 Skills

This category includes both manipulative and computational skills. The pupil is required to perform constructions, computations and solutions which are based on techniques developed in the classroom. Although no decision regarding the approach to the solution is required, accuracy in the use of the technique is essential.

The following are examples of objectives relating to Skills:

- (a) the pupil should be able to produce standard geometrical constructions using ruler, protractor and compasses;
- (b) the pupil should be able to factorize expressions of the form, $ab + ac$, $a^2 - b^2$ and $ax^2 \pm bx + c$;
- (c) the pupil should be able to find solution sets of simultaneous linear equations;
- (d) the pupil should be able to substitute in given formulae.

Examples of test items which illustrate the category, Skills:

- (i) Use ruler and compass only to construct $\triangle ABC$ with $AB = 6$ cm, $BC = 8$ cm and $m(\widehat{ABC}) = 90^\circ$.
- (ii) (78) Substituting the values $a = 3$, $b = -2$, $c = 1$ in the expression: $\frac{ab^2 - c}{c^2 + \frac{a}{b}}$ we get:
- A. -22 B. $+33$ C. -39 D. $+15$ E. $+26$.

2.4.2.3 Comprehension

The category, Comprehension (as defined in the classification) is characterised by three types of behaviour: Translation, Interpretation and Extrapolation. Comprehension of mathematical concepts and terminology is basic to mathematical thinking. Objectives under this category must, therefore, ensure a meaningful use of the concepts and terms. Each kind of Comprehension behaviour is illustrated here.

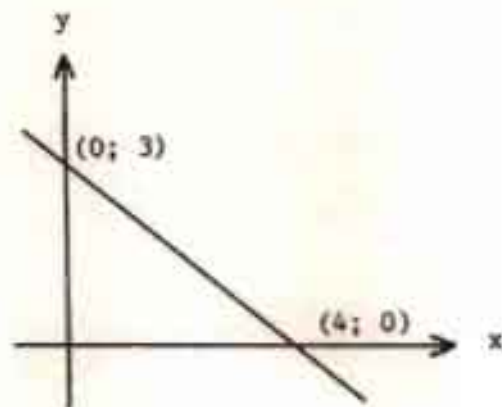
Translation requires the ability to transform one form of communication into another form, e.g. ability to translate from the verbal to the symbolic and vice versa. A great deal of mathematical performance depends on Translation behaviour which is essential to develop a fluent use of mathematical language. Translation behaviour is apparent in the following examples:

- (a) the pupil should be able to represent graphically equations of the form, $y = mx + c$;
- (b) the pupil should be able to translate geometric concepts expressed in verbal form into spatial form; (79)

- (c) the pupil should be able to express in symbolic form a given verbal statement.

Illustrative test items:

(i)⁽⁸⁰⁾



Write down an equation to represent the graph of the given straight line.

(ii)⁽⁸¹⁾

The area of a rectangle is 6 and its perimeter is 10. One of the following equations may be used to find the sides of this rectangle.

A. $x^2 + 5x + 6 = 0$

B. $x^2 - 10x + 6 = 0$

C. $x^2 - 5x + 6 = 0$

D. $x^2 + 10x - 6 = 0$

E. $x^2 - 5x + 5 = 0$

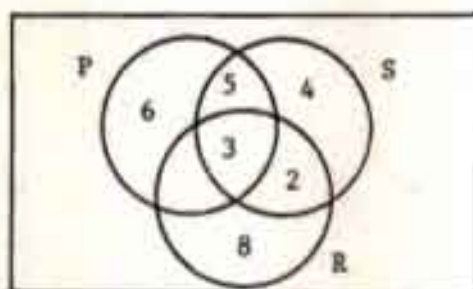
Interpretation requires the ability to illustrate and explain mathematical terms, notation, concepts and principles. It is different from a straightforward translation of a communication in that it implies the recognition of the major ideas included in a communication and their interrelationships. Examples of

interpretation behaviour are:

- (a) the pupil should be able to supply or recognize inferences which may be drawn from a given graph or table of data;
- (b) (82) the pupil should be able to identify the uses of the associative, commutative and distributive laws;
- (c) the pupil should be able to compare related mathematical concepts.

Illustrative test items relating to Interpretation:

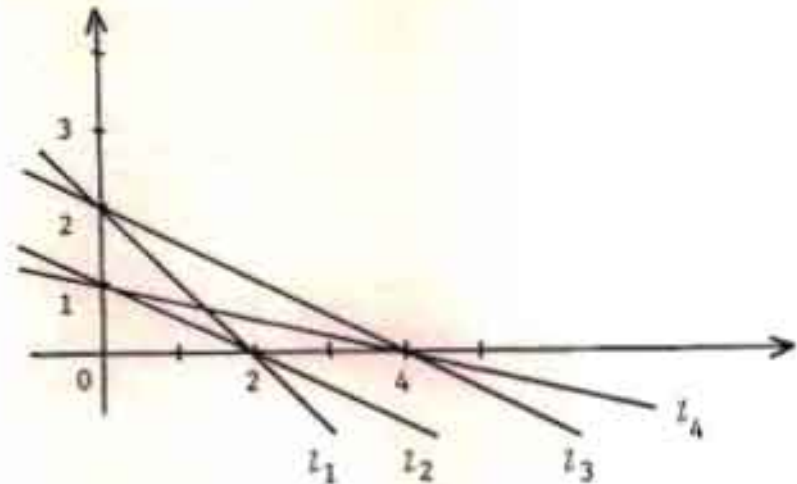
- (i) In the Venn diagram the numerals represent the number of elements in each area.



Find $n[P \cup (R \cap S)]$.

- A. 19
- B. 8
- C. 16
- D. 10
- E. 3

(ii)



Which pair of straight lines has the same gradient?

- A. l_1, l_2
- B. l_2, l_3
- C. l_3, l_4
- D. l_4, l_1
- E. l_1, l_3

Extrapolation involves the ability to extrapolate or extend trends and tendencies beyond the given data. It is an extension of Interpretation behaviour in that the pupil is required to go beyond merely stating the essence of the communication and specify any implications. The following examples of objectives illustrate the category of extrapolation:

- (a) the pupil should be able to perceive the underlying relationship in a sequence and extend it by supplying the next few terms;

- (b) the pupil should be able to fill in (or interpolate) where there are gaps in data, for example, in a given graph;
- (c) the pupil should be able to predict population characteristics of sample data. (In recognizing a pattern he is translating and interpreting the data and in predicting he is going beyond what is given).⁽⁸³⁾

Illustrative test items relating to Extrapolation:

- (i) Fill in the missing numbers in the following sequence:

1, 4, 8, 13, 19, ____, 34, ____.

- (ii) ⁽⁸⁴⁾Examine the following number arrangement:

			1		
			1		1
		1	2		1
	1	3	3		1
1	4	6	4		1

If the rows are continued in the same pattern, what will be the second last number in the fifteenth row?

- A. 1 B. 15 C. 14 D. 16 E. 60
- (iii) ⁽⁸⁵⁾A volume, v , of gas is related to its pressure, p , by the relationship $pv = 16$. When p becomes very small, v will become:
- A. very small B. very large C. almost 16
D. negative E. zero.

2.4.2.4 Selection-Application

This category is usually referred to as Application.

It involves firstly, the correct selection of the methods and principles required for the solution of a problem, and secondly, the accurate application of the selected principles and methods. While the former requires the decision about which principle or method is relevant, the latter requires the decision about the manner in which the selected tools should be organised or used to produce a solution.

If what is learnt is intended for application in real-life situations then the Application category must be regarded as an extremely important aspect of the mathematics curriculum. Application depends on the pupil's ability to transfer learning from one situation to another. Although the ability to transfer also underlies Comprehension behaviour, Application differs from Comprehension in that it presents an unfamiliar situation to the learner.⁽⁸⁶⁾⁽⁸⁷⁾ If a pupil had previously encountered a test item requiring Application behaviour then he need only recall the original situation; in which case, the item will not be testing Application but rather Knowledge, or an aspect of Comprehension.

The following are examples of objectives illustrating the category, Application:

- (a) the pupil should be able to select the most appropriate formula, theorem, method or process to solve a problem;

- (b) the pupil should be able to apply the knowledge and method of solving simultaneous equations to solve a verbal problem ("story problem");
- (c) the pupil should be able to select appropriately and apply the laws of trigonometry to problems involving heights and distances;
- (d) the pupil should be able to apply vector addition and inner product of vectors to geometric problems.

Examples of test items illustrating Application behaviour:

- (i) The height of a triangle is increased by 10% and its base is decreased by 10%. Does its area increase or decrease? If so, by what percentage?
 - A. decrease, 10% B. increase, 10% C. decrease, 1%
 - D. increase, 1% E. area remains the same.
- (ii) Set up a vector diagram and prove that the angle in a semi-circle is a right angle.

2.4.2.5 Analysis-Synthesis

This is a broad category which includes all the behaviours defined under Analysis, Synthesis and Evaluation in the suggested classification.

Analysis involves the breaking down of information in a problem situation and the reorganizing of the parts within the problem. Often it also requires the ability to compare related mathematical concepts and to distinguish between them. Geometric problems, for example, require the careful analysis of given and implied data.

Closely related to Analysis is the Synthesis behaviour which involves the "piecing together" of the relevant parts in order to establish a pattern or structure not clearly visible before. The pupil must be able to recognize the need for certain principles which may, at first, seem unrelated to the problem. Synthesis may even include the ability to reason creatively in mathematics.

Evaluation refers to the ability to check the validity of a solution, proof or generalization.

In general, the Analysis-Synthesis category requires much more than the straightforward application of previously learned principles and concepts. Thus, while Application requires the ability to reproduce previously learned rules and procedures in order to solve unfamiliar problems, Analysis-Synthesis requires the pupil to produce a solution by discovering relationships among certain previously unrelated principles and procedures.⁽⁸⁸⁾ It is necessary to bear this difference in mind when deciding whether to assign a mathematical problem to Application or to Analysis-Synthesis.

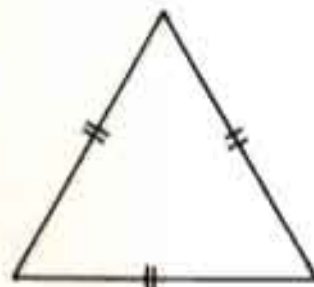
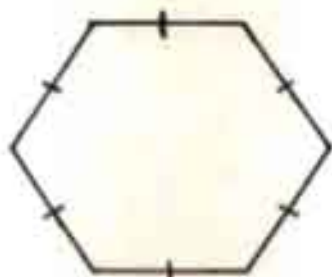
The following are examples of objectives illustrating the category, Analysis-Synthesis:

- (a) the pupil should be able to analyse given and implied information in a geometric problem and establish a relationship;
- (b) the pupil should be able to make mathematical generalizations from a consideration of a variety of results and data;

- (c) the pupil should be able to check the correctness of solutions and proofs by internal analyses of the various steps;
- (d) the pupil should be able to construct a proof or problem new to him.

Examples of test items illustrating the objectives under Analysis-Synthesis:

- (i) (89) Let m and n be any two odd numbers with $n < m$.
The largest integer which divides all possible numbers of the form $m^2 - n^2$ is
- A. 2 B. 4 C. 6 D. 8 E. 16
- (ii) (90)



In the figure, the area of the regular hexagon is 6.
If each side of the equilateral triangle is twice the size of each side of the hexagon then the area of the equilateral triangle is

- A. 3 B. 4 C. 6 D. 9 E. 12

2.4.3 General Comments on the Suggested Classification Scheme

This classification scheme clearly emphasizes cognitive behaviours. It has already been pointed out that most behaviours in mathematics have cognitive origins. In this regard Wilson⁽⁹¹⁾ suggests that "the first concern of evaluation in mathematics learning has been, and will continue to be, cognitive outcomes or achievement". However, it must be stated that, while behaviours in the affective (or even the psychomotor) domain may well have an important bearing on cognitive objectives, research which is necessary in this area is outside the scope of this study.

Since this classification is based on the Taxonomy and other modifications of it, it is implied that those assumptions which apply to the Taxonomy should also apply to this classification. The assumption regarding the hierarchical nature of objectives is apparent in the present scheme. Analysis-Synthesis is, for example, more complex than Application, which is in turn more complex than Comprehension. In this research, Knowledge and Skills are regarded as Lower Level Objectives, while Comprehension, Selection-Application and Analysis-Synthesis are regarded as Higher Level Objectives.

The preparing of test items and the classifying of test items into the various categories of behaviours always present problems in deciding whether a particular item is meant for the one category or another. Therefore, when discussing levels of mathematical performance it is necessary to carefully consider the particular test problem in relation to what the pupil has already been exposed. For example, a problem which may require performance at the Application level for a Standard 8 pupil, may become an instance of recall of knowledge for a more advanced pupil.

The Mathematics Subject Committee⁽⁹²⁾ of the Division of Education of the Department of Indian Affairs has adopted this classification scheme for redrafting the senior secondary mathematics syllabus in terms of instructional objectives. At the time of writing of this thesis the Subject Committee reported that the scheme was being successfully implemented and that the work on the Higher Grade Syllabus was nearing completion.

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CHAPTER THREE

3. CONSTRUCTION OF THE TEST INSTRUMENT

Having decided on a classification scheme of objectives for mathematics learning, the next step was to design test items to measure the extent to which senior secondary pupils were achieving such objectives. The development of test items involved the following steps:⁽¹⁾

- (i) Specifications for the test in terms of instructional objectives.
- (ii) Preparation, review and editing of test items which had to conform to the specifications.
- (iii) Preliminary trial of test items to gauge reliability, validity, difficulty, discrimination, etc.
- (iv) Final selection of test items based on conformity to specifications and on the results of the preliminary try-out.
- (v) Compilation of test and questionnaire in appropriate format for use.

In this chapter the details of the development of the test according to the above plan will be presented and discussed.

3.1 SPECIFICATIONS FOR THE TEST

Test specifications involve a consideration of both the content and the specific objectives, i.e. instructional objectives. A test item will be completely specified in terms of the content area and the specific behaviour which must be exhibited with respect to the particular content.

3.1.1 Selection of Content

Since the target population⁽²⁾ to be tested was selected from standard nine pupils taking mathematics in either the higher or standard grades, the content had to be restricted so that it was within the experience range of this group. It was therefore decided that the content for the test be restricted to the standard eight standard grade syllabus.⁽³⁾ Since the higher grade syllabus contained every topic set down in the standard grade syllabus it was assumed that the content outlined in the standard grade syllabus would be well within the experiences of all the standard nine pupils.

In order to ensure that there was a fair measure of uniformity in pupils' experiences, the problem was discussed with the mathematics teachers at the various schools which were selected to take part in this research project. It was discovered that, in three schools, at least one of the following sections had not been completed (or "covered") in the previous year:

- (i) "5.1.10 Systems of linear equations and inequalities
(in two unknowns)"
- (ii) "5.1.11 Logarithms"

Although the teachers indicated that these sections were completed at the beginning of the following year (i.e. in standard nine) they were omitted for purposes of test construction in this study. The test items were based on the remaining sections of the standard eight standard grade syllabus.

3.1.2 Specific Objectives

The objectives for mathematics learning were discussed in detail in the

previous chapter and a classification scheme, together with illustrative test items, was presented for use in this study. The specific objectives in this classification were stipulated according to the following major categories of behaviours or processes:

- A. Knowledge
- B. Skills
- C. Comprehension
- D. Selection-Application
- E. Analysis-Synthesis

These categories formed the basis for the construction of test items in this study.

3.1.3 The Content-Objectives Grid

Given the content areas and the specific objectives (behaviours or processes) which were to be attained, a content-objectives grid was easily constructed. Such a grid yielded the instructional objectives which had to be considered in constructing the test items. In the construction of the grid each content area of the syllabus was considered in terms of the five categories of objectives. Figure 3.1 below is an illustration of the construction of part of the grid.



CONTENT (Topics from Syllabus) (4)	OBJECTIVES (PROCESSES)				
	A KNOWLEDGE	B SKILLS	C COMPRE- HENSION	D SELECTION- APPLICATION	E ANALYSIS- SYNTHESIS
5.1.1 Sets					
5.1.2 Number Concept					
5.1.6 Algebraic Fractions					
5.1.8 Formulae					

FIGURE 3.1 PART OF THE CONTENT-OBJECTIVES GRID

Provided the content area lends itself to the attainment of the objective in question, each cell in the grid defines certain instructional objectives, e.g. the instructional objective, "ability to recall the definition of intersection", clearly belongs to the first cell in the above illustration.

Since the test instrument was not meant to be an examination for purposes of promotion or certification, it was not necessary to test every section of the selected content. It was, however, essential for this study that every level of behaviour be tested. Therefore, it was decided that an equal number of test items should be constructed for each category of objectives. In a full scale examination, however, it would be essential to use a system of "weighting" which ensures a balance between the objectives and content areas. (5)

3.2 PREPARATION OF TEST ITEMS

3.2.1 Type of Test Item

Objective-type test items framed in such a way as to give only one pre-determined correct answer were preferred. The decision to use this type of test was based on the following advantages it had for test construction in this study:

- (i) a large number of scripts could be rapidly and accurately scored;
- (ii) the objectivity of the marking process ensures a fair degree of reliability;⁽⁶⁾
- (iii) well-constructed objective tests can have an acceptable concurrent validity;⁽⁷⁾⁽⁸⁾
- (iv) objective tests can successfully test a whole range of performance including higher level abilities.⁽⁹⁾

It was also decided that only one form of the objective-type test should be used, viz. the multiple-choice form. The multiple-choice type, which is by far the most popular, consists of the item stem (an introductory question or incomplete statement) and two or more responses (the suggested answers to the questions, or completions of the statement).⁽¹⁰⁾ The responses include one correct response and several incorrect responses, called distractors.

In this study multiple-choice items with five alternative responses were used so that the probability of guessing the correct answer would be minimized.⁽¹¹⁾⁽¹²⁾

3.2.2 Writing the Test Items

In constructing the test items certain basic considerations relating to the reliability and validity of the test, the distractors, the difficulty, the discrimination and the technical aspects had to be borne in mind.

It was assumed that the objectivity in scoring multiple choice-type tests would ensure a reasonable degree of reliability.

Each test item was constructed to conform to the specifications set out in the content-objectives grid in order to ensure a fair degree of content validity and objective validity.⁽¹³⁾⁽¹⁴⁾ It was also recognised that some topics (content areas) readily lend themselves to the achievement of certain objectives, while some objectives are not easily attained through a study of certain topics.

The distractors in each item were made as plausible as possible by compiling them largely on the basis of the errors which pupils were likely to make because they lacked the abilities being tested.⁽¹⁵⁾

Although both the difficulty level⁽¹⁶⁾ and the discriminating power⁽¹⁷⁾ of the test items are attributes which could be accurately ascertained only after the administration of the test, it was useful to keep these aspects in mind during the initial writing of the test items. Based on the author's experience in the teaching and examining of school mathematics, an attempt was made to keep the items within reasonable reach of the target population.

The fact that the pupil's prior knowledge might alter the objective for which an item had been constructed had to be considered. An item on Application could easily become an item of Comprehension or

Knowledge for a pupil who had been exposed to a very similar type of item.⁽¹⁸⁾ Although the chances of this occurring in this study were small because multiple choice-type items were not often used in schools, it was decided that exercises in textbooks (used by the pupils) and the examination papers of the schools concerned should be scrutinized. In this way efforts were made to avoid the reproduction of the types of test items to which pupils might have been exposed.

Careful consideration was also given to certain technical aspects of the format of each test item in order to prevent adverse effects on test reliability. Each item stem was worded or structured as clearly, simply and correctly as possible. The formats of responses for each item were kept as uniform as possible. The correct response positions were varied randomly to avoid any set pattern.

Several other suggestions⁽¹⁹⁾⁽²⁰⁾ made by experts in the field of educational measurement were also taken into account in checking the formats of the test items.

In all, 36 test items with at least 7 in each of the five categories of objectives were constructed.

In the actual construction much difficulty was experienced with items relating to higher level abilities. The items testing lower level abilities were relatively simpler to construct.

3.2.3 Review and Editing of Test Items

Since the initial item drafts were written in pencil with each item on a separate sheet, review and editing were greatly facilitated. Each sheet also carried a code (K = Knowledge, C = Comprehension, etc.) indicating the type of behaviour the item was testing.

Three experienced mathematics teachers (two of whom were teaching standard eight mathematics) were required to work through the test items and to comment on the clarity of the item stems and responses, the appropriateness of the item idea, the plausibility of the distractors and the difficulty level of the items.

Three pupils were also required to work through the test items and to discuss any difficulties they experienced with the author.

On the basis of the discussions with these teachers and the pupils some minor changes were made to seven items. Three other items were considered to be too difficult and hence unsuitable. It was generally agreed that the test of 36 items was too long. All three pupils took more than 130 minutes to complete the test. Since it was the intention to construct a power rather than a speed test, a test which could nevertheless be completed in 90 minutes, the 10 items were omitted. The remaining 26 items were accepted to make up the trial test.

