The Use of Visual Reasoning by Successful Mathematics Teachers: A Case Study

Vishamlal Ramtahal Budaloo

Learner No. 2 000 00076

A thesis submitted for the degree of Doctor of Philosophy in the School of Science, Mathematics and Technology Education (SSMTE)

University of KwaZulu-Natal

Edgewood Campus

Supervisor: Dr. V. Mudaly

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Declaration

I, Vishamlal Ramtahal Budaloo, declare that the research reported in this thesis, except where otherwise indicated, is my original work, and has not been submitted for any other degree or examination at any other university.

_____________ _____________

V.R. Budaloo Date

_____________ _____________

Supervisor Date

Dedication

To My Spiritual Master,

HH Giriraj Maharaj

My wife Rene,

and my daughter, Vandena

Acknowledgements

I am deeply indebted to my Spiritual Master and the Supreme Lord for their inspiration and guidance.

In presenting this thesis, I wish to express my sincere gratitude and appreciation to each of the following individuals for their invaluable assistance:

Dr V. Mudaly, my supervisor for his excellent guidance, constant motivation and words of encouragement from the inception of this research study to the final preparation of this thesis. The expert advice given by Dr Mudaly and his phenomenal insights in Mathematics, have helped me to achieve my goals.

My wife Rene, for her constant motivation and dedication. She has been my pillar of strength during these demanding years. I salute her for her patience and willingness to shoulder all other responsibilities during this period as well. I shall always remain grateful to her for her many sacrifices.

My daughter Vandena, for having the patience to understand me during the period of my study. Her determination to succeed at a gruelling career path herself, kept me focussed during the past four years. She constantly reminded me of the need to have the endurance and determination to succeed.

My colleagues and many friends for their encouragement and constant support.

The teachers of Mathematics who participated in my study, for their cooperation, time and patience during my interviews and classroom observations.

The PhD cohort for the many hours of important debates and most useful feedback.

Dr S. Ramson for his expert editing and brilliant suggestions.

Ethical Clearance Report

UNIVERSITY OF KWAZULU-NATAL™
INYUVESI YAKWAZULU-NATALI

12 October 2012

Mr V R Budaloo 200000076
School of Education and Development
Edgeood Campus

Dear Mr Budaloo

Protocol reference number: HSS/1054/012D
Project title: Exploring the use of visual reasoning as a pedagogical tool by successful mathematics teachers in secondary schools.

Expedited Approval

I wish to inform you that your application has been granted Full Approval through an expedited review process.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number. Please note: Research data should be securely stored in the school/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....
Professor Steven Collings (Chair)

/px

cc Supervisor Dr V Mudaly
cc Academic leader Dr D Davids
cc School Admin. Mrs S Naicker

Professor S Collings (Chair)
Humanities & Social Sc Research Ethics Committee
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban, 4000, South Africa
Telephone: +27 (0)31 260 3587/8350 **Facsimile:** +27 (0)31 260 4609 **Email:** ximbap@ukzn.ac.za / snymanm@ukzn.ac.za

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The Use of Visual Reasoning by Successful Mathematics Teachers: A Case Study Vishamlal Ramtahal Budaloo Student No. 2 000 00076 A thesis submitted for the degree of Doctor of Philosophy in the School of Science, Mathematics and Technology Education (SSMTE) University of KwaZulu-Natal Edgewood Campus Supervisor: Dr. V. Mudaly October 2015 i Declaration I, Vishamlal Ramtahal Budaloo, declare that the research reported in this thesis, except where otherwise indicated, is my original work, and has not been submitted for any other degree or examination at any other university. ________________ V.R. Budaloo ________________ Date ____________ Promoter ___________ Date ii Dedication To My Spiritual Master, HH Giriraj Maharaj My Wife Rene and my Daughter Vandena iii Acknowledgements I am deeply indebted to my Spiritual Master and the Supreme Lord for their inspiration and guidance. In presenting this thesis, I wish to express my sincere gratitude and appreciation to each of the individuals for their invaluable assistance: Dr V. Mudaly, my supervisor for his excellent guidance, constant motivation and words of encouragement from the inception of this research study to the final preparation of this thesis. The expert advice given by Dr Mudaly and his phenomenal insights in mathematics, have helped me to achieve my goals. My wife Rene, for her constant motivation and dedication. She has been my pillar of strength during these demanding years. I salute her for her patience and willingness to shoulder all other responsibilities during this period as well. I shall always remain grateful to her for her many sacrifices. My daughter Vandena, for having the patience to understand me during the period of my study. Her determination to succeed at a gruelling career path herself, kept me focussed during the past four years. She constantly reminded me of the need to have the endurance and determination to succeed. My Colleagues and many friends for their encouragement and constant support. The Mathematics teachers for their cooperation, time and patience during their interviews and classroom observations. The PhD. Cohort for the many hours of important debates and most useful feedback. Mr Ramson for his expert editing and brilliant suggestions. iv Ethical Clearance Report v Turnitin Report vi Abstract Visualization has become increasingly important in view of the advent of technology which allows us to understand an idea at a single glance (Bosveld, 2005). Globally, educationists have been searching for ways to improve students' understanding of mathematics. This has led them to believe that perhaps the focus on mathematics teaching needs to change. Contrary to the view that mathematics can only be presented sequentially, there is another view that mathematics is a multimodal discourse where different modes of representation are necessary. The protagonists of this view believe that visualization when accompanied with algebraic linearity can offer a solution to the worldwide crisis in mathematics. Whilst many of our students experience serious difficulties in mathematics, as testified

Abstract

Visualization has become increasingly important in view of the advent of technology which allows us to understand an idea at a single glance. Globally, educationists have been searching for ways to improve learners' understanding of mathematics. This has led them to believe that perhaps the focus on the teaching of mathematics needs to change. Contrary to the view that mathematics can only be presented sequentially, another view exists that mathematics is a multimodal discourse where different modes of representation are necessary. Whilst many of our learners experience serious difficulties in mathematics, as testified by the education authorities, there are groups of teachers that attain outstanding results in this subject. In this study I chose to explore how this group of people attain success, whilst engaging with the phenomenon of visual reasoning.

The intention of this research was to interpret the kind of meanings that these successful teachers offered in their use of visual reasoning in their practice. Therefore in order to explore the personal, social and learning experiences of these successful mathematics teachers, a case study form of enquiry was employed, with the use of a social constructivist research paradigm, following a qualitative research tradition. A purposeful maximal sampling technique was used to identify five participants. Knowledge is not merely received but constructed by individuals or groups of people who try to make sense of their experiential worlds. This study sought to explore the ways in which these successful teachers constructed and interpreted the knowledge that they passed onto their learners, using the phenomenon of visual reasoning. Furthermore, constructivism as a research paradigm is characterised by plurality and multiple perspectives. Taking this into account, Attribution Theory, Gardner's Multiple Intelligences Theory and Situated Cognition Theory were used as a lens to explain the complex phenomena of visual reasoning and successful teaching.

The results of the study showed that all the participants were actively engaged in using visual reasoning as a pedagogic practice in their mathematics classrooms. Explanations for their actions in terms of using visual reasoning resonated with the literature and the theoretical frameworks that were used.

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Chapter 1
Background

1.1 Introduction

Contentions exist between two schools of thought about the nature of mathematics and how it should be taught. On the one hand, there are groups of mathematicians who believe that mathematics involves symbols and should therefore be taught using language, whilst another group who believe that mathematics is about images and that it should be taught using visual approaches (Zimmerman & Cunningham, 1991). Yet again, there are those that argue that both approaches are necessary. The protagonists of the visual models claim that visual reasoning tends to improve the understanding of mathematics (Eisenberg & Dreyfus, 1991). There has been a vast amount of research conducted and the results seem quite diverse. Visual reasoning is a broad term, encompassing a range of visual strategies that are used to convey meaning to mathematical concepts. According to Lavy (2006, p. 26) ―visualization is a process of construction or use in geometrical or graphical presentations of concepts, or ideas built by means of paper and pencil, a computer software or imagination". It uses pictures, diagrams, and images that could be drawn by hand or through the use of technology. It also incorporates the use of gestures and body language. It is concerned with objectification and representing abstractions from reality. Seen from this perspective, visualization surpasses the biological notion of vision. Mathematicians who advocate this type of reasoning believe that it offers solutions to many of the problems that exist in mathematical understanding. On the other hand proponents of the symbolic model of mathematical reasoning, contend that visual reasoning does not foster logical and coherent thinking. Over the decades, these debates have continued. My research does not claim that one method is better than another, but it may motivate that a combination of methods may be useful to increase understanding.

My focus in this research was to identify successful teachers (through the pass rates of learners) and then identify the visual pedagogical tools they used in their teaching. Whilst success can be defined in many ways, I have used it in the context of the teacher producing good examination results. In this qualitative study, in which I use a constructivist view, I explore the pedagogical models used by successful mathematics teachers, in KwaZulu-Natal, South Africa. In this chapter the rationale for my study will be discussed, the key questions will be outlined, a brief description of the methodology will be presented, and the literature of the differing schools of thought on the teaching of mathematics will be reviewed. The chapter will conclude with an overview of the dissertation.

1.2 Interrogating the Research Problem

In South Africa, the results of mathematics in the National Senior Certificate Examination has often been a concern for many individuals (curriculum developers, academic researchers, tertiary institutions, teachers, recruitment agencies in the corporate world, parents and learners), including myself, as a teacher of mathematics (Ndlovu & Mji, 2012). Despite the urgency for the improvement of mathematics results, all interventionist efforts seem to be relatively unsuccessful. The Senior Certificate Examination reports indicate disappointing results. In the National Senior Certificate, Technical Report (2014, p 68), the results in mathematics for the last 5 years were as follows:

Table 1: National Senior Certificate Averages in Mathematics: (2009 -2013)

Year	2009	2010	2011	2012	2013
% of learners attaining above 30%	46	47.4	46.3	54	40.5

It must be noted that the pass mark in mathematics was taken as 30%. Despite this, the overall averages indicate disappointing results in mathematics. In a list of suggestions in the National Senior Certificate Diagnostic Report (2012, p. 122) it was reported that ―basic mathematical principles and rules need to be practised continually". Furthermore, the report also comments on ways in which teachers could overcome the difficulties learners were experiencing in mathematics. According to the National Senior Certificate Diagnostic Report (2012, p. 127)

> ―(a) Teachers must expose learners to a variety of different questions involving rates of change, including rates of change in practical situations. For this purpose, teachers should use questions that have appeared in past examination papers. With sufficient exposure, learners should gain confidence in answering these questions.

(b) Teachers need to show learners how to extract essential information from the question and how to translate these words into the language of Mathematics".

Similar reports have been made over several years. Despite these diagnostic analyses and recommendations to teachers, results in mathematics have not shown a marked improvement (National Senior Certificate Examinations Technical Report, 2014); The Annual National Assessment Examinations for grade nine learners (2014); Third International Mathematics and Science Study (Mullis, Martin, Foy & Arora, 2012). Currently, some effort is being made nationally and internationally to change the image of mathematics (Govender, 2014). Through the introduction of mathematics education in many pre-service teacher-training modules, it is hoped that the image of mathematics will undergo a positive transformation. Furthermore, by introducing more authentic assessments in the mathematics classroom, it is envisaged that mathematics will become more acceptable to the vast majority of learners. Other initiatives, such as encouraging mathematics learners to participate in mathematics competitions, are designed with the intention of presenting the diverse nature of mathematics. It is also a means of encouraging learners to appreciate the relevance of mathematics in solving problems that plague many of our societies. The view that learners currently have of mathematics is that it is often difficult and only designed for a few learners, possibly only the high achievers. It is possible that the ways that learners view mathematics may be impeding their progress in the understanding of mathematics. Therefore, the study focussed on finding alternate ways, such as using visual strategies to enhance the view of mathematics, since much of the literature seem to suggest the benefits of using visual approaches in the teaching of mathematics (Malaty, 2008; Chiappini & Bottino, 2007).

The search for more effective ways of teaching mathematics is paramount for most mathematics researchers. The advent of technology seems to have given many teachers new hope, in the teaching of mathematics. The influence of technology cannot be ignored. In this regard, there are many research projects which advocate the use of visual approaches in the teaching of mathematics. However, not all research has shown a positive correlation between visual strategies and mathematical understanding (Duncan, 2010) and Palais (1999). A careful analysis of these research projects was conducted, as part of my literature review, in order to understand the context of the research and the use of visual strategies in the teaching of mathematics. The debates surrounding the use of multiple forms of representations are based on deep theoretical foundations. As such, it demanded some exploration. Visual reasoning also has a deep theoretical basis, and has been extensively investigated by psychologists, since it relates directly to learning from a mental perspective. If it has a bearing on learning, ultimately it would mean an adjustment on the part of teaching.

The poor results in South African schools has also created some concern, where the different stakeholders of education felt that it was becoming more difficult to attain success and as a result, researchers have to now constantly look for newer ways to address this concern (Fleisch, 2008). Despite this, there was still a group of very successful teachers who, despite all odds were achieving success in their classrooms. This trend was particularly evident in the school that I had been teaching at, at the time of this study. I posit that this group of teachers were employing some pedagogic tool or tools that ensured success in their classrooms. I explored their pedagogy with a view to determining the extent to which they were using visual reasoning as a pedagogical tool. Visualization, according to Zimmerman and Cunningham (1991, p. 4) is more than “math appreciation through pictures”. They further contend that “one must learn how ideas can be represented symbolically, numerically and graphically and to move back and forth among these modes” (Zimmerman & Cunningham, 1991, p. 4). Visual reasoning cannot simply be seen as the ability to see. I chose to observe the constant iteration between the visual and symbolic aspects of mathematics during the teaching process.

1.3 Rationale for the Study

The rationale for this study has been derived from personal and contextual imperatives. My personal interest derives from the fact that I have been a mathematics educator for the past 29 years. During this period, I have taught at public, ex-model C and private schools. As a result of my experience and the level of my competence, I was head-hunted by these prestigious schools, to serve as a mathematics teacher. I served on an advisory level on most of the management teams at these schools.

Each of these institutions varied in terms of their infrastructure, resources and management styles. The learners in these schools came from divergent backgrounds: lower, middle and upper socio-economic environments. However, in each of these schools, the following common trends were evident: These observations were made, taking into account my position as a senior mathematics teacher. First, there was generally a negative perception of mathematics held by learners. According to Dogan (2012), the way that learners perceive mathematics can affect their understanding of concepts. Furthermore their anxieties could also affect their perception of mathematics and their success in the subject. The second trend that emerged was that some learners seemed to have struggled with mathematics. My passion for the subject, led me to believe that these issues ought to be explored. I strongly believed that, with the shift of focus on multiple forms of representation, visual reasoning had an important part to play in addressing these concerns, since visual reasoning may not have been used extensively.

From the contextual point of view, there was a world-wide and national drive to develop more competent mathematics teachers. According to Darling-Hammond (2005, p. 237),

> –around the world, the importance of education to individual and societal success has increased at a breathtaking pace as a new knowledge-based economy has emerged. As a consequence, most countries have been engaged in intensive reforms of their education systems, and many have focused especially on improving teacher education, recognizing that preparing accomplished teachers who can effectively teach a wide array of learners to high standards is essential to economic and political survival".

During my years of practice, I was often called upon to assist the Department of Education in in-service teacher training. During my presentations and in my interaction with several teachers I observed that there were many differences in the competency levels of these teachers. This was also visible in the results of mathematics at the different schools. I believed that these results were often an indication of the kind of teaching. In my quest to obtain answers to these problems, I chose to focus of those schools that were performing very well in mathematics. The pass rate for mathematics in these schools was over 70% in the grade 12 exit examination, whist the national pass rate fluctuated between 45% and 55%. I also believed that these results were an indication of the kind of teaching that was taking place. In my opinion, these teachers were engaging in successful practice.

It is these personal and contextual imperatives that have impelled me to engage in this study. This study has important ramifications for the improvement of mathematics results, and for informing education policy in mathematics teaching and learning.

1.4 Context of the study

According to Barab and Squire (2004, p. 1)

> –if one believes that context matters in terms of learning and cognition, research paradigms that examine these processes as isolated variables within laboratory or other impoverished contexts of participation will necessarily lead to an incomplete understanding of their relevance in more naturalistic settings".

In order to fully understand the practice of these successful teachers, a study of the context was necessary. Descriptions of the location of the research site, the school and the mathematics team was necessary, since each participant operated within a certain milieu, which was determined by certain dynamics, which ultimately affected the way in which they used visual reasoning in their practice.

1.4.1 Location of the research site

Manor College (pseudonym) is located on the very serene and peaceful suburb along the east coast of South Africa, about 35 kilometres away from the city centre, on a large piece of land, about three kilometres away from one of the national roads. Its lush greenery, neatly trimmed fields and beautiful architecture immediately gave me the sense that I was entering a peaceful environment. The almost one kilometre long entrance to the school, with modern building of the pre-primary and primary schools, stood in strong contrast to the research site. The buildings at the research site were similar to that of a holiday resort. The classrooms were designed in the shape of rondavels with high-gabled roofs. On entering this site one immediately felt a sense of relaxation. As you drove on the tarred road leading up to the beautiful campus, you observed a large swimming pool, tennis courts, and several parking lots with several parking bays.

1.4.2 The Research site

Parking for guests and parents were immediately in front of the reception area. A large glass feature with water cascading immediately drew your attention to the name of the school. Reception was located immediately on entering this beautiful school. A receptionist attended to you as you entered the double glass doors. This was also the block which housed the administration of the school. The Principal, Deputy Principals and staff Offices were located in this building. The classrooms, laboratories, hall, media-centre, sport offices, cafeteria and other specialised rooms formed a large, almost hexagonal enclosure. In the middle of this enclosure was an open field for learners to enjoy their breaks. Several benches surrounded this area. A large sports field was located further away from the classrooms. There were many bathrooms for the learners, ample storage lockers for learners as well. The learners were very friendly and immediately greeted visitors and members of staff on their arrival. Each classroom had its own character. The individuality of the teachers was often seen in their classrooms. Pictures, posters, resources and artefacts decorated most of the classrooms. The buildings were immaculately maintained. The gardens were fully landscaped, with huge trees providing a cool environment. The school was well maintained and very well-resourced, as was evident in the many facilities.

1.4.3 The Mathematics Team

At the time of this research study, I was one of the six members of the Mathematics Team at Manor College. The team comprised of the following members (all pseudonyms used): Valerie, Betty, Nellie, Sam and Karl. The management structure at Manor College did not cater for a head of the department. The management structure essentially comprised a principal, two deputy principals, and a grade controller for each of the five grades from 8 to 12. Subject teachers were grouped into teams, such as English Team, Mathematics Team and so forth. The two deputies were responsible for supervising these teams amongst themselves. In the absence of a traditional head of department, each member of the Mathematics Team was responsible for themselves. It was the philosophy of this College that the teachers that they employed were very responsible and could manage themselves. The Mathematics Team was very structured and met regularly to discuss the various aspects relating to mathematics at the school. Issues such as the allocation of teaching loads, time-tabling, classroom duties, record-keeping, textbooks, stationery requirements and a host of other administrative duties were discussed. There was mutual respect amongst the members of the Mathematics Team and they co-operated with each other very well. They often shared the work load per grade, by assigning grade leaders who would take care of each of the grades from 8 to 12. These leaders were responsible for resource materials, assessments tasks and examinations of their grades. Although resources were selected by these grade leaders, teachers were not compelled to use them. Teachers had the freedom to choose their own resource materials for their learners and teach accordingly. By meeting regularly, teachers covered common aspects of the syllabus and this facilitated common assessments and examinations to be held. The Mathematics Team also attended regular workshops and orientation seminars. The Team worked as a cohesive unit.

1.4.4 The learners at Manor College

The learners who attended Manor College came from affluent homes in the majority of cases. The majority of the parents were quite wealthy as they were able to pay the average school fee of 65000 rand (ZAR) per annum. There were a handful of learners who were granted scholarships and bursaries from the local industry. Because of these bursaries, they were expected to maintain a minimum academic record. The learners were respectful and a fair percentage of them were highly motivated academically and on the sports field. At the same time, there were those learners who struggled academically. It was a common practice for the teachers to provide support lessons during their free time and after school. The school offered boarding facilities; hence approximately fifty learners of the total enrolment of 450 were boarders. Often their parents worked in other provinces or even other countries. Approximately, 75% of the learners did mathematics, whilst the remaining 25% chose mathematical literacy. However, of the 75% of the learners who did mathematics, a further 25% of these learners struggled to cope with the demands of mathematics but refused to choose mathematical literacy for several reasons. However, despite these constraints, the teachers were able to achieve the desired results at the end of the grade 12 exit examinations.

By describing the background, rationale and context of this study, it is hoped that the research problem becomes more meaningful. Furthermore, I hoped that it acted as a motivation for examining the practice of these successful teachers through the use of visual reasoning.

1.5 Research problems and objectives

The critical questions used to substantiate this research are:

1.5.1 Critical research questions

1.5.1.1 What is the nature of visual reasoning and how is it used by successful mathematics teachers in mathematics classrooms?

The reviewed literature indicates that visual reasoning is a broad concept. There are divergent views of this phenomenon and there is no unitary way to define it. According to Natsheh and Karsenty (2014, p. 110) visualization is a “broad term that incorporates a spectrum of cognitive processes”. Presmeg and Balderas (2001, p. 291), on the other hand believes that “the use of visual imagery, with or without drawing diagrams is called visualization”. Against this backdrop, it is very likely that the participants may not necessarily share the same views of visual reasoning. Having established this as my point of departure, I therefore believed it was necessary to explore this phenomenon more deeply so that a common understanding of this phenomenon could be arrived at. It would make sense then, to explore the extent to which this phenomenon was being used by these teachers. Since I chose to focus on successful teachers and their implementation of the phenomenon, I wanted to then ascertain their reasons for using visual reasoning, hence I believed that the second critical question should address this issue.

1.5.1.2 Why is visual reasoning used by successful mathematics teachers?

I believed that the first critical question was relevant and necessary to understand the participants‘ views of visual reasoning. I was of the opinion that only through an in-depth understanding of their views of visual reasoning will I be able to grasp their use of the phenomenon. I also envisaged that the first critical question may only address the practice of these teachers. This study however, was conducted specifically with a view to establishing why these teachers used visual reasoning in their practice. In trying to understand their reasons, I was hoping to gather a glimpse of a deeper understanding of how it contributed to their success.

1.6 Significance of the study

In an attempt to improve the performance of learners in mathematics and bring about a positive change in their attitude towards mathematics, this study chooses to focus on a group of successful mathematics teachers with a view to establishing their good practice in the process of using visualization. Although research has been done on visualization, case studies offer unique advantages which will be explained in detail in chapter three. However, for the purposes of highlighting the significance of this study, case studies will offer the advantage of an in-depth analysis of the actions of these successful teachers. Since case studies focus on depth of understanding rather that a superficial understanding of the phenomena that is being investigated, it will be beneficial to the following individual(s):

- Curriculum-planners and developers with a view to informing them on the degree to which policies that has been introduced are being implemented by teachers and the reasons for such forms of action.
- Subject-advisors and tertiary institutions engaged in teacher-training with a view to designing appropriate in-service and pre-service programmes to support the use of visual reasoning as a complementary pedagogical tool in the teaching of mathematics.
- Curriculum-material designers with a view to incorporating visual approaches in their mathematical content and assessments.
- Teachers with an interest in improving their teaching practices.

1.7 Structure of the dissertation

The proposed study will comprise seven chapters. The summary that follows is designed to give the reader an overview of the subsequent chapters.

Chapter one focuses on the research problem, the key questions, and the need for this study. Studies in mathematics education show that results in mathematics are generally problematic. In an attempt to find some solution to this problem, this study focuses on the phenomenon of visual reasoning as a process and how it is used by successful mathematics teachers. This study seeks to identify successful mathematics teachers with a view to explore how and why they employ the process of visual reasoning. The aim of this study is to gain a deeper understanding into this phenomenon with a view to informing other practitioners and policy makers about the role of visual reasoning in successful mathematics teaching.

The focus of Chapter 2 is on the two significant aspects of this study, visual reasoning and successful teachers. The first part of this chapter examines the notion of visual reasoning. Several aspects of visual reasoning are discussed here, with a view to arriving at an understanding of the concept. Apart from defining visual reasoning, this part of the review also focuses on the benefits and pitfalls of using visual reasoning. The discussion on visual reasoning also focuses on the role of visual reasoning in a context of multiple representations and Gardner‘s Multiple Intelligences Theory. The second part of this chapter focuses invariably on successful teaching. Attributes of successful teachers emerge as a common theme in much of the literature, hence the discussion around the qualities of successful teachers. However, whilst the literature shows much research in the field of visual reasoning, and successful teaching, few studies focus on a combination of these two phenomena. This chapter uses this as a basis to argue for the pursuance of this study.

Chapter 3 outlines the research methodology that will be adopted. The rationale for using a qualitative research design and the choice of research methods is discussed in this chapter. The chapter also engages in discussions rationalising the use of multiple case studies as a research method. It also examines the research methodology and the procedures to be adopted in this study.

Under the banner of constructivism as the research paradigm, three theories were used to understand the phenomenon under investigation, in chapter 4. They are Attribution Theory, Gardner‘s Theory of Multiple Intelligences and Situated Cognition Theory. Each of these theories is discussed in detail with a view to understanding the theory and its relevance to this study.

In chapter 5, two significant discussions are entered into. The first focuses on the participants views of their notion of visual reasoning and success. The purpose of these descriptions is to show how their views about the phenomena were consistent with their practice. The second part of the analysis focuses on the actual practice of the participants and the justification for their actions. Using a narrative approach, thick descriptions of the classroom observations of each of the participants are dealt with individually, with a view to understanding how and why visual reasoning is used. There is an iteration process between what each of the participants believed about visual reasoning and their actual practice. The practice of each of the participants is also examined in relation to the theoretical frameworks.

Chapter 6 forms the second part of the analysis. Whilst chapter five focussed on the individual participants and their practices, in this chapter, my findings are discussed under themes and sub-themes that are largely based on a cross-case analysis of the evidence presented in earlier chapters. Where similarities emerge, it may be used to generalise to other cases.

Chapter 7 concludes this research study by summarising how the critical questions were answered. It also brings to the fore new insights that have emerged through the process of integrating the critical questions with the reviewed literature, theoretical frameworks and the data. In this process a model is presented, which offers new insights into incorporating visual reasoning, as a pedagogy in the teaching of mathematics. The chapter will conclude with recommendations and suggestions for further research.

1.8 Conclusion

This chapter presented the research problem, after alluding to the background, and explaining the rationale for the study. A brief outline of the context of the research was also delineated. The chapter concluded with a brief outline of the structure of the study.

The next chapter will engage with the literature available on the two phenomena that are under scrutiny. A careful examination of the literature will be undertaken to gain an understanding of the notion of visual reasoning and successful teaching.

Chapter 2

Looking into Visual Reasoning and Successful Teaching

2.1 Introduction

The concept of visual reasoning has been the focus for many writers: (Stokes, 2002), Ainsworth and VanLabeke (2004), Elliott and Bills (1998), Konyalioglu, Isik, Kaplan, Hizarci, and Durkaya (2011), Lavy (2006) and Alsina (2000) to name a few. They range from educationists to researchers, psychologists and many other persons and institutions that have an interest in the phenomenon. It is an issue that has been at the heart of educational debates for many years now and is still current.

An important contention emanates from the debates surrounding the nature of mathematics and how it should be taught. The views held by two opposing schools of thought on the nature of mathematics are discussed. In these discussions, an attempt is made to highlight the essential points of differences between these two groups of mathematicians in terms of their use of the symbolic and visual strategies in mathematical understanding and mathematics teaching. A third view, which seeks to show a compromise between these two opposing schools of mathematicians, is also presented. Through an examination of the available literature on these issues it is hoped that a deeper understanding of the concept of visual reasoning will emerge. The phenomenon of successful teaching forms an integral part of the discussion in this chapter. By examining the attributes of successful teaching, it is envisaged that some understanding of the notion of successful teaching can be reached. An attempt will be made to locate the concept of visual reasoning within the context of teaching and learning. The review will focus on examining the notion of successful mathematics teaching and what it generally implies. Attempts will be made to critically discuss the literature available as well as draw the readers' attention to possible areas for further investigation and the need for this research to be undertaken. This has a direct bearing on my study as I wish to understand how successful teachers use visual reasoning in their practice.

2.2 Visual Reasoning and Teaching: The two Schools of Thought

Essentially there are two groups of mathematicians. On the one hand, there are those who believe that mathematics involves symbols and should therefore be taught using language de

Guzmán,(2002), whilst another group who believe that mathematics is about images and that it should be taught using visual approaches (Chiappini & Bottino, 2007; Mencinger & Mencinger, 2004). Woolner (2004, p. 450), confirms that this divide exists between the two categories of mathematicians when she draws our attention to ―a distinction between people who seem to prefer to use verbal abilities and those who prefer visual processing". Similar views have been expressed by Barwise and Etchemendy (1991), as well as Zimmerman and Cunningham (1991). According to Haciomeroglu, Aspinwall, and Presmeg (2010, p. 154) there are ―analytic, visual, and two subtypes of harmonic thinkers, namely, pictorial-harmonic and abstract-harmonic, based on learners' preferences for two cognitive processes: verbal-logical or visual-pictorial". The reference to the different kind of mathematical thinking, further confirms the distinction between the two opposing schools of mathematical thought.

Although Mathematics teaching and learning has been dominated by the techniques favouring the use of formal symbolic language Barwise and Etchemendy (1991), in this type of mathematical teaching and understanding, the focus has been on the use of language and logical statements in producing a linearly coherent type of mathematical argument (White & Mitchelmore, 2010). The dominance of this kind of teaching and learning may possibly be attributed to the nature of mathematical topics that have been generally accepted in the secondary school curriculum. Not all topics in the curriculum, particularly in South Africa can be taught using a visually-centred approach. Topics such as number theory, probability and equations, for example, may not lend themselves to a purely visual approach. Although these aspects can be presented visually to the learners, the arguments that are used in teaching these concepts may depend on the use of mathematical language and logical statements.

On the other hand, the proponents of visualization seemed to favour the use of visual approaches in teaching and learning. Whiteley (2004, p. 1), supports this view when he asserts that ―it is an illusion that mathematical reasoning is done in the brain with language". He confirms the view that mathematical thinking is not necessarily dependent on language and the symbolic form for it to be understood. Rather he adds that visual thinking is ―learned" and as such it can be taught in a similar way to which symbolic lingual arguments are done.

The review above introduces several concepts that have implications for my study of how mathematical knowledge is perceived and presented. These differences, possibly alludes to the differences in the kind of learners that mathematics teachers may face in their classroom, considering that cognition may be classified as verbal-logical or visual-pictorial. It may necessitate the exploration of the efficacy of incorporating visual approaches, which this study wishes to investigate.

Although mathematics teaching and learning has been dominated by techniques using symbolic language, visualization as a technique has also been explored for many decades. According to Novick (2004, p. 307) ―diagrams are among the oldest preserved examples of written mathematics. Geometric diagrams have been found on Babylonian clay tablets dating from around 1700 B.C.E". However, Novick (2004, p. 307) also note that

> ―during the European Renaissance (circa 1455–1620), there was a movement away from using geometry as the foundation of mathematics, and correspondingly a change from perceiving diagrams as fundamentally important in mathematical proofs to perceiving them as unnecessary".

It seems that historically, visualization has contributed significantly to mathematical thinking. This has been made clear by the reference to ―clay tablets" and ―the foundation of mathematics". However, there was a period in the history of mathematics, particularly during the Renaissance, where the prominence of geometry began to wane.

The swing in balance between the proponents of algebraic reasoning and geometers seems to have continued over the decades. This is evident in the statement made by Novick (2004, p. 308) that ―the change from geometry to arithmetic as the foundation of mathematics and logic was solidified in the early 19th century with the discovery of non-Euclidean geometries and the development of Boolean algebra". During this period, the verbal-symbolic argument seemed to have featured very prominently in mathematics reasoning and teaching. Elliott and Bills (1998, p. 46) and Lomas (2002, p. 208), further contend that ―geometric proofs were often relegated to a substantial role in mathematics despite their authenticity". However this view has been challenged by Barwise and Etchemendy (1991). They argued that diagrammatic proofs were equally convincing if care was taken not to use diagrams incorrectly. Sherry (2009) further contends that by denying the significance of diagrams in mathematics, it began to question the foundation of mathematics. Sherry‗s view of the importance of diagrammatic reasoning resonates with my own questionings about our curricula being informed by a purely analytical approach to the teaching of mathematics.

A similar trend has been observed in the South Africa mathematics curriculum. Geometry was considered to be a fundamental component of mathematical thinking and reasoning, dating back to the early 1990‘s until the year 2005, when Euclidean geometry was removed from the mathematics syllabus. It was viewed as unnecessary. However, it was reintroduced into the curriculum (Curriculum Assessment Policy Statement, 2011) as a significant part of the curriculum, comprising one third of the contents of the second paper that grade 12 learners would write during an external exit examinations. This trend in the South African mathematical landscape seems to resonate with the trends that were taking place globally as well. Novick (2004, p. 308) alludes to this when she declares that diagrams are now seen as an important and “an indispensable tool for thinking”.

This constant tussle between the proponents of algebraic thinking and the geometers seem to shape the mathematics at a particular time and during a particular period. Whilst algebraic reasoning seemed to have dominated a period in our mathematics, it would seem that the visual reasoning is rekindling the significance of geometric thinking.

The debates stated above are far from over. In the last decade, many more writers have added their voice to this debate. A renowned professor and Dean Emeritus, Small (2012, p. 21) states that:

> “reasoning is an important element in mathematics learning, whether the learner is in Kindergarten or in Grade 12. Visuals, either with or without accompanying words, can be extremely powerful tools for reasoning and explaining; they are powerful not only because they help us understand more quickly, but also because sometimes they lead us to be more general than when we use specific numerical examples. This can be true at any grade level”.

It would seem that according to Small (2012), diagrams have the potential to simplify mathematical explanations as well as lead to generalizations quickly.

Similar views has been made by Mann and Brown (2007, p. 12) who assert that “diagrams are empirically accessible, spatiotemporal objects”, that contrast with algebraic generalisations. Alshwaikh (2009, p. 1) and Ball and Ball (2007, p. 2), also support this view by asserting that ―powerful images is every learner‗s right". They contend that it is a necessity for every learner of mathematics, by saying that it is every learner‗s right. The effectiveness of diagrams and maps became evident in a study undertaken by Cañas, Novak, and González (2004), where it was found that diagrams require a greater level of understanding than the use of algebraic formulae.

It would seem that, not only are arguments that promote the use of visual reasoning, becoming increasingly common, but that arguments against the use of algebraic reasoning in mathematics are also gaining more support: Jamnik (2001); Hanafin (2104) and Alshwaikh (2009). This trend is now observed in other disciplines such as Science as well. Butcher (2004, p. 1), is of the opinion, that ―text is a very simple type of material given the diverse media options that are increasingly available in learning situations." The reference to text as a ―simple type of material," seems to suggest that the emphasis placed on algebraic, symbolic language seems to be diminishing in science education in relation to the potential that visualization has in terms of supporting new learning. This kind of argument has been strongly advocated by ―the key role of visual reasoning" in addressing modern day science and mathematics teaching, since both these subjects are analytical.

Arcavi (2003, p. 227), believes that there is a move to make visual reasoning a part and parcel of mathematics by incorporating it with algebraic reasoning. The view that ―a diagram is ‗only a heuristic to prompt certain trains of inference‗", has been strongly challenged by Barwise and Etchemendy (Lomas, 2002, p. 209). Hanna (2000, pp. 5-6), further asserts that ―the key role of diagrammatic proof is the promotion of mathematical understanding".

The views of these mathematicians are suggesting that diagrammatic proof should be recognised as autonomous and not relegated to a second degree status. This is clearly evident in many of the proofs of the theorems in Euclidean Geometry as well as in Algebra. The proof of the Theorem of Pythagoras, for example, contains the rigour of algebra in much of its argument. However, the geometry of similar triangles provides this basis for the logical proof that ensues. It would seem that similar arguments have been at the forefront of much debate recently as well.

However, it is observed that while the research reviewed often refers to the uses of visual reasoning and problems associated with its use, little has been done in the way of examining how successful teachers use this phenomenon. Although the phenomenon of successful teachers may be defined in many ways, I identify them it in terms of the pass rates of the learners. I wish to examine the extent to which these successful teachers engage with visual reasoning and how it resonates with the reviewed literature, depicted thus far.

2.3 Visualization versus symbolic logic: Is there a Compromise?

The second reason for visual reasoning receiving much prominence in the literature, concerning the teaching and learning of mathematics, is a view that both the verbal/lingual and the visual/spatial forms of representations have their respective place in mathematics. The emerging view sees visual reasoning as having a complementary role in mathematics. Arcavi (2003, p. 220), supports this view by claiming that ―visualization can accompany a symbolic development". Rahim and Siddo (2009) confirm the dependence of the algebraic logic on the visual form. Similar views have been expressed by Thornton (2001), Ball and Ball (2007), Zimmerman and Cunningham (1991) and Saundry and Nicol (2006). Rösken and Rolka (2006) and Haciomeroglu et al. (2010) refer to the need for both visual and algebraic techniques to be used interchangeably.

I wish to examine the extent to which Cox‘s (2013) findings, on the interdependence of algebraic and geometric techniques, resonate with those of the participants, in this study. Cox (2013, p. 4) in her research found that the use of algebraic rules and geometric strategies such as lines in the Cartesian plane, ―supported learners understanding of similarity", a topic which is prominent in the teaching of geometry.

According to Berthold and Renkl (2009, p. 70) ―during recent years, the substantial increase of multimedia learning environments has raised the question of whether combinations of representations such as diagrams, equations, tables, text, and graphs are actually helpful". As a result of this argument, I believe that the notion of multiple representations is closely associated with the two prominent kinds of representations. However, it must be noted that the term multiple representations is not to be treated synonymously with visual reasoning, although visual reasoning incorporates various forms of representations. From the discussions that follow, it would seem that most researchers are unanimous in their understanding of the phenomenon. They conclude that multiple representations in mathematics often include text, pictures, manipulative modes, diagrams, multimedia and written symbols. Clearly, most of these representations are of the spatial/visual and verbal/lingual types. They therefore offer a compromise to the debates seen earlier about the nature of mathematics. The nature of this compromise as well as the benefits of multiple representations is evident in the discussions that follow.

Berthold and Renkl (2009, p. 70), declare that ―learners acquire more knowledge when they receive multiple representations in the form of text and pictures". Similar claims have been made by Herman (2007), who found that learners preferred the use of multiple forms of representation in solving mathematical problems. Limin Jao (2013) also found that the use of multiple representations promoted a deeper, abstract understanding of mathematical concepts. It would seem that by using both analytical and visual approaches in the teaching of mathematics, learners seemed to have gained a deeper understanding of mathematics, a view supported by Ainsworth and VanLabeke (2004). It seems that this view of the benefits of multiple representations is accepted by many other researchers, namely, Davis (2007), Sconce (2009), Berthold and Renkl (2009), Prusak, Hersckowitz and Schwarz (2012) and Chaudhury (2003).

Panasuk and Beyranevand (2010) ascribe the benefit of multiple representations to a process of transforming a mathematical idea from one form into another. In this process of transformation, the learner is given alternate forms of representations of the same knowledge and may become more inclined to make sense of it.

Contrary to the view that visualization must be used as a complementary teaching process, the findings by Sevimli and Delice (2011) seem to indicate that visualization decreases the use of algebraic representations. These researchers are now claiming that by engaging in visual reasoning, learners are becoming less dependent on algebraic forms of representations.

In a study conducted by Cox (2009, p. 5), it was found that ―visually-based strategies supported learners as they reflected on and sought to improve wholly numeric strategies". Orhun (2007), also found that algebra and visual strategies could not be separated.

It would seem that the symbolic/arithmetic and the visual/spatial forms of representations are crucial components in mathematical reasoning. Moreover, researchers are also propagating the iteration of the visual and analytic/symbolic forms of representation. Rather than simply incorporating these models into teaching, research is suggesting that there be a constant iteration of these representations. According to Stylianou (2002, p. 306)

~~"f~~or most people both visual and analytic thinking may need to be present and integrated in order to construct rich understandings of mathematical concepts and proposed a heuristic description of the processes that take place during problem solving that involves visual representation use, the Visualizer/Analyzer (V/A) model, which views visual and analytic reasoning as complements".

Zazkis, Dubinsky and Dautermann (1996), acknowledges the importance of both these forms of representations in solving problems in mathematics. Rather, than attempting to solve problems purely through analytical or visual methods, an attempt should be made to integrate the two. The model as described below shows how the process of thinking unfolds. It starts with the concept of visualization V1. This can be any visual representation, either in the form of a drawing, or any image or a mental image. This image is then analysed during the process A1. During this process an analysis of the visualised material takes place. The process is repeated and hence there is a constant iteration between the visual and analytical processes (Stylianou, 2002).

The iteration ends as the problem solver comes to a better understanding of the problem he/she was solving. The process can be seen in the following model, figure 2.1 depicting the process.

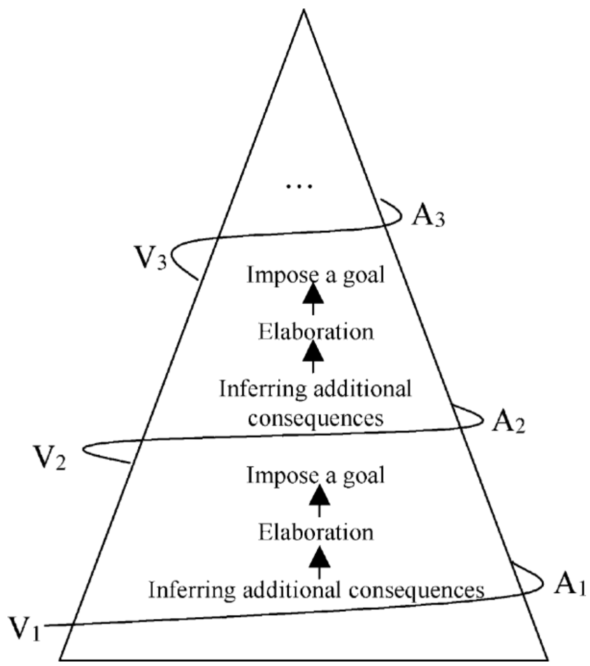

Figure 2.1: The Enhanced Visualizer /Analyser Model (Stylianou, 2002, p.312)

Furthermore, the research indicated that ―mathematicians tend to use visual representations in a very structured and methodical fashion in their problem solving," and that ―the exploration of a visual representation appeared to be an automatic response, or a procedure, in expert mathematicians' behaviour". In my opinion, it would seem that visualization is almost an automatic response to problem solving. It must therefore be seen as an inherent process of problem solving.

The advent of technology has repercussions for teaching and learning. Some of the advantages are captured in the arguments made by Stohl Drier, Harper, Timmerman, Garofalo, and Shockey (2000, p. 71), where it is believed that technology allows topics to be studied in greater depth through ―multiple proofs and more interactive ways", through the use of simulations and probes. Topics that were considered to be difficult and impractical to represent, such as recursion and regression, can be easily visualized with the use of technology.

It would seem that technology in itself offers multiple representations to the educator. The reference to ―multiple proofs and in more interactive ways", suggests that the mathematical curriculum can be delivered in a variety of ways. This will imply that mathematical concepts may be presented in the form of pictures, diagrams, manipulative models, written form, gestures and verbal form. Through the use of technology, this may be executed with ease. It may be argued that the use of technology may also promote multiple representations in the mathematics classroom. This is evident in the statement that ―mathematics educators, should make the best use of multiple representations, especially those enhanced by the use of technology" (Stohl Drier et al., 2000, p. 76). Seen from this perspective, technology also offers the learner the opportunity of combining the analytic and visual approaches. In a research study undertaken by Duncan (2010), it was felt that the use of multiple representations promoted learners' understanding of mathematical concepts and that technology offered the learners the opportunity to engage with different representations. Similar view has been expressed by Jackson, Gaudet, McDaniel and Brammer (2009), Molenje and Doerr (2006) and Nathan and Kim (2007).

Tall (1991, p. 4), summarises the arguments about the need for a compromised situation between the analytical and visual approaches to teaching and learning when he declares that ―the existence of different modes of thought suggests a distinction between intuitive thought processes and the logical thought demanded by formal mathematics. Intuition involves parallel processing quite distinct from the step by step sequential processing required in rigorous deduction”. Whilst there are notable differences in these processes of thinking, each unique in its own way, he is recommending that by integrating the ―right brain which deals with establishing global linkages though the process of visualization, and the left brain which deals with logical inferences”, through the use the symbolic form of mathematics, mathematical understanding will be achieved (Tall, 1991, p. 5).

It would seem that, he is also advocating a hybrid of the two processes in mathematical thinking if the learner is going to achieve success in understanding mathematics.

From the foregoing discussion, it can be seen that visual reasoning is now being viewed as a powerful, complementary tool in the mathematics classroom.

2.4 The benefits and pitfalls of visual reasoning

The third reason for the prominence given to visual reasoning in teaching and learning, is the benefits that it brings to the fore. Whilst many researchers endorse the benefits, others point to some of the dangers of visual reasoning in the classroom.

According to Tall (1991, p. 1) ―visual intuition in mathematics has served us both well and, badly. It suggests theorems that lead to great leaps of insight in research, yet it also can lead up blind alleys of error that deceive”. Based on this statement, it would seem fair to argue that visualization can be beneficial if it is used correctly, or misleading if it is used incorrectly.

The literature points to several benefits that may become evident by engaging with visual reasoning. Hence, the first part of this discussion will be to focus on the benefits that visual reasoning has to offer to the learners.

Tall (1991) found that in the initial stages of mathematical concept formation in calculus visualization proved to be vital. He also concludes that during this stage of exploration, mathematical thinking is enhanced from creating an overall picture, which visualization offers. Seen from this perspective, it seems that visualization is a powerful process in the exploratory stage of mathematical thinking and an indispensable tool. It lays the foundation for reasoning and arriving at intuitive knowledge. It is only later that a more formal rigorous proof follows. This is confirmed by Tall (1991, p. 1), when he argues that “in mathematical research proof is but the last stage of the process. Before there can be proof, there must be an idea of what theorems are worth proving, or what theorems might be true”. Whilst visualization can be both useful as well as misleading, he showed, through the use of visualization in the teaching of calculus, rigour is often the outcome of mathematical teaching. He is advocating that intuition and rigour should work in tandem with each other, rather than in isolation.

Visualization was also found to have a positive effect on learners that struggle with the concept of misperception in mathematical visualization. According to Malone, Boase-Jelinek, Lamb, and Leong (2004, p. 1), after using “manipulatives and a specially developed computer program”, it was found that “only 10%” of the learners misperceived. It would seem that visualization can alleviate the problem of misperception as well.

Butcher (2004), asserts that visualization, promotes learning by guiding the cognitive process. Similar views have been expressed by Yilmaz, Argun, and Keskin (2009, p. 130), in their statement that “a significant aspect of mathematics is concerned with objectification and representing abstractions from reality. These representations generally appear visually or connect with visually sense experiences”. Thus, two common features are evident from these discussions: First, visualization aids in the cognitive development of the learner by the process of objectification or abstraction. In this context, abstraction refers to the process of representing an abstract concept in mathematics to an object, through the use of images. Second, visualization assists in the process of cognition by assisting the learner in arriving at a process of generalization. This is also evident in the statement made by Yilmaz et al (2009), who asserts that through the process of visualization a mental transformation takes place that eventually leads to successful understanding of a concept. This view has been confirmed by Andrade (2011), who found that this process allowed learners to achieve a better understanding of fractions. In their research, Haciomeroglu et al. (2010), found that those learners who did not integrate the visual and analytical processes were often unsuccessful in attaining mathematical understanding. The role of visualization as an important complementary process cannot be over emphasized. Malaty (2008), declares that visualization is an important process in developing causal thinking or the relationship between cause and effect in mathematics which they believe is a fundamental process in mathematical thinking. Seen from this perspective, visualization offers immense benefits. Perhaps this was another reason why I chose to investigate this phenomenon with successful teachers. Perhaps their views and practice of visualization may confirm the claims made in the literature.

The concept of visual imagery and cognition takes on a far deeper analysis when it is examined by DeWindt-King and Goldin (2003, p. 4), who contend that ―various researchers: Bishop (1989); English (1997); Goldin (1992/1987/1998); Goldin and Kaput (1996); Owens (1993); Presmeg (1985/1986/1998); Thomas and Mulligan (1995); Thomas, Mulligan and Goldin (2002), have considered imagistic representation generally, and visual imagery in particular, as a fundamental cognitive system of representation for mathematical learning and problem solving". Similar views have been expressed by Jones (2001) when he declares that visualization contributes to problem solving by using concrete objects to understand abstract mathematical ideas.

A distinction has been made between five kinds of imagery, namely the ―concrete/pictorial imagery, pattern imagery, memory images of symbolic notation, dynamic imagery, and mental operations on or transformations of images". A brief examination of these concepts reveals that ―concrete/pictorial imagery" is an internal image which is represented as an external object or image. ―Pattern imagery" leads to the formation of patterns, which ultimately lead to the process of generalisation. ―Memory images" refers to the visualization of the symbolic notation" and reference is made to the ―visualization of formal mathematical of the formal expressions, whilst in ―dynamic imagery" the image undergoes a change. The fifth type of imagery which is termed ―mental operations or transformations," refers to the act by the imager to transform the image (DeWindt-King & Goldin, 2003, p. 5). They regarded the above-mentioned five characteristics of visual imagery to be regarded as internal constructs of external behaviour.

Visual imagery is seen as a means of constructing knowledge. The representations through gestures, body movement, paper or pen diagrams, pictures, charts and other concrete objects, are the external means of achieving that knowledge construction. A similar view has been expressed by Lavy (2006). However, she uses the term kinaesthetic imagery to replace the term ―dynamic imagery".

A study was conducted by Chang, Sung, and Lin (2007, p. 2225), to help learners to develop their geometric thinking skills. The study revealed that there was a distinct difference in the geometric abilities of those learners who were exposed to visualization. It would seem then, that visual reasoning is beneficial to learners‘ understanding of geometric concepts and helps to ―produce significant enhancement effects on inference and induction abilities", in geometry. Similarly, Brown, Jones, Taylor and Hirst (2004)) claim that visualization is powerful tool in geometry. A prominent mathematician and teacher on problem solving, Professor G.Polya, encouraged his learners to draw a figure to solve a problem. He went on to claim that merely by ―drawing a diagram, it would catapult one towards a solution, even if the problem was not necessarily a geometric one" (Stylianou, 2002, p. 305). The effectiveness of the use of visual images in reasoning has been reiterated here, even if the problem was considered to be non-geometric, more especially in problem solving which is considered to be at the pinnacle of mathematical teaching and learning. Stylianou (2002), speaking about the qualities of a mathematician, declares that one of the most important qualities of a mathematician is the ability to visualise.

Mathematics modelling is a cornerstone of mathematics teaching throughout the world. ―Constructing mathematical models is a process of abstracting reality. Throughout this process, problems must be solved and decisions made. Models, are often constructed in our minds, i.e., mental models" (Waisel, Wallace, & Willemain, 1997, p. 1). The effectiveness of visualization in mathematics modelling is evident, since mathematical modelling is designed around the idea of problem solving which is at the pinnacle of mathematical thinking. The benefits of visualization continue to manifest themselves in all aspects of mathematics. This is evident in deGuzmán, (2002) that since human perception is primarily visual, much of mathematics depends on the visual component. Similar results were found in a study using computer animations in the study of astronomy by focusing on geometric concepts and visual-spatial reasoning, conducted by Martinez and Pérez (2010). The use of visual interactive material in a study conducted by Maschietto, Bartolini, Mariotti and Ferri (2004), proved that it can contribute to mathematical meaning if the material is presented correctly.

Research by Ioannou and Nardi (2009), on this aspect showed that visualization was a very powerful tool in the teaching of other branches of mathematics such as abstract algebra.
Visualization is a powerful tool in teaching and learning and its benefits have been far reaching. Its effects have also been noted in other disciplines such as Geography, Science, Geoscience and other mathematically related fields (Montello, 2004).

Visualization, as seen from the above argument, has the potential to assist those learners with low spatial ability as well. This augurs well for such learners since it offers an alternate form of representation. Should such learners not be inclined towards the other form of representations, visualization offers them this platform to grasp mathematical concepts. Equally important is the use of colour in visualization, which cannot be ignored. In keeping with the positive effects of visualization, Stone (2006, p. 1) recommends that ―colour used well can enhance and clarify a presentation”. It would seem that visualization can offer added advantages in the form of colour graphics and presentations. Silverman (2003) believes that visualization is beneficial to the visual-spatial learner in this day and age, because progress in science depends on those people who can think in images.

The implications of these statements are that visualization does succeed in developing learners who are capable of creative, problem solving. It would seem that the effects of visualization transcend the classroom environment and offer other advantages to learners than mastery of concepts alone. In her study with gifted learners, Silverman (2003) made the following observation: The children with the highest test scores, the ones who went beyond the norms in the manual, achieved these scores by passing visual-spatial items that were designed for children twice their age. They demonstrated excellent auditory-sequential abilities, but their visual-spatial abilities were even more extraordinary.

Silverman (2003) further contends that, there is a need to develop these abilities in the classroom to ensure the success of our learners. It would seem that these researchers are placing strong emphasis on the need for the visual-spatial teaching styles in the current day classroom practice.

The foregoing discussion has emphasized the relevance of integrating the visual-spatial teaching component in the curriculum. However, mathematical diagrams are synonymous with mathematical reasoning and understanding. In this section I point to the many uses of diagrams in mathematical thinking. The literature revealed that many researchers have emphasized the use of diagrams in mathematics. Diagrams, being an important part of the concrete imagery, are crucial to the cognitive process. This point has been emphasized by Puphaiboon and Woodcock (2005), who state that understanding a diagram is part of the thinking process that combines pattern formation with the symbol aspects of mathematics. It is because of this unique combination of cognitive attributes that diagrams are an essential part of mathematical thinking.

