AN EXPLORATION OF FET MATHEMATICS LEARNERS' UNDERSTANDING OF GEOMETRY

BY

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DEDICATION

This work is dedicated

To:

My children, **Tinotenda and Makanaka**

My wife, **Mollen**

My Mother, **Mrs Loice Ngirishi (Ngorima)**

My Father, **Mr Neva Ngirishi;**

Who were the original sources of my inspiration.

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- All the learners who contributed in this work, it was their participation which allowed this study to be completed.

DECLARATION

I, Harrison Ngirishi, declare that this study is my own work, submitted in partial fulfilment of the requirements of the degree of Master of Education at the University of KwaZulu-Natal and that this dissertation has never been submitted at any other university or institution for any purpose, academic or otherwise.

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DATE……………………………………………………………………………………….

As the candidate's supervisor, I agree to the submission of the thesis/dissertation

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Prof Sarah Bansilal

DATE……………………………………………………………………………………….

TABLE OF CONTENTS

FIGURES USED IN THE STUDY

List of Tables Used in the Study

ABSTRACT

This research study explored the FET learners' understanding of geometry. The aim of this research study was to explore how grade 10 and grade 11 learners perform on tasks based on basic geometric concepts. The research also aims to make deductions about the van Hieles' levels of geometry thinking of the learners. The research study will also provide recommendations on how some of the arising issues could be addressed.

The study is framed within the theoretical framework of social constructivism and is located within the qualitative research paradigm. This study was carried out in three high schools in rural KwaZulu-Natal, South Africa. The study used qualitative data analysis methods to analyse data generated through task based worksheets and semi-structured interviews for individuals. A total of 147 learners completed the task based worksheet, of which 74 learners were doing grade 10 and 73 were doing grade 11. Eighteen learners were invited to participate in the interviews after the analysis of the task based worksheets.

The research revealed a lack of understanding of many geometric concepts by the learners. Learners had difficulties with problems involving definitions of geometry terms, interrelationships of properties and shapes, class inclusion and proof type questions. Learners also showed lack of procedural and conceptual understanding. The study also revealed that the majority of the learners involved in the study were operating at the visual level and analysis level of the van Hiele levels of geometry thinking with a few learners able to reason at the informal deduction level.

The research recommends that educators should explain and use relevant geometry vocabulary in their everyday teaching of geometry to try and address the issue of the language barrier, allow learners to work with a diversity of registers of semiotic representations of geometric concepts instead of sticking to the traditional geometric figures only, expose learners to shapes involving common properties (class inclusion) and allow them to come up with their own conclusions, use manipulatives, real life objects and cite real life examples when teaching geometry to make it more relevant to everyday life, use teaching methods that encourage conceptual understanding instead of rote learning and provide learning experiences that improve learners' proof skills.

CHAPTER ONE: INTRODUCTION

1.1 Rationale and motivation of the study

The quality of teaching and learning in South Africa has been a concern for teachers, parents, education officials and ordinary citizens (DoBE, 2011). Many reports and studies point to the poor achievement in mathematics by learners as particularly problematic (DoBE, 2015a; DoBE, 2015b; Reddy, 2005). The 2014 results in mathematics in Grade 12 in the country show that 22.3% of the group who wrote mathematics achieved a pass of 50% and above (DoBE, 2015a, p. 135). It is clear that learners are not performing well in mathematics, most especially in Euclidean geometry (Maree, Aldous, Hattingh, Swanepoel & van der Linde, 2006; DoBE, 2011b; DoBE, 2012e; DoBE, 2012f).

Many researchers have expressed concerns about the poor performance in mathematics, especially in geometry (Lee & Ginsburg, 2009; Mthembu, 2007; Singh, 2006). According to Patkin & Lavenberg (2007) geometry is perceived as the most complex part of the curriculum and students often assume that it does not relate to their daily life. This may be part of the reason why geometry has posed a serious challenge to most learners and teachers in South African schools. Cassim (2006) stated that learners' performance in school geometry, especially in grade 12 has been inadequate, a view that is supported by the 2006 Trends In Mathematics and Science Study (TIMSS) report on learners' poor performance in geometry which reported that South African learners performed worst in Mathematics out of the 50 participating countries and that the weakest area of performance was geometry (Reddy, 2005). De-Villiers (1997) commented that South African learners have been performing more poorly in geometry than in other strands such as algebra. The view was supported by a study by Howie (2001), which reported that South African learners had difficulties dealing with geometry questions and in some cases they were easily distracted by questions testing misconceptions in geometry. Similarly Roux (2003) reported that high school learners' mathematical performance in South Africa appears to be unimpressive, more especially in geometry. It is therefore important that all stake holders pay urgent attention to mathematics teaching and learning, especially geometry, to try and address these challenges.

Geometry was relegated to an optional paper in Mathematics under the National Curriculum Statement (NCS) in grades 10, 11 and 12 (DoE, 2006). Learning Outcome number 3 and Specific Outcome number 2 for grades 10, 11 and 12 were considered optional mathematics assessment standards for examinations (DoE, 2003), and the optional section was geometry. Making geometry optional resulted in many school teachers avoiding it, which then limited their learners' exposure to the type of reasoning associated with geometric understanding. According to Bowie, 2009, geometry was difficult for learners and educators and the mathematics pass rate for most schools was below average as most schools chose not to write the optional paper to boost their results. However in the new curriculum referred to as the Curriculum and Assessment Policy Statement (CAPS), geometry was brought back as a compulsory chapter in 2012 (DoBE, 2011). It is now compulsory for all schools teaching mathematics at grade 10, 11 and 12 to teach geometry to their learners. The diagnostic Grade 12 report on the 2014 national examinations carried out a diagnostic analysis of a sample of grade 12 mathematics scripts and found that the lowest average in that sample for the second mathematics paper was in the question on Euclidean geometry (DoBE, 2015). The report recommended that learners need to spend much time in solving geometry problems so that their skills in this area can be improved. However in my own teaching experience, I have noted that learners are afraid of geometry and prefer to spend most of their time going over the algebra and statistics sections of mathematics. When I tried to probe the reasons for the learners' fear of geometry, some learners reported that the mathematics teachers often skipped the sections on geometry and so they tried to cover the topics by themselves with little success.

Hence there is a need to find out more about the particular areas of geometry that pose challenges to the learners and why these areas pose problems. It is therefore important for teachers and researchers to learn more about geometric concepts that current learners are struggling with. Research in mathematics education will help identify ways to address some of these challenges faced in mathematics. This study was carried out in an attempt to improve the mathematics performance by exploring the knowledge and understanding that FET learners have on selected geometric concepts.

1.2 Focus of the study

This study's primary focus is to explore FET mathematics learners' understanding of geometry. According to Schoenfeld (2000) research purposes in mathematics can be pure or applied, where the pure purpose involves knowing the environment of mathematical knowledge and the accepted wisdom, teaching and learning, while the applied purpose is to use this understanding to improve mathematical instruction. This study intends to fulfil both the pure and applied purpose as it intends to explore FET learners' geometric knowledge and to then make recommendations about how the learning and teaching can be improved.

The study sought to answer the following questions:

1. How do grade 10 and grade 11 learners perform on tasks based on basic geometric concepts?
2. What can be deduced about the van Hiele levels of geometric thought of the learners?

I hope that the study will contribute to an understanding of the challenges encountered by FET learners when solving tasks based on geometry. The study will also come up with some recommendations on ways in which the situation could be improved. By identifying particular issues relating to the poor understanding of geometry, teachers, researchers and other stakeholders can be better informed when trying to address problems.

1.3Outline of the study

This study consists of six chapters, the references and the appendices. The contents of these chapters are briefly highlighted in this section.

- Chapter One
  Chapter one of this study gives an overview of the study, the rationale and motivation for the study. It also provides the focus, the purpose and the outline of the study.
- Chapter Two
  Chapter two provides the literature review relevant to this study in terms of what other researchers say about the teaching and learning of geometry and their findings.
- Chapter Three
  Chapter three describes the theoretical framework underpinning this study which is social constructivism.
- Chapter Four
  The research paradigm and research design appropriated in this study will be discussed in this chapter. The methodological procedures undertaken to complete this study are discussed in this chapter.
- Chapter Five
  This chapter presents the data of this research, the analysis of the data and the interpretation of the findings. Common themes of the results will be grouped together with pertinent issues raised and the main findings.
- Chapter Six
  Chapter six of this study will summarise the main findings of the study and will discuss how the research questions of the study were answered. Other issues arising from the findings will be discussed and recommendations will be made. The possible limitations of this study will also be highlighted in this chapter.

CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

In this section I will first look at the South African mathematics curriculum, followed by the definition of geometry as viewed by other researchers and thereafter look at the role of geometry in the understanding of mathematics. This is then followed by a review of relevant studies carried out in geometry and I also present a review of research issues concerned with misconceptions. Then I will briefly look at the task of teachers in facilitating learning before concluding the review with a discussion of conceptual and procedural understanding in mathematics. For ease of navigation, this review is arranged in terms of the relevant headings.

2.2.1 Mathematics Curriculum in South Africa

The South African curriculum emphasizes the knowledge and values that should be fore-grounded in schools (DoBE, 2011). The NCS was designed to promote learning for understanding and application in the real world as compared to learning in order to pass examinations only. The curriculum aims to cater for learners of different social statuses, different abilities, different physical abilities, different races, gender and intellectual abilities (DoBE, 2011). The NCS also aims to provide access to higher education, to make it possible for students to move smoothly from education institutions to the work place. It also provides employers with an adequate history of a learner's ability in the form of school progress reports (DoBE, 2011).

The new Curriculum and Assessment Policy Statement (CAPS) describes mathematics as consisting of several branches involving symbols, notations, graphs and geometrical figures while having its own language (DoBE, 2011). Mathematics has many operations which work together and include representation of information, observing data, investigating patterns and analysing information (French, 2004). Understanding mathematics makes it easier to adapt to the world around us, as many human activities makes use of mathematics, like family budgets, savings, buying and selling. Geometry is one branch of mathematics that requires our special attention because it extends spatial awareness, it helps develop the skills of reasoning and it informs challenges and stimulation (French, 2004).

The previous South African mathematics curriculums all included geometry in all GET and FET grades, but from 2008 to 2011, geometry was made optional in grades 10-12. The implementation of the new Curriculum and Assessment Policy Statement (CAPS) in 2012 reinstated Euclidean geometry as a compulsory section of mathematics after it had previously been relegated to an optional Paper Three (DoBE, 2011).

In grade 10 geometry, learners are required to revise basic concepts introduced in the previous grades and to investigate the properties of special quadrilaterals. It also includes the revision of problem solving and shapes, which were covered in the previous grades. In grade 11 geometry, theorems on circles are introduced with the assumption that the work covered in the previous grades is known. The breakdown of the topics in the Euclidean Geometry sections indicate that the teaching of geometry should be done in a manner that builds upon the work done in previous grades, for example, in Grade 12 learners need knowledge of similar and congruent triangles, a topic covered in grades 9 (DoBE, 2011).

In the next section I will provide the definition of geometry before looking at some misconceptions generally found in geometry.

2.2.2 Geometry

Geometry involves the relationships and properties of shapes according to Bassarear (2012). Knight (2006) defined geometry as a section of mathematics that has different applications in science and technology which include the construction industry, design and architecture amongst others. This notion is supported by Chambers (2008), who defined geometry as an exploratory field of mathematics that has links with the real world. The relationship between geometry and everyday human activities creates interest in the learning of geometry (Chambers, 2008). Soanes and Stevenson (2009) defined geometry as the branch of mathematics that deals with solids, surfaces, lines, points, angles, properties, measurements and relationships appropriate to them and their positions in space. Egyptians used geometry in their history of land measuring whereas Greeks came up with properties of different shapes (Cooke, 2007).

Discussions in geometry can create an authentic environment in mathematics classrooms making mathematics more interesting (Chambers, 2008). Usiskin (2002) stated that there are two reasons why geometry is important to teach: it connects mathematics with the real world and it enables ideas from other areas of mathematics to be pictured. According to French (2004) there are three reasons why geometry is included in learning and teaching, which are; it extends spatial awareness, it develops the skill of reasoning and it informs challenges and stimulation. Usiskin (2002) also mentioned the importance of geometry to other parts of mathematics such as in the learning of the distributive property which can be related to learning using an area model. Students may get a deeper perspective of the world if they study geometry as it improves their reasoning capacity and relates well to other branches of mathematics (Ozerem, 2012).

According to Piaget (1971) children understand more geometric concepts as they grow and that they need to work with the shapes physically for them to have a deeper understanding. Clement, Swaminathon, Hannibal & Saram (1999) stated that children's understanding of shapes is because of the child's mental abilities and internalised actions whereas van Hiele (1986, 1999) studied how children learn geometric concepts. The formulation of the van Hiele theory has led to much progress in the teaching and learning of geometry.

An important aspect of geometry is the role of proof. A proof is a formal written argument of the complete thinking procedures that are used to reach a valid conclusion. The procedures are supported by a theorem, postulate or definition verifying the validity of each step and explaining why these steps are achievable. According to Mudaly & De Villiers (2004), the underlying principles for including formal proof in geometry in the schools are that it is perceived as a medium for teaching and learning deductive reasoning and is a first encounter with a formal axiomatic system. Serra (1997) stated that a proof in geometry consists of a sequence of formal statements and should be supported by valid reasons. A proof begins with a set of given properties and concludes with a valid conclusion. Likewise De Villiers (2004), also views a proof as a formal written argument of the complete thinking procedures that are used to reach a valid conclusion; the steps of these procedures are supported by a theorem, postulate~~s~~ or definition verifying the validity of each step and explaining why these steps are achievable.

Proofs assist learners in developing reasoning and proving abilities, forming conjectures, evaluating arguments (Christou, Mousoulides & Pitta-Pantazi, 2004). According to Webber (2003), teachers have established a number of functions of proof writing which includes explanation, systematization, communication, discovery, justification, intuition development and autonomy. In a similar way Hanna (2000), described functions of proofs as proving, verification, explanation, systematization, discovery, communication, construction, exploratory and incorporation.

Verification and explanation include the product of the long historical development of mathematical thought and thus are considered as the basic essential functions of proof. Verification indicates a correct statement while explanation refers to a reason why the statement is true and correct. De Villiers (2004) looked at a model for the function of proof as verification. Varghese (2009) lamented that very few learners in his study pointed out explanatory functions to secondary level pre-service teachers. The validity of a guess may be supported by explanation alone, although it does not provide reasons why it may be true. Knuth (2002), stated that systematization is the organization of different outcomes that will constitute a proof. Proofs play a vital role in the discovery of new outcomes or relationships.

Mathematical reasoning can be broadly described as inductive or deductive. In deductive reasoning, valid conclusions are based on previously known facts. Deductive reasoning is a valid form of proof and it is the way in which geometric proofs are written. Given below is an example of deductive argument:

- All quadrilaterals with sides and angles that are equal are squares
- The window has four equal sides and four equal angles.
- Therefore the window is a square.

The first point gives the known definition of a square with its properties and the second statement gives the characteristics of a window as one of the figures identified in the first statement. A conclusion is then reached that the window is a square because it has all the characteristics of a square.

When one is using inductive reasoning, one starts from the particular and proceeds to a general conclusion. Inductive reasoning plays an important role in doing proofs from the observations. Serra (1997) mentioned that inductive reasoning is based on the observation of geometric figures and have a conjecture about them. Mason, Burton & Stacey (2010), stated that a conjecture is a statement which appears reasonable, but whose truth has not been established. A conjecture may be justified on the basis of direct, conditional or indirect proof.

In the modern sense, geometry includes any mathematical system that is developed from a set of statements that are called axioms or postulates. When teaching geometry the teacher must consider various factors like how the learners understand, perceive and think about geometry. As geometry deals with positions, shapes and visualization, it is important for teachers to consider the development of the levels of geometric thinking proposed by the Van Hiele model.

One other aspect in the teaching and learning of geometry is the understanding of misconceptions and errors that learners make when working with geometric concepts. Some of the issues involving misconceptions and errors will now be discussed in the next section.

2.3 Misconceptions and Errors

2.3.1 Misconceptions

I will now provide a brief review of research based on misconceptions. Misconceptions are often formed because learners and the teacher fail to understand each other during the learning process. Michael (2001) defined misconceptions as lack of understanding of content that may prevent a learner from understanding subject matter; while to Swan (2001) misconceptions are parts of the natural process of learning. I concur with Drews (2005), who believed that a misconception can be caused by wrong application of a formula or limited understanding of a particular situation. However Macabre (2005), argues that misconceptions stem from learners' prior knowledge and are very resistant to change. A misconception according to Smith, Disessa & Roschelle (1993), is a conceptual structure constructed by the learner, which makes sense in relation to his/her current knowledge, but which is not aligned with conventional mathematical knowledge. Confrey (1987), supported the same view when he states that constructivists identified a misconception when a relatively stable and functional set of beliefs held by an individual comes into conflict with an alternative position held by the community of scholars, experts and teachers as a whole.

Nesher (1987) stated that a single misconception produces a cluster of errors since mathematical errors and ideas are hierarchical and closely connected. Misconceptions can lead to more complicated and multiple problems if they are not rectified early. Studying misconceptions and errors is extremely important in mathematics, especially in geometry. Setati (2002) argued that mistakes create difficulties for learners, a view that Shannon (2002) agreed with as being a big impediment to meaningful learning.

Misconceptions result in mathematics being viewed as difficult and dull (Confrey, 1987). This creates a negative attitude towards mathematics in what Shaughnessy & Burger (1985), referred to as mistakes being associated with negative feelings. A study by Mestre (1989), found that learners are emotionally and intellectually attached to their misconceptions because they have actively constructed them. According to Makhubele (2014), misconceptions and errors result in the emotional disposition of a set of emotions like fear, anxiety, frustration and rage which often threatens both performance and participation in mathematics.

Learners have many misconceptions in geometry which teachers come across in their daily lessons (Chazan, 1993). Teachers normally present proofs by presenting routine examples on the chalk board or electronic equivalent while learners take down notes at the same time while also trying to make sense of what the teacher is teaching. According to Sweller (1988), when learners are supposed to learn and understand so many concepts and activities at the same time they may experience a cognitive overload which results in inefficient learning. Van Der Sandt & Nieumwoudt (2003) propose that teachers need to help learners to overcome their misconceptions for them to progress in conceptual understanding.

2.3.2 Errors

Closely linked to the issue of misconception is the notion of errors. There are many different ways of describing errors. According to Harper (2010), the word error originally meant to wander or go astray. Harper (2010) defined an error as a deviation from accuracy or correctness, a mistake or a speech belief in something untrue, holding of mistaken opinions and errors are witnesses of a learner's misunderstanding or habits. To Merenluoto (2004), an error could be a mistake caused by a fault, the fault being misjudgement, carelessness or forgetfulness. Nesher (1987) argued that errors are systemic, persistent and pervasive mistakes performed by learners across a range of contexts. Errors are therefore mistakes made by learners due to lack of concentration, poor memory or lack of conceptual understanding.

Luneta (2008) defined errors as signs of the challenges experienced by learners in a learning process. However Swan (2001) regarded an error as a mistake or lack of understanding of the information given. Errors are a result of misconceptions. Errors are therefore symptoms of misconceptions that learners have. Swan's view is supported by Confrey (1990), who believes misconceptions are a result of a wrong perception of a concept.

There are two types of errors which Riccomini (2005) differentiated as systematic and non-systematic errors. Non-systematic errors (also referred to as slips, lapses or unintended mistakes) are mistakes which learners make un-intentionally and they are easy to correct with no assistance (Riccomini, 2005). Van Lehn (1982) referred to non-systematic errors as inconsistent and being less predictable and emanating from fatigue and carelessness. According to Olivier (1989), non-systematic errors can easily be corrected when identified and normally these errors are unintended. Makhubele (2014) stated that a learner who is engaged in a task can be overconfident, impulsive or hurried and this can lead to more careless errors.

In contrast to the non-systematic errors, Riccomini (2005) described systemic errors as symptomatic of a faulty line of thinking causing them to be referred to as misconceptions. These errors are often repeated and thus they symbolise underlying incorrect hypotheses. Systematic errors are procedural and often feature an incorrect routine in an otherwise correct method. According to Brown & Burton (1978), systematic errors are bugs of a procedural nature involving an incorrect routine. Riccomini (2005) believes that these types of errors occur when a learner is faced with a difficult or unfamiliar feature of a task that leads the student to an impasse in which a learner will modify a known procedure and incorrectly apply it to a task.

Understanding the nature of the error and why it was made is a crucial process for learners and teachers when they try to review errors. Watson (1980) conducted a study using the Newman model which stated that all errors can be placed into eight different categories. The eight types of errors according to Watson (1980) are reading ability errors, comprehension errors, transformation errors, process skills errors, encoding errors, motivation errors, carelessness errors and question form errors. Elbrink (2008) also devised his own classification of errors, consisting of four categories namely mechanical errors, application errors, careless errors and order of operation errors.

Booker (1988) came up with a classification of errors that was slightly different from that of the other researchers mentioned above, namely that of two main groups differentiated into mathematical errors and didactic errors. Mathematical errors are made when people (educators or learners) consider valid relationships or deductions as false or vice versa (Booker, 1988). Mathematical errors include but are not limited to omission of essential characteristics in a given class of objects; inclusion of unessential characteristics into the definition when defining mathematical concepts and application of definitions; using hypothesis without testing the proposition in theorem understanding and application and finally to quick and unjustified generalizations made on the basis of observing a few particular cases (Booker, 1988).

To Booker (1988), didactic errors refer to situations when educators' behaviours are opposed or not in line with the normally accepted methods. Some of the educator's errors associated with teaching a class include the following: incoherent structure of teaching content; using complicated examples or examples which are too easy; unsuitable selection of problems for use in aim realisation and understanding the necessity to master the basic skills by students, such as correct calculations, representation and comprehension of the data on graphic representation of geometric figures and inappropriate educator's response to learners' errors (Booker , 1988).

According to Movshovitz-Hadar, Zaslavsky & Inbar (1987), educators may use the different classification techniques to identify repeated errors and mistakes in different topics and within a topic, including geometry. When the persistent tendencies are identified, effort can then be channelled into finding ways and methods to address and correct the misconceptions and errors.

It is essential for teachers to have information about their learners' misconceptions and resulting errors if they are to design effective teaching and learning programmes. Various factors are attributed to misconceptions and the resultant errors. Students' backgrounds and the context within which learning takes place may be sources of misconceptions. Luneta (2008) identified, amongst other factors, a lack of conceptual knowledge, different learning styles, different levels of cognition, lack of prior knowledge of the concept, lack of concentration during instruction and language barriers as some of the reasons why misconceptions are formed.

2.3.3 Some misconceptions and errors in geometry identified in research studies

Luneta & Makonye (2010), in their study on mathematics and science misconceptions, are of the view that although errors and misconceptions are related, they are different. They regard an error as a mistake, slip, blunder or inaccuracy and a deviation from accuracy. According to Luneta & Makonye (2010), errors are visible in learners' artefacts such as written text or speech whereas misconceptions are a result of wrong information regarding a certain aspect which may result from prior knowledge on the given aspect.

Research done by de Villiers (1997), Siyepu (2005) and Roux (2003) found that learners' performance in U.S.A. high schools is poor when it comes to items involving understanding of features and properties of shapes, the very fundamentals of geometry understanding. This was confirmed by Siyepu (2005), whose research study found that secondary school learners in South Africa cannot identify and name shapes like the rhombus, kite, trapezium, parallelogram and triangle. This is consistent with the research done by Ateba (2008), who conducted a research study involving South African and Nigerian learners, with special focus on geometry. The research found that both South African and Nigerian learners had problems in naming shapes stating reasons. Atebe's research found that the majority of learners described shapes entirely by the properties of sides while neglecting their angle properties and many of the learners in his study had inadequate knowledge of geometric terminology.

Research done by Brombacher (2001) and Howie (2001) also confirm the problem that learners can't recognize basic geometrical shapes, can't correctly define basic geometrical shapes, don't know the correct properties of shapes, etc. Clement & Battista (1992) in their research study, found that many high school learners reason a square as not a square if its base is not horizontal. This was also confirmed by Mayberry (1983), whose research reports that some learners in her study had difficulties in recognizing a square with a non-standard orientation. According to research by Marchis (2008), learners recognised a square as not a rhombus and a rectangle as not a parallelogram. According to Marchis (2008), two thirds of the learners in his study could not correctly define basic geometrical shapes nor did they know the correct properties of the shapes. This was also confirmed in the findings of Clements & Battista (1992), who found that only 64% of the 17 year olds in U.S.A. knew that a rectangle is a parallelogram. Therefore Clements & Battista (1992) deduced that the problem is that learners sometimes reason with a concept image rather than with concept definition.

According van Hiele (1999), the ability to recognise and name shapes has been recognized as important for geometric conceptualization. Most research evidence indicate that many high schools learners lack the ability to correctly identify, name and classify many simple geometric shapes (Clements & Battista, 1992; Marchis, 2008; de Villiers, 1997; Siyepu, 2005 and Roux, 2003). This was confirmed by Feza & Webb (2005), who found that many learners have difficulties in perceiving class inclusion of shapes, for example, they might think that a square is not a rectangle.

In a study conducted by Atebe (2008), the task sorting of shapes revealed some important misconceptions about geometric concepts among the learners. Thirty six students participated in his study (18 Nigerians and 18 South Africans). There were 8 learners (2 Nigerians and 6 South Africans) who reasoned that all 4-sided shapes are called squares. There were 4 other learners (3 Nigerians and 1 South African) who reasoned that all 4-sided shapes were called parallelograms. Rectangles, squares and rhombuses were all excluded from the class of parallelograms by all the learners who also failed to perceive squares to be rectangles. Right-angled isosceles triangles were excluded from the class of right angled triangles. Learners were not able to perceive the relationship between the properties of a shape and between different shapes.

2.4. Conceptual Understanding and Procedural Fluency

Mathematical knowledge acquisition is the transformation of knowledge from the forms in which it exists, (contexts, texts and mathematics teachers' heads), into forms that the learner can understand and use. Misconceptions are formed and errors occur when learners fail to incorporate or acquire procedural and conceptual knowledge associated with their concept images. Mathematics education researchers have generated enormous amounts of work which have focused on distinguishing between conceptual and procedural understanding of mathematics understanding (Mamba, 2011; Mestre, 1989; Michael; 2001 and Olivier, 1989). Mathematical procedural knowledge is usually specific to particular tasks, while mathematical conceptual knowledge is often more generic.

Schneider & Stern (2010) defined conceptual knowledge as being achieved by linking existing knowledge to new knowledge or the construction of relationships between pieces of information and procedural knowledge as that which enables people to quickly and effectively solve problems. Schneider and Stern (2010) defined conceptual knowledge as the link between pieces of information or knowledge in a certain domain and procedural knowledge as that which enable a person to follow certain steps to solve a problem. Teachers, who are grounded in the conceptual knowledge of mathematics including geometry, teach in ways that ensure that the learners are exposed to mathematical knowledge and understanding through investigation, exploration, discussions and sharing of ideas (Zakaria & Zaini , 2009). Faulkenberry (2003) regarded conceptual knowledge as one that relates to the principles that refine understanding of mathematics.

Teaching for conceptual understanding is very important for teachers of geometry because it enables them to define and explain geometric concepts in ways that enable learners to understand and articulate geometry in their own but correct ways and this is a central process of the learners' knowledge construction. According to McCormic (1997), conceptual knowledge is acquired through conceptual understanding. The receiver of the knowledge must first understand the concept. In trying to understand the concept, the learner will develop a concept image based on what he/she understands. When a concept is well articulated by the teacher to the learner and the learner understands its meaning, the appropriate concept image is formed. It is at this point that Tall & Vinner's theory of concept image formation interacts with the constructivist theory of knowledge. Errors and the misconceptions responsible for them are aggravated and perpetuated if the learner attempts to incorporate new knowledge into inadequate or defective concept images. The correct mathematical concept that the teacher articulates to the learner is regarded as the concept definition as defined by the community of practitioners, in this context, the mathematicians and mathematics educators.

Although the notion of conceptual understanding seems to occupy the attention of most mathematics education researchers, some researchers have emphasised the role of procedural fluency in acquiring competence in mathematics. Zakaria & Zaini (2009) described procedural knowledge as mastery of computational skills and familiarity with procedures, rules and algorithms for solving problems. Star (2005) as well as McCormick (1997) explained procedural fluency as "knowing how" of mathematical knowledge which involves learners' ability to recall and accurately execute procedures. Cognitive scientists and learning theorists, (Piaget, 1952; Vygotsky, 1978; Bruner, 1966; Gagne, 1985; Skemp, 1971) have identified general frameworks and models of knowledge. Hiebert & Carenter (1992) introduced procedural and conceptual knowledge as being essential knowledge to mathematics. Researchers (Zakaria & Zaini, 2009; Fuller, 1997) acknowledged how understanding the procedural fluency and the conceptual understanding of learners can assist teachers in their teaching and the roles the two kinds of knowledge play in enabling the learners to acquire the skills of problem solving and critical thinking. For Eisenhart, Borko, Brown, Jones & Agard (1993), procedural knowledge enables teachers and learners to justify their solutions to problems but with little or no knowledge as to why a particular method, operation or formula is used to find the solution to the problems.

2.5 The duty of the teacher in developing understanding

Teachers play a crucial role in children's acquisition of geometric knowledge. Effective instruction in geometry requires that teachers develop sound instructional strategies and knowledge of useful resources and activities (Ding & Jones, 2006). Luneta (2008) advised that mathematics teachers should reflect on their connected mathematical knowledge bases and fluidly combine that with their experience and understanding of geometry when teaching. Shulman (1986, p. 9) in his now classic article wrote, "Those who understand knowledge, grow in teaching" and he regarded content knowledge as the mother of all the knowledge bases that a teacher must possess in order to be effective. This idea concurs well with the work of Crowley (1987), who stated that the teacher should understand the level at which his learners are operating at so that instruction can be matched to learners' thinking.

