EXPLORING GRADE ELEVEN LEARNERS' VIEWS ON USING GEOMETER'S SKETCHPAD FOR PROOFS IN EUCLIDEAN GEOMETRY

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DEDICATION

This work is dedicated to:

Reverend G F Shandu, Arch Bishop Reverend S. Chiliza and Mamkhulu, Mama D. G. Sithole.

My mother, Marjorie, my brothers, Mandlenkosi, Andile, my sisters Makhosazana, Thandi, Nokwanda, and my children Thando and Sbu. My cousins Thenjiwe, Mato, Phumzile, Lindiwe, Mzuvelile and Bongiwe.

AND

My late dad Makhosi Walton, my late uncle Alfred Phambanani, my late mom Mary, my late aunt Regina, my late aunt Nobahle, my late grand-father Benjamin, my late grandmothers Trenah, MaRadebe and MaKhumalo, my late uncles Gamakhulu and Sandile, my late brother Themba and my late sisters Joyce, Zoliswa and Mandisa and my late cousin Nthabeleng, who have been my pillars of strength.

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16. To Nhlakanipho Khanyile and Mhlengi Nxele for their technological assistance.
17. To all those whose names I might have omitted.

DECLARATION

I, **Nombulelo Thembile Mbokazi**, declare that the research involved in my dissertation submitted in partial fulfillment of the M. Ed. Degree in Mathematics Education, entitled **Exploring Grade Eleven learners' views on using Geometer's Sketchpad for Proofs in Euclidean Geometry**, represents my own and original work and that it has never been submitted to any other university for any purpose.

Nombulelo Thembile Mbokazi

Date

As the candidate's supervisor, I agree to the submission of this dissertation for submission.

Dr Jayaluxmi Naidoo

Date

ACRONYMS

Abbreviation	Descriptions
AMESA	Association for Mathematics Education of South Africa
CAPS	Curriculum and Assessment Policy Statement
CD	Compact Disc
CPTD	Continuing Professional Teachers' Development
DEP	Dragging Exploration Principle
DG	Dynamic Geometry
DGA	Dynamic Geometry Application
DGE	Dynamic Geometry Environments
DGS	Dynamic Geometry Software
DoE	Department of Education
DSG	Development Support Group
GHOM GHOMs	Geometry Habit of Mind Geometry Habits of Mind
GSP	Geometer's Sketchpad Program
HLT	Hypothetical Learning Trajectory
IQMS	Integrated Quality Management System
NCS	National Curriculum Statement
NCTM	National Council of Teachers of Mathematics
OBE	Outcome - based – Education
PMTs	Pre - service Mathematics Teachers
RNCS	Revised National Curriculum Statement
SACE	South African Council for Educators
SASA	South African Schools' Act

SDT 2-D	Staff Development Team Two Dimensional
3-D	Three Dimensional
ZPD	Zone of Proximal Development

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A	AO1	AO2.1	AO2.2	AO3.1	AO3.2	AO3.3
R	RO1	RO2.1	RO2.2	RO3.1	RO3.2	RO3.3
S	SO1	SO2.1	SO2.2	SO3.1	SO3.2	SO3.3
J	JO1	JO2.1	JO2.2	JO3.1	JO3.2	JO3.3
N	NO1	NO2.1	NO2.2	NO3.1	NO3.2	NO3.3
T	TO1	TO2.1	TO2.2	TO3.1	TO3.2	TO3.3
G	GO1	GO2.1	GO2.3	GO3.1	GO3.2	GO3.3

LEARNERS	**EXAMPLE**
L – Lettie	LO1 – Lettie Observation Question 1
A –Asanda	AO2.1 – Asanda Observation Question 2.1
R – Ridge	RO2.2 – Ridge Observation Question 2.2
S – Sonke	SO3.1 – Sonke Observation Question 3.1
J – Jabu	JO3.2 – Jabu Observation Question 3.2
N – Ntsiki	NO3.3 – Ntsiki Observation Question 3.3
T – Thula	TO1 – Thula Observation Question 1
G – Gabi	GO2.1 – Gabi Observation Question 2.1

A LIST OF LEARNERS WHO PARTICIPATED IN THE INTERVIEWS

Learners	Code	Code	Code	Code
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R	RI1	RI2	RI3	RI4
S	SI1	SI2	SI3	SI4
J	JI1	JI2	JI3	JI4
N	NI1	NI2	NI3	NI4
T	TI1	TI2	TI3	TI4
G	GI1	GI2	GI3	GI4

LEARNERS	**EXAMPLE**
L – Lettie	LI1 – Lettie Interview Question 1
A – Asanda	AI2 – Asanda Interview Question 2
R – Ridge	RI3 – Ridge Interview Question 3
S – Sonke	SI4 – Sonke Interview Question 4
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ABSTRACT

Curriculum planners in South Africa have reinstated geometry in the Curriculum and Assessment Policy Statement (CAPS) which had been relegated to the National Curriculum Statement (NCS). Euclidean Geometry has been taught again in South African schools from Grade 10 since 2012 and the first Matric Examination based on CAPS was written in 2014.

This study reflects on research that has been conducted with Grade Eleven mathematics learners in one of the high schools in one of the townships in Durban in South Africa. It discusses the views of these learners on the use of the *Geometer's Sketchpad Program* (GSP) on learning proofs using the *Geometer's Sketchpad Software* in Euclidean Geometry. The research focused on exploring the concepts of the angle at the centre theorem using both the paper and pencil method and the *Geometer's Sketchpad Software*. The current study's objective was to explore how the *Geometer's Sketchpad Software* can be used as an experimental tool to teach proofs as a means of verification and explanation. The study aimed to incorporate technology in mathematics teaching.

The advantages and disadvantages of using Dynamic Geometry Software (DGS) for example the *Geometer's Sketchpad Software* are discussed. Additionally, DGS in mathematics teaching, experimentation, functions of proof, Driscoll's (2007) Habits of Mind (GHOM's) and teachers' professional development are discussed.

CHAPTER ONE

1. Introduction

1.1 Introduction and Overview

The National Department of Education of South Africa has had many changes since 1994. Due to the first democratic elections which were held in South Africa on 27 April 1994, changes were made in Education by policy makers. These changes aimed, among other things, to redress past inequalities as stated in National Curriculum Statement (NCS) policy (2000).

In 1997 Outcomes-based Education (OBE) was introduced to beat the divisions in the curriculum that prevailed in the past. Due to challenges in implementation, Outcome-based Education was reviewed in 2000. This led to the Revised National Curriculum Statement (RNCS) Grades R - 9 and the National Curriculum Statement (NCS) Grades 10 - 12 (2002). The National Curriculum Statement Grades R - 12 encouraged learner-centredness and mathematics teaching which is based on learners' experiences (Department of Education, 2003). However, some teachers use time constraints that govern the syllabus completion as an excuse to resort to a traditional way of teaching (Mthembu, 2007). There were political redress attempts of past inequalities and tremendous changes in different departments including the Department of Education. In South Africa, there was a paradigm shift from the previous curriculum based on apartheid which was initiated by the National Department of Education. Outcomes-Based-Education called Curriculum 2005 was introduced (DoE, 2010).

A foreword in Curriculum and Assessment Policy Statement Grades 10 - 12 for Mathematics (DBE, 2011), stated that ongoing implementation challenges led to a review of the Revised National Curriculum Statement (2002) which resulted in the Curriculum and Assessment Policy Statement Mathematics, Grades 10 - 12 (2011).

1.2 Rationale and motivation of the study

The re-introduction of Euclidean Geometry in 2012 when the Curriculum Assessment and Policy Statement (DBE, 2011) replaced the National Curriculum Statement (2002) motivated the current study. The first examination in Grade 12 under this new curriculum was written in South Africa in 2014.

Exploring Grade Eleven learners' views on using Geometer's Sketchpad for learning mathematics proofs in Euclidean Geometry seemed relevant and appropriate in Grade Eleven as these learners have had previous background from Grade Ten where they conducted proofs in Euclidean Geometry as per Curriculum and Assessment Policy Statement (DBE, 2011) requirement. This is congruent with the CAPS document which stated that Euclidean Geometry be dealt with in term two for three weeks in Grade Ten. To be more specific, Grade Ten learners need to investigate, make conjectures and prove them (DBE, 2011). The Curriculum and Assessment Policy Statement Grades 10 - 12 (DBE, 2011, p. 34) stipulated that for three weeks in term three, learners need to explore and prove the circle geometry theorems in Grade Eleven. Lastly, learners are expected to use the theorems of the geometry of circles and the existing converses, to solve riders. Therefore, exploring Grade Eleven mathematics learners' views on using *Geometer's Sketchpad* for mathematics proofs in Euclidean Geometry seemed appropriate in Grade Eleven, since geometry proofs were within the curriculum in South African schools. The current study sought to encourage mathematics teachers to incorporate the use of Dynamic Geometry (DG) in mathematics teaching.

The study's background is underpinned by previously conducted studies. Mudaly (1998) conducted a study with the aim to investigate whether learners have a need for explanation and conviction incorporating dynamic geometry software, *Geometer's Sketchpad*. Moreover, the study tested material of the curriculum that was developed as a result of previous theoretical and empirical research. This curriculum material was conceptualised within the different functions of proof. Additionally, Mudaly (2004) conducted a research where he investigated *Geometer's Sketchpad* role as a modelling instrument. This study discussed the impact of *Geometer's Sketchpad* as a problem solving device. Another study was conducted by Govender (2013) which proved Viviani's theorem using *Geometer's Sketchpad*. The study was based on constructions and justifications of generalisation of Viviani's theorem. More specifically, the study investigated how eight Pre-Service mathematics teachers (PMT's) experienced the reconstruction of Viviani's theorem through experimentation, conjecturing, generalising and justifying. In this study, all PMT's exhibited a need for an explanation as to why their equilateral triangle generalisation was always true.

Van Hiele (1986) proposed a sequence of five psychological cognitive levels of geometric thinking on how children learn geometry. These are recognition, analysis, ordering, deduction and rigour. A proof in Euclidean geometry was conducted in the current study thus cognitive levels of geometric thinking were encapsulated though the focus was not on these levels. It seems vital to describe each of these levels which are interdependent in the order in which they are presented in. These are:

- Level 1 - Recognition (Visualisation): wherein learners recognise geometric figures.
- Level 2 - Analysis: wherein learners analyse diagrams' components and properties.
- Level 3 - Ordering (Informal deduction) wherein learners logically interrelate properties and rules.
- Level 4 - Deduction: wherein learners prove theorems deductively, but do not recognise the need for rigour.
- Level 5 - Rigour: wherein learners establish theorems, analyse and compare different self-evident systems. These are interdependent psychological cognitive levels which learners undergo when learning geometry.

De Villiers (1990) mentioned that verification; explanation, systematisation, discovery, communication and self-actualisation are functions of proof. These functions seem essential to teaching of conducting proofs in totality. Each and every one of these will be discussed broadly in Chapter Two. De Villiers (1990, 1991) investigated whether learners displayed a need for explanation and conviction within the context of geometry using the paper and pencil method. Mudaly's (1998) study explored whether learners exhibited a need to be convinced and a need for explanation of why conjectures were true, within the context of dynamic geometry, based on De Villiers' (1990, 1991) studies. De Villiers (2004) motivated me to undertake my study by using *Geometer's Sketchpad* which allowed learners to discover instantaneously whether or not a conjecture is true. The current study intended to similarly to focus on the importance of teaching proof as a means of authentication and explanation within the context of Dynamic Geometry, *Geometer's Sketchpad*.

1.3 Focus of the main study

Freudenthal (1983) in his book Didactical Phenomenology of Mathematical Structures' described mathematical concept phenomenology, structure, or idea on one hand, in its relation to the phenomena for which it was created, and to which it has been extended in the learning process of human beings. Didactical phenomenology on the other hand is a way to show the teacher where the learner might move into the learning process of human beings. He compared mental objects and concept attainment. Freudenthal (1983) explained that concepts are the backbone of our cognitive systems and highlighted that they are not considered as a teaching subject. Furthermore, when learners learn mathematics, they grasp mathematics concepts as intellectual objects and not as physical and concrete objects in the real world. They also carry out these concepts as brain activities. This means learners do not achieve conceptual understanding, but only achieve procedural fluency (Kilpatrick, 2001) where they only master the method of how to solve mathematics problems without any understanding mathematical concepts.

This was linked to the focus of the current study, whose aim was to explore the concepts of 'the angle at the centre of the circle' and of 'subtended by'. These concepts need not be memorised but need to be understood. To be more specific, this study focused on exploring that the angle subtended by the arc at the centre of the circle is double the size of the angle subtended by the same arc on the circumference. Freudenthal (1983) seriously considered concept realisation; a stage he claimed is often neglected in mathematics education.

This study utilised the Dynamic Geometry Software (DGS), the Geometer's Sketchpad Program (GSP), which can be used for teaching mathematics, science and technology. Through the use of the Geometer's Sketchpad Program, learners from Grade Three through to University, learn mathematics in a tangible and visual approach. The present study aimed to investigate whether learners would be able to construct their own knowledge through conducting an empirical proof making use of the Geometer's Sketchpad Program. It, meaning the current study, incorporated a pilot study and main study focusing on exploring Grade Eleven learners' views on using *Geometer's Sketchpad* for mathematics proofs in Euclidean Geometry.

1.4 Research questions

This study addresses two critical questions:

1. What are Grade Eleven mathematics learners' views about the use of *Geometer's Sketchpad* for teaching of proofs in Euclidean Geometry?

2. How can *Geometer's Sketchpad* be used to teach proofs in Euclidean Geometry to Grade Eleven learners?

1.5 Aims and objectives of the main study

The aim of the main study was to investigate Grade Eleven Mathematics learners' views on incorporating the use of Dynamic Geometry Software (DGS), *Geometer's Sketchpad* for proofs in Euclidean Geometry. It was also to explore how *Geometers Sketchpad* can be used to teach proofs in Euclidean geometry to Grade Eleven learners.

1.6 Significance of the study

The present study was conducted in a peri-urban (township) area about 25 km north of Durban city centre in South Africa. It seemed important to conduct the study as it would attempt to add to the knowledge on the teaching of proofs in Grade Eleven using Dynamic Geometry Software (DGS). Dynamic geometry on one hand signifies action as opposed to static. DGS allows one to draw sketches which can be dragged around and transformed and measured using the built-in measuring mode. Static geometry, on the other hand, refers to sketches that are drawn on paper or on chalkboard which are at rest and thus do not move. The study would also add to the literature pertaining to the topic. The data generated in the research project would hopefully serve as an eye opener to teachers who have never used Geometer's Sketchpad. Most importantly, the study would contribute as resource material which is an alternative approach to teaching geometry proofs, as proofs are a challenge to some learners.

1.7 Description of key terms

1.7.1 Proof

A proof is a phenomenon providing a way of being sure of something one is uncertain about whether it is true or not, or as a way of understanding something one already believes to be true (Key Curriculum Press, 2009). It is a written version of the complete notion that is used to arrive at an inference. Each stage of the development is supported by a proposal, theorem, or explanation verifying why the stage is possible. De Villiers (2003) defined proof as a logical argument in mathematics whose purpose is to verify a conjecture, as well as an understanding of why something is true.

Proof is a tool used by mathematicians for verifying, explaining, communicating, systematising and self-realising (de Villiers, 2007). Proving is the individual's procedure to eliminate or create doubts about the observation's truth (Harrel & Sowder, 1998). Furthermore, proving procedure involves discovering with certainty and convincing. However, De Villiers (2012) suggested that because proof has other functions apart from verification, it should instead be defined more broadly as a logical argument denoting how a particular end result can be obtained from other assumed or proven results.

Proof in mathematics at school needs to be scrutinized, as it seems a very important tool in mathematics. Thales of Miletus (600 B. C.), a Greek philosopher, proved the very first proof that, for any chord *AB* in a circle, all the angles subtended by points anywhere on the same semi arc of the circle will be equal (Wesstein, 1999). Learning procedures and proofs without good understanding of why they are important will leave learners ill-equipped to use their knowledge later in life (Curriculum and Assessment Policy Statement, 2011).

De Villiers (1999) described seven different functions of proof which need to be taken into cognisance while teaching proofs. These are: Verification/Justification, communication, explanation/Illumination, discovery, systematisation, self-realisation/intellectual, challenge/fulfillment and lastly memorisation and algorithmisation. These will be discussed broadly in Chapter Two.

1.7.2 Mathematics

Mathematics is a language making use of symbols and notations for describing numerical, geometric and graphical relationships (Curriculum and Assessment Policy Statement, 2011, p. 8). Furthermore, it is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. In addition, it helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making. Most importantly, mathematical problem solving enables us to understand the physical, social and economic world around us, and to teach us to think creatively.

Mathematics is a large and diverse subject which requires an extensive and systematic organisation. Mathematics is often split into four parts, algebra, analysis, geometry and applied mathematics. Mathematics is branch of logic and so can be founded by a development from a careful initial statement of the principle of logic.

1.7.3 Euclidean Geometry

Mathematics comprises Algebra, Trigonometry and Geometry. As the study is based on Euclidean Geometry, different types of geometry will be discussed. Etymologically, geometry means measuring the earth as performed by surveyors (Freudenthal, 1983). Furthermore, mensuration and geometry are required in order to plan buildings and to establish their capacity. Both these are also required for the construction of roads, canals, tunnels, pyramids and fortifications, which as geometrical figures are designed according to geometric principles. There is Euclidean geometry and non-Euclidean geometry, which refer to geometry on a plane and geometry on a sphere respectively.

Geometry in the traditional approach is viewed on one hand, as an endless set of algorithms and theorems where the learners will be given units of information, one after the other, and they memorise a definition of a theorem and a proof without really having a grasp of what the problem was all about (Gattegno, 1980). On the other hand, Nxawe (1995) argued that the Euclidean understanding of Geometry is a logico-deductive system, with a handful of units of knowledge to be mastered or memorised. Both scholars stressed rote memorisation which produces no relational understanding. Breen (1992) highlighted that most of school geometry is taken from six of Euclid's thirteen books, '*Elements*'. Caleb Gattegno (1965) defined geometry as being an awareness of imagery.

It seems important to differentiate between Euclidean and non-Euclidean geometry though the current study is not based on non-Euclidean Geometry. Euclidean Geometry is where geometric entities such as circles, squares, triangles *etc.* are on a plane. In this type of geometry, a line formed when two points are joined, has no limitations. An example of non-Euclidean Geometry is the geometry of the surface of a sphere where lines are arcs or curves. Hyperbolic geometry known as Bolyai-Lobacheskian geometry is an example of non-Euclidean Geometry. Elliptical geometry is another example and, elliptical geometry can be visualised as the surface of a sphere on which lines are regarded as great circles (Wesstein, 1999). Most importantly, the sum of angles of a triangle is greater than 180 degrees in elliptical geometry. All these are static and inactive as opposed to Dynamic Geometry (DG) that will be discussed in the next paragraph.

1.7.4 Dynamic Geometry (DG)

Dynamic in mathematics refers to objects in motion or movement (ideas of motion and change). Thus dynamic geometry refers to vigorous, active and forceful geometry as opposed to static geometry media, which is inactive, fixed, lethargic and stationary like the chalkboard and the projector. Dynamic geometry is a term coined in response to software packages such as *Sketchpad, Geogebra*, and *Cabri*. These software packages act as a sort of electronic ruler and compass. The *Geometer's Sketchpad Program* (GSP) will be discussed at length in the next paragraph as it is the software that will be employed in the current study.

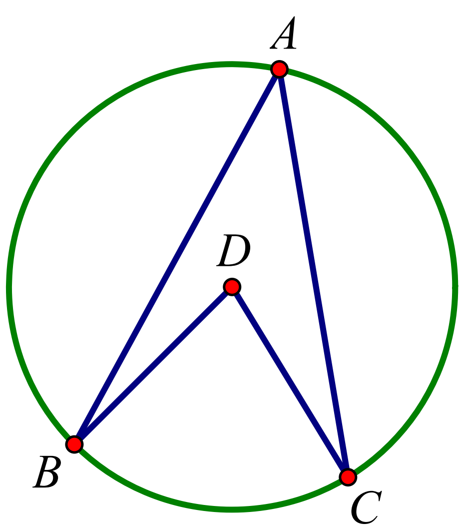
1.7.5 Geometer's Sketchpad

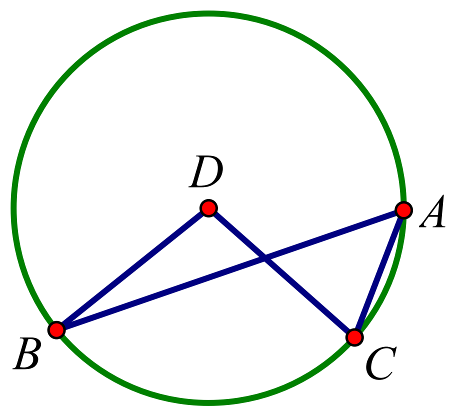
The *Geometer's Sketchpad Program* (GSP) is a software package that enables learners to discover geometric concepts through experimentation and exploration. It is software used for teaching science, mathematics and technology from grade three through university. It provides learners with a tangible and visual way to learn mathematics which increases their engagement, understanding and achievement (Webster, 1997). It enables elementary learners to manipulate dynamic models of fractions, number lines and geometric patterns. The program can help middle school learners to build their readiness for algebra by exploring ratio and proportion, rate of change and functional relationships through numeric, tabular and graphical representations. High school learners can use it to construct and transform geometric shapes and functions from linear to trigonometric, promoting deep understanding. The software can be used daily to illustrate and illuminate mathematical ideas. The latest *Sketchpad* version, *Sketchpad* version 5.05's module, focuses on the Common Core Standards for Grades 3-12. A free scaled-down version can be downloaded and used on the iPod.

Geometer's Sketchpad is a very accurate and valuable tool for visualisation. De Villiers (2003) warned that one needs to be careful not to make false conjectures despite the fact that *Sketchpad* is a very powerful, accurate tool. In an attempt to ensure accuracy, one needs to look at extreme cases and where possible use enlargement or animation facilities of *Sketchpad* to check the validity of conjectures. Hulme (2012) mentioned that GSP encourages a process of discovery where learners first visualise and analyse a problem and make conjectures before attempting a logical explanation (proof) of why their observations are true.

1.8 Theorems

A theorem or lemma in mathematics is a mathematical statement whose truth can be proved on the basis of a given set of axioms according to the American Heritage dictionary of Student Science. A theorem is an idea that has been regarded as true. The current study is based on the angle at the centre theorem. The diagrams associated with the proof of this theorem can be represented in four different forms (du Plessis, 2013).