3.2.4 Compilation of Trial Test, Questionnaire and Answer Sheet

A questionnaire was necessary to obtain the personal details of the pupil (e.g. name, age, sex, grade, etc.), his previous performance in mathematics, and (where possible) intelligence test scores. It was essential that pupils were given detailed instructions on how to answer multiple-choice questions in order to avoid any confusion arising out of a possible lack of experience in answering such questions.⁽²¹⁾ It was also felt that it would be useful to ascertain how pupils reacted to the test as a whole.

The whole trial test booklet was thus compiled in four parts:

- Part I : Details to be filled in by pupils (name, age, sex, grade, etc.);
- Part II : The guide to the test, the test itself and the answer sheet;
- Part III : To be filled in by pupils *after* the test (details regarding reaction to the test);
- Part IV : Details to be filled in by mathematics teacher (overall assessment of pupil's mathematical ability, results of standard 8 examination and intelligence test scores).

The first page of the booklet carried Part I and Part IV. The second page contained the guide to the test. This guide included detailed instructions on how to answer multiple choice questions and an instruction not to guess. The next seven pages contained the trial test items with solid lines (across the page) separating one item from another. The trial test items were arranged according to the order of the categories of objectives, i.e. items testing Knowledge came before items testing Skills and these in turn came before those testing Comprehension, etc. The last page contained the answer sheet and Part III.

3.3 PRELIMINARY TRIAL OF TEST ITEMS

3.3.1 Administration of Trial Test

From amongst 132 pupils taking mathematics (at the school used for pilot testing) at the standard nine level 66 were randomly selected⁽²²⁾ to represent both grades and both sexes. These pupils were informed

of the date and time of the trial test. They were also told that the test was well within their reach and that it was not an examination for purposes of promotion.

The 58 pupils who were present on the day of the trial test were required to take the test during the first four periods. The trial test was administered with the help of two senior mathematics teachers who were given all the necessary information on the administration of the test. No exact time limit was set for the test. The pupils were informed that the test would take about 90 minutes. They were also given time for the completion of the questionnaire (Parts I and III). The test was written under usual examination conditions with the mathematics teachers acting as invigilators.

Problems regarding the administration of the test were discussed with the teachers. The pupils were also asked about any difficulties they might have experienced in understanding the questionnaire, instructions and test items.

3.3.2 Analysis of Trial Test Results

The test was scored manually by constructing a stencil. The scores were not subjected to correction for guessing for several reasons⁽²³⁾ pertinent to this study.

The scores were statistically analysed⁽²⁴⁾ to yield means, standard deviations, reliability and validity coefficients and item analysis data. These formed the bases for the selection of items which made up the final drafts for the test. These aspects of test analysis will be discussed in greater detail in chapter six.

3.3.2.1 Distribution of Scores

The frequency distribution of the scores on the trial test is shown in Fig- 3.2. The analysis of these scores yielded the following:

N	= 58;
Mean	= 11,71;
Standard Deviation	= 4,15;
Standard Error of Mean	= 0,55.

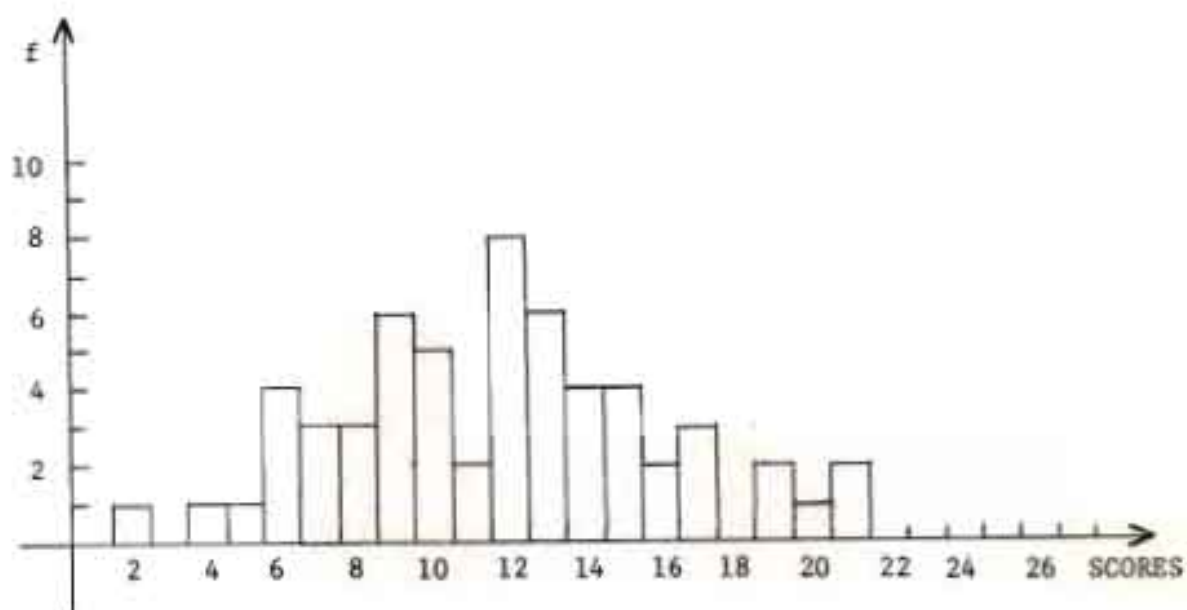


FIGURE 3.2 FREQUENCY DISTRIBUTION OF TRIAL TEST SCORES

In view of the fact that the sample was fairly small and it was drawn from a single school, the distribution of scores was interpreted as being fairly satisfactory. It was expected that the standard error of the mean and the distribution would improve with an improvement in the test and an increase in the sample size.

3.3.2.2 Reliability⁽²⁵⁾

The split-half method was used to calculate test reliability. The Pearson's product-moment correlation was computed for $N = 58$ to yield $r = 0,61$. On using the Spearman-Brown formula for correction of this result, it was increased to $r = 0,76$ ($p < 0,001$).⁽²⁶⁾ It was expected that a larger sample and the re-organization of the test items in pairs of similar difficulty would make this result more dependable.

3.3.2.3 Validity

The test scores were correlated with two sets of criterion scores viz.

- (i) overall assessment of pupil's mathematical ability based on classroom performance, tests, examinations, etc.;
- (ii) results of end of year Standard 8 examination in mathematics.

In the absence of other reliable measures this procedure was accepted as satisfactory for a rough estimate of the degree of validity of the test. The test scores correlated with the above two measures as follows:

- (i) overall assessment of pupil's mathematical ability:
 $N = 58, r = 0,42$ ($p < 0,01$);
- (ii) results of standard 8 examination in mathematics:
 $N = 54, r = 0,50$ ($p < 0,01$).

With an improvement in test quality and increase in sample size, it was expected that these correlations would be more dependable.

3.3.2.4 Item Analysis

The Facility Index and Discrimination Index⁽²⁷⁾ were calculated for each item in order to gauge the difficulty of the items and the extent to which they differentiated between the weaker and brighter pupils. For this purpose the upper 27 per cent and the lower 27 per cent of the scores were used to make the two extreme groups as large and different as possible.⁽²⁸⁾ The item analysis data is presented in Table 3.1.

The average facility index for the first 10 items testing lower level objectives (Knowledge and Skill) was 56,39 while that for the next 12 items testing higher level objectives (Comprehension, Selection-Application and Analysis-Synthesis) was 37,33. Thus, items testing higher level abilities tended to be more difficult than those testing lower level abilities.

The item analysis data provided the basic guide for item selection. However, it was recognised that such data were closely related to the particular sample for which they were calculated. For this reason, other criteria (to be discussed in the next section) were also used in the selection of items for the final form of the test.

TABLE 3.1

ITEM ANALYSIS DATA FOR TRIAL TEST

ITEM	FACILITY INDEX (F)	DISCRIMINATION INDEX (D)
1	63,89	0,50
2	36,11	0,50
3	63,89	0,72
4	41,67	0,39
5	55,56	0,33
6	75,00	0,39
7	61,11	0,44
8	38,89	0,33
9	69,44	0,39
10	58,33	0,28
11	19,44	0,06
12	33,33	0,44
13	41,67	0,72
14	47,22	0,28
15	55,56	0,44
16	19,44	0,39
17	72,22	0,44
18	25,00	0,28
19	50,00	0,56
20	41,67	0,28
21	38,89	0,56
22	52,78	0,28
23	22,22	-0,11
24	5,56	0,11
25	38,89	0,56
26	33,33	0,44

3.4 ITEM SELECTION AND REVISION

The selection of items for the final form of the test involved the consideration of item analysis data, distractors, rating of items according to objectives, and length of test.

3.4.1 Selection based on Item Analysis

It was decided beforehand that items with discrimination indices below 0,20⁽²⁹⁾⁽³⁰⁾ should be rejected or, if necessary, modified. On this basis items 11, 23 and 24 were rejected. It was also found that all three items had very low facility indices (as shown in Table 3.1).

3.4.2 Distractors

An analysis was also made of the number of responses attracted by each distractor. It was found that each of the items 6, 7 and 9 had a distractor which was not chosen by any of the pupils. Since item 6 also had two other distractors with only one response each it was felt that this was a weak item. Although it had a reasonable discrimination index (0,39) it appeared to be a relatively easy item ($F = 75,00$). Item 6 was thus rejected. The other two items were modified in an attempt to make the weak distractors plausible:

(i) Item 7 was presented as follows in the trial test:

Adding $\frac{3m}{b}$ and $\frac{3m}{c}$ we get:-

A. $\frac{3m}{b + c}$

B. $\frac{6m(c + b)}{bc}$

C. $\frac{6m}{b + c}$

D. $\frac{6m}{bc}$

E. $\frac{3mb + 3mc}{bc}$

The weak distractor D was changed to $\frac{9m^2}{bc}$ in order to attract those who multiply numerators and denominators. (31)

(ii) Item 9 was presented as follows in the trial test:

Substituting the values $a = 3$, $b = -2$, $c = 1$

in the expression: $\frac{ab^2 - c}{c^2 + \frac{a}{b}}$ we get:-

A. -22 B. 33 C. -39 D. 15 E. -26

The weak distractor E was changed to +26 and the '+' signs were introduced in the case of B and D to make all distractors uniform. (32)

The '+26' was intended to attract those who calculated:

$$b^2 = (-2)^2 = -4.$$

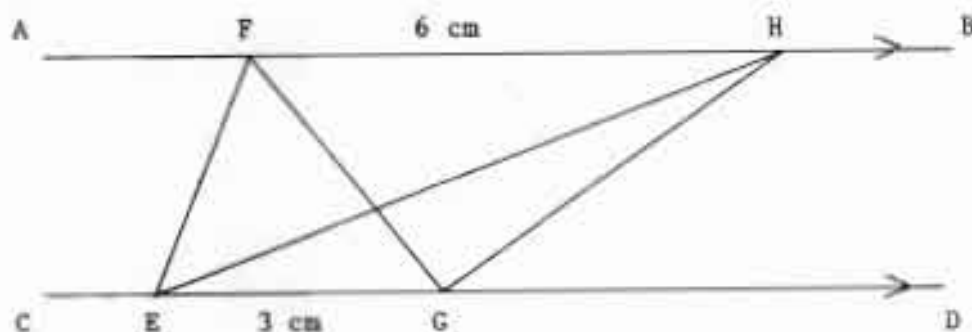
3.4.3 Rating of Items with respect to the Objectives they were Testing

Test items were also selected on the basis of their objective validity. For this purpose, 8 members of the Mathematics Subject Committee,⁽³³⁾ who had had experience in redrafting the syllabus in terms of objectives, were required to rate the test items according to the objectives they were testing.

The items from the trial test booklet were re-arranged so that they were no longer in groups according to the categories of objectives they were testing. A rating sheet⁽³⁴⁾ was prepared. In it the item number, a brief statement of the objective being tested by it, and a suggested classification of the item into one of the five categories of objectives were presented. Each rater was required independently to rate his agreement with the classification of each item in terms of "high", "moderate" or "low". If his agreement was "low" for an item, he was required to give his own classification for that item.

It was found that 12 items received eight out of eight "high" ratings. Items 2 and 11 received 2 "low" ratings each while item 22 received 3 "low" ratings. The remaining items received at most 1 "low" rating each. It was decided that items 2, 11 and 22 should be modified or rejected. Item 11, however, had already been rejected. Since item 2 also had a low facility index ($F = 36,11$), relative to the other items testing Knowledge, it too was rejected.

Item 22 was modified. The item had been presented as follows in the trial test:



In the figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $FH = 6$ cm, $EG = 3$ cm. The area of $\triangle FEG$ is:

- greater than area of $\triangle HEG$.
- less than area of $\triangle HEG$.
- twice area of $\triangle HEG$.
- equal to area of $\triangle HEG$.
- sometimes greater and sometimes less than area of $\triangle HEG$.

It was argued that the item did not require more than the recognition of the fact that "triangles having a common base and lying between the same parallels are equal in area". At most, the item required the ability to interpret a geometric situation. Therefore, it was decided that this item should be modified to demand the higher level ability of Analysis-Synthesis. The raters, subsequently, approved the item in its modified form. ⁽³⁵⁾

3.4.4 Length of Test

It was decided that the final form of the test should be planned for

less than 90 minutes in order to avoid the effects of fatigue that might result from a test of longer duration. From the author's experience with the time taken for the trial test, it was felt that not more than 20 items should be attempted.

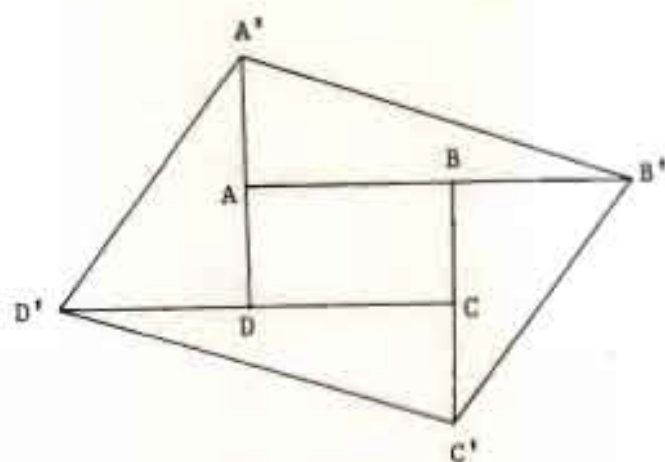
Although 21 items from the trial test were acceptable (three of them having been modified) they were not equitably distributed in terms of the categories of objectives they were testing. The acceptable items were distributed as follows:

Knowledge	4;
Skill	4;
Comprehension	6;
Selection-Application	4;
Analysis-Synthesis	3.

Since an equitable distribution was necessary for a balance among the various categories and for purposes of calculating split-half reliability, it was decided that 4 items in each category would yield the optimal number of items. This meant that the Analysis-Synthesis category needed an additional item while two items had to be omitted from Comprehension.

It was decided that item 24, which was rejected because of the low facility and discrimination indices ($F = 5.56$; $D = 0.11$), should be modified.

Item 24 had been presented in the trial test as follows:



In the figure ABCD is a rectangle. Each side is extended its own length so that $AB = BB'$, $BC = CC'$, $CD = DD'$, $DA = AA'$. If the area of $A'B'C'D'$ is p times as big as the area ABCD then the value of p is:

- A. 5 B. 4 C. 8 D. 2 E. 6

On careful re-examination of the item it was concluded that the proportionality constant " p " presented some difficulty as it introduced a further unknown element. In order to obviate this problem it was decided that a specific value should be given for the area of ABCD and that the pupils should be required to determine the area of $A'B'C'D'$. In this form⁽³⁶⁾ the item would require an analysis and synthesis of the area relationships between the rectangle (with a known area) and the triangles in the figure. This modification, it was expected, would reduce the difficulty level of the item and increase the discrimination index.

In addition, items 14 and 17 were omitted from the 6 which were acceptable for testing Comprehension. This decision was based on the

fact that item 14 had the lowest discrimination index ($D = 0,28$) while item 17 had the highest facility index ($F = 72,22$) among the 6 items (see Table 3.1).

The 20 items, which were finally accepted, included 4 from each of the five categories of objectives. In the final analysis the 20 items were selected from the 26 items of the trial test as follows:

- (a) 4 items (2, 6, 11 and 23) were rejected;
- (b) 2 items (14 and 17) were omitted in order to reduce the number of items under Comprehension;
- (c) 4 items (7, 9, 22 and 24) were accepted with modifications;
- (d) 16 items (1, 3, 4, 5, 8, 10, 12, 13, 15, 16, 18, 19, 20, 21, 25 and 26) were accepted without modifications.

The average discrimination index for the trial test of 26 items was 0,38 while that for the selected 20 items was 0,42. Since this was higher, despite the fact that the 4 items accepted with modifications had low D values (see Table 3.1), it was expected that the selected items would make up a more reliable test. ⁽³⁷⁾

3.5 FINAL FORM OF TEST AND QUESTIONNAIRE

The final selection of the test items having been completed, it was necessary to assemble the materials (instructions, test items, answer sheet and questionnaire) in some meaningful way.

3.5.1 Instructions on Test Booklet

All the instructions relating to the test were clearly and simply

presented on the front page of the test booklet.⁽³⁸⁾

Certain minor changes were made to the original set of instructions contained in the trial test booklet before it was printed in its final form. The first sentence, "This is not an examination" was included in order to dispel any fears and to motivate the pupils. It was also decided, on recommendations by teachers, that the cross (X) be changed to a tick (✓) for purposes of indicating the right answer.

The details on how to answer multiple-choice items included a worked out example. The example itself was made simpler than the one in the trial test booklet in order to save time.

It was felt that the statement, "You will also be required to say what you thought about this test.", would motivate pupils to take the test seriously as their opinions were obviously valued.

3.5.2 Final Form of Test

3.5.2.1 Grouping of Test Items

Although the grouping of test items is usually done according to subject matter,⁽³⁹⁾ it was necessary, in this study, to group them according to objectives. This procedure was considered meaningful⁽⁴⁰⁾ because the research concerned itself with the achievement of objectives. Since the trial test results pointed to an increase in difficulty of items with increase in complexity of objectives, there was also the advantage of arranging the items in an increasing order of difficulty. The latter is also an important consideration in devising an achievement test.

The order of the items had to be considered within each group of 4 items which were selected to test a particular category of objectives. It was decided that the items should be grouped in pairs which were (more or less) of the same level of difficulty. It was expected that this would ensure fairly equivalent forms of the test for purposes of calculation of the reliability coefficient by the split-half method. In the Knowledge category, for example, items 1 and 3, and items 4 and 5 (from the trial test) were grouped because the first two had equal facility indices ($F = 63,89$) while the next two had facility indices 41,67 and 55,56 respectively (see Table 3.1).

3.5.2.2 Format and Layout⁽⁴¹⁾

Having decided on the sequence of the items in this way, it was necessary to reconsider the positions of the correct responses. These positions were randomly varied in order to avoid a set pattern in the correct responses, and hence to minimise any adverse effect on the reliability of the test.

It was also necessary to get the test items into a legible, attractive and economical format in order to ensure that there was no adverse effect on the validity of the test results.⁽⁴²⁾

The alternatives for each item were consistently labelled, (A), (B), (C), (D), and (E). To avoid any confusion, no use was made of these letters in the alternatives themselves. Short alternatives were presented in one line (e.g., items 6, 8) while the longer ones were presented in two lines (e.g., items 5, 7). Others were presented one below another, e.g. items 4, 9.

Each item was clearly numbered and distinguished from the others by two solid lines across the page. The items were printed on one side of the page and the splitting of an item at the bottom of the page was avoided.

3.5.3 Final Form of Questionnaire and Answer Sheet

The final questionnaire and answer sheet (which will be referred to as the data sheet) was printed on both sides of a single sheet of paper in order to facilitate the handling of data. The material was presented in four parts as follows:⁽⁴³⁾

- (A) Details to be filled in by pupils;
- (B) Details to be filled in by mathematics teacher;
- (C) Answer Sheet;
- (D) To be filled in by pupils *after* the test.

The answer sheet (Part (C)) was arranged in two columns, one for odd-numbered items and the other for even-numbered items, in order to facilitate the finding of the two totals for the purposes of calculating split-half reliability. Since the results were computerized, this consideration became unnecessary.

Parts (A) and (D), which had to be filled in by pupils, were made as simple as possible by requiring the pupils to place a tick (\checkmark) in the relevant block.

NOTES AND REFERENCES

1. This plan was adapted from T. Husén (ed.): *International Study of Achievement in Mathematics, Vol. I.* Almqvist and Vicksell, Stockholm, 1967. p. 90.
2. Sampling will be discussed in chapter 4.

