Examining primary school teachers' understanding of teaching geometry through the problem solving approach in Swaziland

by

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ABSTRACT

In this study, I examined primary teachers' understanding of teaching geometry through the problem solving approach, using Ball, Thames and Phelps's (2008) mathematical knowledge for teaching (MKT) framework as a lens, focusing on the knowledge of content and students (KCS) and knowledge of content and teaching (KCT) domains. I collected data in two phases. In the first phase, 34 participants who had been purposefully sampled completed an open-ended questionnaire with tasks adapted from Manizade and Mason (2011), which extracted their MKT in the two domains of decomposing and recomposing the area of polygons. In the second phase, I collected data using lesson observations, semi-structured interviews and lesson plan analysis from two participants, who volunteered from the initial 34, in their respective schools.

The results showed that the participants had limited understanding of teaching geometry through the problem solving approach. The participants demonstrated procedural understanding in most of aspects used to describe MKT in this study. Notably, most participants could not identify the important mathematical ideas necessary for comparing the areas of the parallelogram and the triangle. In addition, they could not identify the misconception in the questionnaire associated with decomposing and recomposing the triangle into a parallelogram to compare their areas. Moreover, during the lesson observations, the participants could not demonstrate most of the practices associated with teaching by means of the problem solving approach, demonstrating instead traditional instructional practices consistent with their descriptions in the open-ended questionnaire and the semi-structured interview. The participants cited the language used in problems, learners' abilities and attitude, their own knowledge and the advantages of the problem solving approach as the factors influencing their conceptions of teaching geometry through the problem solving approach. Arising from these results, I recommend that primary school teachers need to be empowered with knowledge in the relevant aspects of MKT that would enable them to teach geometry through the problem solving approach.

Keywords: Conceptions, knowledge of content and students, knowledge of content and teaching, mathematical knowledge for teaching, the problem solving approach

PREFACE

The work described in this thesis was carried out in the School of Education, in the Cluster of Mathematics and Computer Science Education, at the University of KwaZulu-Natal, from **March 2014 to February 2017** under the supervision of **Goba Busisiwe B.** (Supervisor).

This study represents original work by the author and has not otherwise been submitted in any form for any degree or diploma to any tertiary institution. Where use has been made of the work of others, it is duly acknowledged in the text.

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CCK	common content knowledge
CG	control group
CK	content knowledge
EG	experimental group
GPK	general pedagogical knowledge
KCS	knowledge of content and students
KCT	knowledge of content and teaching
MKT	mathematical knowledge for teaching
PBL	described problem-based learning
PCK	pedagogical content knowledge
PUFM	profound understanding of fundamental mathematics
SCK	specialised content knowledge
SMK	subject matter knowledge

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DEDICATION

I dedicate my thesis to my late father J. F. Ndlandla who despite leaving school in grade 3 ensured that I received the best education.

CHAPTER 1 : INTRODUCTION TO THE STUDY

1.1 INTRODUCTION

Concerns have been expressed regarding the problem of poor performance of most learners in geometry (Sinclair & Bruce, 2015, Steele, 2013). Although a lot has been accomplished in trying to resolve this problem, insufficient attention has been paid to teachers' understanding of meaningful geometry instruction. This is despite the fact that many studies have identified certain instructional strategies in geometry as contributing to the problem (Sinclair & Bruce, 2015; Kuzniak & Rauscher, 2011; Cavanagh, 2008; Roh, 2003; Zacharos, 2006). My study focused on this under-researched aspect of geometry instruction. Consequently, in this descriptive case study, I sought to examine primary school teachers' understanding of teaching geometry through the problem solving approach.

This chapter introduces the study. I present a discussion of my motivation for undertaking the study, followed by a discussion of my personal experience and interest. I proceed to a discussion of the current instructional strategies of geometry and measurement in the context of my study. I then discuss teaching through the problem solving approach, followed by a presentation of the statement of the problem of my study. I present the aims of the study followed by the research questions. I discuss the theoretical orientation of the study, justifying why adopting a qualitative case study approach was appropriate for my study. Thereafter a discussion of the significance of my study and, followed by a scope of my study follows. The chapter concludes with definitions of key terms used in my study.

1.2 MOTIVATION TO UNDERTAKE THE STUDY

Concerns have been expressed about the poor performance of most learners in geometry (Sinclair & Bruce, 2015). In Swaziland, for instance, as a Principal Examiner (PE) of the Swaziland Primary Certificate mathematics examination I have noticed that most candidates experience challenges in attempting geometry items, especially area of polygons items. In this capacity, one of my duties is to compile a report each year on the performance of the candidates per item in the examination. My analyses of responses on the area of polygons item reveals that candidates provide the following common incorrect responses. Firstly, most candidates cannot distinguish between perimeter and area; as a result, the majority calculate the perimeter when required to calculate the area of the polygon. Secondly, most candidates write the correct area formula but fail to substitute the correct dimensions in the formula. Thirdly, for the triangle, most candidates multiply the base by the height but fail to divide the product by two. Fourthly, some candidates do not respond to the items at all. A closer analysis of these incorrect responses reveals that they are conceptual in nature (Kakoma, 2015). Thus, researchers in the field of education have concluded that geometry is challenging to both teach and learn (Kakoma, 2015; Steele, 2013).

Despite geometry being challenging to teach and learn, it is the responsibility of teachers to help learners overcome their challenges in geometry (Shulman, 1986; Ball, Thames & Phelps, 2008). However, research has found that most teachers are struggling to assist learners overcome their challenges in geometry mainly due to two main reasons. The first reason is that most teachers have procedural knowledge of geometry (Steele, 2013). Secondly, due to their procedural knowledge, most teachers are teaching geometry through traditional instructional approaches, focusing on procedural understanding (Cavanagh, 2008; Kuzniak & Rauscher, 2011; Roh, 2003; Zacharos, 2006). Recommendations from educational research have suggested that teaching geometry with an emphasis on conceptual understanding can assist learners to overcome their challenges, thus improving their performance (Huang, 2016; Huang & Witz, 2013). Some researchers have provided evidence supporting the teaching of geometry through the problem solving approach as being one of the instructional approaches promoting conceptual understanding (O'Dwyer, Wang and Shields, 2015; Swafford, Jones & Thornton, 1997). However, little attention has been paid to examining teachers' mathematical knowledge of teaching geometry through the problem solving approach. Even less is known about in-service teachers' understanding of teaching geometry through the problem solving approach.

According to Baumert Kunter, M. Blum, W. Brunner, M. Voss, T. Jordan, A ... and Tsai, (2010. p. 138), "the repertoire of teaching strategies and the pool of alternative mathematical representations and explanations available to teachers in the classroom are largely dependent on the breadth and depth of their conceptual understanding of the subject". Therefore, teachers might fail to teach geometry with an emphasis on conceptual understanding due to inappropriate conceptions of geometry and measurement instruction (Huang, 2016; Lui & Bonner, 2016; Menon, 1998; Baturo and Nason, 1996).

Educational researchers have investigated the reasons for the poor performance in geometry and measurement and found that instruction focusing on procedural understanding is responsible for most learners' difficulties and misconceptions (Cavanagh, 2008; Roh, 2003; Tchoshanov, 2011; Zacharos, 2006). The term 'misconceptions' in this study means learners' "conceptions that produce a systematic pattern of errors" (Smith, diSessa & Rochelle, 1993, p. 119). Despite research evidence indicating that most learners' challenges in geometry emanate from the way it has been taught, researchers have given little attention to teachers conceptions of teaching geometry with an emphasis on conceptual understanding. It was not clear why most teachers persist in teaching geometry for procedural understanding despite having the responsibility of transforming the subject in a comprehensible way for learners (Shulman, 1986). Hence, there is a dire need to improve the quality of teaching geometry at the primary school level. As a starting point towards improving the quality of teaching geometry, I found it imperative to examine primary school teachers' understanding of teaching geometry through the problem solving approach, focusing on the aspect of the area of polygons. In this study, I was concerned about the poor performance of candidates in geometry at the primary school level because at this level the rudiments of abstract mathematical concepts are learned which lay the foundation for future mathematics learning (Ma, 1999). Specifically, the area of polygons – apart from laying the foundation for further learning of higher order area concepts – is useful in introducing abstract mathematical concepts such as distributive law, multiplication of numbers including fractions, and the commutative law of multiplication (Cavanagh, 2008). If learners perform poorly in geometric concepts it implies that they will experience challenges when learning the other mathematical concepts. Only a few studies have focused on examining primary teachers' conceptions of effective teaching in this important area of primary school mathematics. The majority of studies have investigated teachers' conceptions of geometry instruction without focusing on any instructional approach (Daher & Jaber, 2010; Kuzniak & Rauscher, 2011; Scholz, 1996; Smith, 2016; Steele, 2013; Swafford et al., 1997). In addition, most of these studies are from western countries, a different context from my study, with only a few focusing on in-service teachers.

In order to bridge the gap in this under-researched area, I have sought to examine primary teachers' understanding of teaching geometry through the problem solving approach. In the next section, I present a discussion of my personal interest and experience.

1.3 PERSONAL INTEREST AND EXPERIENCE

Having discussed my motivation to undertake this study, in this section I present personal interests and experiences that have motivated me into undertaking this study. I developed a strong interest in teaching mathematics through the problem solving approach after undergoing in-service training on this topic in Japan. I attended this training through a partnership between the governments of Swaziland and Japan, through the Japanese International Cooperation Agency (JICA). The two governments have collaborated in trying to improve the teaching of mathematics in Swaziland. In this partnership, JICA offered in-service training to different stakeholders in primary mathematics education, including teacher education lecturers, primary school teachers, in-service lecturers and primary school inspectors. The main goal of this in-service training was to train the different stakeholders on how to teach mathematics through the problem solving approach with the aim of infusing it to other teachers

During this training, I observed how experienced Japanese teachers planned and taught mathematics through the problem solving approach. I observed that during these lessons the learners were highly motivated and participated actively, sharing their ideas freely. The teacher acted mainly as a facilitator during the lessons, probing learners for better understanding and providing necessary materials. This training changed my conception of effective teaching of mathematics. I was impressed mainly with how the teacher called different learners to present their solution strategies to the whole class. The teacher would select presenters based on the level of difficulty of their solution strategy. Learners with the simplest solution strategy presented first and those with sophisticated solution strategies later. This resulted in the presentation of multiple solution strategies to the same problem, varying in the level of sophistication. During the solution presentation step, each presenter was encouraged to describe in detail their solution strategy, while the other learners asked probing questions for clarification, to enable them to judge the correctness of the presented solution. Takahashi (2008) referred to this practice as the *Neriage*. It was during this step that different learners revealed their misconceptions about the new content, but as the other learners explained and justified their solutions these misconceptions were resolved.

As a result of my experiences of the problem solving approach, I believed that teaching geometry through this approach might improve learners' performance. As recommended by the education policy in Swaziland, I began teaching this approach to student teachers at the Primary Teachers' College where I was a mathematics education lecturer. However, the Swaziland society seems to have a different conception of effective teaching. On numerous occasions I have experienced resistance from student teachers when trying to teach them through the problem solving approach. The student teachers seemed to prefer to be taught through traditional instructional approaches. They resisted finding for themselves ways of solving their own problems. I wondered why they were acting in such a manner. Raymond (1997) provided a possible explanation for this student teacher behaviour saying that they enter college with already formed conceptions of effective teaching from their own school experiences. Therefore, the behaviour I was observing suggested that these students had experienced mathematics instruction through traditional instructional approaches during their own schooling. I encountered the same behaviour during in-service cluster workshops for mathematics teachers that I have facilitated in the Shiselweni region. Teaching mathematics through the problem solving approach has, in recent times, dominated discussions in these workshops. I noticed that most teachers had traditional conceptions of teaching mathematics. For example, some teachers were uncomfortable conceiving of there being multiple solution strategies to the same mathematical task. These teachers expected their learners to use the solution strategies suggested in the teacher's guide. Some teachers were answer focused, instead of focusing on the process to the answer when evaluating learners' work. Chapman (1999) attributed such behaviour by many practicing teachers to lack of mathematical problem solving experiences while they were learners themselves.

In learning the area of polygons when I was in primary school, I was taught through traditional instructional approaches. My mathematics teachers would spend a few minutes of the allocated time demonstrating a procedure we should use in calculating the area of the shapes. After demonstrating the procedure, the teachers would assign many exercises for us to practice using the demonstrated procedure. My teachers were not bothered whether we understood why the procedure worked. My teachers would severely punish those learners who failed to use the procedure correctly. Research indicates that the practice of teaching geometry through traditional approaches has contributed to most learners' misconceptions in geometry (Scholz, 1996; Zacharos, 2006; Cavanagh, 2008). Drawing from the recommendations from existing research in this field, I have assumed that teaching geometry through the problem solving approach will improve learners' performance in geometry. However, there is a challenge as there is little known about the conceptions of teachers regarding this approach in the context of this study.

In the next section, I presented a discussion about the current teaching of geometry.

1.4 CURRENT INSTRUCTIONAL STRATEGIES

In the previous section, I presented my personal interest and experience which spurred me into undertaking this study. Central to this process was my experience in Japan of the teaching of the area of polygons through the problem solving approach which I could see resulted in highly motivated learners compared to the context in Swaziland. Having successfully implemented universal primary education, the government of Swaziland has now shifted focus towards improving the quality of education (Ministry of Education and Training [MoET], 2011). In trying to improve the quality of education, the government through the Ministry of Education and Training (MoET) has introduced a new education sector policy, emphasising that education should capacitate citizens with critical and analytical thinking skills. In addition, it should capacitate citizens with "technical, mathematical and quantitative skills necessary for calculation, analysis and problem solving" (MoET, 2011, p. 7). Thus, the focus of the education system in Swaziland now is to develop people with sufficient problem solving skills to cope with the changing demands of society. Informed by this education and sector policy, the MoET has introduced a new teaching syllabus in primary school mathematics stipulating that mathematics teaching should be strictly learner-centred with the problem solving approach as the principal instructional approach for mathematics (MoET, 2013).

The new mathematics teaching syllabus reaffirmed the teaching of Polya's (1945) problem solving phases and heuristics in stages in all the grades in primary school. The government is correct to emphasise the teaching of mathematics through the problem solving approach because numerous studies have described it as having many advantages for the learner (Lester, 2013; Schoenfeld, 2012; Takahashi, 2008; Taplin, 2006, Cai, 2003; Chapman, 1999). However, in order for learners to benefit from this approach, teachers should have the necessary mathematical knowledge for teaching (MKT). A few researchers have investigated teachers' conceptions of teaching through the problem solving approach (Anderson, 2000; Nantomah, 2010; Donaldson, 2011; Andrews & Xenofontos, 2015). These researchers have tended to classify teachers' conceptions of the problem solving approach into three categories, namely, teaching for problem solving, teaching about problem solving, and teaching through problem solving. They did not investigate teachers' conceptions of teaching through the problem solving approach of a specific topic such as geometry.

In Swaziland, prior to implementing the new teaching syllabus, no research was conducted to establish the teachers' conceptions of teaching through the problem solving approach. However, research has shown that adopting new instructional approaches is a challenge for most teachers (O'Shea & Leavy, 2013). In Ireland, O'Shea and Leavy (2013) reported that teachers struggled to integrate this approach in their teaching despite undergoing thorough training. Restructuring instructional approaches to integrate the problem solving approach requires teachers to reconceptualise their roles (Lester, 2013; Schoenfeld, 2012). According to Polya (1945), the problem solving approach requires teachers to reconceptualise their roles as a facilitator of learning, a role requiring "time, practice, devotion and sound principles" (p. 1). Thus, teachers need to have appropriate conceptions of problem solving, enabling them to design appropriate learning environments for instruction through the problem solving approach (Krulik & Rudnick, 1982; Anderson & Hoffmeister, 2007; Schoenfeld, 2012).

Despite the curriculum documents recommending that the problem solving approach be the main instructional approach, anecdotal evidence suggests that primary teachers have had challenges in assimilating this approach. In my view, the prescribed teacher's guides exacerbate the situation as they present lessons aligned with traditional conceptions of teaching. Most primary teachers rely on these teacher's guides when planning and delivering their lessons. However, research has shown that teaching geometry through traditional instructional approaches contributes most to learners' difficulties and misconceptions (Cavanagh, 2008). Therefore, in this study I propose that teaching geometry through the problem solving approach can improve learners' conceptual understanding, thus improving learners' performance (O'Dwyer et al., 2015).

Schoenfeld (1992) identified four aspects necessary for teaching mathematics through the problem solving approach, namely, the problem solver's knowledge base, problem-solving strategies, monitoring and control, appropriate beliefs and practices. In the context of my study, having introduced the teaching of Polya's (1945) problem solving phases and heuristics over a decade ago into the curriculum, in this study I assumed that both teachers and learners had the necessary problem-solving strategies. However, most in-service teachers did not experience problem solving in their schooling, which may result in them having inappropriate conceptions of teaching geometry and measurement through the problem solving approach. Sakshaug and Wohlhuter (2010) explain that teaching mathematics through the problem solving approach is more challenging for teachers who experienced mathematics through traditional instructional approaches as school learners.

In this study, primary teachers' understanding of teaching geometry through the problem solving approach is contrasted with traditional instructional approaches. Anderson, Sullivan and White (2004) describe mathematics instruction through traditional instructional approaches as instruction that considers mathematics as a collection of facts and procedures to be memorised by the learners. In this approach, the teacher is perceived to be responsible for transmitting knowledge to learners, relying heavily on prescribed textbooks and assigning learners repetitive exercises. In addition, during instruction the learners follow the teacher's instructions, responding correctly to his/her questions without gaining mathematical understanding. Thus, knowledge acquired in this manner is not easily transferable to unfamiliar situations. To restructure the teaching of geometry from traditional instructional approaches requires insight into the teachers' conceptions of teaching geometry through the problem solving approach and the factors underlying those conceptions.

In the next section, I discuss the problem solving approach with the purpose of situating my study.

1.5 TEACHING THROUGH THE PROBLEM SOLVING APPROACH

In the previous section, I presented a discussion of the current instructional strategies employed by most teachers in Swaziland from anecdotal experiences. It emerged from my discussion that adopting the problem solving approach is a challenge for most inservice teachers. In Swaziland, despite policy recommending that the problem solving approach should be the principal means of instruction, some lessons in the official teachers' guides present lessons from the traditional instructional approach perspective. The teachers are expected to transform these lessons into lessons through the problem solving approach.

However, there is less known about their understanding of teaching through the problem solving approach (or lack of).

There are various ways of incorporating problem solving into teaching. Hence, in this section, I attempt to clarify what I mean by teaching mathematics through the problem solving approach. Most mathematical education literature suggests three ways, namely teaching for problem solving, teaching about problem solving and teaching through problem solving. When teaching for problem solving, the focus is on capacitating learners with the skills necessary for solving problems. According to Van de Walle, Karp, Loving & Bay-Williams (2014), the learners learn a skill for the purpose of using it to solve similar problems in that topic. Consequently, Anderson (2000) associates this approach with traditional instructional approaches. Teaching about problem solving focuses on equipping learners with the procedures of problem solving, including Polya's(1945) problem solving strategies (Killen, 2015; Schoenfeld, 2012). In my study, I focus on teaching through the problem solving approach which is using problem solving as a teaching strategy.

Fí and Degner (2012, p. 455) define teaching mathematics through the problem solving approach as an instructional approach engaging learners in problem solving as a means of facilitating the learning of mathematical content and practices. Teaching through the problem solving approach assists learners "to define the problem, then assess and select among possible solutions" (O'Dwyer et al., 2015, p. 3). The problem solving approach is aligned with contemporary instructional approaches, conceiving mathematics as a non-static body of knowledge to be examined and discovered (Anderson et al., 2004). Mathematical instruction through the problem solving approach develops learners' inquiry process skills, and considers problem solving as the primary means to learning mathematics. Moreover, it encourages active learners' involvement during instruction. Learners work in groups solving unfamiliar problems with the teacher facilitating. Instruction through contemporary approaches such as the problem solving approach exposes learners to the view that mathematics is a sense-making activity (Schoenfeld, 2012).

Based on Polya's (1945) problem solving phases, Takahashi (2008) described instruction through the problem solving approach. He explained that the problem solving approach provides learners with opportunities to collaborate in small groups when solving tasks using their existing knowledge. He emphasised that when teaching through the problem solving approach the lesson does not end after learners have presented correct solutions, because the teacher facilitates extensive discussions with the learners. According to Takahashi (2008), this discussion allows learners to refine their solution strategies and overcome their misconceptions. Thus, I assumed that teaching geometry through the problem solving approach could assist learners develop conceptual understanding.

Teaching mathematics through the problem solving approach has various advantages to the learner (Chapman, 1999; Lester, 2013; Schoenfeld, 1992; Schoenfeld, 2012; Takahashi, 2008; Taplin, 2006). According to Schoenfeld (2012), the problem solving approach develops the culture of viewing mathematics a “sense-making” enterprise. In line with the principal goals for learning mathematics, teaching through the problem solving approach develops the culture of solving abstract problems among learners (Taplin, 2006). Teaching through the problem solving approach commences with the teacher presenting a task and learners using their existing knowledge in attempting to solve the task resulting in assimilating and accommodating new knowledge (Van de Walle, et al. 2014). According to Lester (2013) as learners grapple through tasks, they restructure their knowledge resulting “in the meaning making that is central to mathematical activity of all kinds” (p. 255).

Kazemi and Ghoraishi (2012) posited that the problem solving approach promotes the development of adaptable citizens, thus contributing to the practical use of mathematics. According to Lester (2013), allowing learners to develop their own solution strategies intrinsically motivates them. Charles (2009) concurs with Lester (2013) stating that the problem solving approach intrinsically motivates learners and provides opportunities for learners to experience success, spurring them to learn mathematics further. Thus, the problem solving approach has a positive effect on learners’ attitude towards mathematics.

Changing instructional approaches to teaching through the problem solving approach as suggested in the curriculum documents requires teachers to have the necessary MKT. According to Lester (2013), teachers “must be adept at selecting good problems, at listening and observing, at asking the right questions, at knowing when to prod and when to withhold comment, as well as a host of other actions” (p. 261). He argued that considering teachers’ conceptions of effective mathematics instruction and their aims of teaching would ensure a successful change of instructional approaches. In the context of my study, no attention has been paid to teachers’ conceptions of teaching through the problem solving approach, in particular the teaching of geometry. This lack of attention is lamentable because, in my view, the problem solving approach could alleviate the poor performance in geometry and measurement. In order to profit from teaching geometry and measurement through the problem solving approach, I argue that teachers should possess appropriate conceptions. Therefore, there is a need to elucidate their conceptions or lack of conceptions of teaching geometry through the problem solving approach.

In the next section, I present the statement of the problem of my study.

1.6 STATEMENT OF THE PROBLEM

In this section, I present the statement of the problem of my study. There is a problem of unsatisfactory performance of learners in geometry in Swaziland. Internationally, research has indicated that teaching geometry through the problem solving approach might alleviate this problem. As a result, the MoET has introduced a teaching syllabus for mathematics, emphasising the teaching of mathematics through the problem solving approach (MoET, 2013). However, little is known about primary school teachers' conceptions of teaching of geometry through the problem solving approach or lack thereof. Research has identified teachers' conceptions as a principal factor influencing the adoption of new instructional approaches. However, there is a gap in literature regarding primary teachers' understanding of geometry instruction through the problem solving approach. Perhaps, a study using Ball et al.'s (2008) MKT theoretical construct as a lens, examining primary teachers' understanding of teaching geometry through the problem solving approach using a qualitative case study could provide information that could elucidate the situation.

1.7 AIMS OF THE STUDY AND RESEARCH QUESTIONS

In this section, I present the aims and research questions of my study. Fundamentally, the purpose of this descriptive qualitative case study was, firstly, to identify primary school teachers' conceptions of teaching geometry through the problem solving approach in the Shiselweni region (Swaziland) and, secondly, to identify the factors influencing these primary school teachers' conceptions of teaching geometry through the problem solving approach. Informed by the purpose of my study, I formulated two research questions to guide my study:

(a) What are primary school teachers' conceptions of teaching geometry through the problem solving approach in the Shiselweni region (Swaziland)?

(b) What are the factors influencing the primary school teachers' conceptions of teaching geometry through the problem solving approach?

1.8 THEORETICAL ORIENTATION OF THE STUDY

In this section, I present the theoretical orientation of my study. Shulman (1986) used the term pedagogical content knowledge (PCK) to describe the aspects of teachers' knowledge essential for teaching specific mathematical content conceptually. According to Shulman (1986), PCK integrates the understanding of content and pedagogy specific to the teaching profession. Further, PCK encompasses knowledge of learners' difficulties, existing knowledge and misconceptions in the learning of that content. Later, Ball et al. (2008) refined Shulman's description of PCK by differentiating two categories, knowledge of content and students (KCS) and knowledge of content and teaching (KCT), in the MKT framework. According to Ball et al. (2008), KCS integrates the knowledge of learners and the knowledge of mathematics and KCT integrates the knowledge of specific mathematics content and instructional approaches appropriate for teaching that content.

In answering the research questions in my study, I employed Ball et al.'s (2008) MKT as a theoretical lens, for a nuanced examination of the teachers' conceptions of teaching geometry through the problem solving approach. Fundamentally, my study focused on two domains of the MKT framework: KCS, the integration of knowledge of learners and the knowledge of mathematics; and KCT, the knowledge of both specific mathematics content and instructional approaches appropriate for teaching that content. This framework views the different aspects of teacher knowledge as a context specific entity, implying that for each mathematical topic, teachers should possess the relevant instructional approaches necessary for transferring that topic in a comprehensible manner to learners. In addition, the MKT framework for each topic regards the knowledge of learners' misconceptions as springboards for transferring content knowledge in meaningful ways to learners, as teachers should plan in advance instructional activities for overcoming them.

Despite clearly elucidating the different aspects of teacher knowledge essential for teaching towards conceptual understanding, Ball et al.'s (2008) MKT framework does not provide a description of teaching geometry through the problem solving approach. Therefore, in this study I framed the teaching of geometry through the problem solving approach using Takahashi's (2008) problem solving approach framework. Careful examining of teacher knowledge, guided by these frameworks allowed me to gain insight into the primary teacher's understanding of teaching geometry through the problem solving approach.

In addressing the research questions, my study employed a qualitative case study approach (Yin, 2009). This allowed me to gather exceptionally rich, exhaustive and in-depth data from the participant's perspectives in their natural environment (Berg, 2001). Due to the inconsistencies that exist between teachers' conceptions and practice, in my study I collected data in two phases. In the first phase, 34 primary school mathematics teachers completed an open-ended questionnaire with three parts. Part I probed the participants' demographic information, Part II (adapted from Manizade & Mason, 2011) probed the participants' MKT regarding the area of polygons, Part III asked the participants a direct question about the factors that have affected their conceptions of teaching geometry through the problem solving approach. To verify the information supplied by the participants in the open-ended questionnaire, I selected two participants from the initial 34 to participate in a lesson observation while teaching a lesson on the area of polygons and subsequent semi-structured interviews about their MKT related to the area of polygons, adapted from Sothayapetch, Lavonen and Juuti (2013). This provided two case studies.

Mapping the data to the research questions, Research Question (a) maps to Part II of the open-ended questionnaire, lesson observation, semi-structured interview, and lesson plan analysis and Research Question (b) maps to item 11 of the open-ended questionnaire.

In the next section, I present a discussion of the significance of my study.

1.9 SIGNIFICANCE OF THE STUDY

My study is important in the Swaziland context, for providing a description of primary teachers understanding of teaching geometry and measurement through the problem solving approach in primary schools. Focusing at the primary school level of geometry instruction is important because primary school is responsible for inculcating the necessary skills and habits for further learning of other mathematical concepts (Browning, Edson, Kimani, & Aslan-Tutak, 2014). However, most teachers rely on traditional approaches in geometry instruction, resulting in unsatisfactory performance of learners in geometry items. This study provides insight into primary teachers' understanding of teaching geometry through the problem solving approach. Knowing primary teachers understanding of teaching geometry through the problem solving approach and the factors influencing their understanding is a starting point towards improving geometry teaching. This is consistent with Atweh and Ochoa's (2001) assertion that any successful transformation of instructional practices needs to consider teachers' conceptions to ensure that "the reform directly addresses the real problems that they face and empowers them to take control over the change process so that it becomes continuous after special projects cease" (p. 181).

Several different groups in the mathematics education community including the teacher participants, pre-service and in-service educators, curriculum developers, and classroom teachers can benefit from this study. The study provides pre-service teacher educators with knowledge that can inform their practice while trying to provide opportunities for pre-service teachers to improve the teaching of geometry through the problem solving approach. Educators will know which conceptions they need to challenge in their student teachers and those that need development in order to promote the teaching of the area of polygons and mathematics in general, through the problem solving approach. Likewise, the study can provide in-service teacher educators with information that can assist them when designing appropriate training activities for their training programs. Moreover, results from this study represent a starting point for extending teachers' understanding of teaching mathematics through the problem solving approach to other topics in the primary school mathematics curriculum.

The participants involved in the study benefited from their participation in the study through opportunities to reflect on their practice while completing the questionnaire and during lesson observation and interviews with researcher. Curriculum developers might benefit from knowledge of teachers' conceptions regarding teaching geometry through the problem solving approach since this information can inform the design teaching/learning materials. Lastly, this study contributed knowledge about primary school teachers' knowledge regarding the teaching of geometry through the problem solving approach in Swaziland for future researchers in primary school mathematics education.

1.10 SCOPE OF THE STUDY

My study involved some primary school teachers in the Shiselweni region of Swaziland, teaching mathematics in the grades of grade three to grade seven. My study was confined to teachers' understanding within the area of polygons as taught in Swaziland primary school curriculum.

1.11 DEFINITION OF TERMS

In this section, I provided the definitions of some of the key terms as I used them in my study.

1.11.1 Conceptual knowledge

In this study, the term 'conceptual knowledge' is used according to Rittle-Johnson and Alibali's (1999) definition which is: "explicit or implicit understanding of the principles that govern a domain and the interrelations between pieces of knowledge in a domain" (p. 175).

1.11.2 Procedural knowledge

The term procedural knowledge is used to mean: "action sequences for solving problems" (Rittle-Johnson & Alibali, 1999, p, 175).

1.11.3 Teacher conceptions

Education literature has various definitions of teacher conceptions, but in this study, the assertion by Cai (2007) was adopted. Cai (2007) asserts that a teacher's conception can be viewed as "that teacher's conscious or subconscious beliefs, concepts, meanings, rules, mental images and preferences concerning the discipline of mathematics" (p. 266).

1.11.4 Pedagogical content knowledge

Different meanings have been associated with the term 'pedagogical content knowledge' (PCK) by researchers in education. In this study, I used PCK according to Shulman's (1986) definition which encompasses:

> … the most regularly taught topics of one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others … Pedagogical content knowledge also includes an understanding of what makes the learning of specific concepts easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those frequently taught topics and lessons. If those preconceptions are misconceptions, which they

> so often are, teachers need knowledge of strategies most likely to be fruitful in reorganizing the understanding of learners because those learners are unlikely to appear before them as blank slates (pp, 9-10).

1.11.5 Teaching mathematics through the problem solving approach

Teaching mathematics through the problem solving approach refers to an instructional approach "that engages students in problem solving as a tool to facilitate students learning of important mathematics subject matter and mathematical practices" (Fí & Degner, 2012, p. 455). A further definition is given by Takahashi (2008) who defines teaching mathematics through the problem solving approach as being an instructional approach greatly influenced by Polya (1945) which allows learners to construct mathematical knowledge and skills by creatively solving challenging problems by themselves.

1.11.6 Learners misconceptions

According to Smith et al. (1993), research on misconceptions has conceived a wide range of terms to describe learner's conceptions such as preconceptions, alternative conceptions, naïve beliefs, alternative beliefs, alternative beliefs and naïve theories. However, describing learner's conceptions using any of these terms has epistemological differences, originating from "how researchers have characterised the cognitive properties of student ideas and their relation to expert concepts" (Smith et al., 1993, p. 119). In this study, I used the term misconceptions to mean learners' "conceptions that produce a systematic pattern of errors" (Smith et al., 1993, p. 119).

1.12 STRUCTURE OF THE STUDY

In Chapter 1, I presented a brief introduction to my study. I then proceeded to present a discussion of my motivation to undertake the study, followed by a discussion of my personal experience and interest. I proceeded to present a discussion of the current instructional strategies of geometry and measurement in the context of my study. I then presented a discussion of teaching through the problem solving approach followed by a presentation of the statement of the problem of my study. I proceeded to present the aims of the study followed by the research questions. I proceeded to present the theoretical orientation of the study, justifying why adopting a qualitative case study approach was appropriate for my study. I went on to present a discussion for the significance of my study, followed by a presentation of the scope of my study. I concluded the chapter by presenting the definition of key terms as I used them in my study.

In Chapter 2, I present the discussion of literature related to the study. I begin by discussing literature focusing on teacher conceptions followed by literature on conceptions of teaching mathematics through the problem solving approach. I then discuss literature focusing on conceptions of teaching geometry followed by a discussion of literature focusing on common learners' misconceptions in area of polygons. I conclude the chapter by presenting a discussion of how other researchers have examined different aspects of teacher knowledge.

In Chapter 3, I present a discussion of the conceptual framework of the study, comprising the MKT framework framing the different aspects of teacher knowledge in this study. I then discuss Polya's (1945) problem solving phases and Schoenfeld's (1992) mathematical cognition framework. I conclude the chapter by presenting Takahashi's (2008) framework for teaching mathematics through the problem solving approach.

In Chapter 4, I present the methodological approach employed in addressing the research questions of my study. I begin by presenting a justification for locating my study within the interpretivist paradigm. I then proceed to present a discussion of the research design, followed by the population of the study and the sample and sampling procedures. I then discuss the data collection and data analysis procedures. I conclude the chapter by presenting a discussion of how I addressed ethical issues in my study and the limitations of my study.

In Chapter 5, I present the data for my study in two sections, starting with the first research question regarding conceptions of teaching geometry. In the second section I present results addressing the second research question regarding the factors influencing conceptions of teaching geometry. I conclude the chapter by presenting a summary of the results of my study.

In Chapter 6, I discuss the results of my study, linking them with existing literature. I went on and presented a conclusion of my study and recommendations of my study. I concluded this chapter by presenting contributions of my study and autobiographical reflections.

1.13 SUMMARY

In this chapter, I presented the motivation for doing this study, justifying why the study was worthwhile. In the next chapter, I present a review of related literature.

CHAPTER 2 : REVIEW OF RELATED LITERATURE

2.1 INTRODUCTION

In the previous chapter, I discussed my motivation to undertake the study, personal experience and interest, the background my study, statement of the problem, purpose of the study, research questions, significance of the study, limitations of the study and definition of terms. In this chapter, I present a review of related literature focusing on studies of teachers' conceptions of geometry instruction and teachers' conceptions of teaching through the problem solving approach. The review adopts a thematic approach. According to Litchman (2006), reviewing literature in a qualitative research study enhances the understanding of the present state of research, indicating accomplished works and what needs to be accomplished concerning the phenomenon being studied. Thus, doing a literature review assists in situating the present study in the context of existing theories.

2.2 TEACHER CONCEPTIONS

In this section, I present a discussion of literature related to teacher conceptions. The construct 'conception' lacks a precise definition, hence scholars have presented various definitions (Hoz and Weizman, 2008). Hoz and Weizman (2008) defined a conception as "a comprehensive and homogeneous set of ideas about a particular characteristic or feature of that entity" (p. 905). Ponte (1994) defined conception as an "underlying organising frames of concepts, having essentially a cognitive nature" (p. 5). Cai (2007) described conceptions as "teacher's conscious or subconscious beliefs, concepts, meanings, rules, mental images and preferences concerning the discipline of mathematics" (p. 266). According to Cai (2007), teachers' fundamental perspective of mathematics consists of beliefs, concepts, views and preferences about mathematics. In my study, I use the term conceptions according to Cai's (2007) definition. From Cai's (2007) definition of conceptions it is apparent that beliefs are a subset of conceptions, therefore I use them interchangeably.

Acknowledging the complex relationship between teachers' conceptions and practice, in this study I argue that teachers' conceptions influence their instructional actions (Thompson, 1984). According to Wang, Chin, Hsu and Lin (2008) teachers expose their conceptions of mathematics and its instruction during the teaching process. Porte (1992) asserts that teacher's conceptions function as a sieve, either organising the knowledge derived from their experiences or inhibiting the acquisition of knowledge derived from unfamiliar experiences. Thus, Porte (1992) asserts that teachers' conceptions affect their decision making process either positively or negatively, thus shaping the nature of the classroom environment they create. In this study I argue that teachers' conceptions influence their classroom actions. I investigate this by examining primary teachers' conceptions of teaching geometry through the problem solving approach.

Several studies have examined the nature of the relationship between teachers' conceptions and instructional practices. Raymond (1997) examined the relationship between beliefs and practices of six beginning primary teachers in the United States of America. In the study, the primary teachers listed the following factors they considered as influencing their beliefs and practice: their own schooling experiences, their instructional experiences and their pre-service education. The teachers in the study identified their own mathematical conceptions and learners' behaviour as exerting strong influence on their conceptions of instructional practices compared to their pre-service education. In the study, she concluded that teachers with strong traditional instructional conceptions tended to enact traditional mathematics despite holding non-traditional conceptions of mathematics instruction.

Instead of focusing on investigating teacher beliefs as consistent/inconsistent, Zheng (2013) used complexity theory to investigate the connection between beliefs, practice and context of six experienced English as Foreign Language teachers in a Chinese secondary school. In her study, she collected data using interviews, classroom observations and stimulated recall for holistic examination of the participants' beliefs. She found three characteristics of the relationship between teachers' belief and their practices. Firstly, the relationship was complex as beliefs systems could be either in harmony or in conflict. As a result, she suggested that any effort in modifying teachers' beliefs and practice should take into account both the teachers' core beliefs and the nature of their connection. Secondly, the relationship was dynamic; describing it using the term "superficial", as some teachers only described their classroom practice as recommended in curriculum documents but could not demonstrate them in actual classroom teaching. She used the term *token adoption*, to refer to the tendency of some teachers to describe their classroom practice as recommended in curriculum documents, but failing to implement them in actual practice. She perceived this tendency as revealing the dynamic and complex nature of teachers' belief systems. Consequently, she recommended observing teachers in actual practice when examining their beliefs to minimise the act of "token adoption" during curriculum reform. Lastly, teachers resorted to an "eclectic" approach in militating against disequilibrium between their belief systems. She asserted that awareness of this disequilibrium between beliefs systems allowed teachers to review their thinking resulting in reformed practices.

This study provided valuable information about the characteristics of the relationship between teachers' beliefs systems and their practice. As my study focused in examining teachers' conceptions of a particular instructional, it informed my data collection procedures. However, Zheng (2013) study did not examine teachers' conceptions of teaching a particular topic through a specific approach, as I did in this study.

Using Shulman's (1986) conceptualisation of PCK, Hawkins (2012) examined lower primary teachers' knowledge of measurement instruction. However, he did not focus on the PCK of measurement instruction through a specific pedagogy, but used a model to examine the PCK required for measurement instruction. The model represented the relationship between the three categories of Shulman's (1986) conceptualisation of PCK: knowledge of mathematics, knowledge of students, and knowledge of teaching. In the study, he investigated among other things, the impact of teachers' knowledge on their actions.

In addressing the research questions in his study, Hawkins (2012) used a qualitative multiple case study approach with four cases. At the beginning of the study, he interviewed each participant and then observed them while teaching some measurement lessons and pre- and post-lesson interviews. Two weeks after finishing the measurement lessons, the participants completed a reflective questionnaire. He found that the teachers had diverse PCK in the knowledge of mathematics, knowledge of students and knowledge of teaching. After a detailed examination of the differences in the participants' PCK, he found that PCK was influenced by self-efficacy, beliefs and the school culture. He argued that beliefs influenced PCK. Consequently, a teacher holding strong beliefs in one category may fail to enact those beliefs in actual classroom practice, because of insufficient knowledge in another category. This finding was consistent with Park and Oliver's (2008) assertion that PCK for effective instruction was a complex amalgam of all the different aspects of teacher knowledge. Due to the complex relationship between the different aspects of teacher knowledge, Park and Oliver (2008) concurred with Hawkins (2012) that improving one component of PCK may be insufficient in initiating change in behaviour, although this may stimulate the growth in the other components.

In this study, I argue that teachers' conceptions influence their practice. Due to my argument, I sought to examine primary teachers' conceptions of teaching geometry through the problem solving approach. The studies I reviewed in this section provided evidence that, indeed, teachers' conceptions influence their classroom actions. However, they urged caution in that the nature of the relationship between teachers' conceptions and practice was complex and dynamic. However, these studies did not focus on the conceptions of teaching geometry through the problem solving approach, as I did in this study.