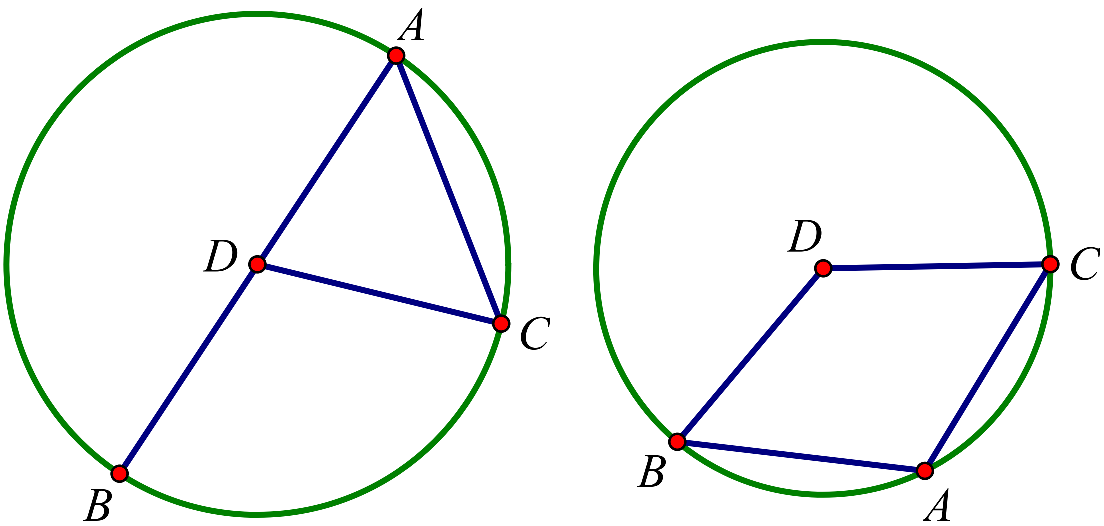


Figure 1 Four different forms of diagrams associated with the proof

of the angle at the centre theorem

In all four different forms of these diagrams, $<BDC = 2<BAC$, but $<BDC$ referred to in the fourth diagram is reflex. An angle subtended by an arc, line or curve in geometry, is the one whose rays pass through the endpoints of the arc, line or curve.

The next chapter presents some of the studies which are pertinent to the current study which modelled some aspects of mathematics. Furthermore, it describes the meaning and importance of proof, functions of proof, reasoning and proof and experimentation. In addition, it discusses some theorems which are pertinent to the pilot study which preceded the main study. Moreover, it elaborates on DGS's limitations, advantages and factors to be taken into cognisance when using DGS. Additionally, it discusses teachers' professional development.

1.9 The outline of the main study

The current study comprises six chapters. It begins with an introduction which discusses background to the study, focus of the study, rationale, aims and objectives of the study and significance of the study. The introduction also clarifies what is entailed in each chapter.

Chapter One describes key terms, proof, mathematics, Dynamic Geometry and a theorem.

Chapter Two discusses different dynamic geometry software packages, which were used to conduct various studies in different topics in mathematics. Different authors argue for and against the use of the different dynamic geometry software packages. The terms proofs, mathematics, and dynamic geometry, are discussed and supported by literature. Importance of proofs and functions of proofs are also discussed.

Chapter Three discusses the research design and methodology that was used in the collection of data. It also discusses the research approach, population and sampling, instrumentation, data collection procedures, data analysis method, theoretical framework, paradigm and research ethics. It also entails, advantages and disadvantages of methodologies used and research techniques that have been used in the field. Furthermore, it discusses how reliability, validity and trustworthiness were ensured in the current study. Lastly, issues, concerns and foci that featured in the design of the instruments are discussed.

Chapter Four discusses Constructivism, the Theoretical Framework in which the current study is embedded. Chapter Five deals in depth, with data analysis. It also discusses the questions that participants were asked and the responses obtained. Lastly, Chapter Six deals with the main findings. It also discusses how the research questions were answered by the current study. The main findings are also presented.

CHAPTER TWO

2. Literature review

2.1 Introduction

Teaching mathematics has proved to be a challenge to most teachers (Naidoo, 2011). Ndlovu (2012) explored pre-service mathematics teachers' (PMT's) knowledge of proof in geometry. The empirical evidence of Ndlovu's (2012) study illustrated that a large percentage of the student teachers had minimal knowledge of geometry regarding proofs. This minimal knowledge emanated from the knowledge of geometry which the student teachers were exposed to, whilst they were in high school. Thus the current study took into consideration that in high school, learners obtain minimal knowledge of proof and the study incorporated the use of the *Geometer's Sketchpad Program* in an attempt to teach mathematics proofs in Euclidean Geometry. Ndlovu (2012) suggested that Dynamic Geometry Application (DGA) may be used to encourage teachers to change their attitude towards geometry. The current study is therefore motivated by Ndlovu's (2012) suggestions to integrate the teaching of proofs with technology, using *Geometer's Sketchpad*.

This chapter discusses the meaning and importance of proof, reasoning and proof, experimentation, examples of some theorems, and different Dynamic Geometry Software (DGS). In addition, some studies closely resembling the proposed study are discussed. The current study builds onto a body of knowledge provided by these studies. Limitations of Dynamic Geometry Software (DGS), and the advantages and disadvantages of using DGS are also discussed in this chapter. Factors to be considered when using DGS and factors retarding computers' full potential are also discussed. Additionally, this chapter also discusses teachers' professional development.

2.2 Proof

2.2.1 The meaning and importance of proof

Proof, is a logical argument which verifies whether a conjecture is true or not and provides reasons why it is true. A conjecture is a speculation or a conclusion formed on the basis of incomplete information. Proof makes use of definitions, axioms, postulates and previously proved theorems to arrive at a conclusion. De Villiers (2002) defined proof as a means of discovery, as a means of exploring, analysing and inventing new results. It seems vital that the various functions of proof are communicated appropriately to learners to make proof more meaningful.

The aspects of proof will be discussed, but it does not necessarily mean they need to be taught in this order. De Villiers (1999) stated that in mathematics, we prove when we either have doubts or we are convinced that the conjecture is true. Additionally, we use counter-examples through quasi-empirical experiments to find contradictions. That is when we want to verify conjectures and to attain conviction. Verification/Justification is the function of proof which is concerned with the truth of a statement or proposition (Mudaly, 1998).

Learners may be convinced that a conjecture is true but, without explanation as to why the conjecture is true, they will still have questions as to why it is true. De Villiers (1999) highlighted that Halmos displayed a need for a logical proof despite the fact that he was convinced by the four colour theorem, computer conducted proof, by Appel and Haken. This shows explanation gives insight. Therefore proof as a means of explanation remains indispensable and irreplaceable.

Proof as a means of communication provides a unique way of communicating mathematical knowledge among mathematicians. Such communication enables other mathematicians to appreciate the proof, refine it, find errors or refute it by counter-examples. Proof as a means of discovery refers to exploring, analysing, discovering and inventing new results (De Jager, 1990 & Schoenfeld, 1986). Systematisation refers to the organisation of a variety of results into a deductive set of theorems and axioms.

Another function of proof is self-realisation. De Villiers (1999) referred to this as proof as a means of intellectual challenge, a proof which serves the function of fulfillment. Any proof on any aspect of mathematics, needs to provide fulfillment. Self-realisation is an aesthetic function of proof where mathematicians find joy in conducting proofs, though there are neither huge benefits nor remunerations.

Renz (1981) and Van Asch, (1993) referred to memorisation and algorithmisation as a function of proof wherein algorithms are memorised. Functions of proof need to be understood and considered in rigorous mathematics teaching and learning, as this would make a proof more meaningful. Proof and reasoning cannot be detached. In the current study learners constructed a proof making use of *Geometer's Sketchpad.* Learners were expected to reason and formulate conjectures which led to a proof, using the measurements they obtained. They were, however, not expected to memorise any algorithms but to measure, find relationships and formulate conjectures through reasoning with the help of *Geometer's Sketchpad* (Renz, 1981 & Van Asch, 1993). The next paragraph discusses how proof and reasoning are linked.

2.2.2 Reasoning and proof

Many mathematicians and mathematics teachers claim that reasoning and proof are central to doing mathematics and to learning mathematics (Ball, Hoyles, Jahnke & Movshovitz-Hadar, 2002). Considering this, it seems true that proof is essential and indispensable in Euclidean Geometry. If learners can prove any aspect of mathematics, they are engaged in doing mathematics through problem-solving and they are thus learning mathematics. One would need to reason while conducting a proof by determining relationships and formulating conjectures from the relationships.

Hersh (2009) stated that the possibility of proof is what makes mathematics what it is, and what distinguishes it from other varieties of systems. Mathematics language is ambiguous and confusing considering the mis-match between home language and school language (Zevenbergen, 2000). This is what makes it different from other disciplines. Proof is conducted in different aspects of mathematics, and without proof, there is no mathematics. Learners need to learn what proof is, to learn mathematics. Hanna (2000) stated that learners who have not learned what proofs are have not learned mathematics. Research has demonstrated that many secondary school learners and university students have difficulty understanding proofs despite the importance given to them by teachers, lectures and scholars (Thompson, Senk & Johnson, 2012).

Numerous studies have shown that some of the most persistent proof-related difficulties noted among secondary school learners and university students, are that they are confused with the meaning of proof, use of empirical examples in proofs, inability to define concepts, proof strategies and how to start a proof and how to monitor one's progress (Thompson, Senk & Johnson, 2012, Korb, 2007). Such difficulties could be caused by misconceptions, the level of cognitive and conceptual understanding of teachers who taught these learners at primary school and the approach used in teaching proofs. These misconceptions need to be eradicated.

Geometric proof depends on inductive and deductive way of thinking. The latter is the way in which geometric proofs are written. A deductive argument example is discussed in the following paragraph:

- All quadrilaterals with four of their vertices on the circumference are cyclic quadrilaterals. Refer to the diagrams that follow.

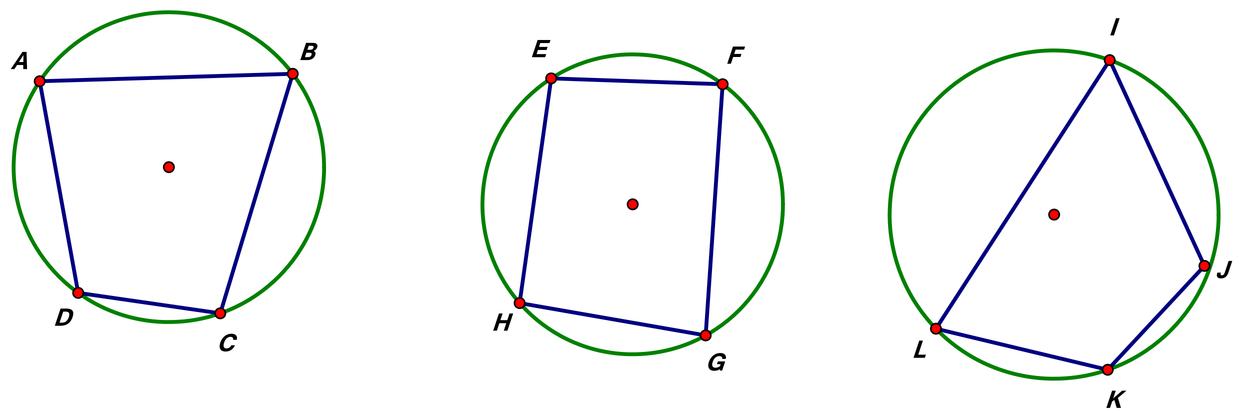


Figure 2: A diagram showing cyclic quadrilaterals

- A rondavel (African-style hut which is a cylindrical shape) with a circular floor is divided by four lines joined on the walls, to cater for the entrance, females' place, males' place and sacred place for burning incense.

Figure 3 shows African huts with a cylindrical shape and conical thatched roofs of grass.

Figure 3: African huts Adapted from Naude (2007: p. 231)

In Figure 4 the space from the door of the rondavel (traditional hut), represents the entrance of the hut. The space on the right-hand side of the hut from the entrance represents the space where men are allowed to sit inside. The space on the left-hand side of the hut from the entrance represents the space where women are supposed to sit. Lastly, the space labeled sacred place, is a space opposite the entrance, used by African family elders to burn incense which is used as a means of communicating with the ancestors. Figure 4 is a diagram that was drawn by me which illustrates the shape of the floor of an African hut.

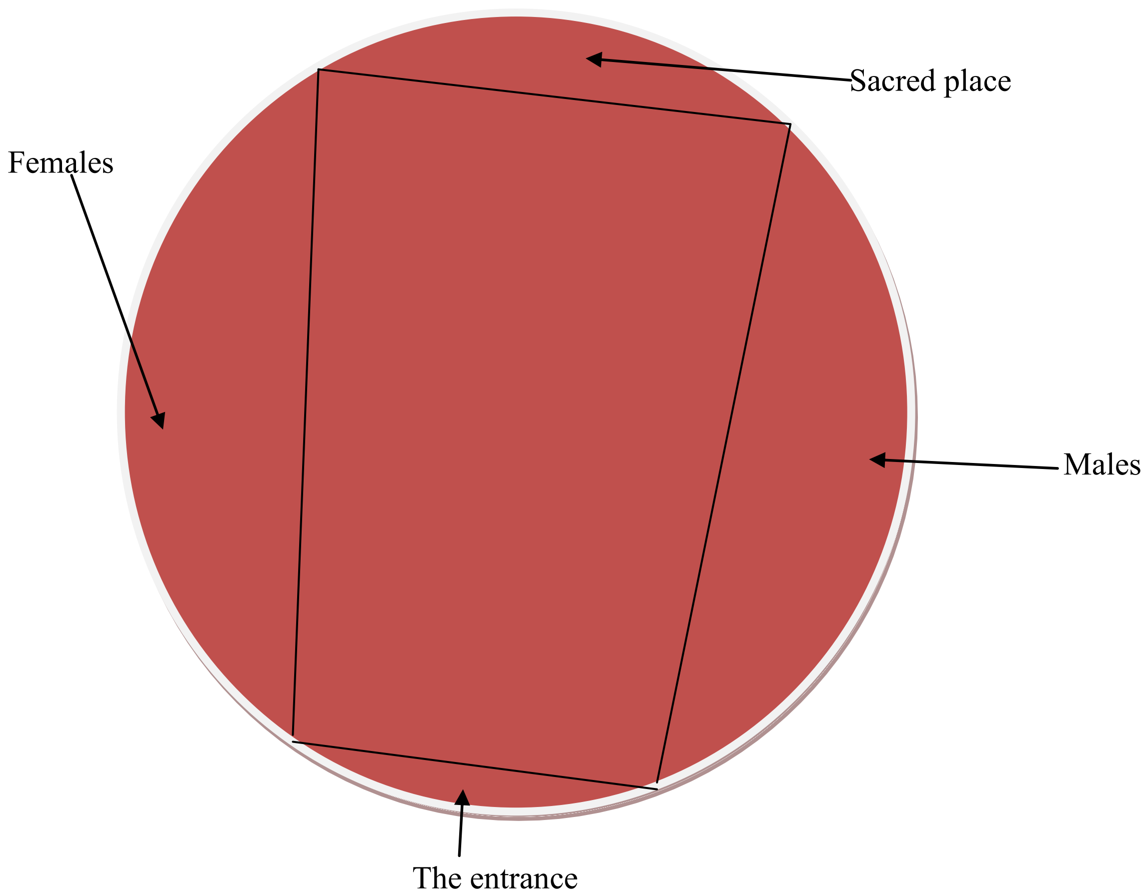


Figure 4: A diagram illustrating the shape of the floor of a hut

The floor of the traditional hut will look more like Figure 4 with no dimensions in real life even though it is divided into the four sectors mentioned above. Therefore the floor is divided by a cyclic quadrilateral, a four-sided figure with four of its vertices on the wall of the rondavel (African traditional hut).

Deductive reasoning made in the last premise is a conclusion arrived at, deduced from one of the properties of a cyclic quadrilateral given in the first premise. The conclusion is that the design made on the circular floor is a cyclic quadrilateral since it has four lines joined on the walls of the rondavel. Inductive reasoning, is the process of observing the given patterns or information. Then from observations one makes generalisations which are called conjectures. A conjecture is a statement which appears reasonable but whose truth has not been established (Mason, Burton & Stacey, 2010). In the current study, learners were expected to formulate conjectures from the measurements they obtained while using *Geometer's Sketchpad.* However, learners had to establish the truth of the generalisations they made with the help of the Dynamic Geometry Software, *Geometer's Sketchpad*.

2.3 Experimentation

Experimentation is used to explore hypotheses where new hypotheses or existing theories are tested with the aim of refuting or supporting them. It is employed when conjectures, generalisations or conclusions are made, based on analogy, intuition and experience obtained through experimental methods. In addition, mathematical statements are visually or numerically evaluated by measurement and accurate geometric construction.

In the current study, the participants conducted an experimental investigation in which they were required to make conjectures based on their mathematics experience. They also formulated conjectures having made use of the *Geometer's Sketchpad Software* as an experimental tool. Participants employed experimentation when they visually evaluated mathematical conjectures on geometric figures that I drew for them to explore.

2.4Theorems

This study highlighted a few theorems which employ experimentation, reasoning and proof. It explored learners' understanding of angles in mathematics. Concurrency of angle bisectors in a triangle will be discussed briefly.

2.4.1 Concurrency of angle bisectors

This theorem in geometry states that the angle bisectors of a triangle are concurrent at the incentre. Bopape, Hlomuka, Magadla, Shongwe, Taylor and Tshongwe (1993) proved this theorem by means of congruency, as in many high school textbooks. All angle bisectors of the triangle have an incentre meaning they all meet at a common point called an incentre, hence concurrent. The angle bisectors of ΔABC in Figure 5 that follows all meet at a common point D

where *D* is the incentre. Figure 5 was drawn by me using *Geometer's Sketchpad* to illustrate the incentre. The incentre is the point where all three angle bisectors of a triangle meet. It is also the centre of the inscribed circle. This is illustrated in Figure 5.

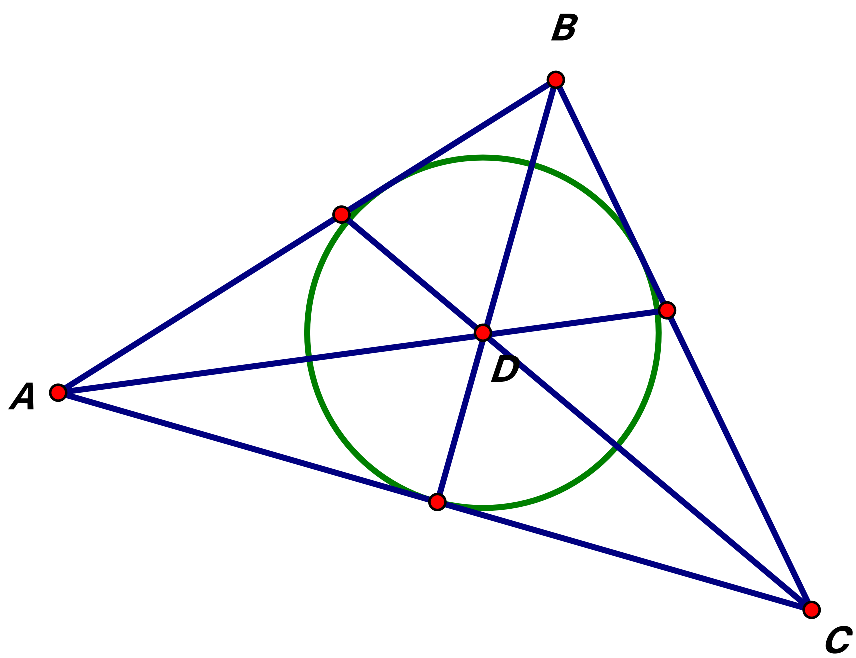


Figure 5: Angle bisectors of ΔABC meet at a common

point D called the incentre, the centre of the inscribed circle

The diagram that is illustrated in Figure 6 helps to prove that the angle bisectors of a triangle are concurrent, making use of congruency. Figure 6 was drawn by me with the help of *Geometer's Sketchpad.* In Figure 6, *<BAC* has been bisected such that *<BAG* = *<GAC. <ABC* has also been bisected such that *<ABH* = *<HBG*. Perpendicular lines have been drawn on *AB,* on *BC* and on AC.

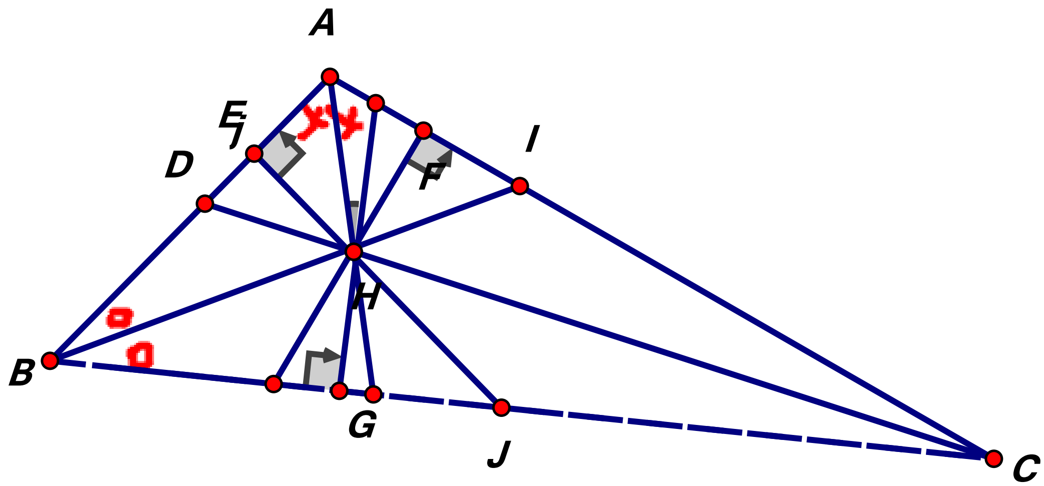


Figure 6: ΔABC drawn with *Geometer's Sketchpad*

illustrates that the angle bisectors are concurrent with the

help of perpendiculars on *AB, BC* and on *AC.*

Given Δ *ABC,* one is required to prove that the bisectors of $< A, < B$ and $< C$ are concurrent. The construction of angle bisectors *AH* and *BH,* and of perpendiculars *HE, HF* and *HG* help one to prove congruency in Δs *AHF* and *AHE.* This leads to the conclusion that *HE* = *HF* where *HE* is perpendicular to *AB* and *HF* is perpendicular to *AC.* In Δs *BHE* and *BHG, HE* = *HG* where *HE* is perpendicular to *AB* and where *HG* is perpendicular to *BC.* In Δs *CHG* and *CHF, HF* = *HG, CH* is common and <*HGC* = <*HFC* = 90°. This leads to the conclusion that <*GCH* = <*HCF* and hence *CH* bisects <*C.* The bisectors of angles *A, B* and *C* meet at *H* and are thus concurrent.

However, instead of proving the concurrency of the angle bisectors using congruency, the idea of equidistance can be developed and used to construct a proof in different types of triangles. The use of the *Geometer's Sketchpad Software* dragging mode can be used to find the incentre. This can be done after measuring the perpendicular distances between the proposed incentre and each of the three sides of the triangle. Each of these perpendicular distances need to be dragged till they are equal, thus equidistant from the incentre. The construction of the perpendiculars and the use of congruency in the proof could be difficult for some learners. Moreover, some learners would not think of any constructions in conducting this proof instead, they would apply congruency inappropriately, if they have not mastered it. Besides, the traditional method of learning proofs encourages learners to copy somebody else's way of reasoning without even understanding what is going on. This also deprives learners of creative thinking and of discovery. Bopape *et al* (1993) had regarded this proof as a proof that was meant for learners doing Mathematics Higher[1] Grade in Grade Eleven known as Standard Nine, at the time. The next paragraph discusses Ceva's Theorem which is linked to concurrency.