The effectiveness of diagrams in facilitating strong mathematical understanding has received much attention as a result of the emerging crises that seems to be facing learners in the United Kingdom. It was found that these learners are failing to grasp basic geometry, in comparison to their counterparts in America and Japan (Puphaiboon and Woodcock, p. 2). It would seem that mathematicians are looking at visualization as a means to aid the crisis in mathematical understanding. However, Romberg (1989) recommended that there was a need to change the form of representation from a symbolic to diagrammatic form. They believed that it would be advantageous to mathematical understanding.

Diagrams offer the flexibility of changing the mode in mathematics teaching from the sequential, lingual to the visual-spatial presentation. This is particularly the case because “all elements of the display are available simultaneously and thus interpretation requires visual reasoning. In decoding diagrams, attention can oscillate between the whole diagram and sections of the diagram (Diezmann, 1996). Thus diagrams offer two purposes: First, unlike other forms of representations, diagrams allow the user to view the problem in its totality since all aspects of the problems are shown on the diagram at the same time. Second, diagrams also offer the user the advantage of switching between different observations by viewing the entire diagram, or parts of it. This flexibility may be crucial in trying to understand mathematical concepts, since it allows the observer the opportunity to focus on the entire image or a part of it. In so doing, the observer may be able to arrive at a solution to a problem.

The significance of diagrams as visual tools in mathematics has also been emphasized by Alcock and Simpson (2004, p. 3) who set out to observe “that visual images are important in mathematical thinking, whether these are imagined, drawn on paper or created using a technological tool”. These visual images that are drawn on paper or created using visual tools are mathematical diagrams. They capture the most important aspects of the mathematical problem and often offer a possible solution in this form. In their study, Konyalioglu et al. (2011) found that visualization made lessons exciting and removed boredom from the learners. At the same time the study showed that most of the learners indicated that they saw the need for visualization, more especially the use of diagrams, whether they were drawn or created by technology.

Dynamic imagery is one of the five characteristics of visual imagery, as discussed before. It is therefore regarded as one of the internal constructs of external behaviour. In studies where dynamic imagery has been used, positive effects have been experienced. In a study conducted by Üstün and Ubuz (2004), when they applied a dynamic software package called Geometers Sketchpad (GSP) in the teaching of geometry, the results showed that the use of dynamic imagery enhanced the learning of geometric concepts. The findings also revealed that these learners also developed a strategy to form connections between the geometrical shapes and also formed a ―hierarchical" relationship between the shapes. The use of computer software in mathematics has made visualization a very powerful tool in mathematics teaching and learning.

According to Palais (1999, p. 648) ―even more exciting for the research mathematician are the possibilities that now exist to use mathematical visualization software to obtain fresh insights concerning complex and poorly understood mathematical objects". The benefits of these advanced software programmes serve to extend the position of visualization as a tool for mathematical insights and further exploration into deeper areas of mathematics. Elliott and Bills (1998), using technology to develop learners visualization, also believed that by using technology it would lead to a greater depth of learners' understanding of mathematics. Bosveld-de Smet (2005) also subscribes to the benefits of using technology when he states that visual presentation is easier than a textual presentation. It must be borne in mind though that the phenomenon of visualization or visual reasoning is being enhanced by the use of technology. However, the concept of visual reasoning or visualization is still the dominant aspect in these and similar studies. He does however caution the readers of the skills that are necessary in designing these software programmes and the level of expertise that is required in being able to effectively use these tools in teaching. Such a finding was also made in a study conducted by Mitchell, Bailey and Monroe (2007), when they discovered that the movement from a traditional pedagogy of the lecture and text book teaching to that of a rich technology presentation may overwhelm some teachers as well.

Increasingly there is a trend for countries throughout the world to participate in surveys that test their academic programmes at schools, particularly in mathematics and science. TIMMS is regarded as one such survey. Japan and Singapore feature very prominently in these surveys. According to Murata (2008, p. 379), it was found that –in comparing videos of teaching as a part of Third International Mathematics and Science Study, that the Japanese instruction made two to four times more use of visual representations than the instruction of other countries". A similar experience is prevalent with Singapore, another high achieving country in terms of TIMMS. Gagatsis and Elia (2005) also found that the use of visual representations in multiple problem-solving, helped the learners in their understanding of mathematics and also helped them to organise the data.

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The National Council of Teachers of Mathematics (NCTM), in the United States of America is constantly looking at ways of improving the teaching and understanding of mathematics. Some of the recommendations that it advanced in order to facilitate the curriculum are discussed by Bills (2001): At all Key stages 1, 2 and 3, the emphasis was on observing, describing and visualizing geometrical shapes. However at key stage 3, the focus was further moved to conduct practical work with a view to visualising the objects and eventually being able to work with them mentally. As far back as the year 2000, the report from The National Commission on Mathematics and Science Teaching for the 21st Century (2000), entitled Before It‘s at Too late, stated that U.S. learners were receiving only a superficial knowledge in their classrooms. Furthermore it was felt that the reason for this level mathematics and science was the as a result of a lack of proper understanding of key mathematical and science concepts and their application to problem solving. Furthermore, these subjects were taught as subjects in their own rights and there was very little collaboration between these disciplines yet they were subjects that dealt with real world problems and issues. The recommendation at that stage was that, –one teaching strategy that could be used to make connections between science and mathematics is visualization" (Harnisch, 2002, p. 1). The results of the study indicated that visualization promoted collaboration between the two subjects. Seen in context then, it would seem that visualization can act as a strong medium in facilitating cross-curricula teaching as well.

Whilst, the literature has shown the potential of visualization to assist in mathematical teaching and learning, it however, does not point to many claims about the pitfalls of visual reasoning. This is confirmed in the statement made by Bresciani and Eppler (2009, p. 3), that

> –despite the notable number of publications on the benefits of using visualization in a variety of fields (ranging from biotechnology to corporate

> communication), few studies have so far investigated the possible pitfalls of graphic depictions used for communication or reasoning".

Apart from research done by Najjar and Balachandar (1998), in the field of education, there have been few researchers who have delved into this aspect of visual reasoning. Three prominent issues around graphic representations have surfaced. They are: ambiguous visualizations, visualizations that appear to be convincing and the familiarity of the observer's previous experiences with visualization. Although not much appears on this area of visualization, attention should be given to the weaknesses that have been researched. The main purpose of examining these pitfalls is to help to improve the quality that will benefit the designer and the user. Furthermore, if these pitfalls point to major discrepancies, alternate forms of representations may be considered. In the case of ambiguous visualizations, it may be possible that a concept may not deliberately be presented in a manner so as to create the misconception, but it may render erroneous reasoning, which may ultimately lead to arriving at incorrect solutions to problems. The use of colour and graphic images do have the potential to create ambiguity. The teacher, however, who is the expert, must edit these visual forms so as to remove such ambiguities or even warn their learners of these deficiencies in the presentations. Furthermore, it may also be possible, particularly in the field of mathematics that a visual image may suggest a phenomenon that may appear convincing, when mathematically it may not be possible. Here too, it is imperative that teachers edit and view these presentations before-hand unless they have a deliberate reason for using the visual as it is. This seems to be the observation made by Alcock and Simpson (2004, p. 3), when they contend that ―various cases have been presented in which learners are shown to ―justify" a mathematical conjecture by generalizing from a single image, failing to see the need to provide a properly-based argument". The danger of such a generalization is that it may be made prematurely, without consideration being given to those cases where the generalization may not be applicable, particularly in mathematics, where erroneous conclusions may be made. This view is confirmed by Lavy (2006) who indicated that it is important to be aware of the dangers of visualization if it is not used properly, particularly though the differences of a geometrical image and its visual presentation. The study also found that learners who depend heavily on visual reasoning, could not reason collectively, by introducing formal definitions and language. However, it must be pointed out that the literature reveals the emphasis placed on sequential, analytical, logical construction off mathematical proofs and its scepticism towards visual arguments. Notwithstanding this bias, it seems fair to assume that learners who have been exposed to several years of such mathematical teaching will invariably be deficient with visually constituted proofs and arguments. Equally important is the kind of instrument that was used to monitor the progress of these learners. If these were fully constituted visual ways of assessing the learners, perhaps the outcome would be different. However, if analytical tools and instruments were being used to measure the success of these learners, then these instruments may be seen to be deficient as measuring instruments.

Opponents of visual reasoning have often also argued that it cannot be classified as real mathematics. According to Arcavi (2003), one of the cognitive difficulties of visual reasoning is that it is demanding. As a result of this, it may not be seen as real mathematics.

However, one of the other pitfalls of visual reasoning is that it requires the learners to come in with a certain visual background. This may not always be the case, since there are some learners who come from very rich visual backgrounds, whilst other learners don‘t. In a classroom context, learners come with a myriad of different backgrounds. A similar finding has been made by Butcher (2004, p. 2), who concludes that ―the interpretation and use of visualizations may be greatly affected by perceptual qualities of the visualization as well as by the expertise of the individual", thus alluding to the difference in backgrounds.

Arcavi (2003) classifies the difficulties in visual reasoning into three categories: In the cultural category, the beliefs of what is regarded as mathematical is either accepted or rejected. The cognitive category considers whether visual reasoning is easier than symbolic reasoning. The social category examines the issue of teaching with a view to understanding the difficulty that some teachers experience in using visual or analytical approaches in their teaching. The cultural belief of the nature of mathematics can create difficulties in employing visual reasoning in the mathematics classroom. Perceptions of what constitutes mathematics, may be affected by the following factors: the dominant thinking of the nature of mathematics at that time, the type of teacher education that is offered and the type of curriculum materials that are being used or prescribed. Eisenberg and Dreyfus (1991) refer to the ways in which knowledge enters schools and how it changes due to the dominant influences that it is subject to. If there is a bias towards the symbolic representation of mathematics, then there is a greater chance that the visual representation will be under-utilised. The counter argument may also be true. This is clearly evident in research conducted by Tall and Watson (2009), with a group of calculus learners. The research showed that the teacher who emphasized visual approaches in her/his teaching of the concept of gradient attained the best results (Tall & Watson, 2009).

Based on the kind of perceptions that the teacher has of mathematical knowledge, and the way in which it should be presented, visualization may be either construed as easier or more difficult to apply in her/his practice. According to Eisenberg and Dreyfus (1991), knowledge that is linear is considered easier and is preferred by many teachers. He also claims that learners will invariably be inclined to accept linearly presented knowledge more readily if they are constantly exposed to it. These factors may hamper the use of visual reasoning by teachers in their practice.

The discussions have shown that although ―visual representations can have beneficial effects, they can also be misleading. The misleading effects and misuse of visualizations have been pointed out most clearly in the area of scientific visualizations" (Bosveld-de Smet, 2005, p. 5). In this case they can easily be manipulated using computer software and graphics. However, it would seem then that the overwhelming majority of research done on visualization believes that visualization offers more solutions than barriers to teaching and learning. Against this backdrop, perhaps these successful teachers have also seen the benefits in their practice. This claim may be corroborated if the results point in the same direction.

2.5 Arriving at an understanding of visual reasoning

Although a great deal of research has been done on visualization, the literature review reveals that researchers have not delved into the concept of visualization as a process that goes beyond perception. Many researchers have focused on the issue of using visualization as an alternate pedagogical practice. This study seeks to go beyond the mere use of visualization as an alternate form of classroom activity. For the purposes of this study, a more comprehensive understanding of the concept of visual reasoning has to be established. However, the literature suggests that the concept of visual reasoning is very broad. According to Costa, Matos, and eSilva (2009, p. 2246), a point further emphasized by Wijk (2006, p. 42) that ―visualization is considered from many points of view", implying that there are several dimensions to visualization as a phenomenon.

According to Yilmaz et al. (2009, p. 131), one possible way of trying to understand visual reasoning is to see it in the context that ―mathematics is a subject that is concerned with objectification and representing abstractions from reality and many of these representations appear to be visual, having roots in visually sensed experiences”. Similar views have been expressed by Thornton (2001) and Lomas (2002). Silverman (2003), in her definition, shares a common understanding with some of the previous writers who have referred to visualization as transforming images in the mind‘s eye. This must be taken to mean that visualization is more than mere perception but based on the individuals‘ experiences of objects and events which are now rooted in the mind. The process unfolds as follows. The visualizer sees an image. This image is then reconciled with a past experience of the incoming visual image. New meaning is then attached to the object that is being visualised.

Another view of visual reasoning has been advocated by Arcavi (2003, p. 215), when he states that ―visualization, as both the product and process of creation, interpretation and reflection upon pictures and images, is gaining increased visibility in mathematics and mathematics education”. Although he concurs with the previous writers, he seems to go beyond the point that they have made, in that he sees visualization as a process of using the mind‘s eye to see the object. He further contends that although technology has evolved to such an extent that we can see almost anything, still, the mental processes that go, can only be perceived with the use of ―cognitive technology” (Arcavi, 2003, p. 216). He thus implies that the extent to which visualization can be seen as a process in human cognition is dependent on how much the person has evolved with regards to visual cognition. Other researchers who share this view are Rahim and Siddo (2009), Lavy (2006), Malaty (2008).

It would therefore seem as though, mathematics depends on a semiotic system of transference of knowledge though representations and signs or symbols. This point is further elucidated when they declare that ―mathematical knowledge is not simply a ready-made product that can be directly introduced into processes of teaching and learning. The new mathematical knowledge will only be actively constructed, in social interaction, by the learner in his or her learning process within an activity” (Chiappini & Bottino, 2007, p. 2).

This implies that the experiential knowledge of the learner has an important effect on his/her learning. Extending this notion, Chiappini and Bottino (2007) further argue that the mathematical object which has an external representation, must be studied in relation to cultural and historical considerations. It is through this process that the external representation of the object is internalised as a mental image of a mathematical construct. According to this argument, the ―social interaction” is a key component of this process, without which mathematical visualization is difficult. The concept of visualization, although very broad, goes beyond seeing. It is the process of interpreting and adding meaning to the representation of mathematical objects, through a semiotic process, as negotiated by the broader community of mathematicians and society.

Other views of visualization are also considered in the ensuing discussions. According to Lavy (2006, p. 26) ―visualization is a process of construction or use in geometrical or graphical presentations of concepts, or ideas built by means of paper and pencil, a computer software or imagination". Gutiérrez (1996, pp. 1-2), believes that ―although Pestalozzi exaggerated in giving visualization the role of the absolute basis of cognition it is true that visualizations is one of its main basis". She also adds that ―visual reasoning, imagination, spatial thinking, imagery, mental images, visual images, spatial images and others", are some of the terms closely used to signify the process of visualization. It would seem then, that the concept is indeed a broad one, the literature uses these terms interchangeably. This thesis also presents this concept as it appears in the literature. Hence, the reference to these concepts in this research as well. Hershkowitz (1989, p. 63) define visualization as ―the ability to represent, transform, generate, communicate, document, and reflect on visual information. It is a crucial component of learning geometrical concepts. Furthermore, a visual image, by virtue of its concreteness, is ―an essential factor for creating the feeling of self-evidence and immediacy". Visualization was also seen as ―process of forming and manipulating images, whether with paper and pencil, technology or mentally, to investigate, discover and understand (Hershkowitz, 1989, p. 63).

In their research on young children's mathematical reasoning through pictures, Saundry and Nicol (2006), believed that by drawing, it assisted learners in problem solving.

According to Diezmann (1995, p. 1), ―a diagram is defined as an abstract visual representation that exploits spatial layout in a meaningful way, enabling complex processes and structures to be represented holistically". Being visual instruments they are essential components of visual reasoning and offer the following advantages: First, they assimilate chunks of information so that the individual does not have to rely on memory to retain them. Second, they allow for connections to be made between the various components. Third, they allow information to be displayed in its totality. Fourth, they facilitate reorganisation thus allowing for new information to emerge. Last, they provide an alternative to words. Because of their significance in visual reasoning, it is necessary to consider the attributes that make it such a powerful visual instrument. It would seem that diagrams are an invaluable asset in visual reasoning, a viable form of multi-representation, which ultimately contributes to problem solving in mathematics. According to Kadunz and Sträßer (2004, p. 241), ―we want to stress that the changing and mutually controlled use of multi-purpose representations is the most characteristic feature of visualization in this text". The multi-purpose representations that they refer to are ―images as potential representations", a view that resonates with other similar definitions.

A somewhat different view on visual imagery is contained in the writing of Hofstadter (2007, p. 27), when he contends that ―it makes one realize that blind people have visual imagery every bit as rich as that of sighted people". This definition therefore focuses on the deep cognitive processes that takes place within the brain, that allow for the transformation of the image into a mental picture. This definition therefore dispels the notion that the object has to be seen before it is transformed into a mental picture.

Visual reasoning can therefore be seen as a two-fold process: Firstly, there is an image which then translates to a mental picture. Secondly, there is a process by which meaning is attached to the image in order to develop some mathematical understanding of a concept. Although the literature does point to some understanding of the concept of visualization, it nevertheless does not discuss the process of creating and interpreting mathematical concepts in full, except for those researchers who approach visualization from the perspective of psychology.

Some attempt has been made by Vinner and Dreyfus (1989) to construct a model of the cognitive processes by using the idea of a concept image. The mental picture, which is the set of all pictures that have ever been associated with the concept in our mind, together with a set of properties associated with the concept in our mind, is defined as the ―concept image." He then assumes that for each concept there are two different cells in the cognitive structure, one for these images and another one for the formal definition of this concept. It is through the process of visualization that connects the visual and symbolic in mathematical problem solving (Figueiras & Deulofeu, 2005). Furthermore, according to Figueiras and Deulofeu (2005), this is achieved through an iterative process. The teacher then assists in the process of cognition by transforming the icon into a mental image, through the process of visualization. Through different representations, the concept is formed in the mind and a mental picture emerges. As an example, a diagram may be drawn to capture the essential parts of the problem. The teacher may then switch the representation of the problem into a symbolic form, such as an analytic form. By using, both the visual and analytic forms of representation, the teacher may be able to arrive at a solution of the problem. In this way a conceptual understanding of the problem may be created. Once the concept has been formed, it may be replicated when a similar problem is posed.

Stylianou (2002) makes a similar claim, that visualization is a process in which a person forms a link between an internal construct and an object perceived by the senses. The reference to the senses implies that it need not only be the sense of sight, but that any of the four other senses may act as an entry points for the transference of information. An ―act of visualization may consist of the construction, on some external medium such as paper of objects or events that the individual identifies with objects or processes in her or his mind" (Stylianou, 2002, p. 306). This act goes beyond merely seeing as has been mentioned in previous parts of this chapter. Rather the emphasis is on these external events or objects that the individual interacts with, through a process of socialization.

The views expressed by the previous authors are reiterated by de Guzmán (2002), who asserts that visualization is a product of our interaction with many people in a social context of mathematics. Visualization must therefore be seen as a complex process that is culturally intertwined, that connects different representations and that goes beyond just seeing. The extent of the influence of ―visual representations were recognised by the psychologist, Vygotsky as powerful cultural tools that support learning within socio-cultural contexts and as such have an important role in children‘s developing understanding" (Worthington, 2005, p. 7). These visual representations in the form of drawings and graphics give the children and opportunity to explore their thinking. At the same time these visual representations also provide a socio-cultural context, since the meaning of these representations, are based on the socio-cultural context in which the child interacts.

The previous discussions indicate that visualization or visual reasoning is a complex phenomenon. It goes beyond perception. It is an integral part of cognition and that a simple, single definition will not be adequate. However, the literature also points to ways in which teachers can use this multiple fold definition of visualization in their teaching, taking into account the most important aspects of the definition as discussed. Since the focus of this study is on successful teachers, it would be appropriate to examine the literature in terms of the actual implementation of visualization in the mathematics classroom. By examining what the literature is suggesting about the effective use of visualization, it could act as a basis for understanding how the participants employed visualization in their practice.

In designing a curriculum, Cuoco, Goldenberg, and Mark (1996, pp. 7-8), suggested that, amongst other ways to advance good thinking,

> ―learners should be visualisers" and that the following aspects of visualization be taken into account as recommendations:
>
> - They should be encouraged to reason in 2-Dimensional and 3-Dimensional space.
> - They should be encouraged to draw tables and graphs.
> - They should be encouraged to draw diagrams.
> - They should be encouraged to think in terms of machines, for example function machines.
> - They should be able to visualize a rate of change.
> - They should be encouraged to perform mental calculations.

In each of the aspects outlined, the intention was to develop a visualization process that would develop and advance good thinking. In the first example quoted, learners were encouraged to think about two-dimensional and three-dimensional arguments without necessarily drawing pictures. The use of an area model could be used to visualize the product of two binomials, for example. Learners could investigate number relationships through the use of Venn diagrams. To foster the visualization process, learners could be encouraged to create diagrams showing input and output process, that are typically used in understanding the concept of a function. In a similar way, the concept of a rate of change could be visualised by observing how gradient changes as it moves along a curved surface. The idea behind visualising calculations was to give the learner the opportunity to work mentally with numbers and see their properties, without necessarily reducing them to the written form. These initiatives were suggested in order to promote visualization. It was hoped that these strategies would advance visual reasoning and promote its use in problem solving.

In discussions on the process of cognition, reference was made to the different kinds of imageries as discussed by DeWindt-King and Goldin (2003). Kinaesthetic imagery refers to a representation that is connected with associated physical actions. Gestures feature prominently as this kind of representation in the literature, more especially in the aspects concerning cognition. Because of the complex nature of gestures and its significance in cognition and visualization, I choose to delve into this aspect separately.

2.6 Gesturing as a visualization Process

According to Vygotsky and Wollock (1997, p. 133), ―a gesture is specifically the initial visual sign in which the future writing of the child is contained as the future oak is contained in the seed. The gesture is a writing in the air and the written sign is very frequently simply a fixed gesture".

Even the protagonist of algebra who believe it to be a sequence of linearly organised steps, must understand that ―what the ontogenesis of the algebraic language suggests is that, in the meaning-making process accompanying the production of signs, the primacy is action—action objectified through different semiotic means, starting from the form of a crude pointing" (Radford, 2003, p. 66). This view has been confirmed by Roth and Thom (2009).

A similar view has been echoed by Edwards (2009), who claims that human thought and mathematical thinking takes place through various levels such as images, bodily movement and gestures. Edwards (2009, p. 129), subscribe to the definition of a gesture as ―the relationship between how the hands move in producing a gesture, and whatever mental representation underlies it, inferred both from the gesture and the accompanying speech". As such, it highlights the role of gestures in mathematical thinking and learning and being visual, an important aspect of visual reasoning. Research conducted by Edwards shows that gestures can enhance learners' understanding (Edwards, 2009).

According to Bakker and Hoffmann (2005), signs are very important in mathematics and science. Without them the study of mathematics and science would be impossible. It seems that gestures form an integral part of mathematical thinking. However, it must be noted that gestures also represent invisible objects that learners are meant to understand. This often poses difficulties to learners who are expected to perceive the object (Bakker, 2007). The notion of an object or the process of objectification is also echoed by Radford, Miranda and Guzmán, (2008).

Furthermore, a diagram is a sign ―which is predominantly an icon of relations and is aided to be so by conventions" (Bakker, 2007, p. 16). More importantly, against the backdrop of visual reasoning, ―a diagram is a complex sign that may include icons, indices and symbols"

(Bakker, 2007, p. 16). In keeping with the definition of a sign given by Vygotsky (1997), a diagram is therefore a visual sign and key part of visual reasoning. ―The power of diagrammatic reasoning is that we are continually bumping up against hard fact. We expected one thing, or passively took it for granted, and had the image of it in our minds, but experience forces that idea into the background, and compels us to think quite differently" (Bakker & Hoffmann, 2005, p. 18).

Nemirovsky and Ferrara (2009) also emphasize the importance of gestures when they declare that different aspects such as gestures, tone of voice, body poise and facial expressions are being considered as important part of human cognition.

In their research investigating the use of visual and analytical methods in the study of calculus, Haciomeroglu et al. (2010, p. 153) found by performing the bodily action of drawing the anti-derivative graph, they had minimised the information that had to be kept in memory ―by changing semiotic resources, thus lessening their cognitive load, suggesting that the semiotic resources, gestures in particular, played a significant role in their mathematical thinking". In this context, the action of sketching the curve, as an alternate form of representation using kinaesthetic imagery, helped the learners in the process of cognition.

According to Murata (2008), the constant use of a certain type of representation eventually becomes part of a mathematics thinking process The representation can vary according to the five types of imageries discussed before. This implies that kinaesthetic imagery too, therefore will form an important part of the learners' mathematical thinking. Again, the relevance of semiotics in learning is advocated. As discussed before, gestures are considered to be part of semiotics.

Radford (2009, p. 91), also confirms this view that the ―construction of mathematical meaning", comes ―from the perspective of multimodality, that is, taking into account the range of cognitive, physical, and perceptual resources that people utilize when working with mathematical ideas". Furthermore, he contends that these modalities incorporate oral, written, pictorial and gesture forms. He further asserts that ―the return of the body is rather the awareness that, in our acts of knowing, different sensorial modalities such as tactile, perceptual, kinaesthetic, etcetera, become integral parts of our cognitive processes" (Radford, Edwards and Arzarello, p. 92). In my opinion, the art of gesturing significantly contributes to the construction of knowledge and cannot be treated as separate entities. Research in both mathematical and non-mathematical domains has examined how gesture acts in the generation as well as the communication of meaning. A similar claim has been made by Williams (2009) that a large portion of communication in mathematics is though gestures. According to Edwards (2005, p. 1-135), a “gesture can be seen as an important bridge between imagery and speech, and may be seen as a nexus bringing together action, imagery, memory, speech and mathematical problem solving”.

This implies that the belief that the mind and the body operate as separate entities in mathematical thinking, is a misnomer and that they are to a large extent dependent on each other. This seems to contradict the view of other philosophers/psychologists and theorists on learning and conceptual development. This argument has been strengthened by researchers such as Arzarello, Paulo, Robutti, and Sabena who “provide rich descriptions of gestures in multi-modal communications, involving learners and teachers in mathematical activity in classrooms” (Williams, 2009, p. 202). They conclude emphatically that “gestural modalities of communication are important not only in showing how mathematics can be learnt but also in understanding the critical mediating role of spatially organised models in this learning process”.

It would seem that human cognition is a multi-modal activity and that gestures form an important component of this process.

The view that thinking is only a process that takes place in the mind has been the focus of debates for several decades. From the rationalists such as Plato, Liebniz and Descartes, to Piaget, the debates about the nature of learning still continue. Unlike the rationalists, “Piaget sought to answer the riddle of the sensual and the conceptual through a combination of empiricism and rationalism: for Piaget, knowledge starts with sense data” (Radford, 2005, p. 2). Plato alludes to the acquisition of knowledge through the senses. However, Radford (2005, p. 2) also contends that “for Piaget, it is the logic-mathematical structures” which will gather the knowledge through the senses “and transform it into abstract thinking”. The emphasis of gestures and signs in mathematical understanding is clear in this study which showed that much of the understanding was achieved through pointing gestures and the use of language as well as the kinaesthetic movement of the pen along the graph (Radford, 2005).

The points raised above are echoed in research conducted by Bjuland, Luiza, and Borgersen (2008, p. 273), who declare that “thinking and mathematical reasoning are not just mental but are realized through semiotic activity. Semiotics is related to the activity of semiosis or the study of semiosis, which is a term that comprises any sign action or sign process". Furthermore, the authors also claim that semiosis is the ability to freely move between ordinary language, mathematical language, mathematical notations, diagrams, graphs, etcetera. Hence the importance of signs as gestures, become evident as key components of visual reasoning. Based on these discussions, it would be fair to conclude that gestures too are an integral component of visualization.

Since the focus of this study is on successful teachers and an examination of their practices, it is mandatory that some consideration be given to this important aspect of the study.

2.7 Successful Teachers

According to Archer (2000, p.3), currently there is considerable interest in mathematics teaching”. This would seem to explain why mathematics teachers are now revisiting the ways in which they teach mathematics. A great deal of research is also being done in mathematics education, with the intention of trying to promote a better understanding of the ways in which mathematics can be presented to learners in a more meaningful way.

Archer (2000, p.3) further contends that

> –the notion of mathematics as a set of procedures to arrive at a right answer is so deeply ingrained in most mathematics teachers, and in teachers of mathematics teachers, that a re-focus on mathematics as a way of making sense of the world will be hard won.”

This statement seems to create the impression that reform in mathematics teaching will be a long drawn out process. Despite these claims, there are some apparent contradictions: The following questions may be asked:

1) Why are the results of some teachers consistently good whilst those of others consistently bad if they are exposed to almost the same set of factors that influence teaching and learning?
2) Why are some teachers classified as more successful, than others, in mathematics teaching?

Archer (2000, p.3), also refers to differences in examination results of neighbouring classes. These apparent contradictions seem to suggest that there are successful teachers, who despite all the odds are making a difference in the lives of their learners. In this discussion an attempt will be made to negotiate and arrive at an understanding of the concept of successful mathematics teaching. This can be achieved by surveying the relevant literature.

In her research, Archer (2000,p.6) mentions –enthusiasm” as one of the elements that may be identified as necessary for successful teaching, when she claims that, the interest of the learners would grow if teachers exhibited this quality of enthusiasm. Morris (2006, p.72) in his research too, alludes to the –enthusiasm of the teachers” in order to effectively bring about change in their learners‘ understanding of concepts. A similar notion has been captured by Morris and Easterday (2008) in their research on the use of technology in mathematics teaching. They found that observations, whether it be ―real or virtual,” does not necessarily mean that it will have an influence of an ―educative value.” Rather, they contend that it is the teacher who ultimately determines the way in which they interact with the material (Morris & Easterday, 2008, p. 50). This statement seems to suggest a broader understanding of the effect of technology in classrooms. It seems to suggest that it is ultimately the teacher who can determine the type of learning that takes place. This view however, may be challenged. Whilst the teacher is a crucial element in promoting learning, the attitudes and the mood of the learners must also be considered. Equally important is the focus on socio-economic factors. The research further explains that the use of technology, such as the iPods were helping teachers to develop their understanding of learner growth, by making learner learning more ―visible”. However, the study found that the teachers themselves were willing to make the change. Part of this change must be attributed to the enthusiasm of the teachers that Morris and Easterday (2008) refer to.

The aspect of enthusiasm is seen as a key attribute by other researchers. According to Low (2011) teachers are not static and continue to learn, most often passing this knowledge onto other people or their learners. This will be as a result of their enthusiasm to achieve the best indicator of their ability. A similar observation was made in a study observing the way successful university professors teach. Four categories were identified by these professors in their understanding of successful teaching. It was found that enthusiasm was another category that was identified by the professors. This category encompassed the affective quality of their teaching, their specialty or subject matter, and their learners. ―Enthusiasm, passion, enjoyment, and love are all words identified in the narratives of the professor” (Rossetti & Fox, 2009, p. 15).

Furthermore, according to Low (2011) the other notable qualities include willingness, offering learners incentives, having an open-mind and offering learners a myriad of teaching methods. These qualities are identified as some of the key aspects of successful teachers. It would seem then that enthusiasm is a foremost requirement of successful teachers. This view is confirmed by Johansen (2011), who believes that a teacher‘s belief, knowledge, training and attitude also determine the kind of teaching that takes place.

Low (2011, pp. 32-33), also believes that there has to be a deep desire to teach which he captures in these statements: ―without passion or ‗a loving heart‘, the teacher would not have the fire in his or her belly, and indeed cannot excel – in fact, (s)he would be just an average or an ordinary teacher".

In a study conducted by Sanders (2002), the following results emerged after eighty schools provided the following information to applicants for mathematics teaching posts: The attributes that featured most prominently were qualification, teaching skills, personal qualities, and value-addedness which referred to other exceptional qualities that included knowledge and skill that went beyond the normal expectations of a mathematics teacher. The survey also indicated that the highest percentages of 94 and 81 were allocated to personal qualities and value-addedness.

Apart from being enthusiastic about the subject, the researchers, quoted above seem to suggest that enthusiasm when accompanied by a need to make teaching more meaningful, relevant and adaptive are some of the qualities of successful mathematics teachers. In this regards, Low (2011) believes that successful teachers find innovative ways of encouraging their learners to remember the mathematical concept. In order to stimulate the right side of the brain they normally use visuals as well as analogies to help their learners to retain important mathematical concepts.

In their research in Kenya, Nyabwa (2008), believe that high academic knowledge, enthusiasm in teaching, and appropriate teaching methods are essential attributes of successful teachers.

They also believed that ―knowledge of how a specific teaching method or strategy impacts learners' learning may help mathematics teachers in selecting teaching methods that enhance the effectiveness, efficiency and quality of learning and teaching" (Nyabwa, 2008, p. 440). Although research has been done on teaching and the learning of mathematics separately, according to, Peterson, Fennema, Carpenter and Loef (1989), there does not seem to be much research integrating teaching with teachers' mathematical content. Hence they conducted a study that investigated the relationship between teachers' pedagogical beliefs, pedagogical content knowledge and learners' achievement in mathematics. The results of the findings indicated that ―teachers' pedagogical content beliefs and teachers' pedagogical content knowledge may be importantly linked to teachers' classroom actions, and ultimately, to learners classroom learning in mathematics" (Peterson, Fennema, Carpenter, & Loef, 1989, p. 36). Zollman, Tahernezhadi, and Billman (2012) also subscribe to the view that most successful teachers have an excellent grasp of their content knowledge. This content knowledge is closely linked to their success in their classrooms and their practice. Similar findings have been made in a study by Tryggvason (2009), of Finnish teachers‗ teaching styles and the reasons why the Finnish were ranked second in Mathematics in the Programme for International Learner Assessment (PISA). The teacher educators stated that they build their teaching on a variety of learning theories. Furthermore, they vary between inductive and deductive approaches, and between theory and practical situations. In order for these teachers to be able to use a variety of learning theories, it seems fair to assume that they had the necessary pedagogical knowledge. The reference to the inductive and deductive approaches also seems to suggest that these teachers also possessed the necessary content knowledge. Inductive and deductive theories cannot be applied unless the teachers possess a strong mathematical content knowledge. Similar views have been echoed by Ference, Clement and Smith (2009).

The view that a negative attitude towards teaching mathematics adversely affects a teacher‗s performance, has also come under scrutiny. Atnafu (2014) made this observation about the attitude of a group of teachers from Pakistan. It was found that the majority of the teachers in Pakistan do not have a positive attitude towards teaching. The study made the following recommendations: to improve motivation, reduce the anxiety and change the attitude of the mathematics teachers. Atnafu (2014) and Bonner (2014) found that even in the case of traditionally undeserved American learners, it was the passion of the teachers that encouraged their learners to be successful. It would seem that the attitude of the teachers can play a significant role in the way in which mathematics is understood by the learners. Also, studies of the teaching styles of professors at a mid-western university, showed that, these professors had a high degree of commitment to teaching (Rossetti & Fox, 2009, p. 14). It would seem that there is a close correlation between successful teachers and their pedagogical content beliefs and their pedagogical knowledge.

Literature on successful teachers, do not show much research being done in the field of successful mathematics teachers. According to Baiocco and DeWaters (1998, p. 133) ―research on teaching excellence has been rudimentary, focusing primarily on the identification of characteristics or traits that mark the outstanding teacher". Hence, the examples that have been quoted in this research emanates from research that tend to focus on the traits of successful teachers. It would seem that these researchers did not focus on the practices of teachers per se. Furthermore available research tends to focus on other subject disciplines. In the absence of much research on the practice of successful mathematics teachers, I tend to focus on research done on successful teachers in other disciplines as well. In their study of successful agricultural and life sciences teachers, Roberts, Conner, Estepp, Giorgi, and Stripling (2012, p. 22), found that, with regards to teaching techniques ―active learning strategies used in a wildlife management course and that lecture, coupled with cooperative learning and inquiry resulted in most learners (77%) showing gains in knowledge". Thus the kind of teaching approach can affect the outcomes of the learners, in terms of the results that they attain. This is dependent on the confidence of the teacher in terms of his pedagogic content knowledge and his pedagogic content beliefs.

The literature is also alluding to other attributes of successful mathematics teachers. In her research, Archer (2000, pp. 22-28) in her findings also reveals that those who were more motivated, made the following comments about their beliefs and practices: that they are generally very friendly, and they develop warm relationships with their learners and other teachers. Similar views have been expressed by Low (2011), when he declares that, by knowing a learner well it will help to facilitate the teaching. According to Roberts et al. (2012, p. 21), ―sensitivity and inclusion are the two key dimensions that describe the teacher's beliefs related to their thoughts and actions. Sensitivity relates to understanding learners' needs, while inclusion refers to the amount of control the learners have over their learning". It would seem that the success of a teacher is judged by her/his ability to accommodate the needs of his learners, whilst also being flexible enough to allow his learners to develop in his/her classroom. In taking account the needs of her/his learners, the teacher tries to relate the subject matter to the learners' lives. In doing so the teacher is also being sensitive. This process contributes to the development of the learners learning, over which she/he has full control. The importance of these qualities of teachers are also evident in a research conducted by (Strahan, 2008), who found that, the teachers who were involved in the study were able to respond to all the needs of their learners as well respond to their learners on an individual basis. The study revealed that the learners had limited skills, and were not willing to take risks. However, with the dedication and determination of their teachers, they were willing to make the shift. Against this backdrop it would be fair to assume that such learners will only develop if they are carefully nurtured. It would seem that learners value ―relationships with their teachers that are based on trust" (Strahan, 2008, p. 6). This view is also confirmed by Rossetti and Fox (2009), who conducted a study which was designed to determine the factors associated with successful teaching. The results showed that the professors valued their relationships with their learners and emphasized the need to work together with their learners. They also placed much emphasis on mentoring their learners and providing them with the necessary guidance during the periods in which the learners were experiencing difficulties.

Seen from this perspective, it is fair to argue that successful teachers possess several traits that set them apart from the general body of teachers. A feature that is almost a pre-requisite for success as a teacher is he/his ability to be caring, and having the best interest of her/his learners at heart. It seems as though all the other essential qualities of successful teachers are fulfilled if the first two qualities mentioned are fulfilled. The quality of being flexible and open-minded, even after acquiring much knowledge and skills is viewed as necessary for a teacher to be recognized as highly successful. According to Low (2011, p. 35), ―teachers who manage the classroom by showing such open mindedness in the teaching process become successful teachers. Indeed excellent teachers learn from others through a variety of ways such as the use of questions, learner involvement and various activities".

The following qualities were noted as necessary by Gehrke (2005), in a research observing the qualities and actions of successful teachers in poor urban schools. Their understanding of successful teaching is a combination of strong academic qualifications, enthusiasm, a strong knowledge base of mathematics, good professionalism, good facilitations skills, having a strong sense of understanding their learners' needs and the effective use of a teaching method. The following qualities were regarded as important: First, successful teachers accept that their views and philosophies may differ from their learners. Second they also accepted that in order to teach in poor urban schools, they required a strong knowledge base and third, successful teachers believe that all children can learn and that environmental factors are not necessarily a reason to lower expectations.

Research with the Mexican American child, has focussed on their poor performance as a result of their lack of interest, language problems, parents' low education and other factors. (Garza & Garza, 2010). This study set out to examine how the teachers' beliefs, perceptions and experiences affected the success or failure of these children. Furthermore it examined the attitudes of four White female teachers on these children. The study revealed that the teachers were highly committed and ensured that the learners performed well in all tasks at any cost. They were willing to make sacrifices for their learners. They spent long hours before and after school, preparing their learners for all the tasks. Based on the information furnished, it is fair to conclude that these teachers seem to have cared for the welfare of their learners, by providing the necessary transportation and tutorials.

In their research, Jeanpierre and Lewis (2007), show, via a case study, how a novice teacher develops into a highly successful mathematics teacher, though the adoption of a programme. Similar to the findings that were made by Garza and Garza (2010), the teacher believed in her learner‘s ability to succeed. She also believed that she was competent to teach mathematics and furthermore, that she possessed the enthusiasm to teach.

In summing up, successful teachers are a combination of enthusiasm, energy, confidence through thorough knowledge of the subject, confidence through thorough pedagogic knowledge and pedagogic beliefs, able to facilitate learning effectively, develop sound rapport with their learners, have a strong sense of understanding of their learners‘ needs and are caring individuals.

Although the discussions on successful teachers focussed primarily on the attributes of such teachers, the intention here was to show the qualities necessary, in order to be classified as a successful teacher. By carefully examining these qualities that set successful teachers apart from the general body of teachers, I am hoping that the actions and the attributes of the participants of this study will resonate with the reviewed literature. Although the literature does not specifically allude to the actions of successful teachers in terms of using visual reasoning as a teaching strategy, the attributes of successful teachers have indicated that successful teachers use a myriad of teaching approaches in their practice. This will include visual reasoning as an approach to successful teaching.

Similarly, the quality of enthusiasm that has been identified as one of the most prominent attributes of successful teachers. The literature points to teachers that have persevered beyond the call of their duties and were willing to try new approaches such as the use of technology in their practice. It could therefore be argued that teachers who subscribe to these criteria are successful. Similarly, by examining the other qualities of successful teachers, as evidenced in the literature review, this study will try to examine the extent to which the participants in this study fulfilled these criteria, with a view to incorporating the phenomenon of visual reasoning.

The following picture, figure 2.2 depicts the essential attributes of successful teachers, as derived from (Low, 2011, p. 43)

It outlines the characteristics of successful teachers. At the apex of the model, is the main criterion of positive teaching. This can only be achieved if the teacher exhibits qualities that are listed in the circles below, namely: passionate teaching, creating a positive learning situation and conveying the message in a convincing way. Positive teaching implies that the teacher will have to develop a positive mental attitude towards teaching. This would imply that teaching must be done with enthusiasm and passion. The enthusiasm of the teacher will create a positive learning environment. The positive attitude towards the subject matter and the learners will assist the teacher in delivering the curriculum in an interesting and exciting manner.

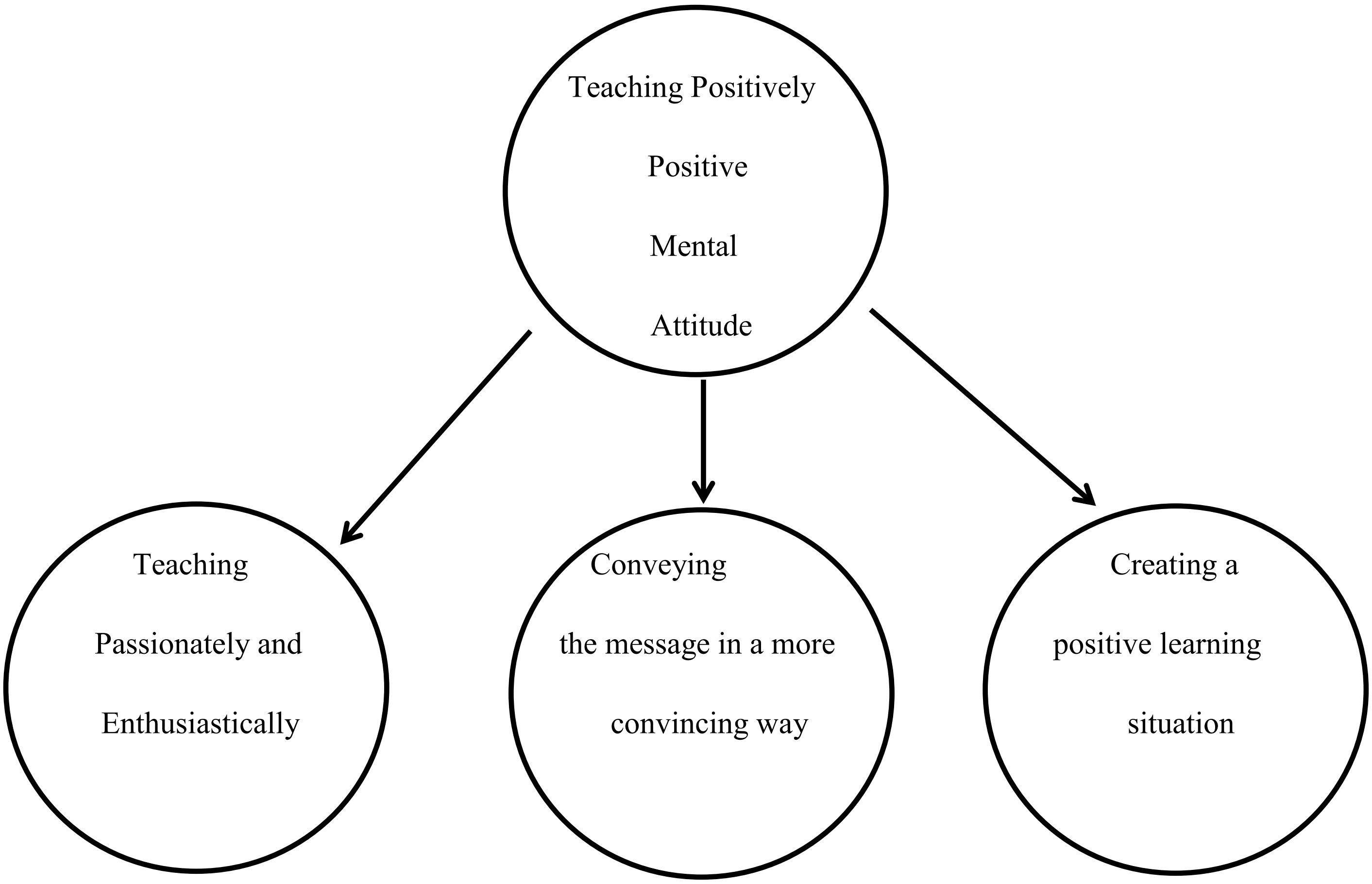

Figure 2.2: The essential attributes of successful teachers (Low, 2011, p. 15)

2.8 Conclusion

In this chapter the essential components of the phenomenon visual reasoning, with particular reference to the main contentions surrounding its position in the mathematics classroom have been discussed and an attempt made to define the phenomenon as it is to be understood in the context of this research. A description of the advantages and pitfalls of visual reason has also been given to present a balanced view of this phenomenon. The qualities of successful teachers as described in the literature were provided and the relevance between the phenomena visual reasoning and successful teachers was discussed. By defining the concept of visual reasoning and placing parameters within which it is to be understood, it paved the way for examining the responses of the participants in terms of their understanding of the concept. Similarly, by examining the qualities of successful teachers, it provided a framework by which the actions of the participants could be analysed in terms of their classroom practice. It also contributed towards arriving at a theoretical definition of the concept of visual reasoning that would assist to provide answers to the first crucial question, namely what is visual reasoning? Arriving at an understanding of successful teachers would pave the way to decide how the participants in this study satisfied these criteria. In the next chapter, an attempt will be made to describe in detail the research design that was employed in order to provide answers to the two critical questions of this study.

Chapter 3

Research Methodology

3.1 Introduction

Visual reasoning as a pedagogy for mathematics has received much attention as reviewed in the previous chapter, with mathematics teachers posturing either for verbal logical reasoning or visualization as a tool for understanding mathematical concepts. Still another group suggests using both tools as complementary. This study wishes to explore the use of visual reasoning by successful mathematics teachers, and aims to gather a detailed understanding of the how successful mathematics teachers actually use visual reasoning in the context of their classrooms. This chapter decribes in detail the research design that was employed and the development of the research instruments, and provides the justification for the use of these strategies. This study called The Use of Visual reasoning by Successful Teachers, attempts to answer the two critical questions: what is the nature of visual reasoning and how is it used by successful mathematics teachers in mathematics classrooms, and why is visual reasoning used by successful mathematics teachers? A qualitative design was chosen because it tries to assign meaning to a human problem that may affect either an individual or a group of people.

Furthermore, this study tries to make interpretations of the meaning of the data (Creswell, Hanson, Plano & Morales, 2007). In this context, the researcher seeks to understand the extent to which successful teachers were using the phenomenon of visual reasoning. This study is underpinned by the social constructivist paradigm. According to social constructivism, individuals seek to understand the world in which they work and live, by developing meanings of their experiences. The intention is to make sense of the meanings others have of the world. These meanings are varied and multiple (Creswell et al., 2007). This study seeks to understand the action of these successful teachers in their practice, using the concept of visual reasoning. Having formalised the paradigm, case studies were then selected as a strategy that would be used as a research method. Within the context of past year examination results, certain schools were identified as very successful (National Senior Certificate, Technical Report, 2014). The research site and participants were chosen using a purposive sampling technique. In this chapter the following aspects will be elaborated: the research question, the general methodological orientation, the types of data collection and the justification for their use, the research instruments used, the research parameters within which the data was collected, and how the concerns of reliability and validity, and ethics were addressed.

Based on the description given by Creswell et al. (2007), I have captured the essentail processes in Figure 3.1 below.

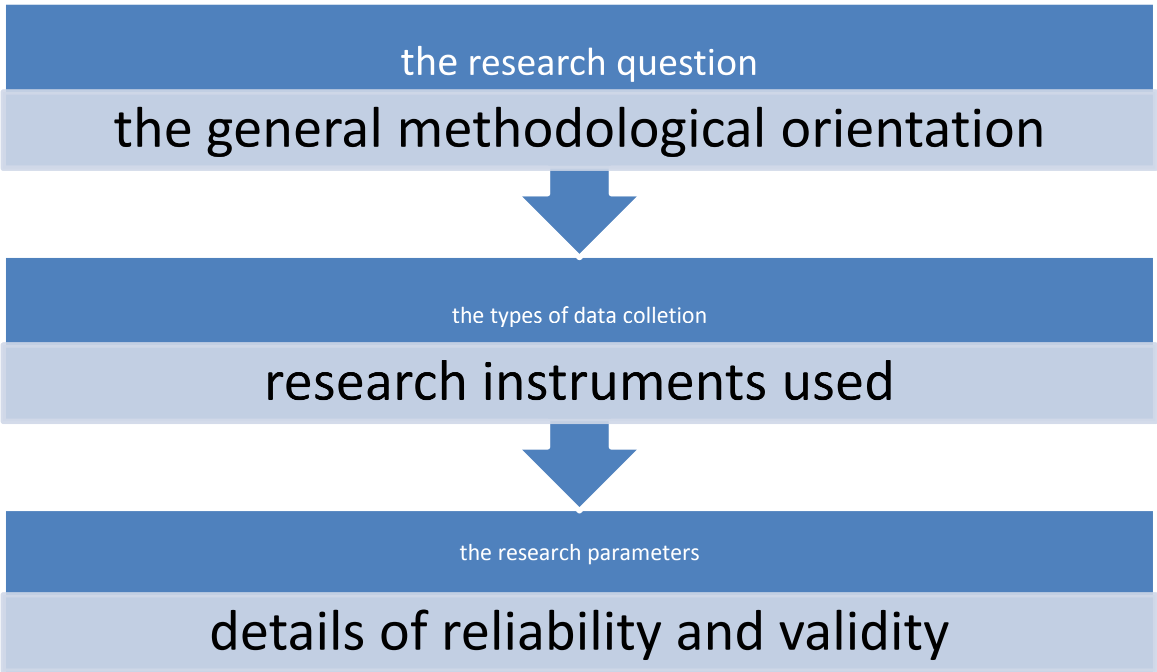

Figure 3.1: Phases of data generation

3.2 The Critical Questions

Although research has been done on successful teachers of mathematics and visual reasoning in mathematics classrooms, in my search for research on a combination of these two components in education, little research was found. It was for this reason that I chose to explore how succesful teachers use visual reasoning in their teaching practice.

This study addresses the following critical questions, which I have created in figure 3.2 below.

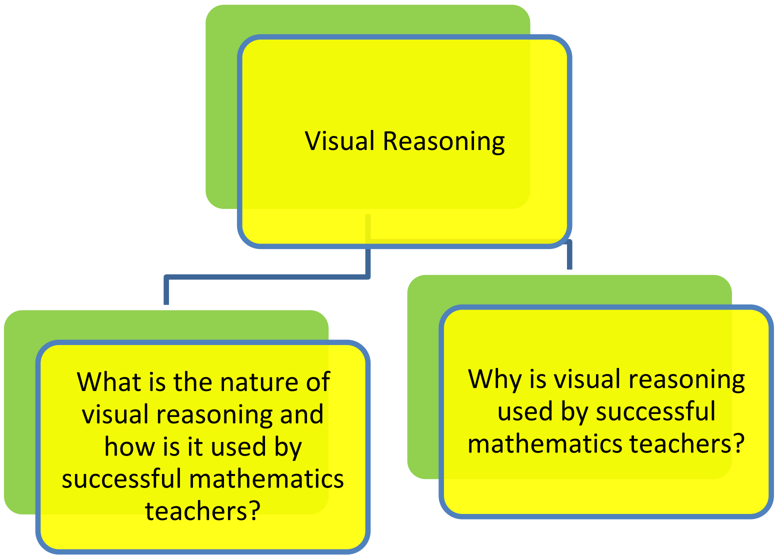

Figure 3.2: Critical questions of the study

The rationale for these questions are as follows: The first focus was to establish a view of the participant's understanding of visual reasoning and how they used it in their mathematics classrooms. The second and more important focus was to arrive at a deeper understanding of why these successful teachers used visual reasoning in their practice.

3.2.1The Social Constructivist Paradigm

The study is underpinned by the social constructivism paradigm. According to Creswell et al. (2007, p. 20) the view of social constructivism is that people develop ―subjective meanings of their experiences" in their social or work contexts. Since these meanings are ―varied and multiple" the researcher looks for ―complexity of views rather than narrow the meanings into a few categories or ideas". Whilst the literature reveals that research has been done on visual reasoning in mathematics classrooms, it remains silent on examining how successful teachers employ it in their pedagogy. Furthermore, research conducted on visual reasoning, were predominantly undertaken using the quantitative paradigm approach. In the context of this study, a qualitative approach would be preferred since it offers more than a unitary explanation for the phenomena that may emerge.