3. See Appendix B.
4. For details of the remaining topics see Appendix B.
5. Wood, R.: "Research into Assessment - The Item Bank Project", In: Association of Teachers of Mathematics: *Examinations and Assessment - Mathematics Teaching Pamphlet No. 14*. Lancashire, 1972. pp. 37-38.
6. "Reliability" refers to the consistency with which the test yields its results (for a detailed discussion see chapter six, section 6.1.2).
7. "Validity" refers to the extent to which the test measures what it sets out to measure (for a detailed discussion see chapter six, section 6.1.3).
8. Fraser, W.G. and Gillam, J.N.: *The Principles of Objective Testing in Mathematics*. Heinemann, London, 1972. p. 144.
9. Ebel, R.L.: "Writing the Test Item." In: E.F. Lindquist (ed.): *Educational Measurement*. American Council on Education, Washington, 1951. pp. 195-196.
10. Ibid., p. 195.
11. Lindvall, C.M.: *Measuring Pupil Achievement and Attitude*. Harcourt, Brace and World, New York, 1967. p. 42.
12. Ebel, R.L.: *Measuring Educational Achievement*. Prentice Hall, New Jersey, 1965. p. 150.
13. Fraser and Gillam, op cit., p. 134.
14. Behr, A.L.: *A Textbook of Educational Method*. J.L. van Schaik, Pretoria, 1971. pp. 128-129.
15. Here the author was guided largely by his own experiences as a teacher of mathematics and as sub-examiner for junior secondary and matriculation mathematics examinations. For illustrative examples see section 3.4.2.
16. The "difficulty level" of an item is determined by the facility or difficulty index which is given by the proportion of the candidates in the extreme groups who get the item right. See Table 3.1 for facility values of items in the trial test. See Appendix E (4) for statistical formulae.
17. "Discriminating power" (or discrimination index) of an item refers to the extent to which an item differentiates between bright and weak candidates. See Table 3.1 for discrimination indices of items in the trial test. See Appendix E (4) for statistical formulae.
18. Avital, S.M. and Shettleworth, S.J.: *Objectives for Mathematics Learning*. The Ontario Institute for Studies in Education, Bulletin No. 3, Ontario, 1968. p. 18.

19. Ebel, R.L.: *Measuring Educational Achievement*. op. cit., pp. 151-170.
20. Lindvall, op. cit., pp. 42-43.
21. Most of the pupils had had experience in writing intelligence tests with multiple choice-type items.
22. Every second name on the class register (where the names appear in alphabetical order) was selected.
23. See section 4.2.4.2 for a detailed discussion.
24. See Appendix E for details of statistical methods.
25. See Appendix E (3) for calculation of reliability coefficients.
26. In order to ensure that this reliability coefficient did not depend entirely on the particular split chosen (viz. odd-numbered items and even-numbered items) the Kuder-Richardson Formula 20 was used (see Appendix E (3.2.2)). This yielded $r = 0,71$ ($p < 0,001$).
27. See Appendix E (4) for details of statistical formulae.
28. Ebel, R.L.: *Measuring Educational Achievement*. op. cit., p. 349.
29. Davis, F.B. cited by Klein, M. Frances: "Use of Taxonomy of Educational Objectives (Cognitive Domain) in Constructing Tests for Primary School", *The Journal of Experimental Education*, Vol. 40, No. 3, 1972. p. 44.
30. Macintosh, H.G. and Morrison, R.B. cited by Behr, op. cit., p. 131.
31. See Appendix C, item 5.
32. See Appendix C, item 6.
33. A committee of the Division of Education of the Department of Indian Affairs (see ref. 92 of chapter 2).
34. See Appendix F.
35. See Appendix C, item 17.
36. See Appendix C, item 18.
37. Ebel, R.L.: *Measuring Educational Achievement*. op. cit., p. 364.
38. See Appendix C.
39. Furst, E.J.: *Constructing Evaluation Instruments*. Longmans, Green and Co., New York, 1958. p. 277.

40. Ibid.
 41. See Appendix C for details.
 42. Furst, *op. cit.*, p. 279.
 43. See Appendix D.
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CHAPTER FOUR

4. SELECTION OF SAMPLE AND ADMINISTRATION AND SCORING
OF THE TEST4.1 SAMPLING4.1.1 Definition of Population

The Population in this study was defined as follows:

All Standard Nine pupils studying Mathematics in Indian High Schools in the Durban and District Area.

The standard nine pupils were chosen, in particular, because they made up the only group of pupils who had completed (by the end of 1973) the first year of mathematics at the senior secondary level under the New System of Differentiated Education.⁽¹⁾

4.1.2 Selection of Sample

Since it was necessary to calculate several correlation coefficients and to produce item analysis data, it was decided that a large sample should be selected from the population in order to produce statistically dependable results.⁽²⁾

Fourteen high schools with pupils taking standard nine mathematics were selected to represent the various urban and sub-urban Indian areas of Durban. One other high school was set aside for pilot testing. Table 4.1 shows the distribution of the 14 schools according to the areas. Eleven of them were mixed schools while three were single sex schools (two for boys and one for girls).

TABLE 4.1

DISTRIBUTION OF SCHOOLS (WITH STANDARD NINE
MATHEMATICS CLASSES) ACCORDING TO AREAS

AREA	NO. OF SCHOOLS IN THE AREA	NO. OF SCHOOLS SELECTED
Reservoir Hills	1	(pilot testing)
Clare Estate	1	1
Sydenham	1	1
Asherville	1	1
Durban Central	4	4
Clairwood	1	1
Merebank	1	1
Chatsworth	9	4
Shallcross	1	1

From each of these areas except Chatsworth every eligible school was selected. In Chatsworth the two schools which were drawing pupils from Umhlatuzana Township and Kharwastan (where there were no senior secondary classes) were included, and two more were randomly selected from the remaining seven. The inclusion of all four schools in Durban Central ensured a representative population because these schools were drawing pupils from several suburbs outside this area.

Information regarding the number of mathematics class units in each school and the total roll of pupils in these classes was obtained by interviews with the school principals and mathematics teachers.

Table 4.2 shows the distribution of pupils according to schools, class units and grades.

TABLE 4.2
DISTRIBUTION OF POPULATION ACCORDING TO SCHOOLS,
CLASS UNITS AND GRADES

SCHOOL	NO. OF MATHEMATICS CLASS UNITS	NO. OF PUPILS IN HIGHER GRADE	NO. OF PUPILS IN STANDARD GRADE
1	3	45	40
2	5	33	118
3	5	62	106
4	3	87	23
5	1	30	-
6	5	75	77
7	6	147	58
8	2	41	22
9	7	196	26
10	3	62	39
11	4	130	3
12	4	44	52
13	4	64	49
14	2	36	36
TOTAL	54	1 052	649

Random selection of pupils was not practicable. It would not only have meant the disruption of all the standard nine mathematics classes but would also have presented the problem of room space for testing. The selection of a small number of pupils from each class would have resulted in the need for more rooms (suitable for testing). Such rooms were not available in most of the schools. Thus random selection was perforce abandoned.

The next alternative was selection by class units, which was preferred by the principals and teachers. Since a large sample was required it was decided that about 50 per cent of all the class units would yield a fairly large but manageable sample.

Since the number of mathematics class units ranged from 1 to 7 per school, selecting strictly half of the class units per school would have resulted in several half-class units. This would have raised the problem of having to select pupils to form half-a-class unit. In order to overcome this problem, the following procedure was adopted in deciding on the number of class units to be selected from each school:

Number of units per school	Number of units to be selected
1 - 2	1
3 - 5	2
6 - 7	3

This procedure also ensured that at least one class unit was drawn from each school.

The stipulated number of units were then randomly⁽³⁾ selected from the total number of units in each school. A total of 27 mathematics class units with 851 pupils was selected (see Table 4.3). However, the final sample was determined by the number of pupils (769) who presented themselves for the test. The remaining pupils who were either absent from school or ill on the day of the test had to be excluded from the selected sample.

TABLE 4.3

DISTRIBUTION OF SELECTED SAMPLE AND FINAL SAMPLE
ACCORDING TO SCHOOLS AND CLASS UNITS

SCHOOL	NO. OF SELECTED CLASS UNITS	NO. OF PUPILS IN SELECTED SAMPLE	NO. OF PUPILS IN FINAL SAMPLE
1	2	65	61
2	2	63	46
3	2	68	61
4	2	72	67
5	1	30	27
6	2	67	62
7	3	91	87
8	1	36	34
9	3	88	84
10	2	71	58
11	2	63	53
12	2	53	52
13	2	55	49
14	1	29	28
TOTAL	27	851	769

In general, it may be said that the sampling procedure was satisfactory and it was consistently applied. The final sample ($N = 769$) reflected a fairly large proportion of the population under study, viz, 45.21%.⁽⁴⁾

Since the locations of the schools used in this study indicate a reasonably good geographic coverage of the Indian areas of Durban (see Table 4.1), the final sample was considered to be representative of the population under study.

4.2 ADMINISTRATION OF TEST

The sample having been selected and the test material having been produced in its final form, it remained for the test to be administered. In this section the various aspects of administering the test will be described. These range from certain preliminary arrangements to the final receipt of the completed test materials.

4.2.1 Preliminary Arrangements

Prior approval for the use of the 14 schools for purposes of this research project was obtained from the Director of Indian Education. The testing was scheduled for October of 1974, which was well before the end of year examinations for standard nine pupils.

Personal visits had to be made to the schools in order to make final arrangements for the administration of the test with the principals and teachers concerned. During these meetings a general plan of the research, the possible value of the results to be obtained, and the selection of the units were explained. Arrangements regarding the following aspects were also discussed and finalised:

- (i) date and time of test;
- (ii) room and seating;
- (iii) invigilation and collection of completed test material;
- (iv) informing pupils of the test.

The date was chosen to fit in with the school programme. It was decided that testing should be done at only one school on a particular day so that each testing session could be personally supervised by the researcher. This made it possible to have all the testing done during

the morning sessions. The exact time for the start of the test was agreed upon. This always coincided with the start of a period in the particular school. Three periods were set aside for the test and questionnaire.

Since the class unit as a whole was selected there were no problems in arranging the room and seating. The pupils could write the test in their own classrooms in the same way as they wrote their examinations.

Invigilation presented no problems as those teachers who were due to teach in the selected classes were required to act as invigilators. They were also responsible for the administration of the test and the collection of the completed test material.

The pupils in the selected class units had to be informed of the date and time of the test. They also had to be told that the test was well within their reach and that it was not meant to be an examination for purposes of promotion.

The preliminary arrangements were successfully completed with the co-operation of all the principals and teachers concerned.

4.2.2 Administering the Test

Test material packs were prepared in advance. A pack was made up of three files labelled 'TEST BOOKLET', 'MATHEMATICS QUESTIONNAIRE AND ANSWER SHEET' (data sheet), and 'PAPER FOR ROUGH WORK'. Each pack carried material for 40 pupils. A note to the invigilator, indicating the testing procedure to be followed, was prominently displayed on the pack. The invigilation and testing procedure in respect of each testing session were personally supervised by the researcher.

In general, the typical testing procedure followed in a class unit involved several steps which are presented here.

- (i) The seating arrangement was checked and adjusted where necessary. Each pupil was handed a data sheet (mathematics questionnaire and answer sheet) and was required to fill in part A.⁽⁵⁾
- (ii) The test booklets were handed out. The pupils were required to read the front page carefully and *not* to turn over that page. They were also given the paper for rough work.
- (iii) The pupils were reminded to use a soft lead pencil, *not* to write on booklet and to use answer sheet (part C of data sheet).⁽⁶⁾
- (iv) Pupils were then allowed to turn over the front page of the test booklet and begin the test. They were allowed about 80 minutes to complete the test. Those who were not able to complete the test within this time were allowed up to a maximum of 10 minutes extra.
- (v) Pupils were finally required to fill in part D.⁽⁷⁾ of the data sheet.
- (vi) The test booklets and data sheets were collected separately and checked to ensure that none were missing. The number of pupils who took the test was recorded.

The data sheets for each class unit were placed in a file which bore the name of the school, the class unit and room number, and the number who took the test. These files were left with the principal so that the required details in part B⁽⁸⁾ could be filled in by the mathematics teachers. The completed data sheets were collected from the school at a later date.

4.2.3 Some Comments on the Administration of Test and Questionnaire

Since no questions were raised by the pupils, it was concluded that they had no difficulty in coping with the instructions on the test booklet. In respect of the test items, apart from a typographical error which was queried by two pupils on the first day of testing, there were no further problems.

The test booklets had to be carefully checked, for pencil marks and writing, after each testing session. Those booklets with marks on them were cleaned in order to prepare them for re-use.

Almost all the pupils completed the test well within 80 minutes and a small number (12) were allowed a further 10 minutes to complete the test. This confirmed that the pupils were given enough time to consider all the items.

Some pupils experienced difficulty with item 4 of Part A of the questionnaire,⁽⁹⁾ which was concerned with the "grade" of the mathematics course. These pupils were assisted and their attention was drawn to the fact that the questionnaire concerned itself only with mathematics. The pupils had no difficulty in filling in part D of the data sheet.

In respect of part B of the data sheet, the standard eight mathematics examination results of a few⁽¹⁰⁾ pupils who had been transferred from one school to another were not available. Several of the scores on GTISA (which has been used "to measure certain aspects of developmental intelligence"⁽¹¹⁾) were also not available.⁽¹²⁾ Since item 4 of part B did not specifically state that standardized scores on GTISA were required, some teachers filled in only raw scores while others filled in both raw and standardized scores. All these scores, therefore, had to be personally checked with those on the pupils' record cards. These precautions were essential because the scores had to be used as a set of criterion scores for evidence of validity of the mathematics test.

In general, the administration of the test and questionnaire was considered to be a success. This may be attributed, *inter alia*, to the following factors:

- (a) the considerable time that was spent on the careful planning of preliminary arrangements,
- (b) the careful construction and presentation of the test material and data sheet, and
- (c) the co-operation received from pupils, teachers and principals.

4.2.4 Scoring the Test

4.2.4.1 Scoring Procedures

All the data sheets were collected from the schools within three weeks of the last day of testing. These were counted and checked to see that the total number of data sheets (769) tallied with the

number of pupils who had taken the test. The data sheets were numbered from 1 to 769 in order to facilitate the recording and retracing of information. The next stage was concerned with the scoring of the test.

The actual scoring procedure was greatly facilitated by the specially prepared answer sheet.⁽¹³⁾ Stencil keys⁽¹⁴⁾ cut out from unused answer sheets, were used for this purpose. The simple method of counting correct responses was not sufficient because information on each item was necessary for purposes of item analysis. Therefore, each response had to be clearly marked right or wrong.

Each answer sheet had to be scanned, before marking, to see whether pupils had marked more than one answer to any of the items. Such responses (only three in this study) were marked wrong.⁽¹⁵⁾ Items omitted were also marked wrong. Since pupils were given sufficient time to consider all the items, a lack of response was assumed to be an indication of failure.

Since there was only one correct answer for each test item as reflected in the scoring keys, there was perfect agreement among the three scorers engaged in the scoring. The scored answered sheets were checked independently for clerical errors.

No attempt was made to count up the correct responses in order to arrive at a total score for each pupil because this could easily be incorporated in the computer programme which had to be designed to solve several of the other statistical problems.

In general, it was expected that the objectivity in scoring attained in this study would contribute positively to test reliability.⁽¹⁶⁾

4.2.4.2 Correction for Guessing

When considering the total score for each pupil on an objective test it is necessary to decide whether or not the score should be corrected for guessing.

In this study, it was decided that the correction for guessing should not be applied to the scores. There were several reasons for this decision. Firstly, certain precautions had been taken in the preparation and administration of the test in order to minimise the probability of guessing:⁽¹⁷⁾

- (i) pupils were warned against guessing;
- (ii) the distractors were made as plausible as possible so that they might be selected by pupils through misinformation or incorrect reasoning;
- (iii) the pupils were allowed sufficient time to consider all the items.

It was felt that, under these circumstances, a correction for guessing would tend to over-correct the scores.

Secondly, it may be argued that, since corrected scores "usually rank students in about the same relative positions as do the uncorrected scores",⁽¹⁸⁾ there was no value in such a correction for this study.

Thirdly, it must be pointed out that:

- (i) the probability of getting a respectable score on an objective test by blind guessing alone is extremely small;⁽¹⁹⁾

- (ii) "correction for guessing complicates the scoring task somewhat and tends to lower the accuracy of the scores". (20)

Correction for guessing does, however, become necessary when items with fewer alternatives are used, e.g. true-false items, or when speed tests are used. Neither of these instances apply in the present study.

NOTES AND REFERENCES

1. Department of Indian Affairs, Division of Education: "The New System of Differentiated Education", I.E. Circular No. 28 of 1972, The Department, Durban, 1972.
2. Moser, C.A. cited by Behr, A.L.: *Methods and Techniques in Educational and Psychological Research*. J.L. van Schaik, Pretoria, 1973. p. 6.
3. Each unit in a school was recorded on a piece of paper. All the pieces were thoroughly mixed and one was picked out, recorded and replaced. This procedure was repeated until the required number of units was obtained for that school. The whole procedure was then repeated for each of the other schools.
4. The total population from which the sample was selected was 1 701. See Table 4.2.
5. Details of part A of the data sheet are presented in Appendix D.
6. For details of part C of the data sheet see Appendix D.
7. Details of part D of the data sheet are presented in Appendix D.
8. For details of part B of the data sheet see Appendix D.
9. Details of part A of questionnaire appear in Appendix D.
10. The scores were not available for 9 pupils from the total sample of 769.
11. National Bureau of Educational and Social Research: *Preliminary Manual for the Group Test for Indian South Africans (GTISA) NB824*. NBESR, Pretoria, 1968. p. 1.
12. These scores were not available for 173 pupils from the total sample of 769.

13. See part C of Appendix D.
 14. The correct response for each item of the test is denoted by ' * ' in Appendix C.
 15. Furst, E.J.: *Constructing Evaluation Instruments*. Longmans, Green and Co., New York, 1958. p. 293.
 16. Wesman, A.G.: "Reliability and Confidence". In: Gronlund, N.E. (ed.): *Readings in Measurement and Evaluation*. Macmillan, New York, 1968. p. 201.
 17. Davis, P.B.: "Item Selection Techniques". In: Lindquist, F.F. (ed.): *Educational Measurement*. American Council on Education, Washington, 1951. p. 271.
 18. Ebel, R.L.: *Measuring Educational Achievement*. Prentice Hall, New Jersey, 1965. p. 227.
 19. *Ibid.*, p. 229.
 20. *Ibid.*, p. 232.
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CHAPTER FIVE

5. DATA PROCESSING AND STATISTICAL ANALYSES OF RESULTS

In the previous chapter the administration of the test and questionnaire and the scoring of the test were discussed. The present chapter will deal with the methods which were used to process and statistically analyse the data obtained from the questionnaire and test. The implications of these results will be discussed in the next chapter.

5.1 DATA PROCESSING

Since the sample was large and it was necessary to calculate correlation coefficients and item analysis data, both of which involved a considerable amount of computation, it was impossible to accomplish these calculations within a reasonable length of time without the aid of a computer. It was, therefore, decided that the statistical problems should be solved by computer. This step necessitated the coding and storing of data on punch cards and the programming of the problems which had to be solved.

Numerical codes were used in transferring data from the data sheets to IBM punch cards. Numerical data were punched directly while other data such as sex, grade and test response had to be coded, e.g. male = 1, female = 2; higher grade = 1, standard grade = 2; correct response on a test item = 1, incorrect response = 0. Since all the data were collected on a single data sheet for each pupil, handling and punching of cards became fairly straightforward. The data for each pupil was punched on a separate card. In all, 769 cards with 35 pieces of information on each were punched. Each card was checked against the

corresponding data sheet for possible errors.

The fact that the data was stored in terms of the smallest unit, e.g. each item response was recorded separately, made it possible to retrieve any or all of the data for answering questions pertinent to this study.

Computer programmes⁽¹⁾ were written in the FORTRAN language to determine the following:

- (i) number of elements in each group, e.g. number of pupils taking higher grade mathematics;
- (ii) means;
- (iii) standard deviations;
- (iv) significance of the difference between means, i.e. the determination of z-scores;
- (v) product-moment correlation coefficients;
- (vi) difficulty index and discrimination index for each test item.

Each of the sub-programmes was tested to see whether it was working by executing the programme for 20 data cards. The results obtained in this way were checked against those obtained by the use of an ordinary electronic calculator and by simple counting where possible.

5.2 DISTRIBUTION OF SAMPLE

The sample (N = 769) was analysed in terms of sex, grade (in mathematics), class grade (in mathematics) and age.

5.2.1 Sex and Grade in Mathematics

Table 5.1 shows the distribution of the sample according to sex and grade. It is clear that the sample contained a larger proportion of males and higher grade mathematics pupils than females and standard grade mathematics pupils respectively.