2.4.2 Ceva's Theorem

Ceva's theorem which was discovered from concurrency states that three lines that join three points, one on each side of a triangle, to the opposite vertices are concurrent if and only if the product of the ratios of division of the sides equals one (Beyer,1987). Proving this theorem using and the paper and pencil method seems complicated. It would be interesting to prove this theorem using *Geometer's Sketchpad* and thus to determine whether it will be less complicated or not and less confusing for the learners, even if it is not in the high school mathematics curriculum in South Africa at the moment. The diagram that was used to prove Ceva's theorem is shown in Figure 7 (Beyer, 1987).

[1] Higher Grade means a level of mathematics higher than Standard Grade in a Senior Certificate recognised by the Council.

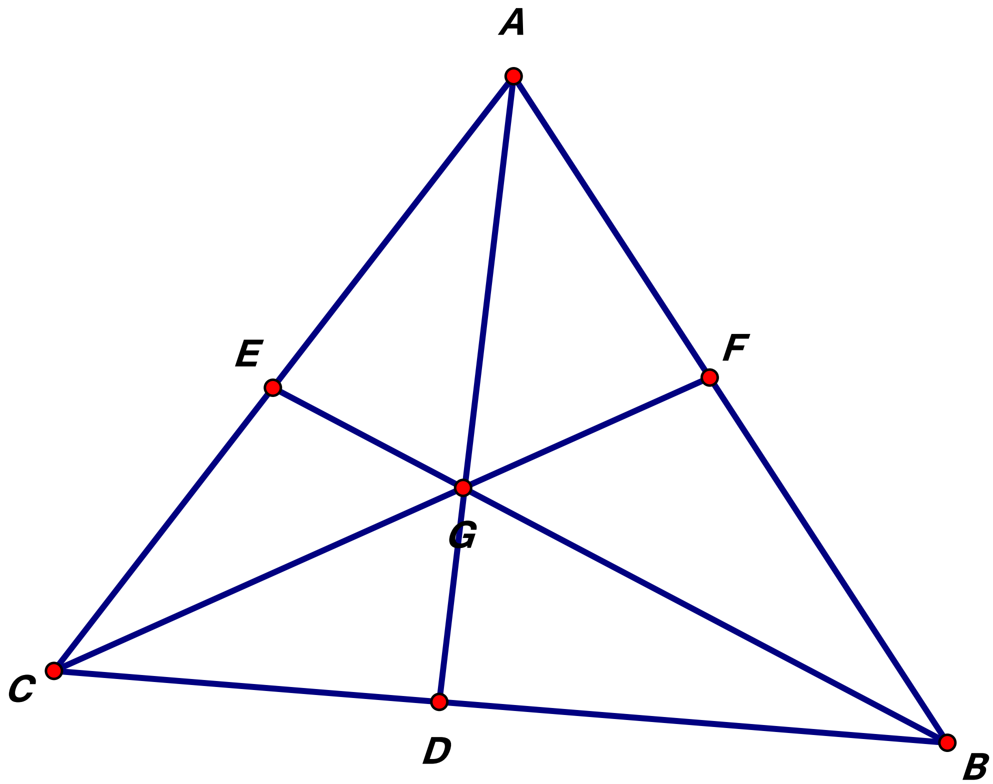


Figure 7: Diagram helping to prove Ceva's theorem

Adapted from Beyer (1987: p. 122)

$\frac{AG.DC.BF}{GD.CB.FA}$ = -1

For Δ *ACD* and transversal *BGE*, we have = $\frac{AE.CB.DG}{EC.BD.GA}$ = -1

Multiplying these equations, one obtains: $\frac{AE.CD.BF}{EC.DB.FA}$ = 1

Assuming the above equation is true, refer to Figure 8.

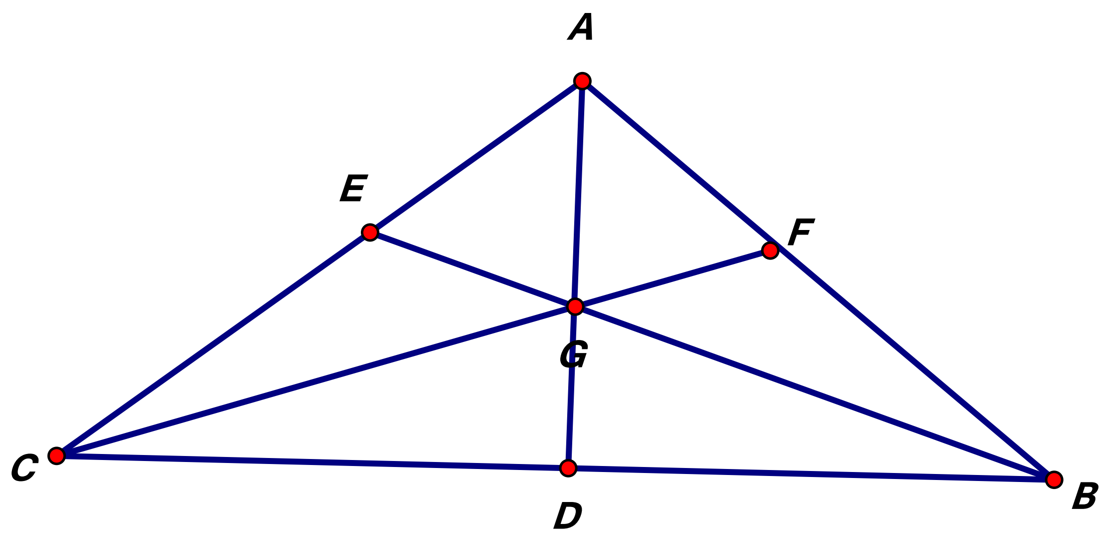


Figure 8: A diagram helping to prove Ceva's

Theorem Adapted from Beyer (1987: p. 122)

Given: $\frac{AE.CD.BF}{EC.DB.FA} = 1$

We need to show that *AD*, *BE* and *CF* are concurrent, to show *A*, *D* and *G* are on the same line.

Looking at Δ*BCF*, we find that *A*, *D* and *G* are three points one on each side of Δ *BCF*. Then the product of the ratios of division given by points *A*, *D* and *G* on each side of Δ *BCF* is: $\frac{BA.FG.CD}{AF.GC.DB}$ = -1

$\frac{AE.CG.FB}{EC.GF.BA}$= -1, then, *AD*, *BE* and *CF* are concurrent.

The converse of Ceva's theorem therefore can be used to prove concurrency. It seems essential to look at alternative ways of proving concurrency, besides using congruency. This proof is an example showing how a proof is conducted from already known theorems to obtain a new theorem. The next paragraph discusses the use of Dynamic Geometry Software (DGS) in mathematics teaching.

2.5 Dynamic Geometry Software (DGS) in mathematics teaching

The incorporation of different types of Dynamic Geometry Software may be used in an attempt to explore different aspects of mathematics. This will be illustrated from different studies which incorporated Dynamic Geometry Software in their exploration.

Some studies illustrated a positive impact of using DGS on enhancing learners' understanding of mathematics concepts. Mudaly (1998) stated that learners developed very high levels of conviction within Dynamic Geometry Environment. The findings displayed that the use of *Geometer's Sketchpad* helped the learners conduct a proof, though they still needed the teacher's guidance. Vygotsky (1978) referred to the zone of proximal development (ZPD) which is the gap between actual and potential development. Potential development is only acquired by a learner after being scaffolded, supported and guided by the teacher. Therefore the use of *Geometer's Sketchpad* employed in the current study will not replace the teacher but will serve as an intervening tool. In another study, Mudaly (2004) illustrated how learners integrated *Geometer's Sketchpad* (GSP) and proved that the perpendicular bisectors of a cyclic quadrilateral are concurrent. This study motivated the current study which started as a pilot study where learners were required to prove that the angle bisectors of a triangle are concurrent using *Geometer's Sketchpad.*

The National Council of Teachers of Mathematics (NCTM), (2000) recommended the use of Dynamic Software Programs to teach geometry. Hollebrands, Conner and Smith (2010) justified this stating that the use of Dynamic Software Programs would support learners' development of formal justifications and proofs. Ndlovu, Wessels and De Villiers (2013) concluded that pre-service teachers needed to be exposed regularly to the new technological environment, instrumental genesis and instrumental orchestration. This proved that mathematics teachers ought to be developed in the use of technology so that they are competent in their attempts to explain to learners how they may integrate technology to enhance mathematics teaching and learning.

Instrumental orchestration is used to describe competencies related to didactic management of the computer environment. While instrumental genesis is linked to van Hiele's levels of geometric thought, instrumental orchestration is linked to van Hiele's phases of instruction.

These phases will be described in paragraph 2.7 where mathematics teachers' development is discussed.

Ndlovu, Wessels and De Villiers (2011) made use of *Geometer's Sketchpad* to conjecture and implement a hypothetical learning trajectory (HLT). The aim was for learners to experience visualisation and multiple representations of calculus concepts on the Cartesian plane with a computer graphic interface. This illustrated that *Geometer's Sketchpad* can be used as a potential tool to model other parts of mathematics besides geometry.

Naidoo (2011) conducted a study which focused on exploring the use of visuals as tools employed by Master[2] mathematics teachers in the mathematics classrooms. Visual tools that are referred to incorporated symbols, diagrams, transparencies, pictures, the use of colour, graphs, mathematics manipulatives, shapes, gestures and any other visual that was considered as a trigger that encouraged the need for interpretation of mathematical ideas. Naidoo (2011) explored Master teachers' views about the use of visuals in mathematics teaching while the current study explored Grade Eleven learners' views on using *Geometer's Sketchpad* for proofs in Euclidean Geometry. The current study is pertinent to this study as it employed the *Geometer's Sketchpad Program* as a visual tool. It also made use of diagrams, symbols and shapes. A key finding of the research conducted by Naidoo (2011) demonstrated that using visuals as scaffolding tools was vital for the effective learning and teaching of mathematics. The *Geometer's Sketchpad Program* in the current study was used as a scaffolding, supporting and guiding tool to explore Grade Eleven mathematics learners' views about the use of *Geometer's Sketchpad* for teaching of proofs in Euclidean Geometry.

Lazarov (2012) presented applets designed with Geogebra. These were designed for in-service teachers training to teach some properties of parabola through examining dynamic constructions. The findings in Lazarov's (2012) study displayed that teachers managed to defeat the anxiety associated with the use of Dynamic Geometry Software. Geogebra can be downloaded free of charge from the internet, thus it is easily accessible. This software could have been used in the current study to teach proofs as it is similar to *Geometer's Sketchpad* whose licence needs to be purchased. I used the *Geometer's Sketchpad Software* as it was the

[2] Master Teacher means an experienced expert teacher with the potential to mentor inexperienced teachers

Dynamic Geometry Software I was exposed to whilst studying towards my BEd Honours degree.

Leung, Baccaglini and Mariotti (2012) discussed means of discernment and reasoning for Dynamic Geometry Environments (DGE) in their paper. This paper focused on the fundamental object of discernment being the invariant. Furthermore, it explained different dragging modalities and four types of variation interaction under the drag mode. Additionally, it introduced a Dragging Exploration Principle (DEP) that might assist to link the realm of DGE to Euclidean Geometry. This paper guided the current study and provided a better understanding of the *Geometer's Sketchpad Program*. It was worth understanding the two broad categories of dragging modality, dragging for testing and dragging for discovering. Moreover, this paper contributed to the current study by clarifying that invariants are linked to geometry. Similarly, Breen (1992) described geometry as the study of the invariant properties of given elements under certain specified groups of transformation. Understanding the software, helped in modelling mathematics aspects and thus in explaining the use of technology and in enhancing the learners' understanding of mathematics proofs.

Trgalova and Jahn (2012) highlighted the way the teachers orchestrated Dynamic Geometry (DG) activities in mathematics classrooms and in teacher training. They found that teachers began to be concerned with didactical and pedagogical issues related to DG integration as opposed to concentrating on technical aspects of mastering Dynamic Geometry Software. Ndlovu, Wessels and de Villiers (2013) mentioned on one hand that instrumental orchestration is used to describe competencies related to didactic management of the computer. Rowlett (2013) on the other hand, highlighted that the pedagogy need, needs to be considered while using technology in mathematics teaching. These studies informed the current study to be vigilant of didactical and pedagogical needs while exploring Grade Eleven learners' views on using *Geometer's Sketchpad* for proofs in Euclidean Geometry.

In another study, Govender (2013) proved Viviani's theorem using the *Geometer's Sketchpad Program*. The study was based on constructions and justifications of generalisation of Viviani's theorem. More specifically, it investigated how eight Pre-Service mathematics teachers (PMT's) experienced the reconstruction of Viviani's theorem through experimentation, conjecturing, generalising and justifying. Govender (2013) investigated how they generalised Viviani's result for equilateral triangles, some polygons and any convex equi-sided polygons.

In this study, all PMT's exhibited a need for an explanation as to why their equilateral triangle generalisation was always true. These teachers were only able to construct a logical explanation after they were guided through a worksheet. Govender's (2013) study findings coincide with Mudaly's (1998, 2004). Both Govender's (2013) and the current studies, made use of task-based activities embedded in a Dynamic Geometry Software (DGS) the *Geometer's Sketchpad Program* (GSP). There are factors that need to be taken into cognisance when using DGS. Some of these are discussed in the next paragraph in no particular order.

2.6. Factors to be considered when using Dynamic Geometry Software (DGS)

Oldknow and Taylor (2003) described the factors, which need to be taken into cognisance, when using dynamic geometry. They stated that firstly, teachers need to be innovative by carefully planning activities making sure that the intended mathematical learning outcome is achieved. Secondly, teacher's influence allows technology to provide opportunities for learners to learn mathematical concepts and principles.

Thirdly, teachers need to know how to use dynamic geometry effectively in selecting subject-specific software to meet particular teaching objectives in mathematics. Fourthly, teachers need to decide how and when to merge the use of dynamic geometry with conventional teaching of geometry. Fifthly, the teachers need to know how to incorporate dynamic geometry effectively into lessons and planning and how to organise classroom dynamic geometry resources effectively to meet learning outcomes in mathematics. These factors need not necessarily be considered in this order, but they all need to be taken into consideration (Oldknow & Taylor, 2003). Consideration of these will ensure understanding of mathematics is enhanced.

DGS has disadvantages which need to be highlighted so as to be able to use it meaningfully. Some of its disadvantages are discussed in the next paragraph.

2.6.1 Disadvantages of using Dynamic Geometry Software in the teaching of mathematics

Dynamic Geometry Software has its disadvantages. De Villiers (2007) put it that dynamic geometry cannot offer a magical solution for learning geometry automatically by simply staring at the beautiful, moving pictures on the screen. Furthermore, concrete skills of physically handling objects are still primarily important. Moreover, it needs not be taken for granted that dynamic geometry clarifies everything. It needs to be used in conjunction with some effective traditional styles of teaching and learning. Most importantly, dynamic geometry software needs to be used in a manner that engages the learner in problem solving instead of being carried away by the splendid and overvalued moving pictures.

Oldknow and Taylor (2003, p. 197) viewed the following as disadvantages of using dynamic geometry software:

- Dynamic Geometry simulates reality. The images produced on the screen are sometimes inaccurate. For example, proportions are sometimes wrong making it difficult to measure lines and angles directly off the screen
- Lines are not always straight due to the resolution and curvature of the screen
- Learners cannot explore everything and teachers need to see to it that they reach the learning outcomes of the day
- The teacher does not utilise all the capabilities of the program
- Sometimes the exploration does not need the technology.

However, DGS does have its advantages, some of which are discussed in the paragraph that follows.

2.6.2 Advantages of using Dynamic Geometry Software in mathematics teaching

Oldknow and Taylor (2003) stated that dynamic geometry allows the teachers and learners to deform shapes dynamically and observe which of their properties change and which remain the same. Furthermore, the dynamic images produced on the screen help learners to form mental images on which to base their understanding of concepts, for example the concept of concurrency. In addition, dynamic geometry enables learners and teachers to demonstrate a wide range of examples without having to draw them physically, but by changing variables. Through its use, learners can discover new ideas such as linking properties of polynomials and formulating new conjectures. As a result, learners obtain greater opportunity to consider general rules, test and formulate hypotheses.

Dynamic Geometry Software provides immediate visual feedback and thus enables learners to recognise quickly when they have made an error, as the software requires learners to construct figures accurately. Consequently, learners are able to undo their mistakes and correct them easily and immediately. Moreover, they can concentrate more on mathematical relationships rather than on the mechanics of construction, enabling the teacher to intervene more productively while maintaining the focus on mathematics. The next paragraph discusses teachers' professional development.

2.7 Mathematics Teachers' Professional Development

The Department of Basic Education (DBE) has policies in place underpinned by certain principles, which govern schools in South Africa to ensure development of teachers. These principles become a reality through the features of the Act. Some principles of these policies will be discussed briefly illustrating how they coincide. These are, South African Schools' Act (SASA, 1996), The Norms and Standards for Educators (1998), National Curriculum Statement (NCS) and South African Council of Educators (SACE, 2000).

The South African Schools' Act (1996) ensures co-operative partnership, democratic governance, quality education, and human rights, redress of past inequalities and social equity and consideration of diversity. This Act, together with The Norms and Standards for Educators (2000), National Curriculum Statement (2002) and South African Council of Educators (2000) ensure that there is a partnership between government, schools and communities. SASA, NCS and SACE ensure that schools adhere to non-racism and non-sexism thereby sharing values on human rights. These three policies are underpinned by the principles of redress of past inequalities and promotion of social equity. Nation building is catered for by NCS and SACE. Two SACE features are admitting teachers to the profession and regulating their qualification. Norms and Standards roles motivate teachers. Teachers are assessors, mediators of learning, specialists, lifelong learners, learning programmes designers and citizenship facilitators and, community and pastoral care givers. Integrated Quality Management Service (IQMS) and

Continuing Professional Teachers' Development (CPTD) have been put in place by the Department of Basic Education. The aims of these are to identify teachers', schools' and district offices' needs for support and development. Support is provided for continued growth. Additionally, it is to promote accountability, to monitor an institution's overall effectiveness and to evaluate teachers' performance. The Staff Development Team (SDT) members, who monitor mathematics, sciences and technology, need to be well equipped in the incorporation of technology in mathematics teaching. The same applies to mathematics, sciences and technology Development Support Groups (DSGs). This will ensure that the teachers are developed such that they are aligned with a modern technological world. Consequently, this will enhance learners' understanding of mathematics and of proofs in Euclidean Geometry.

Despite these policies which develop teachers, there is still a challenge to most teachers in the teaching of mathematics (Naidoo, 2011). The Department of Basic Education (DBE) has in-service training for all teachers, but it seems inadequate. This needs to be supplemented by attending Association for Mathematics Education of South Africa (AMESA) congresses. AMESA is the voice of teachers and it offers presentations for teachers on different teaching strategies by teachers, lecturers and educationists. Not all mathematics teachers are members of the association and some who are members, do not attend the congresses where development is provided and opportunity is afforded to anyone who wishes to present how s/he models aspects of mathematics effectively.

Alternative strategies need to be used when teaching learners to cater for diversity. Ethno mathematics [3] seems to make mathematics interesting. Integration of technology with mathematics is another alternative. Ndlovu (2012) stated that the teacher may use the Dynamic Geometry Application (DGA) to motivate students to learn geometry. The current study made use of the *Geometer's Sketchpad Program* to teach proofs in Euclidean Geometry exploring what Ndlovu (2012) suggested.

Teachers need to be competent in instrumental genesis and instrumental orchestration (Ndlovu, Wessels & de Villiers, 2013). Instrumental genesis [4] describes teachers' competencies in

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[3] Ethno mathematics means the study of the relationship between mathematics and culture

[4] Instrumental genesis. Genesis is a Greek word which means beginning. Instrumental genesis explains how learners may use the computer hardware and software to represent mathematical concepts.

explaining how learners may use computer hardware and software. Instrumental orchestration describes competencies related to didactic management of the computer environment. Instrumental genesis is linked to Van Hiele's (1986) levels of geometric thought while instrumental orchestration is linked to Van Hiele's phases of instruction. In the current study, I needed to be competent in the use of the *Geometer's Sketchpad Program* in order to be able to incorporate it in the teaching of proofs in Euclidean Geometry.

2.8 Conclusion

Based on the research that I have worked with, it is evident that high school mathematics teachers ought to work closely with tertiary institutions to assist them with conceptualising experimental mathematics. This will also help breach the gap between high school and tertiary level mathematics knowledge. Borwein (2005) described experimental mathematics as gaining intuition, discovering new relationships, exploring and suggesting formal proof approaches, testing conjectures, computing lengthy hand derivations and confirming analytically derived results. The next chapter discusses the background of the theoretical framework employed by the main study, which is Constructivism. Aspects of constructivism, social constructivism, its relationship with Curriculum and Assessment Policy Statement and how Constructivism contributed to the main study are also discussed.

CHAPTER THREE

3. Theoretical Framework

3.1 Introduction

The current study explored Grade Eleven learners' views on using *Geometer's Sketchpad* for proofs in Euclidean Geometry. The previous chapter discussed the literature review, that is, some of the previous studies which closely resemble and influenced the current study while this chapter presents the theoretical orientation of the current research study.

The aim of presenting this chapter is to create the context for the theoretical framework that is later established. Furthermore, it is to show through discussion, the coherence between the methodology and theoretical framework. This chapter also discusses the origin and the development of Constructivism, the theoretical framework within which this study is embedded. Constructivism therefore is discussed in detail, to show how it is manifested in the current study.

3.2 Constructivism

3.2.1 Background

Constructivism is a theory of knowledge rooted in psychology, philosophy and cybernetics[5] (Von Glasersfeld, 1989). Furthermore, one of its principles asserts that, knowledge, when received is actively constructed by the perceiving subject. Additionally, the other principle within Constructivism states that the function of cognition is adaptive and does not serve the discovery of ontological reality (Von Glasersfeld, 1989). He explained that to know something is to know what parts it is made of and how they have been put together.

In modern psychology, the concept of cognitive construction was first formed and shaped into a developmental theory by Baldwin (1861-1934) and Piaget (1896-1980). Von Glasersfeld (1989) stated that another source of Constructivism is the analysis of language stimulated by computer science and communication. From the constructivist point of view, meanings are conceptual structures which influence the individual's construction and organisation of the individual's experiential reality (Von Glasersfeld, 1989).

[5] Cybernetics means the science of effective organisation and of communication

Von Glasersfeld (1989) described Constructivism as a psychological theory of knowledge or epistemology which believes that human beings generate knowledge and meaning from their experiences. The formalisation of the Theory of Constructivism is generally attributed to Jean Piaget, a Swiss developmental psychologist and philosopher who explained mechanisms by which knowledge is internalised by learners. Cognitive constructivism is based on his work and he is known for his epistemological studies with the development of thought in children. Piaget's Theory of Cognitive Development proposed that human beings cannot be given information which they immediately understand and use. He believed that human beings must construct their own knowledge and build it through experience.