In the context of this study, the process of teaching and learning must be seen as a complex relationship between the teacher, learner, the didactic material and the social environment. It is anticipated that multiple meanings of these phenomena will emerge out of this study because of this very complex interaction between these components. As such, no single theory would be adequate to explain why and how successful mathematics teachers use visual reasoning in their teaching, hence the focus on multiple theoretical frameworks. Furthermore, these meanings ―are not simply imprinted on individuals but are formed through interactions with others (hence social constructivism) and through historical and cultural norms that operate in individuals' lives" (Creswell et al., 2007, p. 21). These views are confirmed by Riegler, Stewart and Ziemke (2013). The construction of knowledge by these teachers must be seen as a product of their interaction with other persons, namely peers, learners and the many others that they interact with.

Creswell et al. (2007, p. 21) also asserts that within a social constructivist framework, researchers acknowledge that ―their own background" and experience shapes the interpretation ―of what they find". The intention of this research is then to interpret the kind of meanings that these successful teachers offer in their use of visual reasoning in their mathematics classroom. Therefore in order to explore the personal, social and learning experience of successful mathematics teachers, a case study form of enquiry will be employed, within a social constructivist research paradigm, following a qualitative research tradition.

3.2.2The Qualitative Method

A qualitative case study evaluation approach was chosen for this study. It was deemed that this method woud be most appropriate, reasons for which I describe in the this section.

Qualitative research may be used for many reasons: Cresswell et al. (2007, p. 39) declares that when a comprehensive understanding of an event or problem is required, as well as ―the context and settings in which paticipants find themselves" a qualitative study will be the appropriate choice, as ―quantitative research ignores the individuality of the participants". Also qualitative approaches allow researchers to make ―knowledge claims based primarily on constructivist perspectives". Thus a qualitative method underpinned by a constructivist paradigm facilitates the understanding of multiple meanings of individual experiences and ―meanings socially and historically constructed, with an intent of developing a theory or pattern". According to Denzin and Lincoln (2002, p. 3), ―qualitative research is a process of doing inquiry in more natural settings, collecting more situational information and reintroducing discovery as an element of inquiry and in the social sciences particularly, soliciting emic viewpoints to assist in determining the meanings and purposes that people ascribe to their actions".

The choice of a qualitative design for my study may also be justified on the following grounds, as described by the Silverman (2003, pp. 325-329) and depicted in Figure 3.3

Figure 3.3: Other reasons for choosing a qualitative design

In the context of this study, this design will allow me to understand the meanings or explanations offered by these successful teachers without allowing the researcher‘s own background as a mathematics teacher to have any effect on the outcome of the research. The research design will allow for any unanticipated factors to be considered, since it is flexible. The design being iterative, will allow the discovery of the phenomenon of visual reasoning in successful mathematics teachers through the use of multiple research methods in ―collecting, double-checking and verifying their discoveries" (Silverman, 2003, p. 330).

A further justification for the use of a qualitative research design is given by (Caruth, 2013), where it is accepted that qualitative research offers greater depth of understanding than does quantitative research.

A similar view about the benefits of a qualitative research design and the pitfalls of a quantitative research design has been echoed by Dereshiwsky and Packard (1992, p. 1), who manitain that quantitative procedures are –sadly lacking” since quite often –critical variables as attitudes, feelings and emotions have been stripped of much of their meaning and value to the researcher when subjected to often-artificial quantification”

According to Dereshiwsky et al. (1992, p. 3) data from qualitative approaches produces a more in-depth understanding of the phenomenon being evaluated rather than obtaining just –a single, perhaps overly summarized numeric indicator*”.* Lee (2012, p. 404) concurs that qualitative research is more appropriate since it uses multi –methods and practices, and traverses several disciplines.” This method is suited for my study as both semi-structured interviews and observations are being used to gather data.

Similar views have been echoed by Pole (2007), when she states that in social constructivism there is no single reality because knowledge is culture bound and therefore varies according to cultural differences.

Murakami (2013) suggests that quantitave studies are useful for their ease to control variables and to make inferences because of the large volume of data. Qualitative studies can be far more difficult to work with because of the difficulty experienced with ethical issues and the difficulty of arriving at inferences because of the size of the observations. Murakami (2013, p. 83) does however point out that inferences can be made by –creatively using a comparative method”, thus implying the need for the researcher, using the qualitative paradigm, to be taking a more social construcivist rather than empirical stand in their research perspective.

My choice of a qualitative approach was to glean an understanding of the intentions underlying human behaviour in their practice with regards to their execution of their vocational duites. Both critical questions explained earlier attempts to gain an interpretive understanding of human behaviour, namely the action of these successful teachers in their practice of mathematics teaching. Furthermore, qualitative research allows for an investigation to be done in a more natural setting. My study attempts to understand how successful teachers use visual reasoning in their mathematics classrooms. By examning the classroom practice of these educators, as reported by them and indicated in their personal classroom practice over a considerable period of time, this study becomes more natural and the conclusions become more meaningful and authentic.

The instruments used in qualitative methodology allowed me to immerse myself into the social setting to an extent. Furthermore, it allowed me to develop a relationship of trust and respect with the participants in their work.

Human behaviour cannot be observed in isolation. It must be understood in context, taking into account their persoanl biographies, backgrounds, beliefs, values and the contexts within which they work. Each of these participants are a product of these factors.

Having established the paradigm within which the sudy was to be located, the next important aspect of the design was to determine, the kind of strategy that would be used within the qualitative approach. According to Neuman (2006), there are approximately nineteen qualitative inquiry approaches.

3.2.3The Case Study

Stake (1995) identified the processes of case study research as one of the most appropriate strategies. The decision to use case studies was informed by the research and the type of answers it necessitated. Case studies have been largely used in the social sciences and have been found to be especially valuable in practice-oriented fields such as education, management, public administration, and social work: Simons (2009); Kyburz-Graber (2004); Stake (1995) and Cohen, Manion and Morrison (2001). According to Starman (2013, p. 30) “the interpretative paradigm, phenomenological approach, and constructivism as a paradigmatic basis of qualitative research are closely linked to the definition and characteristics of case studies”.

Case studies allow deep probing and analysis as well as follow the life cycle of a case, Cohen et al., (2001) and can allow both generalizations and localized analysis. This is also supported by Simons (2009, p. 21) who describes cases studies as allowing for “deep exploration from multi perspectives”.

This makes it an ideal choice for the purposes of my research, considering that the focus of this research is on an interpretation of the practices of these successful teachers. This view is also echoed by Mesec and Lamovec (1998, p. 383) who states that a case study “allows identification of variables, structures, forms and orders of interaction between the participants in the situation (theoretical purpose), or, in order to assess the performance of work or progress in development (practical purpose)”. Both Mesec et al. (1998) and Cohen et al.

(2001), elaborate that case studies may be regarded at different levels, eg., a person, a group like a class, school, institutions, or even a specific phenomenon. This means that a case study has a wide ambit.

De Vos, Strydom, Fouche and Delport (1998), allude to three types of case studies, namely the intrinsic, instrumental and collective case study. An instrumental case study elaborates on a theory or aims at acquiring a better understanding of a social issue. Against this backdrop, I believe that the choice of an instrumental study, for this study was the most appropriate since the purpose of this study is primarily to eloborate on existing theory as well as gain a better understanding of the concept of effective teaching which has wider implications for society.

Based on the understanding of case studies as discussed, it is fair to conclude that it is the most appropriate choice of methodology in order to facilitate this research. The purpose of this study is to gain a social constructivist understanding of human behaviour, conducted in a more natural setting. My study attempts to understand how successful mathematics teachers use visual reasoning in their mathematics classrooms in this natural setting. The focus on understanding the actions of these teachers could have been done by using a quantative study, however, the rigid instruments that characterise quantitatve methodology did not allow me the flexibility to acquire a deeper, more comprehensive understanding of the justification for their behaviour.

Furthermore, human behaviour cannot be studied by discounting the beliefs, backgrounds, and views of individuals. Also the contexts in which these teachers work ought to be carefully considered. One of the characteristics of case studies is its flexibility to take into account these complex, in-depth multiple perspectives, which this research requires. Against this backdrop, the qualitative case study was selected.

In terms of this study, the cases chosen were five individual teachers from a semi-urban private school, which was located in KwaZulu-Natal. The participants were identified using the purposive sampling technique. Teachers who were considered succesful Mathematics teachers, based on the quality of the results they produced, were selected as the participants for the study. Although success may be defined in many ways, the pass rate of learners was used to identify the five participants. The purpose of this study therefore was to explore the activities of a group of successful teachers with a view to arriving at a thorough understanding of how they incorporated visual reasoning in their practice. The unit of analysis was a group of successful mathematics teachers, who came from a particular school.

3.3 Using Participant Observation

Cohen et al. (2001 p. 186), identify two types of observations within case studies, namely ―participant observation and non-participant observation". They assert that participant observation ―is eminently suitable to many of the problems that the educational investigator faces" and submit the following reasons for their use: that the researcher can observe behaviour in situation and can record notes ―about its salient features"as it unfolds; and, since the ―case study observations take place over an extended period of time" it allows the fostering of a more ―intimate and informal relationship" with the participants in a natural context as compared to the more artificial ones produced by surveys or experiments. In non-participant observation the observer stands aloof from the participants that are being observed (Cohen et al., 2001). The relationship between the partcipant and the researcher is more formal. Based on the nature of the observations that this study warranted, I chose participant observation within case studies.

In this study the advantges of participant observation as described by Cohen et al. (2001) are represented in the following ways: as an educator in the school in which the study was conducted, being part of the research context allowed easy observations and recordings of the behaviour of the participants without being intrusive. It also created the opportunity to take note of special phenomena as they arose allowing for deeper insights, by personally observing, listening, enquiring, probing and writing up notes in an unobtrusive way, whilst still being able to show sufficient interest in the participants. At the same time, the research process was not compromised by my own prejudices.

3. 4 Choosing the sample

Having identified case studies as the method, it is important that an appropriate sampling technique is chosen. This is confirmed by Coyne (1997, p. 623), who states that ―in qualitative research sample selection has a profound effect on the ultimate quality of the research". Creswell et al. (2007, p. 75), identified ―purposeful maximal sampling, ordinary cases, accessible cases and unusual cases". According to Schatzman and Strauss (1973, p. 39), ―selective sampling is a practical necessity that is ‗shaped' by the time the researcher has available to him, by his framework, by his starting and developing interests, and by any restrictions placed upon his observations by his hosts'". The reference to –him" in the quotation is not intentional. It must be understood to include both genders of researchers. They state that the researcher selects people according to the aims of the research. Categories such as age, gender, status, role or function in an organization, stated philosophy or ideology may serve as a starting point. The logic and power of purposeful sampling lies in selecting information-rich cases for study. Information-rich cases are those, from which one can learn a great deal about issues of central importance to the purpose of the research.

For the purposes of this study, a convenient, purposeful sampling technique to identify the participants was employed. Successful teachers are the subject of this research, within the context of secondary schools. Within the context of past year examination results, certain schools have been identified as very successful schools. These details are contained in a document released by the Department of Education at the end of each year, based on the grade 12 examination results (National Senior Certificate Technical Report, 2014). The researcher states that the school that he works at is identified as a –successful school" in the above-mentioned document. Schools that achieved a pass rate of more than 80% in their previous years grade 12 examination were classified as successful schools. In the individual subject ranking as compiled by the Department of Education in the same document, the chosen school was identified as one of the top 10 schools in terms of their results in mathematics. The school has been awarded certificates of excellence for its results in these examinations. These documents may be viewed with written permission. For the purposes of confidentiality the school has not been identified by name. Although the school attained pass rates greater than 80% in these examinations, any disclosure of these results will be a violation of the confidentiality of the school. In defining successful teachers, I gave credence to the results attained by these teachers, based on past year examination results, especially in grade 12. Although this has been one of the criteria that have been used to identify the participants, some teachers may not have been directly involved in the teaching of grade 12 learners, but played a fundamental role in shaping the mathematics of those learners that have attained the necessary success at the end of grade 12. They form part of a highly successful team.

The other consideration in choosing this school is that, learners are exposed to the same exit examination at the end of grade 12, as compared to the majority of schools in the country. The notion of success, in this regard cannot be seen in isolation, but rather in the context of this school achieving good results in the grade 12 examination and hence being recognized as a successful school with successful teachers.

Furthermore, the nature of this study made this an ideal choice. In terms of this research framework, which was designed to gather information-rich cases, the chosen site had fulfilled these important criteria.

3.4.1 Size of the Sample

For the purposes of the scale of this study, five cases were deemed sufficient. Marshall (2013, p. 13) asserts that the ―choice of more than one case tends to dilute the overall analysis and when choosing multiple cases, researchers tend not to choose more than four to five". Hodges (2011, p. 90) asserts that the purpose of a qualitative study ―is not necessarily to predict or generalize" and therefore the size of the sample chosen ―is more about saturation than representation", to obtain as much detail from each case rather than to generalize. Rule, Davey and Balfour (2011, p. 302), also declare that ―the strengths of case study include its ability to generate rich and thick description of phenomenon" and case studies are appropriate because they are manageable in terms of time, resources and number of sites and contexts, and have wide applicability and flexibility.

Given the above considerations and the limited scope of this study, a sample size of five participants was considered to be appropriate.

3.4.2 Participants Engaged

After informal discussions with each of the five participants, about the nature of research, the level of their participation and assurances of confidentiality, they agreed to being participants. Initially, I detected some reservations from some of the participants, considering my position as a fellow mathematics teacher. However after allaying their fears that my purpose was to examine their practices, without being judgemental, they seemed more willing. Whilst I had also been teaching with the participants, at the time of the study, I have shown in a previous section on participant observations, the steps that were taken to minimise the effects of the bias that may have arisen because of my position as a fellow mathematics teacher in the same school, at the commencement of this research study. I thereafter proceeded to obtain the necessary written consent for the participation of these teachers in the study. The headmaster of the school willingly consented to his team of mathematics teachers being part of the study, after explaining the nature of the research and giving him the necessary assurance about the confidentiality of the research and ethical considerations that were made. He was also assured that the focus of the research was on the teachers and the learners would not form part of the study. In this regards ethical clearance regarding the use of learners were not deemed necessary. The manner in which I fulfilled the ethical requirementrs of the study will be further elaborated under section 3.7.

3.5 Data Collection Plan

According to Creswell et al. (2007, p. 129) –new forms of qualitative data continually emerge in the literature, but all forms might be grouped into four basic types of information: observations, interviews, documents and audio-visual materials". According to De Vos, Delport, Fouché, and Strydom (2005, p. 314), –within the context of qualitative research, observations and interviews are usually utilised to collect the relevant data". In an attempt to elicit responses from the participants to each of the research questions, interviews and observation as research methods of data collection were employed.

3.5.1 Interviews

In qualitative research, –interviews are often a predominant mode of data or information collection" (De Vos et al., 2005, p. 287). Interviews allow the resesrcher to undertand the stories of the interviewees (De Vos et al., 2005). In reseach, various types of interviews may be used, –namely standard, in-depth, ethnographic, elite, life history and focus groups" (Cohen et al., 2001, p. 270). This list is not exhaustive and other forms of interviews include: –semi-structured, structured, exploratory, informal conversational and closed quantitative interviews" (Cohen et al., 2001, p. 270). Although there are several types of interviews as is evident, interviews may fall into the categories of being structured or semi-structured. If the interview is structured, the intention is to make comparisons and the interview is more inclined to be a standard interview. However, –if the interview is seeking personalized information and depth of information, the interview is more inclined to be semi-structured" as was the case chosen in this study (Cohen et al., 2001, p. 270).

In my study semi-structured inteviews were conducted with each of the participants. According to Longhurst (2003, p. 103), –semi-structured interviews is about talking with people but in ways that are self-conscious, orderly and partially structured". In general, researchers use this technique in order to gain a detailed picture of a participant's belief about a topic or issue. Furhermore, this method gives the researcher much more flexibility since the researcher has a set of predetermined questions, but the interview is only guided by it and not dictated to by it, giving the researcher the opportunity to delve deep into issues that she/he is investigating. The flexibility of this format enabled the researcher to gain as much information about the participants' understanding of visual reasoning and its use in their mathematics classrooms. These interviews were then transcribed for the purposes of analysis. Copies of the transcripts were sent to the participants to assure them of the confidentiality of their responses. It also offered the participants the opportunity to withdraw from the study, should they feel that their position had been compromised in terms of the confidentiality that I had assured them of. The questions were designed around eliciting responses to the participants' views about visualization, the extent to which they use it in their classrooms and the reasons that they use it as a teaching strategy.

In an attempt to answer the first critical question, a set of interviews (first-round), with each of the participants was held prior to observing their classroom practice (lessons). The purpose of these discussions was to establish the kind of practice (using visual reasoning) the participants used in their classrooms with a view to identifying them during the observation sessions later through video-recorded lessons sessions. Thereafter, a second set of inteviews (second-round), were conducted with a view to establishing the answers to the second critical question, which was to ascertain why these teachers use visual reasoning?

In total five first round interviews were conducted at a venue and time agreed upon by both the researcher and the participants. Because of the demands placed on teachers and the lack of availability of time, scheduling these interviews were often difficult. Furthermore, because of the long distances of the participants from the researchers home, most interviews had to be done at school. These interviews were mainly held after the participants and the researcher had completed the days work and any other extra-mural activity that was expected of them.

Each interview was audiotaped. Each participant had to answer ten questions. The participants were given a copy of the interview schedule before the interview because of their demanding work loads. Furthermore, I was interested in obtaining deep insights into their practice. If they were not given the interview schedules earlier, they may not have given me the responses that would have reflected their complete understanding of the issues that were being addressed. Furthermore, having conducted these interviews at the end of the day, would have meant that they would have to had to spend more time after their busy day. Each interview lasted approximately forty-five to fifty minutes.

Another set of five, second-round, interviews were conducted with the participants. These were conducted following the observation of the participants' practice in their classrooms. These were also semi-structured, thus allowing significant exploration into the partcipants' classroom practice. These questions were more individually structured, with a core framework of questions about the use of visual reasoning. The questions were more specific to how each of the participants used the different tools of visual reasoning in their classrooms. However, since their strategies differed, a uniform interview schedule could not be used, although the essential elements of the interview were the same. Generally, participants answered approximately fourteen to sixteeen questions each. The purpose of these interviews was in response to the observation lessons of the participants. Having observed these lessons, the researcher was interested in establishing reasons for the participants' behaviour in terms of their actual classroom teaching. These questions were designed with the intention of understanding why teachers did, what they did in their classroom practice, with reference to specific incidents and moments in the lessons. As such, the interviews probed the participants for reasons for their actions as explanations of their behaviour in their practice. The interviewees were also given copies of the instrument before the interviews and each interview lasted approximately forty five minutes. The interviews were also transcribed and edited. Both these interview schedules have been entered into the research as final instruments. (See Appendix A and Appendix B resectively)

3.5.2 Observations

Observations refer to the ―process of taking field notes on the behaviour and activities of individuals at the research site" (Creswell et al., 2007, p.181). Observations in qualitative research are used for the purposes of understanding the context of the programmes, to discover insights into aspects that the participant may not have spoken freely about in the interviews, either consciously or unconsciously. Observations also allow the researcher to ―gather data on the physical setting, the human setting, the interactional setting and the programme setting" (Cohen et al., 2001, p. 305). The physical setting focuses on the physical environment of the research site, which may have a bearing on the finding of the research. The human setting refers to the organisation of the people or the structure of the group of people, whilst the interactional setting focuses on the interactions that take place between the individuals concerned. The programme setting examines the resources, pedagogic styles, curricula and related aspects of the phenomenon being examined (Cohen et al., 2001).

Prior to the advent of technology, field notes were the prominent forms of observations. However, the advent of the video recorder has introduced a different form of observation in research.

Rochelle (2000, p. 709) suggests that ―video is becoming the medium of choice for collecting data for educational and social science research projects". Video recording allows the researcher to preserve ―aspects of interation including talking, gesture, eye gaze, manipulatives, and computer displays" and viewing the events over and over again to analyse many subtle cues that may be missed on first observation and analysis. Rochelle's assertion provided the impetus for me to use videography of the class lessons as an observation tool to enable me to more deeply analyse the dynamics of the interactions later.

I obtained permission from the teachers to record the lessons on video. In order to answer the first critical question, and to gather a deeper understanding into the practice of these successful teachers, video-recorded lessons were essential. A series of two lessons each were recorded. The purpose of the observations was to gain a deeper understanding of the practices of the participants with a view to observing the extent to which they were using visual reasoning.

Based on the reasons furnished above, by Rochelle, it can be seen why video recording as a data collection techique is suited to this study. First, this study is focussing on examining the classroom activities of the teacher. Teachers engage in several activities in their actual classroom practice. This normally deals with talking, engaging in the use of getsures, using eye gaze as a technique to keep their learners focussed, using visual resources such as manipulatives, diagrams, video clips and a range of other activities. To capture the range of activities of these teachers, video recording is an appropriate choice as a data collection method. Second, video recording also offered the advantage of repeated observations. A single observation is not enough to examine all the activities of the participants, video recording made it an appropriate tool for this study, considering the range of activities that they engage in. From the point of carrying out an analysis of the data, it proved very useful since it allowed the researcher the opportunity to repeat incidences and elicit further responses from the participants. It was through the repetition that the participants were reminded of their actions and could offer reasonable explanations for their actions. Third, whilst the issue of a ―naturalistic fieldwork", may be debated, in this study, the use of video recordings as an obseravtion method helped to create this naturalistic fieldwork. Since the recordings were done more than once, in each participants' case, there was a greater chance that the field being observed was a little more ―natural", as opposed to performing a single recording. Besides, the duration of the lessons, allowed the learners and the participants to revert to their normal, natural tendencies. Furthermore, the video camera was located somewhere at the back of the classroom, out of the sight of all learners, The recordings were not done by the researcher, but by someone appointed by the researcher. In this way, all necessary precautions were taken by the researcher to ensure that the ethos of the teaching environment of each participant was preserved to a large degree.

A total of ten lessons were observed. Each lesson was approimately forty five to fifty minutes in length. The equipment was set up before the lesson could commence and recording started as soon as the learners came into the classroom. The observations were targeted at the teacher. The video recorder did not focus on the students. The observations were done randomly. The topics varied and the grades that were observed ranged from 8 to 12. Each of the recordings were saved for the purposes of analysis. Prior to the second round of interviews being conducted, each of these recording were analysed with a view to eliciting responses from the participants about their practices. Critical incidences were noted. These incidences and important moments in the lessons were used as prompts in the second round of interviews. The recordings of the participants' interviews have been entered into the reasearch as final research instruments (Appendix C and Appendix D).

3.5.3 Establishng Trustworthiness

Replicability and generalization of results are emphasized in quantitative and positivist research. The terms reliability and validity are associated with this. Morse, Barrett, Mayan, Olson, and Spiers (2008, p. 14) assert that, ―without rigor, research is worthless, becomes fiction, and loses its utility. Hence, a great deal of attention is applied to reliability and validity in all research methods". In qualitative studies however, the understanding and application of the need for reliability and validity differ from the positivist, quantitative studies. In their seminal work in the 1980s, Guba and Lincoln introduced the concept of *trustworthiness* in qualitative studies as a “parallel concept” to reliability and validity. Trustworthiness has four components, namely, *credibility, transferability, dependability*, and *confirmability* (Morse et al., 2008, p. 14). These terms have gained prominence in qualitative enquiry and have become the mainstay “for evaluation of the overall significance, relevance, impact, and utility of completed research”.

According to De Vos et al. (2005), *credibility* refers to the research being conducted accurately. The term triangulation is sometimes used as a validation strategy. This implies that the research is thorough and the parameters of a credible qualitative study are adhered to. Furthermore, the study must be in-depth, closely integrating the many variables and data.

According to Flick (2004), the term triangulation of data refers to combining data from different sources. This data may be collected from different places, at different times and with different people. Although the data may come from these different sources, the data can be verified against each other, thus leading to a process of triangulation. From the discussion, it would seem that these criteria were fulfilled in this study. Data collected from the participants were verified. Transcripts of the interviews, observation videos, as well as the draft report were also made available to the participants for their verification. The objectives, design features, theoretical frameworks and parameters have been clearly adhered to ensure a coherent flow of evidence that can be verified and triangulated by any person wishing to test the validity of the research.

“*Transferability* is an alternate to external validity or generalizability” (De Vos et al., 2005, pp. 346-347). In the qualitative research paradigm, external validity is not a priority. Furthermore, generalisation is not a priority in case study methodology. According to Starman (2013, p. 35) in terms of the case study the “idea of representative sampling and statistical generalizations to a wider population should be rejected, and analytical induction should be chosen instead”. Yet others believe that case studies may be used to make generalizations, and the case study allows the researcher to recognize “the similarities of the objects and issues in different contexts and by understanding the changes as they happen” (Starman, 2013, p. 39).

De Vos et al. (2005, pp. 346-347) asserts that choosing a case strategically can “enhance a study‘s generalizability”. Triangulation of many data sources allows the researcher to “corroborate, elaborate or illuminate the research in question” and “greatly strengthen the studies usefulness for other settings”. This alludes to the concept of internal validity as found in the quantitative studies. However in qualitative studies this is not the same and the concept of *dependability* best replaces this. *Dependability* refers to ―the process of identifying acceptable process of conducting the enquiry so that the results are consistent with the data" (Cohen et al., 2001, p. 120). ―Dependability is fulfilled if the research process is subject to triangulation, member checks and respondent validation (Cohen et al, 2001, p.120). As has already been stated above, during the process of data generation, aspects such as triangulation, member checks with the respondents and respondent validation was carried out on an on-going basis.

In my research I have fulfilled the requirements of validity by encompassing the use of multiple informants, multiple case studies and different forms of data gathering like interviews and observations.

Confirmability, according to De Vos et al. (2005, p. 347), replaces the ―traditional concept of objectivity", where the influence of the researcher is removed and the data itself is examined as objective. In qualitative studies however, as described, there cannot be a situation of total objectivity as the researcher is integrated in the research context. However the study tries to remove situations of observer bias which could influence participant responses.

I addressed the threat of observer bias in my study by declaring and disclosing my bias to the participants. During the year-long engagement with the participants, I disclosed my bias and beliefs about mathematics teaching, and the use of visual thinking in the mathematics classroom. I understood the potential influence it may have on the research findings. One of the methods employed to minimise this was to constantly corroborate the data with the participants. Through this process, it was anticipated that I had minimised the effects of observer bias and satisfied the necessary criteria for *credibility, transferability, dependability*, and *confirmability.*

3.6 Data Analysis

Data analysis refers to the ―process of bringing order, structure and meaning to the mass of collected data". In the context of this study, rich data emerged. This aspect of data collection was also compounded by the volume of data that emerged. It required careful scrutiny and categorisation. In order to address the first critical question, several views emerged on the nature of visual reasoning and successful teaching, making the process very complex and time-consuming. This process seemed to correlate with the views about data collection as –messy, ambiguous, time-consuming" yet simultaneously –creative and fascinating" (De Vos et al. (2005, p. 333).

Data analysis and interpretation in qualitative study, according to Conrad and Serlin (2011, p. 208), –tend to occur more simultaneously in interpretative of analysis". Merriam (1988, p. 123) suggests that –the process of data collection and analysis is recursive and dynamic". The iterative nature of –analysis and interpretation of case study research" is also confirmed in Hesse-Biber and Patricia (2011, p. 269) who suggests that the researcher collects data and analyses it as an –on-going activity", and simply intensifies the analysis when all the –data have been collected". In order to address the second critical question, an iterative approach had to be employed. By examining the two critical questions, the data had to be examined and re-examined several times in order to fully understand how and why visual reasoning was employed by successful teachers in their practice.

Taking into account the kind of methodology being used in this research project, and bearing in mind the processes involved in analysis, an attempt was made to adhere to the above mentioned procedure, seeing data analysis as an on-going process. Immediately, after each set of data had been collected, the data was organised into tables, according to different categories. Accounts given by authors in the literature helped to create these categories. At the same time, the theoretical frameworks (to be discussed in chapter 4), also shaped the kind of categories and themes that were emerging. This process continued as each new set of data emerged, thus making it an iterative process. According to Merriam (1988, p. 133) this process entails, –developing categories, typologies, or themes, as well as searching for recurring regularities in data, which units of information go with each other?"

The use of multiple case studies in this study was supported by Merriam (1998). Cross-case, cross-site or multiple site case studies, involves collecting and analysing data from several cases. After analysing each case on its own, then cases are compared for patterns and trends. This provides a more compelling analysis and has the potential "for generalizing beyond the particular case" (Merriam, 1998, p. 153).

The use of cross-case analysis seemed to be an ideal choice for this research. Hence, themes, similarities and differences had to be sought in the data so as to foster an intensive analysis, by comparing the different cases.

The multiple sources of data and the use of a multiple method study, facilitated triangulation as well. During the process of data collection and analysis, member checks were employed, where the data was taken back to the participants and asked if the results were plausible. This also facilitated the process of internal validity.

Cohen, Manion and Morrson (2011, p. 301), propose six ways to organize and provide written analysis of case studies, namely, “the suspense structure, the narrative report, the comparative structure, the chronological structure, the theory-generating structure and the un-sequenced structure”. This research report was compiled on the basis of the comparative structure format, since each of cases was analysed using different lenses, which Yin (2009) lists as: explanatory, descriptive and theoretical. This provided a richer, well-rounded “account of the case”. The data has been described, and explanations have been offered in terms of the variables that have been identified, based on the theoretical frameworks of Attribution Theory, Gardner‘s Multiple Intelligences Theory and Situated Cognition Theory.

3.7 Ethical Considerations

According to Cohen et al. (2001, p. 50), “much social research necessitates obtaining the consent and cooperation of subjects who are to assist in investigations and of significant others in the institutions or organisations providing the research facilities”. Research ethics refers to the process of conducting the research in a responsible manner. This process is aimed at ensuring that the confidentiality of the participants are not breached in any way. Furthermore, the researcher also accepts to use the information in a scientific manner, and in no way, bring harm to the participants.

Gravetter and Forzano (2003, p. 60) assert that “researchers have two basic categories of ethical responsibility: responsibility to those, both human and non-human, who participate in a project; and responsibility to the discipline of science, to be accurate and honest in the reporting of their research”. Ethical guidelines are considered to be the code of conduct or standards that a researcher evalautes her/his own engagement in the entire research process.

In an effort to ensure no harm to the participants in this study, either physically and/or emotionally, each of the participants were informed of the nature of my research at the outset. Participants were made fully aware of the level of their participation in this study, through an introductory letter issued by myself. They were given the opportunity to withdraw at any time, should they feel that their position was being compromised. I also informed each participant that they would be interviewed twice. The first interview would try to determine their understanding of the phenomonen being investigated. The purpose of the observations was to observe, how visual reasoning was being employed in their practice. The second iterview would offer them an opportunity to explain their actions in terms of their classroom practice, using visual reasoning. Permission was granted by the head of the school, to conduct the research. The participants were made aware of these conditions. In this regard, letters of consent for each of these processes were obtained from the head of the institution as well as from each of the participants. Furthermore, they were also assured that their confidentility would be guaranteed at all times and at no point would their actual names be used. They agreed to the terms of the research (See Appendix E) .

The participants were also given the assurance that the evidence that had been revealed in the study would be recorded, accurately, as it was observed. They were also given the opportunity to correct any data that was supposedly misrepresented.

The head of the school was assured, through a written consent form, that the research was confined to the mathematics staff only, and that learners that may appear in the classroom observations would not be used for the purposes of this study. In such instances, the identity of the learner would be protected by using suitable strategies to conceal their faces.

I believed that honesty to the participants would engender a relationship of trust which would be beneficial to the research.

3.8 Background of the Participants

In the discussion that follows, each of the five participants, three females and two males, will be introduced. A brief decription of their biographies will be furnished. A comprehensive description of their qualifications, teaching experience and background will be examined in chapter 5, which is concerned with analysing the data.

A brief description of the research site will also be traced. This will help the reader to gain a clearer picture of the kind of environment these participants were engaged in their practice.

Fictitious names have been used for the participants, in order to protect their identity. This was negotiated with the participants at the commencement of the research project. The research site will also assume a fictitious name, in accordance with the requirements of research ethics.

3.8.1 Valerie

Valerie, in her early forties, had taught in several schools over the past twenty years. She had taught in public, ex-model C and privates schools during her teaching career. She has taught in Gauteng, Swaziland and KwaZulu-Natal. She holds a Diploma in Education and Further Diploma in Education (FDE). During her teaching career, she has taught Mathematics, Additional Mathematics and Technical Drawing, ranging from grades eight to twelve.

3.8.2 Betty

Betty, a Bachelor of Science Graduate, taught in several schools in a career spanning 33 years. She taught in several public and ex-model C schools prior to her coming to her present school. She is now in her earliy fifties. Apart from obtaining a degree from the Western Cape, she also holds a Higher Degree in Education and a Post Graduate Diploma in Education. She has been primarily concerned with the teaching of Mathematics and more recently, Mathematical Literacy.

3.8.3 Karl

Karl is in his early forties. He has taught at both public and private schools in KwaZulu-Natal, during his nineteen year teaching career. He attained a Degree in Mechanical Engineering and a Teaching Diploma. Karl has enrolled for a Master‘s Degree in Communication Science. He has predominantly taught Mathematics and Mathematical Literacy.

3.8.4 Sam

Sam, in his early fifties, has taught at two other public schools in Kwazulu-Natal, prior to his transfer to his current school. He attained a Bachelor of Arts degree and a Higher Diploma in Education. Throughout his teaching career, he has taught Mathematics and Additional Mathematics except for a short stint of Geography, in his first year of teaching. He has been teaching Mathematics for 28 years.

3.8.5 Nellie

Nellie is the youngest member in the Mathematics Team, who has taught at one other school, before migrating to her current teaching post. She has attained a Bachelor of Science Degree and an Honours Degree in Science. She has taught Science and Mathematics, although much of her teaching load has been made up of Science. She has been teaching now for a period of 3 years.

A summary, of the brief profiles of each participant, is presented in Table 2 below:

Table 2: Summary of the Profile of the Participants

SUMMARY OF THE BRIEF PROFILES OF THE PARTICIPANTS					
NAME	VALERIE	BETTY	KARL	SAM	NELLIE
AGE CATEGORY	40-44	50-54	40-44	50-54	30-34
GENDER	FEMALE	FEMALE	MALE	MALE	FEMALE
QUALIFICATION	Teaching diploma and FDE	B.Sc. and HDE and PGCE	Mechanical Engineering Degree	B. Arts and HDE	B.Sc and Honours in Science
TEACHING EXPERIENCE	20 YEARS	33 YEARS	19 YEARS	28 YEARS	3 YEARS
NO.OF SCHOOLS	5	7	2	2	1
TYPE OF SCHOOL	Public, Ex-model-C, private	Public, Ex model-C	Public and private	Public	Ex model-C

3.9 Problems encountered with Data Collection

Teachers are generally apprehensive if they have to allow ‗outsiders' into their classrooms to observe their teaching practice. This was anticipated. The participants felt that they would be exposing themselves to the outside world and their colleague. They had to be ressured that the intention of the study was not to expose them in any way, but to analyse their behaviour in terms of using visual reasoning in their classrooms. Furthermore, in terms of the confidentiality that I had assured them of, no part of their work would be made available to the outside world, without their prior written consent.

However, during the course of research, unexpected situations do often arise. During observation lessons of Karl and Sam, the video recorder malfunctioned and therefore the video recording of these two participants were destroyed. Another recorder had to be purchased, and the two observation lessons had to be repeated. This resulted in delays in data collecting. Several other incidents also worked against the reseacher in the process of data generation:

Karl had to attend a conference from (13 – 20) May 2013. When he returned, it was the commencement of the June examination at school. He was not in a position to teach a lesson, except revise with the learners. The recordings of his lessons were deferred. They were only done late in August 2013. Nellie was on maternity leave at the beginning of 2013. She had taken leave and only returned on 21 May 2013. Her observation lessons were only scheduled after the July vacation. Valerie, as a grade coordinator was really under a great deal of pressure, performing other adminstrative duties. As a result, her observation lessons were rescheduled as well. This resulted in delays in the process of data generation. Perhaps the most challenging event during data generation was the loss of Betty‘s son. Since he was an ex-learner of the school and the family lived on the Campus, his death had serious implications for my research. His long drawn illness, meant that Betty had to take leave often. Furthermore, his funeral arrangements were made at school and this spread over a number of days. A memorial service was held to honour him. Although the whole event was rather sad and traumatic, Betty eventually, after several weeks got back into teaching and allowed me to continue with the data generation. These events delayed the process of data collection.

Thereafter data generation proceeded fairly well. However, with these unexpected events, much of the data generation was only relegated to the latter part of the year 2013. I was hoping to have collected all data by June 2013. The result of these delays was that the process of analysis was delayed as well. As a reseacher, I had been cautioned about difficulties in data generation and had to adopt a sensible and sympathetic stance during these difficult periods.

The above incidents and difficulties have resulted in delaying the progress of this study. By examining the issues affecting the participants, perhaps it may help to explain the actions of some of the participants, in light of the difficulties that they had experienced. On the contrary, it may, show how despite these difficulties, these participants were able to remain successful in their practice.

3.10 Conclusion

This chapter described the methods that were used in this study. A justification for the use of a qualitative paradigm was provided as well as a motivation for the choice of case studies, as a research methodology, was also presented. The chapter also outlined the process of sampling that was used in the study and briefly explained the data collection instruments. Issues relating to trustworthiness were discussed. Before considering the issue of research ethics, the process of data analysis was outlined. The chapter concluded by identifying the participants to be engaged in the study and some difficulties that were experienced in the course of the study.

In the next chapter I examine the theoretical frameworks that have underpinned this study. Three theoretical frameworks have been chosen for the purposes of this study, namely Attribution Theory, Gardner‘s Multiple Intelligences Theory and Situated Cognition Theory. A detailed analysis of these theories will be conducted, with a view to explaining the research undertaken in this study.

Chapter 4

Theoretical Framework

4.1 Introduction

Social Constructivism is the overarching theory which underpins this study. The social constructivist paradigm offers a tangible explanation to the process of knowledge acquision, which is particularly useful in understanding the practice of teaching as well as explaining empirical evidence gathered in various research studies: Brummitt (2013); Kumari (2014); Zane (2009); Sivan (1986); Belbase (2011) and Yilmaz (2008). Yilmaz (2008, p.162) and Belbase (2011, p. 2), assert that knowlege is not merely ―received” but ―constructed by individuals or groups making sense of their experiential worlds”. This study therefore seeks to explore the ways in which teachers construct and interpret the knowledge that they pass onto their learners, using visual reasoning in their practice. Brummitt (2013, p. 100) also describes how social constructivism provides an explanation about how knowledge is constructed, and ―generated, that is, in relation to an individual‘s unique experiences and ideas”. As a consequence, the process of learning is actually an exercise in ―restructuring preexisting modes of thinking rather than the accumulation of information”. Teaching too, is regarded as a strategy of facilitating ―this thinking, not the imparting of facts” and learners learn ―by looking, listening, and constructing for themselves”. Similar views are echoed by Kumari (2014, p. 32) who asserts that learners also have to construct ―their own knowledge individually and collectively”. In light of these assertions, the tool of visualization can be regarded as important part of the process of engaging the learners and inculcating deeper levels of understanding of mathematics.

Further, with social constructivism there are multiple, constructed realities (Yilmaz, 2008). This view of constructivism is very relevant for this study as it implies that the realities are varied, as well as dispels the notion of a single truth. The literature review reveals divergent views of visual reasoning. This seems consistent with the views of education, in which learners construct knowledge based on their own experiences. It seems plausible that no single view of visual reasoning will suffice and that multiple interpretations of this concept will emerge in the context of education. Furthermore, in terms of this study, the action of teachers and the contexts in which they work cannot be seen in terms of a single theory. The dynamic nature of teaching and learning demands multiple focus on several variables in education, hence the emergence of multiple realities.

By underpinning my study with a social constructivist framework, multiple perspectival views become possible (Yilmaz, 2008). It offers different theoretical frameworks within itself, thus allowing different perspectives from which to conduct my research. In this, it became evident that no single theory could adequately explain the research findings. Thus I identified three theories within the framework of social constructivism to assist my understanding of the data, namely, Attribution Theory, Gardner‘s Theory of Multiple Intelligences and Situated Cognition Theory. Each of these theories have different contributions to show how teaching and learning takes place. The salient characteristics of each of these theories will be discussed in this chapter.

In trying to extricate from the topic, the most important aspects that need to be considered are: successful teachers, visual reasoning and teaching. Whilst the successful teacher engages in teaching, and focusses on her/his learners‘ success, teaching is about the moments of excellence that we need to concentrate on. The moments of excellence may actually be visual in nature. The focus is therefore on those situations that show why these teachers are successful. This situation is illustrated in figure 4.1 which I have created to show the possible relationship between visual reasoning, successful teaching and teaching.

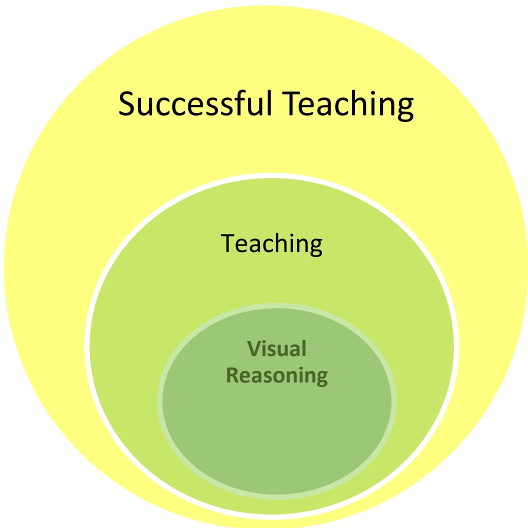

Figure 4.1: The possible relationship between successful teaching, visual reasoning and teaching

The figure shows that whilst teaching is about the moments of excellent the teacher can achieve, it nevertheless is a part of attaining success, hence the bigger cirle. However in attaining these moments of excellence, teachers will use various strategies. In this context, visual reasoning is the strategy that is used in order to attain this success. Hence visual reasoning may be seen as a process that can be used in teaching in order to become a successful teacher.

In presenting the different theoretical frameworks, it will provide a framework for the content, since this is in keeping with my personal idea that visualization will help to comprehend and make it easier to assimilate the discussion.

4.2 The Three Theories

Attribution Theory is concerned with the nature of success and motivation. Success, incorporating the notion of motivation forms the basis of attribution theory. The actions of successful teachers can therefore be explained succintly using this theory. Visual reasoning can be explained by using the concept of Multiple Intelligences. Of the several intellingences through which learning takes place, visual reasoning occupies a prominent position. Visual reasoning is a prominent type of representation since vision often trumps any of the other senses. Therefore, visual reasoning is often inextricably connected to multiple representations as well. Teaching as an embedded, active process is an activity that can be explained using Situated Cognition Theory. As a social process, teaching is embedded in many contexts and hence its dependence on a social theory to explain its intricate nature is crucial to its understanding.

The integration of these three theories in attempting to explain the phenomena of success, visual reasoning and successful teaching are captured in figure 4.2.

It would seem that successful teaching and visual reasoning have elements that are common with each of these theories, which I propose will become evident as the chapter unfolds.

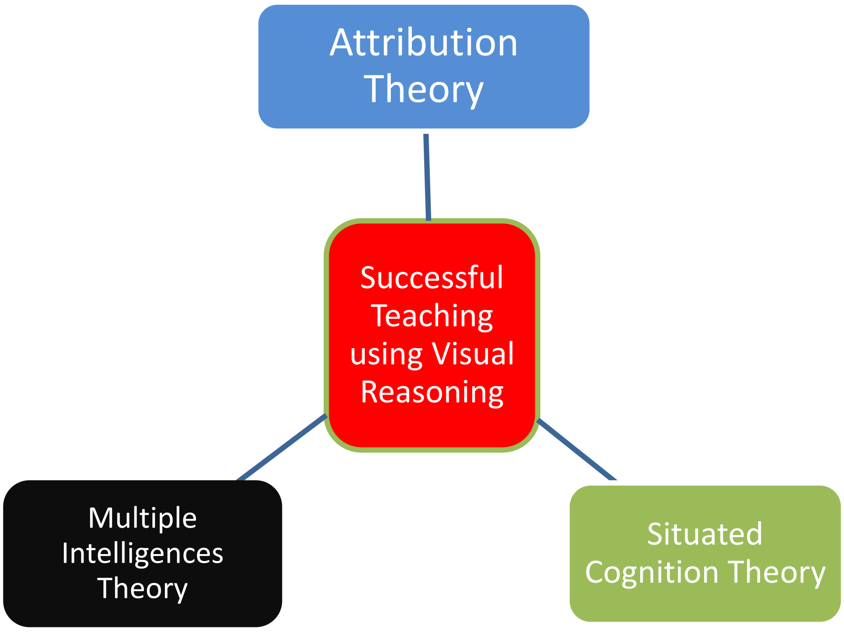

Figure 4.2: Possible relationship between successful teachers who use visual reasoning and the theoretical frameworks of Gardner‘s Multiple Intelligences Theory, Situated Cognition Theory and Attribution Theory

A detailed examination of each theory has been conducted in order to explain phenomena observed in this study.

4.2.1 Attribution Theory

The focus of this study is on successful teachers. Attribution Theory deals directly with the phenomenon of success and motivation. Although it focuses on failure as well, this study will examine the application of Attribution Theory to the notion of success.

This theory will be examined under the following themes, as illustrated in (Figure 4.3): definition of the theory, nature of the theory, its ability to explain the essential phenomena in the study and any shortcomings that may be inherent in the theory.

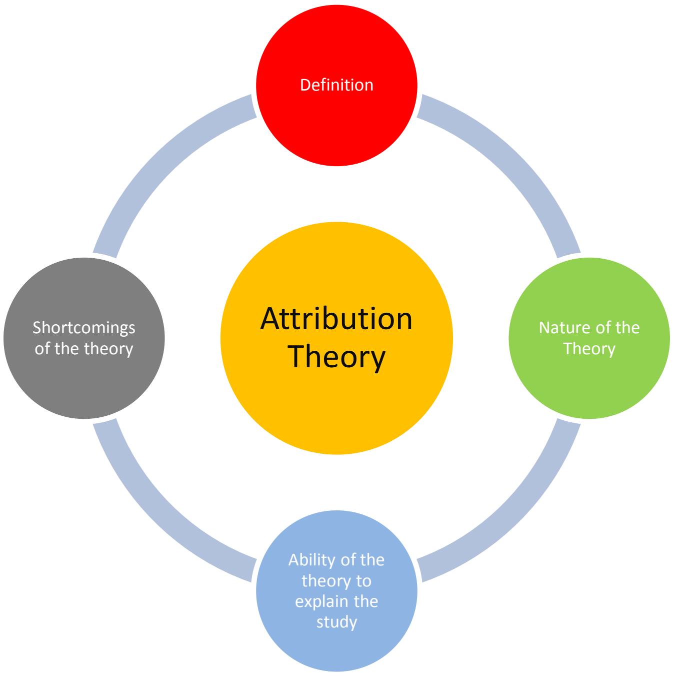

Figure 4.3: Relevant themes under which Attribution Theory will be explained

4.2.1.1 Definition of Attribution Theory

Attribution Theory is a social theory of motivation, based on the notion that causal attributions can influence present and future achievement and motivation (Weiner, 1972). Although there are several definitions of attributions, the internal thinking and external talking process of interpreting and understanding what is behind our own and others‘ behaviour is deemed important (Manusov & Spitzberg, 2008). Similarly, researchers like Shores (2011), Wolters, Fan and Daugherty (2013) and Thang, Gobel, Mohd Nor and Suppiah (2011) assert that Attribution theory‘s main concern is about how people process information and what kind of information they process in order to understand events and act on them.

According to McArthur (2011, p. 32), ―the concept of attribution, is typically an abstract concept". Whilst, it may seem to be abstract in nature, it has useful implications for understanding human behaviour. According to Savolainen (2013, p. 64), ―attribution theorists share an interest in studying how people explain successes and failures by making judgments about someone else's or their own behaviour. They aim to understand the causes to which they attribute behaviour". According to Weiner (2010, p. 29) ―behaviours previously rewarded will be repeated whilst those that were punished, will be avoided". He declares that how one perceives actions and events of the past will influence one‘s future decisions and courses of action.

Other researchers: Martinko, Sikora, and Harvey (2012), Weiner (2008), Zuckerman and Feldman (1984) and Hunter and Barker (1987) have made similar claims about the definition of attribution theory. It would seem that attribution theory has much to offer in terms of understanding human behaviour, more especially, the phenomena of success and failure. On this basis, it was chosen as one of the theoretical lenses that would be used to underpin this study.

4.2.1.2 Nature of the Attribution Theory

According to Manusov and Spitzberg (2008, p. 37),

> ―humans are an inquisitive species: we wonder why and how things occur, and we develop religions, philosophies, and sciences as ways of answering our questions. Such curiosity influences our cultural, societal, interpersonal, and personal lives in intricate ways".

A set of theories, collectively called attribution theories, seek to explain the mental and communicative processes that are involved that explain our behaviour.

According to Attribution theory, the explanations that people tend to make to explain success or failure can be analysed in terms of three sets of characteristics.

4.2.1.2.1 Locus

The cause of success or failure may be internal or external. Here we attribute our success or failure to factors that lie within ourselves or within the environment. An example of an internal attribution is where a person attributes her/his success to her/his innate ability. In an external attribution, the person may attribute some outside factor. One may often hear a reason such as ‗I was not in the correct frame of mind”, implying that some external factor was responsible for a specific behaviour.

4.2.1.2.2 Stability

The cause of failure and success may be stable or unstable. If the cause is stable, then the outcome is likely to be the same if we perform the same behaviour on another occasion. If it is unstable, then the outcome will be different on another occasion. For example, if a person is successful because of hard work and determination, then she/he will believe that success depends on the amount of effort rather than luck or ability. On the other hand, if success was dependant on luck, then the person may not always achieve the desired success. Hence, many people will continue to devote much time and effort to tasks because of their previous experience of success through hard work.

4.2.1.2.3 Controllability

Success or failure may either be controllable or uncontrollable. The former can be altered if we wish to whilst the latter may not be altered easily. Effort may be regarded as a controllable factor. The amount of effort that can be exerted into a task can be adjusted for success to occur. Task difficulty may not be easily altered. Task difficulty is determined by the person or the institution that sets the task. Hence it is regarded an uncontrollable by the person who has to perform the task. Luck is also considered to be uncontrollable, since it is deemed to be controlled by some supernatural force or power.

According to Hunter and Barker (1987, p. 51), ‗people attribute success and failure to four factors: native ability, effort, task difficulty and luck”. There are two general categories of causes, internal and external. Internal causes implicate characteristics of the individual (such as ability, attitudes, mood, and effort) for having caused a particular behavior, whereas external causes implicate external factors (such as the task, other people, or luck) for causing an event or outcome to occur. The distinction between stable, non-variable causes (such as innate ability for internal attributions and inherent task difficulty for external attributions) and unstable, variable causes (such as effort and luck respectively) was combined with internal/external dimension to form a basis for classifying the performance attributions made by individuals (Martinko et al., 2012).

According to (Malle, 2008), these factors can be analysed in the following ways:

Ability is relatively internal and stable over which the person does not have direct control. As a result of its nature, which is more or less internally determined, it is referred to a native ability. Although the term ability is often seen as controversial in research circles, because of the difficulty to authentically quantify and measure it, it is used in the theory. Hence, the term has been used in this context. Task difficulty is external and stable over which the person does not have any control. The reason for this is that the task often is prescribed and the level of difficulty is determined by another person or by the task itself. Effort is internal and unstable over which the person can exercise a great deal of control. It is classified as internal since it is determined by the person.

Depending on the nature of the task and the person, a certain degree of effort may be exhibited. This can be altered by the person depending on the level of success she/he wishes to attain. If a reasonable level of success is required of the task, then the person will put in the relevant amount of effort to make the event or task successful. Other factors may also dictate the amount of effort that may be required, such as incentives, desire for success and recognition for success. Luck is external and unstable over which the person has very little control. Luck has always been considered as some attribute that is provided by a mysterious, external, almost higher order force that cannot be explained on the material platform. These factors appear on three continuums: locus, stability and controllability. According to Savolainen (2013, p. 64), ―locus refers to the location, internal or external, of the perceived cause of a success or failure". A person may ―succeed or fail" because they attribute the cause to themselves, or externally to their environment. Internal factors are ―ability and effort" and external factors are task difficulty and sometimes a perception that something happened by luck, over which they had not much control.

According to Savolainen (2013, p. 64) stability refers to the ―extent to which a given reason for success or failure will change". If the reason for success or failure is considered to be stable, then there is a possibility that the individual will achieve the same level of success, if the behaviour is repeated. If an individual attains success due to persistent effort, then the outcome of performing a similar task will be stable and consistent with the previous behaviour.

Controllability refers to the extent to which an individual has control over the events in a situation. Luck is considered to be an uncontrollable cause of an event, since it is dependent on forces beyond the control of the individual. On the other hand, causes such as effort are regarded as controllable. According to attribution theory, the amount of effort can be determined, before the activity is performed.

Hunter and Barker (1987, p. 51) use similar arguments when they state that, –attributions of success to internal locus (ability, effort) result in increased self-esteem (lack of ability). Attributions to failure to internal locus result in shame (lack of ability) or guilt (lack of effort)". Teachers often attribute success to their efforts and failure to the efforts of their learners. If a learner or a group of learners perform well, then teachers will attribute the learners' success to their (the teachers') efforts. On the other hand if a group of learners perform badly in a task, it is very likely that the teachers will attribute their poor performance to either laziness or lack of sufficient effort on the part of the learners. According to McClure, Meyer, Garish, Fischer, Weir and Walkey (2011, p. 71) –this patterns serves to enhance people's self-esteem and lessen the chance that they feel demoralised when they fail". This is also referred to a self-serving bias, which was found to be common across cultures, age groups and gender. However, it was also found that this principle was less pronounced in eastern and Asian cultures. The basic principle of attribution theory as it applies to motivation is that a person's own perceptions or attributions for success or failure determine the amount of effort the person will expend on the activity in the future. Even Gardner, a protagonist of Multiple Intelligences theory believed that successful people often exert enormous effort (Hunter & Barker, 1987).