Researchers (Chazan, 1993; Heally & Hoyles, 2000) found in their studies that learners faced serious challenges in geometry like lack of comprehension, not understanding geometrical symbols, difficulties in dealing with proofs and lacking strategies to produce proofs. According to Duval (1998), geometric problems are more difficult for learners than dealing with other areas of mathematics like algebra and he encouraged teachers to use modern techniques and methods, like using visual and multimedia tools to help learners understand geometry concepts.

CHAPTER THREE: THEORETICAL FRAMEWORK

3.1 Introduction

The study is framed within the theory of social constructivism and underpinned by the van Hiele model of geometric thought. This chapter will begin by describing constructivism as the theoretical framework. This will be followed by a discussion on social constructivism. Vygotsky's learning approaches will then be discussed as the major theorist among the social constructivists. The contributions of the cognitive constructivists can sometimes go unnoticed and therefore Piaget's learning types will also be discussed as one of the chief theorists among cognitive constructivists. Discussions on the learning types will then be followed by a discussion on the link between social constructivism and mathematics and this study. This chapter will end with a discussion on the van Hiele model of geometry thinking and its implications for this study.

3.2. Constructivism

The study is informed by the constructivism theorem as the national school curriculum is based on the theory of constructivism. The Theories of Piaget and Vygotsky form the foundation of constructivism. According to Watson (2001) in constructivism, the social environment is the source of knowledge where people construct knowledge when they interact with each other or with their experiences. Llewellyn (2005) describes constructivism as a philosophy about how an individual learn, where the learner is embedded in active engagement and is constantly constructing and reconstructing knowledge through environmental interactions. Ernest (2007) states that constructivism is a theory of knowledge with roots in philosophy, psychology and cybernetics, where cybernetics is the science of communication.

According to van de Walle (2007), in constructivism, children construct their own knowledge. Constructivists are of the belief that learners learn best when they construct their own knowledge from their own realities and their way of thinking is a true reflection of their daily experiences including their culture and beliefs (van de Walle, 2007). In constructivism the responsibility of learning lies with the learner. There are two type of constructivism, cognitive and social constructivism (Atherton, 2011). In cognitive constructivism the understanding of an individual learner in his or her developmental stages is important, whereas social constructivism is centred on how meanings and understandings of a learner may increase and improve as a result of social encounters. This study is thus framed within the social constructivism theory which will now be elaborated upon in the next section.

3.2.1 Social Constructivism

According to Ernest (1994) social constructivism emphasizes the social nature of learning, where learners should actively construct or create their own subjective representations of the objective reality. In social constructivism, the teacher is no longer the central figure in the learning process as learners are supposed to take an active role. Learning takes place when the learners construct and transform external, social activities into internal activities.

Social constructivism was developed by Vygotsky and his views on learning approaches forms the basis of social constructivism. According to Vygotsky (1978), learning is a social interaction which plays an elementary role in the process of cognitive development. Vygotsky was opposed to Piaget's assumption that learning is separated from the social context. In contrast to Piaget's understanding of child development, Vygotsky believed that social learning precedes development. Learners need to be given the chance to reflect on their correct and incorrect solutions. It is thus very significant that teachers should not dismiss wrong or incorrect solutions but teachers should rather allow the learners to explain and reflect on how they arrived at their solutions.

Scientific concepts do not come to learners in ready-made forms (Vygotsky, 1986). Vygotsky viewed development as dependant on social interaction and theorized that social learning leads to cognitive development, which he called the Zone of Proximal Development (ZPD). Vygotsky also believed in recognizing the gap between the learners' ability to perform a task under adult supervision and the ability of the student to solve problems independently. Scott (2008) views the cultural version of ZPD as comprising the merging of different elements of learning. This involves a distinction between what Vygotsky described as scientific knowledge and everyday knowledge. Piaget's learning is viewed as an active construction, where prior knowledge is restructured. Learners have to be educated, but they also need to want to be educated. They need to be internally motivated to want to learn, acquire knowledge and to socially construct knowledge in mathematics. Piaget's views on learning will be discussed in the next section

3.2.2 Cognitive Constructivism: Piaget's learning types

According to Piaget (1970), new experiences are either integrated into existing schema (assimilation). Accommodation takes place when existing schema changes to accommodate new ideas. Accommodation and assimilation result in new knowledge being constructed from an individual's experience (Piaget, 1970). In assimilation learners are able to match new ideas to those they already have by seeing and matching similarities (Llewellyn, 2005). Assimilation refers to the use of an existing schema to give meaning to new experience (Llewellyn, 2005). Learners may encounter experiences which contradict what they already know in their internal structures, so they have to change their thinking in order to fit the new information in a process of accommodation (Watson, 2011). Accommodation alters existing schema, thus re-organising one's way of thinking and the mental picture to allow for new knowledge.

Constructivism in mathematics from Piaget's view is important in encouraging the construction of logico-mathematical knowledge. The ability to develop this knowledge is based on the quality of learning that the learner has been exposed to. What the learner has experienced or learnt needs to make sense so that it can be added to the existing knowledge. Thus teaching and learning is enhanced with the use of different learning approaches.

In the next section the importance of constructivism in mathematics education will be discussed.

3.3 Social constructivism and learning mathematics.

The performance of learners in mathematics at FET level has been a worry for the education department. Elmore (2002) compares the levels of what is known theoretically with what is done practically. Because of the poor performance in mathematics at FET level, there is need to prioritize the understanding of mathematics developmental theories, the construction of knowledge and the understanding of learning theories appropriate to the education of mathematics. In mathematics education environments, there is a need for the learning of the construction of knowledge. Learners need to be given opportunities to explore and construct knowledge for themselves. Problem solving activities when used in mathematics lessons will encourage learners to use inquiry methods. In social constructivist mathematics classrooms, collaborative learning through peer interaction should be mediated and structured by the teacher.

The knowledge and experience that the students bring to the classroom is of paramount importance. The teacher in a social constructivist classroom should design learning programmes that are sufficiently flexible to permit development in line with students' enquiries. Due to the interpretivist nature of social constructivism, there are different interpretations of information by different learners; the teacher should allow learners to express themselves, asking questions where they feel confused and allow them to reflect on their learning process. The teacher should also present authentic tasks to contextualize learning through real world and case-based learning environments and should also provide scaffolding at the appropriate time and level.

Learners take an active role in a constructivist classroom, taking responsibility for their own learning process while the teacher acts as a facilitator. Learners are encouraged to develop different lenses to see things in new ways. The teacher designs learning programmes that encourage learners to be more critical thinkers by presenting them with provocative situations.

3.4 Social constructivism and the study

As stated earlier, the study focuses on exploring FET learners' understanding of geometry and their progression on the Van Hiele's levels of geometry thinking. The teaching and learning of geometry is best done when examples are cited from real world contexts. More real world contexts need to be cited because geometric concepts are applied in disciplines such as art, architecture, interior design and science. According to Usiskin (2002), teaching geometry is important in our daily lives, because it connects mathematics with the real world and it enables ideas from other areas of mathematics to be brought into focus. School geometry helps to prepare learners for the outside world where learners are able relate what is learnt at school with their application in the real world.

Social constructivism encourages groups of people to construct their own knowledge from one another and these groups have the ability to work collaboratively in creating small cultures of shared objectives with shared meanings and understanding. When one is involved within such a culture, one is learning all the time about how to be part of that culture in many ways.

The main ideas of social constructivism have been outlined and furthermore Piaget and Vygotsky's approaches in teaching mathematics have also been explained in this chapter. This chapter also linked social constructivism to the teaching and learning of mathematics.

The next section will now discuss the van Hiele theory of geometry thinking as one of the theories underpinning this study.

3.5 The van Hiele model of geometric thought

This study is underpinned by the van Hiele model of geometric thought and I will now present a brief overview of this model. The van Hiele theory was developed by Pierre van Hiele and his wife to explain how children developed spatial geometry concepts (Crowley, 1987). Van Hiele (1986, 1999) postulated the existence of five levels of geometric thought; visual, analysis, informal deduction, formal deduction and the rigour levels. The terms or words used in each van Hiele level describe the kind of thinking that learners are required to engage in at each level. According to the theory, learners move from the most basic level (visual), through to each of the others, while teachers can help by giving them suitable experiences at each level. Very few learners in high school can operate at level 5 of the van Hiele levels (Crowley, 1987).

Level 1: Visual level

At the visual level learners have a very simple concept of space. They see geometric shapes or figures as a complete whole. They recognize geometric figures by their appearance and not by their properties. Learners are able to identify the given shape because they associate the shape with what they know. Burger & Shaughnessy (1986) mentioned that learners see these figures as a whole, without being able to analyse their properties.

Level 2: Analysis Level

At the analysis level, learners are able to analyse shapes in terms in terms of their parts and properties, but are not able to make any connections between different shapes (Mason, 2010). A learner at this level should be able to recognize that a square has four sides and four angles which are all equal and that the diagonals of a square are equal and perpendicular bisectors of each other. However learners placed at this level, may have an incomplete understanding of how properties of shapes relate to each other.

Level 3: Informal Deduction level

At the informal deduction level learners can analyse the properties of figures and can understand relationships between the properties of a figure and relationships between figures. Learners are also able to follow all the logical arguments using the properties of the figures, but they may not be able to create a new proof from scratch.

Level 4: Formal Deduction level

At the formal deduction level learners understand and use the ideas of formal geometry, they understand how important deduction is, and can use it to build up a geometric theory based upon axioms and proofs in the same way that Euclid did. Learners now learn to do formal proofs. They now understand the role played by terminology, definitions, axioms and theorems in Euclidean geometry.

Level 5: Rigor level

At the rigor level learners are expected to work with many axioms, non-Euclidean geometries and compare different systems. Thus geometry is now seen as abstract.

According to Singh (2006), the van Hiele theory is an important theory which asserts that mastery at a particular level is crucial but not sufficient for understanding on a higher level. Crowley (1987) argues that the type of activities given to learners represent the most meaningful consideration in terms of the development of geometric thinking. Instruction should be built around specific content knowledge which is appropriate for the student levels and activities designed to challenge learners at the next level, for example those at level one should provide properties of shapes and those at level two should be encouraged to formulate and engage in deductive reasoning. According to van Hiele, knowledge of geometry is developed as learners move through these levels. The van Hiele theory suggested that one possible cause for learners' failing school geometry was that the curriculum was taught at a level higher than that of the learners' level of understanding. This implies that there is no shared understanding by the learners and the teacher on certain concepts. Learners fails to understand the teacher's instruction and teacher fails to see why learners are struggling. The general characteristics of the Van Hiele Theory are summarized in the table below:

Table 2.1: Characteristics of the Van Hiele theory (Adopted from Crowley, 1987)

Characteristics	Description
The model is sequential	Each level builds on the thinking strategies developed in the previous level. Learners ought to finish all preceding levels before arriving at the next level.
Advancement through the levels	This level is dependent upon achieving the thinking strategies of the previous level. No level should be skipped when planning learning activities for development of spatial thinking.
Not age dependent	All levels at this stage are not dependent on age unlike the way Piaget described development.
Implicit ideas become explicit ideas	At this level, as thinking advances, geometric ideas and concepts that are only implied at a level become the objects of study at another level and so become explicit ideas.
Each level has its own language	This implies that learners reasoning at different levels cannot understand each other's explanation even though they might be describing the same idea or shape; neither can they follow the reasoning of each other.
Instruction should match thinking	If the learner is on one level of thinking and the teacher's language, curriculum content, materials, etc, are on a different level, learners will not understand the language that is being used and as such their progress may be obstructed. Furthermore learning and progress from different levels is more reliant upon instruction and opportunities for learning than upon age.

De Villiers (1996) to some extend supported Crowley's summary of the characteristics of the Van Hiele theory by outlining four characteristics of the theory as summarized by Usiskin (1982). These characteristics are:

a. Fixed order: Learners need to go through all preceding levels to arrive at any specific level, as the levels are hierarchical.
b. Adjacency: the properties which are intrinsic at one level of thought becomes understandable at the next level,

c. Distinction: What is supposed to be correct at one level of thought may not necessarily be correct at another level. At Level 1 a square is something that looks like a box while at Level 3 a square is a special type of parallelogram.
d. Separation: when a learner is reasoning at a different level from the teacher, they cannot understand each other. The learner ends up not understanding the teacher and the teacher has a problem understanding the learner's reasoning. This may lead to the rejection of the learner's answers by the teacher.

The Van Hieles also recognized characteristics of the levels, realising that a learner must progress through the levels in a sequential fashion, that the progression from the lower level to a higher level depends on subject matter and mode of instruction rather than on age, and that each level has its own terminology and its unique method of operation of relations. That is why the Van Hieles proposed sequential phases of teaching to assist learners to move from one level to the other (Usiskin, 1982).

The methodology used to complete this study will be discussed in the next chapter.

CHAPTER 4: METHODOLOGY

4.1 Introduction

I will discuss the methodology of the study in this section. I will begin by explaining the research paradigm and research design before describing the context of the study by looking at the manner in which the research information was collected and the targeted population. This is then followed by a discussion on the data collection instruments and the procedures before explaining the details of the pilot study that was carried out to refine the questionnaires. I will then present a discussion on the intended methods of data analysis, followed by the limitations and ethical considerations that I took into account.

4.2 Research Paradigm

According to Stanage (1987), the word "paradigm" can be traced back to its Greek (paradeigma) and Latin (paradigm) origins, meaning pattern, model or example. Huitt (2011) suggests that a paradigm is one's way of seeing the reality. In educational research, a paradigm means a plan or set of rules that guides a research study and how the information will be interpreted (Mackenzie & Knipe, 2006).

This research used the interpretivist paradigm which studies humans as individuals and takes their environment into account with their unique characteristics (Cohen, Manion & Morrison, 2007). The interpretivist paradigm emphasizes the ability to construct meaning (Ernest, 1994). Interpretivists believe that reality must be interpreted if one is to fully understand it (Bryman, 2001). Olssen (2006) states that interpretivists believe that research must be observed from the direct experience of people rather than being observed from the outside.

Researchers in the interpretivist paradigm seek to understand rather than explain. Knowledge is gained through personal experience and finding meaningful observation of objects. The interpretivist paradigm allows us to understand humans' behaviour and the reason for their actions (Bryman as cited in Grix, 2004).

The data collection and analysis was primarily determined in relation to the contextual settings and the perspectives of the learners. The interpretivist paradigm was used in this research because the interpretivist paradigm studies individuals and their characteristics together with reasons for their actions. This paradigm allowed the researcher to analyse how grade 10 and grade 11 learners perform on selected geometric tasks and observe how they were progressing on the van Hiele levels of geometric thinking, thus investigating the phenomena of the world and humans. It is with this understanding of my role as researcher that I undertook for this study to explore the grade 10 and grade11 learners' understanding of basic geometry concepts by using the van Hiele theory as a framework. It also gave me the opportunity to make sense of some of the misconceptions that learners have and the errors that they make.

4.3 Research Design

In this section I will first look at the definition of a research design and then explain how this research was designed.

Creswell (2009) defines research designs as plans, strategies and procedures for the research, comprising decisions from the underlying world views to the detailed methods of data collection and analysis. According to Luneta (2013), the research design is a road map of how the research will be conducted. The research design includes the method to be used, the data to be collected, where, how and from whom the information will be collected as well as the circumstances under which the information will be collected.

The decision for using a specific research design, as stipulated by Creswell (2009), is influenced by the world views and assumptions of the researcher, personal experiences of the researcher, audience of the study, nature of the research problem, research strategy and methods of data analysis and interpretation. Similarly Schulze (2003) views research design as the choice of research strategy and methods based on the researcher's opinion on how solutions to the research problems may be obtained.

The data of this research is contained within the perspectives of the learner participants and the researcher engaged them through written work (worksheet based questionnaire) and interviews. The intention of this research was to gather data regarding the perspectives of research participants on their understanding of geometric concepts.

The study was a qualitative one, aiming to understand the meaning which informs the human behaviour and it was carried out in a naturalistic setting. According to Borrego, Douglas & Almelink (2009), qualitative research is characterised by the collection and interpretive analysis of written data obtained from surveys, interviews, questionnaires and focus groups. The study utilised questionnaires and interviews to generate the data from three high schools in rural KwaZulu Natal, about 40 km south of Durban.

Inductive and exploratory methods were used in this qualitative research for the purpose of description and exploration of data as well as to gain understanding of how learners think and experience their lives. Data was examined through interpretive analysis for patterns and themes. The qualitative research approach was chosen because qualitative research problems are phrased as research purposes or questions, but not as a hypothesis. Problems are phrased more broadly to answer questions like what, how and why, as demanded by the research questions of this study. Qualitative researchers undertake an in-depth investigation of small, distinct groups and in this case the grade 10 and grade 11 learners of three different schools in the same cluster in KwaZulu Natal participating in this study

4.4 Context of Study

The targeted population of this study was FET learners (grade 10 and 11learners) from three different schools in the same cluster in rural KwaZulu Natal. All the participants consider Zulu as their first language and English as a second language. The schools are all quintile 3 schools, meaning the learners in these schools do not pay school fees and the schools rely on government for funding to meet all their financial needs. All three schools (coded as School A, School B and School C) have been struggling to achieve high pass rates in Matric. The pass rate in grade 12 in the past three years for School A was 68%, 90% and 81% respectively. School B achieved pass rates of 94% and 97% in 2012 and 2013 respectively, dropping down to 53% in 2014. For school C the pass rate was 76% in 2012, moving down to 50% in 2013 and then improving to 87% in 2014. Hence the rates fluctuated with no steady pattern of improvement in any of the schools in the past three years, suggesting that they are experiencing challenges common to most schools in the country.

A total of 147 learners took part in the study from the three schools comprising 73 grade 11 learners and 74 grade 10 learners. The breakdown from the three schools is shown in the table below.

Table 4.1: Distribution of learners according to grades and schools

	Number of Learners		
School	**Grade 10**	**Grade 11**	**Total**
School A	31 learners	31 learners	62 learners
School B	32 learners	23 learners	55 learners
School C	11 learners	19 learners	30 learners
Total	74 learners	73 learners	147 learners

Eighteen (18) learners were selected from the group of 147 learners to participate in the interviews. I selected participants for the interview based on their performance in the task based worksheet and the school in which the participants attended-a process called purposive sampling. Six learners (2 from each school) whose responses displayed a sound knowledge of geometry concepts, six learners (2 from each school) who displayed moderate knowledge and six learners (2 from each school) who displayed limited knowledge of geometry concepts were chosen to participate in the interviews in order to meet specific needs of the researcher. In this way the purposive sampling technique built up a sample that meets the needs of the study (Cohen, Manion & Morrison, 2007 ; Strydo, Fouche & Delport, 2004).

The next section will focus on how the data was collected from the targeted population and the instruments used in the data collection.

4.5 Data collection instruments and procedures

This section will look at the instruments that were used to collect the data which was the questionnaire in the form of a task based worksheet and an interview schedule and how they were used to collect the data. In order to answer the research questions, a data collection plan designed to allow the researcher to collect the data. The table below shows the summary of the data collection plan.

Table 4.2: Data collection plan

Critical Research Question	**Participants**	**Data Collection Method**
1. How do grade 10 and 11 learners perform on tasks based on basic geometric concepts?	Grade 10 and 11 learners	Task based worksheet[questionnaire] Interview schedule
2. What can be deduced about the van Hiele levels of geometric thought of the learners?	Grade 10 and 11 learners	Task based questionnaire

4.5.1 Task based worksheet (questionnaire)

The task based worksheet was designed to provide information to assist the researcher in determining the FET (grade 10 and 11) learners' knowledge of geometry, the van Hiele levels of geometry thinking at which they were operating at and how the learners were progressing through the levels. The task based worksheet (questionnaire) ensured that the learners' written work became the primary data-gathering tool. The purpose of the learners completing the questionnaire was three fold: it allowed the researcher to observe and identify the kind of geometric knowledge the learners have, including the misconceptions and errors if there are any; it allows the researcher to place learners into different van Hiele levels using the written responses they provided and lastly it allowed the researcher to select/identify learners to participate in the interviews. According to Parahoo (2006), an individual's way of thinking is best revealed in the documents they produce.

The questionnaire was divided into two parts, Questionnaire A and Questionnaire B and they appear in Appendix B and Appendix C respectively. Questionnaire A consists of multiple choice questions, requiring knowledge of different aspects on geometry using the required reasoning at different van Hiele levels. Questionnaire A was a modification of Usiskin's instrument used in his study (Usiskin, 1982). Each item was analysed according to the van Hiele levels that it could be matched to. For example, if an item was classified as level 1, it means that the item required reasoning as described in the van Hiele level descriptors appearing in Chapter 3. Table 4.3 below presents a description of items and the van Hiele level of classification of the items (questions) in Questionnaire A.

Table 4.3: The van Hiele levels of the questions in Questionnaire A

Questions [Items]	Type of Question	Van Hiele level of item
Item 1	Identifying a triangle from a group of shapes	Level 1 (Visual level)
Item 2	Identifying a circle from a group of shapes	Level 1 (Visual level)
Item 3	Choosing the correct property of perpendicular lines	Level 2 (Analysis level)
Item 4	Calculating the area of a rectangle	Level 2 (Analysis level)
Item 5	Comparing similar and congruent triangles	Level 3 (Informal deduction level)
Item 6	Choosing the correct property of parallel lines (definition)	Level 2 (Analysis level)
Item 7	Choosing the correct property for an equilateral triangle	Level 2 (Analysis level)
Item 8	What other conclusions can be drawn from a parallelogram [class inclusion]	Level 3 (Informal deduction)
Item 9	Definition of circle	Level 2 (Analysis level)
Item 10	Interrelationships between rectangles and triangles [if there are any]	Level 3 (Informal deduction)
Item 11	Interrelationships between properties of triangles	Level 3 (Informal deduction)
Item 12	Comparing properties of shapes [class inclusion] rectangles, squares and parallelograms	Level 3 (Informal deduction)
Item 13	Interrelationships between shapes: rectangles and other parallelograms	Level 3 (Informal deduction)
Item 14	Interrelationships between properties and shapes (diagonals, squares and rectangles)	Level 3 (Informal deduction)
Item 15	Knowledge of proofs required, proving a rectangle using diagonals	Level 4 (Formal deduction)

The second part of the task based worksheet contains Questionnaire B, which consisted of open ended questions. Each item in questionnaire B was also categorized at different van Hiele levels. If an item is categorized as level 2, then it implies that the reasoning required to solve the question is that described in level 2 of the van Hiele model as described in Chapter 3. The table 4.4 below shows the van Hiele levels of items in Questionnaire B.

Table 4.4: Van Hiele levels of questions in Questionnaire B

Question [Item]	Van Hiele level	Explanation
Question 1	Level 2 (Analysis level)	Identifying equal angles between parallel lines and giving reasons
Question 2	Level 2 (Analysis level)	Finding the missing angle using the sum of angles property of a triangle
Question 3	Level 3 (Informal Deduction level)	Learners are required to have knowledge of the transitivity property and a knowledge of relationships between properties of shapes, equal angles of an isosceles triangle and vertically opposite angles
Question 4	Level 3 (Informal Deduction level)	Using analytical means (calculations) to show that a shape is a parallelogram
Question 5	Level 3 (Informal Deduction)	Showing a square using analytical methods, providing sufficient conditions
Question 6	Level 4 (Formal Deduction)	Proving angles, congruent triangles and parallelogram using deductive reasoning, starting proofs from scratch.

4.5.2 The semi-structured interview

The semi-structured interview schedule in this study was the individual interview schedule which helped to identify some challenges the learners faced when learning geometry. A semi-structured interview schedule is a method of research used in social sciences to collect data. It is more flexible and allows for new questions to be asked during the interview process and it allows the researcher to probe for more information. According to Dowling & Brown (2010), interviews enable researchers to probe difficult issues in greater detail so as to provide clarification and to prompt responses from the participants. The interviews provided important information with respect to learners' knowledge of geometry. During the interview process, the silent probe (pause and wait) method was used. According to Trochim (2006), the most efficient strategy to encourage someone to explain, is to do nothing at all, but just pause and wait. It works because the respondent is uncomfortable with the pause or silence. It suggests to the respondent that you are waiting, and listening to what they will say next, especially where further clarification is required.

The interviews gave the participants the opportunity to express their perceptions and understanding of geometry, and for the researcher to probe the participants' responses. Using interviews to a larger extend minimises the limitations of questionnaires. Eighteen learners were selected for the individual interviews on the basis of how they responded in the Questionnaire, thus the interviews were based on their responses to the task based worksheet.

The learners for the interviews were then classified according to different codes, for example LSA1, for learner number 1 from school A, LSB3 for learner number 3 from school B. The number given to the learner, like LSA1 (1 for learner number 1) was not significant but just a way of identifying the learner from other participants in the same school. When the interviews were recorded, the learners were referred to as Learner 1, Learner 2, etc, but when the transcriptions were made, Learner 1 was then changed to LSA1, and learner 7 was changed to LSB1, depending on the school the learner attends. The table below shows the learners who participated in the interviews and their respective codes.

Table 4.5: List of learners who participated in the interviews and their respective codes

Recorded as	School	Grade	Gender	Code
Learner 1	A	10	Male	LSA1
Learner 2	A	11	Male	LSA2
Learner 3	A	10	Female	LSA3
Learner 4	A	11	Male	LSA4
Learner 5	A	10	Female	LSA5
Learner 6	A	11	Male	LSA6
Learner 7	B	10	Male	LSB1
Learner8	B	11	Female	LSB2
Learner 9	B	10	Female	LSB3
Learner 10	B	11	Female	LSB4
Learner 11	B	10	Male	LSB5
Learner 12	B	11	Male	LSB6
Learner 13	C	11	Female	LSC1
Learner 14	C	11	Female	LSC2
Learner 15	C	10	Male	LSC3
Learner 16	C	11	Female	LSC4
Learner 17	C	10	Female	LSC5
Learner 18	C	10	Female	LSC6

The participants were allocated enough time (15-25 minutes) to respond to questions when they were interviewed. The individual interviews were conducted on a one on one basis involving only the participant and the interviewer. Individual interviews provide the freedom for the participant to talk freely without fear of being ridiculed or judged by their peers.

4.6 Pilot Study

A pilot study is usually conducted to test the reliability and to refine the measuring instruments. It is administered to a small group of participants similar to those to which the actual test will be administered. According to McMillan & Schumacher (1997), it is highly recommended that researchers conduct a pilot study of their questionnaire before using them in the main study. It is thus recommended to locate a sample of subjects with characteristics similar to those who will form part of the main study.

The pilot helps the researcher to determine whether or not the time frame is adequate or whether the directions and items on the questionnaire are clear. The pilot study for this research was conducted at a different school from that of those involved in the main study but belonged in the same cluster. This school was chosen for the pilot study because it provided easy access to the researcher and it was in the same setting as those involved in the main study. Twenty three grade 10 learners and nineteen grade 11 learners were asked to complete the task based worksheet. The results of the pilot study showed that the questionnaire was too long, consisting of 25 multiple choice questions in Questionnaire A and 12 questions in Questionnaire B. The majority of the learners could not finish the paper as they complained that there are too many questions in both Questionnaire A and Questionnaire B. The number of items in Questionnaire A was reduced from 25 items to 15 items, while the numbers of items in Questionnaire B was reduced from 12 to 6 items. All questions on circle geometry were removed from the research instrument, so that the grade 11 won't have an unfair advantage over the grade 10 since circle geometry is covered in grade 11 and grade 12.

4.7 Data analysis procedure

In this section I first describe the process of data analysis in terms of definition and the processes involved before explaining how I will analyse the data of this research.

When data is presented in the form of tables, graphs, diagrams and figures before being analysed to check for similar patterns and themes, this is what Marshall & Rossman (2006) referred to as the process of bringing to order, structure and interpretation to the data.

Data needs to be organised into different categories if there are similarities and differences observed especially when analysing qualitative data (Marshall & Rossman, 2006). Luneta (2013) argues that the data analysis that is informed by the data collected is called inductive analysis. Analysis informed from the themes, patterns and categories that emanate from the literature can be called deductive analysis. According to Luneta (2013) deductive analysis authenticates theory. It is important to analyse information collected in order to arrive at the findings of a research.

According to Cohen, Manion & Morrison (2007), when analysing data, the researcher should strive to analyse and interpret the collected data in terms of the objectives of the research study, respect the anonymity of the participants (as information gathered should be treated in a highly confidential manner) and limit their own biases or personal prejudices towards the study. In order to respect the anonymity of the learners, the names of the learners and their schools will not be divulged.

The data analysis involved analysing learners' responses so that common themes could be identified. The learners' responses for Questionnaire A and Questionnaire B were marked and analysed for common themes. Misconceptions and common errors were observed and pertinent issues raised were noted. In Questionnaire A(multiple choice questions), responses were marked as correct or incorrect but for Questionnaire B, besides looking for final correct answers, the procedures, errors and misconceptions were noted for further analysis.

The validity and ethical issues will now be discussed in the next section.

4.8 Validity Issues

Validity is an important issue in the effectiveness of a research. According to Winter (2000) validity refers to whether an account accurately represents those features that it is intended to describe, explain or theorise. It refers to whether a particular instrument is actually measuring what it is meant to measure. Validity can depend on instrument used, the participants and the extend of triangulation (Winter, 2000). Guba & Lincoln (2005) suggested that the key criteria of validity in qualitative research are credibility, transferability, dependability and conformability. These will now be described below together with how they were achieved in this research.