Piaget (1967) believed that individuals construct new knowledge from their experiences through processes of accommodation and assimilation. When individuals assimilate on one hand, they incorporate the new experience into an already existing framework without changing the framework. This occurs when individuals' experiences are aligned with their internal representation of the world. Accommodation, on the other hand is the process of reframing one's mental representation of the external world to fit new experiences. It can be understood as the mechanism by which failure leads to learning, that is, when we act on the expectation that the world operates in one way but our expectations are violated, we feel as if we have failed. By accommodating this new experience and reframing our model of the way the world works, we learn from the experience of failure (Piaget, 1967). Constructivism suggests that learners construct knowledge out of their experiences. Therefore the learning activities are characterised by active engagements, inquiry, hands-on activities, investigations, problem-solving, experimental design and collaboration with others (Bodner, 1998).

3.2.2 Constructivism concepts

Constructivism suggests that knowledge is not often transferred directly from teaching to learning in a form that can immediately be understood. There are significant qualitative differences in the understandings that different learners develop in the teaching and learning contexts, and it looks as if understanding is mostly different from what the teacher intends (Hiebert & Carpenter, 1992).

Constructivism has key concepts. Those are scaffolding, the zone of proximal development, discovery, problem-based and exploratory learning. Scaffolding refers to a process of guiding the learner from what he knows presently, to what he needs to know (Bruner, 1960). Furthermore, this guidance is provided by the teacher when learners are unable to proceed during teaching and learning. Additionally, it is facilitated to help learners perform just beyond the limits of their ability. Vygotsky (1978) discussed three categories into which learners' problem solving skills fall. These are skills which the learner cannot perform, skills which the learner may be able to perform and skills that the learner can perform with help.

Moreover, Vygotsky (1978) highlighted the zone of proximal development (ZPD)**,** which he described as the gap between actual development and potential development of the learner. I supported the participants by asking them leading questions, considering their previously acquired knowledge. I did this when they seemed not to understand the questions or instructions and thus, unable to proceed. I also rephrased questions in some cases which required that participants formulate conjectures and explain why they are true.

Within Constructivism, learners' understanding usually has to be constructed by their own individual efforts as well as their own mathematical ways of knowing, as they strive to be effective by restoring coherence to the world of their personal experience (Cobb, 1994). Constructivism though, does not imply that learners can make progress on their own without the teacher's help (Orton & Frobisher, 1996). This theory correlates with Bruner's (1960) Scaffolding Theory whereby children need support from their teachers and parents to become independent and mature learners. Considering Constructivism requirements, I supported and guided the learners so that I would be able to explore their understanding of geometry proofs and of angles in mathematics, using *Geometer's Sketchpad.*

An important concept discussed in Constructivism, is problem-based and exploratory learning**.** According to this concept, learners are provided with a task designed from the real world where they follow no particular procedures, and they find a variety of solutions or no solutions at all. In this study, firstly I started by using the question and answer method to attempt to ensure the participants' understanding of the concept of the sum of the interior angles of a triangle and of the exterior angle of a triangle. Secondly, I used the same method to elicit their understanding of radii of the same circle and thus the equality of the angles opposite equal sides. Thirdly, I demonstrated the proof of the angle subtended by the arc at the centre of the circle, is double the size of the angle subtended by the same arc on the circumference.

Another concept entailed in Constructivism is discovery**.** This refers to teaching which is problem-based, thus allowing learners to create their own conceptual understanding, but being guided by the teacher. Human learning of concepts can be studied in the framework of controlled laboratory experimentation (Clarizio, Craig & Mehrens, 1970). In addition, concept-learning refers to a kind of change in human performance that is independent of content subjects such as science, mathematics, language art *et cetera.* Furthermore, concept-learning is simpler than principle-learning since it is prior to principles (Clarizio *et al*, 1970).

To learn a principle or a rule, one must have learned the concepts of which it is composed. Principles can be learned by discovery as opposed to concepts. Clarizio *et al* (1970) stated that learning concepts by definition might be inadequate. Most teachers would maintain that the performing of operations, including observation in the laboratory, is an essential part of the learning situation required for the learning of fully adequate, generalised concepts (Clarizio *et al*, 1970, pp. 236-237). In this study, participants were expected to conduct the empirical proof of the angle at the centre theorem using *Geometer's Sketchpad* at the computer laboratory where I would observe them.

3.2.3 Social Constructivism

Social Constructivism encourages the learner to arrive at his/her own version of truth. This is important for the learner's social interaction with knowledgeable members of society. Wertsch (1997) highlighted that without interacting with the more well-informed society, it is not possible to acquire the meaning of vital symbol systems and how to use them. Von Glasersfeld (1989) mentioned that Social Constructivism takes into consideration the environment and culture of the learner throughout the learning process because the responsibility of learning rests more progressively with the learner. Furthermore, learners must look for meaning and regularity in the order of events.

Gagnon and Collay (1999) proposed six important elements that the Constructivists should consider when designing the learning content. These elements include: developing situations, organising groupings, building bridges, questioning, inviting reflections and arranging exhibits. Gagnon and Collay (1999, p. 7) described these elements as:

- Situation - facilitators should develop a situation for learners to explain events during the learning process. For example, learners may be asked to explain why a conjecture is true;
- Groupings - refer to a process of compiling materials and grouping learners for interactive learning. The instructor should identify a criterion to classify learning materials according to their purpose and design interactive tasks for interactive and active learning. Slow learners could be grouped together with fast learners for improved collaboration and information access;
- Bridging - how teachers incorporate learners' previous experiences into new learning environment. Every individual learner is believed to have some prior experiences related to what is studied in the classroom. Such experiences need to be included in the new training institution to form the basis of new knowledge;
- Questioning - what guiding questions will the facilitators use to introduce the situation or what situation will facilitators set for learners to ask questions during a lecture;
- Reflections - encourage learners to socially reflect on what they have learned. Learners may be given an opportunity to make presentations in groups. Activities such as poster presentations may indicate the level of skills that one has acquired and
- Exhibits - encourage learners to exhibit a record of their thinking by sharing it with other learners.

I created a situation within the current study for participants to explain the relationships between angles and I selected a process for groupings of axioms that would lead to the angle at the centre theorem. In addition, I built a bridge between axioms and theorems participants already knew and the angle at the centre theorem they had to learn. Moreover, I anticipated questions to ask without giving any explanation. Lastly, I encouraged participants to exhibit and share a record of their thinking and solicited participants' reflections about what they learnt from the task using *Geometer's Sketchpad.* Social Constructivism was therefore employed in the current research study.

Constructivists believe in active and interactive learning. This is the creation of an environment where learners discover and construct their own knowledge (Gagnon & Collay, 1999). Scheepers (2000) critiqued Constructivism by pointing out that it is time consuming and that its outcomes are vague and unpredictable. Furthermore, in a situation where agreement is essential, deviating and contradicting thinking and action can cause problems. The principle of Constructivism is thus subjective in nature and as a result, learners are free to develop multiple interpretations from the learning content, although such context may sometimes fail to match expected outcomes.

3.2.4 How Constructivism relates to the philosophy of CAPS Policy Document

The philosophy of the Curriculum and Assessment Policy Statement (CAPS) Grades 10-12 Mathematics policy document (DoE, 2010) is based on the principles of the National Curriculum Statement (NCS) Grades R-12. One of these principles of NCS is based on encouraging an active and critical approach to learning rather than rote and uncritical learning of given truths (DoE, 2010, p. 4). In active and critical approach to learning, learners are actively engaged in problem solving given investigations, for instance where they draw, construct, measure and find relationships, formulate conjectures and explain why conjectures are true. Learners construct their own knowledge by so doing, they discover new information and that is exactly what happens within Constructivism.

An active and critical approach links with one of mathematics specific aims. This aim provides an opportunity to develop in learners the ability to generalise, formulate conjectures and try to justify them (DoE, 2010, p. 8). As learners formulate conjectures and justify them, they make use of their experiences, language and knowledge from their community, thus interact socially. The principle of encouraging an active and critical approach to learning also links with one of mathematics specific skills. This skill states that the learner should use the mathematics process skills to identify, investigate and solve problems creatively and critically (DoE, 2010). Grade 10-12 learners are required to achieve ten content areas so that the link is forged between the Senior Phase and the Higher/Tertiary Education Band. These learners will achieve the skill to solve problems creatively and critically through scaffolding, by being guided and supported by the teacher. Scaffolding is one of the Constructivism concepts through which a learner is supported such that s/he can acquire new knowledge by adding it to knowledge already acquired and make sense of it. Thus Constructivism and Curriculum and Assessment Policy Statement are intertwined. Furthermore, problem-based and exploratory learning and discovery are other concepts of Constructivism which need to be taken into cognisance in mathematics teaching to achieve active and critical learning. This is one of the principles of National Curriculum

Statement Grades R-12 (DoE, 2010). Through Constructivism, the principles and features of NCS can be unlocked through adhering to Curriculum and Assessment Policy Statement Grades R-12 Mathematics (2011).

3.2.5 How did Constructivism contribute to the current study?

The aim of the current study was to explore Grade Eleven learners' views on using *Geometer's Sketchpad* for proofs in Euclidean Geometry. Constructivism contributed to the current study by enabling the learners to formulate conjectures through measuring the lengths of sides and sizes of angles of sketches provided on *Geometer's Sketchpad.* Thereafter, learners were required to determine relationships between sides and angles and to provide reasons for the relationships.

The current research study was problem-based and exploratory. Learners proved that the angle subtended by the arc at the centre of the circle, is double the size of the angle subtended by the same arc on the circumference. They linked already acquired knowledge with new knowledge. Learners knew already that radii of the same circle are equal. They also knew that angles opposite equal sides are equal. Additionally, they already knew that the exterior angle of a triangle is equal to the sum of the two interior angles of a triangle. They explored the relationships between all these aspects to prove the angle at the centre theorem. In conducting the proof, they first used the paper and pencil method and thereafter Dynamic Geometry Software, *Geometer's Sketchpad*.

Constructivism also contributed when I scaffolded learners' understanding. Scaffolding contributed when I guided and supported the learners. I used the *Geometer's Sketchpad Software* to draw diagrams and asked the learners questions that led them to the proof. Learners also measured lengths of radii of the circle and size (s) of angles. This guided them to discover on their own that angles opposite equal sides are equal. The aim was that learners retain this information and understand it better, as they discovered it themselves as they were engaged in problem solving. I did not ask them to prove anything; instead they conducted an investigation which was supposed to lead to a proof. This way Constructivism contributed in the sense that scaffolding enabled learners to conduct the same proof the way they understood it, instead of imitating the teacher or the textbook. Scaffolding is one of the important concepts of Constructivism. In other words, learners constructed knowledge without even being aware they were conducting a proof.

Constructivism links to Driscoll's (2007) Geometry Habits of Mind (GHOM's) as reported by Du Plessis (2013). Therefore, Constructivism contributed to the current study through consideration of Geometry Habits of Mind. GHOMs include the following processes: seeking and using relationships, investigating invariants and effects of transformations, generalising geometric ideas and balancing exploration with deduction (Du Plessis, 2013, p. 43). The current study considered one of Geometry Habits of Mind where participants sought relationships between sides and, between and among angles. It considered seeking and using relationships which involve actively looking within and between geometric figures in one, two and three dimensions (Du Plessis, 2013, p. 44). This GHOM involves thinking about how these relationships can help one's understanding of problem solving.

3.3 Conclusion

Constructivism seems to be essential in mathematics teaching. Most importantly, its concepts need to be considered in mathematics teaching so that a learner learns as a whole. The concepts referred to are scaffolding, problem-based and exploratory learning and discovery. Learners require guidance and support from parents and teachers so as to be able to construct knowledge as they perceive the world. Problem-based and exploratory learning seem to help learners to be engaged in problem solving and thus discover relationships and formulate conjectures on their own. These conjectures lead to proofs in Euclidean Geometry.

The next chapter discusses the methodology employed in the study, the study's design, the researcher's philosophical position, sampling, research instruments, data collection procedures, data analysis method, ethics and how trustworthiness was ensured in the current study.

CHAPTER FOUR

4. Research Methodology

4.1Introduction

In chapter three, Constructivism, the theoretical framework in which the current study is framed, was discussed. The chapter describes and discusses the research design and its aim. It discusses qualitative research as well its aim. The paradigm, in which the current study is embedded, is also described. The current study explored Grade Eleven learners' views on using *Geometer's Sketchpad* for proofs in Euclidean Geometry.

Chapter four defines action research enquiry as defined by different scholars. It clarifies why a pilot study was conducted and explains how the pilot study helped build the main study. This chapter also defines a pilot study and discusses its aims. It also shows my philosophical position as a researcher. The chapter also discusses data collection instruments and procedures, and the data analysis method used in the main study. Lastly, it shows how research ethics were adhered to and how trustworthiness was used to enhance the validity of the main study data.

My general plan of action was to take two days to teach proofs using the paper and pencil method in the current study. My aim was to improve and change my teaching strategy hence the study was underpinned by action research. After observing, monitoring, reflecting, and evaluating the effect of the action, I reviewed my initial plan. Cohen, Manion and Morrison (2011) stated that action research comprises planning, acting, observing and reflecting. My new plan took eight days with minimal or no improvement on how learners conducted proofs using the paper and pencil method. The last plan was to incorporate the use of *Geometer's Sketchpad* which seemed to facilitate huge improvement in learners' conducting mathematics proofs in Euclidean Geometry.

4.2 The research design

The research design expresses the procedures for conducting the study. It includes aspects such as from whom, when, and under what conditions the data will be obtained (McMillan & Schumacher, 2006). This suggests that a research design is the plan, strategy and structure of investigation conceived so as to obtain answers to research questions. Mouton and Marais

(1990) stated that the aim of the research design is to plan and structure a given research project in such a manner that the eventual validity of the research is maximised. Furthermore, the research design is viewed as the arrangement of conditions for collection and analysing data in an approach that aims to combine relevance to the research project.

Qualitative research is a form of social enquiry that focuses on the way people interpret their experiences and the world in which they live**.** The basis of qualitative research lies in the interpretive approach to social inquiry (Holloway, 1997). De Vos (1998) maintained that qualitative study aims to comprehend and interpret the meaning the subjects give to their everyday lives.

4.3 Research Approach

The current research project employed a qualitative action research enquiry underpinned within the interpretive paradigm, and framed by the theory of Constructivism. The study employed action research enquiry as it sought an in-depth understanding of the impact of exploring geometry proofs in mathematics while using dynamic computer software called *Geometer's Sketchpad*. It also sought to understand how learners construct their own knowledge using their mathematics experience.

One of action research's founders is Kurt Lewin who intended to change disadvantaged groups' lives (Cohen, Manion & Morrison, 2011, p. 344). Cohen and Manion (1994, p. 186) defined action research as "a small-scale intervention in the functioning of the real world and a close examination of the effects of such an intervention." It is sometimes called "practitioner based research." (Cohen, Manion & Morrison (2011, p. 344). Action research is also regarded as a form of self-reflective enquiry (Cohen, Manion & Morrison (2011). Cohen, Manion & Morrison (2011, p. 345), defined action research as "a form of collective self-reflective enquiry undertaken by participants in social situations in order to improve the rationality and justice of their own social or educational practices, as well as their understanding of these practices and the situations in which these practices are carried out."

Action research is an approach which seeks to improve education by changing it and learning from the consequences of changes (Cohen, Manion & Morrison (2011). Furthermore, it is participatory, collaborative, self-reflective, understanding the relationships between circumstances, actions and consequences in people's own lives. In addition, it involves collecting and analysing our own judgements, reactions and impressions about what is going on. Moreover, it involves planning, acting, observing and reflecting which can help to define issues, ideas and assumptions more clearly.

Participatory action research's aim is to attempt to help people investigate and change their social and educational realities by changing some of the practices which constitute their lived realities (Atweh, Kemmis & Weeks, 1998). Furthermore, in education, this can be used as means for professional development and improving problem solving in a variety of work situations. Atweh, Kemmis and Weeks (1998) highlighted that participatory action research has self-reflection cycles. It has six key features which state that participatory action research is a social process which is participatory, practical and collaborative. Participatory action research is also emancipatory, critical and recursive, reflexive and dialectical.

The current qualitative action research study aimed at exploring the concept of understanding angles. It was guided by the following research questions:

1. What are Grade Eleven mathematics learners' perceptions about the use of *Geometer's Sketchpad* in the teaching of proofs?
2. How can *Geometer's Sketchpad* be used to teach proofs to Grade Eleven mathematics learners?

The data that addressed the research questions were generated through individual observation of the participants as they proceeded with the task. This involved observation of the proof on the paper using the paper and pencil method, observation of responses on the laptop using *Geometer's Sketchpad,* observation of responses on the completed worksheet and face-to-face, individual, semi-structured interviews for learners.

4.4 Population and sampling

I decided to work with learners from a high school in a peri-urban area (township), north of Durban, due to convenience (purposive sampling). Eight learners, of ages 16-21, were interviewed from a mathematics Grade Eleven class.

I randomly selected eight participants, from a class which studied Mathematics and Physical

Science/Accounting. They were selected from a group of 36 learners in 2014. The learners had been taught proofs in Grade Ten, using the paper and pencil method. They had written one short test during the first term (quarter), half-yearly examination, third term quarterly test and the final examination of the year 2013. At the time of the research project, they had written class tests and the March Quarterly Test in 2014.

4.5 The pilot study

The pilot study increased the validity of the research design, paradigm, and methodology and refined collection methods which I had chosen for the main study.

It also afforded the learners an opportunity to construct their own knowledge in modelling proofs. The ontology associated with this approach is a subjective reality (Falconer & Mackay, 1999). Subsequently, the epistemology is one where the values of the participants as well as the researcher have, become interlaced. Hence the methodologies that lend themselves to subjectivity and interpretation such as individual face-to-face interviews which were employed in the pilot study (Terre Blanche & Durrheim, 1999).

Through conducting the pilot study, employing action research enquiry, I realised I could triangulate within a single data collection instrument (Bertram, Christiansen & Land, 2010). I then learnt I needed to ask the same question in more than one way as this would enhance validity in data collection of the main study.

4.6 The main study

The current study attempted to explore Grade Eleven learners' views on using *Geometer's Sketchpad* for proofs in Euclidean Geometry. The study employed a qualitative action research embedded within an interpretive paradigm. In an attempt to explore Grade Eleven Mathematics learners' views, I gave them a task where they were required to prove that the angle at the centre theorem using the paper and pencil method. Thereafter I discussed their challenges with them. Eventually, I learnt from their mistakes, misconceptions and discussion of their challenges, how to change my teaching strategies. Moreover, I evaluated through studying the consequences of exploring the angle at the centre theorem.

After the learners had proved the angle at the centre theorem using the paper and pencil method, I gave them a task-based experiment where they were required to prove the same theorem using the *Geometer's Sketchpad Program.* They were not told to prove this theorem but given an investigation which was supposed to lead them to the proof of this theorem. They were supposed to measure the size (s) of sides and angles and then formulate conjectures which showed relationships between sides and angles after having measured them.

Most importantly for an action research enquiry, I tried to reflect before conducting the study, during the process of conducting the study and afterwards.

4.7 Researcher's philosophical position in the study

The current study was embedded in the action research inquiry. As action research is collaborative and participatory, I was a participant in this teaching/learning situation. Cohen, Manion and Morrison (2011) defined action research as a self-reflective enquiry which is conducted by participants with the aim of improving their own practices (participatory) and being involved in improving them (collaborative).

I tried by to be objective but, contradictorily, I was a learner as well, and an observer who was subjective as all data generated depended on what was said by the participants. This study employed a qualitative methodology; hence in-depth verbal and textual data were generated. I was a participant and also a learner, who sought to understand the nature of reality. This was evident in the lens with which I viewed the world, through change, improvement and development, which sought to understand deeply Grade Eleven learners' views on using *Geometer's Sketchpad* for proofs in Euclidean Geometry. I also intended to improve participants' and my own understanding of proofs as a means of verification and explanation by using a Dynamic Geometry Software called *Geometer's Sketchpad*. I was a learner and also a facilitator, guiding supporting and scaffolding the learners, while allowing them to explore, discover and formulate conjectures on their own.

Data interpreted were generated from learners' social experiences. I intended to explore how Grade Eleven mathematics learners would construct their own knowledge and mathematics experiences through an experimental process using *Geometer's Sketchpad.* I made use of the habits of mind of seeking and using relationships Du Plessis (2013, p. 43) using *Geometer's Sketchpad*. This Geometry Habits of the Mind (GHOM) seeks to explore relationships between and within geometric shapes (Du Plessis, 2013, p. 44). I allowed the participants to find relationships between angles in the triangles within the circle. Those are relationships between angles opposite equal sides, radii in this case. Consequently, relationships between the angle subtended by the arc at the centre of the circle and the angle on the circumference subtended by the same arc. This GHOM enhanced the learners' and my understanding of the aspects of the angle at the centre proof of the theorem.

I guided, supported and motivated the participants, thereby empowering them through letting them be actively engaged in a problem-based task using the *Geometer's Sketchpad.* Vygotsky (1978) stated that when scaffolding, a teacher supports the learner by arranging a task such that it can be done successfully by the learner. Though the task had been carefully designed such that it guided the participants, I discussed it with them (the participants) before they started investigating. This was to ensure that they understood what they were required to do more especially because they were using *Geometer's Sketchpad* for the first time. I explained how to use the programme and demonstrated key processes. I also encouraged them to be actively engaged in problem solving. I sought to enable the participants to link what they already knew with what they did not know.

Therefore, my philosophical school of thought was the philosophy of Social Construction epistemology of mathematics.

4.8 Research instruments

The current study used two forms of instruments to generate data. These were an observation schedule and a semi-structured interview schedule. Concerning the observations, I observed participants'

- attitude towards proofs (positive or negative?)
- attitude towards using *Geometer's Sketchpad* as a gadget (positive or negative?)
- behaviour (comfortable or uncomfortable)
- competency
- confidence (using Geometer's Sketchpad with ease)

This would help to determine whether the participants were coping or not coping with proving the angle at the centre theorem. Observation is regarded as first-hand data as I see for myself the context and site of the study (Bertram, Christiansen & Land, 2010). Furthermore, I report on what I have witnessed and recorded myself as opposed to what I read from other studies.

Finally, for all the participants selected for the study face-to-face, individual interviews were conducted where use was made of an open-ended, semi-structured interview schedule. All participants were listened to, and all their responses were audio recorded. In instances where more clarity was required, I probed using additional, relevant questions (Patton, 1990). Asking participants questions directly, seemed to help me find responses immediately. This is aligned with Tuckman's (1994, p.372) assertion that "one direct way to find out a phenomenon is to ask questions from the people who are involved in the study in some way". I also probed during interviews to obtain more in-depth textual data. To obtain even deeper data, I established a rapport with the participants.