Figure 4.4 represents the basic causal factors of attribution and the dimension of causes, in which they occur. As seen in figure 4.4, the basic attribution categories can be divided into three categories, namely locus of control, locus of causality and stability. As discussed, locus of causality refers to factors that are either internal or external. These factors may be within an individual's control or out of the individual's control. If the factor can be altered, it is regarded as controllable, otherwise it is regarded as uncontrollable. Stability refers to the extent to which the given reason for success will change. If the reason for success or failure is considered to be stable, then there is a possibility that the individual will achieve the same level of success, if the behaviour is repeated.

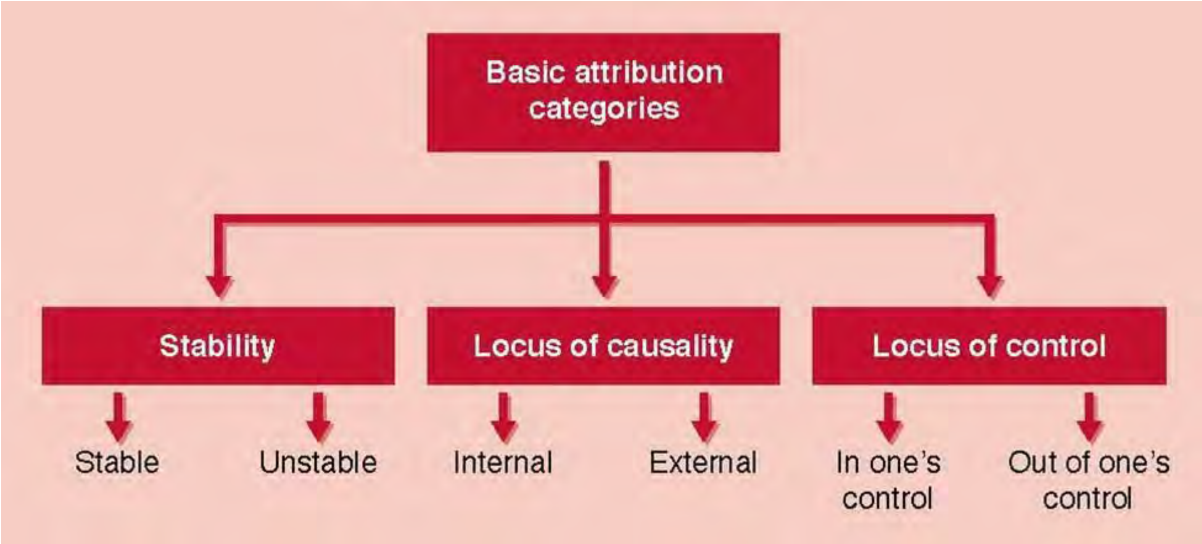

Figure 4.4: Causal factors and dimension of causes [1]

The above mentioned causal factors and dimension of causes can also be represented in table 3.

Table 3: Table showing causal factors and dimension of causes [2]

	Stable	Unstable
Dispositional	Intelligence, judgement, personality, willpower ―He's just not a good singer"	Moods, exertion of effort in a specific task, momentary whims ―She wasn't trying hard enough"
Situational	Institutional factors, economic factors, structures such as race, gender, class ―The judges always go easier on girls"	Coincidence, weather, dumb luck ―You're lucky I had a sore throat today"

[1] http://www.google.co.za/imgres?imgurl. For complete reference see bibliography.
[2] http://www.psychologynoteshq.com/attributiontheory/. For complete reference see bibliography

In table 3, the dispositional factors refer to internal factors such as ability. For example a person may be very successful because of his intelligence, which may be considered as an internal characteristic and stable. This is reflected in the second row and second column of the table). On the other hand, a dispositional factor like effort may be considered to be unstable, since the reason given for his failure can change. If a person believes his success or failure is stable, then the outcome is likely to be the same. In the case of failure, effort would be considered unstable. It can always be argued that a person did not devote sufficient effort into the task. This is reflected in the second row and third column of the table. Situational factors refer to the locus of controllability. In this case, the situation is controlled by some external factor such as preference to a specific gender in a task. This is considered to be very stable as well. If, for example, the judges in an event give preference to girls, it will mean that success for girls is more or less guaranteed. According to Kelley and Michela (1980, p. 476), –since attributions to controllable factors imply that the person can satisfy his goals through his own effort, such attributions should be beneficial in promoting expectations that the goals will be reached". On the other hand, luck may be considered unstable. If a person attributes her/his failure to luck, she/he may claim that she/he was just not lucky enough. In this event, the person may attribute her/his success or failure to something that is uncontrollable. It would seem then, from the explanations given, and from the summaries displayed in the table that the aspect of motivation and success is fairly abstract and that several factors may contribute to a person's success or failure. The intention of this study is to find possible reasons that the participants of this study may be able to furnish for explanations of their behaviour in the teaching of mathematics. Based on these factors, it is arguable then that attribution theory is very appropriate in explaining some of their reasons for their successes in their mathematics teaching.

The aspect of control needs to be analysed in terms of demographics and cultural differences. Whilst most of the factors of attribution theory tend to explain the motivation patterns of persons of western origin, these factors have to be examined differently when dealing with persons of non-western origins. This has been asserted by McClure et al. (2011, p. 72) that –western cultures attribute outcomes relatively more to internal causes, particularly ability, whereas Asian cultures attribute outcomes more to social factors such as family and to external factors such as fate". This aspect of this theory has to be taken into account considering that not all of the participants of this study are the products of western cultures.

Of the five chosen participants, two were of Asian descent. Perhaps the explanations for their behaviour in terms of their classroom practice may resonate with this theory. In an analysis of the data, consideration will be given to this aspect of the theory. However, attribution theory is still significant in explaining human behaviour and motivation, although the causal factors may not necessarily conform to those laid down by the theory, but it nevertheless falls within the domain of attribution theory, which accepts that some cultures subscribe to the theory differently. Essentially, whilst some people may attribute their success or failure to ability or effort, others may attribute these aspects to fate and luck. This is therefore, still within the ambits of attribution theory, albeit different from the generally acceptable premises of the theory.

It has also been shown that gender plays an important part in attribution theory, where ―male learners attributed their success to ability more than female learners" (McClure et al., 2011, p. 77). Whilst this is an aspect of the theory that needs to be taken into account in studying human behaviour, it nevertheless points to the differences in style of attribution of males and females. Therefore attribution theory must take into account differences in culture and gender if it is going to be used to explain behaviour of individuals. The influence of family, peers, culture and teachers is an important consideration in attribution theory if it is going to be used effectively as a tool to explain human behaviour. Since this study includes participants from both genders, the attributions that they make for their actions must be considered in relation to the above. The theory therefore points to the more important aspects of attribution, namely the social influences on human behaviour and not just the internal attributions of the individual. Whilst aspects such as task difficulty may be seen as an external factor in terms of this theory, they may see the influence of family as reasons for success rather than failure. In this regards, attribution theory then, must be seen as having the unique ability to explain behaviour across different cultures, since it is flexible enough to cater for the differences in society. This study will examine the influence of these factors in explaining the success of the participants.

According to Martinko et al. (2012, p. 147), ―a wide body of research indicates that the formation of causal attributions is vital for adapting to changing environments and overcoming the challenges we are confronted with in our daily lives". It gives the individual the opportunity to become successful by repeating the good actions, and avoiding those actions that lead to failure. It is through this that correct or successful behaviour is learnt and reinforced. Hence, the actions of these successful teachers may be viewed as having adapted through the various tasks that they have performed in their daily practices. Hence success is the continuation of behaviour that caused success of the previous events.

Judgements about personality can also be made using attribution theory. In their research, Hareli and Weiner (2000, p. 216), contend that

> –if one describes the success as resulting from his or her high ability, he or she is likely to be seen as arrogant. However, if persons downplay their role in bringing about that success, then they are likely to be perceived as modest”.

This is dependent on how communications of arrogance and modesty are defined. Arrogant communications suggests that one‘s qualities are superior to others and communications of modesty suggest that one‘s qualities are no different to others. The research also showed that women are seen more negatively than men when they communicate arrogant messages. Linked to the notion of arrogance and modesty, research by Hareli and Weiner (2000) indicate that success is more praised when it is achieved by hard work rather than by other causes. In this regard effort seems to allow people equal chances of success as opposed to factors such as ability. This study will examine the extent to which both factors of stability and controllability can explain the behaviour of the participants in their practice.

Against this backdrop, it would seem that Attribution Theory offers a tangible basis under which the behaviour of participants in this study can be examined. Apart from being able to offer possible explanations for the actions of these successful teachers, it may also present other insights into the nature of the participants, which may be relevant to the phenomenon of visual reasoning, which is the subject under scrutiny.

4.2.1.3 Ability of Attribution Theory to explain the study

According to Frieze, Weiner and Weiner (1971), ability, effort and luck are deemed to be more important causal factors than task difficulty, when an individual succeeds whilst others fail. However, if success occurs when others also succeed, then the above-mentioned causal factors are not considered very important.

This suggests that where a person is deemed to be successful whilst others fail, they will attribute their success to ability, effort and luck more than the level of difficulty of the task. The individuals will tend to place more emphasis on their effort and ability. In the case of failure, the individual may explain the failure to the absence of some external factor such as luck. Within the context of mathematics education in South Africa, many teachers are failing to attain reasonable levels of success with their learners. The South African grade twelve examinations bear testimony to this. During the period 2009 to 2012, an average percentage of 47, was obtained in the National Senior Certificate Examinations (National Senior Certificate Diagnostic Report, 2012). Against this backdrop, these successful teachers will attribute their success in these examinations to their better ability, more effort and luck, rather than acknowledge the task difficulty. The focus will therefore be on the amount of effort that they invested into the activity. If most teachers were deemed successful, it is very likely that the causal factors will be deemed to be externally controlled. In this particular case, the task may be considered to be easy, hence others have also succeeded. Whilst ability and effort seem to determine success, task difficulty seems to be a major contributing factor in failure. It would seem therefore that success is attributed to internal sources and failure to external sources. According to Frieze et al. (1971, p. 595) the ―locus of control influences affective reactions to an outcome, and that ego-enhansive and ego-defensive attributional tendencies are elicited in achievement contexts". There is this tendency to attribute success to ability rather than to chance. The reference to ―ego-enhansive tendencies", that accompanies success and ―ego-defensive tendencies" illustrates the point that those who are successful under these conditions will attribute their success to their ability more than they would to the task difficulty, whilst those who fail will attribute the difficulty of the task for their failure. For example, it will not be uncommon to hear statements to the effect that, ―even though the task was difficult, I really worked hard with my learners and that‟s the reason for my success", in the context of a teacher. It would seem that these successful teachers also attribute their success to ability as they declared in their interviews. In examining the actions of the successful participants in this study, these causal factors will serve as a basis by which their actions could be understood.

Whilst success is deemed very necessary in society, it can be accompanied by pride and arrogance. These findings were made by Hareli and Weiner (2000, p. 215) who ―showed that accounts ascribing success to internal, stable, uncontrollable, and desirable causes represent positive given qualities of a person (e.g. intelligence and beauty) that lead to perceptions of arrogance and lack of modesty". Since many of these teachers attributed their success to internal factors, stable conditions, we can detect a sense of arrogance and pride. On the contrary, many of these teachers also attributed success to external factors such as good support from managers and the presence of excellent role models in their lives. At the same time, all these teachers attributed their success to passion for the subject which may be seen as an internal stable factor. Hareli and Weiner (2000, p. 216), make another important point when they say that ―arrogance and arrogant communications emphasize that one‗s qualities and worth are better or superior than those of others". In a way it may seem contradictory. However successful people are those that distinguish themselves from the others. Successful people often exert themselves so that they can be recognized. Hence the actions of these teachers may be justified. According to Hareli and Weiner (2000, p. 217), ―stability and controllability may also have roles in determining perceptions of modesty and arrogance, which have been ignored in prior literature". We believe that arrogance is associated with causes that are not only internal to the person, but also are seen as stable and uncontrollable.

The application of Attribution Theory to explain everyday behaviour of individuals is relevant since the study intends examining the behaviour of successful teachers on an on-going basis, rather than as sporadic acts within the ambits of teaching. This idea has been echoed by Manusov and Spitzberg (2008, p. 47), when they declare that ―their ubiquity in our everyday sense-making means that attributions are ripe for study by people in their everyday lives".

This study examined the actions of these teachers, as they unfolded in their day to day activities, and the reasons they furnished for their actions.

Attribution Theory is concerned with the study of causal attributions which is vital for adapting to changing environments, and as such, it is the ideal choice to explain the behaviour of these successful teachers who are constantly working in very dynamic environments.

Attribution Theory goes beyond explaining human behaviour. In his works, Weiner and Walker (1966), alluded to the significance of motivational factors in memory retention. This aspect of teaching and learning can possibly explain why some teachers are considered to be more successful than others. If teaching is conducted with the intention of increasing the motivation during perception, then the chance of retention and permanent memory is greater. This in effect will lead to the ultimate success of the teacher (Weiner, 1966).

4.2.1.4 Shortcomings of the Theory

Since attributions are contextual and culture bound, the theory may appear to be restricted in terms of explaining the concept of motivation in human beings. Effort, for example, which is defined as an internal factor, may be interpreted differently by different groups of people, depending on the culture of that group. The same is true for ability as a causal factor. Some cultures may refuse to categorise ability as an internal factor and may classify it as external. Often religious and cultural biases may affect the view that people have of these causal factors. For example –Asian and eastern cultures are more situational biased than western cultures" (Manusov & Spitzberg, 2008, p. 45). Asian cultures place more emphasis on the role of society, peers and teachers rather than the individual. Hence it is considered a norm if attributions for success are made to external factors rather than internal factors such as ability and effort. Unless the contextual factors are described fully, Attribution Theory may appear to explain human behaviour incorrectly.

In as much as this may be seen as an alternate perspective in Attribution Theory, the differences in cultures and gender will provide a strong basis in order to better understand the actions of the participants in this study. The presence of both genders in the study, allows attribution theory the opportunity to explain success and failure holistically. In the absence of one of the genders, the study might have been construed as incomplete or biased in favour of that gender. The same is true for the differences in culture of the participants. This study is representative of both eastern and western culture. Again, Attribution Theory, has offered an alternate perspective in these cultures. This, apparent shortcoming, advocates a more convincing case for the use of attribution theory. The theory has already pointed out to the differences in its application in each of these cultures. Hence, it will offer a balanced theoretical explanation for the actions of the participants in this study.

4.2.2 Gardner's Theory of Multiple Intelligences (MI)

The concept of intelligence is complex and multidimensional. Gouws and Dicker (2011, p. 572) suggest that it was –Gardner (cognitive neuroscientist, education researcher and Harvard professor) whose seminal work Frames of Mind in 1983 radicalized the way the concept of intelligence was perceived". His main thrust was that intelligence is to be regarded as multifacted and that each learner has a different capability or strength which must be identified and developed to produce effective education. Other researchers like Blomberg, (2009, p. 163) laud the contribution of Gardner as a ―scientific justification for a more pluralistic pedagogy" while Roberts (2010, p. 241) emphatically declares that Gardner‗s ―Multiple Intelligences has left its indelible mark on pedagogical canon now for more than a quarter of a century" changing the conceptualization of intelligence from the narrow, standard measure of intelligence using the Stanford-Binet IQ Test based on the premise of one type of intelligence that is genetic in nature and thus fixed throughout life.

I lean strongly upon Gardner‗s ideas of Multiple Intelligences as a tool for the analysis of my data as I deem it relevant to understanding the phenomenon of visual reasoning.

In this section Gardner‗s Multiple Intelligence Theory will be referred to as MI and will be discussed as follows: definition of the theory, nature of the theory, ability of the theory to explain the study and shortcomings of the theory. A thorough understanding of Gardner‗s Multiple Intelligences Theory can only be achieved by examining each of the aspects mentioned above. In creating the diagram below, it is hoped that it will help to facilitate the discussion that will follow.

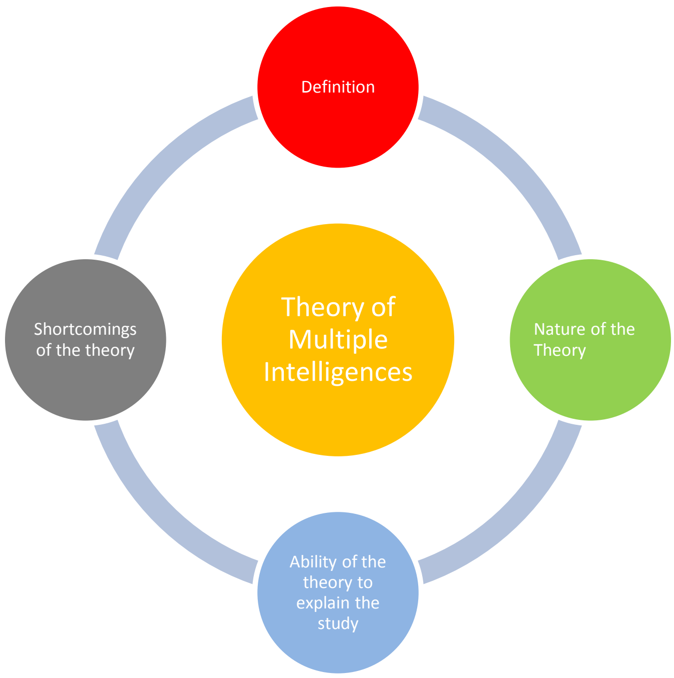

Figure 4.5: Relevant themes under which Gardner‗s Multiple Intelligences Theory will be explained

4.2.2.1 Definition of the theory

Blomberg (2009, p. 164) describes Gardner‘s idea of multiple intelligences as ―analogous to computational capacities for information processing, each is like a separate computer" who also proposes that the theory ―provides a sound framework for mathematics educators".

Gouws and Dicker (2011, p. 569), also claim that MI ―may enable mathematics teachers to improve the performance levels of learners and provide them with the knowledge and skills to enable them to contribute and participate confidently in society".

4.2.2.2 Nature of the theory

Gardner‘s theory according to Gouws and Dicker (2011) revolutionized the traditional view of education which saw intelligence as a one dimensional component. It proposed that intelligence is made up of several components, thus allowing the individual several opportunities by which to learn. Its multiple centred approach, therefore offered teachers of mathematics the opportunities to present mathematics in a variety of ways, because of the nature of human intelligence. Seen from this perspective, it resonates with the literature surrounding visual reasoning. Visual reasoning itself is seen as a multiple form of representing mathematics, through diagrams, pictures, artefacts, technology and gestures. Against this backdrop, visualization in itself offers multiple forms of mathematical presentations.

It was believed that different parts of the brain assimilated knowledge in different ways. Hence the learning was not restricted to a one dimensional path in which knowledge seemed to have flowed, but through multiple sources and pathways. In this regards the old debates surrounding the teaching of mathematics purely symbolically or visually is of little consequence. Rather, this theory welcomes a more diverse teaching style, incorporating many approaches to teaching and learning, of which the visual approach was a crucial element, and a key idea in my study. My exploration of how successful teachers engage with visual approaches in the teaching of mathematics is supported by the call for a more diverse pedagogy that focuses on the individual differences of learners. Andronache, Bocoş, Stanciu, and Raluca (2011, p. 18) assert that schools ―must recognize the diversity of the human beings and inter-individual differences". A key consideration is the ―differentiated treatment of learners" in the ―design of teaching and learning activities" based on Gardner‘s MI

Theory. Gouws and Dicker (2011, p. 572) also assert that teaching and learning is –not an exclusive process, but an inclusive one”. They suggest this means finding the strengths that each learner has and not only identifying –the exceptional gifts in some learners”. To Gouws and Dickers (2011, p. 572) intelligence is –a dynamic process encompassing multiple abilities”.

MI theory lists several types of intelligences which different individuals possess, namely: verbal/linguistic, logical/mathematical, visual/spatial, musical/rhythmic, corporal/kinesthetic, interpersonal, intrapersonal, naturalistic and existential intelligence. Not even identical twins have the same combination of intelligences. According to Andronache et al. (2011, p. 19) the –configuration of intelligences and relations between them will change themselves in response to personal experiences and according to the sense that one associates with them”.

This reference to the changes according to personal experiences highlights the significance of a person‘s experiences in terms of shaping their intelligence. The impact of society cannot be discounted in terms of shaping a person‘s experiences. Hence learning must therefore be seen as embedded in society and the link to the constructivist theory of learning becomes more apparent. The need for understanding learning as situated, also emphasizes the role played by situation cognition theory in understanding learning and hence its application to this study, which will be discussed in greater detail in the next subsection. It would seem therefore, that the theory of multiple intelligences is integrated very closely with situated cognition theory in explaining the process of learning.

Contrary to the belief that human intelligence was fixed, this theory allows educationists the opportunity to believe in the potential of every learner. The theory caters for individual differences, thus necessitating the need for a realignment of pedagogy to include alternate forms of assessment as well as incorporating a variety of teaching styles. It would seem that this theory explains the possibility that every learner has the opportunity to be successful. The means to transmitting knowledge and skill is no longer limited, but flexibly allows the learner the opportunity to gain insights through different means. If a learner does not have a highly developed logical/mathematical ability, this does not imply that all learning will be unsuccessful. The other intelligences may be used to develop the potential to think logically and mathematically. This therefore implies that no learner can be considered to be an absolute failure. Furthermore, alternate forms of assessment must accommodate these differences in learners.

Initially Gardner classified the intelligences into seven main streams. In order to fully understand the complexities of these intelligences, it is important to examine each of these intelligences in greater depth. Other educational and cognitive theorists: Piaget, Bandura, Binet also propose specific categories of intelligence, at least nine of which have been identified as follows: verbal/linguistic, mathematical/logical, musical, visual/spatial, bodily/kinaesthetic, intrapersonal, interpersonal, naturalistic and existential. In the next section, the nature of these categories of intelligence will be elaborated. Taking into account that there are nine prominent modalities of intelligences, I have attempted to capture them in Figure 4.6, based on Gardner's idea of these intelligences. These intelligences may be represented as nine essential components of the central intelligence. Although each of these intelligences appear in different aspects of an individual's being, they collectively contribute to the individual's intelligence.

Figure 4.6: The Nine Modalities of Intelligence (adapted from Gardner, 1983)[3]

[3] http://be-human.org/wp-content/uploads/2008/03/gardners-theory.JPG

4.2.2.2.1 The Verbal/Linguistic Intelligence

This intelligence is largely associated with the development of language. According to Andronache et al.(2011), this type of intelligence enables a learner to be highly proficient with the use of language, both written and spoken. Such learners are able to understand and apply the rules of grammar proficiently, and are able to negotiate the many meanings that are found in all languages.

Mathematics, to a large extent is dependent on language. The development of this intelligence is therefore crucial to a learner‘s understanding of mathematics since many teachers will use language to teach mathematics. Learners, who are well disposed in the discipline, will have the advantage of understanding the demands made by the questions in mathematics. This may not guarantee their understanding of the mathematical concepts, but it may facilitate their understanding of what is required in the concept. Learners, who struggle with language, may find it difficult to understand mathematics, particularly if most of the instruction is done through the spoken medium, rather than the written medium. Although the verbal/linguistic intelligence is predominantly related to the development of language and the use of language in processing information, it is a significant contributor to learners understanding of mathematics.

4.2.2.2.2 Logical-mathematical Intelligence

It is that intelligence that is concerned with problem solving and is often regarded as the measure of ―raw intelligence”. According to Andronache et al. (2011, p. 20) learners who display this intelligence are able to predict, ―formulate, hypothesize and understand causal relationships”. They are able to work with space and shape, conjecture and make predictions based on trends.

Gardner (2006, p. 12) declares that ―logical-mathematical intelligence is supported as well by empirical criteria”. This view is summarized by Gouws and Dicker (2011) who believe that logical-mathematical intelligence is concerned primarily with the use of numbers effectively and to make logical conclusions based on inductive and deductive reasoning.

This intelligence is primarily concerned with the mathematical development of an individual. Many of the processes mentioned above are visual and can be facilitated through visual means. Patterns, trends and causal relationships often require knowledge of tables, diagrams and graphs which are often visual components. It can be concluded that the logical-mathematical intelligence is closely related to the phenomenon of visual reasoning.

4.2.2.2.3 Visual/Spatial Intelligence

Visual intelligence allows an individual to understand and represent concepts using ―colour, line, shape, form and space" (Gouws & Dicker, 2011, p. 581). Spatial intelligence refers to an individual‗s ability to ―perceive the visual spatial world accurately and perform transformations based on these perceptions" (Gouws and Dickers, 2011, p. 581). Although Andronache et al. (2011, p. 20), agrees with much of the definition given above, he goes further in defining spatial intelligence learners as those who ―have the ability to perceive with particular acuity the colours, lines, shapes, space: they may perceive relationships between these elements.".

Both writers are suggesting that through visual intelligence, individuals understand concepts through visual components such as colour, line, shape and space.

The nature of mathematics, to a large extent, makes demand of this intelligence. The emphasis of space and shape in the study of geometry, trigonometry, transformation geometry, analytical and Euclidean geometry, in the high school mathematics curriculum, shows how significant this intelligence is. It would therefore seem that these intelligences are closely integrated and interdependent. The implications for education, becomes highly visible. For effective teaching and learning, teachers have to be able to integrate and engage several intelligences in order to ensure maximal learning is taking place. Alternate assessment, and the use of different intelligences may ensure that the concept is understood, if not through one intelligence, but by many intelligences.

4.2.2.2.4 Musical /Rhythmic Intelligence

According to Gouws and Dicker (2011, p.581) ―musical intelligence refers to pitch, tone, rhythm and emotional expression. There is a distinct correspondence between mathematical patterns and music".

This intelligence too, is closely linked to the mathematical ability of the individual. The link to mathematics means that this intelligence can be used in the teaching of mathematics. The link to mathematical patterns and music can be used to develop the aspect of patterns in mathematics. Educators may be able to exploit this intelligence in promoting mathematical understanding. Topics such as number patterns and trigonometric functions can easily be related to music. Seen from this perspective then, the integrated nature of the theory of multiple intelligences by Gardner allows the educator several opportunities to engage the learners in mathematical thinking.

4.2.2.2 5 Corporal-kinesthetic Intelligence

This intelligence is responsible for special physical movements which incorporate fine motor coordination and balance, often seen in dance, sport and related activities that require the use of the body. In this regards, Andronache et al. (2011, p 20), makes a strong case for Gardner having alluded to it as one of the seven intelligences. He argues that “the assessment of corporal-kinesthetic knowledge may be less intuitive as a problem solving aptitude”. However he further explains that by using the body to “convey an emotion or play a game” displays cognitive skills. Whilst this may seem irrelevant to the study, the notion of bodily movement is closely associated with the concept of gesturing, which has been alluded to in the literature review. Gestures are inextricably associated with visualization, which is the focus of this study.

4.2.2.2.6 Interpersonal Intelligence

Like the corporal-kinesthetic intelligence, this intelligence allows an individual “to review and assess rapidly the status, intentions, motivations and feelings of others” (Andronache et al., 2011, p. 21). Through gestures and facial expressions, interpersonal relationships are understood. Further to the statement made by Andronache, with regards to this intelligence, Gardner (2006, p. 15) posits other attributes of this intelligence, “which permits a skilled adult to read the intentions and desires of others, even when they have been hidden.”.

The reference to gestures and facial expressions indicate the applicability of this theory to this study. Visual reasoning is often conveyed through the process of gestures, as the literature has indicated. The use of facial expressions and gestures contributes significantly in mathematical understanding through visualization. Therefore this kind of intelligence is also a key component in the development of mathematical understanding. Furthermore the reference to “reading the intentions and desires of others” may also be applicable to the tenets of Attribution Theory, since it probes human behaviour and human action which is often based on motivation.

4.2.2.2.7 Intrapersonal intelligence:

In a nutshell, this intelligence tries to access one‘s own feeling, with the intention of guiding one‘s behaviour. According to Gardner (2006, p. 17), ―a person with good intrapersonal intelligence has a viable and effective model of him or her-self". This intelligence is intimate and it is only through language, or observation that it is detected. The success of the participants in this study can be seen through this intelligence. Through discussions and observations of the participants, a more holistic assessment of their characters can be made. Their actions in terms of their practise could better be understood through observing this intelligence.

4.2.2.2.8 Naturalistic Intelligence

This intelligence did not form part of the list of intelligences that Gardner‘s first outlined in his theory. It was however added in at a later stage. ―In this type of intelligence, there is the core capacity to recognize instances as members of a species" (Gardner, 2006, p. 19). In their interpretation of naturalistic intelligence, Gouws and Dicker (2011, p 583), define it as ―the ability to recognize patterns in nature and classify objects; the mastery of taxonomy; sensitivity to features of the natural world, and an understanding of different species".

This intelligence also focuses on the recognition of patterns, more especially in nature. However, in mathematics, the study of patterns is integral and occupies a significant part of the curriculum. Patterns in nature are used in studies in mathematics, for example: the Archimedean spiral, Fibonacci‘s sequence and rates of change problems, involving bacterial growth. Educators can integrate these patterns and exploit these relationships to promote mathematical understanding.

4.2.2.2.9 Existential Intelligence

Initially, Gardner did not propose this as one of the intelligences in his theory. It was subsequently added to the list at a later stage. This intelligence has the ―ability to pose questions about life, death, and ultimate realities like ‗Who are we‘ and ‗What is the meaning of life?'" (Gouws & Dicker, 2011, p. 575). Similar views have been expressed by Andronache (2011, p. 22) that ―existential intelligence is concerned with the sensitivity and the ability to emit and cope with the profound questions about human nature. Roberts (2010) claims that this intelligence enables an individual to seek solutions to problems relating to issues of human existence. Through this intelligence, individuals question their purpose in life and the contribution that they are expected to make to the world. Robert further believes that teachers with this intelligence are interested in addressing deeper issues in education such as the type of education, why it is being given and how it is being delivered.

In the context of this study, issues surrounding mathematics teaching and learning has been at the forefront of many debates and continue to dominate many discussions. Keen mathematics teachers are constantly in search of alternate ways of improving results and understanding in mathematics. They are often troubled by the negative perception of mathematics and make endeavours to change this perception. These teachers are often considered to be the most successful, since they are in a constant state of assessing and evaluating their practice, with the intention of creating better understanding in mathematics.

4.2.2.3 Multiple Intelligences Theory explaining the study

The shifting trends in education have seen a move away from selecting intelligent learners and preparing them for tertiary institutions, but rather on educating the masses. Much emphasis is being placed on individualised training. Hence the focus on Gardner's theory of multiple intelligences becomes more prominent. Teaching must now take into account this differentiated training that adapts to the learner. It is therefore essential that teachers carefully examine the abilities of their learners with an attempt to harness their strengths appropriately. Multiple intelligences theory will require teachers to develop a wide range of talents and abilities. In order to do this, teachers have to significantly alter their methods of delivery and presentations. A variety of teaching strategies needs to cater for the different intelligences of the learners. Their presentation strategies must now accommodate the visual, lingual, spatial, logical, rhythmic, corporal-kinesethetic, interpersonal and intrapersonal intelligences of the learners. It is therefore imperative that teachers, who are going to be successful, will ultimately subscribe to these guidelines, considering the nature of learners currently. The modern learner is an individual, where "individual-centred education focuses precisely on the differences between individuals, on the types of intelligence that each possesses more or less, thus giving equal opportunity to all. Thus teachers use specific strategic systems, means and methods for valuing each individual" (Andronache et al., 2011, p. 25).

Furthermore, the theory allows an explanation of how learners may master a concept. Multiple Intelligences Theory allows a learner to master a concept, not by repeated exposure in the same way, but in different ways in various forms and contexts. In so doing "we can use all relevant intelligences to the topic concerned: this way the theory or the concept will reach many learners" (Andronache et al., 2011, p. 25). This statement implies that teaching must use multiple representations as a strategy in order to successfully satisfy the needs of individual learners. According to Özmantar, Akkoç, Bingölbali, Demir and Ergene (2010, p. 19), –the issue of multiple representations is an important one in mathematics education and attracted the interests of researchers especially within the last three years". The reason for this assertion is that –multiple representations cater for a wider range of learners with different learning styles," as well as –emphasizes different aspects of the same concept" (Özmantar et al, 2010, p 20). This view confirms the view held by Andronache (2011), that repetitions of the same concept in different ways and contexts will reach many more learners. Successful teachers will invariably try to employ all these strategies in their classrooms.

Visual reasoning as a complementary process in mathematics teaching, closely relates to Multiple Intelligences Theory, since it is an alternate form of representation that stimulates the visual/spatial intelligence of the learner and can be seen as a form of multiple representation. This is confirmed by Özmantar et al. (2010, p. 19), that these various –representations of problems serve as different lenses through which learners interpret the problems and solutions." This means that for learners who want to become mathematically astute, flexibility in approaching problems and recognition of diverse relationships and points of view is required.

This resonates with the use of visual reasoning by successful teachers, the phenomenon which is being explored in this study. The literature indicates that visual reasoning is paramount in the teaching of mathematics, since it offers the visual learner the opportunity to understand concepts using visual stimuli. It offers an alternate strategy to teaching and learning. In this way, it creates a different form of representation, thus giving the teacher an opportunity to introduce multiple representations into the mathematics classroom.

According to Bleed (2005, pp. 3-4), the average teenager –watches 22 000 hours of television by the time she/he graduates from high school. Children aged up to six years spend as much time playing with TV, computers, and video games as outside". Bleed (2005, pp., pp. 3-4) declares that although children are subject to the powerful influence of the entertainment industry and visual media, which has introduced a new learner media, –educational institutions have not adjusted to these changes". Hence there are huge calls for visualization to be incorporated into the classroom environment to be in touch with multiple-representations that the learners are now demanding. If learning is going to be authentic, it must be aligned to the world we live in, a world in which visualization is so prominent.

Chick and Vincent (2005) also confirm that the need for multiple representations in mathematics education cannot be ignored any more. A similar claim is made by Ainsworth and VanLabeke (2004), that single representations cannot have maximum effect on learners. The research done by Dong-Hai and Rebello (2009), in science education indicates that the use of multiple representations helped the learners to cope better in physics and in problem solving involving physics. Multiple representations offer advantages in the teaching of mathematics for the following reasons: ―multiple representations target a wider range of learners; the use of multiple representations promotes deeper understanding of the subject as each representation emphasizes the same concept; multiple representations can constrain interpretations" (Özmantar et al., 2010, p. 21). Since mathematics and science share common attributes in terms of conceptual understanding, the results of the studies are applicable to this study as well. According to Özmantar et al. (2010, p. 21), ―when the multiple representations are used for constraining, the purpose is not necessarily to provide new information but to support a learners reasoning about a less familiar one". The correlation between Multiple Intelligences Theory and multiple representations cannot be ignored. It is as if the two theories are used synonymously by researchers. Ultimately it is the role of the teacher in ensuring that she/he can carefully integrate these phenomena successfully. Roberts (2010, p. 242) proposes that it is important to regard MI as ―an attitude toward learning rather than a fixed set of precepts circulated to learners" which will also benefit teachers‘ concepts of intelligence. This suggests that multiple intelligence theory is applicable to both teachers and learners. From a teacher‘s point of view, it is important that they understand the dynamics involved so that they can adjust their pedagogy accordingly. From the learners‘ point of view, an understanding of this theory will give them the opportunity to engage with the different intelligences if they wish to achieve success in a concept.

4.2.2.4 Shortcomings of the Theory of Multiple Intelligences

The Theory of Multiple Intelligences is not easily incorporated in mathematics classrooms for the following reasons: It requires a thorough understanding and necessitates proper pre-service and in-service training for teachers. (Gouws & Dicker, 2011).

Even Dong-Hai and Rebello (2009) arrived at similar conclusions when the found that the correct instructions from instructors would be of assistance to learners. The reference to ‗correct instructions' may be seen as an impediment, particularly if the instructors are not properly trained. They also found that the application of multiple representations more regularly would make the learners more competent in problem solving. The emphasis on regular application of the phenomenon could mean that more time may be required in order to master a concept. In light of the time constraints that teachers often experience, this aspect may be seen as an impediment. This would imply that the teachers need to be equipped with the necessary skills and knowledge to be able to vary their techniques so as to use the multiple intelligences of their learners and incorporate multiple representations. The other criticism from Peariso (2008, p. 8) is that, given that individuals use different intelligences to demonstrate their understanding of concepts, ―how are the intelligence or learning styles going to be objectively or quantitatively assessed"? This places pressure on teachers to be able to find innovative ways of quantifying the different types of intelligences.

According to Roberts (2010, p. 244), many institutions are judged by their performances and productivity. The reality of the situation is that, they are pressured into attaining acceptable learner achievement, which denies the teacher the freedom to use their ―insights and intuitions developed through years of experience". It is not uncommon therefore to see teachers abandoning innovative teaching strategies in exchange for regular, uninteresting methods of presentation. This is summed up succinctly by Roberts (2010, p. 250), when he says that ―the human being in the role of the educator encompasses more than an effective task producer".

Although Gardner seems to have produced a theory that moves away from the orthodox psychometric means of measuring the human intelligence, ―any taxonomy that deals with classifying the human mind should not be regarded as static or exhaustive since our understanding of human cognitive ability is ever expanding" (Roberts, 2010, p. 242). It is implicit therefore that it is not an exclusive explanation of the process of cognition.

4.2.3 Situated Learning Theory

According to Brown, Collins and Duguid (1989), knowledge was regarded as a transferable commodity from teacher to learner. Conventional education was seen to be independent of the social context in which the learner found herself/himself in. However this view was challenged by constructivist theorists who believed that knowledge was constructed through the learner‘s interaction with society. An expansion of the concept of situatedness or embeddedness as is commonly used is explained as follows: ―knowledge, we claim, is partially embedded in the social and physical world. This implies that knowledge is part and parcel of an individual‘s interaction with and within the society she/he is part of. ―These embedding circumstances allow people to share the burden of solving problems efficiently" (Brown et al., 1989, p. 2). This view is also confirmed in Presmeg (2008).

Altrichter (2005, p. 14), in his research is of the view that ―learning is in need of engagement in specific social situations; it draws on them and it is, in some sense, bound to them". This view has also been echoed by Margarisová, Štastná, and Stanislavská (2011), who claim that the concept of Communities of Practice creates an environment for collaborative learning. By engaging with society, the learner is exposed to a community of practice, which allows her/him to understand this knowledge. In the process of acquiring this knowledge there is a change of identity. The learner or newcomer, as is often referred to, becomes conversant with the knowledge and becomes the expert or old-timer as is commonly called in this theory. This view is also confirmed by Ben-Ari (2005, p. 367) that, ―in situated learning, learning takes place within the context of a community of practice".

This theory will be examined on the basis of the following themes: definition of the theory, nature of the nature of the theory, ability of the theory to explain the study and shortcomings of the theory, as I have described in Figure 4.7.

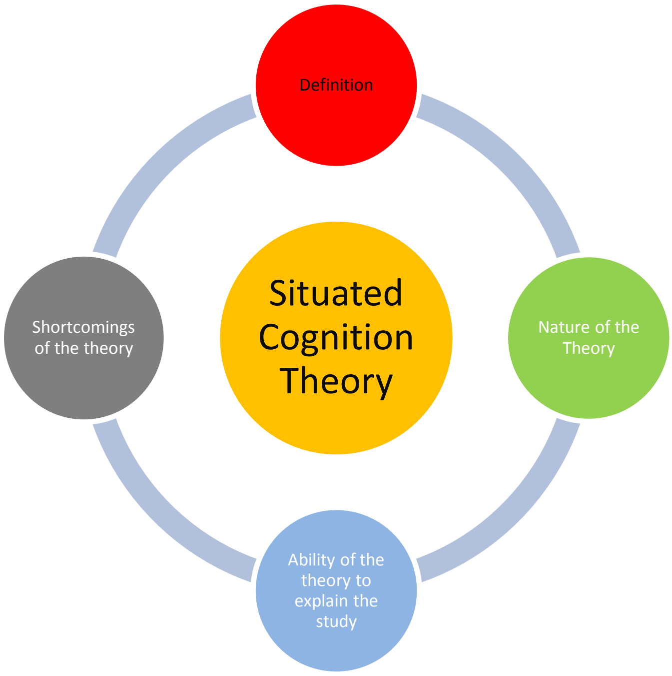

Figure 4.7: Relevant themes under which Situated Cognition Theory will be explained

4.2.3.1 Definition of the theory

Situated cognition theory therefore is largely concerned with the way in which learning takes place in context and the emergence of a new identity. The prominent ideas in situated cognition theory was also addressed by Fuller, Hodkinson, Hodkinson and Unwin (2005, p. 50), in which ‗communities of practice' and ‗legitimate peripheral participation', are discussed. According to Jawitz (2009, p. 603) ―knowledge is distributed amongst the members of a community of practice, and can only be understood with the ‗interpretive support' provided by participation in the community of practice itself", Similar views of the concept of communities of practice have been made by Li, Grimshaw, Nielsen, Judd, Coyte and Graham (2009, p. 3), stating that ―communities of practice are considered to be a type of learning communitites", whilst ‗legitimte peripheral participation' refers to the transfomation of the status of the novice to the expert. This is seen as a movement away from the central focus to a position further away from the centre of focus, hence the term peripheral.

Hence the definition elaborates on the significance of learning, context, communities of practice and identity. Although these aspects of the definition have been already explained, the notion of identity needs further elucidation. An identity may be defined as a set of choices that an individual chooses to make, to transform herself/himself from a stage of apparently little or no knowledge of a phenomenon to a stage of adequately knowing the phenomenon

(Faircloth, 2012). Furthermore, ―it may be important therefore to consider identities as negotiated, fluid, and multiple, rather than achieved, unitary, or consistent events of engagement" (Faircloth, 2012, p. 187).

The effect of making a connection between the social and personal lives and the mathematics they learn can be seen in the learner‗s ability to solve problems. Closely associated with this notion, is the relevance of providing authentic artefacts. According to Lowrie (2004), learning is only meaningful if learners develop a need for it. Furthermore, learners must take ownership of their learning. This is only possible if teachers develop real-life contexts that create realistic learning. This implies that teachers must be willing to make the change in their beliefs and views about context-bound teaching. It would seem then that, unless problem solving is made authentic by the teachers themselves, learners will perceive them as merely an application of the rules of algebra or rote learning of geometrical or trigonometric facts. Hence, problems have to be authentic, and relevant to the learners. The call is then for mathematics to be situated within the context in which the learners finds himself in, if the mathematics is going to be realistic.

Seen from this perspective, it seems as though the focus of situated cognition learning theory is the relationship between individuals and members of a community of practice that only share this knowledge through participation. Hence the definition emphasizes the importance of learning in context.

4.2.3.2 Nature of Situated Learning Theory

A little over a decade ago in 1991, Lave published Situated Learning: legitimate peripheral participation, which became the cornerstone on which the Situated Cognition Theory is based. This is confirmed by Margarisová et al. (2011, p 156), when they declare that ―the framework for the concept of Communities of Practice is based on a publication by Etienne Wenger and his co-authors, considered to be the most recognised authors in this field". The contribution by the above-mentioned authors is confirmed by yet another researcher Ollis (2011), who maintains that Lave and Wenger‗s theory of situated learning has have been at the forefront of much debate for over two decades. According to Ollis (2011, p. 254), ―Lave is referring to our reliance on cognition for understanding ways of knowing, and in a sense, her work is similar to Merleau-Ponty‘s existential examination of somatic knowledge developed though perception and being in the world”.

According to Contu and Willmott (2003, p. 283), ―Situated Learning Theory has emerged during the past decade as an alternative to dominant cognitive perspectives on learning”. Learning is not construed as some independent process which is separated from the world we live in. Rather learning must be seen as ―a pervasive, embodied activity involving the acquisition, maintenance and transformation of knowledge through processes of social interaction” (Contu & Willmott, 2003, p. 285). The nature of Situated Learning Theory is succinctly portrayed by Contu and Willmott (2003) as follows: learning is seen as cognitive process that transfers knowledge from one context to another; learning focuses on the practices of groups rather than the individual; learning is seen as the ability to participate with people with the requisite competence; learning is not only acquiring the skill and knowledge but being able to apply that knowledge that is valued by other members of the group and that leaning is the construction of identities.
A similar view has been expressed by Altalib (2002, p. 4) that ―knowledge is acquired by embedding the subject matter in the experiences of the learner and by creating the opportunity for the learner to interact in the context of real life situations”.

According to Lave (1991), learning must be viewed in the context of the social world. Learning is seen to be a social practice that is constantly taking place as knowledge is passed on. According to Reeves (2004, p. 90), ―the classroom could be described as a contextualised social space where the teacher and the pupils enact teaching and learning on the basis of a set of norms and values that form the particular modus vivendi for the group concerned”. The incumbent becomes proficient at a particular skill and demonstrates these skills through her/his interaction with society. This leads to a process of entering a community of practice and eventually becoming proficient and allowing newer members to enter and continue in this fashion. In the same way, teachers interact with each other at different social spaces, where a set of values and norms are shared.

The reference to ‗peripheral participation‘ needs further clarification. Lave contends that learning must not be seen as a social sharing process but as a process where the person becomes ―a member of a sustained community of practice”. A detailed analysis of these concepts will follow after a closer examination of the theories that surround situated experience or situated activity. According to (Lave, 1991), there is a misconception that situated activity is a single concept. On the contrary, it incorporates other theories such as cognition, language, learning, agency, the social world and their interrelations.

According to Lave (1991, p. 66) ―there appears to be at least three different genres of situated approaches". The first view is that ―researchers should extend the scope of their intrain-individual theory to include everyday activity and social interaction". This is the case because people process and remember information in relation to their interactions with people in a social setting, hence the need for integrating their individual theories with their daily activities. The second view, ―dcates situatedness in the use of language and/or social interaction". Rather than seeing the external world as fixed, it is recognizes the need for constant negotiations, using language as a social activity. It would seem that the first view sees the external world as a fixed world which is highly scientific in nature and devoid of feelings, whilst the second view sees the world as a place of negotiation and making sense of the situations. However, both these views share a common factor, that of ―social world as an object of study" (Lave, 1991, p. 67). The third view integrates the first two views stated above, which claims ―that learning, thinking and knowing are relations among people engaged in activity with, and arising from the socially and culturally structured world" (Lave, 1991, p. 67). This view sees the relevance of integrating the process of learning with the social environment which is mediated through the use of language.

The concept of legitimate peripheral participation must be seen as a process of developing knowledgeable skill and identity. The process unfolds as follows:

> ―Newcomers develop a changing understanding of practice over time from improvised opportunities to participate peripherally in on-going activities of the community. Knowledgeable skill is encompassed in the process of assuming an identity as a practitioner, of becoming a full participant, an old timer" (Lave, 1991, p. 68).

Thus it would seem that this gradual transformation from novice to experts through the process of social integration, ultimately leads to learning and the acquisition of a new identity. According to Seaman (2008, p. 278), the term ―community of practice" was collaboratively coined by Lave and Wenger in an effort to provide a perspective on learning and knowing within a social context. This view is also shared by Altalib (2002), where he subscribes to three levels of learning, each leading to a higher level: the first being the individual learning process, in which the learner becomes a member of the community of practice; the second level is where the whole community ensures the transition from novice to expert and the third level refers to the learning on the level of the entire organisation, where the organisation must be able to accommodate these multiple communities within its sphere. This notion has been reiterated by Fuller et al. (2005), when they explain that the processes, experiences and relationships which form the participant's sense of belonging determines the nature of the subsequent learning. Lave and Wenger have captured this complex notion in their term ‗legitimate peripheral participation'. They explain that, each of its aspects is indispensable in defining the others and cannot be considered in isolation.

An important characteristic of Situated Cognition is that it ―integrates useful principles of sociocultural learning and cognitivism. It makes the important stipulation that information becomes knowledge only in context" (Altalib, 2002, p. 12).

4.2.3.3 Using Situated Cognition Theory to explain the study

Recent theory and research have also recognized this intersection between identity and context as a potentially significant aspect of learner engagement. According to Faircloth, (2012, p. 186), ―learners who intentionally examine the relevance and meaning of school content and learning with respect to their sense of who they are (or want to become) develop an exploratory orientation toward learning that involves actively seeking/processing information". The relevance of learning in context is emphasized here and it acts as a motivation factor to genuine learning. Situated cognition theory therefore is applicable to anyone who is considered to be a learner. Hence, its relevance to the study, since these successful teachers are also learners on an on-going basis. Situation cognition theory has an overlap with attribution theory of motivation. Together they can explain successful behaviour. These theories can easily be correlated with the notion of success of the participants in this study.

The principles of learning as espoused by Lave and Wenger, namely learning in context has relevance to the study for the following reasons: that the participants, although qualified, to a large extent became members of a community of practice. In this regards, their learning extended beyond the theoretical training that they had attained in their tertiary years of study. That knowledge had to be situated in context in order for it to have become more useful and meaningful to each of the teachers. Furthermore the learning did not end, as they were constantly engaging with each other and with the curriculum and pedagogy to find the best position to be in with regards to becoming successful and being recognised as highly successful teachers in their fields.

In terms of a community of practice, there have been vivid indications of newcomers and old timers in this study. The transition through these stages, have to a large extent subscribed to the theory as expounded by Lave and Wenger. However, as has been identified as criticisms in the subsequent section, the theory does point to a dynamic process of power relations at play.

In a research conducted by Jawitz (2009), the author explored how new academics engaged with the assessment practices in their departments, and developed this practice to assess learners' complex tasks, by becoming more confident. They found that though there were power relations that subtly existed, there was sharing of knowledge through a community of practice that gradually enabled them to become more confident in their task of assessing learners' tasks.

In this way, the study further propagates the notion of communities of practice and how leaning takes place through this process. The research showed not only how learning takes place, but the development of confidence, through learning how to become harmonised with the rest of the team in the study. It would seem then, by becoming part of the collective is an important part of learning. Similarly, the participants of this study belong to a community of practitioners. The way they gained confidence through practice can be explained using this theory. The theory points to the significance of learning in context, rather than in isolation and the need for sharing knowledge (Jawitz, 2009).

This is visible when one examines the ways in which the participants interact with each other in terms of becoming harmonised with the team of other successful mathematics teachers, which forms the basis of much discussion in the subsequent chapter, that analyse the data.
With regard to the notion of legitimate peripheral participation and the formation of a new identity, the theory can be used to explain the behaviour and the identity of the successful participants. Through the engagement of the participants in presenting the curriculum, and becoming harmonised with the team, each one of them nevertheless, develops a unique identity. This is clearly evident in this study, in that even though all of the participants are considered to be very successful, yet there are differences in the teaching styles and identities.

Korthagen (2010, p. 98) state that ―Lave and Wenger have greatly influenced existing views of learning and teaching, but relatively little has been written about the implications for the understanding of teacher behaviour and teacher learning, and for the pedagogy of teacher education”. This issue must be seen in conjunction with the morale of teacher education which according to Korthagen (2010, p. 98) is ―currently facing a crisis, given the many research studies showing the disappointing impact of teacher education on teacher behaviour and teacher learning”. Furthermore, ―many researchers from various countries demonstrated that teacher education graduates were facing severe problems trying to survive in the classroom, and were implementing little of what they had learnt during their professional preparation”. Against this backdrop, there are those exceptional teachers who excel against all odds. It would be fair to assume then that even these teachers have had issues that perturbed them in their movement towards success. It would also be reasonable to examine the implications of the three-level model that Lave and Wenger have prescribed to deal with issues surrounding friction between teacher behaviour in practice and teachers‘ practice in theory. This aspect of the participants‘ behaviour must also be taken into account when considering their initial teacher training and implementation in the classroom. According to Korthagen (2010, p. 104), ―learning to teach” is ―a socio-cultural process relying on discursive resources”. The significance of developing a proper understanding through dialogue, within a community of practice, is being emphasized here. In their research, Margarisová et al. (2011, p. 155), argues that there is proof ―that well developed professional learning communities have a positive impact on both teaching practice and learner achievement”. The impact of communities of practice is seen to be an important aspect in producing good teaching practice. To an extent it may offer a viable explanation for the success of the participants in this study

4.2.3.4 Shortcomings of the theory

According to Fuller et al. (2005), although Lave and Wenger‘s work provides deep theoretical insights, it nevertheless has shortcomings.

The first criticism of this theory is found in this discussion by Fuller et al. (2005, p. 51) which contends that

> ―this participatory approach shares one characteristic with the standard paradigm approaches to learning that it was set up to oppose. In placing the

> emphasis on learning as a progression from newcomer to full participant, Lave and Wenger focused their theory of learning on novices and largely ignored the effect on communities when they import _old timers‘ from elsewhere”.

Hence, the main premise underlying this theory that the community of practice moves from novices to _old timers,‘ seems flawed. In this way, no attention was paid to the way in which the learning of experienced people differed from novices. There has been some attempt by the authors of this theory to try to close this theoretical gap. However, –they view the knowledgeable practitioner not only as someone who commands and can apply the necessary knowledge and skills but who, through their membership, has become a full participant in the cultural practices of the community” (Fuller et al., 2005, p. 52). In this way, they have attempted to bridge this theoretical gap.

The second criticism that is levelled against Lave and Wenger‘s theory is that the transition from newcomers to old timers was often presented as smooth and stable. Fuller et al. (2005) suggests that this has been the case in most of the examples used by Lave and Wenger. This contradicts the view that the shift from newcomers to old timers is dynamic, with inherent power struggles.

The third criticism levelled against this theory is that it helps to promote the capitalistic ideology that wants to empower people to such an extent that there is no need for a central authority. Such is he view held by Fuller et al. (2005, p. 53), who claim that –as the concept of _communities of practice‘ is being embraced by a range of occupational fields (e.g. education, health and social care, management), it provides another useful vehicle by which the new capitalism can further its aims”. It would seem that, from an ideological point of view, this theory favours domination by the minority, which is seen as a capitalistic premise.