- ***Credibilty***

Guba & Lincoln (2005) define credibility as the confidence that can be placed in both the data and the analysis while Koul (2008), defines credibility as the believability of the findings. There are a number of instances in which credibility can be enhanced, which include confirmation of conclusions by the researcher from those who participated. According to Merriam (1991), credibility means that the findings should be congruent with what is observed. Credibility can be explained in terms of faithful descriptions and recognisability by other researchers (Koul, 2008).

I achieved credibility in this study by sticking to the learners' written responses and comments given during interviews and giving sufficient attention to bracketing. In this study, much effort was put in to ensure that participants' words and descriptions were authentically presented without additions or deletions throughout the analysis. The presentation of some of the learners' written work in the data analysis section also helped to ensure credibility.

- ***Transferability***

According to Merriam (1991) transferability is the ability to apply the results of the research to another context similar to the one in which the research was conducted in while Koul (2008) defines transferability as the evidence supporting the generalization of the findings to other contexts, involving, different participants, groups and situations

In this study a description of the settings including the targeted population was provided so as to allow other researchers to apply the findings to similar settings. The findings were also described in detail, and some of the learners' responses were inserted including direct quotes taken from the interviews.

- ***Dependability***

Dependability considers whether the research was transparent and clearly written. It must be possible to trace the data and the findings must be consistent (Howitt, 2007). The learners' original written responses can be used to confirm the dependability of this study.

- ***Conformability***

Conformability according to Howitt (2007) is the objectivity or neutrality and control of research bias. In this study conformability was achieved through the data transcripts and data analysis as well as the description of the findings and relating them to the literature.

4.9 Ethical Issues

Ethics show or illustrate what are correct and what are incorrect actions or what moral research procedures are involved in the study.

In the course of the preparation of this study, it was evident that some ethical issues would arise involving the institutions (the schools where the learners were studying) and the participants themselves. I contacted the research office of the university in order to be granted permission to conduct the study. Permission was also sought from the principals, the heads of departments and teachers of the learners involved in the study.

Consent forms were designed and given to the relevant authorities including the parents of the participants. The consent forms informed the participants about their confidentiality and anonymity, the purpose and procedures to be followed. This included the stipulated time frames expected for the completion of each instrument. Participants were also informed of their freedom to withdraw at any stage of the research study without any fear of victimization. According to Cohen & Manion (1994) withdrawal from participation can be done anytime without prejudice to the participant. Participants were also advised that there were no rewards or favours to be gained for their participation.

The ethical clearance for this study was approved by the Department of Basic Education, and by the University of KwaZulu Natal. Permission for this study was also granted by the principals of the three selected schools, the heads of departments and the parents of the participants.

4.10 Limitations

The study was carried out in only three schools in the same cluster which may not be representative of the whole province or country. The schools were similar in character, of the same quintile ranking and all the learners were from one race group. The results of the findings of this research might be different with studies which involve different kinds of schools and learners from different parts of the province. If there is a need to extend the scope of findings then the sample should be increased and learners should be targeted from different parts of the province.

4.11 Conclusion

This chapter presented the methodology used in this study. It outlined the main aspects of the research design and methodological procedures, and explained how the data was collected. The learners' responses from the questionnaires, task based worksheet and the individual interviews were used to provide data. The next chapter discusses the data collected and the results obtained in the process together with evidence from the participants' written work.

CHAPTER 5: RESULTS

5.1 Introduction

This chapter presents the results of this study. The participants' performance in the task-based worksheet was analysed using descriptive statistics and various tables. In this chapter I first present the overall results of the learners for Questionnaire A in Section 5.2 and that of Questionnaire B in Section 5.4. The analysis of the written responses to the questionnaires is supported by the analysis of the interviews.

5.2 Results by item for Questionnaire A.

The questionnaire comprised 15 multiple choice items. Table 5.1 represents the overall results of the learners' responses to the items appearing in Questionnaire A. Thereafter an item by item analysis will follow with learners' interviews used to support the interpretation of the learners' written responses, where possible.

Table 5.1 Learners' Responses to Items in Questionnaire A (n = 147)

ITEMS	A	B	C	D	E	No Response	Percentage of Correct Responses
	Number of responses per item						
1	0	0	**147**	0	0	0	100%
2	0	0	**147**	0	0	0	100%
3	**53**	56	9	8	17	4	36%
4	4	0	**110**	21	4	8	75%
5	18	6	12	11	**91**	9	62%
6	**101**	18	6	3	5	15	69%
7	**81**	5	14	3	32	12	55%
8	11	**57**	5	20	45	9	39%
9	24	44	7	34	**22**	16	15%
10	13	29	**19**	2	60	24	13%
11	11	**53**	12	8	36	27	36%
12	**25**	8	61	14	14	25	17%
13	15	**40**	13	47	11	21	27%
14	69	10	**24**	7	3	34	16%
15	27	27	39	**7**	17	30	4%

The number of responses with the correct answer is indicated by the bold number in the table above.

I will now discuss the learners' responses to each item in greater detail. The discussion of the written responses is supplemented by responses from the interviews wherever possible.

5.2.1 Items 1 and 2

All the learners responded correctly to items 1 and 2, which required them to identify a triangle in item 1 and to identify a circle in item 2. These two items lie within the visual level of van Hiele's levels of geometric thinking. Learners identified the shapes because of their appearance, figures were recognised as a complete whole, and no knowledge of properties was required.

During the interviews, many learners commented that identifying the shapes was very easy:

> **LSA1:** These 2 questions were very easy, this is basic knowledge in which when we started primary school we were taught shapes. I easily recognised the shapes without much thinking.
>
> **LSB1:** Question 1 and 2 were very easy to answer. Even before I started attending school I was taught to identify a circle even though no one gave me a proper definition up to now.
>
> **LSB2:** Questions involving identifying shapes and figures were never a big issue to me because in our everyday lives we make use and interact with these shapes, therefore question one and two were quite easy for me.

5.2.2 Item 3

In item 3 only 56 out of the 147 learners (or 36 %) responded correctly by choosing the option that perpendicular lines intersect to form four right angles. This was a concern. Classified as a level 2 question, on the van Hiele's levels of Geometric thinking, it required learners to identify properties of perpendicular lines without making any connections between different shapes. Of the 147 learners 56 believe that perpendicular lines intersect to form two acute and two obtuse angles. Nine learners believe that perpendicular lines do not intersect at all, suggesting that these learners are confusing properties of perpendicular lines with those of parallel lines. Twenty learners did not respond to this question. Interview responses provided some insight into misconceptions related to perpendicular lines:

> **Researcher:** Can you explain to me in your own words what you understand by perpendicular lines?
>
> **LSA5:** Perpendicular lines are two lines which meet to form 90 degrees, and there is a sign for 90 degrees where they meet.

The learner's response reveals that he has a mental picture of how perpendicular lines are denoted but he did not say anything about the properties. Asked about the meaning of right angles, the learner admitted to a lack of knowledge about its meaning together with other terms like acute angles and obtuse angles and had this to say:

LSA5: It never mattered to me to know about the definition of perpendicular lines besides the fact that I must see 90 degrees where two lines meet for them to be perpendicular.

I thought the definitions were not important as we always see the sign for 90 degrees whenever the lines meet.

This means that the learner lacks the knowledge of the language used in describing geometric figures even though the learner has an appropriate picture in mind of what is meant by perpendicular lines. Another learner's response identified the way that perpendicular lines are often presented in textbooks and activities as influencing his perception that perpendicular lines don't intersect:

> **LSB1:** "At first I was confused by the fact that the perpendicular lines form four right angles, because in some cases the two lines don't cross each other, they just meet to form a T-shape and in this case only two right angles are formed, but according to the options I was given, the most convenient one was the one saying the perpendicular lines intersect to form four right angles".

Learner LSB1 has clearly explained that in his experience the perpendicular lines usually meet to form a "T-shape" leading him to infer that only two right angles are formed at the intersection of two perpendicular lines.

5.2.3 Item 4

Item 4 was satisfactorily answered as 110 learners (75%) responded correctly in calculating the area of a rectangle by multiplying the length by the breadth. However 31 learners responded incorrectly and 8 learners did not respond at all revealing that there are some learners in the FET phase who still face challenges in understanding properties of shapes. All the learners chosen for the interview responded correctly to item 4, and when asked to comment on the item, learner LSA2 commented: "This question was relatively easy; we spend much of our time in the senior phase solving problems involving area, volume and perimeter".

5.2.4 Item 5

In item 5, learners were required to compare similar and congruent figures. This question falls within the informal deduction level (level 3), as learners are required to understand the relationships between properties of different figures. There were 91 learners (62%) who responded correctly. It is of interest that 19 of the learners believe that if two figures are similar but not congruent, then they have equal bases and equal heights, while 12 learners believe that the figures have to have horizontal bases for them to be similar and not congruent. There were 9 learners who did not respond at all.

In the interviews, two learners with correct responses responded as follows, when probed about their opinion of the item:

> **LSA2:** "I had to think about the definition of similar and congruent triangles. So I had to look at the properties of similar triangles which congruent triangles don't have".

> **LSC1**: "It was a case of remembering congruent and similar triangles. I remembered that in similar triangles all corresponding angles are equal, which means the shape will be the same but the size of the sides maybe different, so I had to settle for the last option (e).

These two learners both considered triangles as their figures of reference. Learner LSC1 in her response linked the meaning of equal corresponding angles to the fact that the figures have the same shape but different sizes, showing reasoning that is typical of level 3 (understanding relationships between the properties of a figure). He linked the property to a visual aspect of the condition. It is interesting that learner LSA2 just considered the definitions of congruency and similarity without linking it to a visualisation.

Two other learners who did not give correct responses were also interviewed:

> **Researcher:** Your response to item 5 was not correct; can you explain to me the reasoning behind your response?
>
> **LSB2:** I don't remember the properties of similar triangles but I remembered some of the properties for congruent triangles. The main problem is when we learnt these in class we were just told by our teachers what the properties were and never had the chance to work with the actual figures and shapes on our own to discover these properties. So I just thought the figures must have equal bases and equal heights.
>
> **LSA1:** I never understood similar and congruent triangles, so it was quite a challenge for me to list all the properties of similar triangles and then compare them with those of congruent triangles.

Learner LSA1's response indicate reasoning that is at Level 2 (analysis), not yet at level 3 since he can list the properties of the two types of figures but struggled to compare them. Learner LSB2's response indicates a problem with linking the definitions with the visualisation of properties of the figures. She commented that they were just told the properties without being given the chance to work with the figures and understand the implications of these properties. The learner feels at a disadvantage because he was not given the opportunity to work with the actual figures.

5.2.5 Item 6

Item 6 was a level 2 (analysis level) question, as it requires learners to have knowledge of properties of figures, in this case parallel lines. There were 101 learners (69%) who responded correctly. However it was quite disturbing to note that 18 learners believe that parallel lines are lines which never lie in the same plane and never meet. It shows that some FET learners still struggle to understand properties of parallel lines.

5.2.6 Item 7

Item 7 was a level two (analysis) level question as it requires knowledge of properties of an equilateral triangle. Eighty one learners (55%) responded correctly as they mentioned that an equilateral triangle has three equal sides. However 36 learners believe equilateral triangles have angles of different sizes.

5.2.7 Item 8

Item 8 was correctly answered by 57 learners (38%), who stated that if ABCD is a parallelogram then triangle ABD is congruent to triangle CDB. There were 45 learners who believed that if ABCD is a parallelogram then all the given statements will be true, that is ABCD is equiangular, triangle ABD is congruent to triangle CDB, the perimeter of ABCD is four times the length of AB and that AC is the same length as BD. The learners' responses show lack of understanding for the basic properties of shapes. Here are some of the interview extracts:

> **Researcher:** This item involved properties of a parallelogram, and I am sure in the lower grades you learnt about them. What can you say about this question, did you find it easy to answer or was it very difficult?
>
> **LSC2:** I know parallelograms but I failed to link them to the question, so I had to guess the answer from the given options.
>
> **LSB1:** We have done parallelograms before but the way the question was asked was challenging to me. I even drew my own parallelogram but I failed to create the triangles which they are talking about in the item, then there was this term equiangular which I don't even know its meaning.
>
> **LSA1:** At first I struggled to answer the question, but then I decided to draw the parallelogram and after naming the sides and angles it became easy for me to answer it.

Learner LSC2 could not link the question to the properties of a parallelogram. LSB1 explained that he could not identify the triangles that the question referred to, showing that he had difficulties in translating the verbal (written) representation into the diagrammatic representation, while LSA1 found the question easier because he was able to move between the two representations and the diagrammatic form helped him find the answer. These learners' responses indicate how visualisation of the figures is important when trying to identify the relationships within a figure.

5.2.8 Item 9

In item 9 only 21 learners (15%) were able to give the definition of a circle. 33% of the learners (52 learners) believed that the plane figure produced by drawing all points exactly 6cm from a given point is a square with a side of 6cm. This is a concern because it shows that most learners in the FET phase do not know the difference between a circle and a square.

Learner LSC2 had this to say about item 9:

"This question was confusing to me, I know what a circle is and I can even draw it but the proper definition never crossed my mind. I just took it for granted that a circle is just a shape I know since I was young."

This learner's comments highlight the fact that recognising a shape as a circle is easier than understanding a definition of a circle. For item 2, all the learners identified the circle correctly, yet in this item, only 15% were able to link the given description to the property of a circle that all the points are equidistant from the centre. This result also demonstrates the difficulty in moving from a verbal (written) representation to a diagrammatic representation.

5.2.9 Item 10

Item 10 was a question under the informal deduction (level3) level which involved logical relations. Learners were required to analyse the properties of figures and deduce relationships between the figures. Only 19 learners (13%) responded correctly to this item. 68 learners (77%) believed that there was no answer in the given statements. Since 77 % of the learners failed to respond correctly, it shows that very few learners are able to work with logical relations, which is a skill involved in the informal deduction level of geometric thinking. One of the learners explained the difficulty he experienced in distinguishing between two statements:

> **Researcher:** Can a shape be a rectangle and a triangle at the same time?
>
> **LSC1:** No a figure cannot be a rectangle and a triangle at the same time. When I was answering the question I had problems in choosing between (b) and (c), but I am just realising now that both cannot be true. The options required us to think carefully before we chose our answer.

In the above extract, the learner explains that she struggled to see the relationship between option b) (If 1 is false, then 2 is true) and option c) (1 and 2 cannot both be true). Option (b) expresses a conditional statement and option (c) is actually a negation of the false statement in (b) and is true. The learner first could not see the difference between the statements in (b) and (c) but during the interview realised that it is option (c) (both given statements cannot be true) that was correct.

5.2.10 Item 11

Item 11 also required learners to unpack logical relations between two statements. This item was classified at the informal deduction level of Van Hiele's levels of geometric thinking. Learners were expected to understand the properties of an equilateral triangle and an isosceles triangle. Out of the 147 learners who participated in the study, only 53 learners (36 %) responded correctly to this item, which is if triangle ABC is equilateral, then angle B and angle C are equal. There were 29 learners who did not respond and out of those who responded, 40 believed that there was no correct response in the given options.

5.2.11 Item 12

Item 12 likewise required reasoning at the informal deduction level of van Hiele's levels of geometric thinking. It required learners to understand the interrelationships between the properties of squares and rectangles, specifically the issue of class inclusion. Only 25 learners (17%) were able to tell that all properties of rectangles are properties of all squares. 25 learners did not attempt to answer the item whereas 69 learners believed that all properties of squares are properties of parallelograms. This was a misconception as there are some parallelograms which are not squares. A rectangle is a parallelogram but it is a square.

5.2.12 Item 13

Item 13 was also a question at the informal deduction level of the van Hiele levels of geometric thinking and was based on reasoning about class inclusion. Learners were required to analyse properties of rectangles and parallelograms and understand the relationships between the shapes. Learners were supposed to identify one property that all rectangles have, that some parallelograms do not have, which is diagonals are equal. Only 40 learners responded correctly (27%). It was quite worrying to note that 54 learners believed that in all rectangles, opposite sides are parallel whereas some parallelograms do not have parallel sides. This shows that these learners struggle with the notion of class inclusion.

5.2.13 Item 14

Item 14 was designed to assess reasoning at the formal deduction level of the van Hiele's levels of geometric thinking. Learners were supposed to understand interrelationships between definitions and axioms in the process, coming up with deductive chains of statements constituting proofs. Learners were given three properties of a figure and then asked to deduce the interrelationships between the properties. Only 24 learners (16%) responded correctly, that if a figure is a square then it implies that it's a rectangle and it implies that its diagonals are of equal length. 77 learners (49%) believed that having equal diagonals implies that the figure is a square and it implies that the shape is a rectangle. This is a misconception as there are some figures which have equal diagonals but they are not squares like rectangles. 34 learners did not respond to this question which corresponds to 22%. This item had the highest number of students with no response.

5.2.14 Item 15

Item 15 was set at the formal deduction level, requiring learners to understand the interrelationships between axioms and proofs. Only 7 learners (4%) responded correctly to the item, resulting in this item showing one of the worst performances.

5.3.1 Performance in Questionnaire A according to van Hiele's Levels

The items were grouped according to the van Hiele levels of Geometric thought and the average percentage of correct responses were then determined and recorded in the table below. The average percentage of correct responses was calculated by adding the percentage of correct responses per item at that level and dividing the sum by the number of items at that particular level.

$$\text{Average Percentage of Correct Responses} = \frac{Sum\ of\ Percentage\ Correct\ Responses\ per\ item\ per\ level}{Number\ of\ items\ per\ Van\ Hiele\ Level}$$

Table 5.2: FET learners' performance at each Van Hiele Level for Questionnaire A

Van Hiele Level of Geometric Thought	**Items**	**Short Description**	**Average Percentage of correct responses per Level**
Level 1, Visual Level	1 and 2	Identifying shapes by their appearance	100%
Level Two, Analysis Level	3, 4, 6, 7 and 9	Recognising shapes by their properties	42%
Level Three, Informal Deduction Level	5, 8, 10, 11, 12, 13 and 14	Analyse properties of figures and understand relationships between properties of shapes	30%
Level Four, Formal Deduction level	15	Develop a series of statements and start to understand the importance of deduction and vital role of axioms, theorems and proofs.	4%
Level 5, Rigour level	No items were set at this level	Reason formally about mathematical systems and understand geometric figures which are abstract	N/A

The table clearly shows a decline in the number of correct responses at given Van Hiele levels as we move from the most basic level, the visual level to the formal deduction level. Forty two percent of the learners' responses were correct for items at the analysis level. Thirty percent of the learners' responses were correct for items set at the informal deduction level whereas 4% of the learners' responses were correct for items set at the formal deduction level. No items were set at the rigour level as this is the highest level of thinking in the Van Hiele hierarchy where learners are able to reason formally about mathematical systems and they are in a position to understand geometric figures which are abstract. Learners who portray an understanding at the rigour level are most likely to be enrolled in a geometry course at university level. However it was important to note that as the item van Hiele level increases, the success rate decreases.

5.3.2 Classification of learners into different van Hiele Levels

As indicated in Table 4.3 above, the items in Questionnaire A were grouped into different van Hiele levels depending on the kind of information they required from the learners. The description for placement of questions was given in Table 4.3.

Learners were placed into a given van Hiele level according to their responses in the first questionnaire. This was dependent on the number of questions they were able to answer correctly at that particular level. For a learner to be placed at a higher van Hiele level he/she should first have met all the requirements for the lower levels. However out of the 147 learners, there were about five cases where a learner met the requirements for a higher level having failed to meet the requirements for the lower levels, but these cases were too few to be significant and these cases were then attributed to either guessing or copying. So these learners were placed at a lower level where they met the requirements.

Only item 15 was set at the formal deduction level. Learners were said to be operating at the formal deduction level if they were able to meet all the requirements for level 1, 2 and 3 and also to get item 15 correct. Getting item 15 correct, without meeting the requirements for levels 1, 2 and 3, was not enough for a learner to be placed at the formal deduction level as the learners could have copied. There were only two learners who were classified at this level with respect to their responses in Questionnaire A, and both these learners were interestingly, Grade 10 learners.

Seven items were set at the informal deduction level, and the items were item 5, 8, 10, 11, 12, 13 and item 14. Learners were said to be operating at the informal deduction level if they were able to get three or more items correct at this level, but they should also have met the requirements for the lower levels, in this case levels 1 and 2, however failing to meet the requirements for level 4. There were two reported cases where 2 learners failed to meet the requirements for level 2 placement but they met the minimum requirements for placement into level 3. These two cases were overlooked and the two learners were placed into level 1, as these 2 cases were too few to be significant. Three items were taken as a minimum requirement for level three placements because learners show different levels of competence at a particular van Hiele level. A learner may be beginning to understand and being able to work with problems at the informal deduction level, while another learner may be at an advanced stage of level 3 and thus will be able to work with more complicated problems at that particular van Hiele level.

Five items were at the analysis level of the van Hiele levels of geometry thinking and these were items 3, 4, 6, 7 and 9. A learner was placed at the analysis level (level 2) if he/she was able to meet the requirements for level 1, was able to get two or more items correct at level 2, and was not able to meet the requirements for placement into the informal deduction level.

All the learners who failed to meet the requirements for placement into levels 2, 3 and 4 were then placed into the visual level.

The table showing the raw data of the one hundred and forty seven (147) learners and how they were placed into different van Hiele levels appears in Appendix A.

A summary of all the learners at each van Hiele level with respect to their responses in Questionnaire A, is presented in the diagram below.

Table 5.3 Summary of the van Hiele Levels of the FET learners.

Van Hiele Level	Number of Learners Operating at a Particular Level	Percentage of Learners Operating at a Particular Level
Visual Level [Level 1]	23	16%
Analysis Level [Level 2]	77	52%
Informal Deduction Level [Level 3]	45	31%
Formal Deduction [Level 4]	2	1%
Rigour Level [Level 5]	0	0

The above table shows that 16% of the FET learners involved in the study have not progressed beyond the visual level of the van Hiele's levels of geometric thinking. Even though the van Hiele levels are not age dependent, one would expect learners at the FET stage to be operating at level 3 or 4, as they have been exposed to many opportunities of working with geometric figures and thus are expected to show an advanced knowledge of geometry. These learners have not moved beyond the stage of showing the most basic knowledge of identifying shapes by their appearances as they have a very simple concept of space. It's of great concern that 16% of the FET learners involved in the study don't have any knowledge of parts or properties of geometric figures despite having passed through the first 9 years of formal education.

Slightly more than half of the FET learners involved in the study (52%), were operating at the analysis level of the van Hiele's levels of geometric thinking. These learners understood the properties of geometric figures, they could classify properties of some different shapes and analyse the properties but they could not make any connections between shapes and their properties. The learners at the analysis level were able to investigate, understand, deduce and make generalisations from the properties of the figures.

The table also shows that 31% of the FET learners were operating at the informal deduction level of the van Hiele levels of geometric thinking. These learners were able to analyse and understand the relationships between properties of figures. It is at this level that the learners can start putting the properties of the figure in the correct order and be in a position to follow all the logical arguments. A good example of this was item12, where learners were supposed to have knowledge of squares, rectangles and parallelograms.

Only 2 learners showed competence at the formal deduction level which translates to 1% of the learners involved in the study. Both these learners were from grade 10, one from school A and the other from school B. There was no learner from grade 11 who showed competence at the formal deduction level. This confirms one of the characteristics of the van Hiele model which states that all the levels are not dependent on age.

5.3.3 Comparison of the van Hiele Levels of learners between grade 10 and grade 11

The number of learners per grade, operating at each van Hiele level for Questionnaire A was also recorded and shown in the table below.

Table 5.4 Comparison of learners' van Hiele Levels between grade 10 and grade 11.

Van Hiele Level	Grade10	Percentage	Grade 11	Percentage
Visual	8	11%	15	21%
Analysis	46	62%	31	42%
Informal Deduction	18	24%	27	37%
Formal Deduction	2	3%	0	0%
Rigour	0	0	0	0%

Table 5.4 above shows that 11% of the grade 10 learners involved in the study were operating at the visual level for Questionnaire A, as compared to 21% of the learners in grade 11 who were operating at the visual level. This is worrying as one would expect more grade 11 learners to be operating at higher van Hiele levels than the grade 10 learners as they have been exposed to more geometric tasks. Grade 11 learners are expected to have done geometry tasks from grade 1 up to grade 11 whereas grade 10 learners did geometry from grade 1 up to grade 10, even though progression through the van Hiele levels is not age dependent.

Sixty two percent of the grade 10 learners were operating at the analysis level as compared to 42% of the grade 11 learners. At the informal deduction level 37 % of the grade 11 learners were able to work with tasks given at this level as compared to 24% of the grade 10 learners. There was no grade 11 learner who was able to operate at the formal deduction level whereas 2 learners (3%) from grade 10 were operating at level 4 of the van Hiele levels.

5.3.4 Comparison of learners' performance in Questionnaire A

The performance of both grade 10 and grade 11learners for each item in Questionnaire A was then compared.

Table 5.5 Comparison of learners' performance for each item per Grade

Item	Percentage of Correct Responses	
	Grade 10	**Grade 11**
1	100%	100%
2	100%	100%
3	15%	62%
4	77%	76%
5	64%	72%
6	68%	70%
7	31%	77%
8	18%	47%
9	18%	23%
10	10%	20%
11	25%	58%
12	27%	7%
13	12%	42%
14	27%	10%
15	3%	7%

The table above shows that all the grade 10 and grade 11 learners were able to answer items 1 and item 2 correctly. Items 1 and 2 were questions set at the visual level of the Van Hiele's levels of geometric thinking, showing that none of the learners have problems identifying objects because of their appearances.

In item 3, 62% of the grade 11 learners responded correctly as compared to 15% in grade 10. The item was on choosing the correct definition for perpendicular lines. Interviews with some of the learners showed that most of the learners in grade 10 failed to respond correctly to item 3 because they failed to understand some of the terms used in the options given. Terms such as right angles, acute angles and obtuse angles confused most grade 10 learners, as most of them failed to get the question correct. According to Clement (1999), the knowledge of geometry and geometric reasoning is acquired through active interaction and exploration with shapes. The more exposure a learner gets to geometric activities, the more likely are the chances that the learner will be able to move from one van Hiele level to another. The item was at the analysis level of geometric thinking as it involved properties of a geometric figure.

The performance of learners in item 4 was almost the same for both grades, with 77% of grade 10 learners and 76% of grade 11 learners responding correctly. The high percentage of correct responses is mainly because the area concept is strongly emphasized in the lower grades and those who failed to respond correctly, failed to do so because of calculation errors, like failing to multiply 3 and 12 correctly, rather than a lack of knowledge for the calculation of the area of a rectangle.

In item 5, learners were required to compare properties of similar and congruent triangles. Seventy two percent of the grade 11 learners responded correctly to the item as compared to 64% of grade 10 learners. Item 6 involved describing parallel lines, which implies that it was also at the analysis level of van Hiele's levels of geometric thinking. The percentage of correct responses was high for both grade 10 and 11 learners, with grade 10 having 68% of correct responses and grade 11 having 70% of correct responses. This shows that many grade 10 and 11 learners are able to work with questions set at the analysis level of geometric thinking. Learners did far better in this item than in item 3 where learners were asked to provide a definition of perpendicular lines. An interview with one of the learners, (LSC10), a grade 10 learner, revealed that learners have greater exposure to the term parallel lines than perpendicular lines, mainly because parallel lines are used in many everyday life activities and maths activities, for example parallel sides of a rectangle, square, parallelogram or in a debate whether railway lines are parallel or not. The learner also revealed that the term perpendicular lines is mainly used in academic contexts, like when doing geometry, so learners are more likely to forget its meaning than that of parallel lines.

Item 7 involved describing an equilateral triangle and thus this item was at the analysis level. Seventy seven percent of the grade 11 learners responded correctly as compared to 31% of grade 10 learners. Item 8 required learners to have knowledge of parallelograms and other shapes such as congruent triangles, equiangular shapes and also knowledge of the perimeter of shapes. For the grade 10 learners, 18% of the responses were correct and 47 % of the responses were correct for grade 11 learners.

Overall, the grade 11 learners fared better than the grade 10 learners in Questionnaire A except for items 12, 14 and 15. The three items (items 12, 14 and 15) involved properties of rectangles and squares and the interrelationships between the two quadrilaterals. Item 12 required learners to have knowledge of the properties of squares, rectangles and parallelograms. Item 14 also required a deeper understanding of the interrelationships of the properties of squares and rectangles. The properties of quadrilaterals are covered in grade 10, so both grade 10 and 11 learners had already been exposed to this at the time the data was collected.

In item 12, 21% of the grade 10 learners responded correctly as compared to 7% of correct responses from grade 11 learners. Twenty seven percent of grade 10 learners also responded correctly in item 14 while the correct responses for grade 11 were 10%. Items 12, 14 and 15 were set at the informal deduction level and the formal deduction level, but they all involved quadrilaterals, their properties and interrelationships. The grade 10 learners who managed to respond correctly to items 12, 14 and 15 were all from the same school and this could be because they had a teacher who was able to provide them with teaching and learning activities which allowed them to develop their thinking at the informal deduction level.

Of interest is that t-tests carried out on the responses of Grade 11 and the Grade 10 learners being considered as independent groups, show that the differences in the mean for each of these items were not statistically significantly different. So although the grade 10 learners performed better in some items and grade 11 learners in other items, the differences cannot be explained by virtue of their grade level.

5.3.5 Progression from one van Hiele Level to the next

Table 5.2 indicates that the percentage of correct responses sequentially decreased from the most basic level to the most complicated level, visual level (100%), analysis level (42%), informal deduction (30%) and formal deduction (4%). No questions were set at the rigour level of the van Hiele's levels of geometric thinking. According to Usiskin (1982), the van Hiele levels of geometric thinking are sequential and learners should progress from preceding levels to arrive at the next specific level as the levels are hierarchical. This was shown by the percentage of correct responses at any given level. The fact that the visual level had 100% correct responses, analysis level, 42%, informal deduction level, 30% and formal deduction level, 4%, means that the learners who failed to respond correctly to a level 1 question were less likely to respond correctly to a level 2, 3 or 4 question and hence the percentage of correct responses sequentially decreased from one level to the next.