4.9 Data collection procedures

The current study employed qualitative research. Qualitative research is descriptive and the data collected was in the form of words rather than numbers (McMillan & Schumacher, 2006). The current study made use of observation and interviews as data generation tools. The participants were observed while they were engaged in a problem-based task (an investigation) and the observation schedule was filled in. A template of the observation schedule is attached as Appendix F, at the end of the study.

I compiled open-ended questions with the aim of allowing participants to respond to them. After compiling the questions, I conducted a pilot study to ensure the validity and reliability of each instrument. I ensured that the procedures to generate data were the same to all participants during the pilot study. Moreover, I took special note of any signals suggesting that participants were not comfortable or did not understand the questions. In addition, I evaluated the questions for clarity and intention (McMillan & Schumacher, 2006). An interview schedule relating to conducting the proof of the angle at the centre proof and to research questions, is attached at the end of the study as Appendix G.

4.10 Research ethics

I wrote detailed information in the informed consent letters to learners and parents/legal guardians, in the language they understood, requesting that the learners consider participating in the research study. These letters described the research process and discussed the aim of the research project. The research process requires that participation be voluntary. It (the research) needs to cause no harm or victimisation. In addition, it allows participants to withdraw at any time without any victimisation. It also requires that the participants' identity be protected. Anonymity and confidentiality would be guaranteed by using pseudonyms instead of participants' real names to protect their identity (Rand Afrikaans University, 2002). The learners' institution's real name would not be used; instead a pseudonym would be used.

I also ensured that the data generated would be kept confidential and safely locked away. In the letters, I also explained that the data generated would be destroyed after five years by shredding the documents. All data that were audio recorded would be burned into a compact disc (CD). The CD with that data would be incinerated after five years. Consent was obtained from the participants. I also explained that the data generated, would be used solely for the research study.

I ensured throughout the research process, that the rights of the learners being studied were not compromised in any manner. Bertram, Christiansen and Land (2010, p. 50) emphasised that all research studies follow ethical principles which are:

1. Autonomy: the researcher needs to obtain consent of every person to be part of the study, participation is voluntary and participants have freedom to withdraw at any time.

2. Nonmaleficence: the research should do no physical, emotional or social harm to the research participants or to any other person.

3. Beneficence: the study should be of benefit directly to participants or more broadly to other researchers or the society at large. Anonymity and confidentiality will be guaranteed by using pseudonyms instead of participants' real names to protect their identity.

I wrote a letter to the Department of Education, seeking permission to conduct the research at the specific institution. Another letter was written to the Grade Eleven mathematics teacher to seek permission to use learners as participants in the study. I wrote a letter to the institution's principal seeking permission to conduct a study in the institution. In all these letters, I explained all the ethical issues. All the gatekeepers' letters are attached as Appendix B at the end of the study including the ethical clearance certificate. The ethical clearance certificate is attached as

Appendix A.

4.11 Trustworthiness/Credibility/Validity/Reliability

This study employed trustworthiness as it was embedded in action research enquiry. Cohen, Manion and Morrison (2011) maintained that validity of data in action research enquiry is referred to as trustworthiness. The bases for validity in qualitative research are credibility, transferability, trustworthiness and dependability (Cohen et al, 2011). Validity is not always clearly measurable in a qualitative research; instead trustworthiness serves as a better term to describe the authenticity, validity and quality of the research.

The questions in the interview schedule attached at the end of the study as Appendix G, were formulated such that they attempt to draw rich and in-depth textual information from the participants. Moreover, they were neither stressful nor degrading. To ensure these, they were checked by an expert to see if they were appropriate and adequate to obtain rich data from participants without embarrassing them. The expert, who checked if my interview questions were appropriate, was my mentor who recently graduated for a Doctoral degree in Mathematics Education.

When discussing internal validity in qualitative research, Cohen, Manion and Morrison (2011, p. 185) stated that it requires attention to

- Plausibility,
- The kinds and amounts of evidence required and
- Clarity on the kinds of claims made from the research.

Furthermore, credibility in naturalistic enquiry can be addressed by persistent observation, triangulation and, prolonged engagement in the field, amongst others. The present study made use of persistent observation to ensure trustworthiness. To prevent bias, the study made use of two data generating tools, an observation schedule and a semi-structured interview schedule. This provided triangulation.

I minimised threats to trustworthiness as early as possible in the design stage. This was done by choosing an appropriate time-line. Since there were inadequate resources (two laptops), more time was scheduled for the study. Data generation took four months. A representative sample of eight participants was selected. Appropriate foci to answer research questions were selected, to ensure trustworthiness. I attempted to avoid the withdrawal of the participants from the study by establishing rapport with them.

I repeatedly asked myself three questions to ensure reliability. Cohen, Manion and Morrison (2011, p. 203) mentioned these questions as:

- Would the same observations and interpretations have been made if observations had been conducted at different times?
- Would the same observations and interpretations have been made if other observations had been conducted at the time?
- Would another observer, working within the same theoretical framework, have made the same observations and interpretations?

4.12 Conclusion

This chapter outlined the main aspects of the methodology used in the current study. In addition, the research design and how data were generated, were discussed. Data were generated from observations, the task-based worksheet and semi-structured interviews. The data analysis method was also described in this chapter although the data generated will be analysed in Chapter Five.

The next chapter presents the data analysis of the current study. It discusses the research questions, the data generation plan, data analysis and the themes that emerged.

CHAPTER FIVE

5. Data Analysis

5.1 Introduction

In chapter four, the research methodology of the current study has been discussed. The chapter discussed the research approach, population and sampling, the pilot and main studies, my philosophical position in the study, the research instruments I used, the data collection procedures, research ethics and trustworthiness. In this chapter, Grade Eleven learners' views on conducting proofs using *Geometer's Sketchpad* are clarified. The learners were given a task in which they were asked to prove the angle at the centre theorem. To help them conduct it appropriately, I started with a revision lesson where I discussed geometry basics and demonstrated the theorem they were to prove. Then I observed them as they attempted to prove it using the paper and pencil method and afterwards, they proved the theorem with the help of the Dynamic Geometry Software, *Geometer's Sketchpad.*

This chapter presents the data analysis of the current study. It discusses the research questions, the data generation plan in the form of a table, data analysis and the themes that emerged.

5.2 Presentation of data

The results of the data analysis for the two research questions are presented in this section. The two research questions were:

1. What are Grade Eleven learners' views about the use of *Geometer's Sketchpad* for the teaching of proofs in Euclidean Geometry?
2. How can *Geometer's Sketchpad* be used to teach proofs in Euclidean Geometry to Grade Eleven mathematics learners?

Participants' performance on the experiment they conducted is analysed in this chapter. Two tables are used to show the analyses explicitly. The task-based experiment, observation and semi-structured, face-to-face and individual interviews were used to attempt to answer the two research questions in order to generate data. Table 1 that follows encapsulates how data were generated from each of the eight participants and for each research question.

Data generation plan

Table 1 illustrates the processes that were followed to generate data from the participants through research questions.

Table 1

Research questions	Participants	Data generation method
1. What are Grade Eleven learners' perceptions about the use of *Geometer's Sketchpad* in the teaching of proofs?	Lettie (L); Asanda (A); Jabu (J); Ridge (R); Sonke (S); Thula (T); Ntsiki (N); Gabi (G).	Semi-structured individual face-to- face interviews
2. How can *Geometer's Sketchpad* be used to teach proofs to Grade Eleven mathematics learners?		Task-based worksheet. Semi-structured individual face-to-face interviews.

The current study focused on exploring Grade Eleven mathematics learners' views on using *Geometer's Sketchpad* for proofs in Euclidean Geometry. Data were generated from eight Grade Eleven mathematics learners. These learners conducted an experiment and their performance on task-based experiment is discussed. Two research questions were taken into cognisance whilst the questions in the task-based experiment were designed. The learners' performances on the task were assessed and compared, though no marks were awarded. The results of the experiment which was in the form of an investigation, showed that all eight learners were able to conduct the proof using *Geometer's Sketchpad.* Refer to Figure 9 on this page illustrating TO3.3's work on *Geometer's Sketchpad*. TO3.3 refers to Thula's observation question 3.3. In this question, learners were asked to determine firstly, how angles *BCE* and *BAC* are related. Secondly they were required to explore and state the relationship between angles *ECD* and *DAC*. Thirdly and lastly, learners were expected to explore and explain how angles *BCD* and *BAD* are related.

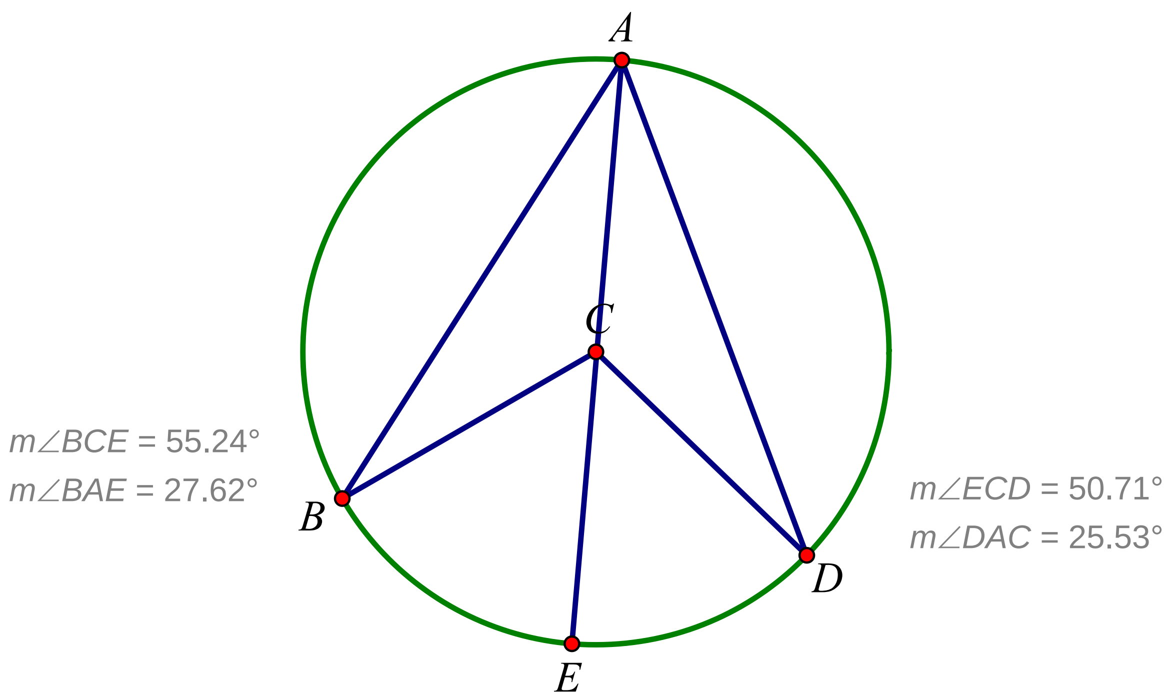
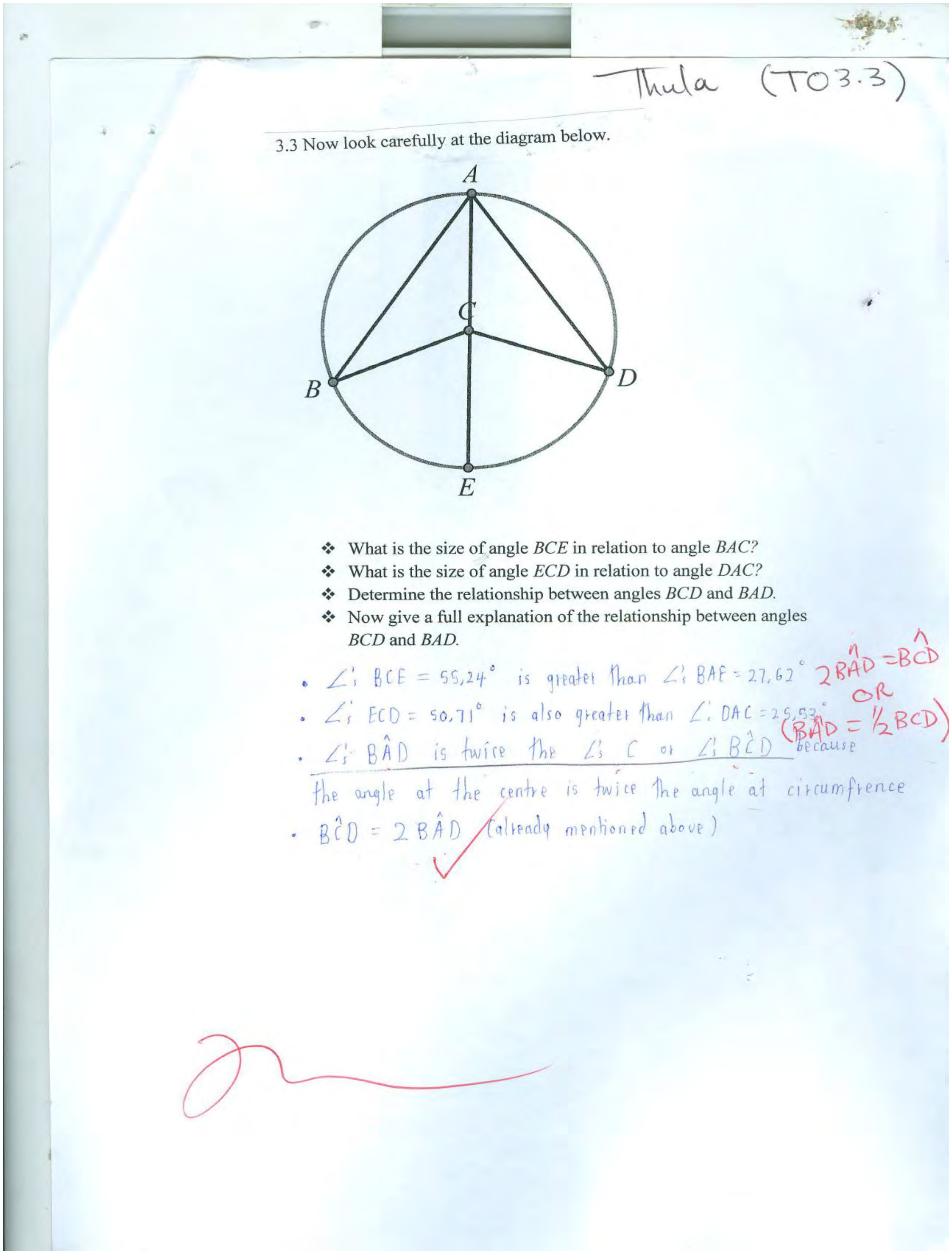


Figure 9: A diagram showing TO3.3's work on *Geometer's Sketchpad*

Each of the participants was required to complete a worksheet. After measuring the sizes of angles *BCE* and *BAC, ECD* and *DAC, BCD* and *BAD* in the worksheet, the participants were required to determine the relationship between the angles, respectively. Lastly, the participants explained the relationship between angles *BCD* and *BAD.*

TO3.3's completed worksheet is shown on page 72 Figure 10. Each participant was required to record measurements in a worksheet after using *Geometer's Sketchpad* to measure sizes of angles. Refer to Figure 10.



Thula (TO3.3)

3.3 Now look carefully at the diagram below.

A

C

B D

E

- What is the size of angle *BCE* in relation to angle *BAC?*
- What is the size of angle *ECD* in relation to angle *DAC?*
- Determine the relationship between angles *BCD* and *BAD.*
- Now give a full explanation of the relationship between angles *BCD* and *BAD.*

- ∠'s BCE = 55,24° is greater than ∠'s BAE = 27,62°
- ∠'s ECD = 50,71° is also greater than ∠'s DAC = 25,53°
- ∠'s $B\hat{A}D$ is twice the ∠'s C or ∠'s $B\hat{C}D$ because the angle at the centre is twice the angle at circumfrence
- $B\hat{C}D = 2B\hat{A}D$ (already mentioned above)

$2B\hat{A}D = B\hat{C}D$
OR
$(B\hat{A}D = \frac{1}{2}BCD)$

Figure 10: A diagram showing TO3.3's completed worksheet

NO3.3's work on *Geometer's Sketchpad* is illustrated. NO3.3 refers to Ntsiki's observed response on question 3.3. Refer to Figure 11.

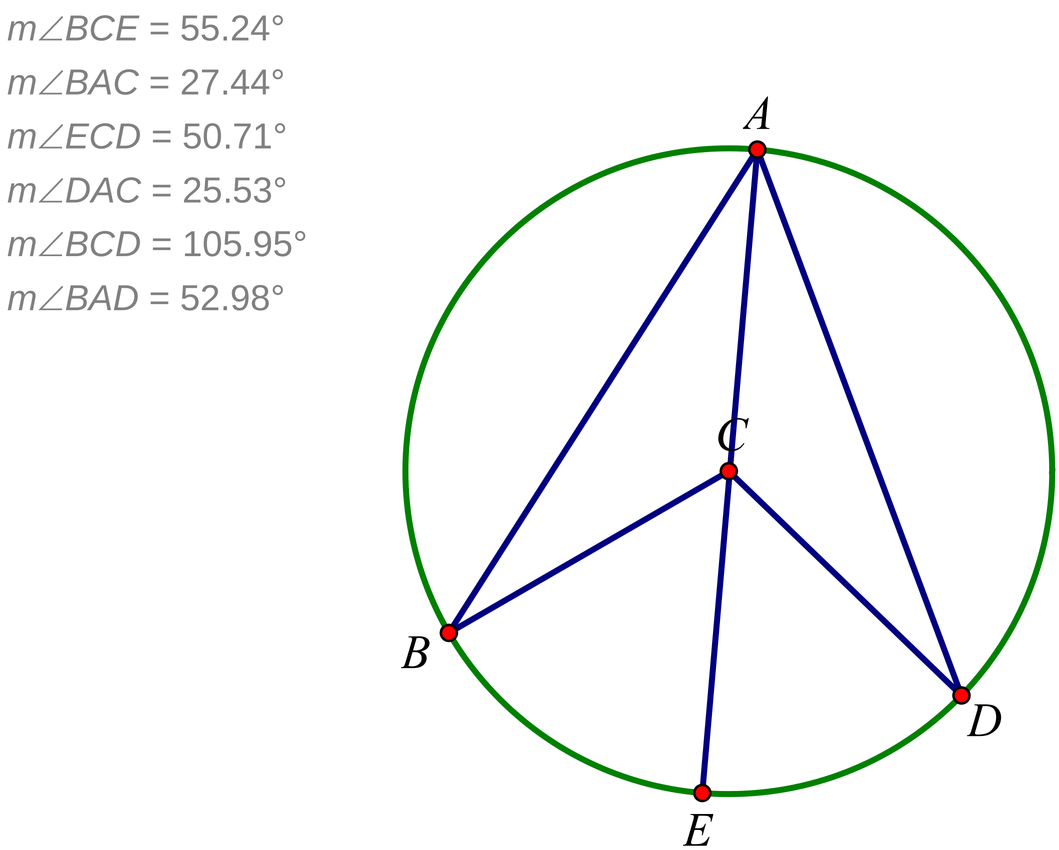
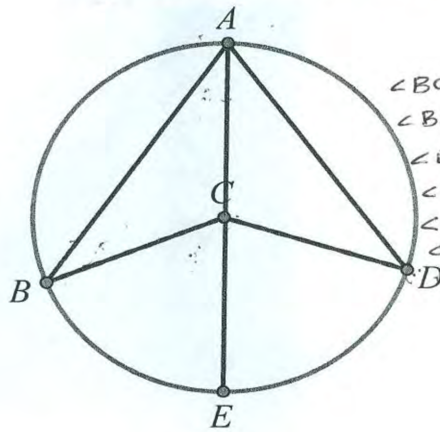


Figure 11: A diagram showing NO3.3's work on *Geometer's Sketchpad*

NO3.3's completed worksheet is shown. She measured the sizes of angles using *Geometer's Sketchpad* and found relationships as required in the worksheet. Refer to Figure 12.

NTSIKI (NO3.3)

3.3 Now look carefully at the diagram below.



∠BCE = 55,24°
∠BAC = 27,44°
∠ECD = 50,71°
∠DAC = 25,53°
∠BCD = 105,95°
∠BAD = 52,98

- What is the size of angle *BCE* in relation to angle *BAC?*
- What is the size of angle *ECD* in relation to angle *DAC?*
- Determine the relationship between angles *BCD* and *BAD.*
- Now give a full explanation of the relationship between angles *BCD* and *BAD.*

* THE ANGLE BCE IN RELATION TO ANGLE BAC IS GREATER.
∠ BCE = 2 BÂC

* THE ANGLE SIZE ECD IN RELATION TO SIZE ANGLE DAC IS
IS GREATER THAN OF DAC.
∠ECD = 2 DÂC

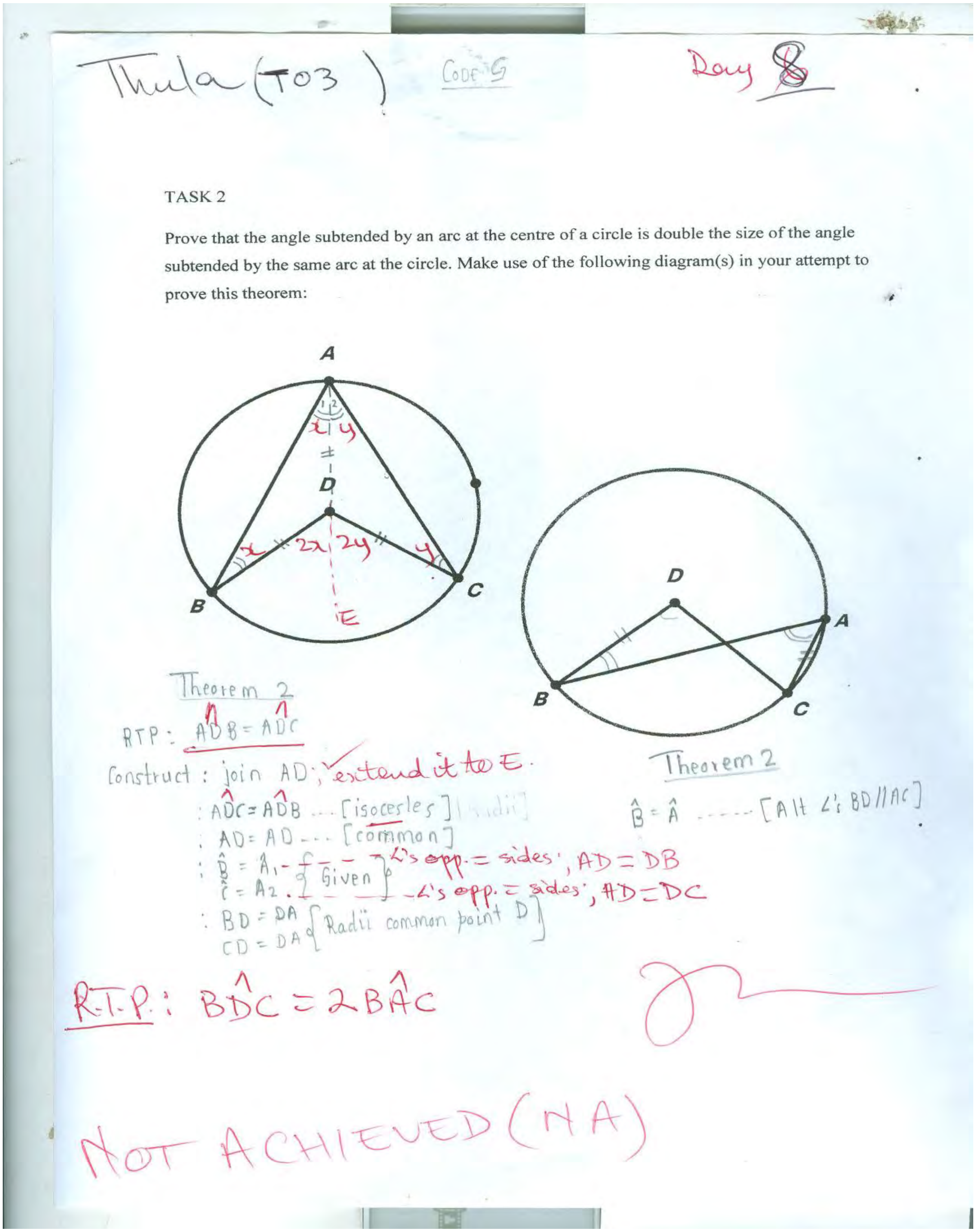
* THE RELATIONSHIP BETWEEN ANGLES BCD AND BAD
THE BC = CD , BA = AC THE RELATIONSHIP THEY
SHARE IS THAT THIS ARE THE RADII WHICH MEANS
∠BCD = 2∠BAD

* THE ANGLE AT THE CENTRE IS EQUAL TO THE TWICE ANGLE
AT THE CIRCUMFERENCE (WHICH LEADS TO BC = CD RADII
AND AC = CD RADII ANGLE THERE ARE MADE BY
RADII AND SIDES EQUAL TO EACH OTHER.)