Fourthly, Situated Cognition Theory has been criticized because Lave and Wenger tend to dismiss the role of teaching in the workplace. Rather they tend to regard the learning of teachers as –participation and a social enterprise” (Fuller et al., 2005, p. 65). Skilled training that teachers receive is ignored in this context.

Lastly, Lave and Wenger do not fully focus on the issue of –conflict and unequal power relations” in their theory (Fuller et al., 2005, p. 66). Their dismissal of these issues may have important consequences on the theory.

4.3 Conclusion

Having examined each of the theories stated above, there appears to be similarities and differences amongst them. There also seems to be areas of intersection of ideas.

In this chapter the theoretical framework that formed the lens though which I analyse my data, has been provided. Attribution Theory describes the phenomenon of success and failure as factors that attempts to explain human behaviour. The three essential features of Attribution Theory were explained, namely locus, stability and controllability. In the context of this study, the causal nature of attribution theory has been used to explain the actions and motivation of the participants, since my research sought to understand the notion of successful teaching. The Theory of Multiple Intelligences focussed on the complex and multi-dimensional nature of intelligence. The theory focussed on the difference in learners and how the different capabilities in learners must be identified and developed. The theory also pointed to the phenomenon of multiple representations. Visual reasoning has been identified as one of the nine modalities of intelligence. However on closer examination, it was shown that most of the other intelligence modalities are also related to visual reasoning. Hence the phenomenon of visual reasoning is closely associated with concept of multiple representations. I therefore wish to understand how these successful teachers integrate visual reasoning, in their practice. The Theory of Situated Cognition learning delved into the relevance of learning in context and focussed primarily on leaning in a community of practice and legitimate peripheral participation. It traced the movement of the novice learner to the expert, thus leading to a change in identity. I focus on how the novice participant eventually becomes the ―old-timer". The role of contextual learning is deemed very significant according to this theory. I wish to understand this phenomenon in the context of this study.

In the next chapter, I attempt to answer the two critical questions of this research study, by examining the practice of the participants in relation to the reviewed literature, whilst using the theoretical lenses that I have discussed in this chapter.

Chapter 5
Analysis-Part 1: Theory and Praxis

5.1 Introduction

The previous chapter focussed on the theoretical frameworks that underpinned this study. It provided the lens through which the data could be analysed.

This chapter examines the participants' views on visual reasoning and successful teaching. In this chapter an attempt will be made to establish a clear understanding of the notion of visual reasoning, as it was understood by each of the participants, the kind of visual reasoning that was used in their classroom practice and the justification for the use of these strategies. This part of the analysis therefore attempts to relate the responses of each of the participants through their narratives to the theoretical frameworks and the relevant literature review. By examining these narratives, the next chapter will focus on attempting to establish common themes and further justification for the actions of these participants. Where no commonalities emerge, the analysis will focus on establishing reasons for these differences. Through this process, it will bring to light a more detailed explanation of the actions of the participants. Whilst case studies as a research methodology do not dwell on the generalization of findings in a study, the emergence of common themes may be used to understand such patterns in other similar populations. On the other hand, the unique nature of findings of individuals adds more credence to case studies, since it fosters and in-depth understanding of the phenomenon under scrutiny.

5.2 Participants' understanding of the notion of visual reasoning

The term visual reasoning needs to be defined in the context in which it is being used. Although there are several definitions of this concept, as it has become clear in the literature review chapter of this study, it is important that a more common understanding of the concept is arrived at in order to use it as a basis to analyse the responses of the participants. Visual reasoning may be defined as

> –the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating

information, thinking about and developing previously unknown ideas and advancing understandings" (Arcavi, 2003, p. 220).

This definition is directly related to this study because of its relevance to mathematics education and the mathematics classroom. The definition alludes to a characteristic of visualization ―as a ‗noun‘ – the product, the visual image". The other characteristic is ―a ‗verb‘ – the process, the activity" (Arcavi, 2003, p. 220). In the context of this study, visual reasoning is seen as the product, namely the visual image in the form of pictures, images and diagrams and the process, which refers to the activity itself, which is to translate and communicate these ideas. Hence visual reasoning must be seen as a complex process, which is not just confined to the act of seeing. It goes beyond the biological interpretation of vision. As a result of the depth of this definition, it immediately discounts any debate concerning the ability of blind people to perform visual reasoning. The process occurs within the mind, thus giving everyone the chance of engaging in visual reasoning. Furthermore, the definition is broad enough to incorporate pictures, images and diagrams using paper and technology or just the mind. This definition has been similarly adopted by Alcock and Simpson (2004, p. 2), who define visual reasoning as ―the tendency to introduce visual images in solving problems", as well as seeing ―visual images as important in mathematical thinking, whether these are imagined, drawn on paper or created using a technological tool".

The discussion that follows, addresses the meaning of visual reasoning as it was understood by the participants of the research study. This will become evident by focussing on their understanding of visual reasoning and examining their use of visual reasoning in their classroom practice.

Valerie was of the view that visual reasoning referred to the notion of the use of concrete objects such as ―match sticks", or ―when we put blocks together and the learners are actually able to figure out, or recognize patterns, like we did with the grade 10 learners, in an investigation where we had cubes that were made with smaller cubes". She also believed that it was ―taking something that is practical and visual and turning into something that is almost a rule". In a mathematical investigation which Valerie asked her learners to engage in, she wanted to establish a mathematical formula or rule that will emerge simply by counting the number of painted sides of a cube.

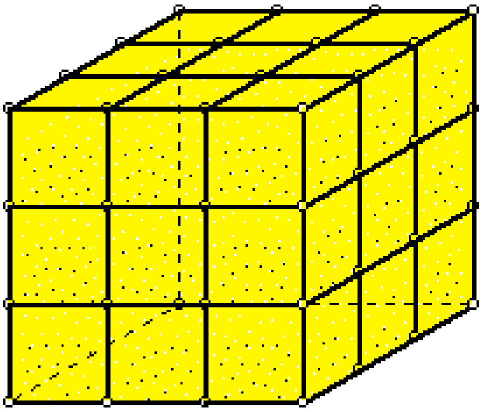

Figure 5.1: A cube as used in the mathematics investigation[4]

In figure 5.1, learners were asked to count the number of sides that would be painted, if the cube was dipped in a bucket of paint. She informed the leaners that the cube was much smaller than the bucket and it could easily be immersed into the bucket. Furthermore, there was enough paint for the cube to be immersed into. Thereafter, the cube would be reduced to a smaller size and the procedure would be repeated. The same procedure had to be followed in bigger cubes. In each case the learners had to count the number of painted sides and investigate this relationship, by drawing up a table and observing important counting patterns, thus leading to a mathematical formula.

She referred to other examples like paper folding to see relationships between reflections and rotations. Whilst pursuing the practical aspect of visual reasoning, she cited examples in mathematics that could be incorporated into her technical drawing class, such as the construction of important lines in concurrency. Whilst she had a broad view of visual reasoning, she nevertheless also stressed the need for visual reasoning in linking up key concepts, such as the roots of a quadratic function, which could be visualized in terms of the x-intercepts of the graph of a particular parabola. In this regard, visual reasoning was a means of representing the end product which made it a useful tool in teaching mathematics. She cited this as a possible reason for teaching the learners how to solve quadratic equations.

[4]http://jwilson.coe.uga.edu/EMT668/emt668.student.folders/SeitzBrian/EMT669/painted.cube/painted.html

Karl's notion of this phenomenon was simple and to the point. He indicated that visual reasoning was "the use of pictures, graphics" or any "multimedia" presentation that had a greater effect than words alone could achieve. He emphasised the extent of visuals in mathematics, stating that pictures often conveyed more information. He quoted the proverb "a picture paints a thousand words", in explaining the extent to which the visual format was more effective. This format cannot be compared to the use of language alone to describe the contents of the picture. In Figures 5.2 (a) and 5.2 (b) Karl showed how a rectangle could be used to simplify the concept of the area of a triangle by drawing in a diagonal.

Figure 5.2 (a): Area of Rectangle = length x breadth

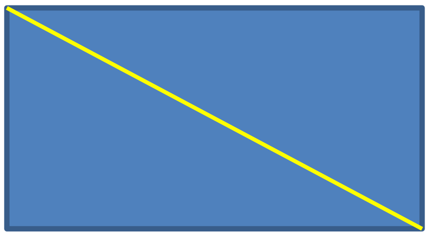

Figure 5.2 (b): Area of triangle = $\frac{1}{2}$ length x breadth = $\frac{1}{2}$ base x perpendicular height

Figure 5.2: Relationship between the area of a rectangle and a triangle

The image in figure 5.2 (a) showed a rectangle, with an area formula given by the product of the length and breadth. When a diagonal was drawn, the area of the rectangle was halved, forming two triangles. The area of each of the triangles was therefore half the area of the original rectangle. Hence the formula for the area of a triangle was half of the product of the length and breadth of the original rectangle. However, because the length and breadth were perpendicular to each other, the formula was modified to half the product of the base and the perpendicular height. The diagram illustrated the derivation of this formula very easily.

Karl saw visual reasoning as a means of enhancing learning. He indicated that, whilst it was possible to describe some concepts, visuals had the potential to deepen the understanding of mathematical ideas very easily. It was as though the learners actually connected with the idea. It brought the idea to life. He cited examples that were often difficult to perceive, yet with the use of visuals, the concept was easily understood. In explaining the parabola as a function, he alluded to its relevance to the conic section. The picture 5.3, depicting the conic section, could help the more advanced learner of mathematics to realise the importance of conic sections. Within the cone, several other functions were contained. For example, a circle could easily be obtained through a horizontal section of the cone. In the same way, it was possible to obtain an ellipse by an oblique section through the cone.

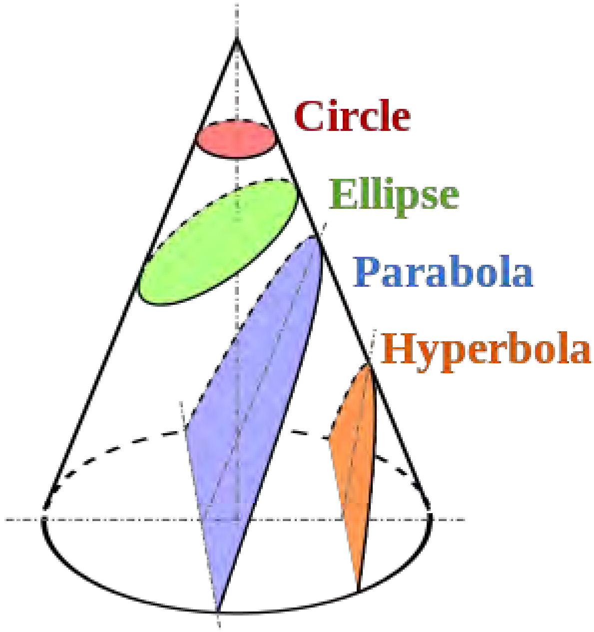

Figure 5.3: Conic Section[5]

Betty's response to her understanding of visual reasoning differed from the views expressed by the other participants. Whilst the other participants immediately responded to the notion of visual reasoning, she indicated that she did not quite comprehend the full meaning of the concept and hence had to research it. Hence her response must be seen in that context.

> ―I've no idea what you mean by visual reasoning. It really bothered me and I looked it up, and there was nothing on Google, which made it clearer to me. If you had to say to me take a guess, I would say visual reasoning is when you put out something on a board that people can see to follow the reasoning".

[5]https://en.wikipedia.org/wiki/Conic section#/media/File:ConicSections.svg

However, after some discussion with the researcher, she soon realised that she was engaging in visual reasoning although she had not quite articulated it in terms of a formal definition. She soon acknowledged the power of visual reasoning in geometry. She added that there was often a ―light bulb moment. It often happens in geometry, because geometry remains so hidden and suddenly you‘ll see the angle that is the key to it all". She also alluded to concrete objects as being part of visual reasoning. Like Valerie, she also believed that the actual object was very necessary in mathematics teaching. Hence seeing, and having an image of the object was necessary for the following reason:

> ―I think visual reasoning is my bug bear, and also with everyone. I think Maths Lit (Mathematical Literacy) has taught me more about teaching mathematics than mathematics did. For instance, I try and make everything hands-on for them. When I did probability with them, I brought the thing with the chips and we recorded it. I feel if they‘ve touched it and experienced it, like the lady said in the conference in Port Natal, they will remember it".

After engaging in further discussion with Betty, she soon realised that her power-point presentations, video clips, and data-projector lessons were all visually centred. Her constant gesturing also contributed to visual reasoning.

Sam had a different understanding of visual reasoning. He seemed to place a great deal of attention on the functioning of the mind and the psychology of learning. He strongly believed

> ―that the mind operates on 3 simple principles: sensory memory, short term memory and long term memory and, if you don‘t have that in your mind, as you teach, how do you expect the kids to remember all the stuff over a long term. So you got to strengthen the sensory memory, and repetition will make it enter the short term memory and the long term memory. Your objective is to push that mathematics into long term memory. But you got to start with the sensory perception. The only 2 senses one uses as a teacher is sight and hearing. The sight is visual".

He further explained that, by asking his learners to visualize their homes and mums‘ faces they were able to describe such objects because of the repetition of these objects in their vision. He drew the analogy from here that through repetitive sight of mathematical procedure, they would remember the processes as well.

Sam was convinced that learning occurred primarily through the visual medium. This was evident in his words, “more important than hearing is sight”. This view seems to resonate with the views of visual reasoning that are held by Arcavi (2003) that vision is the most important faculty of obtaining information and that a large part of the brain is concerned with visual control, perception, movement, form and colours. He also contended that the biological make-up of the eye was much more developed in terms of nerve fibres as compared to the ear, comparing a million fibres in the eye to 50 000 nerve fibres in the ear. This comparison tended to highlight the importance attached to the organs of vision.

To Sam, visual reasoning was the ability to eventually see the object in the mind‘s eye, thus having a permanent mental picture of the object, which he believed was the solution to most learners‘ problems. He believed that the classroom was the place where the learner should be given the opportunity to constantly consolidate their visual experiences in mathematics, through repeated exposure of the concepts.

Although he did believe that pictures, images and technological presentations were important, he seemed to place more emphasis on teaching to form mental pictures. This could be achieved by relating the mathematics to a flow diagram, where the learner could visualise the end product and hence value the process in between. This was evident when he said that “they can see it so when they go into any situation they can close their eyes and visualise the teaching, such as using those mind maps or flow diagrams that we use in our teaching”.

Nellie defined visual reasoning as “anything that a teacher does to teach visually, so anything that I put up on the board, draw any diagrams, or any anything that a learner can look at, to solve a problem in mathematics”. Furthermore she conceded that she “definitely thinks it‘s necessary. I find a lot of our learners nowadays, are visual learners. I do think we need to use a variety of methods still, and that, visuals are not the only way to go”.

In teaching science, which also formed part of her teaching load, she acknowledged the significance of mathematics and found it necessary to employ visual reasoning here as well. She asserted that she found herself “doing a lot of drawings on the board and showing them step by step, with calculations, it really did help with the learning, rather than just talking to them and trying to encourage them to understand that way”. She also alluded to the need for gesturing as a means of keeping her learners focussed on her lessons. “When I talk they switch off completely most of the time so, I generally have to be doing something on the board, or doing something in front of the classroom, whether it‘s with my hands or something”. Like Betty, Nellie also believed in the need for concrete objects when she stated that,

> ―when it comes to Geometry and things, I try to bring in objects. To try and help them, visualize that, especially when it comes to 3D objects, learners kind of battle with 3D objects, so if I bring them in, they can actually visualize it a bit more, hold it up and see what they should be looking at”.

It would seem that all the participants had a common understanding of the concept of visual reasoning, although they seemed to differ in terms of the extent of its definition. Valerie, was of the opinion that visual reasoning focused on the use of concrete and practical tools in mathematics that were visual in nature, such as the match sticks, cubes and constructions that she referred to. Karl‘s view also indicated his focus on the use of visuals, in the form of pictures or any other means that facilitated the use of pictures, which would also include the use of multi-media equipment, promoting pictorial representation of mathematical concepts. Sam‘s use of diagrams, and his emphasis on the role of sight in learning, concurred with the views expressed by Valerie and Karl in terms of the use of the visual components in teaching and learning, although he placed a great deal of emphasis on the role of visualization in enhancing memory, which ultimately leads to learners‘ understanding of mathematical concepts. Betty also concurred with the other participants when she described visual reasoning as the use of pictures in teaching. Nellie believed that visual reasoning related to ―anything that a teacher does to present the material visually, anything on the board, pictures, and diagrams”. All the participants agreed that visual reasoning entailed the use of pictorial representation, whether it was in the form of diagrams, flow charts, multimedia presentations or even concrete objects. It may be possible to claim that the participants‘ understanding of visual reasoning was similar to the definitions discussed in the literature review in this study.

Although gesturing is a prominent form of visual stimulus, as revealed by the reviewed literature, all the participants did not see it as an essential part of visual reasoning, until after they became aware of how they were actually using it in their practice. It was only through the discussions in the interviews that they realized that it was a very powerful visual stimulus. Betty, considered it as a tool for emphasis of mathematical concepts in her teaching. She often felt that it occurred sub-consciously but sometimes it was planned. Similar to Betty, Karl also used gestures to keep the learners focused on concepts that he taught in his lessons, but also echoed the view that his gestures were often unplanned and occurred spontaneously. Nellie shared a similar view about gestures and its uses in her classroom practice. Sam was also of the opinion that he made no conscious effort to integrate gestures in his teaching, but that it often occurred sub-consciously. He did however add that, when he used gestures it was a conscious effort on his part to ensure that the concepts that he taught were understood and that, gestures as a visual stimuli, contributed in fostering good understanding and memory retention. Even Valerie alluded to the use of gestures in her classroom practice as a means of compelling her learners to remain focused on the aspects that she felt were very important. Moreover, she saw the use of gestures as a means of animating her lessons. She like Nellie, Betty and Karl, saw gestures as a means of keeping his learners focused, by preventing them from becoming bored, and providing some sort of entertainment in their practice.

5.3 Participants' Understanding of the notion of successful teachers

Success as a mathematics teacher could be ascribed to many reasons. In this regard, Nyabwa (2008, p. 439) believe that

> ―teachers' possession of high academic qualifications, enthusiasm in teaching, mastery of both procedural and declarative knowledge, and capacity to facilitate learning in classrooms is important, the effective use of an appropriate teaching method or strategy is critical to the successful learning and teaching of mathematics".

Whilst success can be defined in many ways, it has been used in the context of the teacher producing good examination results.

The discussion that follows addresses the meaning of successful teachers as it was understood by the participants in this study. This will become evident by focussing on what these participants viewed as successful teachers and how they achieved this position of success in their classrooms.

Through the interviews and classroom observation with Valerie, it became evident that Valerie was very passionate about teaching mathematics. She attributed her success as a mathematics teacher to her passion for the subject. She believed that it was shaped by several different outcomes. One of these was to transform her under-achieving learners with, marks of 30%, into a mark of 40% or more. Another outcome that she viewed as very positive was changing the view of mathematics, amongst learners, from a negative to a positive one. She also alluded to the ―wow-moment” in the classroom situation which she regarded as another positive outcome. It is fair to assume that a teacher, who is considered to be passionate about his/her vocation, is bound to be enthusiastic. It is clear that Valerie satisfied one of the essential characteristics, namely enthusiasm, which is encompassed in the definition of successful teachers. She defined a successful mathematics teacher as ―one who could inspire someone into actually persevering in Mathematics in any small way. It is disconcerting to hear children wish that they were not doing mathematics”.

Betty attributed her success as a mathematics teacher largely to working with supportive people. She made this claim in respect of all her experiences. From her first teaching post, to her current teaching post, she placed emphasis on the role of supportive Head Masters and mathematics teams. This was evident in her comments when she narrated the following: ―I went to Seaforth High School (name changed), where I spent a year under a very supportive principal and a good Math‘s team”. From there I went to Frans High School (name changed) for I think, 3 and a half years, where we had again, a very supportive principal and a very nice team”. Her view remained the same, even at the school that she taught at, at the time of this study. It suggested that she enjoyed working within a group as opposed to working as an individual, when she stated that ―since 2003 I‘ve been at Manor College (name changed) and I would say that the biggest influence, on me, is working with a competent team of people, willing to work with you and learn and be able to learn from them”.

Part of her success must also be attributed to her industrious nature. She mentioned that she often read and tried to keep in touch with new endeavours. This was in keeping with her nature of trying to be ahead of the pack at all times. Her qualifications in IT are testimony to this. She did however complain that, whilst there was a lot to be done, she was ―frustrated by the lack of time to do that. I would like to be, way more competent on smart boards”. This was the nature of Betty. She enjoyed trying to improve on her skills in all aspects of teaching.

Her determination to succeed drove her to do many things in her classroom practice. She believed that teaching was dynamic and no two groups of learners were exactly the same. This was clear when she explained that she constantly made a concerted effort to treat different groups of learners differently.

> I‘ll use this way and then I‘ll use that way and then I might even go back to the other way. It‘s funny, sometimes you get a sense that something works

> better for some kids and not others. It can‘t bore you because you‘ve got different children all the time,”

This illustrated that Betty was passionate about teaching and did not see it as a mundane job that merely had to be done. She was determined to ensure that her teaching style suited her learners, even if it meant she had to constantly modify it.

She also added that she was not in teaching because of the financial rewards that it offered. Indeed, she claimed, ―but teaching I think, is my love”.

Apart from her passion and industrious nature, Betty also exuded a warmth and care that endeared her to her learners. This was revealed by her life coach, during one of her counselling sessions with him. In her first round interview she disclosed this information.

> I went to this guy and had an assessment. It was interesting. Just through my family history and family values, the women in my family come from a strong, caring background. There‘s a strong caring line through it. That was very interesting and that was revealed. I think that‘s what teaching is about, that strong, caring line that comes through. I decided that I would spend my years in teaching.

Betty was also appraised by her learners at her the school which she taught at. This was a requirement by the institution that all teaching staff be appraised twice a year. The learner appraisals were then analysed by the teacher and presented to the head of the school during the final appraisal. Her learner appraisals generally indicated that ―they hated Math‘s before they came to her, and, ―now they love it”. Betty was probably commenting on the comments that her learners made to her during these appraisals. These appraisals are available or scrutiny.

Betty‘s success is summarized in her comment

> ―I think I care, I think I work hard, I think I go the extra mile, I‘m prepared to do that extra lesson, I don‘t shout at children, I don‘t bully them into understanding, I‘m hopefully trying to build their confidence, and I think, I try and make a difference and keep myself updated and up to speed as well”.

It would seem that Betty was prepared to go to great lengths to ensure that she attained the highest levels of success as a teacher. Her enthusiasm, coupled with her dedication and industrious nature, as well as her ability to facilitate learning, probably contributed to her success as a teacher.

Karl was of the opinion that success as a mathematics teacher was the ability to maximise the performance of learners in mathematics. If a teacher was able to obtain the best effort from a learner, irrespective of the actual level, then that teacher was considered to be successful. A teacher was not considered successful, simply by the number of A and B symbols that he produced, was the opinion that Karl expressed. This was clear in his statement

> ―My notion, I think in the simplest form is, if a Mathematics teacher, through some mechanism or mechanisms is able to allow the learner to perform at maximum potential, then I think, that would be the definition of a successful mathematics teacher".

He also believed that

> ―I do understand that not that not every learner would be the typical A learner or the B learner but if a learner has the potential to succeed at 60% and perhaps at a current state in time is only functioning at 40, and if a teacher is able to bring that learner from 40 to 60 and if 60 is that learner‘s threshold or potential then that would be my simple definition of a teacher who is successful in terms of Mathematics"

He also attributed part of his success as a mathematics teacher to his ability to introduce new ideas into his teaching, thus making teaching a dynamic process. He was of the opinion that the subject matter in mathematics was essentially very static, and that it had to be repackaged in innovative and interesting ways that would appeal to the learners.

Passion for the subject and compassion for the learners were regarded as the main elements for a successful teacher according to Karl. Whilst teaching was very dynamic it required the teacher to be abreast of the changes that took place. He believed that this placed pressure on the teacher to vary his methods in order to arrive at the same results. The exciting changes in the digital arena also helped to make teaching a very exciting and stimulating career. In order to be a successful teacher, a teacher had to be passionate about his subject, or else he/she would face the risk of becoming frustrated and obsolete, was the view held by Karl. Although not every learner would be a high achiever, the teacher‘s responsibility was to equip them with the best possible skills to help them to cope, was also the views that Karl shared about successful teachers.

Sam too, like the other participants was passionate about the job from the outset. This is evident in his statement ―be that as it may, my passion for teaching took over while I was at University". As a result of his passion for the subject, he developed a unique approach to teaching. His practice was based on inculcating inquisitiveness in the minds of his learners. This was evident when he said that he

> ―believes in the simple principle of asking myself, ‗why' every time I do something. I think, it has created a path for me in mathematics, which creates curiosity. When I teach a lesson in class, let‘s say I‘m doing products, I ask myself why am I teaching products to these kids"?

As a result of this teaching strategy, he was able to allow his learners to perceive mathematics as a relevant aspect of their lives.

Sam‘s passion for the subject, and the need to extract the best performances from his learners, ensured that he focussed on understanding human cognition. He claimed that it was very important for him that his learners understood mathematics. He believed that his learners would never forget the mathematical procedures if it eventually entered the long term memory of the learner. His obsession with this notion was evident in his statement ―your objective is to push that Maths into long term memory". Apart from being passionate about teaching mathematics, he pointed out the importance of being dynamic in the mathematics classroom as well.

Nellie‘s past experience with mathematics in high school, up to grade nine was problematic for her. It was only after a change of teacher in grade nine, that she began enjoying mathematics. She saw that change of teacher as a significant point in her mathematics career from which

> ―my marks improved, and I was actually starting to show more of an interest in mathematics again. So I think I always kept that in mind, I didn‘t want to

be one of those teachers that stands in front of the classroom and just talks to the board”.

Like the other participants, Nellie was determined to make a difference in the lives of her learners. Hence her passion for her job was evident. The satisfaction from achieving the desired results has driven her to engage with mathematics differently. This is when she explained that

―I think I can understand when the kids are battling because I‘ve been there myself, I understand when they are struggling to grasp the concept and I really want to carry on working on that concept until they actually understand it, because then it‘s like a light bulb switching on in their head, that they finally understand”.

An important aspect that was missing in the responses of all the participants, was the idea that a successful teacher should know her/his mathematics. This aspect was clearly mentioned in the literature review as one of the necessary attributes of successful teachers. Although the participants did not mention this aspect, it was evident from their qualifications, years of teaching and their level of confidence, that they had in fact known their subject matter well. This was evident in the observation lessons that followed.

There seems to be consensus about what the participants regard as the criteria for successful teachers. ―Enthusiasm” was cited as the main criteria in determining what makes a successful teacher. It would seem like all the participants were indeed passionate about their practice. Furthermore, all the participants spoke about the need to make teaching more meaningful by effectively using the appropriate teaching method or strategies.

5.4 Teacher’s context, background and the use of visual reasoning

This section examined how each of the participants used visual reasoning in their practice. This was based on what they had said or did in their classrooms. These descriptions were substantiated on the basis of the data collected through the interviews and through classroom observation lessons. Each of the participants was first identified and then located within the classroom context in which they taught. An analysis of each of the participant‘s background was conducted with a view to understanding their classroom practice. A brief description of their academic qualifications, teaching experience and their views about successful teachers and visual reasoning were also conducted. All these factors were taken into account to fully understand the kind of visual reasoning that was being used and the rationale for its use.

5.4.1 Valerie as a successful mathematics teacher

Valerie studied at the Edgewood College of Education. She was one of the first groups of learners that were given an opportunity to pursue a Technical Course of study. She majored in Mathematics, Electronics and Mechanics. Whilst she held a four-year teaching Diploma, she also held a Certificate in Further Education (ACE) in Mathematics.

Valerie had a large number of years of teaching experience. She taught at different schools, ranging from public to former ex-model C and even private schools. An elucidation of these types of schools is necessary. In South Africa, prior to the Department of Education having integrated all the provincial education departments, under a central unified education department, many provincial departments had categorised their schools as public, private and ex-model C. Public schools were administered by the Department of Education. The Department was responsible for all aspects of these schools, since the school fees that these institutions levied were minimal. Ex-model C schools were better equipped in terms of facilities, the number of teachers and resources. The State allowed these schools to employ their own teachers depending on their needs and the needs of the school to improve on the ratio of learners to teachers. These additional teachers were paid by the schools themselves since they were financially very stable because of the higher school fee structure that they adopted. Private schools were independently administered and funded. The Department of Education simply administered these schools in terms of curriculum and policy procedures in terms of national education. Valerie taught at Goodwood Boy High School (name changed), Nelson High School (name changed), Northway Girls High School (name changed), St Patricks School, in another province in South Africa, and a school in Swaziland, before she joined Manor College (name changed). She had also been exposed to the different examining bodies such as the Department of Education, the Cambridge Board and Independent Examination Board (IEB). Her teaching career spanned a period of twenty years. In view of her experience, she had a wide range of exposure to learners, syllabi, examination styles and administration systems. The length of her career ensured that she had been through different curricula changes in the South African system as well. She taught the syllabi that ended in 2005, 2008 and 2013, amongst others. She had also been exposed to different types of resources. With the private and ex-model C schools, she conceded that resources were not an issue, whilst with the public schools, resources were often an issue. Valerie‘s descriptions of her teaching experience and background indicated that she had come from a diverse background.

Valerie taught both technical drawing and mathematics. Apart from teaching these subjects across the grades from grade eight to twelve, she was also the head of grade, for grade twelve learners. She was also the convenor of the Learner Representative Council, which was a Liaising Body between the learners and the Management Team. As the Head of Grade, she was expected to organise all the major functions for the grade twelve learners as well as cater for their academic, cultural, sporting and psychological needs in terms dealing with the various issues at school. This placed immense pressure on her as a teacher because she had a huge workload.

Her classroom was well resourced, like most of the classrooms at this private school, with a standard size white board (1m by 3m) at the front, a smart board, located at the left side of the classroom, in the front, a data projector, a laptop, electric fans, adequate lighting, approximately 25 desks and chairs, a teacher‘s table and chair, several notice boards around the perimeter of the classroom as well as steel cupboards and bookshelves for her resource material. She also had a private room (ante-room) which was used to store her resources and consumables. Her classroom had many displays of learners‘ work, particularly in technical drawing. The room was also well ventilated and had a sliding door that led to open a vacant field.

Valerie conceded that whilst in the Independent Examination Board (IEB) schools, learners focussed on the reasons for performing different mathematical operations, the learners in most of the other schools were merely interested in the mechanical aspects of mathematics without really wanting to understand the reasons for these steps. She felt that there were demands for rote learning as opposed to learning with understanding. In her opinion, this was the reason for the scarcity of learners who were capable of thinking effectively in mathematics. This difference was largely because of the ways in which assessments were done in the IEB schools. In the (IEB) system, assessments were based on contextual teaching and a problem solving pedagogy. Hence the assessments were primarily geared towards a greater understanding of the mathematical concepts, rather than a mechanical application of these concepts.

In her understanding of the concept of visual reasoning, she also alluded to reasons for not being able to use visual reasoning as much as she would have liked to. She acknowledged that the lack of time was often a reason for not wanting to engage too much with visual reasoning. In this regard, she used it often in grade 10, in her additional mathematics class, since she was not so constrained by time. She quoted examples of where they might be used more effectively, especially with functions in graph theory and in geometry using constructions. She also maintained that where learners were streamed, it would have made it possible to try different methods to encourage them to think differently. The lack of visual resources also made it very difficult to use visual approaches as much as was necessary.

She believed that visual reasoning would certainly benefit the high and low achieving learners. In the case of the high achievers, she believed that it was necessary to promote high order thinking and develop critical thinkers. In the case of the low achievers, she believed it gave them an opportunity to understand a concept at a different level, therefore empowering them as well. She also believed that these two categories of learners –responded positively to something that is just not rote”.

She was of the opinion that visual reasoning helped to create the –wow” moment. In her opinion, it creates the moment when certain difficult concepts were actually understood, whilst it was often not the case in the –chalk and talk” approach. She had seen learners respond positively when visual reasoning was used. The use of visual and concrete objects, immediately brought light to the learners who were struggling with certain concepts. It helped to transform an abstract idea into a concrete one.

With regards to the degree of success that she attributed to the use of visual reasoning in her practice, Valerie, believed that it certainly helped to make her a more successful mathematics teacher. Whilst it may not have worked for all the learners, it certainly did for many. She also believed, given the necessary training in the use of visual reasoning, she could only become more successful in teaching mathematics. She attributed her success to the effective use of concrete and visual reasoning as opposed to merely the written, lingual arguments that were traditionally used in the mathematics classroom. In this regards, she expressed her disappointment with the syllabus, where these was little or no use made of concrete objects such as geo-boards. She saw this as an invaluable tool in the teaching of fundamental concepts.

Due to her training as a teacher in the technical field, Valerie showed a keen interest in the use of concrete objects and the need for a visual, practical delivery of her lessons. Her Diploma focussed on technical subjects like electronics and technical drawing. Her diploma was geared mainly around the subjects that required a technical skill. This required that the learner be able to draw and perceive objects in different planes. Hence her training was based on a hands-on kind of curriculum.

The following analysis shows how Valerie used visual reasoning in her practice. In a lesson on Rational Functions in a Grade 10 Additional Mathematics class, Valerie proceeded with the lesson as follows: She introduced a rational function using a sketch on the white board to explain key concepts of regular graphs, such as the intercepts with the axes, asymptotes and stationary points where applicable, since a rational function is the quotient of two other functions. The immediate use of a sketch of a previous graph ought to have given the learners a sense of the topic under discussion. The visual stimulus was extremely powerful and easily allowed the learners to relate to the basic concepts of graph theory. The reviewed literature has already pointed to the benefits of using a visual stimulus.

Valerie also gestured a great deal. She used gesturing for different purposes: firstly as a means of representing a picture (as in the case of holding her hands horizontally to suggest a horizontal asymptote) and secondly as a means of ensuring that her learners focussed on her discussion (by using her fingers to point to learners or draw their attention to an aspect of her teaching that required their undivided attention). This was emphasized in her statement

> ―I do know that for me it‘s a focus thing, so when you are pointing and you actually gesturing a lot, a lot of kids will actually concentrate far more because they looking at something. So for me it‘s more an animation and I know I learn better when I‘m seeing somebody doing something".

Valerie renders a possible explanation for her gesturing during her lessons. Perhaps her background and academic qualifications have led her to use this mechanism of knowledge transmission in her practice. Having received a very technically orientated training, it seems plausible that she finds gesturing to be an unconscious action. Gesturing therefore forces her learners to focus as well as conveys mathematical ideas to learners. In terms of the reviewed literature, a gesture is a powerful visual stimulus which signals the formation of a picture and thus conveys a meaning of a concept(s).

Her justification for the use of diagrams is found in her words

> ―Because I think for me it‘s to put it into perspective so we‘ve done all this, we‘ve done all the factorising of higher order equations, etcetera, and they don‘t know where, so I always use diagrams to show them I‘m factorising not just to find a root, actually the reason I‘m finding a root, is it‘s on the x-axis. So I use it a lot to put it into perspective. The answer doesn‘t make sense if it‘s just theory".

Based on the observations, Valerie uses visual reasoning extensively in the forms of diagrams, pictures and gestures.

She then proceeded to use the data projector and software to introduce a rational function. The presentation was made up of several slides, which were in different colours showing different pictures of graphs and tables. Whilst using the data projector and slide presentation, she simultaneously used the white board to explain concepts such as the intercepts of the graphs, asymptotes and others critical points on the graph. She also used gestured a great deal to draw the attention of the learners to critical points of discussion. For example when she equated the y-variable to zero, in order to find the x-intercept, she pointed to this zero to emphasize the significance of zero and explains why this was being done. Refer to figure 5.4 below.

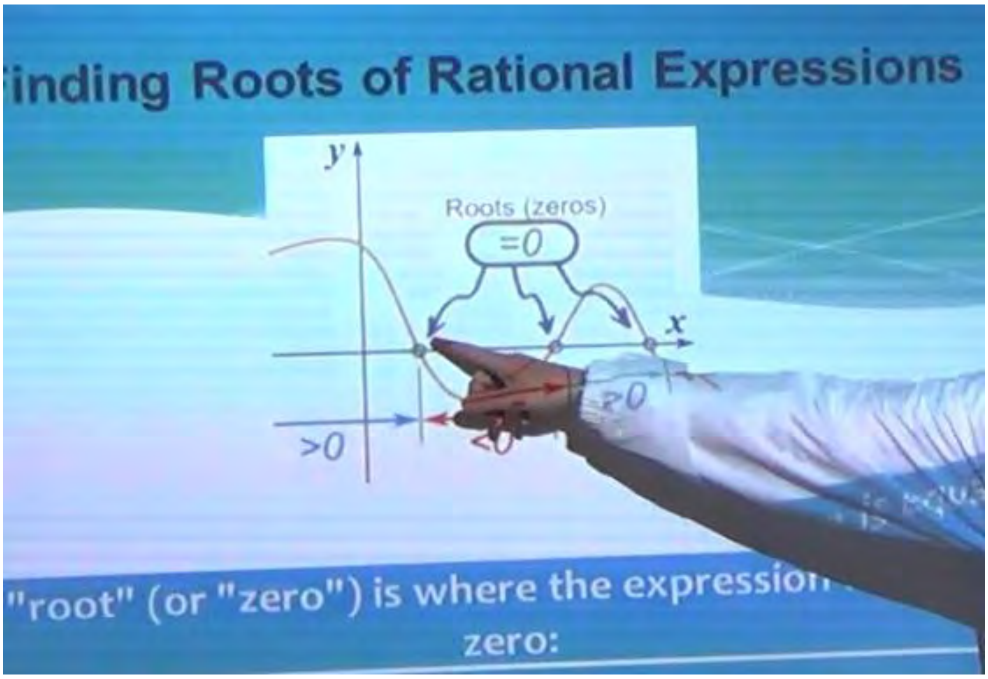

Figure 5.4: Video clip showing Valerie Gesturing

Using gesturing and the slides she was also able to emphasize other aspects of graphs such as: where the functions were positive, zero and negative. By using hand gestures and pointing to the relevant portions of the curve, the learners could see the relevance of her explanations. There was a constant iteration between the slides, the white board and gestures. In the same way she continued to explain the asymptotes of the graphs. During the course of her discussion she often used different colours of pens. It became clear that she used different coloured pens to illustrate the difference between her questions, solutions and notes or summaries. She sometimes circled parts of a problem to emphasize its significance. For example, in the fraction: $\frac{3}{x-2}$, she circled $x-2$ to show why it represented a vertical asymptote if x was equal to 2. Before she eventually drew the graphs, she used gestures again to indicate how she calculated gradient as the rise up and a walk. She then proceeded to give the learners a practice exercise in determining the critical points of a few graphs which she wrote on the white board. Valerie then wrote up the solutions in a different coloured pen. During the discussion of the solution to the exercises, she used techniques that were similar to the ones she had been using throughout the lesson. Whilst providing solutions to the exercises, she prepared a summary of the key points that were necessary when finding the salient features of a graph. This was done using a different coloured pen. This is indicated in the picture below (figure 5.5)

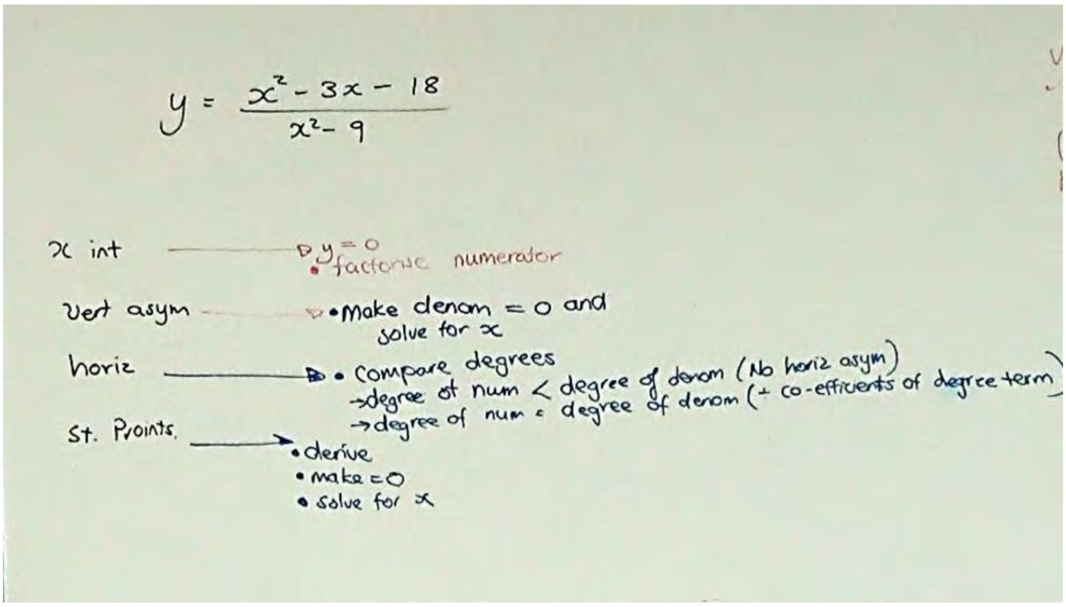

Figure 5.5: Summary of Valerie's notes on the whiteboard using different coloured pens

She then proceeded to drawing the graph on a Cartesian Plane which she drew on the whiteboard. The set of axes were labelled and numbered. After plotting all relevant points and asymptotes on the set of axes, different coloured pens were used to differentiate between the intercepts and the asymptotes. She also used the scientific calculator to generate a table of values that would be necessary to draw the graph more accurately. All learners worked on their calculators. She gestured to the different functions on the calculator (which she held up in front of the classroom), by pointing at the relevant keys, to generate the table of values that were necessary. Thereafter the graph was sketched.

Whilst a great deal of mathematics was introduced and many concepts were presented, the objectives of the observations were to ascertain the extent of the use of visual reasoning and the reasons for their use. Whilst she has offered explanations for the use of gestures, she also had reasons for using other visual components in her lesson. These reasons were determined from the interview conducted with her subsequent to the lesson.

She justified the use of a static slide presentation, as opposed to a dynamic presentation, on the basis that

> ―I don‘t think it would be more effective I tell you why, it‘s because I‘ve used video clips before, for example quadratic theory, when we were doing completing the square and it‘s, it‘s such a nice way that it‘s actually been done on this video clip and they just completely missed the whole point. So for me it was easier, if I was doing it, as a series of static slides rather than a dynamic presentation.

Furthermore, in her opinion dynamic presentations such as video clips were simply presentations made by other teachers in most cases. Through her experience with video clips, she felt that they were simply a repetition of what she was doing. Rather, it would be more beneficial to learners to view them ―in their own time” and then ―work through it because they can stop”. The video can be used more effectively by the learner at home because he/she can stop or rewind the presentation until it made sense to them. She also disapproved of video clips because ―you require knowledge that maybe you don‘t have”. She believed that these video presentations assumed that learners have acquired a certain level of mathematical ability. If they have not, then the video clip did not have any value to the learner.

She also justified the use of the data projector and the use of slides on the basis that it allowed her the opportunity to vary the lesson." And also because my subject is very much chalk and talk". She furthermore explained the need for varying presentation strategies to accommodate the dynamic nature of the learner, which she believes is different every year".

Yet another reason for doing electronic presentations, was that it offered Valerie the opportunity to upload her material onto the school‘s shared folder", which learners can access" at a later stage.

Whilst it offers the learner the opportunity for easy access, Valerie also saw another reason for its use. She believed that because they‘ve heard it, they‘ve seen it and now they can access it again the more senses that they use, the better".

The nature of examinations, and the focus on graphical and geometric interpretations, has also made the shift to visual reasoning necessary, according to Valerie. She was of the opinion that much of the recent examinations, particularly in grade 12 had been focussing on graphical and geometric interpretations. This meant that teachers had to adapt their teaching to cater for this change in curriculum demands.

In rationalising the use of colour in her power-point presentation and on the white board, Valerie pointed that it was done to emphasize key aspects and get her learners to focus on a specific aspect at a time. She firmly believed that your, brainwaves work on alpha ….. your actual waves and certain colours actually emphasise that part of your brain". She also believed in using a lot of red because that also opens up the whole Mathematical thing"

In explaining the use of arrows in her working on the white board, she cited the need for learners to be able to follow her discussions. She was of the opinion, that when several concepts are introduced, they may tend to confuse the learners. Hence there was a need for the use of arrows, especially in explaining problems involving several steps.

Valerie‘s lesson was animated, with much gesturing. She contended that this style of presentation was necessary so that they will remember it like they will remember every word of every song". She was positive that her animated presentations would contribute to the learners‘ understanding and retention of concepts.

In a subsequent lesson with the same class on the absolute value function, similar strategies were used. Valerie used the visual reasoning to a large extent in her mathematics teaching.

However, much of Valerie‘s actions could be justified in terms of the theoretical frameworks that have been identified in this study. Her use of multiple representations was directly related to Gardner‘s Theory of Multiple Intelligences. According to this theory, not all learners learn in the same way due of their biological genetic make-up. Different faculties of the brain may be used to acquire knowledge. Whilst presenting mathematics using language and symbols alone may appeal to the verbal/linguistic learner, it will not benefit the visual/spatial learner. Hence different approaches have to be made available to the learners in order for them to understand the different concepts in mathematics. Multiple representations offer this viable alternative. The constant iteration between the visual, lingual and the body kinaesthetic intelligences ensures that the learners remain focussed in Valerie‘s lessons. Gardner advocated using as many intelligences as possible, to enhance learning. The variations in terms of colour, and the use of highlighting techniques such as arrows, circles and other kind of markings, help to promote learning by appealing to the different intelligences. Her use of gestures, as a body kinaesthetic intelligence, coincided with the theory of multiple intelligences, since it offered her learners the opportunity to use another aspect of their intelligence in trying to make meaning of mathematical concepts. Furthermore, it resonated with the concept of multiple representations, which allowed her the opportunity to present her subject matter using alternate methods. Valerie also accepted that by using as many senses as possible, it was beneficial to the learner.

In terms of her success, Valerie identified her passion for the subject, as one of the contributory factors. Attribution Theory which offers explanations for human behaviour, classifies a human quality such as being passionate as an internal dimension and stable. According to the theory, if a person is successful, it is more than likely that they will attribute it to an inherent quality. It is for this reason that Valerie believes that she is successful.

5.4.2 Betty as a successful mathematics teacher

Betty, being a graduate of the University of Cape Town during the period 1976 to 1980, majored in Psychology and Geography, with Mathematics at second-year level. Betty who was an enthusiastic and dynamic teacher was determined to further her academic qualifications. Hence, she pursued a Higher Diploma in Education and a Post-Graduate Certificate in Education. After joining Manor College, her current place of employment, she continued with her studies. However, much of her energies were devoted to improving her skills in the area of IT. In this regards, she obtained the full International Computer Driving Licence (ICDL), a certificate in computer applications, after doing several modules in Computer Science.

Her teaching career began in Ketteldorp High School (name changed), in the Northern Cape Region. Having moved to approximately five schools in Northern and Western Cape, she then transferred to schools in Durban until she eventually took up a teaching post at Manor College, which was the school that she was at, during the time of this research. Betty was the most experienced, of all the teachers, in this study, with a total of thirty-three years of teaching experience in mathematics.

During her teaching career, Betty gained the necessary experience of being in public, ex-model C and private education. As such she had experienced learners in terms of different socio-economic backgrounds. The span of her teaching career ensured, that she was exposed to several syllabi changes, during the course of her years. This vast experience has made her very knowledgeable in the subject and in teaching in general. Betty has also been an active member of Naptosa, a Teacher Union. This position enabled her to keep abreast of the changes in education. It would seem that Betty was well-equipped in terms of having acquired the necessary teaching skills and many other professional qualifications that would have benefitted her in her career.

Betty‘s teaching load was made up of Mathematics and Mathematics literacy. She taught mathematics across the grades, ranging from grades eight to grade eleven. Her Mathematical Literacy teaching class had been across grade ten to twelve. Having attended all the necessary training in the teaching of Mathematical Literacy, she was often seen as the expert in Mathematical Literacy. Over and above her duty as a mathematics teacher, she was also the Mistress in charging of the Boarding House, which the school catered for. This duty ensured that she was fully occupied after school and on the weekends, taking care of the needs of the learners that board on the premises. Her residence was also on the campus.

Her classroom was well resourced with two standard white boards, one in the front and another at the back of the classroom. She also had a fully fitted data projector, a laptop, electric fans, adequate lighting, approximately 25 desks and chairs for learners, a teacher‘s workstation, two cupboards to store her resources, several notice boards with information of the daily, weekly, termly assessment requirements, notices and so on. She had several resource textbooks which she constantly referred to. This was evident in daily discussions with other teachers. The room also had a feel of mathematics in it, with charts of mathematical quotations, posters of the Casio calculator and other posters in mathematics and general inspirational pictures and posters.

Whilst she was bold and honest about her understanding of visual reasoning as being minimal, she later conceded that she had been using it in her teaching. She conducted her own research into what visual reasoning entailed. She reached the conclusion that it related to vision and the use of pictures to follow a particular kind of reasoning in mathematics. She also argued that not all of the mathematics could be reduced to pictures. She quoted an example of a topic in the grade-nine syllabus, that she believed that could not be reduced to pictorial reasoning. Products, she believed did not easily relate to a geometric interpretation. Whilst an alternate argument can be raised, it is not necessary. This opinion will not alter the findings of this study.

Whilst she could not readily employ the use of visual reasoning in her algebra teaching, she conceded that she was attracted to geometry teaching. This was evident in her statement that, ―I like doing geometry. I like it because, it is about diagrams, and I like drawing big ones and putting colours on it".

With regards to the use of visual reasoning in the mathematics classroom, Betty was of the view that it provided a means of stimulating the learners, whilst it made the lessons more exciting. She believed that learners were easily bored and saw the use of visual media as an alternate form of presenting data. This became clear when she stated that,

> ―I think more and more, a lot of our children's worlds are visual. TV, phones, internet, are what they are good at. I think just talking to them, they don't hear you anymore, and I think that just writing it on the board is also ineffective."

Betty eventually conceded that she had been using visual reasoning in her classroom practice, more than she had given herself credit for. This been something that she did unconsciously without too much of pre-planning. She illustrated this when she stated that ―now that I'm talking to you, I'm thinking that I'm using it a lot".

She also animated the teaching of algebra, by telling stories, thus forming associations with objects in the real world and mathematical concepts. When she wanted to emphasize the importance of the minus sign in multiplying out several terms in a bracket, she likened the minus sign to a rhino, which was considered to be a very dangerous animal. This was done with the intention of associating a dangerous step in a mathematical computation with a dangerous situation, such as being confronted by a rhino. The reference to ―I had Malcolm (name changed) scan, my textbooks”, suggest that she did use visual means of presenting lessons. In so doing, she was engaging in teaching using visual reasoning.
Betty also believed in the need for teachers to constantly keep their learners entertained, through visual reasoning, when she declared that,

> ―I think it makes it more interesting. I think it puts pictures in the children‗s brains. They can remember and I think it creates the interest, especially, when a lot of it‗s very boring, when you sit here and you just multiply out brackets like we are doing with the Grade 9's”.

The constant reference to ―it” illustrated the emphasis she placed on the significance of visual reasoning in her practice.

Visual reasoning could also be an impediment in classroom practice, according to Betty. She stated that it tended to slow down the work, since it took a great deal of time to convert notes into visual format. Whilst she conceded to this, she also saw it as a positive factor. She asserted that it helped her to slow down the pace of her lessons. She believed that her lessons are fast-paced and visual reasoning actually helps her to lower the pace.

The analysis that follow, attempts to show how Betty used visual reasoning in her practices.

In a lesson on Financial Mathematics in a grade ten class and in a subsequent lesson on Probability in the same class, Betty proceeded with the lessons as follows. A short synopsis of the lessons is important, since it will show the extent to which she has visual reasoning in her teaching.

Betty‗s lesson on financial mathematics revealed the following: The lesson on finance commenced with a projected worksheet of questions that were given to the learners the day before. It was to be completed as a homework exercise. After settling the learners down, Betty started the lesson by correcting the examples that were projected on the whiteboard. During this period she used gestures a great deal and body language was evident. She also varied the colour of her pens as she performed different mathematical operations. She then used her hand held calculator to explain rounding off as a mathematical operation. Thereafter, she referred to the picture of the calculator on a chart to demonstrate the mathematical functions such as storing and rounding (figure 5.6)

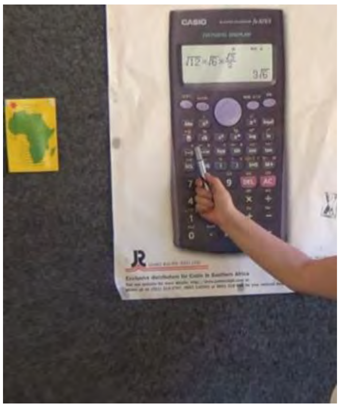

Fig 5.6: Betty gestures to the chart depicting the Casio Calculator

She then changed her slide to deal with inflation. Having described inflation, she distributed a worksheet on inflation, in which she used a newspaper cut-out to explain the ill effects of inflation on medical aids in our country. Her body language suggested her views about the effects of inflation on medical aid costs. To emphasize the influence of inflation over a period of time, she used gestures to denote this concept. Her hands showed a constant decrease in the amount of disposable income that a person will have, since the rate of salary increases was lower than the rate of medical aid cost increases. She then explained population growth using the compound interest formula and gave the class an application exercise to complete in class, whilst she went around addressing individual learner‘s concerns.