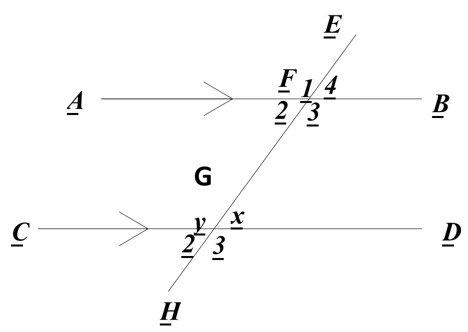
5.4 Results for Questionnaire B

Questionnaire B consisted of 6 open-ended questions unlike Questionnaire A which only consists of multiple choice items. The results for each of these items are presented together with interview responses where relevant.

5.4.1. Question 1 analysis

Question one in Questionnaire B was as shown below.

1. Consider the diagram drawn below.



1.1 Name two angles which are equal to x and explain why they are equal.

(i)____________________Reason _____________________________________

(ii) ___________________ Reason _____________________________________(4)

1.2 Name two angles which are equal to y and explain why they are equal.

(i)______________________ Reason ____

(ii) ______________________Reason ____(4)

Comment

Most of the learners responded correctly to this item which was set at the Analysis level of Van Hiele's levels of geometric thinking. The question required learners to have knowledge of properties of geometric shapes, in this case parallel lines and angles between the parallel lines. Learners were supposed to identify angles which were equal to x and y and to give reasons for their answers. The question carried 8 marks and the summary of the learners' performance is shown in the table 5.6 below.

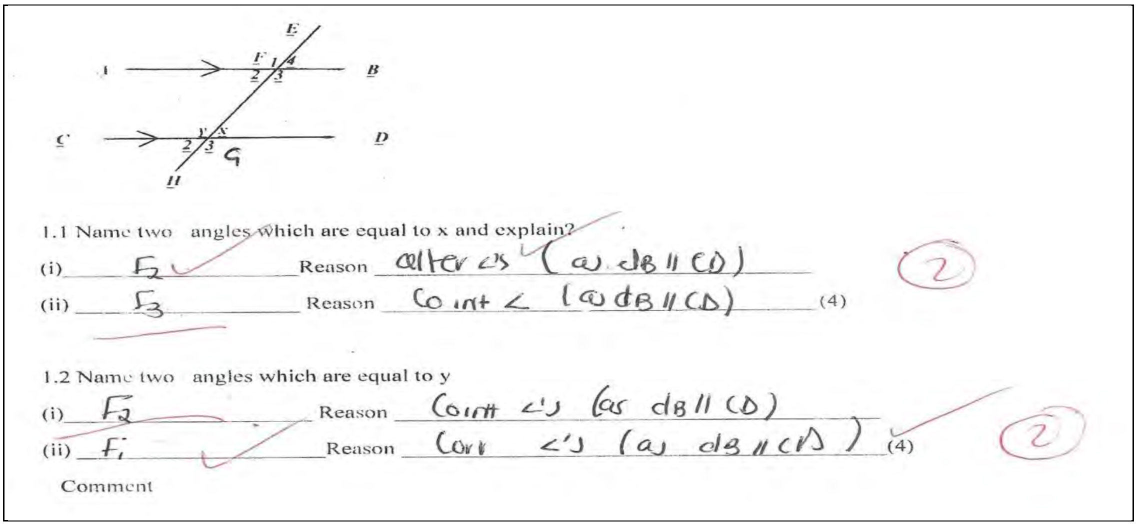
Table 5.6 Summary of learners' performance in question one.

Number of Marks	Number of Learners	Percentage of Learners
1	0	0%
2	23	16%
3	8	5%
4	11	8%
5	5	3%
6	47	32%
7	4	3%
8	49	33%

Most of the learners involved in the study were able to identify two other angles which were equal to x and y. However many learners experienced problems in coming up with correct reasons as to why the named angles were equal to x and y. The results in Table 5.6 show that 11 learners, which correspond to 7% of the learners, scored 4 marks out of the possible 8 marks in Question 1. Eight of these learners identified the equal angles without giving the reasons why they were equal by simply looking at the angles. These learners then failed to give the correct reasons as to why the angles were equal. These learners were operating at the visual level of the van Hiele's levels of geometric thinking as they identified equal angles because of their appearance and not because of their properties.

The other three learners who scored 4 out of the possible 8 lost marks for various reasons like failing to identify the correct angles. One such learner was learner LSC3 whose response to question one is shown in the figure below.

Figure 5.1: Learner LSC3's response to question 1.

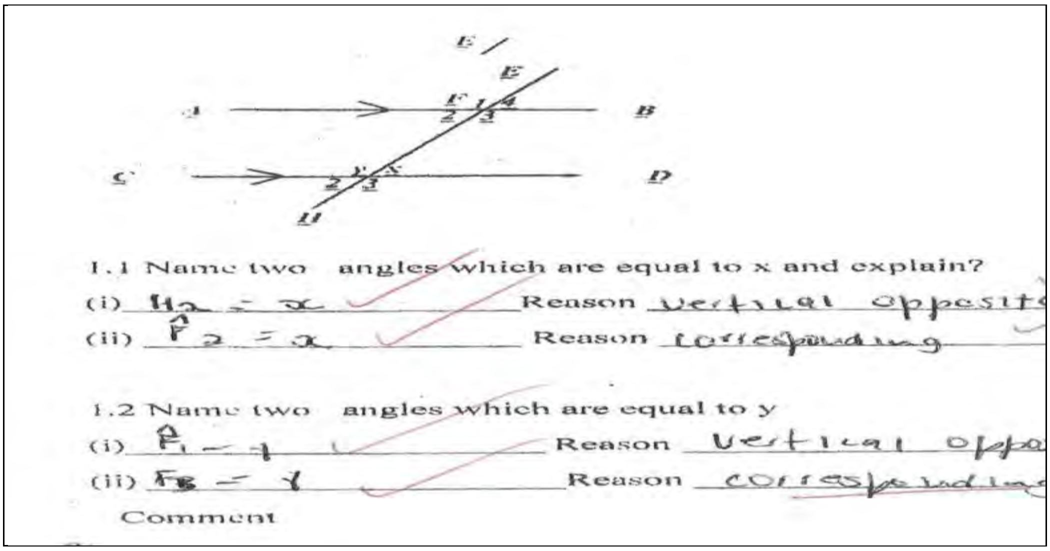


The learner wrote $F_3 = x$, and the reason this learner gave was because they are co-interior angles between parallel lines. He correctly identified the pair as forming co-interior angles but had a misconception about the relationship between co-interior angles formed between two parallel lines. Co-interior angles between parallel lines are supplementary (they add up to 180^{o}). The learner recognised the co-interior angles but was not so clear about the relationship between co-interior angles between parallel lines. The learner's lack of understanding of co-interior angles was revealed in the interview below:

> **Researcher:** "In both questions 1.1 and 1.2, you gave one correct answer and one wrong answer. In the wrong responses you gave the same reason of co-interior angles. Can you explain to me why you chose this and if possible the meaning of co-interior angles."
>
> **LSC3:** "To be honest I just thought since alternating angles between parallel lines are equal then co-interior angles will also be the same. I am still able to recognise that angle F3 and x are co-interior but I can't remember what will be the relationship between them. When we were taught geometry, the terms were never explained to us, we were just told the angles were equal because they are alternating or because they are vertically opposite and that was that."

Most of the learners who scored 6 or 7 marks out of the possible 8 in Question one, failed to give the correct reasons for their answers. One such learner was learner LSB2, whose response to question 1 is shown in the figure below.

Figure 5.2: Learner LSB2's response to question 1



The figure shows that learner LB2 responded correctly to Question 1.1(ii) but his reason for 1.1 (ii) was incorrect. Learner LB2 also failed to give the correct reasons for Question 1.2. This learner said angle $\widehat{F}_1$= y because they are vertically opposite instead of saying because they are corresponding angles, indicating a lack of deeper understanding of the terms used in geometry even though he recognised angles which were equal.

The results show that 68% of the learners involved in the study were able to score 6 or more marks in question one. This shows that more than 50% of the learners were able to work at the analysis level of the van Hiele's levels of geometric thinking as this question requires learners to think at the analysis level. The results also show that 29% (42 learners) of the learners were at the visual level as they only managed to identify equal angles without understanding why the angles were equal.

5.4.2 Question 2 analysis

Question 2 was set at the analysis level of the Van Hiele's levels of geometric thinking as it required learners to have knowledge of the sum of angles of a triangle. Angle C belongs to triangle ABC while at the same time it is also an angle in triangle CDE. A learner needed to first calculate angle C in triangle ABC and then use it to calculate x in triangle CDE. In both cases a learner needed to understand the property of sum of angles of a triangle. The question carried a total of 3 marks. The question is shown below.

Question 2

Find the value of x in the diagram below. (3)

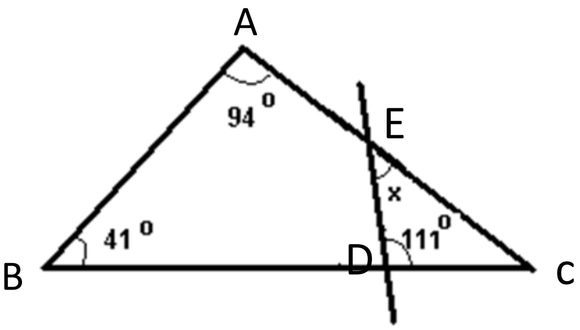
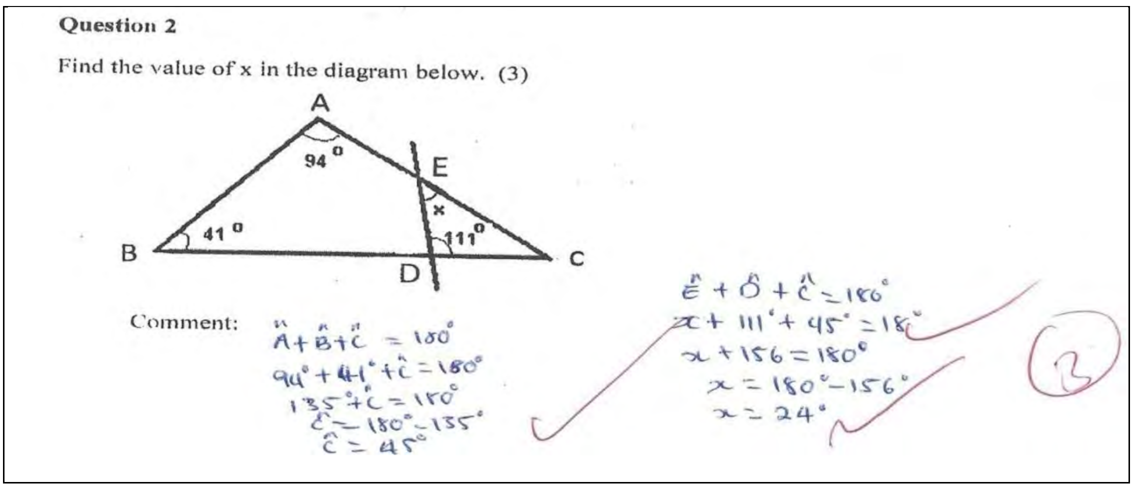


Table 5.7 Summary of the learners' performance in question 2

Number of Marks	Number of Learners	Percentage of Learners
0	77	52%
1	2	1%
2	1	1%
3	67	46%

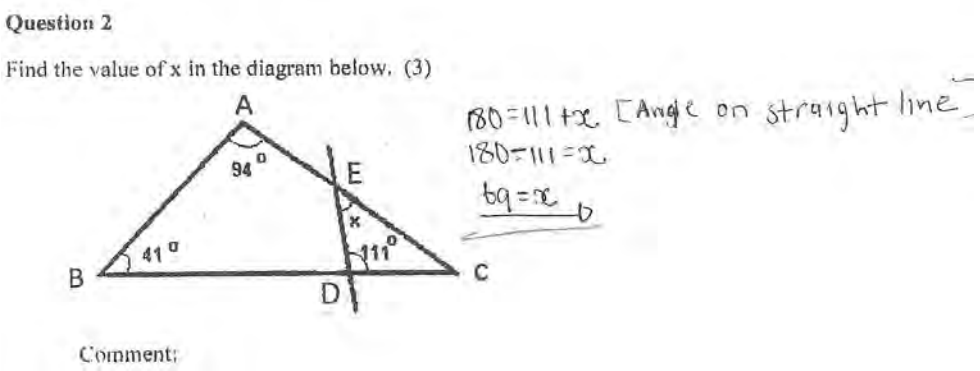
The table shows that 46% of the learners involved in the study answered this question correctly, thus managing to get all the marks. The responses in this item were a bit worrying as most of the learners in one of the schools produced one or more of the same answer. There might have been some element of copying on the part of learners for the answers to be similar. Learner LA2 was one of the learners who managed to get all 3 marks from question 2 and the responses which this learner gave are shown in the figure below.

Figure 5.3 Learner LSA2's response to question 2



Learner LSA2 managed to discover that there are two triangles involved in solving for x, that is triangle ABC and triangle EDC and one was supposed to use the property of sum of angles of a triangle to solve for x. Out of the 62 learners who participated in school A, 41 learners produced almost the same steps as learner LSA2. In school B almost all the learners also produced answers which were similar, raising questions on whether there was an external influence to the learners' responses. There were 77 learners who failed to score a mark in question 2 and that corresponds to 52% of the learners involved in the study. Most of these learners had a common misconception as shown in the responses given by learner LSC1 in the figure below.

Figure 5.4 Learner LSC1's response to question 2



The learners with this type of misconception believed that angle $C\hat{D}E$ and angle $D\hat{E}C$ lie on a straight line and the sum of angles on straight line gives 180 degrees. Out of the 77 learners who failed to get this question correct, nearly 50 of them had the same misconception as learner LSC1. An interview with learner LSC1 regarding question 2 resulted in the following observations.

> **Researcher**: "When you added $C\hat{D}E$ and $E\hat{D}C$, you said they must give 180 degrees. Can you please explain the reason why you responded in that way?"
>
> **Learner LSC1:** "When we were learning properties of geometric shapes, I still remember one property which says the sum of angles on a straight line add up to 180º. If we check angles $C\hat{D}E$ and $E\hat{D}C$, they are both lying on the straight line ED, so if we add them they must give us 180º"
>
> **Researcher:** Don't you think that there was a kind of misunderstanding of the concept of straight lines? Can you please look at angles $B\hat{D}E$ and$E\hat{D}C$, and also angles $D\hat{E}C$ and angle $D\hat{E}A$ and tell me what you think about the whole concept of angles on straight line?
>
> **Learner LSC1:** (long pause). I think I might have made a mistake. The angles have to be at the same point and when you put the angles together they must form a straight line like in the case of angles $B\hat{D}E$ and$E\hat{D}C$.

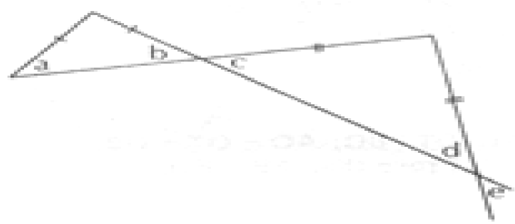
The other learners who failed to get marks from question 2 believed that angle $E\hat{D}C$ and $D\hat{E}C$ were the equal angles of an isosceles triangle. However there was no isosceles triangle given in the diagram and the learners seemed to have made additional assumptions so that they could get to a solution.

5.4.3 Question 3 analysis

Question 3 was set at the Informal deduction level of the Van Hiele's levels of geometric thinking as it required learners to analyse properties of the figures and understand relationships between properties of the shapes. The question is shown in the figure below.

Question 3

Prove, giving reasons, that $\hat{\boldsymbol{a}} = \hat{\boldsymbol{e}}$. **(4)**



Learners were required to have knowledge of the properties of isosceles triangles and angles formed by intersecting lines, mainly vertically opposite angles. Learners were also supposed to have an understanding of the transitivity property of angles. According to Ryan (2008), the transitivity property states that if two angles are congruent to a third angle, then all the angles are congruent to each other. Stephan & Clements (2003, p.5) outlined transitivity as the understanding of the following points:

> If angle A is equal to angle B and angle B is equal to angle C then angle A is equal to angle C. If angle A is greater than angle B and angle B is greater than angle C, then angle A is greater than angle C, and, if angle A is less than angle B and angle B is less than angle C, then angle A is less than angle C (Stephan & Clements, 2003).

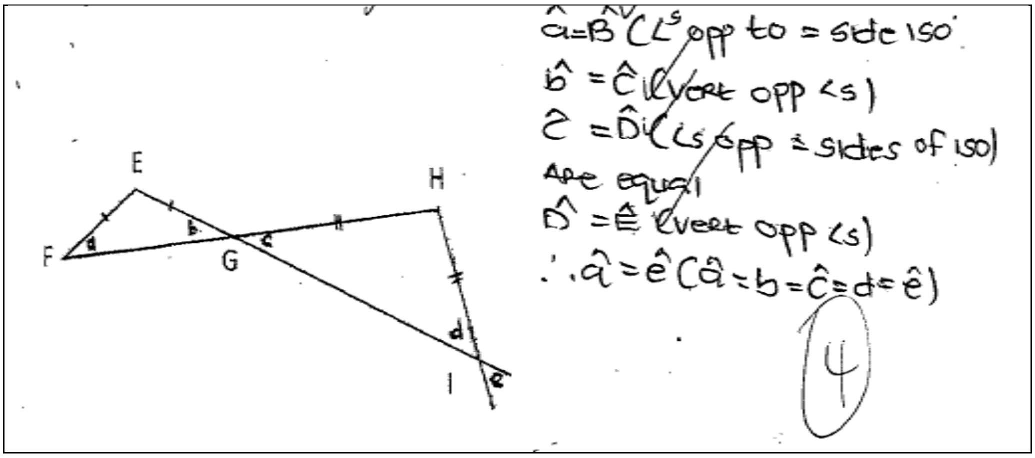
The table below shows how the learners performed in this question out of a possible 4 marks.

Table 5.8 Summary of learners' performance in Question 3

Number of Marks	Number of Learners.	Percentage of Learners
0	72	49%
1	9	6%
2	6	4%
3	2	1%
4	58	40%

Of the learners involved in the study 40% gave correct reasons for their answers in Items 3 thus getting full marks. Most of these learners responded in almost the same way as shown in the figure 5.5 below for learner LSA16.

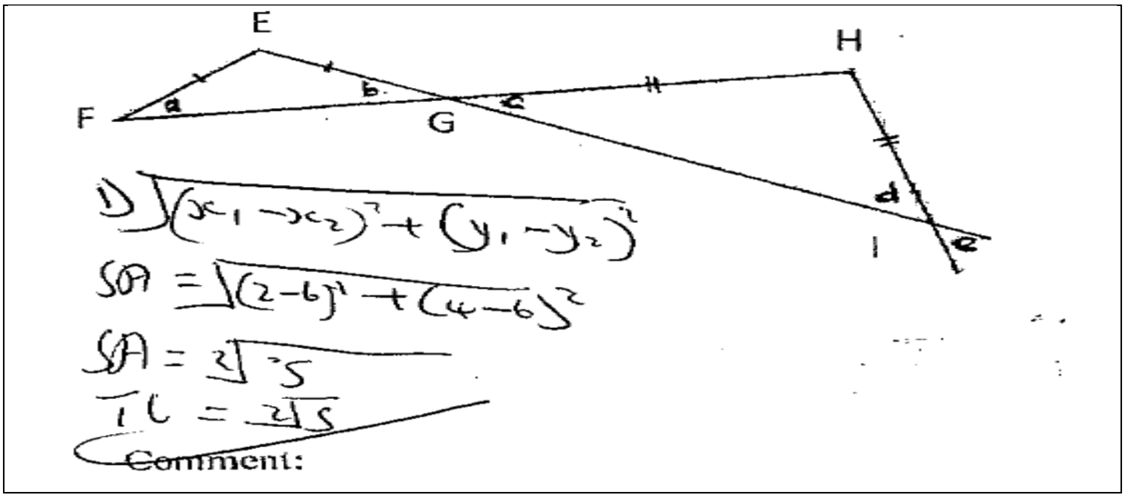
Figure 5.5: Learner LSA16's response to Item 3



This learner acknowledged that triangles FEG and triangle GHI were isosceles triangles since two of their sides were equal and hence two angles were equal. The learner recognised that vertically opposite angles formed at the intersections of lines EI and FH and lines HI and EI, are equal. Using these properties, the learner was then able to prove that angle â is equal to angle ê.

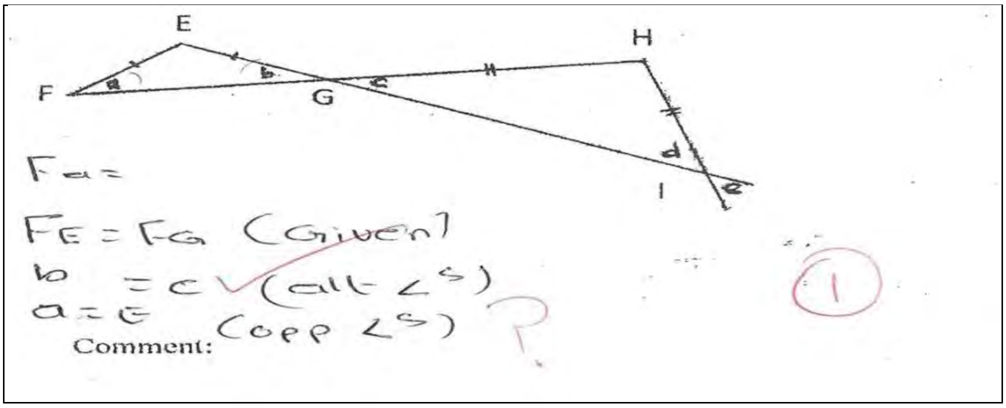
The 72 learners who scored 0 marks in question 3 did not even attempt to answer the question or showed a lack of conceptual understanding. These learners lacked knowledge of the requirements of the question and ended up using the distance formula which was not relevant as no coordinates were given in the diagram provided. The figure below shows the response of one of the learners who lacked conceptual understanding of the properties of the given triangles and how they relate to each other.

Figure 5.6: Learner LSB7's response to item 3



The learners who scored one mark were only able to identify 2 angles which were the same but could not give a reason as to why the angles are equal. One such learner was learner LSA5 whose response is shown in the figure below.

Figure 5.7: Learner LSA5's response to question 3



This learner managed to identify that side FE = EG as given on the diagram but he failed to use that property to find equal angles from the isosceles triangle. The learner also managed to state that angles b and c are equal even though the reason for them being equal was wrong. The learner said they were equal because they were alternating instead of saying because they were vertically opposite. The learner then reached a conclusion without giving enough information. However the responses from the learners indicated that 45% of the learners were now able to work at the informal deduction level as 45% managed to score 2 or more marks out of 4 marks.

5.4.4 Question 4 analysis

Question 4 was set at the informal deduction level of the van Hiele's levels of geometric thinking. It required learners to prove that the shape is a parallelogram. The question is shown below.

Question 4

Quadrilateral SALT with vertices S(2;4), A(6;6), L(5;4) and T(1;2) is shown in the figure alongside

Show that SALT is a parallelogram. (3)

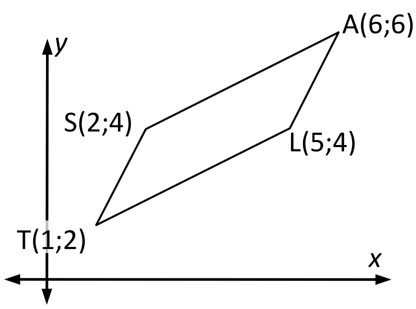
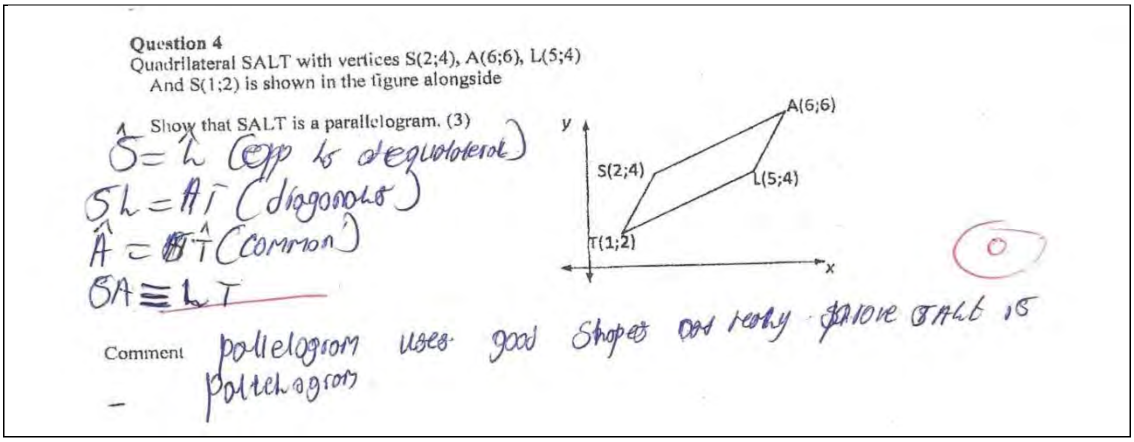


Table 5.9 Summary of learners' performance in question 4

Number of Marks	Number of Learners	Percentage of Learners
0	139	95%
1	5	3%
2	2	1%
3	1	1%

Most of the learners who scored zero in question 4 showed properties which were not necessary to prove that the shape is a parallelogram. One such learner was learner LSA10, whose response is shown in the figure below.

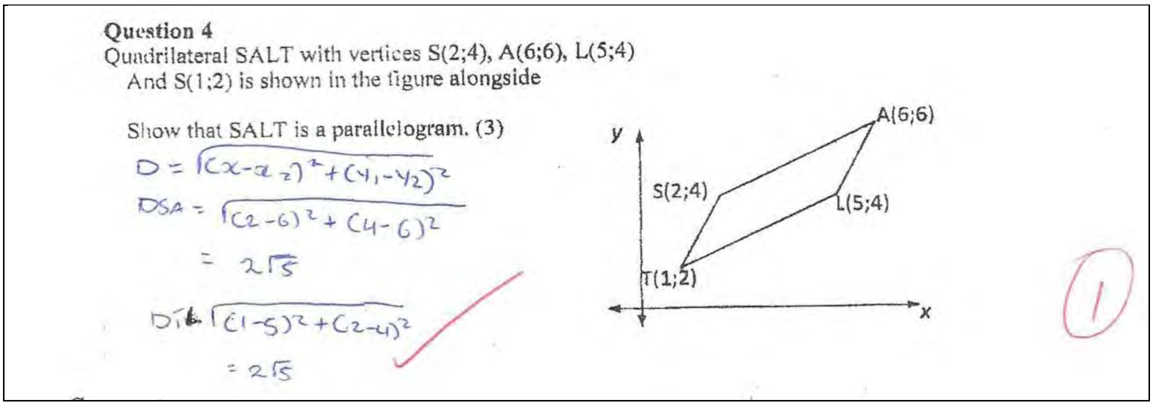
Figure 5.8: Learner LSA10's response to question 4



Learner LSA10 stated that the opposite angles of the shape SALT were equal without providing any calculations or explanations. The question required learners to do some calculations to justify the answers. Hence only mentioning that opposite angles are equal was not enough. Learner LSA10 and a number of other learners also mentioned that the diagonals SL and AT were equal and that was why the shape SALT was a parallelogram. This was incorrect as in this shape the diagonals are not equal and this could have been shown if the learner had tried to calculate the length of the diagonals. Not all parallelograms necessarily have diagonals which are equal in length. Parallelograms such as a square and a rectangle have equal diagonals but not this particular shape (SALT).

Learners who received one mark in question 4 did not provide sufficient information to show that the shape (SALT) was a parallelogram. One such learner was learner LSC 12, whose response to question 4 is shown in the figure below.

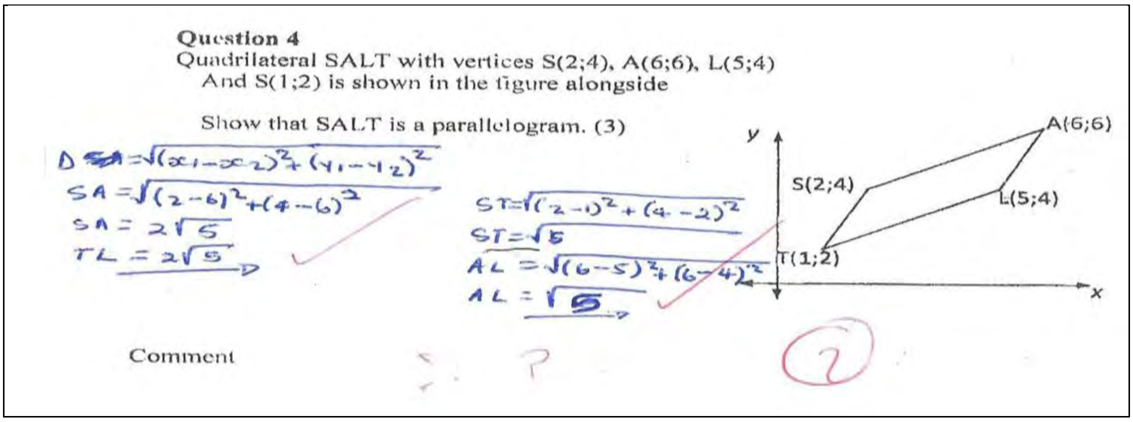
Figure 5.9: Learner LSC12's response to question 4



The figure showed that the learner used the distance formula to calculate the length of one set of opposite sides. Proving that one set of opposite sides are equal was not sufficient to prove that the shape SALT is a parallelogram as all the opposite sides must be equal. The learner showed that SA = TL = $2\sqrt{5}$, but did not calculate ST and AL.