Figure12: A diagram showing NO3.3's completed worksheet

The proof was based on the angle at the centre theorem. It seems important to note that these learners could not prove this theorem appropriately while using the paper and pencil method. This happened despite the different teaching strategies that I used to demonstrate the proof. Fifty percent of these learners could not prove this theorem appropriately even after extensive revision, remedial work, application of the theorem, and conducting the same proof three times. Proofs conducted by learners, TO3 and NO3 are shown. The proof was conducted for the third time using traditional method by these learners. Refer to Figure 13 and Figure 14.

TO3's traditional proof is illustrated in Figure 13.


Thula (TO3) Code 5 Day 8

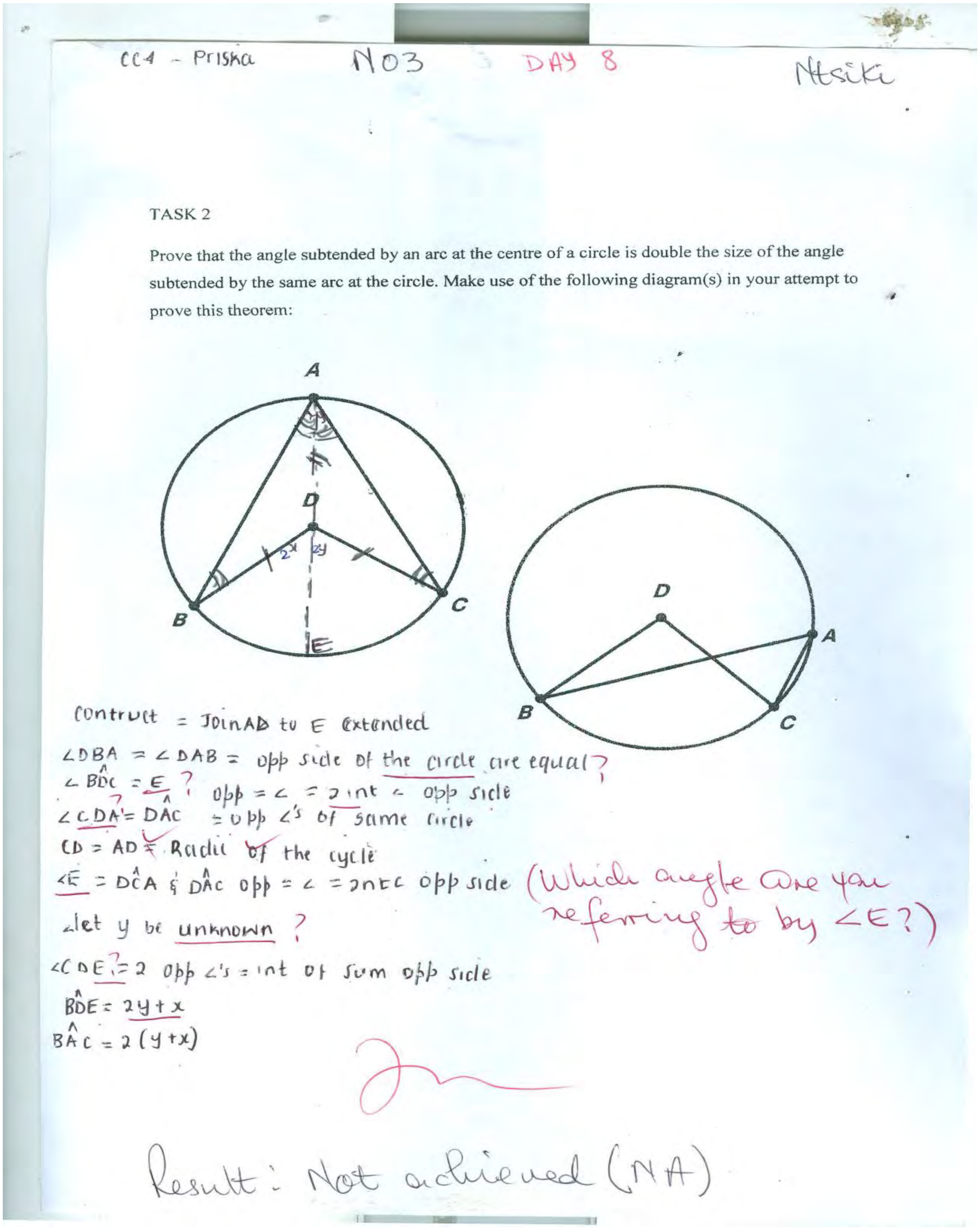
TASK 2

Prove that the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle. Make use of the following diagram(s) in your attempt to prove this theorem:

Figure 13: A diagram showing TO3's traditional proof

of the theorem

NO3's traditional proof using the paper and pencil method is illustrated in Figure 14.



CC4 - Priska NO3 DAY 8 Ntsiki

TASK 2

Prove that the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle. Make use of the following diagram(s) in your attempt to prove this theorem:

Contruct = Join AD to E extended

∠DBA = ∠DAB = opp side of the circle are equal ?

∠ BD̂C = E ? opp = ∠ = 2 int ∠ opp side

∠ CDA = DÂC = opp ∠'s of same circle

CD = AD = Radii of the cycle

∠E = DĈA & DÂC opp = ∠ = 2 int ∠ opp side (Which angle are you referring to by ∠E?)

let y be unknown ?

∠CDE = 2 opp ∠'s = int of sum opp side

BD̂E = 2y + x

BÂC = 2 (y + x)

Result: Not achieved (NA)

Figure 14: A diagram showing NO3's traditional proof

Based on the data presented, it is evident that *Geometer's Sketchpad* enhanced the learners' understanding of proofs. This is attributed to *Geometer's Sketchpad's* built-in tools which were used by the learners to measure and hence deduce relationships of the angles. Consequently, this led to the proof of the angle at the centre theorem.

The task-based experiment required that the learners prove that the angle at the centre of the circle equals double the size of the angle subtended by the same arc on the circumference. The investigation was designed such that it gradually led the learners to this proof, making use of axioms and theorems learnt in previous grades. Table 2 illustrates what the learners were required to investigate.

Table 2: Learners' correct responses expressed as a percentage

Table 2 illustrates the learners' correct responses on each question that was asked. 100% is a percentage which shows that participants' responses were all correct. While the learners were proving the angle at the centre theorem using the paper and pencil method, their baseline performance was, as follows: In Question 1; correct responses were 50%, in Question 2.1; 52%, in Question 2.2; 60%, in Question 3.1; 51%, in Question 3.2; 50% and in Question 3.3, correct responses were 50%. Some named angles using two letters and some discussed angles which did not exist. While justifying their arguments, the learners provided inappropriate reasons.

Question	**Task - based questions**	**Correct responses as** a **%**
1	▪ Determining the relationship between radii of the same circle and angles opposite the radii.	100%
2.1	▪ Exploring the relationship between angles opposite equal sides (Grade 8)	100%
2.2	▪ Investigating the relationship among	100%

	the three interior angles of a triangle by determining their sum. (Grade 8).	
3.1	▪ Determining the relationship of the exterior angle of a triangle and the two interior opposite angles of a triangle (Grade 8) ▪ the relationship between the angle subtended by the arc at the centre of the circle and the angle subtended by the same arc on the circumference (Grade 11).	100% 100%
3.2	▪ Investigating the relationship between angles opposite equal sides in a triangle (Grade 8) ▪ the exterior angle of a triangle and the two interior opposite angles of a triangle (Grade 8)	100% 100%

3.3	▪ Exploring the relationship between the angle subtended by the arc at the centre of the circle and the angle subtended by the same arc on the circumference (Grade 11) ▪ Explanation of the relationship of the angle subtended by the arc at the centre of the circle and angle subtended by the same arc on the circumference.	100% 100%

Chapter four of the current study clarified that pseudonyms were used to protect the learners' identity. I observed the learners while they were responding to the questions on the task. A copy of the observation schedule is attached as Appendix F at the end of the current study. I assessed the task but awarded no marks. A copy of the task-based worksheet is attached as Appendix E at the end of the current study. After observing learners' responses when completing the worksheet, I noticed some learners' responses were common.

I interviewed the learners after having observed them while they conducted the experiment. The interview which comprised four questions was designed as a semi-structured, face-to-face and individual interview. I had planned to interview one learner per day. Due to favourable circumstances, I interviewed two learners on Day One and three learners on Day Two. Then I interviewed one learner per day on three days. The interview schedule is attached as Appendix G at the end of the current study. The table explaining the coding used in the current study is on pages 10-11 of the current study. Examples: NO3.3 refers to learner recorded as Ntsiki observation question 3.3. LI1 is a code representing learner recorded as Lettie interview question1.

Some learners' interview responses were common as will be observed in the discussions in the next paragraphs.

Interview questions

The interview comprised four questions. The first question focused on how learners felt about using *Geometer's Sketchpad.* The second one focused on why the learners felt the way they did. The third question aimed at finding out if the learners encountered any problems while using *Geometer's Sketchpad.* The fourth question attempted to extract learners' perceptions about the use of the *Geometer's Sketchpad Program.*

Question 1: How do you feel after having done this task, using *Geometer's Sketchpad?*

When asked how they felt after using *Geometer's Sketchpad,* all eight learners exhibited emotions of happiness and excitement. They all displayed feeling good about using *Geometer's Sketchpad.* I actually observed that they all looked excited and interested while conducting the experiment they were given, although they seemed tense when they initially started. This was shown clearly by the reasons they provided when responding to the next question.

JI1 articulated, "*I feel great*. *Sketchpad encourage me to do maths*, *Euclidean Geometry*". JI1 is a code representing learner recorded as Jabu interview question 1. TI1 confidently stated, **"***I feel very excited and I enjoyed the task*. *It inspired me in such a way that it encourages me to want to prove more theorems using the Sketchpad".* This proved that motivation is essential for learning and *Geometer's Sketchpad* does not only help learning, but it also instills motivation. Incorporating the use of *Geometer's Sketchpad* in the teaching of proofs in Euclidean Geometry instills (Kilpatrick's, 2001) productive disposition, the love of mathematics for what it is worth, in learners. LI1 justified her feeling good by saying, "*I feel good and I believe that what I have done is true. When you doing theory, you don't have proof. Now I have done practicals which approve of the theory. Yes, Sketchpad made me sure of what I have done is the truth".* Mudaly *(*1998*)* found that learners had developed very high levels of conviction after having used *Geometer's Sketchpad.*

When NI1 was asked how she felt after having used *Geometer's Sketchpad,* she excitedly responded, "*Happy*". AI1 responded thus, "*Excited*, *never done it before*. *I've gained a lot of experience*". RI1 excitedly shouted, "*I feel very happy*". SI1 screamed, "*I feel excited*!" Lastly, GI1 confidently stated, "*Well*, *I feel good even though not used to it*".

Question 2: What makes you feel that way?

It was interesting to observe that all eight learners displayed a positive attitude towards the use of *Geometer's Sketchpad.* This was deduced from the fact that all eight learners concentrated on the task and that they were all engaged in problem solving. When asked why they felt excited about the use of *Geometer's Sketchpad,* learners gave different reasons some of which were similar.

JI2 and NI2 responded that they were happy because it is easy to use *Geometer's Sketchpad.* When NI2 was asked why she felt happy after using *Geometer's Sketchpad,* she excitedly responded, "*Because having done the task, using Sketchpad I understand that when using a Sketchpad, it is easy to measure and find angles easier than to prove in an old way, you personally calculate lengths, size of angles which sometimes you don't get answers right when calculating yourself and other sometimes you got wrong answers*". JI2, when asked why she felt great, she responded, "*Challenge not hard*, *measuring is easy*". When probing, I found that learner JI2 meant that she found it easy to do the investigation as it is easy to measure lengths and sizes of angles using *Geometer's Sketchpad*. AI2 highlighted that she had never used *Geometer's Sketchpad* before but she gained a lot of experience. She added that she was happy because *Geometer's Sketchpad* would help her in her studies in class. AI2 stated, "*Happy because Sketchpad helped me with my studies in class*". The use of *Geometer's Sketchpad* made learners more interested in conducting proofs. This is consistent with Idris (2009) who claimed that the use of *Geometer's Sketchpad* increases learners' interest in geometry and enhances understanding.

GI2 stated, "*It makes me work harder. I like challenges because they make me work harder to prove myself that nothing can beat me*". Learner recorded as Gabi interview question 2, GI2 justified her happiness by stating that she likes challenges and using *Geometer's Sketchpad* makes her work harder. To her, using *Geometer's Sketchpad* appeared a challenge due to the fact that she had never used it before. Otherwise she did manage to use it. The use of *Geometer's Sketchpad* encouraged her to work harder not only in mathematics but also in class.

RI2, SI2 and TI2 stated that the use of *Geometer's Sketchpad* is practical. RI2 clarified, "*Sketchpad is fast, needs no deep thinking and it's practical*". SI2 responded, "*Doing the task, it's like I have expand the horizon of learning because I enjoy practical work. Besides that, I have add my knowledge in Euclidean Geometry*". TI2, referring to Thula's response to interview Question 2, stated, "*It is because it is more easier to prove the theorems practically than doing it on your own on a paper*".

I was more interested when LI2, learner recorded as Lettie interview question 2 stated, "*When doing theory, you don't have proof. Now I have done practicals which approve of the theory*". After probing, learner LI2 responded that she meant to say practicals prove the theorem which is in the form of a theory which she heard from her teacher. This links to one of the functions of proof, verification/justification. Mudaly (1998) highlighted that verification is the function of proof which is concerned with the truth of a statement or a proposition. Lettie had conducted an empirical proof herself which verifies that the angle subtended by the arc at the centre of the circle is double the size of the angle subtended by the same arc on the circumference. What she actually meant here was that, when one conducts a mathematics proof in Euclidean Geometry using the paper and pencil method, one has no empirical proof.

Question 3: Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*? Explain.

When all eight learners were asked if they encountered any problems while conducting the experiment using *Geometer's Sketchpad*, they responded differently. Three learners, TI3, LI3 and RI3 mentioned that they did not experience any problems whatsoever. TI3 stated, "*No, I did not experience any problems using the Sketchpad because I was measuring sides and angles and finding relationships between sides and angles. But I had problems when I proved on my own on a paper the angles at the centre theorem*". LI3 stated, "*No problems*". RI3 responded thus, "*Didn't find any problems cause it was practical. I measure angles and sides and concluded from there*".

Five learners, SI3, GI3, NI3, JI3 and AI3 stated that they did encounter some minimal problems. SI3 reiterated, "*Yes*, *when measuring the angles I have discover my carelessness*. *The laptop did not measure some angles the way they are asked*. *I had to start on the other end*, e.g. *measuring <BCD as <D*". GI3 stated, "*Well*, *for me it was a bit harder*, *didn't know how to measure*, '*cause at school we don't use Sketchpad*". NI3 responded, "*Yes*, *but not a serious problem*, *it was a minor problem where I could not be able to find the length of CA*. *Then I got help by measuring it distance between two points*, *then the problem was solved*. *Because as I was measuring CA, couldn't give me the length because it highlighted the whole line from C to E. That made me learn that there difference between length and distance in the number 3.1*". NI3 was referring to Question 3.1 in the experiment the learners conducted on *Geometer's Sketchpad*. NI3 had encountered a problem while measuring the length of *CA*. When she was attempting to measure it, line segment *ACE* was highlighted on *Geometer's Sketchpad.* She only managed to measure the length of *CA* by opting for the distance between points *C and A* to be able to get the length of *CA*. She did this by clicking on point *A* and point *C*. The diagram that follows shows the diagram NI3, learner recorded as Ntsiki interview question 3, was referring to. Refer to Figure 15.

3.1 In the next diagram, which is Figure 15, measure the following:

- The length of *CA*
- The length of *CB*
- The size of angle *BAE* and of angle *ABC*
- The size of angle *BCD*
- What do you notice about the size of angle *BCD?*

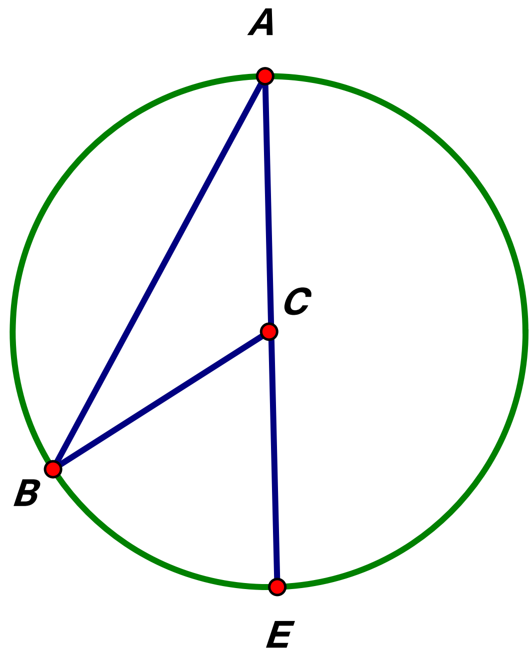


Figure 15: A diagram given to learners to measure

lengths and angles using *Geometer's Sketchpad*

NI3 explained that she encountered a problem when she tried to measure the length of *CA* in Task 3 Question 3.1. She ended up measuring the distance between *C* and *A* to find the length of *CA*. Consequently, she was happy to learn of the difference between length and distance. RI3 and NI3 seemed to have encountered no problems but to have gained experience.

JI3 stated, "*Yes*, *not knowing how to measure*, *but ended up knowing*". AI3 reiterated, "*Failed to delete*, *measuring lengths*. *Laptop was a challenge*. *Didn't know how to measure though at last I did it at last*". These learners seemed to have encountered minimal problems as they soon were competent with the use of *Geometer's Sketchpad.*

It occurred to me that the naming of angles in different ways seems important. This manifested itself in the problem SI3 experienced. SI3, in his response, stated that *Geometer's Sketchpad* measuring tool, did not measure some angles the way they were asked, for some strange reason. As a result, he had to measure the same angle starting from the other end. He gave the example of where he had been asked to measure <*BCD* and 'angle' was not highlighted by *Geometer's Sketchpad,* and thus he did not get the size of the angle. Subsequently, he measured the same angle as <*DCB* and he got the measurement right. He encountered that problem in Task 3 Question 3.3, of the experiment. This proved how learners manage to use gadgets automatically without any help, but using their experience in mathematics.

Question 4: Did the use of *Sketchpad* help you*?* How did it help you, if it did*?*

All eight learners claimed to have been helped by the use of *Geometer's Sketchpad.* Five learners, SI4, LI4, NI4, AI4 and JI4 stated in their responses that *Geometer's Sketchpad* helped them prove the angle at the centre theorem. SI4, learner recorded as Sonke interview question 4 appreciated that he managed to work on his own while using *Geometer's Sketchpad.* He stated, "*Yes*, *because I managed to do the task I was given*. *I measured sides and angles and found the relationship between angles*. *It also help me to prove angle centre theorem*". LI4 reiterated, "*Yes*, *Sketchpad made me sure of what I have done is the truth*. *Sketchpad has proved that the theory I have learnt is really true*. *Sketchpad has helped me understand maths better*. *Sketchpad has helped me prove the angle at the centre theorem*". This means it was verified to her through empirical proof that the angle subtended by the arc at the centre of the circle is double the size of the angle subtended by the same arc on the circumference.

NI4 stated, "*Yes*, *because if I was proving using old method*, *when you prove yourself by referring to what I've been told*, *or just looking at angle*, *it would have took me time to arrive at an answer for me to be able to find the sizes and the length*. *So*, *meaning it is easy using Sketchpad because Sketchpad have tools for measuring. The measuring helped me to get the sizes and length of the angles using the Sketchpad. As I was measuring, I got the relationship* *between the angles that angle at the centre is twice angle at the circumference, <BCD = 2<BAD in the task that I was given. Measuring helped me prove the theorem. Sketchpad is so useful because it's easy to prove using it than using the old method".*

AI4 responded thus, "*Sketchpad made it easy for me. It helped me prove that the angle at the centre is equal to twice the angle, the angle in the circumference*". JI4 explained, "*Yes. Sketchpad helped me find that <BCD = <BAD. To measure sizes of angles and find relationship between angles. Sketchpad is easy to use".*

GI4 and TI4 stated that the use of *Geometer's Sketchpad* helped them understand the angle at the centre theorem better. GI4 stated that she appreciated *Geometer's Sketchpad's* built-in tools. She was happy that she did not have to bring any tools to measure. This denoted the importance of *Geometer's Sketchpad* built-in tools and facilities which draw sketches, delete objects, measure quantities and animate objects. These are learner GI4's direct words, "*Yes, it's did help me. Sketchpad has tools. Don't have to bring any tools. Helped me understand how to measure and find relationships. Helped me understand angle at the centre is twice the angle at the circumference". TI4 also clarified how the use of Geometer's Sketchpad helped him. He* explained, "*Yes, it helped me to understand the angle at the centre better than before and to know some of the terminologies of geometry which I did not know before, like 'subtended by'. Sketchpad is more advance than the method my teacher uses. It is easy to use and understand because the Sketchpad has tools to draw and measure. It can help me pass mathematics because it is practical. I enjoy practical work than theoretical work*". The use of *Geometer's Sketchpad* seemed to have instilled love of mathematics in him and motivated him to pass. RI4 mentioned, "*Sketchpad is easier, fast and visual, don't think deeply".* When I asked him what he meant by *visual,* he confidently responded*, "When you working with something you see*".