TABLE 5.1

DISTRIBUTION OF SAMPLE ACCORDING TO SEX
AND GRADE (IN MATHEMATICS)

	MALES	FEMALES	TOTAL	% OF TOTAL
Higher Grade	306	130	436	56,70
Standard Grade	223	110	333	43,30
Total	529	240	769	-
% of Total	68,79	31,21	-	100

5.2.2 Class Grade⁽²⁾ in Mathematics

Pupils were required to indicate whether they were taught in a mathematics class with

only higher grade pupils,

or only standard grade pupils,

or mixed higher and standard grade pupils.

Table 5.2 shows the distribution according to class grade in mathematics and the grade of mathematics course taken by individual pupils.

TABLE 5.2

DISTRIBUTION OF SAMPLE ACCORDING TO CLASS
GRADE IN MATHEMATICS AND GRADE OF
MATHEMATICS COURSE

	HIGHER GRADE	STANDARD GRADE	TOTAL	% OF TOTAL
Only Higher Grade pupils	195	-	195	25,36
Only Standard Grade pupils	-	162	162	21,07
Mixed Higher and Standard Grade pupils	241	171	412	53,57
Total	436	333	769	100

A fairly large proportion of the sample was taught mathematics in classes with a mixture of higher and standard grade pupils.

5.2.3 Age

The average age of the standard nine mathematics pupils who made up the sample was found to be 16,57 years ($N = 769$, $SD = 0,987$, $SE_{\text{mean}} = 0,036$). The confidence interval⁽³⁾ for the mean was found to be $16,57 \pm 0,093$ ($p < 0,01$). This was indicative of the homogeneity of the sample population in respect of age.

5.3 DISTRIBUTION OF TEST SCORES

Table 5.3 shows the frequency distribution of the test scores while Figure 5.1 provides a corresponding histogram.

TABLE 5.3

FREQUENCY DISTRIBUTION OF TEST SCORES

TOTAL SCORE	FREQUENCY
0	1
1	9
2	23
3	37
4	69
5	92
6	104
7	104
8	88
9	57
10	57
11	40
12	21
13	17
14	26
15	13
16	6
17	1
18	3
19	1
20	0

$N = 769$; $\bar{x} = 7,41$; $SD = 3,29$

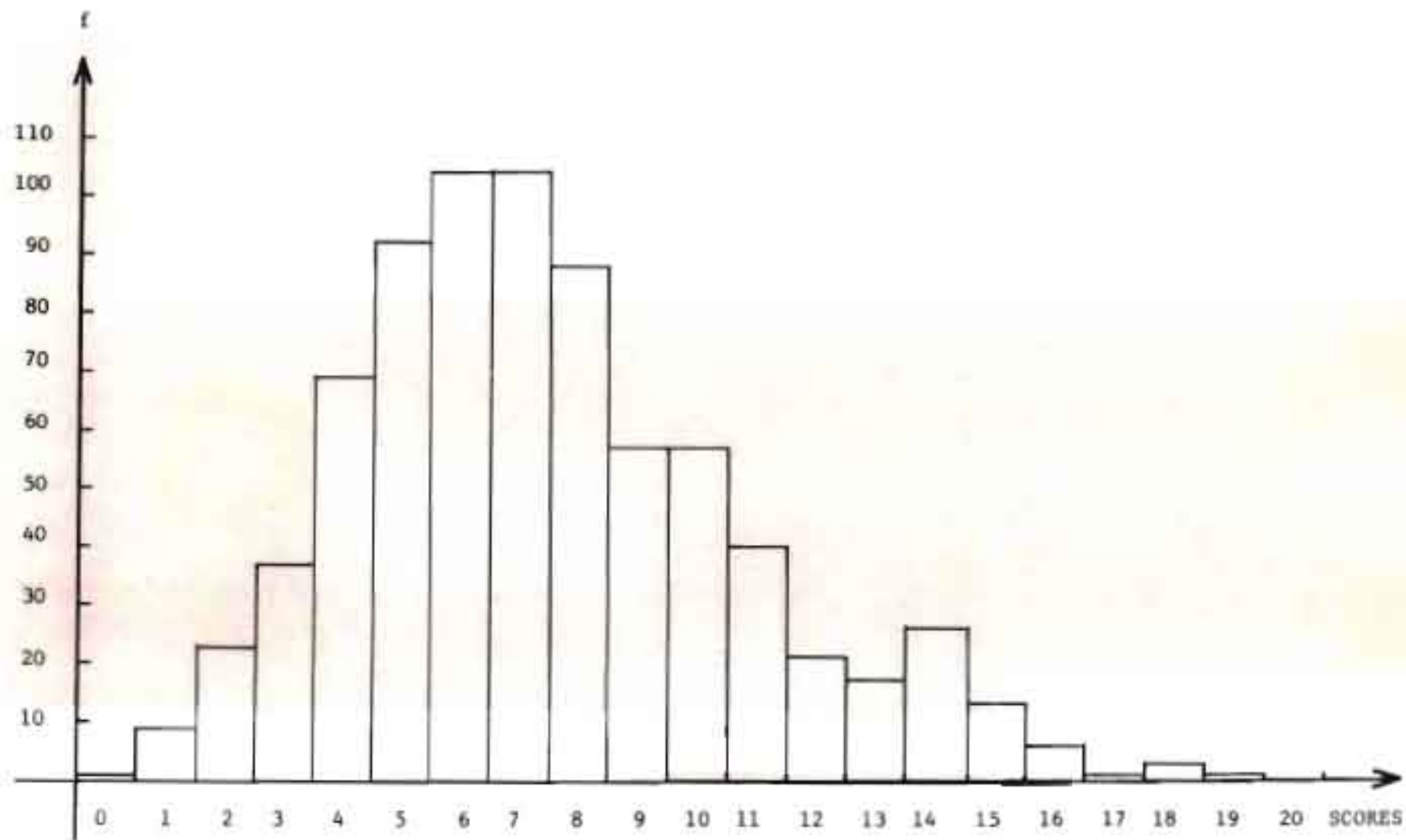


FIGURE 5.1 FREQUENCY DISTRIBUTION OF TEST SCORES

$N = 769$ $\bar{x} = 7,41$ $SD = 3,29.$

The scores ranged from 0 to 19. The standard error of the mean was found to be 0,12 and this yielded a confidence interval⁽⁴⁾ (for the mean) of $7,41 \pm 0,31$ ($p < 0,01$).

In Figure 5.2 the frequency distribution of test scores is shown in terms of class intervals.

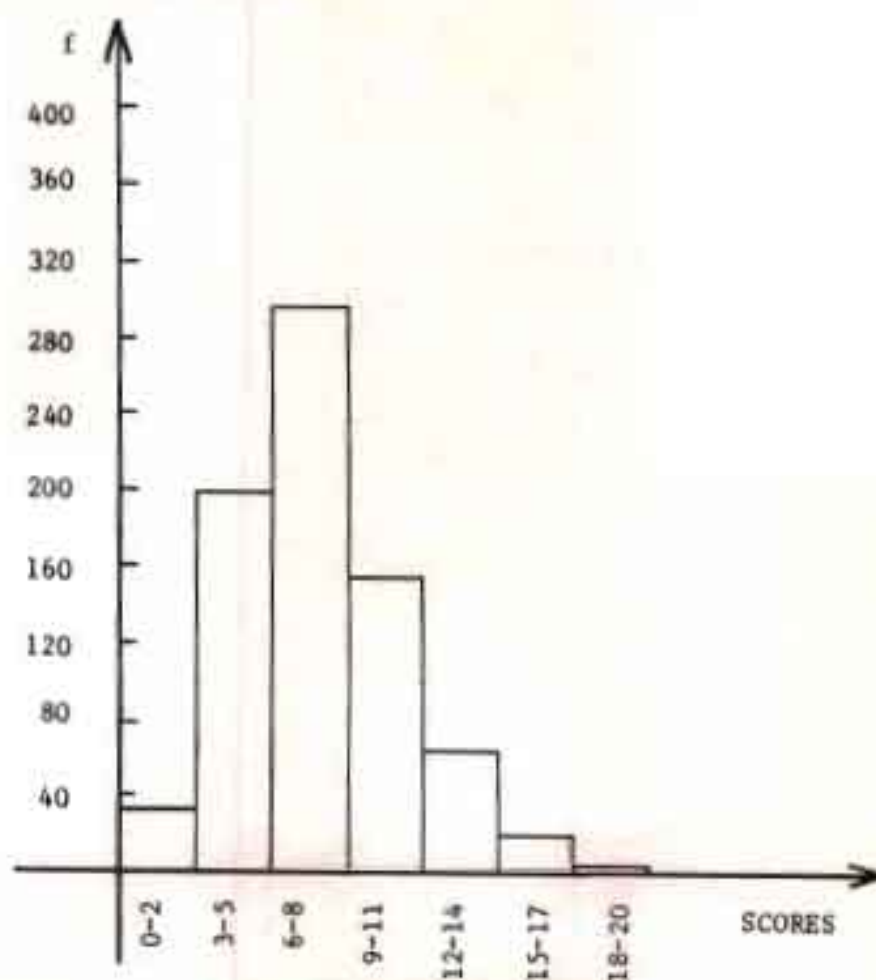


FIGURE 5.2 FREQUENCY DISTRIBUTION OF TEST SCORES
ACCORDING TO CLASS INTERVALS (N = 769)

5.4 RELIABILITY OF TEST

The split-half method and the Kuder-Richardson Formula 20 were used to calculate the test reliability.

5.4.1 Split-Half Reliability

The test items had been organized in such a way that even-numbered and odd-numbered items were balanced as far as possible in respect of the abilities they were testing and the difficulty level of the items. It was, therefore, possible to treat the two halves as equivalent tests for purposes of correlation.

The Pearson's product-moment correlation⁽⁵⁾ was calculated to yield $r = 0,505$ for $N = 769$ and $SD = 3,29$. Since this value of r was the measure of the reliability of a test half as long as the actual test, it was corrected by the use of the Spearman-Brown formula⁽⁶⁾ to yield $r = 0,671$ ($p < 0,001$).

5.4.2 Kuder-Richardson Formula 20⁽⁷⁾

Since the test was not speeded it was possible to apply the Kuder-Richardson Formula 20 in order to obtain a second value for the reliability coefficient. It must be pointed out that while the split-half method depends on the particular test split chosen, the Kuder-Richardson Formula 20 depends on the proportion of candidates responding to each test item.⁽⁸⁾

The value of r obtained in this way was $0,645$ ($p < 0,001$).

5.4.3 General

Each of these methods yielded a reasonably high reliability coefficient which was indicative of the internal consistency of the test.

For $r = 0,671$ the standard error of measurement for the test was found to be $1,887$ which was smaller than the estimated standard error of measurement ($1,932$) for a 20-item test.⁽⁹⁾

5.5 VALIDITY OF TEST

Measurements of validity are provided essentially by measurements of correlation. In order to determine the validity coefficient of the mathematics test, the pupils' scores on the test were correlated with the following three sets of criterion scores:

- (i) teacher's overall assessment of pupils' mathematical ability based on classroom performance, tests, examinations, etc.;
- (ii) results of end of year standard eight examinations in mathematics;
- (iii) full scale (combined) score on GTISA.⁽¹⁰⁾

It was recognised that the criterion scores were not perfectly reliable measures of the attributes⁽¹¹⁾ which this test was attempting to measure. In order to compensate for any possible unreliability at least two measures had to be combined⁽¹²⁾ to yield a correlation with the "true" criterion score.⁽¹³⁾⁽¹⁴⁾ It was therefore necessary to calculate the intercorrelations of the above three sets of scores and the scores on the test. Table 5.4 shows the intercorrelations.⁽¹⁵⁾

TABLE 5.4

INTERCORRELATIONS OF MATHEMATICS TEST SCORES
AND CRITERION SCORES

	a	b	c	d
a. Teacher's overall assessment of pupils mathematical ability	-	0,396	0,296	0,462
b. Scores on standard eight mathematics examination	0,396	-	0,430	0,471
c. Full scale (combined) score on GTISA	0,296	0,430	-	0,417
d. Total Score on Mathematics Test	0,462	0,471	0,417	-
N	769	760	596	769
MEAN	42,20	42,61	112,43	37,06
SD	15,99	15,83	13,05	16,44
All correlations are significant ($p < 0,01$ and also $p < 0,005$).				

The correlation between the mathematics test score and the "true" criterion score was then found as follows:

$$(i) \quad r_{d(ab)} = \frac{r_{da} \cdot r_{db}}{r_{ab}} = 0,550 \quad (p < 0,001)$$

$$(ii) \quad r_{d(bc)} = \frac{r_{db} \cdot r_{dc}}{r_{bc}} = 0,457 \quad (p < 0,001)$$

$$(iii) \quad r_{d(ac)} = \frac{r_{da} \cdot r_{dc}}{r_{ac}} = 0,651 \quad (p < 0,001).$$

The high relationships between the total scores on the test and the criterion scores were indicative of the degree of validity of the test.

5.6 ITEM ANALYSIS DATA

Item analysis data provides quantitative information regarding the difficulty and discriminating power of each test item. The Facility Index (F) and the Discrimination Index (D) were calculated for each item.⁽¹⁶⁾ The F and D values are presented in Table 5.5.

TABLE 5.5

THE FACILITY INDEX (F) AND DISCRIMINATION
INDEX (D) FOR EACH TEST ITEM

ITEM	F	D
1	51,9	0,45
2	51,4	0,56
3	44,2	0,34
4	51,7	0,42
5	64,7	0,49
6	56,5	0,65
7	40,4	0,57
8	52,6	0,50
9	33,4	0,46
10	26,0	0,28
11	31,7	0,48
12	24,8	0,26
13	52,6	0,38
14	20,9	0,26
15	22,6	0,33
16	50,5	0,34
17	22,8	0,12
18	17,1	0,24
19	23,3	0,32
20	35,3	0,51
$\bar{F} = 38,72$		$\bar{D} = 0,40$

The average facility index for the first 8 items (1 - 8) testing the lower level (LL) objectives (Knowledge and Skill) was found to be 51,68 while that for the next 12 items (9 - 20) testing the higher level (HL) objectives (Comprehension, Selection-Application and Analysis-Synthesis) was 30,08.⁽¹⁷⁾ The difference between means (as shown in Table 5.6) was significant⁽¹⁸⁾ at $p < 0,001$. This indicated that the items testing lower level objectives were significantly easier than those testing higher level objectives.

TABLE 5.6

DIFFERENCE BETWEEN MEANS OF F VALUES FOR LL
AND F VALUES FOR HL

PART OF TEST	N (ITEMS)	MEAN F	DIFFERENCE	Σd^2	t
LL	8	51,68	21,60	376,915	4,7401 ($p < 0,001$)
HL	12	30,08		1417,217	

The average discrimination index for the first 8 items testing the lower level (LL) objectives was found to be 0,50 while that for the next 12 items testing the higher level (HL) objectives was 0,34. The difference between the means (as shown in Table 5.7) was significant at $p < 0,001$. This indicated that the items testing lower level objectives were significantly more discriminating than those testing higher level objectives.

TABLE 5.7

DIFFERENCE BETWEEN MEANS OF D VALUES
FOR LL AND D VALUES FOR HL

PART OF TEST	N (ITRMS)	MEAN D	DIFFERENCE	Σd^2	t
LL	8	0,50	0,16	0,0656	3,9528 (p < 0,001)
HL	12	0,34		0,0760	

5.7 DIFFERENCE BETWEEN MEAN SCORES

5.7.1 Mean Scores on Total Test for Different Grades and Class Grades

Mean scores on total test were calculated for the following groups:

- (i) pupils taking higher grade mathematics (H);
- (ii) pupils taking standard grade mathematics (S);
- (iii) pupils from classes with only higher grade mathematics pupils (HO);
- (iv) pupils from classes with only standard grade mathematics pupils (SO);
- (v) pupils from classes with mixed higher and standard grade mathematics pupils (HS);
- (vi) higher grade pupils from the HS group (H(HS));
- (vii) standard grade pupils from the HS group (S(HS)).

Table 5.8 shows the comparisons of the mean scores on the test for the different groups.

TABLE 5.8

COMPARISONS OF MEAN SCORES ON TOTAL TEST FOR
THE DIFFERENT GROUPS ACCORDING TO GRADES AND
CLASS GRADES

GROUP	N	MEAN (\bar{x})	SD	DIFFERENCE BETWEEN MEANS	z-SCORE
H	436	42,60	17,01	12,79	12,106 (p < 0,001)
S	333	29,81	12,23		
HO	195	45,23	17,28	15,41	9,900 (p < 0,001)
SO	162	29,82	11,94		
H(HS)	241	40,46	16,49	10,66	7,443 (p < 0,001)
S(HS)	171	29,80	12,50		
HO	195	45,23	17,28	4,77	2,921 (p < 0,005)
H(HS)	241	40,46	16,49		
SO	162	29,82	11,94	0,02	0,015 (p > 0,05)
S(HS)	171	29,80	12,50		

The difference between the means of SO and S(HS) was not significant (p > 0,05). For all the other groups which were compared the differences were significant (p < 0,005).⁽¹⁹⁾

5.7.2 Mean Scores on Parts of Test for Total Sample

Mean scores were calculated for total sample on the following two parts of the test:

- (i) first 8 items testing lower level objectives;
- (ii) next 12 items testing higher level objectives.

Table 5.9 shows the comparison of the means for the different parts of the test in terms of the levels of objectives.

TABLE 5.9

COMPARISON OF MEAN SCORES ON PARTS OF TEST
(ACCORDING TO OBJECTIVES) FOR TOTAL SAMPLE

PART OF TEST	N	MEAN (\bar{x})	SD	DIFFERENCE BETWEEN MEANS	z-SCORE ⁽²⁰⁾
Lower level objectives	769	50,390	22,322	22,226	29,616
Higher level objectives	769	28,164	15,762		($p < 0,001$)
The correlation between two parts = 0,446 ($p < 0,005$)					

The mean score on the items testing lower level objectives was significantly higher than the mean score on the items testing higher level objectives.



5.8 INTERCORRELATIONS OF SUBSCORES ON MATHEMATICS TEST

Subscores were calculated for the following parts of the test according to the objectives which the items were testing:

- | | | |
|-------|-------------------------|------------------|
| (i) | Knowledge | (items 1 - 4); |
| (ii) | Skill | (items 5 - 8); |
| (iii) | Comprehension | (items 9 - 12); |
| (iv) | Selection-Application | (items 13 - 16); |
| (v) | Analysis-Synthesis | (items 17-20); |
| (vi) | Lower Level Objectives | (items 1 - 8); |
| (vii) | Higher Level Objectives | (items 9 - 20). |

Table 5.10 shows the intercorrelations of subscores on the mathematics test and Table 5.11 shows the significance levels for the differences between certain coefficients of correlation.

TABLE 5.10

INTERCORRELATIONS⁽²¹⁾ OF SUBSCORES ON THE MATHEMATICS TEST

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
(i) Knowledge	-	0,319	0,258	0,213	0,217	0,786	0,329
(ii) Skill		-	0,334	0,215	0,269	0,836	0,392
(iii) Comprehension			-	0,224	0,266	0,367	0,733
(iv) Selection-Application				-	0,206	0,263	0,692
(v) Analysis-Synthesis					-	0,301	0,669
(vi) Lower Level Objectives						-	0,446
(vii) Higher Level Objectives							-
N	769	769	769	769	769	769	769
MEAN (%)	49,32	51,46	26,36	35,34	22,79	50,39	28,16
SD	23,17	24,92	20,08	19,81	16,71	22,32	15,76
All correlation coefficients are significant at $p < 0,01$							

TABLE 5.11

SIGNIFICANCE OF DIFFERENCE BETWEEN CERTAIN
CORRELATION COEFFICIENTS SHOWN IN TABLE 5.10

DIFFERENCE	t-VALUES ⁽²²⁾	LEVEL OF SIGNIFICANCE
$r_{(i)(vi)} - r_{(i)(vii)}$	19,45	$p < 0,001$
$r_{(ii)(vi)} - r_{(ii)(vii)}$	21,29	$p < 0,001$
$r_{(iii)(vii)} - r_{(iii)(vi)}$	14,18	$p < 0,001$
$r_{(iv)(vii)} - r_{(iv)(vi)}$	15,66	$p < 0,001$
$r_{(v)(vii)} - r_{(v)(vi)}$	13,02	$p < 0,001$

5.9 THE CANDIDATES' REACTION TO THE TEST

Part D of the questionnaire⁽²³⁾ required the candidates to indicate their preferences, after the test, as follows:

- (i) Did you think the test as a whole was too hard?
Too easy? About right?
- (ii) If you were taking a mathematics examination and you wanted your knowledge of mathematics to be tested as fairly and thoroughly as possible would you prefer this kind of test? The ordinary kind of test? A mixture of both?