In the next section, I present a discussion of literature concerning teachers' conceptions of teaching mathematics through the problem solving approach.

2.3 TEACHERS' CONCEPTION OF TEACHING MATHEMATICS THROUGH PROBLEM SOLVING

Having discussed literature dealing with the relationship between a mathematics teacher's conceptions and practice in the previous section, in this section I discuss teachers' conceptions of teaching mathematics through the problem solving approach. Fí and Degner (2012) defined teaching through the problem solving approach as "a pedagogy that engages students in problem solving as a tool to facilitate students learning of important mathematics subject matter and mathematical practices" (p. 455). Much research on problem solving has focused on teachers' conceptions of problem-solving with a few studies investigating teachers' conceptions of problem solving as a pedagogical approach, which is what I have done in this present study.

As a background for discussing teacher's conceptions of teaching mathematics through the problem solving approach, I begin this section by discussing studies investigating teachers' conceptions of problem solving. Cai and Lester (2005, p. 221) described problem solving as "an activity requiring the individual to engage in a variety of cognitive actions, each of which requires some knowledge and skill, and some of which are not routine". According to Lester (1994, p. 668), problem solving is "an extremely complex form of human endeavour that involves much more than the simple recall of facts or the application of well-learned procedures".

Grouws, Good and Dougherty (1990) ascertained through interviews the problem solving conceptions of 25 junior high teachers from eight different schools. They found that the teachers had four different conceptions of problem solving. Firstly, some conceived problem solving as being synonymous with word problems. They noted that this group disregarded the cognitive demand of a problem and concentrated on the ease of translating the problem into symbols, with the textbook as the main source of problems. Secondly, some conceived problem solving as searching for answers to problems. For this group, engaging in problem solving meant using a problem solving model successfully to obtain correct solutions, hence they regarded their learners' solution procedures as problem solving. Thirdly, some participants conceived of problem solving being a hands-on activity involving solving real problems. Consequently, they expected their learners to apply acquired problem solving skills in overcoming their daily life challenges. Lastly, some participants conceived problem solving as solving abstract problems, involving higher order thinking, and generating multiple solution strategies for the same problem. In this study, I adopted this last conception of problem solving. Despite providing an understanding of the different teachers' conceptions of problem solving, their study was limited because it did not focus on conceptions of teaching mathematics through the problem solving approach.

A few studies have tried to investigate teachers' conceptions of teaching mathematics through the problem solving approach. However, most of these studies did not investigate teachers' conceptions within a specific topic, such as area of polygons. In addition, they did not use the MKT construct as a lens. Using a mixed methods approach, Anderson (2000) investigated primary mathematics teachers' problem solving beliefs and practices in New South Wales in Australia. In the first portion of her study, she used a closed-ended questionnaire to extract the problem solving conceptions and practices of 162 primary school teachers. In the second portion of her study, she selected two participants from the initial 162, to participate in interviews and observations conducted in their respective schools. Results from her study indicated that the participants implemented teaching through the problem solving approach in varying degrees. She found that 4% reported very traditional approaches, 11% traditional approaches, 9% contemporary, 7% very contemporary and 69% demonstrated both traditional conceptions and contemporary conceptions. In addition, the results revealed that teaching experience did not have a significant influence on the participants' teaching practices. However, the grade level had a significant influence on the teaching practices, as the results revealed that middle primary school teachers demonstrated mostly traditional practices, emphasising procedural understanding compared to lower grade teachers who preferred contemporary approaches.

She reported that these results showed that participants were predominantly teaching *for* and *about* problem solving instead of teaching *through* the problem solving approach. The participants stated numerous reasons for not teaching through the problem solving approach, despite believing that it was beneficial to the learner. She categorized the constraints impeding the participants from teaching through the problem solving approach as being “those related to teachers themselves, to the learners, to the school culture, and to the education system” (p. 328). She found that these constraints were difficult to overcome, however extra knowledge acquired in postgraduate education seemed to help in overcoming them. She identified three factors as influencing the participants’ conceptions of teaching mathematics through the problem solving approach. These factors were the participants’ actual conceptions about problem solving, their own knowledge of problem solving, and their assimilation of advice concerning instruction through problem solving and, lastly, their use and comprehension of curriculum documents.

In a different study, Nantomah (2010) investigated the conceptions of the problem solving approach of 107 junior secondary school teachers in the Upper East region of Ghana using a qualitative case study. His study had a limitation, as he did not focus on teachers’ conceptions of instruction as a specific topic area through the problem solving approach. In addition, he focused on junior secondary school teachers, a different level from my study which focuses on primary school level. Despite these limitations on the study, it increased my understanding of teachers’ conceptions in the African context. He reported that the participants had four conceptions of problem solving informing their classroom actions. These conceptions were, firstly, that some participants conceived problem solving as a process of solving difficult mathematical tasks for which there was no apparent and immediate method of finding the solution. Secondly, problem solving was conceived of as solving mathematical word problems, which involve real life situations. Thirdly, it was conceived of as accepting a challenge and striving hard to resolve a mathematical problem. And lastly, problem solving was regarded as solving open-ended problems which led to open investigations and multiple solutions.

He observed that the participants experienced different challenges in their attempts to teach mathematics through the problem solving approach. Some of these challenges related to the participants' understanding of teaching mathematics through the problem solving approach. This indicated the need for examining their understanding of teaching mathematics through the problem solving approach using a framework that considered the different aspects of teacher knowledge. He found that the participants had limited understanding of problem solving to enable them to create effective problem solving classroom contexts. Furthermore, the teachers' own conceptions about effective mathematics instruction and the challenges in planning of problem solving lessons affected their implementation of the problem solving approach. These factors related to the participants' PCK of problem solving.

He reported other factors unrelated to teacher knowledge of problem solving in the study that affected the participants in their attempts to implement the problem solving approach. These factors were time constraints, insufficient problem solving exercises in instructional materials, the higher order skills demanded by the problem solving approach from the learners, the learners' low linguistic abilities, and lastly, insufficient contact time allocated to mathematics lessons in the schools' teaching schedule. In a different study, Anderson et al. (2004) identified the level of cognitive development of the learners at the grade being taught, the school philosophy concerning teaching and learning of mathematics, and time limitations as the factors that influences teachers' conceptions of the problem solving approach.

Andrews and Xenofontos (2015) examined English and Cypriot prospective primary teachers' conceptions of problem solving, but did not focus on conceptions of teaching a particular topic through the problem solving approach. Furthermore, the study focused on prospective primary teachers, a different focus group to the present study that focused on in-service primary teachers. However, the study enriched my understanding of the problem solving phenomenon in the primary school. Moreover, as a comparative qualitative study across two countries, the study also provided vital knowledge as it revealed the culturally specific nature of teachers' problem solving conceptions. In the study, they claimed that their "analyses have shown that concepts typically thought to have common meanings cross-culturally invoke subtly different responses in different cultures" (p. 294). As a result, both cohorts articulated both convergent and divergent views concerning the nature of problem solving. Both groups construed problem solving as being "a process during which problem solvers attempt to understand what is required, extract relevant information, select and implement an appropriate mathematical procedure before comparing the solution with the problem's expectations" (p. 293). However, regarding the influence of the learners basic arithmetic skills in problem solving outcomes, the cohorts had divergent views. Compared to their Cypriot counterparts, the English prospective teachers believed that the learners' competency on basic arithmetic skills was a prerequisite to successful problem solving. They argued that such a conception was unlikely to lead to problem solving as pedagogy. The Cypriot prospective teachers described mathematical problem solving as a *process,* an observation they inferred might be a result of having acquired this perspective in their problem solving course. While the English prospective teachers mentioned the aspects of problem solving such as "understanding, *knowing basic mathematics, adapting and applying*" (p. 293) which was in accordance with their country's mathematics teaching objectives.

Despite bringing in the influence of cultural influence on the conceptions of problem solving instruction, their study had limitations in the context of my study. A major limitation of the study resulted from collecting data through semi-structured interviews only, which raised trustworthiness issues with the findings. As already discussed in Section 2.2, the relationship between conceptions and practice was sometimes inconsistent, therefore it was recommended to verify accuracy of responses through observation of practice. For instance, there was a need to observe in real classroom what the Cypriot prospective teachers meant by describing mathematical problem solving as a *process* as the researchers inferred it might be a description they learned during their classes.

Emerging from the literature reviewed, it is evident that mathematics teachers have various culturally specific conceptions of mathematics problem solving implying the need for separate studies for each context. In the reviewed studies, the participants had varied conceptions of teaching mathematics through the problem solving approach. However, in the majority of the studies, the participants regarded teaching mathematics through problem solving approach as developing the learners' problem solving skills, equipping them with skills for resolving their daily life (Anderson, 2000). Furthermore, the teachers had insufficient PCK necessary for implementing the problem solving approach in their teaching. Teaching mathematics through problem solving approaches requires teachers to reformulate their roles to prepare effective problem solving environments.

The studies I discussed in this section increased my understanding of teachers conceptions of the teaching through the problem solving approach. However, as I already revealed, these studies tended to investigate teachers conceptions of teaching through the problem solving approach without focusing in the teaching of a specific topic, such as the area of polygons in the primary school level. None of these studies used Ball et al.'s (2008) MKT as a theoretical lens in investigating the teachers' conceptions, as I did in my study. Moreover, these studies were conducted in a different context to my study. Teacher knowledge is contextual (Depaepe, Verschaffel and Kelchtermans, 2013) hence, there is a need to investigate teachers' conceptions of problem solving in the context of Swaziland. In addition, the reviewed studies revealed that teachers faced various challenges in their attempt to teach mathematics through the problem solving approach.

In the next section, I present a discussion of literature on teachers' conceptions of geometry instruction.

2.4 TEACHERS' CONCEPTIONS OF GEOMETRY INSTRUCTION

Having discussed the literature on teachers' conceptions of teaching mathematics through the problem solving approach, in this section, I present a discussion of literature on teachers' conceptions of geometry instruction. In this study, one of my aims was to examine primary teachers' conceptions of teaching geometry through the problem solving approach and not just the teaching of mathematics through this approach. Therefore, in addition to reviewing literature dealing with teachers conceptions of teaching mathematics through the problem solving approach, it was imperative that I review literature on teachers' conceptions of geometry instruction. In this section, I discuss literature focusing on teachers' conceptions of geometry instruction.

Clements and Battista (1986) defined geometry as "the study of objects, motions, and relationships in a spatial environment" (p. 29). A few studies have examined the kinds of teacher knowledge associated with geometry instruction (Steele, 2013). Previous research in geometry has focused mainly on teachers' conceptions of geometry content, neglecting the conceptions of geometry instruction, especially through the problem solving approach, as I have done in my study. Sinclair and Bruce (2015) argued that teachers' conceptions of geometry influence their interpretation of school geometry learning outcomes. Due to the scarcity of studies focusing on in-service primary teachers' conceptions of geometry and measurement, focusing on area of polygons, I also included studies focusing on primary pre-service teachers and secondary teachers in the review in this section.

Scholz (1996) investigated the conceptions of geometry and its instructional approaches among pre-service secondary teachers in their final year of study in the United

States of America in two stages. In the first stage, ten teachers' participated in a card sort, with an interview, a journal and video tasks and four of these teachers participated in the second stage of the study, which involved a lesson observation while teaching geometry lessons. In the video task, each participant analysed the instructional practices of experienced teachers. In the study, she found that the classroom observations, apart from ascertaining the participants' geometry conceptions and their instructional practices, allowed for the examination of the connection between the participants' conceptions of geometry and their instructional practices. Informal pre- and post-classroom observation interviews allowed the participants to clarify their statements or actions. Lastly, document analysis gathered detailed information about the participants' activities and classroom actions.

In the study, she found that the connection between the participants' geometry conceptions and their instructional approaches was complicated. She observed that the participants mentioned their conceptions of geometry whenever they described their geometry instructional approaches. In addition, their conception that geometry was linear influenced their conception of its instructional approaches. The participants developed their linear conception of geometry from their own school geometry experiences, their own conceptions of effective geometry instruction and lastly, their conception that the textbooks had ideal sequencing of geometry concepts, therefore geometry instruction should conform to the textbooks. Consequently, for instruction the participants depended on the textbooks for instructional content. In addition, some participants believed a task had only one acceptable solution strategy while others believed a task had multiple solution strategies. Furthermore, the participants' conceptions of geometry influenced their conceptions of its instructional approaches, while their teaching of geometry affected their geometry conceptions. She concluded that the textbook determined the link between the participants' geometry conceptions and its instruction.

Regarding the nature of geometry, she reported that the participants had different conceptions. First, some construed geometry as a subject providing skills necessary for problem solving while a minority understood geometry as a subject providing algorithms necessary for problem solving. Second, some construed geometry as providing a means of describing the world visually. Lastly, some conceived of geometry as providing a useful link between various mathematical concepts, especially algebra. She noted that most participants' asserted that their unsatisfactory geometry knowledge eroded their confidence during instruction. Among the reasons stated by the participants as to why their knowledge was unsatisfactory was that they had not yet had the experience of teaching it. This implied that the participants believed that their geometry knowledge would improve after they have had the opportunity of teaching it.

In the study, she reported that the participants had varied conceptions of geometry instructional approaches, which were influenced by their views concerning the roles of the teacher during instruction. These roles encompassed the teachers' conduct, knowledge of geometry, ability to explain the content clearly and their awareness of meaningful ways of instruction. However, during their classroom instruction the participants referred constantly to the textbook, an act she interpreted as indicating their weak geometry knowledge. There was dissonance between their conceptions and practice, as most mentioned that the teachers should provide learners with manipulative activities and involve them actively during instruction. During the classroom observations, she found that the participants were teaching towards procedural understanding. Most participants believed that geometry instruction should be contextual, considering learners experiences; as a result, they intended to make geometry useful in their learners' daily lives.

This study provided useful information about participants' conceptions of geometry and its instruction. However, this study had a limitation as it focused on conceptions of geometry and its instruction at the secondary level. In addition, the study did not investigate the conceptions of geometry instruction through a particular approach, such as teaching through the problem solving approach. Moreover, the study did not include the area measurement aspect of geometry, such as the area of polygons in the primary school level.

In a similar study, Daher and Jaber (2010) investigated the conceptions of geometry instruction at primary school. Unlike Scholz (1996), their study focused on 52 geometry teachers in the primary school level in a particular school in Israel. In their study, Daher and Jaber (2010) interviewed each participant about their conceptions of geometry and its effective instruction. They reported that the participants had varied conceptions of geometry, stressing the importance of using manipulatives during geometry instruction. In addition, they mentioned that manipulatives influenced the success or failure of the participants' instructional strategies. However, the participants did not elaborate on how these manipulatives influenced the success or failure of their instructional strategies. Furthermore, they found that the participants' "educational and life experiences, their use of tools, and the abundance of tools they can use in their teaching" influenced their conceptions of geometry (p. 152). Moreover, they found that the participants believed geometry teaching was important at the primary school level because it had some connection to other educational fields, while others believed it was a source of motivation in mathematics learning.

Similar to Scholz's (1996) study, Daher and Jaber's (2010) study provided information about teachers' conceptions of geometry, but their study focused on teachers' conceptions of geometry instruction without focusing on a specific instructional strategy such as the problem solving approach. Moreover, these studies did not adopt Ball et al.'s (2008) MKT framework in investigating the participants' conceptions as I did in my study. Emerging from both studies was the usefulness of involving learners and the use of manipulatives during geometry instruction; however, they did not provide a framework of how to include them during instruction.

A few studies investigated teachers' conceptions using a framework that differentiated the different aspects of teacher knowledge, such as the PCK or MKT constructs (Steele, 2013; Swafford et al., 1997; Yeo 2008). Yeo (2008) investigated how PCK influenced the instructional approach used by a participant, who was a beginning mathematics teacher, in teaching area and perimeter of composite flat shapes to primary school learners. In the study, he collected videotaped data of five lessons taught by the participant and recorded field notes. His data analysis concentrated on the participant's actions and decisions during the lessons, providing knowledge about the interaction between content knowledge (CK) and PCK in the topic. He found that the participant's classroom actions demonstrated that the participant had strong CK and PCK, as the participant's sequenced appropriately the content in the lessons. In addition, the participant chose appropriate tasks, used effective questioning techniques and focused on connecting the area and perimeter concepts during instruction. Moreover, the participant used appropriate language during lessons and contextualized the lesson tasks, resulting in learners relating to the tasks. Further, the participant effectively interrogated learners' alternative solutions during lessons, encouraging them to present unique solutions that differed from their textbooks. Lastly, the participant reported that group work allowed the learners to develop conceptual understanding of the content.

This study provided a picture of lesson on area of polygons by a teacher with strong PCK. Hence, he concluded that effective instruction of area of polygons required both strong CK and PCK. Spurred by this conclusion, in my study I argue that a teacher's conception of effective geometry instruction influences their instructional practices. The goals of the his study were close to the goals of my study, however it had a limitation in that it was a single case study, therefore its results could not be generalized. In addition, his study did not focus on investigating the interaction of the participant's CK and PCK within a specific instructional approach, such as the problem solving approach. As result, some of the actions regarded as appropriate in that study contradicts practices of teaching area of polygons through the problem solving approach. For instance, the practice of explaining a procedure to learners before they do tasks is frowned upon in teaching through the problem solving approach. Secondly, this study accepted definitions promoting procedural understanding such as defining the area of a rectangle as "length times breadth". Thus far no study has focused on teaching geometry through the problem solving approach, indicating a gap in the literature.

Swafford et al. (1997) examined the influence of improved geometry knowledge of 49 in-service primary school teachers to their instructional approaches after engaging the participants in intervention training. In the training they engaged the participants in a session on Van Hiele's levels of geometric development and geometry instruction through the problem solving approach. They found that because of the intervention course the participants' improved their geometry CK, reporting the following practices as demonstrating their improved geometry knowledge and instructional approaches: (1) The participants prepared more learner-centred lesson plans providing opportunities for learners to use concrete activities; (2) on numerous occasions before teaching a new concept they considered learners' prior knowledge, as they claimed it revealed the amount of knowledge the learners' had about the new concept; (3) the participants were keener to experiment with new ideas and instructional approaches, thus improving the quality of their instruction; (4) the teachers were more confident in their capabilities to stimulate and react to learners' higher order geometric thinking, easily tackling concepts not covered in the prescribed textbook.

From their study, they concluded that training the participants' in the problem solving approach positively influenced their conceptions of effective geometry instruction, as they expected similar goals from their geometry instruction. This study provided information supporting my own assumption that teaching geometry through the problem solving approach could improve learners' performance. As this study demonstrated, teachers with conceptions of teaching geometry through the problem solving approach taught geometry in context, connecting it with other mathematical concepts, and used divergent questions that promoted learners critical thinking abilities.

Despite providing valuable information, one major limitation of this study was the teachers being observed after the intervention course; hence, there was a possibility of the teachers acting out their practices as per the expectations of the intervention course. Moreover, the study did not focus on the aspect of geometry involving area of polygons. Furthermore, their study adopted the Van Hiele levels of cognitive development as theoretical framework, a different framework to my study. However, this study added weight to my argument that there is a need to examine teachers' understanding of teaching geometry through the problem solving approach.

Menon (1998) investigated directly the effect of teachers' CK on instruction. In the study, he found that participants with weak CK could not teach geometry with integrity but focused on teaching geometry with an emphasis on procedural understanding. When compared with Swafford et al.'s (1997) study, the weak geometry knowledge of the teachers in his study constrained them to design teaching/learning activities that inhibited the development of higher order thinking skills among their learners. Reinke (1997) in a different study, with pre-service teachers, on the topic of area and perimeter, found similar results to Menon (1998). Reinke (1997) found that primary pre-service teachers had difficulty in calculating the perimeter and area of shaded geometric shapes. These studies highlight the importance of teacher geometric knowledge in the teaching of geometry in primary school.

Anderson and Hoffmeister (2007) reported on a study of an intervention course designed to enhance the subject matter knowledge (SMK) of primary school mathematics teachers' towards conceptual understanding. The intervention course focused on problem solving, examination of learner thinking and discussion of research in a connected manner. They found that most participants had difficulty in area and perimeter concepts with only 21% demonstrating conceptual understanding while the rest showed no understanding at all.

Most of the reviewed studies tended to investigate conceptions of geometry instruction in general, without focusing on the conceptions of geometry instruction within a specific instructional approach such as the problem solving approach. The few studies that included the teaching of geometry through the problem solving approach did not focus on investigating teachers' conceptions of teaching geometry through the problem solving approach intervention courses, but investigated the influence of teachers' knowledge of the problem solving approach on their instructional practice after acquiring its knowledge from intervention courses. Therefore, there is a gap in literature on studies investigating teachers understanding of teaching geometry through the problem solving approach at the primary school level focusing on area of polygons.

In the next section, I present a discussion of literature on common learners' misconceptions in area of polygons.

2.5 COMMON LEARNERS' MISONCEPTIONS REGARDING THE AREA OF A POLYGON

In this section, I present a discussion of literature focusing on common learners' misconceptions in the area of polygons. Awareness of the common misconceptions related to a specific concept forms an important aspect of teacher knowledge (Depaepe et al., 2013). In this study, 'misconception' means learners' "conceptions that produce a systematic pattern of errors" (Smith et al., 1993, p. 119).

Zacharos (2006) conducted a survey among elementary school learners in Greece on how manipulatives provided to learners and certain instructional strategies influenced their conceptualisation of area measurement strategies. The survey involved 106 learners from the same school in their last grade of primary school divided into two groups, the experimental group (EG) and the control group (CG). The EG was subjected to conceptual instructional strategies of decomposing/recomposing shapes and overlapping, highlighting the underlying principles of these strategies. He subjected the CG to the same content area as EG but their regular teachers taught them through instructional strategies focusing on mastering the formula. After completing the lessons, both groups completed tasks involving area measurement and comparison different from those encountered in their lessons while being interviewed. The interview focused on matters of teaching.

In the study he found that the EG used innovative solution strategies while the CG persistently used solution strategies producing incorrect responses. The wrong solution strategies demonstrated by the CG included measuring the wrong sides when intending to use the formula. In addition, they used the formula even in cases where it was not applicable. As a result, he concluded that "the combination of lengths in formulas which contain multiplication is not actually meaningful in the context of area measurement"

(p. 234). From the results of his study, he observed that the early arithmetisation of area measurement by the learners in the CG created challenges for them, as it obstructed the acquisition of conceptual understanding of its meaning. Concerning the nature of the manipulatives, he noted that the type of manipulative made available to learners in area measurement activities influenced their conceptualisation of the concept. He recommended that teachers should provide their learners with manipulatives with common attributes in area measurement activities. This study supports my argument that teaching geometry with an emphasis on conceptual understanding minimised learners' difficulties and misconceptions.

Cavanagh (2008) examined the types of learners' errors and misconceptions associated with the calculation of areas of polygons and found three closely tied misconceptions regarding the areas of rectangles, triangles and parallelograms. Firstly, the learners were confusing area and perimeter. Secondly, the learners were using the slant height of a shape instead of the perpendicular height when calculating area. Lastly, the learners lacked conceptual understanding of the connection between areas of rectangles and triangles. In the study, he found that learners failed to comprehend relationship between the area of a triangle and the area of a rectangle (the fact that the area of a triangle is equal to half the area of a rectangle with the same base and perpendicular height).

In the study, he collected data in two phases from two groups of learners from two mixed ability schools in Australia. In the first phase, 43 learners participated in the study. Firstly, their teachers taught them eight lessons of 50 minutes each, focusing on strategies for calculating areas of plane shapes. In the study, the researcher did not observe the teaching of the lessons in person, but obtained information from the classroom teachers. For lesson content, which included illustrations and practice exercises, the classroom teachers relied on the textbook. After finishing the lessons, the learners were tested on the content of the lessons with results indicating a poor performance on the items requiring the calculation of the area of a right-angled triangle. A majority of the learners tried to find the area of the triangle by marking a square centimetre grid inside the triangle. This strategy resulted in complications for the learners as part squares resulted after they had drawn the grid inside the triangle. Others failed to obtain the correct area of the triangle due failure to divide by two, that is, the product of the base and the height of the triangle, or by finding the product of the three lengths of the triangle.

In the second phase of the study, he interviewed three boys and three girls from each school about two weeks after the initial phase. He found that the learners could find the areas of the shapes in the tasks, but failed to justify why their responses were correct when prompted to do so. In addition, most learners were able to perform the second task correctly, involving the comparing of areas of a right-angled triangle and parallelogram. However, one learner argued that it was impossible to compare the areas of the two shapes since they were different.

Based on the results of his study, as way of conceptually teaching the concept of area of polygons, he suggested giving leaners enough opportunities to master the underlying principles of area measurement before introducing them to algorithms. In addition, he suggested that in tasks involving the counting of units, the learners should demarcate the regions themselves. This would allow the learners to comprehend the underlying ideas necessary for deriving the formula. In addition, he cautioned that the correct application of a formula did not always equal conceptual understanding of the area concept. Lastly, he suggested a sequence for teaching the area of polygons, namely, as a rectangle/square followed by composite rectangular/square shapes, then parallelograms and lastly triangles. He justified this order by arguing that the area of a parallelogram was a prerequisite for the area of a triangle since cutting a parallelogram along the diagonal produces two identical triangles, each with an area that is half its initial size. Therefore, teachers should ensure that learners construct meaningfully the concept of perpendicular height of the triangle by either drawing the triangles within rectangles or drawing the triangles in various orientations.

Huang and Witz (2011) investigated the impact of three different instructional strategies in developing primary school learners' mastery of formulas and competency in solving problems involving area measurement. In addition to the treatment, the subjects were interviewed to gain insight about the impact of the instructional strategies on the learners, and mastery of the formulas for area. They described the three instructional strategies as encompassing traditional instructional approaches emphasising the arithmetisation of area, instructional approaches focusing in geometric motions emphasising the basic concepts of formula, and instructional strategies focusing in connecting both the arithmetisation area formula and the geometric motions. All three groups were taught through the problem solving approach. The teachers were required to initiate classroom discussions where learners would suggest/predict solutions to tasks and explain their solution strategies using their knowledge.

In this study, they found that the learners who were taught through instructional strategies combining both the arithmetisation area formula and the plane geometric motions showed remarkable “improvement in mathematical judgements and explanations which require conceptual understanding of area measurement” (p. 10). The learners who were taught through traditional instructional strategies emphasising the arithmetisation of area and those taught through instructional approaches focusing in plane geometric motions did not exhibit any improvement in their mathematical decisions and explanations. Confirming this result was the interview data that revealed that all three groups were conversant with “the importance of using the geometric operations of decomposition, recomposition, and superimposition to measure areas” (p. 10). They reported that the interview results showed that the learners taught through traditional strategies relied more on arithmetisation of area and the group taught through instructional approaches focusing in plane geometric motions mostly relied on geometric operations, while the group taught through instructional strategies focusing in connecting both the arithmetisation area formula and the geometric motions relied more on arithmetisation and geometric computations.

They concluded that for conceptual understanding of area measurement formula, both arithmetisation and geometric computations were essential. In particular, they suggested that during instruction conceptual understanding of area formula should be enhanced by connecting it with plane geometry topics and encouraging learners “to justify mathematical ideas and verbally explain their reasoning about area measurement” (p. 11).

In this subsection, I discussed literature investigating instructional approaches contribution to learners’ misconceptions in areas of polygons. In this review, it was apparent that focusing on teaching area of polygons towards procedural understanding only contributed to most learners’ misconceptions. Moreover, the review showed that teaching geometry, in particular area of polygons, through contemporary instructional approaches such as the problem solving approach enhanced learners mastery of the concept. However, so far, the literature has not given any attention towards investigating teachers’ understanding of teaching geometry with an emphasis on conceptual understanding. Hence, my study remedied this situation by examining primary teachers understanding of teaching geometry through the problem solving approach, focusing on area of polygons.

In the next section, I discuss how other researchers have examined the different aspects of CK for teaching.

2.6 HOW HAVE OTHER RESEARCHERS EXAMINED ASPECTS OF TEACHERS' KNOWLEDGE?

In this section, I discuss the methodologies used by other researchers in examining the different aspects of teacher knowledge. The purpose of my study was to examine primary teachers' understanding of teaching geometry through the problem solving approach, focusing on area of polygons.

Scholars have asserted that examining accurately the different aspects of teacher knowledge is not an easy endeavour. Most studies have examined the different aspects of teacher knowledge using quantitative approaches through subjects responding to multiple-choice items (Steele, 2013). According to Steele (2013), this approach has drawn criticism for two reasons. Firstly, it fails to provide the subjects' opportunities to justify their reasoning regarding their chosen responses. Secondly, it does not reveal the interactions between the different aspects of MKT.

To address these criticisms, Steele (2013) described a process he undertook in constructing open-response tasks aiming to investigate accurately the different aspects of MKT for geometry and measurement instruction. To ensure that the resulting tasks distinguished clearly between the different aspects of MKT, he used the MKT construct as a theoretical lens. In addition, he constructed the tasks adhering to three design values, namely, ensuring that the teachers examined real mathematical tasks learners were likely to perform, considering matters of actual teaching including lesson planning, and evaluating learners' responses involving the connections between different aspects of teacher knowledge associated with geometry. In other words, the tasks examined the dimensions of teacher knowledge pertaining to the reasoning behind accurate and inaccurate responses, taking into account specific misconceptions and recognising agents for change in their knowledge.

He considered the ability to create diverse representations of a mathematical concept, connecting those representations, and selecting appropriate representations for specific learners as the principal knowledge aspect of teaching different mathematical concepts. Based on this view, he asserted that investigating this type of knowledge provided essential information regarding teacher knowledge. Moreover, examining these diverse aspects of teacher knowledge provided the possibility of revealing critical aspects of the teacher's abilities normally difficult to ascertain. It also provides data on how to improve these teacher abilities. He claimed that examining accurately teacher knowledge entailed more than the design of the tasks, but included issues related to the nature of lens used to judge the correctness of responses. Therefore, he suggested that accompanying task design with rubric construction was essential as rubrics facilitated the identification of specific teacher competencies and predicting future behaviour. In addition, rubric construction should focus on aspects of teacher knowledge revealed when answering the tasks and the possibility of extending solution's strategy.

He demonstrated his three design principles by constructing six open-response tasks, which he used to examine the mathematical knowledge of geometry and measurement instruction of 25 primary school teachers, focusing on common content knowledge (CCK) and specialised content knowledge (SCK). Among the subjects, he found that the tasks worked according to his design expectations, offering insights on how the knowledge in the two domains interacted. Consequently, he recommended that researchers intending to examine MKT should base their instruments on relevant knowledge of learners and teacher professional development.

This study contributed knowledge on how to design open-ended response tasks with the potential for effectively examining the different aspects of teacher knowledge using the MKT framework as a lens. However, he analysed his tasks quantitatively by scoring the responses, which was a limitation. My study focused on analysing open-ended tasks through content analysis. In addition, he demonstrated his design principles on tasks focusing on CCK and SCK of geometry and measurement instruction, a different focus to my study. Steele (2013) acknowledged that due to the situated nature of teacher knowledge, relying on one type of instrument was not sufficient for accurately examining aspects of teacher knowledge.

Prior to Steele's (2013) study, Manizade and Mason (2011) designed open-ended tasks using Delphi methodology. Unlike Steele (2013), they used PCK as a theoretical lens. However, before using the PCK construct, they operationalised the construct by synthesising different PCK definitions from literature with the aim of incorporating within PCK the multi-dimensional nature of teacher knowledge related to geometry. Consequently, they defined PCK as encompassing four categories, namely, knowledge of networks of big mathematical ideas, knowledge of learning principles explaining learners' developmental ideas, knowledge of common learners' misconceptions and topic specific challenges, and knowledge of appropriate representations and effective instructional approaches for teaching that topic. They considered defining PCK with these categories as reflecting the various learning outcomes in the content of decomposing and recomposing of geometric shapes.

They perceive SMK as preceding PCK; hence, they argued, it is impossible to investigate PCK in teachers lacking SMK. They designed their tasks based on this PCK definition. They argued that using the Delphi methodology allowed them to design an open-ended response instrument with high validity as it specifically focused on a specific content area and was rigorously validated by the experts involved in the study. In my study, I adopted this instrument because it focused on examining PCK in the same content as this study, area of polygons by decomposing and recomposing. In addition, I adopted this instrument because it focused on examining the situated perspective on PCK. However, in addition to this instrument, due to the dynamic nature of teacher knowledge, in actual teaching there is a possibility for some teachers to answer correctly the paper-pencil items but fail to apply such knowledge in actual teaching, an act Zheng (2013) described as "token adoption". For this reason, I supplemented the instrument with lesson observation, lesson plan analysis, and semi-structured interviews. My study also took a different focus as it examined aspects of PCK related to a particular instructional approach such as the problem solving approach.

From these studies, it is apparent that a nuanced way of examining aspects of teacher knowledge was to employ the MKT construct as a theoretical lens, using open-ended tasks. Both studies argued that open-ended tasks provided participants with an opportunity to justify their responses thus revealing their reasoning behind their responses. In my study, I extended knowledge from these studies by using Ball et al.'s (2008) MKT framework to examine primary teachers' understanding of geometry instruction through the problem solving approach. In addition, I analysed the data through content analysis to capture the teachers' perspectives of teaching geometry through the problem solving approach. I also complemented open-ended tasks with semi-structured interviews and lesson observation to gain deep insight into the primary teacher participants' understanding of teaching geometry through the problem solving approach, from their own description.

2.7 SUMMARY

In summary, my review of related literature in this chapter revealed a gap in literature on studies examining primary teachers' understanding geometry instruction through the problem solving approach. Most of these studies focused either on conceptions of teaching mathematics through the problem solving approach without focusing on a specific topic or on conceptions of geometry instruction without focusing on a specific instructional approach. In addition, none of the studies used Ball et al.'s (2008) MKT as a framework as a lens. Hence, my study contributes knowledge about primary teachers' understanding of teaching geometry through the problem solving approach, using the MKT construct as a lens.

In the next chapter, I present a discussion of the conceptual framework underpinning the study.

CHAPTER 3 : CONCEPTUAL FRAMEWORK OF THE STUDY

3.1 INTRODUCTION

My goal in this study was to examine primary teachers understanding of teaching geometry through the problem solving approach. Therefore, I particularly focused on the teachers understanding of integrating the problem solving approach in teaching geometry content. Based on the goal of my study, I considered the teachers understanding of teaching geometry through the problem solving approach under the different aspects of teacher knowledge. As a result, I include in the first section of this chapter frameworks describing the different aspects of teacher knowledge. In the second section, I discuss frameworks describing the teaching of mathematics through the problem solving approach. Before discussing the theoretical underpinning of teaching mathematics through the problem solving approach, I discuss Polya's (1945) problem solving phases to situate the construct. I conclude the chapter by presenting the theoretical model linking the different constructs in my study.

In the next section, I discuss the different frameworks describing the different aspects of teacher knowledge.

3.2 FRAMEWORK FOR THE DIFFERENT ASPECTS OF TEACHER KNOWLEDGE

In this section, I present a discussion of the theoretical framework describing the different aspects of teacher knowledge. Shulman (1986) described teacher knowledge as "the amount and organisation of knowledge per se in the mind of the teacher" (p. 9). According to Shulman (1986), a teacher should possess different kinds of knowledge that enables him/her to "explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and practice" (p. 9). He classified the types of teacher knowledge into three sets: - SMK, PCK and curricular knowledge. Among these three sets, my study focused on PCK, a set he described as encompassing

> … the most regularly taught topics of one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others … Pedagogical content knowledge also includes an understanding of what makes the learning of specific concepts easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of strategies most likely to be fruitful in reorganizing the understanding of learners because those learners are unlikely to appear before them as blank slates (pp. 9-10).

Shulman (1987) refined his earlier conceptualisation of CK for teaching into seven sets encompassing content knowledge, general pedagogical knowledge (GPK), curriculum knowledge, PCK, knowledge of learners and their characteristics, knowledge of educational contexts, and "knowledge of educational ends, purposes, and values, and philosophical and historical grounds" (p. 8). In addition, he described GPK as the kind of teacher knowledge associated with the "broad principles and strategies of classroom management and organisation that appear to transcend subject matter" (p. 8). He described PCK as "that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 8). Among these sets of CK for teaching, he noted that PCK pronounced clearly the different kinds of knowledge required for teaching. According to Shulman (1987), PCK represented "the blending of content and pedagogy into an understanding how particular topics, problems, or issues are organized to the diverse interests and abilities of learners, and presented for instruction" (p. 8). Therefore, he regarded PCK as the set accurately discriminating the comprehension of mathematical content of other experts from that of teachers.

Shulman's (1986) conceptualisation of the different kinds of teacher knowledge provided a solid theoretical foundation, enhancing my understanding of teacher knowledge for effective teaching. However, Ball et al. (2008) argued that Shulman (1986) described PCK ambiguously, resulting in various interpretations from educational researchers. Park and Oliver (2008) concurred with Ball et al. (2008), arguing that "…individuals within any group of educational stakeholders, researchers, teacher educators, teachers or others, are likely to interpret the nature of PCK differently thus endangering a variety of meanings"

(p. 262). Ball et al. (2008) added that, despite the high interest shown by educational researchers in PCK, few articles focused on mathematics education. Thus, they argued, "… the field has made little progress on Shulman's initial charge to develop a coherent theoretical framework for CK for teaching. The ideas remain theoretically scattered, lacking clear definition" (p. 394). Furthermore, they argued that Shulman's conceptualisation of CK for teaching lacked empirical support from research. Consequently, several mathematical researchers proposed refinements to Shulman's conceptualisation of CK for teaching.

Ball et al. (2008) refined Shulman's conceptualisation of CK required for teaching by adopting a practice-based approach considering the daily demands of teaching, focusing on "the mathematical knowledge needed to carry out the work of teaching mathematics" (p. 395). They described this multi-dimensional nature of teacher knowledge using the theoretical construct, MKT. According to Ball et al. (2008), for effective teaching, a teacher requires mathematical knowledge deeper than they would normally need for their daily activities for the following reasons: firstly, teachers are required to analyse learners' errors and misconceptions, in addition to evaluating the correctness of their solution strategies. Secondly, in most instances, teaching requires teachers to make decisions promptly during the teaching process, an act that Mason and Spencer (1999) referred to as "knowing to act in the moment" (p. 143). Ball et al. (2008) noted that learners could not tolerate watching their teacher stumble over the mathematical content they were supposed to learn – learners sometimes give unfamiliar responses to their teachers which teachers need to be ready to deal with. Thirdly, teachers should possess knowledge of relevant algorithms for clarifying and justifying to the learner the suitability of these algorithms in specific cases among other reasons. This knowledge, according to Ball et al. (2008), incorporates ability, ways of thinking, and insight necessary for effective teaching.

In their MKT framework, Ball et al. (2008) distinguished two kinds of SMK: CCK, the mathematical competency useful in solving everyday life problems that any educated person might have and SCK, the mathematical knowledge and proficiency exclusively required for teaching. In addition, they differentiated two aspects of PCK: KCS, the interaction of knowledge of learners and the knowledge of mathematics and KCT, the knowledge of both specific mathematics content and instructional approaches appropriate for teaching that content. Figure 3.1 illustrates the relationship between the different aspects of knowledge in the MKT framework.

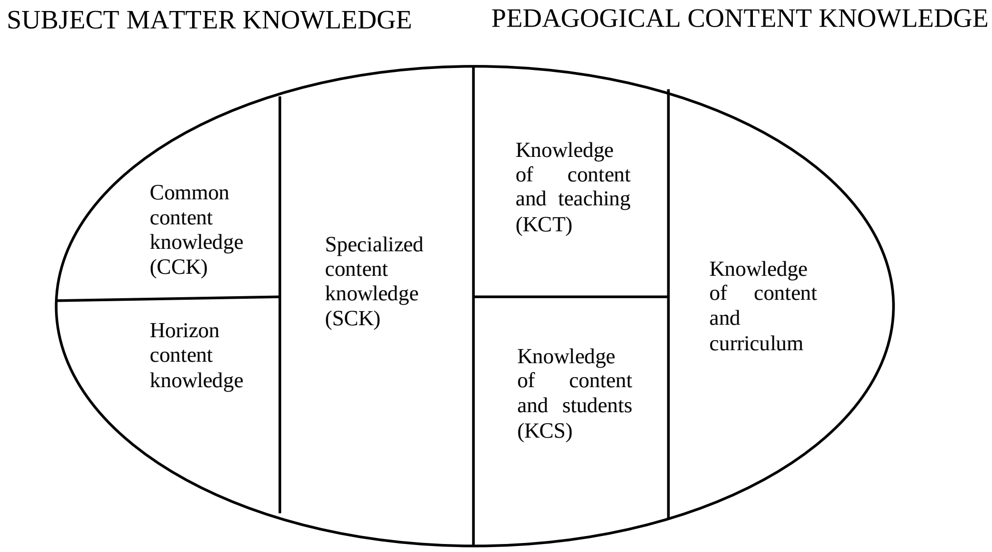


Figure 3.1: Domains of mathematical knowledge for teaching
Source: Ball et al., 2008, p. 403

Among the different aspects of the MKT framework, PCK determines the quality of teaching, greatly influencing learner achievement (Baumert et al., 2010). In addition, Brodie and Sanni (2014) argue that PCK exerts a strong influence on the acquisition of CK while preceding its acquisition in certain circumstances. Among the different categories of the Ball et al. (2008) MKT framework, my study focused on the KCS and KCT categories.