5.3 Introduction to themes

The current study is a qualitative study embedded in an action research enquiry. As it is important to triangulate data generated in a qualitative research study, I triangulated by making use of constant observation and semi-structured interviews. This allowed me to gain various perspectives learners have about the use of *Geometer's Sketchpad* to conduct mathematics proofs in Euclidean Geometry.

All the learners who participated in this study claimed that the use of *Geometer's Sketchpad* helped them to prove the angle at the centre theorem. Evidence to this effect is shown by the excerpts that follow. These excerpts were extracted from the learners' face-to-face and individual interview responses. LI4 responded, "… *Sketchpad has helped me prove the angle at the centre theorem"*. AI4 mentioned, "…*Sketchpad made it easy for me*. *It helped me prove that the angle at the centre is equal to twice the angle in the circumference*". SI4's response, "…*It also help me to prove angle centre theorem*". JI4 stated, "…*Sketchpad helped me find that* <*BCD is equal to twice* <*BAD."* NI4 reiterated, "…*As I was measuring*, *I got the relationship between angles that angle at the centre is twice angle at the circumference*, <*BCD is equal to twice* <*BAD in the task that I was given*".

TI4 stated, "…*Yes*, *it helped me to understand the angle at the centre theorem better than before*". GI4 responded thus: "…*Helped me understand angle at the centre is twice the angle at the circumference"*. RI4, learner recorded as Ridge interview question 4 responded differently. He stated, "*Sketchpad is easier*, *fast and visual*, *don't think deeply*". When I asked what he meant by visual, he confidently explained, "*When you working with something you see*".

5.3.1 Themes

The task-based experiment and the interviews helped me identify common themes about the learners' perceptions about conducting mathematics proofs in Euclidean Geometry, using *Geometer's Sketchpad.* Themes that I identified were:

1. *Geometer's Sketchpad* ease of use.
2. *Geometer's Sketchpad* as a motivational tool.
3. *Geometer's Sketchpad* assists in conducting mathematics proofs

The three themes are discussed in the three paragraphs that follow.

5.3.2 *Geometer's Sketchpad* ease of use

The study required that I improve my teaching strategies through the use of an action research enquiry. I had revised all necessary proof tools but no participants managed to conduct the proof appropriately. Proof tools referred to here were angles opposite equal sides in a triangle and the exterior angle of a triangle. I had also considered the conditions involved in the sketches. These conditions stated by du Plessie (2013, p. 44) and referred to as:

- The same chord must subtend both angles
- An angle at the centre, and
- Another angle on the circumference.

My beliefs and my goals affected my teaching of proofs. I believed that a Grade Eleven mathematics learner would have achieved the necessary basics s/he had learnt in previous grades. While assessing task 2, I observed that the participants had failed to recognise the exterior angles in another form. I interpreted that as memorisation of the relationships of angles, not understanding. Additionally, I learnt how vital it is that the learners are exposed to diagrams in different forms. This links with transformation of diagrams in mathematics.

It was only after the learners had proved the angle at the centre theorem using *Geometer's Sketchpad* that they conducted the proof appropriately. *Geometer's Sketchpad's* built-in tools helped learners measure the lengths of sides of triangles and sizes of angles accurately. Then they formulated conjectures from the measurements. Leong (2003) stated that the 'measure' menu allows learners to measure lengths, angles, *et cetera.* The 'measure' menu also enables learners to perform computations. These measures help in the observations for relationships. *Geometer's Sketchpad* built-in features made conducting of mathematics proofs easier than conducting proofs using the paper and pencil method. These features help learners measure and find relationships between and within figures and to make conjectures. They thus developed geometrical ideas which led to appropriate proof.

5.3.3 *Geometer's Sketchpad* as a motivational tool

Geometer's Sketchpad proved to be a motivational tool to learners. Learners who had challenges with conducting mathematics proofs in Euclidean Geometry were motivated extrinsically by the use of *Geometer's Sketchpad.* Hulme (2012) highlighted that the use of enthusiasm for geometric exploration. Following are excerpts extracted from learners' interviews' responses, which serve as evidence that *Geometer's Sketchpad* proved to be a motivational tool. AI2 stated, "…*Happy because Sketchpad helped me with my studies in class*". GI2 responded thus, "…*It's because it makes me work harder*". JI1 confidently stated,

"…*Sketchpad encourage me to do Maths*, *Euclidean Geometry*". TI1 reiterated, "**…***It inspired me in such a way that it encourages me to want to prove more theorems using the Sketchpad*".

When the learners were asked what made them feel happy, AI2 and GI2 responded thus, respectively, "…*Happy because Sketchpad helped me with my studies in class*" and "…*It's because it makes me work harder*". The learners were also asked if the use of *Geometer's Sketchpad* helped them. They were also expected to explain how the software helped them, if it did. GI4, learner recorded as Gabi interview question 4 responded with contentment, "*…Eh….Sketchpad helped me love maths better than before*".

5.3.4 *Geometer's Sketchpad* assists in conducting mathematics proofs

As discussed previously, learners could not conduct the proof appropriately when they were using the paper and pencil method. It was only after the learners were conducting the proof using *Geometer's Sketchpad* that they all managed to conduct the proof appropriately. All these learners were using *Geometer's Sketchpad* for the first time. Excerpts extracted from the interviews were identified as evidence that *Geometer's Sketchpad* enabled the learners to conduct the proof appropriately. Refer to these excerpts: LI4 stated, "…*Sketchpad has helped me prove the angle at the centre theorem*". AI4 articulated with relief, "…*Sketchpad made it easy for me. It helped me prove that the angle at the centre is equal to twice the angle in the circumference*". SI4 boldly mentioned, "…*Yes***,** *it also help me to prove angle centre theorem*". JI4 responded, "…*Yes*, *Sketchpad helped me find that <BCD is equal to twice <BAD*". NI4 explained, "*As I was measuring*, *I got the relationship between the angles that angle at the centre is twice angle at the circumference*, *<BCD is twice <BAD in the task that I was given*".

TI4**'**s response denoted improvement in understanding proofs, "…*Yes*, *it has helped me to understand the angle at the centre theorem better than before*". Lastly GI4 stated, "*Yes*, *it's did help me*. *Helped me understand angle at the centre is twice the angle at the circumference*".

The Dynamic Geometry Software, *Geometer's Sketchpad,* also seemed to assist learners to understand concepts in Euclidean geometry. Learners, after conducting the investigative task, using *Geometer's Sketchpad* as an experimental tool*,* understood the term 'subtended by' as a concept. TI4 stated, "…*Yes*, *it helped me to understand the angle at the centre better than before and to know some of the terminologies of geometry which I did not know before*, *like* ‘*subtended by’* ”. NI3, from the challenge she encountered while measuring, learnt of the difference between ‘length’ and ‘distance’. She mentioned, “…*That made me learn that there difference between length and distance”.*

The use of the *Geometer’s Sketchpad Program* proved to develop the capability to seek relationships between and within sides and angles in diagrams. One observed learners becoming actively involved in looking for these relationships. Furthermore, this involves thinking about how these relationships can help one’s understanding of problem solving. The use of *Geometer’s Sketchpad* encouraged the learners to conduct more mathematics proofs using the *Geometer’s Sketchpad Program.* Incorporating the use of *Geometer’s Sketchpad* also instilled love of mathematics.

5.4 Conclusion

It was exciting to watch the learners conduct the experiment on their own. However, they needed my help as they knew nothing about the different tools they needed to use in *Geometer’s Sketchpad.* Furthermore, I had designed the problem-based task such that it enabled the learners to be actively involved by giving clear instructions.

I learnt a lot from the learners’ views about the use of *Geometer’s Sketchpad* to conduct proofs. Meaningful learning became effective whilst learners were all actively engaged in problem solving as compared to when the learners proved the angle at the centre theorem using the paper and pencil method. This became evident when all eight learners obtained 100% in the task to prove the angle at the centre theorem while using *Geometer’s Sketchpad.* On the contrary, they had obtained less than 59% when they were proving the same theorem using the paper and pencil method. The proof seemed abstract to them as they struggled to recognise exterior angles of triangles in the proof. They also provided incorrect and/or irrelevant reasons for their arguments. Consequently, they ended up with a distorted inference.

The use of *Geometer’s Sketchpad* eradicated the abstractness the learners experienced and provided them with visualisation. The do it yourself approach proved to be very effective as compared to demonstrations done by the teacher where learners are expected to imitate the teacher. Learners are human beings who naturally have their mathematical perceptions which need to be nurtured to enhance their understanding of proofs. Despite all the revisions of basic

Euclidean geometry I did, repeated demonstrations of the angle at the centre theorem, 50% of the learners were unable to prove the angle at the centre theorem appropriately. I had hope that after showing them their mistakes and explaining repeatedly what they should have done, the learners would improve their conducting the proof.

The next chapter is the final chapter of this study. It presents the main findings of the current study, how the current study answered the research questions, the current study's limitations and how the current study contributed to mathematics education.

CHAPTER SIX

6. Conclusion

6.1 Introduction

The current study sought to explore Grade Eleven learners' views on using *Geometer's Sketchpad* for proofs in Euclidean Geometry. This chapter discusses the main findings of the current study. It also summarises how I, the researcher achieved the objectives of the current study. This is denoted explicitly in the way each of the research questions was answered. Additionally this chapter highlights the possible limitations of the current study. Furthermore, it discusses the significance of incorporating Dynamic Geometry Software like *Geometer's Sketchpad* in mathematics teaching of proofs.

6.2 The main findings

Though the current study was not a comparative study, the empirical evidence showed that all learners only succeeded in conducting the proof appropriately when they were using *Geometer's Sketchpad.* The empirical evidence also revealed that the learners found it easier to conduct proofs using *Geometer's Sketchpad* than using the paper and pencil method. The learners started proving the angle at the centre theorem using the paper and pencil method. Several remedial works were involved together with the application of this theorem. Despite all this, learners still had challenges. Some learners formulated correct conjectures and justified them with inappropriate reasons. There were those who could not conduct the proof appropriately despite repeating it three times after it had been demonstrated to them, but managed to apply it when they were solving riders. I used different teaching strategies due to the nature of the study, and conducted an action research enquiry as I had aimed to improve and thus change my teaching strategy.

These were the research questions which the main findings of the current study sought to answer:

1. What are Grade Eleven learners' views about the use of *Geometer's Sketchpad* for teaching of proofs in Euclidean Geometry?

2. How can *Geometer's Sketchpad* be used to teach proofs to Grade Eleven mathematics learners?

6.3 The research questions

6.3.1 What are Grade Eleven Mathematics learners' views about the use of *Geometer's Sketchpad* for the teaching of proofs in Euclidean Geometry?

Learners had numerous perceptions about the use of *Geometer's Sketchpad* which all denoted learners' positive attitude towards its use. Hulme (2012) noted that the learners had overwhelmingly positive attitudes towards the use of *Geometer's Sketchpad.* The following excerpts taken from interview transcripts serve as evidence. Learners perceived that using *Geometer's Sketchpad* makes mathematics easier. Learners N12 and J12 pointed out that it is easier to measure lengths and sizes of angles using *Geometer's Sketchpad* rather than to prove theorems using the traditional paper and pencil method. Following are JI2 and NI2 excerpts: JI2 explained, "…*Challenge not hard*, *measuring is easy*". When probing, I found that the learner meant to say that the task I had given to them was not difficult, due to that they had to solve it through measuring using the built-in tools in *Geometer's Sketchpad,* which was easy. NI2 mentioned, "… *Because having done the task*, *using Sketchpad I understand that when using a Sketchpad, it is easy to measure and find angle size rather to prove in an old way*. *You personally calculate lengths*, *size of angles which sometimes you don't get answers right when calculate yourself and other sometimes you got wrong answers*".

Some learners perceived the use of *Geometer's Sketchpad* as a practical exercise which makes conducting proofs easier. For example learners T12 and L12 stated that it is easier to prove the theorems practically than doing it on your own, using the paper and pencil method. TI2 said, "*It is because it is easier to prove the theorems practically than doing it on your own on a paper*". LI2 emphasized, "…*When you doing theory, you don't have proof. Now I have done practicals which approve of the theory".*

Some learners stated that they enjoyed practical work as learner TI4 stated, *"...It is more advance than the method my teacher uses. It is easy to use and understand because Sketchpad has tools to draw and measure. It can help me to pass mathematics because it is practical. I enjoy practical work than theoretical work".* Furthermore R12 viewed *Geometer's Sketchpad* as fast, and as a tool that needs no deep thinking. Most importantly, he stated *Geometer's Sketchpad* is visual. Learners remember what they saw as opposed to what they heard. The excerpt that follows, is learner RI2's response to interview question 2. "…*Sketchpad is easier, fast, visual, needs no deep thinking".*

It is not just that some learners enjoyed practical work like learners A12 and S12. They also gained more learning strategies. Learner AI2 mentioned that *Geometer's Sketchpad* helped her broadly across her studies. For learner S12 the use of *Geometer's Sketchpad* added more information to what he knew in Euclidean Geometry. AI2's response, "…*Happy because Sketchpad helped me with my studies in class".* SI2 proudly stated, "…*Doing this task, it's like I have expand the horizon of learning because I enjoy practical work. Besides that, I have add my knowledge in Euclidean Geometry".* Learner LI2 excitedly mentioned that practicals prove the theory they learn in class which has no proof as seen in the excerpt that follows. LI2 stated, *"…When you doing theory, you don't have proof. Now I have done practicals which approve of the theory".*

Learner GI2 highlighted that *Geometer's Sketchpad* motivated her to work harder as seen in the excerpt: "…*It's because it makes me work harder".*

In the current study, learner LI4, learner recorded as Lettie interview question 4 displayed a need for conviction denoted in the excerpt:
LI4 reiterated, "…*Yes, Sketchpad made me sure of what I have done is the truth. Sketchpad has proved that the theory I have learnt is really true. Sketchpad has helped me understand maths better. Sketchpad has helped me prove the angle at the centre theorem".* This concurs with Mudaly (1998) who found that learners exhibited a need of conviction in a study where he used *Geometer's Sketchpad*.

Learners enjoy doing the tasks they can manage and they are encouraged to look to the future positively. Learner SI4 gives such evidence in this excerpt, "…*Yes, because I managed to do the task I was given. I measured sides and angles and found the relationship between angles. It also help me prove angle centre theorem. I would recommend the use of Sketchpad because it's the beginning of tertiary".*

It is evident that learners prefer the incorporation of Dynamic Geometry Software *Geometer's Sketchpad* while conducting proofs in Euclidean Geometry to using the traditional method. *Geometer's Sketchpad* is fast, as RI2 mentioned earlier. This concurs with learner NI4 in this excerpt, *" ....Yes, because if I was proving using old method, when you prove yourself by referring to what I've been told, or just looking at angle, it would have took time to arrive at an answer for me to be able to find the sizes and the length. So, meaning it is Sketchpad because Sketchpad have tools for measuring. The measuring helped me to get the sizes and length of the angles using the Sketchpad. As I was measuring, I got the relationship between the angles that angle at the centre is twice angle at the circumference, <BCD is equal to twice <BAD in the task that I was given. Measuring helped me to prove the theorem. Sketchpad is so useful because it's easy to prove using it than using the old method".*

The use of *Geometer's Sketchpad* enhanced learners' understanding of geometry concepts as learner TI4 reiterated in this excerpt, "…*Yes, it helped me to understand the angle at the centre better than before and to know some of the terminologies of geometry which I did not know before, like 'subtended by".* Besides the proof, the other concepts learners needed to understand were, angle subtended by the arc at the centre of the circle and angle subtended by the same arc on the circumference. Learners enjoyed the fact that *Geometer's Sketchpad* has built-in tools that help them measure and manage to find relationships. Learner GI4 was one of those learners. She mentioned, "…*Yes, it's did help me. Sketchpad helped me love maths better than before. Sketchpad has tools. Don't have to bring any tools. Helped me understand how to measure and find relationships. Helped me understand angle at the centre is twice the angle on the circumference".*

It was gathered from the learners that the use of *Geometer's Sketchpad* is easy to use and understand because it is practical. Therefore they enjoyed conducting the experiment using *Geometer's Sketchpad.* SI4 viewed the use of *Geometer's Sketchpad* as the beginning of tertiary. The use of *Geometer's Sketchpad* convinced LI4 that it is really true that the angle subtended by the arc at the centre of the circle is double the size of the angle subtended by the same arc on the circumference. RI4 appreciated the fact that *Geometer's Sketchpad* is easier, fast and visual. GI4 was so thrilled that she did not have to bring any mathematical tools since *Geometer's Sketchpad* has built-in tools. TI4 viewed *Geometer's Sketchpad* as an advanced gadget that would help him pass mathematics as he enjoys practical work.

6.3.2 How can *Geometer's Sketchpad* be used to teach proofs in Euclidean Geometry to Grade Eleven mathematics learners?

The *Geometer's Sketchpad Program* can be used for drawing diagrams and allowing the learners to measure the dimensions of the diagrams. Then from the measurements the learners can formulate conjectures which can lead to proofs. The dragging mode in *Geometer's Sketchpad* can help convince the learners of axioms and theorems. It also helps the learners discover invariants and different forms of diagrams. Invariants in mathematics refer to mathematical objects which remain unchanged when transformations are applied to them.

Learners prefer practical work to theoretical work. This is shown in the excerpts: SI2 responded thus, "*Doing this task*, *it's like I have expand the horizon of learning because I enjoy practical wor*k". NI2 responded, "*.... using a Sketchpad, it is easy to measure and find angle size rather than prove in an old way*". TI2 stated, "*....it is more easier to prove the theorems practically than doing it on your own on paper*". TI4 highlighted, "*...I enjoy practical work than theoretical work*". RI3 put it "*Didn't find any problems because it was practical*". So, it seemed useful to let learners conduct an experiment practically by themselves, using *Geometer's Sketchpad*.

Furthermore learners found it easier working with *Geometer's Sketchpad.* RI4 stated, "*…Sketchpad is easier...*"JI4 mentioned, "*…Sketchpad is easy to use*". NI4 put it, "*...So*, *meaning it is easy using Sketchpad because Sketchpad have tools for measuring*". TI4 reiterated, "*...It is easy to use and understand because Sketchpad has tools to draw and measure*". Therefore providing them with ready-made sketches on *Geometer's Sketchpad* and asking them to measure sides and angles and to determine relationships and formulate conjectures, leads to a proof. All learners in the current study measured accurately despite the fact that they were not acquainted with the *Geometer's Sketchpad Program.* They also managed to prove the angle at the centre theorem appropriately whilst using *Geometer's Sketchpad.* Hoyles (1995), Lampert (1988) and Olive (1998) agreed that the inherent features of software such as *Geometer's Sketchpad* help learners to easily explore, create and verify conjectures. De Villiers (1998) and Scher (1999) highlighted that this exploration subsequently leads to proof.

Mudaly (1998) conducted a study using *Geometer's Sketchpad* where he aimed to investigate whether learners have any need for conviction and explanation. Through the use of *Geometer's*

Sketchpad, the Grade Nine learners managed to find out the point where to build the house so that the total sum of the distances from P to all three beaches was a minimum. The use of *Geometer's Sketchpad* illuminated geometrical ideas and appropriated a proof. Mudaly (2004) explored the role and use of *Geometer's Sketchpad* in high schools. His aim was to investigate the concurrency of perpendicular bisectors of a triangle at the circumcentre irrespective of the shape of the triangle. Learners were expected to dynamically explore and observe if perpendicular bisectors were concurrent in a cyclic quadrilateral, in a non-cyclic quadrilateral and in a triangle. Learners found that the bisectors of a cyclic quadrilateral are concurrent and that those of a triangle are always concurrent at the circumcentre. *Geometer's Sketchpad* helped appropriate the proof of concurrency despite the fact that concurrency was not within the mathematics curriculum at the time in South Africa.

Govender (2013) conducted a study where he investigated how the pre-service mathematics teachers experienced the reconstruction of Viviani's theorem through experimentation, conjecturing, generalising and justifying using *Geometer's Sketchpad*. The results showed that experimental exploration in a dynamic geometry context was required in order that student teachers reconstruct Viviani's generalisation for equilateral triangles. All pre-service mathematics teachers exhibited a need for explanation as to why their conjecture generalisation was always true. Similarly with Mudaly (1998) learners displayed a need for explanation and managed to give a logical explanation after being guided.

6.4 Limitations

The current study had a small sample from only one institution and from only one class, Grade Eleven. The participants were chosen from one of the peri-urban areas (township) north of Durban in South Africa. Therefore its results and findings were obtained from only one institution. Thus, the results of the current study cannot be generalised. Different results may probably be obtained in various contexts with larger samples.

Other research approaches besides qualitative action research could have been employed. The theoretical framework which I chose, Constructivism, within which the current study is embedded, is not the only position that a researcher can choose though it seemed relevant to the current study. More and other research instruments besides observation and individual face-to-face interviews could have been used for triangulation.

6.5 How the current study is beneficial to mathematics education

The incorporation of Dynamic Geometry Software like the *Geometer's Sketchpad Program* in mathematics teaching seems to be helpful. Designing investigations (tasks) with ready-made sketches on *Geometer's Sketchpad* based on learners' experiences, on the curriculum and beyond, can be used to teach proofs. Mays (2003) stated that *Geometer's Sketchpad* Dynamic Software illuminates mathematics and advances learners' comprehension. Furthermore, *Geometer's Sketchpad* can be used to build mathematical models and to investigate them. It can be used to build and investigate objects, figures, diagrams and graphs. The current study will provide teachers with another teaching strategy which is aligned with modern technology.

Stols, Mji and Wessels (2008) highlighted that *Geometer's Sketchpad* is a powerful teaching tool. The technology principle, as one of the six principles, states that technology "influences mathematics taught and enhances learners' learning" (National Council of Teachers of Mathematics (2000, p. 24). However, technology will not replace teachers but would bring reform in mathematics teaching. Teachers are the key agents of change, assisted by the incorporation of technology in mathematics teaching.

6.6 Conclusion

The *Geometer's Sketchpad Program* may be used to build mathematical models and to investigate them. It can be used to build and investigate objects, figures, diagrams and graphs. Laborde (2002) stated that teachers take time to accept incorporation of technology in mathematics teaching. Despite all that, Dynamic Geometry Software needs to be incorporated as the learners enjoy using technology. Besides, the use of Dynamic Geometry Software such as *Geometer's Sketchpad* enhances understanding of proofs in Euclidean geometry. It needs to be taken into cognisance that the use of technology in mathematics teaching requires teachers who are competent in using technology to demonstrate mathematics concepts. Additionally, the incorporation of *Geometer's Sketchpad Software* does not replace a teacher but, serves as a scaffolding and motivational tool that helps mathematics teaching and learning.