Table 5.12 shows the candidates' reaction to the test.

TABLE 5.12

ANALYSIS OF CANDIDATES' REACTION TO THE TEST

	NUMBER	PERCENTAGE	SIGNIFICANCE ⁽²⁴⁾
(i) 'Too hard'	7	0,91	$\chi^2 = 699,02$ (df = 2, p < 0,001)
'Too easy'	174	22,63	
'About right'	588	76,46	
	769	100,00	
(ii) This kind preferred	223	29,00	$\chi^2 = 291,52$ (df = 2, p < 0,001)
Ordinary kind preferred	82	10,66	
Mixture of both preferred	464	60,34	
	769	100,00	

A significantly high proportion of the candidates claimed that the test was "about right". A significantly high proportion of the candidates also preferred a mixture of the ordinary kind and the multiple choice-type test.

NOTES AND REFERENCES

1. A listing of the computer programme is presented in Appendix G.
2. "Class Grade" refers to one of three types of classes in which the pupil was taught mathematics:

- (i) a class with only higher grade pupils;
 - (ii) a class with only standard grade pupils;
 - (iii) a class with both higher and standard grade pupils.
3. See Appendix E (1.2).
 4. The details of statistical calculations are presented in Appendix E (1.2).
 5. The statistical formula is presented in Appendix E (3.1).
 6. The statistical formula is presented in Appendix E (3.2.1).
 7. The formula is presented in Appendix E (3.2.2).
 8. Fraser, W.G. and Gillam, J.N.: *The Principles of Objective Testing in Mathematics*. Heinemann Educational Books, London, 1972. p. 132.
 9. See Appendix E (3.2.3).
 10. The Group Test for Indian South Africans (GTISA) is used "to measure certain aspects of developmental intelligence, - that is, the inherent intellectual potential that has developed under environmental influences up to the day of testing". (National Bureau of Educational and Social Research, 1968).
 11. Details of the classification of objectives used in the construction of the mathematics test are presented in chapter two, section 2.4.
 12. The details of the statistical formula used for this purpose are presented in Appendix E (3.3).
 13. Fraser, W.G. and Gillam, J.N., op. cit., p. 133.
 14. Cureton, E.E.: "Validity". In: Lindquist, E.F. (ed.): *Educational Measurement*. American Council on Education, Washington, 1951. p. 680.
 15. The Pearson's product-moment correlation formula was used (see Appendix E (3.1)).
 16. Details of the statistical methods are presented in Appendix E (4).
 17. This distinction between higher and lower level objectives has already been made in chapter two, section 2.4.3.
 18. The determination of significance levels is presented in Appendix E (2.3).
 19. The determination of significance levels for uncorrelated data is presented in Appendix E (2.1).

20. The z-score refers to difference between means for correlated data for large samples (see Appendix E (2.2)).
 21. The Pearson's product-moment correlation formula was used (see Appendix E (3.1)).
 22. See Appendix E (3.5).
 23. The details of the questionnaire are presented in Appendix D.
 24. See Appendix E (5).
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CHAPTER SIX

6. DISCUSSION OF RESULTS AND CONCLUSIONS

The previous chapter dealt with the methods of data processing and the statistical analyses of the results. In this chapter the findings from these analyses will be discussed.

6.1 QUALITY OF THE TEST INSTRUMENT

The quality of a test may be judged

- (i) by studying the test itself and the written specifications used in developing it, and
- (ii) by a statistical analysis of the data it provides.

In the present study the former is evident from the development, trial and final administration of the test (as outlined in chapters three and four). In this section the statistical analyses of the test scores will be discussed as evidence of the quality of the test.

6.1.1 Distribution of Test Scores

The data concerning the distribution of the scores on the final test have been presented in section 5.3. The fact that the scores (as shown in Fig. 5.1) range from a low of 0 to a high of 19 on the twenty-item instrument, with a relatively high standard deviation of 3.29, shows that the scores vary widely. This variability is indicative of the test's effective discrimination among the candidates.^{(1) (2)}

A standard deviation of one-sixth of the range between the highest possible score and the expected chance score is considered to be quite

satisfactory and generally the larger it is the better the test.⁽³⁾ The standard deviation of 3,29, which is greater than the expected value ($\frac{20-4}{6} = 2,67$), can thus be regarded as a very satisfactory value.

Although the mean of 7,41 (37,05%) indicated that the test as a whole was a little on the difficult side, it compared favourably with similar experimental tests, e.g.

- (i) the Secondary School Examinations Council⁽⁴⁾ reported a mean of 25,9% for a Secondary Modern group and 38,5% for a Grammar School group on a twenty item multi-facet test;
- (ii) the IEA study⁽⁵⁾ reported a highest mean of 26 (37,68%) on a sixty nine item multiple choice-type test.

6.1.2 Reliability

Reliability is often regarded as the "most significant statistical measure of a classroom test".⁽⁶⁾ Statistical evidence is not only central to establishing the reliability of the test but also has some value in attesting to its validity.

While expertly constructed standardized tests yield reliability coefficients as high as 0,90, achievement tests used in schools and colleges often show reliability coefficients of 0,50 or lower.⁽⁷⁾ In the IEA study for example, reliability coefficients on the seventy-item test ranged from a low of 0,732 to a high of 0,958 for the different countries.⁽⁸⁾

In general, how high or how low a reliability coefficient one is willing to accept in any given case will "depend upon the practical values which are involved in that particular case".⁽⁹⁾ In this regard Kelly⁽¹⁰⁾ arrived at, *inter alia*, a minimum correlation of 0,50 for evaluating the level of group accomplishment.

Since this study is primarily concerned with group accomplishment, viz. the attainment of objectives in mathematics learning by a group of standard nine pupils, a reliability coefficient of 0,50 or higher is acceptable. This would therefore mean that the reliability coefficients, 0,671 and 0,645 ($p < 0,001$) found by the split-half method and by the Kuder-Richardson Formula 20 respectively, are reasonably high values attesting to the internal consistency of the test.

Further, it must be pointed out that these values were obtained on a relatively short test of 20 items. It is generally claimed that an increase in the number of items - provided the added items are equal in quality to those of the original test - tends to increase the reliability.^{(11) (12)} In fact, the doubling of the length of the test used in this study should yield a high reliability coefficient of 0,80 (as predicted by the Spearman-Brown formula⁽¹³⁾).

The absolute consistency of the test is given by the standard error of measurement. The standard error of 1,887 (based on $r = 0,671$ in this study) is indicative of the accuracy with which the test instrument measures. This relatively high value must be attributed to the high variability of the test scores ($SD = 3,29$). Moreover, this value is less than the estimated standard error of measurement

(1,932) for a 20-item test.⁽¹⁴⁾ Under these circumstances even a high reliability coefficient of 0,80 (for the same SD) can reduce the standard error by only 0,416 to 1,471. It may also be argued that "good tests may have larger probable errors of measurement than poor tests"⁽¹⁵⁾ because there is a greater variability (which is desirable) in the scores.

Apart from the statistical evidence produced here, claims for test reliability may also be made on the basis of the following considerations (made in this study):

- (i) the perfect marker reliability that was attained due to the objectivity in scoring;
- (ii) the elimination of administrative unreliability by ensuring uniformity in testing conditions, e.g. all candidates were given enough time to consider all the items;
- (iii) the matching of items according to difficulty;
- (iv) the selection of items for the final test on the basis of discrimination, difficulty and distractors.

6.1.3 Validity

While reliability which is indicative of the degree of internal consistency of a test is a prerequisite for validity, it is not a sufficient condition.

Since the essential question of validity of a test is how well it measures whatever it sets out to measure, validity must always be seen

in terms of the purpose for which the test is used.⁽¹⁶⁾ In this study the claims for validity rest upon

- (i) the soundness and appropriateness of the procedures for developing the test instrument, and
- (ii) the statistical evidence relating to the reliability coefficient and the correlations between the test scores and the "true" criterion scores.

The details of the procedures regarding the development of the test instrument have already been presented in chapter three. There were two considerations which were directly concerned with the validity of the test. Firstly, the test items were prepared, reviewed and selected after trial only if they conformed strictly to the specifications as set out in the content-objectives grid (see Fig. 3.1). Secondly, the final selection of items was based on, *inter alia*, the rating of the items (according to the objectives they were testing) by eight judges. These and other considerations, based on appropriateness of items in terms of item analysis data, should be interpreted as satisfactory attempts at ensuring the "immeasurable" aspects of validity.

When providing statistical evidence for validity, the latter is usually interpreted as "an estimate of the correlation between the raw test scores and the 'true' (that is, perfectly reliable) criterion scores".⁽¹⁷⁾ In the present study the validity coefficients ranged from 0,457 to 0,651 ($p < 0,001$). The relevant data are presented in Table 5.4 in section 5.5.

These values compare very favourably with those obtained for the Group Test for Indian South Africans (GTISA). When the total scores on GTISA were correlated with the examinations the resulting validity coefficients ranged from 0,36 to 0,60.⁽¹⁸⁾ Similarly, in a Schools Council Study, correlations between experimental mathematics tests and the GCE and CSE examinations ranged from 0,523 to 0,647.⁽¹⁹⁾ Therefore, the validity coefficients obtained for the mathematics test in this study are considered to be acceptable.

Furthermore, in respect of the validity of parts of the test, intercorrelations (as presented in Tables 5.10 and 5.11) of subscores on the mathematics test were obtained. Firstly, the scores on each of the sets of items testing Knowledge and Skills correlated significantly more highly with scores on items testing lower level objectives than with items testing higher level objectives ($p < 0,001$). Secondly, the scores on each of the sets of items testing Comprehension, Selection-Application and Analysis-Synthesis correlated significantly more highly with scores on items testing higher level objectives than with items testing lower level objectives ($p < 0,001$).

These findings suggest that the first 8 items on lower level objectives were testing Knowledge and Skills while the next 12 items on higher level objectives were testing Comprehension, Selection-Application and Analysis-Synthesis. Thus the two sets of items were testing two distinctly different components, or levels.

6.1.4 Item Analysis

Item analysis, which provides quantitative information about difficulty and discriminating power of the test items, is usually carried out after the trial test (as was done in this research) in order to aid in

the selection of items for the final form of the test.

However, in this study, item analysis data has also been gathered after the administration of the final form of the test for two reasons:

- (i) to produce evidence concerning the quality of the test, and
- (ii) to provide data to aid in selection and modification of items in the event of future research (based on this test).

The relevant data are presented in Tables 5.5, 5.6 and 5.7.

However, it must be pointed out that "statistical data are at best merely a valuable guide in putting a test together and cannot take the place of scholarship, ingenuity, and painstaking effort on the part of the item writer".⁽²⁰⁾ Therefore, when judging the quality of a test, several of the other factors and considerations (mentioned in chapter 3) in the development of the test items, must be taken into account. Further, like all other statistical data concerning the test, item analysis depends on the characteristics of the sample of examinees tested.⁽²¹⁾

6.1.4.1 Difficulty

The facility indices (as shown in Table 5.5), which indicate the difficulty of the items, range from 17,1 to 64,7 with a mean of 38,72. This data suggests that while the test as a whole was a little on the difficult side it was not composed of only very difficult or very easy items. If this had been the case then the reliability of the scores would have been very low.⁽²²⁾

While items of middle difficulty are desirable in achievement tests, some of these low and high facility indices were unavoidable because the test was designed to test different types of abilities. For this reason, also, no attempt was made to select items on the basis of middle difficulty.

These results which yielded a mean score of 37,05 and a mean difficulty of 38,72 substantiate the claim that "the mean score on a test is determined completely by the mean difficulty of the items composing it".⁽²³⁾

The data (as shown in Table 5.6) suggests that the items testing lower level objectives were significantly easier than those testing higher level objectives ($p < 0,001$). Since no attempt was made in the development and selection of items to regulate the difficulty values, this finding would appear to support the claim by Bloom *et al.*⁽²⁴⁾ that "it is more common to find that individuals have low scores on complex problems and high scores on the less complex problems than the reverse".

In general, the candidates' reaction to the test was favourable (as presented in Table 5.12). The data shows that a significantly high proportion of the candidates regarded the test as being "about right".

6.1.4.2 Discrimination

The discrimination indices (as shown in Table 5.5) indicate the extent to which the items differentiate between the top 27 per cent and the bottom 27 per cent on the total score. These D values range from 0,12 to 0,65 with a mean of 0,40. All the items yielded positive D values and 95 per cent of them yielded D values of 0,24 and higher.

Since each of the upper and lower 27 per cent criterion groups was

fairly large ($N = 208$), the item analysis data presented here must be regarded as being reliable.⁽²⁵⁾

In view of the fact that, in this study, only items with D values of 0,20 and higher were considered acceptable (as suggested by Davis,⁽²⁶⁾ and Macintosh and Morrison⁽²⁷⁾) the above data suggest that the test items were fairly highly discriminating. The fact that the standard deviation was large may also be attributed to the item discrimination.

The data presented in Table 5.8 suggests that the higher grade pupils performed significantly better than the standard grade pupils ($p < 0,001$). Since under the New System of Differentiated Education, the higher grades contain better pupils than the standard grades, this finding lends support to the discriminating power of the test.

While the test as a whole yielded a high mean discrimination index of 0,40, the data presented in Table 5.7 suggests that the items testing lower level objectives were significantly more discriminating than those testing higher level objectives ($p < 0,001$). This finding may be attributed to the difficulty levels of the two sets of items. Since items of 50 per cent difficulty are maximally discriminative,⁽²⁸⁾ it is expected that the lower level items ($\bar{D} = 51,68$) would be more discriminating than the higher level items ($\bar{D} = 30,08$).

6.1.5 General Conclusions

In general it may be concluded that a reasonably reliable and valid paper-and-pencil instrument was constructed to test a range of abilities or objectives in mathematics (as defined by the test items) which go beyond simple recall and manipulative skill.

The discussions in the preceding sections also suggest that it is possible to devise test items which test at least two taxonomic levels of mathematical ability. A Schools Council study⁽²⁹⁾ has reported similar findings for mathematics while Klein⁽³⁰⁾ has come to similar conclusions for geography.

6.2 DISCUSSION OF FINDINGS FROM THE TEST SCORES AND CONCLUSIONS

Since the test was designed to measure a range of abilities in mathematics as set out in the suggested classification of objectives, performance scores on this test must be indicative of the attainment of these objectives.

In this section some of the major findings arising out of the administration of the test instrument to the sample population will be discussed. The claims for validity of the conclusions resulting from these findings must rest largely upon the quality of the evaluation instrument, the evidence for which has already been presented and argued.

6.2.1 Performance according to Levels of Objectives

It has already been shown (in section 6.1.3) that the items testing lower level objectives (less complex items) were predominantly testing Knowledge and Skills while the items testing higher level objectives (more complex items) were predominantly testing Comprehension, Selection-Application and Analysis-Synthesis.

In order to determine the extent to which the pupils in the sample population attained the objectives in mathematics the following null hypothesis was tested:

that there was no difference between performance on items testing lower level objectives and performance on items testing higher level objectives.

The observed data as presented in Table 5.9 showed that the mean score on the items testing lower level objectives was significantly higher than the mean score on the items testing higher level objectives ($p < 0,001$). On the basis of this finding the null hypothesis was rejected ($p < 0,001$). Stated otherwise, this means that the lower level objectives in mathematics were significantly easier to attain than the higher level objectives. This is in agreement with an earlier finding that items testing lower level objectives were significantly easier than those testing higher level objectives ($p < 0,001$), which pointed to the equivalent nature of item complexity and item difficulty.

These findings tend to support the assumption regarding the hierarchical structure of a taxonomic classification of objectives in at least two levels. It has already been shown that several research studies (reviewed under 2.2.4 in chapter two) also support this assumption in respect of the Taxonomy.

It must also be pointed out that, in the absence of a classification of clearly defined objectives for mathematics learning at the senior secondary level, there probably has been a tendency for teachers and examiners to emphasize the easily tangible and measurable lower level objectives and to avoid the less tangible higher level objectives. Moreover, it has been claimed that the extent to which "objectives near the evaluatory extremity (i.e. higher level objectives) are practicable is clearly dependent on the level at which the teaching takes place". (31)

6.2.2 Performance According to Class Grades

An analysis of the distribution of the sample population according to class grades (as shown in Table 5.2) revealed that a large proportion of the sample was receiving instruction in mathematics in mixed classes with both higher and standard grade pupils. This led the present researcher to investigate the extent to which the pupils were attaining the objectives in mathematics in terms of the class grades. For this purpose the performances of the various groups were compared (as shown in Table 5.8).

As expected, a comparison of the performances of the higher and standard grades showed that the higher grades performed significantly better than the standard grades ($p < 0,001$).

In order to compare performances in respect of class grades the following pairs of groups were considered:

(i) H(HS) (higher grade pupils from mixed higher and standard grade classes)

and H0 (higher grade pupils from only higher grade classes);

(ii) S(HS) (standard grade pupils from mixed higher and standard grade classes)

and S0 (standard grade pupils from only standard grade classes).

In accordance with the above comparisons the following null hypotheses were tested:

- (i) that there is no difference in mean performance between H0 and H(HS) on the mathematics test;
- (ii) that there is no difference in mean performance between S0 and S(HS) on the mathematics test.

In terms of the observed data in Table 5.8, the first null hypothesis was rejected ($p < 0,005$). The H0 group performed significantly better than H(HS) group. This difference in performance may be attributed to the sole difference resulting from the composition of the groups for purposes of instruction, i.e. the H0 group was taught in classes with only higher grade pupils while the H(HS) group was taught in classes with mixed higher and standard grades. Owing to the difficulties involved in coping with instruction at two different levels for the two ability groups, it would seem that teachers of the HS group, tended to keep the instruction at the level of the standard grades and failed to differentiate between the two groups.

The second null hypothesis was, however, accepted ($p > 0,05$). There was no significant difference in performance between the S0 and S(HS) groups. This finding is in agreement with the argument presented above. Since instruction in the HS group seems to be aimed at the standard grade level and that in the S0 group is obviously confined to the standard grade level, it may be expected that there would be no difference in performance between the S(HS) and S0 groups.

On the basis of the above findings in respect of the population under study, it is concluded that the performance of the higher grade pupils taught mathematics in mixed higher and standard grade classes tends to be adversely affected.

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CHAPTER SEVEN

7. RECOMMENDATIONS AND PROBLEMS FOR FUTURE RESEARCH

The present research study has given rise to several implications for mathematics education in South Africa, and it has also revealed certain problems for future research and investigation.

7.1 IMPLICATIONS AND RECOMMENDATIONS7.1.1 Objectives

This research has clearly underlined the need for the stating of objectives, and it has demonstrated especially the value of formulating objectives in the development of mathematics test items. Mathematics teachers should be encouraged to study the implications of objectives for mathematics education in general and achievement testing in particular.

7.1.2 Syllabuses

If mathematics examinations are to test "abilities of educational value and not just a series of topics written into a syllabus",⁽¹⁾ then syllabus constructors must first spell out worthwhile instructional objectives (apart from the aims). The next logical step would then be to include in the syllabus only that content material which lends itself to the attainment of the objectives. Since this has not always been the case, pupils, teachers and examiners have been prone to interpreting aspects of the syllabus in widely varying ways. Under these circumstances it is reasonable to expect that the abilities tested in the final examinations are not necessarily those developed in the pupils.

It is thus recommended that (until such time syllabus constructors stipulate clearly defined objectives) the mathematics subject panels of the departments of education use a scheme of objectives (e.g. the one suggested in this study) to redraft the syllabuses in terms of objectives. In this way each topic would be seen in terms of the desirable behaviours it evokes. Such a two-dimensional (content-process) treatment of the syllabus would be invaluable in devising learning experiences and constructing test items. In addition, a weighting scheme should form the basis for a correct balance of items in terms of content and process in the examination.

7.1.3 Instructional Methods

The present research has revealed that there is a great need for mathematics teachers to become aware of the higher level objectives and to devise learning experiences to develop in their pupils the appropriate abilities. In this regard Krathwohl⁽²⁾ suggests that "the learning environment must give major emphasis to the more complex objectives if significant growth is to take place in these objectives".

While the instructional objectives themselves suggest certain methods of teaching, research needs to be done in order to provide "practical assistance to the teacher by trying to find instructional methods which most efficiently and effectively permit achievement of these objectives".⁽³⁾

7.1.4 Examinations and Tests

Since examinations have such a powerful influence on teaching and learning, it is essential to consider the kinds of thinking that tests and examinations demand. It has already been shown that examinations can be improved by basing them on clearly spelt-out instructional

objectives. This will in turn place the teaching and learning of mathematics on a sounder basis. With a content-process blueprint in mathematics as a common source of reference for both teachers and examiners it would be reasonable to expect that final examinations will demand the kinds of abilities which the teachers have attempted to develop in their pupils.