According to Ball et al. (2008), a teacher with strong KCS anticipates in advance possible learner behaviour and sources of learning difficulties during instruction, in particular, learners' prior conceptions and common misconceptions regarding the learning of specific mathematical concepts. They posited that such knowledge enables teachers to select appropriate learning activities for their learners. In constructing instruments for examining KCS, they suggested that "we ask questions that require interpretation of students' emerging and inchoate thinking, that present the thinking or expressions typical of a particular learner, or that demand sensitivity to what is likely to be easy or challenging" (p. 401). Depaepe et al. (2013) observed that a majority of the scholars hold the view that "…knowledge of students (mis)conceptions and knowledge of instructional strategies and representations" (p. 22) were the principal aspects of CK for teaching. In this study, KCS describes the aspects of teacher knowledge required when teaching primary school geometry through the problem solving approach.

Ball et al. (2008) describe KCT as the category of teacher knowledge combining the understanding of mathematical content with instructional strategies. They state that a teacher with strong KCT has mastery of the mathematics content required for designing instruction, including structuring the content at the learners' cognitive level. This kind of knowledge encompasses the ability to select appropriate illustrations for the different stages of a lesson, fully engaging the learners in the content. In addition, this kind of teacher knowledge allows the teacher to select the most effective instructional approaches for particular topics based on their merits. They argue that teachers require explicit knowledge of mathematics and knowledge of instructional strategies influencing learners' achievement of instructional goals when making decisions. Moreover, the teacher needs to have the ability to harmonise the mathematical content on offer, and the teaching options and goals at stake. They illustrated this point by highlighting that during lesson deliberations "a teacher must decide when to pursue for more clarification, when to use a student remark to make a mathematical point, or when to ask a new question or pose a new task to further student learning" (p. 401). They clarify the characterisation of KCT by stressing that it is a blend of knowledge of specific mathematical algorithms and competency with the instructional norms regarding the instruction of that specific content. In this study, Ball et al.'s (2008) KCT describes the aspects of teacher knowledge required for teaching geometry through the problem solving approach.

Silverman and Thompson (2008) propose a theoretical framework extending the MKT useful in identifying teachers' characteristics supporting the teaching of mathematical content towards conceptual understanding. According to Silverman and Thompson (2008) a teacher

> (1) has developed a KDU [key developmental understanding] within which that topic exists, (2) has constructed models of the variety of ways students may understand the content (decentering); (3) has an image of how someone else might come to think of the mathematical idea in a similar way; (4) has an image of the kinds of activities and conversations about those activities that might support another person's development of a similar understanding of the mathematical idea; (5) has an image of how students who have come to think about the mathematical idea in the specified way are empowered to learn other, related mathematical ideas.(p. 508)

They emphasised that a teacher attuned to teach conceptually has images of how best a learner can meaningfully learn the content. Thus, as the teacher thinks about the content she/he wants to teach, she/he should visualise a learner attempting the content, overcoming some challenges and failing in some. In addition, a teacher with strong MKT always thinks about what he/she intends the learner to master in constructing the desired understanding and the types of interactions conducive for the learner to construct such understandings.

Silverman and Thompson (2008) enriched my understanding of the features of teachers with good understanding of mathematical instruction. Ball et al. (2008) and Silverman and Thompson (2008) focused on operationalising PCK, a different focus from my study. In my study, I use the MKT construct to examine primary teachers' understanding of teaching geometry through the problem solving approach.

Prior to Ball et al. (2008), Ma (1999) used the theoretical construct, profound understanding of fundamental mathematics (PUFM), to describe aspects of teacher knowledge essential for teaching specific mathematical concepts conceptually. She developed this construct from results of her comparative study of United States of American and Chinese primary teachers' knowledge of teaching specific topics. She described teachers with PUFM as having more than just a vast knowledge of primary mathematics, but also having a comprehension that was "deep, broad, and thorough" (p. 120). She defined PUFM as "the awareness of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics and the ability to provide a foundation for that conceptual structure and instil those basic attitudes in students" (p. 124).

She described four characteristics defining teachers with PUFM of a specific topic, namely, connectedness, multiple perspectives, basic ideas and longitudinal coherence. She asserted teachers with PUFM purposefully teach mathematics topics as a unified body of knowledge, linking all related concepts, from simple to abstract. In addition, they emphasise multiple perspectives on ideas and solution strategies which includes assessing their advantages and disadvantages, as well as explaining them. Moreover, such teachers demonstrate mathematical attitudes and are predominantly cognisant of the elemental yet important concepts and values of mathematics, prompting learners to engage in genuine mathematics activity. Lastly, they have deep understanding of the whole primary school mathematics curriculum, such that, they "exploit an opportunity to review crucial concepts that students have studied previously. They also know what students are going to learn later, and take opportunity to lay the proper foundation for it" (p. 122). She regarded multiple perspectives, basic ideas, and longitudinal coherence as the types of linkages leading to breadth, depth, and thoroughness, resulting in diverse understanding in mathematics.

Ball et al.’s (2008) MKT framework informed my description of the different aspects of teacher knowledge required for teaching geometry through the problem solving approach in this study. I adopted this framework for two reasons. Firstly, this framework clearly differentiates aspects of PCK, thus providing a rigorous way of examining mathematical knowledge for teaching. Secondly, the construct MKT provides evidence linking PCK and learners’ academic achievement, which is consistent with my underlying assumptions in undertaking this study (Ball & Hill, 2008). I operationalised the definition of MKT to encompass nine aspects of teacher’s knowledge, namely: knowledge of important mathematical ideas, topic specific challenges, and interpreting learners emerging and incomplete ideas. In addition, it encompasses recognising and articulating learners’ misconceptions, sequencing content of the topic and selecting suitable problems for teaching the topic. Further, it encompasses selecting appropriate representations to illustrate the topic, knowing the aims for learning the topic, and knowledge of effective instructional approaches for teaching that topic area (Ball et al., 2008). Operationalising the MKT construct in this manner extends its scope to accommodate the various mathematical outcomes in the content of area of polygons in geometry. Among the different perspectives on MKT, in this study I adopted the situated perspective, considering teacher’s knowledge “as knowing-to-act within a particular classroom context, typically acknowledging that the act of teaching is multi-dimensional in nature and that teachers’ choices simultaneously reflect mathematical and pedagogical deliberations” (Depaepe et al., 2013, p. 22).

In my study, I examined teachers’ understanding of teaching geometry through the problem solving approach using Ball et al.’s (2008) MKT as a lens. However, Ball et al.’s (2008) MKT framework describes the different aspects of knowledge required for mathematics teaching without the provision of how this knowledge should be organised for effective lesson delivery through the problem solving approach. Hence, there is a need to discuss a framework describing instruction through the problem solving approach.

In the next section, I discuss theoretical underpinning of instruction through the problem solving approach in my study.

3.3 FRAMEWORK FOR PROBLEM SOLVING

In the previous section, I discussed the MKT theoretical construct, describing the different aspects of teacher knowledge in my study, focusing on KCS and KCT categories. The purpose of my study is to examine primary teachers understanding of teaching geometry through the problem solving approach, focusing on these two categories. Therefore, there is a need to link these different aspects of teacher knowledge with theory underpinning instruction through the problem solving approach. In this section, I do so starting with Polya's (1945) four-step problem solving model, followed by Schoenfeld's (1992*)* mathematical cognition framework, then the problem-based learning model, concluding with Takahashi's (2008) Japanese problem solving approach framework.

3.3.1 Polya's (1945) problem solving model

Before discussing the theory underpinning teaching through the problem solving approach, in order to situate the study in context and for comprehension of problem solving as a construct, it is necessary to first discuss Polya's (1945) four-step problem solving model. The first step is: understand the problem. He considered this step as necessary because "It is foolish to answer a question that you do not understand" (p. 5). In addition, the learner should be willing to solve the problem. As a result, he stressed that the teacher should select the problems cautiously at the cognitive level of the learners. In trying to understand the problem, he recommended giving learners enough opportunity to comprehend the problem by restating it verbally in their own words. The learner should be allowed to identify the key words or phrases, the given information, the question and the circumstance(s) under which it would be solved. Teachers should ask the learners questions such as: "What are the key words or phrase(s) of the problem? What is the given information in the problem? What does the problem require? What are the circumstance(s) under which the problem will be solved?" Learners should be encouraged to examine the given problem exhaustively from all dimensions. Lastly, learners should draw any diagram associated with the problem, indicating all the given information and what is required by the problem. The learner should use the acceptable representation of information while taking care of the correct operation signs.

He described two approaches to understanding a problem; being acquainted and working for understanding. In trying to become acquainted with the problem, he recommended reading the problem intensively to gain deep insight into its key elements. Having a deep insight of the problem could trigger the recall of relevant concepts in the mind of the learner. In working for understanding, he suggested starting by identifying the key elements of the problem; examining them exhaustively from all dimensions while determining their nature of connection. He asserted that prior knowledge plays a vital role in helping the learner in their understanding of the given problem.

The second step in Polya's (1945) problem solving model is: devise a plan. He described this step as being the most difficult, asserting that "the main achievement in the solution of a problem is to conceive the idea to a plan" (p.8). He describes a plan as having some information or idea of the "calculations, computations, or constructions" required for finding the solution (p. 8). Hence, the teacher should have conceptual understanding of the content in order to offer, without interfering, appropriate hints to the learner. Moreover, he asserts that the teacher relies on KCS when developing good hints for the learners. Thus, he contends that good hints "are based on past experience and formerly acquired knowledge" (p. 8). In order to stimulate the learners to conceive their own plan of solving the problem, he recommended asking the learners questions such as "Do you know a related problem? Here is a problem related to yours and solved before, could you use it?" (p. 9). Asking such questions, he argued, could trigger a chain of thoughts that might assist learners in developing the required plan. If there was still no breakthrough to conceiving the plan to solving the problem, he suggested that the learner should try to rephrase or translate the problem, thus a teacher may ask the learner the question: "Could you restate the problem?"

The third step in Polya's (1945) problem solving model is: carrying out the plan. Before taking this step, he recommended that the learner should examine the plan, ensuring the consideration all the dimensions of the problem. Secondly, the learner should establish firmly the linkages between key elements and all necessary facts supplied. At this stage, he suggested asking the learner a question such as: "But can you see clearly that the step is correct?" To ensure that the learner remembers his/her plan, he stated that learners should develop their own plan, with minimum assistance, to derive gratification. The learner implements his/her plan to solve the given problem at this step.

The fourth step in Polya (1945) problem solving model is: looking back. He asserts that closing their books after finding the correct solution to a problem denies the learner an enlightening opportunity because "by looking back at the completed solution, by reconsidering and re-examining the result and the path that led to it they could consolidate their knowledge and develop their ability to solve problems" (p. 14). In addition, this helps the learners to identify errors in the solution. In addition, reflecting on the solution motivates the learners, more especially when they have worked through the problem with integrity, providing the satisfaction of achievement. Moreover, learners might discover advanced efficient solution strategies. After finding the initial solution, he implored teachers to instil in their learners the culture of exploring other possible solution strategies. He asserted that exploring other solution possibilities refines learners' solution strategies and teachers can instil this culture by asking questions such as: "Can you check the result? Can you check the argument? Can you derive the result differently? Can you see it at glance? Can you use the result, or the method, for some other problem?"

He recommended that learners improve their reflection skills by examining the solution from different facets, linking new information with previous experiences or examining the parts of the solution in detail, to name but a few. Apart from enabling learners to attain logical knowledge, which was readily available, he asserted that examining the solution in such a manner, also improved the learners' aptitude for problem solving.

The discussion of Polya's (1945) problem solving model reveals the influence teacher knowledge has on the success of problem solving. In the discussion, it emerged that teacher knowledge influences the teacher decision-making process in trying to create a classroom environment conducive for problem solving. For instance, the teacher should decide when to intervene during problem solving and what type of intervention they should provide to their learners to avoid interference. Concerning the questioning skill of the teacher, he recommended adopting a flexible approach eliciting multiple solution strategies from the learners and asking learners questions they could have asked themselves.

3.3.2 Schoenfeld's mathematical cognition framework

In this section, I discuss Schoenfeld's (1992) mathematical cognition framework (knowledge base, problem solving strategies, monitoring and control, beliefs and affects, practices). Unfortunately, Polya (1945) described problem solving as a non-scientific enterprise and described his problem solving strategies in a non-functional form disregarding aspects such as knowledge and the application of mathematics in practical settings (Kilpatrick, 1987). In response to this criticism, Schoenfeld (1992) developed a mathematical cognition framework informed by his intensive study of epistemology and pedagogy of mathematics education. In his framework, Schoenfeld (1992) adopted a broader view of mathematics considering it as a discipline of patterns. Moreover, he considered the mathematics enterprise as "an act of sense-making; and cognitive apprenticeship and 'cultures of learning' " (p. 337) and that the main goal of teaching mathematics is to develop mathematically thinking learners. As a result, his mathematical cognition framework considered aspects such as the knowledge base, problem solving strategies, monitoring and control, beliefs and affects, and practices as major factors influencing problem solving outcomes.

He described the knowledge base as one of the aspects determining problem solving outcomes that encompassed both mathematical 'tools' and 'knowledge' about the problem. In addition, it encompassed ways of retrieving the knowledge or implementing it. He argued that failure to apply a correct solution strategy during problem solving might be due to either disregarding it or not knowing it. Further, he warned that sometimes the individual's knowledge base may consist of misconceptions and errors, which the teacher should be aware of, as the learner may apply these during problem solving. Furthermore, he distinguished routine procedures from algorithmic procedures, in that when implemented properly, algorithms always yield the required solution while routine procedures are useful but without always delivering the desired solution.

In addition, he identified knowledge of problem solving strategies (heuristics) as a critical aspect in developing mathematical cognition. However, he argued that Polya's (1945) description of problem solving strategies (heuristics) was not authoritative; hence, "it did not provide the amount of detail that would enable people who were not already familiar with the strategies to implement them" (p. 353).

Moreover, Schoenfeld (1992) identified monitoring and control (self-regulation) skills, a subset of the term metacognition, as influencing problem solving outcomes. He asserted that monitoring and control encompassed "Monitoring and assessing progress online and acting in response to the assessments of the online progress…" (p. 355). He argued that this aspect allowed learners to monitor the appropriateness of their solution strategies during problem solving. As a result, using feedback from monitoring and control, learners are able to discard unfruitful solution strategies and formulate alternative efficient options. He recommended that allowing learners to solve problems in small groups while facilitating might improve learners' monitoring and control skills.

He identified beliefs and affects as one of the aspects that can influence problem solving outcomes. He conceived mathematics learning as “an act of sense- making that is socially constructed and socially transmitted” (p. 340). He posited that beliefs about mathematics and its understanding influence the interpretation of instructional goals. Since learners develop the bulk of their mathematical beliefs in the classroom environment created by their teachers, he argued that teacher’s beliefs influence problem solving outcomes of their learners. Consequently, he recommended providing classroom environments allowing learners to conceive problem solving as the main goal for doing mathematics. Furthermore, he argued, “One develops one’s point of view by the process of acculturation by becoming a member of the particular community of practice” (p. 344). Hence, he suggested immersing learners in classroom environments promoting mathematical thinking to foster their problem solving culture.

Schoenfeld (2012) used the term ‘didactical contract’ to describe the principles governing the teacher/learner relationship in productive classrooms. He asserted that the nature of this relationship affects the learner’s responses during classroom discourse. He made an example of a class focusing on ‘answer getting’ which tends to breed learners responding to teacher’s questions without explaining them. On the other hand, a class focusing on mathematical sense-making produces learners who feel obliged to explain the thinking informing their responses.

He discussed four key characteristics of classroom environments nurturing mathematical sense-making among the learners, namely, “problematizing, agency and authority, accountability, and resources” (p. 594). He argued that problematising the classroom environment cultivates among learners the conception that learning mathematics involves “exploring structure, pursuing generalizations, and abstractions, and variations” (p. 595). Allowing learners to work on problems in groups and then sharing their solution strategies with the whole class, through discussion focusing on the process to solution strategy, develops the correct conception of doing mathematics. In addition, the teacher encourages different solution strategies, developing the belief of multiple solution strategies to the same problem among learners. He discussed agency and authority as a second characteristic of classroom environments fostering mathematics as sense-making activity. He described a mathematical productive classroom context as giving authority to learners for solving mathematical problems by themselves, resulting in new, unfamiliar, solution strategies and generalisations. In addition, it fosters a sense of agency among the learners. The third characteristic he discussed was accountability to the discipline. Being accountable to the discipline implies that learners should develop their skills of interpreting the world through the eyes of a mathematician. Therefore, teachers should socialise learners to be accountable to the discipline, assuming the responsibility of evaluating the correctness of their solution strategies by themselves using mathematical norms and values. Being accountable to the discipline enhances the learners' ability to communicate with authority. The role of the teacher is to stimulate appropriate behaviour. Lastly, he stated that the availability of resources contributes to shaping classroom environments fostering mathematics as a sense-making activity. He regarded doing mathematics as a sense-making activity requiring active involvement of learners such as making predictions, inferences, hypotheses, and selecting sensible ideas, which requires the selection and use of appropriate tools.

Schoenfeld's (1992) mathematical cognition framework enhanced my understanding of problem solving as an act of sense-making. However, it does not provide a description of a lesson structure through the problem solving approach. It describes the different aspects necessary for successful problem solving, providing an understanding of a classroom discourse through problem solving. This framework considers the influence of problem solver's mathematical knowledge, problem solving strategies, monitoring and control, appropriate beliefs and practices to the problem solving outcomes. In addition, this framework explains the specific teacher knowledge as described by Ball et al. (2008). Thus, Schoenfeld's (1992) mathematical cognition framework elaborates the concept of problem solving in my study, arguing that teachers should provide their learners with authentic mathematical problem solving environments. Hence, in my study I argue that teachers' conceptions influence successful teaching of geometry through the problem solving approach.

Having discussed Schoenfeld's (1992) mathematical cognition framework in this section, in the next section I discuss instructional approaches for teaching through problem solving.

3.4 TEACHING THROUGH THE PROBLEM SOLVING APPROACH

The purpose of my study was to examine primary teachers understanding of teaching geometry through the problem solving approach. In this section, I discuss approaches to problem solving as an instructional approach.

3.4.1 Problem-based learning

Hung, Jonassen and Liu (2008) described problem-based learning (PBL) as a highly inventive instructional approach adopted in education around the 1990s, originating from medical education. According Hung et al. (2008), constructivist principles of learning underpin PBL, describing it as a learner-centred approach. In addition, PBL engages learners in solving real life problems as a means of knowledge construction. Moreover, in this approach, learners “construct content knowledge and develop problem-solving skills as well as self-directed learning skills while working towards a solution to the problem” (p. 486). They state that PBL assumes that meaningful learning results from solving real life problems, producing life-long learners. Furthermore, knowledge construction in PBL is both an individual and social process resulting from interacting with the environment. Hence, the environment and the available tools in which learning occurs play a critical role, considering different viewpoints to the same problem. They argued that contextual learning results in knowledge that is “more meaningful, more integrated, better retained, and more transferable” (p. 488). Moreover, solving real life problems presents the goal for learning mathematics, thus enhancing learners’ motivation.

They argue that in PBL, the teacher assumes a facilitating role, supporting and modelling thinking processes, ensuring efficient group functioning. In addition, this approach involves limiting the interrupting of learners, and giving straightforward responses, but giving direction by probing learners for deep understanding during lessons.

They described a PBL classroom discourse as follows: in the first step, working in groups of five to eight, the learners try to understand the problem by assessing the kind of knowledge they would require in solving successfully the problem. After comprehending the problem, they state their learning outcomes. In addition, they identify the hypotheses or conjectures they need to construct, the type of knowledge needed for further comprehension of the facets of the problem and determine the kind of learning activities that would equip them with skills necessary for solving the problem.

In the second step, referred to as self-directed study, each learner finishes their learning tasks by gathering and studying knowledge sources in preparation for reports to the group. After completing their reports, each learner presents their solution strategies to the group. Using knowledge from the presentations of individual learners, the group reformulates their hypothesis. Lastly, the learning session (normally a week) concludes by learners summarising and synthesising their knowledge of the problem.

Having discussed a PBL framework in this section, in the next section I discuss Takahashi (2008) framework for teaching through the problem solving approach.

3.4.2 Takahashi's (2008) Japanese problem solving approach

In this section, I discuss Takahashi (2008) framework for teaching through the problem solving approach. Takahashi (2008) described the Japanese problem solving approach as a pedagogical approach underpinned by Polya's (1945) four phases of problem solving – understanding the problem, devising a plan, carrying out the plan, and looking back. According to Takahashi (2008), the Japanese problem solving approach allows learners to construct mathematical knowledge and skills by creatively solving challenging problems by themselves. When compared to other problem-based pedagogies, the Japanese problem solving approach differs in that, apart from developing the learners' problem solving skills, it also develops specific mathematical knowledge, skills and processes (Takahashi, 2008). Nunokawa (2005) elaborated that in teaching through the problem solving approach, "what we expect our students to obtain through problem solving is a mathematical content and how that new content is related to the mathematical knowledge they already have" (p. 332). Hence, Takahashi (2008) stated that some researchers referred to this approach as "structured problem solving" because its instructional goals include mastering mathematical content and the development of problem solving strategies and skills. Thus, teachers used problems for enhancing their learners' understanding of mathematical content and developing their skills of learning and applying mathematics.

According to Takahashi (2008), lessons through the Japanese problem solving approach commence with the teacher presenting a problem, and then giving learners opportunities to examine the problem in detail i.e. understanding the problem. Moreover, he explained that the teacher should present a problem with the potential of providing learners with "the ability to learn something new after they have solved the problem by using their existing knowledge and skills … which is the goal of the lesson" (p. 11). Thus, selecting a suitable problem requires sound KCS as described by Ball et al. (2008). According to Nunokawa (2005), the selected problem should provide a link between known and unknown knowledge. In addition, the problem should prompt learners into restructuring their mathematical knowledge towards the intended lesson objectives.

Takahashi (2008) explained that in step two, after understanding what the problem entails, the teacher should instruct the learners to devise a plan for solving the problem, using their prior knowledge. In step three, he stated that learners solve the problem using their devised plan. Additionally, at this step, teachers not only expect correct solution strategies from the learners, but also incorrect solution strategies and naïve solution strategies, resulting from inappropriate application of prior knowledge and misconstructions. At this step, Nunokawa (2005) recommends providing learners with all relevant tools that would assist them in executing their plan and gaining more knowledge about the problem.

Lastly, in step four, Takahashi (2008) stated that the learners should evaluate the effectiveness of their solution strategies, a step called "looking back". He stated that the Japanese teachers referred to this step as *Neriage* because their lessons did not end after each learner has presented their solution strategies, but "Japanese teachers facilitate extensive discussion with students, which is called *Neriage*, by comparing and highlighting the similarities and differences among students', solution approaches" (p. 4). Therefore, for this step to be successful, the teacher should design in advance sound guidelines for facilitating effective discussion of the anticipated learners' multiple solution strategies, including the ones resulting from misconceptions. During the *Neriage* stage, the teacher organises learners' solution strategies, assisting in their refinement in order to master the mathematical content. At this stage, the teacher focuses learners' thinking on the main concepts and thought processes necessary for achieving the lesson objectives. He asserted that the learners should persevere through a problem by themselves, providing them with opportunities to relate their prior knowledge to the new content.

The development of new mathematical knowledge among the learners is not an accidental process when teaching through the problem solving approach. Nunokawa (2005) suggested that in order to enable the learners to appreciate and recognise the connection between the new knowledge and their prior knowledge, teachers should choose a problem that would present learners with opportunities of discovering the need or advantage of the newly acquired knowledge and the deficiencies of their prior knowledge in solving the problem. Takahashi (2008) suggested that teachers might initiate the *Neriage*, by requesting learners to examine the different solution strategies presented for the problem for similarities and differences. Moreover, while examining the different solution strategies, the teacher should focus learner attention on investigating the advantages and shortcomings of each solution strategy. He argued that comparing the different solution strategies compelled learners to think deeply about the problem, resulting in the development of new mathematical knowledge. Furthermore, the *Neriage* provided teachers with opportunities for teaching their learners' new mathematical knowledge based on their solution strategies, a process requiring much effort in terms of the teacher's preparation. In summarising the lesson, the teacher should stretch and challenge the presented solution strategies and ideas for the learners to establish their relationship, thus helping them master the lesson's objectives. In addition to that, the learners are able to reflect on the knowledge they have newly mastered in the lesson.

In my study, Takahashi's (2008) problem solving approach framed the teaching of geometry through the problem solving approach as it best describes my own conceptions of effective geometry instruction. For effective teaching of geometry through the problem solving approach, my discussion of this framework shows that teachers need good MKT.

Having discussed the theoretical underpinnings of my study, Figure 3.2 presents the theoretical model depicting the connection between the MKT construct and teaching through the problem solving approach construct. As shown in this figure, my study focused on two domains of the Ball et al. (2008) MKT framework, the KCS and KCT, for a nuanced examination of primary teachers' understanding of the problem solving approach. As my study focused on the MKT of a specific instructional strategy, Takahashi's (2008) problem solving approach elaborates on the aspect dealing with understanding effective instructional strategies.

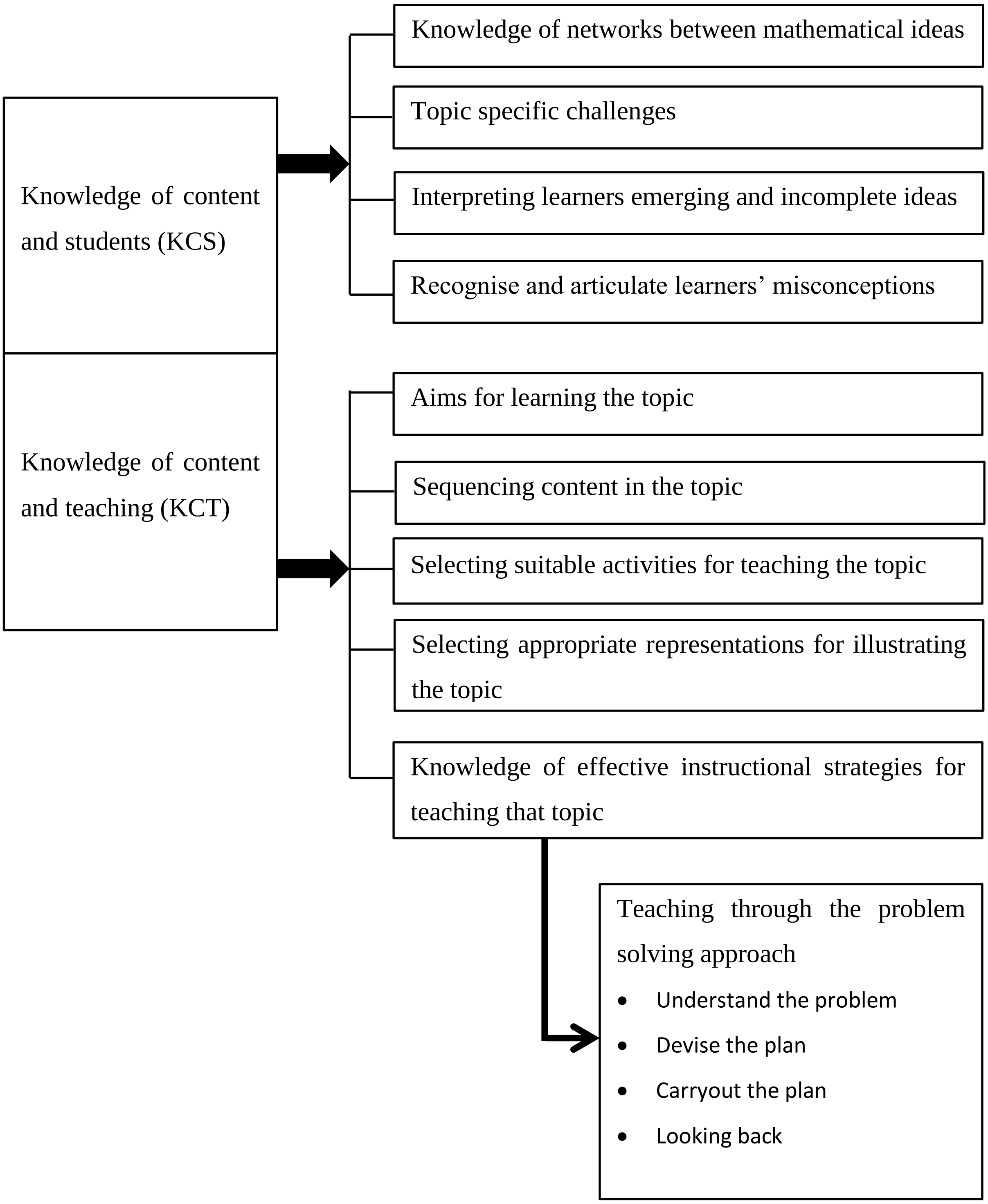


Figure 3.2: The theoretical model of the study
Source: Ball et al., 2008; Takahashi, 2008; Polya, 1945

3.5 SUMMARY

In this chapter I discussed the theoretical underpinning of my study and operationalisation of the MKT construct underpinning my study. Ball et al.’s (2008) MKT framework describes the various aspects of CK for teaching; however, this framework does not provide descriptions of these aspects in relation to teaching through the problem solving approach. Hence, it was necessary to discuss Schoenfeld’s (1992), PBL model, and Takahashi’s (2008) model of instruction through the problem solving approach, starting with Polya (1945) for comprehension of problem solving as a construct.

In the next chapter, I present the methodological approach I employed in trying to answer the research questions in my study.

CHAPTER 4 : METHODOLOGICAL APPROACH

4.1 INTRODUCTION

In the previous chapter, I discussed the theoretical frameworks informing my study, arguing that successful designing of geometry lessons through the problem solving approach required sound MKT. The purpose of this chapter is to present the methodological approach I employed in answering the research questions. Creswell (2003) described methodology as a "strategy or plan of action [which] links methods to outcomes [and] governs our choice and use of methods" (p. 5). Hays and Singh (2012) described methodology as a philosophy of science encompassing the actual educational research practice. Therefore, in this chapter I discuss the outline and justification of the research design of my study. I begin the discussion by stating my philosophical position regarding knowledge construction and how it influences my choice of research design, justifying its appropriateness in the context of examination of the research problem of the study. I describe the population and sampling procedures I adopted when selecting the participants of the study, justifying why they are appropriate for the study. I describe the data collection instruments, as well as the procedure I followed in collecting the data. Furthermore, I discuss issues related to rigour and trustworthiness, related to instrument design and data collection procedures in the study. I conclude the chapter by discussing ethical issues and the limitations of the study.

4.2 RESEARCH DESIGN

In this section I present the research design of my study. Educational research is located within the social sciences, traditionally comprising two competing research paradigms. Guba and Lincoln (1994) defined a research paradigm as being a "basic belief system or world view that guides the investigator, not only in choices of method but in ontological and epistemologically fundamental ways" (p. 105). As a result, Hays and Singh (2012) suggested that educational researchers should state upfront the research paradigm adopted as it influences the choice of the research methodology employed in addressing the research problem. To address the research questions effectively, I located my study within the interpretivist paradigm. Interpretivists' researchers "seek to construct knowledge through social interactions as well as to understand how individuals construct knowledge" (Hays and Singh, 2012, p. 41). According to Check and Schutt (2012), the constructivist paradigm broadens the interpretivist paradigm by considering reality as a socially constructed entity. Saunders, Lewis and Thornhill (2003) elaborated that in the constructivist paradigm, each individual interprets their environments differently, thus influencing their behaviour and the way they interact with each other. In terms of this paradigm, Saunders et al. (2003) argue that understanding the phenomenon being studied results from examining the participants' subjective interpretations of the world with the aim of gaining insight and understanding of their intentions, actions and interpretations from their perspectives. Locating my study under the interpretivist paradigm was appropriate as Guba and Lincoln (1994) argued that "human behaviour, unlike that of physical objects, cannot be understood without reference to the meanings and purposes attached by the human actors to their activities" (p. 106). The interpretivist paradigm allowed me to gain insight regarding the participants' understanding of the teaching geometry through the problem solving approach from their own descriptions.

According to Guba and Lincoln (1994), research paradigm assumptions influence the choice of research design adopted in answering the research questions. Nieuwenhuis (2007) defined a research design as the strategy linking the research paradigm assumptions with criteria for selecting the participants, data collection and data analysis procedures for a study. The interpretivist paradigm is associated with qualitative approaches (Check & Schutt, 2012). According to Mavhunga (2012), investigating teacher knowledge is a difficult enterprise due to its multi-dimensional nature. Thus, for the present study I chose a qualitative approach (Creswell, 2003). Berg (2001) defined qualitative research as "the meanings, concepts, definitions, characteristics, metaphors, symbols, and descriptions of things" (p. 3). Qualitative research aims to provide "an in-depth description and understanding of human experience" (Lichtman, 2006, p. 8), which is similar to the goal of the present study which sought to describe primary teachers' understanding of teaching geometry through the problem solving approach. Creswell (2003) added that investigation of the phenomenon in a qualitative approach takes place in its natural setting through various methods yielding textual data. Depaepe et al. (2013) recommended that the best place to examine aspects of teacher knowledge was the school or inside the classroom where teachers use their knowledge. Hence, a qualitative approach was the most appropriate for addressing the research questions in the present study, since using various data collection methods accommodated the multi-dimensional nature of the aspects of teacher knowledge. Besides, “qualitative techniques allow researchers to share in the understandings and perceptions of others and to explore how people structure and give meaning to their daily lives” (Berg, 2001, p. 7).

There are many research designs congruent with qualitative approaches. Among these various research designs, I selected a case study design as the most appropriate for answering the research questions in this study. Baxter and Zack (2008, p. 544) defined a qualitative case study as “an approach to research that facilitates exploration of a phenomenon within its context using a variety of data sources”. A qualitative case study seeks deep understanding from defined settings, namely, the participants’ conceptualisation of the phenomenon (Nieuwenhuis, 2007). According to Berg (2001), a case study focuses on systematically collecting rich data about the participant(s) to allow the researcher to gain in-depth insight regarding their behaviour. Therefore, choosing a qualitative case study design in this study allowed for the gathering of exceptionally rich, exhaustive and in-depth data (Berg, 2001). A qualitative case study was the most appropriate for answering the research questions in this study as it provided comprehensive data from a defined region of the participants’ understanding of teaching a specific concept and through a particular instructional approach, in their natural environment. Moreover, due to the situational nature of the phenomenon, a qualitative case study design provided means of examining it within its context, i.e. the participants understanding of teaching geometry through the problem solving approach, from the participants’ accounts. Furthermore, a qualitative case study design ensured a holistic examination of the phenomenon through various lenses, thus accommodating the multiple aspects of teacher knowledge.

According to Yin (2009), case study designs can be exploratory, explanatory or descriptive. In this study, I chose a descriptive case study design as the most appropriate for answering the research questions. A descriptive case study, according to Zainal (2007) aims to describe the unobstructed phenomena emerging from the data collected, describing the data as collected. Congruent with a descriptive case study designs, which requires the researcher to present upfront the theoretical framework guiding the study (Berg, 2001), I discussed in detail the theoretical framework underpinning this study in Chapter 3 of my study. Berg (2001) stated that in a descriptive case study, prior to commencing the study, the researcher must determine the unit of analysis of the study. In this study, the unit of analysis were the primary teachers in the Shiselweni region in Swaziland.

In this section I presented the research design of my study. In the next section I present the population of my study.

4.3 POPULATION

Having presented a description of the research design of my study in the previous section, in this section I present the population of my study. A qualitative case study design seeks deep understanding of the participants' conceptualisation of the phenomenon from bounded settings (Nieuwenhuis, 2007). My study was located in the Shiselweni region, found in the southern part of Swaziland. The Shiselweni region is comprised predominantly of rural schools. My study took place in primary schools that were mostly teaching practice sites for student teachers from the local teacher training college where I work. Due to this fact, the learners were used to intruders observing them during their learning.

Due to the shortage of primary school teachers, the population consisted of four different groups of teachers. The first group comprised teachers with university degrees in primary education plus a primary teacher diploma. The second group, which was the largest, consisted of teachers with a primary teacher diploma. The third group consisted of teachers with tertiary qualifications but not linked to teaching. Lastly, the fourth group consist of high school leavers without any formal training in teaching.

In Swaziland, the teaching policy advocates that primary school teachers should teach all the subjects offered in their schools, regardless of their subject specialisation in training. However, some primary schools have adopted the concept of team teaching, more especially in the upper primary grades.

In this section, I presented the description of the population of my study. In the next section, I present the sample and sampling procedure I adopted in my study.

4.4 SAMPLE AND SAMPLING PROCEDURE

Having presented the description of the population of my study in the previous section, in this section I present the sample and sampling procedure. Nieuwenhuis (2007) defined sampling as "the process used to select a portion of the population of the study" (p. 79), adding that, in a case study the primary aim for sampling is to identify participants with a potential of providing rich data sources for answering the research questions. In this study, I sampled participants in two phases. In the first phase, through non-probability and purposeful sampling procedures, I selected 34 participants to complete an open-ended questionnaire (Check & Schutt, 2012). According to Nieuwenhuis (2007), purposive sampling decisions aims at obtaining rich data sources for addressing the research questions. In my study, purposive sampling was appropriate because my goal was to obtain data from experienced mathematics teachers.

Of the many purposeful sampling strategies, I determined that criterion sampling (Nieuwenhuis, 2007) was most appropriate for selecting participants for the study. In criterion sampling, a predetermined criterion informs the selection of the participants. As a result, in selecting the participants in the study, I considered primary school teachers in the Shiselweni region, teaching mathematics in any of the grades from grade three to grade seven. I considered these teachers for the following reasons: firstly, from the situated perspective of teacher knowledge, Brodie and Sanni (2014) argued that practice heavily affects in-service teachers' knowledge, resulting in them developing more understanding of the concepts they teach regularly. Hence, I considered teachers who were teaching or had recently taught the concept of area of polygons, as rich sources of data. Secondly, in primary schools in Swaziland, the concept of calculating the area of polygons using standard units was taught in these grades. Thirdly, in these grades, learners were introduced to calculating the area of polygons using formulas. Lastly, the learners start learning problem solving strategies including Polya's (1945) problem solving phases in grade three. I felt that these teachers were potentially rich sources of information for answering the research questions. The sample of 34 participants for this phase was appropriate as it provided a broader understanding of the phenomenon. Further, it allowed for quantitative analysis of the open-ended questionnaire data (Hsieh & Shannon, 2005).

In the second phase of my study, through self-selection, two participants, Zane and Patrick volunteered to partake in a lesson observation and subsequent semi-structured interviews (Saunders et al., 2003). The participants indicated their willingness to participate in this phase by writing their names and contact details in the open-ended questionnaire. The second phase of my study was necessary to provide data triangulating the participants' descriptions of their conceptions of teaching geometry through the problem solving approach in actual classroom actions. Therefore, the two participants were appropriate for a detailed analysis of their classroom actions as they provided two cases for this phase (Yin, 2009).

Although some scholars have criticised results from case studies for lacking generalisability due to small sample sizes, Nieuwenhuis (2007) argued that such criticism was misdirected, as generalisability is not the aim of case studies. In the same vein, my goal in this study was not to generalize, but to gain deep understanding of the participants understanding of the phenomenon from their own perspectives.

In this section, I presented the description of the sample and sampling procedures. In the next section, I present the description of my data collection procedure.