Betty started the lesson on probability by showing the learners a ten minute video, which she downloaded, on the introduction of probabilities. The video was a lesson taught by another person. The speaker used a coloured pen to write as he spoke. The speaker also used objects such as cards and dice to explain the basics of probability. Betty used different coloured pens when she wanted to define a concept, for example, the definition of probability, was done in red. She then issued a worksheet which learners used to perform an application exercise. She often gestured for the sake of emphasizing key concepts. Betty then allowed the learners to attempt the exercise in class and corrected these examples. Again body language was used effectively. Betty wanted to show the chance of something being very likely or less likely. Her hands moved between the intervals of zero and one, on the probability scale, that she drew on the board. The lesson continued in this way. Towards the end of the lesson, she drew a two-way contingency table. She indicated that the topic would be continued in the following lesson.

An examination of her lessons, and transcripts of her interviews, will be used to analyse her teaching practice, in terms of the ways in which she used visual reasoning. These actions and explanations will be viewed through the lens of the theoretical frameworks, used in this study, in order to arrive at an understanding of these actions.

She explained the use of gestures by saying that she wasn‘t ―even aware that she was doing it". She further explained that she thought that they ―were entertainers more than ever before", and that ―children were very bored if you just stood there and talked to them because they saw TV, and marvellous actors". They saw this as an opportunity" to watch other than just hear the information". She pointed to several reasons for the use of gestures. It was used as a means of emphasizing parts of the lesson that she would have considered important for their understanding of the topic. She also believed that it was necessary to be dramatic in the mathematics classroom as it was a means of keeping the learners interested in the lesson. Her reference to television and actors indicated that mathematics teaching ought to change to maintain the focus of the learners. She also stressed the need of using different techniques for communicating and teaching. The reference to boredom through talk indicated the need for alternate forms of lesson presentations, hence, the need for them to ―watch" as well. She further explained that gesturing was an ―extension of herself", indicating that it was a sub-conscious activity, although she later conceded that there are instances when she believed that gestures were planned. However, on closer examination of the lessons, it was evident that she used gestures a great deal and that body language was strong in her teaching. Although she may have believed that it was a sub-conscious action, on many occasions, her gesturing was closely related to the way in which cognition took place. When she spoke of a rise in inflation her hands moved upwards naturally. This action would seem to suggest that it was not sub-conscious but rather directly linked to her understanding of the concept of increase and her actions were emphasizing this notion. In this way she had given her learners the opportunity of hearing and seeing the concept. This resonates with the principle of multiple representations and the theory of multiple intelligences. She also argued against the use of static presentations in the classrooms, since she perceived them to be boring. This supported her stance that gestures were necessary in order to animate the lessons in order to sustain the attention of the learners. This was evident when she stated ―it‘s important that children just don‘t see a static presentation". According to the definition of visual reasoning, cited in the literature, Betty was using visual reasoning, since gestures are considered to be important component of visualization.

Betty‘s use of the data projector can be summarised in the arguments that follow: In the main she was of the opinion that it was an alternate form of presentation which was very necessary in the teaching of mathematics. It created variety in terms of teaching methods, and it helped to alleviate boredom on the part of her learners. She felt strongly about using visual stimuli in her teaching. For her it was like having ―a TV in your classroom", which created enthusiasm in the learners.

She justified the use of posters and pictures on the same basis. This was evident in her statements that ―it puts pictures in children‘s brains, they can remember" otherwise ―it is boring so you trying to put a little bit of lightness in, you are also trying to hook a memory". Moreover, she saw the use of these posters and pictures as a useful aid in enhancing the remembrance of the concepts that she taught.

She furnished another reason for the use of a poster of the calculator. She felt that the poster facilitated their understanding, by representing the object. She argued that she ―didn‘t think they‘ll understand unless they saw it". She was convinced that an even more appropriate visual tool would have been the calculator simulator, which was a projected version of the calculator through the data projector. This was evident when she said that ―I think it would be better if I had one of those simulators here". This related to her earlier comment about the use of concrete objects in her teaching and its effectiveness in the understanding of mathematical concepts. Betty also believed that the call in mathematics teaching was for the use of more visual reasoning because ―we are constantly being told that our children are more visual and they‘ve got shorter attention spans".

Through the use of the projector, posters and pictures Betty believed that she was ―trying to make it like the real world with their phones and their TV‘s and their computers. She also conceded that she had been ―lagging behind" in terms of the changes that were necessary to update her pedagogy. It would therefore be fair to argue that Betty engaged in visual reasoning for the purposes of promoting understanding, creating exciting lessons, creating variety, eliminating boredom and creating authentic learning in the classroom.

Betty argued that visual reasoning also offered other benefits in her teaching. She claimed that the data projector –slowed her down” by limiting the amount of available space on the chalkboard/whiteboard. Prior to her using the projector, she felt that she merely –rattled away”, by filling up the board with mathematical calculations and notes. She saw the use of the data projector as a technique that prevented teachers from continually talking and writing on the chalk/white board, thus signalling the use of alternate presentations or multiple forms of representations that the reviewed literature has made reference to. Whilst it helped Betty to reduce the pace of her teaching, it also offered the benefit of saving time in her teaching. This was possible because some of her work could be done at home. The results or solutions would simply be projected on a screen, thus saving time, by not having to write the entire solutions to exercises that were given as homework. This was evident when she stated that –you also don‘t waste time on irrelevant stuff you know”.

She emphasized the need for mathematics to be authentic by engaging with resources that learners saw in their real world, such as newspapers. Her reason for using a newspaper article on budgets could be found in her words –we always saying Maths isn‘t in the real world. However this section on probability is. I really think we should bring it to the fore to engage the interest of the learners”.

Like Valerie pointed in her interview, besides being used as an alternate form of presentation, visual reasoning could be reinforced by repetition. The learners now have the advantage of watching these videos several times in the comfort of their homes. This was the view held by Betty when she argued for its use by saying that if –a child doesn‘t understand you say, please bring me a flash disc. Go watch this at home”.

She justified the use of dynamic software on the basis that

> –I wanted them to be keen to do probability and not to be scared of it because my fear around probability is I last did it at Varsity and hated it. I‘m desperately up-skilling myself as well. It‘s also just making them a bit more interested, keeping them going a bit you know, same voice same thing every day, hear someone different, enjoy it”.

Apart from dynamic software being useful for the purposes of simplifying difficult mathematical topics because they may have been presented by someone who is an expert in a topic, they offered excitement and variety in terms of presenting the topic differently. The variety is created by hearing someone different, or by doing something differently. She also explained that dynamic presentations had the distinct advantage of keeping learners enthused because the learners identified with the technology, thus making their learning authentic.

She explained that coloured pens and colourful video clips highlighted information that was more relevant than others, as in the case of the probability scale in figure 5.7

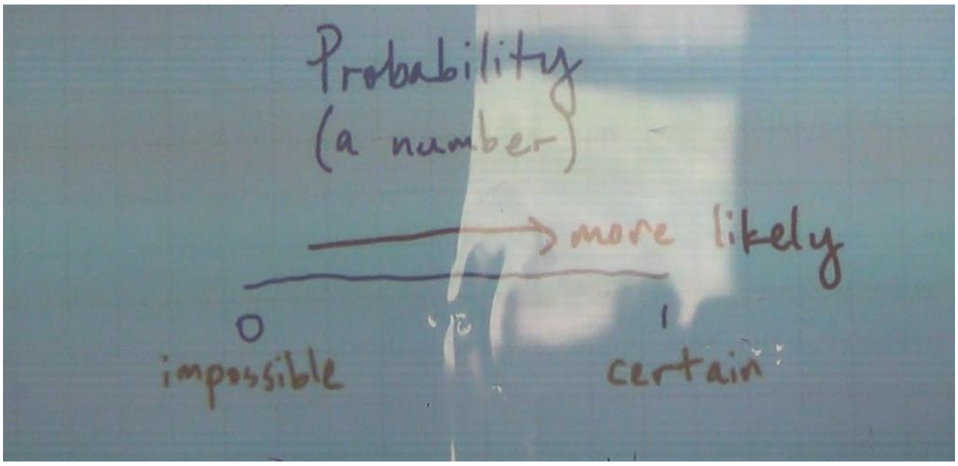

Fig. 5.7: Video clip showing different colours on the probability scale

In her teaching she also circled important information, which she claimed was done for emphasis. She explained that this practice was done with the intention of trying to improve the learners' understanding of concepts. This was evident in her statement —you know in Geometry, in the colouring in of the equal angles and all of those different things, they just really to help as much as you can".

The theoretical frameworks in this study seem to be able to explain much of Betty's action in terms of using visual reasoning in her teaching. Her reliance on a good team, and the need to have good role models, seemed to suggest that Betty's attribution was external but stable. Hence her motivation to continue to learn and strive to be the best that she could be was not attributed to an internal dimension as in the case of most people that are successful. According to Attribution theory, internal attributions can often be construed as arrogant if the attribution is made to the ability of the successful person. However if the attribution is made to effort, then it may be construed as modest. If the attribution is made to an external factor, to explain success, then it is fairly rare. People who fail generally attribute their failure to external factors such as task difficulty. In terms of this theory then, it would seem that Betty was modest even though she was successful. Apart from making internal attributions, such as her passion for the subject, she tended to focus on external factors that made her successful. In trying to maximize the potential of her learners to understand mathematics, she used multiple representations in her teaching. By engaging with multiple representations, she allowed her learners the opportunity to use their different intelligences to understand the subject matter. Although this applies to learning more than it does to teaching, her ability to recognize the need for alternating presentation styles would seem to indicate that she was aware of Gardner‘s Multiple Intelligences Theory. According to this theory, in as much as it is to be used by the learner, teachers too, must be aware of its use so as to develop their learners‘ potential.

Betty‘s constant reference to the real world and the need for mathematics to be made authentic through the use of visual reasoning can easily be justified on the grounds of Lave‘s Situated Cognition Theory, which has been dealt with in the previous chapter. This theory has been founded on the premise that cognition does not take place in isolation, but it is also the engagement of the individual with society that creates knowledge. Knowledge is then seen to be situated and relevant. Betty strove constantly to achieve this goal. This was evident in the choice of authentic teaching material such as the newspaper article, the use of technology and the kind of examples that she used in her teaching. Whilst dealing with the topic of probability, she touched on important aspects such as gambling and its dangers. Her discussion on issues relating to medical aids and salary increases also made the topic on financial mathematics very authentic. Lave‘s reference to the community of practice is clearly seen in the case of Betty as she evolves into a successful teacher. Initially she was the inexperienced teacher, who either through training or consultation with her peers, was able to learn new subject matter. This knowledge was then practiced, either through her own efforts or again through interacting with her peers until she became the old timer that is referred to in this theory. Her transition from a novice teacher to an expert in the field must be seen in the context of this theory.

The intersection of her attribution for success, the use of multiple intelligences and the search for relevant authentic knowledge is all cohesively brought together through the use of visual reasoning.

5.4.3 Karl as a successful mathematics teacher

Karl, unlike the rest of the teachers that are part of this study, did not study towards a teaching degree. He qualified with a degree in Mechanical Engineering, from the University of Karville (name changed), hoping to go into industry. He indicated that it was very difficult to find employment in the engineering sector and therefore he decided to pursue a career in teaching. It was much later that he completed a relevant teaching diploma. In terms of further qualifications, Karl had initiated a study in Digital Literacy, under the banner of a Masters in Communication Science degree, through the University of Zanda (name changed), at the time of this study. However, this degree was incomplete at that stage. He also indicated that he had found other interests in the digital area but would like to complete the previously-mentioned qualification at some later stage.

In terms of his teaching experience, Karl had taught for a period of nineteen years. He has taught grades seven to twelve during this period. At the time of this study, he had been teaching in the senior phase, namely, grades ten, eleven and twelve.

He taught at both government and private schools. He had been exposed to a range of learners, coming from different socio-economic backgrounds. Whilst there were those learners came from affluent households, there were those who came from low-income households. The schools themselves varied in terms of their resources. In the government schools, he faced the challenges of inadequate resources, whilst in the private schools resources did not pose a serious problem.

The length of his career has ensured that he has been through the different curriculum changes in the South African system of education. These changes took place in the years 2005, 2008 and 2013.

Karl taught Mathematics and Mathematical Literacy. His teaching load in mathematics ranged from grades eight to twelve. He taught Mathematical Literacy at grade ten and eleven levels. Apart from teaching mathematics, he initiated many projects in school, using technology as a basis and attended several workshop and seminars on the use of technology. He presented many talks on the use of technology in teaching. At the time of this research, he initiated the introduction of an internationally used calculator, called the Texas instrument into the school. All mathematics teachers were trained in the use of this device. The device was being piloted on a group of learners. The mathematics team was waiting for the results of this pilot project.

His classroom, like the other participants‘ classrooms, was also well resourced. It was equipped with a standard white board, in the front of the classroom, a fully fitted data projector, a laptop, electric fans, adequate lighting, approximately 25 desks and chairs for learners, a teacher‘s workstation, two cupboards to store his resources, several notice boards with information of the daily, weekly, termly assessment requirements, notices and other necessary teaching equipment. There were several resource textbooks which he constantly referred to. The classroom walls and notice boards were covered with mathematics posters. His classroom was very conducive to the teaching and learning of mathematics. The atmosphere of the room seemed to have conveyed this idea.

Karl believed that the main reason for his success as a mathematics teacher was largely through the efforts of good mentors, whom he encountered on his journey as a teacher. These mentors included subject advisors as well as seasoned educators. This was evident when he stated that

> ―most of the successes rely on the fact that I‘ve had good mentors along the way. From my first day in the classroom, I was mentored by seasoned mathematics educators who spent many years teaching, and then, in particular, in recent years, the Maths subject advisor. I suppose mistakes that other educators made alerted me to the fact that certain things will not work because they have been tried and tested and because they failed".

It would also seem that he also learnt from the mistakes of other teachers, who may have expressed their failures to him. He could have also observed these mistakes in the course of his development.

He also indicated that his digital experience and background influenced him to participate in the Innovative Teacher Forum Awards. His passion for using digital media to change the face of teaching saw him win the Innovative Teacher of the year Award, a competition that was initiated by the Microsoft Foundation. He initiated a programme to upload, grade twelve, mathematics content onto cellular phones. This victory, against international competition gave Karl the opportunity to be recognized by the Department of Education. He was acknowledged by subject advisors who wanted to use his skill in teacher training for the curriculum changes that were taking place at the beginning of the year 2008. Although the Department of Education did not embrace Karl‘s achievement and incorporate it into schools, nevertheless, the publicity that he derived out of this experience earned him a great deal of accolades.

Being the digitally inclined person that he was, I was curious to engage with him in this regards. His notion of visual reasoning was simple and to the point. He indicated that visual reasoning was the use of pictures, graphics or any multimedia presentation method that had more effect than words alone. He emphasised the extent of visuals in mathematics, stating that pictures often conveyed more information. This format could not be compared to the use of language to describe the contents of the picture. Furthermore, he made reference to gesturing as a form of visual reasoning.

Karl saw visual reasoning as a means of enhancing learning. He indicated that, whilst it was possible to describe some concepts, visuals made mathematical ideas very easy to understand. It was as though the learners actually connected with the ideas. It brought the ideas to life. According to Karl, visual reasoning obviated the need for detailed explanations. The use of visuals offered the advantages of colour, three dimensional effects, rotational properties and the various transformational properties. He saw these properties of visual reasoning being very useful

> ―in terms of the Grade 12 syllabus, with calculus maxima and minima, where the concept of volume and surface area still poses a great deal of difficulty to some learners. They find it difficult to see a drawing on a flat piece of paper as representing a three dimensional shape".

These properties made visual reasoning a good medium of instruction. Karl always made use of the digital technology that was available at school. This often ranged from using the overhead projector, the digital projector, three-dimensional models and gesturing. He supplemented his teaching with video clips from various sources on the internet.

Karl also indicated that visual reasoning had the potential to impede teaching, if it was not carefully used. In this regards he suggested that there was a need for careful editing and adjustment of the selected subject material. However, when that was done in advance, success was inevitable. This was evident in his words ―when I use visual reasoning as a tool, it accelerates and enhances the learning experience". He further asserted that

“I think this is a rule: it has the potential to be an impediment to learners who are very well developed spaciously. If a learner is able to visualize the content immediately, then to that learner, the graphic may just be a waste of his time because, once he has grasped the concept, and this occurs in a few seconds or few minutes, then the learner may appear to be bored”.

Whilst visual reasoning did have the potential to impede teaching and learning, it had more to offer, according to Karl. It created more excitement and promoted research for the teacher. It meant that teachers had to search various tools that promoted visual reasoning. Karl saw this as a very exciting process. He also believed that it was very necessary to enhance the learners' experience of mathematics. As much as it was time-consuming, it was also very rewarding. This point of view was seen in his words

“it makes it exciting, it promotes research, it enables me or creates a stimulus for me to go onto the net and research the various tools that are been used by fellow educators across the globe, and implement or attempt to implement some of the concepts which I think, may work in this environment, in our country”.

He strongly believed that it was the responsibility of the teacher to expose the learners to the most recent trends in teaching and learning. The increasing emphasis on technology made it impossible for teachers to ignore this call to change the way they taught. Hence they had to embrace the changes that were taking place.

He also observed that, learners who were less developed spatially, were more inclined to benefit from visual reasoning. The choice of colour and the graphics enhanced such learners' understanding of certain concepts. He saw this as a “light bulb” moment. It created that moment of understanding to these learners.

Furthermore, Karl saw the use of visual reasoning as a time-saving strategy, and an alternate platform to the traditional lecture-method of teaching. In the teaching of three dimensional shapes, he argued that “it also saved time in terms of finding the appropriate models”, which he believed were “hard to find”, since they don't “just exist”, but by “using 3D images, it helped the process nicely”.

He was convinced that learners had embraced digital teaching and suggested it as the way forward. This was closely linked to his academic qualification and experience in promoting digital teaching. Another view that Karl shared about the benefits of visual reasoning was that visual reasoning offered the benefits of extending the learner beyond the curriculum. He believed that this benefit could only be gauged once the learner had left the school, and entered the tertiary world. In that environment the advantages of that extension often became evident. He cited the example of "the basic $y = x^2$", and how it allowed

> "the learner to understand that while the parabola is in two dimensions, there are parabloids that exist in three dimensions. Showing them the X, Y and Z axes as a graphic suddenly expands their thinking into another dimension", and how "some of them are waiting to be taken to that level".

Observing his lessons on the quadratic function, he introduced the topic by a picture of the basic curve of the parabola on the data projector. The slides were very brihtly coloured, as indicated in figures 5.8 and 5.9

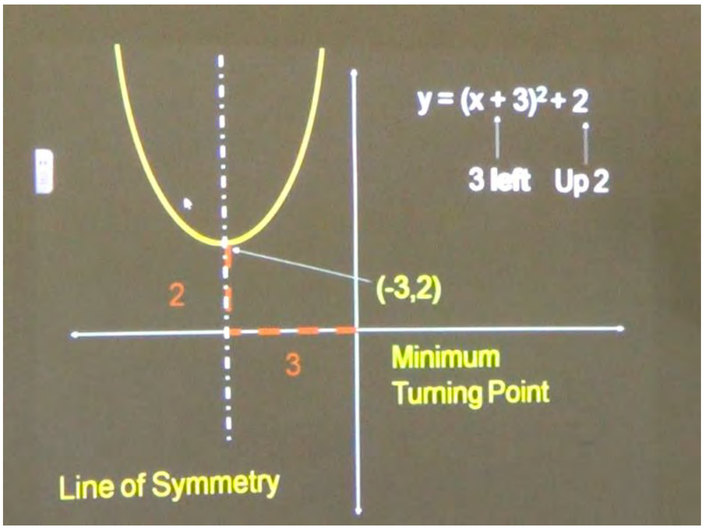

Figure 5.8: Slide showing the basic parabola

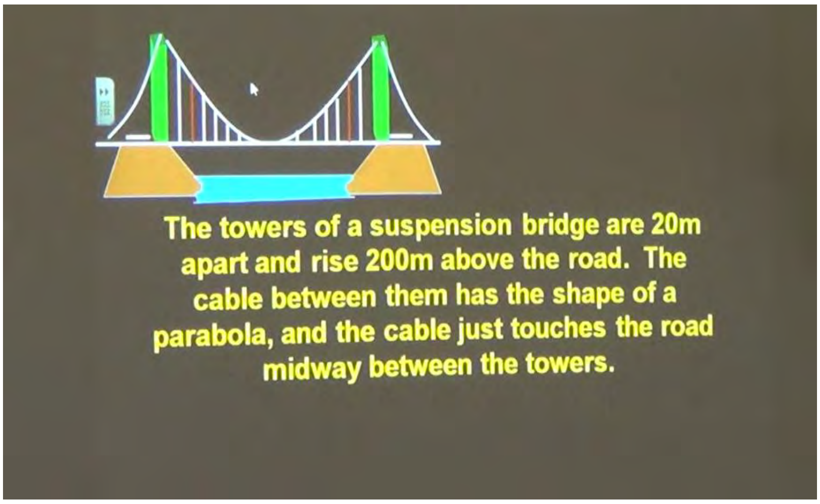

Figure 5.9: Slide showing mathematical modelling

He used dynamic software to introduce the equation of the parabola. As he rotated the slides, the equation appeared as if it was being typed in at that moment. This can be seen on the video clip of this lesson.

He used different colours to write down the defining equations of the different curves. Whilst encouraging learners to draw different curves and find the defining equations, Karl used his hands to gesture as well. He used his hand movements to show the different transformations that were possible. The use of the digital projector was supplemented with the use of the white board, on which essential calculations were performed. These transformations were enhanced by the use of arrows, drawn in different colours. To emphasise key points, colour was used to highlight these essential details. In an example, depicting the real world situation, the use of the dynamic software made the situation more relevant. In this regards, Karl mentioned two lights, which he showed as flashing lights. The learners appreciated this real life depiction. This effort to simulate the real situation, made the problem more meaningful and relevant to the learners.

Whilst these visual tools were used very effectively, he also encouraged learners to visually recall properties of the quadratic function such as the minimum and maximum values of these functions. He therefore also encouraged a visualization of the function in the minds of the learners. There was a constant iteration between the visual and calculation aspects of the graphs under discussion. Learners were able to see the relevance of the quadratic function in their daily lives, through the reference to the Mac Donald advertisement. Karl also used the visuals reasoning to extend his learners. His reference to conic sections and the subsequent derivation of the different functions were designed to help the learners to understand the quadratic function in more detail.

An analysis of Karl's actions will explain why he used visual reasoning in the way he did. He justified the use of gestures on the following basis

> ―Well, I think over the years I've started to realise that much of teaching is conveyed vocally, I do agree. However, emphasis, I realise lies in with body language. With experience I've realised that gesturing emphasizes key points in the lesson".

He therefore saw it as a means of highlighting key points in the lesson. In this regards he asserted that ―it somehow helps to reinforce the learners' memory and recollection of the important points that I do highlight so, in other words it could be something similar to using a highlighter on a page". His reference to memory and recollection clearly indicated that he used visual reasoning with the intention of assisting the learner in mathematical cognition.
He also believed that the role of the mathematics teacher had evolved and that it was necessary to keep the learners focussed by being seen as an entertainer. He had a similar view to Betty when he argued that the current learners struggled to concentrate on the subject matter without being easily distracted. Although he also believed that gesturing was a spontaneous activity on his part, he accepted that he thought that it was very useful for promoting understanding of concepts. This was evident when he said ―I think that it definitely helped the learners understand the concept better". He cited another reason for the use of gesturing. He argued that ―the whole idea behind the picture is that they attach a picture to the concept itself". He referred to the gesture as a picture that the learner would remember. In the process of remembering the gesture, the learners would remember the mathematical concept that was associated with it. In his teaching he moved his hands horizontally to indicate horizontal shifting of the graph. He believed that his learners would remember the concept because they would remember the gesture associated with it, thus making gesturing a tool for visual reasoning.

During his lessons, Karl drew diagrams on the white board, explaining that

I find this especially true in Mathematics that the more pictures the more diagrams are used the more effective it become in conveying a concept and because I believe most of our learners are visual it helps to convey the message quite clearly”.

This view coincided with his earlier view that learners were changing in terms of their ability to concentrate on lessons and remaining focussed for prolonged periods of time. He further claimed that seeing that 3 D picture in front of them at that moment in time in the Maths lesson brings the live integration of the Mathematics with the real life”. He believed that pictures had the ability to make learning authentic since they brought the reality into the classroom. He further justified the use of pictures by arguing that there are times when the model if available would be better, but I found that in most cases it is difficult to obtain the model at hand”. Therefore Karl used pictures as a means of reasoning in mathematics. The use of pictures by Karl was further justified on the grounds that learners could see very easily that Mathematics plays a very vital role in the real working world, in the real life scenario and they could appreciate it more readily”. To Karl the relevance of an application of mathematics was seen as very important. Pictures and concrete objects could easily create this reality in the minds of learners, as in the case of figure 5.10, which learners could identify with in their lives.

Figure 5.10: Mathematical modelling, the parabola in real life

Besides perceiving mathematics for its utility value, some of the learners in the class who were desiring careers in science and in engineering mathematics as well could see the beauty of Maths at this young age in the real working world". His qualification in engineering had equipped him to see mathematics being used in engineering and related fields, which also contributed to the realm of aesthetics and art. When he was asked about the relevance of using concrete objects in his mathematics classroom to perform mathematical discoveries, he argued that if ―a discovery is made by the individual themselves it will be an investigation they will never forget for the rest of their lives". Karl was convinced of the use of pictures and concrete objects as a means of promoting mathematical understanding, except that it was being done through the use of technology.

He justified the use of the data projector on the basis that ―there's more opportunity to show lot of colour, I could walk around the class, be it different advantage points and rotate from one slide to the next".

He used colour because he ―believed that by using those vibrant colours it helped to perhaps deepen the little passages, so learners could have a better recall of the work done". Furthermore, when they were given any ―example their minds would go back to those colour graphics". His reference to deepen little passages and recall, seemed to have strengthened his earlier arguments that the effective use of pictures and colour, acted as strong tools for visual reasoning. His further justification for the use of colour was found in his statement ―as I said there is maybe, a digital imprint that takes place in the brain when they watch slides of that nature. As I've said earlier it helps just to solidify, accentuate perhaps, and reinforce the concept being taught".

During his teaching Karl often used different strategies to highlight key concepts such as underlining, ringing and alternating between colours on the white board. He justified this action on the basis that ―by them observing me underline which at that moment would be deliberate I underline, highlight or ring in the hope that when the learners are working with their questions they would mimic what I've done". The intention behind this action was simply to ―reinforce the concept ―.

Karl also felt strongly about animating his mathematical presentations. He believed that

> ―the greater the extent to which we could animate our lessons, I think the greater the impact to the class. Sometimes I use that animation as a key point. In this case it was the light. Sometimes it could be the collapsing of a bridge

or whatever. It‘s deliberately done with the main purpose of allowing the learner to recall with purpose when I do present the scenario again at a later stage".

He used animations as a mean of enhancing the memory and recollection of the concept that he had taught.

Karl was of the opinion that dynamic software and projection techniques were necessary because of ―a trend of diminishing concentration span of our current generation of learner". He was of the view that

> ―we have to do a bit more to achieve the same results that we got 15 to 20 years ago. The content has not changed but the calibre of the learner has changed and hence we‘ll have to do a bit more than what we normally did in the past to achieve the same type of result".

It would seem that the emphasis on keeping the learners entertained in the mathematics classroom was great. Teachers had to make every effort to try to incorporate this notion into their teaching, was the view that Karl seemed to have held. This view had also been echoed by other participants.

A careful analysis, in terms of the theoretical frameworks that have been chosen for this study will reveal that Karl had made increasing use of Gardner‘s theory of Multiple Intelligences. It would seem that by varying his presentations, he had been given both the visual and non-visual learners the opportunity to understand the topic. The use of gestures also helped to emphasise the elements of transformational geometry and its application to functions. By allowing the learners to draw their own functions, he was creating an opportunity for the learners to engage with other aspects of their cognition. In terms of Gardner‘s theory, the use of several cognitive structures of the brain is preferred since learning does not necessarily take place through a single medium. Rather, Gardner emphasised the need for presenting information in several different ways. In this regard, it seems that Karl was enacting Gardner‘s theory.

According to attribution theory, the theory focusses primarily on the notion of success and failure as a motivational tool, hence making it an ideal choice for this study. It may also attribute cause for the behaviour of these successful teachers and explain why they act in the way they do, which could be internal or external.

Karl attributed his success to both internal and external factors. His reference to the many mentors that he had, seemed to suggest that these were external factors that accounted for his success, whilst his reference to his passion for the subject seemed to indicate an internal factor that he attributed for his success. He attributed his success to his digital literacy which could be seen as an internal factor. Even in his teaching Karl attributed his success to the use of visuals, which he claims he created. Because he was seen as a successful teacher, it acted as a motivational tool. He believed that it promoted research and necessitated introducing new strategies in teaching. Hence the element of motivation became clear. He attributed internal factors to the learners who over-achieve, by stating that there exist learners who will achieve, no matter what the situation is. He believed that such learners were intrinsically motivated, and hence excelled because of this. At the same time, he attributed the success of his low -achieving learners to his efforts as a teacher. He believed that it was his efforts that ensured their success.

The theory of situated cognition learning is also applicable in this case. In Lave‘s opinion, learning should not be seen ―as a process of socially shared cognition that results in the internaliation of knowledge by individuals, but as a process of becoming a member of a sustained community". She advocated that ―newcomers" through a ―community of practice" become ―master practitioners within a community of practice" (Lave, 1988, p. 71).
Karl‘s reference to the help that he derived from good mentors illustrates this theory clearly. In this case, Karl is seen as a newcomer according to Lave and Wenger, through a ―community of practice" becomes a ―master practitioner". He firmly believed in the need for knowledge to be situated and relevant in order for it to be meaningful. In this regard he spoke of the use of concrete objects and creating the real world within the confines of the classroom. His use of the MacDonalds logo and authentic pictures of bridges, are yet another example of learning as situated cognition, and not simply a cognitive process that took place in a vacuum. Not only was his own learning situated, but the learning of his learners was also dealt with in a similar manner.

Karl‘s reference to the need to vary his teaching methods, by introducing multimedia in his presentations, indicated that he subscribed to Gardner‘s Theory of Multiple Intelligences. He also placed a great deal of emphasis on the different kinds of learners, possibly referring to the different intelligences that learners have. He alluded to visually enhanced learners and non-visual learners. It would seem that he acknowledged that not all learners learnt in the same way. In so doing, he was accepting the tenets of Gardner‘s Theory, that learning could take place through any one or more of the intelligences that a person possessed. He argued that there was a constant need for teachers to be seen as entertainers, thus implying that teachers engage the different intelligences in their mathematics classroom.

Thus, it would seem that his success as a mathematics teacher could easily be seen through the lens, of each of the theoretical frameworks that have been identified for this study.

5.4.4 Sam as a successful mathematics teacher

Sam had set his sights on becoming a lawyer. Due to financial constraints, he was forced to abandon his dream and pursue a career in teaching because of a bursary that he received. He admitted that his passion for teaching developed during his years at university. During his first year at university, his studies were interrupted by a boycott which forced him seek employment as a locum tenens at a school in Velmo (name changed) for a period of six months before he could return to university the following year. After completing a Higher Education Diploma at the University of Karville (name changed), he majored in Mathematics at the University of The South (name changed), where he obtained a Bachelor of Arts Degree. Thereafter he taught at Velmo Secondary School (name changed) for seventeen years, before he accepted a position at Manor College, where he remained. Sam had been teaching for a total of twenty-seven years, at the time of this research study. Despite the length of his career, he was exposed to two types of schools, namely a public and a private school. The length of his career ensured that he had been through the different curriculum changes in the South African education system. He taught the syllabi that ended in 2005 (Outcomes Based Education), 2008 (National curriculum Statement) and 2013 (The South African schools Curriculum), amongst others. Whilst resources were not so readily available in the public school, Sam admitted that it had not been the case with his present school. The lack of resources in his former school may possibly account for Sam‘s inexperience with technological devices, and audio-visual equipment in the main. This was evident in his reluctance to use technology.

Sam taught Mathematics and Additional Mathematics in the senior grades only. Apart from teaching mathematics, he was actively involved at different committees at schools, such as the staff committee and The Higher Education Committee, which was a committee that represented the staff at Manor College at the Head Office level. He spent several hours after school providing extra lessons to learners needing assistance in mathematics.

Sam‘s classroom was well resourced, with approximately 25 desks and chairs, a teacher‘s desk an table, two steel cupboards, a standard white board, laptop, data projector, ceiling fans, and display boards that were bare.

Sam‘s understanding of the notion of visual reasoning could be gathered from his statement ―I think that‘s where your visual thing comes in. I believe in the operation of the mind. When I‘m teaching I must know how the kids mind operate". Sam was convinced that the primary focus of teaching was to ensure that the content knowledge and skills taught were eventually transferred to the learners‘ long term memory. He believed that the learner would then be able to access this knowledge whenever there was a need, hopefully for its use in problem solving. He identified the sense of hearing and sight to be the most important in the classroom situation. This was evident when he stated that ―you got to start with the sensory perception" and that ―the only two senses that one uses as a teacher is sight and hearing". Multiple intelligences Theory dictates that all intelligences have the potential to contribute to leaning. However, perhaps the sense of sight is fundamentally the most important in perceiving the world around us. He argued by combining this aspect of sight with repetition of skills and knowledge, understanding of concepts would be natural. In this way, he argued that having used a visual strategy, and repeating this visualization would lead to remembrance which would have indicated that learning had taken place.

There may be counter arguments about learning through sight, as is often argued in studies of blind people, since the visual stimulus is absent. However, Sam seemed to have focussed on the importance of the formation of a mental picture of the concept. This was the focus of his argument. This was evident in his statement that

> ―more important than hearing is sight. They can see it, so when they go into any situation they can close their eyes and visualise the teaching, such as using those mind maps or flow diagrams that we use in our teaching".

Although the concept of reasoning and cognition falls within the domain of psychology, it is possible to postulate that most people perceive most of their world through the sense of vision and that visual reasoning is therefore an integral component of learning. Knowledge gained in this way could easily be applied in the real life context, was Sam‘s philosophy of teaching. He argued that if he could “teach them the flow of the work in one picture, they would visualise that picture anywhere they are in in life”. Learning, according to Sam, ultimately had to be situated and not be seen as an abstract cognition that took place in the mind alone. Hence learning according to him was also a social process, since it had to be applied in the reality.

He furnished several reasons for his success as a mathematics teacher. First he identified passion as an important factor in determining the success of a mathematics teacher. Second, he believed that a successful teacher was able to maximise the academic potential of his learner. The successful teacher, according to Sam

> “will be able to extract the best out of the kids, not only in terms of exam results, but also in terms of future critical and in-depth thinking. I think the only way you can do that, is by being a teacher that they are looking forward to, a dynamic teacher and a passionate teacher”.

His belief about success was summarised in his statement that “half our battle was won”, by being passionate about the subject.

Sam, like the other participants believed in the need for mathematics to be relevant and to be seen as a cohesive entity. He was of the opinion that much of mathematics teaching was compartmentalised, hence the learners did not see the end product and the relevance of mathematics. He argued that the problem which many learners experienced in mathematics was a lack of connection with the topic with other topics and with reality. Furthermore he argued that these learners were not shown the relevance of their mathematics as he points out in the example:

> “the reason for solving quadratic equations is to be able to find the x-intercepts of the parabola and the reason for finding the x-intercepts of the parabola is that you can draw it. Why do we draw the parabola? Because our whole life is interpreted on graphs, whether you are studying economics, or pure mathematics or engineering, it revolves around a graph”.

Sam also attributed his success to the learners themselves. If learners performed well in mathematics, he viewed this as contributing to his success. As a teacher in the private school, he was subject to a mid-term and annual learner appraisal of his ability as a mathematics teacher. In this regards, the appraisal received from his learners indicated his outstanding ability as a teacher. He attributed his success to creating a happy clientele. Through their success, he was able to measure his success.

Sam also attributed other reasons for his success. This was evident in his statements “I love being with kids and I think that a subject like Maths, keeps me on the hop all the time. I also strongly believe that you learn all the time. In Mathematics you can never stagnate”. He loved working with children as well as being at the cutting edge of knowledge in his subject. He saw mathematics as a very dynamic subject that constantly called for the engagement of the teacher. In this regards, he attended many conferences and mathematics workshops to keep abreast of the changes in mathematics education. He read widely and would often discuss important mathematical trends during staff meetings and staff functions.
In his practice, it was evident that Sam was passionate about teaching. This resonated with his views on good and effective teaching and the desire to be highly successful as well.

The analysis that follows examines how Sam used visual reasoning in his practice. In a lesson on Probability in a grade ten class, the following observations were made. First, he started the lesson by drawing a diagram, illustrating the different places learners may find themselves in if they had won the lotto. He immediately gestured with his hands to indicate the total outcomes and continued to use gestures to indicate other aspects of the choices that the learners would have to make if they won the lotto. He thereafter proceeded to explain how the lotto was played. This was followed by several diagrams (in three-dimension) of a lotto ball set machine, as it appeared on the television, as is indicated in the figure 5.11

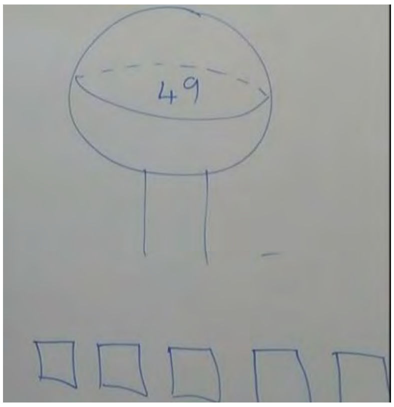

Fig 5.11: Three dimensional drawing of a lotto ball set

The entire game was represented with pictures. The learners were actively involved in answering questions about the game. They seemed very enthusiastic and actively involved in the lesson as it unfolded. Sam continued to use gestures and drew several more diagrams, such as coins (in three-dimension), in figure 5.12

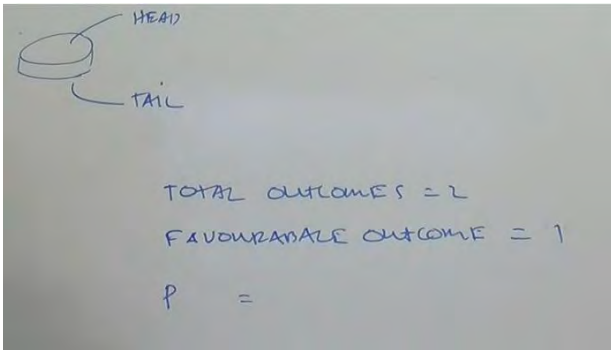

Figure 5.12: Three dimensional drawing of a coin

He also used arrows and drew a two way table to deal with a more complex problem on probability. The learners were also given worksheets from which examples were done and corrected before they were finally asked to complete a practice exercise in class.

Sam explained that everything in his practice was designed with the purpose of ensuring learning took place, through repetition, via the senses of hearing and seeing as he indicated ―I think it‘s instilled in me like you know you remember the first discussion I told you that everything I teach, I think of the three memories". He did not view gesturing as being a separate aspect of his teaching. To Sam, it was all contributing to his ultimate goal, which was to make the entire learning experience a visual one. He justified his actions by stating that ―the diagrams and the gestures come in because of the, the sight. The two senses you use in the classroom are sight and hearing". He admitted that, although his actions sometimes seemed to have been sub-conscious, he often gave it conscious thought as he indicated in his statements, ―I think basically I want them to create that sensory memory and I consciously do it, but maybe it appears sub-conscious". He also conceded that gestures, on their own, had the special advantage of ―keeping their attention. If you are a teacher that just goes through the motions with no body language, no voice modulation, you become boring. Then you lose your kids. I think that gestures are important". Like the other participants, he also justified the use of gestures primarily for ensuring that learners were attentive during lessons. He too, like the other participants, shared the opinion that teaching had to be made exciting and that teachers had to perform roles as entertainers, in their practice. In this regards body language and gestures created this kind of excitement for the learner.

The use of arrows and circles around parts of his work, on the white board were used for emphasis. He explained that the intention here was to ―direct them to some particular thing". Furthermore ―you write, highlight, you circle, you put an arrow here, you, do these things because it drives them to that point". He reasoned that it was a way of drawing his learners‘ attention to important mathematical concepts.

He also explained that he used pictures because it resembled the learners‘ real life experience. This was evident when he argued that it was a means of ―trying to create reality but you creating reality, from the head". Sam was particularly serious about the need for learners to have formed mental pictures of everything in their lives. In this way, he was advocating visual reasoning as going beyond just seeing an object. More specifically, that object had to be internalised. Without unnecessarily trying to speculate about matters that are relating to psychology, it is possible to postulate that Sam‘s notion of visual reasoning was rooted in a deeper understanding of human cognition. Although he did not engage specifically with technology, he made use of diagrams, pictures, highlighting techniques and body language to help his learners to create mental pictures of every concept that he taught. In terms of the definition of visual reasoning, he was also engaging in visual reasoning, similar to the other participants.

Although he claimed not to have used the data projector in the teaching of concepts, ―he does use it to project examination papers and their memorandums". He did not particularly prefer the use of dynamic software and video clips because, to him ―a video clip will simply be somebody else doing what I‘m doing". Sam was convinced that technology that repeated the actions that he was capable of performing on the white board, were superfluous. In this regard, he did not object to multiple representations, rather he was of the opinion that it could be done without the use of technology. Technology, to him was not a different visual tool. It merely presented the same visual material differently. Whilst an alternate argument can be raised, it is not necessary. This opinion will not alter the findings of this study. He did contend that teaching strategies had to constantly change in order to keep the learners enthused. His reference to ―when you are teaching you can‘t be boring, you got to basically keep the attention, it‘s becoming more difficult nowadays you know that", illustrated the point that there was a need for constant engagement by the teacher to ensure that she/he delivered exciting lessons to cater for the dynamic nature of the learner.

Sam‘s teaching can also be viewed through the lens of the theoretical frameworks that have been used in this study. Using constructivism as its basis, Attribution Theory, the Theory of Multiple Intelligences and Situated Cognition Theory can be used to justify Sam‘s classroom practice. Sam‘s reference to the need for repetition, before learning eventually took place, seemed to suggest that knowledge had to be constructed. This process was only possible through using mental pictures that represented the real world. His reference to problem solving, emphasized the need for authentic learning. This resonates with the theory of Situated Cognition which states that in order for leaning to be meaningful, it must be socially constructed. In this context, Sam was of the belief that by repeating learning that took place through the visual and auditory processes, through engagement with the real world, in problem solving contexts, real knowledge was created. The repetition could also be seen as the community of practice that the learners would engage in before they became adept at the subject matter. His own development as a successful teacher subscribed to this theory. He started as a novice teacher and trough his studies and engagement with other mathematics teacher and experts in the field, he evolved into a successful teacher as well. His engagements with his learners, his colleagues and the different workshops and conferences have acted as the community of practice that Lave and Wenger refer to.

He attributed his success to both internal and external factors. When he declared that his success was due to his passion for teaching, he was making an internal attribution. Such an attribution style suggests that people feel good about themselves and their capacity for success. It is referred to as an optimistic attribution style. According to research on this theory, males attributed success to ability more than females. Whilst it is true that internal factors can signal a sense of arrogance, according to the literature, it is also true that Asian people often attribute their success to society more than to themselves, according to the study. Seeing that Sam was an Asian teacher, it was very likely that he also attributed his success to external factors such as the community or society. This was evident in the discussions held with him. In much of the interviews he spoke of ―we” as opposed to ―I‘. Furthermore, he also attributed his success to his learners and his clientele more than he did himself. This was evident when he stated that it was important to maintain a ―happy clientele”.

Sam's reference to the use of diagrams, gestures and different highlighting techniques, as well as, his use of the data projector, indicated his subscription to the theory of Multiple Intelligences. His reference to the dynamic learner that had to be entertained was indicative of his awareness of the theory of Multiple Intelligences that learning could take place through the different intelligences. His teaching was designed with the understanding that learners could learn through these different intelligences. Therefore he used visual, auditory, body-kinaesethetic movements, verbal, and logical strategies in his teaching practice.

Sam's success as a mathematics teacher can therefore be explained through the theoretical frameworks that have been used in this study.

5.4.5 Nellie as a successful mathematics teacher

Nellie obtained her Bachelor of Science Degree (Hon), with majors in Science. Although her focus was primarily in Science, she nevertheless pursued some courses in Mathematics. She did not intend pursuing a career in teaching. However, by engaging in practical work in Science, it created the interest in teaching. Although she had been teaching for approximately six years, she had only recently started teaching mathematics. She estimated the number of years to be approximately two to three years, at the time of this study. Her first teaching - experience, was at primary school. She attributed her interest in mathematics to her experiences in this particular school. Her words ―It was at a primary school, so I actually felt that it was a good grounding for me", seems to convey her sentiments about her first few years of teaching mathematics. The mathematics classes were streamed in that particular school. The learners were therefore graded according to their ability in his subject. Nellie was given the weakest set of learners to teach. The term weak may be understood in several ways. However, the context in which it is used here is to suggest that these learners performed poorly in mathematics. As such, most of them had actually failed mathematics, implying that they obtained marks of less than forty percent in most of their assessments. Nellie was able to relate to the level at which they found themselves in terms of mathematics, since she had experienced difficulties with the subject, as a high school learner. Eventually, she was able to raise the performance of her learners to that of a pass.

Having relocated, she then sought employment at other schools. She was shortlisted and eventually appointed at Manor College (name changed), where she taught Mathematics and Science, although most of her teaching load was made up of Science. Her other duties included administering sport and participating in other committees at school. She also conducted regular extra lessons after school for learners who experienced difficulty in Mathematics.

In terms of the nature of her subjects, she used two classrooms. When she was teaching Science, she had access to a fully fitted Science Laboratory. Mathematics was taught in her classroom which was well resourced as well. She had a standard size white board, approximately 25 desks and chairs, a teacher‘s table and a chair, adequate lighting, ceiling fans, a data projector, a laptop, bookshelves, cupboards, several resource textbooks and resource materials. Each class was also equipped with internet facilities. Her classroom was adequately ventilated. The walls of her classroom displayed learners‘ work and other important notices for learners, as well as reminders of important due dates of assessments and other related issues.

Nellie‘s teaching load was made up primarily of grade eight and nine learners. She enjoyed teaching these classes and learners were pleased with her dedication. His was evident in the appraisals that learners had to perform of their teachers, on a regular basis, at this school. Her learners‘ appraisals always indicated that her learners valued her as a teacher. The appraisals had a section that allowed the learners to write general comments about their teachers. Nellie‘s learners commented positively on her ability as a teacher.

Nellie was of the opinion that a significant number of learners were visual learners. This change in dynamics meant that teachers had to adapt their style of teaching in order to accommodate these learners. In this regard, she referred to the need for a variety of methods in the teaching of mathematics. She did concede however, that she would be keen on using more visual strategies in her classroom practice. She admitted to using the white board considerably. However, her teaching style also focused on the use of three dimensional models as well. Her reference to the use of Power Point presentations, three dimensional models and gestures in her teaching, implied, that she was fully aware of the notion of visual reasoning. This was evident when she stated that ―you can use more colours that way and grab the learners‘ attention". She did acknowledge the significance of using pictures, colours, gestures and other multi-media presentations in order to enhance the understanding of learners.

Nellie felt that her passion for teaching mathematics explained much of her success as a mathematics teacher. This was evident in her words ―I love teaching them, I actually was a bit nervous to teach the Grade 8‘s and 9‘s mathematics, but I‘m surprised how much I actually enjoy it”. Her passion for her learners and the subject was clearly evident. She wanted to be able to make a difference to the lives of her learners, as was seen in discussions with her and her learners‘ appraisals. She was keen on varying her presentations styles so that it would contribute to her success as a teacher, since she believed that

> ―you get those extremely exceptional learners that can just pick up anything just by listening to. I find that with the majority of learners nowadays, you actually have to use visual examples and show them on the board. It takes a bit of time and then they eventually get it”.

She firmly believed that her success as a teacher was measured by the success of her learners, which she justified as follows:

> ―I do think teachers are expected to get results nowadays, so, achieving those results is definitely important. Encouraging your learners to pass mathematics is also very important. I generally think that my learners must set themselves goals, and if I can get them to those goals that they want, then I would consider myself successful”.

She was also of the opinion that if she could change the perception of learners towards mathematics, it contributed to her success as a teacher. In this regard she asserted that ―if I can see a learner suddenly starting to show an interest in mathematics, where there was none before, I would also consider that to be very successful”.

The analysis that follows, attempts to show how Nellie used visual reasoning in her practice. In a lesson on data handling, Nellie was focusing on the significance of the histogram in calculating the estimated mean. This was a follow up lesson, from one in which, she had already discussed the basics of statistics. The histogram was projected onto the white board via the data projector. The horizontal axis depicted the South African Rand saved on petrol whilst the vertical axis depicted the frequency. The histogram was blue, against a white background and all labels were in black. Nellie proceeded to calculate the estimated mean for the learners. She then projected a new slide with grouped data, showing frequency, class midpoints and the estimated total, which was derived by multiplying the number in the class midpoint column with the corresponding number in the frequency column. The calculation for the estimated mean was done on the bottom the slide. She then explained the calculation.

For the class midpoint column, she gestured the idea of the midpoint, with her hands, as seen in figure 5.13

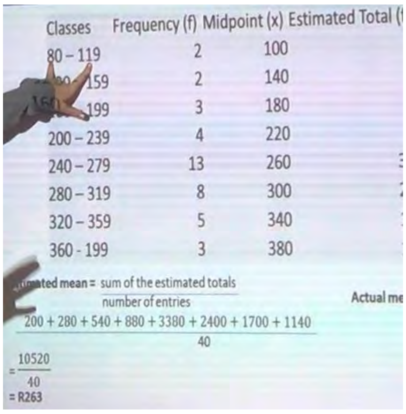

Figure 5.13: Nellie gesturing to finding the class midpoint

She placed her fingers on 80 and 119 and showed how she arrived at 100 as the midpoint of that interval. She repeated the procedure for each number until that column was competed. She then proceeded to show how the next column was created by pointing to the numbers in that column and multiplying it by the numbers in the frequency column, as in figure 5.14.

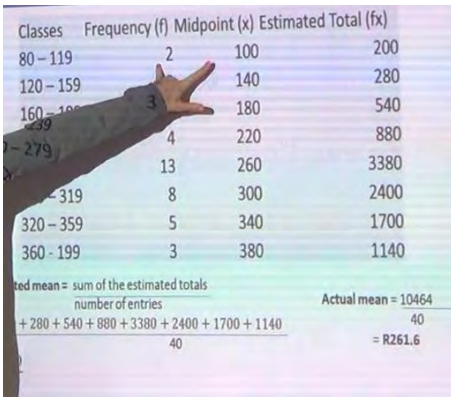

Figure 5.14: Nellie gestures to the calculation of the last column using the previous columns

In this way by gesturing, she explained the calculation of the estimated mean.

Nellie then went on to discuss polygons as a new section. She introduced polygons my means of diagrams. She also gestured a great deal to try to compel learners to understand that a polygon had to be a closed figure. Thereafter the learners were shown a video clip on polygons, which she downloaded. The video clip defined polygons and named the different types of polygons. Different slides were presented to accommodate the different types of polygons. Learners had to answer a short exercise on the different types of polygon. The answers were revealed at the bottom of each slide after the learners answered. Thereafter she referred to worksheets and continued to explain the concepts of interior and exterior angles. Again, she used hand gestures to point to the meaning of these terms in diagrams that were drawn. In this way, Nellie was able to investigate the sum of the exterior angles of a triangle. She then proceeded to investigate the same concept with other polygons. To convince the learners further of the validity of this fact, she showed the learners a video clip, called Maths Doctor, which she downloaded. The clip investigated the link between the number of sides of a polygon, the number of triangles that could be formed and the sum of the interior angles. The video showed a teacher splitting the polygons into triangles and finding the angle sum of the different polygons. The teacher used different colours to highlight the angles, drew construction lines joined vertices, and investigated the property up to a seven sided polygon. After learners viewed the clip, Nellie drew a table that summarized the discovery. The table comprised the following columns: number of sides, name of polygon, sum of interior angles, sum of exterior angles. An extra column was drawn to inert the shape so that the learners could divide the shape into triangles. Nellie insisted that learners drew diagrams to assist them in the counting process. Judging from the responses of the learners, during the observation, it was evident that the majority of the learners were able to able to arrive at the necessary conclusions about the sum of the interior and exterior angles of any polygon after completing the table.

An analysis of Nellie's actions revealed that she used a number of visual strategies in her practice. These strategies revealed the use of visual reasoning in her teaching. From the outset, Nellie used a great deal of gestures to encourage the learners to focus on key aspects of her lesson. Her justification for the use of gestures was that it allowed her learners the opportunity to understand the concepts by trying to

> ―grab attention a bit more than if you just stand up. It might be a bit boring just to stand up hands by your side so I think if you wave your hands around you might just grab the, the learners attention" .

Her constant use of diagrams helped the learners to reason visually as well. Diagrams, she felt had the potential to capture arguments with the least amount of effort. They made explanations easier for the teacher and easier for the learners to understand. She explained the use of diagrams as follows:

> ―I find that a lot of learners respond a lot better to visual aids, so diagrams are going to just tie the information together. I think I was one of those learners that wanted to see something on the board. I couldn‘t just listen and take in information. I had to be able to see it and hear the explanation explained on a diagram which is why I think I stick to that now".

It would seem that, her own experiences of Mathematics had an influence on the way she taught this subject. Furthermore, she explained that the visual and auditory process had to be seen as complementary processes.