Objective tests, e.g. multiple choice-type items, can play an increasingly important role in mathematics examinations in measuring the various instructional objectives. An analysis of the candidates' reaction to the multiple choice-type test items in this study (as presented in Table 5.12) revealed that a significantly high proportion of the pupils (60,34%) preferred a mixture of the ordinary kind and the multiple choice-type test. Moreover, 29 per cent of the sample population preferred the multiple choice-type test. It is thus recommended that future mathematics examinations should include a greater percentage of multiple choice-type items.

Item-writing is undoubtedly an exacting task, as has been shown by this and other studies. It is an art which requires "an uncommon combination of special abilities. It is mastered only through extensive and critically supervised practice".⁽⁴⁾ Further, it demands "imagination and ingenuity in the invention of situations that require exactly the desired knowledge and abilities".⁽⁵⁾ It must be recognized that not every mathematics teacher is endowed with such specialized abilities. However, these teachers - and hence the pupils - stand to benefit tremendously if opportunities are available whereby they can share expertly written items. In this respect it is recommended that the establishment of an item bank, on the same lines as suggested by Wood's reports of the NFER study,^{(6) (7)} should be given serious

consideration. It has also been claimed that "with assistance available to teachers for understanding how clearly stated objectives should influence the classroom activities and evaluation practices, it may be possible that the use of a bank of test items which represent the range of defined cognitive behaviours will result in a greatly improved educational program in the future".⁽⁸⁾

An increasing understanding and use of tests based on instructional objectives will enable teachers to appraise more meaningfully the levels of performance in terms of the different objectives. On the basis of such diagnostic information, "balanced emphasis can be laid in the instructional programmes on those objectives that are more difficult to achieve".⁽⁹⁾

7.1.5 Differentiated Education

The New System of Differentiated Education has been designed "to ensure that children receive the education that suits their skills, interests, abilities and aptitudes".⁽¹⁰⁾ In the senior secondary phase this system provides for extensive differentiation "by offering various fields of study and in certain approved subjects (e.g. mathematics), within the fields of study, at a higher as well as at a standard level. This then implies that the subjects which are taught at two levels will also be examined at two levels during and at the end of this phase".⁽¹¹⁾ The implication here is that for effective differentiation the two levels should be taught separately.

In this study, however, it was found that 53,5% of the sample population from the 14 high schools was taught mathematics in classes with mixed higher and standard grade pupils. A comparison of the performances of the various groups (as discussed in 6.2.2) led the researcher to conclude

that the teaching of mathematics to mixed higher and standard grade classes was detrimental to the performance of the higher grade pupils who scored significantly lower than those taught in only higher grade classes.

It is thus recommended that, for effective differentiation, mathematics should be taught separately at the two different levels.

Another major problem is that of differentiating between the standard and higher grade syllabuses for purposes of instruction and evaluation. It is suggested that there should be a difference in emphasis according to the levels of objectives as stipulated in a content-objectives weighting scheme. For example, a particular topic may be more heavily weighed under Knowledge and Skills for the standard grade than for the higher grade, while the same topic under Analysis-Synthesis may be given more emphasis in the higher grade than in the standard grade. Fig. 7.1 is presented as an illustration.

		OBJECTIVES					
		CONTENT	A	B	C	D	E
HIGHER GRADE	Topic I	10	20	30	30	10	100
	Topic II	15	25	30	25	5	100
	Total	25	45	60	55	15	200
STANDARD GRADE	Topic I	20	30	25	20	5	100
	Topic II	25	35	25	13	2	100
	Total	45	65	50	33	7	200

FIGURE 7.1 EXAMPLE OF CONTENT-OBJECTIVES WEIGHTING SCHEME SHOWING THE DIFFERENCES IN EMPHASIS. (A, B, ... E REFER TO THE CATEGORIES OF OBJECTIVES SUGGESTED IN THIS STUDY).

It needs to be emphasized that it is not desirable to teach and examine (on the standard grade syllabus) for lower level objectives to the exclusion of all else. This is in keeping with the views expressed in respect of the O and A levels in England.⁽¹²⁾ This study has also demonstrated that it is possible to test for the entire range of objectives.

7.2 SOME PROBLEMS FOR FUTURE RESEARCH

Certain research problems have emerged from this study, which have fallen outside its scope because of its limitations and assumptions.

The results of this study support only in part the hierarchical structure of a taxonomic classification of objectives, i.e. on two levels. A more detailed investigation will be necessary to suggest a

hierarchical order among all the levels of objectives. This will entail construction of an extended evaluation instrument with a greater number of items at each level and the validation of these items in terms of the objectives they test. One of the major problems in this connection is that the final judgement regarding the relevance between the item and the objective it tests will depend almost entirely on expert opinion. This is so because "strictly empirical data on this aspect of validity is almost impossible to obtain".⁽¹³⁾ One approach to the problem may be that proposed by Wood:⁽¹⁴⁾ a factor-analytic study of the item responses, which would cluster items into groups possessing similar characteristics. In this way the speculation about the behaviour which each cluster is supposed to evoke will be placed on a sounder basis.

The test developed for this research may not necessarily be of any prognostic value as its predictive validity still needs to be determined. For this purpose a follow-up study of the sample population would be necessary.

The present study has revealed a difference in the attainment of objectives in mathematics between the two ability groups in standard nine. The question arises whether there are such differences between the various standards or forms in the senior secondary phase. Research must be designed to determine the extent to which the development of the pupil's mathematical ability from standard eight to standard ten influences the attainment of objectives. Such findings will provide useful feedback for future curriculum planning.

In this study, only the cognitive abilities in mathematics have been considered. Krathwohl⁽¹⁵⁾ has stressed the importance of affective objectives by stating that "nearly all cognitive objectives have an

affective component". Research needs to establish the extent to which these objectives are appropriate for school mathematics and whether any provision must be made for their testing.

In fine, it is hoped that this study will serve to stimulate further research. Such research can only lead to a better understanding of the "objectives" approach and its role in examinations, curriculum development and the learning process.

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-

APPENDIX AA SUMMARY OF THE TAXONOMY OF EDUCATIONAL OBJECTIVES:
COGNITIVE DOMAIN. (BLOOM, B.S. (ed.), 1956)

Six major categories of cognitive behaviour are identified in the *Taxonomy*: Knowledge, Comprehension, Application, Analysis, Synthesis, and Evaluation. The behaviours are assumed to be hierarchical and cumulative. The first category is concerned with recall or recognition of knowledge, and the latter five are called the intellectual skills and abilities or the higher mental processes. All the major categories except application are further broken down into more explicit and discrete behaviours. Through this breakdown, twenty-one separate behavioral categories are defined in the cognitive domain of human behaviour.

1.00 KNOWLEDGE

1.10 Knowledge of Specifics

- 1.11 Knowledge of Terminology
- 1.12 Knowledge of Specific Facts

1.20 Knowledge of Ways and Means of Dealing with Specifics

- 1.21 Knowledge of Conventions
- 1.22 Knowledge of Trends and Sequences
- 1.23 Knowledge of Classifications and Categories
- 1.24 Knowledge of Criteria
- 1.25 Knowledge of Methodology

1.30 Knowledge of the Universals and Abstractions in a Field

- 1.31 Knowledge of Principles and Generalizations
- 1.32 Knowledge of Theories and Structures

2.00 COMPREHENSION

- 2.10 Translation
- 2.20 Interpretation
- 2.30 Extrapolation

3.00 APPLICATION4.00 ANALYSIS

- 4.10 Analysis of Elements
- 4.20 Analysis of Relationships
- 4.30 Analysis of Organizational Principles

APPENDIX A (Continued)5.00 SYNTHESIS

- 5.10 Production of a Unique Communication
- 5.20 Production of a Plan or Proposed Set of Operations
- 5.30 Derivation of a Set of Abstract Relations

6.00 EVALUATION

- 6.10 Judgments in Terms of Internal Criteria
 - 6.20 Judgments in Terms of External Criteria.
-

APPENDIX B

COMMON BASIC SYLLABUS FOR MATHEMATICS (STANDARD GRADE)
FOR THE SENIOR SECONDARY SCHOOL. (DEPARTMENT OF INDIAN
AFFAIRS, DIVISION OF EDUCATION, 1973)

1. GENERAL REMARKS

- 1.1 The syllabus content which is prescribed for Std. VIII, is here taken as pre-knowledge to the syllabus for Stds. IX and X. This knowledge is again used either directly or indirectly in the syllabus for the last two school years and thus also in the Matriculation examination, but during these years the emphasis will fall on the actual syllabus for Stds. IX and X.
- 1.2 In tests and examinations more stress should be laid on insight and understanding than on mechanical reproduction of formal knowledge.
- 1.3 Where applicable, the slide rule may be used.

2. GENERAL AIMS

- 2.1 To acquaint the pupils with the part played by mathematics in the modern world in which man is constantly required to handle quantitative and spatial aspects of situations.
- 2.2 To contribute to the general education of the pupils with special emphasis on the development of logical thought and of systematic accurate and neat methods of working.
- 2.3 To cultivate appreciation for the structure and the continuous theme of each section of the syllabus as well as for the underlying relation between certain sections.
- 2.4 To acquaint pupils with and train them in mathematical methods of thought and work.
- 2.5 To give pupils a clear insight into, and a thorough knowledge and understanding of those basic mathematical principles which will prepare and equip them for daily life and further study.

3. REMARKS

- 3.1 In all sections of the subject pupils must be guided to tackle and solve each problem or theorem systematically by:
 - 3.1.1 giving close attention to the data and what is required;

APPENDIX B (Continued)

- 3.1.2 taking account of all facts and theorems that can possibly serve as a key to the solution, irrespective of the section of mathematics in which they appear;
 - 3.1.3 starting with the most obvious method and then considering other possibilities;
 - 3.1.4 comparing the different methods of solution and making a choice;
 - 3.1.5 giving close attention to necessary and sufficient requirements with respect to formulation and reasoning when writing down the solution.
- 3.2 Pupils must be trained in the correct use of notation and terminology, the exact formulation of statements and the making of valid deductions.
- 3.3 As a variety of solutions is possible for most problems, pupils must be trained to consider each solution carefully in order to make sure that there is not a better one.
4. EXAMINATION PAPERS AND ALLOCATION OF MARKS
- 4.1 Standard VIII - To be decided on by each department.
 - 4.2 Standards IX and X (omitted in this Appendix).
5. SYLLABUS FOR STANDARD VIII
- 5.1 Algebra
 - 5.1.1 Sets (Consolidation)
 - 5.1.1.1 The set concept.
 - 5.1.1.2 Elements of a set, the empty set, subset, universal set, complement of a set, intersection and union of set, Cartesian product.
 - 5.1.1.3 Venn diagrams and their applications as an aid to illustrate the solution of problems.
 - 5.1.2 Number Concept
 - 5.1.2.1 An outline of the structure of the system of real numbers as developed from the natural numbers with emphasis on the irrational and real numbers.

APPENDIX B (Continued)

- 5.1.2.2 Relations of order between numbers and the relevant laws; operations with numbers, rules for operations, closure in respect of operations.
- 5.1.2.3 The number line. One-to-one correspondence between points on the number line and real numbers.
- 5.1.3 Products of the following types by inspection:
 - 5.1.3.1 $(a \pm b)(c \pm d)$
 - 5.1.3.2 $(ax \pm b)(cx \pm d)$
 - 5.1.3.3 $(ax \pm b)^2$
 - 5.1.3.4 $(ax + b)(ax - b)$
- 5.1.4 Factors of polynomials of the following types:
 - 5.1.4.1 $ax \pm bx \pm ay \pm by$
 - 5.1.4.2 $ax^2 \pm bx \pm c$ and $ax^2 \pm bxy \pm cy^2$
 - 5.1.4.3 $a^2 - b^2$
 - 5.1.4.4 These types with the inclusion of a common factor.
- 5.1.5 L.C.M. of polynomials by factorisation only.
- 5.1.6 Algebraic fractions
 - 5.1.6.1 Simplification
 - 5.1.6.2 Operations
- 5.1.7 Solution Sets
 - 5.1.7.1 Determining the solution sets of linear equations in one unknown with numerical and literal coefficients.
 - 5.1.7.2 Determining the solution sets of linear inequalities in one unknown with numerical coefficients only.

APPENDIX B (Continued)

5.1.8 Formulae

5.1.8.1 Changing the subject of a formula.
(Simple examples only).

5.1.8.2 Substitution in formulae.

5.1.9 Relations and functions

5.1.9.1 Relation. The mapping according to a given rule of the elements of one set on the elements of another set.

5.1.9.2 Sets of number pairs.

5.1.9.3 The function concept.

5.1.9.4 Representation of number pairs by points in the Cartesian plane.

5.1.9.5 The function defined by $y = mx + c$ and its graphical representation; intercept and gradient.

5.1.10 Systems of linear equations and inequalities

5.1.10.1 Algebraic solution of systems of linear equations in two unknowns.

5.1.10.2 Graphical illustration of the solution sets of systems of linear equations in two unknowns.

5.1.10.3 Graphical illustration of the solution sets of systems of inequalities in two unknowns.

5.1.11 Logarithms

5.1.11.1 Definition of a^n for n an integer.
($a > 0$).

5.1.11.2 Use of logarithmic tables for:

5.1.11.2.1 Multiplication

5.1.11.2.2 Division

5.1.11.2.3 Raising to a power (fractional indices excluded).

APPENDIX B (Continued)

5.2 Synthetic Geometry

(N.B. Although all theorems must be proved, formal proofs of theorems in examinations must be limited, and only proofs of theorems (but not of their converses) denoted by an asterisk in the following list will be required. However, applications (including constructions), of any definition, axiom or theorem in this list can be set).

- 5.2.1 If two lines intersect, the sum of any pair of adjacent angles is equal to 180° , and conversely, if the sum of two adjacent angles is 180° , the outer arms form a straight line. (Axiom).
- * 5.2.2 When two lines intersect the vertically opposite angles are equal. (Theorem).
- 5.2.3 Two lines are parallel if and only if an intersecting transversal forms equal corresponding angles. (Definition).
- 5.2.4 If a transversal intersects two lines, these two lines are parallel if and only if alternate angles are equal. (Theorem).
- 5.2.5 If a transversal intersects two lines, these two lines are parallel if and only if the sum of the interior angles on the same side of the transversal is 180° . (Theorem).
- 5.2.6 Lines which are parallel to the same line are parallel to each other. (Theorem).
- * 5.2.7 The exterior angle of a triangle is equal to the sum of the interior opposite angles. (Theorem).
- * 5.2.8 The sum of the angles of a triangle is 180° . (Theorem).
- 5.2.9 The concept of congruence.
- 5.2.10 If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, the two triangles are congruent. (Axiom).
- 5.2.11 If three sides of one triangle are respectively equal to the three sides of another triangle, the triangles are congruent. (Axiom).
- 5.2.12 If two angles and a side of one triangle are respectively equal to two angles and the corresponding side of another triangle, the two triangles are congruent. (Theorem - without proof).

APPENDIX B (Continued)

- * 5.2.13 In an isosceles triangle the angles opposite equal sides are equal and conversely, if two angles of a triangle are equal, the sides opposite them are equal. (Theorem).
- 5.2.14 If in two right-angled triangles the hypotenuse and one side of the one are respectively equal to the hypotenuse and one side of the other, the triangles are congruent. (Theorem).
- 5.2.15 Definitions of: quadrilateral, parallelogram, rhombus, rectangle, square and trapezium.
- * 5.2.16 The opposite sides and angles of a parallelogram are equal and, conversely if the opposite angles or sides of a quadrilateral are equal, the quadrilateral is a parallelogram. (Theorem).
- * 5.2.17 A diagonal of a parallelogram bisects the area of the parallelogram. (Theorem).
- * 5.2.18 A parallelogram and a rectangle on the same base and between the same parallels have equal areas, (Theorem) with the following corollaries:
 - 5.2.18.1 The area of a parallelogram = base \times height.
 - 5.2.18.2 The area of a triangle = $\frac{1}{2}$ base \times height.
 - 5.2.18.3 The area of a trapezium = $\frac{1}{2}$ (sum of the lengths of the parallel sides) \times (the perpendicular distance between the parallel sides).
- * 5.2.19 The diagonals of a parallelogram bisect each other, and conversely, if the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram. (Theorem).
- 5.2.20 If two opposite sides of a quadrilateral are equal and parallel, the quadrilateral is a parallelogram. (Theorem).
- 5.2.21 The lengths of the diagonals of a rectangle are equal to each other. (Theorem).
- 5.2.22 The diagonals of a rhombus bisect each other at right angles, and bisect the angles of the rhombus. (Theorem).
- * 5.2.23 The line segment joining the mid-points of two sides of a triangle is parallel to the third side, and its length is equal to half the length of the third side. (Theorem).

APPENDIX B (Continued)

- 5.2.24 The line drawn through the mid-point of one side of a triangle, parallel to the other side, bisects the third side. (Theorem).
- 5.2.25 If three or more parallel lines make equal intercepts on a given transversal, they make equal intercepts on any other transversal. (Theorem).
- 5.2.26 The theorem of Pythagoras and its converse.
- 5.2.27 Calculations in connection with lengths, areas and volumes of right prisms on the following bases: triangle and rectangle; of right pyramids on rectangular bases, right cylinder and cone on circular bases, and the sphere. Direct calculations with the formulae concerned. Only simple numerical applications where the data and the results are rational numbers. (Deduction of formulae is not required).
-

A P P E N D I X C

MATHEMATICS TEST BOOKLET

READ CAREFULLY:

1. This is not an examination. It is a mathematics test designed to test your knowledge, skill, understanding and ability to apply in mathematics. The whole test can easily be completed within 80 mins.

2. HOW TO ANSWER:

- (a) The actual writing down of the answer is simple. Each question is supplied with 5 probable answers marked (A), (B), (C), (D), (E). Only ONE of them is correct. When you have carefully thought about the question select the correct answer and put "✓" on your answer sheet in the relevant block. Use a pencil.

If you selected (D) then your answer sheet for the particular question will look like this:

A	B	C	✓	E
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- (b) If you change your mind erase and place "✓" correctly and clearly.
- (c) If you are uncertain do not guess but leave a blank. Guessing will be of no help.
- (d) You will be supplied with paper for rough work. Please do not tear any part of this booklet or write on it.
- (e) Here is an example done for you:

In the equation $\frac{x+10}{3} = \frac{3x+2}{2}$, x is equal to:-

- (A) $\frac{6}{7}$ (B) $\frac{7}{2}$ (C) -2 (D) $\frac{2}{7}$ (E) 2

ANSWER SHEET :

Eg.	A	B	C	D	✓
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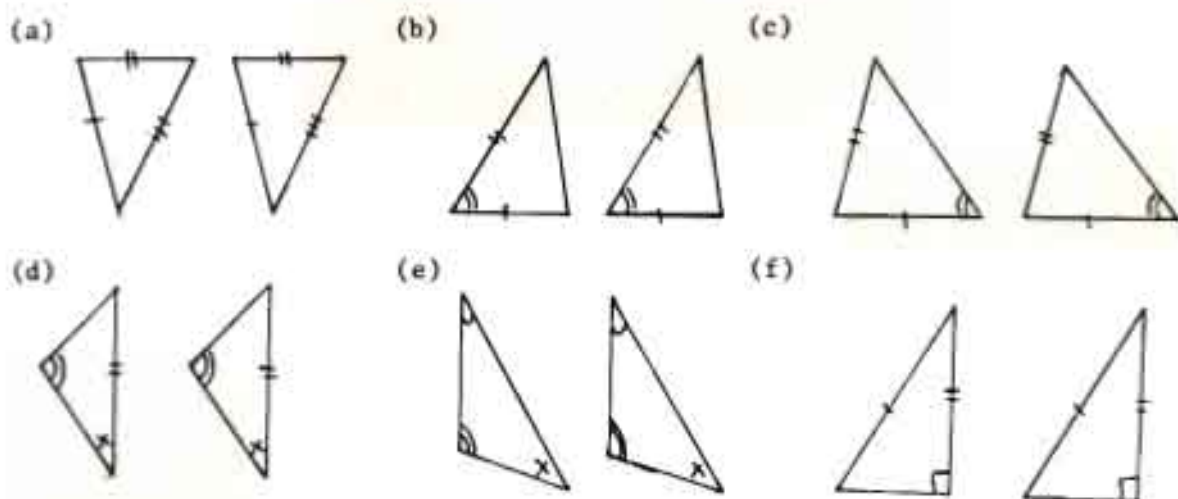
3. You will also be required to say what you thought about this test. Please be frank as your answers will be of great help.
4. If you are not sure of anything on this page please ask the teacher.