Some learners after finding the length of the four sides did not make any further statements and they were allocated two out of the possible three marks. An example of such learners is learner LSA21, whose response to question 4 is shown in the figure below.

Figure 5.10: Learner LSA21's response to question 4



The learner managed to calculate and show that ST =AL= $\sqrt{5}$ and that SA=TL= $2\sqrt{5}$. However the learner did not thereafter conclude that because the opposite sides are equal, the shape therefore is a parallelogram.

The learner who scored three marks, managed to show that side SA is parallel to side TL, and that side ST is parallel to side AL by showing equal gradients.

5.4.5 Question 5 analysis

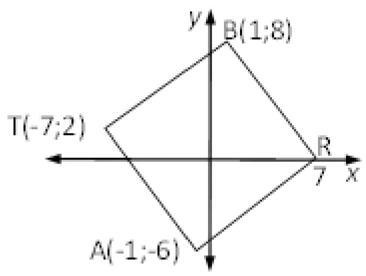
The question had two parts, 5.1 and 5.2. The first one (5.1) required learners to write down the coordinates of point R which was lying on the x axis and the second one involved proving that the quadrilateral was a square. The question is given below.

Question 5

Below is a quadrilateral BRAT. Use the quadrilateral to answer the following questions.

5.1 Write down the coordinates of R. (1)

5.2. Is BRAT a square? Why or why not? (3)



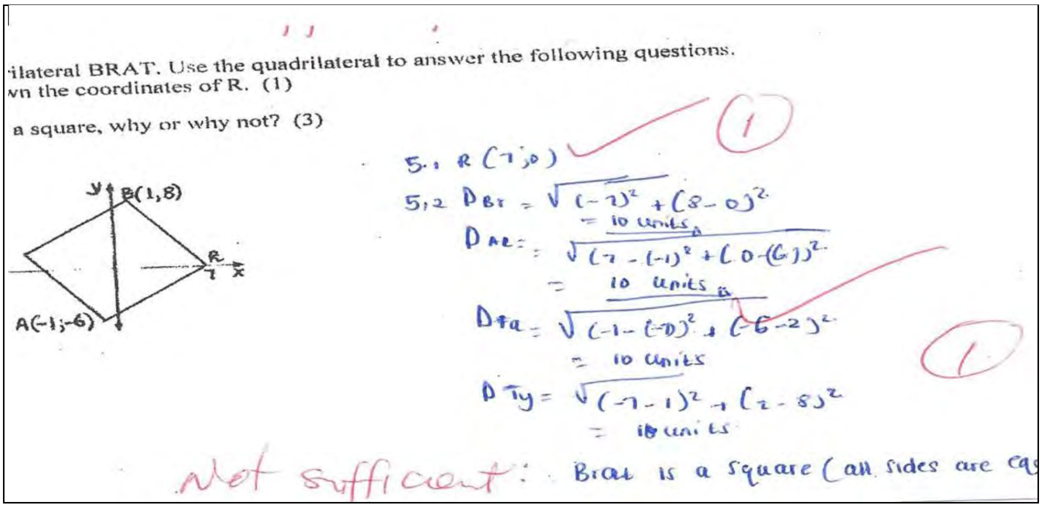
The question was set at the informal deduction level of the Van Hiele's levels of geometric thinking. Learners were supposed to understand the relationships between the properties of the shape BRAT in order to prove that it is a square. Most of the learners scored one or two marks in question 5 because they only proved necessary but not sufficient conditions. The results for Question 5 are summarised in the table below.

Table 5.10 Summary of learners' performance in question 5

Number of Marks	Number of Learners	Percentage of Learners
0	74	50%
1	64	44%
2	7	5%
3	1	0,5%
4	1	0,5%

Most of the learners' responses showed that learners lacked the knowledge of necessary and sufficient conditions for a quadrilateral to be a square. The table shows that 99% of the learners were unable to establish sufficient conditions, with many proving the property of four equal sides. However a quadrilateral with all sides equal can be a rhombus but not a square, where angles may not be 90°. This then implies that the property that all sides are equal is a necessary but not sufficient condition for a shape to be a square. A good example of one such learner was learner LSC11 who only proved that the sides are equal and then concluded that the shape is a square.

Figure 5.11 Learner LSC11's response to question 5

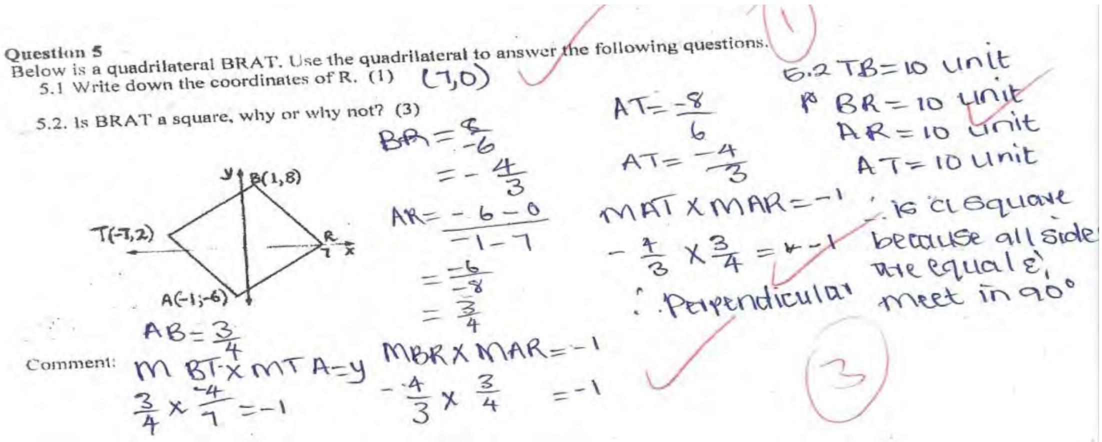


The response by learner LSC11 showed that the learner understands the properties of a square in isolation as he failed to make connections between the properties so as to prove that the shape BRAT is a square. To him if the sides are all equal, then it means that the shape is a square. The extract below was taken from the interview conducted with Learner LSC11.

> **Researcher:** Can you describe to me what you understand by a square and some of its properties.
>
> **Learner LSC11:** When I was growing up I knew that a square has equal sides, so I did not think about other properties, the first thing that came to my mind was to show that all sides are equal and that's what I did.
>
> **Researcher:** Can you look at these two rhombuses that I am drawing on this graph and tell me what you think, whether you still agree with your definition or needs some refining.
>
> The researcher draws a square and a rhombus whose angles are not 90 degrees.
>
> **Learner LSC11:** Mmmm (long pause). It seems like the one with angles 90° is the square, the other rhombus cannot be a square because the angles are not right angles even though the sides are equal. I therefore think it is not enough to prove that a shape is a square by showing that the sides are equal, one needs to show that all angles are right angles.

The table also shows that only 0.5% of the learners involved in the study managed to prove all the sufficient properties for the shape BRAT to be a square. One such learner was learner LSA11 whose response to question 5 is shown in the figure below.

Figure 5.12: Learner LSA11's response to question 5



Learner LSA11 showed that all sides are equal to 10 units and that all the angles were 90º by showing that the sides were meeting at 90º, thus they were perpendicular to each other. However if the learner first proved that all sides are equal, then that would make the figure a rhombus and then it was sufficient to show only one angle equal to 90º, because that would then imply that all angles are 90º.

There are several necessary conditions for a shape to be called a square, and all of them must be satisfied for the shape to be a square and these are: the shape must lie in a plane shape; the four sides must be equal in length; each of the shape's interior angles must be equal to the others and equal to 90º and the sides must be joined at the ends (Swartz, 1997).

There are different sets of sufficient conditions for a shape to be square. For example, one set would be that the shape must be a rectangle with a pair of adjacent sides equal. Another set of sufficient conditions could be that the figure should be a quadrilateral with all four sides equal and an angle equal to 90º.

Distinguishing between necessary and sufficient conditions is a level 3 (informal deduction level) question on the van Hiele's levels of geometric thinking. According to van Hiele theory such reasoning is evident in learners who are working at the informal deduction level of the model. This level requires learners to understand the properties and relationships between the properties of figures and relationships between figures.

The other learners who received 2 marks were only proving that the angles are equal to 90º without showing that the sides are equal. When all the angles are right angles, the shape can be a rectangle, where all the sides might not be necessarily equal. Hence the property of four equal angles is a necessary property of a square as in the case of a rectangle but it is not sufficient. A lack of such kind of reasoning shows that the learners were working with properties in isolation, failing to make connections between them, thus operating at the analysis level of the van Hiele's levels of geometric thinking.

Other students struggled because they were unable to use calculations to find the relationships between the sides and the angles of the shape. These learners failed to calculate the correct gradients for the sides so as to prove that either the sides were equal or parallel or that the angles were 90º. This means that these learners lacked computational skills and familiarity with procedures, rules and algorithms for proving that the shape was a square. A further analysis of the results showed that those learners who managed to find the gradients failed to use the value of the gradients to deduce that the relevant angles were 90º.

5.4.6 Question 6 analysis

Question 6 was set at the formal deduction level of the van Hiele's levels of geometric understanding. The question required learners to understand and use the ideas of formal geometry to understand interrelationships between definitions, theorems and proofs. The learners were supposed to come up with deductive chains of statements constituting proofs. The question required learners to understand and use the ideas of formal geometry. The learners were supposed to analyse the properties of the figure and understand the relationships between properties of the shape which contained parallel lines, isosceles triangles and a parallelogram. The question is shown below.

Question 6

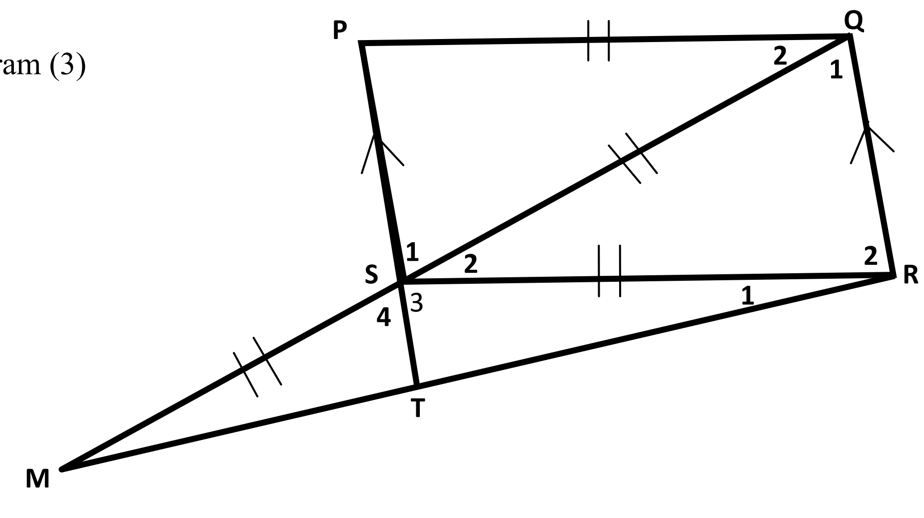
In the diagram below, PQ = QS = SR = SM and PS // QS.

Prove that:

6.1. $\hat{R}_2 = \hat{P}$ (2)

6.2. ΔPQS ≡ ΔRSQ (3)

6.3. PSRQ is a parallelogram (3)



A summary of the learners' performance in this question is represented in the table 5.11 below.

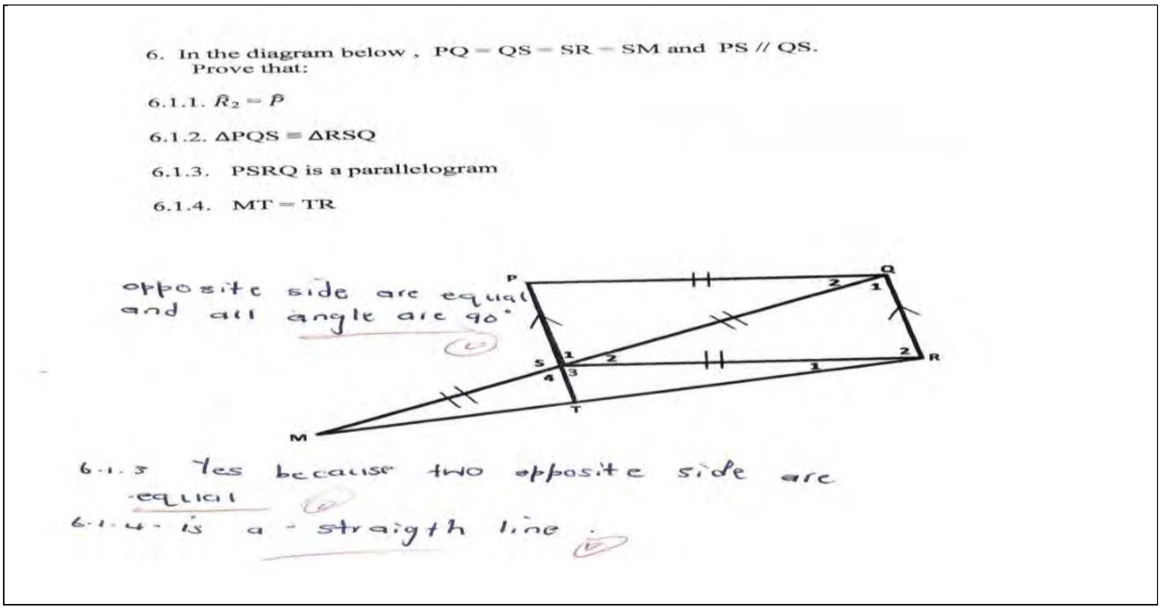
Table 5.11 Summary of learners' performance in question 6

Number of Marks	Number of Learners	Percentage of Learners who scored that particular mark
0	114	77%
1	28	19%
2	3	2%
3	0	0%
4	1	1%
5	1	1%
6	0	0%
7	0	0%
8	0	0%

This table shows that 77% of the learners who participated in the study got zero out of the possible 8 marks. This means that 77% of the learners involved in the study failed to prove congruent triangles even though this work is covered in grade 9 according to the new CAPS document and also failed to prove that a given shape is a parallelogram. Quadrilaterals are covered at grade 10 but most of the learners still failed to provide the proofs.

Most of the learners did not attempt to answer question 6 and those who tried to answer it, such as learner LSC9, shown below in Figure 4.13, showed that they did not recognise the requirements of the question and possible routes to the solution. The response shows very low engagement with the properties of geometric figures.

Figure 5.13: Learner LSC9's response to question 6


6. In the diagram below , PQ = QS = SR = SM and PS // QS.
Prove that:

6.1.1. $R_2 = P$

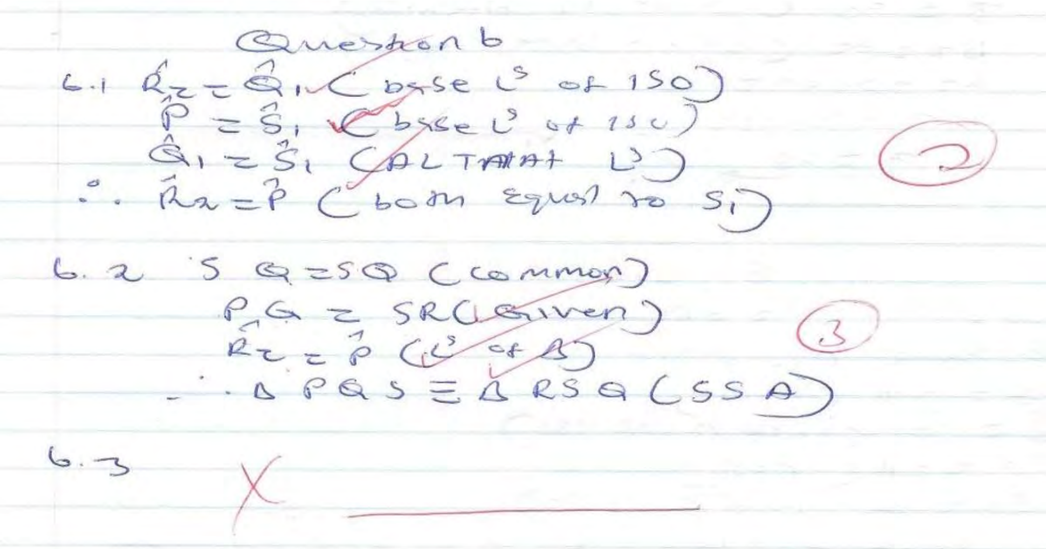
6.1.2. ΔPQS ≡ ΔRSQ

6.1.3. PSRQ is a parallelogram

6.1.4. MT = TR

The learners, who scored 3 marks or more out of the possible 8 marks, did not give all the necessary and sufficient properties which are required to prove the congruent triangles and the parallelogram. In question 6.1, the learners were supposed to use the transitivity property of angles to prove that $\hat{P}$ = $\hat{R}_2$. Learners were first supposed to prove that $\hat{P}$ = $\hat{S}_1$ [equal angles of an isosceles triangles], then to show that $\hat{S}_1 = \hat{Q}_1$ [alternating angles], and then lastly to show that $\hat{Q}_1$ = $\hat{R}_2$. According to the transitivity property $\hat{P}$ is then equal to $\hat{R}_2$. Learner LSC3 managed to show the transitivity property and also proved the congruent triangles but she failed to prove the parallelogram. The response to question 6 by learner LSC3 is shown in the figure below.

Figure 5.14: Learner LSC3's response to question 6


Question 6
6.1 $\hat{R}_2 = \hat{Q}_1$ (base ∠s of iso)
$\hat{P} = \hat{S}_1$ (base ∠s of iso)
$\hat{Q}_1 = \hat{S}_1$ (ALTANAT ∠s)
∴ $\hat{R}_2 = \hat{P}$ (both equal to S_1)
2
6.2 SQ = SQ (common)
PQ = SR (given)
$\hat{R}_2 = \hat{P}$ (∠s of Δ)
∴ ΔPQS ≡ ΔRSQ (SSA)
3
6.3
X

The learner managed to prove that triangles PQS and RSQ are congruent by proving that two angles and one side of triangle PQS, are equal to two angles and a corresponding side of triangle RSQ. The learner was able to start the proofs from scratch by working with different properties and linking the interrelationships between the properties. Thus this revealed that the learner was working at the formal deduction level of the van Hiele's levels of geometric thinking. However she was unable to apply the fact of the congruency to deduce that PSRQ was a parallelogram.

5.4.7 The van Hiele Levels for the learners using Questionnaire B

The learners were grouped into different van Hiele levels depending on the degree of competence they displayed when responding to the different items in Questionnaire B. The items themselves were also set at different van Hiele levels as indicated in the previous sections.

In Question 1.1 learners who scored 3 or 4 marks out of the possible 4 marks were considered to be operating at the analysis level of the van Hiele levels for that item, because for them to get three or four marks, they needed to identify equal angles and provide reasons for them being equal. Two marks or less implies that the learner is at the visual level as either the learner would have identified equal angles but failed to give valid reasons or they failed to even identify the equal angles. The same process of categorizing learners was employed for question 1.2 as the questions were almost identical, using the same system of marks for the same van Hiele level.

In Question 2, learners who scored two or three marks out of the possible three marks, were considered to be operating at the analysis level as they would have shown competence in identifying the property of the sum of angles of a triangle and then using substitution to get the value of x.

In Question 3, learners who scored 2 or more marks out of the possible four marks were considered to be operating at level 3 as they would have shown that at least three angles were equal with reasons provided for their answers. For the learner to prove that three angles were equal, he/she would at least have to link two or more properties of triangles, for example, $\hat{a} = \hat{b}$ because they are equal angles of an isosceles triangle; then $\hat{b} = \hat{c}$, because vertically opposite angles are equal. Learners who scored one mark or no mark in Question 3 were considered to be operating on levels below level 3 (informal deduction level) as they would have used one property only or would have failed to use a property to prove that two angles are equal.

In question 4, learners who scored two or three marks were categorized as operating at level 3 of the van Hiele levels as they would have identified the sufficient conditions for the figure to be a parallelogram. Learners who scored one mark identified a property but failed to prove all the sufficient conditions and those who scored zero were grouped as operating below the informal deduction level.

In Question 5, learners who scored two or three marks out of the possible four marks were considered as operating at the informal deduction level, as they would have shown the sufficient properties for the figure to be a square.

Question 6 had three items which were all set at the formal deduction level. Learners were required to show knowledge of proofs by using the interrelationships between properties of shapes and the definitions of shapes. In question 6.1, a learner was found to be at level 4 if he/she managed to score 2 marks. In questions 6.2 and 6.3, learners categorized as operating at level 3 if they scored two or three marks out of the possible 3 in each of these questions.

Questionnaire B had three level 4 questions, which were questions 6.1, 6.2 and 6.3. A learner was classified as operating at the formal deduction level (level 4), if they managed to get one or more level 4 statuses in the three level 4 questions. Those learners who achieved one level 3 status, were identified as just starting to show competence at level 3.Those who achieved 2 or 3 level 4 statuses, showing understanding required at level 4, were thus classified as operating at an advanced stage of level 4. However, for learners to be classified as operating at level 4 across the instrument, they would have to meet all the requirements for the lower levels otherwise they would be placed at a lower level and their ability to get the level 4 questions would then be attributed to external factors like copying rather than their ability.

Questionnaire B had three level 3 questions, which were questions 3, 4 and 5. Learners were finally categorized as operating at the informal deduction level across Questionnaire B if they managed to get one or more level 3 statuses in these items. This was construed because learners could be showing different competence levels at the same van Hiele level, in this instance, the informal deduction level, with some just beginning to operate at the level (those with one or two level 3 statuses) and others showing more advanced understanding at level three.

Learners who failed to get level 3 or level 4 status using the above criteria in questions 3, 4, 5 and 6 were either operating at the analysis level or the visual level. Questions 1.1, 1.2 and question 2 required thinking at the analysis level and a learner was given a level 2 status if he/she got 2 or more level 2 statuses but failed to meet the requirements for level 3 and 4. However a learner who had one level 2 status but managed to get some marks in the items which needed thinking at higher levels, was also classified as operating at level 2.

The remaining learners were then categorized as operating at the visual level after failing to show competences at the analysis, informal deduction level and the formal deduction level.

The raw data for the classification of learners into different van Hiele levels using the above criteria is shown in the Appendix section.

The table below shows a summary of the van Hiele levels of all the grade 10 and 11 learners.

Table 5.12: Learners' van Hiele Levels from Questionnaire B

	Number of Learners			
Grade	**Visual Level**	**Analysis Level**	**Informal Deduction Level**	**Formal Deduction**
10	35	29	8	2
11	15	11	47	0
Total	50	40	55	2
Percentage at Each level	34%	27%	38%	1%

Table 5.12 shows that 34% of the FET learners involved in this study were still operating at the visual level of the van Hiele levels of geometric thinking, a worrying observation as they are expected to work at higher levels of the van Hiele model after spending nine years of primary education, which one assumes, would have given them exposure to geometric figures and problems.

Of the learners involved in the study, 27% were operating at the analysis level. The learners at this analysis level were able to form a network of interconnected facts and properties; they understood the terminology and were able to formulate definitions used in geometry. These learners were able to produce knowledge of parallelograms, squares and triangles and their different properties in the form of sides, angles and diagonals.

The percentage of learners found to be working at the informal deduction level was 38%. This implies that out of the 147 learners participating in the study, around 55 learners were able to analyse properties of figures and understand the relationships between properties of shapes. Table 5.12 also confirms that sixty one percent of the learners were not in a position to follow all the logical arguments and did not show evidence of deductive thinking.

Only two learners showed evidence of competence at the formal deduction level and these corresponded to 1% of the total number of learners involved in the study. These two learners were able to come up with deductive chains of statements constituting proofs.

The van Hiele levels of grade 10 and grade 11 learners were also compared for Questionnaire B by looking at the number of learners operating at each van Hiele level for both grades and the results were recorded in the table below.

Table 5.13: Comparison of learners' van Hiele Levels between grade 10 and grade 11

Van Hiele Level	**Grade 10**	**Percentage**	**Grade 11**	**Percentage**
Visual	35	47%	15	21%
Analysis	29	39%	11	15%
Informal Deduction	8	11%	47	64%
Formal Deduction	2	3%	0	0%
Rigour	0	0	0	0%

The number of grade 10 learners who were operating at the visual level was higher (35 learners) than that of grade 11 (15 learners). For the grade 11 learners, 21% were operating at the visual level as compared to 47% of the grade 10 learners. At the informal deduction level, 64% of the grade 11 learners were able to show a level of competence at this level, whereas 11% of the grade ten learners were operating at the informal deduction level. No learner from grade 11 showed competence at the formal deduction level. However two grade 10 learners were able to solve problems at the formal deduction level.

The table below shows the group statistics for the data.

Table 5.14: Group Statistics for Questionnaire B

	Grade	Number	Mean	Std. Deviation
Question 2	10	74	0.58	1.182
	11	73	2.22	1.304
Question 3	10	74	0.62	1.372
	11	73	2.96	1.634
Question 4	10	74	0,72	0.472
	11	73	1,67	0,812
Question 5.2	10	74	0.55	0.577
	11	73	0.59	0.684
Question 6.1	10	74	0.9	0,443
	11	73	0,00	0
Question 6.2	10	74	0.49	0,602
	11	73	0.01	0.117
Question 6.3	10	74	0	0
	11	73	0	0

In terms of specific questions, even though the average marks of the grade 11 learners were higher than the average marks of the grade 10 learners in all questions except for questions 6.1 and 6.2, the difference was only statistically significant for three items. These statistics are presented below.

The differences in the mean for the data were statistically different for Question 2, Grade 11's (Mean (M) = 2.22, Standard Deviation (SD) = 1.30) and Grade 10's (M = 0.58, SD = 1.18; t(143) = -7.97; p = 0.00). The effect size was large (eta squared = 0.3)

The differences in the mean for the data were statistically different for Question 3 with respect to Grade 11's (M = 2.96; SD = 1.63) and Grade 10's (M = 0.62, SD = 1.37; t(143) = -9.34 ; p = 0.00). The effect size was large (eta squared = 0.31).

The differences in the mean for the data were statistically significantly different for Question 6.2, grade 11's (M = 0.01, SD = 0.117) and Grade 10's (M = 0.49; SD = 0.602; t(145) = 6.584; p = 0.00). The effect size was large (eta squared = 0.23). This result is interesting because the grade 10's did significantly better than the grade 11's even though both means were very low. This might have been caused by the different learning opportunities between the grade 10 and grade 11 learners. The learning opportunities for grade 11 learners might have been poorer in that section which could be due to a variety of reasons, such as the competence of the teacher, the absence of the teacher, school disruptions or even a lack of teaching resources.

5.4.8 Overall Van Hiele Levels of the FET Learners

The findings from Questionnaire A were combined with the findings from Questionnaire B, and these are presented below.

Table 5.15: Overall van Hiele Levels of FET learners

Van Hiele Levels	Percentage of Learners at Each Level	
	Questionnaire A	Questionnaire B
Visual Level	16%	34%
Analysis Level	52%	27%
Informal Deduction Level	31%	38%
Formal Deduction Level	1%	1%
Rigour Level	0%	0%

The results in the table show that the majority of the FET learners who participated in the study were operating at the first three levels of the van Hiele model of geometry thinking (the visual level, the analysis level and the informal deduction level). Sixteen percent of the learners were still at the visual level for Questionnaire A and 34% were still operating at the visual level for Questionnaire B. There were more learners (52%) operating at the analysis level for Questionnaire A than those operating at the same level for Questionnaire B (27%).

At the informal deduction level, 31% of the learners in the study showed a degree of competence required at this level for questionnaire A and 38% of the learners showed a degree of competence required at this level in Questionnaire B. At this level the learners were able to deduce the properties of a figure and recognize the classes of figures and thus classify figures according to their properties. At this level the learners should understand that a quadrilateral having equal sides is a necessary but not sufficient property for a rhombus to be a square. It is at this level that the learners were able to understand that it is sufficient for a quadrilateral to have two pairs of opposite sides equal in order to be a parallelogram, and that it is not necessary to show that all possible properties of a parallelogram hold in order for a quadrilateral to be considered as a parallelogram.

One percent of the grade 10 and 11 learners operated at the formal deduction level for both Questionnaire A and Questionnaire B. This implies that 99% of the learners in the study could not understand or use the ideas of formal geometry to understand interrelationships between axioms, definitions, theorems and proofs. The majority of the learners thus could not come up with deductive chains of statements constituting proofs.

Table 5.15 also shows that 16% of the FET learners in the study were operating at the visual level of the van Hiele model of geometry thinking for Questionnaire A and 34% for Questionnaire B. These learners had a very simple concept of space, seeing geometric shapes as a whole with no knowledge of parts or properties. It is worrying that FET learners are still operating at the visual level having passed through primary education, which should have given them practice in identifying shapes and their properties.

5.4.9 Conclusion

The results have shown that the majority of FET learners involved in the study are operating at the first three levels (visual level, analysis level and informal deduction level) of the van Hiele model of geometry understanding. It is quite worrying to observe a number of FET learners still at the visual level. The results showed that visual images and diagrams play a crucial role in the understanding and learning of geometric figures and their properties.

The next chapter will now discuss how the research questions were answered, the issues that were raised and the misconceptions and errors which were discovered. Suggestions and ways of addressing the issues will also be discussed.

CHAPTER 6: DISCUSSION, RECOMMENDATIONS AND CONCLUSION

6.1 Introduction

The previous chapter presented an analysis of the learners' scripts and the interviews. The focus was on the knowledge and understanding of geometric concepts the learners displayed and the van Hiele levels at which the learners were operating at. In this final chapter, a summary of the findings will be presented first, followed by the conclusions drawn and then valid recommendations will also be suggested.