The use of video clips and the data projector allowed the learners to see a range of presentations that would help them to understand the concepts better. The explanation given by Nellie for varying her presentation techniques, were focused on the need to create dynamic lessons so that learners would not easily be bored. She elaborated that learners often got bored with listening to the same teacher, and that introducing other voices helped her to maintain the attention of her learners. She believed this was necessary to ―keep the kids entertained and so you have to change the strategy sometimes". Furthermore, she believed that

> ―it‘s not the same things that we grew up with. They have technology from when they are 2, 3 years old they already working on Ipads and visual games that are bright colourful grabbing their attention that‘s just how they‘ve grown up. So I think we have to do our bit to adapt to that".

She felt that teachers had to be seen as entertainers more than just teachers of a curriculum and because of the influence of technology on their lives. It would seem fair to assume that Nellie saw visual reasoning as an important component of teaching and an integral component of her teaching style, because

> ―you are just hoping that the learners are going to retain the information a bit better than, than what they would have by just working out of their textbooks".

The theoretical frameworks of attribution theory, situated cognition theory and multiple intelligences theory seemed to be able to justify Nellies practice as a successful mathematics teacher. Nellie attributed her success to her passion and catering for the needs of her learners. Thus attribution theory provides an ideal framework from which her motivation can be analyzed. According to this theory, people attribute their successes to internal factors and failure to external factors. However if an attribution is made to qualities such as effort and inherent qualities such as passion for a subject, it may be regarded as stable, but internal. Nellie also attributed her success to her learners. This is regarded as an external attribution. In this way, Nellie is also attributing her successes to other factors. Her reference to the use of a variety of teaching strategies can easily be explained through the framework of Gardner‘s Multiple Intelligences Theory, which proposed the use of several intelligences in trying to achieve success in the classroom. According to Gardner, all learners do not learn in a specific way and hence the different intelligences of learners must be used when designing and presenting lessons. Indeed this has been the case with Nellie, who applied a variety of different approaches in her classroom practice. Thus she tried to appeal to the range of learners who used different intelligences to learn. The emphasis was not only on the verbal lingual learner, but also on the visual/spatial and the body kinesthetic type of learner. Nellie emphasized the need for learning to be made authentic, through the use of real, tangible mathematics, when she spoke of using mathematical objects in her practice. Her views and subsequent actions in this regards can find its explanation in the theory of situated cognition. According to this theory, learning had to extend beyond human cognition only. Learning had to be seen as being part and parcel of a social process as well. Learning that was acquired was only meaningful when it was socially shared and used in everyday real contexts. Hence the use of the real world examples and videos contributed to making her lessons more authentic. Nellie‘s development as a teacher was also exemplified by this theory. Her constant negotiations with fellow colleagues, who were regarded as old-timers, were indicative of the notion of the community of practice that this theory refers to. We observe how Nellie, as the newcomer, according to Lave and Wenger, was able to acquire the necessary skills and soon become the old-timer, in her practice. This was evident when she indicated her fear to teach grade eight and nine learners initially. However, it soon became clear that her learners perceived her as a highly successful teacher. Her learner appraisals indicated this quality in Nellie. It would seem that her practice can be explained ideally by the theoretical frameworks that have been selected for the purpose of this study.

Chapter 6

Analysis-Part Two: Emerging Themes

6.1 Introduction

Stake (1995) advocates four forms of data analysis and interpretation in case studies. In the first, aggregation, the researcher seeks collections of instances or events, hoping that relevant themes emerge. In direct interpretation, the researcher only draws from a single instance and attempts to create some meaning or understanding. In pattern generalisation, the researcher tries to develop patterns using the different categories. In the final stage, which he refers to as naturalistic generalizations, the researcher develops generalizations from the case under study and extrapolates them to the population of cases.

In this study, whilst every effort was made to conserve the naturalistic aspect, under the circumstances it was not possible for several reasons. As soon as the participants were aware that they were being recorded during data collection, either in the interviews, or in their classroom pratice, the element of a naturalistic enquiry diminished.

Whilst the previous chapter focused on direct interpretation, by drawing from individual cases, this chapter focuses on aggregation, which seeks the collection of these instances, hoping that relevant themes will emerge. The chapter also focuses on developing patterns and generalizations aimed at extrapolating them to the population of cases. The analysis that follows is based on the interactions of the researcher with the participants through pre-observation semi-structured interviews, classroom observations and post-observation semi-structured interviews. The purpose of the first round of interviews was to ascertain the understanding of visual reasoning as it was understood by these successful teachers. The purpose of the classroom observation was to examine the extent to which these successful teachers used visual reasoning in their actual practise. These observations were conducted with a view to trying to establish whether their practice coincided with their notion of visual reasoning. The post-observation interviews were conducted with a view to understanding, in greater depth the use of visual reasoning in their practise. It was envisaged that through these data collection strategies, the critical questions of this research will be answered. As a result some themes emerged without much searching. These themes were justified on the basis of the theoretical frameworks that were used to underpin this study. The divergent responses pointed to the unique individual differences that existed between the participants.

6.2 Other attributes of successful teachers

Previous discussions have tried to link the biographical attributes of each of the participants to the notion of successful teachers. The discussion also tried to show how the participants fulfilled the first criteria as stipulated by Sanders (2002). Whilst qualities such as personal and professional qualities, are important attributes for successful teachers, they are also deemed necessary requirements for all teachers. However, teaching skills and value addedness, which refers to extra skills and knowledge beyond that which might be expected of a mathematics teacher, are regarded as important attributes for a successful teacher according to Sanders (2002). Archer (2000, p. 6) and Morris and Easterday (2008, p. 49), cite ―enthusiasm” as a key element that is necessary for successful teaching. They also add that, enthusiasm when accompanied by a need to make teaching more meaningful, relevant and adaptive are some of the qualities of successful mathematics teachers. It would seem that the views expressed by these researchers have a common point of departure. It seems that exceptional teaching is a pre-requisite for successful teachers.

When the participants were probed about their understanding of success as mathematics teachers, they all echoed similar sentiments namely: passion, love for the subject, love for children, the ability to change the negative perceptions of mathematics, and the ability to inspire learners to achieve their best in the subject. These qualities point to a group of enthusiastic teachers as defined above. They were not only interested in delivering the mathematics curriculum, but each was determined to inculcate a deep understanding of the mathematical concepts, and a love for the subject. Sam, for example was determined to create inquisitive mathematics learners. This was evident when he stated that ―we can still extract the best out of that kid not only in terms of exam results, but in term of future thinking, you know critical thinking”. His teaching focussed on developing critical thinkers that would use their knowledge, in the tertiary phase or in the working environment. Valerie also subscribed to a similar view when she stated that ―we could be teaching a lot more thinkers than we do now”. Valerie, Betty and Nellie focussed on changing the negative perception that their learners had of mathematics. ―I‘ve had them dancing like I know what I‘ doing”, was how Valerie felt about changing the negative perception of her learners about mathematics. Betty, after 33 years of teaching mathematics declared that ―I‘m not bored with grade 8, I love grade 8 maths”. This indicated her sincere dedication, commitment and passion for her learners and for her job. To Nellie ―90% of the hard work was to try to encourage them to have a positive attitude towards maths". Karl was passionate about –helping learners to move closer to the career goal". He believed that –not every learner will be an achiever, but if we can propel them to a level where they would semi-achieve or even pass semi-achievement, then I think we would have done well".

All the participants expressed their willingness to accomplish these goals in their teaching. Moreover, each one of them, wanted to inspire their learners to give of their best in the subject.

Success as defined by the researchers mentioned above called for dynamic, exciting and stimulating presentations or any method that would enhance learners learning of a particular concept. Each of the participants expressed strong views about the need for creating dynamic lessons. Karl, for example, was emphatic that it was –not about proceeding from A to B, but how you get there". He further declared that if –we could get the learners from A to B in a very exciting manner, then why not"? Karl was of the view that presentations that were exciting would create more enthusiasm and enhance learning. He further argued that –technology serves a very, very important purpose in education. It really makes life easier. Ordinarily learners would suffer with colour not being at their disposal. The excellent quality of images is something that our learners gravitate towards". He also saw the –element of fun", that dynamic lessons had the potential to create. It was similar to choosing a movie in black and white, or in colour. He explained that the colour just added to the enjoyment of the movie. In much the same way, dynamic lessons made conceptual understanding much more exciting. He summed up his views of dynamic teaching when he state that –the content has not changed but the calibre of the learner has changed and hence we'll have to do a bit more than what we normally did in the past to achieve the same type of result". Even Betty subscribed to the same view as Karl when she declared that –they've got TV, they've got movies and they exposed to it all the time". Like Karl she also believed that –we can be seen as a boring subject, which is why I will clown around and make jokes, you know I want them to be happy here". She believed that these measures were necessary –to give them a positive feel for the subject, I suppose better results at the end of the day and better understanding".

Valerie introduced new ideas in her practice although she didn't –know if it would work, but was willing to try". It would seem that all the participants were determined to offer their learners dynamic and exciting presentations. In order to accomplish this, they felt the need to vary their presentation techniques and use multiple forms of representation in their practise.

This was evident in their practice. Although they all stressed the need for multiple representations, most of this took the form of a variety of visual strategies, hence illustrating the potential of visual reasoning in creating dynamic presentations. Clearly, these teachers demonstrated the extra skills and knowledge that was required in order for them to be classified as successful teachers. Sanders (2002) referred to this attribute as value addedness, an attribute that distinguishes a successful teacher form the general body of teachers.

Attribution theory is founded on the assumption that people want to understand and explain the events or outcomes in their lives, including what happens within academic settings (Weiner, 1983).

According to attribution theory people who are considered to be successful, attribute this quality as an inherent trait and seek recognition from within, as opposed to failure which they attribute to some external factor or factors. In this study, all the participants accepted that their success could be internally attributed. They attributed their success to effort and passion for the subject. Karl and Betty, Valerie and Nellie attributed part of their success to others factors such as good mentors or good teachers that they experienced as learners, whilst Sam also attributed his success to his learners. The reference to good mentors and supportive teachers could be further explained using attribution theory. ―Success due to help from others leads to the appraisal that the person is not responsible for the outcome" (Hareli, 2000, p. 217). Furthermore, according to Hareli (2000, pp. 216-217),

> ―accounts for success are seen as more arrogant if they describe an internal cause rather than an external one. Arrogance and arrogant communications emphasize that one‗s qualities are worth better or more superior to those of others. Modesty, in contrast, communicates that one has qualities or values that are not different from those of others".

In light of the above, and having established the nature of each of the participants, through a long term engagement would them, it would be fair to declare that their attributions were done out of modesty.

An analysis of the data also revealed certain themes with regards to the ways in which these teachers defined visual reasoning, how they used it and the reasons for their use.

6.3 Understanding visual reasoning and how it was used

In attempting to answer the first critical question, the participants were asked to present their views on the concept of visual reasoning and how they used it. The responses seem to suggest the emergence of different themes, which may invariably be linked to the underlying theoretical frameworks as well as the relevant literature. The first theme that emerged was on the nature of visual reasoning.

6.3.1The nature of visual reasoning

It would seem that most of the participants were referring to the use of diagrams, pictures, manipulatives, gesturing and digital media techniques when describing visual reasoning. Valerie made the following comments: “using something that is concrete and practical that is visual and arriving at a generalisation from this”, whilst Karl made the following comment: “the use of pictures or graphics to convey a concept to learners”. Betty “guesses that it is something that learners can see to follow reasoning, such as pictures”. Nellie subscribed to a similar definition when she said that it is “anything that a teacher does to present the material visually, anything on the board, pictures and diagrams”. Sam had a different definition of the concept when he declared that, “it‘s simply the use of sight”. This was very evident in both his interviews and in his practise. Although he may not have used digital media in his teaching, he emphasized the use of diagrams and pictures, which he preferred to, draw on his own. It may be argued that Sam was not using visual reasoning if he claims that visual reasoning is only about sight. We would then be assuming that visual reasoning is absent in blind learners. From the discussion, it seems as though, visual reasoning was understood to be the use of visuals means to convey the meaning of a concept. It included the use of diagrams, pictures, manipulative models and the use of digital technology in the form of videos, slide shows, Power-Point presentations and any other visual means. This view about the nature of visual reasoning seems to be corroborated by the literature. Alshwaikh (2009, p. 1) claims that

> “mathematics is a multimodal discourse where different modes of representation and company communication are used such as verbal language, algebraic notations, visual forms and gestures”.

Surprisingly though, the literature is quite extensive on the use of gestures as a tool that is prominent in visual reasoning. However, none of the participants seemed to have mentioned this in their preliminary interview on their definition of visual reasoning. However, it became fairly obvious to the researcher that all the participants saw this as an unintentional act, although they seemed to have used it extensively in their classroom practice. It was only after discussions with them that they became aware of its direct importance in their practice.

6.3.2 The need for multiple representations

Another prominent theme that seemed to have emerged from the data was the notion of the need for multiple representation in the mathematics classroom in order to achieve success. According to Gouws and Dicker (2011, p. 569), Gardner‘s theory of ―Multiple Intelligences has shown that all learners can learn successfully when they have opportunities to process information in their own way. Gardner‘s theory therefore provides a useful foundation for understanding individual differences”.

Blomberg (2009) believes that Gardner wants teachers to use his theory in catering for the individual differences of their learners. A similar view seems to be portrayed in the literature. Ainsworth and VanLabeke (2004, p. 243), in their study of learning with multiple representations and the use of dynamic representations found that ―dynamic representations are almost never presented in isolation but are combined with textual description of the situations, pictures or other forms of representation”.

Ainsworth and VanLabeke (2004, pp. 249-251) ascribe the following reasons for the use of multiple representations:

> ―First, multiple representations provide complementary information when a single representation would be insufficient to carry all the information about the domain. Second, the use of multiple representations is to develop a better understanding of a domain by using one representation to constrain their interpretation of a second representation. Third, multiple representations can support the construction of deeper understanding when learners abstract over representations to identify what are shared invariant features of a domain and what are properties of individual representations".

The theory allows teachers to accommodate nine different intelligences in a variety of approaches to mathematics teaching and learning. The multiple approaches that Gardner mentions in his theory, was evident in the approach of the participants. They too, shared the view that the learner in this day and age had to be kept entertained through different means. They made a concerted effort to vary their lessons through the use of visual reasoning. In the discussion that follows it becomes clear that even whilst using visual reasoning, they focussed on strategies to make the visual stimuli attractive. Such views have been echoed by Valerie, Karl, Betty and Nellie. These claims have been made by all the participants. They seemed to think that changing teaching strategies was a key ingredient to successful teaching. Nellie, for example, referred to herself as a ―learner that required to see something on the board, and couldn‘t just listen and take in information". She was alluding to the difference between visual and auditory learners. This implied that teachers had to recognize these differences and cater for the divergent types of learners. In this case, the one form of representation could complement the other form of representation. She also added that it was important to teach with the objects where possible, but in reality this was not the case. Hence, according to Nellie, diagrams could be used as one form of representation to constrain another form of representation. Valerie also believed that multiple representations gave the weaker learners the opportunity to grasp a concept, which may not have been possible using a single representation. It would seem that she was also advocating the need for variation of teaching strategies, using visual and non-visual domains. Her rationalisation for using the body kinaesthetic intelligence was that gestures helped the learners to remember the important aspects of a concept by associating the concept with a ―move." Karl too, was of the view that the use of a variety of teaching strategies, which ultimately translated to introducing the visual component, helped the learners to remember the concepts that were taught. His reference to ―discovery and a hands on approach with Geometry", which he favoured over a Power Point presentation also illustrated the view that some forms of representations offered a greater opportunity for learning by constructing a deeper understanding of the concept. Betty‘s statement that ―I think most of them enjoy it as a break to just very serious hard core maths", indicated that she saw visual strategies as an alternate form of representation as compared to the singular form of representation in mathematics.

Gouws and Dicker (2011, p.568) also allude to the very diverse nature of the South African classroom situation, thus making it imperative that no singular mode of teaching may achieve the best outcome. This is evident in his statement

> “The South African classroom currently presents a number of new challenges to people working in the field of education. Learners in the same classroom may come from diverse backgrounds, may possess various levels of background knowledge and may have various learning styles”.

Against this background in the South African school context, Gardner‘s Multiple Intelligence Theory creates opportunities for teachers and encourages them to think of ways of teaching mathematics that address learners‘ multiple intelligences.

6.3.3 Visual reasoning as a complementary teaching process

In discussing the use of visual reasoning in their classroom practice, the participants concurred on some aspects while differed on others. A common response was that visual reasoning was necessary. However, visual reasoning was seen as an important complementary process. Valerie, like Sam used visual reasoning to illustrate the significance of mathematical concepts to end product. Her statement “for me visual reasoning is when I‘m teaching something, I can show them where it‘s going”. Betty, Karl and Nellie saw visual reasoning as offering the divergent learner different opportunities to understand a concept. Sam, for example saw visual reasoning as more than a complementary process. To him it was crucial. He regarded visual reasoning as a process of forming pictures in the mind, as a cognitive process. This process went beyond just using alternate ways of presenting mathematics. He conceded that visual strategies were important in achieving this, but ultimately, the mathematical concept had to be visualised in the mind. To him, all of mathematics had to be related in the form of flow diagram that led to a particular end. Hence the learners had to see the concept as part of a bigger picture.

This view resonates with the literature that states that visual reasoning should be seen as a complementary process. In a study conducted by Cox (2009, p. 5), it was found that “visually-based strategies supported learners as they reflected on and sought to improve wholly numeric strategies”.

The notion of visual reasoning being seen as a complementary process also resonates with the theoretical framework of multiple intelligences as propounded by Gardner. According to Aborn (2006, p. 84)

> ―through an awareness of their intellectual strengths and challenges, and their specific profiles of intelligence, learners can begin to learn strategies to use their intellectual strengths in support of their learning, while compensating for those areas that are more challenging”.

In this way the process of visual reasoning may be seen as complementary process in teaching and learning.

6.3.4 The associated pitfalls of visual reasoning

Although all participants seemed to have favoured the use of visual reasoning as an integral part of their pedagogy, some pitfalls became evident, which featured as another theme. One of the main criticisms with its use has been the implication on time. It was often regarded as time consuming. Valerie, for example stated that ―there are a lot of kids that want to think and then we can just draw and we can fold paper and we can just time out. Time constraints also play a part”. In her discussion of the use of visual reasoning, she emphasized the need for working with practical things, which she felt constituted visual reasoning to a large extent. However, she believed that the lack of time prevented her from pursuing this kind of pedagogy. Karl sometimes saw the use of visual strategies and visual reasoning as time-consuming because ―finding the appropriate models” were ―hard to find”. He also felt that using visual strategies impeded the lessons if the material was not ―selected and edited before it was presented”. This implied that much time had to be dedicated to ensure the successful use of visual strategies and visual reasons. He indicated that it had the potential to slow down the pace of the lessons if the learners were spatially developed. In an experiment that he conducted on a group of learners, he found that those who grasped the concept quickly were bored. He felt that ―the lesson was too slow”. Betty also alluded to the lack of time as an important factor in trying to integrate visual strategies and visual reasoning into her teaching. Her comment ―I just find the Math‘s syllabus increasingly crowded and less room to enjoy”. She also believed that it was time consuming. Invariably the focus was on pushing ahead with the syllabus. Because of the demands that visual strategies made on the teacher in terms of time, there was a tendency not to use it too readily. Nellie was direct about the extra time that was necessary when visual strategies were used in her pedagogy. Her statement ―you actually have to use visual examples and show them on the board and, it takes a bit of time”, seemed to capture her opinion succinctly.

The relevant literature seems to add credibility to these comments. Najjar and Balachandar (1998) believe that issues such as costs and the time taken to create visual lessons have a bearing on the use of visual reasoning. Again the issue of time featured as a key contributing element in the reluctance to use visual strategies and visual reasoning more readily. Although all the participants did not consider the cost and difficulty in developing lessons that were biased toward the use of visual reasoning, Valerie alluded to this aspect when she stated that ―there‘s not enough there, we aren‘t exposed enough and I think there isn‘t enough scope to learn". Perhaps she was referring to the costly nature of producing such materials and the time and effort that such materials required in terms of their preparation.

These criticisms also resonate with the theoretical framework of attribution theory. According to attribution theory, whilst success may be attributed internally in most cases; failure is often attributed to some external factors. In this way the person finds a reason for his inability to act. Such was the case here as well. Although all the participants used visual strategies and visual reasoning extensively, they seemed to have attributed time constraints and exorbitant costs to not being able to use it as much as they would have liked to, thus making this an external attribution.

6.4 The praxis of successful teachers using visual reasoning

The preliminary interviews conducted with the participants seemed to have concurred with the observed lessons. The participants‘ beliefs about visual reasoning became evident during the observations. Such was clearly the case with all the participants. Valerie demonstrated her ability to use visual reasoning in her classroom. Her lessons included graphs, tables, pictures, gestures and the use of computer application software. Karl too, used digital media extensively, was highly graphic in his presentation, used colour extensively and varied between still and dynamic software. His focus on practical work and a hands-on approach to leaning significantly coincided with his views on visual reasoning and the passion that he had for digital media and its use. The correlation between his qualification and his practise was evident in these observations. Sam‘s use of pictures, diagrams and tables also showed the correlation between his views on visual reasoning and his practice. He used diagrams extensively and used gestures significantly in his teaching. Although he steered away from the digital media, he nevertheless was able to conduct dynamic lessons. Betty also used pictures, manipulatives, tables, graphs, video clips, still projection techniques and gestures to deliver dynamic lessons. Being a novice teacher, Nellie was adept at using technology, diagrams, pictures, tables and gestures effectively in her lessons.

Thus, it would seem that all participants were using visual strategies and visual reasoning in their practise.

6.5 Establishing reasons for using visual reasoning

Having examined the extent to which visual reasoning was used by the participants, the focus was then on establishing reasons for the use of these strategies. The classroom observations revealed a close correlation between the participants' views of visual reasoning and the ways in which they actually used it in their practise. In this way, the first critical question has been addressed. However, in order to answer the second critical question, a thorough investigation of the kind of reasoning that these successful teachers employed in their use of visual reasoning, was necessary. The answer to this was found in an analysis of the responses of these teachers to a post-observation semi-structured interview. Participants were interviewed with a view to trying to determine the exact reasons for their actions in terms of using visual reasoning in their classroom. Whilst several reasons have emerged, for the use of visual reasoning by the participants, certain themes have emerged.

6.5.1 Visual Reasoning contributes to success in the classroom

In terms of attributing success to the use of visual reasoning, all the participants were unanimous in their views. They all agreed that visual reasoning helped them to achieve their success with learners, although they may not have been able to achieve the same level of success with every learner. Valerie believed that it would be very successful to all learners if they were trained in that particular way from an earlier age. She also believed that the syllabus was transforming towards using geometry to discover the fundamentals of algebra. In this regards, visual reasoning would become more necessary in all classroom practice. Karl was of the view that visual reasoning did not only help the bright learners, but also the more challenged-learner. The advantage that visual reasoning offered to the bright learner was that, it extended them beyond the syllabus. He quoted the example of the parabola and its rotation to a three-dimensional shape. Sam's approach to teaching was also visual. He used the visual approach, since he believed that 70% of all learning was through the visual medium. He also believed that the visual approach forced the learner into remembering concepts, which was crucial to all learning. Nellie attributed her success to visual reasoning. She also believed that this kind of reasoning helped to make the understanding of key concepts easier. In its absence, learners would struggle. Betty saw visual reasoning as a means of improving learners‘ understanding of concepts, by attaching images to the brain.

6.5.2 Gesturing as a Powerful form of Visual Reasoning

All participants testified to the necessity of visual reasoning, more especially in the way of gestures, to enable their learners to focus on key concepts. The statement made by Valerie, ―for me it‘s just about focussing on a particular thing," emphasised this point. A similar comment has been made by Karl when he reiterated, I think it‘s very important because the gesturing helps to highlight key points in the lesson". Betty shared a similar view in her statement", it‘s possibly just for emphasis". Sam‘s comment, ―I think gestures in the sense that it keeps them alert," emphasised the use of visual reasoning through gestures.

Karl and Sam also alluded to other reasons for the use of gestures in their practise. They believed that gestures, apart from emphasising key points, also contributed to memory retention. According to Sam it gives, ―something that they can remember for a long time". Karl, in a similar manner also believed that it offered the benefit of memory retention that enabled learners to answer questions in examinations. He believed that there was often a time lapse between the teaching of concepts and an examination of these concepts, in the school situation. Hence, gestures were useful in memory retention, thus contributing to the success of the teacher and the learner.

The belief that gestures offered an alternate to boring presentations in the classroom, also emerged as a reason by all the participants. Betty succinctly stated

> ―that we are entertainers more than we ever were before".

This view was also corroborated by Karl and Sam. This view was closely related to the notion of diminishing attention span of learners‘ that Karl referred to in his comment, ―I‘ve realised that the concentration span as compared to a learner of 10 years ago has diminished quite considerably with our current generation". A similar view has been expressed by Betty as well. These statements seemed to confirm the need for teachers to develop more exciting lessons.

The views mentioned above resonate with the relevant literature or gestures. According to Radford (2009, p. 13)

> thinking is not something strictly mental. I advocated instead for a sensuous conception of thinking – one in which gestures and bodily actions are not the ephemeral symptoms announcing the imminent arrival of abstract thinking, but genuine constituents of it".

Whilst this may be seen as a motivation for the use of gestures in teaching, on the contrary,

> in normal learning settings, gestures, considered in isolation, have a very limited cognitive scope. The cognitive possibilities of gestures can only be understood in the broader context of the interplay between the various sensuous aspects of cognition as they unfold against the background of social praxes" (Radford, 2009, p. 2).

Thus the literature is pointing that, whilst gestures are key elements of teaching and learning, it nevertheless cannot be used in isolation. A similar view has been propounded by the participants as well. This will also account for the responses made by the participants. They stressed the importance of gestures in their practice, as teachers. However, they did not see it as the only tool that contributed to successful teaching.

6.5.3 Visual Reasoning creates a realistic classroom environment

Visual reasoning offered the alternative to the traditional ‗talk and chalk‘ presentation of classroom lessons. With the focus on alternate ways of teaching in a technologically advanced world, visual reasoning gives the learner the opportunity to come into close contact with the real world.

Betty‘s words confirmed this when she stated that ―I think it‘s important that children just don‘t see a static presentation. I think as I said they‘ve got TV, they‘ve got movies and they exposed to it all the time so you‘ve got to try and be a bit more interesting in the classroom".

The participants also justified the use of diagrams, pictures, manipulatives and digital media on the basis of this reason. When Betty stated that

> ―I‘m just hoping it makes it more their world you know more technologically, I‘m trying to engage technology I‘m trying to make it more like their real world experience with their phones and their TV‘s and their computers, I think I‘m still lagging behind but just anything to keep the children engaged",

we can understand that visual reasoning was used to create a realistic, authentic classroom environment. Karl, who used digital media significantly more than the other participants, also believed that

> ―technology is not the solution it‘s the use of technology rather the effective use, or the implementation of the role of the ICT that makes the difference. We did not use it just because it‘s a data projector, it helps me convey what I need to convey, that‘s the reason why I have an inclination towards it".

He strongly believed that there was a need for bringing the reality into the classroom and visual reasoning offered this option to the teacher.

Sam‘s use of diagrams and tables was also as a result of the need to ―to create reality, but by you creating reality from your head from seeing something in the past but you can draw it". To Sam, creating a picture of real object was more important than bringing the reality into the classroom through a video presentation. He argued that it was always not possible to bring the real object to the classroom. Furthermore, by the use of diagrams, tables and pictures it would enable learners to develop the object in the mind through repetition. Although he shared a different view about the need for reality in the classroom, he also saw no other alternate than visual reasoning to create this reality in the mathematics classroom.

The constant search for a balance between relevant, realistic and exciting lessons seems to be at the forefront of most educational discourses on effective and successful teaching. The participants seemed to have embraced these strategies effectively through the use of visual reasoning.

In trying to create a dynamic, realistic classroom environment, even visual reasoning had to evolve. This view has been raised by Valerie when she stated that ―they have been watching several videos because the minute you put something on, then they go onto their cell phone, or they talk or they seriously are not watching the video". The point that she was making here was that videos may have been over-used and that they are no longer appealing to learners. This seemed to suggest that even though visual reasoning was employed as a means of offering an alternate to the traditional talk and chalk method, it had to evolve as the times changed. Every effort had to be made by the teacher to ensure that this practice was adhered to. Hence the participants themselves have ensured that they were employing all possible strategies to make their learners more focussed. The use of colour, still and dynamic software, colourful video clips and different means of emphasising key concepts was required. The use of flashing lights in Karl‘s power-point presentation, for example, was to create a realistic classroom environment. Even Karl shared the view of Valerie that visual presentations were changing rapidly and there was a need for the teacher to be able to adapt to these changes. His statement ―okay so a lot is going to happen very soon, that 3D model we spoke about could be printed in class, who knows even while the lesson is on, it could be generated so each learner can feel and have a good grasp of what is been discovered if it‘s a 3D model", consolidated the view that even visual reasoning had to adapt to the changing times to keep the classroom realistic and authentic.

6.6 Conclusion

This chapter examined the data with a view to developing patterns and making generalisations to the larger population of similar studies. The direct interpretation of the individual cases that were dealt with in the previous chapter was aggregated with the intention of extracting common themes. In so doing, it was hoped that points of similarities and differences would emerge. The pre-observation interviews allowed the researcher to understand the participants' views of the phenomena of visual reasoning and successful teaching. During the observation of the participants' classroom practice, the researcher was given an opportunity to examine the extent to which the participants were engaging with visual reasoning. The post-observation interviews allowed the researcher to delve into the participants' justification for the use of visual reasoning.

The nature of visual reasoning emerged as a common theme. It would seem that despite the differences in terms of the backgrounds of the participants, they seemed to have concurred on the nature of visual reasoning with a common understanding of the phenomenon as incorporating the use of diagrams, pictures, manipulatives, gestures and digital media techniques. This view resonated with the reviewed literature. The participants also agreed on the need for multiple representations in their classroom practice in order to be successful. They believed that differences in learners' abilities necessitated a variety of lesson presentation techniques. This view resonated with the literature and the theoretical framework of Multiple Intelligences as proposed by Gardner.

Whilst they believed in the need for visualization, it became evident that they saw visual reasoning as a complementary process. This theme resonated with the literature review and the theoretical framework of Multiple Intelligences.

The participants offered several reasons as a justification for their use of visual reasoning in their practice. However, certain common reasons emerged out of this discussion. Visual reasoning was seen to be contributing to their success in the classroom. Although gestures are regarded as an important component of visual reasoning, they emerged as an important factor in determining their success.

The participants agreed that visual reasoning created an authentic and realistic classroom environment. This view subscribed to the theoretical framework of Situated Cognition Learning, where authentic learning on takes place when it occurs in a social context.

Each of the themes were analysed in relation to the literature review and the theoretical frameworks that have been used in this study.

Chapter 7
Emerging Insights

7.1Introduction

In chapter five, I presented the findings of this study by engaging in two aspects: First, the focus was on establishing the participants‘ views of the phenomena of visual reasoning and successful teaching. Second, an attempt was made to reveal the praxis of the participants and a justification for the use of visual reasoning in their teaching. Through this engagement, this chapter addressed the two critical questions:

1. What is the nature of visual reasoning and how is it used by successful mathematics teachers in their mathematics classrooms?

2. Why is visual reasoning used by successful mathematics teachers?

Whilst chapter five focussed on direct interpretation, by drawing from individual cases, chapter six focussed on aggregation, which sought the collection of these instances, by establishing common themes. The following themes emerged out of the data: the need for multiple representations, visual reasoning as a complementary process, visual reasoning creates an authentic classroom environment and the attributes of successful teachers.

7.2 Emerging Insights: Emergence of a new model

An examination of the data, through the lens of the theoretical framework and the relevant literature, indicated certain trends. In terms of the study, each of these theories has a unique contribution to make in terms of explaining the phenomena of successful teaching.

However, situated at the centre or the heart of these theories is the phenomenon visual reasoning. Each of the theories is inextricably bound to this concept. Based on these theories I extend the idea to create a new model, posited in Figure 7.1. This model is presented with a view to offering a tangible explanation of successful teaching through the use of visual reasoning. Figure 7.1 captures the essential feature of this proposed model.

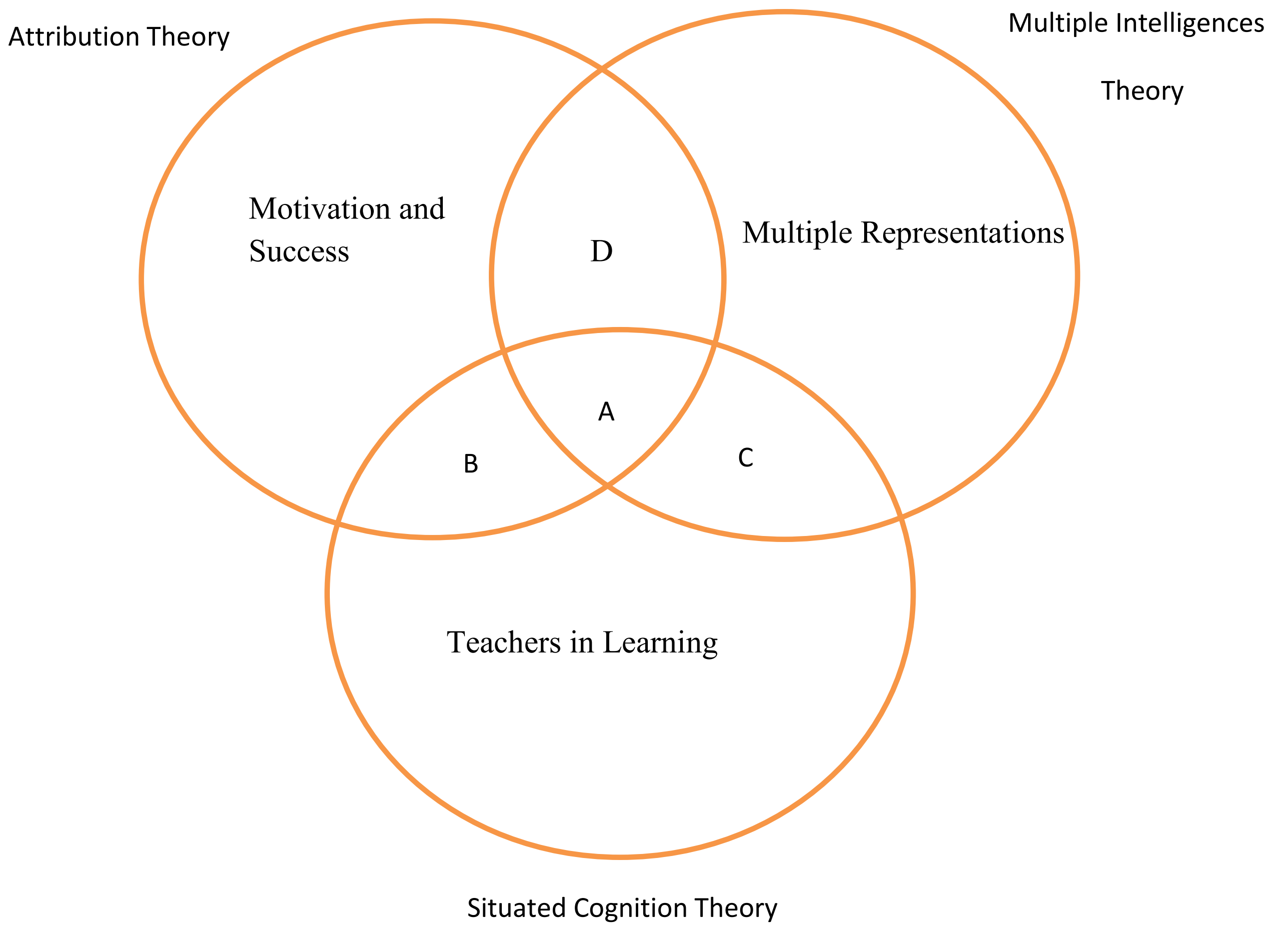

Figure 7.1: Integrating the theoretical Frameworks through Visual Reasoning

Each of the regions A, B, C and D are explained briefly as follows:

A: Is the intersection of the success in the classroom through motivation (Attribution Theory), teaching in context (Situated Cognition Theory) using (Multiple Intelligences Theory). The means to achieve this intersection is through Visual Reasoning.

B: Development of identities according to Situated Cognition Theory is also motivation (Attribution Theory) for genuine learning. Also successful teachers are in need of each other.

C: Multiple Intelligences as a leaning theory must also be situated.

D: Furthermore successful people according to Attribution Theory tend to exert much effort through the use of multiple intelligences (Multiple Intelligences Theory).

In order to understand this model, an in-depth analysis of the four regions described above, needs to be undertaken.

According to Manusov and Spitzberg (2008, p. 37) attribution theories evolved out of the desire to answer the questions such as ―how and why things happen as they do"? This theory seems to be able to explain the actions of these successful teachers on the following basis. The motivation for the success of the participants could either be an internal attribution or an external attribution. Manusov and Spitzberg (2008) further state, that people generally make internal attributions for their success. Similarly, McClure (2011, p. 71) believes that ―this pattern serves to enhance people's self-esteem and lessens the chance that they feel demoralised when they fail". Furthermore, ―it is generally assumed that outcomes perceived as internally controlled produce greater magnitudes of typical expectancy shifts than outcomes perceived as externally controlled" (Frieze et al., 1971, p. 591). In the context of this study, the participants attributed their success to internal factors such as ability and effort. They believed that if they were consistent in terms of the effort, they would achieve success.

Whilst the participants attributed their success to internal factors, many of them also attributed their success to external factors as well. Their dependence on role models and others on their journey to success could be explained on this basis. However, the theory views attributions that are internal to be arrogant and those that are external as modest. In the context of this study, the attribution of the participants, were both internal and external. In the absence of the external attributions, it would have been logical to conclude that the participants were arrogant about their success. The implications of such a finding would not augur well for other teachers who were yearning to become successful. On a more positive note, ―attributing outcomes to controllable causes such as effort increases motivation and perseverance" (McClure et al., 2011, p. 71).

According to Savolainen (2013, p. 64) ―the interpretation of the past, such as the perceived causes of prior events, determines what actions will be taken in the future". It would seem that because the participants were successful in the past, this success seemed to have influenced their success at the time of the research and will continue to in the future. Furthermore, Savolainen (2013, p. 64) states that ―causes such as effort are considered to be controllable, whereas luck is uncontrollable". Clearly the participants did not believe in luck in terms of their success, but rather on effort. This was demonstrated in their in their work.

Weiner (1966) concluded that motivation affected the way in which material was stored or retained in memory. In essence this study showed that the greater the strength of motivation during the perceptual stage, the greater was the retention in memory. It would seem then that memory retention could also be enhanced through effective motivation and arousal. This research was directly related to attribution theory of success and failure. It is not surprising that Sam, with his deep insight into education alluded to the strong correlation between memory retention and perceptual ability. He emphasized the need for repetition in terms of perception and strong motivation in this regards. According to Sam, this would guarantee, permanence in terms of memory retention. Karl and Betty also alluded to the need to make the presentations in mathematics exciting so that it could foster greater memory retention. The reference to exciting presentations could be likened to the strong perceptual motivation that Weiner referred to.

Seen from this perspective, visual reasoning, through its manifold variety would be seen as a strong perceptual motivation for learning. Hence, the intersection of the areas denoting attribution theory and visual reasoning (A), can be seen in figure 7.1. Furthermore, visual reasoning could be seen as an instrument of providing powerful motivation in the process of learning, from the view of the teacher who is providing the stimulus and from the perspective of the learner who is learning the mathematical concept. The success of the teacher also depends on how easily accessible she/he make mathematics to the learner.

According to Wares (2012, p. 123)

> ~~-n~~o longer is the purpose of education simply to pick out those learners who are intelligent, on one or another definition, and give them special access to higher education. Rather, the purpose of education now is to educate an entire population for we cannot afford to waste any minds".

This was the motivation for Gardner introducing the theory of multiple intelligences. As a result of this view of education, mathematics educators too believe that different learners also learn mathematical concepts differently.

Seen from this perspective, this theory has primarily been designed to facilitate better teaching with the ultimate aim of developing better learners of mathematics. I subscribe to the view given by Blomberg (2009) that Multiple Intelligences Theory was designed to cater for the individual differences of learners. Against this background, this theory was chosen as one of the lenses through which the actions of the successful participants could be analysed. Not only does it apply to the way in which learners acquire mathematical concepts, but it allows the educator to develop pedagogy, so that she/he can ensure maximal participation takes place. Furthermore according to Wares (2012, p. 123) ―in a mathematics classroom it is essential to provide opportunities for learners to understand mathematical concepts in a variety of ways".

The reference to ―variety of ways" suggests that teachers need to ensure that they used as many ways to present the subject matter as possible. Visual strategies and visual reasoning offered them the opportunity to engage in multiple representations. Seen from the perspective of the learner, there are at least 9 different intelligences that can facilitate learning. These are regarded as points of entry. Since these points of entry differ in terms of the kind of information that can be processed, it affords the learner different ways to assimilate knowledge. The different intelligences ranging from verbal, mathematical, spatial, musical, kineaesthethic, interpersonal, intrapersonal, naturalistic and existential allows different individuals to assimilate knowledge using any of these faculties or intelligences. Because of the range of faculties, the methods of presentation could therefore be varied. Visual reasoning, which focuses on the visual or spatial faculty, is seen as an alternative to the traditional verbal/ linguistic form of presentation done in the mathematics class. Within the domain of visual strategies there are several other means of representation such as pictures, diagrams, graphs, tables, gestures and objects that could be used to explain mathematical concepts. As a result of the divergent options in terms of presentation of the subject material, assessment strategies also had to change.

In terms of figure 7.1, visual reasoning is one the means of providing teachers the opportunity to engage with the multiple intelligences of their learners. It is through this medium that these successful teachers are able to maximise their potential of their learners, hence the intersection of these two areas, namely multiple intelligences and visual reasoning (A).

The area denoted by section D is the intersection of Multiple Intelligences and Attribution Theory. Gardener believed that successful people often exerted much effort through the use of multiple intelligences. In the context of this study, it became evident that these successful teachers used multiple intelligences extensively. This was evident in the multiple strategies that were employed in their practice. Furthermore, the focus on interpersonal and intrapersonal intelligences also means that this theory has implications for aligning itself with attribution theory. ―Interpersonal intelligence denotes a person's capacity to understand the intentions, motivations and desires of other people and consequently, to work effectively with others", whilst ―Intrapersonal intelligence involves the capacity to understand oneself"

(Wares, 2012, p. 123). These intelligences focus on individual, in trying to understanding how and why they do what they do. This notion is closely related to Attribution Theory.

The theory of multiple intelligences attempted to explain how these successful teachers employ this perspective, through visualization, in their teaching. Attribution Theory attempted to explain the motivational attributes of these successful participants. However, successful teachers develop from novices to experts through a process. This process is explained by a theory called Situated Cognition Theory.

According to this theory, learning takes place in a context. This is important for two reasons:

First, from the point of view of these successful teachers, they had to ensure that the kind of teaching they engaged in was relevant and authentic. Through their use of multiple representations of visual reasoning, they had to engage their learners with the real world. Through this social engagement, the knowledge became meaningful. The learner then developed an identity. The identity is one of a successful learner.

Second, this theory also had implications for the participants themselves. These successful teachers themselves were also learners at one stage in their lives. Their graduation to the position of successful practitioners meant that they were subject to a process of development. Their learning was subject to different contexts. This is referred to as a community of practice. It was through this social engagement that they developed into highly skilled teachers. The result of this endeavour was the development of a new identity. In this case the novices developed into successful teachers, hence the change in identity.

Equally important in the Theory of Situated Cognition, is the role of the context of learning. This is determined by the environment in which the learner finds herself in. In the context of this study, the environment must also be seen as contributing to the success of these teachers. The affluent nature of the school and the parent community must be seen as a positive influence on the development of the staff and may also be seen as a contributory factor to the success of these teachers. The school was well resourced and this would have given the teachers more opportunity to engage with these resources. Unlike their counter parts in impoverished schools, where the focus would have been on trying to obtain the necessary rudimentary resources, these teachers were in a position to focus on areas of teaching and learning to a greater extent. It is possible to argue that the school environment provided them with the resources for them to become successful as mathematics teachers. Although, this argument may be contradicted, it does afford another dimension to the success of these teachers. Perhaps further studies could be extended to impoverished schools to examine the success of visual reasoning in the absence of resources.

In figure 6.1, through the process of visual reasoning, and multiple representations, the learners develop an understanding of mathematical concepts. However this learning must be embedded through social interaction. The intersection of these two areas is represented by C. However, the teachers also evolve from novices to experts. This is accomplished through their motivation and effort. The intersection of these two areas is represented by area B.

It would seem that real success is attained at area A, which is the convergence of all three areas. This is the area where visual reasoning is the means by which, teachers who are highly motivated, achieve success. This is attained through the process of engaging in multiple representations, using the concept of multiple representations in teaching mathematical concepts that are socially and contextually bound.

This model therefore attributes successful mathematics teaching to a combination of different theories that uniquely define and demarcate it, whilst at the same time integrates it with the central phenomenon of visual reasoning. The results indicate, by engaging with visual reasoning extensively, these teachers attained a high level of success.

7.3 Conclusion

In South Africa, the results of mathematics in the National Senior Certificate Examination, has often been a concern for all stakeholders in education. Despite the urgency for the improvement of mathematics results, all interventionist efforts seem to be relatively unsuccessful. The view that learners have of mathematics is that it is often very difficult and only designed for a few learners. This study therefore focussed on finding alternate ways of teaching mathematics, such as using visual strategies to enhance the views of mathematics, since much of the literature seem to suggest the benefits of using visual approaches in the teaching of mathematics. This study examined the engagement of successful teachers through their use of visual reasoning in their practice.

I have argued that visualization contributes to successful teaching. The results of the study revealed positive findings in terms of the role of visualization in successful mathematics teaching. As the study progressed, it became evident that the notion of success was closely integrated with the Theory of Attribution. On further investigation, it was found that success was only attainable if the learning was situated within a social context. The use of multiple representations facilitated the process of visual reasoning by allowing the teacher to exploit the multiple modalities of learning within learners, since human cognition is regarded as multimodal.

From my observation, it may be proposed that the best or most successful teachers are those who find themselves located within the region where the three theoretical frameworks converge. However, situated at the very centre of that point of convergence is the concept of visual reasoning. I propose that this forms the basis of a new model for successful mathematics teaching. However, whilst I argue that visual reasoning has enabled these teachers to attain a high level of success, further research may be necessary to establish whether the success of these teacher may be attributed to the use of visual reasoning only.

It is envisaged that the proposed model will benefit learners, teachers, curriculum designers, subject advisors, and curriculum-materials designers in the following ways: improved learner performance in mathematics through a multi-modal presentation of mathematics; it will inform curriculum design, policy, training of mathematics teachers, and the teaching of mathematics, and an improvement in the image of mathematics that should positively motivate both teachers and learners alike. This model which focuses on satisfying the needs of the individual, may contribute significantly to the notion of learner-centred learning, and can also be applied to other disciplines at school and tertiary levels. Visual reasoning must be seen as the means to achieving this end.

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Appendix A

UNIVERSITY OF KWAZULU-NATAL

Interview Schedule
Pre- Classroom Lesson Observation Interview
Semi Structured Questions

Co-researchers Pseudonym: ________________________________

Researcher: ____

Date: ______________________

Time: _____________________

Venue: _____________________

Start time: ________________

End time: _________________

Preamble:

Thank you for volunteering to participate in this study. I want to assure you that the interview and anything you share will remain confidential. Please note that you may withdraw at any time from the study. As I explained in the introductory letter, the study is about your practice as a mathematics teacher, more particularly, the extent to which you use visual reasoning as a teaching tool in your classrooms. The questions that I will be asking will need you to describe, in as much detail as possible, your classroom experience. Please feel free to ask me to clarify what I mean at any time.

Background:

This interview schedule will be used to determine the extent to which successful mathematics teachers' use visual reasoning in their classrooms and the reasons they furnish for using such a teaching tool.

Questions:

1. Tell me about your teaching career very briefly, taking into account your background, teacher training experience, further qualifications and teaching experience up to now.

2. You have been identified as a very successful mathematics teacher. Can you tell me something about yourself that enabled you to reach this position?

3. How would you define a successful mathematics teacher?

Why do you think you have been identified as a successful teacher?

4. What do you understand by the term visual reasoning?

5. Do you think that visual reasoning is necessary in the mathematics classroom?

6. Do you use visual reasoning in your mathematics classroom? How and to what extent do you use it?
What are some of the examples of visual reasoning that you use?
If not using: why do you not use visual reasoning? Do you find that it impedes your teaching? Why? Will you be willing to try?

7. How does the use of visual tools influence the teaching of mathematics?

8. Do you think that learners benefit from the use of visual reasoning? Why?

9. What would you attribute to your success as a mathematics teacher?

10. Would you attribute the use of visual reasoning, in your practice, to your success as a mathematics teacher? Or vice versa: Do you attribute your success to the use or non-use of visual reasoning? Why?

Thanks for your input.

Appendix B

Interview Schedule
Post- Classroom Lesson Observation Interview
Semi Structured Questions

Co-researchers Pseudonym: ________________________________

Researcher: ____

Date: _____________________

Time: ____________________

Venue: ____________________

Start time: ________________

End time: _________________

Preamble:

Thank you for volunteering to participate in this study. I want to assure you that the interview and anything you share will remain confidential. Please note that you may withdraw at any time from the study. As I explained in the introductory letter, the study is about your practice as a mathematics teacher, more particularly, the extent to which you use visual reasoning as a teaching tool in your classrooms and why. Please feel free to ask me to clarify what I mean at any time.

Background:

This interview schedule will be used to determine the reasons for successful mathematics teachers using visual reasoning in their mathematics classrooms.

Questions:

1. I noticed that you used gestures as a means of emphasizing key points in your teaching. By that I mean, you often illustrate significant points in your lessons, by using your hands and body language. Can you explain why you choose to use such gestures?
2. Does this occur consciously or are you unaware that it occurs? In other words do you ever plan the use of such gestures?
3. How effective do you think gestures are in terms of promoting understanding?
4. Are there some standard gestures that you use in most of your lessons?

5. You have used pictures/diagrams in your teaching, for example you have used a picture of a bridge, to explain the use of the parabola in a mathematics modeling problem? Why did you do that?
6. Does the picture replace the actual object? Do you think it would be better if we have the object or model before us?
7. How do you plan or select which picture/diagram/manipulative to use in class? Do you ever consider the type of learner you have? So can these pictures/diagrams/manipulatives change each year because your learners change?
8. What was your expected outcome?
9. Do you think that this could be done differently?
10. I noticed that you used paper, scissors and a ruler to do this investigation on the angle sum of a triangle. Why did you choose to use this as a means of conveying your message?
11. Did you think it is the most effective way of presenting that information?
12. If you used a video clip on the sum of the angles of a triangle, do you think it would be more effective?
13. Why do you use the data projector in your teaching? For example, you had a worksheet on the projector?
14. Do you think it was necessary to use the projector for this? Explain
15. Do you think that changing your strategies in presenting lessons is important? Why?
16. Can you explain why you use diagrams in your teaching?
17. I noticed that your power point presentations use a great deal of colour. Can you explain the significance of colour in these presentations?
18. You sometimes ring or underline, highlight some of the work on the board. Is there a particular reason for this?
19. You have used animations in your presentations, is there a particular reason for this?
20. Why do you use dynamic presentations rather that still presentations?
21. What is your expected outcome behind such presentations?

Thank you for your input.

Appendix C

Sample of interview 1 with one of the participants

Interview between Researcher and Karl -19 March 2013

R: Thank you Mr Pillay for your participation in this study. I've sent you a letter explaining the nature of the research which I'm sure you have read.

Karl: I have received it.

R: Just to reiterate, my study is exploring the use of visual reasoning by successful maths teachers. You probably are aware that a great deal of research has been done and is still been done in the field of visual reasoning.

Karl: Ya

R: But my focus is on the successful educator as opposed to those experiencing difficulties. You'll find that there's a lot of research talking about problems with visual reasoning rather than those that are talking about success with it, hence you've been identified as a successful maths teacher.

Karl: That's correct

R: In arriving at this conclusion, I have examined the results of maths at this school, both internally and externally and come to the conclusion that this Institution has a great academic record and the maths results are also very impressive. I believe that you are one of the very successful maths teachers that have been selected to teach at this school. You also no doubt aware, that results in maths are always a problem and hence research is constantly been done to try and improve the image of maths both nationally and internationally.

Karl: Yes, I am aware.

R: Okay. The phenomenon that I am investigating is visual reasoning. What I wish to explore is the extent to which you either use or do not use visual reasoning in your maths classroom as a mathematics practitioner. I wish to assure you that your comments will be treated within the strictest sense of confidence. The school and its context will remain anonymous and all this research belongs to the University and will be destroyed after the study is completed. A transcription of this interview will be made available to you. Should you not agree with any part of this you can feel free to inform me. Also participation is voluntary and at any time, should you feel the need to withdraw from this study, you are most welcome to do that. Permission has also been obtained from the Head of this school to conduct this study. So once again, thank you Mr Pillay for your time. Reverting to the interview, I wish to establish a few points about your views about visual reasoning, what is your understanding of successful maths teaching and how this links up to your success as a maths teacher. Please be as candid as you can. Question1, is asking you to tell me about your teaching career very briefly, taking into account your background, teacher training experiences, your further qualifications and teaching experience up to now.

Karl: Good Afternoon Mr Budaloo. Thank you for this opportunity to be part of your research. I think it‘s very relevant, very necessary especially in our country and the climate in which education finds itself in, at the moment. Just as a background, as you requested, I have been in the profession for 19 years now. In terms of teacher training, I come from a mechanical Engineering background which simply means that after I did qualify, I went into teaching immediately, teaching Mathematics from day 1 up until now. I have been teaching Grade 7 up to Grade 12, but predominantly in the FET phase over the recent years. In terms of further qualifications, subsequent to the engineering qualifications, I did initiate a study of further study in digital literacy, digital communication, under the banner of a Master in Communication Science degree which has not been completed as yet. But I have started it and have found other interests in the digital area. I suppose, I may cover that a bit later.