4.5 DATA COLLECTION

In this section, I present a description of the data collection procedures I employed for answering the research questions. According to Yin (2009), qualitative case studies focus on examining the phenomenon through multiple data sources. In this study, I collected data in two phases using an open-ended questionnaire, lesson observation, semi-structured interviews and lesson plan analysis to collect data. Depaepe et al. (2013) suggested that using data collection tools such as classroom observations, interviews and lesson plans could effectively examine the situated aspects of teacher knowledge. Besides, Steele (2003) asserted that relying on one type of instrument was not sufficient for accurately examining aspects of teacher knowledge due to its situated nature. Open-ended questionnaire were not common with qualitative case study designs, however in this study, they were necessitated by the Ball et al.'s (2008) MKT theoretical framework informing this study and the literature review. Ball et al.'s (2008) MKT framework and the literature review recommended that instruments for effective examining of aspects of teacher knowledge should include open-ended tasks. To be specific, Ball et al. (2008) recommended that tasks examining KCS should include items requiring teachers to interpret learners' "emerging and inchoate thinking, that present the thinking or expressions typical of particular learners, or that demand sensitivity to what is likely to be easy or challenging" (p 401). Furthermore, Ball et al. (2008) recommended that instruments for examining KCT, should include items asking teachers both the comprehension of specific mathematical content and instructional approaches influencing the learning of that content.

In the first phase, 34 participants completed an open-ended questionnaire consisting of three parts (Appendix F). Part I of the open-ended questionnaire focused on extracting participants' demographic information such as name (which was optional, only those willing to participate in the second phase indicated their names), school, gender, number of full years teaching experience, highest qualification achieved including major, mathematics class teaching and number of in-service training days attended in the last two years. I adapted Part II items of the open-ended questionnaire from Manizade and Mason (2011). In this part, I focused on examining the participants' conceptions of geometry and measurement instruction, focusing on the aspect of decomposing and recomposing the polygons (Manizade & Mason, 2011). In Part III of the questionnaire, I used an item adopted from Anderson (2000), asking the participants to describe the factors they considered as influencing their conceptions of teaching geometry through the problem solving approach.

In my literature review, it emerged that the relationship between teachers' conceptions and practice was dynamic and not always linear (Zheng, 2013). Consequently, she recommended that in the study of teachers conceptions, observations were essential to minimise data resulting from "token adoption". Besides, due to the tacit nature of teachers' knowledge, Mavhunga (2012) asserted that sometimes teachers concealed ideas or thoughts they regarded as unpopular. Informed by these recommendations, in the second phase, I observed the two participants while they taught a lesson on area of polygons. During the lesson observations, I was able to experience the participants' actual classroom practices. Nieuwenhuis (2007) defined an observation as an organised procedure for capturing participants' actions and incidences without interacting with them. In this study, during the lesson observations, I employed participant observation, collecting data about participants' actual classroom practices in their natural environment, their respective classrooms. During an observation, the researcher can assume different roles depending on the aim for the observation (Check & Schutt, 2012). In my study, I assumed an observer as participant role, which allowed me to observe the participants without disturbing their natural environment (Hays & Singh, 2012).

In my study, I was concerned about gathering observation data that reflected the actual participants' practices of teaching area of polygons. As a result, to avoid conducting the lesson observations with pre-conceived ideas about their MKT, I did not read their Part II and Part III responses in the open-ended questionnaire, until after the observations.

Moreover, during the lesson observations, I used an observation schedule to focus my attention on the goals of my data collection. I developed the observation schedule informed by my conceptual framework. During each lesson, I looked for all or some of the following practices as adapted from Polya (1945), Takahashi (2008) and Donaldson (2011). With each key feature, I had in mind the accompanying questions to direct my data collection.

1. Understand the problem:
    (a) Does the teacher present a problem linking the known to the unknown?
    (b) Does the teacher confirm learners' prior knowledge related to the lesson?
    (c) Does the teacher give learners opportunities to identify key words in the problem?
    (d) Does the teacher give learners opportunities to identify extra information in the problem?
2. Devise a plan:
    (a) Does the teacher give learners opportunities to predict/suggest/hypothesise solution strategies?
    (b) Does the teacher offer, without interfering, appropriate hints to the learner?
    (c) Does the teacher give learners opportunities to restate or translate the problem?
    (d) Does the teacher give learners opportunities to state as many solution strategies to the problem as possible?
3. Solve the problem:
    (a) Does the teacher allow learners to solve the problems in groups and prove correctness of their predictions using their preferred solution strategies?
    (b) Does the teacher provide learners with tools/materials necessary for executing their plan?
    (c) Does the teacher move around class monitoring progress in groups and assisting where necessary without interfering?
    (d) Does the teacher identify unique ideas?
4. Looking back:
    (a) Does the teacher provide learners with opportunities to evaluate effectiveness of their solution strategies?
    (b) Does the teacher provide learners with opportunities to discuss extensively their solution, strategies, comparing and highlighting the similarities and differences among their solutions strategies?

(c) Does the teacher assist learners refine their solution strategies?

I recorded in my field notes any instances where the teacher exhibited any of the behaviours. I also video recorded each lesson observation to ensure accuracy of transcriptions. In addition, I took notes for constructing questions I asked the participant at the end of the lesson, enhancing the understanding of the participant's actions. After each lesson observation, I conducted a follow-up interview with each participant to gain more insight about each participant classroom decisions and actions that emerged during the lesson.

After completing each observation, I interviewed the two participants using a semi - structured interview protocol (Appendix G) in their respective schools. Interviewing the participants increased my understanding of their conceptualisation of the phenomenon. I adapted the semi-structured interview protocol from Sothayapetch et al. (2013). Nieuwenhuis (2007) explained that in a semi-structured interview, the researcher asks the participant a sequence of predetermined open-ended questions. In addition, the researcher might probe the participant as necessary to ensure clear and contextual data, as "interviewers are permitted … to probe far beyond the answers to their prepared and standardized questions" (Berg, 2001, p. 70). Consequently, semi-structured interviews provide rich information that assists in understanding the phenomenon as perceived by the participants. In this study, each interview protocol with each participant lasted about ten minutes. Interview protocols were audio-recorded, ensuring accuracy of the data collected. A field log recorded a comprehensive description of the activities of the researcher at each phase. All data collection in this study took place in the participants' natural environment, the school or their classrooms.

In this section, I presented the description of my data collection procedures. In the next section, I present a discussion of how I analysed the data I collected in my study.

4.6 DATA ANALYSIS

In this section, I present a discussion of how I analysed the data collected in my study. In this study, to answer the research questions, I collected data through open-ended questionnaire, lesson observation, lesson plan analysis and semi-structured interviews. However, Litchman (2006) explained that there are no standard procedures for analysing qualitative data compared to quantitative research due to its richness. Hence, to analyse qualitative data effectively requires creativity and flexibility to capture its richness.

After collecting lesson observation and interview data, I transcribed the data verbatim. In addition, I numerically numbered each questionnaire script for identification purposes. According to McMillan and Schumacher (2010), qualitative data analysis is an inductive process involving inductively sorting data into sets and identifying emerging themes. Therefore, in this study, data analysis involved identifying rich patterns and emerging themes from all the data yielded by the multiple sources. Initially, in this study I adopted Hsieh and Shannon's (2005) content analysis strategy to analyse data from the various sources according to the research questions. To analyse the data answering the first research question, I used Ball et al.'s MKT framework. I created a list of broad codes elucidating clearly the different aspects of teacher knowledge essential for teaching area of polygons, focusing on KCS and KCT. Using Ball et al.'s (2008) MKT framework allowed me to focus on the relevant aspects of teacher knowledge. The code list comprised the following aspects:

1. Knowledge of important mathematical ideas in area of polygons;
2. Knowledge of learners' difficulties in area of polygons;
3. Knowledge of learners' emerging and incomplete ideas in area of polygons;
4. Knowledge of recognising and articulating learners' misconceptions in area of polygons;
5. Knowledge of sequencing area of polygons content;
6. Knowledge of selecting suitable activities for teaching area of polygons;
7. Knowledge of selecting appropriate representations to illustrate the area of polygons content;
8. Knowledge of the aims for learning area of polygons; and
9. Knowledge of area of polygons instructional strategies through the problem solving approach.

Within these different codes, during the analysis I focused in searching for themes and patterns, judging whether they were aligned or not with practices associated with teaching through the problem the problem solving approach, or not. From the conceptual framework, I extracted some of the practices associated with teaching through the problem solving approach in the observation schedule (Appendix G). The observation schedule was most useful in analysing the participants' knowledge of area of polygons instructional strategies through the problem solving approach. In presenting and discussing the results of my study, I used the codes list as subheadings.

To ensure a rigorous data analysis process, I created an analytical framework mapping the different aspects of teacher knowledge described in the code list with the different data sources as shown in Table 4.1.

Table 4.1: An analytical framework showing the mapping of the different aspects of teacher knowledge with data sources

Aspect of MKT	Data sources
Knowledge of content and students	
Knowledge of identifying important mathematical ideas in area of polygons	Item 8.1, Item 9.1
Knowledge of articulating learners challenges in area of polygons	Item 9.2, semi-structured interview
Knowledge of Interpreting learners emerging and incomplete ideas in area of polygons	Item 8.2, lesson observation
Knowledge of Recognising and articulating alternate conceptions learners about area of polygons	Item 8.3, Item 9.2
Knowledge of content and teaching	
Knowledge of Articulating aims for learning area of polygons	Semi-structured interview
Knowledge of Sequencing area of polygons content	Item 10.2, lesson observation
Knowledge of Selecting suitable activities for teaching area of polygons	Item 10.1, semi-structured interview
Knowledge of Selecting appropriate representations for illustrating the area of polygons content	Item 8.4, Item 9.3, semi-structured interview, lesson observation
Knowledge of area of polygons instructional strategies through the problem solving approach	Lesson observation, lesson plan, semi-structured interview

For the second research question, I inductively analysed the participants' responses to Part III of the open-ended questionnaire for emerging themes and patterns.

In this section, I presented a discussion of my data analysis procedures. In the next section, I present a discussion of how I addressed issues of quality in my study.

4.7 QUALITY OF THE STUDY

In this section, I present a discussion of how I addressed issues of quality at each stage of my study. In my study, I made a concerted effort to address issues related to the quality of my study. Qualitative researchers have different conceptions of validity and reliability in qualitative case study (McMillan & Schumacher, 2010). However, there is some consensus on the use of terms such as "rigor, trustworthiness, credibility and goodness, to name a few" (Hays & Singh, 2012, p. 217) to parallel such concepts in quantitative research. In my study, I adopted the term trustworthiness to refer to the concepts of validity and reliability. McMillan and Schumacher (2010) defined validity in qualitative research as "the degree of congruence between the explanations of the phenomenon and the realities of the world" (p. 330). According to Cohen, Manion and Morrison (2007, p. 149), reliability in qualitative research encompasses "fidelity to real life, context- and situation-specificity, authenticity, comprehensiveness, detail, honesty, depth of response and meaningfulness to the respondents". Hence, qualitative research "values subjective meaning of a research problem and context as well as collaboration between researcher and participant in constructing and understanding knowledge" (Hays & Singh, 2012, p. 33). Considering this, Baxter and Zack (2008) suggested that providing a detailed description of a qualitative case study methodology enables readers to assess the quality of the work. Further, providing a detailed description of the methodology "…may transport readers to the setting and give the discussion an element of shared experiences" (Creswell, 2003, p. 196). Data collection and analysis occurred concurrently in the study to avoid missing opportunities of enriched data collection. An audit trail including raw data that was collected was kept / is being kept in a secured pace as per the University of Kwa-Zulu Natal regulations. I elucidated clearly under the appropriate heading any biasness from me that influenced my decisions. My supervisor, an experienced qualitative researcher, rigourously scrutinised all the phases of my study.

I adopted open-ended questionnaire items from Manizade and Mason (2011) and Anderson (2000). I adopted Part II of the questionnaire items from Manizade and Mason (2011). Manizade and Mason (2011) reported that using Delphi methodology established the content validity of these items as experts with vast experience in designing instruments in the field of mathematics education examined the appropriateness of the items in measuring PCK in the topic of area of polygons. Furthermore, Manizade and Mason

(2011) claimed that aspects of content and construct validity were achieved by the elaborate literature review which also informed their decision-making during item construction. They rigorously addressed trustworthiness issues in terms of credibility, transformability, dependability, and confirmability. Hence, I adopted these items with confidence in my study. Since these items were designed to ascertain the same phenomenon as my study, I adopted them without altering their content except for their numbering. I adopted Part III of the questionnaire items from Anderson (2000) to ascertain primary teachers' views regarding the factors they attribute as influencing their conceptions of teaching geometry through the problem solving approach. Hence, overall the questionnaire was appropriate for ascertaining the participants' conceptions of teaching geometric area through the problem solving approach.

The participants completed the questionnaire in the absence of the researcher. Cohen et al. (2007, p. 334) assert that completing a questionnaire in the absence of the researcher is beneficial because it allows the participants to respond to the questionnaire in private, at their own pace, and in a familiar environment without "the potential threat or pressure to participate caused by the researcher's presence". Thus, the participants completed the questionnaire in their natural environment without any pressure. Using the literature and conceptual framework, I developed the codes for analysing data. My supervisor, an experienced mathematical educator, verified the accuracy of the codes.

I ensured quality of the lesson observation by using an observation schedule with the predetermined codes I had developed from literature on teaching mathematics through the problem solving approach. Using the observation schedule ensured that I focused on relevant aspects of the teachers' actions during the lessons. In addition, the observation schedule ensured consistency of the observations since I conducted lesson observations at two sites. Saunders et al. (2003) described the observer effect as a major threat to reliability during observations. In my study, I adopted the observer as participant role to minimise this effect ensuring minimal interactions with the classroom environment.

According to Nieuwenhuis (2007), in qualitative data collection, the researcher acts as a research instrument. During the lesson observations, I used my vast experience gained through observing student teachers during their teaching practice, further minimising observer biasness. Furthermore, I solicited the help of a schoolteacher to video record the lesson observations to ensure an accurate record of incidences that transpired during the lessons. Using a teacher, who was a member of each school, minimised obstructions in the classrooms since learners were familiar with them. In addition, I accorded each participant an opportunity to verify accuracy of transcriptions and results of the analysis. Furthermore, I asked my peer with vast experience in educational research to review and discuss my preliminary analysis. Later my supervisor, another experienced educational researcher, audited my research methods and data collected, thus improving the dependability of the study (Elo, Kääriäinen, Kanste, Pölkki, Utriainen, & Kyngäs, 2014).

Apart from the lesson observation, I interviewed the two participants using a semi-structured interview protocol to gain more insight into their understanding of teaching area of polygons. I interviewed the participants using a semi-structured interview protocol adapted from Sothayapetch et al. (2013). Sothayapetch et al. (2013) developed the items for examining PCK and GPK of primary school science teachers in electric circuits. An intensive literature review on PCK informed the development of these items (Sothayapetch et al., 2013). For this study, I adapted the PCK items and overall the items were appropriate for ascertaining the participants' conceptions of teaching geometric area.

I interviewed each participant in a relaxed and secluded environment, allowing them to express freely their conceptions of teaching geometry. In addition, I commenced each interview by thanking each participant for according me the opportunity to observe his or her lessons to ease any tension. I asked each participant demographic questions to verify information supplied in the questionnaire. During the interview, I allowed each participant freedom to express their perspective regarding the requirements of each item. In addition, I avoided ambiguous and suggestive questions, and provided clarity where necessary. Where the participant gave a vague response, I probed for clarification.

The semi-structured interview protocol ensured that I asked consistent questions at the two interview sites. I audio-recorded each interview ensuring that I stayed focused on the conversation and capturing an accurate record our conversation. To transcribe each interview data, I engaged the services of a professional and experienced transcriber. I verified the accuracy of the transcriptions after the professional transcriber had finished. In addition, I allowed each participant an opportunity to verify the accuracy of their views as reflected in the data transcriptions.

During the data analysis, my supervisor verified my code list as an accurate description of the different aspects of teacher knowledge as described in the conceptual framework. In addition, I engaged an experienced university lecturer to analyse the data within these codes separately. We later compared the results from our independent analysis and found that we agreed on 97% of the items. Where we had disagreements, we resolved them through discussion until we reached a consensus. Lastly, my supervisor audited the appropriateness of the research methods I employed in my study, thus improving the dependability of the study (Elo et al., 2014).

In this section, I presented a discussion of how I addressed issues of quality at each stage of my study. In the next section I present a discussion about how I addressed ethical issues in my study.

4.8 ETHICAL ISSUES

In this section, I present a discussion about how I addressed ethical issues in my study. In my study, I engaged the participants through voluntary participation. Voluntary participation was appropriate for my study considering that I interacted with the participants in their natural environment during the data collection stage. Consequently, informed consent was necessary to protect the integrity of the participants and their schools (Berg, 2001). Before commencing data collection, I had to comply with the University's Humanities and Social Sciences Research Ethics Committee requirements for ethical clearance (Appendix A). The University's Humanities and Social Sciences Research Ethics Committee, before granting ethical clearance, required that I obtain informed consent from relevant gatekeepers and the participants, explaining clearly that their participation in the study was voluntary; hence, they were free to withdraw from my study without any negative consequences. Furthermore, they required assurance that I would ensure anonymity and confidentiality for the participants. The Committee also required that I explain how data would be stored.

In accordance with the University's Humanities and Social Sciences Research Ethics Committee's requirements for granting me ethical clearance, I sought permission to conduct my study from the relevant gatekeepers through letters. In such permission letters, I articulated the focus of my study and the rights of the participants (such as voluntary participation, the time during which the study would be conducted and what was required of them in the study). In the letters, I assured the participants regarding confidentiality on any information they would divulge to me and that I would use pseudonyms for their names and schools in which they teach. I sought permission to conduct my study in the participating schools from the MoET through the Director of Education (Appendix B). After obtaining informed consent from the Director of Education, I sought informed consent from the principals of the participating schools (Appendix C). After obtaining informed consent from each principal of the participating school, I proceeded to seek informed consent from participating teachers (Appendix D). For the learners with whom I conducted the lesson observations, I sought consent from their parents or guardians (Appendix E). I did not collect any data before obtaining ethical clearance as per the University's Humanities and Social Sciences Research Ethics Committee Policy. After fulfilling their requirements, the University's Humanities and Social Sciences Research Ethics Committee granted me ethical clearance (attached after the front page of this dissertation).

Having presented a discussion of how I addressed ethical issues in my study in this section, in the next section I present the limitations of my study.

4.9 LIMITATIONS OF THE STUDY

In this section, I present a discussion about the limitations of my study. A major limitation of my study resulted from the voluntary nature of participation. Since my study was small sample size case study, results are not generalizable over the region. Another limitation of my study concerned maintaining rigor in the data collection, as data involved participants completing an open-ended questionnaire in their school environment in my absence. Therefore, there was a possibility of participants seeking help from their colleagues not involved in my study. This may have contaminated the data; however, the lesson observation and semi-structured interview corroborated the data. Most of the older participants with more experience in teaching mathematics I had targeted declined to complete the open-ended questionnaire, stating that it was too difficult for them, yet it only covered mathematical content in the curriculum they were expected to teach.

The number of lesson observations was limited to one for each participant due to the focus of my study. Lesson observations were only possible when the participant was teaching the area of polygon concept. Finally, semi-structured interviews and an observation schedule may attract human subjectivity. However, I exercised caution to minimise this subjectivity as much as possible.

4.10 SUMMARY

In this chapter, I presented the methodological approach I adopted in answering the research questions. My presentation focused on research design, population, sample and sampling procedure, data collection procedures, data analysis, quality issues, ethical issues and the limitations of my study. In the next chapter, I present the results of my study.

CHAPTER 5 : RESULTS

5.1 INTRODUCTION

Having presented the methodological approach in the previous chapter, in this chapter I present the results of my study. In this study, my intention was to answer the following research questions:

(a) What are primary school teachers' conceptions of teaching geometry through the problem solving approach in the Shiselweni region (Swaziland)?

(b) What are the factors influencing the primary school teachers' conceptions of teaching geometry through the problem solving approach in the Shiselweni region (Swaziland)?

I present the results in this chapter in two sections according to the research questions: conceptions of teaching geometry and factors influencing these conceptions. In the first section, I present results addressing the first research question: What are primary teachers' conceptions of teaching geometry through the problem solving approach? I centred my presentation on results from the analysis of the 34 participants' responses from the open-ended questionnaire, and the lesson observation, semi-structured interview and lesson plan analysis with two participants.

In the second section, I present results addressing the second research question: What are the factors influencing the primary school teachers' conceptions of teaching geometry through the problem solving approach? I based the presentation of the results in this section on the analysis of participants' responses to item 11 in the open-ended questionnaire.

5.2 CONCEPTIONS OF TEACHING GEOMETRY

In this section, I present results from the analysis of the different sources of data describing the participants' conceptions of teaching geometry through the problem solving approach from analysis of the different sources of data. I present the results under the following headings as described in my analytical framework in Section 4.6.

1. Knowledge of important mathematical ideas in area of polygons;

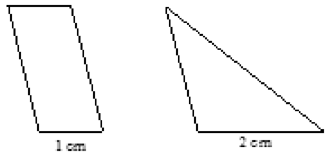
2. Knowledge of articulating learners' difficulties in area of polygons;
3. Knowledge of interpreting learners' emerging and incomplete ideas in area of polygons;
4. Knowledge of recognising and articulating learners' misconceptions in area of polygons;
5. Knowledge of sequencing area of polygons content;
6. Knowledge of selecting appropriate activities for teaching area of polygons;
7. Knowledge of selecting appropriate representations to illustrate the area of polygons content;
8. Knowledge of the aims for learning area of polygons; and
9. Knowledge of area of polygons' instructional strategies through the problem solving approach.

In the next subsection, I present results from the analysis focusing on the participants' conceptions of teaching geometry.

5.2.1 Knowledge of important mathematical ideas in area of polygons

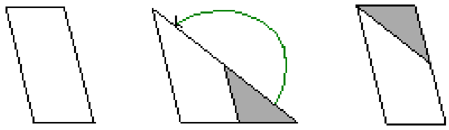
When teaching geometry through the problem solving approach, awareness of the network of mathematical ideas learners might use during lessons is an essential aspect of teacher knowledge. Charles and Carmel (2005) defined a mathematical idea as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p. 10). Therefore, teachers who understand mathematical ideas conceive of mathematics as being a connected set of ideas while those without this understanding conceive of mathematics as being a set of incoherent concepts, skills, and facts (Charles & Carmel, 2005). The literature on the problem solving approach recommends teaching content in a connected manner, compelling learners to draw from all their experience during lessons. Drawing from this argument, I found that examining teacher's knowledge of mathematical ideas related to area of polygons an important aspect of understanding their conceptions of geometric teaching. Consequently, in my study, I examined the participants' knowledge of mathematical ideas that learners might use in learning area of polygons using Items 8.1 (Figure 5.1) and 9.1 (Figure 5.2) from the open-ended questionnaire.

8. Ms. Wilson asked her seventh grade class to compare the areas of the parallelogram and the triangle below. Both of the shapes have the same height.

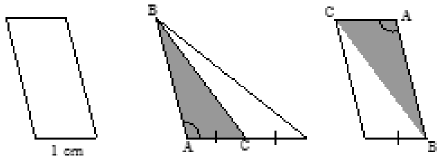


Two groups of pupils in Ms. Wilson's class came to the correct conclusion that the areas are the same. However, their explanations were different. The groups of pupils used the following diagrams to explain their answer.

Solution of group 1



Solution of group 2



Based on what you know as a mathematics teacher:

8.1 What are some of the important mathematical ideas that the pupils might use to answer this question correctly?

Figure 5.1: Item 8 activity

Item 8.1 was based on Item 8 which presented an activity that required learners to compare areas of the parallelogram and the triangle with the same height. In Item 8.1 the participants were required to state the important mathematical ideas that learners might use in comparing the areas of the two shapes accurately. Participants' responses to this item provided an insight into understanding their conceptions regarding what they considered important ideas in learning area of polygons. I coded the participants' responses thematically according to whether they focused on conceptual or procedural understanding of area of polygons.

The results of the analysis of participants' responses to this item indicated that a majority of the participants (65%) stated mathematical ideas promoting procedural understanding consistent with traditional instructional approaches. To compare the areas of the two polygons the participants emphasised that learners would use the area formula for triangle and parallelogram or count unit squares in both shapes. They ignored the fact that the dimensions of the shapes were not provided in the activity, except that they have the same heights. Those who mentioned the ideas of decomposing and recomposing the shapes could also use the formula easily. The following responses illustrate this point.

> *To get the area of a triangle they can use the conventional method, which is* $\frac{1}{2}bh$*. If they can't do that the*[y] *can join the corners of the triangle to form a square then count the number of squares there then divide by 2.* (P1)

> *The learners have to count the part squares and divide them by 2 and add them to the full squares in a triangle or irregular shape. They have to draw another triangle on a given triangle to form a rectangle and then use the formula area =* $\frac{b \times h}{2}$ *or* $\frac{l \times w}{2}$ *to get the correct answer.* (P4)

These responses suggested that participants focused on early arithmetisation of the process of comparing the areas of the two polygons, consistent with traditional approaches. The participants insisted on using the formulas despite that the fact that the information provided was insufficient for the grade level. Besides, the activity did not require the learners to quantify the area of each shape.

A few participants (26%) mentioned ideas with elements consistent with problem solving principles of comparing the areas of the two shapes. According to these participants, in order to compare the areas of the two shapes correctly, learners should understand the concept 'area' and properties of the shapes. The following two responses provided evidence illustrating this point.

> *They need to understand what is meant when they say area is the amount of flat space covered in a boundary. This means that shapes might differ but the area may be the same.* (P20)

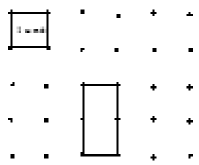
> *An area of a shape is not determined by how the shape is but by the amount of space it occupies on the surface, e.g. a square and a rectangle can have the same area but their shape is not the same.* (P19)

These responses showed that the participants had some conceptual understanding of what it entailed to compare the areas of the two shapes.

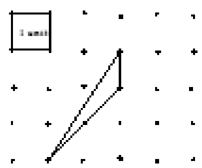
Item 9.1 examined a different aspect of mathematical ideas associated with area of polygons. This item was based on learners' activities as shown in Figure 5.2.

9. Ms. Mason is planning her seventh grade school geometry unit on area. She has a diverse population of pupils with different levels of geometric development. Ms. Mason gathered a set of activities related to the concept of area that she believes would address different levels of geometric development. She wants to include the following activities in her unit.

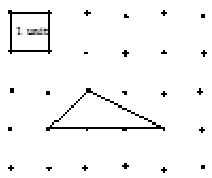
Activity L: Find the area of the rectangle below using the given unit



Activity M: Find the area of the triangle below using the given unit



Activity N: Find the area of the triangle below using the given unit
For each activity, please answer the following.



9.1 What are some of the important mathematical ideas that the learners might use to solve these problems correctly?

Figure 5.2: Item 9 activities

In this item, the participants were required to state mathematical ideas that learners might use when calculating the areas of a rectangle and triangles drawn on a grid. In the activity, the learners were provided with a square unit as a unit of measure.

As with Item 8.1, I coded the participants' responses based on whether they supported problem solving principles in finding the areas of the three shapes, or not. The results of the analysis of the participants' responses revealed that the participants had mixed conceptions of the mathematical ideas that learners might use in finding the areas of the three shapes. The analysis revealed three themes: counting unit squares, decomposition/recomposition and using a formula. A majority of the participants (56%) mentioned that before counting the unit squares inside the shapes, particularly in the triangles, learners should decompose and recompose the triangles into rectangles/squares as illustrated by the following responses from some of the participants.

> *Activity L the learners may simple count the number of units inside the shape. In the other activities, they must join the shape such that it make a rectangle, count the number of squares within them and divide by two.* (P32)

> *Activity L, the pupils will count the number of squares and get the area. In activity M and N they should be able to calculate fractions and extend the triangles.* (P2)

According to these participants, decomposing and recomposing the triangles would make using the unit of measure easy for the learners, as they would eliminate the difficult of calculating part squares. In addition, learners would use their prerequisite knowledge of calculating areas of square/rectangle in calculating the areas of the triangles. I viewed this action as valuing learners' prior knowledge during their learning that had some essential elements of teaching through the problem solving approach.

Ten (29%) participants mentioned that learners would use ideas of counting the unit squares when finding the areas of the three shapes. P15 illustrated this point by mentioning that, *"It is to count the number of squares covered by the shapes".* Such responses indicate that these participants were inclined to teach areas of the shapes through traditional approaches.

The third theme emerging from the analysis involved using the formula when finding the areas of the triangles. As an example, P8 mentioned "*The concept of finding the area of triangles using the formula*". Considering the types of triangles in the activity, in particular Activity M, primary school learners lack the capacity to use a formula. Thus, mentioning the use of a formula to calculate areas of triangles drawn on a grid indicates that the participants had traditional conceptions of teaching area.

P9 mentioned that the learners should be able to tessellate with a polygon as a necessary prior conception for finding area of polygons drawn on a grid. Tessellation with a polygon is one of the relevant important mathematical ideas necessary in learning areas of polygons drawn on a grid.

The analysis revealed that the participants lacked an understanding of the important mathematical ideas related to area of polygons. On both tasks, participants mentioned the use of a formula, which suggested that they focused on procedural thinking. In the next subsection, I present results from the analysis of data probing participants' ability to articulate learners' difficulties in area of polygons.

5.2.2 Knowledge of articulating learners' difficulties in area of polygons

In addition to being conversant with important mathematical ideas learners would use in learning area of polygons, teachers should also be knowledgeable about what learners would find difficult in learning area of polygons. In this study, Item 9.2 in the open-ended questionnaire (see Figure 5.2 for the relevant activities) examined the participants' ability to identify what learners would find difficult when learning the content of calculating area of polygons drawn on a grid. Moreover, the participants were required to justify their responses. I coded the participants' responses based on their choice of activity or activities they anticipated would be difficult to their learners and the justification for the anticipated difficulty. The results from the analysis of participants' responses revealed that most participants (85%) identified activities M and N, involving area of triangles, as the concepts learners would find difficult. To justify their choices, the participants stated, firstly, that in a triangle it was difficult to count the part squares. Secondly, they stated that some learners might count part squares as full squares. Thirdly, they stated that some learners might have difficult in decomposing and recomposing the triangle. Lastly, they mentioned that it was difficult to use the formula $A = \frac{1}{2}bh$ in a triangle. The following responses from some of the participants illustrate this point.

> *I think that it could be difficult for them to master the skill of counting the units or to make them complete unit squares*" (P19).
>
> *The learners may not be able to find the units in polygons like the triangle because the triangle is half of the rectangle so some of the units in the triangle will be half and thus confused the learners because they will be not able to count the halves.* (P 24)
>
> *Some learners may be taught to complete the shape of a triangle and make either a rectangle or parallelogram. Learners may fail to complete triangle by making them very big thus getting a wrong answer when dividing by 2.* (P15.
>
> *The learners might forget to divide by 2 after multiplying the length by the height.* P30

P5 identified a unique challenge that learners might face while attempting the activities, in that learners might confuse the perimeter and the area of the polygons. As a result, P5 mentioned that the learners might count the dots around the shapes as area. P17 mentioned explicitly that learners must desist from using the formula when attempting the activities but must count unit squares.

The rationale put forward by the participants for choosing the area of the triangles as concepts learners would find difficult revealed their conceptions of teaching area of polygons. Teachers attuned to teaching area of polygons through the problem solving approach regarded these reasons as providing opportunities for their learners. Additionally, teachers with traditional conceptions of teaching area of polygons were more likely to regard these reasons as challenges, since they are concerned with how they could easily explain the concept to the learners. This contrast provides an insight into understanding the participants' conceptions of teaching geometry. Therefore, in this item the participants showed that they had traditional conceptions of teaching area of polygons.

In the next sub section, I present results regarding participants' ability to interpret emerging and incomplete ideas in the learning of area of polygons.

5.2.3 Knowledge of interpreting learners' emerging and incomplete ideas in area of polygons

During instruction through the problem solving approach, teachers should understand learners emerging and incomplete ideas in order to offer learners with challenges in solving the problem appropriate hints without interfering (Polya, 1945). In addition, teachers should be able to judge the plausibility of learners' solution strategies. In this study, Item 8.2 required the participants to give a plausible explanation as to why Ms. Wilson was unable to judge the correctness of the solutions provided by groups 1 and 2. The participants' responses on what they thought prevented Ms. Wilson from evaluating the correctness of the groups' responses provided an insight into their own ability to interpret learners' ideas during the learning of area of polygons (see Figure 5.1 for the activity).

The results from the analysis of the participants' responses revealed that the participants had a limited understanding of the content of decomposing and recomposing. Despite the item not requiring participants to evaluate the groups' solutions, a majority (38%) did. However, a closer analysis of the reasons they put forward for the outcomes of their evaluation were incorrect, revealing their limited knowledge regarding decomposing and recomposing the triangles. I classified the responses from those participants who stated that the solutions were correct into four categories. The first category consisted of those participants who justified their responses. The following excerpt illustrates this point.

> *Both answers are correct since the first group was able to change the triangle to be a parallelogram, and this parallelogram is similar or congruent to the [original] parallelogram. The second group was able to change the triangle too, to be a parallelogram which also congruent to the given parallelogram [original].* [Own emphasis] (P15)

Unfortunately, as I have already alluded to, this evaluation was incorrect, indicating poor CK of the participants. The second category consisted of the participants who stated that both groups were correct without justifying their response. The third category consisted of a response from P12 who stated that only group 2's solution was correct saying: "*Only the solution of group 2 is correct because the triangle was cut at the centre to form the parallelogram*". The following response from P6 provides evidence to illustrate a fourth category consisting responses from those participants who stated that both solutions were wrong.

> *Group 1 is wrong because, it is like they were by cutting the parallelogram they were trying to get the height of the parallelogram. According to me where they cut it is not the height. Group 2 is wrong again is wrong because it is like they wanted to get half the parallelogram, height formula for parallelogram is bh.* (P6)

The responses from this group of participants revealed that they have limited understanding of the concept of decomposing and recomposing triangles into a parallelogram. Apart from evaluating the groups' responses which was unsolicited in the item, their evaluation was incorrect.

The second theme emerging from the analysis showed that participants described how the two groups obtained their solutions instead of explaining why Ms. Wilson failed to evaluate the groups' solutions. As an example, one participant wrote "*In both solutions they divided the triangle to form a parallelogram. Whereas to compare area they needed to find different area of shapes separately because they have different bases*" (P22). This group too, exposed their diminished understanding of decomposing and decomposing of the triangle through their explanations, as they were all incorrect. A further three (9 %) did not respond to this item at all. Maybe they found this item too difficult for them.

Only nine (26%) participants recognised that Ms. Wilson failed to judge the correctness of the groups' solutions due to insufficient knowledge of comparing the areas of the two shapes. For instance, P14 mentioned that, "*Ms Wilson lacks knowledge on how* *to find area of the two shapes, that is why she is not sure*". In this theme, the participants' responses demonstrated that they recognised the importance of teacher knowledge in evaluating learners' responses. It might be true that Ms Wilson failed to evaluate the correctness of the responses due to lack of knowledge.

Overall, the results in this item showed that the participants had insufficient knowledge of comparing the areas of the two shapes. In addition, a majority did not recognise the role played by teacher's CK in evaluating learners' responses. These findings are consistent with their expectation of the ideas they expressed that the learners would use when comparing the areas of the two shapes. A majority of the participants in section 5.1.1 mentioned that they expected the learners to compare the areas of the two shapes using the formula. Only a few participants identified the correct reason, insufficient knowledge, which prevented Ms. Wilson from evaluating the solutions.

In the next section I present results on the participants' ability to recognise and articulate learners' misconceptions in area of polygons.

5.2.4 Knowledge of recognising and articulating learners' misconceptions in area of polygons

Apart from knowing, what learners would find difficult in learning area of polygons, teachers should also be aware of misconceptions learners bring in the learning of area of polygons. According to Smith et al. (1993) a misconception is a learner's "conception that produces a systematic pattern of errors" (p. 119). Smith et al. (1993) asserted that misconceptions interfere with the learning of new content. Hence, teachers should identify them in advance in order to provide learners with opportunities to overcome them. In this study, I extracted the participants' ability using Item 8.3 in the open-ended questionnaire. This item was based on activity 8 (see Figure 5.1) which presented a specific context and a question in the semi-structured interview which probed the participants' ability or lack of ability to identify learners' misconceptions related to area of polygons.

Item 8.3 required the participants to ascertain whether each groups' response represented any mathematical misconception or not, justifying their responses pertaining to a specific context. The participants' responses to this item revealed the participants' own misconceptions regarding comparing the area of the triangle and the parallelogram. The participants who responded by "yes" were required to state the underlying misconception that contributed to the learners error while those that responded with "no" were required to explain how the two groups differed in their thinking. The results from the analysis in this item revealed that no participant identified the misconception associated with decomposing and recomposing the triangle into a parallelogram in group 2's solution. Although, the areas of the recomposed parallelograms in both groups' solutions were equal, they resulted from different reasoning from the two groups. The reasoning behind group 1's solution was correct while the reasoning behind group 2's solution carried a mathematical misconception. Group 2 assumed that angle A = angle C in the recomposed "parallelogram" which was wrong. Thus, the resultant shape was not a parallelogram.

The analysis of the participants' responses revealed three themes. In the first category theme, a majority of the participants (44%) stated that there was no misconception in the groups' solutions. In this theme, some participants justified their responses while some did not. Here is an excerpt which illustrates this point:

> *There is no misconception. Seemingly, there is no much difference because both groups know that triangles can be made into quadrilaterals. The only slight difference is that they have cut the triangles in different positions to form the quadrilaterals.* (P12)

By responding that there was no misconception in the groups' solution, these participants revealed their own misconceptions about comparing the areas of the two shapes.

In the second category theme, nine (26%) participants stated that there was a misconception in either one or both group's solution. I distinguished two subcategories from this group of participants. The first subcategory consisted of participants who mentioned that both groups had a mathematical misconception while the second subcategory consisted of participants who identified group 1's solution as representing a misconception. P31 mentioned that "*Yes there is a misconception. Maybe the teacher have taught them in two ways (using the two methods) and the teacher did not deliver the lesson clearly*". P26 stated: "*The mathematical misconception is to convert the triangle into a parallelogram which is not accurate though seen possible*". However, this group of participants failed to identify the relevant misconception associated with the solution of group 2, indicating their inaccurate conceptions of comparing areas of triangles and parallelogram by decomposing and recomposing.

The third category theme consisted of participants who failed to identify any alternate conception. Participants in this category explained how the groups obtained their solutions as shown by P27's response: "*They change triangles into parallelograms*".

However, the interview results with the two participants showed that they understood common misconceptions associated with area of polygons. During the interview, I asked the two participants to state any common alternate conceptions they have noticed during their teaching of area of polygons. Zane mentioned that learners tend to confuse the formula for finding area of a rectangle with that of finding the area of a triangle as shown in the following transcript extract:

> *Yes, there is. They tend to mix the formula for finding area of a rectangle with that of a triangle. Meaning that they can't even know the shapes, because, if I ask them to find the area of a rectangle, then they say half base times height. So that is where I have discovered a problem among the learners. So, they do not make a difference between what formula is used for finding area of rectangle [ehh] which one is used to find the area of a triangle. So they tend to mix the two* (Appendix K, L112-L117).

On the other hand, Patrick mentioned that a common learners' misconception in the learning of area of polygons was learners failing to identify the relevant dimensions when calculating the area of a triangle using the formula. Overall, the results of the analysis revealed that the participants had insufficient knowledge of decomposing and recomposing area of polygons. In the next section, I present results regarding the participants' knowledge of aims for learning area of polygons.

5.2.5 Knowledge of the aims for learning area of polygons

During the interviews I asked each participant what they thought were the main aims for learning area of polygons. Both participants considered that being able to solve daily life problems was the main reason for learning area of polygons. They supported their views by examples. Zane mentioned:

> *… maths is done everywhere, at home, at school. So teaching about area might help them, maybe in future. [ehh] For example, let's assume maybe the adult they want to furniture their houses, they were putting tiles there. So in order to know how many tiles are needed they have to know the concept of area because they have to multiply length times breadth finding the number of tiles needed to cover that surface.* (Appendix K, L77-L81)

The next interview transcript extract illustrates what Patrick considered to be the aims for learning area of polygons:

> *… the main aims is for them to be able to share things or even to be, even to use that information in our daily life experiences. For example, another teacher was being told to buy a plot, which he was given the area, it is so many metres squared then he was not there, but he was charged a very large sums of money. Then he consulted what –*

> *how much is this area? Then I told her, this is something like this, then he discovered. [ohh] I can be robbed here then he moved on*. (Appendix L, L88-L93)

The analyses revealed that the participants considered the learning of area of polygons to be an important concept for problem solving. They regarded the learning of area of polygons as useful for solving daily life problems, which is consistent with goals for teaching geometry through the problem solving approach. However, they did not mention the use of area of polygons in developing other mathematical concepts, and laying the foundation for further learning of area concepts. In the next subsection, I present results from the analysis of data probing participants' ability or lack of ability to sequence area of polygon content.