The researcher was guided by the constructivist theory which is underpinned by the belief that learners gain knowledge by constructing knowledge themselves. Under constructivism the responsibility of learning lies with the learner.

The misconceptions, errors and mistakes the learners make, are an important part in the process of teaching and learning. According to Mamba (2011) misconceptions and errors are crucial in education because they form part of learners' conceptual structures that often negatively interact with new concepts and affect learning because misconceptions generate errors. Educators should regard errors as a clue to help discover what learners already know and how they have constructed such knowledge (Borasi, 1994; Mamba, 2011).

This research was also guided by the van Hiele theory of geometric thinking. By analysing the learners' scripts and interviewing them, it was possible to understand the kind of geometric knowledge the learners had and this then enabled me to place them at certain van Hiele levels of geometric understanding and geometric competency levels. The conceptual framework of the van Hiele model of learning geometry was based on five levels of geometric thinking. According to the van Hiele levels, a learner passes through five levels, where the learner cannot achieve one level of thinking without having passed through the previous levels.

The findings, conclusions and recommendations of this study were a result of the above mentioned theoretical assumptions and conceptual perspectives while answering the research questions: How do grade 10 and 11 learners perform on selected geometric tasks and what can be deduced about the van Hiele levels of geometric thinking of the learners?

6.2 Answer to Research Question 1: How do FET learners (grade 10 and 11 learners) perform on tasks based on basic geometric concepts?

Some aspects of Research Question 1 have been discussed already in Chapter 5, since an item- by-item analysis of the learners' responses was presented. However a brief summary of the overall performance of the learners is first provided. This is followed by a discussion of some issues raised and some common misconceptions and errors made by the learners. Thereafter I discuss the differences between grade 10 and 11 learners' performances.

6.2.1 Results of Questionnaire A

In Questionnaire A, learners did very well in items 1 and 2 where the percentage of correct responses was 100%. These two items required learners to identify a triangle and a circle from given sets of shapes. Items 1 and 2 required the most basic knowledge of geometry where objects and figures are recognised by their appearance, not by analysis of parts or properties. Responses from the interviews with some of the learners indicated that one does not need any formal education in order to be able to identify geometry shapes and figures. One such response is shown below from learner LSB1.

LSB1: Questions 1 and 2 were very easy, even before I started attending school I was taught to identify a circle even though no one gave me a proper definition up to now.

Learners' responses during the interviews also showed that learners found it easy to identify shapes and figures because they made use of them in their everyday lives. It must be noted that these were the only two items in which learners confessed to linking geometry to their real life experiences. In other questions the opposite happened, there was a general outcry at the lack of real life contexts to support some of the geometry contexts. This issue will be discussed further at a later stage in this chapter.

Learners also performed satisfactorily in items 4, 6 and 7 in Questionnaire A, as in all these items the percentage of correct responses was above 50%. Items 4, 6 and 7 required knowledge of properties of shapes and definitions of different geometric shapes such as rectangles, parallel lines and equilateral triangles. In item 4 learners were required to use their knowledge of the area of a rectangle and it was revealed from the interviews that most learners were exposed to challenges and questions of this nature at the primary school stage and hence found it easy to deal with. This is supported by Patkin & Lavenberg (2007), who state that using tried and tested activities designed to promote and develop geometric thinking, help learners to master geometric concepts.

In item 6 learners needed knowledge of parallel lines and in item 7 learners needed knowledge of equilateral triangles. Sixty nine percent of learners had item 6 correct and 55% of learners had item 7 correct. In these two items, exposure to tasks of this nature at primary school level, may have helped learners. However there were also learners who provided wrong responses and in the interviews, these learners blamed the lack of understanding of the terms used and the lack of a pictorial or diagrammatic view of the shapes mentioned. These factors will also be discussed in detail later in this section.

There were also a reasonable number of correct responses in item 5 in Questionnaire A, which required knowledge of similar and congruent figures (62% of correct responses). Comparing properties of different shapes needed thinking at a higher level. Congruent and similar shapes are covered in the senior phase (grade 8 and 9), so learners were expected to perform well. Those who failed to respond correctly, showed a lack of knowledge of the definitions of similar and congruent shapes.

Learners performed poorly in items 3, 8, 9, 10, 11, 12, 13 and 14, where the percentages of correct responses were 36%, 39%, 15%, 13%, 36%, 17%, 27% and 16% respectively. Of these items, items 3 and 9 required knowledge of properties of shapes while for the rest of the items, learners were required to analyse properties of figures and understand the relationship between properties of different shapes. Some of the issues which arose will also be looked at later in this chapter.

Learners performed most poorly in item 15 where the percentage of correct responses was 4%, corresponding to only 7 learners out of the147 learners involved in the study. Item 15 require knowledge of proofs and the use of formal geometry to understand the interrelationships between definitions and proofs. Learners were then required to come up with deductive chains of statements that constitute proofs.

6.2.2 Results of Questionnaire B

In questionnaire B learners did fairly well in question 1. Most of the learners were able to find equal angles but some struggled to give reasons to justify why the angles were equal. Some of the reasons given by the learners indicated that they lacked knowledge and understanding of definitions of some of the words and terms used in geometry, such as co-interior, corresponding and vertically opposite angles. One such learner was LSC3 whose response to one of the questions in the interview is given below:

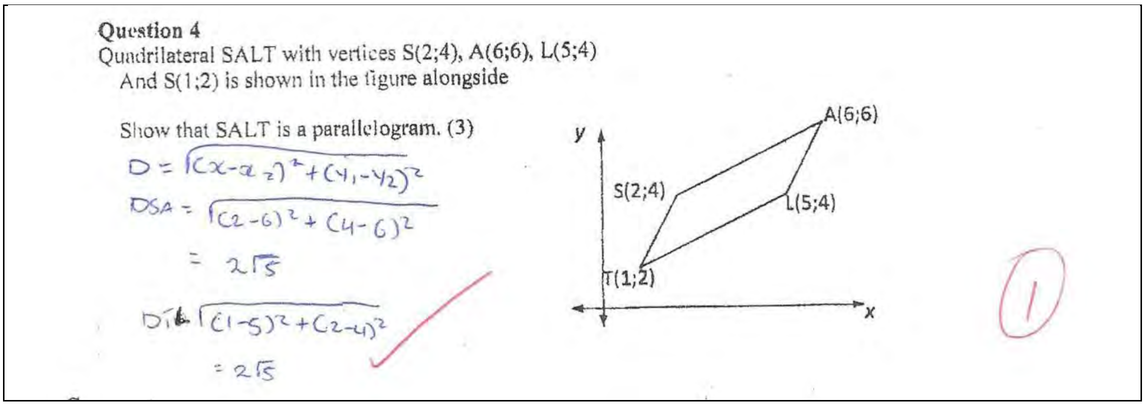
LSC3: To be honest I just thought since alternating angles between parallel lines are equal then the co-interior angles will also be equal. I am still able to recognise that F3 and x are co-interior but I can't remember what will be the relationship between them. When we were learning geometry, we never learn the meaning of the terms and words.

This kind of response shows a lack of deeper understanding of the terms used in geometry which led to the misconception that co-interior angles are equal. This confirms the finding of a study done by Atebe (2008), involving South African and Nigerian learners, with special focus on shapes in geometry. Atebe (2008) found that South African and Nigerian learners had problems in providing reasons when naming geometric shapes.

The analysis of the questionnaire B results also showed that less than 50% of the learners answered questions 3 and 4 correctly, which required learners to use knowledge of properties of shapes. Question 2 required learners to use the sum of angles of a triangle while question 3 required learners to use the transitivity property of angles, properties of isosceles triangles and vertically opposite angles. Learners who did not get marks in question 2 seemed to have misconceptions about angles on a straight line. Those who did not get marks in question 3 showed a lack of knowledge of the transitivity property.

Learners struggled to give adequate answers in questions 4 and 5 in Questionnaire B. These questions required learners to have knowledge of properties of quadrilaterals (parallelograms and squares), and to use analytical methods to show that the shape was a parallelogram in question 4 and that the shape was a square in question 5. Learners who scored no marks in both questions showed that they lacked conceptual understanding of the geometric concepts and those who scored one or two marks in both questions showed that they lacked knowledge of the sufficient properties for them to state that the shape was a parallelogram or a square. An example of one such learner was learner LSC12, whose response to question 4 is shown below.

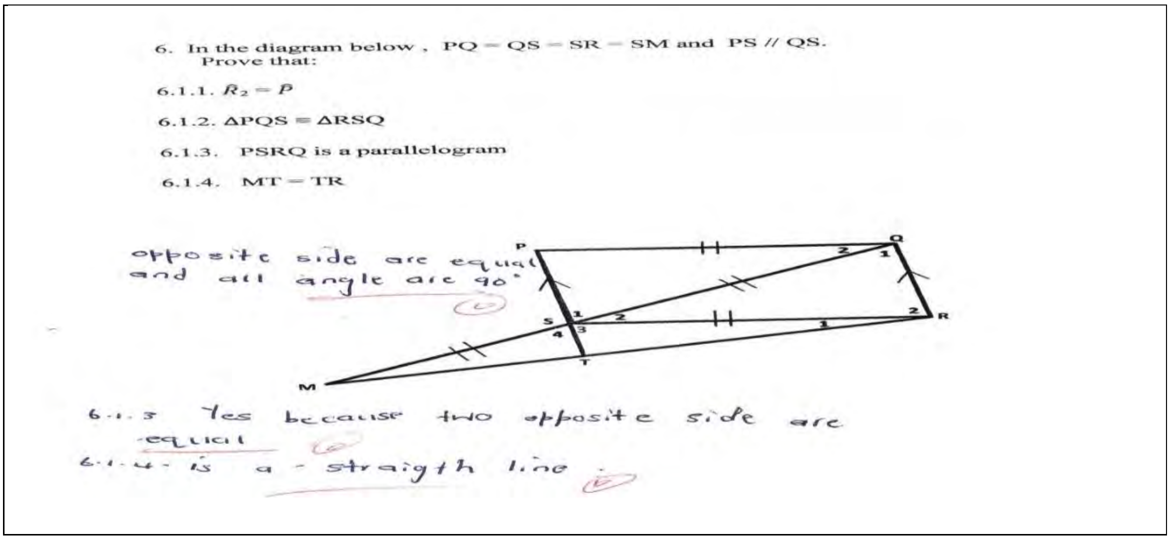
Figure 6.1: Learner LSC12's response to question 4



Having one set of equal opposite sides is a necessary but not sufficient condition for a quadrilateral to be a parallelogram. Studies conducted by de Villiers (1997), Roux (2003) and Siyepu (2005), found that learners in South African schools perform poorly when it comes to questions involving an understanding of features and properties of shapes.

Learners performed most poorly in question 6 in Questionnaire B, with 114 learners not even attempting to answer the question or attempting the question but their responses showed that they had no clue as to what was required by the question. All the sub questions in question 6 required learners to show knowledge of proofs and the use of deductive reasoning since no analytical methods were required. Only five learners scored 2 or more marks in question 6 with no learner getting the total of 8 marks. The figure below is the response from one of the learners who showed no evidence of the kind of knowledge required by the question.

Figure 6.2: Learner LSC9's response to question 6



Research has shown that learners have difficulties with the notion of proof (Senk, 1989; Usiskin, 1982). This could be explained in terms of the van Hiele theory, stating that in order for learners to understand proofs, they need to be on the van Hiele ordering level. So the analysis of the results in the previous chapter showed that most of the learners were operating at the analysis level. Schoenfield (2000) states that proof is not meaningful until the entities manipulated in the proof (like the plane figures and their properties) are meaningful.

In Questionnaire B as in Questionnaire A, some pertinent issues were raised, together with misconceptions and errors identified in the learners' written responses. These will now be discussed in greater detail in the next section.

6.2.3 Comparing the performance of grade 10 and grade 11 learners

Table 5.5 in Chapter 5, shows that the grade 11 learners performed better than grade 10 learners in items 3, 5, 6, 7,8, 9, 10, 11, 13 and 15 in Questionnaire A, while the grade 10 learners did better than the grade 11 learners in items 12 and 14.

In the analysis of results for Questionnaire B, Table 5.14 with the group statistics show that the mean marks for grade 11 were higher than the mean marks for grade 10 learners in questions 2, 3, 4, and 5. In question 6, the group mean for the grade 10 learners was higher than the mean mark for the grade 11 learners. This was mainly because the grade 11 learners did not even attempt to answer question 6 or those who attempted; their answers were totally wrong answers due to a lack of conceptual understanding. Although the average marks for the grade 11 learners were higher than the average marks for the grade 10 learners in all questions except questions 6.1 and 6.2, the differences were only statistically significant in question 2, 3 and 6.2.

6.3 Answer to Research Question 2: What can be deduced about the van Hiele levels of geometric thinking of the learners?

6.3.1 Van Hiele levels according to Questionnaire A and Questionnaire B

The van Hiele levels of geometric thinking are regarded as a good descriptor of current performance in geometry and a reasonably good descriptor of future performance (Usiskin, 1982). This section will look at the van Hiele levels of the learners as revealed by the analysis.

The data analysis of this study showed that the majority of the grade 10 and grade 11 learners who participated in this study were operating at the visual, analysis and informal deduction levels (refer to Table 5.4, Table 5.13 and Table 5.15).

Table 6.1: Learners' van Hiele Levels

Van Hiele Level	Percentage of Learners Operating at a Particular Level	
	Questionnaire A	Questionnaire B
Visual Level	16%	34%
Analysis Level	52%	27%
Informal Deduction Level	31%	38%
Formal Deduction Level	1%	1%
Rigour	0%	0%

The table shows that the majority of the learners were operating at the first three van Hiele levels, with only 1% of the learners operating at the formal deduction level.

The data analysis also revealed that for learners to reach a higher van Hiele level they must first pass through the lower (preceding) levels, hence the criteria used to place a learner at a higher van Hiele level require that the learner first meet the requirements for placement at the lower levels. From Table 6.1 above, it can be deduced that of the 84 % of learners who reached the analysis level in Questionnaire A, 32% have proceeded to the informal deduction level and 1% managed to reach the formal deduction level. Similarly for Questionnaire B, of the 66% of learners who reached the analysis level, 39% have proceeded to the informal deduction level and 1% has reached the formal deduction level.

It was disturbing to see that many grade 10 and 11 learners were still operating at the visual level after spending years in primary school. These learners have not moved beyond the recognition of shapes and mentioning of properties and showed no evidence of knowing how the properties are connected. These findings concur with the findings of the studies by Siyepu (2005), and Atebe & Schafer (2011), whose studies indicated that the majority of the learners were found to be operating at the pre-recognition level and that a very small number of the students operated at the van Hiele level 2. This is consistent with research done by Mateya (2008) who carried out a study on grade 12 students and found that out of 50 students who participated in the study, 19 (38%) were at the pre-cognition level, 11 (22%) were at van Hiele level 1, 13 (26%) at the van Hiele level 2 and 4 (8%) were at the van Hiele level 3. Similarly Senk (1989) and Usiskin (1982), indicated that many secondary school learners are on the visual or analysis levels of the van Hiele levels.

The findings that most of the learners were operating at the visual and analysis level mean that most of the learners involved in this study were operating at levels of geometry thinking which are lower than the mathematics curriculum requirements. Usiskin (1982) mentioned that the van Hiele level 3 is a "guidepost" for the learner to be able to master the art of proof in geometry. According to Usiskin (1982), many learners were found to be unfamiliar with basic terminology or geometric ideas even after many years of geometry lessons.

The study also revealed that the ability of the learners to work with problems at the van Hiele levels 3 and 4 is still a challenge as only a few were able to show competence at these levels. This was evident from the learners' responses to questions which involved proofs or needed learners to carry out more than two steps. The results showed that the majority of learners found it difficult to use deductive reasoning to prove that angles were equal, that triangles were congruent and that a shape was a parallelogram in question 6 for Questionnaire B. To be able to cope with the demands of the axiomatic system and proofs as required by the curriculum, learners need to be on the ordering level of the van Hiele levels. To be at the ordering level learners must have passed through the analysis and visual levels.

The analysis of the results also revealed that sometimes learners have some degree of competence but not much, for example, a learner was able to show that the shape in question 4 in Questionnaire B was a parallelogram but failed to prove that the shape in question 5 was a square. Yet both questions required thinking at the same van Hiele level. This shows that reasoning at a van Hiele level is not static but constantly developing; hence a learner can show some competence at a level but still struggle with other aspects at the same level. It must be noted that the van Hiele levels are not there to make judgements but are there to help diagnose barriers to progression that teachers need to address to their learners in order to improve. Some of the reasons why Van Hiele levels are important are discussed in the next section.

6.3.2 Differences in van Hiele levels according to Questionnaire A and Questionnaire B

Table 6.1 above, showed that there were differences in the van Hiele levels of the FET learners according to Questionnaire A and Questionnaire B, even though some of the differences were minor. Sixteen percent of the learners were at the visual level for Questionnaire A while 34% of the learners were at the visual level for questionnaire B. Questionnaire A consisted of multiple choice items, which means that when learners look at the given options, it is easier for them to compare and contrast. This also helps them to remember the correct option. However sometimes learners could have simply guessed the answer as there is a 20% chance of getting it right for those items with 5 options and a 25% chance of getting it right for those items with 4 options. This was different with Questionnaire B where learners were expected to work out their own answers and could leave questions unanswered if they didn't know the answers.

Fifty two percent of the learners were at the analysis level for Questionnaire A and 27 % were at the analysis level for Questionnaire B. As was explained in the visual level, those learners who managed to guess correct answers in Questionnaire A were placed at the analysis level in the first instrument, hence making the number of learners operating at the analysis level greater than that at the analysis level for Questionnaire B. It became more difficult to guess the correct answer at the informal deduction level as the choices given required learners to think and reason at the informal deduction level. Thirty one percent of the learners were at the informal deduction level for Questionnaire A as compared to 38% of the learners for Questionnaire B.

The differential results obtained from the two instruments are not surprising considering that the issue has been raised previously by other researchers. Smith and de Villiers (1989) found that the results of the level placements for the 1465 grade 9 and grade 10 learners were different according to the two test instruments, one of which was a modification of the Usiskin instrument (Usiskin, 1982), which was also true for this study. In this study, Questionnaire A was a modification of the same instrument. The authors attributed the differences in placement to the mathematical process involved, for example, some learners did better when asked to draw shapes than when they were asked to identify shapes where orientation and shape varied.

6.3.3 Comparison between grade 11 and grade 10 learners according to the van Hiele levels

Table 5.13 shows that 47% of the grade 10 learners were at the visual level as compared to 21% of the grade 11 learners, while 39% of grade 10 learners were at the analysis level as compared to 15% of the grade 11 learners. Eleven percent was at the informal deduction for grade 10 while 64% of the grades 11 were at the informal deduction level. Only 3% of the grade 10 learners were at the formal deduction level whereas no grade 11 learner was at the formal deduction level. Overall it shows that the grade 11 learners performed better than the grade 10 learners in terms of the van Hiele levels. This might have been due to the exposure and experience of geometric concepts that they had compared to the grade 10 learners who have spent a fewer number of years in school and hence have had less exposure to geometric concepts.

6.4 Emergence of Pertinent Issues

6.4.1 Conceptual Issues Identified

The following conceptual issues were identified from the learners' written responses and interviews: straight lines, procedural issues, class inclusion, proof and representations. These will now be discussed in detail in the next section.

i. Problems with Class Inclusion

The analysis of data for Questionnaire A reveals that learners struggled with items involving class inclusion problems. There are many problems involving class inclusion when one is dealing with geometric shapes. Class inclusion means sorting and classifying different shapes according to their properties and appearance and learners should be able to identify different shapes and their properties. Learners who failed to identify and classify shapes struggled with proofs and had many misconceptions and made a lot of mistakes.

In items 12, 13 and 14 in Questionnaire A, learners were supposed to use the knowledge of class inclusion in order to respond correctly. In item 12, knowledge of properties of rectangles, squares and parallelograms was required. Learners needed to understand that a square has all the properties of a rectangle and parallelogram, hence some parallelograms are rectangles, and some rectangles are squares. In other words one can say that all squares are rectangles and all rectangles are parallelograms, thus all squares are also parallelograms. However the opposite is not true, as not all parallelograms are rectangles and not all rectangles are squares.

The inability of most learners to deal with problems involving class inclusion was shown by the low percentage of correct responses in item 12 (17% of correct responses), item 13 (27% of correct responses) and item 14 (16% of correct responses). The learners failed to link all the properties of a parallelogram to the properties of a rectangle and to the properties of a square. They also failed to link the properties of a rectangle to the properties of a square. In one of the interviews, learner LSC1 had this to say:

LSC1: It never crossed my mind that a square can be called a rectangle and a rectangle can be called a parallelogram. Whenever I looked at a rectangle, I always expect to see 2 longer opposite sides which are equal and 2 shorter opposite sides which are also equal, of course with all angles being equal to 90º.

This response shows that the learner has confused facts about the properties of shapes and no knowledge of the link between the shapes, thus no knowledge of class inclusion. This finding is supported by the findings of the study done by Feza & Webb (2005), who found that many learners have difficulties in perceiving class inclusion of shapes, for example, they might say a square is not a rectangle. Studies by de Villiers (1997), Roux (2003) and Siyepu (2005), all found that learners' performance in South African schools is poor in items involving an understanding of features and properties of shapes- the very fundamentals of geometry understanding. Research by Marchis (2008), revealed that learners are not or are rarely taught about class inclusion. Understanding class inclusion is important because it enables learners to establish families of shapes, with common properties.

ii. Difficulties in changing from one semiotic representation to another

The results from Questionnaire A show that some learners struggled with items 3 to 15. These items were given in the natural language, in the form of explanations and definitions of certain geometric concepts. From the interviews conducted with the learners, it became clear that the grade 10 and 11 learners struggled to answer items 3 to 15 because the items focused on the properties of geometric figures without the geometric figures being shown. According to Duval R (2006), the characteristic feature of mathematical activity is the simultaneous mobilisation of at least two registers of representation, or the possibility of changing from one register to another at any moment. Geometric figures arise in a register of multifunctional representation; in this case the learners were given properties and definitions in the natural language (discursive representation) and were required to relate it to the non-discursive representation (geometric figures). If one wishes to analyse the difficulties in learning mathematics, it is of paramount importance to study the conversion of representation (Duval, 2006).

The fact that learners struggled to answer items 3 to 15 without the diagrammatic representation, emphasizes the role played by visualization in the teaching and learning of mathematics. The pictorial images or diagrams for the shapes were not provided. Most geometric concepts are learnt using diagrams and shapes. Arcavi (2003) defines visualisation as the ability of interpretation and reflection upon pictures, images and diagrams in minds, with the purpose of depicting information. It is an aid to an understanding or means towards an end and so one can therefore speak about visualisation as a concept or a problem but not necessarily as a diagram (Presmeg, 2006). Visualisation refers to mental images of a problem, and to visualise a problem means to understand a problem in terms of a diagram or visual image (Presmeg, 2006). According to Presmeg (2006), the visualisation process is one which involves visual imagery with or without a diagram, as an essential part of the solution. Teaching mathematics especially geometry, should include the use of diagrams or visual images to help develop an understanding of conceptual knowledge.

The issue of visualisation was emphasised during the interview with learner LSC11 when the responses to question 5 of questionnaire B were discussed. The learner was being probed about whether all quadrilaterals with equal sides were squares. I presented him with two figures with equal sides: one square with equal angles and the other one a rhombus with only opposite equal angles. The learner quickly explained why the one with unequal angles was not a square, something he did not see when the diagrams were not provided.

Some learners struggled in the items, ranging from items 3 to 15 of Questionnaire A because they failed to change from the natural language which was given, into geometric figures in what Duval (2006) referred to as changing from one semiotic representation to another. According to Ozerem (2012), in a learner's learning process there are some key factors such as network, images, word, anecdotes, case in point, formal principles and explanations help in understanding geometric facts. This implies that for learners to understand geometric concepts and facts, which will eventually improve their skills, educators need to use images, words, explanations, principles and anecdotes to make it easier for learners to understand rather than memorise geometric concepts and facts.

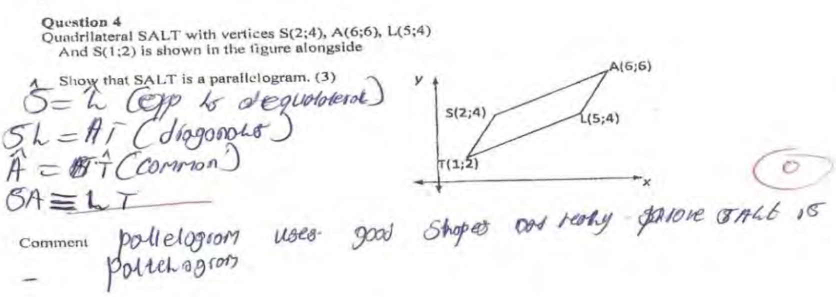
A study by Aydogan (2007) showed that geometrical concepts are acquired by means of using figures, and different names should be assigned to concepts and prototypes. Assigning different names to the concepts, prototypes and non-critical properties of the concepts has an important effect on geometric thinking. Forming different hierarchical relationships between different types of shapes is important for developing connections between shapes and properties.

Some learners struggled in items 3 to 15 because they had the concept image (a mental picture or image) but lacked the concept definition (a specific definition of a shape or its properties), hence they failed to link the two representations.

iii. Lack of procedural fluency

The results of this study showed that some learners lack knowledge of the correct procedures when working with geometry problems involving two or more steps, hence learners lack procedural understanding of concepts. According to Kilpatrick, Swafford & Findell (2001), procedural fluency is learners' skills in carrying out procedures in a manner that is accurate. It also involves the skill of knowing when to use procedures. Lack of procedural fluency was shown by learners' responses to questions in Questionnaire B especially in the responses to questions 4 and 5 where learners had to prove a parallelogram and a square using analytical methods. A learner would write down the correct formula for distance or gradient, but failed to simplify and show calculations in a coherent and logical manner until a conclusion was reached. Some learners knew that they needed to show that the opposite angles are equal and that opposite sides must also be equal for a quadrilateral to be a parallelogram, but they lacked the procedural knowledge of how to do it. An example of such is shown in learner LSA6's response to question 4 which is shown in the figure below.

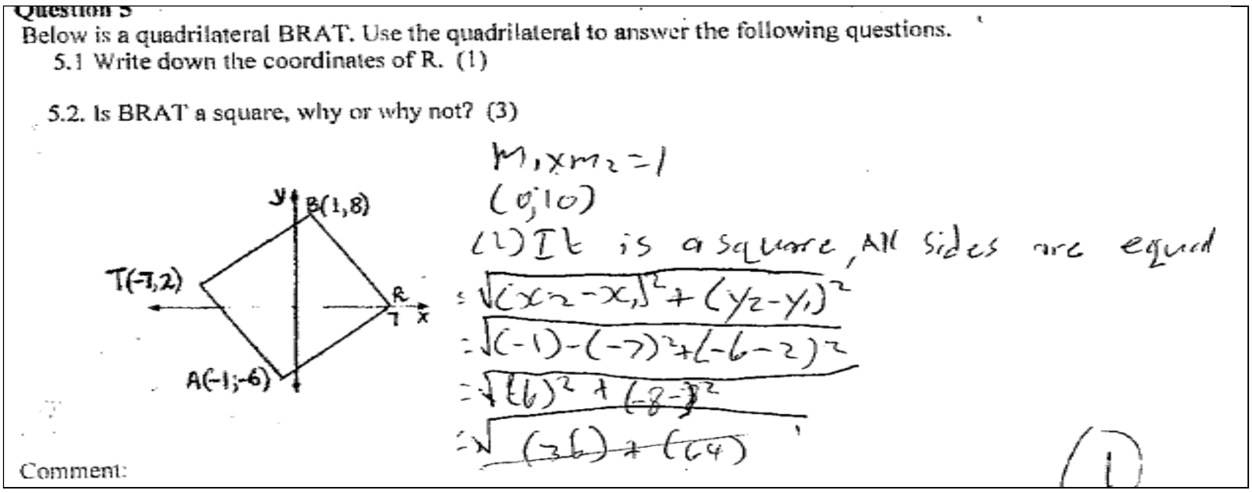
Figure 6.3 Learner LSA6's response to question 4



This learner knew that to prove a parallelogram he/she needed to show that the opposite sides and opposite angles are equal but did not know what steps to use to show it. In fact the learner concluded by just mentioning the equal sides and angles without giving valid reasons as to why they were equal.

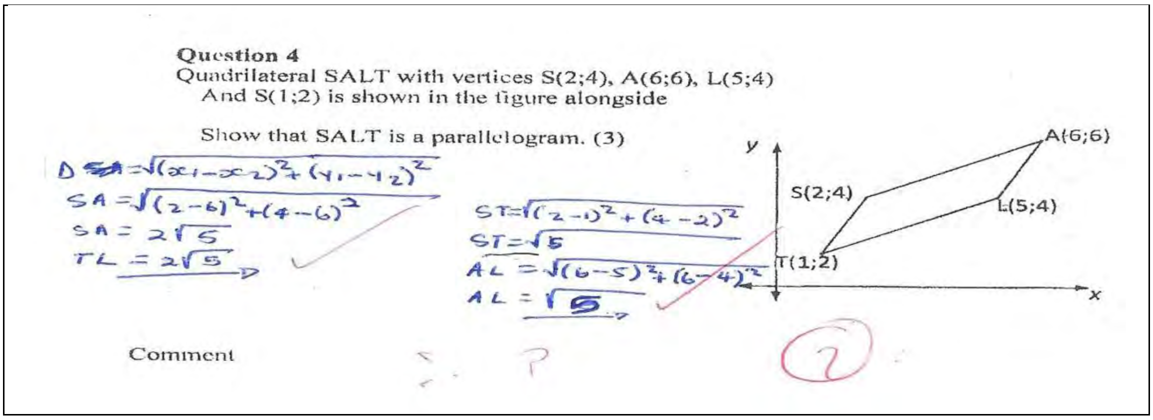
Another example of a response from a learner lacking procedural fluency is shown in the figure below. The learner wanted to prove that all sides are equal by calculating the lengths of the sides. He wrote down the distance formula correctly but substituted wrongly into the formula and could not simplify to get a simplified answer.