R: So you have an interesting background.

Karl: And, and I have taught in a government school setup as well as in a private school, so I‘ve got fairly sufficient experience, number of years of experience in both school types.

R: Mr Pillay you have been identified as a very successful maths teacher as I‘ve pointed out to you earlier on. Can you tell me something about yourself that enabled you to reach this position so that you‘ve been employed by this particular institution.

Karl: I think most of the successes rely on the fact that I‘ve had good mentors along the

way. From my first day in the classroom, I was mentored by seasoned mathematics educators who spent many years teaching, and then, in particular, in recent years, the Maths subject advisor, and also having had the opportunity to be mentored by you Mr Budaloo. I recall being used as a lead teacher at the teacher training for the new curriculum. I have learnt a lot from you and really admire the way that you able to teach concepts. So yes, having good mentors was definitely the main contributor to my success as an educator in the classroom, having learnt from their experiences. I suppose mistakes that other educators made alerted me to the fact that certain things will not work because they have been tried and tested and because they failed, it alerted me to the fact that certain systems, certain mechanisms simply do not work. But yes, definitely been mentored by seasoned educators have been the backbone of my success, I would think.

R: Anything else, that you think, which may have catapulted your recognition in the Department of Education, as you have been recognised as one of the lead teachers in mathematics.

Karl: I think been digitally inclined, using the cell-phone, using the digital projector, data projector in the classroom is something that I really enjoy. In the year 2007 there was a competition that we entered under the banner of Microsoft Innovative Teacher Forum Awards and I used the cell-phone as a mechanism to impart mathematical knowledge and for that I was recognised in this country as well as globally. I suppose that was the catalyst, that saw Crawford becoming interested in my expertise and I think that is perhaps, the single most important factor that allowed me to become part of this institution.

R: So you mentioned digital inclination quite a bit already.

Karl: Yes.

R: I am pretty sure that the investigation or the research that you‘ve done through Microsoft was something that was also very exciting. Do you want to tell me a little more about that project?

Karl: Well essentially, I suppose there is still a need because there is massive digital divide between the haves and have-nots.

R: Agreed.

Karl: And, the problem is that, in our country, I believe that quality education should not be a privilege, it should be a right of every learner. And given that fact that we have so many under-qualified educators, even in the rural areas and having those learners write the same exams as learners that come from urban schools, really created a divide in the sense that many in rural areas were not performing well because of a lack of access to quality material, a lack of internet perhaps. I think that this is a very worrying problem at the moment. The idea was to try to reach the learners as far as possible in the most remote areas and studies indicate that most of our learners, even in rural areas, do have basic entry level cell-phones. So essentially what was done was the presentation of the mathematics content in a manner that was user friendly, using colour graphics and in a language that could be understood, quite clearly by second language speakers. That was the driving force, behind putting the mobile project together and trying to access or enable our learners to access the information at no cost. It was a free service, something done purely out of the intent of helping our learners advance their skills in mathematics. So that in a nutshell, is the project.

R: It must have been quite a phenomenal project because you were granted recognition for that as a South African.

Karl: (agrees)

R: So obviously you would have competed against fellow South Africans, in terms of a competition. Was there any sort of initiative made by the department subsequently to that, to push forward the agenda?
Initially, when I did do a presentation to the leaders of the province, in education, prior to me winning the competition, I did get a very negative response.

R: Okay

Karl: But subsequent to the fact that this project drew worldwide acclaim, their standpoint did change where the department was willing to embrace the idea but since then nothing further did ensue, it was acknowledged and accoladed but in terms of implementation there was nothing forthcoming.

R: That is a pity.

Karl: Well, I would think so.
R: Considering the fact that countries abroad recognised the achievement and here, in South Africa they didn‘t. Really you it‘s a great opportunity to doing something like that.

Karl: In fact there were 24 international judges that agreed that this was the way that South Africa should move in.

R: Okay.

Karl: But it was not so clearly noticeable by our own authorities.

R: So these were international judges?

K: Yes 24 international judges

R: International Judges. Okay. So the competition must have been of real note?

Karl: It was. The competition was very gruelling. It was a 4 day competition that allowed us to have our project on display for a period of 8 hours per day. Visitors were allowed to enquire or ask about the project. At no point did we know who the judges were because they could pose as visitors.

R: Okay

Karl: They never identified themselves, they came as part of the normal audience.

R: Right

Karl: It was only after the competition that we realised who the judges were which meant that we had to be quite clear and specific in what we were explaining throughout the 4 day competition.

R: It must have been quite a gruelling experience.

Karl: Yes

R: Okay Mr Pillay, how would you define a successful Mathematics teacher? What your notion?

Karl: My notion, I think in the simplest form is, if a Mathematics teacher, through some mechanism or mechanisms is able to allow the learner to perform at maximum potential, then I think, that would be the definition of a successful mathematics teacher. I do understand that not that not every learner would be the typical A learner or the B learner but if a learner has the potential to succeed at 60% and perhaps at a current state in time is only functioning at 40, and if a teacher is able to bring that learner from 40 to 60 and if 60 is that learner‘s threshold or potential then that would be my simple definition of a teacher who is successful in terms of Mathematics

R: That‘s interesting because, similar sort of view-points were given by others.
Karl: Okay.

R: Why do you think that you have been identified as a successful teacher?

Karl: I‘m not really sure because, I did not fill in a form, I was not shortlisted, but I suppose, it is because of my passion for the subject, and allowing such learners the opportunity to reach their potential. I have taught at schools where my learners predominantly came from disadvantaged backgrounds, where some of them have been single parents. I would think that the work that I put in, in allowing some of these learners to reach their potential, could have been one of the areas that has allowed me to be classified as a successful teacher. Also, I suppose the analysis of my learners‘ results may also be an important factor. (*clapping sounds in the background*) So I think, it‘s essentially allowing the potential of the learner to rise to the fore and the good results. Perhaps, I think the innovative ways that I may be using in my class (*pages turning*), that has been advertised in public now, could be another reason why I may be viewed as a successful Maths teacher, trying new, things all the time.

R: That‘s great. What I wish to establish now, is what you understand by the term or concept visual reasoning?

Karl: To me the concept in terms of my understanding, would be that the use of pictures or graphics to convey a concept to an audience, in this particular case, would be the learners. It is perhaps the use of multimedia. It is said that, a picture is worth a thousand words.

R: Yes

Karl: So sometimes I think, in three pictures, it makes it equivalent to me speaking three thousand words, it does save time as well. As an example, you can ask a learner

or an adult to explain what the word spiral means. It is almost impossible to explain the word spiral without a person gesturing to it, using their hands, to explain what the concept of a spiral is. So that's a typical example of how powerful a visual medium could be. So in essence the use of the multimedia, whichever form allows a concept to be grasped, understood and applied would be the idea behind what I would classify as visual reasoning. I'm not sure if you wanted a rigorous definition?

R: No, not really.

Karl: That is according to my understanding of the concept.

R: As you understand it.

Karl: As I understand it.

R: Okay, that'll be fine
Do you use visual reasoning in your Maths classroom? How and to what extent

do you use it?

Karl: Yes I do use visual reasoning. In the lower Grades, in grade 9 as an example, the concept of area of a triangle, which is equal to half the product of the base multiplied by the height, somehow comes alive when the learner sees graphically, on a slide, the rectangle being divided into a half with a diagonal. Being a Maths teacher yourself, I am sure you understand the concept. So when the learner sees that happening, somehow we see the lights go on, the understanding of the concept comes in. In terms of the Grade 12 syllabus, with calculus maxima and minima, the concept of volume and surface area still poses a great deal of difficulty to some learners. They find it difficult to see a drawing on a flat piece of paper as representing a three dimensional shape.

R: Okay

Karl: In terms of the corners being chopped of, they have difficulty seeing that, that corner is the height, but when the graphic is shown the learners grasp the concept with greater ease.

R: So when you said the graphic, did you mean the physical model or even a projection of the object?

Karl: Well it's a projection.

R: Okay.

Karl: Because, the power that a projection has, is that we can rotate the models.

R: Yes.

Karl: With the click of a button.

R: Yes.

Karl: And it also saves time in terms of finding the appropriate models, which I'll be honest, is hard to find.

R: Yes.

Karl: They don't just exist, I've got to create them, physically build them, but I found that by using 3D images, it helps the process quite nicely.

R: Do you use the data projector?

Karl: Yes I do use the data projector.
R: With the the 3D images?

Karl: With 3D images, power-pointing, you-tube as well, but not all the time because there are certain parts of you-tube that you'll use and not necessarily the entire clip. So just the clip to show the rotation of the solid, that would be necessary at times.

R: This this is probably very relevant to you. Do you find that visual reasoning impedes your teaching or does it slow down your lessons?

Karl: Well in my personal experience if it's used wisely it will not impede.

R: Okay.

Karl: When I use visual reasoning as a tool, it accelerates and enhances the learning experience.

R: Okay.

Karl: I haven't noted impediments, but there is potential for impediments to occur, hence it's important to carefully select and edit and adjust the material before the lesson is presented. So when that is done before hand I've experienced quite a good success rate in the implementation of tools that enhance the visual experience of the learner.

R: I'm sure, if you research the model before hand, your answer makes sense.

Karl: Yes it does make sense.

R: Ya

Karl: And just to add Mr Budaloo, it‘s not necessary at times to do a full clip.

R: Ya

Karl: Sometimes just a one-minute clip makes a whole world of difference to the learners‘ perception and understanding of what is to follow in the lesson.

R: Do you have any example of a situation where you felt that visual reasoning was actually slowing down the progress or can you site an example in the syllabus, that you think you‘ve made use of that can actually slow you down.

Karl: It maybe.

R: How can it slow you down?

Karl: I think this is a rule: it has the potential to be an impediment to learners who are very well developed spaciously. If a learner is able to visualize the content immediately, then to that learner, the graphic may just be a waste of his time because, once he has grasped the concept, and this occurs in a few seconds or few minutes, then the learner may appear to be bored.

R: Okay.

Karl: As a lesson progresses, seeing the cube in different colours may not interest him, because he has done it already. In a class where in the majority of learners have
excellent spatial components in their thinking then the visual component of the of the lesson may be an impediment, it has potential. In certain cases, where learners are familiar or perhaps saw an inclination to understanding the spatial component, then I would not readily go into the visual reasoning *music in the background*
However, there may be an extention to the concept which I may consider, if time will allow it.

R: Okay. You mentioned a developed spatial component.

Karl: Yeah.

R: Do you accept that there are different kinds of learners in the classroom (closes door to shut off the noise from outside), those who are more developed in terms of spatial orientation and those who are less developed?

Karl: From my experience, I would say yes.

R: Okay.

Karl: I‘ve noticed that there are different levels at which learners would grasp concepts, and from experience, I believe that there is a difference.

R: Okay

Karl: Because I see it.

R: That was a very interesting comment that you made in terms of the level of spatial development in the learner. It is an unusual way of looking at the situation.

Karl: I have experienced that.

R: Okay.

Karl: I did experiment with it just to confirm within myself that they will be bored.

R: Okay *laughing* quickly tell me about it. Quote an example.

Karl: It was more with the data handling.

R: Okay?

Karl: Data handling, it was a clip on mindset learning.

R: Yes?

Karl: But do bear in mind that these are learners are fairly well developed. I wanted to experiment the idea of teaching a concept digitally for the entire lesson, where I would just stand and observe.

R: Okay?

Karl: I did not stand or sit in the front, I was behind, at the back of the class the where learners could not see me observing them. The pace of the lesson was too slow. The amount of the time that it took to progress to where they ought to be, was quite a boring experience to most of the learners and hence I had to shelve it.

R: Okay so there you have an example of how it can impede yourprogress.

Karl: *Words muffled*

R: Mr Pillay, how does the use of visual tools influence the teaching of Maths?

Karl: It makes it exciting, it promotes research, it enables me or creates a stimulus for me to go onto the net and research the various tools that are been used by fellow educators across the globe, and implement or attempt to implement some of the concepts which I think, may work in this environment, in our country. If it works well, it works, if it doesn‘t work then I would know. But it definitely, it creates a great interest for me as an educator to do on-going research to the lessons that we taught 20 years ago on volume and surface area. It changes every year, and the way I taught it last year is different, the new graphics, new visuals and with advancement of technology there‘s much more that the computer boss are able to present that allows us to enhance the teaching experience. So definitely, it‘s very exciting. It takes up time, but I think it‘s rewarding as well, when as an educator, you know that you‘ve exposed the learner to cutting edge methods and graphics and are keeping them up to speed with the current technology that‘s available.

R: You have spoken of the positive influence of visual tools. Can you think of any

negative influences besides the ones that you quoted before?

Karl: Negative. Not really.

R: Not really

Karl: I would say that the only time it will be impede, in my scenario, would be when the learner is spatially developed.

R: Okay.

Karl: I have always noticed an interest in learners who are less developed spatially, an interest into what‘s going on the colour graphics. I must add that the graphics are in colour, not black and white and it‘s colourful and brightly coloured at times. I think it helps to enhance the power behind the graphics, that is, the choice of a good colour combination.

R: Mr Pillay do you think that learners benefit from the use of visual reasoning and if you think they do, why? You go there and present it and you have a certain perception that it will benefit them, but from their point of view, do you think they actually benefit?

Karl: I do believe they benefit. When a lesson is been taught, I‘m very mindful of facial expressions and there is marked change in a learner‘s interest in the concept when a relevant graphic or a picture is shown. You, literally see, the light bulb go on and it stimulates interest. Also, I think it helps in the sense, that as I said earlier, a picture is worth a thousand words. So a concept could be conveyed to a learner in a matter of seconds that would have ordinarily taken much more time in the absence of a visual tour. So definitely, it‘s time saving, it‘s interesting to the learner, it‘s an alternate platform to reach them all, as opposed to chalk and talk about the white board. I believe these are very strong plusses for that: it increased the concentration that they would show and it helped to save time. The time saved there could be used in in new applications.

R: As far as you concerned, you have seen the positive influence?

Karl: Yes, I have seen it.

R: On learners.

Karl: I would believe that visual reasoning is the way to go, and, in inverted commas, it has a bright future

R: *laughing* interesting.

Karl: And I believe that educators need to embrace this concept because it is something that our learners love. They simply love it. Our learners are very digitally inclined now, so reaching them on a digital platform is half the battle that is won.

R: It makes perfect sense, particularly from your background, because you come from a very digital background.

Karl: Yes.

R: As well as your engineering degree qualification and also in mechanical engineering which is also a hands-on, concrete kind of engineering that you are dealing with. Also, your research project which was based on digital learning itself, was so successful.

Karl: Ya.

R: I can see the kind of reasons why you are a strong proponent of visual reasoning. Would you attribute your success as a Mathematics teacher to visual reasoning? What would you *clearing throat* sorry let me clarify, what would you attribute your success as a Maths Teacher to?

Karl: I think it‘s more a co-operation of passion and compassion, passion for the subject.

R: Right.

Karl: And compassion for the learner who has a career goal in mind and being the liaison, or the gap or the connector to where they need to be and that is quite a challenge. But being passionate in helping learners get to the career goals, career paths and also being very considerate, in the sense that not every learner, will be an achiever. But, if we can propel them to a level where they would semi achieve or even by pass semi achievement, then I think, we would have done well. So coming back, having passion for the subject and having compassion for the learners, and I think now that we are so influenced or exposed to digital media and tools as an extra component, that would help to make this a very stimulating career. It‘s very dynamic it‘s not static, while the message has not changed the methods must change. The way you derive for calculus, we know, you have to arrive at your answer, by following a set of rules, which have to be followed. Results will not change but the way we arrived with that result is something that you could obviously look at, as an-going research. Also, there are tools that are available, as an example, years ago if you had to show or try to convey to a learner the process of taking a flat piece of paper and converting it to a box, you would have had to physically for them to see it. Now, with advent of the digital projector, at a click of a button, and in a few seconds that idea is conveyed almost instantaneously. So it‘s a powerful medium that we have and this creates, I suppose, a greater passion, and greater interest. Also, with the number of research papers being published on the benefits of virtual learning, virtual communication, and digital learning, it would make sense to embrace that and to enjoy embracing it, and that‘s what I do, I do enjoy as well.

R: Ya certainly, I can see that, because, obviously you‘ve read a number of research papers as well.

Karl: Ya.

R: You are also at the cutting edge of technology as well, so it‘s quite evident that you enjoy doing it. And I think it also brings in a sort of third dimension where, before you had the subject, and you had to execute it in a certain way, now you have innovative ways of trying to do it.

Karl: Absolutely.

R: So it re-instils that enthusiasm, it gives it a new kind of birth so to speak. Otherwise, as you would say, education would become static after a while.

Karl: Absolutely and it‘s also not about proceeding from A to B, it‘s how you get there.

R: Yes, agreed.

Karl: So how you get there, if we could get the learners from A to B in a very exciting manner, then why not.

R: Why not?

Karl: Absolutely, if you have the tools available.

R: It certainly is. You are doing a remarkable job of convincing the learners about that. Would you attribute the use of visual reasoning in your practice to your success as a maths teacher or do you attribute the non-use of visual reasoning to your success?

Karl: It's not a black and white answer, but to categorise the two components: There will always be learners who will achieve, no matter what, in the absence or presence of a visual learning environment. These learners will achieve any way, in such cases the visual tools will be used to take them beyond the syllabus and could be used as an extention.

R: Okay.

Karl: It is not easily measureable in a school situation, to what extent they would have benefitted from the exercise. It is only gauged once the learner leaves the school environment and goes into the tertiary or the real world, where we later on realise the advantages of that extention. For the weaker learner, definitely, if success is defined by the results we produce, then that weaker learner, from my experience, in most cases, had a mark improvement of the understanding of the concept.

R: Okay.

Karl: The problem is to apply that understanding under a test condition. This becomes the problem. Under non-test conditions and under relaxed conditions the learners do understand the concept. There are times when the learners have a fairly good grasp of the concept, yet they somehow fail to convince us. Very often, after a poor test result, the learner will indicate that they were aware of the mistake that they made.

R: Okay.

Karl: So that in itself tells you something. The learner was familiar with what the concept Entailed, but because of an incorrect substitution or incorrect factorisation or some step along the way that was incorrectly performed, it impacted negatively on the final results. So to sum it up, I would say more yes than no.

R: You eluded to some test performance. And if you had to examine the test performance, it normally would be a pencil or pen and paper sit-down test.

Karl: Ya.

R: Your notion of using the visual tool was some visual instrument that you would have used or something like that.

Karl: Ya.

R: So there is this lack of connection between the two.

Karl: Absolutely.

R: You experienced that problem, because the technology or the visual reasoning that has been used there, is not immediately transferred into a sort of a written kind of situation.

Karl: You are quite correct. I‘m glad you used the word not immediately, because some of these concepts perhaps, will take a phase, maybe the GET phase, for the learner to finally catch it in FET phase. Some of these benefits are not immediately apparent. It does not mean that it does not work.

R: That is so true.

Karl: It has to be quantified over a period of time.

R: You also mentioned something about using it as an extention, which you‘ll see at post matric or tertiary level. Do you want to quote an example to clarify this?

Karl: A very quick example would be the parabola.

R: Okay?

Karl: The basic $y = x^2$, and then you allow the learner to understand that while the parabola in two dimensions there are parabaloids that exist in three dimensions and to show them the X,Y and Z axes as a graphic suddenly expands their thinking into another dimension. Some of them are waiting to be taken to that level.

R: Yes.

Karl: Because such is the desire and interest that it has created. So that‘s just one very quick example of extending learners to third axis.

R: Yes.

Karl: There‘s a three dimensional concept to show them that graphs can also exist.

R: Yes.

Karl: There are real applications in engineering and other spheres that use the basis of the 3D graph to make calculations or predictions.

R: Very interesting examples. Your engineering background helps you to train those kids in a certain way as well, because you are aware of the applications of those things that you‘ve done in in degree. It does influence the way in which you present your material.

Karl: Even, the example of a bridge.

R: Yes.

Karl: We can show them a picture of a bridge and tell them that before the bridge is built, calculations have to be done and based on these calculations, the designer had to determine the strength and weaknesses of the structure, based on finding derivatives, based on calculus theory. The bridge could be analysed and before the bridge is built weaknesses could be identified as opposed to building the bridge and allowing it to collapse. So learners start to see just from a purely a 3D picture of a bridge, the mathematics that was involved. Yes they can visualise a bridge but somehow putting up a slide of a major suspension bridge and showing how weaknesses could have been determined way before the prototype was made, may be beneficial as well.

R: Mr Pillay thank you so much for all your insight in terms of the questions that I have asked you. It seems that we have come to the end of my questions. If there‘s anything interesting that you want to share at this point, because of your vast experience in terms of engineering and the international competition that you engaged in, you can kindly tell me.

Karl: I used the concept of digital communication as a testing tool where I now set tests on the mobile platform and when a concept is taught in class I refer the learners to a to a site and ask them to whip out their cellular phones. They go onto the site and you take a test on the concept, a multiple choice test based on the concepts taught. Obviously, the way I will test a question will determine if the learners understand the foundational concepts. When a test is taken, in an instant the learner gets the feedback, the percentage that they achieved and the answers would appear on the form in an instant together with an explanation so a learner can go back and reinforce the concepts. I am thinking of formally introducing it as a test component at the school, and I believe that the head of the institution, Mr Loots is keen to explore a paperless test component, purely on an experimental basis, to see how it would pan out. If, all goes well for this institution, they are on the cutting edge of many things and I believe that the mobile testing component could be an area that we would implement and hopefully practise in the weeks and months to come.

R: That should be something to look forward to.

Karl: I need to ask if you would be available to moderate some of these tests?

R: By all means, certainly.

Karl: Ya.

R: You‘ll be using the cellular phone.

Karl: Cellular phones with graphics colour. Also, determine as a spin-off of this research, whether the colour component impacts on a learner‘s understanding of a concept.

R: I‘ll be very keen to know what the outcomes of the results are. Once again, Mr Pillay thank you very much for all your time and effort and for having sacrificed your afternoon, and making some contribution to this research. As soon as the results of this research are published I‘m sure you‘ll be one of those few people who‘ll definitely want to read it.

Karl: Absolutely.

R: If it turns out to be.

Karl: Well thank you for the opportunity Mr Budaloo and I wish you well on your research endeavours. I‘m confident that this research topic will make international acclaim, given the passion that you‘ve put into it and whatever further assistance or interview that you may require, please feel free and know that I‘m available and now that we‘ve spoken we will initiate further discussion and I will keep you up to date of the latest trends that I‘m trying to follow in my classroom and let‘s apply our minds together perhaps to enhance the teaching experience for our learners.

Karl: Thank you.

R: Okay.

Appendix D

Sample of interview 2 with one of the participants

Interview with Researcher and Karl on 26 March 2014

R: Thank you Mr Pillay for giving up your time to have the discussion with me. I know it‘s a holiday and that you sacrificed your afternoon and your family, but I really appreciate it. As you are aware this study involves three stages of data collection. In the first round I had an interview with you where I tried to gather your views on visual reasoning and how you use it.

Karl: Okay

R: The second part of the data collection plan was the observation of video recording of your teaching, where we observed your teaching with special focus on visual reasoning itself and we observed two of your lessons.

Karl: Yes I do remember Mr Budaloo

R: Now we want to analyse your actions and try to understand what you do in these lessons. We are interested in what goes on in that head of yours. So if you can explain to us why you do certain things. The focus obviously is on the visual aspects but we are keen to know what actually goes on in maybe your gesture and the use of the projector and the use of anything that you are doing here, that is visual. I‘ve selected specific episodes from your lessons

Karl: Okay

R: I‘ll try and focus your attention to that and then if you could may look at that and also answer these questions that I have in front of you.

Karl: Okay

R: The first question is, I noticed that you use gestures as a means of emphasising key points in your teaching, by that I mean you often illustrate significant points in your lessons by using your hands and body language. Can you explain why you choose to use such gestures? I can show you a few examples of gestures but you probably know them by now for example in minute 2 right at that point there, minute 2, minute 3, minute 4, (refer the participant to points in the video clip)you actually pointing to something there.

Karl: Well Mr Budaloo I think over the years I‘ve started to realise that much of what is said is, is conveyed vocally. I do agree but emphasis, I realise lies in, lies with body language. With the experience I‘ve realised that gesturing focus on key points in the lesson.

R: Okay

Karl: It somehow helps to reinforce the learner‘s memory and recollection of the important points that I do highlight, so in other words it could be something similar to using a highlighter on a page.

R: Okay

Karl: I found that gesturing and using body language has become very effective, more particularly in the day that we live in, where educators are probably been more performers at times.

R: Yes

Karl: For our learners you keep them aware, because in many instances I‘ve realised that the concentration span as compared to a learner of 10 years ago has diminished quite considerably with our current generation. So I feel that gesturing and the body movements helps to allow the learners to stay focused and at times bring back their concentration in some learners.

R: That definitely makes sense because other teachers mentioned the same thing about being performers because somehow the attention span of the children seem to be very short so you have to keep the learners interested in the subject matter.

Karl: Yes for sure.

R: Thank you for that Mr Pillay. Does this occur consciously or are you aware that it occurs, in other words do you ever plan the use of these gestures?

Karl: Well I haven‘t given it thought, but I would say it‘s unplanned, it‘s purely spontaneous. I suppose the stimulus comes from observing the learners. I there‘s a blank on the face or there seems to be some form of confusion. I suppose the correct gesture spontaneously comes out to help them enforce that idea, and help that moment in time.

R: It definitely makes sense as well, because just by observing the transformations in that parabola

Karl: Yes

R: You used your hands a lot to show movements to the right, to the left, up, down and so on.

Karl: Hmm.

R: And I think that definitely helped the learners understand the concept better.

Karl: And quite often I do mention to learners that when they are placed with a similar question in an exam situation they will think of my gesturing in class to help them to initiate towards a correct solution.

R: I think you even mentioned that.

Karl: Yes

R: The whole idea behind the picture is that they attach a picture to the concept itself.

Karl: Absolutely

R: Okay, how effective do you think gestures are in terms of promoting understanding?

Karl: I think it's very important because the gesturing helps to, as you said highlight key points in the lesson.

R: Okay

Karl: And I found that over the years, especially the recent years, when I do question learners, and they do indicate to me that, when the lesson was taught, you'd have to gesture at times or the gesture done in at that moment in time helps that learner to recollect the way forward in particular exam type question.

R: Okay, are there some standard gestures that you use in most of your lessons.

Karl: There could be standard gestures but I'm not aware of it.

R: Yes.

Karl: It's not like I sit down and say, these are the 10 standard gestures out of my toolkit that I'll use, I think it just happens on the spur of the moment.

R: Yes, it's quite evident, because when you are dealing with shifts then you seem to use your hands.

Karl: Yes

R: When you are talking about horizontal shifts you use your hands to show side by side movement, when you talking about vertical shifts then you use your entire body and move your entire arm.

Karl: Yes

R: up or down

Karl: I have to extend my body so that we can see the vertical stretch.

R: Yes I see can that, that was quite evident. Mr Pillay you have used pictures and diagrams in your teaching, for example you have used a picture of a bridge in here to explain the use of a parabola in Mathematics modelling problems.

Karl: Hmmm

R: Why did you do that?

Karl: Well Mr B it is often said that a picture is worth a thousand words and I find this especially true in Mathematics that the more pictures the more diagrams are used the more effective it become in conveying a concept and because I believe that most of our learners are visual it helps to convey the message quite clearly.

R: Okay

Karl: In most cases

R: Okay

Karl: So it is very effective. I firmly believe that pictures really help our learners to have a better grasp of the subject matter at hand.

R: Yes, as opposed to maybe just talking about a bridge. So now they'll see the bridge constructed in front of them.

Karl: For sure

R: And some of them, because of their experience, may never have seen a bridge like the one you talking about.

Karl: It's quite correct, you are quite correct. I've been in some scenarios where some learners were not aware of what I was talking about until the picture was shown to them. In this particular environment most of the learners would have been familiar but I suppose seeing that 3 D picture in front of them at that moment in time in the Maths lesson brings the live integration of the Mathematics with the real life

R: Yes

Karl: Was an important end result. It would have been effective I would believe.

R: Definitely, but does the picture replace the actually object. Do you think it would be better if we had the object or a model instead?

Karl: There are times when the model, if available would be better, but I found that in most cases it is difficult to obtain the model at hand.

R: Yes

Karl: An example

R: construction of a bridge like that

Karl: when I taught technical drawing, sometimes it's virtually impossible to obtain certain 3D objects and have them sectioned or cut in half or cut at an angle. So, I find in times that we living technology serves a very, very important purpose in education it really makes life much easier. Ordinarily learners would suffer. With colour at our disposal and the excellent quality of our images, it is something our learners gravitate towards.

R: It‘s almost real

Karl: Absolutely.

R: Because of technology and 3D effects and not 2D planes that you are showing it was like virtually having the object in front of them.

Karl: Absolutely

R: And you could have easily rotated that bridge if you wanted to.

Karl: Yes there are certain programmes that allows us to rotate assuming the learner wants to know what‘s happening behind.

R: Behind?

Karl: Its really symmetrical so they said well they said seeing is believing and if rotated they could see it. In some cases I do the rotation so learners believe.

R: I see you did rotations later on when you spoke about conics and such things.

Karl: Correct, just to reinforce the concept

R: How do you plan or select these pictures or diagrams or manipulatives that you use in your class? Do you ever consider the type of learner you have? Can these pictures or manipulatives or diagrams change each year because your learners change?

Karl: Firstly the selection of the diagrams and pictures and images involves a lot of time and research.

R: which is very evident here.

Karl: It‘s vital and you quite correct, every year the learner composition is never the same.

R: That‘s definitely true.

Karl: So as a result the lesson plan and the images used would vary from year to year. In some cases learners aren‘t able to grasp the concept without the need for an image but in some cases learners desperately struggle to visual, visualise what is going on. So if a learner or a class would be more at a grasping end, it makes it easier for me to extend them, that is to 3D models in certain cases, but to answer the question that you asked last so can these pictures or diagrams.

R: change

Karl: change, yes most definitely it has to change because I haven‘t had a case where the learners were the same every year

R: Exactly the same

Karl: I think you know at our school as well at Crawford we rotate the classes.

R: Yes true

Karl: Some years we have the stronger class, some years we have the weaker classes. So we cannot use the same lesson preparation

R: Definitely

Karl: For 10 years.

R: Beside if you look at, maybe this one here, McDonalds

Karl: Yes the McDonalds

R: If you had to use that maybe 10 years from now who knows what the McDonalds will become?

Karl: Absolutely

R: It may not mean anything to those learners

Karl: Correct or maybe by then McDonalds might replace the parabola with a paraboloid. Who know Mr B, who knows?

R: *laughing* okay. What was your expected outcome behind using these pictures and manipulatives?

Karl: Essentially to show to our learners that Mathematics is not an abstract concept.

R: It‘s brilliant yes.

Karl: It‘s something that is integrated into daily living, daily life and most of the learners had never seen the double parabola in McDonalds before, and after the visual, started to realise that Sir, now we see the relevance. When I had a discussion with them after the lesson, I indicated to them that when you working on a macro scale the equation becomes very helpful and significant in terms of costing.

R: Yes

Karl: Or structures that need to be designed and supports and so on. So learners could see very easily that Mathematics plays a very vital role in the real working world in the real life scenario and they could appreciate it more readily as opposed to the showing them a parabola, a double parabola is an extension of a single parabola in something that they could see.

R: That‘s brilliant actually. They could see the relevance to the real life immediately.

Karl: Yes

R: Because you taught it and then you showed them a bridge. You taught it. You showed them fully that they can relate to things that are in the real world.

Karl: Absolutely

R: Because often you hear learners say, where is this going help me in life? You brought it right into that class.

Karl: As a result of the questions being asked, quite often year after year, I started to bring in the technology and started to show to them and also to explain to the learners as an example if they could leave school at Grade 11, if they desired to go to a field of medicine or maybe Engineering can they become engineers and they say no. I say yes, that is why you need to know the basics, you need to know the foundational Mathematics to help you cope with the more advanced material later on, but having appreciation for the work now the content now drives them to become hungry for more knowledge perhaps….

R: yes

Karl: And especially with some of the learners in the class who are desiring careers in science and in engineering mathematics as well, they could see the beauty of Maths at this young age in the real working world.

R: That‘s another important point that you teach some the beauty of Mathematics apart from the utility value of mathematics.

Karl: Absolutely

R: Look at how beautiful the sign McDonalds is just by merging to parabolas?

Karl: Correct

R: Do you think this could be done differently?

Karl: Well for sure I have been teaching for quite a few years now

R: Yes

Karl: And before the advancement of technology it was taught differently, namely the chalk and talk approach.

R: I still remember that.

Karl: We just had a single coloured chalk and some of our schools barely had chalk. So just with a green chalk board and white chalk we are able to create a concept. At the same time I believe that in that generation learners were forced to visualise, they were compelled to catch it because there was no alternative. Nowadays learners have multiple alternatives, more textbooks have been presented with colour graphics.

R: Yes

Karl: The software relevance, the CD‘s, there‘s online visitation, there‘s you-tube, so learners nowadays have multiple ways of trying to gain better understanding. Back in the day it was done differently. I must admit strange though in many cases it was done differently but with greater with success

R: It‘s an important point to mention there.

Karl: Because learners were forced to visualise, they were compelled, there was no other alternative. If they never grasped it, then they failed. That was something they were not willing to do.

R: Important point you raised there. Okay I noticed that you use paper, scissors and a

ruler to do the investigation on the angle sum of a triangle, why did you choose to use this as a means of conveying your message?

Karl: Well Mr B, I‘ve always inculcated this in my Grade 8 Mathematics class, especially that if a discovery is made by the individual themselves, it will be an investigation they will never forget for the rest of their lives.

R: Okay

Karl: Neither will they forget the results, more specifically, its outcome, how they made the discovery on their own.

R: That‘s so true.

Karl: Obviously as a facilitator but I can say this with quite a deep conviction that they will never forget that activity for the rest of their lives. I‘m quite sure when they have grand children they will still be able to remember the angle sum of a triangle done in that class.

R: That‘s true because, generally whenever you are doing an experiment, that tends to stay longer than when someone that just speaks it out.

Karl: Absolutely and also it brings in an element of fun.

R: Yes

Karl: The learners are concentrating, suddenly they are cutting, they are pasting they are looking at their friends, there‘s some form of an excitement that comes about.

R: Yes even if you observing whilst you are talking there they all engaged in something useful and they are enjoying it.

Karl: Yes in fact they cannot wait to start the activity. I was trying to restrain them at the, the same time but those that were a bit more enthusiastic perhaps started to go ahead of the pack

R: Yes and they actually started rotating this triangle

Karl: Yes

R: We are I think, at question number 11

Karl: Yep.

R: Did you think it is the most effective way of presenting this information?

Karl: In terms of discovery and a hands-on approach I think, with Geometry I would say yes, I would say yes as opposed to being told or doing the rotation, or the power point presentation. The fact that we were able to have done the rotation hands-on I think would have been most effective. They were 100% committed to being part of the experiment as opposed to me just rotating it my way.

R: I quite enjoyed watching that video because I‘ve seen the angle sum of a triangle being taught differently and this was yet another approach where you cut three triangles and you placed them in such a way and they concluded that the angle some of a triangle is180 degrees. I‘ve seen people just placing them on one straight line and use parallel lines and other techniques.
So this was another different take on the angle sum of a triangle, it‘s also very good.

Karl: And I thought quite straight forward

R: Yes

Karl: At least I believe it was, maybe the least complicated way.

R: Yes it‘s very logical.

Karl: I tried not to use parallel lines and not to complicate things.

R: Yes

Karl: But as an add-on, after these learners could see, when I brought in the parallel lines we could have used the same theory again.

R: Yes and besides parallel lines would have required some additional knowledge.

Karl: and some learners yes

R: Wouldn‘t have known.

Karl: Exactly

R: If you used a video clip on the sum of the angles of the triangle, do you think it would be more effective? You don‘t have to agree you can disagree?

Karl: Well it, it could have been perhaps as effective but as I said when a learner does the answer for themselves…..

R: Yes

Karl: I‘m sure you as an individual if you do something hands-on for yourself and it works I, I think you‘d have a great appreciation as opposed to being told, as opposed to watching it been done or see somebody else do it but in fact you use your own hands, your own skills. Also at the same time it allowed me to walk around observing the learners to see if they could follow my instruction, if they could understand what they were doing. So it also helped me gage whether it was an effective form and from my observation I think it was the best possible way for this particular class at that moment in time. I would definitely say so because I had given it some thought. As I said I do research for my lessons before-hand to decide what works best for this particular class. For the Grade 8 class next year maybe different, maybe a visual presentation is sufficient or more than sufficient perhaps.

R: Judging from the responses of the learners they were quite enthusiastic and they seemed to have enjoyed it and I doubt there was any sign of discipline problems, nothing.

Karl: No I think they were too busy engaging.

R: They were very quiet because they were so actively involved in the discussion. Okay Mr Pillay why do you use the data projector in, in your teaching for example you had a worksheet on a projector in one of them, one of the slides you actually put a worksheet on.

Karl: Okay I feel as if the data projector there‘s more opportunity to show a lot of colour. I could walk around the class, be it different advantage points and rotate from one slide to the next.

R: Okay

Karl: As opposed to being in the front all the time. It gave me the opportunity or it does give me the opportunity to be at the back of the class, watch, observe the learners while they watching the power point slide. From the back I could see more clearly were they do not know who I‘m looking at, at the given particular time.

R: Yes that‘s good.

Karl: As opposed to being in the front, where they are all sitting, in front of me, if I look at a learner they are aware that I‘m looking at them specifically but being at the back I could see whose doing what and they do not know at which point I‘m observing which particular learner.

R: That‘s a completely different view point of how effective you know the use of the data projector is, it gives you that freedom to move around.

Karl: Yes and also the, the quality of the graphics

R: Yes

Karl: Mr B it makes you want to look at the image.

R: Yes it‘s attractive

Karl: Yes exactly I carefully use the colour combination if you look at this particular screen shot here I‘ve got triangles in bright yellow, bright blue and bright purple and somehow the learners just grasp it.

R: It definitely captures their attention and then you were using I think you used more….

Karl: To observe I was able to change the colour of the background as well

R: Yes, in fact that‘s one of my questions that are coming up

Karl: It‘s deliberate

R: Your calculations even those were all colour coordinated in such a way that are attractive

Karl: And I believe that by using those vibrant colours it helps to perhaps deepen the little passages

R: Yes

Karl: So learners could have a better recall of the work done, given any example their minds will go back to those colour graphics.

R: right

Karl: And be able to vividly remember or recollect those images and help them to somehow maybe navigate through the current problem. I do it because I have observed that it has made an impact for the good for the majority of my learners.

R: In fact that was an interesting point another one of the participants raised the same thing about colour. She uses a certain colour and she didn‘t know why but she said she used it and she found it the most effective and I engaged in a discussion about that and she tended to agree. Okay do you think it is necessary, do you think it was necessary to use the projector in this case, question number 14?

Karl: Well Mr B we have technology at our disposal. You may have a bicycle? Have you owned a bicycle before?

R: Yes

Karl: You could have used a bicycle to come visit me to have our discussion but I would not ask you why you did not use the bicycle? So something‘s very similar, where it‘s a more advanced form of technology, it‘s how we use technology that makes a difference

R: yes

Karl: Technology is not the solution it‘s the use of technology rather the effective use of, the implementation of the role of the ICT that makes the difference. We did not just because it‘s a data projector, it helps me convey what I need to convey,that‘s the reason why I have information towards it.

R: I notice you change your strategies in presenting lessons, question number 15. Do you think changing your strategies preparing or presenting lessons is important or judging from the fact that you‘ve said that you change?

Karl: Yes

R: Over the years

Karl: I have changed over the years. Who knows, maybe in 5 years from now the approach may be totally different.

R: Yes

Karl: There may be something that‘s in the offing and the power point is going to be replaced at this stage by quite a few other alternatives, like prezzie there was something also called power tune. These alternatives present an interesting angle to using visual presentations.

R: Okay

Karl: in the classroom, so most definitely they are changing the landscape of the ICT platform. Right now I would expect some major changes to be taking place perhaps I‘m just thinking about the 3D data projector coming out you know?

R: Projector

Karl: and they talking about the 3D printer coming out

R: printer as well

Karl: So a lot is going to happen very soon that 3D model we spoke about could be printed in class, who knows it will, even while the lesson is on, be generated, so each learner can feel and have a good grasp of what is been discovered if it‘s a 3D model.

R: Yes

Karl: or perhaps even if they had to rotate their 2D parabola to have a parabaloids. Maybe the 3D printer in the classroom situation, you could have a portable printer that will be able to do that. So, changing strategies obviously would happen.

R: Okay can you explain why you use diagrams in your teaching? When you did the parabola you drew your own parabola on the side.

Karl: Essentially I was trying to show to the learner that in Mathematics it‘s a picture, if an individual is to take out the words from a printer.

R: Yes

Karl: and thrust it to the diagram and that‘s of a picture, that picture is worth a thousand words. It helps to explain some things, which only becomes more easily understandable once the picture is there. As an example a learner may not understand which angle is been asked for until the diagram is drawn.

R: Okay, so true

Karl: I believe that drawing the diagram or interpreting the question and translating that question into a diagram makes the question that much easier to work with. I suppose it makes it less intimidating

R: Yes

Karl: Because our children would gravitate more towards the diagram. I suppose to some words, so by putting in the diagram would contributes to the anxiety being less in some way.

R: Yes

Karl: where they can see certain things a bit more readily and err quite often I do indicate to learners that err if the diagram is drawn then more than half of the battle is won

R: Because a lot of them think in pictures

Karl: Absolutely

R: Okay

Karl: They are visual learners I‘ve always maintained that

R: We spoke about this briefly earlier on I noticed that your power point presentation was a great deal of colour and obviously you take a lot of time to, to go and you know prepare all of those slides using colour and so on. Can you explain the significance of colour in your presentation because you wouldn‘t be doing it if it wasn‘t significant?

Karl: Mr B if you go to the movies today

R: Okay

Karl: And one of the movies showing right now is the Need for speed I think it is

R: Okay

Karl: Similar to the Fast and furious type of a movie

R: It‘s a black and white version of the movie and a colour version it‘s quite obvious that you are going to watch the colour

R: yes

Karl: Okay, because of the enjoyment factor and the quality of the graphics

R: Yes

Karl: I said, there is maybe digital in printing that takes place in the brain when they watch slides of that nature, so as I‘ve said earlier, it helps just to solidify, accentuate perhaps reinforce the concept being taught and so often when I do revision I‘ll tell my class do you remember the red slide with the red that is for revision

R: Okay

Karl: Or do you remember the blue slide, so certain sections under revision conditions are tagged as colour of slides

R: Okay

Karl: So learners would know okay red is for revision all in all but red is also danger

R: Oh yes

Karl: So we looking for revision slide you doomed so learners start to gravitate towards that, it brings a whole new dimension to the lesson in the class

R: It also keeps your audience captive as well

Karl: absolutely

R: You sometimes ring or you underline or you highlight some of the work on the board is there a particular reason for that like when you working with the equation of that parabola the left of the white board you, you ringed it, sort of you went around it with your hand you gestured around it

Karl: Yes this is where data projection comes in because, if you recall 10, 15 years ago we never had the data projector so we relied on body movements, we relied on highlighting and underlining as a means to allow the learners to somehow remember what is been done, our learners learn by observing

R: Okay

Karl: So by them observing me underline which at that moment would be deliberate I would underline, highlight, ring in the hope that when the learners are working with their questions they would mimic what I‘ve done

R: Okay

Karl: And very often I would reinforce the concept and ask them to highlight as an example with the Grade 8 learners when they do like and unlike terms

R: Okay

Karl: and ask them to do like terms sometimes I do tell them it‘s not necessary to have 3 lines if you got 3 ....to have 3 lines and you know and put the likes up together in colour form I tell them to use different colours highlight different colours with sorry the like with different colours like I expert with red and the whites go with blue and so on

R: Okay

Karl: They simply add the colours so, on one line they can answer that

R: So they not going overboard

Karl: Absolutely so it saves time it‘s, it‘s at our disposal, nothing prevents the learner from highlighting on the question paper

R: yes

Karl: Or you rewriting the question and highlighting where we can follow what they done

R: Alright it‘s an interesting perspective because it‘ll save a lot of time where in Grade 8 we used to teach them column wise

Karl: Absolutely

R: And Grade 9, go horizontally and then they get confused

Karl: And I find that the learners find the highlighting better because they firstly the, the thrill of highlighting is something they enjoy always and just to see the, the colours in like terms in adding and subtracting makes it that much enjoyable I think as well

R: Yes

Karl: Less, less intimidating

R: That‘s it, you use highlighting I use shapes so, I‘ll ring all the X squared in navy maybe ring them

Karl: Yes

R: And all the term in X is with triangles so the same idea

Karl: Okay

R: of highlighting but in using a shape or something. Okay you have used animations in your presentation like in that bridge example if you can remember

Karl: Yes

R: you actually had a flashing light

Karl: I was throwing some light onto the situation

R: So is it a particular reason for animating

Karl: Yes for sure I think the greater the extent to which we could animate our lessons I think the greater the impact to the classroom because sometimes I would use that as a key point later on perhaps I may have in a particular reason previously in 3, 4 lessons down the line I would have highlighted them the deliberate movement perhaps like in this case if it was the light sometimes it could be the collapsing of a bridge or whatever it could be but it‘s deliberately done with the main purpose of allowing the learner to recall with purpose when I do present the scenario again at a later stage

R: Makes sense

Karl: Makes sense

R: Then I‘ve also observed that you prefer dynamic presentations because, most of your slides were not still you actually in that slide itself, you got the equation to come on then you got arrows to come in, you‘ve got dotted lines that come in, into the presentation so it‘s not a still boring presentation even though the parabola can be like a boring topic itself but you brought in all those things

Karl: Yes I feel that by having a dynamic presentation it helps the learners to be a bit more involved in the lesson because they watch they not sure which direction the parabola may come flying next okay top bottom left. I think what would have been better I have been working on this where I use sound effects as well

R: Oh that‘ll be great

Karl: with one of the lessons the Grade 11‘s ....Geometry when the maybe the tangent is coming in from an angle they come in with a boom or a thunder something comes in

R: *can‘t hear audio clearly*

Karl: Yes or a thud so in some cases it‘s a stunt work for ferrari taking off you know so the sound effects come with the dynamic movements and I think that gives you the very well rounded effects in terms of the sound and the visuals.

R: So it‘s almost like sitting in a movie house

Karl: Well that‘s what I was trying to mimic because to bring in some form of excitement and as I said very soon power point would either evolve or maybe replace by a few other more what‘s user friendly but

R: Trendy

Karl: trendy but maybe packages that are a bit more advanced

R: advanced

Karl: Yes

R: I think Prezzie is doing quite well

Karl: prezzie is well but prezzie as well, the slide flies around more than objects here in power point you can let the objects fly

R: Well then the last question you more or less answered what was the expected outcome behind the presentation but you‘ve already covered that by using dynamic presentation to create this whole new

Karl: I think there was, the, the main goal is to obviously convey the concept but to keep the learner engaged as much as possible during the lesson

R: That‘s true

Karl: I think your studies you also reveal in your literature survey that there has been a trend of a diminishing concentration span of our current generation of learners hence we have to do a bit more to achieve the same results that we got 15, 20 years ago and as you know it‘s always our fault as educators it‘s never the child‘s fault so what are we doing as the educators. This is one possible way to show to you that, it‘s an effort, the content has not changed but the calibre of the learner has changed and hence we‘ll have to do a bit more than what we normally did in the past to achieve the same type of result

R: And also because their I think, their worlds itself is so different, they, they bombarded by technology and they want something as exciting

Karl: So I think using this platform allows them to be comfortable on a platform that they comfortable with

R: That‘s true . Okay Mr Pillay thank you so much for your time and your very sincere responses. Like I said earlier on thank you so much for making a sacrifice of coming here and meeting me and you could have very well been doing something else somewhere so, hope you enjoy the rest of your afternoon is there anything you want to say?

Karl: Well Mr B it was an absolute pleasure I think the research would be used quite critically for future research is what we have to say and thank you for giving me the opportunity to be part of your research programme it‘s much appreciated *phone ringing in the background*

R: Okay no you can answer that call. Thank you.

Appendix E

UNIVERSITY OF
KWAZULU-NATAL

Introductory Letter

Dear ___________________________________

The purpose of this letter is to introduce myself, explain the research I am undertaking and to invite you to participate in the study.

My name is Visham Budaloo, an educator at Manor College. I have registered for a PhD at UKZN in 2012 and the aim of the research is to explore visual reasoning as a pedagogical tool in successful mathematics teaching. The issue that I plan to explore is how successful mathematics teachers use visual reasoning in their classrooms.

The analysis of this study is to form my thesis submission to the University of KwaZulu Natal for the award of the degree, Doctor of Education. In raising awareness around successful mathematics teachers, the study will create a space for other teachers to try to improve their practice. The study will also add to the growing body of research in mathematics education.

As I mentioned above, the purpose of this letter is to invite participation in the study. It is anticipated that the research will be undertaken by using multiple data methods such as in-depth semi-structured interviews, video recordings and document analysis, The sources of all information and opinions provided will remain anonymous to everyone barring myself. Please note that there is absolutely no binding commitment to continuing if at any stage you would prefer to discontinue.

In replying to this e-mail, please feel free to ask any questions. I thank you for your time and look forward to your reply.

Yours sincerely

Visham Budaloo

Letter of Consent from Principal: Interview

Dear ______________________________________

I am currently a PhD learner at the University of KwaZulu-Natal. There is tremendous interest throughout the world in the way mathematics is being taught at schools and universities. My interest is in the area of Mathematical Visualization. This research that I am conducting will assist us in making the teaching and understanding of Mathematics much better. As part of my research, I would like to interview a few educators in their classrooms, with particular focus on the educator only.

If you agree, s/he will be interviewed for approximately 60 minutes and will be digitally recorded. The data from the interviews will only be used for my research purposes and will not be used for any other purpose without their consent. The recordings will be lodged with the university authorities. Participation is voluntary and your staff is not obliged to answer all the questions that I ask him/her and is free to withdraw from the process at any time. Please note that no real names will be used in any material that I write up and every attempt will be made to keep the material confidential.

Thank you for your assistance. If you require any further information, please contact me, or my supervisor Dr Vimolan Mudaly at 031 2603682.

Yours sincerely

____

Mr V. R. Budaloo
(0845800861)

DECLARATION

I, ______________________________________ (full name of Principal) confirm that I understand the contents of this document and agree to allow my educators to participate in the above mentioned research study. I understand that the school name will not be used in any write-up and that the educators‘ responses will be treated confidentially. I also understand that he/she will not be under any threat to participate and is at liberty to withdraw from the study at any time.

Signature: ______________________________ Date: ______________

UNIVERSITY OF KWAZULU-NATAL

Letter of Consent from Mathematics Educator: Interview

Dear ____________________________________

I am currently a PhD learner at the University of KwaZulu-Natal. There is tremendous interest throughout the world in the way mathematics is being taught at schools and universities. My interest is in the area of in Mathematical Visualization. This research that I am conducting will assist us in making the teaching and understanding of Mathematics much better. As part of my research, I would like to interview a few educators on their understanding and use of visual reasoning in their classrooms.

If you agree, you will be interviewed by me at the school. The interview will take approximately 60 minutes and will be digitally recorded. The data from the interview will only be used for my research purposes and will not be used for any other purpose without your consent. The recordings will be lodged with the university authorities. Participation is voluntary and you are not obliged to answer all the questions that I ask you and you are free to withdraw from the interview at any time. Please note that no real names will be used in any material that I write up and every attempt will be made to keep the material confidential.

Thank you for your assistance. If you require any further information, please contact me, or my supervisor Dr Vimolan Mudaly at 031 2603682.

Yours sincerely

____

Mr V. R.Budaloo
(0845800861)

DECLARATION

I, _____________________________________ (full name of educator) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to participating in the research project. I understand that my real name will not be used in any write-up and that my responses will be treated confidentially. I also understand that I will not be under any threat to participate and is at liberty to withdraw from the study at any time.

Signature: _____________________________ Date: ______________

Letter of Consent from Principal: Video Recording

Dear ______________________________________

I am currently a PhD learner at the University of KwaZulu-Natal. There is tremendous interest throughout the world in the way mathematics is being taught at schools and universities. My interest is in the area of Mathematical Visualization. This research that I am conducting will assist us in making the teaching and understanding of Mathematics much better. As part of my research, I would like to video-record a few educators in their classrooms, with particular focus on the educator only.

If you agree, s/he will be recorded for the duration of a lesson. The data from the recordings will only be used for my research purposes and will not be used for any other purpose without their consent. The recordings will be lodged with the university authorities. Participation is voluntary and your staff is not obliged to answer all the questions that I ask him/her and is free to withdraw from the process at any time. Please note that no real names will be used in any material that I write up and every attempt will be made to keep the material confidential.

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Signature: ______________________________ Date: ______________

Letter of Consent from Mathematics Educator: Video Recording

UNIVERSITY OF KWAZULU-NATAL

Dear ____________________________________

I am currently a PhD learner at the University of KwaZulu-Natal. There is tremendous interest throughout the world in the way mathematics is being taught at schools and universities. My interest is in the area of Mathematical Visualization. This research that I am conducting will assist us in making the teaching and understanding of Mathematics much better. As part of my research, I would like to video-record a few educators in their classrooms, with particular focus on the educator only.

If you agree, one of your lessons will be video-recorded. The data from the recording will only be used for my research purposes and will not be used for any other purpose without their consent. The recordings will be lodged with the university authorities. Participation is voluntary and your staff is not obliged to answer all the questions that I ask him/her and is free to withdraw from the process at any time. Please note that no real names will be used in any material that I write up and every attempt will be made to keep the material confidential.

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Signature: ________________________