Technology incurs more costs in schools and in some areas crime is rife. This might be the challenge that some institutions may encounter. In case of financial constraints in schools, teachers may purchase laptops for themselves and the school buys only data projector instead of computers. This might expose learners to technology even if they do not use computers individually. Furthermore, this will serve as an alternative teaching strategy that would enable learners to fit in the technological world and working environment. Awe (2007) affirmed that learners can still understand the visual aspects of geometry even if the teacher has a few computers.

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APPENDICES

APPENDIX A

Ethical clearance

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

04 December 2014

Ms Nombulelo Thembekile Mbokazi 207523455
School of Education
Pietermaritzburg Campus

Protocol reference number: HSS/0674/013M
New project title: Exploring Grade Eleven Learners' views on using the Geometer's sketchpad for proofs in Euclidean Geometry.

Dear Ms Mbokazi

Approval - Change of project title

I wish to confirm that your application in connection with the above mentioned project has been approved.

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach/Methods must be reviewed and approved through an amendment /modification prior to its implementation. In case you have further queries, please quote the above reference number. Please note: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

Best wishes for the successful completion of your research protocol.

Yours faithfully

....

Dr Shenuka Singh (Chair)

/px

cc Supervisor: Dr Jayaluxmi Naidoo
cc Academic Leader Research: Professor PJ Morojele
cc School Administrator: Ms B Bhengu and Mr SN Mthembu

Humanities & Social Sciences Research Ethics Committee
Dr Shenuka Singh (Chair)
Westville Campus, Govan Mbeki Building
Postal Address: Private Bag X54001, Durban 4000
Telephone: +27 (0) 31 260 3587/8350/4557 **Facsimile:** +27 (0) 31 260 4609 **Email:** ximbap@ukzn.ac.za / snymanm@ukzn.ac.za / mohunp@ukzn.ac.za
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APPENDIX B

Consent for the participants

……………

School of Education, College of Humanities, University of KwaZulu-Natal,

Edgewood Campus,

Dear Participant

INFORMED CONSENT LETTER

My name is Nombulelo Thembile Mbokazi. I am a Masters candidate studying at the University of KwaZulu-Natal, Edgewood campus, South Africa. I am interested in exploring Grade Eleven learners' views on using the Geometer's Sketchpad for proofs in Euclidean Geometry. To gather the information, I am interested in asking you some questions.

Please note that:

- Your confidentiality is guaranteed as your inputs will not be attributed to you in person, but reported only as a population member opinion.
- The interview may last for about 45 minutes to 1 hour.
- Any information given by you cannot be used against you, and the collected data will be used for purposes of this research only.
- Data will be stored in secure storage and destroyed after 5 years.
- You have a choice to participate, not participate or stop participating in the research. You will not be penalized for taking such an action.
- Your involvement is purely for academic purposes only, and there are no financial benefits involved.
- If you are willing to be interviewed, please indicate (by ticking as applicable) whether or not you are willing to allow the interview to be recorded by the following equipment:

Equipment	**Willing**	**Not willing**
Audio equipment		
Photographic equipment		
Video equipment		

I can be contacted at:

Email: bulembokazi@gmail.com

Cell: 0729959630

My supervisor is Dr. Jayaluxmi Naidoo who is located at the School of Education, Edgewood campus of the University of KwaZulu-Natal.

Contact details: email: naidooj2@ukzn.ac.za   Phone number: +27312601127.

You may also contact the Research Office through:

Ms P Ximba (HSSREC Research Office)

Tel: 031 260 3587

Email: ximbap@ukzn.ac.za)

Thank you for your contribution to this research.

CONSENT FORM FOR PARTICIPANTS: Exploring Grade Eleven Learners' views on using the *Geometer's Sketchpad* for proofs in Euclidean Geometry.

DECLARATION

I……………………………………………………………………… (full names of participant) hereby
confirm that I understand the contents of this document and the nature of the research project, and I
consent to participating in the research project.

I understand that I am at liberty to withdraw from the project at any time, should I so desire.

SIGNATURE OF PARTICIPANT DATE

……………………………………… …………………

SIGNATURE OF PARENT (If participant is a minor) DATE

………………………………………… …………………..

APPENDIX C

Turnitin certificate

Exploring grade eleven learners' views on using the geometer's sketchpad for proofs in euclidean geometry

ORIGINALITY REPORT

12%	9%	5%	7%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

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8	. "ME627 Developing Thinking in Geometry ITM5038525 ISBN1412911699 ", Open University, .	<1%

APPENDIX D

Editor's report

Angela Bryan & Associates

6 La Vigna
Plantations
47 Shongweni Road
Hillcrest

Date: 13 November 2015

To whom it may concern

This is to certify that the Masters dissertation: Exploring grade eleven learners' views on using Geometer's Sketchpad for proofs in Euclidean Geometry written by Nombulelo Mbokazi has been edited by me for language.

Currently an English teacher at a private Secondary school, Angela has a Bachelor's degree specialising in English and Psychology. Her clients include academics from a number of universities, some of which are UKZN, Medical School, Rhodes and NWU. She has edited numerous articles for oversea publications including several translations from foreign languages.

Please contact me should you require any further information.

Kind Regards

Angela Bryan

angelakirbybryan@gmail.com
0832983312

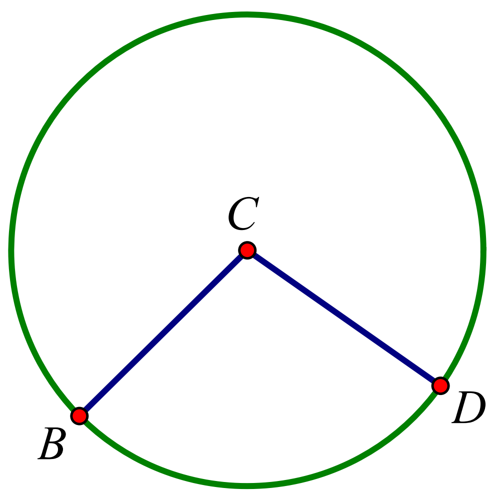
APPENDIX E

Research instruments

Task-based worksheet conducted on the *Geometer's Sketchpad*

Understanding angles

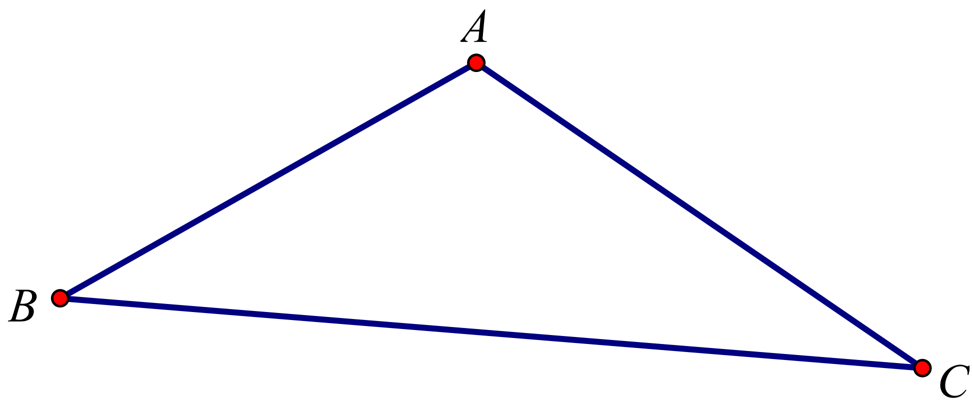
1. Open the *Sketchpad* and you will find a diagram provided by your teacher that shows the figure that follow.



❖ Measure the lengths of *CB* and *CD* and write them down. What do you notice?

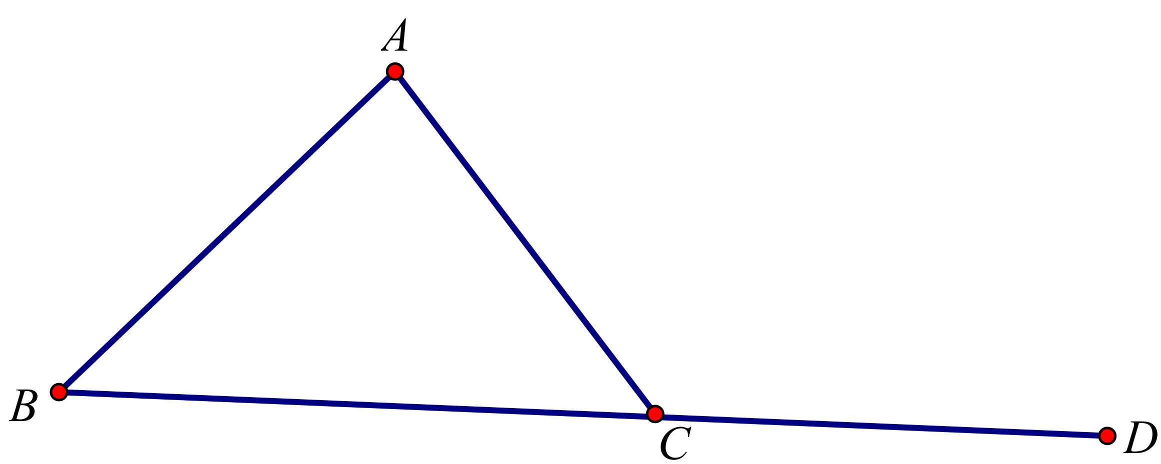
2. Further Exploration

   2.1 Look at the triangle provided. Measure the lengths of *AB* and *AC*. Then measure the size of
   interior angle *ABC* and of *ACB*. What do you notice?



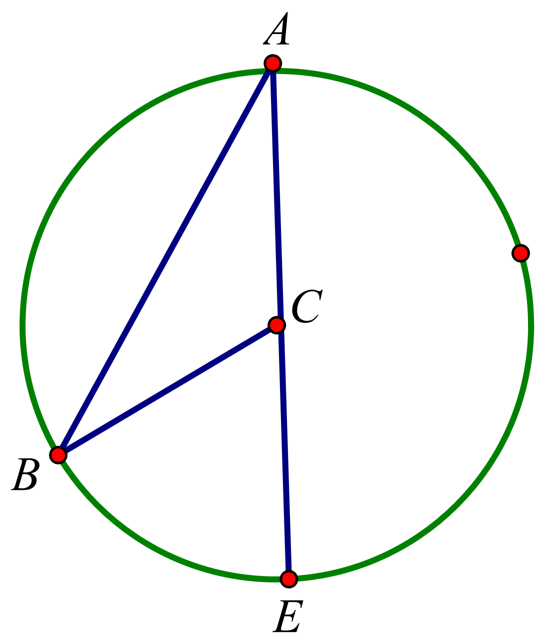
- Is there any relationship between these sides? If you did find a relationship, can you explain what the relationship is?
- What about the angles?
- What can you conclude about these sides and these angles?

2.2 In the next diagram provided, measure the sizes of angles *ABC, BAC, ACB* and *ACD.*



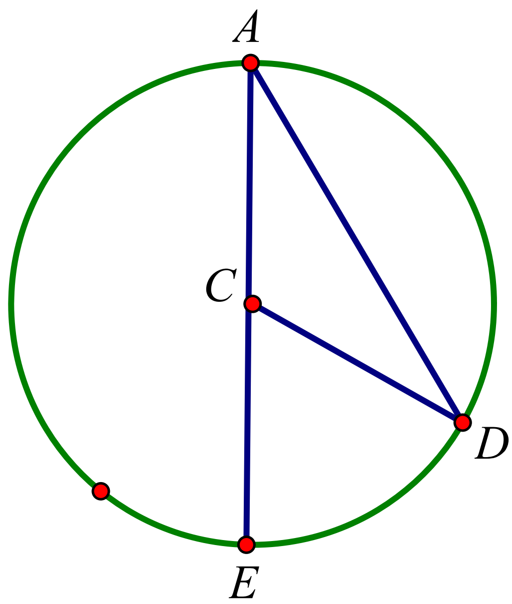
- What do you observe about angles *ABC, BAC* and *ACB*?
- What do you notice about angles *ACB* and *ACD*?
- Is there any relationship between angle *ACD* and the other interior angles? If you did
  find a relationship, can you explain what the relationship is?

3.1 In the next diagram, measure the following:



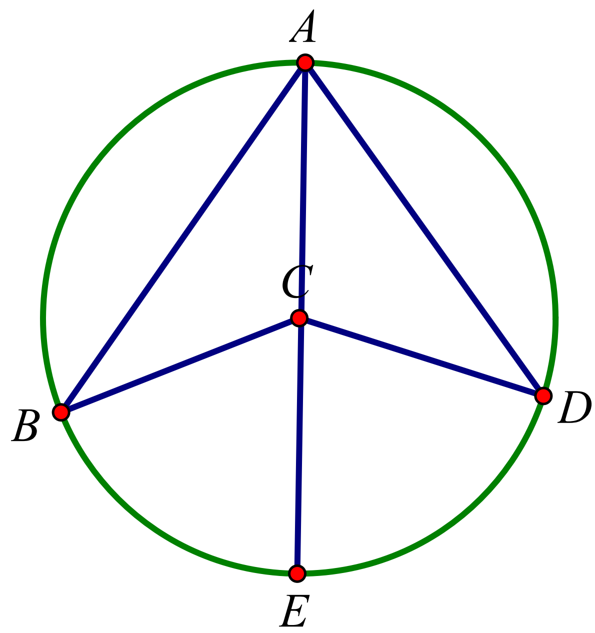
- The length of *CA*.
- The length of *CB*.
- The size of angle *BAE* and of angle *ABC*.
- The size of angle *BCD*.
- What do you observe about the size of angle *BCD?*

3.2 In the diagram that follows, measure



- The size of angle *CAD*.
- The size of angle *CDA*.
- The size of angle *ECD*.
  What do you notice about angle *ECD* and the interior angles of triangle *ACD?*

3.3 Now look carefully at the diagram that follows:



- What is the size of angle *BCE* in relation to angle *BAC?*
- What is the size of angle *ECD* in relation to angle *DAC?*
- Determine the relationship between angles *BCD* and *BAD.*

Now give a full explanation of the relationship between angles *BCD* and *BAD*

APPENDIX F

Observation schedule

This schedule was used by me to observe the learners whilst they conducted the experiment using the *Geometer's Sketchpad* and while assessing the learners' responses to the task-based work sheet.

Attitude towards Geometer's Sketchpad	Positive/Negative
Confidence	Comfortable/Uncomfortable
Competence	Competent/Incompetent
Measuring	Accurate/Inaccurate
Conjectures formulated	Correct/Incorrect
Preferred method of teaching proofs	Traditional method/Incorporation of Dynamic Geometry Software (DGS), the Geometer's Sketchpad Software
Challenges with the use of Geometer's Sketchpad Software	Encountered/Not encountered

APPENDIX G

Interview schedule

EXPLORING GRADE ELEVEN LEARNERS' VIEWS ON USING *GEOMETER'S SKETCHPAD* FOR PROOFS IN EUCLIDEAN GEOMETRY.

Semi - structured Interview schedule

Schedule for individual interview for all eight learners who participated in the study after they finished conducting the experiment using the *Geometer's Sketchpad.*

Name: ....

NB: This interview requires your honest opinion and all details will remain confidential.

1. How do you feel after having done this task, using the *Geometer's Sketchpad?*

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2. What makes you feel that way?

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3. Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*? Explain.

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4. Did the use of the Geometer's Sketchpad Program help you in conducting the proof? How did it help you, if it did?

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APPENDIX H

Interview transcripts

INTERVIEW BETWEEN LETTIE AND RESEARCHER

RESEARCHER: How do you feel after having done this task, using the *Sketchpad*?

LI1 **:** I feel good and I believe that what I have done is true.

RESEARCHER **:** What makes you feel that way?

LI2 **:** When you doing theory, you don't have proof. Now I have done practicals which approve of the theory.

RESEARCHER **:** Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*? Explain.

LI3 **:** No problems.

RESEARCHER **:** Did the use of the *Sketchpad* help you in conducting the proof? How did it help you, if it did?

LI4 **:** Yes, *Sketchpad* made me sure of what I have done is the truth. *Sketchpad* has proved that the theory I have learnt is really true. *Sketchpad* has helped me understand Maths better. *Sketchpad* has helped me prove the angle at the centre theorem.

INTERVIEW BETWEEN ASANDA AND RESEARCHER

RESEARCHER **:** How do you feel after having done this task, using the *Sketchpad*?

AI1 **:** Excited never done it before. I've gained a lot of experience.

RESEARCHER **:** What makes you feel that way?

AI2 **:** Happy because *Sketchpad* helped me with my studies in class.

RESEARCHER **:** Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*? Explain.

AI3 **:** I failed to delete and to measure lengths. Laptop was a challenge. Didn't know how to measure, though at last I did it.

RESEARCHER **:** Did the use of the *Sketchpad* help you in conducting the proof? How did it help you, if it did?

AI4 **:** *Sketchpad* made it easy for me. It helped me prove that the angle, the angle at the centre is equal to twice the angle the angle in the circumference.

INTERVIEW BETWEEN JABU AND RESEARCHER

RESEARCHER **:** How do you feel after having done this task, using the *Sketchpad*?

JI1 **:** I feel great. *Sketchpad* encourage me to do Maths, Euclidean Geometry.

RESEARCHER **:** What makes you feel that way?

JI2 **:** Challenge not hard, measuring is easy.

RESEARCHER **:** Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*? Explain.

JI3 **:** Yes. Not knowing how to measure, but ended up knowing.

RESEARCHER **:** Did the use of the *Sketchpad* help you in conducting the proof? How did it help you, if it did?

JI4 **:** Yes. *Sketchpad* helped me find that angle *BCD* is equal to twice angle *BAD.* To measure sizes of angles and find relationship between angles. *Sketchpad* is easy to use.

INTERVIEW BETWEEN RIDGE AND RESEARCHER

RESEARCHER **:** How do you feel after having done this task, using the *Sketchpad*?

RI1 **:** I feel very happy.

RESEARCHER **:** What makes you feel that way?

RI2 **:** *Sketchpad* is fast, needs no deep thinking and it's practical.

RESEARCHER **:** Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*? Explain.

RI3 **:** Didn't find any problems because it was practical. I measure angles sides and concluded from there.

RESEARCHER **:** Did the use of the *Sketchpad* help you in conducting the proof? How did it help you, if it did?

RI4 **:** *Sketchpad* is easier, fast, visual, don't think deeply.

RESEARCHER **:** (probing). What do you mean by visual?

RI4 **:** When you working with something you see (confidently).

INTERVIEW BETWEEN SONKE AND RESEARCHER

RESEARCHER **:** How do you feel after having done this task, using the *Sketchpad*?

SI1 **:** I feel excited.

RESEARCHER **:** What makes you feel that way?

SI2 **:** Doing this task, it's like I have expand the horizon of learning because I enjoy practical work. Besides that, I have add my knowledge in Euclidean Geometry.

RESEARCHER **:** Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*`? Explain.

SI3 **:** Yes. When I measure the angles, I have discover my carelessness. I measured angles not asked. The laptop did not measure some angles the way they were asked. I had to start on the other end, example, measuring angle *BCD* as angle *DCB*.

RESEARCHER **:** Did the use of the *Sketchpad* help you in conducting the proof? How did it help you, if it did?

SI4 **:** Yes, because I managed to do the task I was given. I measured sides and angles and found the relationship between angles. It also help me to prove angle centre theorem.

RESEARCHER **:** What would you say about the use of the *Sketchpad* in high schools.

SI4 **:** I would recommend the use of *Sketchpad* because it's the beginning of tertiary.

INTERVIEW BETWEEN THULA AND RESEARCHER

RESEARCHER : How do you feel after having done this task, using the *Sketchpad*?

TI1 : I feel very excited and I enjoyed the task it inspired me in such a way that it encourages me to want to prove more theorems using the *Sketchpad.*

RESEARCHER : What makes you feel that way?

TI2 : It is because it is more easier to prove the theorems practically than doing it on your own on a paper.

RESEARCHER : Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*`? Explain.

TI3 : No, I did not experience any problems using the *Sketchpad* Because I was measuring sides and angles and finding relationships between sides and angles. But I had problems when I proved on my own on paper, the angles at the centre theorem.

RESEARCHER : Did the use of the *Sketchpad* help you in conducting the proof? How did it help you, if it did?

TI4 : Yes, it helped me to understand the angle at the centre better than before and to know some of the terminologies of geometry which I did not know before, like 'subtended by'.

RESEARCHER : What would you say about the use of *Sketchpad* in high schools?

TI4 : It is more advance than the method my teacher uses. It is easy to use and understand because the *Sketchpad* has tools to draw and measure. It can help me to pass Mathematics because it is practical.

I enjoy practical work than theoretical work.

INTERVIEW BETWEEN NTSIKI AND RESEARCHER

RESEARCHER : How do you feel after having done this task, using the *Sketchpad*?

NI1 : Happy.

RESEARCHER : What makes you feel that way?

NI2 : Because having done the task, using *Sketchpad* I understand that when using a *Sketchpad,* it is *easy to* measure and find angle size rather to prove in an old way. You personally calculate lengths, size of angles which sometimes you don't get answers right when calculating yourself and other sometimes you got wrong answers.

RESEARCHER : Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*? Explain.

NI3 : Yes, but not a serious problem. It was a minor problem where I could not find the length of *CA* then I got help by measuring it distance between two points, then the problem was solved. Because as I was measuring *CA,* it couldn't give me the length because it highlighted the whole line from *C* to *E*. That made me me learn that there difference between length and distance in the number 3.1.

RESEARCHER : Did the use of the *Sketchpad* help you in conducting the proof? How did it help you, if it did?

NI4 : Yes, because if I was proving using old method, when you prove yourself by referring to what I've been told, or just looking at angle, it would have took time to arrive at an answer for me to be able to find the sizes and the length. So, meaning it is easy using *Sketchpad* because *Sketchpad* have tools for measuring. The measuring helped me to get the sizes and length of the angles using the *Sketchpad.* As I was measuring, I got the relationship between the angles that angle at the centre is twice angle at the circumference, $<BCD = 2<BAD$ in the task that I was given. Measuring helped me to prove the theorem. *Sketchpad* is so useful because it's easy to prove using it than using the old method.

INTERVIEW BETWEEN GABI AND RESEARCHER

RESEARCHER **:** How do you feel after having done this task, using the *Sketchpad*?

GI1 **:** Well**,** I feel good even though not used to it.

RESEARCHER **:** What makes you feel that way?

GI2 **:** It's because it makes me work harder. I like challenges because they make me work harder to prove myself that nothing can bit me.

RESEARCHER **:** Did you experience any problems while conducting the experiment using *Geometer's Sketchpad*? Explain.

GI3 **:** Well, I did experience some problems. For me it was a bit harder. Didn't know how to measure because at school we don't use *Sketchpad.*

RESEARCHER **:** Did the use of the Sketchpad help you in conducting the proof? How did it help you, if it did?

GI4 **:** Yes, it's did help me. *Sketchpad* helped me love Maths better than before. *Sketchpad* has tools. Don't have to bring any tools. Helped me understand how to measure and find relationships. Helped me understand angle at the centre is twice the angle at the circumference.

RESEARCHER **:** What would you say about the *Sketchpad* to any person asking you?

GI4 **:** *Sketchpad* makes you feel good because you measure and find answers without bringing any mathematical tools.