5.2.6 Knowledge of sequencing of area of polygons content

One of the critical aspects of teacher's knowledge emerging from the problem solving instruction literature relates to the ability to sequence mathematical content properly. Sequencing content properly when teaching through the problem solving approach, apart from ensuring that learners are taught content at their cognitive level, ensures that they draw from their existing knowledge during lessons. In this study, Item 10.2 in the open-ended questionnaire ascertained participants' ability to sequence the activities in Item 9 (see Figure. 5.2 for activities). In this item, the participants were required to justify why they would sequence the activities in their chosen order. The analysis of the responses provided deep insight into understanding the participants' conceptions of geometric instruction. Teachers with a good understanding of teaching geometry through the problem solving approach were more likely to arrange the activities based on the opportunities that they would present to the learners, how the learners could apply their existing knowledge in new situations.

I coded responses based on both their choice of activities and the rationale they provided for sequencing the activities in that order. The results of the analysis indicated that the majority of the participants (88%) mentioned that they would sequence the activities in the order L, N and M. They justified this order by stating that it was from simple to complex. As an example to illustrate this point, P4 mentioned, "*I would start with L, then N and finally M to gradually get learners into clear understanding*". In addition, P5 mentioned, *"I think we can start with L followed by N and M be the last one. Reason being that L does not need much think as compared to M and N".* Moreover, P7 mentioned, *"Activity L, Activity N and Activity M. This order starts from the simple to complex".* However, some participants did not include all the three activities. For instance, P13 included only activity L while P24 included activities L and M in that order. A further two (6%) did not give the order but mentioned that they would start from simple to complex. When analysing the reasons advanced by the participants for justifying their order, it was clear that they did not think much about the value of starting with activity L except that for them it was simple for the learners. Thus, this indicated that the participants had traditional conceptions of teaching area of polygons.

P25 reported a different order of the three activities, beginning with L followed by M and lastly N. P25 stated that the activities were arranged in the correct order in the original item. While all the participants who responded to this item were in unison that they would start with activity L, two (6%) participants did not respond to this item.

I observed a similar pattern during the lesson observation with the two participants. Zane commenced his lesson by defining area followed by area of a polygon with straight edges then the area of polygons with slanting edges. Patrick also commenced his lesson by asking learners to define area, followed by area of rectangles on a grid, then area of rectangles by formula. During the lesson, Patrick proceeded to discuss with the learners the relationship between rectangle/square and the right-angled triangle before they could find the area of right-angled triangle on a grid. After discussing the area of right-angled triangle on a grid, Patrick introduced the area of right-angled triangle by decomposing and recomposing into a rectangle/square then using a formula. He concluded his lesson by introducing the area of an isosceles triangle on a grid and then the area of an isosceles triangle by decomposing and recomposing into a rectangle/square then using a formula.

In the next subsection, I present results from the analysis of the participants' ability to select appropriate activities for teaching area of polygons.

5.2.7 Knowledge of selecting appropriate activities for teaching area of polygons

Teaching geometry through the problem solving approach entails learners solving tasks to learn new geometric content (Van de Walle et al., 2014). Therefore, teachers should have the ability "to select high-quality tasks that allow children to learn the content by figuring out their own strategies and solutions" (p. 13). In this study, Item10.1 examined participants' expertise in selecting appropriate activities for teaching the concept of geometric area to a heterogeneous class (see Figure 5.2 for activity). In this item, the participants were required to identify, with justification, appropriate activities they would use to teach the concept. As they responded to the item, the participants revealed their conceptions of activities they considered appropriate for teaching geometric area conceptually. I coded the participants' responses considering their rationale for choosing the activities. Participants attuned towards teaching geometry through the problem solving approach would justify their choices considering the challenge the activity would pose to their learners. In addition, they would consider the prerequisite knowledge needed to solve that challenge. As they justified their selection of activities, participants revealed their conceptions of teaching area of polygons (Sakshaug & Wohlhuter, 2010).

The analysis revealed that most participants mentioned that learners should acquire necessary prerequisite knowledge before progressing to difficult activities. Out of the 34 participants who responded to this item, 22 (65%) mentioned that they would include activity L as it provided the necessary prerequisite knowledge for doing the other activities. Within this group, some participants mentioned that they would include activity N after activity L and reserve activity M for enrichment. The following responses from some participants provided evidence to illustrate this point. P4 mentioned, "*Activity L would be appropriate to put learners into perspective, thereby being able to work the others involving the triangles*". Moreover, P22 stated that activity L and N should be included because "*they are much simpler and can be easily related to the unit square given. M is more complex for lower level of developed learners; it should be included later when L & N has been mastered*". Furthermore, P20 mentioned, "*I can omit activity M and use it as an enrichment activity since it is more challenging even if it is drawn on a grid board*".

The results from this analysis indicated that the participants had the desired knowledge for selecting activities for teaching geometric area. However, only two (6%) participants justified their selection by explicitly mentioning their ability to offer problem solving opportunities for the learners. For instance, P5 stated:

> *I think activity M and N reason being that are more challenging to the teacher as well as the pupils. It makes one to think, read & practice it before giving it to pupils. To pupils it gives them wide opportunity to think & talk, help each other raise confidence to others, and use of their hands sometimes.*

P5's response indicated that he ignored the value of including activity L, which provided prerequisite knowledge necessary for solving activity M, and N. However, he recognised the challenge presented by activities M and N, as he mentioned that it compelled the teacher to prepare in advance for the lesson. Overall, a majority of the participants mentioned appropriate reasons for selecting activities they would include in their lessons on calculating area of polygons drawn in a grid.

5.2.8 Knowledge of selecting appropriate representations for illustrating area of polygons content

Van de Walle et al. (2014) defined a representation as “a kind of tool, such as a diagram, graph, symbol, or manipulative, that expresses a mathematical idea or concept” (p. 21). The literature revealed that teachers should be able to state the advantages and disadvantages of representations they would use to illustrate the concept of area of polygons. Successful teaching of geometry through the problem solving approach requires teachers to select representations with the potential of stimulating critical thinking among learners (Takahashi, 2008). In addition, the activity should prompt learners to restructure their mathematical knowledge towards the intended lesson objectives (Nunokawa, 2005). Data addressing this aspect of the participants’ knowledge came from responses to Item 9.3 in the open-ended question and lesson observation with the two participants.

Item 9.3 was based on activities in Item 9 shown in Figure 5.2. In this item, the participants responded to this question, “What are some of the strengths or limitations of this task? Would you change/adapt this activity? If yes, how would you change/adapt the activity? Why?” I coded the responses to this item focusing on the rationale the participants provided for their choices and whether it was consistent with the problem solving approach or not.

The analysis revealed that a majority of the participants (34%) considered the practical nature of the activities to be a strength that made the activities appropriate for problem solving. The following two responses illustrate this point. P2 mentioned, “*It widens the thinking level of the learner in geometry and the learner will be able to solve problems that he/she may come across*”. In addition, P1 stated, “*The strength of this task is that it provokes the child’s thinking on geometry. It promotes problem solving skills to the kids*”. Interestingly, P5 mentioned that the strength of the activities was that it compelled teachers to prepare in advance for lessons. According to Participant 5, the activities also compelled the teacher to prepare all relevant materials for the lessons. The results of the analysis suggested that the participants had some understanding of the activities appropriate for teaching area of polygons through the problem solving approach.

Concerning the limitations of the activities, the analysis revealed the difficulty in finding the area of the triangle to be a major theme. The participants put forward various reasons why this was a limitation for the activities. Some of the reasons they cited included the following. First, learners might have difficulty in identifying partially full squares. Second, learners might have difficulty in using the area of a triangle formula correctly especially on the scalene triangle. The following responses illustrate these points. "*The tasks have limitations because it becomes difficult for learners to count* $\frac{1}{4}$ *units and* $\frac{1}{2}$ *units*" (P5). P13 stated "*The limitations: 1) Triangle is scalene so* $\frac{1}{2}bh$ *does not apply. 2) Counting of squares not easy since no full squares because we say A= full squares +* $\frac{1}{2}$ *squares*" (P13). According to P13 the area formula for the triangle, $\frac{1}{2}bh$ was not applicable to a scalene triangle. P13 exposed his limited understanding of the concept of finding the area of a scalene using the formula as the formula is applicable. In addition, a majority of the participants regarded the counting of part squares in the triangles as a limitation of the activities. This observation indicated that the participants had traditional conceptions of teaching area of polygons. Teachers with problem solving conceptions of teaching geometry regard the counting of part squares in the triangle as providing a challenge for their learners to solve. This result was inconsistent with the results the participants gave as strengths of the activities. According to Takahashi (2008), when teaching through the problem solving approach, learners should be given activities with a potential of providing them with "the ability to learn something new after they have solved the problem by using their existing knowledge and skills… which is the goal of the lesson" (p11).

The participants suggested adaptations of the activities. However, not all the participants provided their adaptations of the activities. The analysis revealed that most participants suggested that they adapt the activities by introducing the formula of area of a triangle. A majority of the participants (18%) suggested that introducing the formula would simplify the activities. Hence, according to them, it was necessary to define the height and the breadth to enable the learners to use the formula correctly. The participants mentioned two formulas, $\frac{1}{2}\text{bh}$ and number of full squares $+\frac{\text{number of part squares}}{2}$. The following responses from some participants are examples that illustrate this point. "*I would change* *this activity and use the formulas* $\frac{1}{2}$bh *in order to get an accurate answer instead of the square units*" (P26). In addition, P20 mentioned,

> *I could accept activity L and for activity M and N, I could use the concept that is applied when teaching area of irregular shapes like foot, palm of hand which says area of irregular shapes = no of complete squares plus number of incomplete square divided by 2.* (P20)

Furthermore, P17 mentioned, *"I would help the learner understand the formula with the help of a grid"*. Clearly, this data indicated that the participants reported adaptations of the activities promoting traditional instructional approaches.

Two (6%) participants mentioned that they would adapt the activities by adding more triangles and quadrilaterals, while others stated that they would adapt the activity by improving the grid to find the area of the rectangle by counting unit squares more easily. A significant number of participants (15%) misconstrued 'adapt' to mean 'accept'; hence they mentioned that they would adapt this approach in their teaching.

The suggestions provided by the participants for adapting the activities as stated by the majority were consistent with their traditional conceptions of area of polygons instruction as revealed by the analysis in Section 5.1. There was inconsistency between what the participants regarded as strengths of the activities and how they mentioned they would adapt the activities.

5.2.9 Knowledge of area of polygons instructional strategies through the problem solving approach

I present descriptions of the participants' instructional strategies for teaching area of polygons under the following headings, describing teacher's actions at different stages of a lesson: (a) Understand the problem, (b) Devise a plan, (c) Carry out the plan, and (d) Look back/reflection.

I collected data describing the participants' instructional strategies through Item 8.4 in the open-ended questionnaire, and the lesson observation, semi-structured interviews and lesson plan analysis with the two participants. In this section, I begin by presenting results of Item 8.4 of the open-ended questionnaire before describing instructional strategies of the two participants involved in the lesson observation, semi-structured interview and lesson plan analysis.

5.2.9.1 Open-ended questionnaire results

Item 8.4 was based on Item 8 (shown on Figure 5.1) which required the participants to state their instructional strategies/tasks they would use in their next instructional period to improve learner understanding when teaching area of polygons. This item avoided favouring certain instructional strategies. As the participants described their instructional strategies/tasks they would use in the next lesson, they revealed their conceptions of instructional strategies of teaching area of polygons. I coded the participants' responses based on their description of instructional practices and whether it was consistent with principles of the problem solving approach or not. The dominant theme from the participants' (65%) description of their instructional strategies involved participants explaining the concept of finding the area of polygons to the learners. In explaining the concept of finding the area to learners, some participants reported that they would focus on decomposing and recomposing the shapes. In addition, some participants reported that they would draw the shapes on a grid to help them explain the concept. Furthermore, one participant mentioned teacher knowledge, in that the teacher should understand the concept of area and include shapes with measurements outside, using relevant material. The following three responses illustrate this theme.

> *Step 1. Pupils would be reminded of the polygons from triangle to decagon and then be asked to draw them. Step 2. Pupils will be asked to draw some quadrilaterals they know. Step 3. Pupils will then be reminded on how to find the area of a rectangle by multiplying the length by the height. Step 4. Pupils can then find the area of a triangle by dividing the rectangle by two to get two triangles and each triangle's area is half the area of a rectangle. Pupils will then use the concept of multiplying length by height and divide the product by two. Step 5 Pupils will then use the concept of finding the area a right-angle and triangle to find the area of polygon where they will divide a polygon into two to get a rectangle and a triangle.* (P28)
>
> *To find the area of a right-angled or isosceles triangle, I would introduce the concept of finding area of a square and rectangle then use their relationship to find the area of right-angled and isosceles triangles.* (P10)
>
> *The instructional objectives for the next instructional period are: a) develop formulas for the areas of triangle. b) Solve problems by utilising formulas for the areas of triangles, parallelogram…* (P26)

These responses showed that the participants focused on transmitting knowledge of comparing the area of polygons to their learners, a strategy consistent with traditional instructional approaches. In addition, the participants mentioned that the formula was consistent with the mathematical ideas they mentioned in Item 8.1 as learners would use this in comparing the areas of the polygons.

A minority of the participants mentioned problem solving as their instructional strategy for the next lesson. However, as they elaborated on their strategy they revealed elements associated with traditional instructional approaches as illustrated by the response from P8.

> *As a teacher, you need to know that in problem solving you the teacher need to explain the problem to the learner, making sure they understand what is expected of them from the question, thereafter you let them go. You allow the learners to plan for themselves and carry out their plan, then after you check and evaluate what they have done.* (P8)

These findings in general suggested that the participants had limited understanding of teaching area of polygons through the problem solving approach as they described instructional strategies consistent with traditional approaches.

In the next subsection, I present results from the two cases of the two participants from the lesson observations, semi-structured interviews and lesson plan analysis regarding their knowledge or lack of knowledge of area of polygons instructional strategies through the problem solving approach. I compared these results with the open-ended questionnaire results to verify their accuracy.

5.2.9.2 Zane's knowledge of area of polygons instructional strategies through the problem solving approach

At the time of the study, Zane had a primary teacher diploma majoring in Languages and currently, he was pursuing Bachelor of Arts degree in Humanities. He had nine years teaching experience, teaching mathematics from grade 5 to grade 7. Zane was confident about his mathematics teaching as he told me that he has been producing quality results in mathematics in the Swaziland Primary Certificate (SPC) examination every year. At his school, he was the panel leader for mathematics responsible for monitoring the quality of mathematics teaching.

The analysis revealed that the participants preferred teaching geometry through traditional instructional approaches. The participants used the concept of finding the area of rectangle/square in developing the concept of finding the area of other polygons such as triangles. However, in using this relationship, they did not give learners opportunities to find this relationship themselves. The participants explained this relationship and further demonstrated it to the learners.

5.2.9.2.1 Understand the problem

Zane commenced his lesson by telling his class the lesson topic, which was "finding the area of irregular shapes" and asked his learners to define area. After two learners attempted to define area (for example Learner 1: *"Area is the space inside the shape that that that are meant to cover the surface"*) without success, he offered to assist in defining area and defined area as shown in excerpt 1:

> **…** *Area is the number of square units needed to cover a surface. Yes, how many squares are needed to cover a surface, for instance let us look at this class looking at the floor? The floor acts like a surface; let us assume now that we want to put the floor tiles here. How many tiles do we need to cover this surface? The total number of tiles can be area, the number of tiles needed to cover the surface...* (Appendix I).

Excerpt 1 showed that Zane's definition of area was inaccurate, but this definition suggested that he intended to provide learners with a procedural way of finding the areas of the shapes in the lesson. Zane commenced his lesson consistently with his description during the interview. During the interview, Zane mentioned that "… *when I start teaching about area of a triangle, I want the learners first to know what an area is, before we proceed*". (Appendix K, L31-L32).

This interview transcript extract provides evidence that Zane tried to revise prior knowledge in his area of polygons teaching. However, Zane did not revise with his learners the concept of finding the area of rectangles/squares, which they learned, in the previous grade. In addition, Zane did not introduce any problem in his lesson for the learners to examine with the intention to understand it, identifying its key elements.

5.2.9.2.2 Devise a plan

The next step in teaching through the problem solving approach involves allowing learners to use their prior knowledge to develop a plan for solving a problem. In the lesson I observed, Zane did not provide any opportunities for his learners to devise a plan for finding areas of irregular shapes. Instead, he explained the procedure for finding the areas of the irregular shapes to his learners. Except 2 illustrates this point

> *So we can find the area by using the counting method... So we find the area by using the counting method it means you count the number of squares [eh] ... that cover that surface. You get that?* (Appendix I)

This excerpt shows that Zane did not provide opportunities for his learners to apply their prior knowledge in conceiving a plan for finding the area of the irregular shapes, but explained the procedure to his learners. Excerpt 3 provides another episode where Zane denied his learners an opportunity to apply their own knowledge. In this episode, Zane defined the meaning of irregular shapes without eliciting from his learners their definitions. After drawing an irregular shape on a grid board, Zane drew the attention of his learners to the shape and said

> *Let us look at the figure on the board, this one is irregular, it does not have a fixed shape. You get that. Something that is irregular looks like a stone because once you see a stone there is no length there is no breadth. So those shapes are irregular, they have no fixed shape. You get that.* (Appendix I, except 3)

Clearly, this excerpt demonstrated that Zane dominated proceedings in this lesson. He gave his learners the procedure for working out the exercises he gave them.

5.2.9.2.3 Carry out the plan

Zane presented three tasks for the learners to practice using their newly acquired knowledge. He called individual learners to work out the exercises on the chalkboard in turns while the others watched. In all the three tasks, before learners could attempt them, he cautioned them to be careful of the part squares. In addition, he explained the method he expected his learners to use when calculating the areas of the irregular shapes. For instance, in the first task he assigned his learners to calculate the area of the shape shown in Figure 5.3. He explained the procedure as laid out below.

Figure 5.3: Task 1 figure

> **Teacher**: *Yes now there is a mixture of full squares and a half squares [see figure 5.3] So now how can you find the area where by there are full squares and also half squares? Now remember you don't count a square which is not full as one, a full square is regarded as one [drawing full square on chalkboard] this one is a full square but when it is a half square [drawing a half square] now it is not fully. Remember if you are buying bread, a half bread and another half bread. It makes one loaf. Do you get that?*
> **Class**: *Yes*
> **Teacher**: *So now, it means if you put a half and another half it makes one unit. Do you get that*?
> **Class**: *Yes*
> **Teacher**: *So how to find … the area of this figure? Who can tell me? Yes Learner 5*
> **Learner 5**: *You start by counting full squares and then the half* squares.
> **Teacher**: *You start by counting the full squares. Is she correct?*
> **Class**: *Yes*
> **Teacher**: *You start by counting full squares so now let us try with the full squares. So you say full squares plus now how many are the halves*? [Writes full squares + $\frac{1}{2}$ ___ on the chalkboard]. *So now let us start with the full squares who can come and find us or count the number of full squares, starting with the full squares only. Learner 6 can you please come count the full squares, only the full squares*
> **Learner 6**: *one, two, three … twenty-four.*
> **Teacher**: *How many full squares?*
> **Class**: *24 full squares*
> **Teacher**: *Is she correct?*
> **Class**: *Yes* (Appendix I, excerpt 5).

This excerpt illustrated clearly that Zane dominated proceedings in his lesson, which was contrary to practices associated with the problem solving approach. Moreover, excerpt 5 revealed that Zane's instructional goal for teaching the area of irregular shape was towards the learners mastering the algorithm $\mathrm{A} = \text{full squares} + \frac{\text{number of part squares}}{2}$.

In addition, he never provided his learners with materials to use while they were attempting the exercises. However, for each task he assigned his learners, he insisted on giving them the algorithm to use when finding the area of the shapes as illustrated in the next except,

> *Now let us look at this one, this irregular shape, now area is the number of square units needed to cover a surface. Now how many squares are within this figure? Can someone come and count the number of squares, which are within this …* [pointing at a learner 3], *just count* (Appendix I, excerpt 4)

The interview data corroborated this result. During the interview, Zane mentioned that his preferred instructional strategy for teaching area of polygons was the formula, as indicated in the interview transcript extract below.

> *All right, when there is no grid. I give them the formula. So in the area of a triangle, for example – they may be given a height and base. So now, I introduce the, the formula half base times height. Then I show them the base and height, so they must be able to identify base and height while they are given the formula. Children are very smart; they can be in a position to find area using the formula.* (Appendix K, L70-L74)

In the interview transcript extract, Zane mentioned that he gave his learners the formula and showed them the height and base when calculating areas of triangles. In addition, when I asked him to state his preferred instructional strategy resulting in learners calculating the area of polygons easily, he confidently said *"... is to use the formula*" (Appendix J, L86). This data suggested that Zane had traditional conceptions of teaching the area of polygons. In the next excerpt, I present a clear example illustrating that Zane had traditional conceptions of teaching area of polygons. Zane gave the learners four tasks to evaluate if they understood his explanations.

> *Yes this is area and remember that area is the number of square units needed to cover the inside surface so that is area. Let us try the last one so that I see if you all understand what we are talking about. Who wants to try? Who wants to try? Who wants to try? Who wants to try to find the area of this one? Learner 8. Start with the full squares. Let us watch whether she is doing the right thing.* (Appendix I, excerpt 6).

Zane instructed learners to work out the tasks in turns on the chalkboard. However, he did not encourage the learners to explain their working on the chalkboard. After each individual learner had finished working each exercise on the chalkboard, Zane asked the class whether the solution was correct or not. Zane concluded his lesson by emphasising the definition for area as being "the number of square units needed to cover a surface" (Appendix H, except 7). Contrary to what he mentioned during the interview, that he supported learner thinking during the calculation of area by allowing them to work in groups, there was no group work during the lesson observation. He mentioned during the interview that:

> *Yes, I try because when you are a teacher, sometimes you teach a concept and learners tend not to understand what you are teaching. So, if I see that this one doesn't understand, I used to make groups. Believing that learners learn easily from one to another. So if, maybe they don't understand, I try to organise them according to their groups. Then they able to explain to one another then they try*. (Appendix K, L48-L52).

5.2.9.2.4 Looking back/reflection

Zane did not encourage his learners to explain their solutions when they showed their working on the chalkboard, neither did he ask for other possible options for doing the same task. Even when learners had difficulty in computing the area using the algorithm he explained to them prior to working out the tasks, he did not ask them to state their source of difficulty. If a learner presented a correct response to a task, Zane would ask the rest of the learners to clap hands in applauding that learner. The except below provides an example of this type of episode from the lesson illustrating this point,

> **Teacher**: Is she correct? Is she correct?
> **Class**: Yes
> **Teacher**: Yes, she is correct. Now add seventeen plus three point five to find the total area of the shape.
> **Learner 9**: [reluctant to find total area of shape]
> **Teacher**: Now seventeen plus three point five (writing on the chalkboard 17 + 3.5)
> **Learner 9**: 17 + 3.5 = 20.5 square units
> **Teacher**: Yes, she gets 20.5 units. Is she correct?
> **Class**: Yes
> **Teacher**: Yes, let us clap hands for her. (Appendix I, excerpt 7).

As seen from this excerpt, Zane did not probe why L9 was reluctant to find the sum 17 + 3.5. In addition, L9 did not explain her working to the class and Zane did not persuade her to explain her response. As soon as she got the correct answer, Zane instructed the class to clap hands.

The analysis provided evidence demonstrating that Zane had traditional conceptions of teaching area of polygons. His instructional strategies were consistent with the descriptions of the lesson as provided in the textbook.

5.2.9.3 Patrick's knowledge of area of polygons instructional strategies through the problem solving approach

During the period of the study, Patrick had a primary teacher diploma with majors in science and mathematics. He had four years teaching experience at his school, teaching mathematics and science in grade 6 and grade 7. He was also the chairperson of the local cluster of schools responsible for mathematics.

5.2.9.3.1 Understand the problem

Patrick commenced his lesson in a similar way to Zane. In the introduction to the lesson, after writing the lesson preamble including the date, class, subject and topic, he asked the learners to define area. L1 defined area as *"Area is the amount of space covered by a flat object"* (Appendix J, excerpt 1). Patrick reiterated this definition and proceeded to ask his learners what are the units for measuring area

> *Area is the amount of space covered by a flat object or by a two-dimensional object that is area. What else can you say? What is area? What is area? It is a boundary the amount of space in a given boundary yes we have learned about area even in grade 4 even in grade 5 even in grade 6 we have learned about area. [Uhhmm] What are the units for area? What are the units for area? Units for area? Yes.* (Appendix J, excerpt 2)

Seemingly, Patrick wanted to ensure that his learners had a clear understanding of area and the units for measuring area before they could calculate area. This practice was consistent with teachers aiming to teach towards conceptual understanding. In addition, in his first task Patrick instructed his learners to calculate the areas of a square and a rectangle respectively drawn on a grid as shown in Figure 5.4. In this task, he expected his learners to use their prerequisite knowledge, involving calculating the area of area of rectangles and squares by both counting unit squares and the formula.

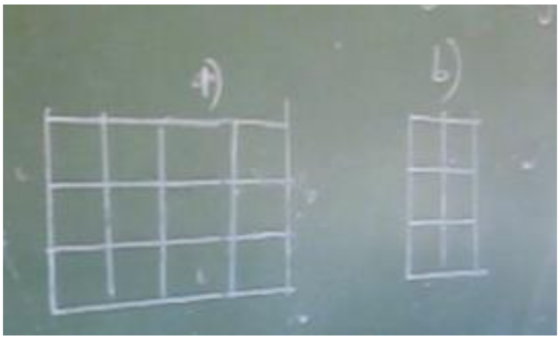


Figure 5.4: Task 1 activity

> *Ok I have got these two shapes of mine here (drawing these shapes on the chalkboard) These are my shapes here.* [Labelling the shapes as a) and b)]. *You may be asked we learned about this in grade 5. How can one find the area of these two shapes eh shape a) and shape b) what would be the area in shape a)? We can count the squares. What would be the area? Learner 3 what would be the area of shape A?* (Appendix J, excerpt 3).

In this activity, Patrick demonstrated that he wanted his learners to retrieve their prior knowledge of area of polygons. However, Patrick spoiled this good work by telling his learners how to calculate the area of the shapes.

> *How can one find the area of these two shapes eh shape a) and shape b) what would be the area in shape a)? We can count the squares. What would be the area? Learner 3 what would be the area of shape A?* (Appendix J, excerpt 4).

As shown in the excerpt, Patrick asked the learners a thought provoking question eliciting learners' ideas on how they could find the area of the shapes, however he did not fully exploit the benefit of the question by answering it himself. He told the learners to find the area of the shapes by counting the unit squares. This is a practice associated with conceptions of traditional instructional approaches.

To introduce the new content, calculating "area of right-angled triangles", Patrick used a teaching/learning aid of a square drawn on a grid as shown in Figure 5.5.

Figure 5.5: Patrick's teaching/learning aid used in introducing area of right-angled triangle

Patrick's introduction of the new content of area of triangles was consistent with the description he gave during the interview. During the interview I asked him to describe how he introduced the concept of area of a triangle. He mentioned that:

> *Area of triangle, learners are to know the area of a rectangle or a square, which is in a form of squares, and they are to count the number of squares, then move on to – Yes. So that... And you might find – so that they may be given, and also may be given the length of the rectangle and the width of the rectangle. That's all I can say about the introduction of the topic. They are to know the length and the width and be able to multiply them.* (Appendix L, L43-L48)

Later during the interview, I rephrased the question and Patrick reiterated that in teaching the area of a triangle, he focused on the area of the rectangle or square, using that concept to develop the concept of finding the area of a triangle. He emphasised that:

> *Now, the greatest thing I have to emphasis on pupils are to know the base which is the length and the height, which is the width of a – they are to know how to calculate the area of a rectangle or a square. If they are not given the...[ehh] whether, they are given the, the squares, they are to count if they are given length. They are to be able to identify the length, they are to identify the length, and they are to be able to identify the width. Then multiply those two, then divide the answer by two.* (Appendix L, L78-L83)

This interview extract and the observation suggested that Patrick had firmly developed conceptions of teaching area of triangles, connecting it to the areas of squares/rectangles. However, Patrick did not fully exploit this connection as he explained this relationship to his learners. He did not provide learners with opportunities to explore this relationship themselves.

5.2.9.3.2 Devise a plan

As I have already mentioned, Patrick did not provide opportunities for learners to devise a plan for finding the area of right-angled triangle. Patrick told the learners the relationship between the square and the right-angled triangle that resulted after cutting the square along the diagonal as illustrated in the excerpt below.

> ***Teacher**: It means that two halves of the triangles make this big square the shape was cut here* [pointing towards the previously drawn square on the chalkboard]. *So it means that if we are given one half of the triangle which is the… if we are given a triangle and we are asked to find the area it is very easy we are going to make this triangle a rectangle. If here, it is 4cm and here it is 5 cm and you can think here if we are to to put it together here, (demonstrating using the two triangles) put it together here. It is now a full triangle and it is six centimetres multiply by six centimetres. One, two, three ... six centimetres multiply by one, two, three ... six centimetres. It is six by six. Six centimetres multiply by six centimetres that why we got the 36 cm^2 for the full square here if we are given. But if it is a triangle you wanted the area of one* [removing one triangle] *we are going two divide the 36 cm^2 by two because this is a half is that clear?*
> ***Class**: Yes*
> ***Teacher**: We then we divide by two and then we got 18 cm^2* (Appendix J, except5).

In the excerpt above, it is clear that Patrick focused on teaching the learners the concept of finding the area of a right-angled triangle by connecting it to the area of a square for procedural understanding. However, this instructional approach was contrary to the instructional approach he mentioned during the interview. In the interview, he mentioned that he taught area of triangles through the discovery method. It may be that Patrick considered showing the learners that the triangle was half a rectangle or square with the same height as discovery approach. If that was the case, it showed poor understanding of the discovery approach on his part.

5.2.9.3.3 Carry out the plan

During the lesson I observed, Patrick gave his learners tasks to practice the knowledge he had taught them. He did not provide any material for the learners to use while attempting the tasks. Moreover, he provided the learners with the procedure for working out the tasks. The following excepts provide episodes illustrating this practice.

> *… If you are given a triangle and a right-angled triangle and you are asked to find the area, make sure that you make it a complete rectangle or a square then multiply the length times the width then you take that after dividing by two….*
> (Appendix J, excerpt 5).
>
> ***Teacher**: What is the first step?*
> ***Class**: You complete the ….*
> ***Teacher**: complete the shape and make it a …*
> ***Class**: complete the shape into a full square*
> ***Teacher**: You complete the shape into a full square or rectangle yes* (Appendix J, excerpt 6).

In both excerpts, Patrick told the learners that in order to find the area of a right-angled triangle they should first complete the shape into a rectangle or a square then use the formula. Learners' participated in the lesson by responding to his questions or completing his statements as shown in excerpt 6. In the lesson, Patrick used questioning to confirm whether learners understood his explanation of procedures. Furthermore, there was no group work witnessed during the lesson. The learners were working individually on the chalkboard. Before working the tasks on the chalkboard, Patrick did not provide learners with relevant materials or worksheets to use while working on the tasks. This data provided evidence suggesting that Patrick had traditional conceptions of teaching area of triangles.

5.2.9.3.4 Looking back/reflection

When teaching through the problem solving approach, in this step, the learners present their solution strategies (Takahashi, 2008). Those learners who are not presenting should examine in detail the solution strategies presented for identifying similarities and differences. However, in Patrick's lesson I observed that the learners were working out the tasks using the method prescribed by the teacher. In addition, Patrick did not encourage his learners to explain their solution strategies to the whole class. As soon as a learner completed working an exercise on the chalkboard, Patrick would ask the rest of the learners to state whether it was correct or wrong. Nevertheless, Patrick presented the solution on behalf of those learners who presented on the chalkboard. Excerpt 7 provided evidence to illustrate this practice,

> *Yes, this is correct. You are to complete this triangle and make it a rectangle or a square then multiply the length times the width. Our length is six, our width is two, two times six is twelve and because we are not talking about the whole triangle the whole rectangle we are talking about a triangle which is the half of the rectangle then we are going to divide this thing (12 cm) by two and get six centimetres…*(Appendix J, excerpt 7).

In addition, Patrick did not encourage his learners to think of other solution strategies, besides ones he had demonstrated. In the interview, Patrick emphasised that;

> *Now, the greatest thing I have to emphasis on pupils are to know the base which is the length and the height, which is the width of a – they are to know how to calculate the area of a rectangle or a square. If they are not given the...[ehh] whether, they are given the, the squares, they are to count if they are given length. They are to be able to identify the length, they are to identify the length, and they are to be able to identify the width. Then multiply those two, then divide the answer by two.* (Appendix L, L78-L83)

From this extract it is evident that Patrick described his instructional strategies similarly to how he practiced in his classroom, emphasising procedural understanding of area of polygons. Evidence from these data sets suggests that Patrick has traditional conceptions of teaching area of polygons.

5.3 FACTORS INFLUENCING CONCEPTIONS OF TEACHING GEOMETRY

To ascertain the factors influencing the participants' conceptions of teaching geometry through the problem solving approach, I used one item in the open-ended questionnaire, that is, Item 11. In the Item the participants responded to the question: "What are the factors that influence your understanding of teaching geometry through the problem solving approach?" Thirty-one participants responded to this item. The analysis revealed that the participants identified various factors. In presenting results involving the factors cited by the participants as influencing their conceptions of teaching geometry through the problem solving approach, I classified them into four sections: the nature of problems, factors related to teacher's knowledge, factors related to the learner, and advantages of teaching through problem solving.

5.3.1 The nature of problems

Some participants (16%) mentioned that the language used in presenting some problems influenced their conceptions of teaching geometry through the problem solving approach. According to the participants, the language affected learners' comprehension of the problem, as most learners were not familiar with the language used in area measurement in geometry. Regarding the language used in some problems, P14 mentioned,

> *One of the factors is the language used. Learners do not understand a difficult language unless simple language is used. Also the vocabulary becomes a factor students are not familiar with vocabulary used in mathematics e.g. base, height. In a problem solving they fail to recognise which side is the base and which is the height. (P14)*

In this response, P14 pointed out that learners fail to recognise the base and height in the shapes. P21 elaborated on the influence the language used in some problems by mentioning that "*the language used in the problem – in order to solve problems you must able to understand the terms and be able to apply their understanding as well as which operation to be used",* yet "*Some learners cannot interpret some of the words" (P16).*

5.3.2 Factors related to the learners' abilities

My analysis revealed three factors related to learners characteristics' mentioned by the participants as influencing their conceptions of teaching geometry through the problem solving approach. First, seven (23%) participants mentioned that the learners' abilities influenced their conceptions of teaching geometry through the problem solving approach. For instance, P28 mentioned that the learners should *"have to be able to visualise, give some reasons, estimate and be able to talk to each other"* (P28). P20 shared the same sentiments as P28 concerning learners' ability to visualise by mentioning, "*It [learners] is the level of the learners to think abstractly…"* (P20) [my own emphasis]. Apart from the learners' ability to think abstractly, P15 mentioned, "m*ost learners are not critical thinkers they want to be spoon fed all the time* (P15).

The second factor raised by the participants (10%) concerned the learners understanding of area. According to the participants, learners have difficulties understanding the concept of area. P7 provided evidence to illustrate this point, "*Learners find it hard to understand the meaning of area".* Another participant, P8 mentioned,

> *Also some don't understand that you can use the same length and width of a given shape to formulate different shape. The same length and width of any shape give the same area of any different kind of shape. Its not about the shape of the figure but the dimensions they have.* (P8)

P8 raised an important point concerning the conservation of area, a concept important in decomposing and recomposing shapes, but his justification was inaccurate revealing his lack of understanding of area.

The third factor, mentioned by three (10%) participants related to the learners' attitude. According to the participants, some learners have a negative attitude towards mathematics. The next response from P7 provided evidence to illustrate this point, "*Some learners have negative attitude towards mathematics* (P7). Another participant mentioned that some "l*earners are very lazy to read…"* (P16), and hence this attitude influenced their conceptions of teaching geometry through the problem solving approach. P22 raised a different dimension regarding learners' attitude. According to P22, "*Pupils who can't find answers on their own without being guided can be demotivated thus leading to hatred of the subject as a whole".*

5.3.3 Factors related to teacher's knowledge

My analysis revealed that some participants (19%) mentioned some factors related to teacher's knowledge. According to these participants, teaching geometry through the problem solving approach required thorough preparation from the teacher, which entailed understanding the content to be taught and effective instructional approaches for teaching that content. One participant stated:

> *To teach area of polygons need the teacher to first understand what he/she is going to teach. Be well prepared before going to teach the concept. Bring relevant teaching aids and teaching materials. Choose relevant teaching method (learner-centred). When teaching or during instructional time, let the pupils discover for themselves the outcomes of the problem*. (P32)

P34 emphasised this point but added that the teacher should use relevant teaching/learning aids, saying: "*Relevant teaching/learning aids should be utilised. It challenges the teacher to do thoroughly research".* In addition, P26 mentioned, "*mastering the formula and understanding when to apply the formula"*. However, these participants did not mention how the need for thorough preparation influenced their conceptions of teaching geometry through the problem solving approach. Again, another participant revealed a different dimension of teacher knowledge that involved knowing the learner characteristics. P31 mentioned, "*The teacher must know the weak part of the learners. The teacher must prepare where the learner is weak. Use teaching aid that will make the learn clear"*. According to P31, knowing the learner's strengths and weaknesses would allow the teacher to use materials appropriate to increase the learners' understanding of the content.

Four (12%) participants suggested instructional approaches they considered effective for teaching area of polygons through the problem solving approach. According to P30, "*The pupils have to manipulate and see the polygons in order to find their areas".* In addition, P9 stated:

> *Pupils must be given the task to work out on their own first. Teachers must allow their different opinions/solutions and have them discussed. Most pupils have hardships when it comes problem solving, so teacher must encourage pupils to read and understand the problem first before attempting it. pupils must not be taught a method of finding of triangle but they shall use whatever ways only it will give a correct answer.* (P9)

Again, these participants did not mention how these instructional approaches influenced their conceptions of teaching geometry through the problem solving approach. However, P24 recognised that problem solving promoted multiple solution strategies, but advised that it should not be used in geometry instruction.

> *Problem solving come with different strategies to solve a problem so in order for a teacher to make sure that the pupils follow the concept or geometry a routine strategy is advisable so that the learners will understand every steps taken to complete that problem.(P24)*

This response indicated that the participant understood some of characteristics of problem solving, but was against integrating this approach in his teaching because he focused on procedural goals for geometry learning, focusing on the steps for completing the task.

P17 provided a unique response by mentioning that most teachers avoided teaching problem solving lessons because they lacked knowledge. P17 mentioned, "*Learners are told to skip this topic because of teachers not understanding it or teachers fail to teach"*. This participant highlighted that lack of sufficient knowledge of problem solving instruction influenced teachers' conceptions of teaching geometry through the problem solving approach.

5.3.4 School factors

In addition to factors related to the nature of the problems, factors related to the learner, and factors related to teacher's knowledge, the analysis revealed a theme category of factors related to the school (16%). The analysis revealed three themes related to school factors, namely, class size, time allocated to problem solving and availability of teaching materials. P15 mentioned class size and time allocated to problem solving, saying "*The number of pupils in the class, time frame given to the topic problem solving"* (P15). P5 put it succinctly stating how time constraints influenced his conceptions of teaching geometry through the problem solving approach, "…*Time allocated to the subject as a whole, e.g. we cut work to finish the lesson"* (P5). According to P5, the time constraints forced teachers to omit some of the content in order to finish the lesson within the stipulated time.

5.3.5 Advantages of teaching through problem solving

Instead of stating the factors affecting their conceptions of teaching geometry through the problem solving approach, some participants (29%) mentioned the advantages of problem solving. I considered these factors because they revealed the participants' conceptions of teaching through the problem solving approach. I analysed the advantages mentioned by the participants using open coding. The analysis revealed three theme categories: increasing learner understanding, increasing abstract thinking of learners and increasing knowledge retaining by learners. According to P6, "*The problem solving approach is the best method because it makes the pupils to stretch their thinking a little bit. The problem solving approach moves the pupils from the box and they think abstractly"*. In addition, P2 mentioned, "*The pupils are able to develop their skills and that it provoke their thinking"*. Clearly, these participants considered the problem solving approach as a superior method for improving learners' critical thinking skills.