TEST BEGINS HERE

1. If a, b are integers which one of the following is NOT true?

- (A) $a + 0 = a$
 (B) $b \times 0 = 0 \times b = 0$
 (C) $0 \div b = 0$
 (D) $a - a = 0$
 *(E) $b \div 0 = 0$

2. Here are 6 pairs of triangles with equal sides and angles marked.



Which of the pairs are congruent?

- (A) a b c d (B) a b e f (C) a b d e
 *(D) a b d f (E) a c d f

3. Which of the following relations are functions?

$$P = \{(1;0); (2;1); (3;2)\} \quad Q = \{(1;2); (1;3); (2;4)\}$$

$$R = \{(1;1); (2;1); (3;2)\} \quad T = \{(1;1); (2;1); (2;2)\}$$

- (A) P, Q only *(B) P, R only (C) P, T only
 (D) R, T only (E) Q, T only

4. Which one of the following statements about a parallelogram is NOT true?

A quadrilateral is a parallelogram if and only if:

- (A) the opposite sides are equal.
 (B) the opposite angles are equal.
 (C) the diagonals bisect each other.
 (D) a pair of opposite sides is parallel and equal.
 *(E) the diagonals are equal.

5. Adding $\frac{3m}{b}$ and $\frac{3m}{c}$ we get:-

- (A) $\frac{3m}{b+c}$ (B) $\frac{6m(c+b)}{bc}$ (C) $\frac{6m}{b+c}$
 *(D) $\frac{3mb+3mc}{bc}$ (E) $\frac{9m^2}{bc}$

6. Substituting the values $a = 3$, $b = -2$, $c = 1$ in the expression:

$$\frac{ab^2 - c}{c^2 + \frac{a}{b}} \quad \text{we get:-}$$

- *(A) -22 (B) +33 (C) -39 (D) +15 (E) +26

7. If $x = \frac{a-b}{c^2}$ then

- (A) $a = \frac{x-c^2}{b}$ (B) $c = \sqrt{a-b-x}$
 *(C) $a = c^2x + b$ (D) $a + c^2x = b$ (E) $c^2 = \frac{x}{a-b}$

8. Consider the following expressions:

- (a) $m^2 + 6mn + 9n^2$
 (b) $m^2 - 9n$
 (c) $m^2 - mn - 6n^2$
 (d) $2mn + 6n^2$

$(m + 3n)$ is a common factor for *only two* of the expressions. Which two are they?

- (A) a, c *(B) a, d (C) b, c (D) b, d (E) c, d





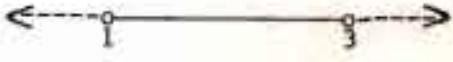
9. The statement:

"For any two consecutive whole numbers their product minus the smaller number is equal to the square of the smaller number."

can be expressed mathematically, if p is a whole number, as:

- (A) $p(p + 1) - (p + 1) = (p + 1)^2$
 (B) $p(p + 1) - p = (p + 1)^2$
 (C) $p(p + 1) - (p + 1) = p^2$
 (D) $(p - 1)(p + 1) - (p - 1) = (p - 1)^2$
 *(E) $p(p - 1) - (p - 1) = (p - 1)^2$

10. The solution set for $(x|x-2 \leq 1) \cap (x|2x > 2)$ where $x \in$ (real numbers) may be illustrated on the number line as:

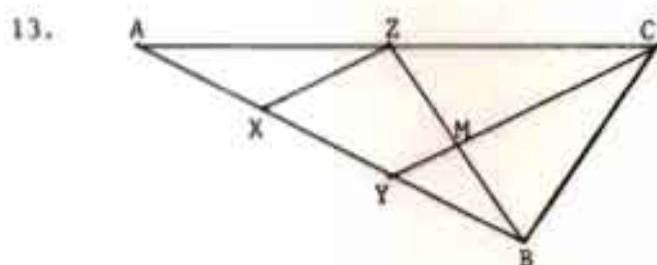
- (A) 
- (B) 
- *(C) 
- (D) 
- (E) 

11. The area of a rectangle is 6 and its perimeter is 10. One of the following equations may be used to find the sides of this rectangle.

- (A) $x^2 + 5x + 6 = 0$
 (B) $x^2 - 10x + 6 = 0$
 *(C) $x^2 - 5x + 6 = 0$
 (D) $x^2 + 10x - 6 = 0$
 (E) $x^2 - 5x + 5 = 0$

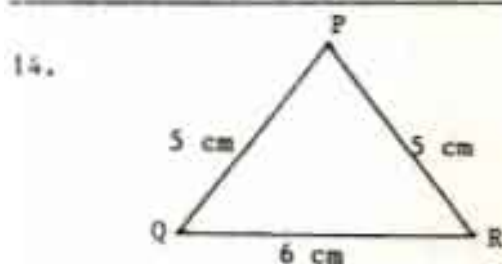
12. If $I = \{\text{integers}\}$, $N = \{\text{natural numbers}\}$,
 $Q = \{\text{rational numbers}\}$, $R = \{\text{real numbers}\}$ and
 $No = \{\text{whole numbers}\}$ then one of the following subset
relations has the correct order.

- * (A) $N \subset No \subset I \subset Q \subset R$
(B) $N \subset No \subset I \subset R \subset Q$
(C) $N \subset I \subset No \subset Q \subset R$
(D) $N \subset I \subset No \subset R \subset Q$
(E) $N \subset No \subset Q \subset I \subset R$



In the figure, $AX = XY = YB$
and $\overline{XZ} \parallel \overline{YC}$. Which one of
the following is NOT true?

- (A) $AZ = ZC$ * (B) $YM = \frac{1}{3}YC$
(C) $YM = \frac{1}{2}YC$ (D) $XZ = \frac{1}{2}YC$
(E) M is the mid point of ZB .



Comparing the areas of these two triangles.

- (A) ΔPQR has the smaller area
(B) ΔXYZ has the smaller area
* (C) area of $\Delta PQR = \text{area } \Delta XYZ$
(D) area $\Delta PQR = \frac{1}{2}$ area ΔXYZ
(E) no conclusion can be made

- 15.

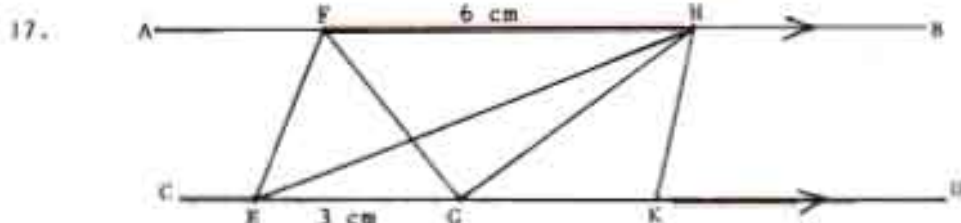


The length of the diagonal of a
square is 16 cm. The area of the
square in square centimetres is

- (A) 4 (B) 8 (C) 16 (D) 64 * (E) 128

16. In a survey in Durban, 100 housewives were questioned about the use of tea, coffee and milo. It was found that 59 use tea, 38 use milo, 4 use all three, 12 use tea and coffee, 19 use tea and milo and 14 use coffee and milo. How many use coffee only?

(A) 44 (B) 32 (C) 9 (D) 22 (E) 40



In the figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. $FH = 6$ cm, $EG = 3$ cm and $EG = GK$. ONE of the following conclusions is NOT true. Which is it?:-

- (a) area $ECH = \frac{1}{2}$ area FGH
 (b) area $GKH = \frac{1}{2}$ area $EFHK$
 (c) area $FGH =$ area EHK
 (d) area $KGH =$ area EFH
 (e) area $GKH = \frac{1}{3}$ area $ECHF$

(A) b (B) d (C) a (D) e (E) c

18.

In the figure, ABCD is a rectangle. Each side is extended its own length so that $AB = BB'$, $BC = CC'$, $CD = DD'$, $DA = AA'$. If the area of ABCD is 2 then the area of $A'B'C'D'$ is:-

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

19. Study the following number sentences and discover a pattern:

$$1 + 2 = \frac{2 \times 3}{2}$$

$$1 + 2 + 3 = \frac{3 \times 4}{2}$$

$$1 + 2 + 3 + 4 = \frac{4 \times 5}{2}$$

$$1 + 2 + 3 + 4 + 5 = \frac{5 \times 6}{2}$$

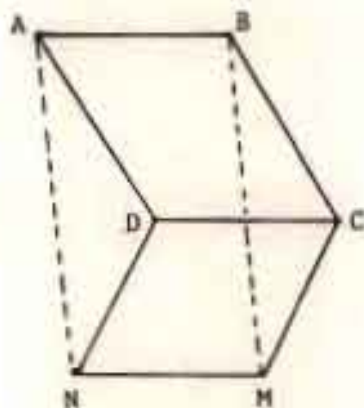
Now, using this pattern the sum of all the whole numbers between 16 and 30 (including both 16 and 30) can be written as:

(A) $\left(\frac{30 \times 31}{2} - \frac{15 \times 16}{2} \right)$ (B) $\left(\frac{30 \times 31}{2} - \frac{16 \times 17}{2} \right)$

(C) $\left(\frac{29 \times 30}{2} - \frac{15 \times 16}{2} \right)$ (D) $\left(\frac{29 \times 30}{2} - \frac{16 \times 17}{2} \right)$

(E) $\left(\frac{29 \times 31}{2} - \frac{15 \times 17}{2} \right)$

20.



In the figure, $ABCD$ is a parallelogram and $DCNM$ is a rhombus. Only 3 of the following steps (given here without reasons) are necessary to form the proof that $ABMN$ is a parallelogram:

(a) $\overline{AB} \parallel \overline{NM}$ and $AB = NM$

(b) $\overline{AB} \parallel \overline{DC}$ and $AB = DC$

(c) $\overline{DC} \parallel \overline{NM}$ and $DC = NM$

(d) $\overline{AD} \parallel \overline{BC}$ and $AD = BC$



The 3 steps placed in the correct logical order for the proof are:

(A) d c a (B) b c a (C) b a c

(D) b a d (E) a b c

(C)

ANSWER SHEET

Eg.	A	B	C	D	<input checked="" type="checkbox"/>
-----	---	---	---	---	-------------------------------------

1	A	B	C	D	E
---	---	---	---	---	---

2	A	B	C	D	E
---	---	---	---	---	---

3	A	B	C	D	E
---	---	---	---	---	---

4	A	B	C	D	E
---	---	---	---	---	---

5	A	B	C	D	E
---	---	---	---	---	---

6	A	B	C	D	E
---	---	---	---	---	---

7	A	B	C	D	E
---	---	---	---	---	---

8	A	B	C	D	E
---	---	---	---	---	---

9	A	B	C	D	E
---	---	---	---	---	---

10	A	B	C	D	E
----	---	---	---	---	---

11	A	B	C	D	E
----	---	---	---	---	---

12	A	B	C	D	E
----	---	---	---	---	---

13	A	B	C	D	E
----	---	---	---	---	---

14	A	B	C	D	E
----	---	---	---	---	---

15	A	B	C	D	E
----	---	---	---	---	---

16	A	B	C	D	E
----	---	---	---	---	---

17	A	B	C	D	E
----	---	---	---	---	---

18	A	B	C	D	E
----	---	---	---	---	---

19	A	B	C	D	E
----	---	---	---	---	---

20	A	B	C	D	E
----	---	---	---	---	---

(D) To be filled in by pupils AFTER the test.

PLACE "✓" IN THE RELEVANT BLOCK

1. Did you think the test as a whole was

too hard too easy about right ?

2. If you were taking a mathematics examination and you wanted your knowledge of mathematics to be tested as fairly and thoroughly as possible would you prefer

this kind of test the ordinary kind of test a mixture of both ?

APPENDIX E

STATISTICAL METHODS

1. Standard Deviation and Standard Error of Mean.

$$1.1 \text{ Standard Deviation (SD)} = \sqrt{\frac{\sum d^2}{N}}$$

where d = deviation from the mean

= $x - \bar{x}$ (where x is the score and

\bar{x} the mean = $\frac{\sum x}{N}$)

and N = the number of scores (cases).

$$1.2 \text{ Standard Error of Mean (S}_{\bar{x}}) = \frac{SD}{\sqrt{N-1}}$$

(Downie and Heath, 1970. p. 161)

The standard error of mean yields a confidence interval for the mean as follows:

$$\bar{x} \pm (2,58) S_{\bar{x}} \quad (p < 0,01)$$

(Downie and Heath, 1970. pp. 164-165)

This interval enables an unbiased estimation of the population mean (μ) from the sample mean (\bar{x}),

e.g. given $\bar{x} = 7,41$ and $SD = 3,29$

(i.e. $S_{\bar{x}} = 0,12$ for $N = 769$) (See Section 5.3)

$$\bar{x} \pm (2,58) S_{\bar{x}} = 7,41 \pm 0,31.$$

Thus, with repeated samples of the same size, the probability that the sample mean will fall in the interval $7,41 \pm 0,31$ is greater than 0,99 (i.e. $p < 0,01$)

2. Differences between Means

Since all samples in this study were considered to be large ($N > 160$) the z -score was computed to test significance of the difference between means (Downie and Heath, 1970. p. 178).

APPENDIX E (Continued)

The z-score is interpreted by the use of the normal probability tables (Downie and Heath, 1970. pp. 303-309). The z-score is calculated differently for uncorrelated data and correlated data.

2.1 Differences between Means - Uncorrelated Data for Large Sample (z-Test)

Firstly, the standard error of the difference between two means, \bar{x}_1 and \bar{x}_2 is defined by the following formula:

$$S_{D_x} = \sqrt{S_{\bar{x}_1}^2 + S_{\bar{x}_2}^2}$$

$$\text{where } S_{\bar{x}_1} = \frac{(SD)_1}{\sqrt{N_1 - 1}}, \quad S_{\bar{x}_2} = \frac{(SD)_2}{\sqrt{N_2 - 1}}$$

are the standard errors of the two sample means. (Downie and Heath, 1970. p. 172).

Secondly, the z-score is given by

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{S_{D_x}}$$

(Downie and Heath, 1970. p. 172.).

From normal probability tables for $z > 2,58$ and $z > 3,30$ it is seen that $p < 0,01$ and $p < 0,001$ respectively.

e.g. The difference between mean scores of H and S on the total test (see section 5.7.1) yielded $z = 12,06 > 3,30$. This difference is significant for $p < 0,001$. Thus the null hypothesis that there is no difference between mean scores of H and S is rejected at $p < 0,001$ level of significance.

2.2 Differences between Means - Correlated Data for Large Sample (z-Test)

When two sets of scores for the same sample are considered the data are said to be correlated. In this case the standard error of the difference between the two means, \bar{x}_1 and \bar{x}_2 is defined by:

$$S_{D_x} = \sqrt{S_{\bar{x}_1}^2 + S_{\bar{x}_2}^2 - 2(r)(S_{\bar{x}_1})(S_{\bar{x}_2})}$$

APPENDIX E (Continued)

Where $S_{\bar{x}_1}$ and $S_{\bar{x}_2}$ are the standard errors of the means, and

r = coefficient of correlation for the two sets of scores.

The z -score is then given by

$$z = \frac{|\bar{x}_1 - \bar{x}_2|}{S_{D_{\bar{x}}}}$$

$$= \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{S_{\bar{x}_1}^2 + S_{\bar{x}_2}^2 - 2(r)(S_{\bar{x}_1})(S_{\bar{x}_2})}}$$

(Downie and Heath, 1970. pp. 174-175).

2.3 Differences between Means - Uncorrelated Data for Small Sample (t-Test)

When the means of two sets of uncorrelated data for small samples are compared the t -test is used. Firstly, the standard error of the difference between the means \bar{x}_1 and \bar{x}_2 is defined by:

$$S_{D_{\bar{x}}} = \sqrt{\frac{Id_1^2 + Id_2^2}{N_1 + N_2 - 2} \left(\frac{1}{N_1} + \frac{1}{N_2} \right)}$$

where $d_1 = \bar{x}_1 - x_1$, $d_2 = \bar{x}_2 - x_2$, and

N_1 , N_2 are the two sample sizes.

The t -score is then given by

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{S_{D_{\bar{x}}}}$$

APPENDIX E (Continued)

$$= \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sum d_1^2 + \sum d_2^2}{N_1 + N_2 - 2} \left(\frac{1}{N_1} + \frac{1}{N_2} \right)}}$$

for $(N_1 + N_2 - 2)$ degrees of freedom.

(Downie and Heath, 1970. pp. 181-182).

3. Correlation Coefficients

3.1 Pearson Product-Moment Correlation Coefficient.

The machine formula for the Pearson product-moment correlation coefficient is given by

$$r = \frac{\sum xy - \frac{\sum x \sum y}{N}}{\sqrt{\left[\sum x^2 - \frac{(\sum x)^2}{N} \right] \left[\sum y^2 - \frac{(\sum y)^2}{N} \right]}}$$

where x, y are pairs of scores and N = number of pairs.

(Downie and Heath, 1970. p. 93).

3.2 Reliability Coefficient

The reliability coefficient of a test indicates the consistency with which it yields its results. The following two methods have been used to obtain the reliability coefficients in this study:

- (i) the split-half method;
- (ii) Kuder-Richardson Formula No. 20.

3.2.1 The Split-Half Method

This method is used to simulate the effect of two equivalent tests by dividing the given test into halves, e.g. even-numbered items and odd-numbered items (as was the case in this study). A Pearson product-moment correlation is computed between the

APPENDIX E (Continued)

scores on the even-numbered items and the scores on the odd-numbered items to yield the coefficient of internal consistency. Since this coefficient measures the reliability of a test which is half as long as the actual test the correlation is corrected by using the Spearman-Brown formula,

$$r = \frac{2r_{oe}}{1 + r_{oe}}$$

where r_{oe} = reliability coefficient obtained by correlating the scores on the odd-numbered items with the scores on the even-numbered items.

(Downie and Heath, 1970. p. 244).

3.2.2 Kuder-Richardson Formula 20

The Kuder-Richardson formula No. 20, which also yields a coefficient of internal consistency, is easily applied to item analysis data.

This formula is defined as follows:

$$r_{tt} = \frac{k}{k-1} \left[1 - \frac{\sum_{i=1}^k p_i q_i}{(SD)^2} \right],$$

where r_{tt} = reliability of the total test,

k = number of items in the test,

$(SD)^2$ = the variance of the test

p_i = proportion of candidates who correctly responded to the i -th item, and

q_i = $1 - p_i$.

(Thorndike in : Lindquist, 1951. p. 587).

APPENDIX E (Continued)

3.2.3 The Standard Error of Measurement

The standard error of measurement provides an indication of the accuracy of the test scores and is estimated by the following formula:

$$Se = SD \sqrt{1 - r}$$

where Se = standard error of measurement
 SD = standard deviation of the test
 r = reliability of the test.

Se = $0.432 \sqrt{k}$, where k = number of items on the test, is also regarded as a good estimate of the standard error of measurement.

(Downie and Heath, 1970. pp. 247-248).

3.3 Validity Coefficient

The validity coefficient of a test indicates the extent to which it measures what it sets out to measure. Measurements of validity are essentially measurements of correlation between the scores on the test and criterion scores in the attribute which the test attempts to measure. Since criterion scores are themselves often not reliable measures it is necessary to have two parallel, but not necessarily equivalent sets of criterion scores. Given a set of scores x (whose validity is to be estimated) and two sets of corresponding criterion scores a and b, the validity coefficient or the correlation between the test and the "true" criterion score is defined by

$$r_{x,ab} = \frac{r_{xa} \cdot r_{xb}}{r_{ab}}$$

where r_{xa} = correlation between the test and criterion score a,

r_{xb} = correlation between the test and criterion score b,

r_{ab} = correlation between the criterion score a and criterion score b.

(Cureton in : Lindquist, 1951. p. 680).

APPENDIX E (Continued)

3.4 Levels of Significance for Correlation Coefficients

The levels of significance for correlation coefficients are obtained from a table of values, e.g. (Edwards, 1967, Table VI, p. 426).

3.5 Significance of the Difference between two Correlation Coefficients for Correlated Data

When determining the significance of the difference between two r 's (r_{12} and r_{13}) where a variable 1 is correlated with a variable 2 and a variable 1 is also correlated with a variable 3 and all measurements are made upon the same sample, the relationship between the pairs of r 's has to be taken into account. In this case a t -Test is used as follows:

$$t = \frac{(r_{12} - r_{13}) \sqrt{(N-3)(1+r_{23})}}{\sqrt{2(1-r_{12}^2-r_{13}^2-r_{23}^2+2r_{12}r_{13}r_{23})}}$$

which is interpreted by entering the t -table of probability with $N-3$ degrees of freedom.

(Downie and Heath, 1970. p. 234).

4. Item Analysis

Item analysis provides quantitative information in respect of the difficulty and discriminating power (the extent to which an item differentiates between the weaker and brighter candidates) of each item. The Facility Index (F) and Discriminating Index (D) are calculated for each item.