Figure 6.4: Learner LSC6's response to Question 5



There was also evidence of a lack of logic and connections in most of the learners' responses. Learners who were proving a parallelogram in item 4 of Questionnaire B by showing that opposite sides are parallel and equal, would simply calculate the length of the sides without giving the connecting statement about the relationships between the sides They would also not give a conclusion as to whether, from what the leaner had done, the shape was a parallelogram or not. An example of a response showing no concluding statement is shown in the figure below.

Figure 6.5: Learner LSC10's response to question 5



Learners need to have conceptual and procedural understanding of geometric concepts to engage fully and efficiently with geometric tasks. If a learner has only learnt procedures without understanding, it can be difficult to get him or her to engage in activities that help to understand the actual reasoning behind the procedure (Kilpatrick, Swafford & Findell, 2001). Also, when learners practise procedure without understanding, there is a danger that they will practise incorrect procedures which makes it more difficult for them to learn the correct procedures.

iv. Learners have problems with proof questions

The learners' responses to item 15 of Questionnaire A and question 6 of Questionnaire B suggest that learners struggle with geometric questions requiring formal deduction reasoning and proofs. Only seven learners answered item 15 in Questionnaire A correctly, while only five learners scored 2 or more marks out of the possible 8 marks in Questionnaire B from the 147 learners who took part in the study. Many learners did not even attempt to answer question 6 in Questionnaire B.

Many researchers have found that learners have difficulties with solving proof problems (Healy & Hoyles, 2000; Moore, 1994; Weber, 2004). If a learner lacks knowledge of definitions, he is likely to face challenges with proof questions (Moore, 1994). Lack of conceptual understanding also leads to learners failing to solve proof type questions together with other factors such as inadequate concept images. Failure to solve proof problems can also be attributed to being unable to understand and use mathematical language and notations and not knowing how to begin the proofs. Writing a proof in a domain requires one to understand the concepts in that particular domain.

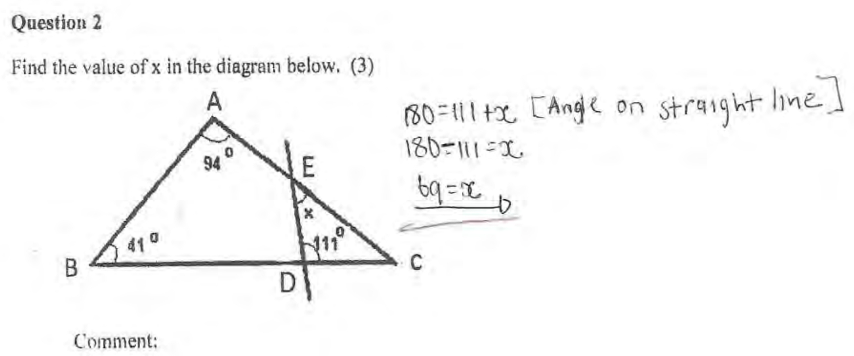
These results support the findings of Clements & Battista (1992), who found that in the United States, elementary and middle school learners fail to learn basic geometry concepts and geometry problem solving techniques, making them woefully under-prepared for the study of more sophisticated geometric concepts and proofs.

De Villiers (2004) stated that proofs help with explanations, with intellectual challenge and with systematization. Learners experience problems in proof questions because proofs are mainly given as finished products in textbooks and this does not challenge learners to think deductively (de Villiers, 2004).

v. Misconceptions

The study identified a few misconceptions that learners seem to have developed. One misconception involved the meaning of angles lying on a straight line. This was observed in responses to question 2 of Questionnaire B as shown in the figure below.

Figure 6.6 Learner LSC1's response to question 2 of Questionnaire B



The learner indicated that angle $C\hat{D}E$ and angle $D\hat{E}C$ add up to 180º because they lie on a straight line. This is a big misconception as it shows that the learner lacks conceptual understanding of geometric concepts. According to Leinhardt, Zaslavsky & Stein (1990), misconceptions are incorrect features of learners' knowledge that are repeatable and explicit. Dikgomo (1994) views misconceptions as conceptual difficulties that learners experience which may hinder the understanding of mathematical concepts.

In this instance the learner knows the concept that angles on a straight line add up to 180º, but is applying it wrongly. The angles on the straight line must be adjacent to each other and when they are put together they must form 180º, a concept that the learner failed to implement.

Another misconception that emerged was the properties of the angles on the same side of a transversal between parallel lines. The learner's (LSC1) misconception was that co-interior angles formed at the intersections were equal instead of being supplementary.

6.4.2 Language Barrier

The issue of language as a barrier to learning and understanding of geometric concepts was revealed when the researcher was conducting interviews and from the learners' responses to items 3, 5, 7, 8, 9 and 15 in Questionnaire A. Learners indicated a lack of knowledge for the definitions and meanings of terms and words like right angles, acute angles, obtuse angles, intersect, similar triangles, congruent triangles, parallel lines, equiangular, bisect and diagonals. Some of the responses in the interviews showing language challenges are given below:

LSB1: We have done parallelograms before but the way the question was asked was challenging to me. I even drew my own parallelogram but I failed to create the triangles which they are talking about in the item, then there was this term equiangular which I don't even know its meaning.

Learner LSB1 was responding to a question from the researcher regarding item 8. Learner LSA1 also had this to say when asked about his response to item 5:

LSA1: I never understand the definition for similar and congruent triangles, so it was a challenge for me to list all the properties of similar triangles and comparing them to those of congruent triangles.

Learner LSB2 had this to say about item 15:

LSB2: I guessed the answer for the question because I don't even know the meaning of the words diagonal or bisect so I was not able to compare the two statements.

These responses show that the learners experience problems with the language which hinder their progress in the learning of geometry. This is supported by Sanders (1994), who stated that learners have difficulties in understanding the language used in geometry. A study by Henderson & Wellington (1998) revealed that language is the greatest barrier to learning mathematics in many African countries, including South Africa. The South African Curriculum uses English as the medium of instruction and this has left many learners in schools struggling to cope with the language that is foreign to them. Setati & Adler (2001) argue that the teaching and learning of mathematics in a classroom where the language is not the learners' main language is complicated since mathematics has elements that are similar to learning a new language because of the presence of specific registers and sets of discourses.

6.4.3 Real life contexts issues

Some learners indicated during the interviews that they found it difficult to understand geometry because geometry is a chapter that they are forced to do in class which did not have any relevance to their everyday lives. This is a big issue because teachers often teach geometry in isolation from reality, without linking it to real life contexts or without using realistic mathematics education approaches.

According to Freudenthal (1988), a realistic mathematics education approach is a mathematical approach which emphasises that the teaching learning of mathematics must be connected to reality and made up of human activity. Real life contexts refer to the use of knowledge connecting content to desired real world outcomes to demonstrate its practical value. According to Dickinson & Hough (2012), Netherlands is now considered to be one of the highest achieving countries in the world in international mathematics tests because they use realistic mathematics education (RME), which enables more students to understand mathematics and engage with it.

6.4.4 The implications of the van Hiele theory in the teaching and learning of geometry

Understanding the van Hiele levels of geometric understanding enables teachers to identify the general direction of the students' learning and the level at which they are geometrically operating at. De Villiers (2004) argued that teachers' presentation of material ought to be within a certain level that is close to where the learners are at, so that the learner will understand what is being taught and progression to the next level will be facilitated

Patkin & Lavenberg (2007) made suggestions for using examples of tried and tested activities designed to promote and develop geometric thinking. The activities must be based on visual illustrations taken from the learners' environments and must incorporate both natural and man-made examples which attempt to bridge the concrete to the abstract. The pedagogical and didactic functions of these activities are to offer interesting and unusual mathematical experiences, encourage mathematical engagement through experience and inquisitiveness, develop the learner's ability to cope with the problems taken from their daily environments, reduce anxiety of the subject and create opportunities for geometric activities for pupils who often find geometry difficulty (Patkin & Lavenberg, 2007). Some questions should comply with the levels of the van Hiele theory, for example a picture from the real world can lead to questions at the first and second level of the van Hiele theory, such as questions relating to the identification of basic shapes and recognition of their features. Patkin & Lavenbeg (2007) also suggest that geometric studies provide a rich environment for the purpose of developing mathematical thinking, developing thinking skills, using intuition and developing spatial orientation and an acquaintance with the real environment. Van Hiele (1999) asserts that mathematical activities developed from the environment enhance an acquaintance with and the inculcation of mathematical processes and implementation. They also improve verbal communication in general and mathematical communication in particular. Teachers must provide context for meaningful exploration and learning, while at the same time enhancing mathematical communication and engaging pupils with the beauty of geometric design.

6.4.5 Recommendations of the study

The analysis of the data and the findings of this study lead to the following general recommendations to try and address some of the issues raised, and to help deal with misconceptions and errors. Addressing these issues can improve the teaching and learning of geometry and help improve performance in mathematics. The recommendations are presented in the discussions below.

- **Educators should use relevant vocabulary to try and address the issue of the language barrier.**

Learners showed a lack of understanding for geometric terms like diagonals, right angles, co-interior angles, corresponding angles, etc. An understanding of geometric terms will help learners to understand the requirements of the questions which will make it easier for learners to cope with the demands of the question. Teachers should not only identify assumptions, hypothesis and concepts for geometric statements, but should also explain and show the role of definition and the meaning of the terms used. Teachers should equip learners with the relevant vocabulary in accordance with the van Hiele teaching model.

- **Learners should be exposed to different shapes, especially those with common properties and address issues on class inclusion.**

This is recommended because learners showed a lack of knowledge on class inclusion, when they failed to relate squares to rectangles and to parallelograms. Exposure to many different shapes will help learners to compare their properties and come up with their own conclusions for example, a square is a rectangle because it has all the properties of a rectangle, therefore all squares are rectangles but not all rectangles are squares. Squares lie within the class of rectangles and rectangles lies within the class of parallelograms. Learners must make conclusions that squares and rectangles are parallelograms but there are other parallelograms which are not squares and which are not rectangles.

- **Educators must give opportunities for learners to work with a diversity of registers of semiotic representation in geometry in order to enhance conceptual understanding of geometric terms.**

Educators rarely take diversity of representations into account when teaching geometry, with most educators normally showing an over-reliance on geometric figures in explaining geometric concepts to learners. This makes it difficult for learners to apply geometric concepts when they are given geometric tasks in other forms of representations, like the natural language (definition), graphic representations or algebraic representations and this can lead to the formation of misconceptions. Educators need to expose their learners to a variety of representations so that they can learn to express themselves and to switch from one representation to another. This is supported by Ozerem (2012), who stated that for learners to understand geometric concepts and facts which will eventually improve their skills, educators need to use lots of images, words, explanations, principles and anecdotes which will make it easier to understand than memorizing geometric concepts.

- **Educators should use or cite real life examples when teaching geometry like using road signs.**

This recommendation was made because learners indicated that they find it hard to understand geometric concepts because geometry seems to be isolated from the real world and they don't see how it relates to their everyday lives. Citing real life examples in the teaching and learning of geometry will help in making it more relevant and understandable. Freudenthal (1988) explains the realistic mathematics education approach as a theory which emphasises the teaching and learning of mathematics connected to reality as well as made up of human activity like road signs in the teaching and learning of geometry. Siyepu & Mtonjeni (2013) carried out a study on the use of road signs in teaching geometry and found that geometric concepts such as triangles, circles, rectangles, lines, squares, etc, can be taught using South African road signs. Their study also found that using real life examples enhance learners' conceptual and critical thinking skills.

- **Educators should put effort into improving learners' conceptual understanding.**

Educators should use teaching methods which promote conceptual understanding rather than rote learning. The results of the study showed that some learners had misconceptions involving geometric concepts like that of angles on a straight line. Some applied geometric properties in incorrect situations and some even failed to solve items because of a lack of conceptual understanding. Geometry allows learners to analyse and interpret the world around them as well as equip them with tools they can apply to other areas of mathematics. Teaching for conceptual understanding means that the teachers will encourage learners to use concepts as much as possible and associate the new terms with diagrams, representations and symbols so that they can connect them easily with the newly learnt geometric terms. The teaching process must play a pivotal role in connecting newly learnt concepts with what is already known.

Educators must use manipulatives in teaching geometry to reduce the prevalence of misconceptions and errors and thus enhance conceptual understanding. The van Hiele theory strongly emphasises the use of manipulatives in geometry to facilitate the transition from one level to the next (van Hiele, 1999). Manipulatives are physical or concrete objects that are used as teaching tools to engage students in the hands-on learning of mathematics. Touching and manipulating concrete objects improves proficiency in knowing positions or locations in space and the structure and its properties.

- **Educators should design learning programmes and provide learning experiences which help learners improve their proof skills**.

This recommendation is made because learners showed little or no knowledge of solving proof-type questions. Geometry teaching naturally aims to improve learners' deductive thinking and reasoning through proofs. According to Moore (1994), there are seven major sources of learners' difficulties in doing proofs and these are: no knowledge of definitions; having little intuitive understanding of concepts; inadequate concept images for doing proofs; being unable or unwilling to generate and use their own example; not knowing how to use the definitions to obtain the overall structure of proofs; being unable to understand and use mathematical language and notation, and not knowing how to begin proofs. Educators should discourage learners from memorizing proofs without understanding the inter-connectedness between one statement and the next. Educators are supposed to expose learners to many proof-type questions. Exposing learners to experiences involving proof should help the learners to perfect the art of responding to proof-type questions.

6.4.6 Limitations

As with every research study, this study has its own limitations. The study was conducted in only three schools in the same cluster and district. If a similar study is carried out in many schools from different clusters and districts, the results may be different. The results of this study are only relevant to the FET learners (grade 10 and grade 11 learners) of the three schools involved in the study and different schools may produce different results. The level placement of the learners was different for the two instruments. Hence further studies need to be conducted to find out more about how and when such differences occur.

6.4.7 Concluding Remarks

The purpose of the study was to explore learners' knowledge and understanding of geometry and the deductions that can be made about their van Hiele levels of geometry thinking. The study also aimed to provide descriptions of some of the misconceptions and errors and any other issues arising from the study. This study can assist teachers and curriculum developers to help learners overcome some of their misconceptions and issues discussed in this study. Educators should understand the van Hiele theory and use it in their teaching of geometry

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APPENDICES

Appendix A

Raw Data Classification into van Hiele levels, Questionnaire A

Questionnaire A

Learner	GRADE	Questions and the van Hiele level on the learner																														Overal Level
		1	level	2	level	3	level	4	level	5	level	6	level	7	level	8	level	9	level	10	level	11	level	12	level	13	level	14	level	15	level	
		C		C		A		C		E		A		A		B		E		C		B		A		B		C		D		
1	11	1	1	1	1	1	2	1	2	0		1	2	1	2	1	3	0		1	3	1	3	0		1	3	0		0		L3
2	11	1	1	1	1	1	2	1	2	0		0		1	2	1	3	0		0		0		0		0		0		0		L2
3	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		0		0		0		1	3	0		0		L3
4	11	1	1	1	1	1	2	1	2	1	3	1	2	0		0		0		0		0		0		1	3	0		0		L2
5	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		0		1	3	0		1	3	0		0		L3
6	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	0		1	2	1	3	0		0		1	3	0		0		L3
7	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		0		1	3	0		1	3	0		0		L3
8	11	1	1	1	1	1	2	0		1	3	1	2	1	2	1	3	0		0		1	3	0		1	3	0		0		L3
9	11	1	1	1	1	0		0		0		0		1	2	1	3	0		0		0		0		0		0		0		L1
10	11	1	1	1	1	0		0		0		0		0		0		0		0		1	3	0		1	3	0		1	4	L1
11	11	1	1	1	1	0		0		0		0		0		0		0		0		0		0		0		0		0		L1
12	11	1	1	1	1	1	2	0		0		0		0		0		0		0		1	3	0		1	3	0		0		L1
13	11	1	1	1	1	0		1	2	1	3	1	2	1	2	0		0		0		0		0		1	3	0		0		L2
14	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		0		1	3	0		1	3	0		0		L3
15	11	1	1	1	1	0		0		0		1	2	1	2	0		0		0		1	3	0		1	3	0		0		L2
16	11	1	1	1	1	0		1	2	0		1	2	1	2	1	3	0		0		1	3	0		0		0		0		L2
17	11	1	1	1	1	0		0		0		0		0		0		0		1	3	0		0		0		1		0		L1
18	11	1	1	1	1	0		1	2	0		1	2	1	2	0		0		0		0		0		0		0		0		L2
19	11	1	1	1	1	0		1	2	1	3	1	2	1	2	1	3	0		0		0		0		0		1	3	0		L3
20	11	1	1	1	1	0		1	2	1	3	1	2	1	2	0		1	2	0		1	3	0		0		0		0		L2
21	11	1	1	1	1	0		1	2	1	3	1	2	1	2	0		1	2	0		0		0		0		0		0		L2
22	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		1	3	0		0		0		0		0		L3
23	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		1	3	0		0		0		0		0		L3
24	11	1	1	1	1	1	2	0		1	3	1	2	1	2	1	3	0		0		1	3	0		1	3	0		0		L3
25	11	1	1	1	1	1	2	1	2	0		1	2	0		0		0		0		0		0		0		0		0		L2
26	11	1	1	1	1	0		1	2	1	3	1	2	1	2	0		0		0		1	3	0		0		0		0		L2
27	11	1	1	1	1	1	2	0		1	3	1	2	1	2	1	3	0		0		1	3	0		1	3	0		0		L3
28	11	1	1	1	1	0		1	2	0		1	2	1	2	1	3	0		0		1	3	0		0		0		0		L2
29	11	1	1	1	1	0		1	2	0		0		0		0		0		0		1	3	0		1	3	0		1	4	L1
30	11	1	1	1	1	1	2	0		1	3	1	2	1	2	1	3	0		0		1	3	0		0		0		0		L3
31	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		0		1	3	0		1	3	0		0		L3
32	11	1	1	1	1	0		0		1	3	1	2	1	2	0		0		0		0		0		1	3	0		0		L1

Questionnaire A

Learner	GRADE	Questions and the van Hiele level on the learner																														Overal Level
		1	level	2	level	3	level	4	level	5	level	6	level	7	level	8	level	9	level	10	level	11	level	12	level	13	level	14	level	15	level	
33	11	1	1	1	1	1	2	0		0		0		0		0		0		0		0		0		0		0		0		L1
34	11	1	1	1	1	0		1	2	1	3	1	2	1	2	0		0		0		0		0		1	3	0		0		L2
35	11	1	1	1	1	1	2	1	2	1	3	0		0		0		0		0		0		0		0		0		0		L1
36	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	0		1	2	0		0		0		0		0		0		L2
37	11	1	1	1	1	1	2	0		1	3	1	2	1	2	1	3	0		1	3	0		0		0		0		0		L3
38	11	1	1	1	1	0		1	2	1	3	1	2	1	2	0		0		0		0		0		0		0		0		L2
39	11	1	1	1	1	0		0		0		0		1	2	1	3	0		0		0		0		0		0		0		L1
40	11	1	1	1	1	0		1	2	0		1	2	1	2	0		0		0		1	3	0		0		0		0		L2
41	11	1	1	1	1	0		1	2	1	3	0		1	2	1	3	0		1	3	0		0		0		0		0		L3
42	11	1	1	1	1	0		1	2	1	3	1	2	1	2	0		1	2	0		0		0		0		0		0		L2
43	11	1	1	1	1	0		1	2	1	3	0		1	2	0		0		0		0		0		0		0		1		L2
44	11	1	1	1	1	0		0		0		0		1	2	1	3	0		0		0		0		0		0		0		L1
45	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		0		1	3	0		1	3	0		0		L3
46	11	1	1	1	1	0		1	2	1	3	1	2	1	2	0		1	2	0		1	3	0		0		0		0		L2
47	11	1	1	1	1	0		1	2	1	3	1	2	1	2	1	3	0		1	3	0		0		0		0		0		L3
48	11	1	1	1	1	0		0		0		0		0		0		0		0		0		0		0		0		0		L1
49	11	1	1	1	1	1		1	2	1	3	0		1	2	1	3	0		0		0		1	3	0		0		0		L3
50	11	1	1	1	1	0		0		0		1	2	0		0		0		0		0		0		0		0		0		L1
51	11	1	1	1	1	0		0		0		0		0		1	3	0		1	3	1	3	1	3	0		0		0		L1
52	11	1	1	1	1	1	2	0		0		0		0		1	3	0		0		1	3	0		0		1	3	0		L1
53	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		0		1	3	0		1	3	0		0		L3
54	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	1	3	0		0		0		1	3	1	3	0		0		L3
55	11	1	1	1	1	1	2	1	2	1	3	1	2	0		0		0		0		0		0		0		0		0		L2
56	11	1	1	1	1	1	2	1	2	1	3	1	2	1	2	0		1	2	0		1	3	0		0		0		0		L2
57	11	1	1	1	1	0		1	2	1	3	1	2	1	2	0		1	2	0		0		0		1	3	1	3	0		L3
58	11	1	1	1	1	1	2	1	2	0		0		1	2	1	3	0		1	3	0		0		0		0		0		L2
59	11	1	L1	1	L1	1	L2	1	L2	1	L3	1	L2	1	L2	1	L3	0		0		1	L3	0		1	L3	0		0		L3
60	11	1	L1	1	L1	1	L2	1	L2	1	L3	1	L2	1	L2	0		1	L2	0		0		0		1	L3	0		0		L2
61	11	1	L1	1	L1	1	L2	1	L2	1	L3	0		1	L2	0		0		0		1	L3	0		0		0		0		L2
62	11	1	L1	1	L1	0		1	L2	0		1	L2	0		0		0		0		1	L3	0		1	L3	0		1	L4	L2
63	11	1	L1	1	L1	0		1	L2	0		1	L2	1	L2	0		0		0		0		0		0		0		0		L2
64	11	1	L1	1	L1	1	L2	1	L2	0		0		0		0		0		0		0		0		0		0		0		L2
65	11	1	L1	1	L1	1	L2	1	L2	1	L3	0		1	L2	1	L3	0		1	L3	0		0		0		0		0		L3
66	11	1	L1	1	L1	1	L2	1	L2	1	L3	0		1	L2	1	L3	0		0		1	L3	0		0		0		0		L3
67	11	1	L1	1	L1	0		1	L2	1	L3	1	L2	1	L2	0		1	L2	0		0		0		0		0		0		L2
68	11	1	L1	1	L1	1	L2	1	L2	0		1	L2	1	L2	0		0		0		0		0		0		0		0		L2
69	11	1	L1	1	L1	0		1	L2	0		0		1	L2	0		0		0		0		0		0		0		0		L2

Questionnaire A

Learner	GRADE	Questions and the van Hiele level on the learner																														Overal Level
		1	level	2	level	3	level	4	level	5	level	6	level	7	level	8	level	9	level	10	level	11	level	12	level	13	level	14	level	15	level	
70	11	1	L1	1	L1	0		0		0		1	L2	1	L2	1	L3	1	L2	0		0		1	L3	0		0		0		L2
71	11	1	L1	1	L1	1	L2	1	L2	1	L3	0		1	L2	1	L3	0		0		0		0		0		0		0		L2
72	11	1	L1	1	L1	1	L2	1	L2	1	L3	1	L2	1	L2	1	L3	0		0		0		0		1	L3	0		0		L3
73	11	1	L1	1	L1	1	L2	1	L2	1	L3	1	L2	1	L2	1	L3	0		0		0		1	L3	1	L3	0		0		L3
74	10	1	L1	1	L1	0		1	L2	0		1	L2	1	L2	1	L3	0		1	L3	0		1	L3	0		1	L3	1	L4	L4
75	10	1	L1	1	L1	0		1	L2	0		1	L2	0		0		0		0		1	L3	1	L3	0		1	L3	0		L3
76	10	1	L1	1	L1	1	L2	1	L2	1	L3	0	L2	1	L2	1	L3	0		0		1	L3	1	L3	1	L3	1	L3	0		L3
77	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
78	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	1	L2	1	L3	0		0		1	L3	1	L3	1	L3	1	L3	0		L3
79	10	1	L1	1	L1	1	L2	1	L2	0		0		1	L2	1	L3	0		1	L3	1	L3	0		0		0		0		L3
80	10	1	L1	1	L1	0		0		1	L3	1	L2	0		0		1	L2	0		0		0		0		0		0		L2
81	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		1	L2	0		0		0		0		0		0		L2
82	10	1	L1	1	L1	1	L2	1	L2	1	L3	0		1	L2	0		0		0		0		0		0		0		0		L2
83	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
84	10	1	L1	1	L1	0		0		0		0		0		0		0		0		1	L3	0		0		0		0		L1
85	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
86	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
87	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
88	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
89	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
90	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
91	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
92	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
93	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	1	L2	0		0		0		0		0		0		0		0		L2
94	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	1	L2	0		0		0		0		0		0		1	L3	0		L2
95	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	1	L2	0		0		0		0		0		0		1	L3	0		L2
96	10	1	L1	1	L1	0		1	L2	0		0		1	L2	0		1	L2	0		0		0		0		0		0		L2
97	10	1	L1	1	L1	0		1	L2	0		1	L2	1	L2	0		0		0		0		0		0		0		0		L2
98	10	1	L1	1	L1	1	L2	0		0		0		0		0		0		0		0		0		0		0		0		L1
99	10	1	L1	1	L1	0		1	L2	1	L3	0		1	L2	1	L3	0		0		0		1	L3	1	L3	0		0		L3
100	10	1	L1	1	L1	1	L2	0		1	L3	0		1	L2	1	L3	0		0		1	L3	1	L3	1	L3	1	L3	0		L3
101	10	1	L1	1	L1	0		1	L2	0		1	L2	0		0		0		0		1	L3	0		1	L3	0		1	L4	L2
102	10	1	L1	1	1	1	L2	1	L2	1	L3	0		0		1	L3	0		0		1	L3	0		0		1	L3	0		L3
103	10	1	L1	1	L	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
104	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2

Questionnaire A

Learner	GRADE	Questions and the van Hiele level on the learner																														Overal Level
		1	level	2	level	3	level	4	level	5	level	6	level	7	level	8	level	9	level	10	level	11	level	12	level	13	level	14	level	15	level	
105	10	1	L1	1	L1	0		1	L2	0		1	L2	1	L2	1	L3	0		1	L3	1	L3	0		1	L3	1	L3	1	L4	L4
106	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		1	L2	0		0		0		0		0		0		L2
107	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	1	L2	0		0		0		0		0		0		1	L3	0		L2
108	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
109	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		1	L2	0		0		0		0		0		0		L2
110	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	1	L2	1	L3	0		0		1	L3	1	L3	1	L3	1	L3	0		L3
111	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
112	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
113	10	1	L1	1	L1	1	L2	0		0		0		0		0		0		0		0		0		0		1	L3	0		L1
114	10	1	L1	1	L1	1	L2	0		0		1	L2	0		0		1	L2	0		0		0		0		0		0		L2
115	10	1	L1	1	L1	1	L2	0		0		0		0		0		0		0		0		0		0		0		0		L1
116	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	1	L2	0		1	L2	0		1	L3	0		0		0		1	L4	L2
117	10	1	L1	1	L1	1	L2	0		0		0		1	L2	0		0		0		0		0		0		1	L3	0		L2
118	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	1	L2	1	L3	0		0		1	L3	1	L3	1	L3	1	L3	0		L3
119	10	1	L1	1	L1	0		0		0		1	L2	0		1	L3	0		1	L3	1	L3	1	L3	0		0		0		L3
120	10	1	L1	1	L1	0		1	L2	0		0		0		0		0		0		1	L3	0		0		0		0		L1
121	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
122	10	1	L1	1	L1	1	L2	1	L2	1	L3	0		1	L2	1	L3	0		0		1	L3	1	L3	1	L3	1	L3	0		L3
123	10	1	L1	1	L1	0		1	L2	0		0		0		0		0		0		0		0		0		0		0		L1
124	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
125	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
126	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
127	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
128	10	1	L1	1	L1	0		0		0		1	L2	0		0		0		1	L3	1	L3	0		0		0		0		L1
129	10	1	L1	1	L1	0		1	L2	1	L3	0		0		0		1	L2	1	L3	0		1	L3	0		0		0		L3
130	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
131	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
132	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
133	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
134	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		L2
135	10	1	L1	1	L1	0		1	L2	1	L3	0		1	L2	1	L3	0		0		0		1	L3	0		0		0		L3
136	10	1	L1	1	L1	0		1	L2	0		1	L2	0		0		0		0		1	L3	1	L3	1	L3	1	L3	0		L3
137	10	1	L1	1	L1	0		1	L2	0		1	L2	1	L2	1	L3	1	L2	0		1	L3	0		0		0		0		L2
138	10	1	L1	1	L1	0		1	L2	1	L3	0		1	L2	1	L3	0		0		0		1	L3	0		0		0		L3
139	10	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		1	L2	0		0		0		0		0		0		L2