The second theme factor mentioned by the participants related to the problem solving approach increasing the learners understanding. P4 mentioned, "*Learners can easily understand the practicality of the problem solving approach than the general operational method"* (P4).

The third theme factor, the participants mentioned that learners retained more knowledge acquired through the problem solving approach. The following two responses provided evidence to illustrate this point, "*One factor is that the learner once understood* *the concept, does not forget. The pupil is able to adopt thinking skills"* (P3). P11 on the other hand mentioned, *"Once the learners have hands-on experience, they do not forget the lesson"*. These participants regarded problem solving as an important instructional approach useful in helping learners retain their mathematical knowledge for longer.

The analysis revealed four theme categories as factors influencing the participants' conceptions of teaching geometry through the problem solving approach, the language used in the problem, factors related to the learner, factors related to teacher's knowledge and advantages of teaching through the problem solving approach.

5.4 SUMMARY

In summary, in this chapter I presented the results in two sections, conceptions of geometry teaching, and factors influencing their conceptions according to the research questions. Overall, the results of this study showed that the participants had traditional conceptions of teaching geometry, as revealed by their emphasis on procedural goals on most of the aspects of their MKT.

A majority of the participants (65%) identified important ideas focusing on procedural teaching of area of polygons, such as the formula, counting unit squares and decomposing and recomposing aimed at using the formula easily.

Most participants reported that learners would face difficulties in learning area of triangles. According to the participants, the learners' difficulties would emanate from the difficulty in using the formula and counting the part squares in the triangles. Mostly, these difficulties indicated that the participants focused on teaching area of the polygons towards procedural understanding.

All the participants (100%) failed to identify the misconception associated with decomposing and recomposing the triangle in Item 8.3 in the open-ended questionnaire. This result indicates that the participants had insufficient knowledge of decomposing and recomposing shapes. This result was consistent with the one related to their knowledge of important mathematical ideas learners would use in comparing the two shapes, where they reported ideas promoting procedural goals.

The results indicated that a majority of the participants did not recognise the importance of teacher's knowledge base in evaluating learners' responses. Instead of giving an accurate analysis of why Ms. Wilson failed to evaluate the correctness of the groups' solutions, they explained how the groups obtained their solutions giving inaccurate explanations in the process.

The results showed that the participants recognised the aims for learning area of polygons. According to the participants, the main aim for learning area of polygons was solving daily life problems. However, the participants failed to identify aims related to laying the foundation for further study in geometry and the introducing of other concepts in mathematics. Again, this result was consistent with the one related to important mathematical ideas in the learning of area of polygons. These results suggested that the participants focused on teaching area of polygons towards procedural goals, as they did not focus in connecting the other concepts in mathematics, and other subjects.

The results showed that the participants would sequence the area of polygons sequence from simple to complex. However, these results did not provide conclusive evidence to whether the participants focused on procedural goals or conceptual goals. In both conceptions, the ability to sequence content is very important.

The results showed that most participants (65%) selected activities based on their potential of providing prerequisite knowledge for doing other activities in a lesson. Therefore, the participants had the desired conceptions of selecting activities suitable for teaching geometry through the problem solving approach.

The results showed there was inconsistency between the rationale they gave for the strengths of the activities and the rationale for adapting them. A major result in this aspect of MKT was that the participants reported they would introduce the formula to simplify the activities, an action typifying traditional conceptions of teaching area of polygons.

The results for instructional strategies for teaching area of polygons indicated that the participants had strong traditional conceptions of teaching area of polygons. All the data revealed a consistent traditional conception of teaching area of polygons. In the open-ended questionnaire, most participants (65%) described instructional strategies that involved explaining the concept of comparing the area of the triangle and the parallelogram to the learners. In addition, during the lesson observation, I observed that both teachers dominated the proceedings of their lessons, prescribing procedures for finding the areas of the figures to their learners. First, they did not use well-structured tasks when introducing the new concept to their learners. Secondly, they did not allow opportunities for learners to examine in detail the relationship between their existing knowledge and the new task. Third, they did not allow their learners to devise a plan for finding the area of the new shapes. Fourth, they did not use group work and did not provide relevant materials for learners to use while working through tasks. Lastly, they neither promoted multiple solution strategies nor encouraged their learners to explain their solution procedures to the class.

Clearly, these results indicated that the participants had limited understanding of teaching geometry through the problem solving approach.

The results addressing the second research question revealed that the participants reported four factors as influencing their conceptions of teaching geometry through the problem solving approach: the language used in the problem, factors related to the learner, factors related to teacher's knowledge and factors related to the school. However, some participants reported the advantages of teaching geometry through the problem solving approach as a factor influencing their conceptions.

In the next chapter, I present the discussion of the results, conclusion, recommendations, contributions and my autobiographical reflection.

CHAPTER 6 : DISCUSSION OF RESULTS, CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

In this study, I extended the understanding of the different aspects of MKT by examining teachers understanding of teaching geometry through the problem solving approach. I pursued a different goal from previous studies, by interrogating the participants' knowledge in the various aspects of Ball et al.'s (2008) MKT framework focusing on the KCS and KCT of teaching area of polygons through the problem solving approach. In addition to the MKT framework, I integrated Takashi's (2008) problem solving approach framework for a holistic examination of teachers' understanding of instruction through the problem solving approach.

In this chapter, I present a discussion of the results of my study. In this study, I argue that teachers' actions in the classroom depend on their conceptions of effective mathematics instruction (Nunokawa, 2005). I found that the participants had limited understanding of teaching geometry through the problem solving approach. In particular, the participants identified important mathematical ideas through focusing on procedural goals for teaching geometry. The participants identified the nature of problems, teacher's knowledge, the learners' characteristics and advantages of teaching through problem solving as factors that influence their conceptions of teaching geometry through the problem solving approach. In the next section I present the results in two sections; conceptions of geometry instructions and factors influencing the participants' conceptions of geometry instruction. It was apparent from the results related to the different aspects of MKT that the participants' CK of area of polygons was incoherent.

In the next section I present a discussion of the results of my study.

6.2 CONCEPTIONS OF GEOMETRY TEACHING

In this section I present a discussion of the results. I present the discussion of results addressing my first research question under the headings: knowledge of important ideas, knowledge of articulating learners' difficulties, knowledge of learners' emerging and incomplete ideas, knowledge of recognising and articulating learners' misconceptions, knowledge of sequencing content, knowledge of selecting appropriate activities for teaching area of polygons, knowledge of the aims for learning the content, and knowledge of area of polygons instruction through the problem solving approach (Ball et al., 2008). As I discuss the results I critique them in terms of practices associated with teaching area of polygons through the problem solving approach as described in Section 3.4 and the observation schedule (Appendix G). Overall, the results of my study revealed that participants had traditional conceptions, predominantly focusing on teaching area of polygons towards procedural understanding.

6.2.1 Knowledge of important mathematical ideas in area of polygons

The results showed that most participants identified important mathematical ideas associated with traditional conceptions of teaching area of polygons. Most participants identified mathematical ideas, which involved following procedures such as area formula and counting unit squares. In Item 8.1 of the open-ended questionnaire I asked the participants to state important mathematical ideas learners might use to compare the area of the triangle and a parallelogram both with the same height. The activity required the participants to demonstrate conceptual understanding of recomposing and decomposing polygons. Therefore, in this item the participants were expected to state the important ideas learners would use in order to decompose and recompose the triangle into the parallelogram correctly. The item required the learners to compare the areas of the two shapes at the concrete level using the ideas of congruence and equivalence (Manizade & Mason, 2011). After successfully recomposing and decomposing of the triangle, participants had to decide whether the shapes had equal areas or not; direct comparison could be used. Quantification of the areas was not required. The results showed that most had the strong intention of comparing the areas of the two shapes by first quantifying its area using a formula or by counting unit squares. Besides, in the context of the study the curriculum did not require learners to know the area formula of both triangle and parallelogram. As Zacharos (2006) points out, "the combination of lengths in formulas which contain multiplication is not actually meaningful in the context of area measurement" (p. 234). In particular, the early arithmetisation of area contributes to most learners' difficulties in area of polygons (Cavanagh, 2008). According to Huang, and Witz

(2013), mastering formulas in area measurements is not equivalent to conceptual understanding; hence, some learners cannot equate the product to area concept.

Apart from mastering the procedures for computing area, learners "should also know why the sequence of steps in the computation makes sense" (Ma, 1999, p 108). Stephan and Clements (2003) identified four important mathematical ideas involved in learning area of shapes, "(1) partitioning, (2) unit iteration, (3) conservation, and (4) structuring an array" (p. 10). In addition, Huang, and Witz (2013) differentiated the important ideas essential for learning area into two, namely, mastering the concept of area and mastering the concept of area measurement. Huang and Witz (2013) extended this set of ideas by adding the "acquisition of shapes, measure, and computation of measure" (p. 11). Conservation of area is important because if learners can conserve area they can understand conceptually the process of decomposition and recomposition of shapes (Stephan & Clements, 2004). In addition, they stand a better chance of connecting the area of the other polygons to the area formula of rectangle/square (Zacharos, 2006). Focusing on these ideas enhances learners' understanding of the underlying principles of area (Cavanagh, 2008). Unfortunately, none of the participants mentioned these ideas in this study.

From this result, it is apparent that the participants had procedural understanding of area of polygons. Anderson and Hoffmeister (2007) assert that in order for teachers to teach towards conceptual understanding they should also have conceptual understanding of the content. Teachers with conceptual understanding of the concept approach geometry instruction in context, connecting geometric concepts to other mathematical concepts (Swafford et al., 1997). In addition, Shulman (1987, p14) argues that teachers should not just understand but "comprehend critically the set of ideas to be taught" (p. 14). According to Shulman (1987), comprehending the set of ideas critically means understanding it in multiple ways, including its connection to other ideas within the subject and other subjects in the curriculum. Hiebert and Carpenter (1992) explain that a mathematical idea is understood deeply "if it is linked to existing networks with stronger or more numerous connections" (p. 67). Lessons through the problem solving approach commences with the teacher presenting a problem to the learners who have the ability of connecting known and unknown knowledge (Nunokawa, 2005). Baturo and Nason (1996) provided a sensible explanation of the results. According to Baturo and Nason (1996), teachers emphasising algorithmic processes of finding area have diminished knowledge of area measurement.

Thus, they are more likely to experience challenges in their teaching of connecting area of polygons to other concepts in the mathematics curriculum. These results resonate well with results from earlier studies (Steele, 2013).

6.2.2 Knowledge of articulating learners' difficulties in area of polygons

According to Ball et al. (2008), teachers should determine in advance when assigning a task whether learners are likely to find it easy or difficult. Silverman and Thompson (2008) explain that a teacher intending to teach towards conceptual understanding envisions how best a learner could meaningfully learn the content. This encompasses thinking about the content, visualising the learner persevering through the content, succeeding in some aspects and failing in some. In doing so, teachers draw from their own understanding of the content and the knowledge of their learners' capabilities. The result in this study in this aspect of the participants' knowledge provided further evidence that the participants had procedural understanding of area of polygons content. Most participants reported that learners would find it difficult calculating the area of triangles using the formula and counting unit squares. Baturo and Nason (1996) referred to these difficulties as insignificant issues when focusing on teaching area conceptually. According to Cavanagh (2008), most learners' difficulties in area tasks emanate from their lack of conceptual understanding of the connection between areas of rectangles and triangles.

The planning and conducting of productive area of polygons lessons required that the participants demonstrate deep understanding of the difficulties their learners would face. This assists the teacher in preparing appropriate instructional experiences for learners. In this study, the participants mentioned challenges related to adding and subtracting areas of the shapes, indicating that their teaching goal was towards procedural understanding. The area of the triangle provides an opportunity for the learners to arrange the triangles into areas of squares or rectangles. The main goal for teaching area of polygons through the problem solving approach is to let learners experience these challenges and help them develop means of overcoming them (Charles, 2009).

6.2.3 Knowledge of interpreting learners emerging and incomplete ideas in area of polygons

According to Ball et al. (2008), teachers should be able to interpret emerging and incomplete learners' ideas in specific topics, as they face different solutions during instruction. Teachers can rely on this aspect of knowledge during instruction to determine learners' online thinking, whether their thinking is consistent with the demands of the tasks or at least in the right direction, thus providing formative feedback to learners. Apart from interpreting learners emerging and incomplete ideas during instruction, "a teacher must interpret students' written work, analyse their reasoning, and respond to the different methods they might use in solving a problem" (Kilpatrick, Swafford & Findell, 2001, p. 370). Jacobs and Empson (2016) conceptualise this teachers' ability as responsive teaching, explaining that it is important because it enables teachers to make informed prompt decisions about "what to pursue and how to pursue it" during teaching (p. 185). This aspect of teacher knowledge is particularly significant when considering that the problem solving approach promotes multiple solution strategies (Takahashi, 2008).

The results in this aspect of the participants' MKT in my study showed that the participants failed to identify the importance of their own knowledge in evaluating learners' response. Instead of explaining why Ms. Wilson was unable to evaluate the groups' responses, they inaccurately described the solution strategies used. This result was more critical when considering the teachers' ability to be responsive during teaching (Jacobs & Empson, 2016). So far, these results pointed towards the direction that the participants had procedural understanding of area of polygons. In addition, this result had a bearing on the participants' ability to conduct instruction through the problem solving approach as instruction through the problem solving approach demands that the teacher facilitates learners into organising their solution strategies, assisting them to refine them in order to master the lesson's objectives (Takahashi, 2008). Ball et al. (2008) argue that teachers should possess knowledge of relevant algorithms for clarifying and justifying to the learner the suitability of these algorithms in specific cases, among other reasons.

6.2.4 Knowledge of recognising and articulating learners' misconceptions in area of polygons

In the beginning of this study, I assumed that teaching geometry through the problem solving approach would assist in minimising learners' misconceptions thus improving their performance. A first step towards addressing learners' misconceptions in area of polygons is awareness of them. Therefore, teachers' ability to identify and articulate learners' misconceptions in area of polygons formed a key component of my study. During instruction, the teacher should create learning opportunities that can help learners overcome their misconceptions. In addressing learners' misconceptions in area of polygons, teachers rely on their own understanding of area of polygons content, but this becomes impossible if the teacher has weak CK (Jadama, 2014). The results in this study showed that all the participants failed to identify the misconceptions associated with decomposing and recomposing the triangle in activity 8 (see Figure 5.1). These results were not surprising considering that the participants had already demonstrated their procedural understanding of area of polygons. Ball et al. (2008) considered the ability to recognise and describe common misconceptions in a specific topic as a key aspect of teachers' knowledge.

Being able to recognise and describe learners' misconceptions in area of polygons negatively affects teachers' ability to choose appropriate learning activities (Jadama, 2014). In addition, learners may apply this faulty knowledge during problem solving (Schoenfeld, 1992). Teachers should promptly recognise learners' misconceptions in area of polygons in order to design instructional experiences with the potential of helping learners overcome these misconceptions (Takahashi, 2008). In fact, the main goal of instruction through the problem solving approach is assigning of tasks to learners with the potential to reveal most of their misconceptions in a specific topic.

According to Cavanagh (2008), most learners' misconception in areas of polygons result from learners' failing to comprehend the relationship between the area of a triangle being equal to half the area of a rectangle with the same base and perpendicular height. Learners lacked the conceptual understanding of the connection between areas of rectangles and triangles Cavanagh (2008).

6.2.5 Knowledge of sequencing area of polygons content

An important aspect of teacher knowledge is their ability to sequence content for effective instruction. In this study, almost all the participants (88%) mentioned that they would sequence the activities from simple to complex. The rationale they provided for sequencing the activities did not suggest that they focused on providing problem solving opportunities for their learners. Teaching through the problem solving approach required teachers to sequence lesson content in such a way that learners link their existing knowledge to the new content (Takahashi, 2008). The goal is for learners to learn new content in a connected manner.

6.2.6 Knowledge of selecting appropriate activities for teaching area of polygons

According to Lester (2013), effective teaching of area of polygons through the problem solving approach requires the teacher to have the capability of designing and selecting appropriate tasks. The tasks used by a teacher are central in determining the outcomes of instruction, especially "the level of challenge of those tasks determines learning opportunities for students" (Wilhelm, 2014, p. 636). According to Takahashi (2008) tasks appropriate for teaching through the problem solving approach should have multiple solution strategies at different cognitive levels. In addition, they should provide learners with the opportunity to engage in productive struggle with their mathematical ideas (Van de Walle et al., 2014). Moreover, these tasks should allow the learners to appreciate and recognise the connection between the new knowledge and their prior knowledge (Takahashi, 2008). Furthermore, these tasks should allow the learners to discover the need or advantage of their newly acquired knowledge and the deficiencies of their prior knowledge in solving the problem (Nunokawa, 2005). Ball et al. (2008) argue that in order to select appropriate activities for specific classes, teachers need to be knowledgeable about both the content and their learners' abilities. According to Ma (1999), problems with multiple solution strategies function as a link, assisting learners in connecting diverse mathematical ideas.

The results in my study showed that most participants (65%) indicated that they would include activity L, because it would be straightforward for their learners. The participants' reasoning behind this selection indicated that they lacked an understanding of the nature of the tasks supporting instruction through the problem solving approach.

According to Baumert et al. (2010), a challenging activity spurs learners to use their experiences when attending to the problem, thus encouraging them to make connections between their existing knowledge and new knowledge. Thus, Polya (1945) stressed that teachers should select the problems cautiously at their learners' cognitive level.

6.2.7 Knowledge of selecting appropriate representations for illustrating area of polygons

According to Ball et al. (2008), teachers should have knowledge that allows them to appraise the "instructional advantages and disadvantages of representations used to teach a specific idea" (p. 401). Moreover, Cai and Lester (2005, p. 221) assert that representations used by the teacher during a lesson affect the representations used by their learners during problem solving. In this study, I was particularly interested in external representations used by the teacher in presenting an activity to the learners. In teaching area of polygons through the problem solving approach, representations provide learners with things they can "explore, reason, and communicate as they engage in problem-based tasks" (Van de Walle et al., 2014, p. 21). The results of my study showed that most participants suggested adaptations that could simplify the activities for their learners, such as introducing the formula of area of a triangle. However, focusing on eliminating the challenge in the activities and focusing on direct instruction reveals that the participants had weak knowledge of activities supporting instruction through the problem solving approach (Sakshaug & Wohlhuter, 2010). Thus, this result was consistent with the participants' procedural knowledge of area of polygons as revealed by their lack of knowledge of important ideas in area of polygons. There was inconsistency between what the participants regarded as strengths of the activities and how they mentioned they would adapt the activities (Sakshaug & Wohlhuter, 2010).

6.2.8 Knowledge of the aims for learning area of polygons

Teachers should be conversant with the aims for learning specific content as that influences the classroom climate they prepare for their learners (Lester, 2013). According to Ball et al. (2008, p. 401), teachers make instructional decisions about "which student contributions to pursue and which to ignore or save for a later time" drawing from this knowledge. Therefore, the learning environment reflects the teachers' conceptions about goals for learning that content (Schoenfeld, 1992). In this study, the results showed that the participants reported one aim for learning area of polygons, namely, solving daily life problems. However, they did not consider many other productive aims for learning area of polygons in primary school, such as introducing other mathematical concepts (Cavanagh, 2008). One possible explanation for this result was that the participants did not recognise the need for teaching mathematics as a connected body of knowledge. This typifies traditional conceptions of teaching mathematics (Anderson et al., 2004). This was true when considering their instructional strategies as it focused on procedural understanding of area of polygons. According to Schoenfeld (2012), the broad aim for learning mathematics is to develop mathematically thinking learners, describing 'mathematically thinking' as conceiving mathematics as a sense-making activity.

6.2.9 Knowledge of area of polygons instructional strategies through the problem solving approach

In this study I argue that teaching geometry through the problem solving approach can help learners develop conceptual understanding. I have based my argument on Schoenfeld's (2012) assertion that the problem solving approach focuses on teaching mathematics concepts towards conceptual understanding. In this approach, learners use their existing knowledge in understanding tasks, formulating strategies for solving the tasks, solving the tasks using their own strategies and presenting solutions to tasks to the class for discussion (Takahashi, 2008). However, for teachers to integrate this instructional approach into their daily teaching routines they need conceptual understanding of the content and conceptions aligned with teaching through the problem solving approach (Lui & Bonner, 2016). In this study, it appeared from the questionnaire results that the participants had procedural knowledge in many aspects of the MKT related to area of polygons examined.

The classroom observations and semi-structured interviews provided opportunities to verify the open-ended questionnaire results, as I was able to witness the participants' actual instructional strategies for area of polygons. The instructional strategies I observed during the lesson observations with the two participants, and the descriptions of the participants' instructional strategies provided by them in the open-ended questionnaire results, were consistent. Notwithstanding the limitation in the number of lesson observations, from the data in this study it is apparent that the participants had traditional conceptions of teaching area of polygons. The open-ended questionnaire results indicated that the participants described instructional strategies focusing on explaining the procedures for finding the area of the shapes consistent with traditional instructional approaches to teaching area of polygons (Cavanagh, 2008). These results were consistent with research on the influence of teacher's understanding of content and instructional practice indicating that teachers with procedural understanding tend to teach through traditional instructional approaches (Lui & Bonner, 2016).

Instruction through the problem solving approach focuses in engaging learners in classroom activities promoting problem solving (Schoenfeld, 2013). In this study, I concentrated my attention on the following four practices related to teaching geometry through the problem solving approach as described by Takahashi (2008): understanding the task, devising a plan to solve the task, solving the task using the devised plan, and looking back/reflection. As discussed at the beginning of this study, I focused on these practices because in the context of my study the participants had already taught most of Polya's (1945) problem solving phases and heuristics. During the observation, I could not witness any of the classroom practices associated with teaching through the problem solving approach as described by Takahashi (2008).

In my study, the results indicated that both participants confirmed learners' prior knowledge to varying degrees. Zane revised the definition of area only and did not revise the concept of finding the area of rectangles and squares yet this was important prerequisite knowledge in his lesson. He defined area as "*the number of square units needed to cover a surface".* This definition provided evidence suggesting that he focused on teaching area of polygons towards procedural understanding. Patrick revised most of the relevant concepts related to his lesson with his learners. He did this at various stages of the lesson. In the introduction, he asked his learners to define area and they did so correctly. He further gave his learners tasks requiring them to use their existing knowledge of calculating area of squares and rectangles drawn on a grid and using the formula respectively. However, he did not allow his learners' to use this knowledge in understanding the tasks for the lesson. One possible explanation was that Zane had weak CK of area, as he was a languages specialist. Patrick on the other hand specialised on mathematics in his training; therefore, his CK was more coherent than Zane. I argue this way because teachers draw from their own conceptions of the content in their classroom practices (Cai, 2007).

The results show that both participants, in presenting the new content, did not present well-defined tasks. Teaching through problem solving commences with the teacher presenting a challenging task(s) compelling learners to use their prior knowledge in understanding the problem (Takahashi, 2008). In addition, the task(s) should challenge the learners to reorganise their knowledge and create the need for new knowledge. Both participants presented tasks for aiding their explanations; hence, they did not give their learners opportunities to explore the tasks using their existing knowledge. According to Nunokawa (2005), as learners explore the task, they restructure their knowledge linking the known and the unknown knowledge. Therefore, the teacher should give the learners enough time to explore the task, identifying the key words or phrases, the given information, the question and the circumstance(s) under which it will be solved (Polya, 1945). Both participants missed this important step. Patrick did probe learners' prior knowledge before introducing new concepts, for example before finding the area of the triangle he asked the class to state the properties of the triangle they knew, but, like Zane, he did not use it for learners to understand the new knowledge. One possible explanation for their action was that they had firmly developed traditional instructional conceptions.

Developing multiple solution strategies for solving a task is a critical step in instruction in the problem solving approach (Schoenfeld, 2012). The results showed that the participants, instead of providing opportunities for their learners to develop plans for working out the tasks they assigned, explained the procedures themselves. According to Roh (2003, p. 1), allowing learners to develop their own plans for solving tasks provides them with opportunities to connect "their conceptual knowledge with their procedural skill". Polya (1945) asserts that for this step to be successful, the teacher should have conceptual understanding of the content in order to offer, without interfering, appropriate hints to the learner as they rely on their existing knowledge. Leaners should predict/hypothesise/suggest solution strategies to the problem (Takahashi, 2008). The participants' practice of explaining procedures of solving the tasks to their learners was congruent with traditional conceptions of teaching area of polygons (Lui & Bonner, 2016). This result suggests that the participants' strong traditional conceptions of area of polygons instruction prevented them from recognising the value of letting their learners develop as many plans as possible for solving tasks.

Consistent with their strong traditional conceptions of teaching area of polygons, the participants did not provide their learners with autonomy to work out the tasks using their preferred strategies. According to Wheatley (1991), allowing learners the autonomy to use their own preferred solution strategies accommodates their different abilities, a major advantage of this approach. Even if learners use naïve solution strategies, the problem solving approach provides opportunities for them to refine these strategies during class discussions (Takahashi, 2008). The participants did not provide their learners with relevant tools that could assist them to execute their plan and gain more knowledge about the problem. These results were contrary to Daher and Jaber's (2010) study who found that teachers valued the contribution of manipulatives to the success of their geometry instruction strategies. Schoenfeld (2012) supported the use of manipulatives during instruction through the problem solving approach, arguing, "Doing mathematics means being actively engaged (as mathematicians are!) – making conjectures, exploring issues, seeing what makes sense. And it means doing so with an expanded and appropriate set of resources" (p. 597). Similarly, Zacharos (2006) found that the nature of manipulatives made available to learners in area measurement activities influenced their conceptualisation of the concept.

The participants did not allow the learners to work in pairs or groups while attempting the tasks. The major strength of teaching through the problem solving approach is allowing the learners to participate actively in small groups in finding solutions to tasks, which promotes deep understanding of the concepts (Ing et al. 2015). Moreover, while explaining and justifying their views in small groups, the learners' level of motivation increased (Wheatley, 1991). Both participants did not move around class monitoring each learner's progress while they attempted the tasks they assigned, assisting learners where necessary. Jacobs and Empson (2016) described this practice as an important aspect of the problem solving approach as it provides the teacher with an opportunity to attend to each learner's thinking, stretching it through probing questions. It also allows the teacher to identify unique ideas from the learners for discussion. The failure for both participants to monitor each learner as they tried the tasks resulted in them calling learners to work out tasks on the chalkboard without seeing their solution strategies in advance. One possible explanation for this observation was that the participants had limited knowledge of instruction through the problem solving approach. It was at this step that teachers probe learners about their solution strategies. Teachers use their own understanding of the content to push learners thinking in working out the problems, instead of providing them with readymade solutions, so that they have to scaffold their thinking (Lui & Bonner, 2016). This result was not surprising as it is consistent with the finding that the participants lacked conceptual understanding of area of polygons. One plausible explanation was that the participants themselves lacked conceptual understanding.

While working on the tasks on the chalkboard, both participants did not encourage their learners to think of different solution strategies. According to Ma (1999), when learners solve a problem, using different strategies allows them to connect various mathematical ideas. The results in this test suggest that the participants believed there was only one correct way of solving a mathematical task, consistent with traditional instructional conceptions.

After solving a problem, Takahashi (2008) described looking back/reflection as an important practise of the problem solving approach. A major aspect of looking back/reflection encompasses "overseeing the entire solution process … choosing the most appropriate strategy, determining whether sub-goals have been met, assessing whether the chosen strategy leads towards the final solution, and making sure that one answers the question being asked" (Donaldson, 2011, p. 92). Moreover, this step allows the learners to recognise their own misconceptions and mistakes, thus improving their solution strategies (Polya, 1945). The results of my study showed that the participants did not encourage their learners to reflect back on their solution strategies. In addition, they did not encourage their learners to explain their solutions while presenting them on the chalkboard. A possible explanation for not allowing the learners to reflect on their solution strategies is that the teachers themselves did not believe that it was important to reflect on their own solution strategies. Both participants moved on to the next task as soon as learners presented the correct solution. Schoenfeld (2012) criticised this teacher action because it tends to breed learners focusing on "answer getting" at the expense of mathematical sense-making.

Overall, the results from this study show that the participants had traditional conceptions of teaching geometry content. From the results it is apparent that the participants' traditional conceptions of teaching area of polygons emanates from their procedural understanding in the various aspects of MKT examined in the study. Noteworthy, the results showed that the participants lacked understanding of the problem solving approach as pedagogy, thus describing and enacting traditional instructional strategies. In geometry, in particular area measurement, research indicates that traditional instructional strategies are responsible for causing most learners' difficulties and misconceptions (Baturo & Nason, 1996; Cavanagh, 2008; Huang & Witz, 2013).

In the next section, I present a discussion of the factors identified by the participants as influencing their conceptions of teaching geometry through the problem solving approach.

6.3 FACTORS INFLUENCING CONCEPTIONS OF GEOMETRY INSTRUCTION

In this section, I present the discussion of the factors identified by the participants as influencing their conceptions of teaching geometry through the problem solving approach. To gain more insight regarding the participants' understanding of teaching geometry through the problem solving approach from their perspectives, I elicited their views on the factors they considered to be influencing their conceptions. As they described how these factors influenced their conceptions of teaching geometry through the problem solving approach, they revealed their own understanding of the problem solving approach. In my study, I grouped these factors into five categories: the language used in the problem, learner characteristics, teachers' own knowledge, the school, and advantages of the problem solving approach.

According to the participants, some of the language used in geometry tasks in the textbook prevented the learners from understanding the tasks. This problem stems from the fact that in teaching through the problem solving approach, the learners should comprehend the problem first. This result was similar to Anderson (2000) and Nantomah (2010) who also found that language was an important variable for successful teaching through the problem solving approach. In Anderson's (2000) and Nantomah's (2010) studies, the participants identified language as one of the constraints in implementing the problem solving approach. In the African context, Nantomah (2010) specifically identified learners' poor linguistic abilities as a factor that constrained teachers from implementing teaching through the problem solving approach. Apart from preventing the learners from comprehending tasks, language affects learners in terms of their ability to express their ideas during classroom discussions.

In addition to language, the participants identified some characteristics related to their learners as influencing their conceptions of teaching geometry through the problem solving approach. According to the participants, these characteristics were learners' understanding of the area concept and their attitude towards mathematics. The participants mentioned that learners had difficulty in understanding geometry, especially area concepts.

In addition, they mentioned that their learners lacked abstract and creative thinking. According to the participants, in order to teach geometry through the problem solving approach, the learners should be able to communicate their ideas, think abstractly and think critically. Roh (2003) concurred with the participants and asserted that for successful instruction through the problem solving approach, the learners should be "skilled in problem solving, creative thinking, and critical thinking" (p. 1). Schoenfeld (1992) added that the learners should be competent in problem solving strategies. However, this result was surprising considering that Polya's (1945) problem solving phases and heuristics were part of the curriculum. Schoenfeld (2012) argued that it is the role of teachers to instil these abilities in their learners. This result has implication in the teaching of Polya's (1945) problem solving phases and heuristics.

The literature recognises the influence exerted by learners' attitudes in teachers' conceptions of instructional approaches and their practices. Raymond (1997) found that learners' characteristics had the strongest influence on teachers' conceptions of instructional practices. In teaching through the problem solving approach, the participants in Nantomah (2010)'s study also identified the learners' negativity as influencing their conceptions of the problem solving approach. From my anecdotal evidence, I have also experienced the influence of learners' negative attitudes towards mathematics which is evident through their refusal to participate in classroom discussions. Learners' difficulties in mastering area concepts are well documented in literature. Stephan and Clements (2003, p14) assert: "measurement sense is more complex than learning the skills or procedures for determining a measure" (p. 14).

In addition, in my study the participants mentioned factors related to their own knowledge as influencing their conceptions of teaching geometry through the problem solving approach. The participants indicated that preparing geometry lessons through the problem solving approach required intensive preparation from the teacher, which involved understanding the content, the appropriate pedagogical approach of that content, relevant instructional material, knowing learner weaknesses and strengths. Indeed, to successfully implement the teaching of area of polygons through the problem solving approach requires time and preparation from the teacher (Hung, 2011). O'Shea and Leavy (2013) reported similar results in their study, reporting that participants in their study were reluctant to implement teaching through the problem solving approach due to their conceptions that planning a problem solving class was a demanding task.

An important result of my study related to a response from one of the participants who mentioned that "*Learners are told to skip this topic* [problem solving] *because of teachers not understanding it or teachers fail to teach"* [own emphasis]. This result provides further evidence indicating that in the context of my study there is a deeper problem than knowing how to teach through the problem solving approach. It seemed the problem related to the participants' knowledge of problem solving and heuristics, identified by Schoenfeld as one of the factors influencing successful problem solving.

Furthermore, the participants reported factors related to the school as influencing their conceptions of teaching geometry through the problem solving approach. According to the participants, class size, time allocated to problem solving and availability of teaching materials influenced their conceptions of geometry instruction. According to the participants, time constraints forced them to omit some of the content in order to finish the lesson within the stipulated time. Hung (2011) supports this result, stating that instruction through the problem solving approach receives criticism for being resource-intensive.

Lastly, the participants reported that the advantages of the problem solving approach influenced their conceptions of teaching geometry through the problem solving approach. According to the participants, the problem solving approach improved learners' understanding, abstract thinking and knowledge retention. This result is true with the problem solving approach as Schoenfeld (2012) asserts that as learners struggle through the content they develop deep understanding of the content; as a result they do not need to memorise procedures.

No participant mentioned their own conceptions of effective geometry instruction as a factor affecting their conceptions of teaching geometry through the problem solving approach. Raymond (1997) found that teachers' own conceptions of effective instruction influenced their conception of instructional strategies. Anderson (2000) concurs with Raymond, finding also that participants' own conceptions of teaching through the problem solving approach influenced their implementation of the problem solving approach. Further, no participant mentioned the textbook as influencing his or her conceptions of teaching geometry through the problem solving approach. Yet research has identified the textbook as factor influencing teachers' conceptions of effective instruction (Daher & Jaber, 2010). Raymond (1997) found that teachers with firmly developed traditional conceptions of mathematics were inclined to propagate traditional mathematics despite holding non-traditional conceptions of mathematics instruction.

6.4 CONCLUSION

In this section, I present the conclusion of my study. In my study, I argued that teachers' conceptions of effective teaching influenced their instructional practices. Concerned by the continued poor performance of most learners in geometry, especially area of polygons, and conforming to the recommendations from the MoET curriculum policies, I assumed that teaching geometry through the problem solving approach could alleviate the situation. However, there was a challenge, as there was little known about primary teachers' understanding of teaching geometry through the problem solving approach in the context of my study. Previous studies had focused either on conceptions of teaching mathematics through the problem solving approach without focusing on a specific topic or on conceptions of geometry instruction without focusing on a specific instructional approach. In addition, none of the studies used Ball et al.'s (2008) MKT as a framework or as a lens. Therefore, my study focused on examining primary teachers' understanding of teaching geometry through their use of the problem solving approach. My intention was to provide information describing the primary teachers' conceptions of teaching geometry through the problem solving approach, and the factors they considered as influencing their conceptions. In addressing the research questions in my study, I collected data in two phases. In the first phase, 34 primary mathematics teachers in the Shiselweni region completed an open-ended questionnaire consisting of three parts. In addition to the first part which explored the participants' demographic information, the second part examined the participants MKT related to area of polygons and the third part required the participants to state the factors they considered as influencing their conceptions of teaching geometry through the problem solving approach. To triangulate the questionnaire data, in the second phase, I collected data through lesson observation, lesson plans and semi-structured interviews from two participants from the initial 34, in their school environment. During the lesson observation I witnessed the actual instructional strategies used by the participants in teaching area of polygons.

Next, I present a summary of the results connected to each of the central research questions. The first research question was "What are primary school teachers' conceptions of teaching geometry through the problem solving approach in the Shiselweni region (Swaziland)?" The results indicate that the participants had limited understanding of teaching geometry through the problem solving approach. The participants showed weak knowledge of area of polygons on the following aspects of the MKT framework including important mathematical ideas related to learning area of polygons, emerging ideas, misconceptions, appropriate tasks, and appropriate representations. In this study, I found that the teachers had limited understanding of teaching geometry through the problem solving approach.

The results of my study showed that the participants had procedural understanding in the following aspects of MKT related to teaching geometry through the problem solving approach; important mathematical ideas, interpreting learners emerging and incomplete ideas, recognising and articulating learners misconceptions, aims for learning area of polygons, selecting appropriate activities, selecting representations, and instruction through the problem solving approach. In these aspects of the MKT, the results showed that the participants had certain conceptions identified as being responsible for most learners' difficulties and misconceptions (Cavanagh, 2008). A majority of the participants (65%) identified important ideas emphasising procedural teaching of area of polygons, such as a formula and counting unit squares. Additionally, they could not identify the critical challenges likely to be experienced by learners in attempting area of polygons tasks in my study. Of note, some participants mentioned that learners would have trouble in using the area formula of a triangle in a scalene triangle because it was not applicable. In addition, no participants could identify the misconception associated with decomposing and recomposing the triangle into the parallelogram in task 8.3 (see Figure 5.1). Moreover, the results indicated that a majority of the participants failed to recognise the importance of a teacher's knowledge base in evaluating learners' responses. Regarding the aims for learning the area of polygons content, the results showed that the participants identified the aims necessary for solving daily life problems. They neglected the importance of linking area concepts to other concepts in the curriculum, such as laying the foundation for further study in geometry and introducing of other concepts in mathematics (Ma, 1999). The results showed that the participants would sequence the area of polygons content from simple to complex. However, these results did not provide conclusive evidence as to whether the participants focused on procedural goals or conceptual goals as in both goals the ability to sequence content is important. The results showed that the participants would adapt the activities in task 9 (see Figure 5.2) by introducing the formula to simplify them. This result provided strong evidence suggesting that the participants focused on teaching area of polygons towards procedural understanding. In selecting activities for their instruction, the results showed that most participants (65%) would select activities based on their potential to provide prerequisite knowledge for doing other activities in a lesson. This reason was not sufficient for judging whether the participants intended to select the activities for procedural or conceptual goals.

During the observation of the two participants, I observed that they taught area of polygons content through traditional instructional strategies. Their instructional strategy was consistent with their conceptions of teaching area of polygons as revealed by the results from the other aspects of their MKT. Even after confirming most of his learners existing knowledge, one of the participants, Patrick did not capitalise on that knowledge to allow his learners to devise a plan for solving the problem. It seemed the participants construed effective teaching of area of polygons as explaining procedures for finding the area of polygons rather than giving learners opportunities to use their existing knowledge to develop their own procedures for finding the areas of the polygons. Furthermore, the participants did not provide manipulatives for their learners during the observed lessons, contrary to Daher and Jaber's (2010) findings. In primary school, Daher and Jaber (2010) found that teachers attributed the success and failure of their instructional approaches to the use of manipulatives. The observation results confirmed both the interview and the questionnaire results, suggesting that this tool was useful in extracting the participants' MKT of area of polygons.

At the beginning of my study, I had hoped to observe the teacher enabling learners to employ Polya's (1945) first three problem solving phases when working on the tasks they were assigned, as the participants had covered them. These phases were: understand the problem, devise a plan and carrying out the plan. A response from one of the participants sums up the state concerning the teaching of problem solving lessons: "*Learners are told to skip this topic* [problem solving lessons] *because of teachers not understanding it or teachers fail to teach"* [own emphasis]. Possibly the participants' weak CK of geometry prevented them from giving their learners' autonomy in developing their plans and explaining their solution strategies to the whole class. Secondly, the participants may not have been aware that Polya's (1945) phases of problem solving were applicable to all the topics in the curriculum. My results confirm the results from other studies that have reported that teachers have weak knowledge of geometry and that most geometry instruction is through traditional instructional approaches. In fact, the participants mentioned that they lacked the knowledge of how to teach through problem solving strategies. Overall, the results showed that the participants had inadequate understanding of teaching geometry through the problem solving approach.

The second research question was “what are the factors influencing the primary school teachers’ conceptions of teaching geometry through the problem solving approach in the Shiselweni region (Swaziland)?” To address this research question, I asked the participants directly in the open-ended questionnaire to state the factors they thought influenced their conceptions of teaching geometry through the problem solving approach. It was clear from the results that the participants regarded problem solving as an important aspect of mathematics. The participants identified various factors influencing their ability to implement problem solving practices in their teaching. I grouped the factors into those related to the language used in the problem, learner characteristics, their own knowledge, the school, and advantages of the problem solving approach.