In order to compute the F and D values the scores are arranged from the highest to the lowest. The upper 27 percent and the lower 27 percent of the scores are used to make the two extreme groups as large and different as possible (Ebel, 1965. p. 349). For each item the number of correct responses from each of the upper and lower groups is counted.

The Facility Index and Discrimination Index for each item are given by the following formulae:

$$F = \left[\frac{(R_u + R_e)}{2n} \right] 100 \quad ,$$

APPENDIX E (Continued)

$$D = \frac{R_u - R_e}{n}$$

- where F = Facility Index,
 D = Discrimination Index,
 R_u = number of correct responses for upper group,
 R_e = number of correct responses for lower group, and
 n = number of candidates in each group.

(The above formulae have been adapted from the steps outlined by Ebel, 1965. p. 347 and Husén, 1967. p. 101).

5. Chi Square (χ^2)

The Chi square statistic is a test of significance which compares observed frequencies (O) with expected frequencies (E).

The general formula for chi square is given by

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

- where O = observed frequency,
 E = expected frequency, and
 n = number of frequencies.

The χ^2 is used to test the null hypothesis that the observed frequencies do not differ from the expected frequencies by chance. The level of significance is read from probability tables for (n - 1) degrees of freedom. (Downie and Heath, 1970. pp. 197-199) and p. 311).

APPENDIX F

LETTER TO RATERS AND RATING SHEET

UNIVERSITY OF DURBAN-WESTVILLE
 Faculty of Education
 Division of Mathematics Education

July 1974

Dear Colleague

I am grateful to you for participating in this research programme and for making time available on your busy schedule.

As you will already know, the need for clearly defined objectives in the teaching of mathematics can hardly be over-emphasised. It is the purpose of this research to:

- (i) suggest a suitable classification of objectives for use in the teaching of mathematics at the senior secondary level;
- (ii) design a mathematics test to ascertain to what extent our pupils are achieving such objectives;
- (iii) make recommendations in respect of curriculum development in secondary school mathematics with reference to objectives, content, methodology and assessment.

I want to assure you that your participation in this programme will contribute in some way to your own thoughts about mathematics education.

There are 3 parts to this questionnaire:

- I A classification[†] of objectives is presented and each is briefly explained and terms are clarified. You need to study these as you will be required to judge test items on this basis.
- II The test items relating to school mathematics are presented. The Rating Sheet provides a suggested classification of test items according to Part I. You need to say whether your agreement with each classification is *High*, *Moderate* or *Low* by ticking on the Rating Sheet that is provided. If *Low*, give your own classification in terms of A, B, C, etc. of Part I. Please be frank in your evaluation.
- III Personal details: name, qualification, experience, etc. All details given by you will be treated strictly

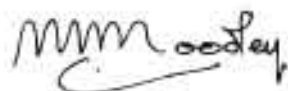
APPENDIX F (Continued)

confidential. You should also kindly treat the questionnaire and test items as confidential.

Since the pupils in the senior secondary classes will soon be taking the test, your assessment of the items will be invaluable in assisting with the selection of the items.

Please feel free to put to me any thoughts or queries you may have. I hope to continue this dialogue when the research is complete as you will no doubt be interested in the findings.

Yours sincerely

A handwritten signature in dark ink, appearing to read 'M. Moodley'. The signature is fluid and cursive, with a long horizontal stroke at the end.

M. MOODLEY

LECTURER : DIVISION OF MATHEMATICS EDUCATION

† The classification is presented in chapter two section 2.4.1.

APPENDIX F (Continued)

RATING SHEET

ITEM CODE	STATEMENT OF OBJECTIVES AND SUGGESTED CLASSIFICATION	AGREEMENT : USE TICK			IF LOW, GIVE OWN CLASSIFICATION - USE A, B, C, ETC.
		HIGH	MODERATE	LOW	
1A	Knowledge of property of 0 and operations (A)				
7B	Skill in adding fractions (B)				
23C	Analysis of data and construction of solution (E)				
21D	Apply Venn diagram and set concepts to problem (D)				
22E	Analyse data and evaluate area relationship (E)				
3F	Knowledge of conditions for congruency of triangles (A)				
13G	Transform verbal material to symbolic form (C)				
5H	Knowledge of theorems on parallelogram (A)				
6I	Skill in solving equations in one unknown (B)				
20J	Apply theorem of Pythagoras and formula for area of a triangle (D)				
17K	Extrapolate a number pattern (C)				
18L	Apply theorem of Pythagoras and formula for area of square (D)				

APPENDIX F (Continued)

ITEM CODE	STATEMENT OF OBJECTIVES AND SUGGESTED CLASSIFICATION	AGREEMENT : USE TICK			IF LOW, GIVE OWN CLASSIFICATION - USE A, B, C, ETC.
		HIGH	MODERATE	LOW	
19M	Apply mid point theorem and its converse (D)				
15N	Translate from symbolic form to graphic form (C)				
90	Skill in substitution (B)				
14P	Extrapolation of a number pattern (C)				
2Q	Knowledge of set concepts and set notation (A)				
8R	Skill in changing subject of formula (B)				
12S	Translate verbal content to symbolic form (C)				
4T	Knowledge of concepts of relation and function (A)				
10U	Skill in factorising (B)				
11V	Translate from graphic form to symbolic form (C)				
16W	Interpret subset relations for numbers (C)				
24X	Analyse data to establish area relationship (E)				
25Y	Analyse pattern and construct solution (E)				
26Z	Construction and evaluation of a proof based on parallelograms (E)				

APPENDIX G

COMPUTER PROGRAMME

BURKHUUGH5 B=5700 FORTRAN COMPILATION (MARK XVI,U.00),

TEST /WEDMOON

=====

DIMENSION X(40),N(40),L(40),SX(40),LS(800),LA(800),S(800,35),
 * UEM(40),MN(10),NK(5),NUM(40),LL(800,20),RAB(30,30),H(800,10),
 * L3(800),L4(800),L5(800),SAB(30,30),LX(800,20)

C THE CODE USED IS AS FOLLOWS:
 C 7 = SCORE ON NON VERBAL TEST (GTISA)
 C 8 = SCORE ON VERBAL TEST (GTISA)
 C 9 = SCORE ON FULL SCALE TEST (GTISA)
 C 10 = TEACHER'S ASSESSMENT
 C 11 = SCORE ON STD. 8 EXAMINATION
 C 12 = SCORE ON EVEN-NUMBERED ITEMS OF MT (MATHS TEST)
 C 13 = SCORE ON ODD-NUMBERED ITEMS OF MT
 C 14 = TOTAL SCORE ON MT
 C 15 = SCORE ON KNOWLEDGE ITEMS OF MT
 C 16 = SCORE ON SKILL ITEMS OF MT
 C 17 = SCORE ON COMPREHENSION ITEMS OF MT
 C 18 = SCORE ON SELECTION-APPLICATION ITEMS OF MT
 C 19 = SCORE ON ANALYSIS-SYNTHESIS ITEMS OF MT
 C 20 = SCORE ON LOWER LEVEL ITEMS (15+ 16) OF MT
 C 21 = SCORE ON HIGHER LEVEL ITEMS (17 + 18 + 19) OF MT
 C 22 = 14 FOR MALES
 C 23 = 14 FOR FEMALES
 C 24 = 14 FOR HIGHER GRADES (H)
 C 25 = 14 FOR STANDARD GRADES (S)
 C 26 = 14 FOR ONLY HIGHER GRADES (HD)
 C 27 = 14 FOR ONLY STANDARD GRADES (SD)
 C 28 = 14 FOR MIXED HIGHER AND STANDARD GRADES (HS)
 C 29 = 14 FOR HIGHER GRADE MALES
 C 30 = 14 FOR HIGHER GRADE FEMALES
 C 31 = 14 FOR STANDARD GRADE MALES
 C 32 = 14 FOR STANDARD GRADE FEMALES
 C 33 = 14 FOR HIGHER GRADE FROM HS (H(HS))
 C 34 = 14 FOR STANDARD GRADE FROM HS (S(HS))
 C 35 = AGE

NN=769

N27 = 208

M=35

UU 48 I=1,7

48 MN(I)=NN

DU 10 I=1,M

10 N(I)=NN

UU 20 J=1,NN

READ(2,30) (L(I),I=1,M)

30 FUHMAT(13,12,311,212,613,2211)

DU 35 I=14,33

K=I-13

35 LL(J,K)= L(I)

L3(J)=L(3)

L4(J)=L(4)

L5(J)=L(5)

S(J,35)=L(2)*1.0

DU 40 I=1,6

```

40  CALL CUND(L(1+7),S(J,1),N(1),0,200)
    DU 50 I=1,3
    S(J,6+I)=S(J,I)+S(J,I+3)
50  N(6+1)=N(1)+N(3+1)
    S(J,10)=L(6)*1.0
    CALL CUND(L(7),S(J,11),N(11),0,111)
    PE = 0.0
    PU = 0.0
    P6 = 0.0
    P8 = 0.0
    P10 = 0.0
    P11 = 0.0
    DU 60 I=7,16
    PU = P0 + L(2*I)*1.0
60  PE = PE + L(2*I + 1)*1.0
    PT = P0 + PE
    S(J,12) =PE*10.0
    S(J,13) =PU*10.0
    S(J,14)=PT*5.0
    IF(S(J,14).EQ.0.0) J0=J
    IF(S(J,14).EQ.0.0) J1=L(1)
    CALL SUM(L,14,17,P1)
    CALL SUM(L,18,21,P2)
    CALL SUM(L,22,25,P3)
    CALL SUM(L,26,29,P4)
    P5=PT-(P1+P2+P3+P4)
    S(J,15)=P1*25.0
    S(J,16)=P2*25.0
    S(J,17)=P3*25.0
    S(J,18)=P4*25.0
    S(J,19)=P5*25.0
    S(J,20)=(P1+P2)*100.0/8.0
    S(J,21)=(P3+P4+P5)*100.0/12.0
    DU 400 I=14,33
    IF(L(3).NE.1) GU TO 401
400  IF(L(1).EQ.1) P6 =P6 +1.0
    GU TO 402
401  N(22)=N(22)-1
402  DU 403 I=14,33
    IF(L(4).NE.1) GU TO 404
403  IF(L(1).EQ.1) P8 =P8 +1.0
    GU TO 405
404  N(24)=N(24)-1
405  DU 406 I=14,33
    IF(L(5).NE.1) GU TO 407
406  IF(L(1).EQ.1) P10=P10+1.0
    GU TO 408
407  N(26)=N(26)-1
408  DU 410 I=14,33
    IF(L(5).NE.2) GU TO 411
410  IF(L(1).EQ.1) P11=P11+1.0
    GU TO 412
411  N(27)=N(27)-1
412  P12=PT-P11-P10
    S(J,22)=P6*5.0
    P7=PT-P6
    S(J,23)=P7*5.0
    S(J,24)=P8*5.0
    P9=PT-P8
    S(J,25) =P9 *5.0
    S(J,26) =P10*5.0
    S(J,27) =P11*5.0
    S(J,28) =P12*5.0

```

```

IF(L(34),NE,1) N(29)=N(29)+1
IF(L(34),NE,2) N(30)=N(30)+1
IF(L(35),NE,1) N(32)=N(32)+1
IF(L(35),NE,2) N(33)=N(33)+1
H(J,1)=S(J,10)
H(J,2)=S(J,11)
H(J,3)=S(J,14)
H(J,4)=S(J,20)
H(J,5)=S(J,21)
20 CONTINUE
WRITE(3,653) J1
653 FORMAT(///20X,"J1=",I4)
N(23)=NN-N(22)
N(25)=NN-N(24)
N(28)=NN-N(26)-N(27)
N(31)=NN-N(29)-N(30)
N(34)=NN-N(32)-N(33)
WRITE(3,640) (N(I),I=22,34)
640 FORMAT(//10X,"NU. OF MALES=",I3//
* 10X,"NU. OF FEMALES=",I3//
* 10X,"NU. OF STUDENTS IN HIGHER GRADE=",I3//
* 10X,"NU. OF STUDENTS IN STANDARD GRADE=",I3//
* 10X,"NU. OF STUDENTS IN HIGHER GRADE CLASS ONLY=",I3//
* 10X,"NU. OF STUDENTS IN STANDARD GRADE CLASS ONLY=",I3//
* 10X,"NU. OF STUDENTS IN MIXED CLASS HIGHER AND STANDARD=",I3//
* 10X,"NU. OF STUDENTS WHO FOUND TEST EASY=",I3//
* 10X,"NU. OF STUDENTS WHO FOUND TEST HARD=",I3//
* 10X,"NU. OF STUDENTS WHO FOUND TEST ABOUT RIGHT=",I3//
* 10X,"NU. OF STUDENTS WHO PREFER THIS KIND OF TEST=",I3//
* 10X,"NU. OF STUDENTS WHO PREFER ORDINARY TEST=",I3//
* 10X,"NU. OF STUDENTS WHO PREFER A MIXTURE OF THE TWO TYPES=",I
JJ=0.0
DU 300 I=1,33
DU 300 J=1,NN
IF(S(J,14) /5.0.EQ.(34-I)) GO TO 310
GU TO 300
310 JJ=JJ+J/J
DU 311 K=1,20
311 LX(JJ,K)=LL(J,K)
300 CONTINUE
WRITE(3,625)
625 FORMAT(////30X,"I T E M A N A L Y S I S ")
WRITE(3,626)
626 FORMAT(//10X,"I",12X,"RH",10X,"RL",14X,"FI",15X,"DI")

DU 700 I=1,20
RH=0.0
RL=0.0
DU 800 J=1,N27
800 RH = RH + LX(J,1)*1.0
DU 810 J=15,NN
810 RL = RL + LX(J,1) * 1.0
DX1=N27*2.0
FI=(RH+RL)/DX1
UX2=N27*1.0
DI=(RH-RL)/UX2
700 WRITE(3,710) I,RH,RL,FI,DI
710 FORMAT( 8X,I3,2(10X,I3),2(12X,F6.3))
WRITE(3,146)
146 FORMAT(//)
DU 39 J=1,NN
CALL COND2(L4(J),L3(J),SA1,S(J,14),1,1,MN(1))
CALL COND2(L4(J),L3(J),SA2,S(J,14),1,2,MN(2))

```

```

CALL COND2(L4(J),L3(J),SA3,S(J,14),2,1,MN(3))
CALL COND2(L4(J),L3(J),SA4,S(J,14),2,2,MN(4))
CALL COND2(L5(J),L4(J),SA5,S(J,14),3,1,MN(5))
CALL COND2(L5(J),L4(J),SA6,S(J,14),3,2,MN(6))
S(J,29)=SA1
S(J,30)=SA2
S(J,31)=SA3
S(J,32)=SA4
S(J,33)=SA5
S(J,34)=SA6
39  CONTINUE
WRITE(J,41) (MN(I),I=1,7)
41  FORMAT(7(10X,I3))
WRITE(J,511)
511  FORMAT(////18X,"TEST",16X,"NUMBER")
DO 500 I=1,20
NUMBER=0
DO 520 J=1,NN
NUMBER=NUMBER+LL(J,I)
520  CONTINUE
500  WRITE(J,510) I,NUMBER
510  FORMAT(20X,I2,20X,I3)
WRITE(J,514)
514  FORMAT(////18X,"SCORE",15X,"FREQUENCY")
NUM(I)=0.0
DO 530 I=1,21
K=I-1
DO 540 J=1,NN
ST = S(J,14)/5.0
IF (S(J,14).EQ.200.0) ST=0.0
540  IF (ST.EQ.K*1.0) NUM(I)=NUM(I)+1
530  WRITE (J,513) K,NUM(I)
513  FORMAT(20X,I2,20X,I3)
WRITE (J,666)
666  FORMAT(////10X,"SIGNIFICANCE OF DIFFERENCE BETWEEN MEANS")
WRITE(J,666)
666  FORMAT (////10X,"CODE",12X,"MEAN",13X,"SDEV",14X, "SEM")
DO 610 I=7,35
SI = 0.0
NM=N(I)
IF (I.GT.28) NM=MN(I-28)
DU=NM*1.0
DO 120 J=1,NN
SCORE=S(J,I)
IF (S(J,I).EQ.200.0) SCORE=0.0
120  SI=SI+SCORE
SX(I)=SI
IF (DU.LE.1.0) GO TO 610
X(I)=SX(I)/DU
SUM=0.0
DO 650 J=1,NN
SCORE=S(J,I)
IF (S(J,I).EQ.0.0) GO TO 650
IF (S(J,I).EQ.200.0) SCORE=0.0
DM=X(I)-SCORE
SUM = SUM + DM*DM
650  CONTINUE
SDEV = SQRT(SUM/DU)
DME = SDEV /SQRT(DU - 1.)
DEM(I) =DME
WRITE (J,555) I, X(I),SDEV,DEM(I) ,SX(I)
555  FORMAT (10X,I2,3(12X,F7.3),10X,F10.3)
610  CONTINUE

```

```

        WRITE(3,667)
667  FORMAT(/ / 10X,"CODE",10X,"CODE",14X,"SEUM",13X,"Z=SCORE")
      UU 670 I=7,33
      UU 670 J = I + 1,34
      SEUM = SQRT(UEM(I)*DEM(I) + DEM(J)*UEM(J))
      IF(SEUM.EQ.0.0) GO TO 670
      Z = ABS(X(I) - X(J))/SEUM
670  WRITE(3,660) I,J,SEUM,Z
660  FORMAT(2(12X,I2),2(13X,F6.3))
      WRITE(3,667)
667  FORMAT(///30X,"C O R R E L A T I O N S ")
      WRITE(3,660)
668  FORMAT(/ / 10X,"CODE",10X,"CODE",16X,"R1",20X,"R2",16X,"R3",14X
      * "CORCOR")
421  NR(1)=N(10)
      NR(2)=N(11)
      NR(3)=N(14)
      NR(4)=N(20)
      NR(5)=N(21)
      UU 580 N=1,5
      UU 580 I=7,11
      RXY=0.0
      RXX=0.0
      RYY=0.0
      SAA=0.0
      SBB=0.0
      UU 582 J=1,NN

      IF(S(J,I).EQ.200.0) S(J,I)=0.0
      IF(H(J,K).EQ.200.0) H(J,K)=0.0
      SA=H(J,K)
      SB=S(J,I)
      IF(S(J,I).EQ.0.0) SA=0.0
      IF(H(J,K).EQ.0.0) SB=0.0
      RXY=RXY+SA*SB
      RXX=RXX+SA
      RYY=RYY+SB
      SAA=SAA+SA*SA
      SBB=SBB+SB*SB
582  CONTINUE
      NS=AMIN1(N(I),NK(K))
      DD=NS*1.0
      IF(DD.EQ.0.0) GO TO 580
      IF(K.EQ.5) J=21
      IF(K.EQ.4) J=20
      IF(K.EQ.3) J=14
      IF(K.EQ.2) J=11
      IF(K.EQ.1) J=10
      R1=RXY-RXX*RYY/DD
      R2=SAA-RXX*RXX/DD
      R3=SBB-RYY*RYY/DD
      IF(R2.EQ.0.0.OR.R3.EQ.0.0) GO TO 580
      RIJ=R1/SQRT(R2*R3)
580  WRITE(3,910) J,I,K1,R2,K3,RIJ
      UU 560 I=12,21
      UU 560 K=1,5
      IF(UD.EQ.0.0) GO TO 560
      DO 560 K=12,21
      SXY=0.0
      UU 581 J=1,NN
      IF(S(J,I).EQ.200.0) S(J,I)=0.0
      SIK=S(J,I)*S(J,K)

```

```

      SXY=SXY+SIX
581  CONTINUE
566  SAB(I,K)=SXY
      RAB(I,K)=SAB(I,K)-SX(I)*SX(K)/DU
560  CONTINUE
      DU 570 I=12,21
      UD 570 K=12,21
      RXY=RAB(I,K)/SQRT(RAB(I,I)*RAB(K,K))
      IF(RAB(I,I).EQ.0.0.OR.RAB(K,K).EQ.0.0) GO TO 570
570  WRITE(J,910) I,K,RAB(I,K),RAB(I,I),RAB(K,K),RXY
910  FORMAT (2(12X,I2),4(10X,F10.3))

920  STOP
     END

```

```

      SUBROUTINE COND(L,S,N,I,J)
      IF (L.EQ.I.OR.L.EQ.J) GO TO 10
      S=L*1.0
      GO TO 20
10   S=0.0
      N=N-1
20   RETURN
     END

```

```

      SUBROUTINE SUM(L,N,M,S)
      DIMENSION L(1)
      S=0.0
      DO 10 I=N,M
10   S=S+L(I)*1.0
      RETURN
     END

```

```

      SUBROUTINE COND2(L1,L2,S,A,I,J,N)
      IF(L1.NE.I.OR.L2.NE.J) GO TO 10
      S=A
      GO TO 20

10   S=0.0
      N=N-1
20   RETURN
     END

```

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