Questionnaire A

Learner	**GRADE**	**Questions and the van Hiele level on the learner**																														**Overal Level**
		1	**level**	**2**	**level**	**3**	**level**	**4**	**level**	**5**	**level**	**6**	**level**	**7**	**level**	**8**	**level**	**9**	**level**	**10**	**level**	**11**	**level**	**12**	**level**	**13**	**level**	**14**	**level**	**15**	**level**	
140	**10**	1	L1	1	L1	1	L2	1	L2	1	L3	0		1	L2	1	L3	0		0		1	L3	1	L3	1	L3	1	L3	0		**L3**
141	**10**	1	L1	1	L1	0		0		0		0		1	L2	0		0		0		1	L3	1	L3	0		0		0		**L1**
142	**10**	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		**L2**
143	**10**	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		**L2**
144	**10**	1	L1	1	L1	0		1	L2	1	L3	1	L2	0		0		0		0		0		0		0		0		0		**L2**
145	**10**	1	L1	1	L1	1	L2	1	L2	0		1	L2	1	L2	1	L3	1	L2	1	L3	0		1	L3	1	L3	1	L3	0		**L3**
146	**10**	1	L1	1	L1	0		0		0		1	L2	1	L2	1	L3	0		1	L3	1	L3	1	L3	0		0		0		**L3**
147	**10**	1	L1	1	L1	0		1	L2	0		1	L2	1	L2	0		0		1	L3	1	L3	0		0		0		0		**L2**

Raw Data Classification into van Hiele Levels from Questionnaire B

	Questionnaire B Data																			
		QUESTIONS																		
Learner	GRADE	1.1	level	1.2	level	2	level	3	level	4	level	5.2	level	6.1	level	6.2	level	6.3	level	Overall Van Hiele level
1	11	4	L2	4	L2	3	L2	4	L3	1		2	L3	0		0		0		level 3
2	11	4	L2	4	L2	3	L2	2	L3	1		1		0		0		0		level 3
3	11	4	L2	4	L2	3	L2	4	L3	1		1		0		0		0		level 3
4	11	4	L2	4	L2	3	L2	4	L3	0	L	1		0		0		0		level 3
5	11	4	L2	4	L2	3	L2	4	L3	1		1		0		0		0		level 3
6	11	2		4	L2	3	L2	4	L3	3	L3	1		0		0		0		level 3
7	11	4	L2	4	L2	3	L2	4	L3	1		0		0		0		0		level 3
8	11	0		2		0		4	L3	0		0		0		0		0		level 1
9	11	2		2		3	L2	4	L3	0		1		0		0		0		level 1
10	11	0		3	L2	3	L2	2	L3	0		1		0		0		0		level 3
11	11	4	L2	2		0		4	L3	0		0		0		0		0		level 2
12	11	0		2		0		2	L3	0		1		0		0		0		level 1
13	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
14	11	4	L2	2		3	L2	4	L3	0		1		0		0		0		level 3
15	11	0		3	L2	3	L2	3	L3	0		0		0		0		0		level 3
16	11	2		4	L2	3	L2	4	L3	0		0		0		0		0		level 3
17	11	2		2		3	L2	0		0		2	L3	0		0		0		level 2
18	11	2		2		0		0		0		0		0		0		0		level 1
19	11	2		4	L2	0		0		0		0		0		0		0		level 1
20	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
21	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
22	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
23	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
24	11	0		2		0		0		0		0		0		0		0		level 1
25	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
26	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
27	11	0		2		0		1		0		1		0		0		0		level 1
28	11	2		4	L2	3	L2	4	L3	0		0		0		0		0		level 3
29	11	4	L2	4	L2	3	L2	1		0		1		0		0		0		level 2
30	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
31	11	4	L2	2		3	L2	4	L3	0		1		0		0		0		level 3
32	11	3	L2	4	L2	3	L2	4	L3	0		2	L3	0		0		0		level 3
33	11	4	L2	2		2		0		0		0		0		0		0		level 1
34	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
35	11	4	L2	4	L2	0		0		0		0		0		0		0		level 2
36	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
37	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
38	11	4	L2	2		3	L2	2		0		0		0		0		0		level 2

	Questionnaire B Data																			
		QUESTIONS																		
Learner	GRADE	1.1	level	1.2	level	2	level	3	level	4	level	5.2	level	6.1	level	6.2	level	6.3	level	Overall Van Hiele level
39	11	2		0		0		0		0		0		0		0		0		level 1
40	11	4	L2	4	L2	1		4	L3	0		1		0		0		0		level 3
41	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
42	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
43	11	2		0		0		0		0		0		0		0		0		level 1
44	11	2		2		3	L2	4	L3	0		1		0		0		0		level 2
45	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 2
46	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
47	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
48	11	3	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
49	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
50	11	2		2		0		0		0		0		0		0		0		level 1
51	11	2		2		0		0		0		0		0		0		0		level 1
52	11	2		0		0		0		0		0		0		0		0		level 1
53	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
54	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
55	11	4	L2	2		3	L2	4	L3	0		1		0		0		0		level 3
56	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
57	11	3	L2	3	L2	3	L2	4	L3	0		0		0		0		0		level 3
58	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
59	11	4	L2	4	L2	3	L2	4	L3	0		2	L3	0		0		0		level 3
60	11	4	L2	4	L2	3	L2	2		0		3	L3	0		0		0		level 3
61	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		Level 3
62	11	2		4	L2	0		3	L3	0		0		0		0		0		level 2
63	11	2		0		0		0		0		0		0		0		0		level 1
64	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
65	11	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3
66	11	4	L2	4	L2	0		4	L3	0		1		0		1		0		level 3
67	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
68	11	4	L2	4	L2	3	L2	4	L3	0		1		0		0		0		level 3
69	11	1		3	L2	0		0		0		0		0		0		0		level 1
70	11	3	L2	4	L2	0		0		0		0		0		0		0		level 2
71	11	4	L2	4	L2	3	L2	2		0		1		0		0		0		level 2
72	11	4	L2	4	L2	3	L2	4	L3	0		2	L3	0		0		0		level 3
73	11	4	L2	4	L2	3	L2	0		0		1		0		0		0		level 2
74	10	4	L2	4	L2	0		0		0		0		0		0		0		level 2
75	10	2		4	L2	3	L2	4	L3	0		1		0		0		0		level 3
76	10	4	L2	4	L2	0		1		0		1		0		2	L4	0		level 2
77	10	2		4	L2	0		0		0		1		0		1		0		level 2

	Questionnaire B Data																			
		QUESTIONS																		
Learner	GRADE	1.1	level	1.2	level	2	level	3	level	4	level	5.2	level	6.1	level	6.2	level	6.3	level	Overall Van Hiele level
78	10	2		4	L2	0		1		0		0		0		1		0		level 2
79	10	2		4	L2	1		4	L3	0		0		2	L4	2	L4	0		level 4
80	10	2		4	L2	0		0		0		1		0		1		0		level 2
81	10	2		0		0		0		0		1		0		1		0		level 1
82	10	4	L2	4	L2	0		0		0		0		0		0		0		level 2
83	10	2		4	L2	0		0		0		1		0		2	L4	0		level 2
84	10	0		2		0		0		0		0		0		0		0		level 1
85	10	2		2	L2	0		0		0		0		0		0		0		level 1
86	10	2		4	L2	0		0		0		1		0		0		0		level 1
87	10	2		3	L2	0		0		0		0		0		0		0		level 1
88	10	1		1		0		0		0		0		0		0		0		level 1
89	10	2		3	L2	0		0		0		0		0		0		0		level 1
90	10	1		2		0		0		0		0		0		0		0		level 1
91	10	2		4	L2	0		0		0		0		0		0		0		level 1
92	10	2		3	L2	0		0		0		1		0		0		0		level 1
93	10	2		4	L2	0		0		0		1		0		1		0		level 2
94	10	1		2		0		0		0		1		0		0		0		level 1
95	10	1		2		0		0		0		1		0		1		0		level 1
96	10	1		1		0		0		0		0		0		1		0		level 1
97	10	0		2		0		0		0		0		0		0		0		level 1
98	10	0		2		0		0		0		0		0		0		0		level 1
99	10	3	L2	4	L2	3	L2	1		0		0		0		0		0		level 2
100	10	3	L2	3	L2	0		4	L3	0		0		0		0		0		level 3
101	10	2		4	L2	0		1		0		1		0		0		0		level 2
102	10	4	L2	2		3	L2	4	L3	0		1		0		0		0		level 2
103	10	2		3	L2	0		0		0		1		0		1		0		level 2
104	10	2		4	L2	0		0		0		1		0		1		0		level 2
105	10	2		0		0		0		0		0		0		0		0		level 1
106	10	2		4	L2	0		1		0		1		1		1		0		level 2
107	10	2		4	L2	0		0		0		1		0		1		0		level 2
108	10	2		4	L2	0		0		0		1		0		0		0		level 1
109	10	2		0		0		0		0		0		0		0		0		level 1
110	10	2		4	L2	0		1		0		1		0		1		0		level 2
111	10	0		3	L2	0		0		0		1		0		0		0		level 1
112	10	2		1		0		0		0		0		0		0		0		level 1
113	10	0		2		0		0		0		0		0		0		0		level 1
114	10	0		2		0		0		0		0		0		0		0		level 1
115	10	0		2		0		0		0		0		0		0		0		level 1
116	10	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 3

	Questionnaire B Data																			
		QUESTIONS																		
Learner	GRADE	1.1	level	1.2	level	2	level	3	level	4	level	5.2	level	6.1	level	6.2	level	6.3	level	Overall Van Hiele level
117	10	0		2		0		0		0		0		0		0		0		level 1
118	10	2		4	L2	0		1		0		1		0		1		0		level 2
119	10	0		2		0		0		0		0		0		0		0		level 1
120	10	4	L2	2		3	L2	0		0		2	L3	0		1		0		level 3
121	10	2		4	L2	3	L2	0		0		1		0		1		0		level 2
122	10	3	L2	3	L2	0		4	L3	0		0		0		1		0		level 3
123	10	2		0		0		0		0		0		0		0		0		level 1
124	10	2		4	L2	0		0		0		1		0		1		0		level 2
125	10	2		4	L2	0		0		0		1		0		1		0		level 2
126	10	2		4	L2	0		0		0		1		0		0		0		level 1
127	10	2		4	L2	0		0		0		1		1		0		0		level 1
128	10	2		0		3	L2	0		0		1		0		1		0		level 2
129	10	2		3	L2	0		0		0		0		0		0		0		level 1
130	10	2		4	L2	0		0		0		1		0		1		0		level 2
131	10	2		2		0		0		0		1		0		1		0		level 1
132	10	2		2		0		0		0		1		0		1		0		level 1
133	10	2		4	L2	0		0		0		1		0		0		0		level 1
134	10	2		4	L2	0		0		0		0		0		1		0		level 1
135	10	4	L2	2		3	L2	0		0		1		0		1		0		level 2
136	10	3	L2	3	L2	0		4	L3	0		0		0		0		0		level 3
137	10	4	L2	4	L2	3	L2	4	L3	0		0		3	L4	2	L4	0		level 4
138	10	4	L2	4	L2	3	L2	0		0		2	L3	0		0		0		level 3
139	10	2		4	L2	0		0		0		1		0		1		0		level 2
140	10	0		3	L2	3	L2	0		0		0		0		0		0		level 2
141	10	2		4	L2	0		0		0		1		0		1		0		level 2
142	10	2		4	L2	3	L2	0		0		2	L3	0		1		0		level 3
143	10	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 2
144	10	2		4	L2	0		0		0		1		0		1		0		level 2
145	10	4	L2	4	L2	3	L2	4	L3	0		0		0		0		0		level 2
146	10	2		2		0		0		0		0		0		0		0		level 1
147	10	1		2		0		0		0		0		0		0		0		level 1

Appendix B:

Ethical Clearance

UNIVERSITY OF
KWAZULU-NATAL
INYUVESI
YAKWAZULU-NATALI

21 October 2014

Mr Harrison Ngirishi (209528282)
School of Education
Edgewood Campus

Protocol reference number: HSS/0987/014M
Project title: Van Hiele's levels of Geometric Thought: Exploring FET mathematics learners' level progression

Dear Mr Ngirishi,

Full Approval – Expedited Application

In response to your application received on 17 August 2014, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol have been granted **FULL APPROVAL**.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number.

PLEASE NOTE: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

....

Dr Shenuka Singh (Chair)

/ms

Cc Supervisor: Dr S Bansilal
Cc Academic Leader Research: Professor P Morojele
Cc School Administrator: Mr Thoba Mthembu

Humanities & Social Sciences Research Ethics Committee
Dr Shenuka Singh (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Telephone: +27 (0) 31 260 3587/8350/4557 Facsimile: +27 (0) 31 260 4609 Email: [illegible] / [illegible] / [illegible]
Website: www.ukzn.ac.za

1910 - 2010
100 YEARS OF ACADEMIC EXCELLENCE

Founding Campuses: Edgewood Howard College Medical School Pietermaritzburg Westville

Informed consent letter for the Principals whose learners are participating in the research project

School of Education,

College of Humanities,

University of KwaZulu-Natal,

Edgewood Campus,

INFORMED CONSENT LETTER

The Principal

Sir *I* Madam

Academic research: Request for permission to conduct a research study in your school.

My proposed research title is: **<u>An Exploration of FET Mathematics Learners' Understanding of Geometry.</u>**

My name is Mr. H Ngirishi from the School of Education, University of KwaZulu-Natal, Edgewood campus. I am an educator who is currently studying towards a Masters Degree in Education in the field of Mathematics Education at the University of KwaZulu Natal. Your school has been identified through voluntary inclusion process as a possible site of research for this project to produce some data that will help us understand situated realities of schooling and its impact on learner performance.

The learners of your school are being invited to participate in this research project that is aimed at exploring learners' understanding of geometry. As it is mandatory according to UKZN, in order for me to complete this degree and my theses I am entailed to do research work that needs to be conducted in a school. The purpose of this study is to explore learners' understanding of geometry, to identify the van Hiele level at which most learners are operating at and how the FET learners are progressing according to the levels.
I humbly seek permission to conduct the above-mentioned research study in your school.

I would like to conduct this research from October 2014 to March 2015. I intend to conduct interviews after school hours with a sample of 30 learners per grade from grade 10 to 12

The school and learners who partake in this study will do so, on a voluntarily basis and confidentiality and anonymity will be ensured. The participants have no obligation to participate in this research and may withdraw from it at any point. I also hereby undertake that the name of your school or the learners will not be mentioned in the subsequent theses. I will ensure that normal learning and teaching is not disrupted in any way whatsoever whilst conducting this research study. I will share my findings and feedback on this research with you and your staff members and the learners. The information acquired from this research study, will be accessible to the Department of Education, as well as school managers. I hope that the information gained from this research will be of great help to you and your learners and that together we might find solutions for our current problems we face in the teaching and learning of geometry.

For further information regarding this study, feel free to contact my supervisor, Dr Sarah Bansilals who lecturers at the University of KwaZulu Natal, Edgewood Campus, who can be contacted on 031- 2603451.

The reply could be sent to me by e-mail at: ngirishih@gmail.com

Thank you for your assistance in this matter

Yours faithfully

H. Ngirishi (Mr.) Student number: 209528282

Contact No's : 079 847 5761

Work : 031 900 5030

DECLARATION

I……………………………………………………………………… (full names of School Management Team participant) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my school's participating in the research project.

I understand that the learners are at liberty to withdraw from the project at any time, should they desire to do so.

I hereby consent to the audio recording of the interviews: YES/NO

SIGNATURE OF PRINCIPAL DATE

Informed consent letter for learners participating in the research project

School of Education,

College of Humanities,

University of KwaZulu-Natal,

Edgewood Campus,

INFORMED CONSENT LETTER

My proposed research title is: **An Exploration of FET Mathematics Learners' understanding of Geometry.**

My name is Mr. H Ngirishi from the School of Education, University of KwaZulu-Natal, Edgewood campus. I am an educator who is currently studying towards a Masters Degree in Education in the field of Mathematics Education at the University of KwaZulu Natal. Your school has been identified through voluntary inclusion process as a possible site of research for this project to produce some data that will help us understand situated realities of schooling and its impact on learner performance.

You as learners are being invited to participate in this research project that is aimed at exploring learners' understanding of geometry. It is mandatory according to UKZN, that in order for me to complete this degree and my theses, research work needs to be conducted in a school. The purpose of this study is to explore learners' understanding of geometry, and to also identify the van Hiele level at which FET learners are operating at and the causes of the progression from one van Hiele level to another.
I humbly seek permission to conduct the above-mentioned research study with you as learners.

I would like to conduct this research from October 2014 to March 2015. I intend to conduct interviews after school hours with a sample of 30 learners per grade from grade 10 to 12

Your participation is completely on a voluntarily basis and confidentiality and anonymity will be ensured. The participants have no obligation to participate in this research and may withdraw from it at any point. I also hereby undertake that your names and that of your school will not be mentioned in the subsequent theses. I will ensure that normal learning and teaching is not disrupted in any way whatsoever whilst conducting this research study.

I will share my findings and feedback on this research with you and staff members and the principal. The information acquired from this research study, will be accessible to the Department of Education, as well as school managers. I hope that the information gained from this research will be of great help to you as learners and that together we might find solutions for the current problems we face in the teaching and learning of geometry and mathematics at large.

For further information regarding this study, feel free to contact my supervisor. My supervisor is Dr Sarah Bansilals who lecturers at the University of KwaZulu Natal, Edgewood Campus, who can be contacted on **031- 2603451.**

The reply could be sent to me by e-mail at**: ngirishih@gmail.com**

Thank you for your assistance in this matter

Yours faithfully

H. Ngirishi (Mr.) Student number: 209528282

Contact No's : 079 847 5761

Work: 031 900 5030

You may also contact the Research Office through:

Mr. P. Mohun

HSSREC Research Office,

Tel: 031 260 4557 E-mail: mohunp@ukzn.ac.za

DECLARATION

I………………………………………………………………………… (full names of leaner) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my participating in the research project.

I understand that I am at liberty to withdraw from the project at any time, should I so desire.

I hereby consent to the audio recording of my interviews: YES/NO

SIGNATURE OF PARTICIPANT DATE

Informed consent letter for parents whose learners are participating in the research project

School of Education,

College of Humanities,

University of KwaZulu-Natal,

Edgewood Campus,

INFORMED CONSENT LETTER

My proposed research title is:

An Exploration of FET Mathematics Learners' understanding of Geometry.

My name is Mr. H Ngirishi from the School of Education, University of KwaZulu-Natal, Edgewood campus. I am an educator who is currently studying towards a Masters Degree in Education in the field of Mathematics Education at the University of KwaZulu Natal. Your child's school has been identified through voluntary inclusion process as a possible site of research for this project to produce some data that will help us understand situated realities of schooling and its impact on learner performance.

Your child is being invited to participate in this research project that is aimed at exploring learners' understanding of geometry, the van Hiele level at which FET learners are operating at and their progression on the van Hiele's levels of geometric thought.
As it is mandatory according to UKZN, that in order for me to complete this degree and my theses, I am entailed to do research work that needs to be conducted in a school. The purpose of this study is to explore learners' understanding of geometry, and to also identify the van Hiele level at which FET learners are operating at and the causes of the progression from one Van Hiele level to another.

I humbly seek permission to conduct the above mentioned research study with your children.

I would like to conduct this research from October 2014 to March 2015. I intend to conduct interviews after school hours with a sample of 30 learners per grade from grade 10 to 12

Your child's participation is completely on a voluntarily basis and confidentiality and anonymity will be ensured. The participants have no obligation to participate in this research and may withdraw from it at any point. I also hereby undertake that your children's names and that of your school will not be mentioned in the subsequent theses. I will ensure that normal learning and teaching is not disrupted in any way whatsoever whilst conducting this research study.

I will share my findings and feedback on this research with you and staff members and the principal. The information acquired from this research study, will be accessible to the Department of Education, as well as school managers. I hope that the information gained from this research will be of great help to your children and that together we might find solutions for the current problems we face in the teaching and learning of geometry.

For further information regarding this study, feel free to contact my supervisor. My supervisor is Dr Sarah Bansilals who lecturers at the University of KwaZulu Natal, Edgewood Campus, who can be contacted on 031- 2603451.

The reply could be sent to me by e-mail at: ngirishih@gmail.com

Thank you for your assistance in this matter

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HSSREC Research Office,

Tel: 031 260 4557 E-mail: mohunp@ukzn.ac.za

DECLARATION

I………………………………………………………………………… (full names of parent) hereby confirm that I understand the contents of this document and the nature of the research project, and I consent to my child's participating in the research project.

I understand that my child is at liberty to withdraw from the project at any time, should she/he so desire.

I hereby consent to the audio recording of my child's interviews: YES/NO

SIGNATURE OF PARENT DATE

……………………………………….......................................

…………………………

APPENDIX C

Task Based Worksheet

Van Hiele's levels of Geometric Thought: Exploring FET mathematics learners' level progression.

Dear Student

Thank you for agreeing to take part in this study. Please remember that your responses in this worksheet will be treated with the strictest confidence. Please answer the following questions in the spaces provided. The diagrams are not drawn to scale. Please fill in your details for research purposes.

Your Name:……………………………………………………………………….
Grade ……………………………………………

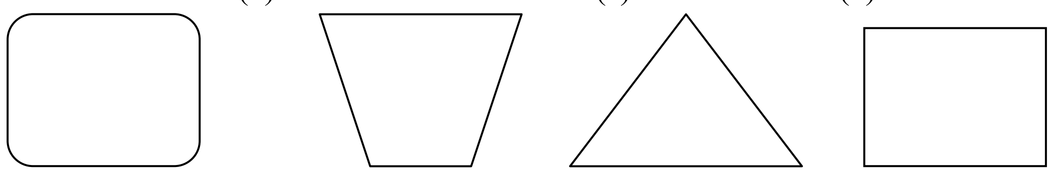
<u>The research instrument consist of Questionnaire A and Questionnaire B. Questionnaire A consist of multiple choice questions where as Appendix B is structured.</u>

Answer all questions.

Questionnaire A: 45 minutes

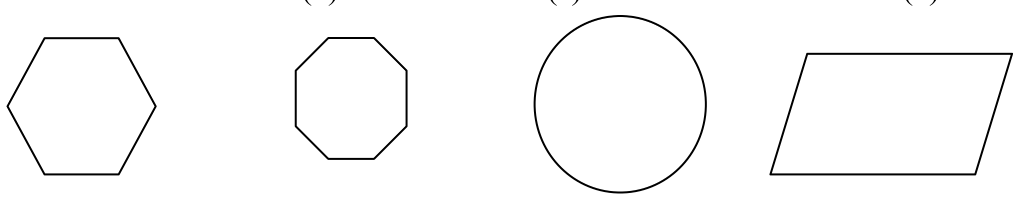
1. Identify a triangle from the following shapes.

(a) (b) (c) (d)



2. Of the following shapes, which one is a circle?

(a) (b) (c) (d)



3. Perpendicular lines
   (a) Intersect to form four right angles
   (b) Intersect to form two acute and two obtuse angles
   (c) Do not intersect at all
   (d) Intersect to form four acute angles
   (e) None of the above
4. The area of a rectangle with length 3cm and width 12cm is
   (a) $18cm^2$
   (b) $72cm^2$
   (c) $36cm^2$
   (d) $15cm^2$

(e) $30cm^2$

5. If two figures are similar but not congruent they
   (a) Have equal bases and equal heights
   (b) Have the same height
   (c) Both have horizontal bases
   (d) Have a different shape but the same size
   (e) Have a different size but the same shape
6. Parallel lines are lines
   (a) In the same plane which never meet
   (b) Which never lie in the same plane and never meet
   (c) Which always form angles of 90 degrees when they meet
   (d) Which have the same length
   (e) None of the above
7. An equilateral triangle has
   (a) All three sides the same length
   (b) One obtuse angle
   (c) Two angles having the same measure and the third a different measure
   (d) All the three sides of different lengths
   (e) All three angles of different measure.
8. Given that ABCD is a parallelogram, which of the following statements is true?
   (a) ABCD is equiangular
   (b) Triangle ABD is congruent to triangle CDB
   (c) The perimeter of ABCD is four times the length of AB
   (d) AC is the same length as BD
   (e) All of the above are true.
9. The plane figure produced by drawing all points exactly 6 cm from a given point is a
   (a) Circle with a diameter of 6cm
   (b) Square with a side of 6cm
   (c) Sphere with a diameter of 6cm
   (d) Cylinder 6cm high and 6cm wide
   (e) Circle with a radius of 6cm.
10. Here are two statements: Statement 1. Figure F is a rectangle

    Statement 2. Figure F is a triangle

    Which is correct?
    (a) If 1 is true, then 2 is true
    (b) If 1 is false, then 2 is true
    (c) 1 and 2 cannot both be true
    (d) 1 and 2 cannot both be false
    (e) None of (a)-(d) is correct
11. Here are 2 statements. Statement S: Triangle ABC has three sides of the same length

    Statement T: In triangle ABC, angle B and angle C are equal

Which statement is true?

(a) Statement S and T cannot both be true
(b) If S is true, then T is true
(c) If T is false, then S is true
(d) If S is false, then T is always false
(e) None of the above is correct

12. Which is true?
(a) All the properties of rectangles are properties of all squares
(b) All properties of squares are properties of all rectangles
(c) All properties of rectangles are properties of all parallelograms
(d) All properties of squares are properties of all parallelograms
(e) None of (a)-(d)

13. What do all rectangles have that some parallelograms do not have?
(a) Opposite sides equal
(b) Diagonals equal
(c) Opposite sides parallel
(d) Opposite angles equal
(e) None of (a)-(d)

14. Here are three properties of a figure:

Property D: It has diagonals of equal length

Property S: It is a square

Property R: It is a rectangle

Which is true?

(a) D implies S which implies R
(b) D implies R which implies S
(c) S implies R which implies D
(d) R implies D which implies S
(e) R implies S which implies D

15. Here are two statements

Statement A. If a figure is a rectangle, then its diagonals bisect each other.

Statement B. If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

(a) To prove that Statement A is true, it is enough to prove that Statement B is true
(b) To prove that Statement B is true, it is enough to prove that Statement A is true
(c) To prove that Statement B is true, it is enough to find one rectangle whose diagonals bisect each other

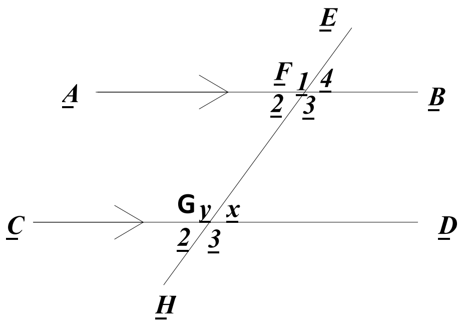
(d) To prove that Statement B is false , it is enough to find one non-rectangle whose diagonals bisect each other

(e) None of (a)-(d) is correct

Questionnaire B
Answer all questions

Question1

Consider the diagram drawn below.



1.1 Name two angles which are equal to x and explain?

(i)____________________Reason ______________________________________

(ii) ____________________ Reason ______________________________________(4)

1.2 Name two angles which are equal to y

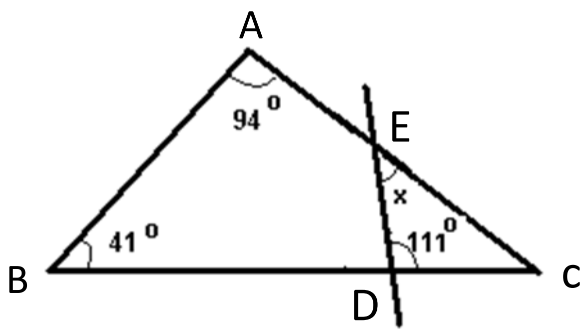
(i)______________________ Reason ____

(ii) ______________________Reason ____(4)

Comment

Question 2

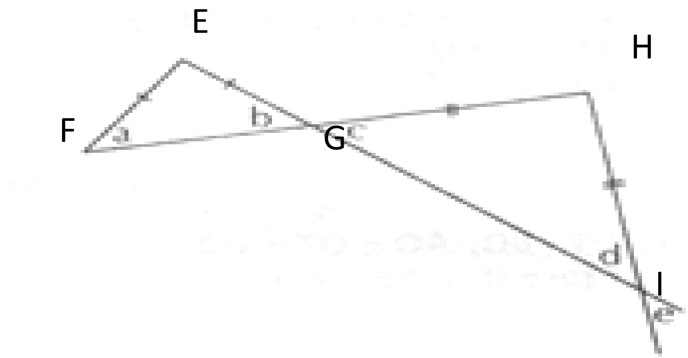
Find the value of x in the diagram below. (3)



Comment:

Question 3

Prove, giving reasons, that $\hat{a} = \hat{e}$. **(4)**

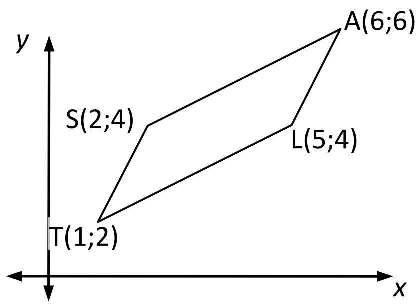


Comment:

Question 4

Quadrilateral SALT with vertices S(2;4), A(6;6), L(5;4)
And T(1;2) is shown in the figure alongside

Show that SALT is a parallelogram. (3)



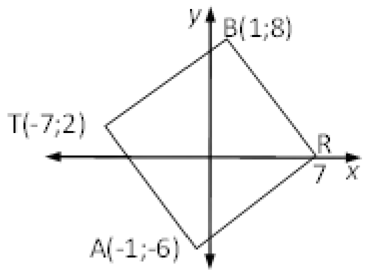
Comment

Question 5

Below is a quadrilateral BRAT. Use the quadrilateral to answer the following questions.

5.1 Write down the coordinates of R. (1)

5.2. Is BRAT a square, why or why not? (3)



Comment:

Question 6

In the diagram below, PQ = QS = SR = SM and PS // QS.

Prove that:

6.1. $\hat{R}_2 = \hat{P}$ (2)

6.2. ΔPQS ≡ ΔRSQ (3)

6.3. PSRQ is a parallelogram (3)