In relation to the language used in the problem, a critical factor was the technical language used in area measurement, such as height and base, used in stating area of polygons problems. This was coupled with the learners’ abilities, which encompassed their ability to communicate their ideas clearly, think abstractly, and think critically during problem solving, and their negative attitude towards mathematics. The participants felt they lacked sufficient knowledge, which was critical in preparing and delivering lessons through the problem solving approach, as they believed that teaching through the problem solving required intensive preparation. School cultural factors included class sizes, time allocated to problem solving and availability of teaching materials. Lastly, the participants construed the problem solving approach as an important instructional approach because it improved learners’ understanding, abstract thinking and knowledge retention.

The participants own knowledge of problem solving was the most significant issue raised by the participants as influencing their conceptions of teaching geometry. The participants raised this influence in three distinct ways. First, intensive preparation, which involved sound understanding of the content as well as relevant materials essential for effective teaching of the geometry content. Second, understanding the learners’ abilities to ensure that they were taught content at their cognitive level. Third, understanding how they could plan the geometry content for instruction through the problem solving approach. Schoenfeld (1992) conceded that problem solving was challenging for both the teacher and the learner, but when implemented properly yields excellent results.

6.5 RECOMMENDATIONS

In this section, I present the recommendations of my study. My study revealed some important issues relating to teachers' understanding of teaching geometry through the problem solving approach. I think future research should focus on the primary teachers' knowledge of problem solving strategies. Despite introducing Polya's (1945) problem solving phases and heuristics over a decade ago in the curriculum, none of the participants included those practices in their lessons. In fact, one participant indicated in my study that teachers skipped those lessons because they lacked the necessary knowledge for teaching it. Secondly, the participants reported that their learners lacked the essential problem solving skills to enable them to teach through the problem solving approach. Arising from this, I recommend that thorough research is needed to ascertain teachers' understanding of problem-solving strategies. In addition, research is necessary to examine the impact on learners of teaching Polya's (1945) problem solving phases and heuristics. Such a study should be conducted at a national level.

Without further research into primary teachers CK of geometry, it would be impossible to teach geometry through the problem solving approach. From the results of my study, it seems that participants' CK (or lack thereof) played a crucial role in shaping their instructional strategies. Therefore, both pre-service and in-service training should focus on providing prospective and in-service teachers with appropriate experiences of geometry, focusing on the different aspects of Ball et al.'s (2008) MKT framework.

From a policy point of view, teachers should be considered as important agents for implementing curriculum innovations; hence, they should be thoroughly equipped with the necessary skills. In my study, the participants understood the problem solving approach as an effective instructional approach but lacked understanding of the relevant aspects of MKT related to it.

6.6 CONTRIBUTIONS

In this section, I present the contributions of my study. Apart from describing the participants' understanding of teaching geometry through the problem solving approach, my study made three major contributions regarding teachers' MKT. First, the participants of my study were in-service teachers teaching mathematics in primary school. As revealed in my study, primary school teachers have difficulties teaching geometry meaningfully, especially area of polygons. My study provided knowledge that one of the reasons for this difficulty may be due to teachers' weak CK. Second, my study extended the understanding of MKT by focusing on teachers' knowledge of a specific concept through the problem solving approach. Although researchers such as Steele (2013) examined teachers MKT in geometry and measurement, they focused on developing items for ascertaining CCK and SCK without focusing on any particular instructional approach. Third, most investigations into teachers' MKT have adopted a quantitative approach. However, my study adopted a qualitative approach using Ball et al.'s (2008) MKT framework as a theoretical lens, increasing the originality of my study. In that regard, I hope that my study will appeal to other mathematical educators' interested in gaining situated knowledge about teachers understanding of MKT of specific concepts through particular instructional approaches.

6.7 AUTOBIOGRAPHICAL REFLECTION

In this section, I present the autobiographical reflection of my study. Engaging in this study was a learning experience for me as an upcoming mathematical researcher. Grappling with the study has taught me to persevere towards my goals as embarking on the study was not an easy exercise. Somehow, undertaking this research was a personal fulfilment as it increased my understanding of the problem solving approach as pedagogy. It also fulfilled my desire to understand primary teachers' instructional strategies in teaching geometry.

My study helped me to do an introspection of my own conceptions of effective mathematics instruction. From now onwards I will pay particular attention to helping my students change their conceptions of effective mathematics instruction.

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APPENDICES

APPENDIX A Ethical clearance

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

09 November 2015

Mr Sibusiso Sandile Ndlandla (214584436)
School of Education
Edgewood Campus

Dear Mr Ndlandla,

Protocol reference number: HSS/1533/015M
Project title: Teachers' conceptual understanding of teaching the area of a triangle through the problem solving approach in primary schools in Swaziland: A case study of Shiselweni Region

Full Approval – Expedited Application

In response to your application received on 19 October 2015 and amendment on 21 October 2015, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol have been granted **FULL APPROVAL.**

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Dr Shenuka Singh (Chair)

/ms

Supervisor: BB Goba
Academic Leader Research: Professor P Morojele
School Administrator: Ms Tyzer Khumalo

Humanities & Social Sciences Research Ethics Committee
Dr Shenuka Singh (Chair)
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APPENDIX B **Informed consent letter for Director of Education, Ministry of education and training, Swaziland.**

School of Education
College of Humanities
University of KwaZulu-Natal
Edgewood Campus
Pinetown

My proposed research title is:
Teachers' conceptual understanding of teaching the area of the triangle through the problem solving approach in Swaziland

THE DIRECTOR OF EDUCATION
Ministry of education and training
P. O. Box 96
Mbabane
Dear Sir / Madam
I am a registered University of KwaZulu-Natal student doing a Master's degree in Mathematics Education (full thesis) 2015 to 2016 academic years. My name is Ndlandla Sibusiso S. and my student number is 214 584 436. As a requirement for completion of my studies, I need to conduct a research study in the area of mathematics education. The purpose of this study is to learn from your mathematics teachers what their conceptual understanding is of teaching the area of a triangle through the problem solving approach. It is not designed to assess the performance of the teachers in the schools in your region. I am particularly interested in the Nhlangano zone. I envisage these schools will provide rich data for the study. The schools will participate in this study voluntarily.

I am seeking your permission to conduct this research in schools in your region. The study will be conducted sometime between the last week of August 2015 and the second week of September 2016. In each school, I will request all teachers with a minimum of two years mathematics teaching experience who teaches any of the grades from grade 3 to 7 to participate in the study. I will ask them to first complete an

APPENDIX B CONTNUED

open-ended questionnaire about their conceptual understanding of teaching the area of a triangle through the problem solving approach. Teachers who are willing to volunteer to participate in classroom observations and interviews will indicate this on the questionnaire. Two teachers from different schools will be selected to participate in the classroom observation and interview. The selected teachers will be requested to prepare a lesson plan and deliver a lesson on “area of a triangle” which the researcher will observe. I will then ask questions related to their lesson. In order for me to remain focused and ensure an accurate record of what we will discuss during the questioning session, I will seek permission from the teachers to record the interview. The teachers will participate in this study voluntarily.

Confidentiality will be upheld to the highest degree possible. Pseudonyms will be used for their names and the school in which they teach when reporting the results of the research. Moreover, I undertake to share my results and thesis with your office and the participating schools. I hope the recommendations of this study will contribute positively to the improvement of teaching of mathematics in your region.

Survey documents and audiotapes will be kept in a secure place until the stipulated period as per University of KwaZulu-Natal regulations guiding such, at which time they will be erased and destroyed. Any queries or questions about the research should be directed to my supervisor:

Goba B.B.
University of KwaZulu-Natal
Tel no: +27 73 848 3377
Email: gobab@ukzn.ac.za

Mr. Ndlandla Sibusiso S.
Ngwane Teachers College
P.O. Box 474
Nhlangano
Contact no: +268 76138961
Work: +268 22078466/7
Email: ndladndlass@gmail.com

APPENDIX B CONTNUED

The Government of the Kingdom of Swaziland

Ministry of Education & Training

Tel: (+268) 2 4042491/5
Fax: (+268) 2 404 3880

P. O. Box 39
Mbabane, SWAZILAND

17th September, 2015

Attention:
Head Teachers:

Christ the King Primary	Hlatikulu Central Primary	Galile Community Primary
Single Tree Primary	Nsongweni Primary	Qinisweni Primary
Tfokotane Primary	Ngwane Practising Primary	Evelyn Baring Primary

THROUGH
Shiselweni Regional Education Officer

Dear Colleague,

RE: REQUEST FOR PERMISSION TO COLLECT DATA FOR THE UNIVERSITY OF KWAZULU-NATAL STUDENT – MR. SIBUSISO SANDILE NDLANDLA

1. Reference is made to the above mentioned subjects.

2. The Ministry of Education and Training has received a request from Mr. Sibusiso S. Ndlandla, a student at the University of KwaZulu-Natal, that in order for him to fulfill his academic requirements at the University of KwaZulu-Natal, he has to collect data (conduct research) and his study or research topic is: *Teachers' Conceptual Understanding of Teaching the Area of a Triangle through the Problem Solving Approach in Primary Schools in Swaziland.* The population for his study comprises of two teachers from the above mentioned schools. All details concerning the study are stated in the participants' consent form which will have to be signed by all participants before Mr. Ndlandla begins his data collection. Please note that parents will have to consent for all the participants below the age of 18 years participating in this study.

3. The Ministry of Education and Training requests your office to assist Mr. Ndlandla by allowing him to use above mentioned schools in the Shiselweni region as his research sites as well as facilitate him by giving him all the support he needs in his data collection process. Data collection period is one month.

Regards,

DR. SIBONGILE M. MTSHALI-DLAMINI
DIRECTOR OF EDUCATION AND TRAINING

MINISTRY OF EDUCATION TRAINING 2015 -09- 17 BOX 39 MBABANE SWAZILAND

cc: **Regional Education Officer – Shiselweni**
Chief Inspector – Primary
9 Head Teachers of the above mentioned schools
Mr. Sabelo Mthembu

APPENDIX C **Informed consent letters from principals of participating schools**

School of Education
College of Humanities
University of KwaZulu-Natal
Edgewood Campus
Pinetown

INFORMED CONSENT LETTER

The Principal

Dear Sir / Madam

Academic research: Request for permission to conduct a research study in your school.

My proposed research title is:
Primary school teachers' understanding of teaching geometric area through the problem solving approach in Swaziland

Dear Sir/Madam

I am a registered University of KwaZulu-Natal student doing a Master's degree in Mathematics Education (full thesis) 2015 academic year. My name is Ndlandla Sibusiso S. and my student number is 214 584 436. As a requirement for completion of my studies, I need to conduct a research study in the area of mathematics education. The purpose of this study is to learn from your mathematics teachers what their conceptual understanding is of teaching the area of a triangle through the problem solving approach. It is not designed to assess the performance of your school in mathematics. I am particularly interested in your teachers because of their vast experience and deep knowledge of primary school mathematics. So I envisage that this deep knowledge in mathematics will provide rich data for the study. Your school participating in this study will be completely voluntary.

APPENDIX C CONTNUED

I am seeking your permission to conduct this research your school. The study will be conducted sometime between the last week of August 2015 and the second week of September 2016. Firstly, I will request all the teachers with a minimum of five years mathematics teaching experience in your school, who teaches any of the grades from grade 3 to 7 to participate in the study to complete a self-administered questionnaire about their understanding of teaching area of polygons trough problem solving approach. Teachers who are willing to volunteer to participate in classroom observations and interviews will indicate this on the questionnaire. One teacher may be selected to participate in the classroom observation and interview. The selected teacher will be requested to prepare a lesson plan and deliver any lesson on "area of polygons" (area of rectangle/ square, area of irregular shapes and area of a triangle) which the researcher will observe. I will then ask questions related to their lesson. In order for me to remain focused and ensure an accurate record of what we discuss during the questioning session, I further seek permission from the teacher to record the interview. The teachers will participate in this study voluntarily.

Confidentiality will be upheld to the highest degree possible. Pseudonyms will be used for your name and the school and the names of the teachers when reporting the results of the research. Moreover, I undertake to share my results and feedback on this research with you and your staff. I hope the recommendations of this study will contribute positively to the improvement of teaching of mathematics.

Survey documents and audiotapes will be kept in a secure place until the stipulated period as per University of KwaZulu-Natal regulations guiding such, at which time they will be erased and destroyed.

Any queries or questions about the research should be directed to my supervisor:

Goba B.B.
University of KwaZulu-Natal
Tel no: +27 73 848 3377
Email: gobab@ukzn.ac.za

Mr. Ndlandla Sibusiso S.
Ngwane Teachers College
P.O. Box 474
Nhlangano
Contact no: +268 76138961
Work: +268 22078466/7
Email: ndladndlass@gmail.com

APPENDIX C CONTINUED

DECLARATION

I...MICAH TSABEDZE...(Full names of Head teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school participating in the research project. I understand that I am at liberty to withdraw it from the project at any time, should I so desire.

SIGNATURE OF HEAD TEACHER: ...M Tsab...

DATE: ...5 – 8 – 15...

TEOKOTANI COMMUNITY SCHOOL
P.O. BOX 1497 NHLANGANO
DATE: 5–8–15
PHONE: 237 0182

DECLARATION

I...NURSE N. SIMELANE...(Full names of Head teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school participating in the research project. I understand that I am at liberty to withdraw it from the project at any time, should I so desire.

SIGNATURE OF HEAD TEACHER: ...

DATE: ...06 . 08 . 15...

CHRIST THE KING PRIMARY SCHOOL
THE PRINCIPAL
P.O Box 44 Hlathikhulu
Tel 2176130

APPENDIX C CONTNUED

9

DECLARATION

I......MALINSA DAVID VUSIE......(Full names of Head teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school participating in the research project. I understand that I am at liberty to withdraw it from the project at any time, should I so desire.

SIGNATURE OF HEAD TEACHER:..

DATE: ....2015-08-05.

CENTRAL PRIMARY P.O. BOX 104 HLATIKULU
TEL: 21 76187
05 AUG 2015
SWAZILAND
HLATIKULU • • •

9

DECLARATION

I......VUSANI P. MOTSA..(Full names of Head teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school participating in the research project. I understand that I am at liberty to withdraw it from the project at any time, should I so desire.

SIGNATURE OF HEAD TEACHER: P.O. Box 164, Nhlangano......

Single Tree Primary School
Tel: 2207 8847

DATE: 04/08/15..........................

APPENDIX C CONTINUED

9

DECLARATION

I..DUNCAN.......MZIMA.................................(Full names of Head teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school participating in the research project. I understand that I am at liberty to withdraw it from the project at any time, should I so desire.

The Head Teacher
Eqinisweni Primary School
POBox 176 Nhlangano
Tel: 22250004
DATE

SIGNATURE OF HEAD TEACHER: ....

DATE: 05/08/15.........

9

DECLARATION

I......Marks Mavuso...(Full names of Head teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school participating in the research project. I understand that I am at liberty to withdraw it from the project at any time, should I so desire.

GALILEE PRIMARY SCHOOL
P.O. BOX 40

SIGNATURE OF HEAD TEACHER: ....

DATE: ....04/07/2015.........

APPENDIX C CONTINUED

9

DECLARATION

I..ADELAIDE DLAMINI..........(Full names of Head teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school participating in the research project. I understand that I am at liberty to withdraw it from the project at any time, should I so desire.

SIGNATURE OF HEAD TEACHER: ....

DATE: 4/08/2015..........

THE HEAD TEACHER
NSONGWENI PRIMARY SCHOOL
P.O. BOX 138, NHLANGANO
PHONE: 207 8558
DATE: 4/08/2015

9

DECLARATION

I.....EPHRAEM PIET DLAMINI..............(Full names of Head teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school participating in the research project. I understand that I am at liberty to withdraw it from the project at any time, should I so desire.

SIGNATURE OF HEAD TEACHER: ....

DATE: ..27/05/2016..

The Head Teacher
NGWANE PRACTISING
27 MAY 2016
P.O. Box 92
NHLANGANO
PRIMARY SCHOOL

APPENDIX C CONTNUED

DECLARATION

I. MUNTU J. DLAMINI (Full names of Head teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school participating in the research project. I understand that I am at liberty to withdraw it from the project at any time, should I so desire.

SIGNATURE OF HEAD TEACHER: ……………………………………………

DATE: 24/05/2016

EVELYN BARING PRIMARY SCHOOL
P. O. BOX 47
NHLANGANO
24 MAY 2016
TEL. 207 8421
SWAZILAND

APPENDIX D **Sample of informed consent forms from participants**

School of Education
College of Humanities
University of KwaZulu-Natal
Edgewood Campus
Pinetown

INFORMED CONSENT LETTER

My proposed research title is:
Teachers' conceptual understanding of teaching the area of the triangle through the problem solving approach in Swaziland

Dear Sir/Madam

I am a registered University of KwaZulu-Natal student doing a Master's degree in Mathematics Education (full thesis) 2015 academic year. My name is Ndlandla Sibusiso S. and my student number is 214 584 436. As a requirement for completion of my studies, I need to conduct a research study in the area of mathematics education. The purpose of this study is to learn from you, about your understanding of teaching the area of polygons through the problem solving approach. It is not designed to assess your teaching.

I am seeking your permission to participate in this research because of your vast experience and deep knowledge of primary school mathematics. This deep knowledge in mathematics will provide rich data for the study. Participating in this study will be completely voluntary. The study will be conducted sometime between the last week of August 2015 and the second week of September 2016. You will be asked to first complete a questionnaire about your understanding of teaching area of polygons trough problem solving. If you are willing to volunteer to participate in classroom observations and interviews you will indicate this on the questionnaire. You may be selected to participate in the classroom observation and interview. Once selected you will be requested to prepare a

APPENDIX D CONTNUED

lesson plan and deliver any lesson on “area of polygons” (area of rectangle/ square, area of irregular shapes and area of a triangle) which the researcher will observe. I will then ask questions related to your lesson. In order for me to remain focused and ensure an accurate record of what we discuss during the questioning session, I further seek permission to record the interview.

Confidentiality will be upheld to the highest degree possible. Pseudonyms will be used for your name and the school in which you teach when reporting the results of the research. Moreover, I undertake to share my results and feedback on this research with you and your administration. I hope the recommendations of this study will contribute positively to the improvement of teaching of mathematics in the region.

Survey documents and audiotapes will be kept in a secure place until the stipulated period as per University of KwaZulu-Natal regulations guiding such, at which time they will be erased and destroyed.

Any queries or questions about the research should be directed to my supervisor:

Goba B.B.	Mr. Ndlandla Sibusiso S.
University of KwaZulu-Natal	Ngwane Teachers College
Tel no: +27 73 848 3377	P.O. Box 474
Email: gobab@ukzn.ac.za	Nhlangano
	Contact no: +268 76138961
	Work: +268 22078466/
	Email: ndladndlass@gmail.com

DECLARATION

I HOPEWELL ZIKALALA ....(full names of teacher) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my participating in the research project. I understand that I am at liberty to withdraw from the project at any time, should I so desire.

I hereby consent to the audio recording of my interviews: YES/NO

SIGNATURE OF PARTICIPANT: ....

DATE: 06-08-2015

APPENDIX E **Sample informed consent forms from guardians of learners involved in observations**

Dear Parent/Guardian

My name is Ndlandla Sibusiso S. and my student number is 214 584 436. I am a registered student of the University of KwaZulu-Natal student doing Masters in Mathematics Education (full thesis) 2015 to 2016 academic years. As a requirement for completion of my studies, I need to conduct a research study in the area of mathematics education. The purpose of this study is to gain and understanding of the mathematics teachers' conceptual understanding of teaching the area of a triangle through the problem solving approach. It is not designed to assess the performance of your child in mathematics. I hereby invite you to give consent for your child to participate in this study. Your child's involvement in the study will be participating during a lesson observed by the researcher, which will be videotaped. The school Principal and the Regional Education Officer have granted their permission for this study at your school. The study will be conducted sometime between the last week of October 2015 and the end of November 2016. It will be during his/her usual mathematics lesson time. Your consent for this is voluntary, and you are not obliged to grant it.

Confidentiality will be upheld to the highest degree possible. Pseudonyms will be used for your child's name and your child's school when reporting the results of the research. Either you as a parent or guardian of the participant or your child's school will incur no costs. Moreover, I undertake to share my results and feedback on this research with your child's school. Videotapes will be kept in a secure place until the stipulated period as per University of KwaZulu-Natal regulations guiding such, at which time they will be erased and destroyed.

If you wish your child to participate in this study, kindly sign this form and your child should return it to the researcher on the day of the first day of the lesson observation.

APPENDIX E CONTNUED

Any queries or questions about the research should be directed to my supervisor:

Goba B.B.	Mr. Ndlandla Sibusiso S.
University of KwaZulu-Natal	Ngwane Teachers College
Tel no: +27 73 848 3377	P.O. Box 474
Email: gobab@ukzn.ac.za	Nhlangano
	Contact no: +268 76138961
	Work: +268 22078466/7
	Email: ndladndlass@gmail.com

DECLARARTION FOR CONSENT

I....Jabulane J. Mamba.....(Full names of parent/guardian) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my child's participating in the research project. I understand that she/he is at liberty to withdraw from the project at any time, should she/he so desire.

SIGNATURE OF PARENT/GUARDIAN: ....J.J. Mamba.................................

DATE: ..09/09/15........

APPENDIX F **Teacher questionnaire**

We are interested in your views on how you teach your learners how to calculate the area of a triangle. There are no “right” or “wrong” answers to the questions. Your opinion is valuable to us.

Your responses will remain confidential. Pseudonyms will be used for your name and school when reporting the results of this study to ensure confidentiality. Thank you for completing the questionnaire.

Any queries or questions about the research should be directed to my supervisor:

Goba B.B.
University of KwaZulu-Natal
Tel no: +27 73 848 3377
Email: gobab@ukzn.ac.za

Mr. Ndlandla Sibusiso S.
Ngwane Teachers College
P.O. Box 474
Nhlangano
Contact no: +268 76138961
Work: +268 22078466/7
Email: ndladndlass@gmail.com

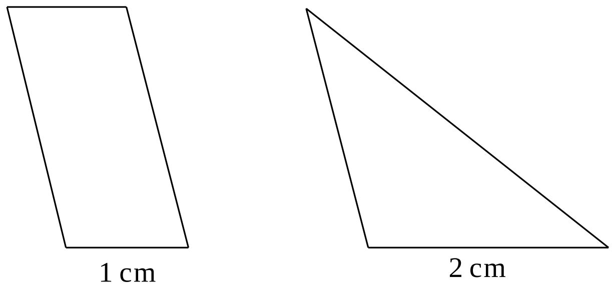
Please answer each of the following questions:

Part I. Background information

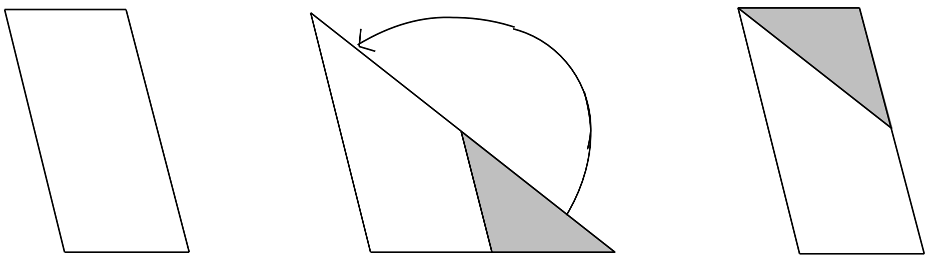
1. Name (optional – only include your name if you are willing to be participate in a classroom observation and follow up interview at a later date):
____
2. School:____
3. Gender:- male / female (circle one)
4. How many whole years have you been teaching? ______________________
5. What is your highest qualification achieved? What was your major?
____
6. Which mathematics class(es) do you teach this year?
____
7. How many in service days have you attended in the last two years(include whole staff development)____
How many of these in service days have had a mathematics focus?
____

Part II. PCK

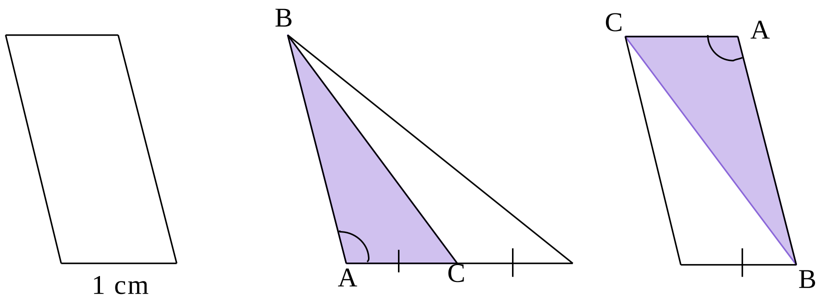
8. Ms. Wilson asked her seventh grade class to compare the areas of the parallelogram and the triangle below. Both of the shapes have the same height.



Two groups of pupils in Ms. Wilson's class came to the correct conclusion that the areas are the same. However, their explanations were different. The groups of pupils used the following diagrams to explain their answer.



Solution of group 1



Solution of group 2

Based on what you know as a mathematics teacher:

8.1 What are some of the important mathematical ideas that the pupils might use to answer this question correctly?

8.2 Ms. Wilson is not sure that both their explanations are correct. What do you think? Why?

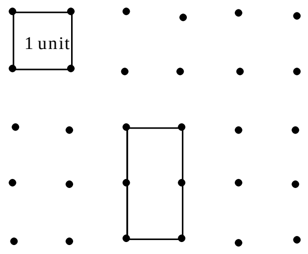
8.3 Does either group represent a mathematical misconception? If yes, what underlying mathematical misconception leads the pupils to this error? If no, how do these two groups differ in their thinking?

8.4 What instructional strategies and/or tasks would you use during the next instructional period? Why?

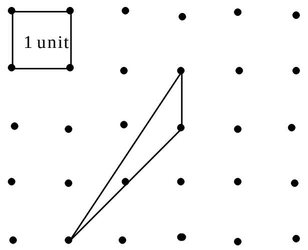
If your answer does not fit in the space provided, please you use the additional page provided to complete your answers.

9. Ms. Mason is planning her seventh grade school geometry unit on area. She has a diverse population of pupils with different levels of geometric development. Ms. Mason gathered a set of activities related to the concept of area that she believes would address different levels of geometric development. She wants to include the following activities in her unit.

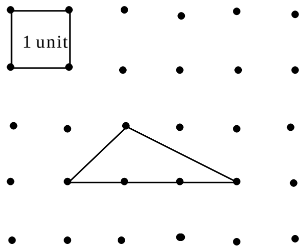
Activity L: Find the area of the rectangle below using the given unit



Activity M: Find the area of the triangle below using the given unit



Activity N: Find the area of the triangle below using the given unit



For each activity, please answer the following.

9.1 What are some of the important mathematical ideas that the learners might use to solve these problems correctly?

9.2 What difficulties and/or common misconceptions related to this topic might the learners have as they solve these problems? Explain your answers.

9.3 What are some of the strengths or limitations of this task? Would you change / adapt this activity? If yes how would you change / adapt the activity? Why?

10. Ms. Mason is planning her seventh grade geometry unit on area. She has a diverse population of learners with different levels of geometry development. Ms. Mason gathered a set of activities related to the concept of area that she believes would address different levels of geometric development; she wants to include activities L, M, and N as described in the preceding item 9 into her unit.

10.1 Which of these activities would be appropriate to include? Why or why not?

10.2 In what order should these activities be presented? Explain why you choose to put these activities in this particular order.

Part III.

11. What are the factors that influence your understanding of teaching geometry through the problem solving approach?

APPENDIX G **Classroom observation schedule**

1. Understand the problem
    (a) Presenting a problem linking the known to the unknown
    (b) Confirming prior knowledge
    (c) Giving learners opportunities to identify key words in the problem
    (d) Giving learners opportunities to identify extra information in the problem
2. Devise a plan
    (a) Giving learners opportunities to use their prior knowledge to predict/suggest/ hypothesize solution strategy.
    (b) Offer without interfering, appropriate hints to the learner
    (c) Giving learners opportunities to restate or translate the problem
    (d) Giving learners opportunities to state as many as possible solution strategies to the problem
3. Solve the problem
    (a) Allowing learners to solve the problems in groups/ prove correctness of their predictions using their preferred solution strategies
    (b) Providing learners with tools necessary for executing their plan
    (c) Move around groups monitoring progress and assisting where necessary without interfering.
    (d) Identifying unique ideas from the learners
    (e) Encouraging multiple solution strategies
4. Looking back
    (a) Providing learners with opportunities to evaluate effectiveness of their solution strategies
    (b) Providing learners with opportunities to discuss extensively their solution strategies, comparing and highlighting the similarities and differences among their solutions strategies
    (c) Assisting learners refine their solution strategies

APPENDIX H **Teacher interview protocol**

Thank you for allowing me to observe your class. I have some questions I would like to ask you related to the classroom lesson and some general questions. Would you mind if I taped the interview? It will help me stay focused on our conversation, and it will ensure an accurate record of what we discussed.

Part I. Personal information

1. How long have you been teaching in this school?
2. What is your highest qualification achieved? What was your major?
3. In what grade do you teach now?
4. Do you have another position besides your teaching role?

Part II. PCK

5. Please describe how you start teaching the topic area?
6. Do you always follow the textbook to teach the learners? Do you use other methods?
7. How do you support learner thinking through teaching the area?
8. How do you teach your learners to learn about calculating the area? In what ways?
9. In your own opinion what are the main aims for learners when learning about area?
10. What is your classroom technique to easily teach the calculation of the area?
11. From your own point of view, what are the main reasons for why the learners should learn the content or concepts regarding the area?
12. How do you teach the concepts? How do you prevent learners' misconceptions?
13. How do you know that learners understand the idea or concepts you teach? In what way do you find out?
14. What other resources do you recommend to the learners in order to learn about area of a triangle? Newspapers, library, internet?

Appendix I **Zane lesson observation transcription**

Teacher: [writing the grade and subject on the chalkboard].

Teacher: Good morning class

Class: Good morning teacher

Teacher: What is the date today class?

Class: The date today is 10^{th} November 2015

Teacher: Today we are going to deal with area of irregular shapes. By the way, what is area? Can you tell me what area is? What do you know about the term area? What is area?

Learner 1: Area is the space inside the shape that that that are meant to cover the surface.

Teacher: A good try. Let us clap hands for him.

Class: [Clapping hands together with teacher].

Teacher: Who wants to try to define area?

Learner 2: Area is the number of units needed to cover a surface …

Teacher: let us help him. Area is the number of square units needed to cover a surface. Yes how many squares are needed to cover a surface, for instance let us look at this class looking at the floor? The floor acts like a surface; let us assume now that we want to put the floor tiles here. How many tiles do we need to cover this surface? The total number of tiles can be area, the number of tiles needed to cover the surface. Do you get that?

Class: Yes

Teacher: So we can find the area by using the counting method [teacher writing definition on chalkboard]. So we find the area by using the counting method it means you count the number of squares [eh]… that cover that surface. You get that?

Class: Yes

Teacher: So we can find the area of figures like regular shapes [teacher writing on the chalkboard] and we can also find the area of irregular shapes [teacher writing on the chalkboard]. But today we want to focus on finding the area of irregular shapes by using the counting method. For instance, let us have this drawing 

Teacher: Let us look at the figure on the board, this one is irregular, it does not have a fixed shape. You get that. Something that is irregular looks like a stone because once you see a stone there is no length there is no breadth. So those shapes are irregular, they have no fixed shape. You get that.

Learners: Yes

Teacher: Now let us look at this one, this irregular shape, now area is the number of square units needed to cover a surface. Now how many squares are within this figure? Can someone come and count the number of squares, which are within this … [pointing at a learner 3], just count.

Learner 3: Counting inside squares

Teacher: How many squares?

Class: 15 square units

Teacher: So 15 square units, so this means that the area of this figure is 15 what?

Class: Square units

Teacher: That is good Learner 3. Now let us try another one. Now l will draw a different shape or a different figure from this one, now. Let us look at this [drawing shape on the chalkboard ]. Now let us look at this shape finding area. Now what do you notice here? Are the squares similar if you look at this figure?

Class: No

Teacher: Learner 4 what do you see?

Learner 4: I see that there are half of squares and a mixture of full squares

Teacher: Yes now there is a mixture of full squares and a half squares. So now how can you find the area where by there are full squares and also half squares? Now remember you don't count a square which is not full as one, a full square is regarded as one [drawing full square on chalkboard] this one is a full square but when it is a half square[drawing a half square ] now it is not fully. Remember if you are buying bread, a half bread and another half bread. It makes one loaf. Do you get that?

Class: Yes

Teacher: So now it means if you put a half and another half it makes one unit. Do you get that?

Class: Yes

Teacher: So how to find eh the area of this figure? Who can tell me? Yes Learner 5

Learner 5: You start by counting full squares and then the half squares.

Teacher: You start by counting the full squares. Is she correct?

Class: Yes

Teacher: You start by counting full squares so now let us try with the full squares. So you say full squares plus now how many are the halves? [Writes full squares + $\frac{1}{2}$ ___ on the chalkboard]. So now let us start with the full squares. who can come and find us or count the number of full squares, starting with the full squares only. Learner 6 can you please come count the full squares, only the full squares

Learner 6: one, two, three … twenty-four.

Teacher: How many full squares?

Class: 24 full squares

Teacher: Is she correct?

Class: Yes

Teacher: Alright let us give him [sic] [teacher clapping hands]

Class: Clapping hands

Teacher: Yes they are twenty..?

Class: 24 full squares

Teacher: there are 24 full squares plus … now how many halves now those squares that are half fully, how many half squares, how many half squares? Learner 7 how many half squares? Those that are not fully

Learner 7: one, two, three, four, five, six

Teacher: Is he correct?

Class: Yes

Teacher: It means now that $24+\frac{1}{2}(6)$. Six. Do you get that?

Class: Yes

Teacher: They are six. So it means that we are going to find the product here [pointing at 1/2(6)] and add it to 24. Do you get that?

Class: Yes

Teacher: So now it is 24 plus half of six. What is half of six?

Class: Three

Teacher: Half of six is?

Class: Three

Teacher: Which makes what? Twenty-four plus three is?

Class: Twenty-seven

Teacher: Twenty-seven square?

Class: units

Teacher: Twenty-seven square units. So that is the area of this figure. Do you understand class?

Class: Yes

Teacher: We all understand?

Class: Yes

Teacher: Yes this is area and remember that area is the number of square units needed to cover the inside surface so that is area. Let us try the last one  so that I see if you all understand what we are talking about. Who wants to try? Who wants to try? Who wants to try? Who wants to try to find the area of this one? Learner 8. Start with the full squares. Let us watch whether she is doing the right thing.

Learner 8: [Counting aloud] one, two, three … sixteen.

Class: [Murmurs]

Teacher: No you raise up your hand.

Learner 8: [recounting] one, two, three … sixteen, seventeen.

Teacher: seventeen full squares?

Learner 8: [writes $17\times\frac{1}{2}$ on chalkboard and proceed to count half squares on figure] $17\times\frac{1}{2}$ 6 [simplifying it to] 17 + 3 = 20.

Teacher: Is she correct?

Class: No [but undeceive]

Teacher: Is she correct? Is she correct?

Class: [Mumbling]

Teacher: Let us clap hands for her try

Class: [Capping hands]

Teacher: Is she correct?

Class: No

Teacher: Who wants to correct? Who wants to correct her? Where is the problem? Where did she go wrong here? (Pointing at learner 8 solution) eh where did she go wrong?

Class: Half

Teacher: [eh]

Class: Half

Teacher: Half

Class: Yes

Teacher: What is wrong with half?

Class: [Mumbling]

Teacher: [eh…] she was supposed to add the number of full squares angitsi kambe[is it so] with the half squares so now here it is should be 17 plus instead of this operation[pointing to the multiplication] then you count how many halves. Do you get that?

Class: Yes

Teacher: Do you get that?

Class: Yes

Teacher: So how many halves? Are they six?

Class: seven

Teacher: Are they six?

Class: seven

Teacher: There are seven halves? So now this is $17+\frac{1}{2}(7)$. What is half of seven? Work out this for us. Who can work out this for us? Who can find half of seven? Who can find half of seven? Half of seven? Yes Learner 9 half of seven. Let us watch

Learner 9: $\frac{1}{2}(7)=3.5$

Teacher: Is it three point five? Is she correct?

Class: Yes

Teacher: Is she correct? Is she correct?

Class: Yes

Teacher: Yes, she is correct. Now add seventeen plus three point five to find the total area of the shape.

Learner 9: [reluctant to find total area of shape]

Teacher: Now seventeen plus three point five (writing on the chalkboard 17 + 3. 5)

Learner 9: 17 + 3. 5 = 20.5 square units

Teacher: Yes, she gets 20.5 units. Is she correct?

Class: Yes

Teacher: Yes, let us clap hands for her.

Class: [clapping hands]

Teacher: Very good. The answer is 20.5 square units so that is the area of this figure the inside of this shape. Do you get that?

Class: Yes

Teacher: Do you have any problem? Do you understand?

Class: Yes

Teacher: So now, I want to see if you have understood. Let me have three people, only three, two girls and one boy, so that we make sure that you know what area is. Let us try this one. Try this one

 using the counting method. Just count, counting method count to find the area, where are the others? Yes learner 10. Let us give her a chance, use the counting method, just count the full squares and the half squares to find the area.

Learner 10: one, two, three … forty four.

Teacher: write forty four those are the full squares

Learner 10: $44+\frac{1}{2}$

Teacher: Plus how many half squares?

Learner 10: one, two, three … six

Teacher: plus six

Learner 10: $44+\frac{1}{2}(6)$

Teacher: Write times six here [pointing in between the $\frac{1}{2}$ and 6)

Learner 10: $44+\frac{1}{2}\times 6$

Teacher: Forty four plus working it down here [showing Learner 10 space on the chalkboard]

Learner 10: $44+\frac{1}{2}\ 3$

Teacher: Half of six

Learner 10: 44 + 3 = 47 square units

Teacher: Is she correct? Is the answer 47?

Class: No

Teacher: Is the answer 47?

Class: No/yes

Teacher: Who wants to work something different? You have something different from this? Yes learner 11, do you have something different?

Learner 11: Ushiye sikwele sinye [She left one square over there]

Teacher: She left one square? Yes, learner 12 come and do it. Very good

Learner 12: $44+\frac{1}{2}\times 3$

Teacher: How many half squares?

Learner 12: [recounting from figure] $44+\frac{1}{2}\times\ 6$

Teacher: Half of 6

Learner 12: 44 + 3

Teacher: 44 + 3 now

Learner 12: 44 + 3 = 47 square units

Teacher: 47 square units. Is the answer correct?

Class: No

Teacher: [he….]

Class: No

Teacher: But you don't want to come and do it? [he…]

Class: No

Teacher: Is the answer correct?

Class: No

Teacher: [giggling] Gaga gaga [teacher laughing]. Yes Learner 13. Let us see if there is something different. Now use this side. Come this side.

Teacher: Start by counting the full squares.

Learner 13: [Counting aloud] One, two, three… forty five.

Teacher: forty five square units. Write forty five here

Learner 13: 45

Teacher: Plus

Learner 13: $45+\frac{1}{2}$

Teacher: Go and count the halves

Learner 13: [Counting aloud] one, two, three…six

Teacher: [ehmm]

Learner 13: $45+\frac{1}{2}\times\ 6$ = 45 + 3 = 48 square units

Teacher: Is she correct? She get 48 square units. Is she correct?

Class: Yes

Teacher: Yes. Thank you very much lets clap hands for her

Class: [clapping hands]

Teacher: Yes to wind up can you please tell me what area is? What did we say about area? What area is? What did we say about area? Yes Learner 14.

Learner 14: Area is…

Teacher: Area is? What did we say is area? What is area?

Learner 14: Area is the number of square units needed to cover a surface.

Teacher: Thank you very much area is the number of square units needed to cover a surface. So we are done with area At home you try to look read more about area so that tomorrow we continue. Do you get that?

Class: Yes. Thank you

Appendix J **Patrick lesson observation transcript**

Teacher: What is the date today?

Class: 12th November 2015

Teacher: Area of isosceles and right-angled triangles this is what we are going to talk about today. The area of isosceles and right-angled triangles

Class: The Area of Isosceles and right-angled triangles

Teacher: Yes

Teacher: Ok what is meant by the word area? What is area? What is area? Yes

Learner 1: Area is the amount of space covered by a flat object.

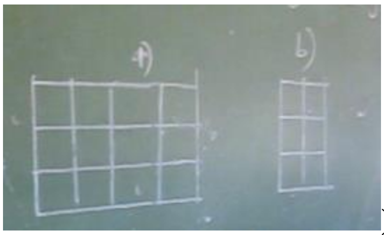
Teacher: Area is the amount of space covered by a flat object or by a two-dimensional object that is area. What else can you say? What is area? What is area? It is a boundary the amount of space in a given boundary yes we have learned about area even in grade 4 even in grade 5 even in grade 6 we have learned about area. [Uhhmm] What are the units for area? What are the units for area? Units for area? Yes

Learner 2: Centimetre squared

Teacher: It is centimetre squared can be centimetre squared, metre squared and the like. Is that clear?

Class: Yes

Teacher: Ok I have got these two shapes of mine here (drawing these shapes on the chalkboard



) These are my shapes here. a) and b) [labelling the shapes]. You may be asked we learned about this in grade 5. How can one find the area of these two shapes eh shape a) and Shape b) what would be the area in shape a)? We can count the squares. What would be the area? Learner 3 what would be the area of shape A?

Learner 3: We can count the squares

Teacher: We can count the squares. What would be the area here? What would be the area?

Learner 3: 12

Teacher: 12 what?

Learner 3: 12 unit squares

Teacher: 12 unit squares 12 cm^2. Ok b)? Learner 4

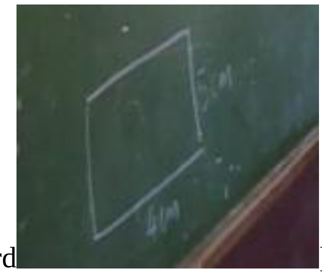
Learner 4: Six

Teacher: Six what?

Learner 4: Six centimetres squared

Teacher: Yes six cm^2. We just count the number of squares. Is that correct?

Class: Yes

Teacher: But we also looked at how can we find the area of shapes like this we are not counting the squares now but there is a formula we used to find the area of shapes like this [drawing a 4cm by 5 cm rectangle on chalkboard ] if we are given 4 cm here and 5 cm here? What should be the area? How can we find the area? Learner 5, how are we going to get the area?

Learner 5: length times width

Teacher: It will be length times width. We are going to say what times what? [Hee]

Learner 5: Four times five

Teacher: It is length times width. Four times five that gives us what?

Class: Twenty

Teacher: Twenty cm^2. [Learners joining in chorus] Yes. Today we are going to look ok yes today we are going to look at the areas of triangles especially the right-angled triangle the right-angled triangle and the isosceles triangle [picking this shape ]. Do you see this? [Pointing the shape]

Class: Yes

Teacher: do you see this? [Pointing the shape)

Class: Yes teacher

Teacher: Ok who can find the area? What would be the area of this shape here? [Pointing the shape] who can help me find the area of this shape here? Learner 6

Learner 6: Six times six

Teacher: what is the area? I want the area. Learner 7

Learner 7: 36

Teacher: [hee]

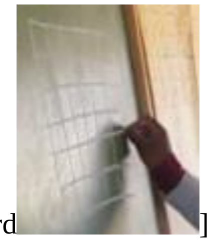
Learner 7: 36

Teacher: Is it 36?

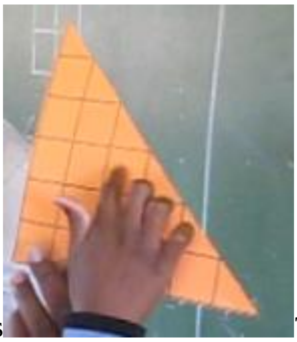
Class: Yes

Teacher: 36 what?

Class: 36 cm^2

Teacher: it is one, two, three… thirty-six. 36 cm^2 ok it is 36 cm^2. This is the shape here [drawing a similar shape on the chalkboard ] Ok now it is 36 cm^2. Where is my pair of scissors?

Class: There

Teacher: Ok now I am going to cut this shape of mine here diagonally. Let me cut it diagonally [cutting it diagonally ]. I am cutting it diagonally. I have got two shapes now, what do we call these types of shapes ?

Class: Diagonal shape

Teacher: What are these types of triangles?

Class: Triangles

Teacher: What do we call these types of triangles? Learner 8

Learner 8: Right-angled triangle

Teacher: Right-angled what?

Class: Right- angled triangles

Teacher: Right-angled triangle. This is a right- angled triangle is that clear? Ok Now I want someone to come up front and try to find the number of squares in one of these triangles here. Learner 9 come. Try to find the number there are half squares here. Count

Learner 9: [Counting aloud] one, two, three… eighteen.

Teacher: How many are they?

Learner 9: Eighteen

Teacher: They are eighteen. Who can come upfront and count these ones? These ones here? Learner 10 come. These one has got eighteen what about this one?

Learner 10: [counting number of squares on shape] Eighteen

Teacher: Eighteen, eighteen, eighteen this is a right-angled triangle. What other triangles do you know? Learner 11 stands up!

Learner 11: Equilateral triangle

Teacher: Equilateral triangle. What are the characteristics of an equilateral triangle? Learner 12

Learner 12: All sides are equal

Teacher: All the sides are equal. What else? Learner 13

Learner 13: All the angles area equal.

Teacher: All the angles are equal. Is that so?

Class: Yes

Teacher: Ok another type of triangle yes Learner 14 stand up

Learner 14: Isosceles triangle

Teacher: The isosceles triangle. What are the characteristics of an isosceles triangle? Learner 15

Learner 15: Two sides are equal

Teacher: Stand up

Learner 15: Two sides are equal

Teacher: Only two sides are equal. Only two sides are equal. Ok what else? Learner 16

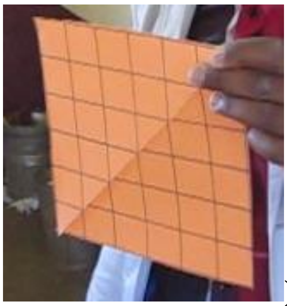
Learner 16: Scalene triangle

Teacher: Nooo they are not we want the other characteristics of the [eh eh] isosceles triangle. Learner 17: Two angles are equal

Teacher: Two angles are equal [joined by learners]. Ok let us get back to this topic of ours. The half here is 18 and the other half is 18. Is that clear?

Class: Yes

Teacher: It means that two halves of the triangles make this big square the the shape was cut here [pointing towards the previously drawn square on the chalkboard]. So it means that if we are given one half of the triangle which is the… if we are given a triangle and we are asked to find the area it is very easy we are going to make this triangle a rectangle. If here it is 4cm and here it is 5 cm and you can think here if we are to to put it together here (demonstrating using the two triangles

) put it together here. It is now a full triangle and it is six centimetres multiply by six centimetres. One, two, three … six centimetres multiply by one, two, three … six centimetres. It is six by six. Six centimetres multiply by six centimetres that why we got the 36 cm^2 for the full square here if we are given. But if it is a triangle you wanted the area of one [removing one triangle] we are going two divide the 36 cm^2 by two because this is a half is that clear?

Class: Yes

Teacher: We then we divide by two and then we got 18 cm^2. Here we are given a triangle of this nature. The height is four centimetres and the base is five centimetres. You are to make the triangle a full rectangle and know that this is your length and this is your width. What is the length of this rectangle now? What is the length of this rectangle 

?

Class: Five centimetres

Teacher: five centimetres and the width is what?

Class: Four centimetres

Teacher: It is four centimetres. Ok the formula says length times width so we are talking about five times four this gives us?

Class: 20 cm^2

Teacher: is this the answer?

Class: No

Teacher: is this the answer?

Class: No

Teacher: No it is not. How are we going to get the answer here? Learner 17

Learner 17: Divide by two

Teacher: We are going to divide the 20 by two because we are talking about this half only not the other half. It is 20 divide by two, which gives us what?

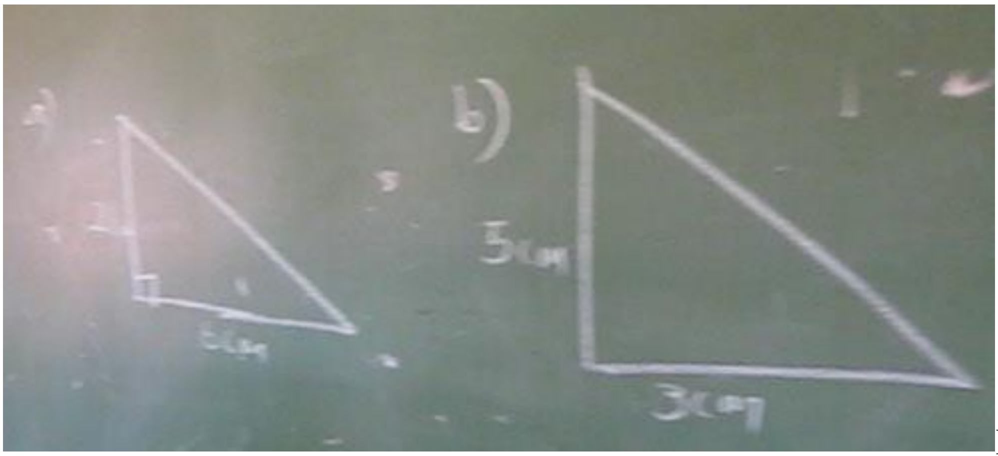
Class: 10

Teacher: 10 what?

Class: 10 cm^2

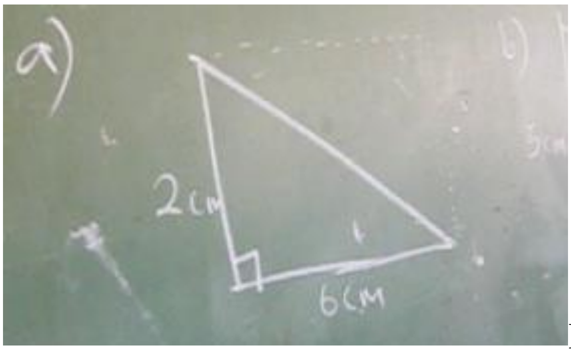
Teacher: 10 cm^2

Teacher: Where is my duster? [searching for duster] 10 cm^2 Ok If you are given a triangle and a right-angled triangle and you are asked to find the the the area, make sure that you make it a complete rectangle or a square then multiply the length times the width then you take that after dividing by two. Ok let us look at this another right-angled triangle [drawing two right-angled triangle on the chalkboard



] Learner 18 come up front and calculate the area of the triangle a) come up front and calculate the area of the triangle a)

Learner 18: [Attempting to calculate the area of the shape and teacher interjects



]

Teacher: What is the first step?

Class: You complete the ….

Teacher: complete the shape and make it a …

Class: complete the shape into a full square

Teacher: You complete the shape into a full square or rectangle yes

Learner 18: 2 [erases] 2 x 6 = 12 cm^2 [thinking about next step]

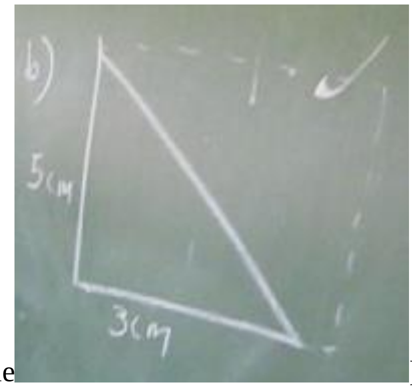
Teacher: Go and sit down Learner 19 come and help him. Help this friend

Learner 19: $\frac{12}{2}$ = 6 cm^2

Teacher: Is this correct?

Class: Yes

Teacher: Yes, this is correct. You are to complete this triangle and make it a rectangle or a square then multiply the length times the width. Our length is six, our width is two, two times six is twelve and because we are not talking about the whole triangle the whole rectangle we are talking about a triangle which is the half of the rectangle then we are going to divide this thing(12 cm) by two and get six centimetres. Learner 20 come upfront and calculate this one for us. Calculate b) for us

Learner 20: [Completing the triangle to make a rectangle 

]

Teacher: hawu ize iyojika lena? [Hey it turns that far?] [Pointing to side not touching its vertex]

Learner 20: [Erasing that straying side and drawing it properly] $5 \times 3 = 16$ $\frac{16}{2} = 16\,\text{m}^2$

Teacher: What are you doing? Learner 21 come and help her

Learner 21: $5 \times 3 = 15$ $\frac{15}{2} = 6\frac{1}{2}$ cm^2

Teacher: Is she correct?

Class: No

Teacher: This is not correct. Learner 22 come and help us it is not correct. People are unable to calculate how come?

Learner 22: 5 cm × 3 = $\frac{15}{2}$ = 7.5 cm^2

Teacher: is this correct?

Class: Yes

Teacher: Read the answer Learner 22.

Learner 22: seven point five centimetres

Teacher: Yes, centimetres what?

Learner 22: Seven point five centimetres squares

Teacher: Why did you put a comma here? here?

Learner 22: Put a comma…

Teacher: Yes it is seven point five centimetres squared or you can write the answer like $7\frac{1}{2}$ cm^2. Is that clear?

Class: Yes

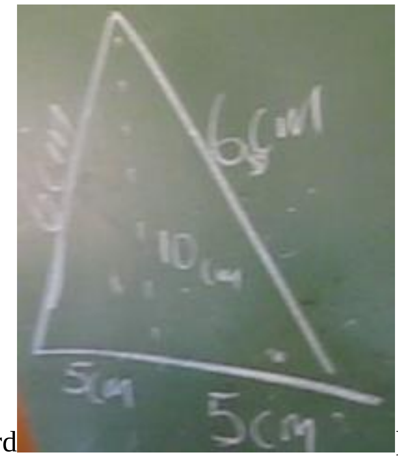
Teacher: [Ya] now we move on from the triangle of which is a right-angled triangle and look at the triangle, which is isosceles triangle. What did we say are the characteristics of an isosceles triangle? Learner 23

Learner 23: Two sides are equal

Teacher: Two sides are equal. Learner 24

Learner 24: Two angles are equal

Teacher: Two angles are equal. Ok this is my two this is my two isosceles triangles I have here isosceles triangle 1 and isosceles triangle 2[showing the class two isosceles triangles]. Ok this one is like this [drawing the isosceles triangle on the chalkboard



] Here it is it is like this. This is my isosceles triangle. How are we going to find the area of an isosceles triangle? It is easy. We are going to do what we did with the right-angled triangle but it is slightly different from the triangle, which is a right-angle. Ok let us cut this triangle into two equal right-angled triangles and we are going to cut it here at the the centre and this is the perpendicular line that meets the base at ninety degrees

. Now let me cut it [cutting isosceles triangle into two equal parts] Ok what type of triangle is this one? What type of triangle is this one

?Learner 25

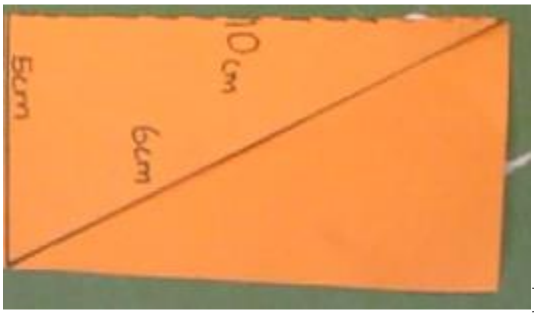
Learner 25: Right-angled triangle

Teacher: What?

Learner 25: Right-angled triangle

Teacher: Right-angled triangle. What about this one? What about this one? Learner 26

Learner 26: Right-angled triangle

Teacher: This is another right-angled triangle. We want to find it was an isosceles triangle it was like this angitsi beyikanje it was like this now it is like this now let us find the area of this triangle, what are we going to do? We are going to change the triangle and make it a a a rectangle. Let us change it and make it a rectangle. The isosceles triangle and make it a rectangle this is our isosceles triangle [demonstrating on chalkboard 

] now it is a.. Yes, do you see the shape on the board?

Class: Yes

Teacher: We have changed the isosceles triangle to be what?

Class: Rectangle

Teacher: Rectangle we have changed the isosceles triangle to be a rectangle it is now a rectangle of this nature [pointing to formed rectangle on chalkboard] of this nature of this nature which is a rectangle which is 10 cm here, 5 cm here, 6 cm here. Is it difficult to find the area of the rectangle now?

Class: No

Teacher: [He!]

Teacher: How can we find the area of this rectangle now? Learner 27 come up front and find us the area of that rectangle and calculate the area of this rectangle. Here is the space baba. Here is chalk. Find the area of this rectangle

Learner 27: $10 \times 6 \times 5 = 30 + 30\ \frac{30}{20} = 12\text{cm}^2$

Teacher: Write the answer down here

Learner 27: 12cm^2

Teacher: Learner 27 says the answer here of that rectangle is 12 cm^2. Go and sit down. Is this correct?

Class: No

Teacher: Is this correct?

Class: No

Teacher: What is the answer? Who can help us here? Leaner 28 come and help us. What is the formula for the area of a rectangle?

Class: Length times width times height

Teacher: What?

Class: Length times width times height [mumbling] Length times width times height [giggling]

Teacher: Did we speak about volume? That is the formula for volume

Class: [Haaa]

Teacher: [Heya]

Learner 28: 5 × 10 = $\frac{50}{2}$ = 25 cm^2

Teacher: Is this the answer?

Class: No

Teacher: Learner 28 says the area of this rectangle is 25 cm^2. The area of this rectangle is 25cm^2. Is he correct?

Class: No

Teacher: Why? Learner 29 come and help us. The area of this rectangle

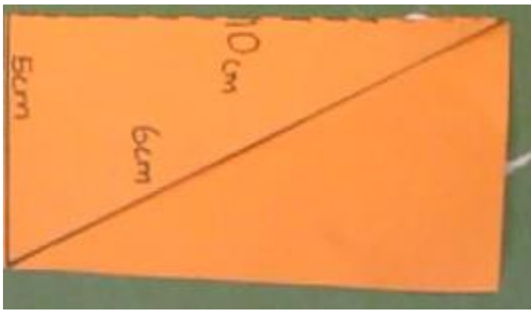
Learner 29: 10 × 5 = 50 cm^2

Teacher: Learner 29 says the area is 50 cm^2 is this correct?

Class: Yes

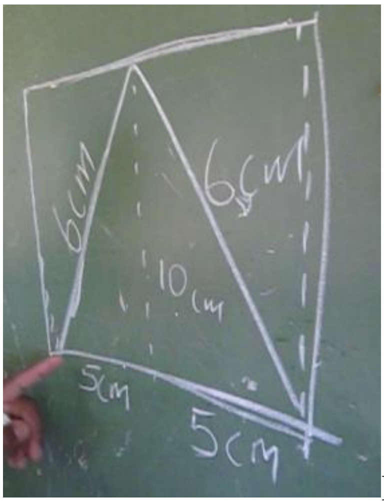
Teacher: Is this correct?

Class: Yes

Teacher: [Heya] it is correct. I don't understand why Learner 28 divided this by two because we want the area of this rectangle 

here which its base is 10 and its width is 5 cm we got 50. It length times width lo height nitsatsephi?[where did you get it?]. Length times width 10 x 5 = 50 Ok our shape was like this [demonstrating using triangles pinned on chalkboard 

]. We were having isosceles triangles of this nature, listen even in this isosceles triangles in isosceles triangles we can complete the isosceles triangle and make it a square or a rectangle by doing like this [demonstrating on chalkboard



] yes by doing like this. This is 5 cm this 5 cm this 6 cm this 6 cm 10 cm so what will be the area of this rectangle here? [Pointing to shape on chalkboard]. We want the area of this whole rectangle here. How are we going to get that area? How are we going to get that area? What is the length of this part? From here to here?

Class: 5 cm

Teacher: It is 5?

Class: It is 10 cm

Teacher: It is 10 cm. what is the formula of area of a a a of a of a rectangle?

Class: Length times width

Teacher: It is length times width. Our length is 5 cm

Class: 5 cm

Teacher: 5 cm and our height is?

Class: 6 cm, 10 cm

Teacher: It is 10 cm because 6 is not our height because it is slanting. Our height is..

Class: 10 cm

Teacher: This line is equivalent to this one ayalingana[they are equal] and now we able to find the area of this figure. Learner

Learner: 10 x 10

Teacher: 10 x 10 which gives us what?

Class: 100

Teacher: 100 what?

Class: Centimetres squared

Teacher: 100 cm^2 because we are talking about the area of a triangle we wanted the area of this triangle not of the whole rectangle we are going to divide this answer by two. We are going to say

$$\frac{100}{2}$$

Class: 50

Teacher: Two uyakangaphi ku 10[how many times does two go into 10]?

Class: Akayi [it does not go]

Teacher: How many times does two goes into 10?

Class: 5

Teacher: Then how many times does two go into zero?

Class: Zero

Teacher: Zero. This is 50 cm^2 the 50 we got here is the 50 we got where it is already erased oh here this 50 is equivalent to this one. Is it clear?

Class: yes

Teacher: let me give you another one. Learner 30 this one is for you

. Learner 30 this one is for you. Calculate the area of this triangle here. Here is chalk.

Learner 30: 7 × 3 = 21

Teacher: What is the first step before writing the formula Learner 30? What is the first step?

Learner 30: [completing the triangle to form a rectangle]

Teacher: Which is the base? Show us the width

Learner 30: [showing the width on the figure]

Teacher: Show us the length

Learner 30: [Showing the length]

Teacher: It starts from where and ends where?

Learner 30: [showing length on the figure]

Teacher: What is the length of the width? The length of the width is what? [Ehh]

Learner 30: [Showing the slant height] 7 cm

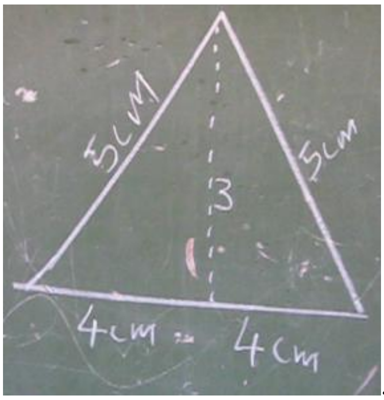
Teacher: From here to here is seven

Learner 30:

Teacher: From here to to here it is seven. From here to here is seven? From here to here is four. From here to here to here is

Learner 30: Ten

Teacher: Learner 31 come up front and help this friend of yours. Go and sit down

Teacher: Yes our length is 3 cm Yes our width is equivalent to the line which is perpendicular to the base which is 4 cm. Our width is 4 cm because we want the area of a triangle that's why we divided by 2 because we not talking about the area of this whole rectangle. Ok who is going to do this for us? Where is the duster? Learner ha Learner 32. Learner 32 come up front and help find for us this area of this 

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Learner 32: [Tries to complete the shape to form a rectangle but incorrectly]

Teacher: [Hawu hawu]

Learner 32: [Erases and starts afresh drawing it correctly this time] 4 + 4 = 8. 8×3= 64, $\frac{64}{2}$ [erases and writes] 64 cm^2

Teacher: Hurry up!

Learner 32: [Pointing at 64 cm^2]

Teacher: is this the answer? Sit down.

Class: Nooooo!

Teacher: is this correct?

Class: Noooooo!

Teacher: Learner 33 come and help her. She cannot multiply 8 times 3. 8 times 3 imagine she cant. 8 times 3 she can't, multiplication is done in grade 3.

Learner 33: $4 \times 4 = 16 \times 3 = \frac{25}{2} = 10\frac{1}{2}$ cm^2

Teacher: Sit down. Is this correct?

Class: Noooooo!

Teacher: Learner 34 come and help us. Ngumhlolo [it is unheard of]

Learner 34: $8 \times 3 = \frac{23}{2} = 11$ cm^2

Teacher: is this correct?

Class: Noooooo!

Teacher: People cannot multiply. Learner 35. Kwentenjani ke [what is the matter?]

Learner 35: $8 \times 3 = \frac{32}{2}$ [erases the $\frac{32}{2}$ and writes] $\frac{24}{2} = 12\ cm^2$

Teacher: He says that he says that the answer here is 12 cm^2 is he correct?

Class: Yesssss

Teacher: People are unable to multiply eight times three, eight times three. Our base here is 8 because 4 + 4 = 8 and our height which is the width is 3 cm 8×3 gives us 24 the 24 we are going to divide it by two and then get the 12 since we are not looking for the area of the whole rectangle but the area of the triangle. Thank you. You may go for break

Appendix K Zane interview transcript

Interviewer: Thank you for allowing me to observe your class. I have some questions I would like to ask you related to the classroom lesson and some general questions. Would you mind if tape the interview?

Zane: Yes, you can tape the interview.

Interviewer: Okay, Thank you...[ehh] because it help me stay focused on our conversation, and it will ensure I have an accurate record of what we discussed.

How long have you been teaching in this school?

Zane: [Ehh] In this school I've been teaching for nine(9) years, since 2006, so it's nine years in teaching profession. Yes.

Interviewer: Okay, What is your highest qualification achieved?

Zane: Currently I have the Primary Teacher's Diploma, but I am trying to get the degree now. Enrolling at the university currently.

Interviewer: What are your majors in the Primary Teacher Diploma?

Zane:[Ehhh] Unfortunately, I majored in languages, but when I came to the school, I taught maths.

Interviewer: Okay, Thank you. In what grade do you teach now?

Zane: I'm teaching grade five(5) and grade seven(7), both maths - Mathematics.

Interviewer: Okay, Thank you. Do you have any other position besides your teaching role?

Zane: No, currently I am just an ordinary teacher. I am not- I do not have any position. I hope maybe next time.

Interviewer: Are you not a sports teacher or conducting the choir, something like that?

Zane: No, I am a panel leader for maths. So I used to check tests, which are given to learners. So its I who check if they are correctly.

Interviewer:[Ohh] there is a panel at this school?

Zane: Yes, there is a panel.

Interviewer: And you are the head of the maths panel?

Zane: The maths panel. Yes.

Interviewer: Okay, thank you. Now please describe how you start teaching the topic Area of a Triangle.

Zane: [Ehh] When I start teaching about area of a triangle, I want the learners first to know what an area is, before we proceed.

Interviewer: Okay, what else do you do besides.

Zane: Even to know how to find the area, the different methods that we use to find the area, but they have to know area first, then we continue with the methods of finding area.

Interviewer: Okay, do you always follow the textbook to teach the learners?

Zane: Sometimes. It is not always whereby you find me [ehh] using the textbook. Sometimes I consult other books, because the information is not complete in the textbook. So I consult other book, so that I get more information for the learners.

Interviewer: Okay, can you give me an example of one book, or other thing that you use beside the textbook.

Zane: [ehh] For now, though I will not mention the name properly, but I do have a book [ehh] the old one it is with me. I can show you if you want to see it. [brief giggle] I have forgotten the name.

Interviewer: Okay, how do you support the learner thinking through teaching the area of a triangle?

Zane: Yes, I try because when you are a teacher, sometimes you teach a concept and learners tend not to understand what you are teaching. So, if I see that this one doesn't understand, I used to make groups. Believing that learners learn easily from one to another. So if, maybe they don't understand, I try to organise them according to their groups. Then they able to explain to one another then they try.

Interviewer: Okay, thank you T. How do you teach your learners [ehh] to learn about calculating the area of a triangle?

Zane: Yes, learning of a triangle is difficult because [ehh] they can use the counting method, also they can use the formula – half base times height, but I usually start with the counting method. Teaching them that when counting they have to concentrate on the full squares and those half squares separately. So they start by counting the full squares and the half squares separately then they combine the two scores which will give them the total area of that figure.

Interviewer: Okay, let's say now there are no squares. How do you teach that one?

Zane: In the case where there will be no squares, it means there are halves and that is an estimation because you cannot get the exact answer if there are no full squares. So they use the halves and multiply how many halves are there, [ehh] the answer will not be an accurate one, since there are no full squares. So I recommend the answer which is close though it is not accurate.

Interviewer: Okay, thank you T. I mean when there is no grid.

Zane: When there is not grid?

Interviewer: Yes, just a dimensions given.

Zane: Alright, when there is no grid. I give them the formula. So in the area of a triangle, for example – they may be given a height and base. So now, I introduce the, the formula half base times height. Then I show them the base and height, so they must be able to identify base and height while they are given the formula. Children are very smart, they can be in a position to find area using the formula.

Interviewer: Okay, Thank you. In your opinion, what are the main aims of learners when learning about area?

Zane: Yes, [ehh] maths is done everywhere, at home, at school. So teaching about area might help them, maybe in future. [ehh] For example, let's assume maybe the adult they want to furniture their houses, they were putting tiles there. So in order to know how many tiles are needed they have to know the concept of area because they have to multiply length times breadth finding the number of tiles needed to cover that surface.

Interviewer: Okay, Thank you. [ehh] What is your classroom technique to easily teach the calculation of area of a triangle?

Zane: Area of a Triangle [ehh] My technique?

Interviewer: Yes

Zane:...is to use the formula.

Interviewer: Ohh it is the formula?

Zane: Yes, I use the formula. [ehh] Sometimes, when I introduce them I start with what they know about the... especially the area of a rectangle they know its length times breadth. Then I draw the diagonal lines [ehh] in that triangle then that, so that it form a triangle now. So it means now there are two triangles now. Now, within a rectangle, so now I start by the rectangle then I continue [ehh] to triangle.

Interviewer: Okay, Thank you T. From your own point of view, what are the main reasons why the learners should learn the content or the concepts regarding the area of a triangle, now?

Zane: [ehh] In the Primary level, it helps them so that maybe when they are getting into Secondary Schools or High school. They do have the base because maths tends to be difficult in the High school. So once they do not catch the concept of area in the primary they won't be able to make it in their High school. So, the Primary level is where I make sure they do have the concept. Teaching them how to find area, what is area. So that not becomes new when they reach it in their Secondary School.

Interviewer: Okay, how do you prevent [ehh] where there are misconceptions when teaching the concept of the area of a triangle?

Zane: Yes, to prevent this, I teach one at a time. I do not teach area of a rectangle, area of a triangle, area of a square at the same time. So to prevent this in a week, maybe I will take one week to talk about triangle. Because, learners tend to mix if you teach them about the concept- about different concept within a week. So we have to be specific, teach about one concept at a time then later you can touch on the other aspects like, bo'Rectangle and squares.

Interviewer: Okay, is there any common error that you have noticed when teaching the area of polygons?

Zane: Yes, there is. They tend to mix the formula for finding area of a rectangle with that of a triangle. Meaning that they can't even know the shapes, because, if I ask them to find the area of a rectangle, then they say half base times height. So that is where I have discovered a problem among the learners. So, they do not make a difference between what formula is used for finding area of rectangle [ehh] which one Is used to find the area of a triangle. So they tend to mix the two.

Interviewer: Okay T, then how do you avoid that one? Do you help them overcome that mixing?

Zane: Yes, to avoid that one – it needs a teacher to go back and classify those shapes because, it means they lack knowing the shapes. So you have to go back and teach them. Now, a rectangle is different from a triangle. So that's the problem is, they don't know which one is which.

Interviewer: Okay, thank you T. Now, how do you know that the learners understand the idea or the concepts you teach?

Zane: [ehh] No, when- I know when they understand [ehh] according to their positive response they give me as a teacher. So once I teach, once they are able to respond positively, everyone in the class, maybe three(3) quarters of the class are able to answer it correctly. Now, I am sure because, now they have absorbed the concept.

Interviewer: Okay, by raising up their hand?

Zane: By Yes, they indicate by raising up their hands. So, they just rush for- so that the teacher can point at them so you can see now that they all understand now.

Interviewer: Okay, do you confirm individually?

Zane: Yes, I do so. [ehh] I do confirm individually because you find that sometimes there are those students who need special attention. So, if I see that now this one needs me, which I call it a 'conference with the teacher' so, I call them, they see me at the stuff room then I try to help that particular learner.

Interviewer: Okay, then the last thing. What are the reasons do you recommend to learners in order to learn about the area of a triangle?

Zane: The reasons to recommend?

Interviewer: [ehh] yes, resources.

Zane: Resources?

Interviewer: That you recommend to your learners?

Zane: Okay, there are many different resources that children can use in order to upgrade their learning, but I prefer in our days, learners to consult internet because, our children now they can learn through internet. So, a child who is exposed to internet performs better than those who are not exposed to internet. So, they can consult internet.

Interviewer: Okay, Thank you T. Thank you for your time.

Zane: Thank you.

Interviewer: Thank you very much.

Length of Audio recording: 11 minutes and 32 seconds

Appendix L **Patrick's interview transcript**

Interviewer: Thank you for allowing me to observe your class. [ehh] I have some questions I would like to ask you related to the classroom lesson and some general questions.

Patrick: Yes.

Interviewer: Would you mind if I tape the interview?

Patrick: I wouldn't mind, you can tape it.

Interviewer: [ohh] It will help me stay focused on our conversation and it will ensure an accurate record of what we have discussed.

Patrick: Okay.

Interviewer: How long have you been teaching in this school?

Patrick: I have been teaching in this school for three (3) years – Three (3) and a half years. Yes.

Interviewer: Okay

Patrick: Three(3) and a half years.

Interviewer: Okay, what is your highest qualification achieved, T?

Patrick: My highest qualification is a Primary Teacher's Diploma, I got it at Ngwane College.

Interviewer: Okay, what were your majors?

Patrick: I majored in Maths and Science which are Pure Sciences.

Interviewer: Okay, and what grade do you teach now?

Patrick: Now, I am teaching grade seven(7) science, grade six(6) science and grade six(6) mathematics.

Interviewer: Okay, do you have, do you have another position besides your teaching role, for example sports teacher, music teacher?

Patrick: Yes, I have. [uhmm] another position I'm holding is a – I am the Chairman of the Cluster Mazombizwe, Mbukwane, of the schools Mazombizwe, Mbukwane, Tfokotani, Jopha, and Mahamba. I am the Chairman of those, of the Mahamba Zone Cluster.

Interviewer: Okay, what are the activities of that cluster you Chair?

Patrick: The activities of the Cluster is to share ideas which are educational and some of them – and playing other types of sports together and doing debates. [uhmm] and all the stuff.

Interviewer: Is Mathematics one of the things you discuss - how to teach mathematics?

Patrick: Where?

Interviewer: In the Cluster.

Patrick: In the Cluster?

Interviewer: Yes, where you Chair.

Patrick: Yes, we are discussing also that, but is just that I have started to be the Chairman some few months back, some few months backs and we have been [uhmm] trying to create a bank account, so that our cluster can move easily so that all the schools can subscribe, so that there is lot of money which will be used in different activities in our cluster.

Interviewer: Okay, Thank you T. Now, please describe how you start teaching the topic area of a triangle.

Patrick: Area of triangle, learners are to know the area of a rectangle or a square which is in a form of squares and they are to count the number of squares, then move on to – Yes. So that.. And you might find – so that they may be given, and also may be given the length of the rectangle and the width of the rectangle. That's all I can say about the introduction of the topic. They are to know the length and the width and be able to multiply them.

Interviewer: Okay, do you always follow the textbook to teach the learners?

Patrick: In most of the time I don't.

Interviewer: [ohh] do you use other methods?

Patrick: Yes, I use other methods.

Interviewer: Can you please explain them for me?

Patrick: [uhh] other method were the discovery method in some topic is one of the methods I used. [uhh] Because, [uhmm] you may find that the spiral approach is better to use it. In the sense that, another topic when you are talking about maybe [uhmm] let me say the – we are talking about numbers, writing them in numerals. Then the following, writing them in words then so you may combine those topic and may find that you can speak once and they are able to, to do another thing besides being given what they are to do.

Interviewer: Okay, thank you. Do you support learner thinking through teaching the area? How do you support learner thinking through teaching the area of a triangle?

Patrick: How do I support learner thinking? [uhm]

Interviewer: Yes, how do you support learner thinking while you are teaching the area, so that they can grasp the concept.

Patrick: Yeah, if they are – if I have to give them a little bit of work to do as we are discussing on the board and discover some errors, they are – they come across. [uhmm] I am going to explain that error that – I am going to explain the error to the pupils so that they can see all of them that this is not the correct way of doing this thing, it is done like that. And that you may find that you may give the pupils, you may find that I sometimes give pupils [uhmm] prerequisite knowledge which something they learned in grade five(5) then starting by working towards it then given them something which is current without teaching them, but expecting them to give, to give relative answers which in applying what they have learned in the other grades and also help me to, to correct the learners in that way.

Interviewer: Thank you, Thank you T. Then how do you teach your learners to learn now, about calculating the area of a triangle?

Patrick: Now, the greatest thing I have to emphasis on pupils are to know the base which is the length and the height, which is the width of a – they are to know how to calculate the area of a rectangle or a square. If they are not given the...[ehh] whether, they are given the, the squares, they are to count if they are given length. They are to be able to identify the length and they are to identify the length and they are to be able to identify the width. Then multiply those two, then divide the answer by two.

Interviewer: Okay, is there anything you want to add?

Patrick: [uhh] I don't think so.

Interviewer: Okay, Okay, Thank you. In your opinion, what are the main aims for learners on learning about area?

Patrick: [uhh] The main aims is for them to be able to share things or even to be, even to use that information in our daily life experiences. For example, another teacher was being told to buy a plot which he was given the area, it is so many metres squared then he was not there, but he was charged a very large sums of money. Then he consulted, what –how much is this area? Then I told her, this is something like this, then he discovered. [ohh] I can be robbed here then he moved on.

Interviewer:[brief giggle] Okay, thank you. Thank you. Now what, what is your classroom technique to easily teach the calculation of the area of a triangle?

Patrick: [uhh] Classroom technique [uhh], [ai] My classroom technique is to move from simple to complex, from concrete to abstract.

Interviewer: Okay, can you make that one in relation to the teaching of the area?

Patrick: Yes, I moved from concrete to abstract by using the, the shapes. Which has got squares in it, then cut it into two equal triangles. Then tell pupils to find the area of those triangles. Then they were able to count the number of squares. So that, they were able to see that –[ohh], half of this triangle is from the, it can be gotten by dividing the number of the whole squares of the rectangle.

Interviewer: Okay, into two equals parts?

Patrick: Into two equal parts.

Interviewer: Okay, from your own point of view T, what are the main reasons for why learners would learn the content or concepts regarding now, the area of a triangle?

Patrick: [uhh] for the learners to know is for them to share things. In order for them to share things which are - they are using in everyday life, maybe, if they are to cut [uhmm], [ehh], a material from home economics if they are given the area, they are to know how much are they to cut the, the, the material.

Interviewer: Okay, if I get you well, you mean it helps them in other subjects?

Patrick: Yes, It's also helps them in the other subject.

Interviewer: Okay, is there anything you can say about Maths –Mathematics?

Patrick: What?

Interviewer: About Mathematics, is there where the learners can use the concept of area of a triangle?

Patrick: In?

Interviewer: In Mathematics.

Patrick: In Mathematics? [uhmm] not really, because, in Mathematics, we are always solving – given problems, but practically we are not. Practical problems are being solved in other subjects like Home Economics, for example in Mathematics you may given that – this boy is having a chocolate which is four(4) centimetres, by five(5) centimetres. Then the other one is given a chocolate five(5) centimetres by eight(8) centimetres. How much more chocolate is this one having to the other one. That all you can say in Mathematics, but in Home Economics you may be given the real thing then you are given –then you are asked to cut the, that's given and asked to, to cut that given area.

Interviewer: Okay, how do you teach the concepts - that is area?

Patrick: The concept?

Interviewer: Yes.

Patrick: [uhmm] The concepts of area [uhmm], it is taught...

Interviewer: Maybe, you can added, how do you prevent the learners' misconceptions or errors that you know when teaching the area of a triangle?

Patrick: [ehh] When – to prevent misconceptions it is, it is [uhmm] very important when writing – when giving the pupils the, the triangle. Given them all the sides of the triangle so that they are able to pick the, make them able to pick the length and the width because, if you are always going to given them they will not be able to pick the length and the width. So if you train them to, to be able to identify the length, able to identify – so they are able to identify the height which is can also be the width. That one can make them realise that the width is this and the length is this. Errors are then minimised by that way.

Interviewer: Okay, Okay how do you know, know that the learners understand the idea or concepts you teach?

Patrick: Okay, if I know. I give more examples on the board then if most of them and even the slow learners are able to identify at least the length and the width. It is now better if they are now able to at least identify, the slow learners. But, if the one that are smart are - if they are able to calculate and come up with the answer, those ones I can, make sure I am able – I have –I taught them

Interviewer: You, you give them some work?

Patrick: Yes, I give them some work and evaluate it.

Interviewer: Okay, thank you T. What other resources do you recommend to the learners in order to learn about the area of a triangle? For example, do you recommend them to use newspapers, go inside the library or to the internet?

Patrick: [uhmm] The library we got here I also encourage them, they are taking books – English books and also Mathematics books. They are here in the library, but most of them are a little bit tougher than them, since they are from Amer- from America. And they are and...

Interviewer: They are donated books?

Patrick: They are donated books, Yes. America and Rome there. So but, their enjoying. The problem is that pupils thought that Mathematics is never studied.

Interviewer: [ohh]

Patrick: Yes, they do. I even asked them today, did you study Mathematics. Bathi, “how can you study Mathematics”? [brief giggle] I say...

Interviewer: Okay, okay T. Thank you. Thank so much for your time.

Length of Interview: 14 minutes and 12 seconds.

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